Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Lehrstuhl für
INFORMATIK I
Algorithmen & Komplexität

Institut für Informatik

Morphing Graph Drawings in the Presence of Point Obstacles

SOFSEM 2024

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Marie Diana Sieper   Alexander Wolff   **Johannes Zink**

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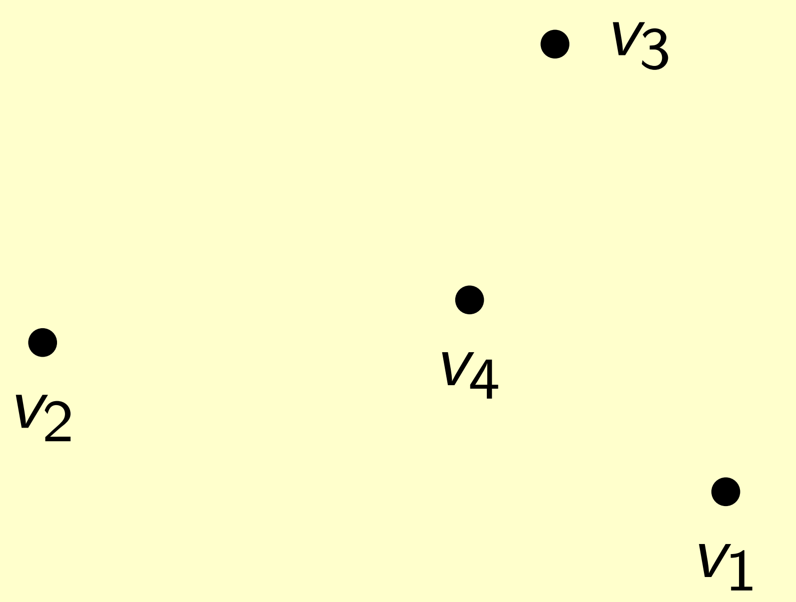

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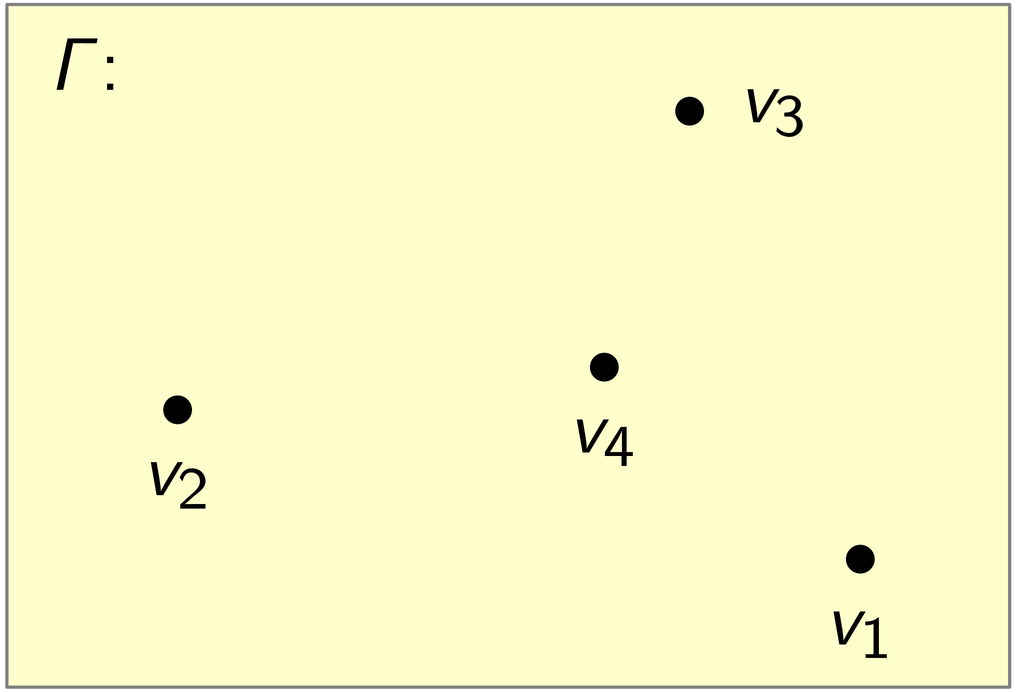

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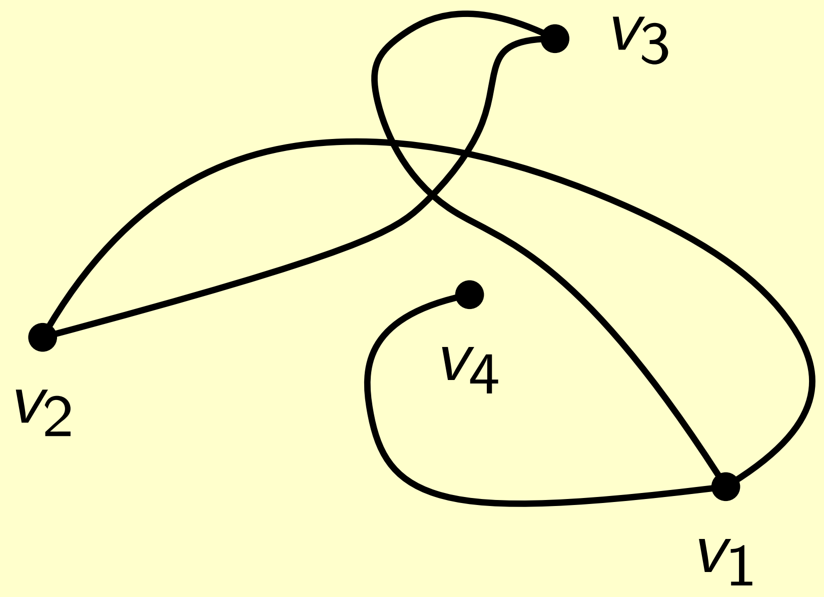

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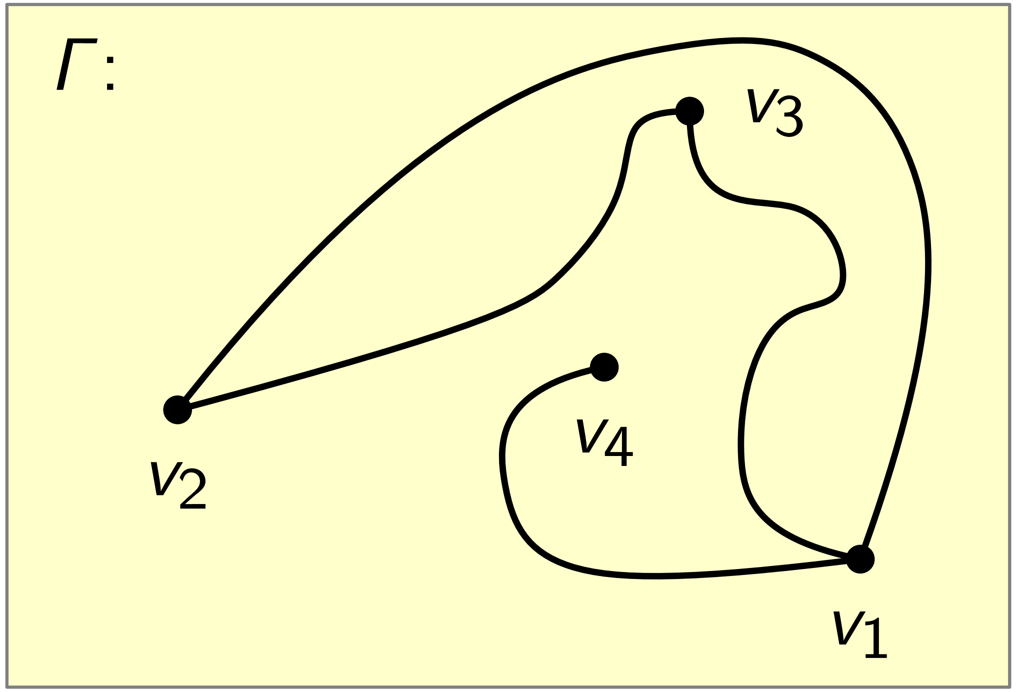

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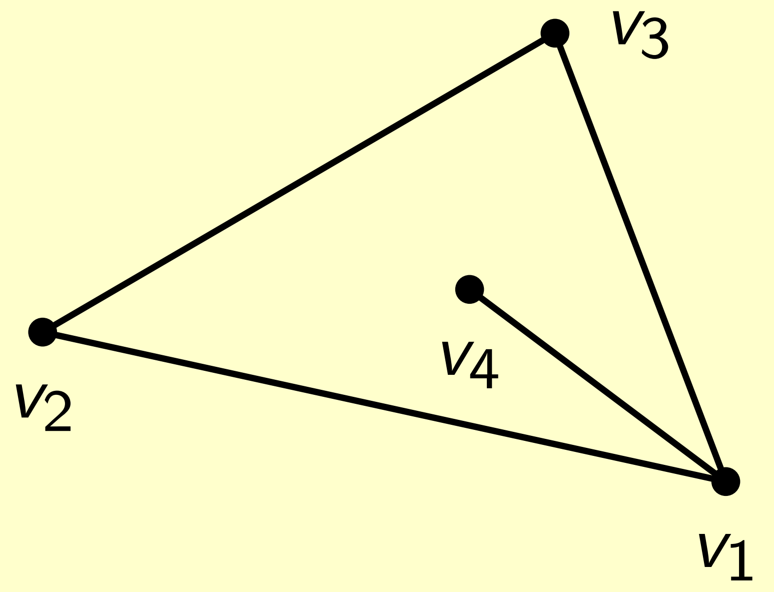

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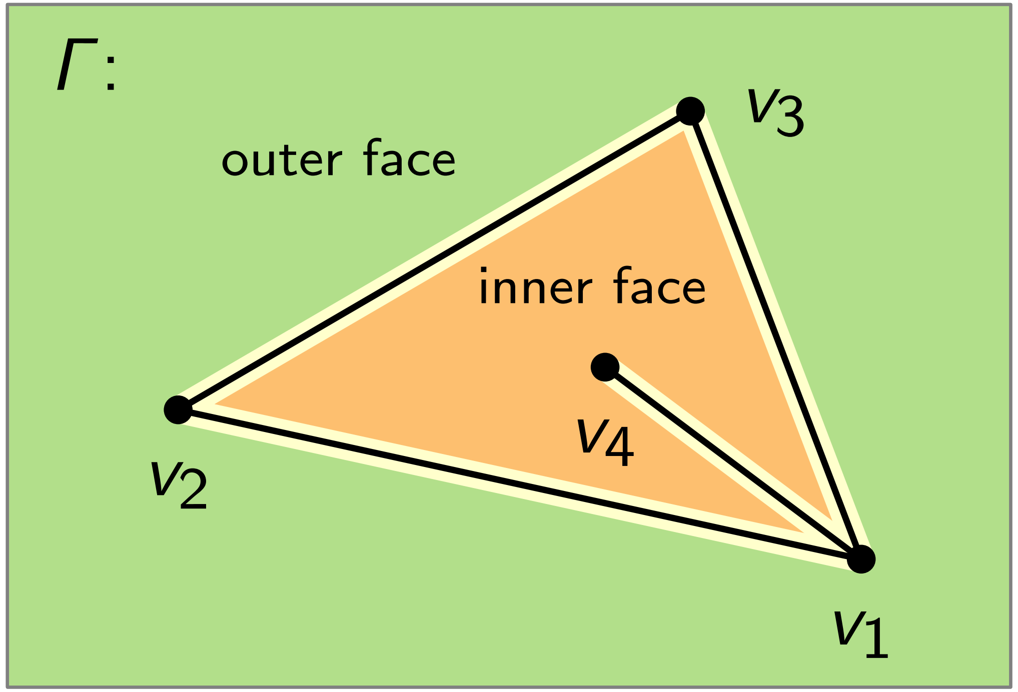

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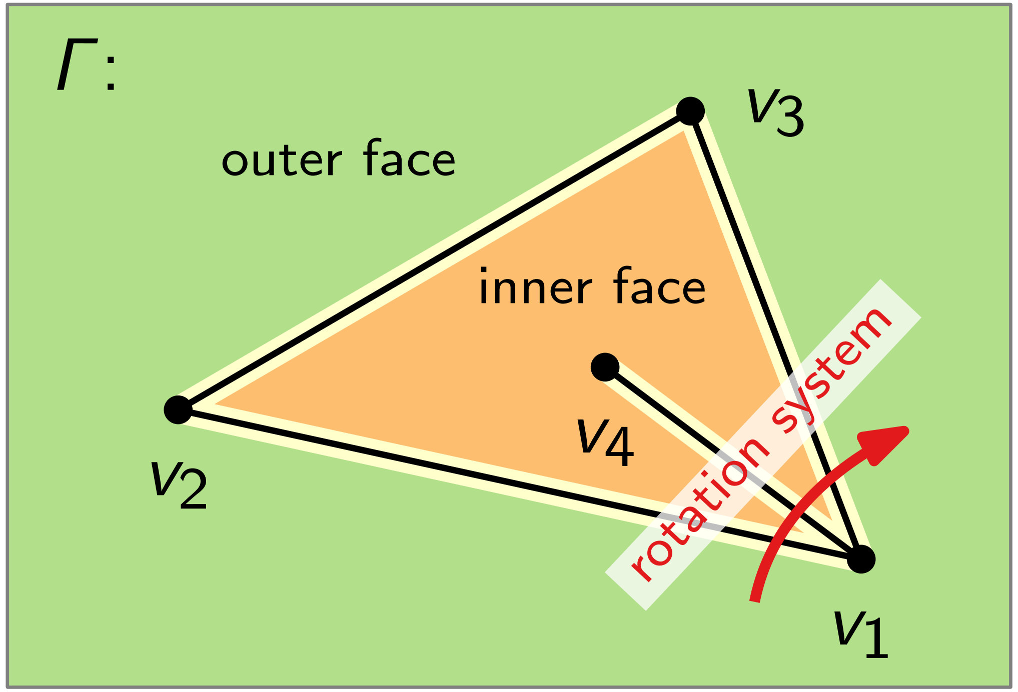

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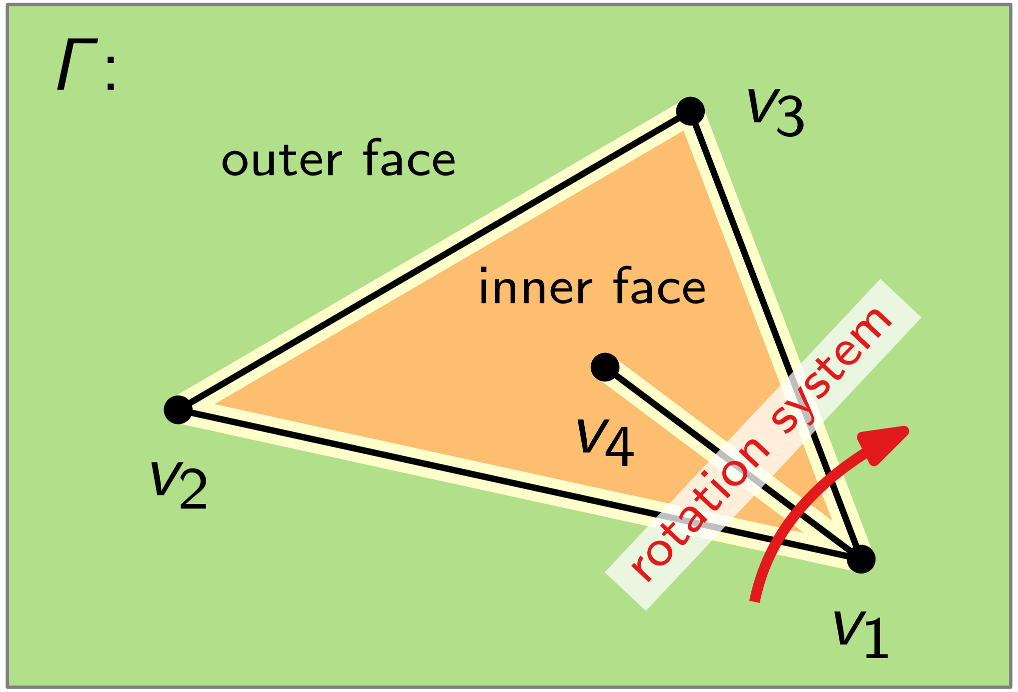

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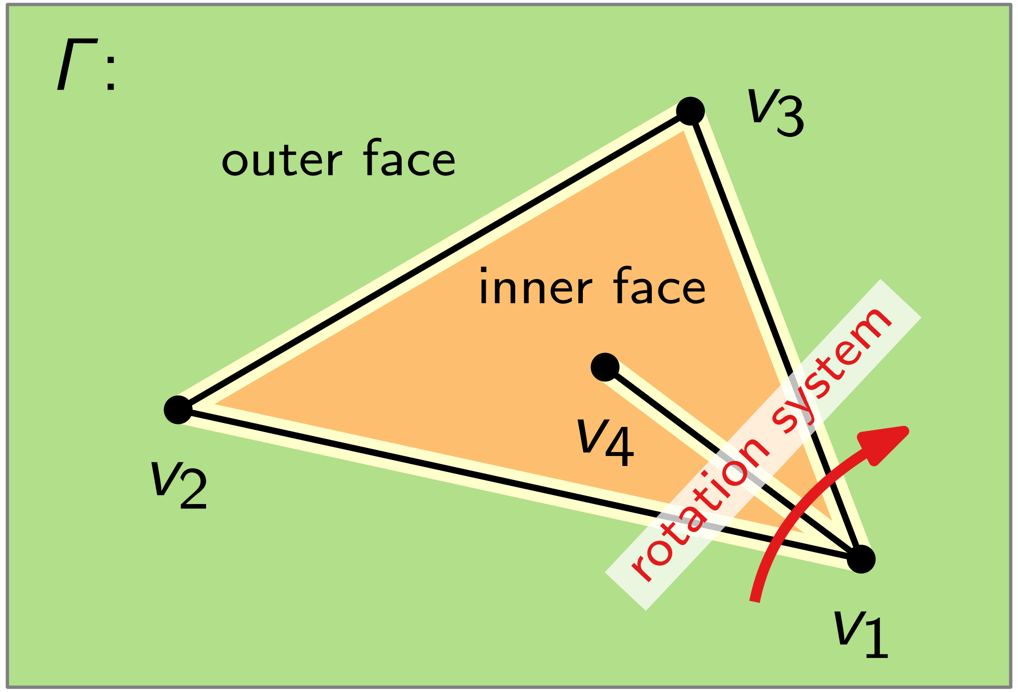

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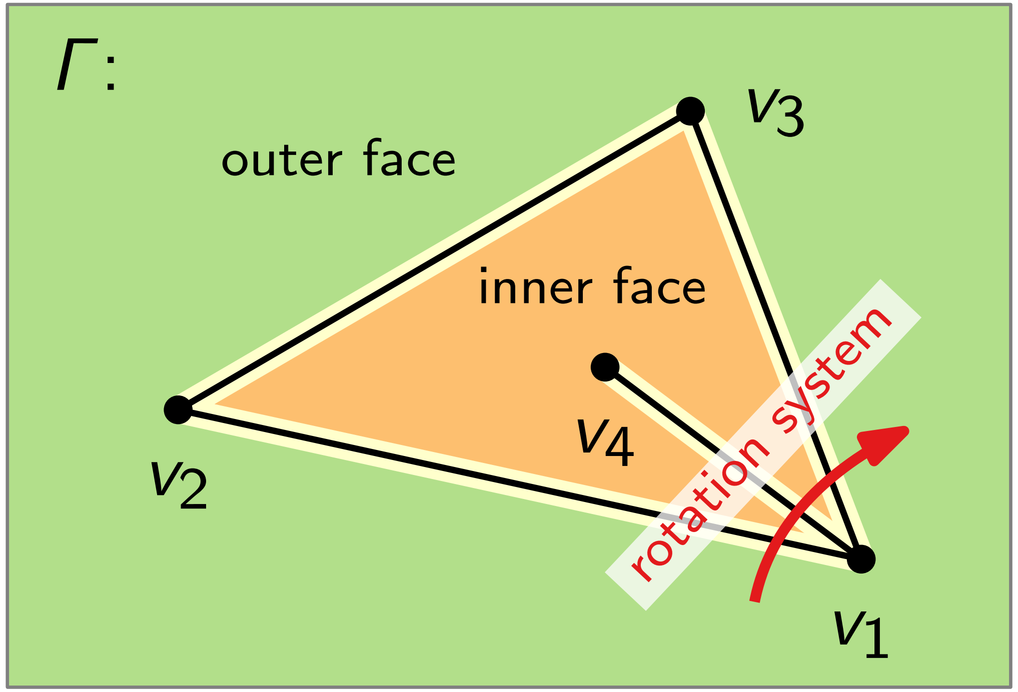

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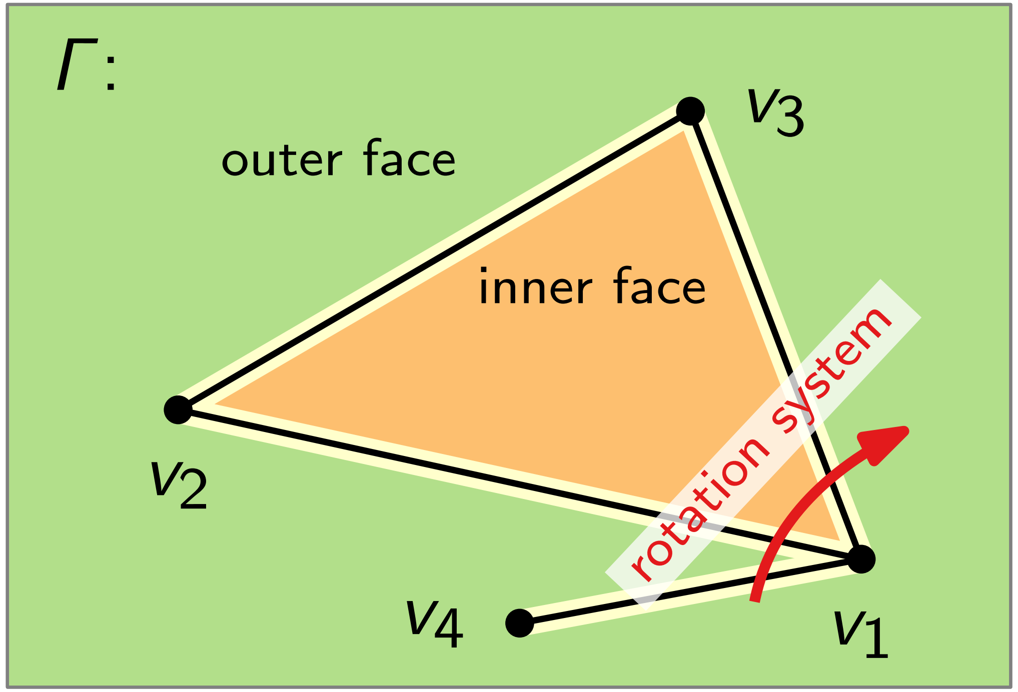

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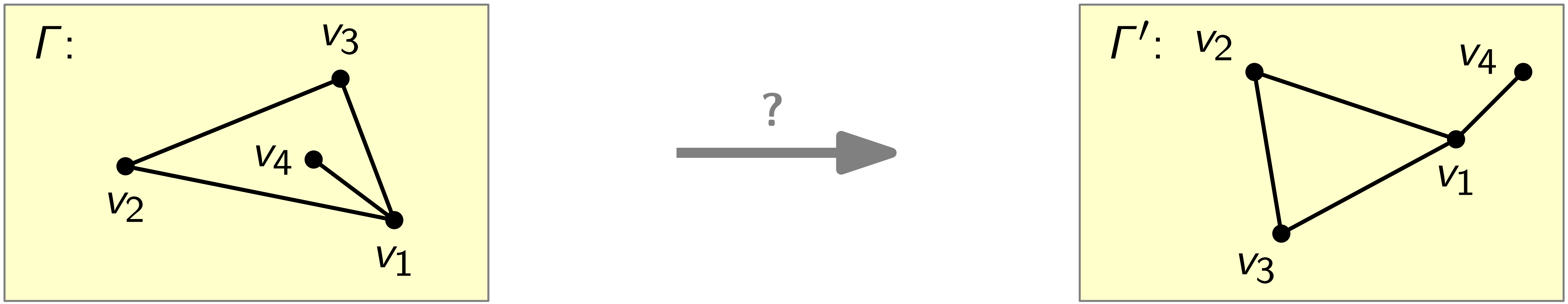

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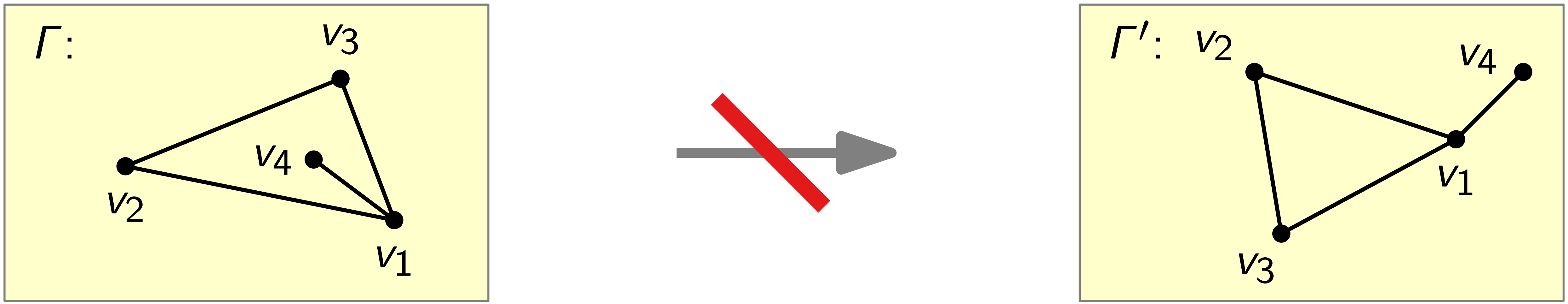

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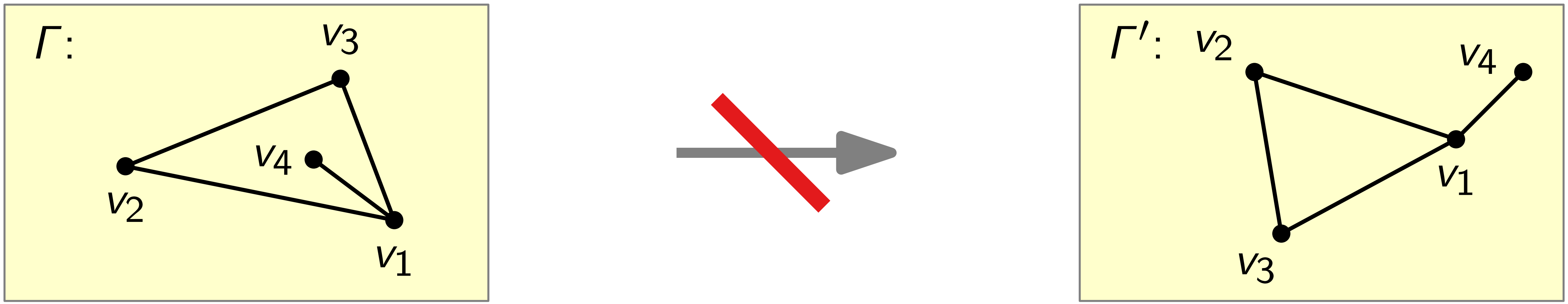

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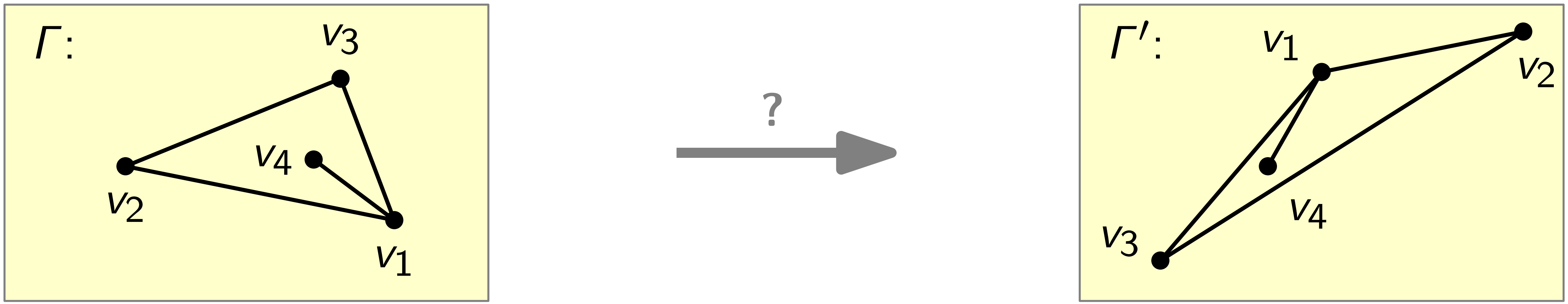

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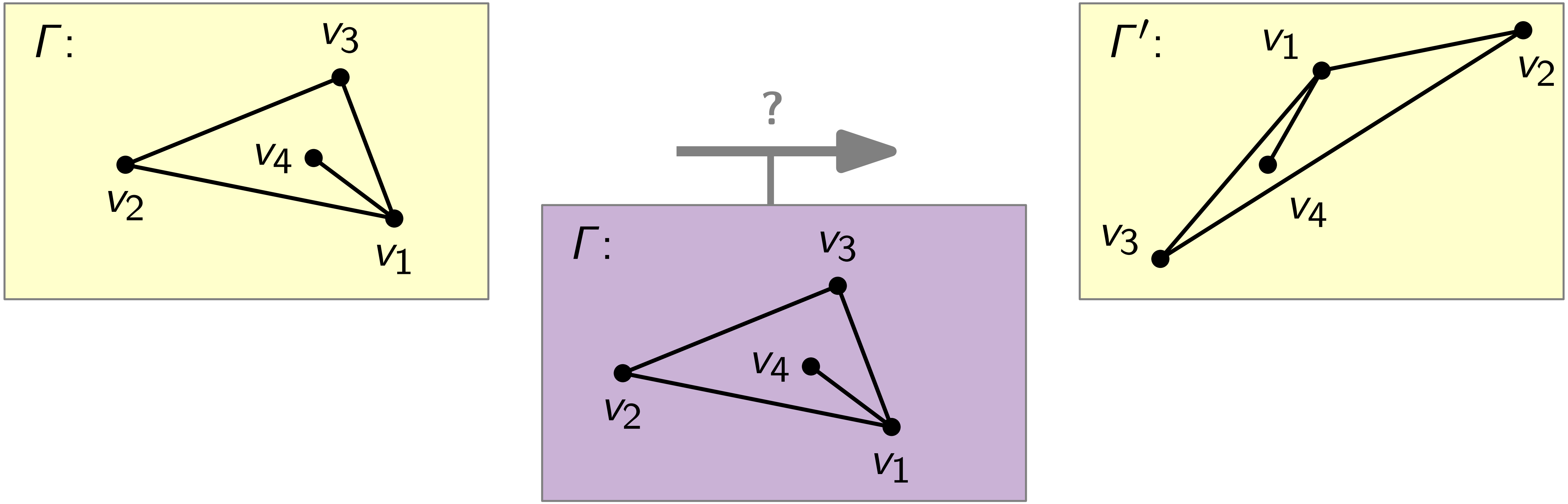

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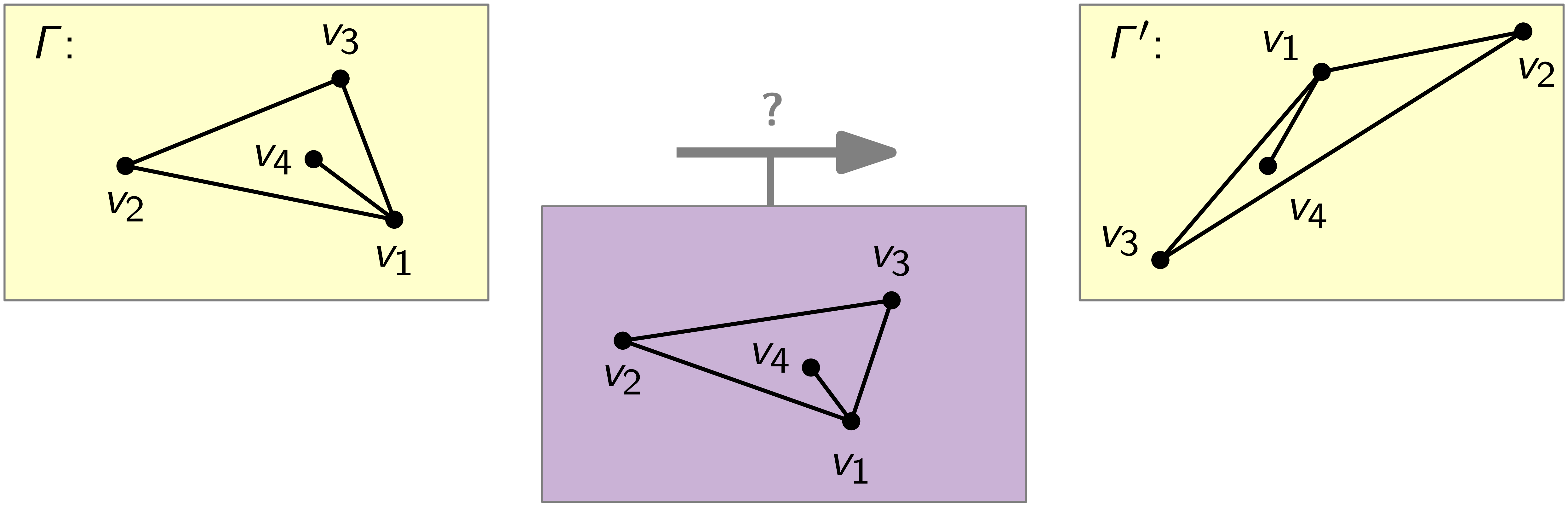

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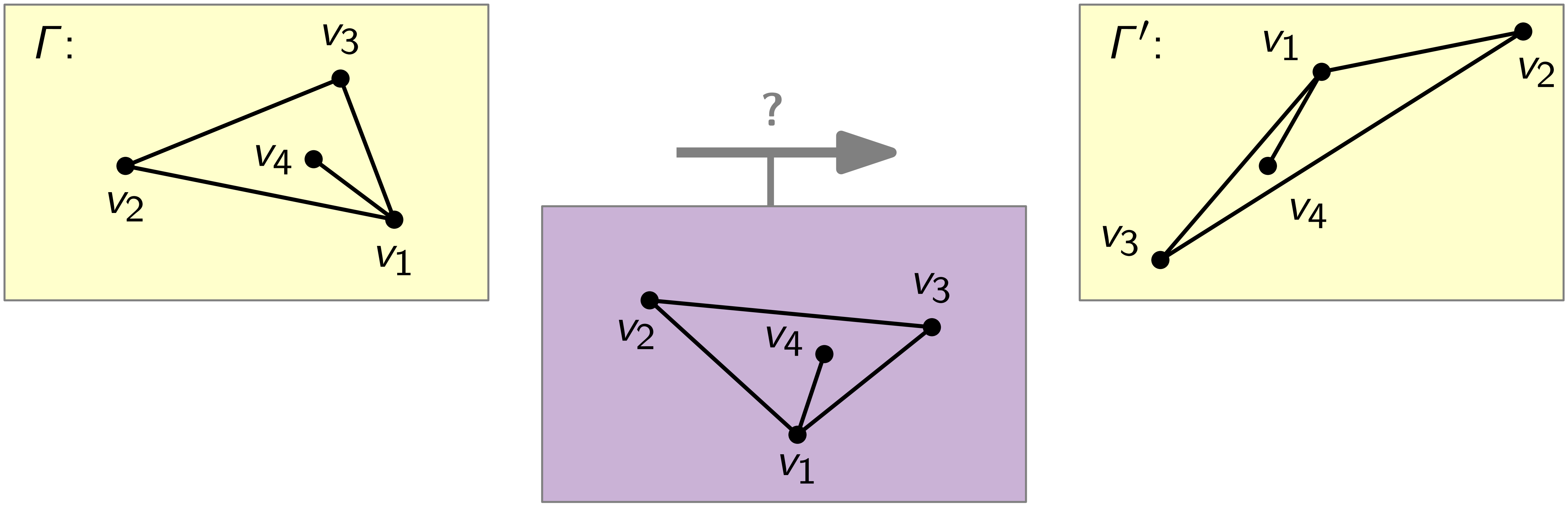

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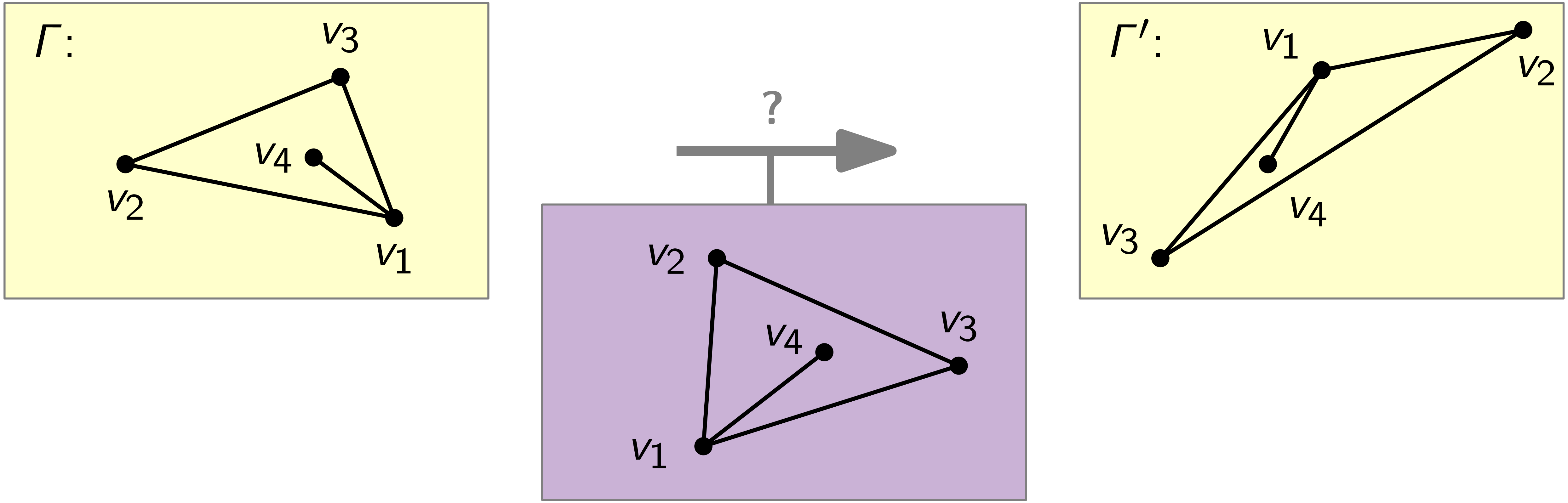

