Complexity of Scheduling Few Types of Jobs on Related and Unrelated Machines

Martin Koutecký and **Johannes Zink**

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FACULTY
OF MATHEMATICS
AND PHYSICS
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Introduction: Scheduling

SCHEDULING old fundamental task in combinatorial optimization

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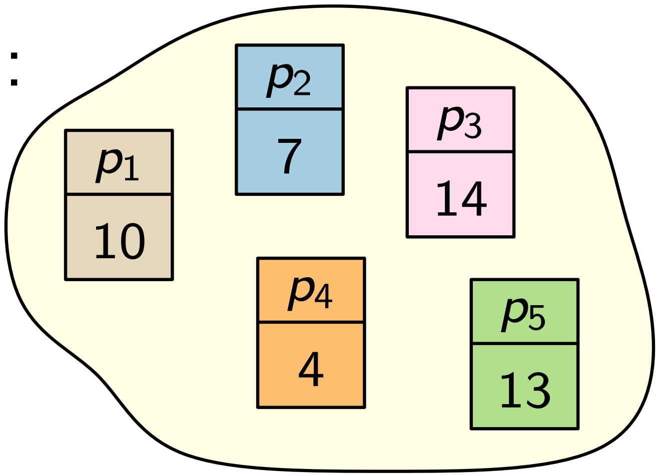

set of n jobs with individual processing times

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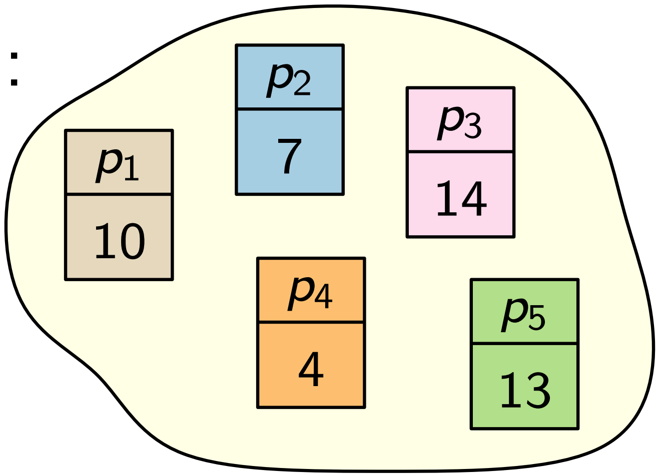

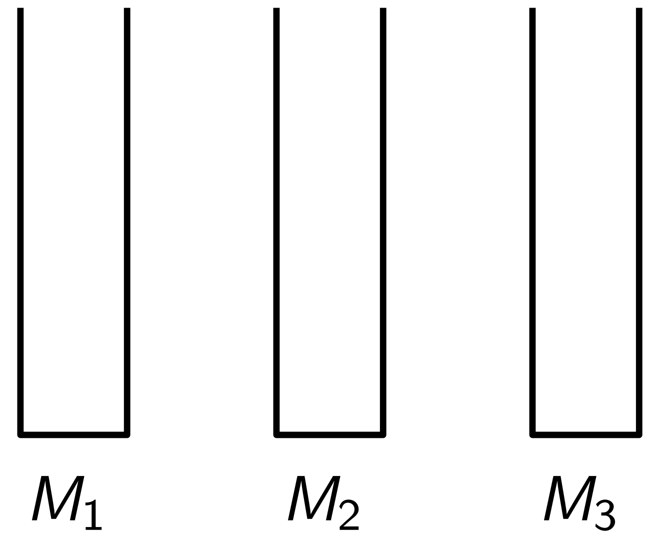

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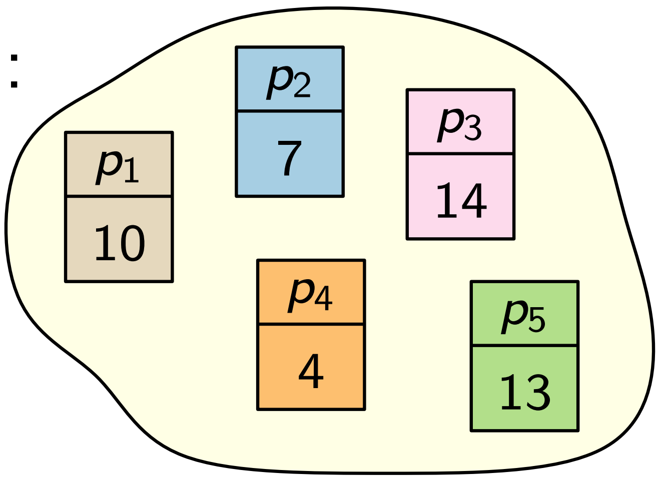

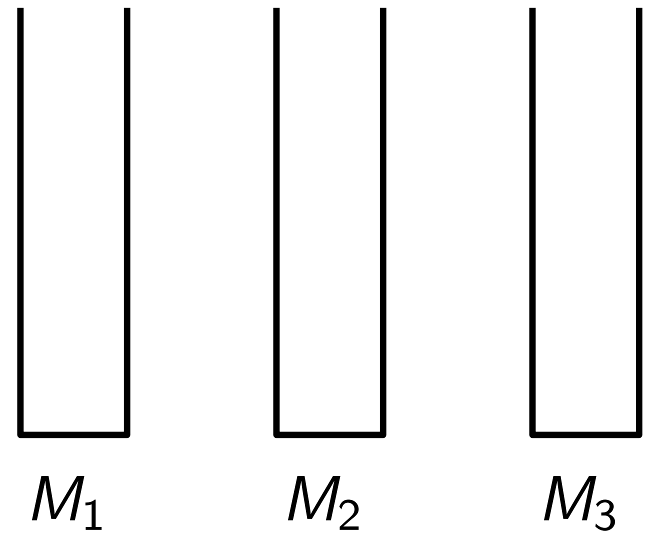

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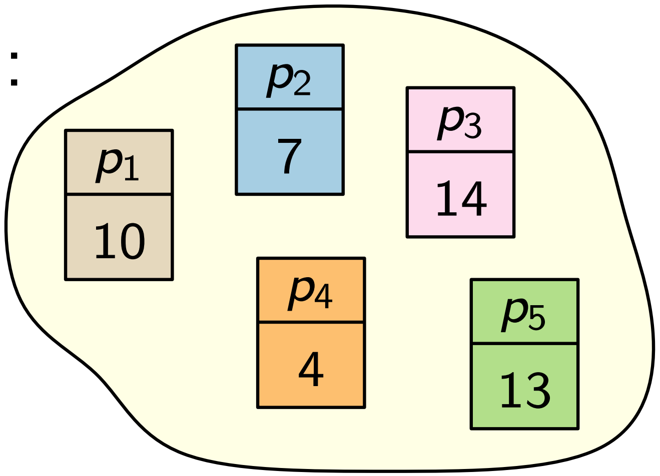

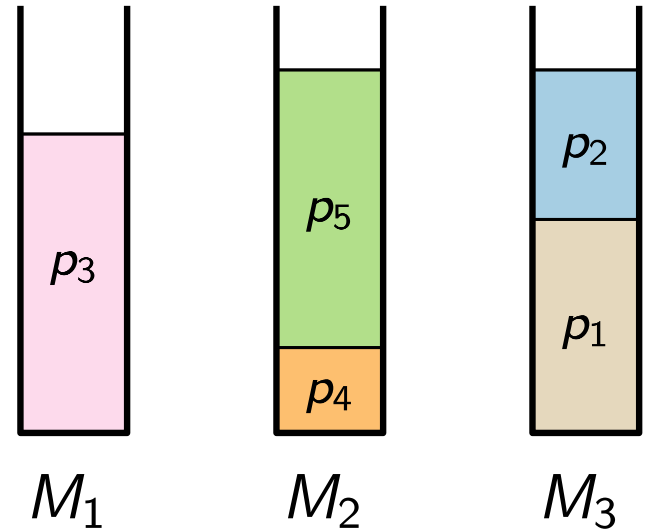

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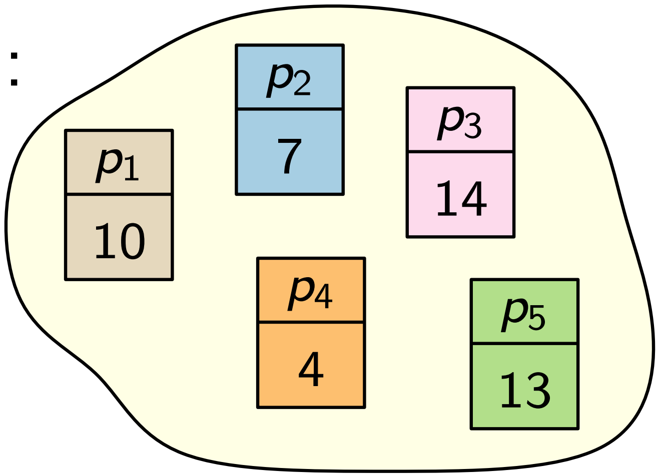

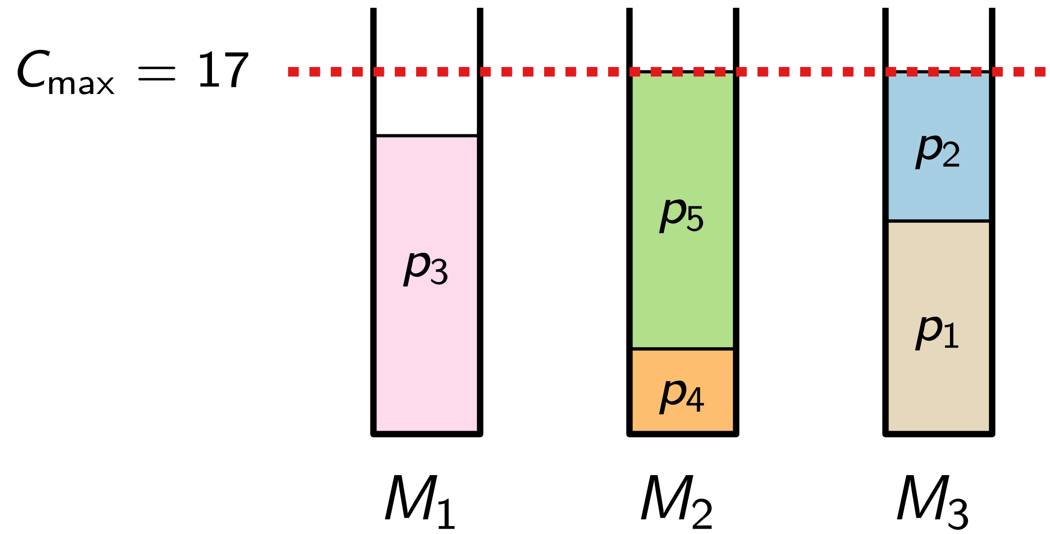

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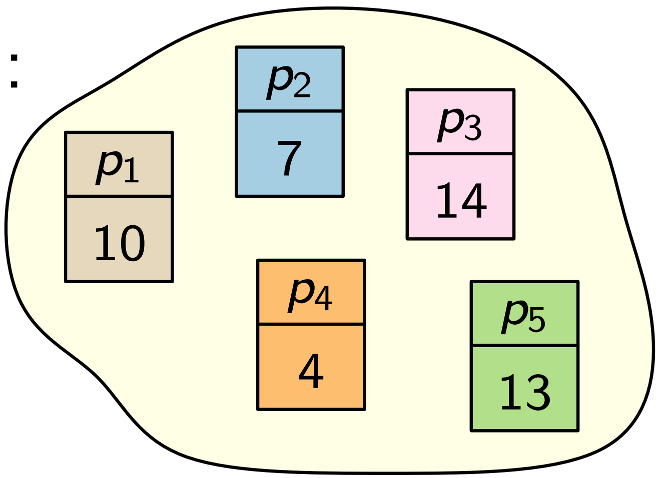

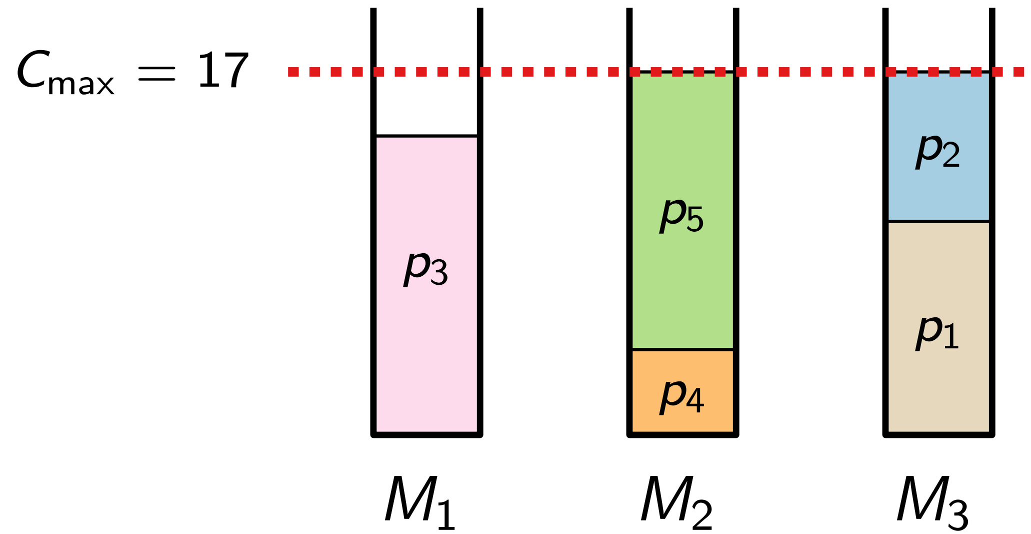

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$P||C_{\max}$

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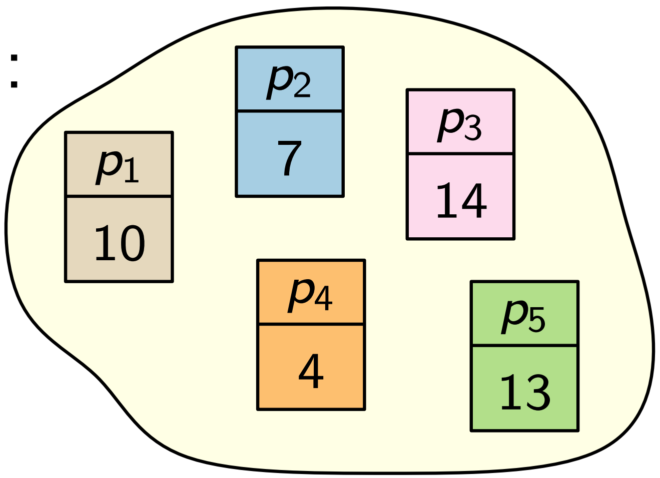

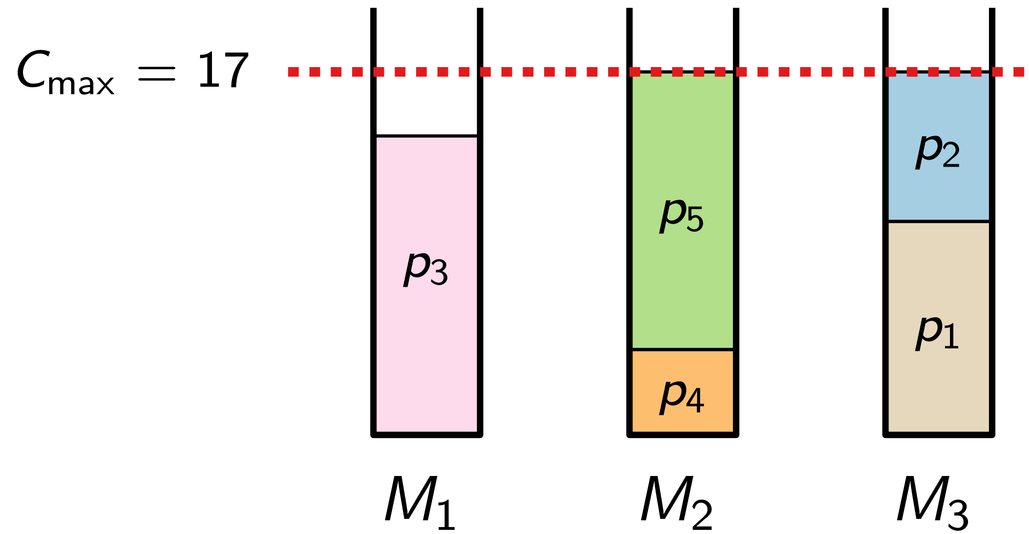

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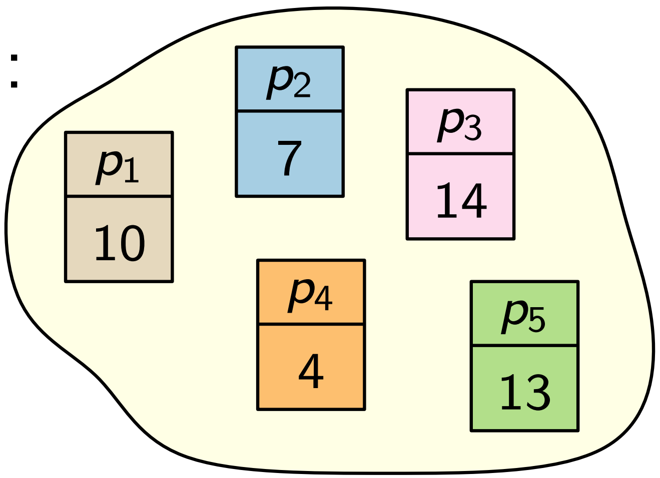

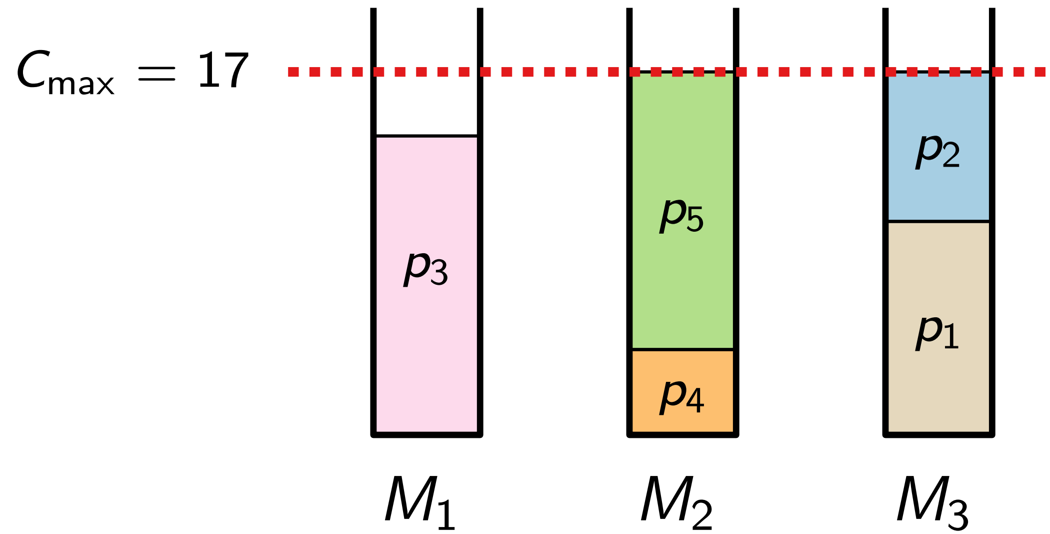

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$P||C_{\max}$ is NP-complete by reduction from BIN PACKING.

