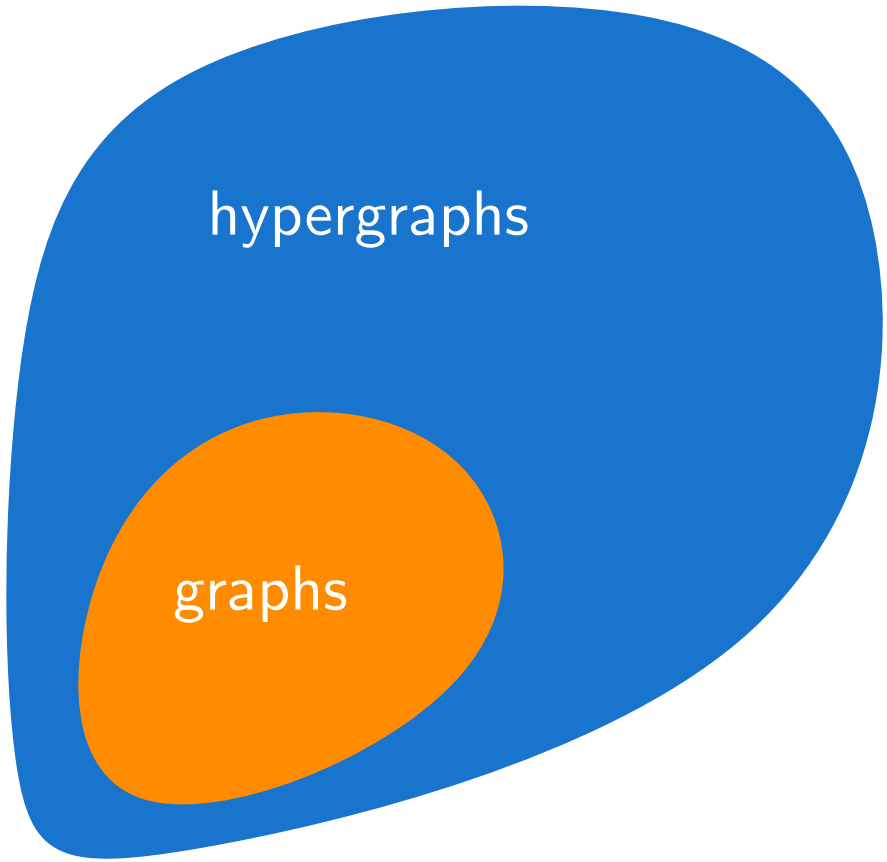

Survey on graph and hypergraph drawing

Dagstuhl Seminar on
Low-Dimensional Topology
André Schulz and Alexander Wolff
August 2019

What is Graph Drawing?

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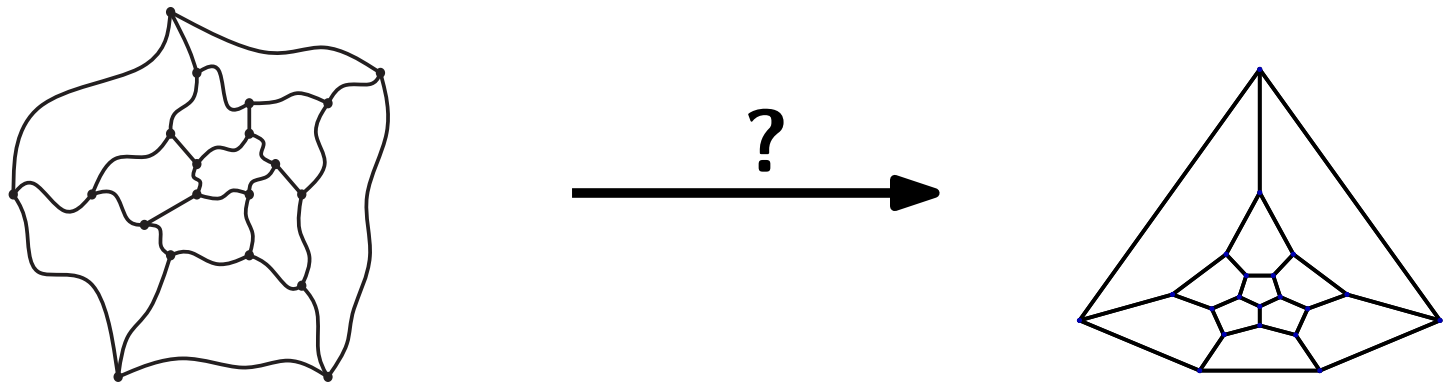

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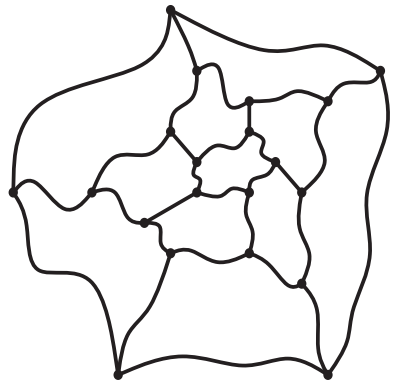
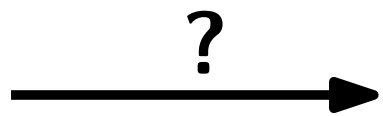
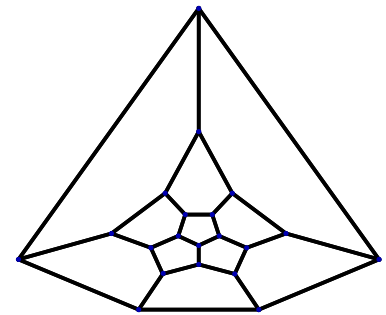


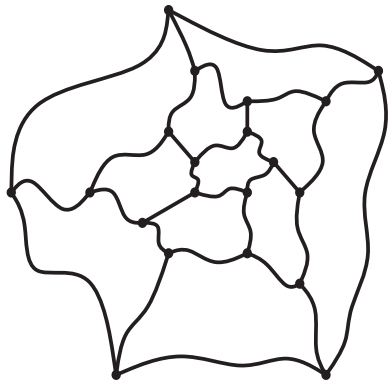




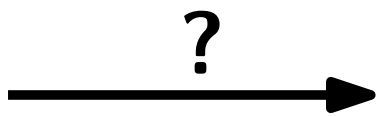
abstract (combinatorial) graph

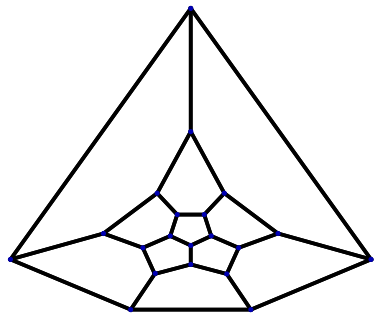
drawing (e.g. node-link diagram)

What is Graph Drawing?



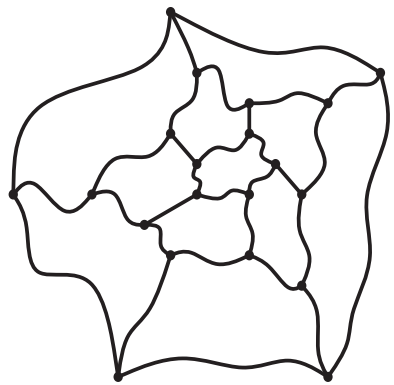
abstract (combinatorial)
graph



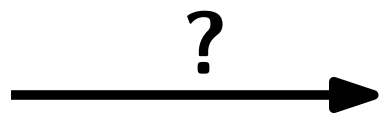


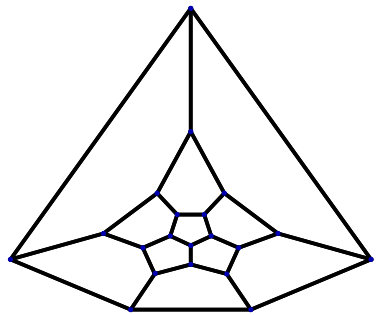
drawing
(e.g. node-link diagram)

What is Graph Drawing?



abstract (combinatorial) graph

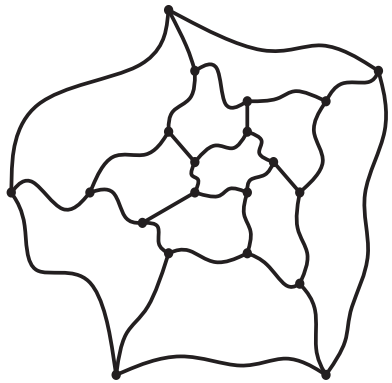




drawing (e.g. node-link diagram)

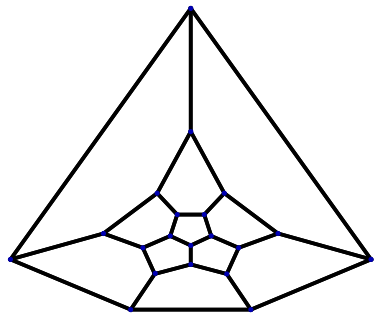
Goal: Algorithm guarantees a (provable) geometric quality measure in the worst case.

What is Graph Drawing?



?

ALGORITHM



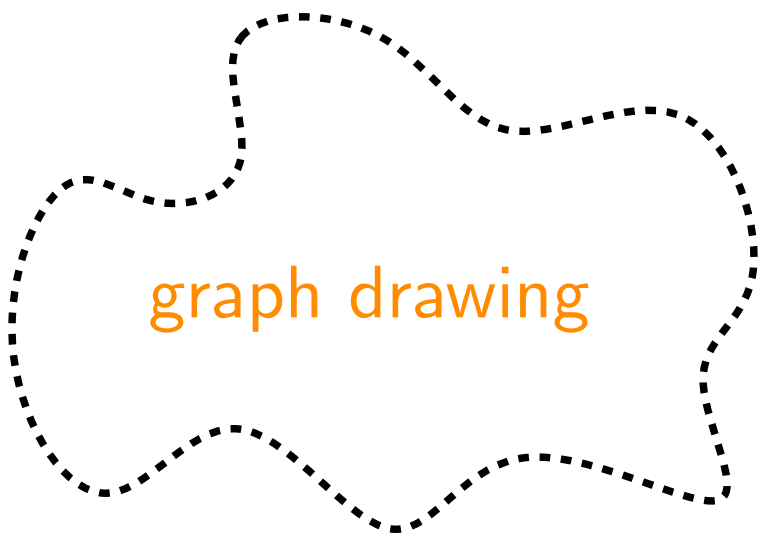
abstract (combinatorial) graph

drawing (e.g. node-link diagram)

Goal: Algorithm guarantees a (provable) geometric quality measure in the worst case.

Evaluation is (usually) not task-driven.

The Many Dimensions of Graph Drawing



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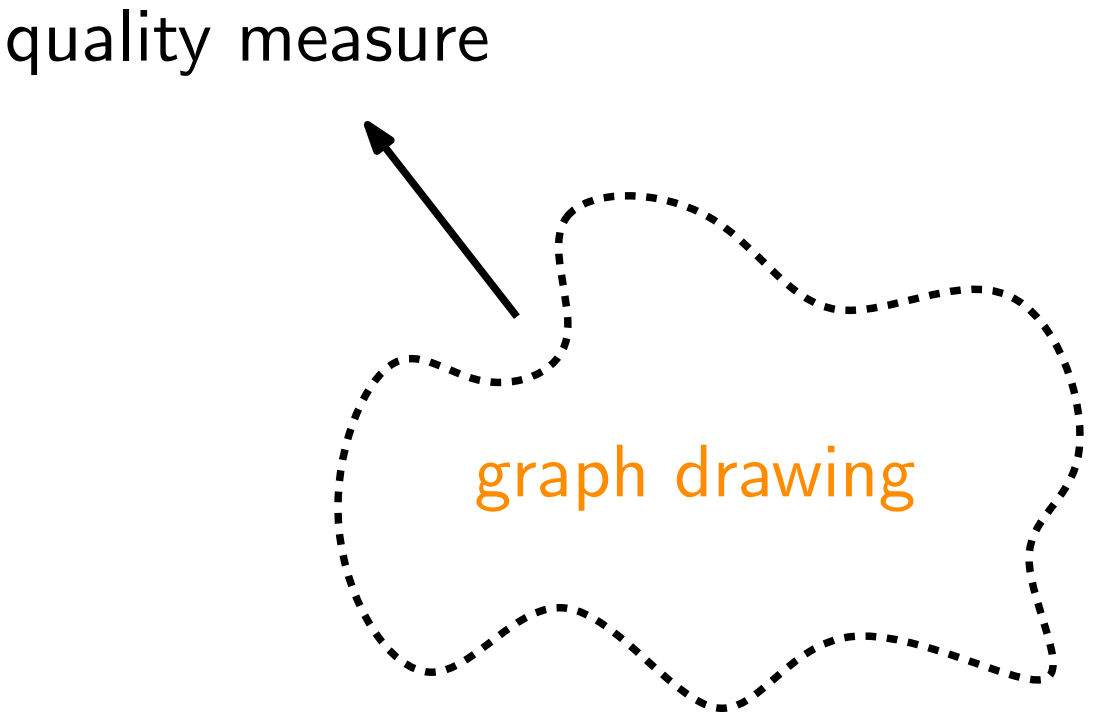

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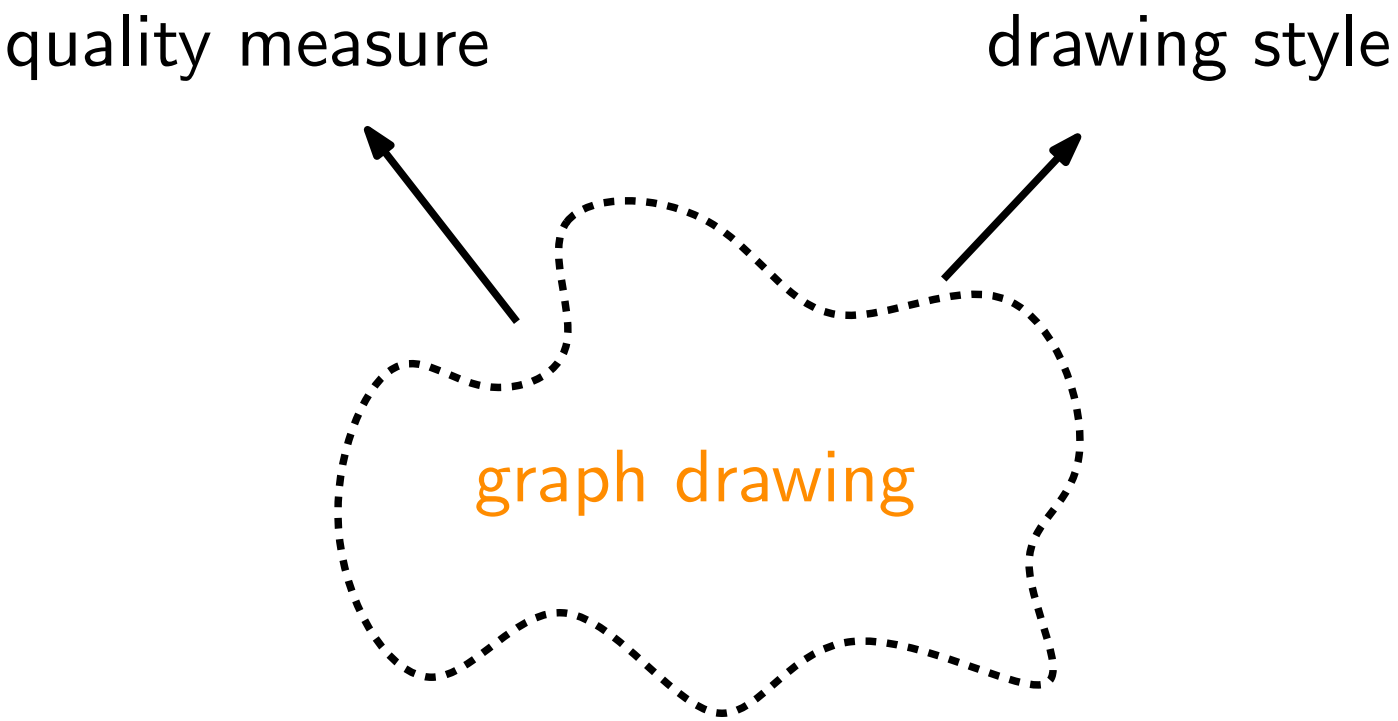

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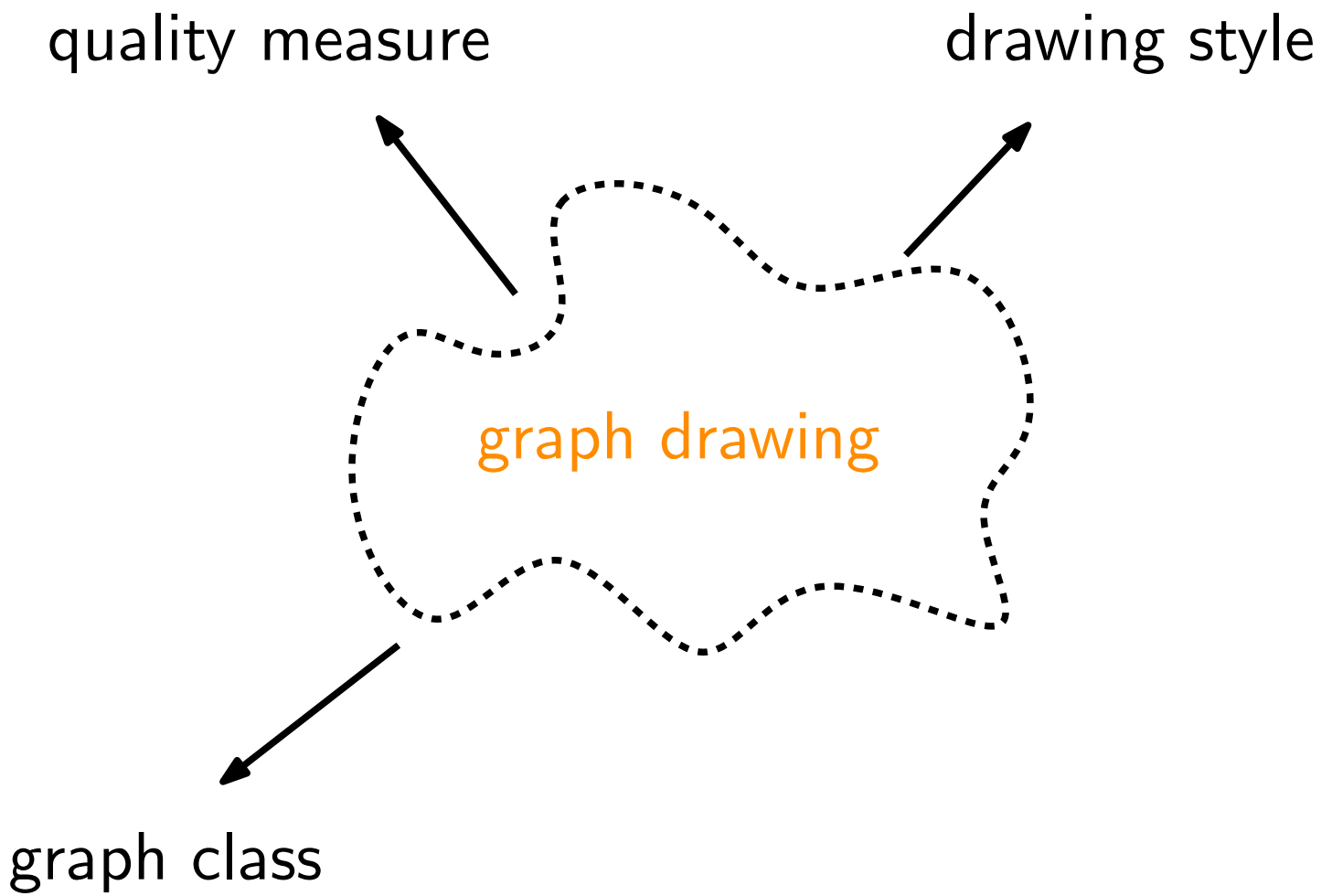

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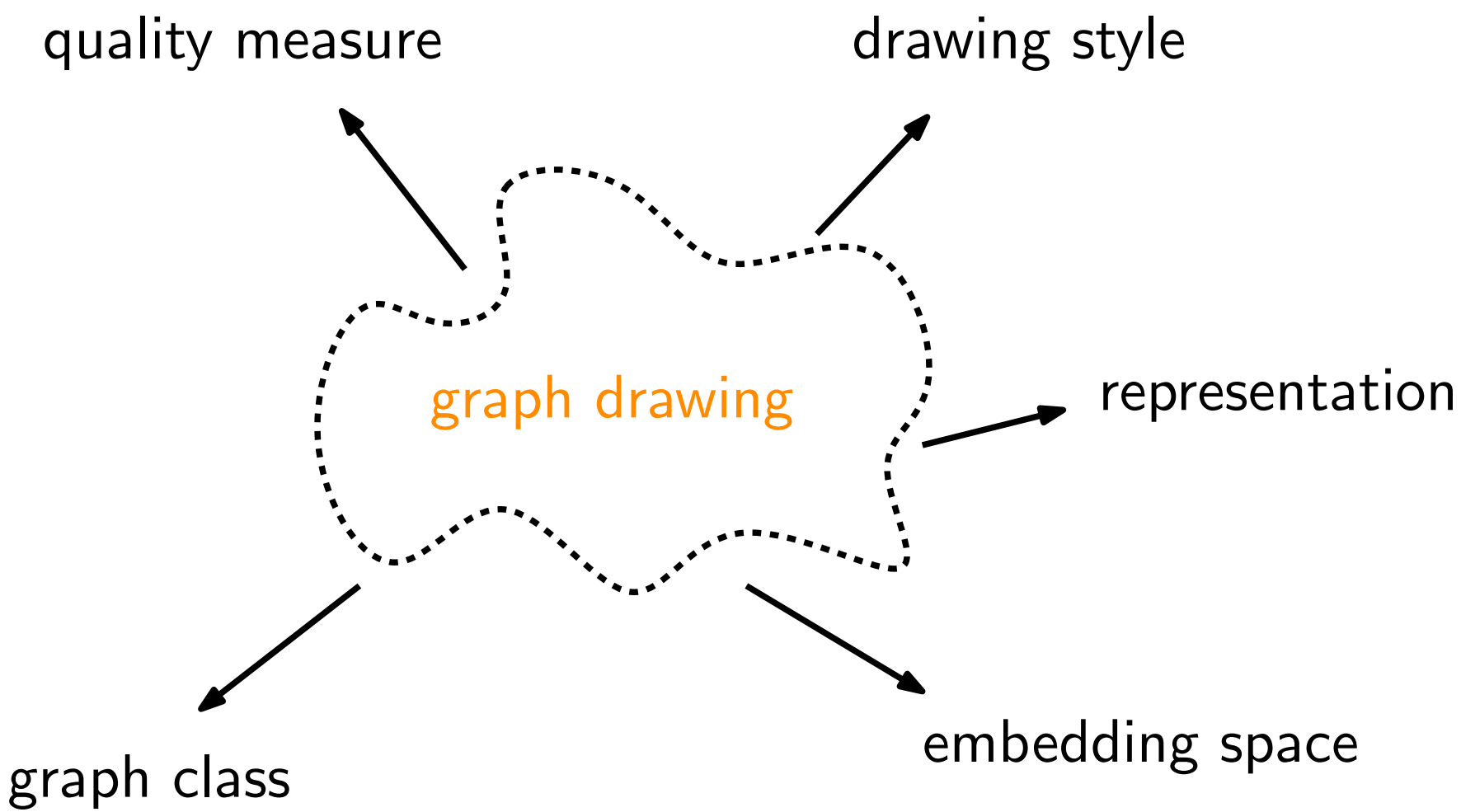

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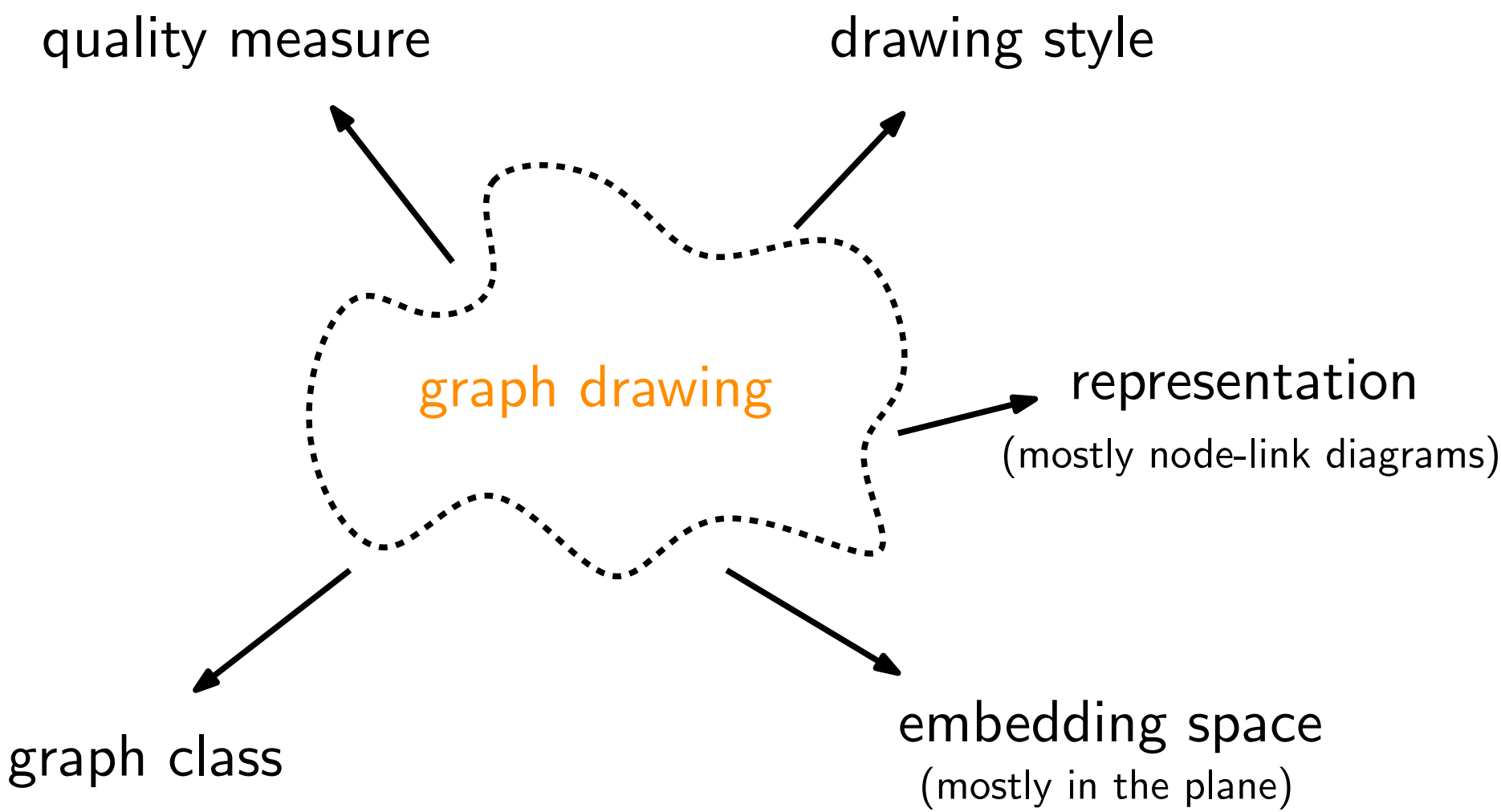
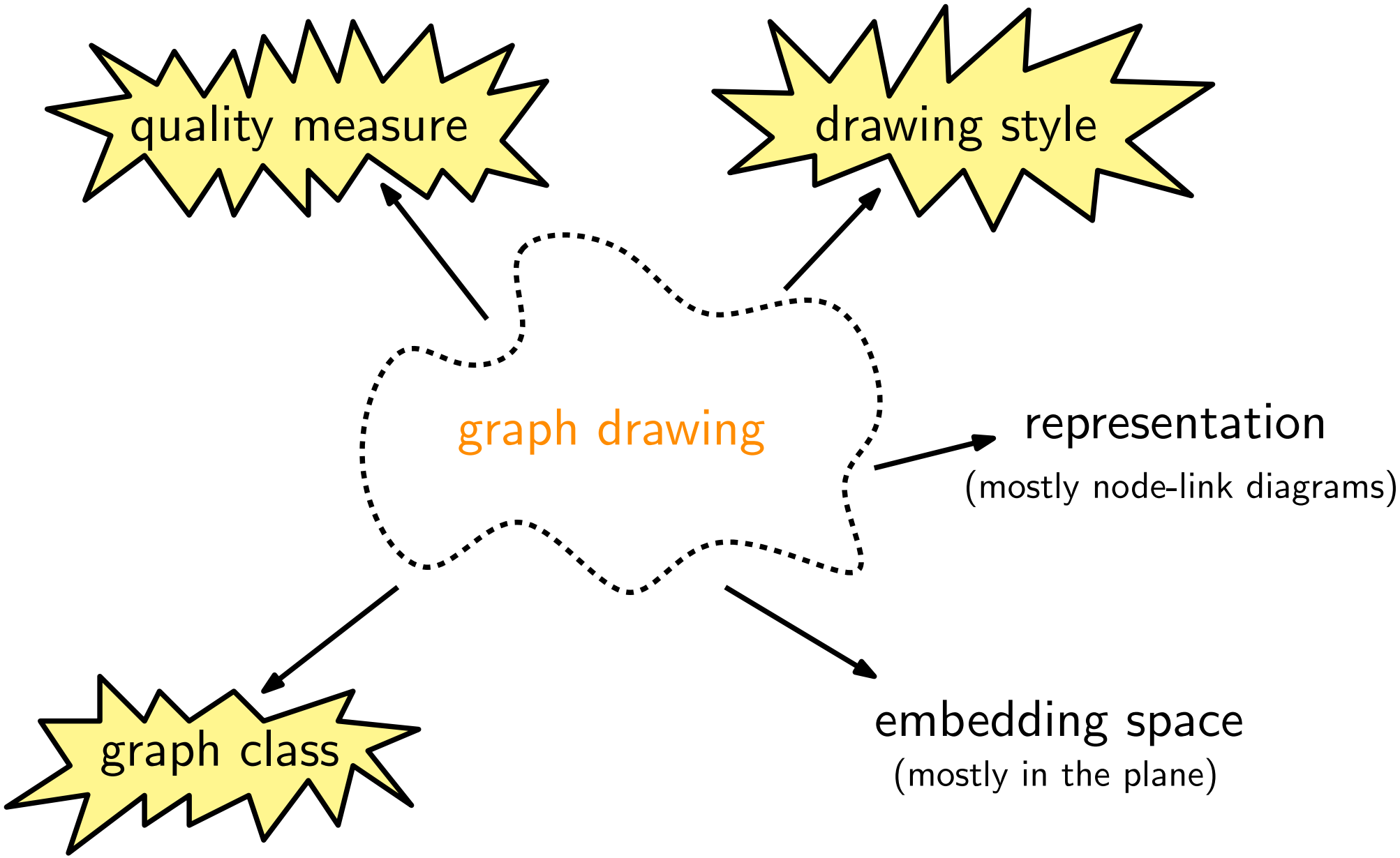


The Many Dimensions of Graph Drawing



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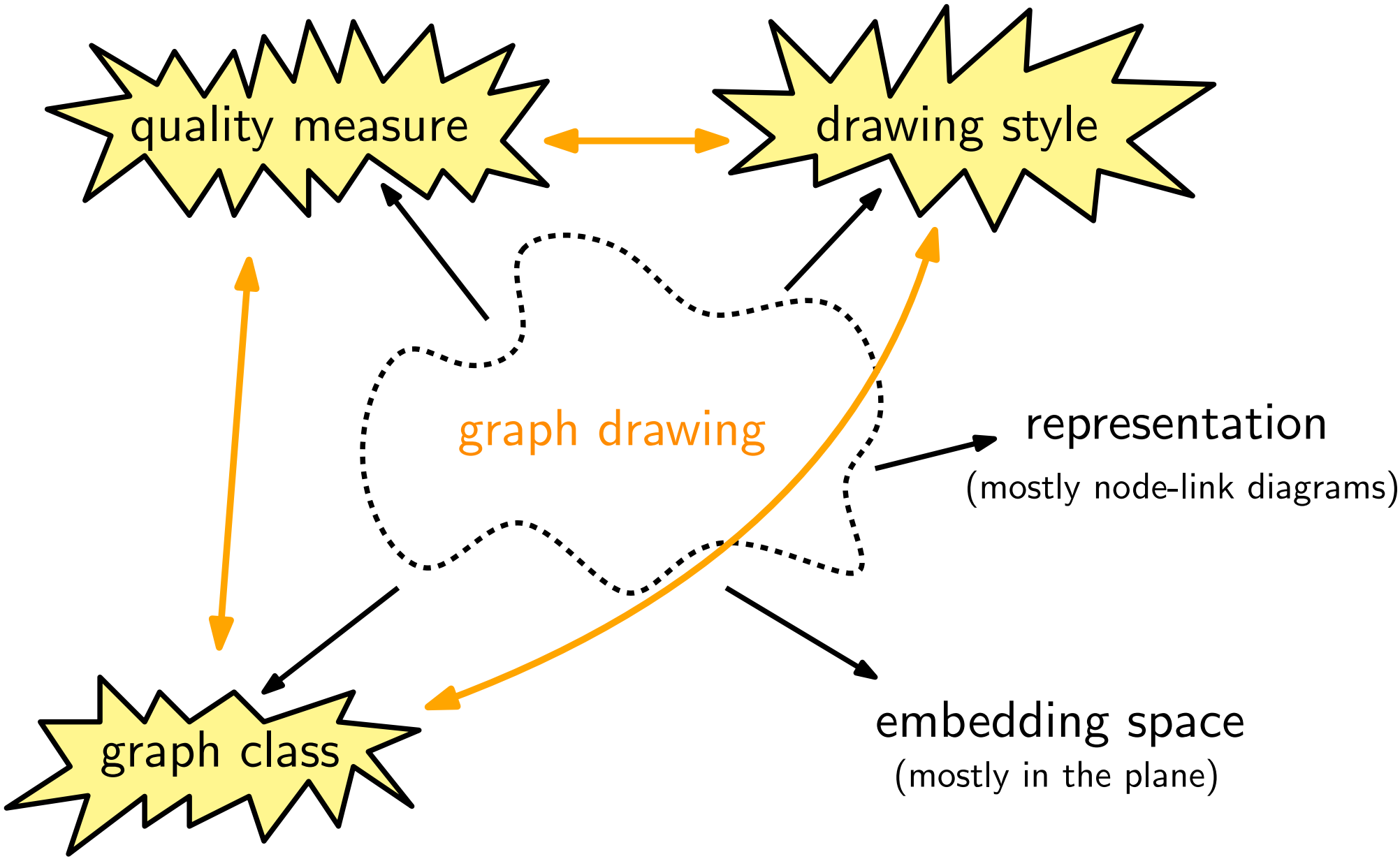
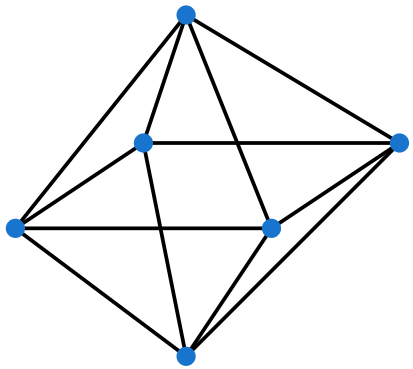
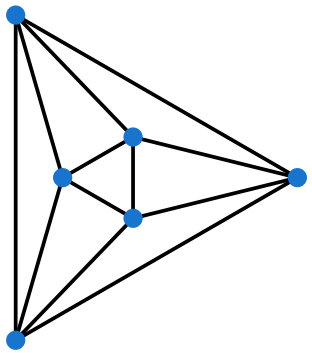
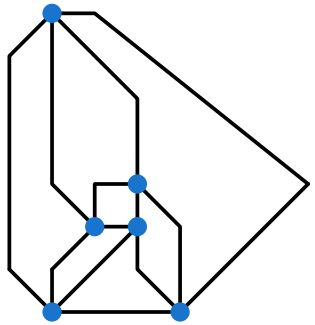


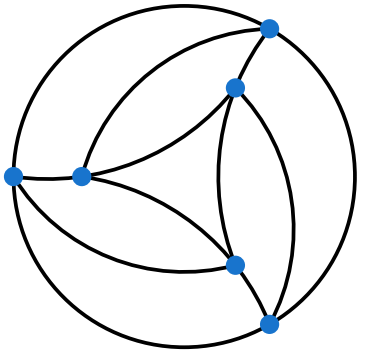
Drawing Styles

Drawing Styles

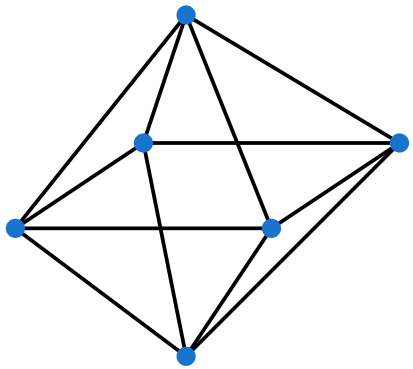
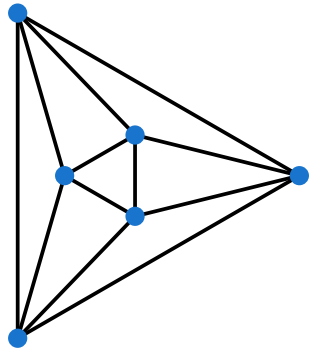
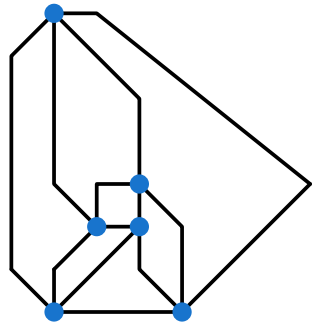
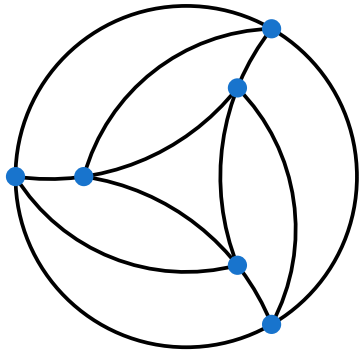






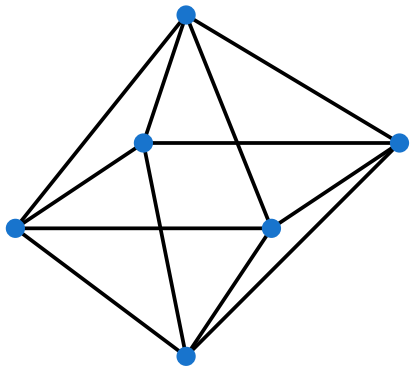
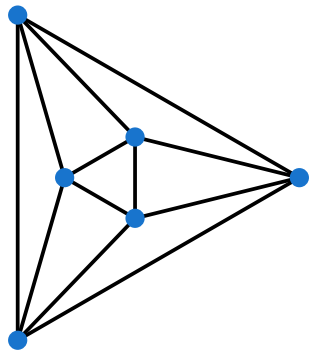
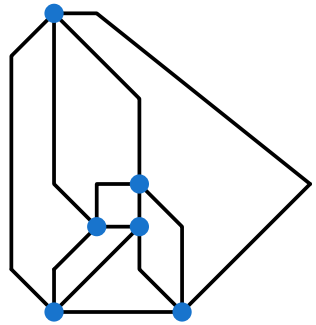
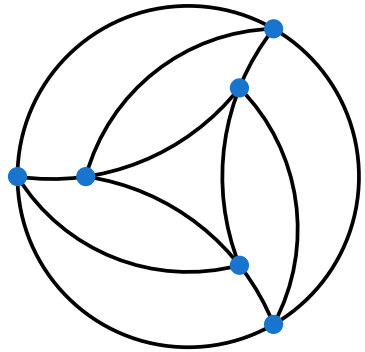


Drawing Styles

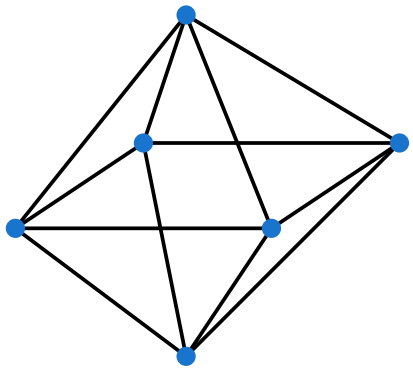
- straight-line vs. curved

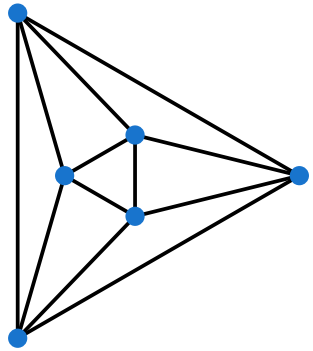
Drawing Styles

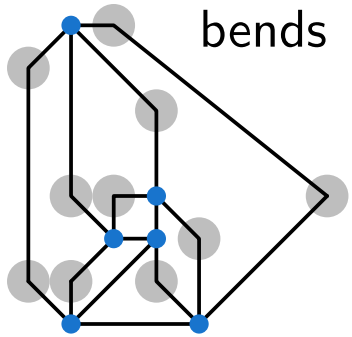





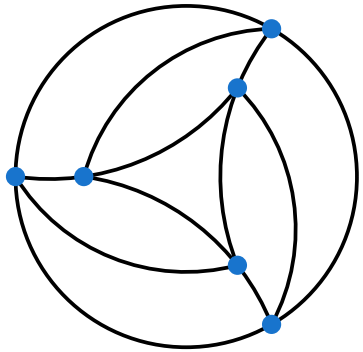
- straight-line vs. curved
- straight-line vs. polyline

Drawing Styles



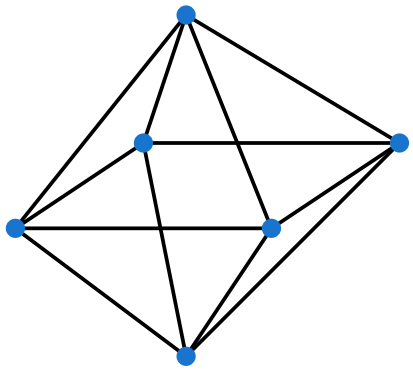
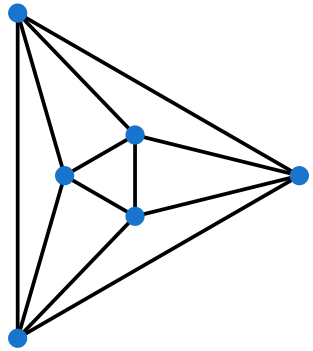
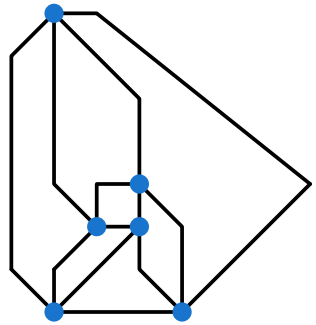
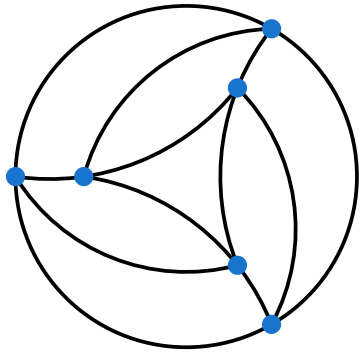






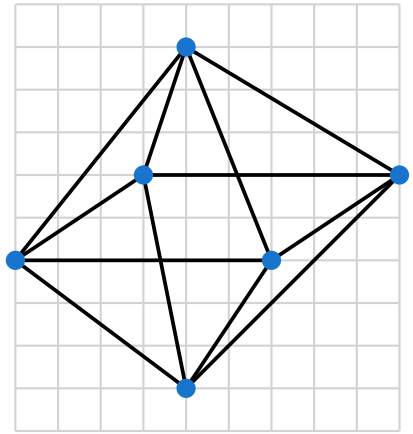
- straight-line vs. curved
- straight-line vs. polyline

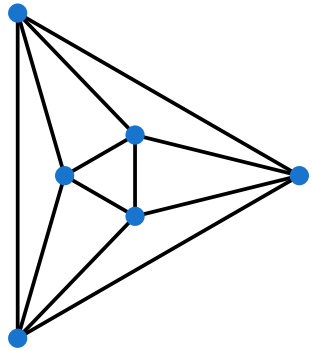
Drawing Styles

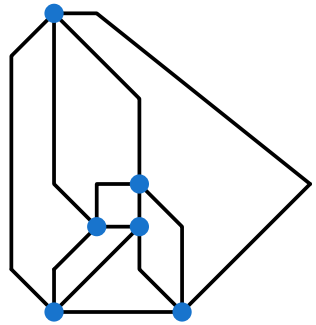





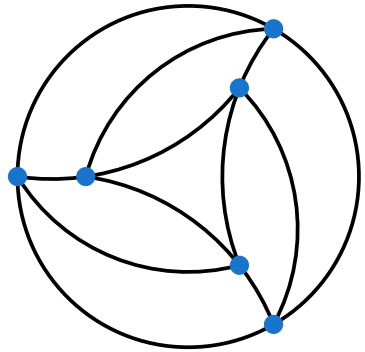
- straight-line vs. curved
- straight-line vs. polyline
- restricted slopes

Drawing Styles



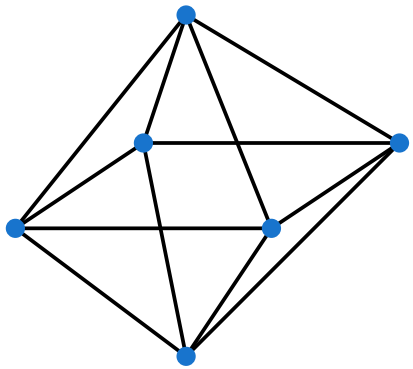
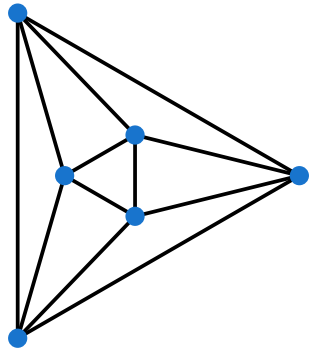
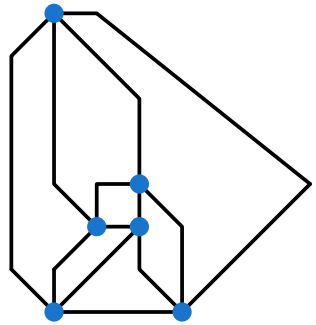
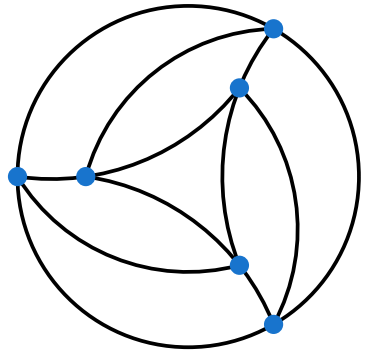






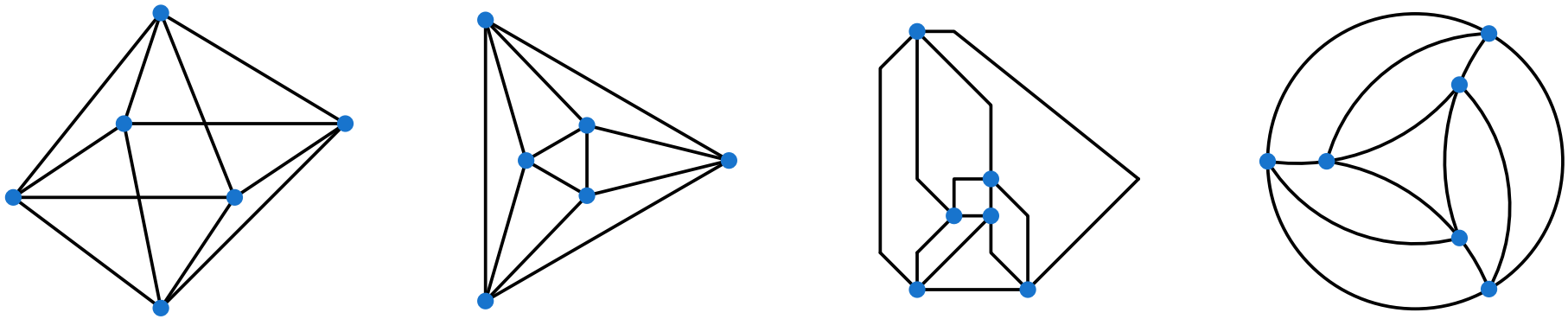
- straight-line vs. curved
- straight-line vs. polyline
- restricted slopes
- restricted to grid points

Drawing Styles

- straight-line vs. curved
- straight-line vs. polyline
- restricted slopes
- restricted to grid points
- directed drawings

Drawing Styles



- straight-line vs. curved
- straight-line vs. polyline
- restricted slopes
- restricted to grid points
- directed drawings
- monotone drawings, confluent drawings, partial edge drawing, radial drawings, thick drawings, Lombardi drawings, ...

