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Optimal Simplification of Building Ground Plans

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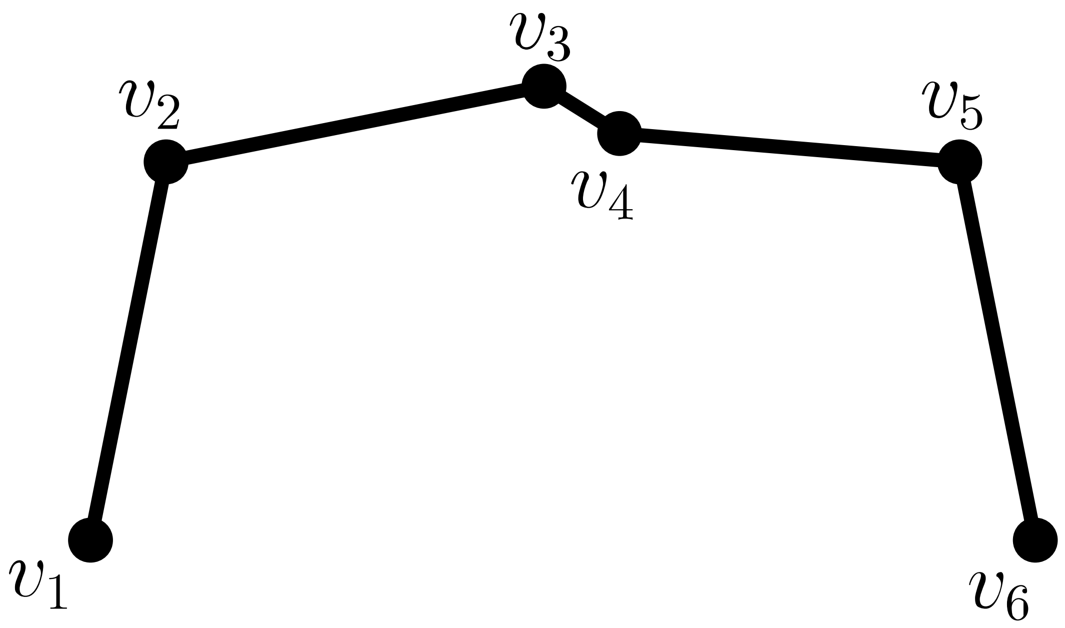
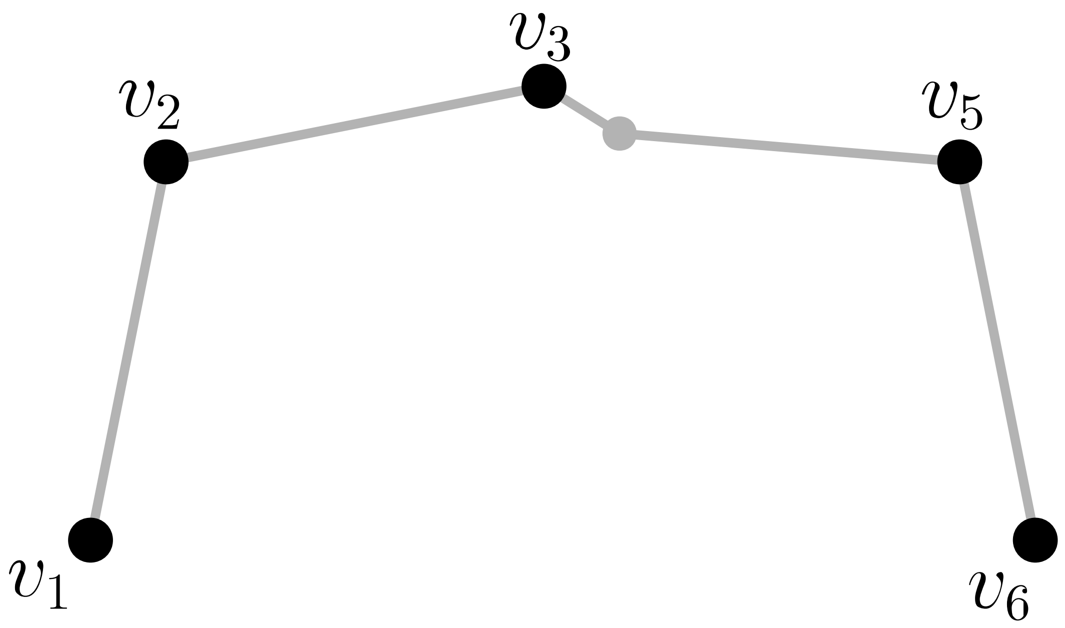
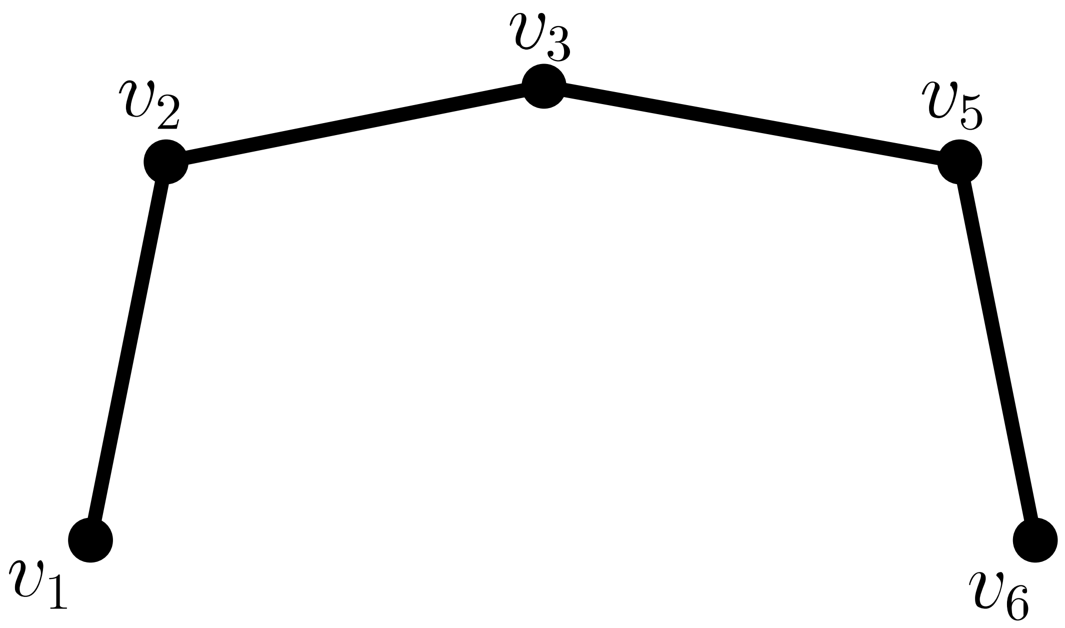
Line Simplification