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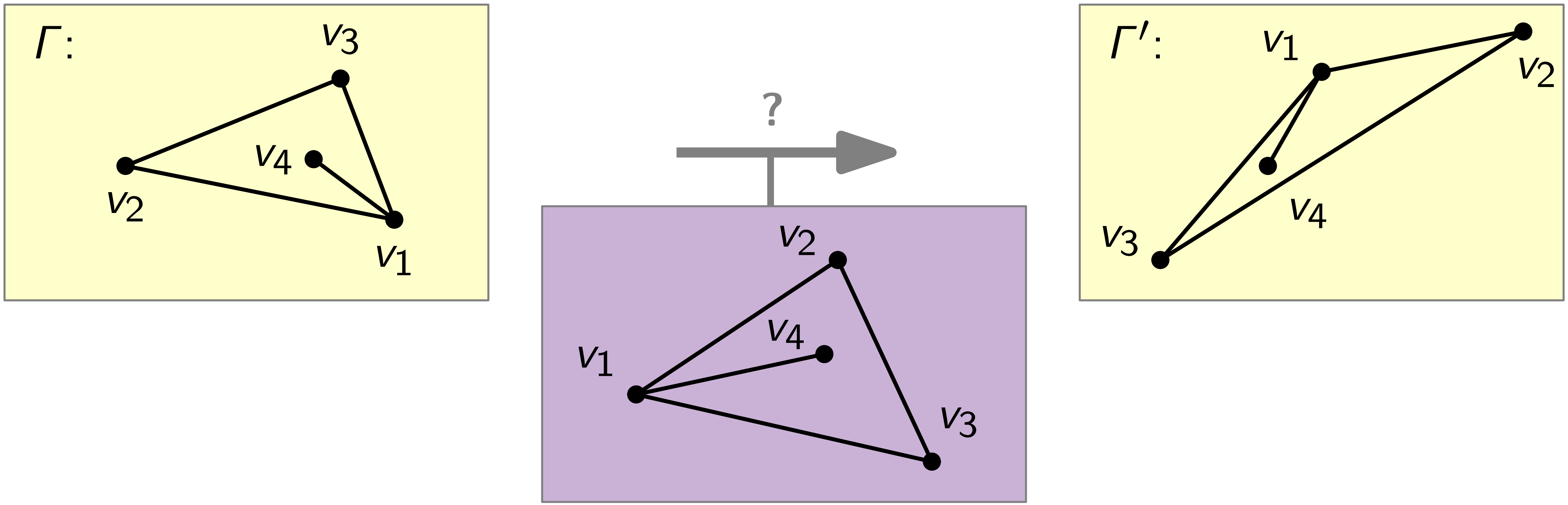

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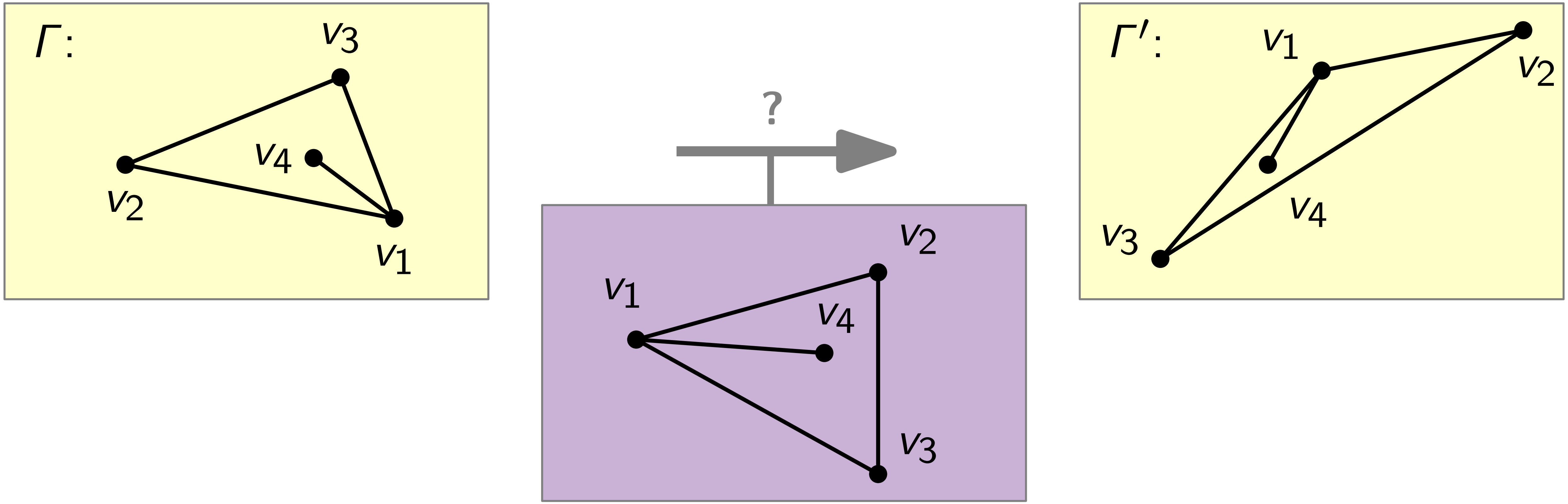

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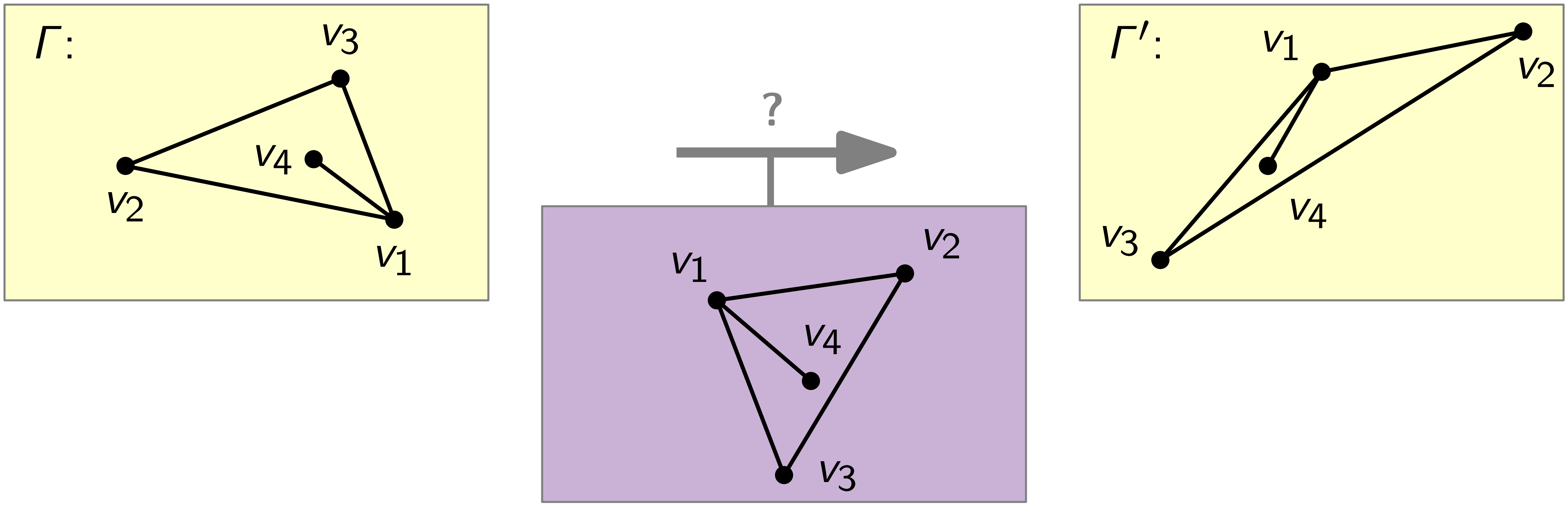

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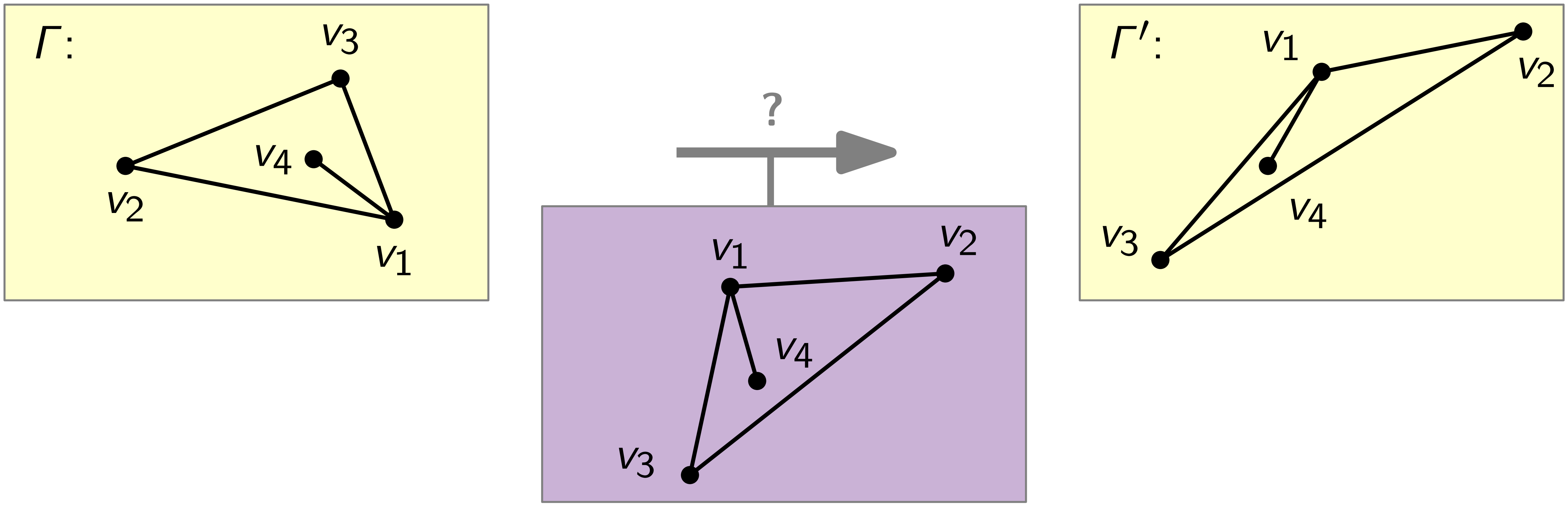

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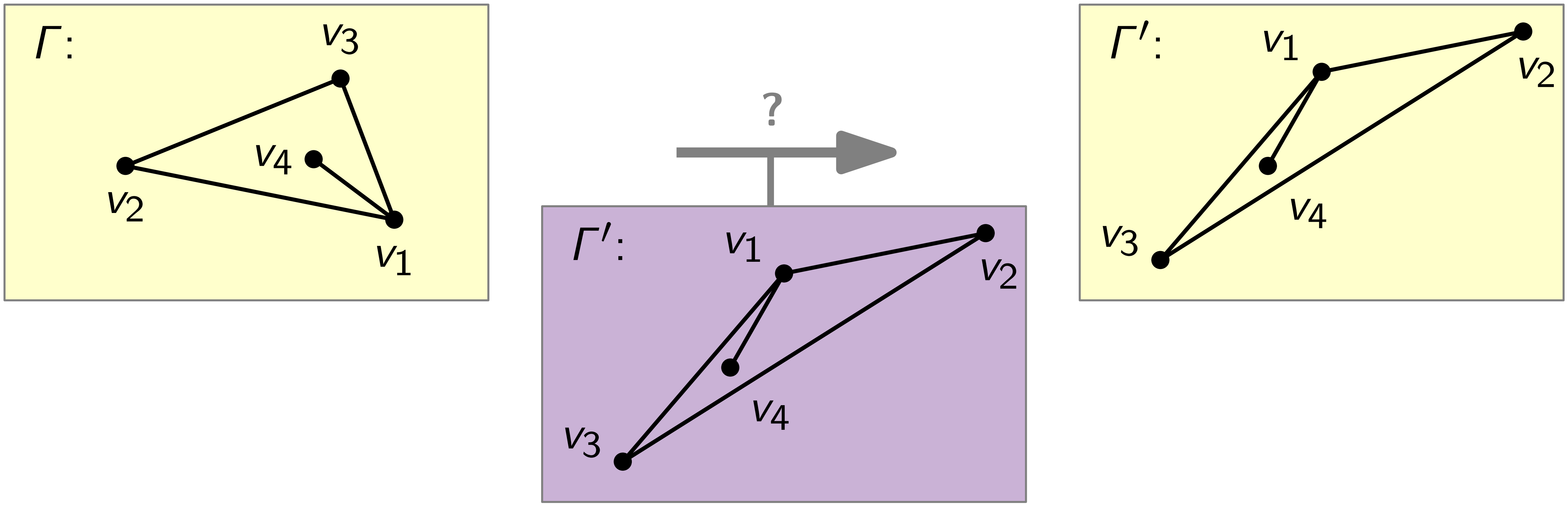

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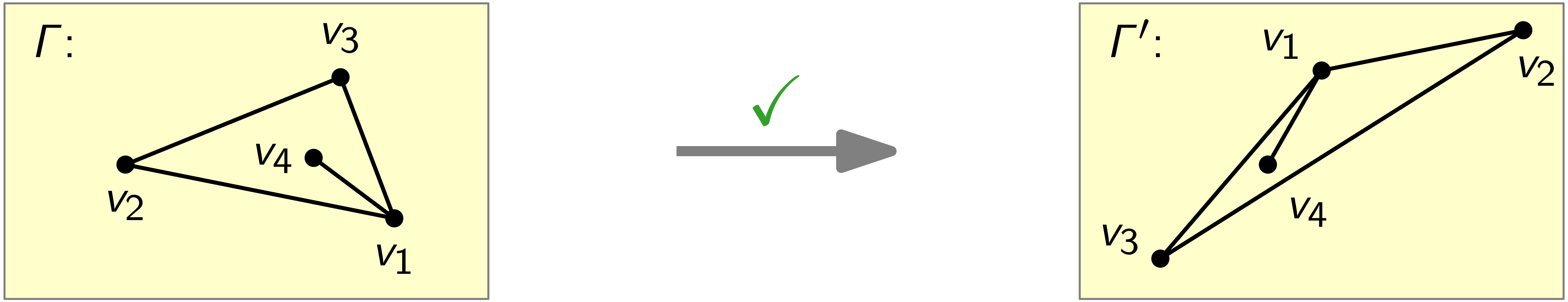

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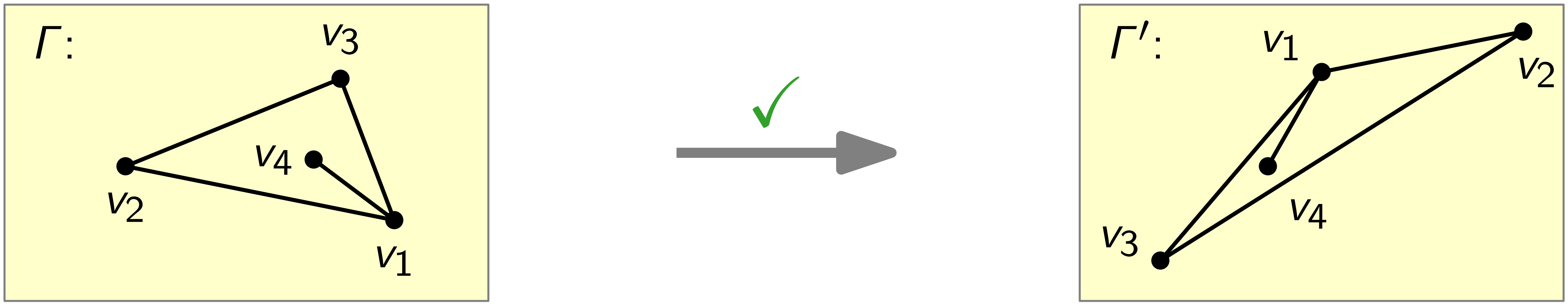

Theorem: It is sufficient that Γ and Γ' have the same planar embedding.
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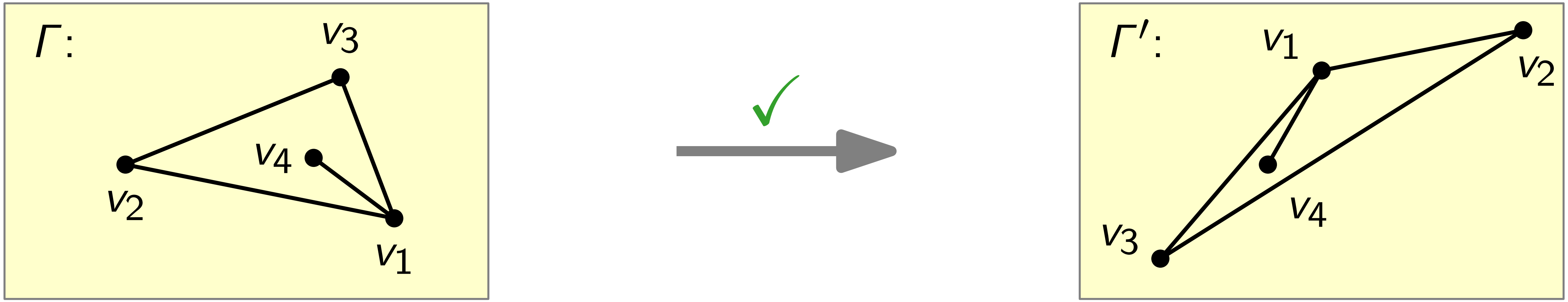

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Note: Checking if two planar drawings have the same planar embedding is in P.

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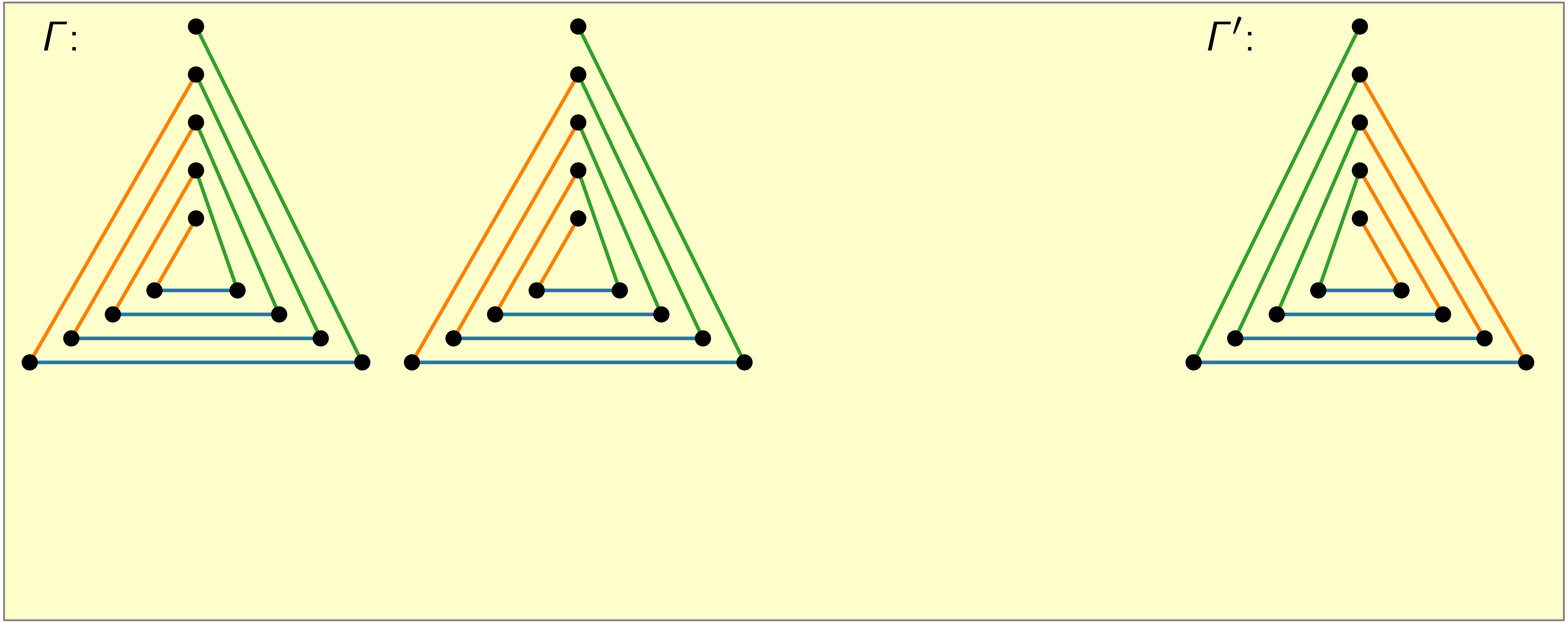

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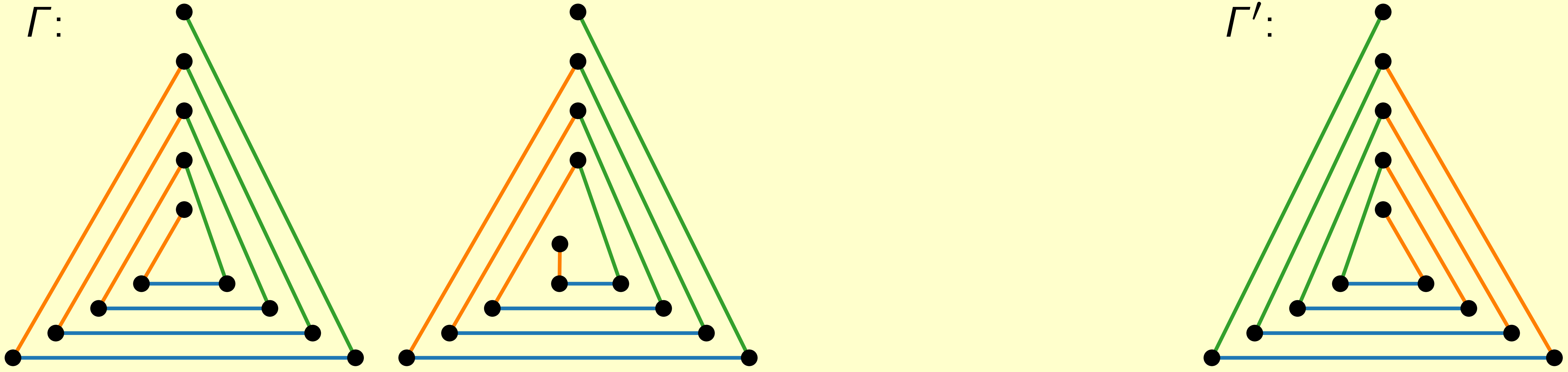

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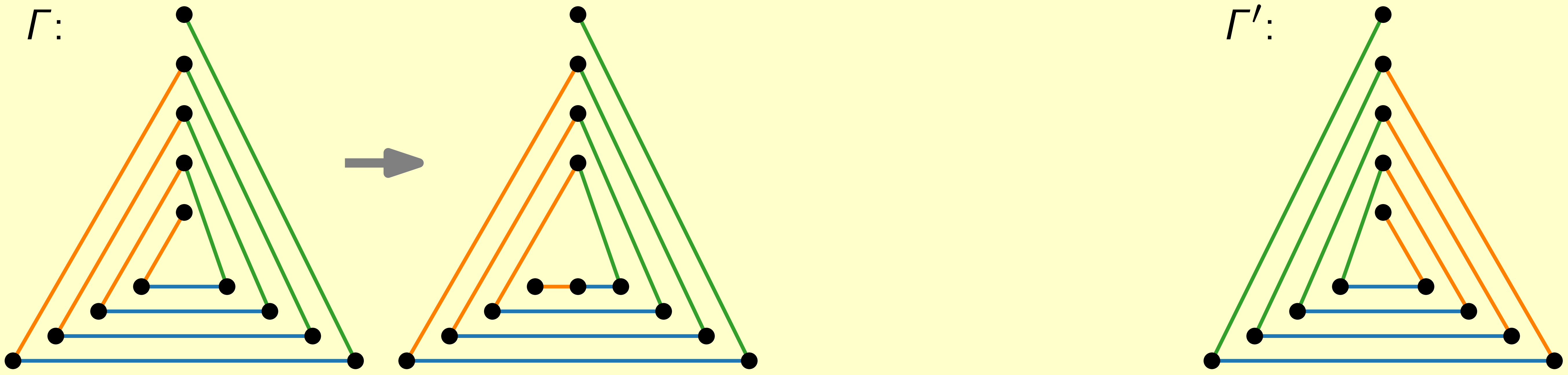

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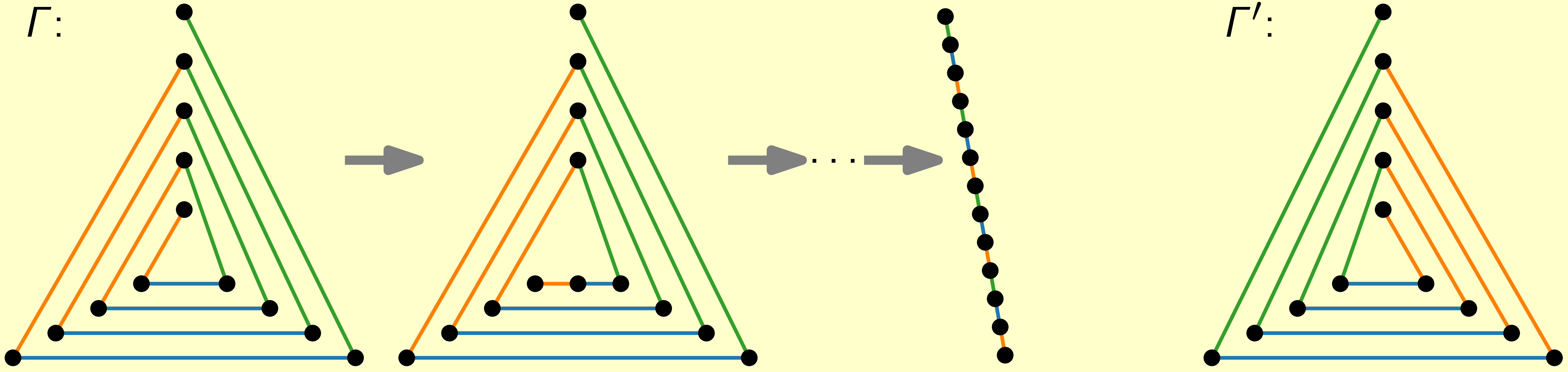

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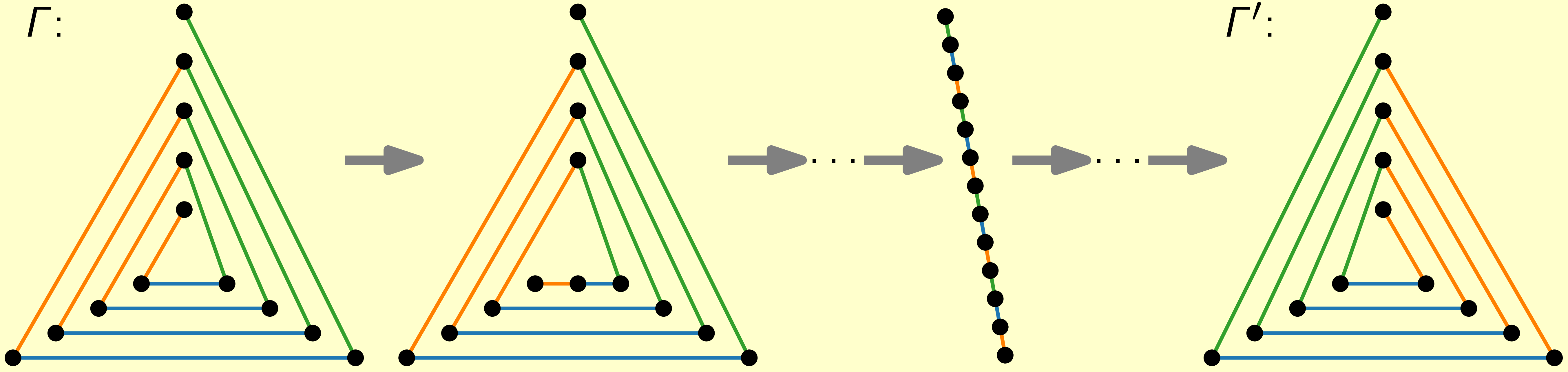

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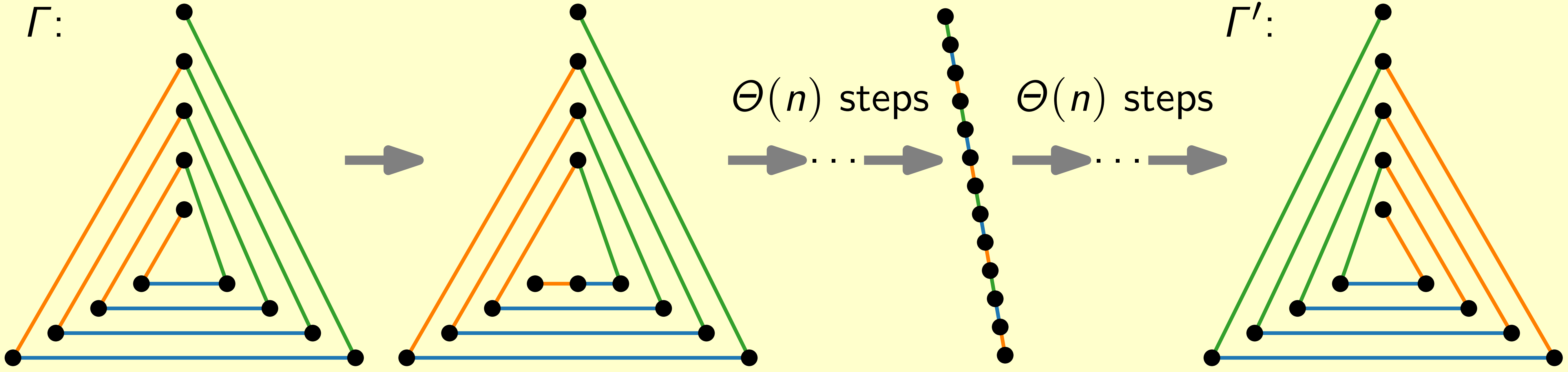

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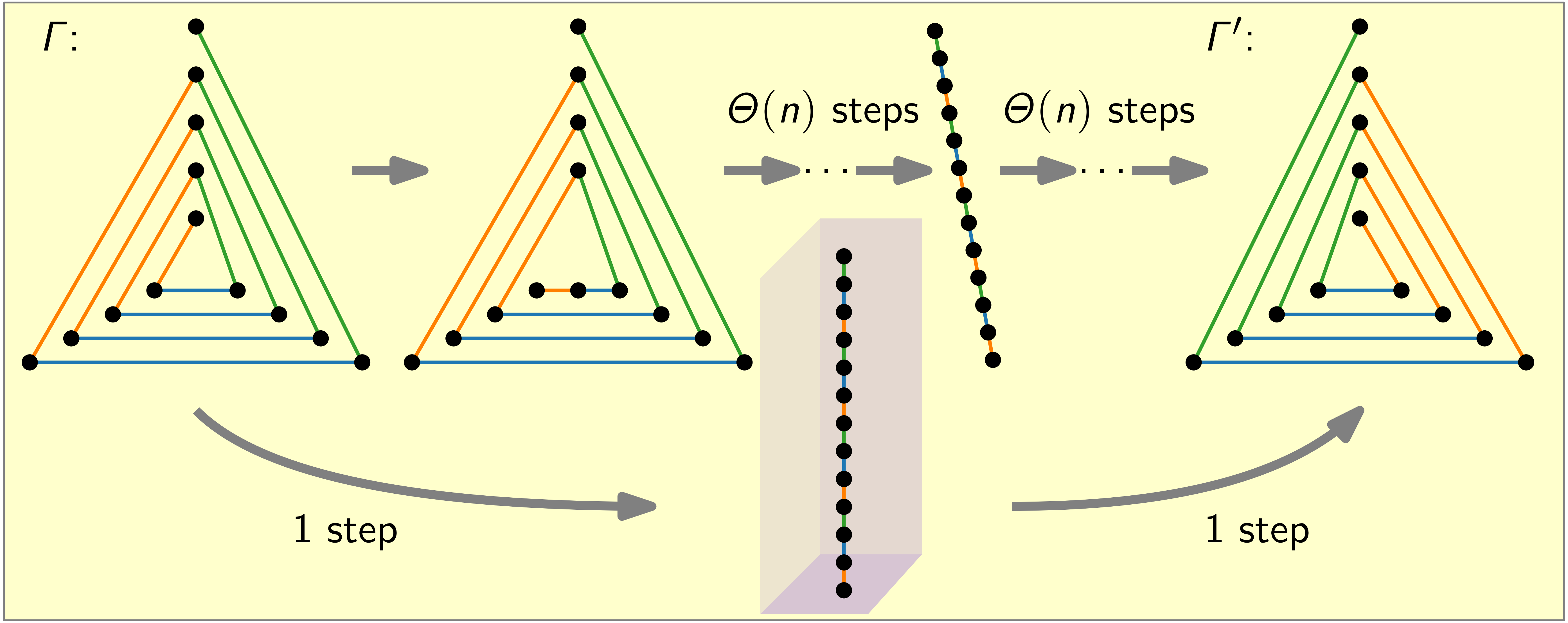

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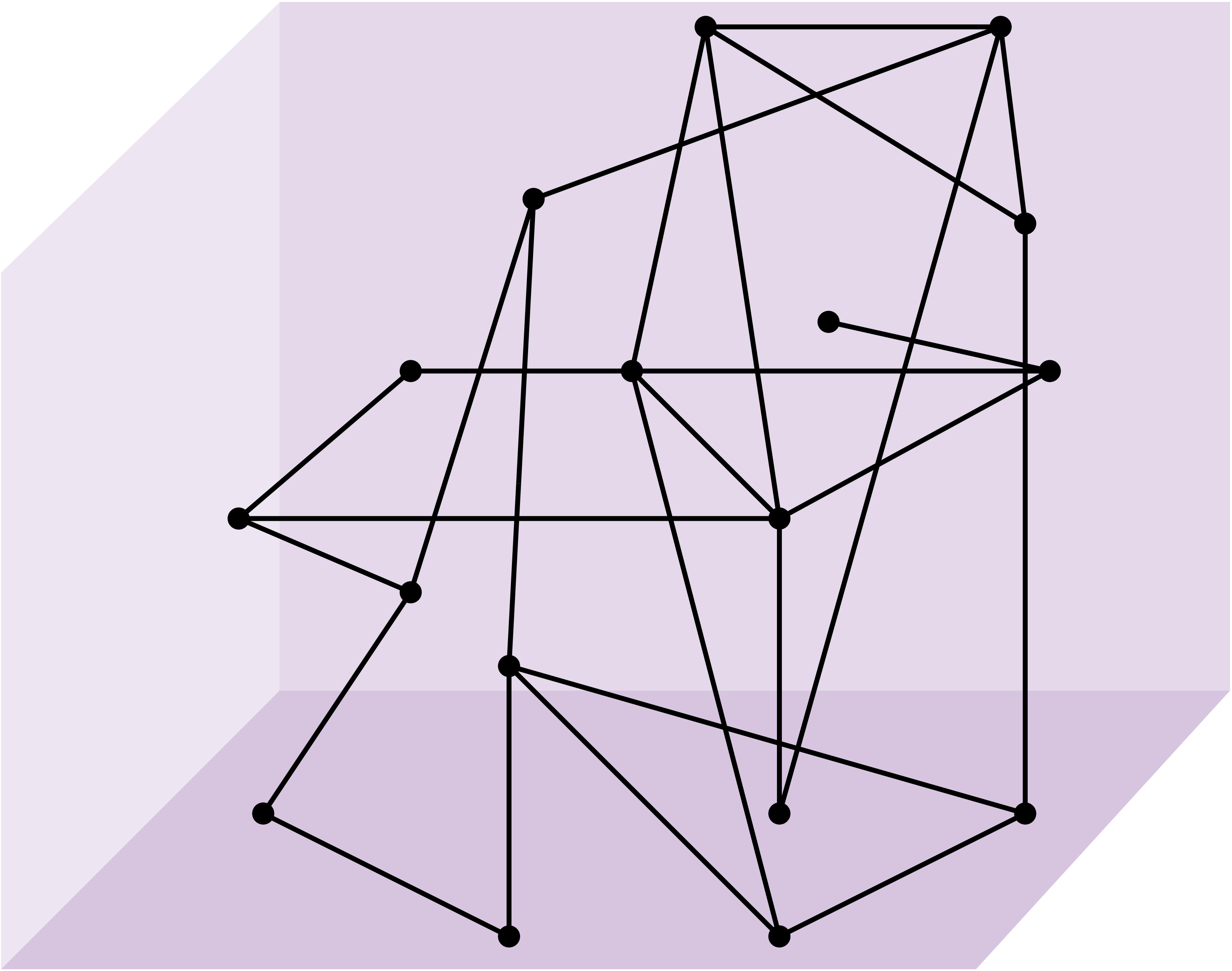

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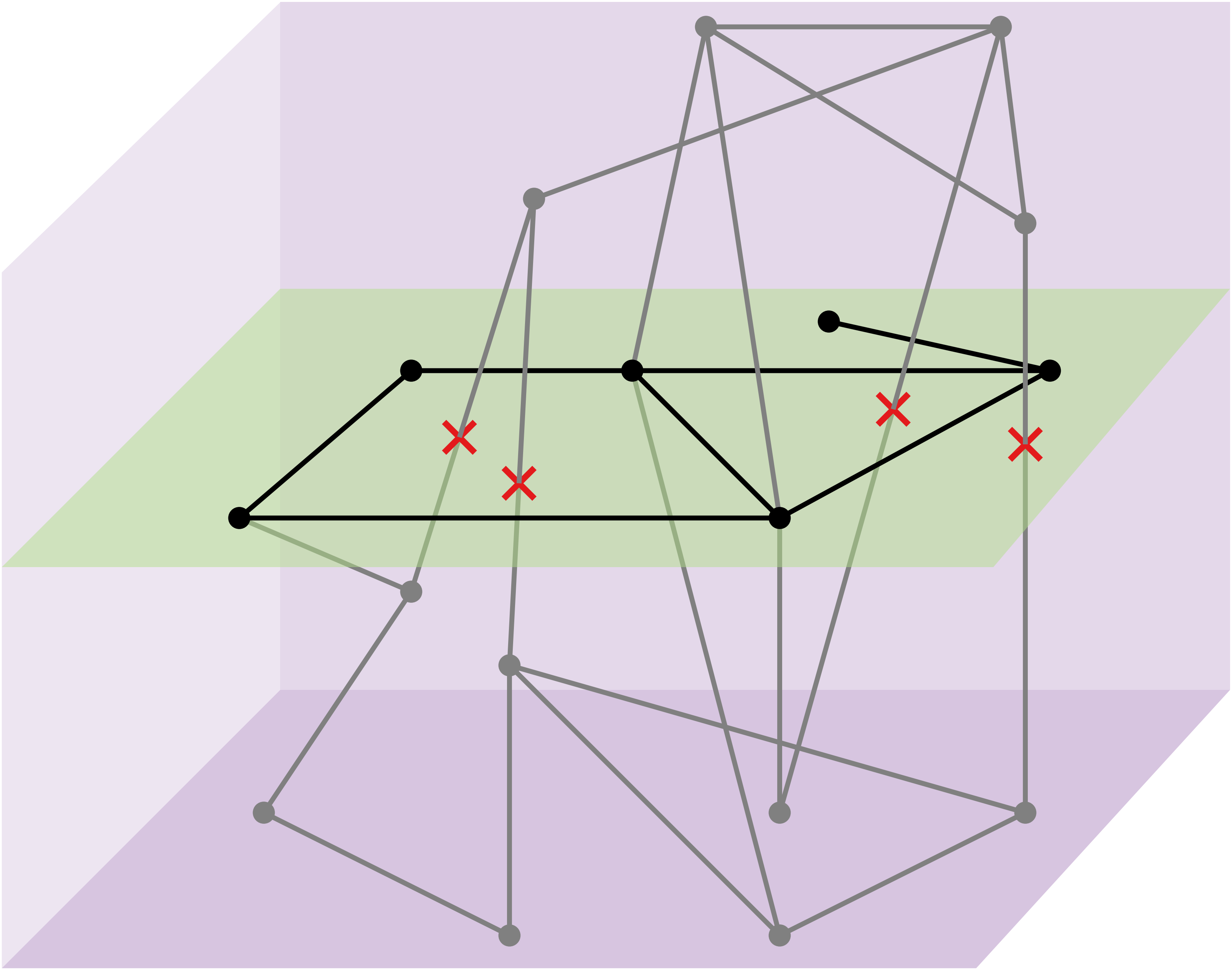

