The Segment Number: Algorithms and Universal Lower Bounds for Some Classes of Planar Graphs

WG 2022

Ina Goeßmann, Jonathan Klawitter, Boris Klemz, Felix Klesen, Stephen Kobourov, Myroslav Kryven, Alexander Wolff and Johannes Zink

Motivation

What makes a *good* graph drawing?

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main topic of this talk

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segment number $\text{seg}(\Gamma) = \#$ line segments of Γ

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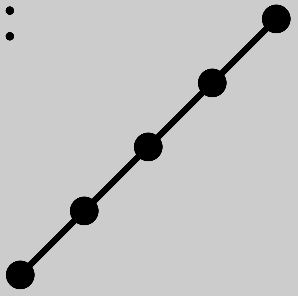

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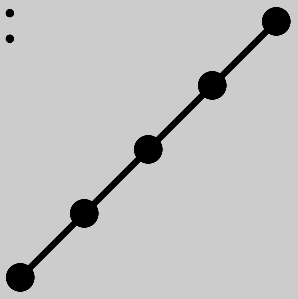

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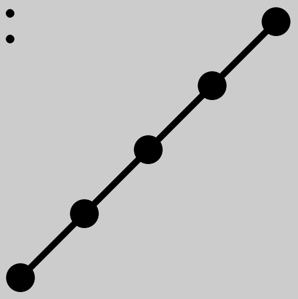

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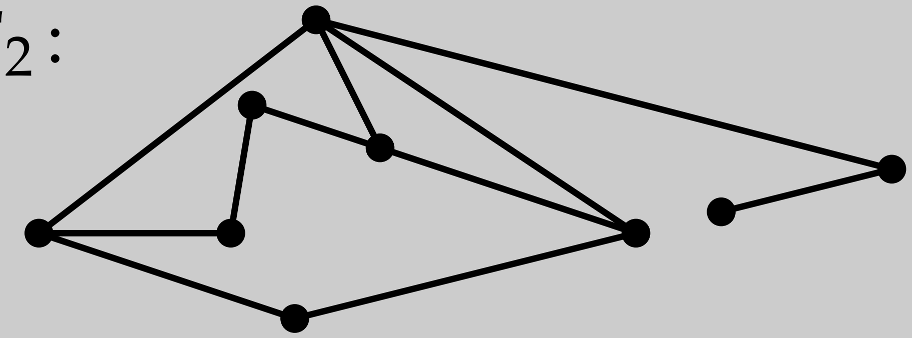

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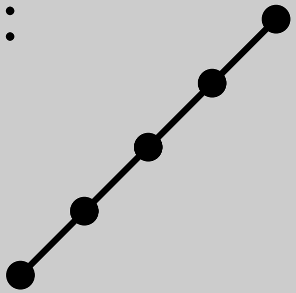

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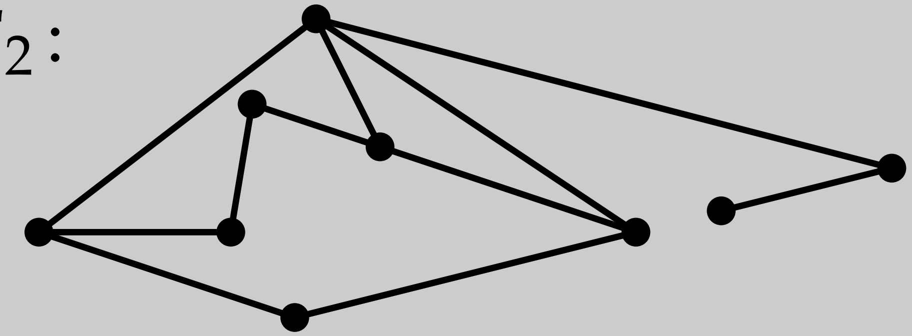

$\text{seg}(\Gamma_2) = 10$

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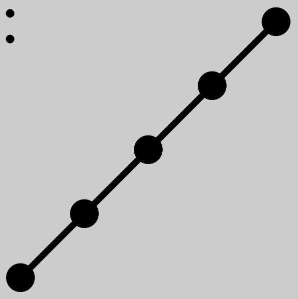

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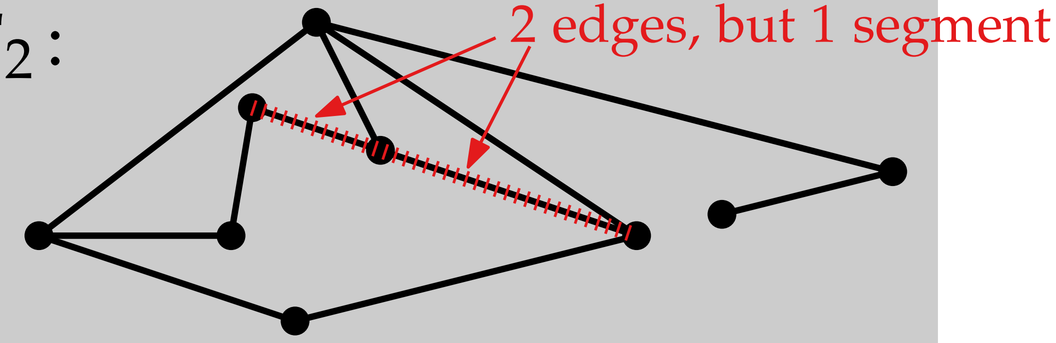

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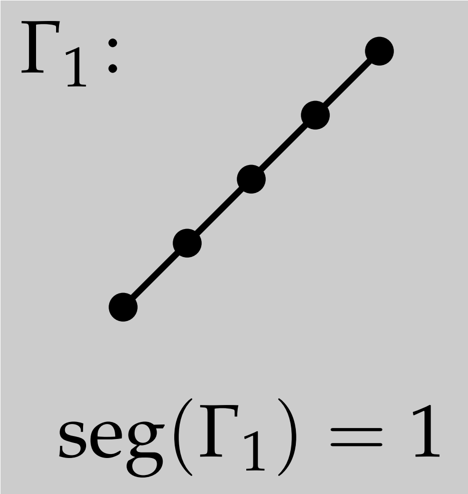

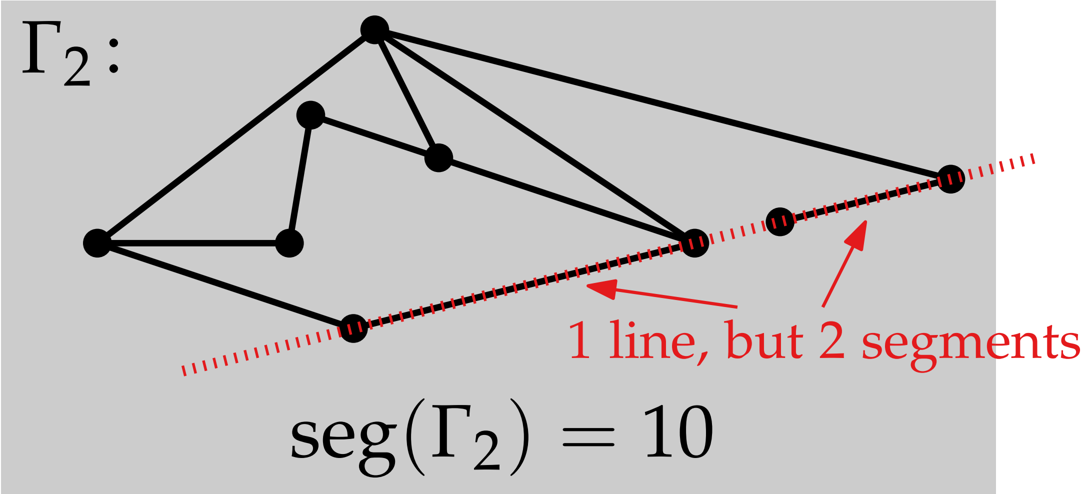

line cover number $(\Gamma_2) = 9$

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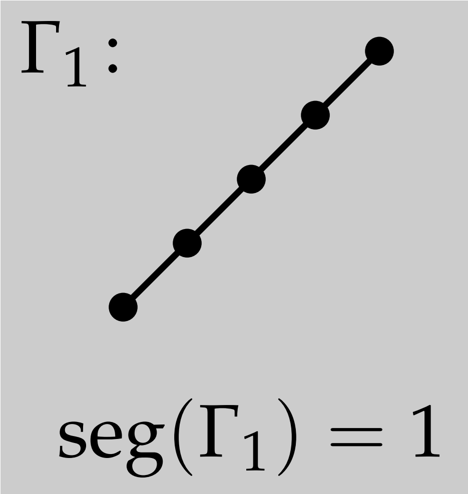

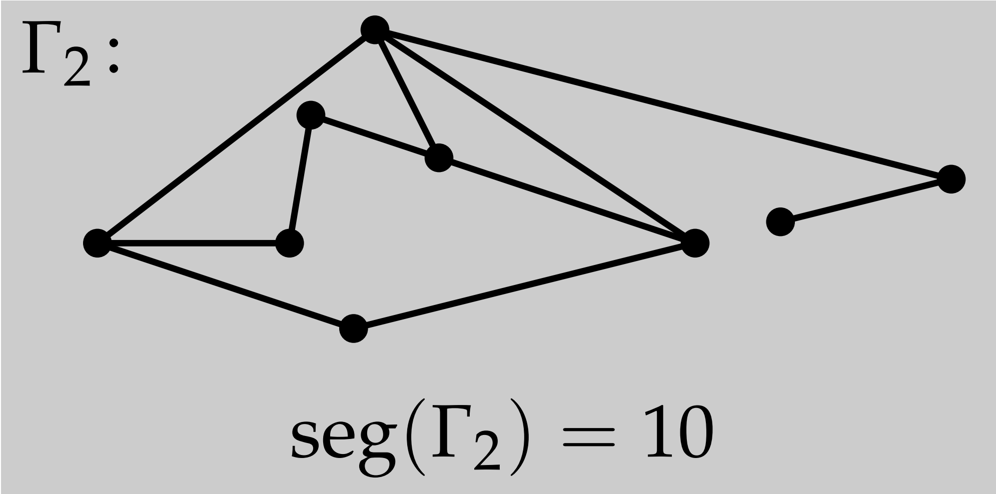

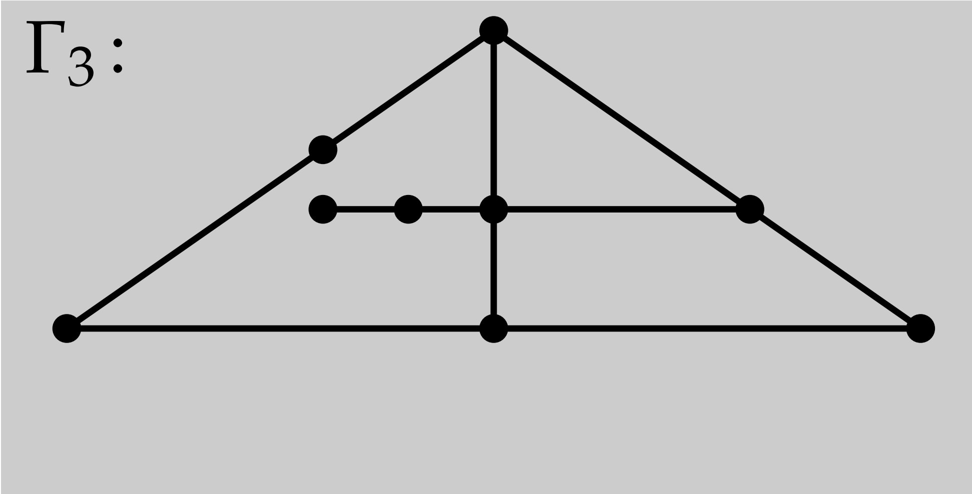

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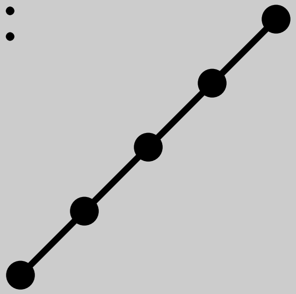

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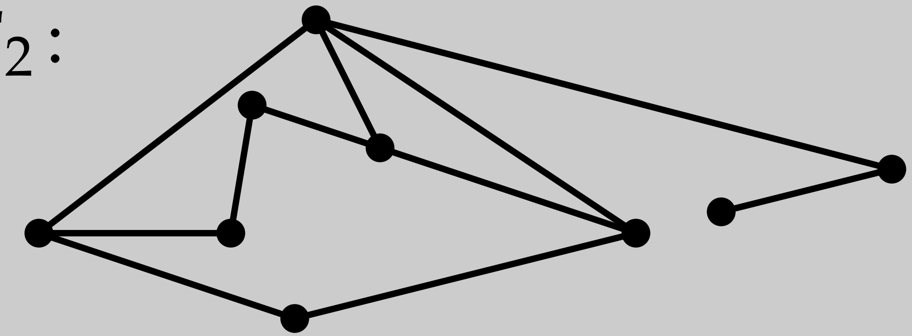

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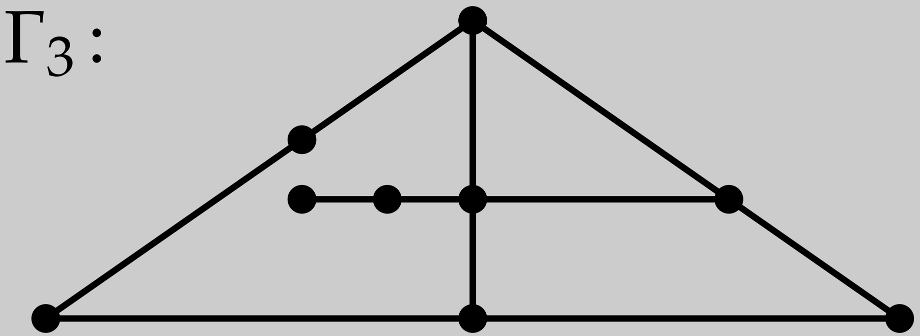

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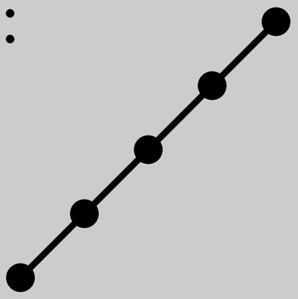

$\mathrm{seg}(\Gamma_1) = 1$

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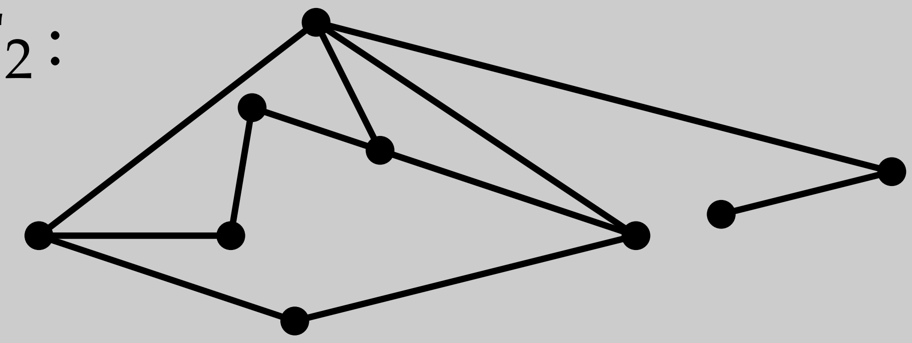

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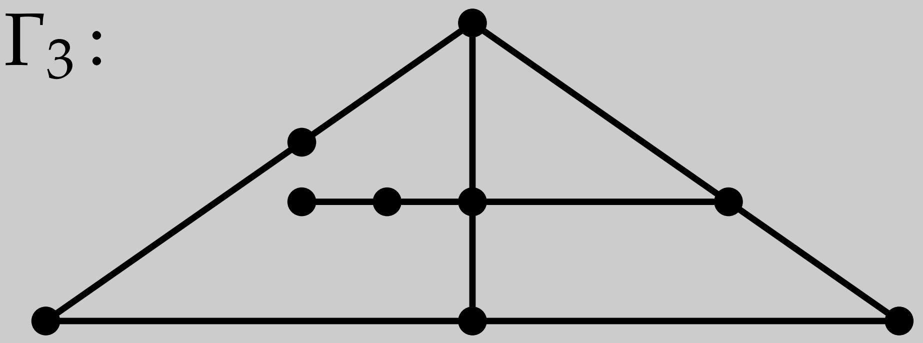

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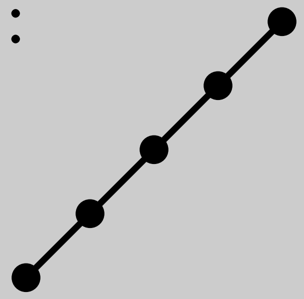

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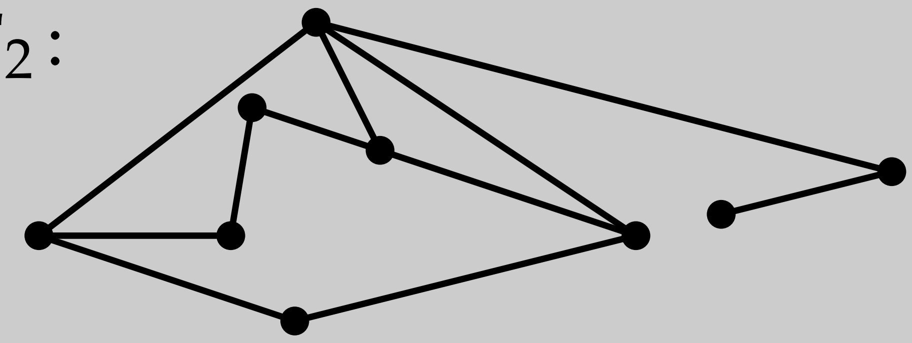
Γ_2:



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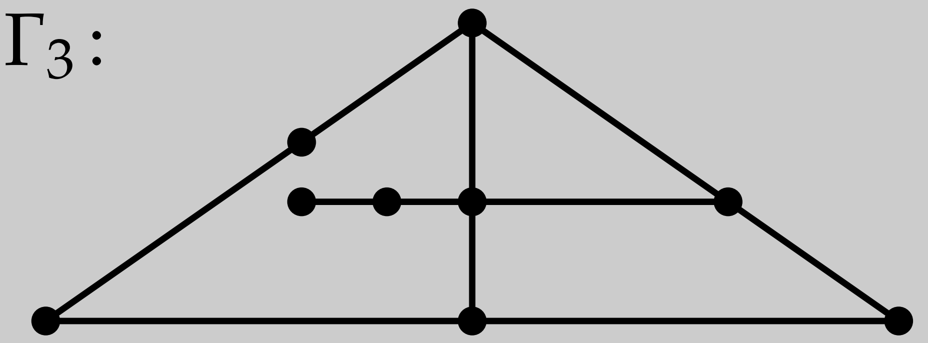

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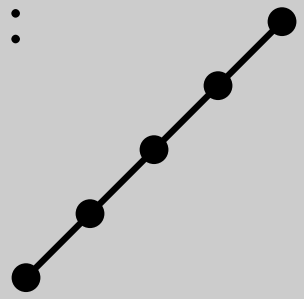

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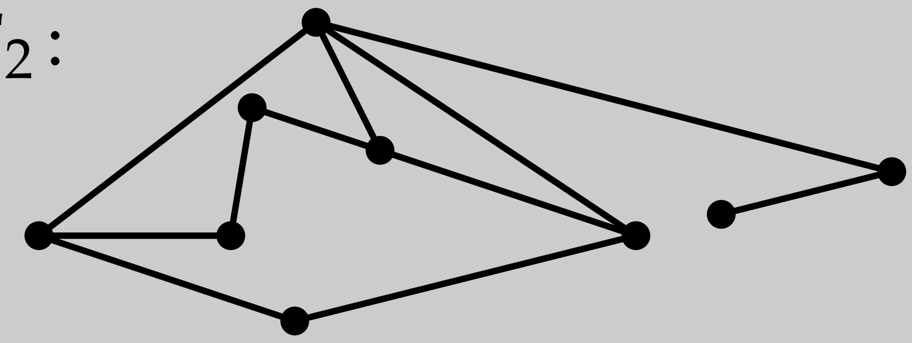

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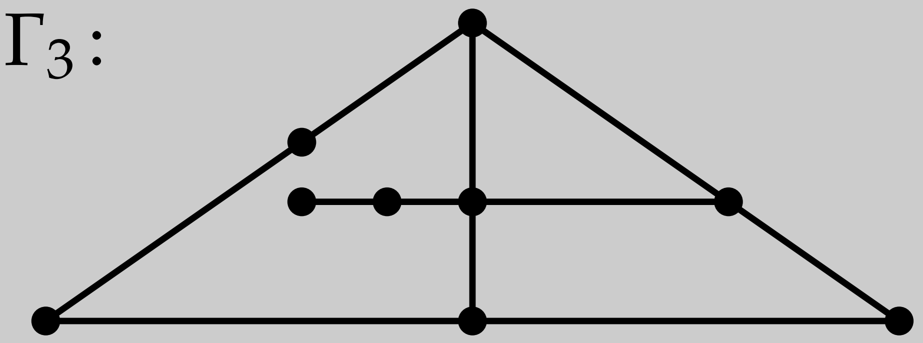

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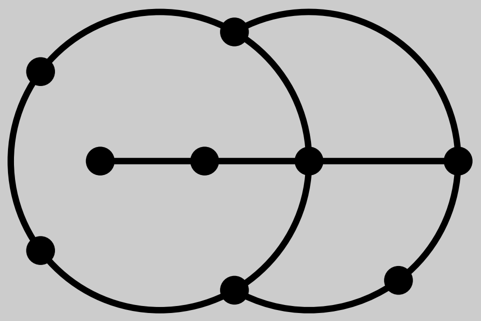

$\text{arc}(\Gamma_4) = 3$

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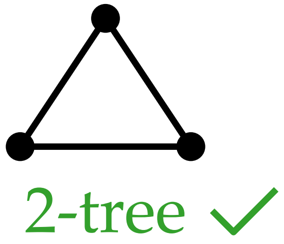

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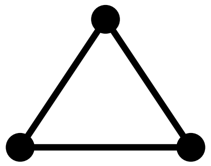
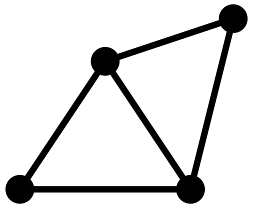


2-tree ✓



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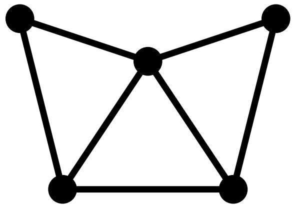

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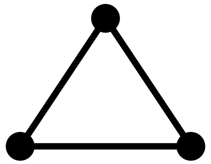

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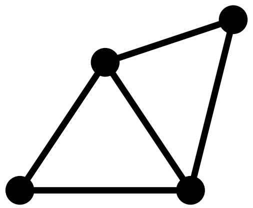

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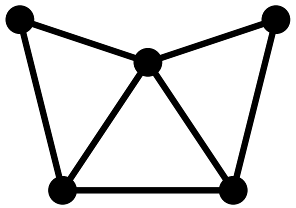

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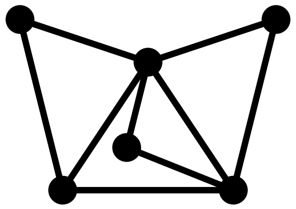

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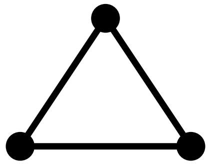
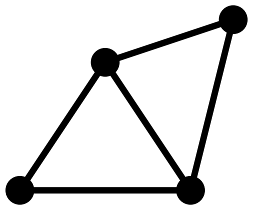


2-tree ✓



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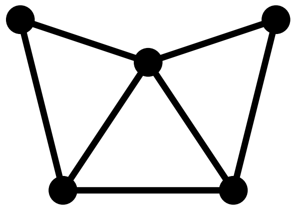

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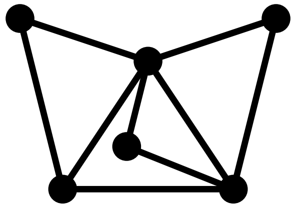

