Julius-Maximilians-
UNIVERSITÄT WÜRZBURG

Lehrstuhl für
INFORMATIK I
Algorithmen & Komplexität

Institut für Informatik

Simplification of Polyline Bundles

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Aalto University, Finland

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Universität Konstanz, Germany

Johannes Zink
Universität Würzburg, Germany

Agenda

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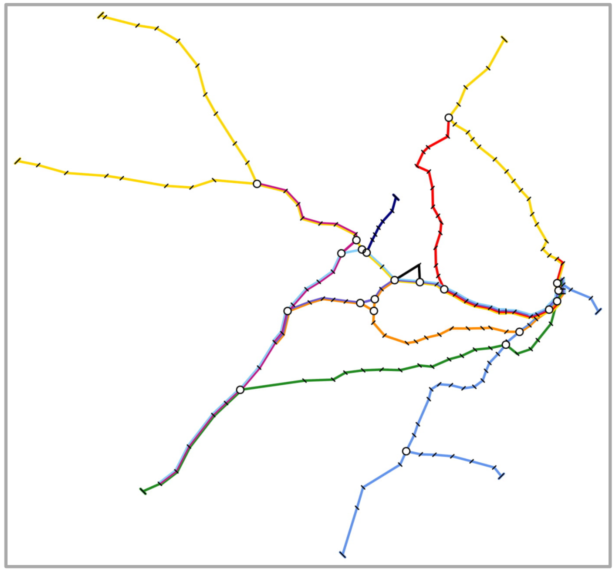

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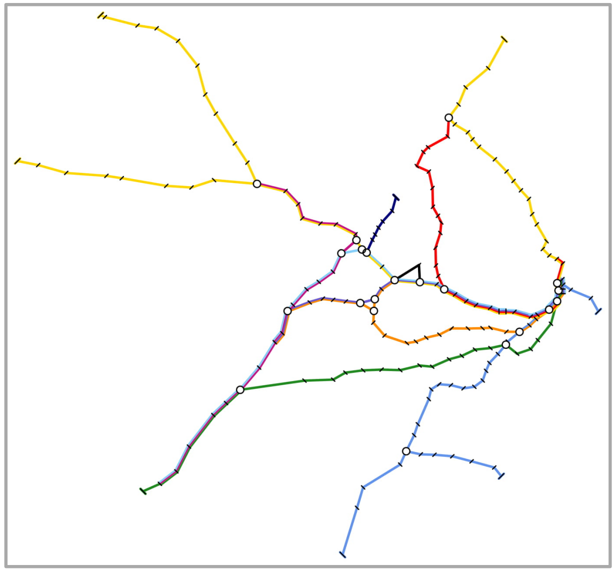

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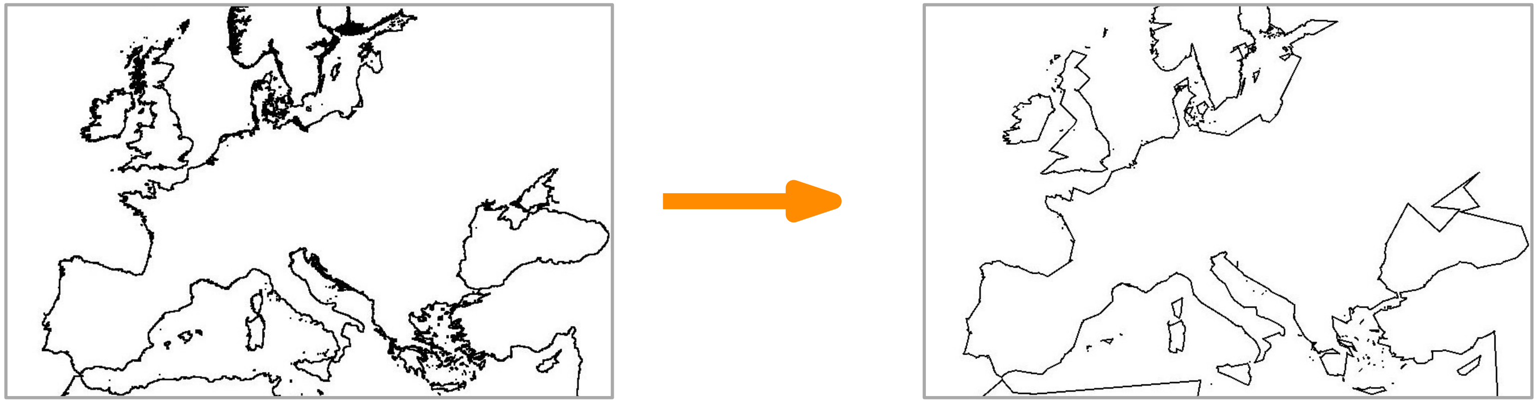

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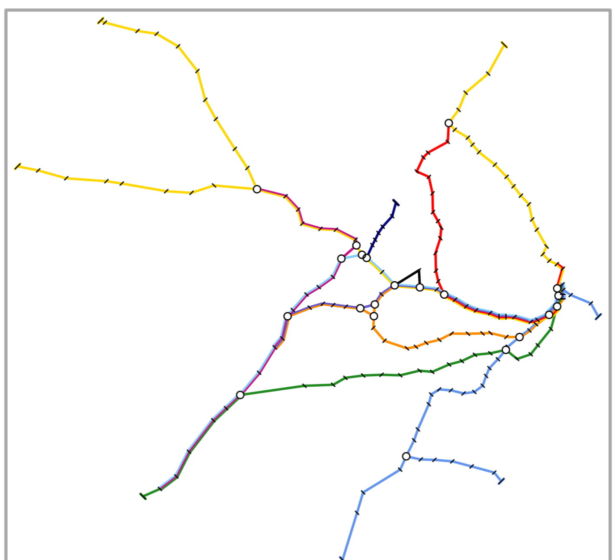

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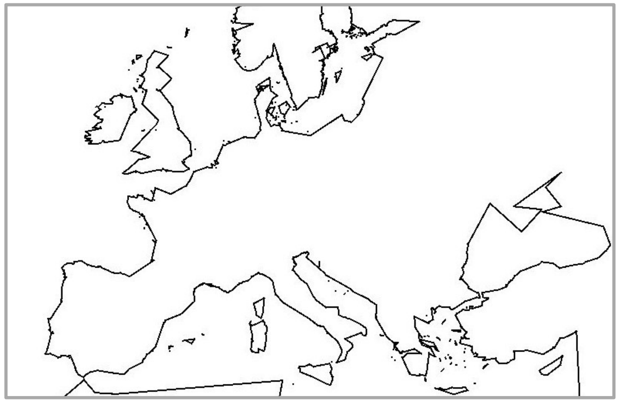

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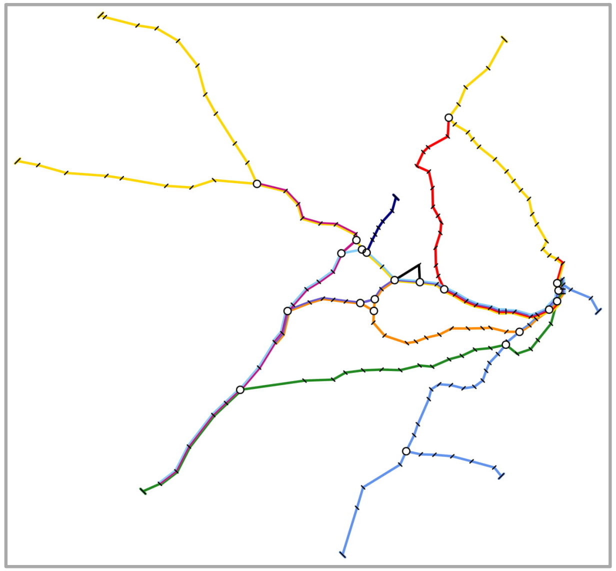

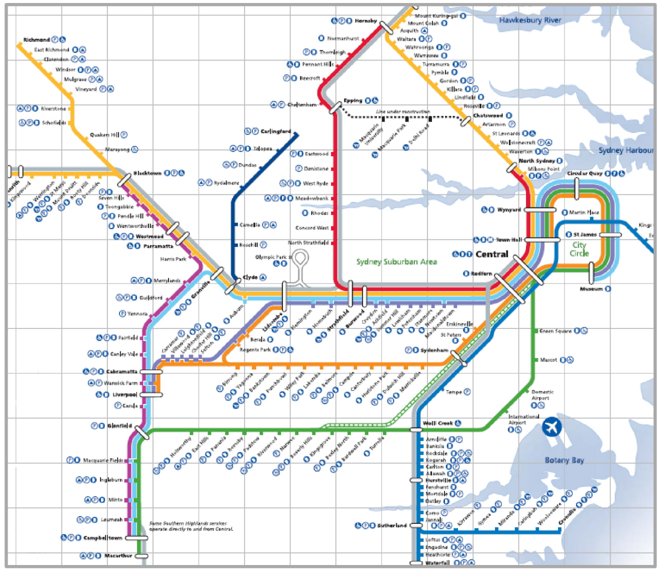

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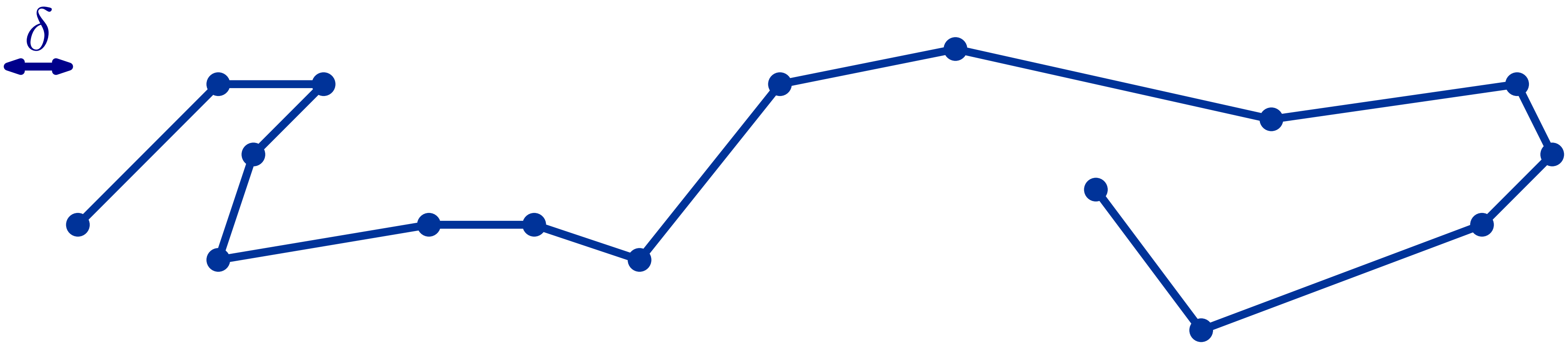

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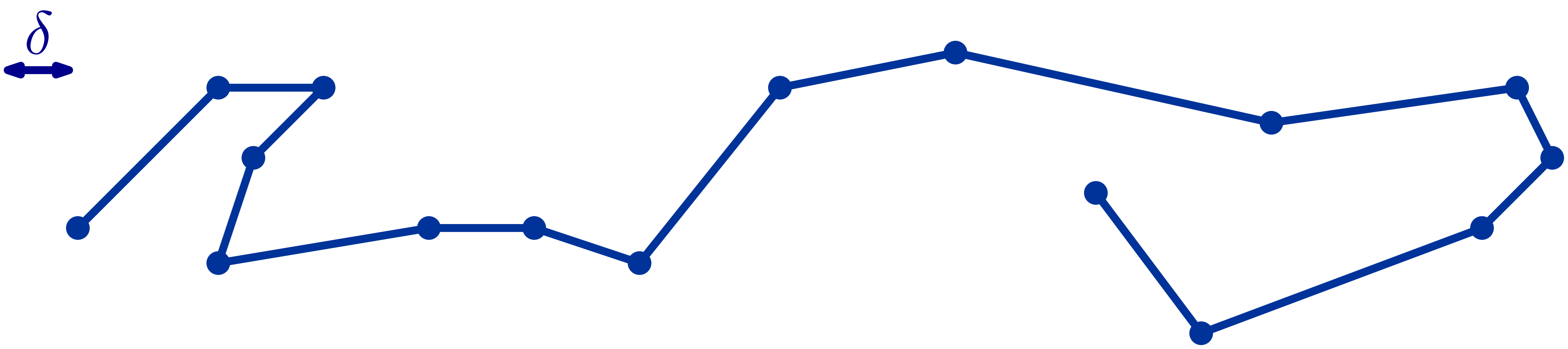

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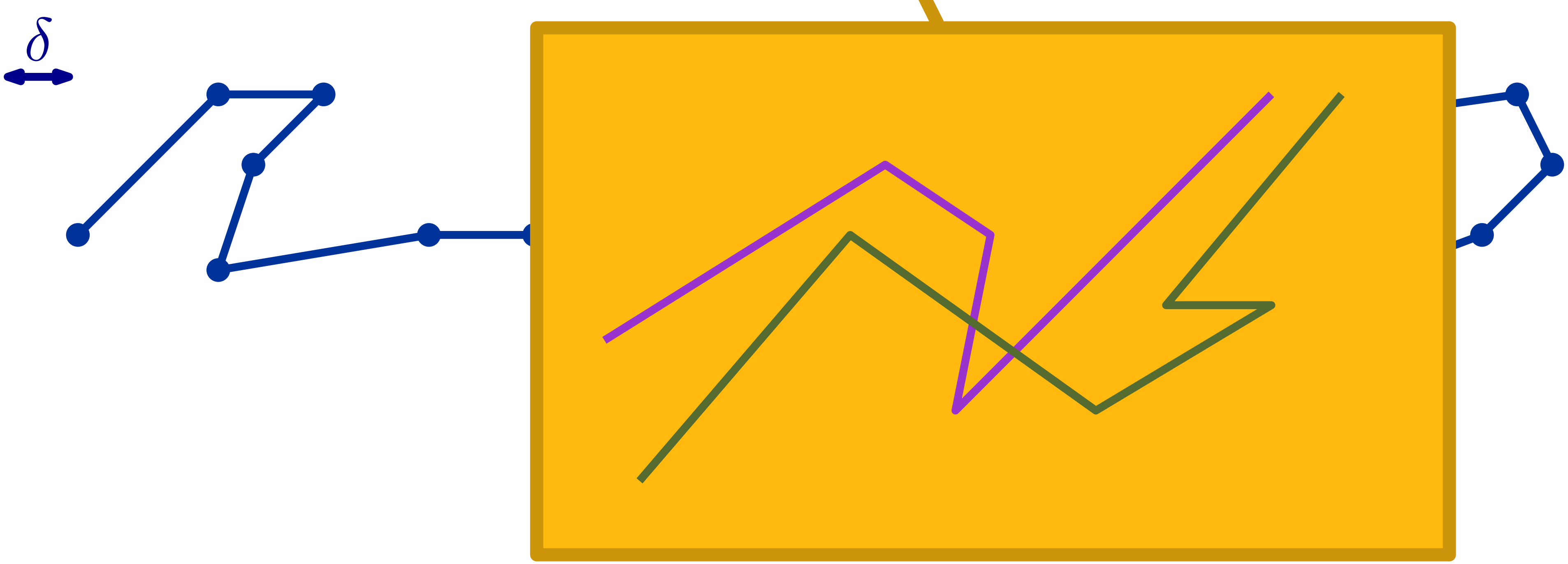

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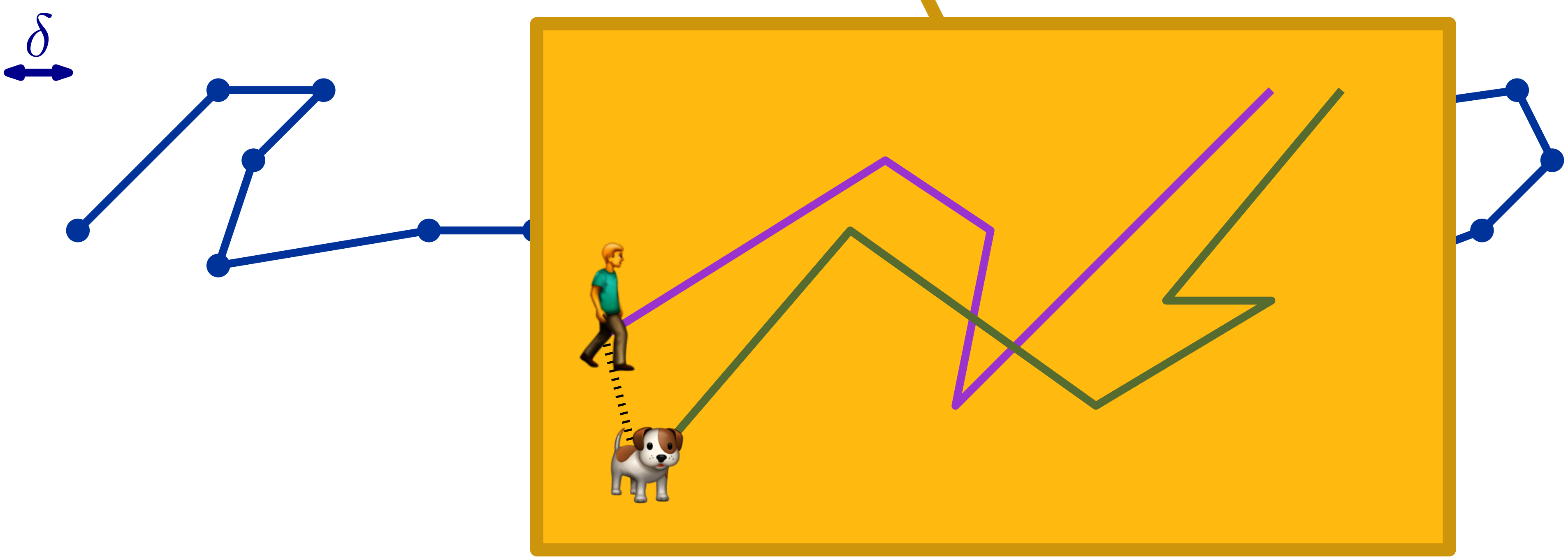

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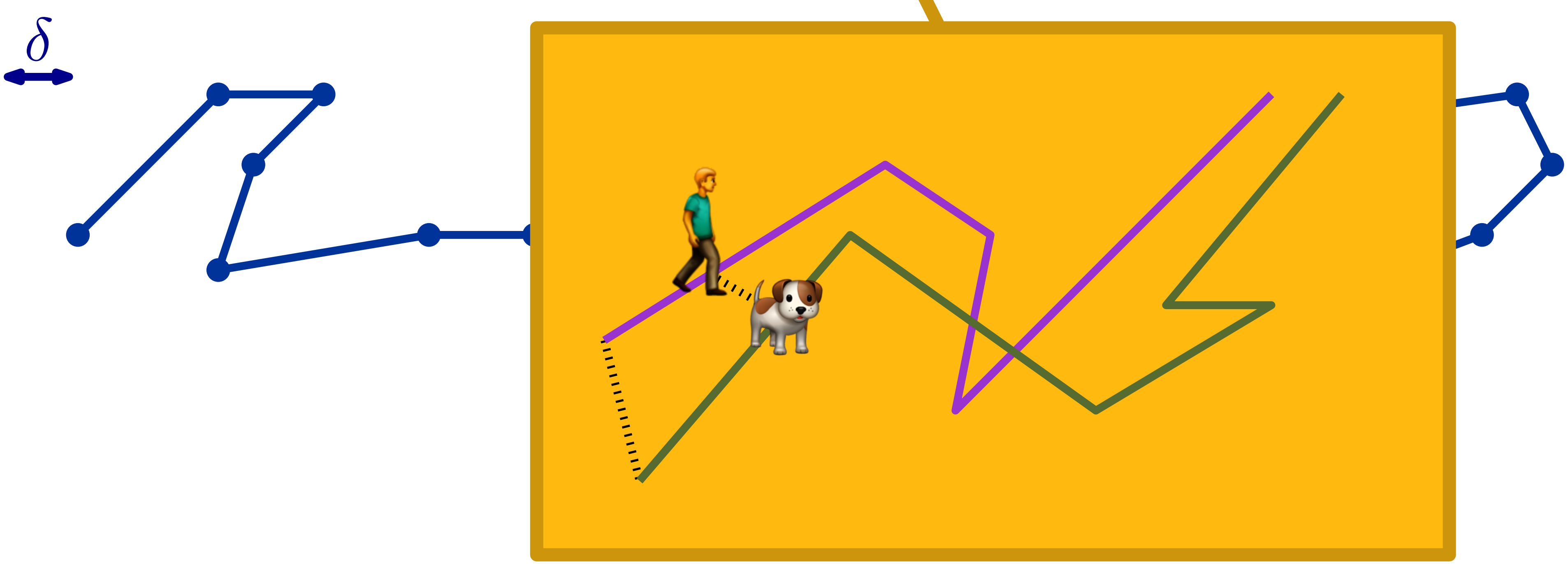

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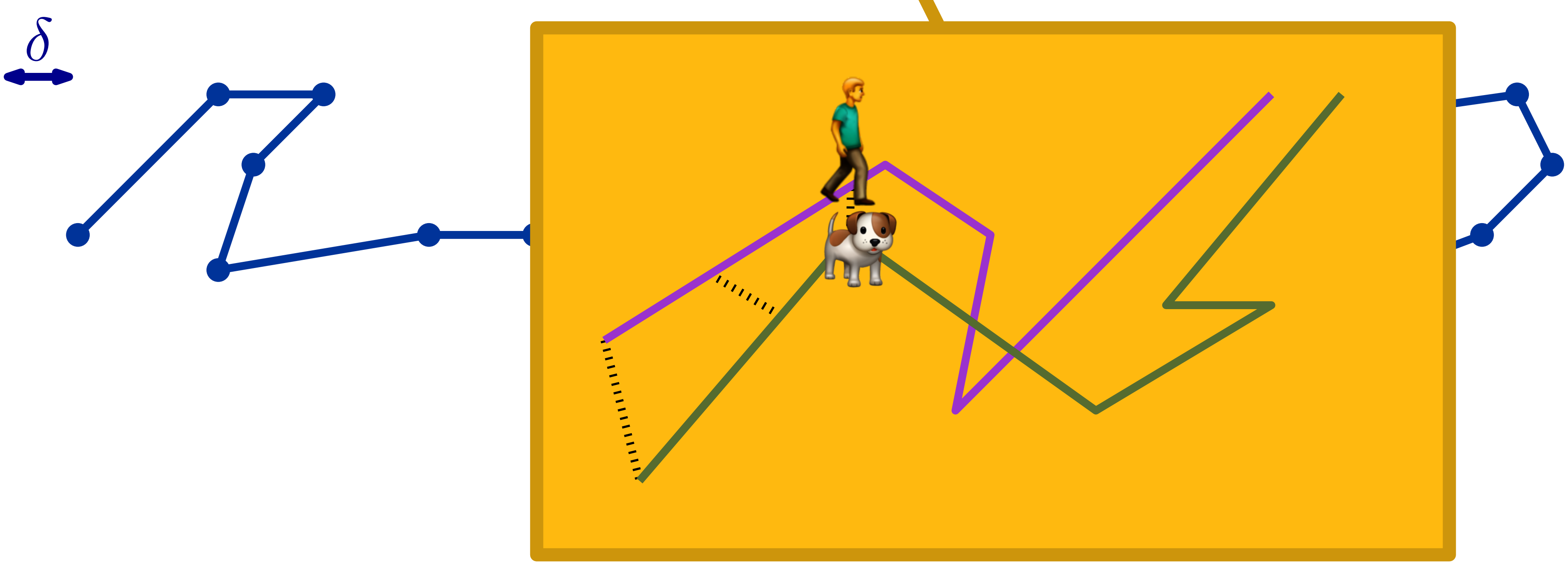

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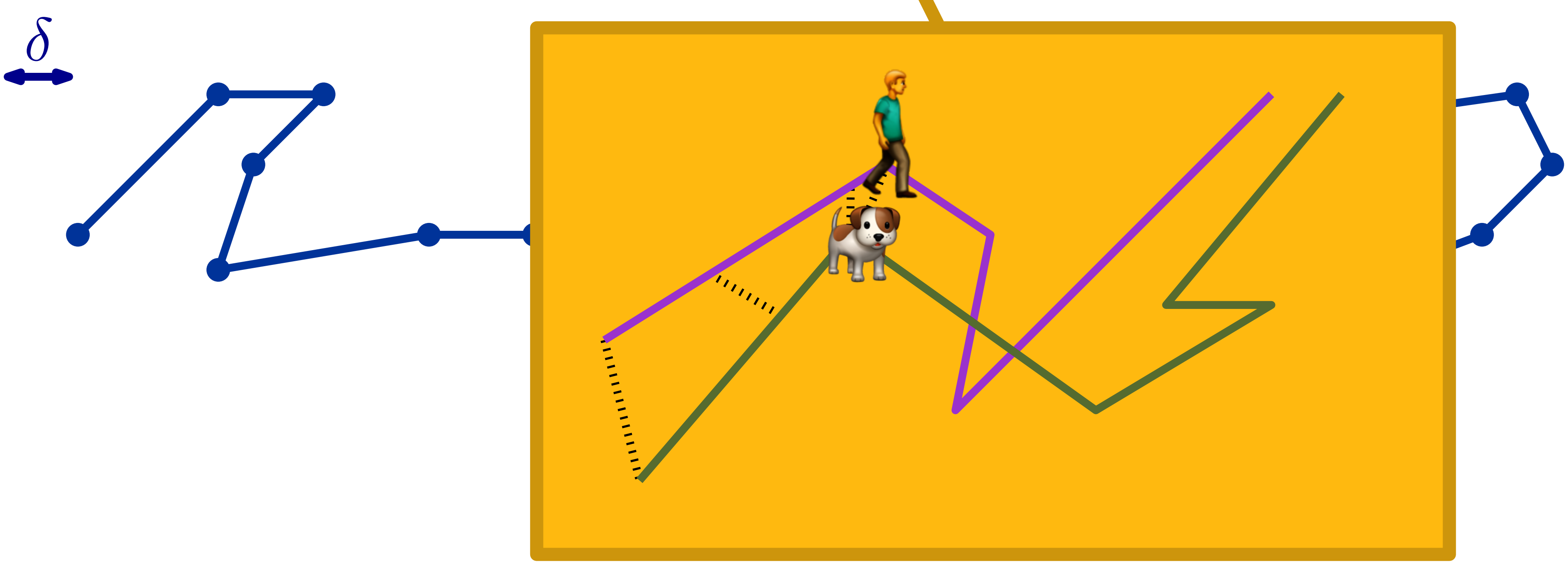

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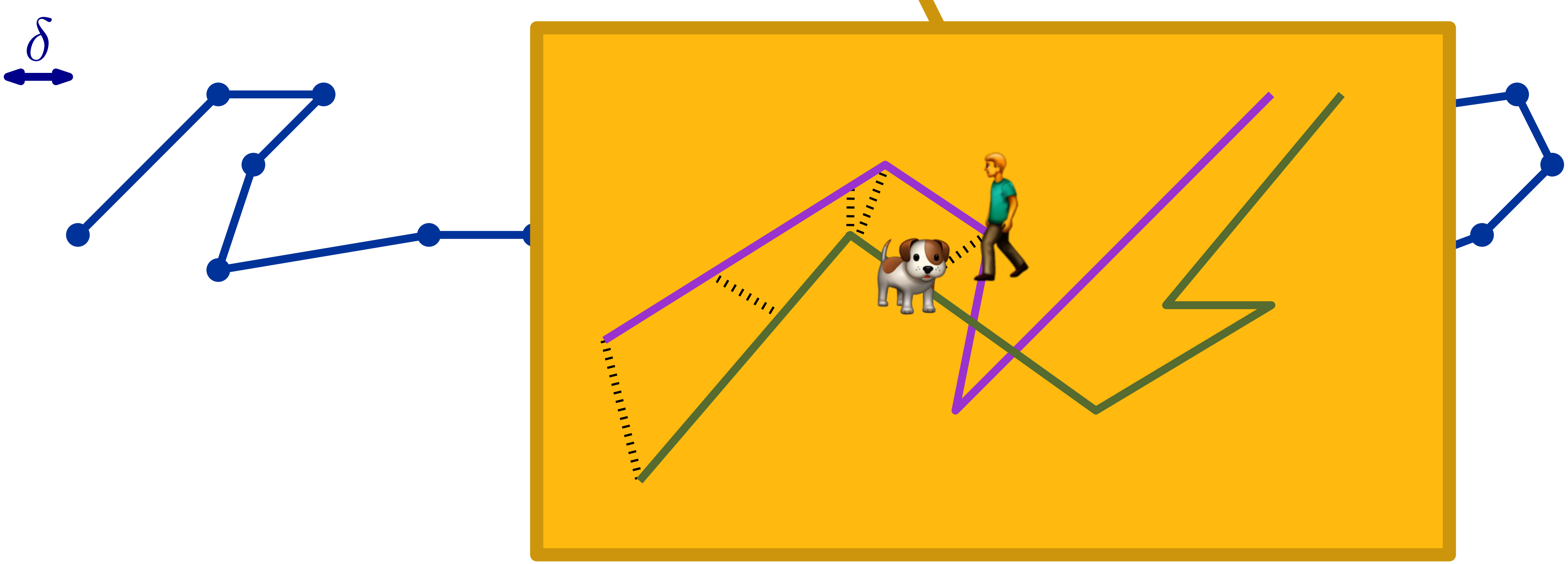

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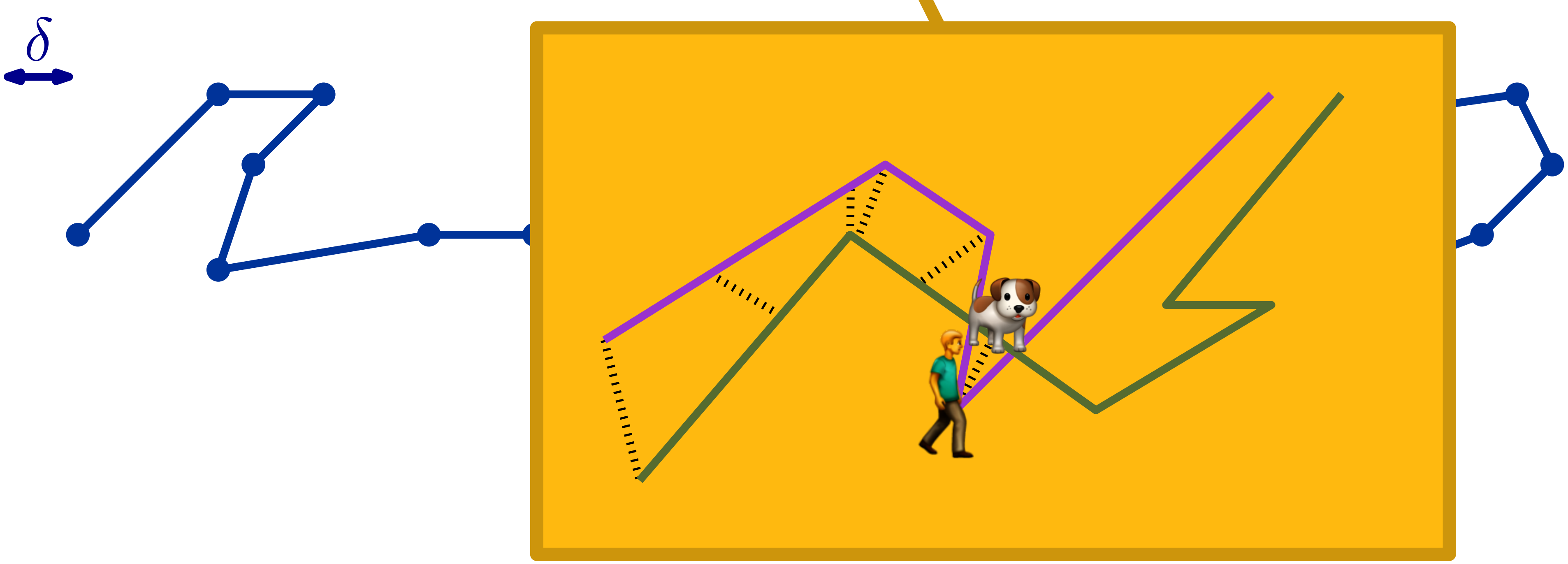

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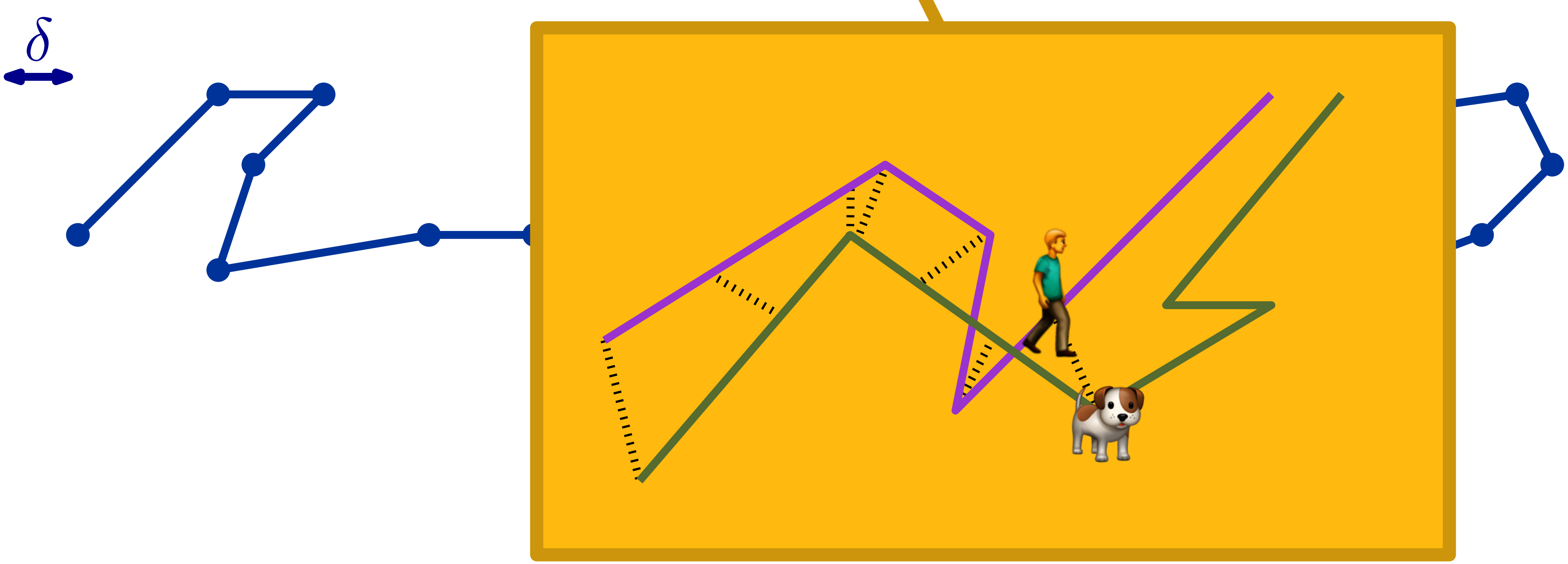

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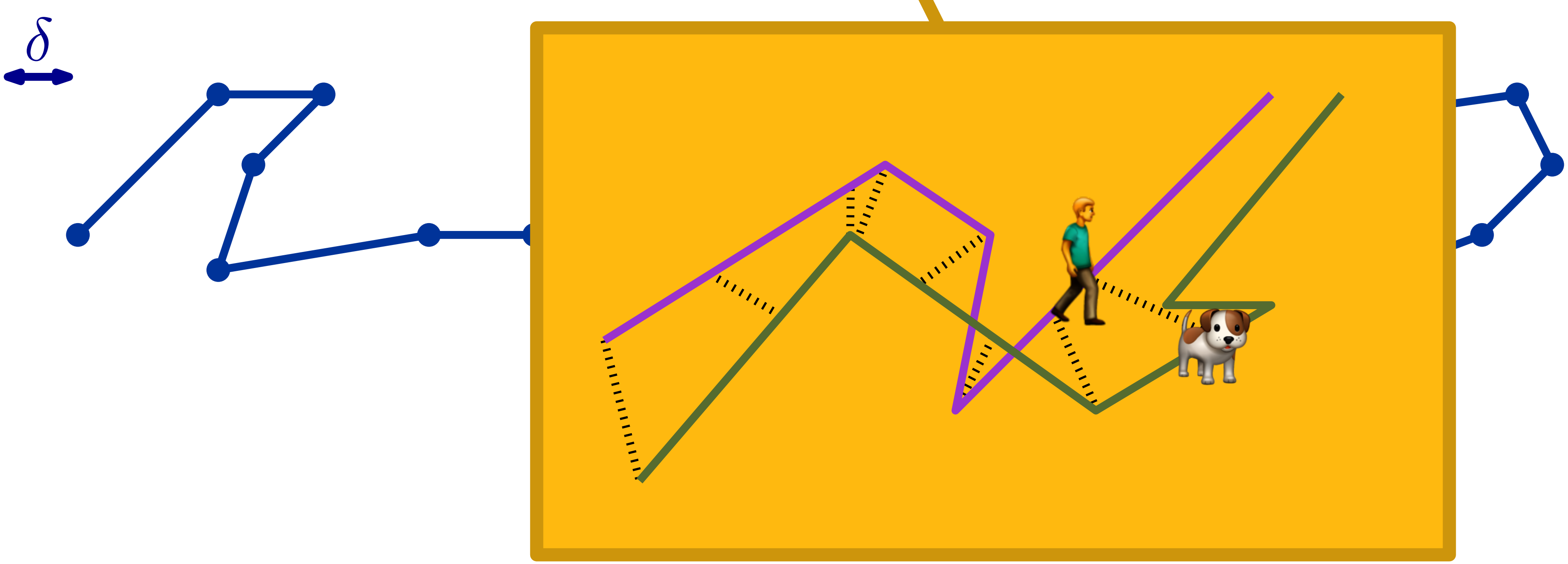

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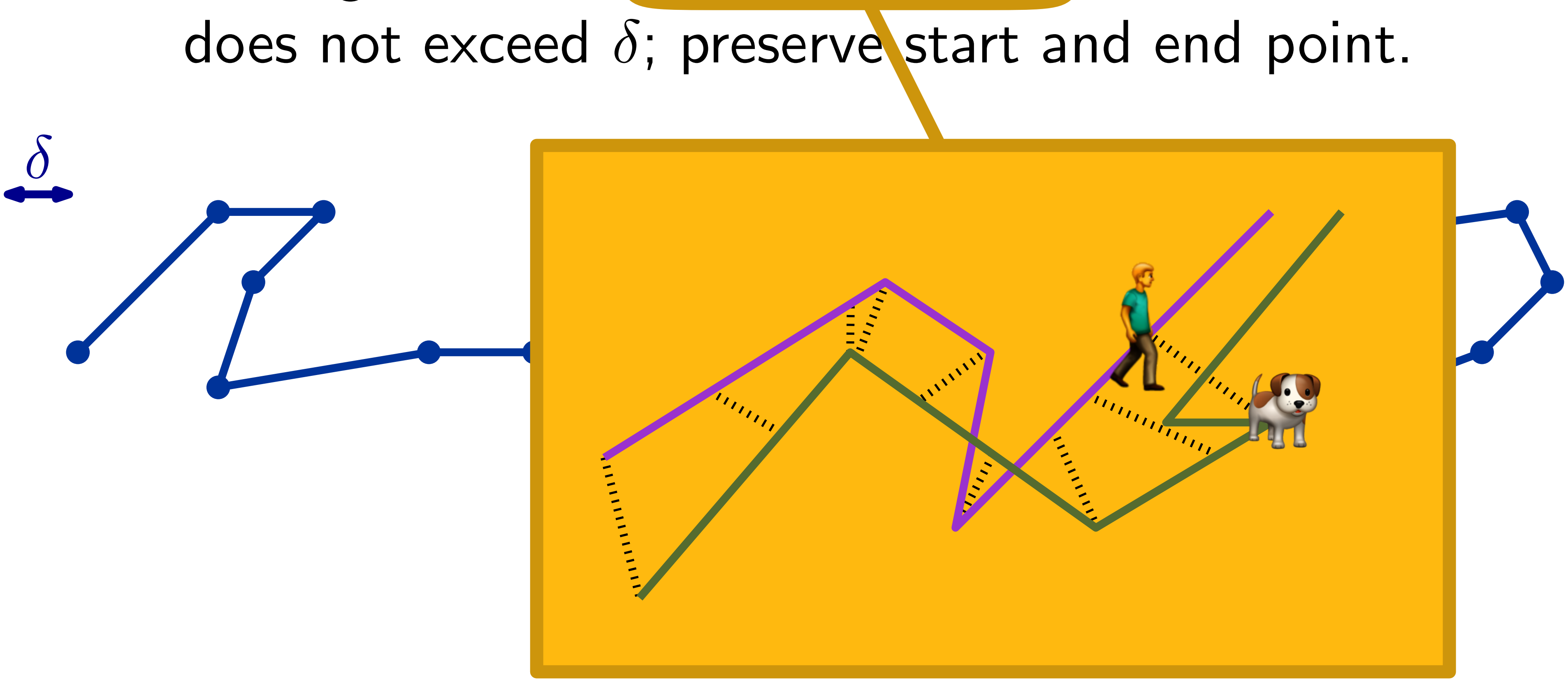

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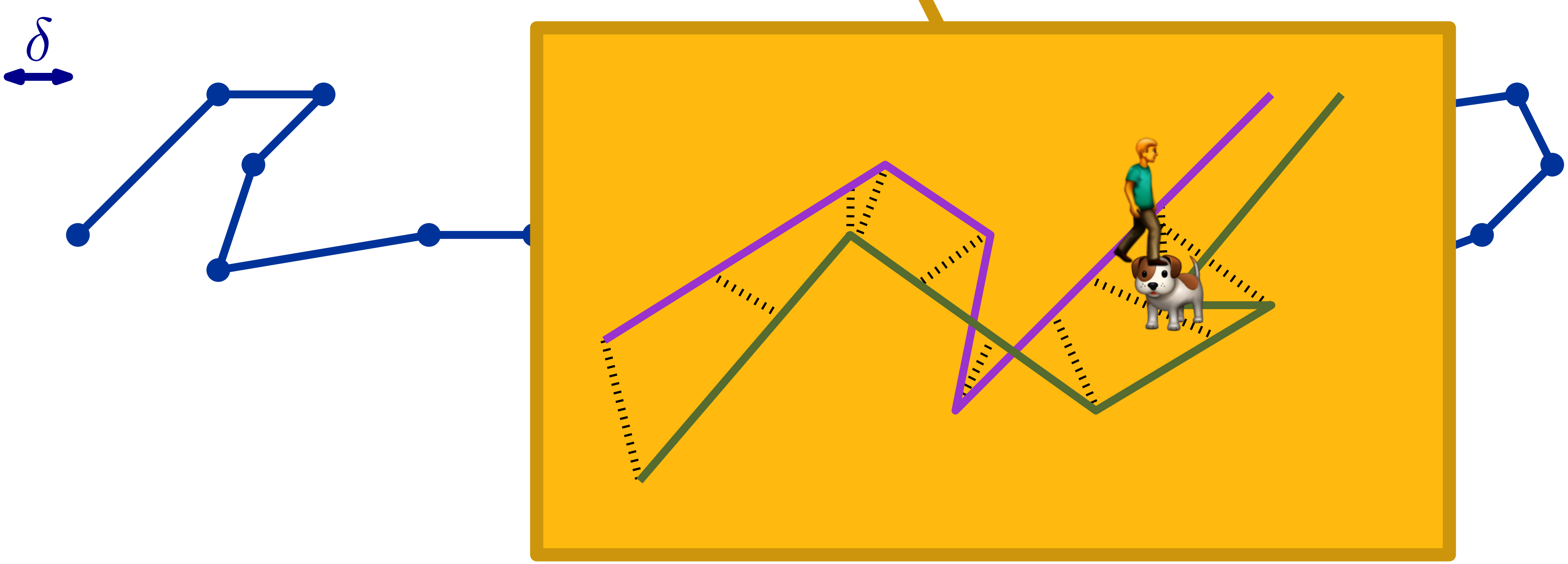

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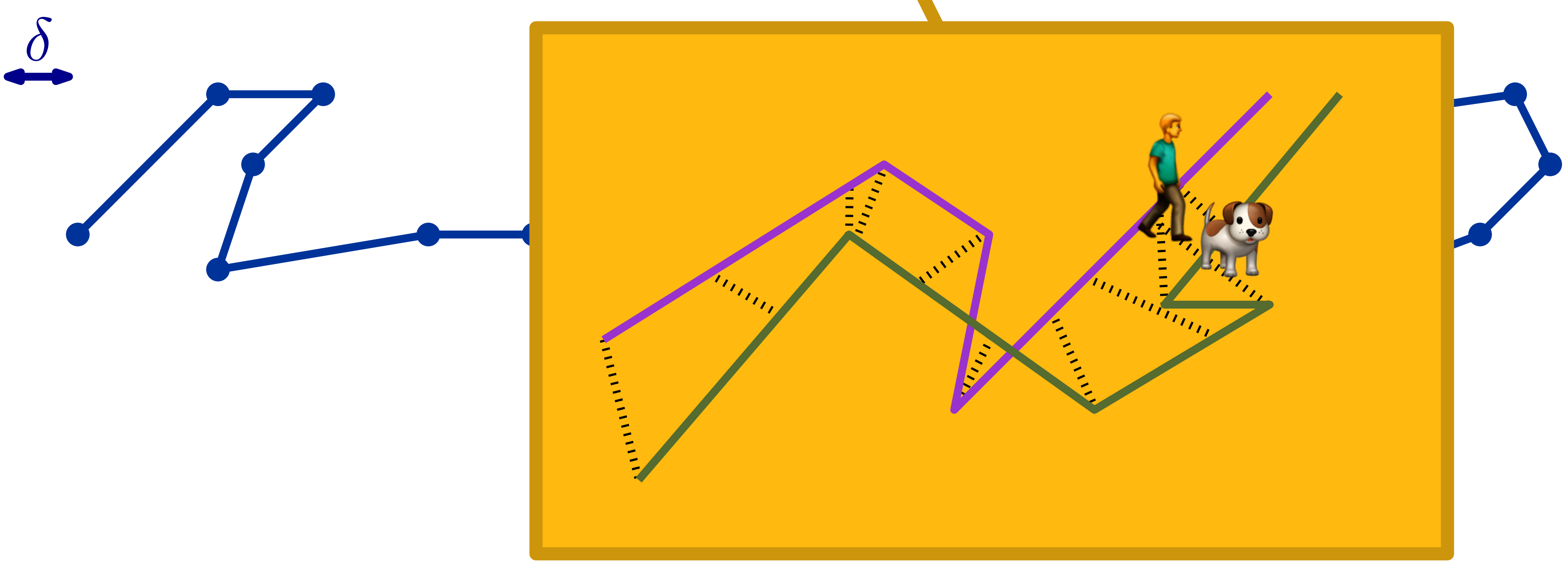

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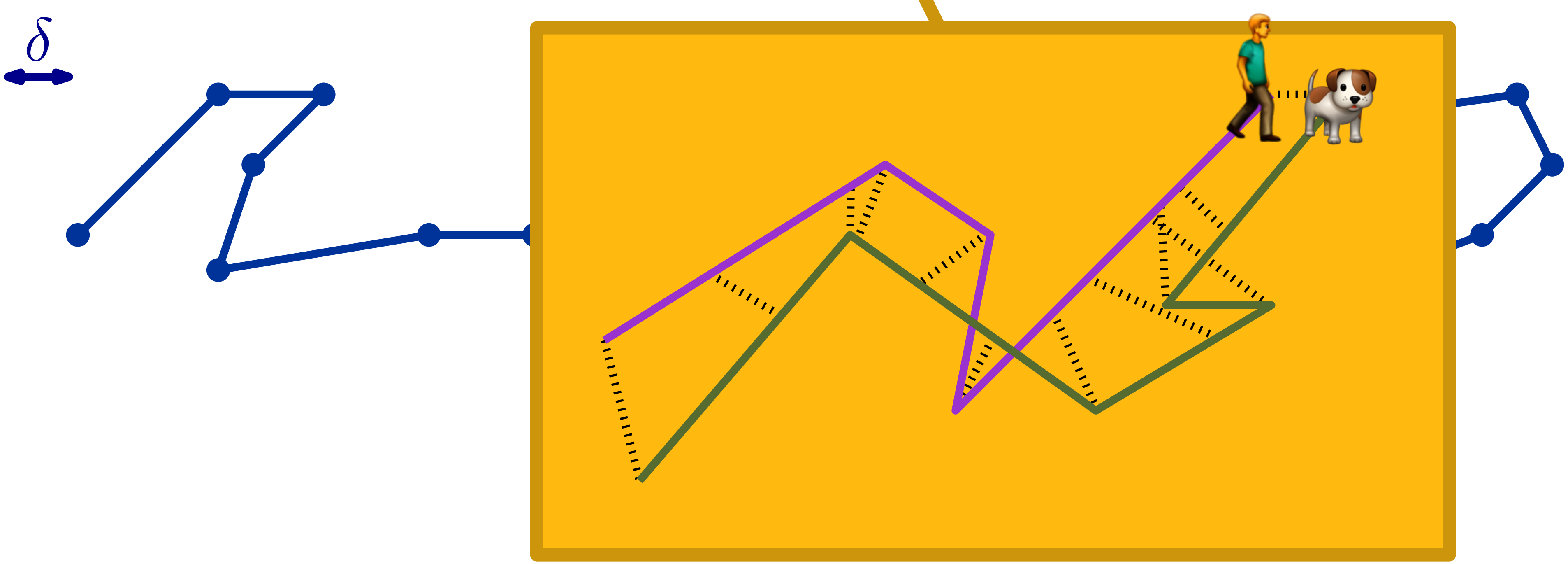

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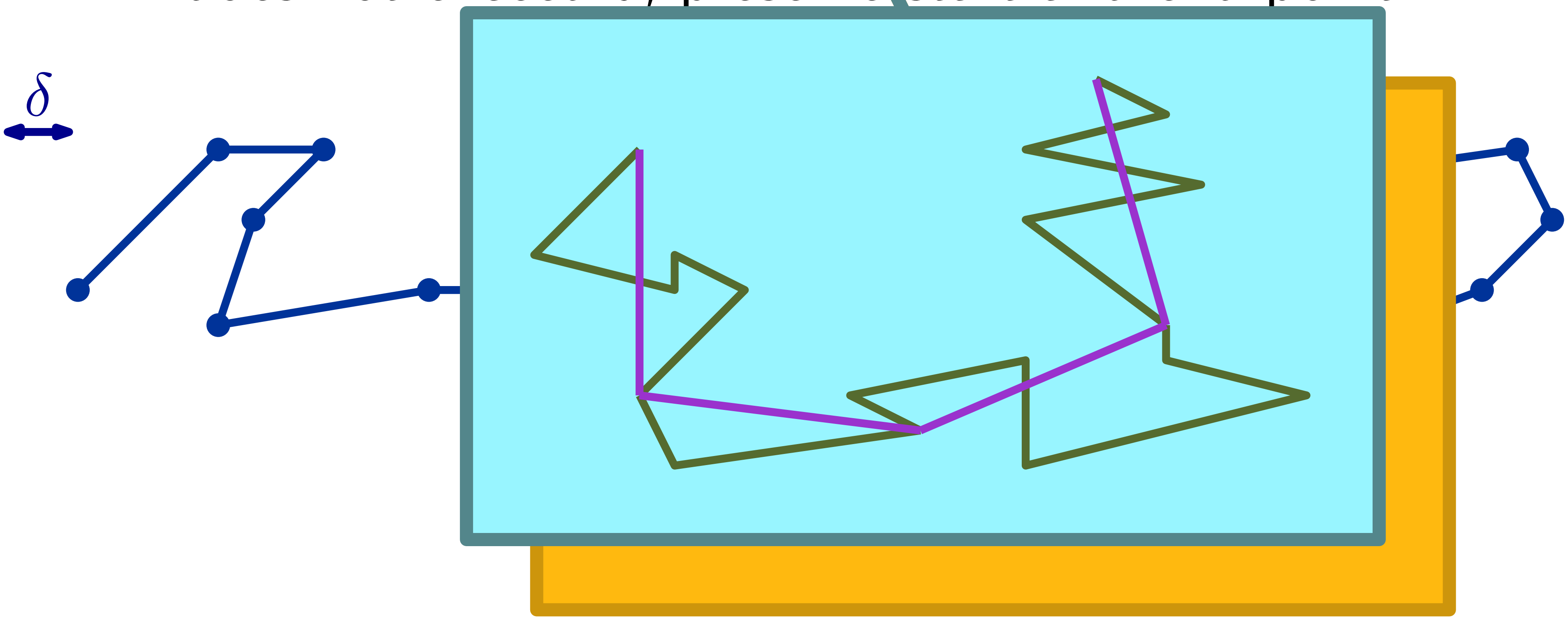