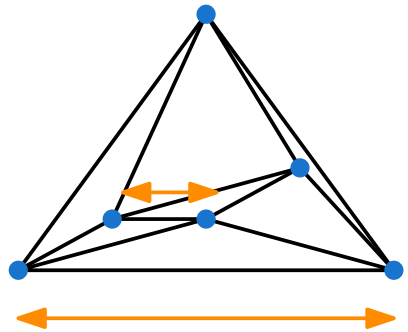
Classical Measures

Classical Measures

- vertex resolution

Classical Measures

- vertex resolution $= \dfrac{\text{maximal distance between two vertices}}{\text{minimal distance between two vertices}}$



Classical Measures

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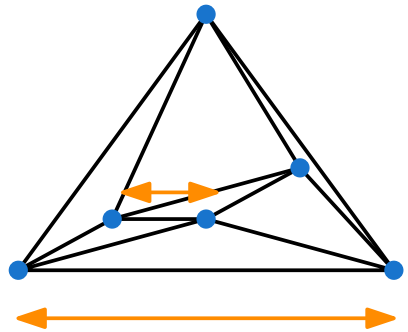

goal: small vertex resolution

Classical Measures

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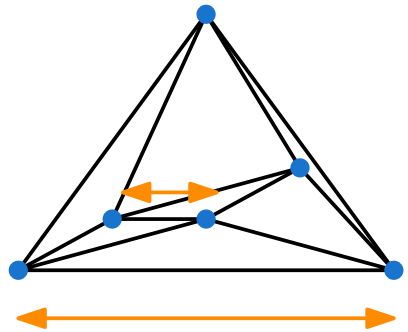
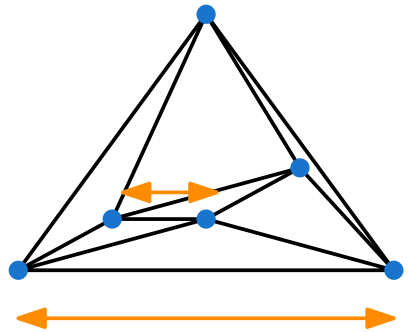


goal: small vertex resolution

- angular resolution

Classical Measures

- vertex resolution $= \dfrac{\text{maximal distance between two vertices}}{\text{minimal distance between two vertices}}$

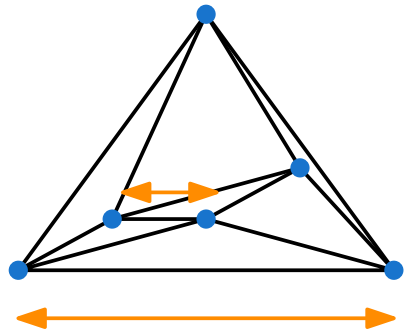


goal: small vertex resolution

- angular resolution = size of the smallest angle

Classical Measures

- vertex resolution $= \dfrac{\text{maximal distance between two vertices}}{\text{minimal distance between two vertices}}$



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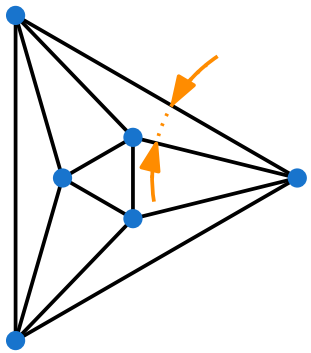
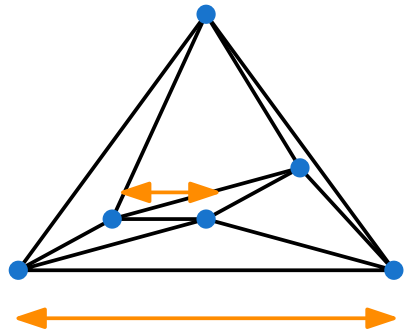


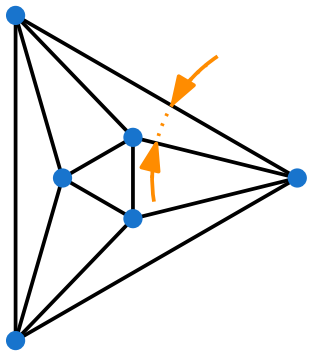
Classical Measures

- vertex resolution $= \dfrac{\text{maximal distance between two vertices}}{\text{minimal distance between two vertices}}$



goal: small vertex resolution

- angular resolution = size of the smallest angle



goal: large angular resolution

More Measures

More Measures

- grid size

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- grid size $=$ area of the drawing using integer grid points

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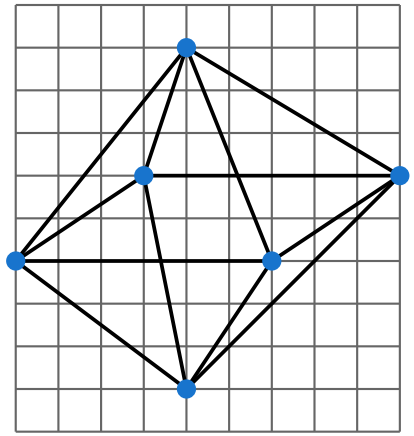

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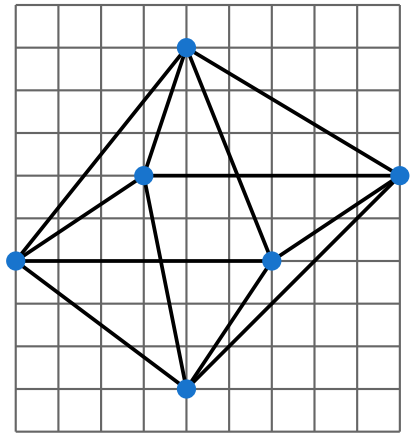

goal: small grid size

$\rightarrow$ implies good vertex and angular resolution

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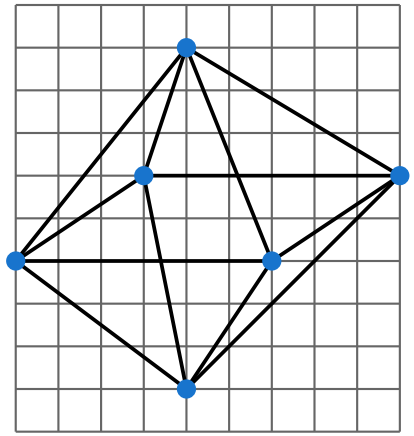

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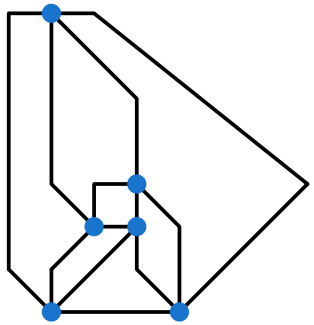
- number of bends



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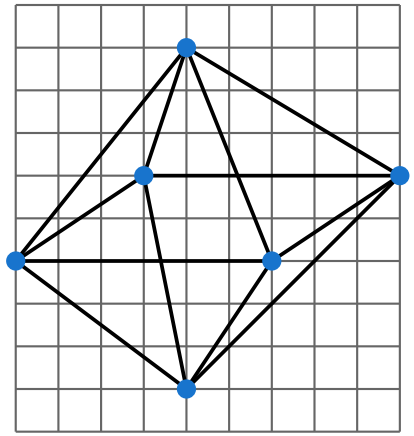

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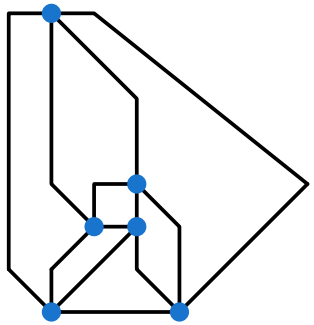

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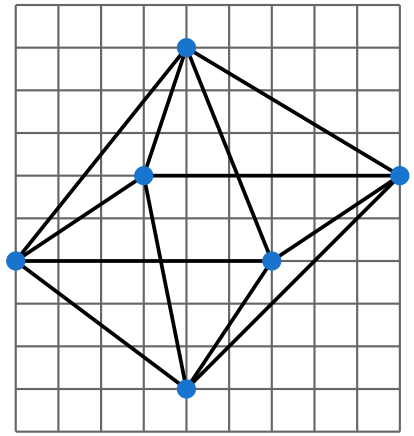

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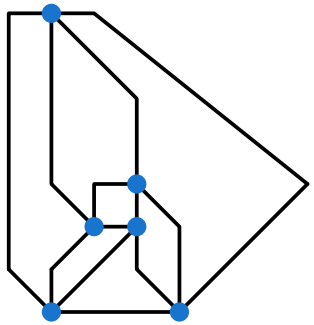

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More Measures

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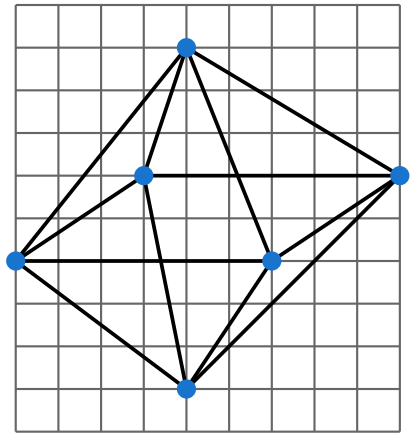

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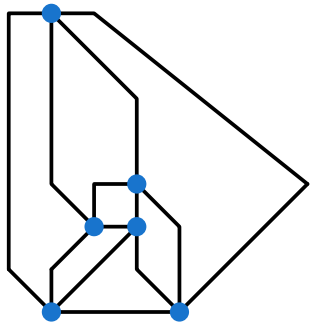

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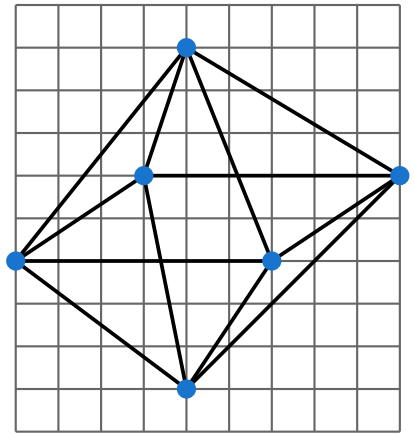

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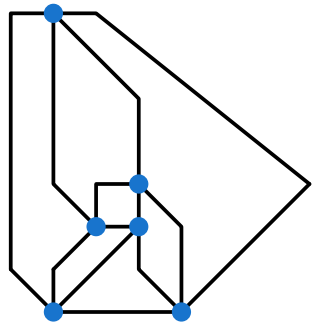

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Improving on one measure often decreases another measure!

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- planar graphs (can be drawn without crossings)
- trees (connected, no cycles)
- triangulations (maximal planar)
- planar 3-trees
- outerplanar graphs
- serial-parallel graphs
- k-connected graphs
- ...

Prominent Graph Classes by Example

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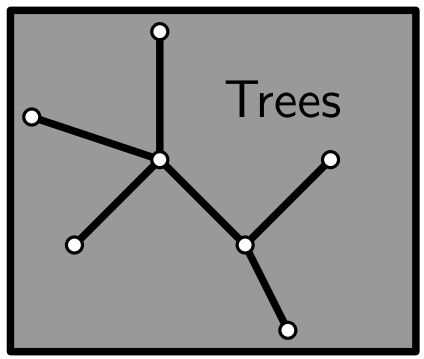

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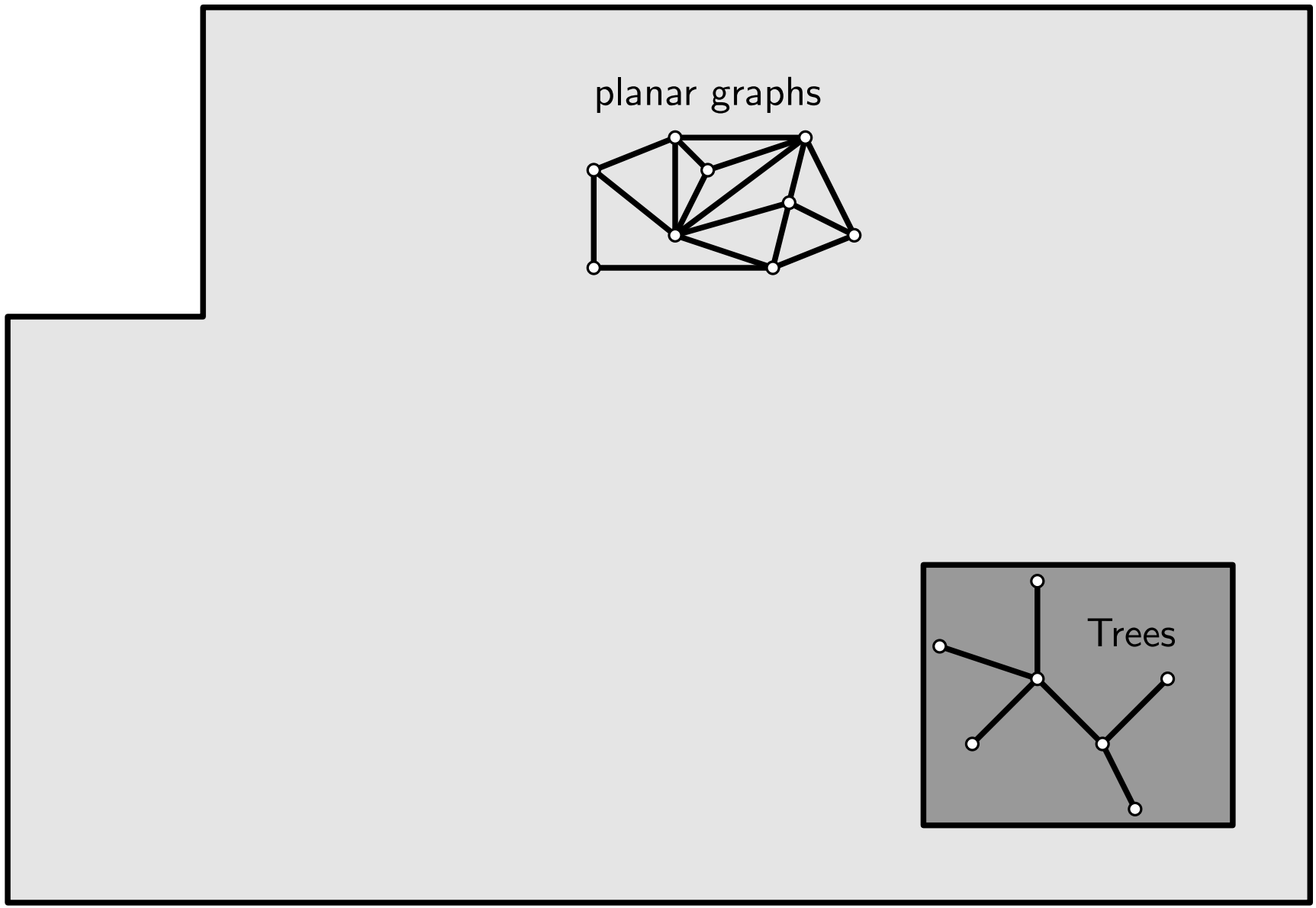

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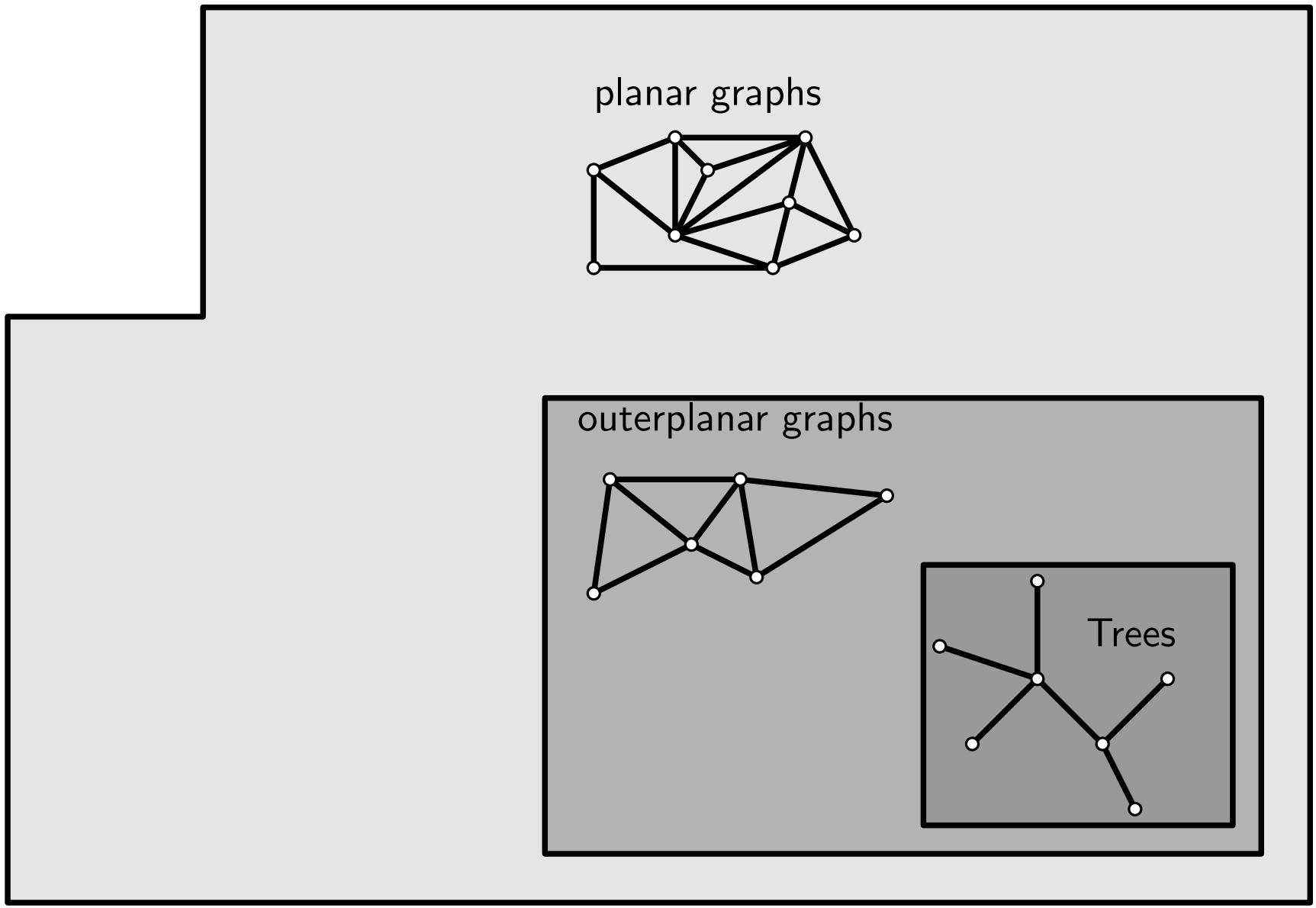

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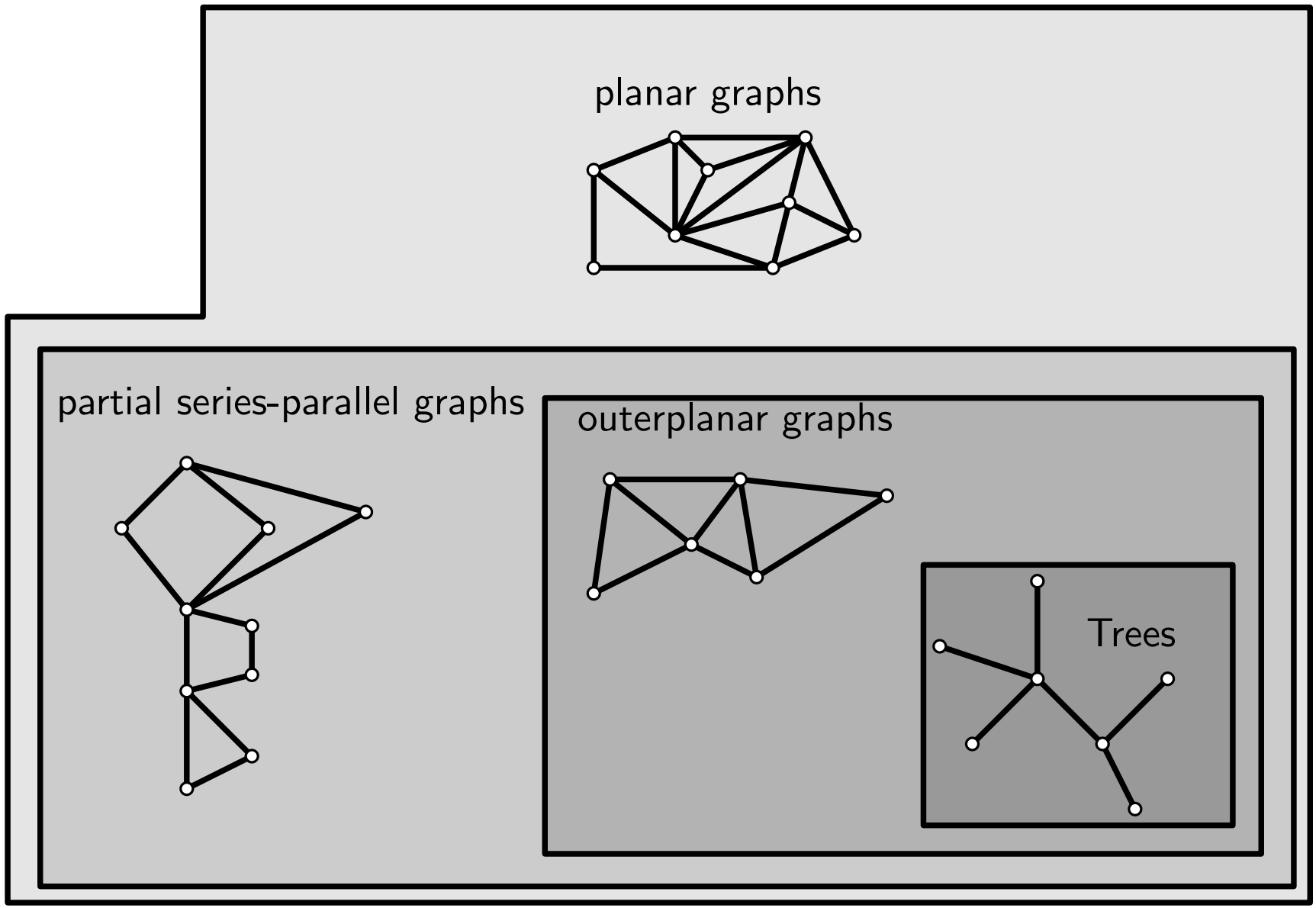

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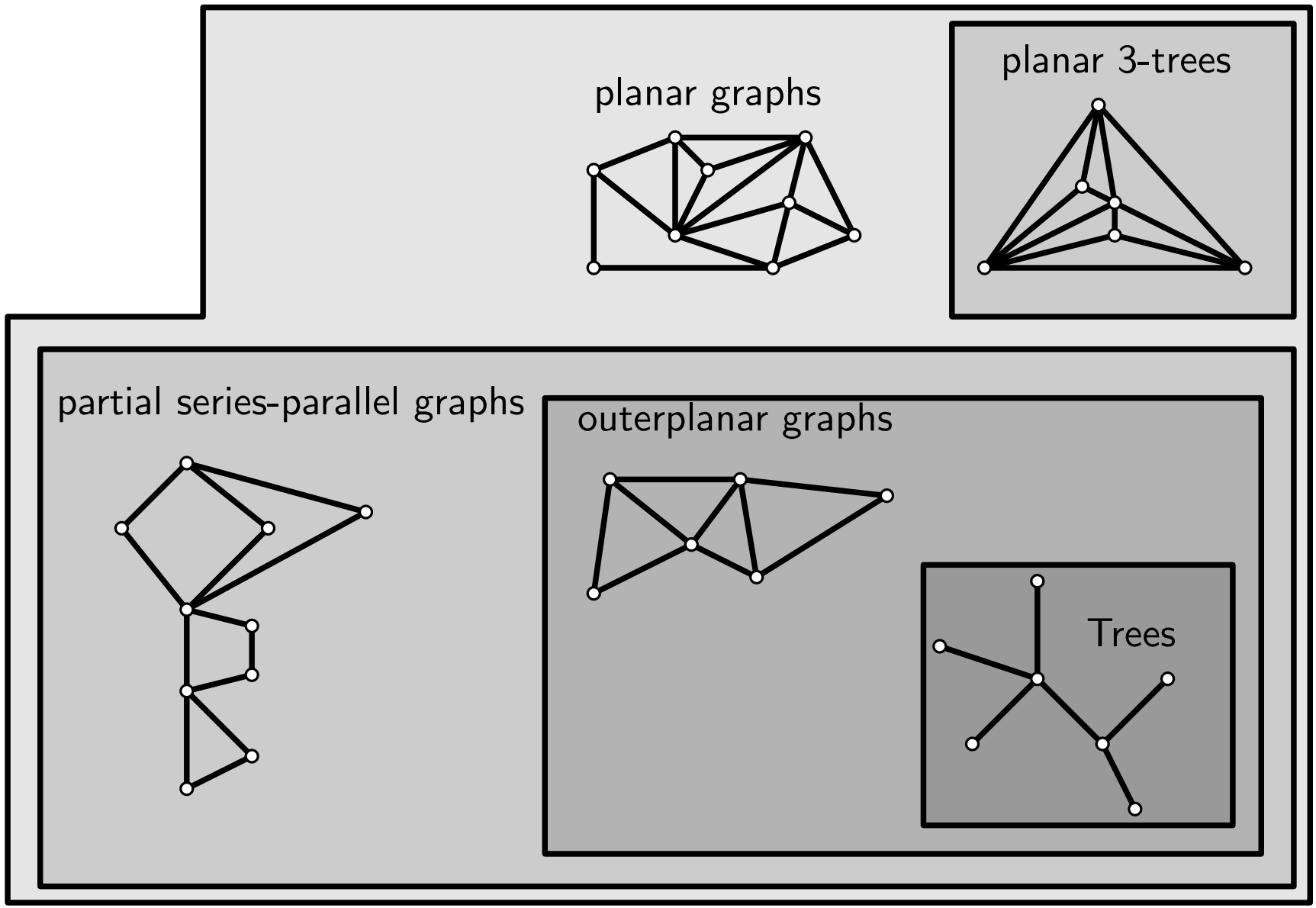

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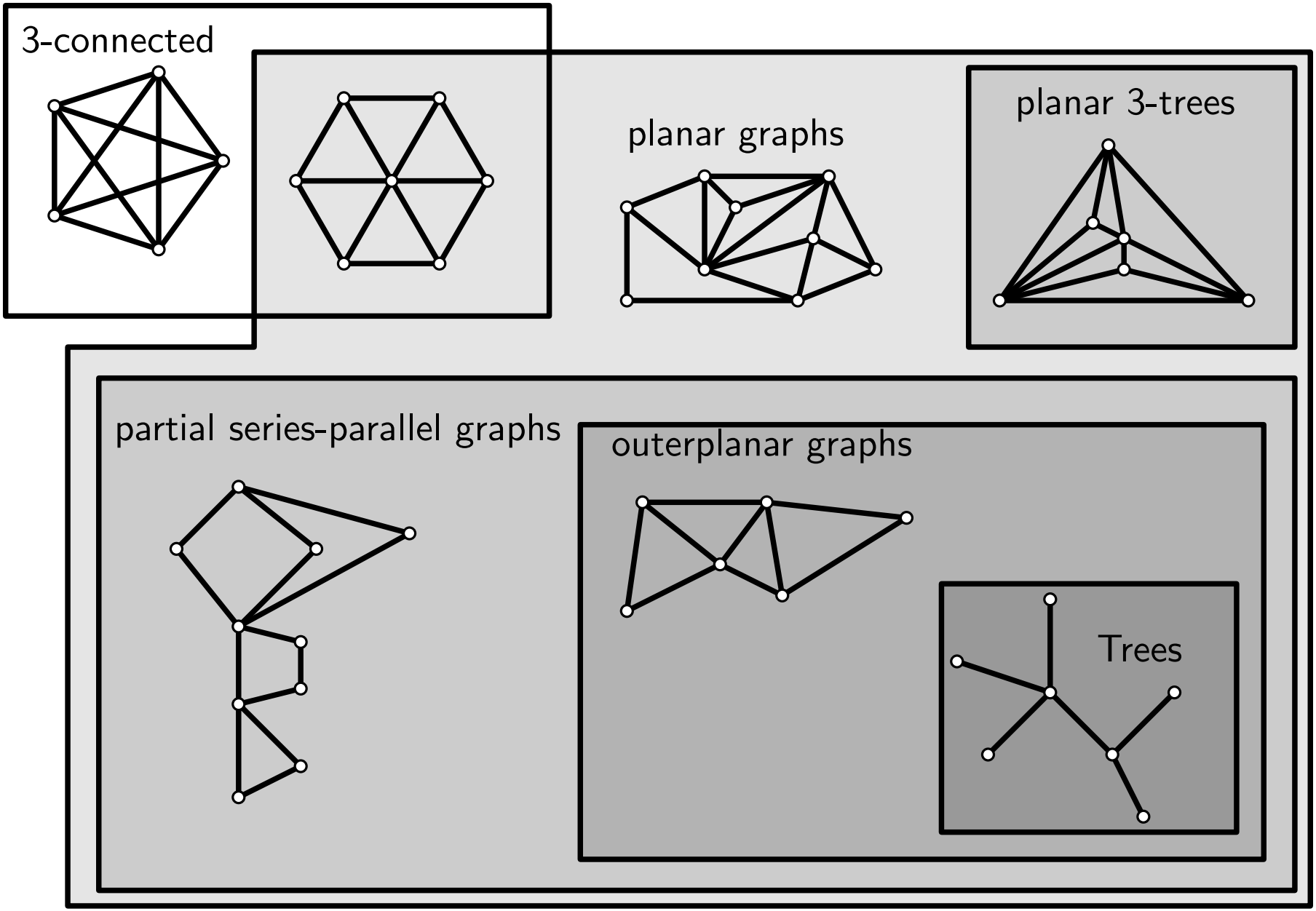

Standard Techniques I

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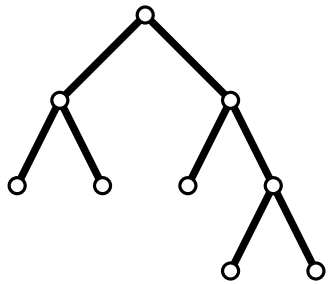
Binary trees:



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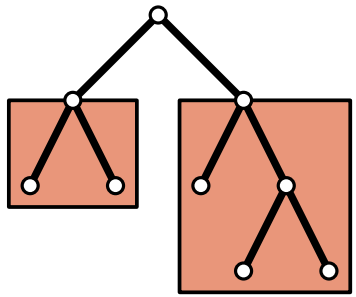

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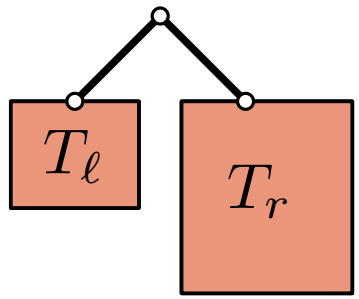

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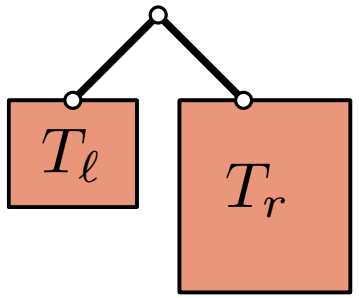

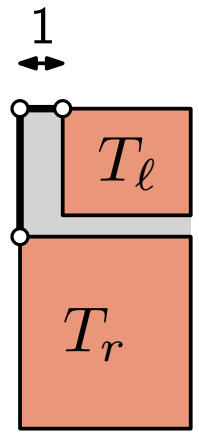

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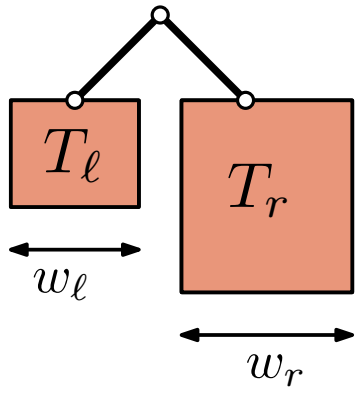

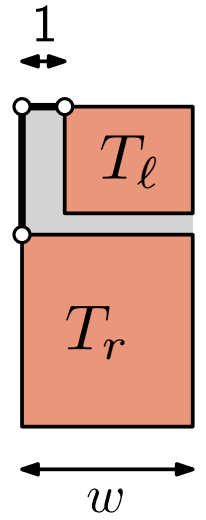

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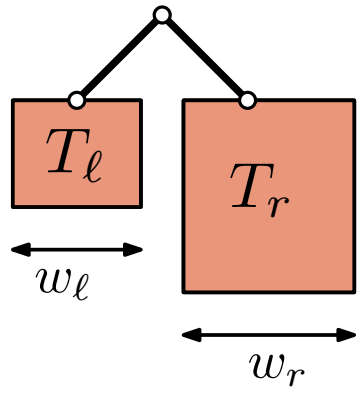

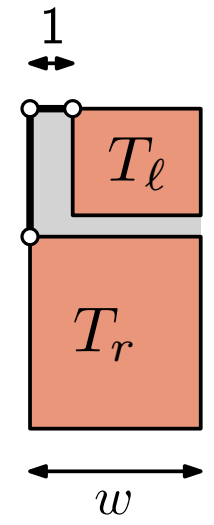

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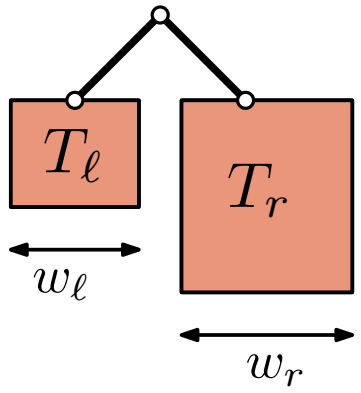

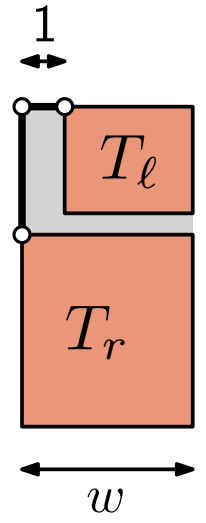

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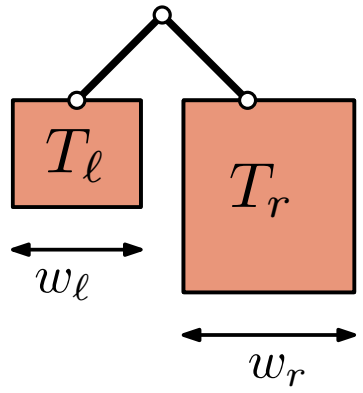

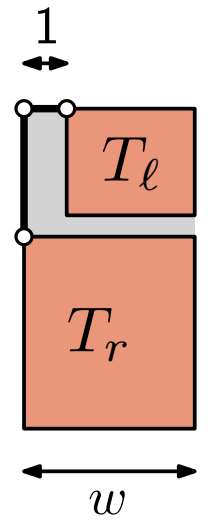

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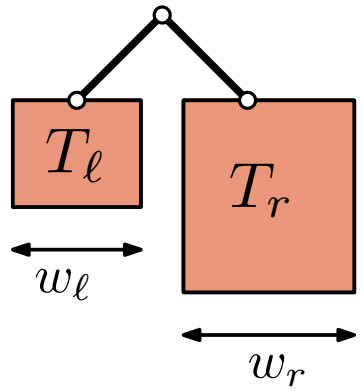

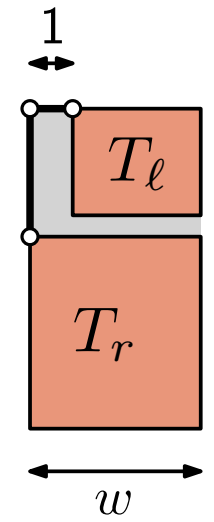

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$\Rightarrow$ Area $O(n \log n)$ for upward grid drawing.

[Crescenzi, Di Battista, Piperno '92]

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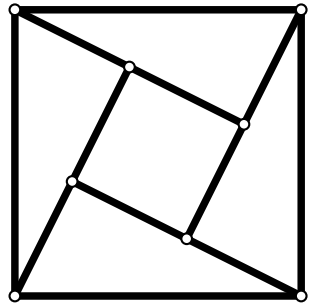

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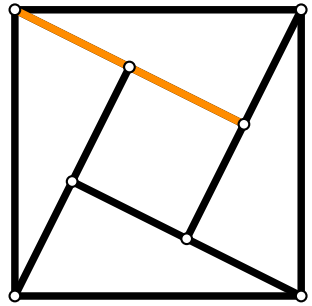

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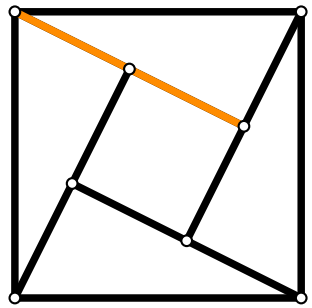

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Planar 3-trees can be drawn with $2n - 4$ segments.