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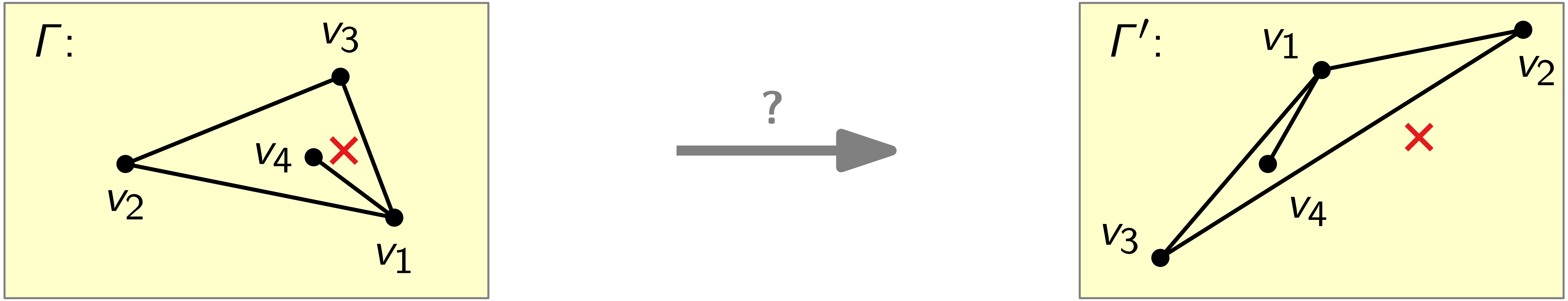

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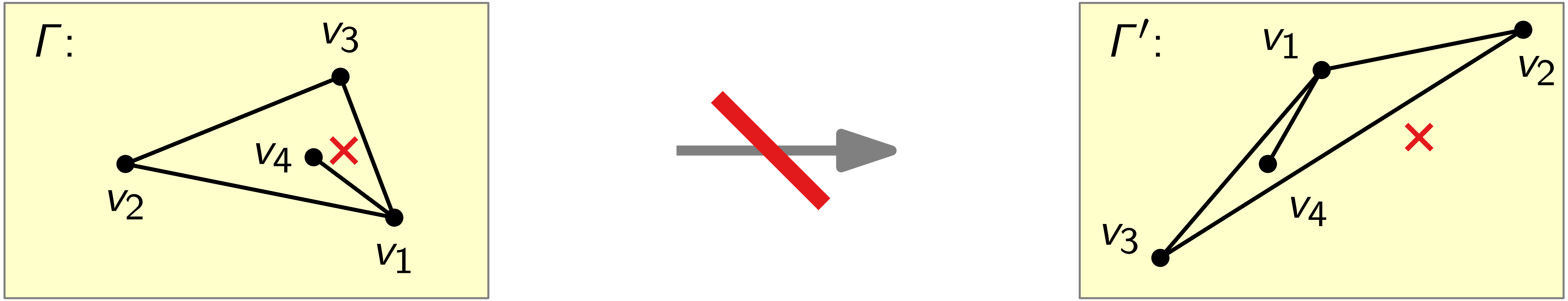

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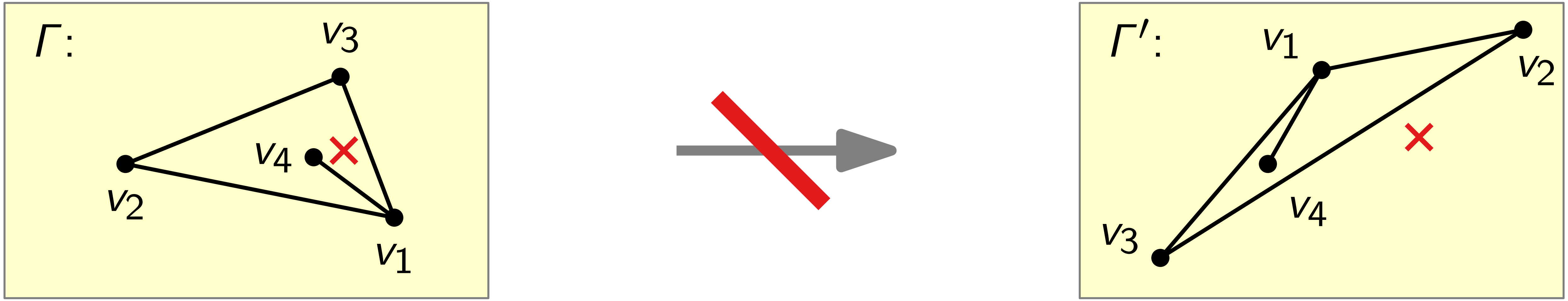

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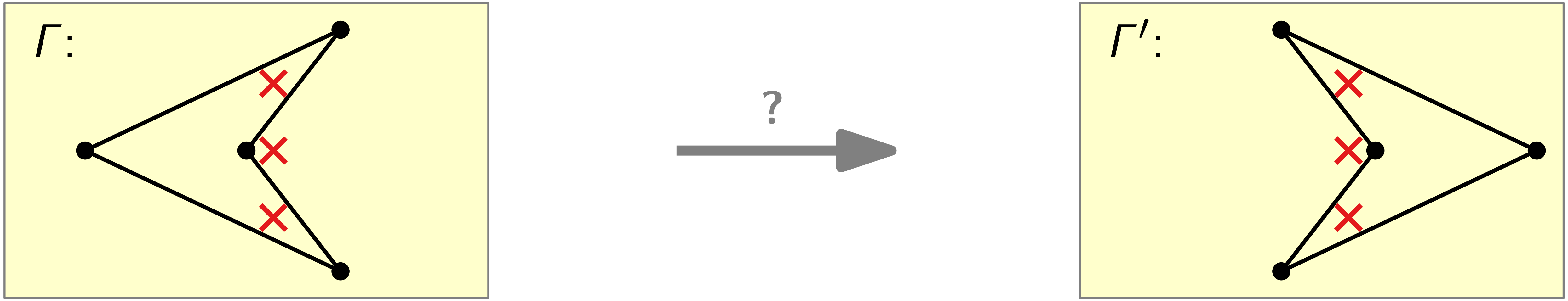

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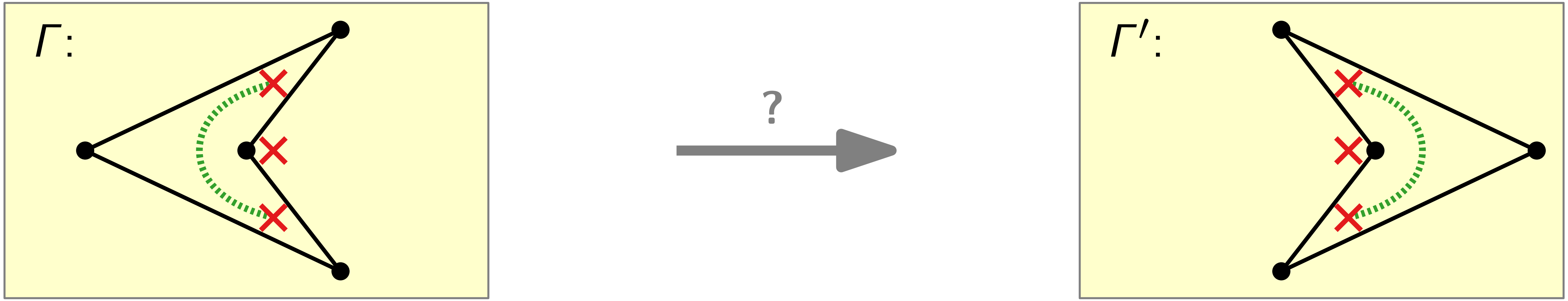

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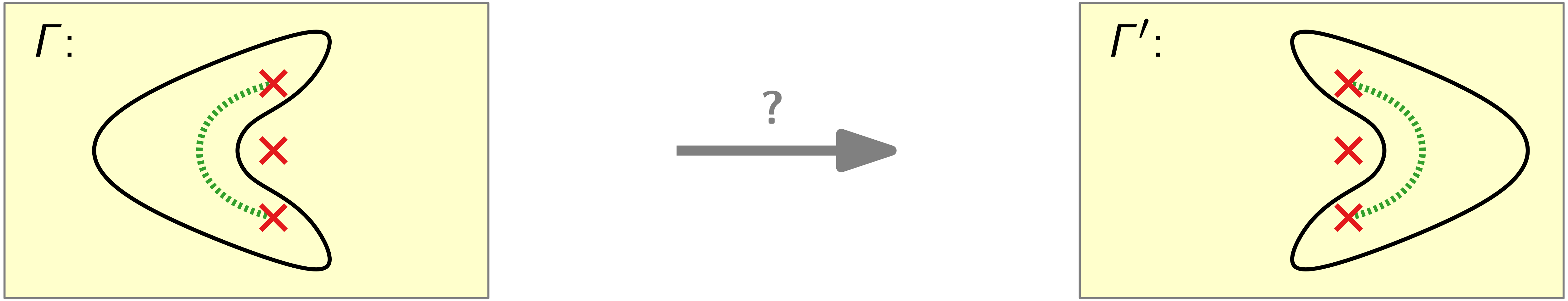

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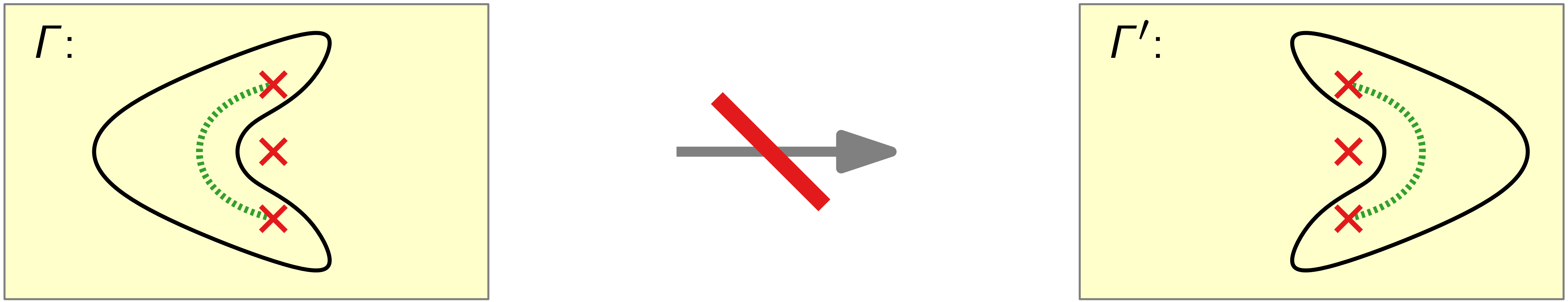

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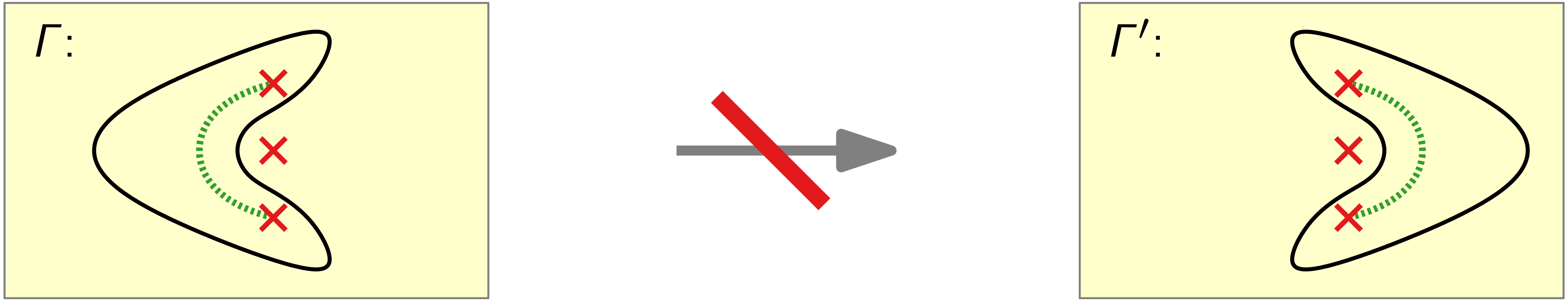

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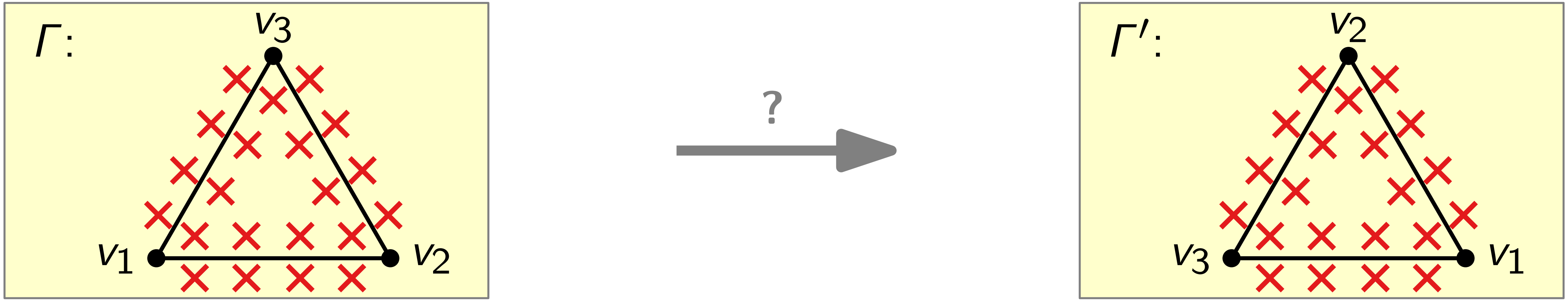

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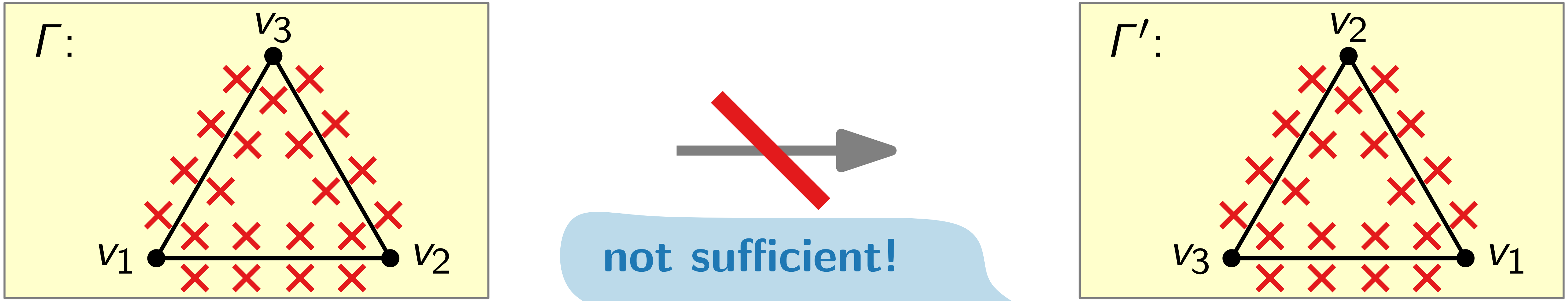

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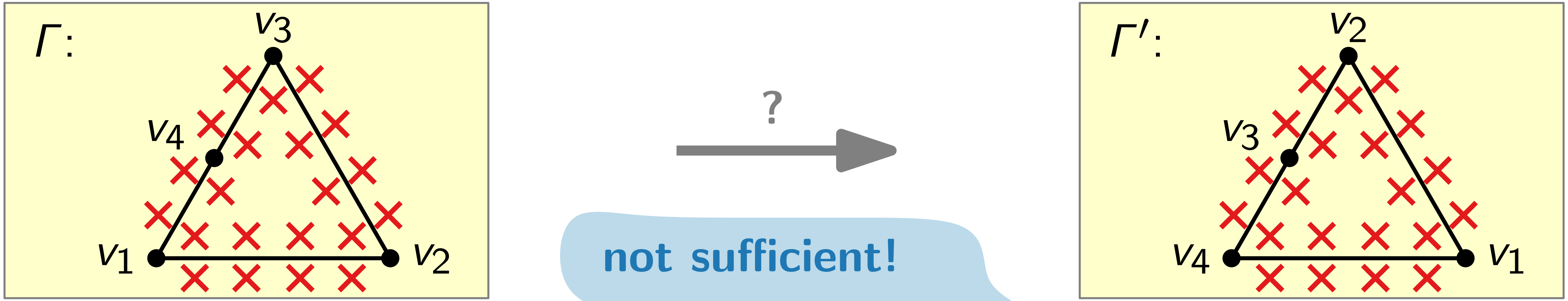

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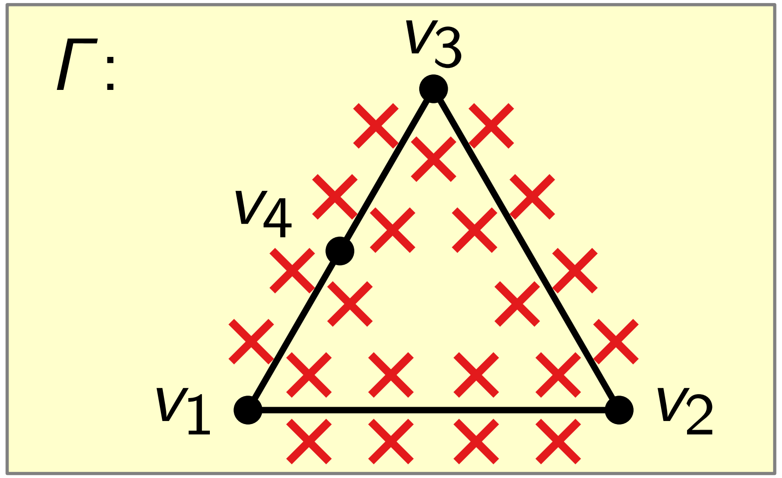

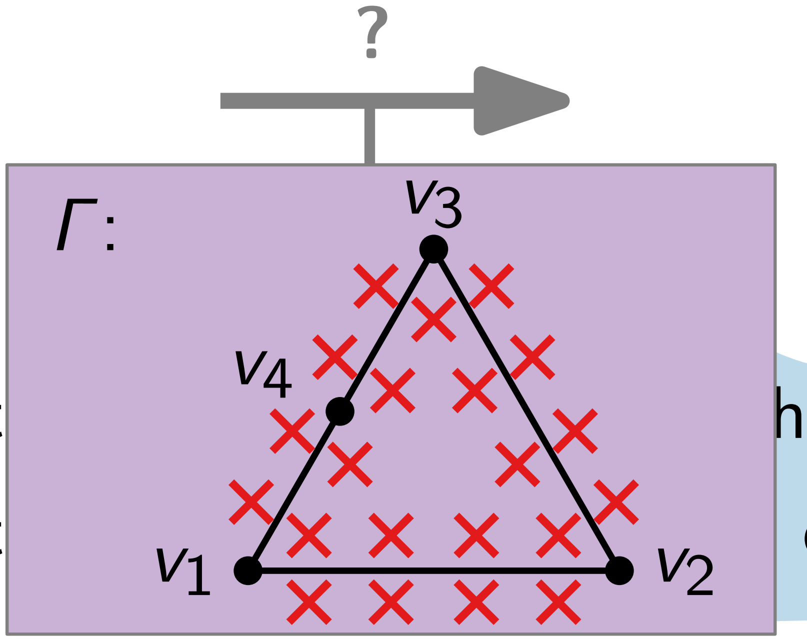

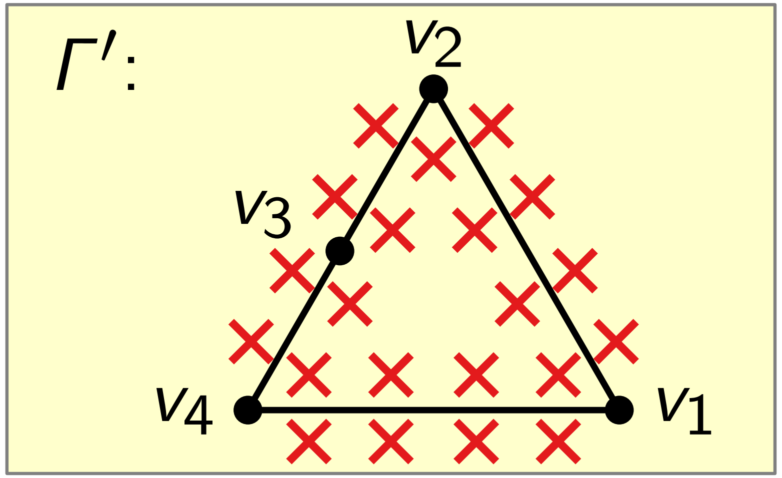

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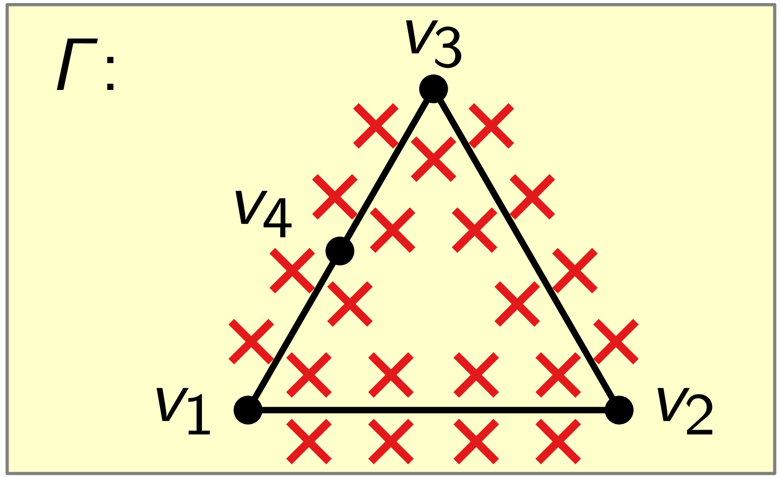

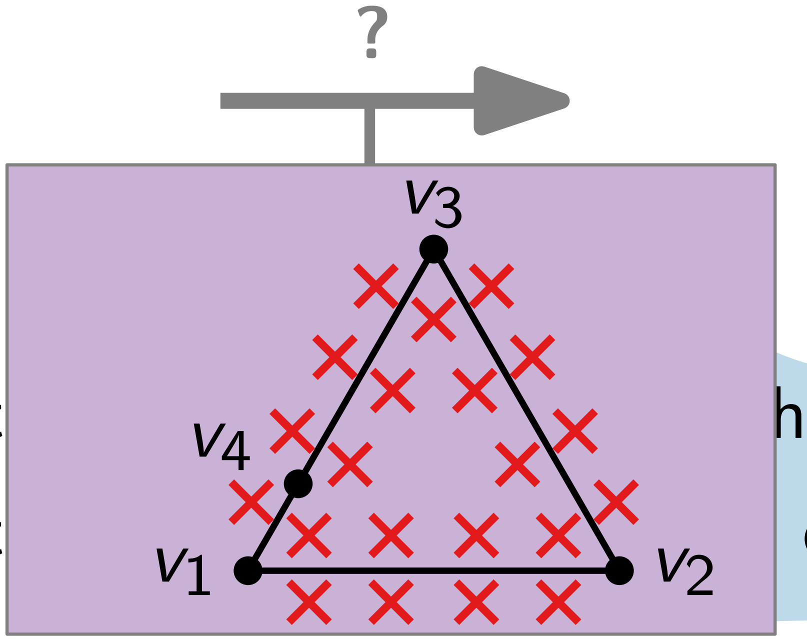

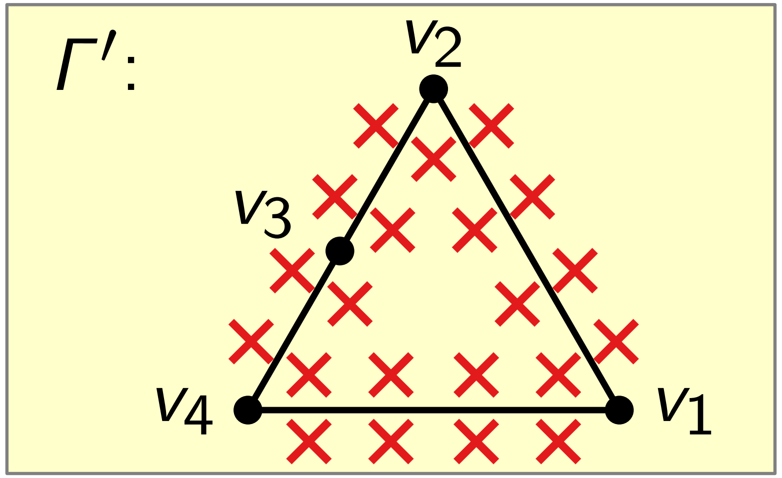

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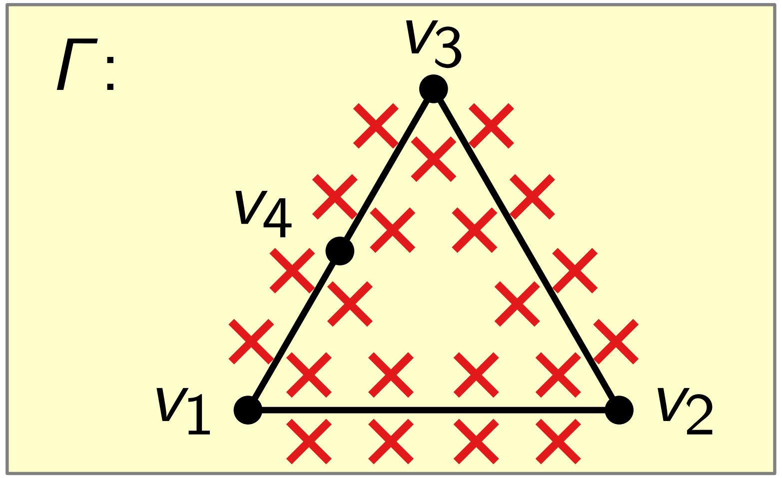

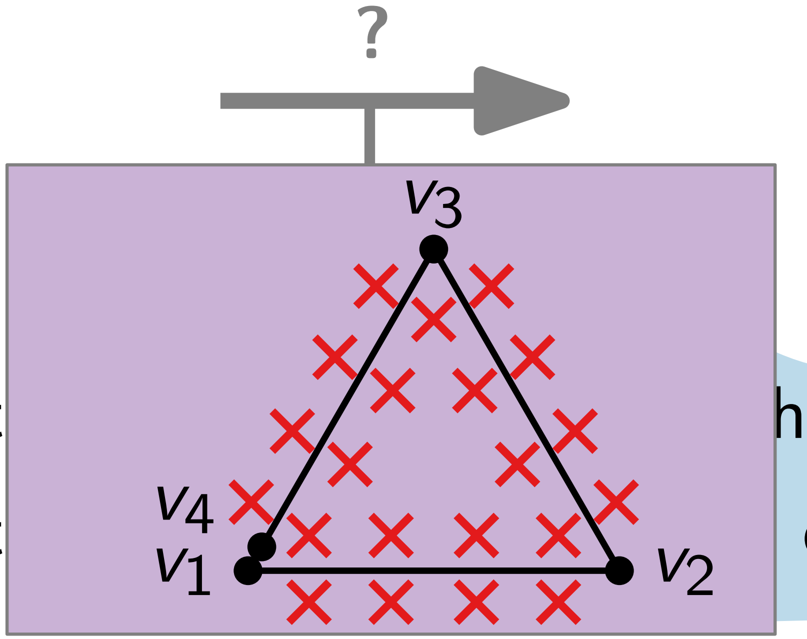

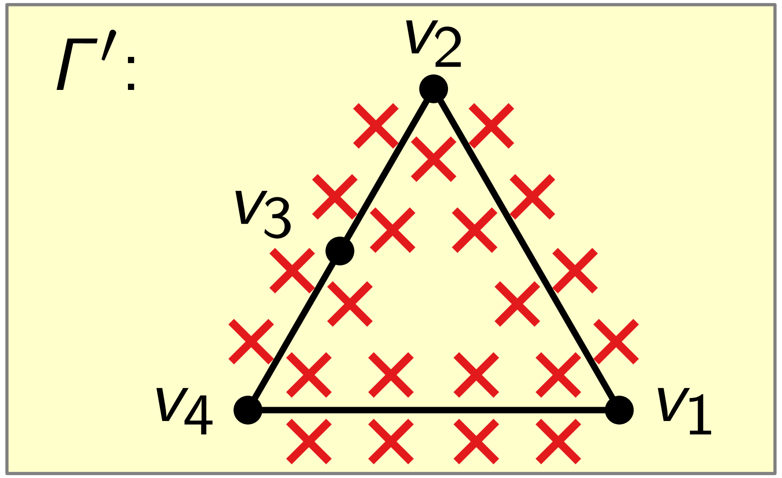

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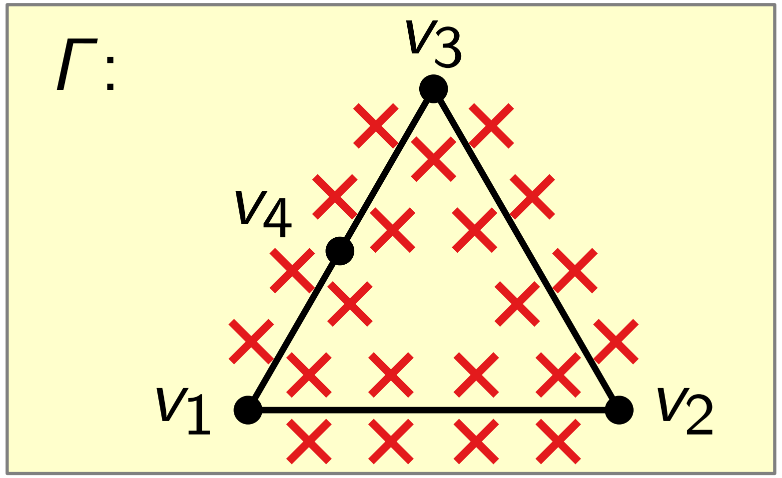

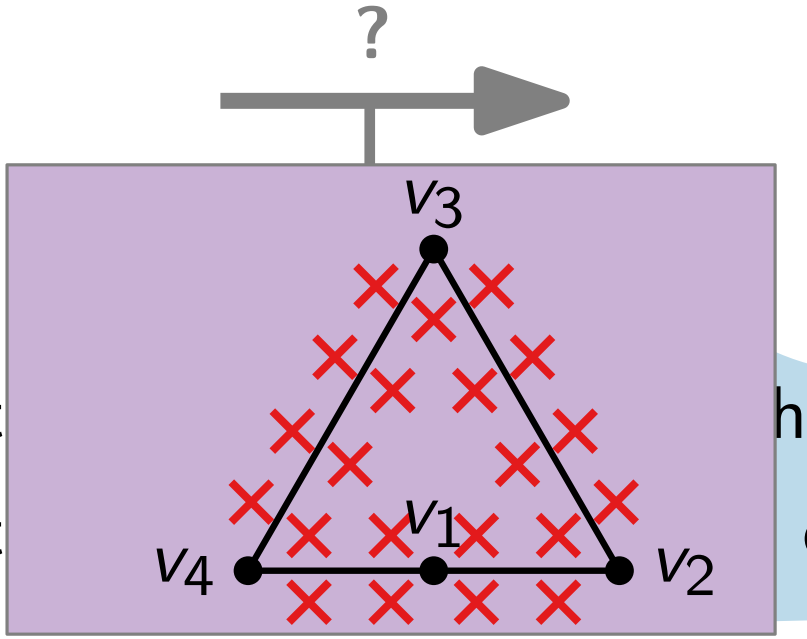

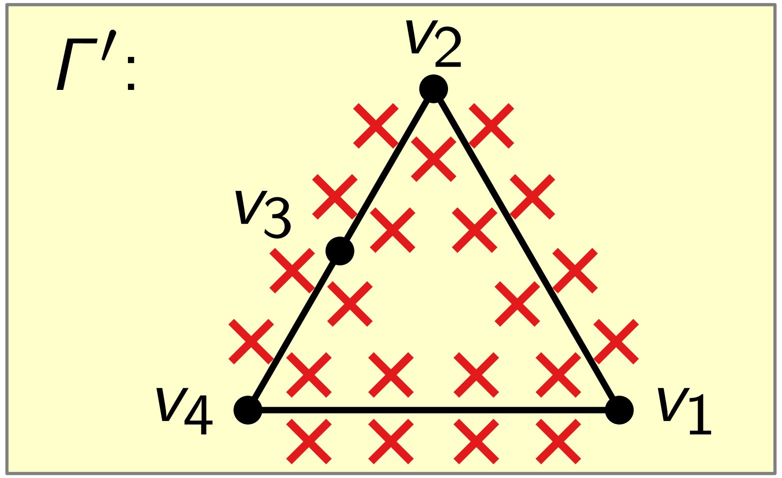

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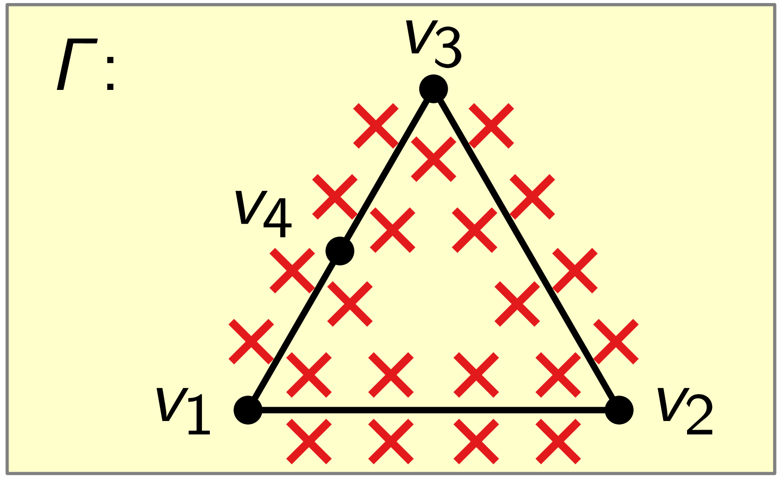

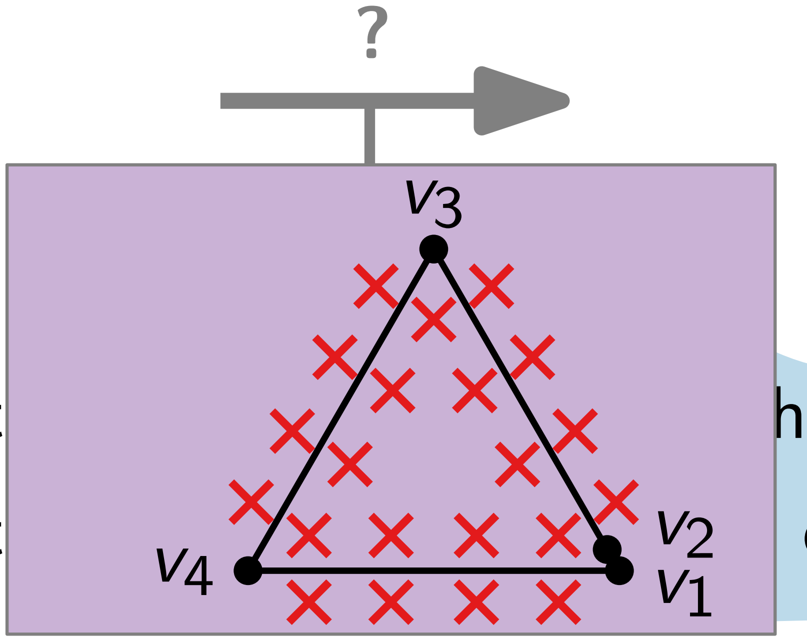

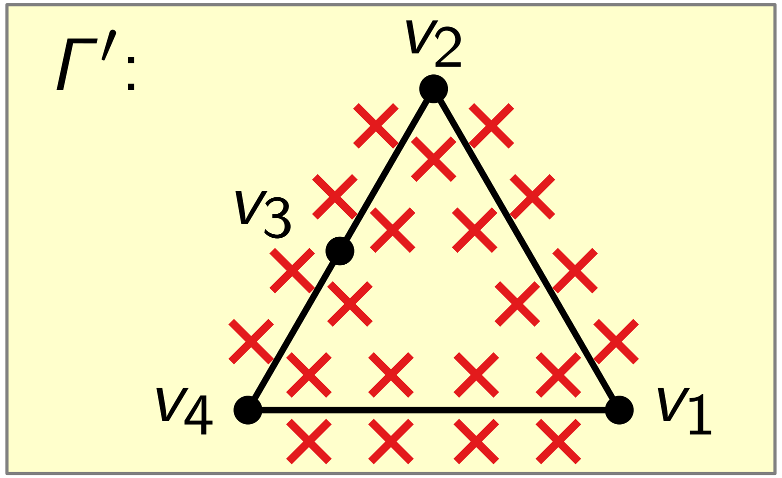

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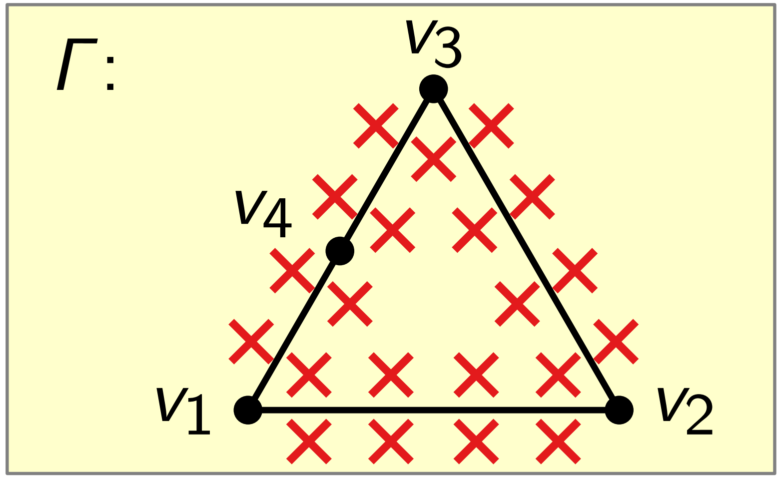

