Universität Konstanz

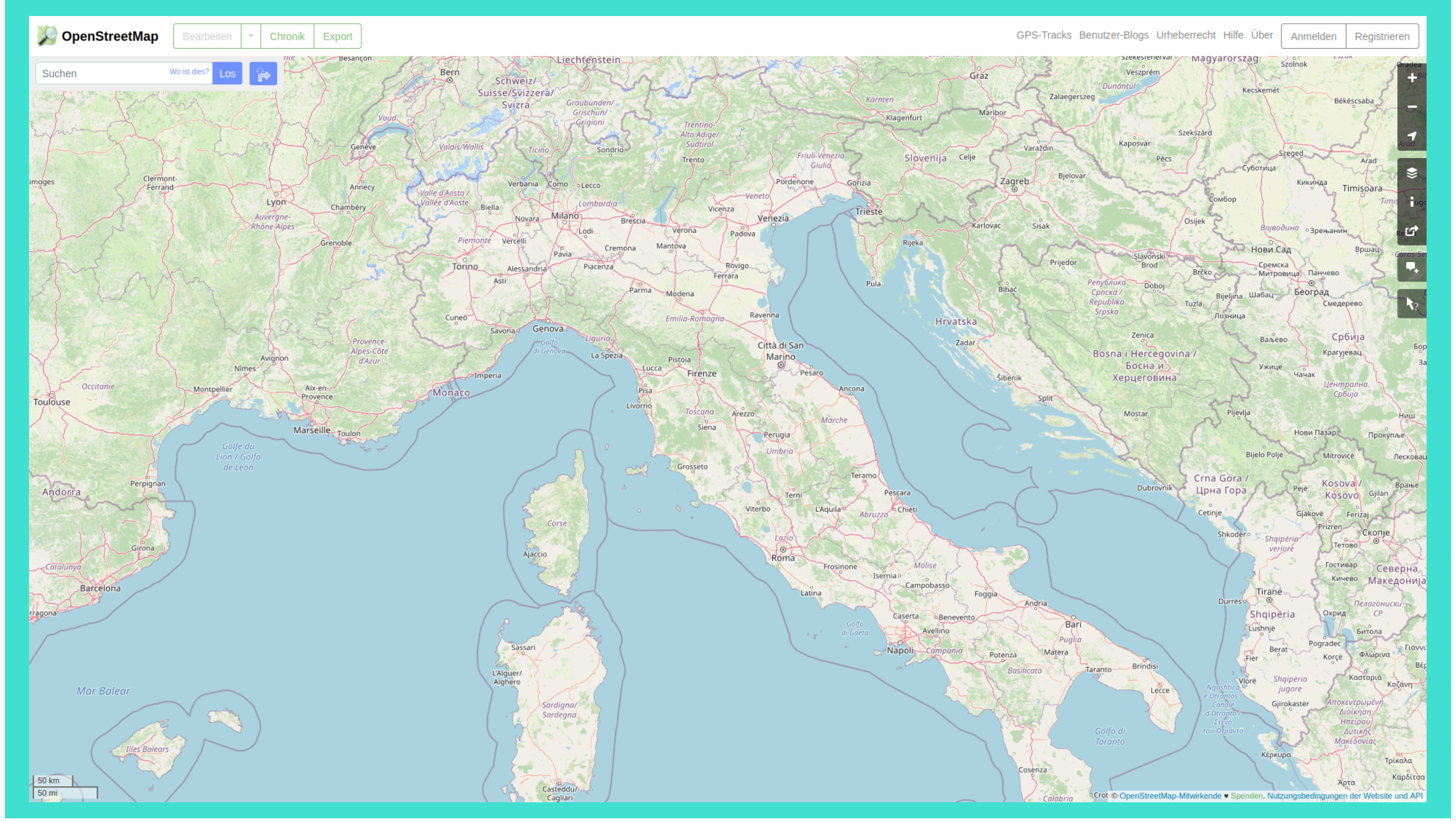
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Local-Fréchet Polyline Simplification Has Subcubic Complexity in Two Dimensions

EuroCG 2022

Sabine Storandt and Johannes Zink

Motivation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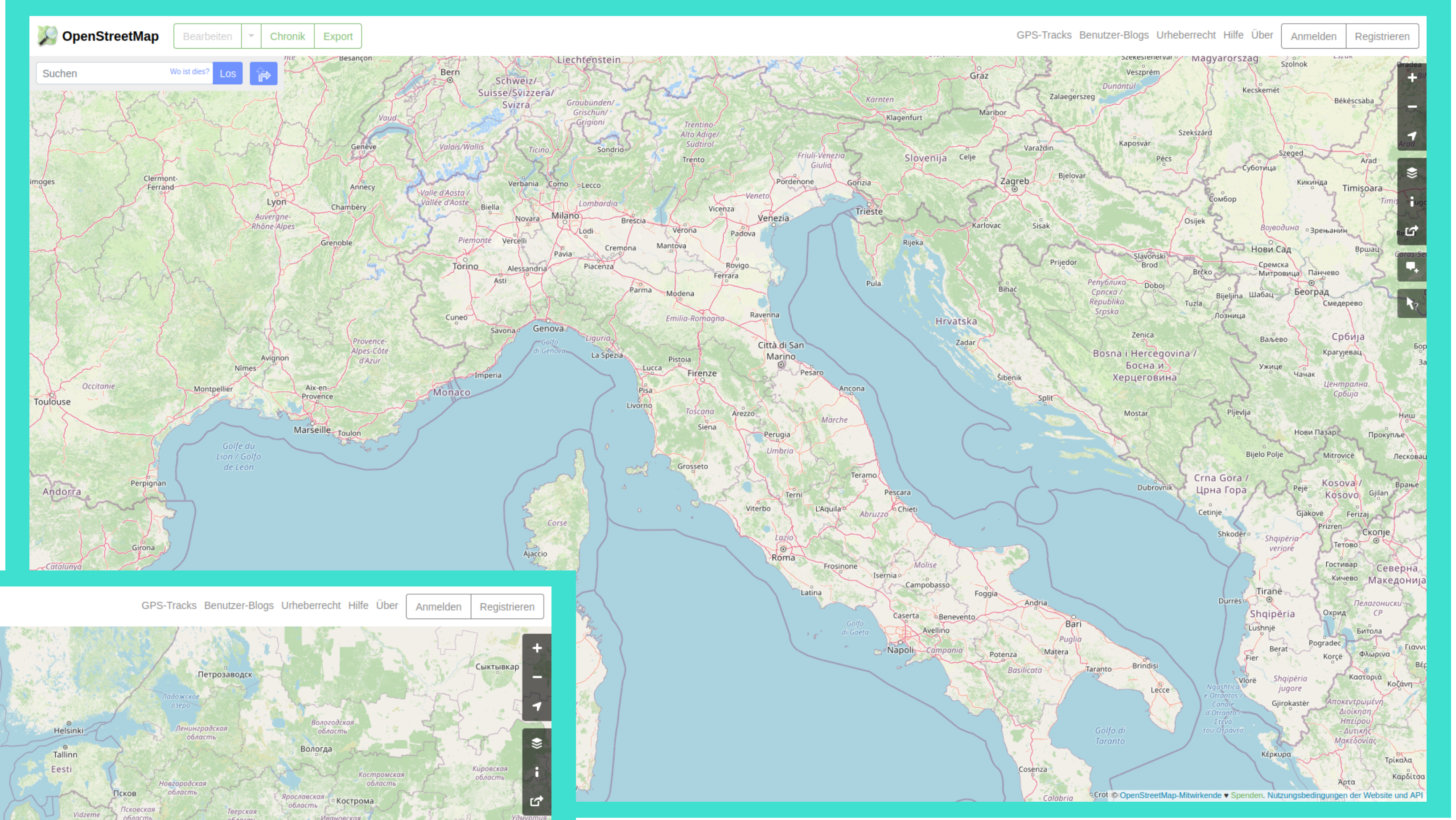
Chronik
Export
GPS-Tracks
Benutzer-Blogs
Urheberrecht
Hilfe
Über
Anmelden
Registrieren
Suchen
Wo ist dies?
Los
50 km
50 mi

Motivation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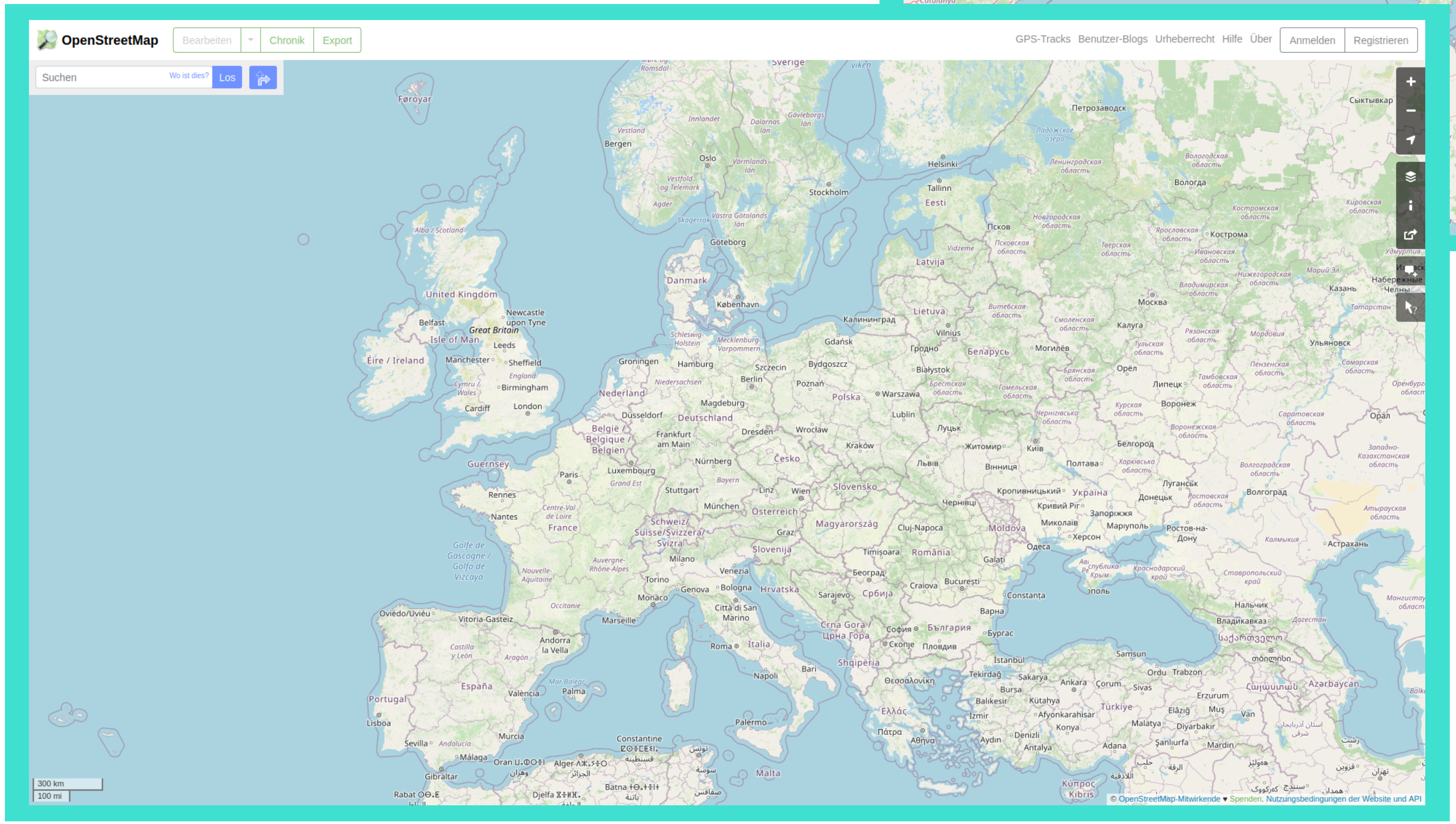
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300 km
100 mi
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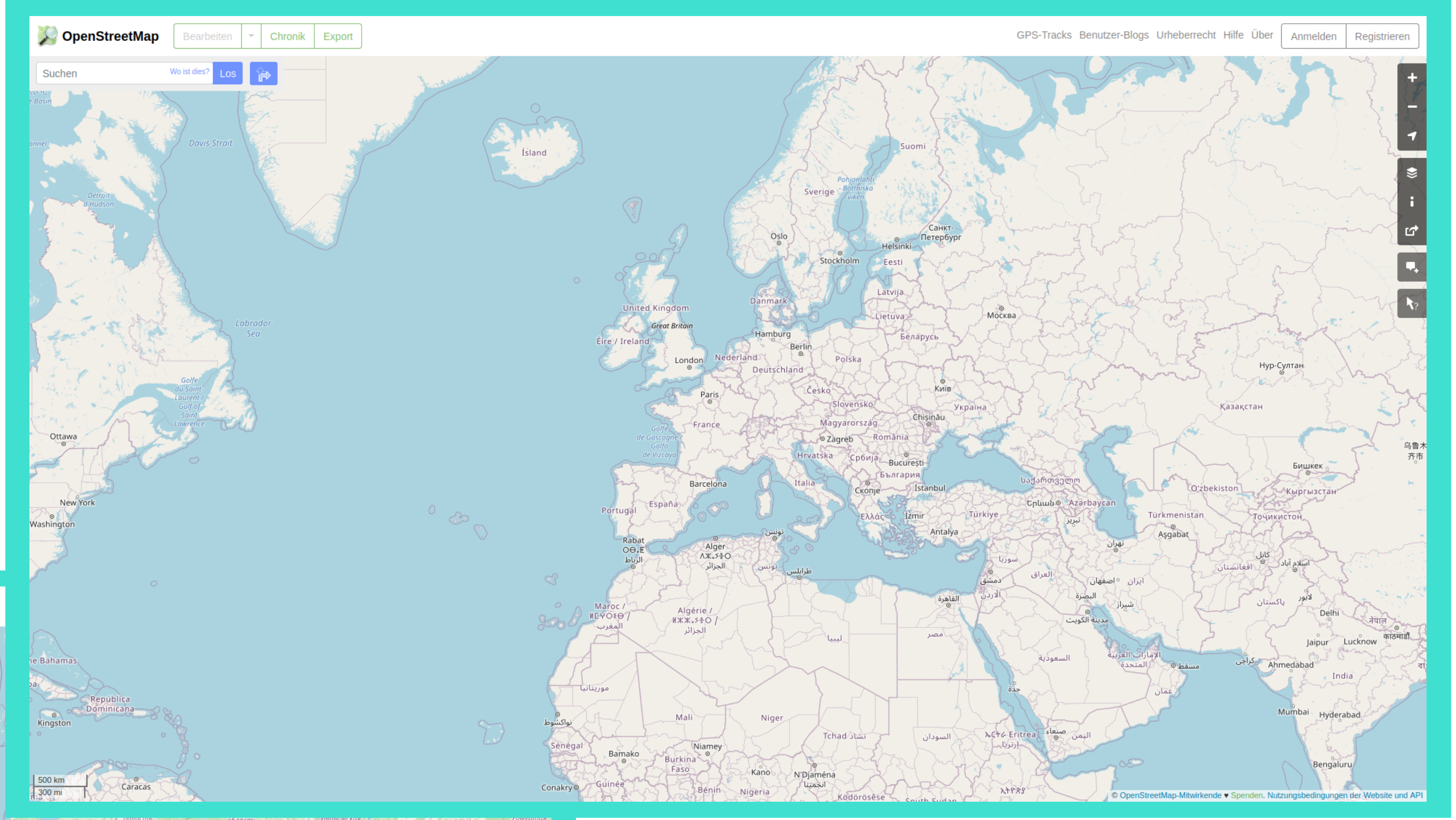
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Wo ist dies?
Los
500 km
300 mi
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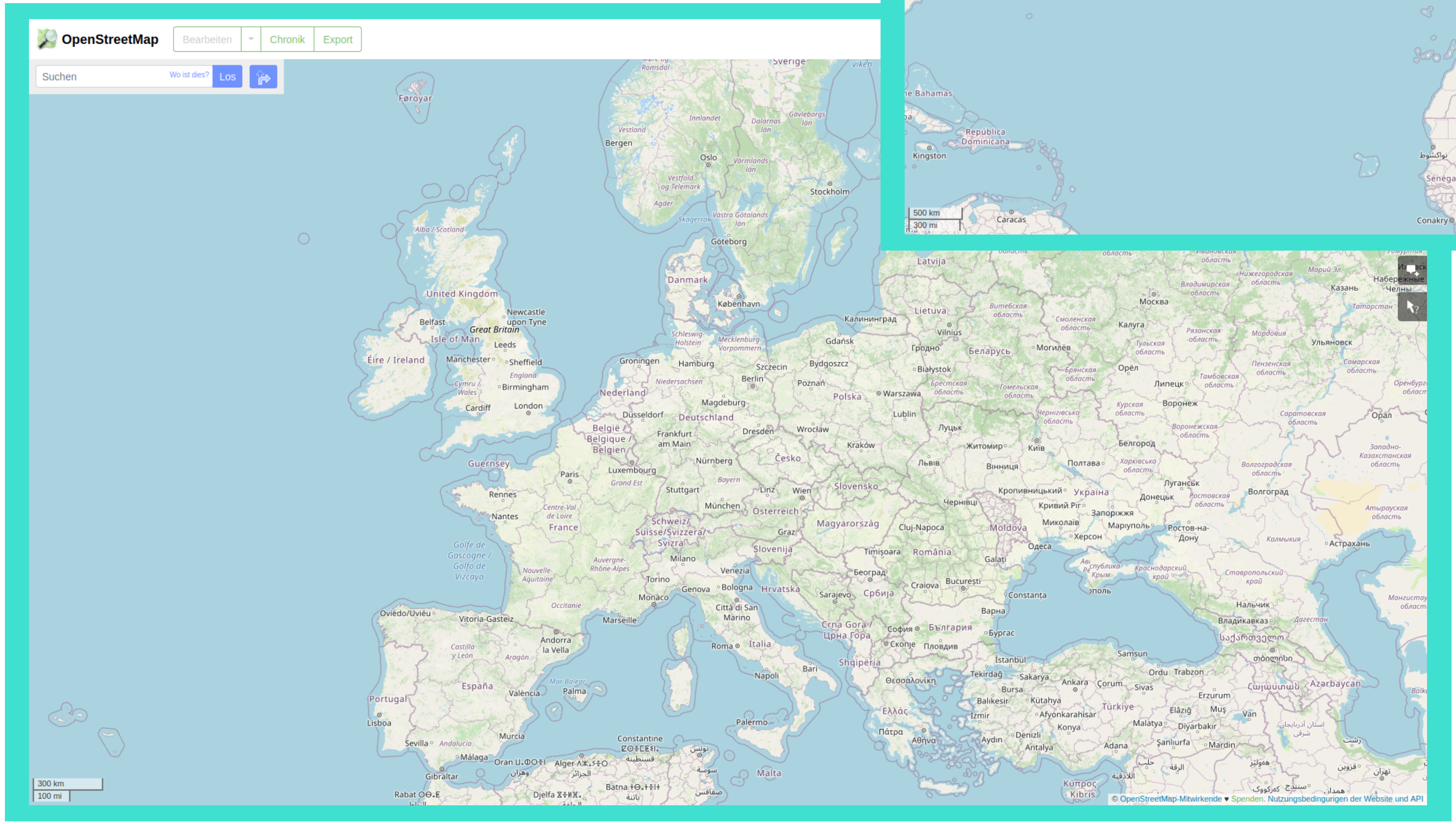
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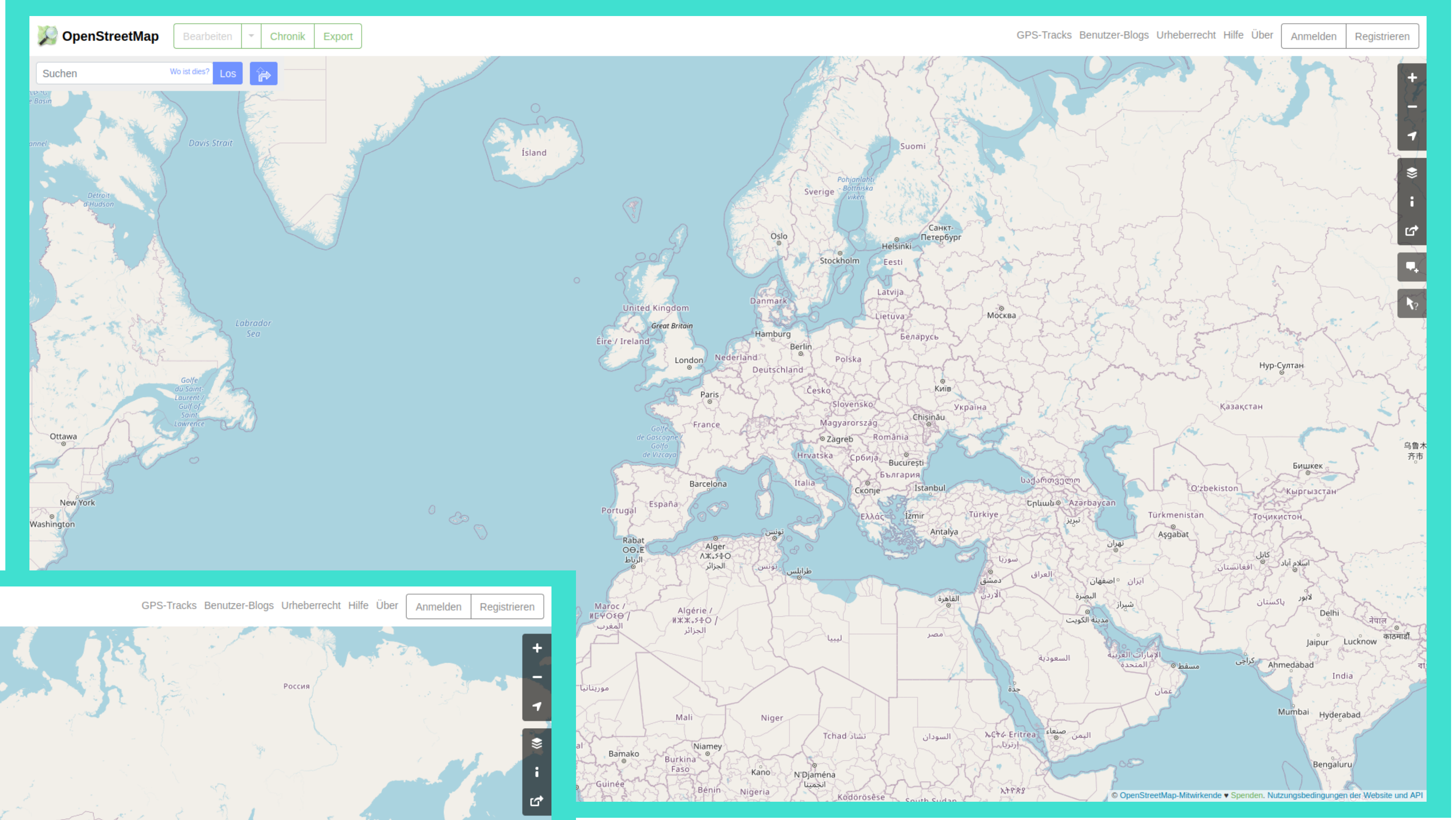
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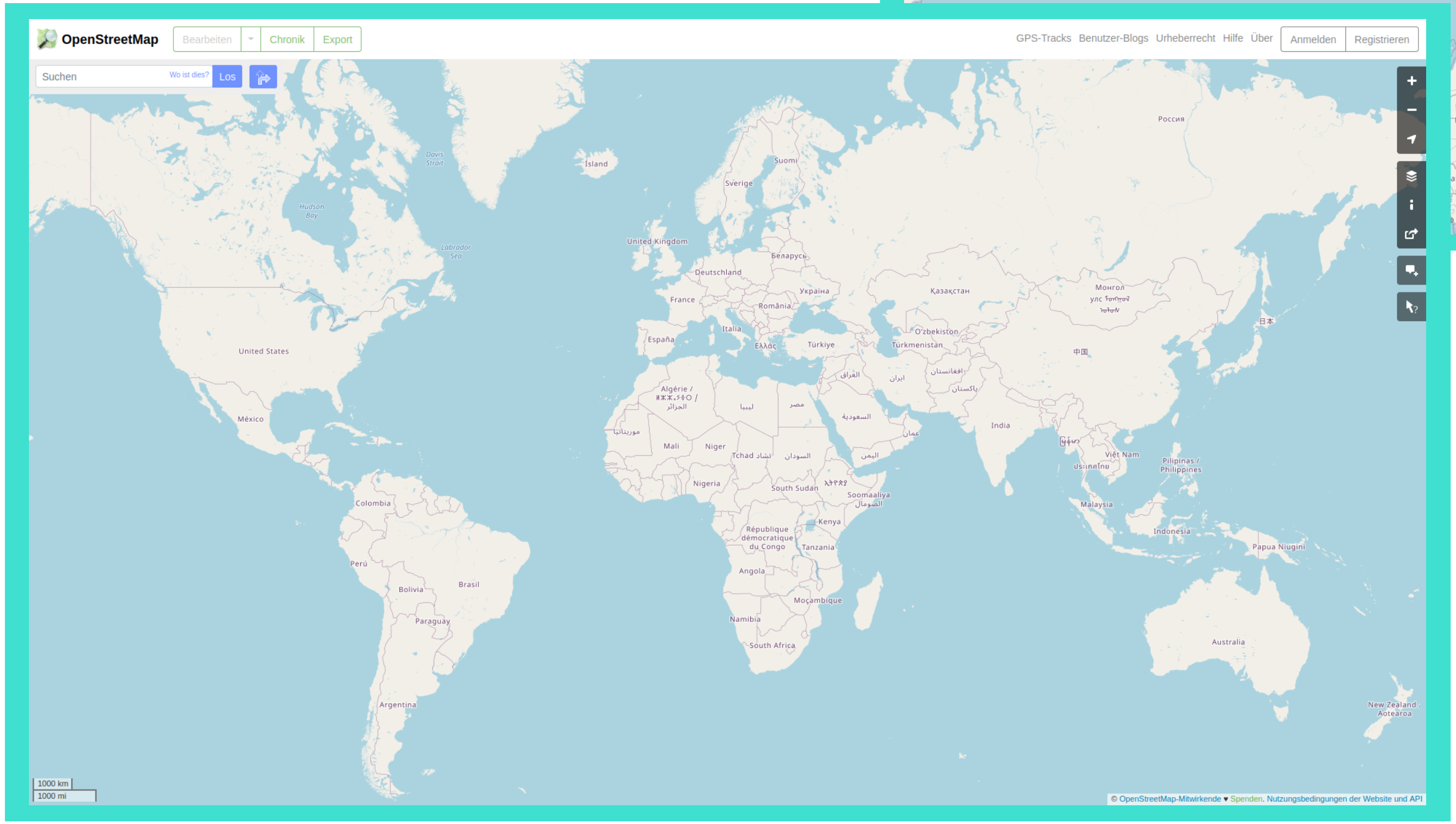
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Definition: Polyline Simplification

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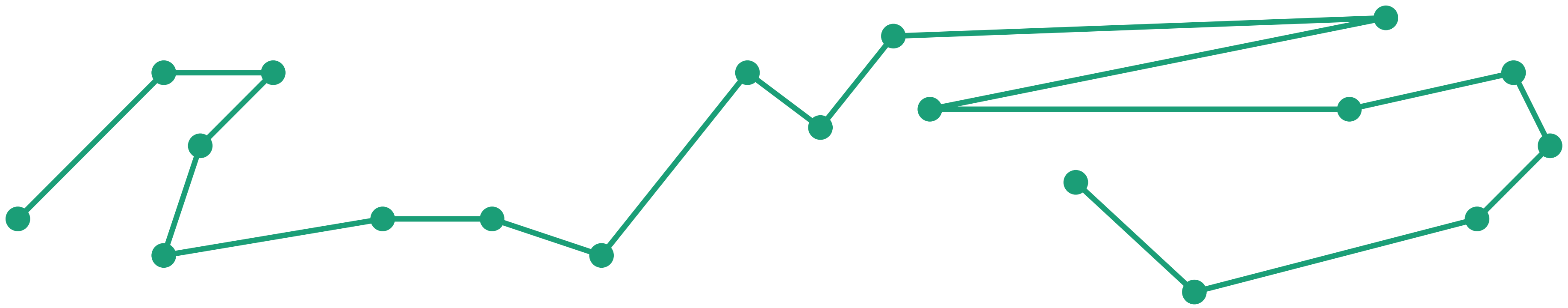

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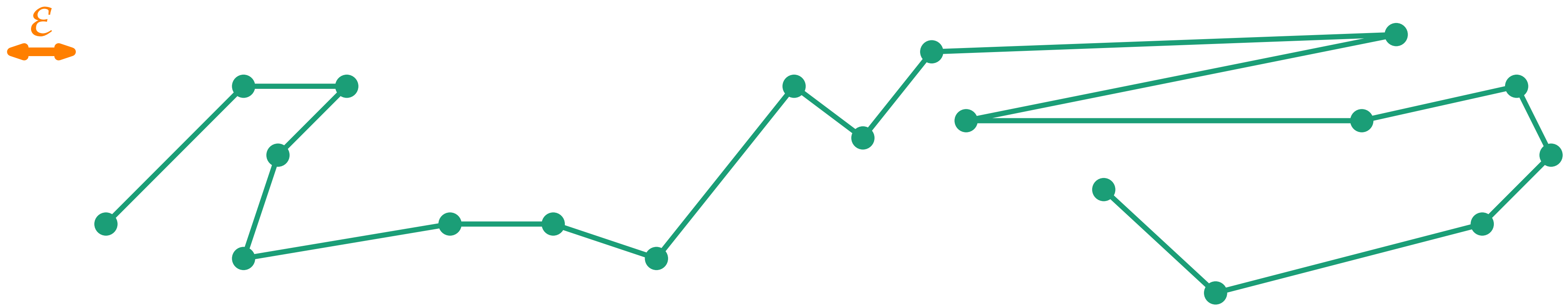