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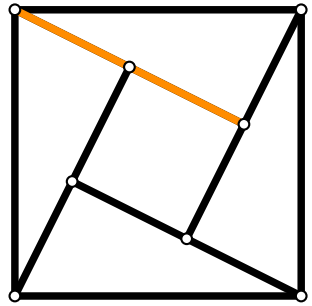

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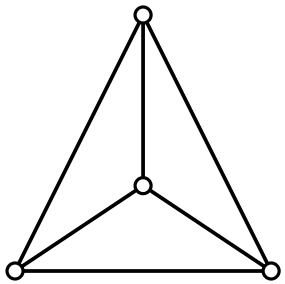

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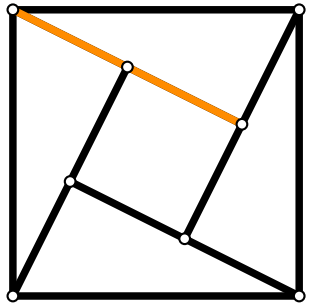

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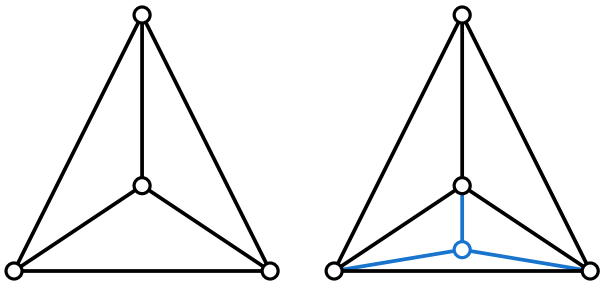

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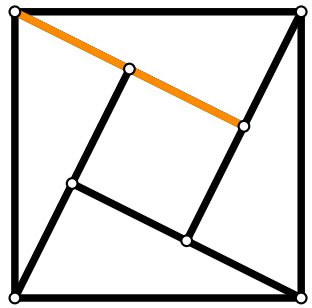

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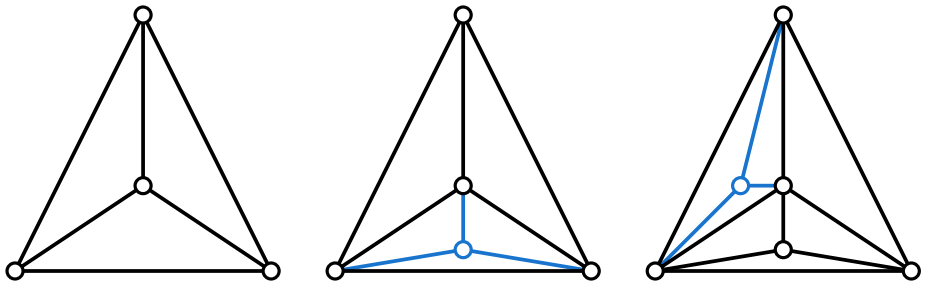

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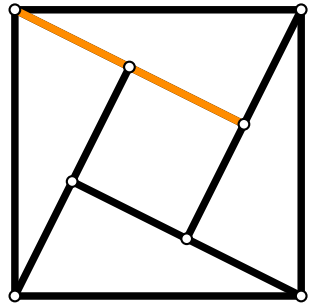

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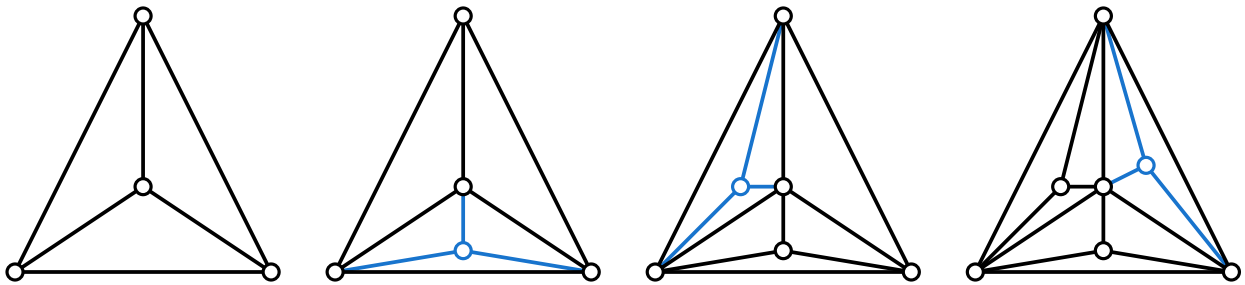

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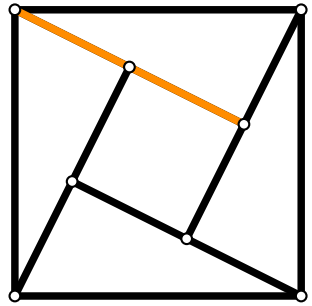

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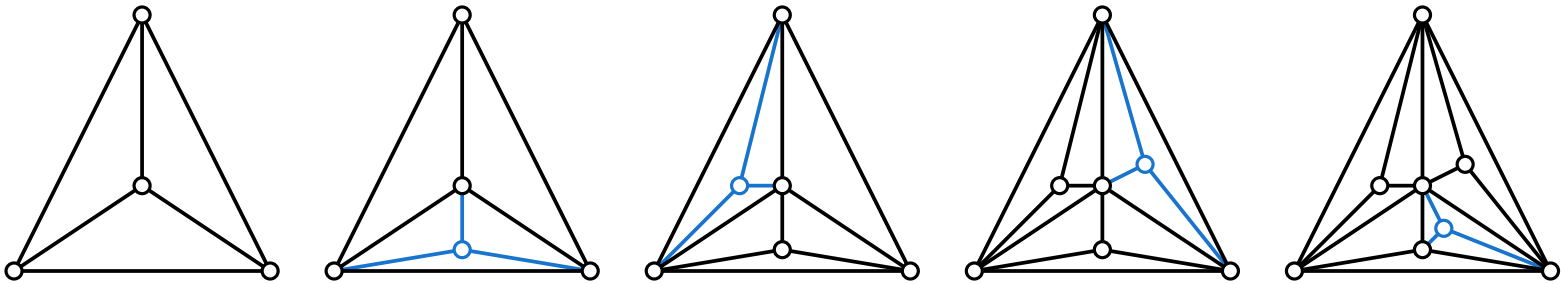

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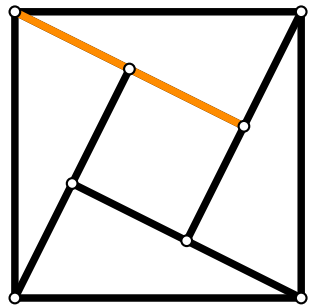

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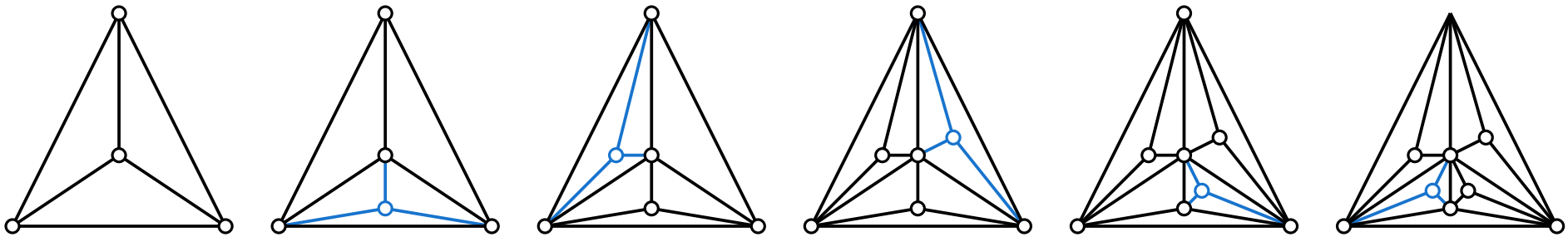

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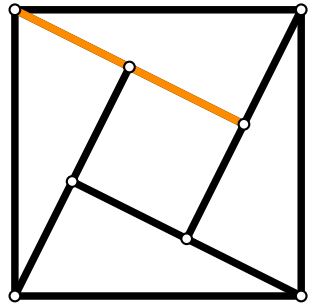

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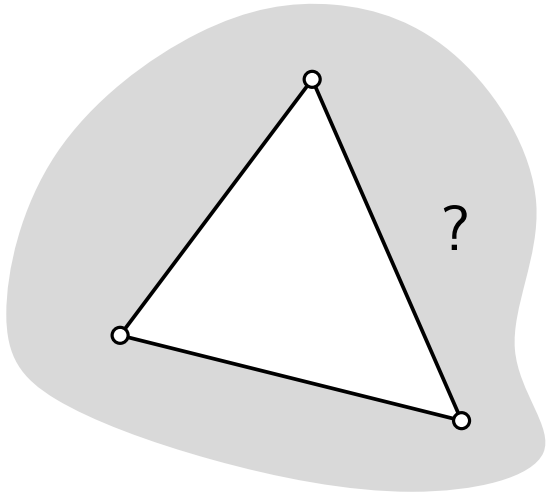

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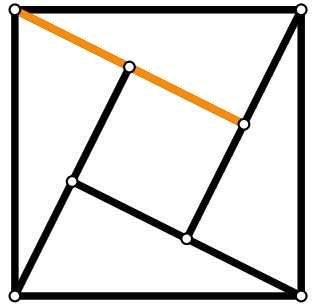

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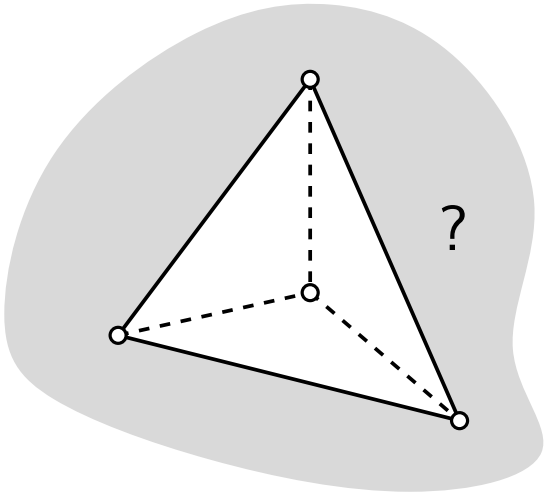

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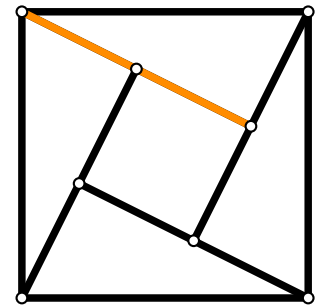

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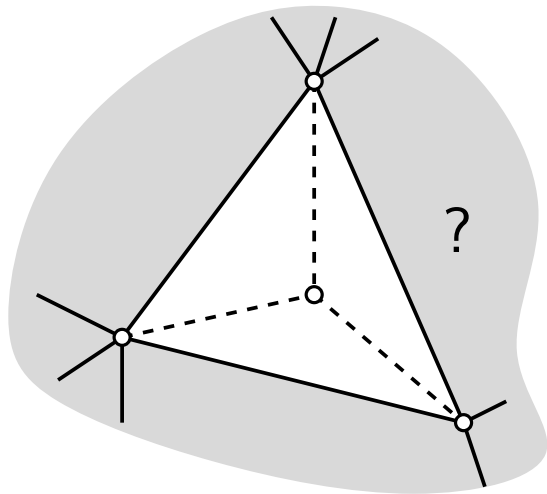

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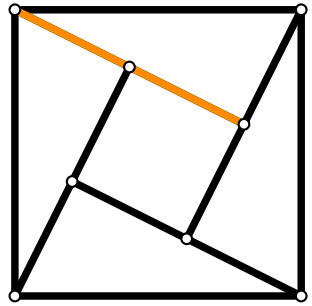

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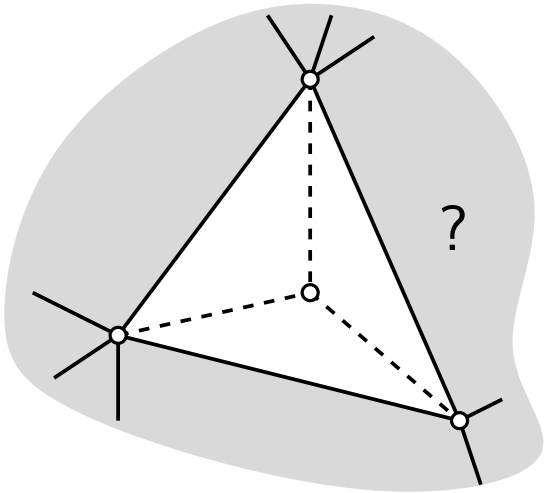

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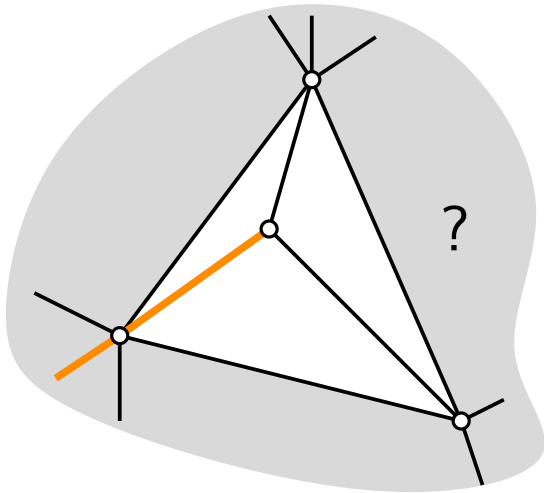

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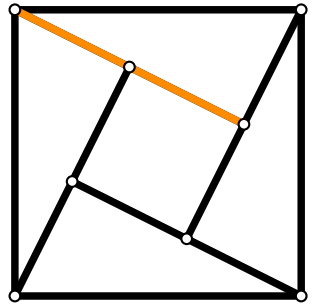

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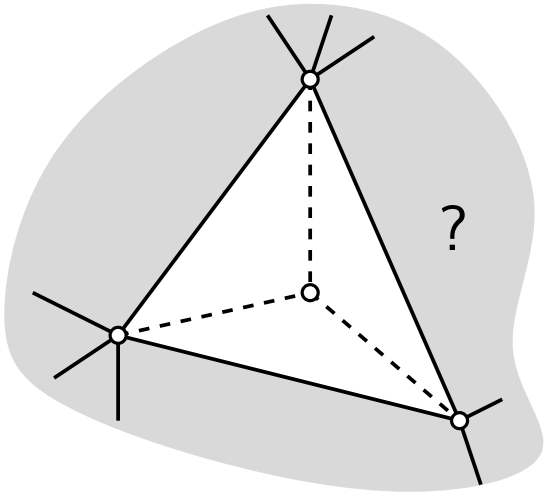

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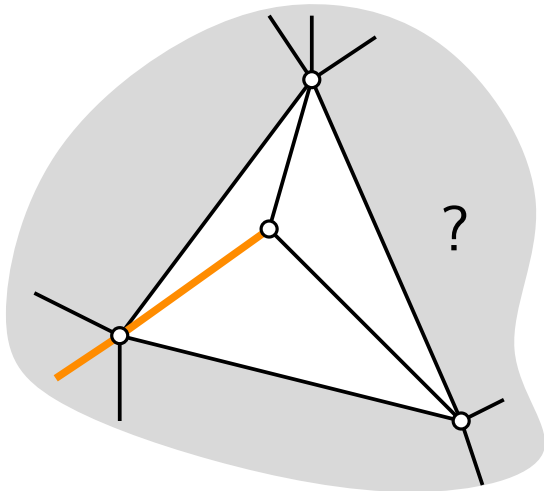

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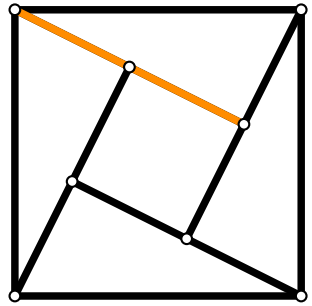

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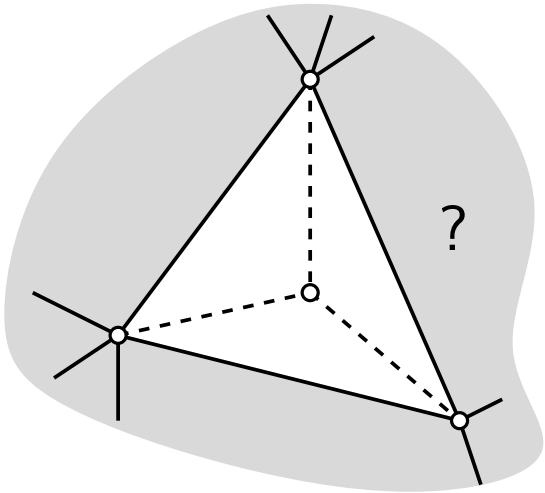

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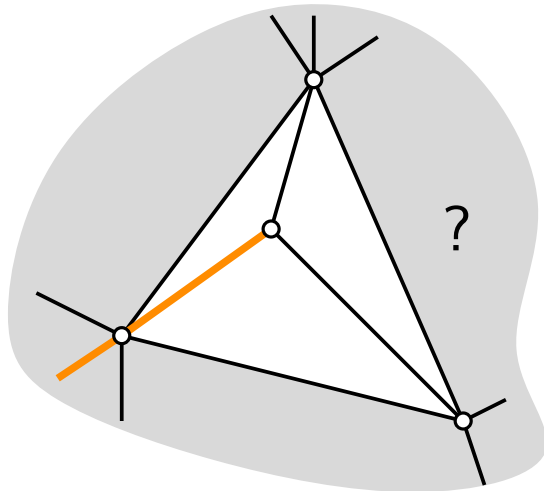

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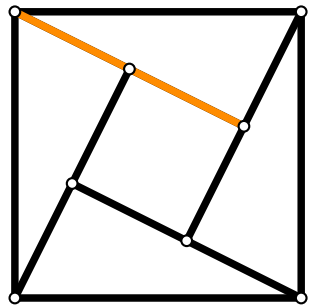

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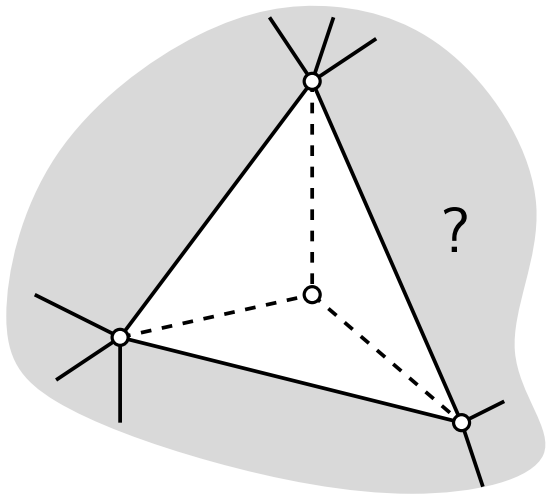

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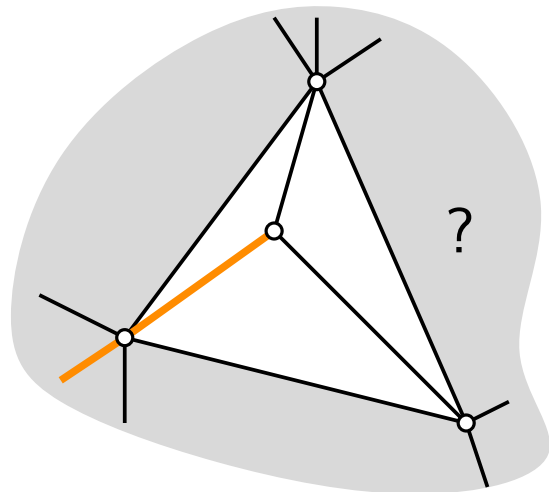

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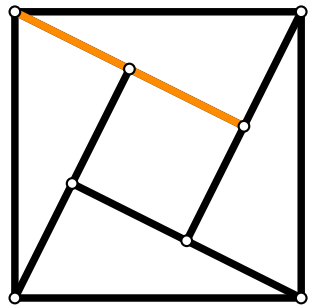

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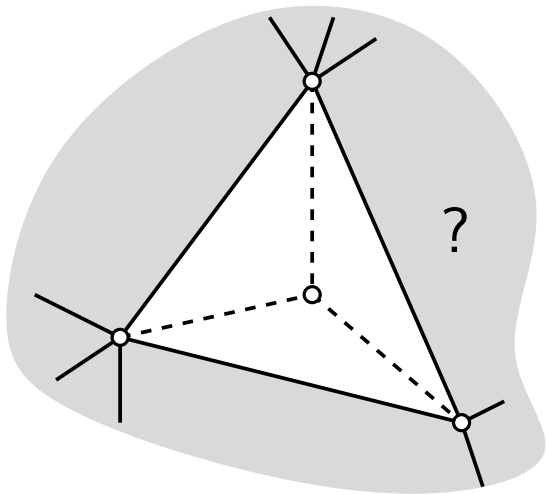

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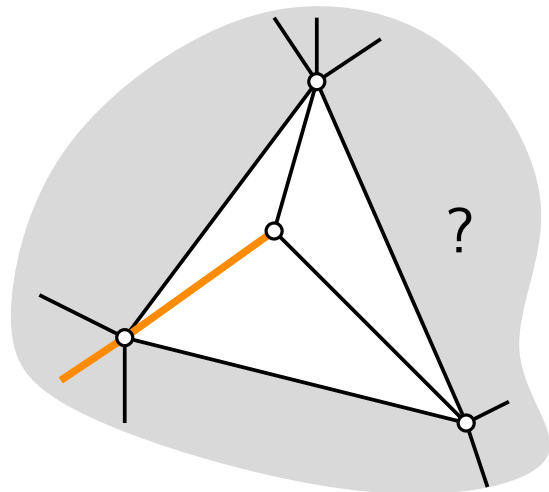

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Fruchterman Reingold '92

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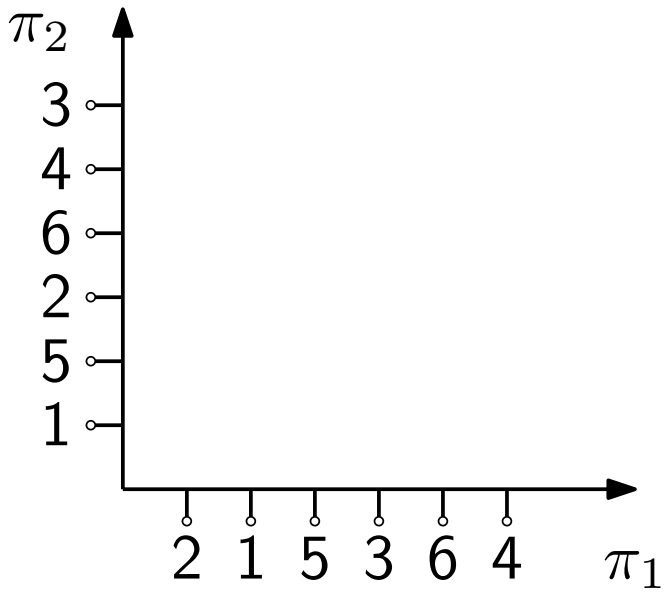

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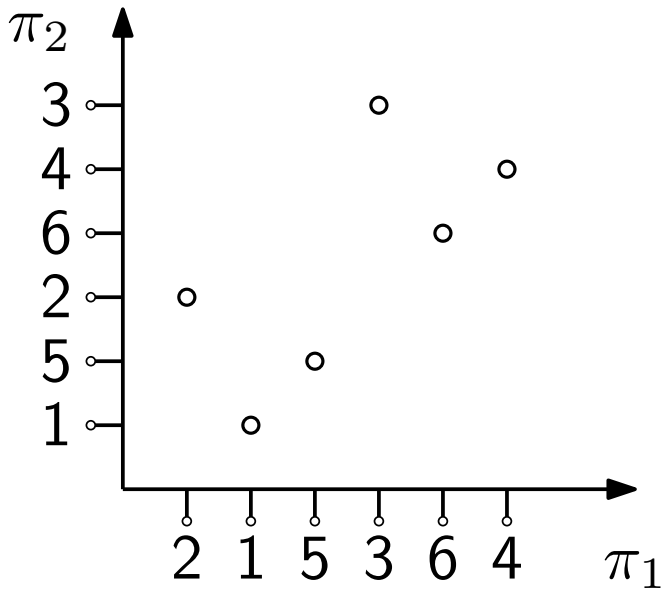

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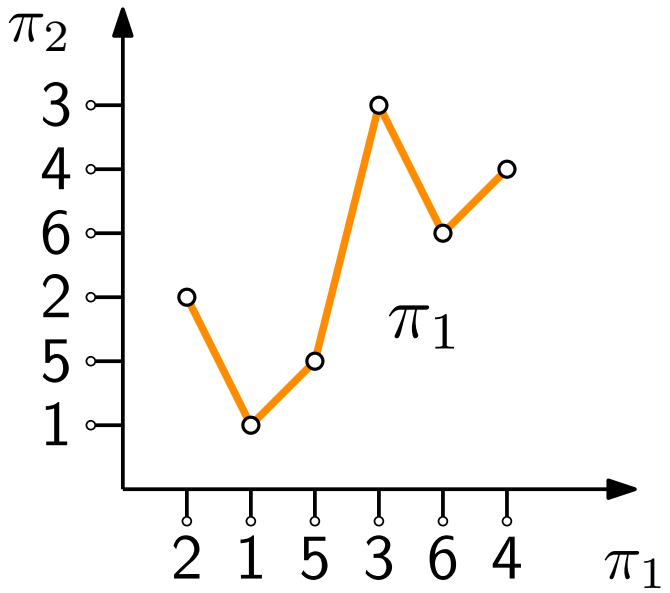

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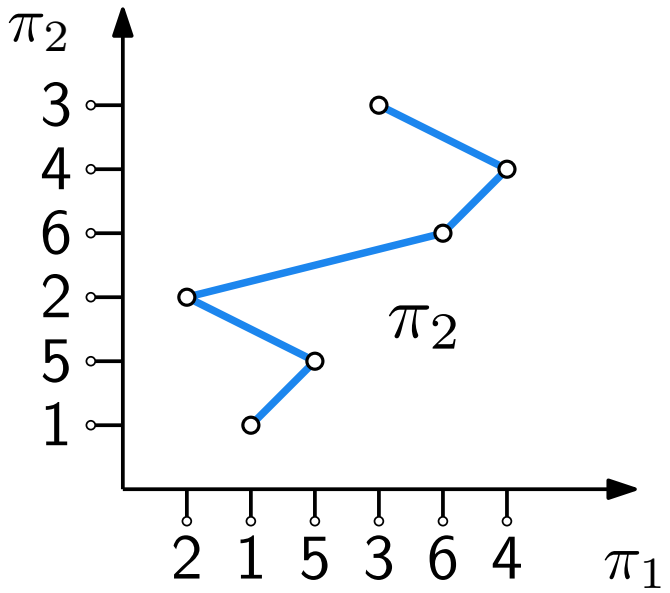

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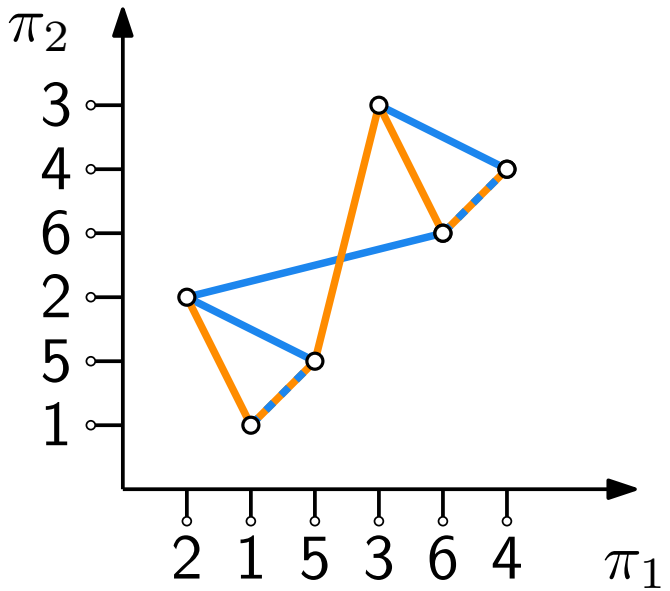

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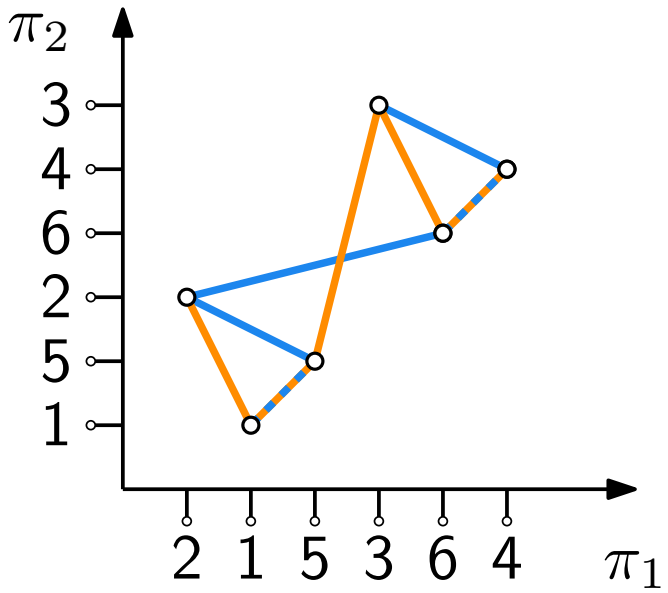

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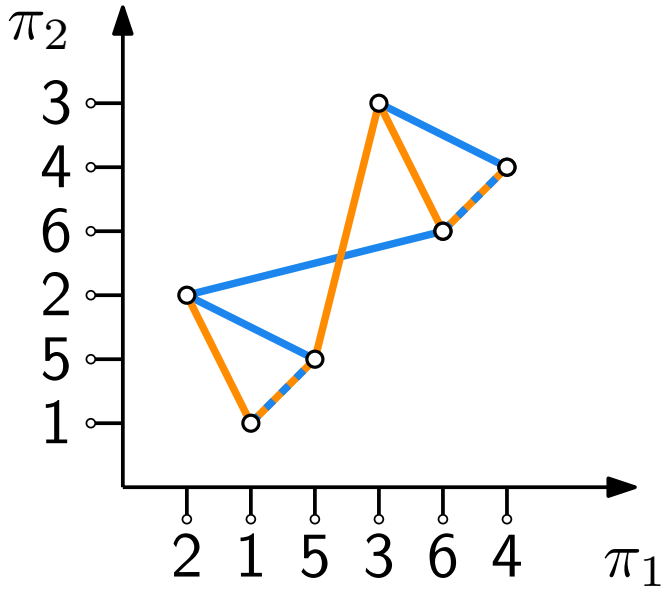

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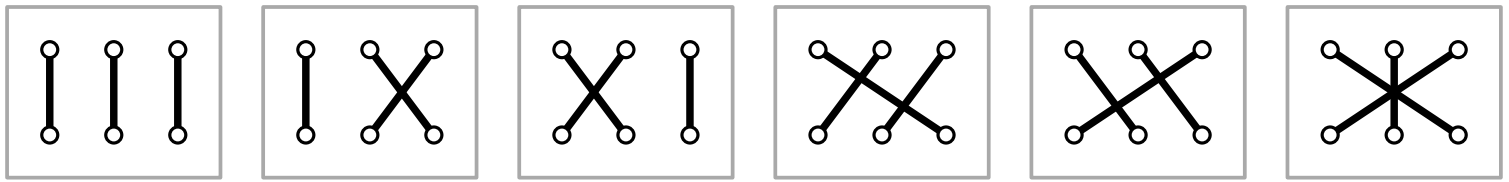

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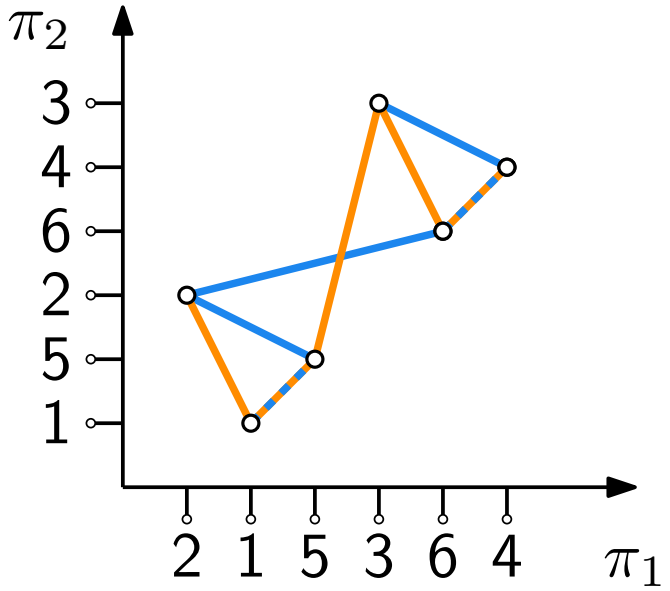

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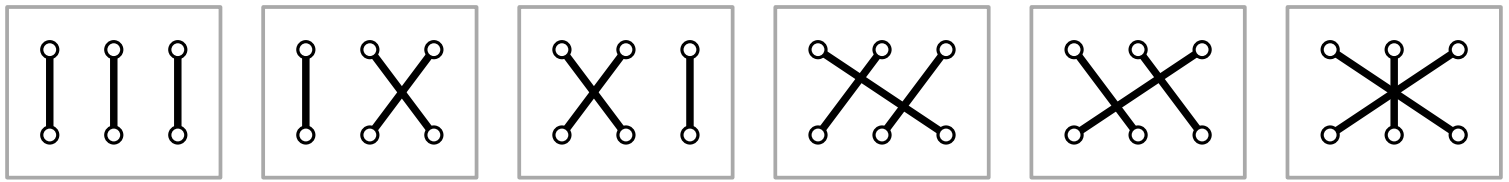

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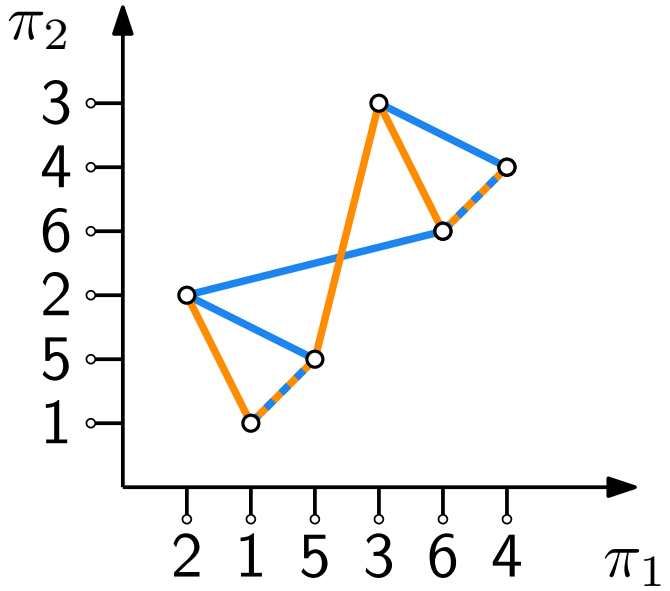

Six Matchings: NO

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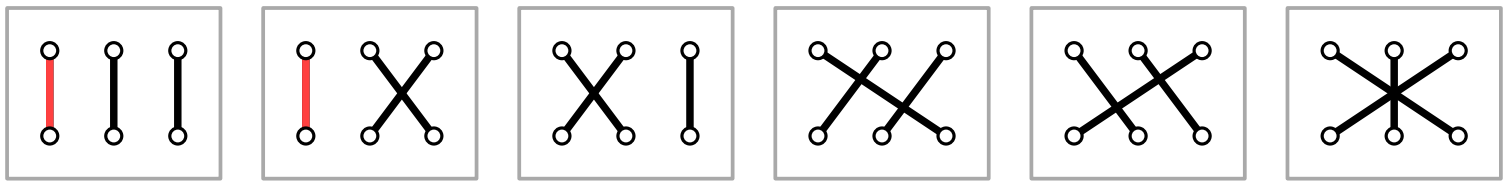

In a $K_{3,3}$-drawing at least two edges cross. For every pair of edges one matching contains these.

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Alternative Solution: [Angelini et al. '14]

- linear number of linear moves per vertex (worst-case opt.)
- complicated

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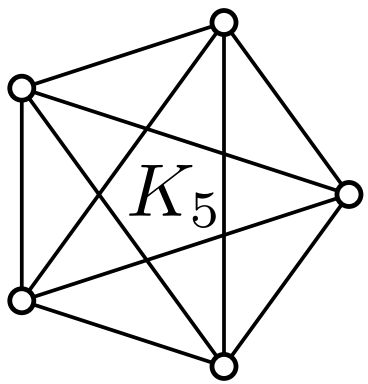
$\rho_3^2(K_5) = ?$



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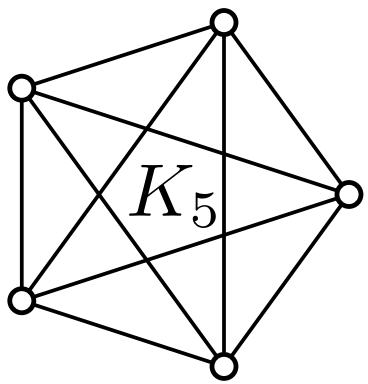

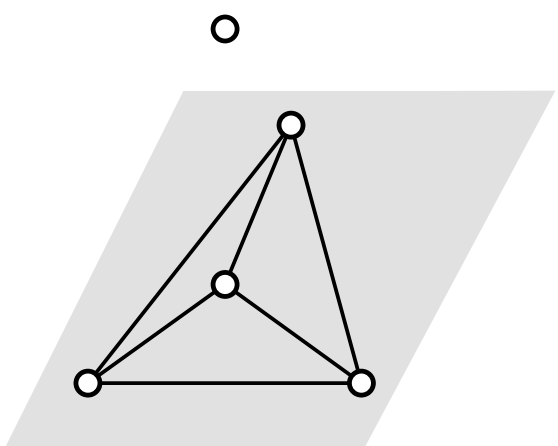

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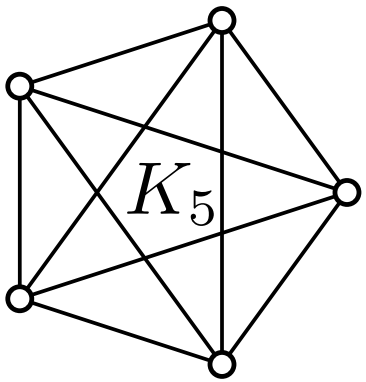

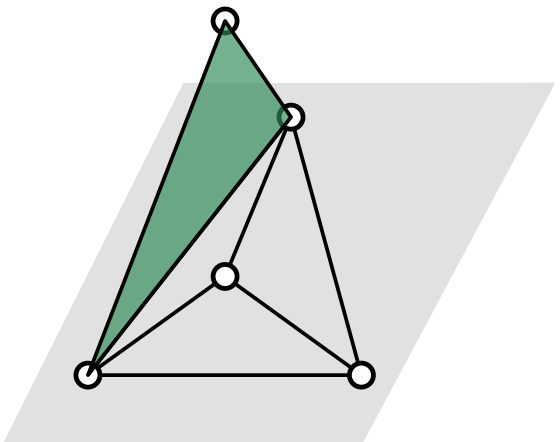

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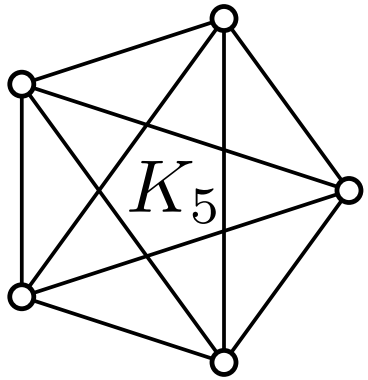

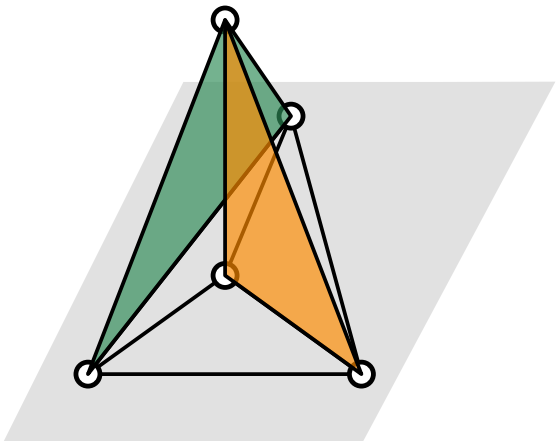

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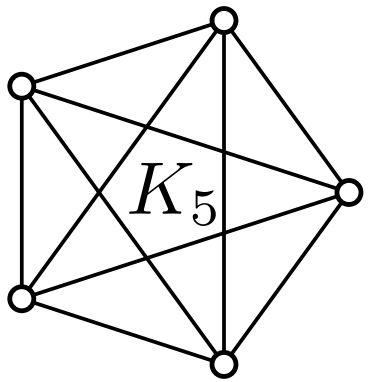

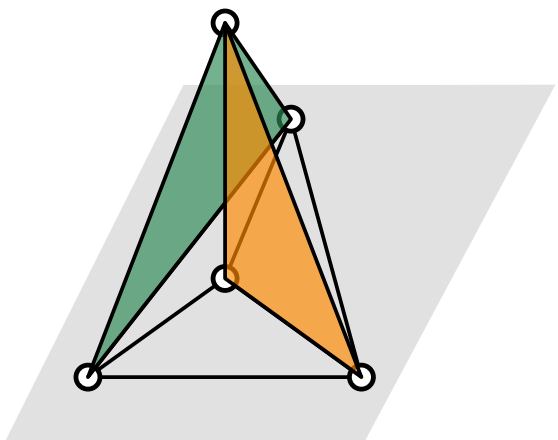

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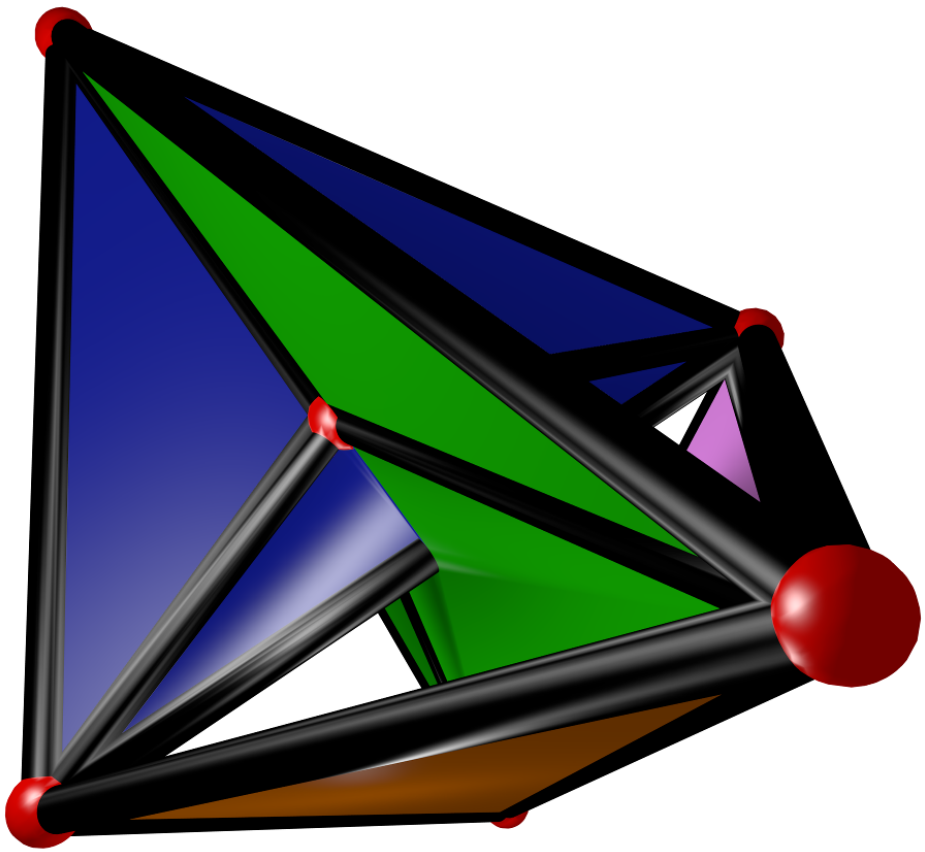