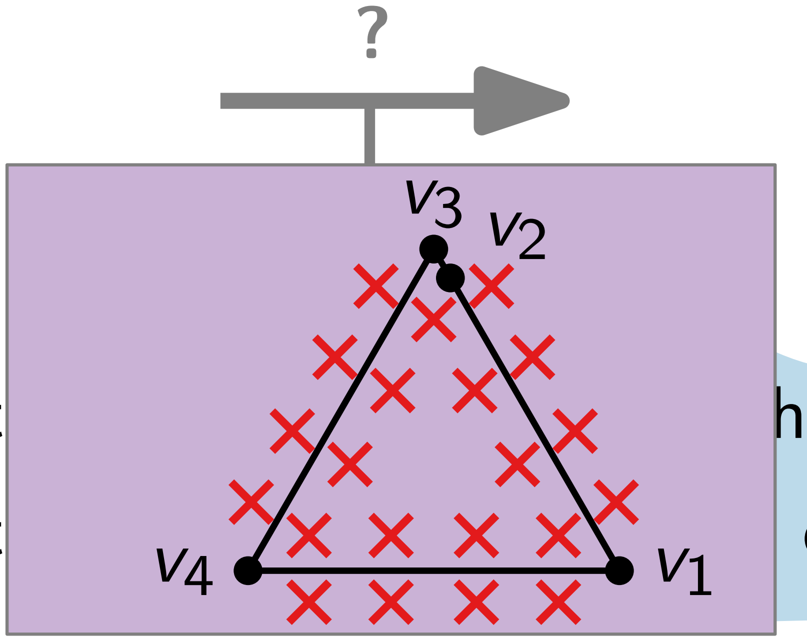

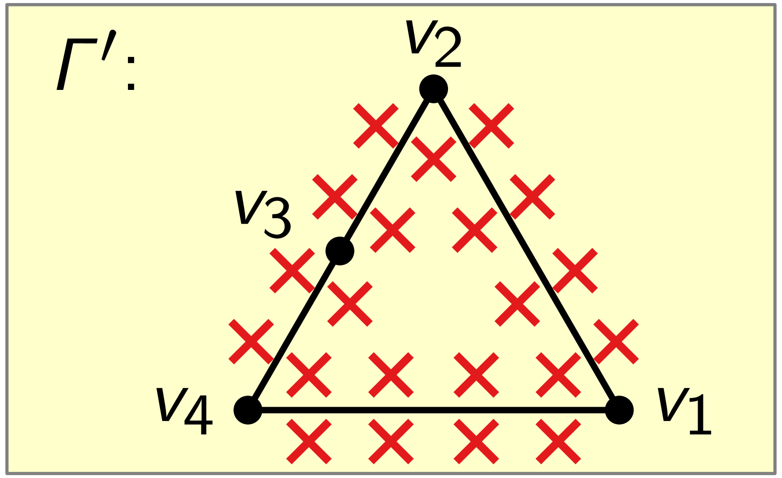

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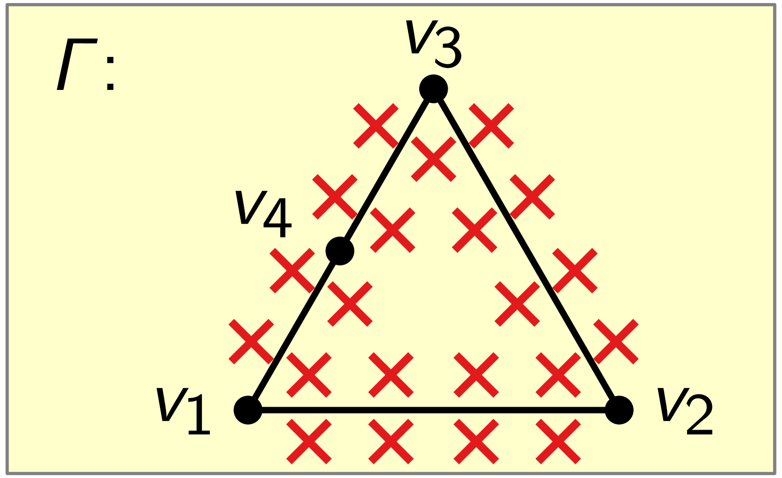

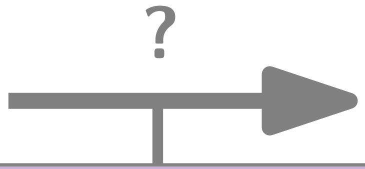

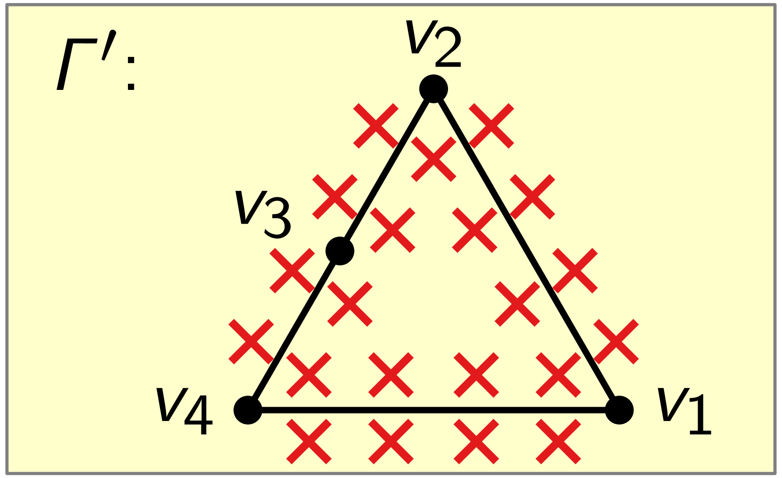

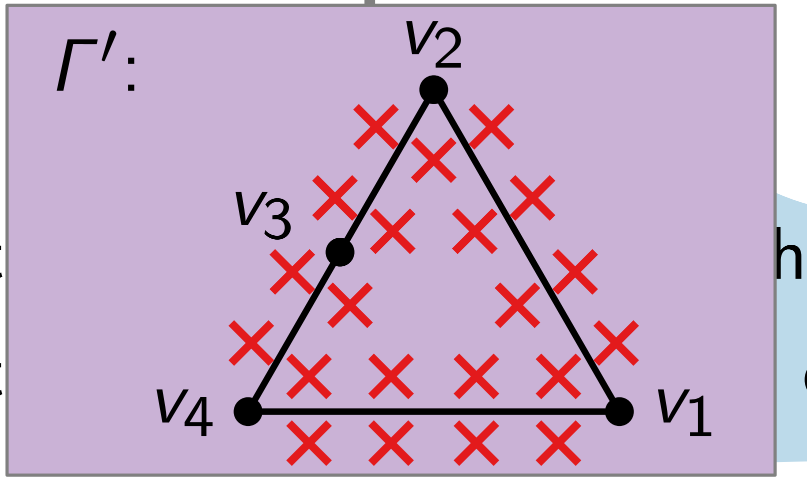

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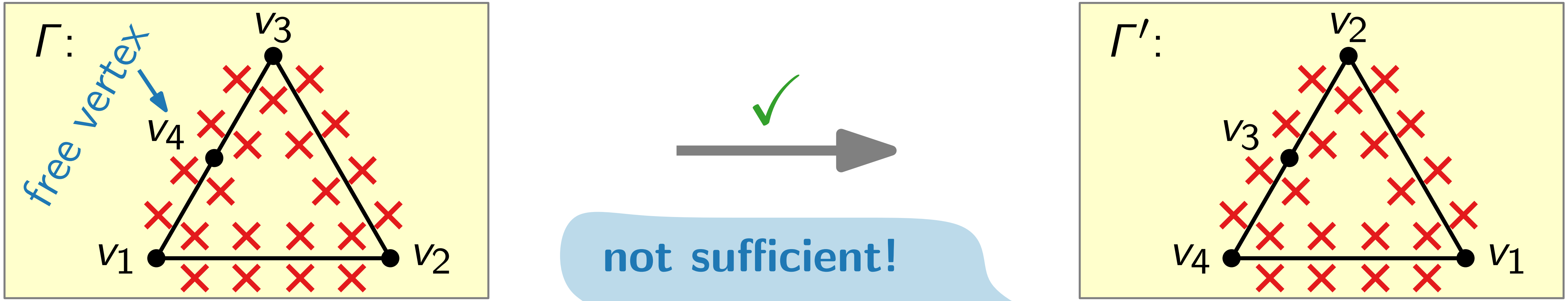

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- G is drawable with some desired properties if and only if I is a yes-instance.

The difficulty in this case:

- We need to construct two drawings Γ and Γ' based on I.
- Γ and Γ' need to exist regardless of whether I is a yes-instance or not.

Complexity of Morphing with Point Obstacles

Main Theorem: It is NP-hard to decide whether there exists an obstacle-avoiding planar straight-line morph in $\mathbb{R}^2$ between Γ and Γ'.

What a "typical" NP-hardness reduction in graph drawing looks like:

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- There is always an obstacle-avoiding continuous deformation from Γ to Γ'.
- There is an obstacle-avoiding planar straight-line morph iff I is a yes-instance.

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Proof idea.

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- Γ and Γ' are identical except for the positions of four vertices.

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Proof idea.

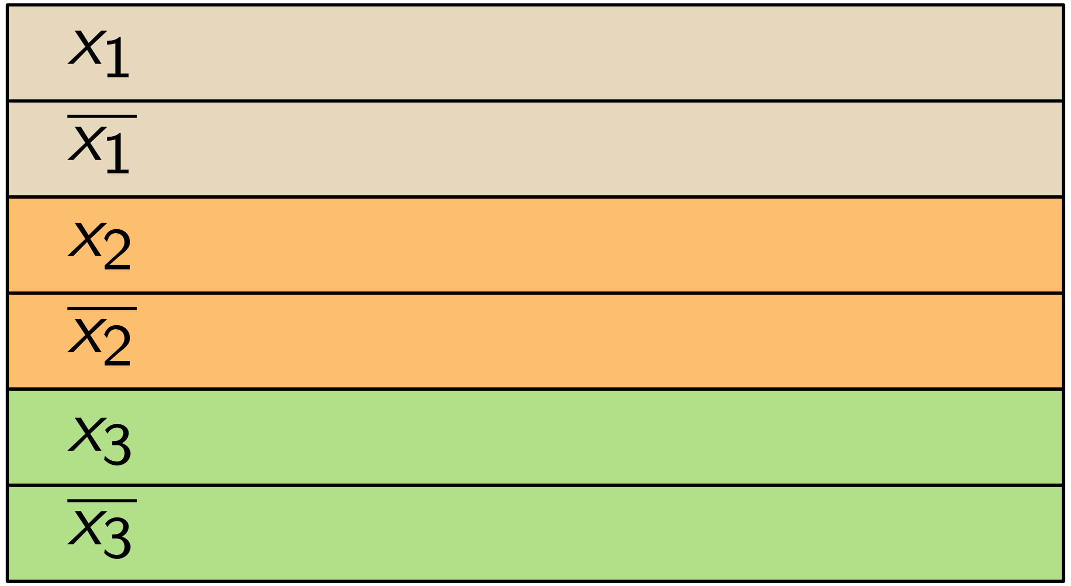
- Reduction from 3-SAT.
- We construct Γ and Γ' based on a given Boolean formula in CNF.
- Γ and Γ' are identical except for the positions of four vertices.
- The obstacles are arranged to form a grid-like tunnel structure.

Complexity of Morphing with Point Obstacles

Main Theorem: It is NP-hard to decide whether there exists an obstacle-avoiding planar straight-line morph in $\mathbb{R}^2$ between Γ and Γ'.

Proof idea.

- Two rows for each variable (one per literal).



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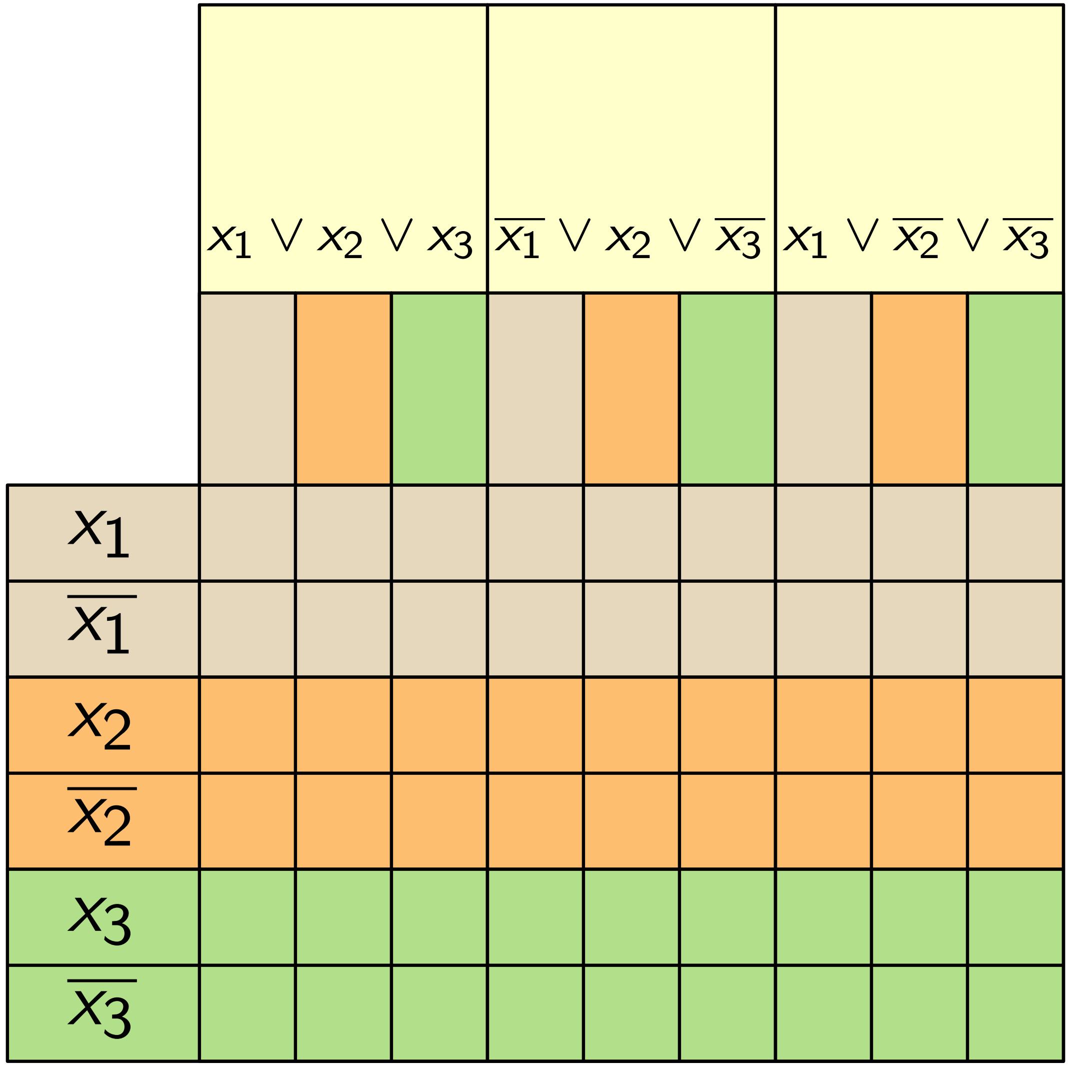
Proof idea.

- Two rows for each variable (one per literal).
- Three columns for each clause (one per literal).



Complexity of Morphing with Point Obstacles

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Proof idea.

- Two rows for each variable (one per literal).
- Three columns for each clause (one per literal).
- Split gadget if same literal in row & column; crossing gadget otherwise

	$x_1 \lor x_2 \lor x_3$			$\overline{x_1} \lor x_2 \lor \overline{x_3}$			$x_1 \lor \overline{x_2} \lor \overline{x_3}$		
x_1	S						S		
$\overline{x_1}$				S					
x_2		S						S	
$\overline{x_2}$					S				
x_3			S						
$\overline{x_3}$						S			S

Complexity of Morphing with Point Obstacles

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Proof idea.

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	$x_1 \vee x_2 \vee x_3$			$\overline{x_1} \vee x_2 \vee \overline{x_3}$			$x_1 \vee \overline{x_2} \vee \overline{x_3}$		
x_1	S	C	C	C	C	C	S	C	C
$\overline{x_1}$	C	C	C	S	C	C	C	C	C
x_2	C	S	C	C	C	C	C	S	C
$\overline{x_2}$	C	C	C	C	S	C	C	C	C
x_3	C	C	S	C	C	C	C	C	C
$\overline{x_3}$	C	C	C	C	C	S	C	C	S

Complexity of Morphing with Point Obstacles

Main Theorem: It is NP-hard to decide whether there exists an obstacle-avoiding planar straight-line morph in $\mathbb{R}^2$ between Γ and Γ'.

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- Two rows for each variable (one per literal).
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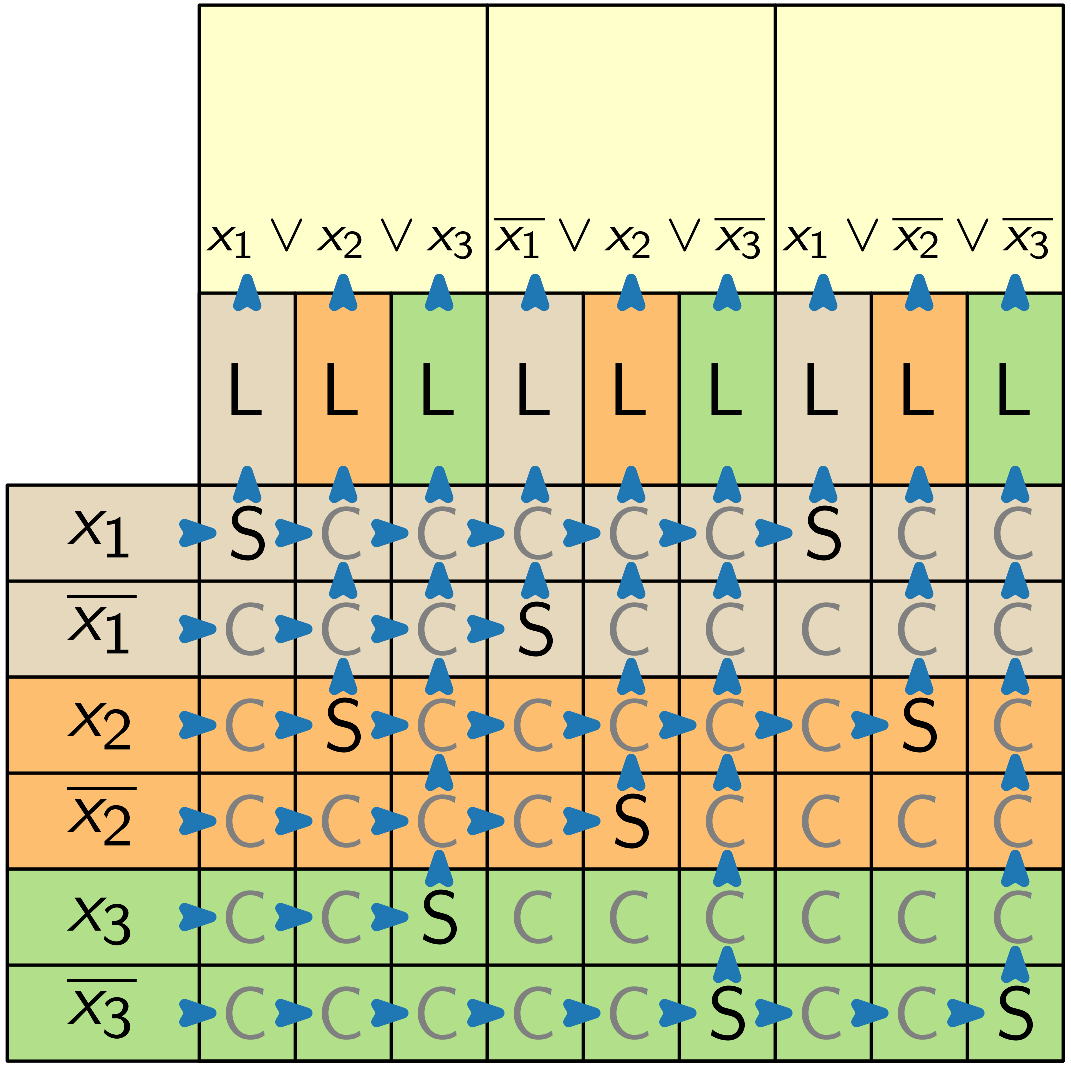
- Split gadget if same literal in row & column; crossing gadget otherwise.
- Free vertices can be passed from variable gadgets along rows and columns to literal gadgets.



Complexity of Morphing with Point Obstacles

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Proof idea.

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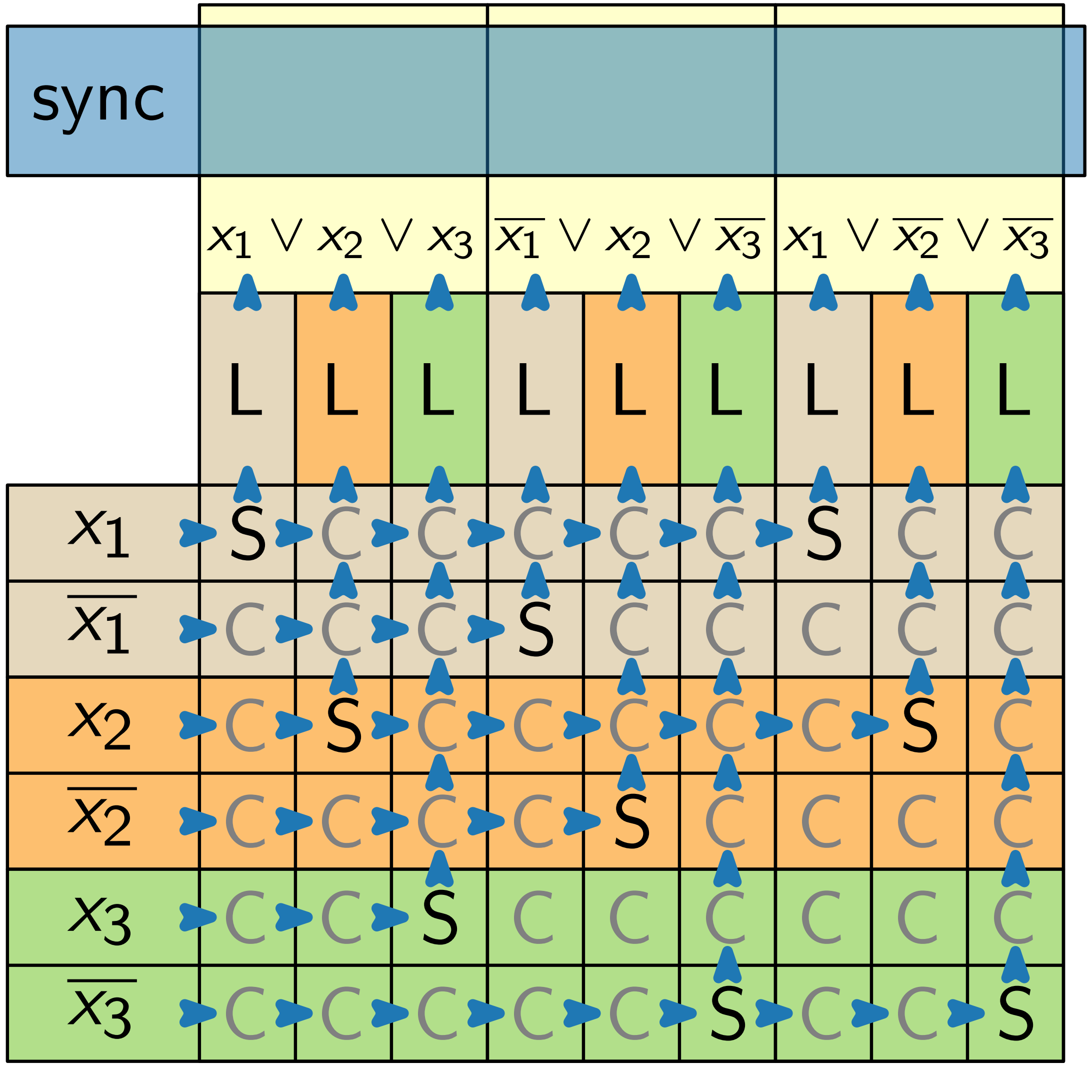
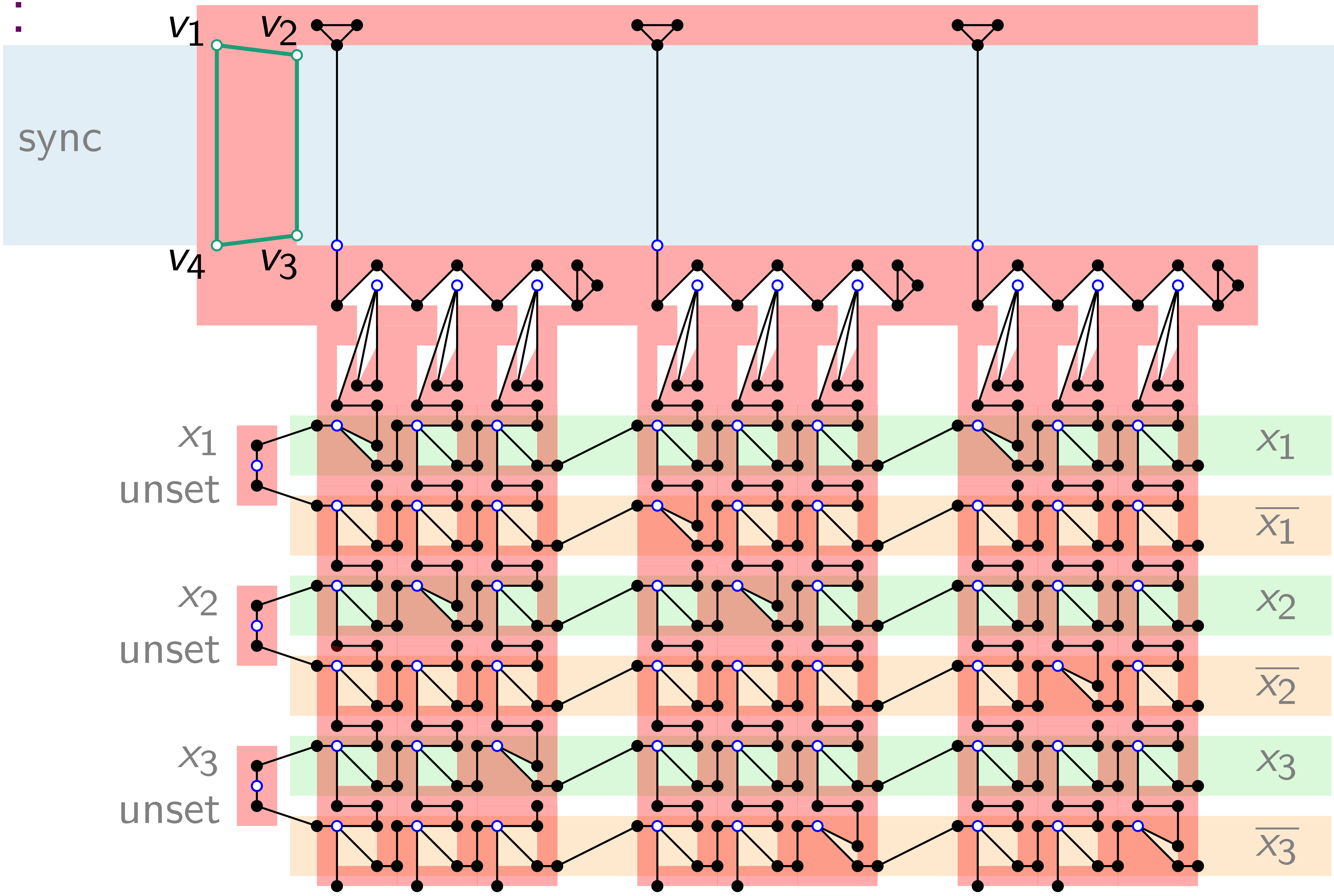
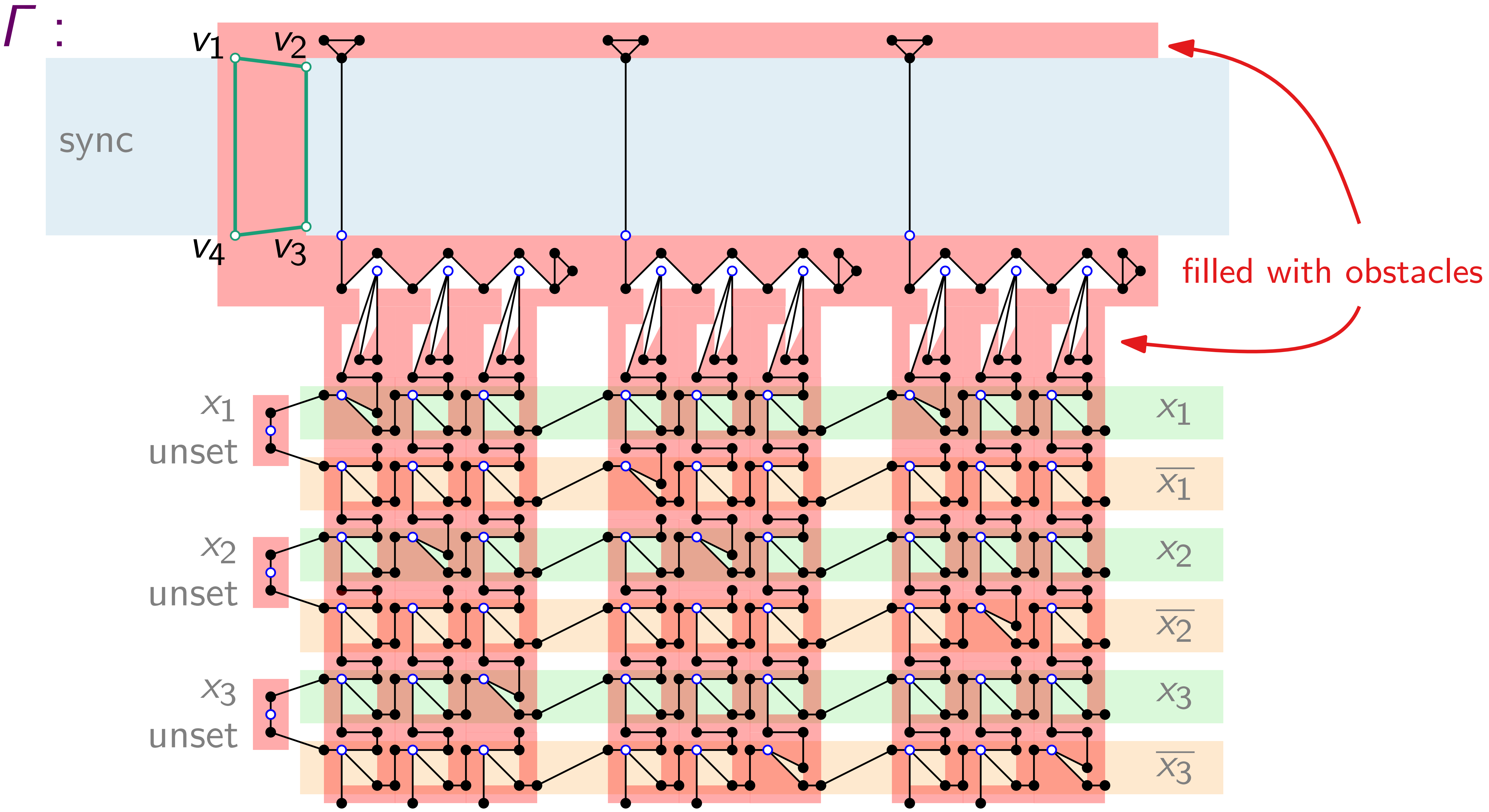
- Three columns for each clause (one per literal).
- Split gadget if same literal in row & column; crossing gadget otherwise.
- Free vertices can be passed from variable gadgets along rows and columns to literal gadgets.
- Synchronization gadget assures consistent assignment of variables.

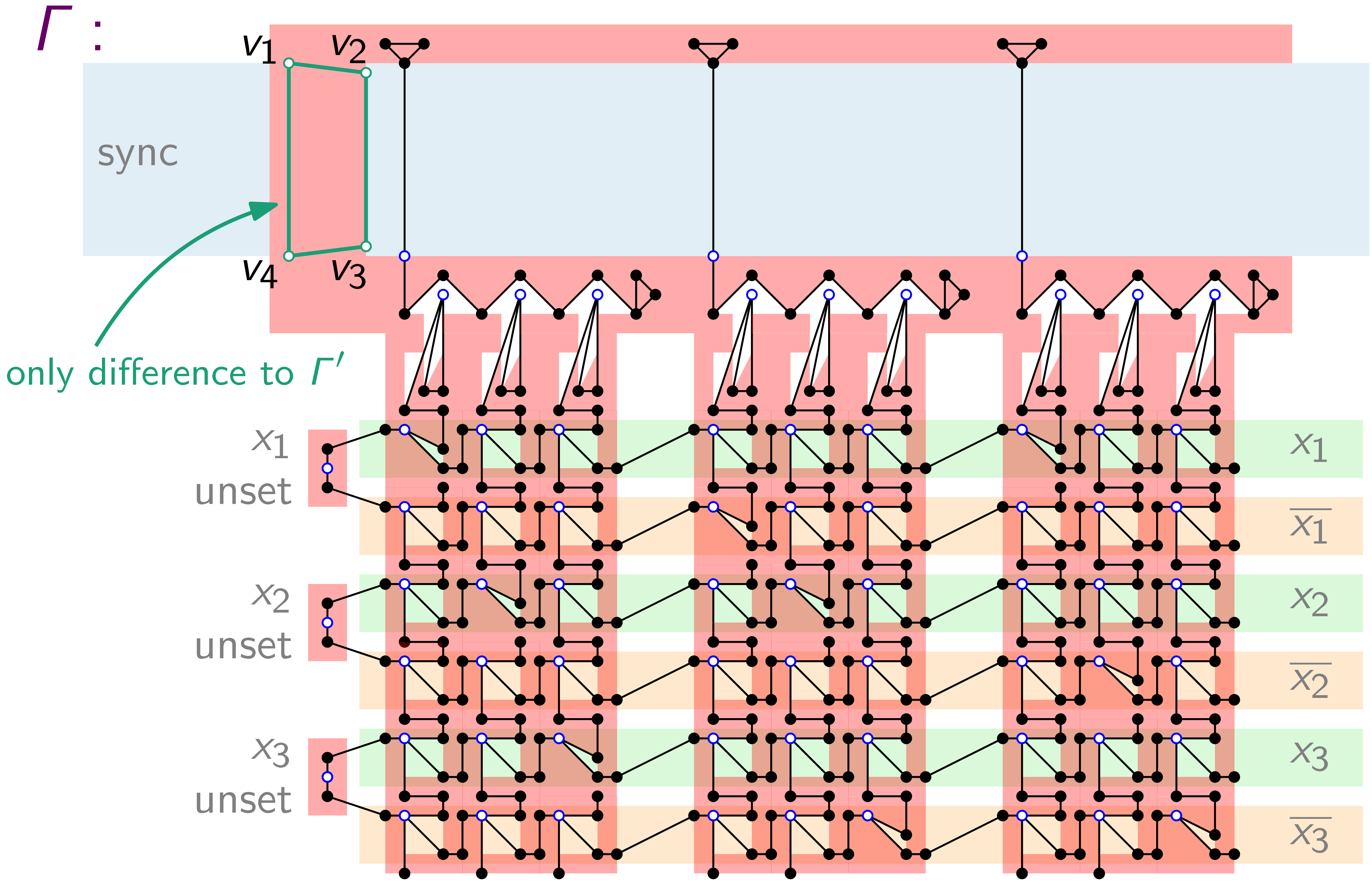


Γ :

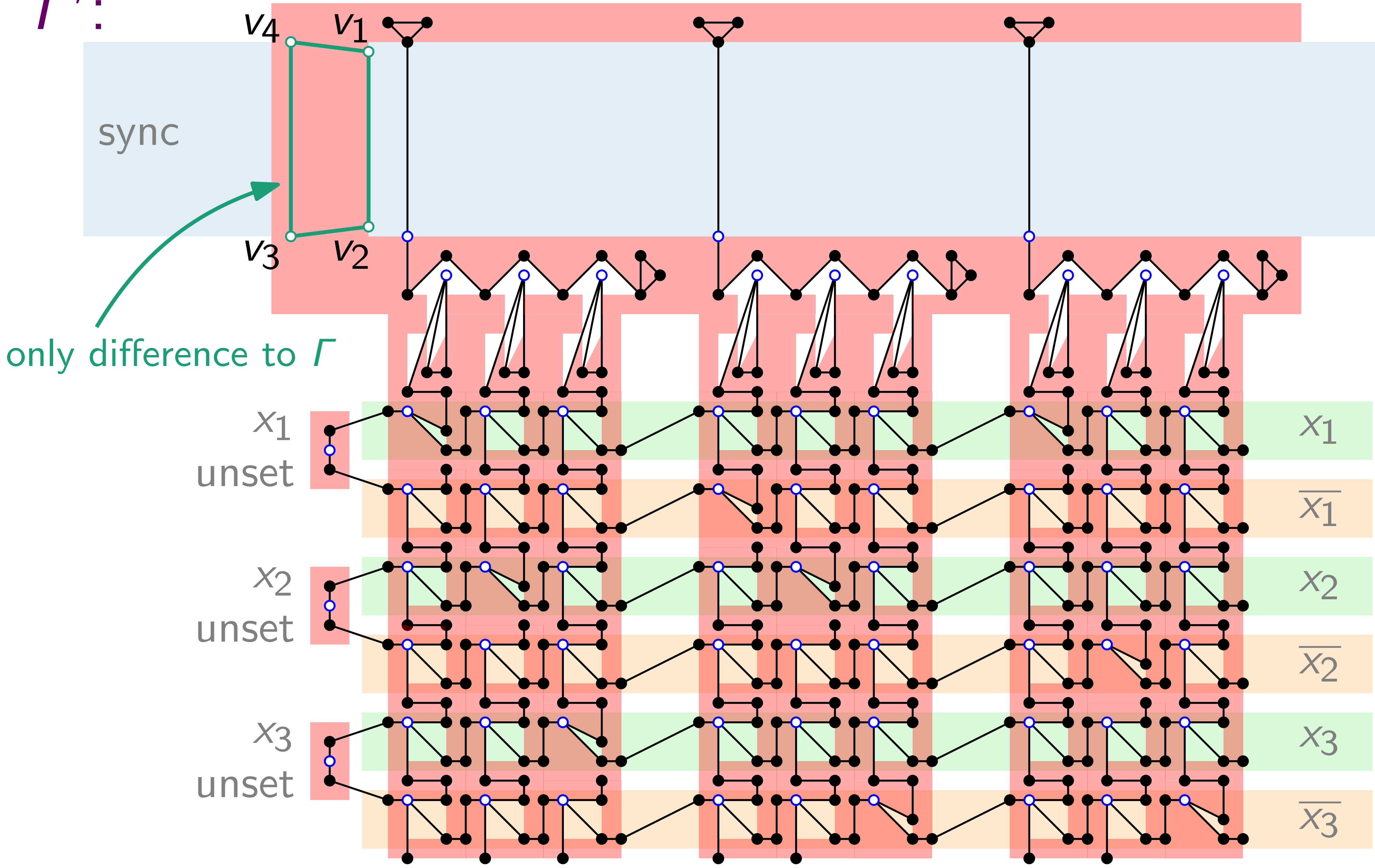


v_1
v_2
v_3
v_4
sync
x_1 unset
x_2 unset
x_3 unset
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$

