Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Drawing Graphs on Few Circles and Few Spheres

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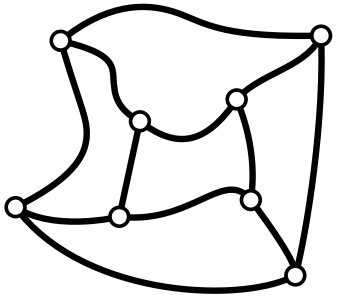
Motivation

Given
a planar
graph,...

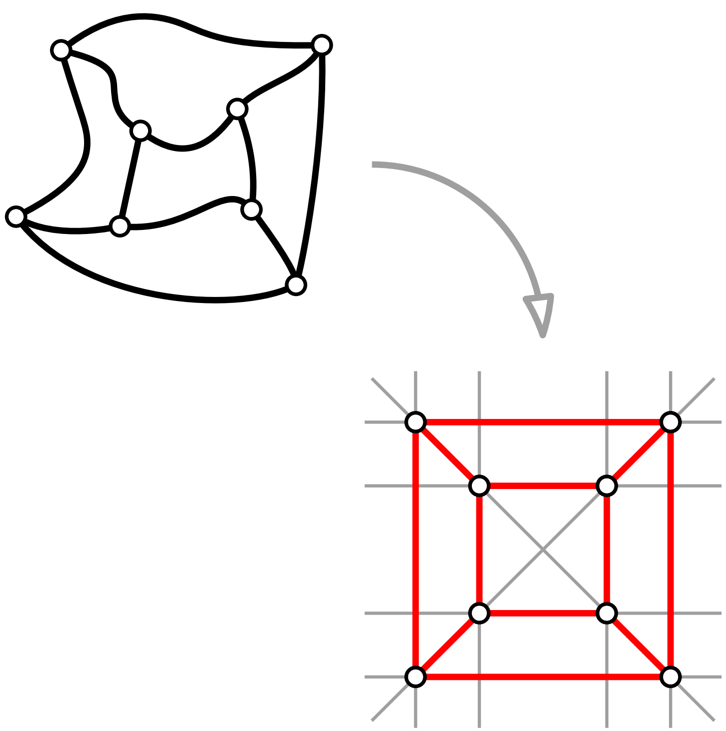


Motivation

[Chaplick et al., 2016]

Given a planar graph,...

...find a straight-line drawing with as few lines as possible that together cover the drawing.



10 lines

Motivation

[Chaplick et al., 2016]

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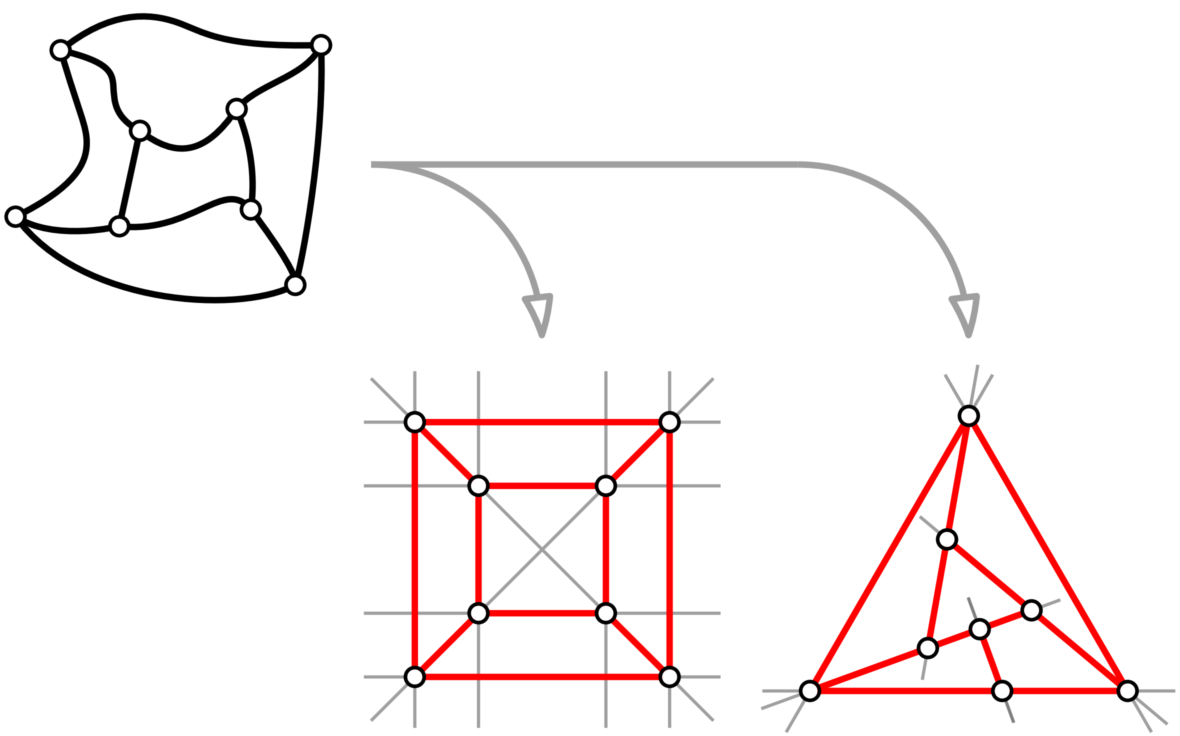


10 lines

7 lines

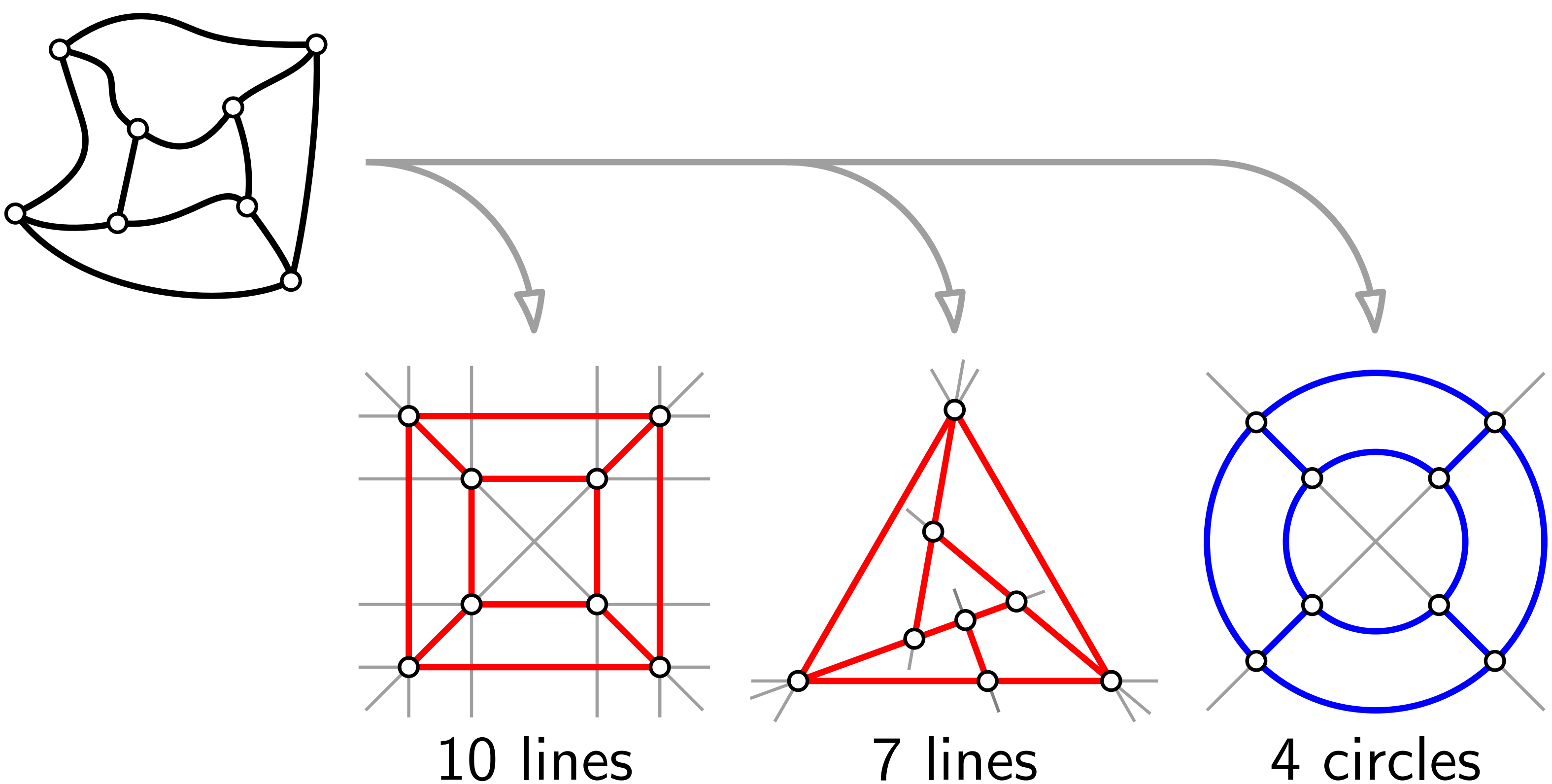
Motivation

[Chaplick et al., 2016]

Given a planar graph,...

...find a straight-line drawing with as few lines as possible that together cover the drawing.

...find a circular-arc drawing with as few circles as possible that together cover the drawing.



Motivation

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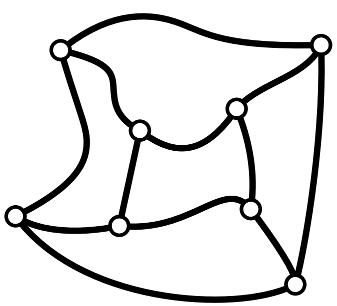
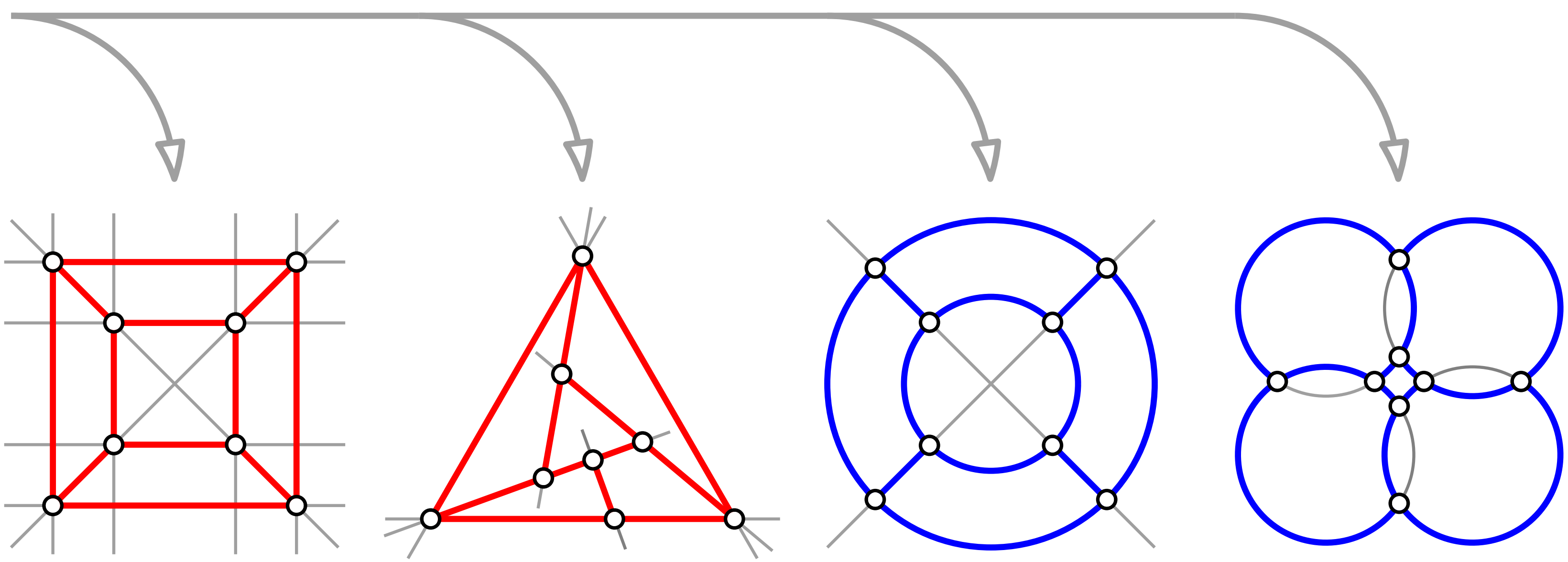
Given a planar graph,...	...find a straight-line drawing with as few lines as possible that together cover the drawing.	...find a circular-arc drawing with as few circles as possible that together cover the drawing.