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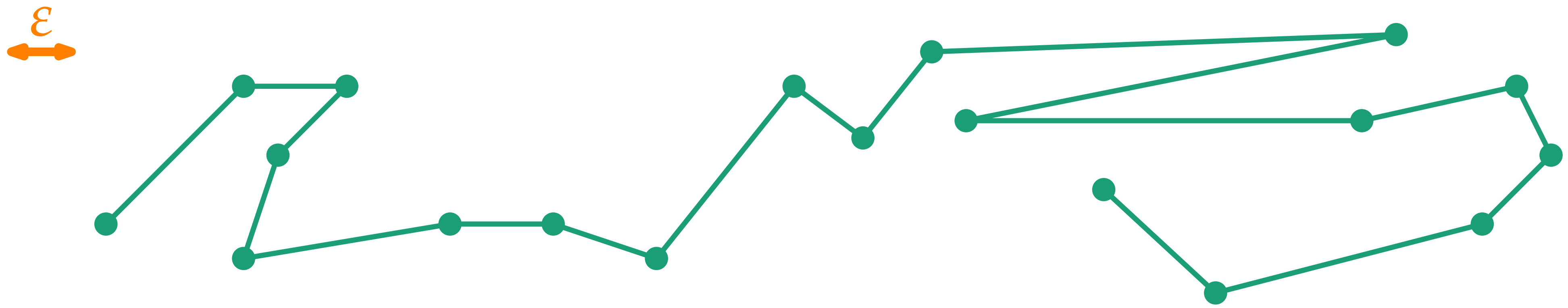

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distance measure for comparing two curves

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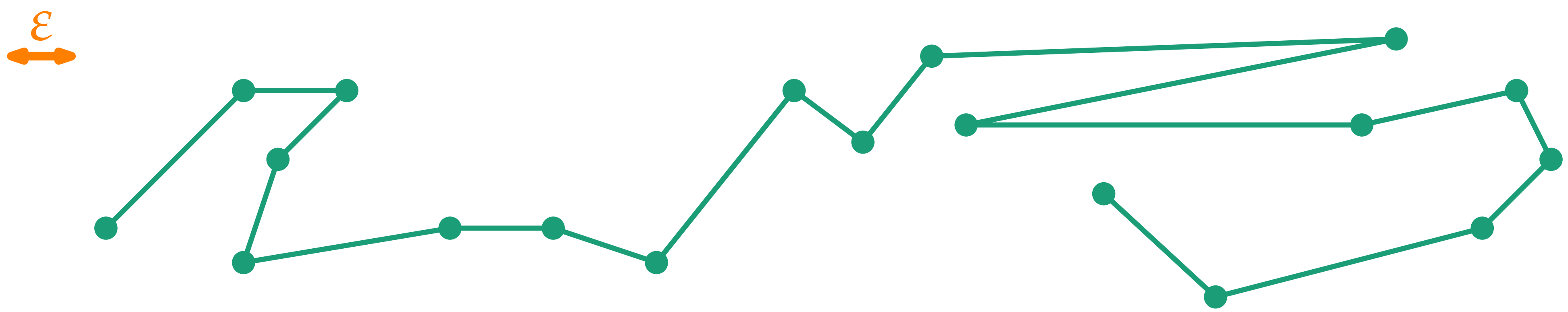

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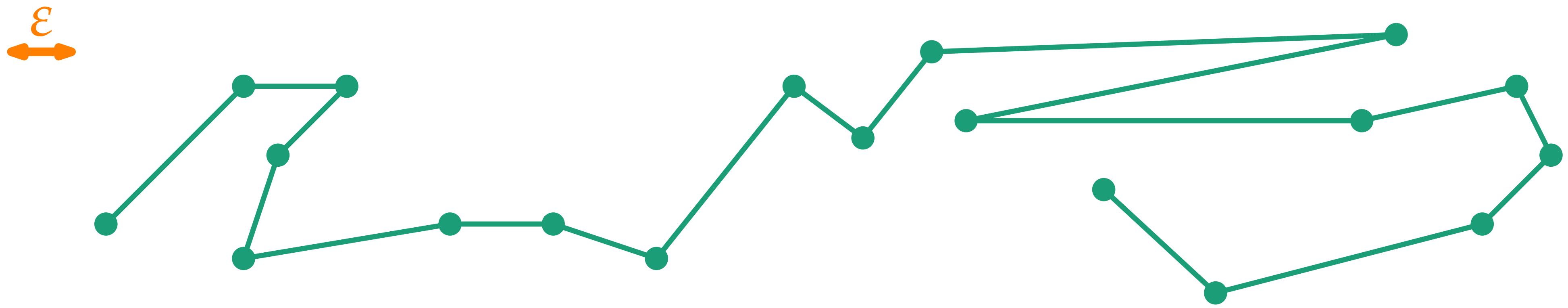

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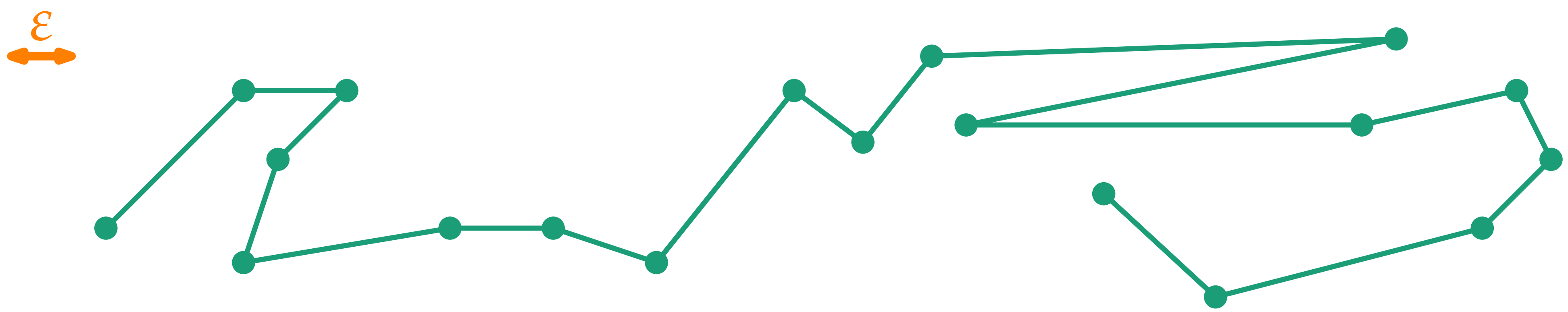

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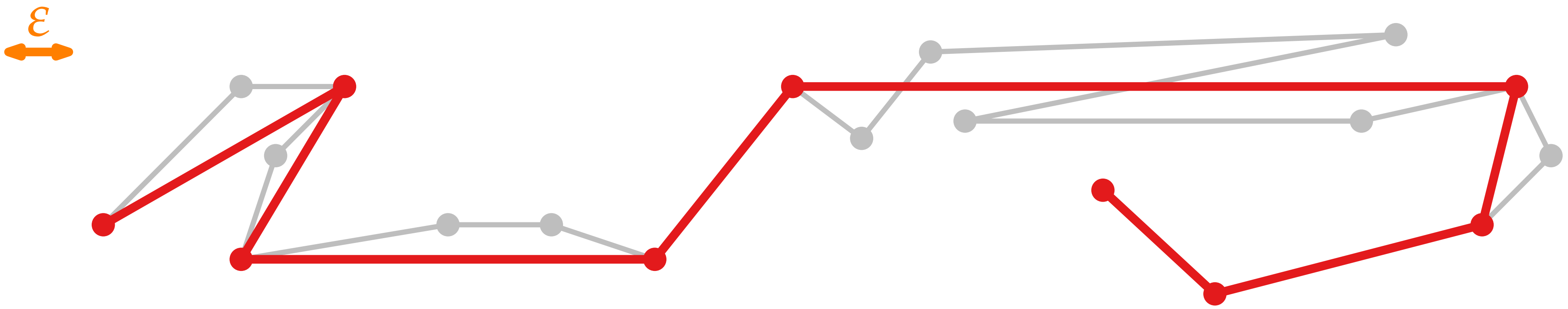

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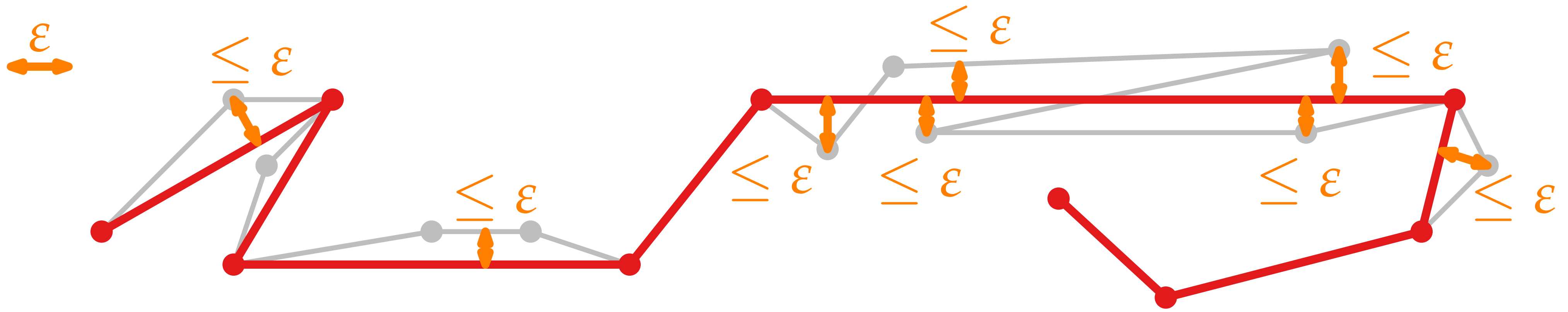

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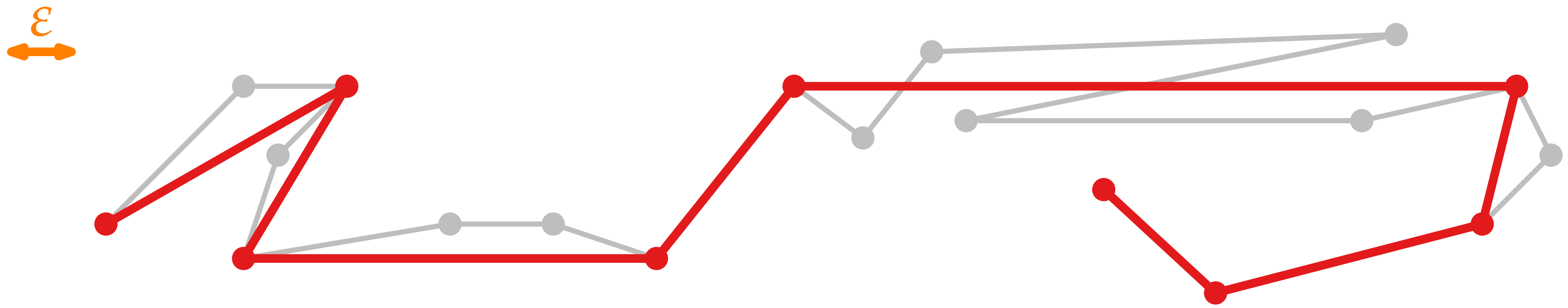

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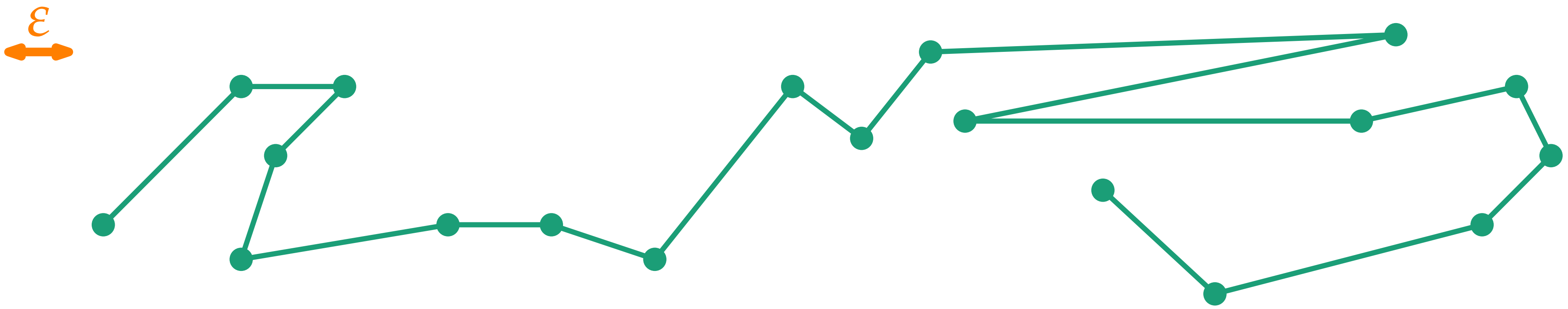

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Find: M

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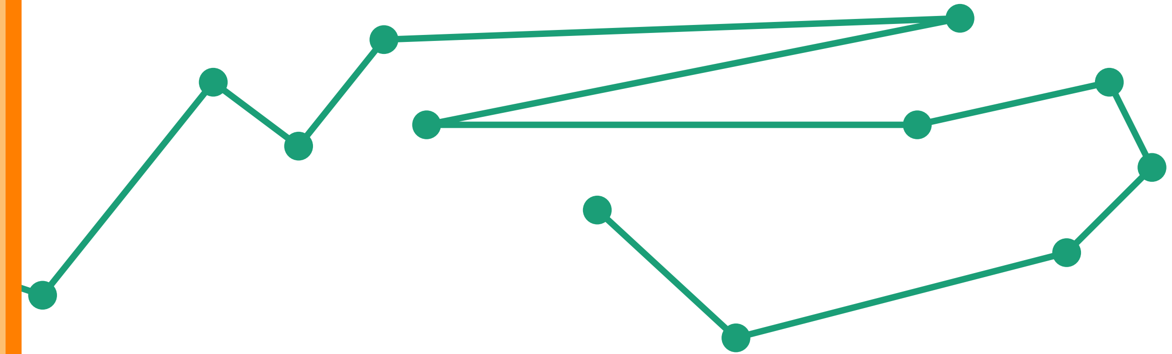
Typical measure



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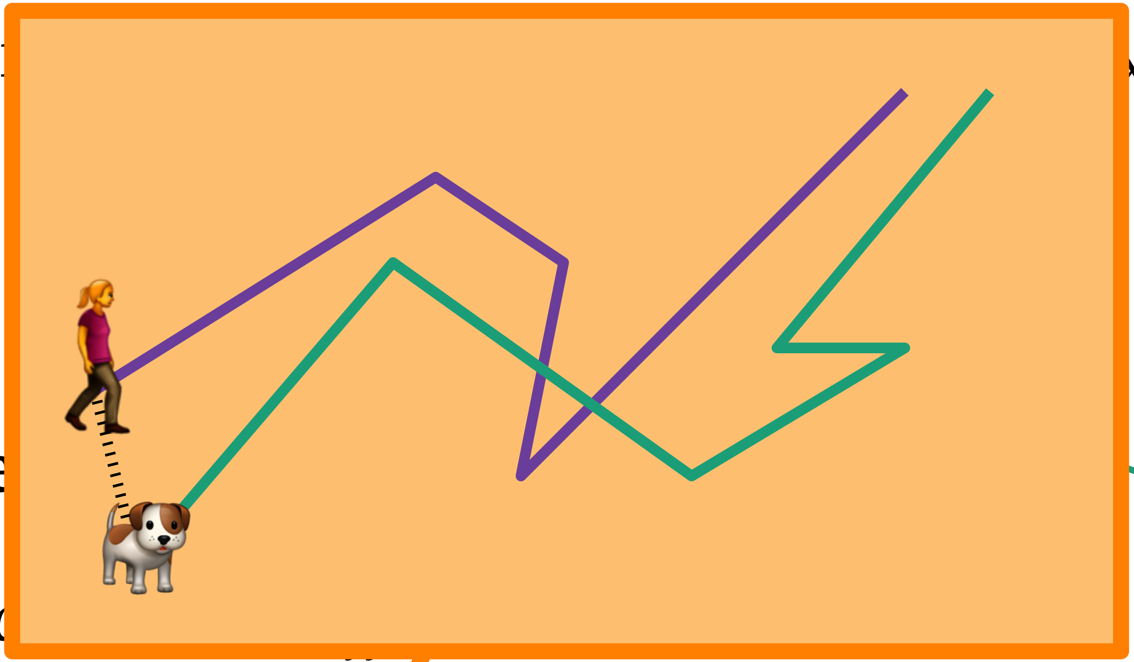

x$(P', P) \leq \varepsilon$ (keep start and end point).

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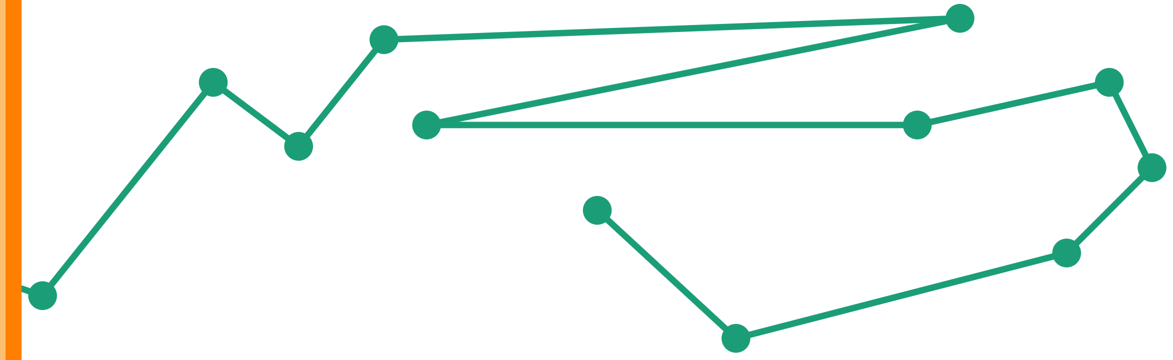

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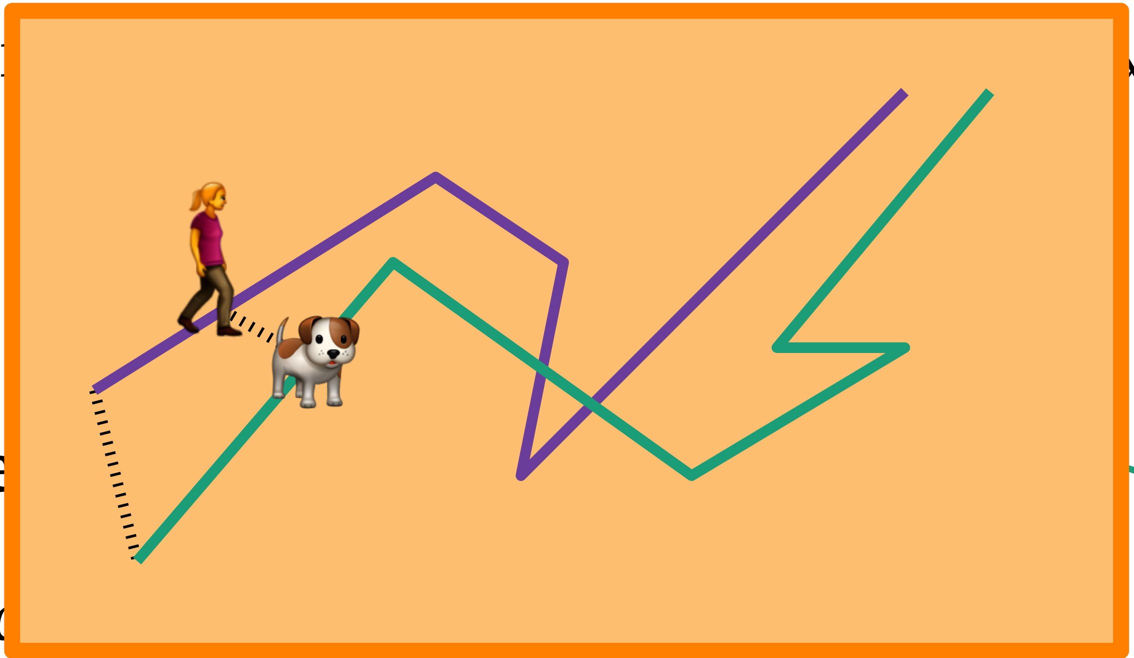
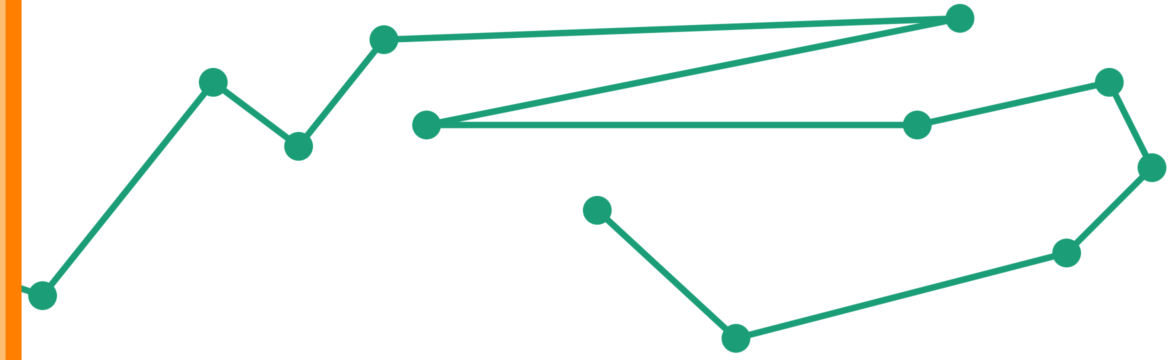
Find: M ... $_X(P', P) \leq \varepsilon$ (keep start and end point).





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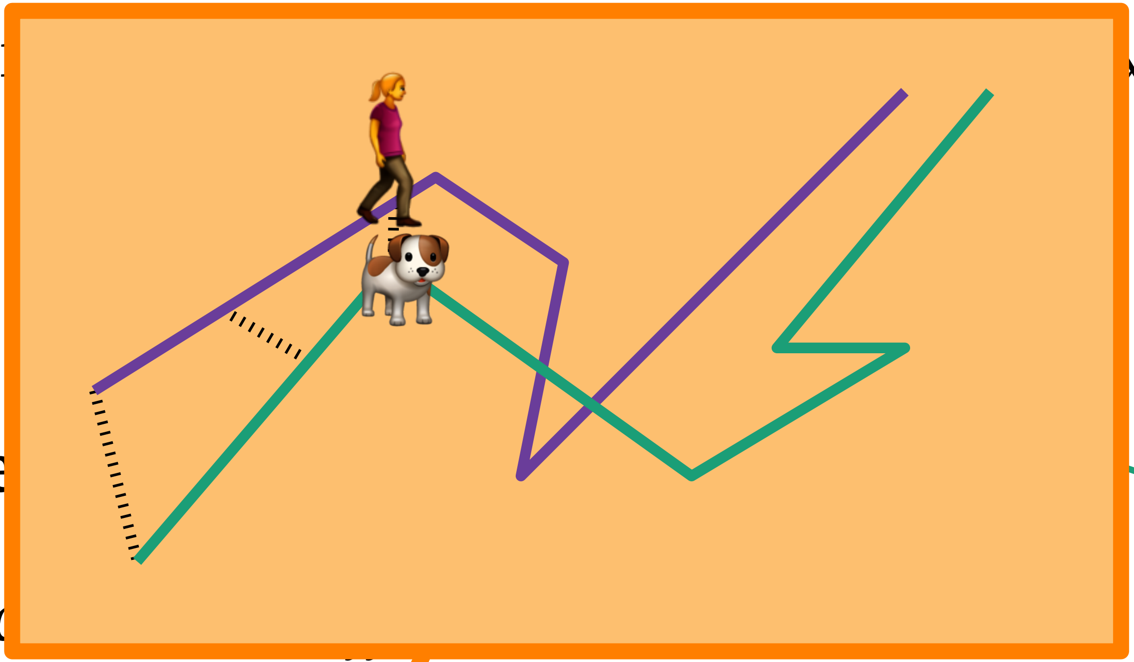

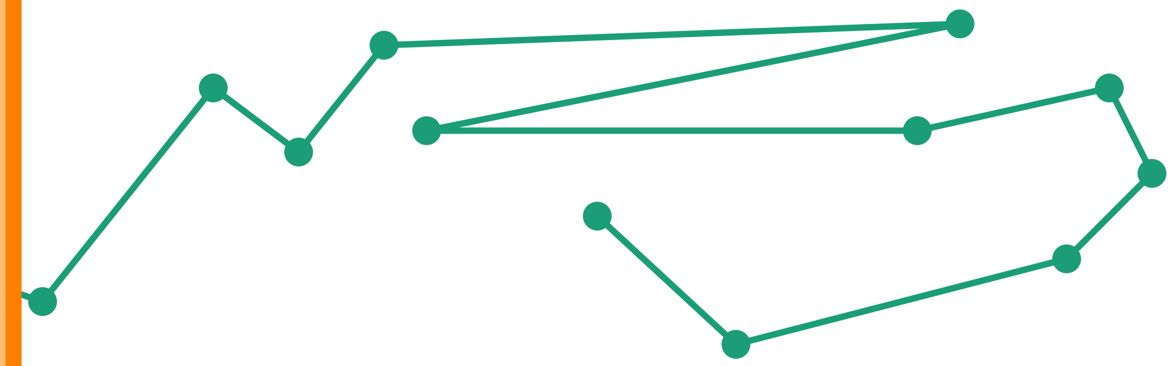

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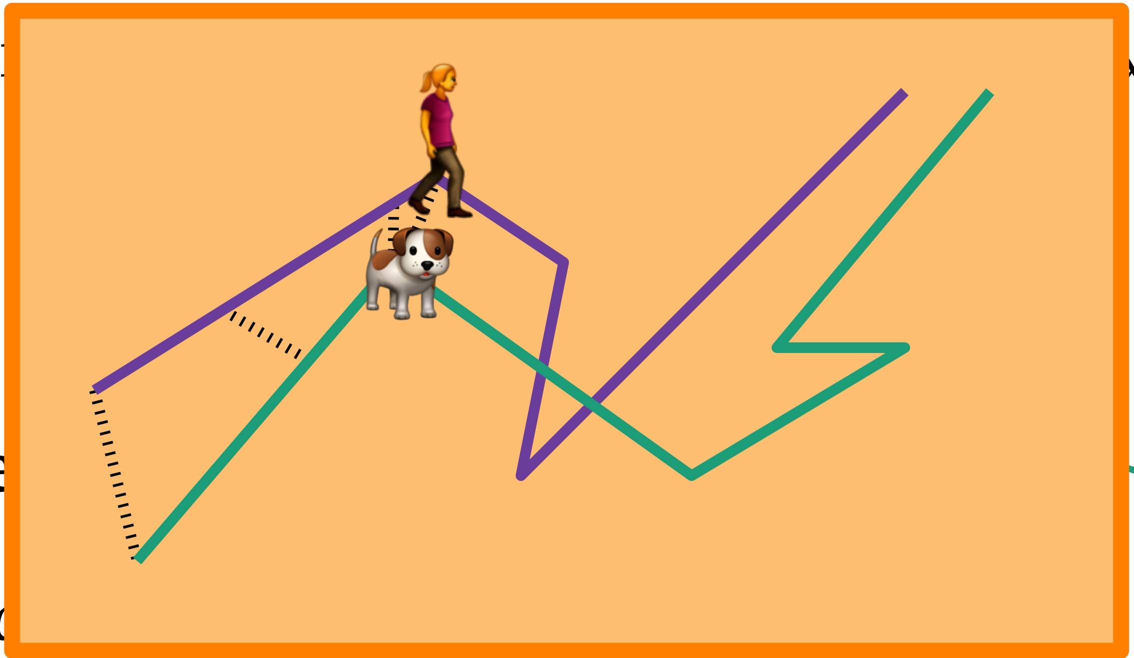

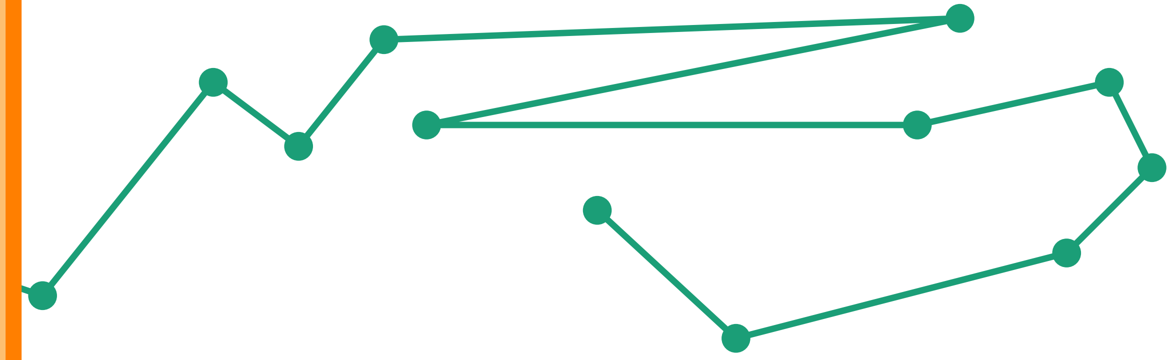

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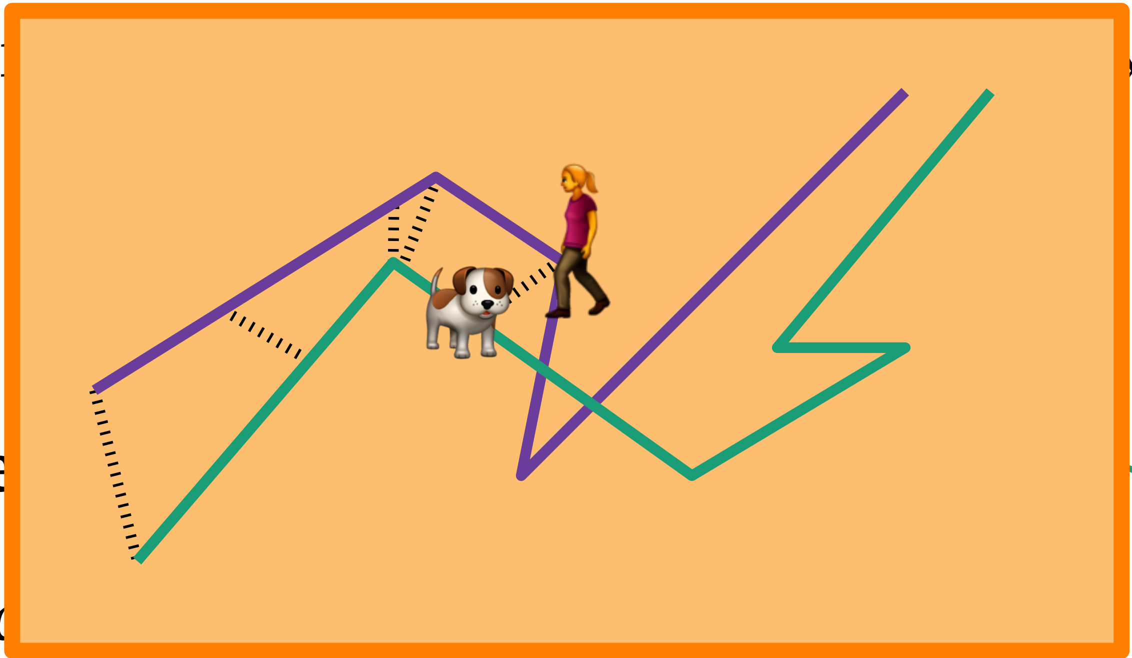