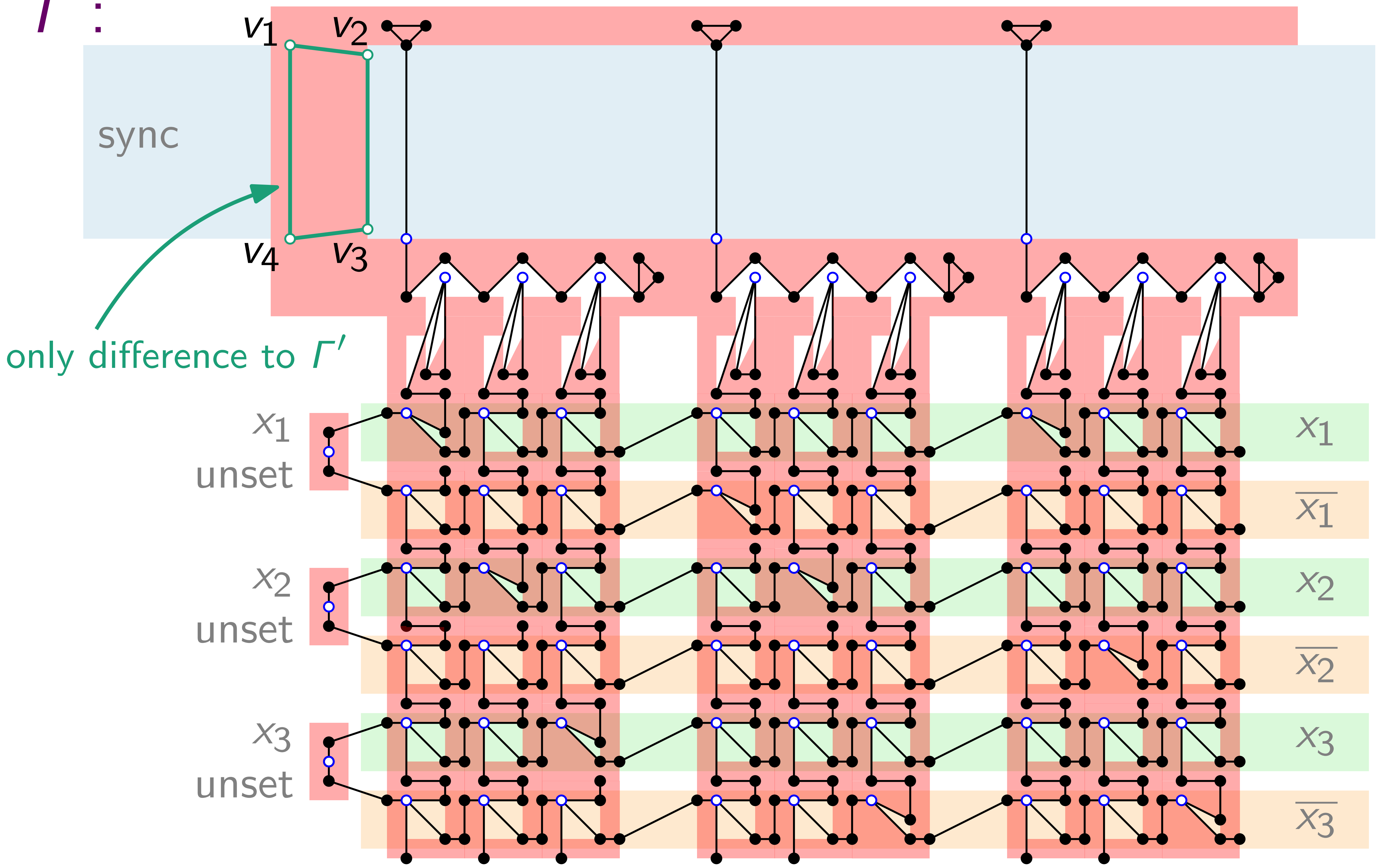

Γ :
v1
v2
v3
v4
sync
filled with obstacles
x1
unset
x2
unset
x3
unset
x1
$\overline{x_1}$
x2
$\overline{x_2}$
x3
$\overline{x_3}$

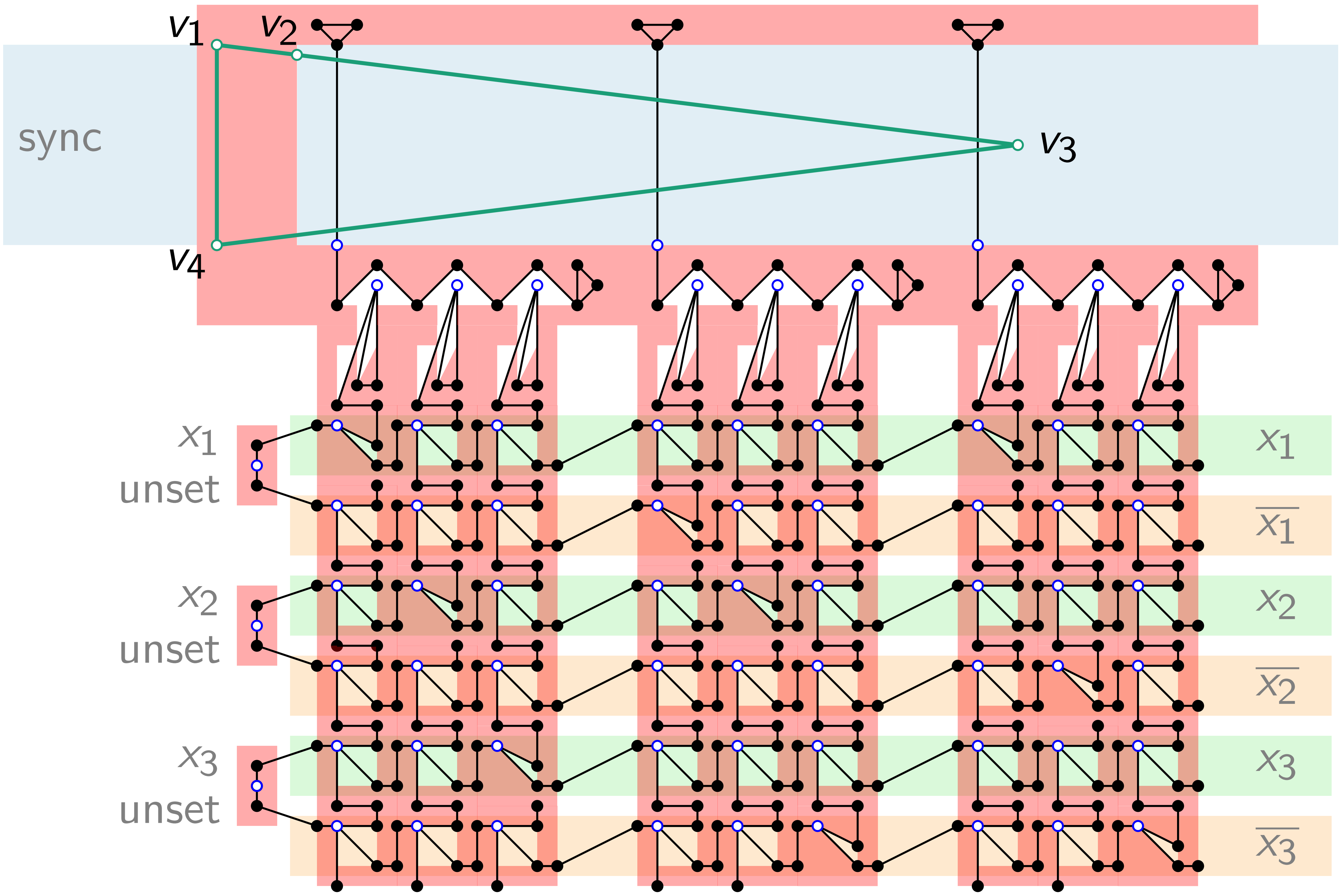

Γ :
v1
v2
v3
v4
sync
only difference to Γ′
x1
unset
x2
unset
x3
unset
x1
x̄1
x2
x̄2
x3
x̄3

Γ':


v_4
v_1
sync
v_3
v_2
only difference to Γ
x_1
unset
x_2
unset
x_3
unset
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$

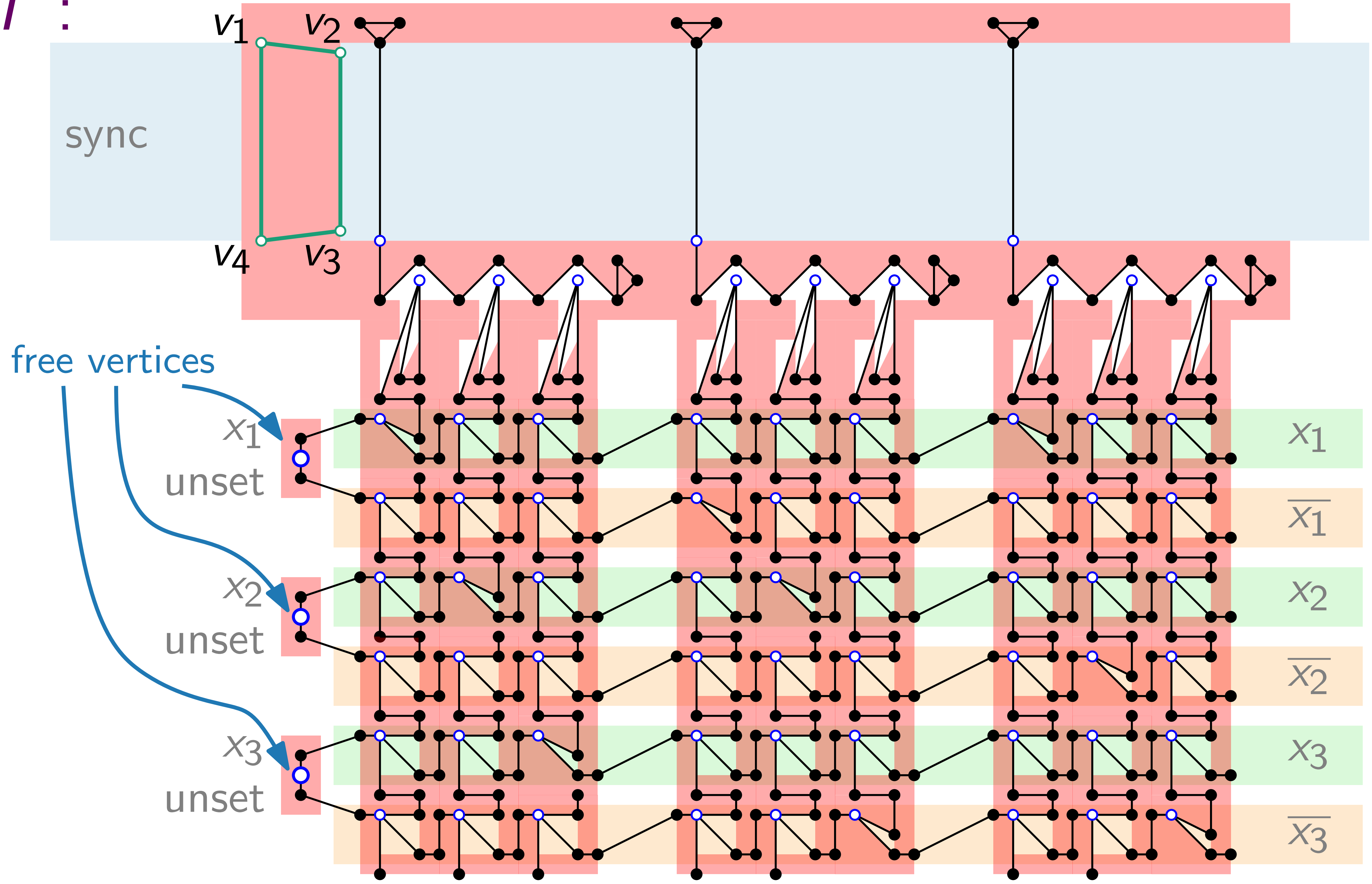
Γ :


v_1
v_2
v_4
v_3
sync
only difference to Γ'
x_1
unset
x_2
unset
x_3
unset
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$

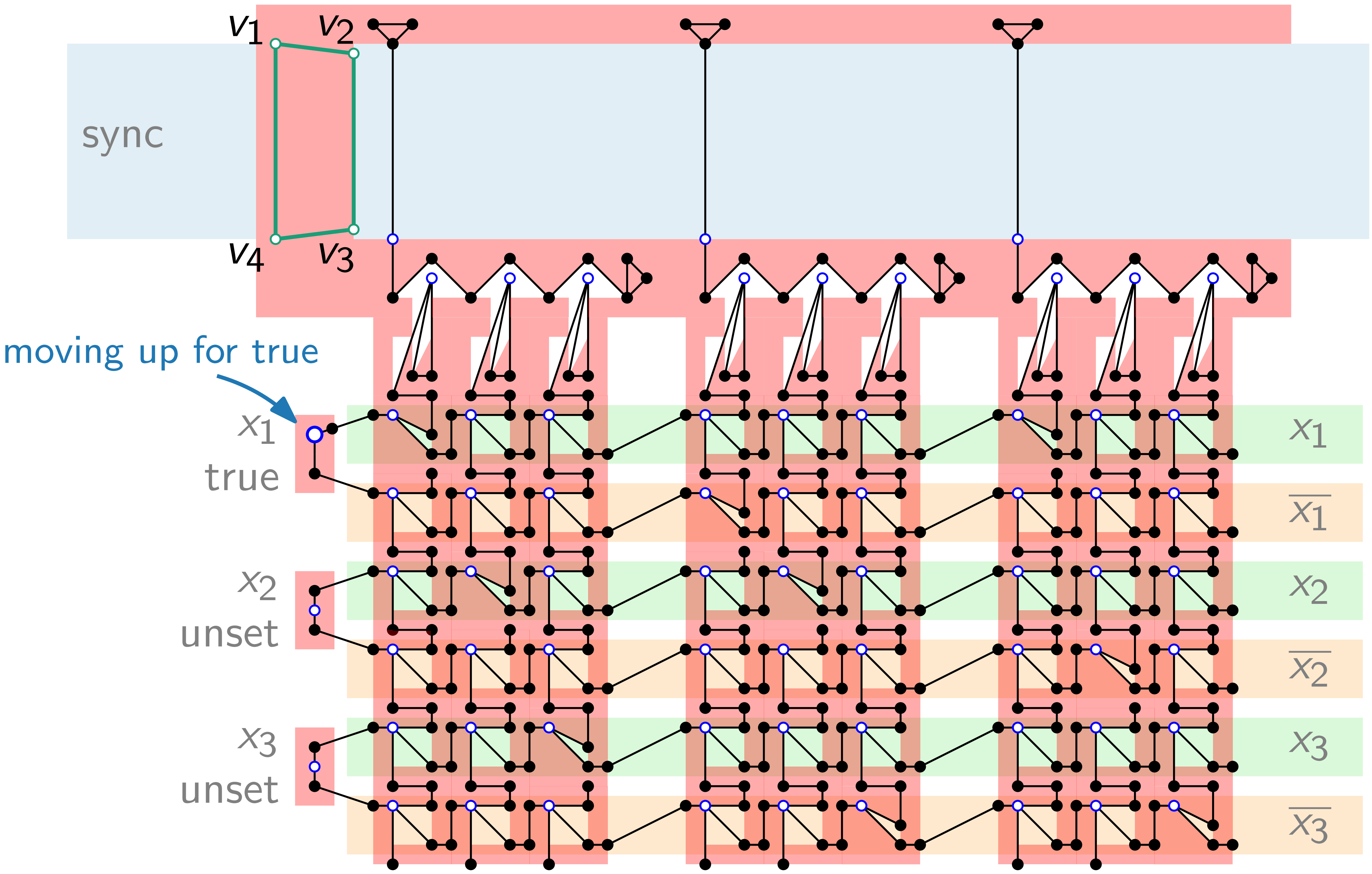


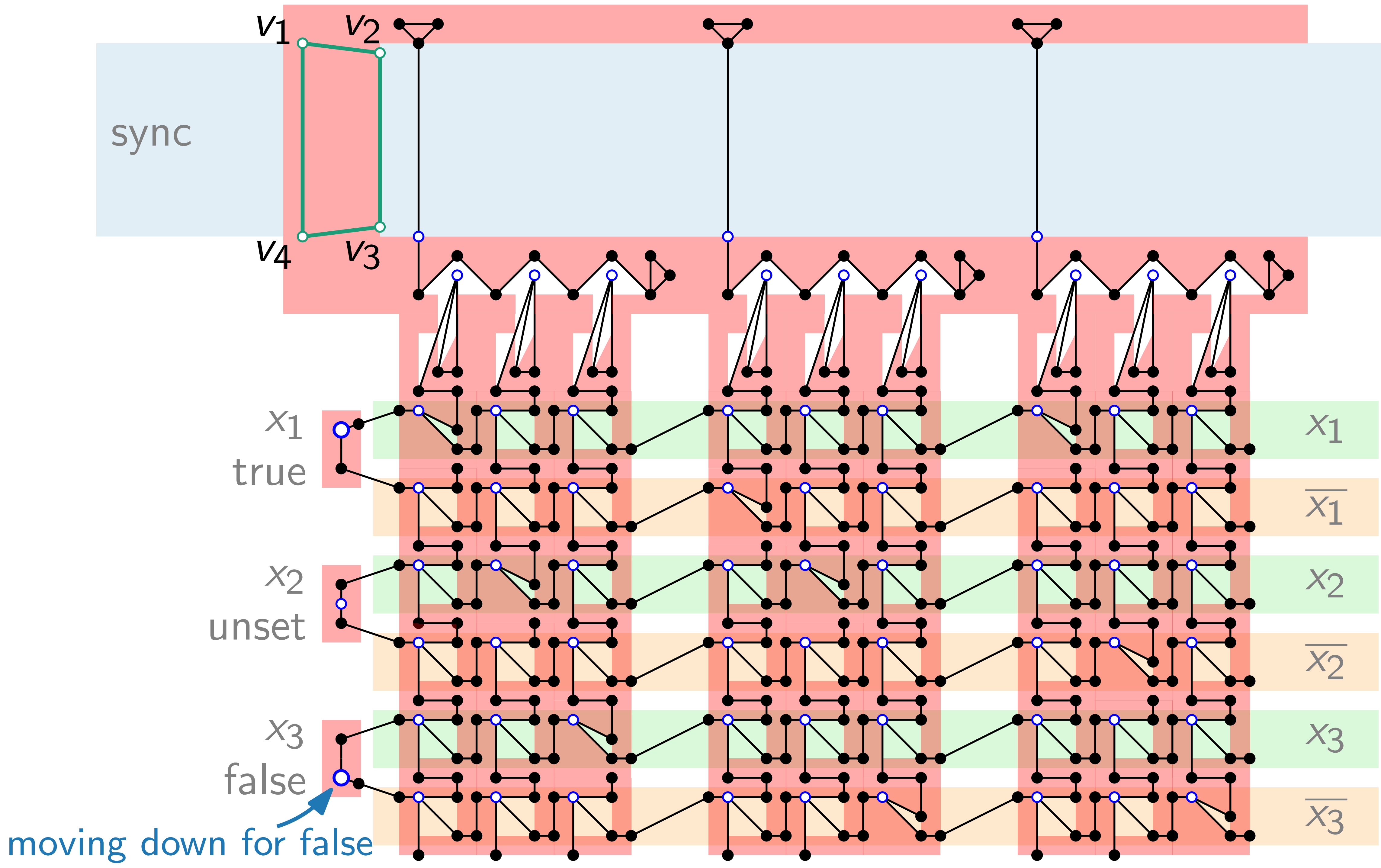
v_1
v_2
v_3
v_4
sync
x_1 unset
x_2 unset
x_3 unset
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$

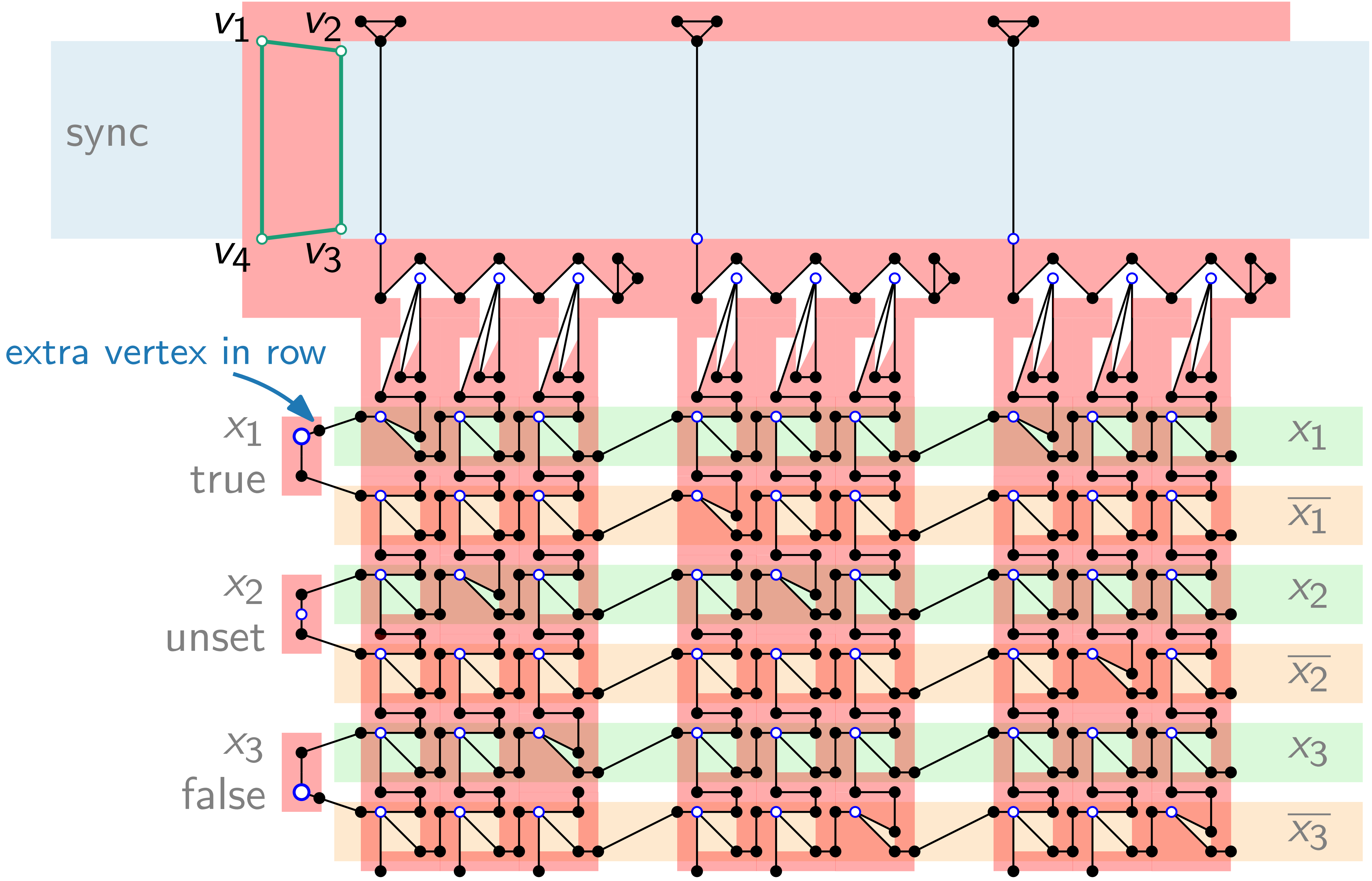
Γ :



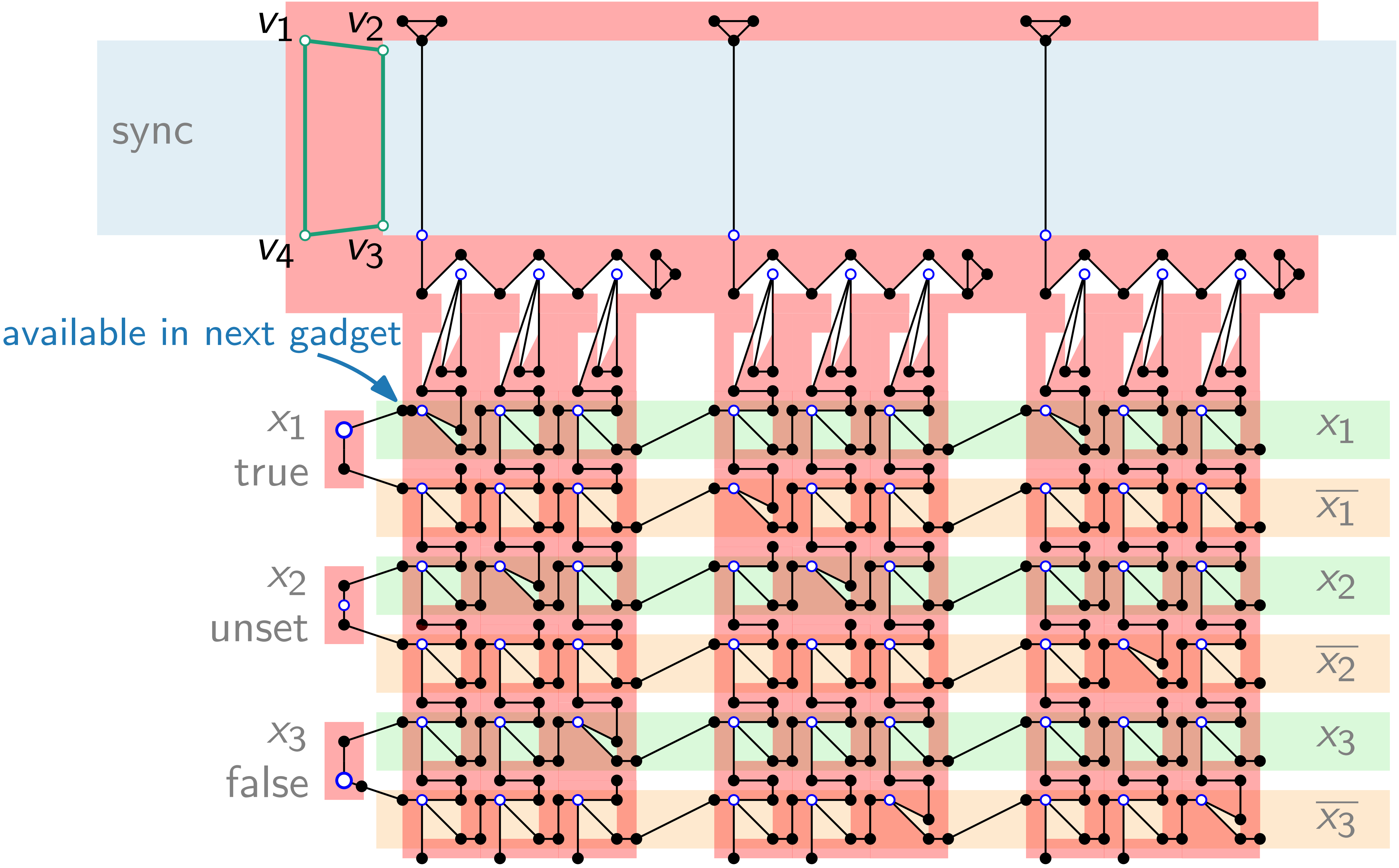
v1
v2
v4
v3
sync
free vertices
x1
unset
x2
unset
x3
unset
x1
x̄1
x2
x̄2
x3
x̄3


sync
v_1
v_2
v_3
v_4
moving up for true
x_1
true
x_2
unset
x_3
unset
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$

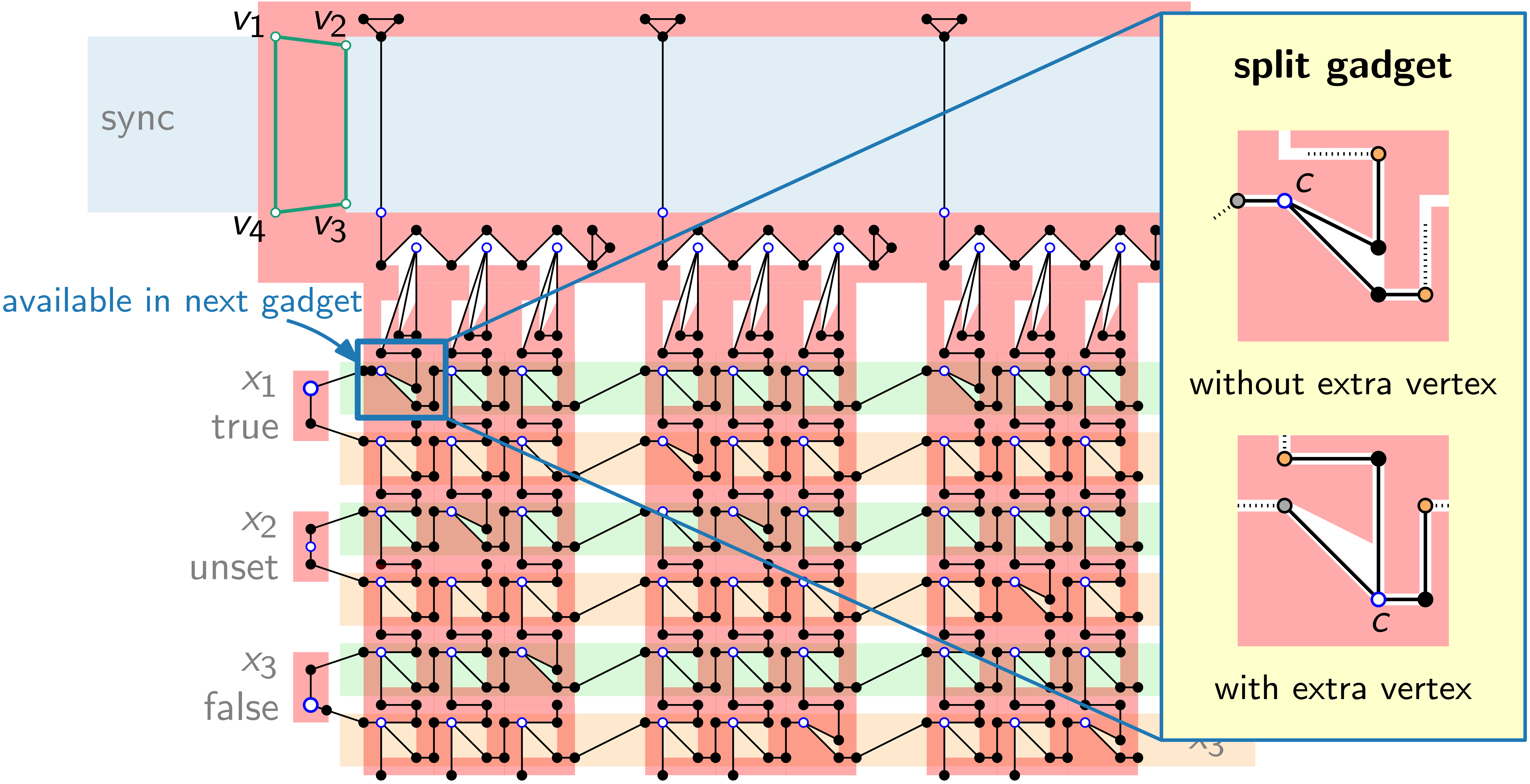

sync
v_1
v_2
v_3
v_4
x_1
true
x_2
unset
x_3
false
moving down for false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$



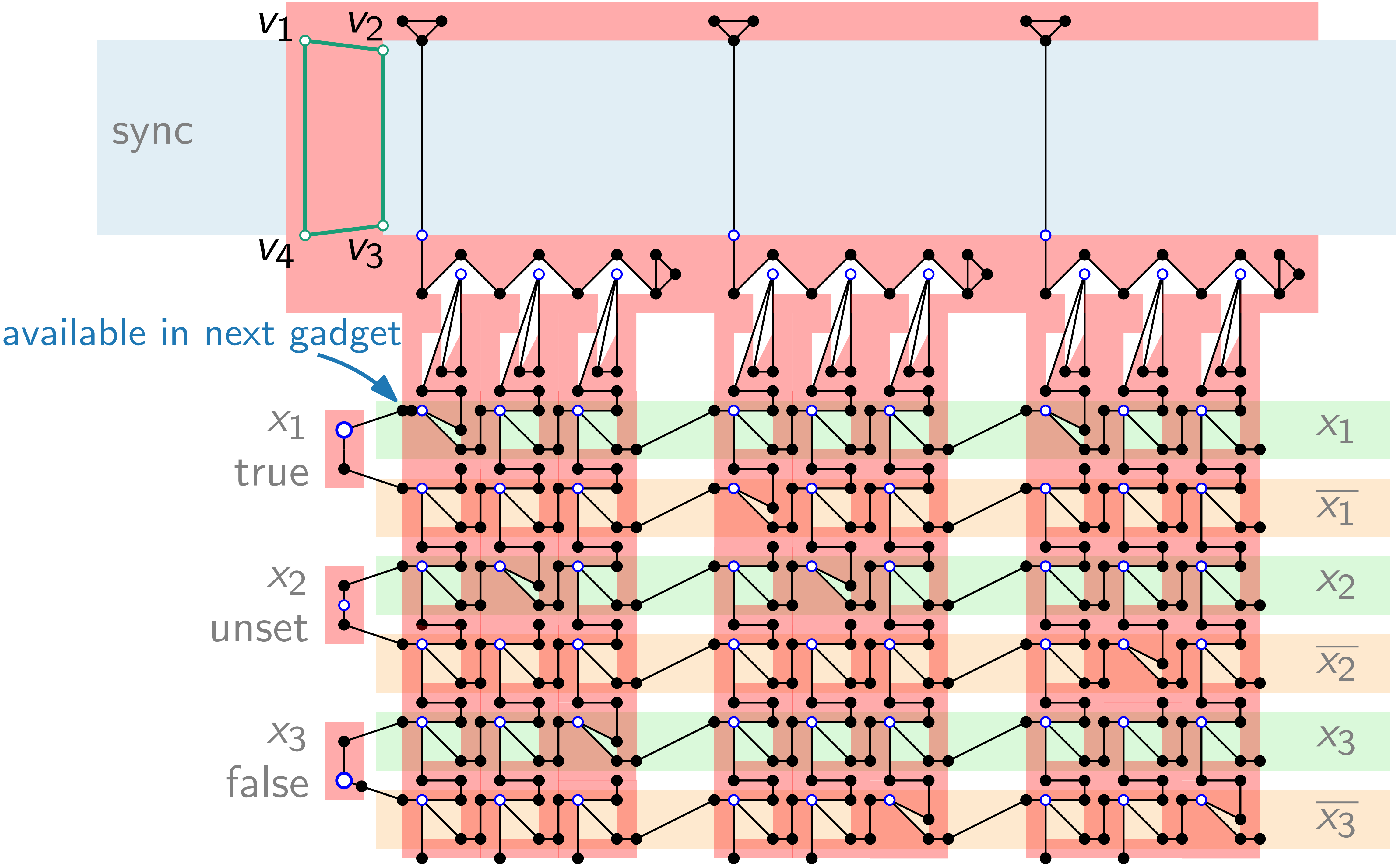
v_1
v_2
v_3
v_4
sync
extra vertex in row
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$



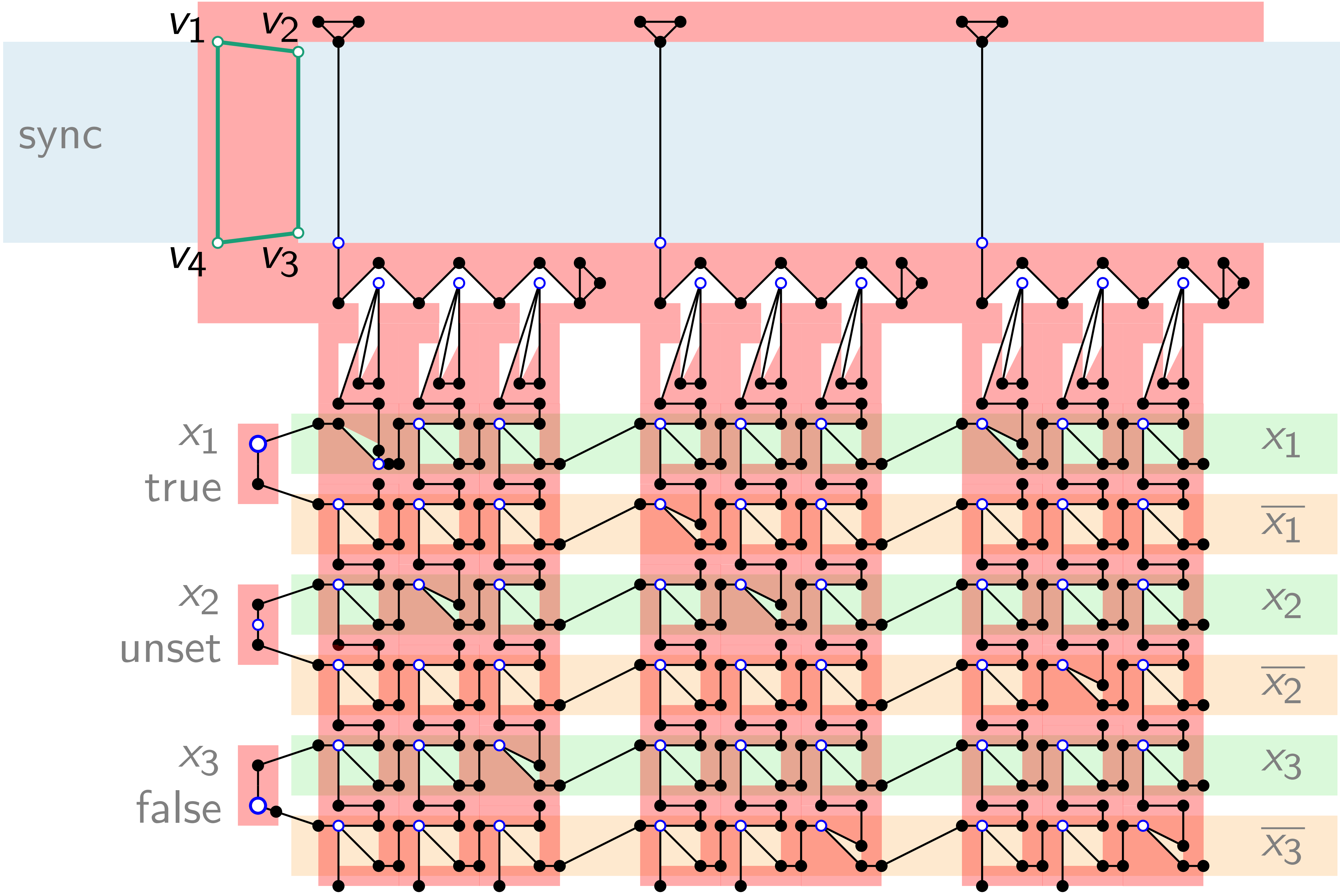
v1
v2
v3
v4
sync
available in next gadget
x1
true
x2
unset
x3
false
x1
x̄1
x2
x̄2
x3
x̄3