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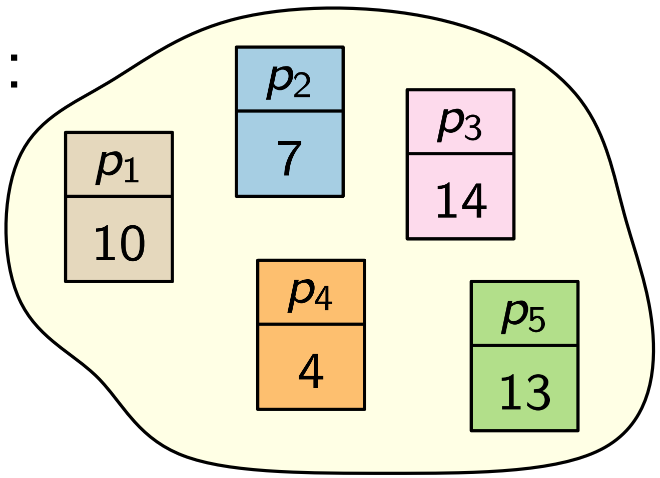

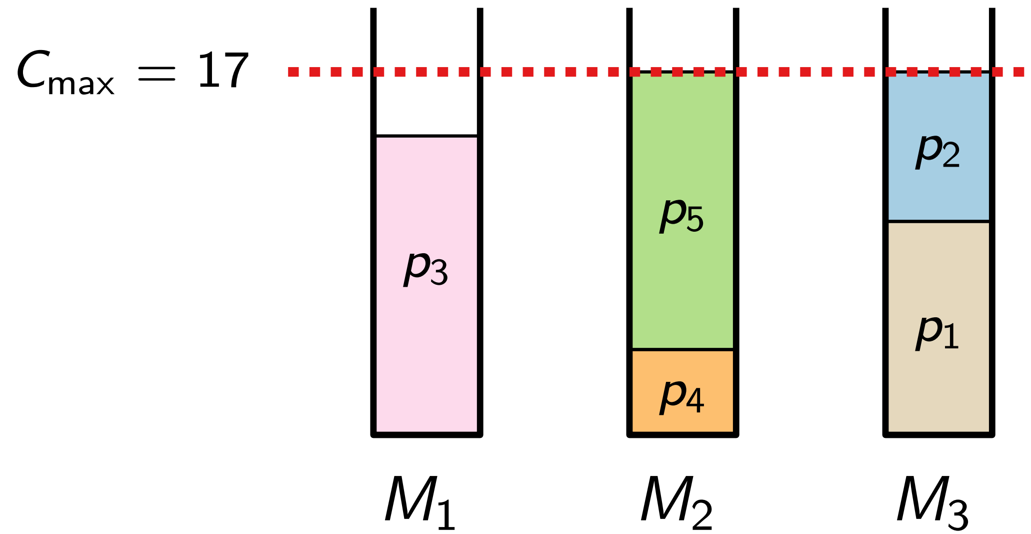

natural numbers

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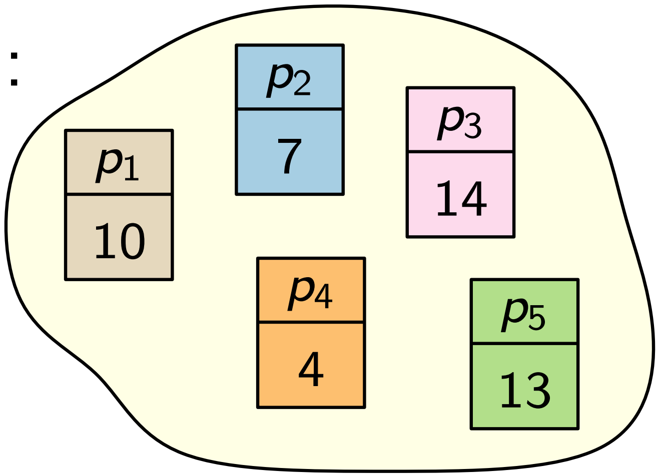

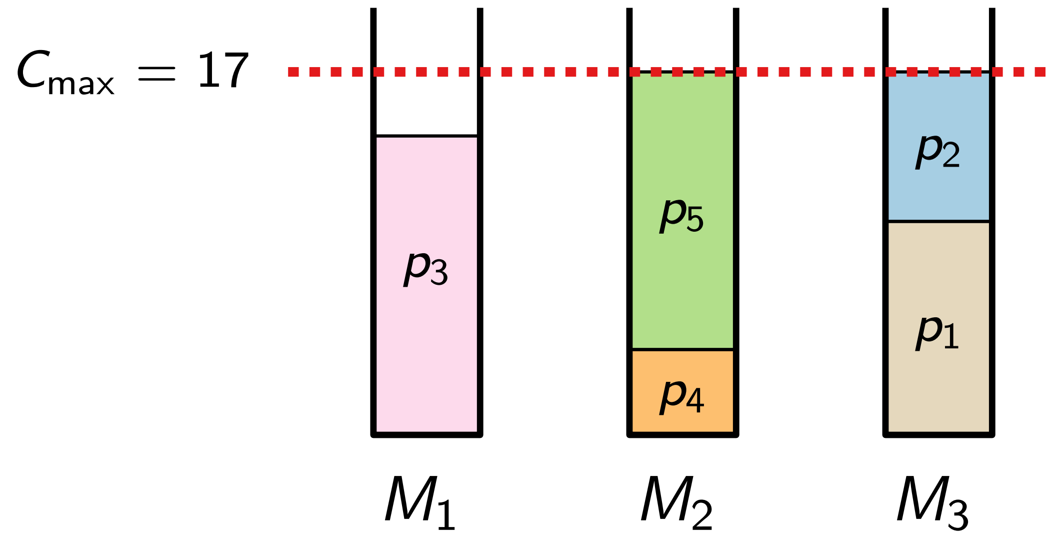

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bins of size $C_{\max}$
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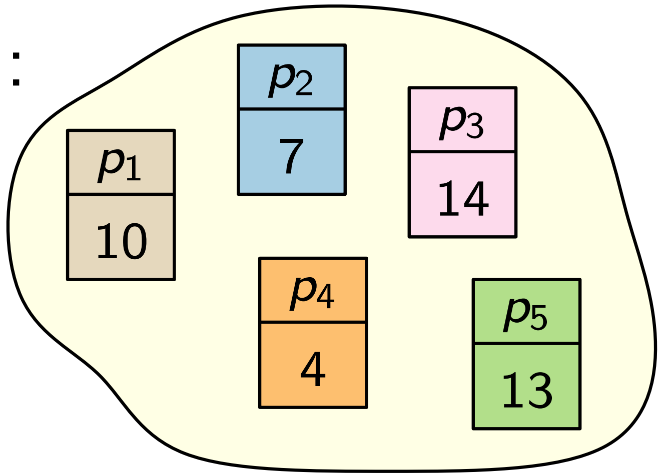

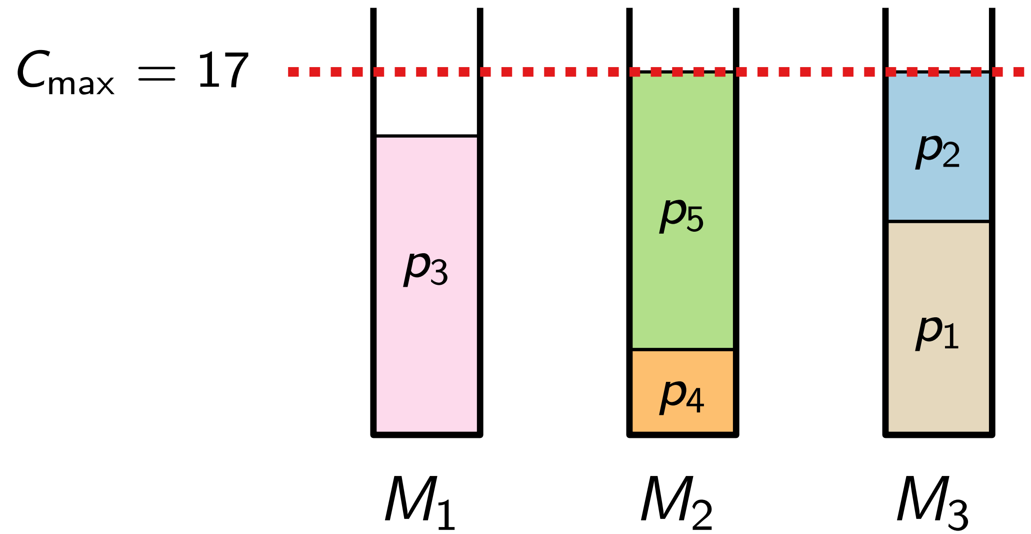

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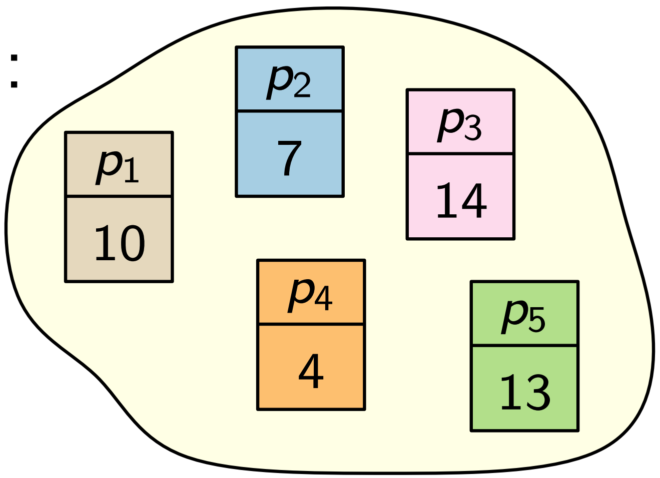

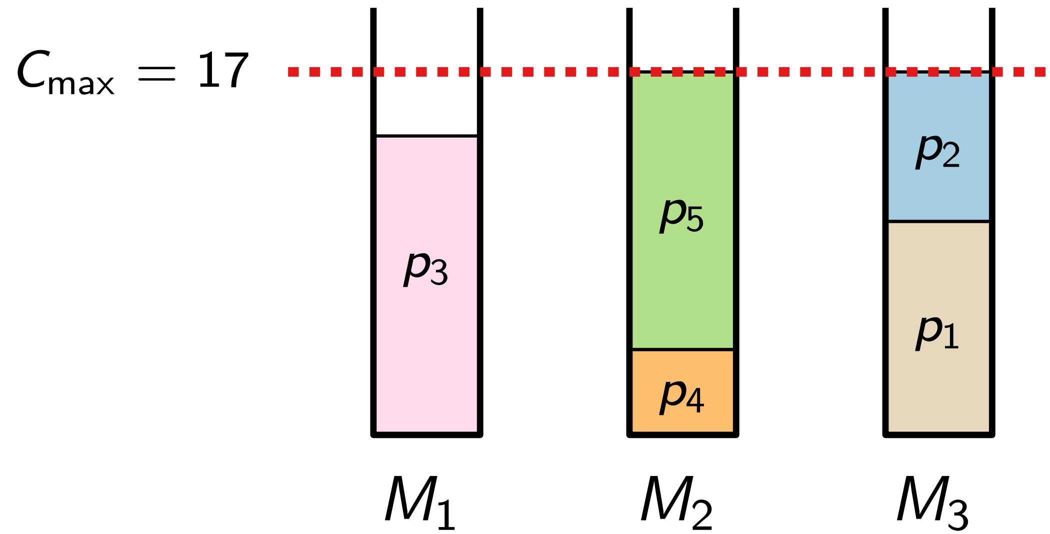

natural numbers — bins of size $C_{\max}$

set of n ~~jobs with individual processing times~~ — m ~~identical machines~~

numbers $\rightarrow$ bins — no bin overfull

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makespan minimization

identical machines

Introduction: Scheduling Few Job Types

So how to simplify our model? What about . . .

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BIN PACKING NP-hard f. const. #bins, even for 2 (PARTITION)

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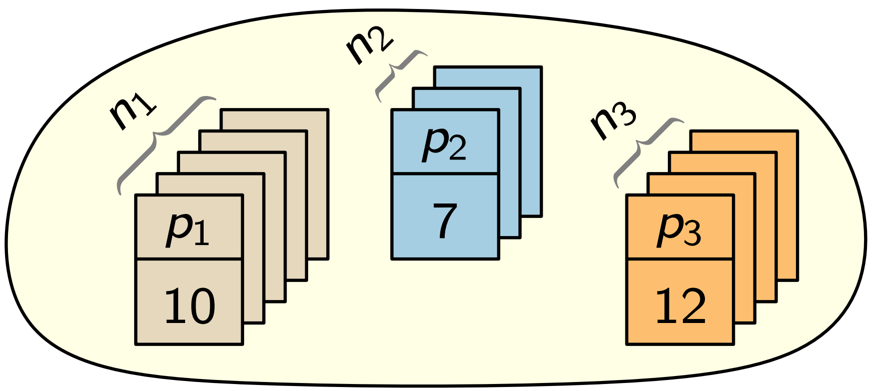
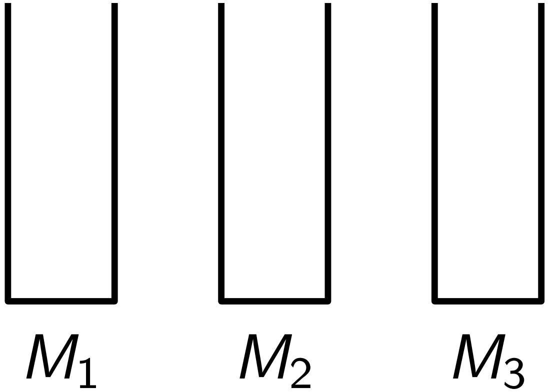


k job types with $n_1, \ldots, n_k$ identical jobs each



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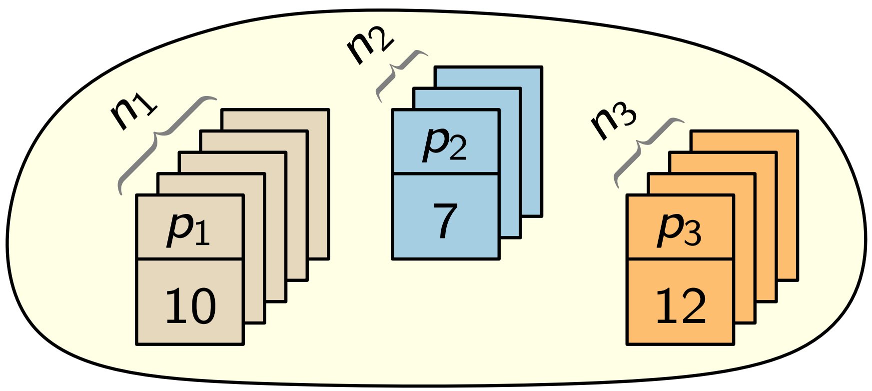

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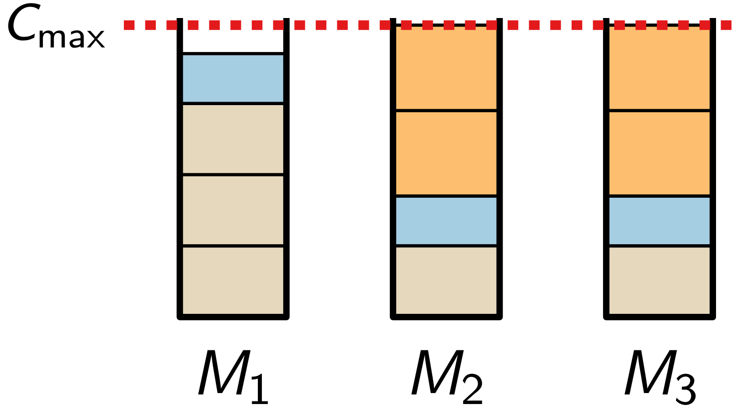

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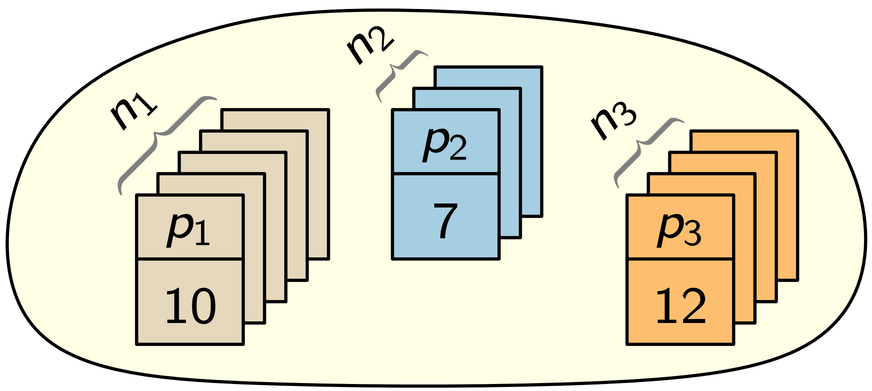

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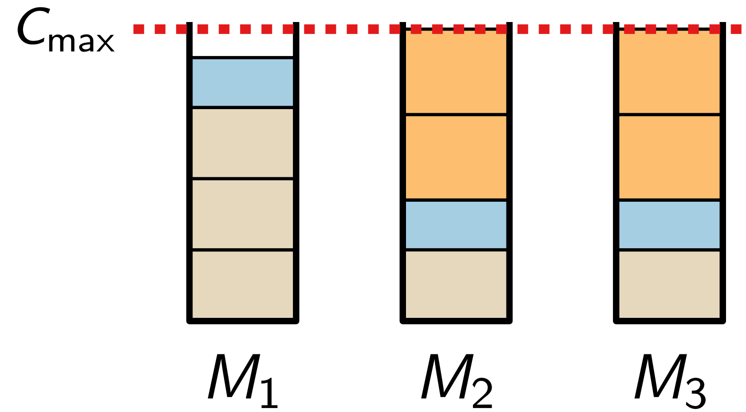

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$P|HM|C_{\max}$

identical machines

high multiplicity of jobs

makespan minimization

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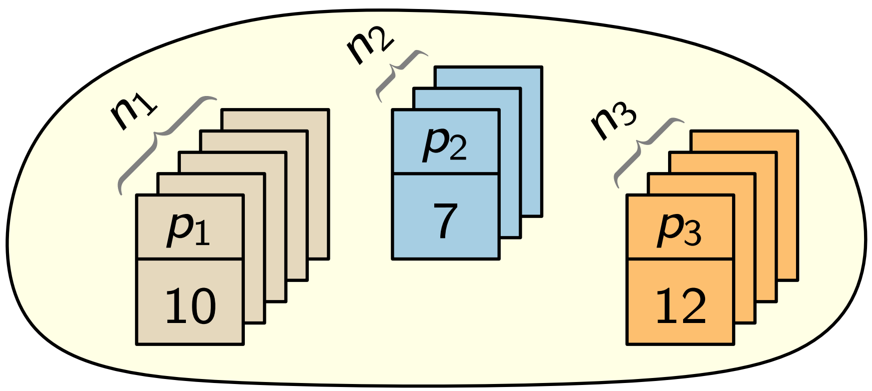

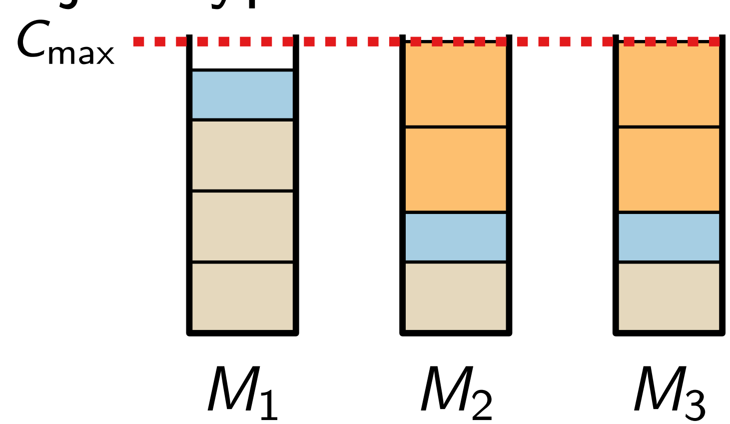

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$P|HM|C_{\max}$ is poly. time solvable (k const.) [Goemans, Rothvoss '14].

makespan minimization

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Scheduling Models

$* | * | C_{\mathsf{max}}$: objective function is makespan minimization

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Scheduling Models

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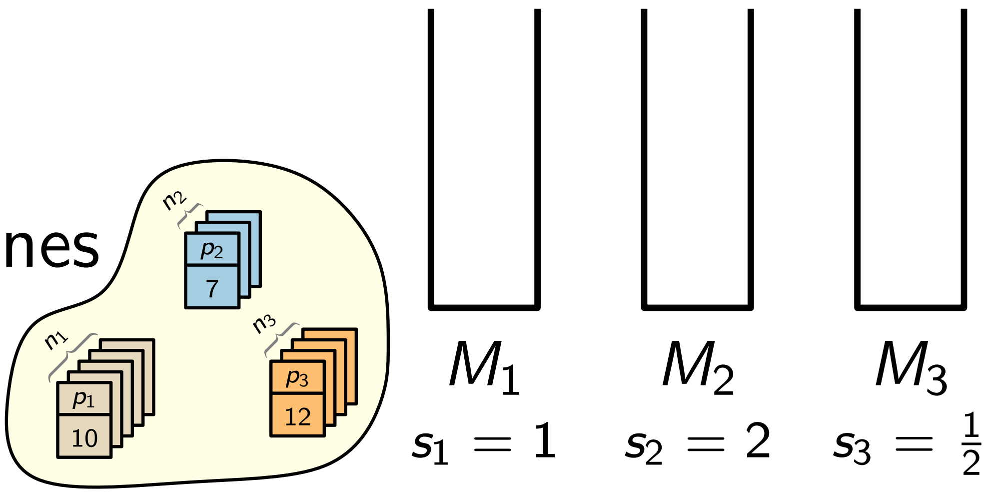
- each machine M_i has a speed s_i
- needs time p_j/s_i