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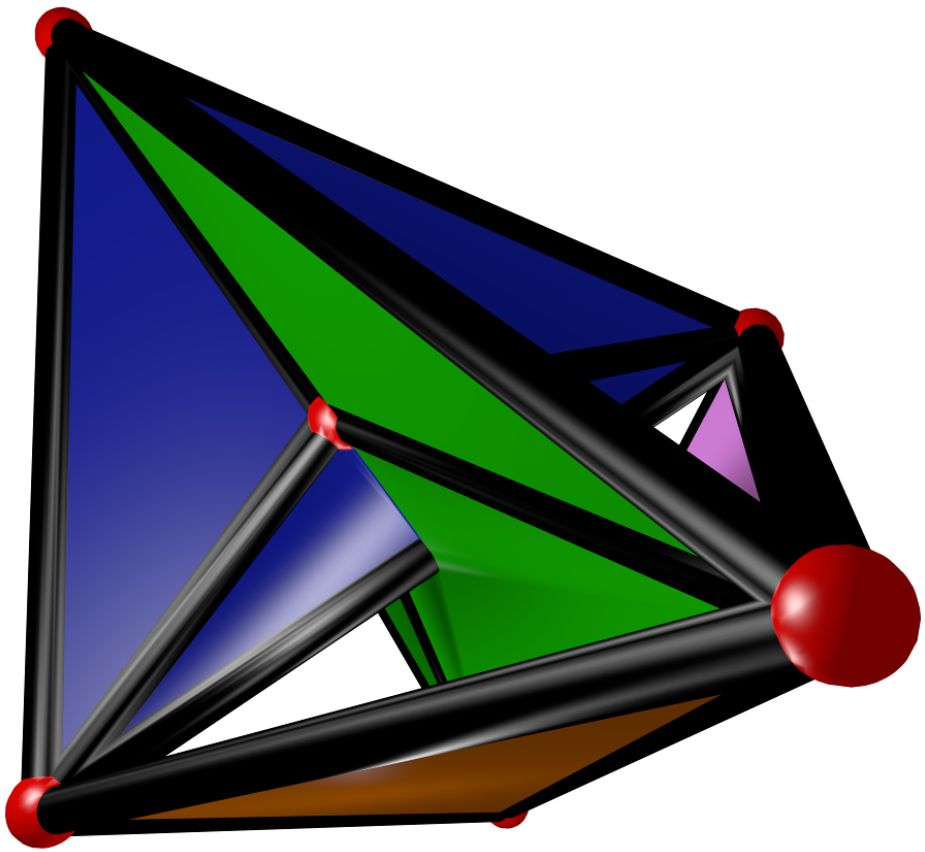

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For any planar graph G,
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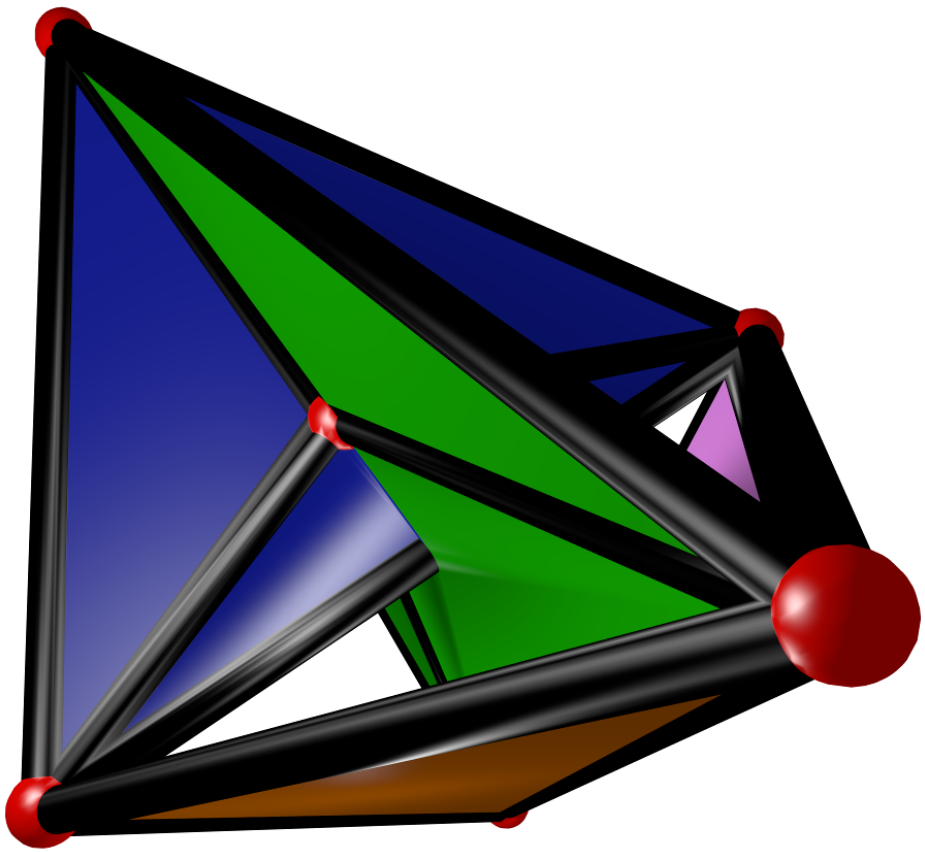

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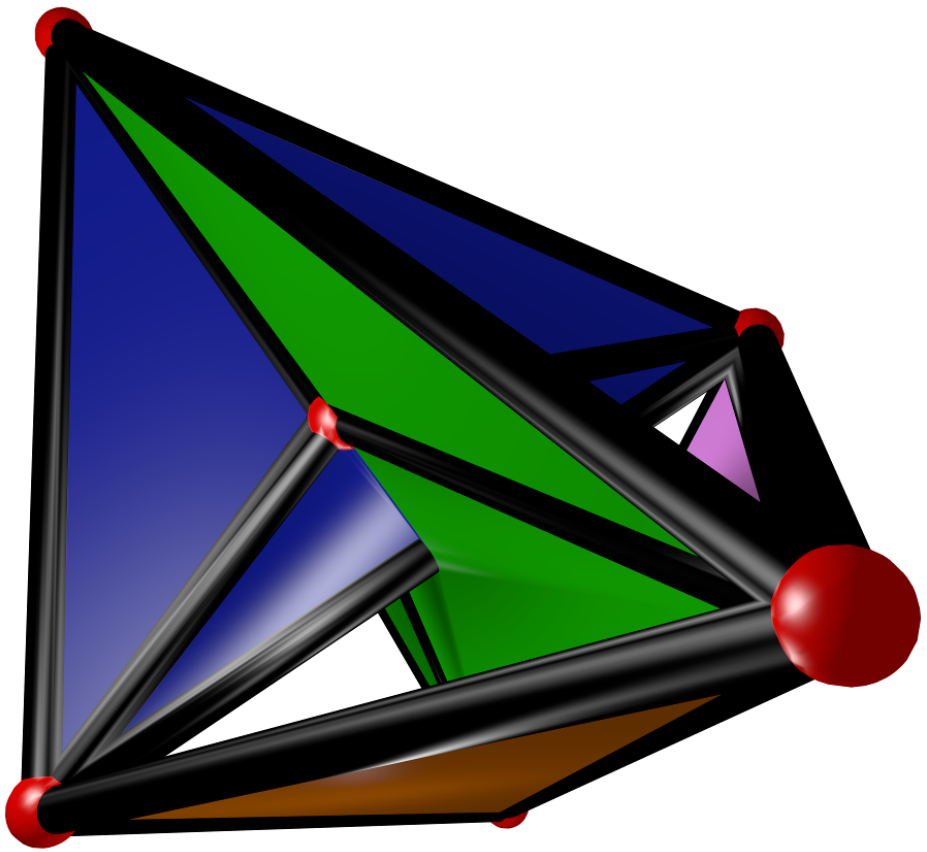

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Note: $\rho_3^2(K_n) \in \Theta(n^2)$.

$$\binom{n}{2}/6 \lessapprox \rho_3^2(K_n) \lessapprox \binom{n}{2}/3$$

Affine Cover Numbers

Let G be a graph and $1 \leq m < d$.

Affine cover number $\rho_d^m(G)$:

minimum number of m-dimensional hyperplanes in $\mathbb{R}^d$ s.t. G has a crossing-free straight-line drawing that is contained in these planes.

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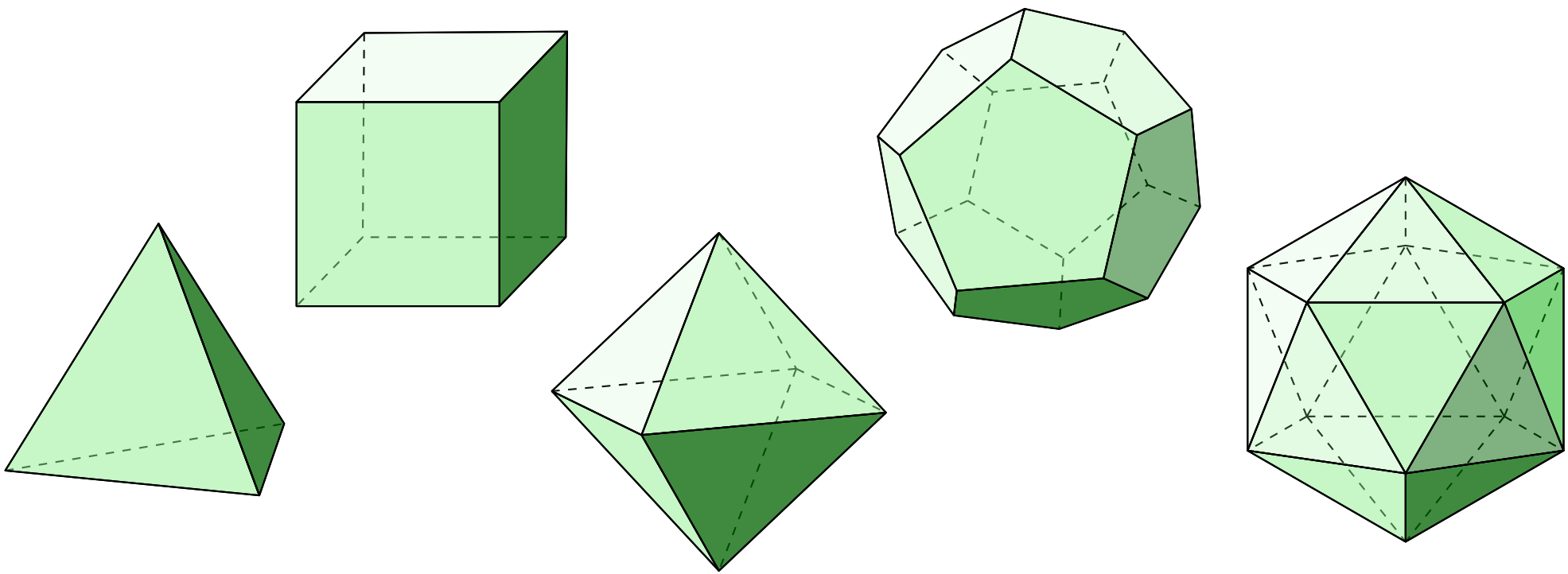
[WADS'17 & GD'19]

Unfortunately, *each* of these numbers is NP-hard to compute :-(

Line Cover Numbers of the Platonic Solids

$G = (V, E)$	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert F\rvert$	$\rho_2^1(G)$	$\rho_3^1(G)$	$\pi_2^1(G)$	$\pi_3^1(G)$
tetrahedron	4	6	4				
cube	8	12	6				
octahedron	6	12	8				
dodecahedron	20	30	12				
icosahedron	12	30	20				



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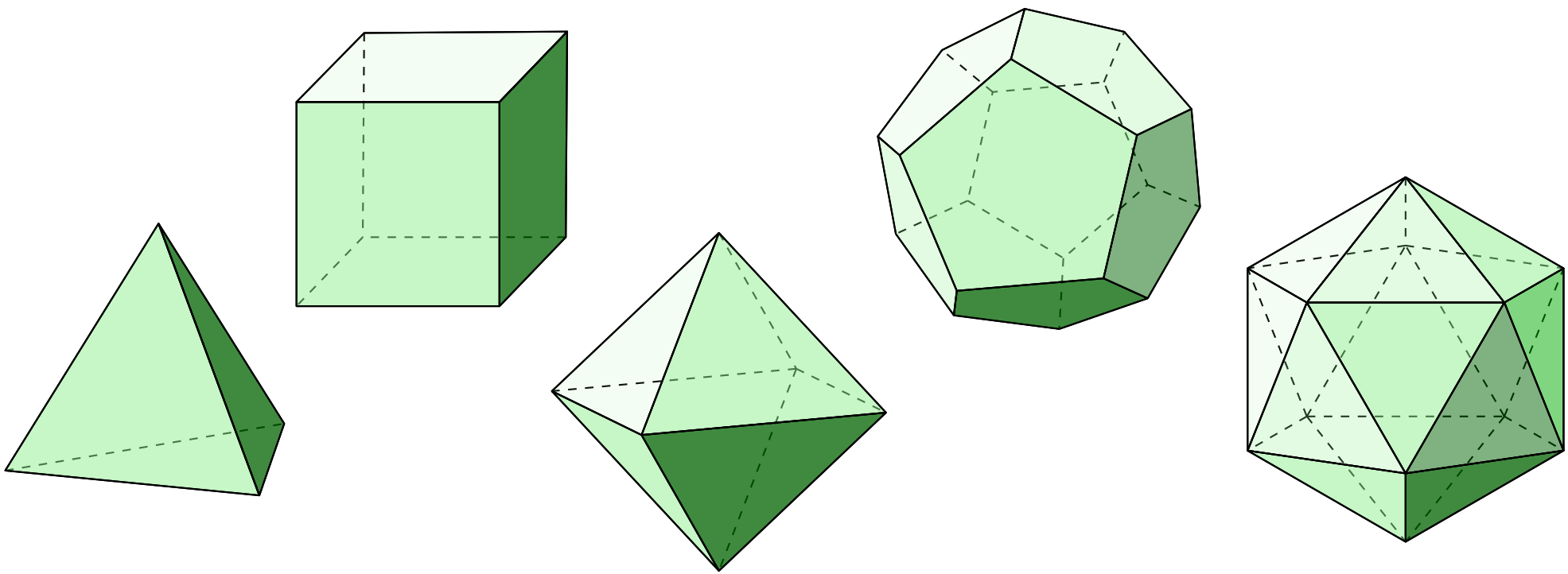

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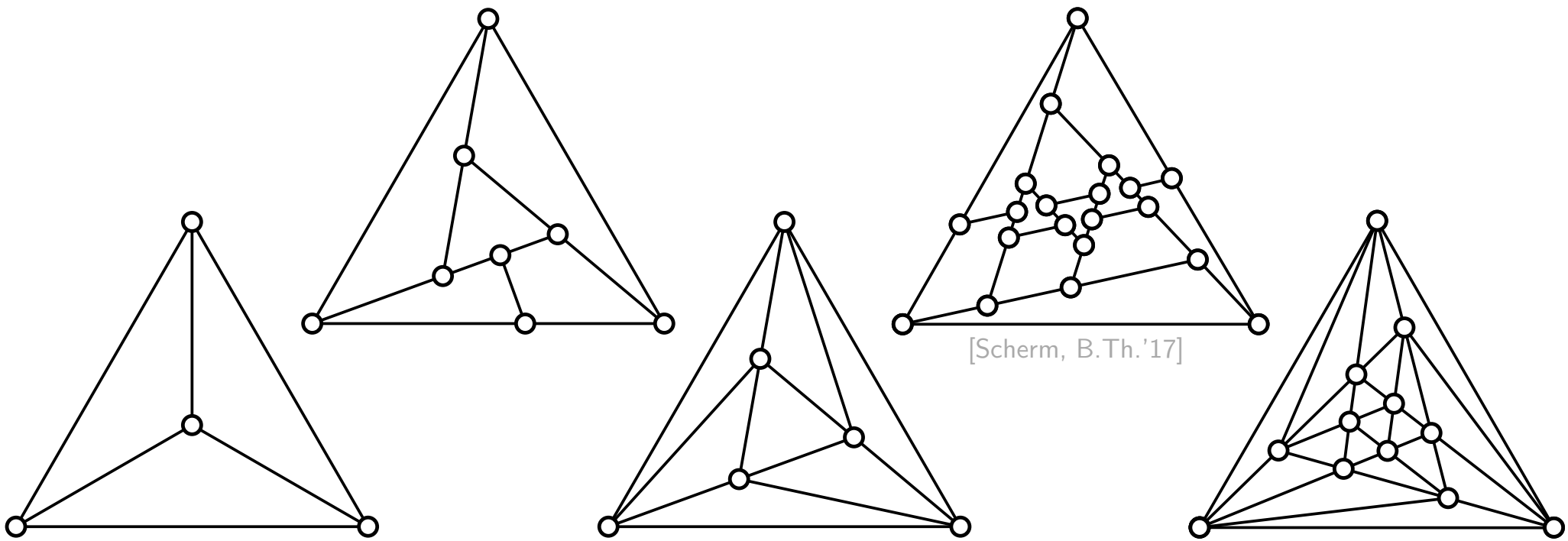

[Scherm, B.Th.'17]

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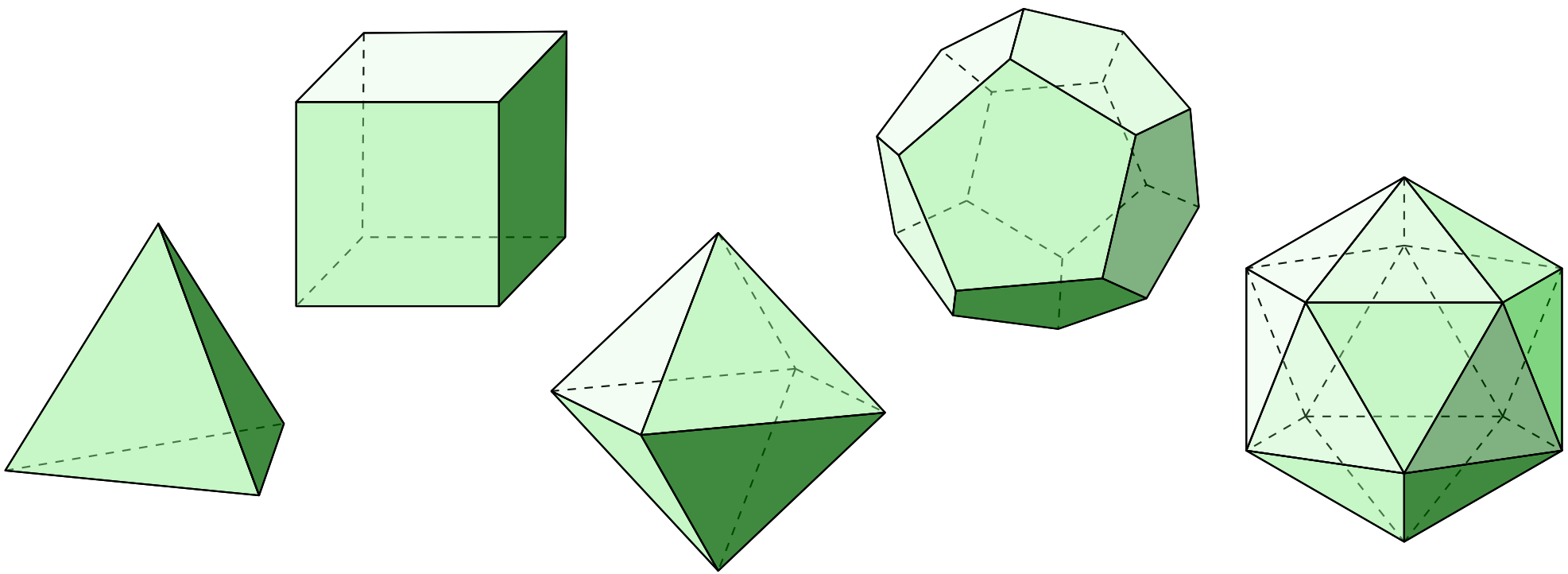

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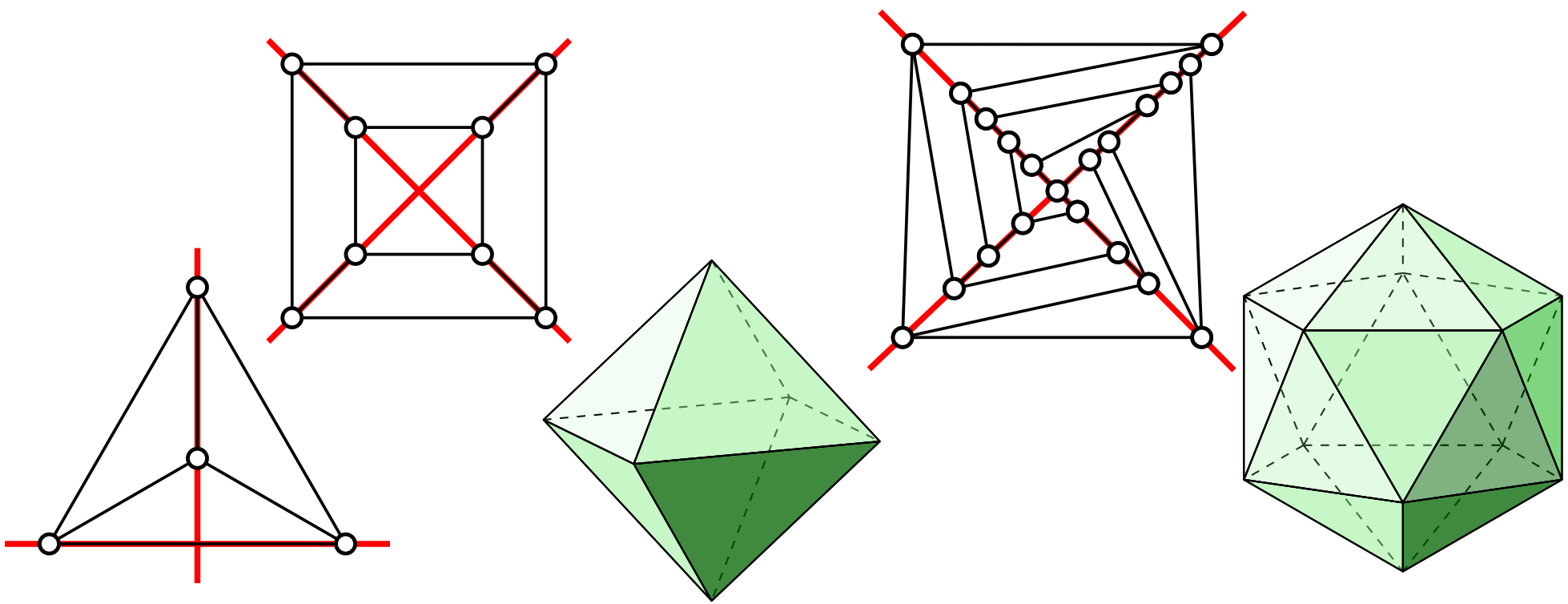

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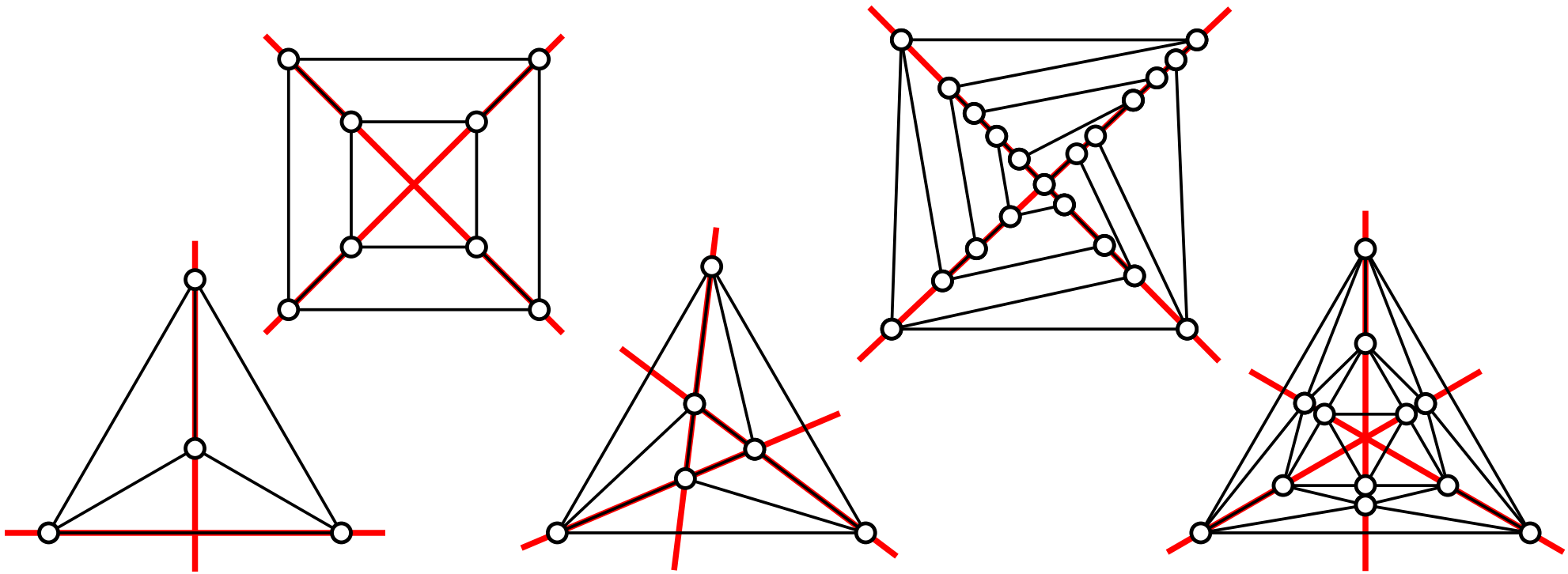

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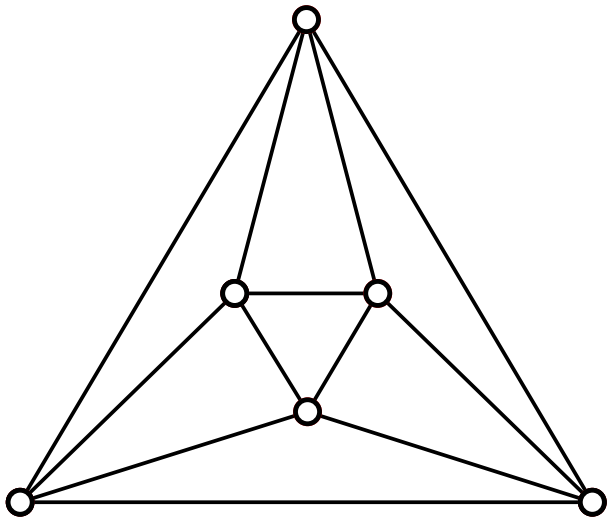

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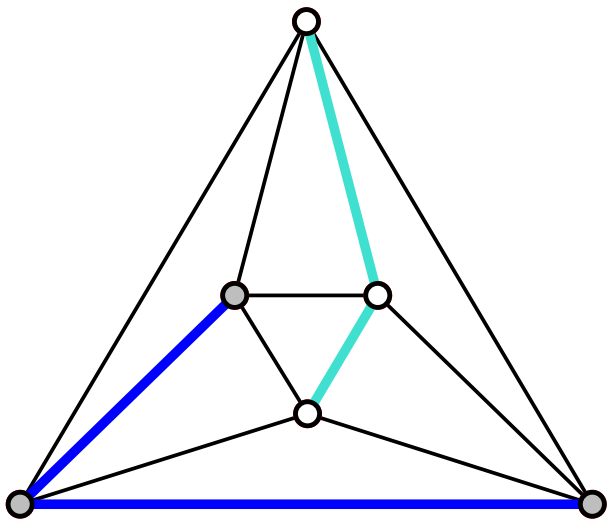
dodecahedron	20	30	12	9...10	9...10	2	2
icosahedron	12	30	20	13...15	13...15	3	



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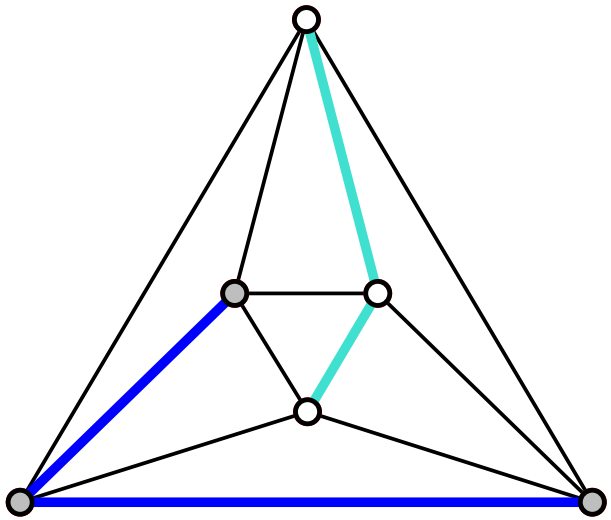

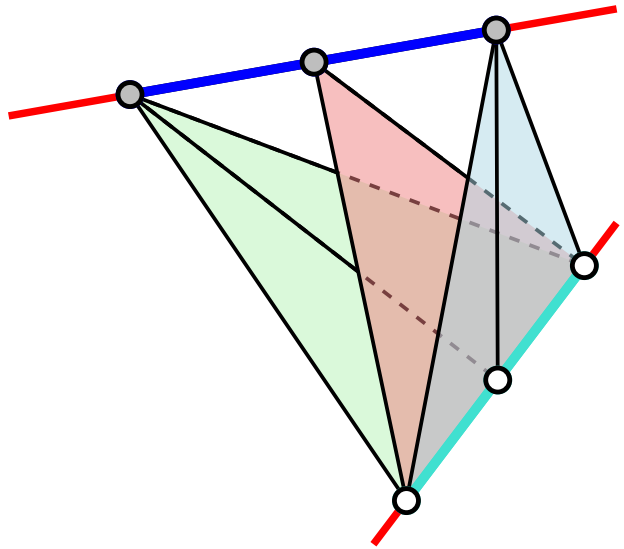

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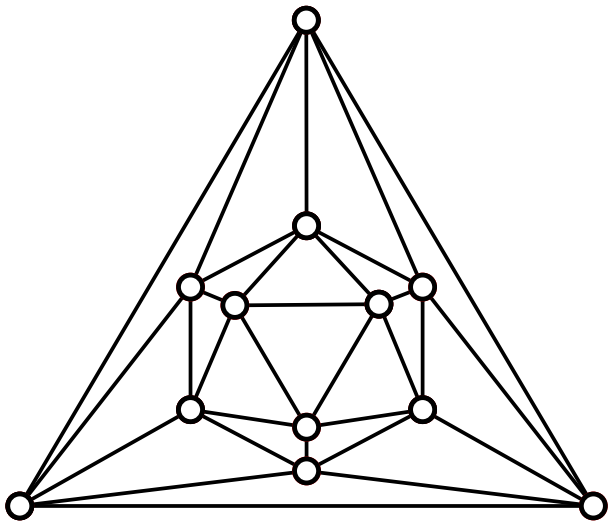

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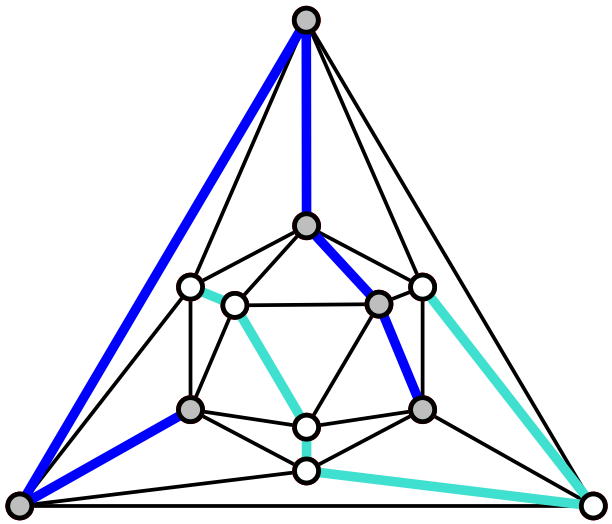

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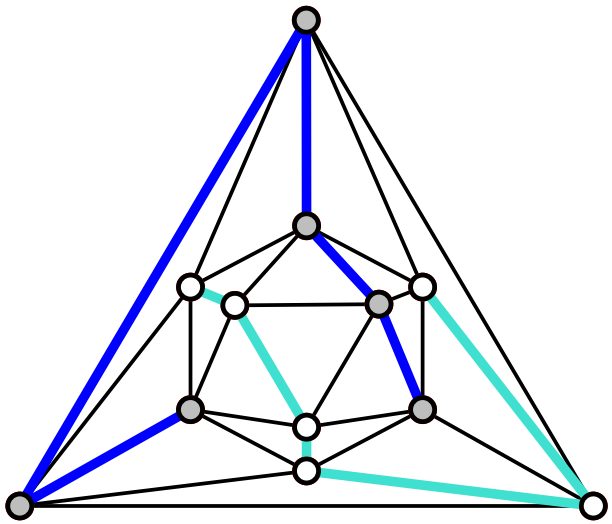

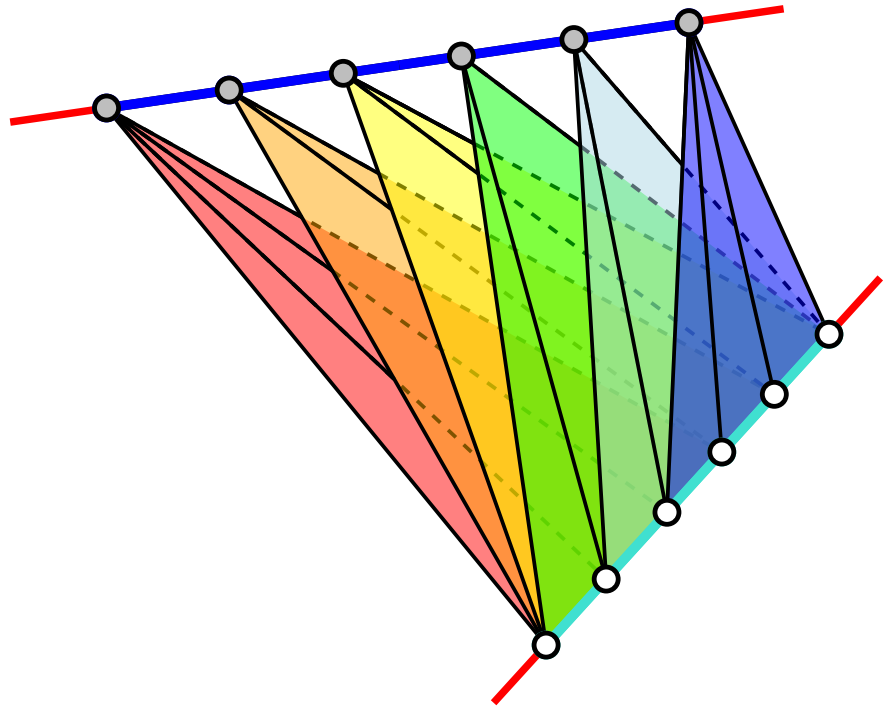

Contact Representations in 3D

[Evans, Rzążewski, Saeedi, Shin, W., GD'19]

Many settings have been studied...

For example:

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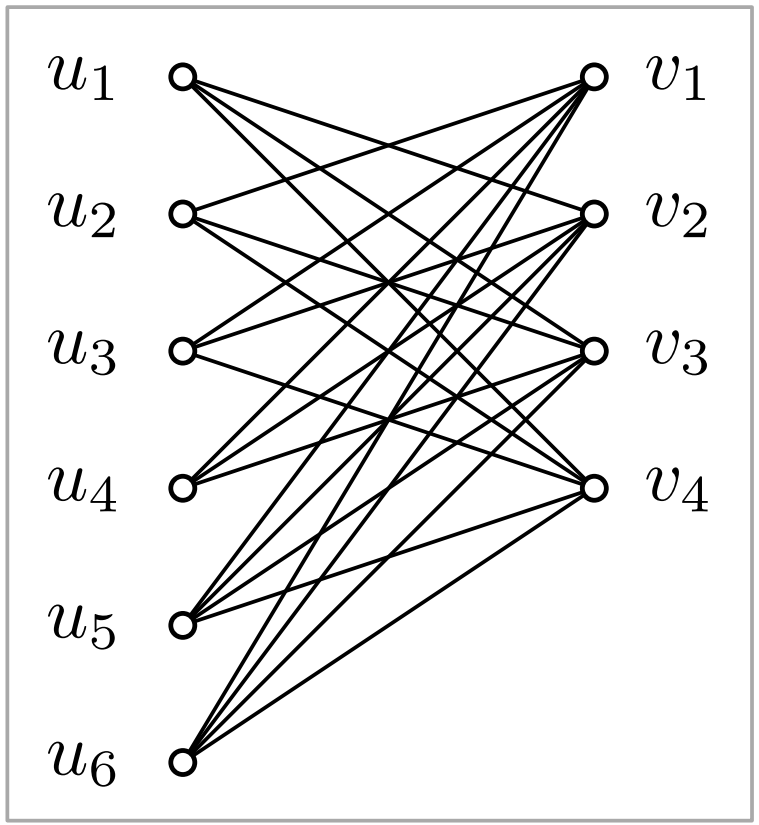

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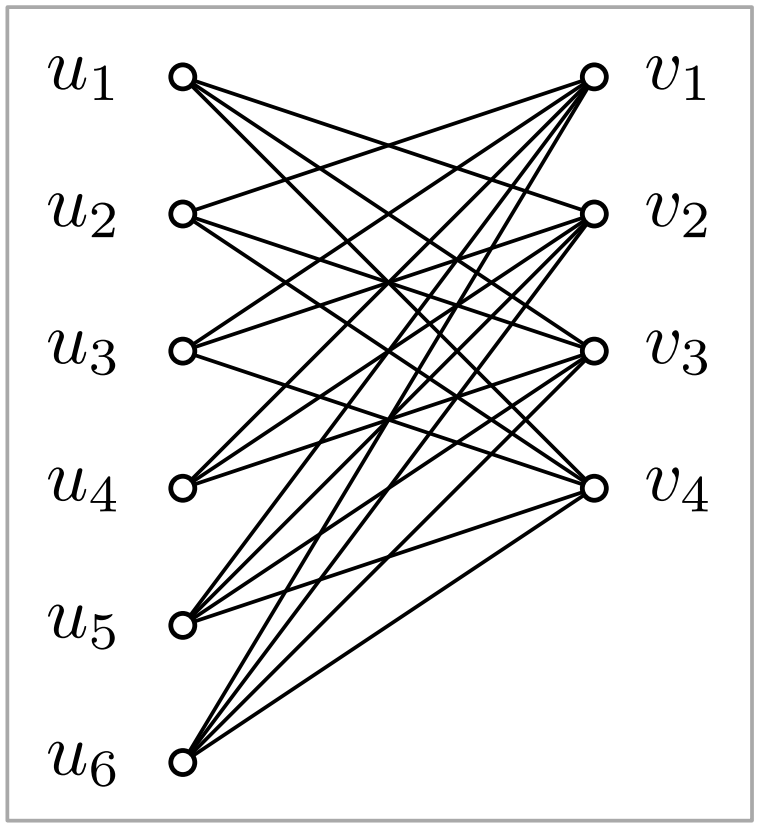

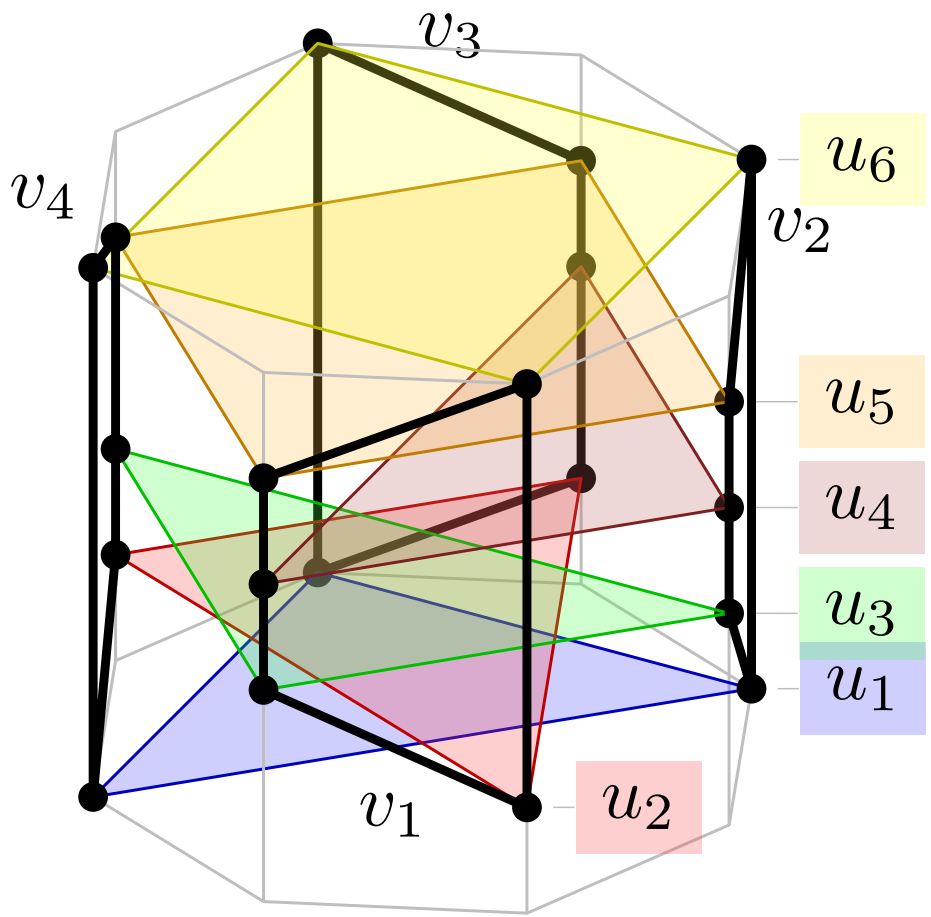

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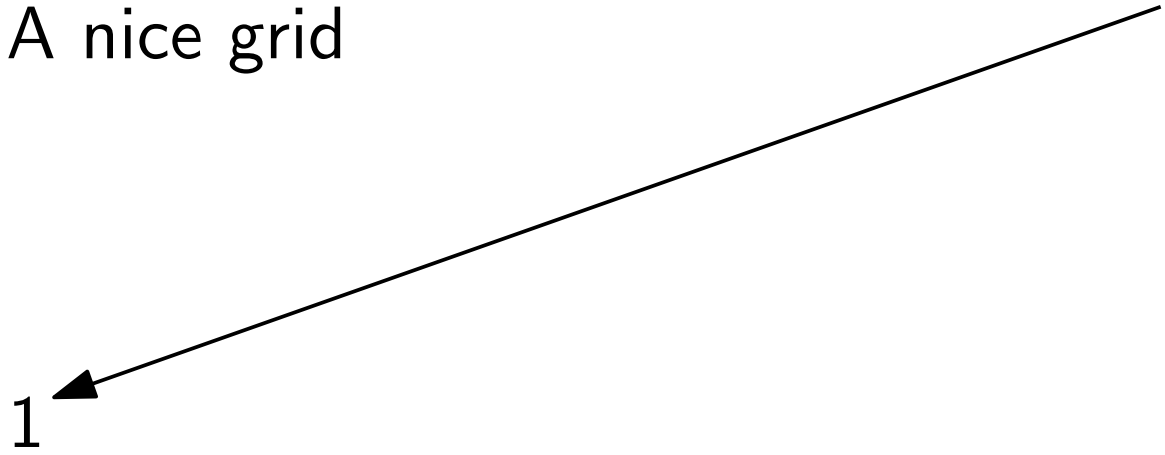