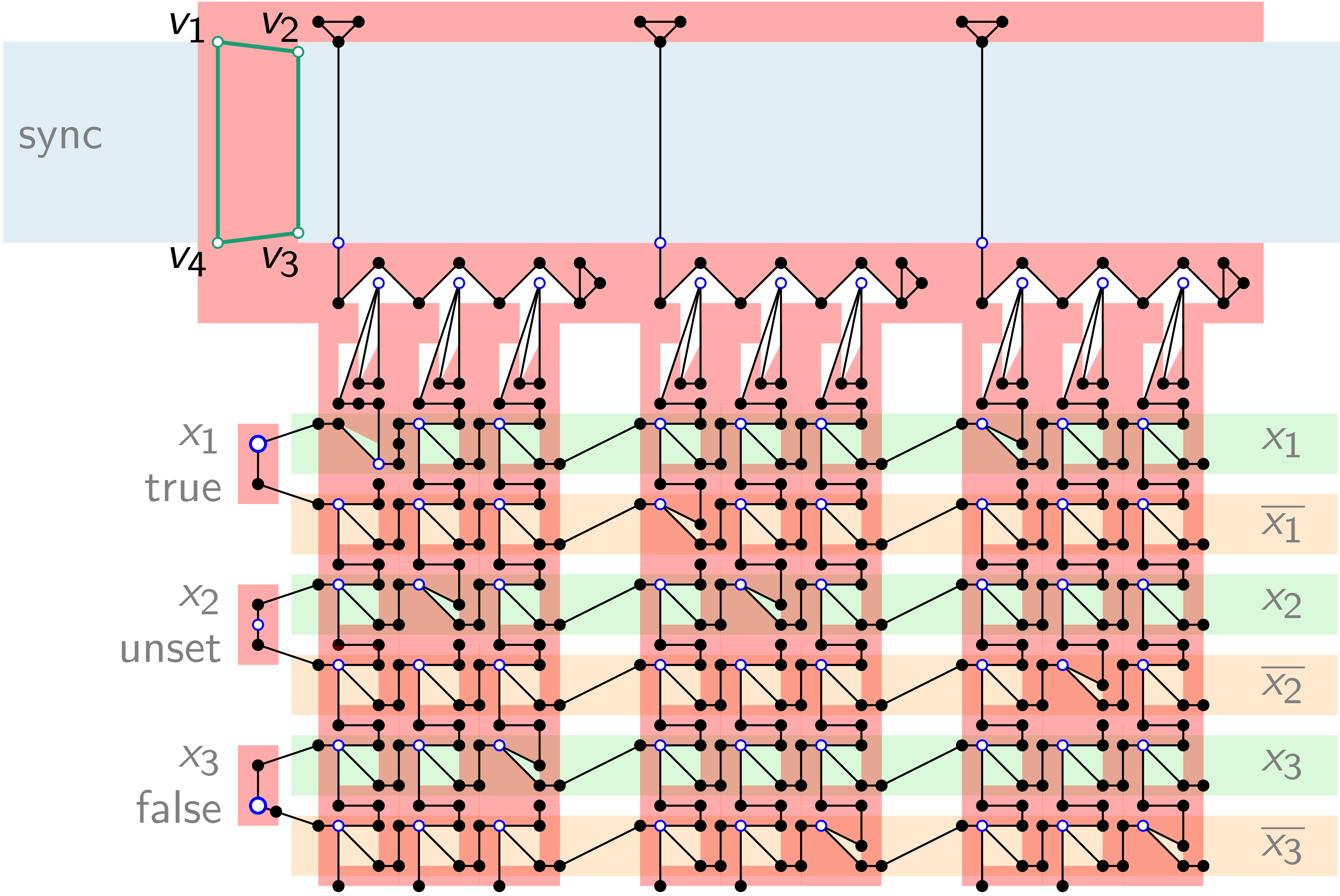
v_1
v_2
v_4
v_3
sync
available in next gadget
x_1
true
x_2
unset
x_3
false
split gadget
c
without extra vertex
c
with extra vertex



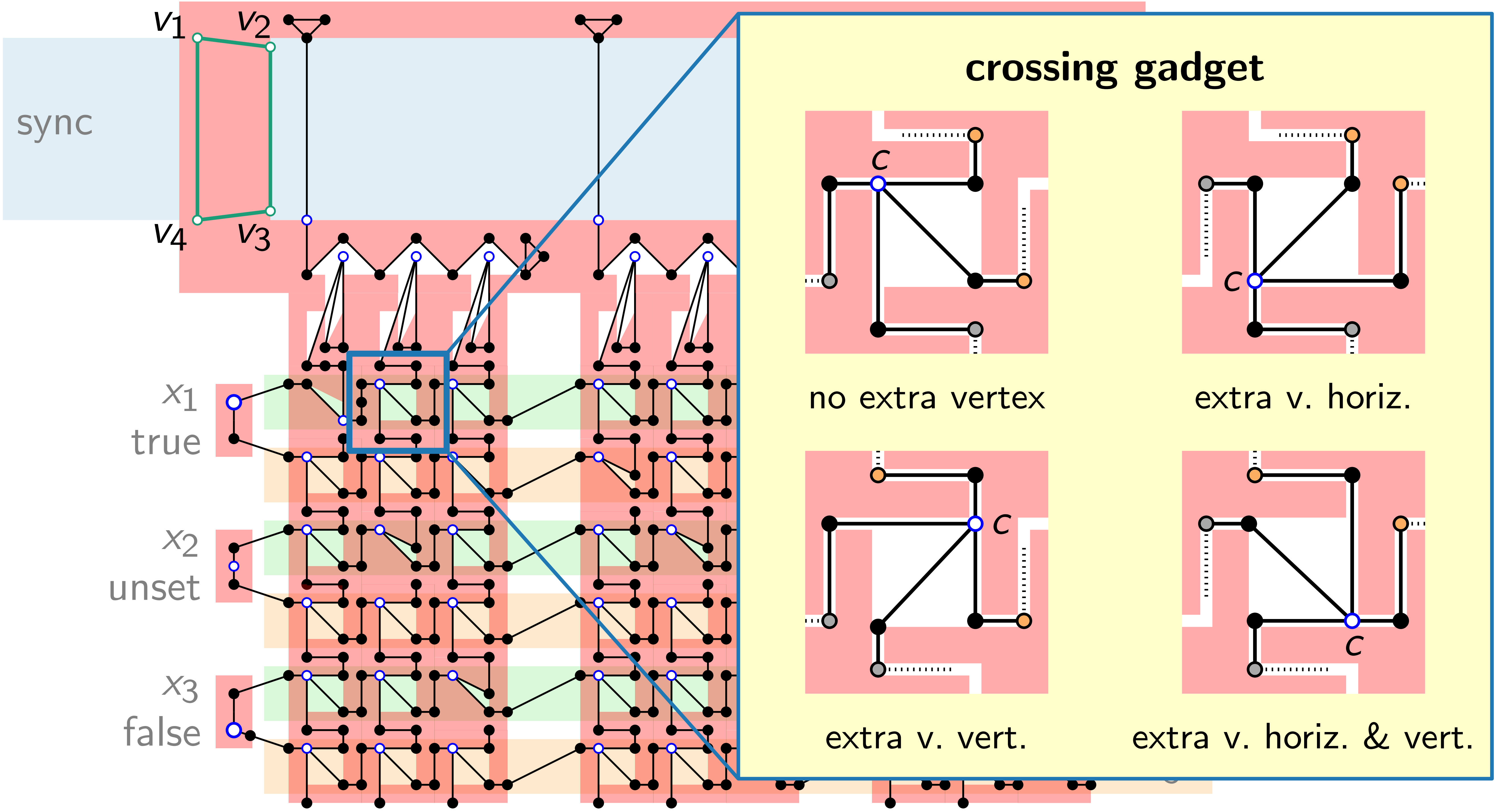
v_1
v_2
v_3
v_4
sync
available in next gadget
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$



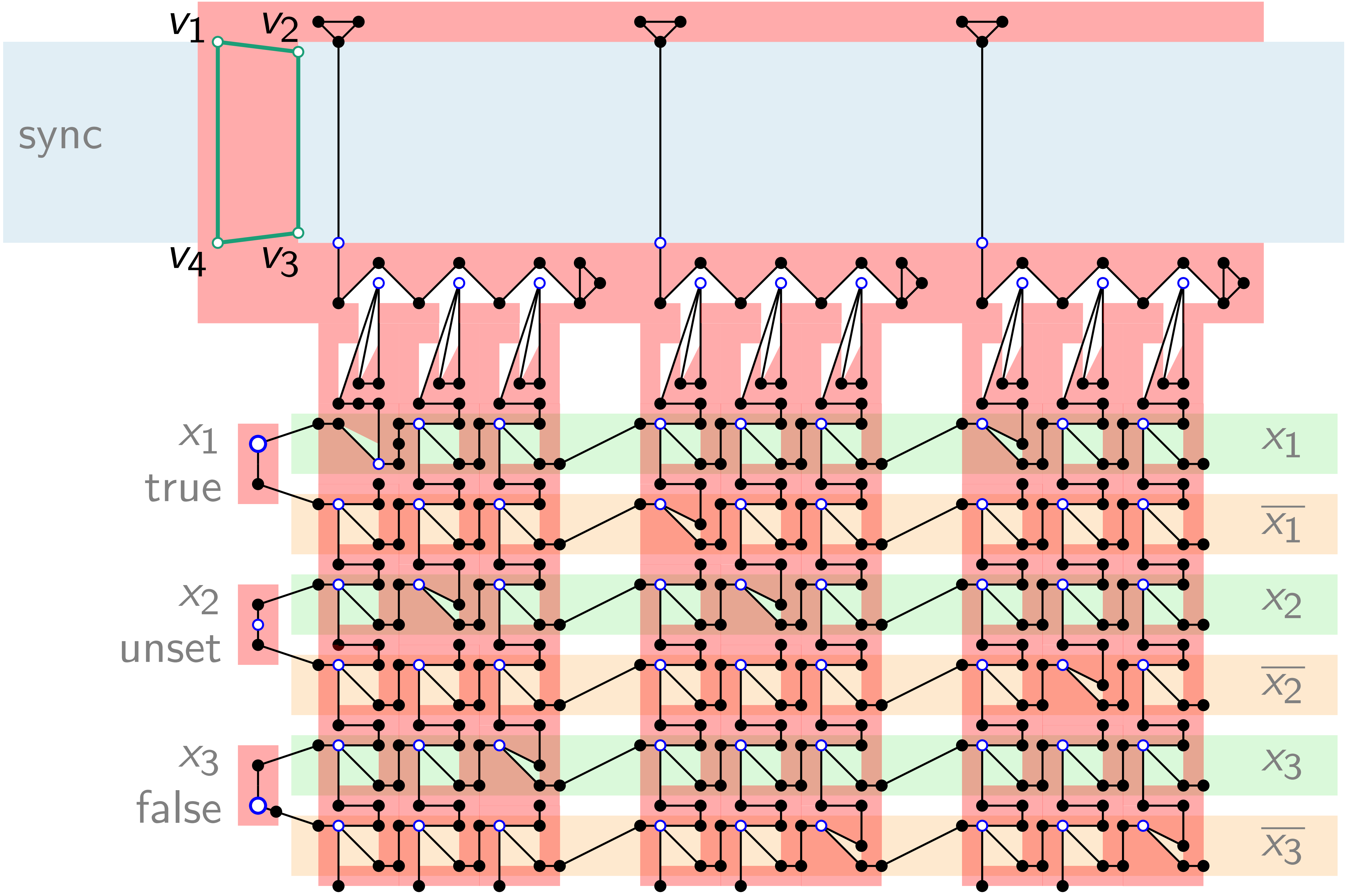
v_1
v_2
v_3
v_4
sync
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$

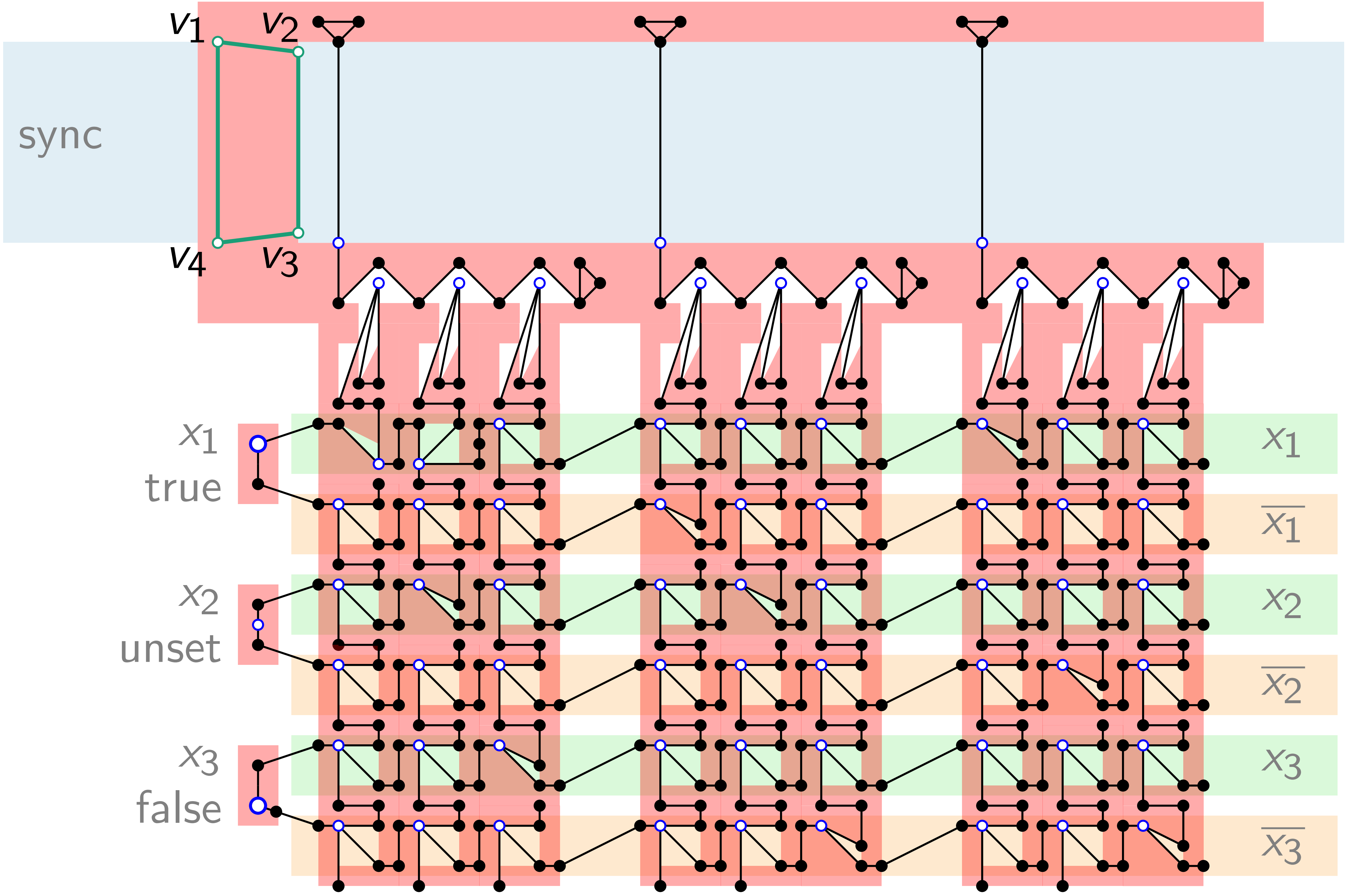


v_1
v_2
v_3
v_4
sync
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$

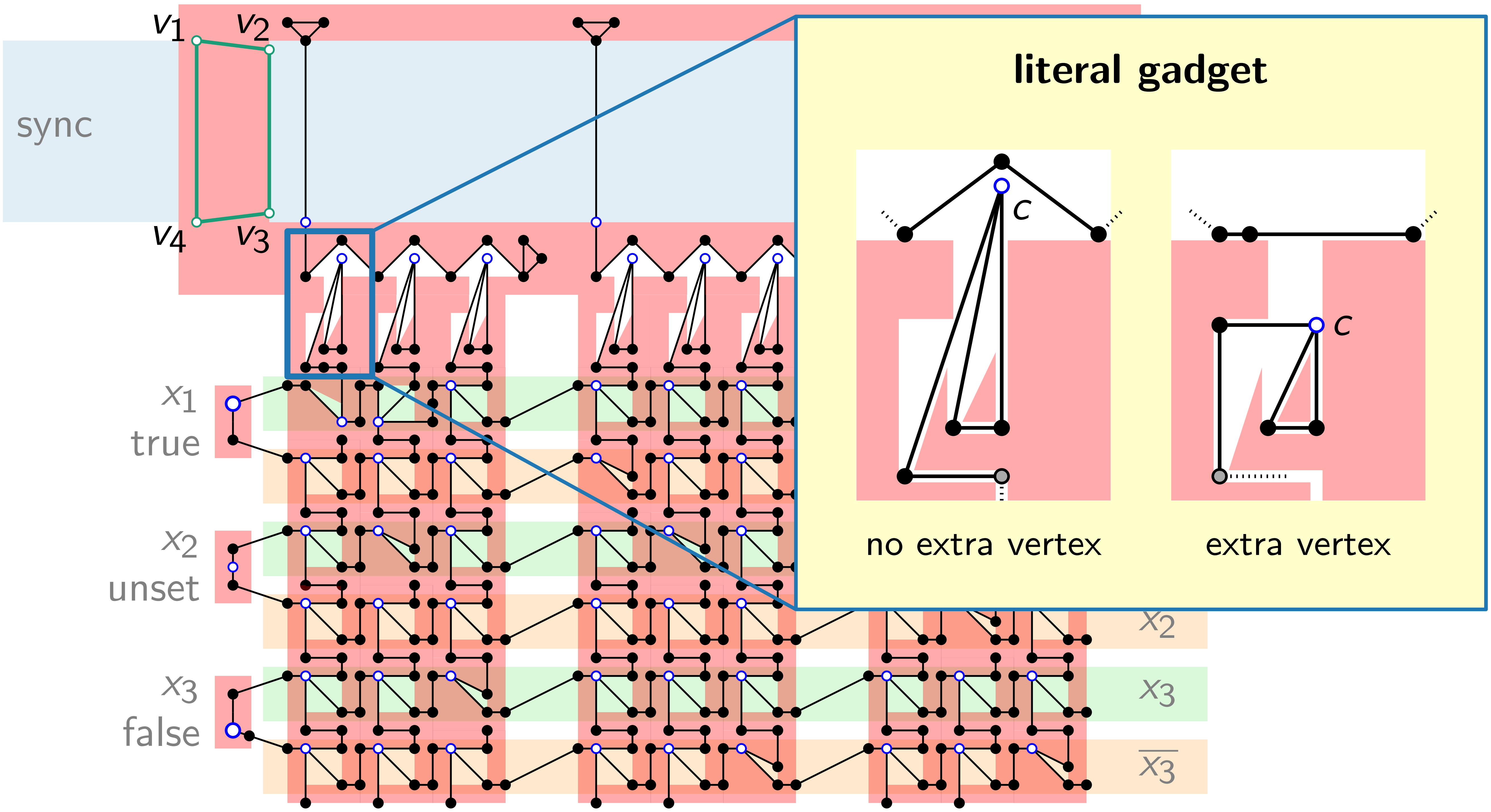


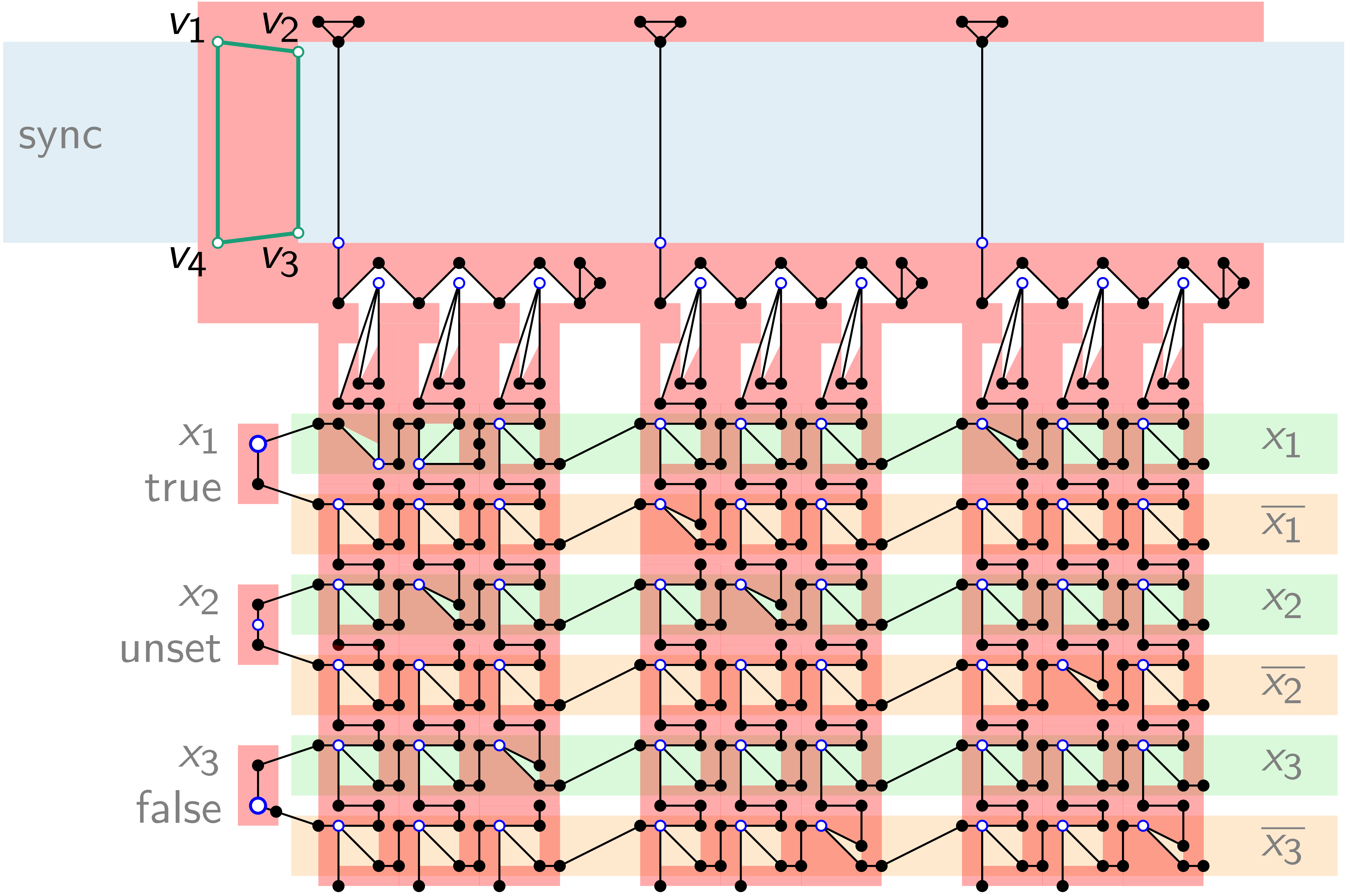
v1
v2
v3
v4
sync
x1
true
x2
unset
x3
false
crossing gadget
c
no extra vertex
extra v. horiz.
extra v. vert.
extra v. horiz. & vert.

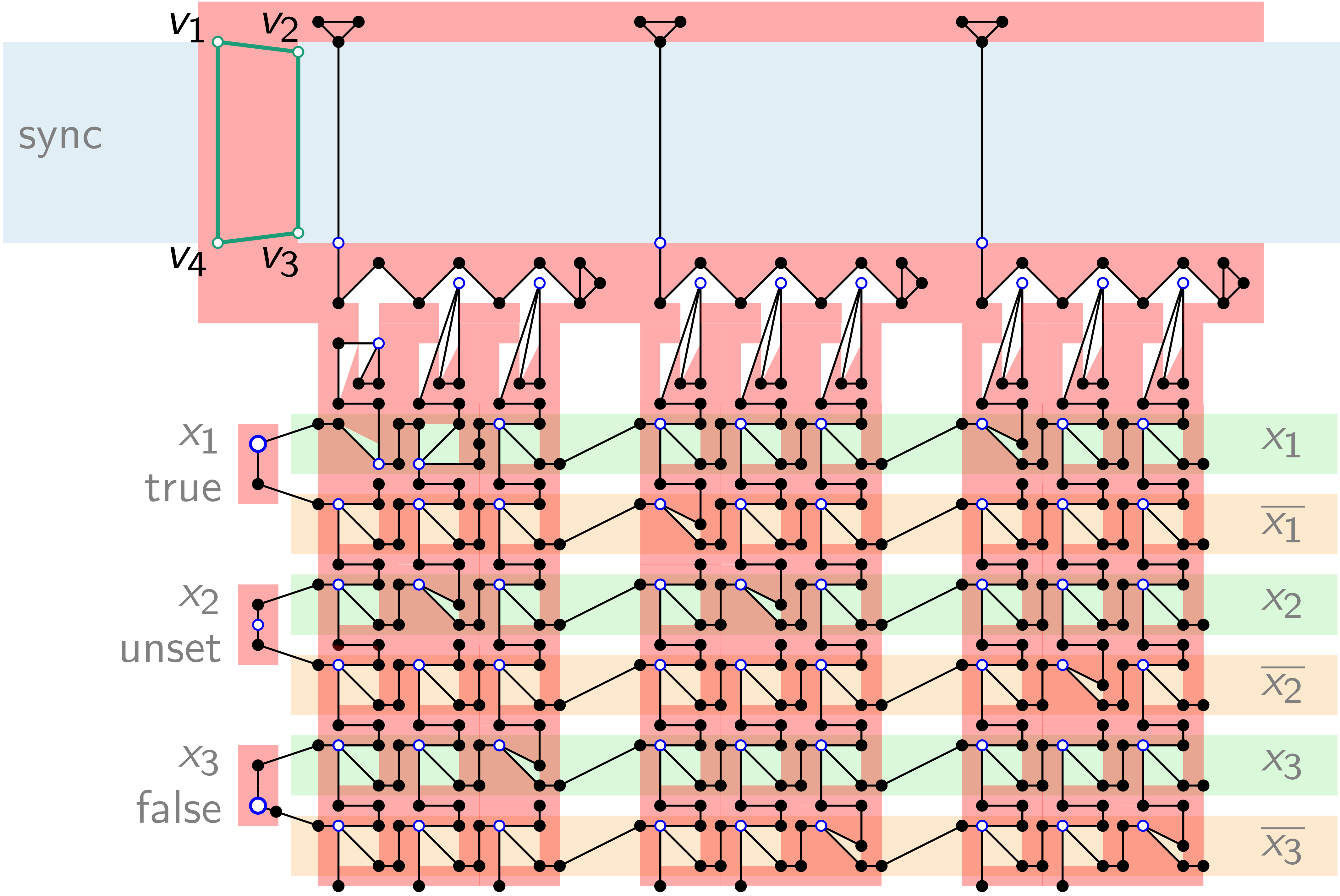

sync
v_1
v_2
v_3
v_4
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$



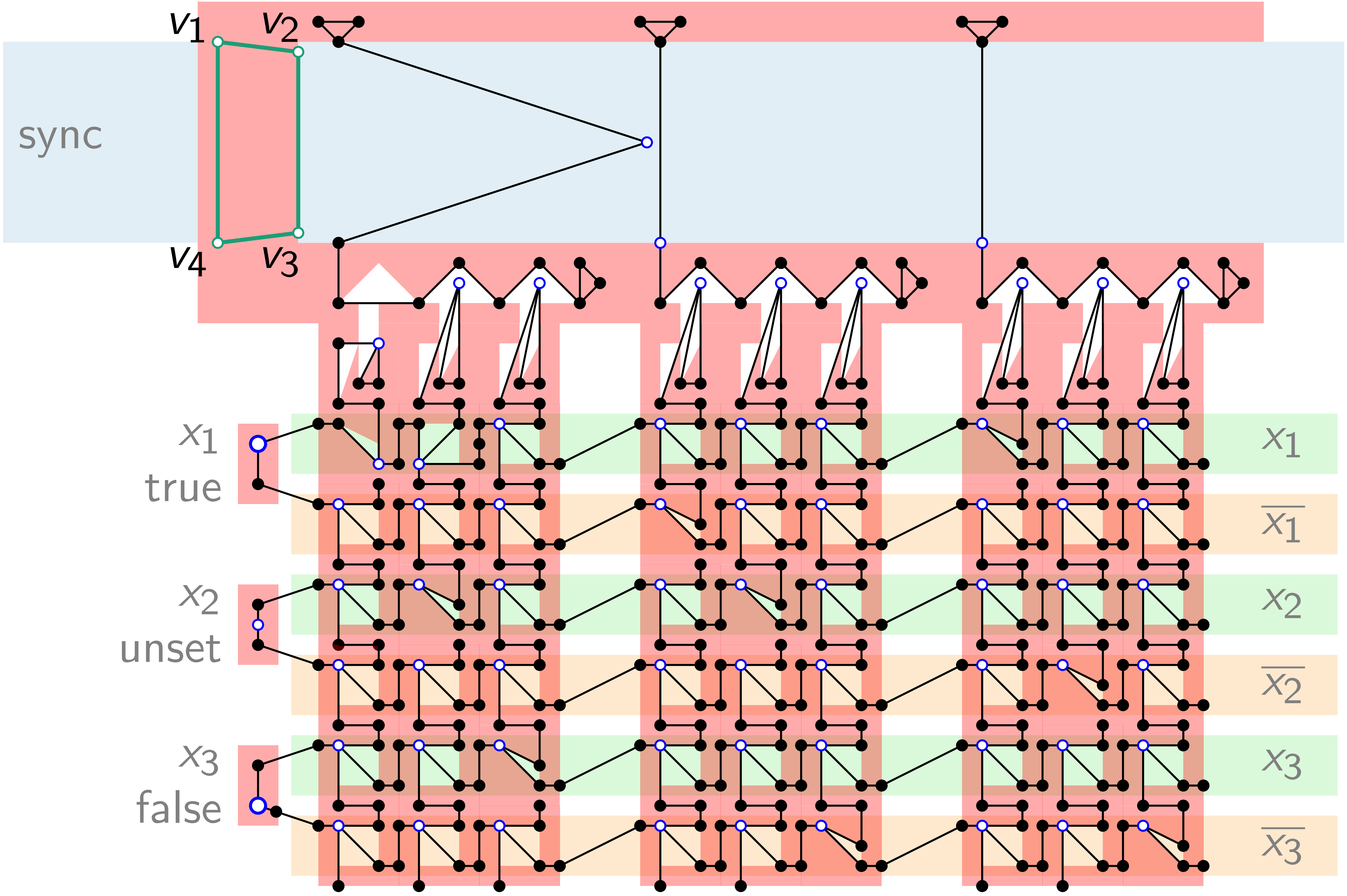
v_1
v_2
v_3
v_4
sync
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$


sync
v_1
v_2
v_3
v_4
x_1
true
x_2
unset
x_3
false
x_2
x_3
$\overline{x_3}$
literal gadget
c
no extra vertex
c
extra vertex

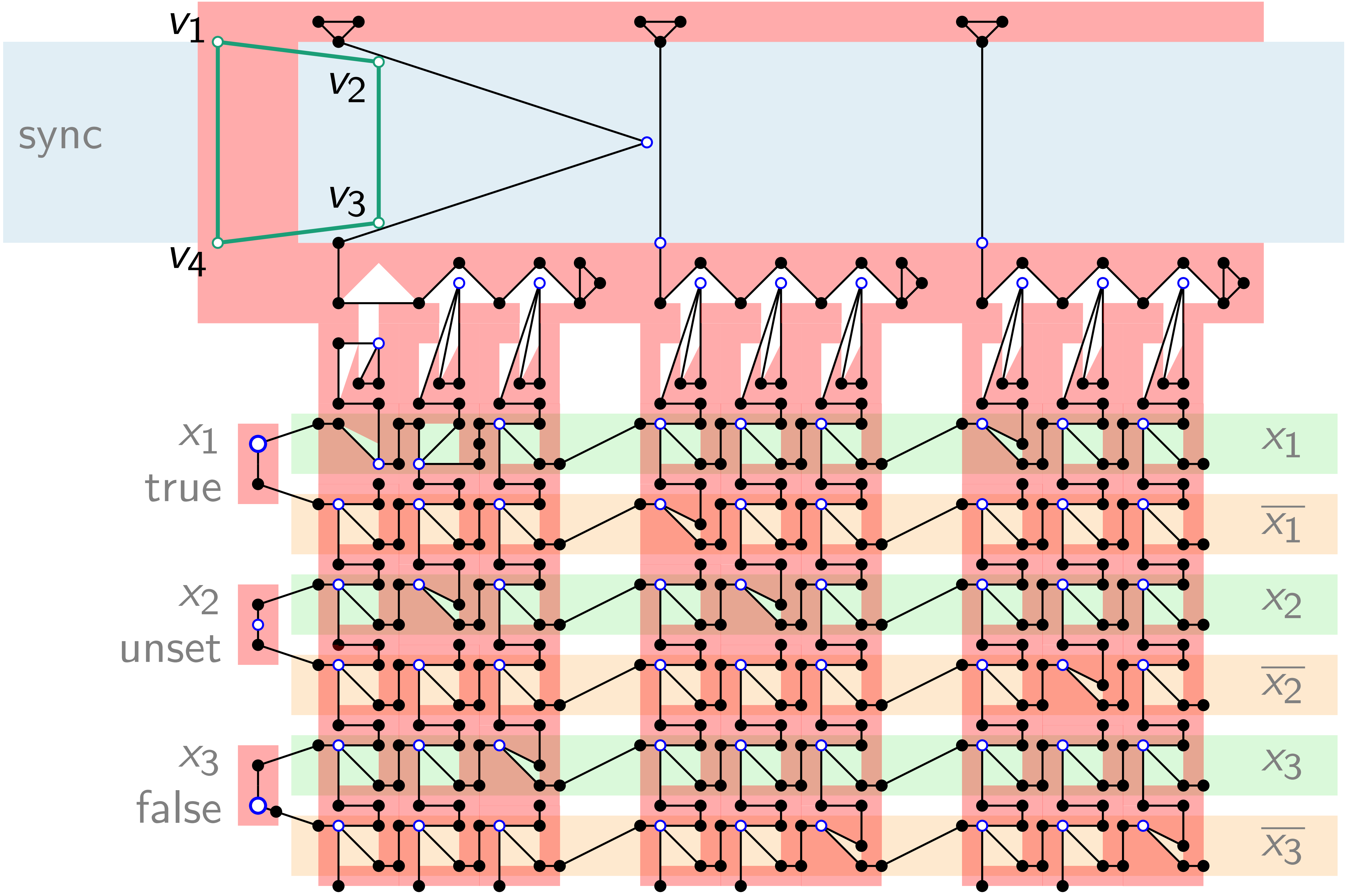

sync
v_1
v_2
v_3
v_4
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$



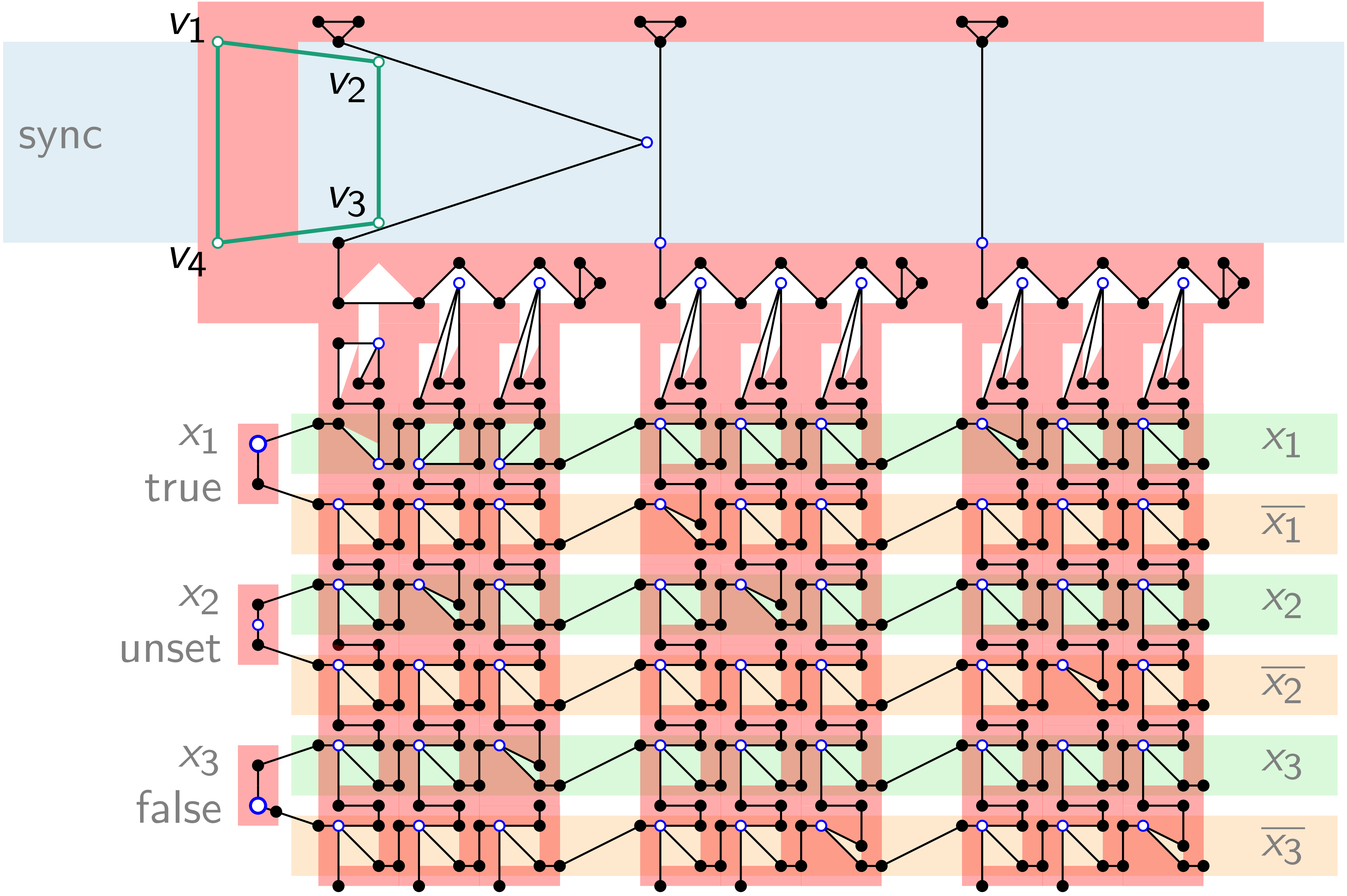
v_1
v_2
v_4
v_3
sync
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$