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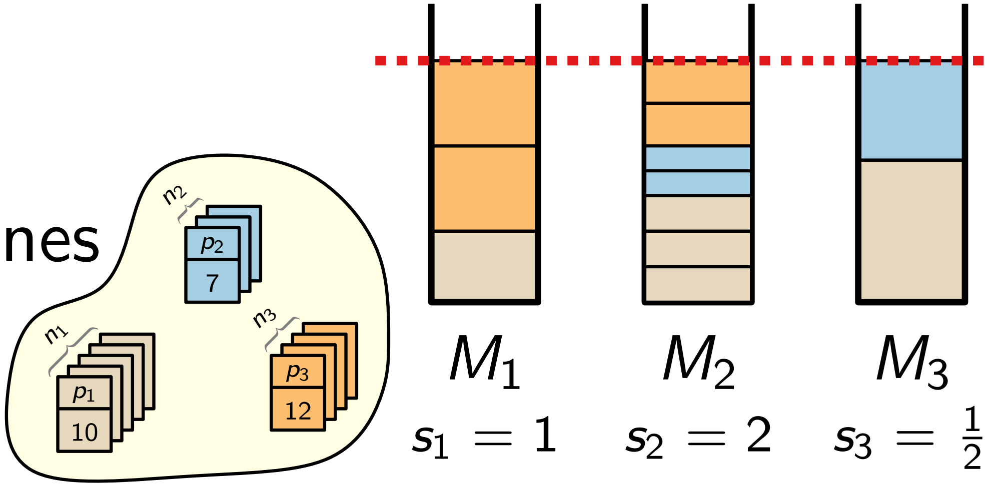

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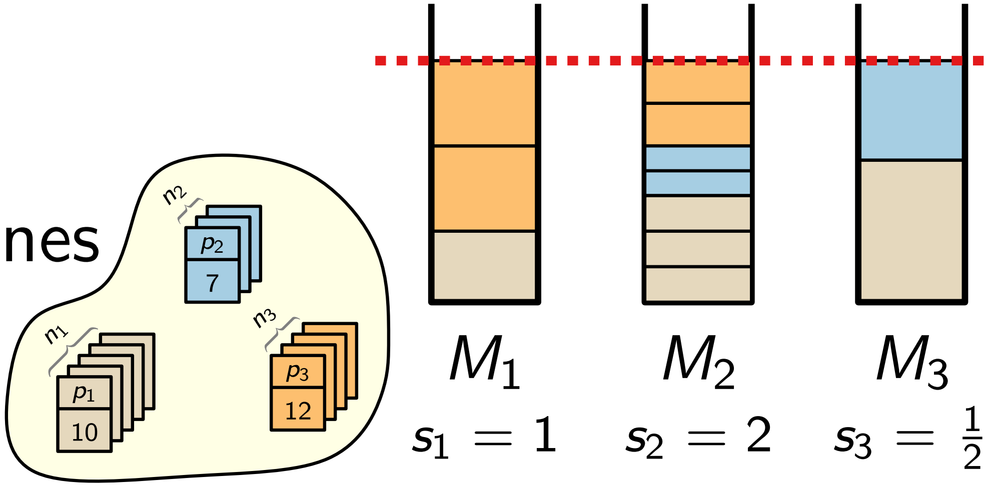

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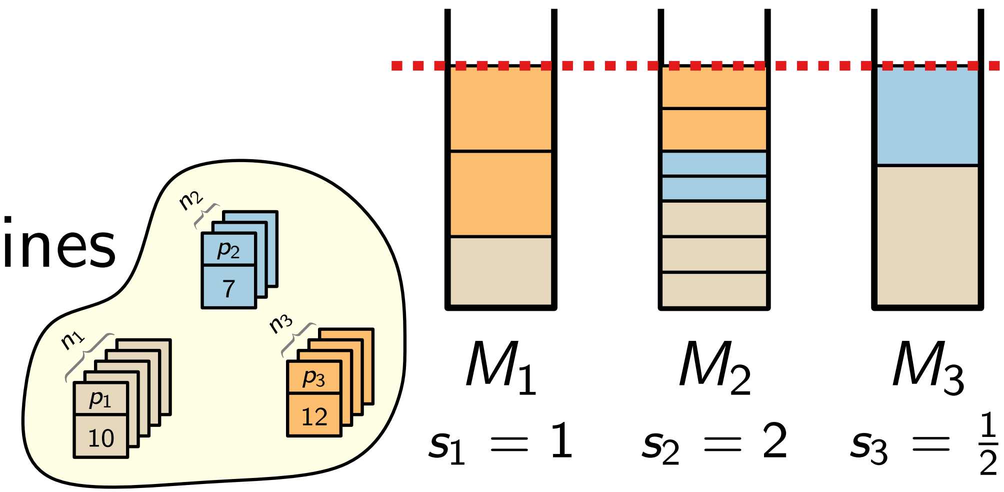

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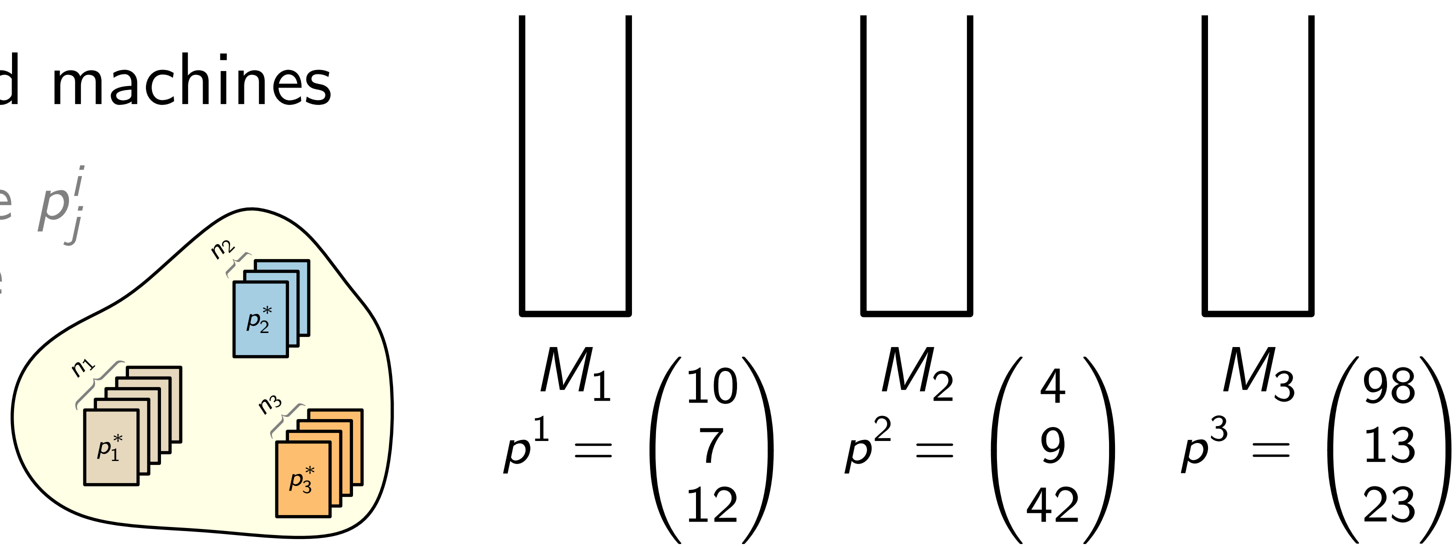

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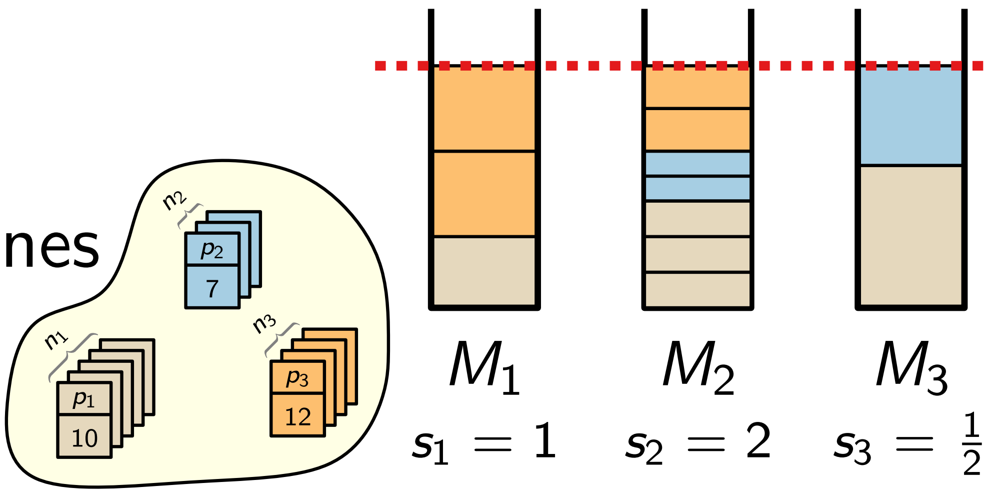

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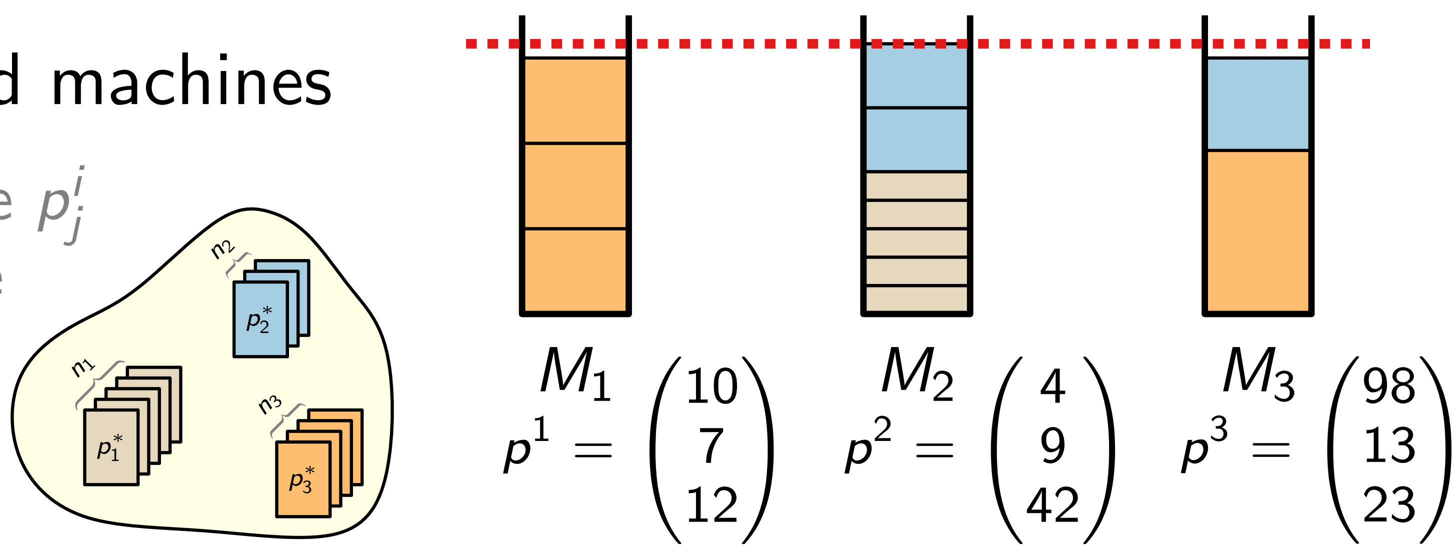

Parametrized Complexity

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- A problem is *fixed-parameter tractable* (FPT) in k if it can be solved by an algorithm of runtime $f(k) \cdot n^{O(1)}$.
  (f: some computable function, n: input size)

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- W[1]-hard problems are unlikely to be FPT.

Our Contribution

P : identical machines

Q : related machines

R : unrelated machines

HM : high multipl. jobs

$C_{\max}$: min. makespan

$P||C_{\max}$: NP-complete

$P|HM|C_{\max}$: poly. time

k : # job types

Our Contribution

So, what if we don't use identical machines?

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$Q|HM|C_{\max}$ is NP-hard (already for $k = 6$).

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Cutting Stock is NP-hard already with 8 item types.

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$\{R, Q\}||C_{\max}$ is in XP parameterized by k, and
$\{R, Q\}|HM|C_{\max}$ is in XP param. by k if job sizes given unary.

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1. hardness reduction

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1. hardness reduction

$Q||C_{\max}$ is W[1]-hard parameterized by k even for unary input.

$P||C_{\max}$ is FPT parameterized by k.

2. obtaining FPT

$\{R, Q\}||C_{\max}$ is in XP parameterized by k, and
$\{R, Q\}|HM|C_{\max}$ is in XP param. by k if job sizes given unary.

Hardness Reduction: Overview

BIN PACKING (tight instances)
[known to be NP-complete]

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$\Big\downarrow$ reduces to

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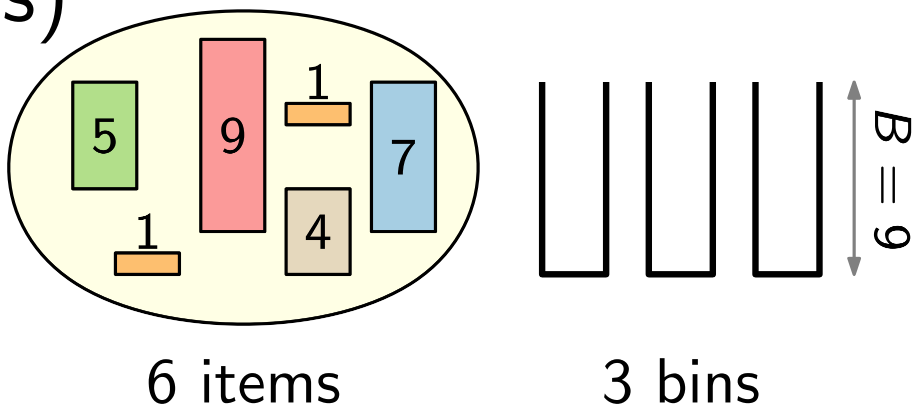

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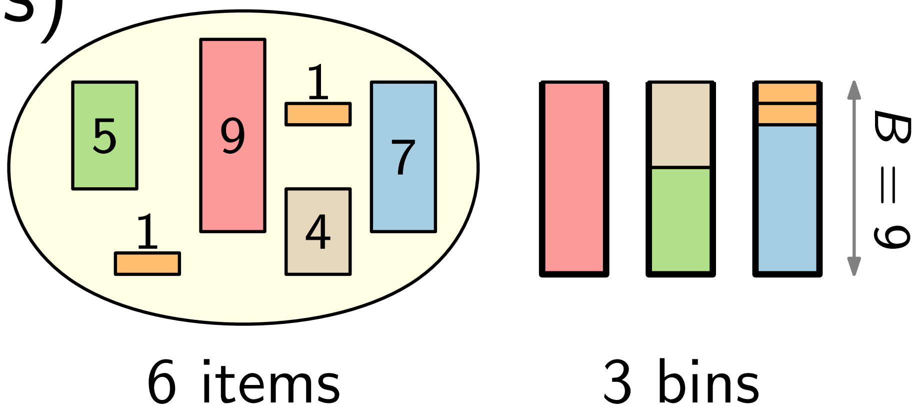

↓ reduces to

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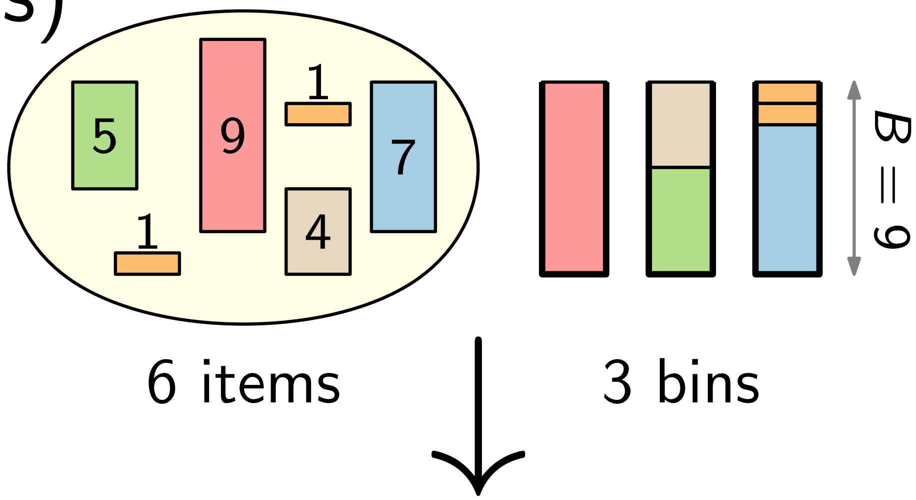

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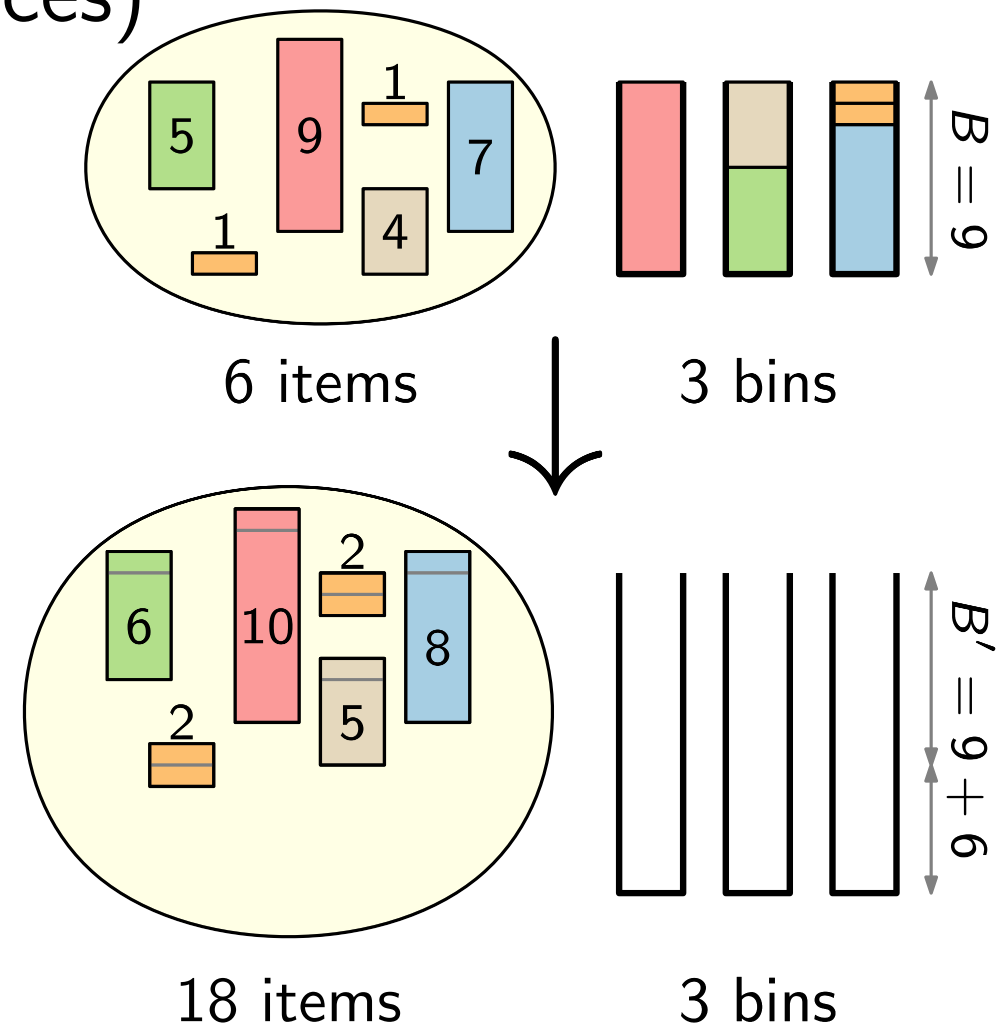