10 lines 7 lines 4 circles 4 circles

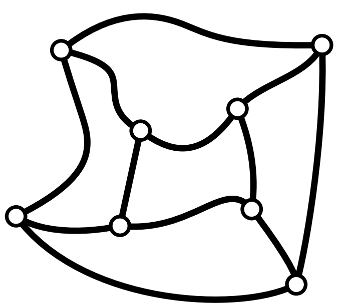
Motivation

[Chaplick et al., 2016]

Given a planar graph,...

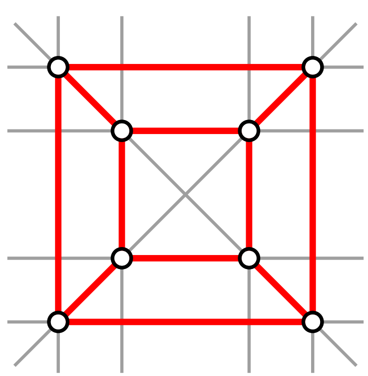
...find a straight-line drawing with as few lines as possible that together cover the drawing.

...find a circular-arc drawing with as few circles as possible that together cover the drawing.

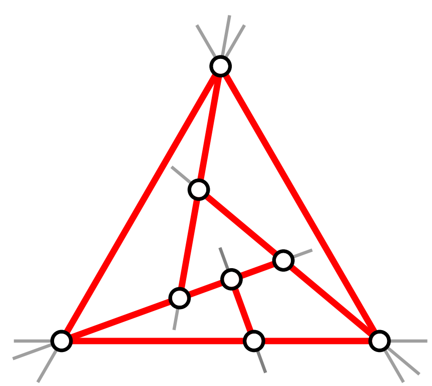


Advantages:

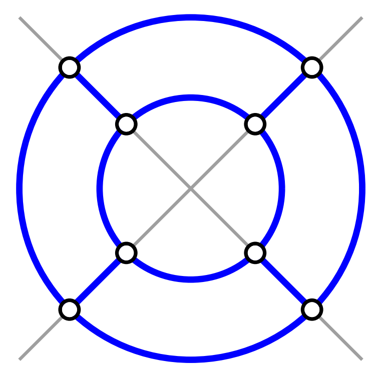
- Smaller visual complexity



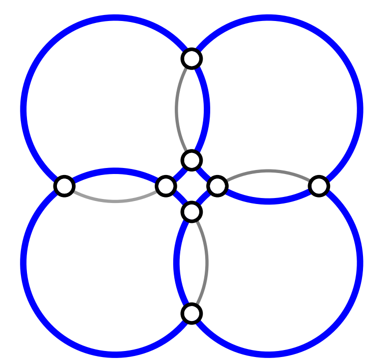
10 lines



7 lines



4 circles



4 circles

Motivation

[Chaplick et al., 2016]

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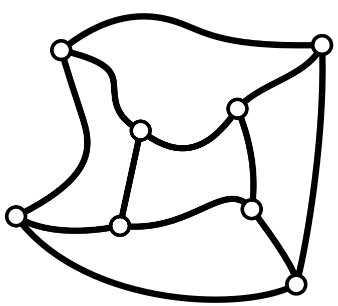
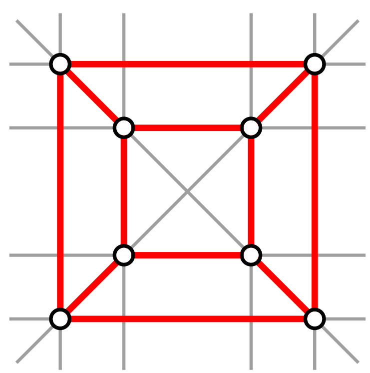
...find a straight-line drawing with as few lines as possible that together cover the drawing.

...find a circular-arc drawing with as few circles as possible that together cover the drawing.

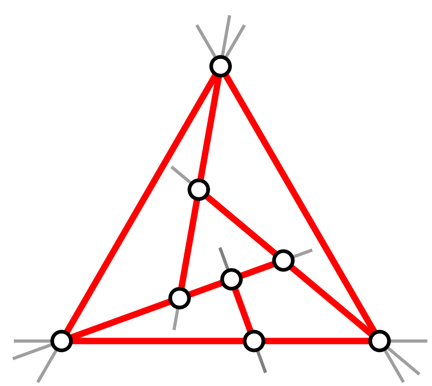


Advantages:

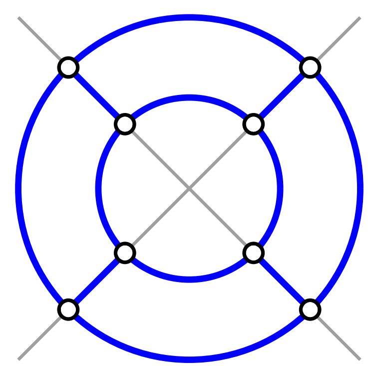
- Smaller visual complexity
- Better reflects symmetry



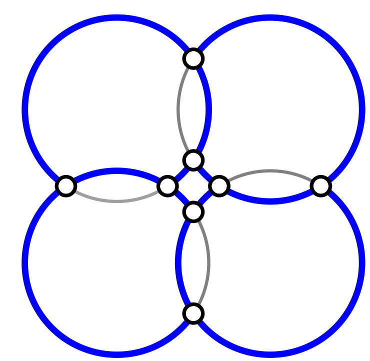
10 lines



7 lines



4 circles



4 circles

Outline

Affine Covers* & Spherical Covers

[* Chaplick et al., 2016]

Let G be a graph, and let $1 \leq m < d$.

Def. The *affine cover number* $\rho_d^m(G)$ is the minimum number of m-dimensional hyperplanes in $\mathbb{R}^d$ such that G has a crossing-free straight-line drawing that is contained in these planes.

Affine Covers[*] & Spherical Covers

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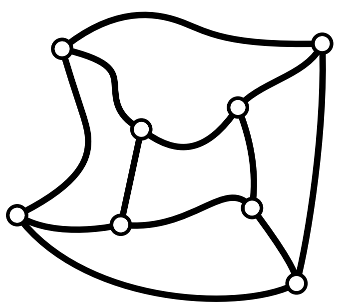

$\rho_2^1(cube) =$

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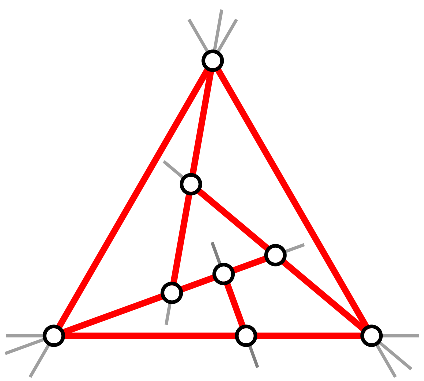

$$\rho_2^1(cube) = 7$$

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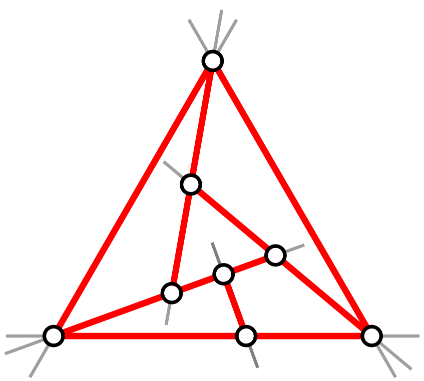

$$\rho_2^1(cube) = 7$$

Def. The *spherical cover number* $\sigma_d^m(G)$ is the minimum number of m-dimensional spheres in $\mathbb{R}^d$ such that G has a crossing-free circular-arc drawing that is contained in these spheres.

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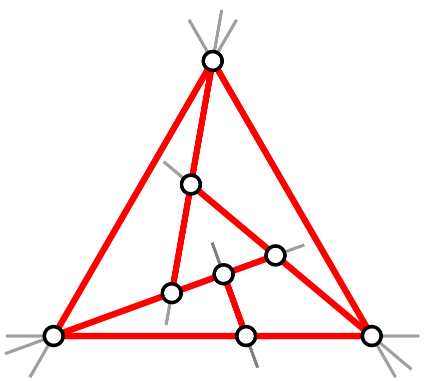

$\rho_2^1(cube) = 7$

$\sigma_2^1(cube) =$

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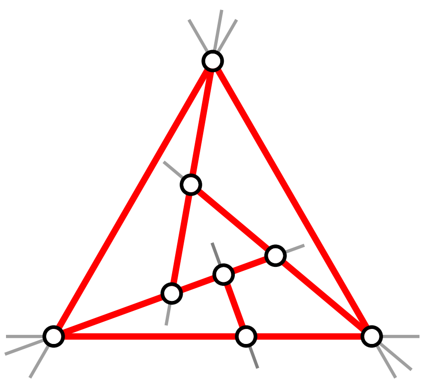
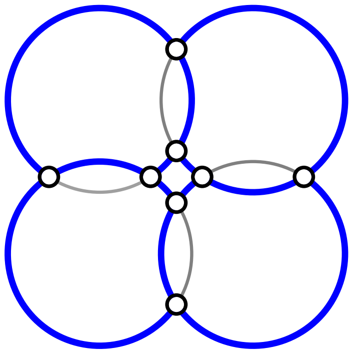


$\rho_2^1(cube) = 7$



$\sigma_2^1(cube) = 4$

Def. The *spherical cover number* $\sigma_d^m(G)$ is the minimum number of m-dimensional spheres in $\mathbb{R}^d$ such that G has a crossing-free circular-arc drawing that is contained in these spheres.

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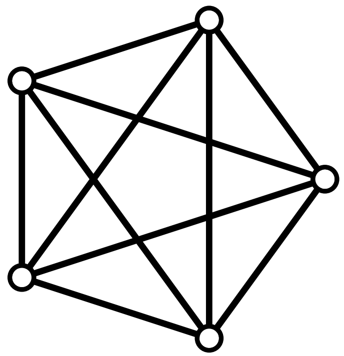

$\rho_3^2(K_5) =$

Def. The *spherical cover number* $\sigma_d^m(G)$ is the minimum number of m-dimensional spheres in $\mathbb{R}^d$ such that G has a crossing-free circular-arc drawing that is contained in these spheres.

Affine Covers[*] & Spherical Covers

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Let G be a graph, and let $1 \leq m < d$.

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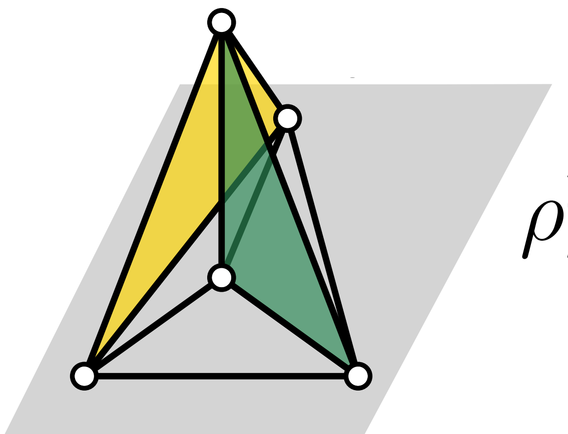

$\rho_3^2(K_5) = 3$

Def. The *spherical cover number* $\sigma_d^m(G)$ is
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Affine Covers* & Spherical Covers

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Let G be a graph, and let $1 \leq m < d$.