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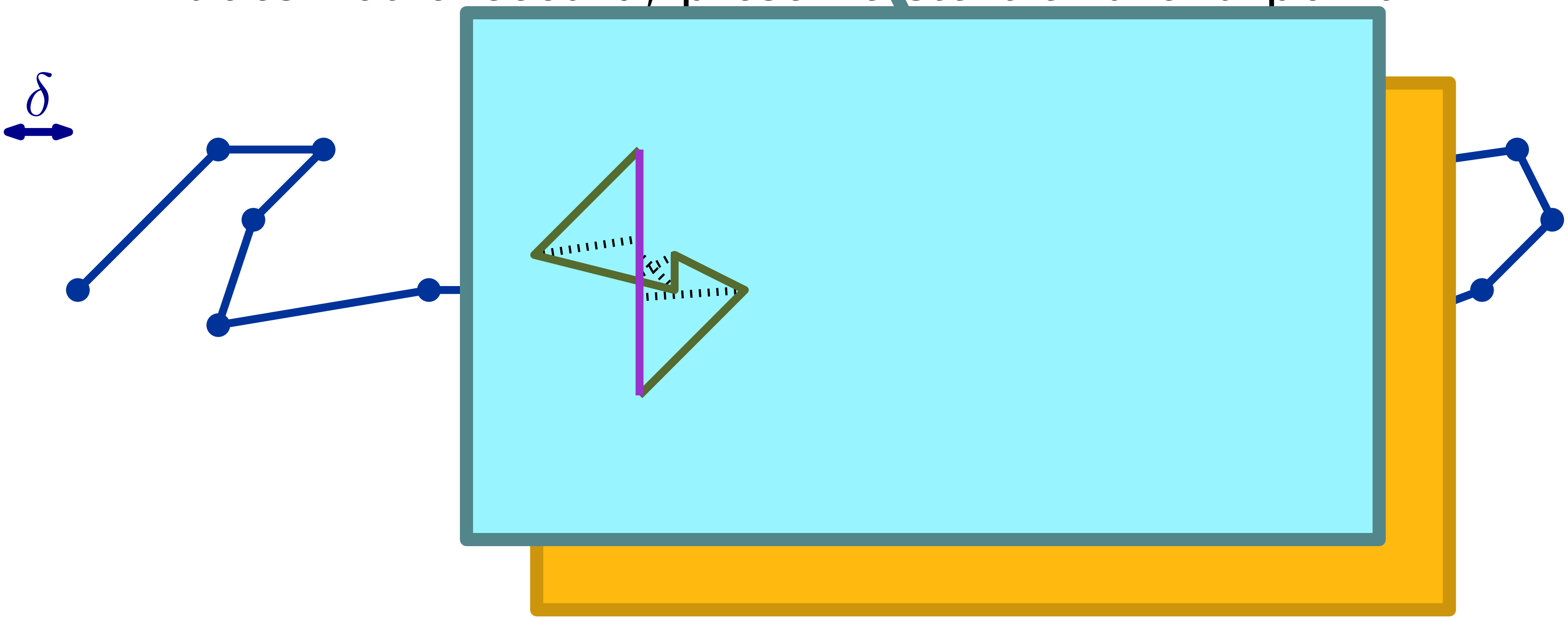

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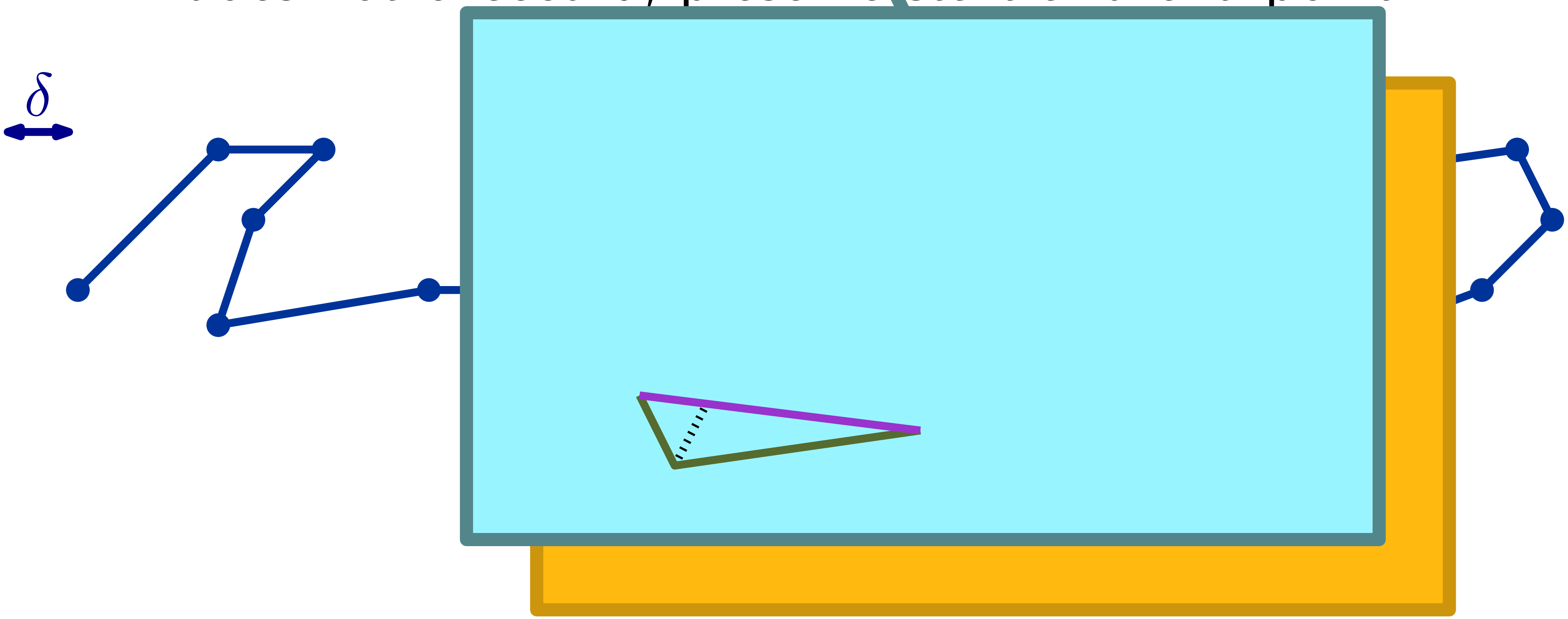

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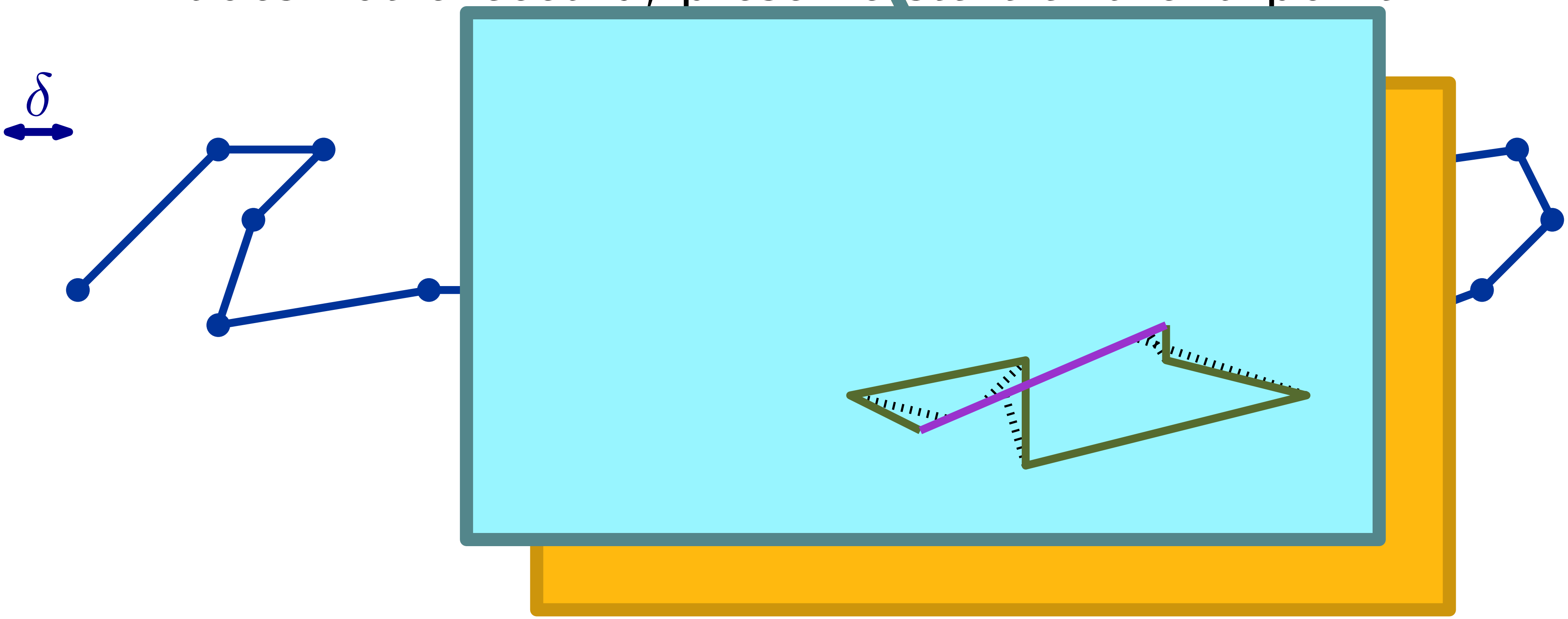

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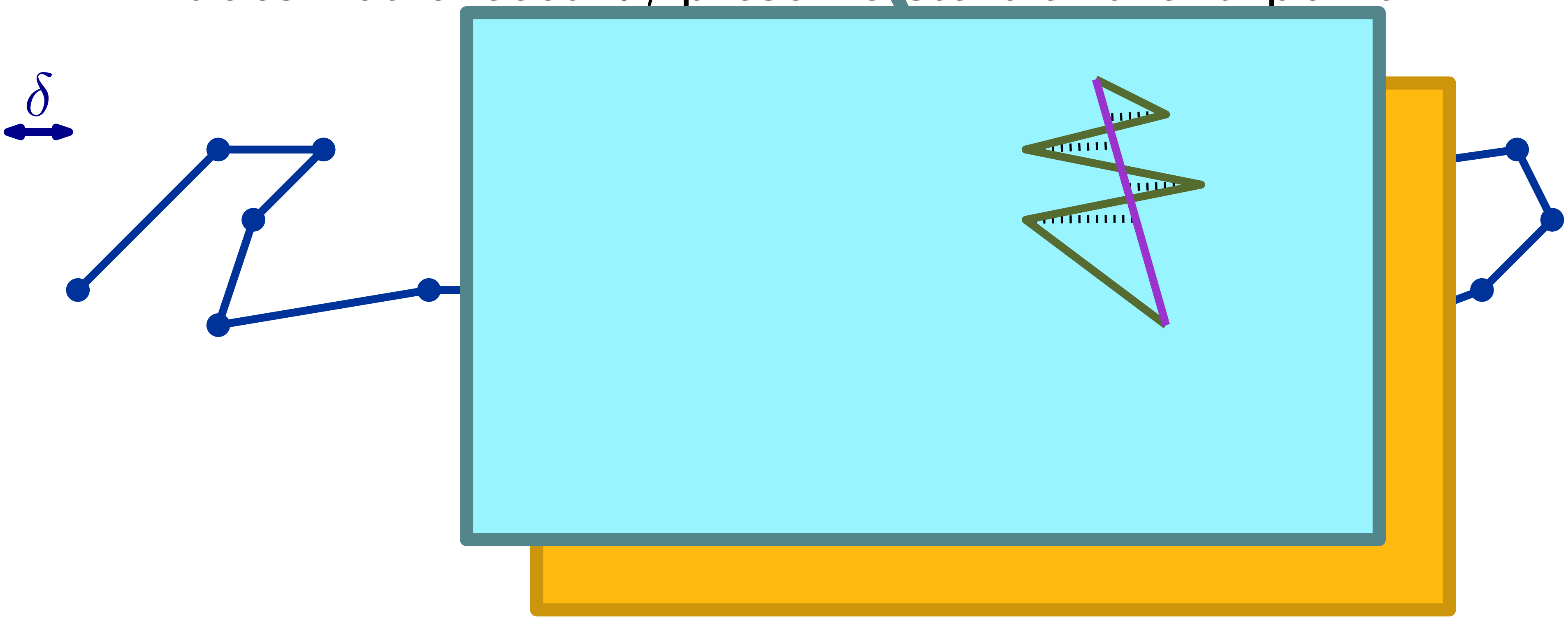

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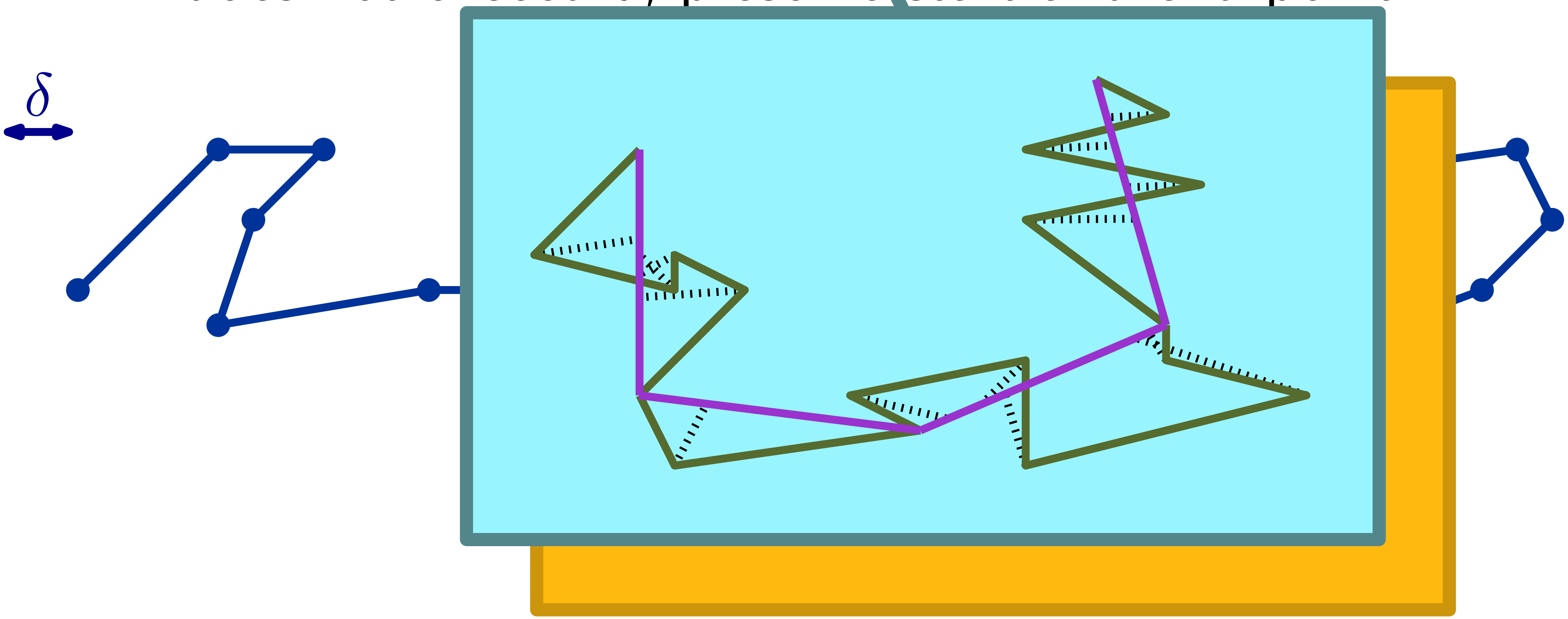

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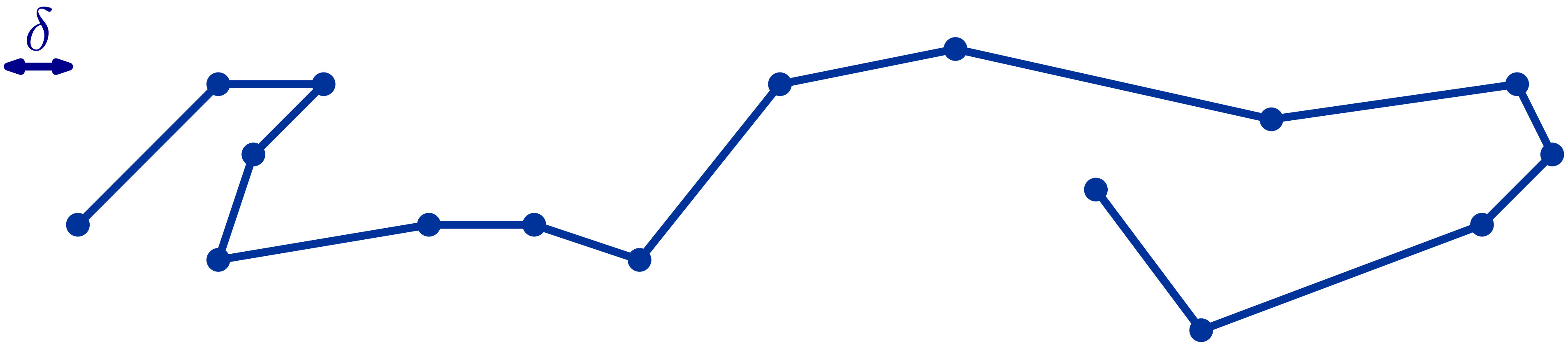

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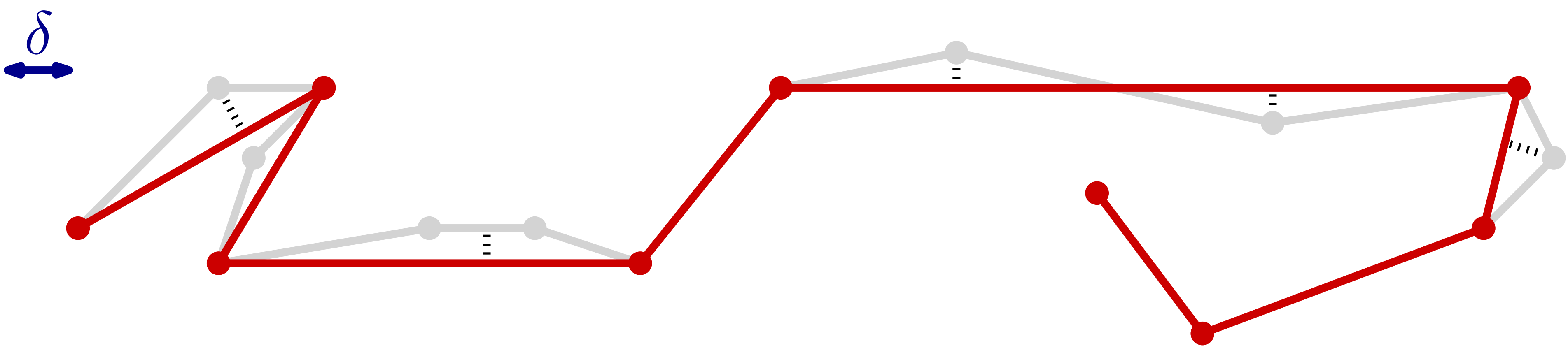

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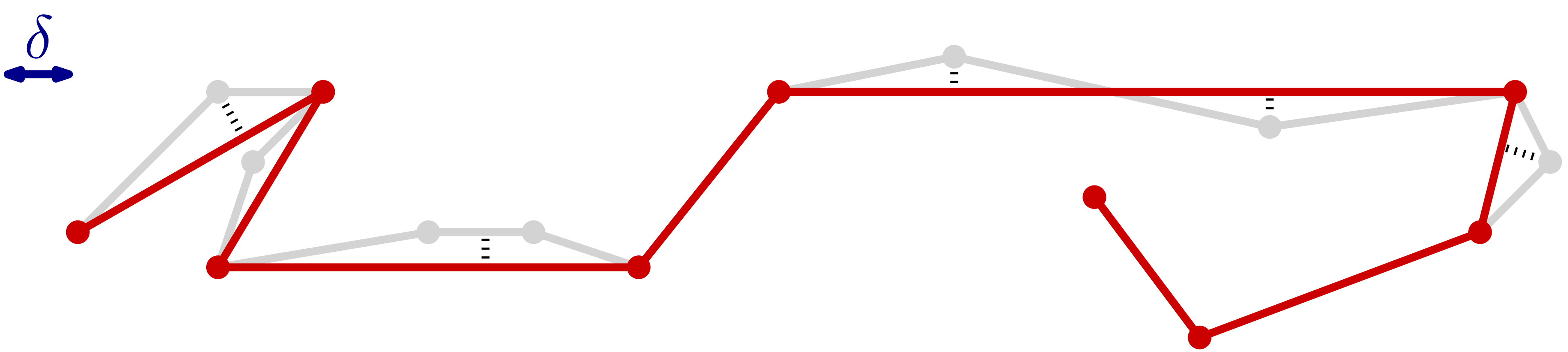

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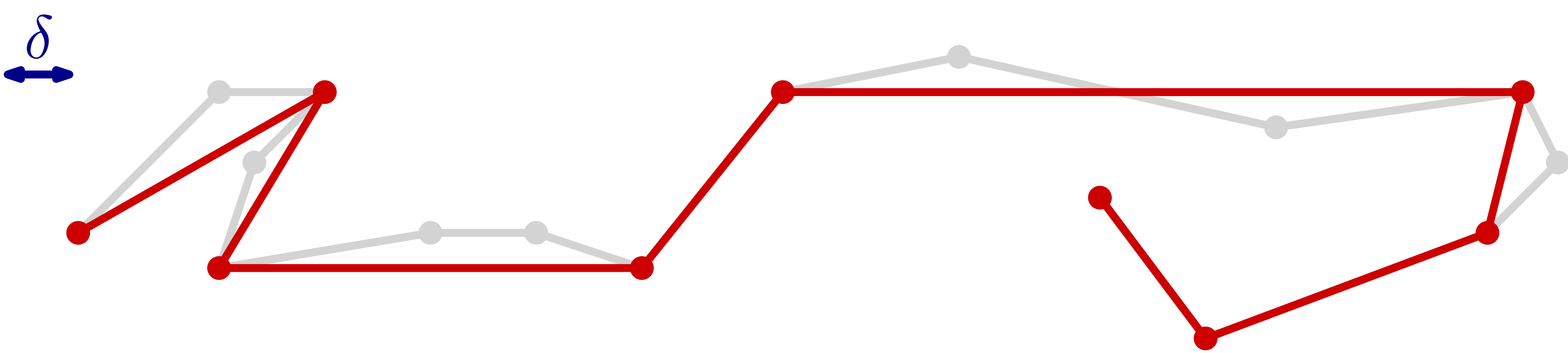

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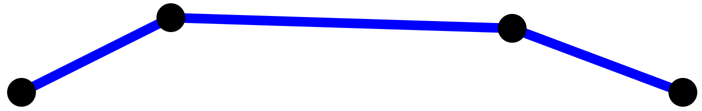

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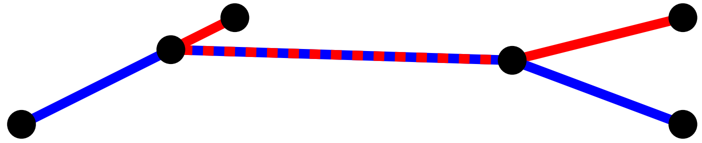

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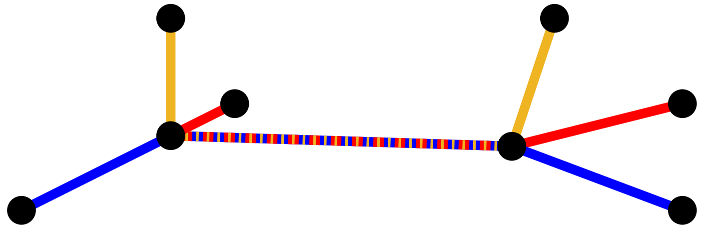

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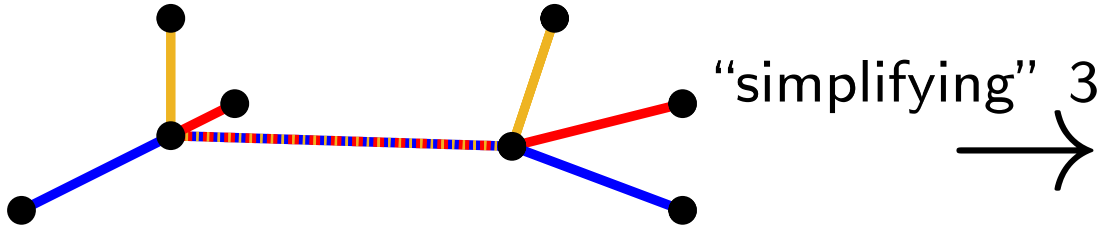

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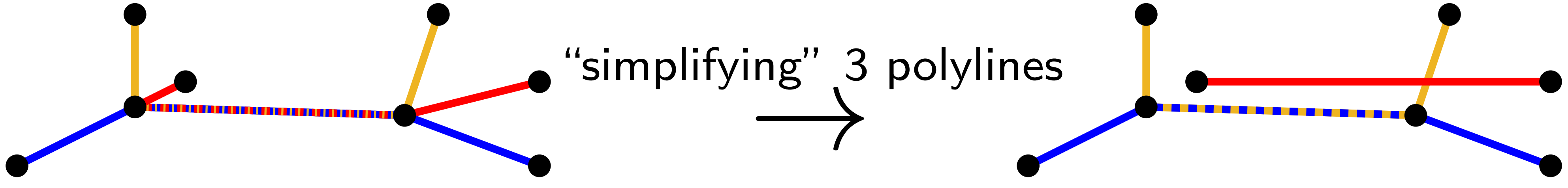

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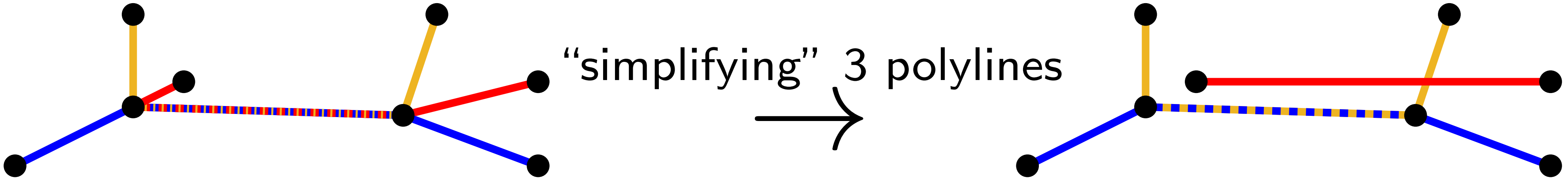

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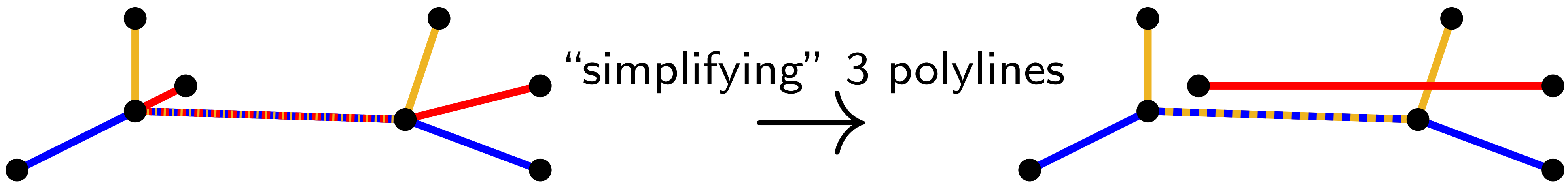

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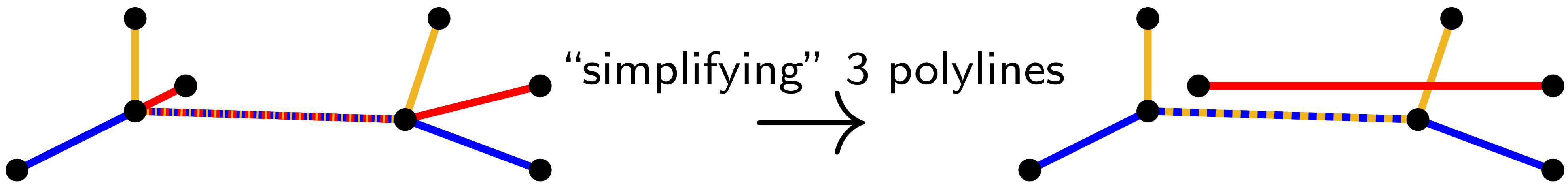

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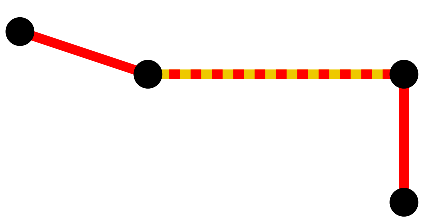

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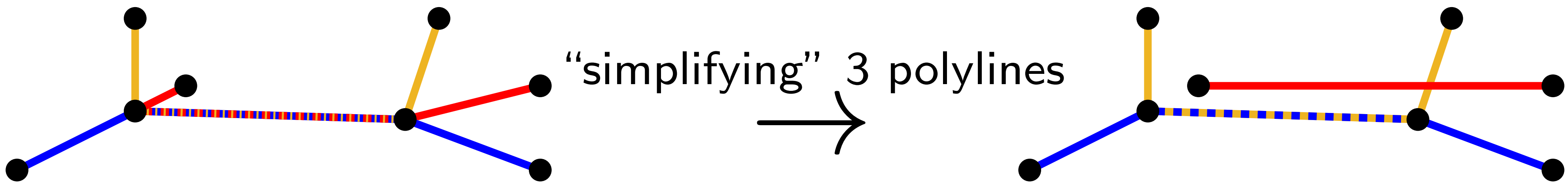

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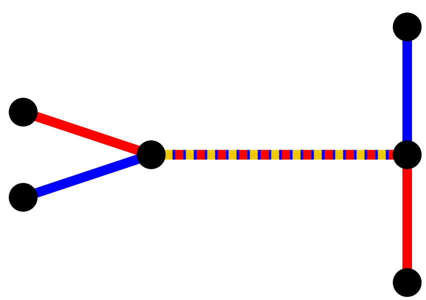

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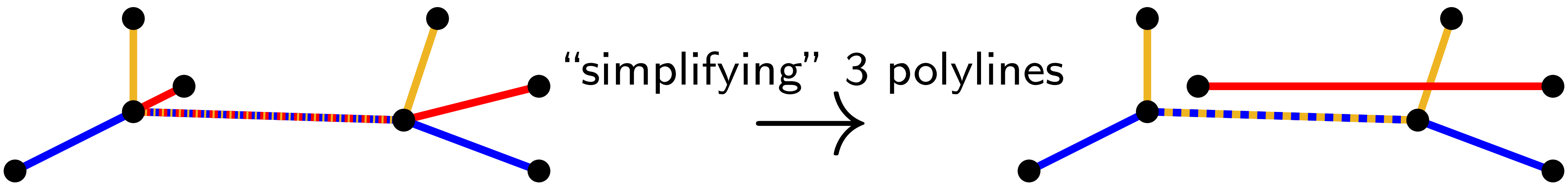

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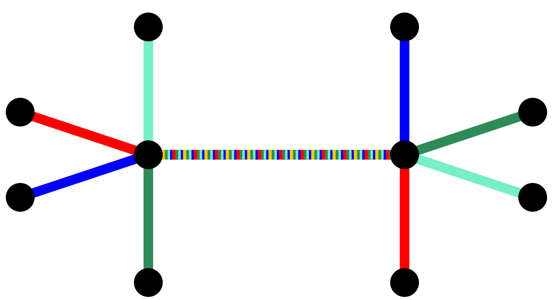

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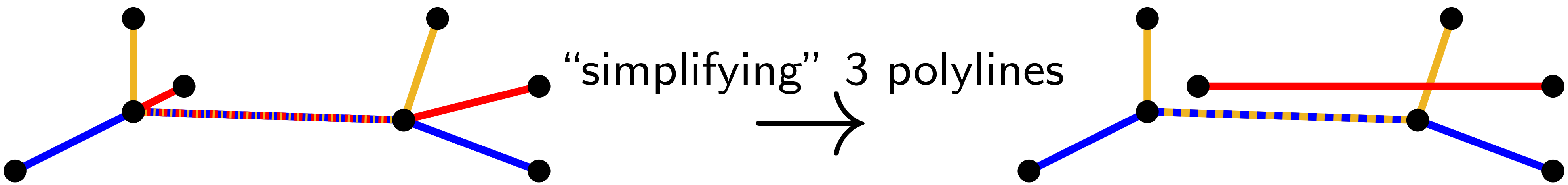

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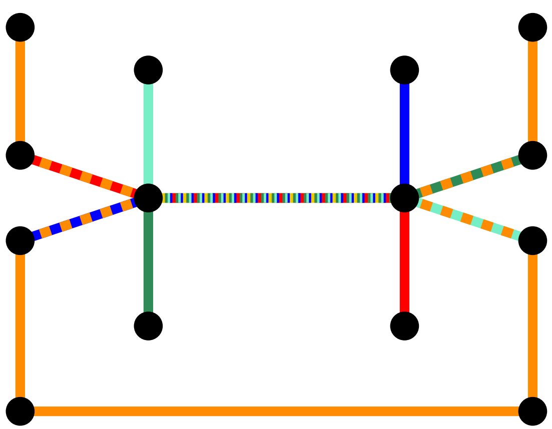

14 bends, 14 segments

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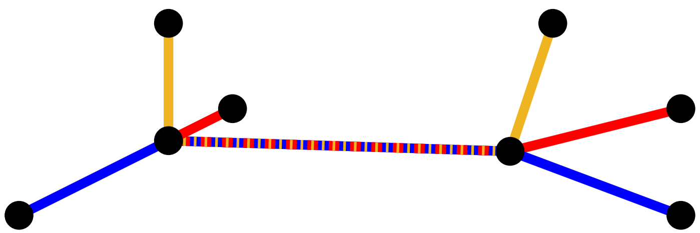
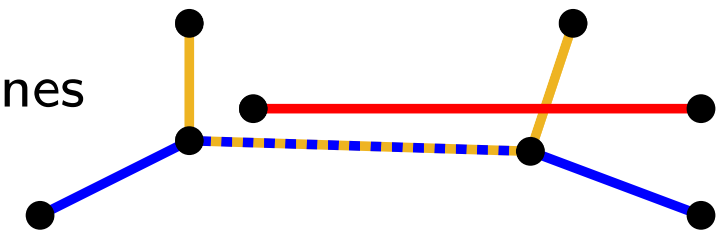


“simplifying” 3 polylines ⟶



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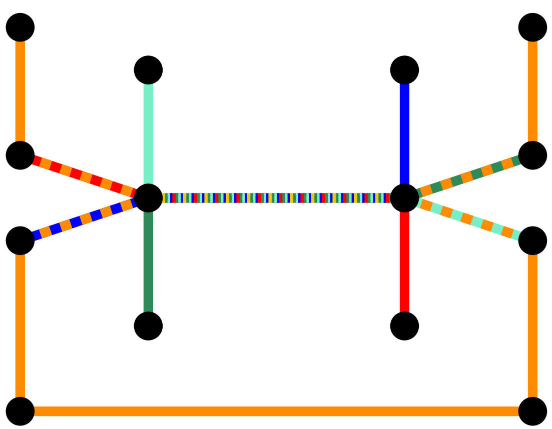


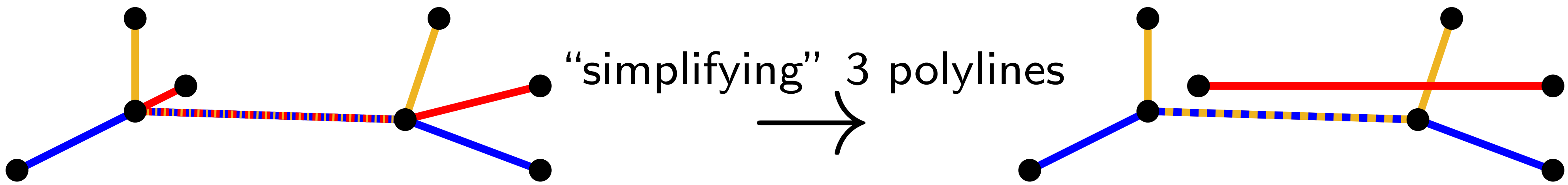
“simplifying” 6 polylines ⟶

14 bends, 14 segments

Why Not Simplifying Independently?

A bend or segment might once be kept and once be discarded

⇒ misleading picture of the reality



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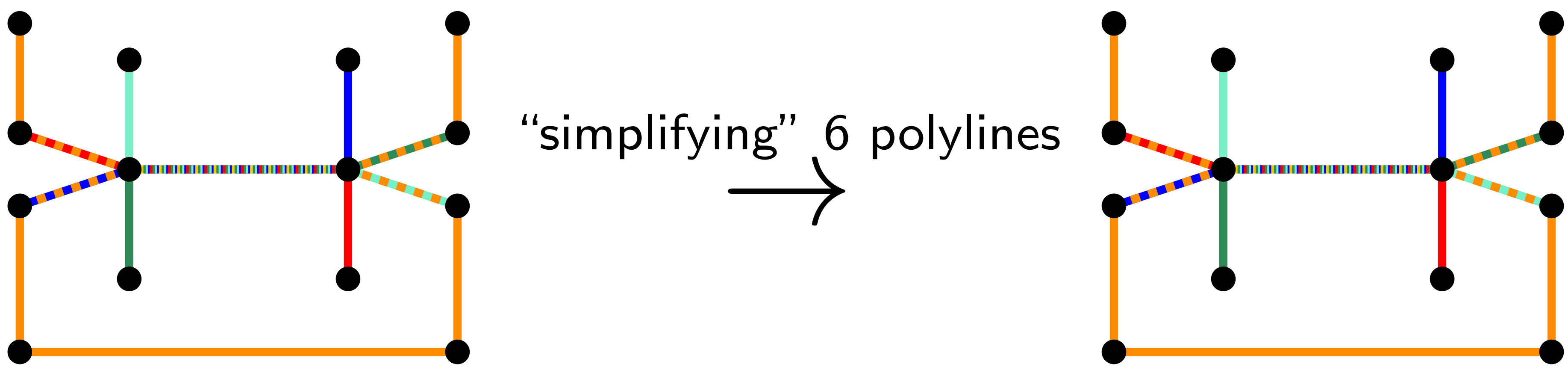

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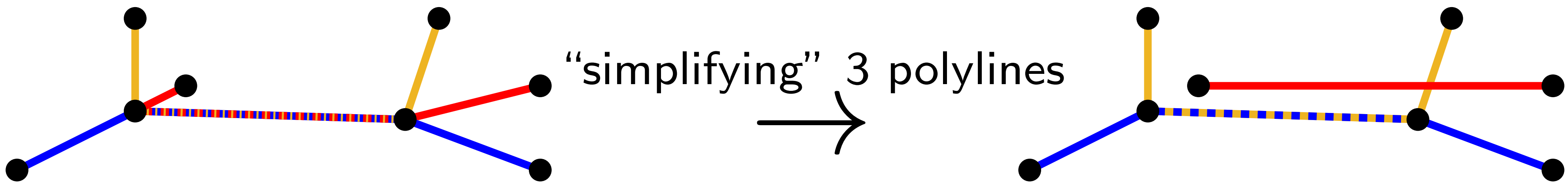

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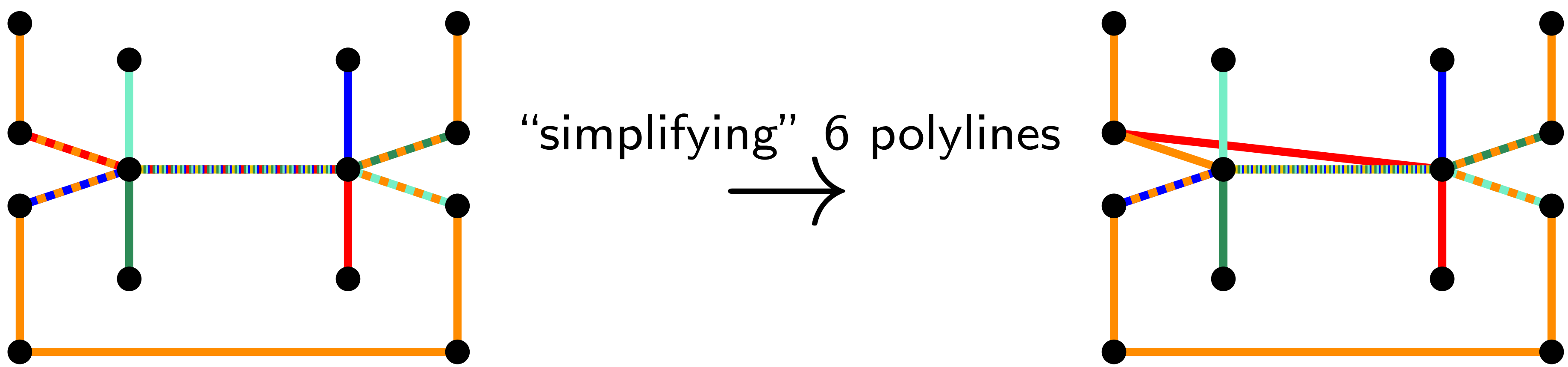

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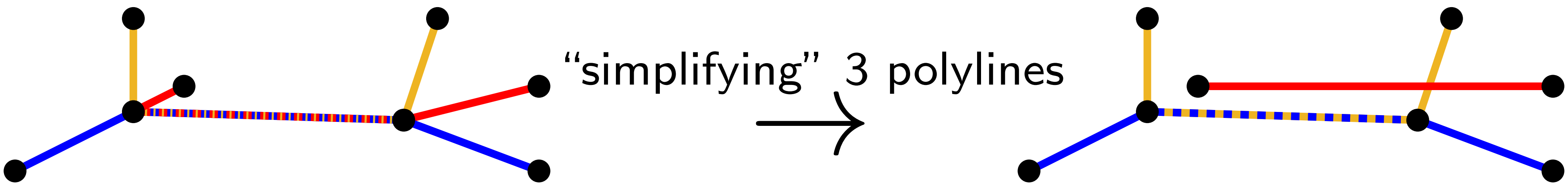

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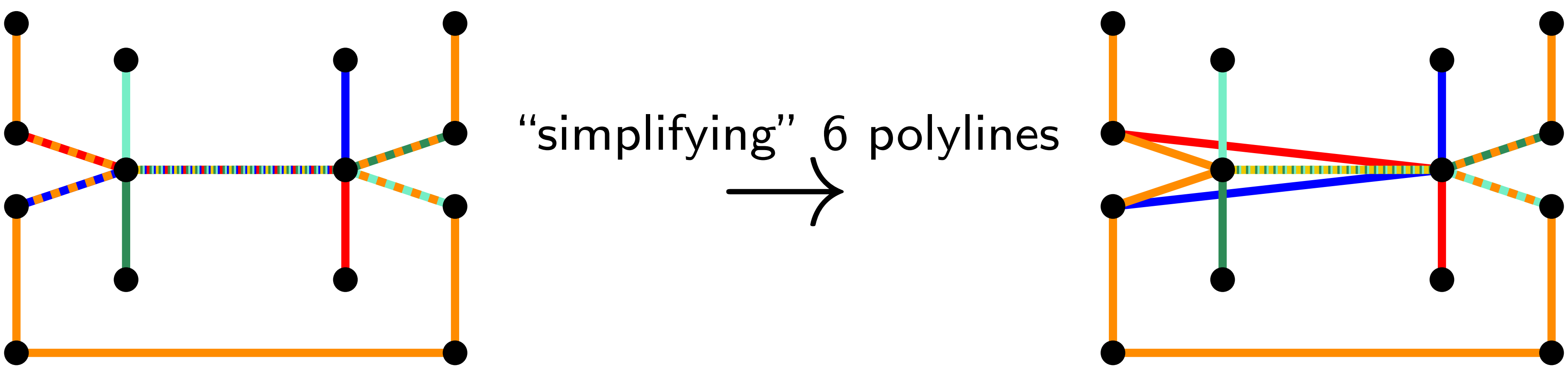

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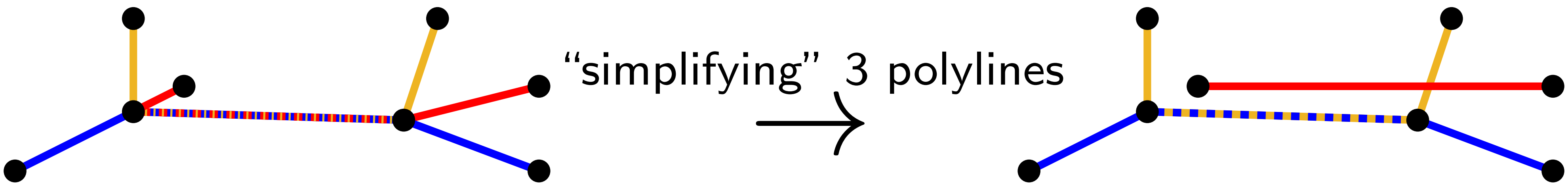

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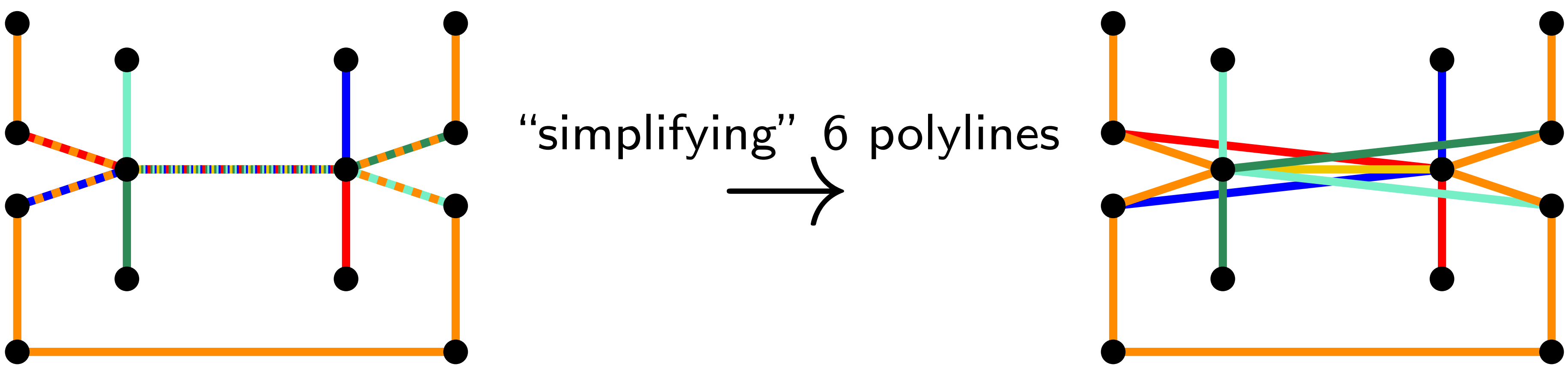

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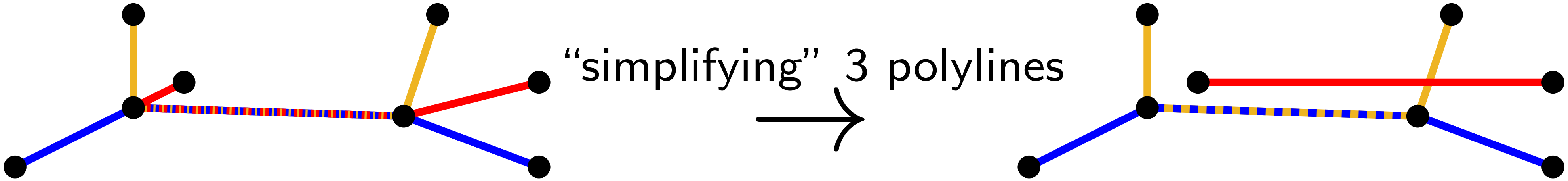

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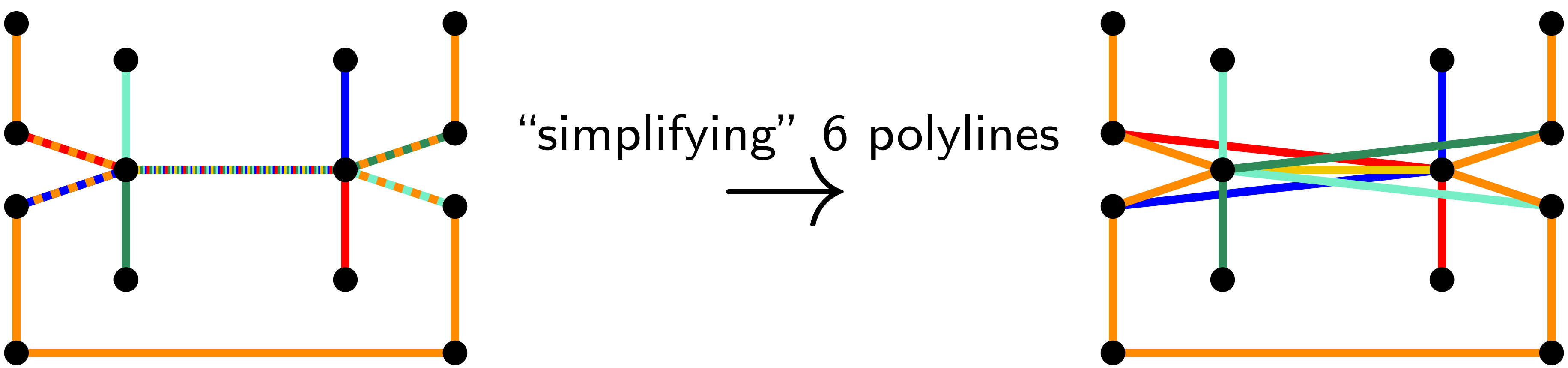

14 bends, 14 segments

14 bends, 18 segments

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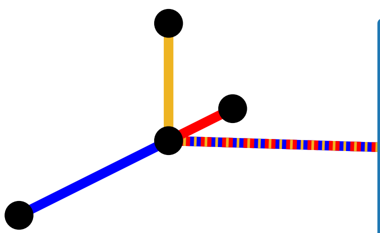

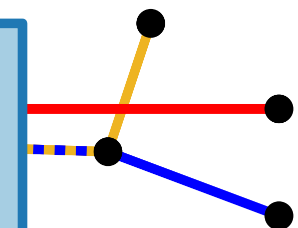

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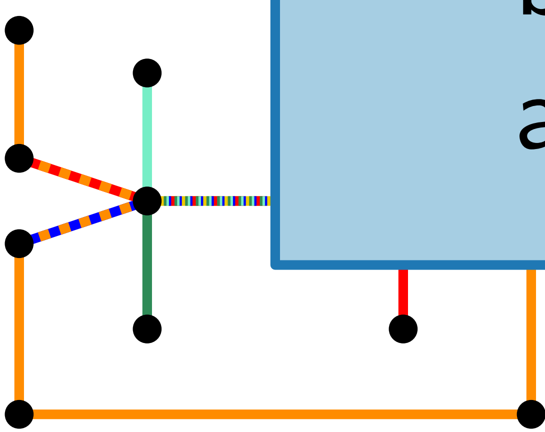