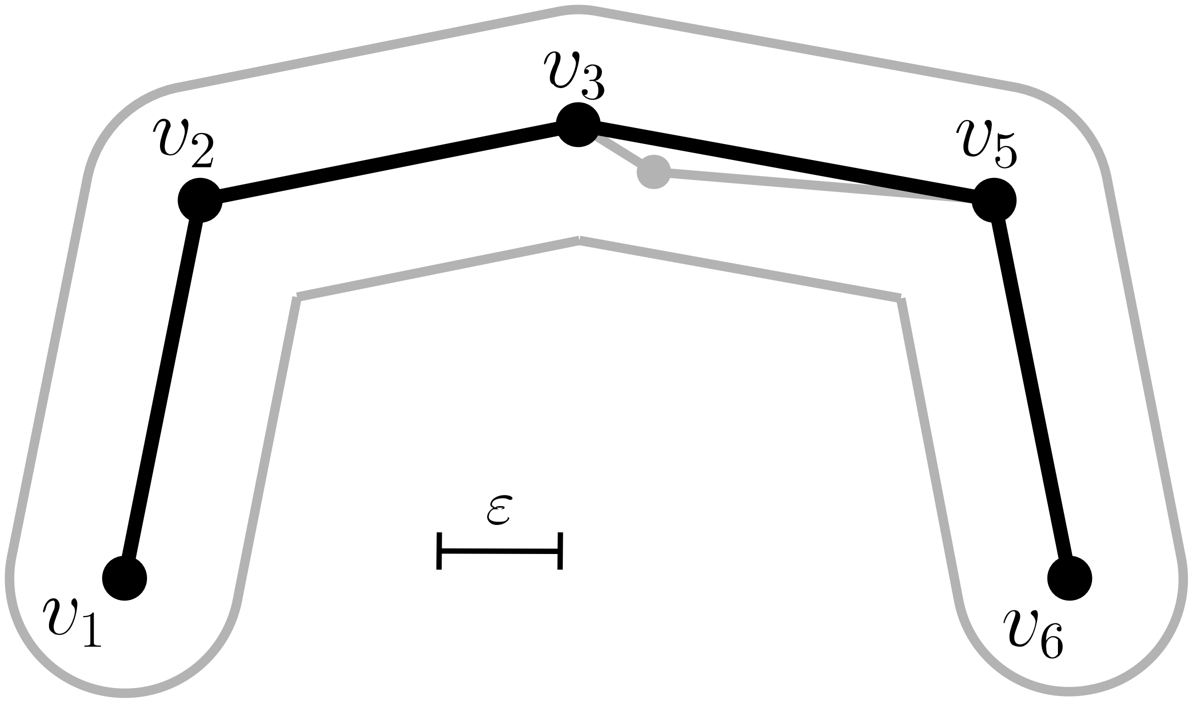
Line Simplification



Line Simplification



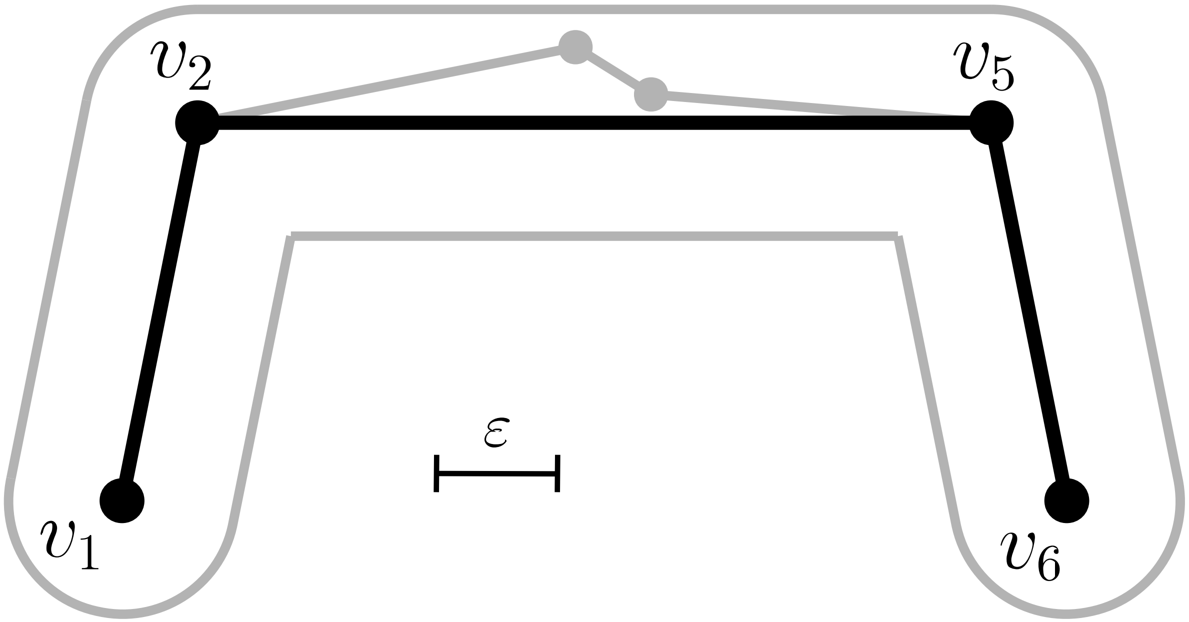
Line Simplification



Bandwidth criterion:

For each line segment (v_i, v_j) of the simplified line, the vertices $(v_{i+1}, v_{i+2}, \dots, v_{j-1})$ of the original line must be within ε distance.

Line Simplification



Basic optimization approach:

Find a simplified line that satisfies the bandwidth criterion and has a minimum number of vertices.

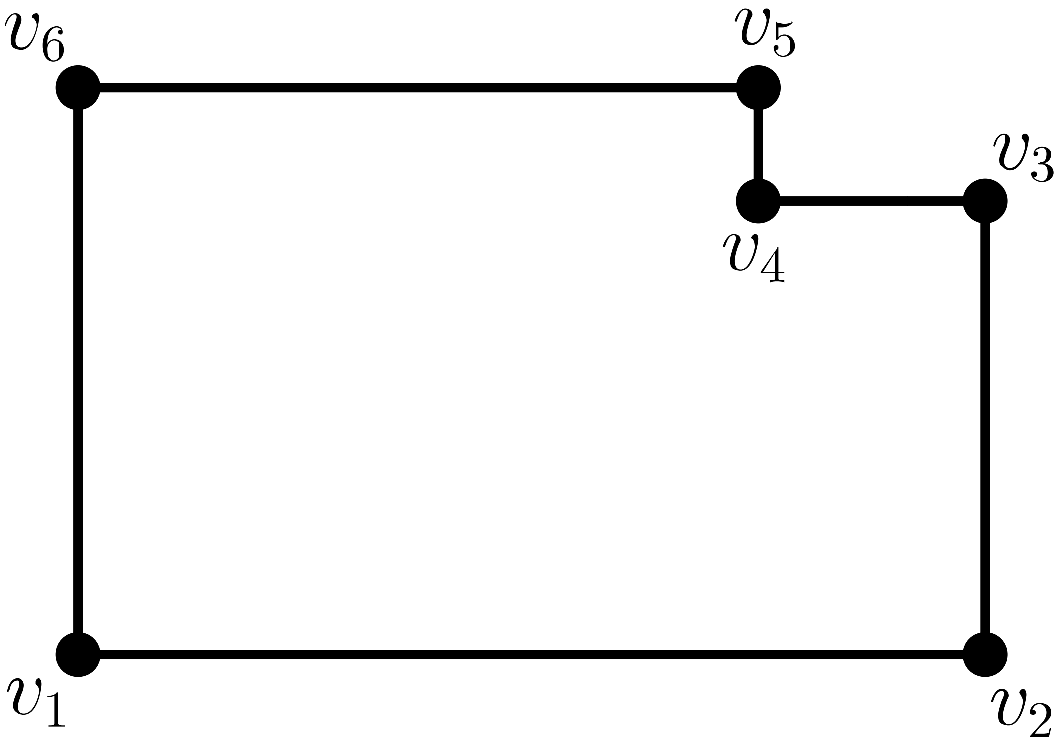
Line Simplification

- Optimization approach allows to apply different constraints and optimization criteria.

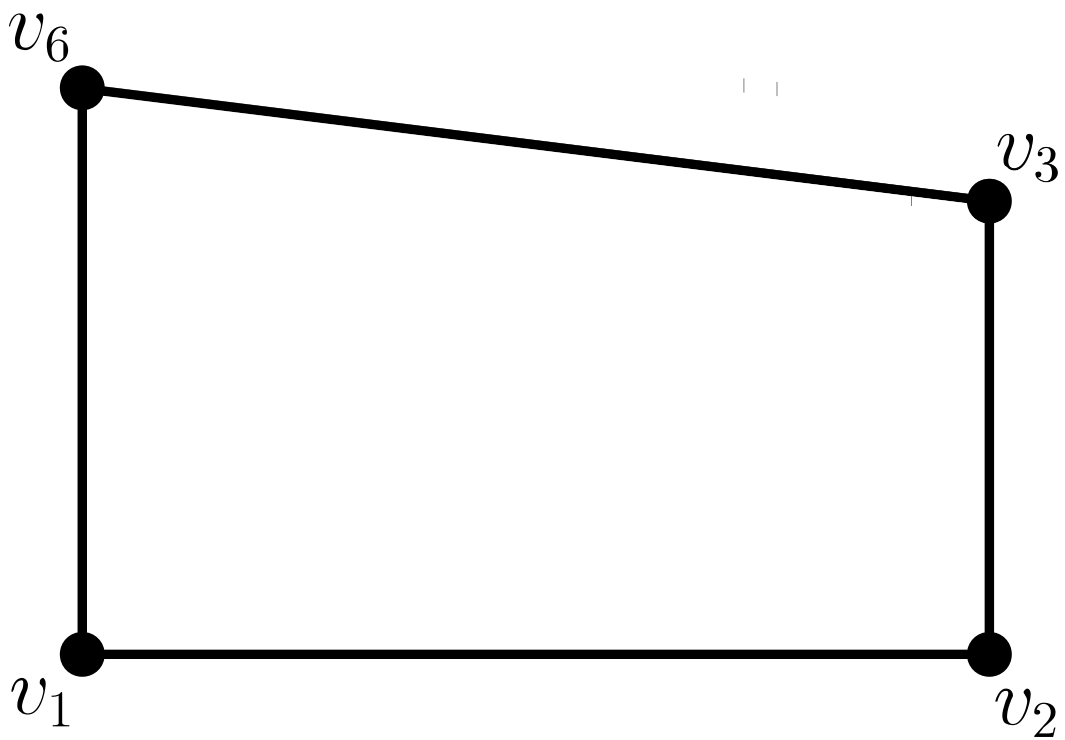
- Solutions exist to preserve
  - topological relationships (deBerg et al., 1998)

  and to minimize changes of
  - distances, (Gudmundson et al., 2007)
  - angles, (Chen et al., 2005)
  - areas. (Bose et al. 2006)