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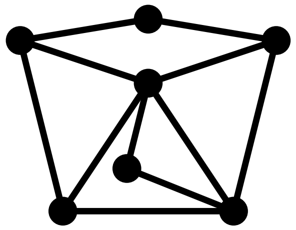

not a 2-tree ✗

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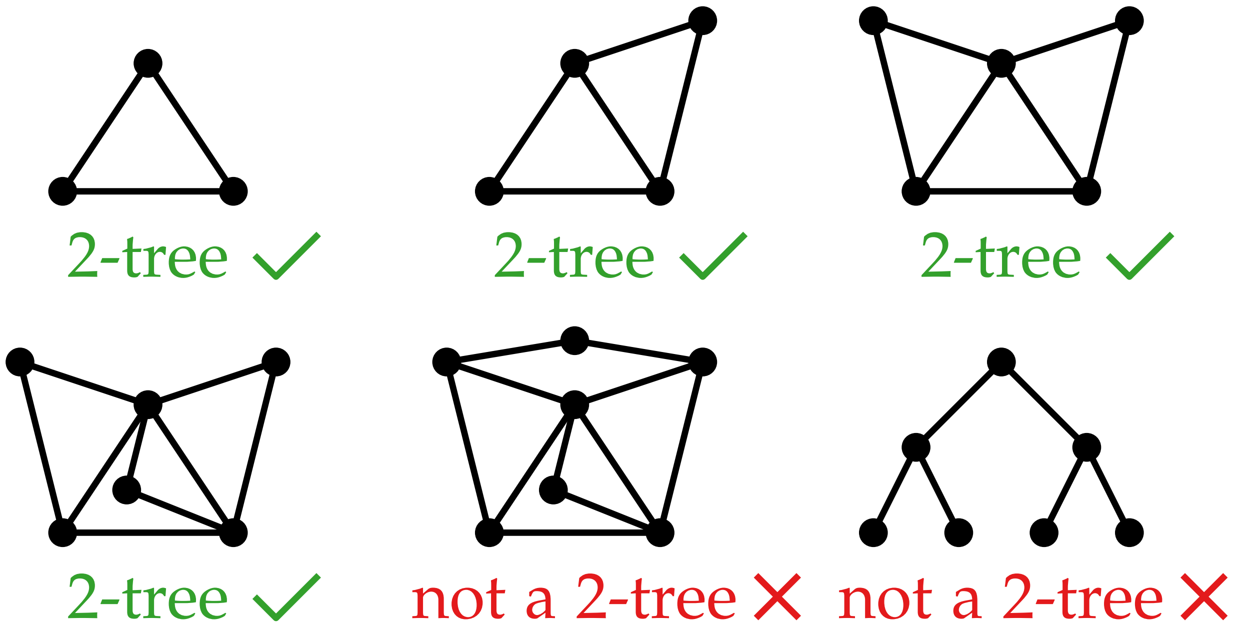

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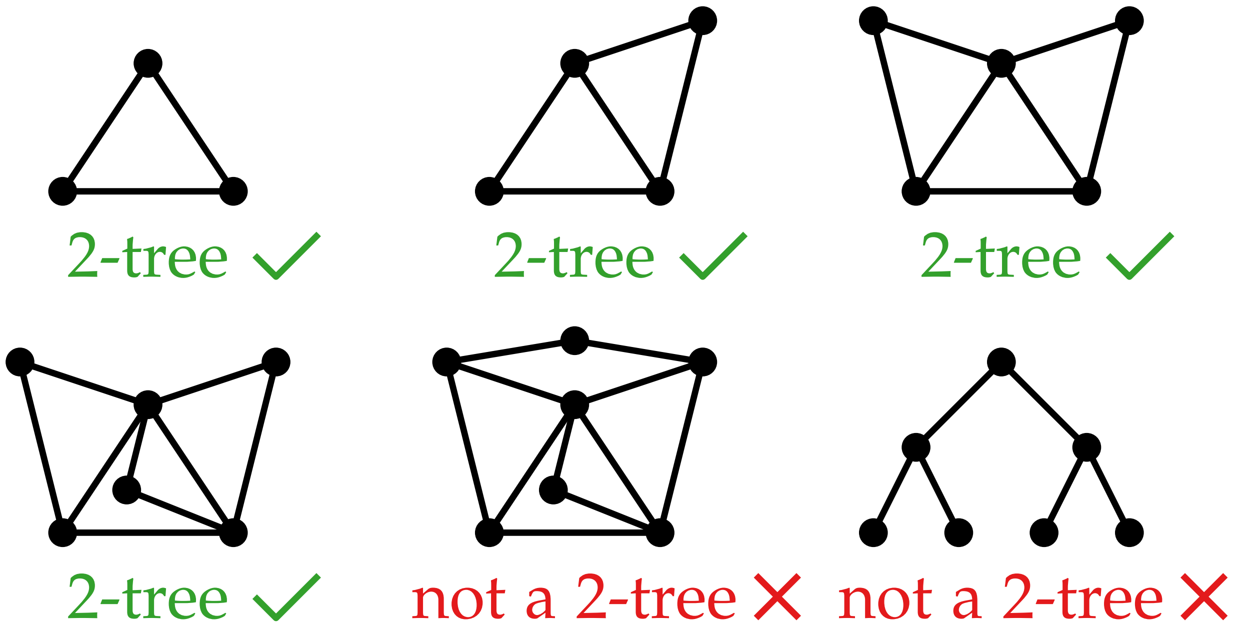

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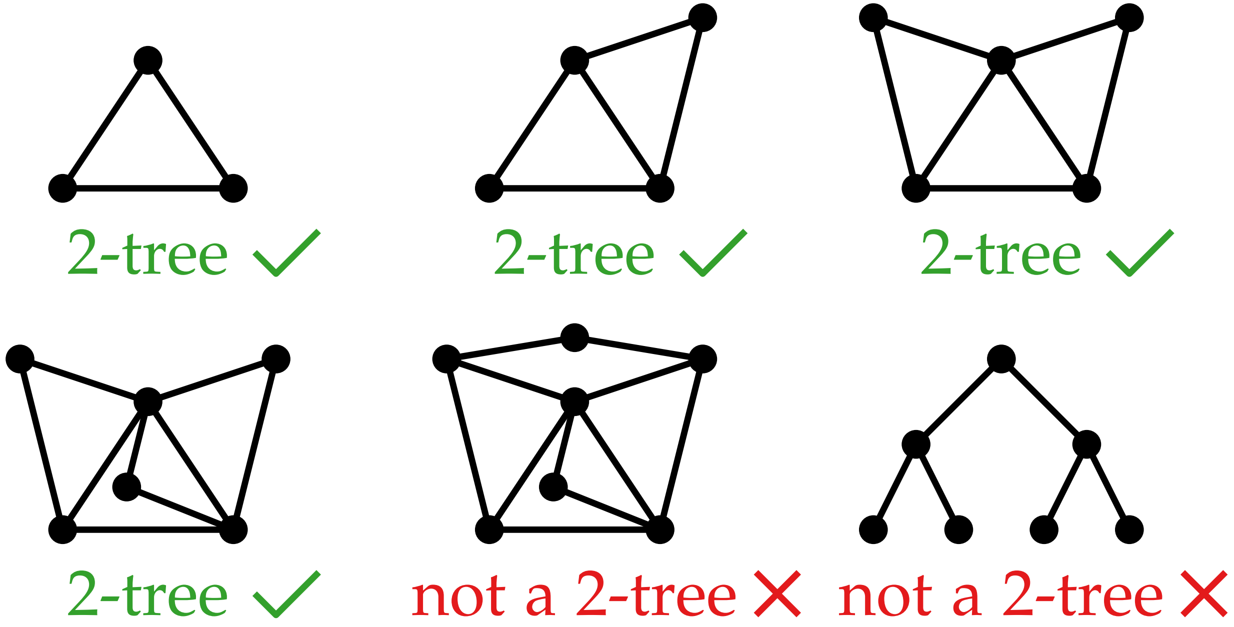

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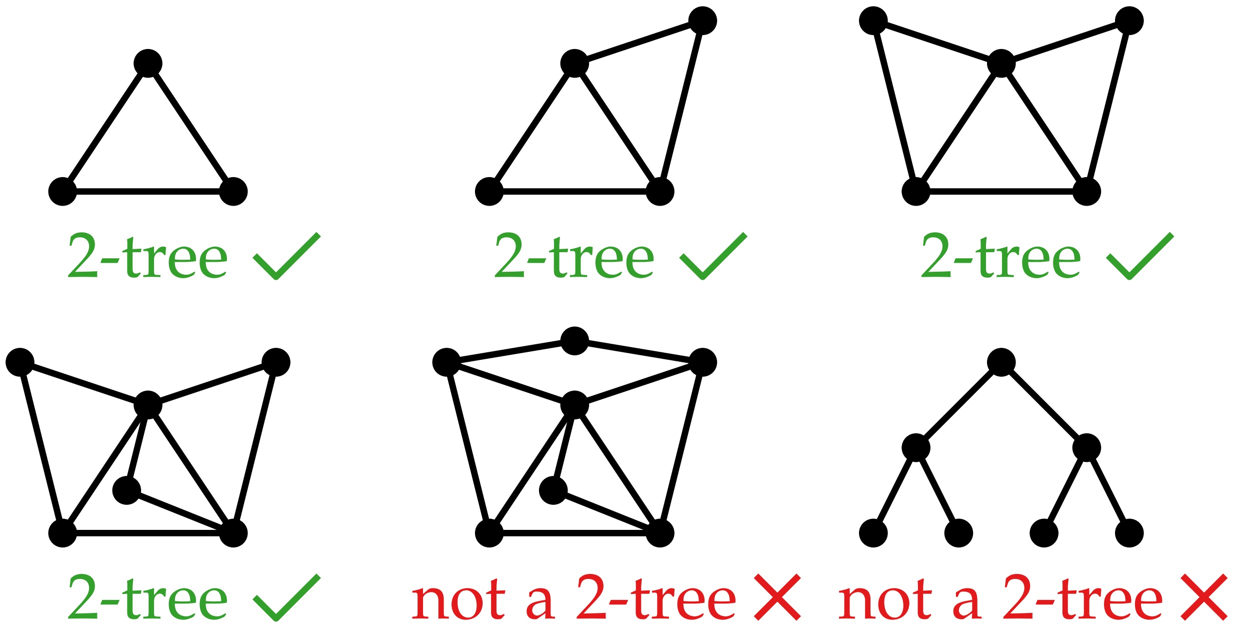

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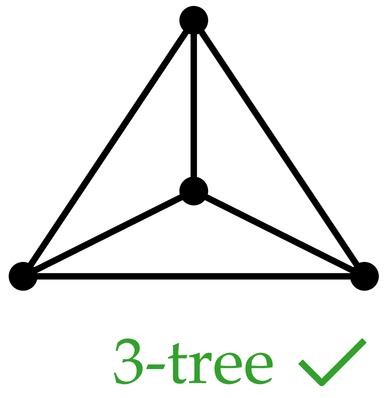

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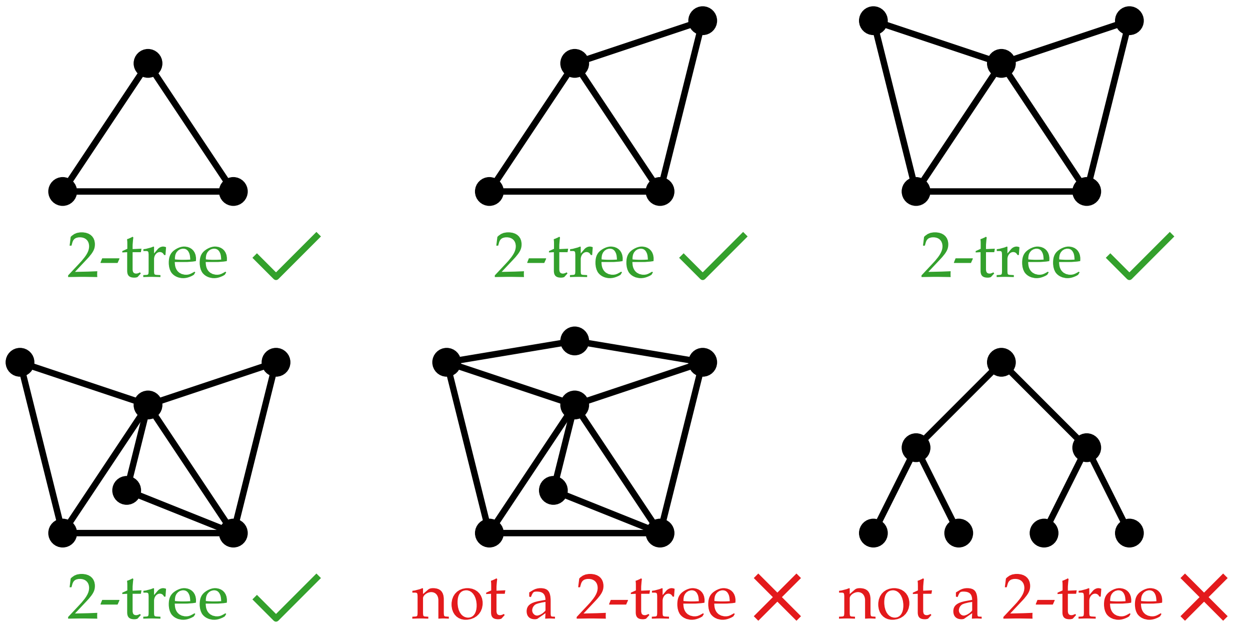

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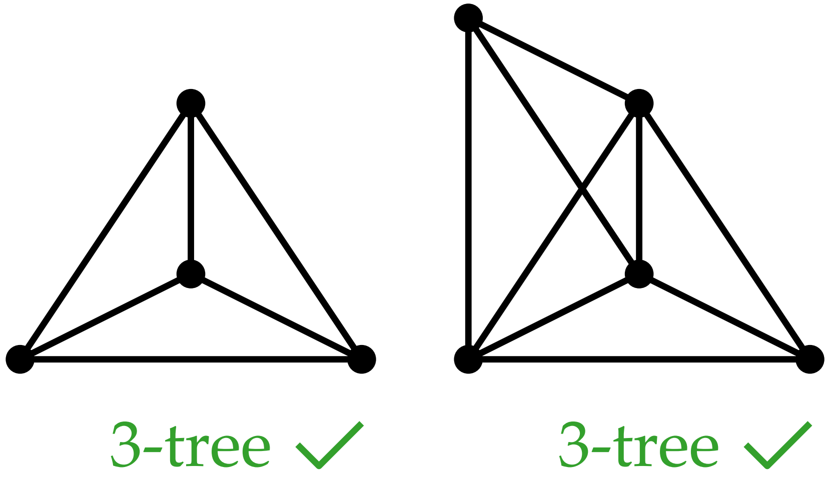

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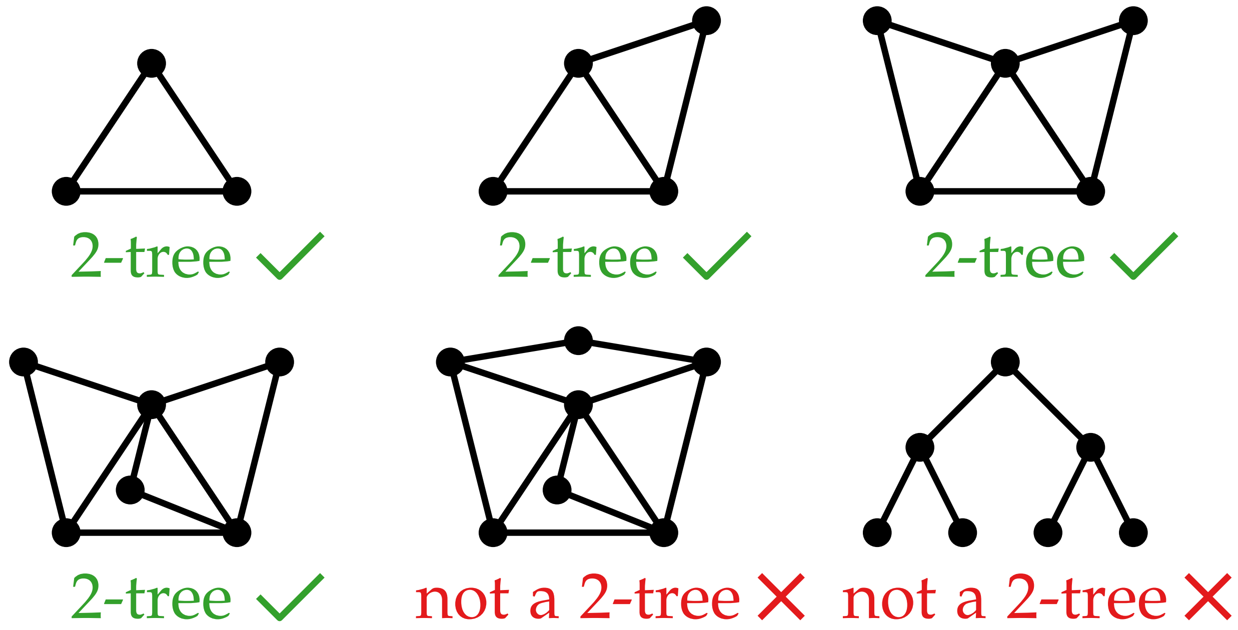

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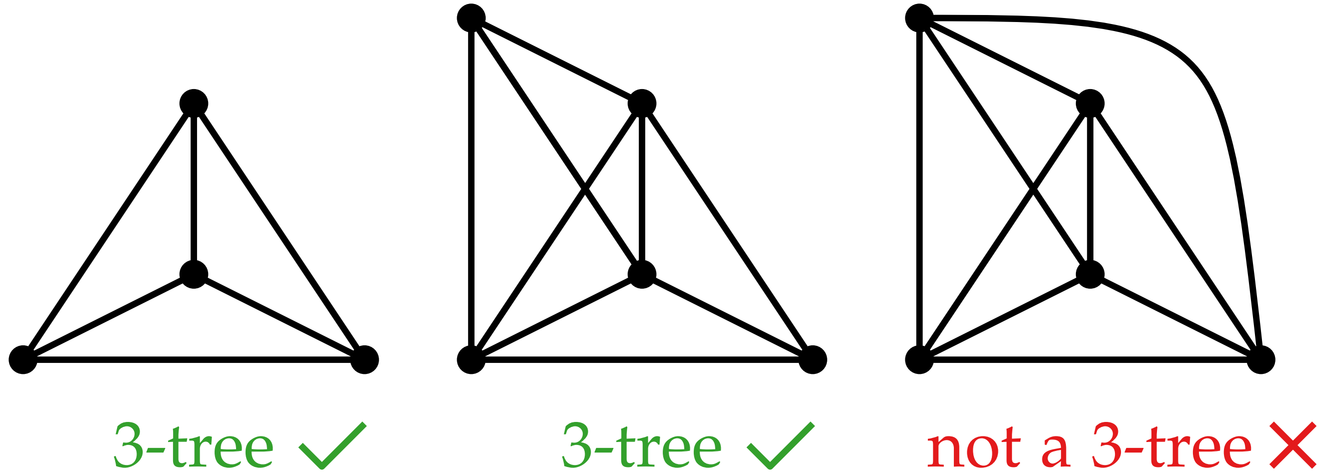

Definition: Outerplanar Graphs and Outerpaths

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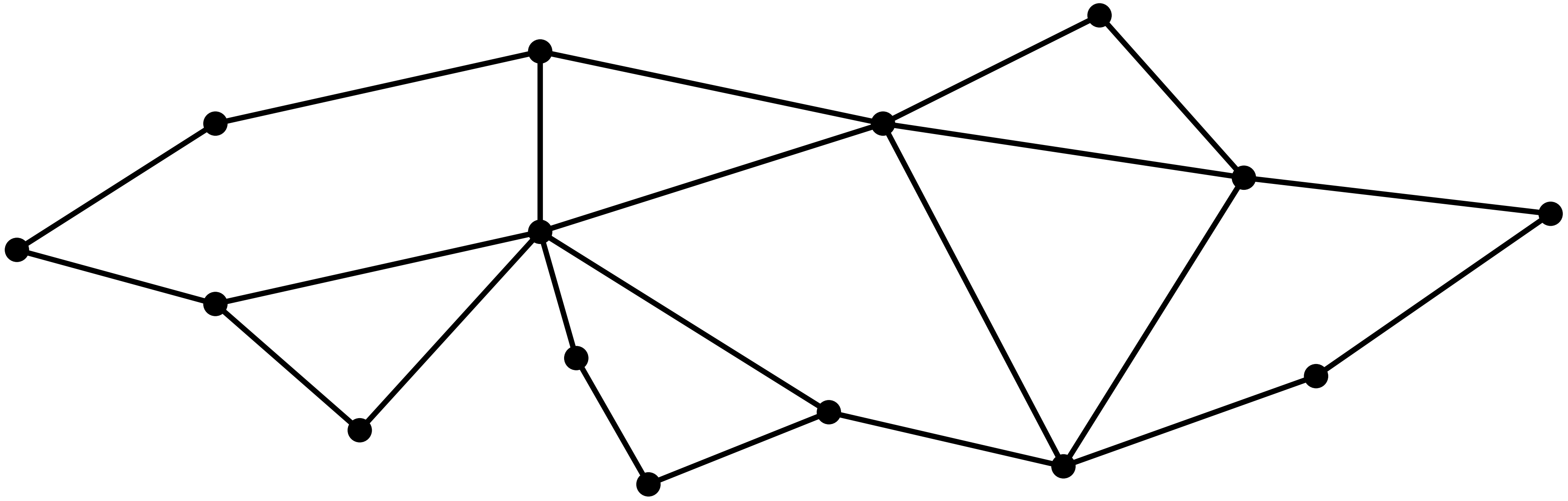
In an *outerplanar* graph drawing, all vertices lie on the outer face



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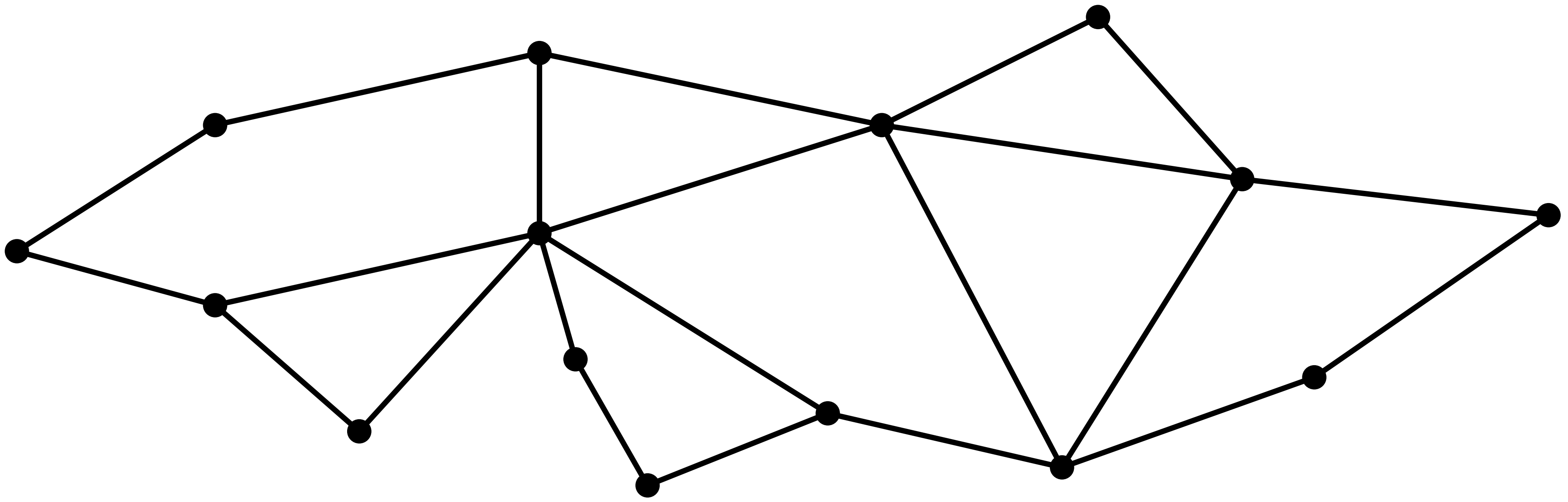
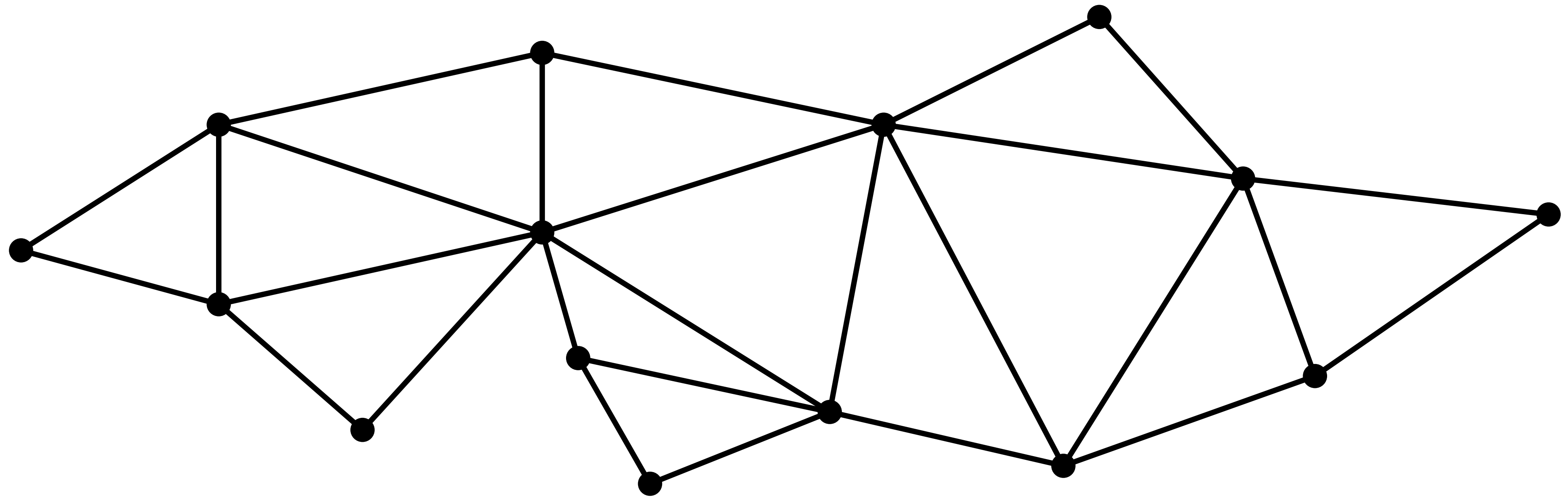


…and it is *maximal* if we cannot add an edge without violating outerplanarity.