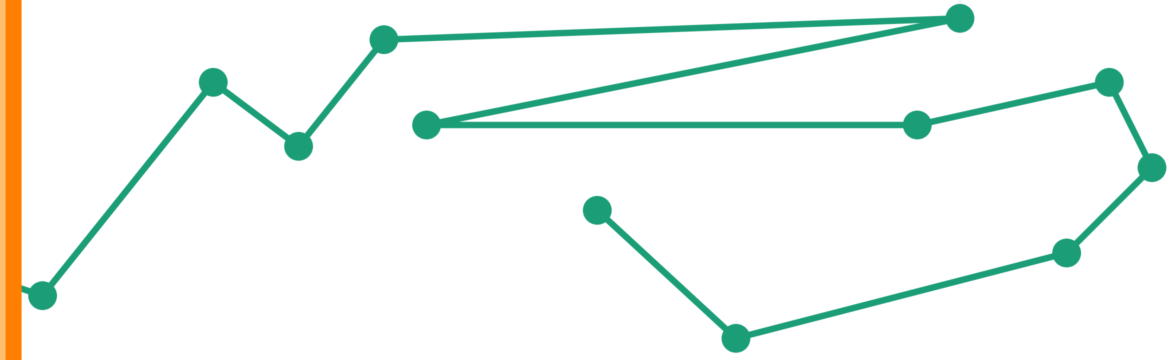

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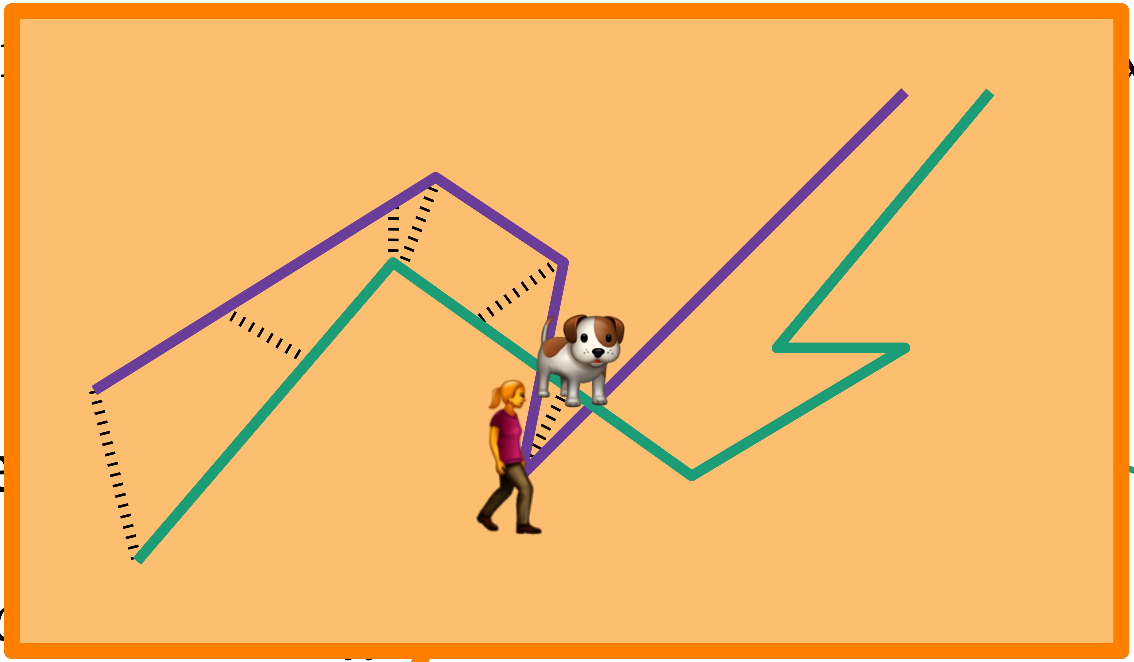
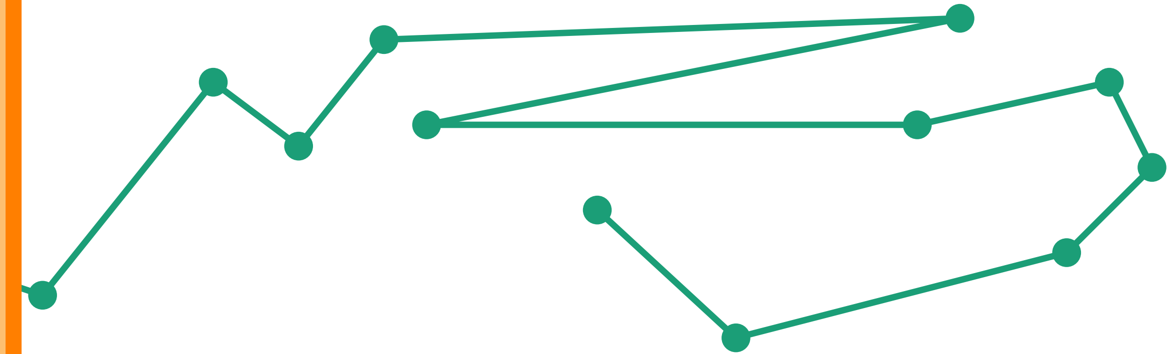
Find: M  $_x(P', P) \leq \varepsilon$ (keep start and end point).



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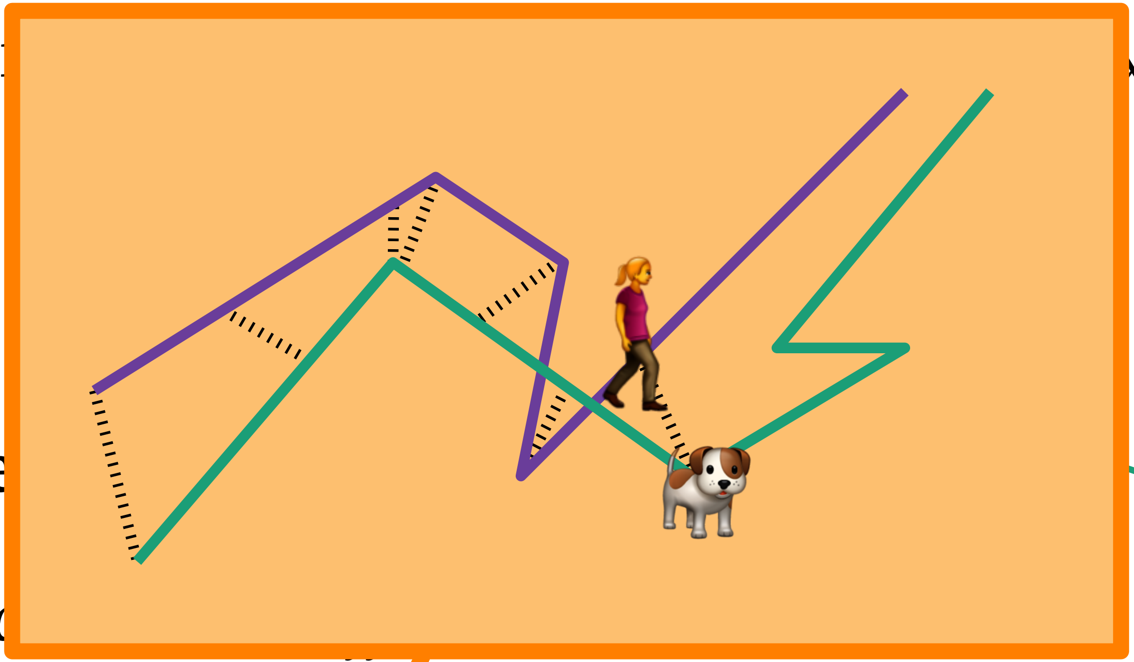

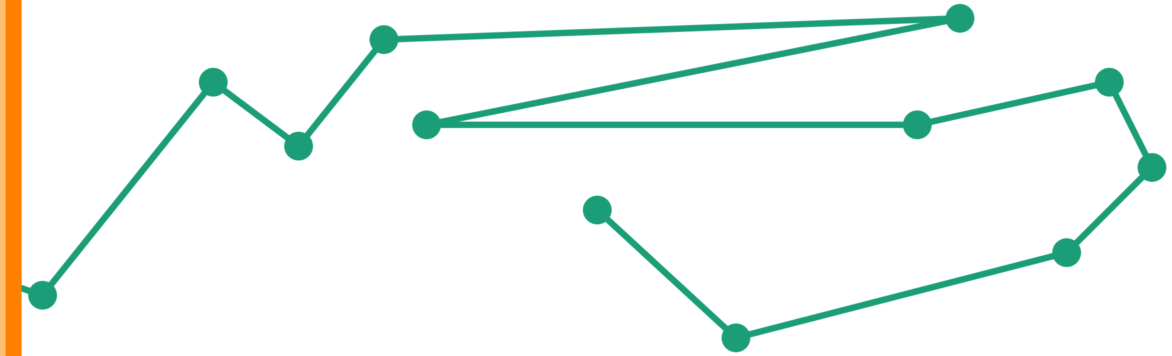

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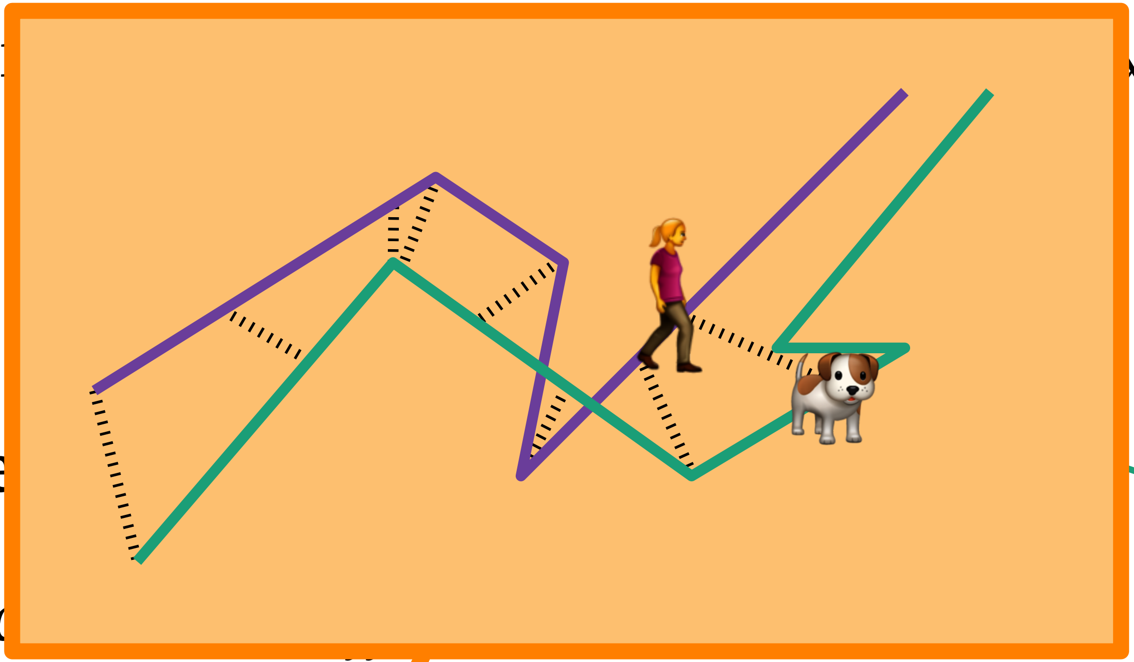

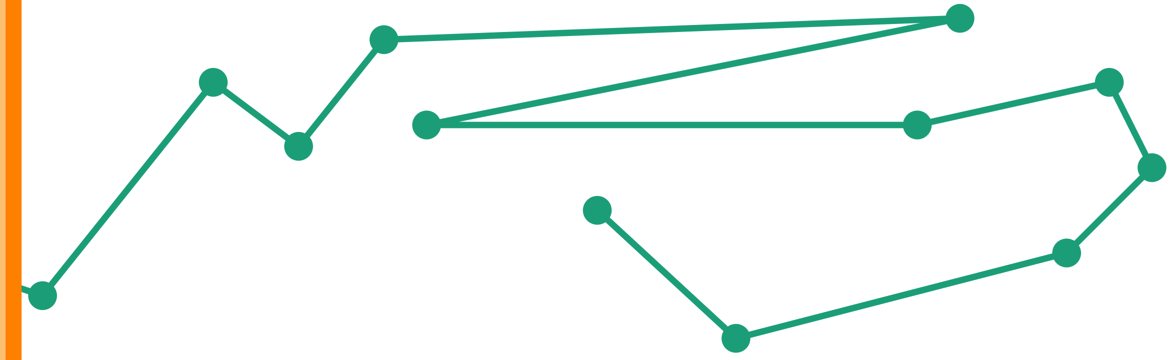

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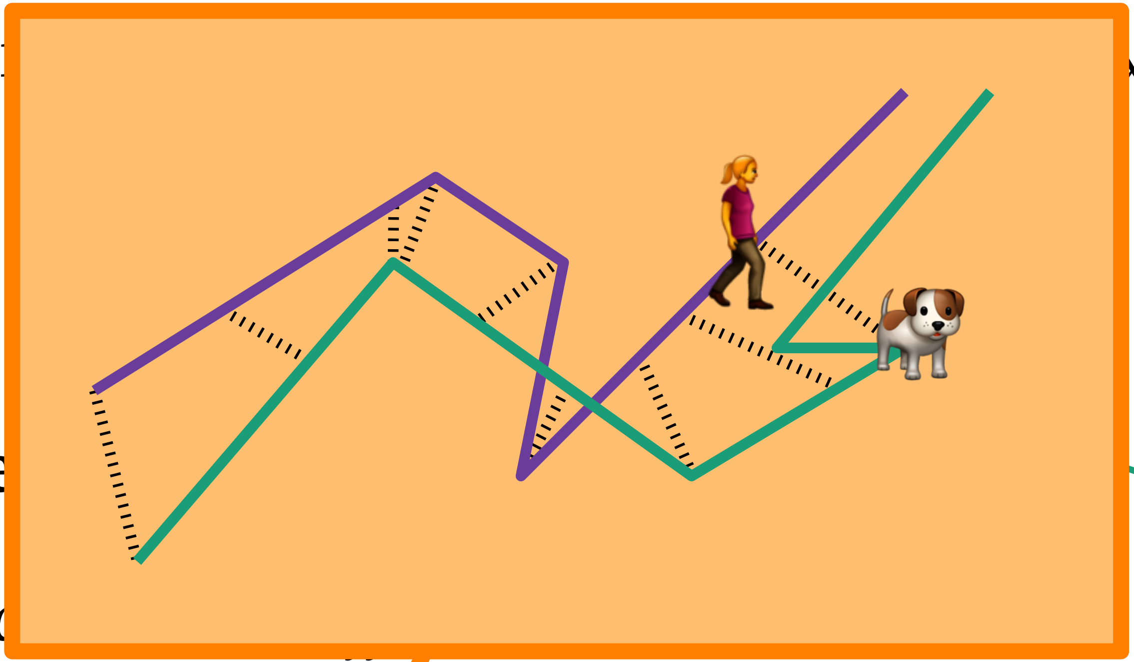

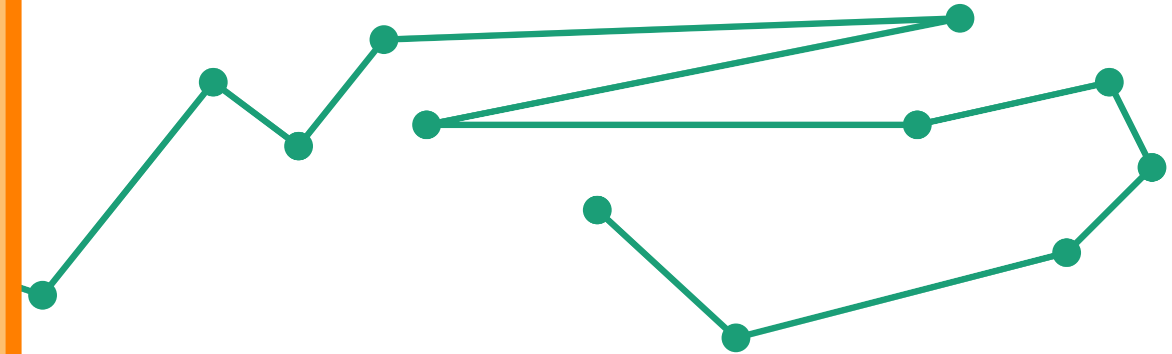

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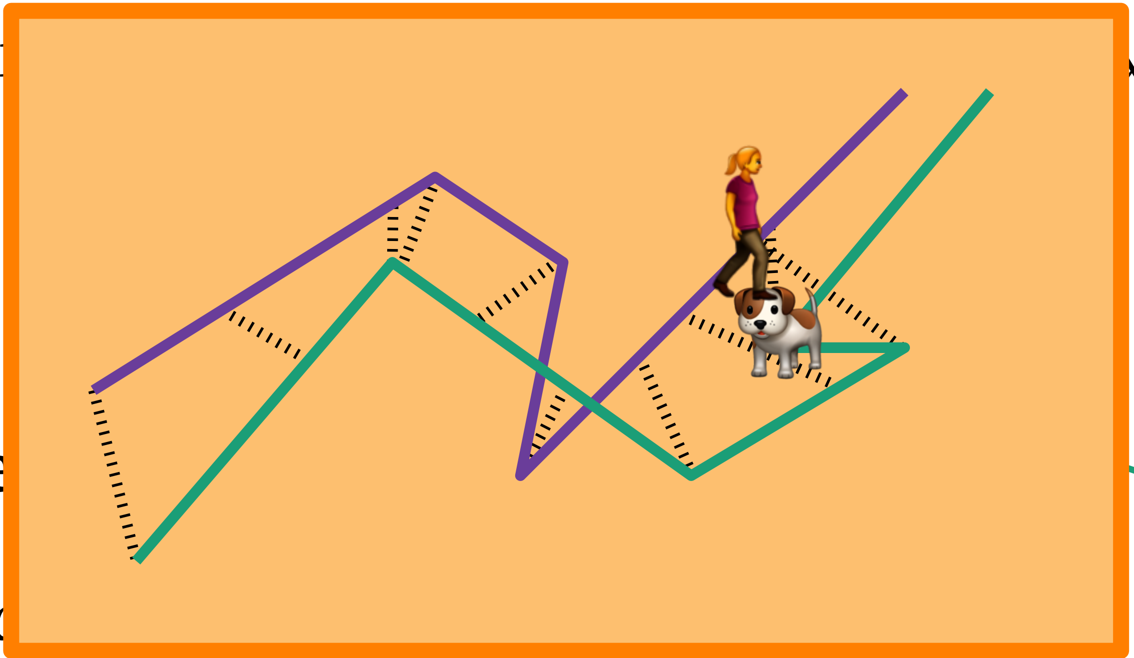

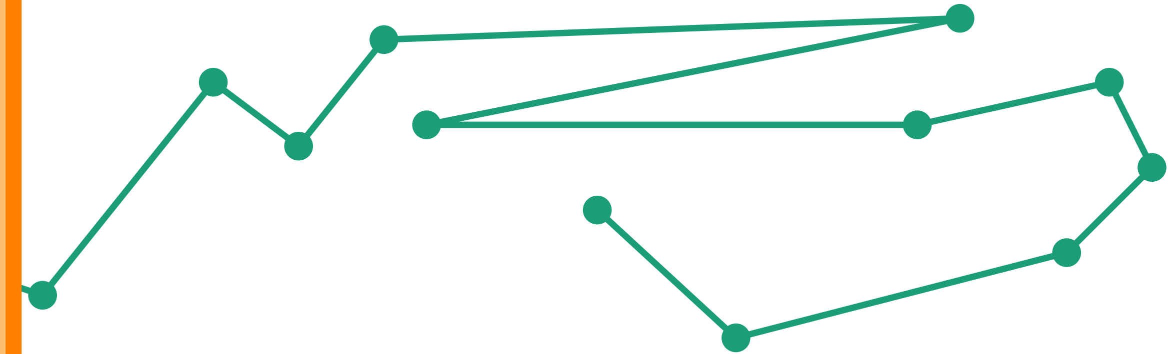

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- *Local Fréchet distance:* for each line segment $\overline{uv} \in P'$: $d_{\mathrm{F}}(\overline{uv}, P[u,v]) \leq \varepsilon$
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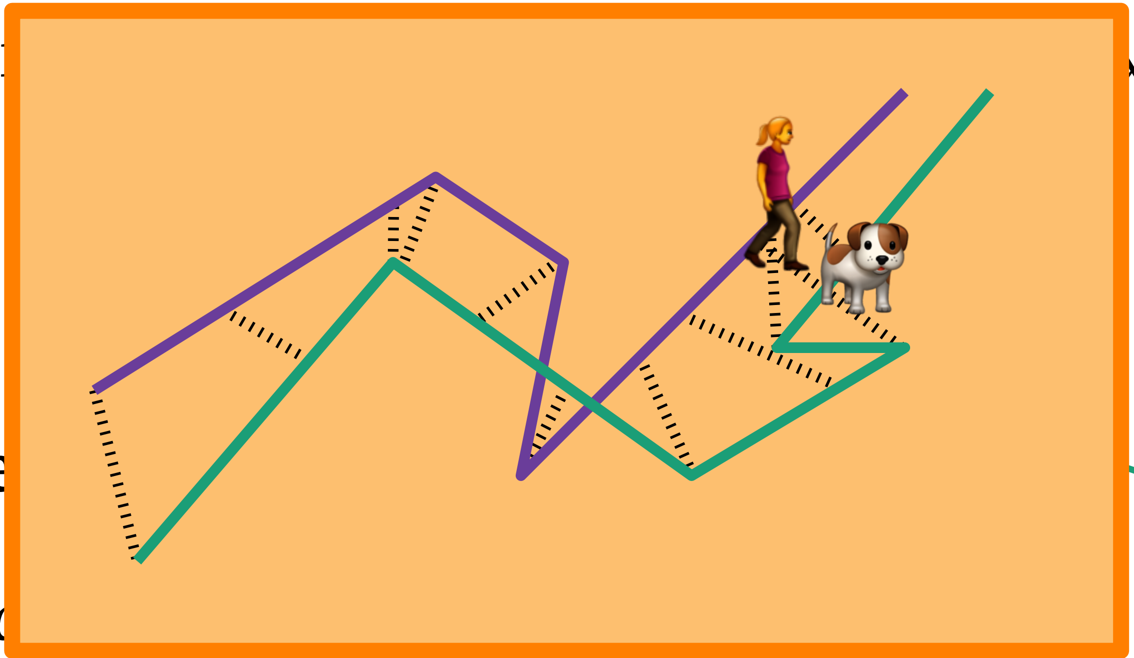
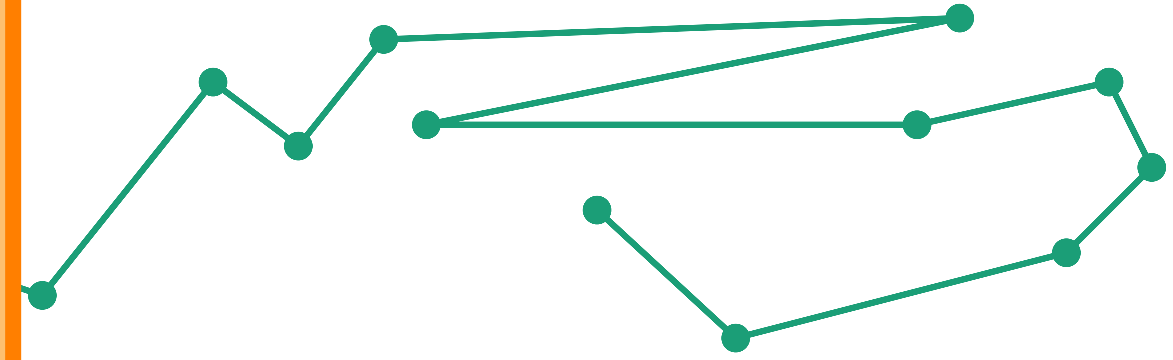
Definition: Polyline Simplification

Given:
- polyline P as a sequence of n points in the plane
- distance threshold ε

Find: M $_X(P', P) \leq \varepsilon$ (keep start and end point).





Typical measure

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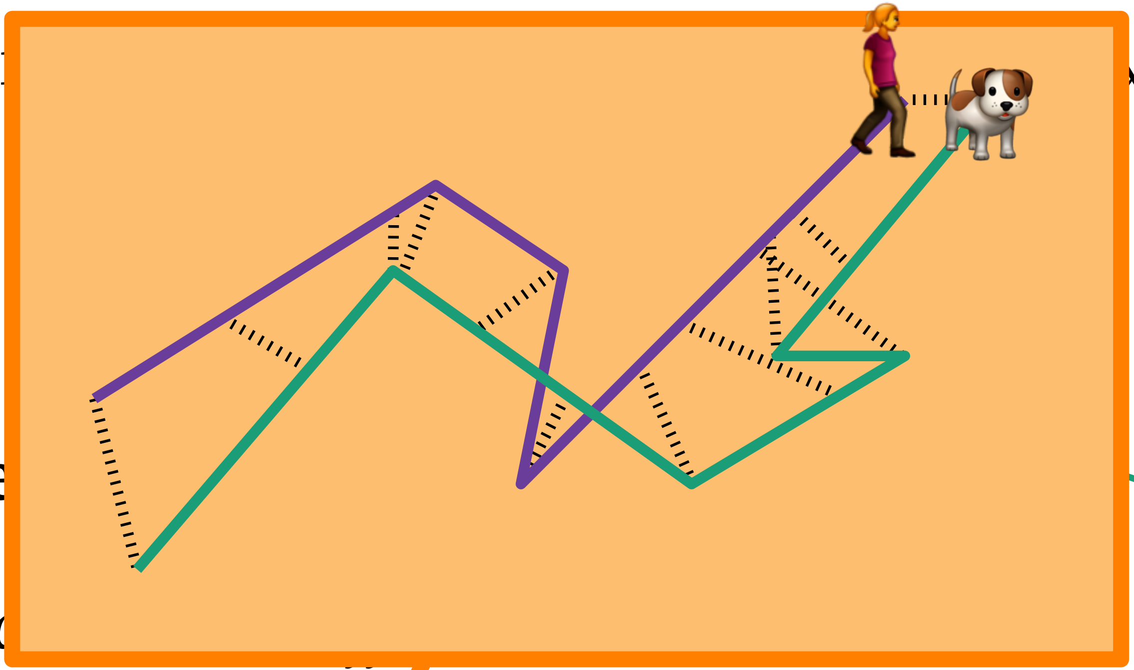
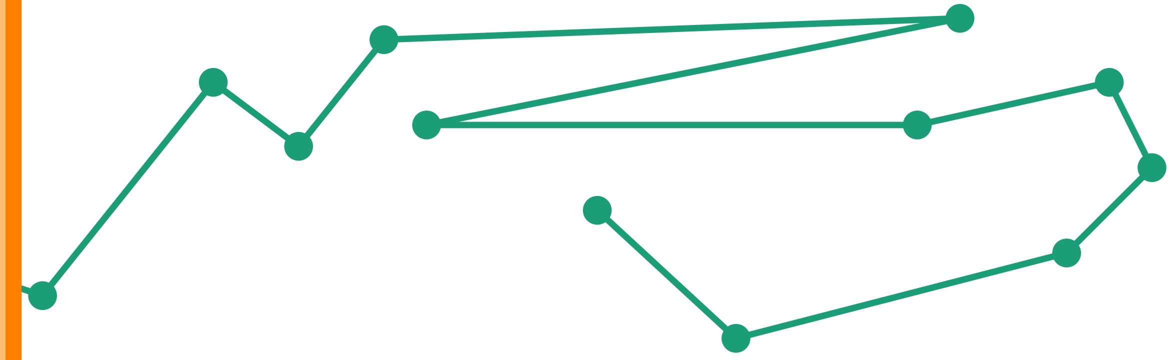
Find: M  $_X(P', P) \leq \varepsilon$ (keep start and end point).