Building Simplification

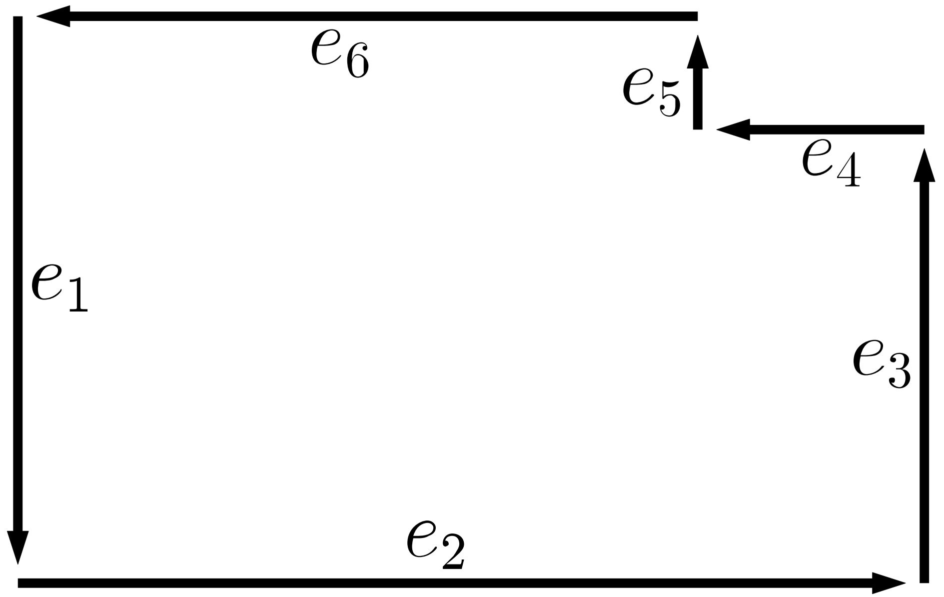


Building Simplification



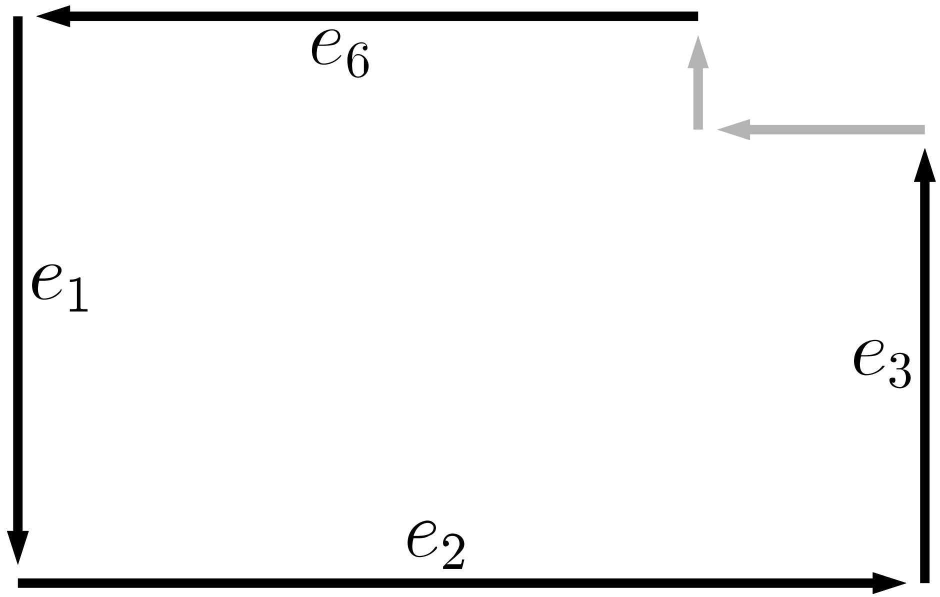
- When reducing a building to a subsequence of vertices, shape regularities will get lost!

Building Simplification



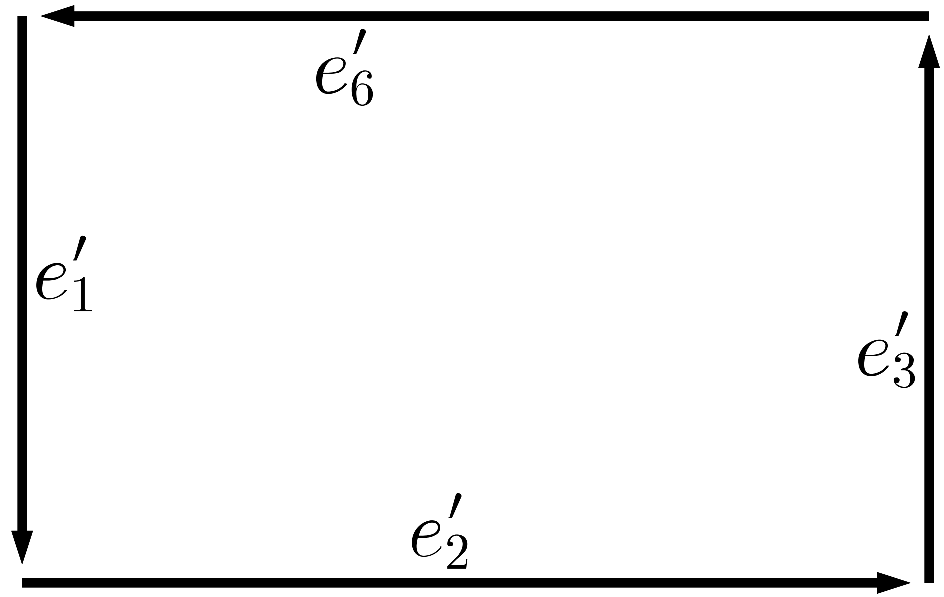
- Idea: Reduce a building to a subsequence of its edges.

Building Simplification



- Idea: Reduce a building to a subsequence of its edges.

Building Simplification

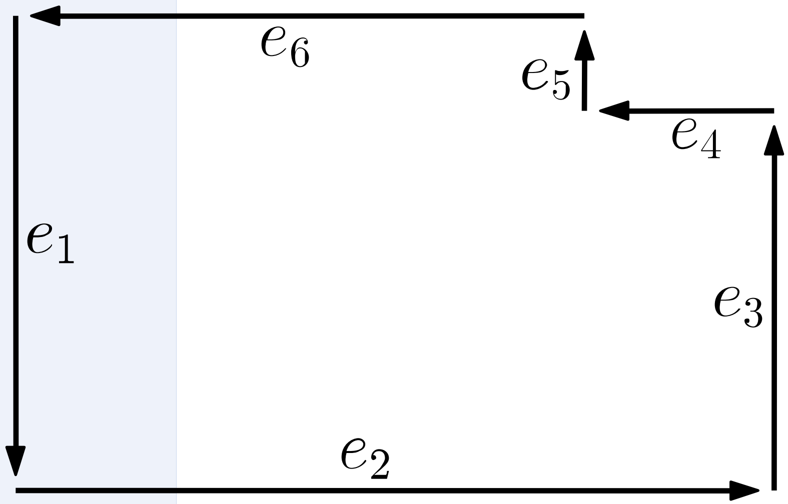


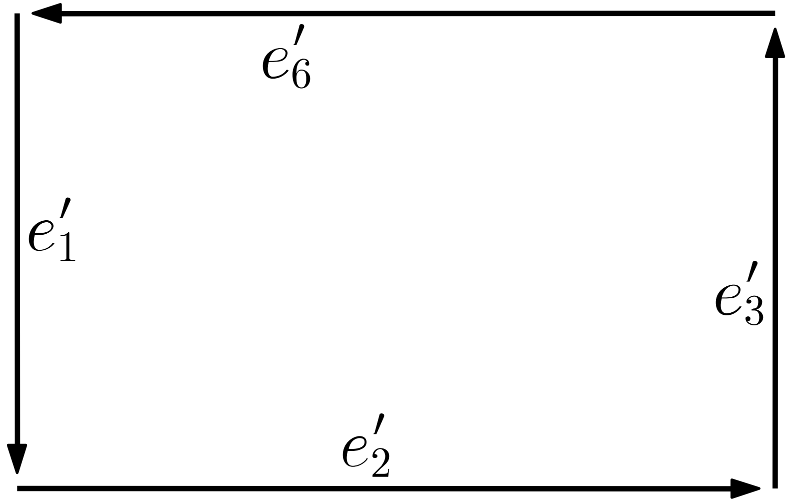
- Idea: Reduce a building to a subsequence of its edges.
- Basic problem:
  - Output polygon must be simple.
  - User-defined error tolerance ε must not be violated.
  - Number of edges is to be minimized.