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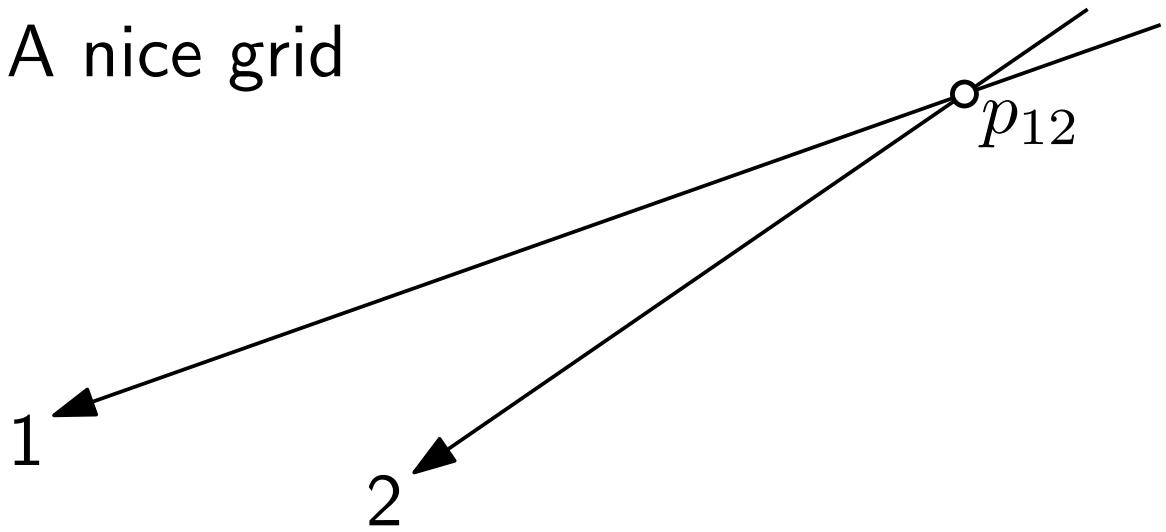

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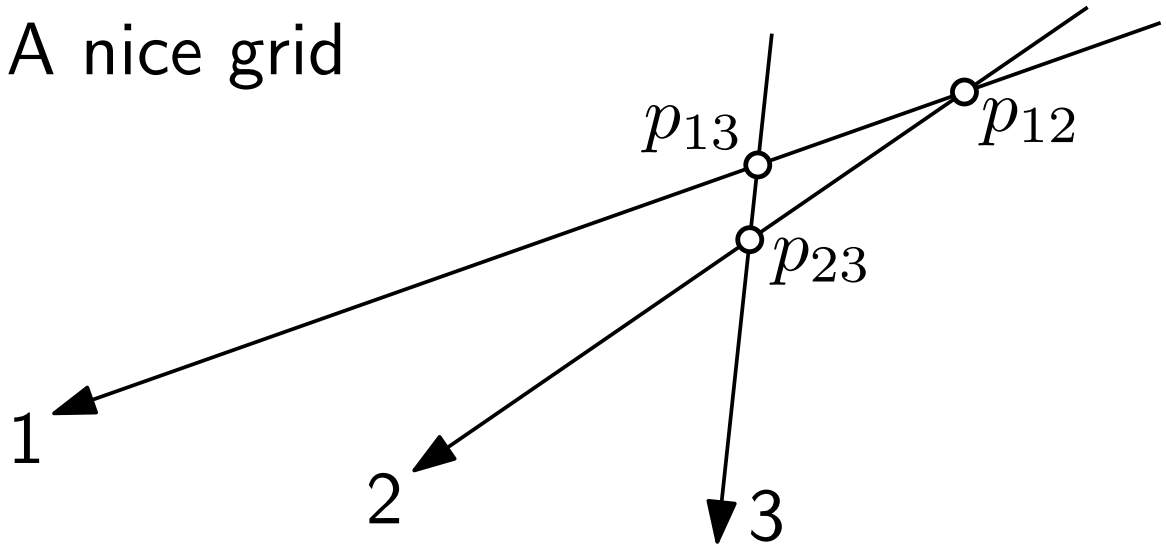

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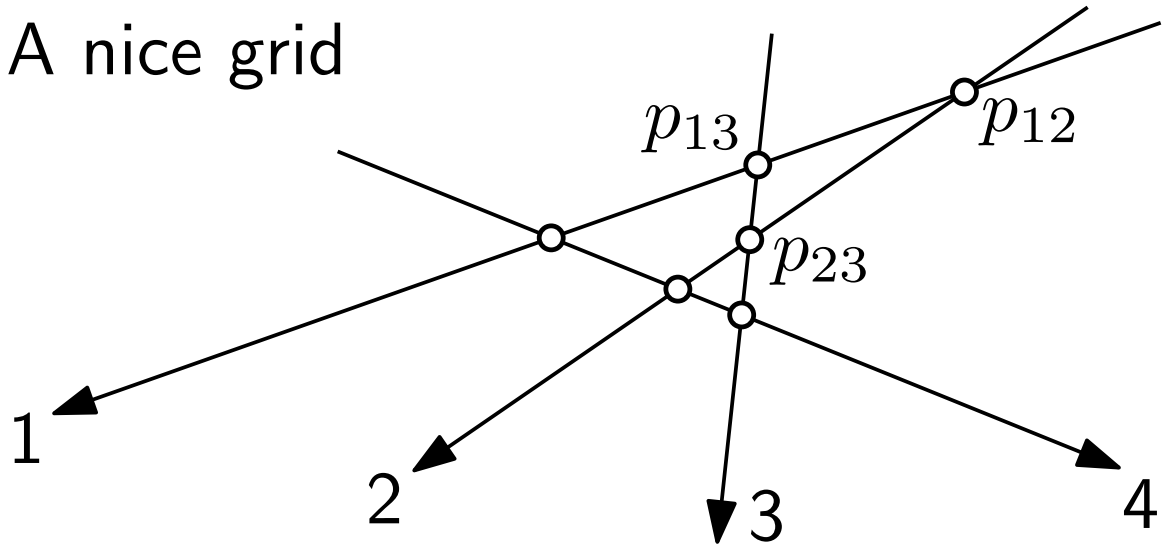

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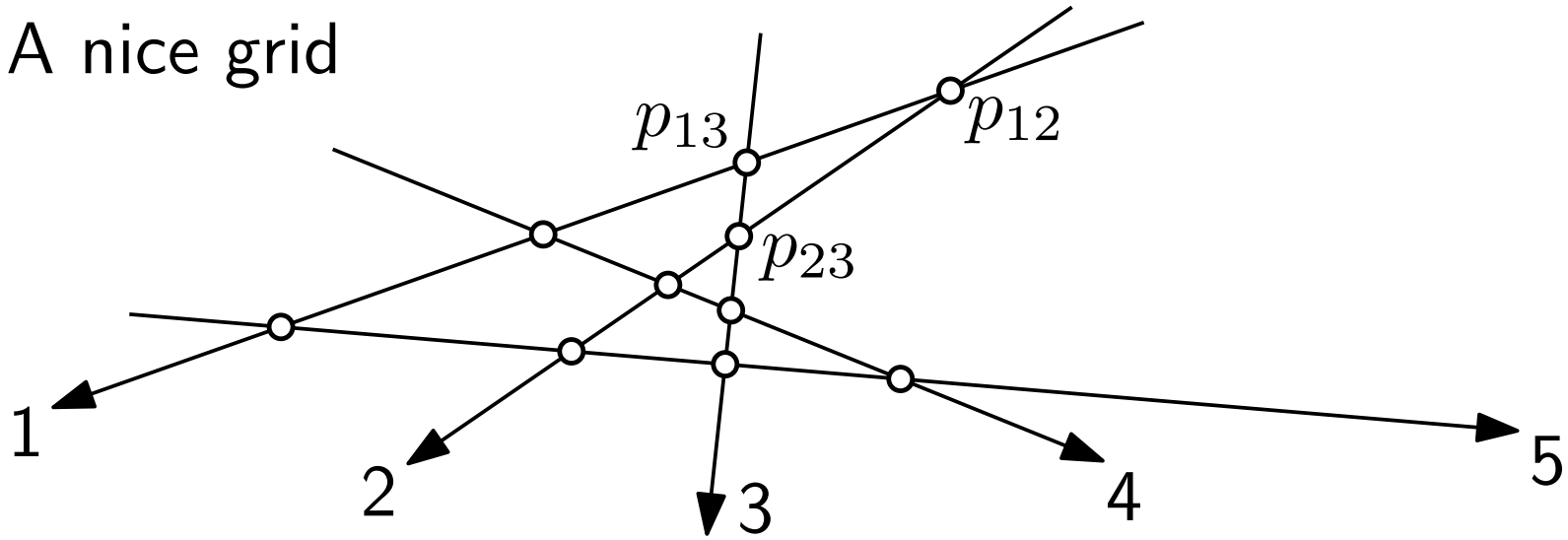

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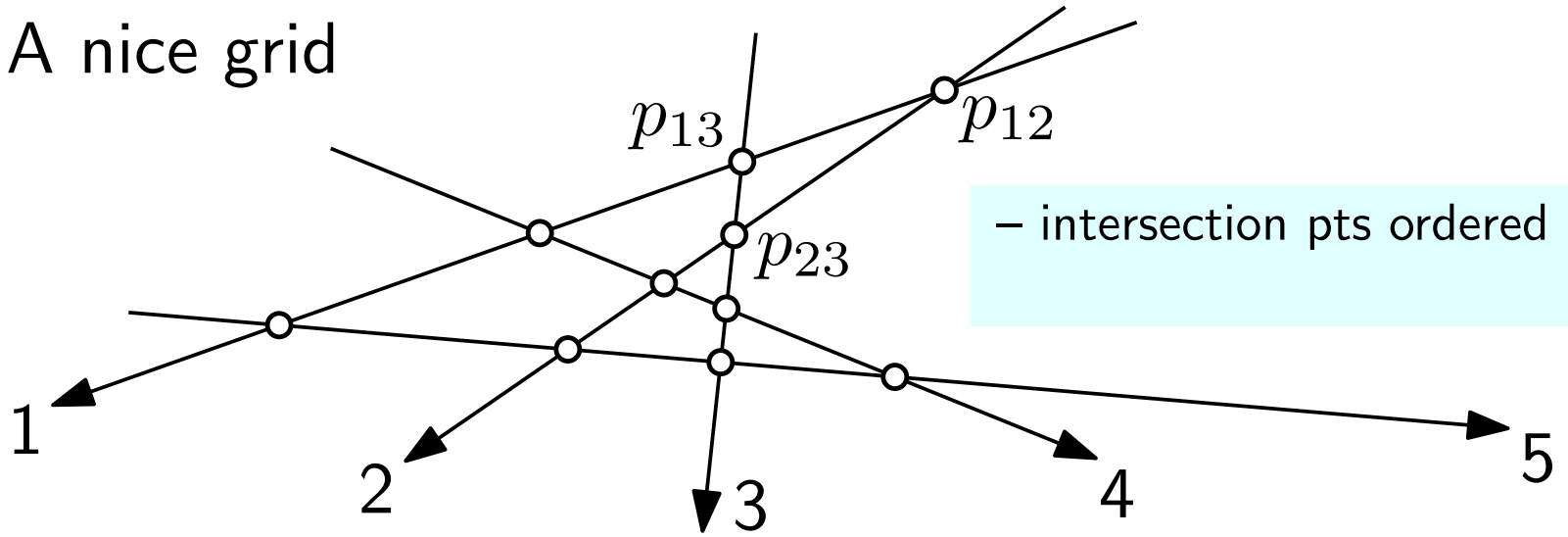

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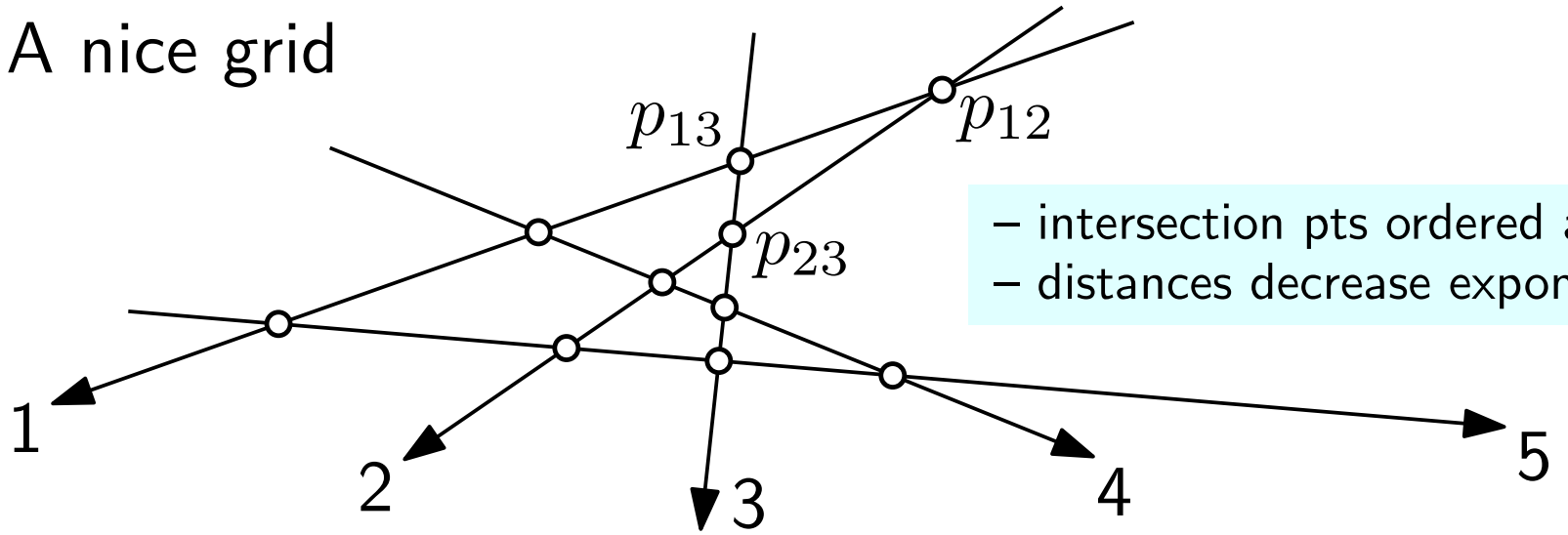

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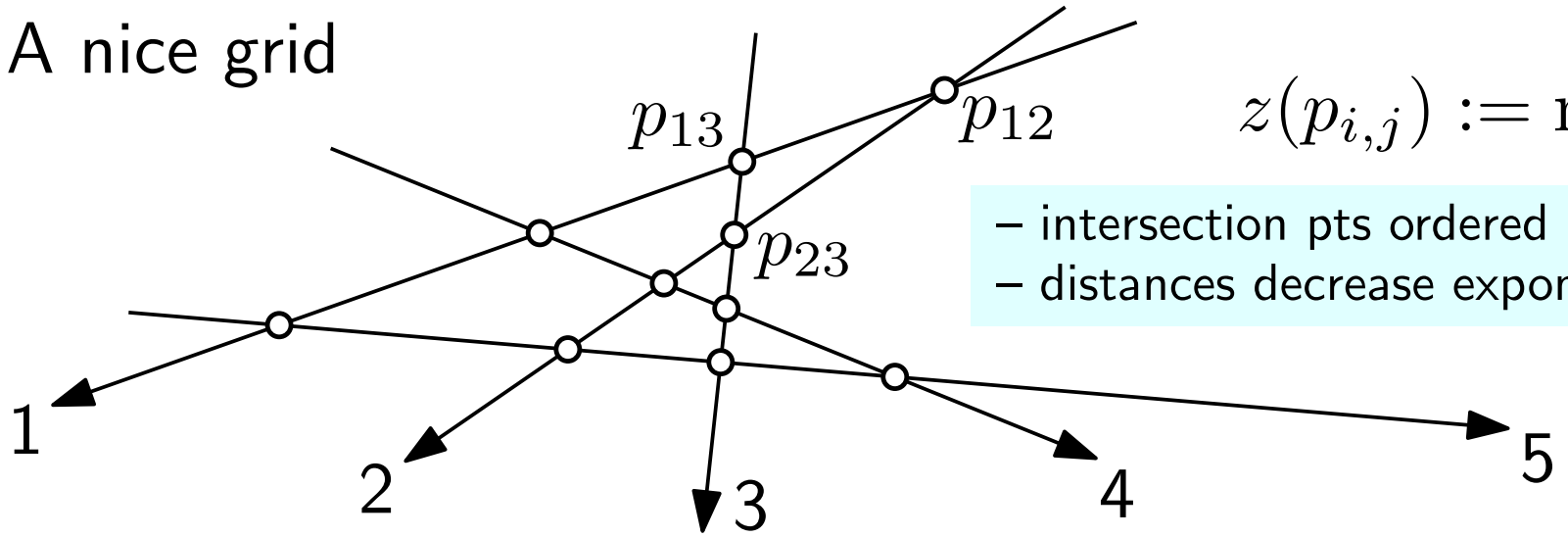

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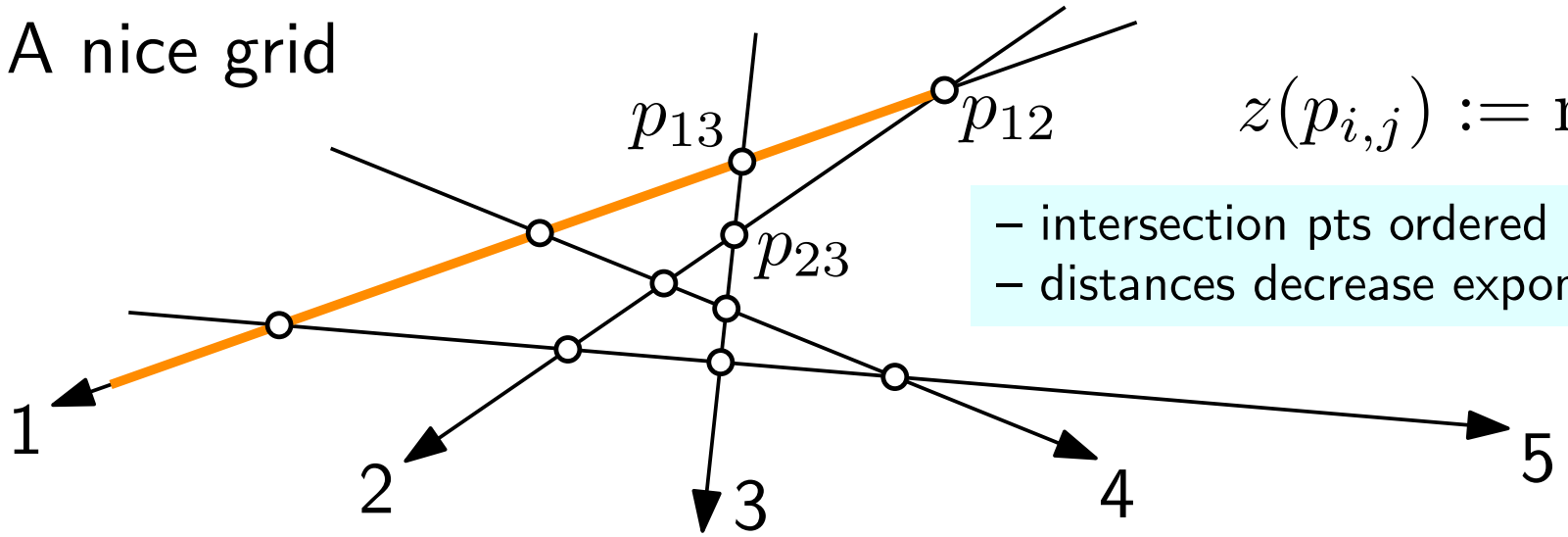

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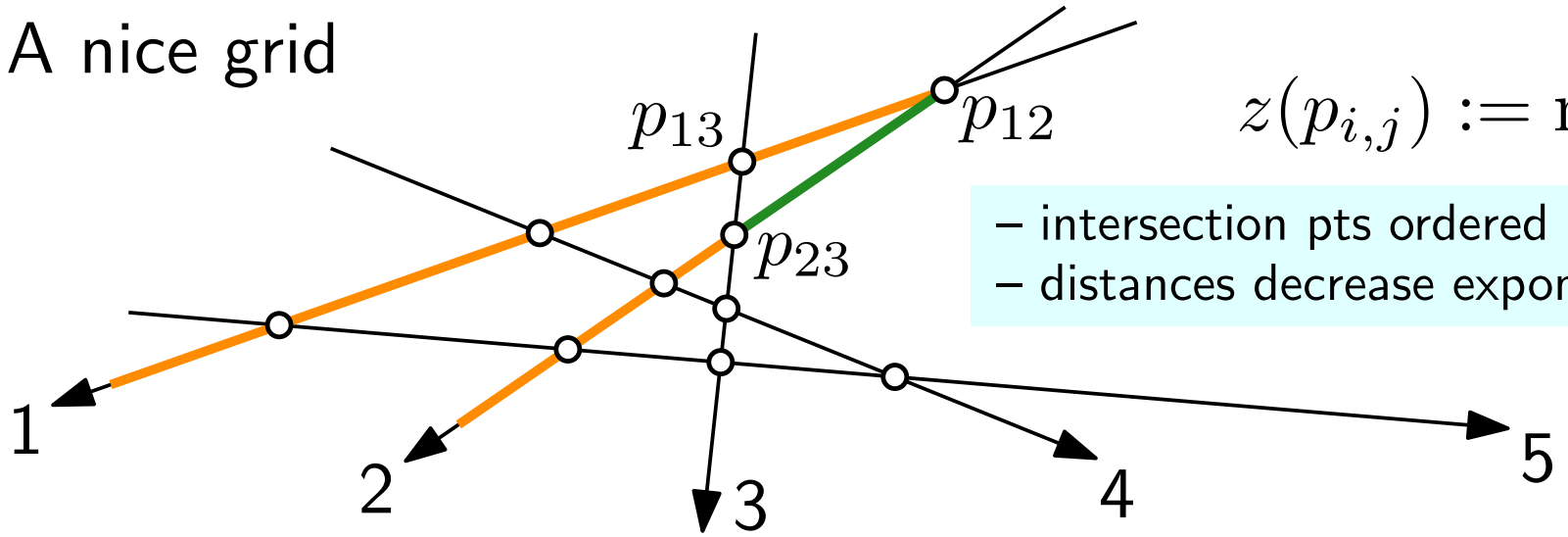

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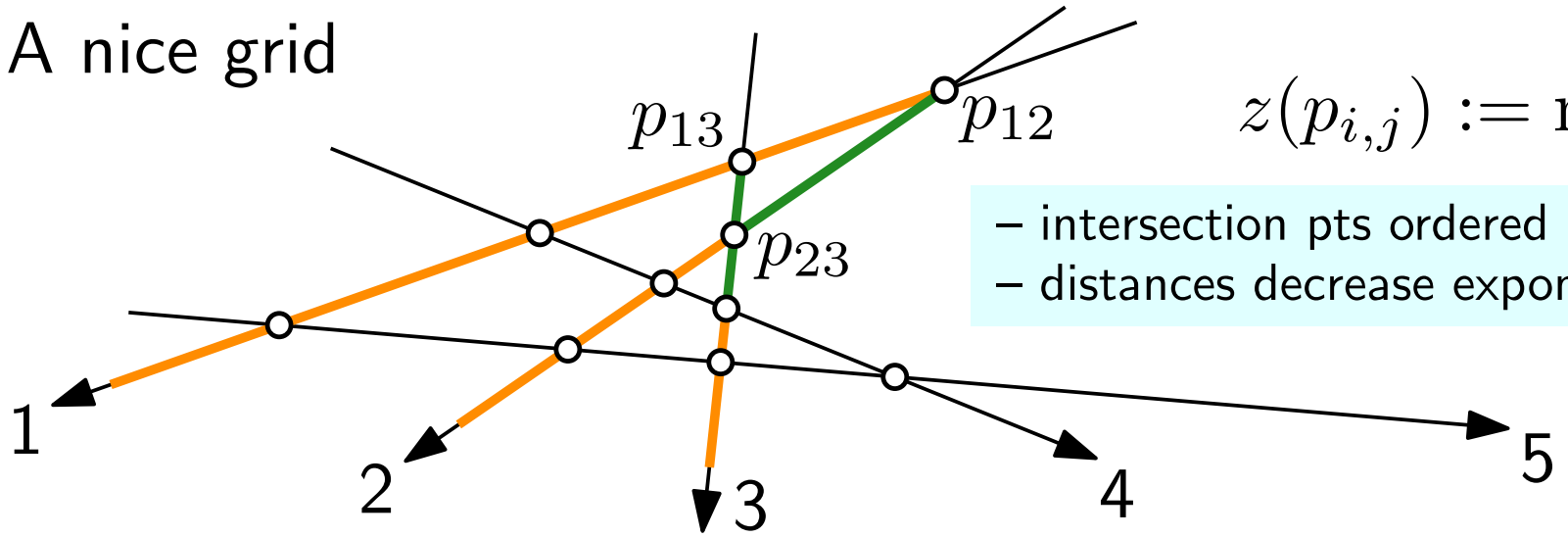

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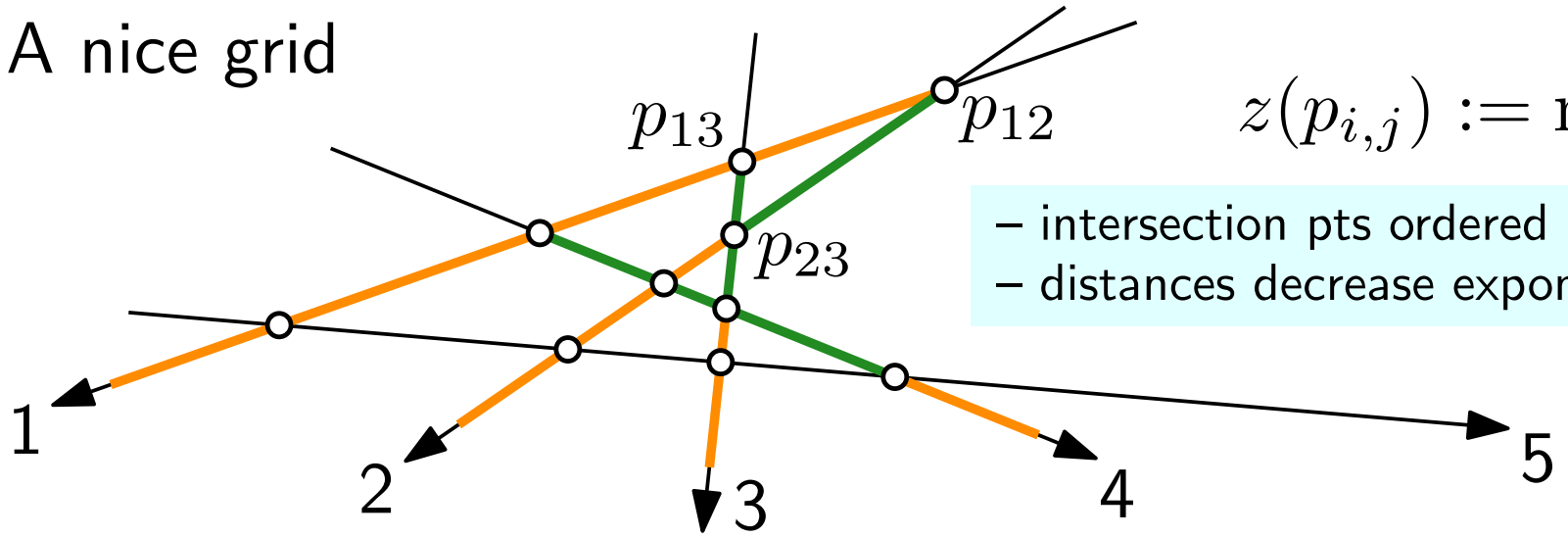

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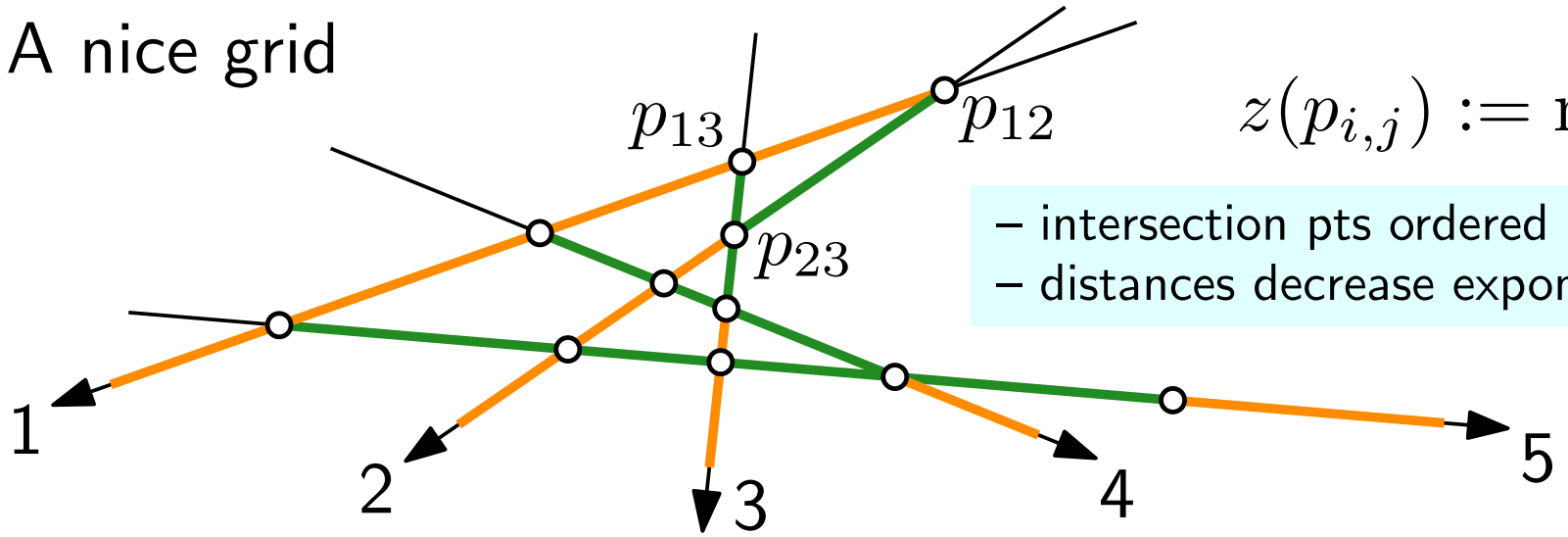

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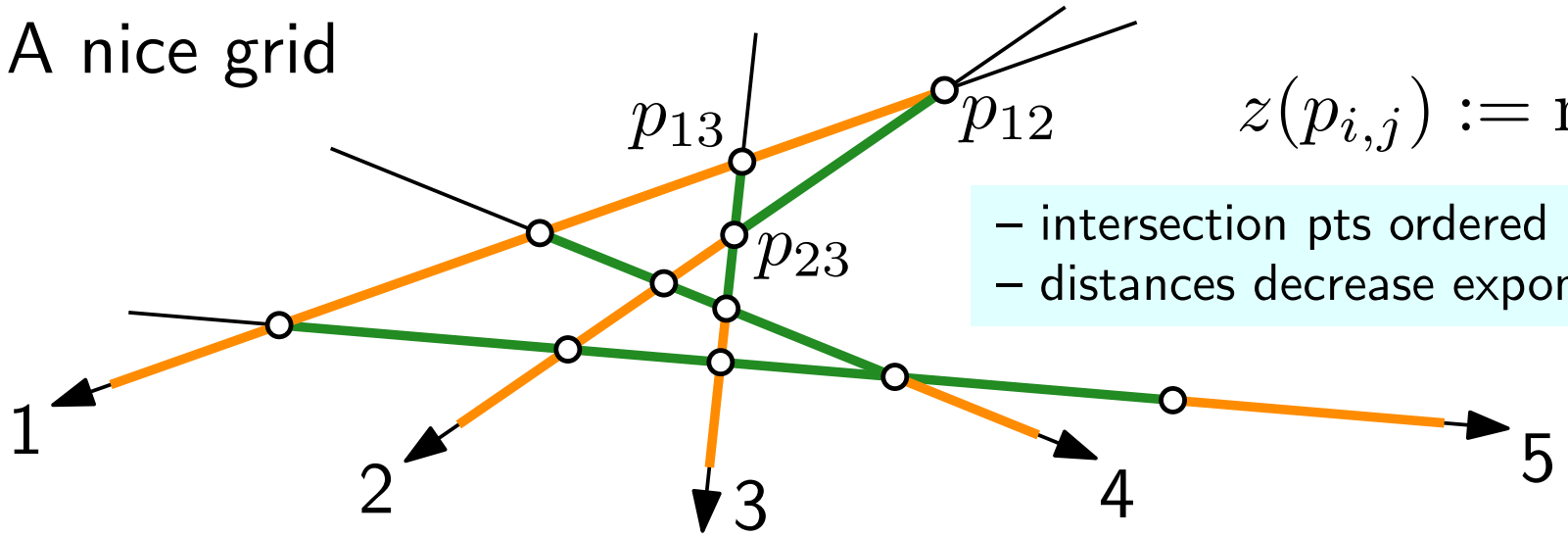

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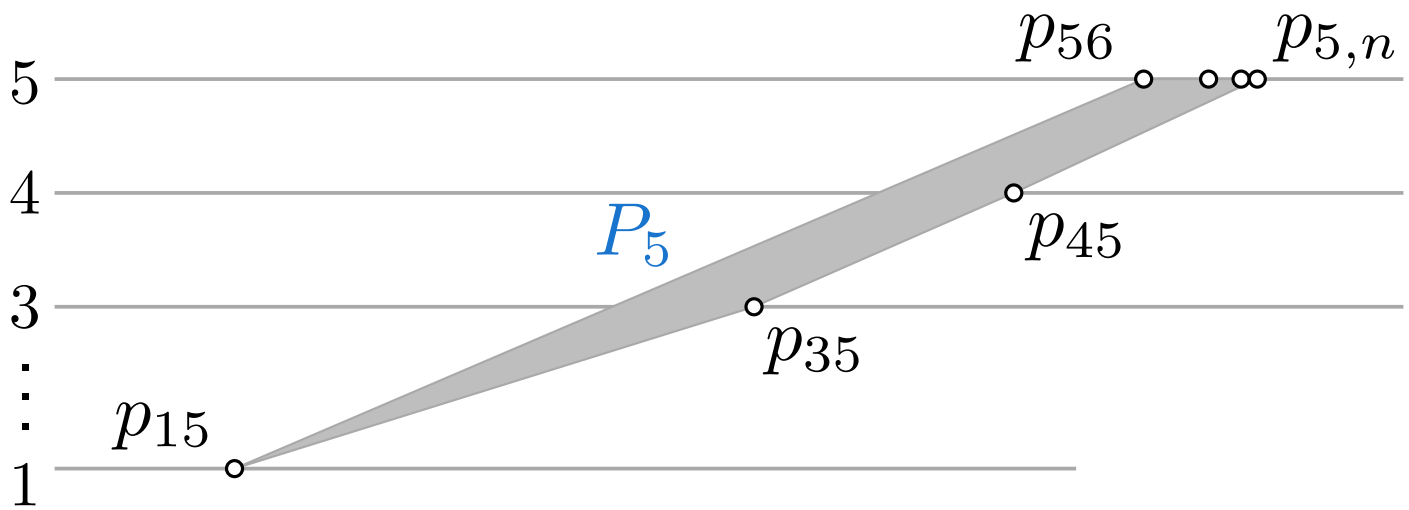

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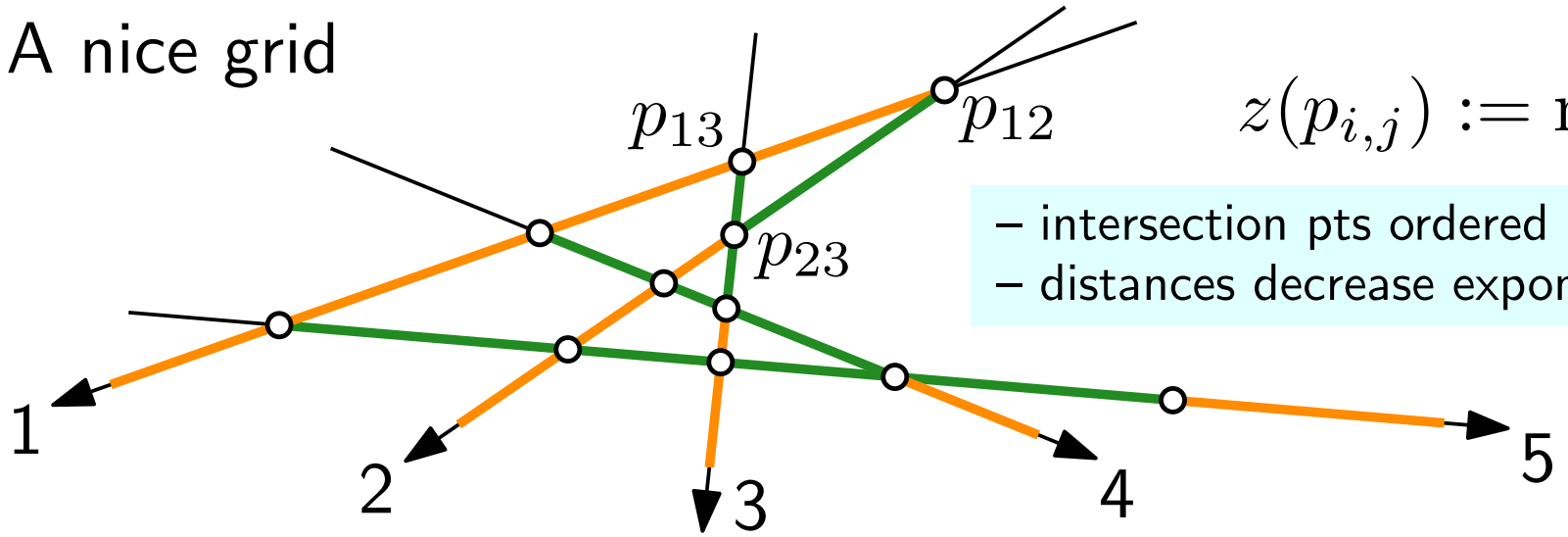

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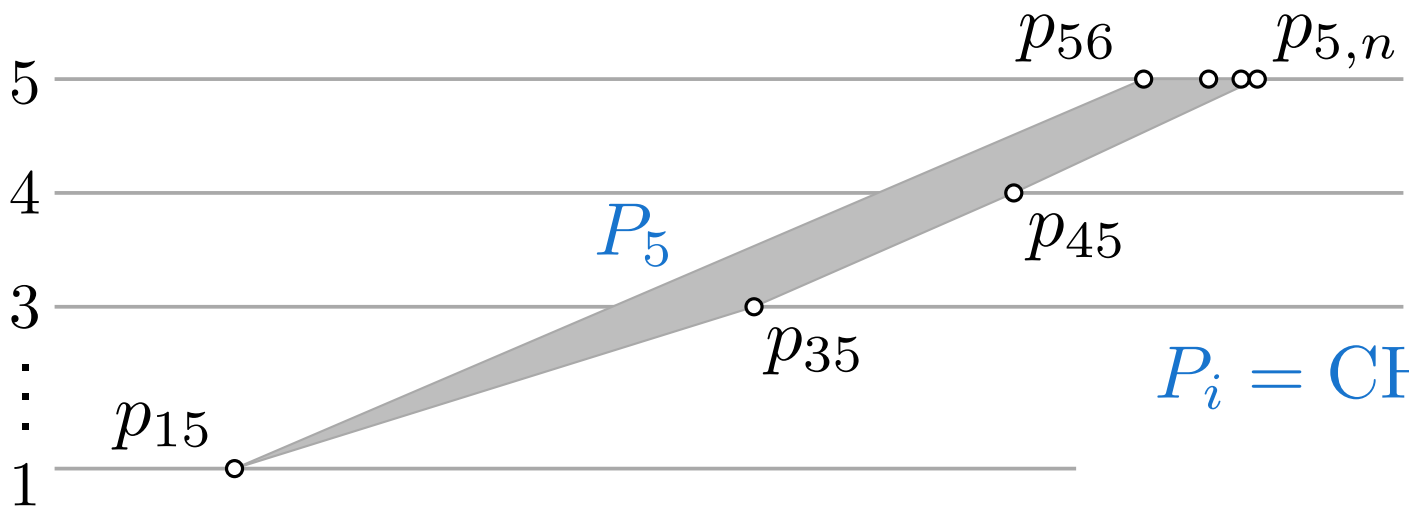

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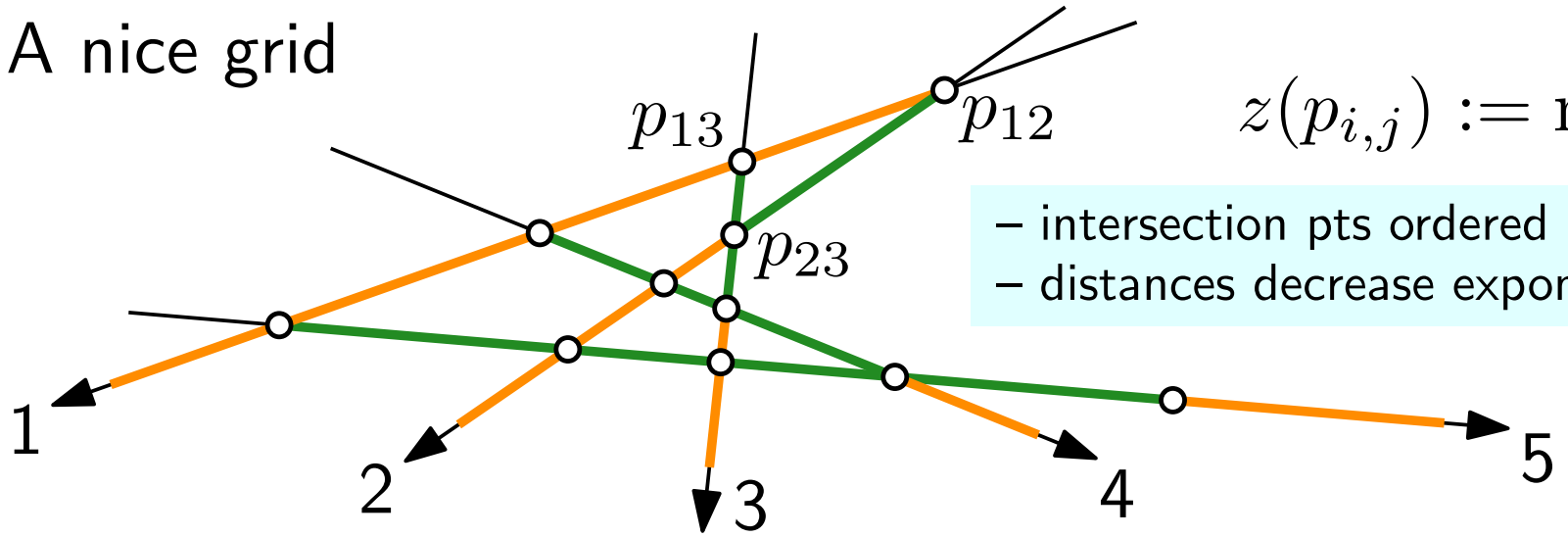

