Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Computing Height-Optimal Tangles Faster

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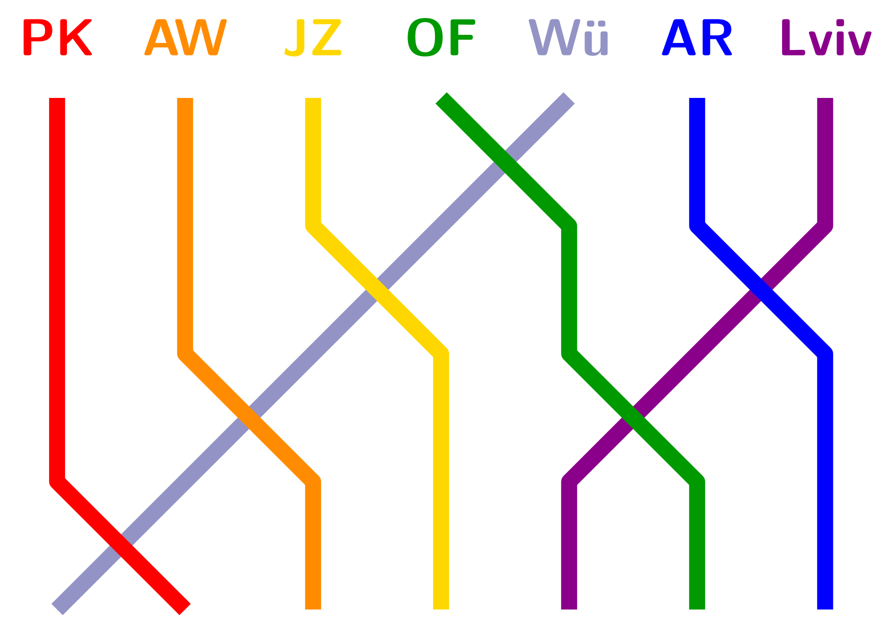
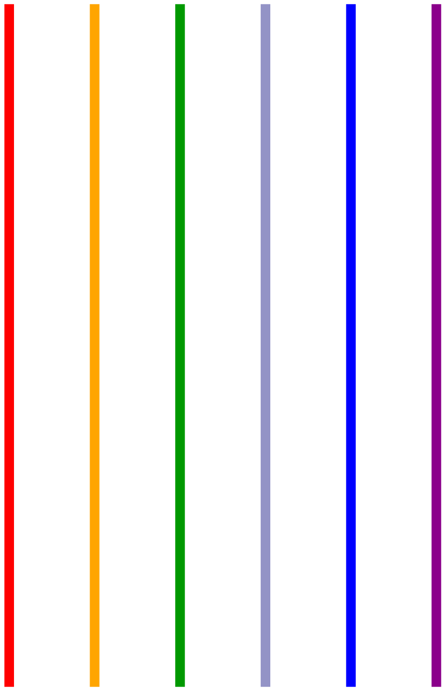


Introduction

Given a set of n
y-monotone wires



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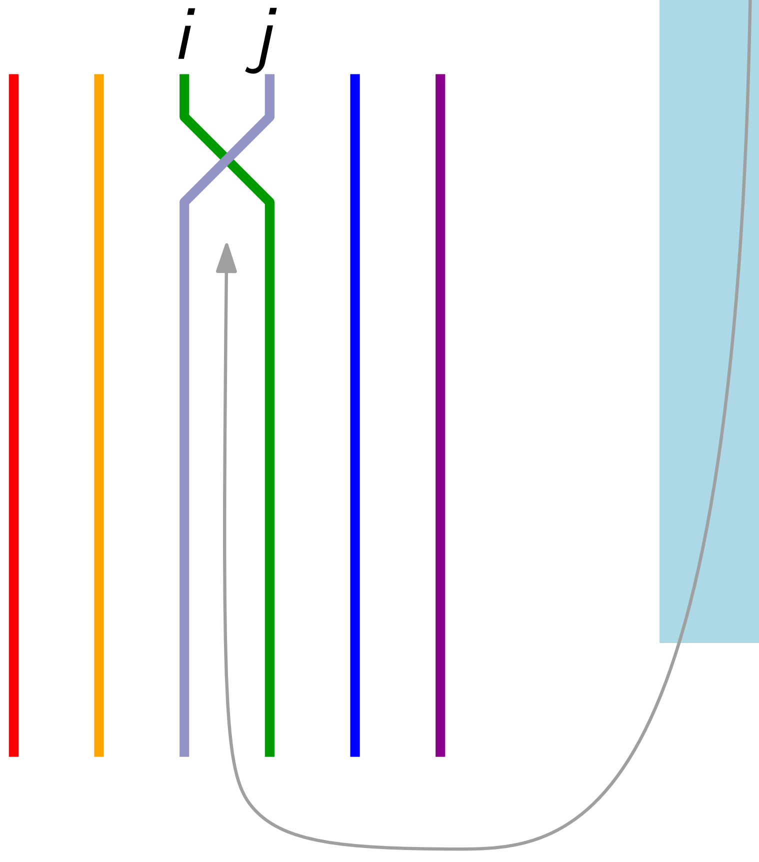

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swap ij

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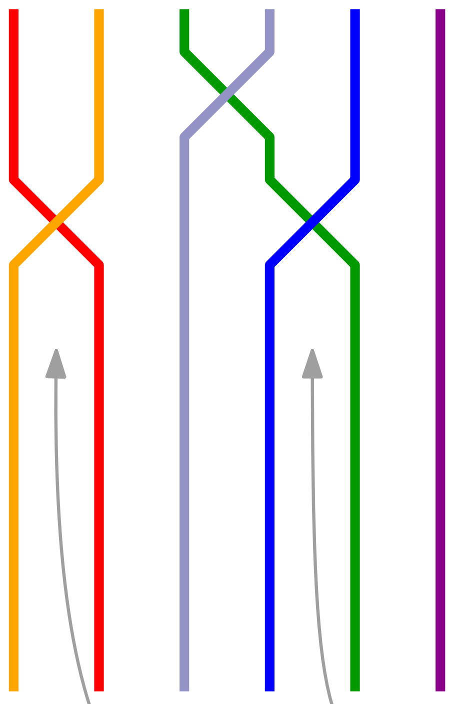

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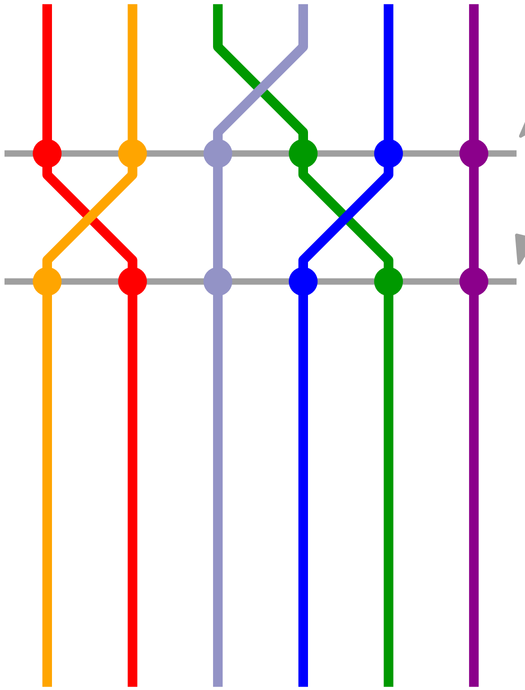

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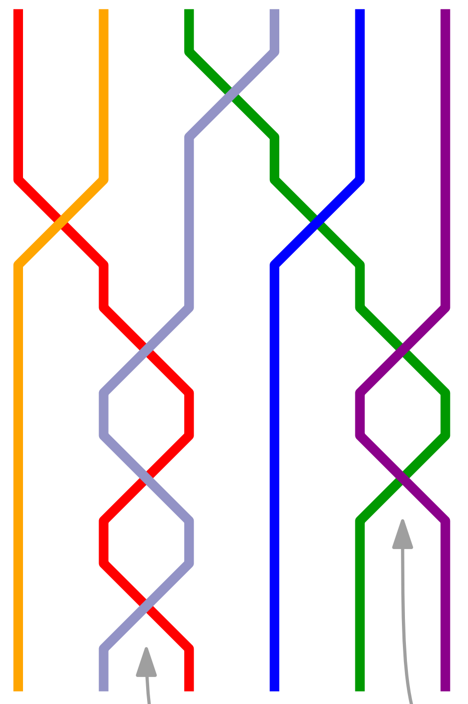

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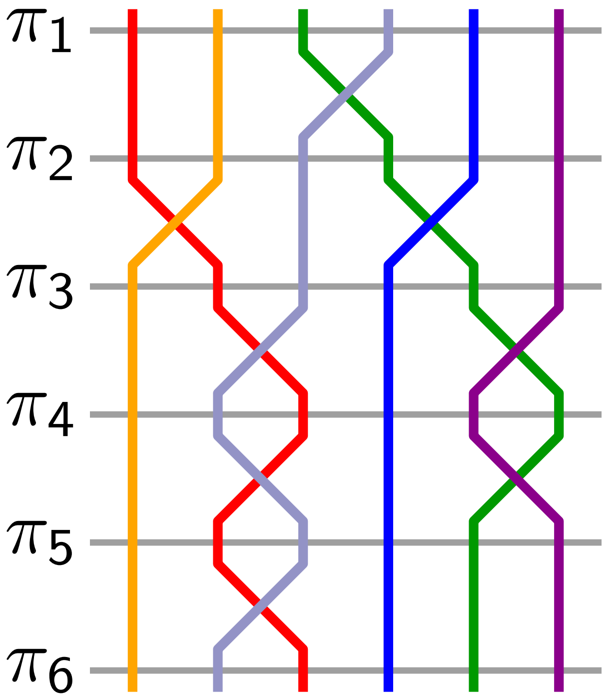

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tangle T of *height* $h(T)$

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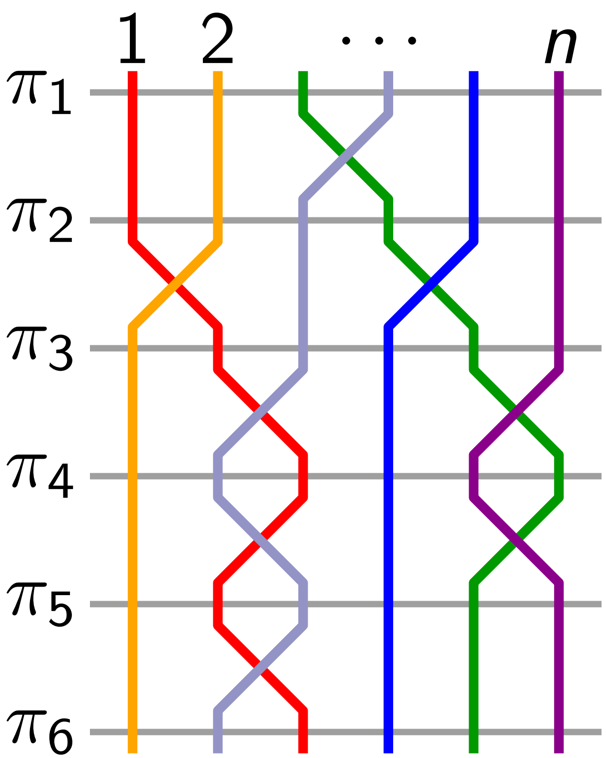

