Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG
Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems
Institute for Informatics

Simplification of Polyline Bundles

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Aalto University, Finland

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Universität Konstanz, Germany

Johannes Zink
Universität Würzburg, Germany

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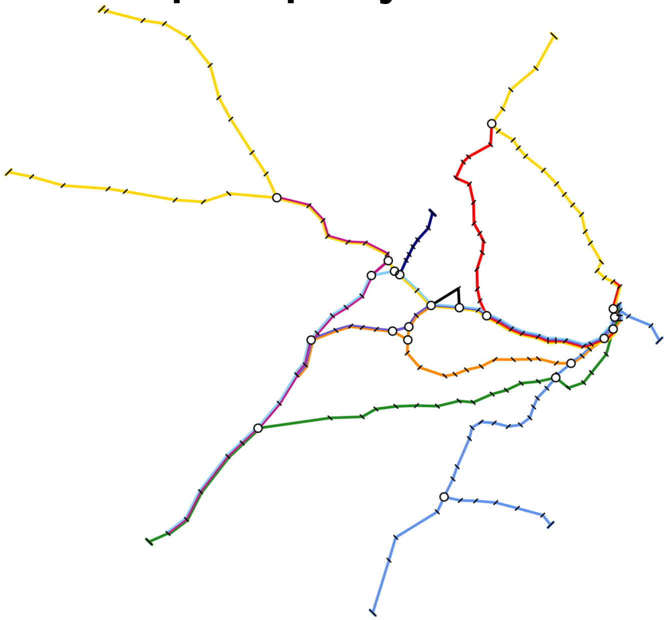

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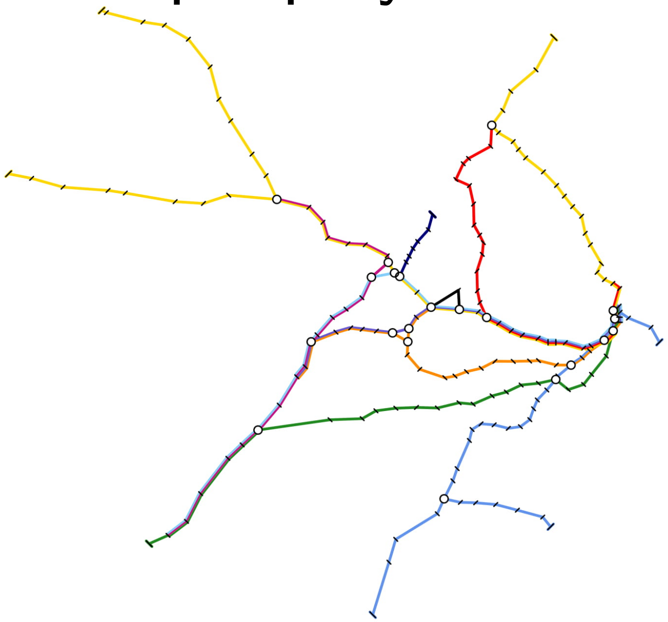

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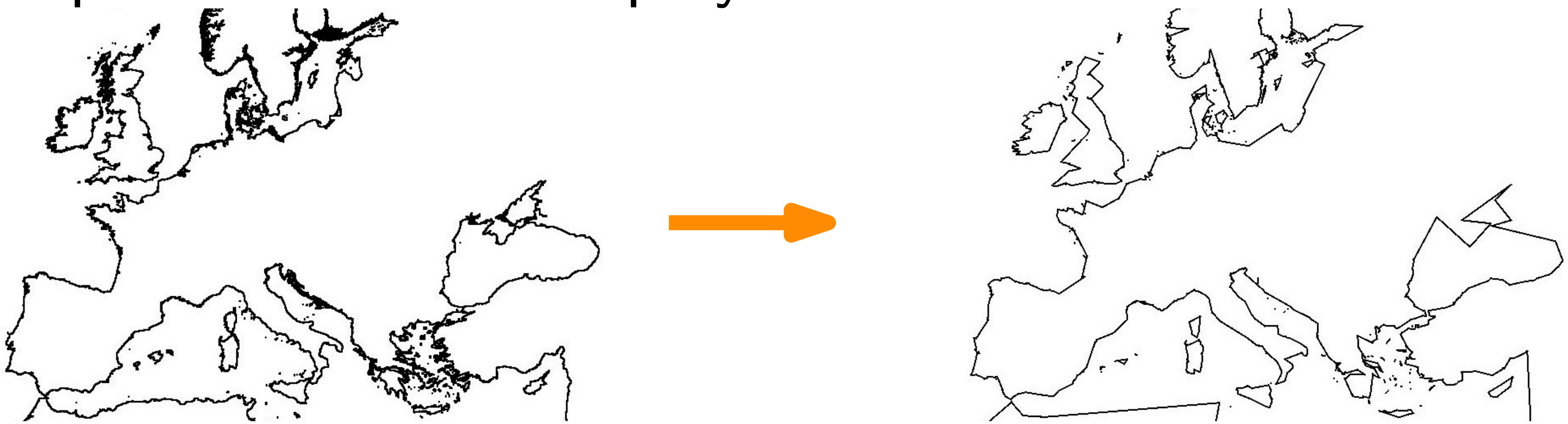

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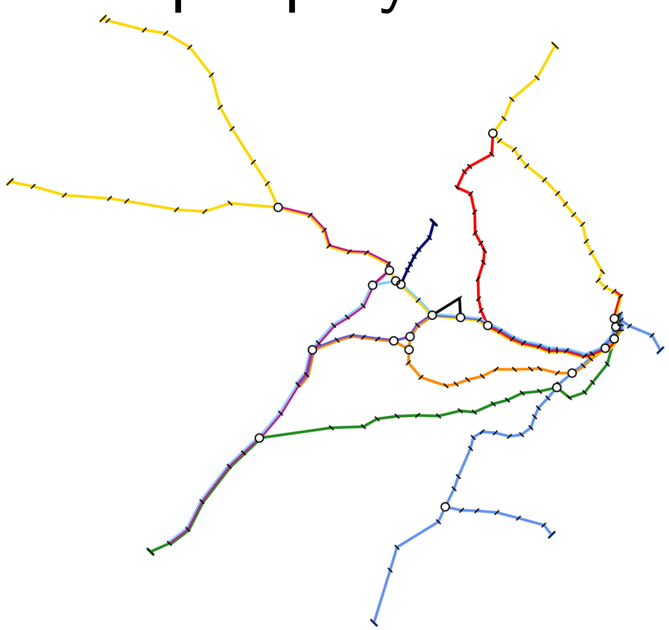

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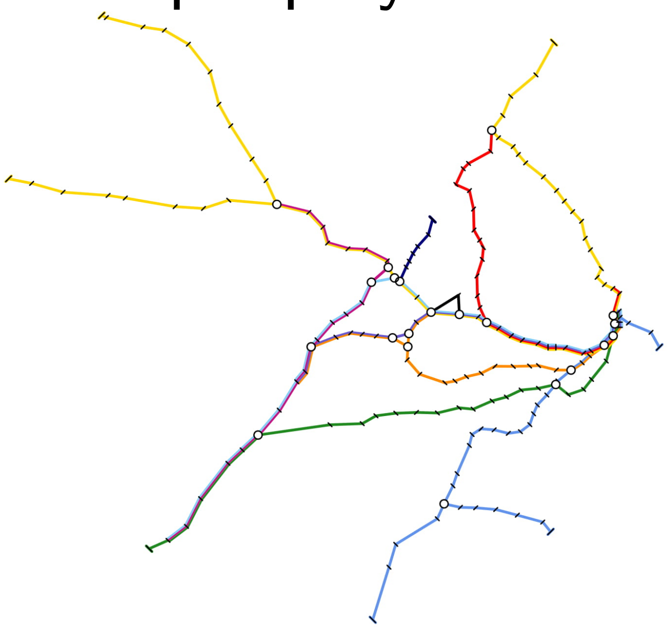

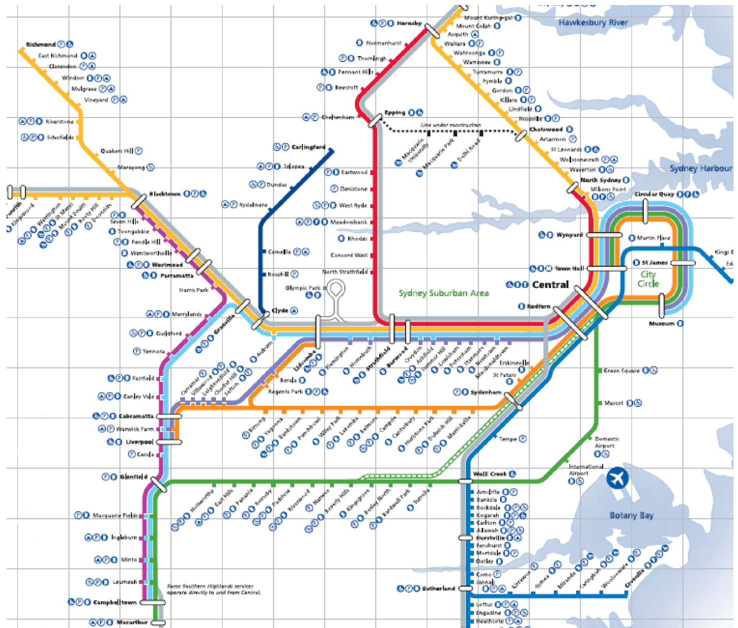

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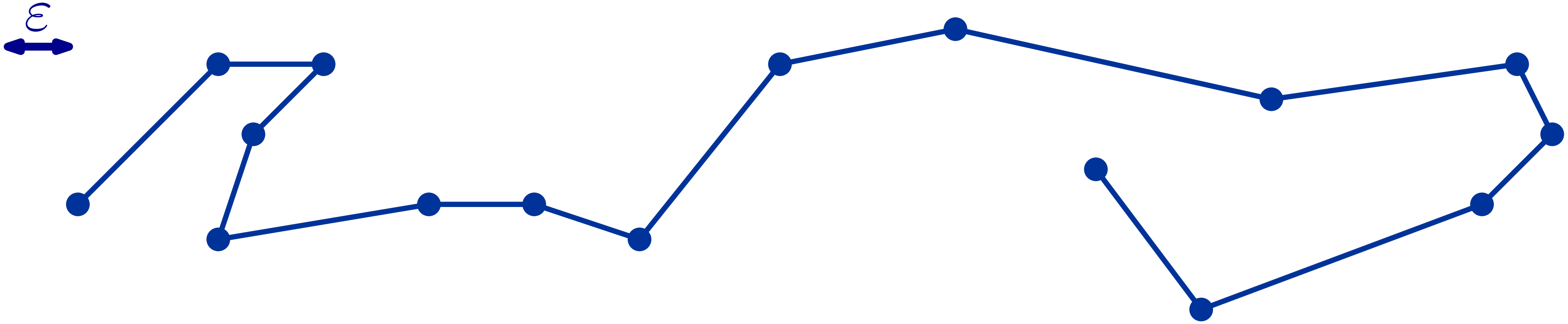

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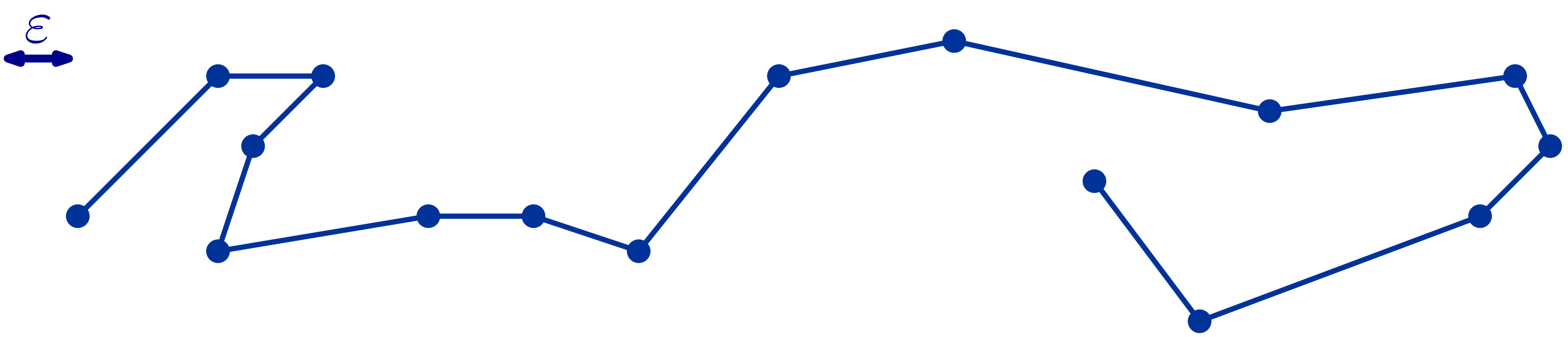

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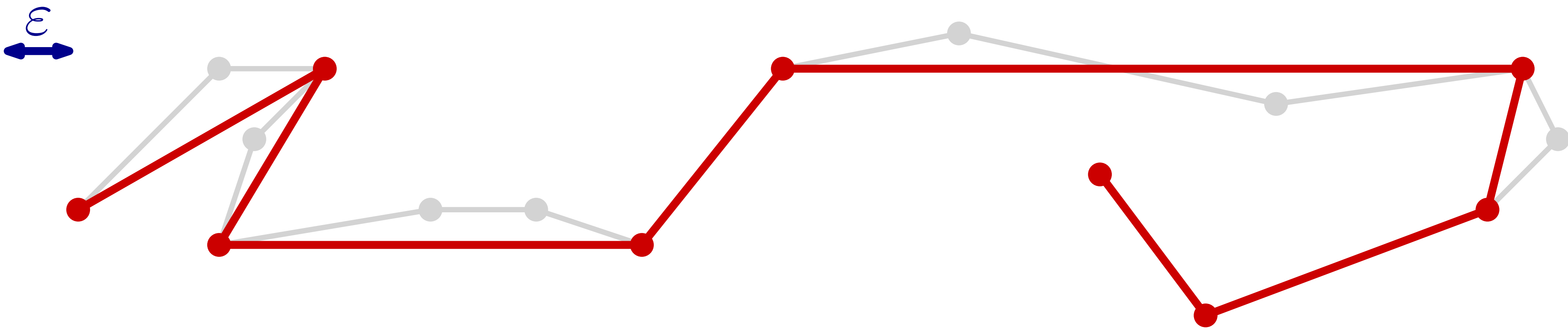

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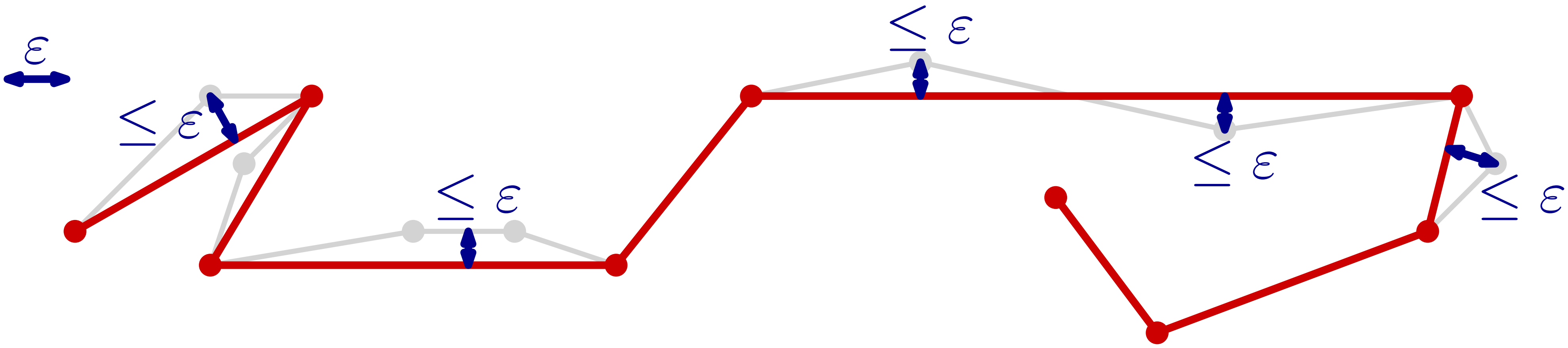

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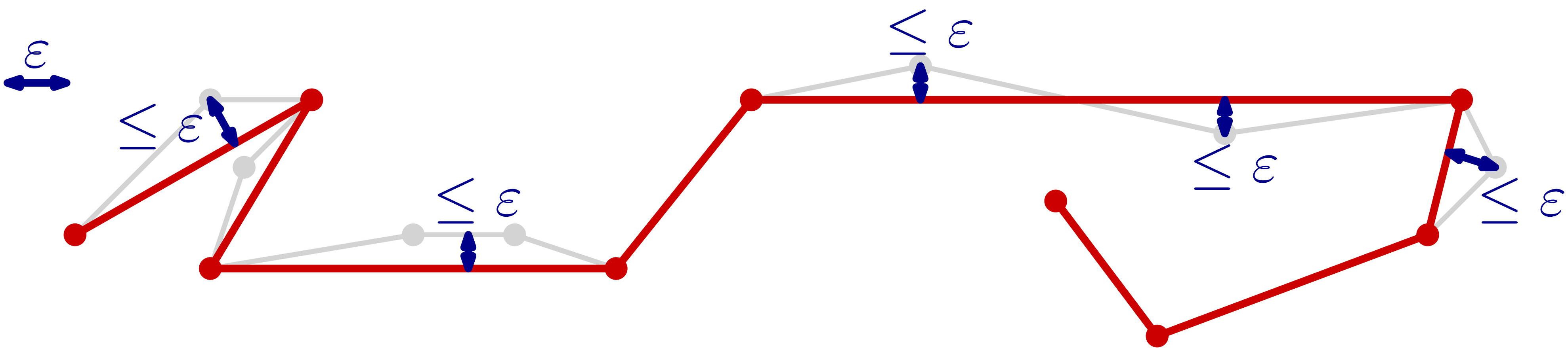

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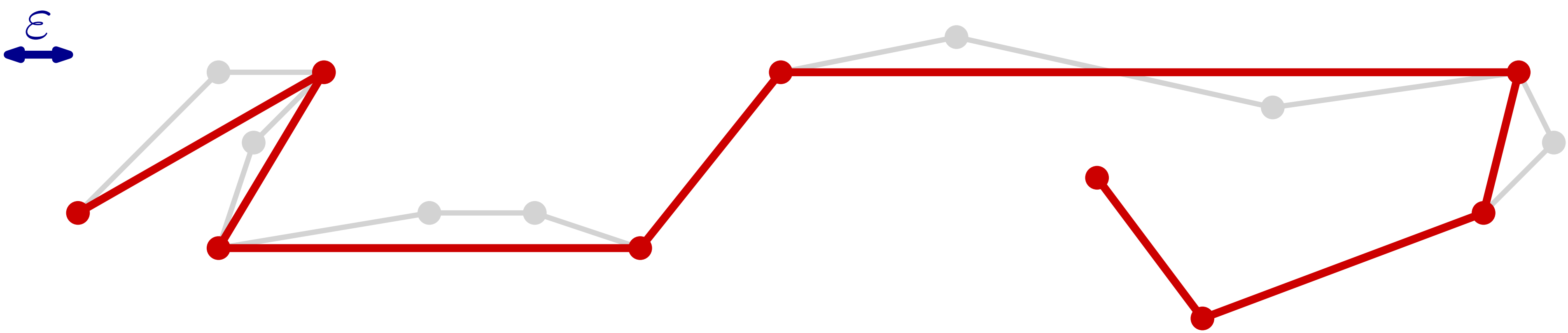

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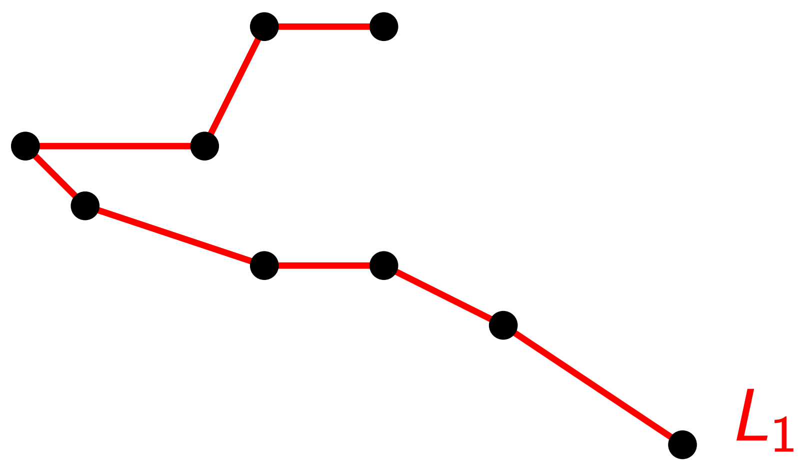

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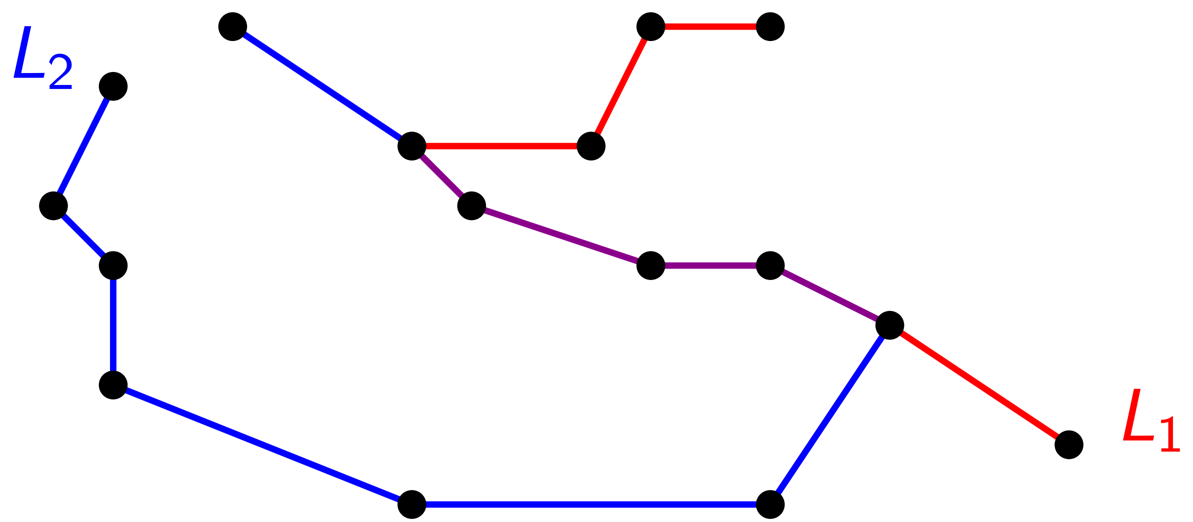

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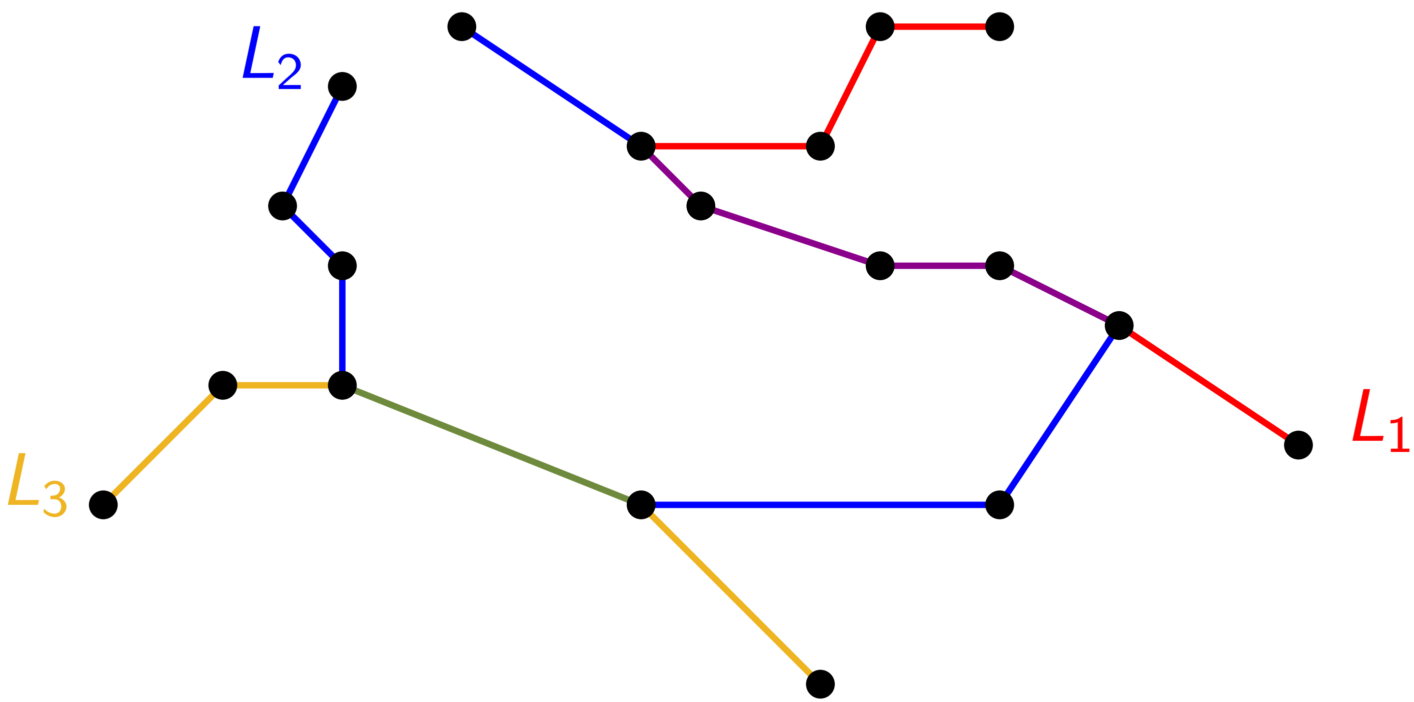

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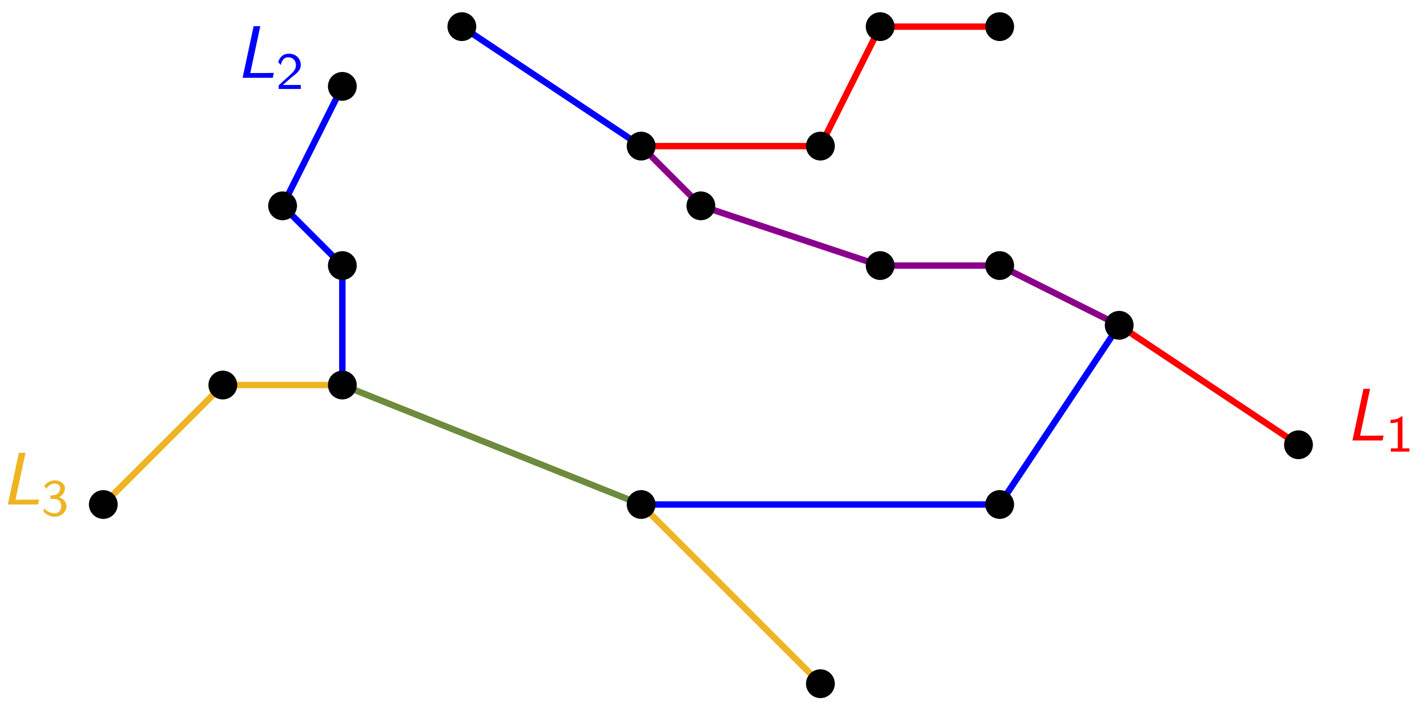

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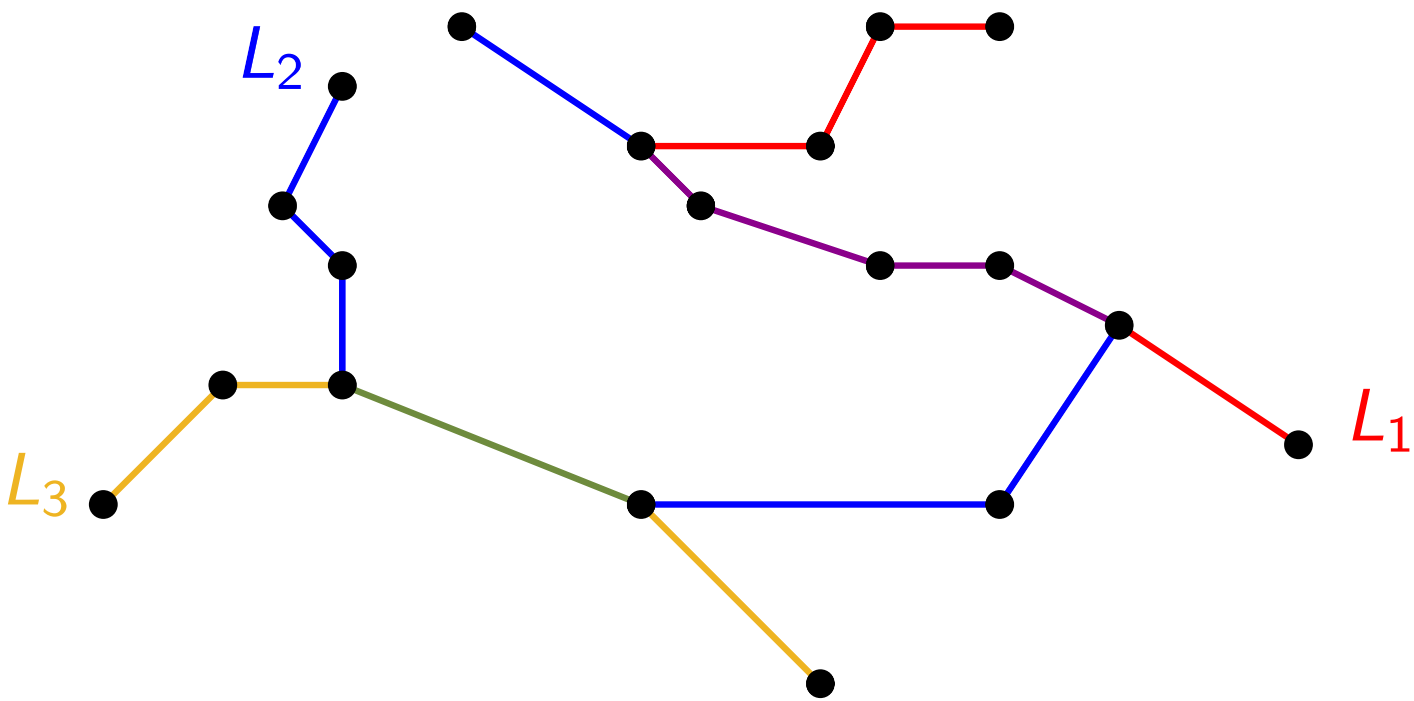

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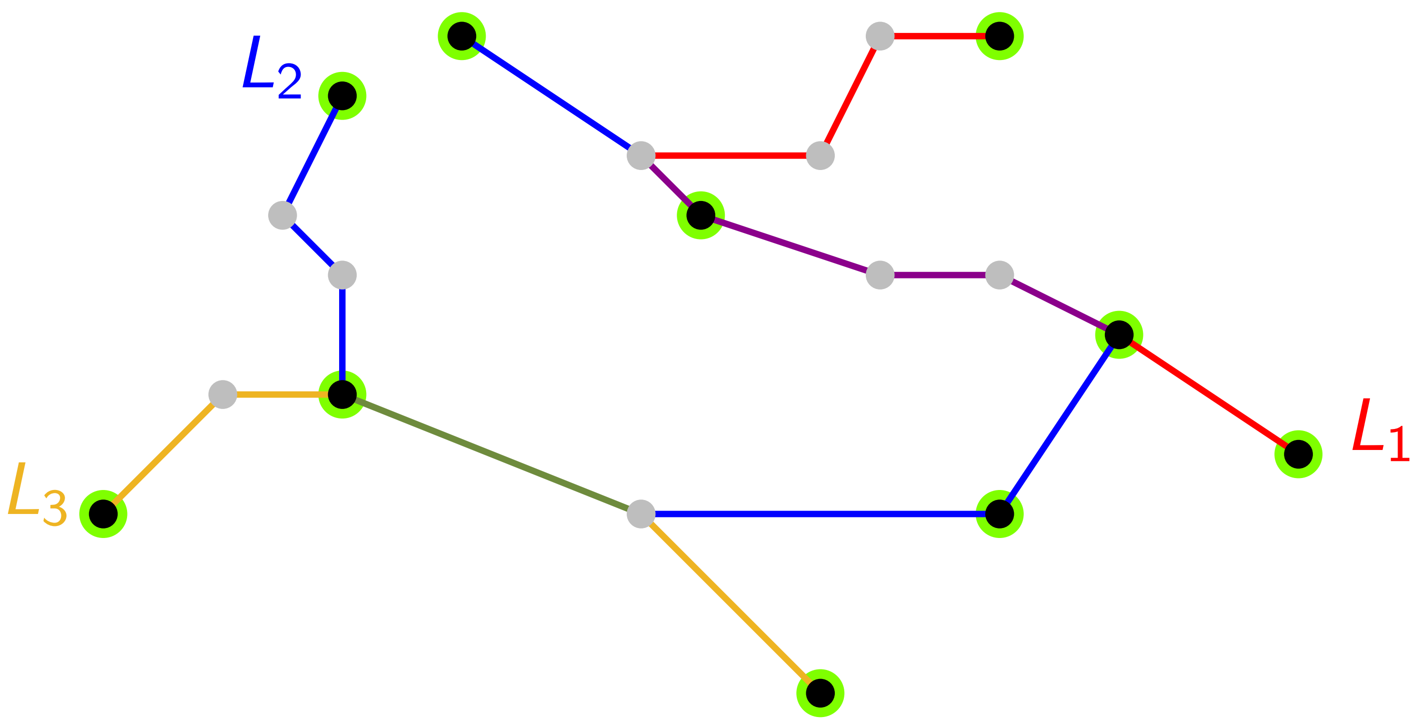