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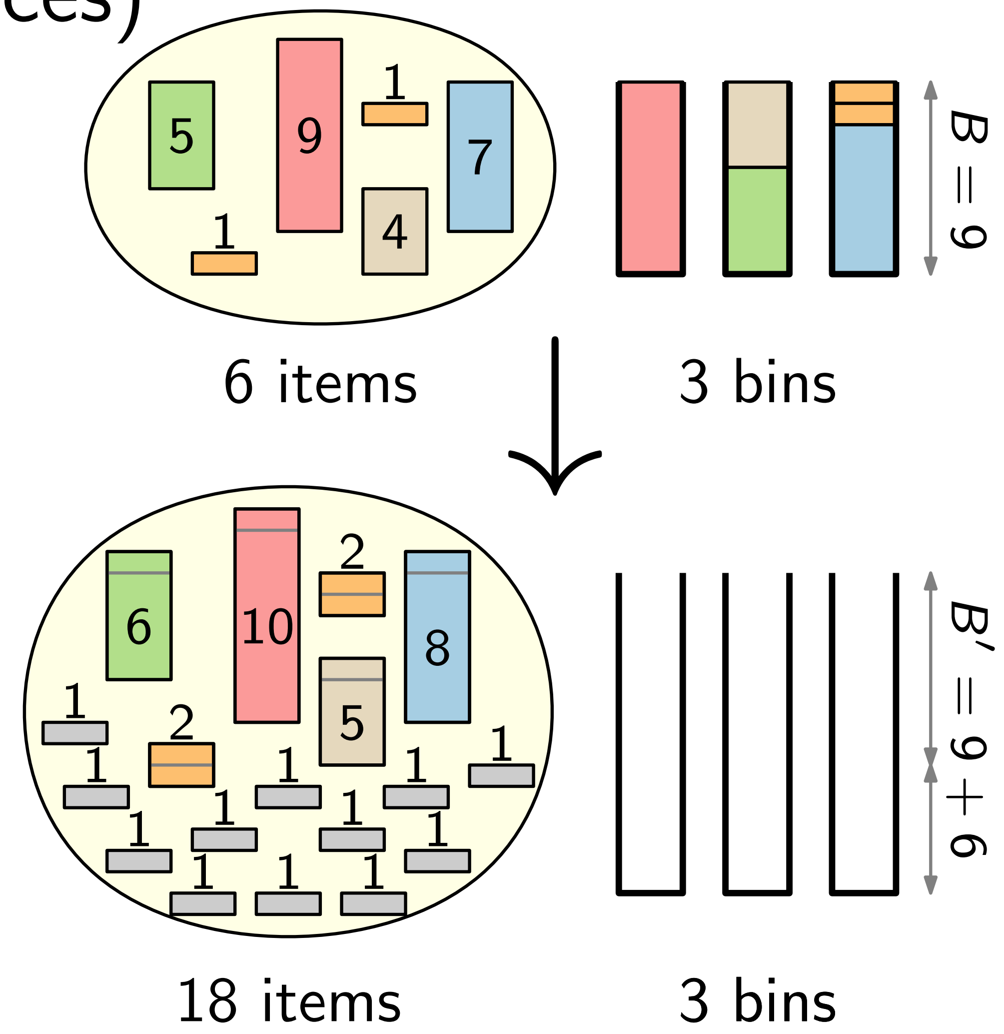

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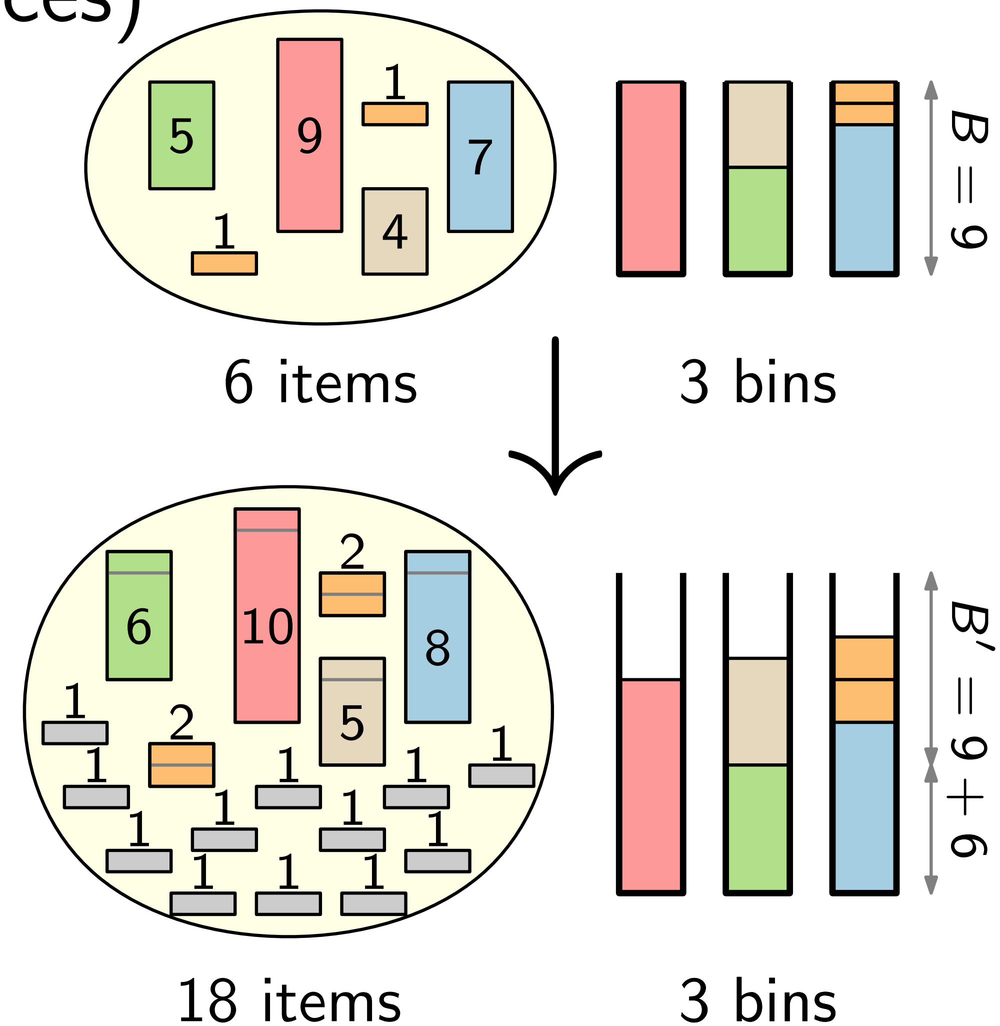

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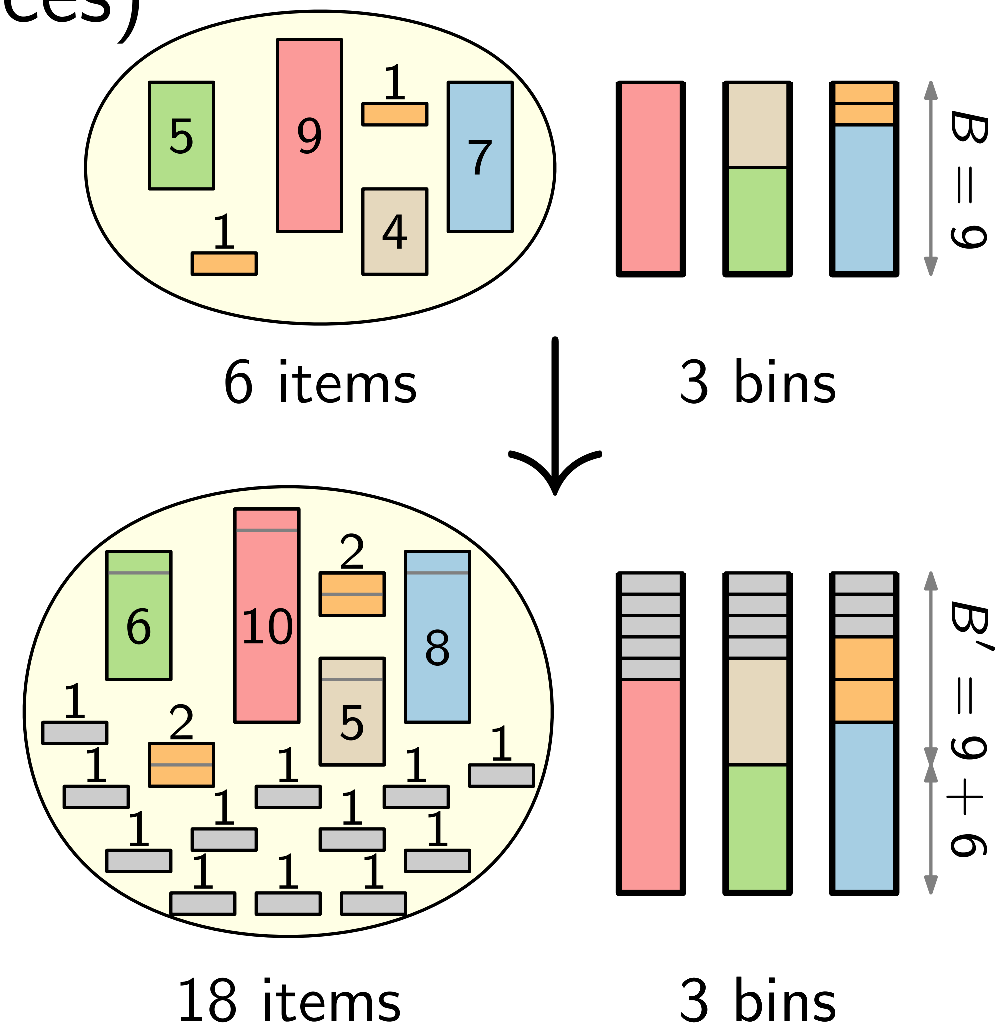

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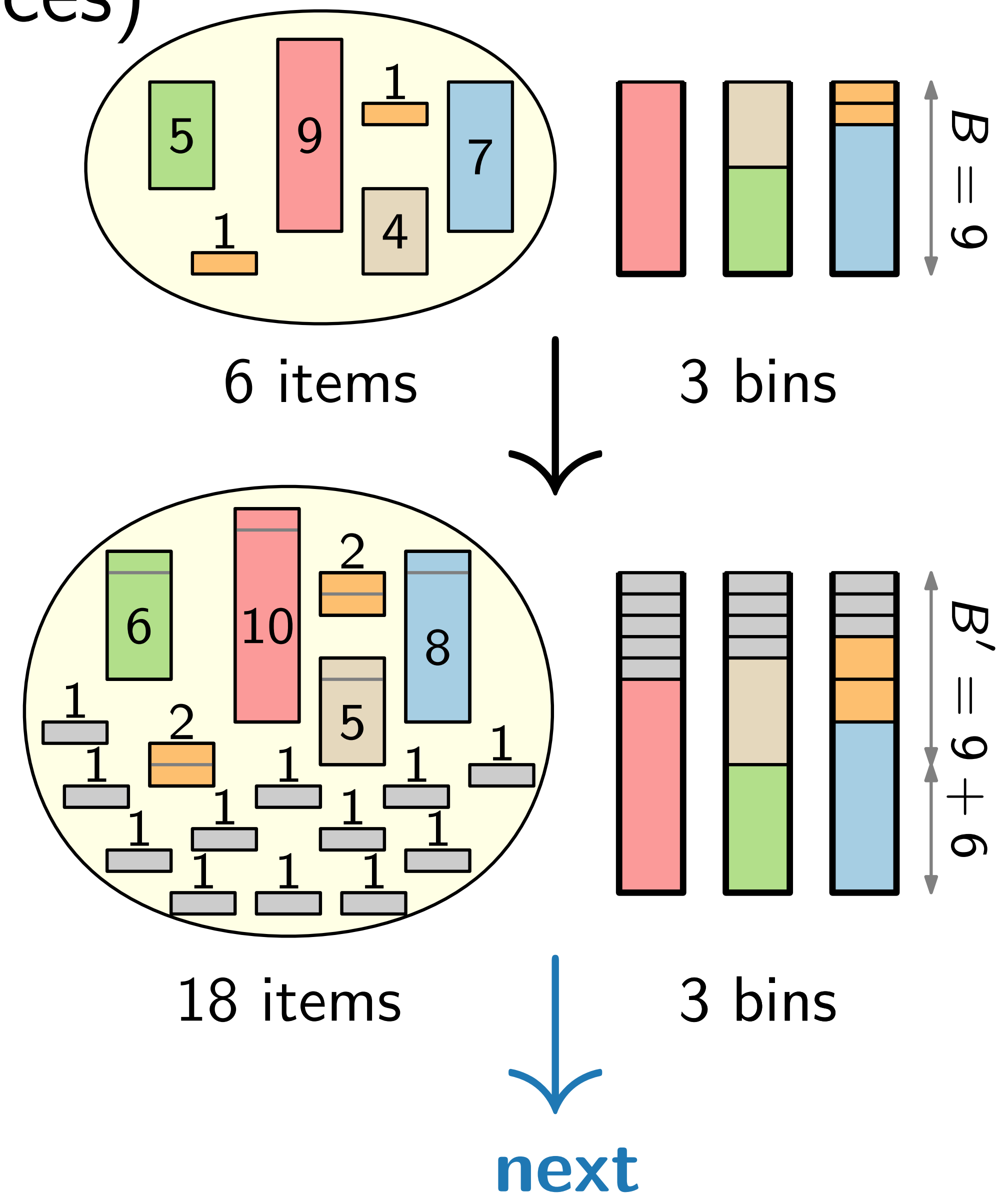

Balanced Bin Packing $\to Q|HM|C_{\max}$

Balanced Bin Packing instance:

- multiset of numbers $\{a_1, \ldots, a_m\}$
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- $A := \sum_{i=1}^{m} a_i = kB$

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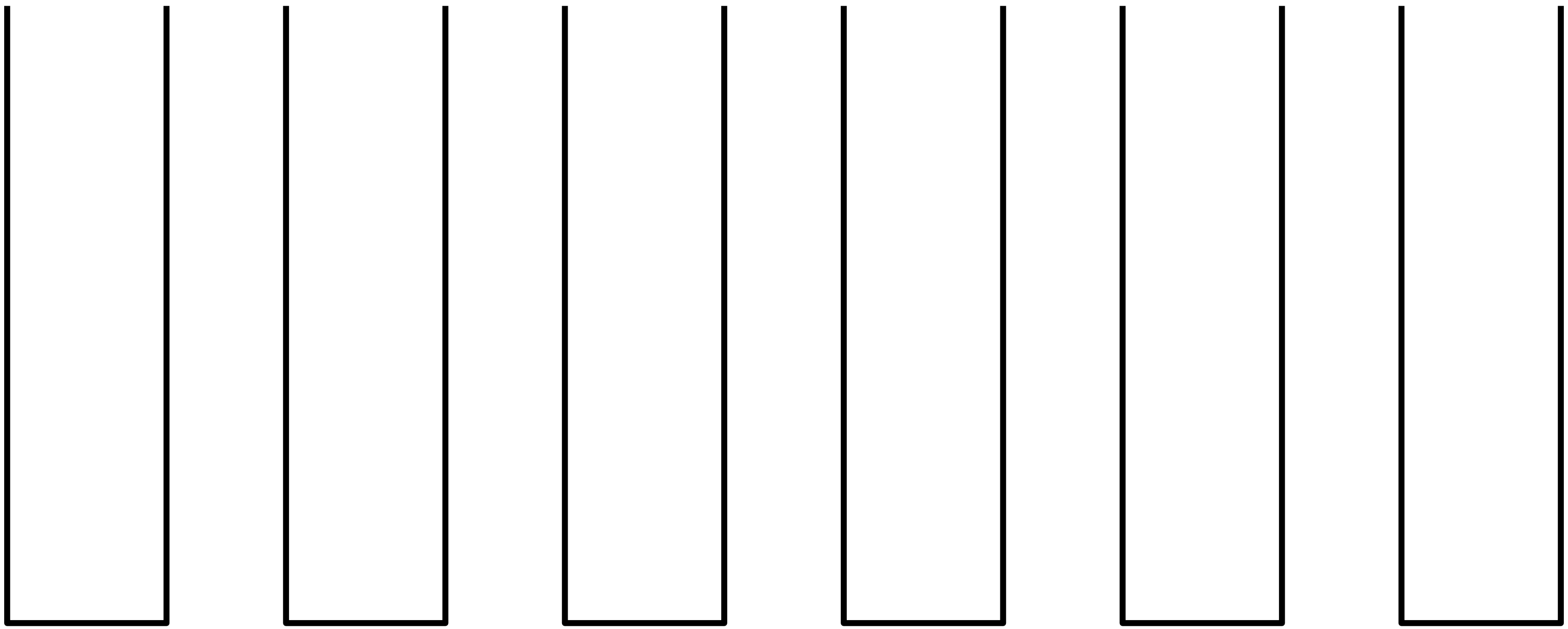

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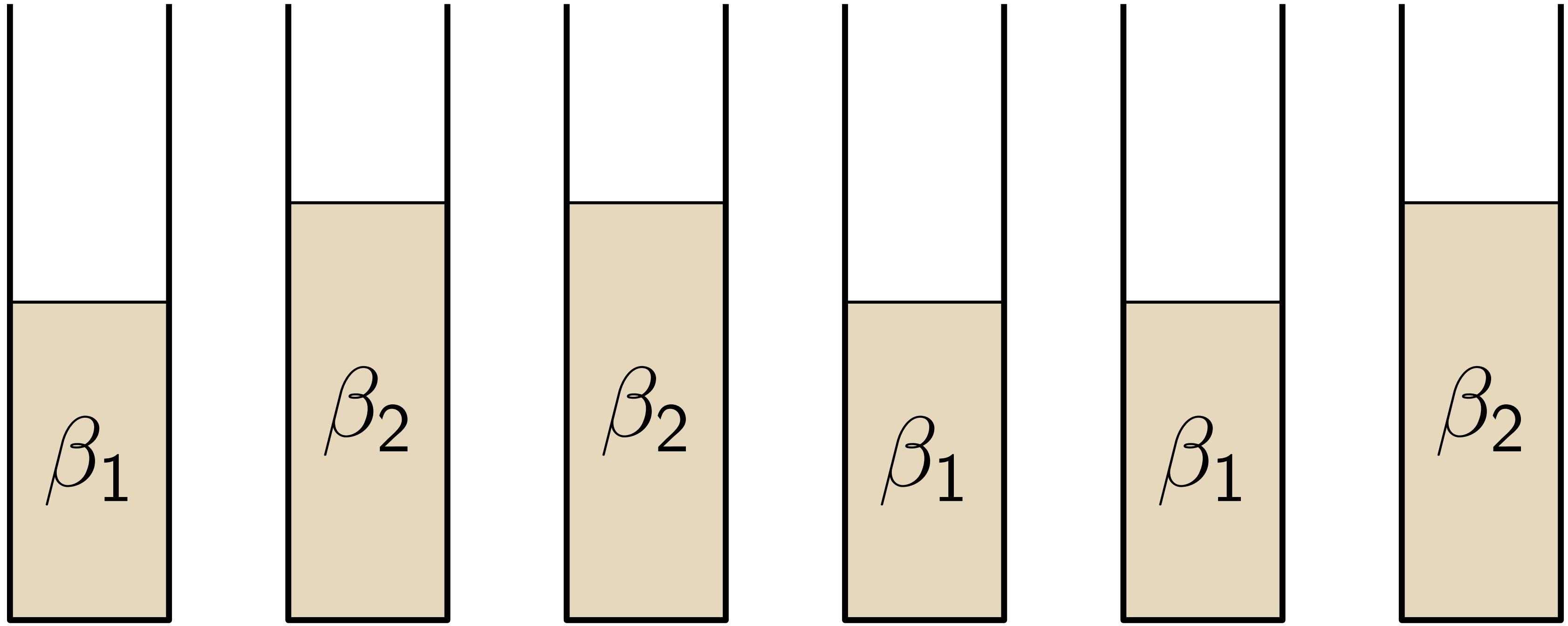

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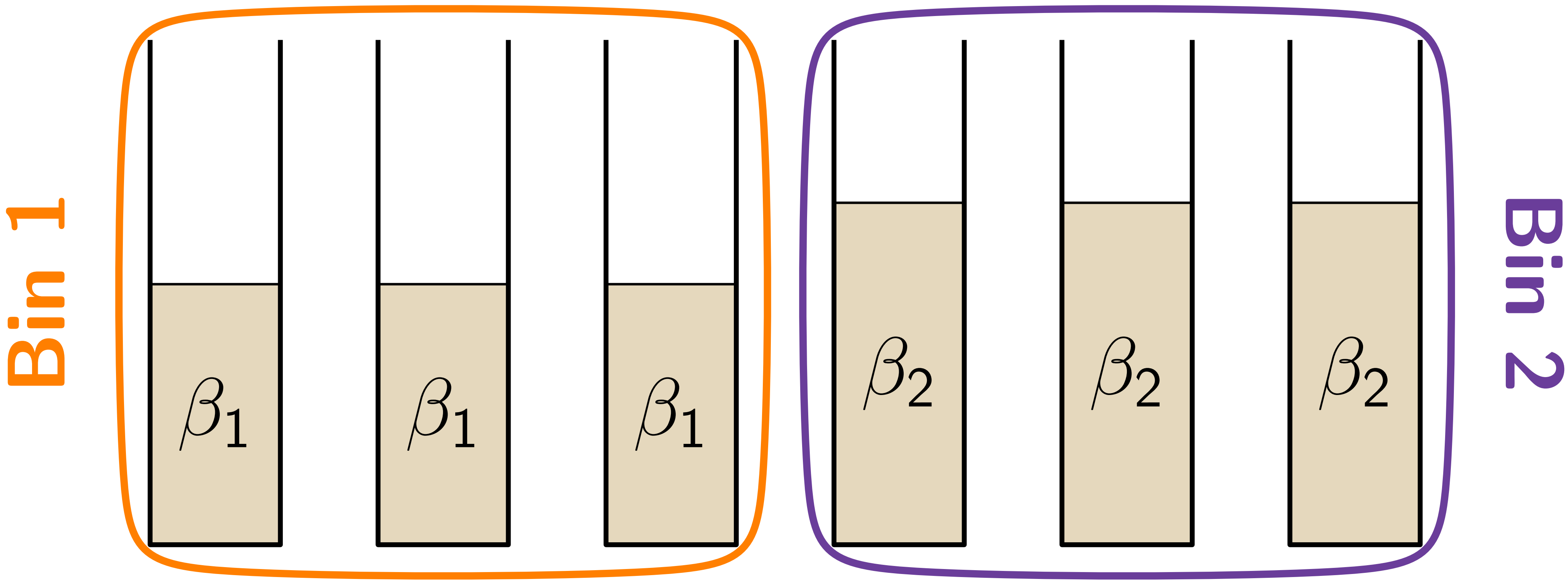