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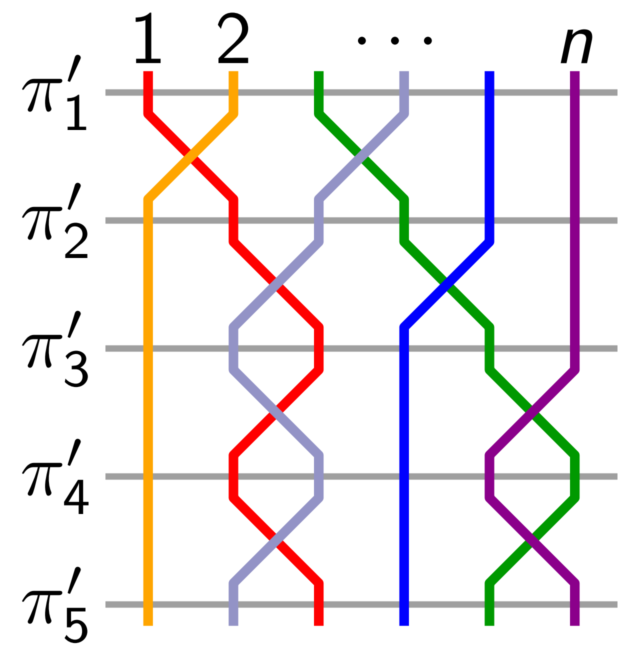

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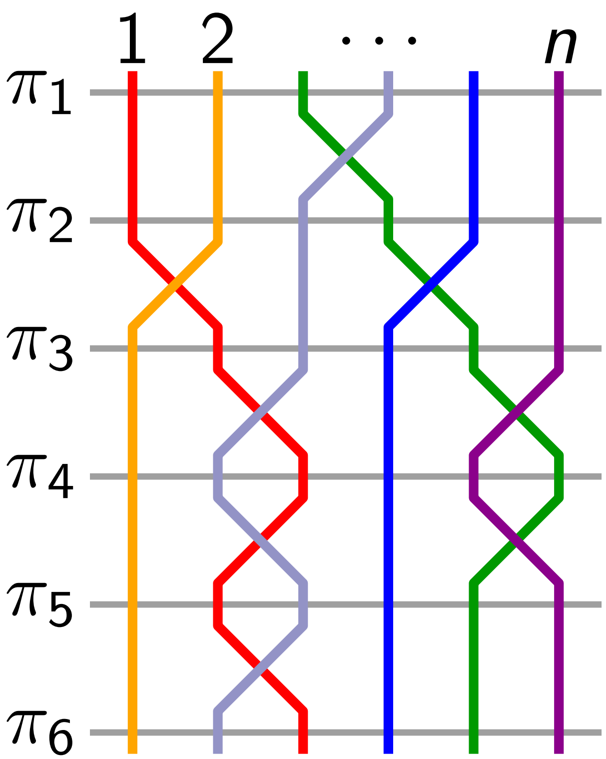

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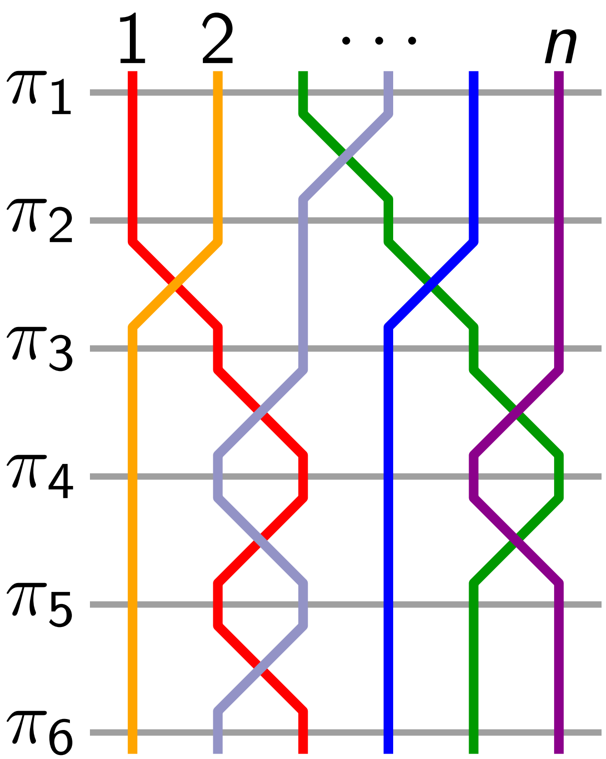

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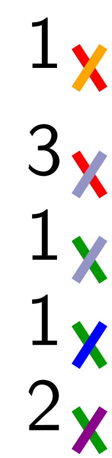

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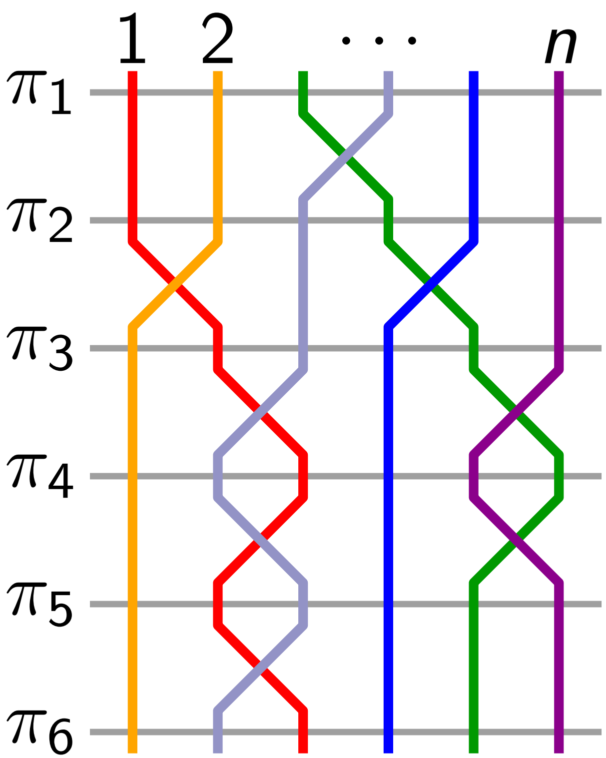

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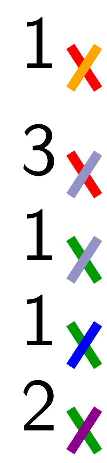

Tangle $T(L)$ *realizes* list L.

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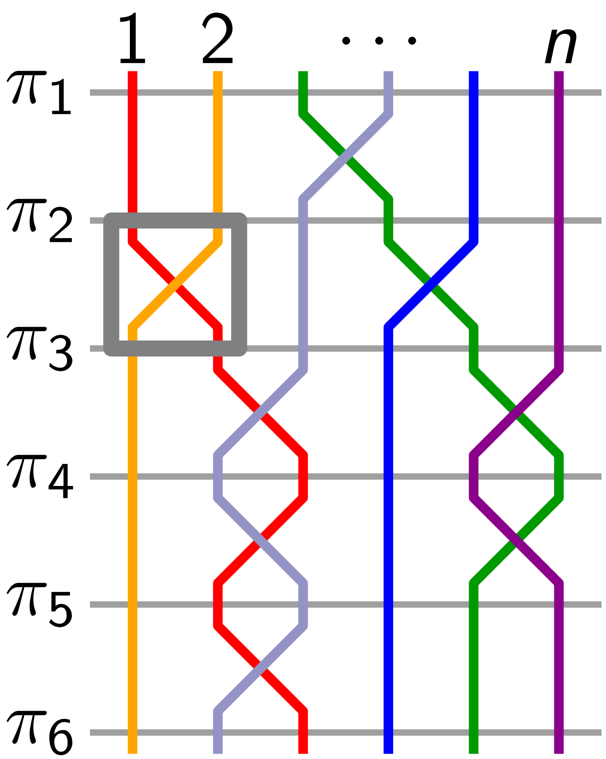

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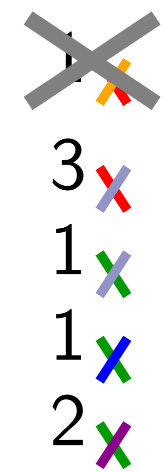

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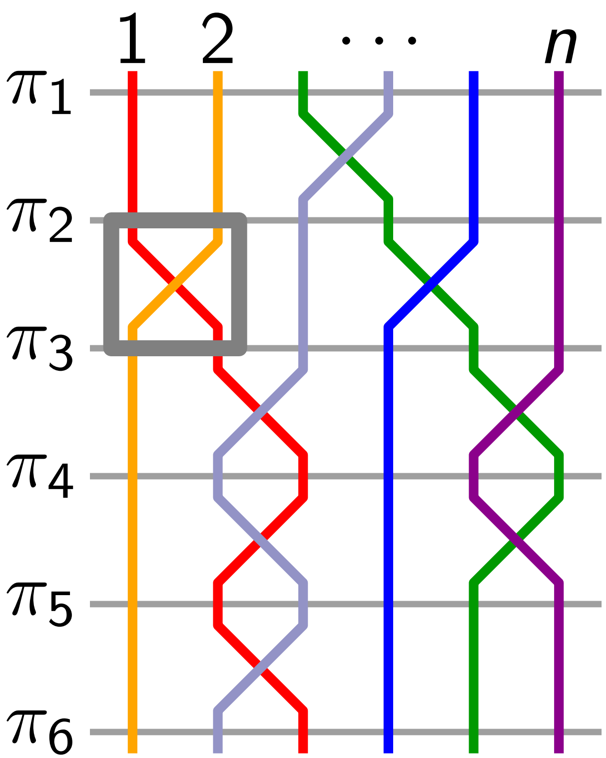