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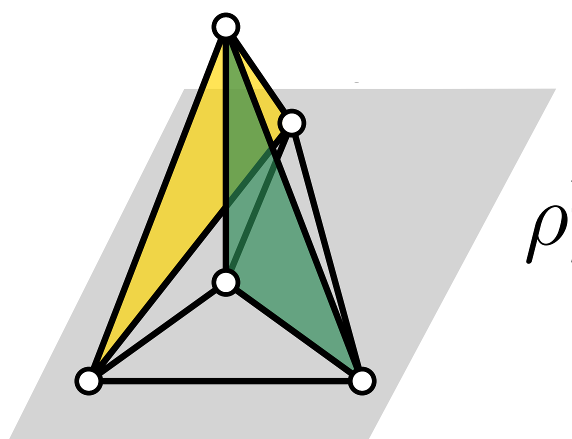
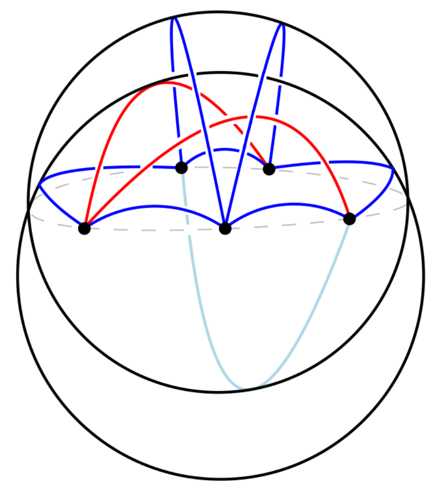


$\rho_3^2(K_5) = 3$



$\sigma_3^2(K_5) = 2.$

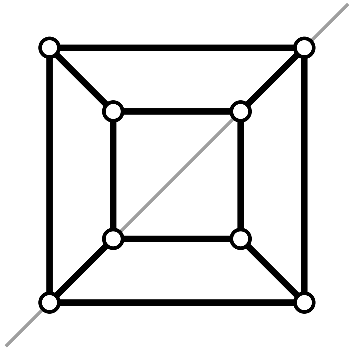
Def. The *spherical cover number* $\sigma_d^m(G)$ is the minimum number of m-dimensional spheres in $\mathbb{R}^d$ such that G has a crossing-free circular-arc drawing that is contained in these spheres.

Segment Number and Arc Number

Def. The *segment number* of G, $\mathrm{seg}(G)$, is the minimum number of line segments formed by the edges of G in a straight-line drawing.

[Dujmović, Eppstein, Suderman, Wood CGTA'07]

1 line,
2 segments



Segment Number and Arc Number

Def. The *segment number* of G, seg(G), is the minimum number of line segments formed by the edges of G in a straight-line drawing. [Dujmović, Eppstein, Suderman, Wood CGTA'07]

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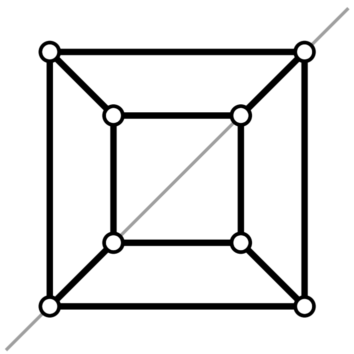


Def. The arc number *of* G, arc(G), is the minimum number of arcs formed by the edges of G in a circular-arc drawing. [Schulz JGAA'15]

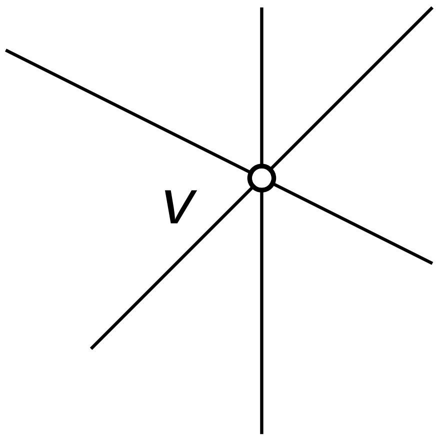
Outline

Combinatorial Lower Bounds on ρ^1_2 and σ^1_2

[Chaplick et al., 2016]

Let G be a graph.

Obs. 1 Any vertex v of G lies on $\geq \lceil \deg(v)/2 \rceil$ lines.

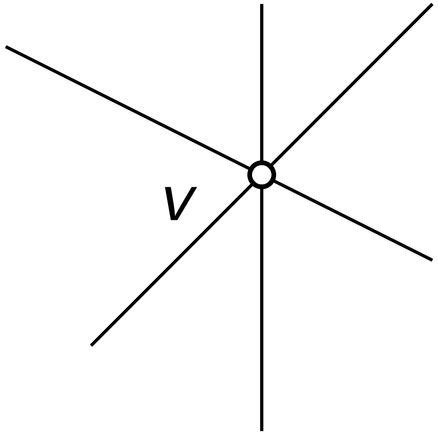


Combinatorial Lower Bounds on ρ_2^1 and σ_2^1

[Chaplick et al., 2016]

Let G be a graph.

Obs. 1 Any vertex v of G lies on $\geq \lceil \deg(v)/2 \rceil$ lines.



$$\implies \binom{\rho_2^1(G)}{2} \geq \sum_{v \in V(G)} \binom{\left\lceil \frac{\deg v}{2} \right\rceil}{2}$$

Combinatorial Lower Bounds on ρ_2^1 and σ_2^1

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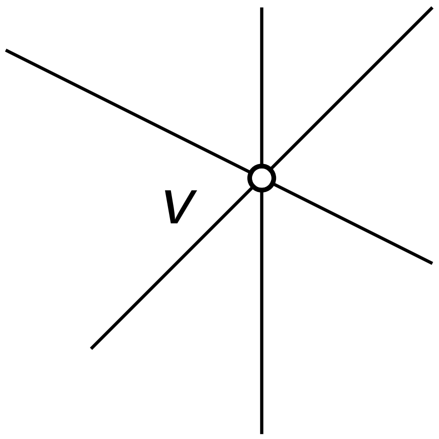
Obs. 1 Any vertex v of G lies on $\geq \lceil \deg(v)/2 \rceil$ lines.



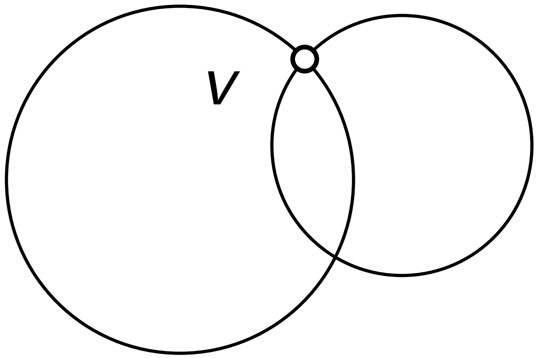
$$\implies \binom{\rho_2^1(G)}{2} \geq \sum_{v \in V(G)} \binom{\left\lceil \frac{\deg v}{2} \right\rceil}{2}$$

$$\implies \rho_2^1(G) \geq \frac{1}{2}\left(1 + \sqrt{1 + 8 \sum_{v \in V(G)} \binom{\left\lceil \frac{\deg v}{2} \right\rceil}{2}}\right)$$

Combinatorial Lower Bounds on ρ_2^1 and σ_2^1

Let G be a graph.

Obs. 2 Any vertex v of G lies on $\geq \lceil \deg(v)/2 \rceil$ circles.



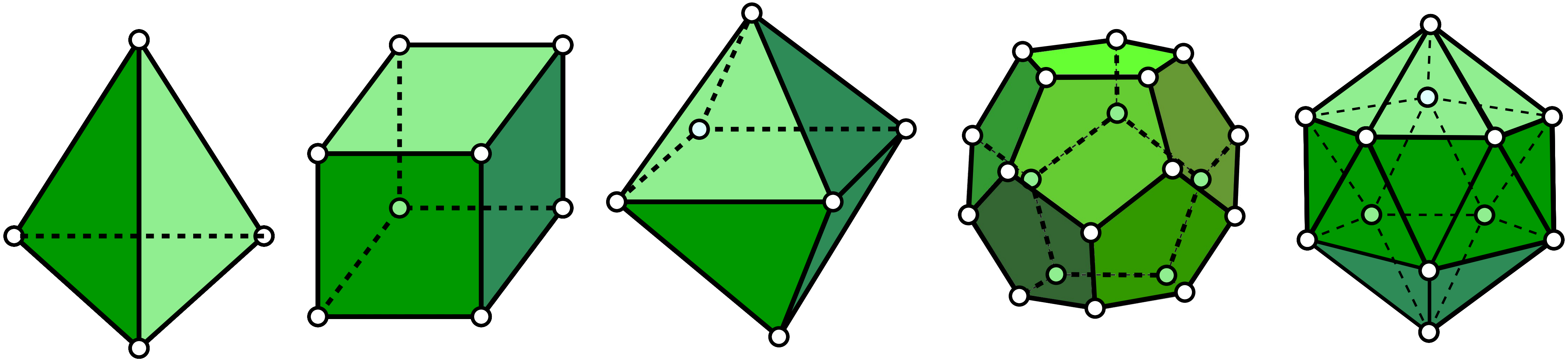
$$\Longrightarrow 2\binom{\sigma_2^1(G)}{2} \geq \sum_{v \in V(G)} \binom{\left\lceil \frac{\deg v}{2} \right\rceil}{2}$$

$$\Longrightarrow \sigma_2^1(G) \geq \frac{1}{2}\left(1 + \sqrt{1 + 4 \sum_{v \in V(G)} \binom{\left\lceil \frac{\deg v}{2} \right\rceil}{2}}\right)$$

Outline

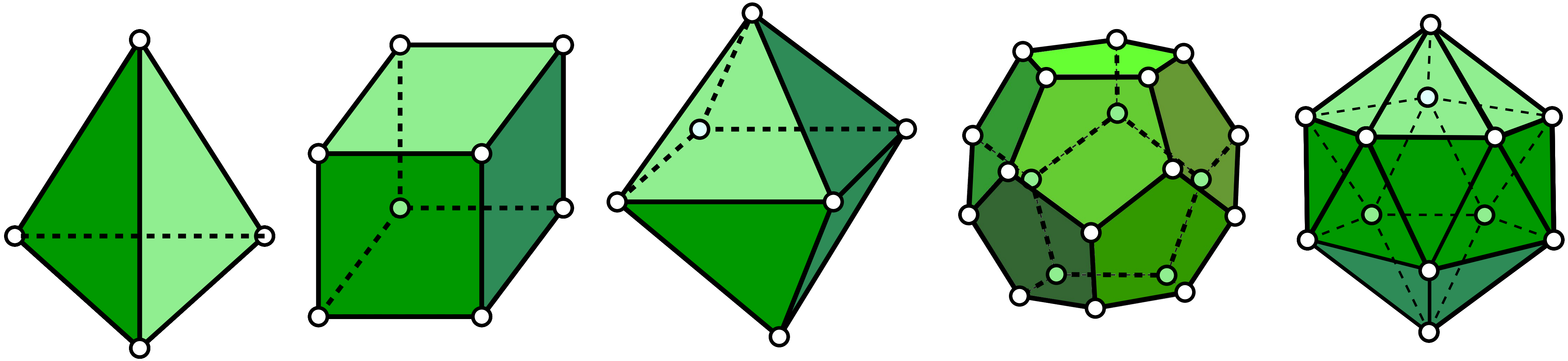
Platonic Solids: Affine Cover Numbers

$G = (V, E)$	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert F \rvert$	$\rho_2^1(G)$	seg(G)	$\sigma_2^1(G)$	arc(G)
tetrahedron	4	6	4				
octahedron	6	12	8				
cube	8	12	6				
dodecahedron	20	30	12				
icosahedron	12	30	20				



Platonic Solids: Affine Cover Numbers

$G = (V, E)$	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert F \rvert$	$\rho^1_2(G)$	$\mathrm{seg}(G)$	$\sigma^1_2(G)$	$\mathrm{arc}(G)$
tetrahedron	4	6	4				
octahedron	6	12	8				
cube	8	12	6				
dodecahedron	20	30	12				
icosahedron	12	30	20				