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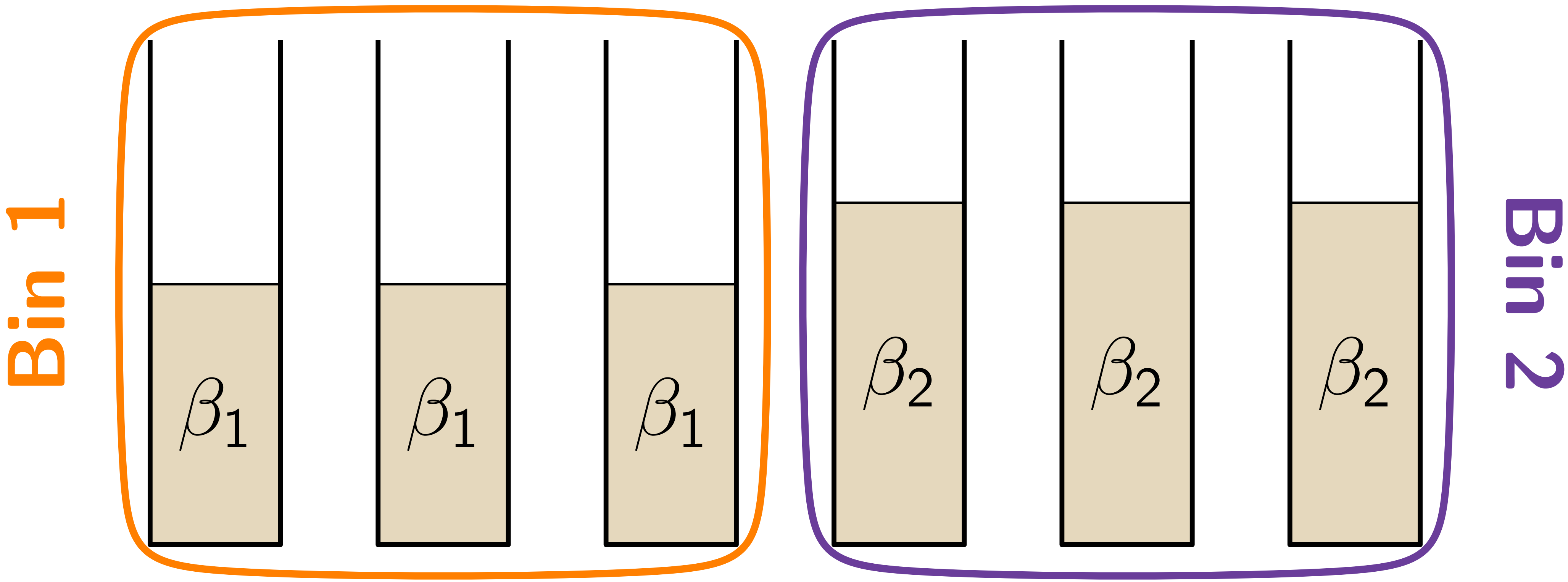

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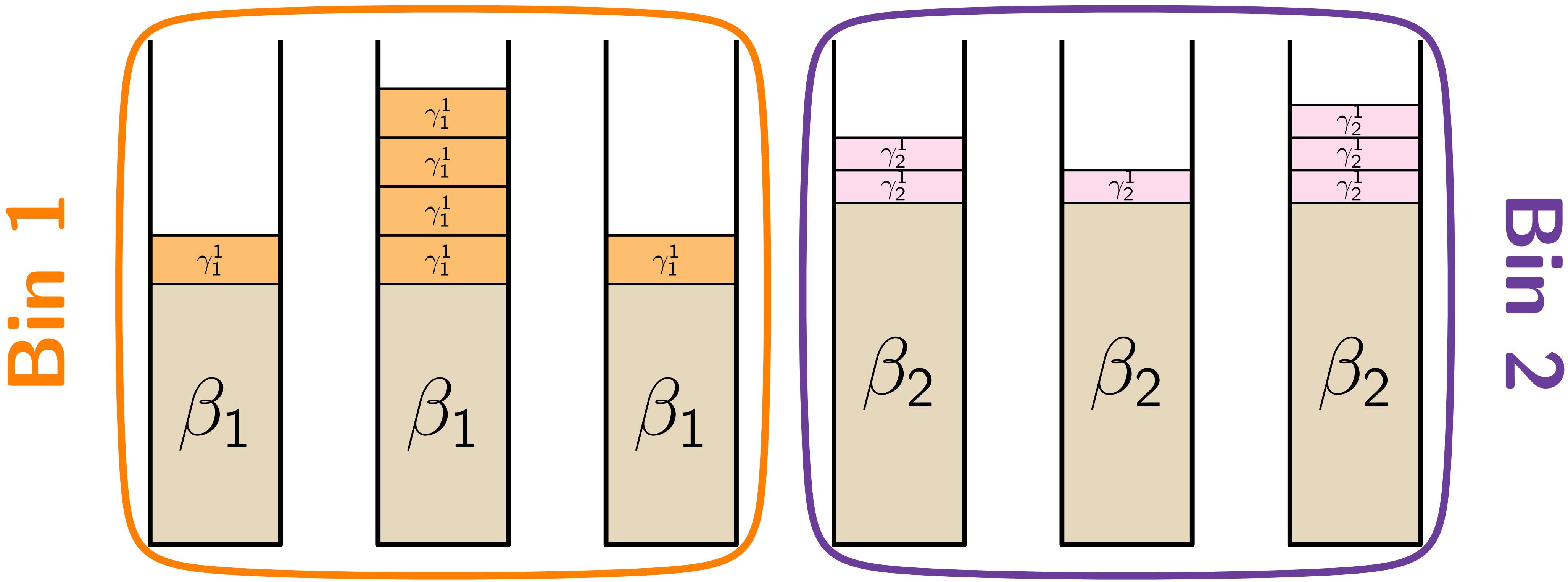

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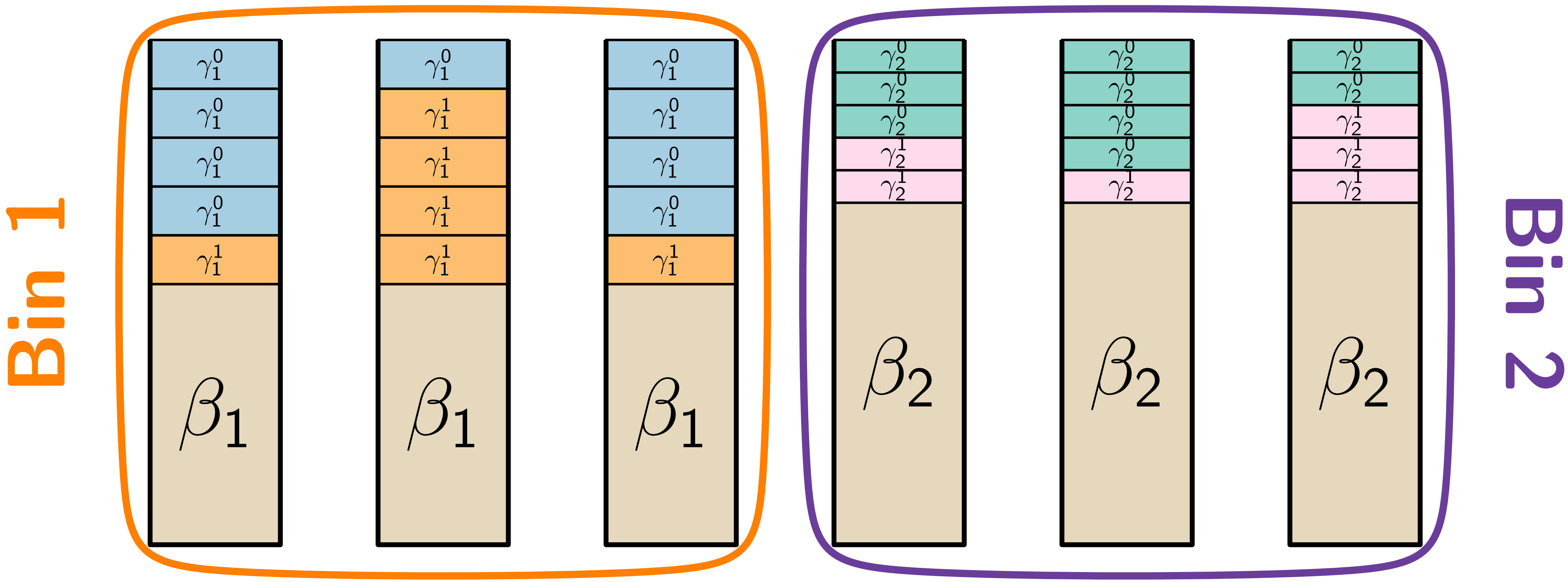

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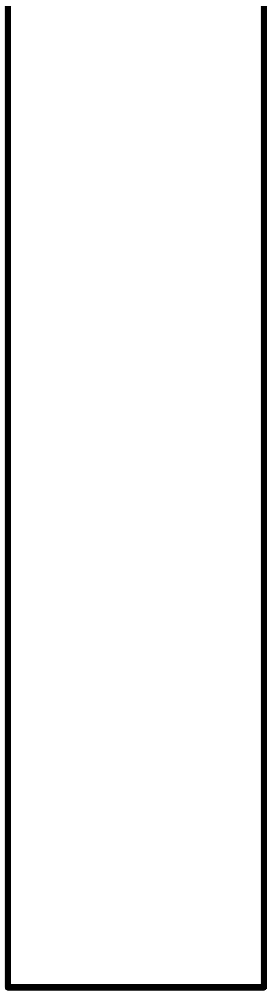

BALANCED BIN PACKING $\to Q|HM|C_{\max}$

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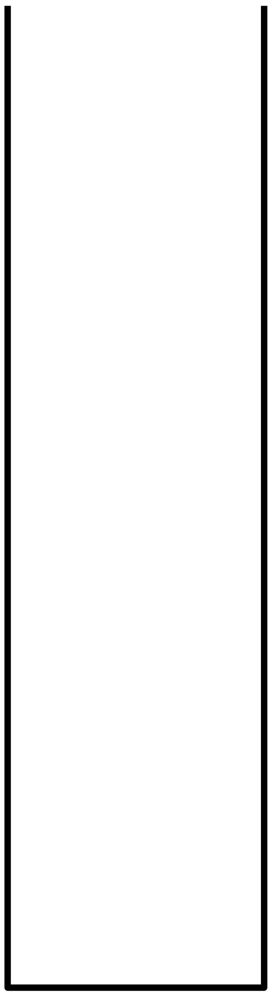

Machine M_i

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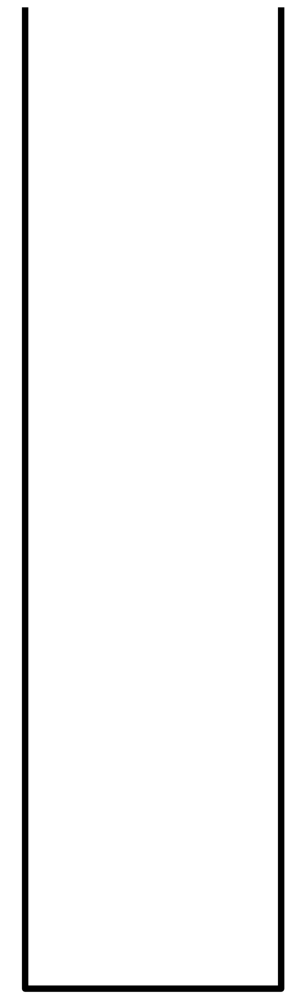

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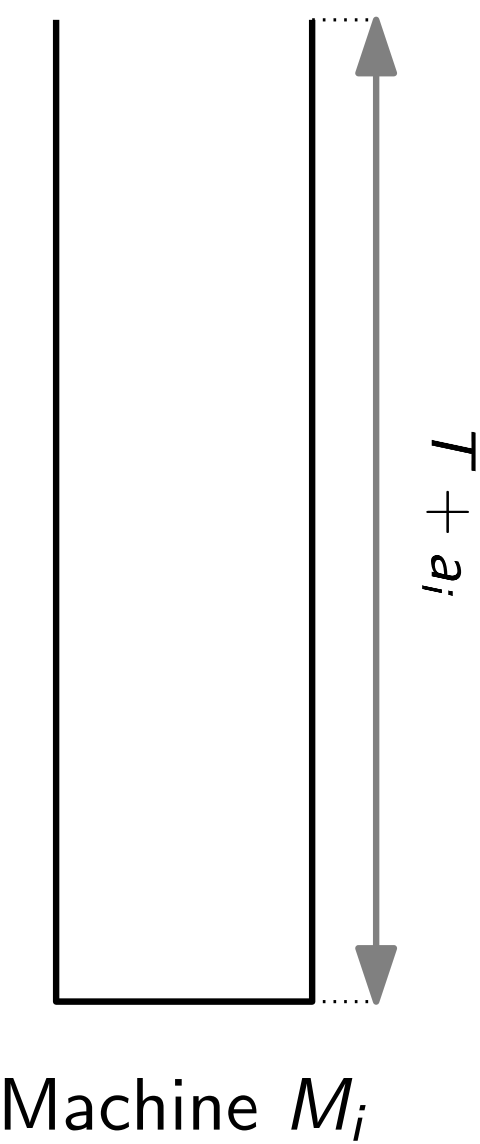

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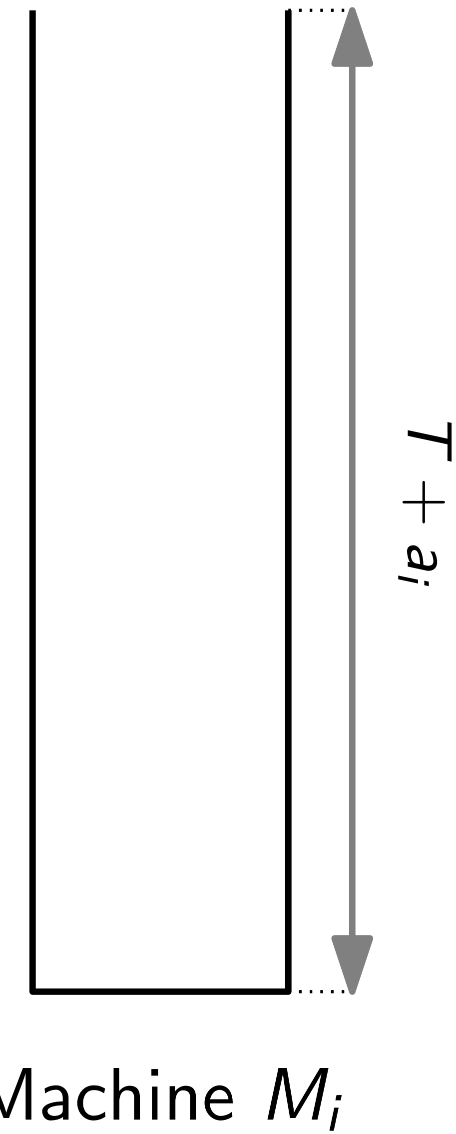

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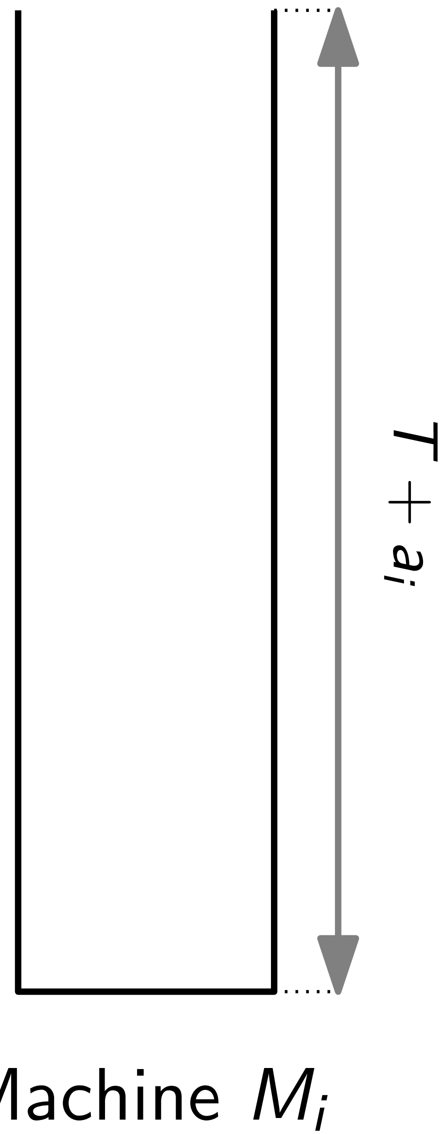

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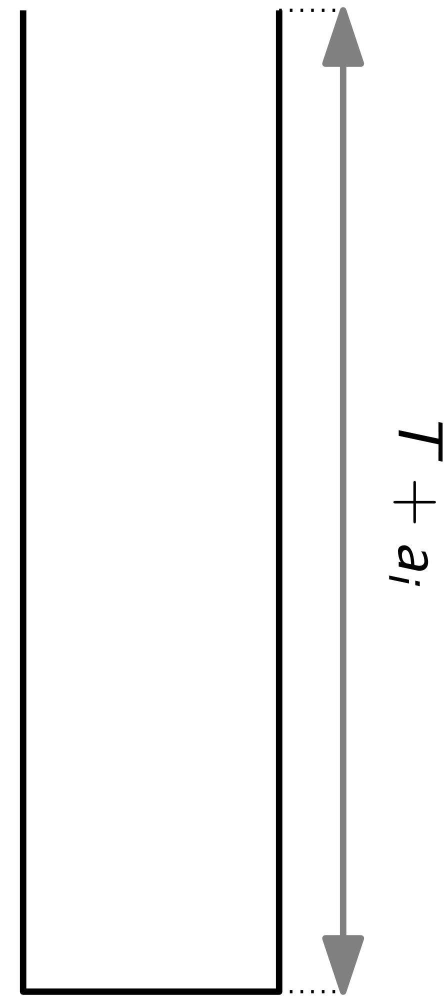

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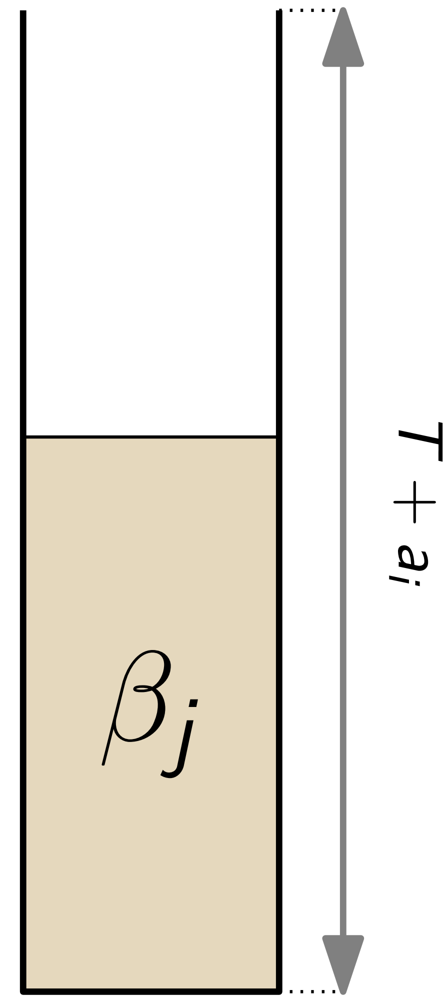