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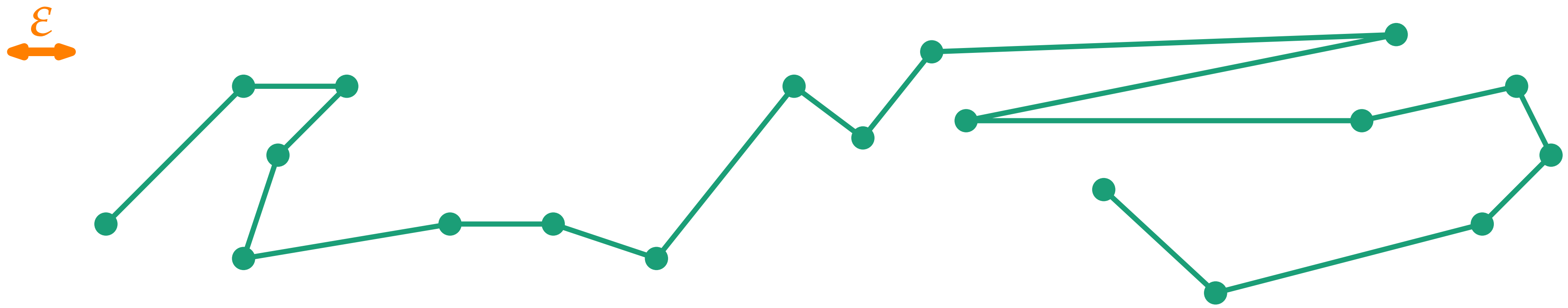

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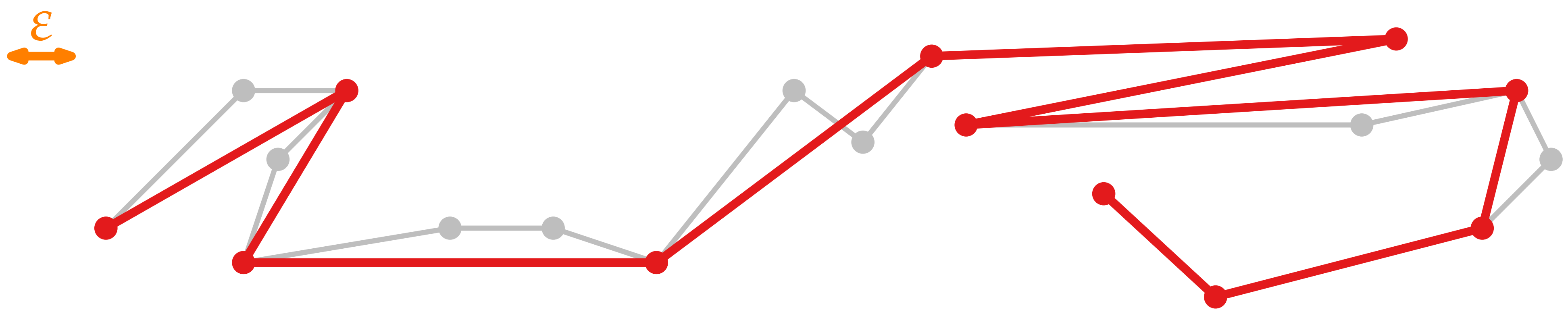

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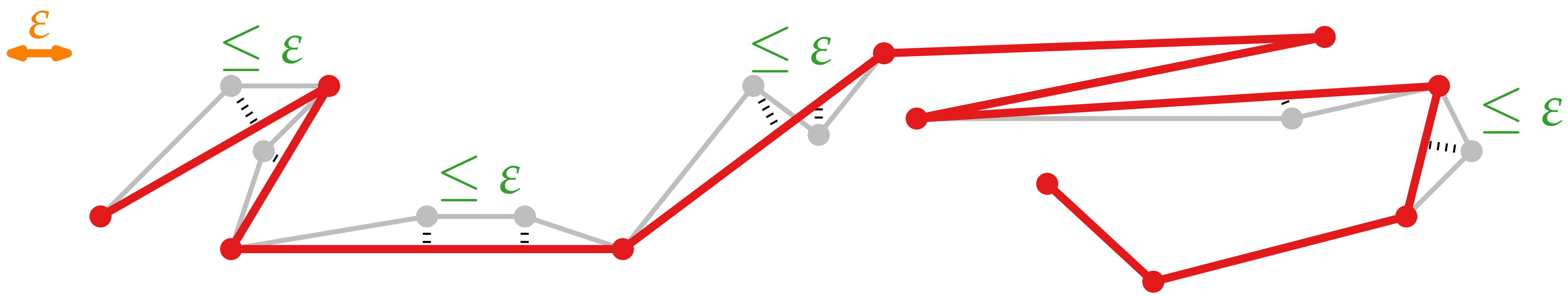

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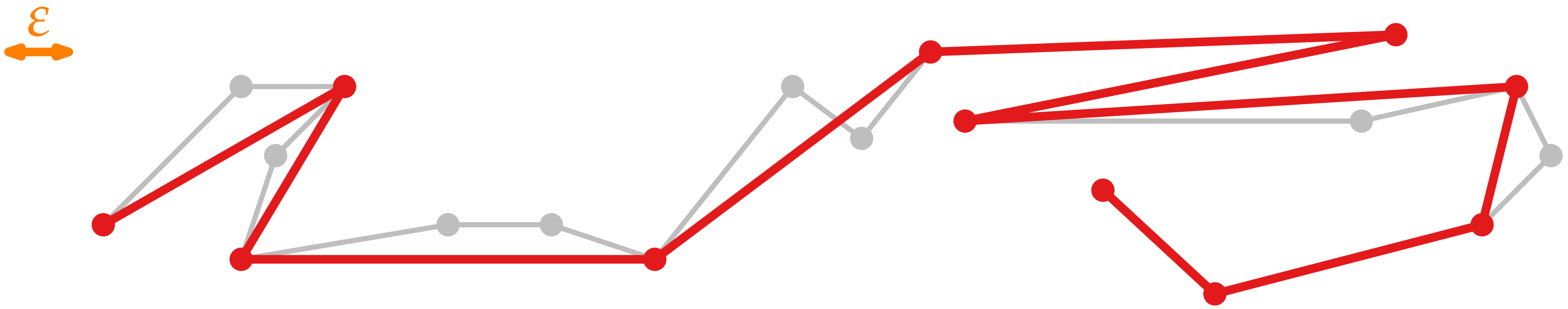

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Related Work & Contribution

Running time for finding an optimal polyline simplification …

Related Work & Contribution

minimum size

Running time for finding an optimal polyline simplification . . .

Related Work & Contribution

minimum size

Running time for finding an optimal polyline simplification …

… under the local Hausdorff distance:

Related Work & Contribution

minimum size

Running time for finding an optimal polyline simplification . . .

. . . under the local Hausdorff distance:

- $O(n^3)$ [Imai, Iri ’88]

Related Work & Contribution

minimum size

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Related Work & Contribution

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Related Work & Contribution

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Algorithmica (2005) 42: 203–219
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Algorithmica

Near-Linear Time Approximation Algorithms for Curve Simplification[1]

Pankaj K. Agarwal,[2] Sariel Har-Peled,[3] Nabil H. Mustafa,[2] and Yusu Wang[2]

5. Conclusions. In this paper we presented near-linear approximation algorithms for curve simplification under the Hausdorff and Fréchet error measures. We presented the first efficient approximation algorithm for Fréchet simplifications of a curve in dimension higher than two. Our experimental results demonstrate that our algorithms are efficient.

We conclude by mentioning a few open problems:

(i) Does there exist a near-linear algorithm for computing an ε-simplification of size at most $c\kappa_M(\varepsilon, P)$ for a polygonal curve P, where $c \geq 1$ is a constant?
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(iii) Is there any provably efficient exact/approximation algorithm for curve simplification in $\mathbb{R}^2$ that returns a simple curve if the input curve is simple.

2005

Related Work & Contribution

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2005

Polyline Simplification has Cubic Complexity

Karl Bringmann
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35th International Symposium on Computational Geometry (SoCG 2019).
Editors: Gill Barequet and Yusu Wang; Article No. 18; pp. 18:1–18:16
Leibniz International Proceedings in Informatics
LIPICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

The classic $\hat{O}(n^3)$[1] time algorithm by Imai and Iri [18] was designed for Local-Hausdorff simplification. By changing the distance computation in this algorithm for the Fréchet distance, one can obtain an $\hat{O}(n^3)$-time algorithm for Local-Fréchet [15]. There are improvements for Local-Hausdorff simplification in small dimension d [19, 8, 6]; the fastest running times are $2^{O(d)}n^2$ for L_1-norm, $\hat{O}(n^2)$ for L_∞-norm, and $\hat{O}(n^{3-\Omega(1/d)})$ for L_2-norm [6].

2019

Related Work & Contribution

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2019

Journal of Computational Geometry jocg.org

ON OPTIMAL POLYLINE SIMPLIFICATION USING THE HAUSDORFF AND FRÉCHET DISTANCE*

Marc van Kreveld,† *Maarten Löffler,*† *and Lionov Wiratma*†‡

Abstract. We revisit the classical polygonal line simplification problem and study it using the Hausdorff distance and Fréchet distance. Interestingly, no previous authors studied line simplification under these measures in its pure form, namely: for a given $\varepsilon > 0$, choose a minimum size subsequence of the vertices of the input such that the Hausdorff or Fréchet distance between the input and output polylines is at most ε.

Table 1: Algorithmic results.

	Douglas-Peucker	Imai-Iri	Optimal
Hausdorff distance	$O(n \log n)$ [19]	$O(n^2)$ [10]	NP-hard *(this paper)*
Fréchet distance	$O(n^2)$ *(easy)*	$O(n^3)$ [17]	$O(kn^5)$ *(this paper)*

n: vertices in the input polyline; k: output complexity of the simplified polyline

2020

Related Work & Contribution

minimum size

Running time for finding an optimal polyline simplification …

…under the local Hausdorff distance:

- $O(n^3)$ [Imai, Iri ’88]
- $O(n^2 \log n)$ [Melkman, O’Rourke ’88]
- $O(n^2)$ time [Chan, Chin ’96]

…under the local Fréchet distance:

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Related Work & Contribution

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our contribution

Related Work & Contribution

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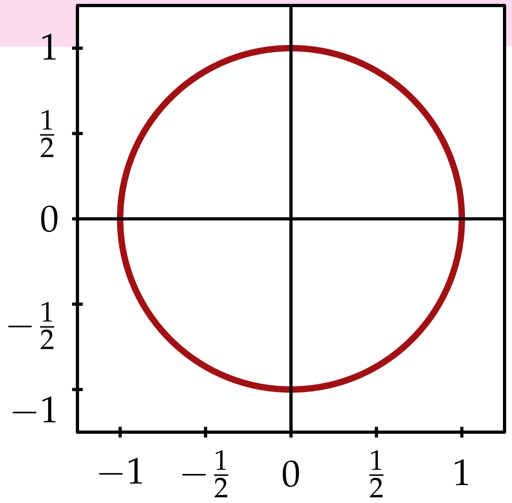
our contribution

L_p:

$p = 2$



Related Work & Contribution

minimum size

Running time for finding an optimal polyline simplification …

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… under the local Fréchet distance:

L_2 norm

- $O(n^3)$ [Imai, Iri '88] → $O(n^3)$ [Godau '91]
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L_1 norm

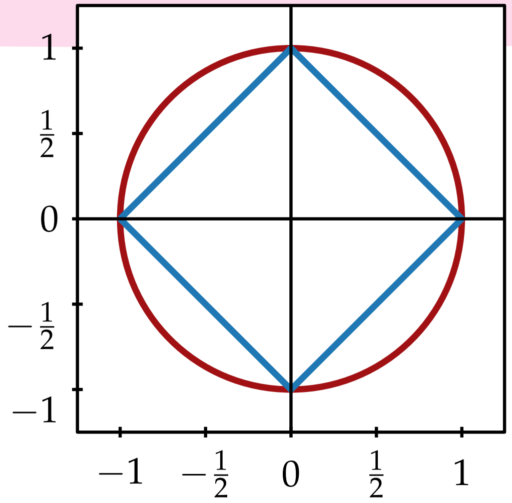
- $O(n^2)$

our contribution

L_p:

$p = 2$

$p = 1$



Related Work & Contribution

minimum size

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L_1 norm

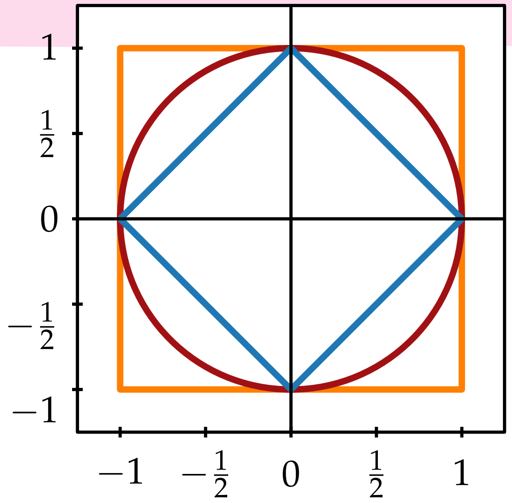
- $O(n^2)$

L_∞ norm

- $O(n^2)$

our contribution

L_p:
$p = \infty$
$p = 2$
$p = 1$



Related Work & Contribution

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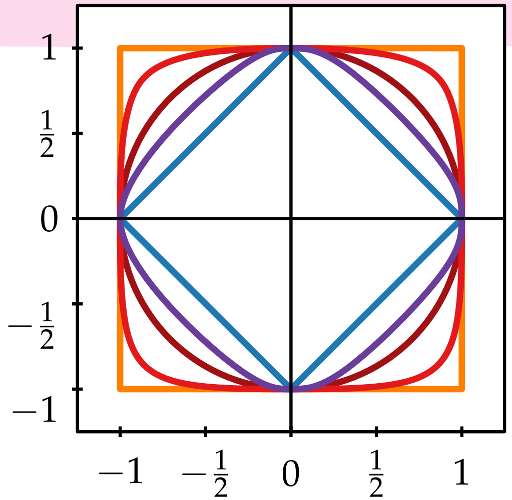
■ $O(n^2)$ time [Chan, Chin '96]	**L_1 norm**	■ $O(n^2)$
	L_∞ norm	■ $O(n^2)$
	$L_{p\in(1,\infty)}$ norm	■ $O(n^2 \log n)$

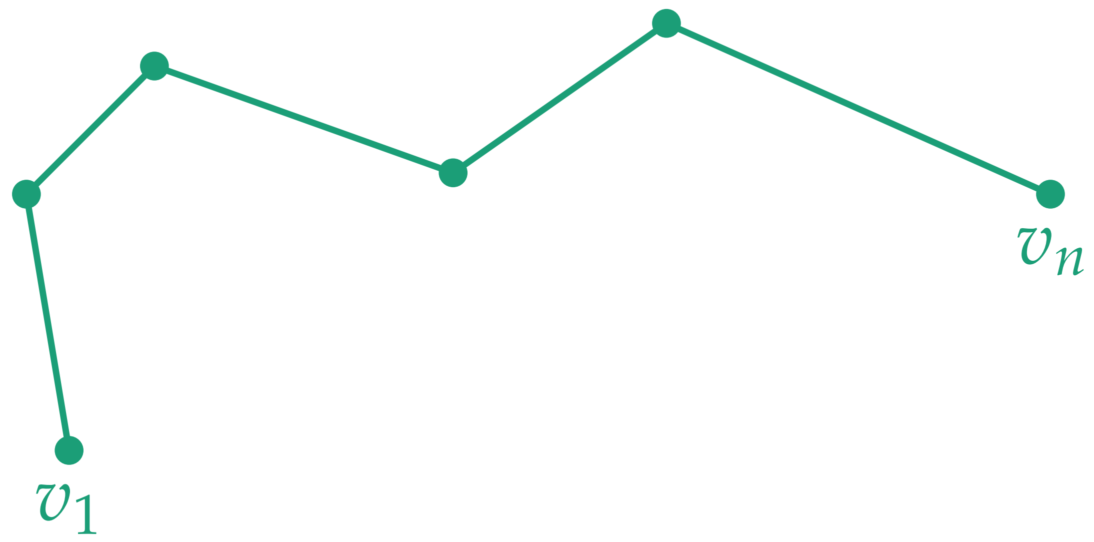
our contribution

L_p:
$p = \infty$
$p = 4$
$p = 2$
$p = 1.5$
$p = 1$



Algorithm by Imai and Iri

- proceeds in two phases



Algorithm by Imai and Iri

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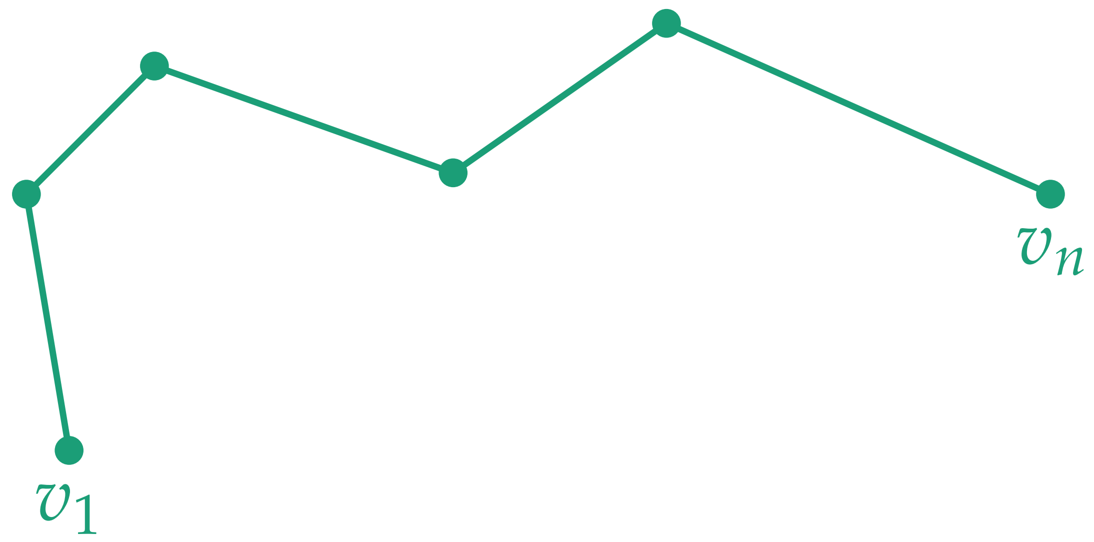

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- First phase:
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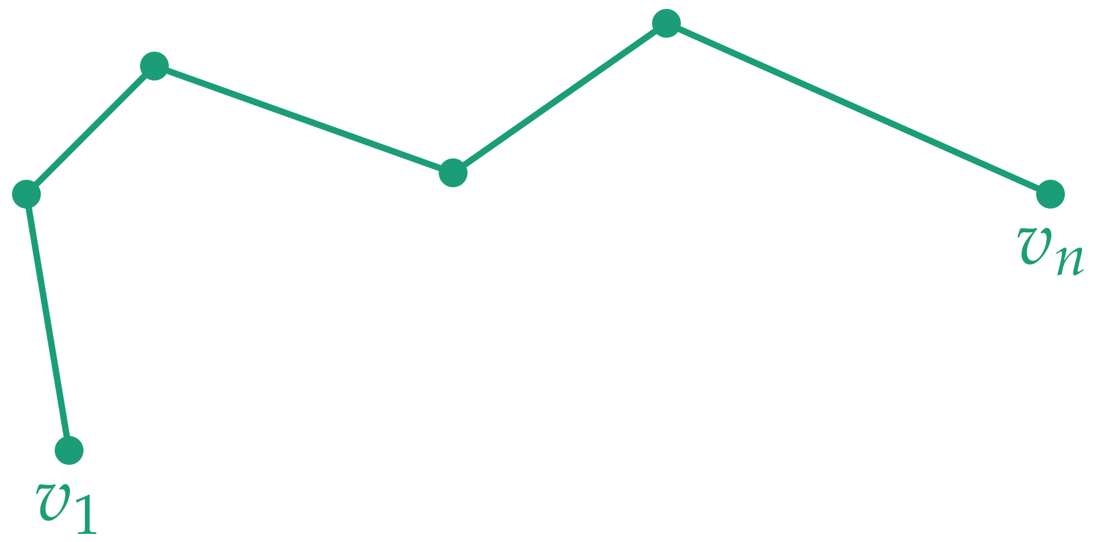

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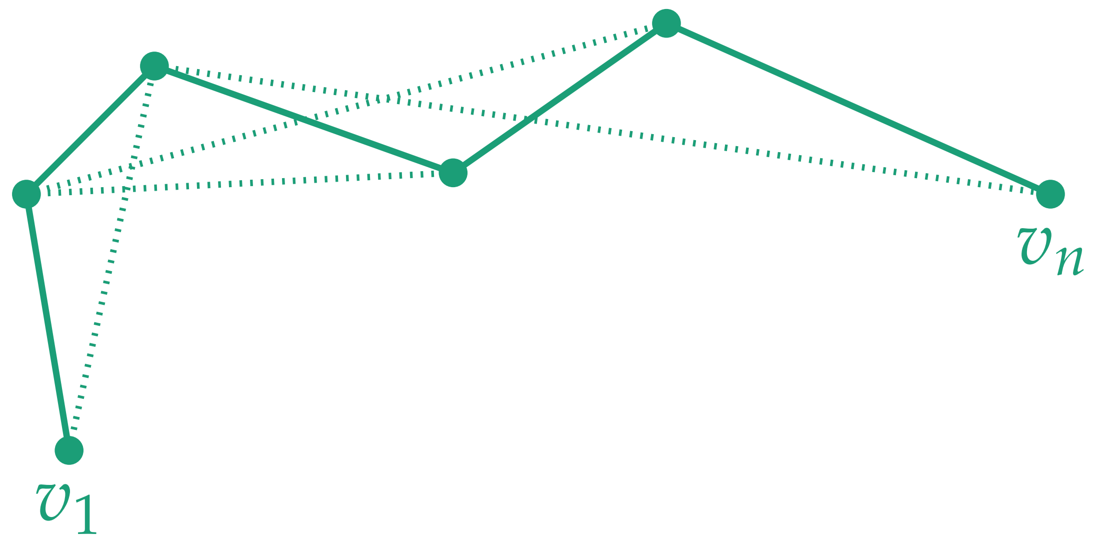

Algorithm by Imai and Iri

- proceeds in two phases
- First phase:
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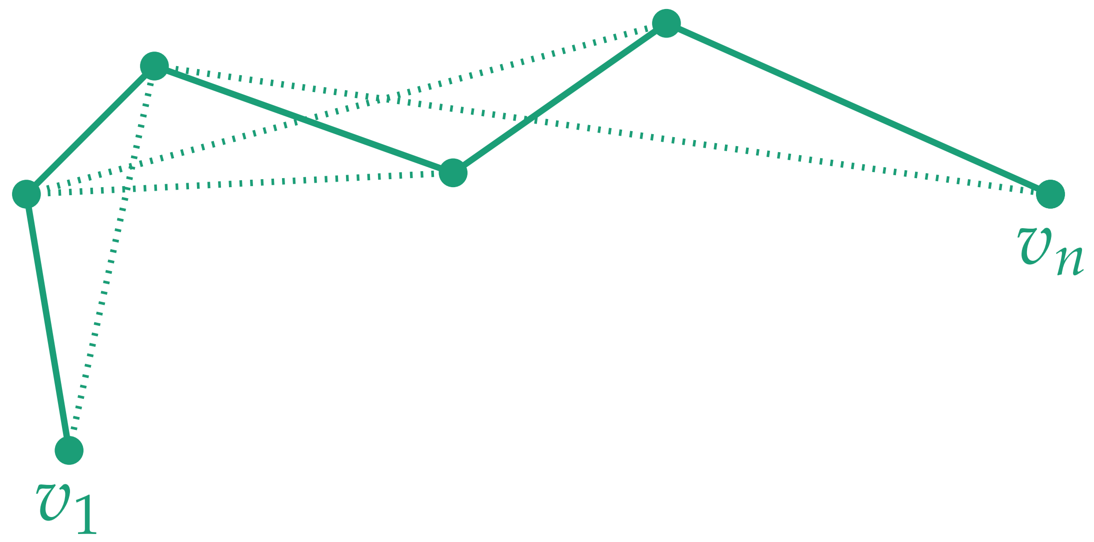
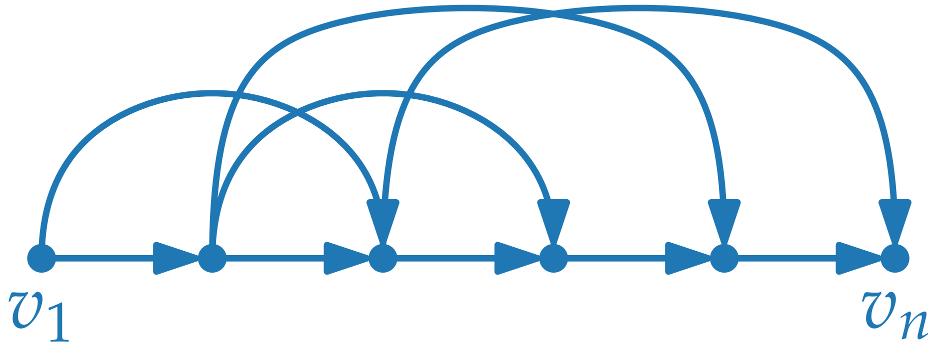
  - build shortcut graph





Algorithm by Imai and Iri

- proceeds in two phases
- First phase:
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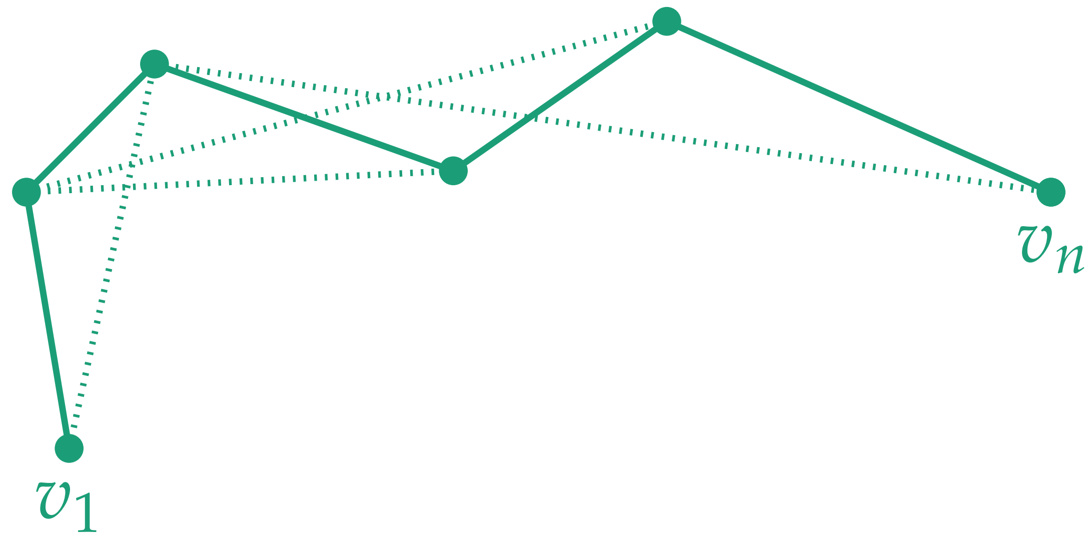
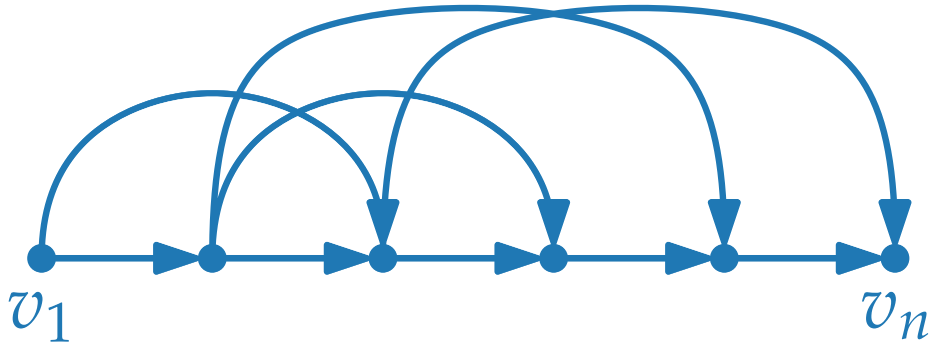
  - build shortcut graph
- Second phase:





Algorithm by Imai and Iri

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- First phase:
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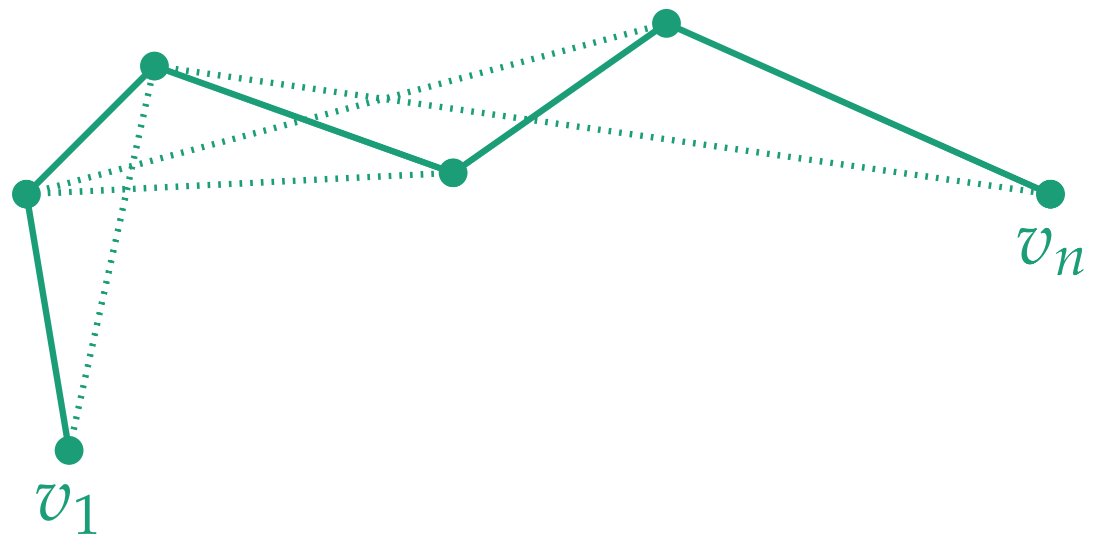
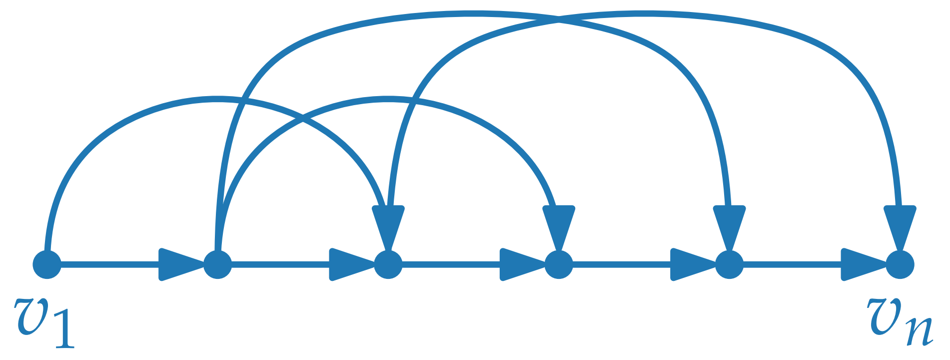
- Second phase:
  - find shortest path in shortcut graph