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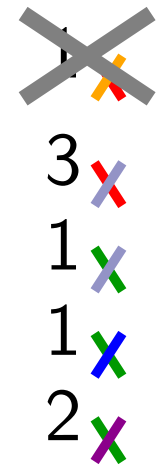

not *feasible*

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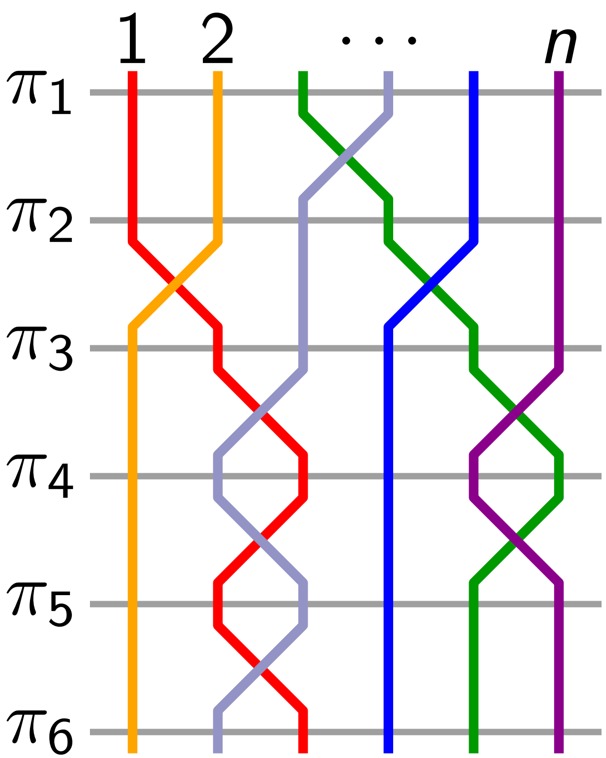

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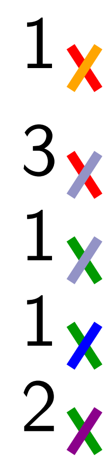
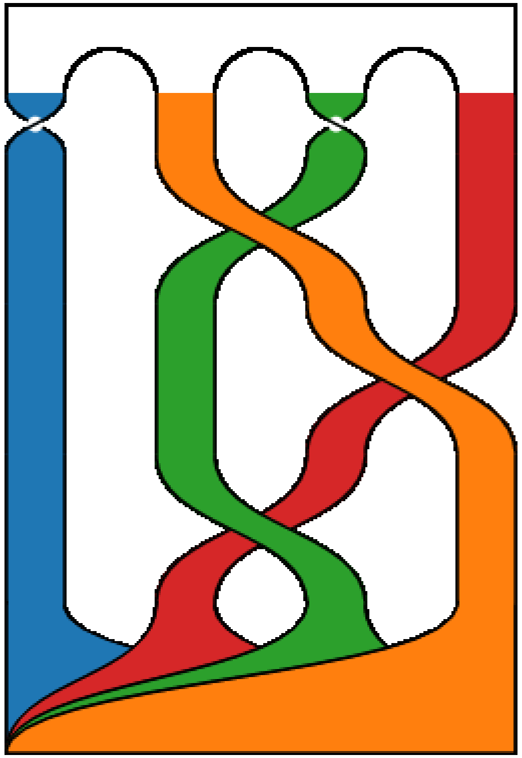


Tangle $T(L)$ *realizes* list L.

A tangle $T(L)$ is *height-optimal* if it has the minimum height among all tangles realizing the list L.

Related Work

- *Olszewski et al.* Visualizing the template of a chaotic attractor.
  GD 2018



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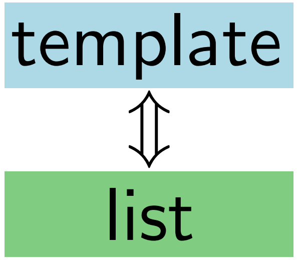

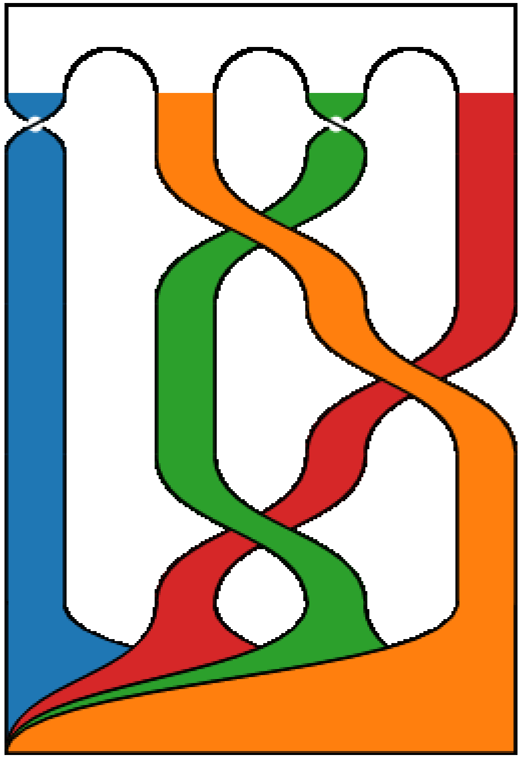

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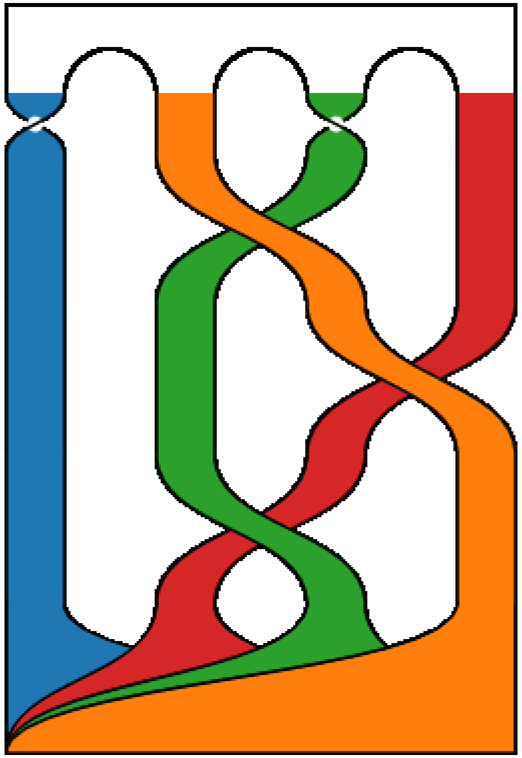
Algorithm for finding optimal tangles



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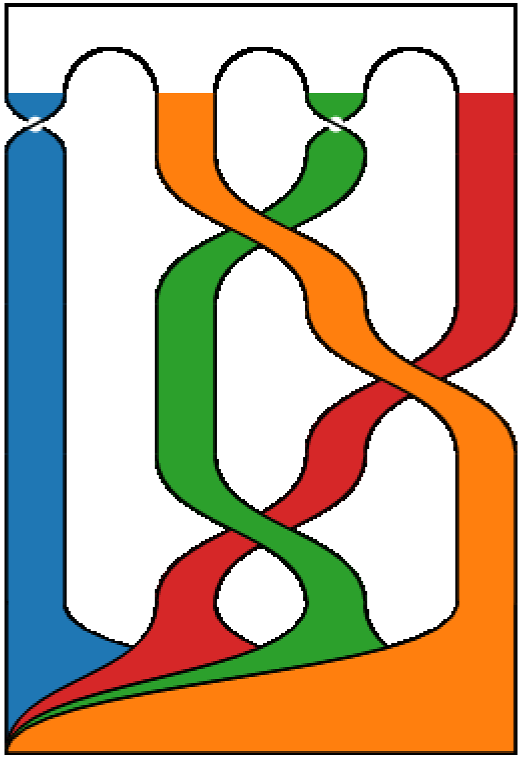
template
$\Updownarrow$
list

Algorithm for finding optimal tangles	*Complexity* **?**



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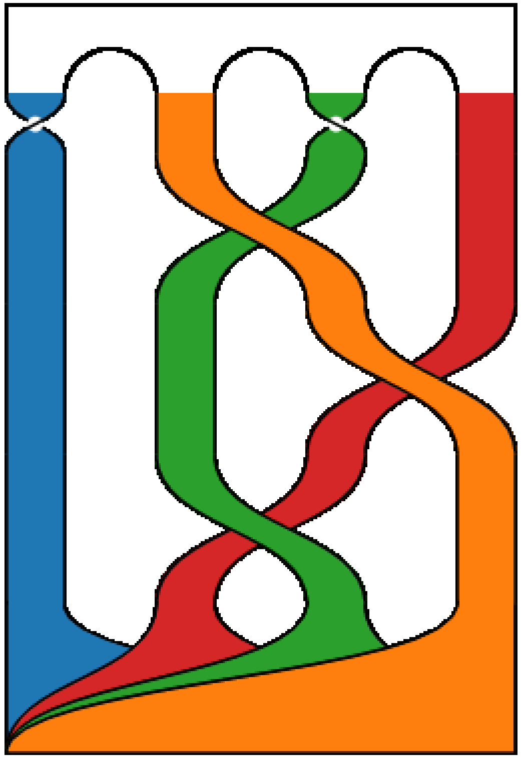

- *Wang.* Novel routing schemes for IC layout part I: Two-layer channel routing. DAC 1991

Given: initial and *final* permutations

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template
⇕
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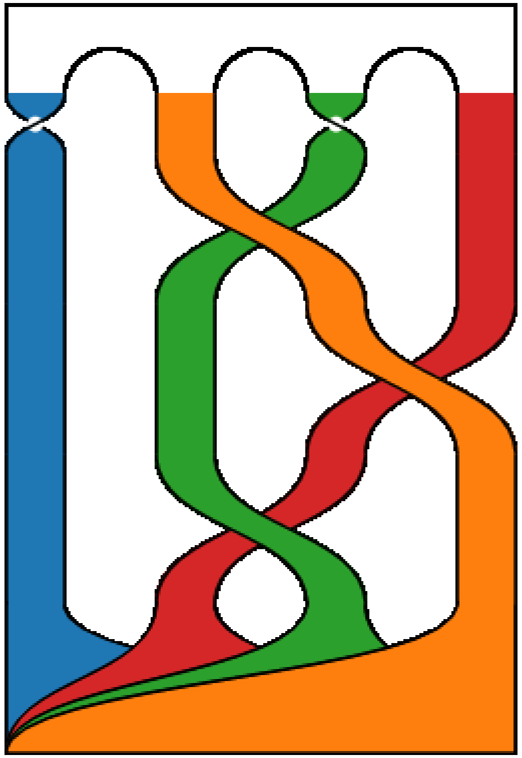

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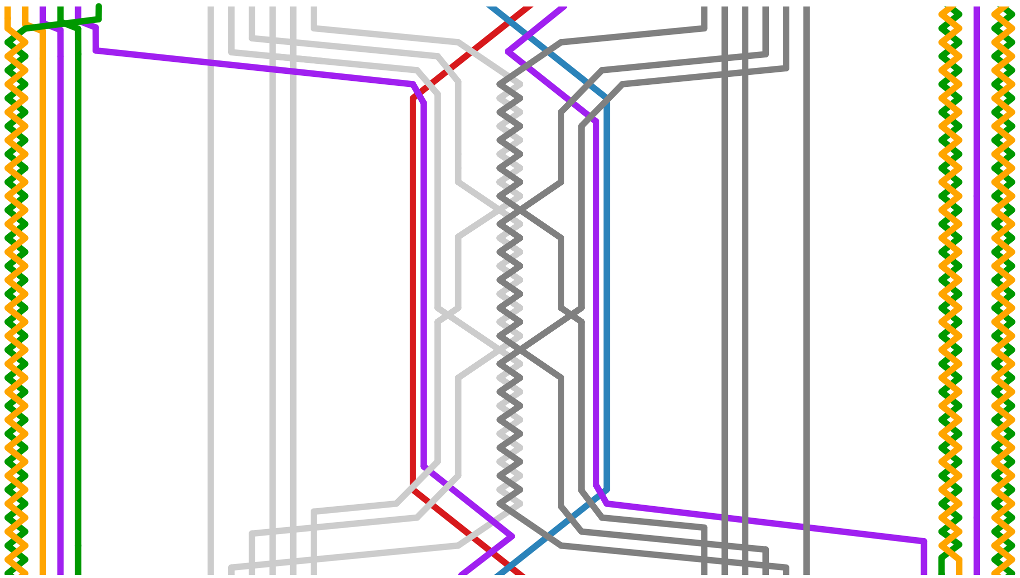
Given: initial and *final* permutations

- *Bereg et al.* Drawing Permutations with Few Corners. GD 2013

Objective: minimize the number of *bends*

Overview

- **Complexity:** NP-hardness by reduction from 3-PARTITION.