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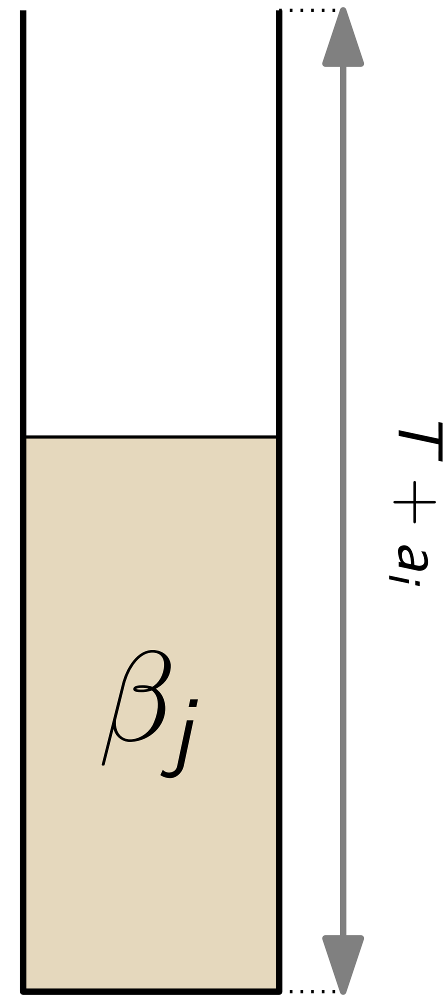

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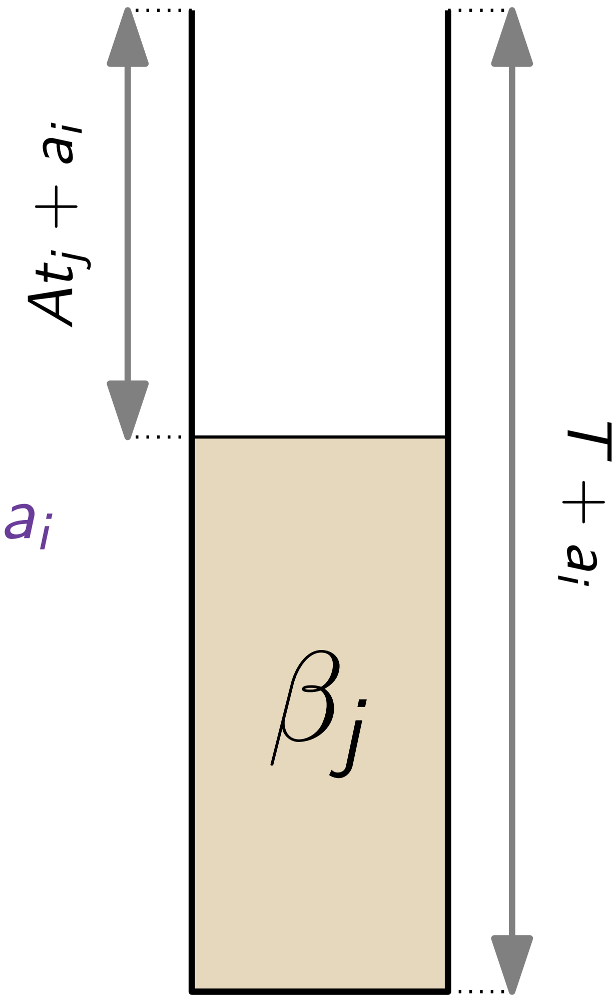

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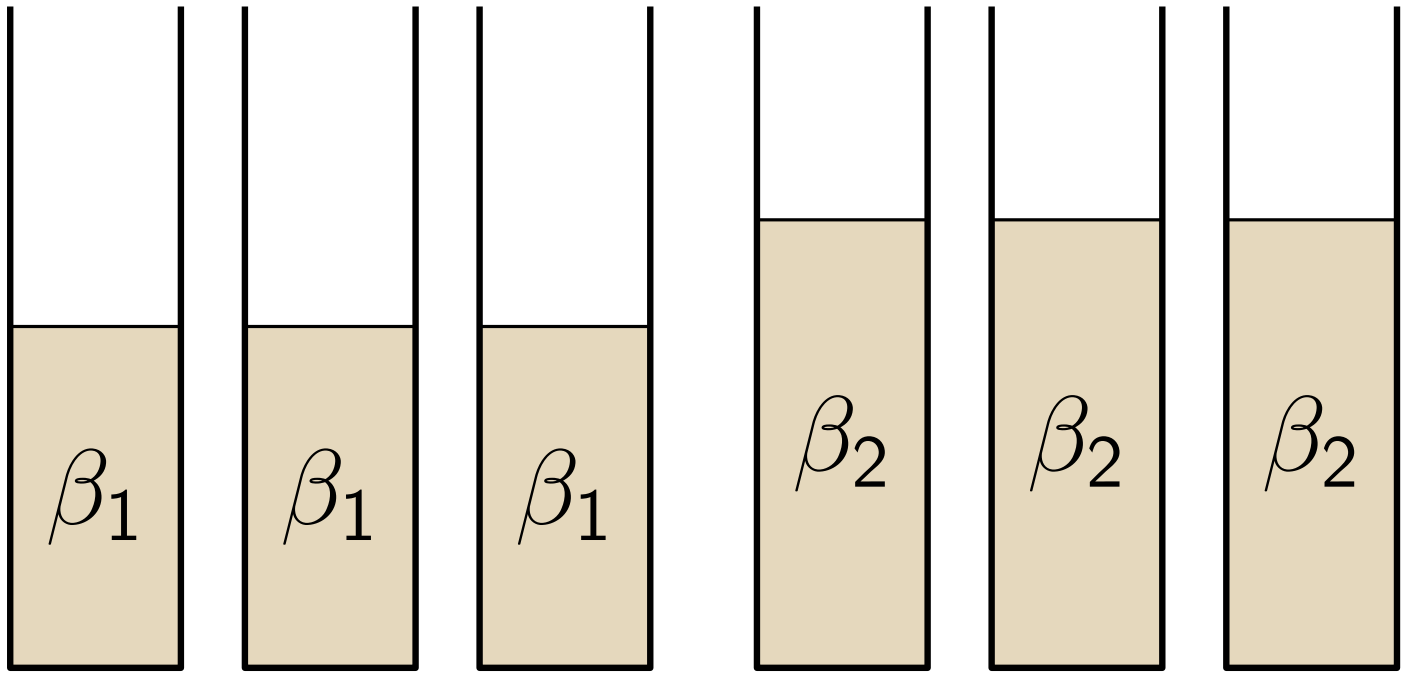

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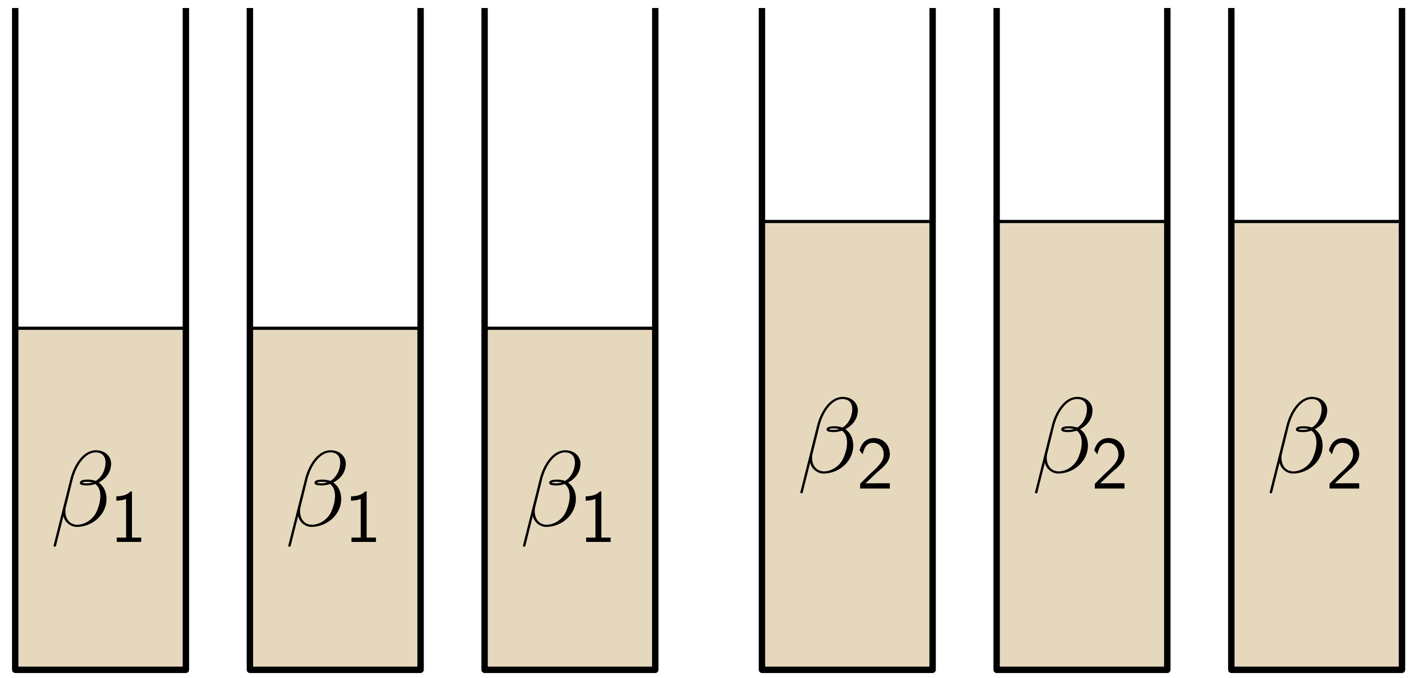

- We have mA γ-type jobs

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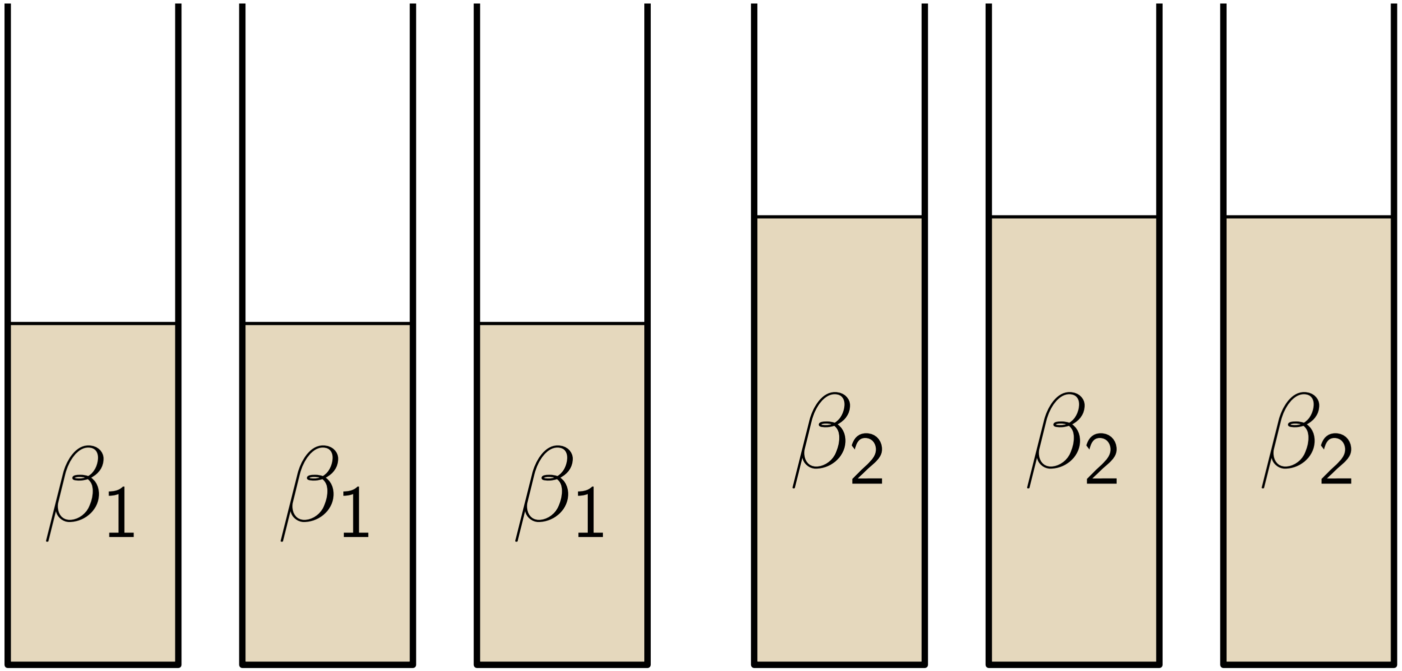

- We have mA γ-type jobs
- Any machine can execute $\leq A$ γ-type jobs

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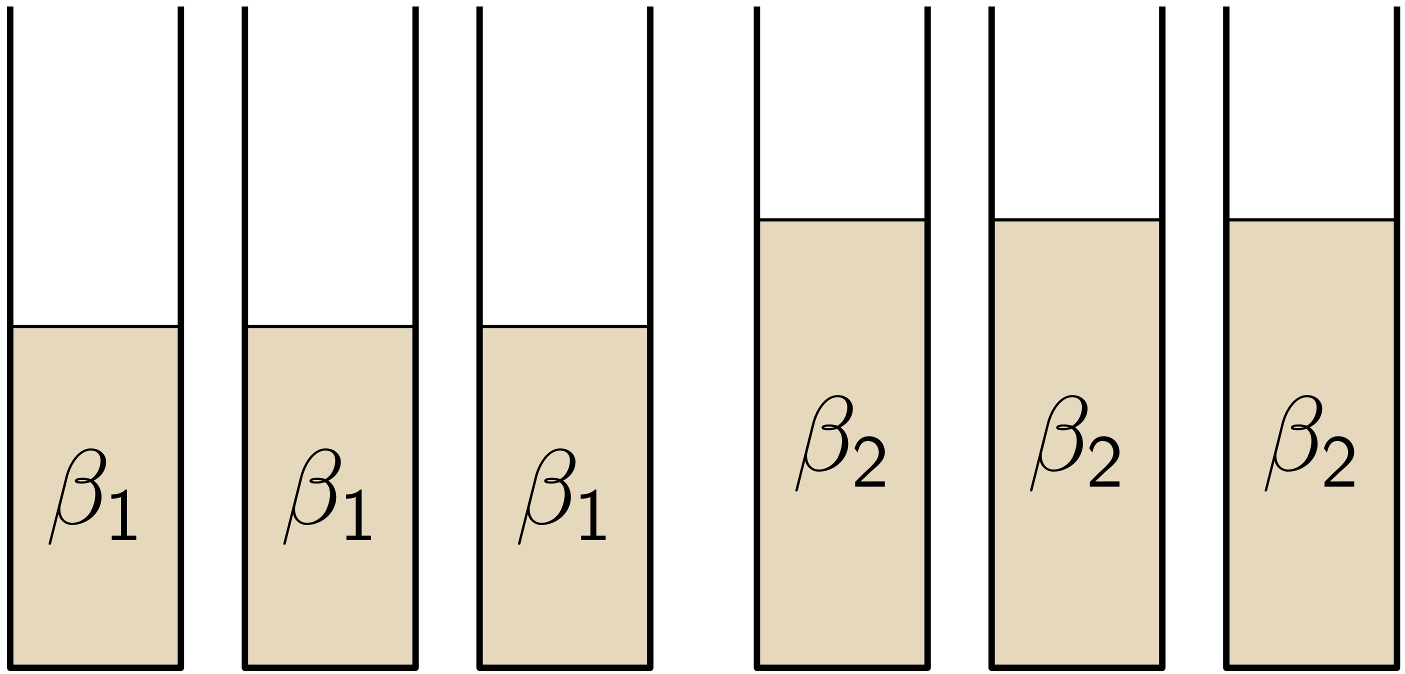

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$\Rightarrow$ exactly A γ-type jobs / machine

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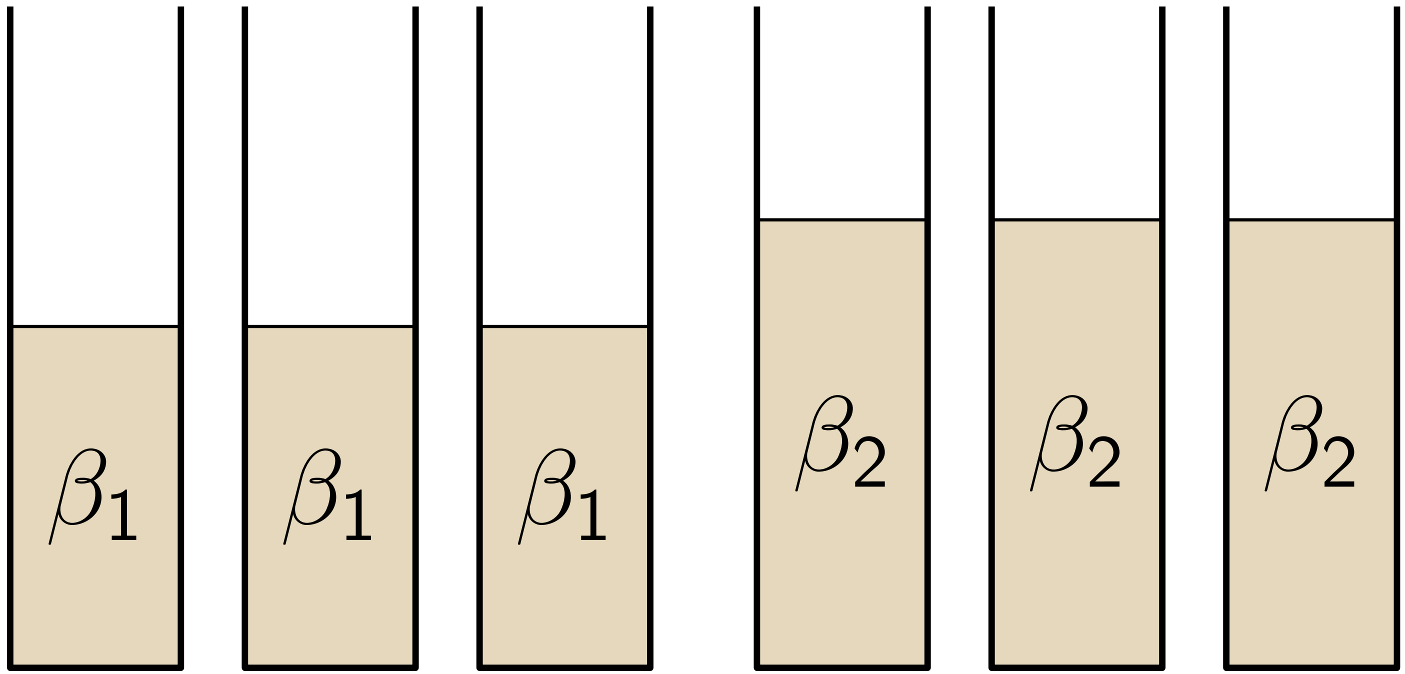

- We have mA γ-type jobs

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- As $\gamma_1^\times > \gamma_2^\times$, only machines with β_1 can host A $\gamma_1^\times$ jobs

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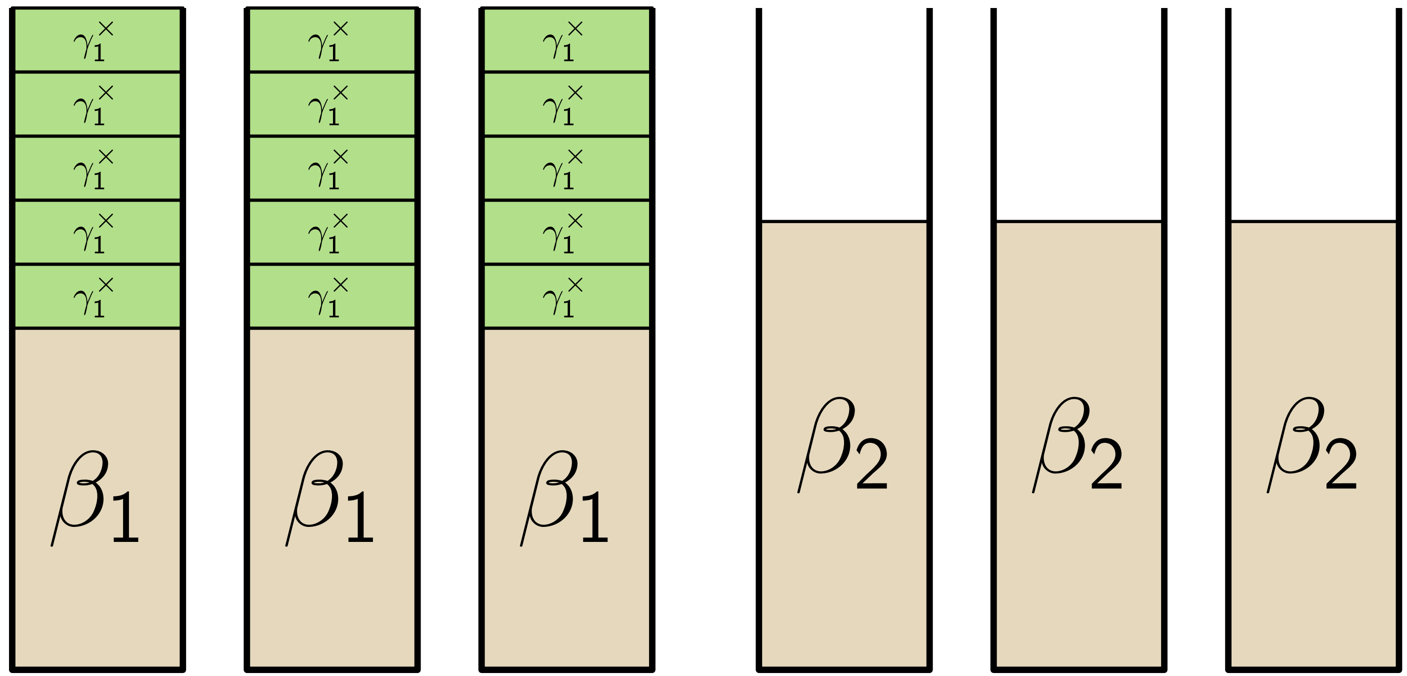