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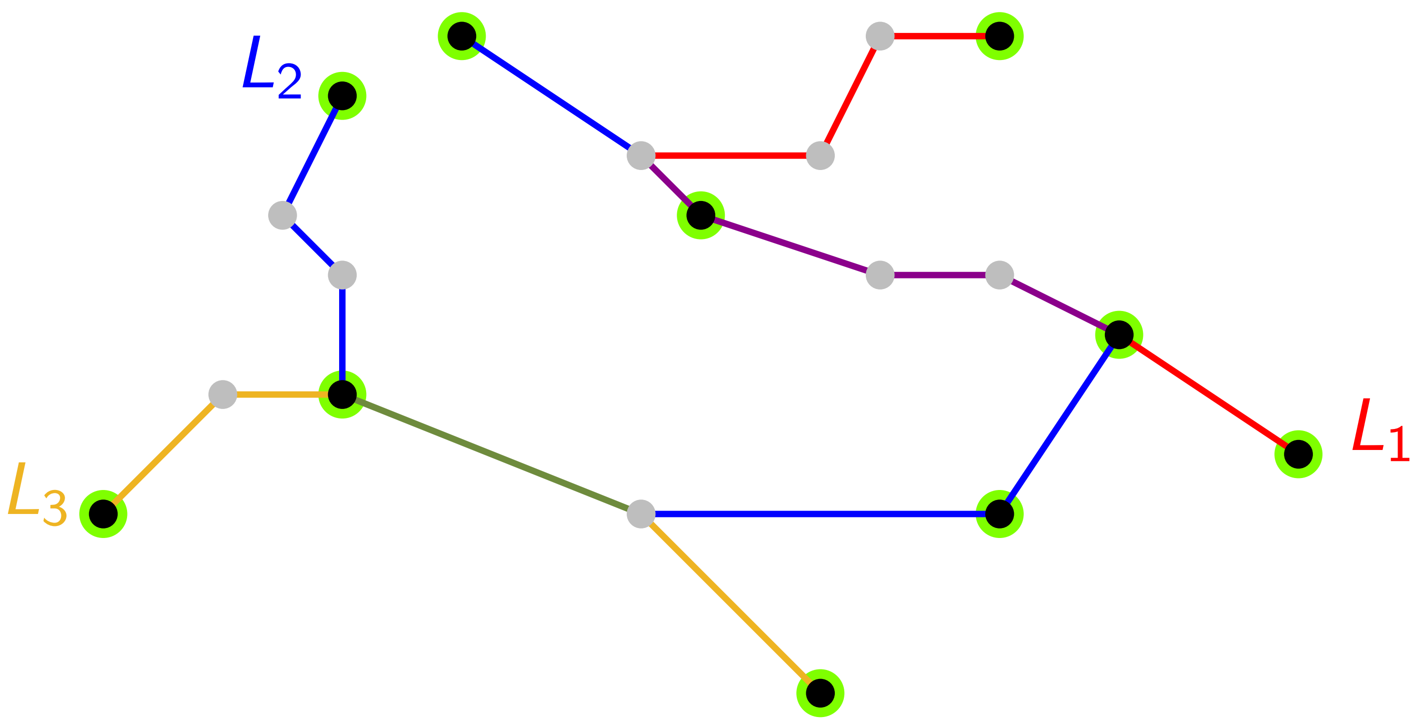

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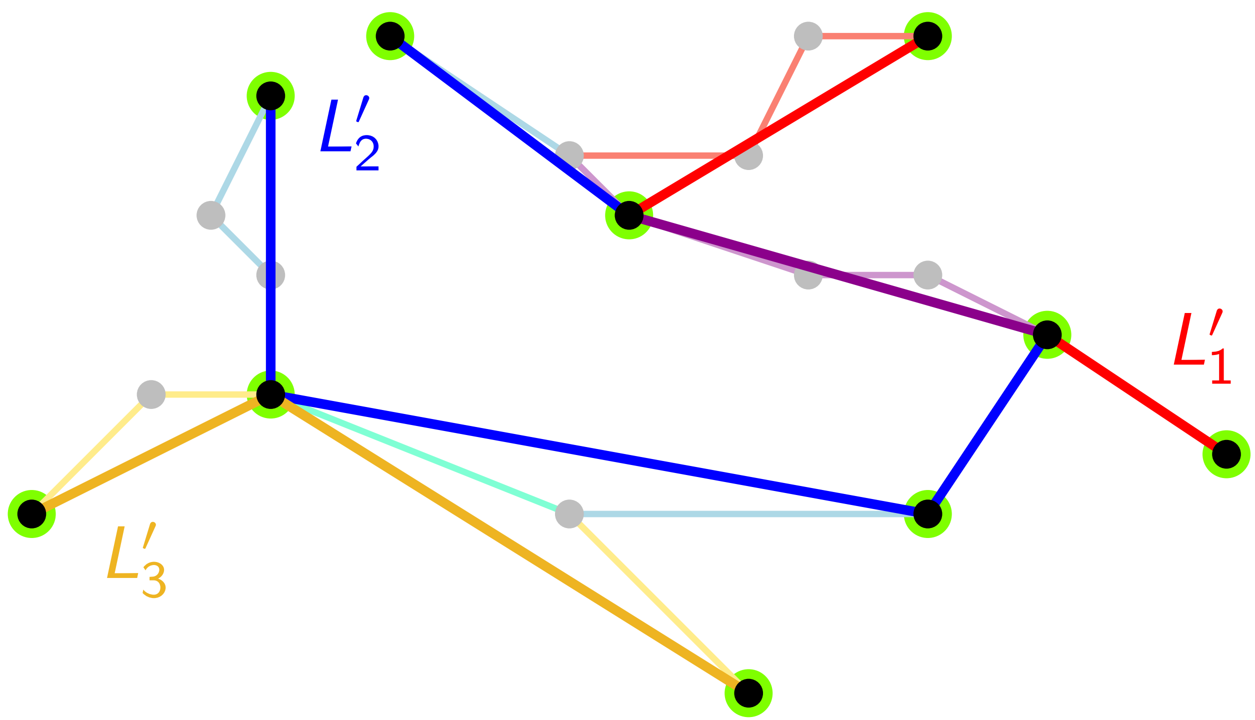

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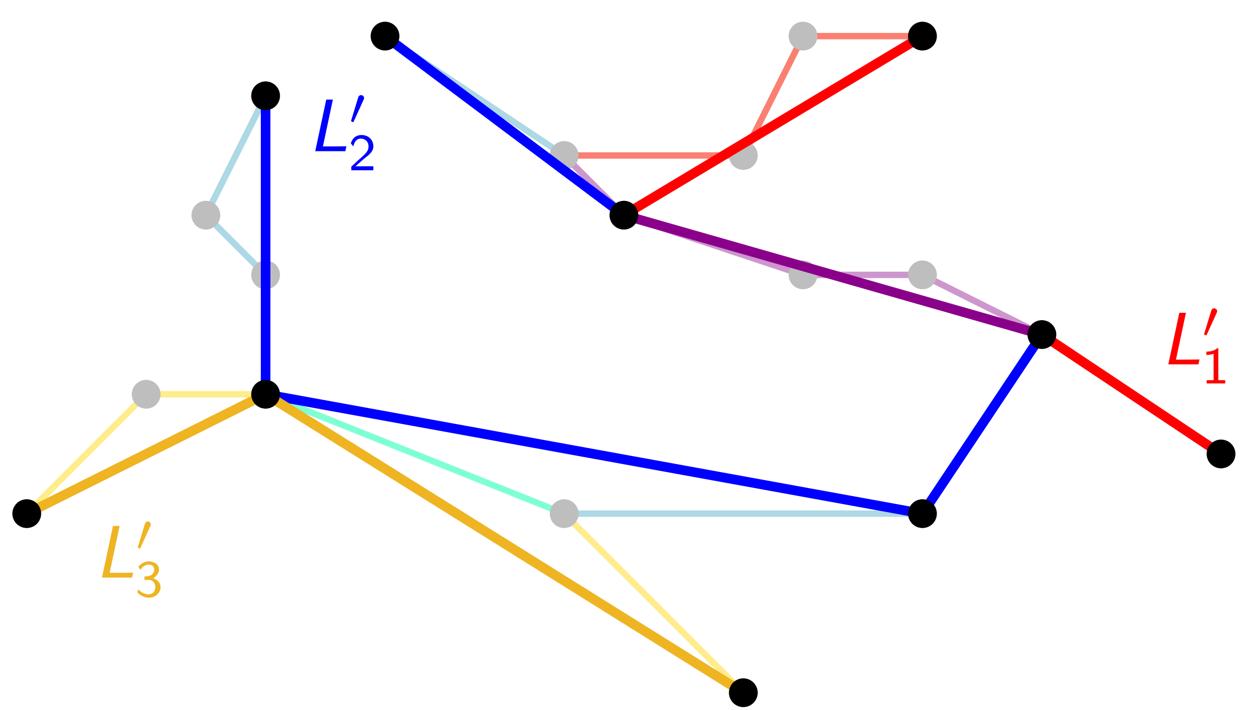

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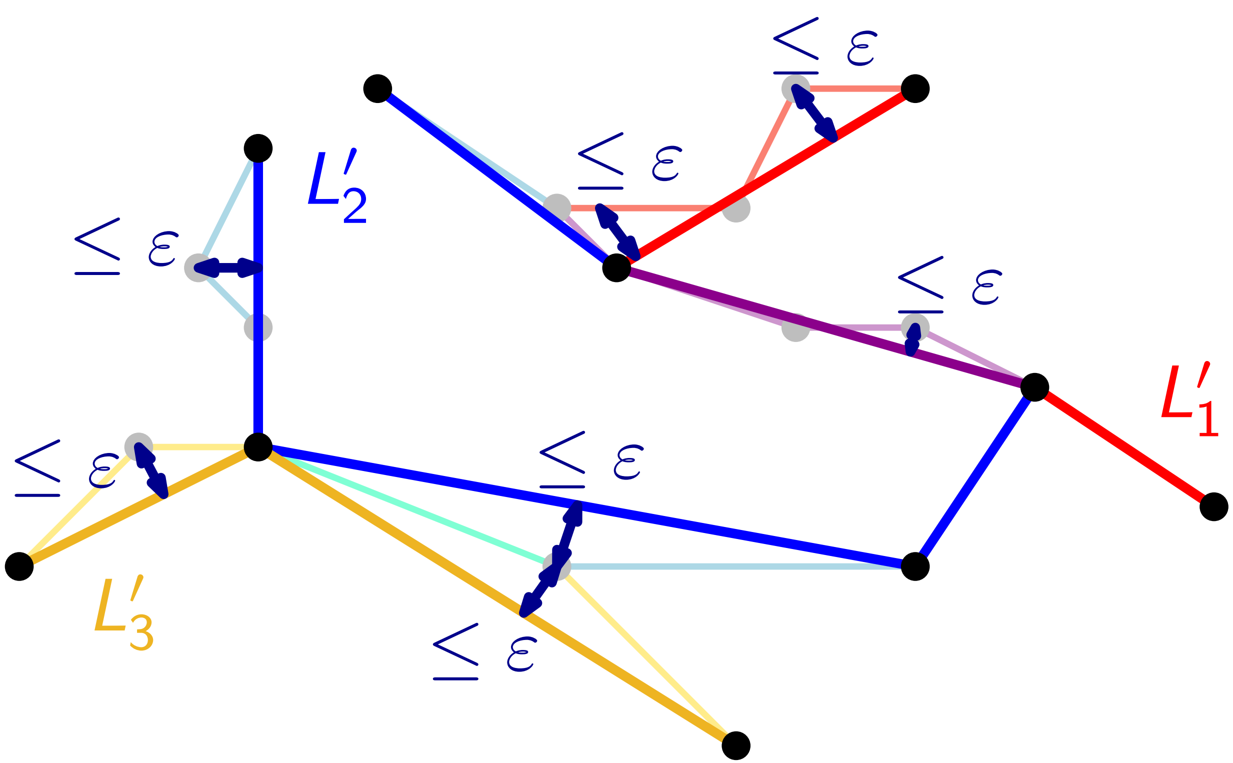

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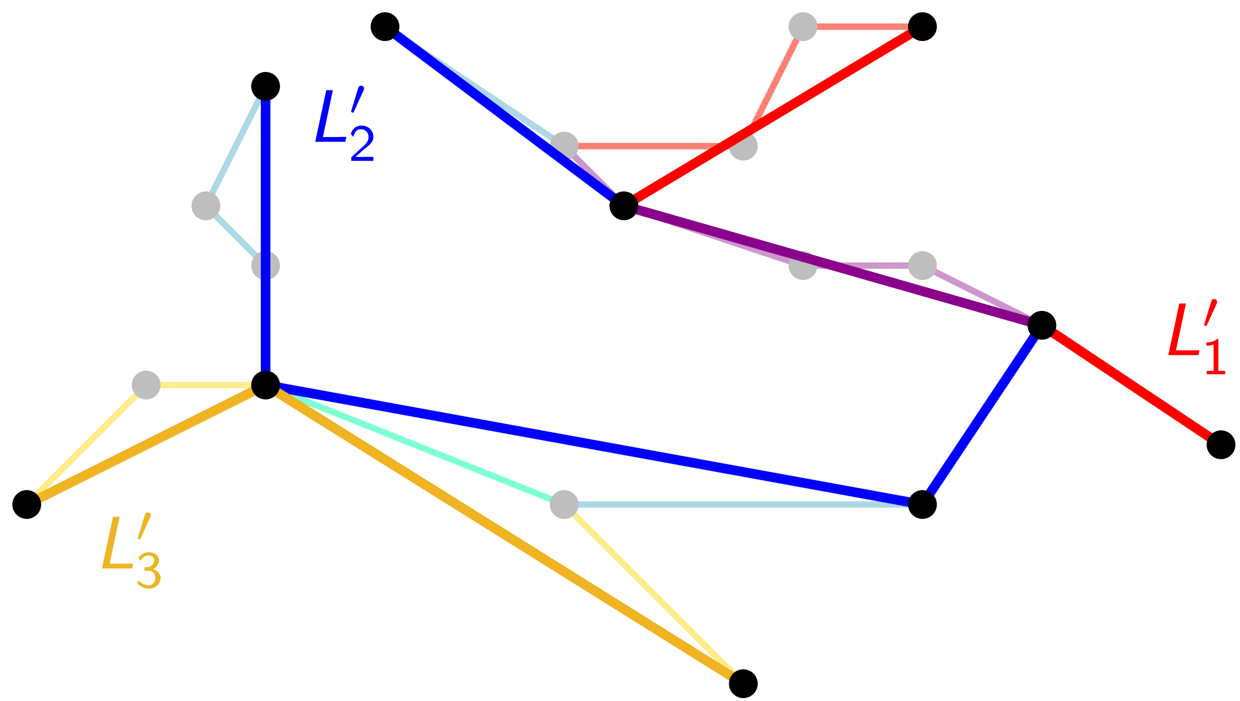

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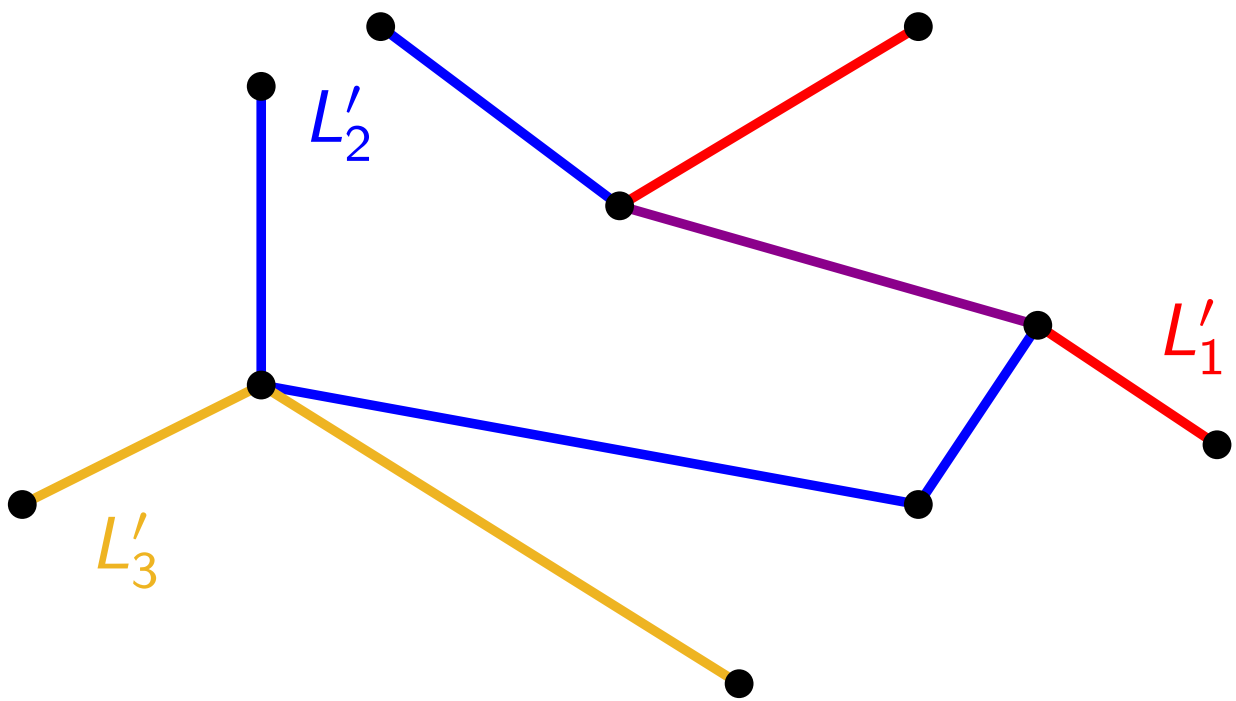

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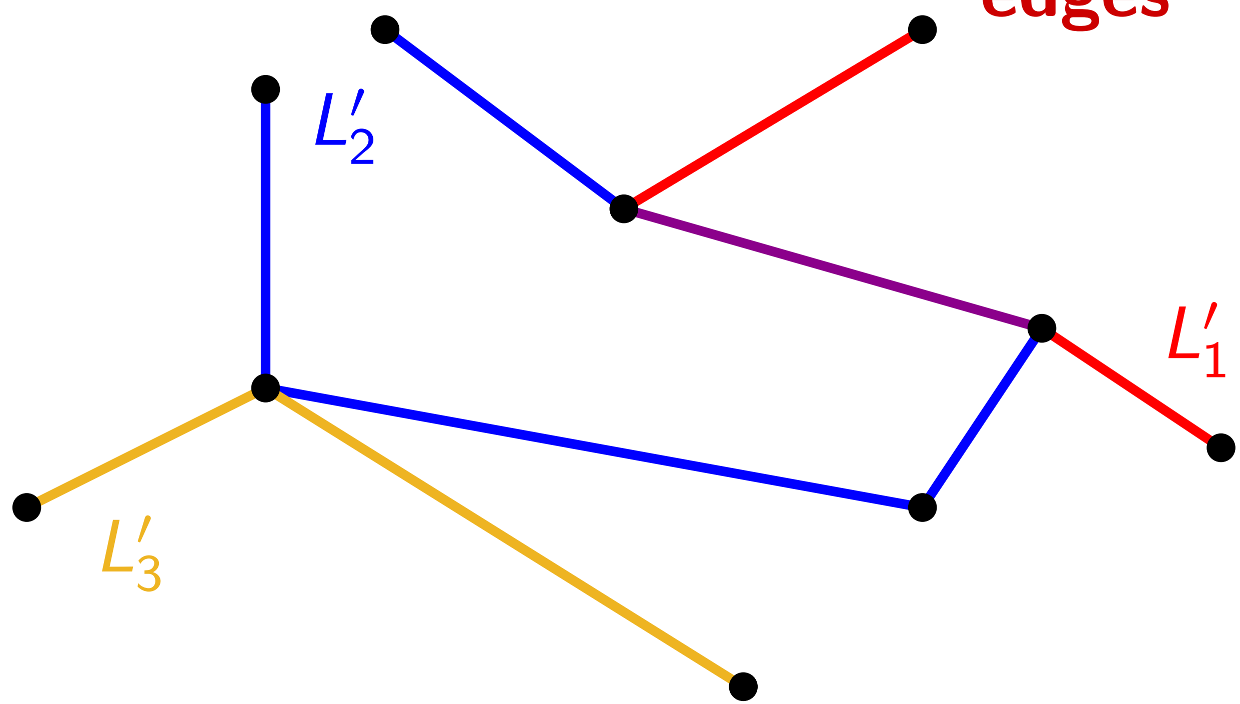

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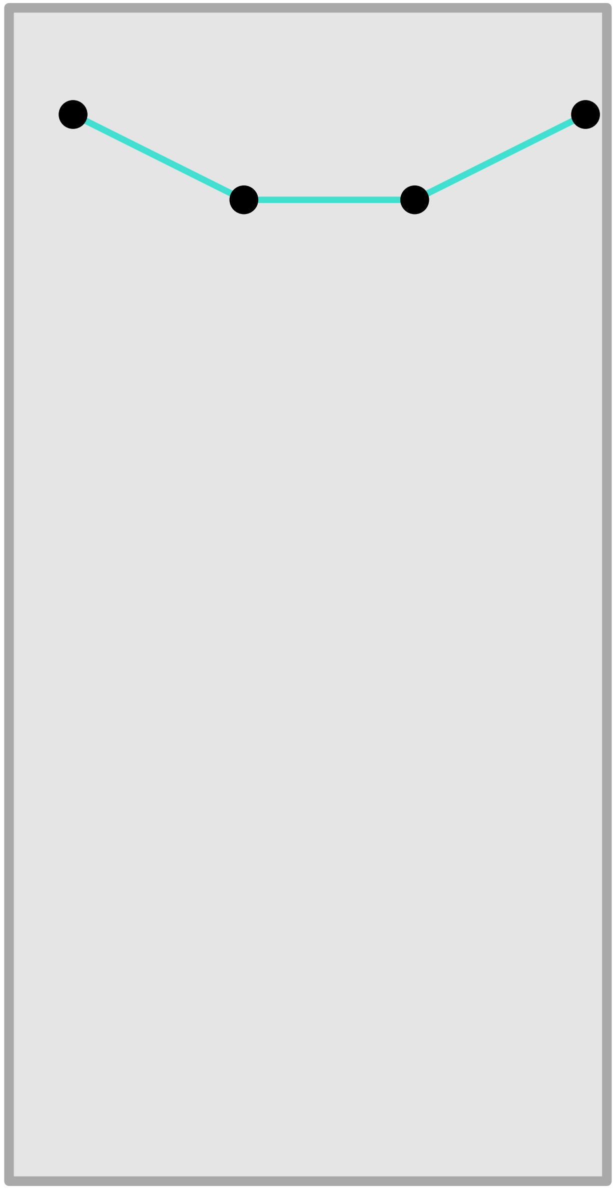

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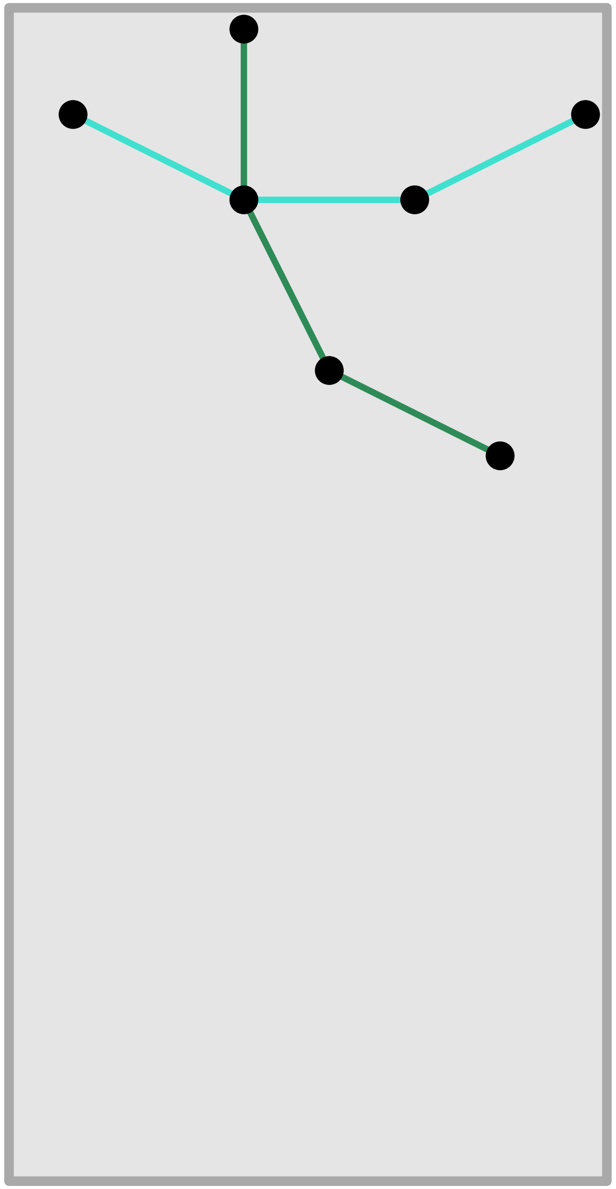

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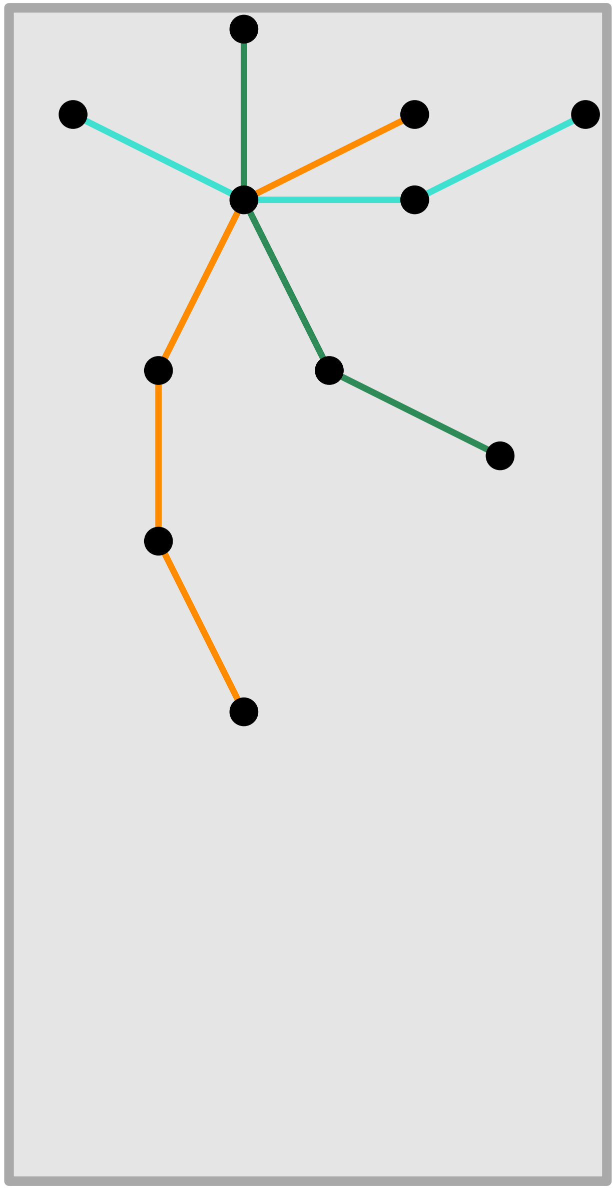

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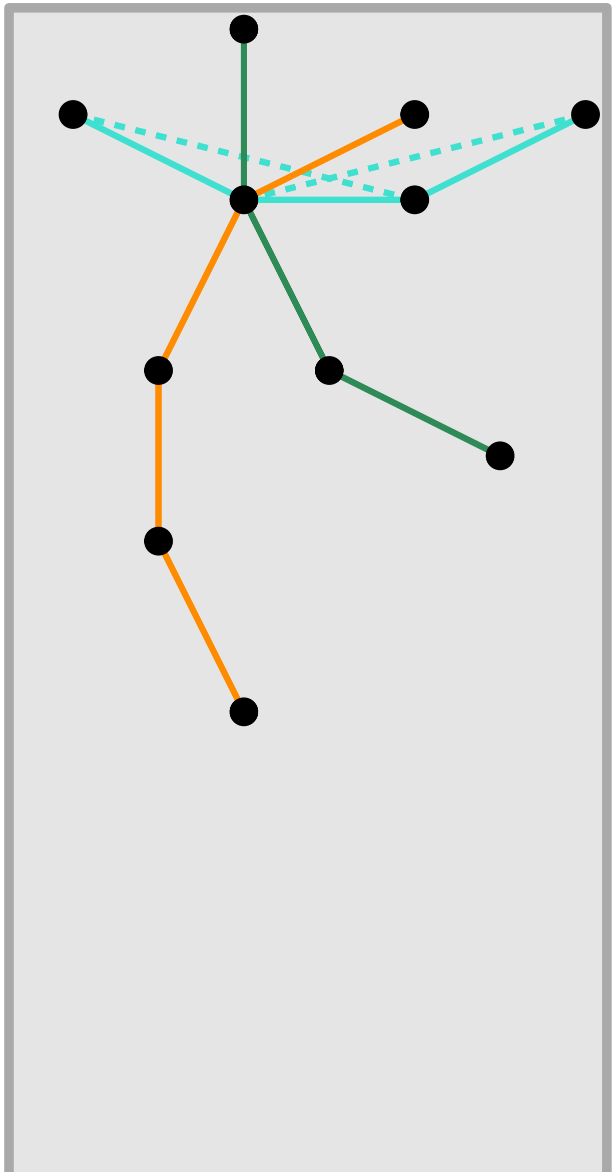

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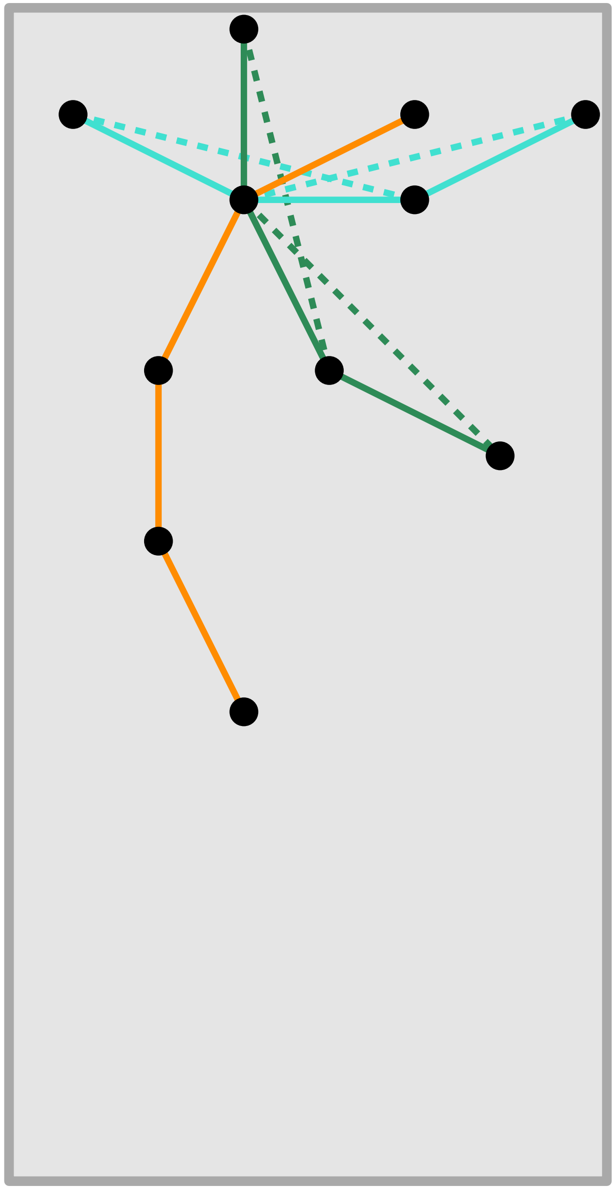

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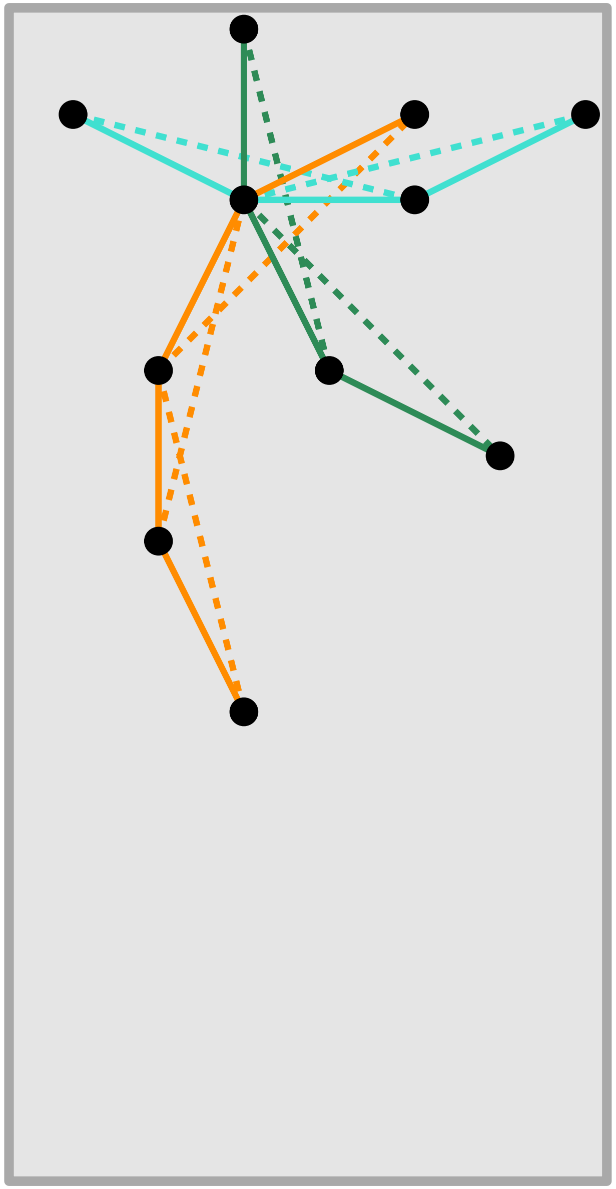

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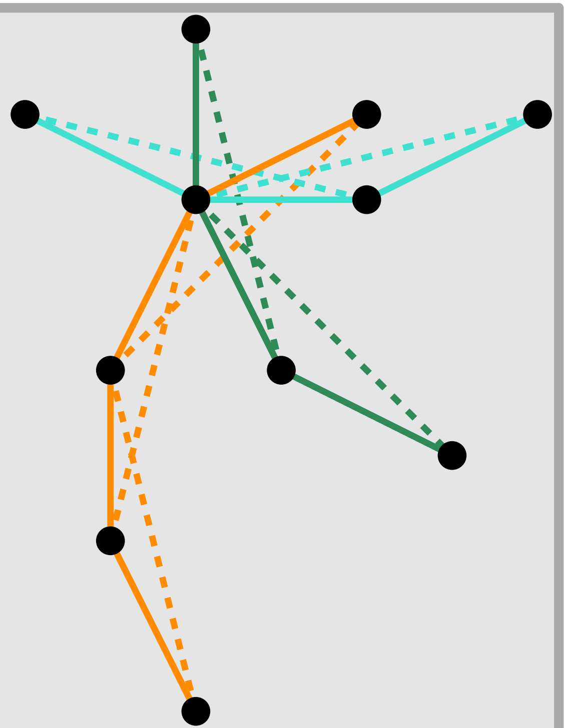

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11 vertices
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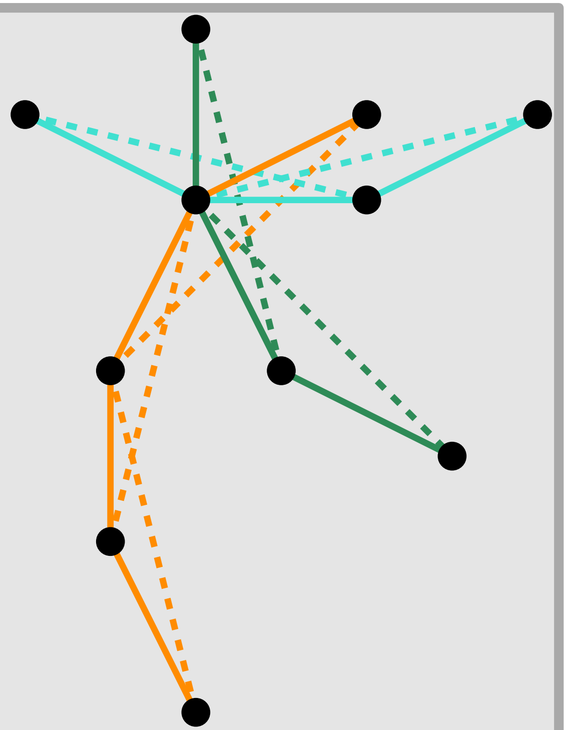

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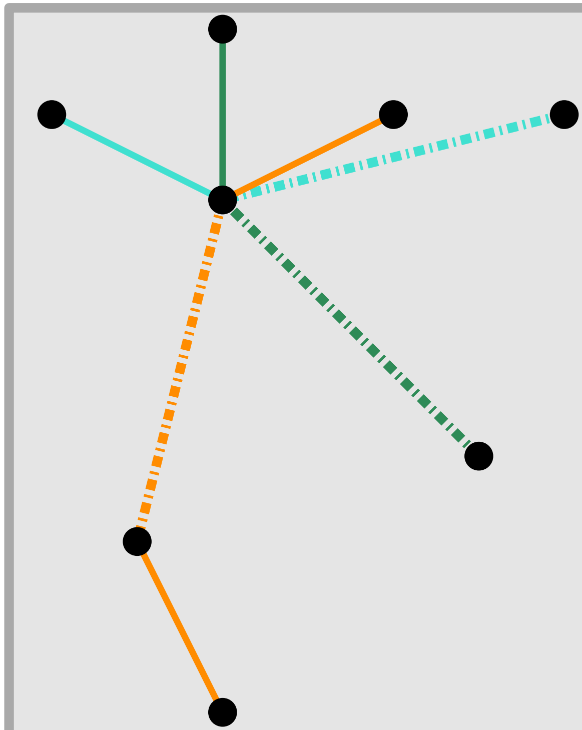