Definition: Outerplanar Graphs and Outerpaths

In an *outerplanar* graph drawing, all vertices lie on the outer face



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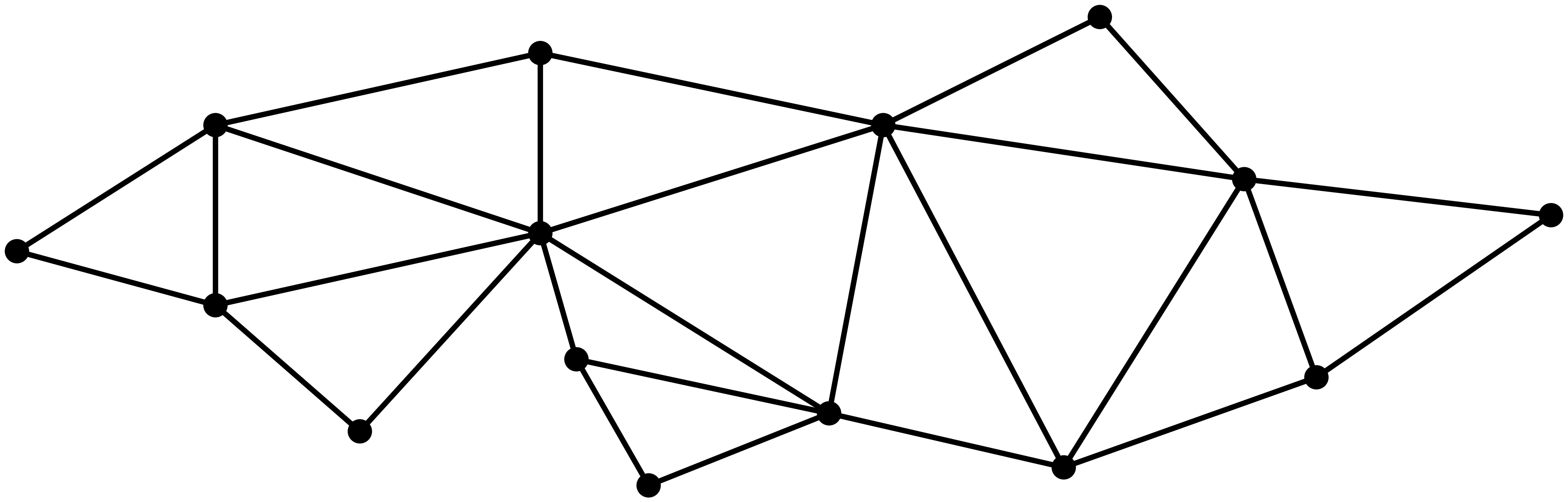

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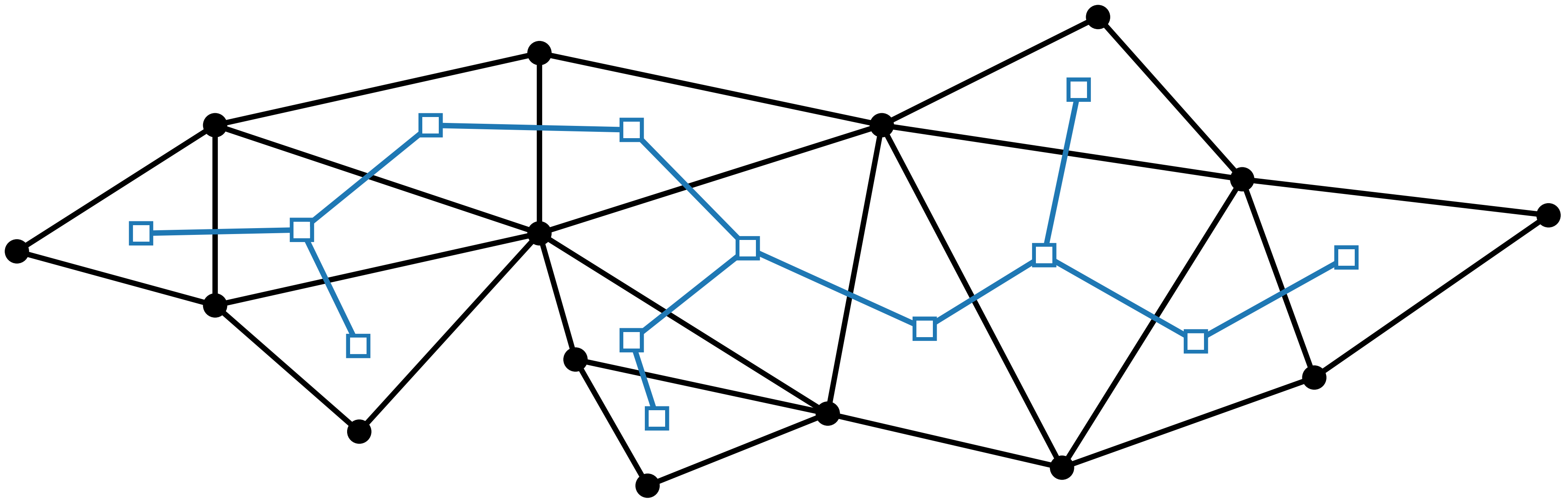

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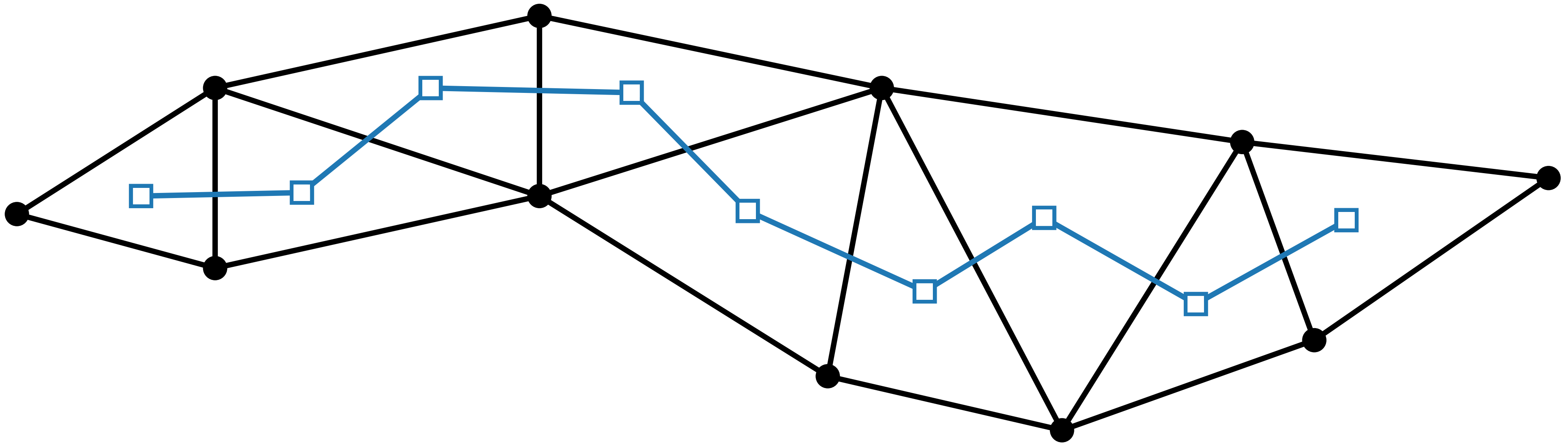

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If the weak dual is a path, the graph (drawing) is called an *outerpath*.

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Dujmović, Eppstein, Suderman, Wood
[CGTA '07] introduce the segment number

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planar 3-tree				$2n-2$
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existing results:

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- first universal lower bounds for ...

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planar 3-tree				$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$

existing results:

Contribution

- first universal lower bounds for ...
  - maximal outerpaths

seg(G)	universal lower bound	existential upper bound	existential lower bound	universal upper bound
planar conn.	$\nu/2$	$\nu/2$	$2n-2$	$\frac{8n-14}{3}$
tree	$\nu/2$	$\nu/2$	$\nu/2$	$\nu/2$
maximal outerpath	$\lfloor n/2 \rfloor + 2$		n	n
maximal outerplanar			n	n
2-tree			$3n/2-2$	$3n/2$
planar 3-tree				$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$

existing results:

new results:

Contribution

- first universal lower bounds for ...
  - maximal outerpaths
  - max. outerplanar graphs & 2-trees

seg(G)	universal lower bound	existential upper bound	existential lower bound	universal upper bound
planar conn.	$v/2$	$v/2$	$2n-2$	$\frac{8n-14}{3}$
tree	$v/2$	$v/2$	$v/2$	$v/2$
maximal outerpath	$\lfloor n/2 \rfloor + 2$		n	n
maximal outerplanar	$(n+7)/5$		n	n
2-tree	$(n+7)/5$		$3n/2-2$	$3n/2$
planar 3-tree				$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$

existing results:

new results:

Contribution

- first universal lower bounds for ...
  - maximal outerpaths
  - max. outerplanar graphs & 2-trees
  - planar 3-trees

seg(G)	universal lower bound	existential upper bound	existential lower bound	universal upper bound
planar conn.	$v/2$	$v/2$	$2n-2$	$\frac{8n-14}{3}$
tree	$v/2$	$v/2$	$v/2$	$v/2$
maximal outerpath	$\lfloor n/2 \rfloor + 2$		n	n
maximal outerplanar	$(n+7)/5$		n	n
2-tree	$(n+7)/5$		$3n/2-2$	$3n/2$
planar 3-tree	$n+4$			$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$

existing results:

new results:

Contribution

- first universal lower bounds for ...
  - maximal outerpaths
  - max. outerplanar graphs & 2-trees
  - planar 3-trees
- ... and examples using few segments

seg(G)	universal lower bound	existential upper bound	existential lower bound	universal upper bound
planar conn.	$v/2$	$v/2$	$2n-2$	$\frac{8n-14}{3}$
tree	$v/2$	$v/2$	$v/2$	$v/2$
maximal outerpath	$\lfloor n/2 \rfloor + 2$		n	n
maximal outerplanar	$(n+7)/5$		n	n
2-tree	$(n+7)/5$		$3n/2-2$	$3n/2$
planar 3-tree	$n+4$			$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$

existing results:

new results:

Contribution

- first universal lower bounds for ...
  - maximal outerpaths
  - max. outerplanar graphs & 2-trees
  - planar 3-trees
- ... and examples using few segments

seg(G)	universal lower bound	existential upper bound	existential lower bound	universal upper bound
planar conn.	$v/2$	$v/2$	$2n-2$	$\frac{8n-14}{3}$
tree	$v/2$	$v/2$	$v/2$	$v/2$
maximal outerpath	$\lfloor n/2 \rfloor + 2$	$\lfloor n/2 \rfloor + 2$	n	n
maximal outerplanar	$(n+7)/5$	$\frac{5n+24}{13}$	n	n
2-tree	$(n+7)/5$	$\frac{5n+24}{13}$	$3n/2-2$	$3n/2$
planar 3-tree	$n+4$	$n+7$		$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$

existing results:

new results:

Contribution

- first universal lower bounds for …
  - maximal outerpaths
  - max. outerplanar graphs & 2-trees
  - planar 3-trees
- … and examples using few segments
- (near) optimal algorithms and bounds for …

seg(G)	universal lower bound	existential upper bound	existential lower bound	universal upper bound
planar conn.	$v/2$	$v/2$	$2n-2$	$\frac{8n-14}{3}$
tree	$v/2$	$v/2$	$v/2$	$v/2$
maximal outerpath	$\lfloor n/2 \rfloor + 2$	$\lfloor n/2 \rfloor + 2$	n	n
maximal outerplanar	$(n+7)/5$	$\frac{5n+24}{13}$	n	n
2-tree	$(n+7)/5$	$\frac{5n+24}{13}$	$3n/2-2$	$3n/2$
planar 3-tree	$n+4$	$n+7$		$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$

existing results: [light green] [light yellow]

new results: [green] [yellow]

Contribution

- first universal lower bounds for ...
  - maximal outerpaths
  - max. outerplanar graphs & 2-trees
  - planar 3-trees
- ... and examples using few segments
- (near) optimal algorithms and bounds for ...
  - planar 3-connected 4-regular graphs

seg(G)	universal lower bound	existential upper bound	existential lower bound	universal upper bound
planar conn.	$v/2$	$v/2$	$2n-2$	$\frac{8n-14}{3}$
tree	$v/2$	$v/2$	$v/2$	$v/2$
maximal outerpath	$\lfloor n/2 \rfloor + 2$	$\lfloor n/2 \rfloor + 2$	n	n
maximal outerplanar	$(n+7)/5$	$\frac{5n+24}{13}$	n	n
2-tree	$(n+7)/5$	$\frac{5n+24}{13}$	$3n/2-2$	$3n/2$
planar 3-tree	$n+4$	$n+7$		$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$
planar 3-conn. 4-reg.	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	n	$n+3$

existing results:

new results:

Contribution

- first universal lower bounds for ...
  - maximal outerpaths
  - max. outerplanar graphs & 2-trees
  - planar 3-trees
- ... and examples using few segments
- (near) optimal algorithms and bounds for ...
  - planar 3-connected 4-regular graphs
  - cactus graphs

seg(G)	universal lower bound	existential upper bound	existential lower bound	universal upper bound
planar conn.	$v/2$	$v/2$	$2n-2$	$\frac{8n-14}{3}$
tree	$v/2$	$v/2$	$v/2$	$v/2$
maximal outerpath	$\lfloor n/2 \rfloor + 2$	$\lfloor n/2 \rfloor + 2$	n	n
maximal outerplanar	$(n+7)/5$	$\frac{5n+24}{13}$	n	n
2-tree	$(n+7)/5$	$\frac{5n+24}{13}$	$3n/2-2$	$3n/2$
planar 3-tree	$n+4$	$n+7$		$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$
planar 3-conn. 4-reg.	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	n	$n+3$
cactus	$v/2+\gamma$	$v/2+\gamma$	$v/2+\gamma$	$v/2+\gamma$

existing results:

new results:

Contribution

- first universal lower bounds for ...
  - maximal outerpaths
  - max. outerplanar graphs & 2-trees
  - planar 3-trees
- ... and examples using few segments
- (near) optimal algorithms and bounds for ...
  - planar 3-connected 4-regular graphs
  - cactus graphs

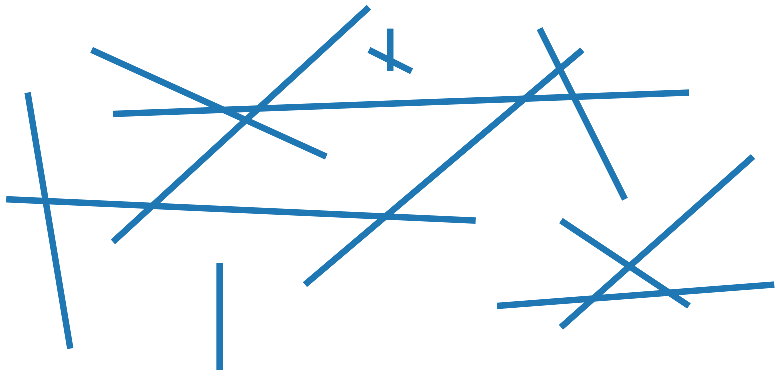
in this talk

seg(G)	universal lower bound	existential upper bound	existential lower bound	universal upper bound
planar conn.	$v/2$	$v/2$	$2n-2$	$\frac{8n-14}{3}$
tree	$v/2$	$v/2$	$v/2$	$v/2$
maximal outerpath	$\lfloor n/2 \rfloor + 2$	$\lfloor n/2 \rfloor + 2$	n	n
maximal outerplanar	$(n+7)/5$	$\frac{5n+24}{13}$	n	n
2-tree	$(n+7)/5$	$\frac{5n+24}{13}$	$3n/2-2$	$3n/2$
planar 3-tree	$n+4$	$n+7$		$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$
planar 3-conn. 4-reg.	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	n	$n+3$
cactus	$v/2+\gamma$	$v/2+\gamma$	$v/2+\gamma$	$v/2+\gamma$

existing results:

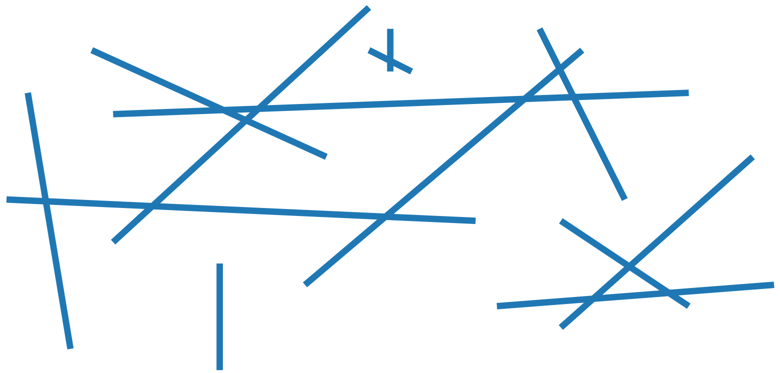
new results:

Generalizing Line Segments and Circular Arcs



arrangement of
line segments

Generalizing Line Segments and Circular Arcs



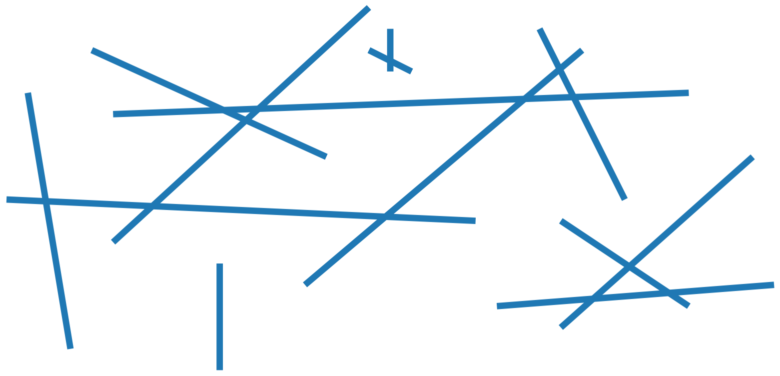
arrangement of
line segments

⇓

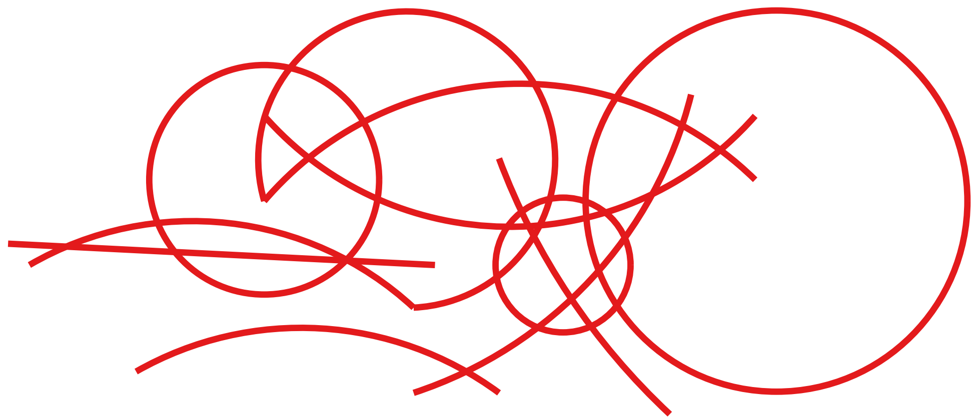
arrangement of
pseudo segments

($\leq$ 1 crossing per pair of curves)

Generalizing Line Segments and Circular Arcs

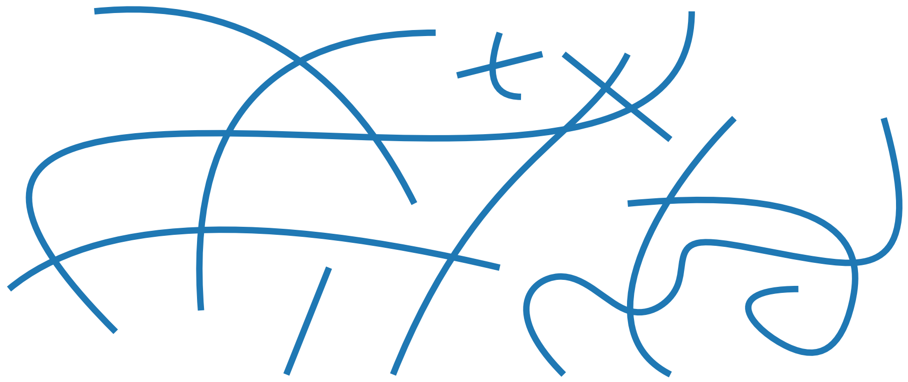


arrangement of
line segments



arrangement of
circular arcs

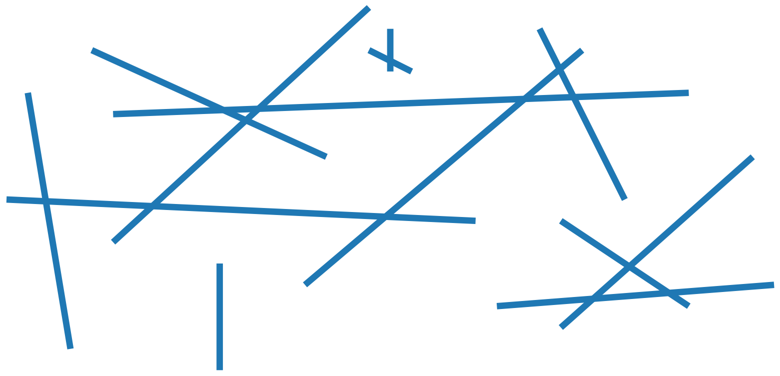
⇓



arrangement of
pseudo segments

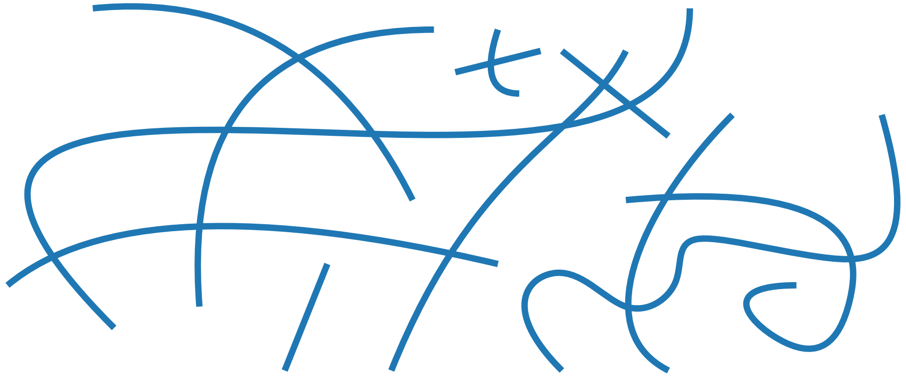
(≤ 1 crossing per pair of curves)