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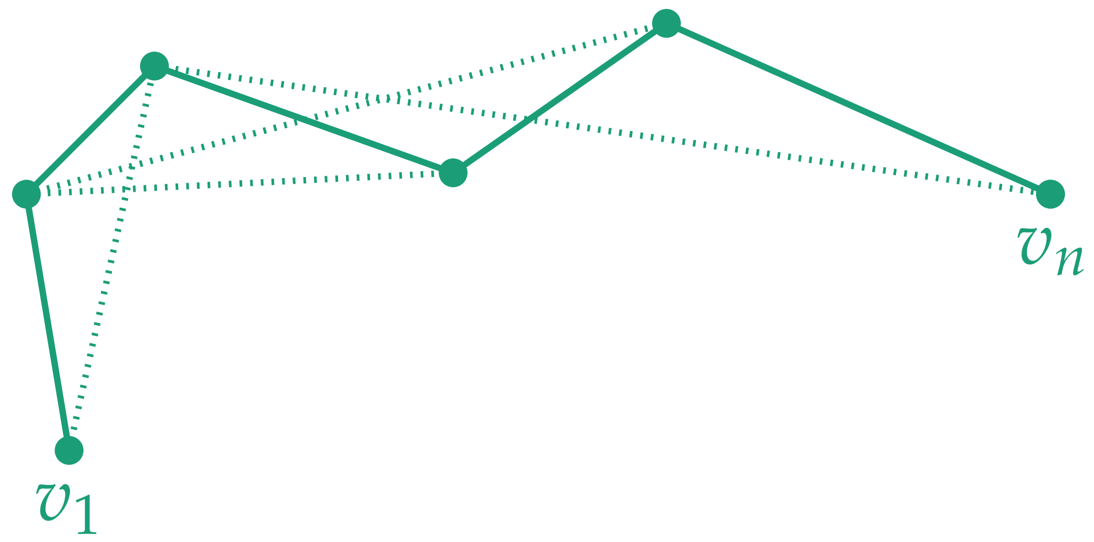

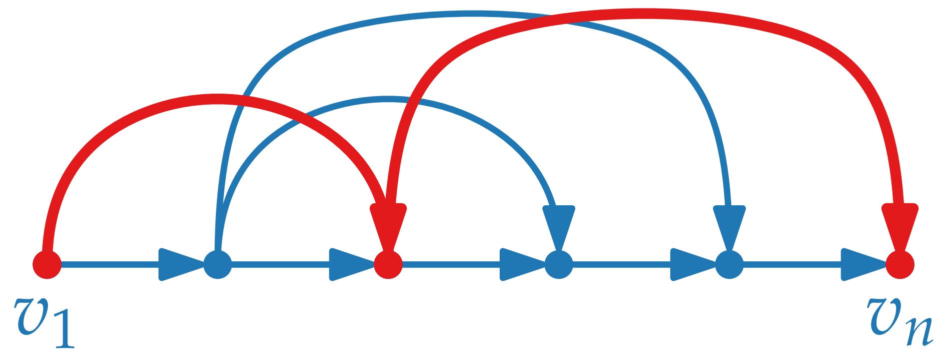

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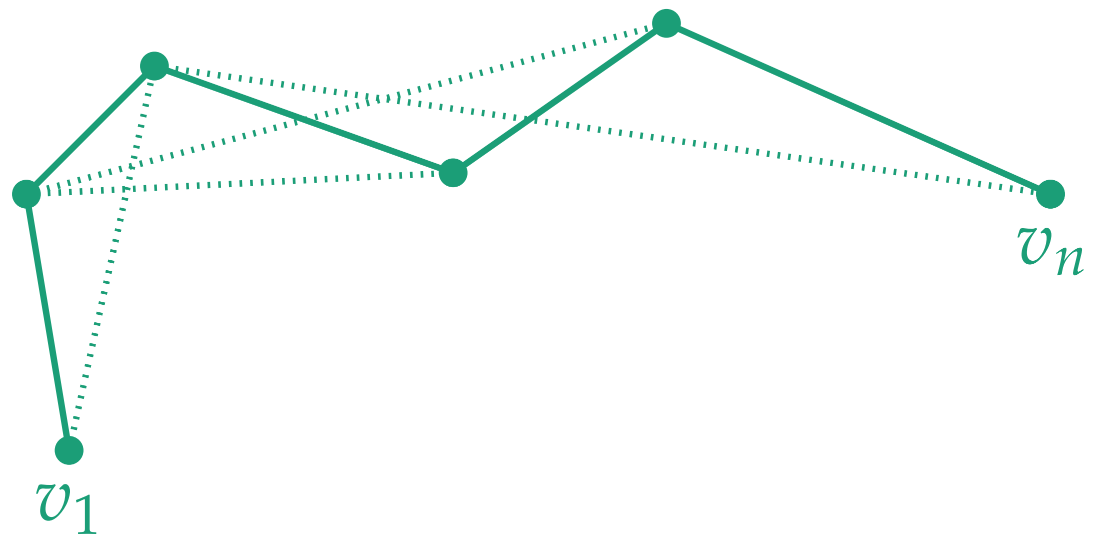

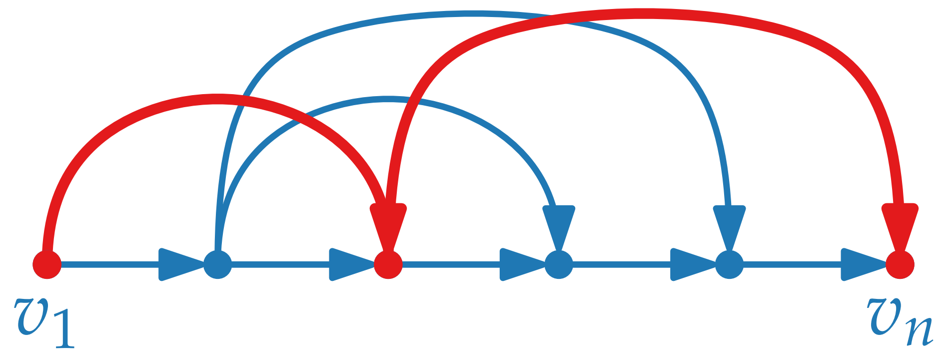

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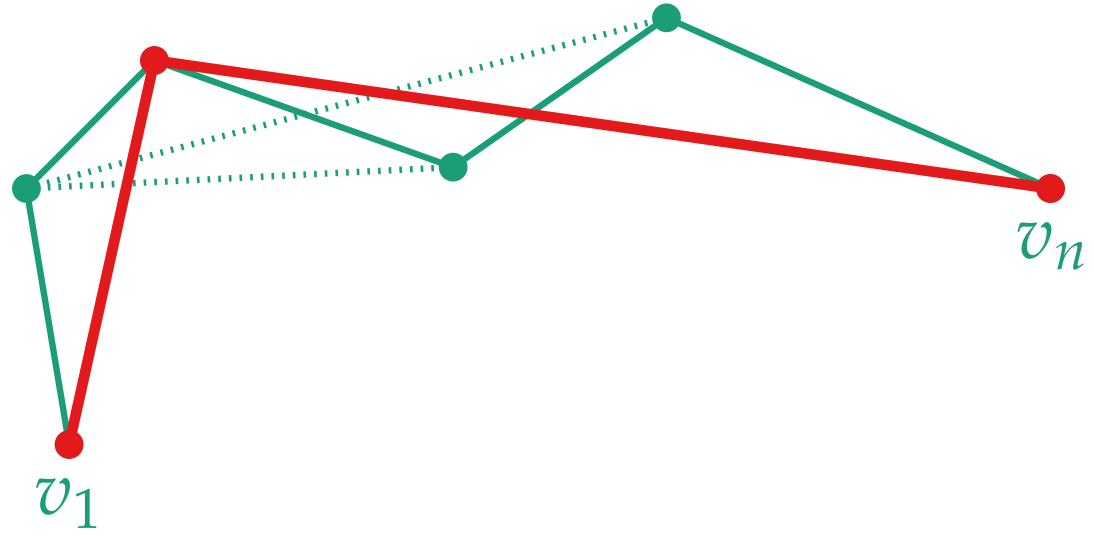

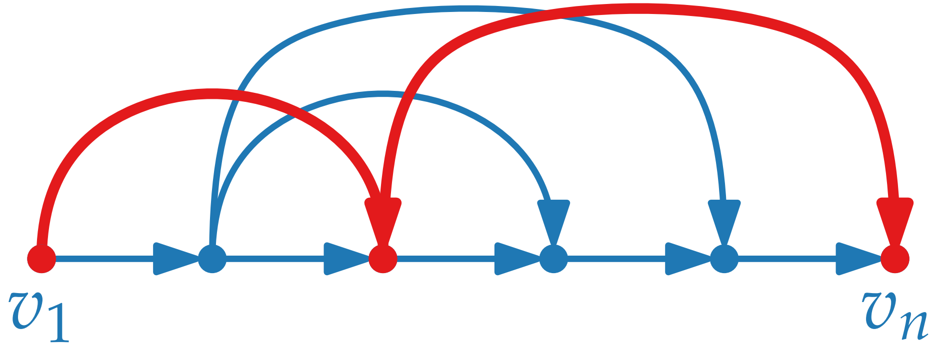

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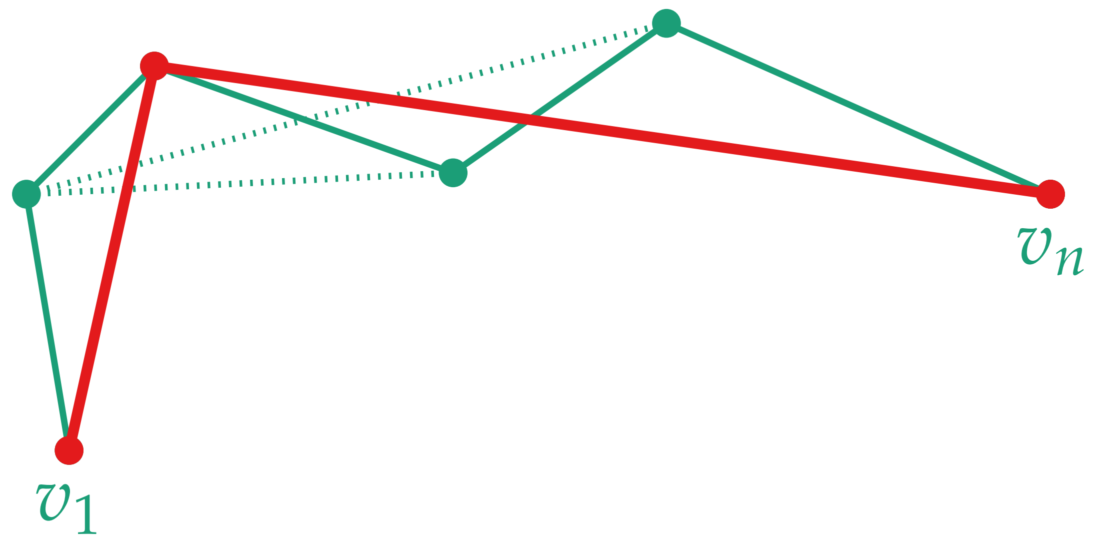

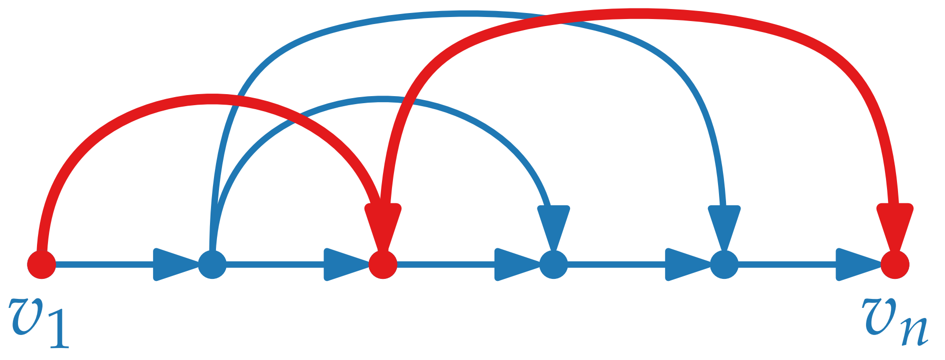

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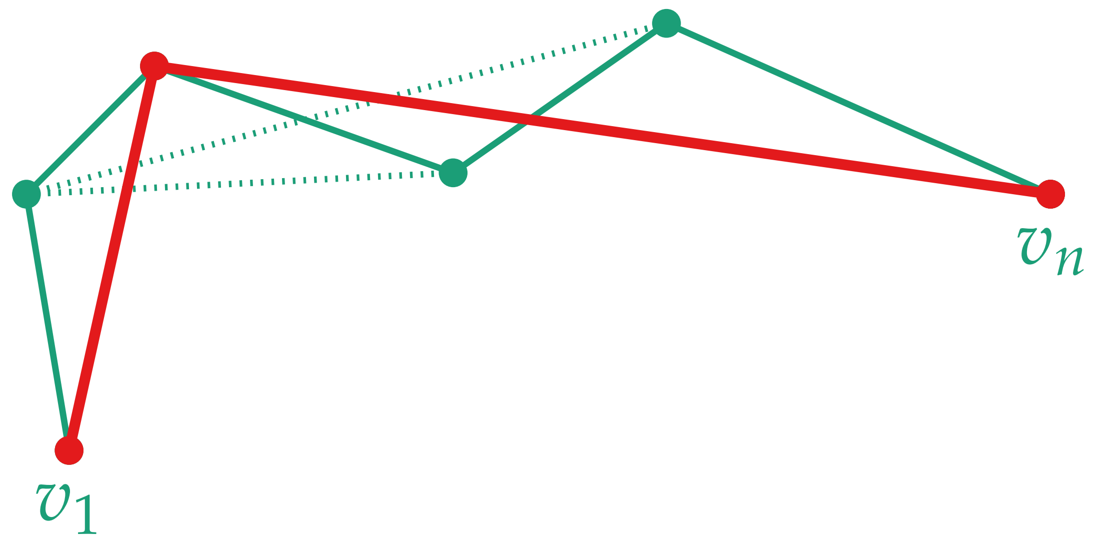

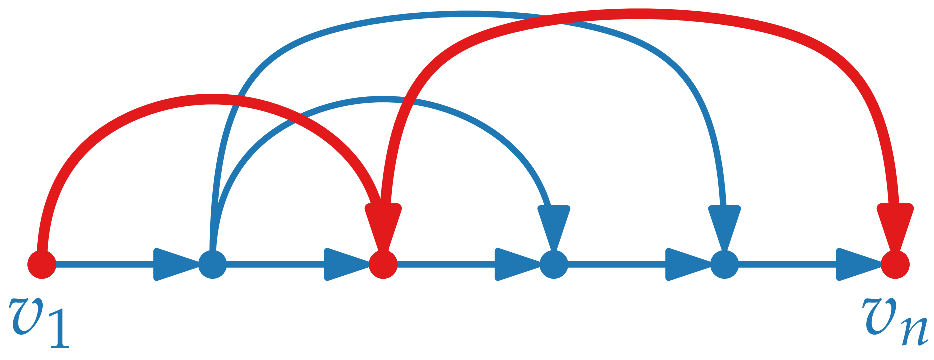

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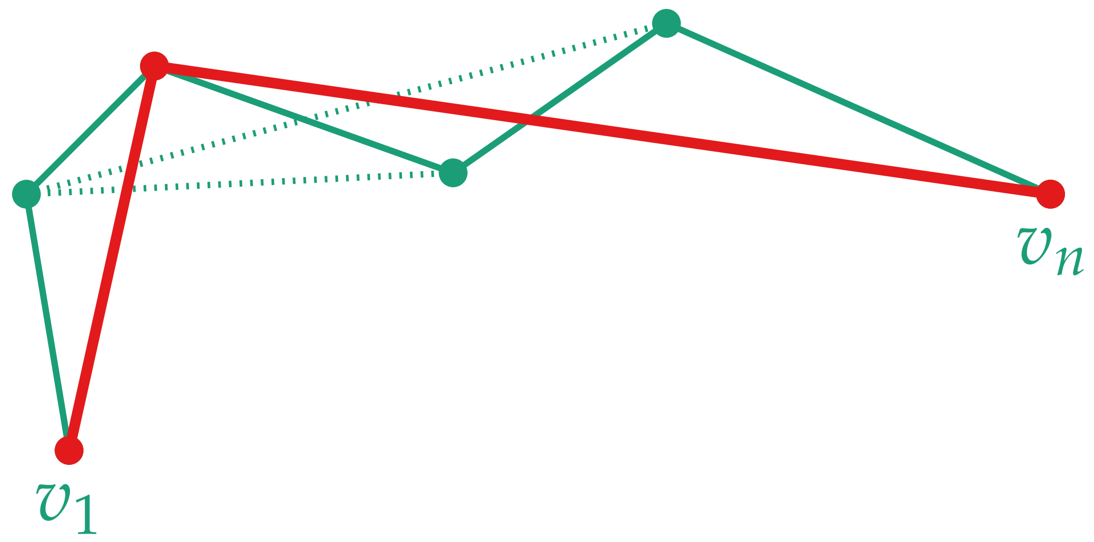

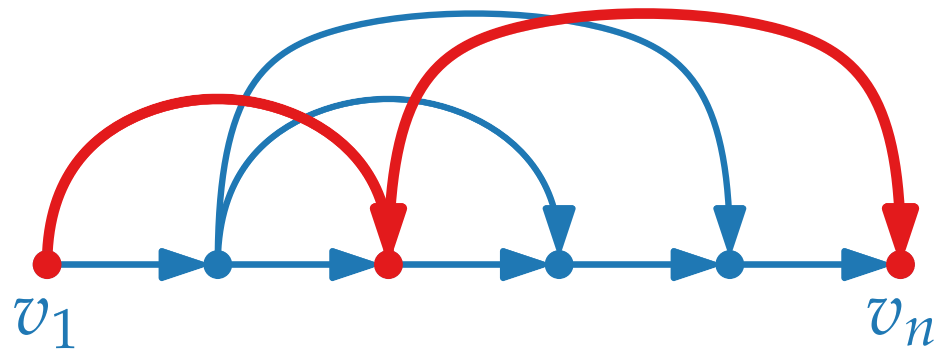

$\Rightarrow$ total running time $O(n^3)$

Algorithm by Melkman & O'Rourke

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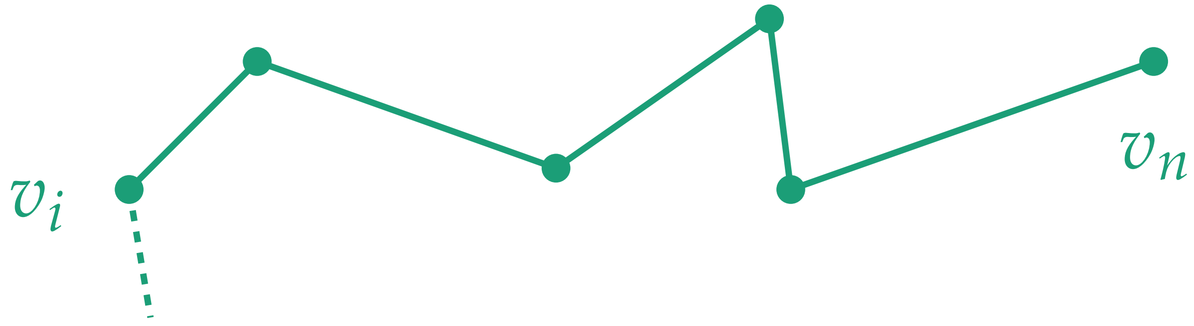

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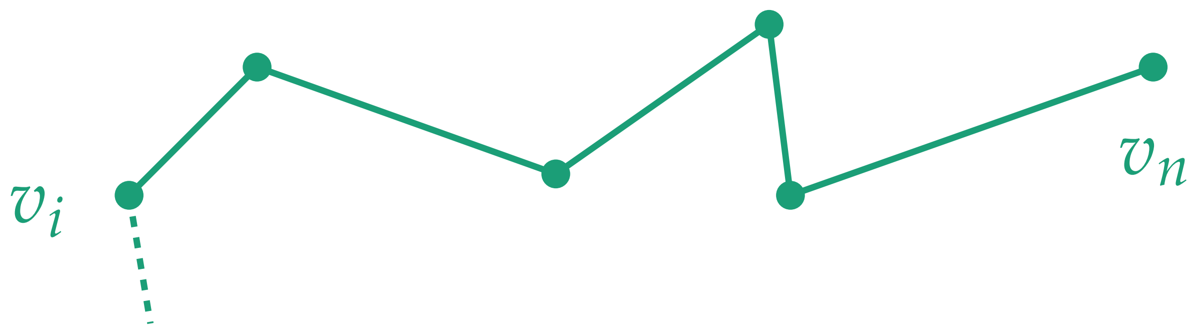

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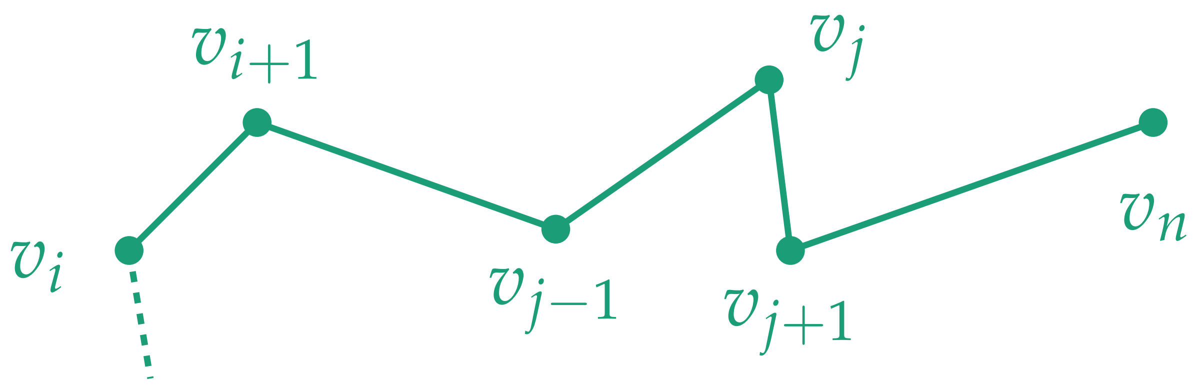

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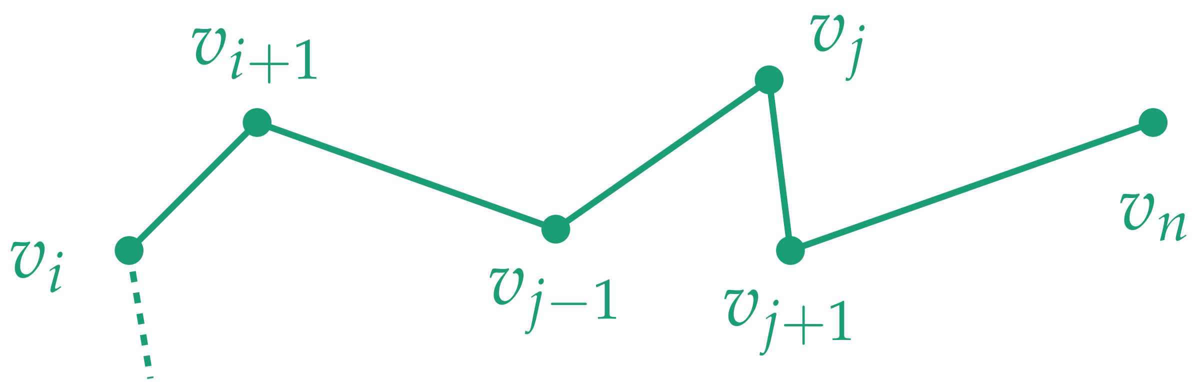

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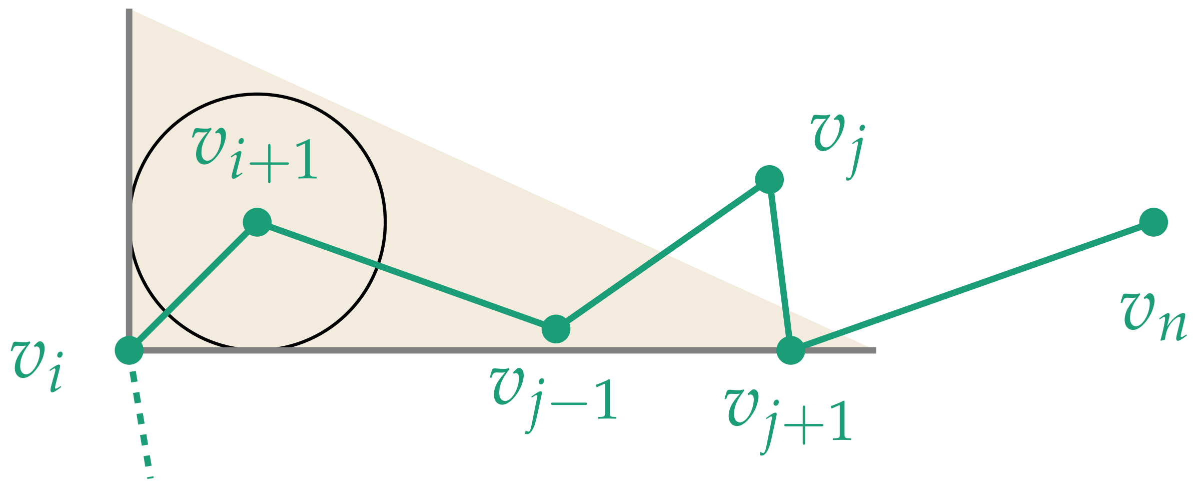

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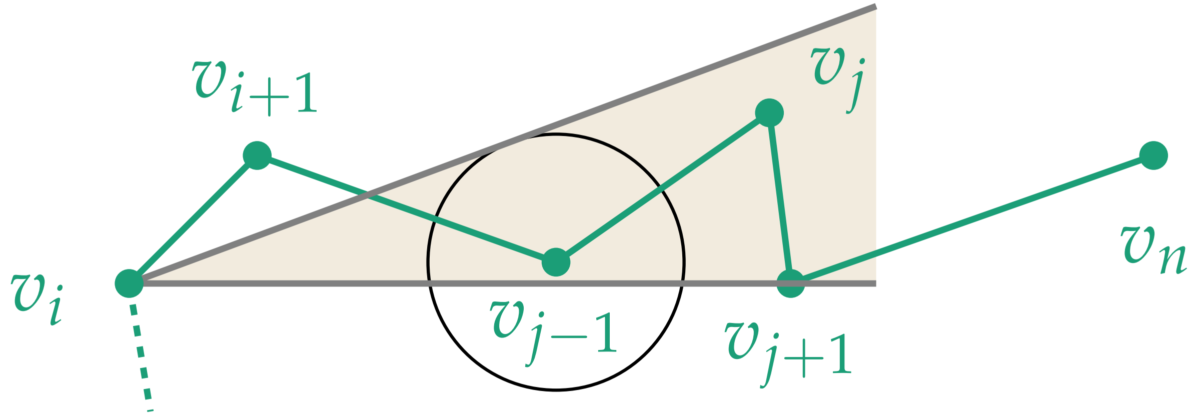

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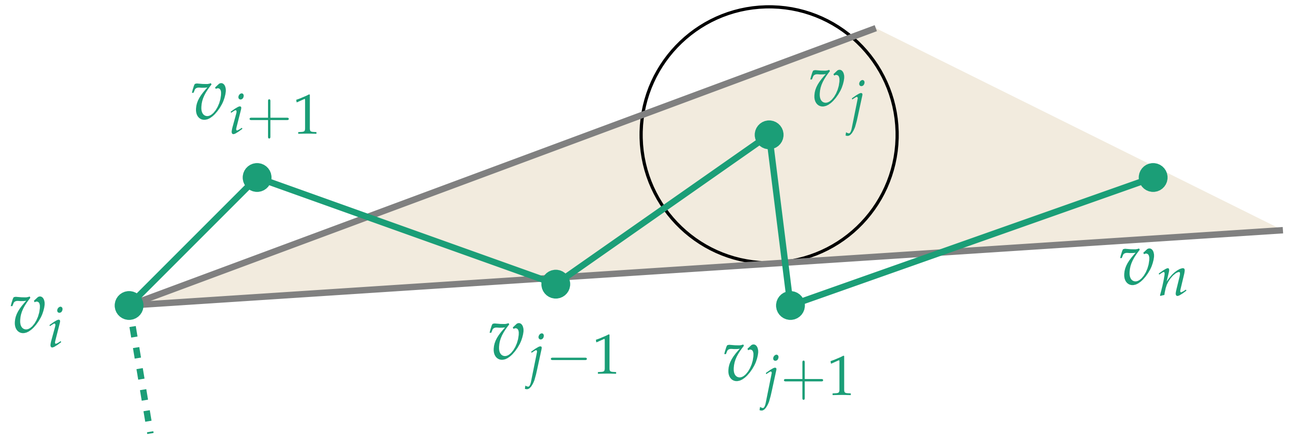

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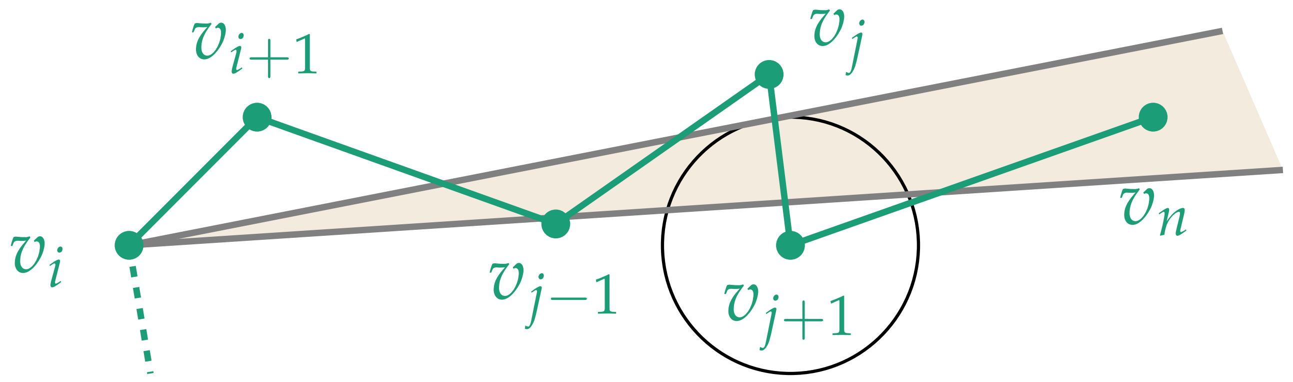