Platonic Solids: Affine Cover Numbers

$G = (V, E)$	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert F \rvert$	$\rho_2^1(G)$	$\text{seg}(G)$	$\sigma_2^1(G)$	$\text{arc}(G)$
tetrahedron	4	6	4				
octahedron	6	12	8				
cube	8	12	6				
dodecahedron	20	30	12				
icosahedron	12	30	20				

Recall Obs. 1:

$$\rho_2^1(G) \geq \frac{1}{2}\left(1 + \sqrt{1 + 8 \sum_{v \in V(G)} \binom{\lceil \frac{\deg v}{2} \rceil}{2}}\right)$$

Platonic Solids: Affine Cover Numbers

$G=(V,E)$	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert F\rvert$	$\rho^1_2(G)$	seg(G)	$\sigma^1_2(G)$	arc(G)
tetrahedron	4	6	4	≥ 4			
octahedron	6	12	8				
cube	8	12	6				
dodecahedron	20	30	12				
icosahedron	12	30	20				

Recall Obs. 1:

$$\rho^1_2(tetrahedron) \geq \frac{1}{2}\left(1+\sqrt{1+8\cdot 4\binom{\lceil \frac{3}{2}\rceil}{2}}\right) \geq 3.37$$

Platonic Solids: Affine Cover Numbers

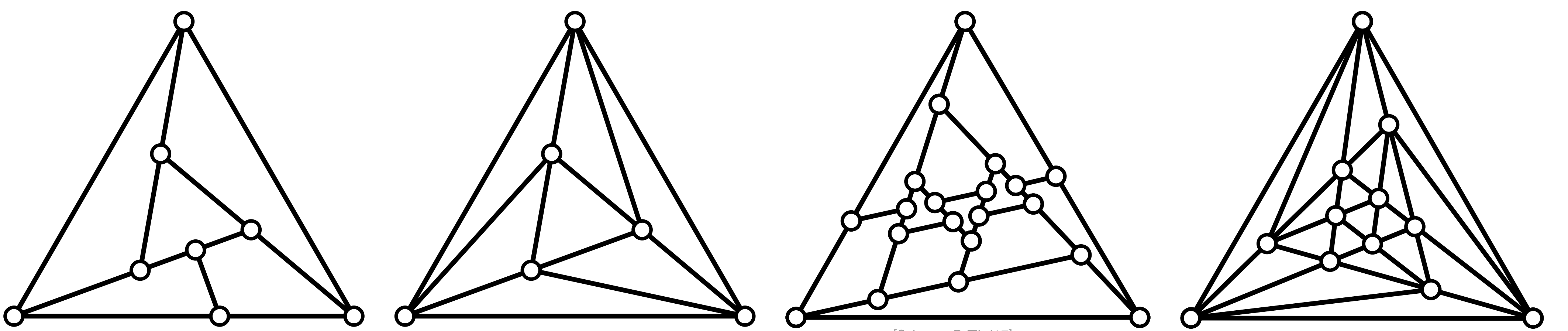
$G=(V,E)$	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert F\rvert$	$\rho^1_2(G)$	seg(G)	$\sigma^1_2(G)$	arc(G)
tetrahedron	4	6	4	≥ 4			
octahedron	6	12	8	≥ 4			
cube	8	12	6	≥ 5			
dodecahedron	20	30	12	≥ 7			
icosahedron	12	30	20	≥ 9			

Recall Obs. 1:

$$\implies \rho^1_2(G) \geq \frac{1}{2}\left(1 + \sqrt{1 + 8 \sum_{v \in V(G)} \binom{\lceil \frac{\deg v}{2} \rceil}{2}}\right)$$

Platonic Solids: Affine Cover Numbers

$G = (V, E)$	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert F \rvert$	$\rho_2^1(G)$	$\text{seg}(G)$	$\sigma_2^1(G)$	$\text{arc}(G)$
tetrahedron	4	6	4	6			
octahedron	6	12	8	9			
cube	8	12	6	7			
dodecahedron	20	30	12	$9 \dots 10$			
icosahedron	12	30	20	$13 \dots 15$			



[Scherm, B.Th.'17]

Arguments: We use the number of nested cycles and the internal degree of the outer face.

Platonic Solids: Segment Numbers

$G = (V, E)$	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert F \rvert$	$\rho_2^1(G)$	seg(G)	$\sigma_2^1(G)$	arc(G)
tetrahedron	4	6	4	6			
octahedron	6	12	8	9			
cube	8	12	6	7			
dodecahedron	20	30	12	$9 \ldots 10$			
icosahedron	12	30	20	$13 \ldots 15$			

Platonic Solids: Segment Numbers

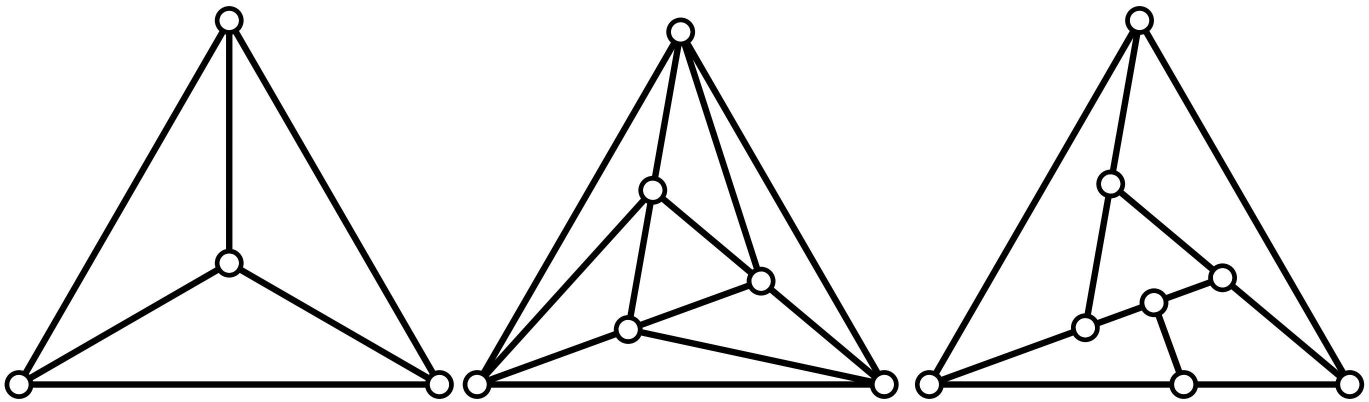
$G = (V, E)$	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert F\rvert$	$\rho_2^1(G)$	seg(G)	$\sigma_2^1(G)$	arc(G)
tetrahedron	4	6	4	6	≥ 6		
octahedron	6	12	8	9	≥ 9		
cube	8	12	6	7	≥ 7		
dodecahedron	20	30	12	$9 \dots 10$	≥ 9		
icosahedron	12	30	20	$13 \dots 15$	≥ 13		

Trivial bound:

$\rho_1^2(G) \leq \text{seg}(G)$

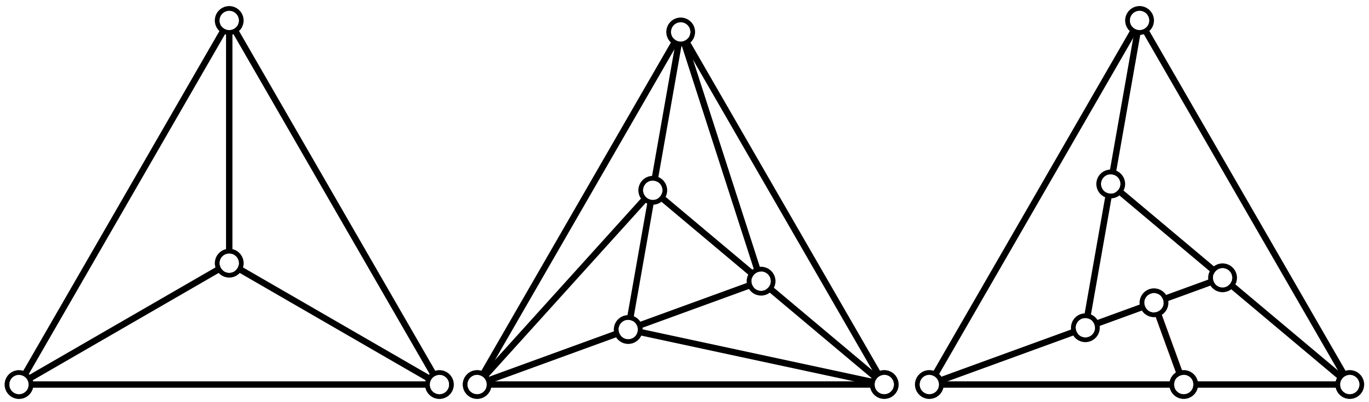
Platonic Solids: Segment Numbers

$G = (V, E)$	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert F\rvert$	$\rho^1_2(G)$	seg(G)	$\sigma^1_2(G)$	arc(G)
tetrahedron	4	6	4	6	6		
octahedron	6	12	8	9	9		
cube	8	12	6	7	7		
dodecahedron	20	30	12	$9 \dots 10$	≥ 9		
icosahedron	12	30	20	$13 \dots 15$	≥ 13		



Platonic Solids: Segment Numbers

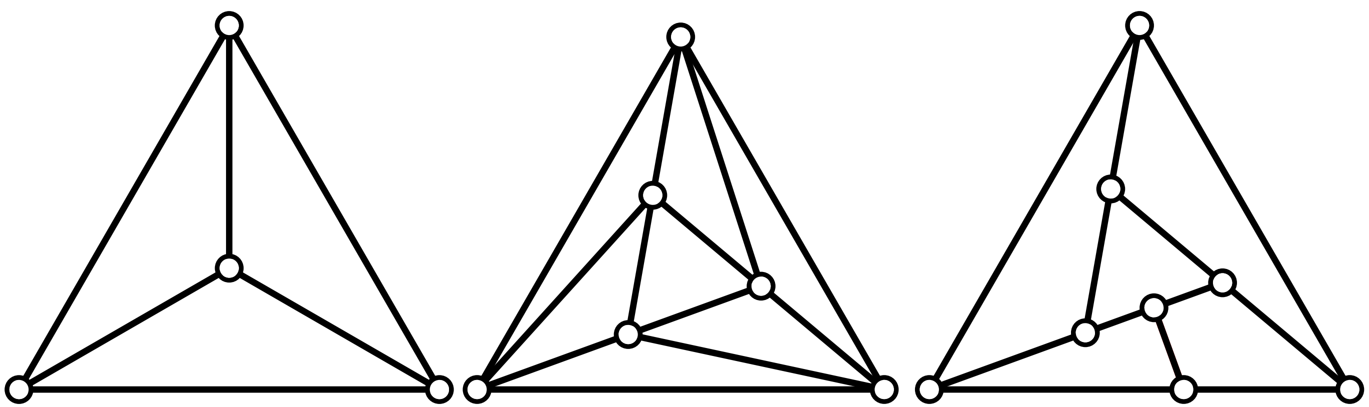
$G = (V, E)$	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert F\rvert$	$\rho^1_2(G)$	seg(G)	$\sigma^1_2(G)$	arc(G)
tetrahedron	4	6	4	6	6		
octahedron	6	12	8	9	9		
cube	8	12	6	7	7		
dodecahedron	20	30	12	$9 \dots 10$	≥ 9		
icosahedron	12	30	20	$13 \dots 15$	≥ 13		



ILP (For fixed embedding.) Find *locally consistent* angle assignment with maximum number of π-angles.

Platonic Solids: Segment Numbers

$G = (V, E)$	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert F \rvert$	$\rho^1_2(G)$	seg(G)	$\sigma^1_2(G)$	arc(G)
tetrahedron	4	6	4	6	6		
octahedron	6	12	8	9	9		
cube	8	12	6	7	7		
dodecahedron	20	30	12	$9 \ldots 10$	≥ 13		
icosahedron	12	30	20	$13 \ldots 15$	≥ 15		



ILP (For fixed embedding.) Find *locally consistent* angle assignment with maximum number of π-angles.