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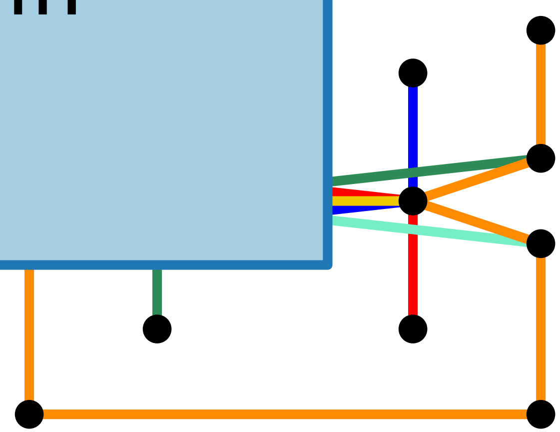

14 bends, 18 segments

⇒ Simplify consistently! (keep or discard a bend or segment in all polylines)

Agenda

1. Motivation and Introduction
2. **Problem Definition**
3. Hardness of Approximation (+ Proof Sketch)
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5. Summary

Our Generalization

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- Given: set $\mathcal{L} = \{L_1, \ldots, L_\ell\}$ of polylines possibly sharing bends and segments,

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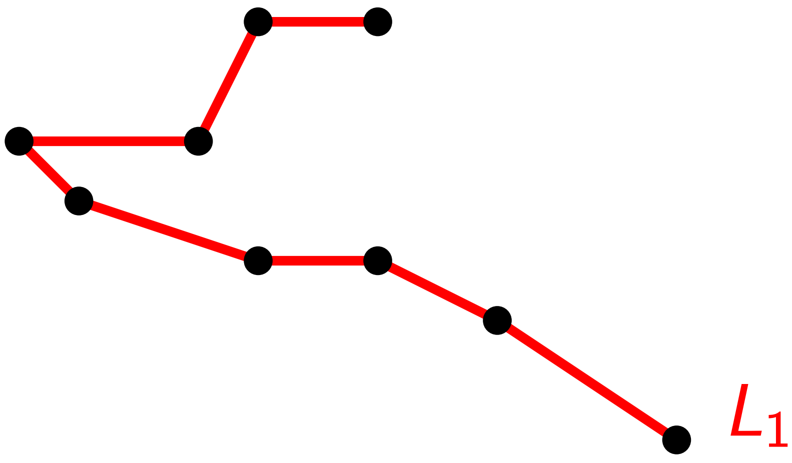

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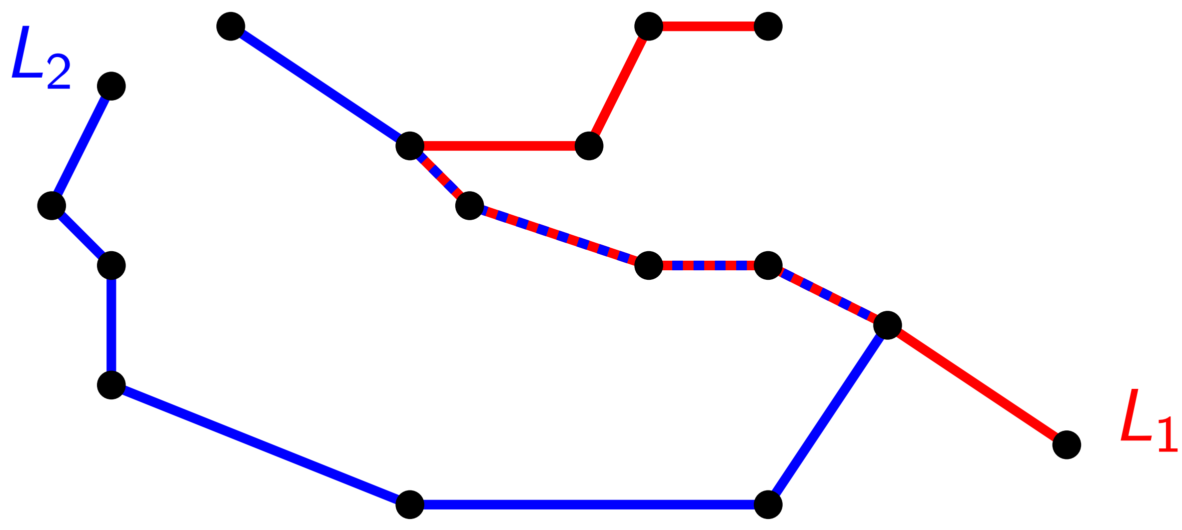

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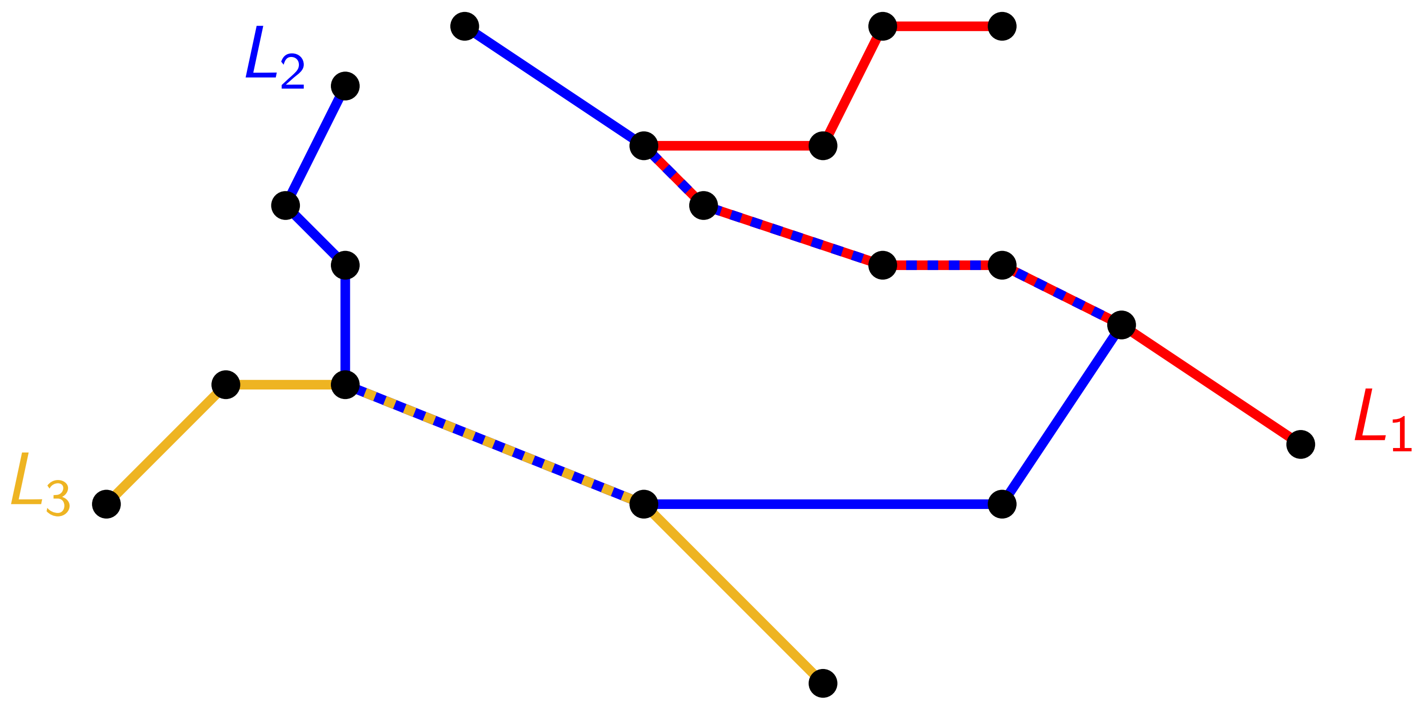

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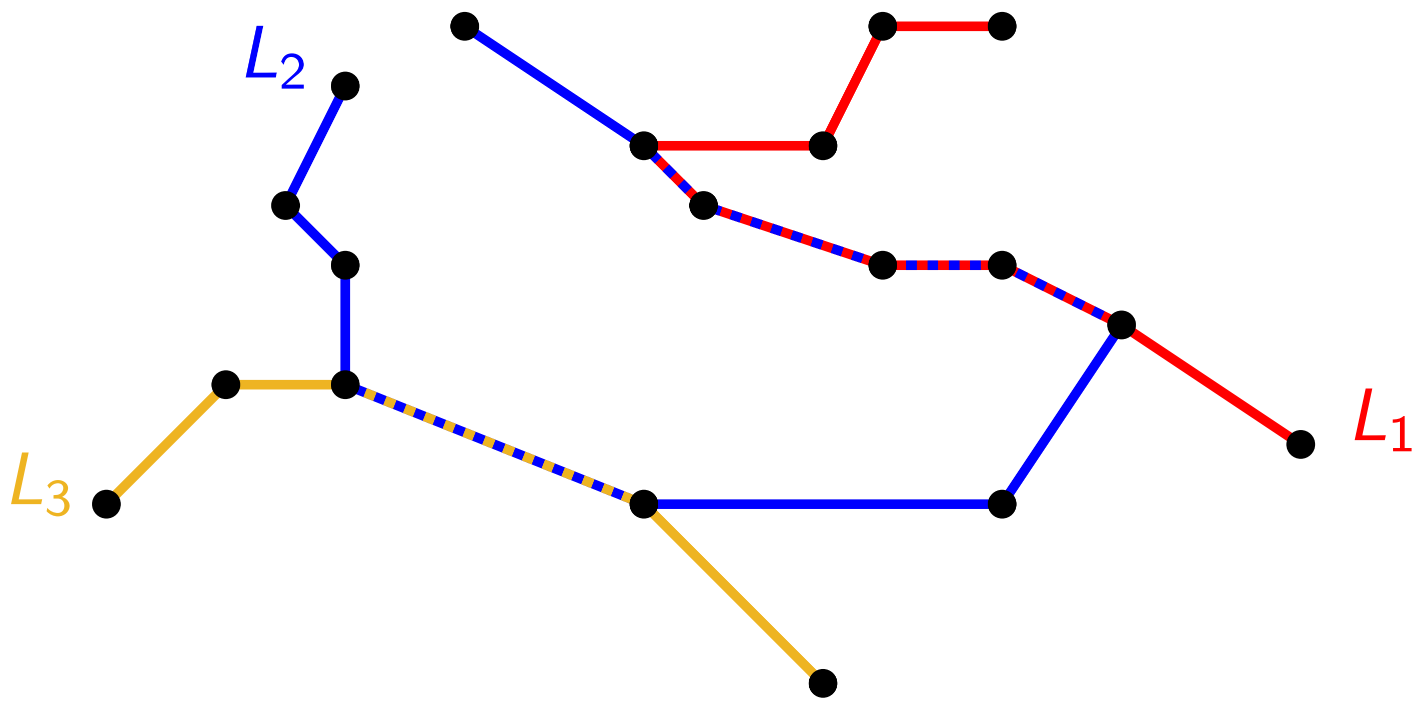

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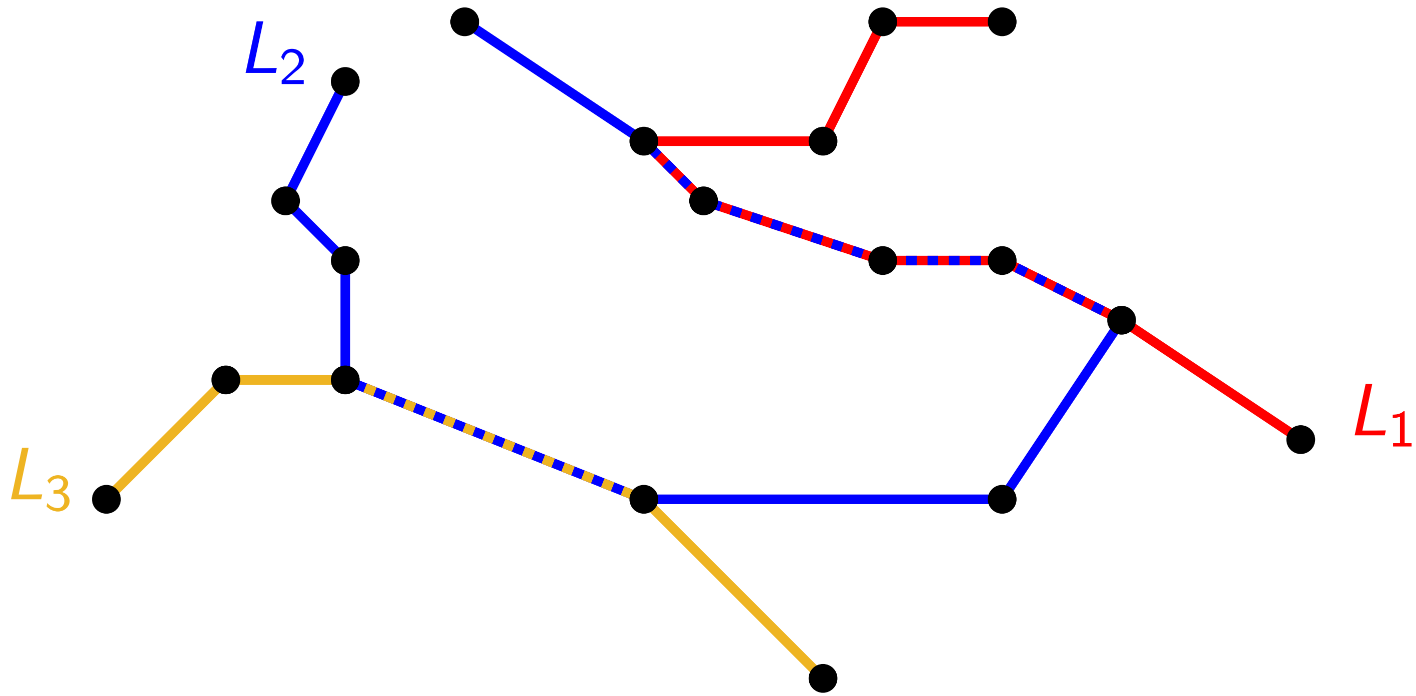

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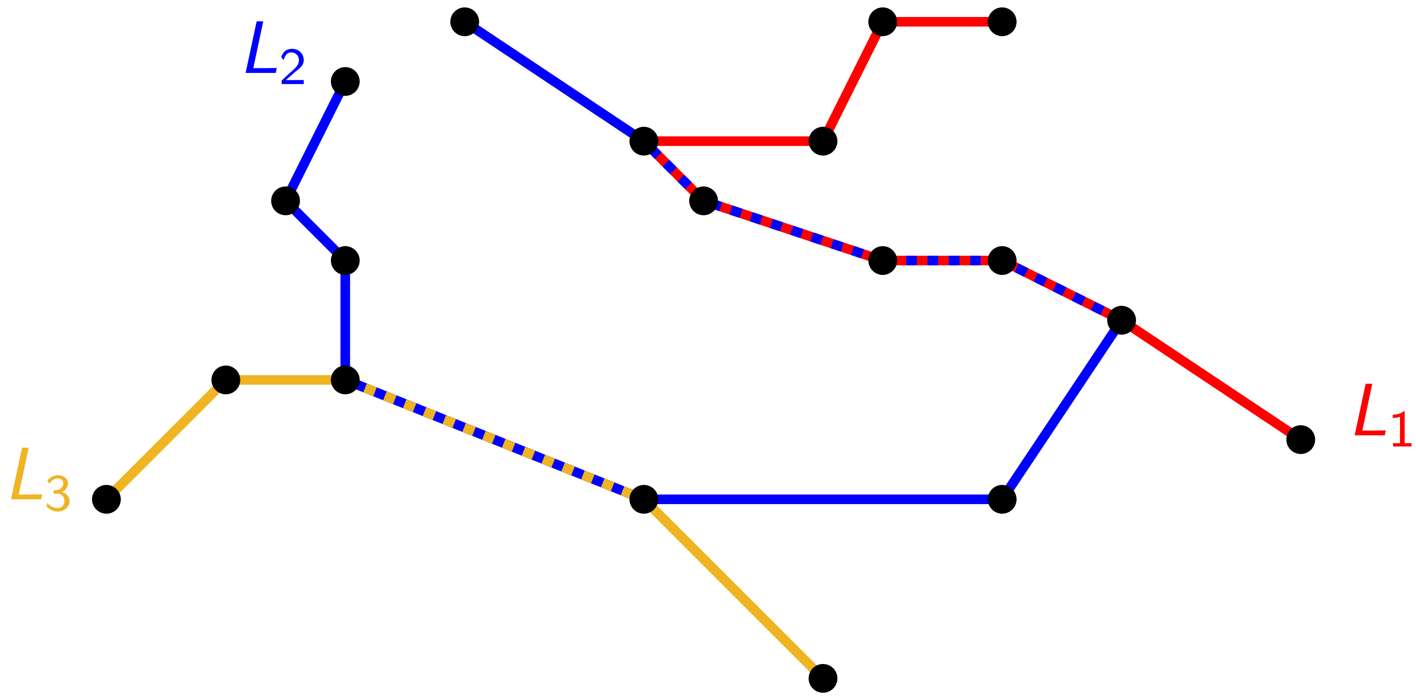

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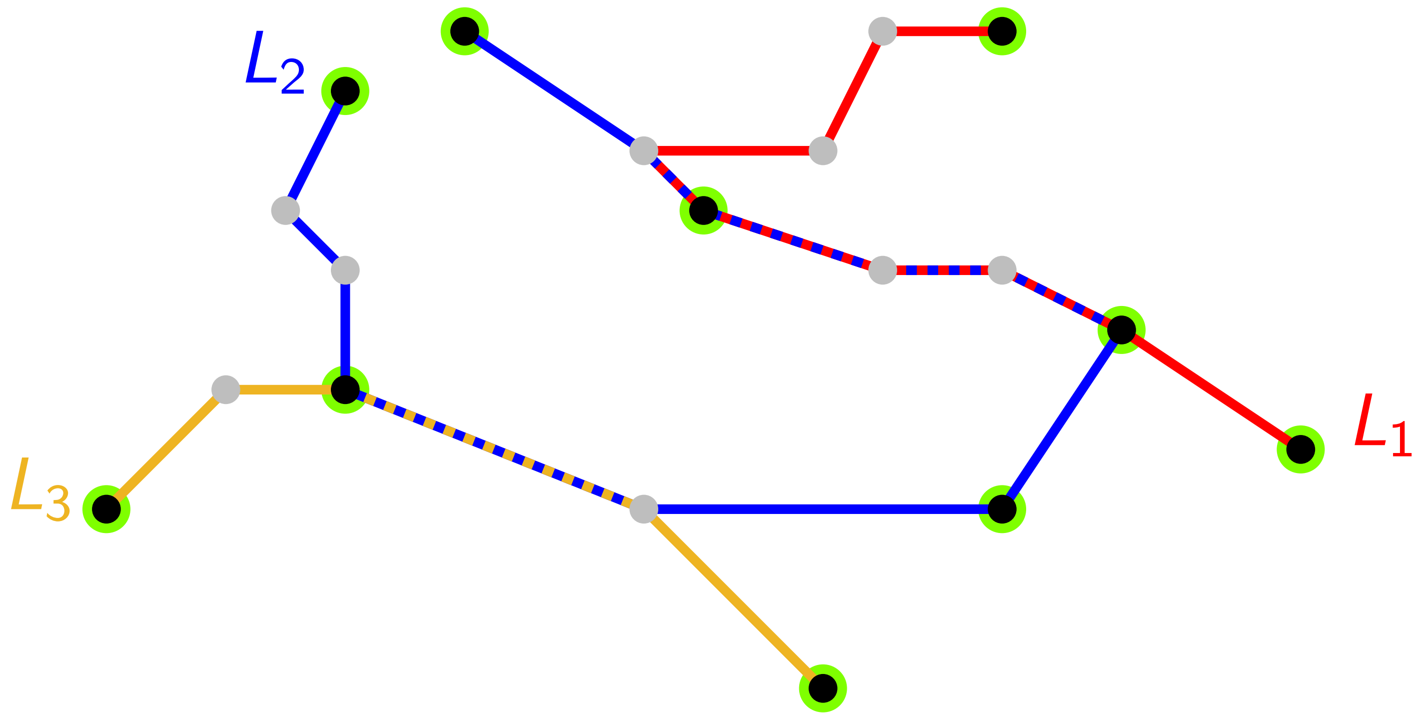

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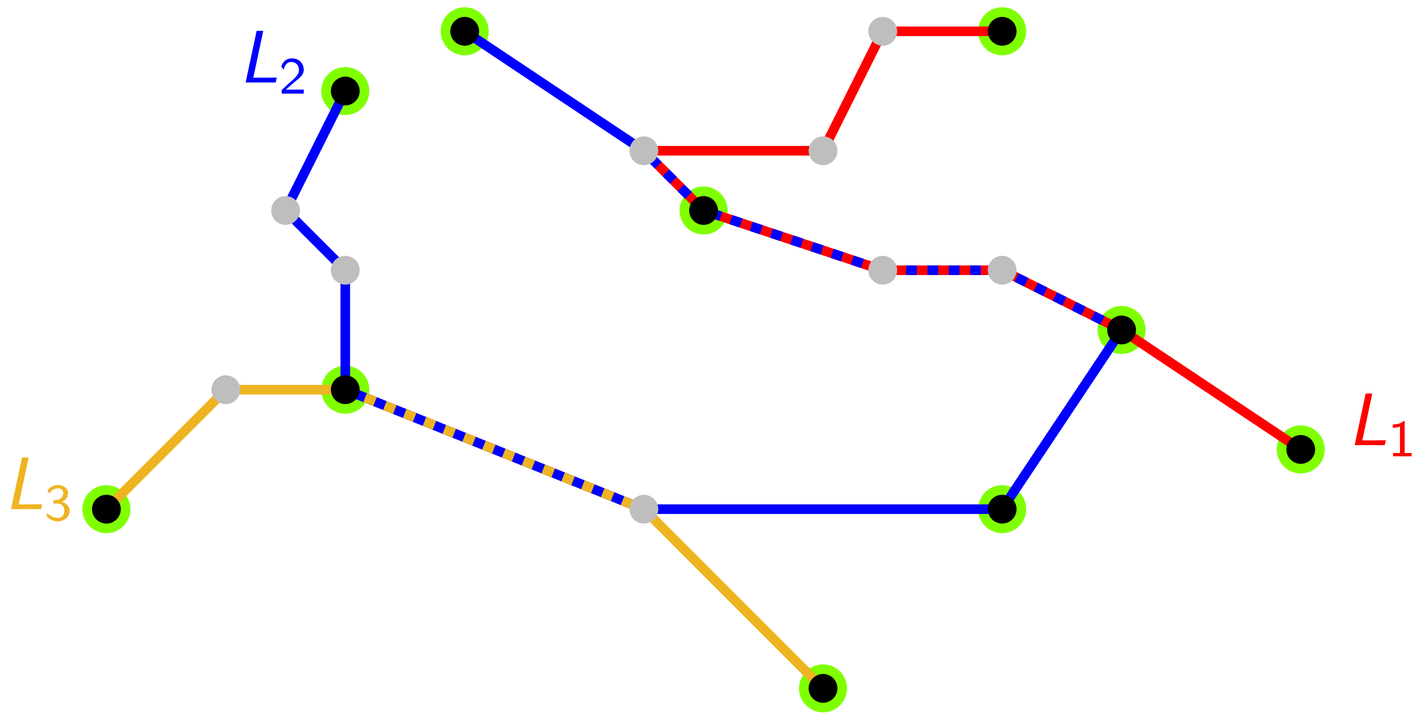

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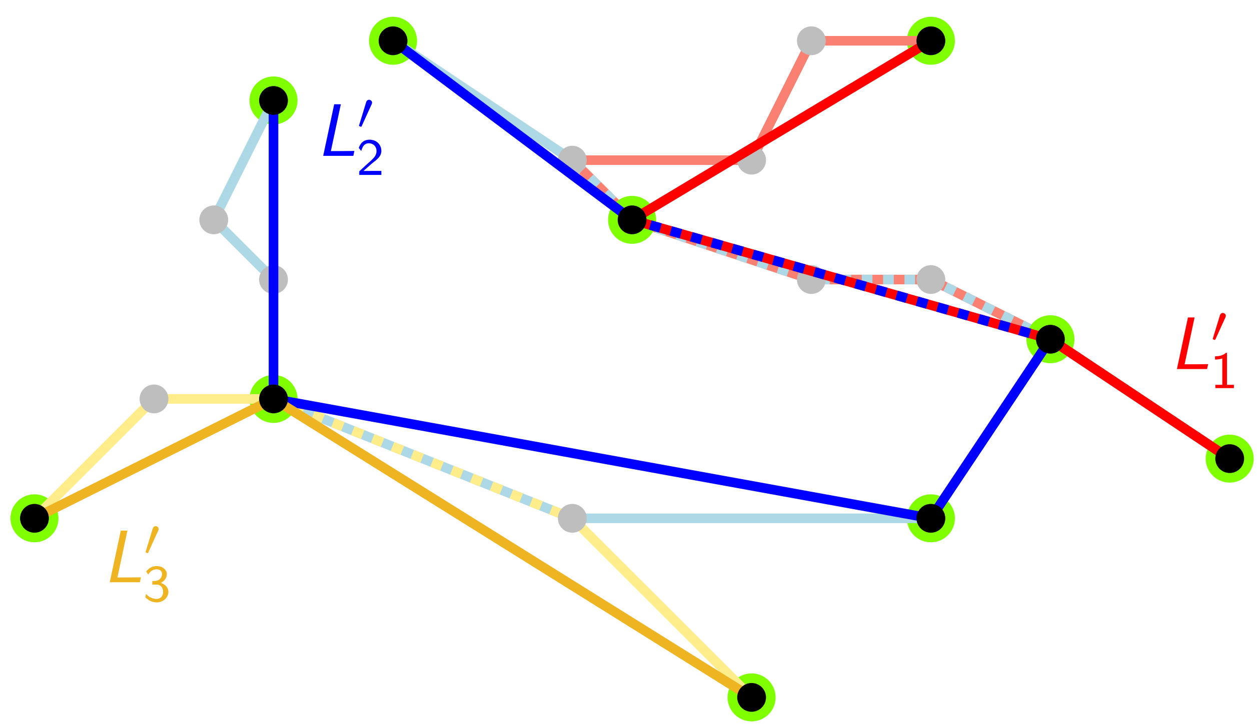

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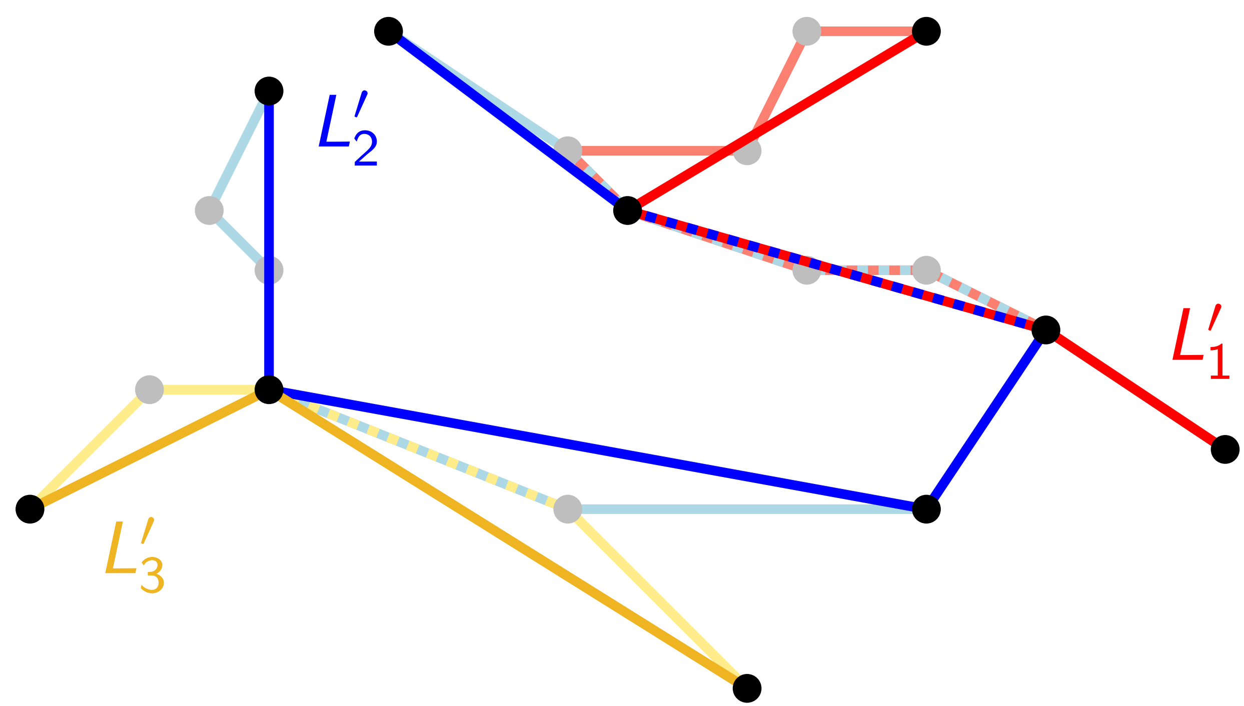

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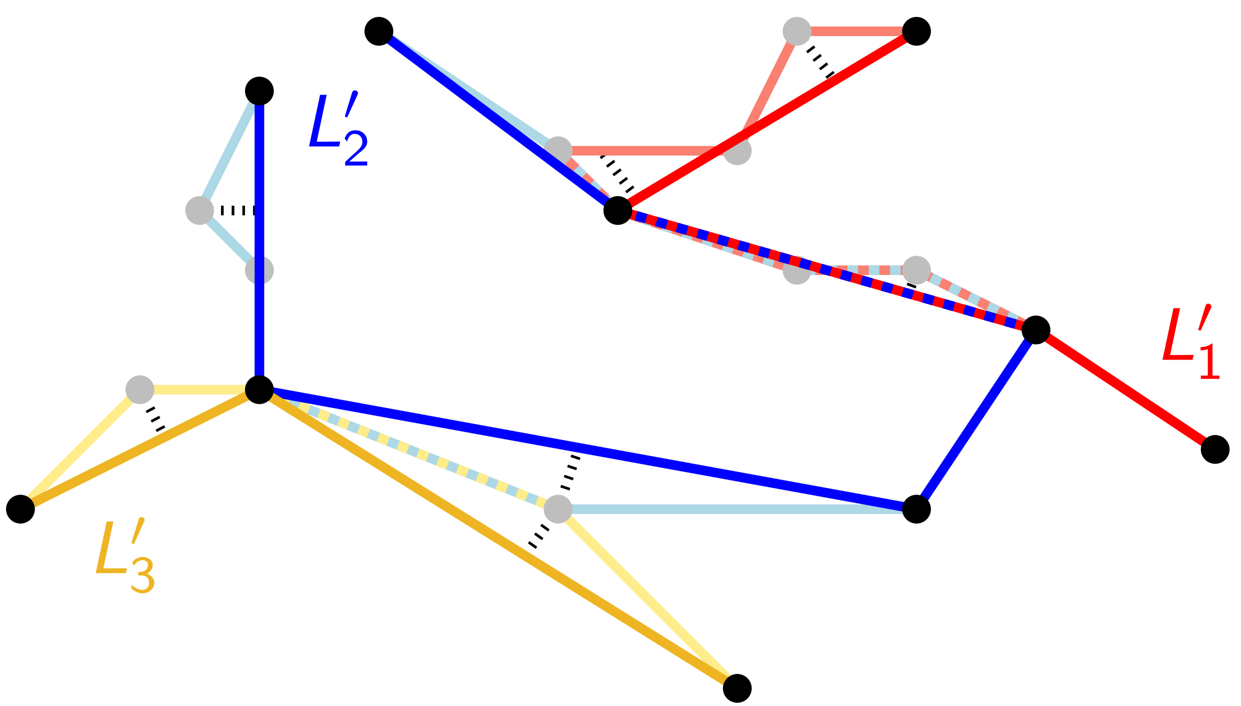

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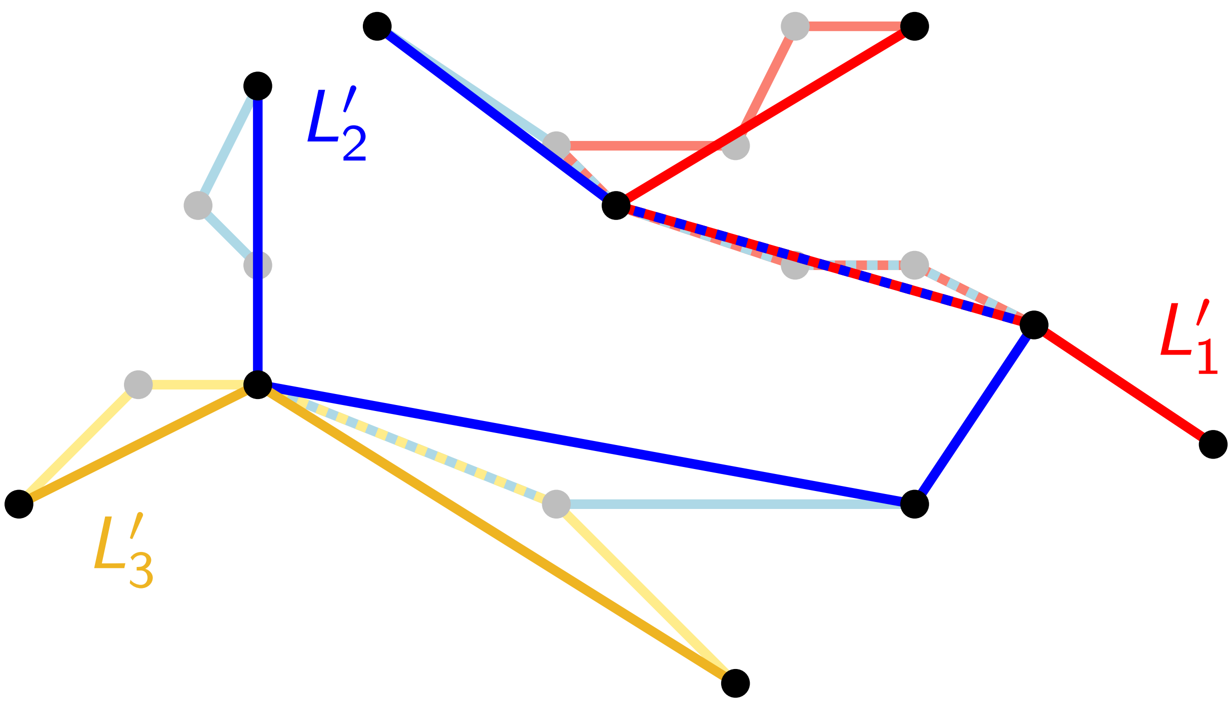

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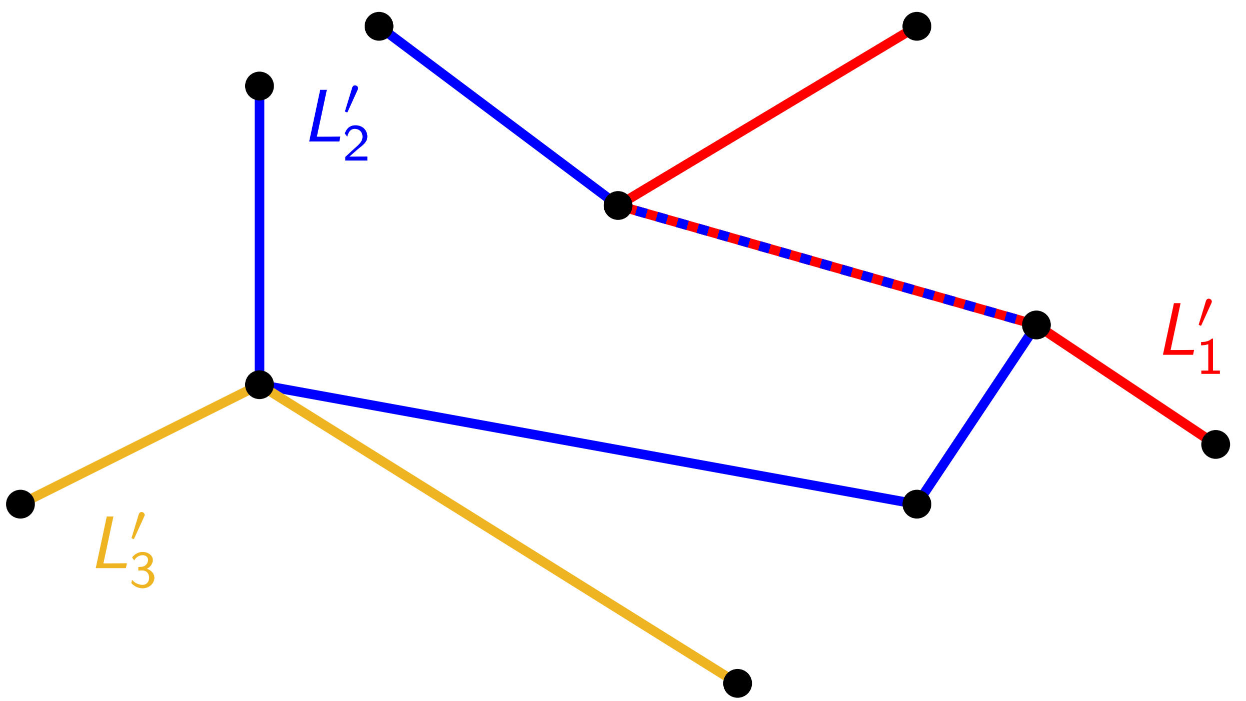

Agenda

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3. **Hardness of Approximation (+ Proof Sketch)**
4. Bi-Criteria Approximation (+ Proof Sketch)
5. Summary

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Theorem 1:
Simplifying a bundle of polylines is NP-hard to approximate within a factor of $n^{1/3-\varepsilon}$ for any $\varepsilon > 0$ even for 2 polylines.

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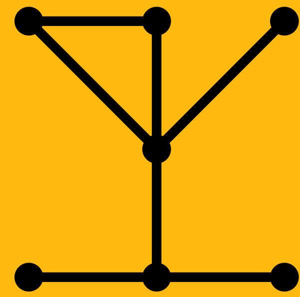

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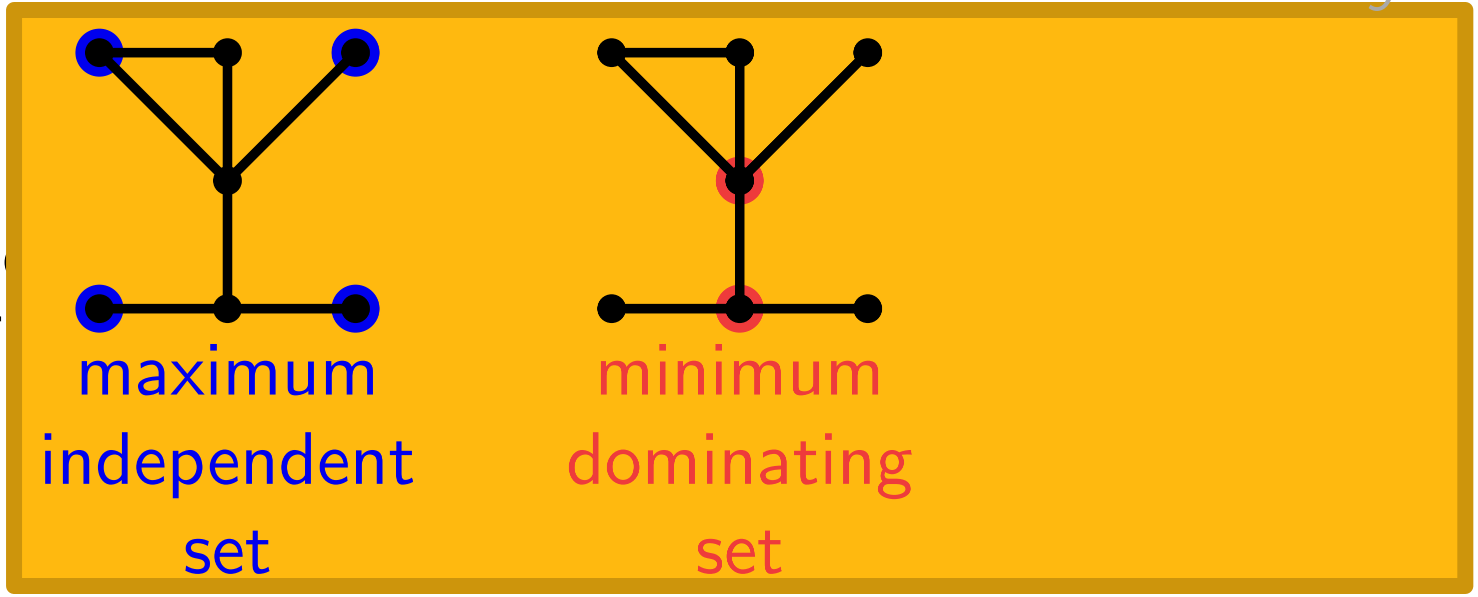

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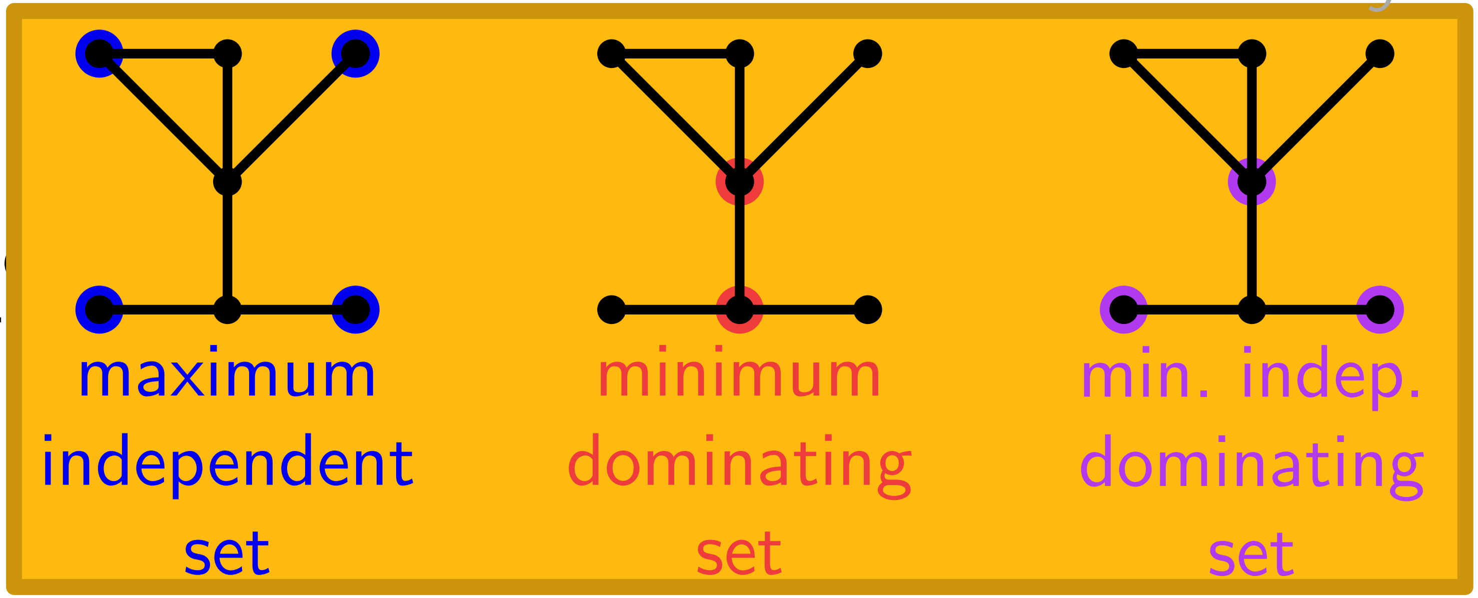

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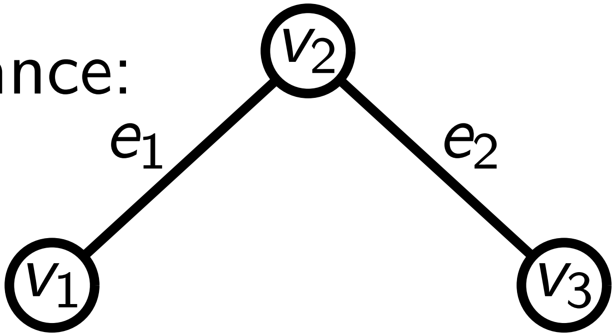
- Connecting all vertex gadgets and connecting all edge and neighborhood gadgets gives us 2 polylines

Full Example

MIDS instance:



$\hat{n} = 3$

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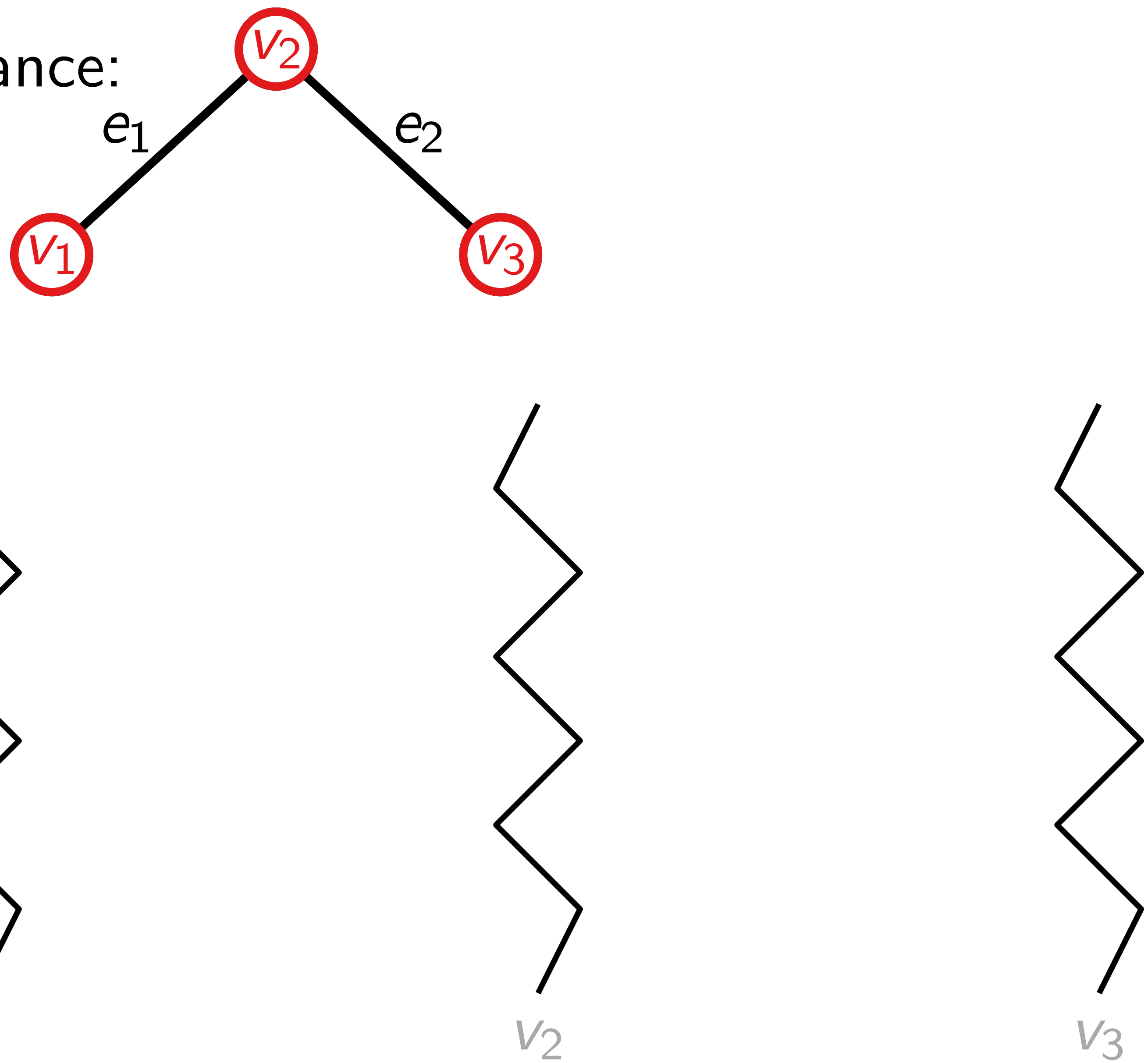

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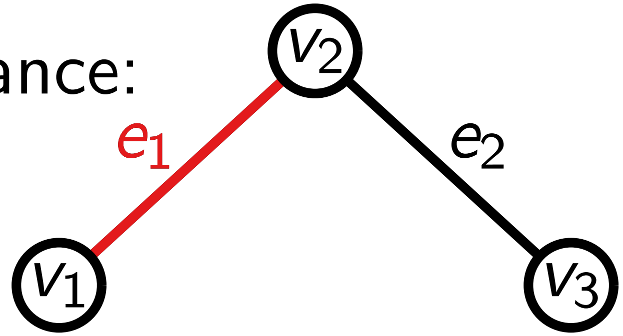

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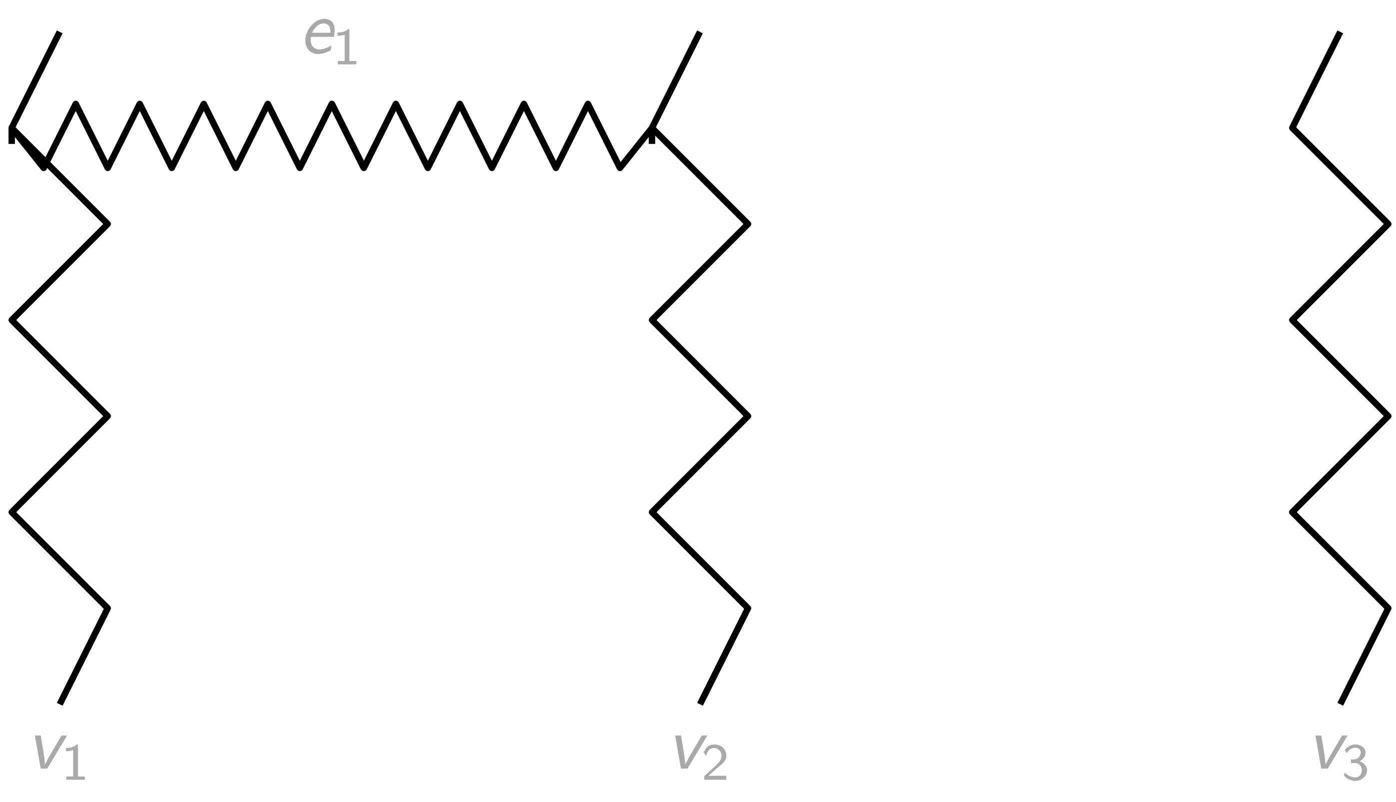