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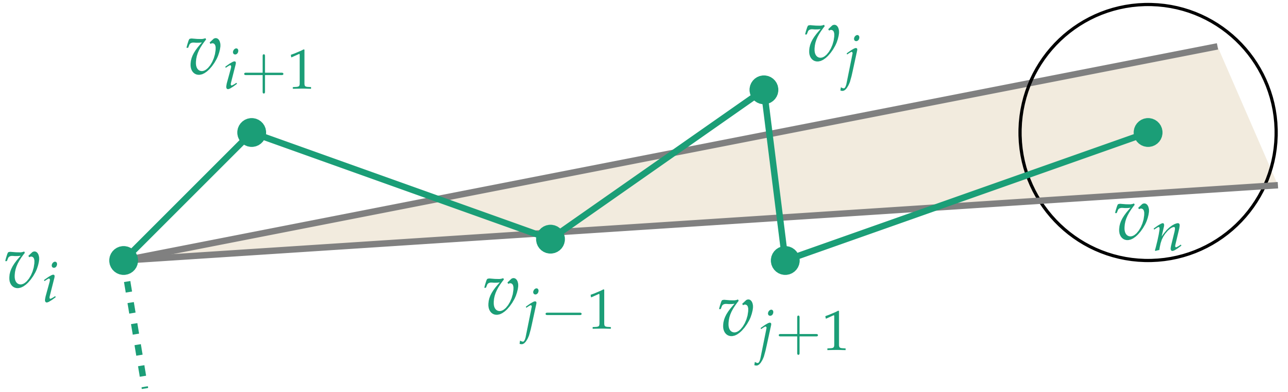

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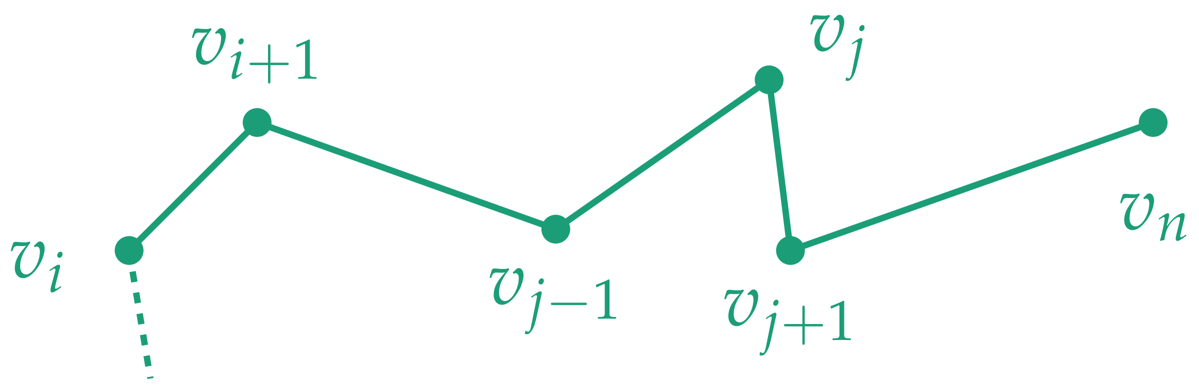

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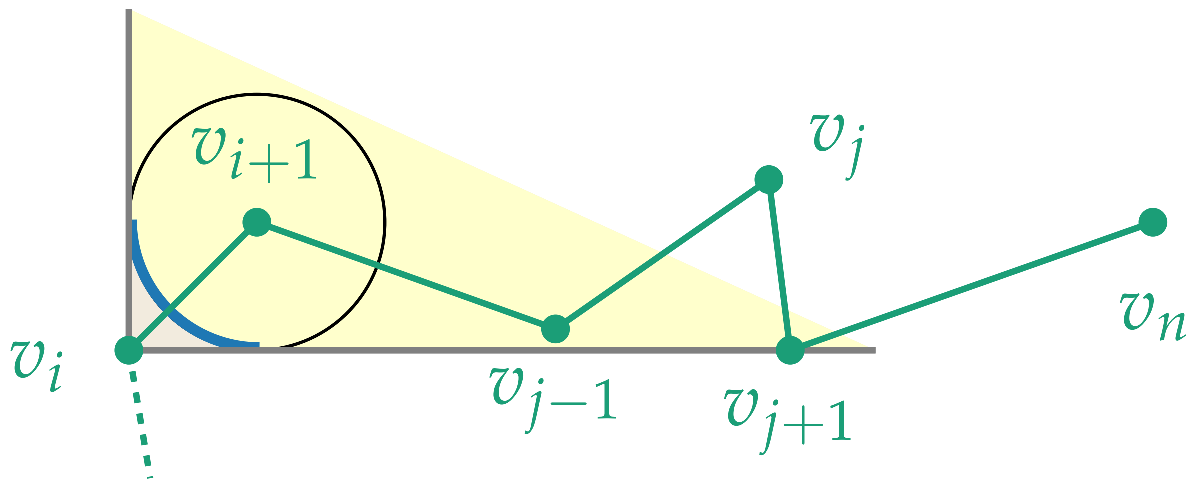

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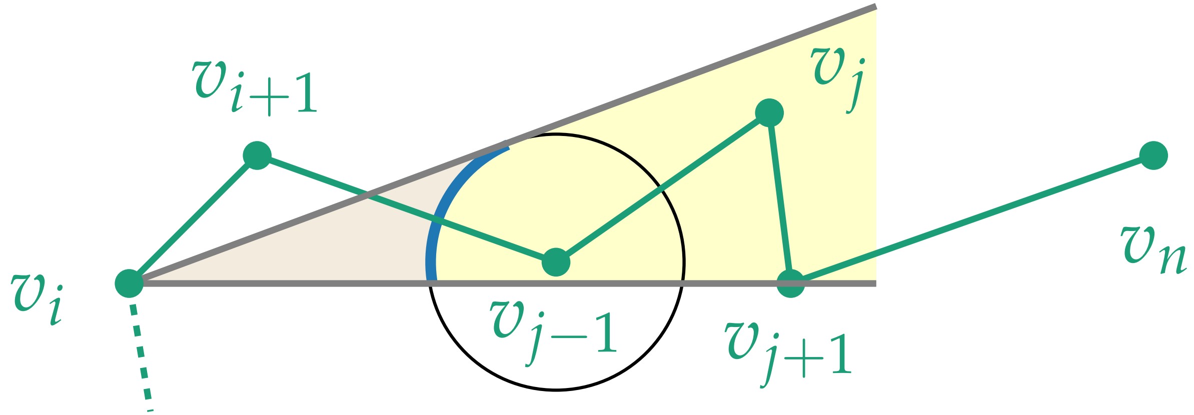

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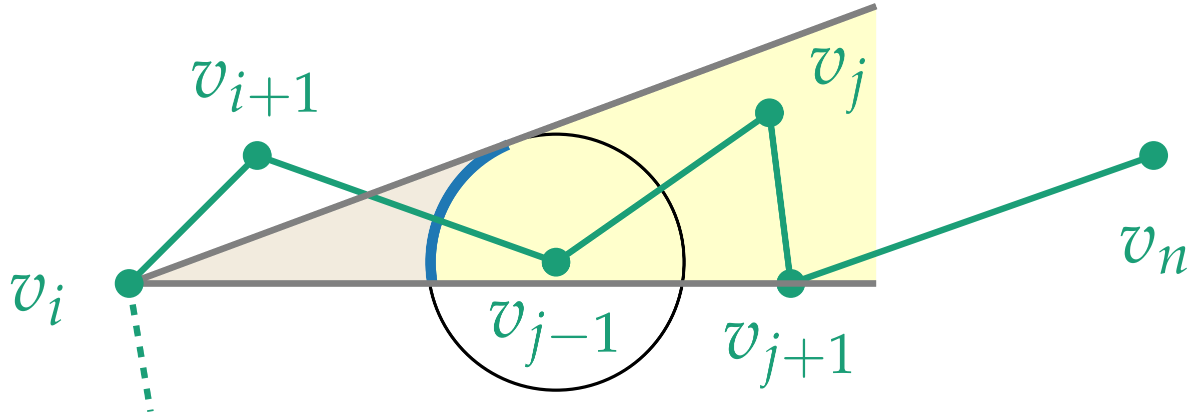

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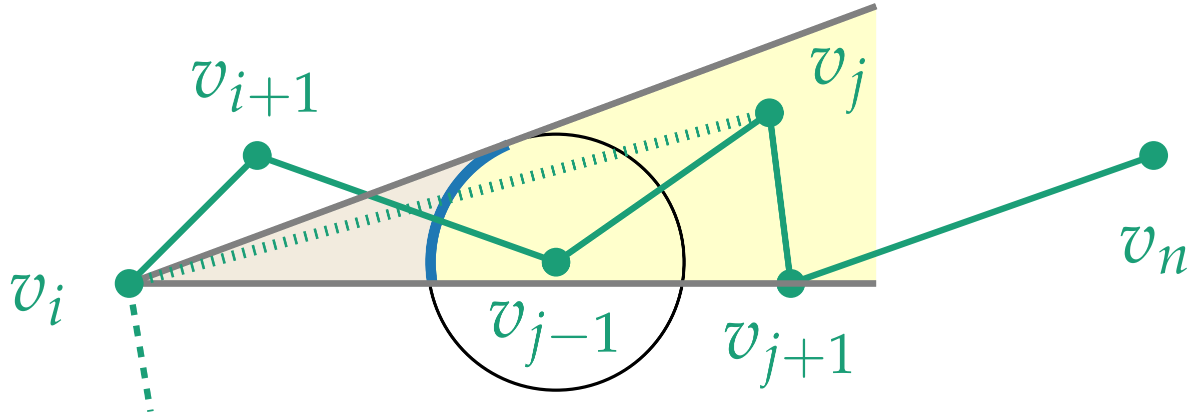

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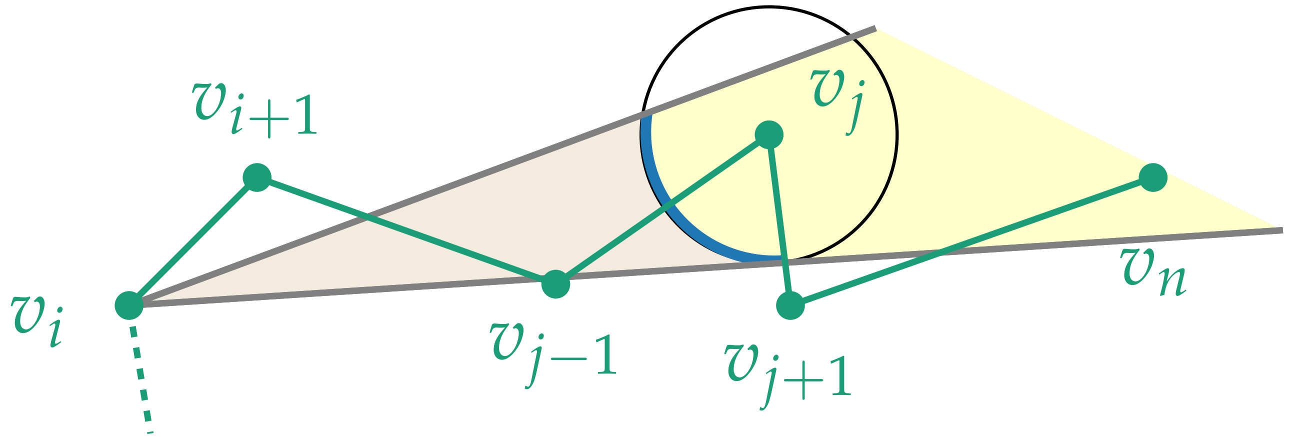

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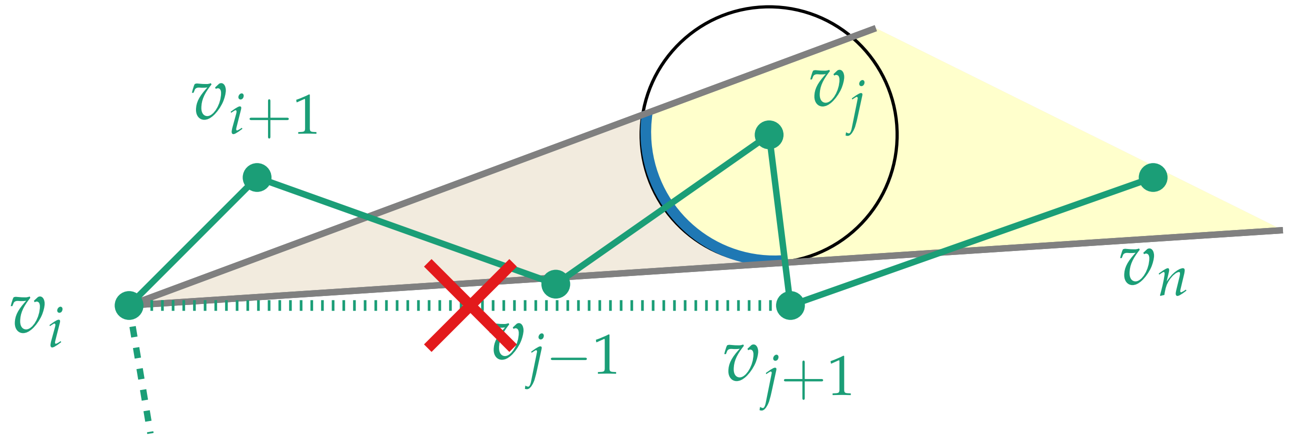

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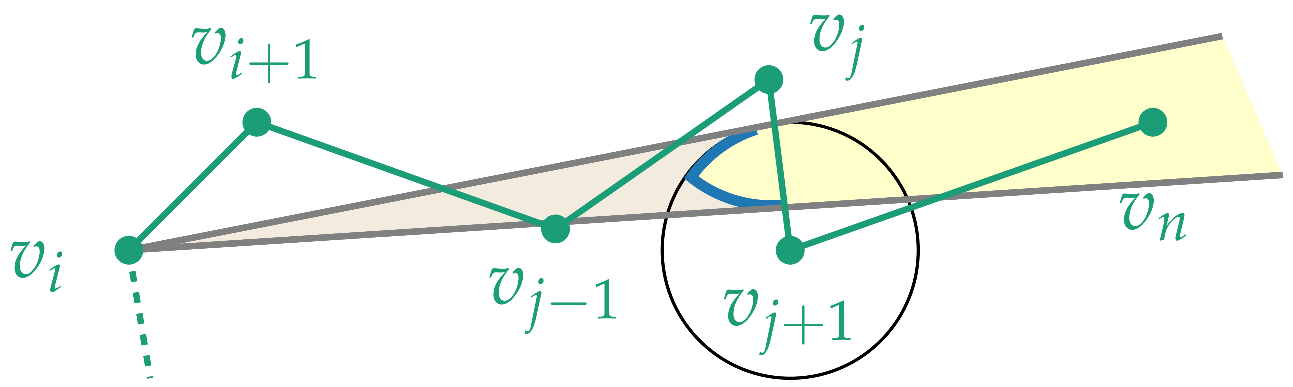

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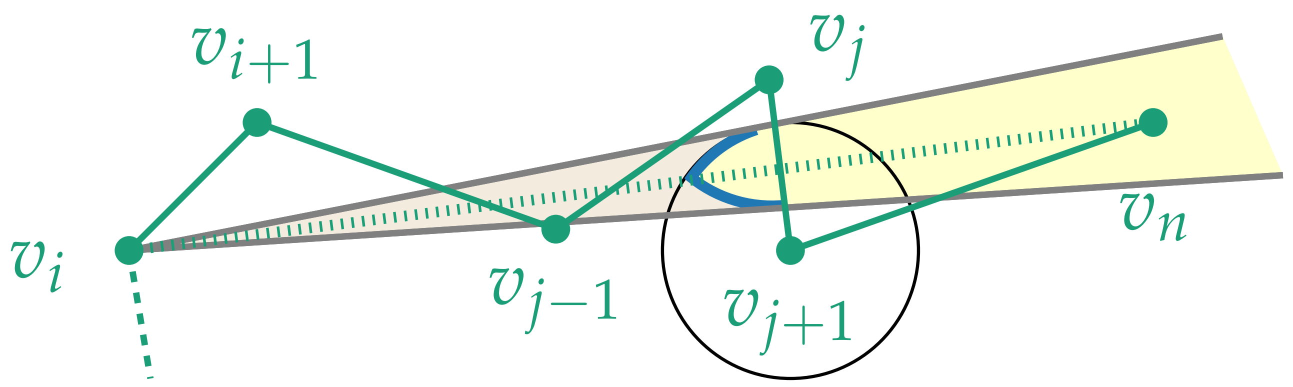

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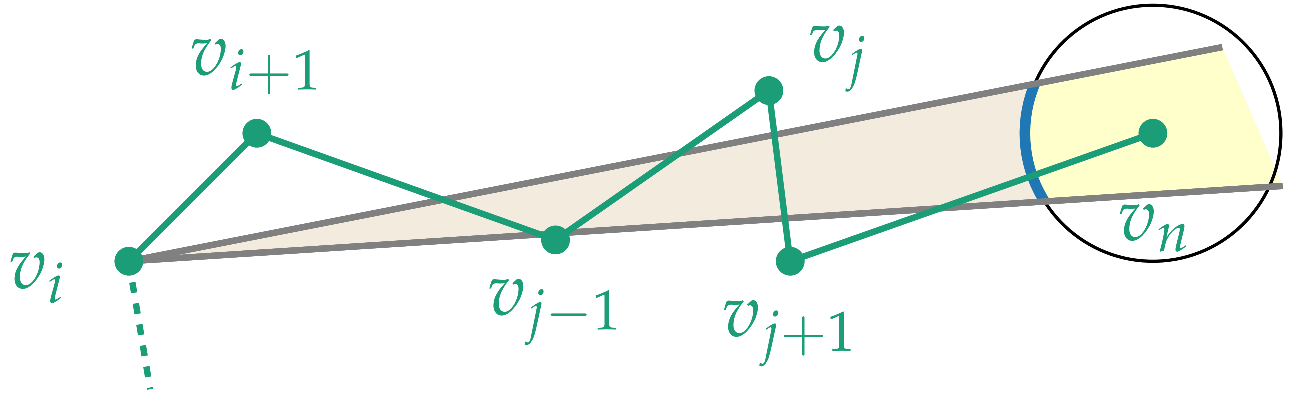

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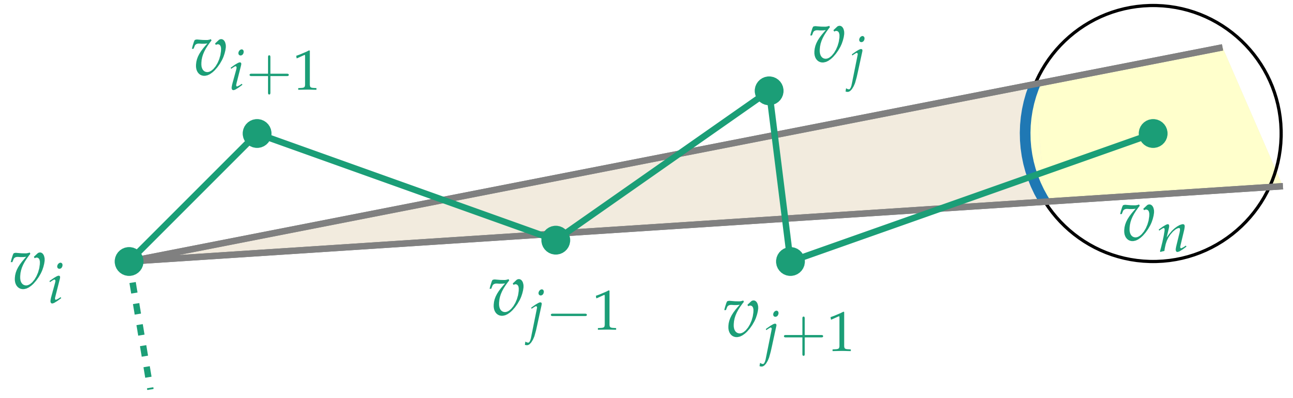

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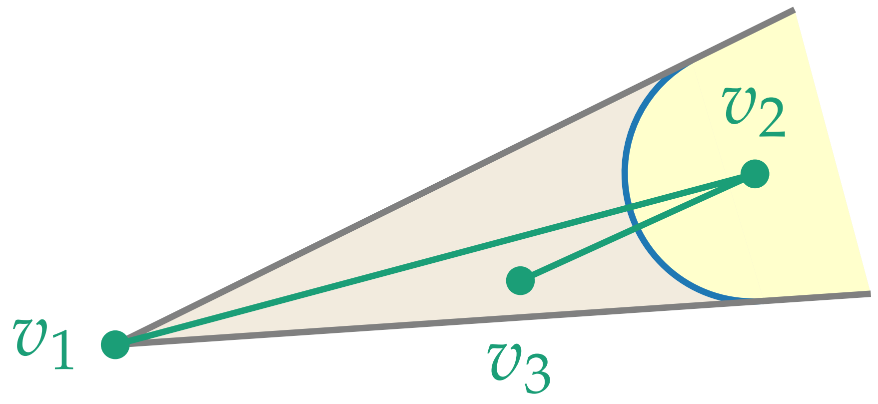

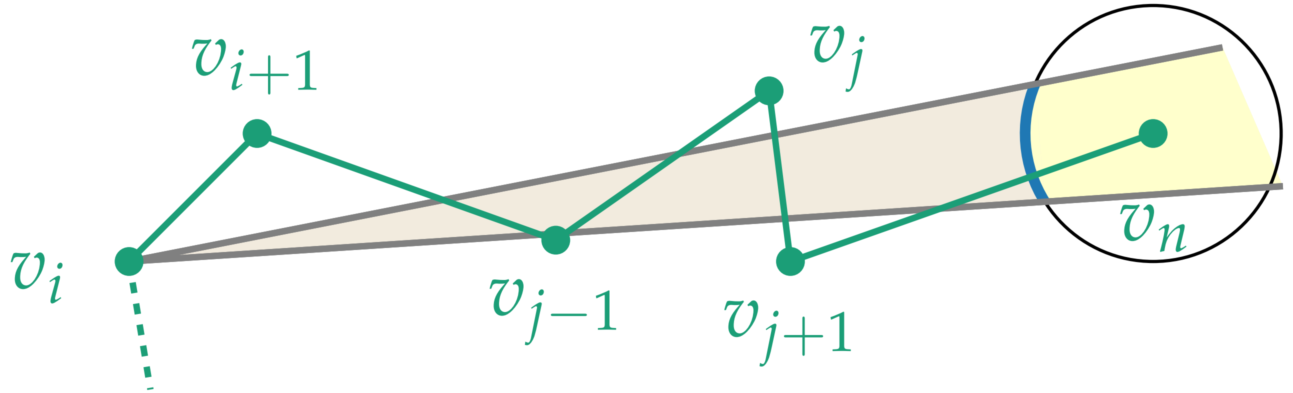

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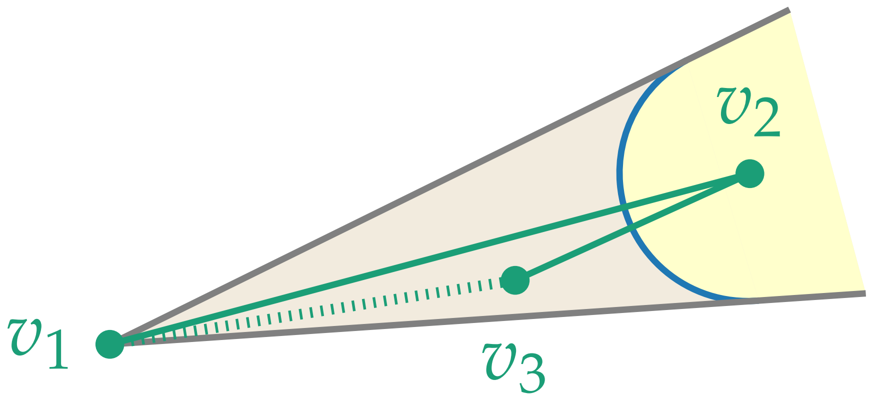

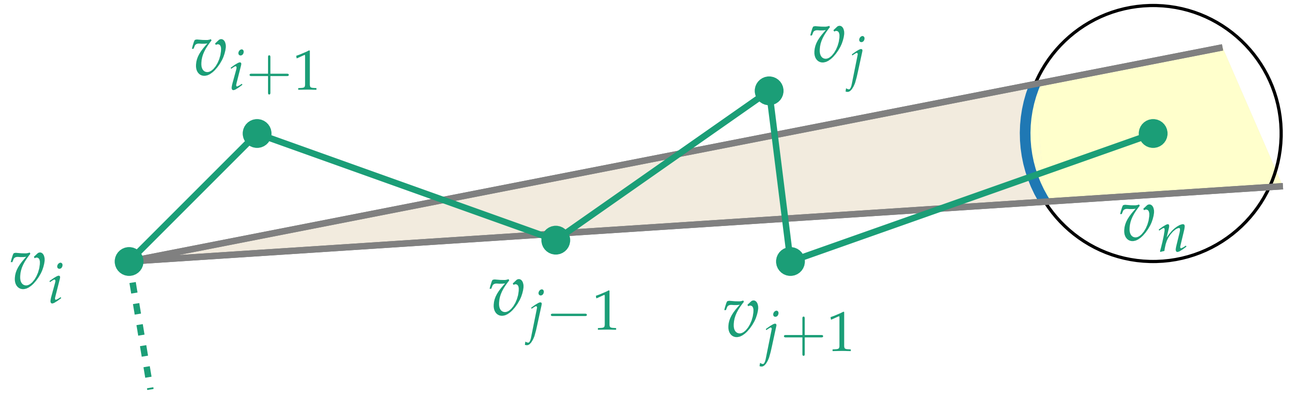

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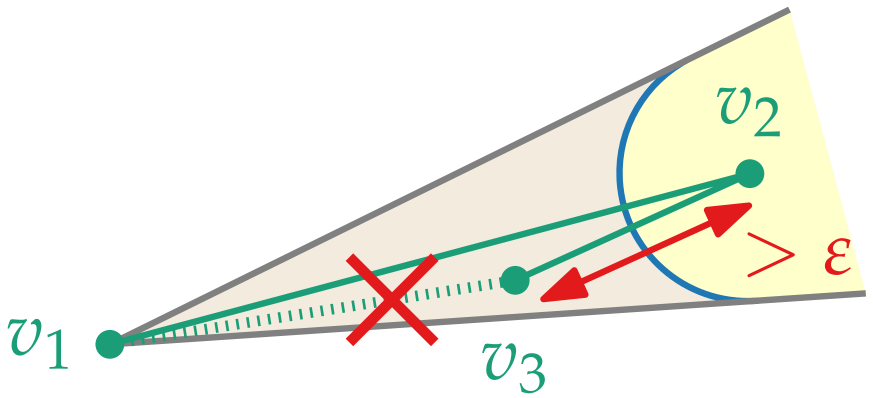

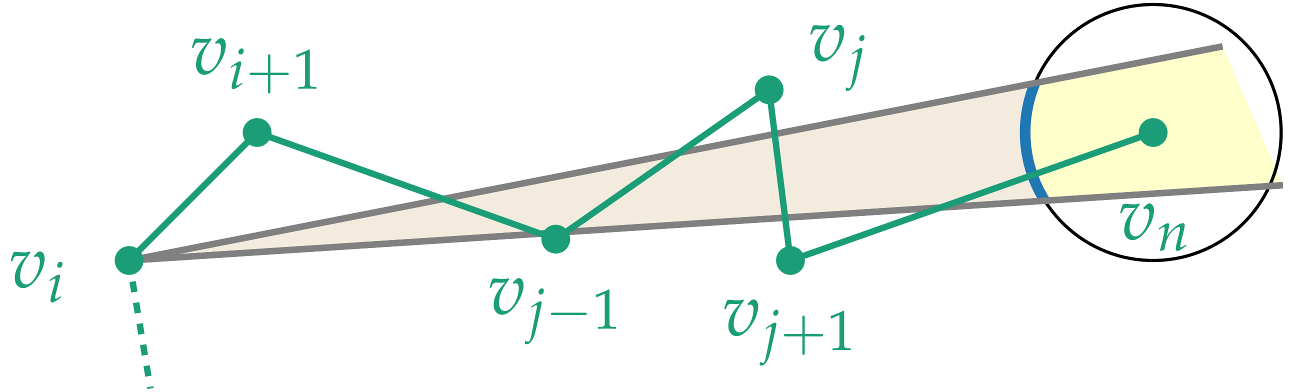

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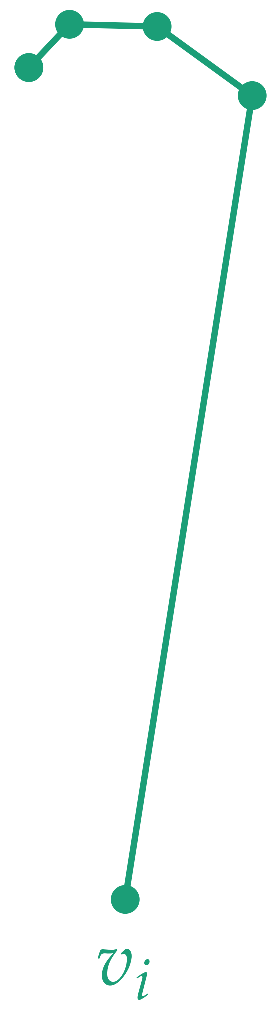