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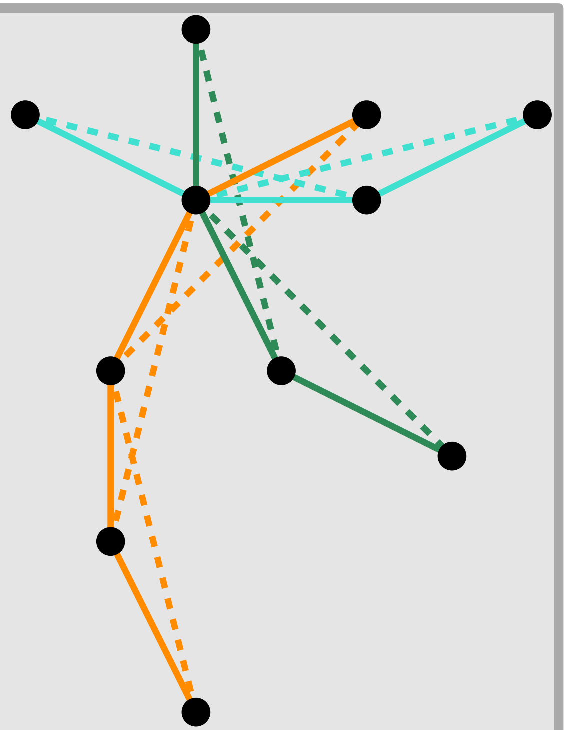

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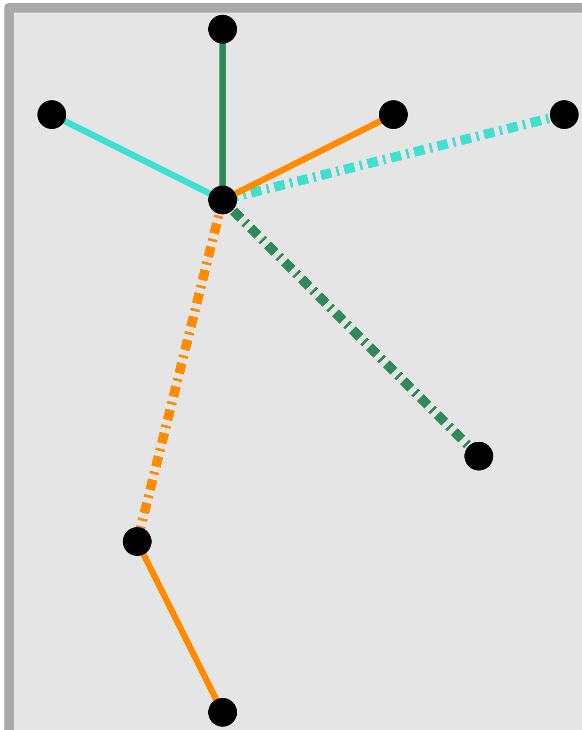

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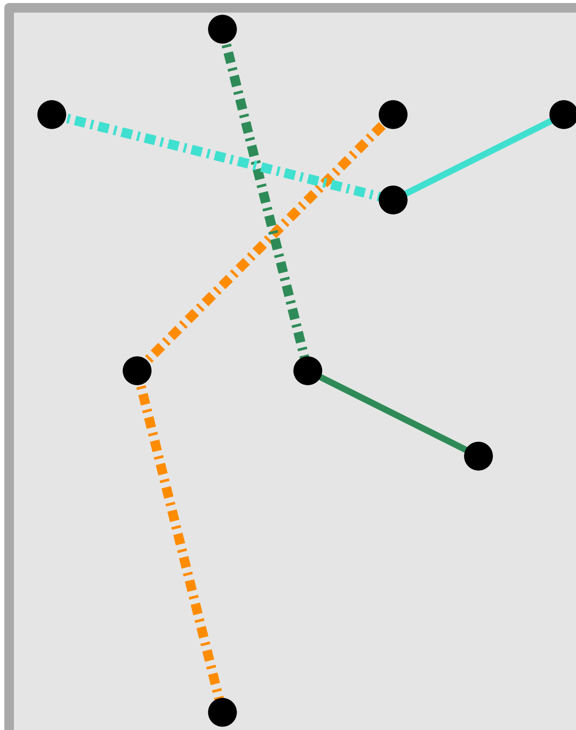

optimal for
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9 vertices
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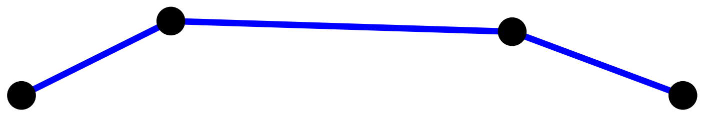

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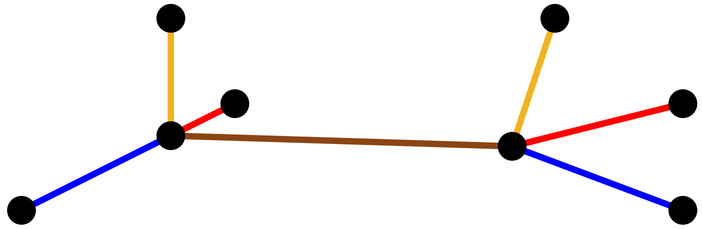

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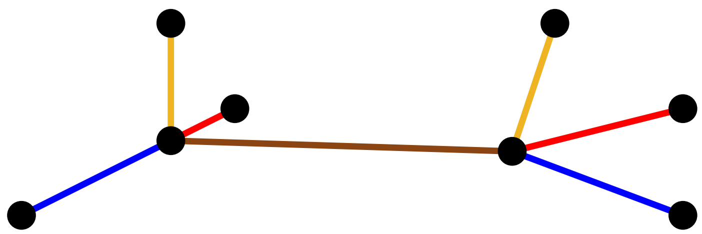

$\xrightarrow{\text{“simplifying” 3 polylines}}$

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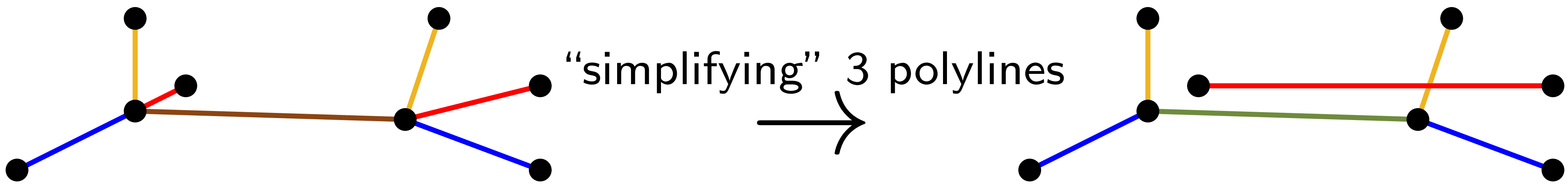

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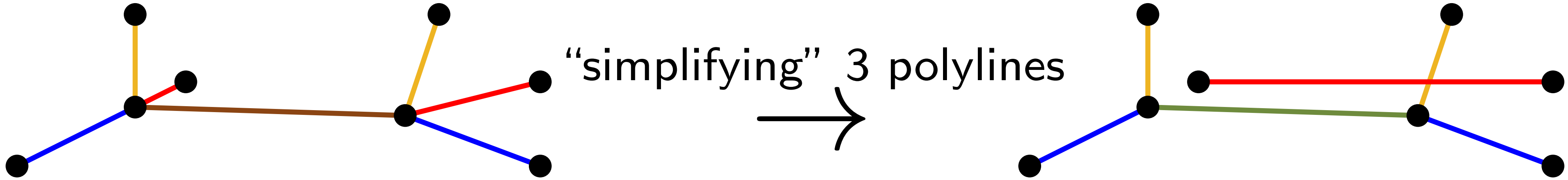

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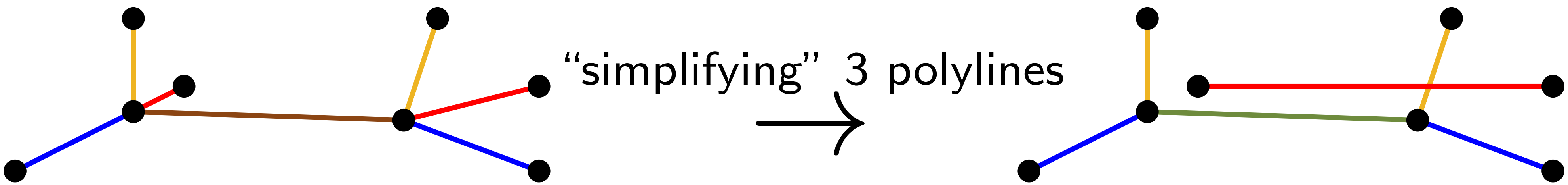

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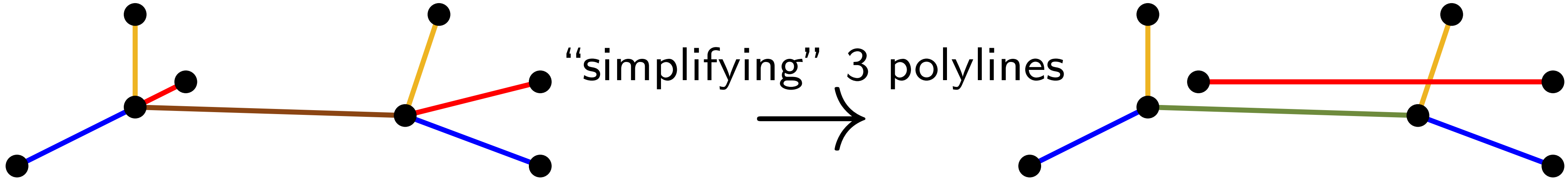

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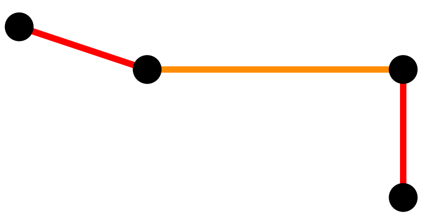

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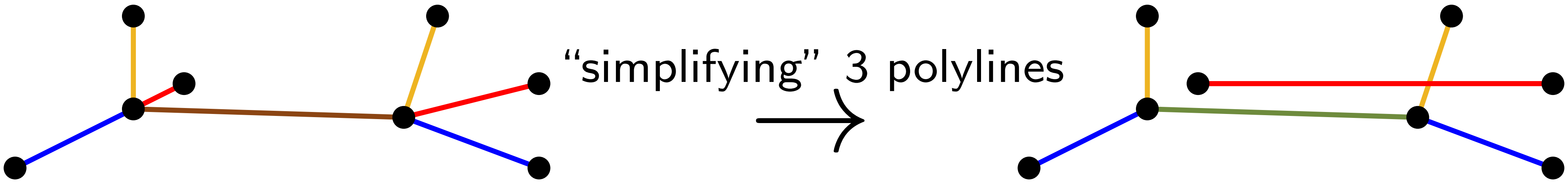

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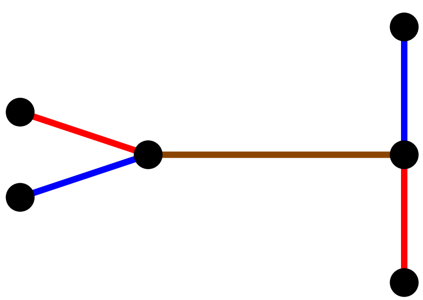

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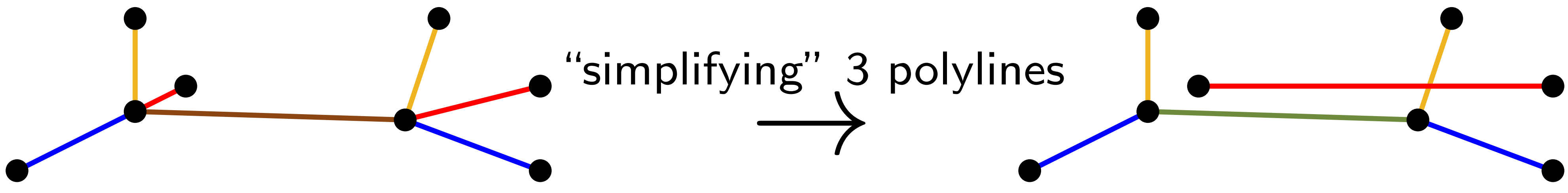

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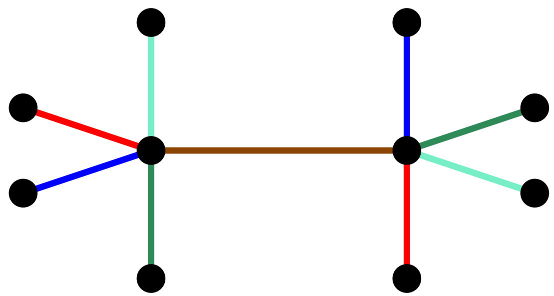

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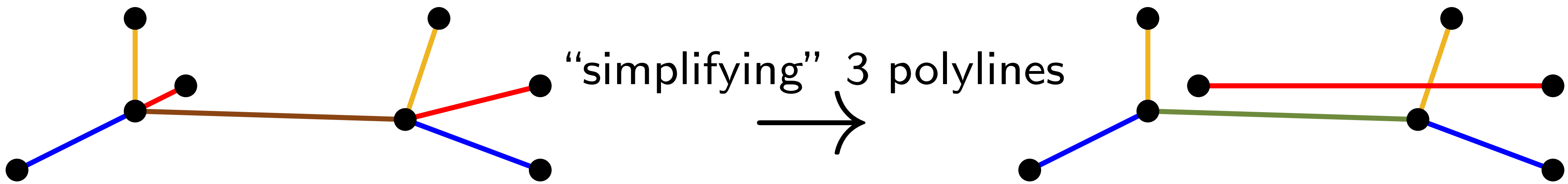

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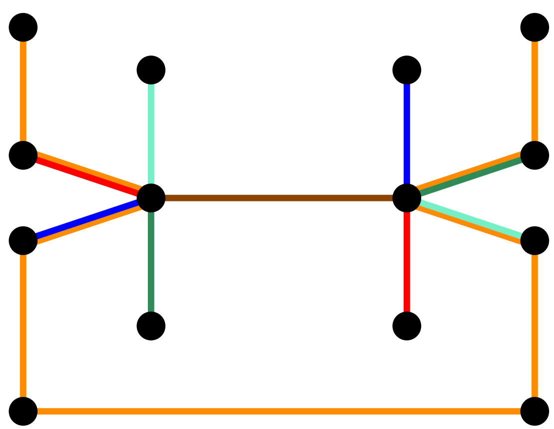

14 vertices, 14 edges

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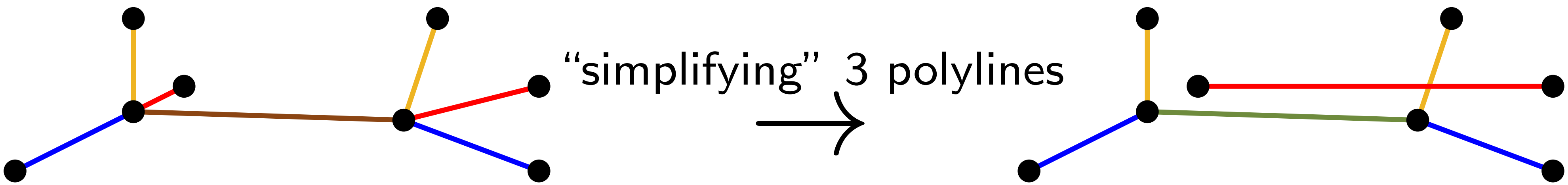
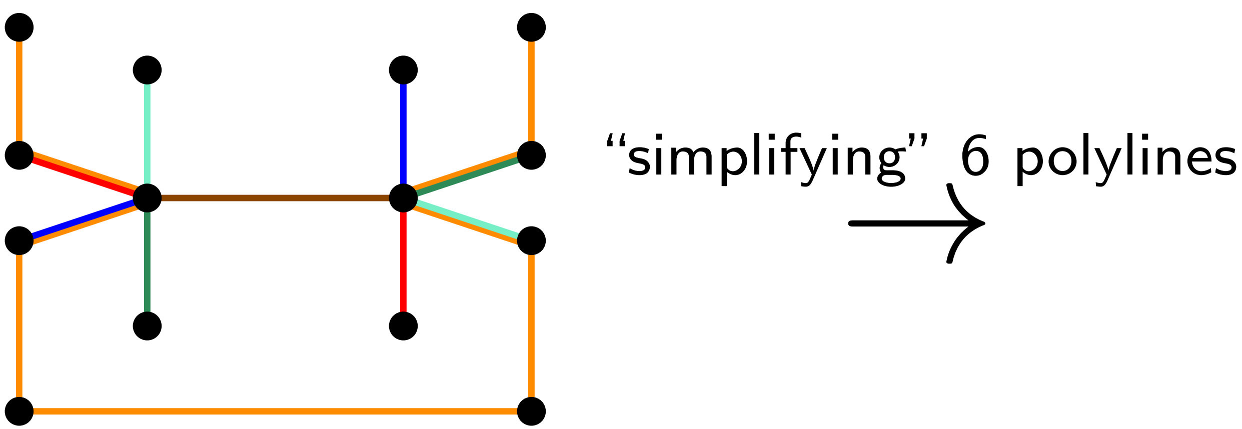


$\Rightarrow$ the total complexity might even increase



14 vertices, 14 edges

Why Not Simplifying Independently?

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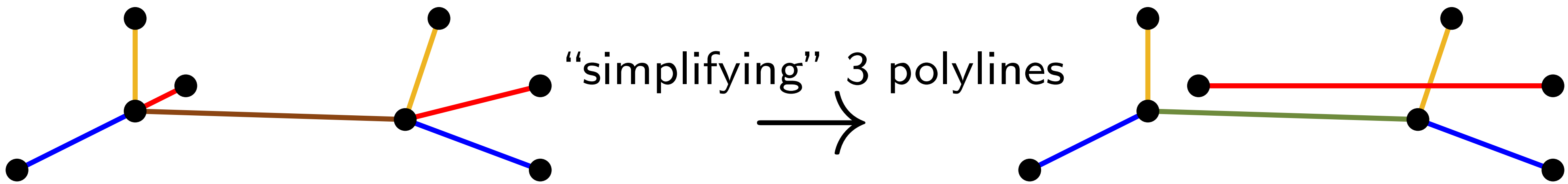

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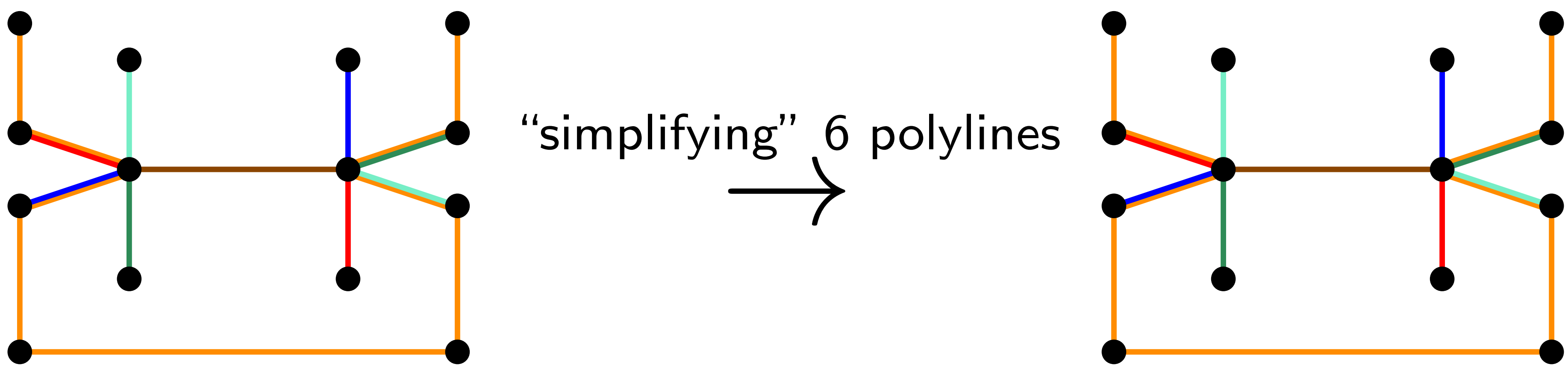

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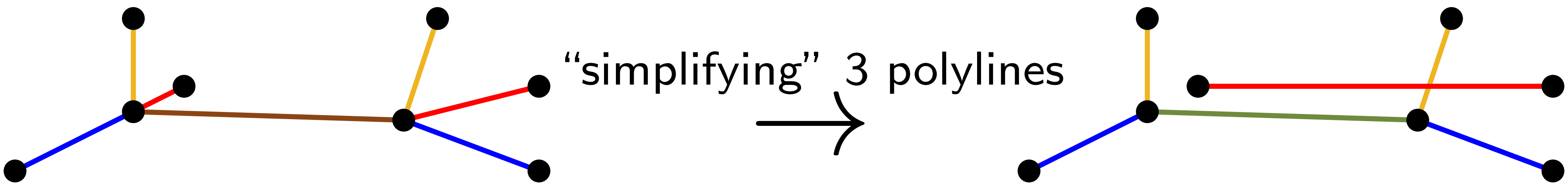

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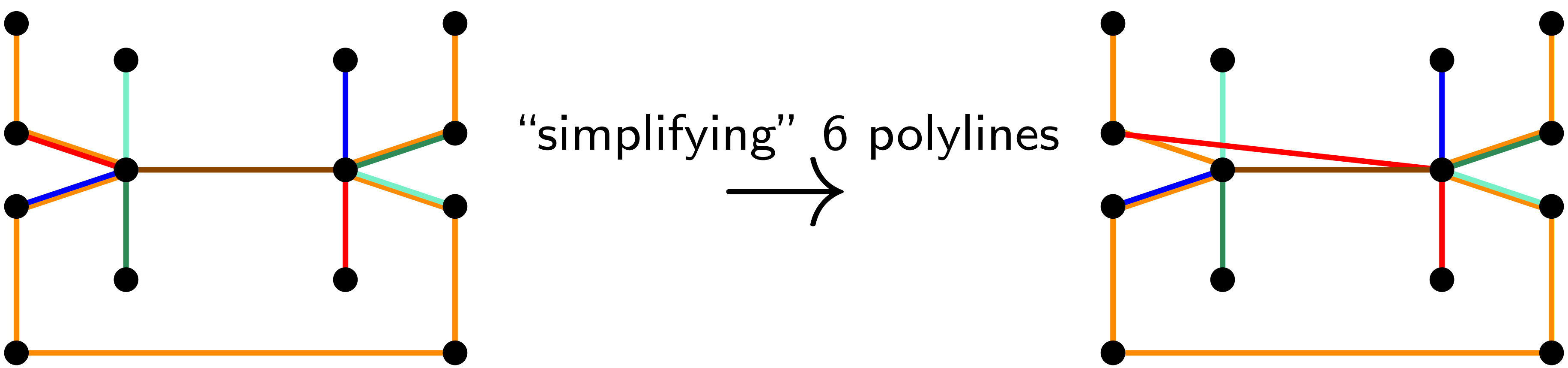

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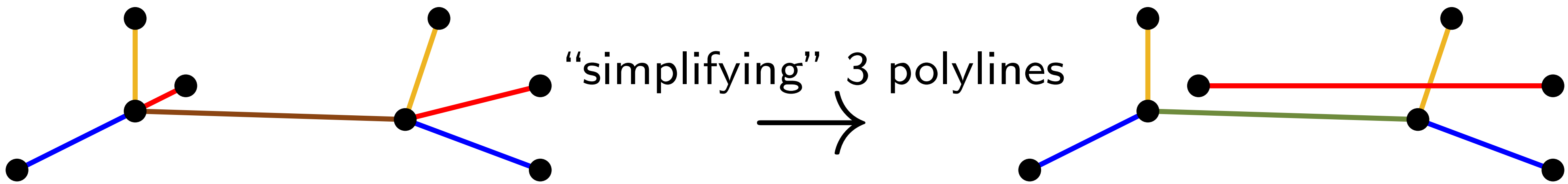

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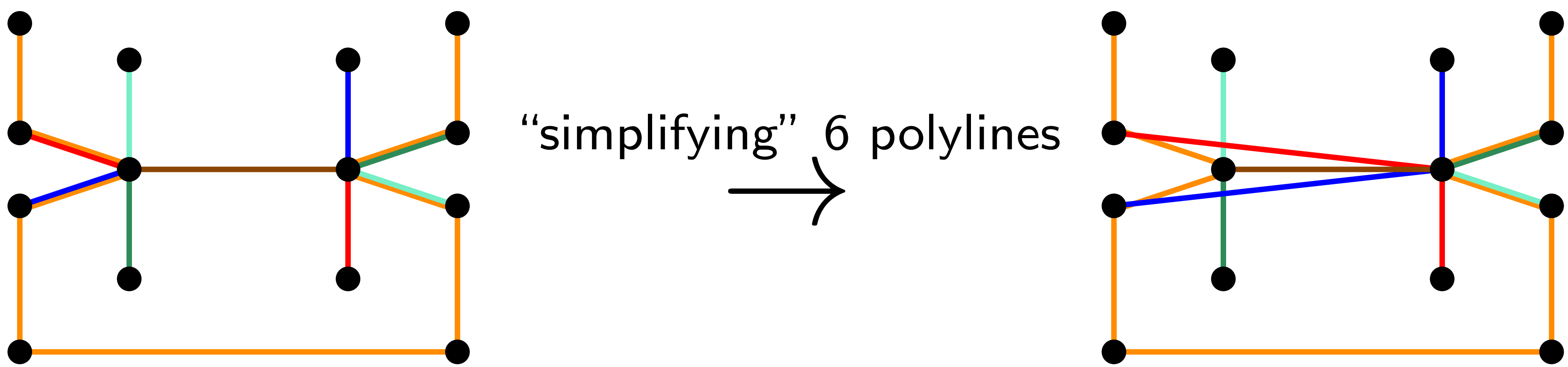

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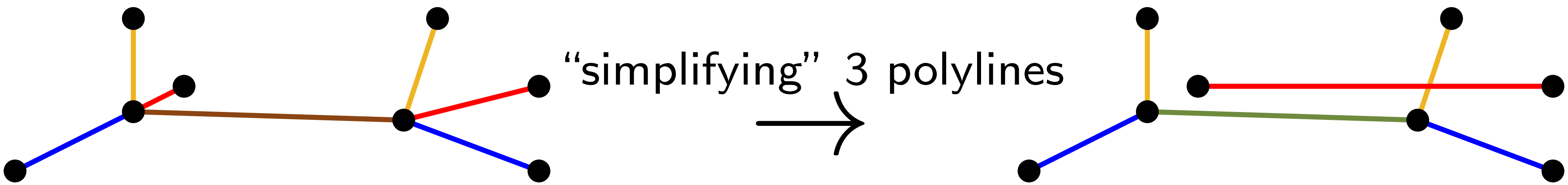

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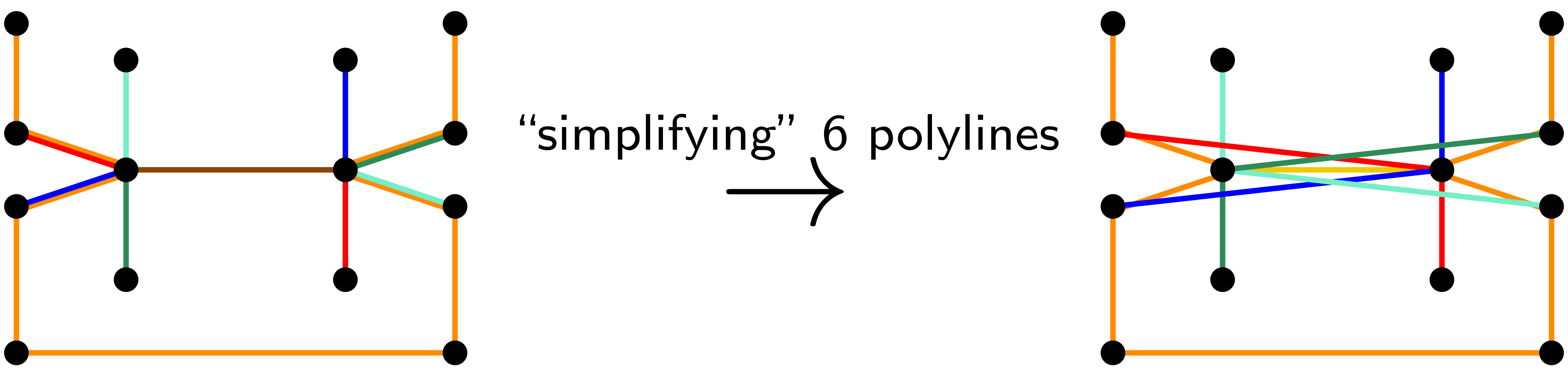

14 vertices, 14 edges

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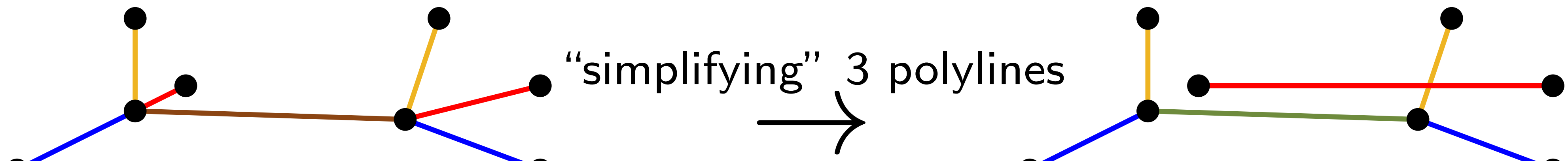

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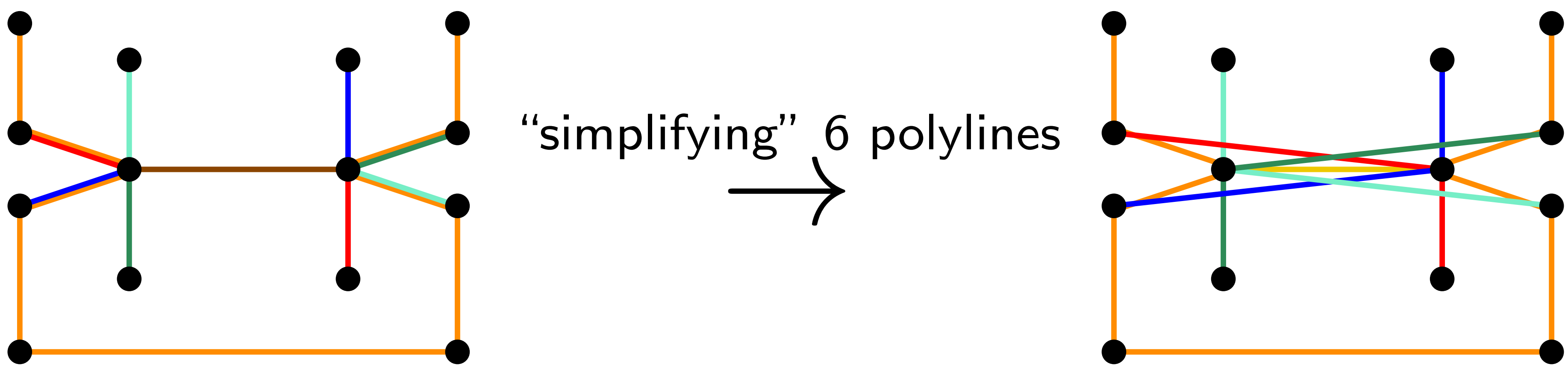

14 vertices, 14 edges

14 vertices, 18 edges

NP-Hardness

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Theorem 1:
Simplifying a bundle of polylines is NP-hard for the goals $\textsc{Min-Vertices}$ and $\textsc{Min-Edges}$ even for 2 polylines.