Generalizing Line Segments and Circular Arcs



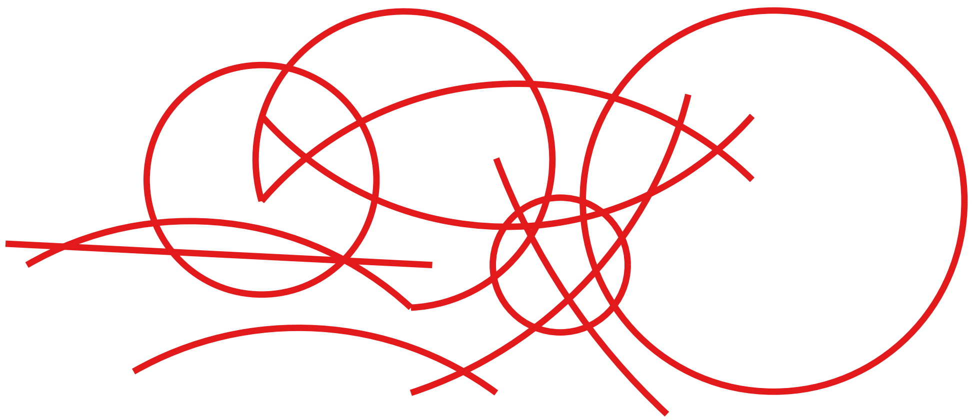
arrangement of
line segments

$\Downarrow$



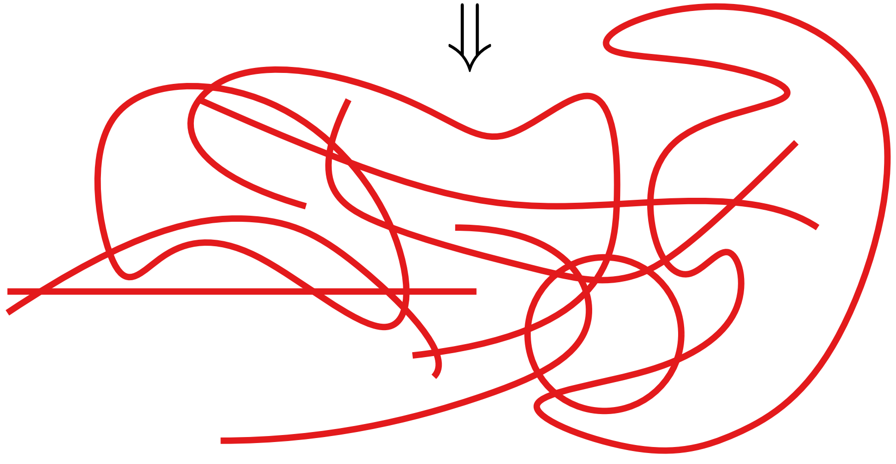
arrangement of
pseudo segments

($\leq$ 1 crossing per pair of curves)



arrangement of
circular arcs

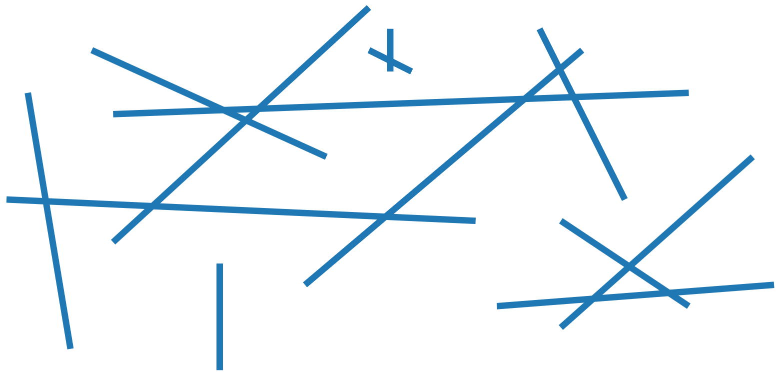
$\Downarrow$



arrangement of
pseudo arcs

($\leq$ 2 crossings per pair of curves)

Generalizing Line Segments and Circular Arcs

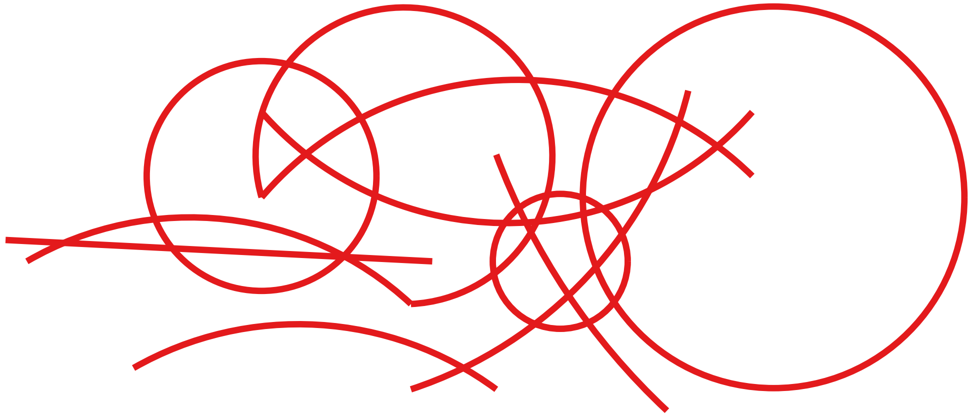


arrangement of
line segments

⇓

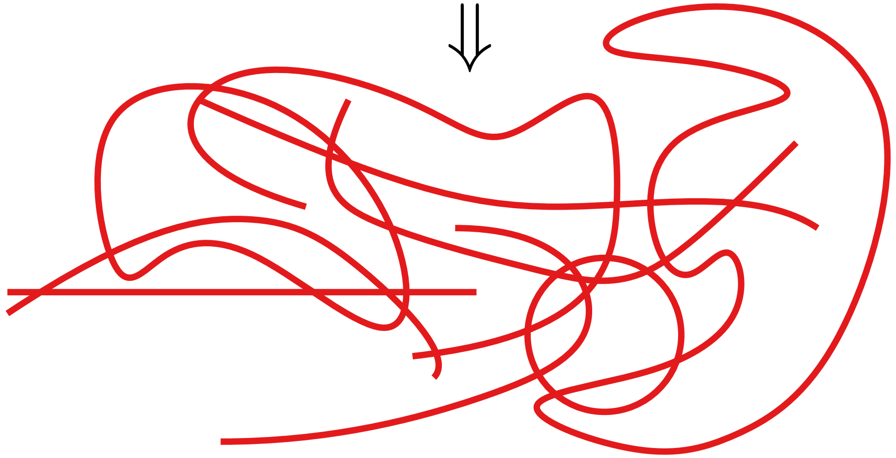
arrangement of
pseudo segments

(≤ 1 crossing per pair of curves)



arrangement of
circular arcs

⇓



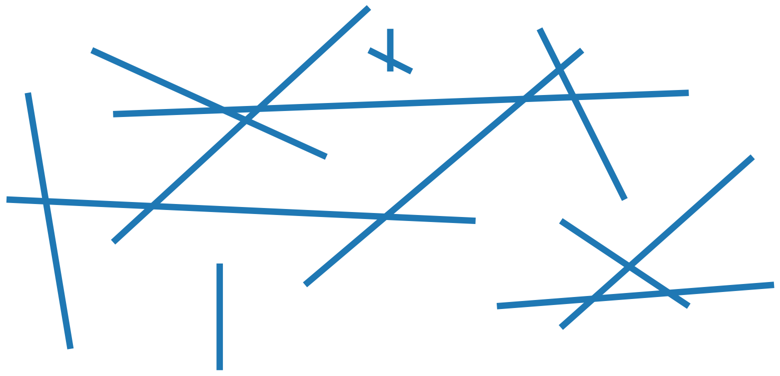
arrangement of
pseudo arcs

(≤ 2 crossings per pair of curves)

arrangement of
pseudo k-arcs

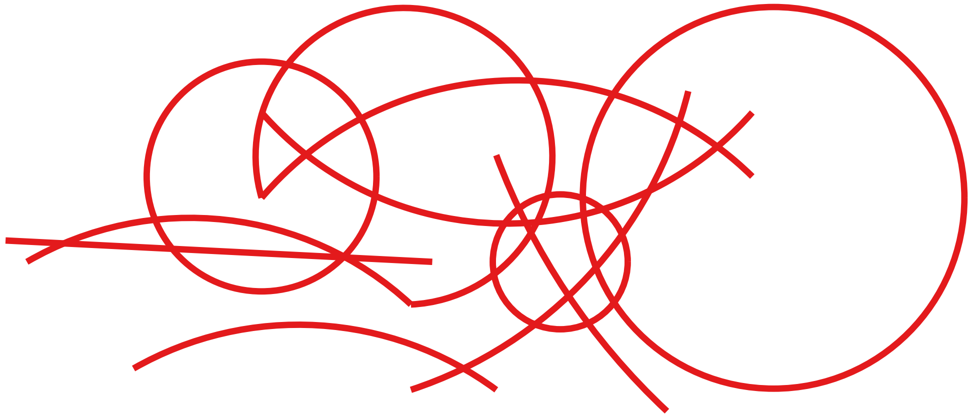
(≤ k crossings per pair of curves)

Generalizing Line Segments and Circular Arcs



arrangement of
line segments

$\Downarrow$

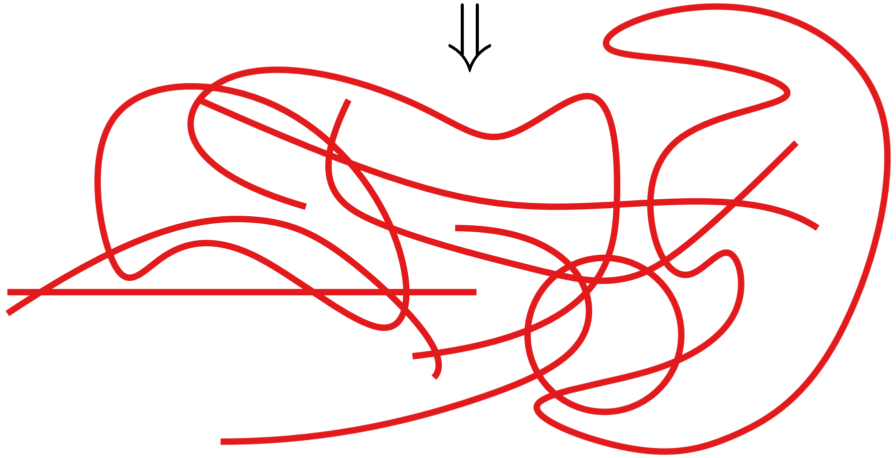


arrangement of
circular arcs

$\Downarrow$

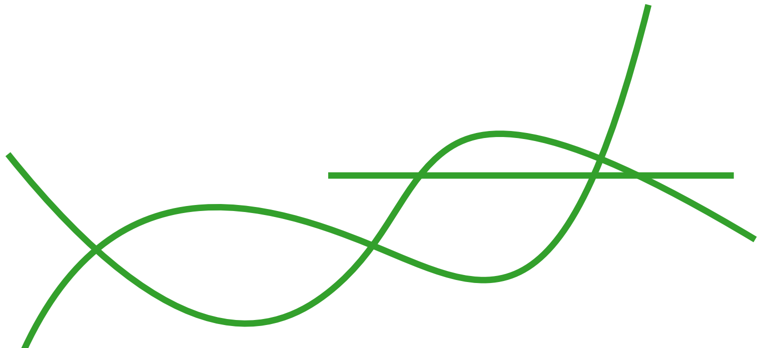
arrangement of
pseudo segments

(≤ 1 crossing per pair of curves)



arrangement of
pseudo arcs

(≤ 2 crossings per pair of curves)

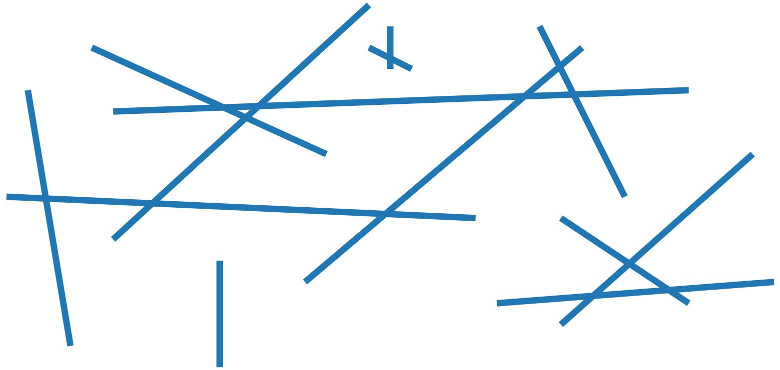


$k = 3$

arrangement of
pseudo k-arcs

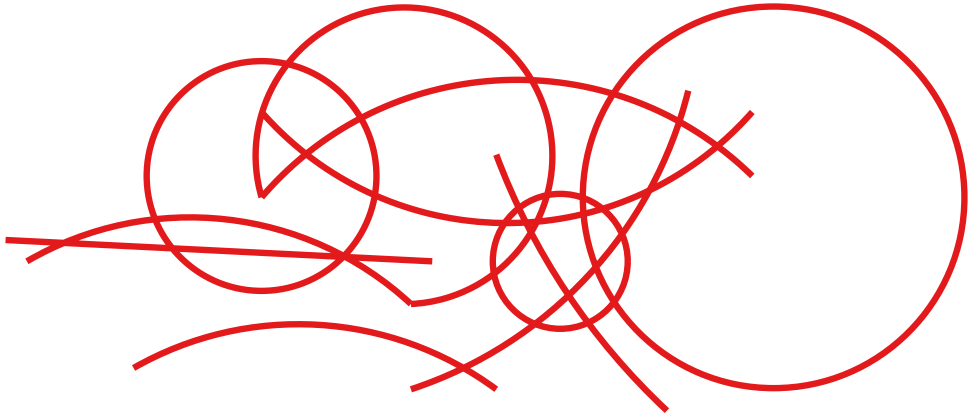
($\leq k$ crossings per pair of curves)

Generalizing Line Segments and Circular Arcs



arrangement of
line segments

$\Downarrow$



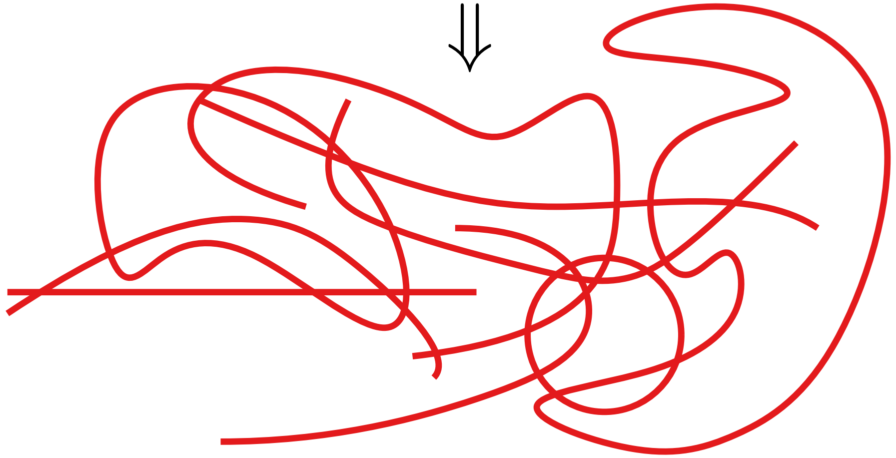
arrangement of
circular arcs

$\Downarrow$

$k = 6$

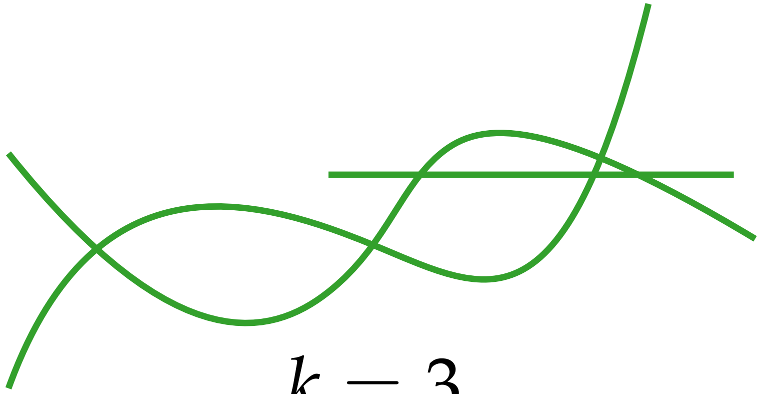
arrangement of
pseudo segments

(≤ 1 crossing per pair of curves)



arrangement of
pseudo arcs

(≤ 2 crossings per pair of curves)



$k = 3$

arrangement of
pseudo k-arcs

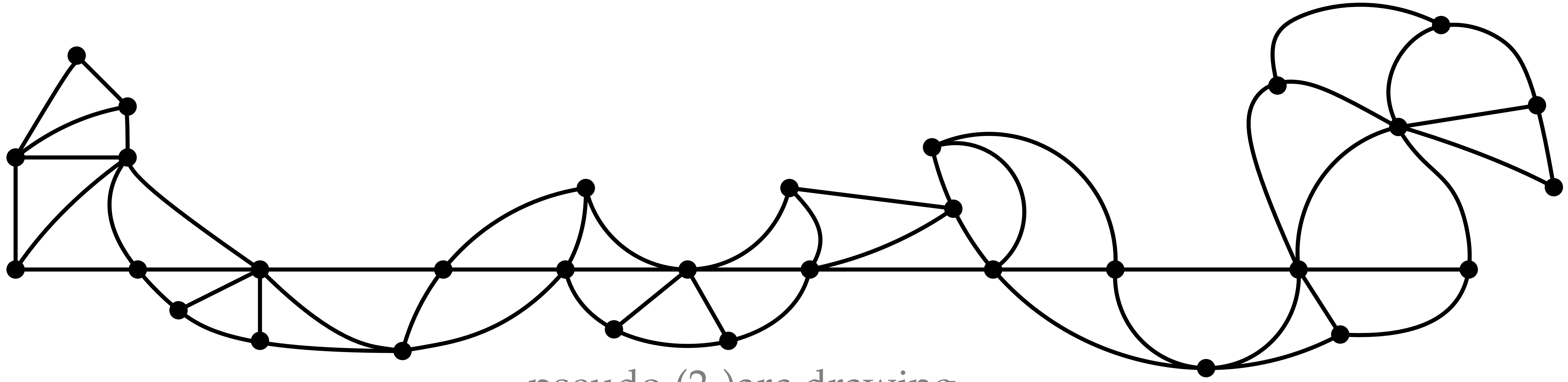
($\leq k$ crossings per pair of curves)

Maximal Outerpaths

Given: planar pseudo k-arc drawing Γ of a maximal outerpath G with n vertices

Maximal Outerpaths

Given: planar pseudo k-arc drawing Γ of a maximal outerpath G with n vertices

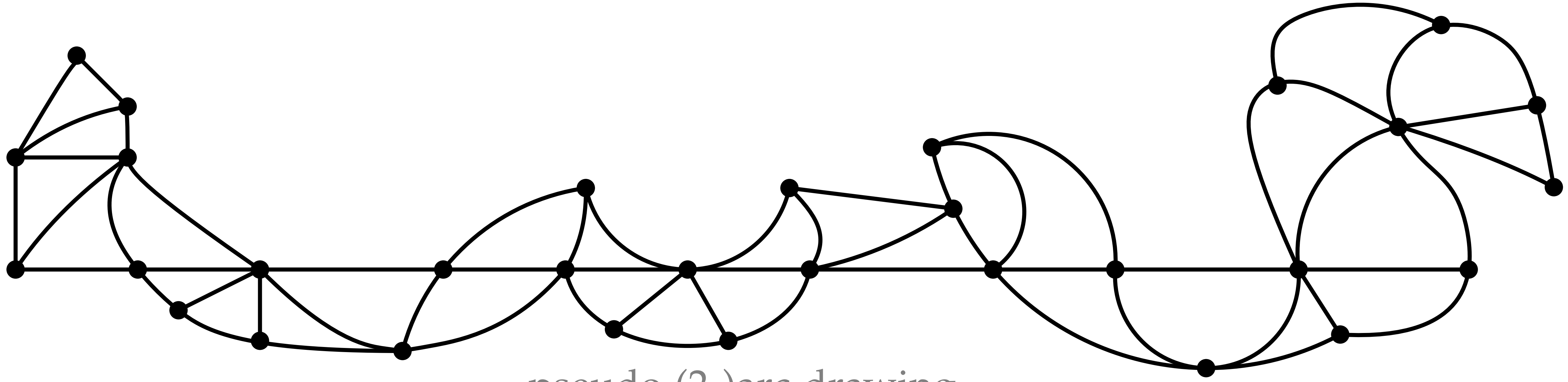


pseudo (2-)arc drawing

Maximal Outerpaths

Given: planar pseudo k-arc drawing Γ of a maximal outerpath G with n vertices

Show: Γ has at least $f(k, n)$ pseudo k-arcs



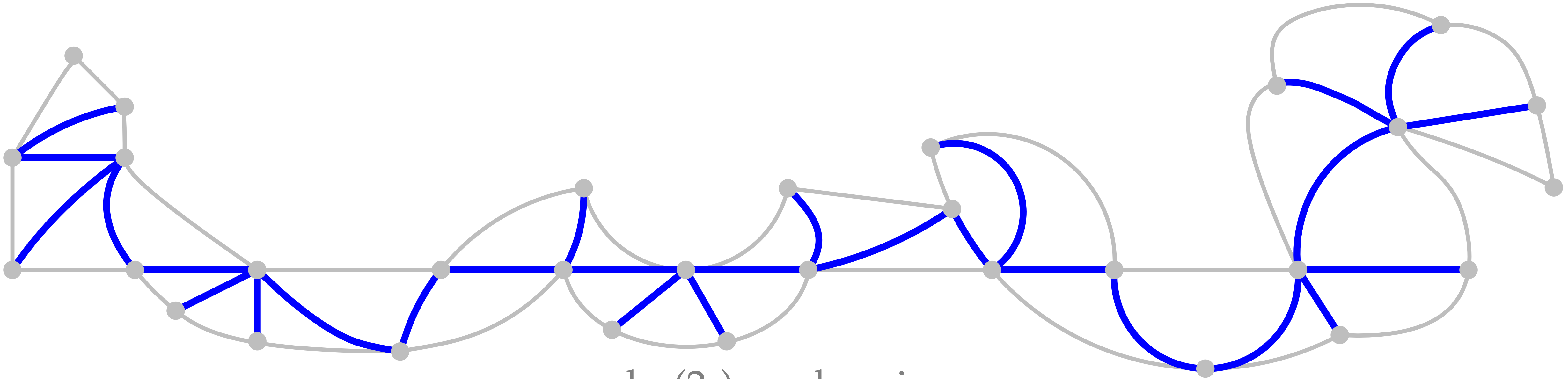
pseudo (2-)arc drawing

Maximal Outerpaths

Given: planar pseudo k-arc drawing Γ of a maximal outerpath G with n vertices

Show: Γ has at least $f(k, n)$ pseudo k-arcs

Proof Idea: G has $n - 3$ inner edges



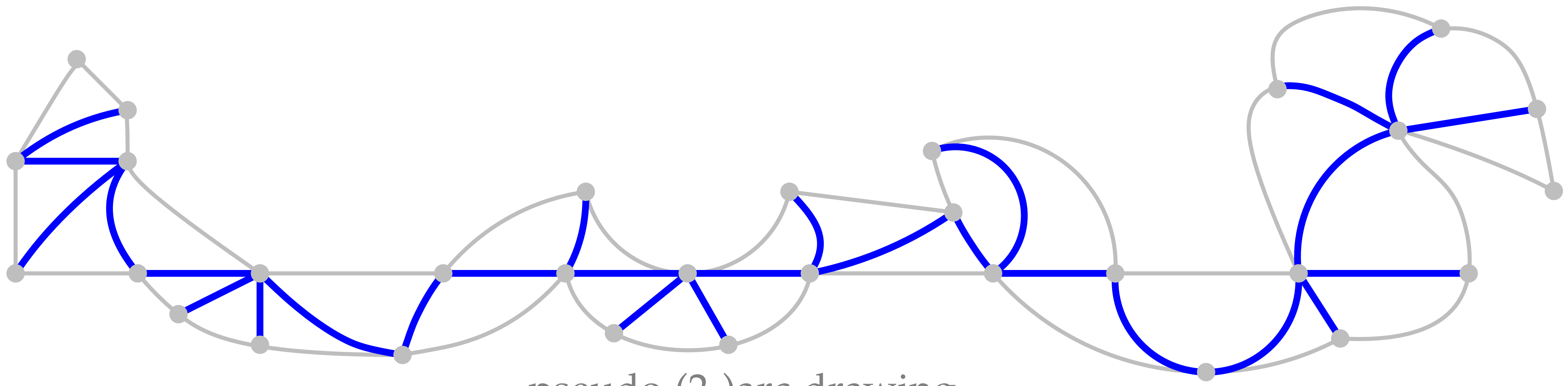
pseudo (2-)arc drawing

Maximal Outerpaths

Given: planar pseudo k-arc drawing Γ of a maximal outerpath G with n vertices

Show: Γ has at least $f(k, n)$ pseudo k-arcs

Proof Idea: G has $n - 3$ inner edges $\rightarrow$ charge them to pseudo k-arcs



pseudo (2-)arc drawing

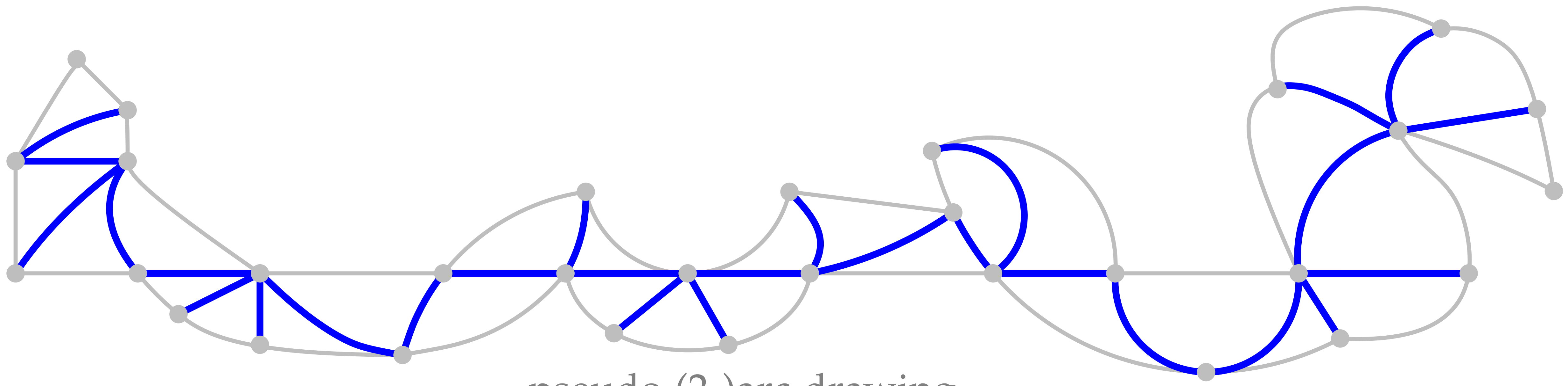
Maximal Outerpaths

Given: planar pseudo k-arc drawing Γ of a maximal outerpath G with n vertices

Show: Γ has at least $f(k, n)$ pseudo k-arcs

Proof Idea: G has $n - 3$ inner edges $\rightarrow$ charge them to pseudo k-arcs

Round-1 assignment: for each pseudo k-arc, charge its k first inner edges to itself