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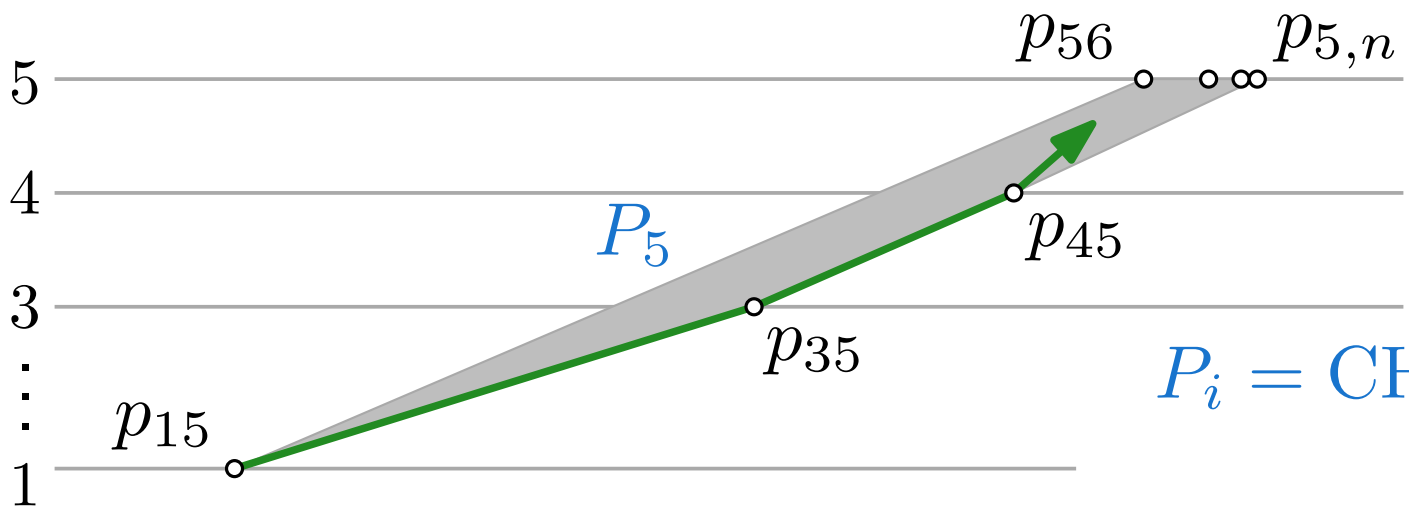

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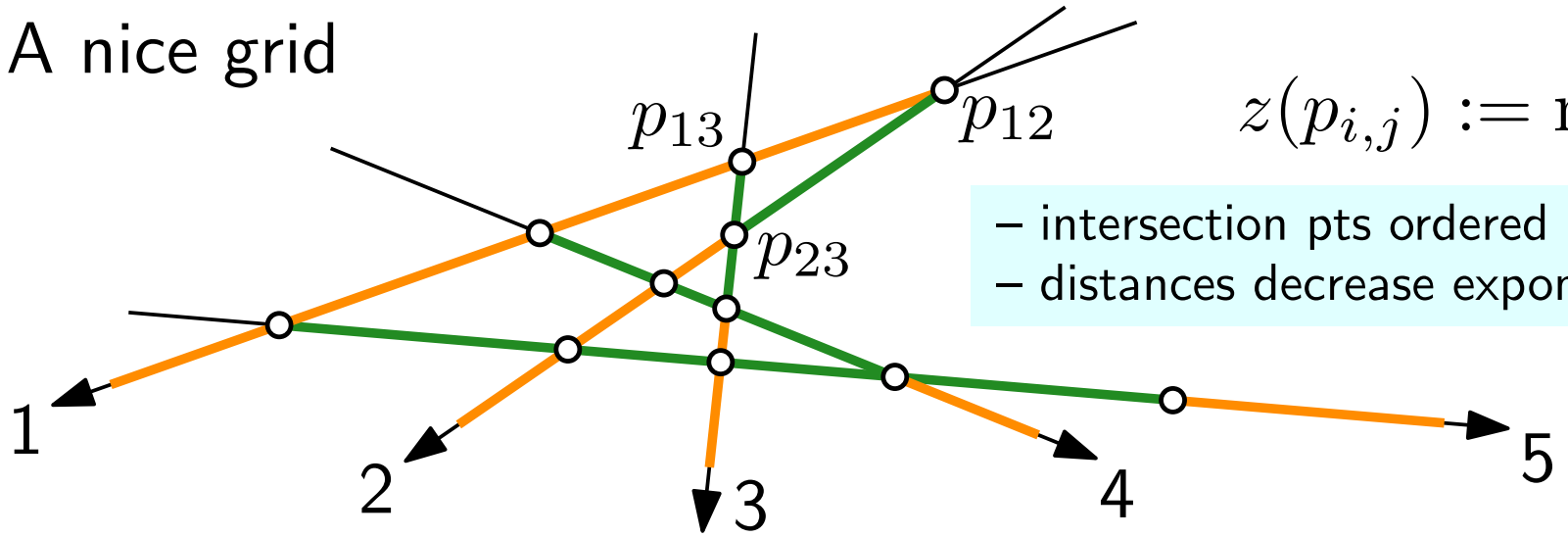

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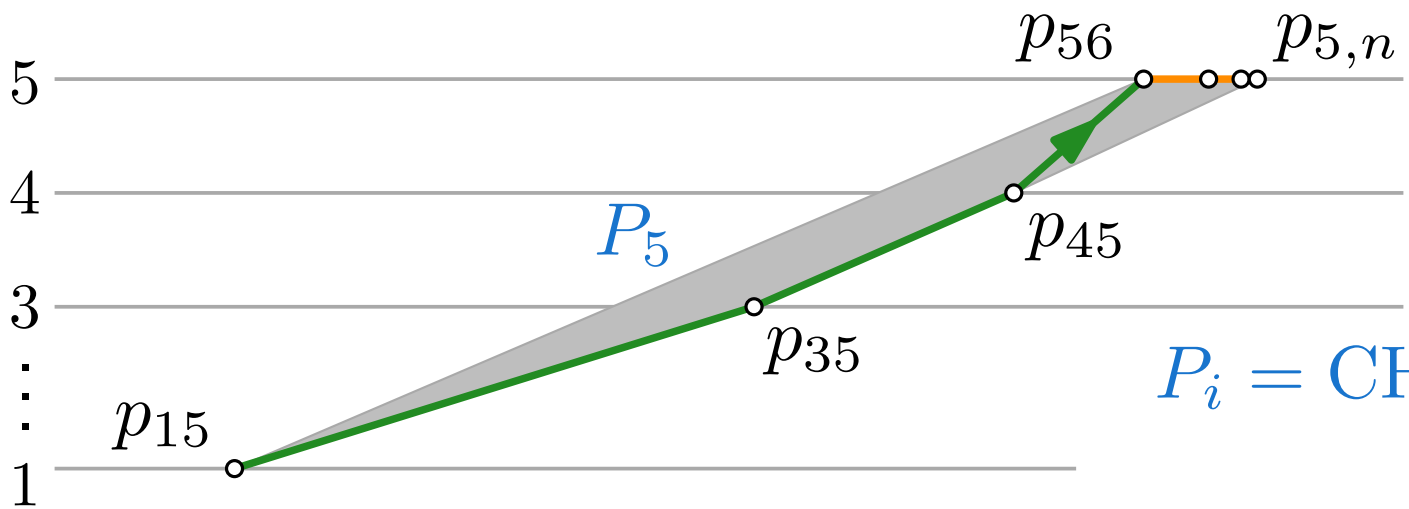

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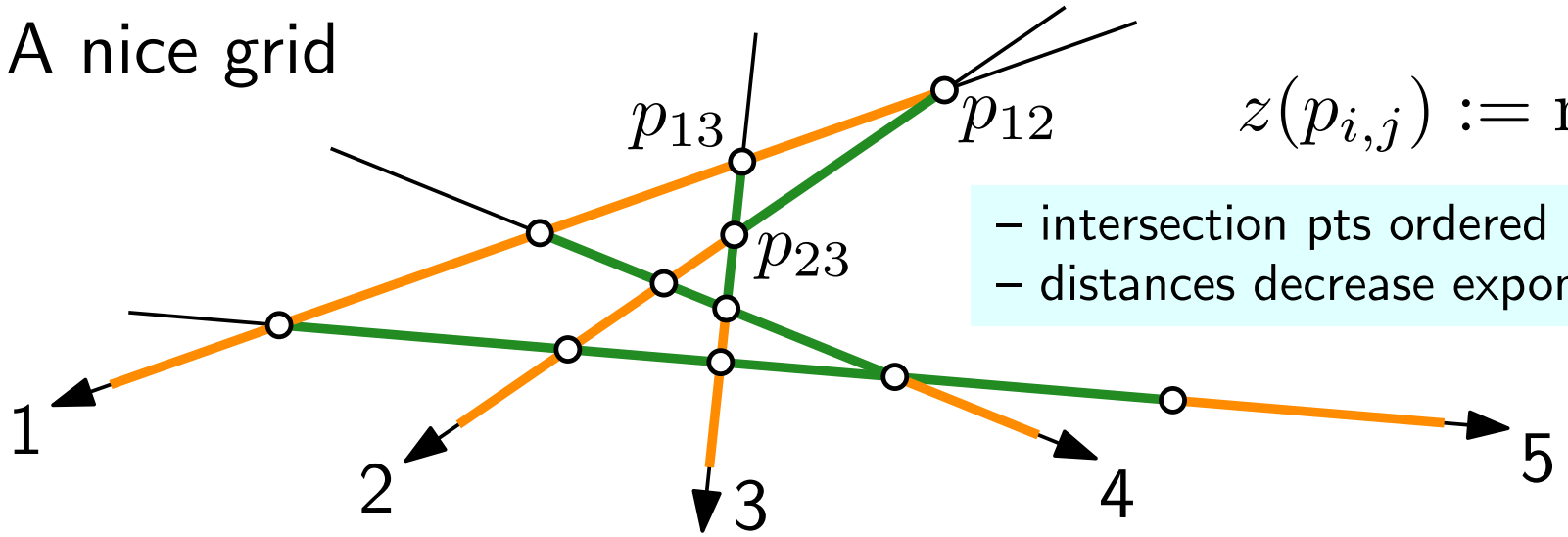

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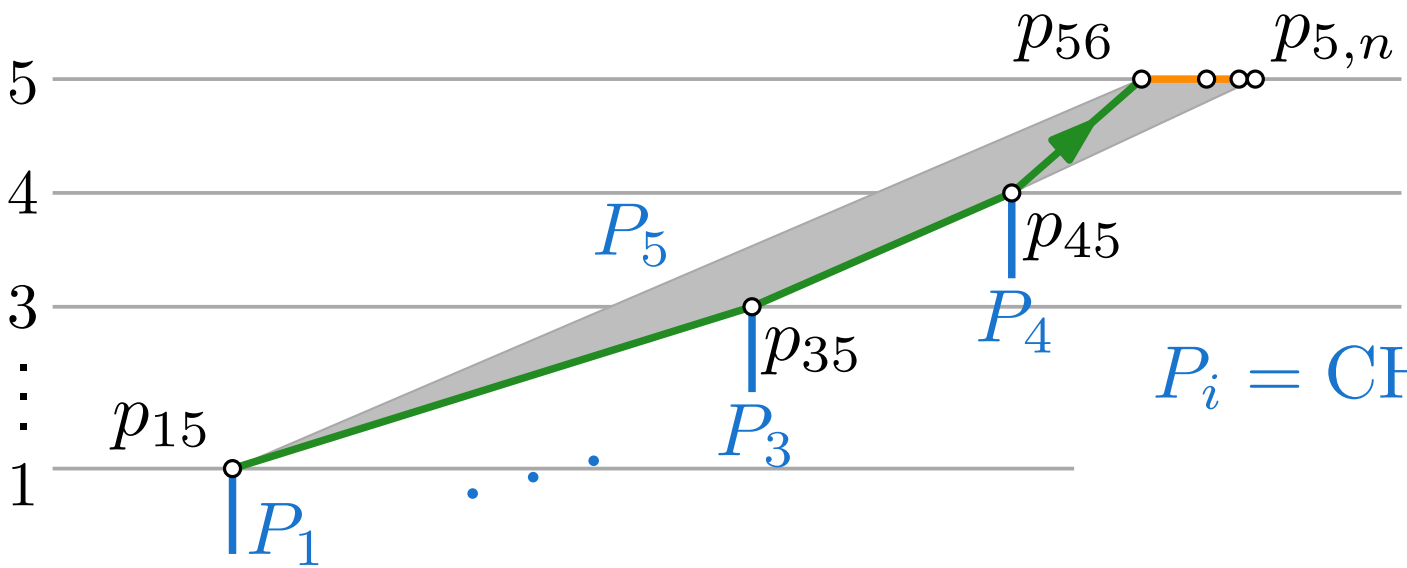

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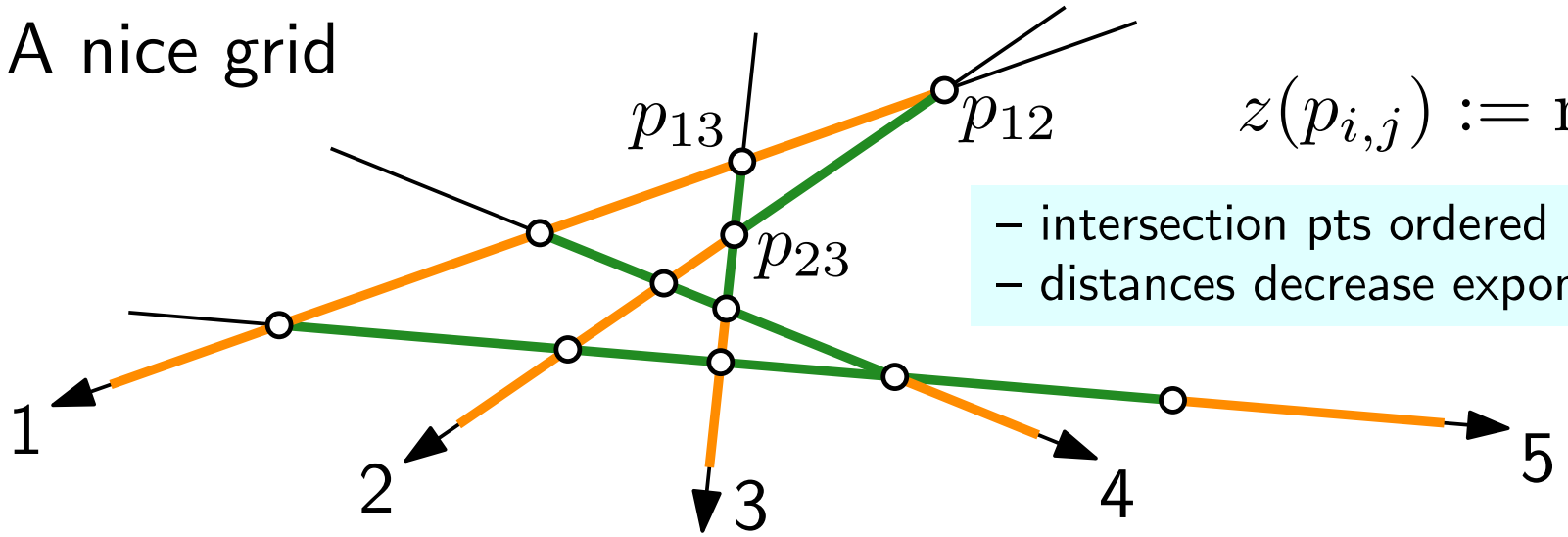

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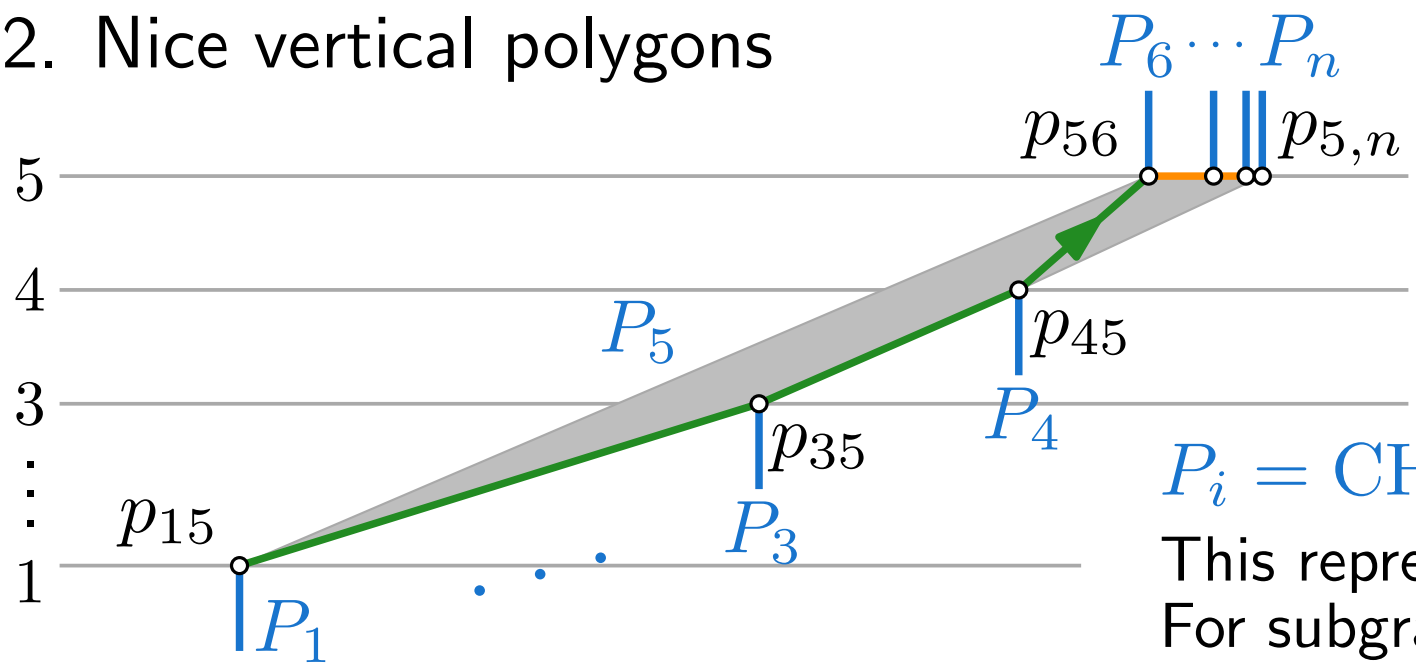

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This represents K_n.
For subgraphs, remove contacts.

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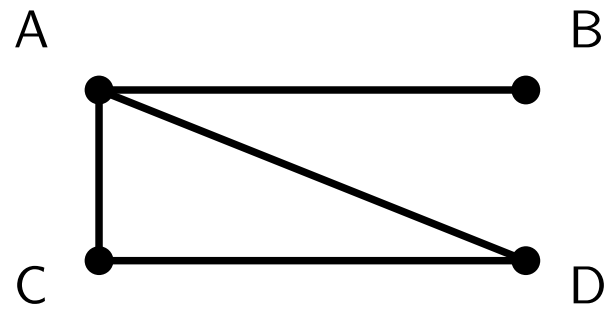

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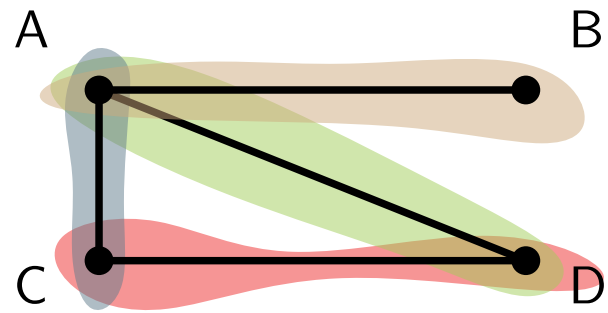

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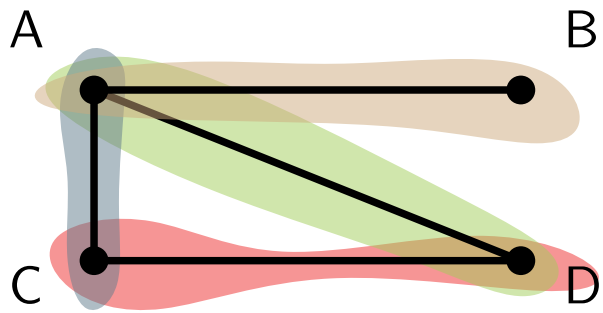

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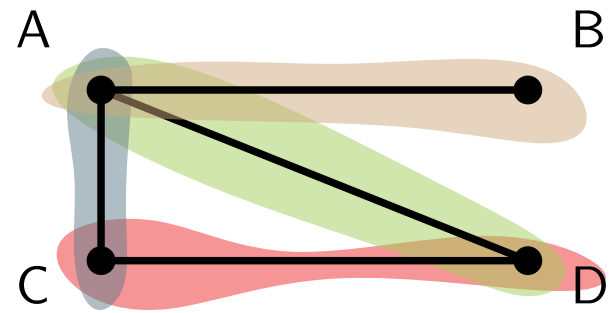

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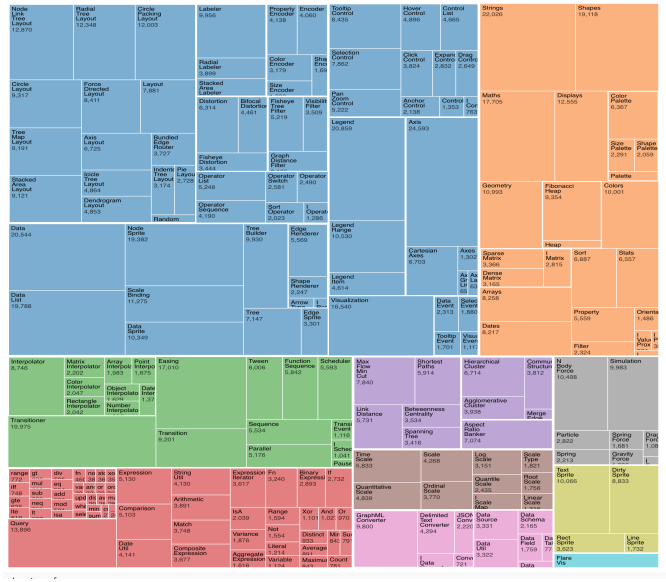
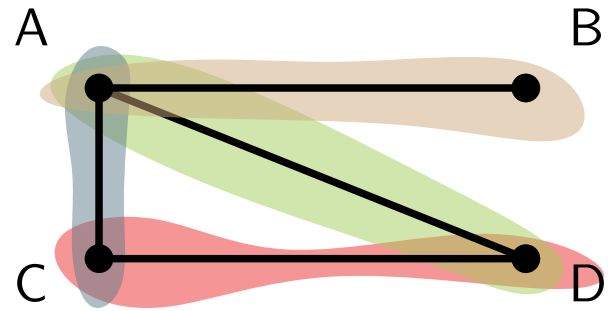


Graphs vs. Sets

- Graphs are defined by a set of edges, which are sets of two elements.

$$\{A, B\}, \{C, D\},$$
$$\{D, A\}, \{A, C\}$$



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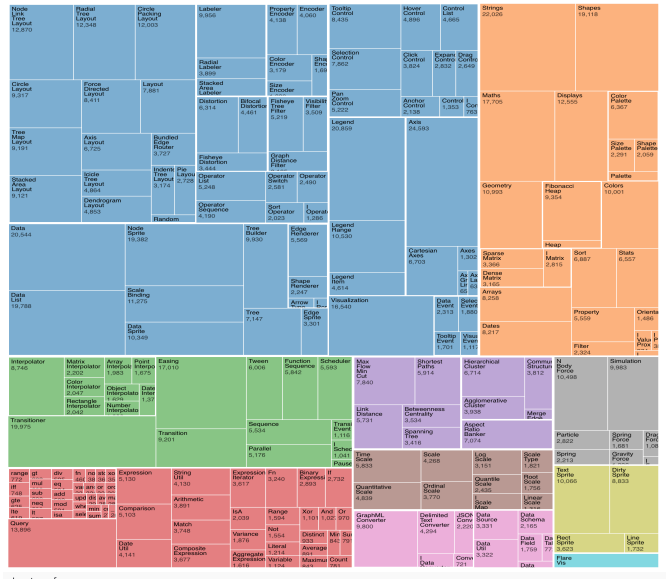

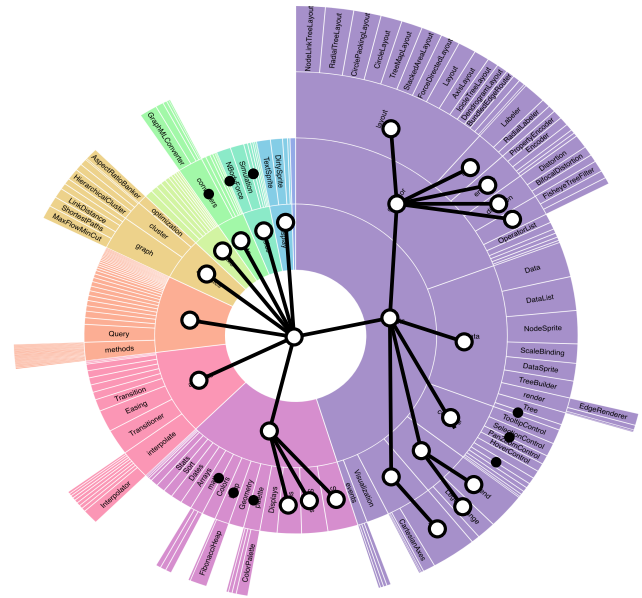

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$V = \{\text{black, red, green, yellow, blue, white, orange}\}$

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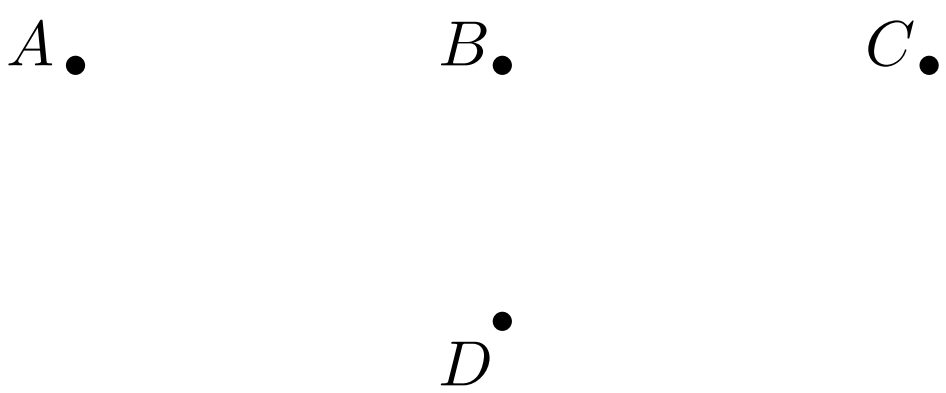

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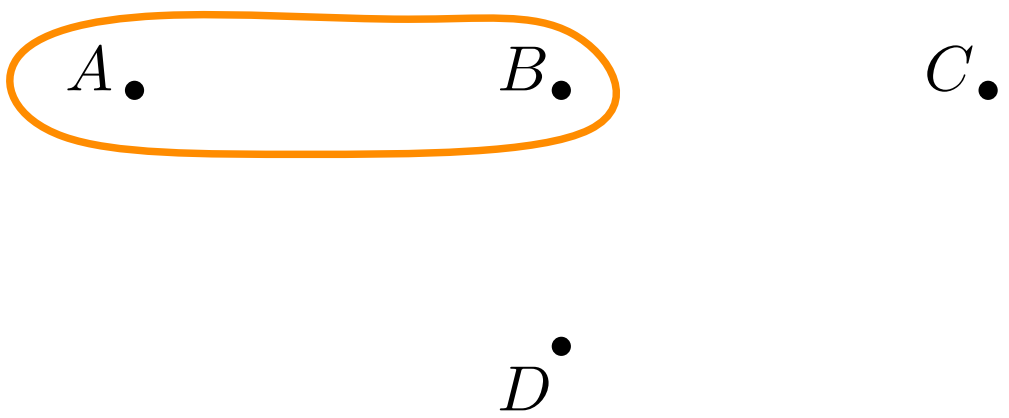

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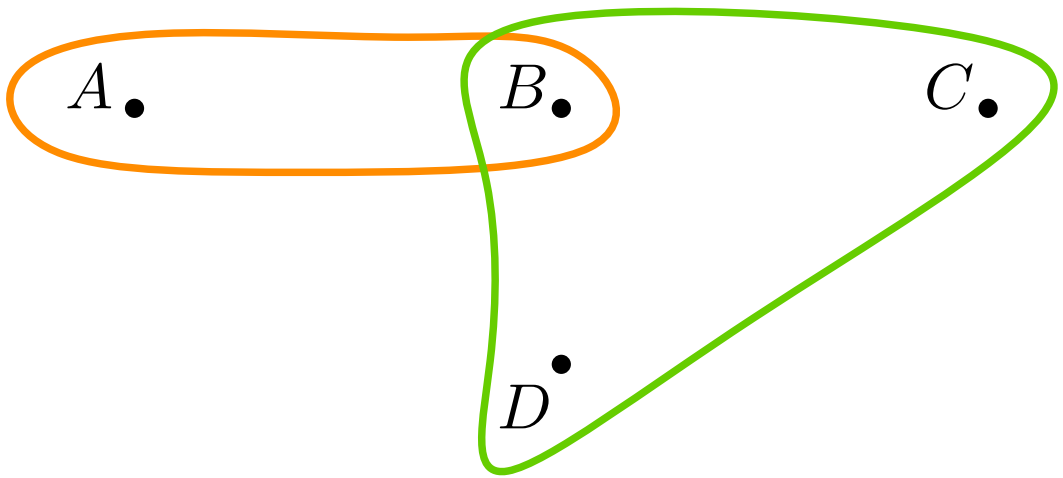

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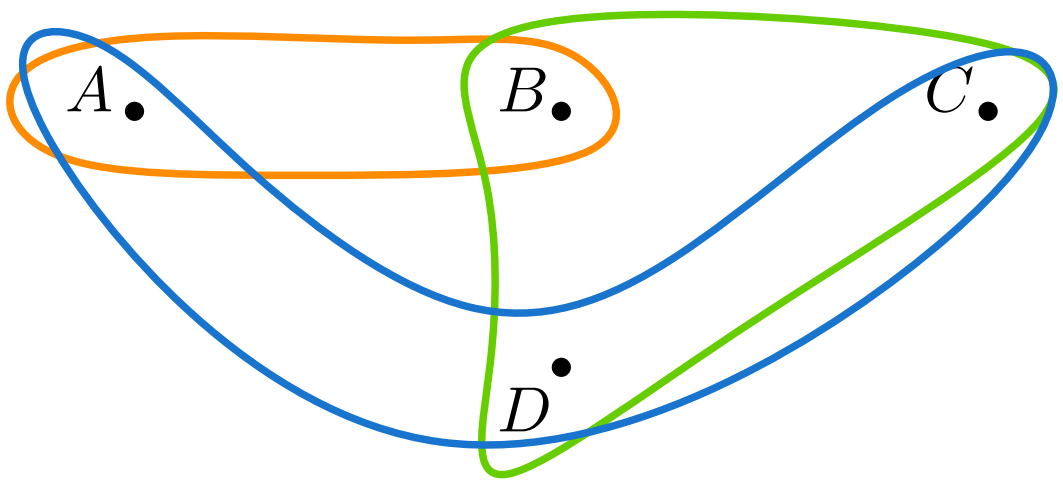

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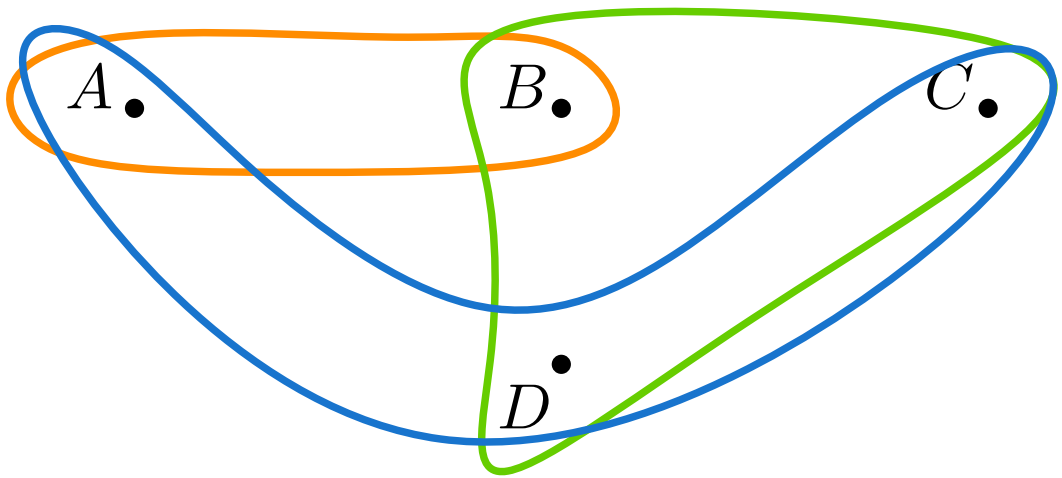

- spring embedder algorithm by Bertault and Eades 2000

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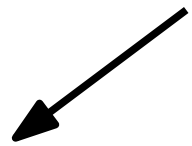

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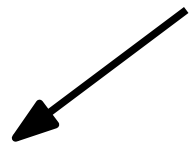

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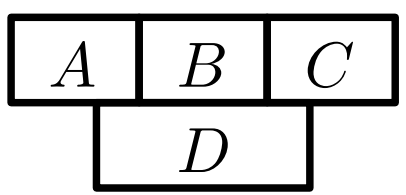

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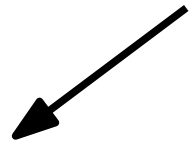

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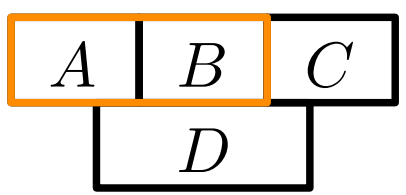

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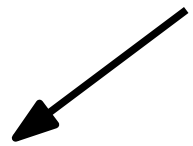

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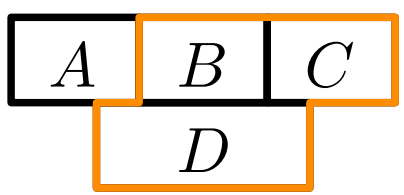

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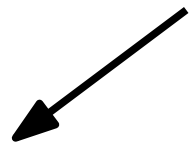

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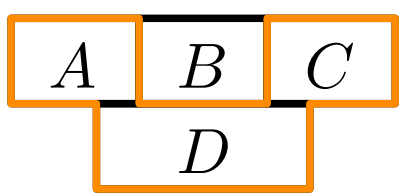

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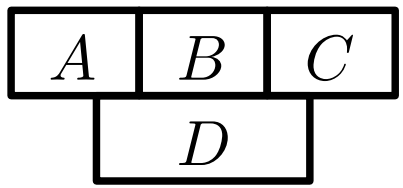

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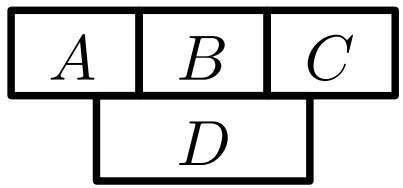

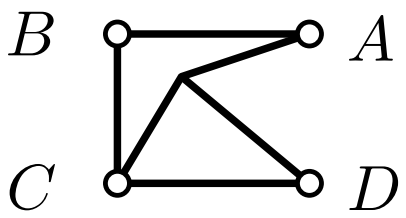

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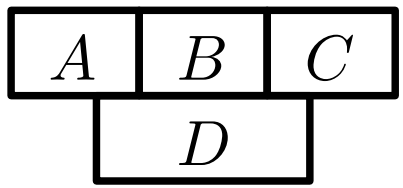

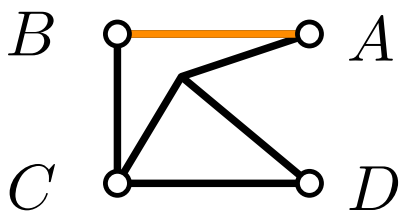

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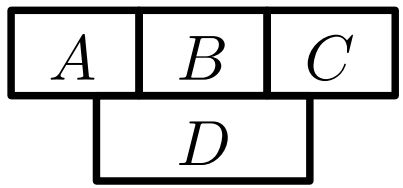

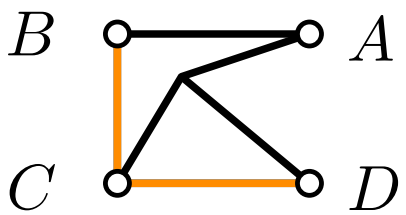

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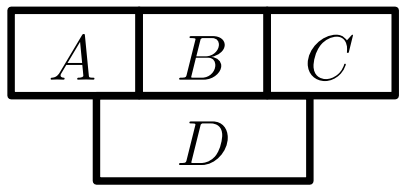

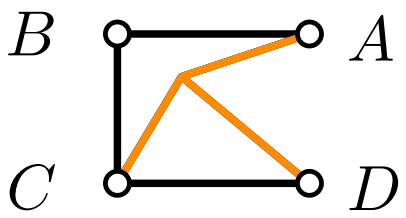

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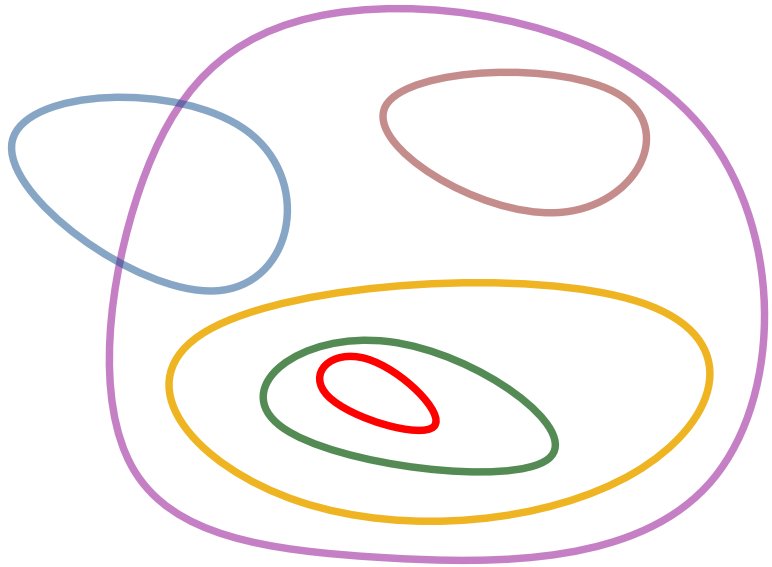

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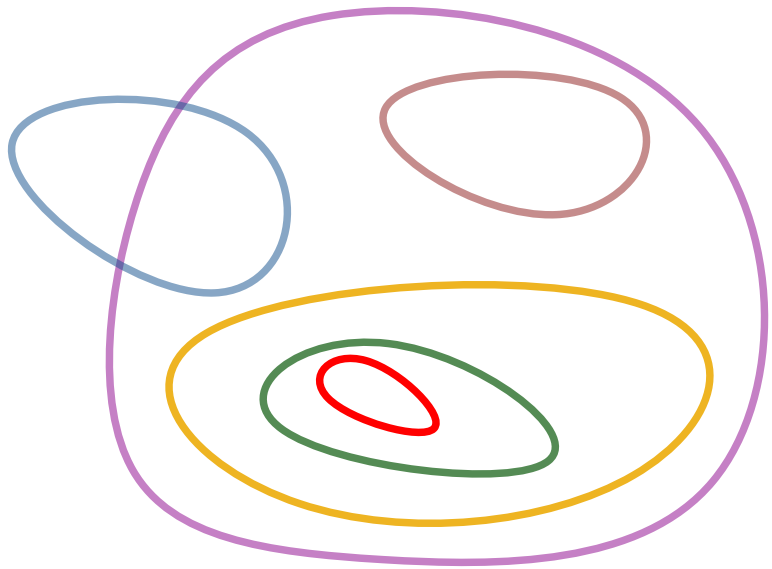

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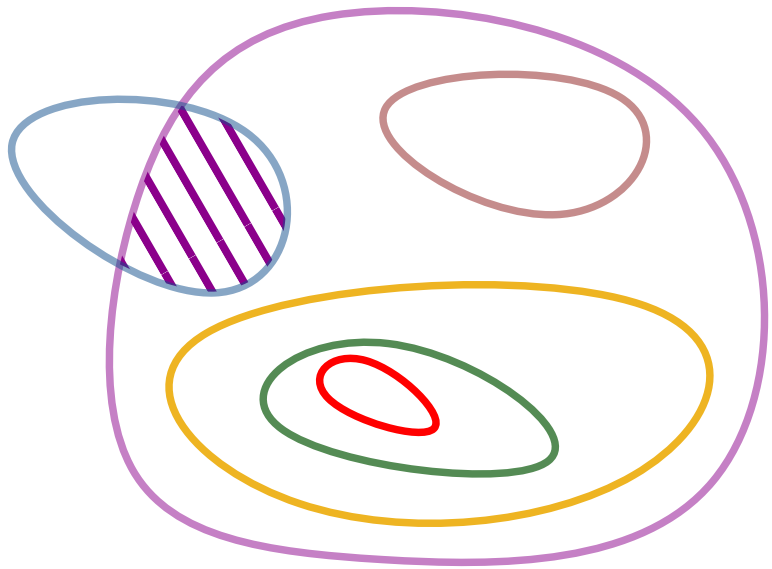