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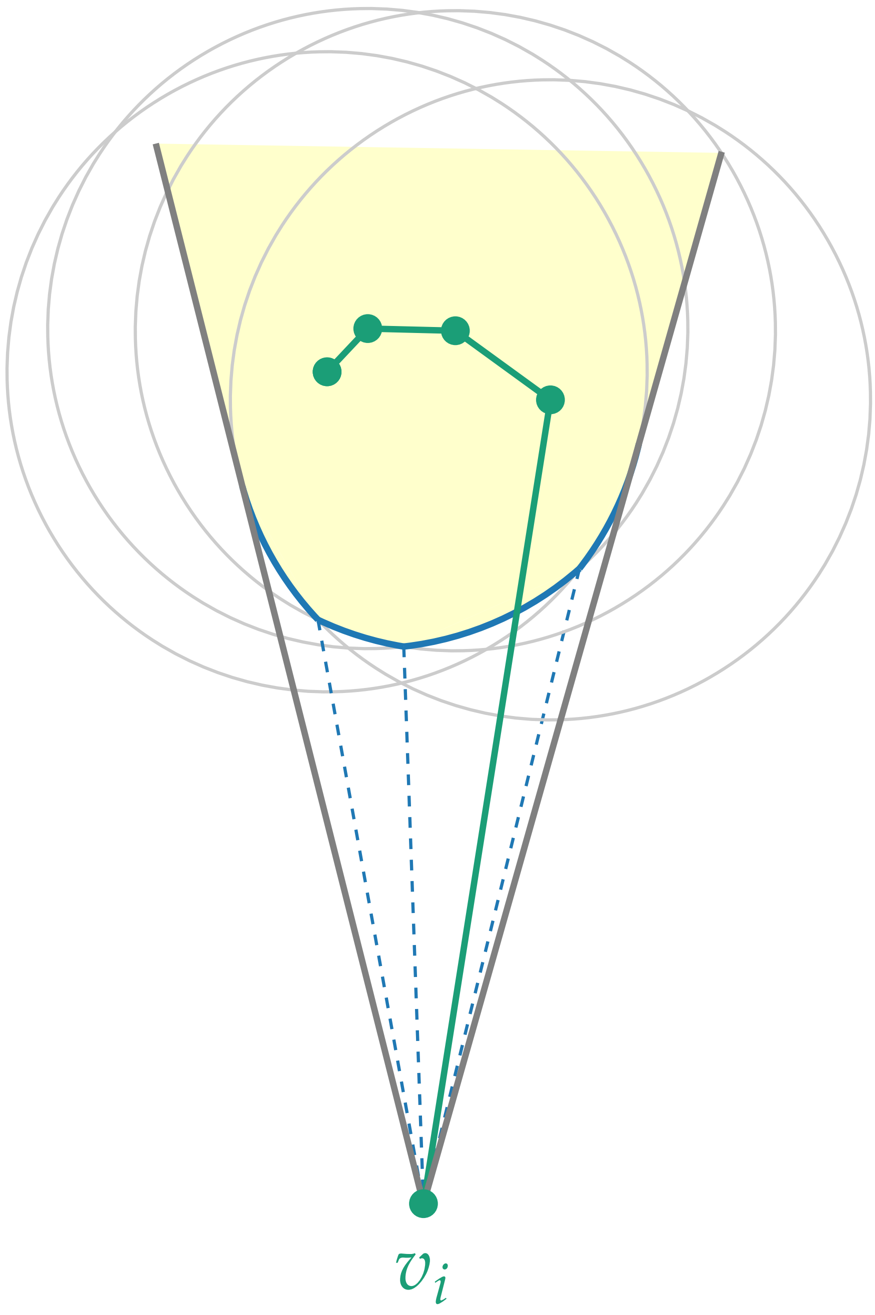

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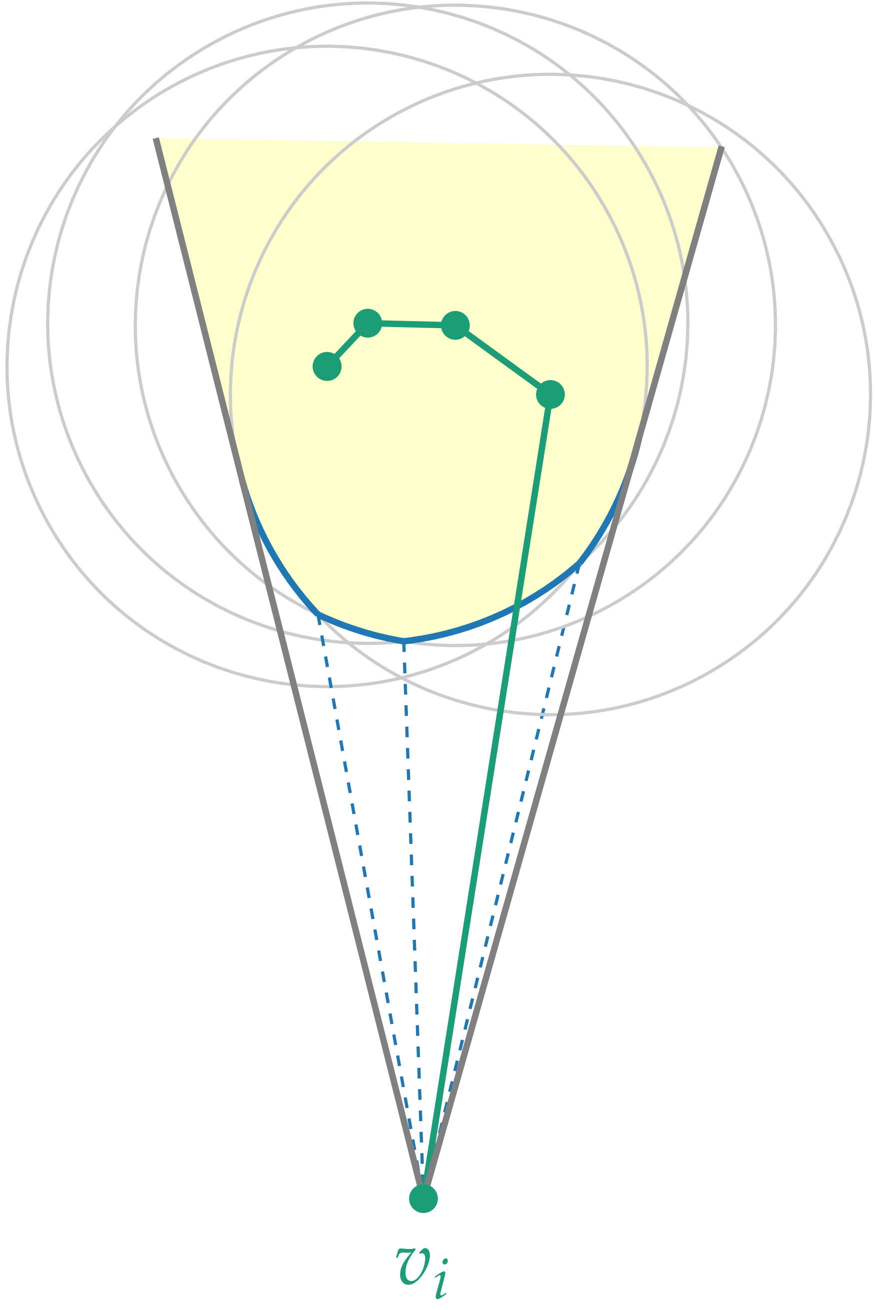

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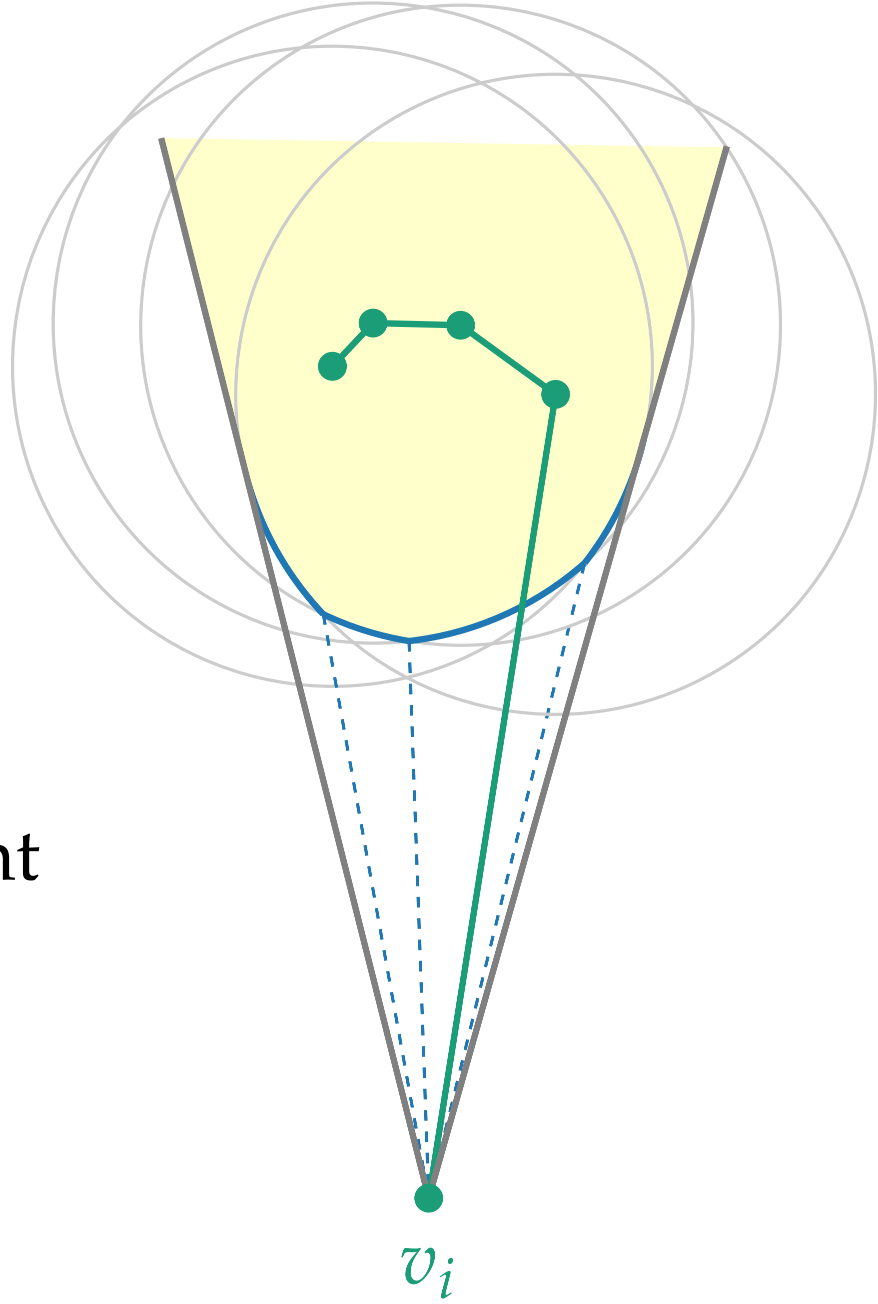

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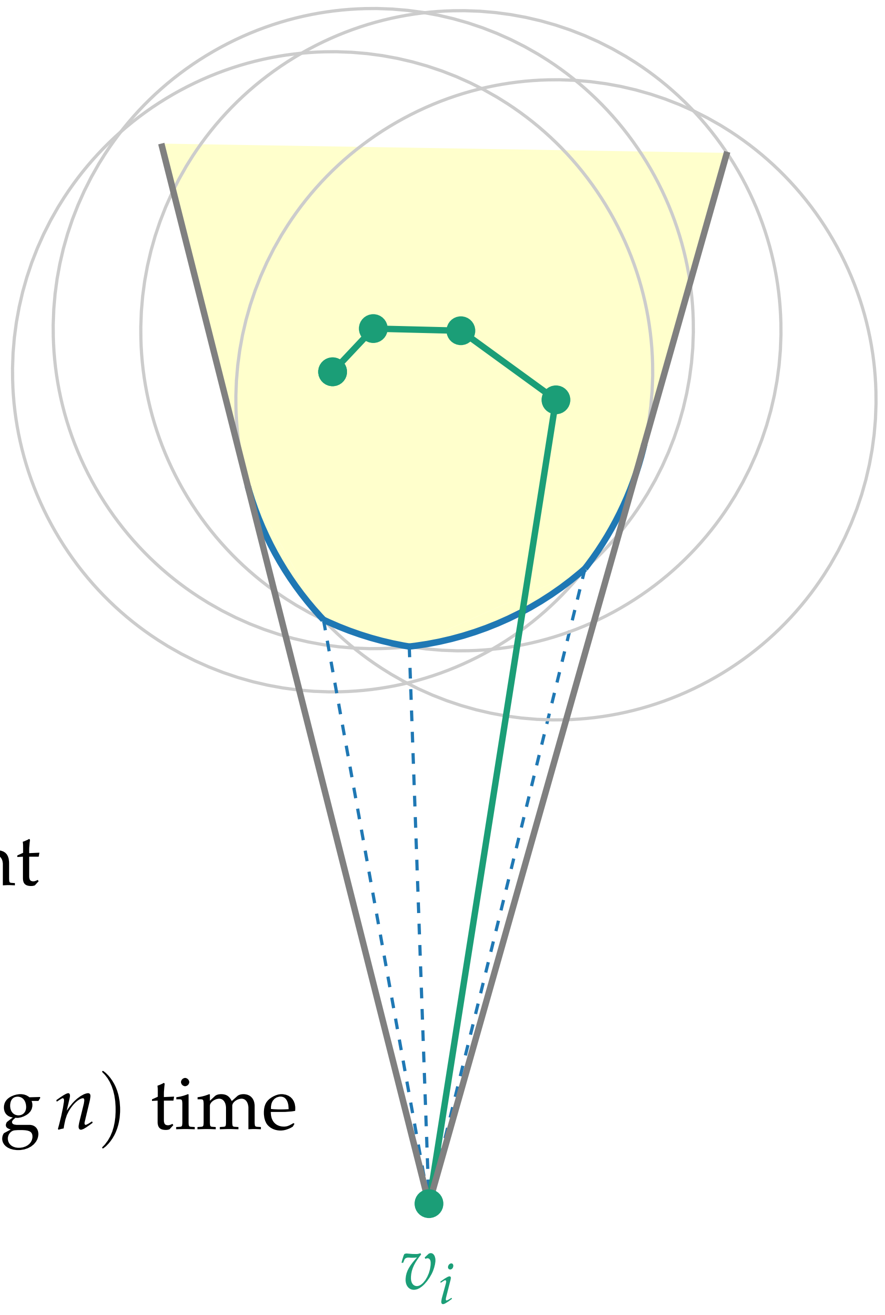

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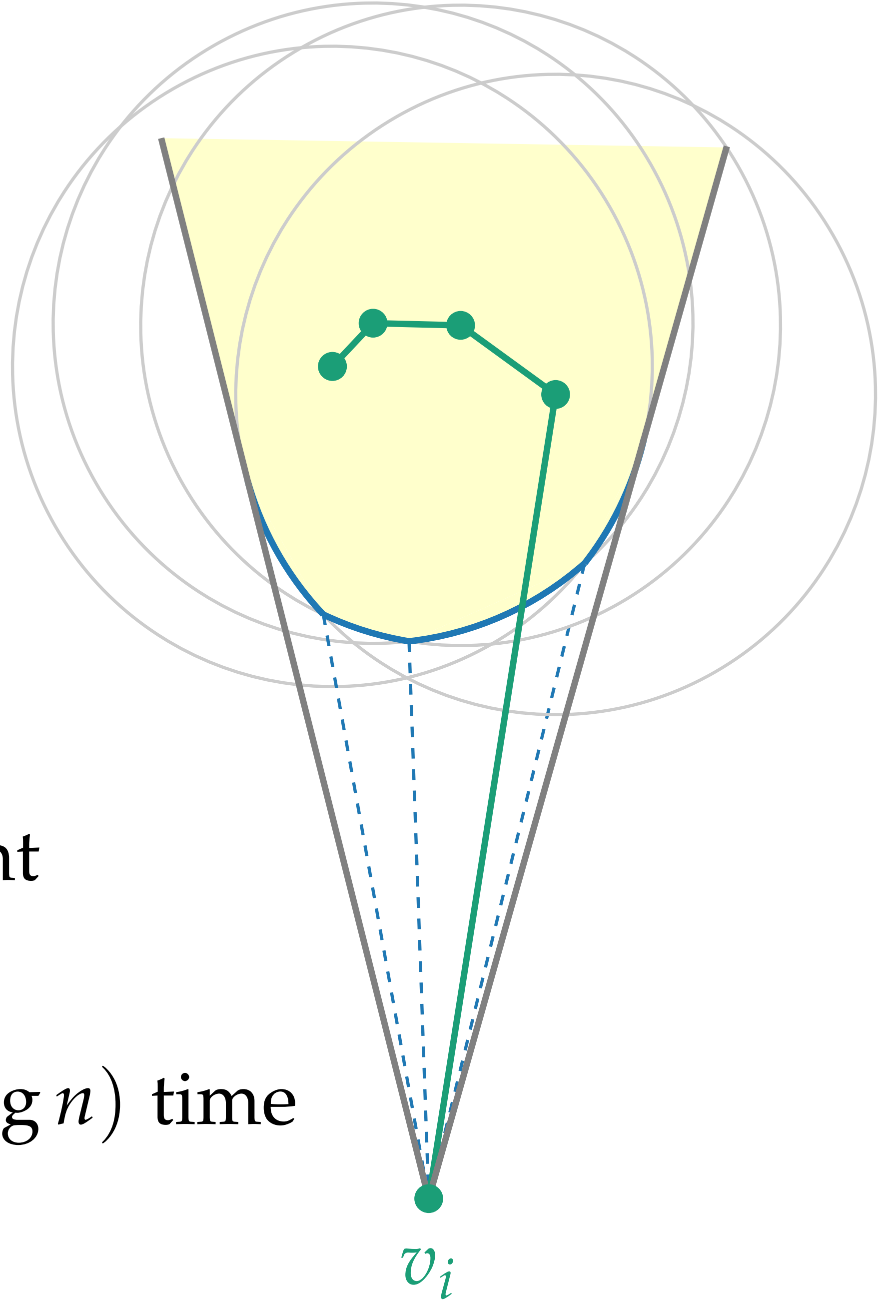

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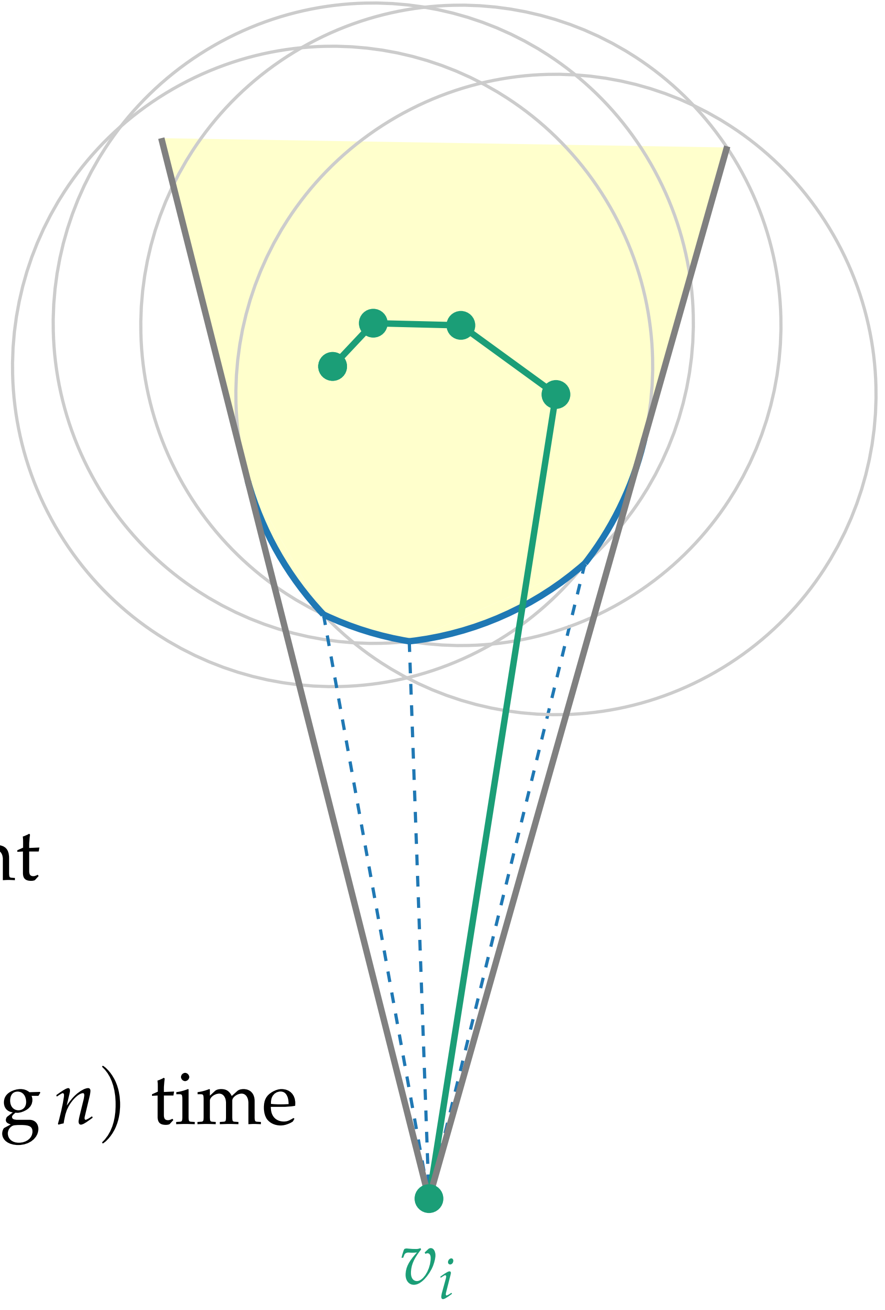

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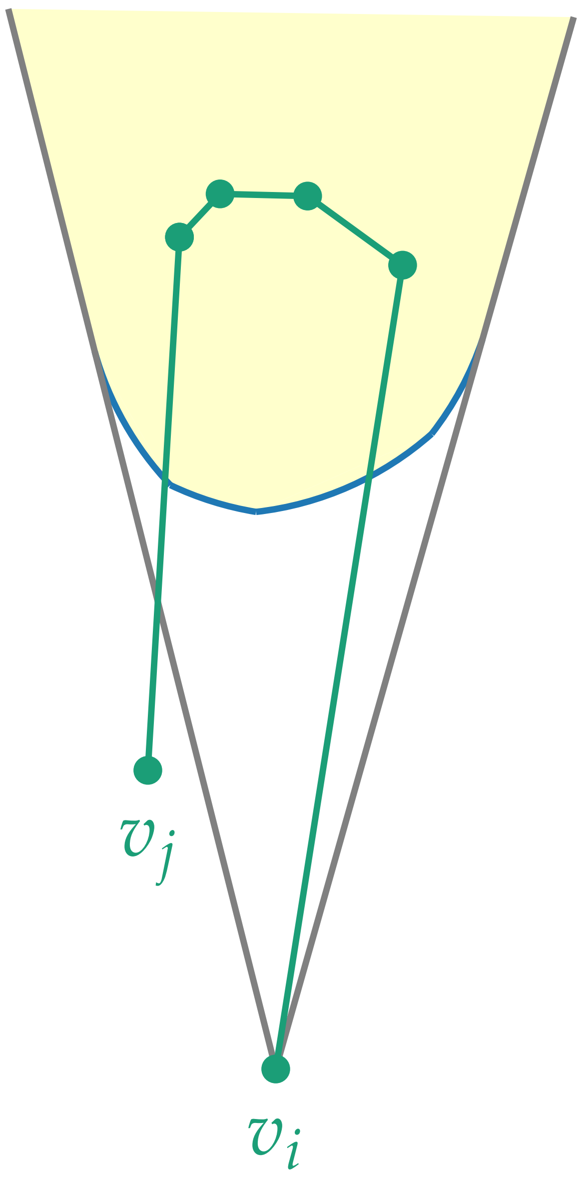

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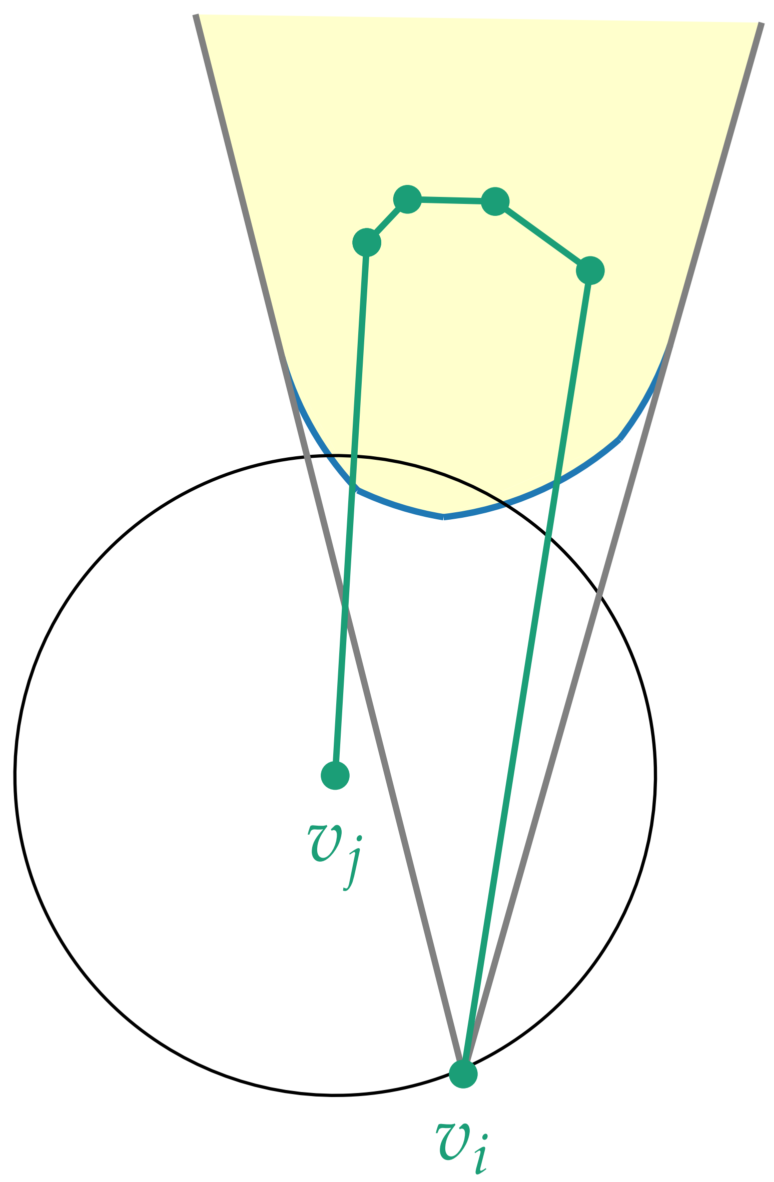

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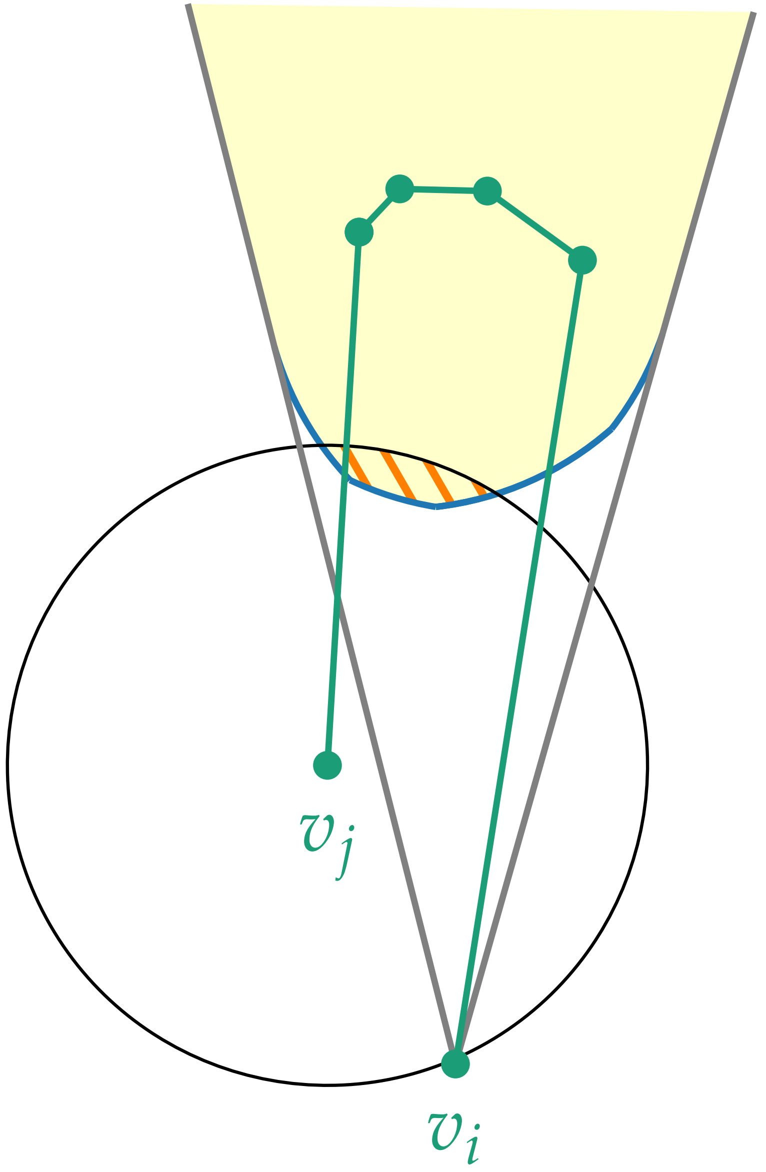

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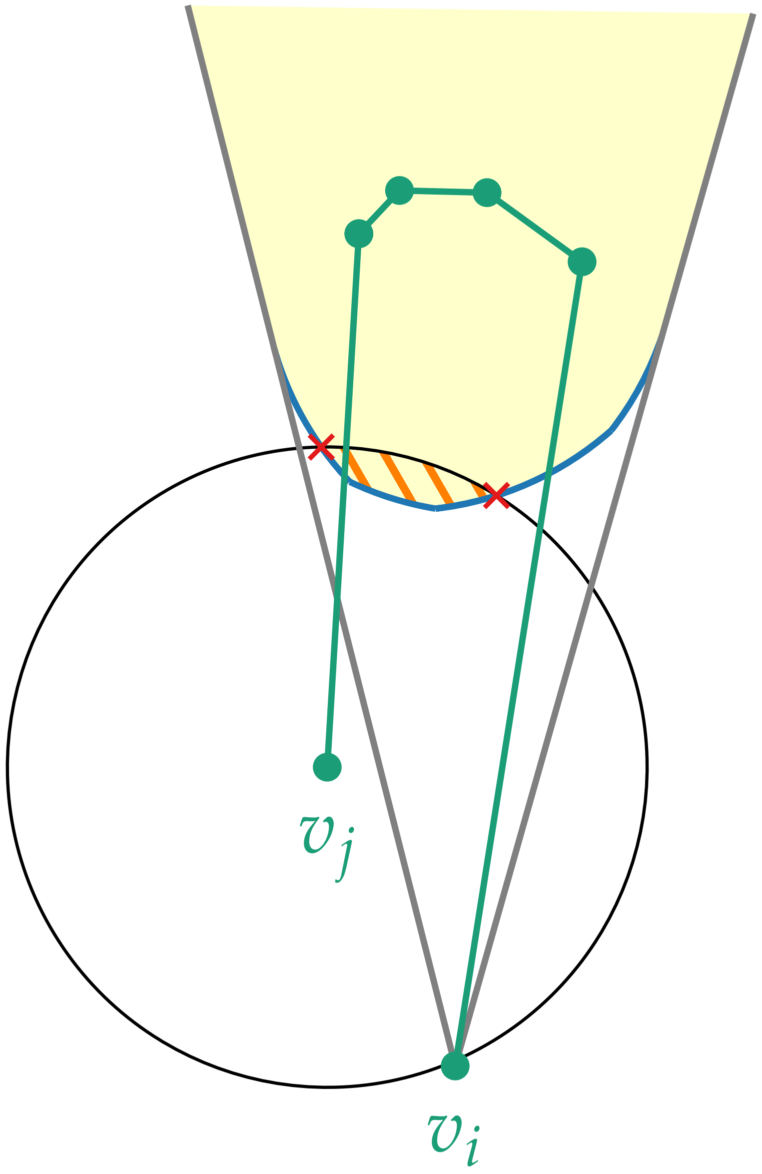