pseudo (2-)arc drawing

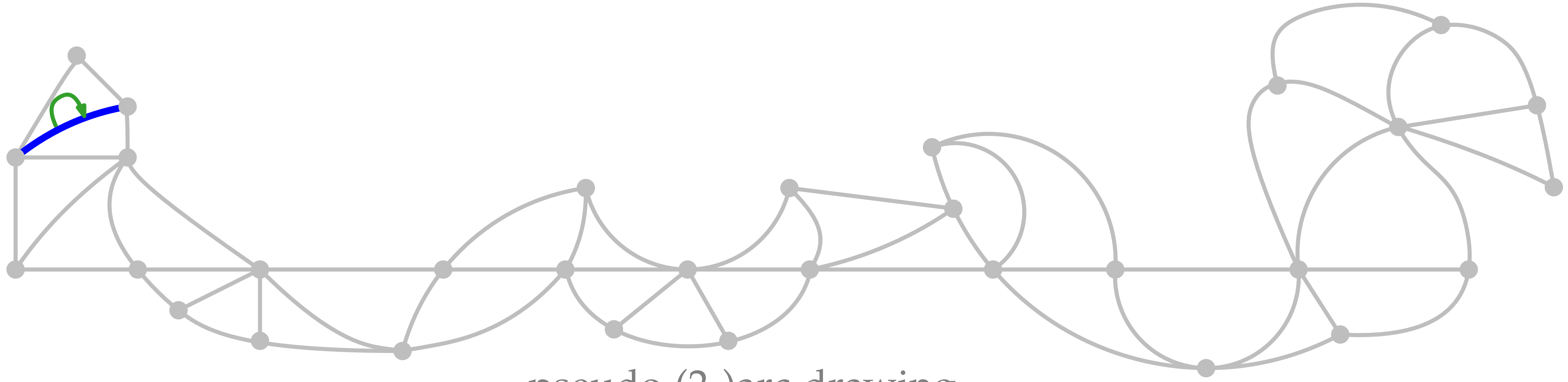
Maximal Outerpaths

Given: planar pseudo k-arc drawing Γ of a maximal outerpath G with n vertices

Show: Γ has at least $f(k, n)$ pseudo k-arcs

Proof Idea: G has $n - 3$ inner edges $\rightarrow$ charge them to pseudo k-arcs

Round-1 assignment: for each pseudo k-arc, charge its k first inner edges to itself



pseudo (2-)arc drawing

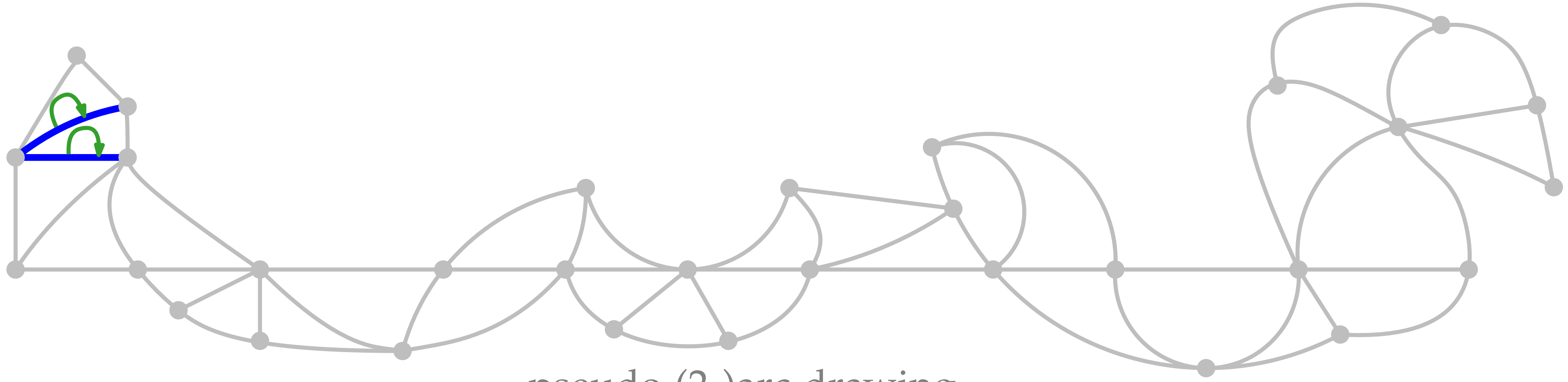
Maximal Outerpaths

Given: planar pseudo k-arc drawing Γ of a maximal outerpath G with n vertices

Show: Γ has at least $f(k,n)$ pseudo k-arcs

Proof Idea: G has $n-3$ inner edges $\rightarrow$ charge them to pseudo k-arcs

Round-1 assignment: for each pseudo k-arc, charge its k first inner edges to itself



pseudo (2-)arc drawing

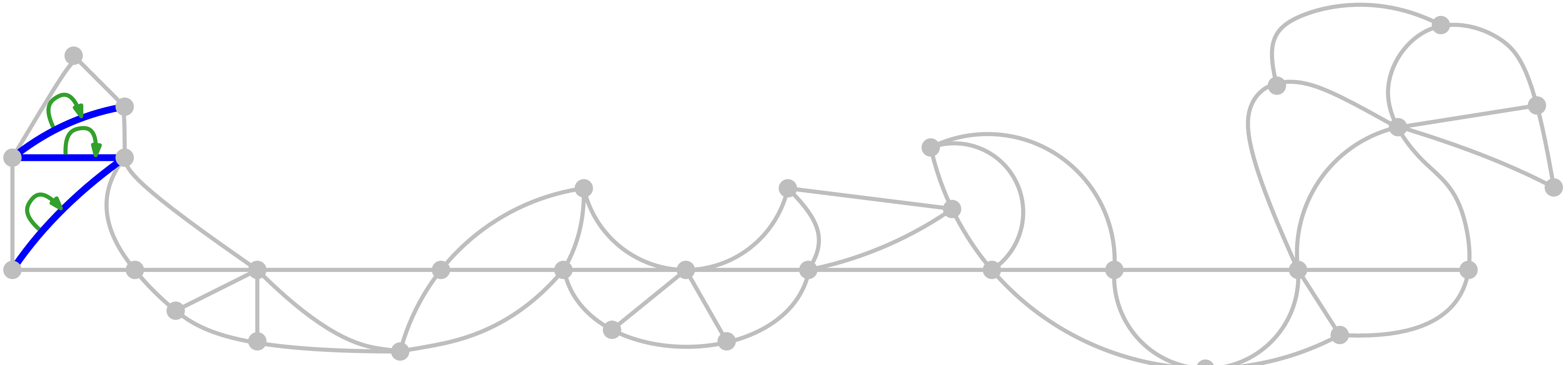
Maximal Outerpaths

Given: planar pseudo k-arc drawing Γ of a maximal outerpath G with n vertices

Show: Γ has at least $f(k, n)$ pseudo k-arcs

Proof Idea: G has $n - 3$ inner edges $\rightarrow$ charge them to pseudo k-arcs

Round-1 assignment: for each pseudo k-arc, charge its k first inner edges to itself



pseudo (2-)arc drawing

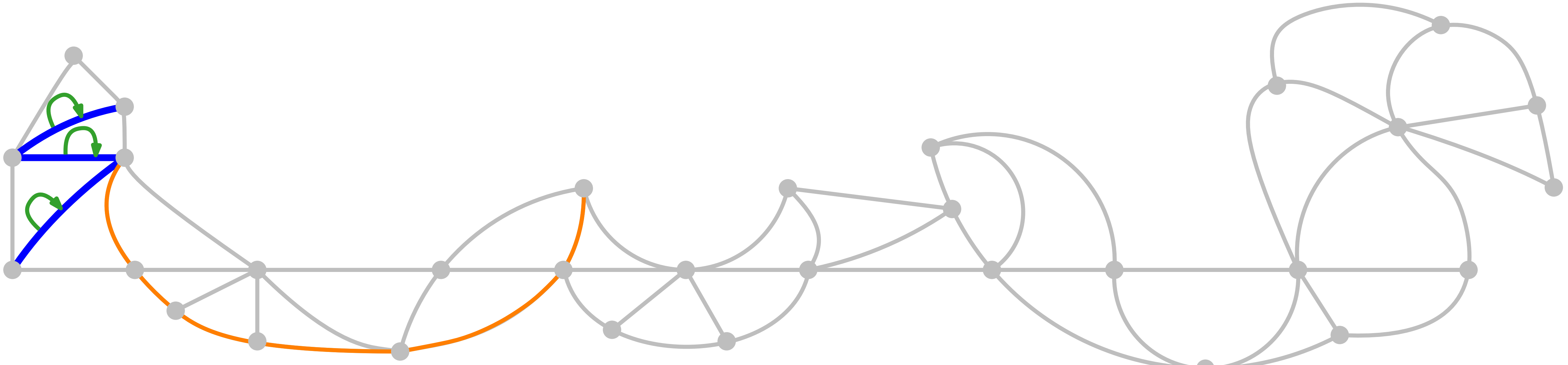
Maximal Outerpaths

Given: planar pseudo k-arc drawing Γ of a maximal outerpath G with n vertices

Show: Γ has at least $f(k, n)$ pseudo k-arcs

Proof Idea: G has $n - 3$ inner edges $\rightarrow$ charge them to pseudo k-arcs

Round-1 assignment: for each pseudo k-arc, charge its k first inner edges to itself



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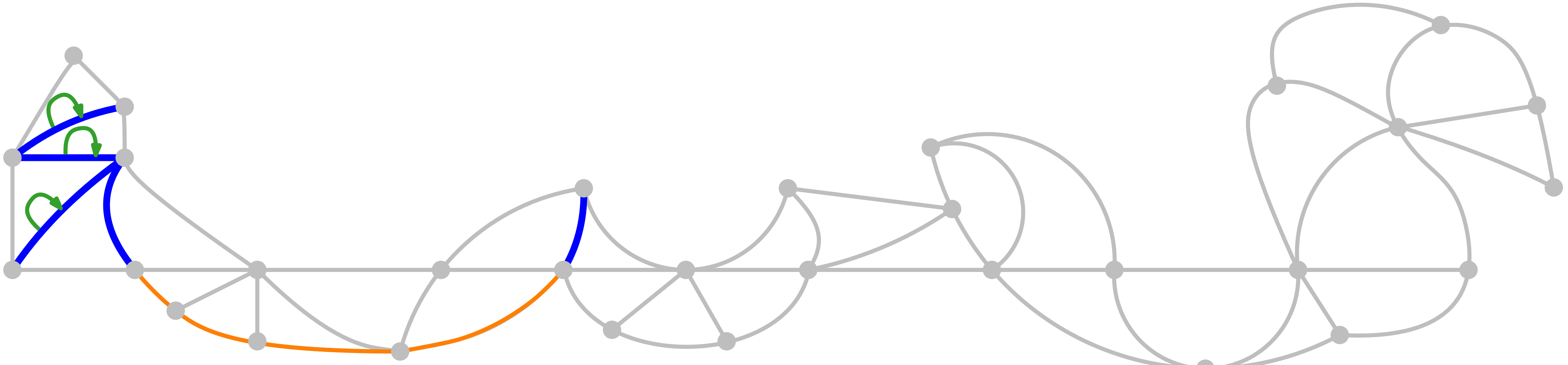

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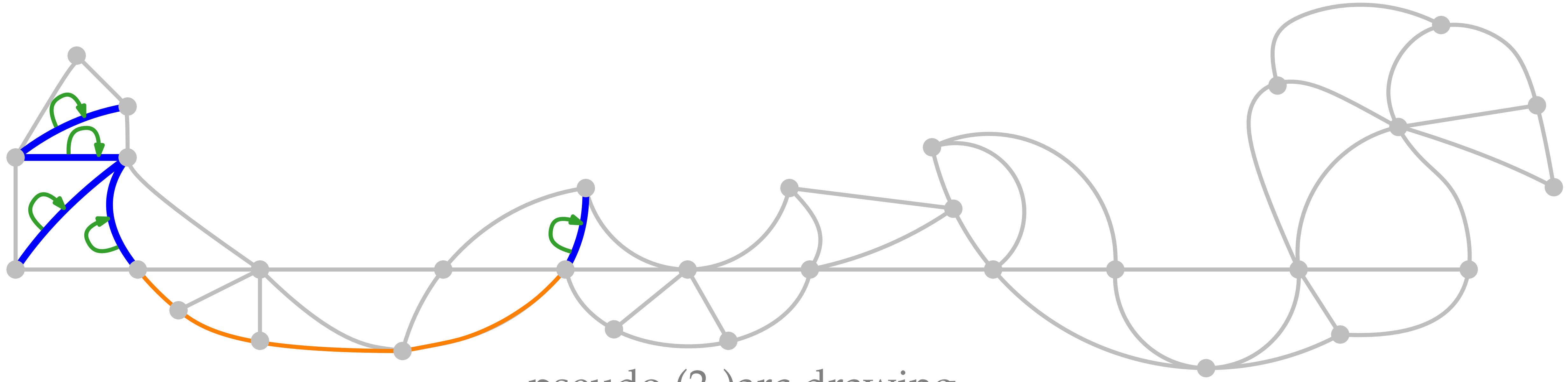

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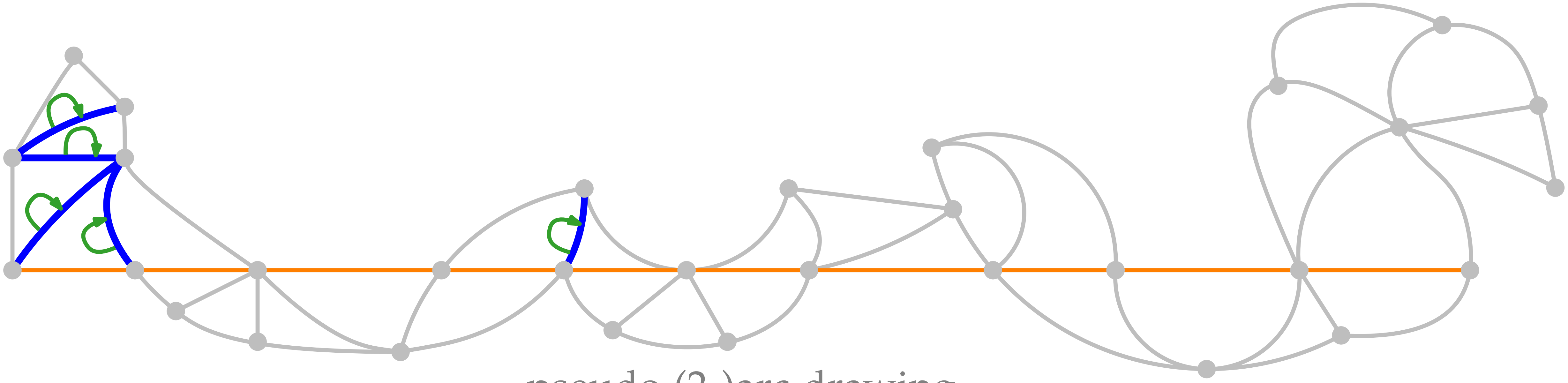

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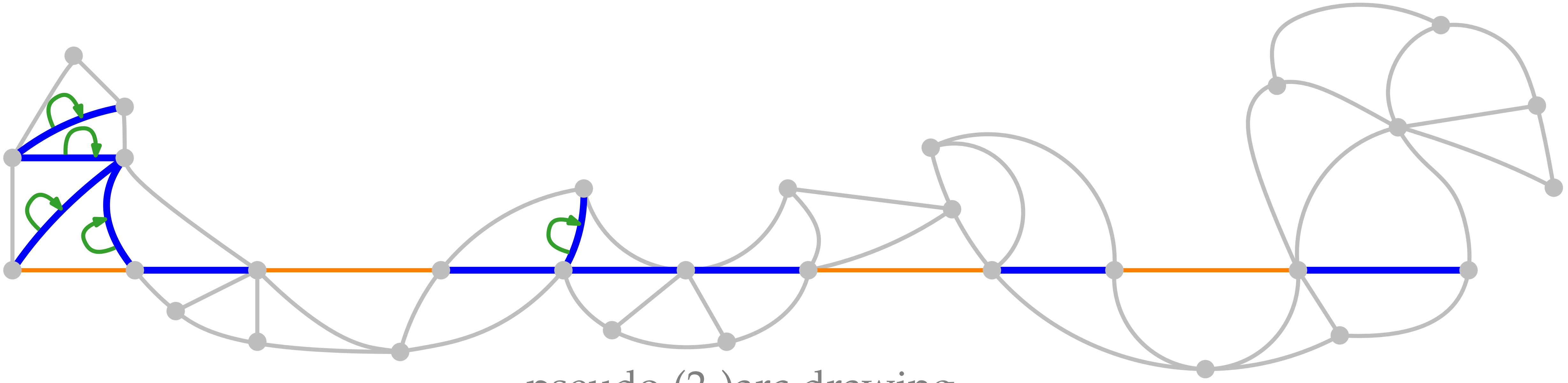

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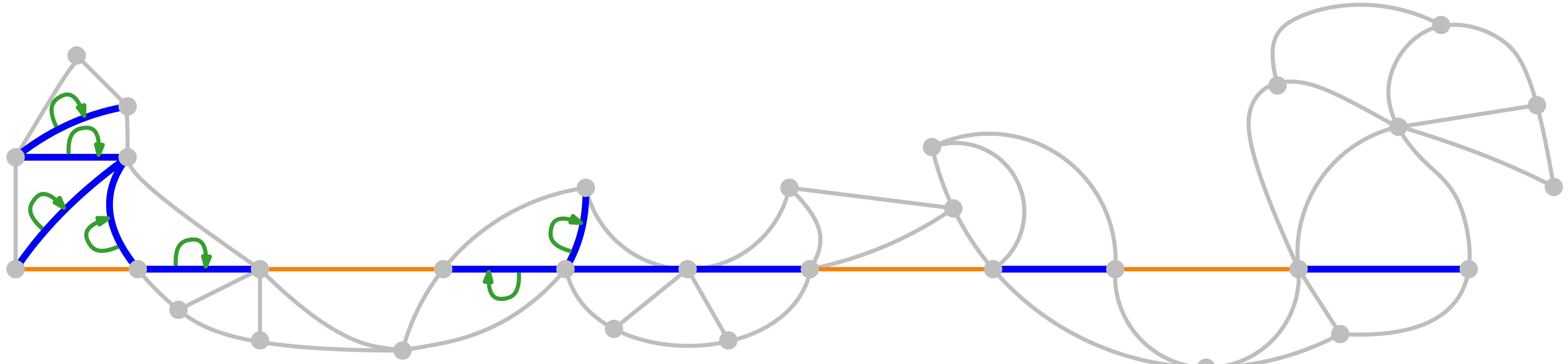

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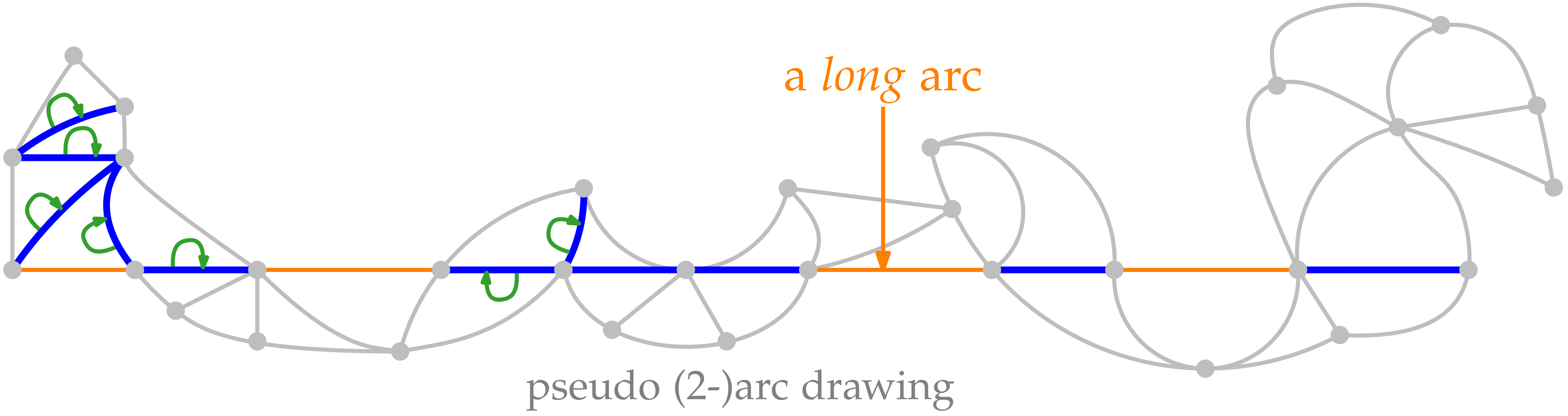

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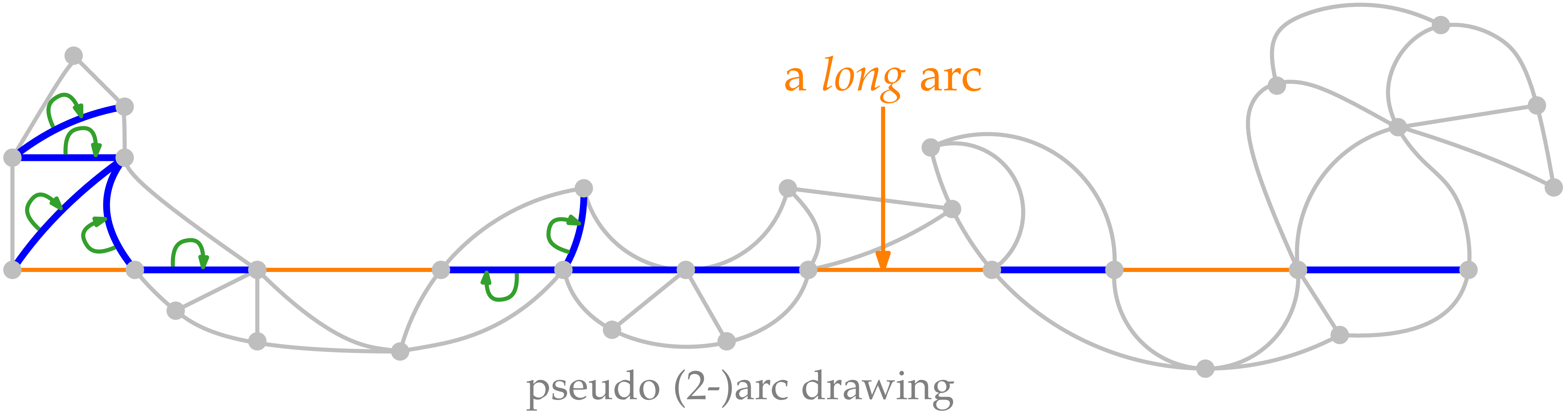