- **New algorithm:** using dynamic programming; asymptotically faster than [Olszewski et al., GD'18].

$$O\left(\frac{\varphi^{2|L|}}{5^{|L|/n}} n\right) \longrightarrow O\left(\left(\frac{2|L|}{n^2}+1\right)^{\frac{n^2}{2}} \varphi^n n\right)$$

- **Experiments:** comparison with [Olszewski et al., GD'18]

Complexity

Theorem.
TANGLE-HEIGHT MINIMIZATION is NP-hard.

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Given: Multiset A of $3m$ positive integers.

Complexity

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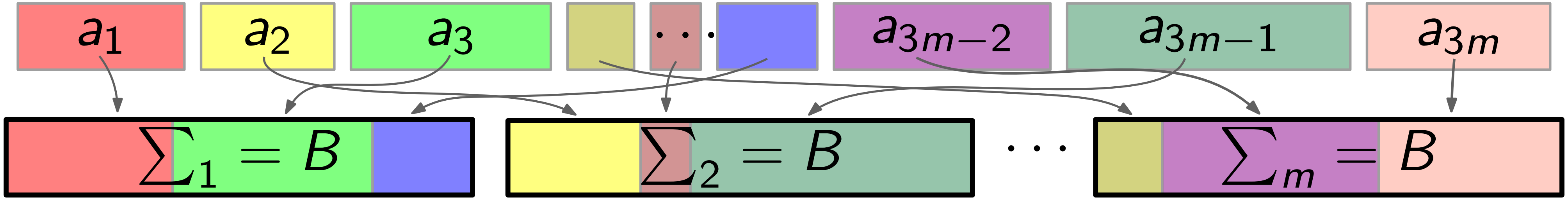
Proof.
Reduction from 3-PARTITION

Given:	Multiset A of $3m$ positive integers.
Question:	Can A be partitioned into m groups of three elements s.t. each group sums up to the same value B?



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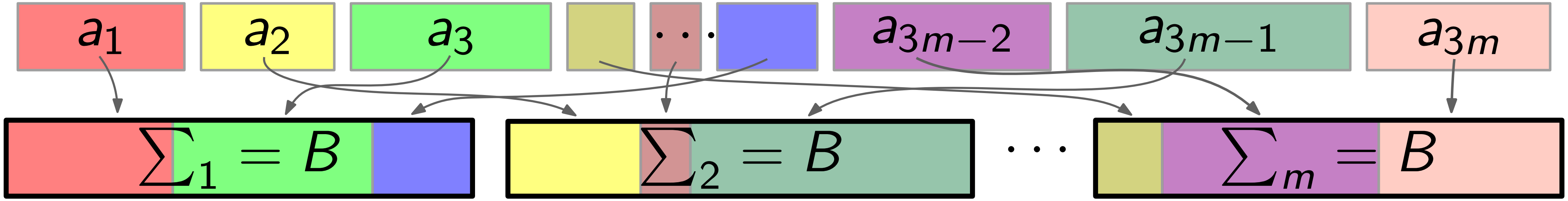

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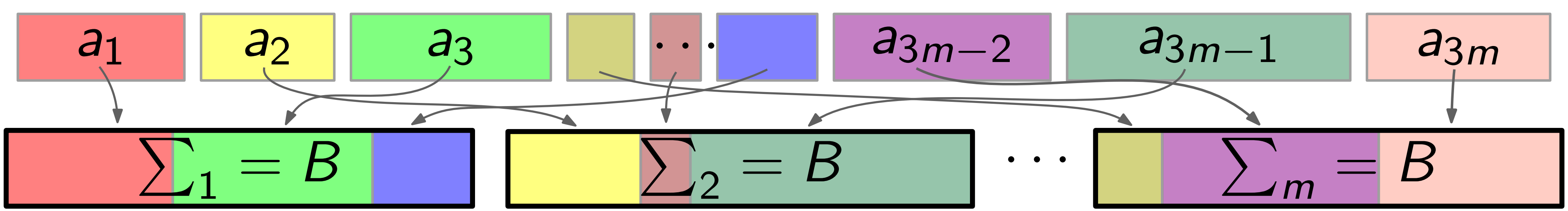

Given: Instance A of 3-PARTITION.

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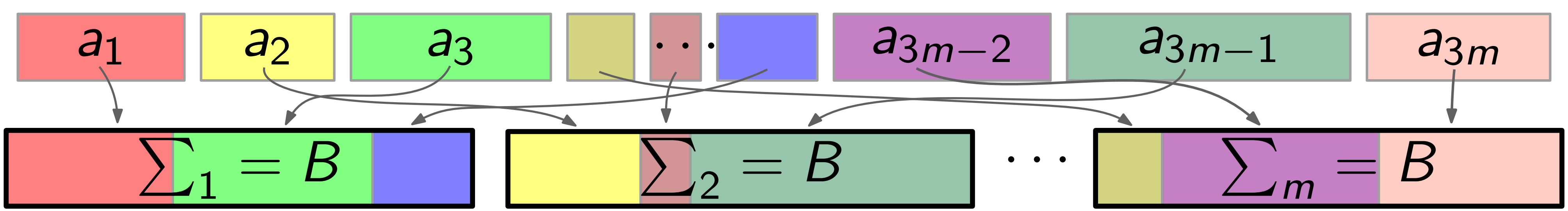

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Task: Construct L s.t. there is T realizing L with height at most $H = 2m^3(\sum A) + 7m^2$ iff A is a yes-instance.

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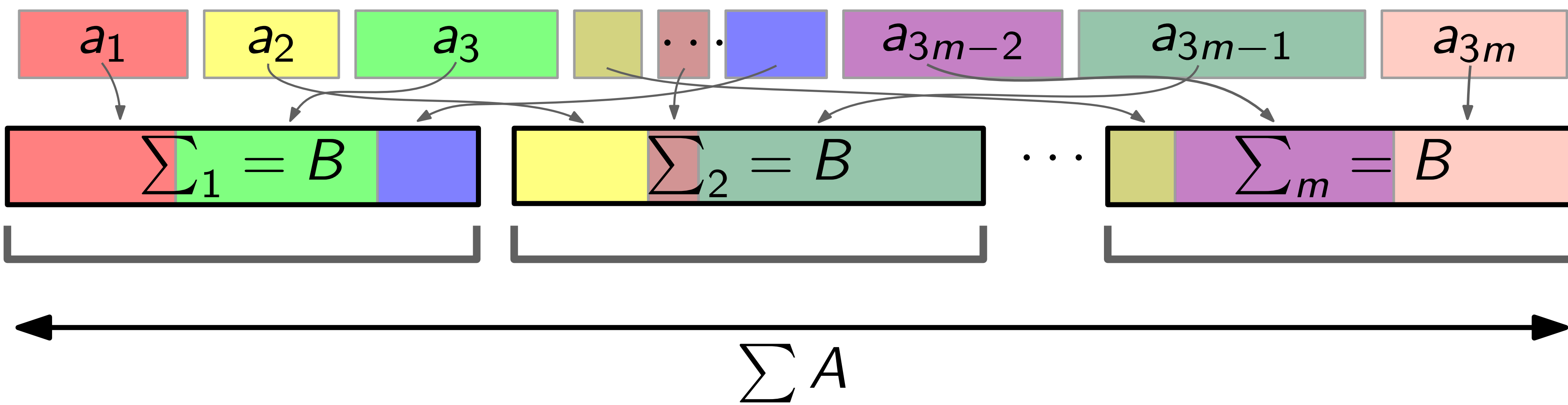

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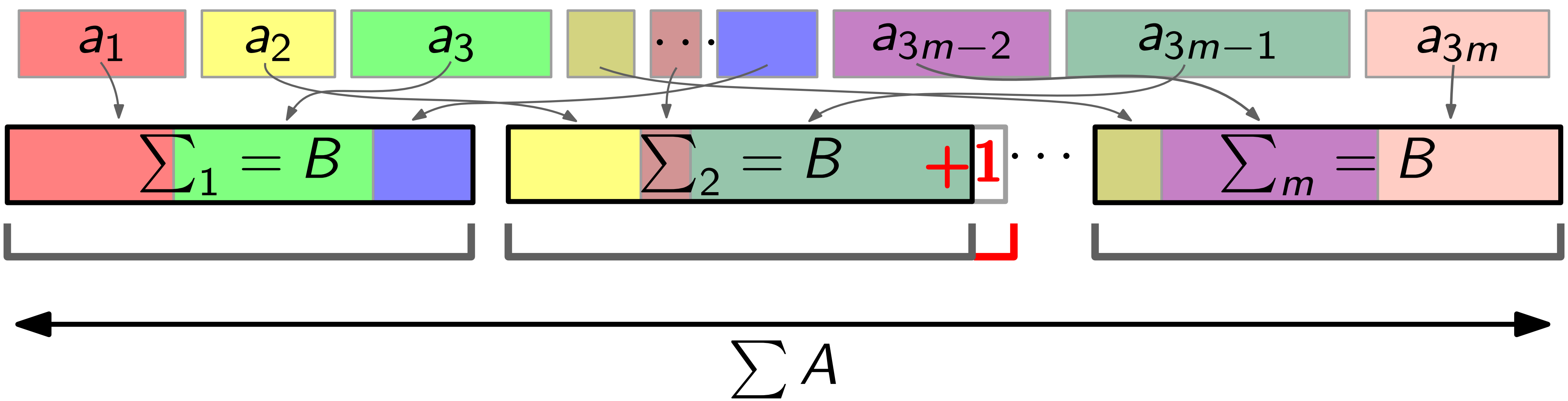