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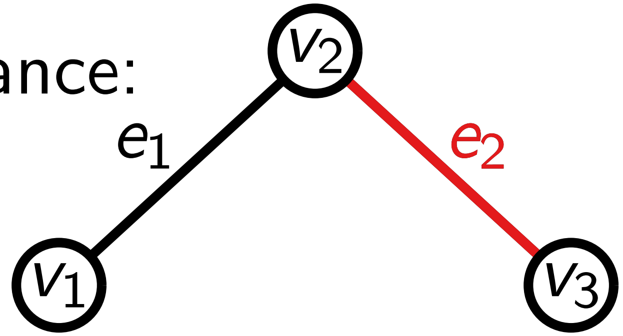

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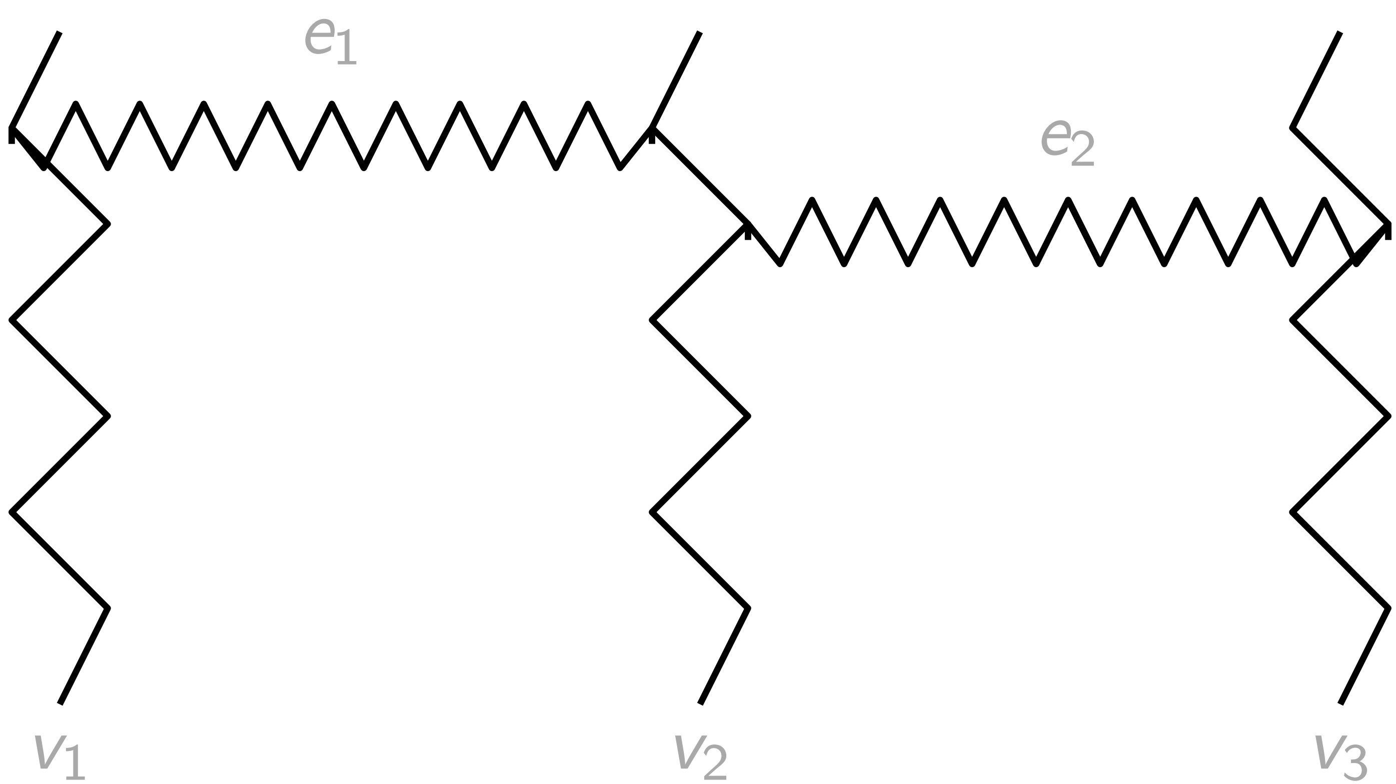

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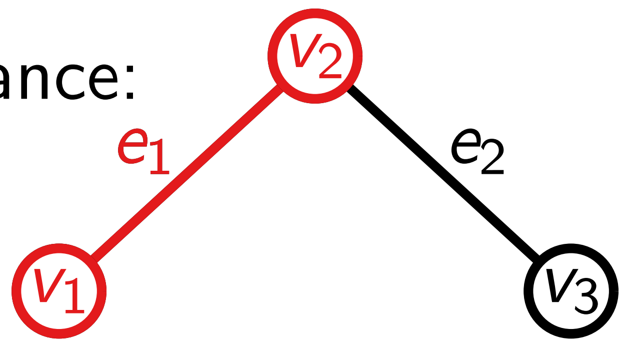

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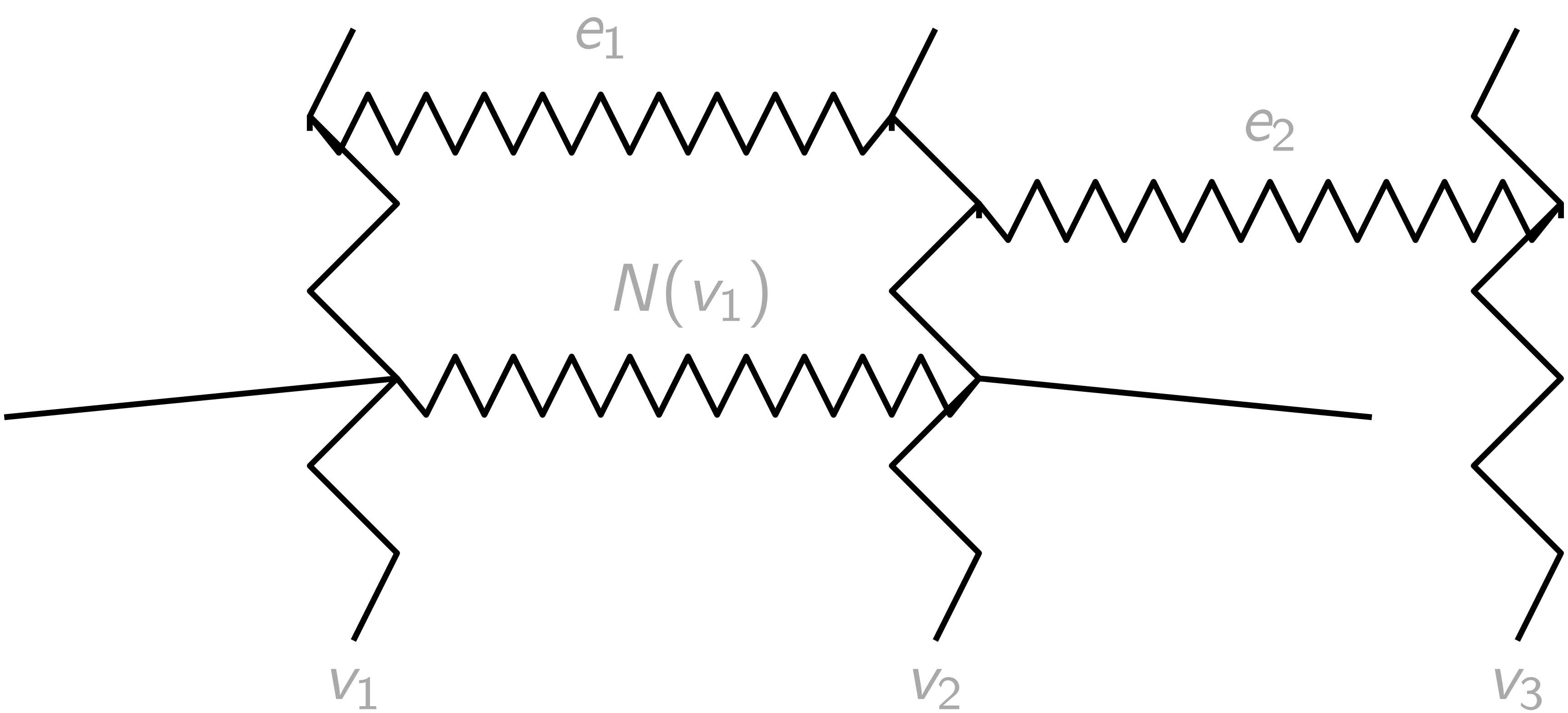

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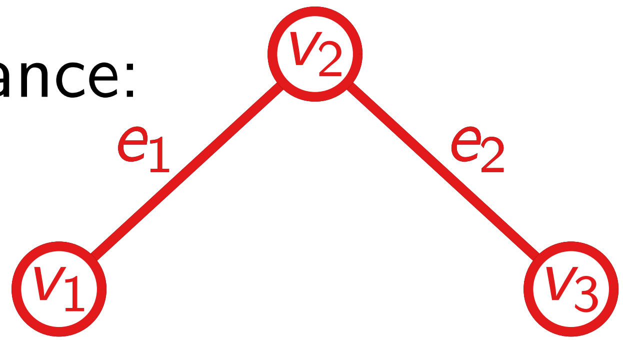

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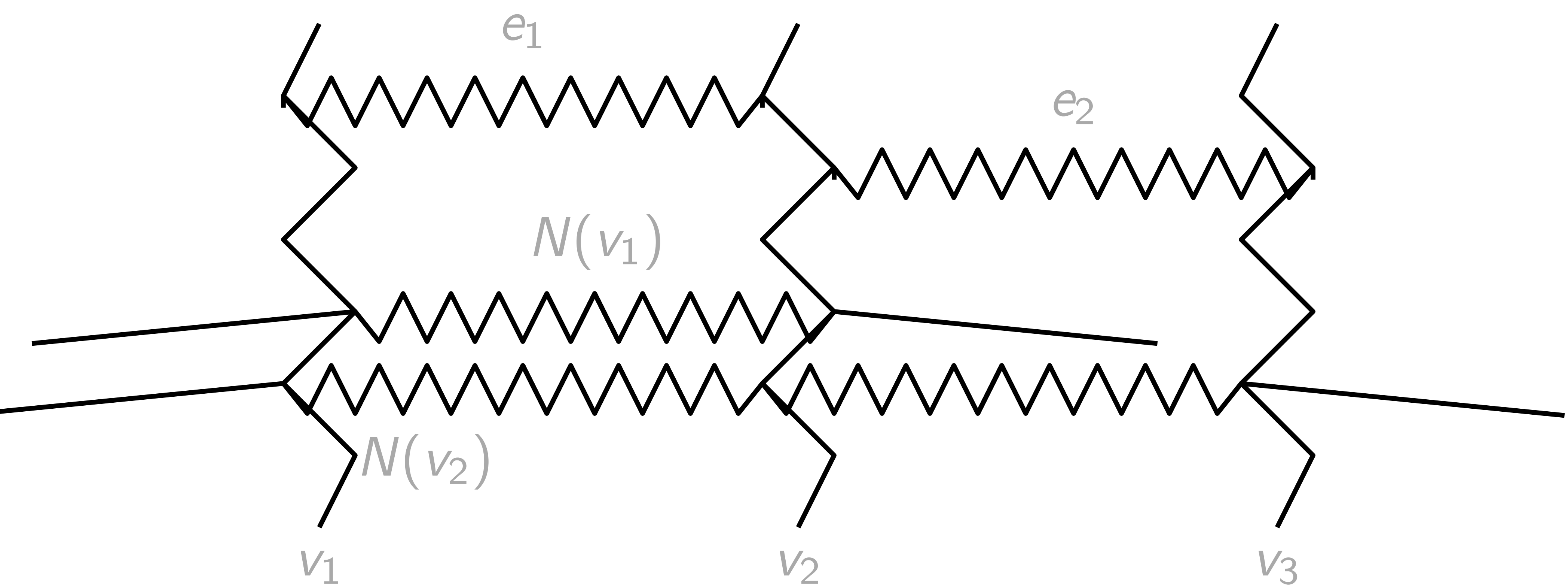

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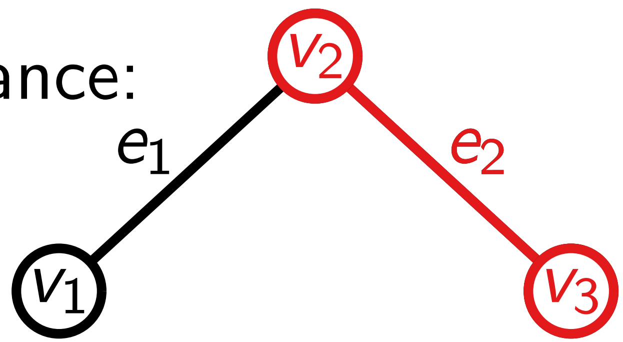

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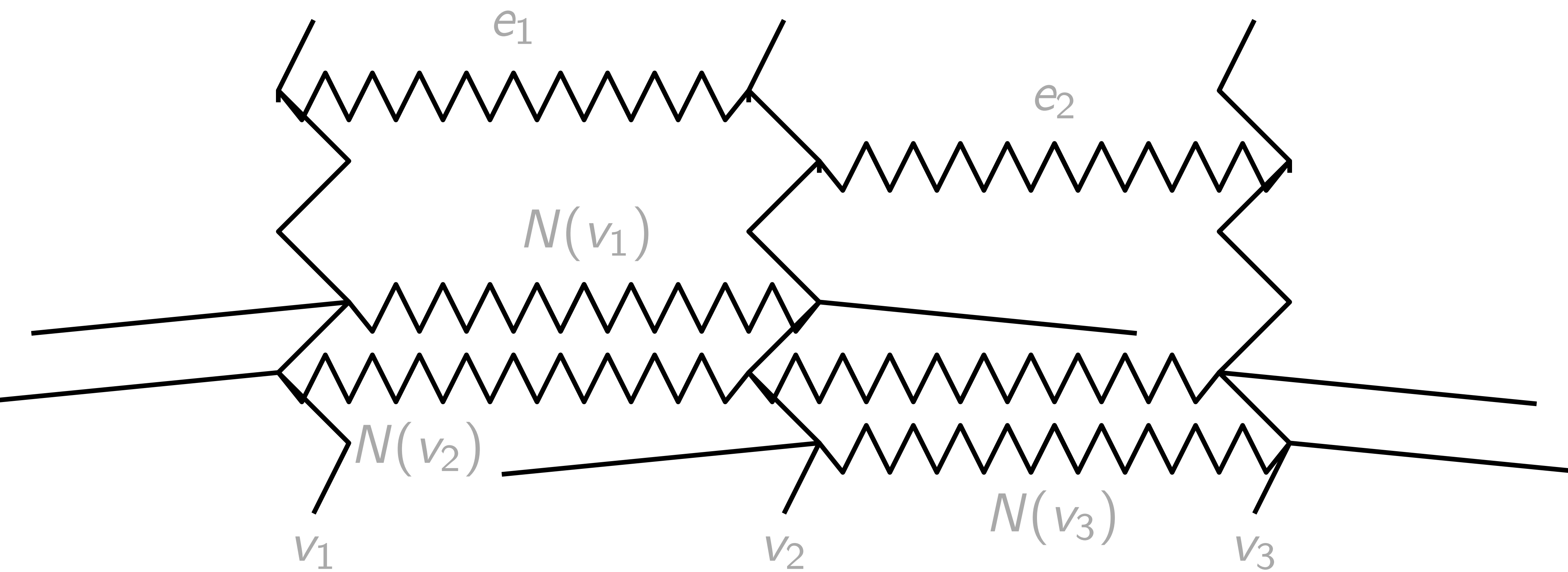

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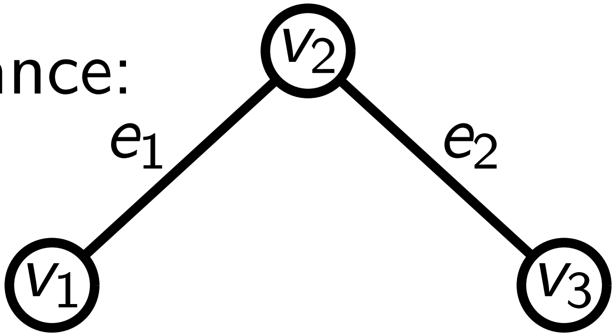

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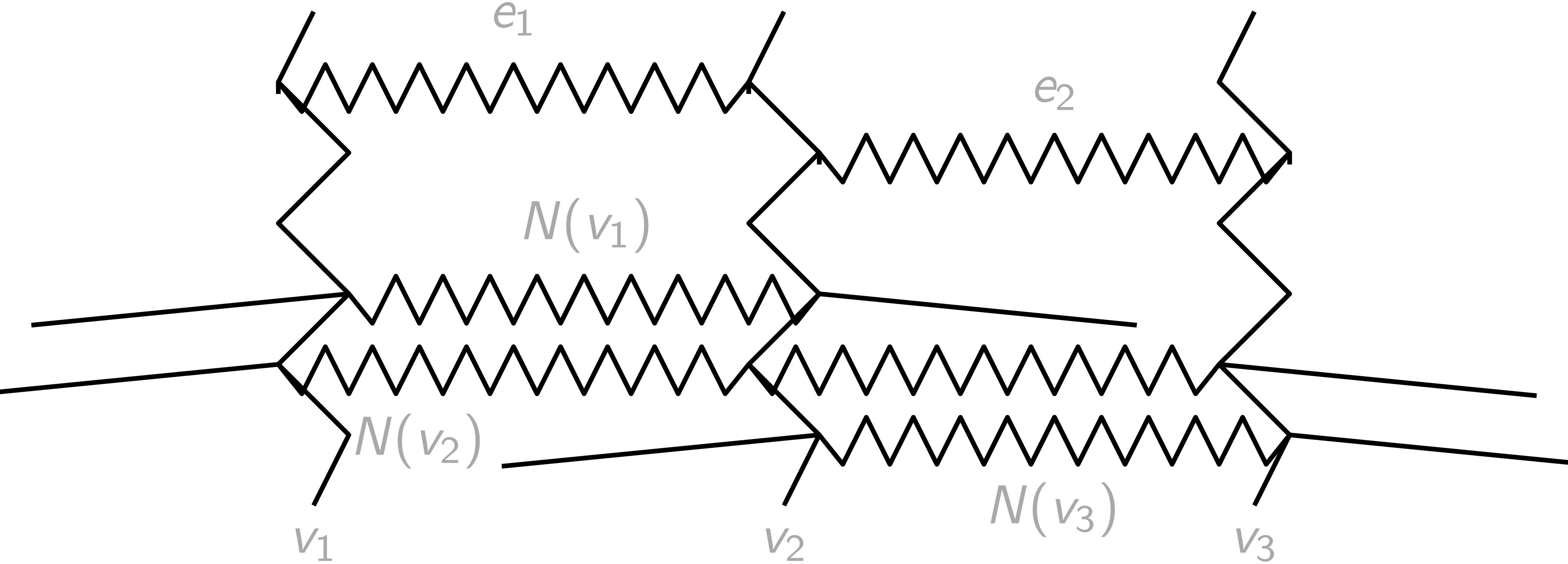
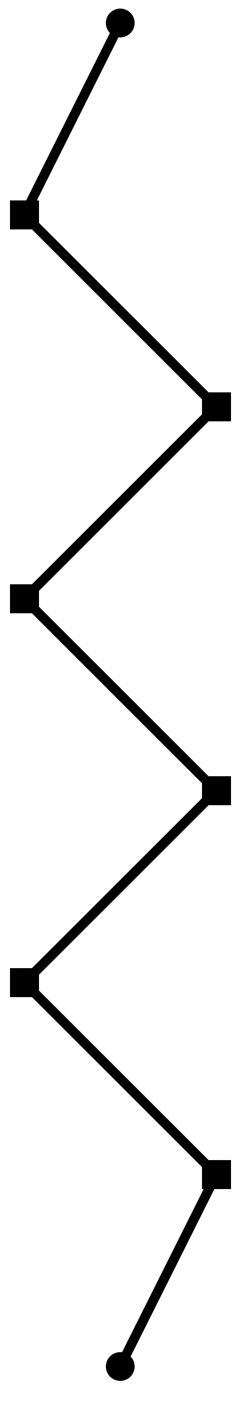


Vertex Gadget



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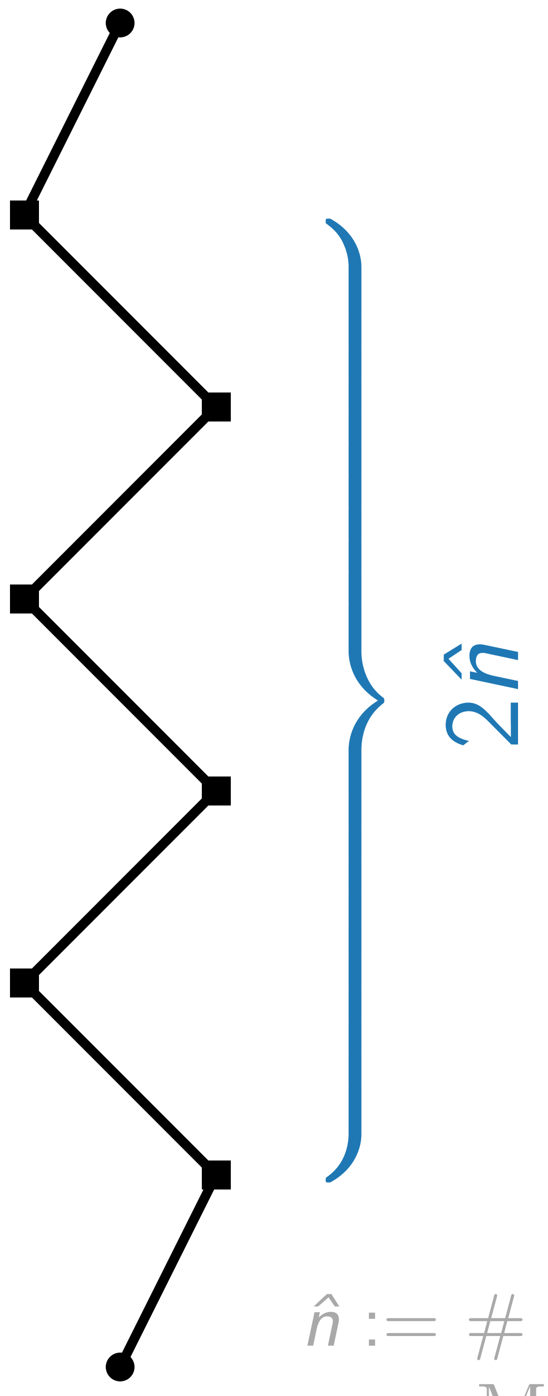
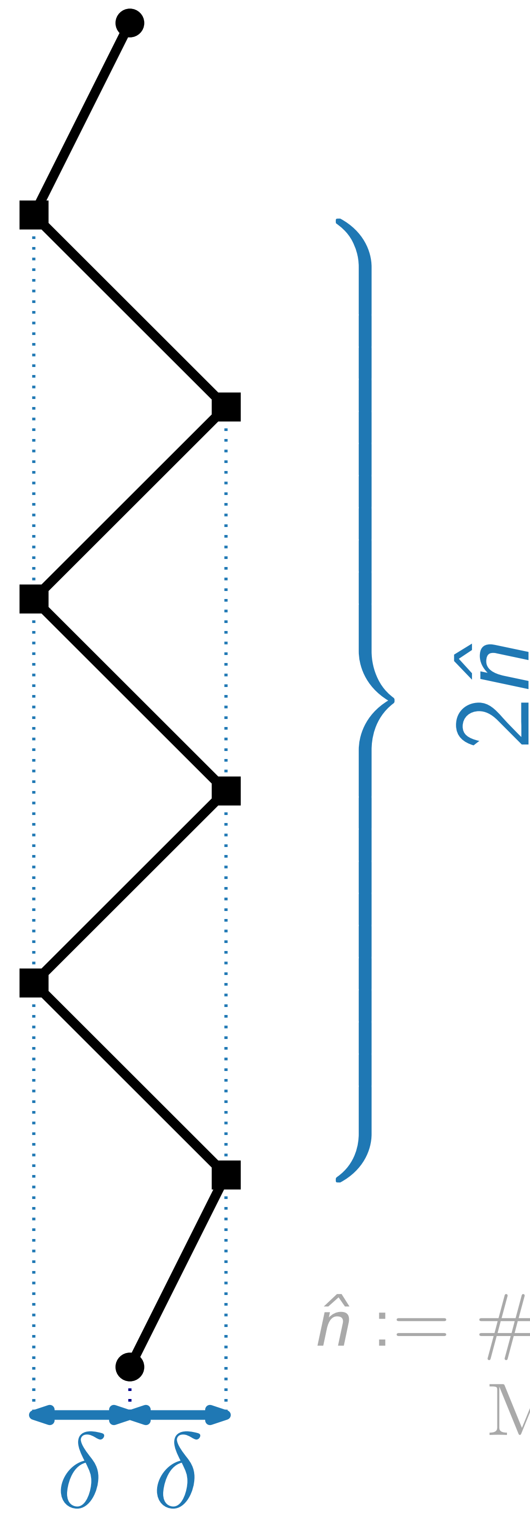


$\hat{n} := \#$ vertices in MIDS graph

Vertex Gadget



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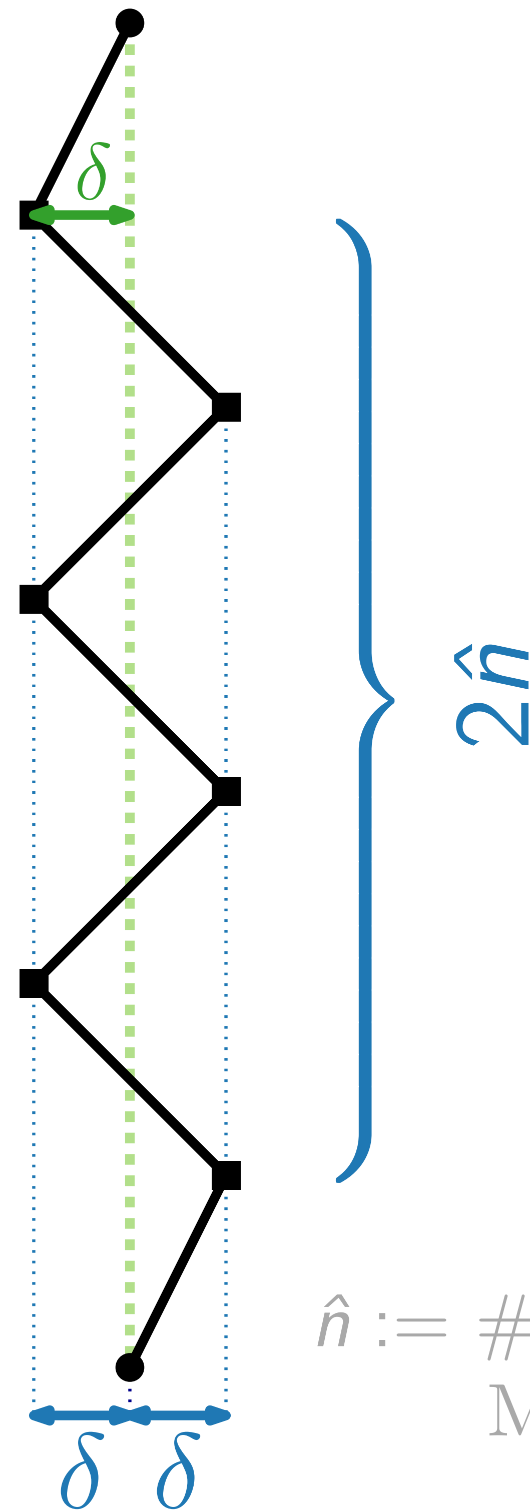

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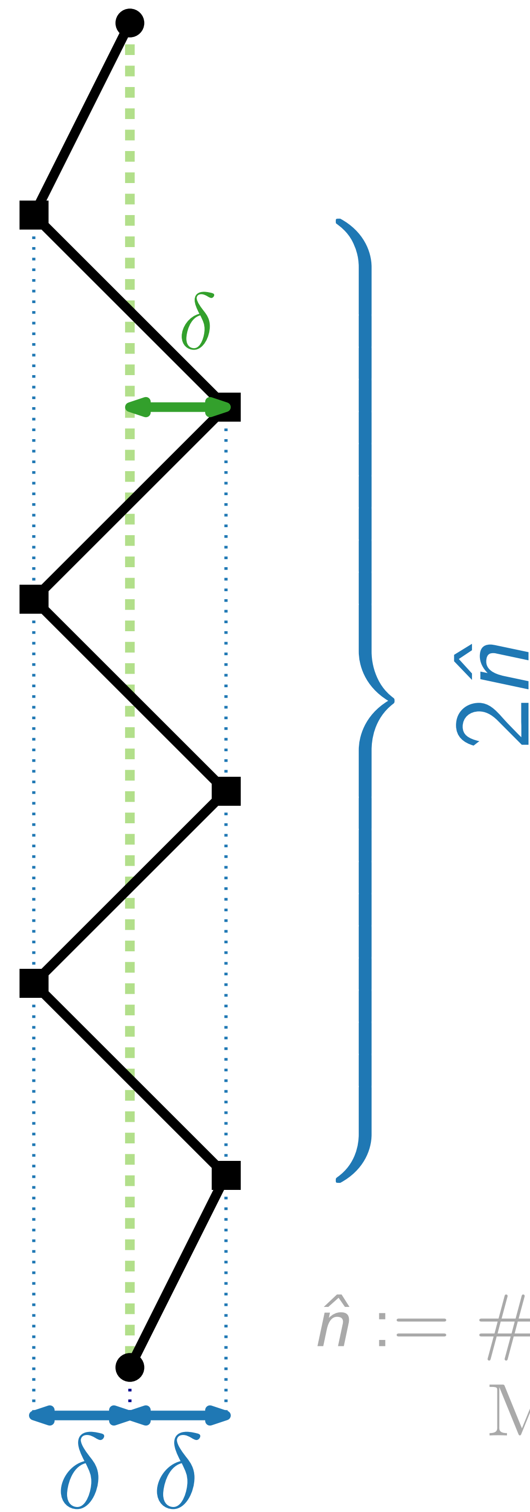

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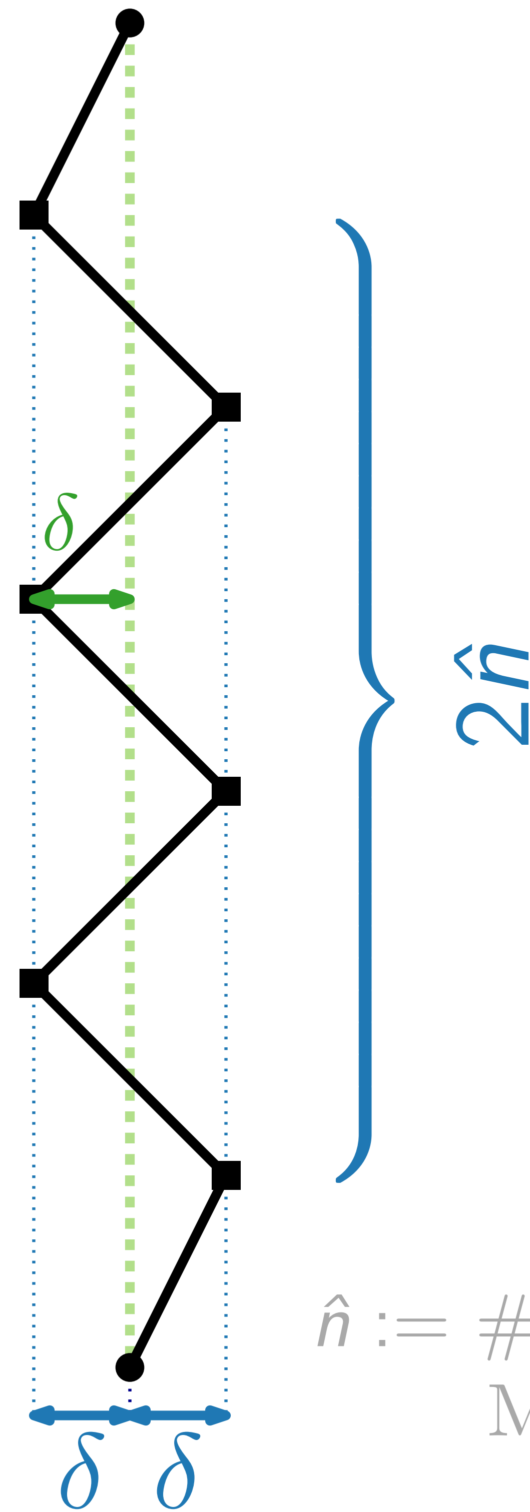

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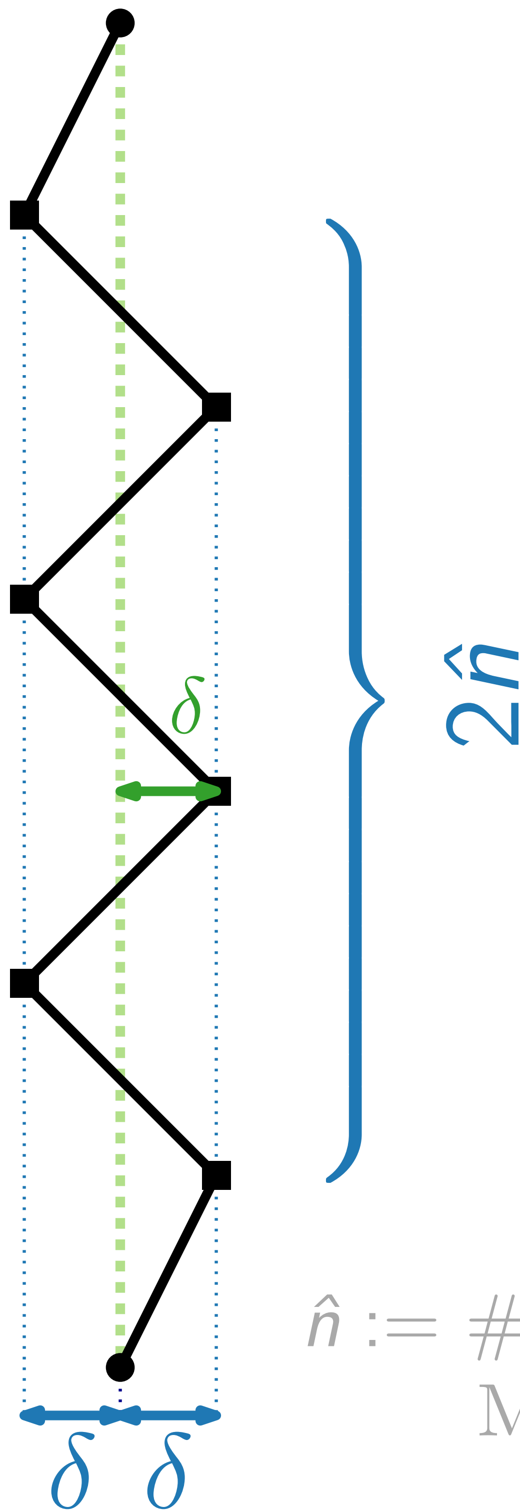

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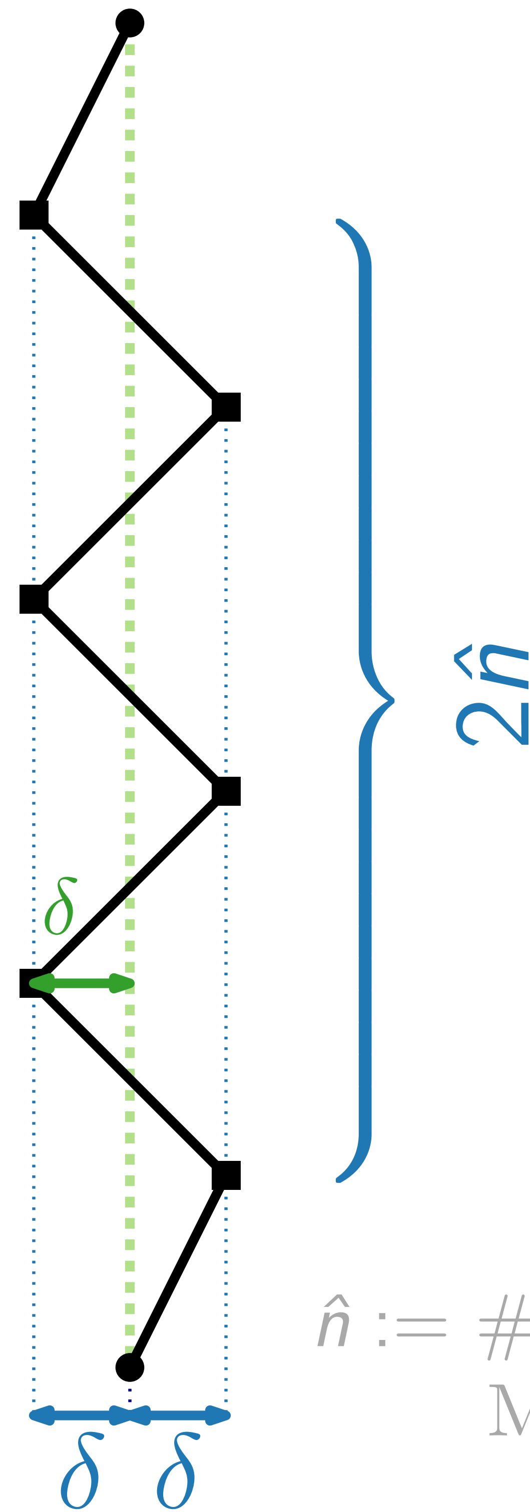

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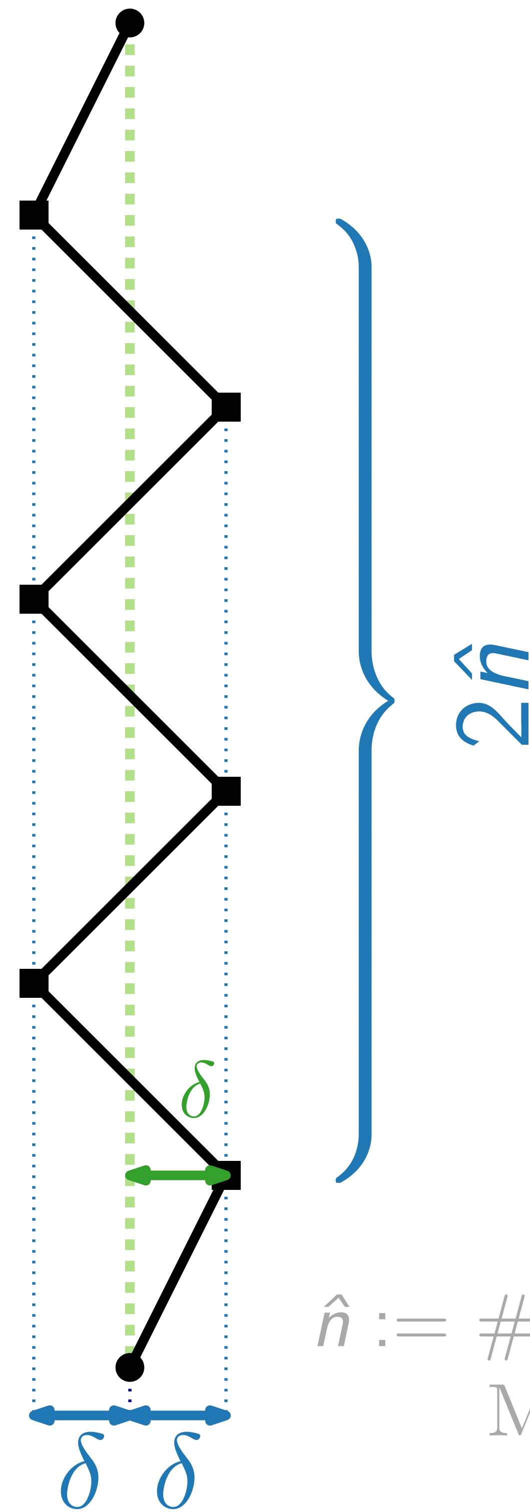

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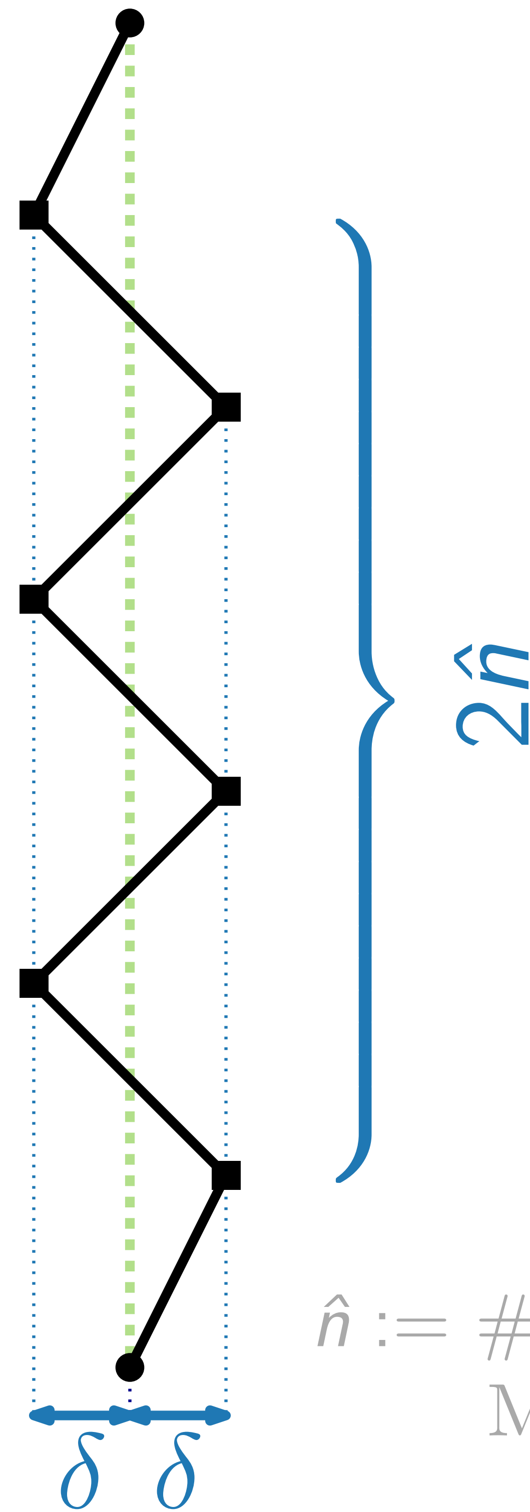

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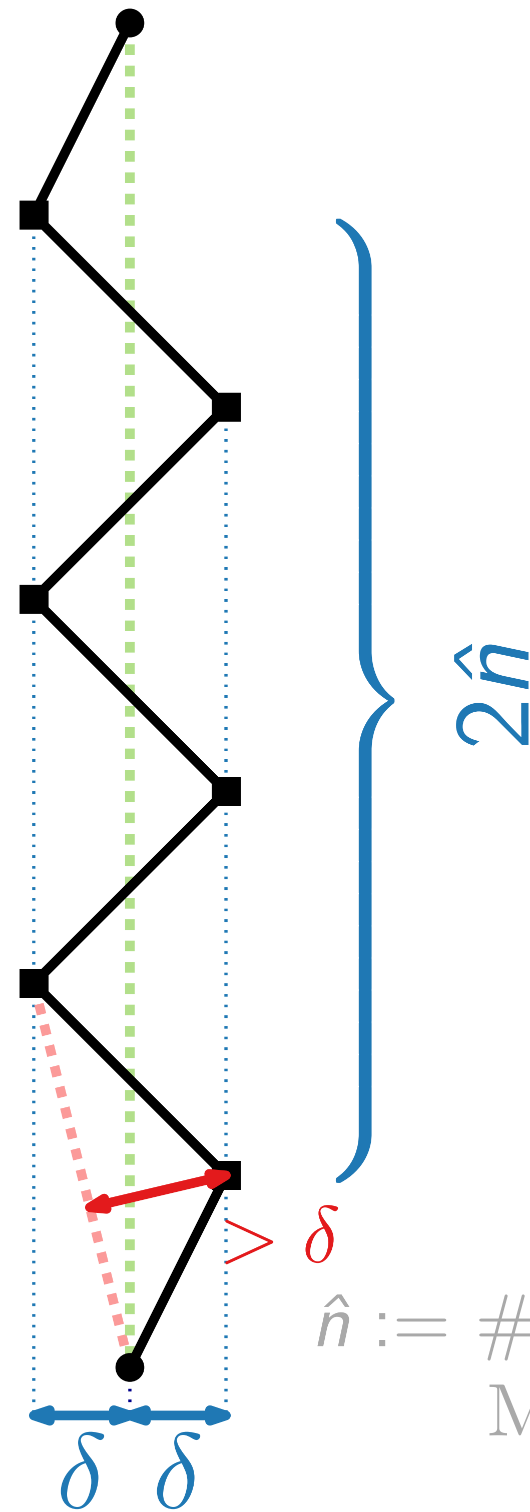

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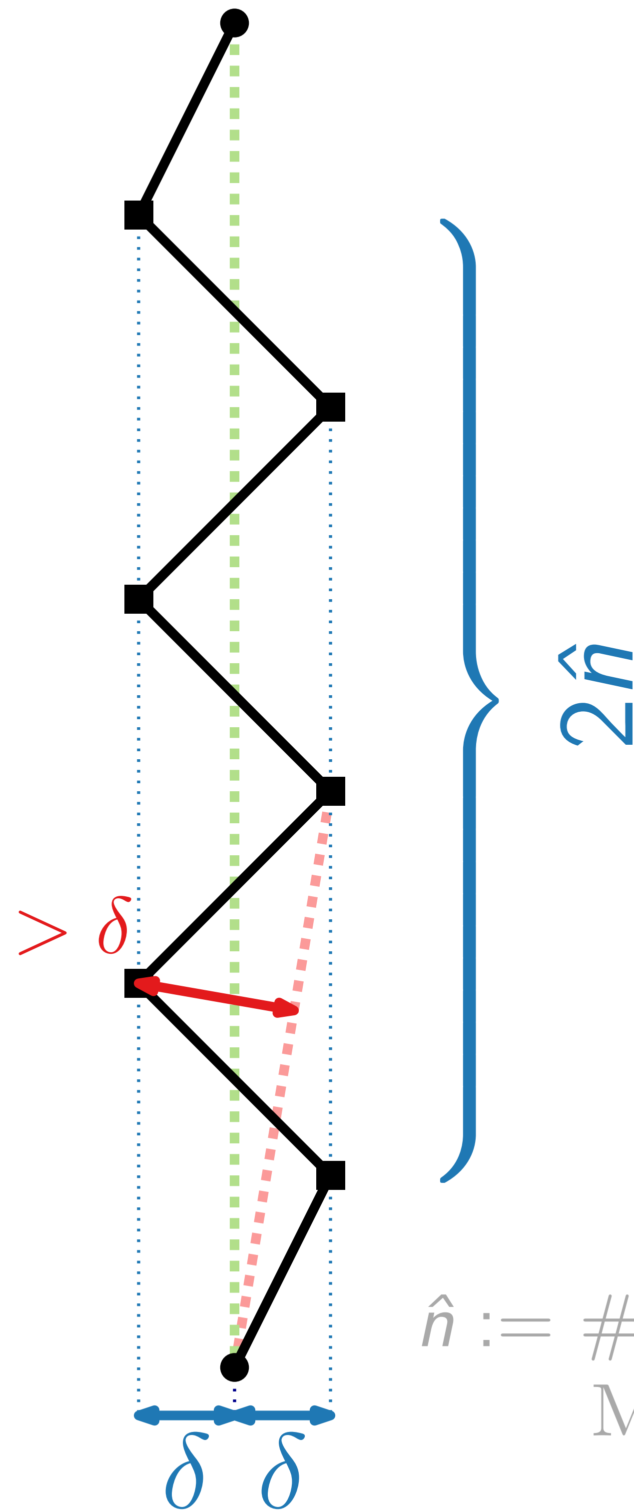

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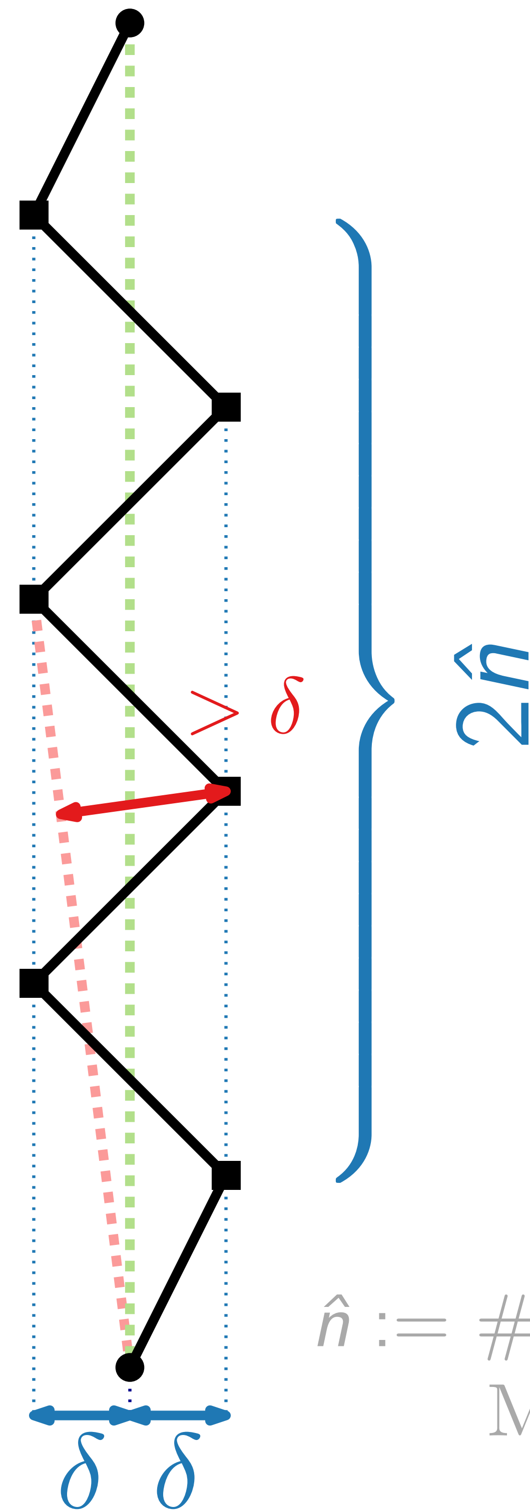