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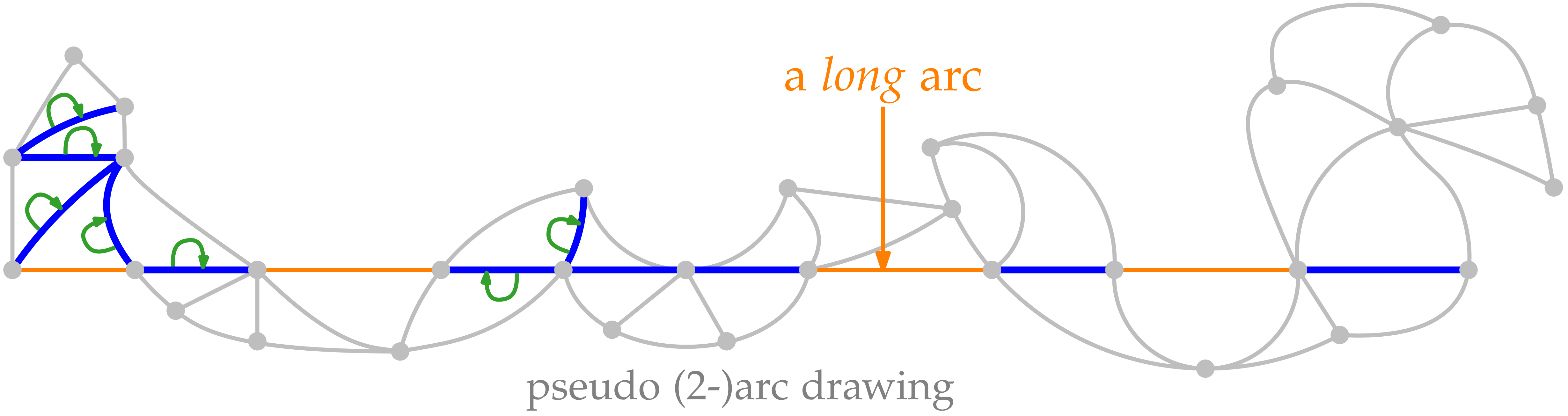

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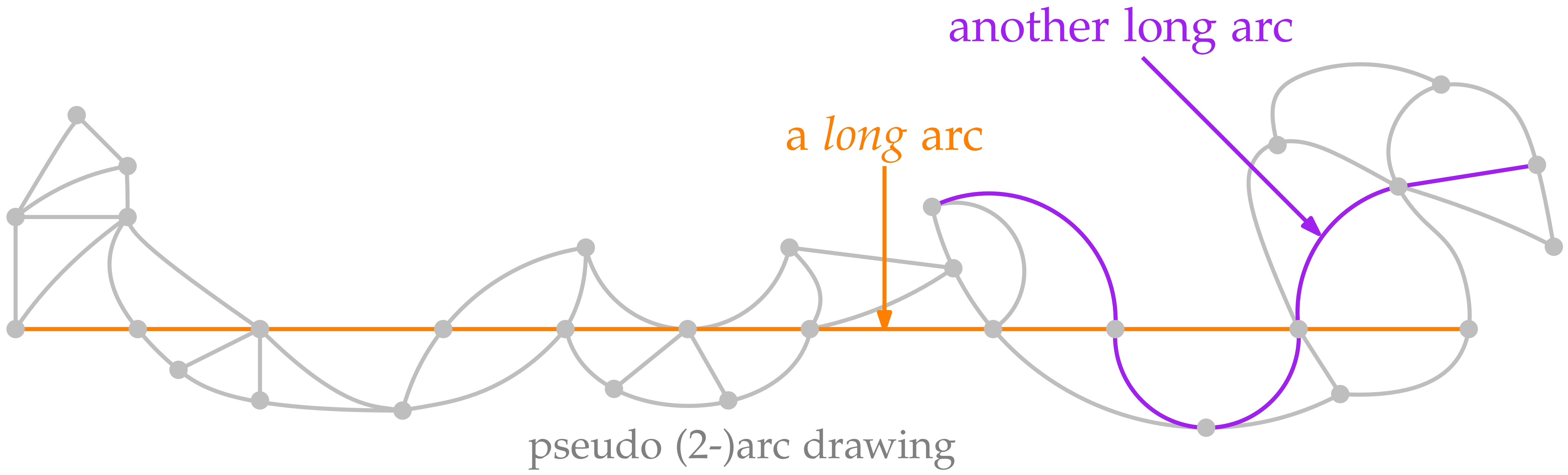

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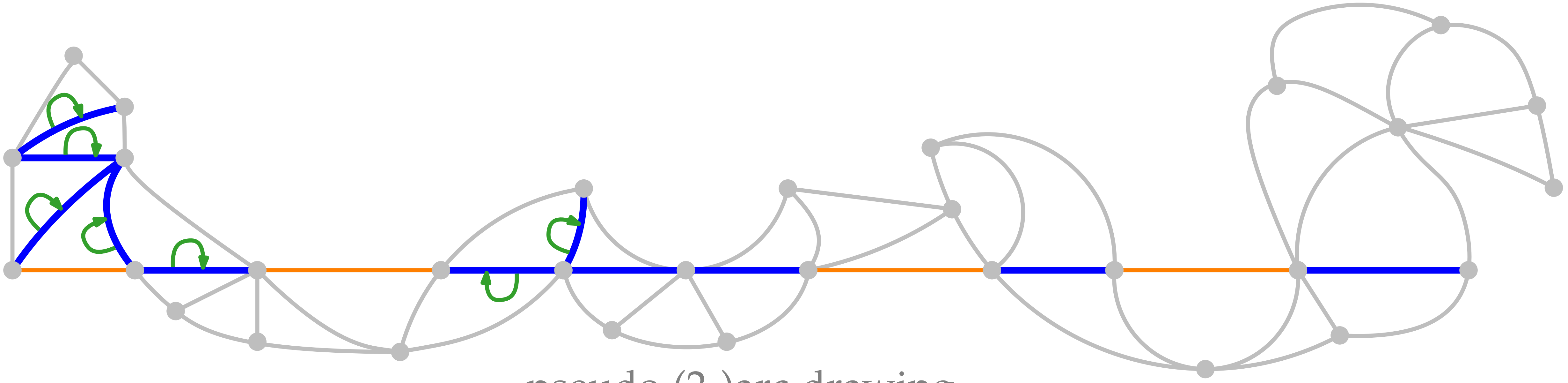

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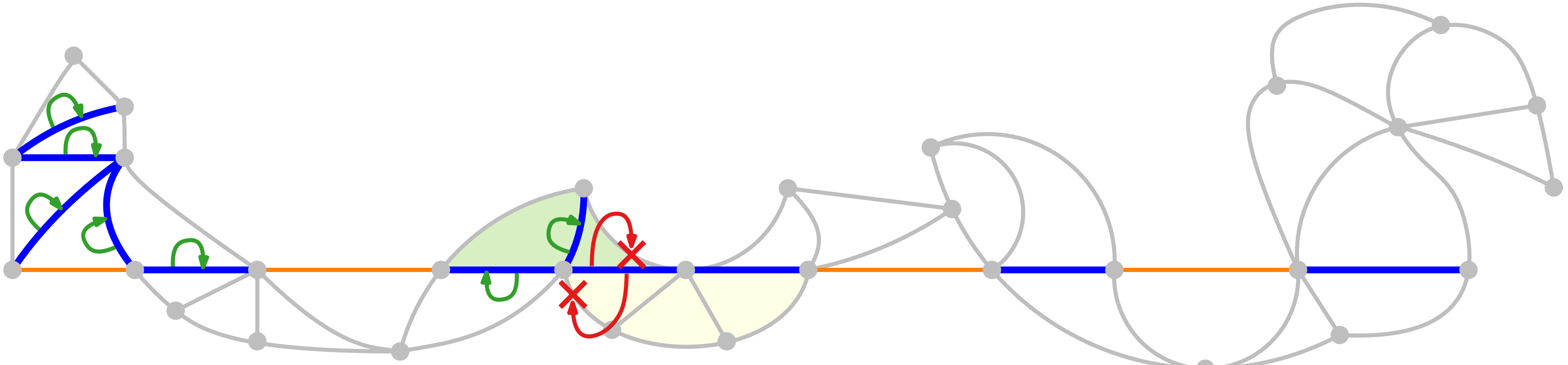

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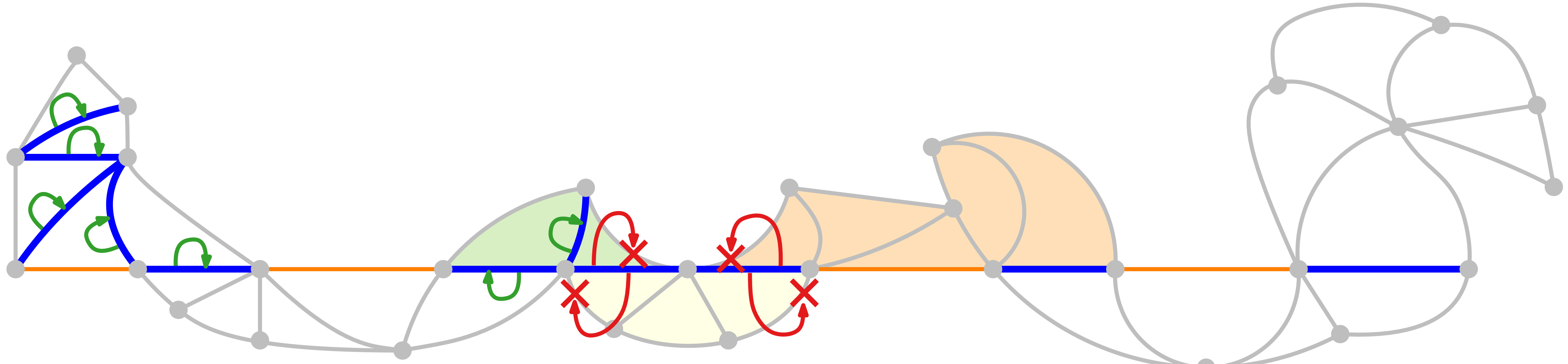

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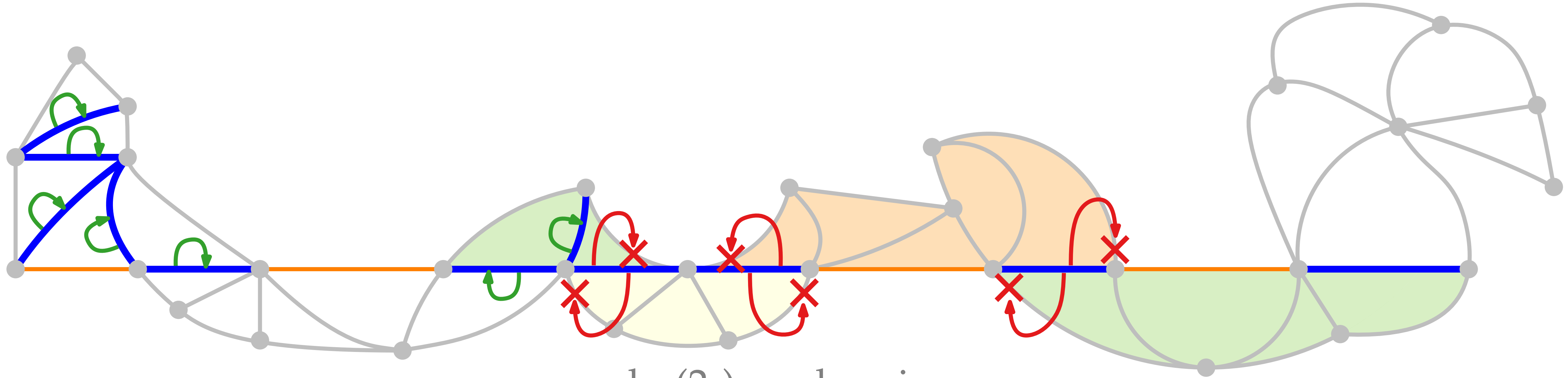

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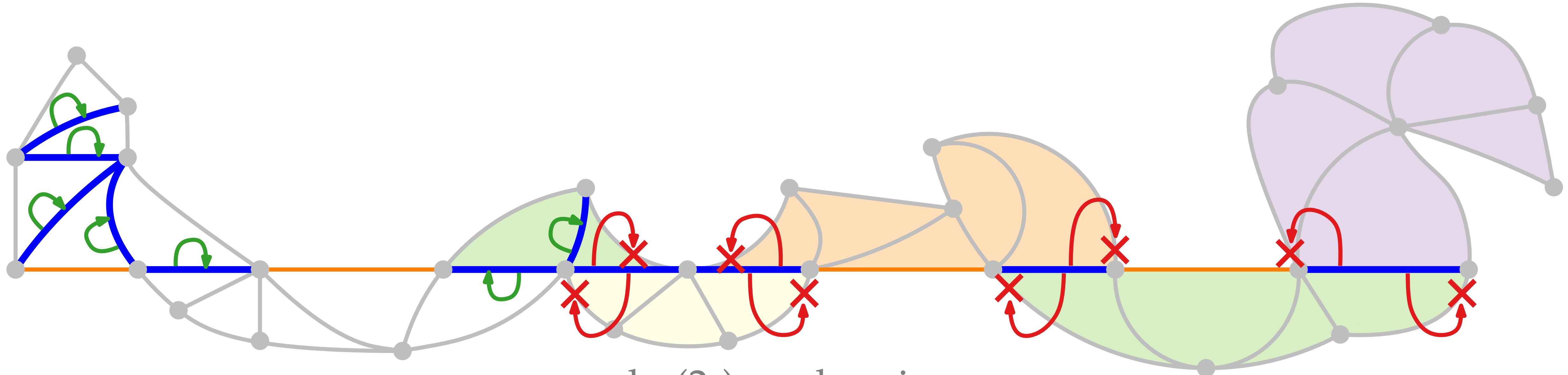

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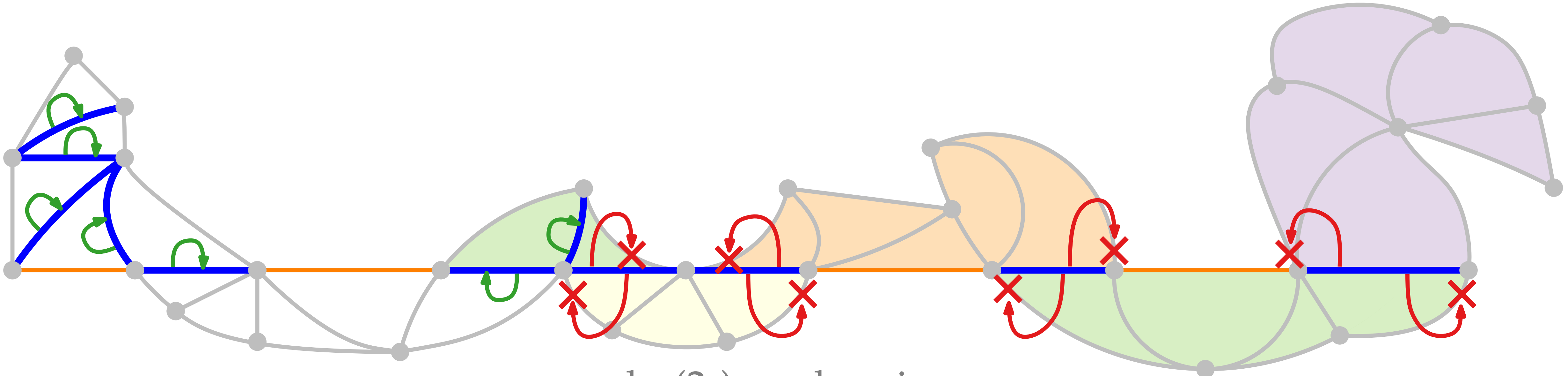

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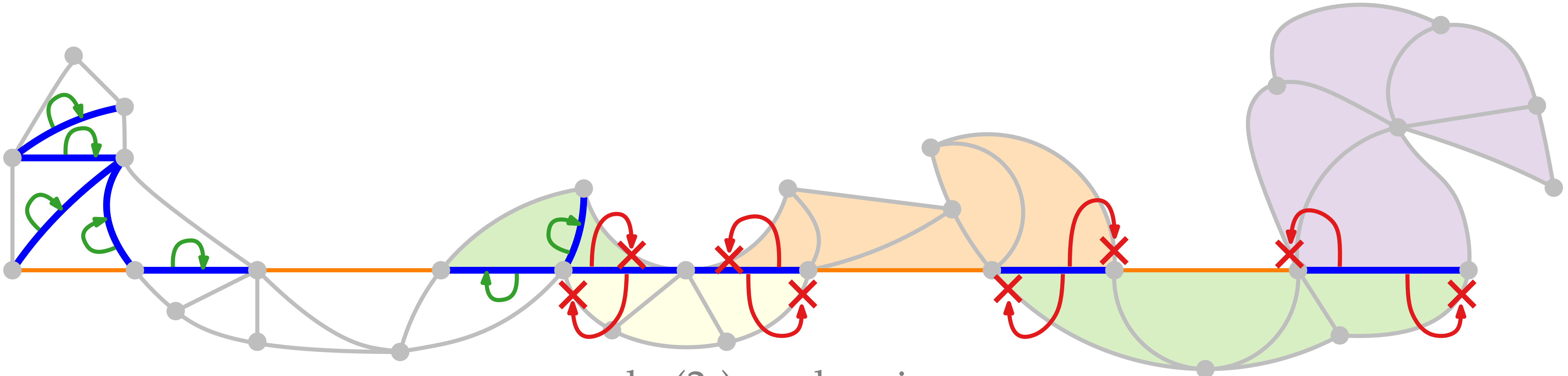

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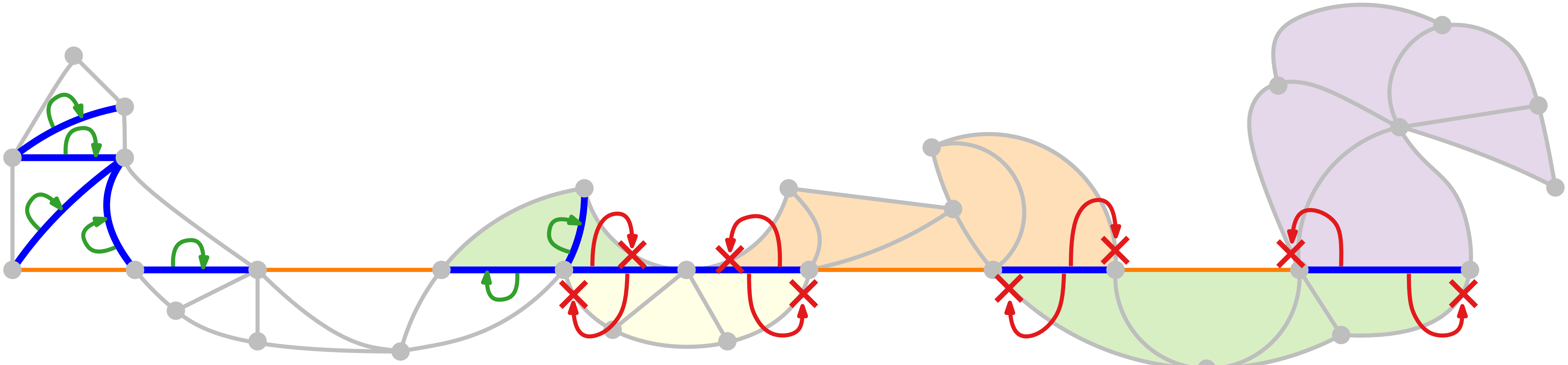

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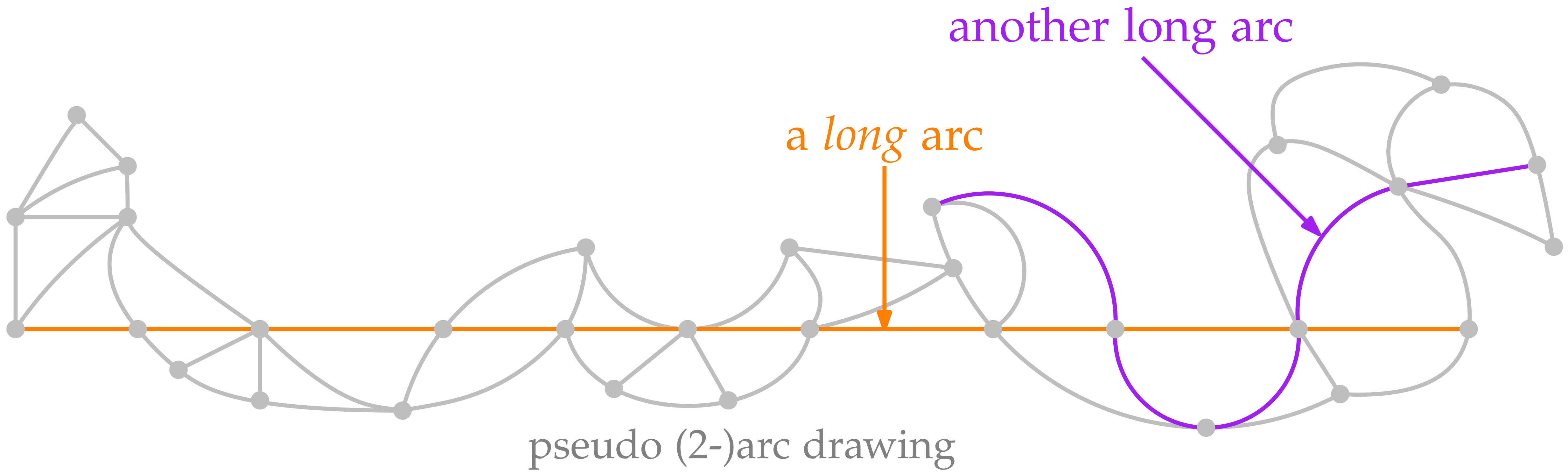

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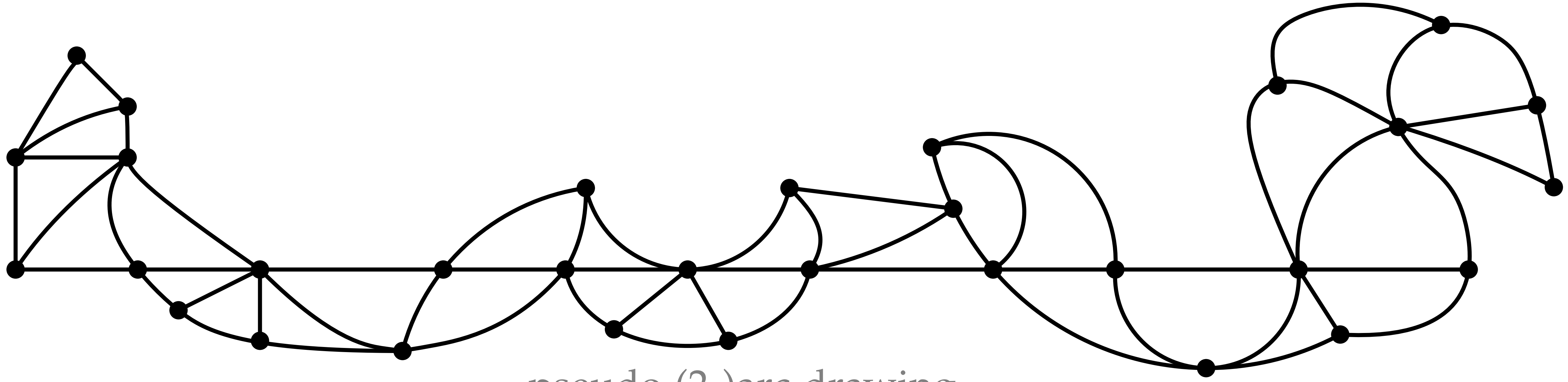