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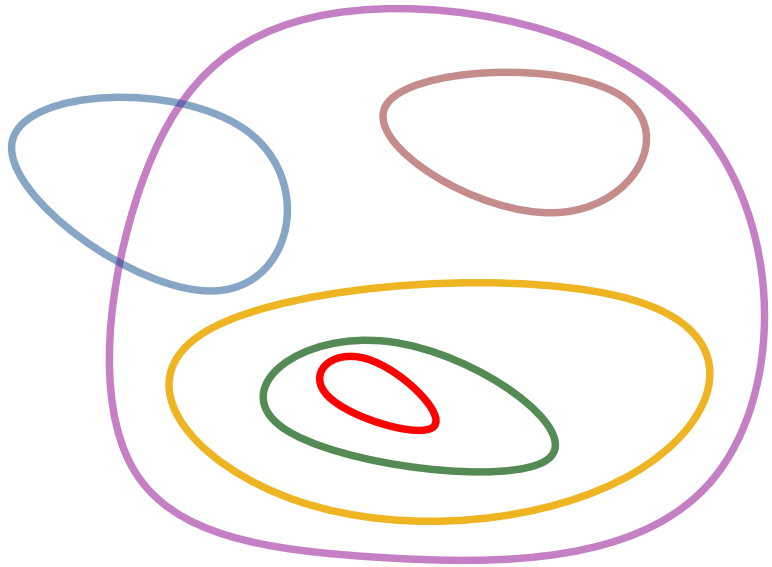

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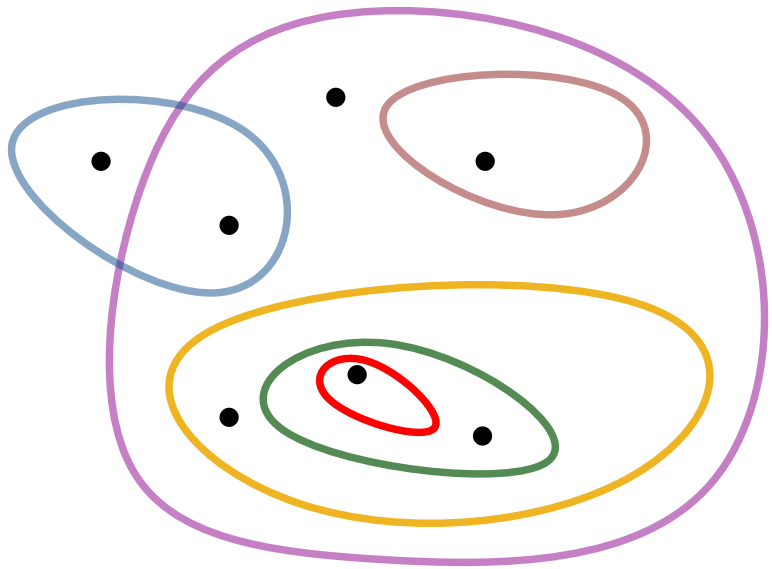

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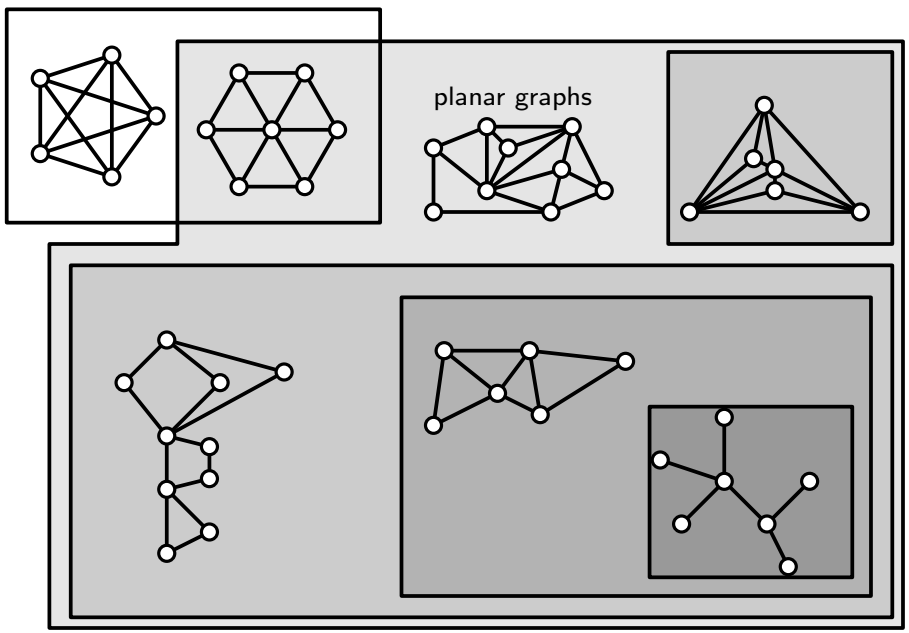

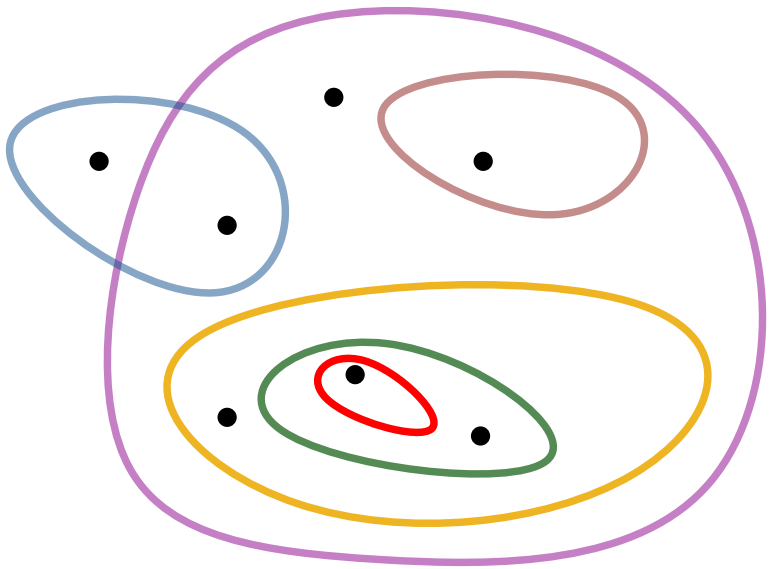

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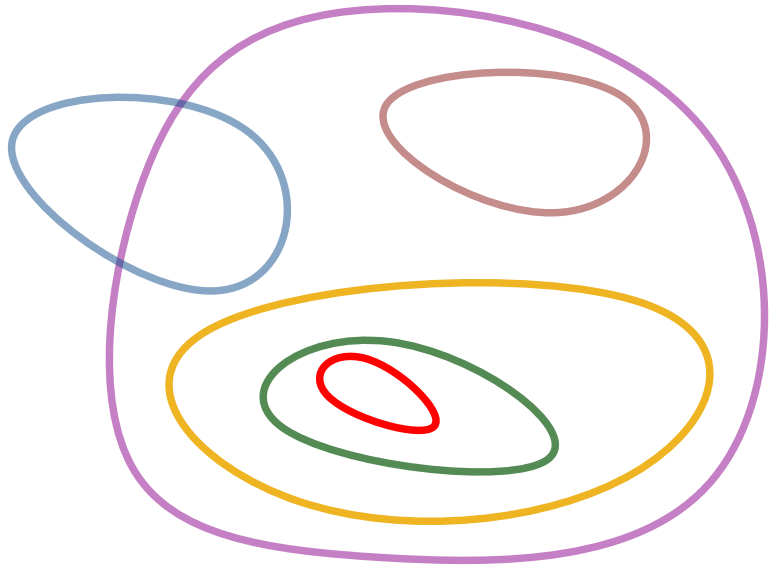

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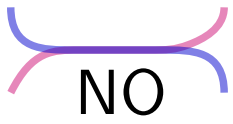

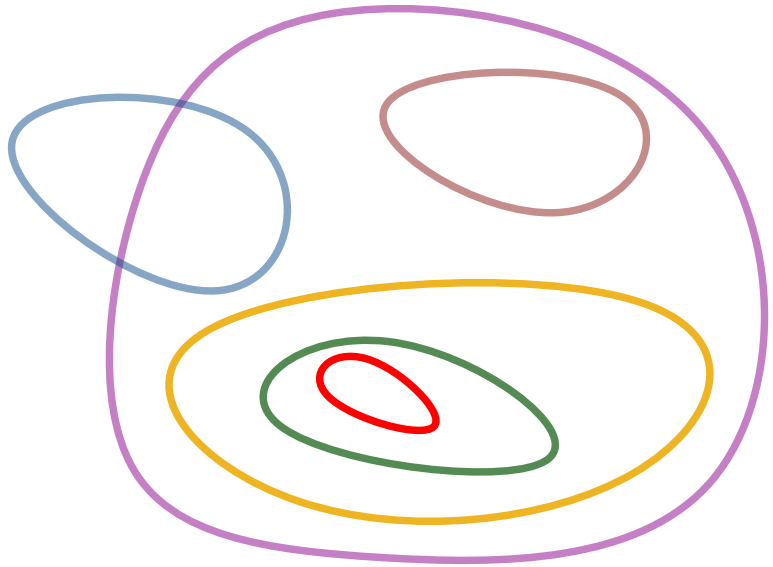

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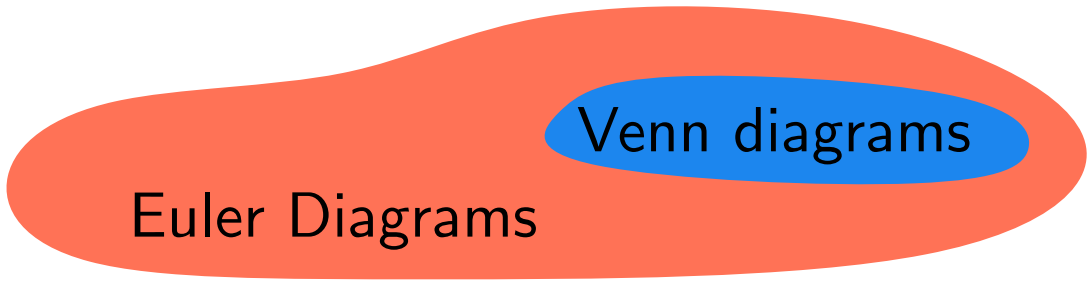
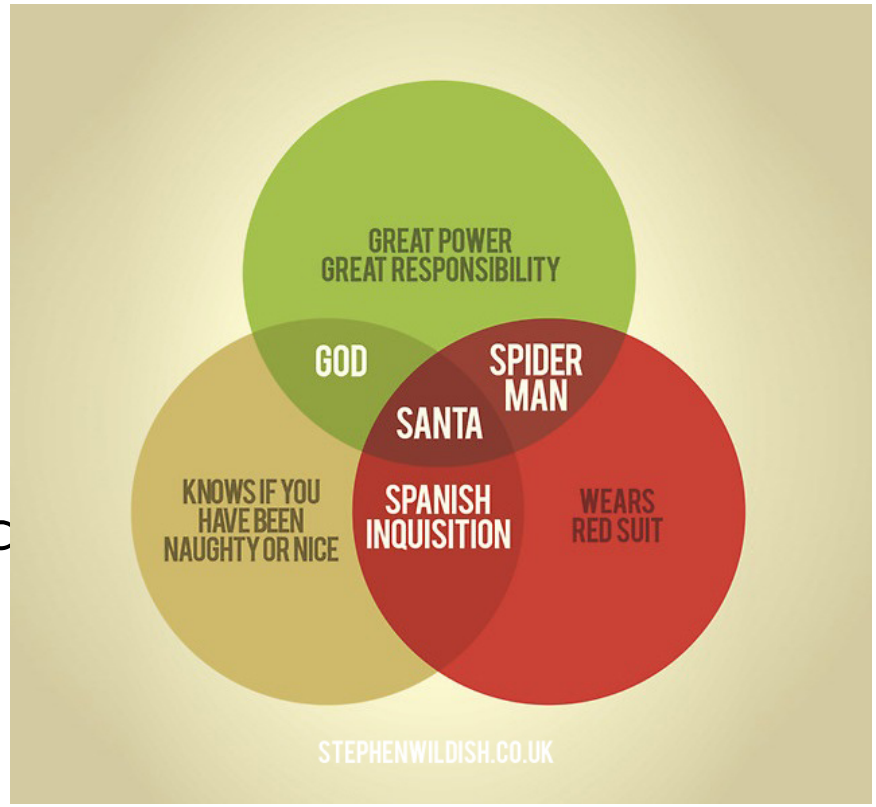
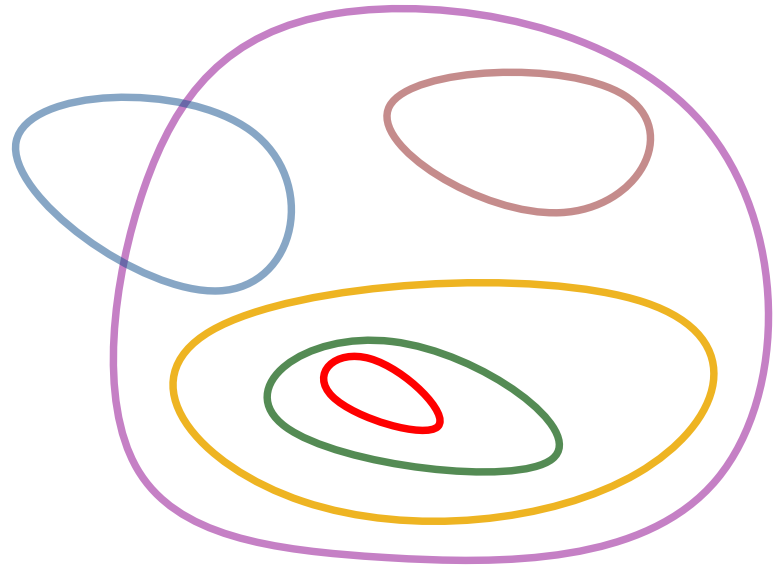
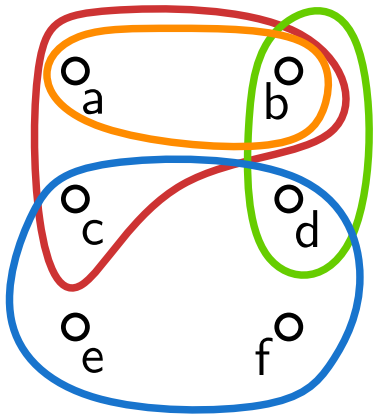






Edge-based methods

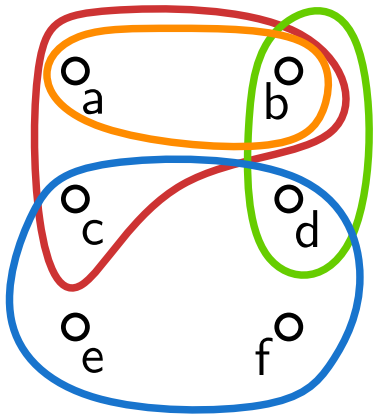
Edge-based methods



subset-based drawing

Edge-based methods

Examples:



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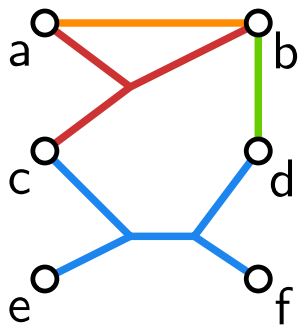

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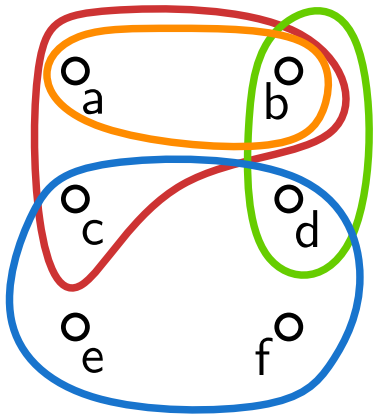

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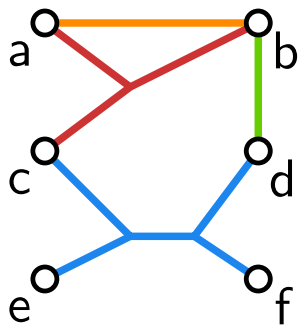

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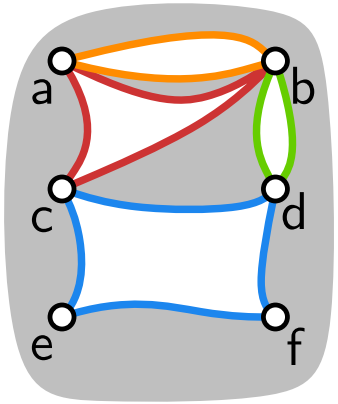

Zykov representation

Edge-based methods

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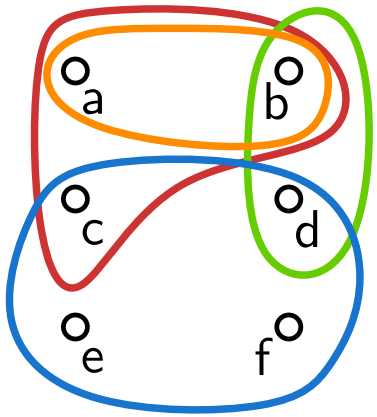

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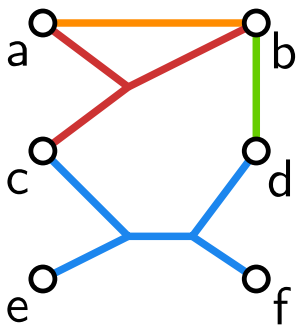

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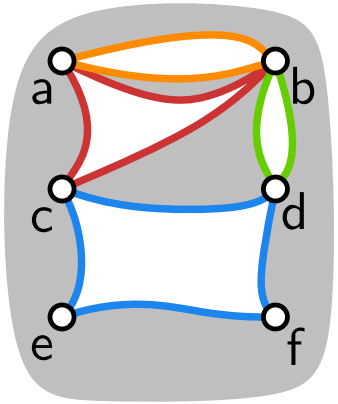

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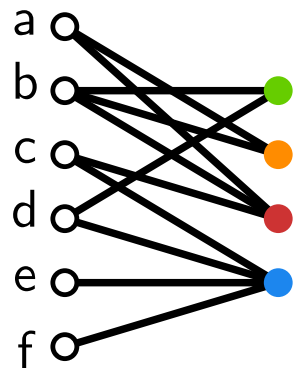

incidence representation

Edge-based methods

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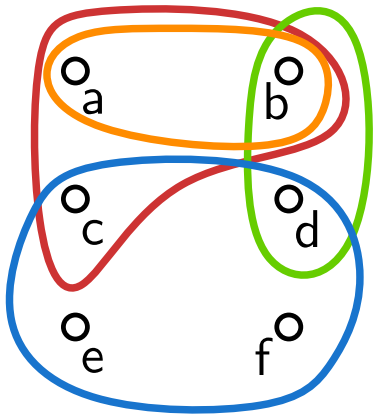

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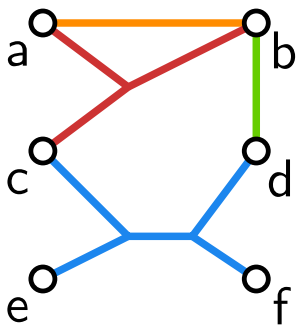

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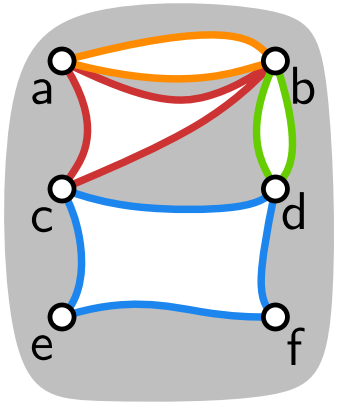

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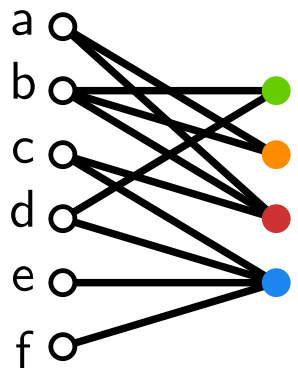

incidence representation

planarity is equivalent in all three models

Edge-based methods

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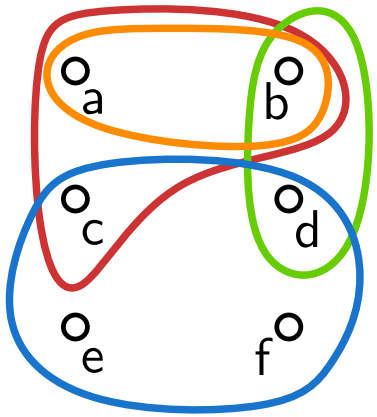

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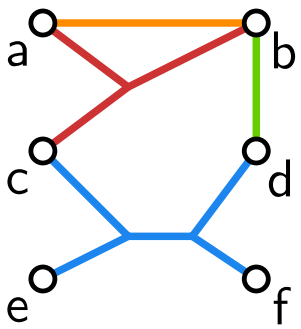

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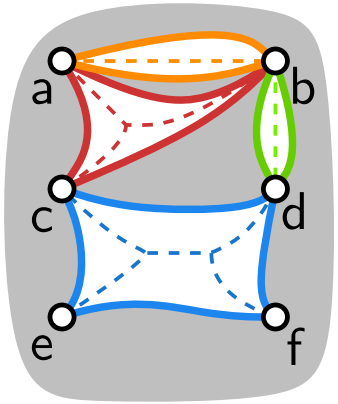

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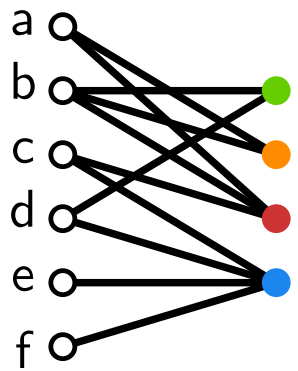

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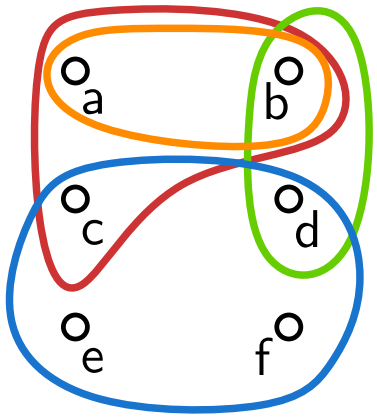

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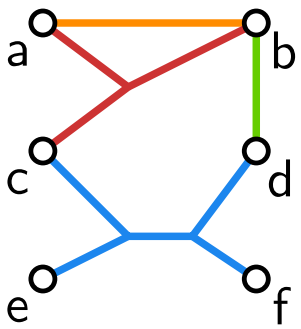

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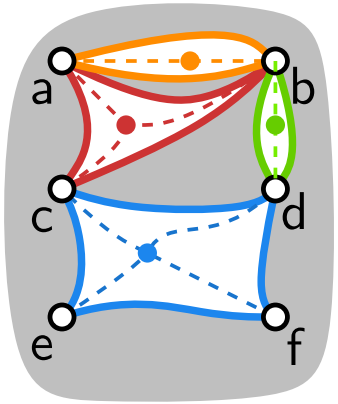

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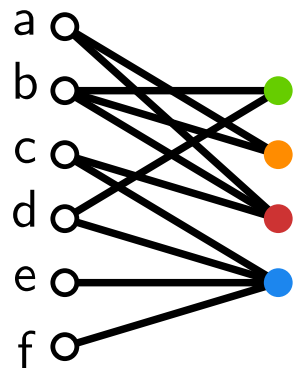

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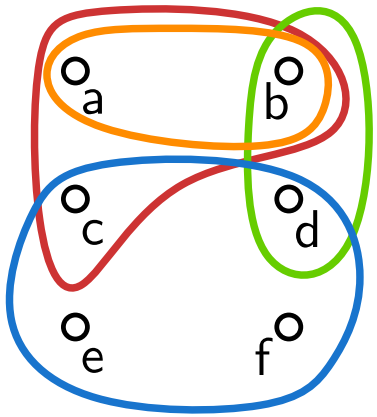

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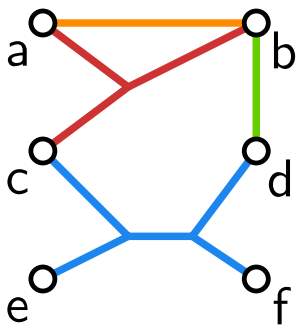

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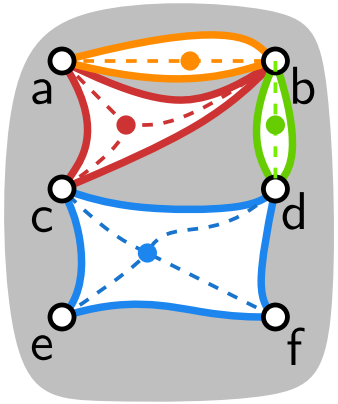

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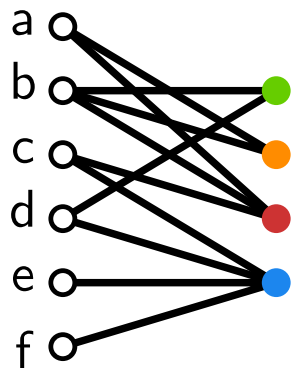

incidence representation

planarity is equivalent in all three models $\in$ P

"planarity"
NP-complete

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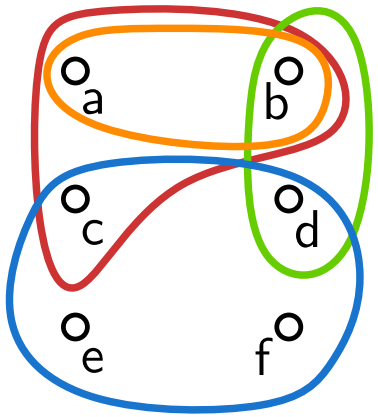
Examples:



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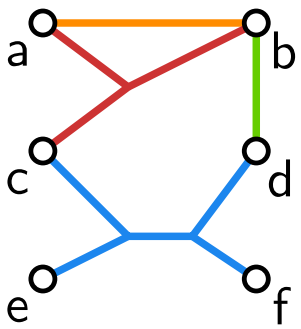

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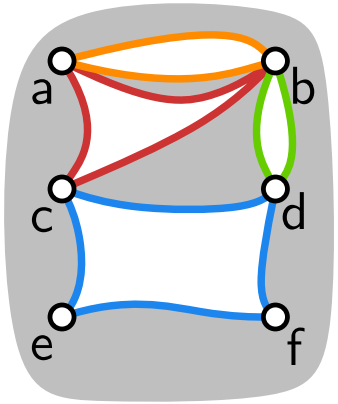

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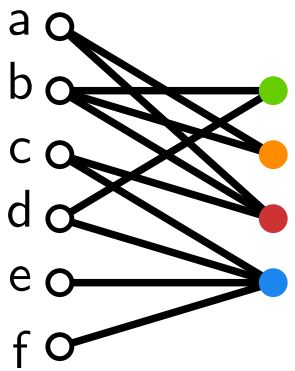

incidence representation

"planarity" NP-complete

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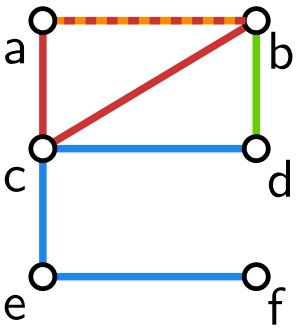
Def.: Support of a hypergraph is a graph such that every hyperedge induces a connected subgraph.



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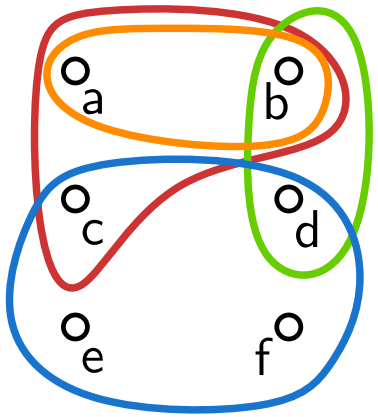
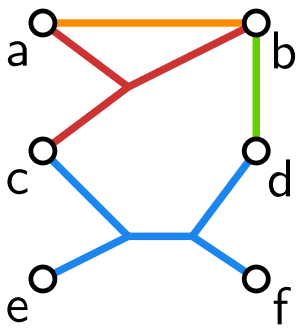


subset-based drawing



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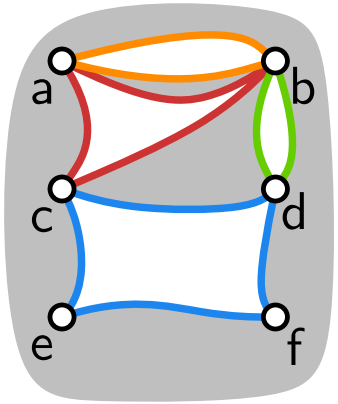

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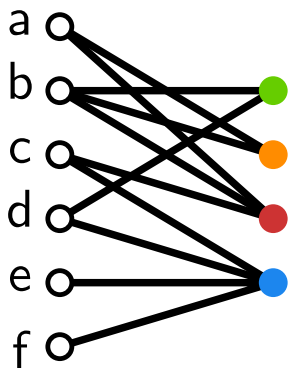

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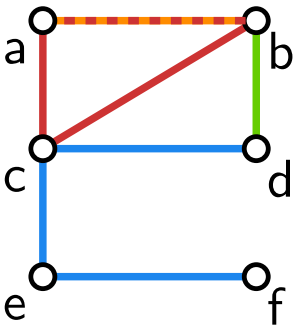

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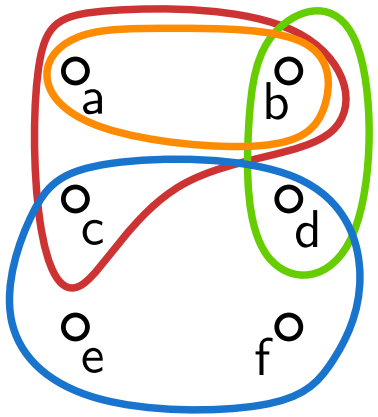

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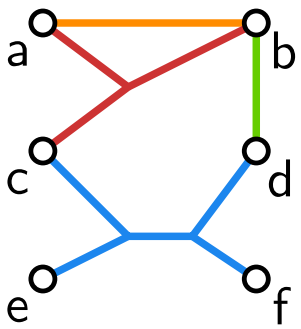

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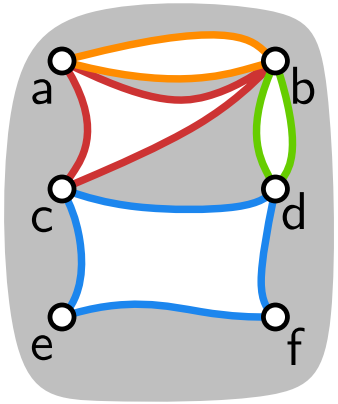

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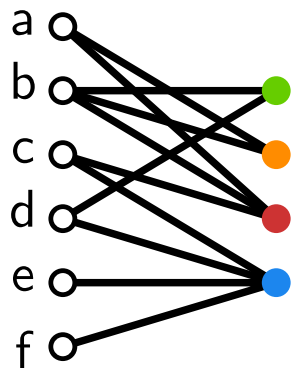

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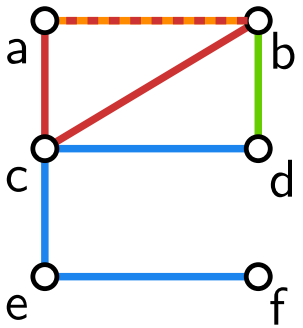

Test for cycle-, tree-, or cactus-support is feasible.

Contact Representations for Hypergraphs in 3D

[Evans, Rzążewski,
Saeedi, Shin, W., GD'19]

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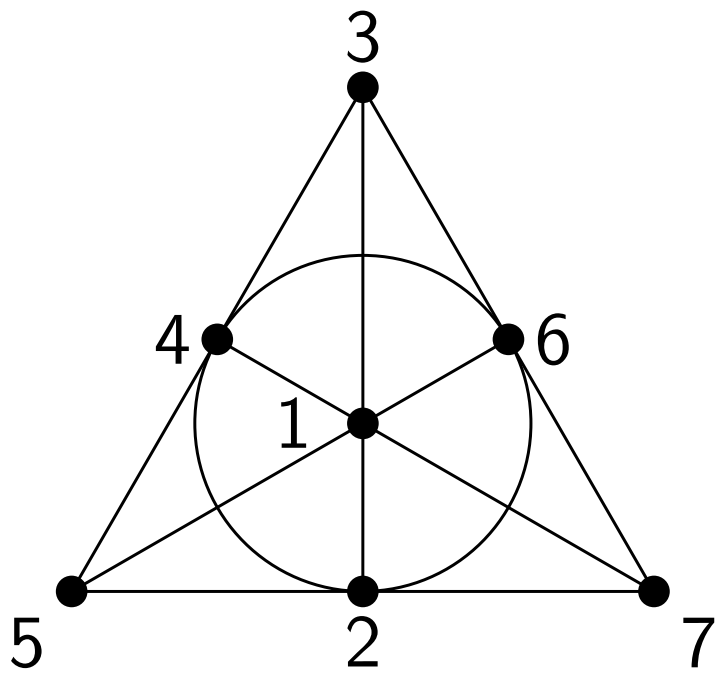
The Fano plane $S(2, 3, 7)$:



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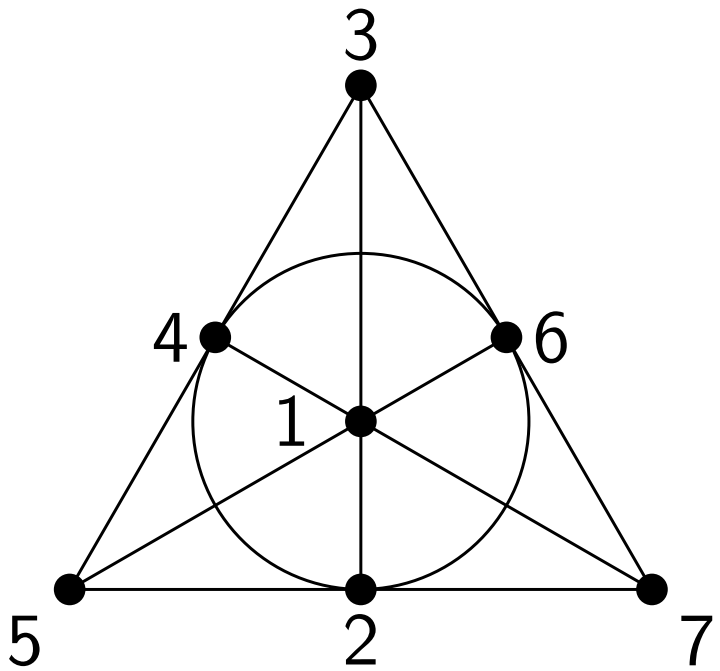

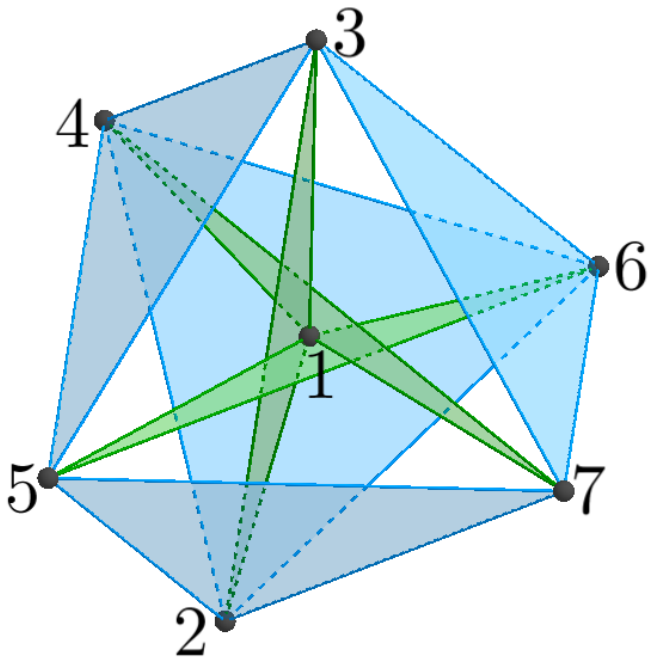

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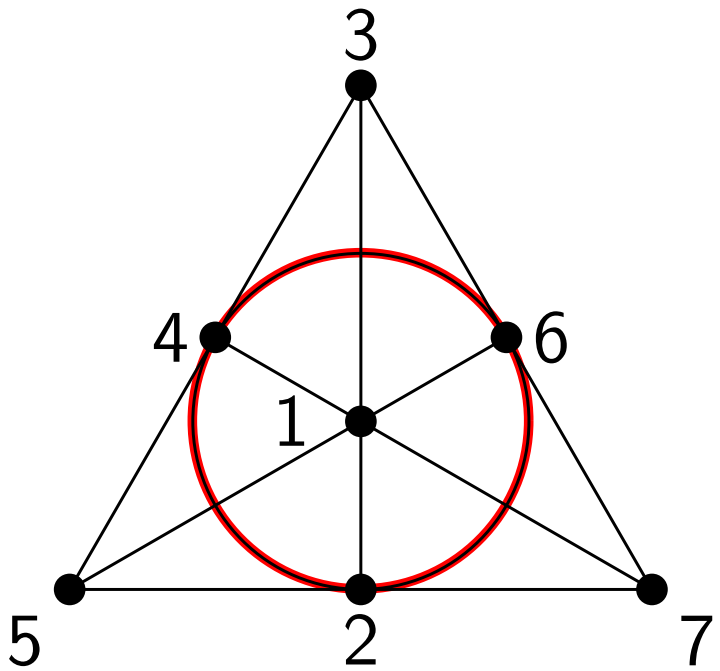

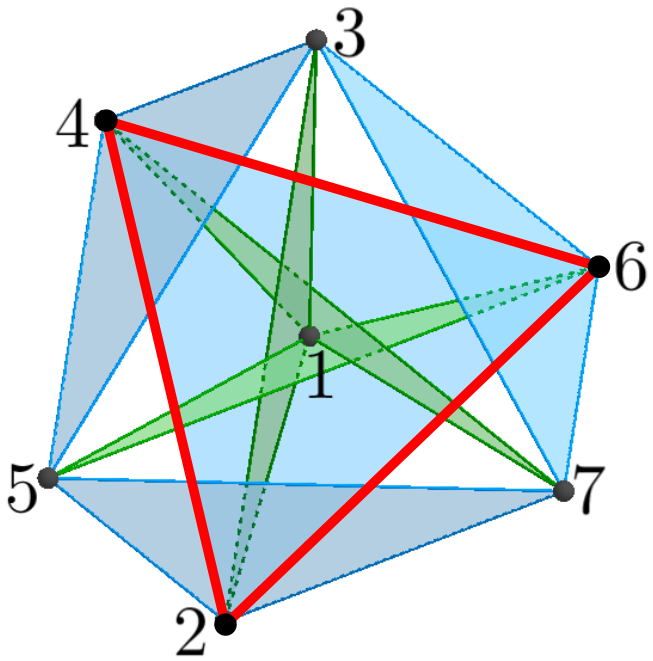

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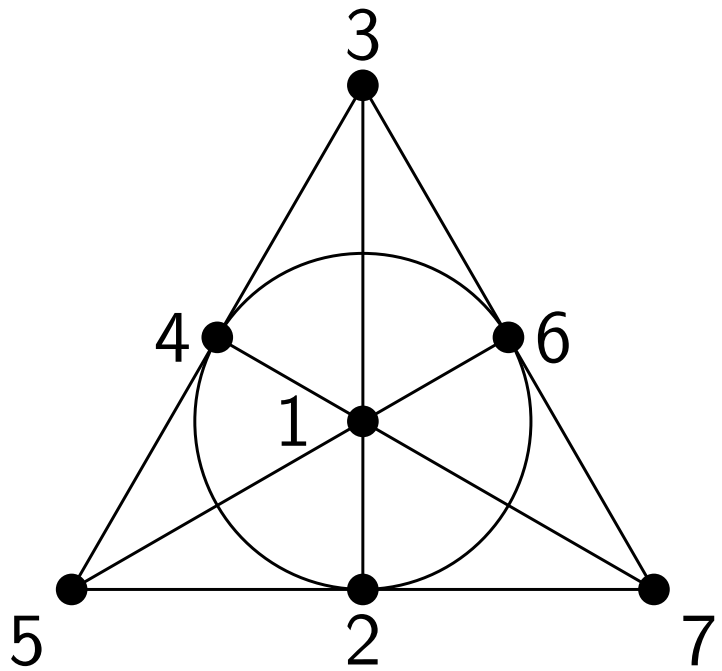

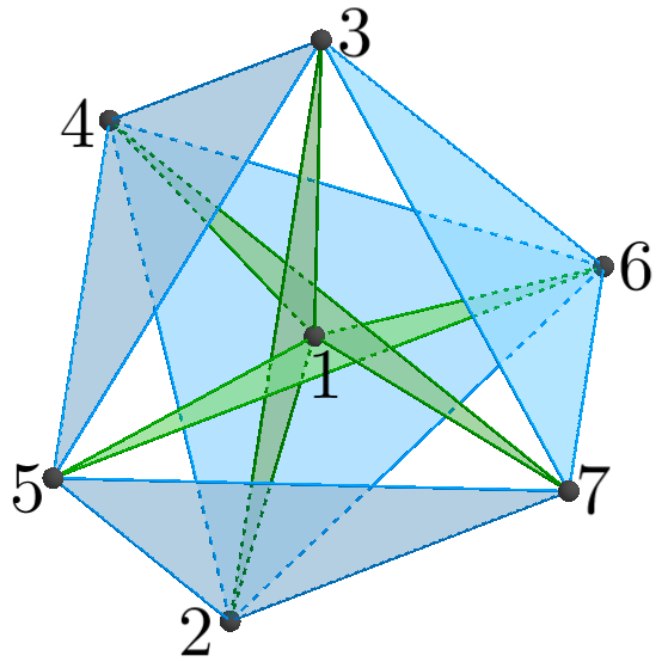

Theorem. [Carmesin/Pardon]

For any $n \geq 6$, the 3-uniform complete hypergraph with n vertices, $\mathcal{K}_n^3$, *cannot* be realized by touching triangles in 3D.