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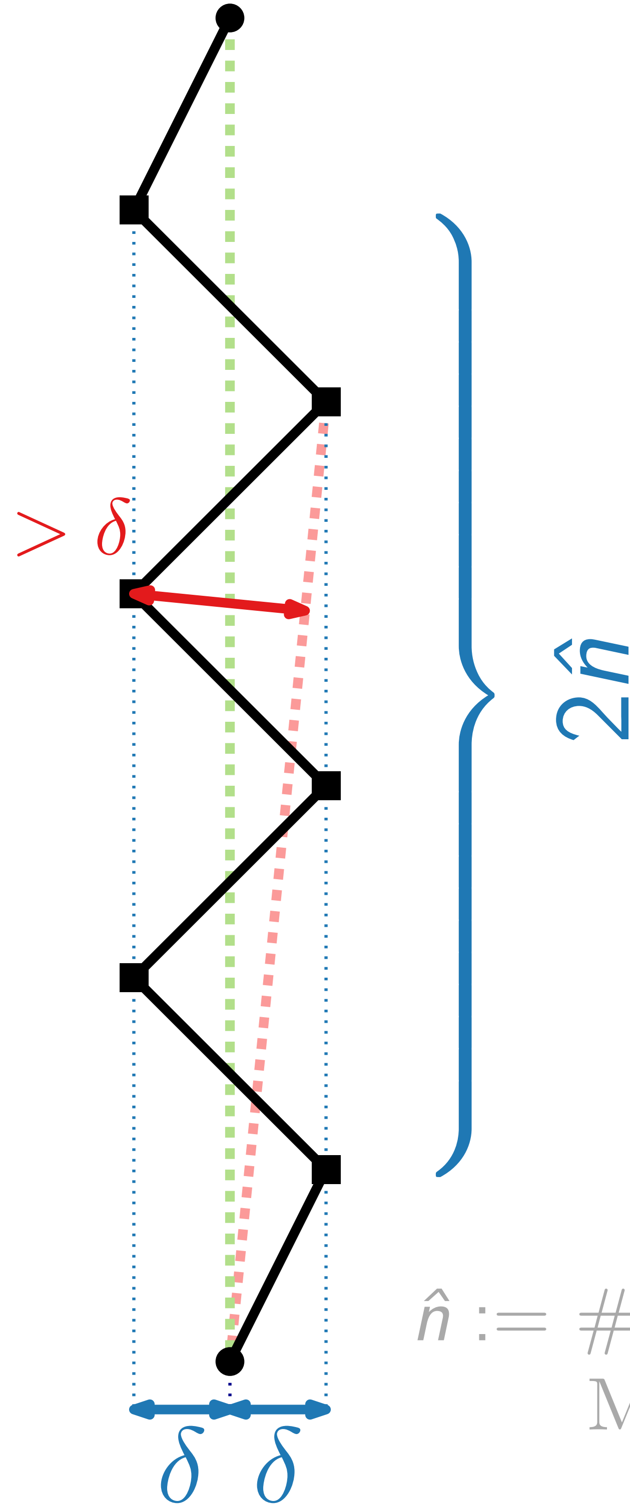

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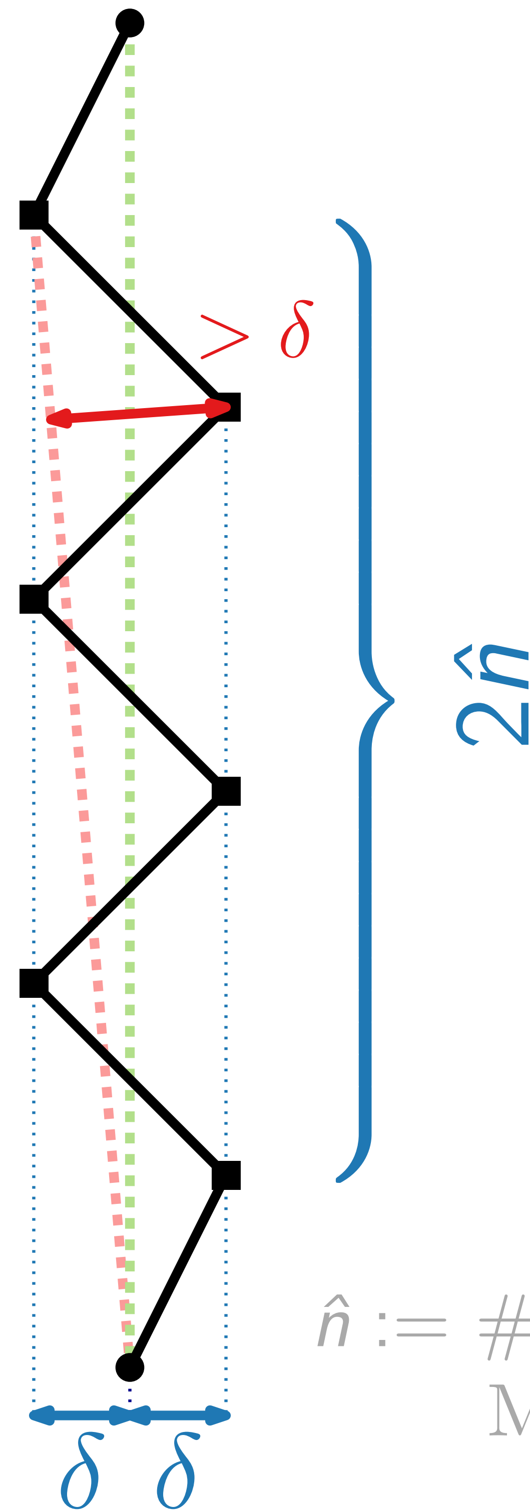

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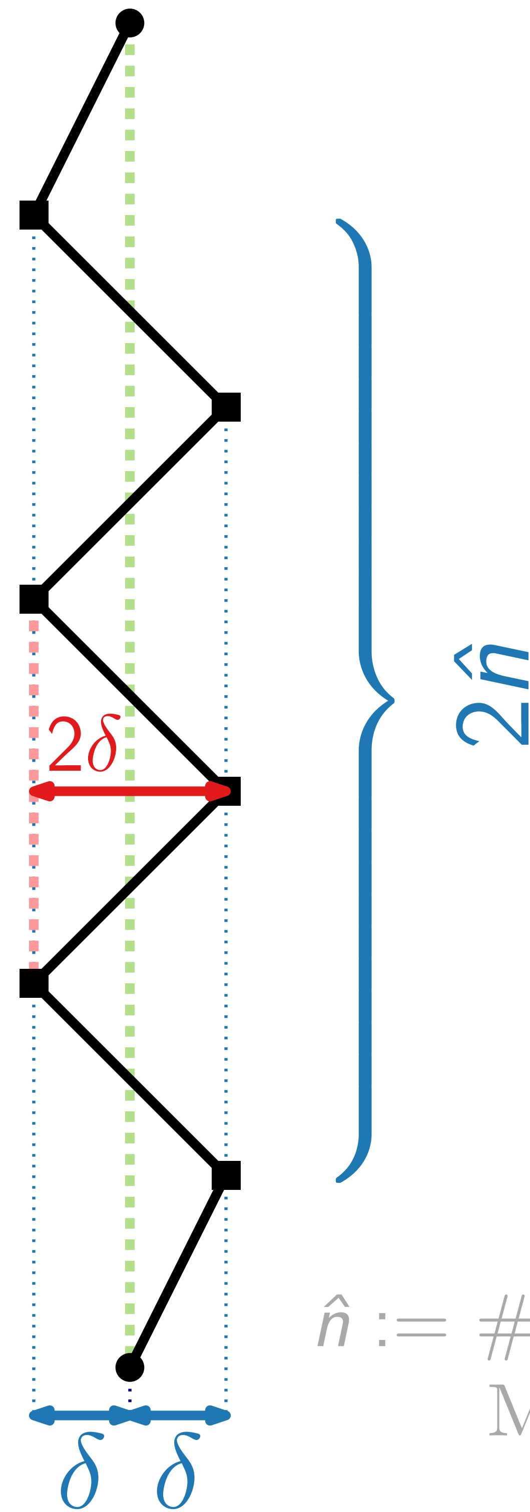

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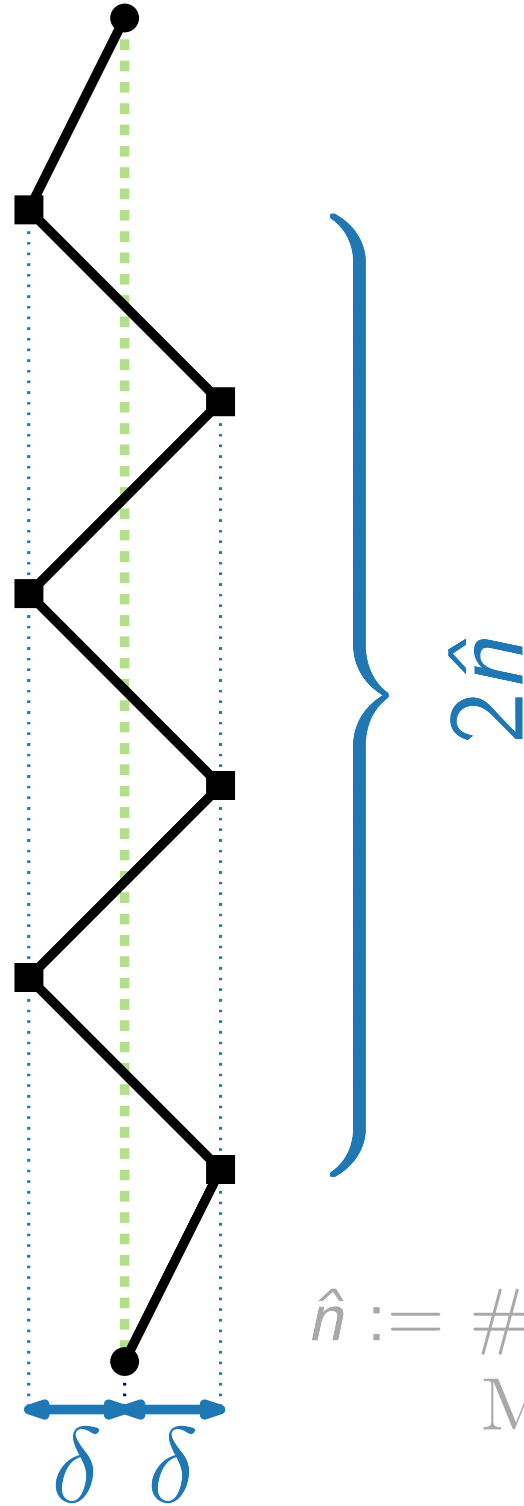
Vertex Gadget

Interpretation:



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Vertex Gadget

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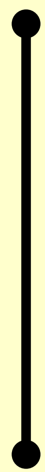
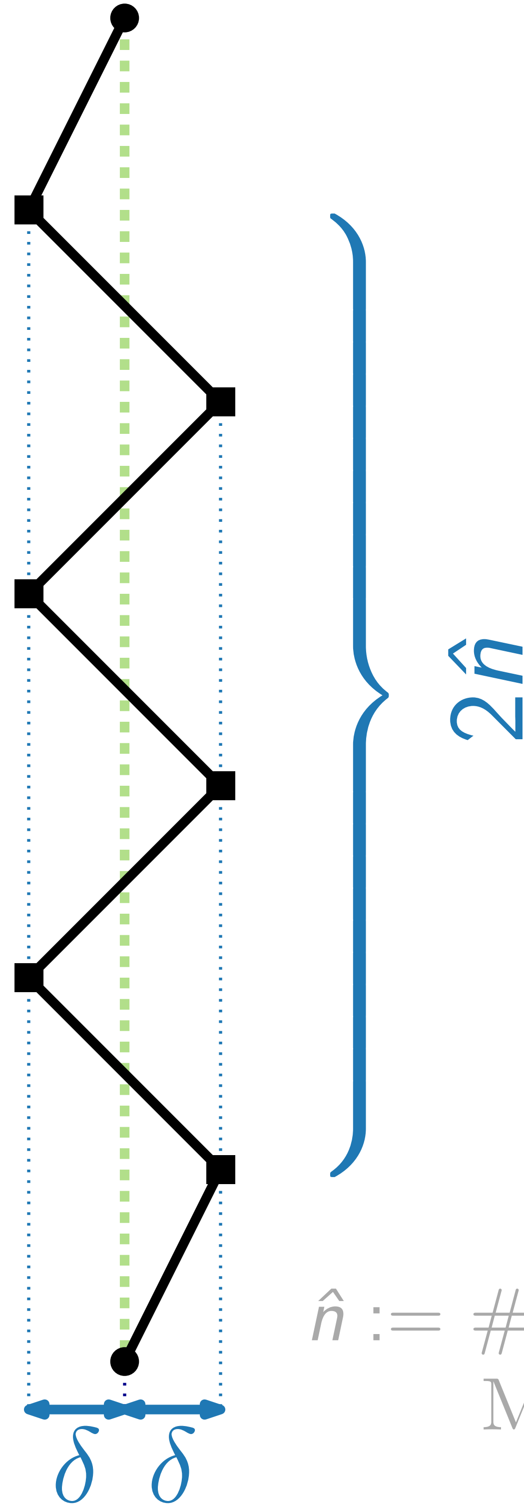


vertex is **not** in the MIDS



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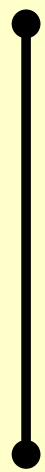

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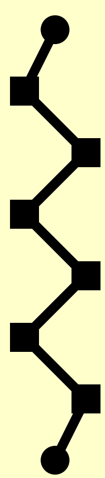

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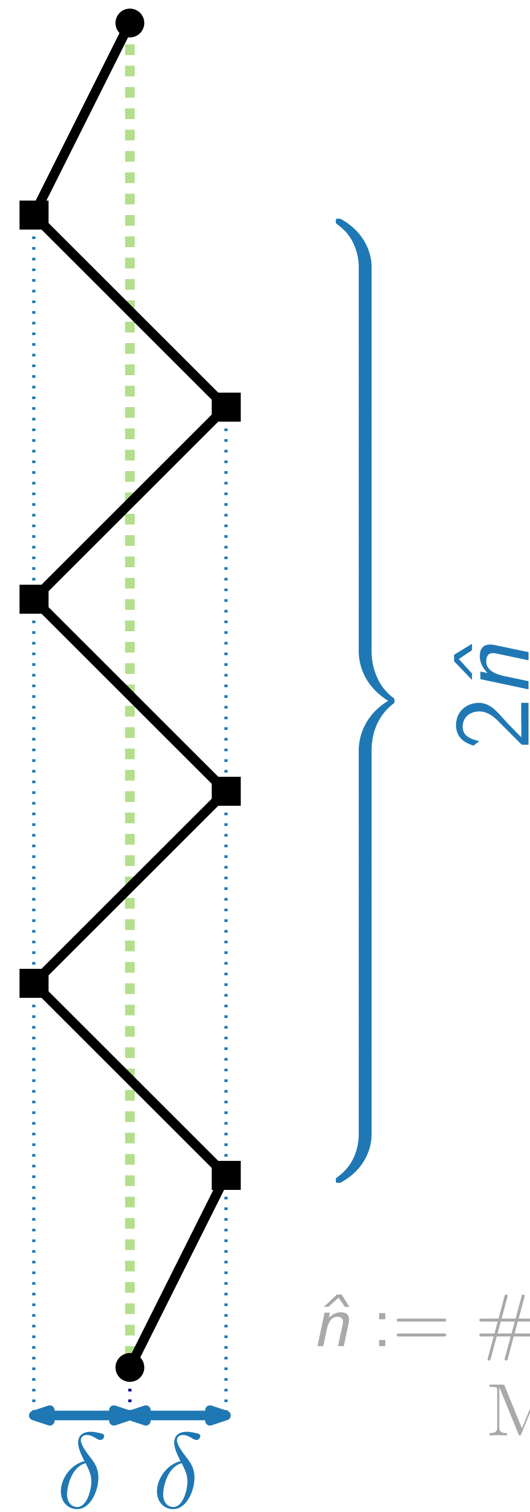

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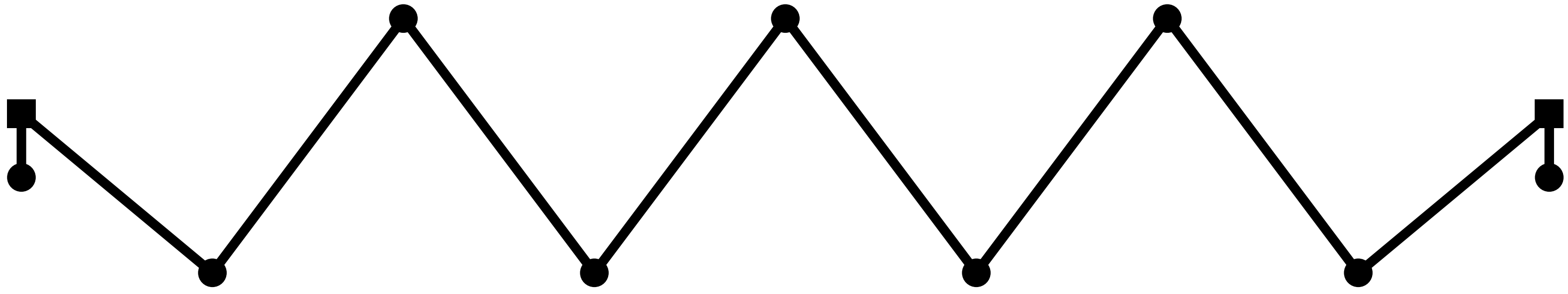

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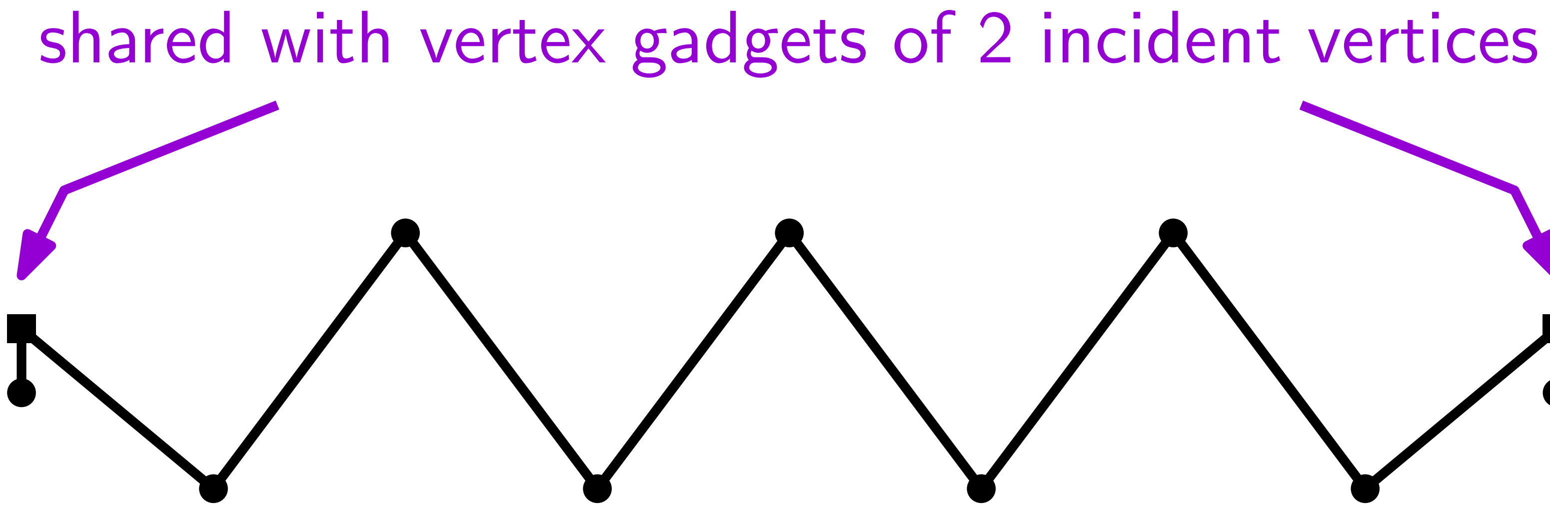

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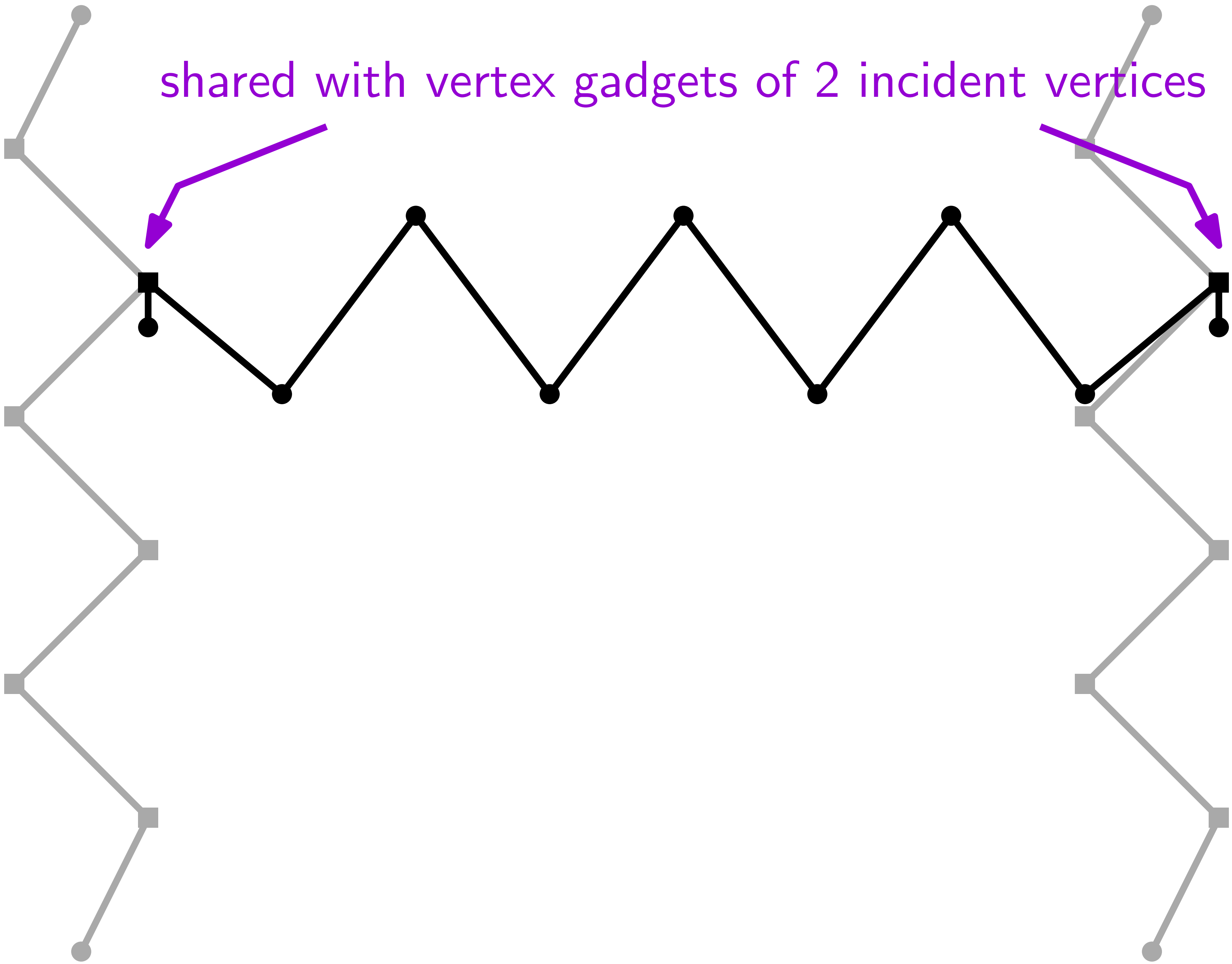

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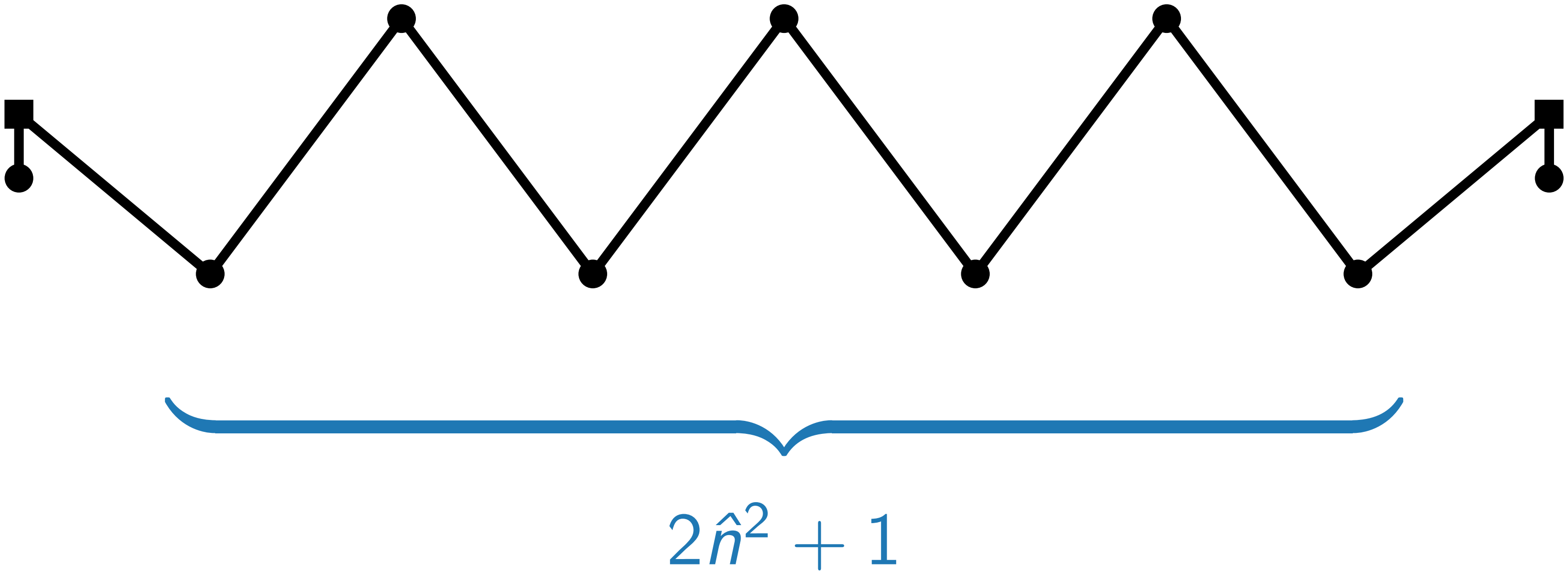

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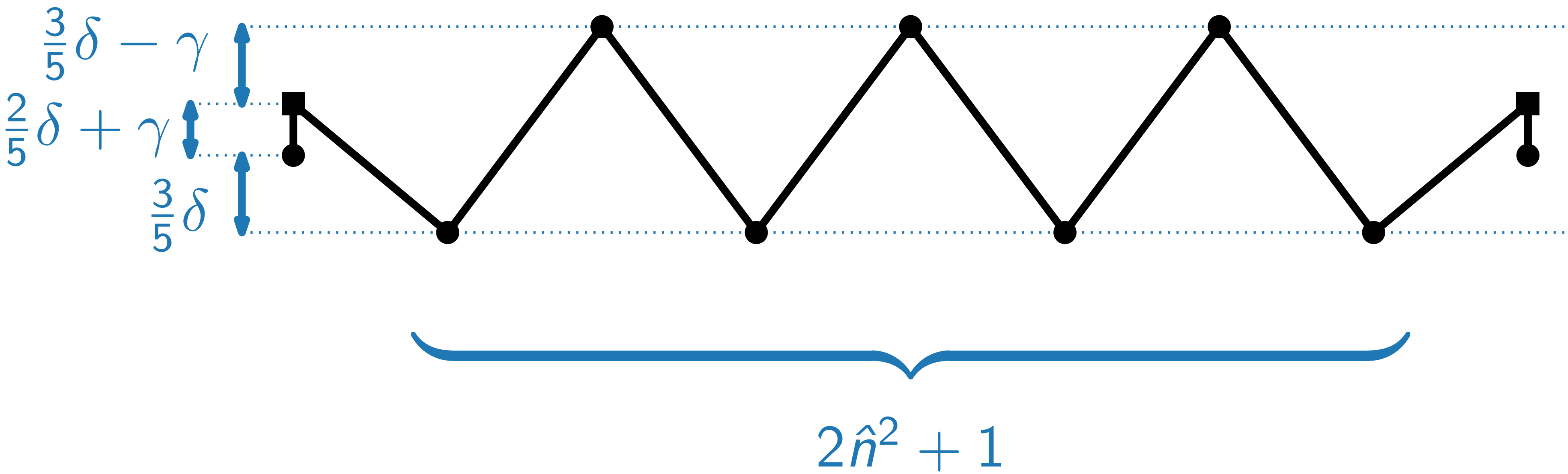
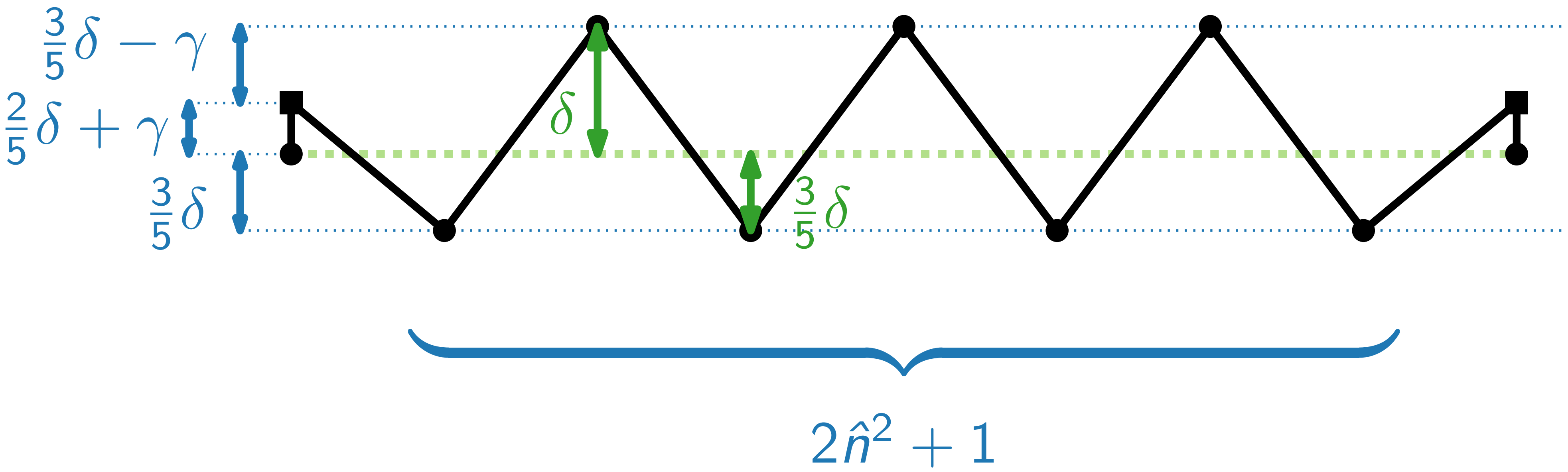


$\frac{3}{5}\delta - \gamma$
$\frac{2}{5}\delta + \gamma$
$\frac{3}{5}\delta$
$2\hat{n}^2 + 1$

Edge Gadget



$\frac{3}{5}\delta - \gamma$
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$\frac{3}{5}\delta$
δ
$\frac{3}{5}\delta$
$2\hat{n}^2 + 1$

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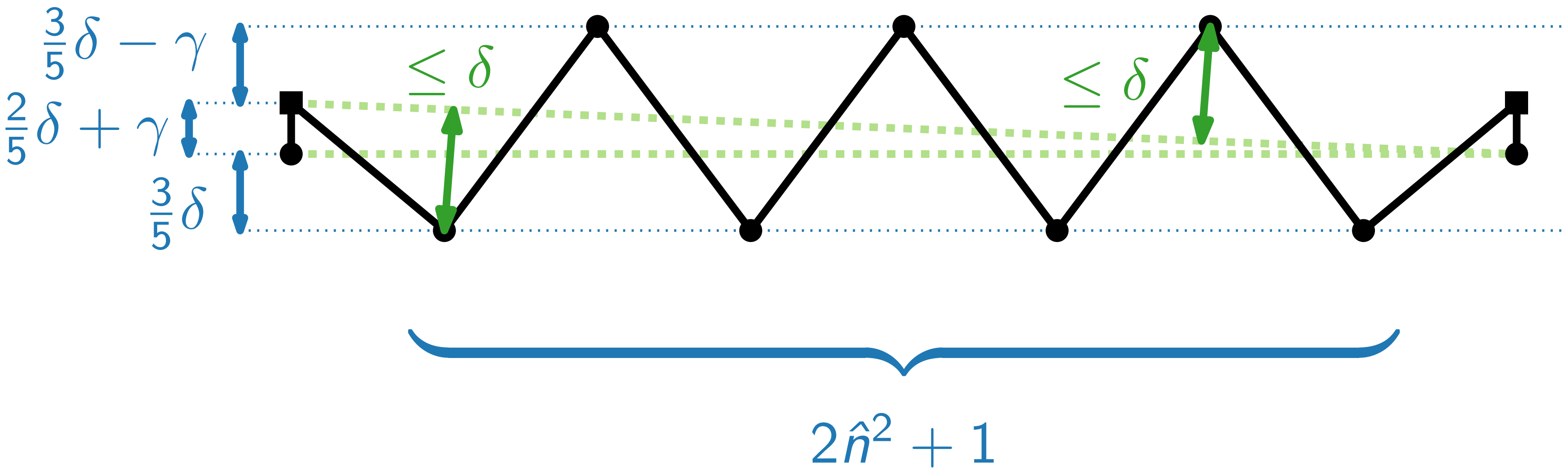
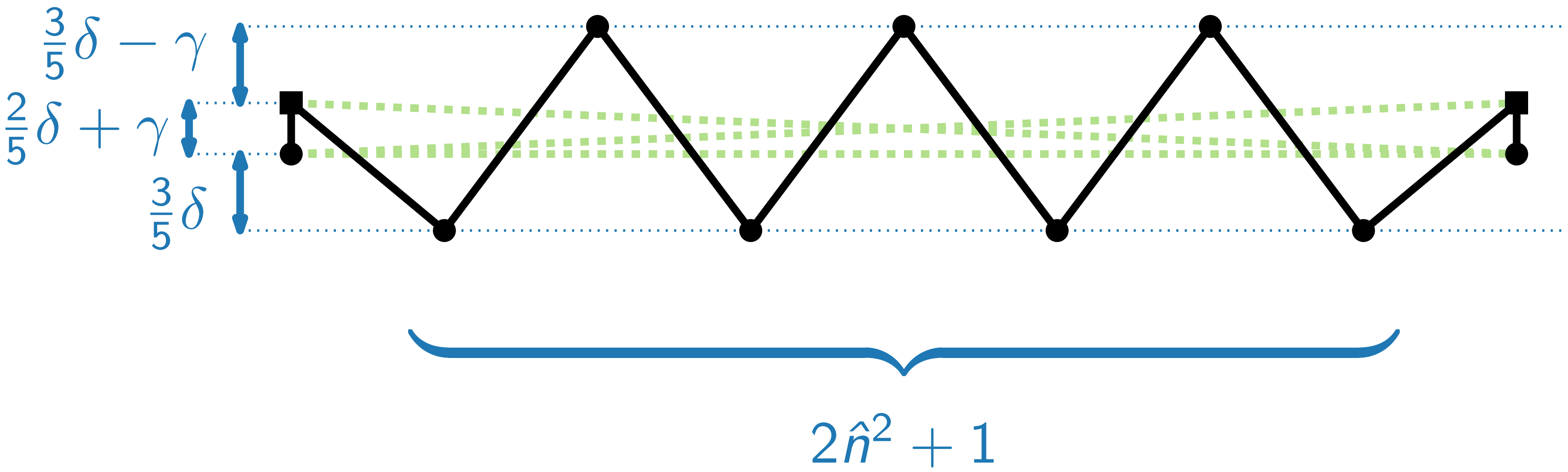


$\frac{3}{5}\delta - \gamma$
$\frac{2}{5}\delta + \gamma$
$\frac{3}{5}\delta$
$\leq \delta$
$\leq \delta$
$2\hat{n}^2 + 1$

Edge Gadget



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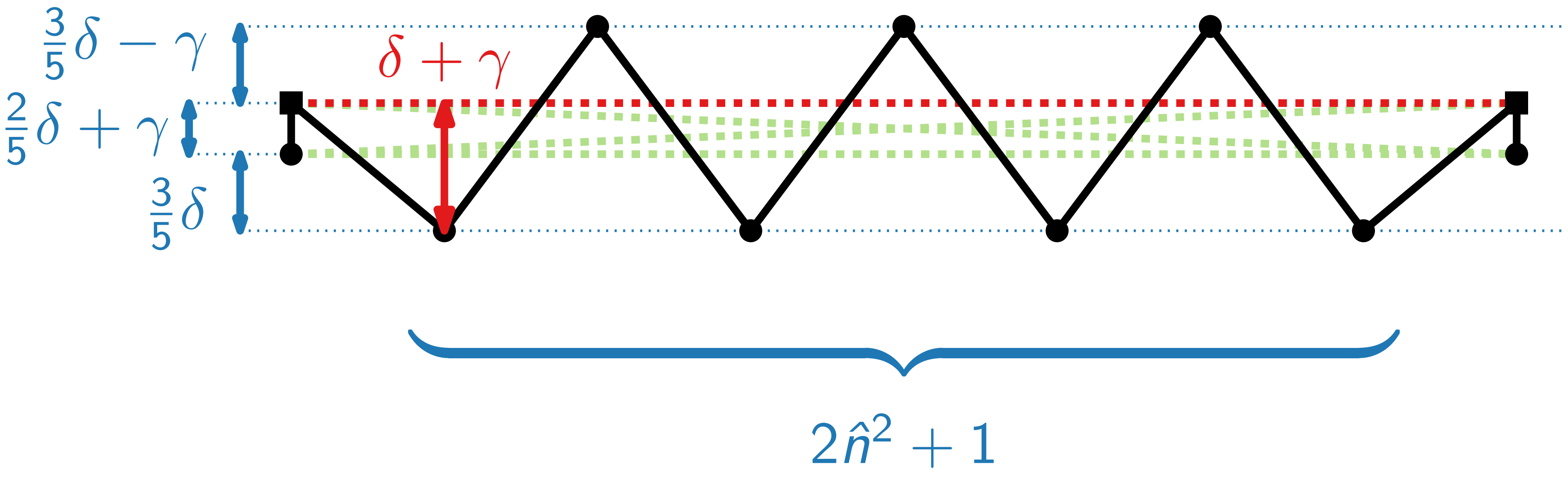

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$\delta + \gamma$
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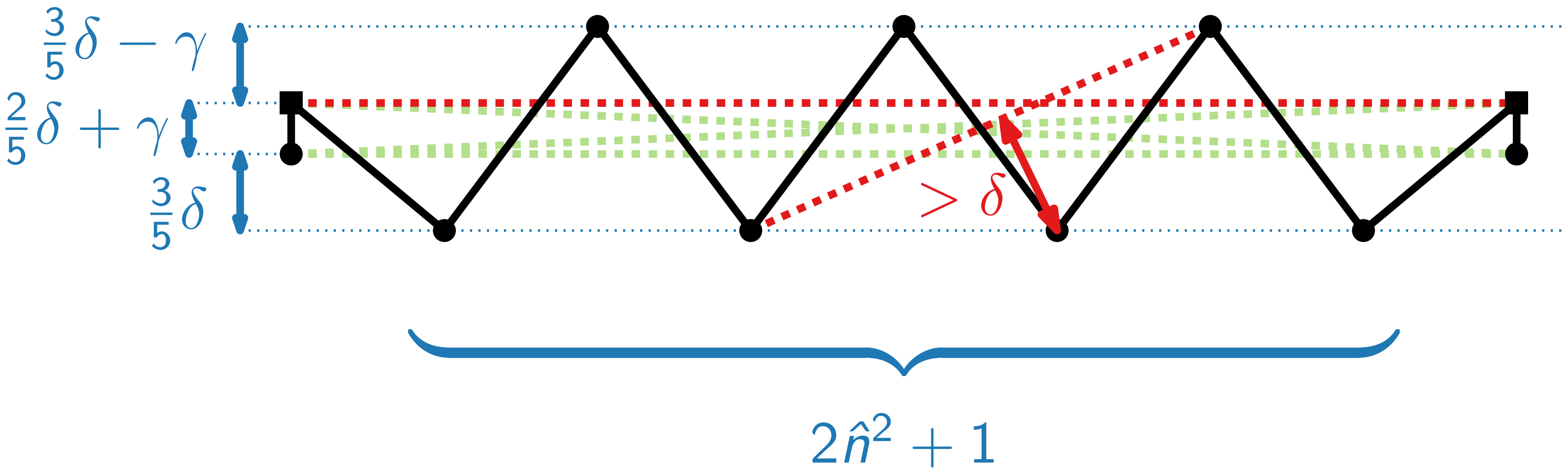

$\frac{3}{5}\delta - \gamma$
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$\frac{3}{5}\delta$
$> \delta$
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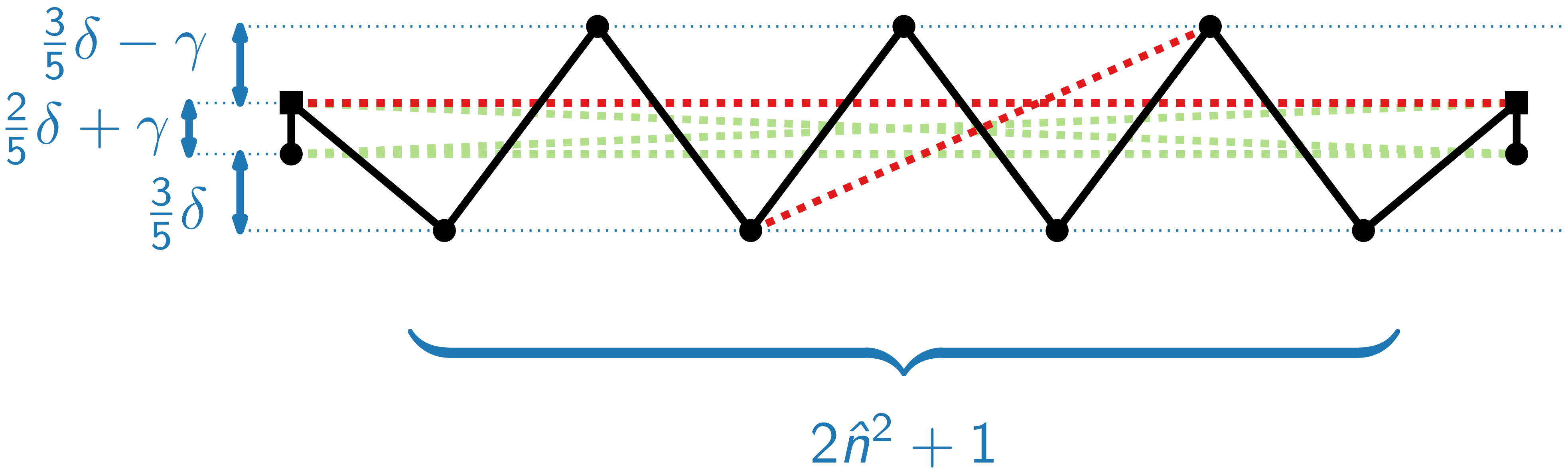

- not taking a "long" shortcut is very expensive

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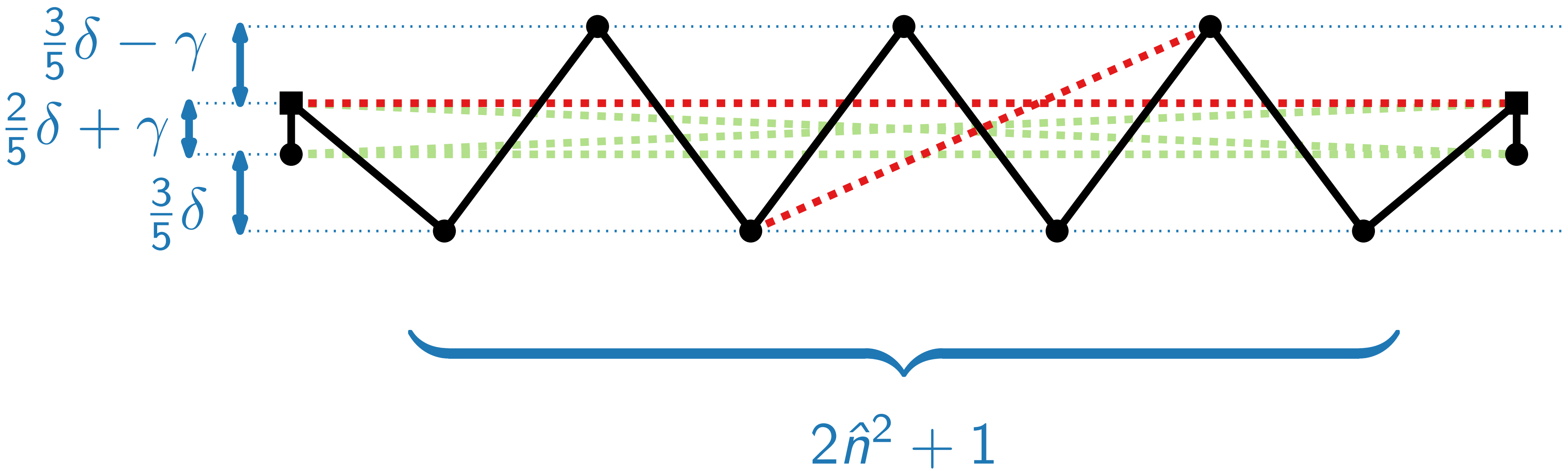

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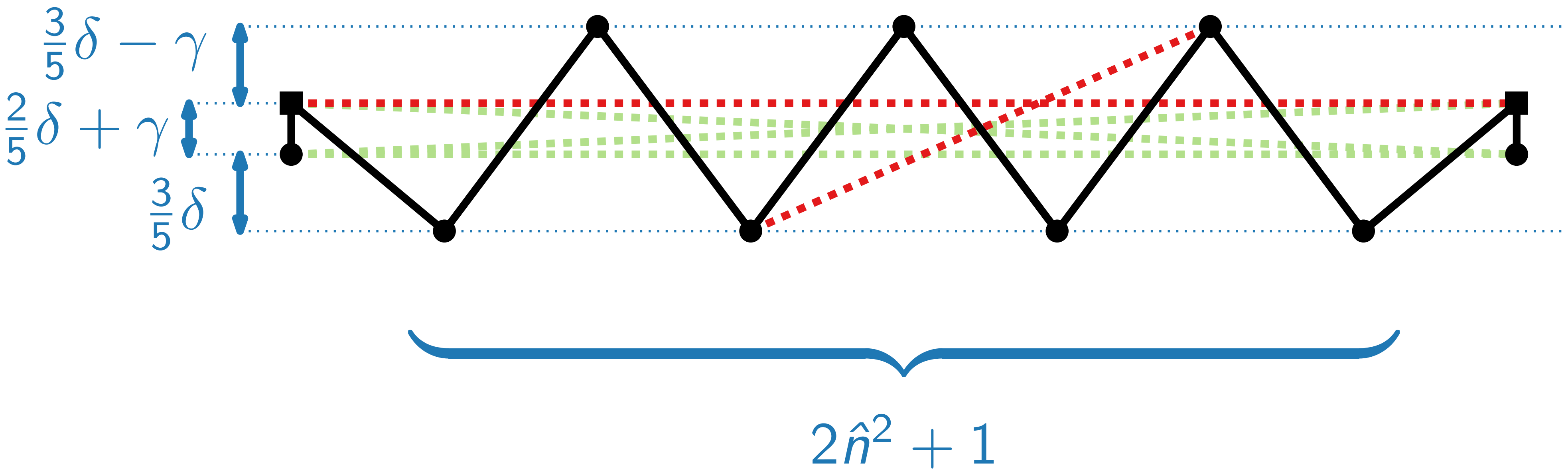
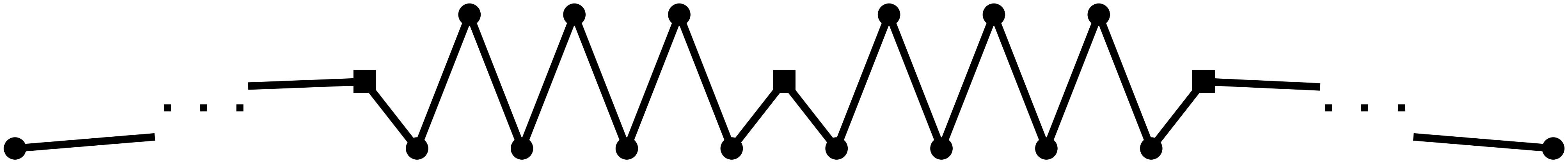


- not taking a “long” shortcut is very expensive
- take a “long” shortcut $\Rightarrow$ take a shortcut of a vertex gadget

$\Rightarrow$ guarantees independent set property

Neighborhood Gadget



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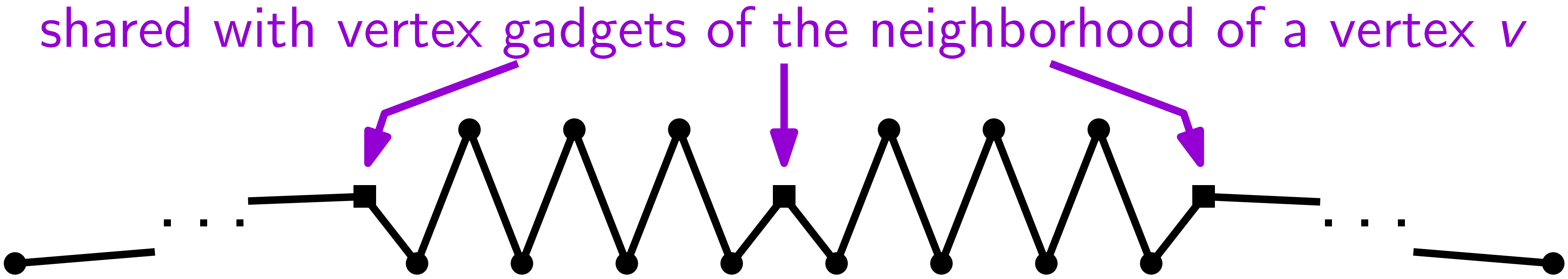

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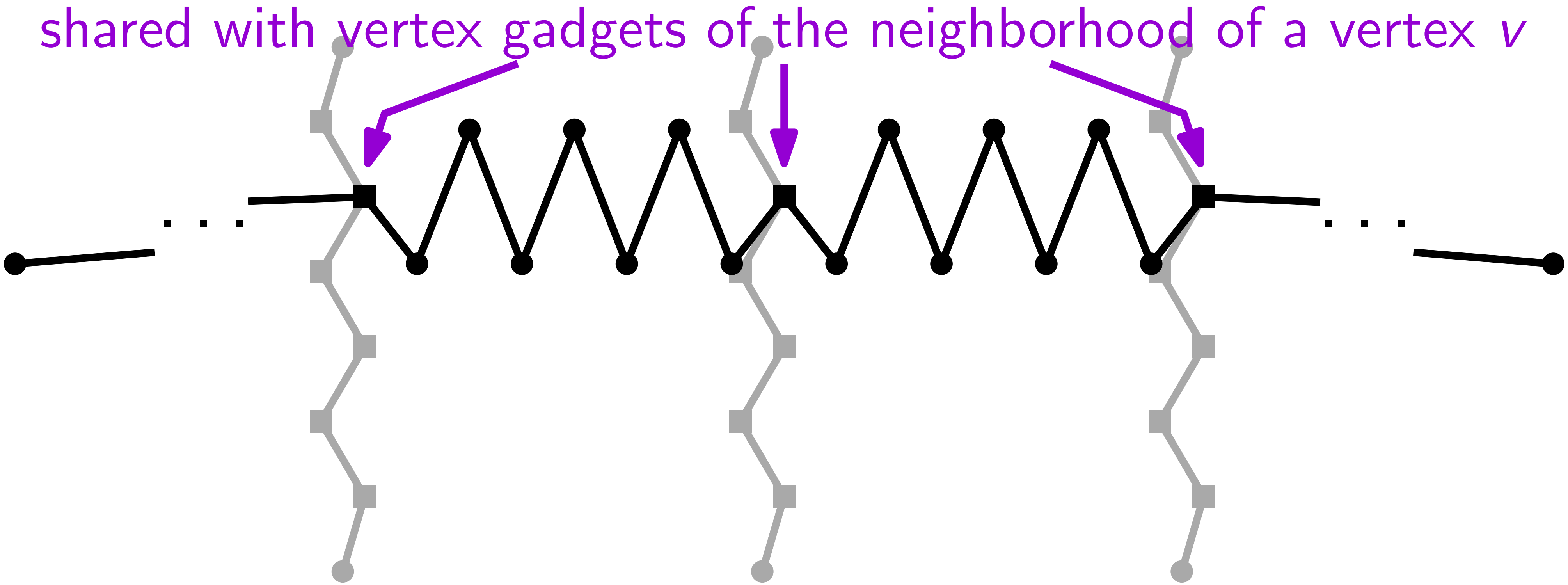