Building Simplification

Definition:

- A *shortcut* is a pair of edges (e_k, e_l) .
- Applying the shortcut (e_k, e_l) implies to omit edges $(e_{k+1}, e_{k+2}, \ldots, e_{l-1})$.





Outline of our Paper

- Formal problem statement
- Complexity : The problem is NP-hard, if we require simple polygons as outcome
- An efficient algorithm for a relaxed problem
- An exact approach by integer programming
- An efficient heuristic
- Outline of an exact, fixed-parameter algorithm
- Experimental results

Outline of this Talk

- Formal problem statement
- Complexity : The problem is NP-hard, if we require simple polygons as outcome
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- Outline of an exact, fixed-parameter algorithm
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Problem Statement

Given

- a simple polygon $P = (e_1, e_2, \dots, e_n)$
- an error tolerance $\varepsilon > 0$

find a polygon $P' = (e'_{i_1}, e'_{i_2}, \dots, e'_{i_m})$

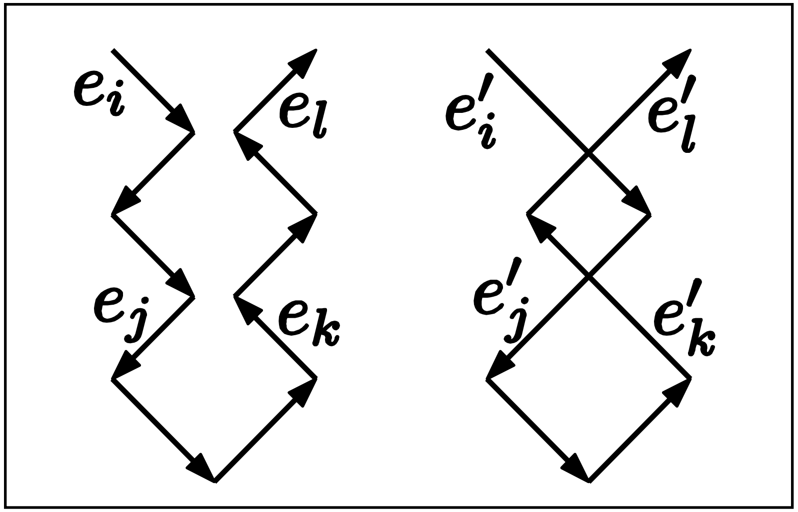
with $i_1 < i_2 < \dots < i_m < n$ such that

- P' has a minimum number of edges and
- the three requirements R1- R3 hold (as follows).

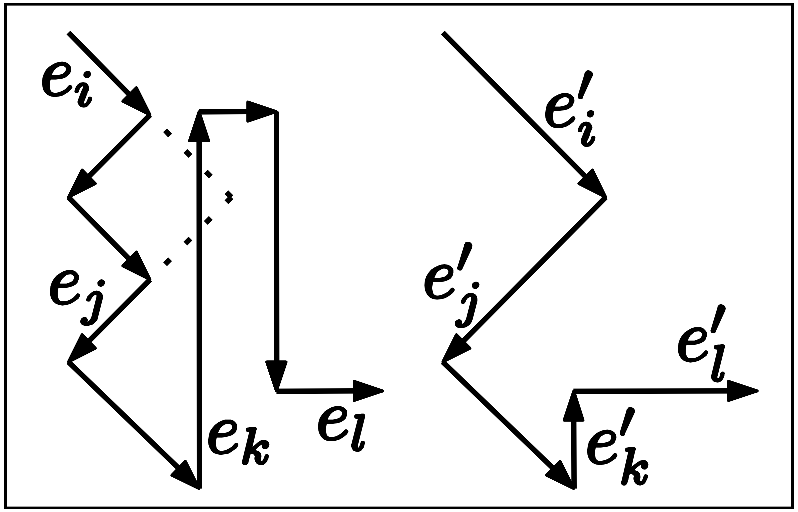
Problem Statement

Requirement R1:

- P' is simple.



infeasible



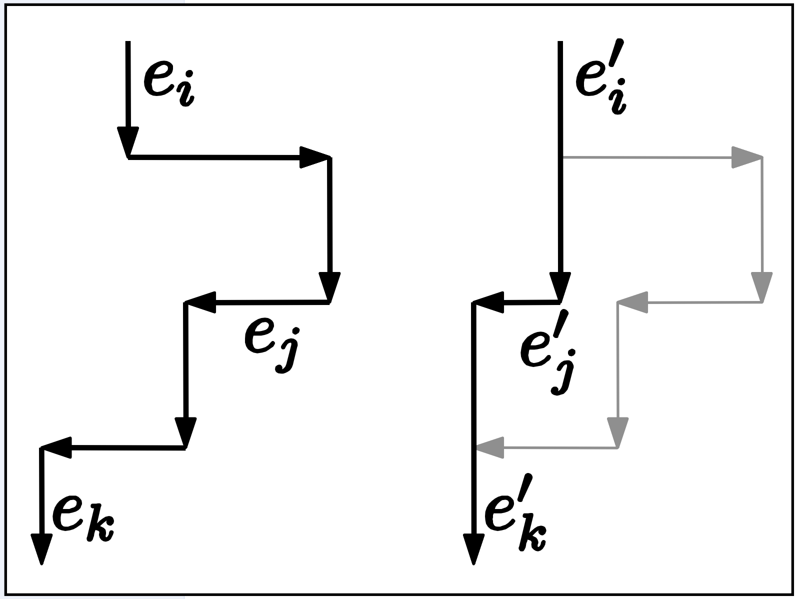
feasible

Problem Statement

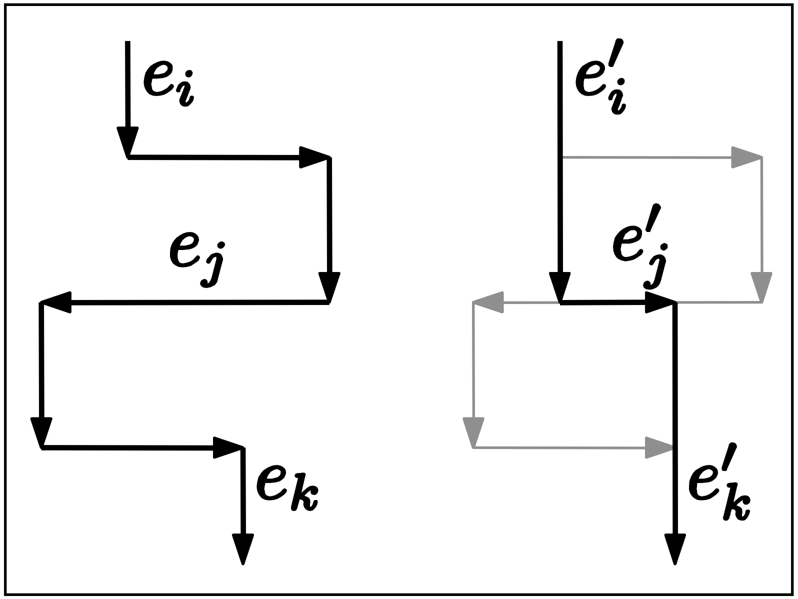
Requirement R2:

- For $j = 1, \dots, m$ it holds that e_{i_j} and e'_{i_j}

  a) intersect and
  b) have the same directed supporting line.



infeasible (a)



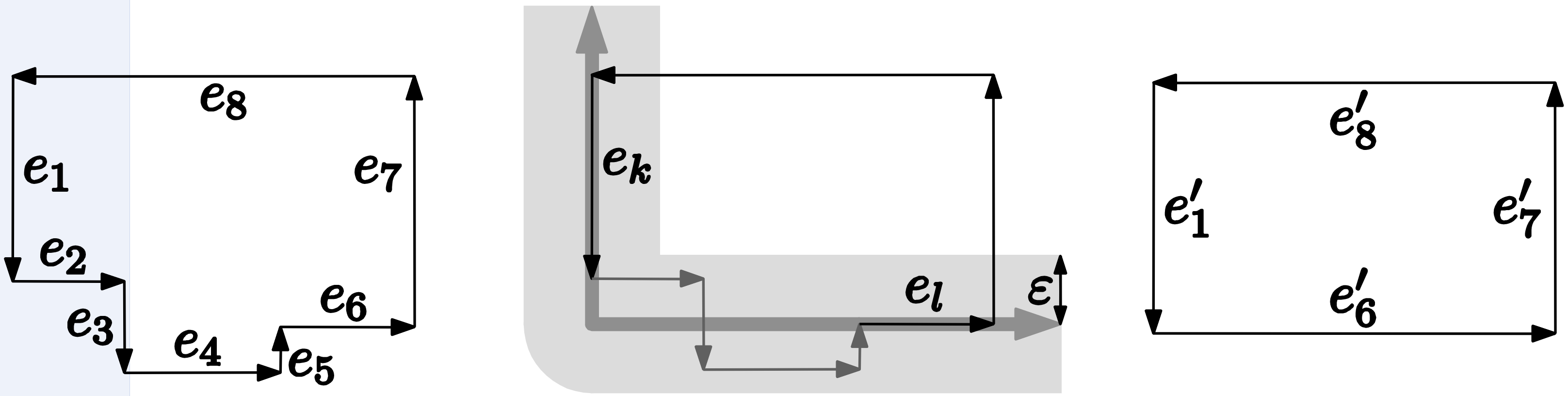
infeasible (b)

Problem Statement

Requirement R3 (similar to bandwidth criterion):

- for each pair of consecutive edges (e'_k, e'_l) in P'

  a) the sequence $(e_{k+1}, e_{k+2}, \dots, e_{l-1})$ is within an ε - buffer of the L-shape defined by (e_k, e_l).