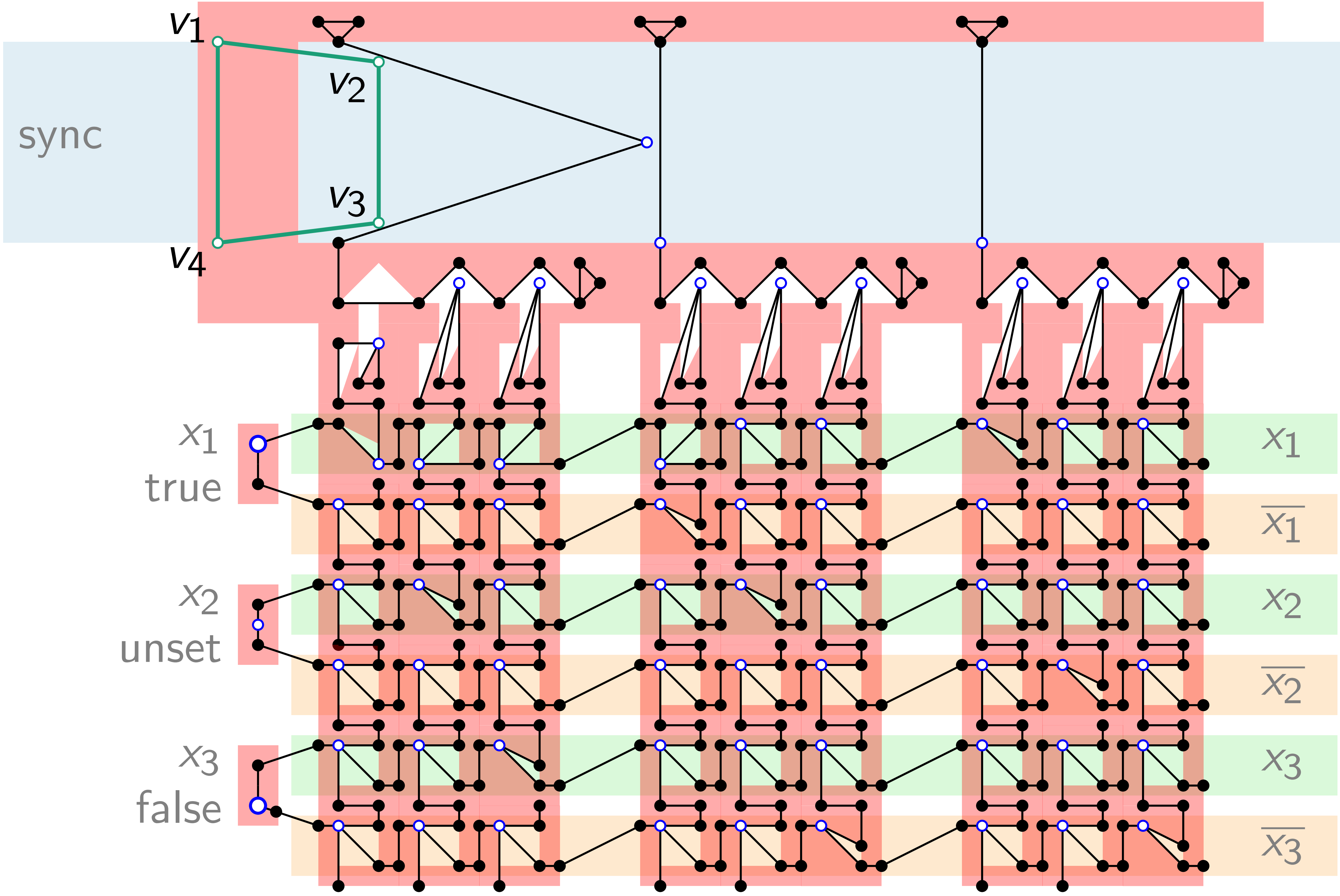
v_1
v_2
sync
v_4
v_3
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$



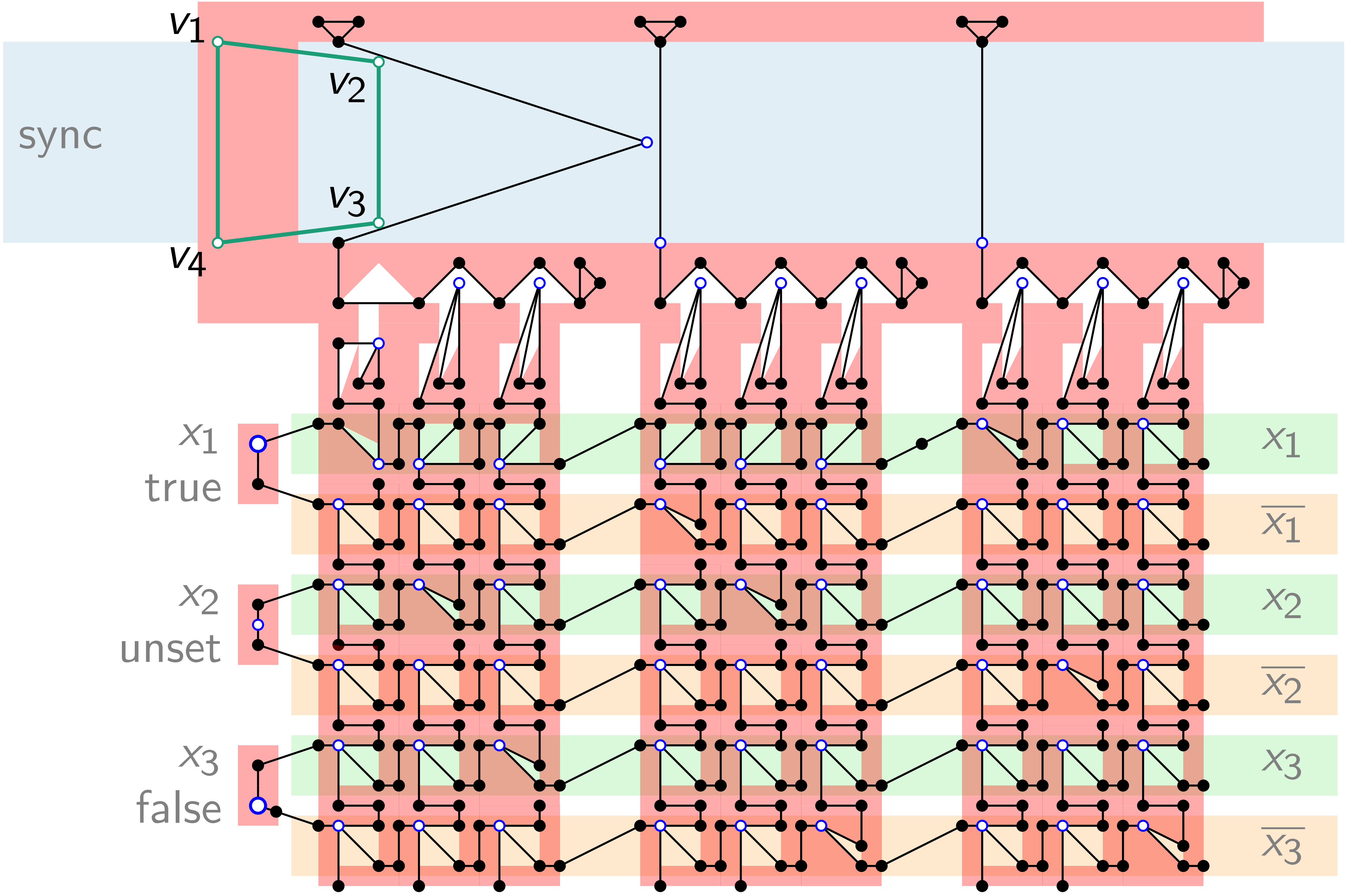
v_1
v_2
v_3
v_4
sync
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$

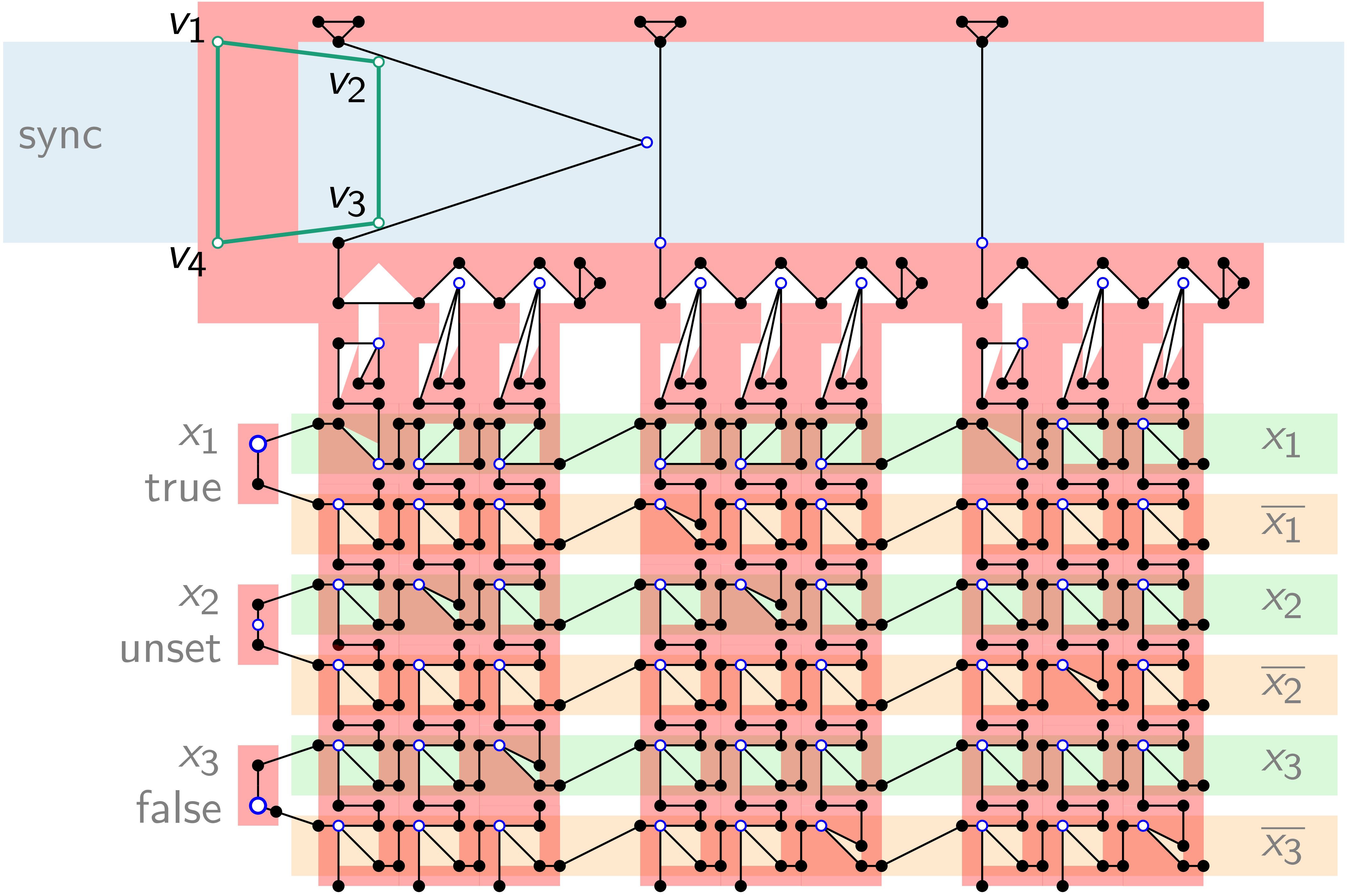


v_1
v_2
v_3
v_4
sync
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$

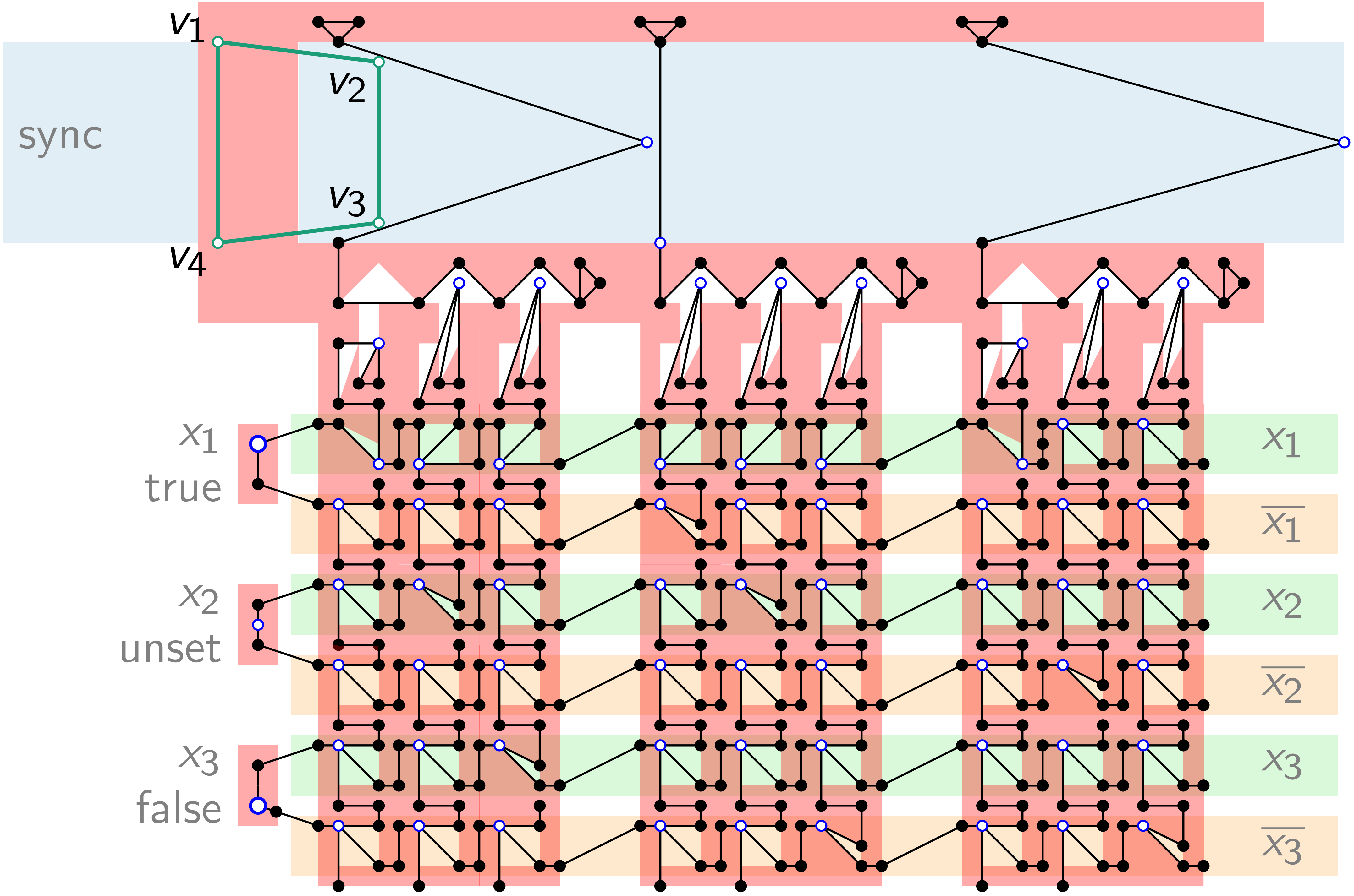


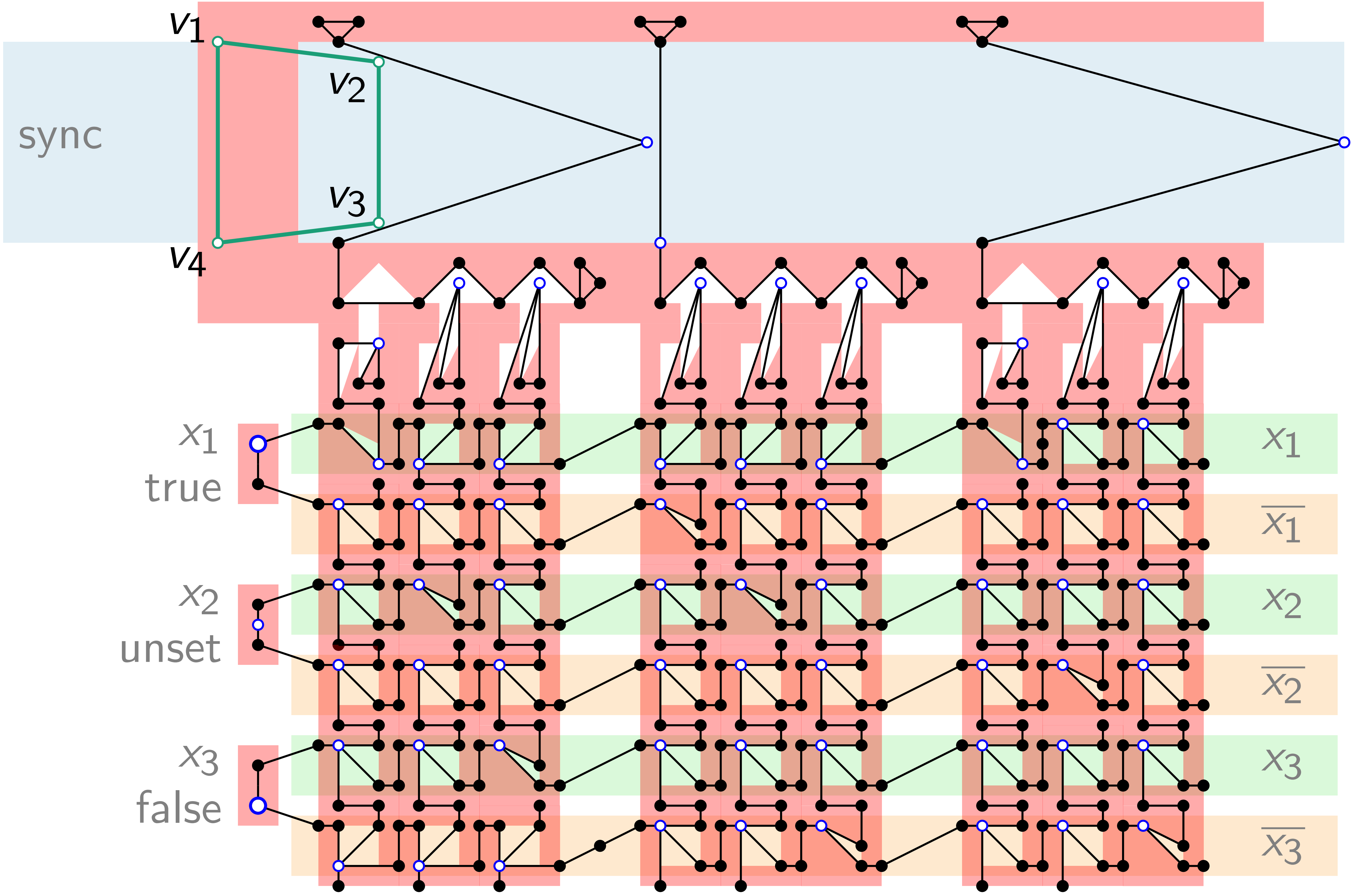
v_1
v_2
v_3
v_4
sync
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$


sync
v_1
v_2
v_3
v_4
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$

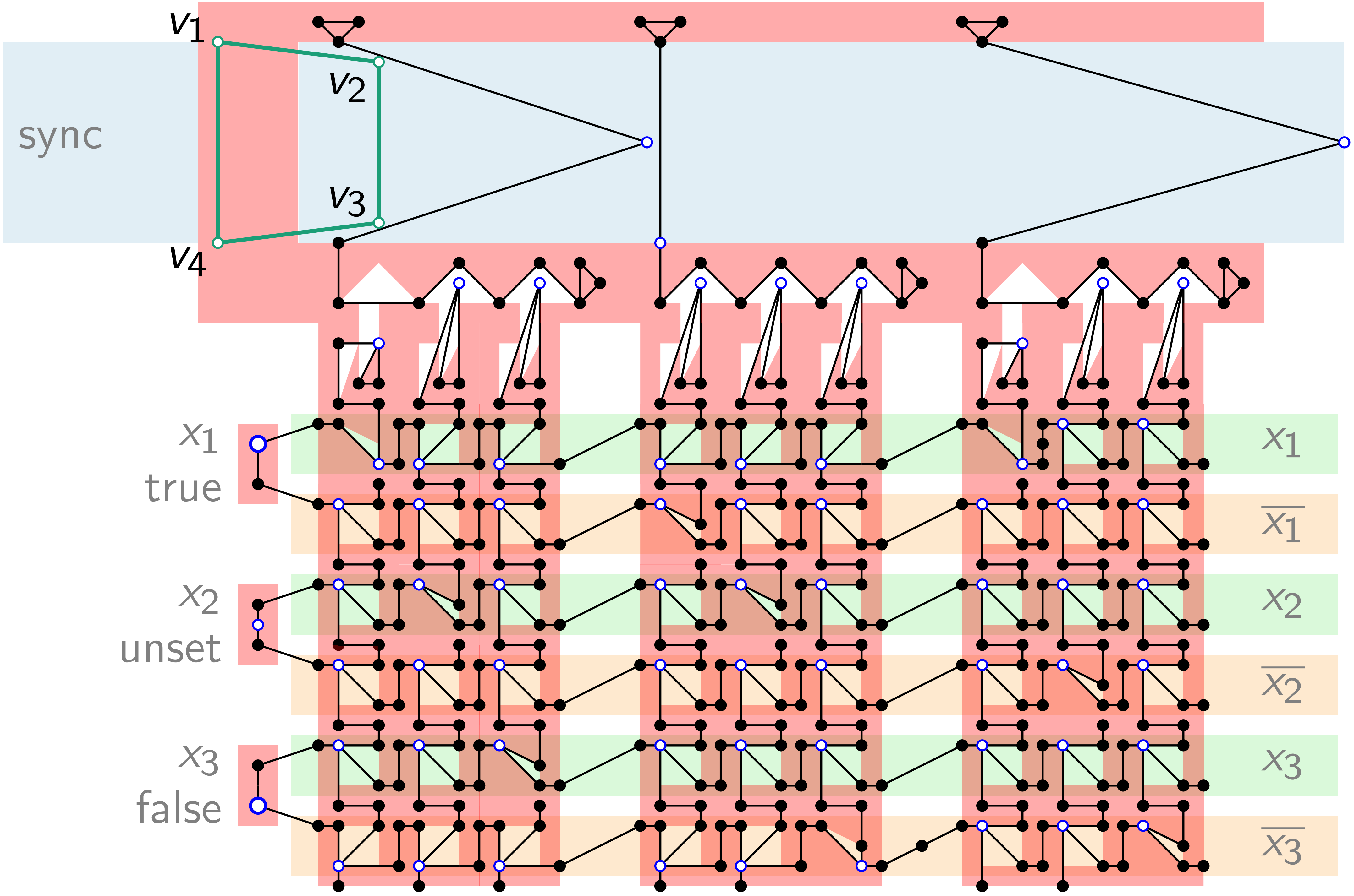


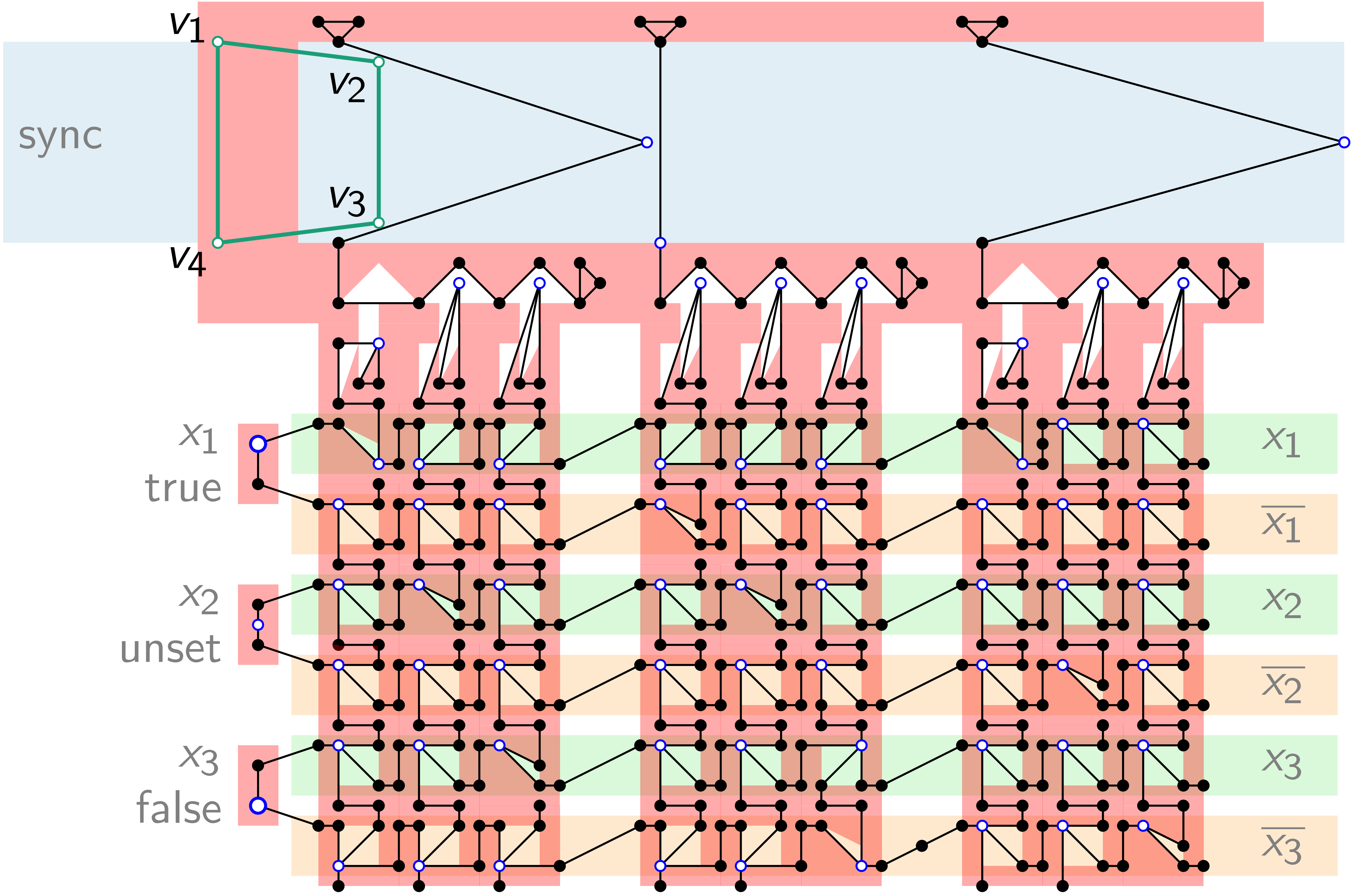
v_1
v_2
v_3
v_4
sync
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$


v_1
v_2
v_3
v_4
sync
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$



v_1
v_2
v_3
v_4
sync
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$


v_1
v_2
sync
v_3
v_4
x_1
true
x_2
unset
x_3
false
x_1
$\overline{x_1}$
x_2
$\overline{x_2}$
x_3
$\overline{x_3}$



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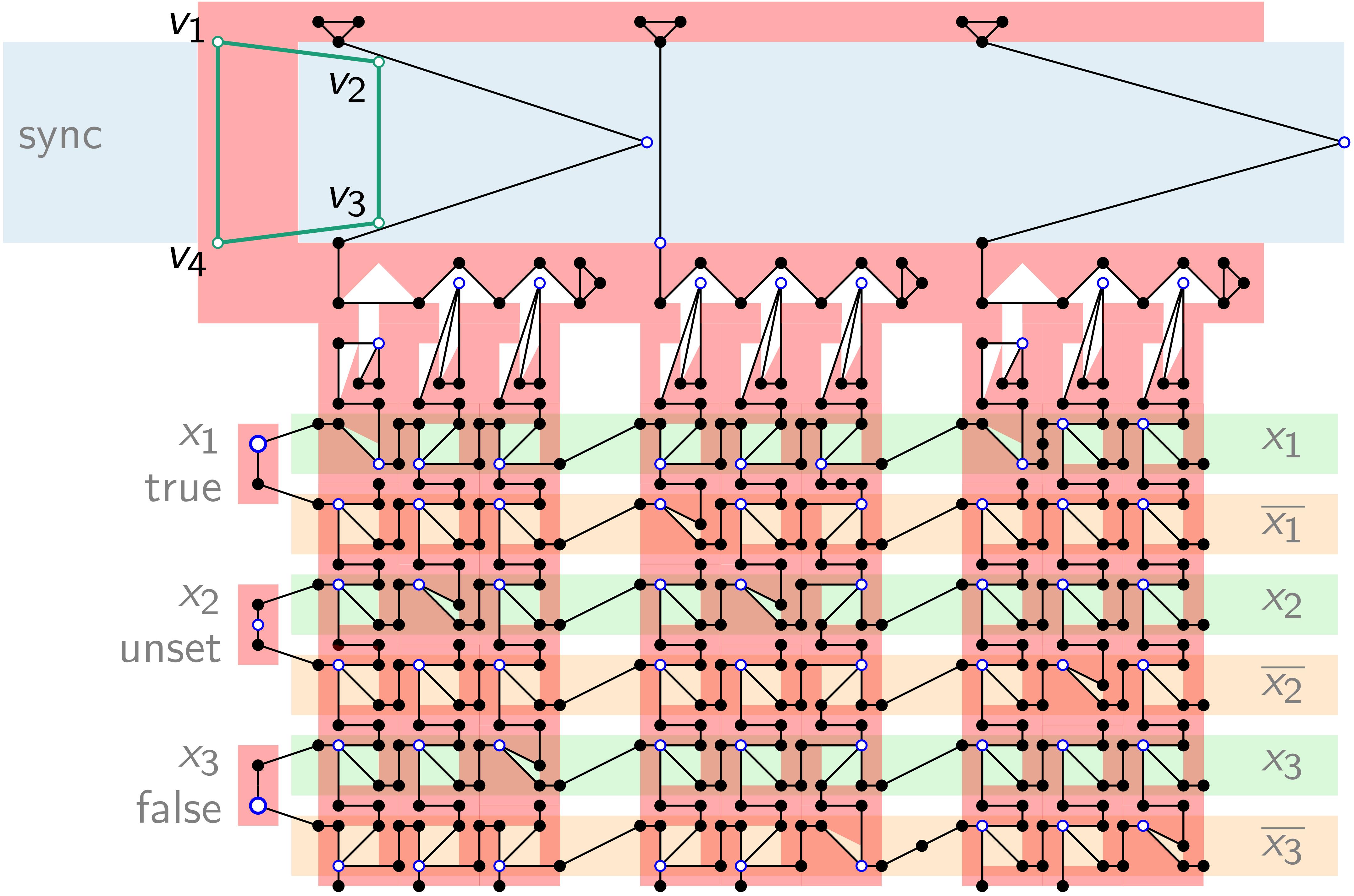

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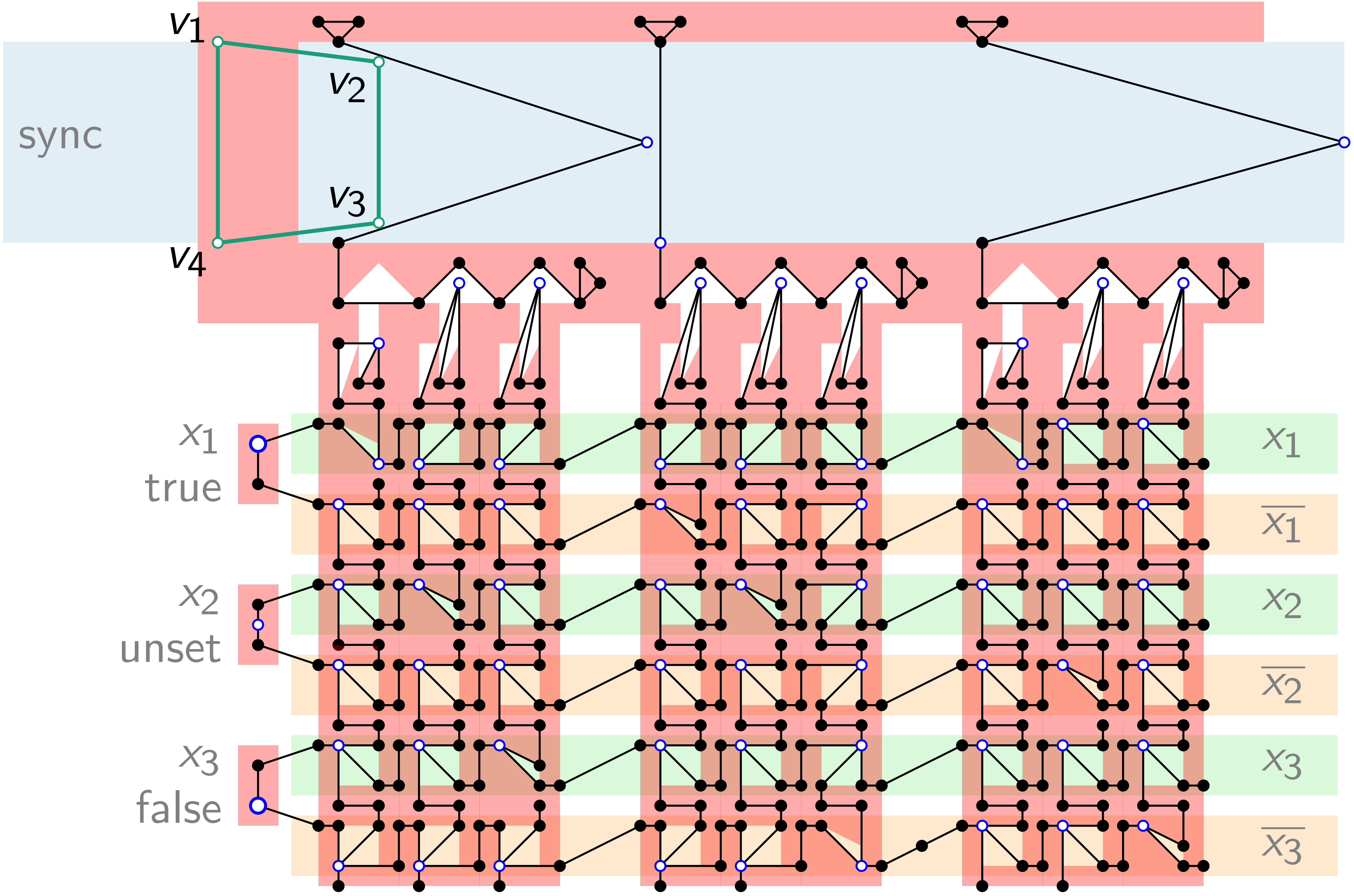
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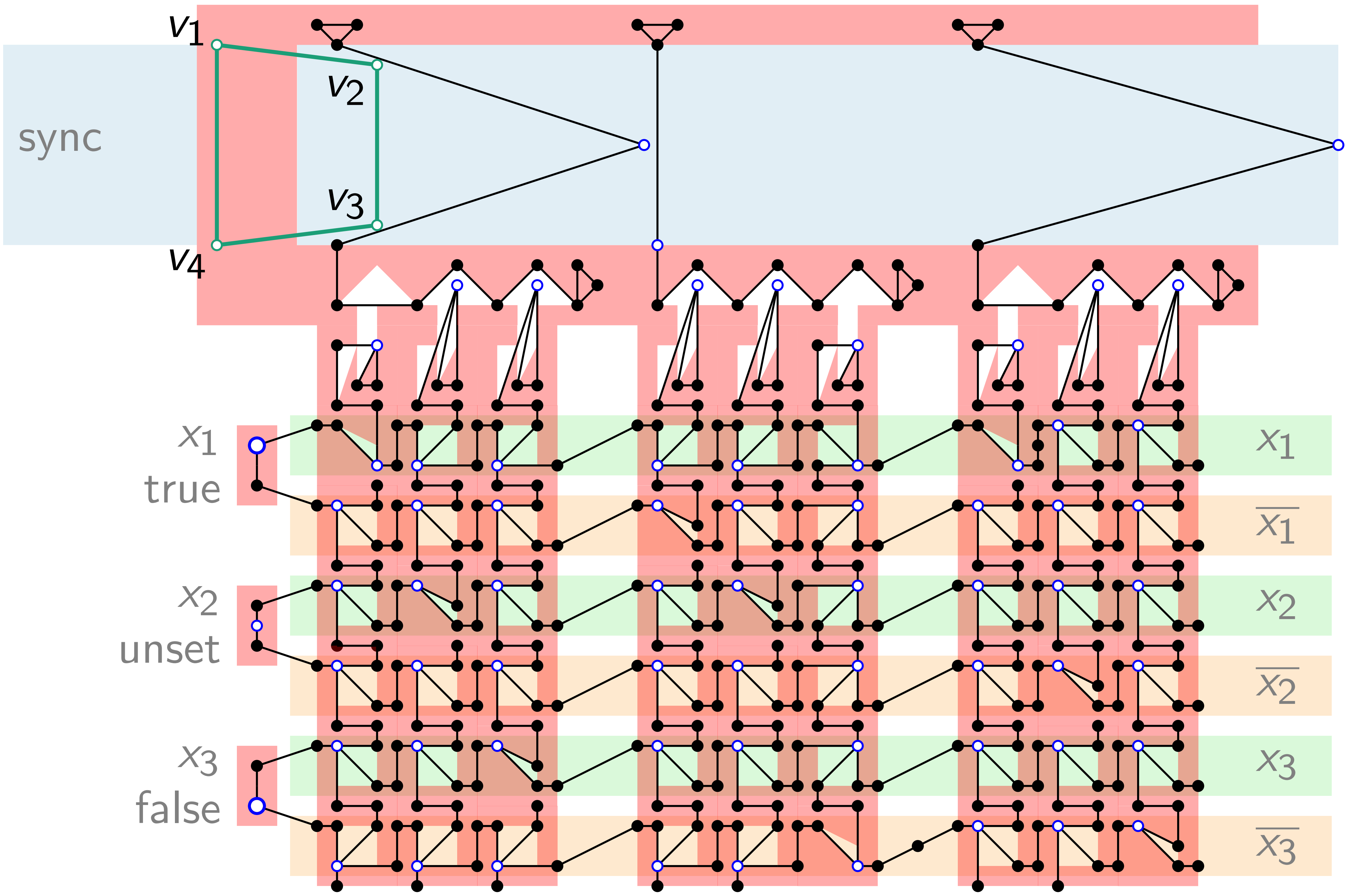
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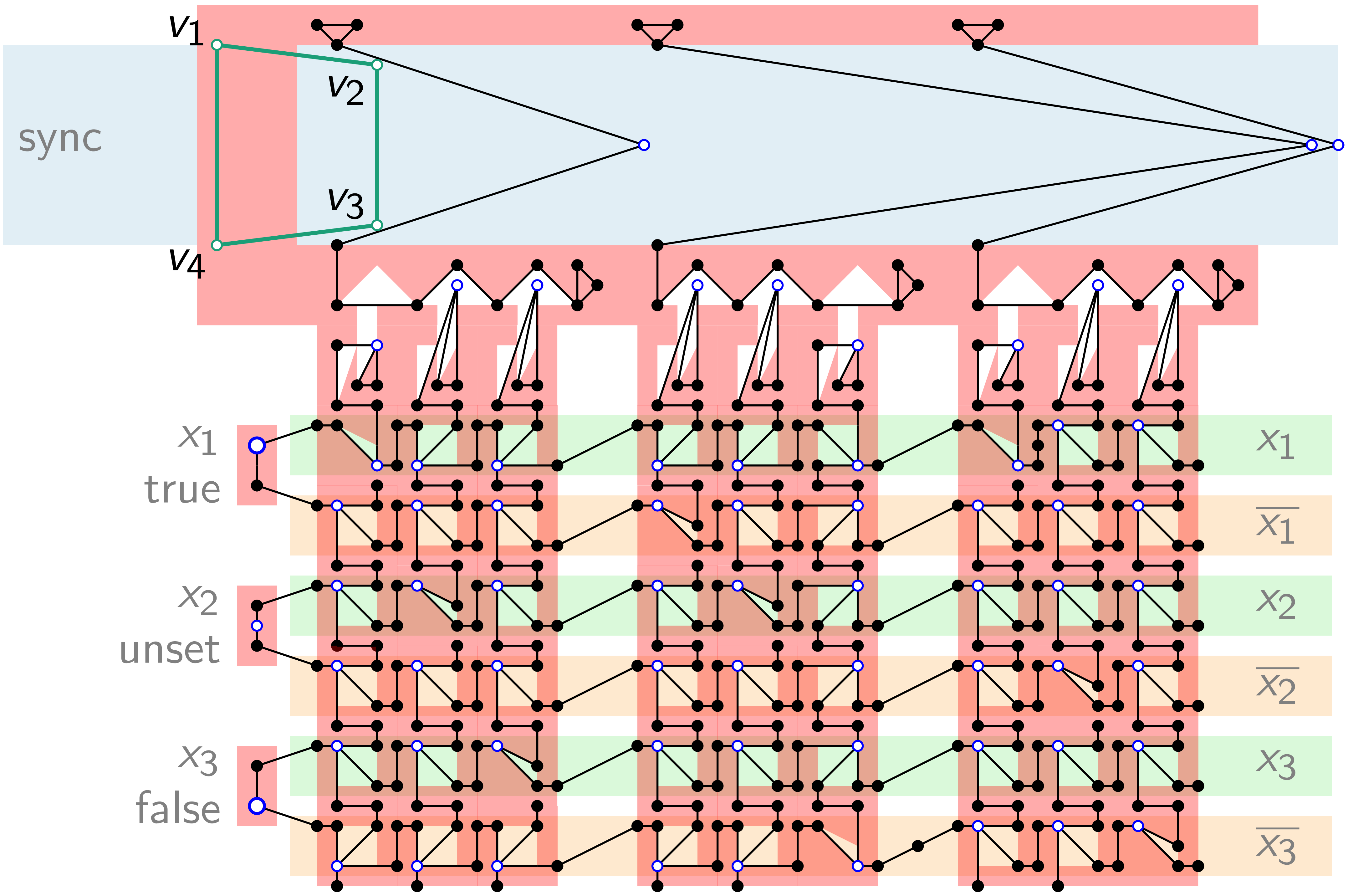