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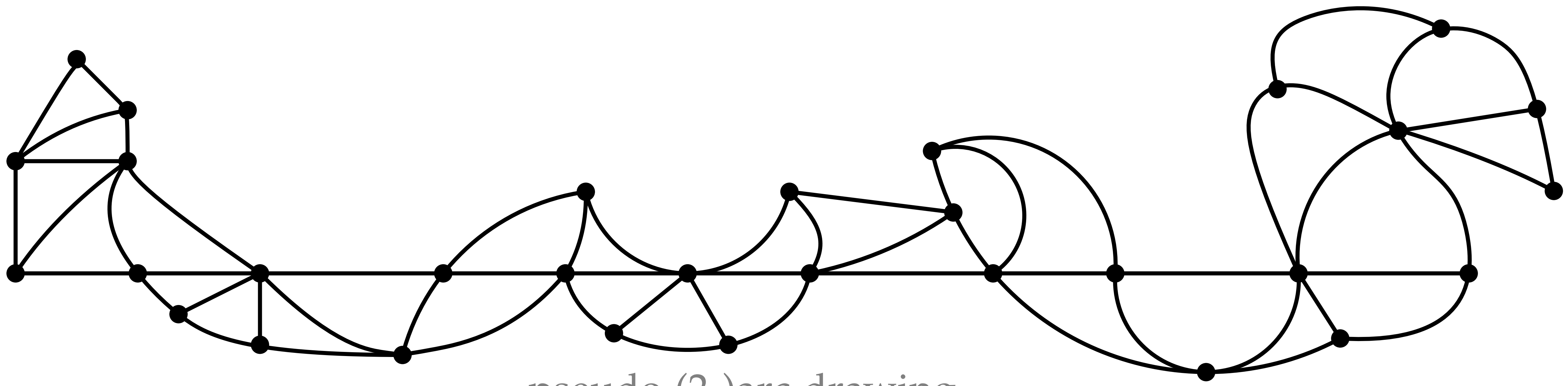

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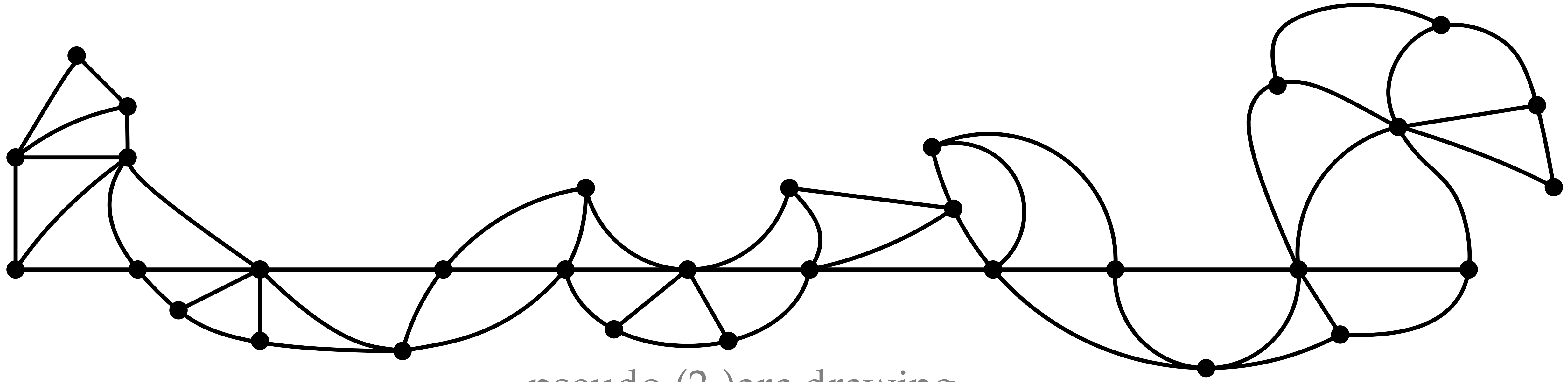

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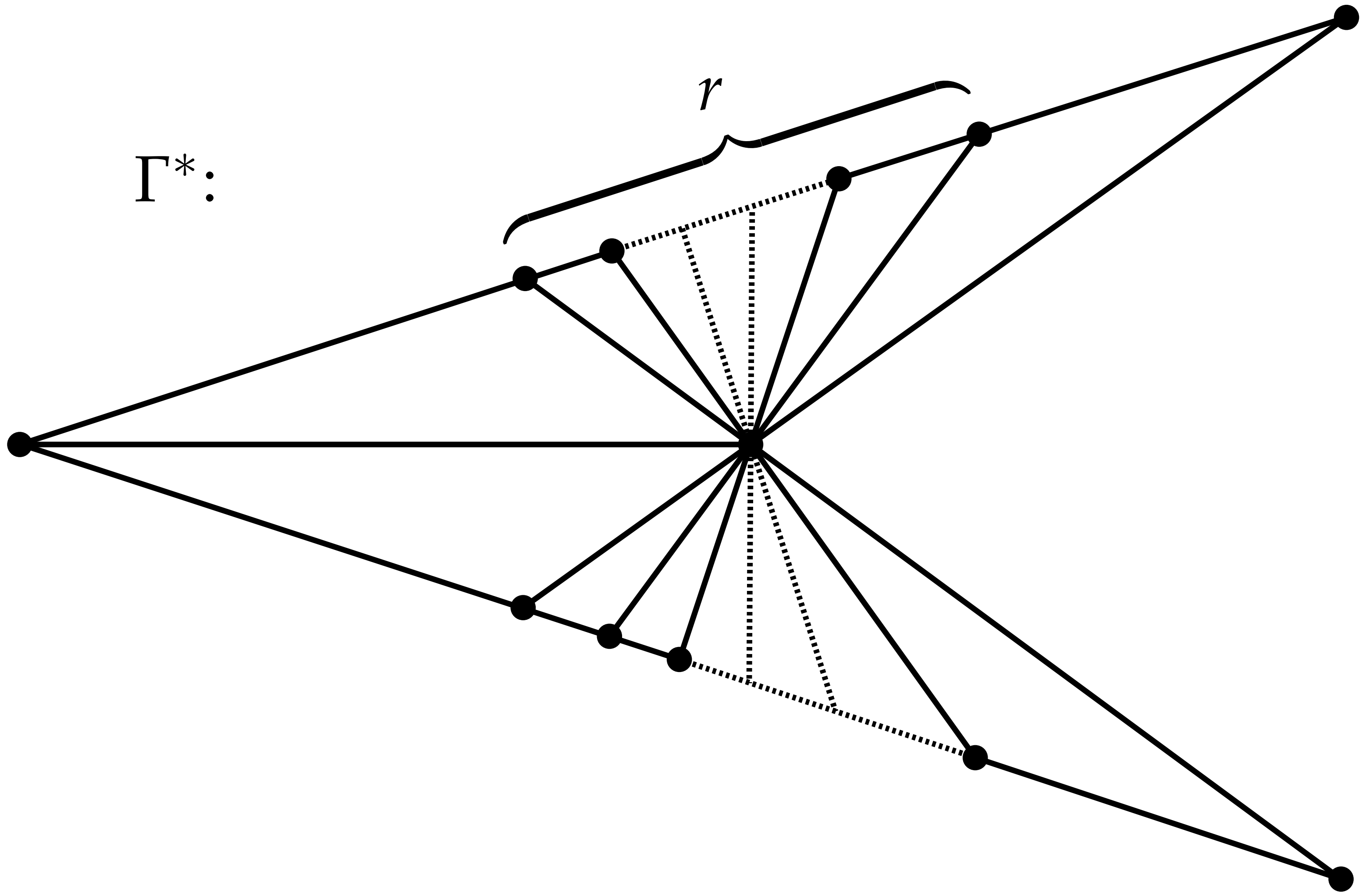

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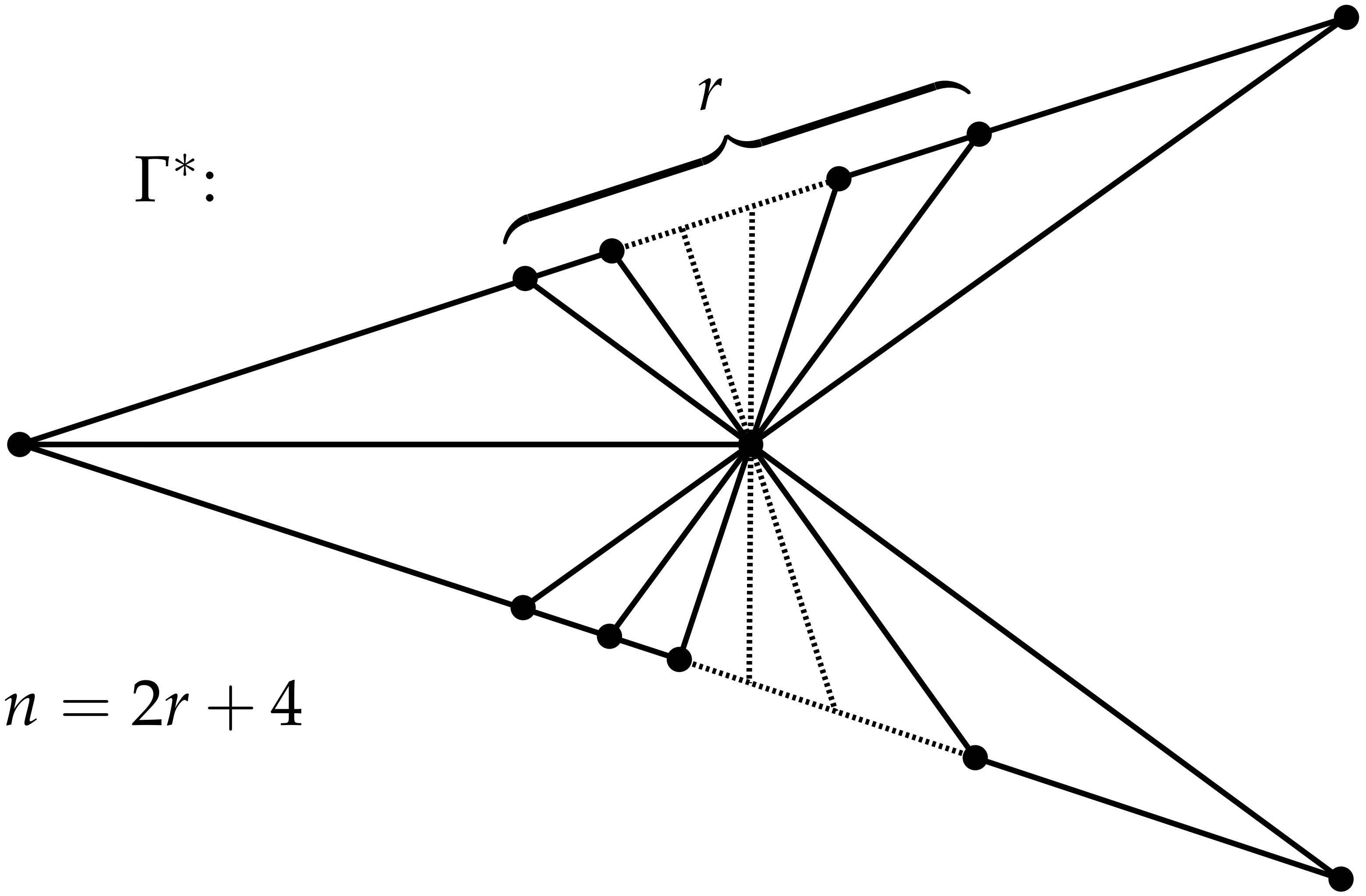

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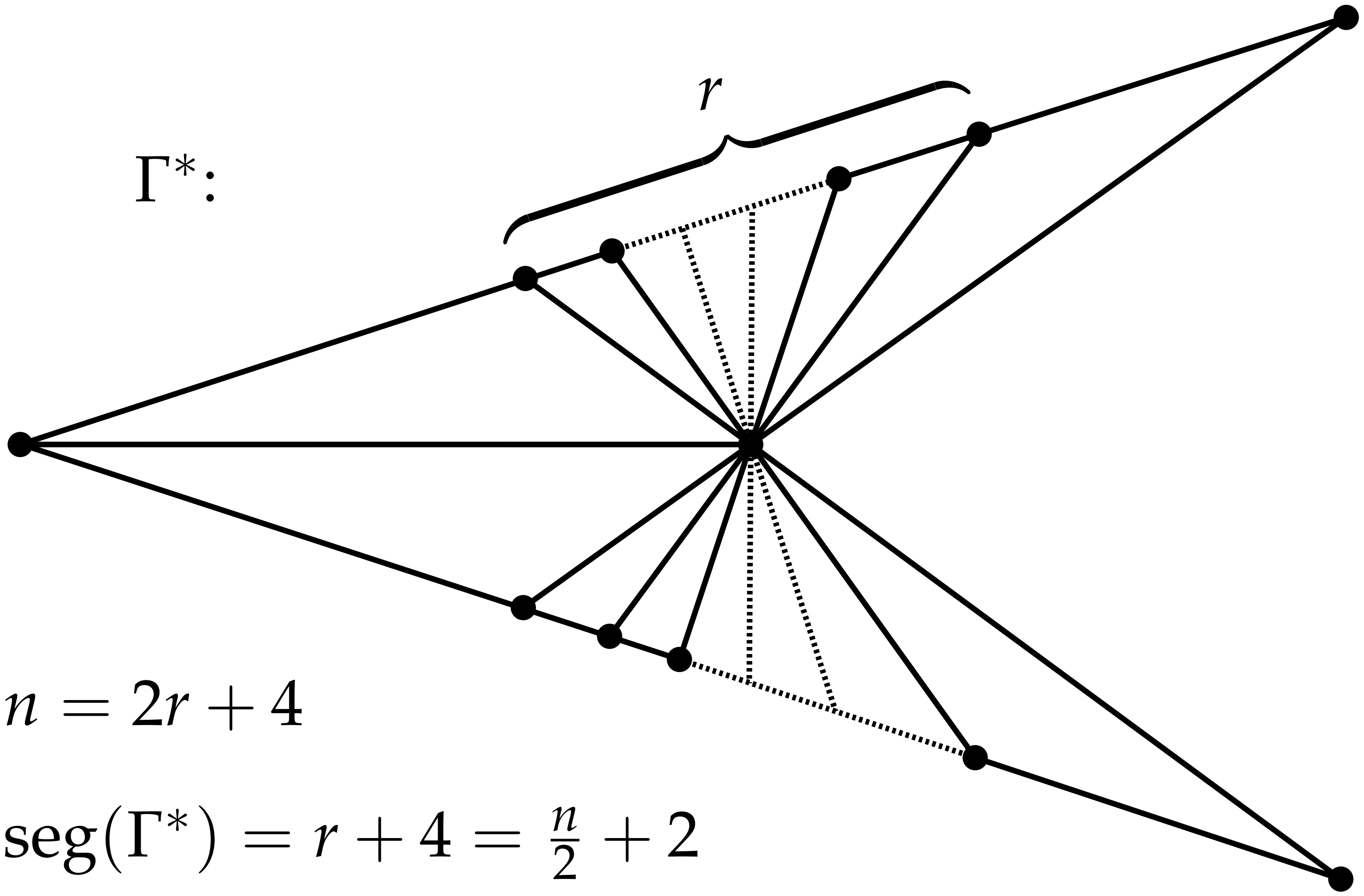

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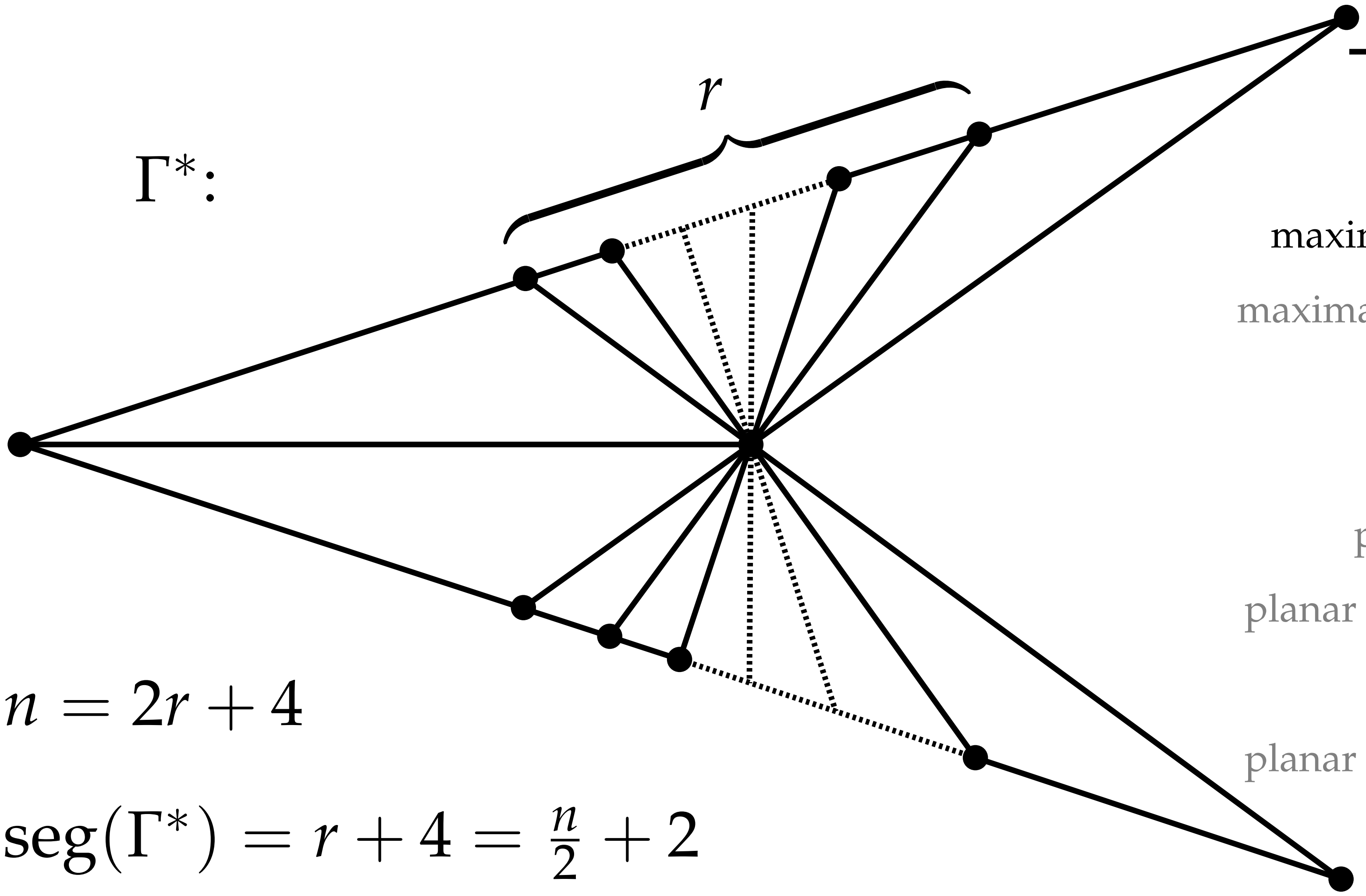

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seg(G)	universal lower bound	existential upper bound	existential lower bound	universal upper bound
planar conn.	$\nu/2$	$\nu/2$	$2n-2$	$\frac{8n-14}{3}$
tree	$\nu/2$	$\nu/2$	$\nu/2$	$\nu/2$
maximal outerpath	$\lfloor n/2 \rfloor + 2$	$\lfloor n/2 \rfloor + 2$	n	n
maximal outerplanar	$(n+7)/5$	$\frac{5n+24}{13}$	n	n
2-tree	$(n+7)/5$	$\frac{5n+24}{13}$	$3n/2-2$	$3n/2$
planar 3-tree	$n+4$	$n+7$		$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$
planar 3-conn. 4-reg.	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	n	$n+3$
cactus	$\nu/2+\gamma$	$\nu/2+\gamma$	$\nu/2+\gamma$	$\nu/2+\gamma$

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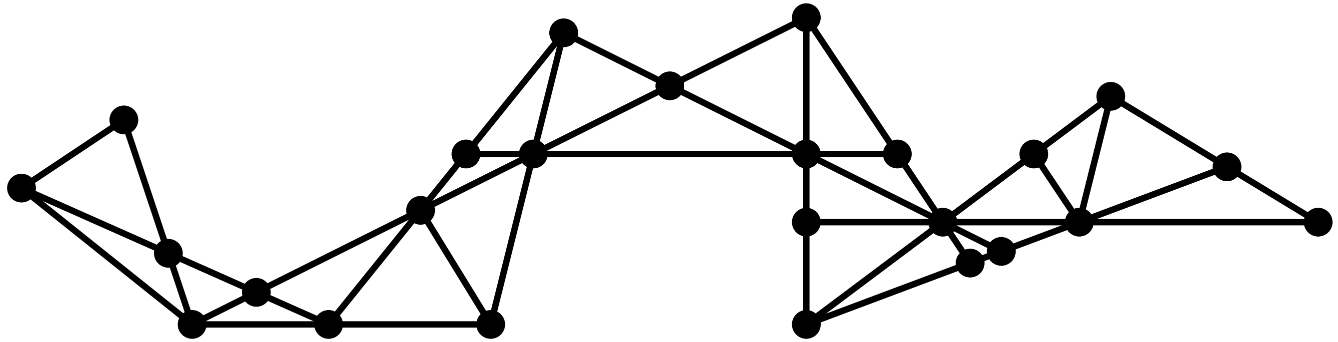

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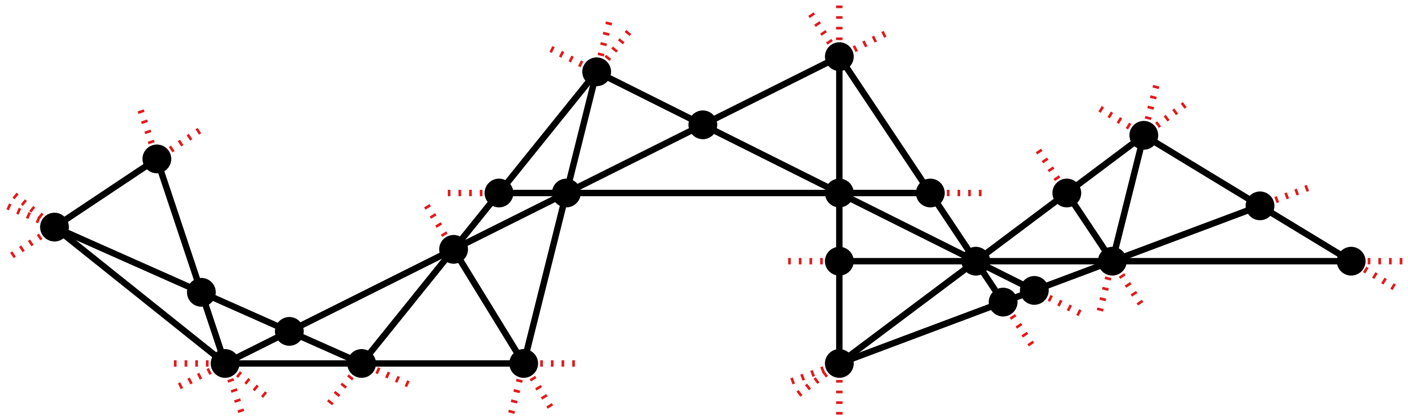

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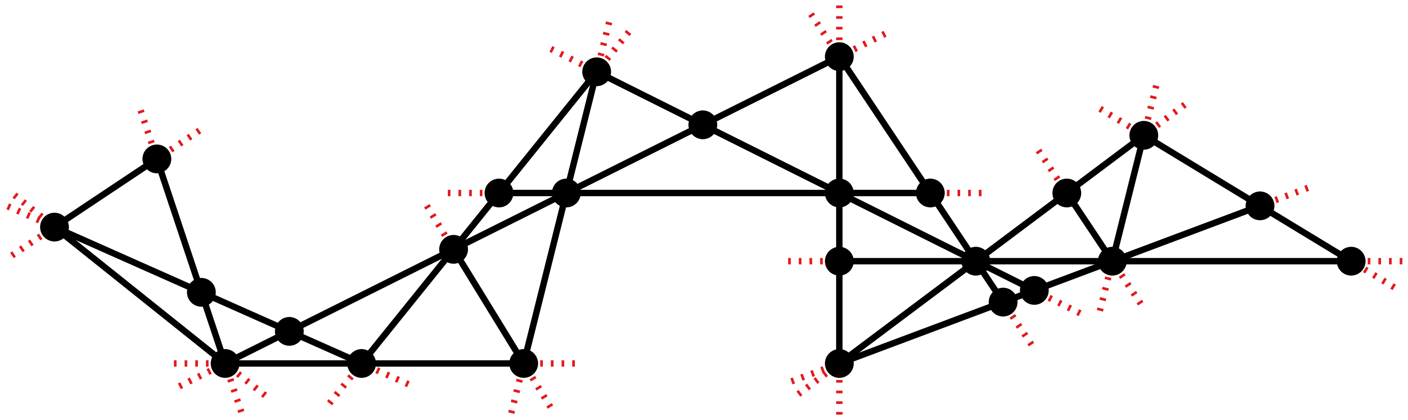