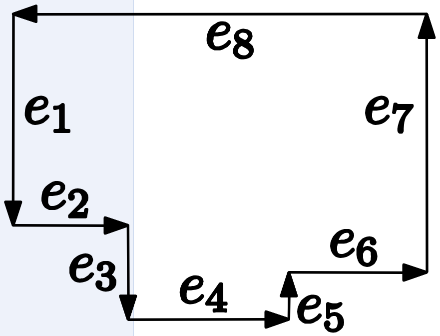


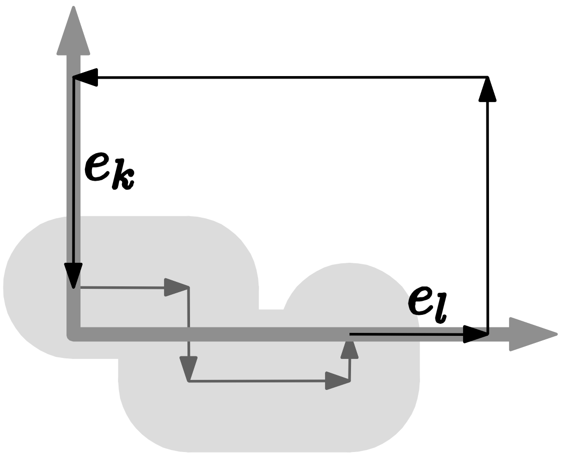
Problem Statement

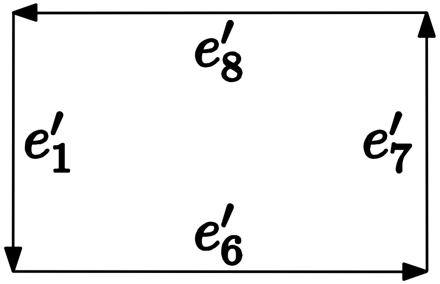
Requirement R3 (similar to bandwidth criterion):

- for each pair of consecutive edges (e'_k, e'_l) in P'

  b) the L-shape defined by (e_k, e_l) enters and leaves the ε - buffer of the sequence $(e_{k+1}, e_{k+2}, \dots, e_{l-1})$ exactly ones.

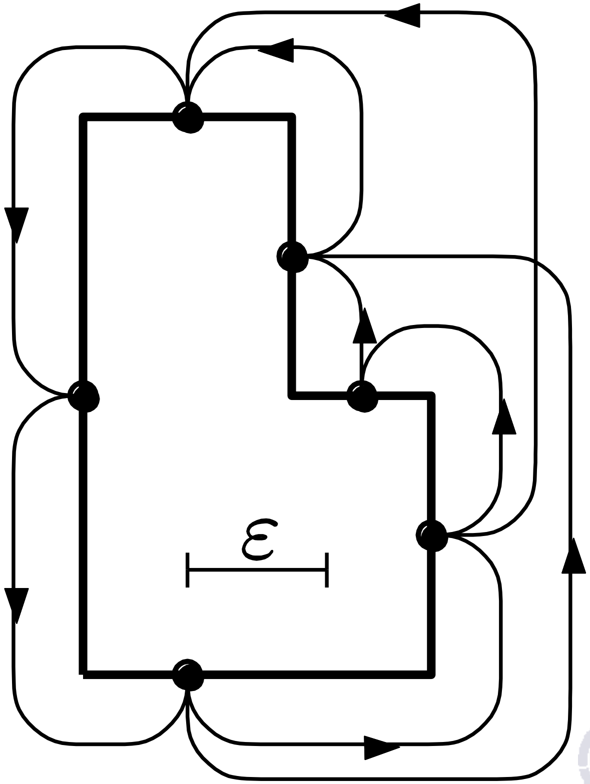






An Efficient Algorithm for a Relaxed Problem

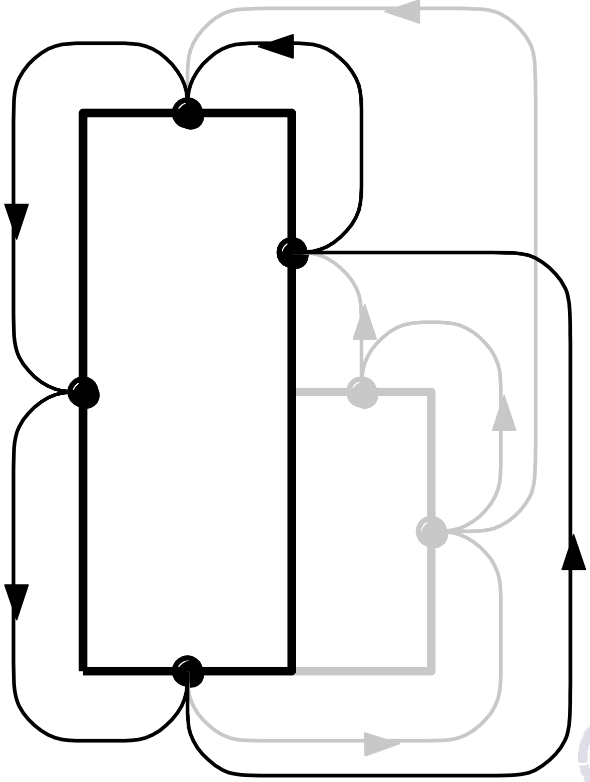
1. Construct the shortcut graph $G_{\mathbf{scut}}(E, S)$ that contains a node for each edge of P and an arc for each shortcut that satisfies requirement R3 (the bandwidth criterion).



An Efficient Algorithm for a Relaxed Problem

2. Find the shortest cycle in $G_{\mathbf{scut}}$.

- The shortest cycle in a digraph can be found in $\mathcal{O}(mn)$ time (Itai & Rodeh, 1978).



An Efficient Algorithm for a Relaxed Problem

2. Find the shortest cycle in $G_{\mathbf{scut}}$.

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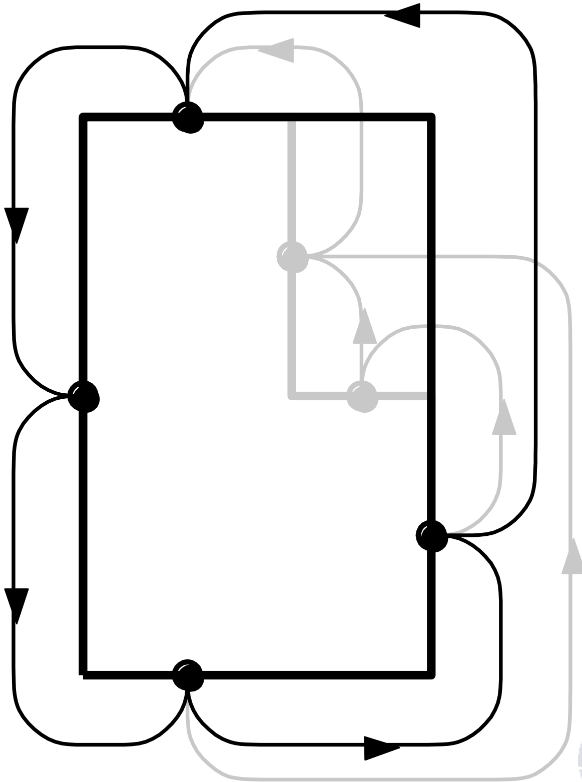
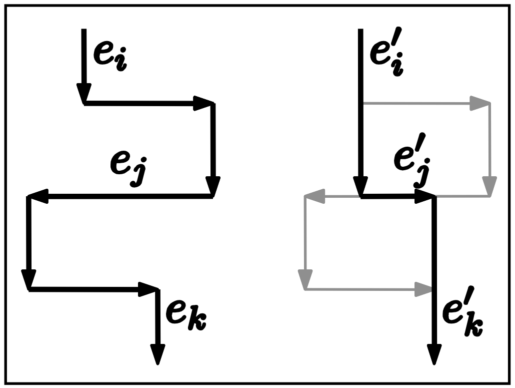


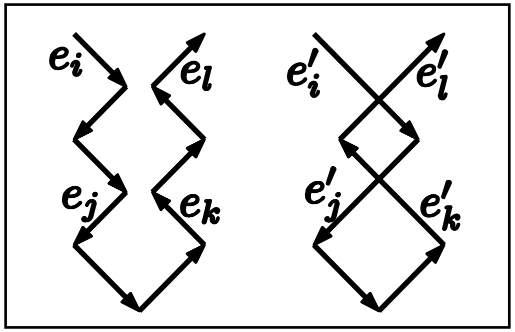
An Efficient Algorithm for a Relaxed Problem

The obtained cycle yields a simplified building, but:

- An edge may change its direction.