$\Rightarrow$ Lower bounds for the minimum number of segments in the corresponding drawing.

Platonic Solids: Segment Numbers

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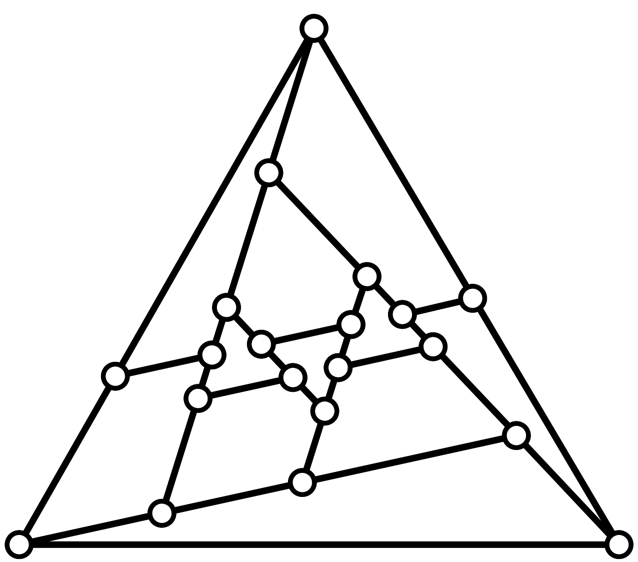
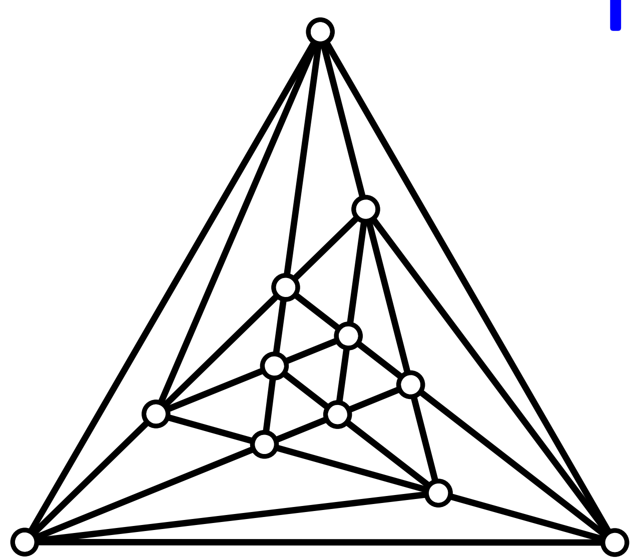
icosahedron	12	30	20	$13 \dots 15$	15		



13 segments



15 segments

ILP (For fixed embedding.) Find *locally consistent* angle assignment with maximum number of π-angles.

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Platonic Solids: Spherical Cover Numbers

Platonic Solids: Spherical Cover Numbers

$G = (V, E)$	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert F \rvert$	$\rho^1_2(G)$	$\text{seg}(G)$	$\sigma^1_2(G)$	$\text{arc}(G)$
tetrahedron	4	6	4	6	6	≥ 3	
octahedron	6	12	8	9	9	≥ 3	
cube	8	12	6	7	7	≥ 4	
dodecahedron	20	30	12	$9 \dots 10$	13	≥ 5	
icosahedron	12	30	20	$13 \dots 15$	15	≥ 7	

Recall Obs. 2:

$$\Longrightarrow \sigma^1_2(G) \geq \frac{1}{2}\left(1 + \sqrt{1 + 4 \sum_{v \in V(G)} \binom{\left\lceil \frac{\deg v}{2} \right\rceil}{2}}\right)$$

Platonic Solids: Spherical Cover Numbers

$G = (V, E)$	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert F \rvert$	$\rho_2^1(G)$	seg(G)	$\sigma_2^1(G)$	arc(G)
tetrahedron	4	6	4	6	6	3	
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cube	8	12	6	7	7	≥ 4	
dodecahedron	20	30	12	$9 \ldots 10$	13	≥ 5	
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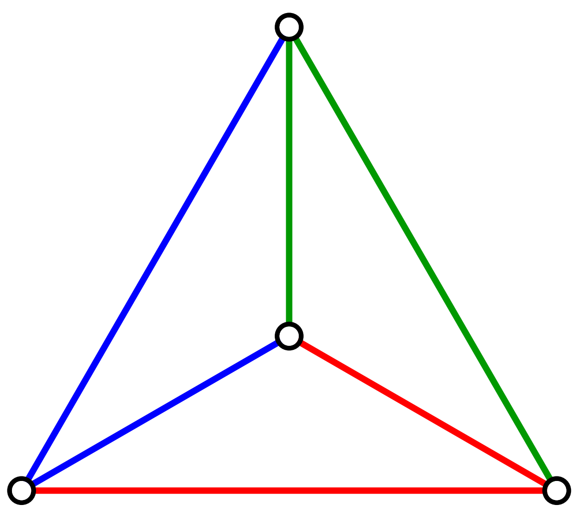

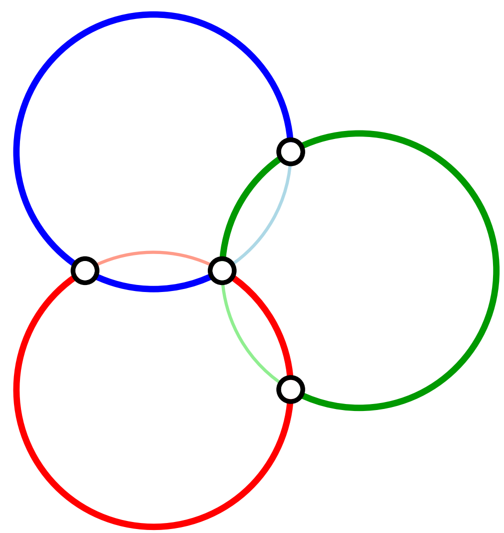

Platonic Solids: Spherical Cover Numbers

$G = (V, E)$	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert F \rvert$	$\rho^1_2(G)$	seg(G)	$\sigma^1_2(G)$	arc(G)
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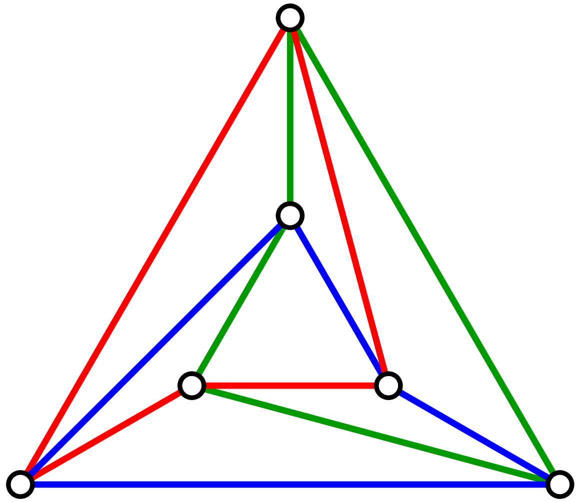

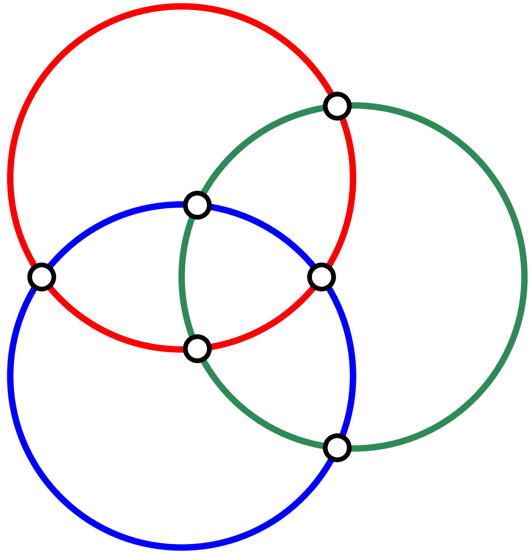

Platonic Solids: Spherical Cover Numbers

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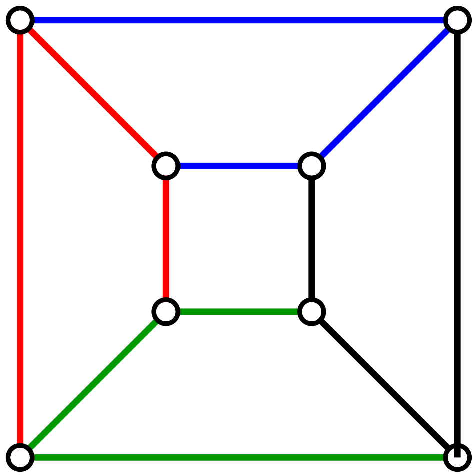

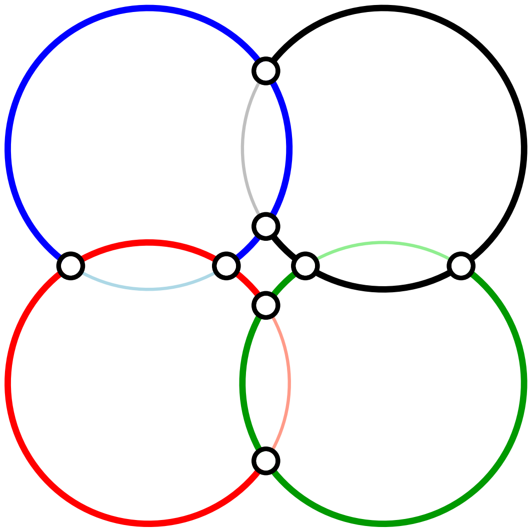

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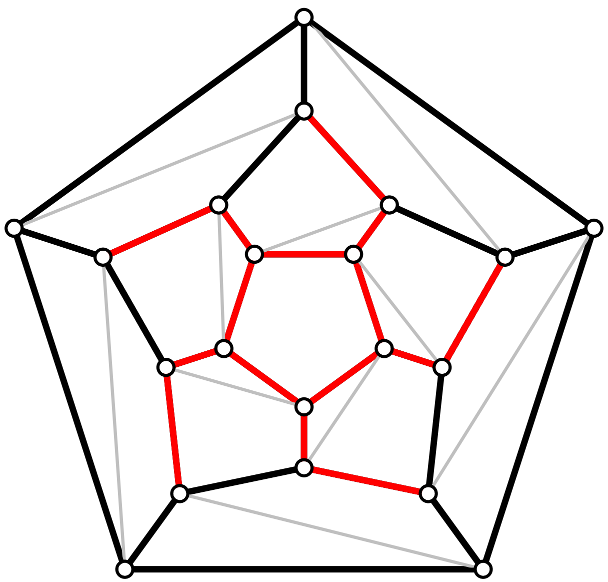

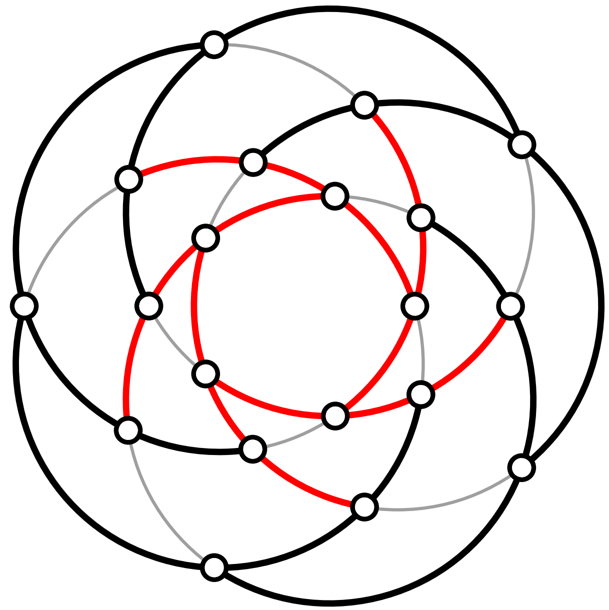

[André Schulz, JGAA'15]