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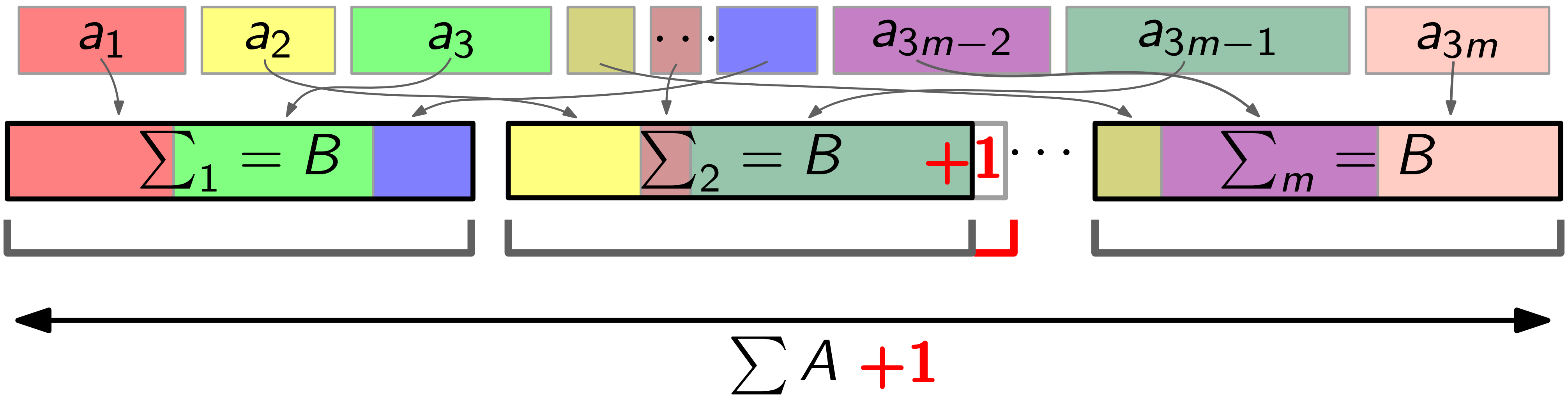

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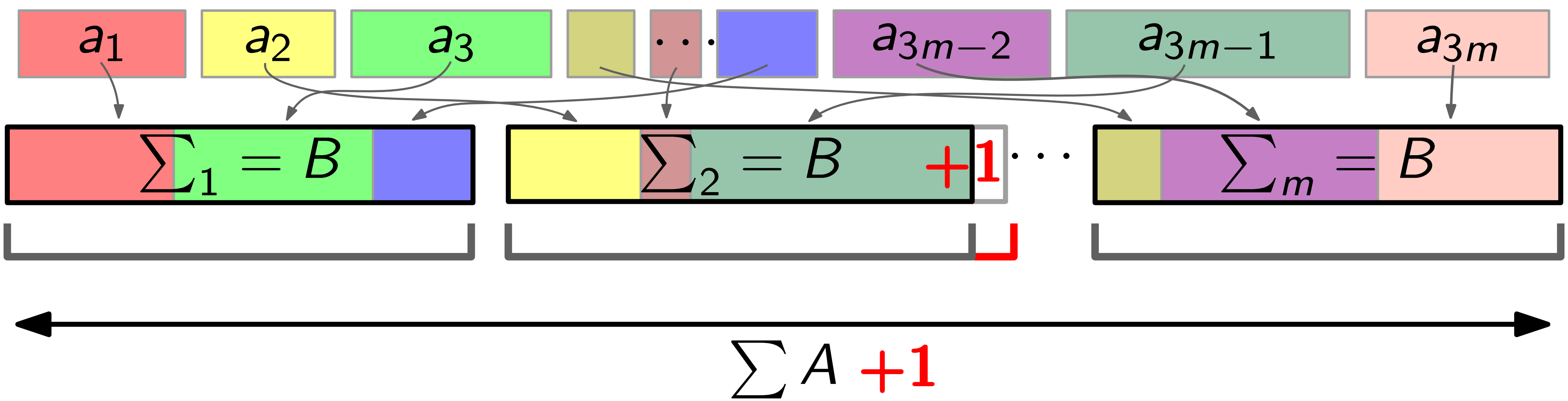

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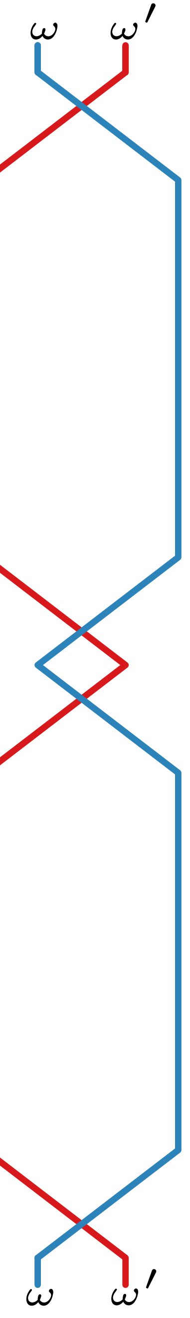
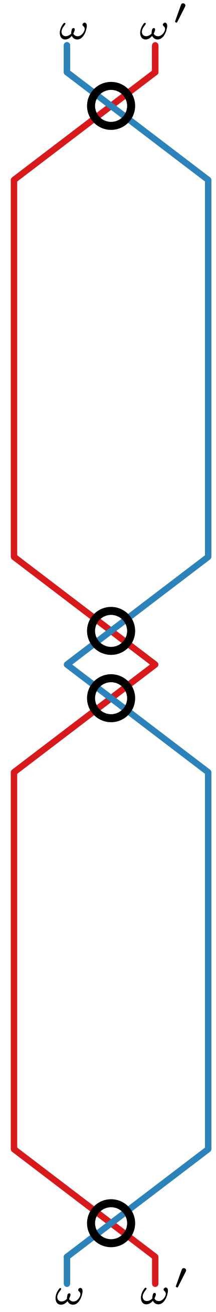
Transforming the Instance A into a List L



Transforming the Instance A into a List L

$2m$ swaps



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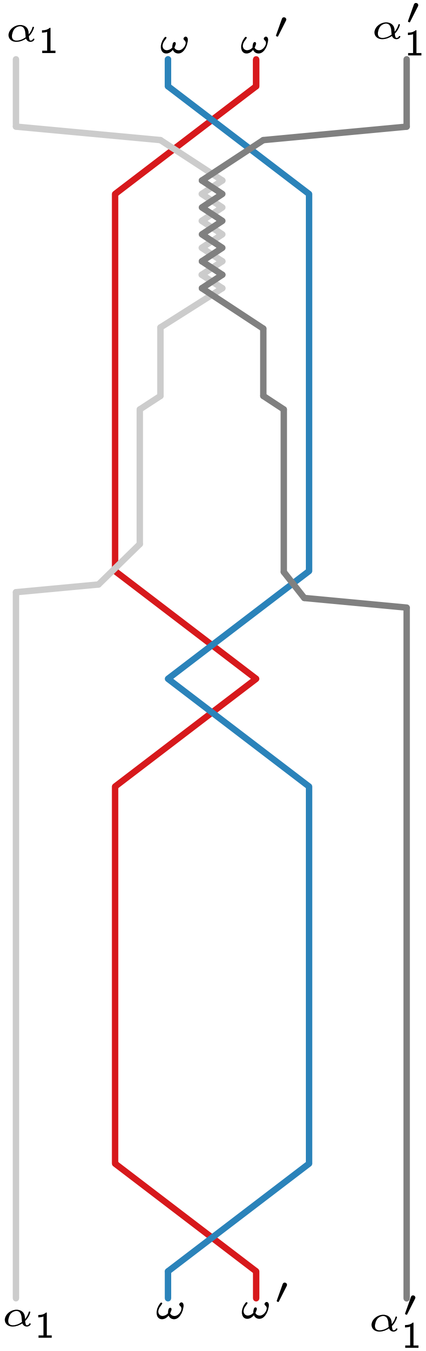

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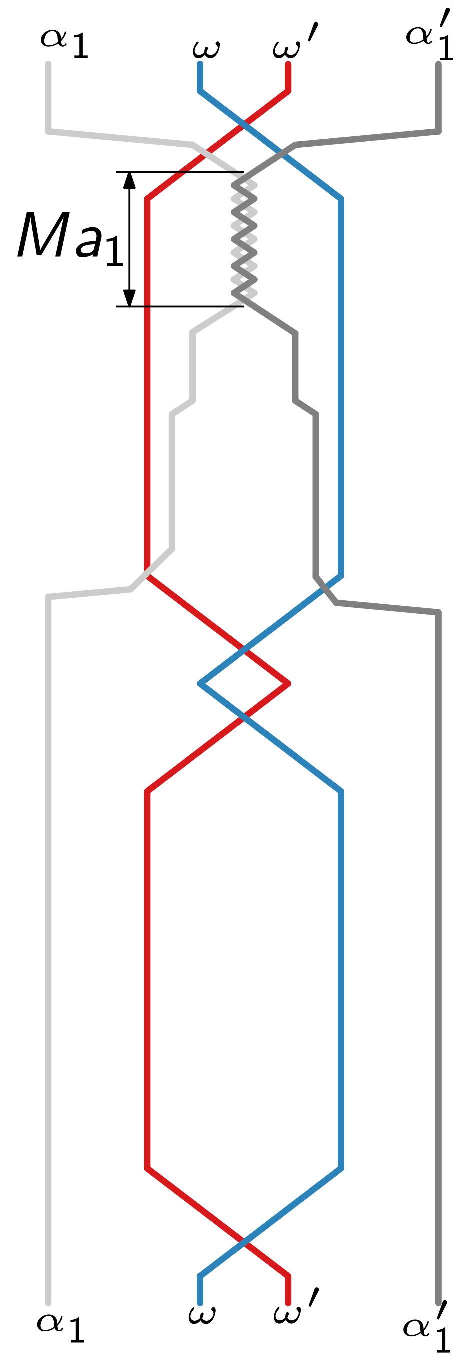
$$M = 2m^3$$



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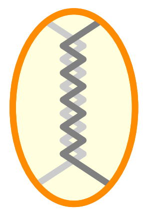