- The simplicity requirement may be violated.



An Efficient Algorithm for a Relaxed Problem

The obtained cycle yields a simplified building, but:

- An edge may change its direction.
  - Can be solved with a simple extension of the shortest cycle approach.
- The simplicity requirement may be violated.
  - Renders the problem NP-hard.

An Exact Approach by Integer Programming

- Integer programming is a special combinatorial optimization problem:

  Given an $m \times n$ integer matrix A,
  an m - vector of integers b,
  an n - vector of integers c,

  minimize $z = c^T \cdot x$

  subject to $A \cdot x \geq b$, $x \geq 0$ with $x \in \mathbb{Z}^n$.

- Many problems can be transformed into this form.

- Existing solvers can be applied (CPLEX, lp_solve).

An Exact Approach by Integer Programming

- Variables:

  $x_s \in \{0, 1\}$ for each shortcut $s \in S$

  with $x_s = 1$ if and only if s is selected.

An Exact Approach by Integer Programming

- Minimize $\sum_{s \in S} x_s$

  subject to

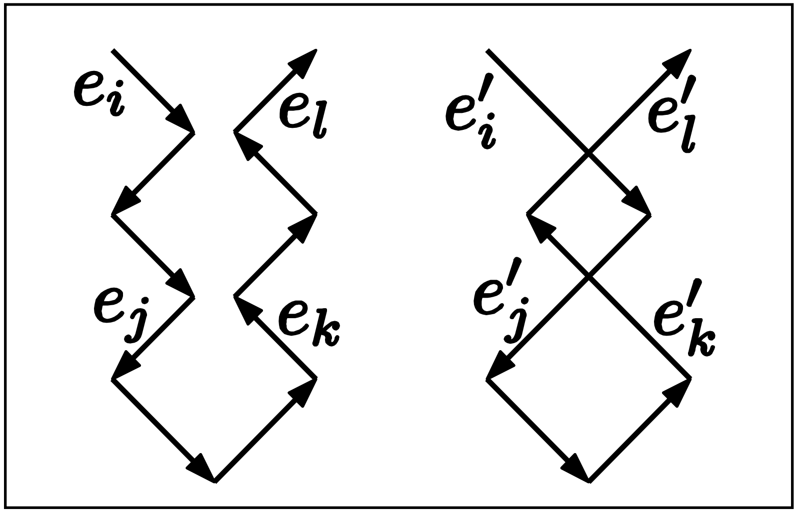
  $$\sum_{s \in \{(e_i, e_k) \in S \mid i \leq j < k\}} x_s = 1 \quad \text{for each edge} \quad e_j \in E$$

  For each edge e_j of the original building, there is one shortcut omitting e_j or starting at e_j .

An Exact Approach by Integer Programming

From each pair of conflicting shortcuts $s, t \in S$ do not select more than one.

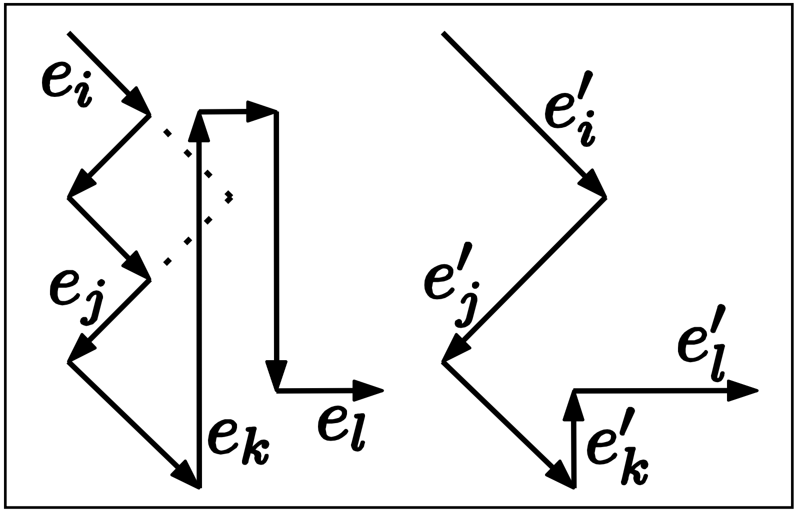
$$x_s + x_t \leq 1$$



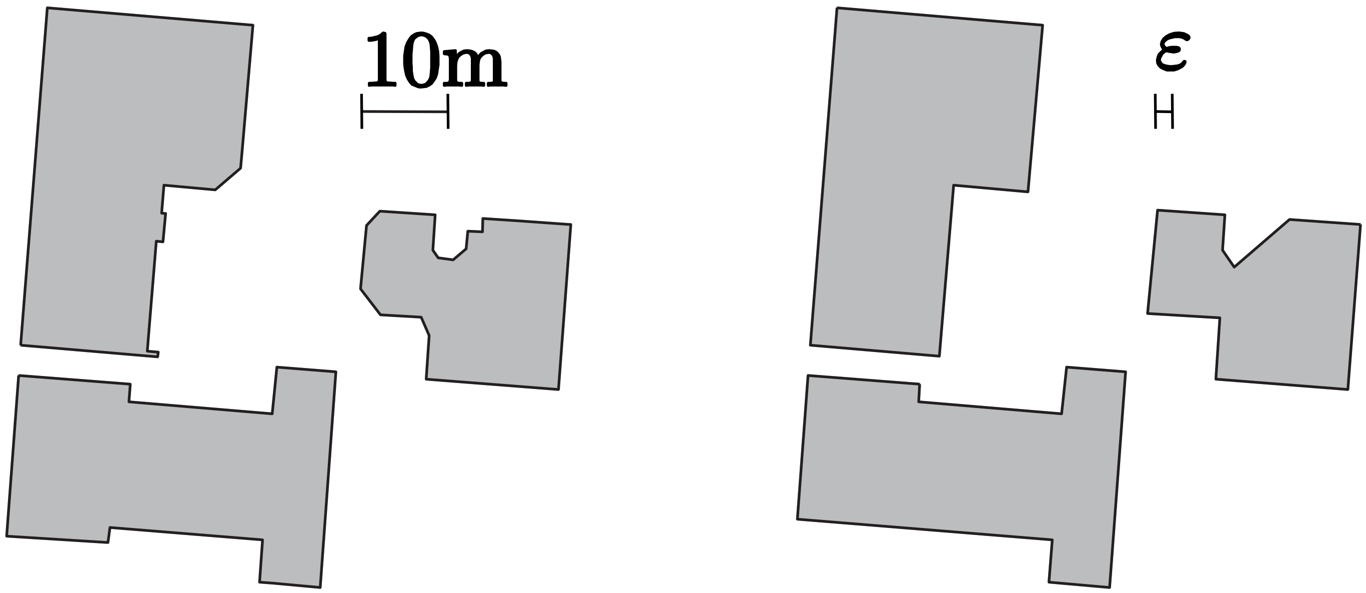
An Exact Approach by Integer Programming

A shortcut $s \in S$ that implies an intersection with edge $e \in E$ must only be selected together with a shortcut that omits or sufficiently shortens e.