Platonic Solids: Spherical Cover Numbers

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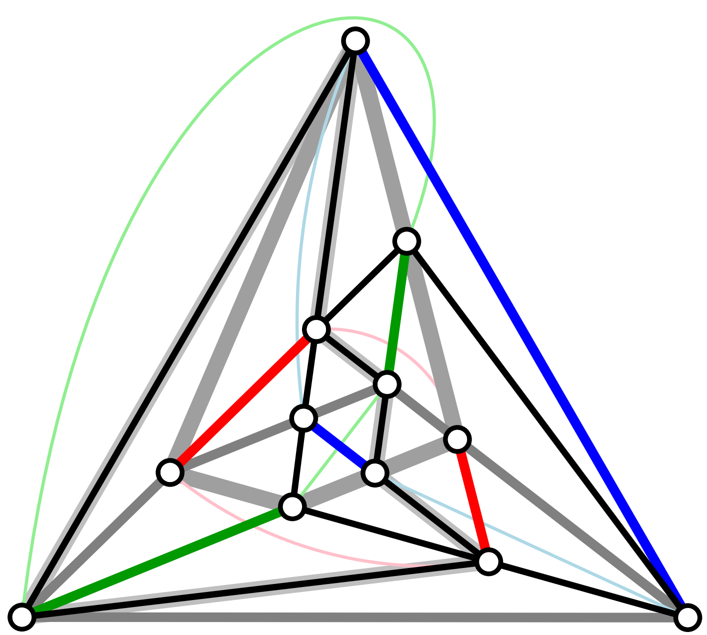

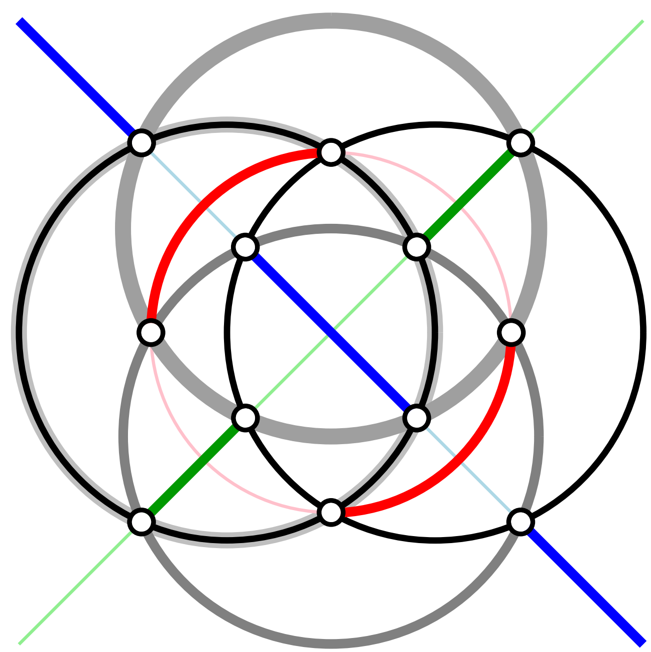

Platonic Solids: Spherical Cover Numbers

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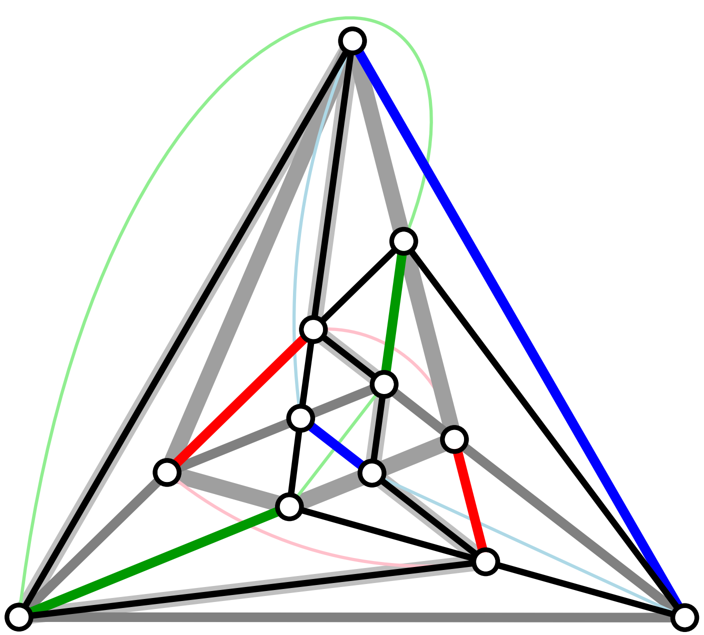

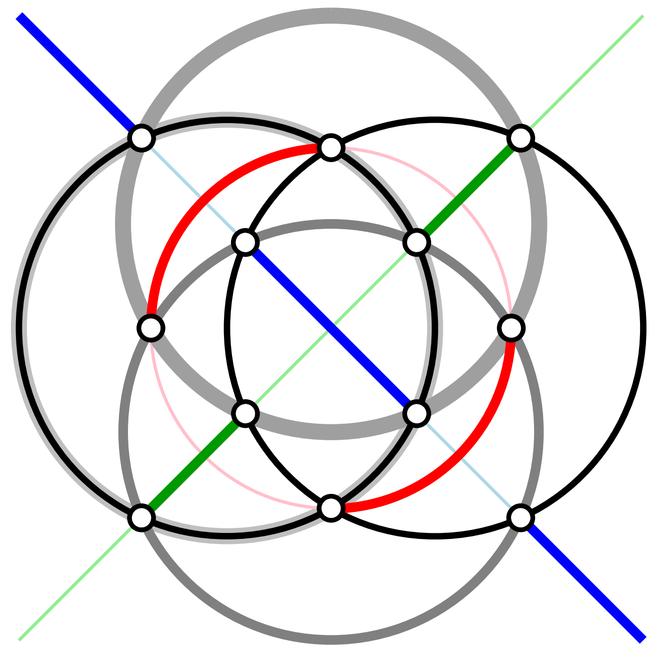

7 circles / 10 arcs

Platonic Solids: Arc Numbers

$G = (V, E)$	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert F\rvert$	$\rho_2^1(G)$	seg(G)	$\sigma_2^1(G)$	arc(G)
tetrahedron	4	6	4	6	6	3	
octahedron	6	12	8	9	9	3	
cube	8	12	6	7	7	4	
dodecahedron	20	30	12	$9 \ldots 10$	13	5	
icosahedron	12	30	20	$13 \ldots 15$	15	7	

Platonic Solids: Arc Numbers

$G = (V, E)$	$\|V\|$	$\|E\|$	$\|F\|$	$\rho^1_2(G)$	$\text{seg}(G)$	$\sigma^1_2(G)$	$\text{arc}(G)$
tetrahedron	4	6	4	6	6	3	≥ 3
octahedron	6	12	8	9	9	3	≥ 3
cube	8	12	6	7	7	4	≥ 4
dodecahedron	20	30	12	$9 \dots 10$	13	5	≥ 5
icosahedron	12	30	20	$13 \dots 15$	15	7	≥ 7

Trivial bound:

$\sigma^2_1(G) \leq \text{arc}(G)$

Platonic Solids: Arc Numbers

$G = (V, E)$	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert F \rvert$	$\rho_2^1(G)$	$\mathrm{seg}(G)$	$\sigma_2^1(G)$	$\mathrm{arc}(G)$
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octahedron	6	12	8	9	9	3	≥ 3
cube	8	12	6	7	7	4	≥ 4
dodecahedron	20	30	12	$9 \ldots 10$	13	5	≥ 10
icosahedron	12	30	20	$13 \ldots 15$	15	7	≥ 7

Trivial bound:

$\sigma_1^2(G) \leq \mathrm{arc}(G)$

Obs: For any graph G, $\mathrm{arc}(G) \geq \#\big(\text{odd-deg. vtc. of } G\big)/2$

[Dujmović, Eppstein, Suderman, Wood CGTA'07]

Platonic Solids: Arc Numbers

$G = (V, E)$	$\|V\|$	$\|E\|$	$\|F\|$	$\rho^1_2(G)$	$\mathrm{seg}(G)$	$\sigma^1_2(G)$	$\mathrm{arc}(G)$
tetrahedron	4	6	4	6	6	3	3
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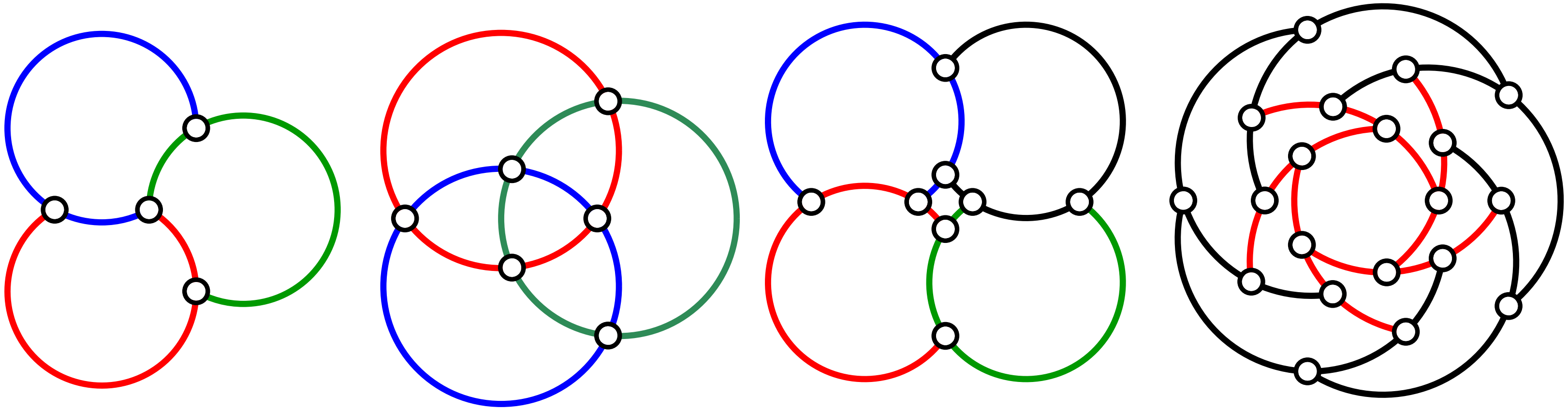

Platonic Solids: Arc Numbers

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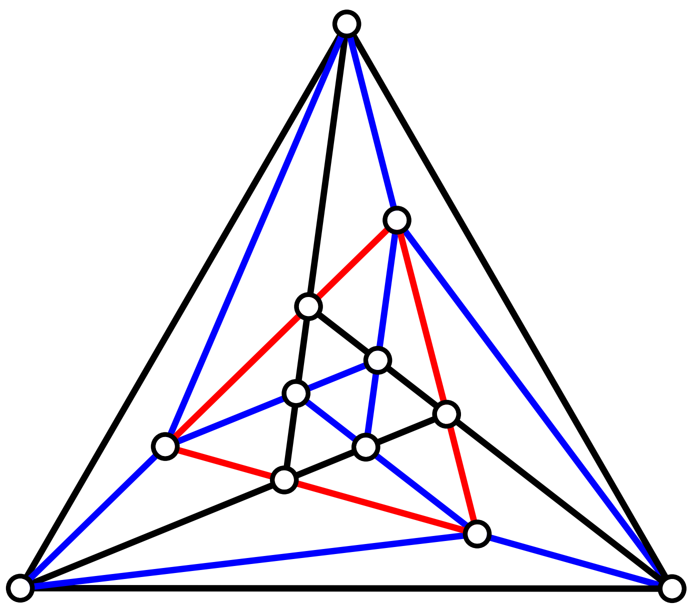
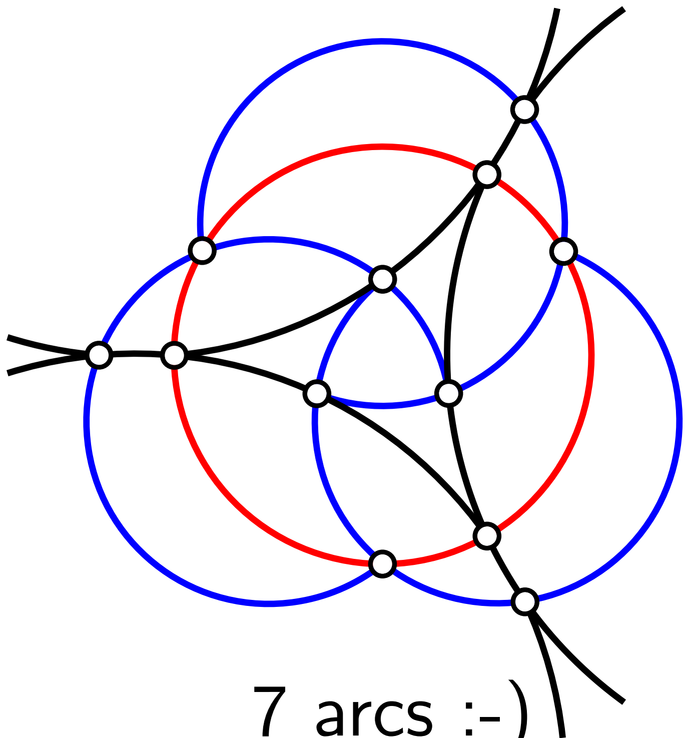
dodecahedron	20	30	12	9 . . . 10	13	5	10
icosahedron	12	30	20	13 . . . 15	15	7	7