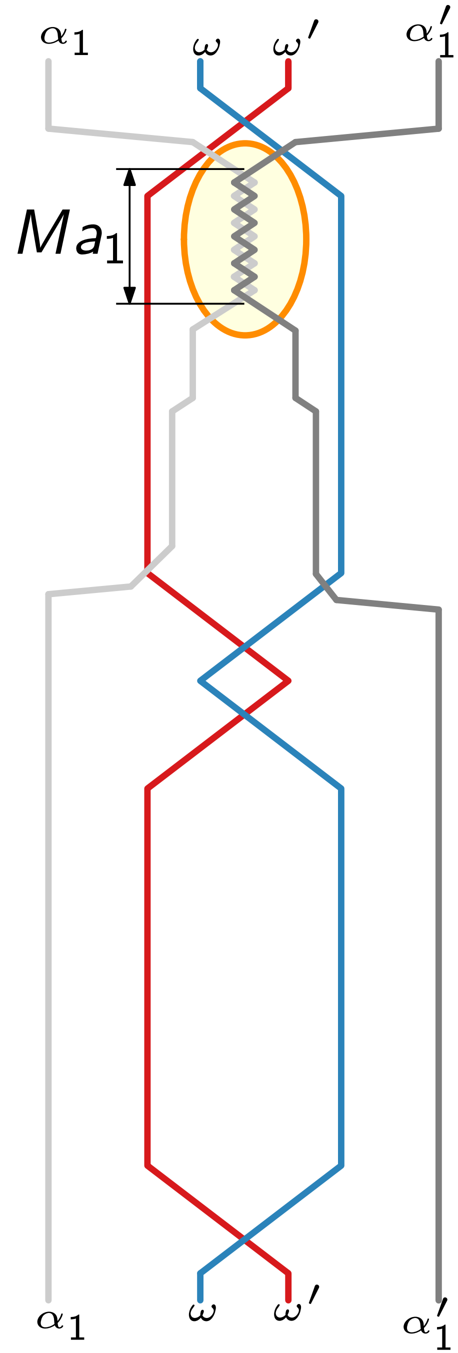

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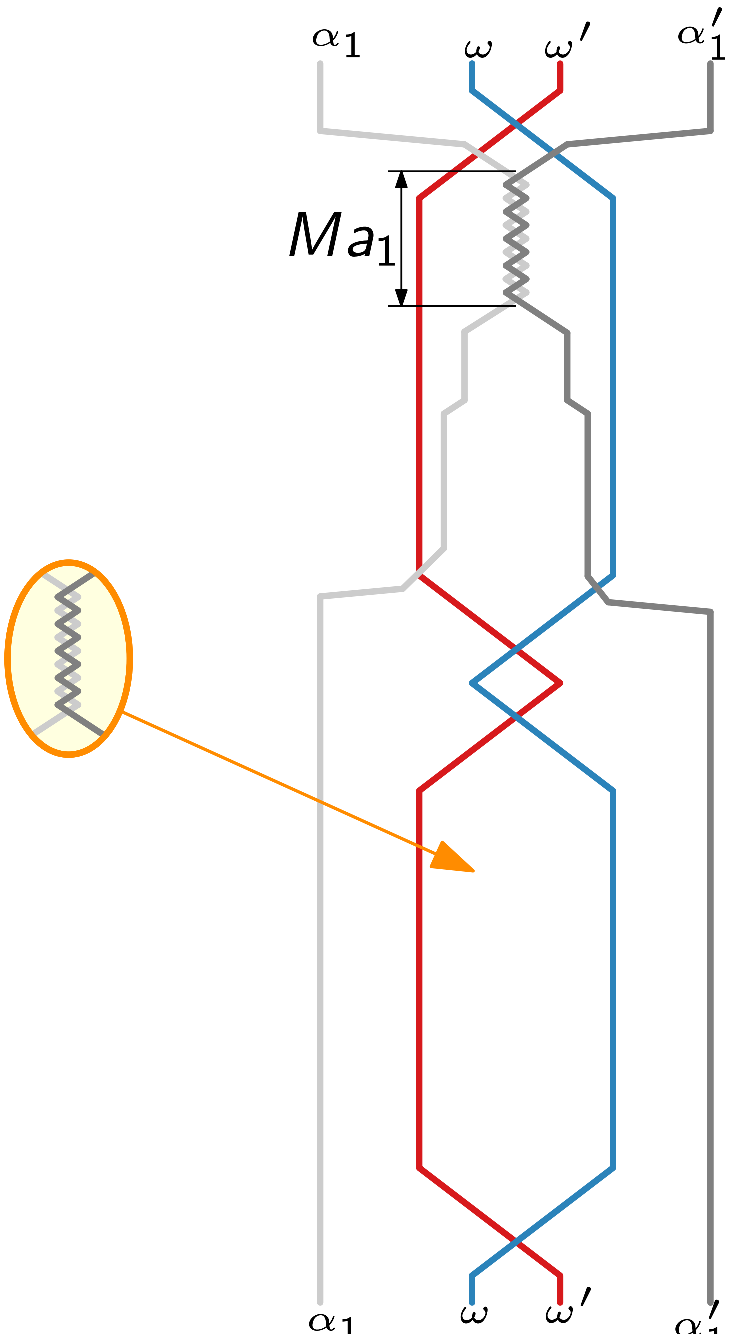

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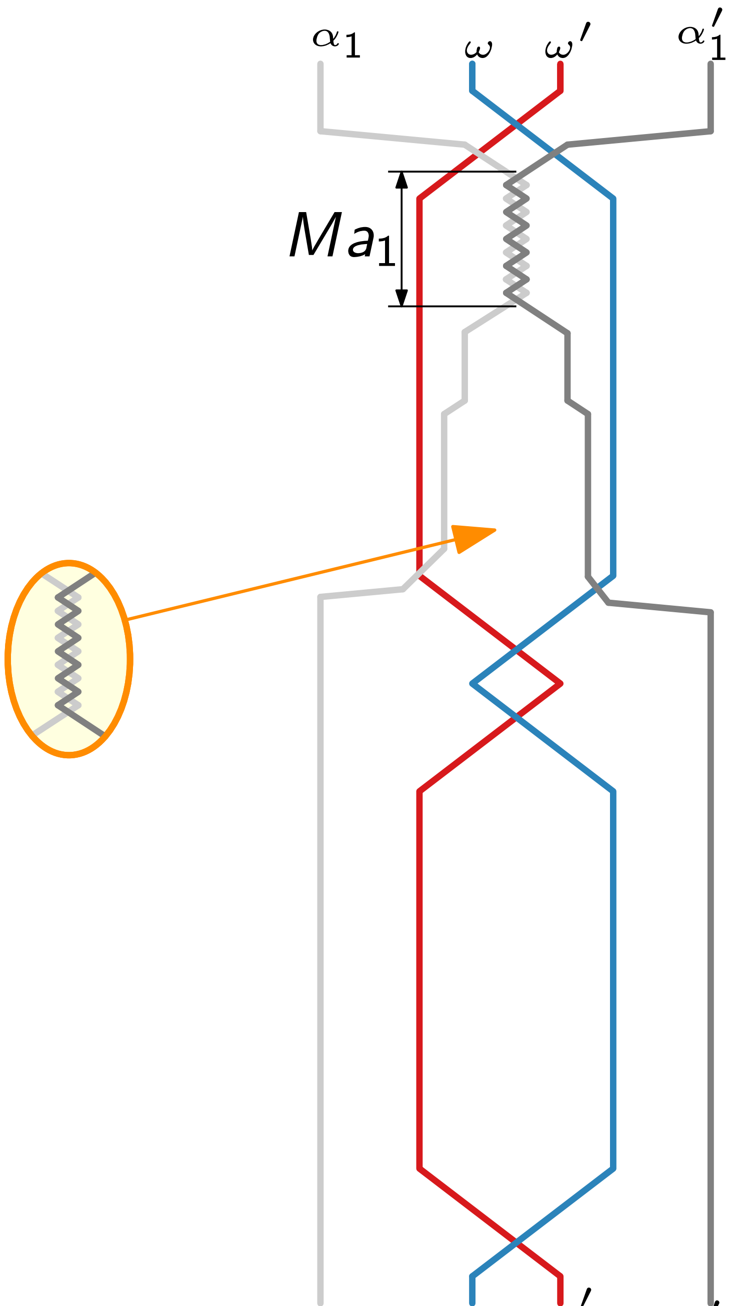

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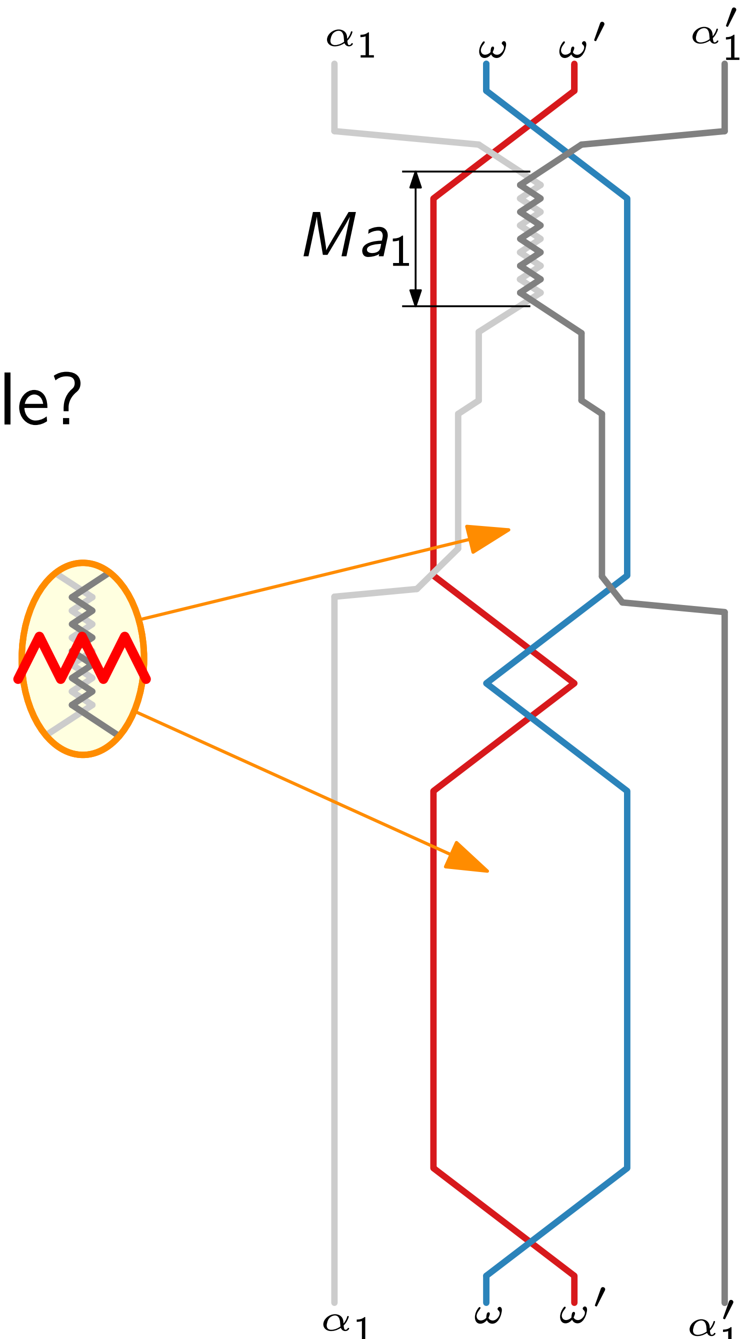
What is not possible?