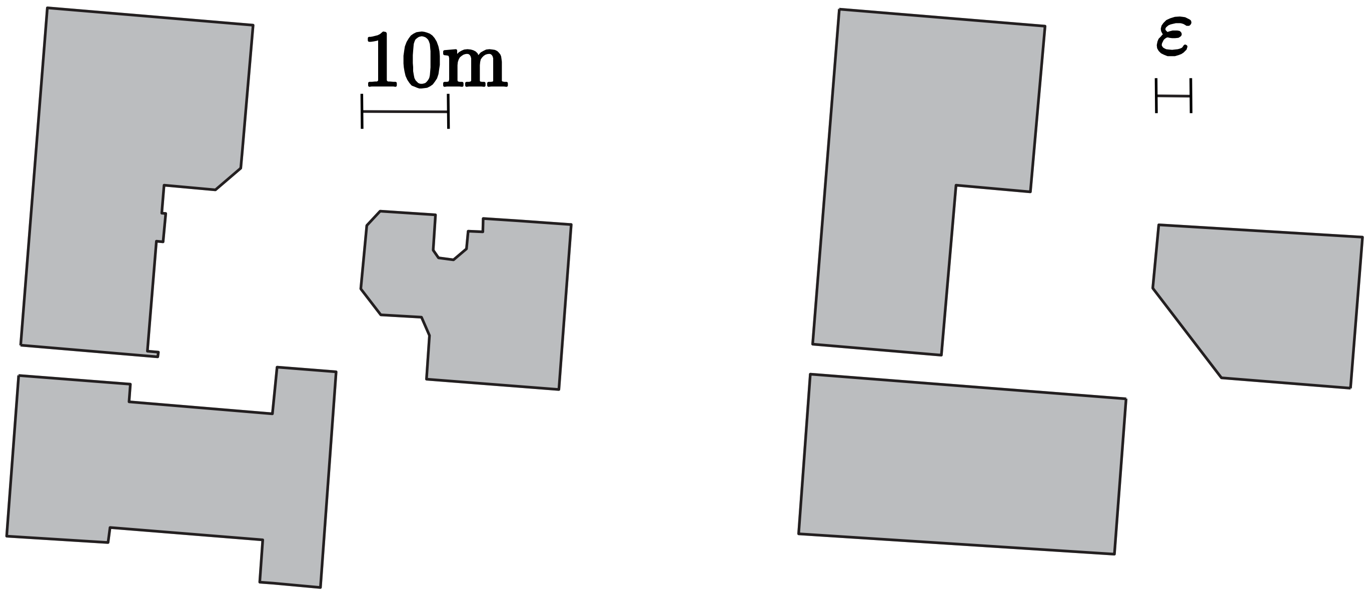
$$x_s \leq \sum_{t \in S_{s,e}} x_t$$



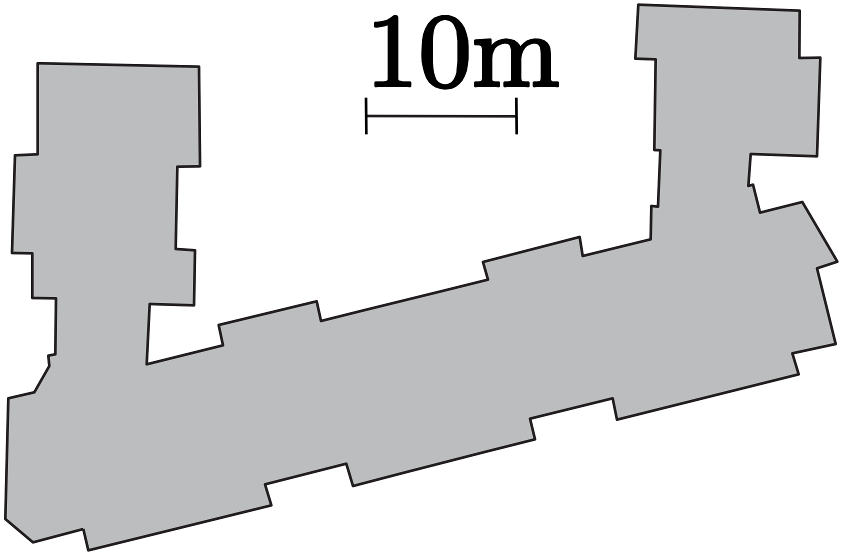
Results

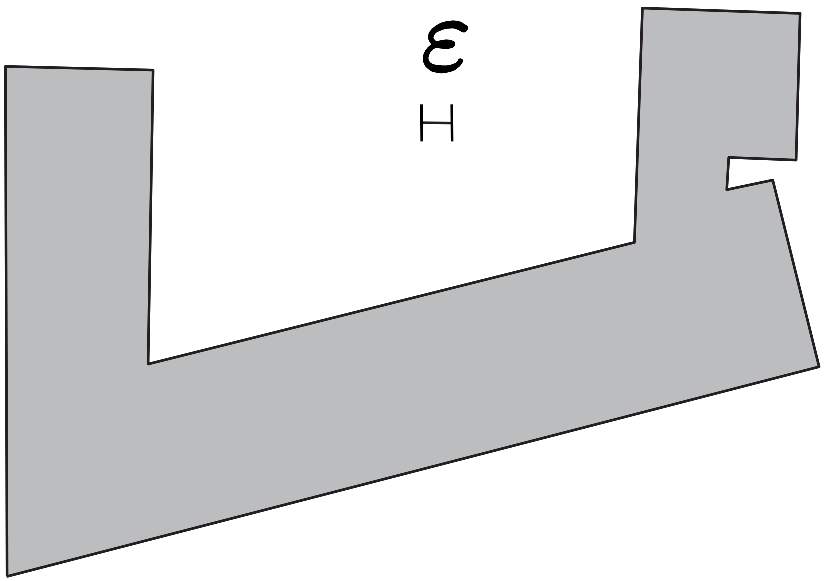


Results



Results

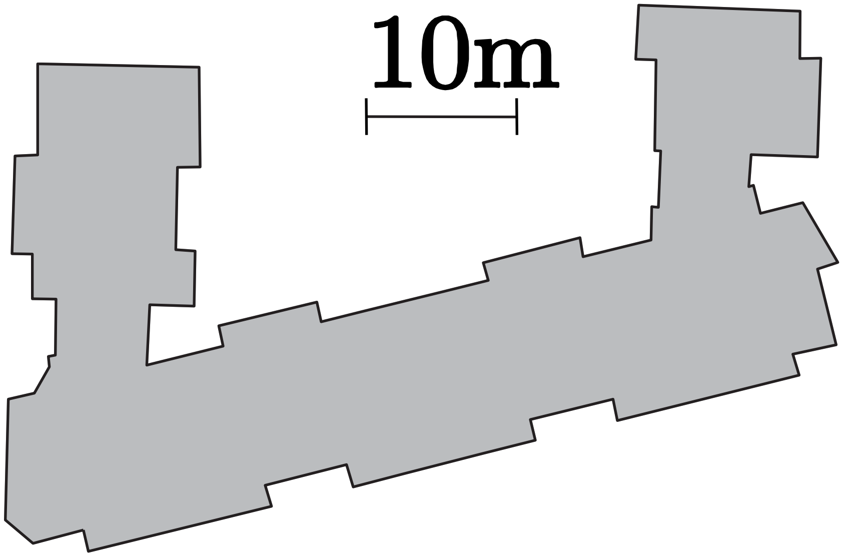


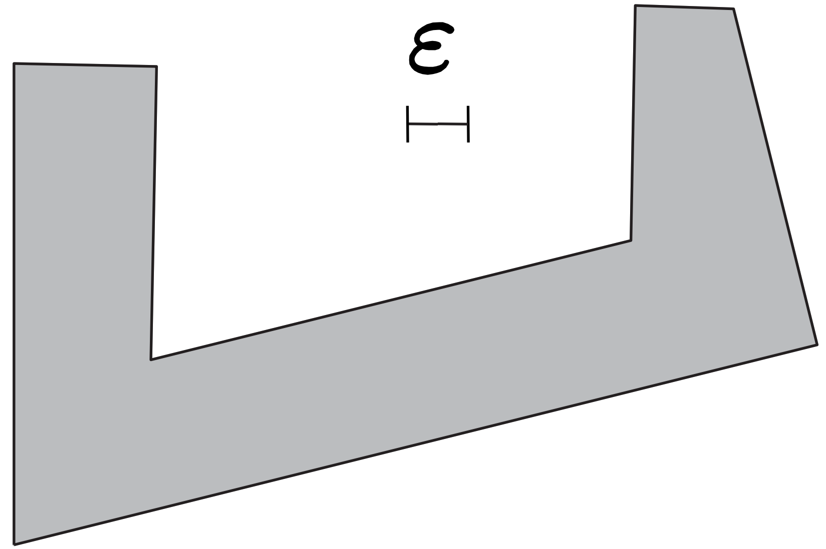


Processing time:

CPLEX	0.01s
lp_solve	0.22s

Results

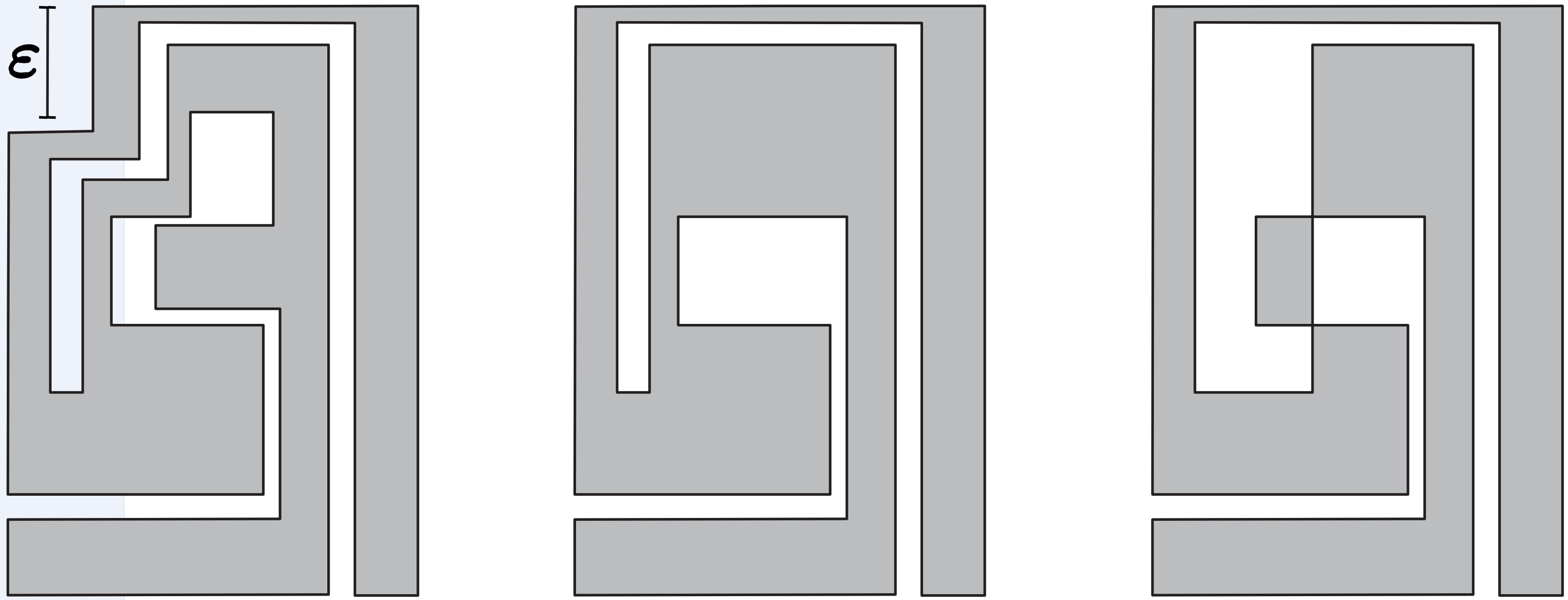




Processing time:

CPLEX	0.01s
lp_solve	0.17s

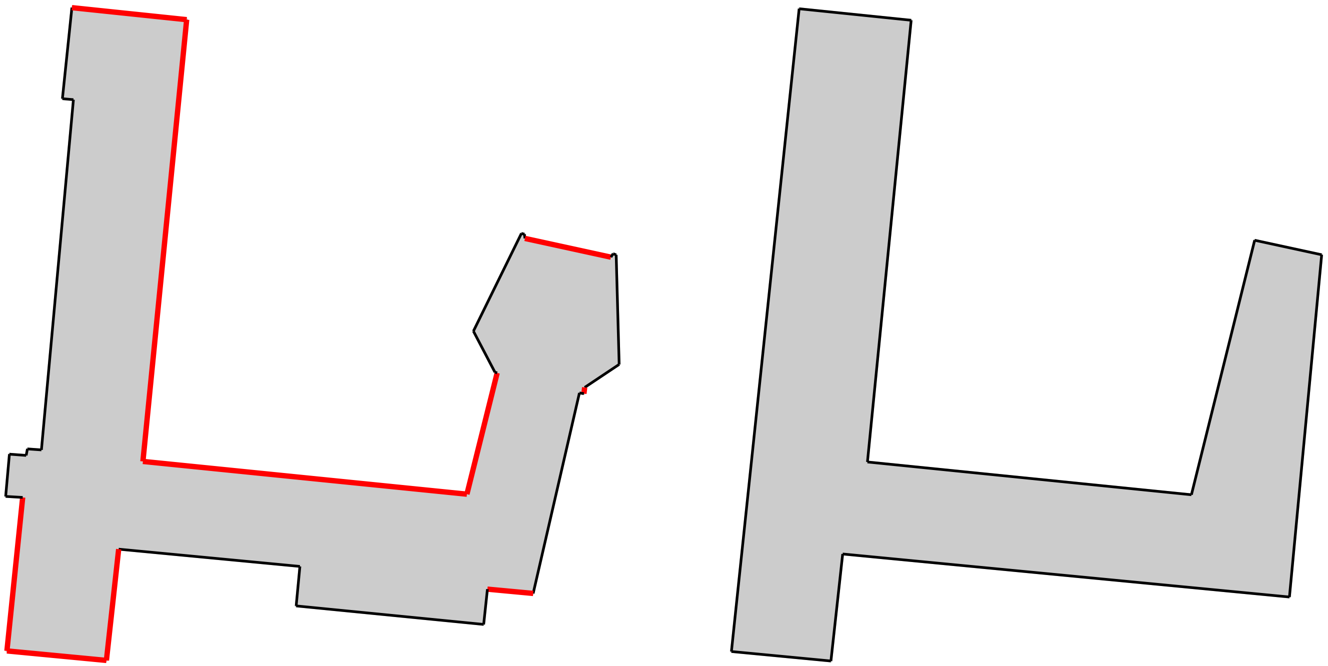
Results



Exact solution

Solution of relaxed problem

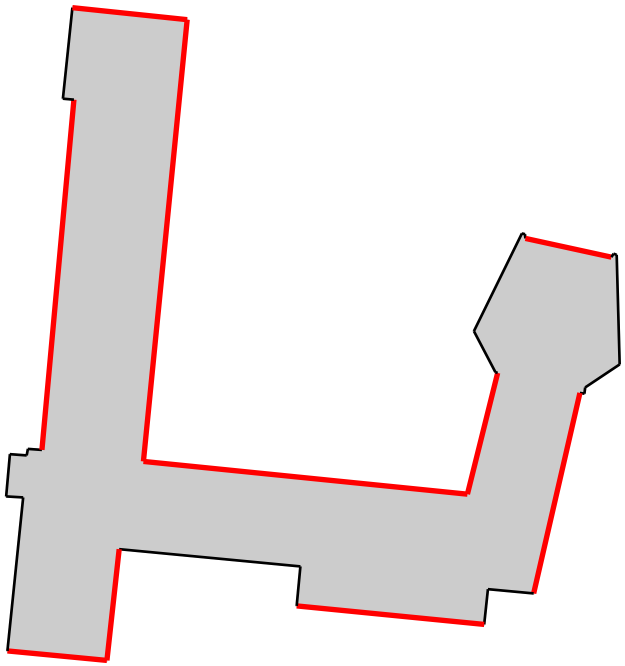
Results



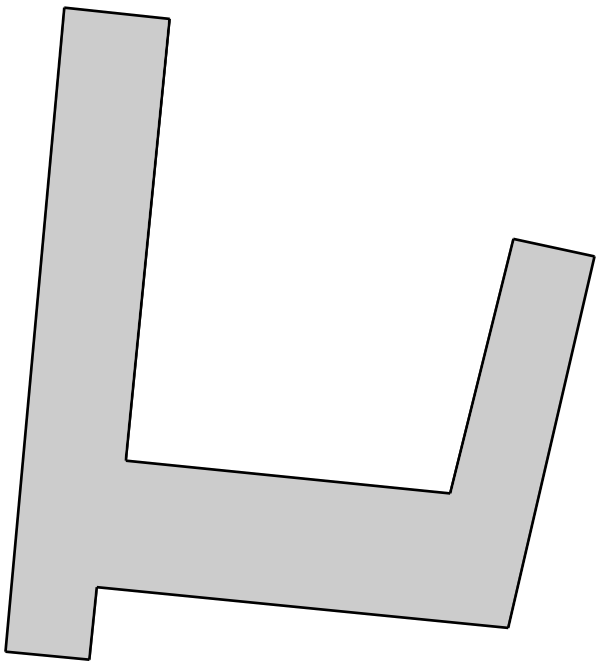
Input

Result with minimum number of vertices

Results



Input



Result of minimum cost when charging a high cost for selecting short edges

Conclusion

- We presented a new method for building simplification that yields results with a minimum number of line segments subject to several basic requirements.
- The method ensures a limited positional error ε .
- The method ensures a simple polygon as output.
- Future research is needed to find an appropriate cost function that better reflects the quality of a generalized building.

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