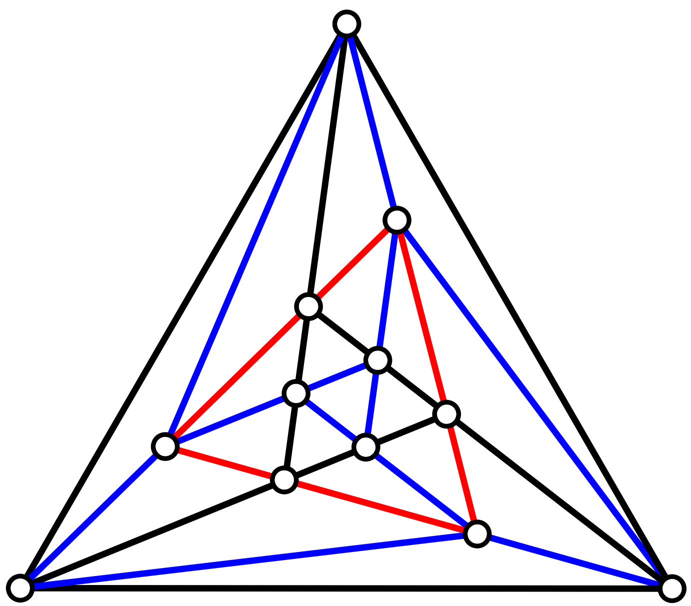


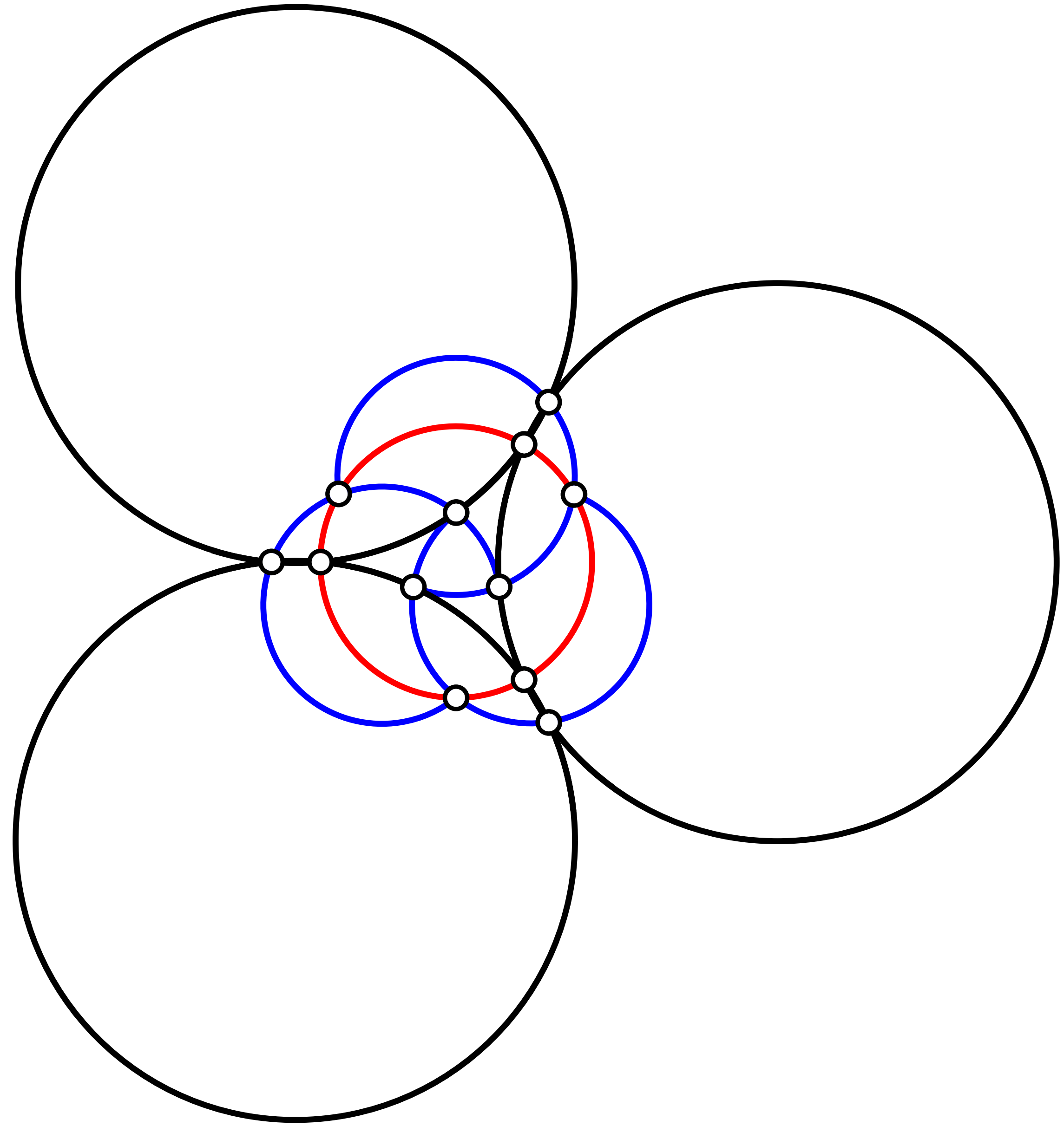


7 arcs :-)

Platonic Solids: Arc Numbers

$G = (V, E)$	$\|V\|$	$\|E\|$
tetrahedron	4	6
octahedron	6	12
cube	8	12
dodecahedron	20	30
icosahedron	12	30





Platonic Solids: Arc Numbers

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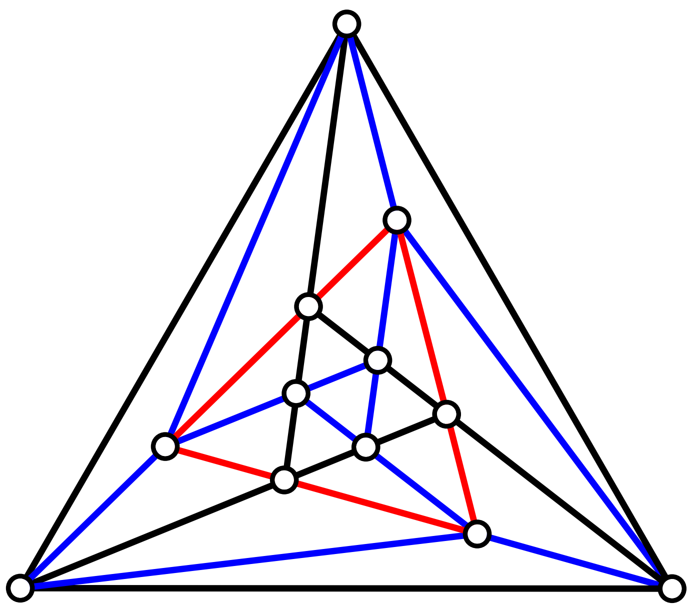

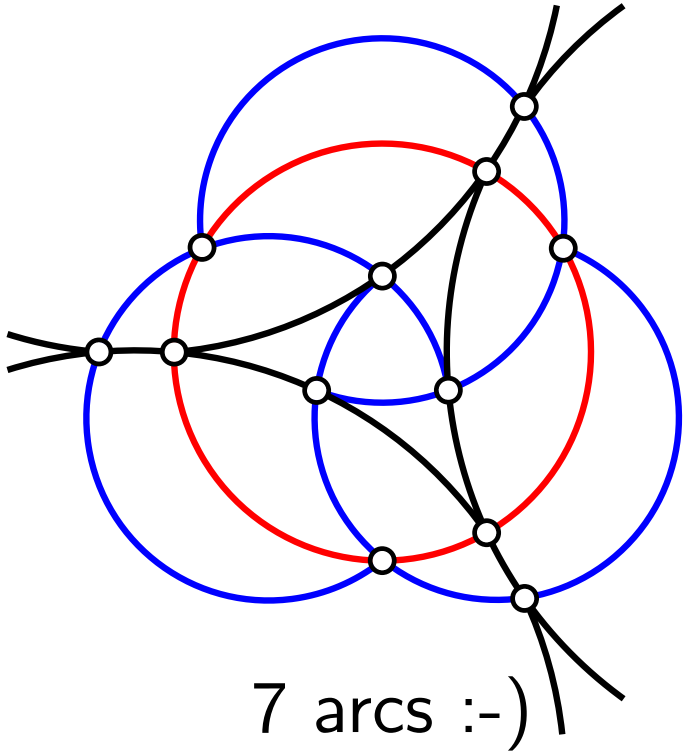

How to draw?

7 arcs :-)

Platonic Solids: Arc Numbers

$G = (V, E)$	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert F \rvert$	$\rho^1_2(G)$	$\text{seg}(G)$	$\sigma^1_2(G)$	$\text{arc}(G)$
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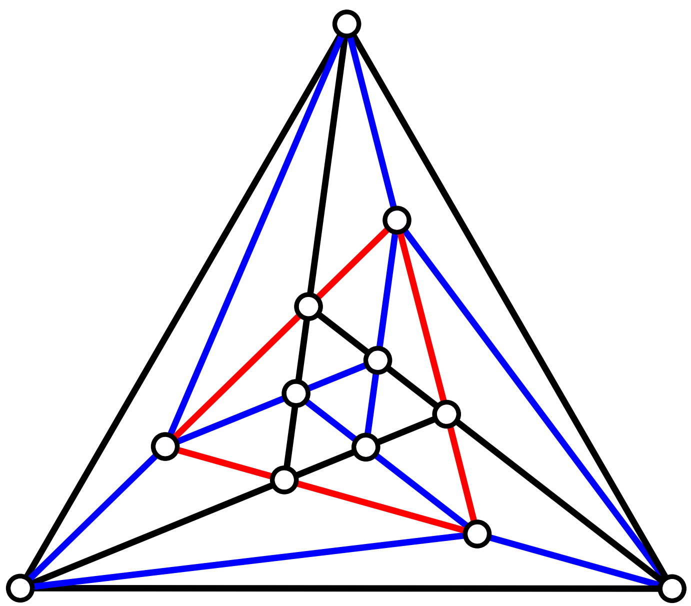

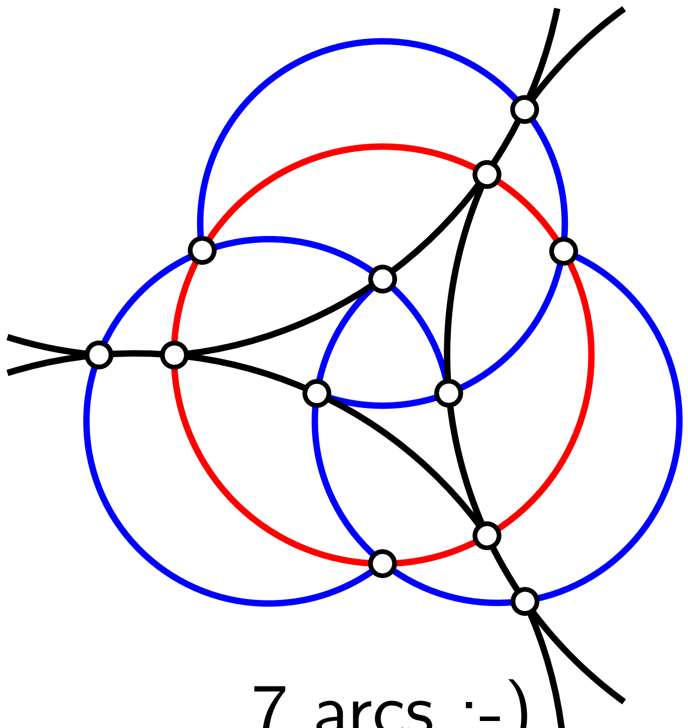

7 arcs :-)

How to draw?