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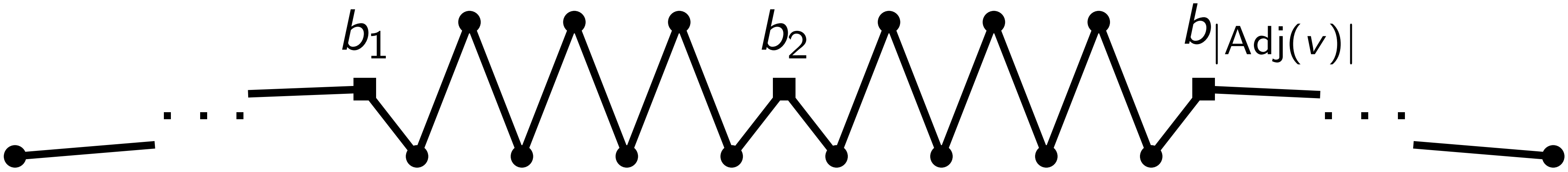

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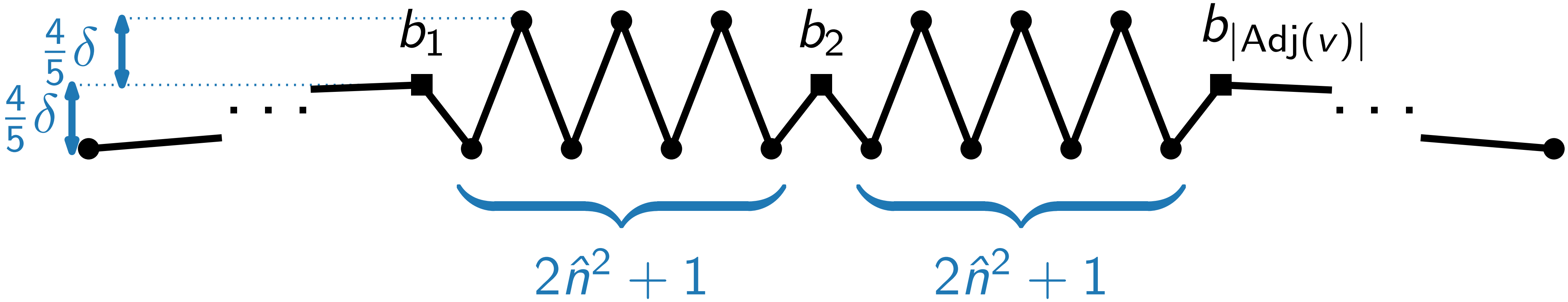

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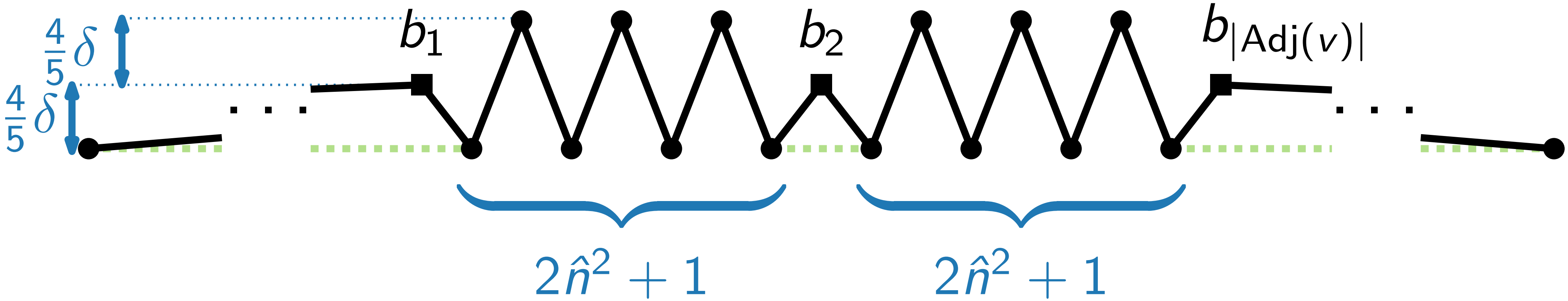

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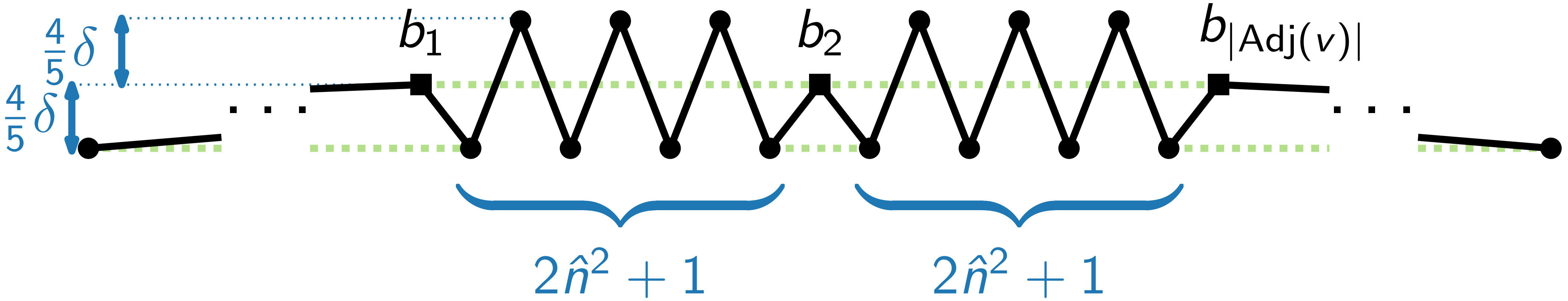

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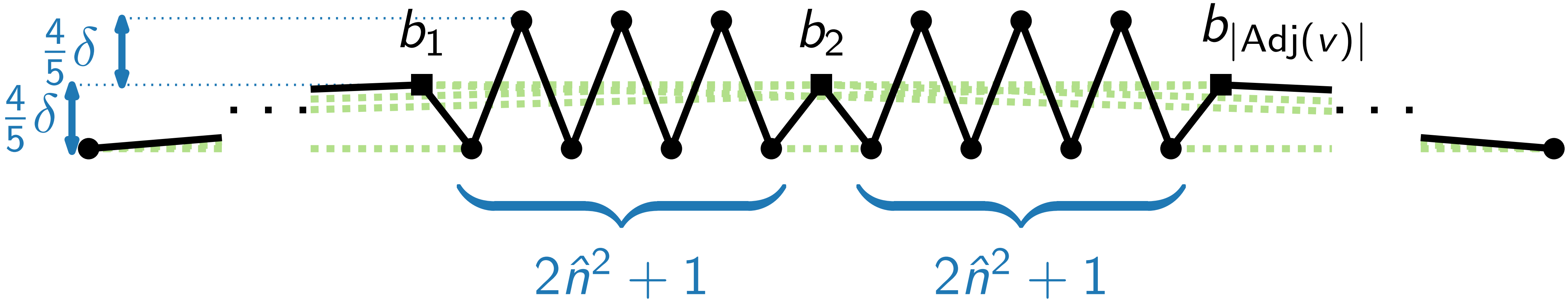

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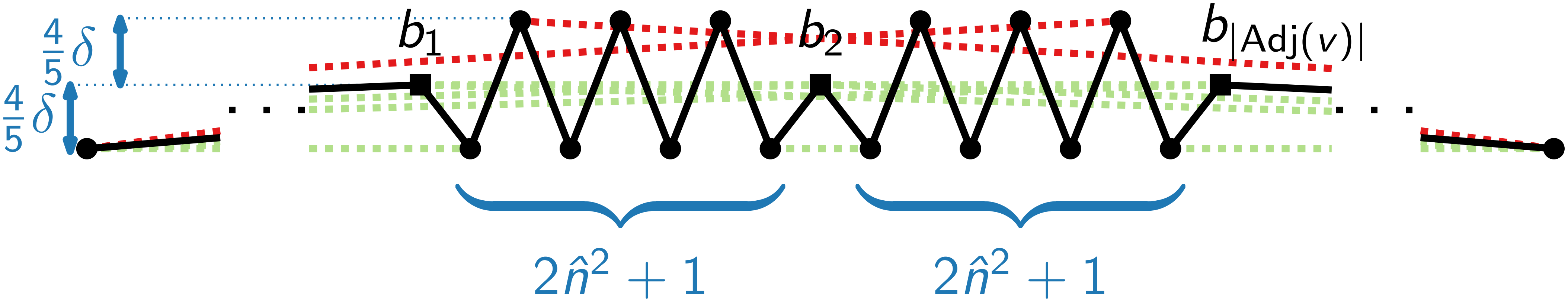

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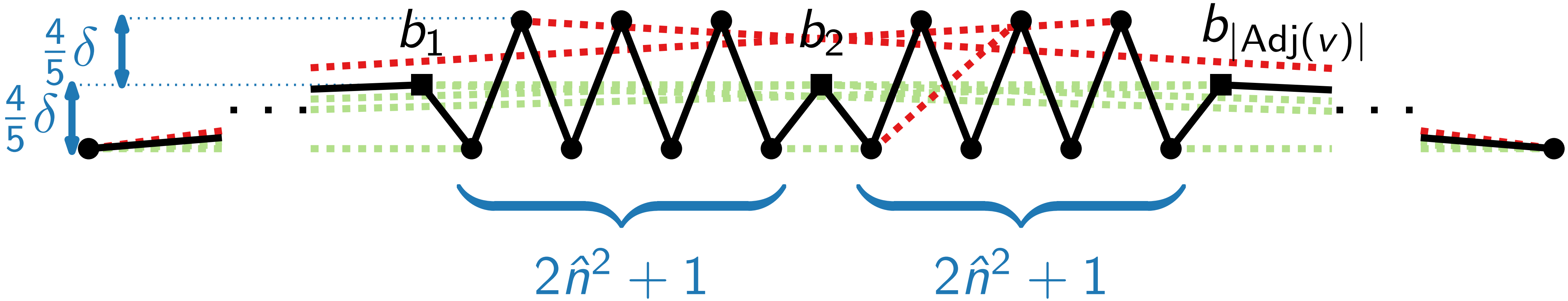

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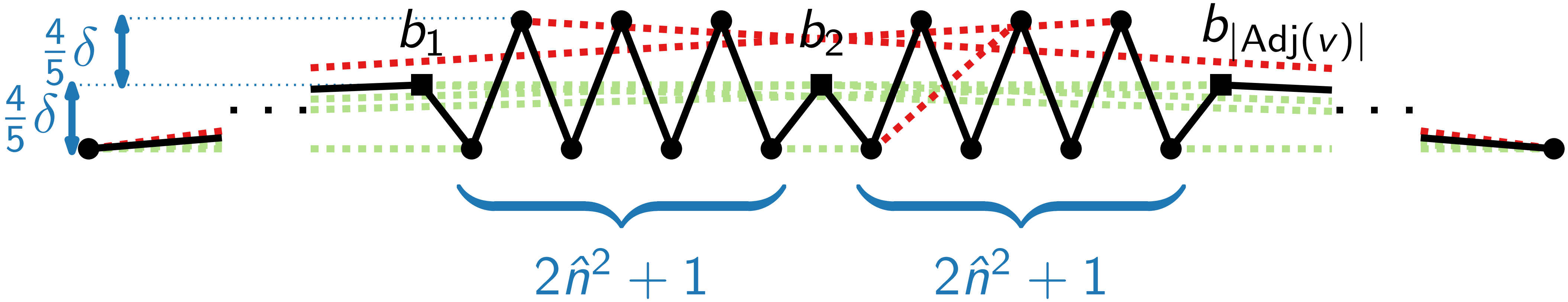

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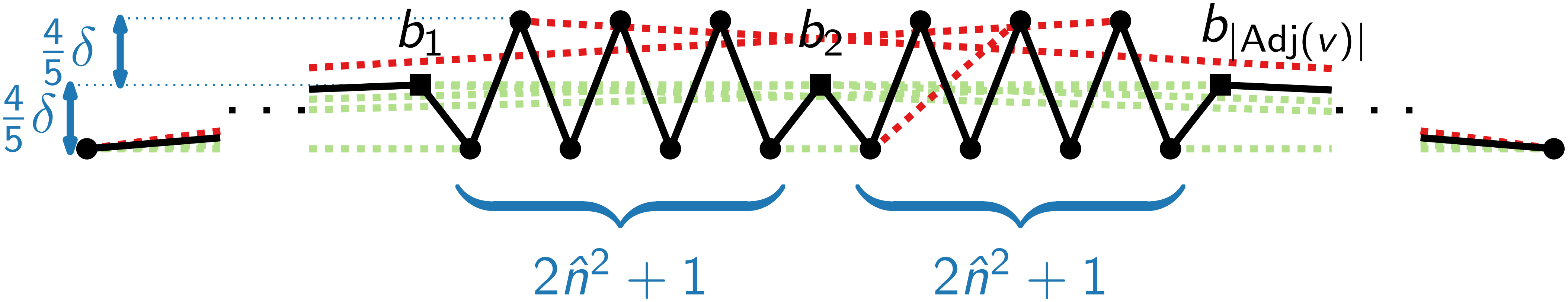

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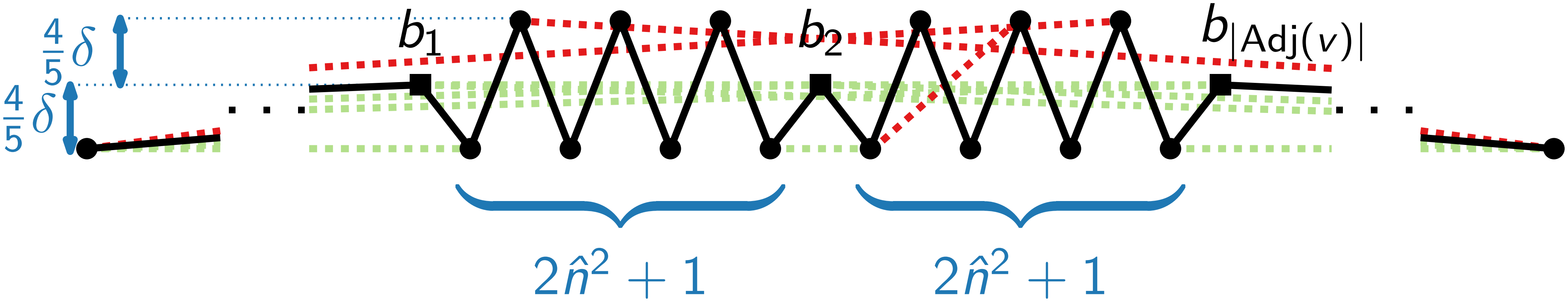

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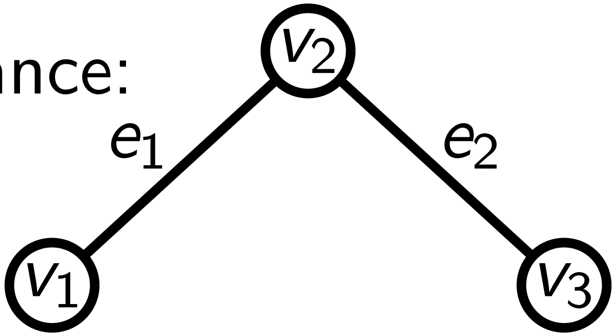
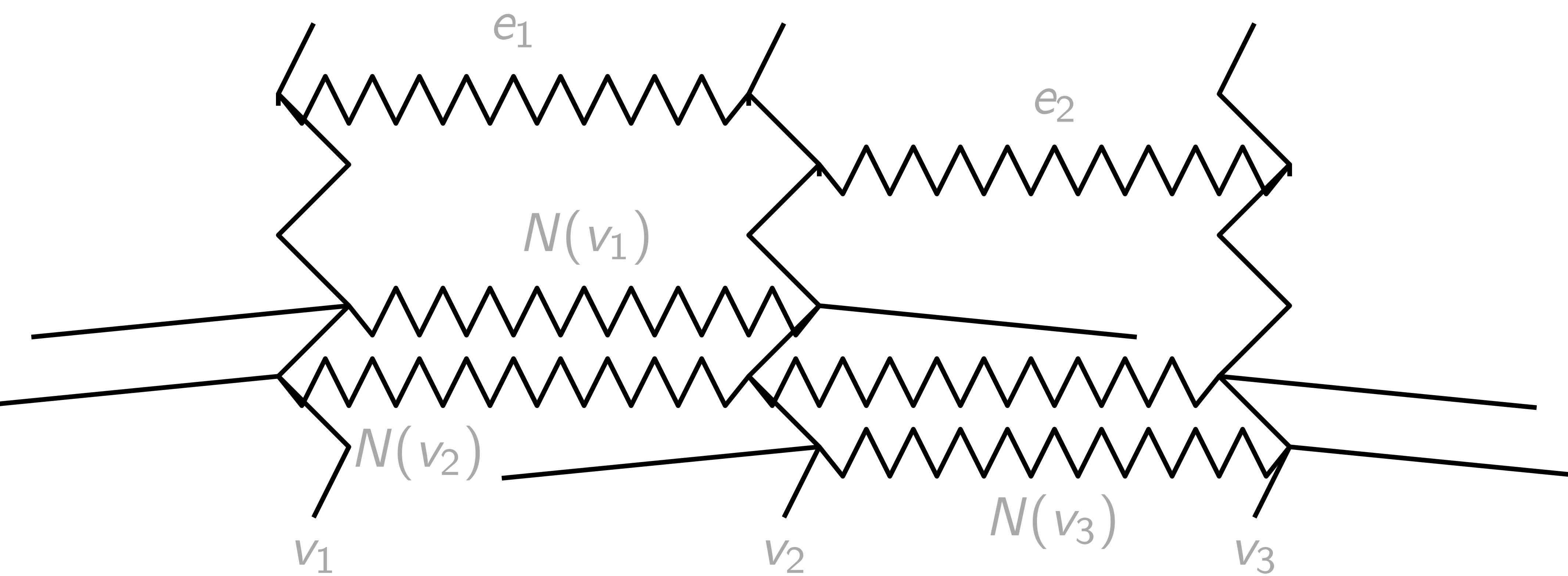
$\Rightarrow$ guarantees dominating set property

Full Example

MIDS instance:



$$\hat{n} = 3$$
$$2\hat{n} = 6$$
$$2\hat{n}^2 + 1 = 19$$



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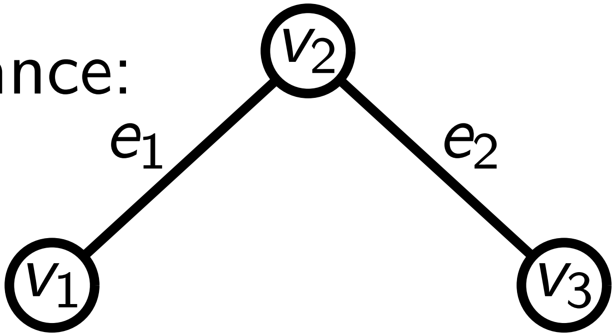

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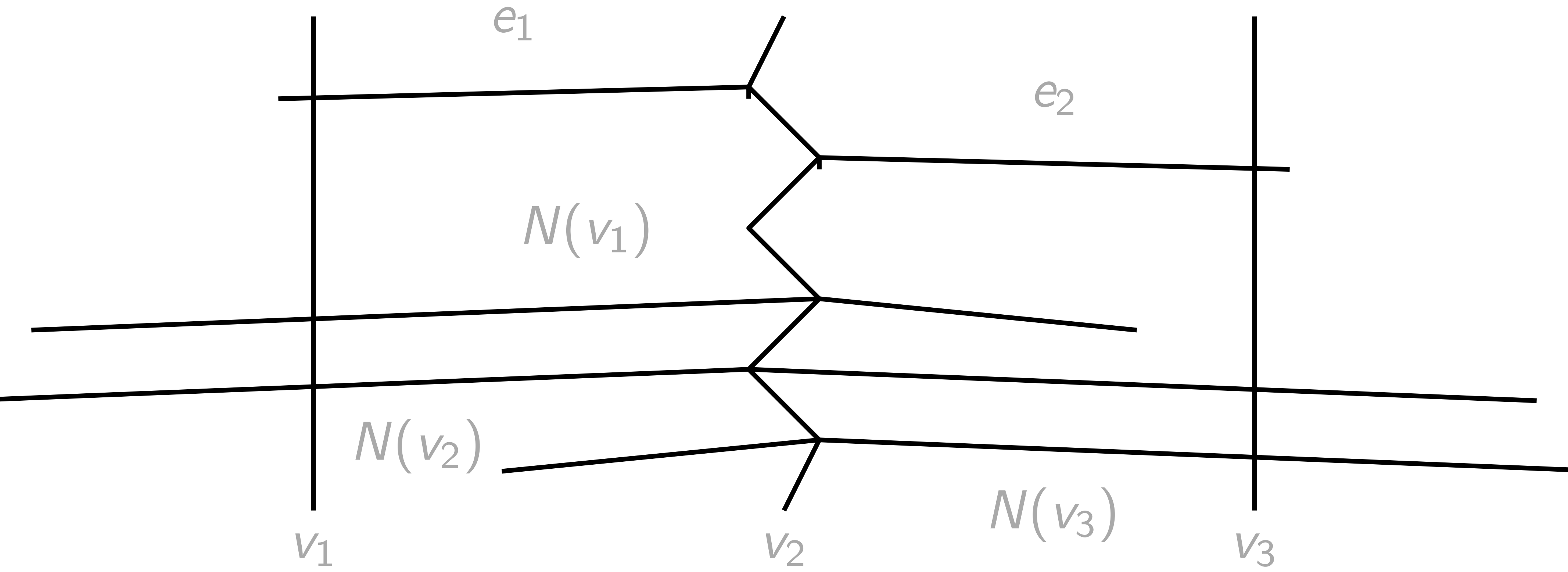

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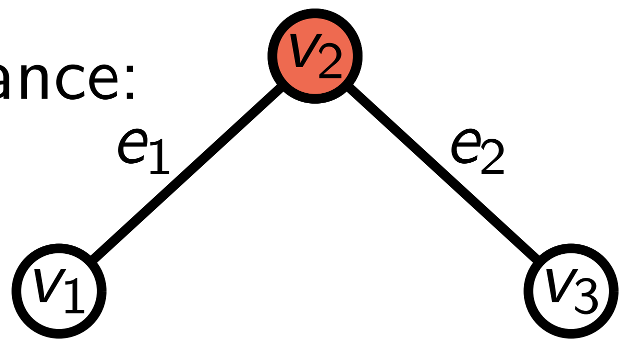

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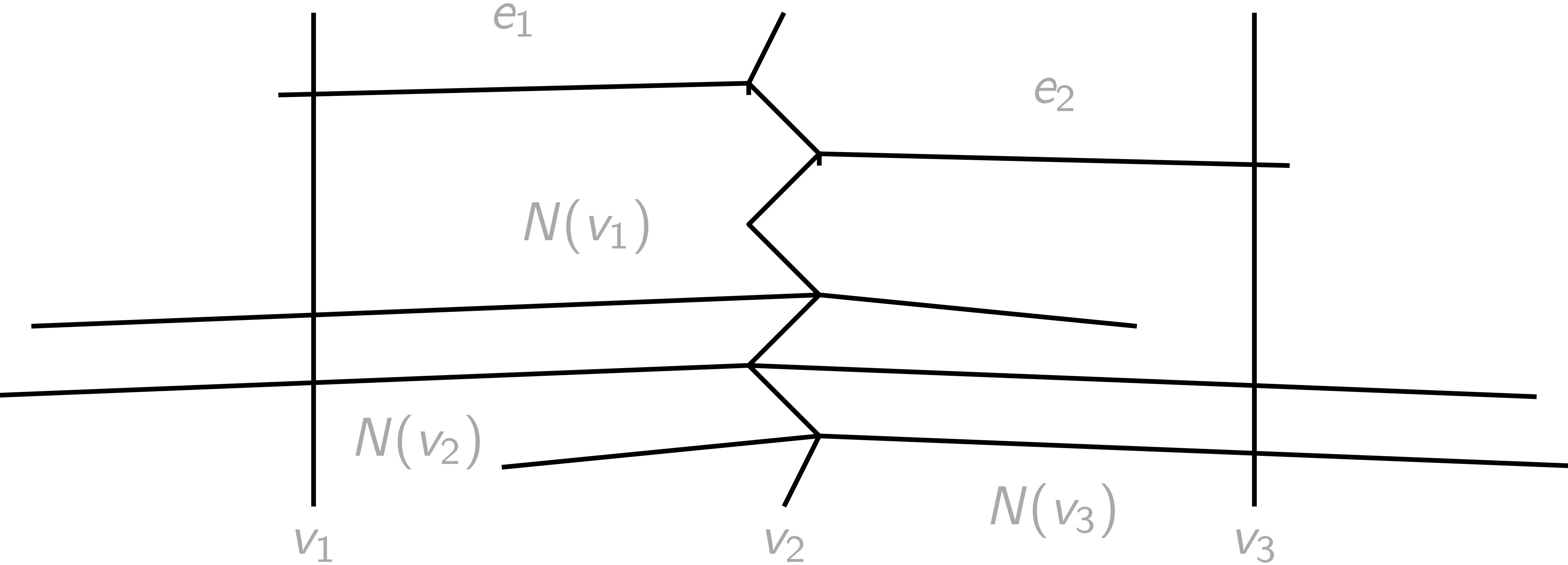

Agenda

1. Motivation and Introduction
2. Problem Definition
3. Hardness of Approximation (+ Proof Sketch)
4. **Bi-Criteria Approximation (+ Proof Sketch)**
5. Summary

Bi-criteria Approximation

- Reason for this strong inapproximability: high sensitivity towards “bad” choices for keeping or discarding a bend

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Theorem 2:
There is a bi-criteria $(O(\log(\ell + n)), 2)$-approximation algorithm for simplifying a bundle of polylines.
$\ell = \#$ polylines, $n = \#$ bends

Bi-criteria Approximation

instance of polyline bundle simplification (PBS)

Bi-criteria Approximation

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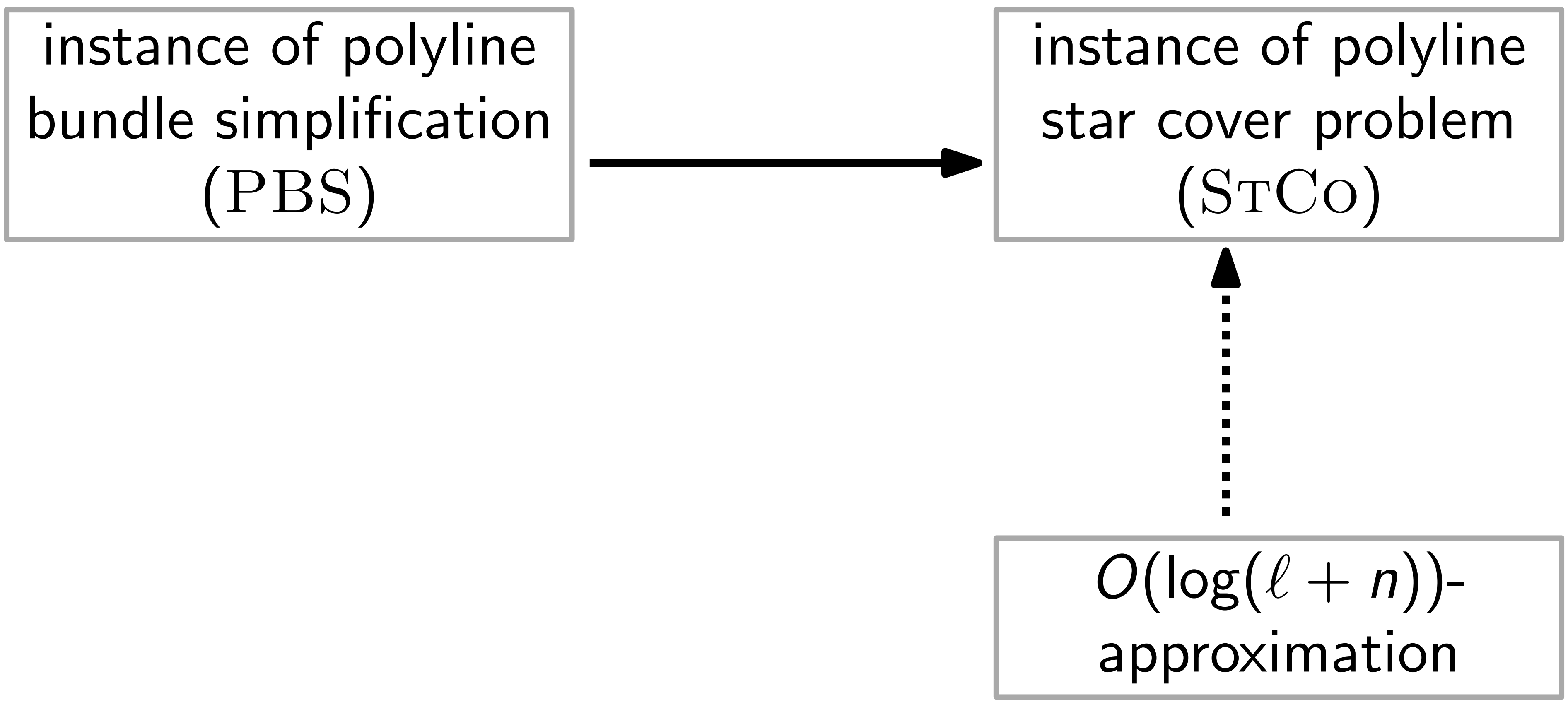

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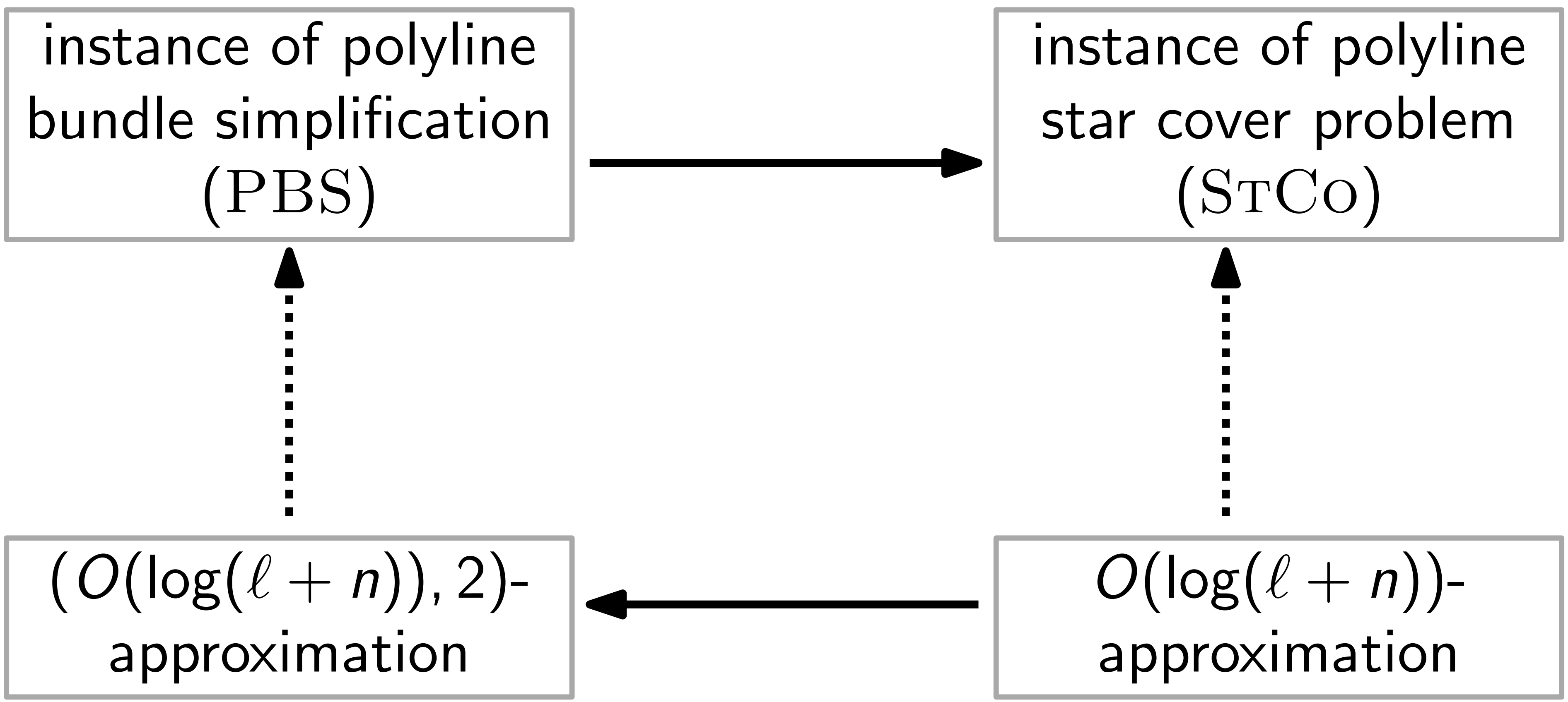
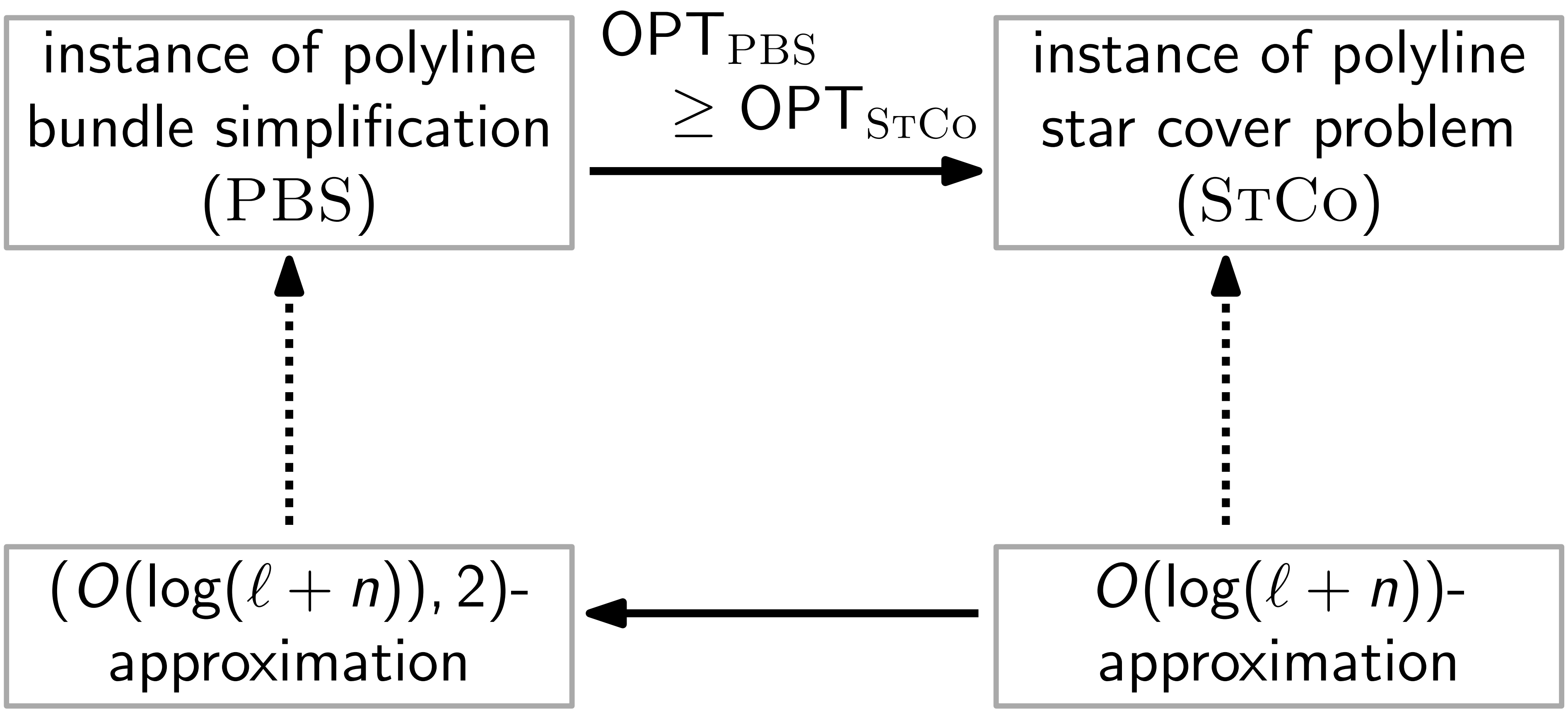


Bi-criteria Approximation



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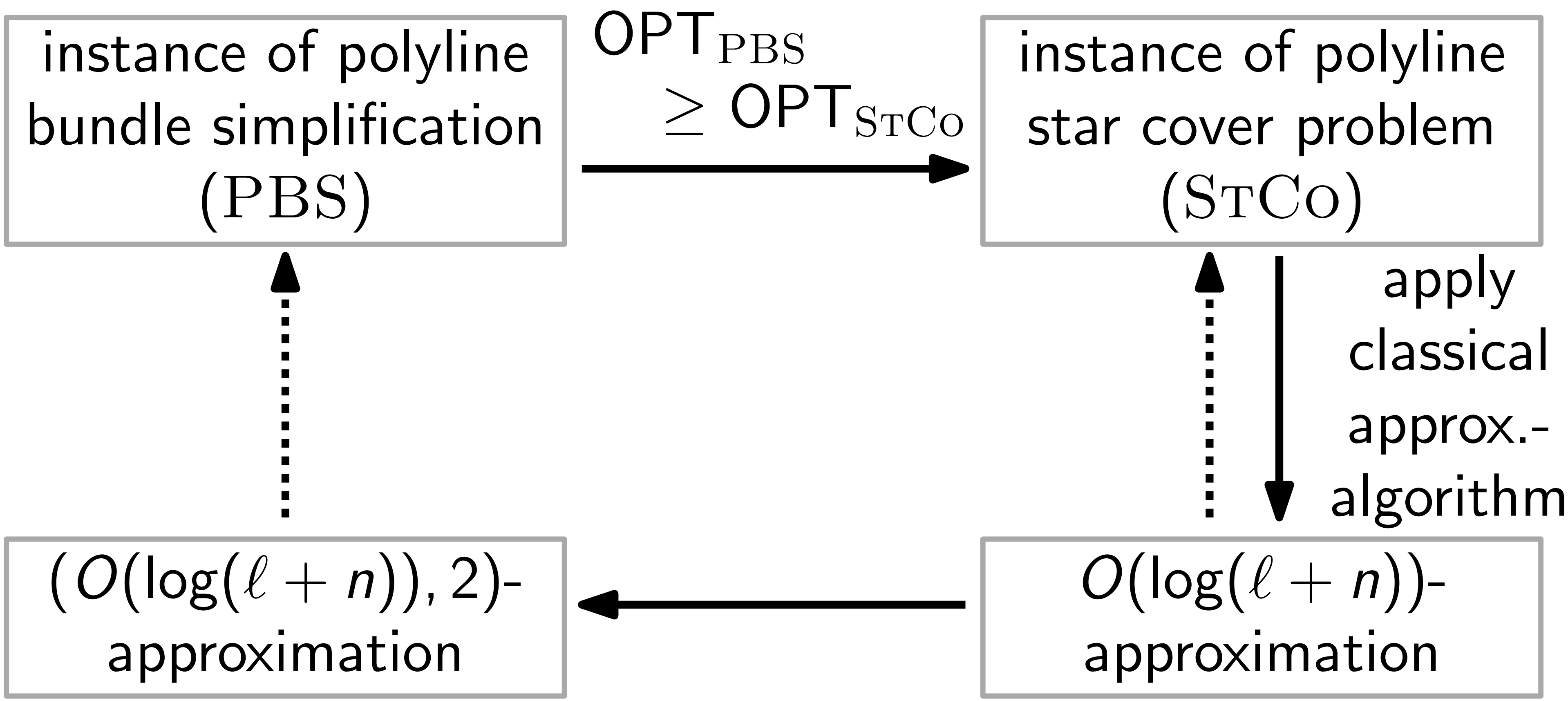

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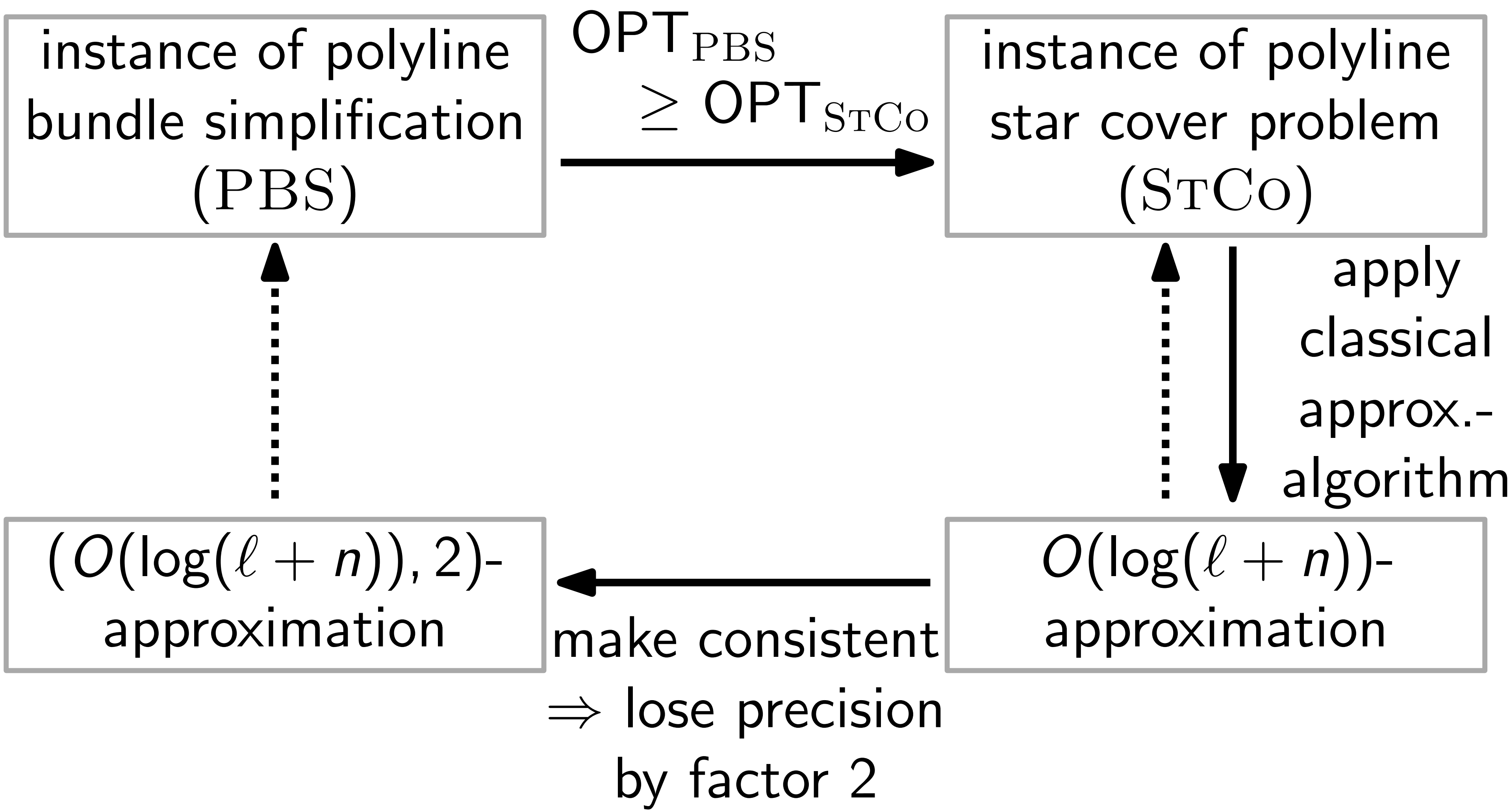

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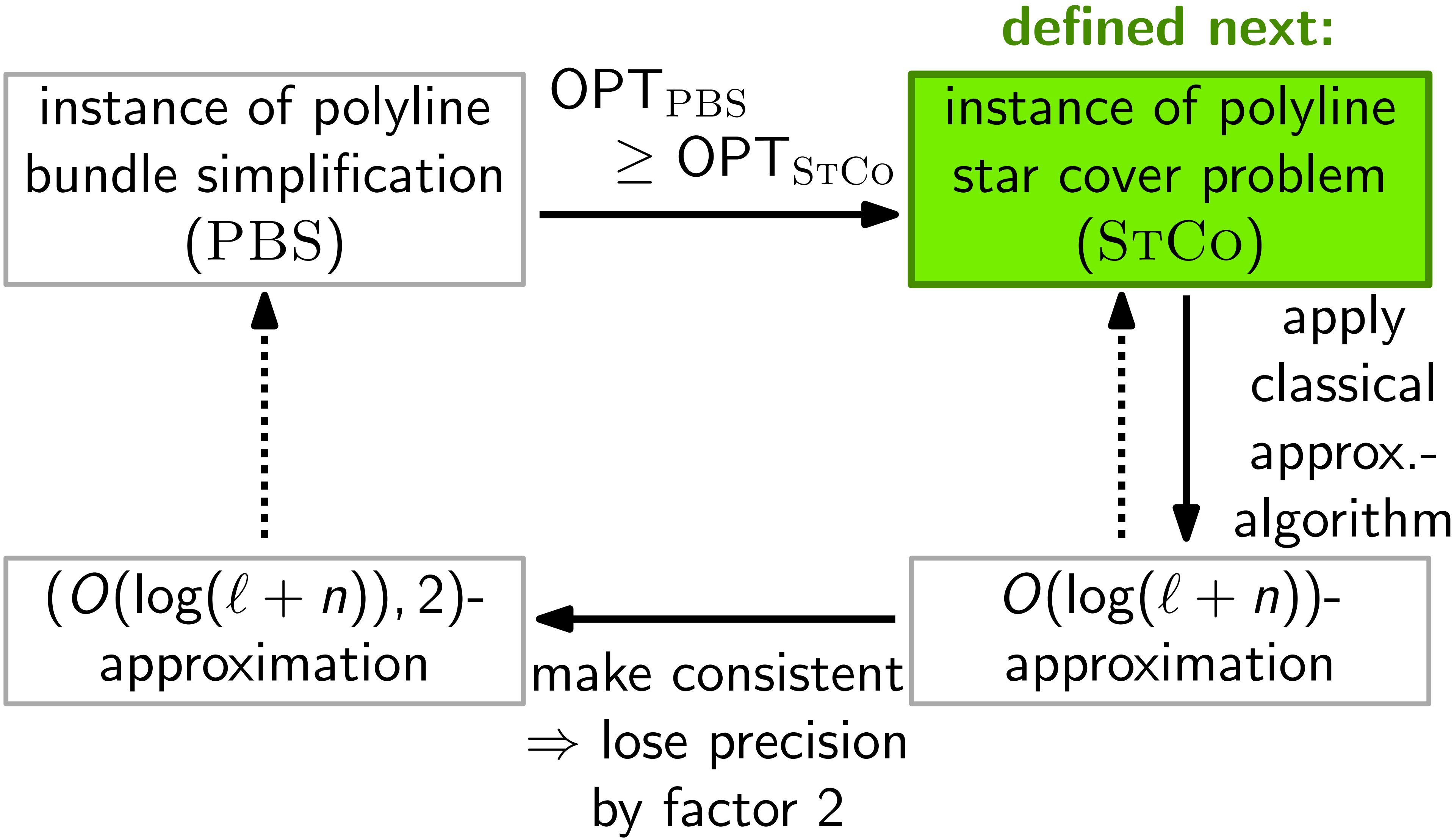
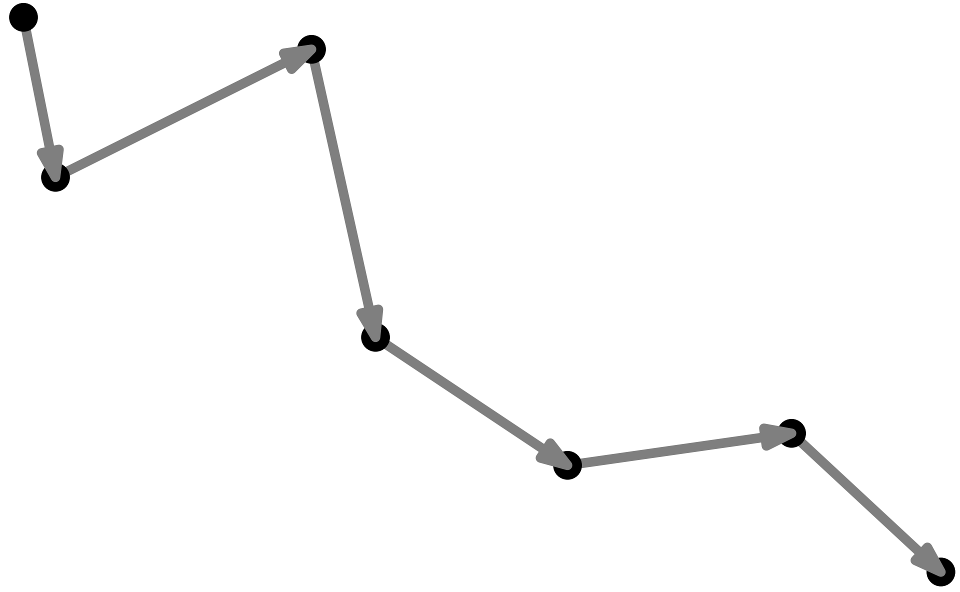


Polyline Star Cover Problem



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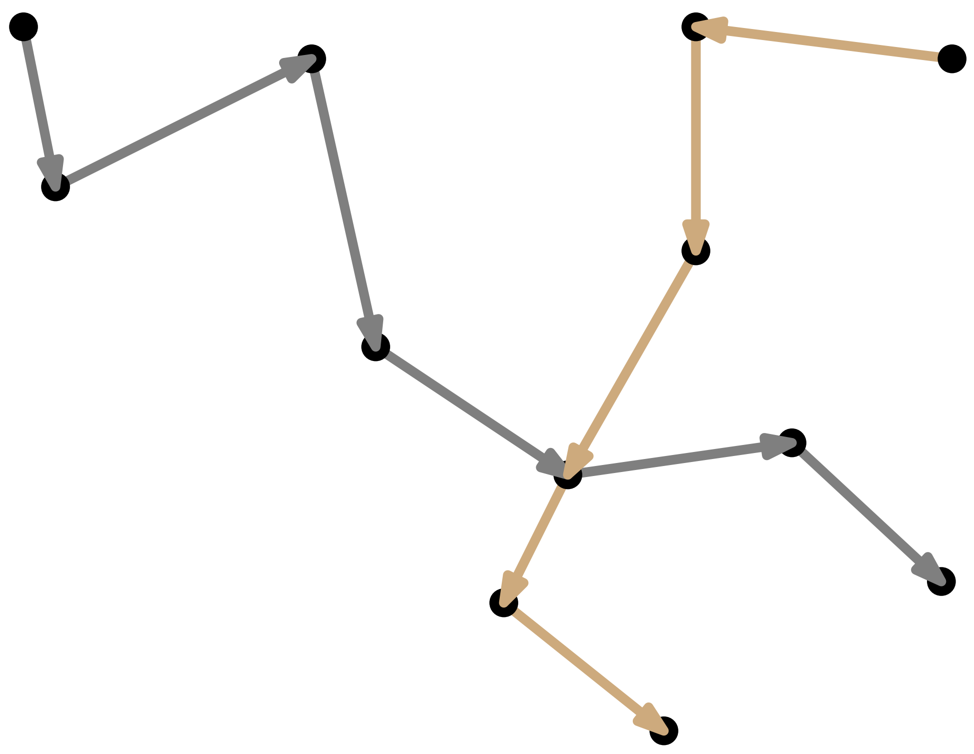