split



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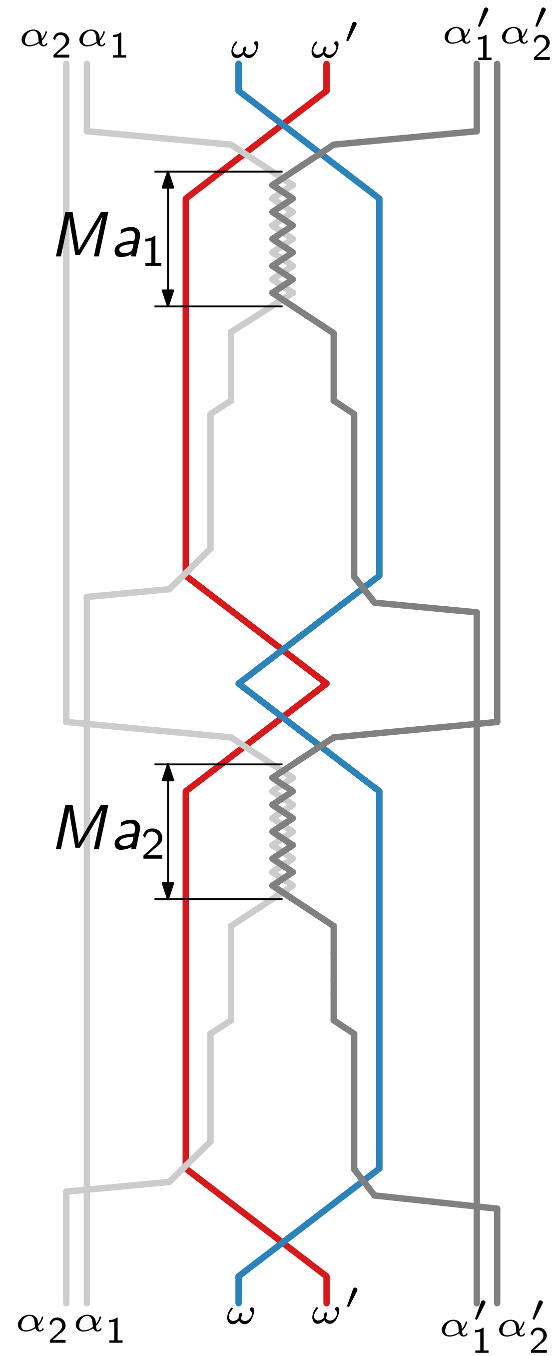
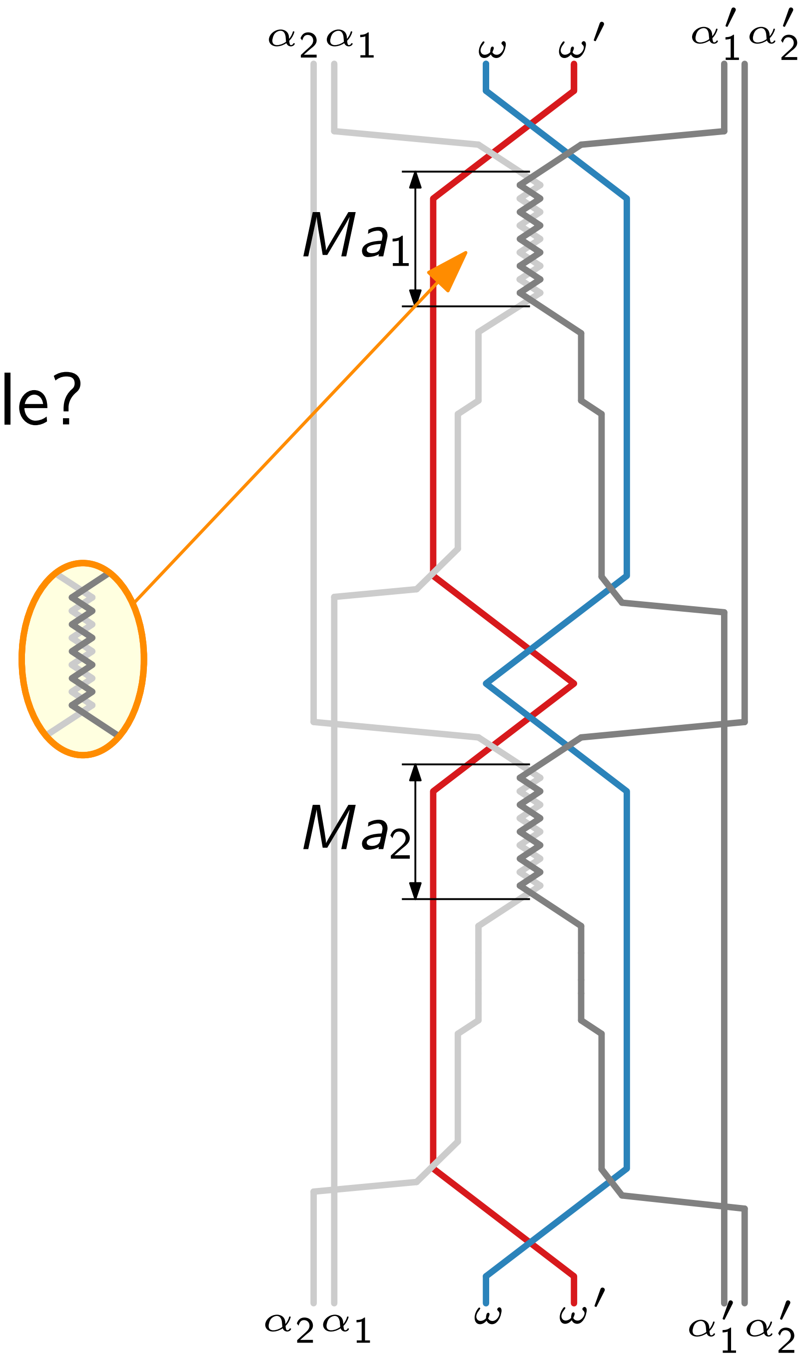


Transforming the Instance A into a List L

$M = 2m^3$

What is not possible?

put it on the same level
with other α-α' swaps



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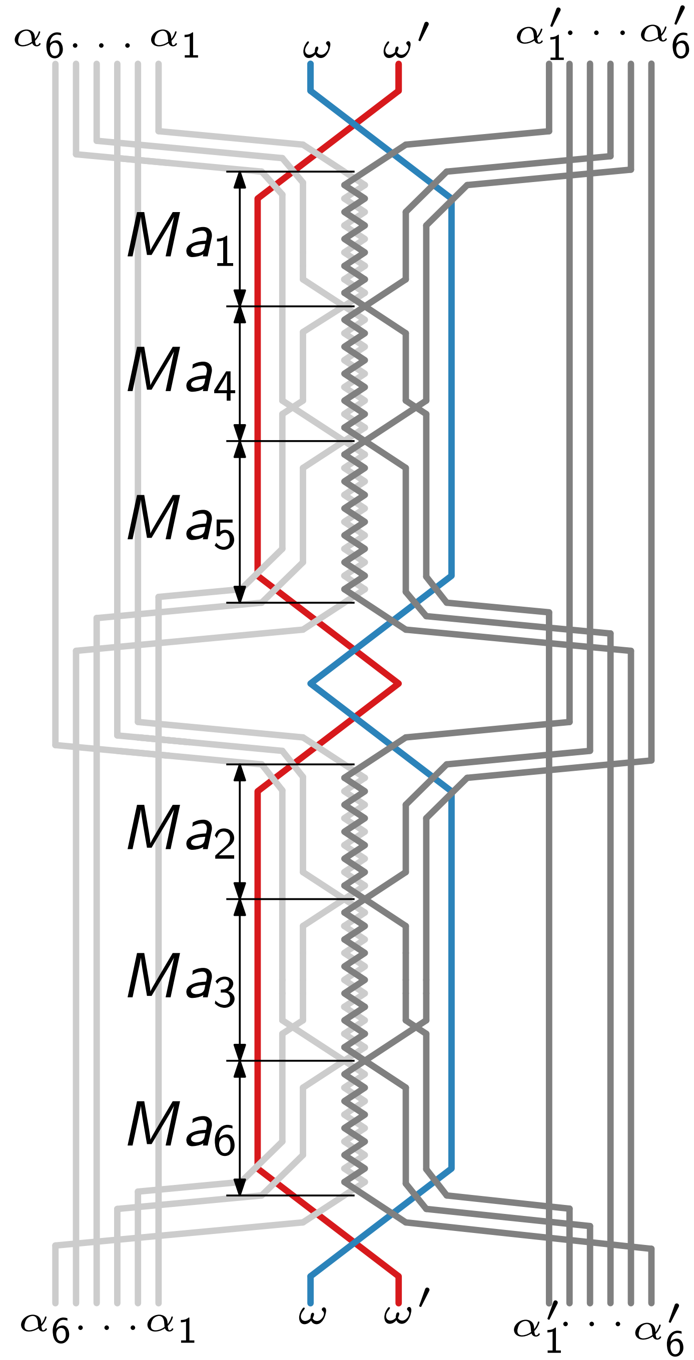
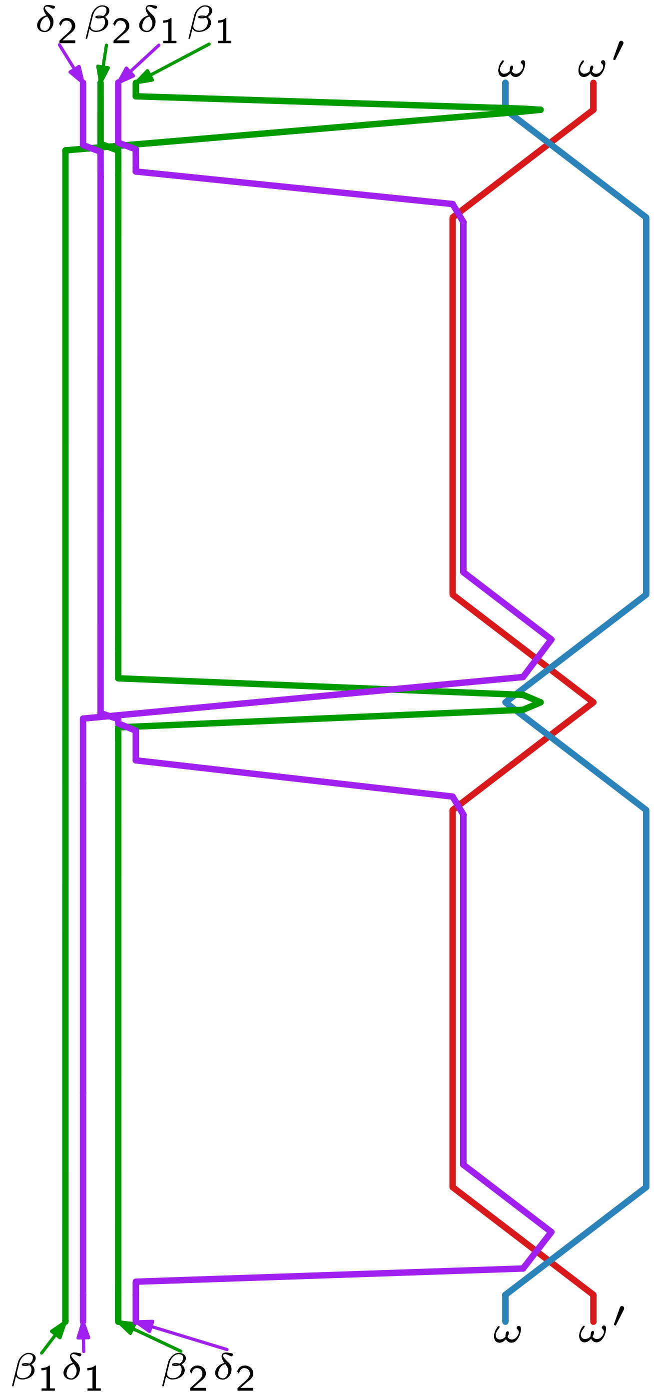