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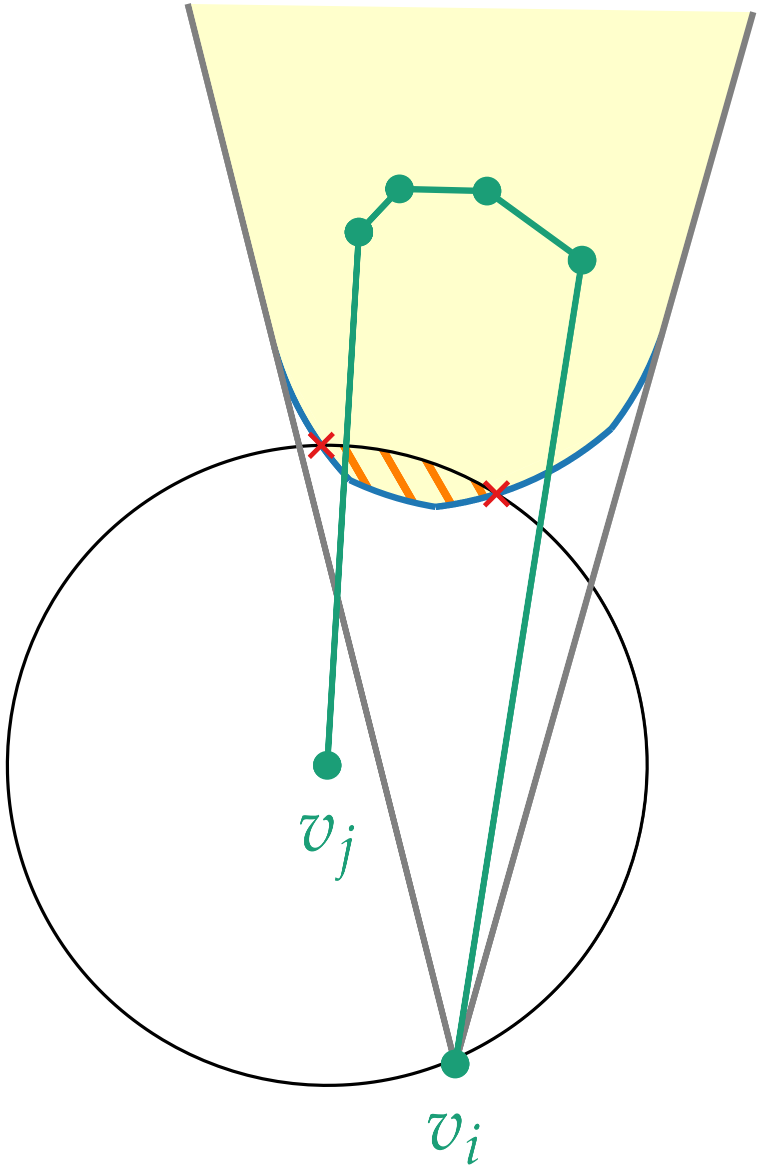

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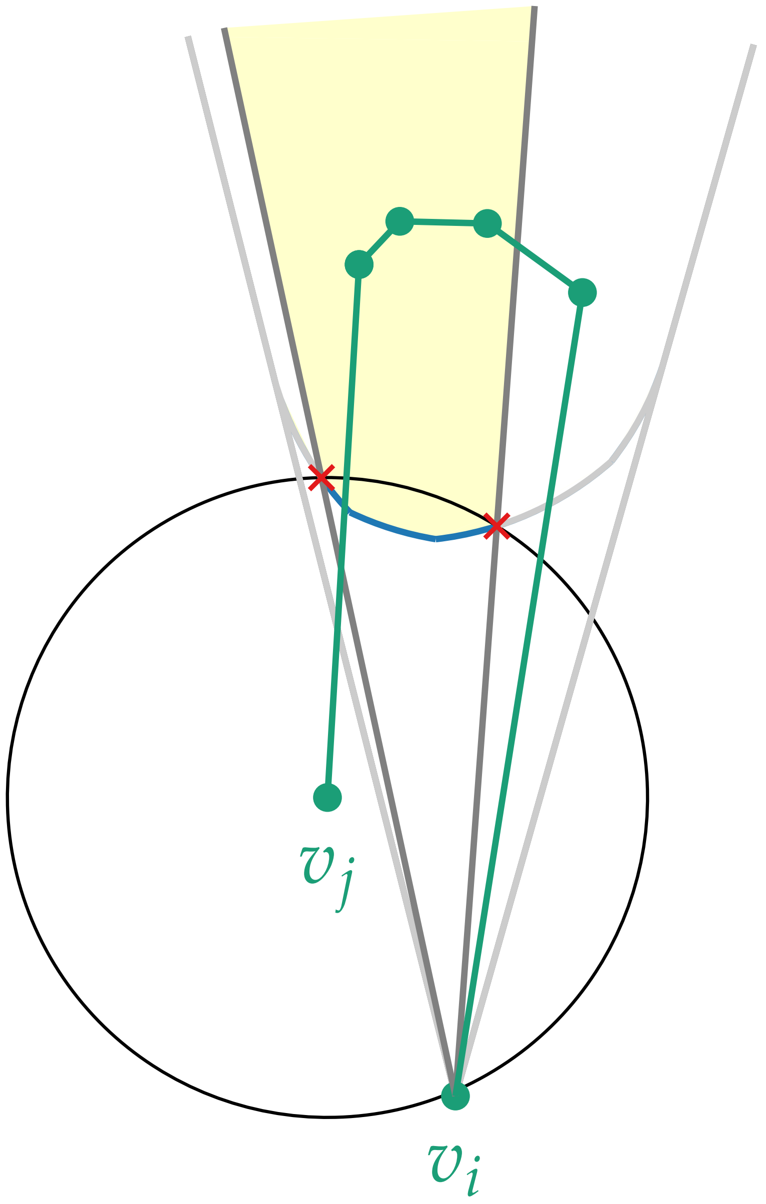

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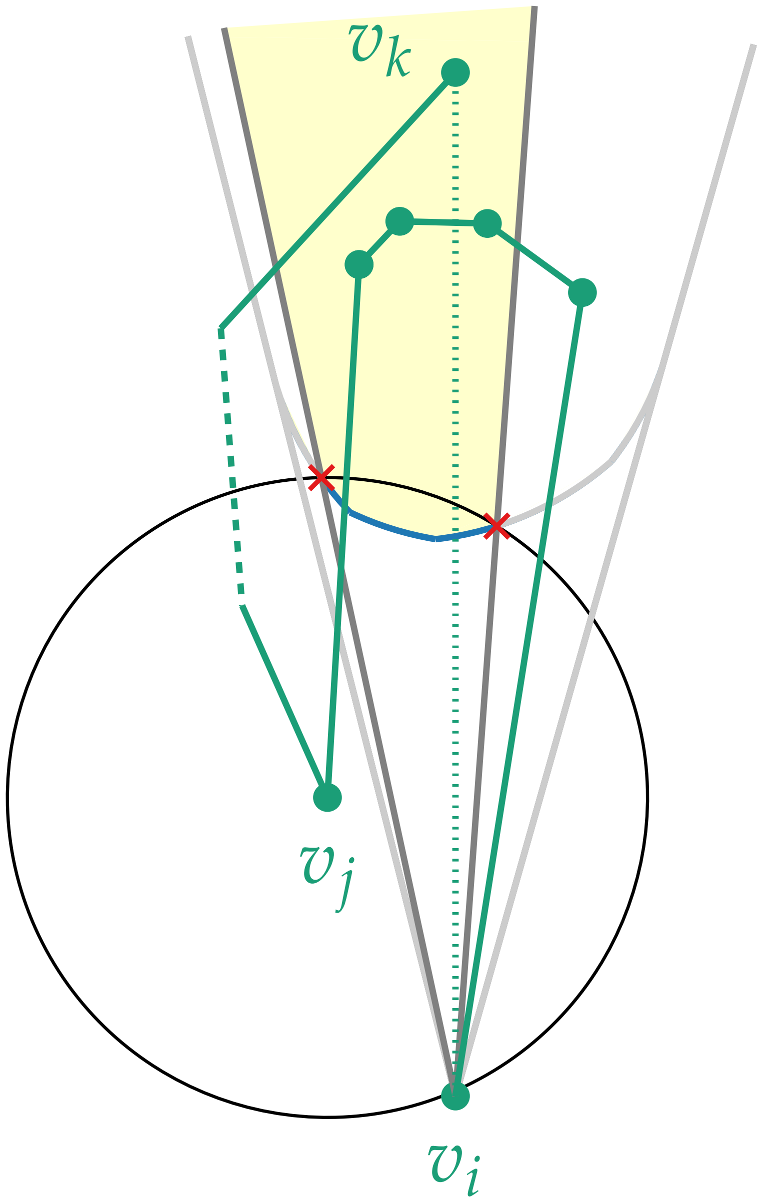

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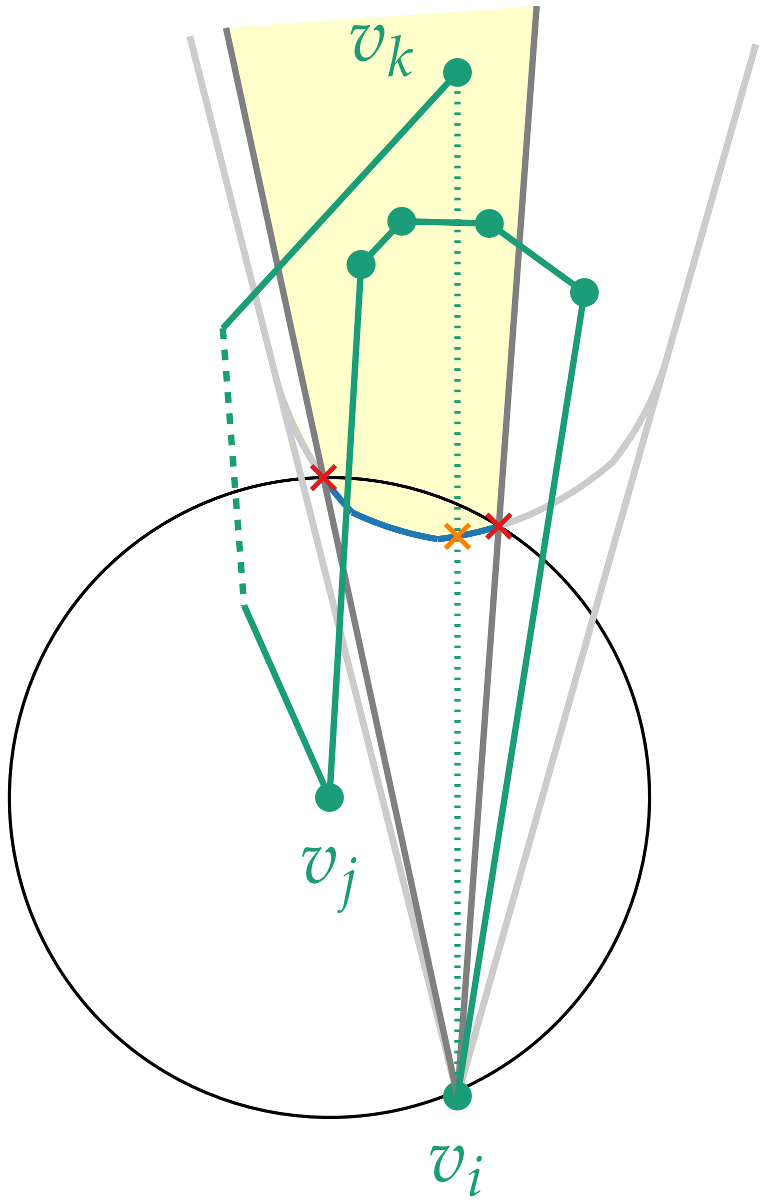

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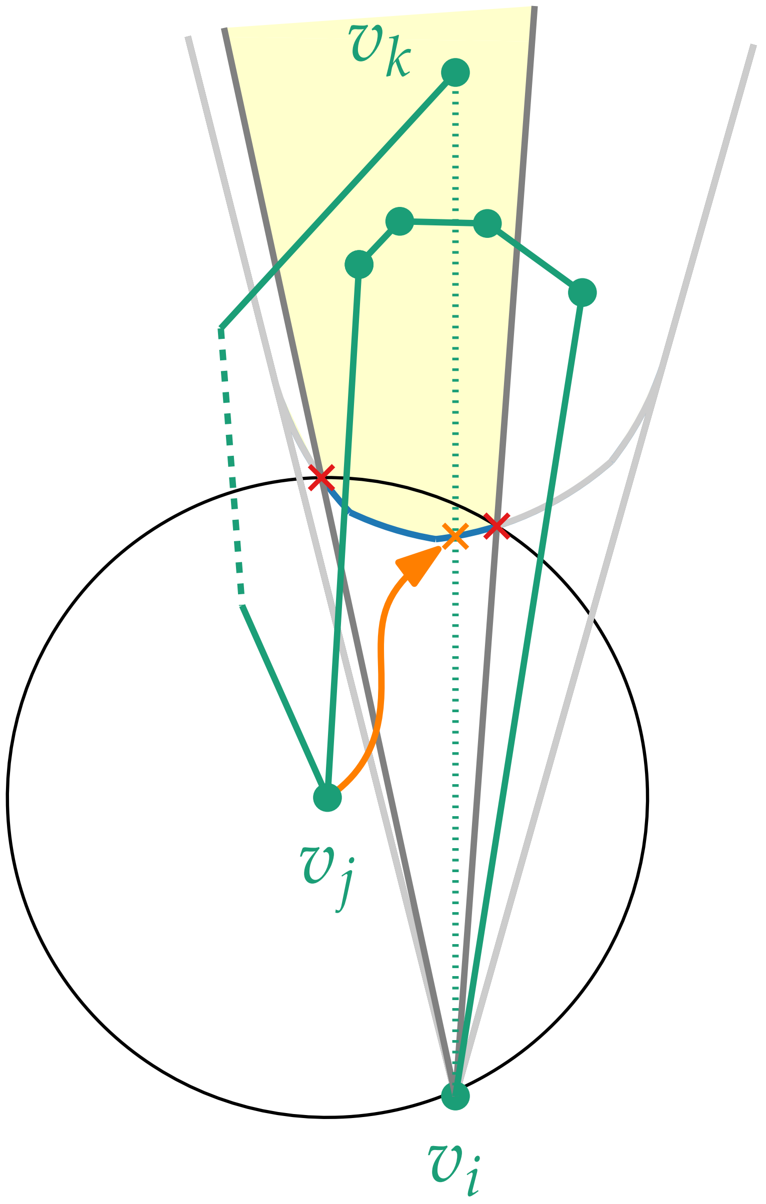

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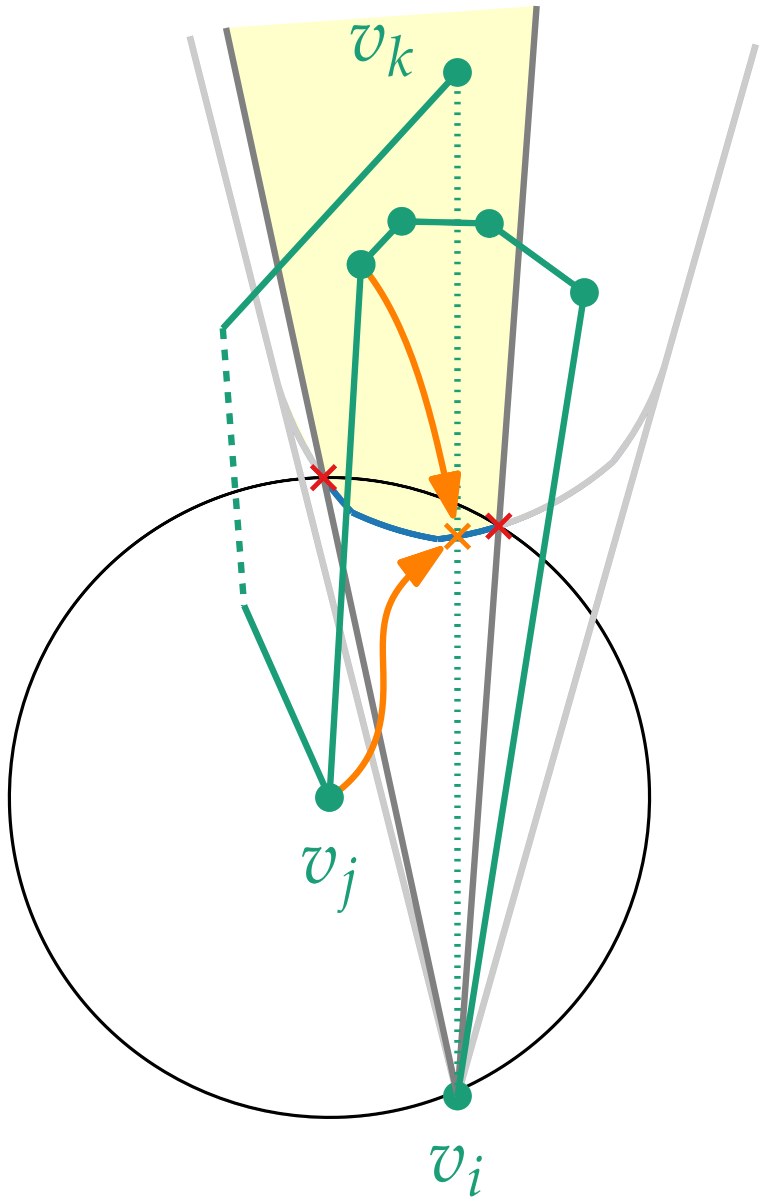

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Corollary: Polyline simplification under the local Fréchet distance can be done in $O(n^2 \log n)$ time in the L_p norm where $p \in [1, \infty]$.

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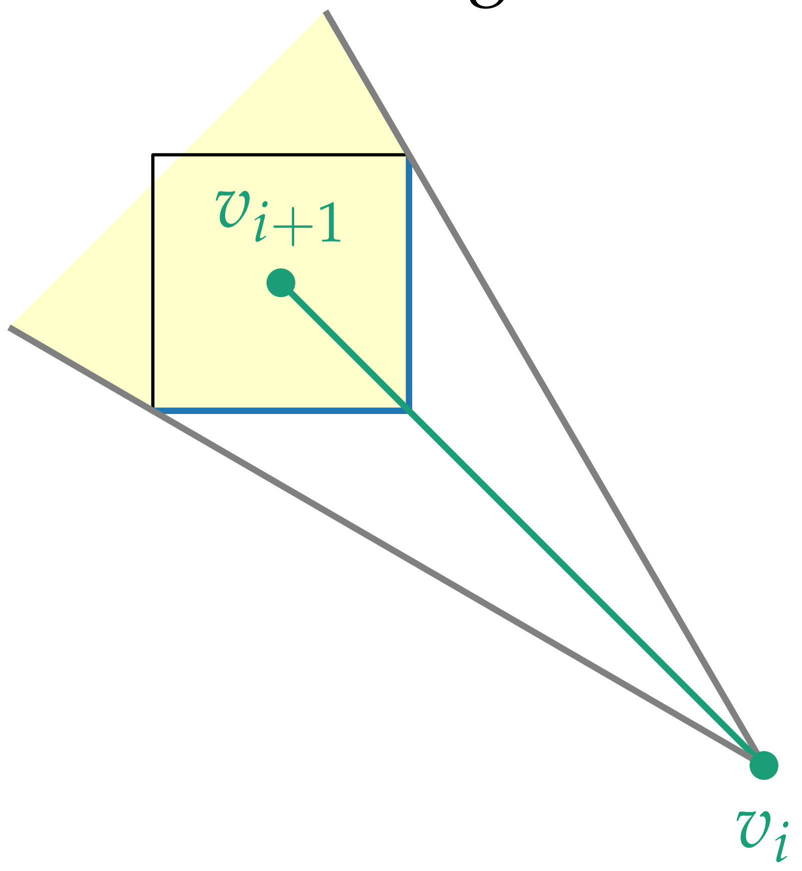

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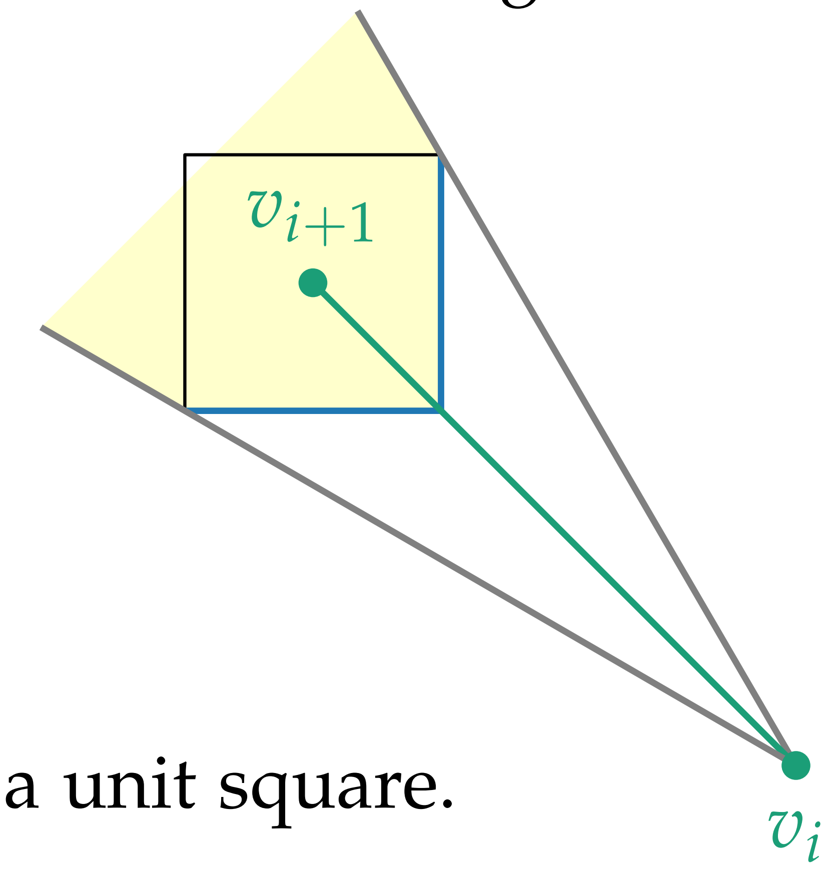

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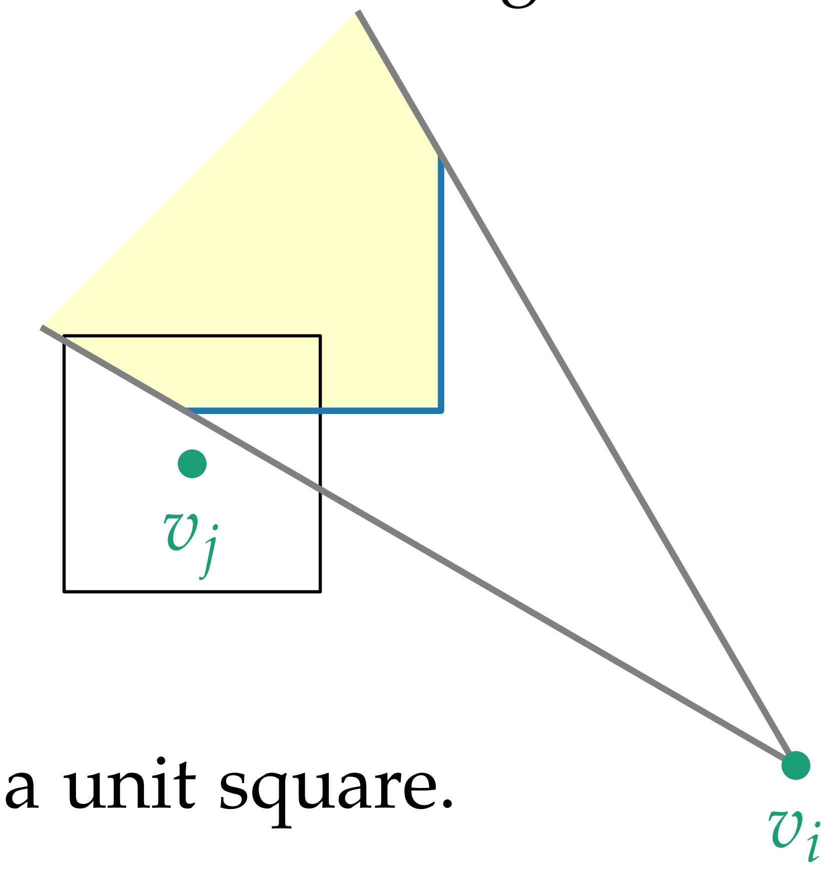

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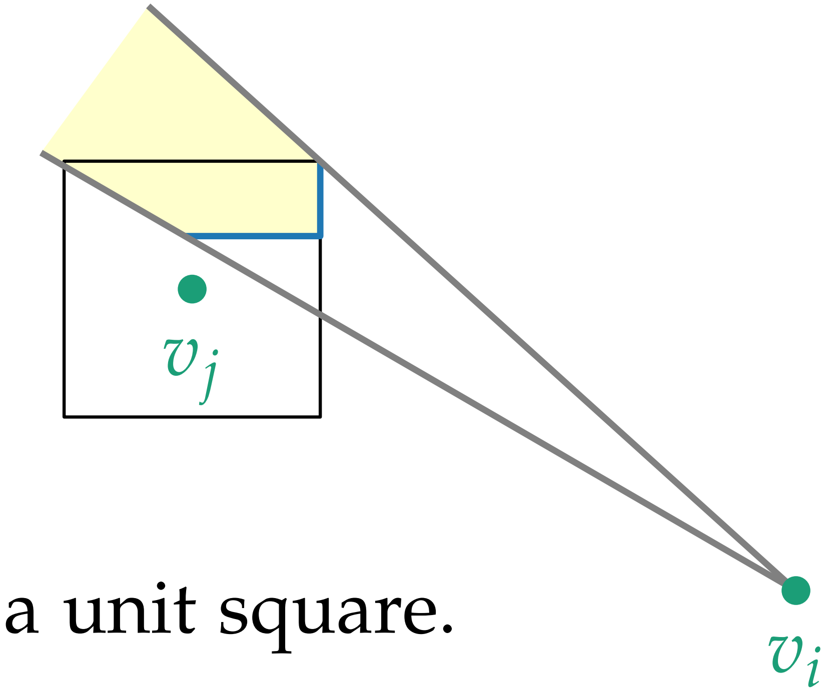

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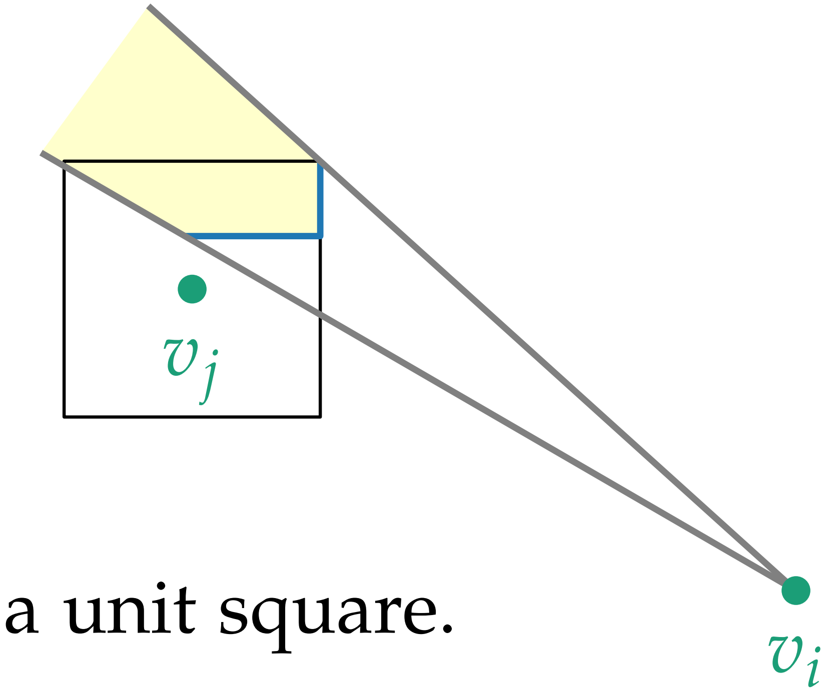

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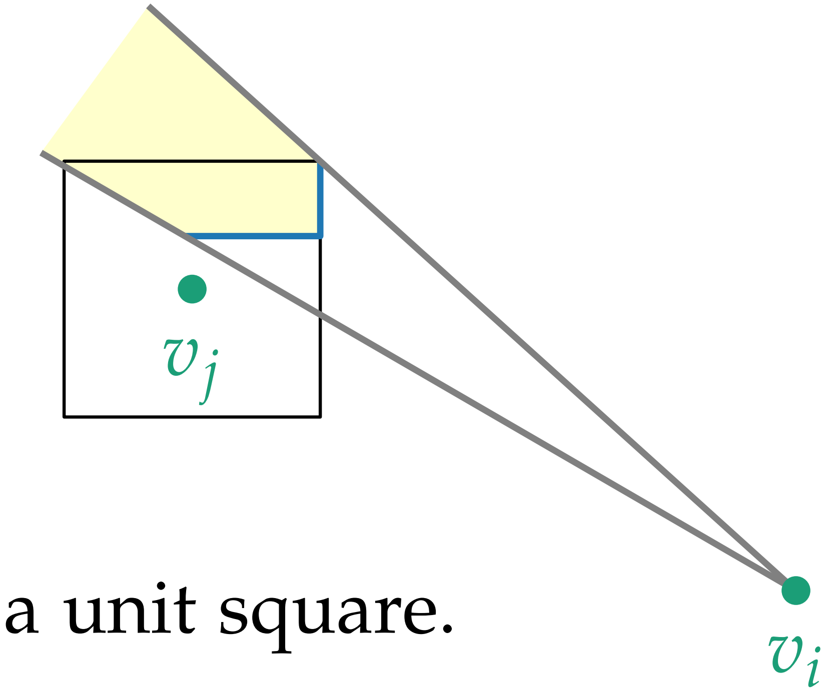

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