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- We use a polyline as a gadget for each variable and each clause. We connect them serially. $\rightarrow$ second polyline

Variable-Gadget

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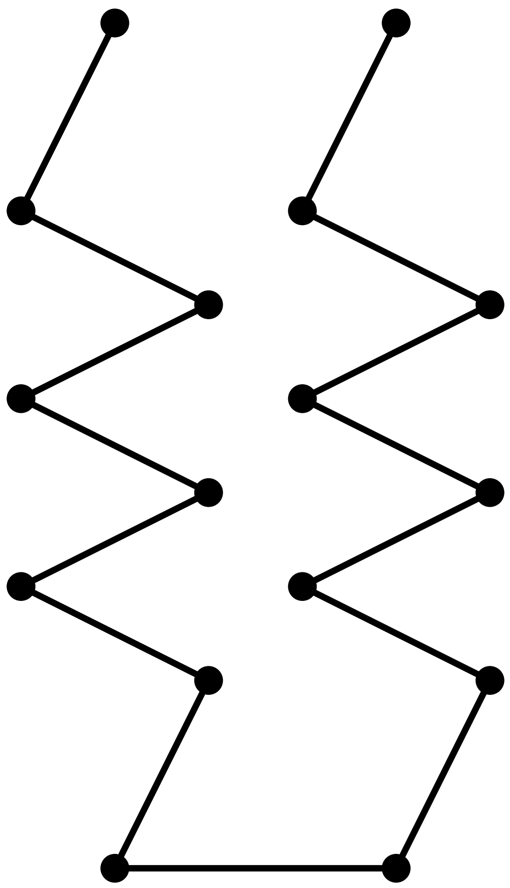
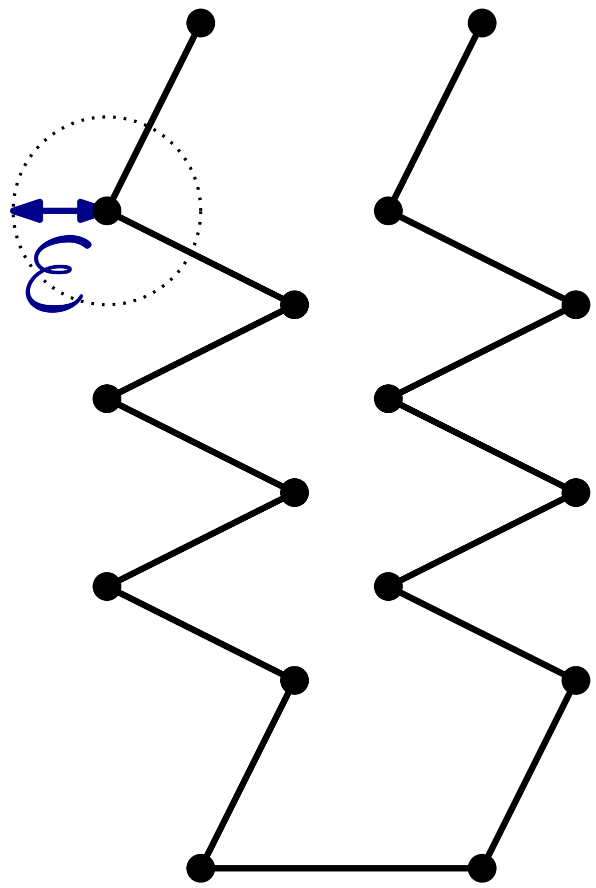


Variable-Gadget



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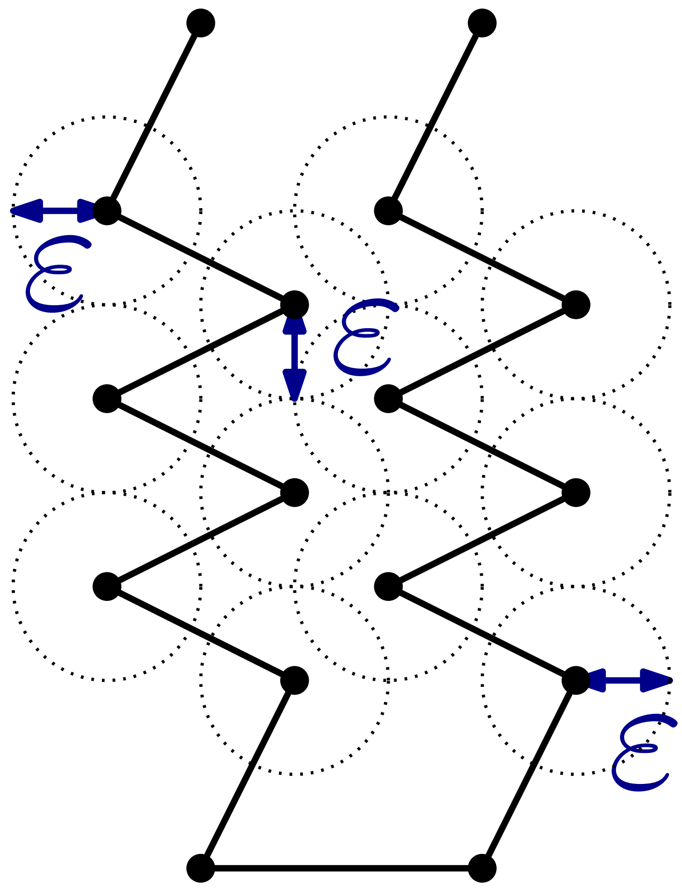

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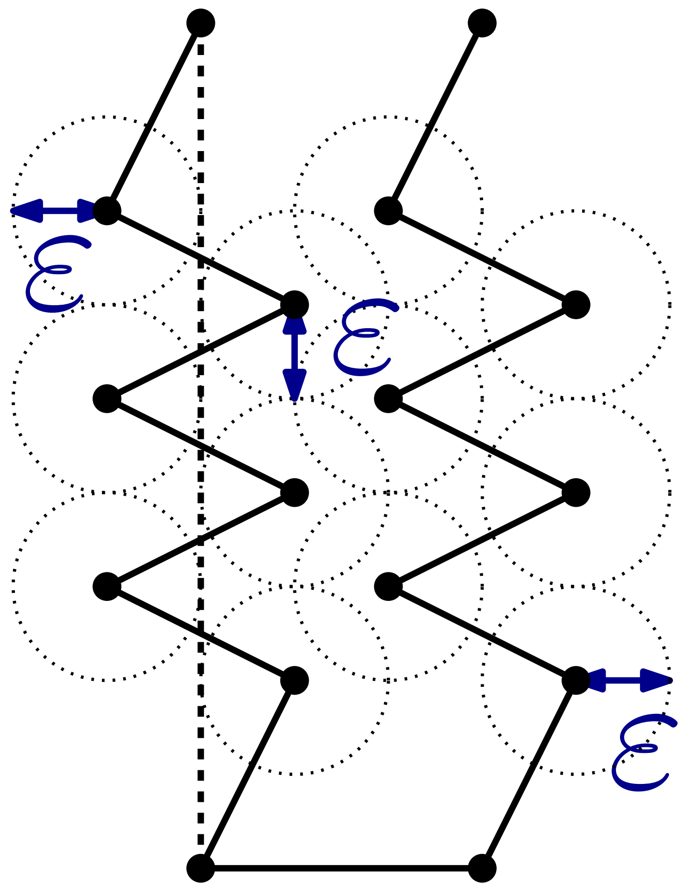

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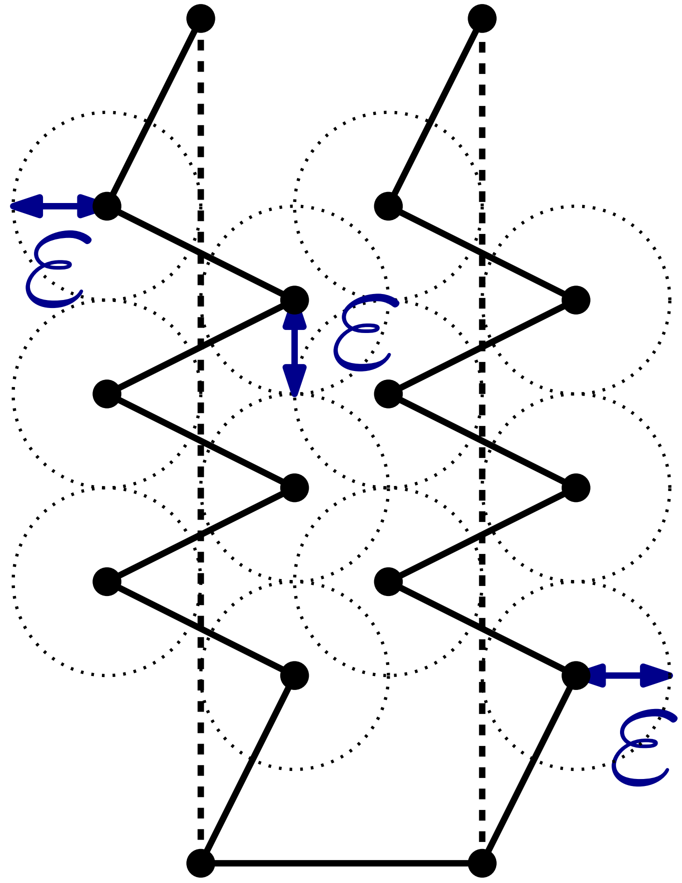

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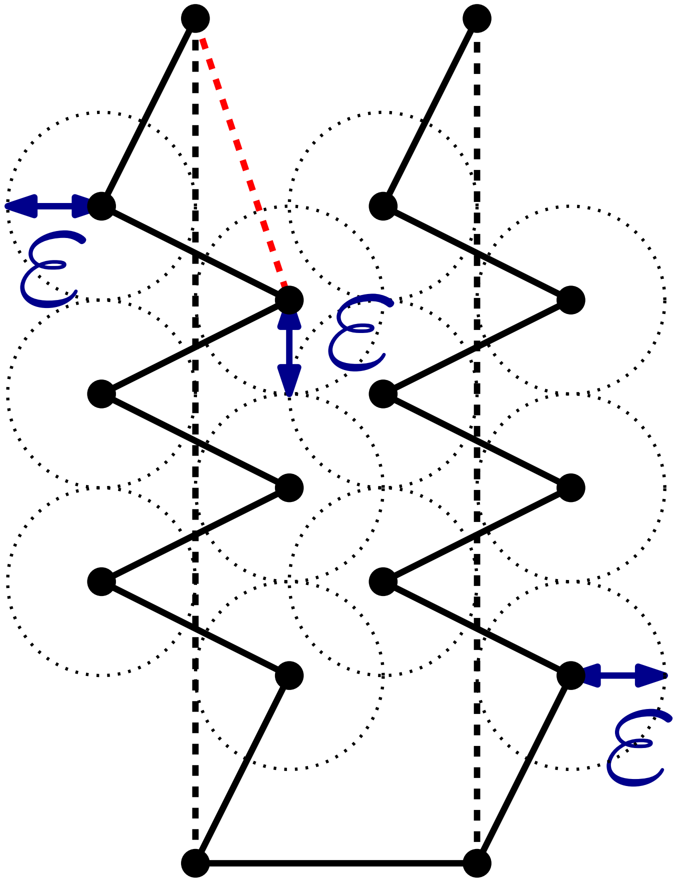

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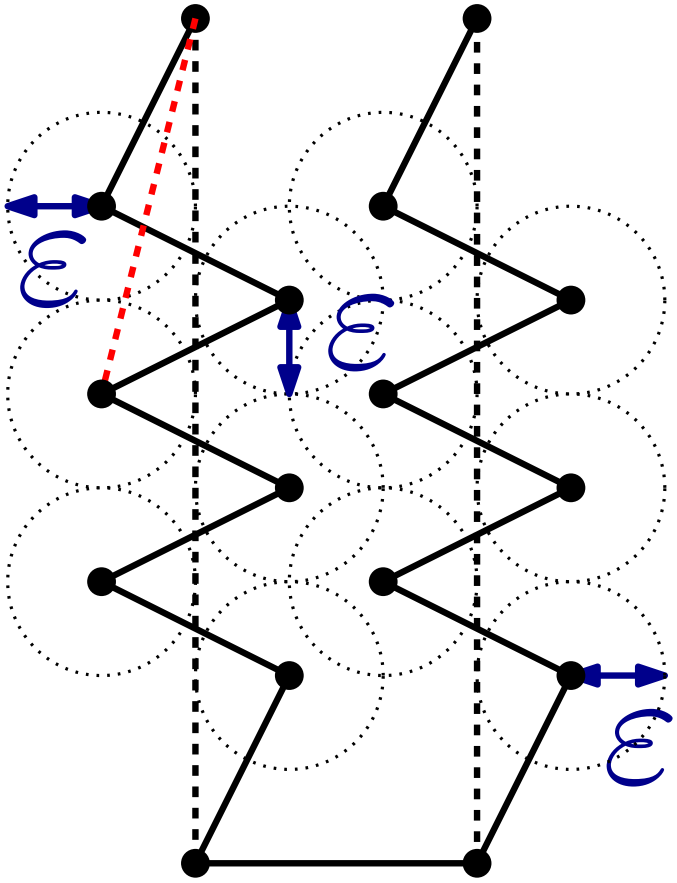

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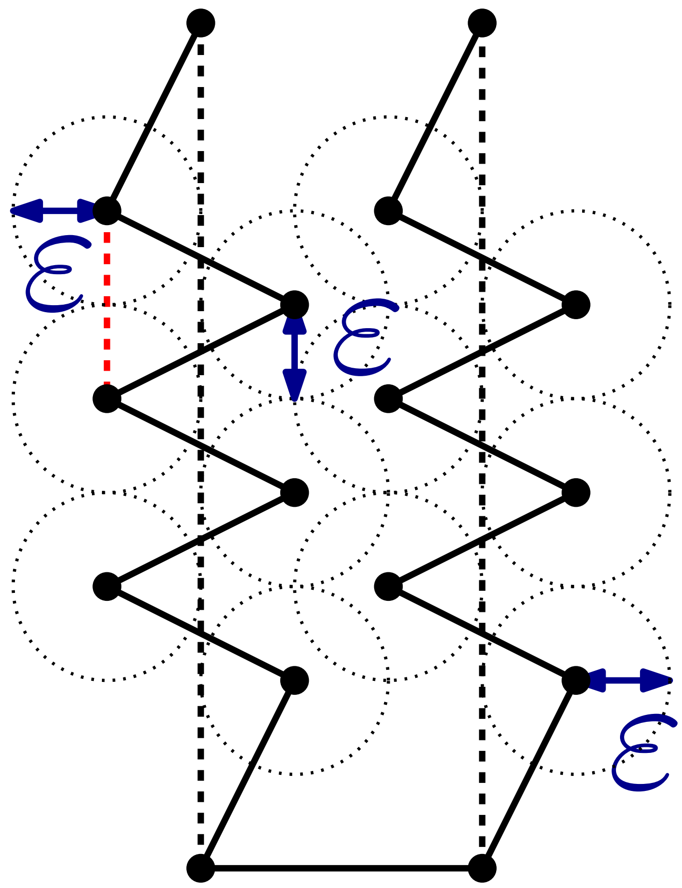

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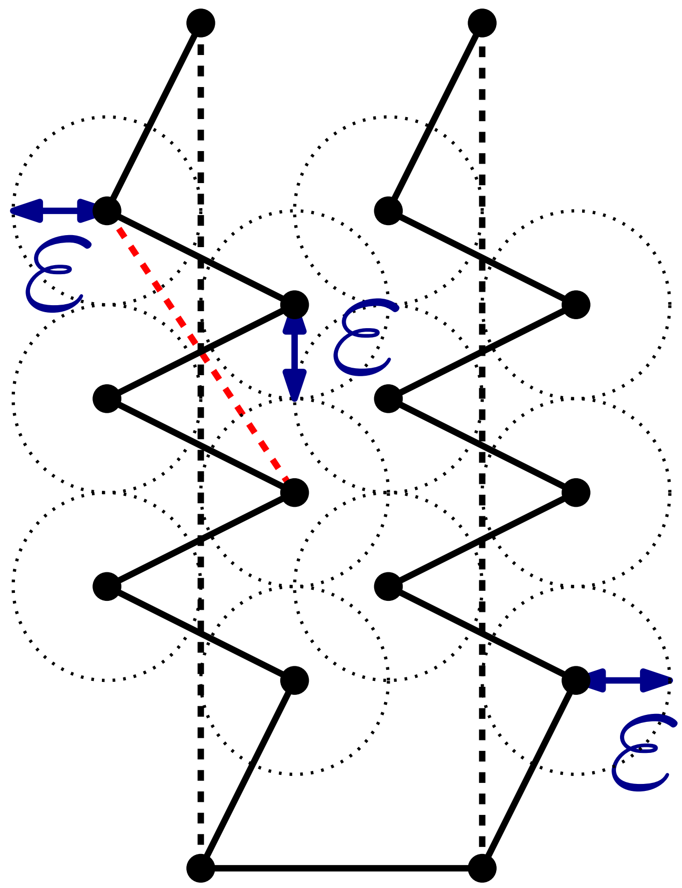
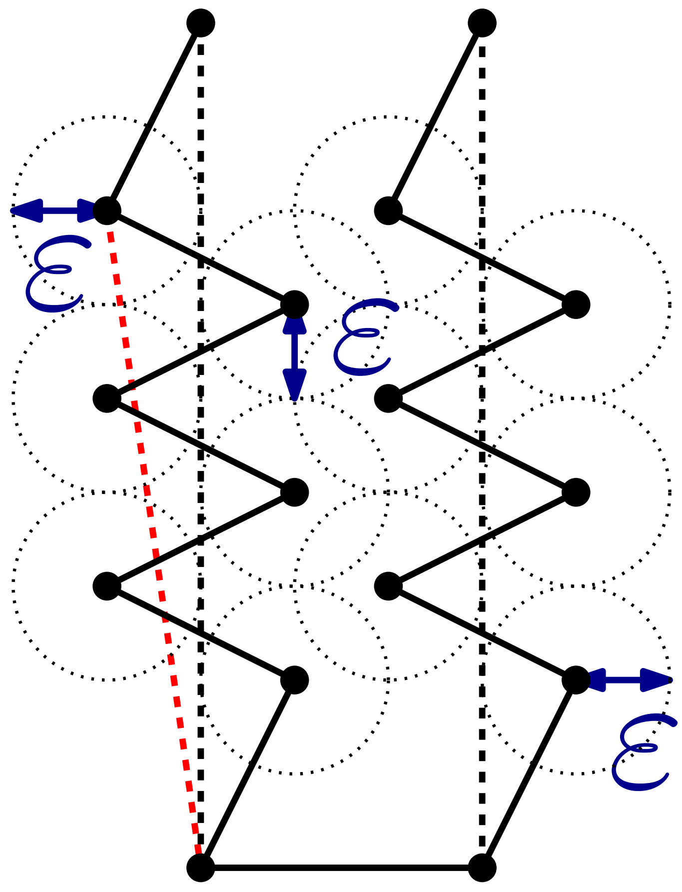


Variable-Gadget



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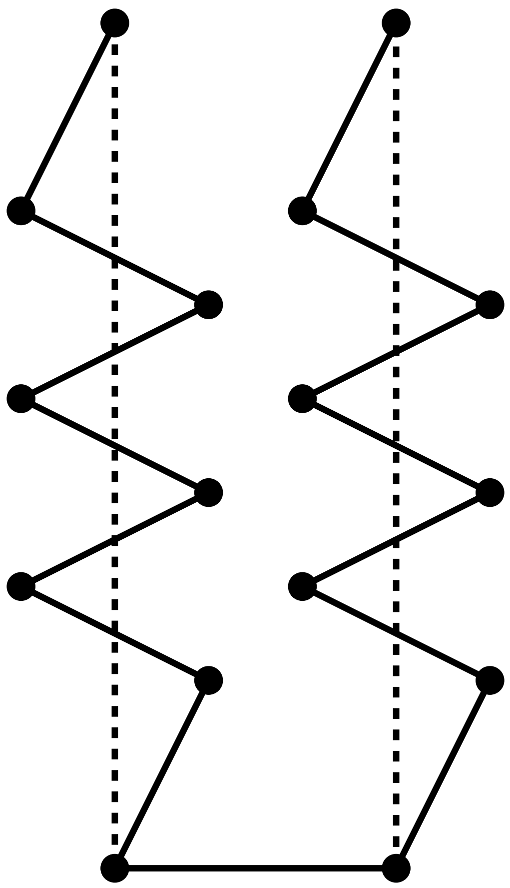

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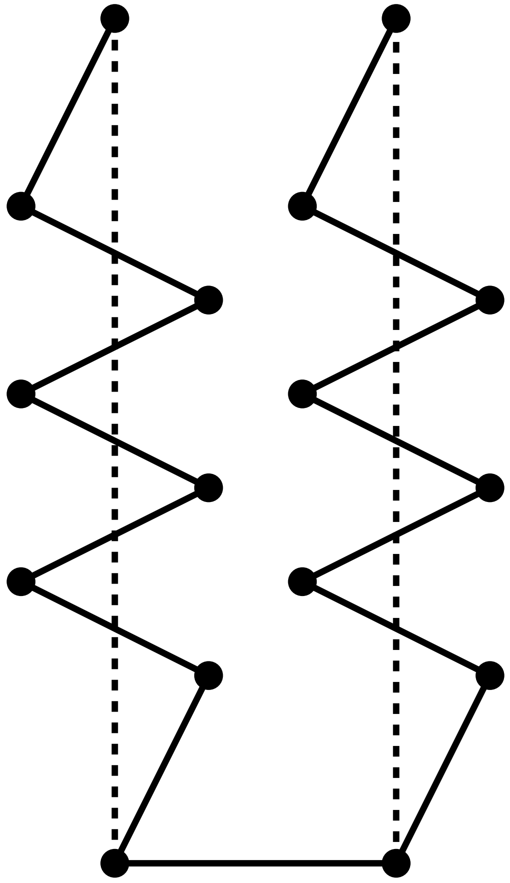
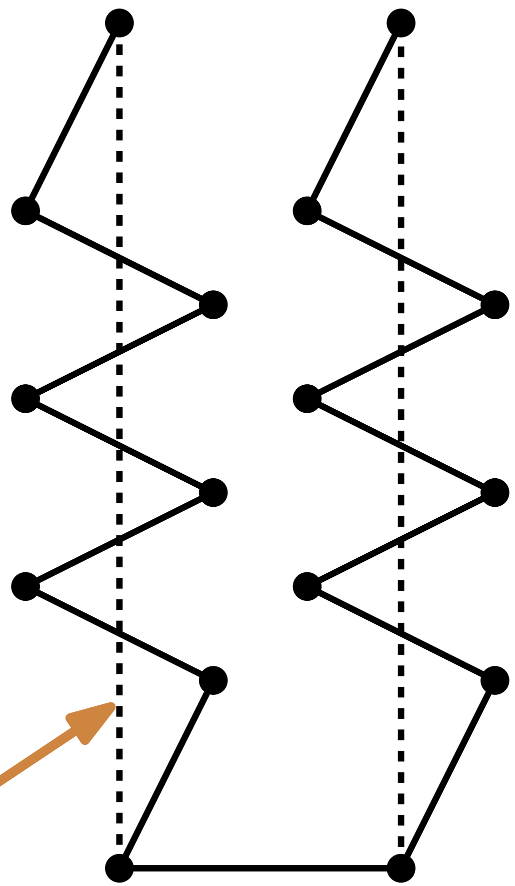


Interpretation:

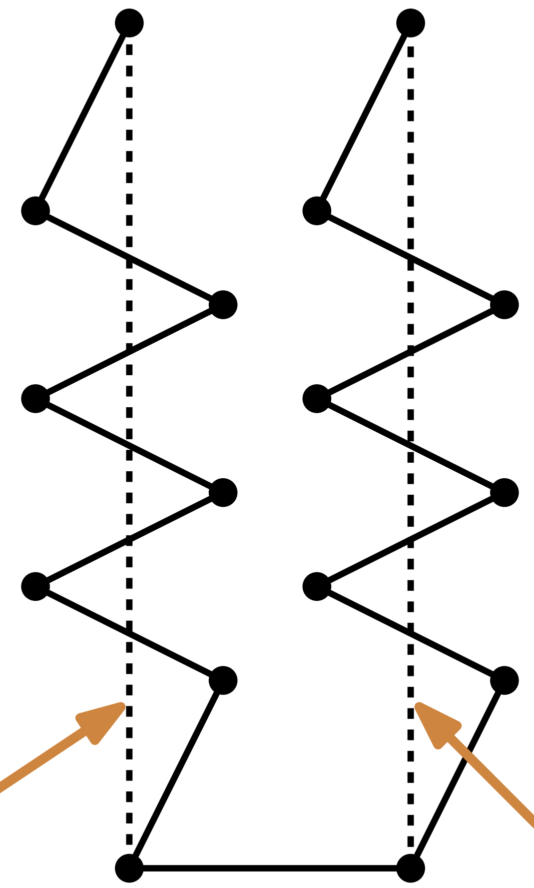
Variable-Gadget



Interpretation:

shortcut taken $\Leftrightarrow$ variable set to true

Variable-Gadget

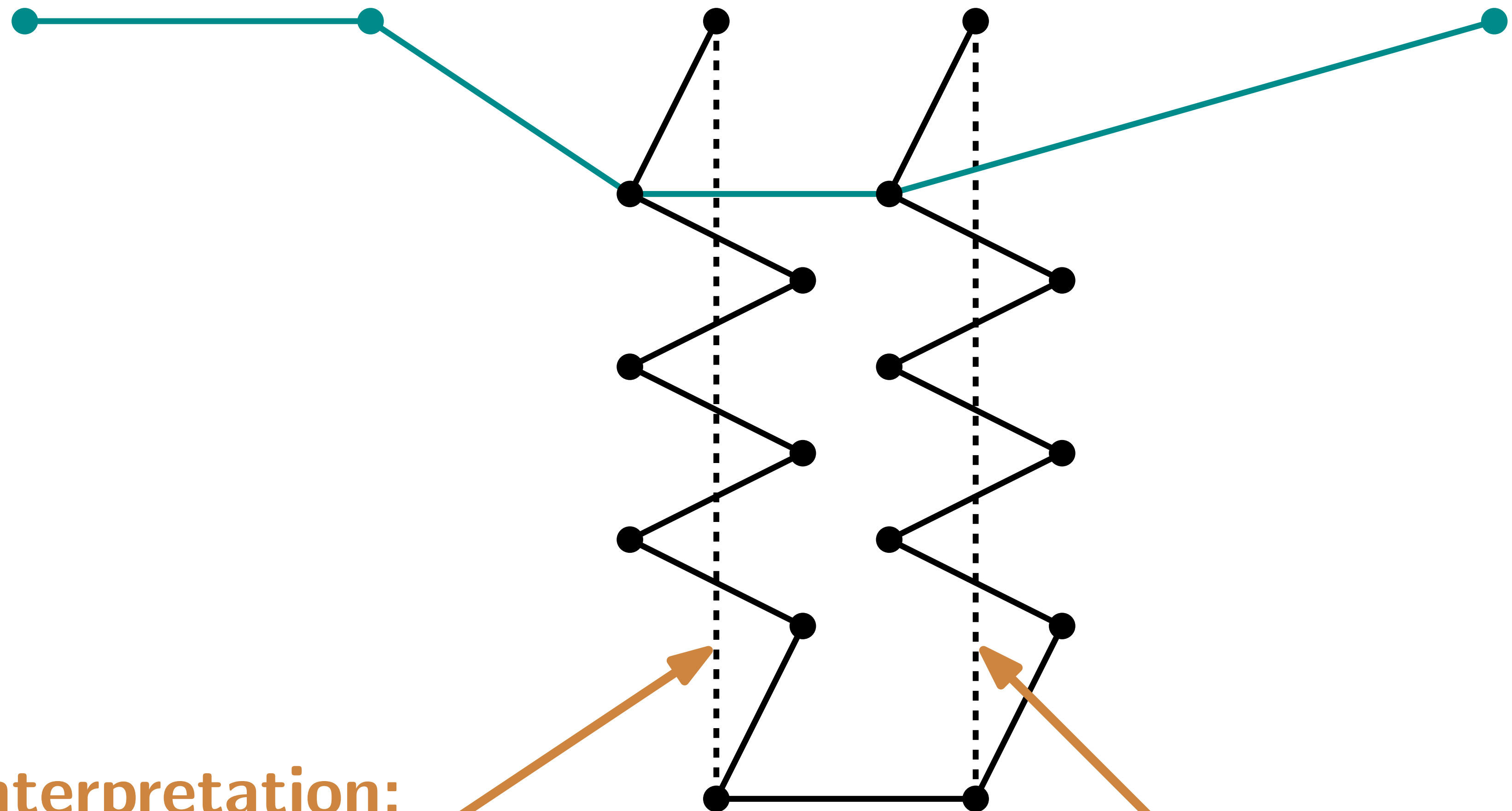


Interpretation:

shortcut taken ⇔ variable set to false

shortcut taken ⇔ variable set to true

Variable-Gadget



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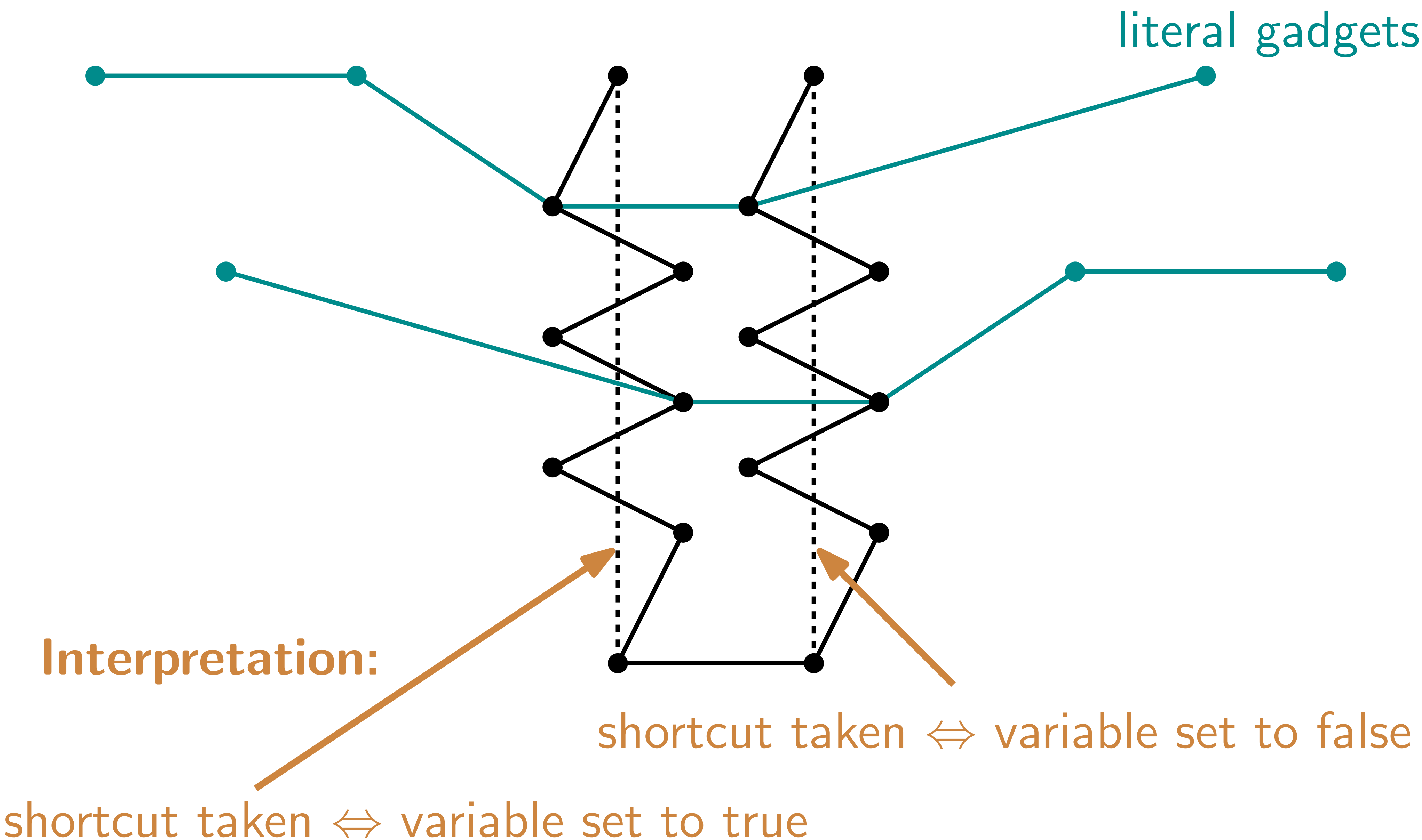
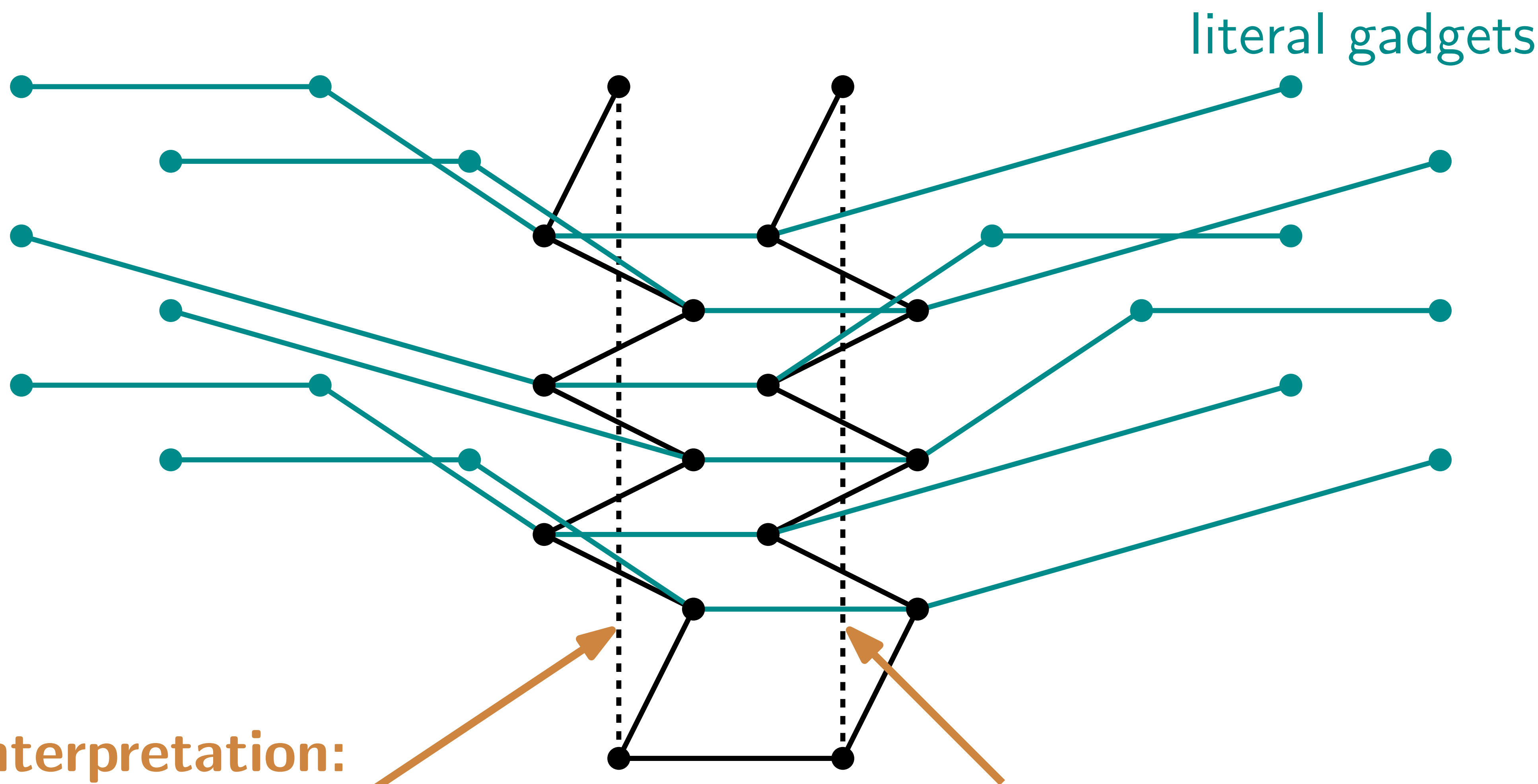


Variable-Gadget



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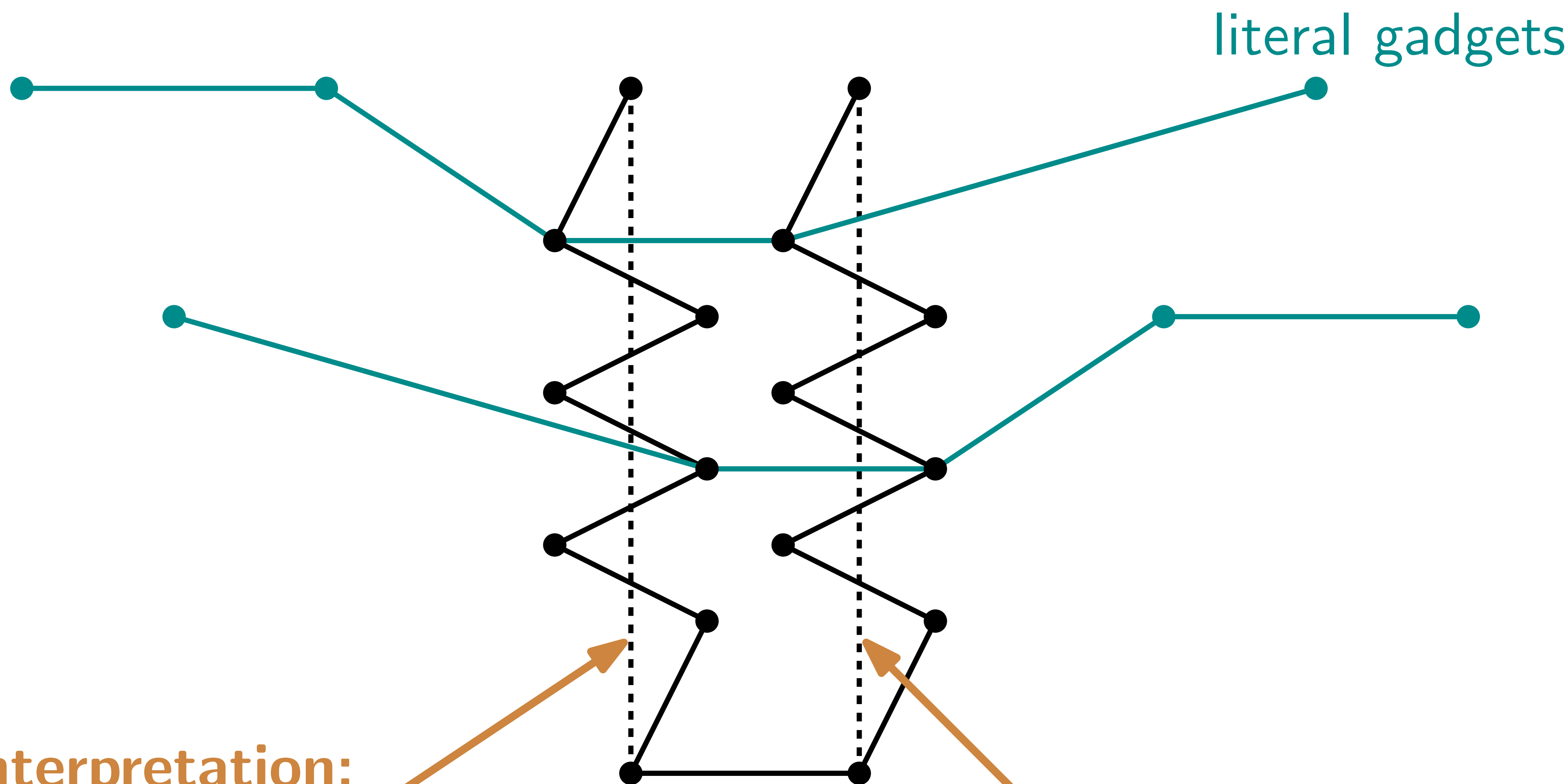
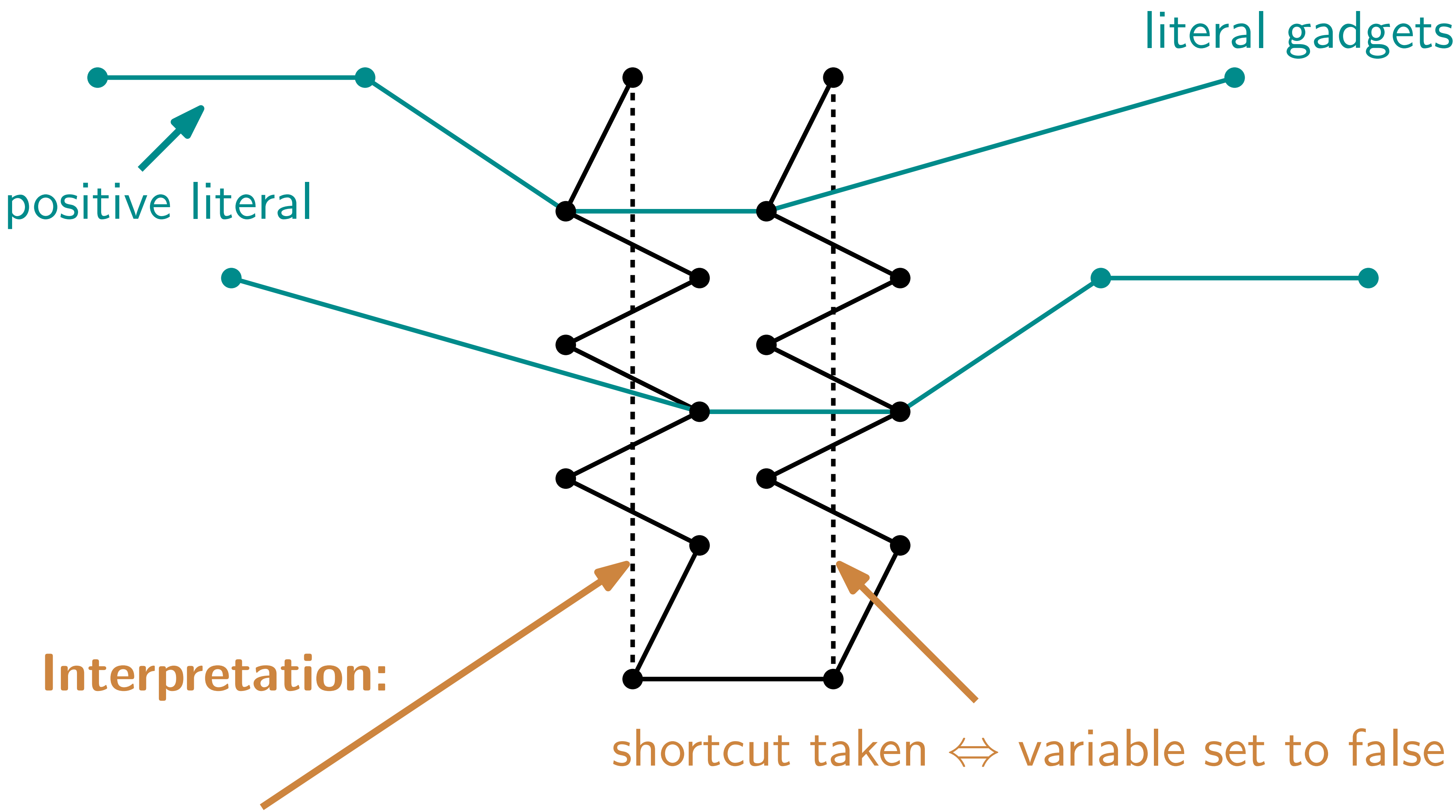