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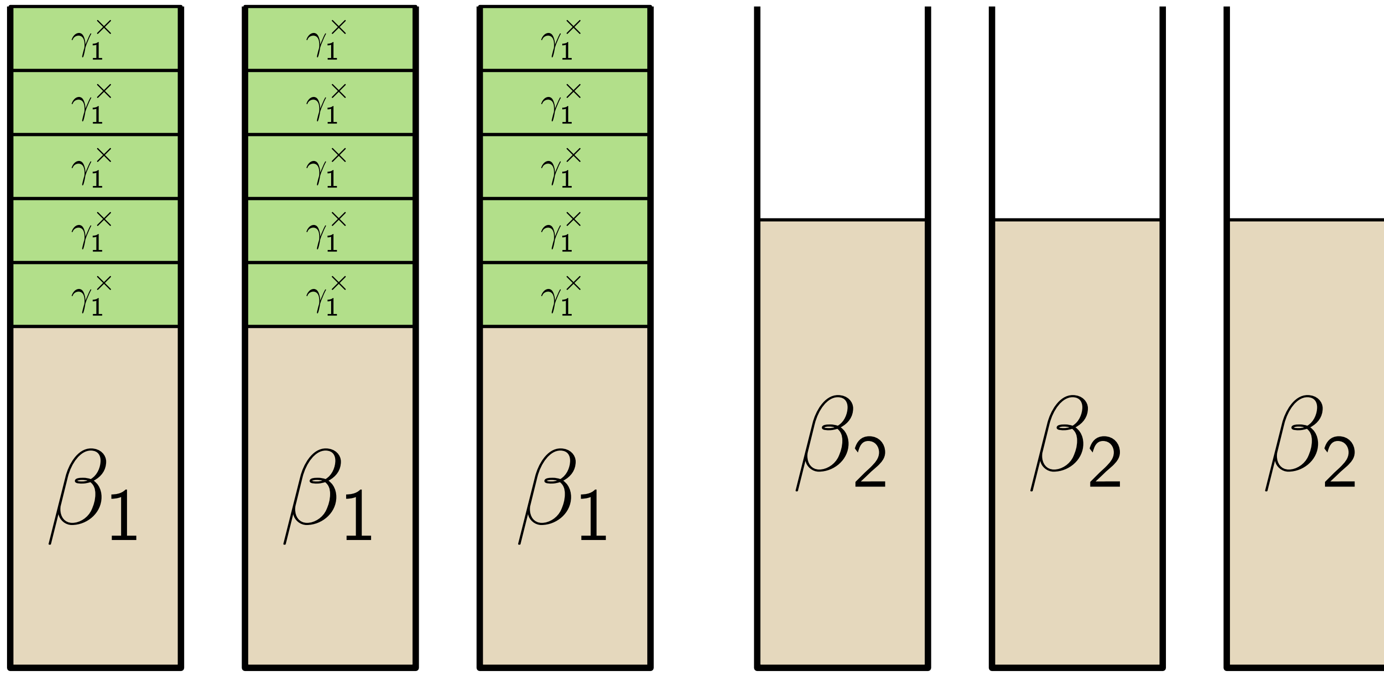

- We have mA γ-type jobs
- Any machine can execute $\le A$ γ-type jobs

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- As $\gamma_1^\times > \gamma_2^\times$, only machines with β_1 can host A $\gamma_1^\times$ jobs
- Only machines with β_2 remain for $\gamma_2^\times$, and so on

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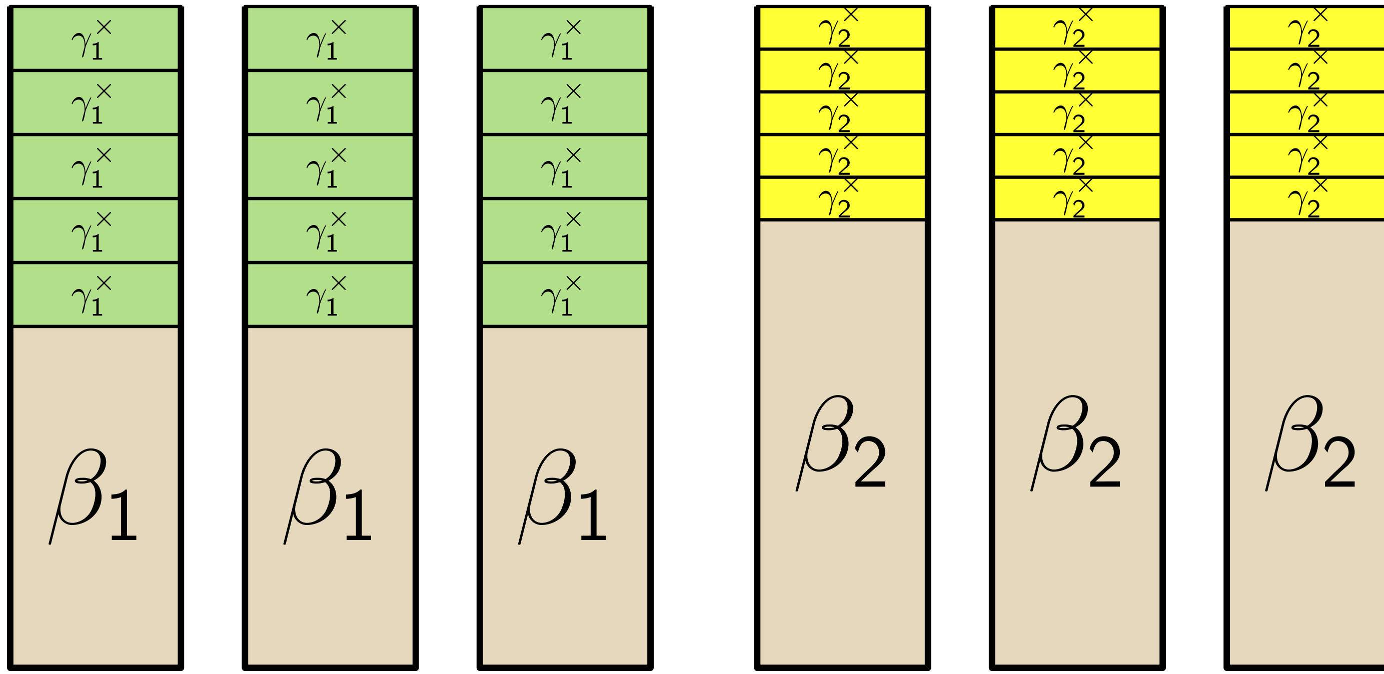
- k bins of size B
- $A := \sum_{i=1}^{m} a_i = kB$

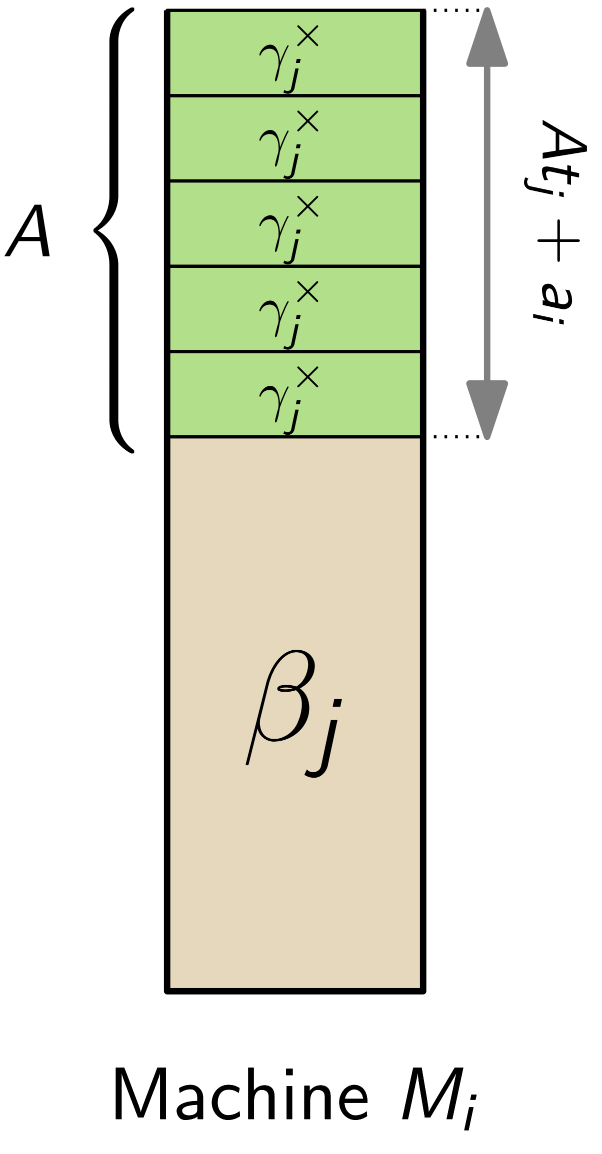


- We have mA γ-type jobs
- Any machine can execute $\leq A$ γ-type jobs

$\Rightarrow$ exactly A γ-type jobs / machine

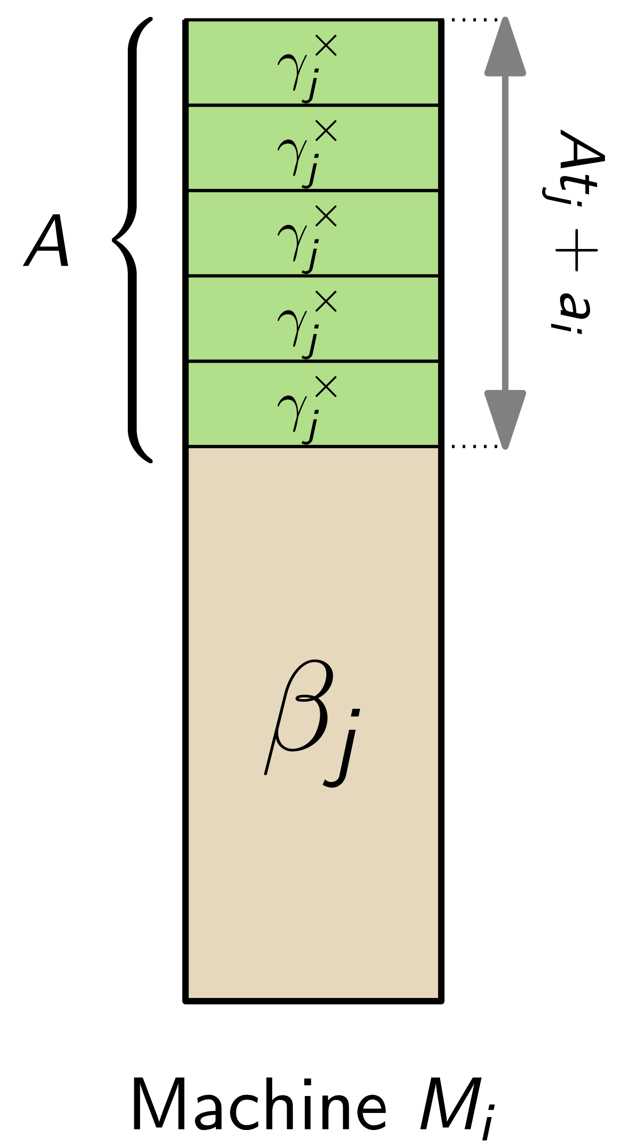
- As $\gamma_1^\times > \gamma_2^\times$, only machines with β_1 can host A $\gamma_1^\times$ jobs
- Only machines with β_2 remain for $\gamma_2^\times$, and so on

Balanced Bin Packing $\to Q|HM|C_{\max}$



Balanced Bin Packing $\to Q|HM|C_{\max}$

Distinguishing between γ_j^0 and γ_j^1:



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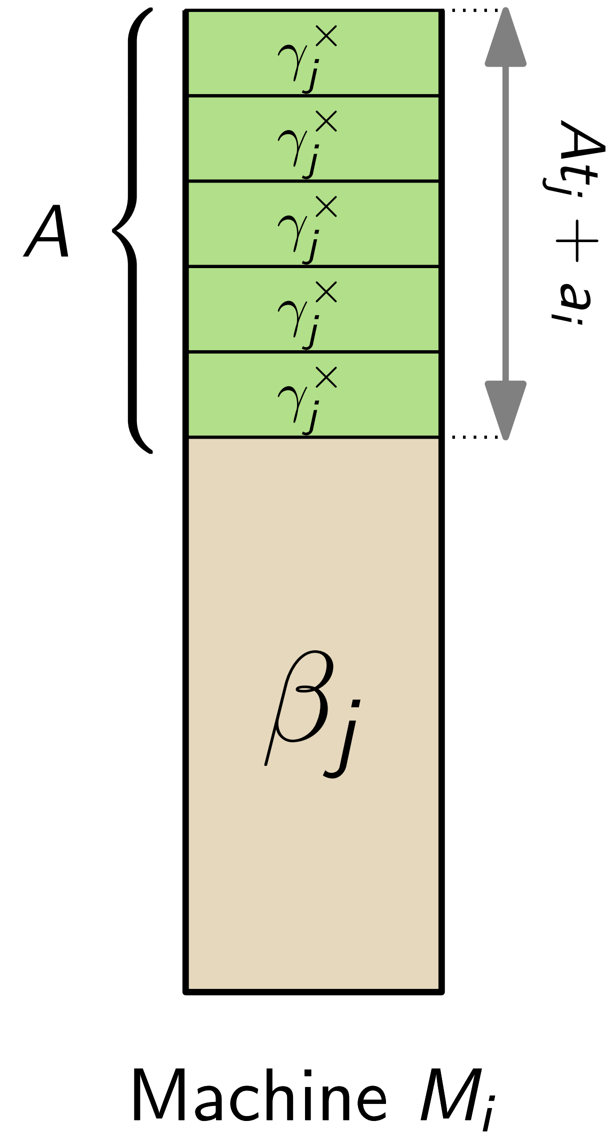
Distinguishing between γ_j^0 and γ_j^1:

- $p_{\gamma_j^0} = t_j$



Balanced Bin Packing → $Q|HM|C_{\max}$

Distinguishing between γ_j^0 and γ_j^1:

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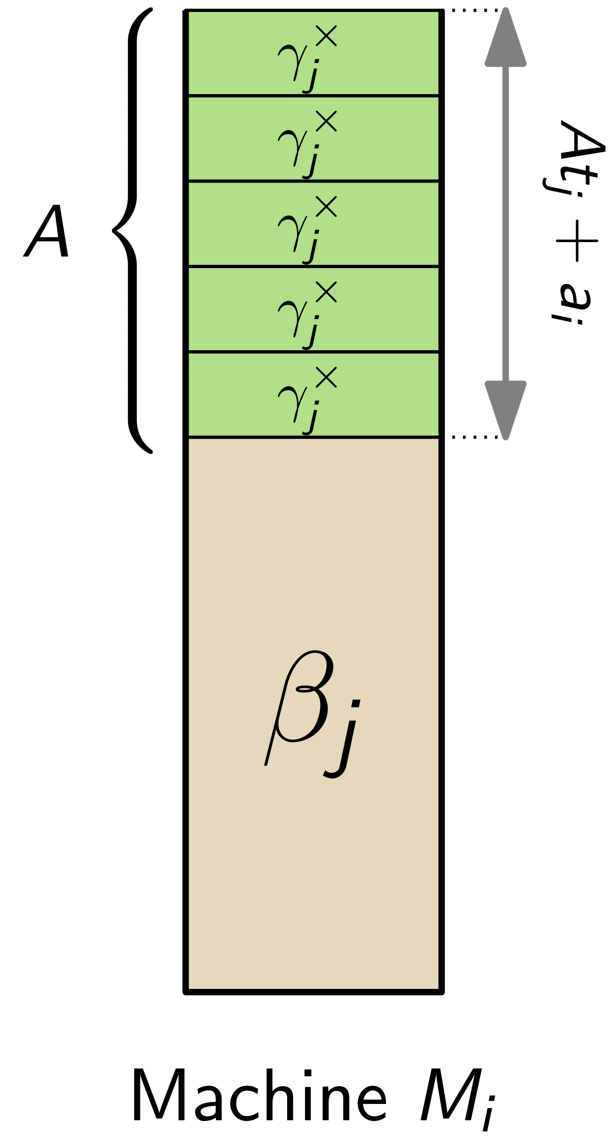

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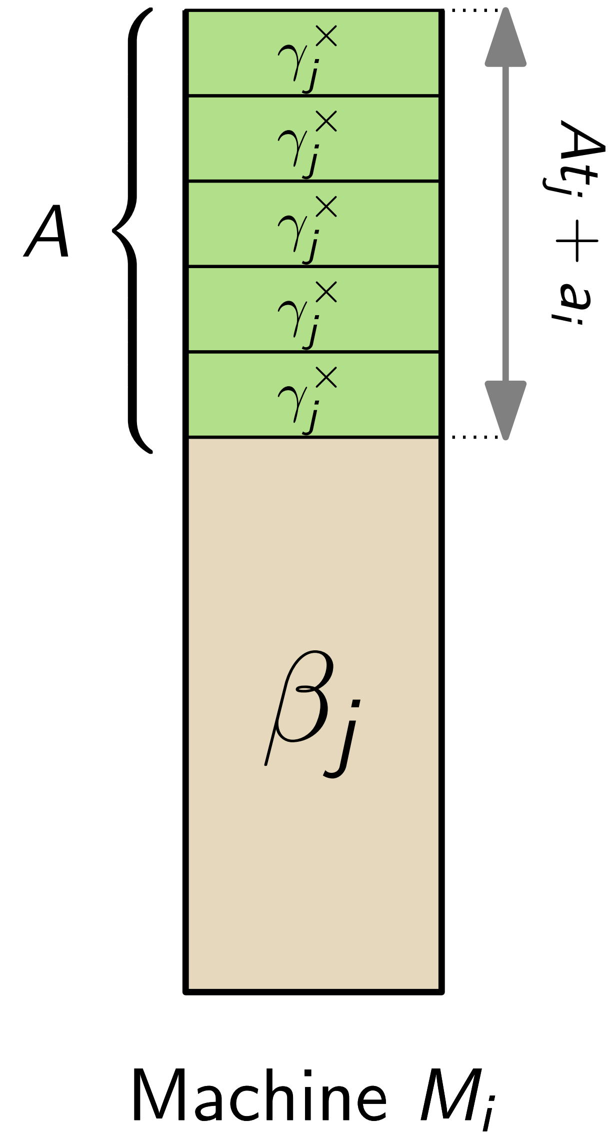
$\Rightarrow \; \leq a_i$ jobs of type γ_j^1 per M_i