Solve a system of quadratic equations.

Platonic Solids: Arc Numbers

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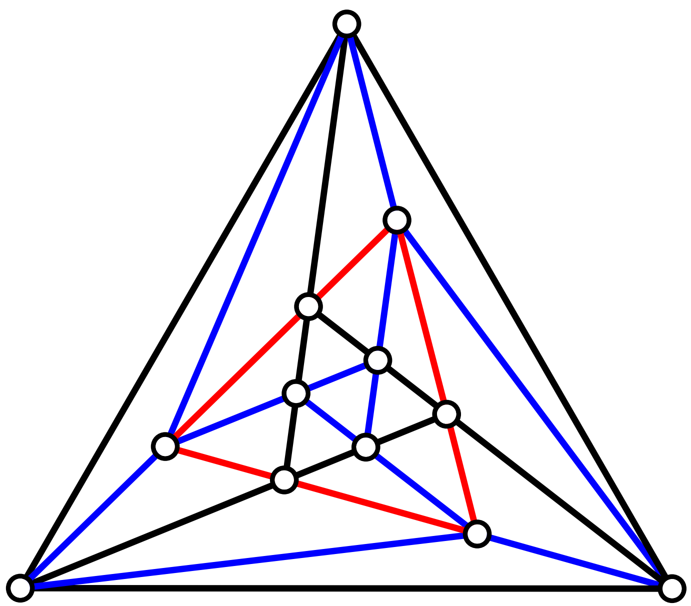
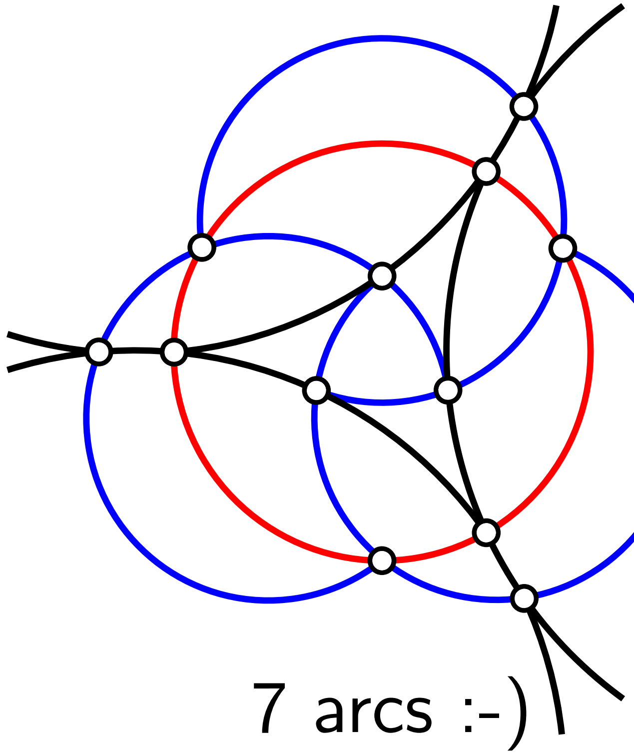




7 arcs :-)

How to draw?

Solve a system of quadratic equations.

Solution exists!

Outline

Motivation

Formal Definitions

A Combinatorial Lover Bound

Platonic solids

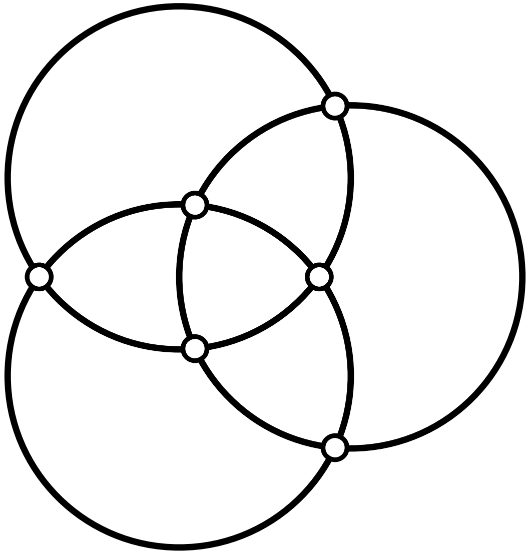
- affine cover number
- segment number
- spherical cover number
- arc number

Lower Bounds for σ_d^1 w.r.t. Other Parameters

Open Problem

Lower Bounds for σ_d^1 w.r.t. Other Parameters

For any $d \geq 1$ and any graph G, the following bounds hold:



Edge-chromatic # $\sigma_d^1(G) \geq \chi_e(G)/3$,

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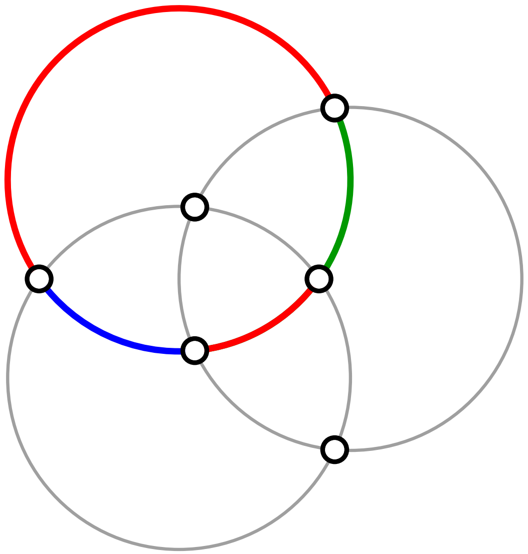

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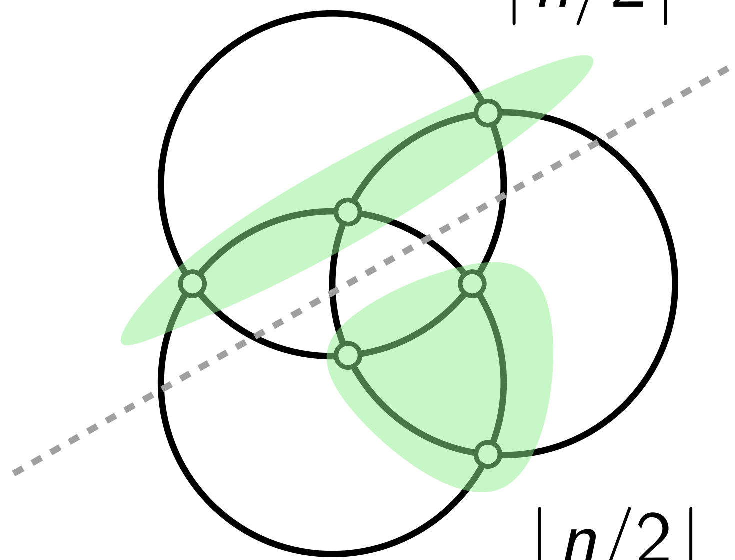

Lower Bounds for σ_d^1 w.r.t. Other Parameters

For any $d \geq 1$ and any graph G, the following bounds hold:

Edge-chromatic #	$\sigma_d^1(G) \geq \chi_e(G)/3$,
bisection width	$\sigma_d^1(G) \geq \mathrm{bw}(G)/2$,
linear arboricity	$\sigma_d^1(G) \geq \frac{2}{3}\,\mathrm{la}(G)$,
balanced separator	$\sigma_d^1(G) \geq \mathrm{sep}_W(G)/2$,
for almost all G cubic	$\sigma_d^1(G) > n/10$,
treewidth	$\sigma_d^1(G) \geq \mathrm{tw}(G)/6$.

Outline

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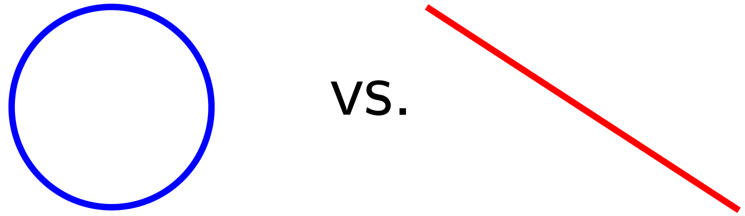
Lower Bounds for σ_d^1 w.r.t. Other Parameters

Open Problem

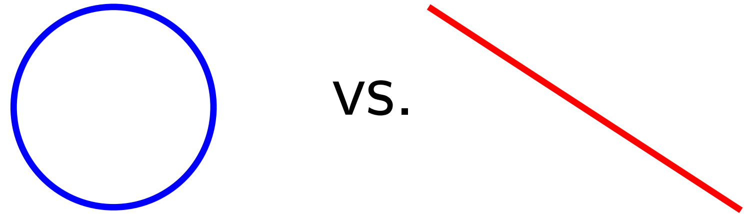
Open Problems



Problem 1:

Is there a family of planar graphs whose circle cover number grows asymptotically more slowly than their line cover number?

Open Problems

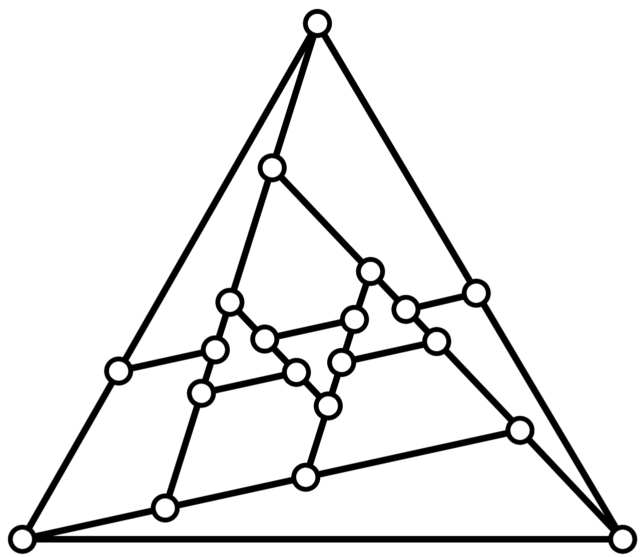


Problem 1:

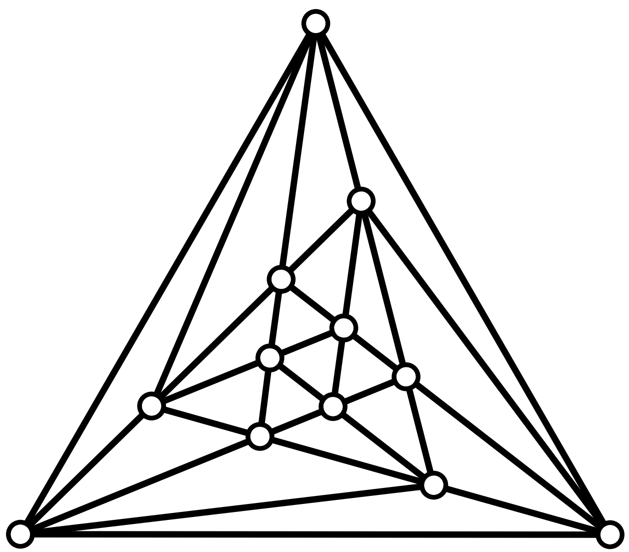
Is there a family of planar graphs whose circle cover number grows asymptotically more slowly than their line cover number?

Problem 2:

Determine the line cover number for the dodecahedron and icosahedron.



9...10



13...15