Interpretation:

shortcut taken ⇔ variable set to false

shortcut taken ⇔ variable set to true

Variable-Gadget



Interpretation:

shortcut taken ⇔ variable set to false

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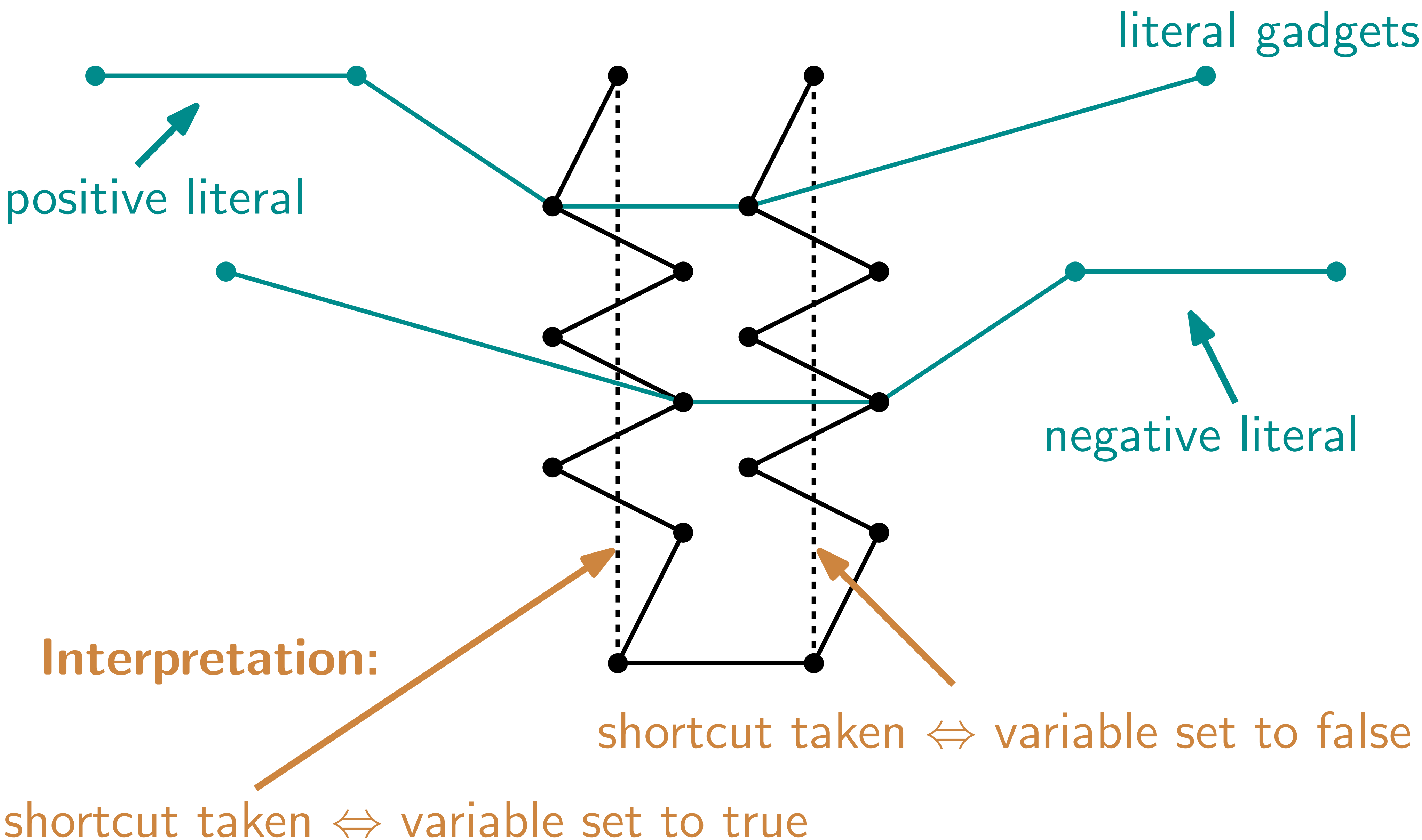
Variable-Gadget



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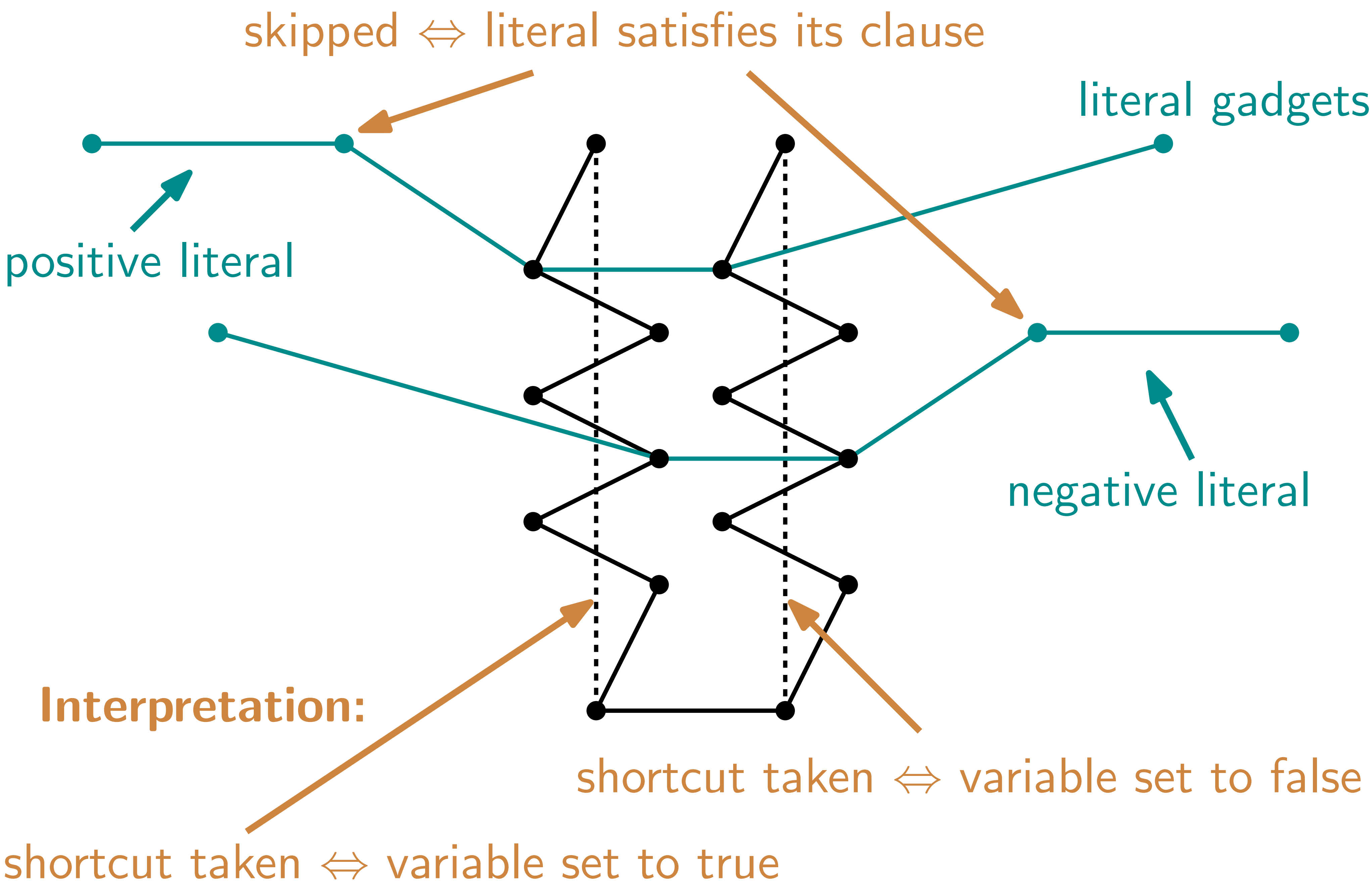

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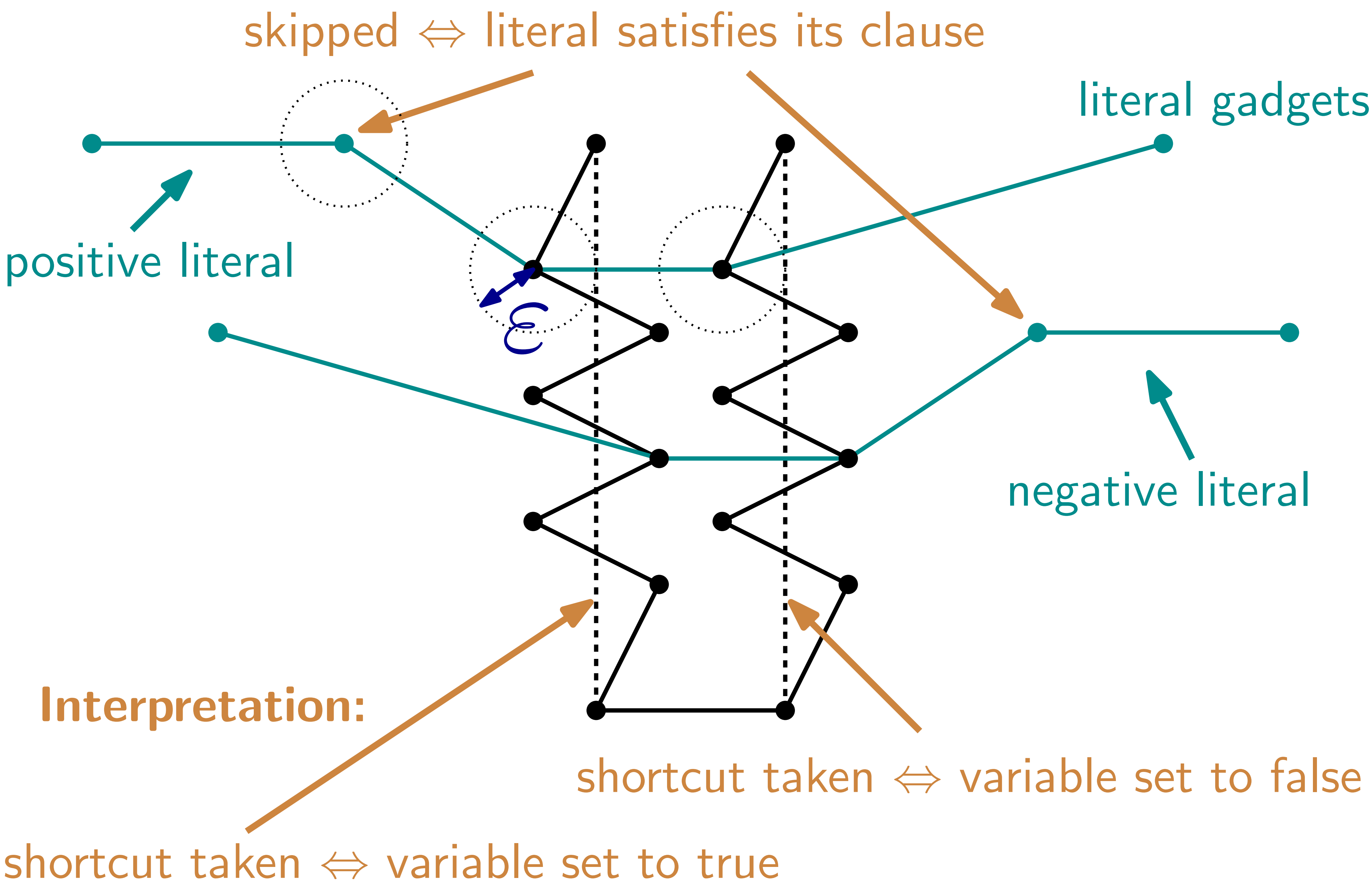

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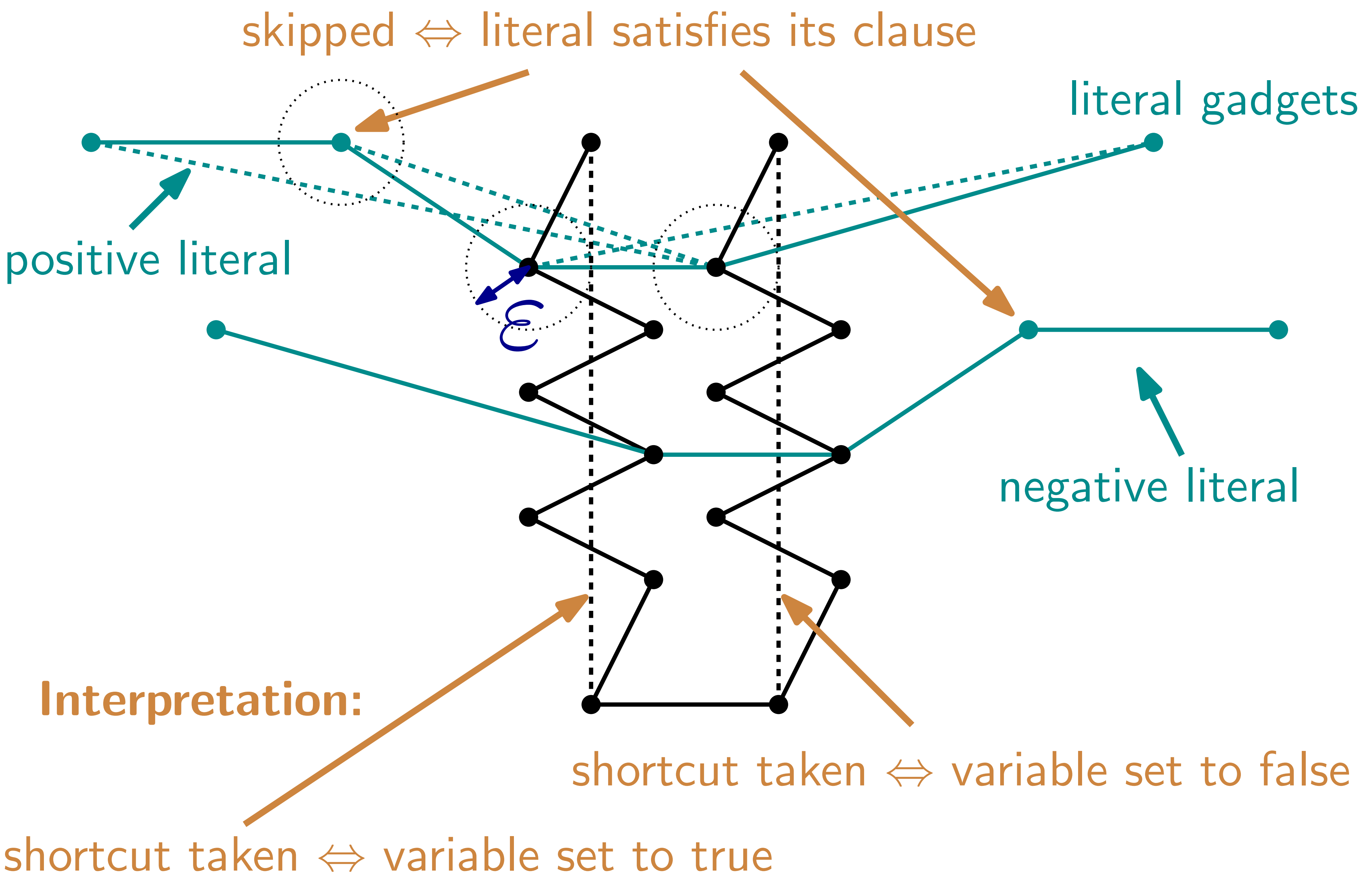

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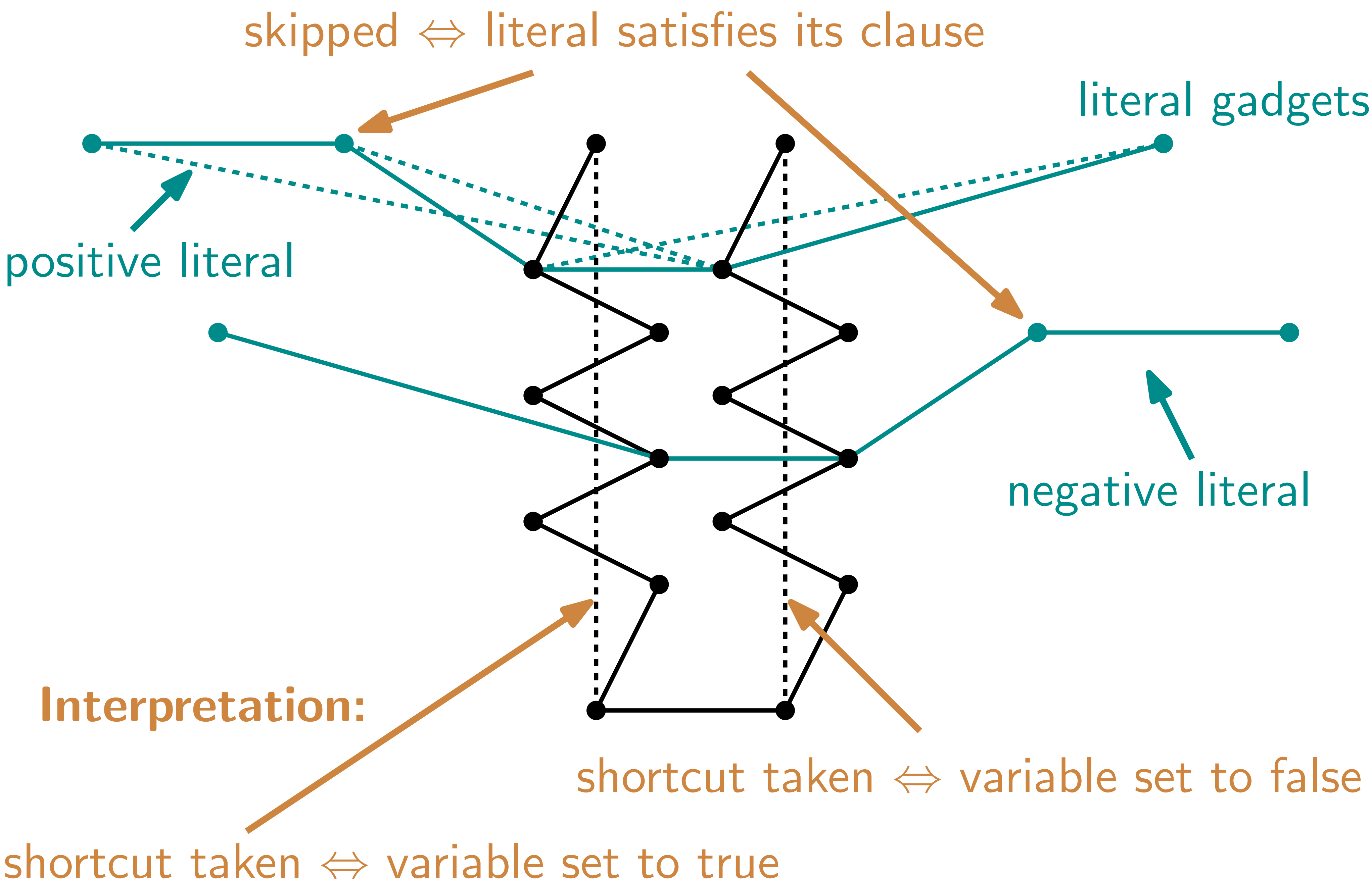

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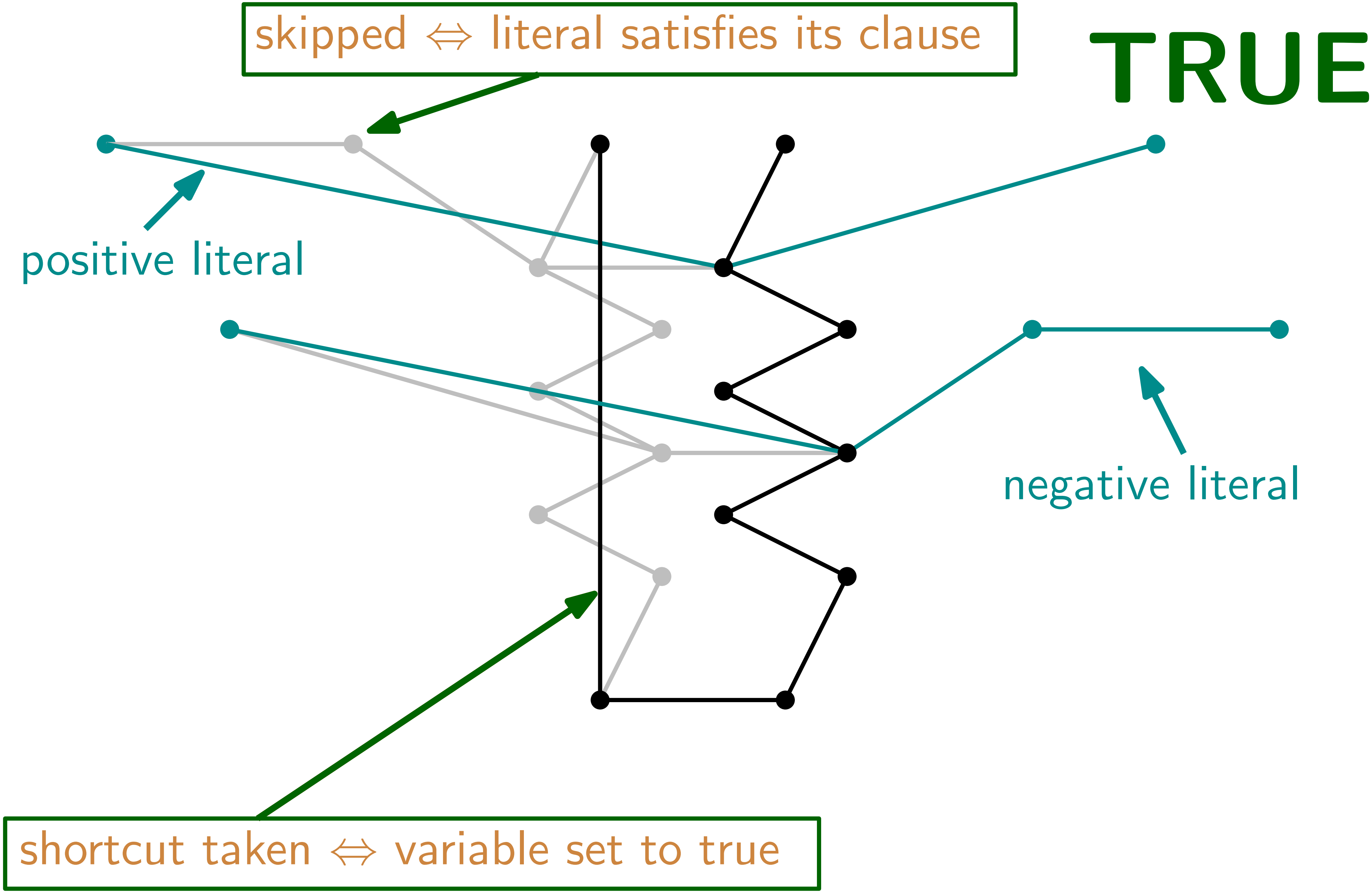

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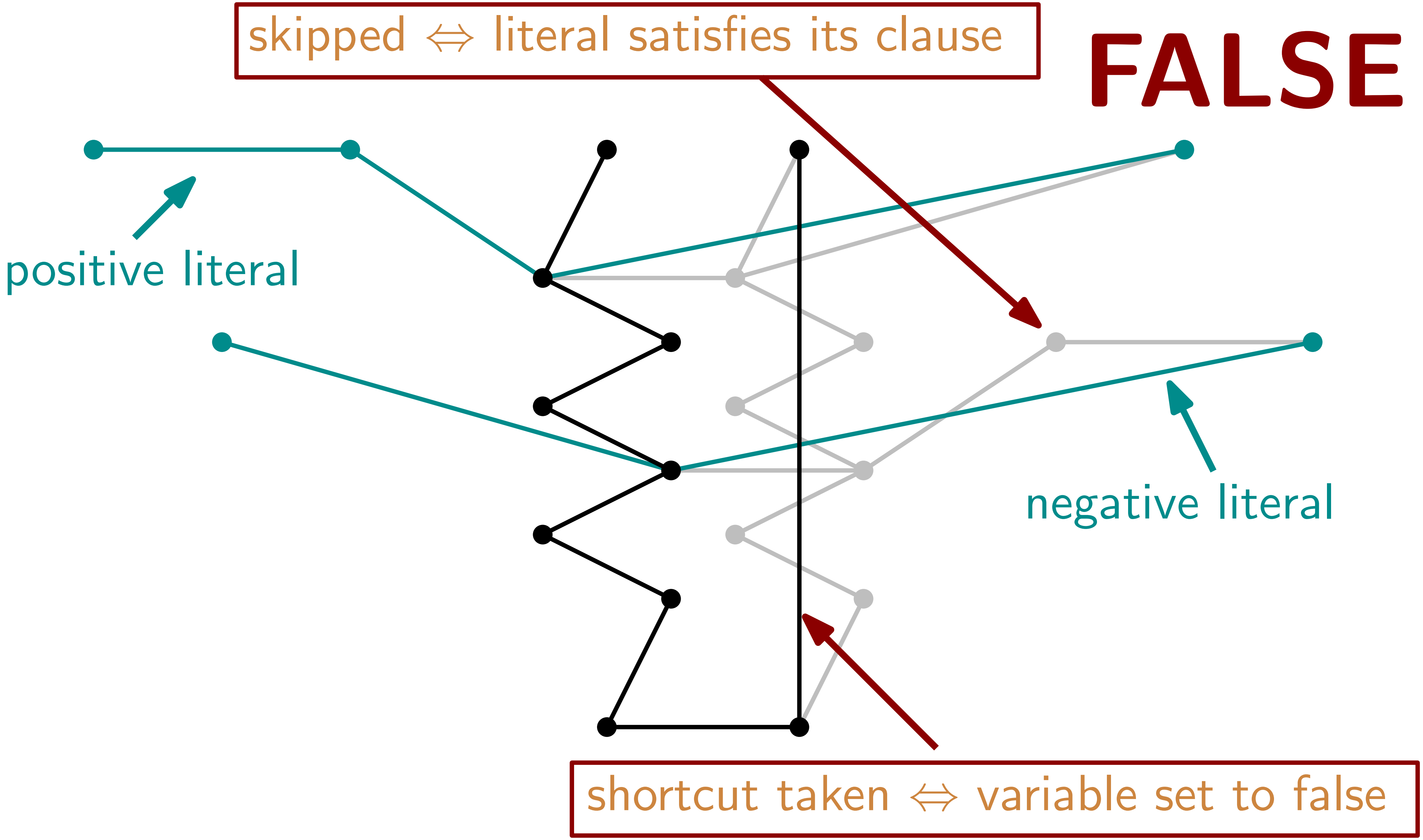

Clause-Gadget

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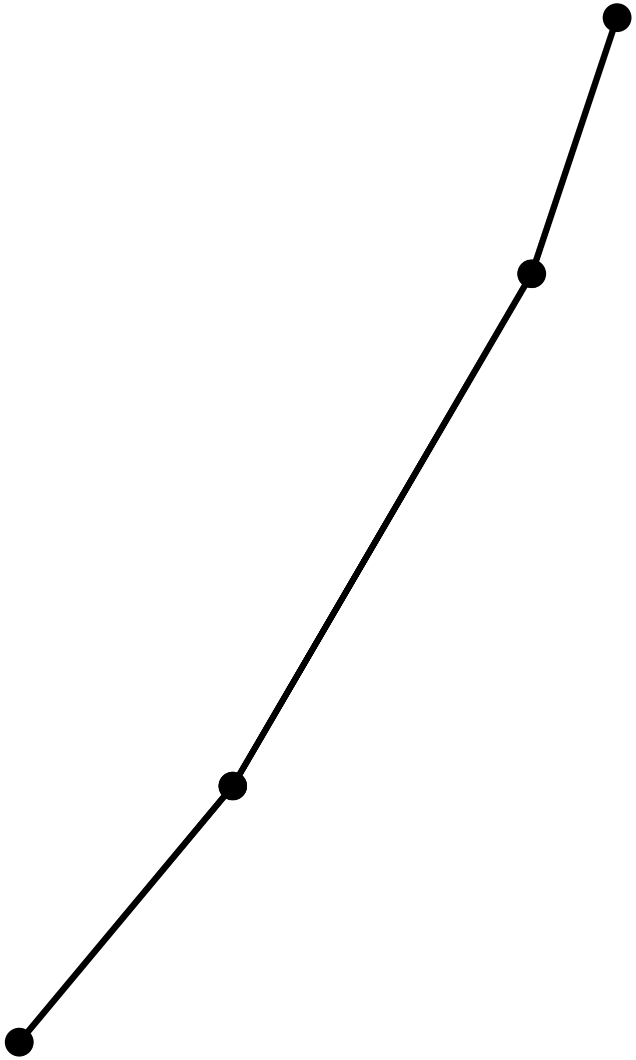

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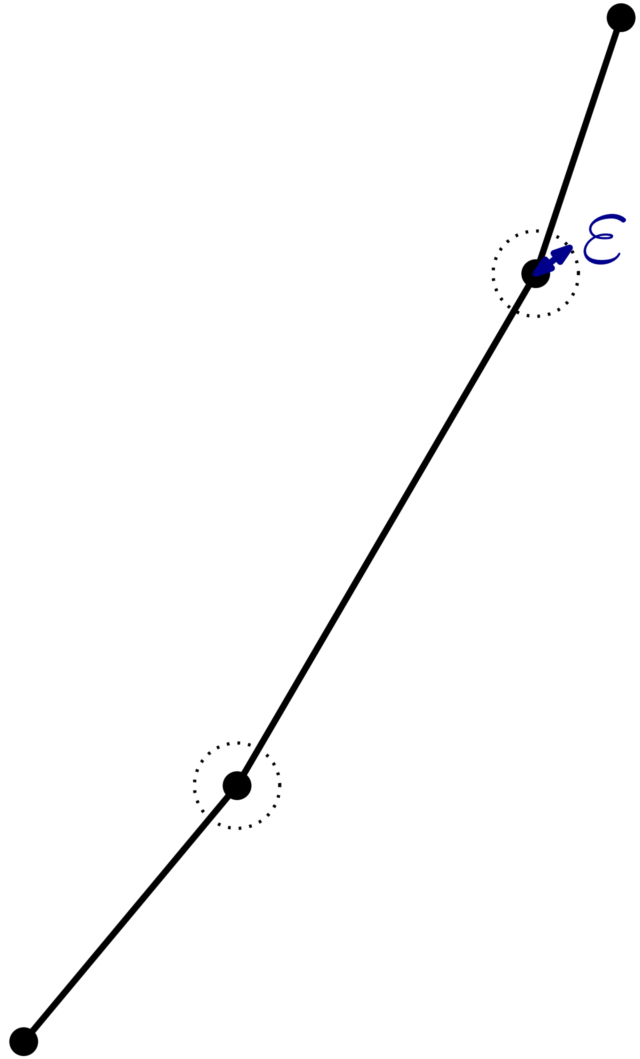

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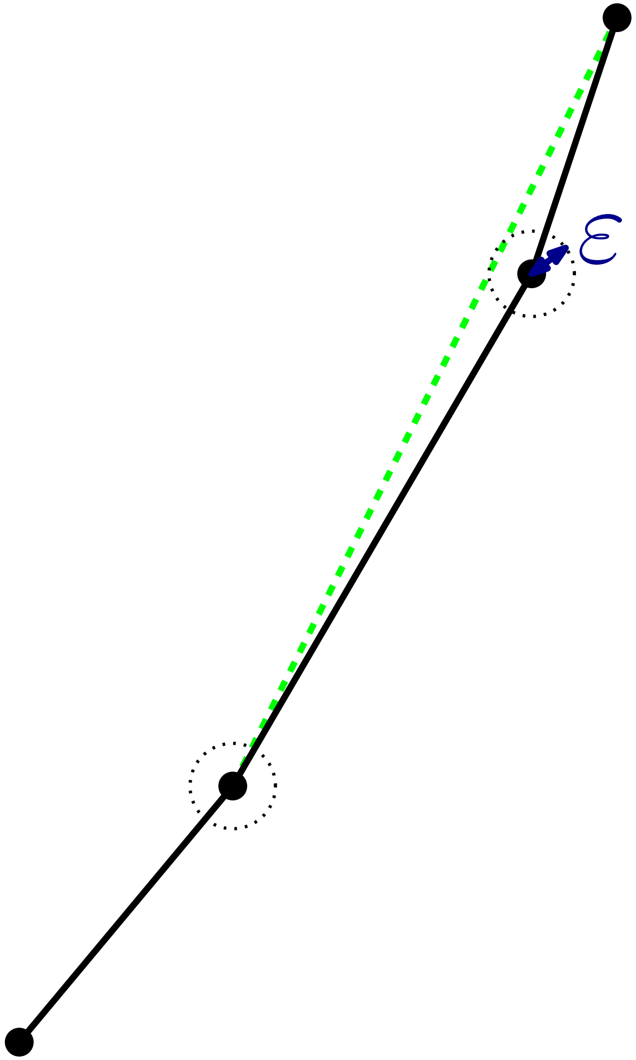