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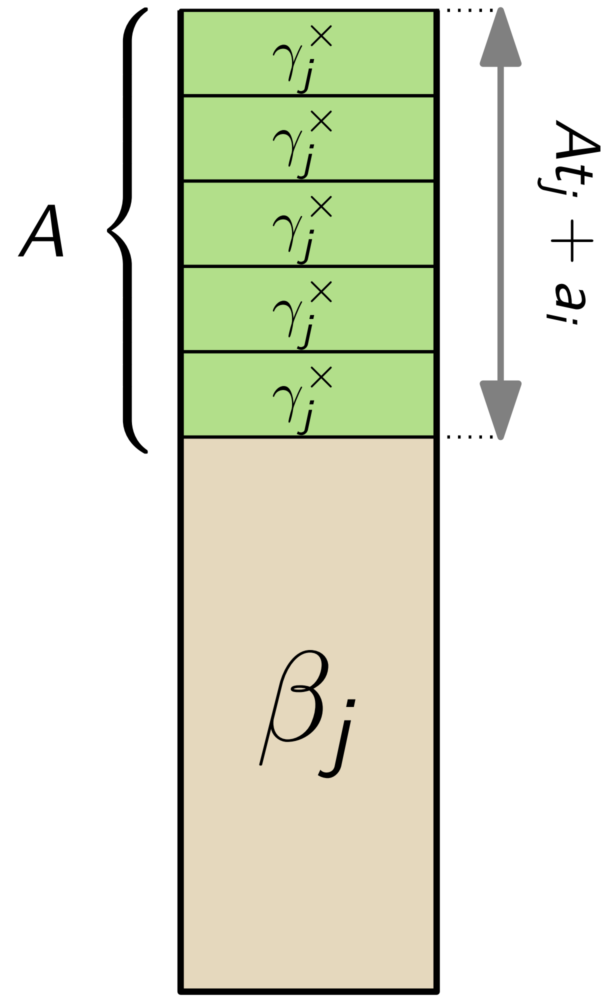

Machine M_i

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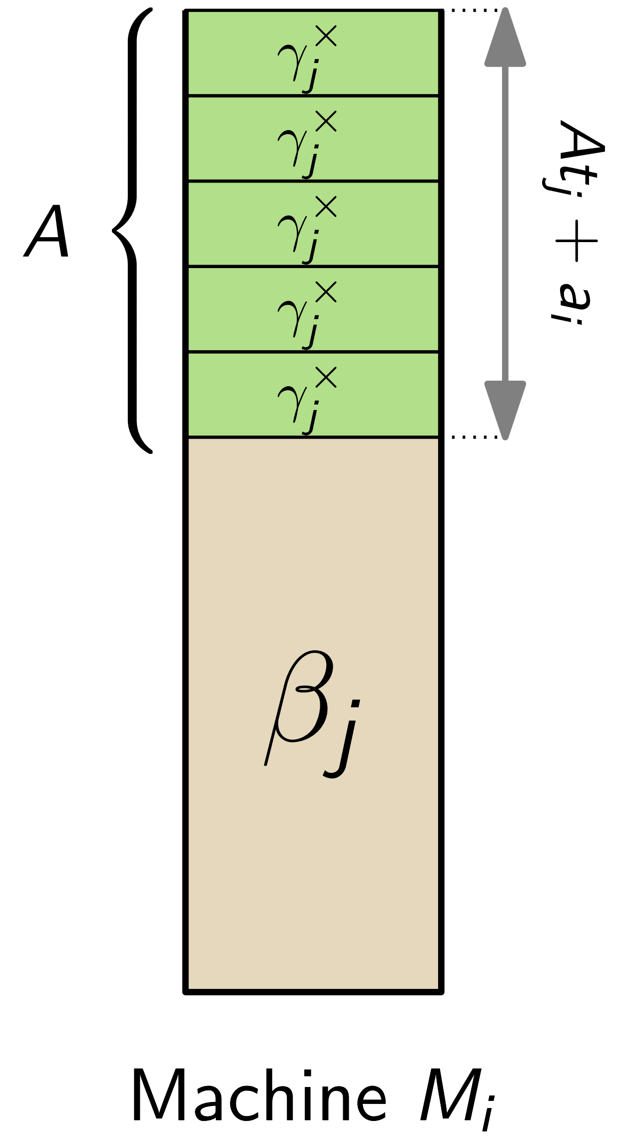
- We have $A = \sum_{i=0}^{m} a_i$ jobs of type $\gamma_\times^1$ in total.



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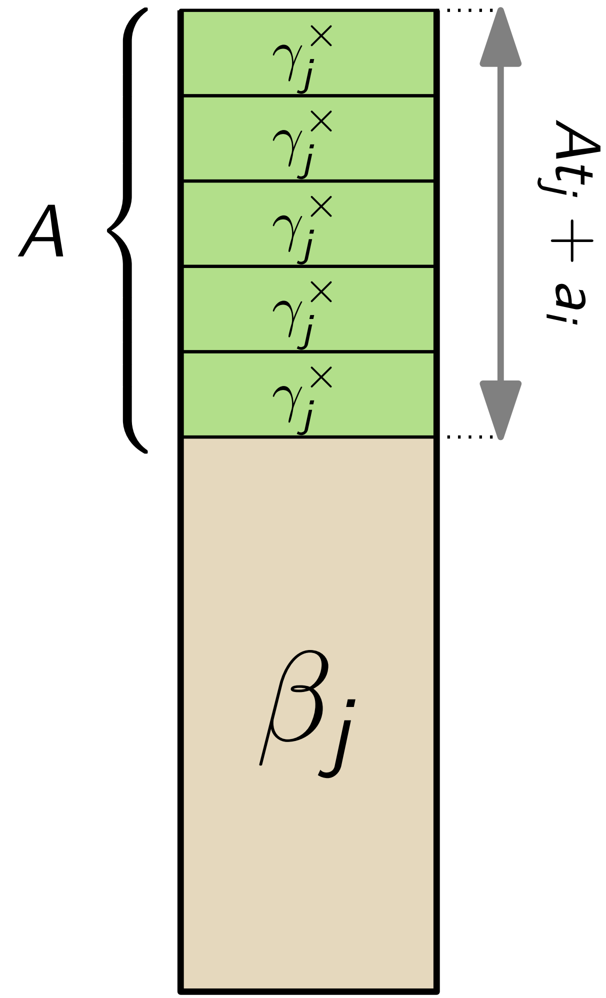
- We have $A = \sum_{i=0}^{m} a_i$ jobs of type $\gamma_\times^1$ in total.

$\Rightarrow$ M_i has exactly a_i jobs of type γ_j^1



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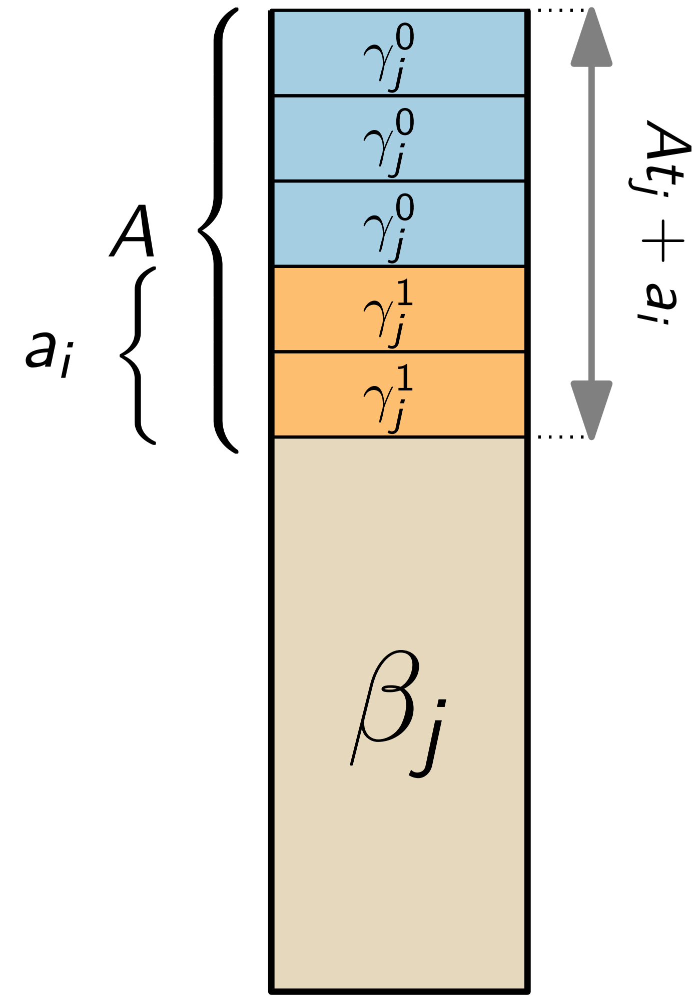

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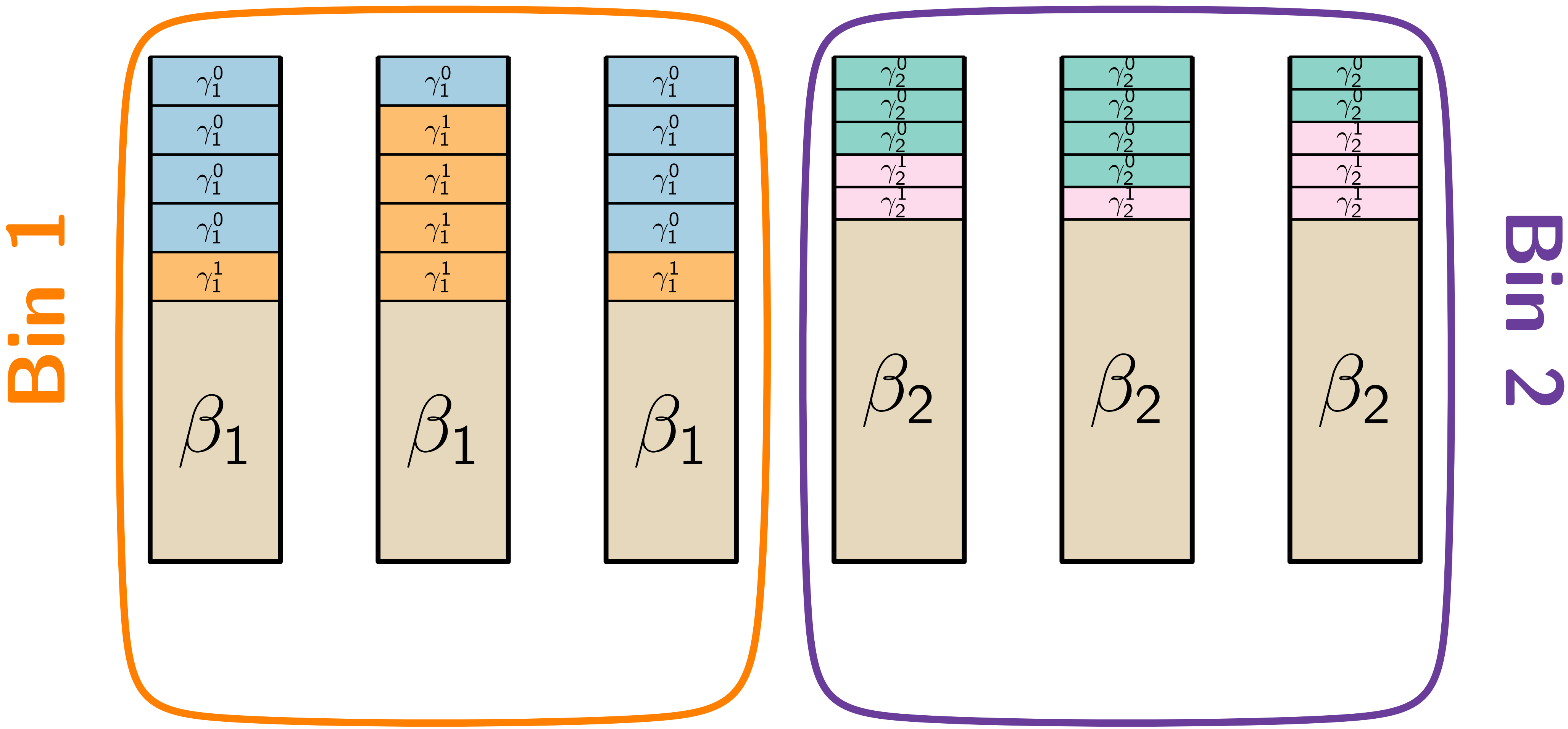
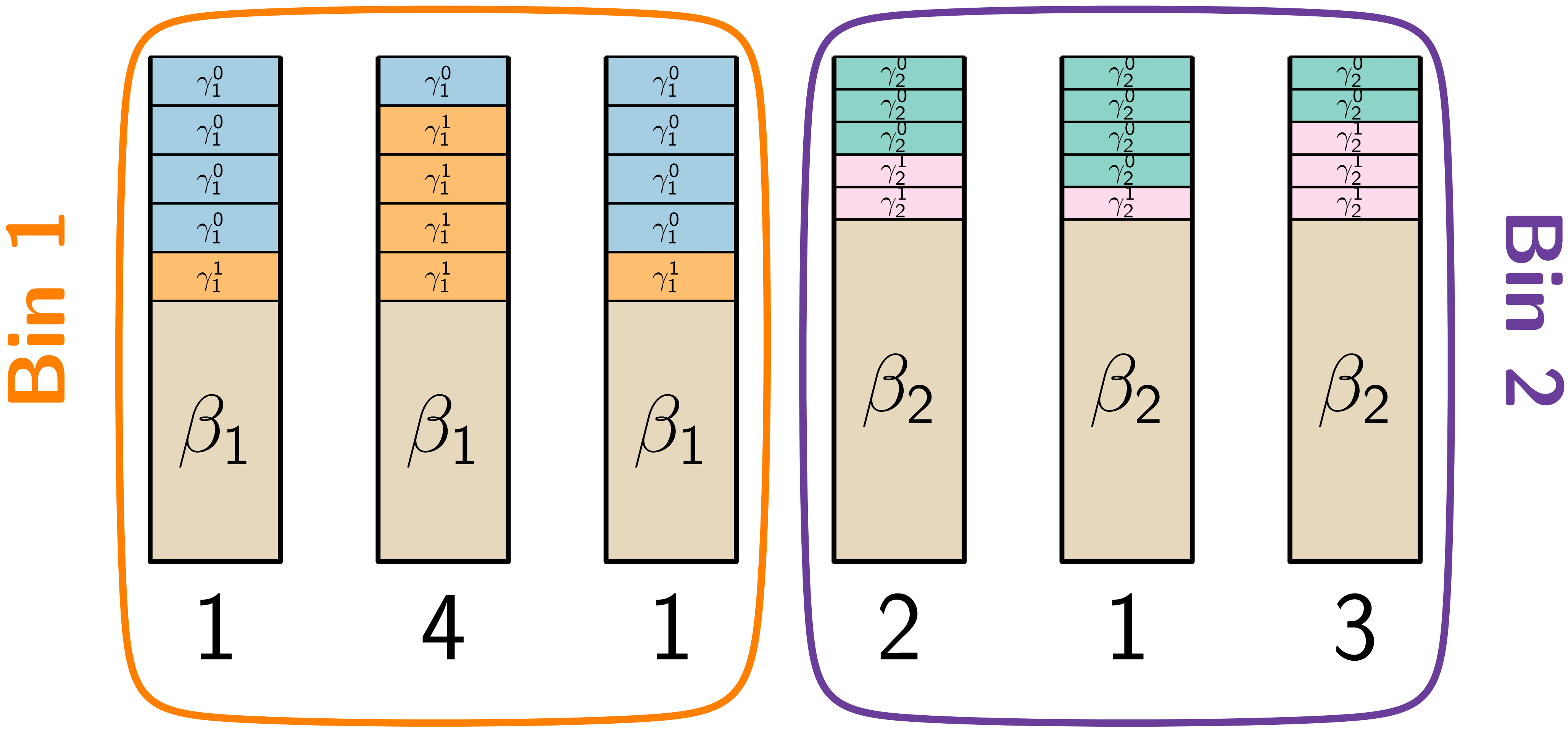
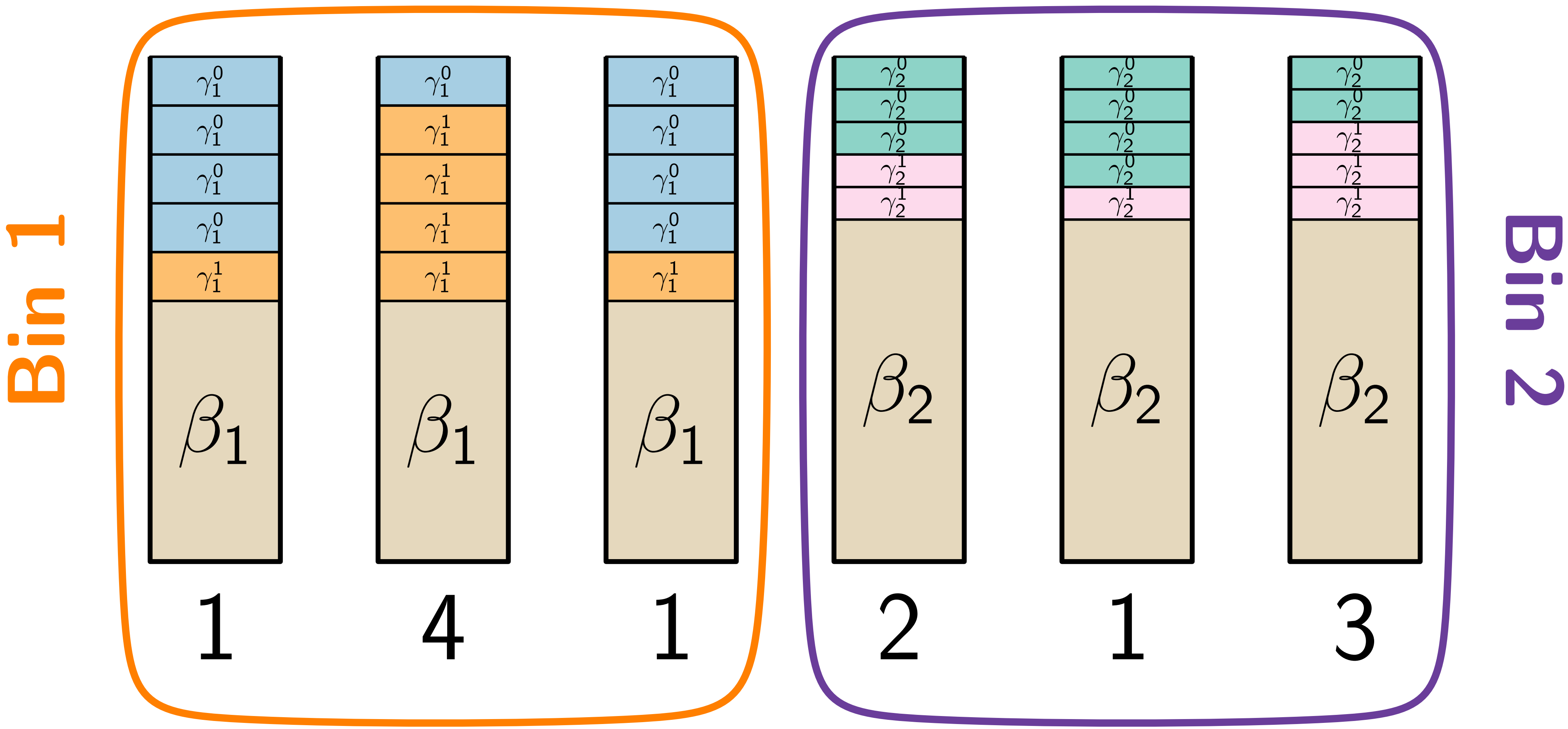


Balanced Bin Packing $\to Q|HM|C_{\max}$



Balanced Bin Packing → $Q|HM|C_{\max}$



For $k = 2$ bins, Balanced Bin Packing is already NP-hard (→ Partition), there we use $2 \cdot 3 = 6$ job types.

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$\{R, Q\}||C_{\max}$ is W[1]-hard parameterized by the number of job types even for unary input.

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Beside $C_{\max}$, most of our hardness results also hold true for the objectives ℓ_2 and $\sum w_j C_j$.

Fixed-Parameter Tractability

$P||C_{\max}$ is FPT parameterized by the number of job types k.

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m $(\mathbf{p}, C_{\max})$

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Proof Idea:

assignment of jobs to machines

- Use algorithm by Goemans and Rothvoss, which has runtime $(\log p_{\max})^{f(k)} \cdot \text{poly}\log(n) \leq g(k) \cdot p_{\max}^{o(1)} \cdot \text{poly}\log(n)$.

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m $(\mathbf{p}, C_{\max})$

Summary

	$P\|HM\|\ldots$	$Q\|HM\|\ldots$	$R\|HM\|\ldots$
$C_{\max}$			

P : identical machines

Q : related machines

R : unrelated machines

HM : high multipl. jobs

$C_{\max}$: min. makespan

ℓ_2 : min. time under ℓ_2-Norm

$\sum w_j C_j$: min. sum of weighted completion times

k : # job types

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known before
new
open

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C_{max}	poly. time for const. k	NP-hard for $k \geq 6$	NP-hard for $k \geq 4$	FPT in k	XP in k	W[1]-hard	XP in k	W[1]-hard
ℓ_2	?	NP-hard for $k \geq 6$	NP-hard for $k \geq 7$	?	XP	W[1]-hard	XP	W[1]-hard

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Open Problems

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- Is Cutting Stock W[1]-hard when input is given in unary?