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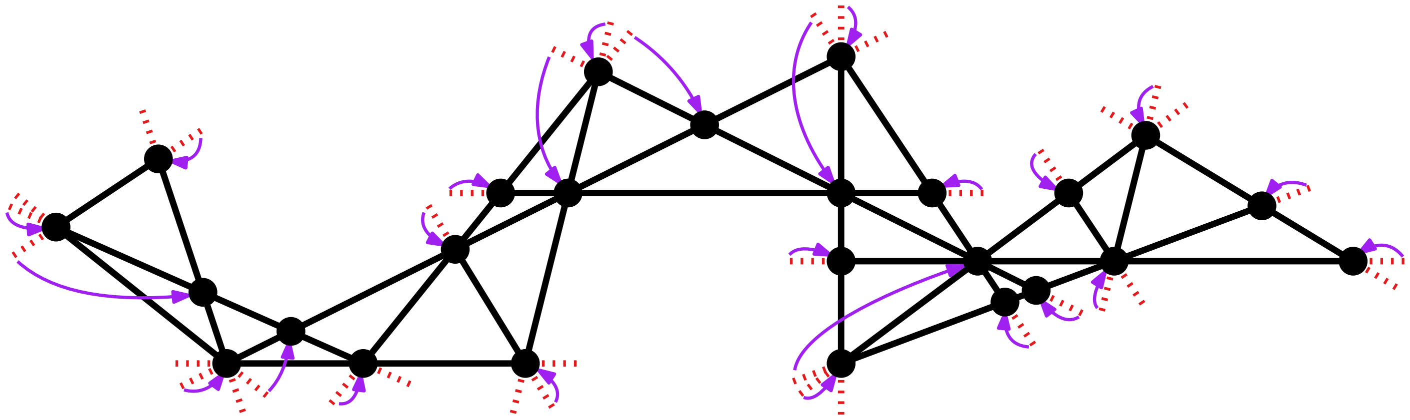

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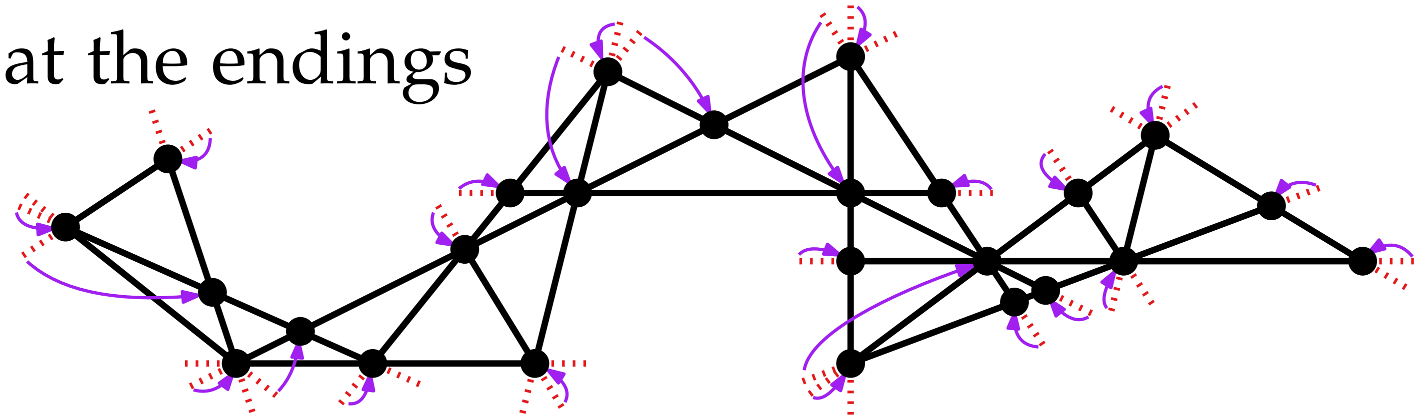

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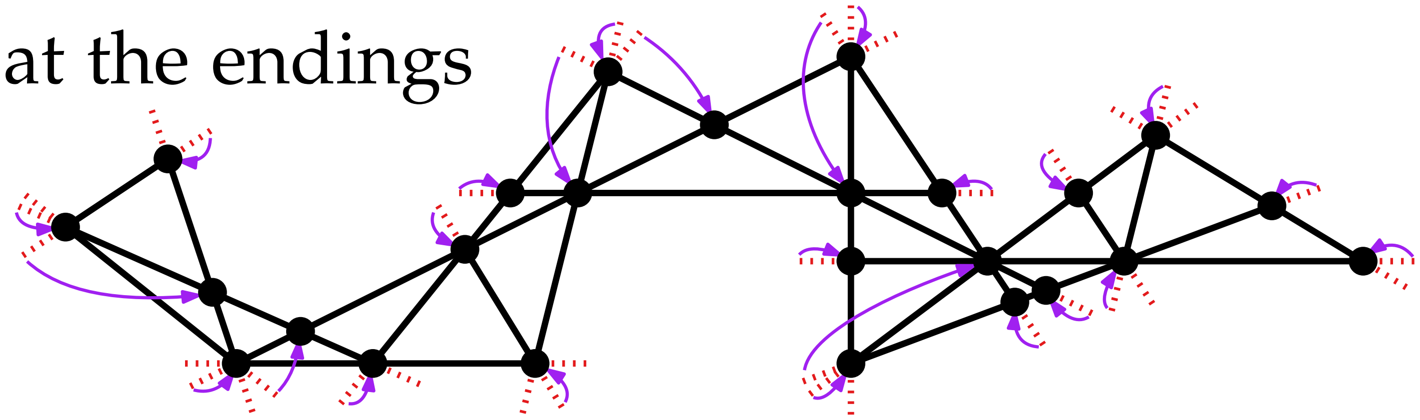

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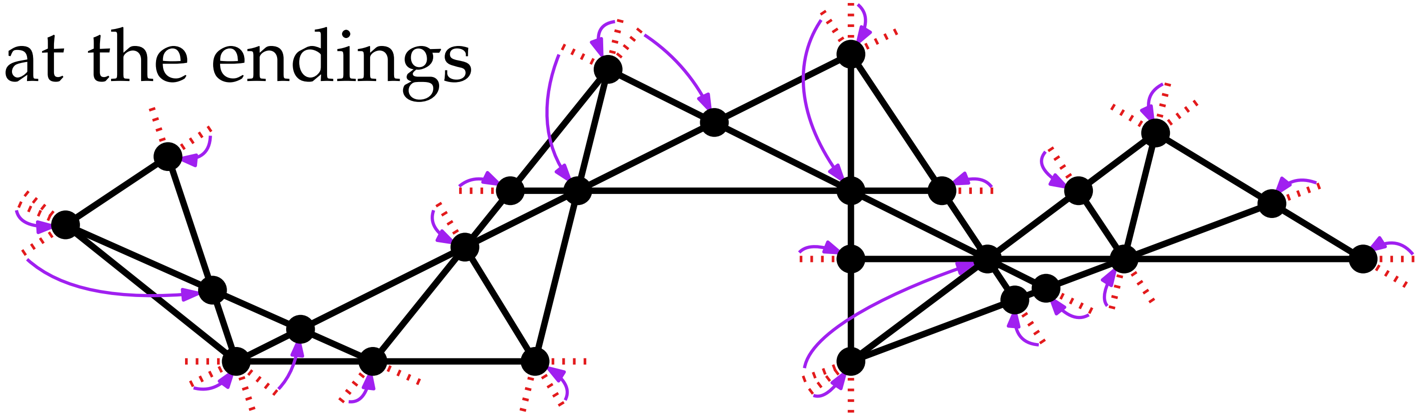

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Maximal Outerplanar Graphs and 2-Trees

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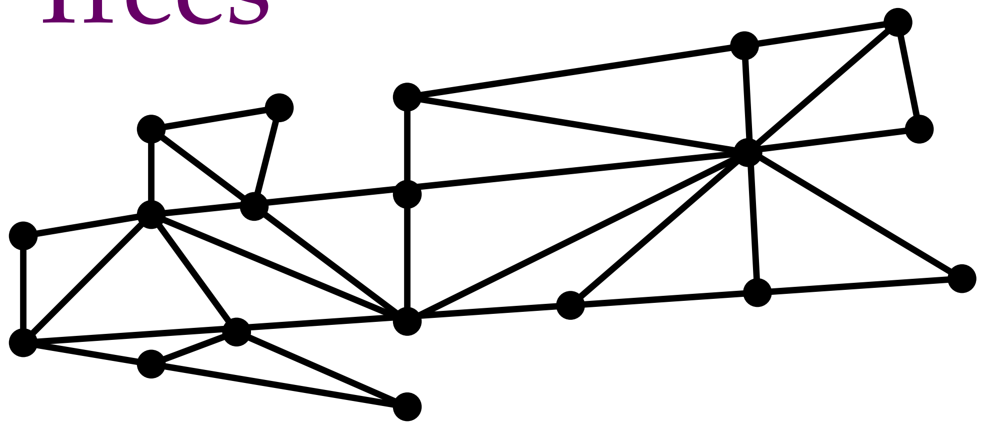

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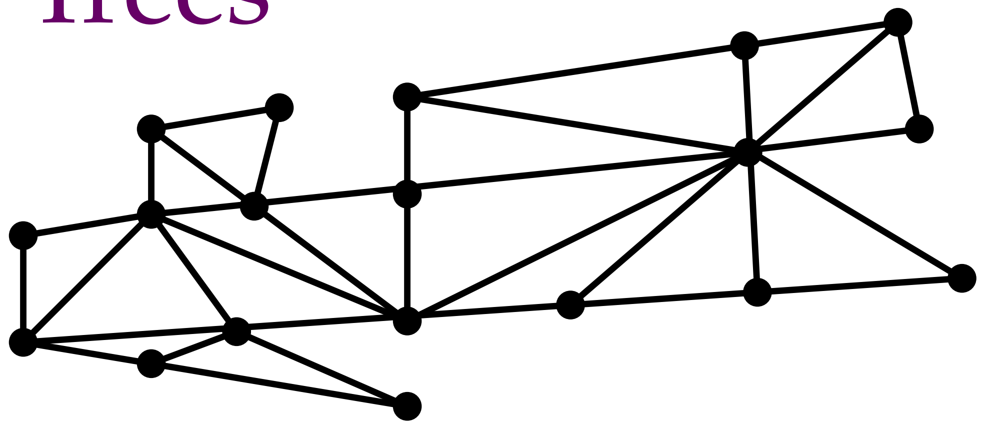

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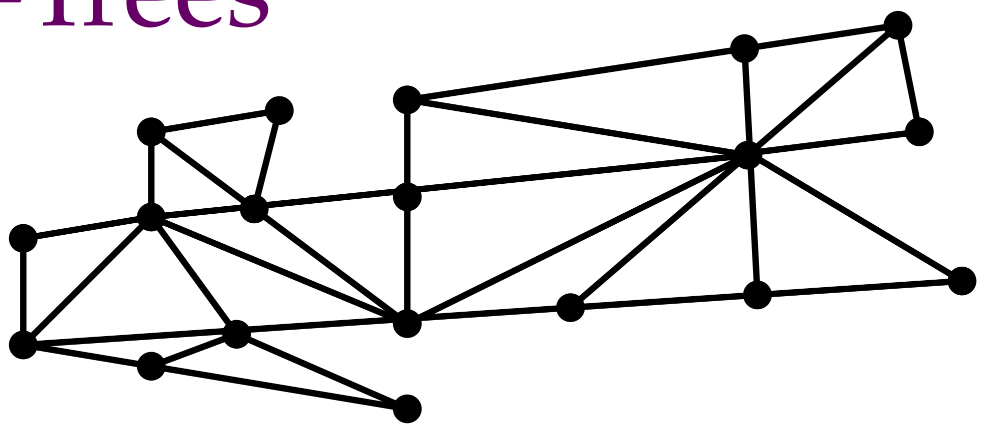

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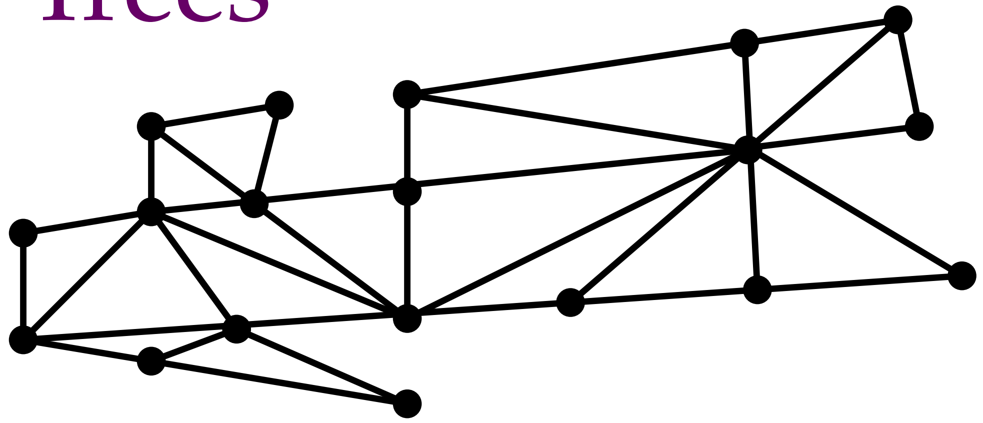

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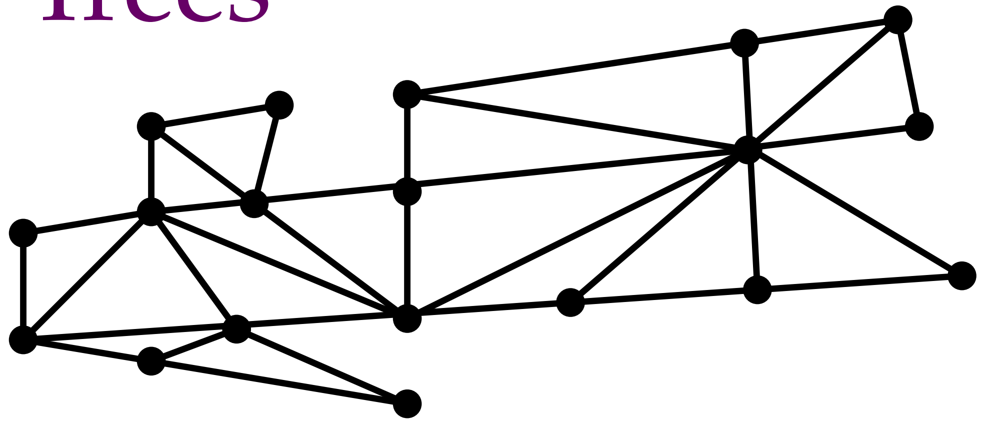

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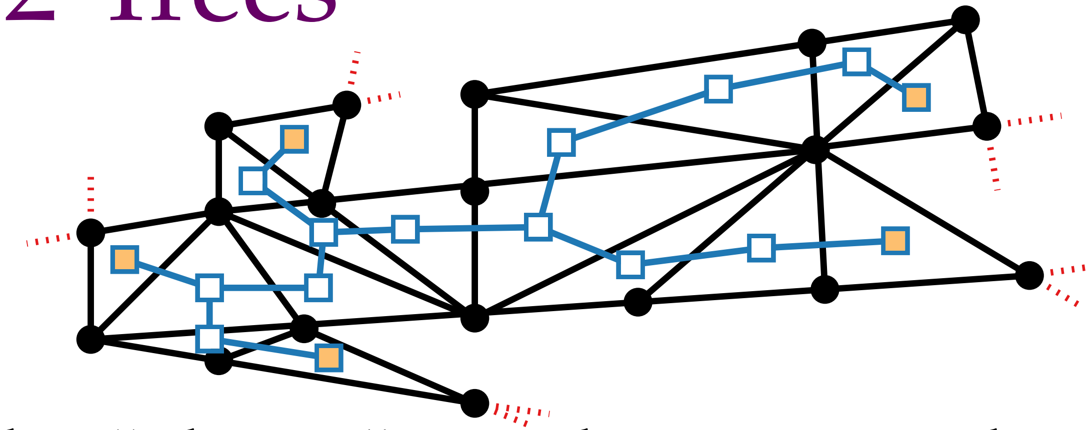

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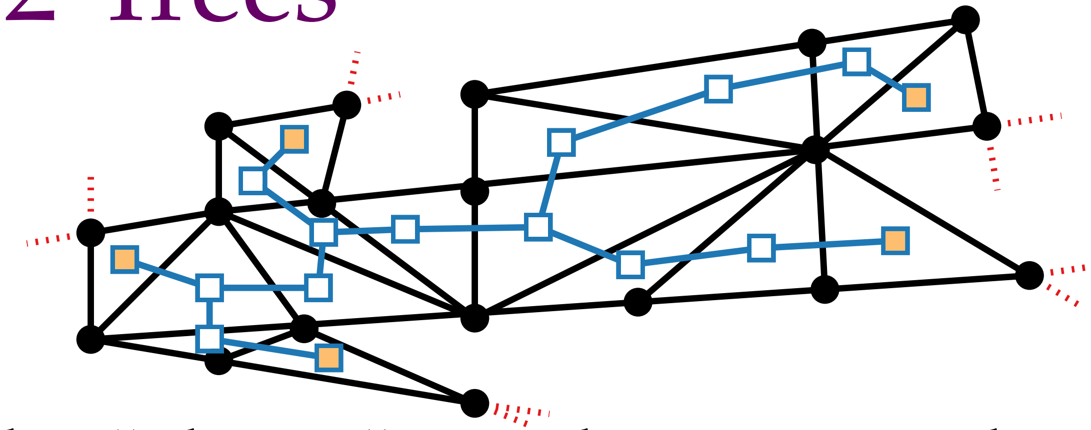

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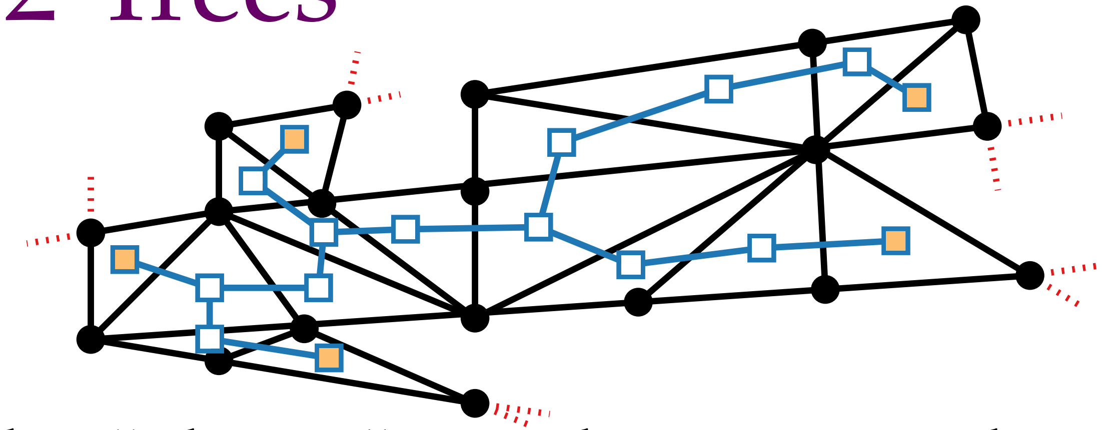
Maximal Outerplanar Graphs and 2-Trees



For a max. outerplanar graph G, $\mathrm{seg}(G) \geq \frac{n+7}{5}$.

the following argument also extends to 2-tree

Proof Idea:

- a max. outerplanar drawing can be obtained by "gluing" together outerpaths
- count ports using a case distinction:

(C1) any leaf of the weak dual tree provides 2 ports $\Rightarrow$ done for $\geq \frac{n+7}{5}$ leaves

(C2) for $< \frac{n+7}{5}$ leaves, we "glue" together $< \frac{n+7}{5}$ outerpath drawings

use the port assignment from the alternative proof for max. outerpaths

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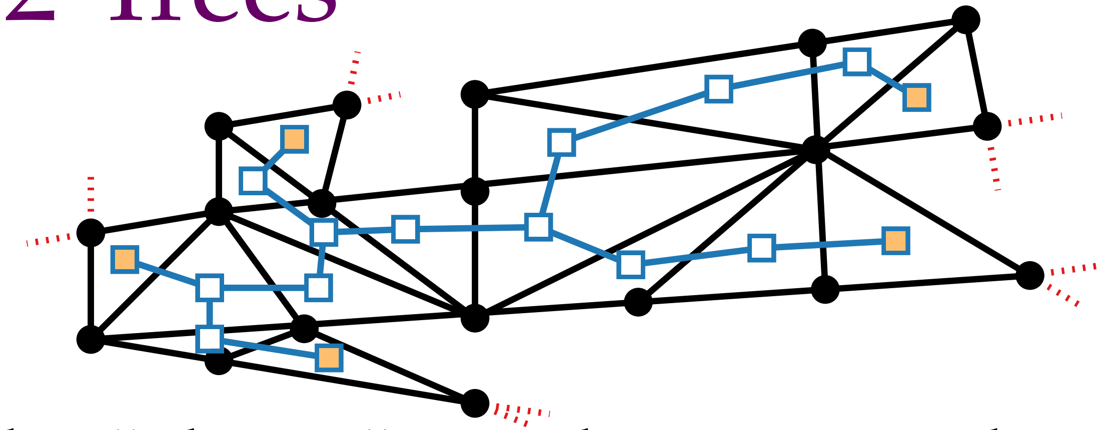

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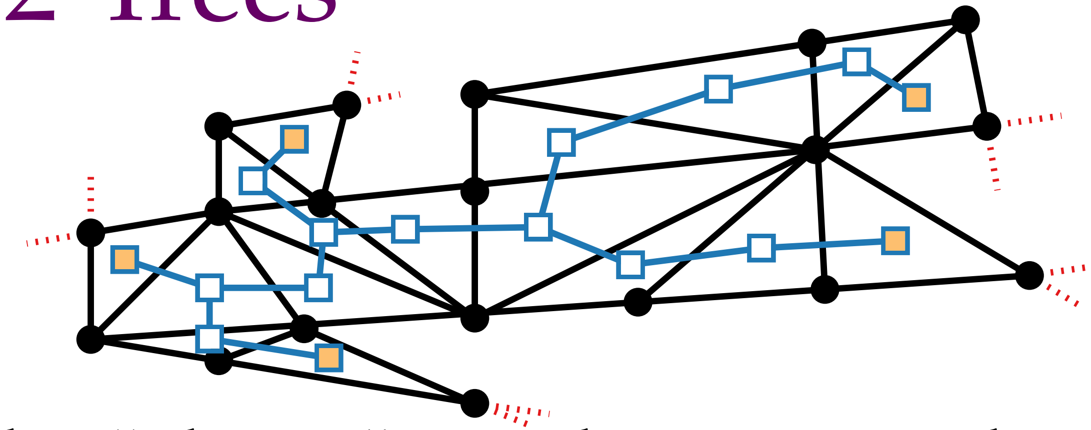

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$\Rightarrow \mathrm{seg}(G) \geq \frac{n+7}{5}$ $\square$

Summary and Open Problems

seg(G)	universal lower bound	existential upper bound	existential lower bound	universal upper bound
planar conn.	$\nu/2$	$\nu/2$	$2n-2$	$\frac{8n-14}{3}$
tree	$\nu/2$	$\nu/2$	$\nu/2$	$\nu/2$
maximal outerpath	$\lfloor n/2 \rfloor + 2$	$\lfloor n/2 \rfloor + 2$	n	n
maximal outerplanar	$(n+7)/5$	$\frac{5n+24}{13}$	n	n
2-tree	$(n+7)/5$	$\frac{5n+24}{13}$	$3n/2-2$	$3n/2$
planar 3-tree	$n+4$	$n+7$		$2n-2$
planar 3-conn.	$\sqrt{2n}$	$O(\sqrt{n})$	$2n-6$	$5n/2-3$
planar 3-conn. 3-reg.	$n/2+3$	$n/2+3$	$n/2+3$	$n/2+3$
triangulation	$\Omega(\sqrt{n})$	$O(\sqrt{n})$	$2n-2$	$\frac{7n-10}{3}$
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- Where does it help using pseudo segments/arcs instead of segments/arcs?