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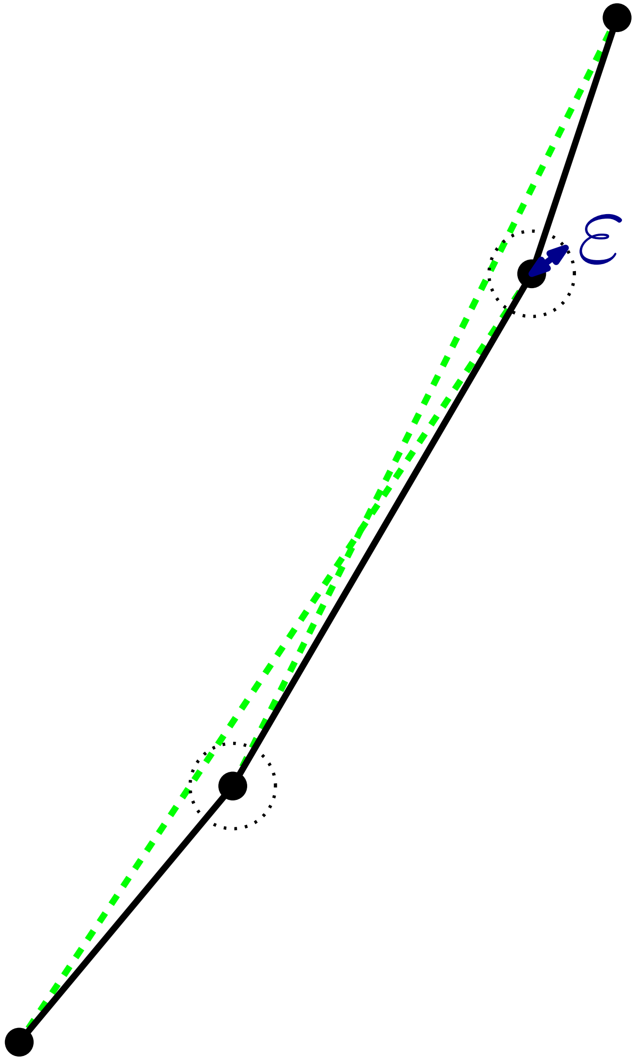

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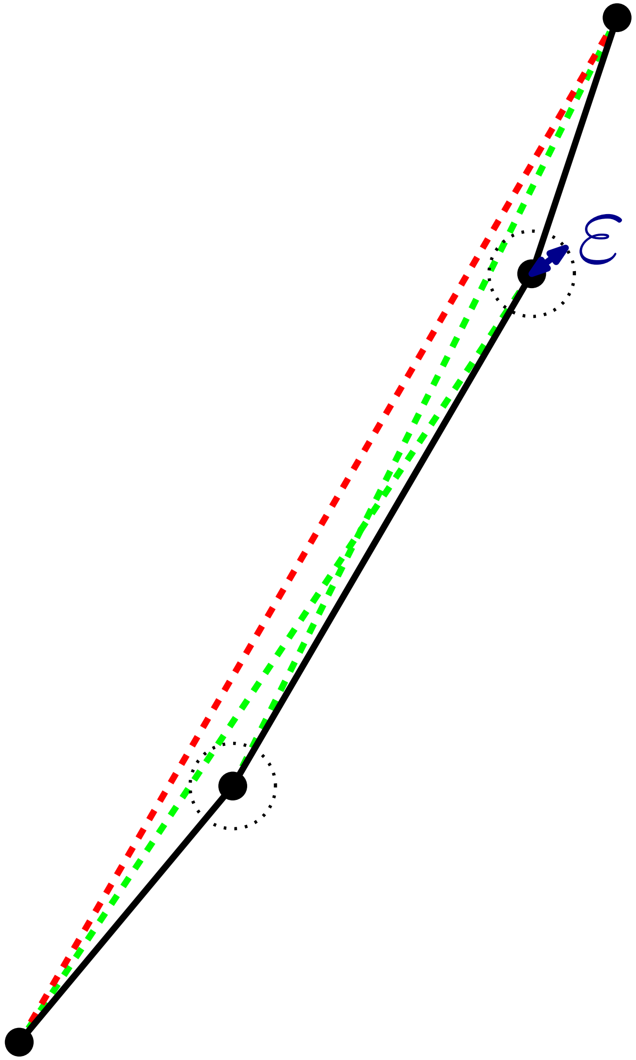

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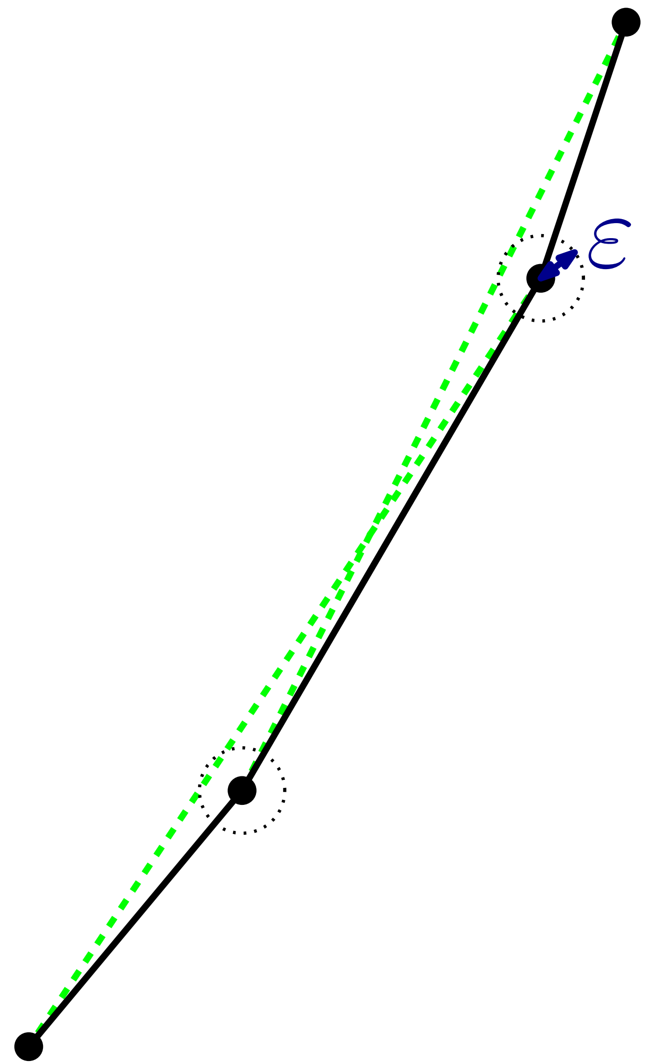

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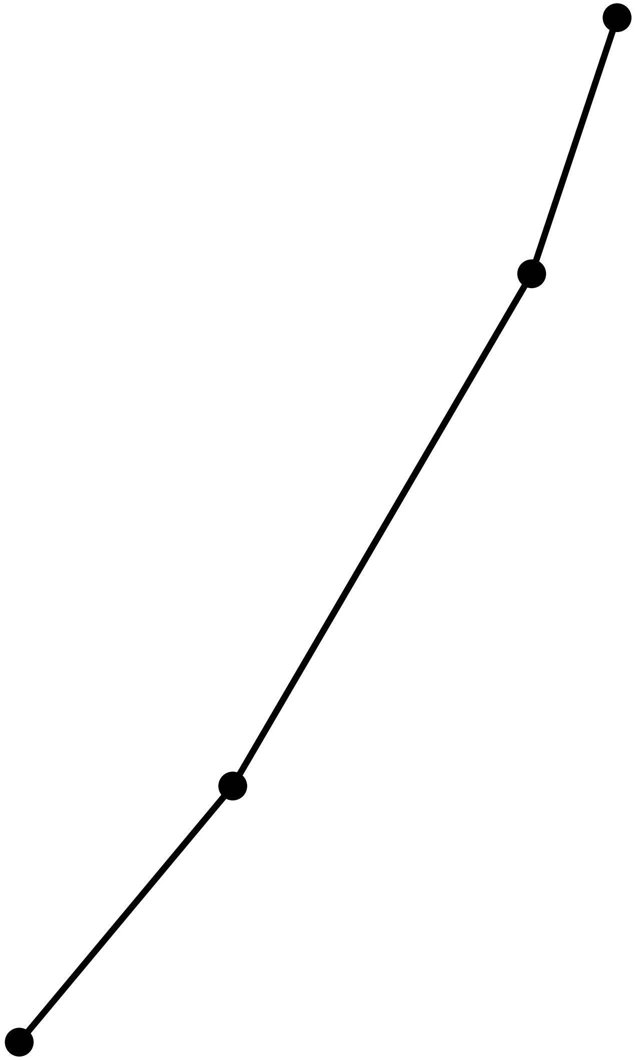

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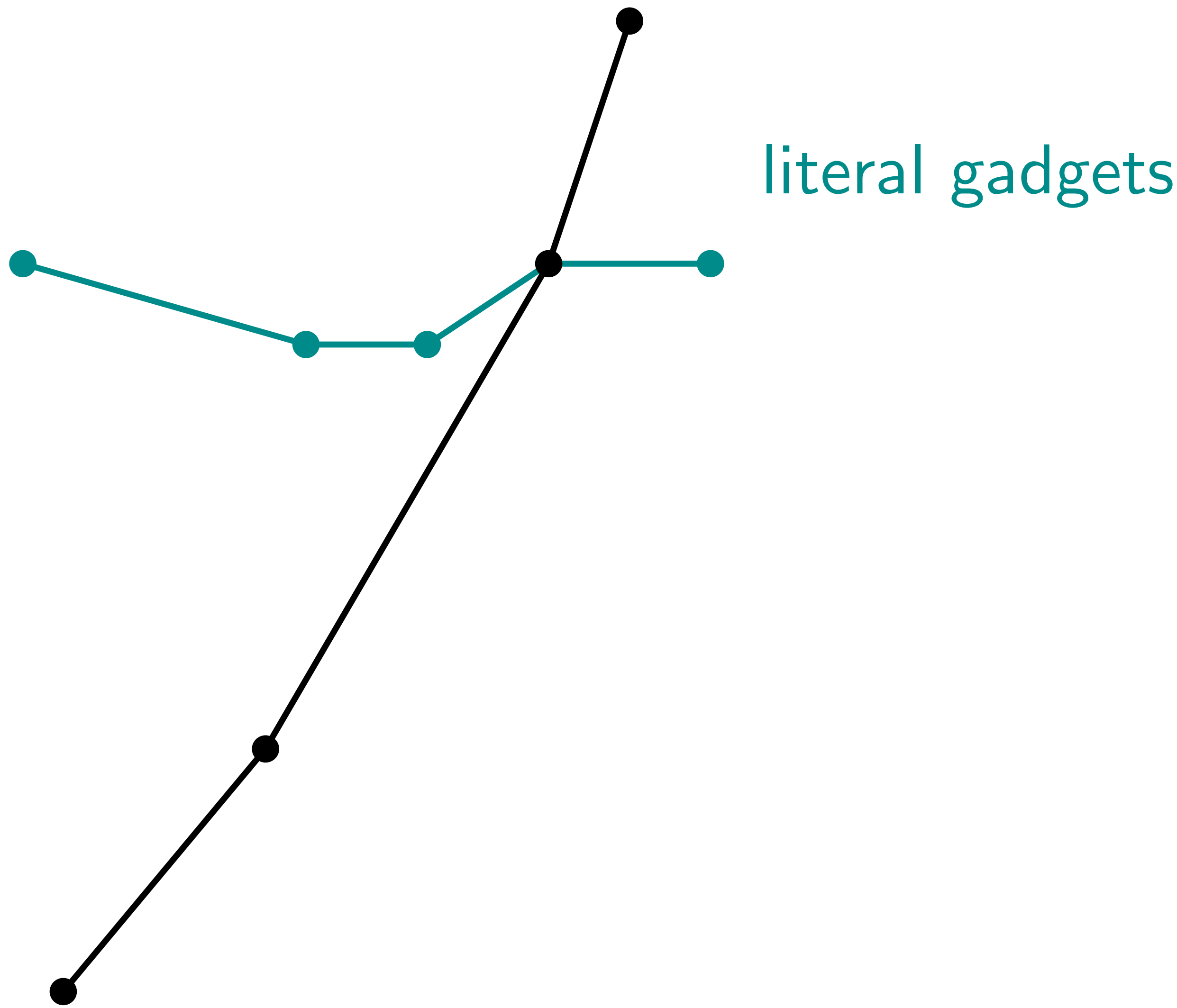

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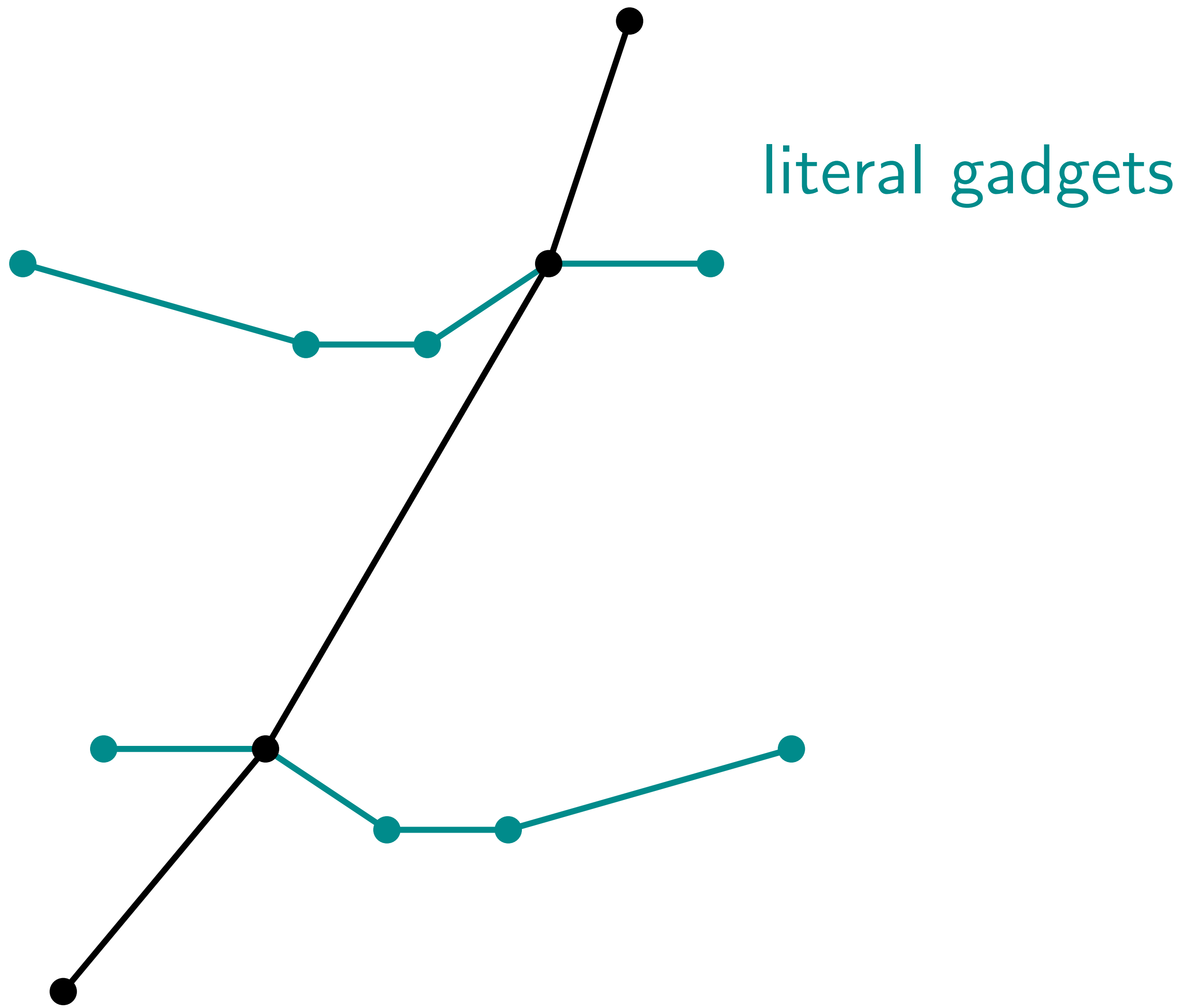
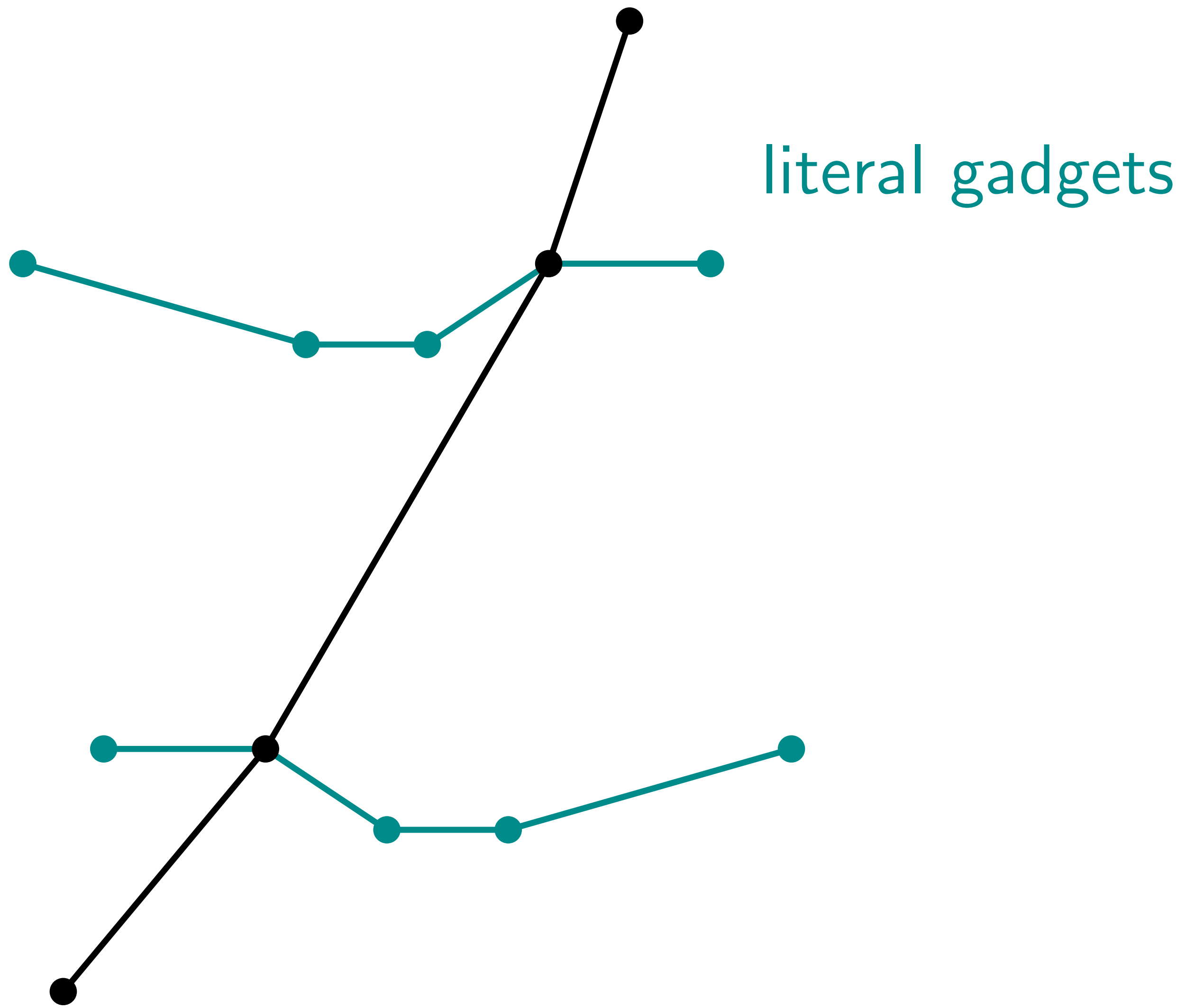


Clause-Gadget

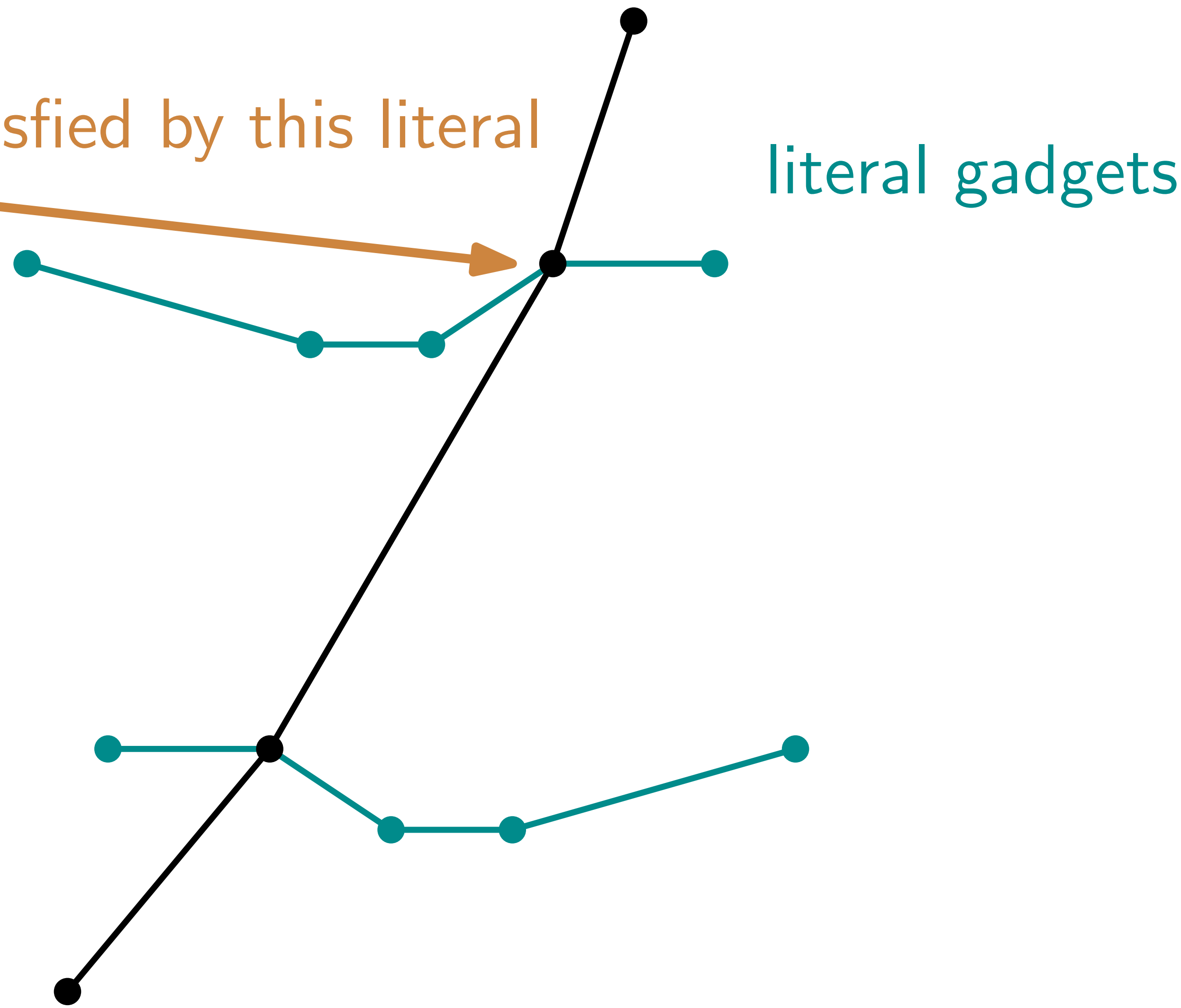
Interpretation:



Clause-Gadget

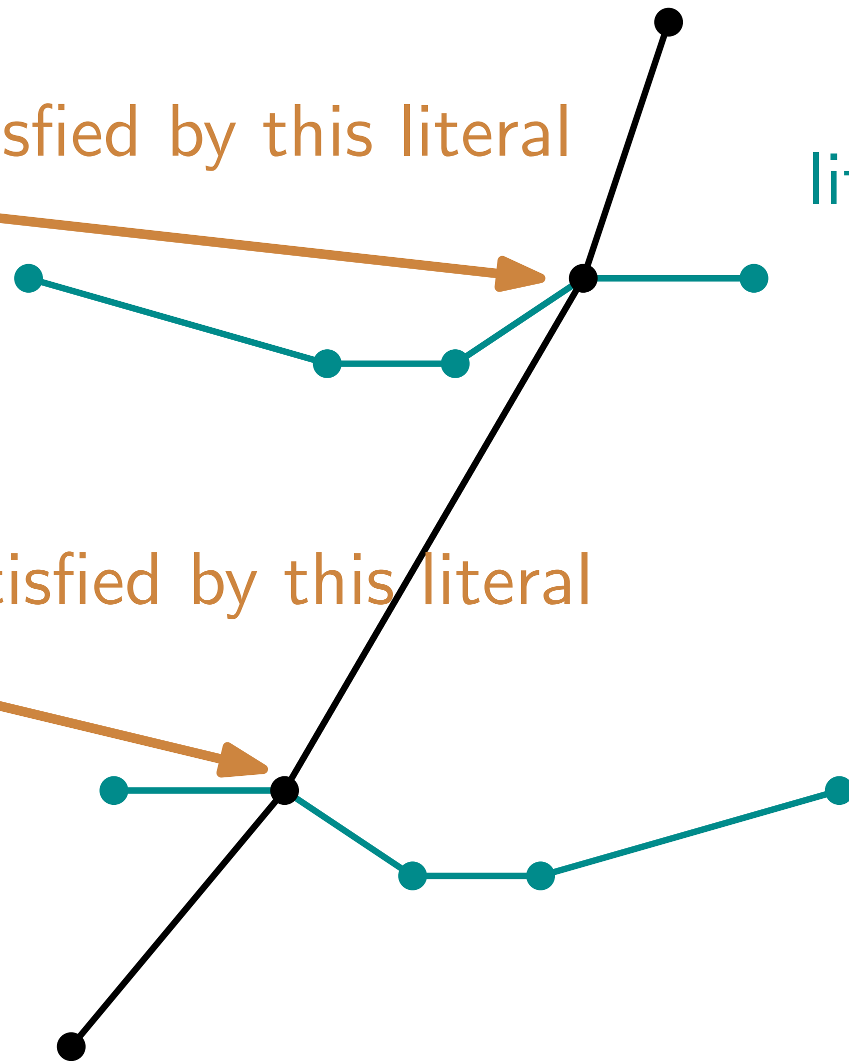
Interpretation:

skipped $\Leftrightarrow$ clause satisfied by this literal



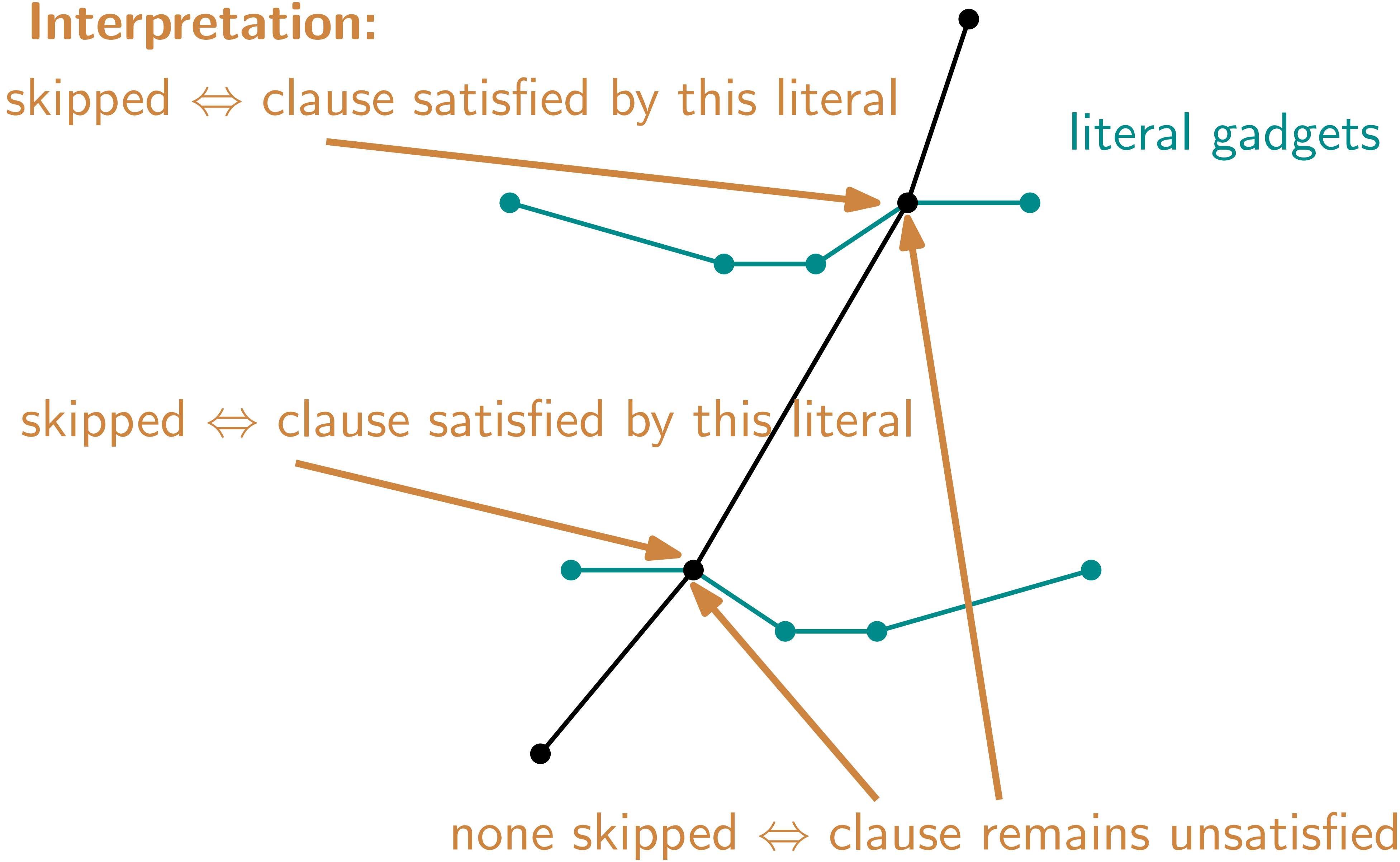
Clause-Gadget

Interpretation:



Clause-Gadget

Interpretation:



Full Example

Full Example

$(x_1 \lor x_2) \land$
$(\neg x_1 \lor x_3) \land$
$(\neg x_3)$

Full Example

$(x_1 \vee x_2) \wedge$

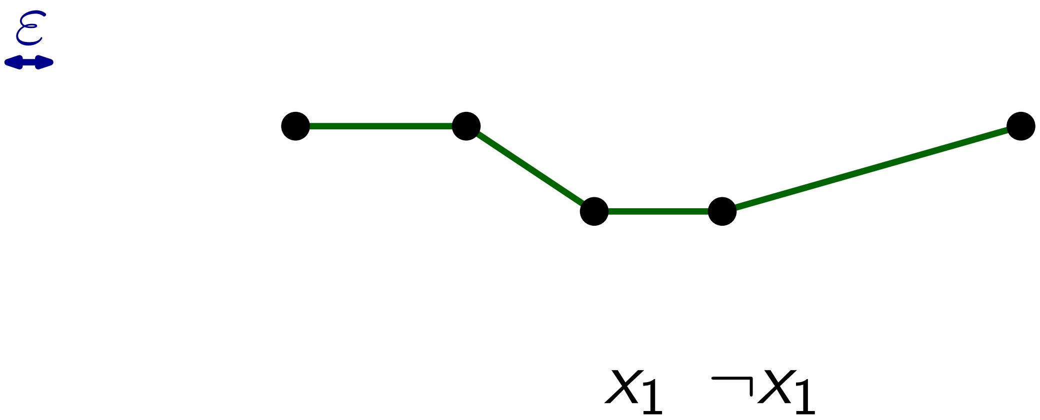
$(\neg x_1 \vee x_3) \wedge$

$(\neg x_3)$

$\xleftrightarrow{\varepsilon}$

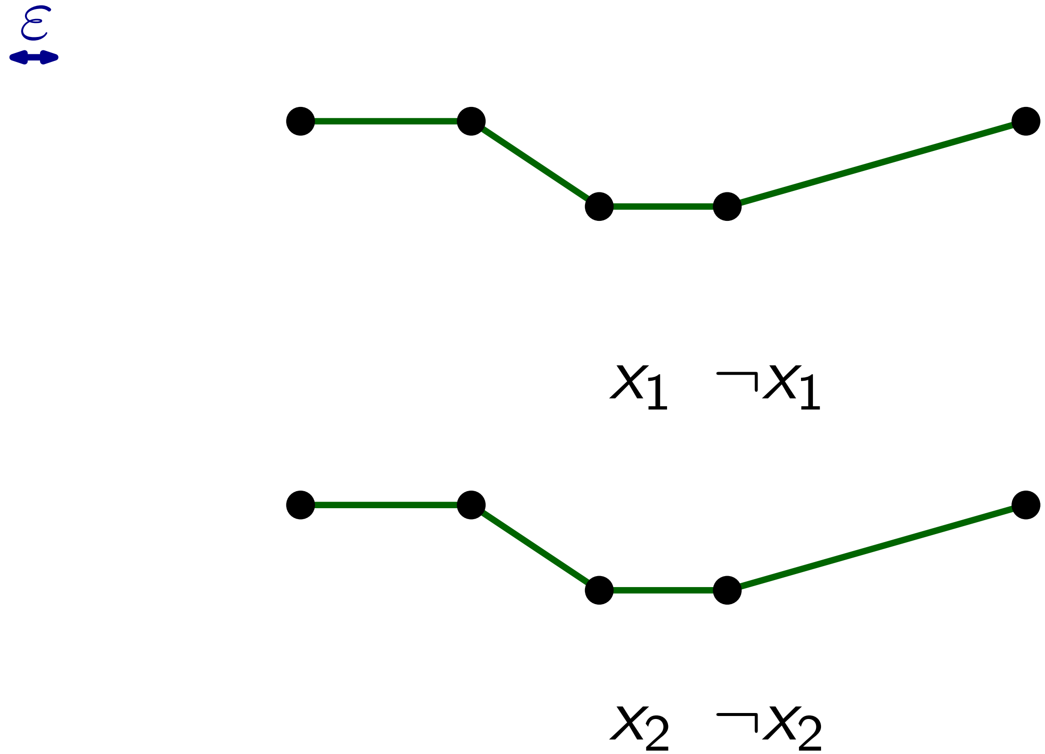
Full Example

$(\boxed{x_1} \vee x_2) \wedge$
$(\neg x_1 \vee x_3) \wedge$
$(\neg x_3)$



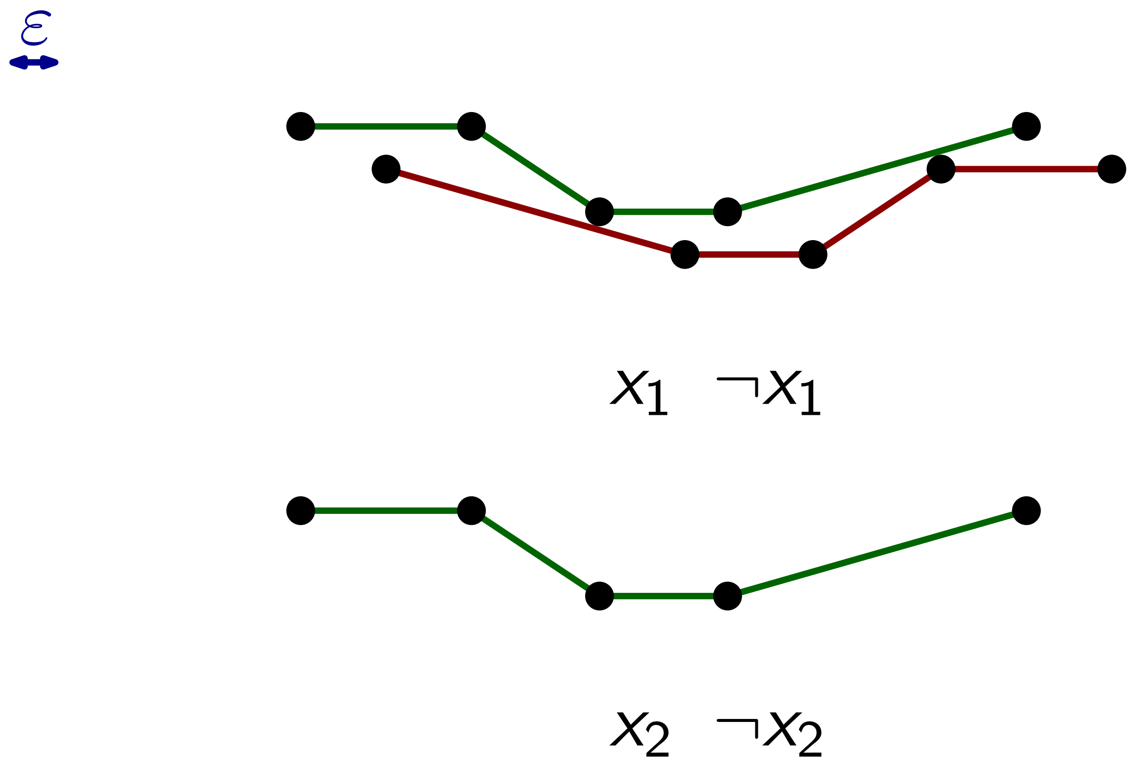
Full Example

$(x_1 \vee \boxed{x_2}) \wedge$
$(\neg x_1 \vee x_3) \wedge$
$(\neg x_3)$



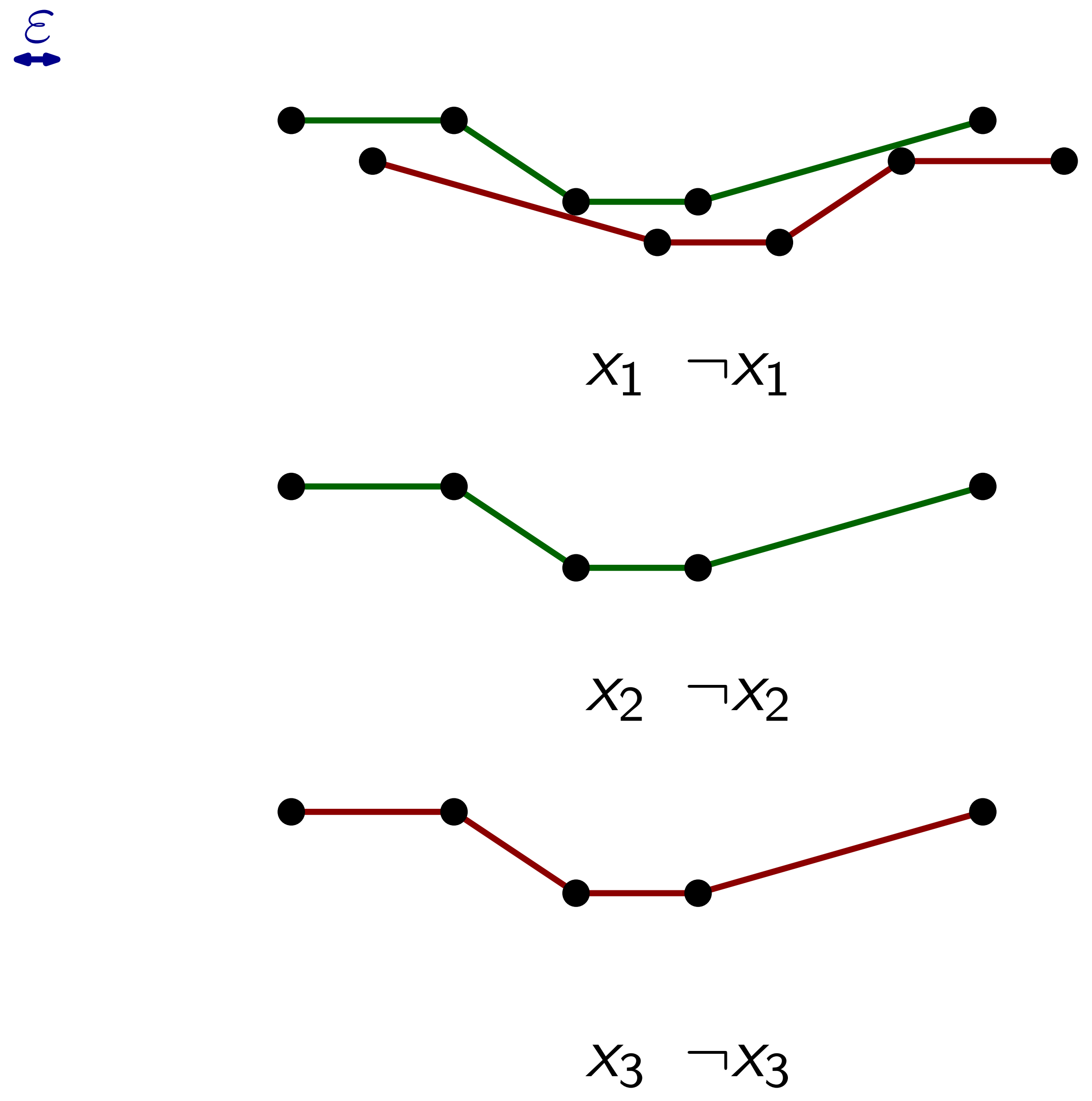
Full Example

$(x_1 \lor x_2)\land$
$(\boxed{\neg x_1} \lor x_3)\land$
$(\neg x_3)$



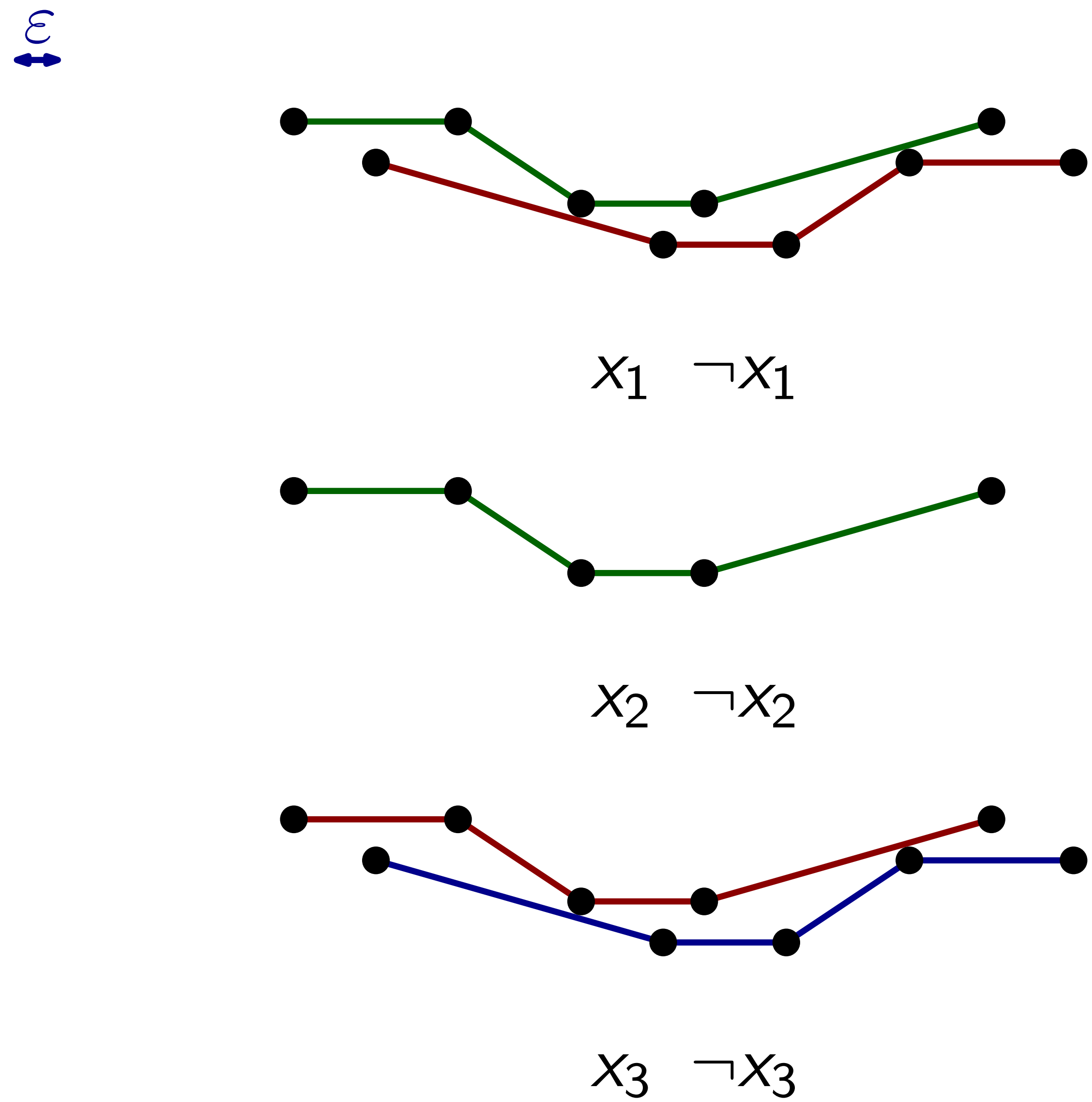
Full Example

$(x_1 \lor x_2) \land$
$(\neg x_1 \lor \boxed{x_3}) \land$
$(\neg x_3)$



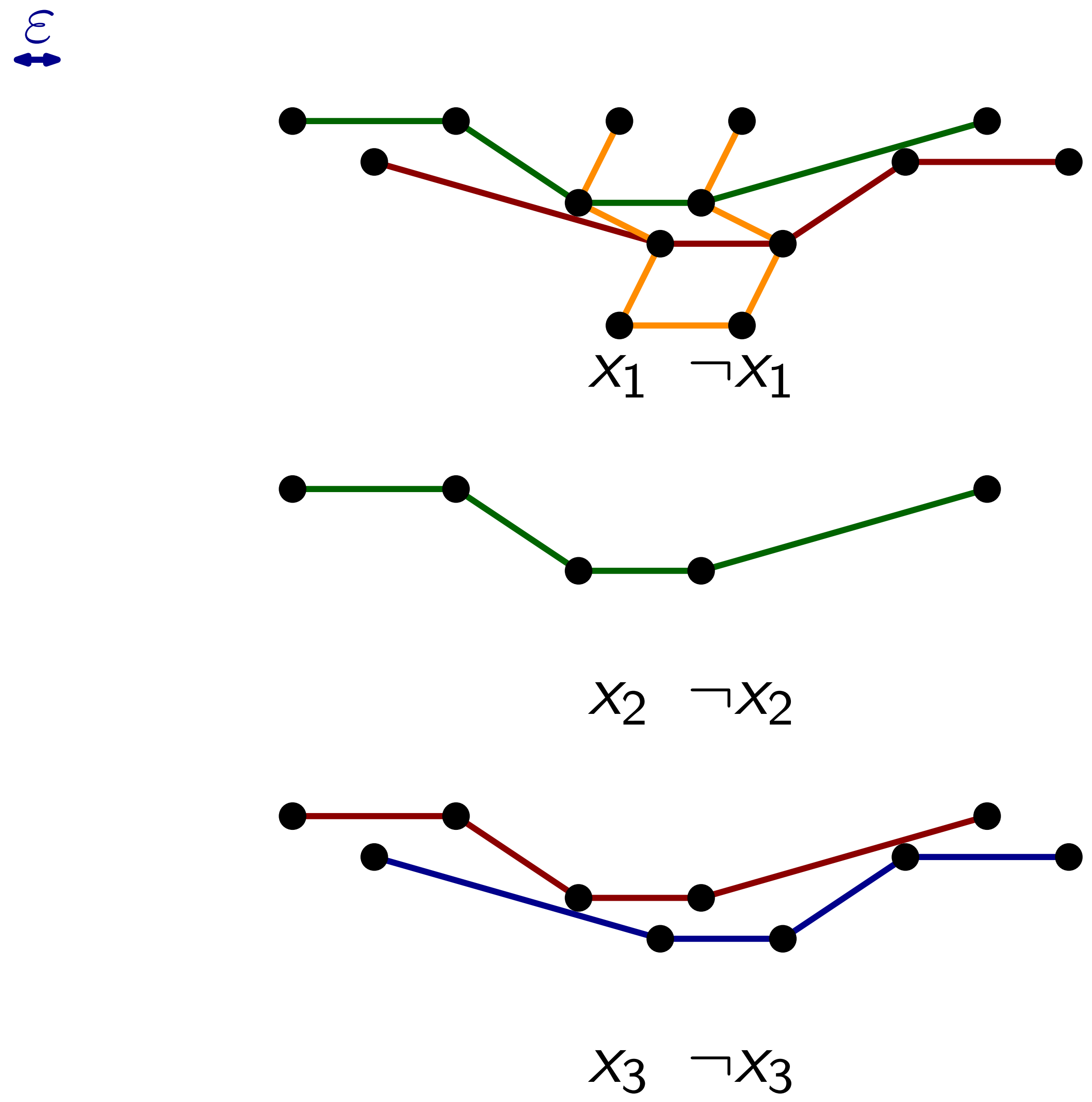
Full Example

$(x_1 \vee x_2) \wedge$
$(\neg x_1 \vee x_3) \wedge$
$(\boxed{\neg x_3})$



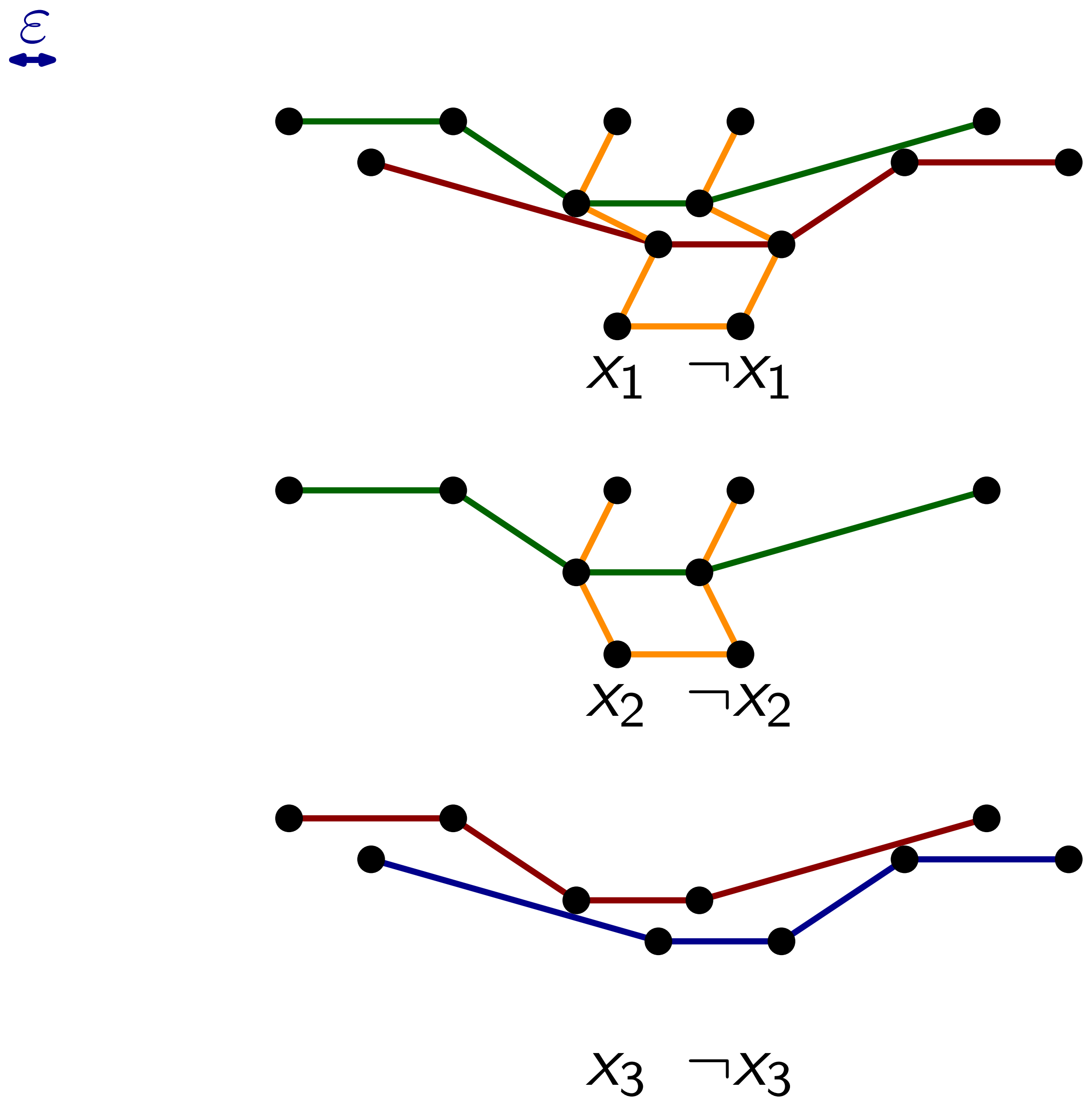
Full Example

$(\boxed{x_1} \vee x_2) \wedge$
$(\boxed{\neg x_1} \vee x_3) \wedge$
$(\neg x_3)$



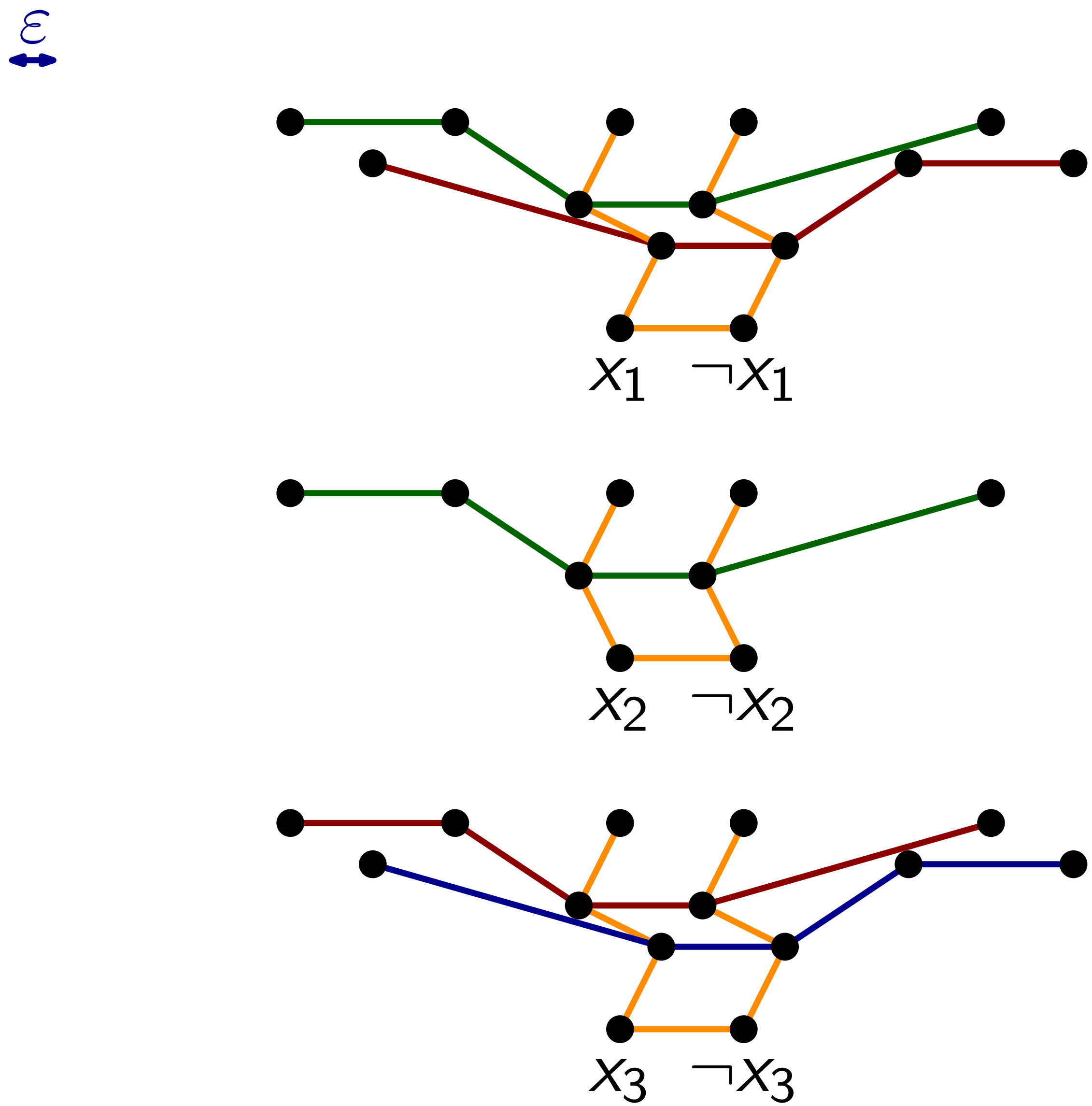
Full Example

$(x_1 \vee \boxed{x_2})\wedge$
$(\neg x_1 \vee x_3)\wedge$
$(\neg x_3)$



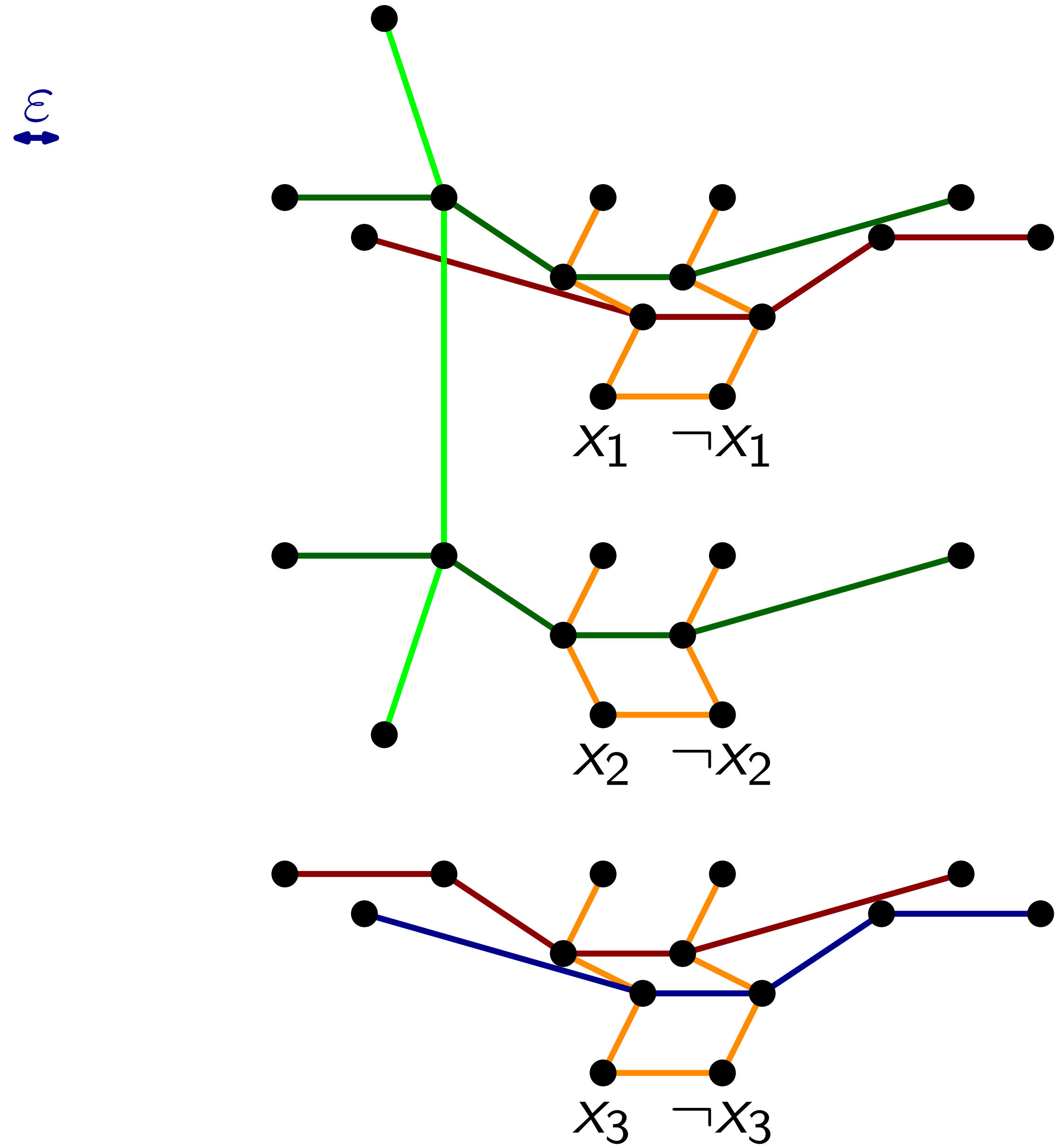
Full Example

$(x_1 \vee x_2) \wedge$
$(\neg x_1 \vee \boxed{x_3}) \wedge$
$(\boxed{\neg x_3})$



Full Example

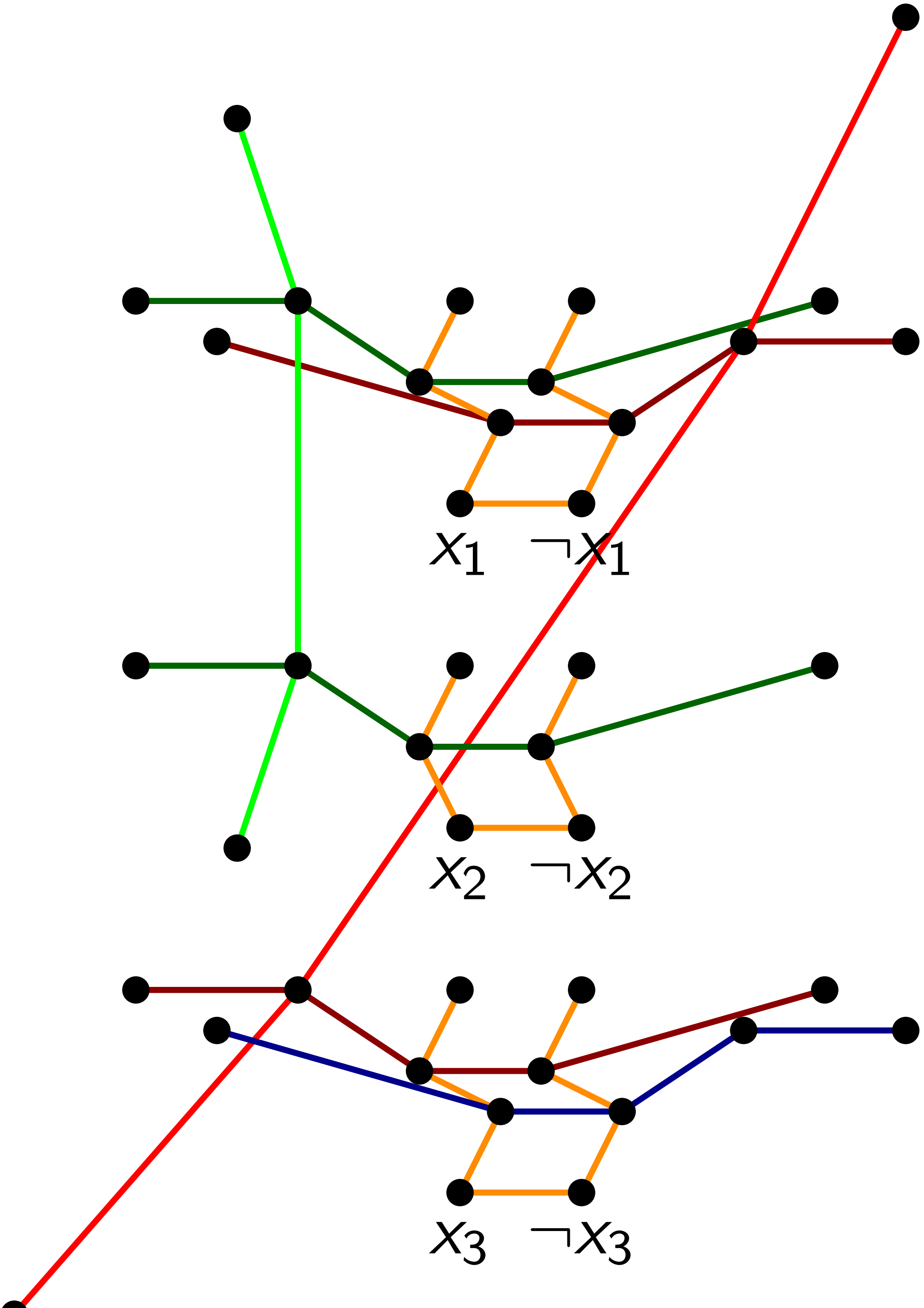
$\boxed{(x_1 \vee x_2)}\wedge$
$(\neg x_1 \vee x_3)\wedge$
$(\neg x_3)$



Full Example

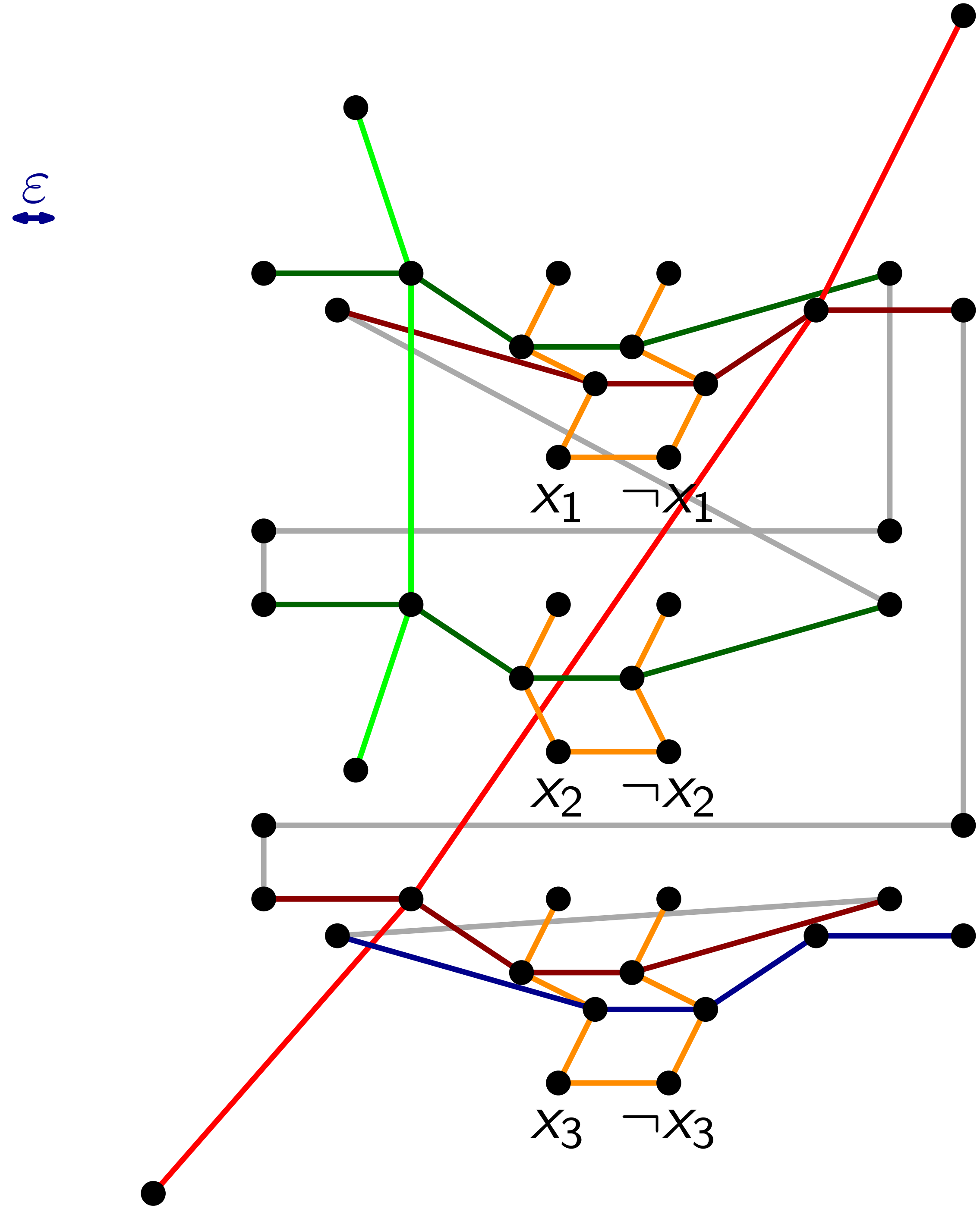
$(x_1 \lor x_2) \land$
$\boxed{(\neg x_1 \lor x_3)} \land$
$(\neg x_3)$

ε



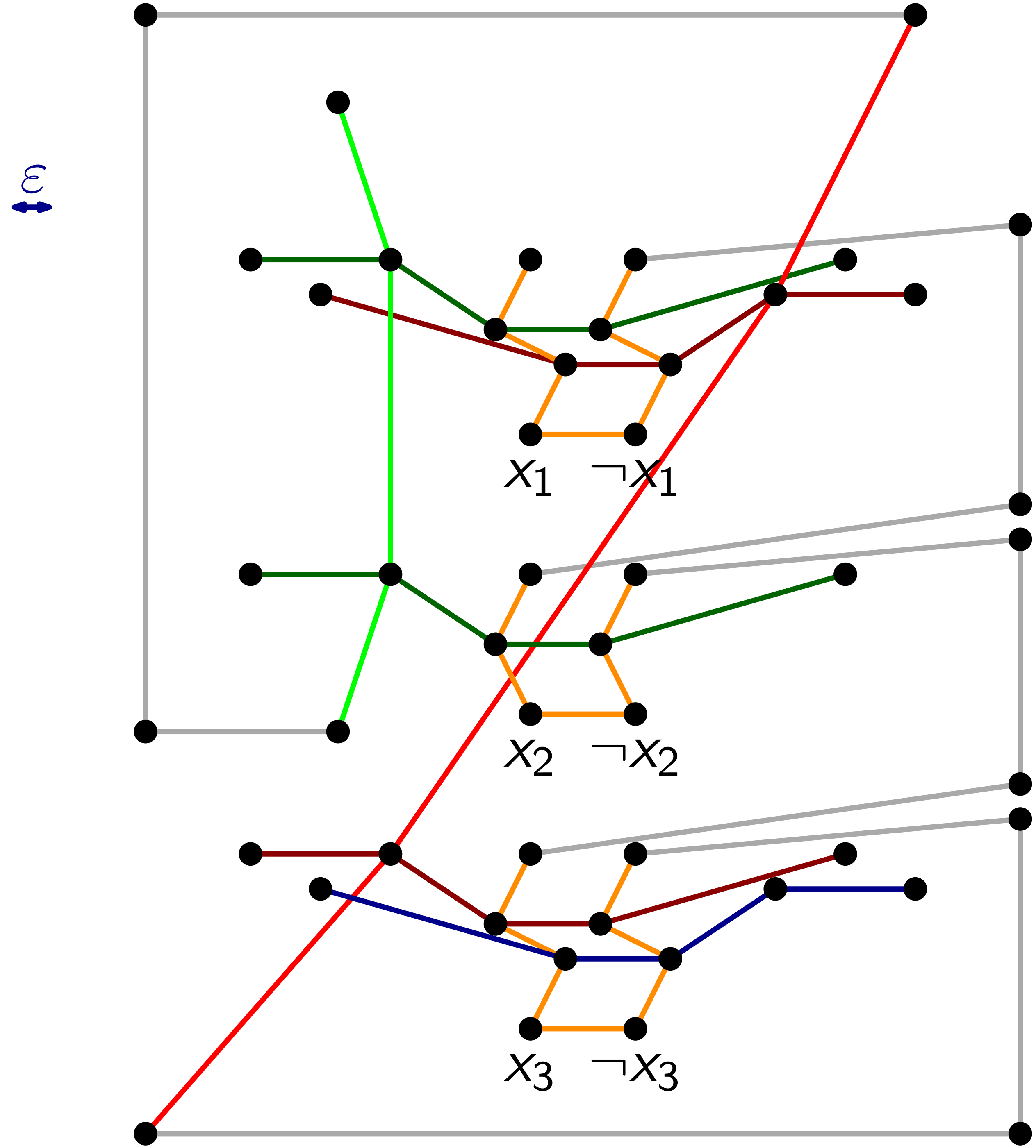
Full Example

$\big(x_1 \lor x_2\big) \land$
$\big(\neg x_1 \lor x_3\big) \land$
$\big(\neg x_3\big)$