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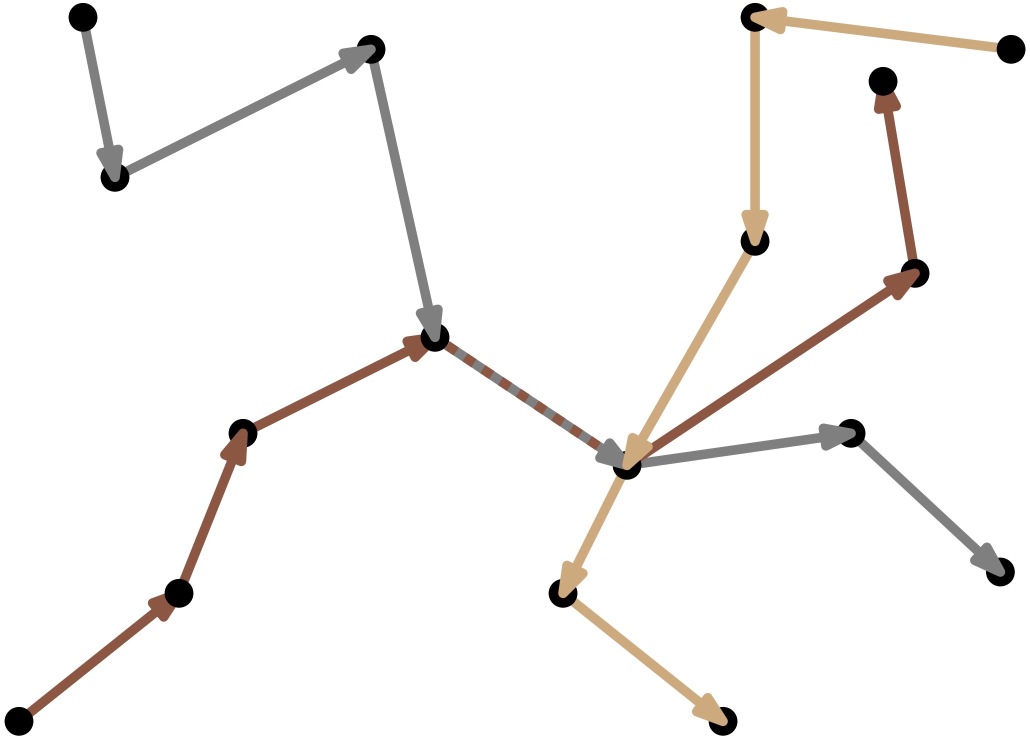

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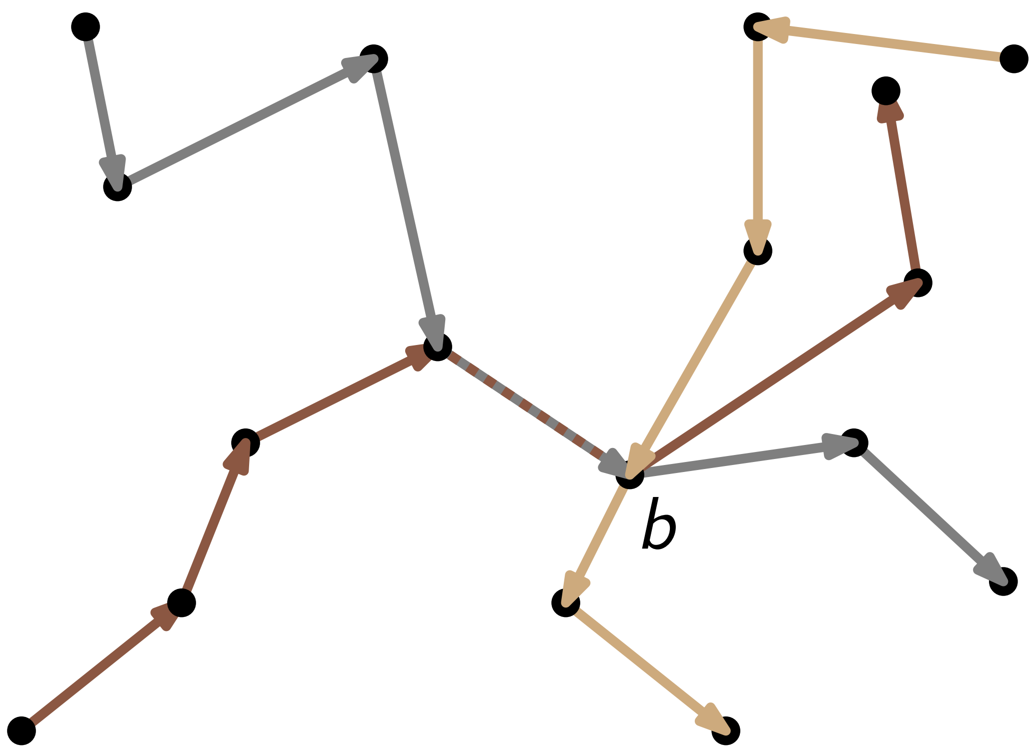

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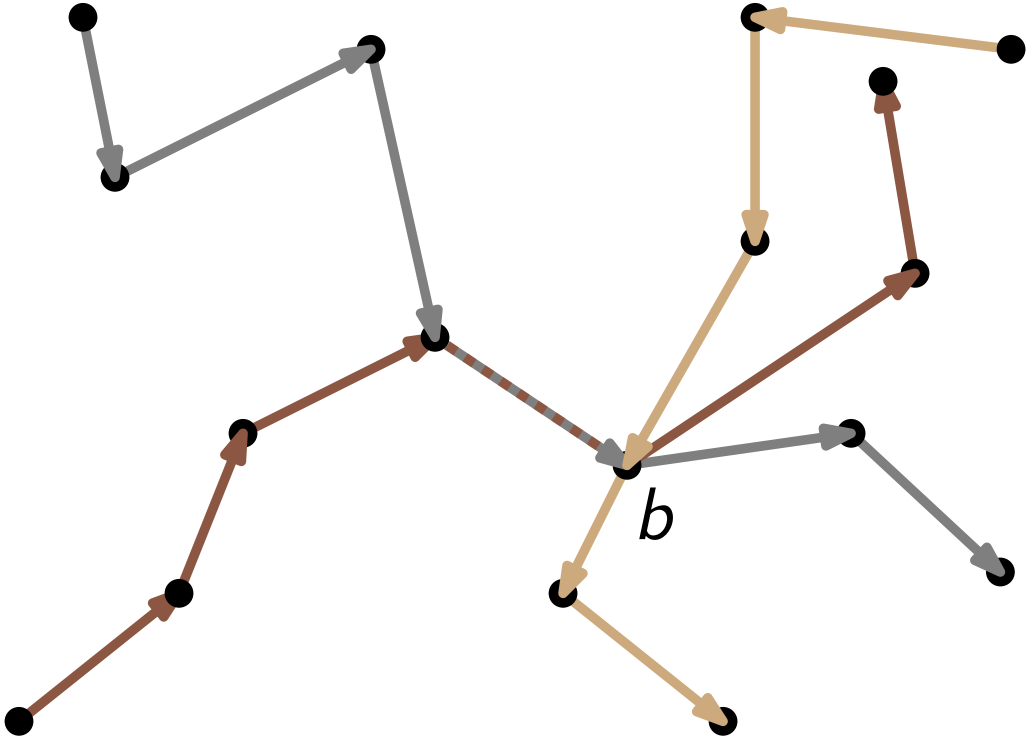
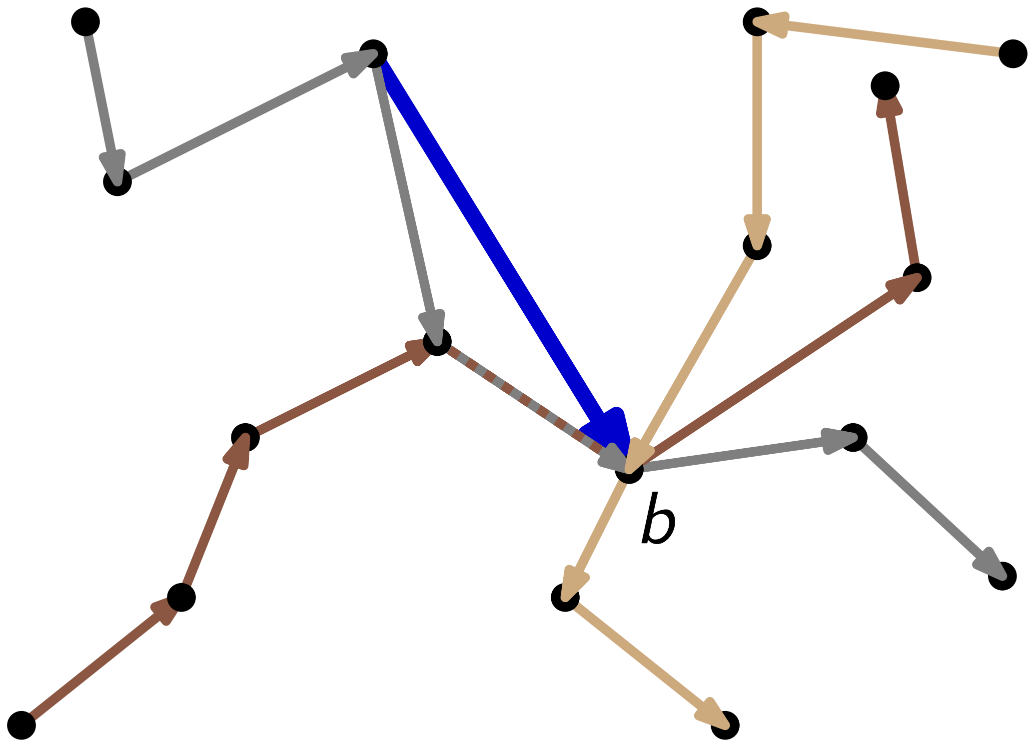


star around b

Polyline Star Cover Problem



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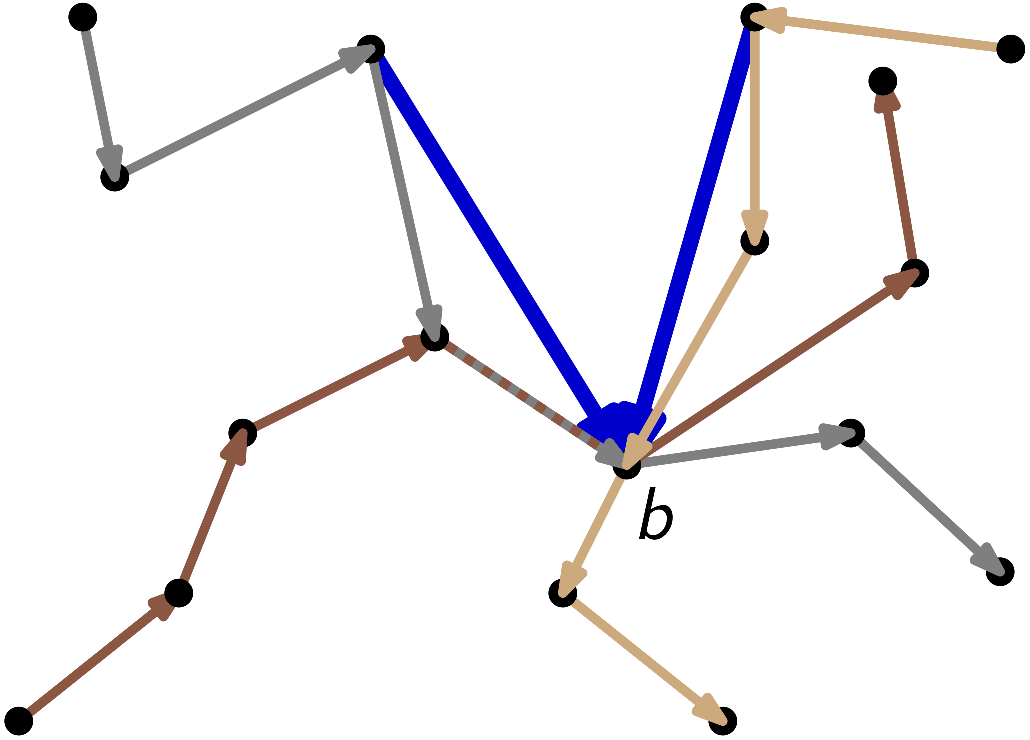

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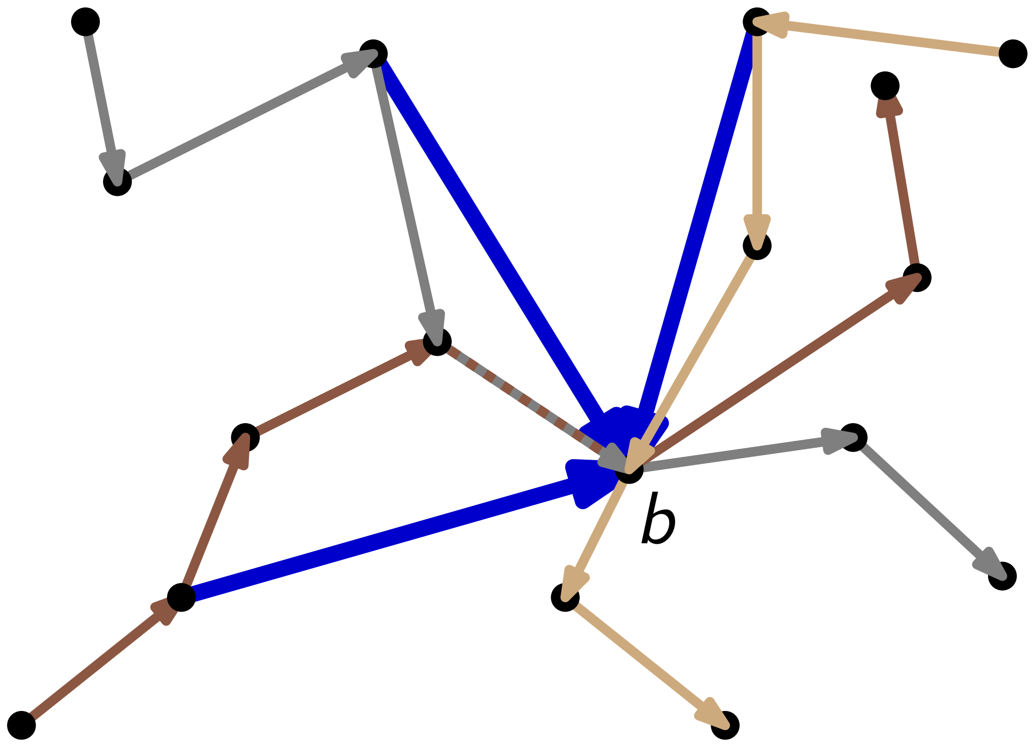

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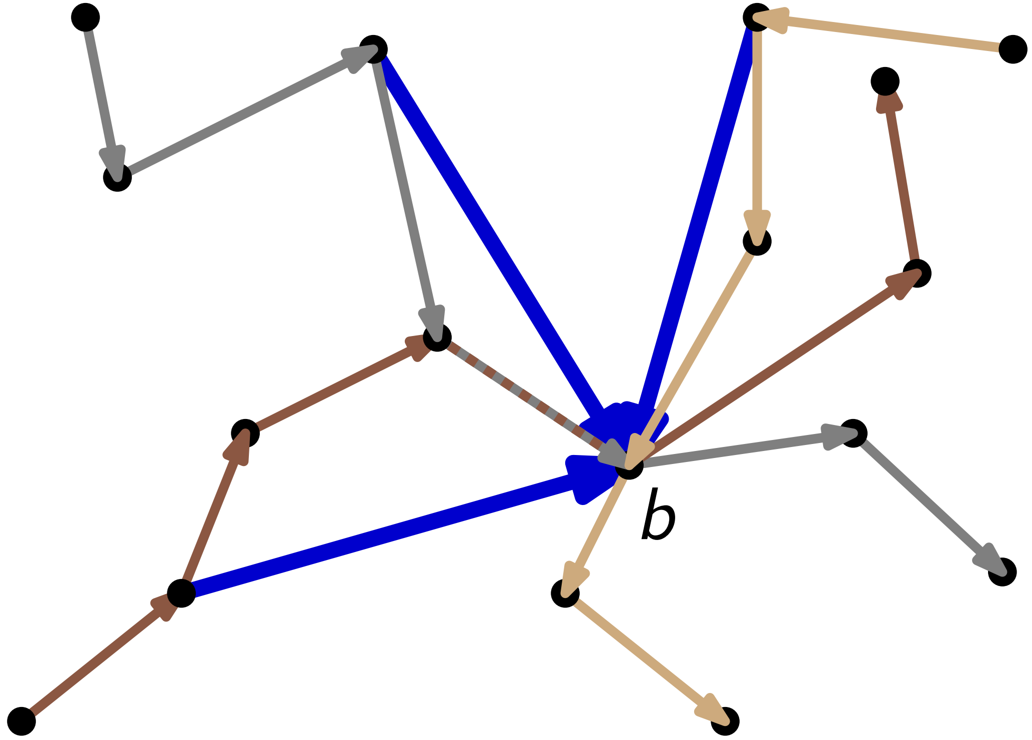

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Polyline Star Cover Problem
(for an instance of PBS)

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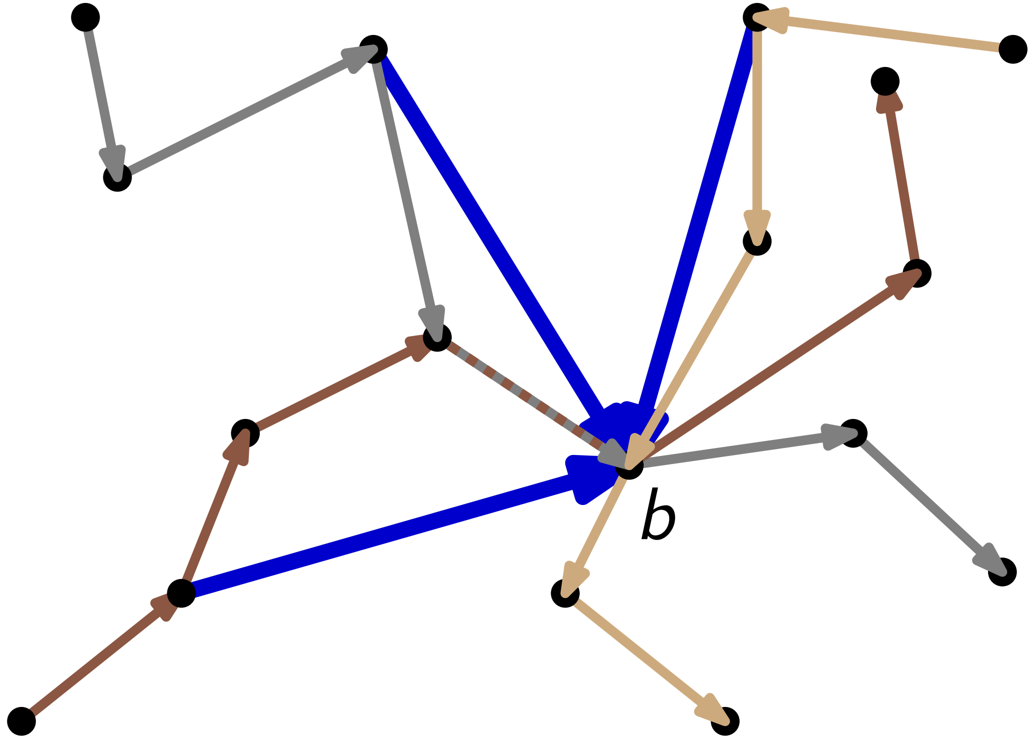

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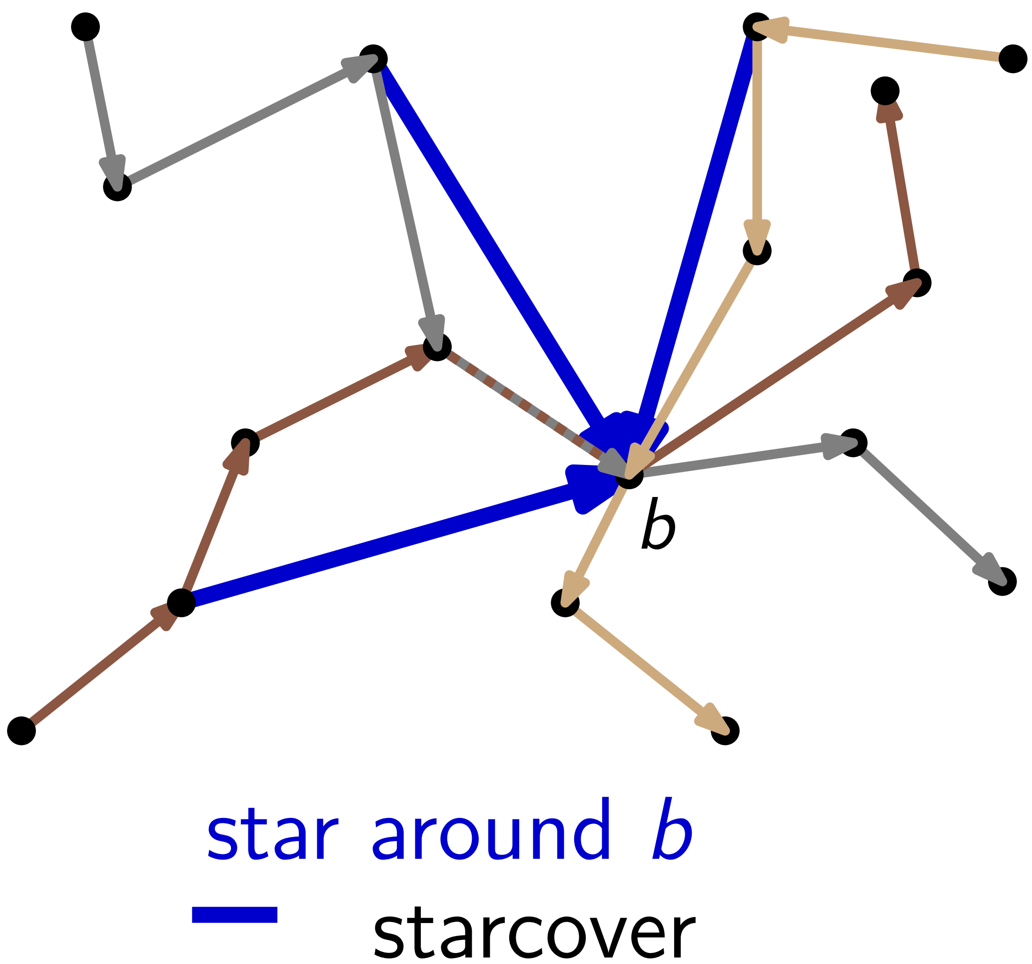
Given: all stars

Find: minimum size set of stars covering all polyline–segment pairs

Polyline Star Cover Problem



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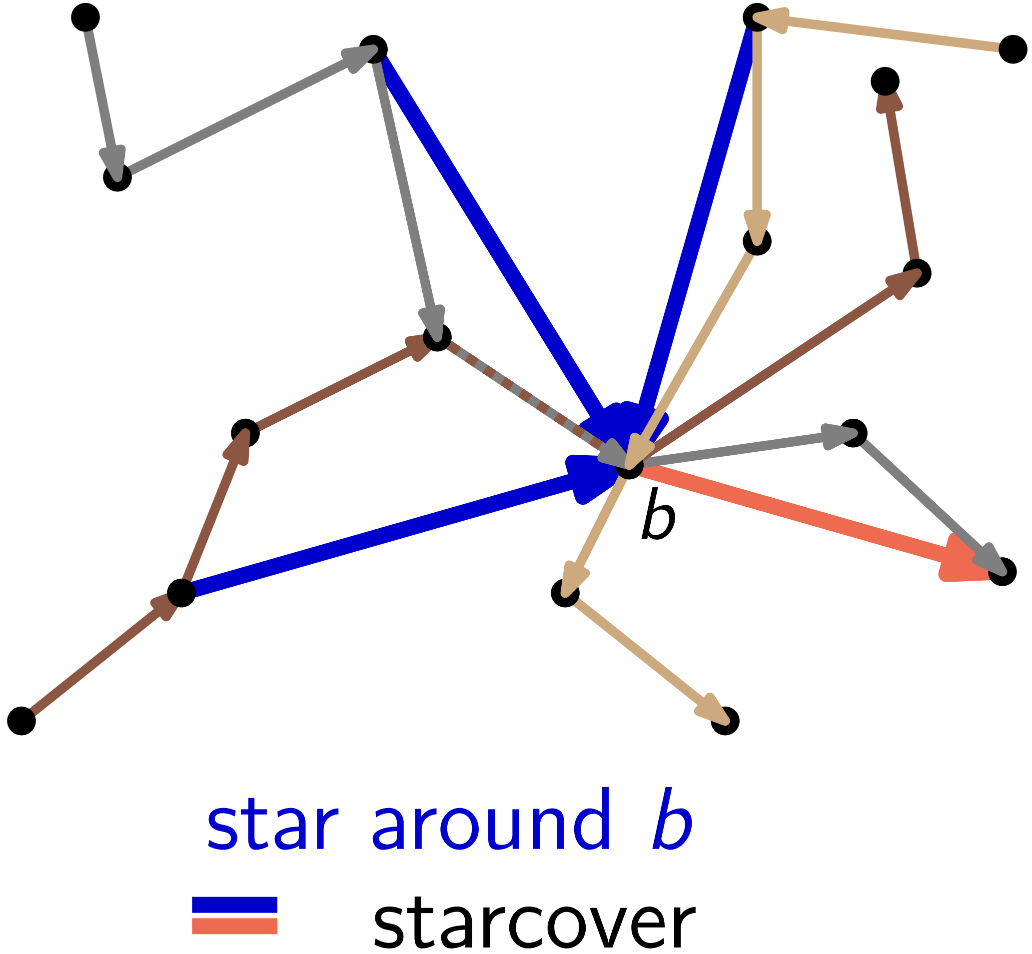

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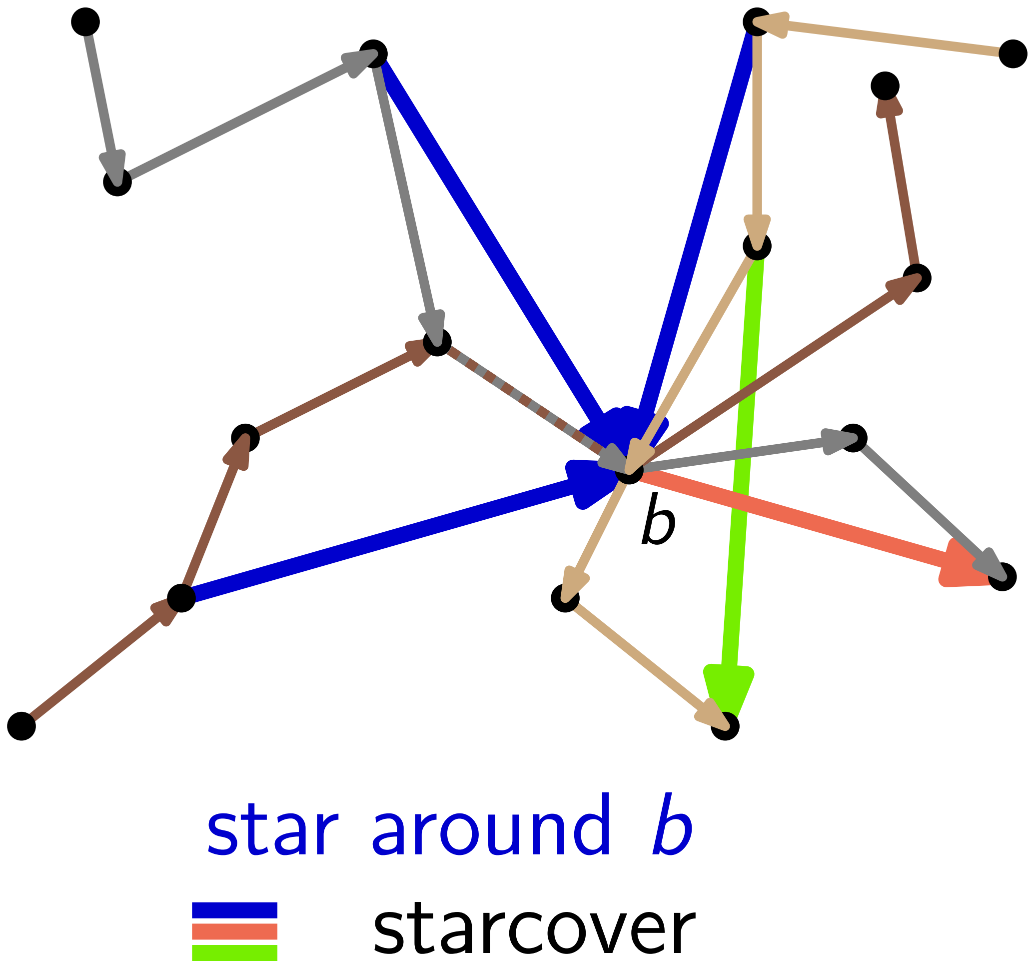

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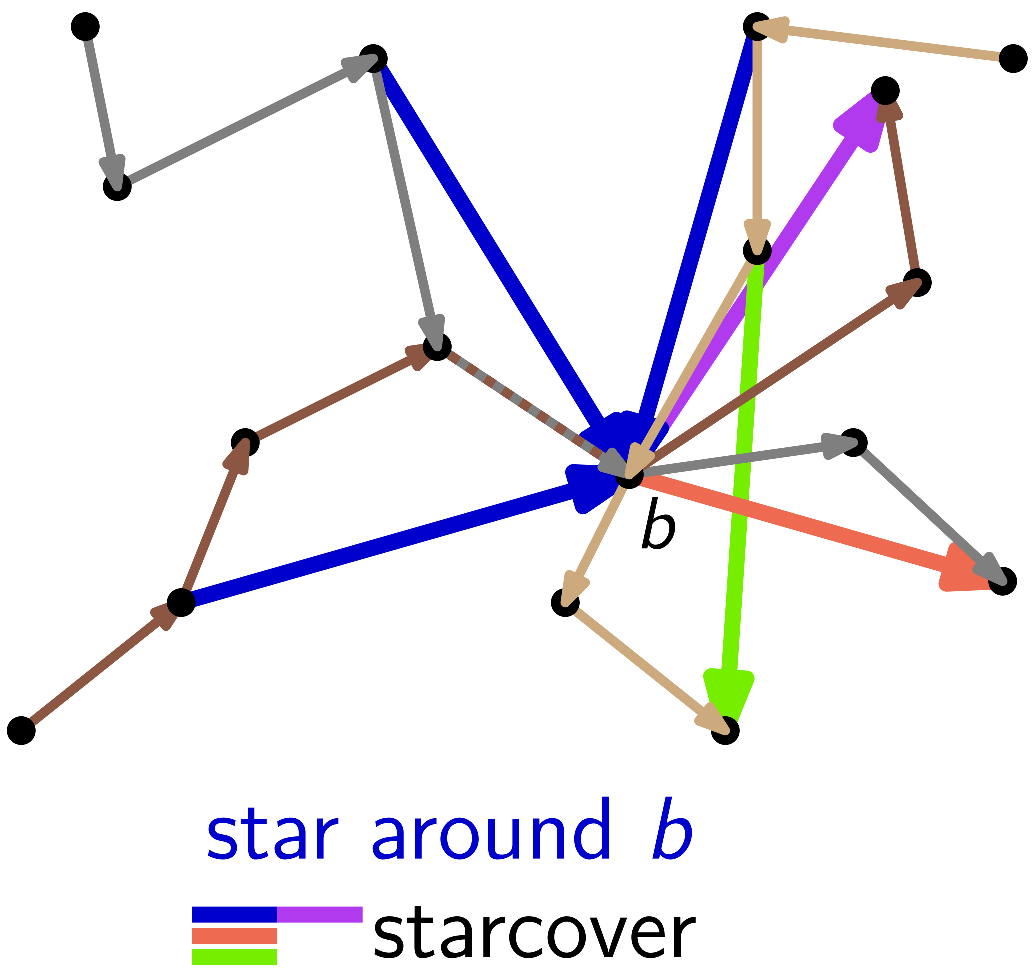

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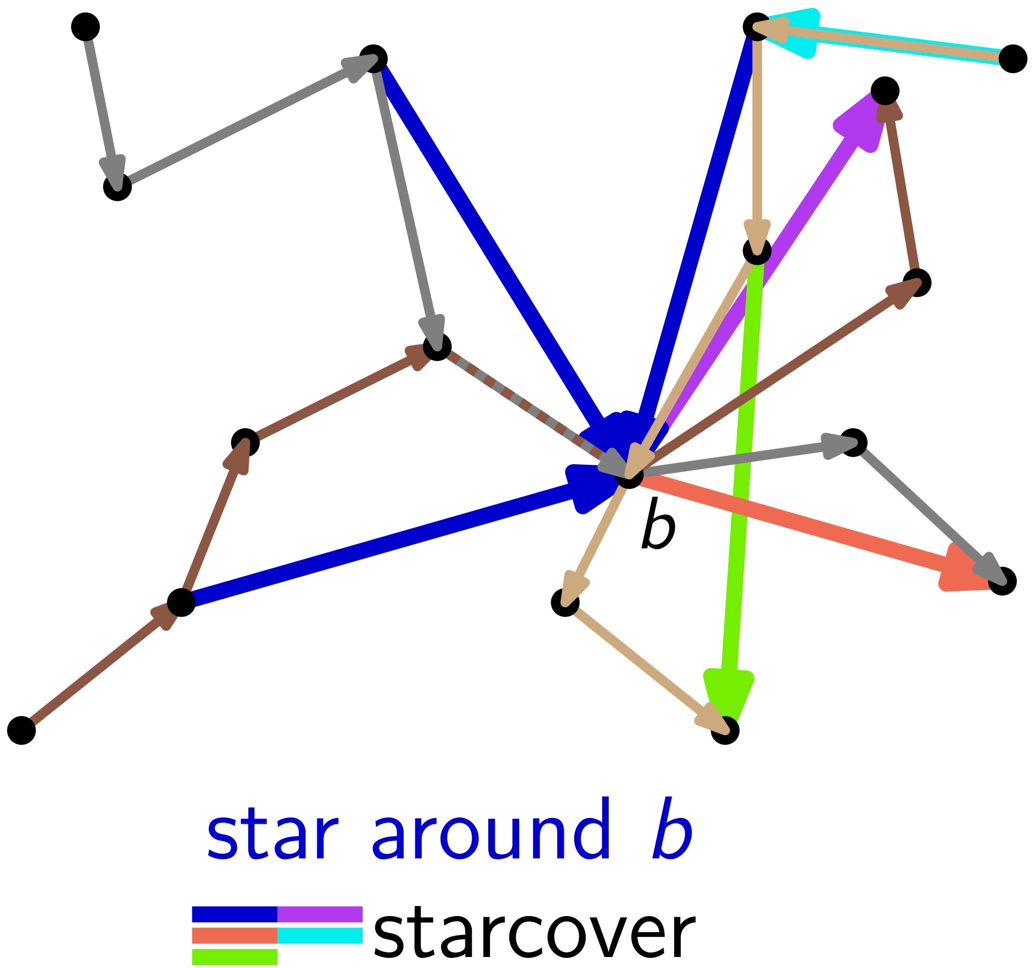

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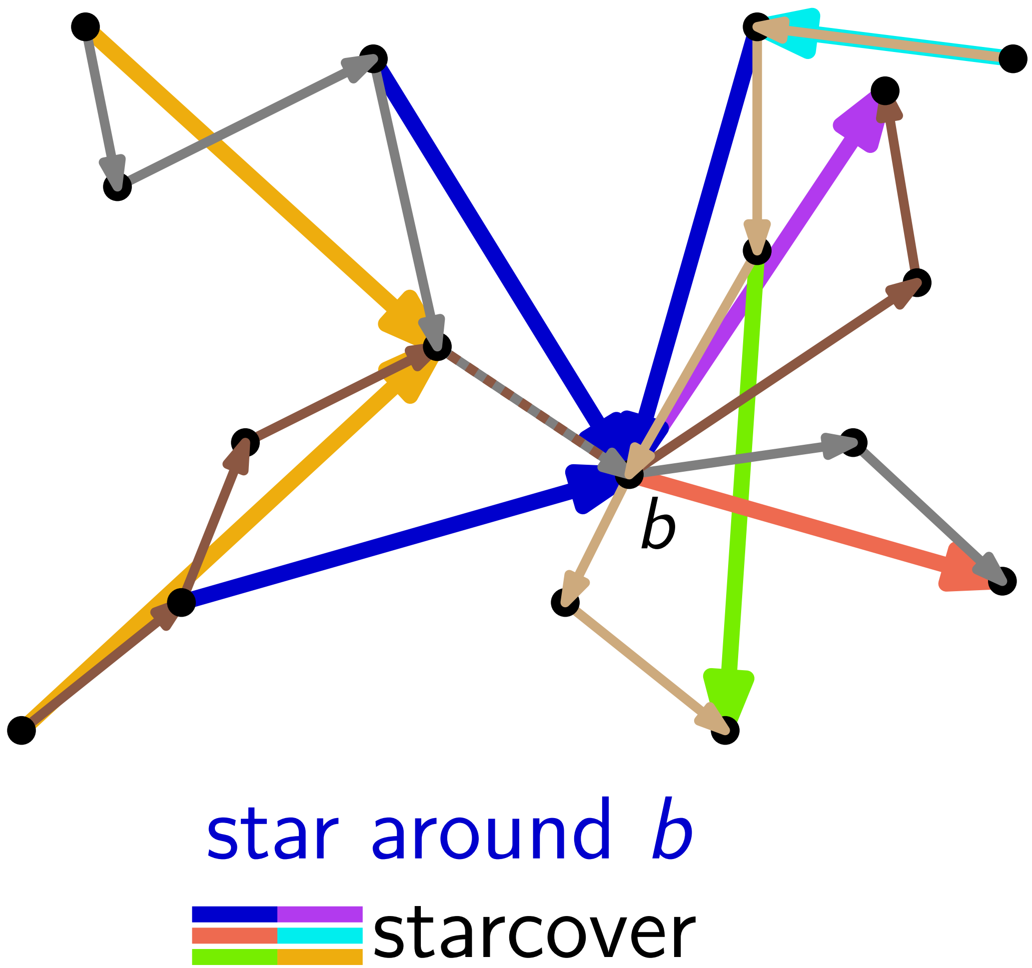

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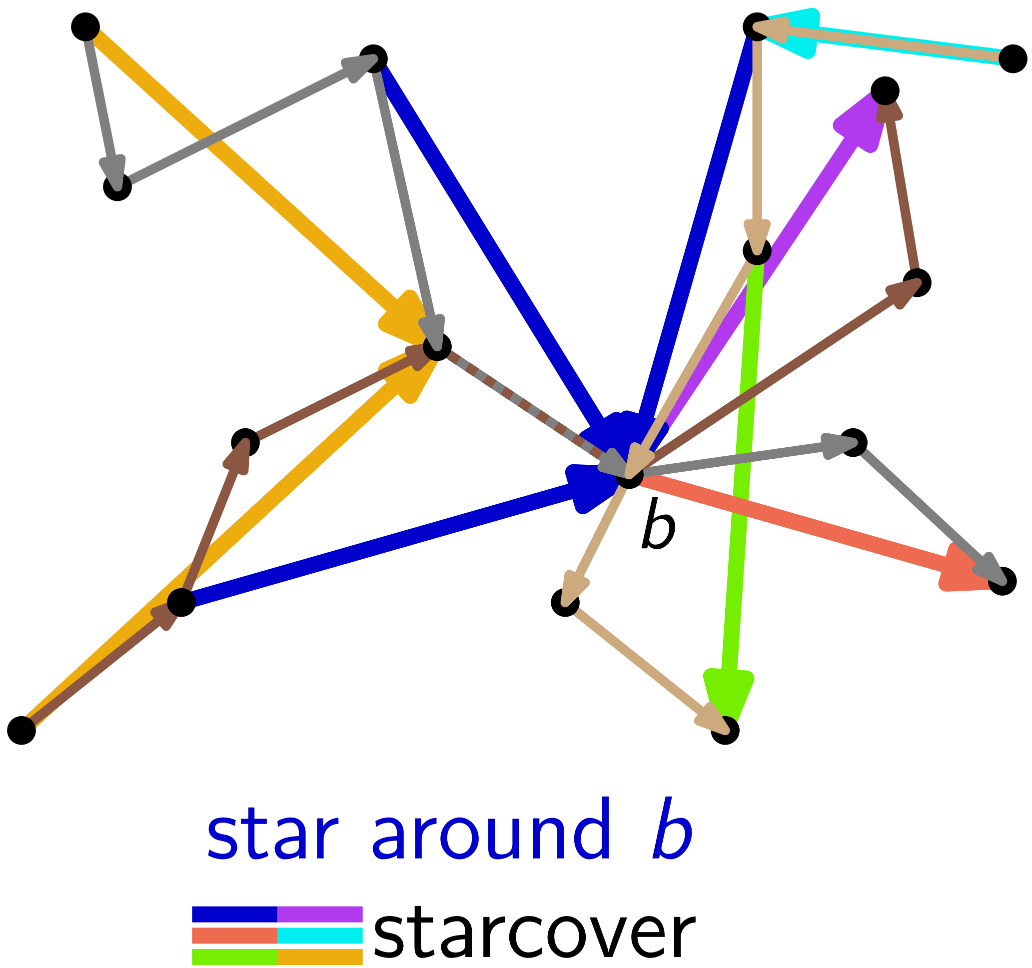

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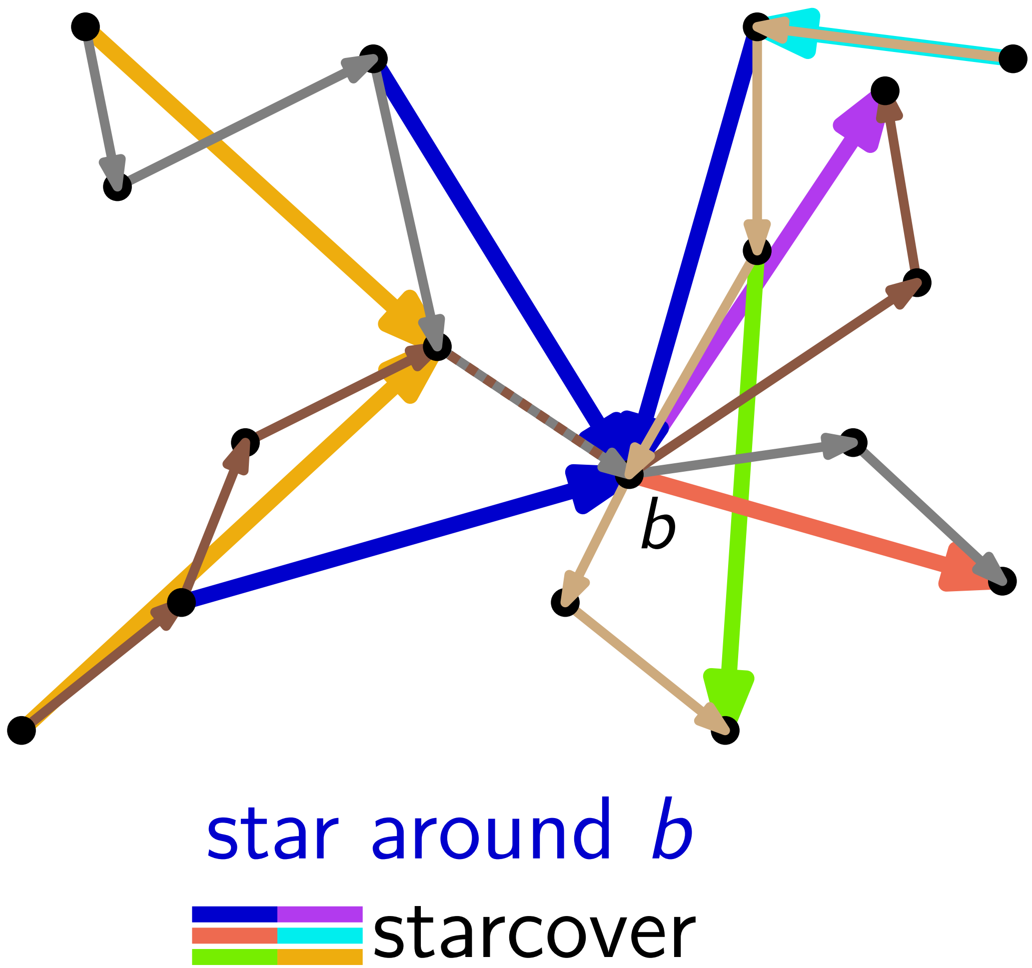

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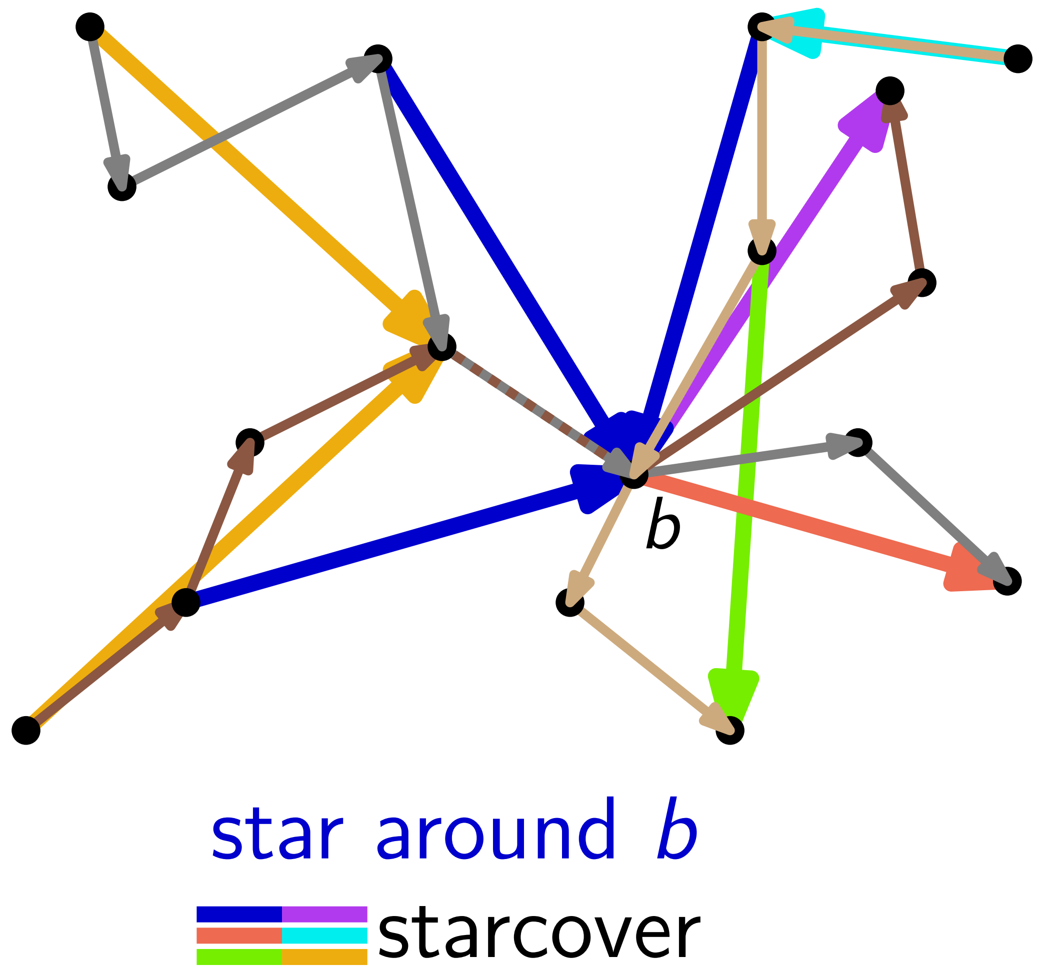

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- any star covers $\leq n \cdot \ell$ polyline–segment pairs

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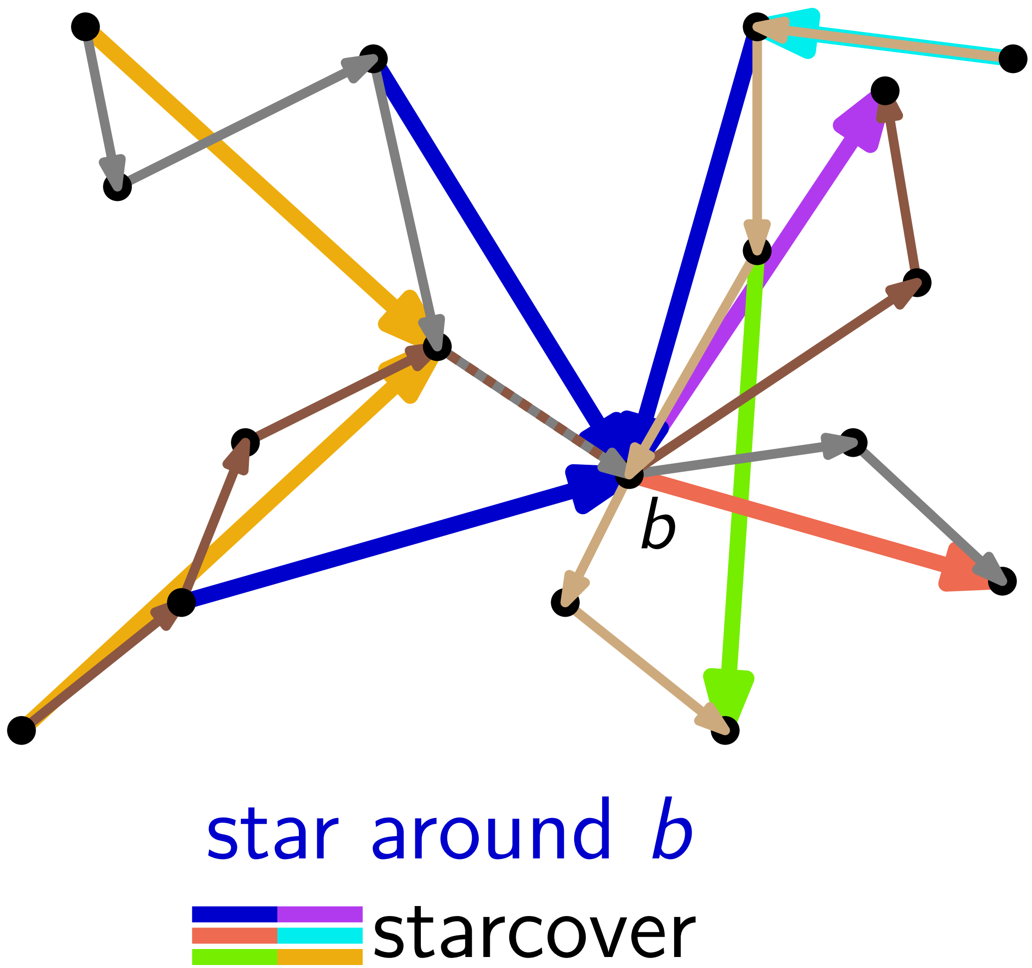