Contact Representations for Hypergraphs in 3D

The Steiner triple system $S(2, 3, 9)$:

1 2 3	1 5 9
4 5 6	2 6 7
7 8 9	3 4 8
1 4 7	1 6 8
2 5 8	2 4 9
3 6 9	3 5 7

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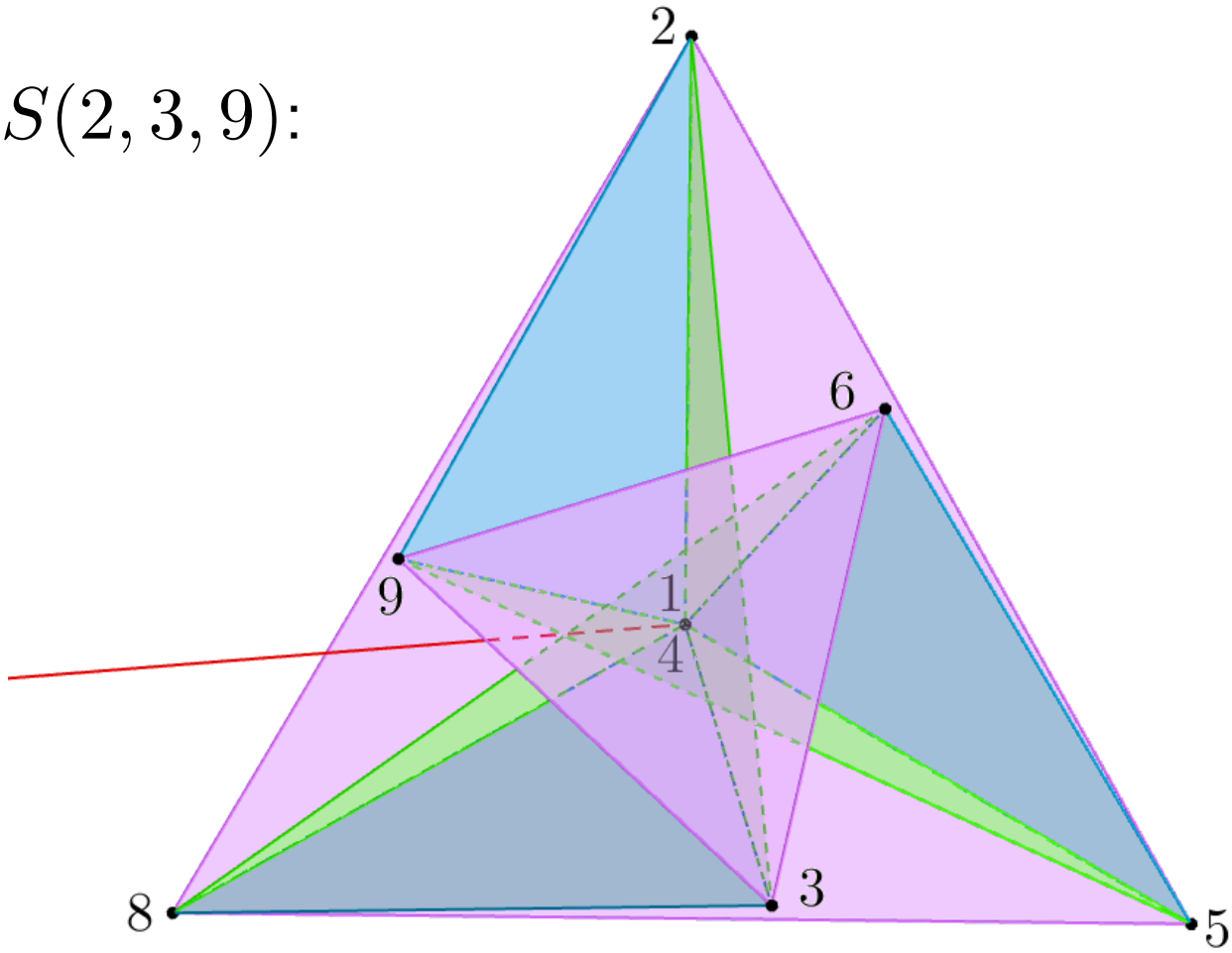

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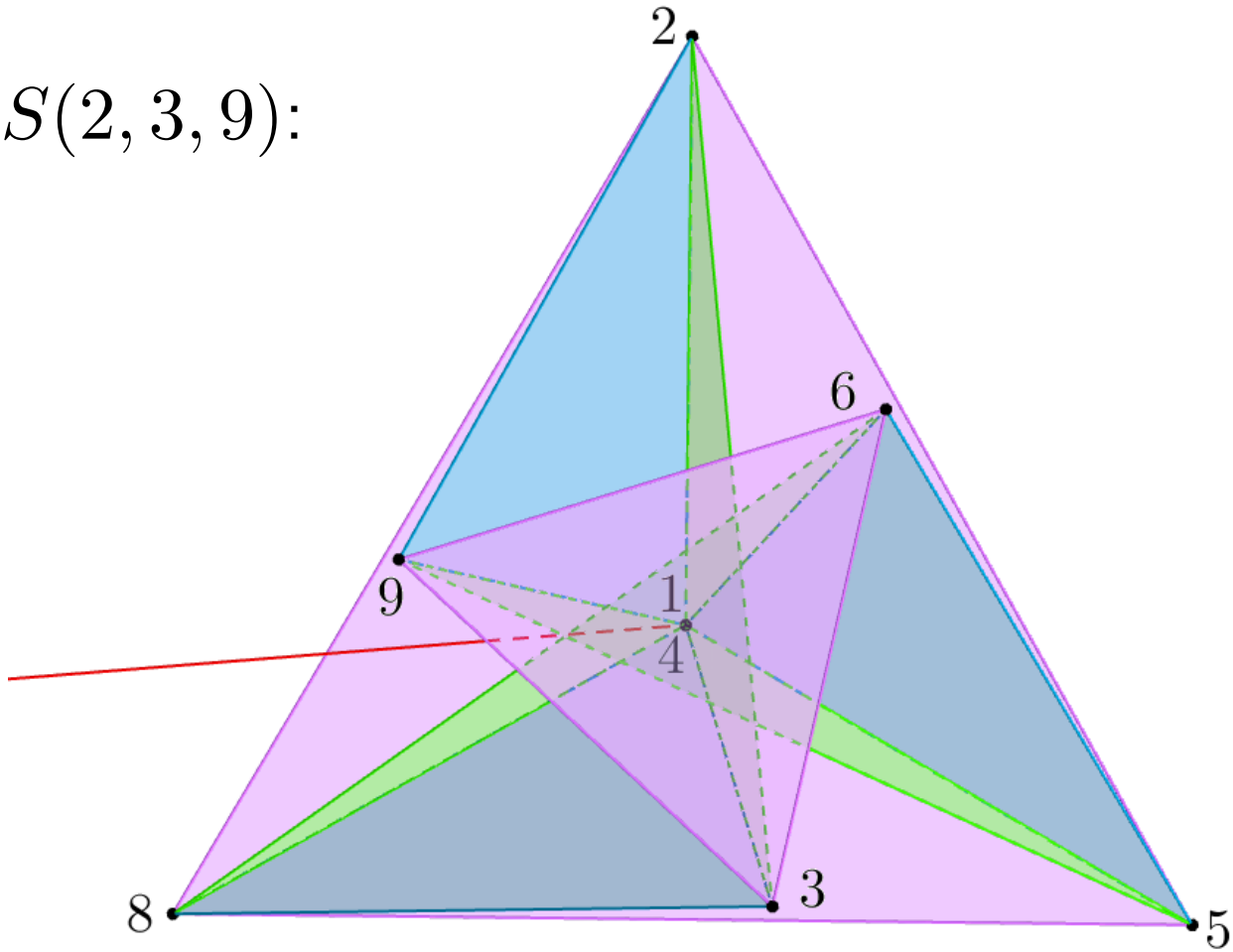

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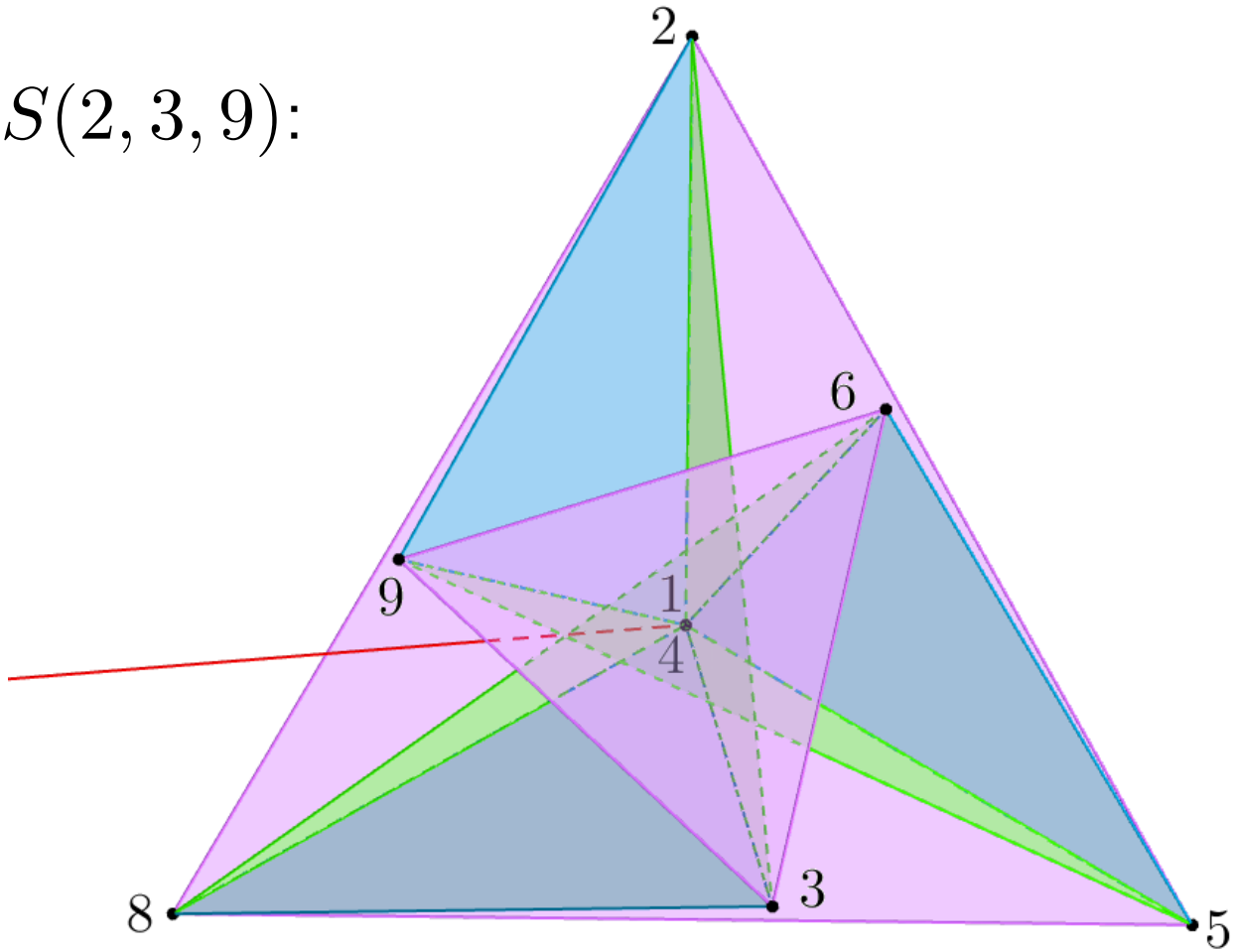

Contact Representations for Hypergraphs in 3D

The Steiner triple system $S(2, 3, 9)$:

1 2 3	1 5 9
4 5 6	2 6 7
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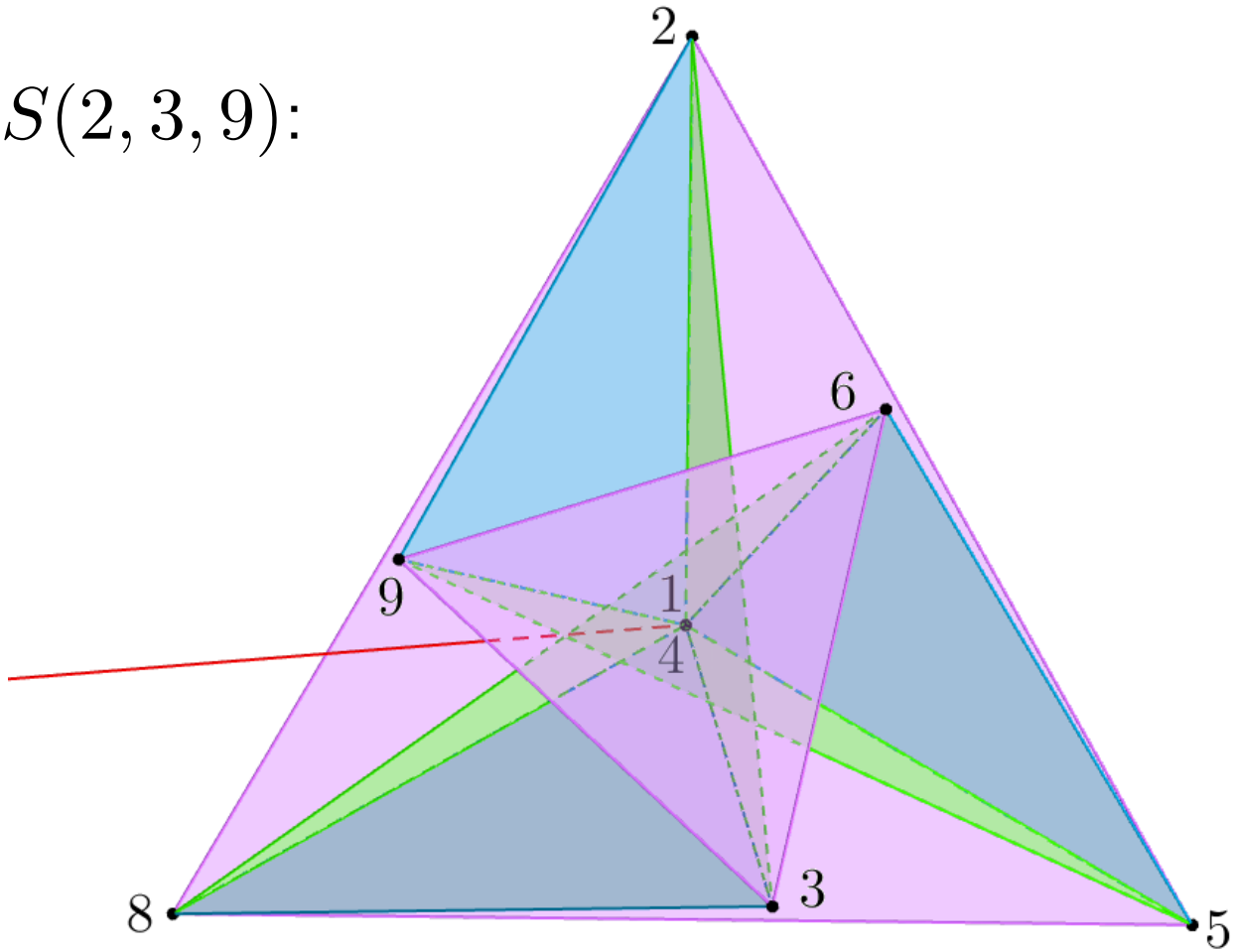

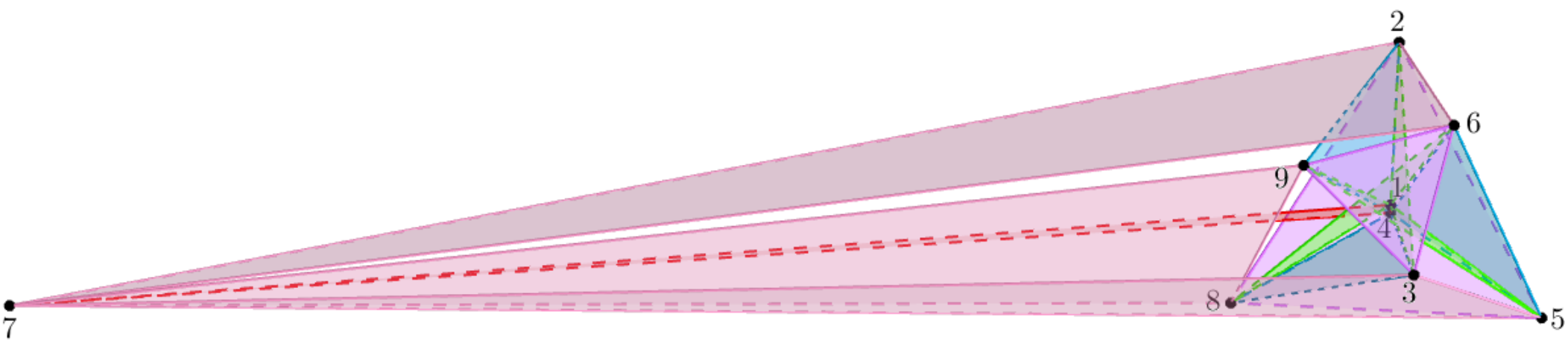

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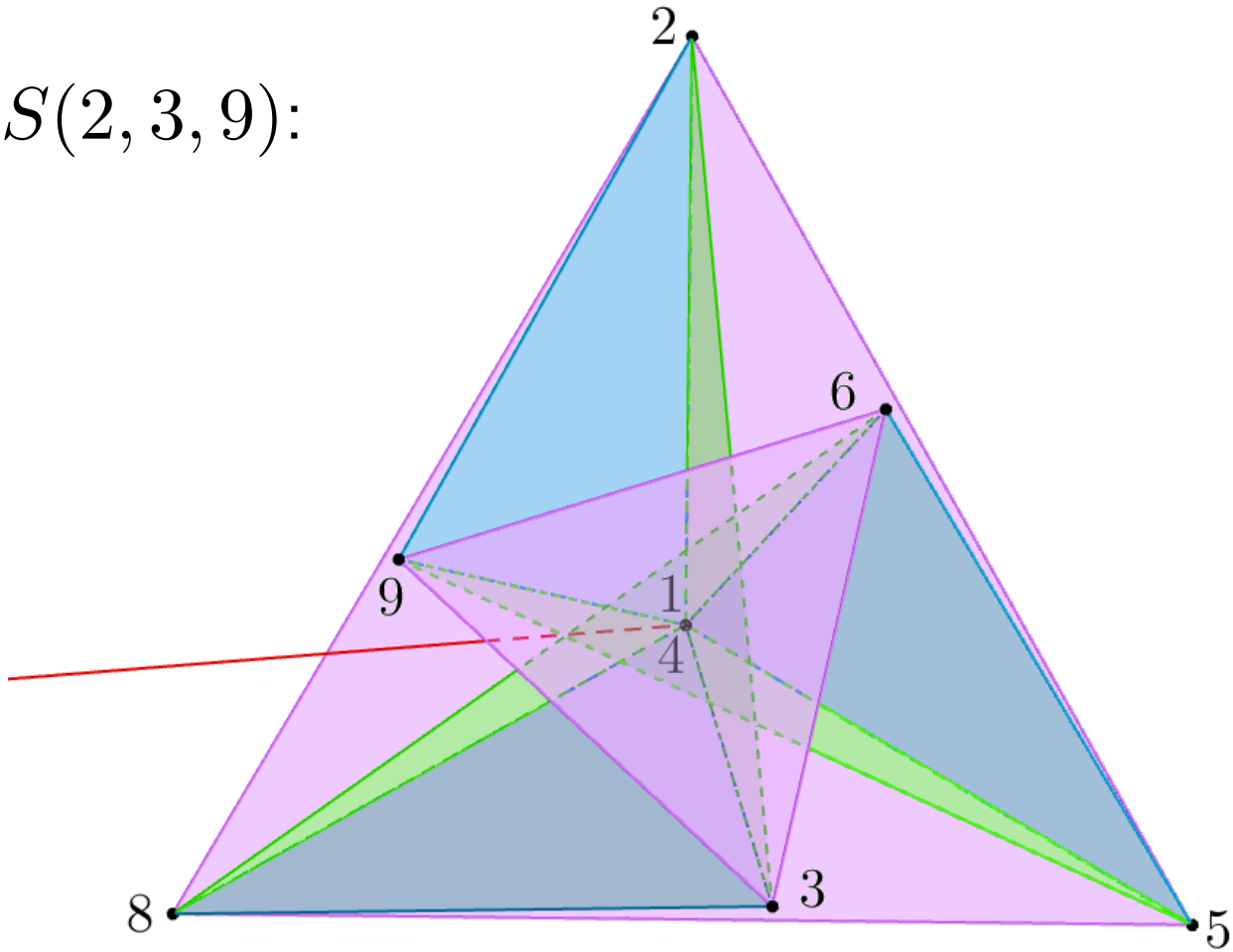
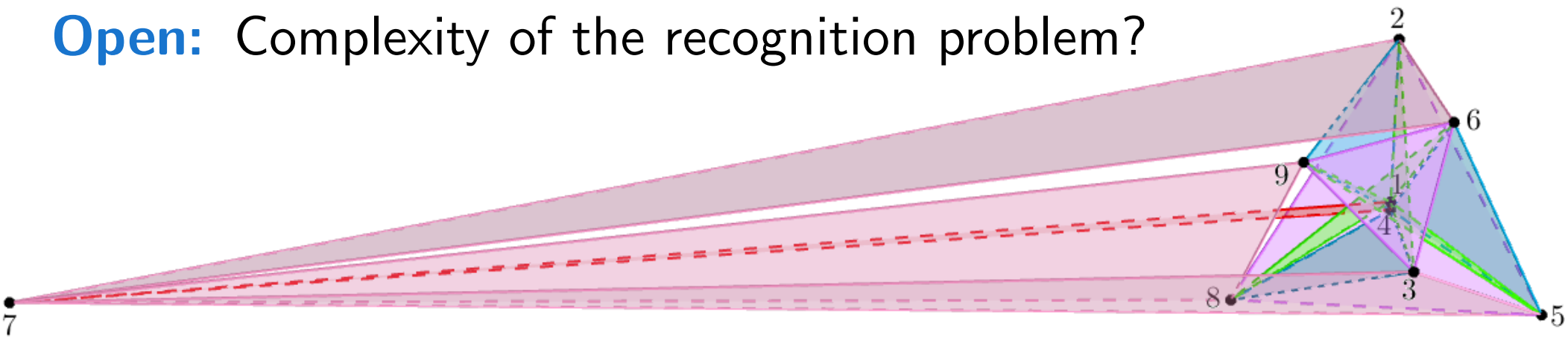


Open: Complexity of the recognition problem?



Steiner Quadruple Systems

$S(3,4,8)$	
1 2 4 8	3 5 6 7
2 3 5 8	1 4 6 7
3 4 6 8	1 2 5 7
4 5 7 8	1 2 3 6
1 5 6 8	2 3 4 7
2 6 7 8	1 3 4 5
1 3 7 8	2 4 5 6

Steiner Quadruple Systems

$S(3,4,8)$	
1 2 4 8	3 5 6 7
2 3 5 8	1 4 6 7
3 4 6 8	1 2 5 7
4 5 7 8	1 2 3 6
1 5 6 8	2 3 4 7
2 6 7 8	1 3 4 5
1 3 7 8	2 4 5 6

Theorem. The Steiner quadruple system $S(3,4,8)$ does not admit a contact representation by quadrilaterals.

Steiner Quadruple Systems

$S(3,4,8)$	
1 2 4 8	3 5 6 7
2 3 5 8	1 4 6 7
3 4 6 8	1 2 5 7
4 5 7 8	1 2 3 6
1 5 6 8	2 3 4 7
2 6 7 8	1 3 4 5
1 3 7 8	2 4 5 6

$S(3,4,10)$		
1 2 4 5	1 2 3 7	1 3 5 8
2 3 5 6	2 3 4 8	2 4 6 9
3 4 6 7	3 4 5 9	3 5 7 0
4 5 7 8	4 5 6 0	1 4 6 8
5 6 8 9	1 5 6 7	2 5 7 9
6 7 9 0	2 6 7 8	3 6 8 0
1 7 8 0	3 7 8 9	1 4 7 9
1 2 8 9	4 8 9 0	2 5 8 0
2 3 9 0	1 5 9 0	1 3 6 9
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1 3 7 8	2 4 5 6

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3 4 6 7	3 4 5 9	3 5 7 0
4 5 7 8	4 5 6 0	1 4 6 8
5 6 8 9	1 5 6 7	2 5 7 9
6 7 9 0	2 6 7 8	3 6 8 0
1 7 8 0	3 7 8 9	1 4 7 9
1 2 8 9	4 8 9 0	2 5 8 0
2 3 9 0	1 5 9 0	1 3 6 9
1 3 4 0	1 2 6 0	2 4 7 0

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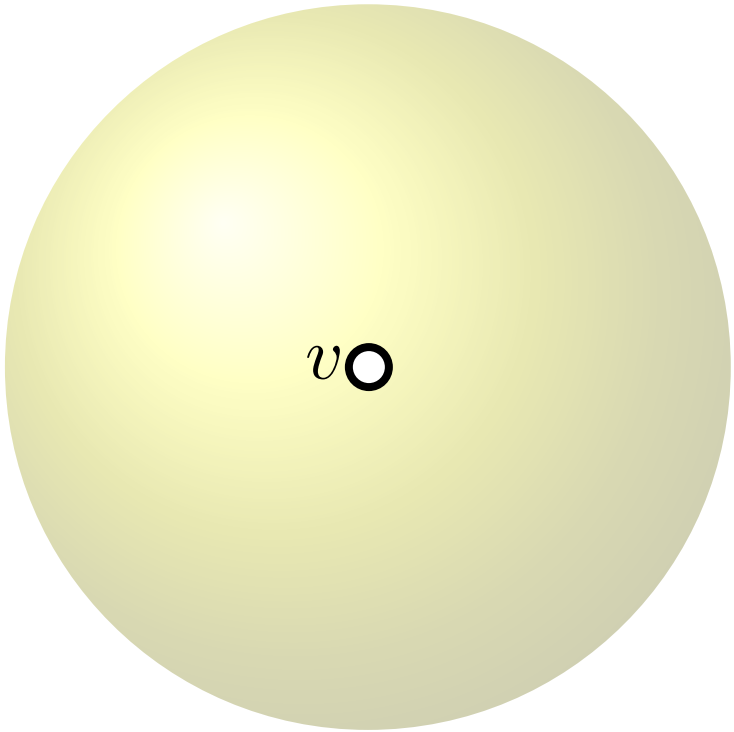
Theorem. No Steiner quadruple system admits a contact representation by *convex* quadrilaterals.

Steiner Quadruple Systems



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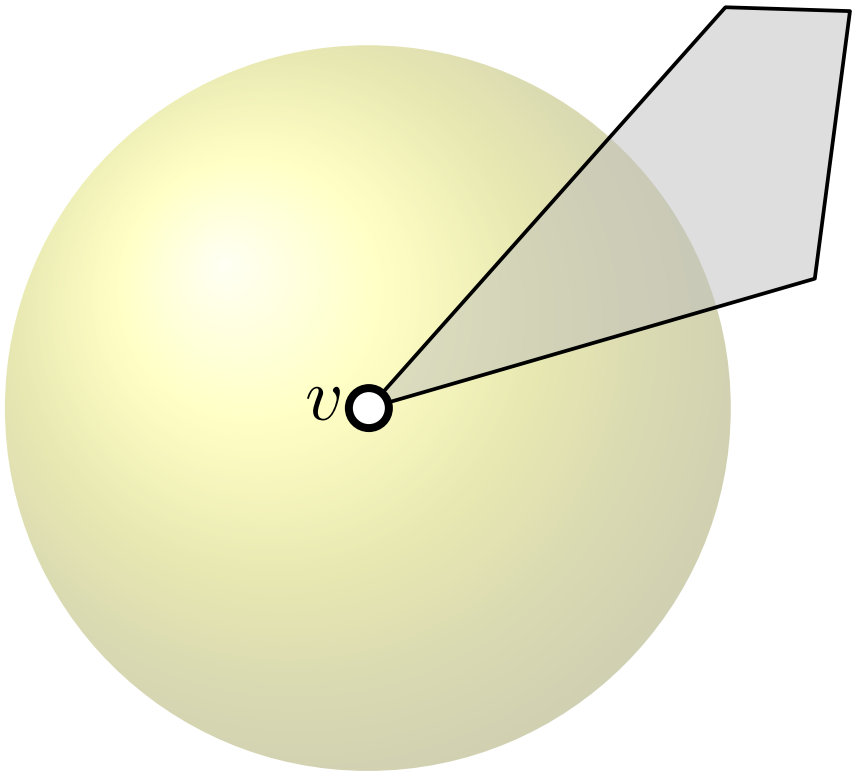

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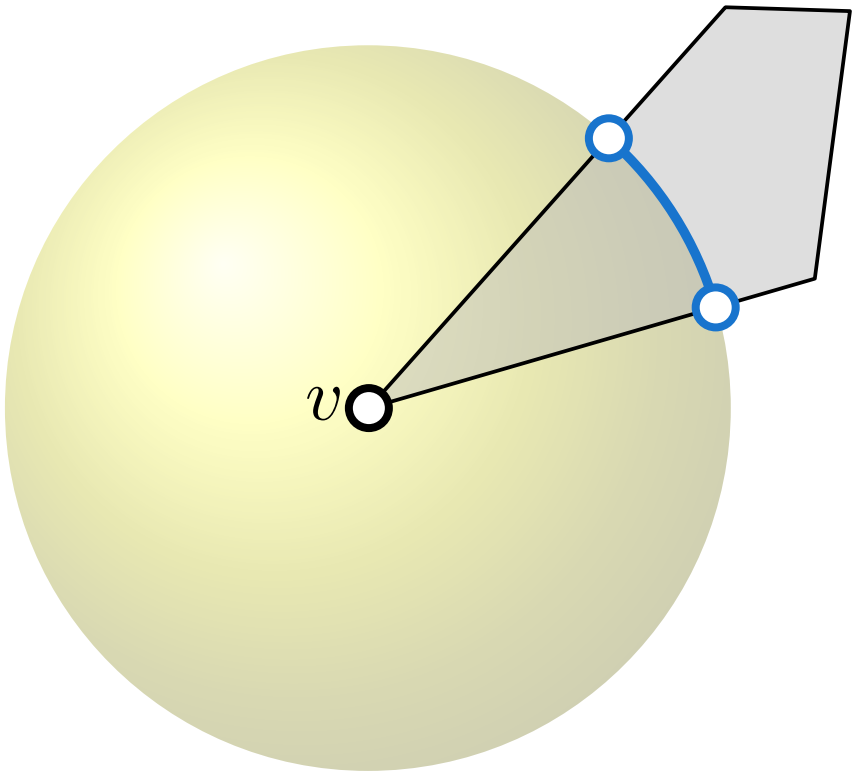
Steiner Quadruple Systems

Dey and Edelsbrunner [DCG'94]



Link graph of v:

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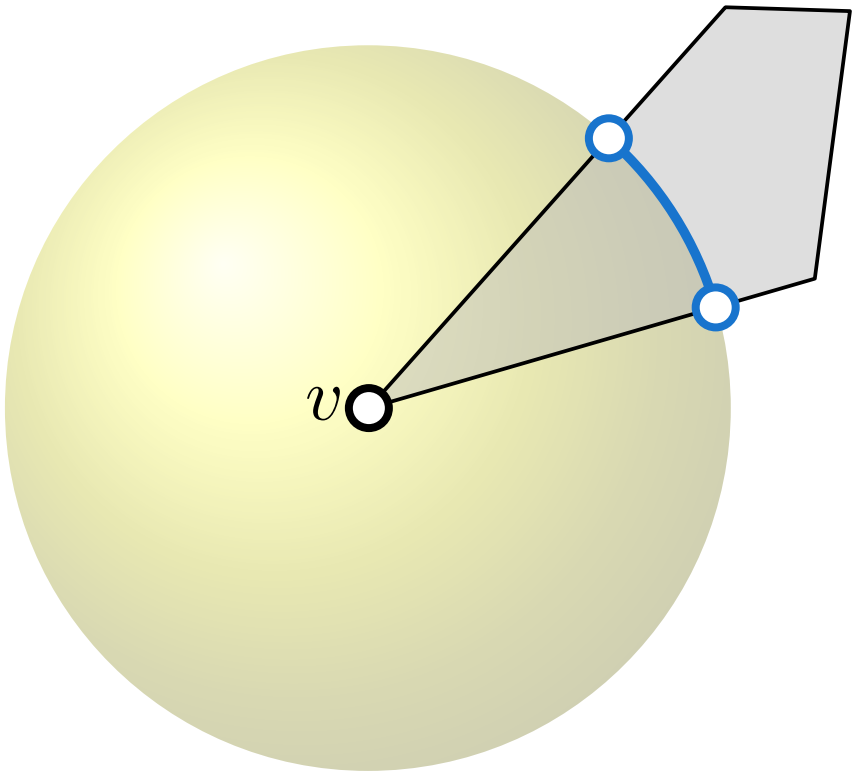

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– edge for each hyperedge

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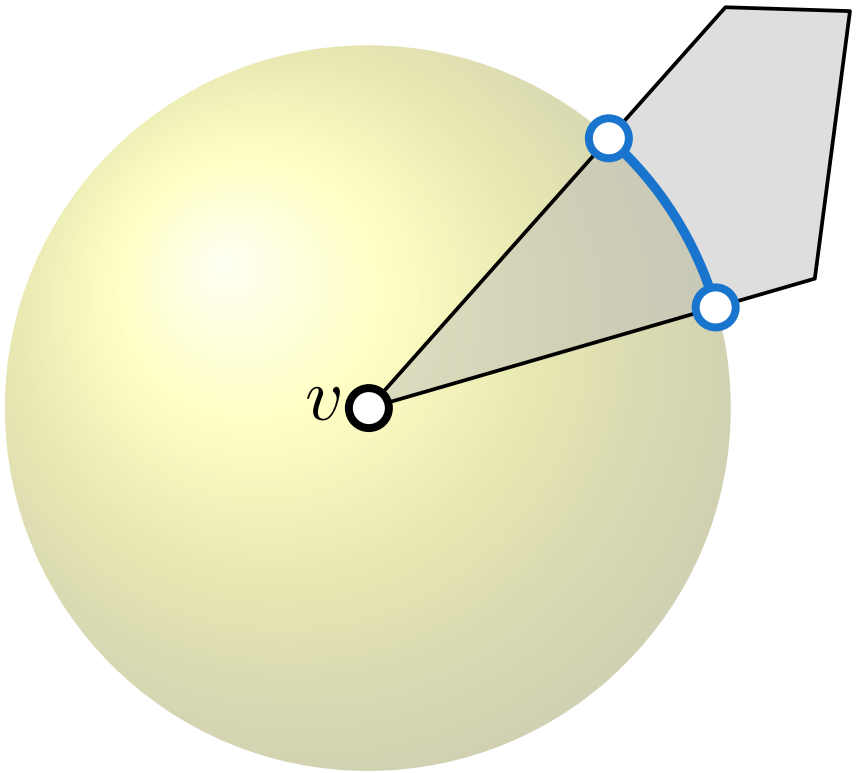

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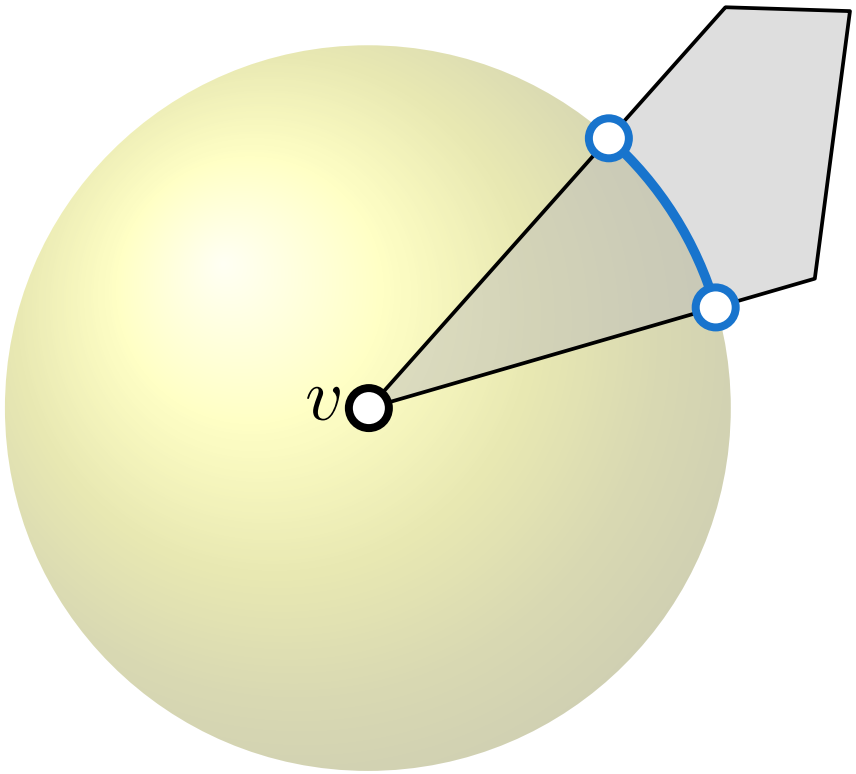

Link graph of v:

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Here:

$\#$ vertices $= n - 1$

$\#$ edges $= (n-1)(n-2)/6$

Trick: split quadrilaterals!

$\Rightarrow$ $\#$ edges $= (n-1)(n-2)/3$

$\Rightarrow$ link graph not planar for $n > 8$.

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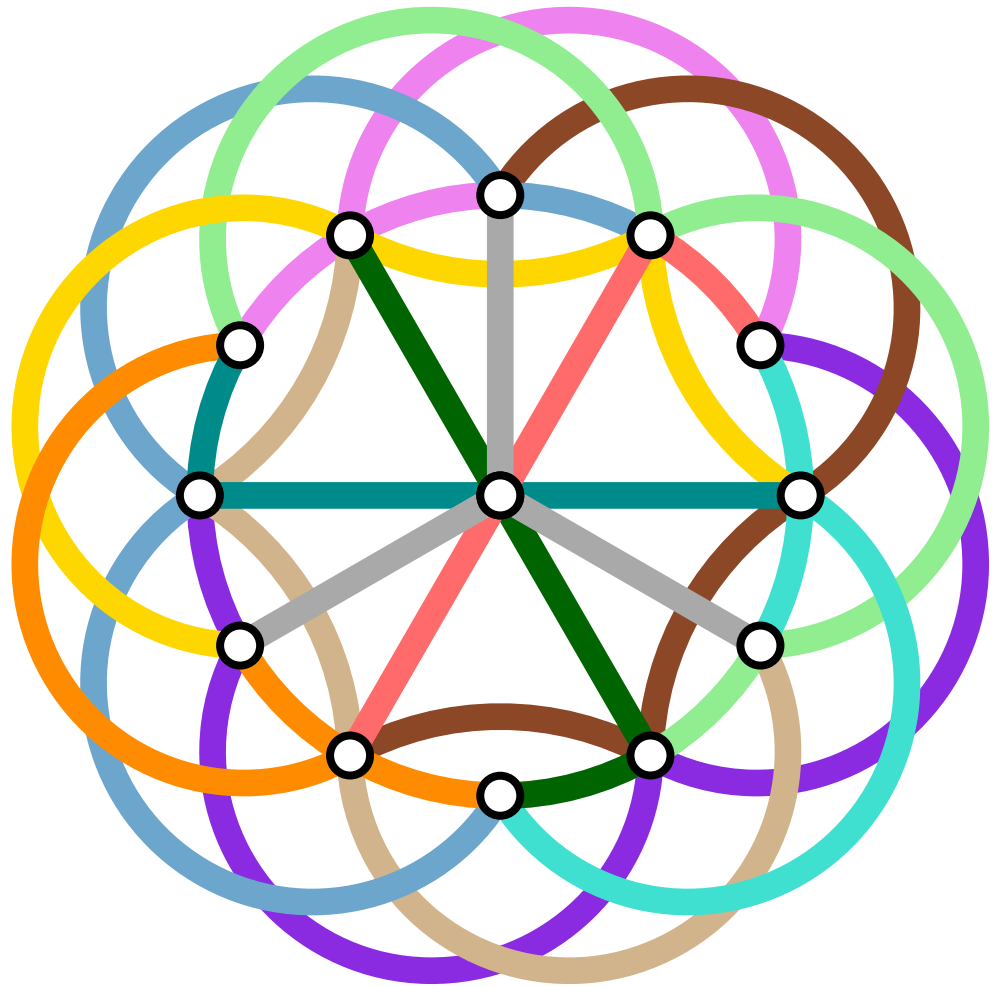
Discrete Projective Plane

Open: Can the second smallest projective plane, $PG(3)$ $(= S(2, 4, 13))$, be represented by touching (convex) quadrilaterals?

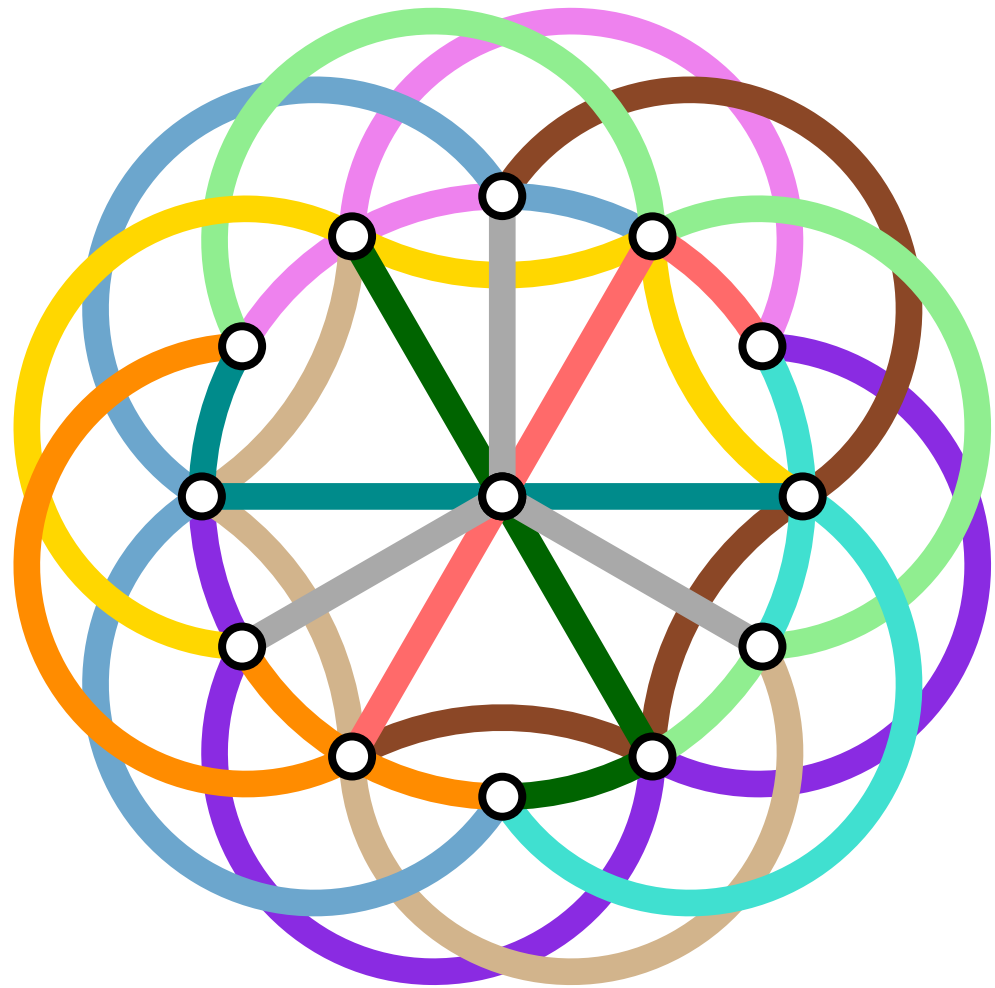
A	B	C	D
A	1	2	3
A	4	5	6
A	7	8	9
B	1	4	7
B	2	5	8
B	3	6	9
C	1	5	9
C	2	6	7
C	3	4	8
D	1	6	8
D	2	4	9
D	3	5	7



Discrete Projective Plane

Open: Can the second smallest projective plane, $PG(3)$ $(= S(2, 4, 13))$, be represented by touching
– (convex) quadrilaterals?
– tetrahedra?

A	B	C	D
A	1	2	3
A	4	5	6
A	7	8	9
B	1	4	7
B	2	5	8
B	3	6	9
C	1	5	9
C	2	6	7
C	3	4	8
D	1	6	8
D	2	4	9
D	3	5	7



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  - clustered planarity
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  - right-angle-crossing drawings
  - universal point sets
  - topological drawings
  - representation as contact/intersection graphs
  - layered drawings
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- So I hope to talk to you more about Graph Drawing – maybe at the Winter School in Teheran next year?