Full Example

$(x_1 \vee x_2) \wedge$
$(\neg x_1 \vee x_3) \wedge$
$(\neg x_3)$



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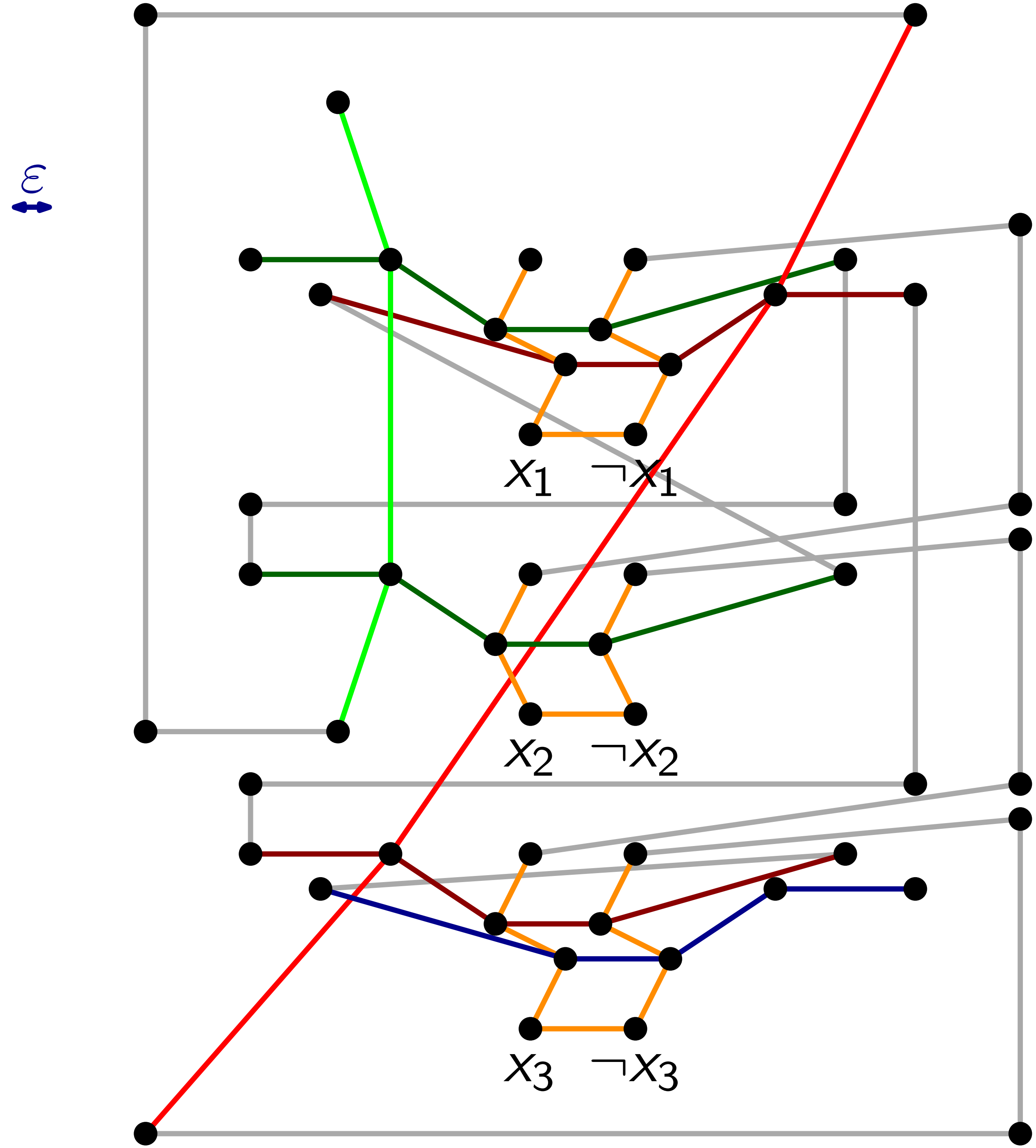

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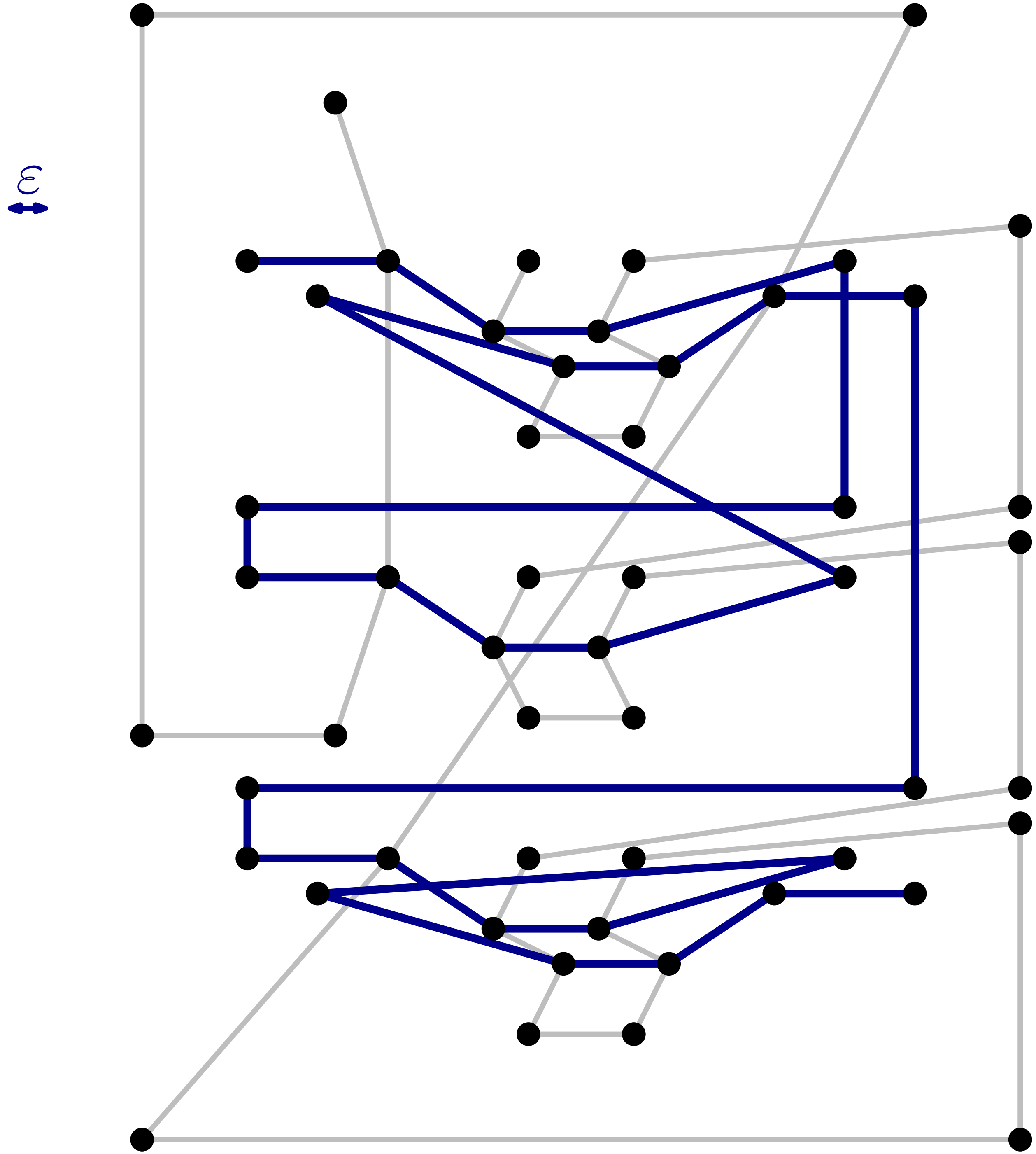

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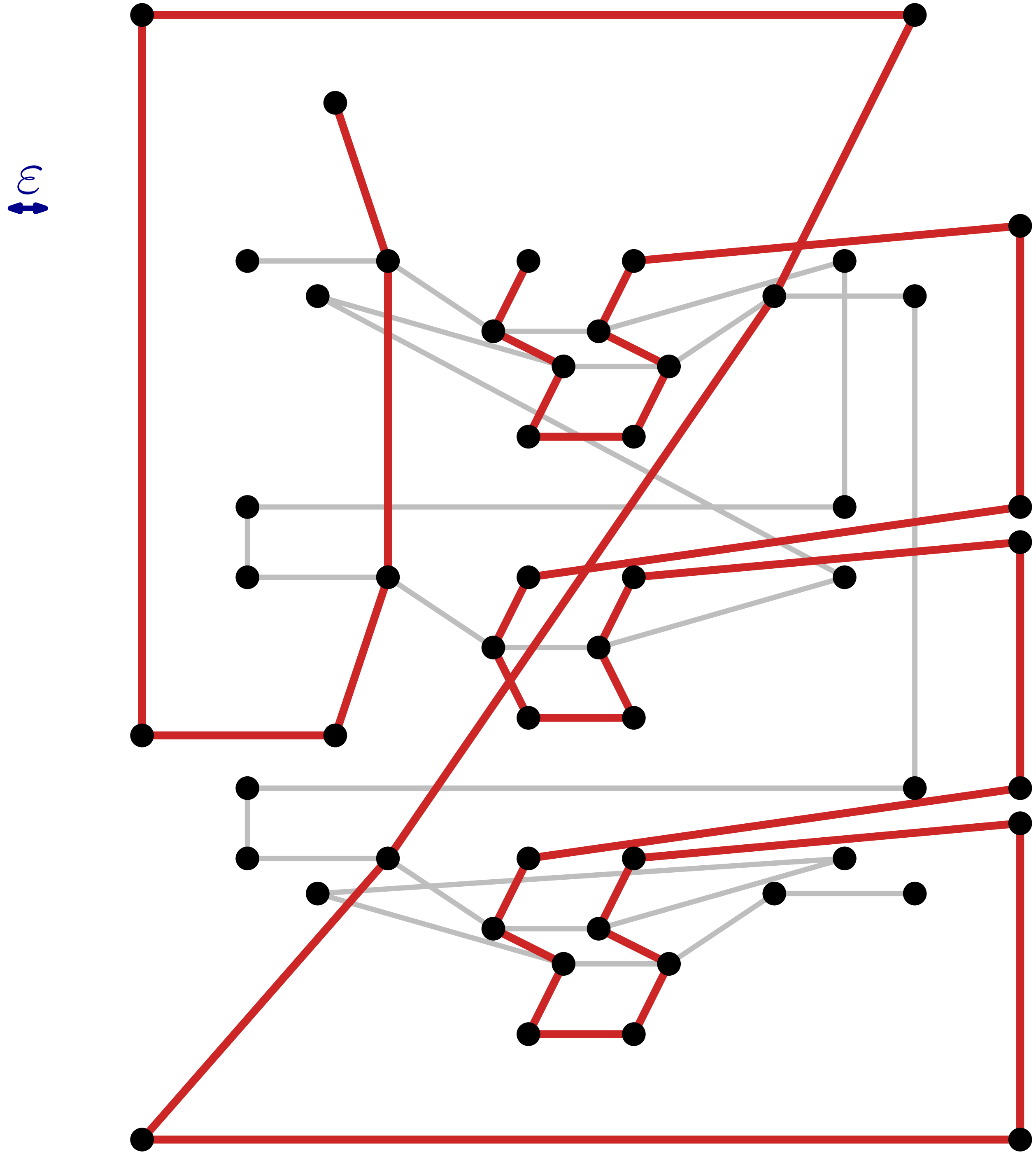

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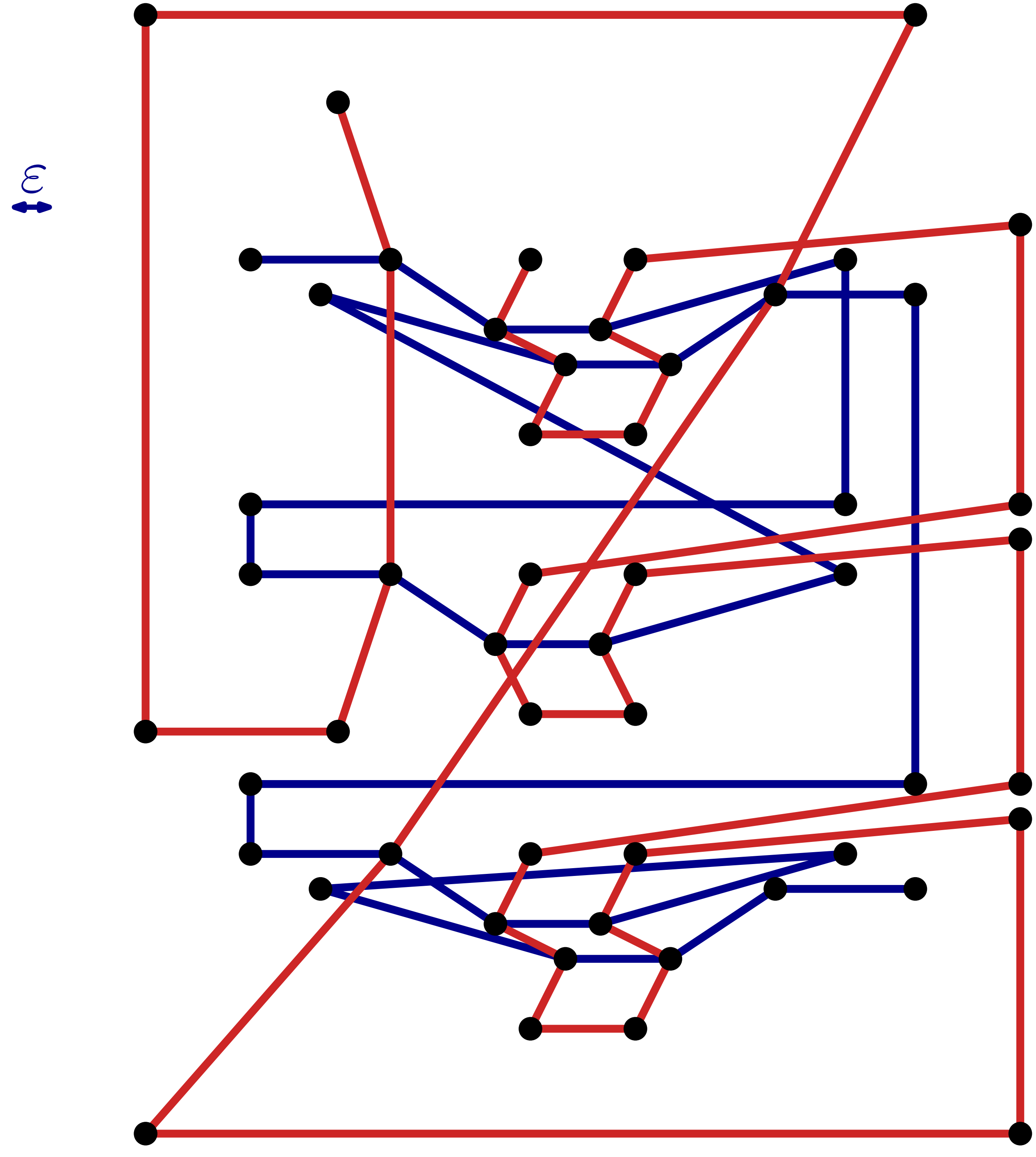

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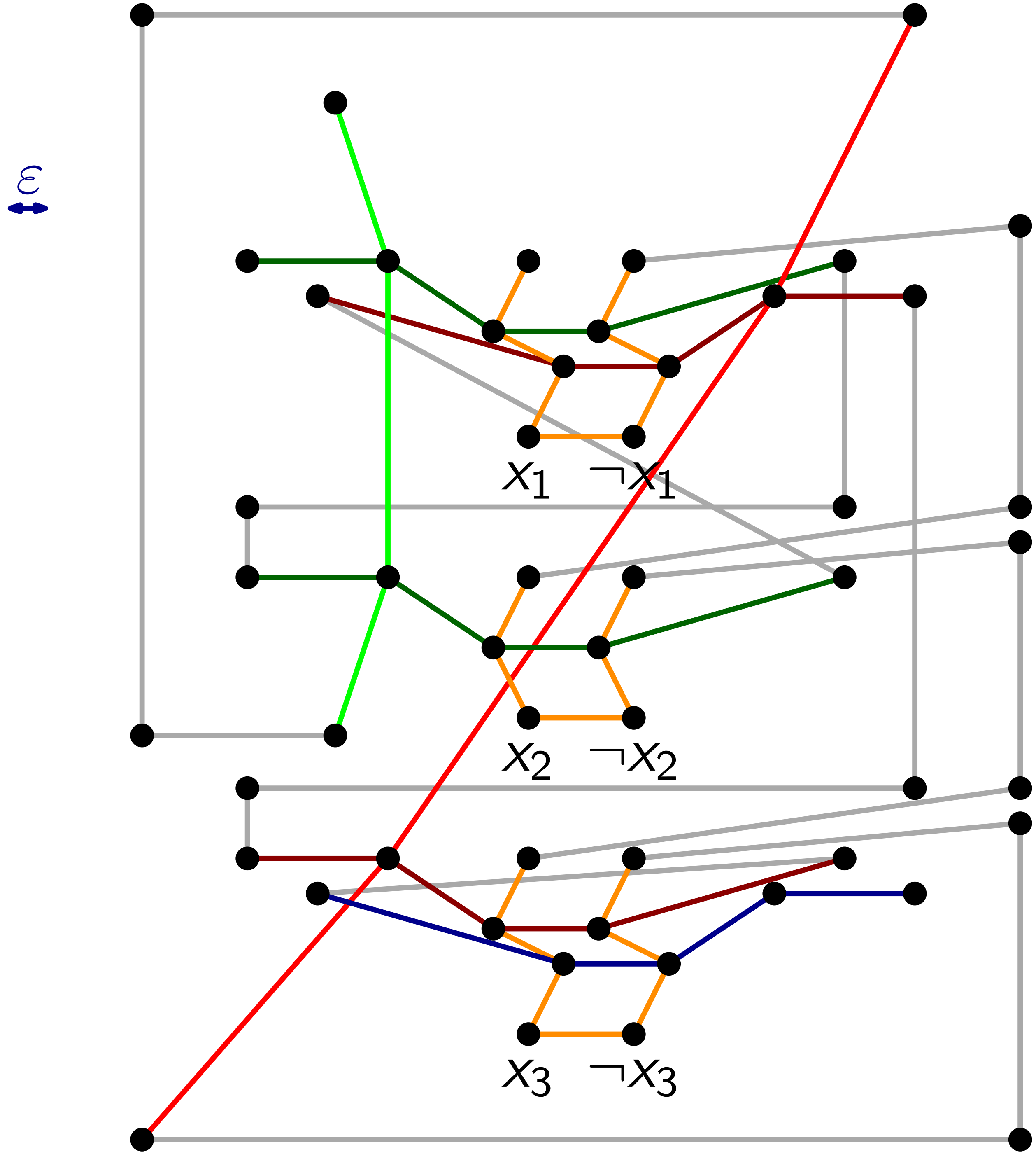

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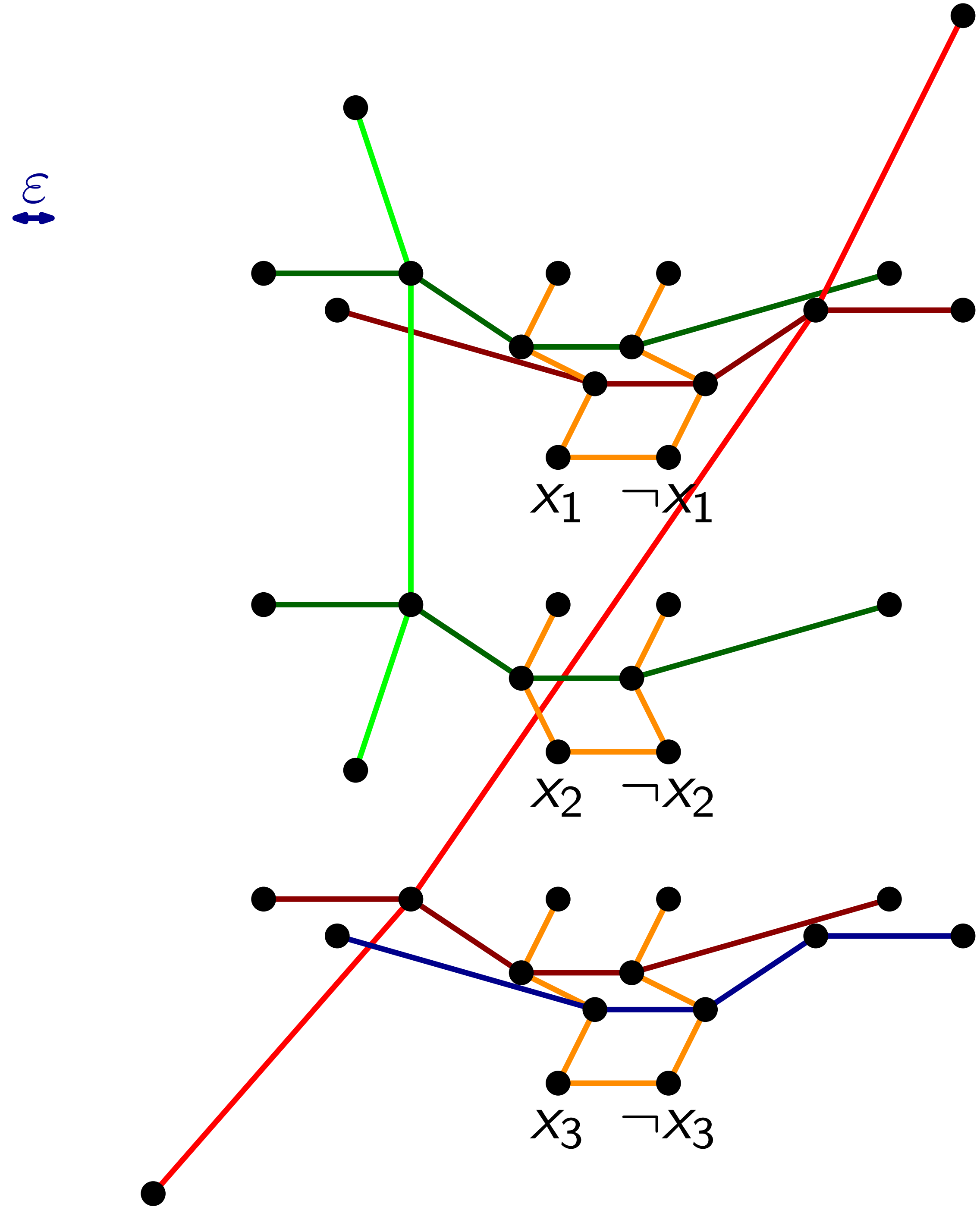

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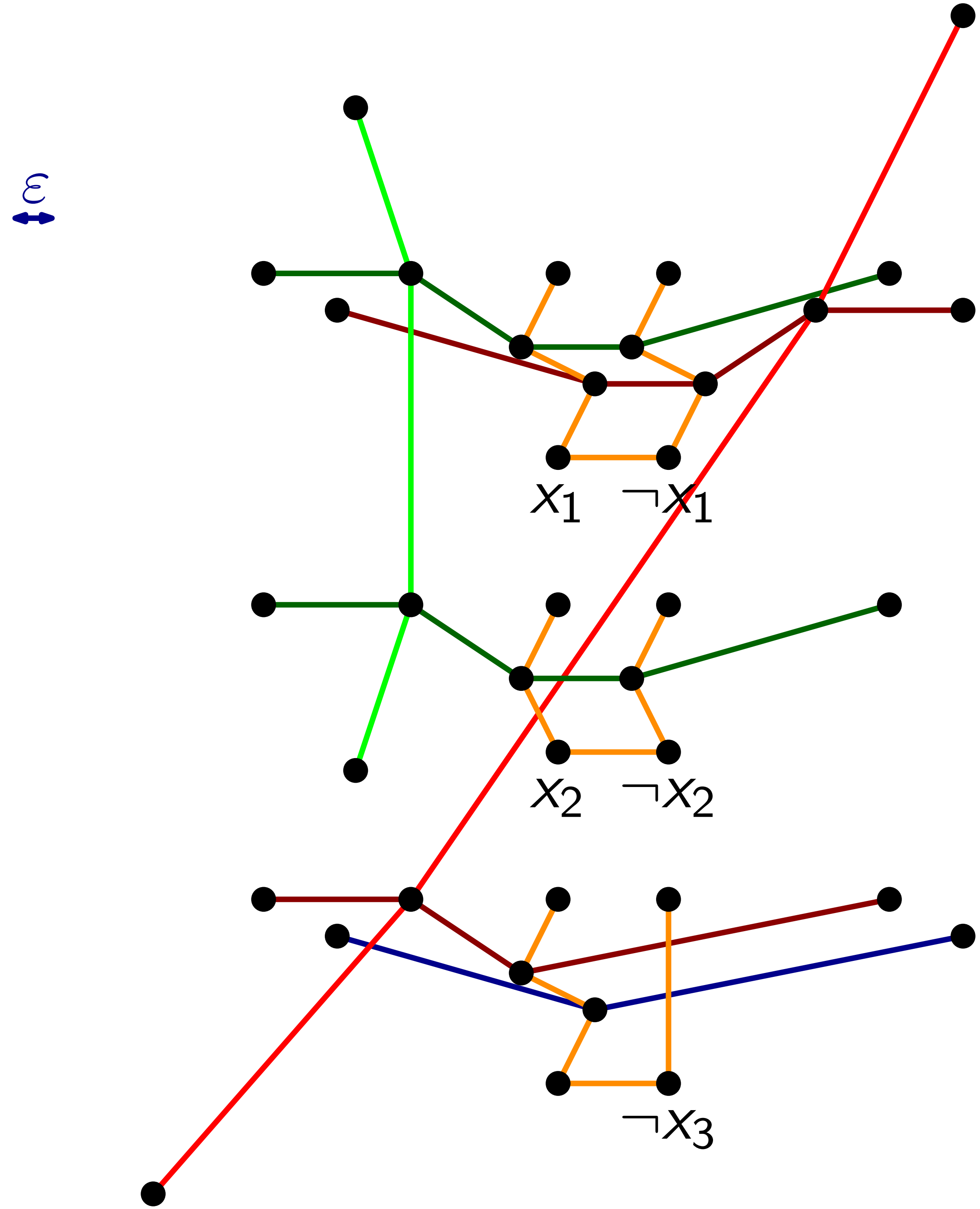

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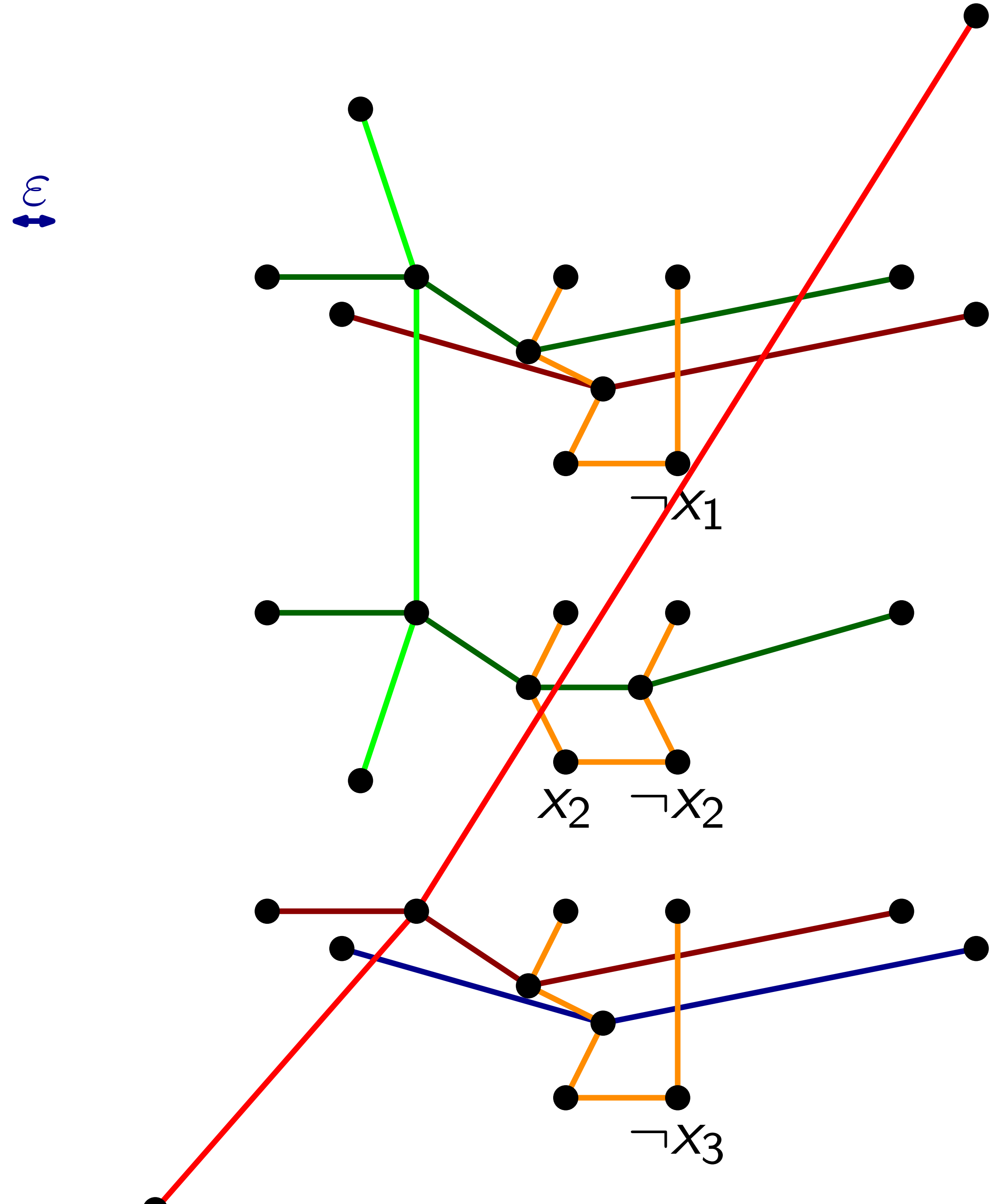

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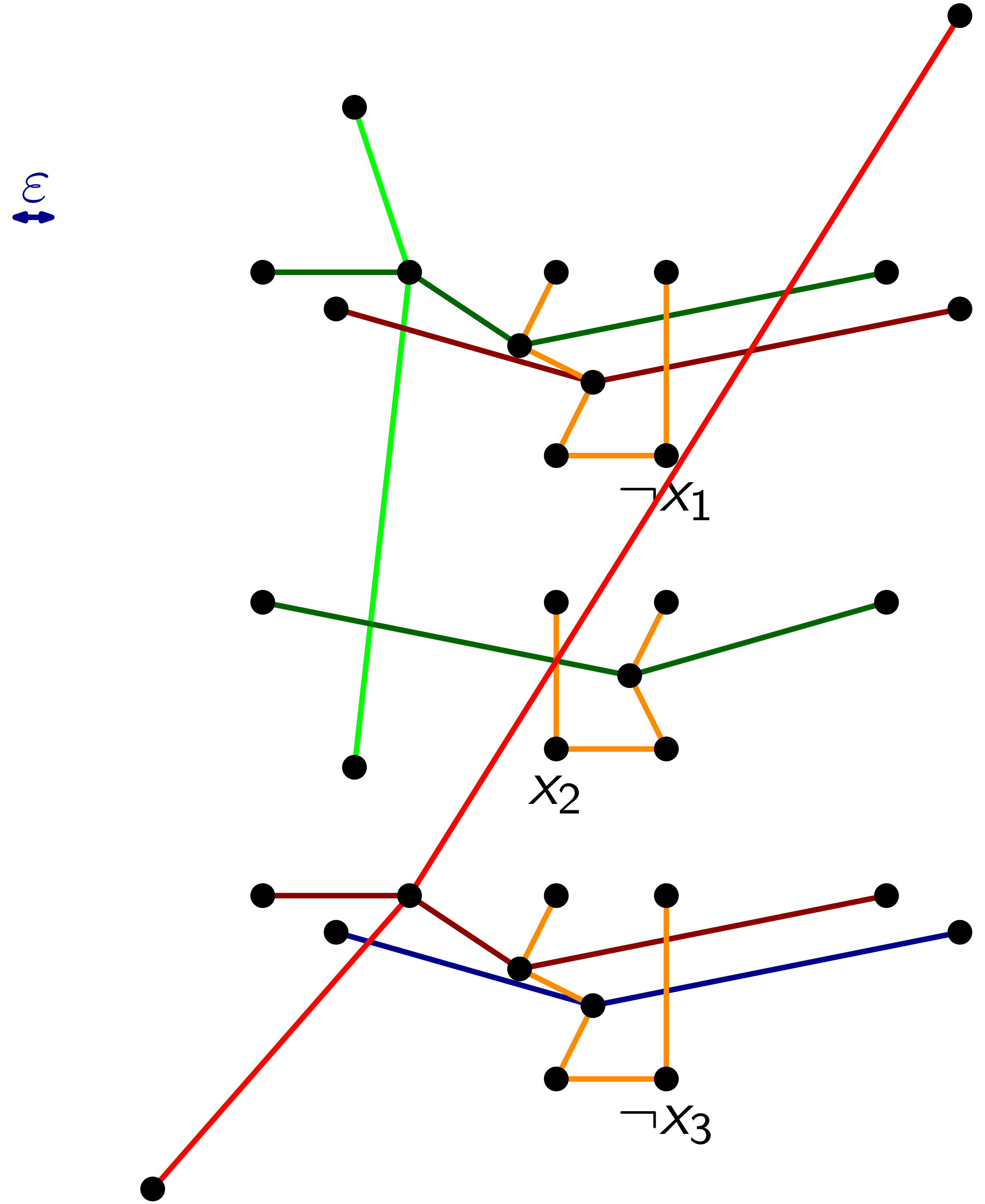

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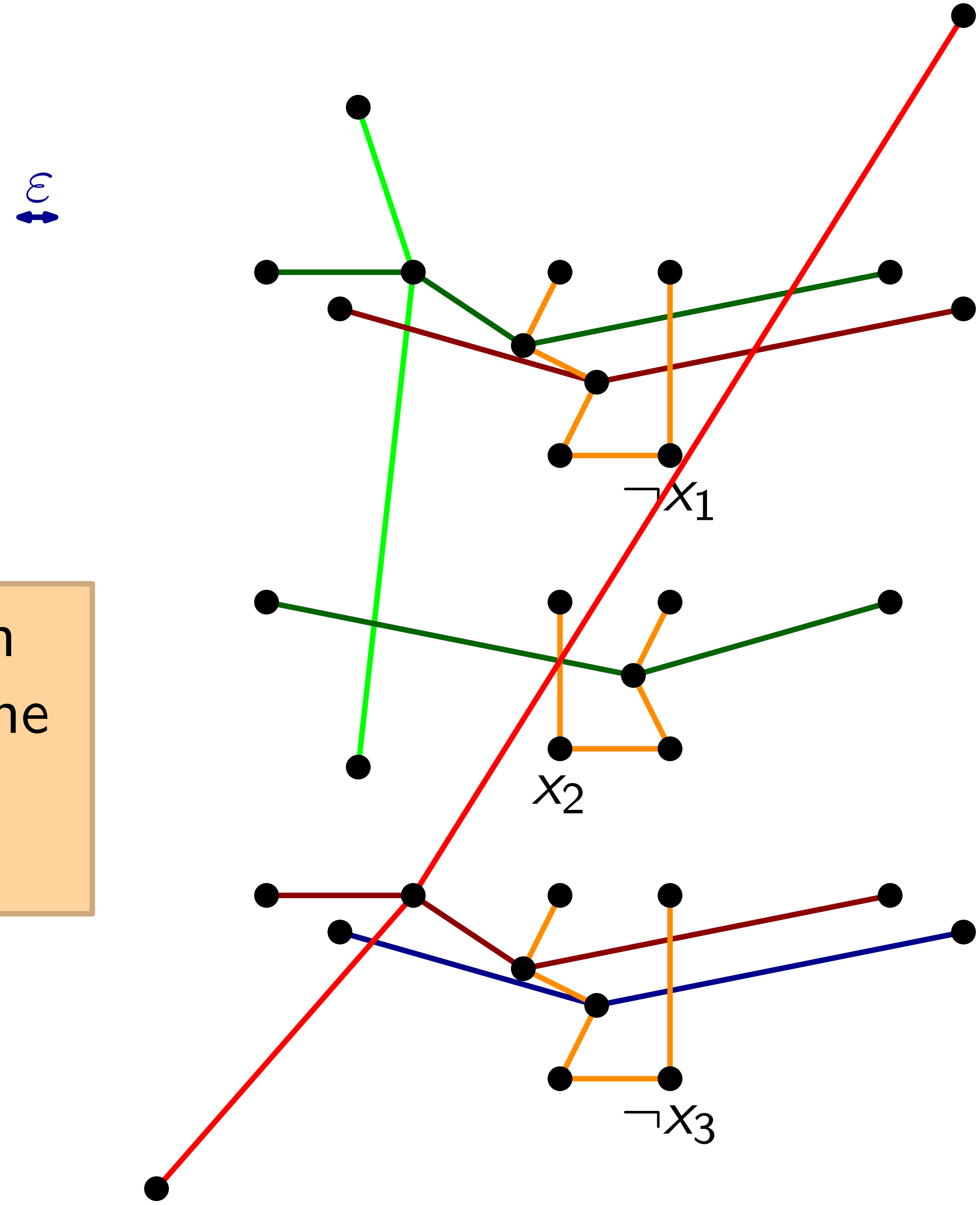

We can even obtain APX-hardness by the reduction from MAX-2-SAT

Fixed-Parameter Tractability

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- Running time in $O(2^k \cdot \ell n^2)$

$k := |V_{\text{shared}}|, \quad \ell := \#$ polylines, $\quad n := \#$ vertices

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Goal 1: Minimize the number of vertices
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- Since there is no PTAS, is there a constant factor approximation algorithm?