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$\Rightarrow$ standard greedy set cover yields $O(\log(n \cdot \ell)) = O(\log(n + \ell))$ approximation

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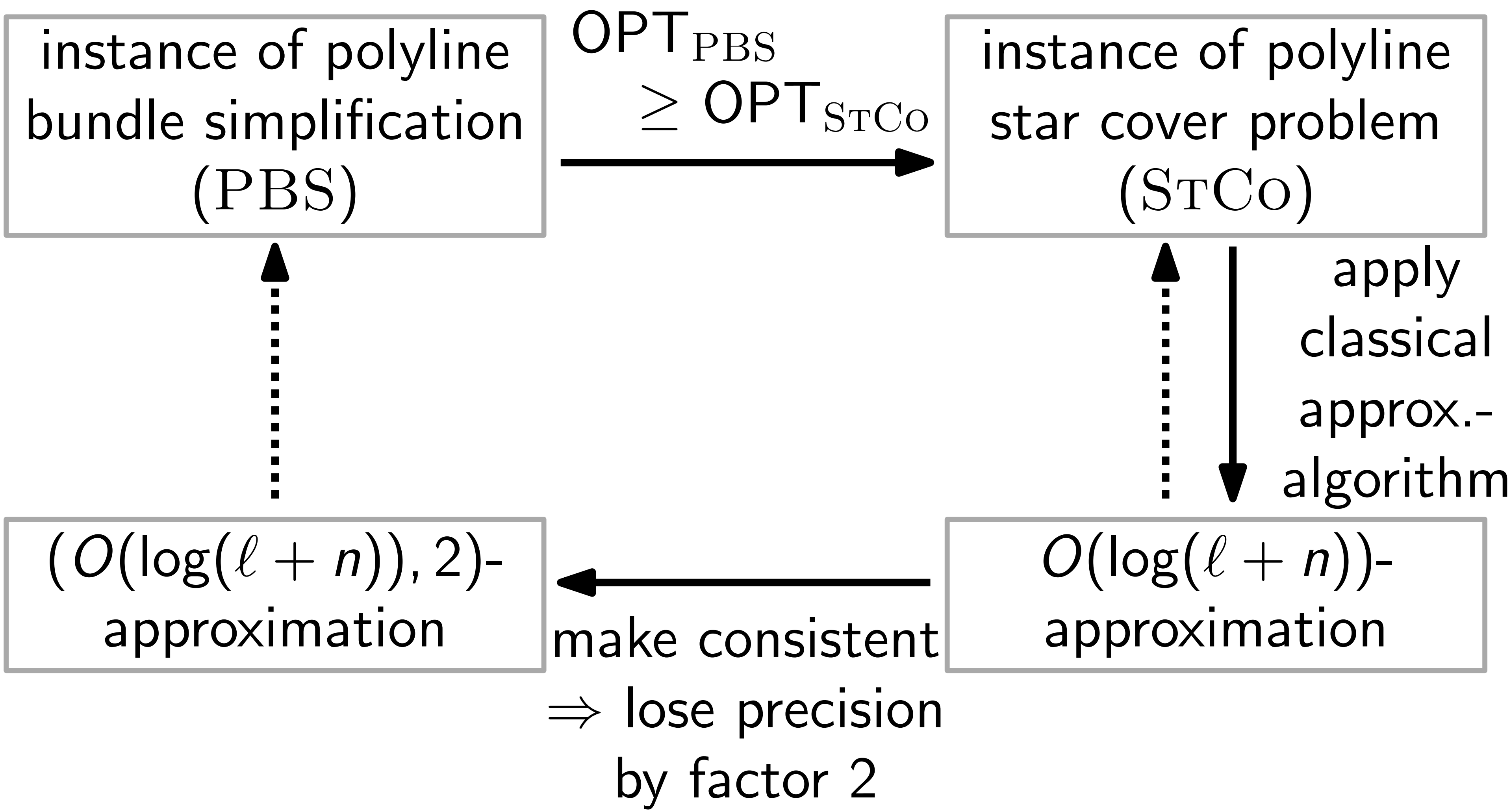

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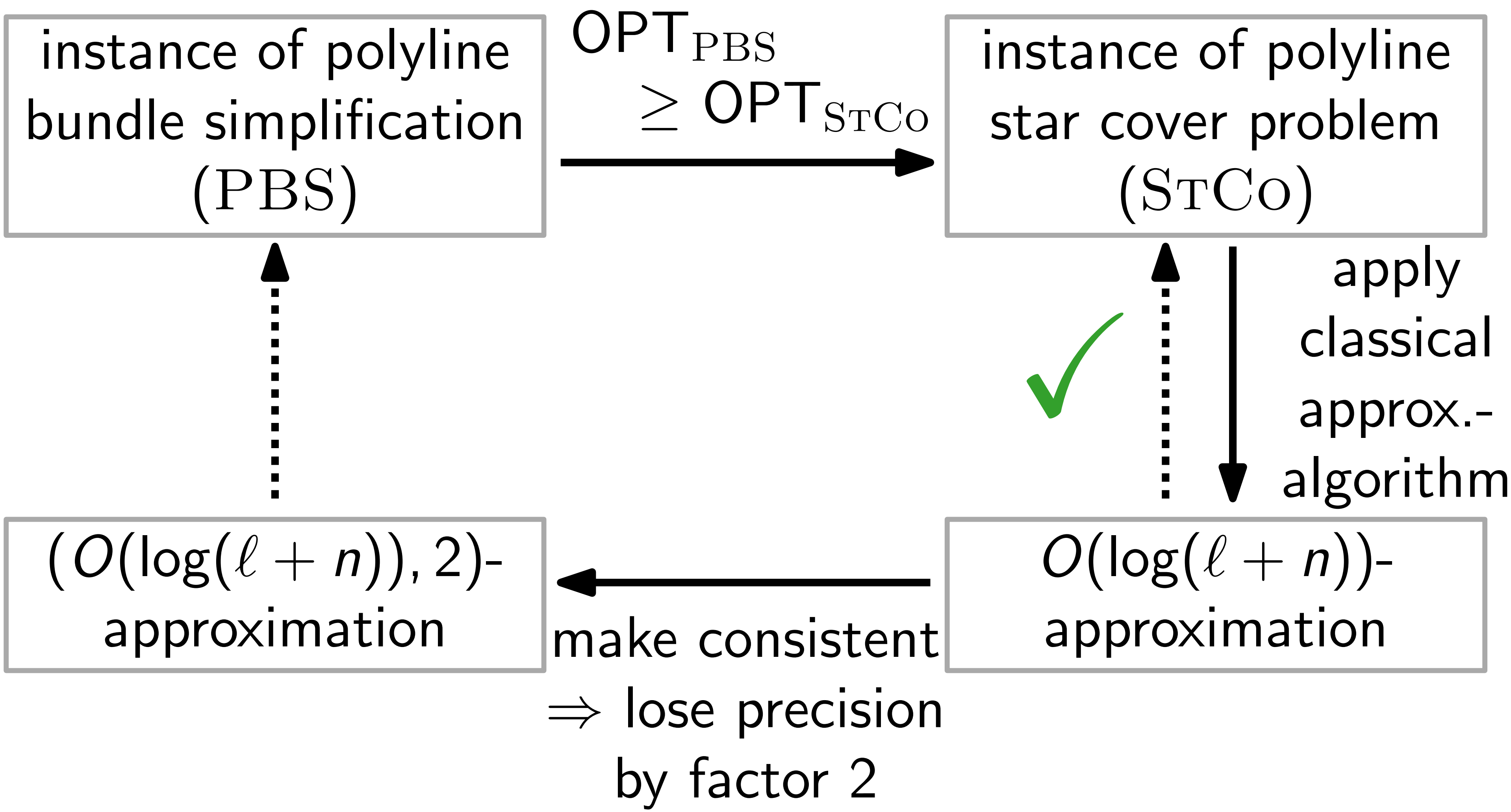

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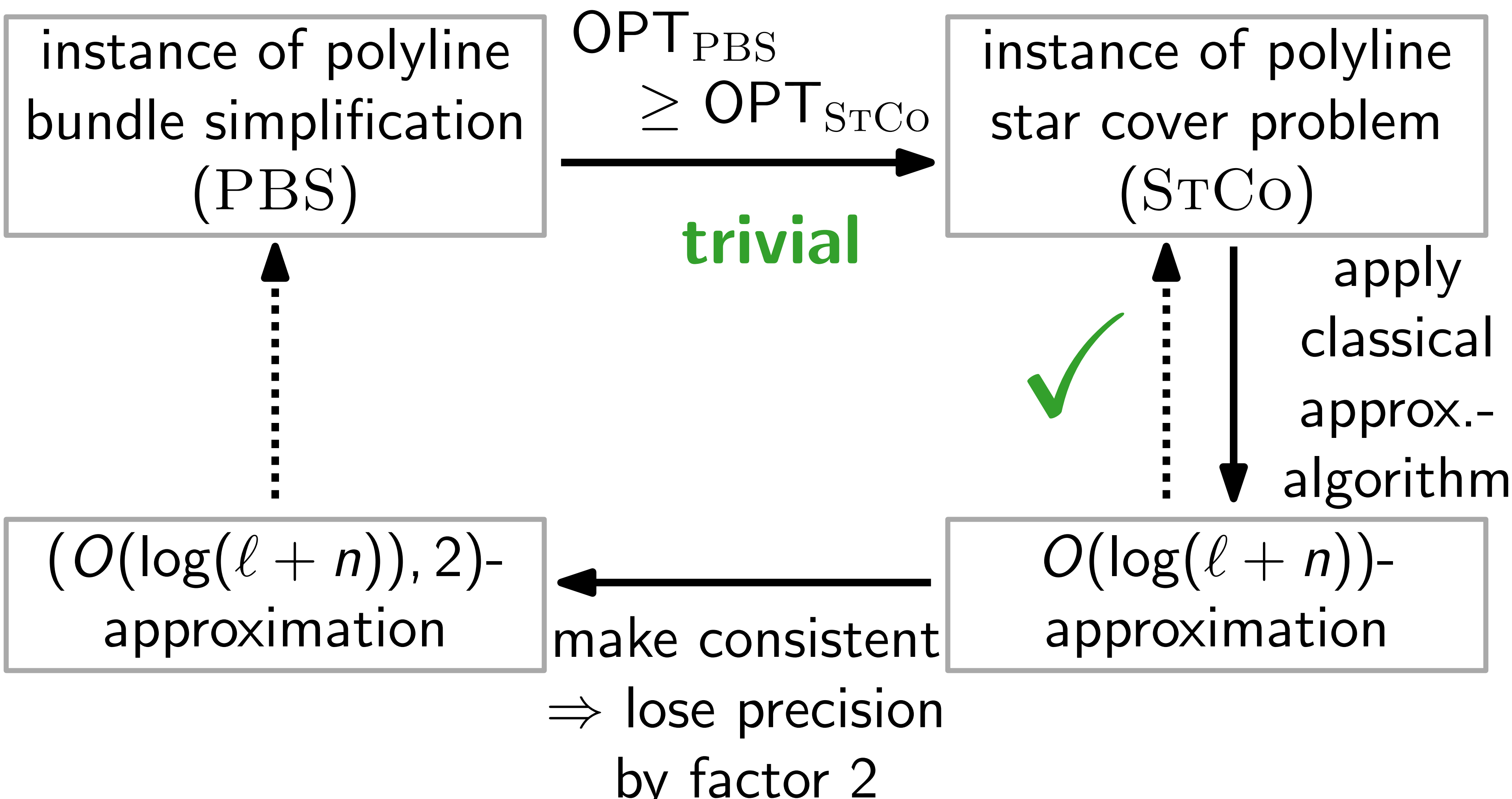

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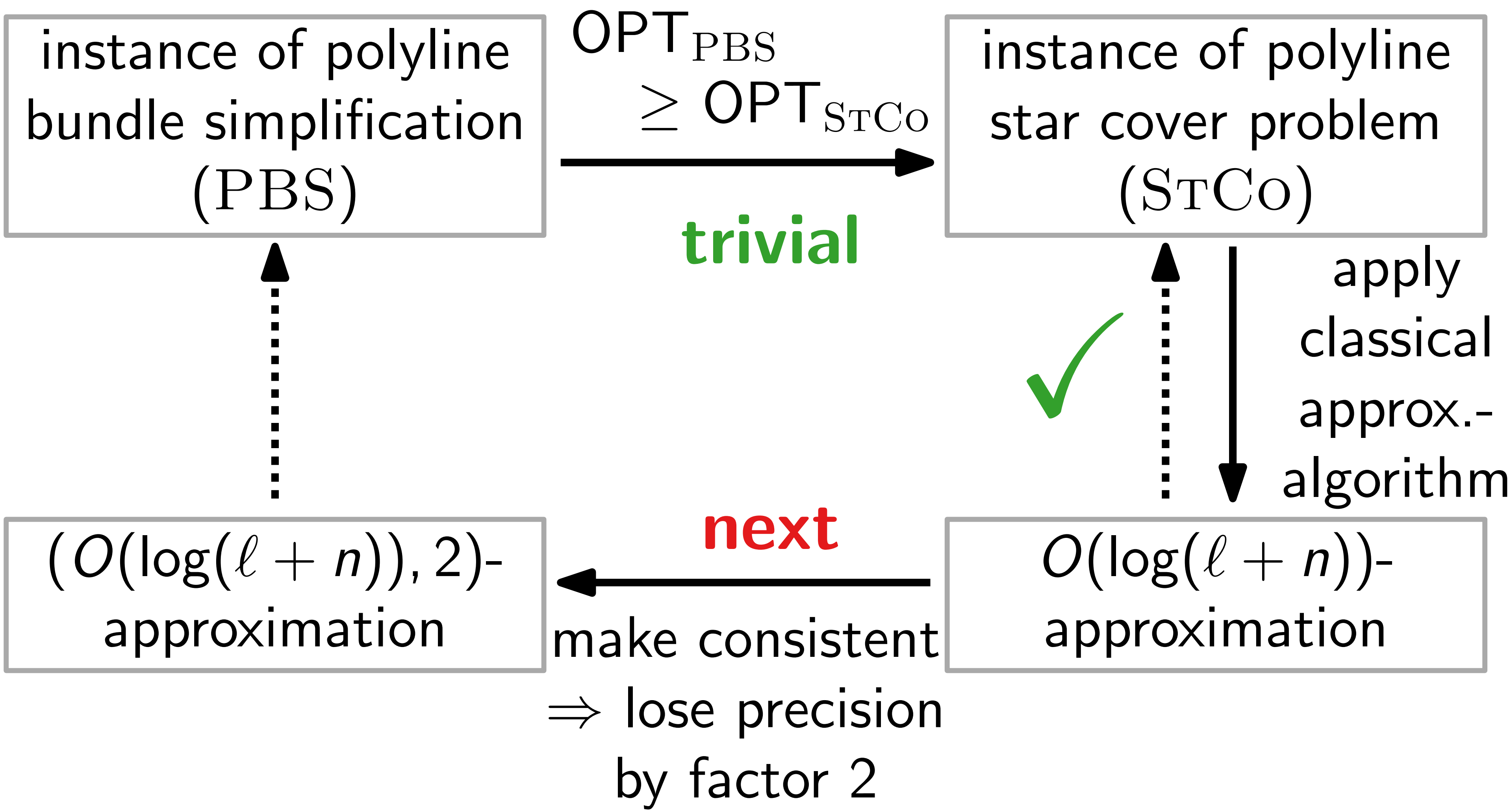
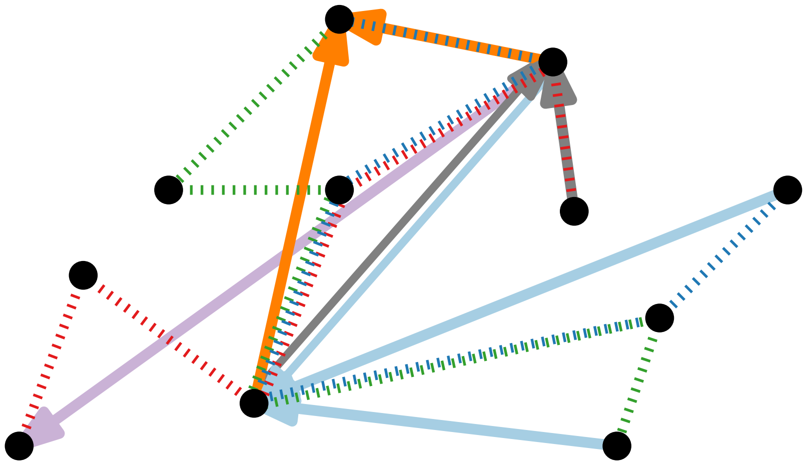


StCo solution → PBS solution



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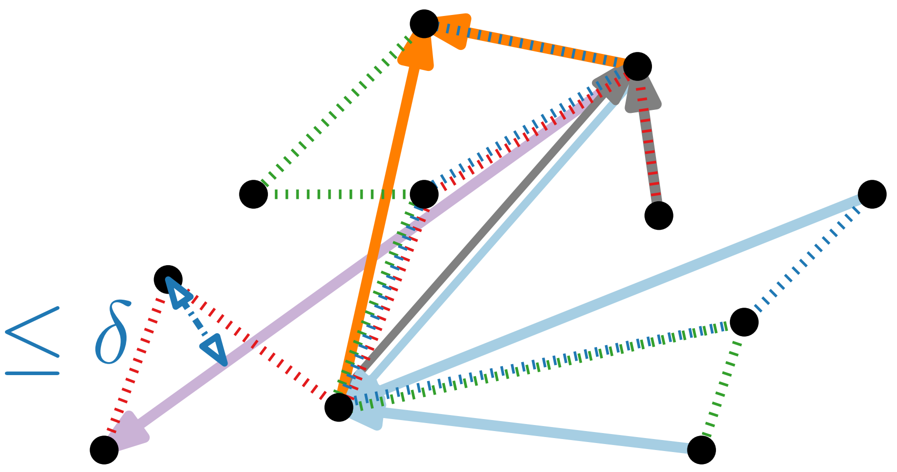

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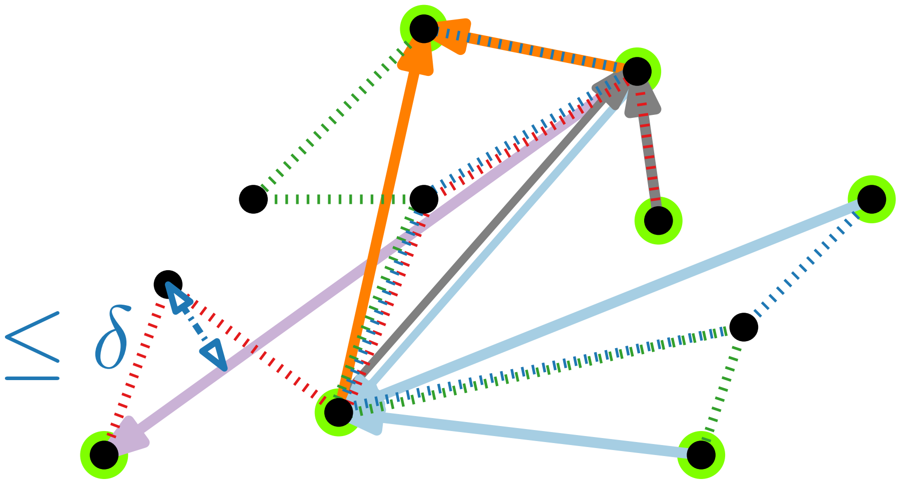

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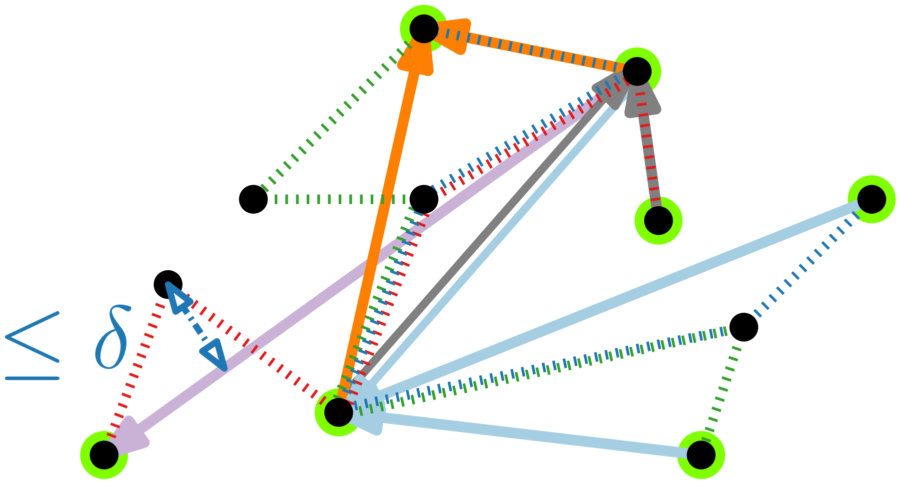

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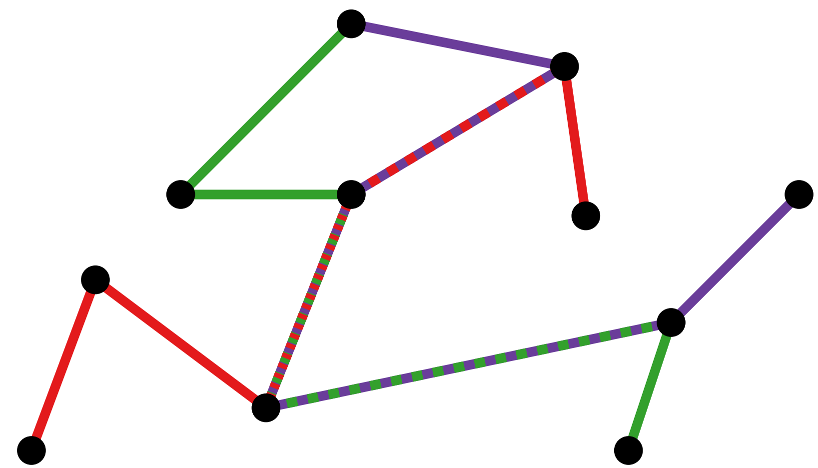

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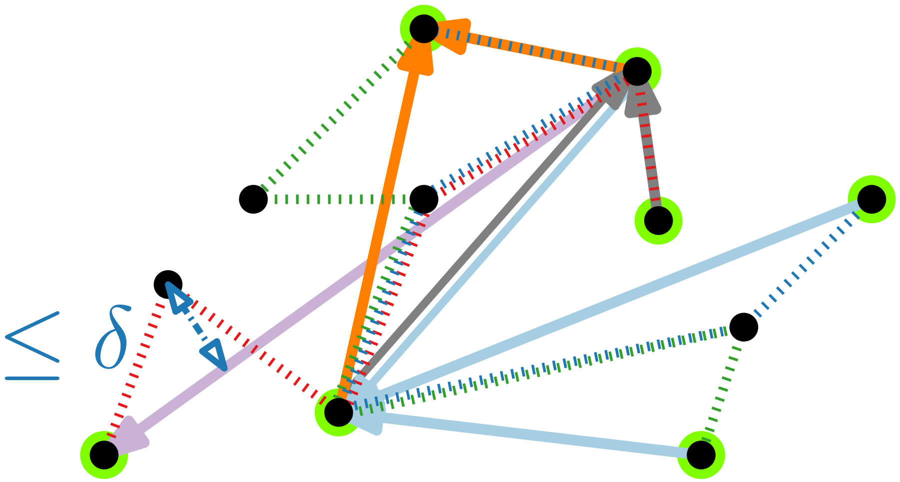

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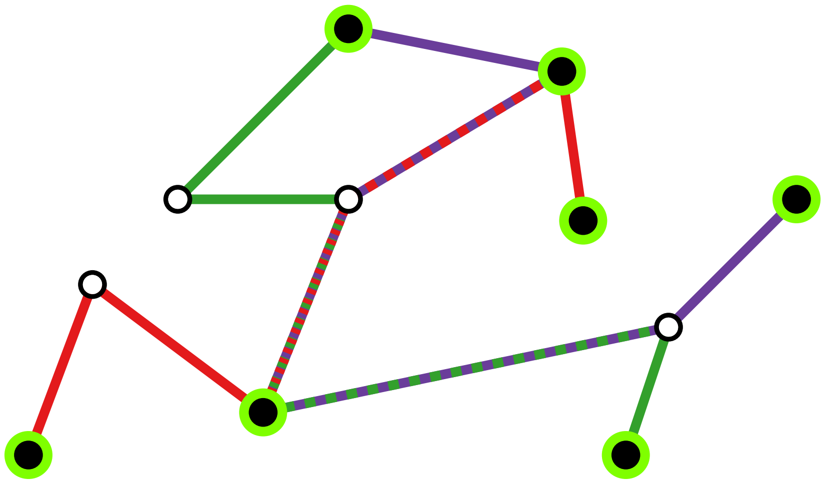

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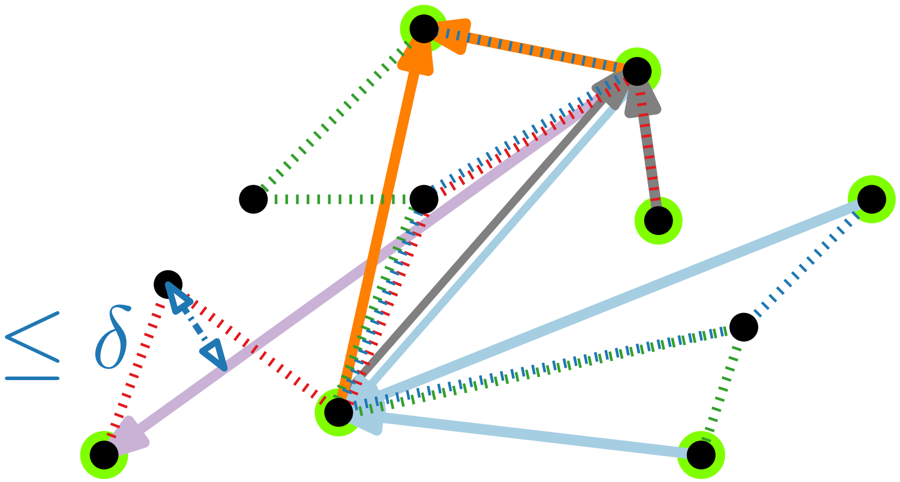

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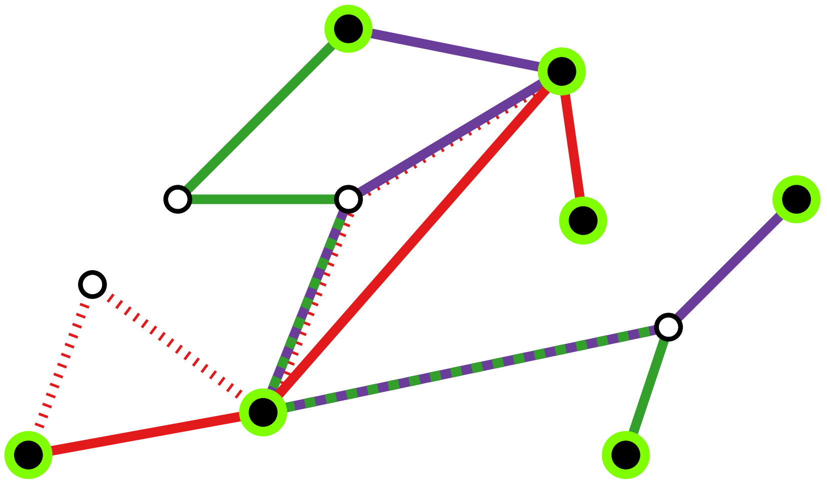

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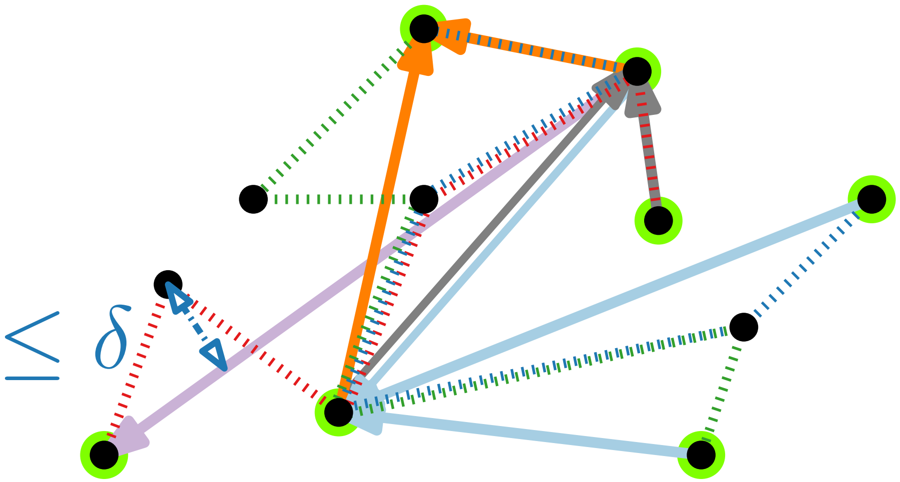

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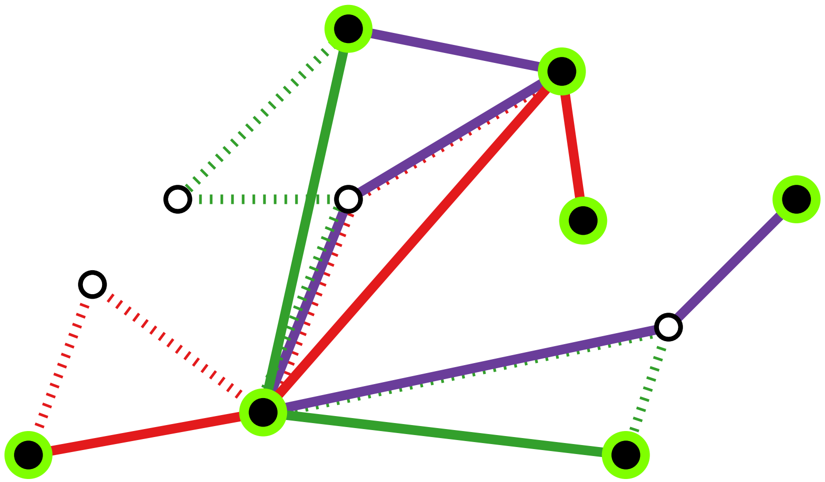

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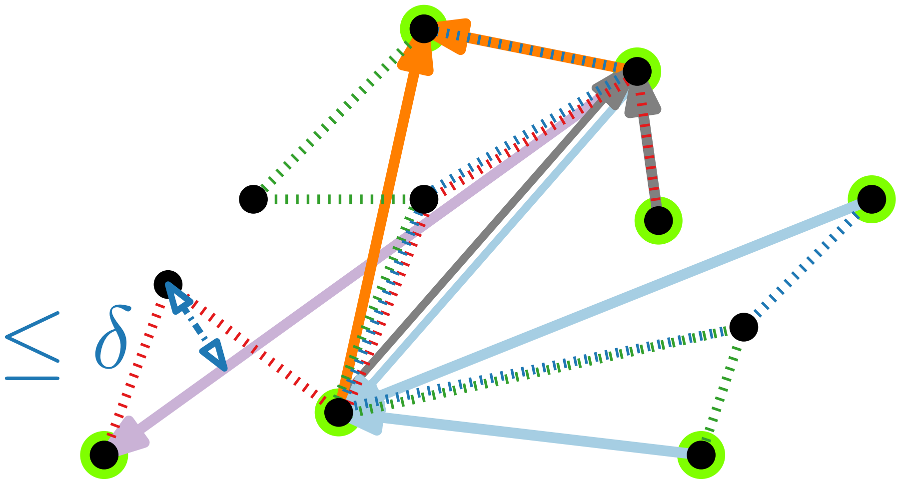

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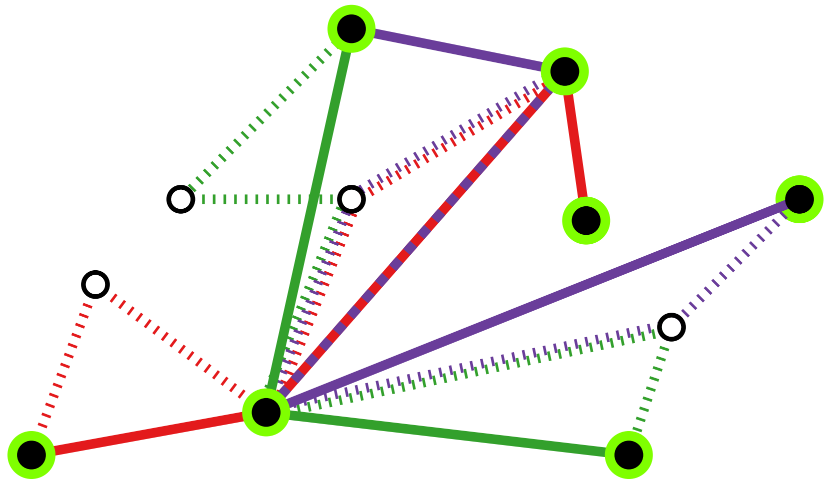

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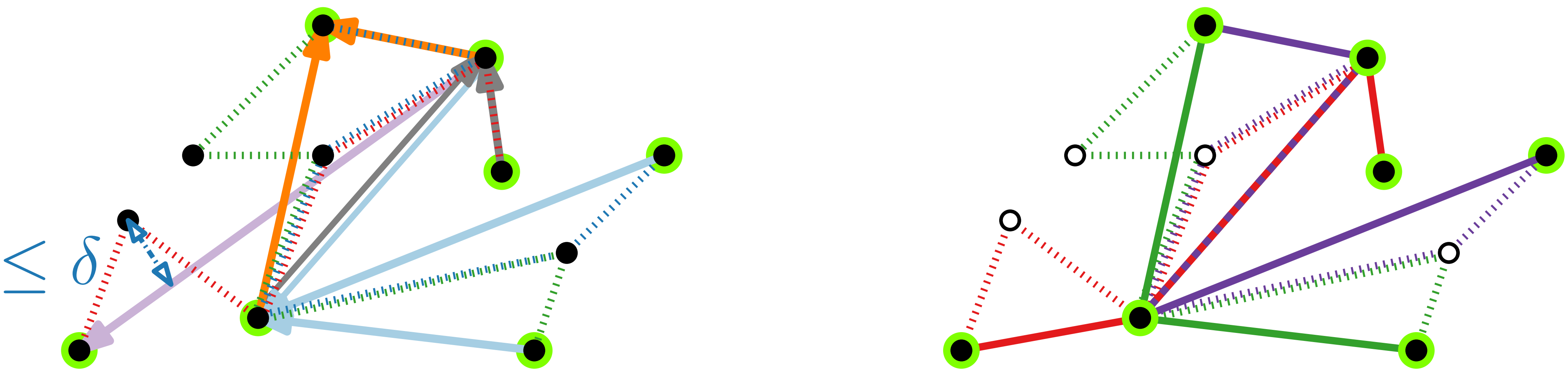

StCo solution $\Rightarrow$ PBS solution

Lemma: [Agarwal et al.]
If $d_{\text{Fréchet}}((b_y, b_z), (b_y, \ldots, b_i, \ldots, b_j, \ldots, b_z)) \leq \delta$,
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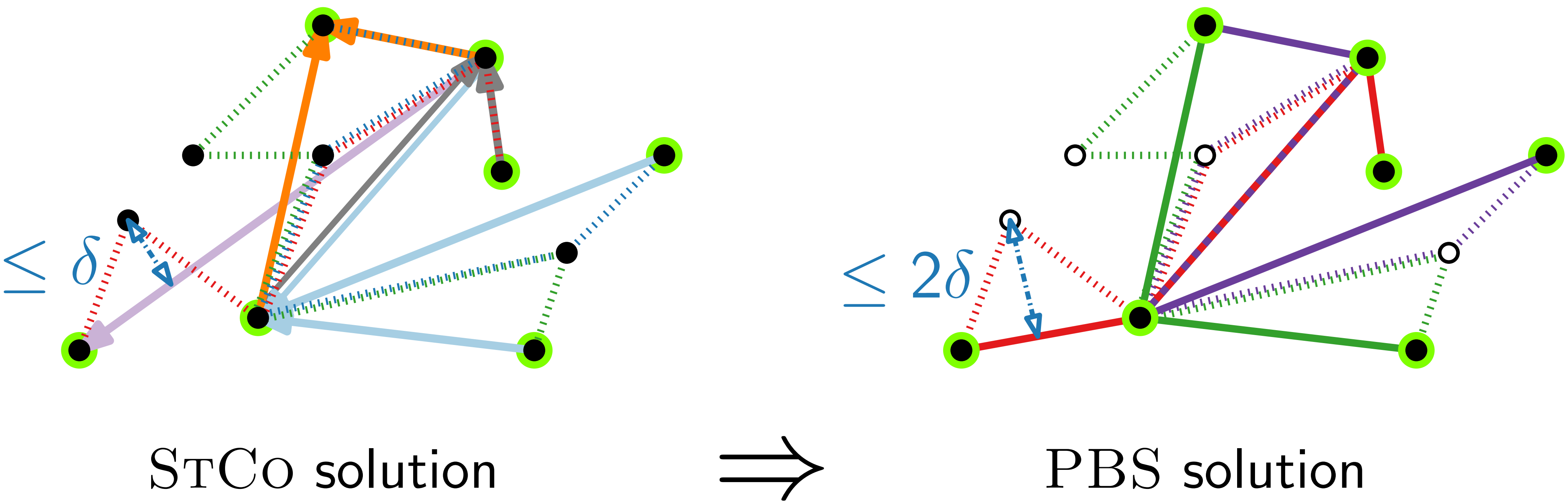

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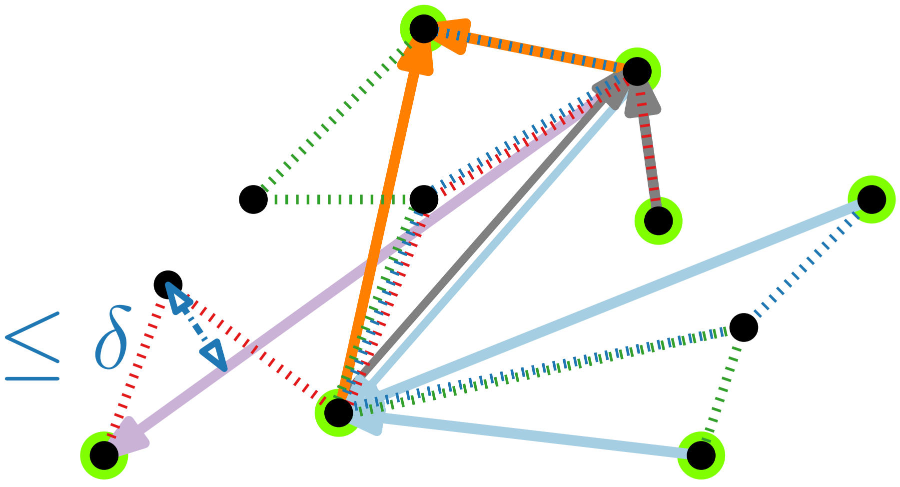

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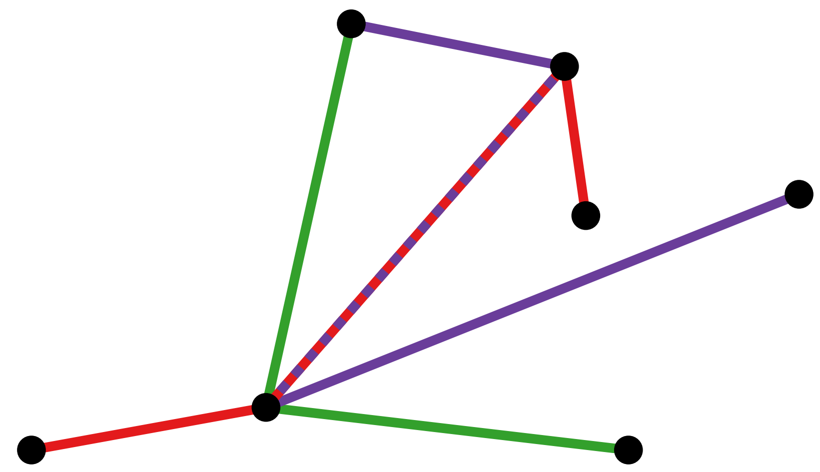

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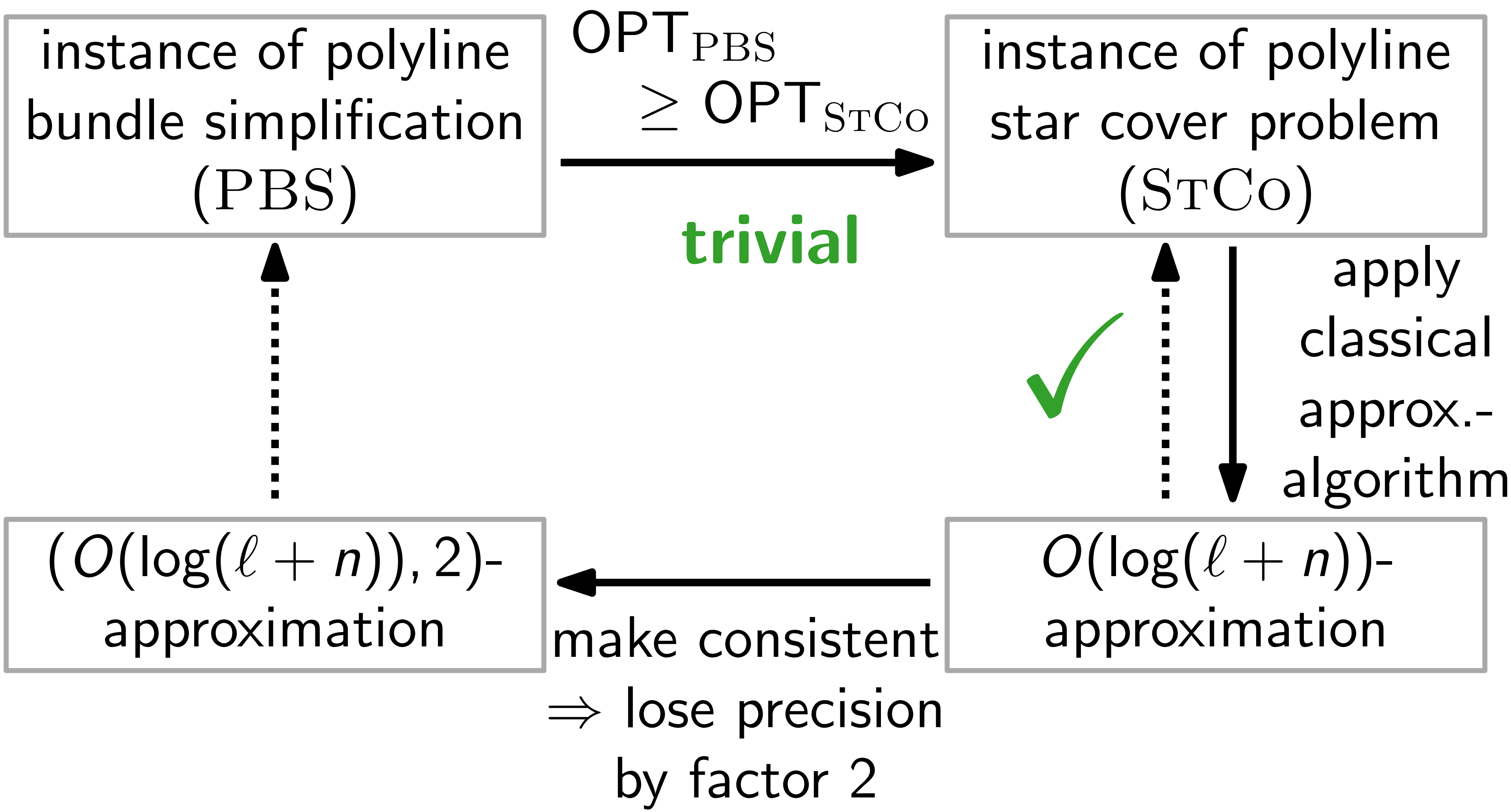

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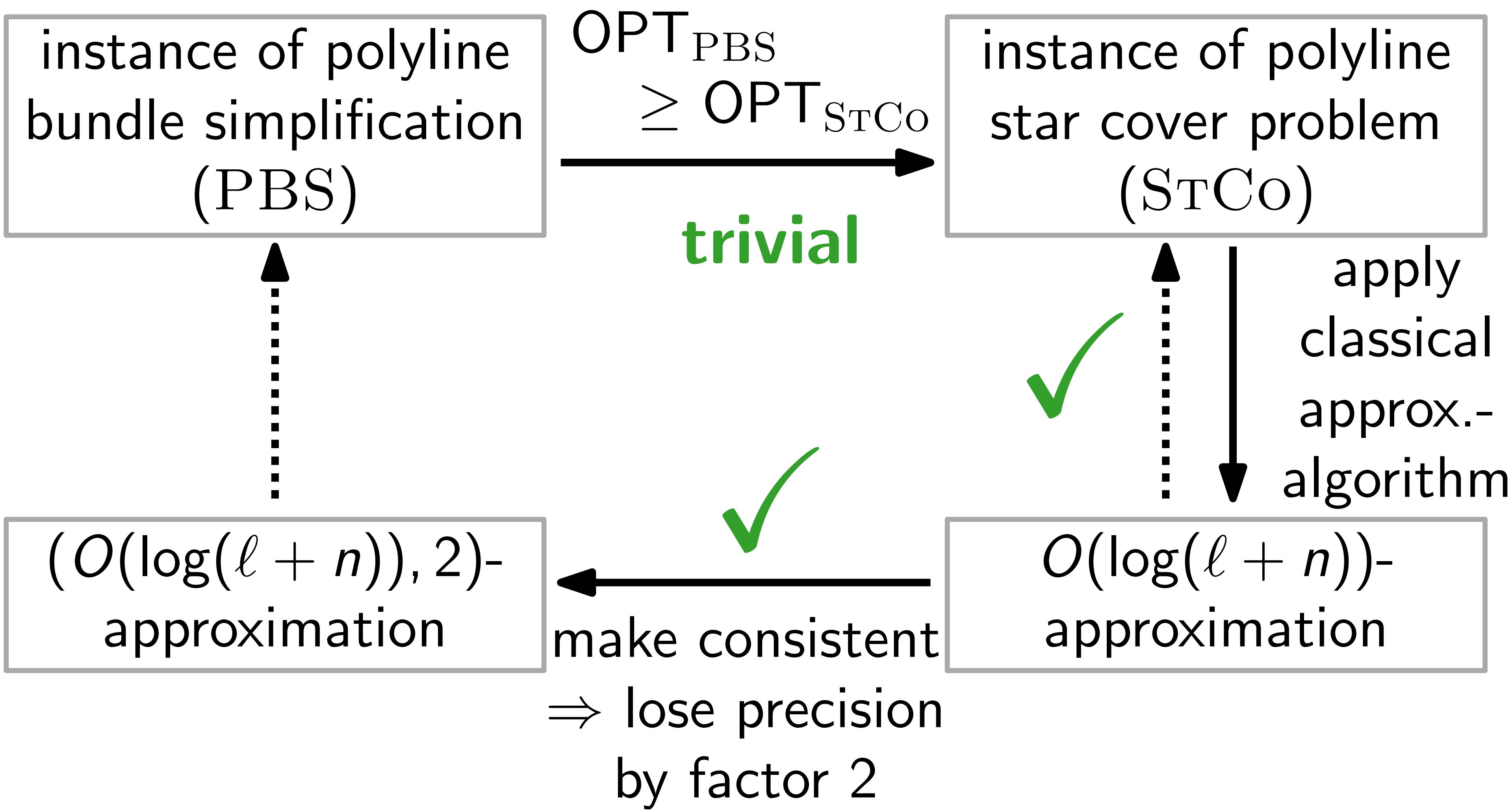

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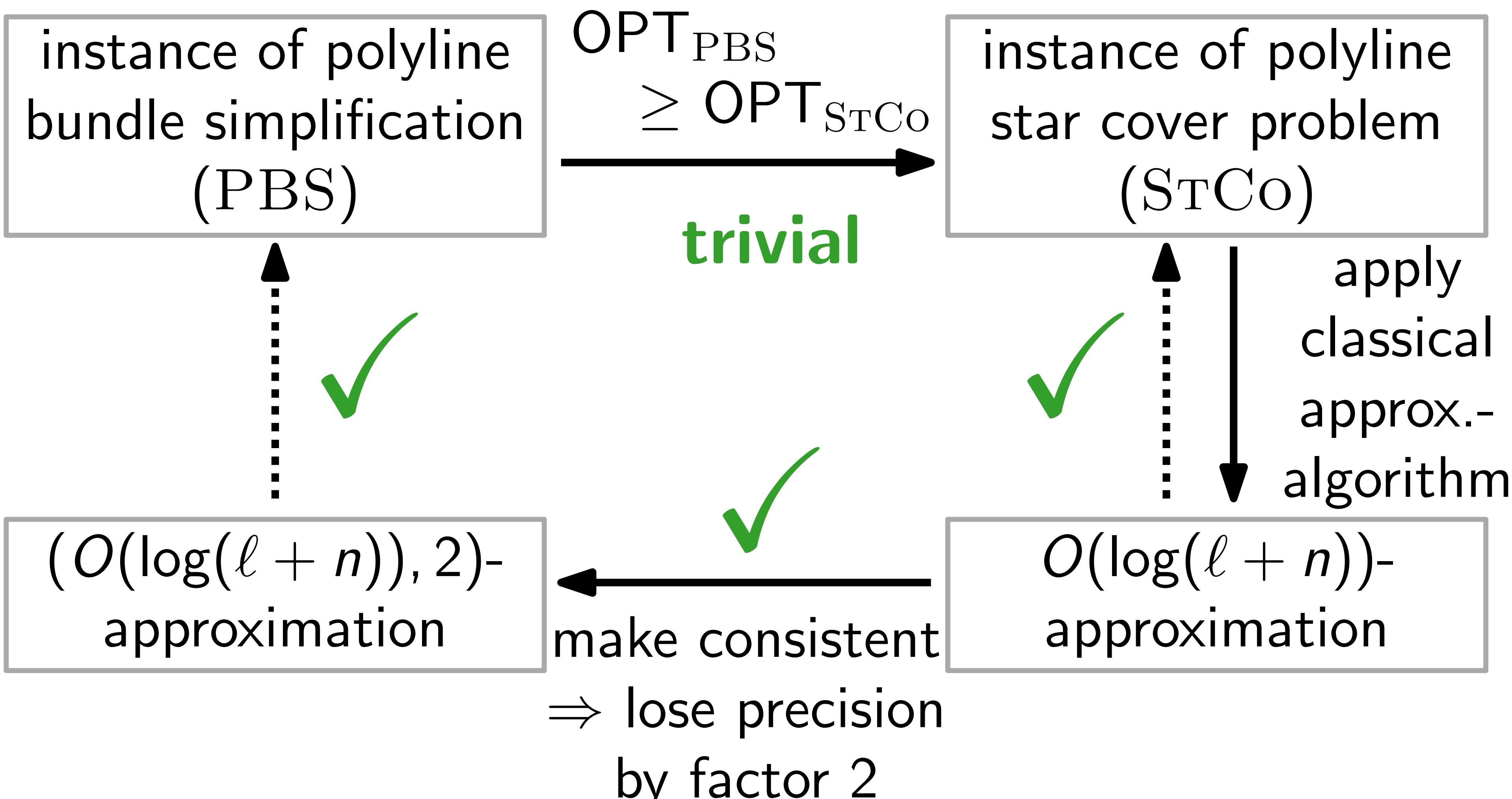

Agenda

1. Motivation and Introduction
2. Problem Definition
3. Hardness of Approximation (+ Proof Sketch)
4. Bi-Criteria Approximation (+ Proof Sketch)
5. **Summary**

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