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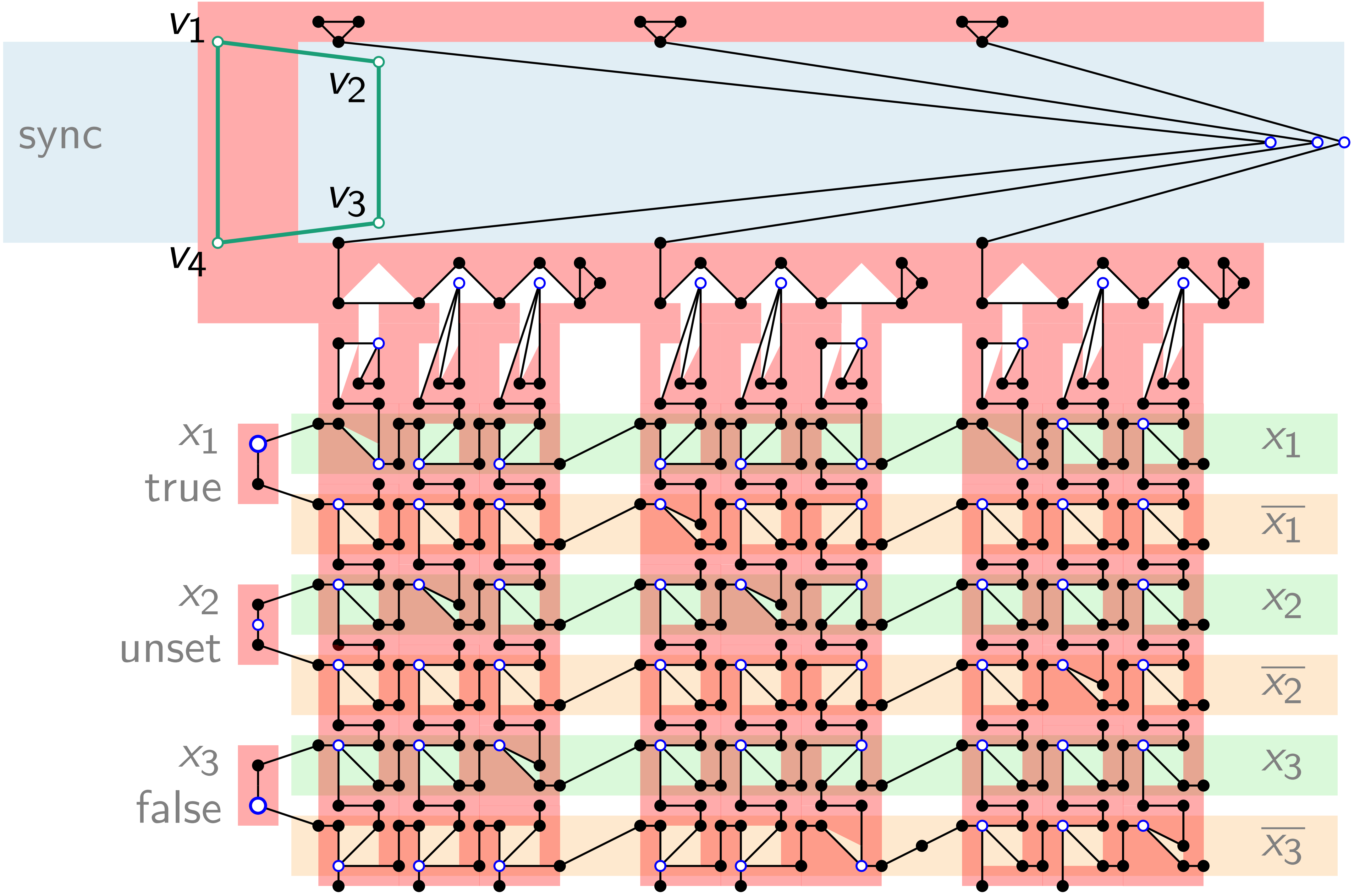

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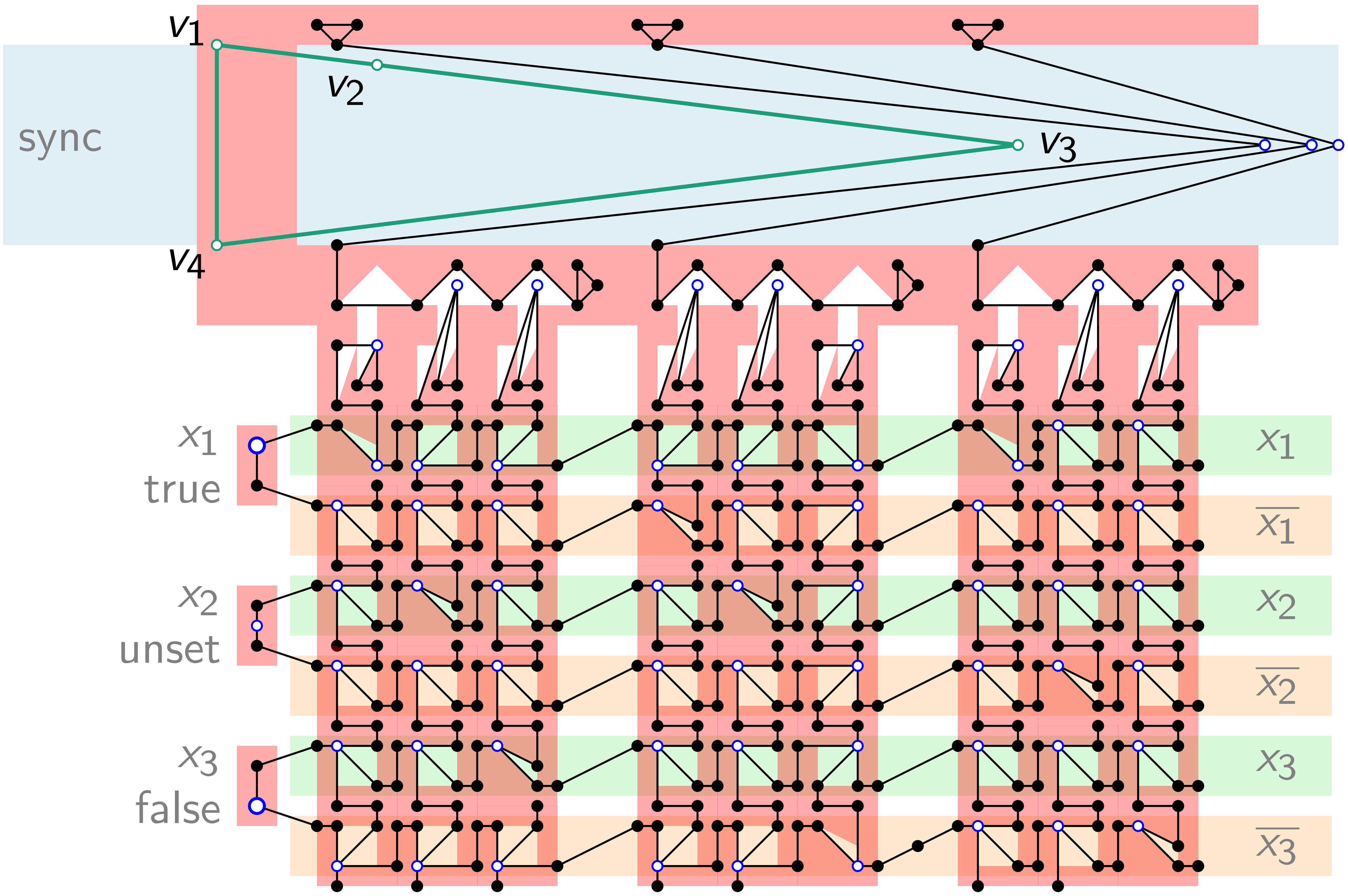
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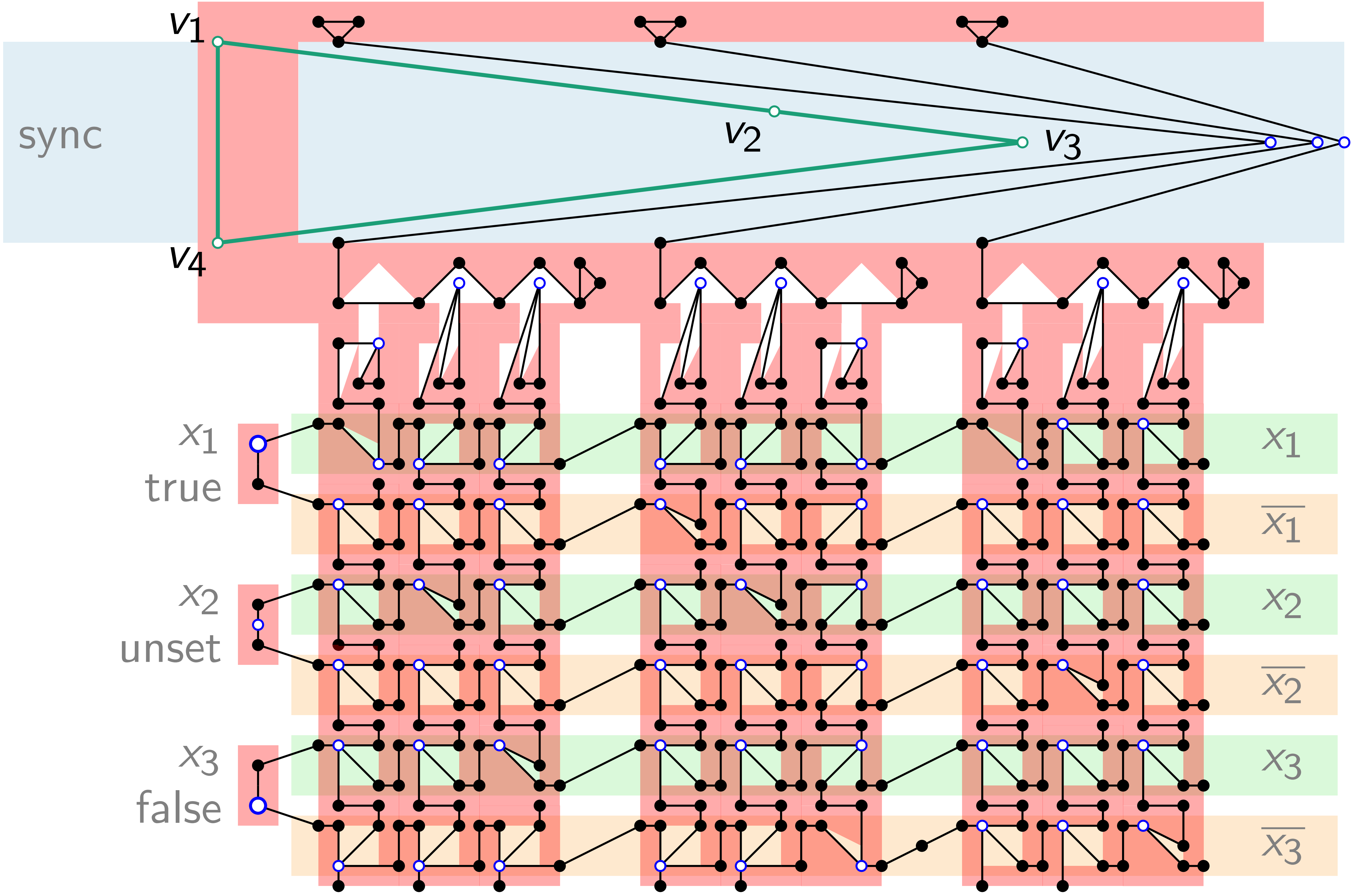

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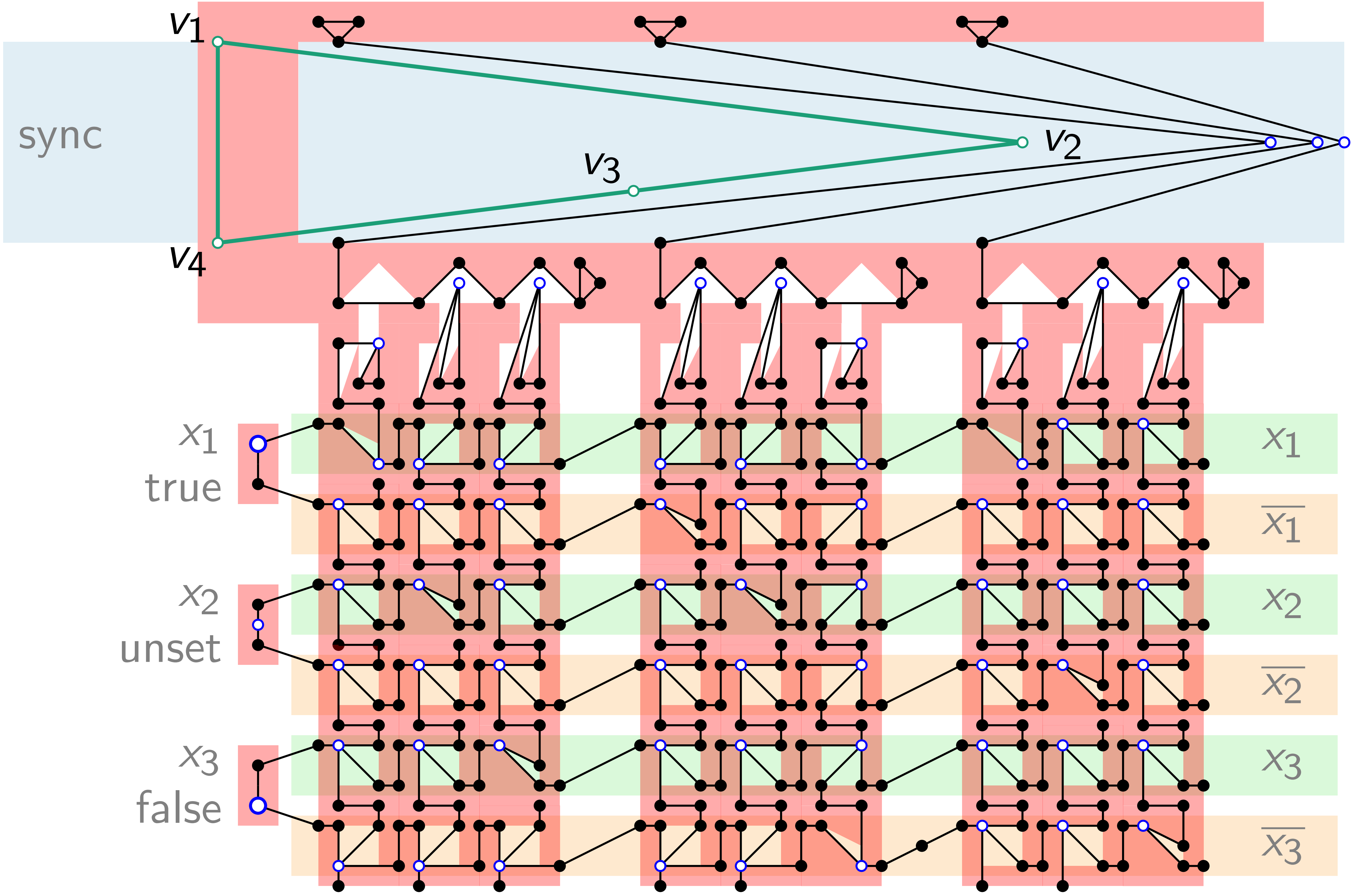

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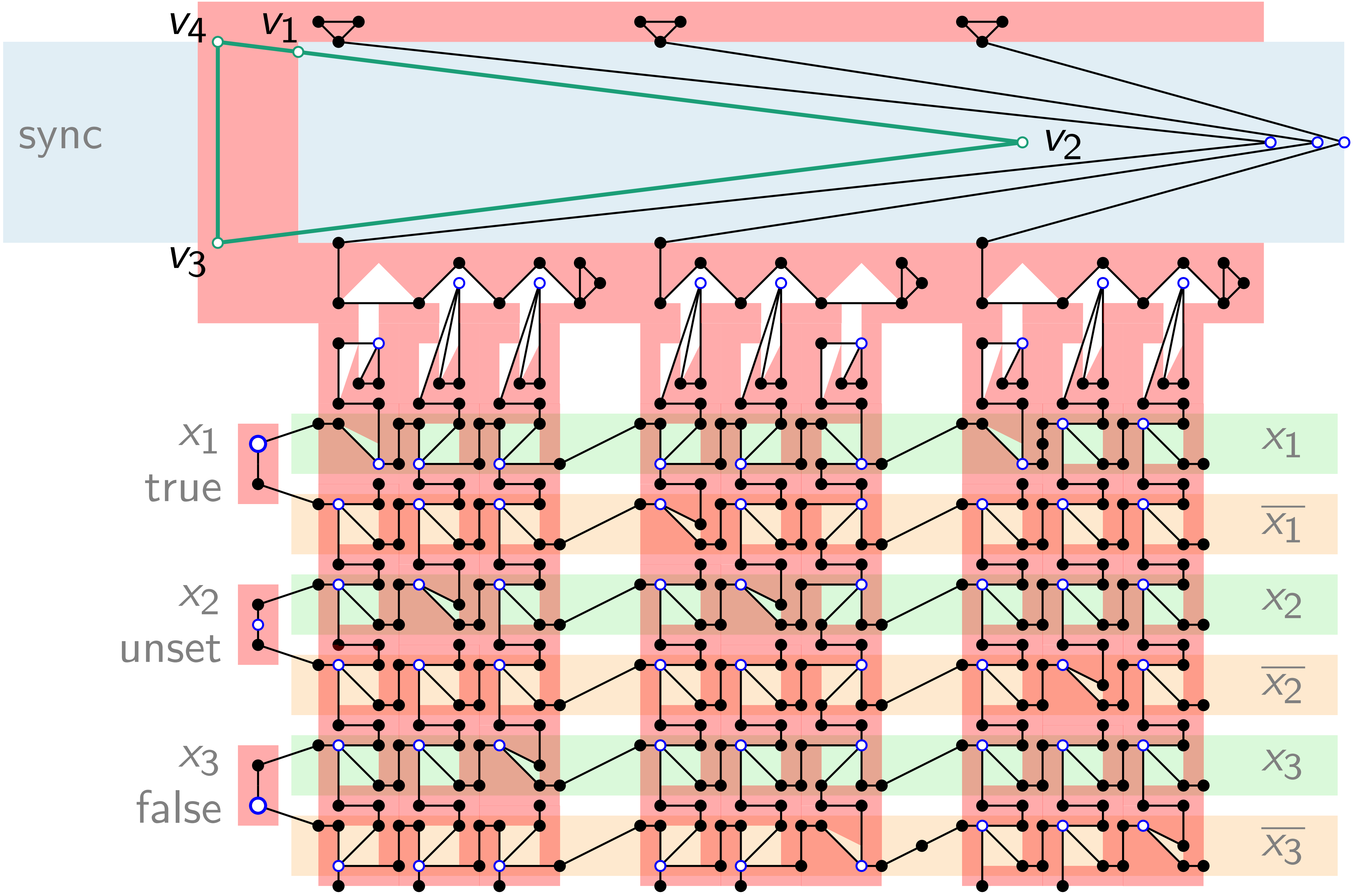

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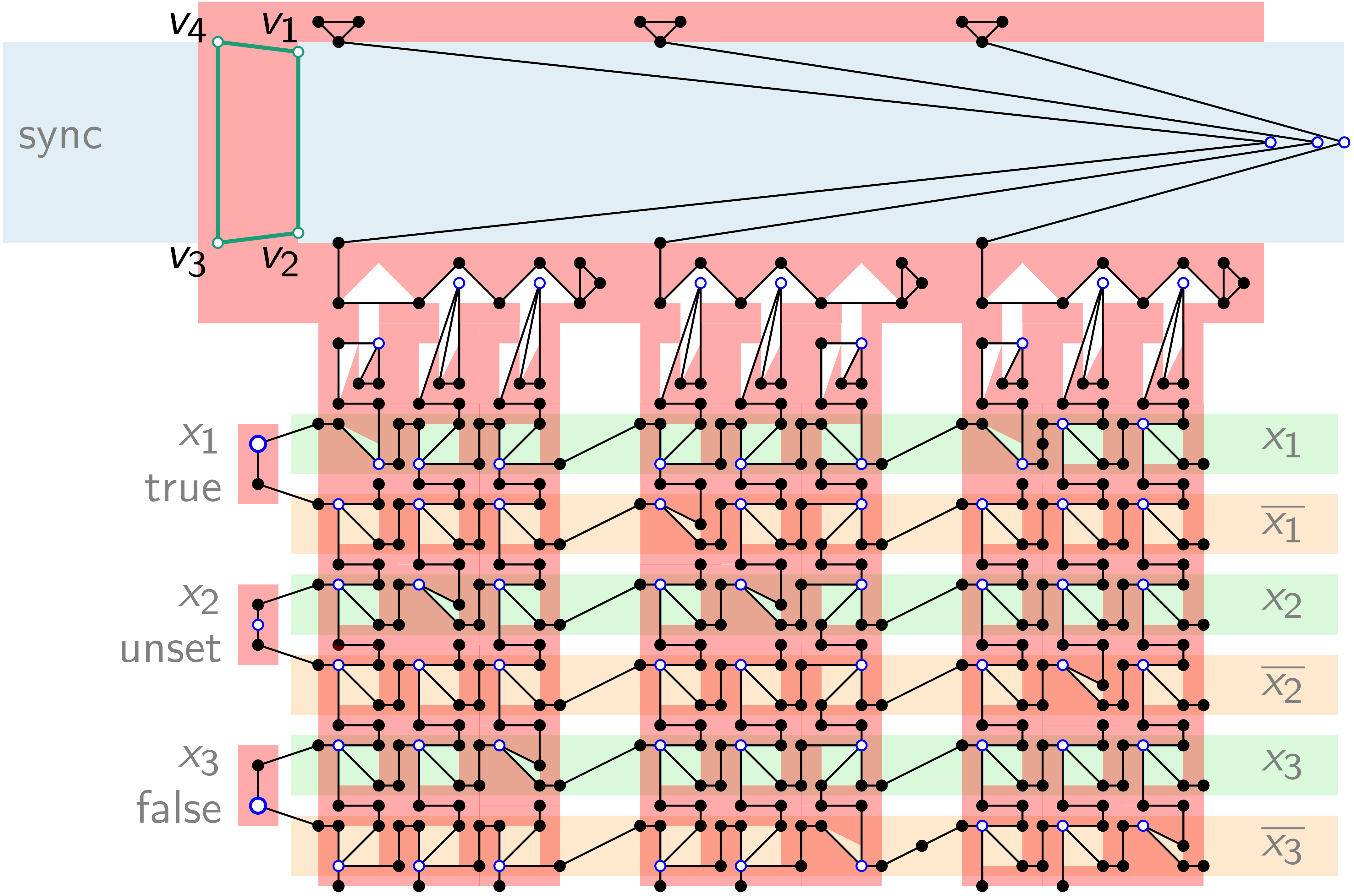

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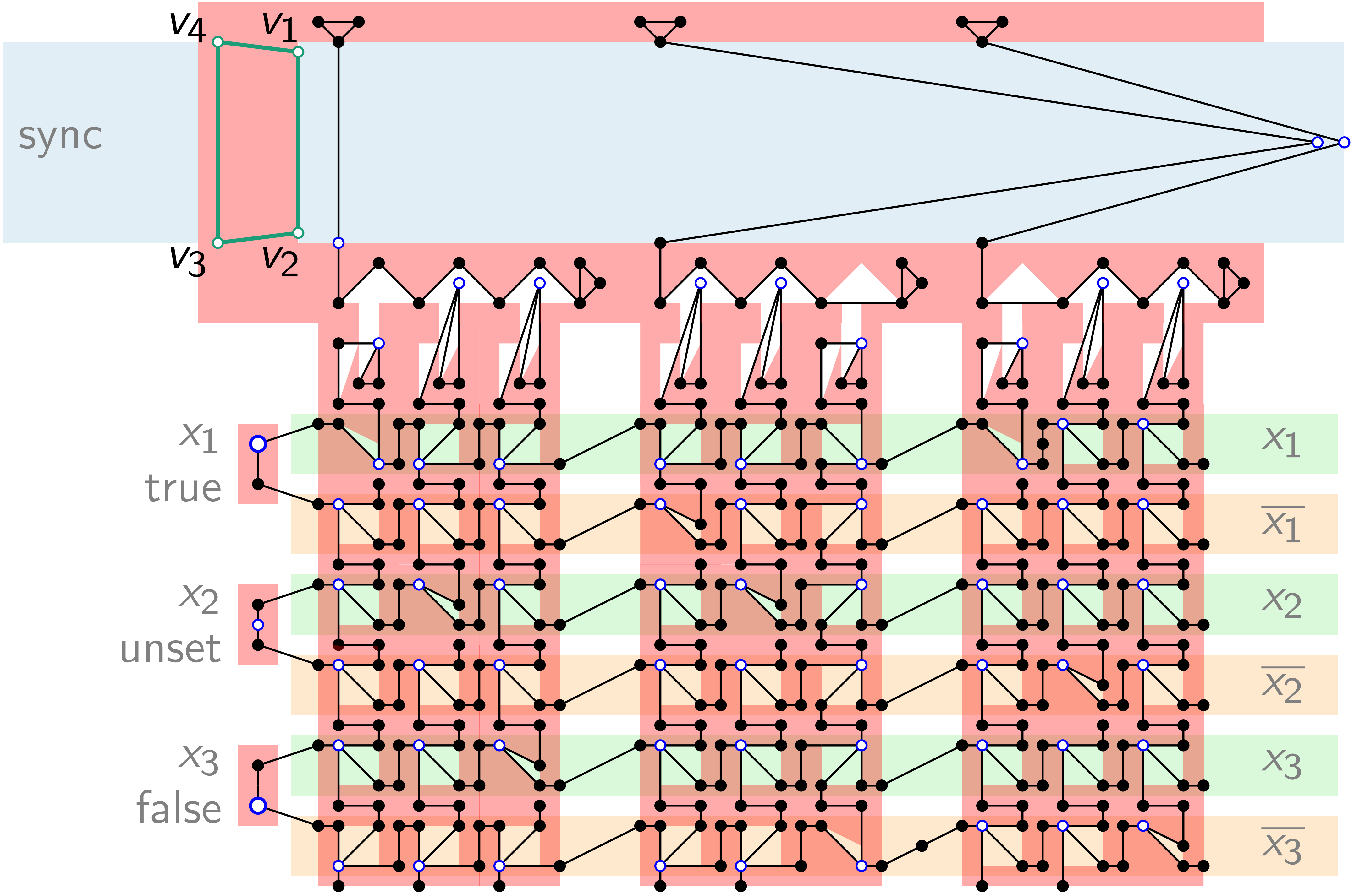
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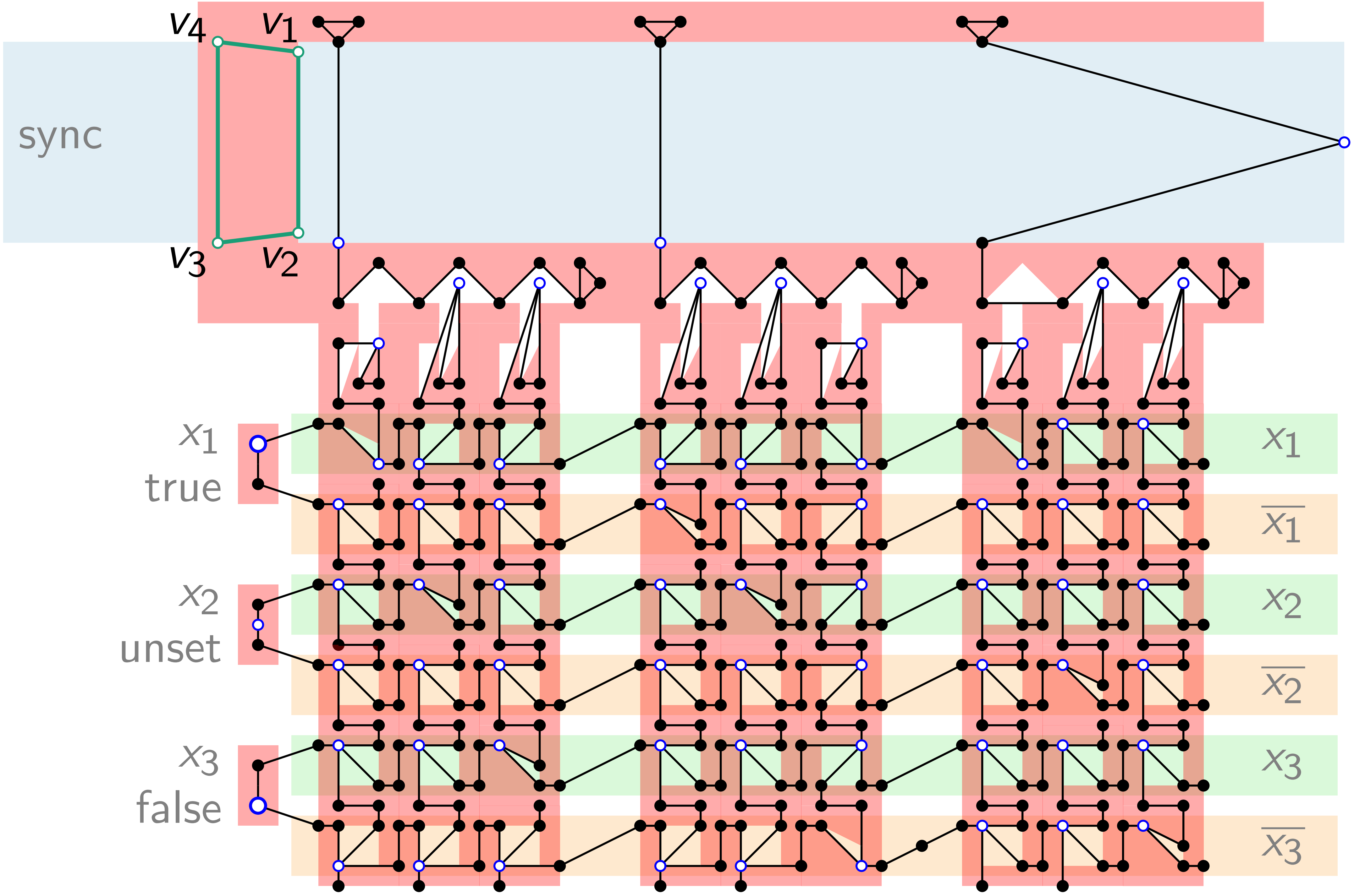

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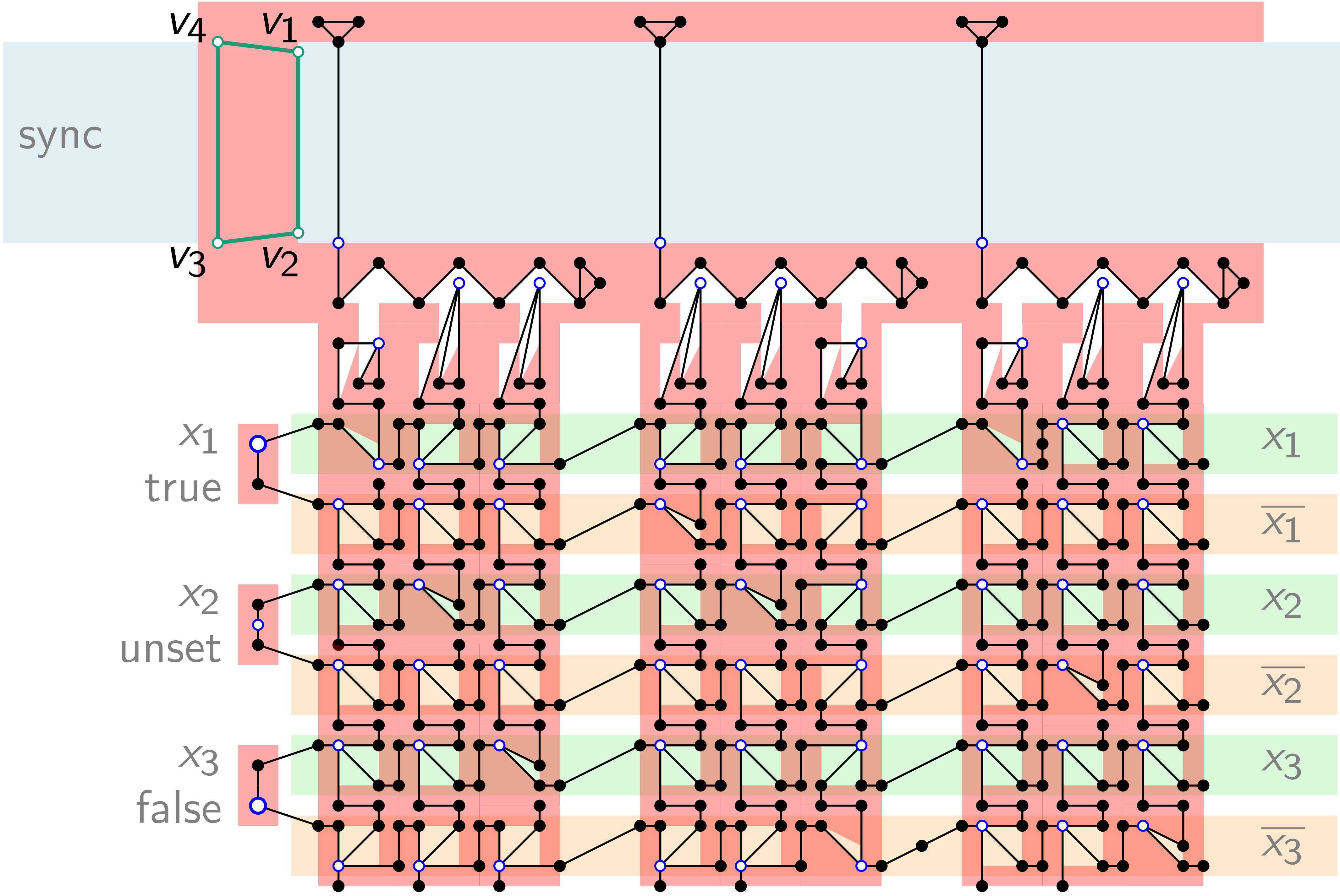

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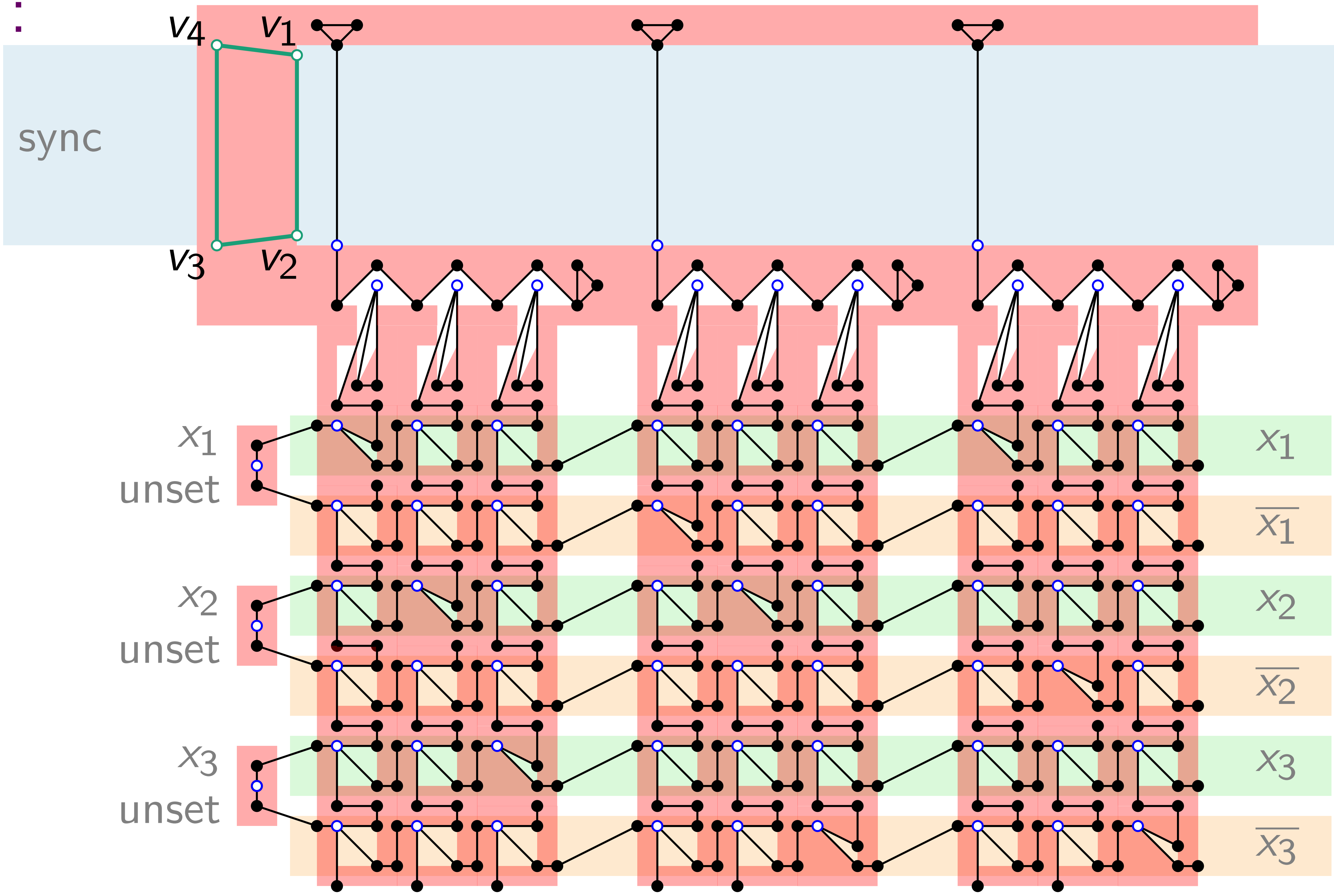

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- What if we restrict the number of piecewise linear morphs?
- Given two drawings of the same graph, how many obstacles are necessary and sufficient to block them? Can this be computed efficiently?