Making Sure That the “Pockets” Can’t Be Squeezed



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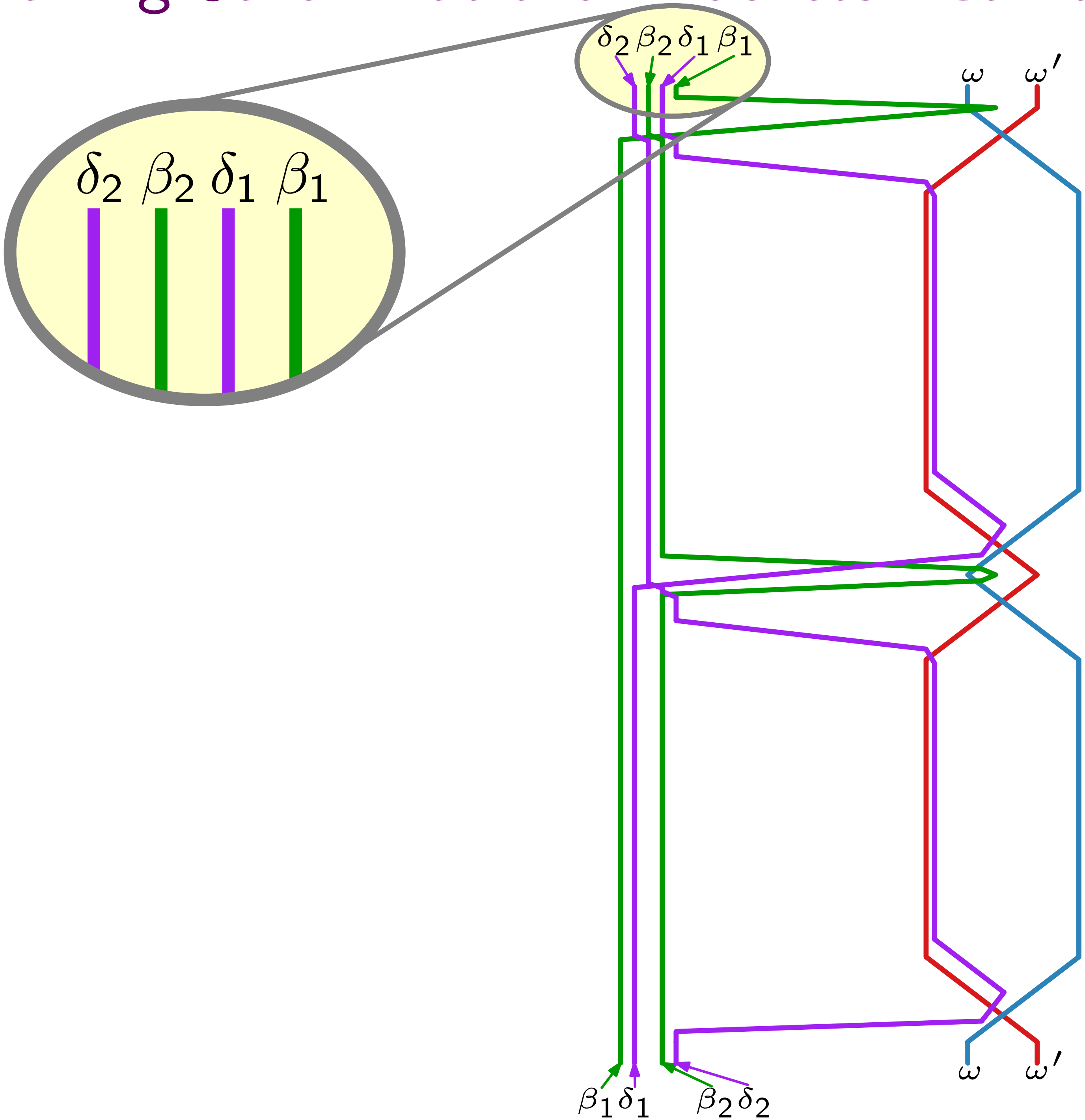

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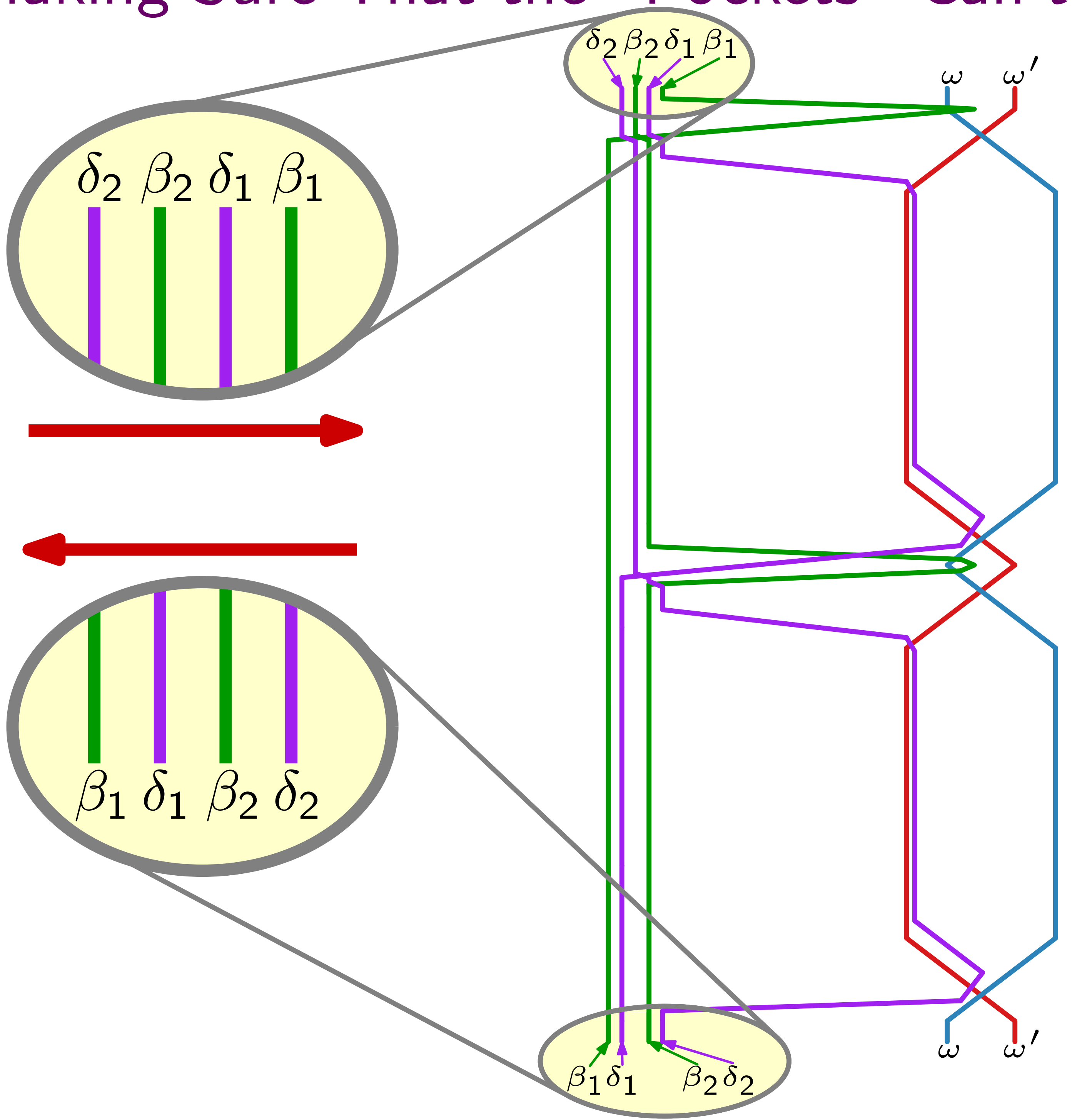

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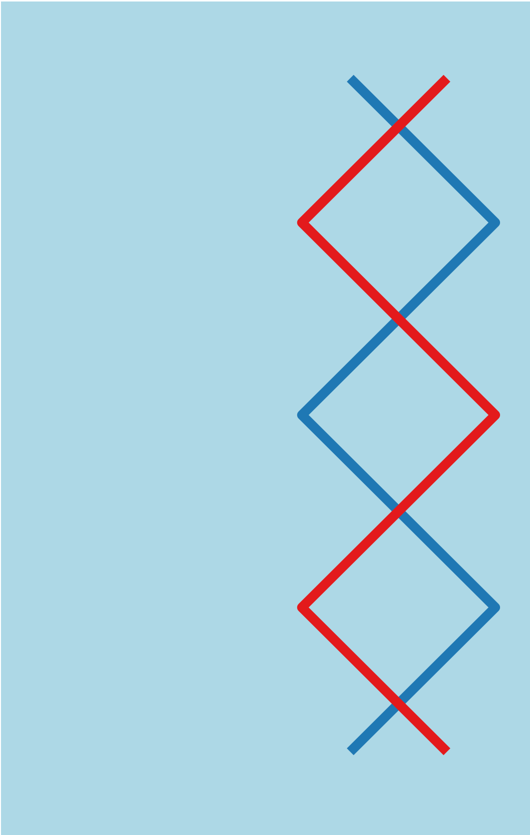

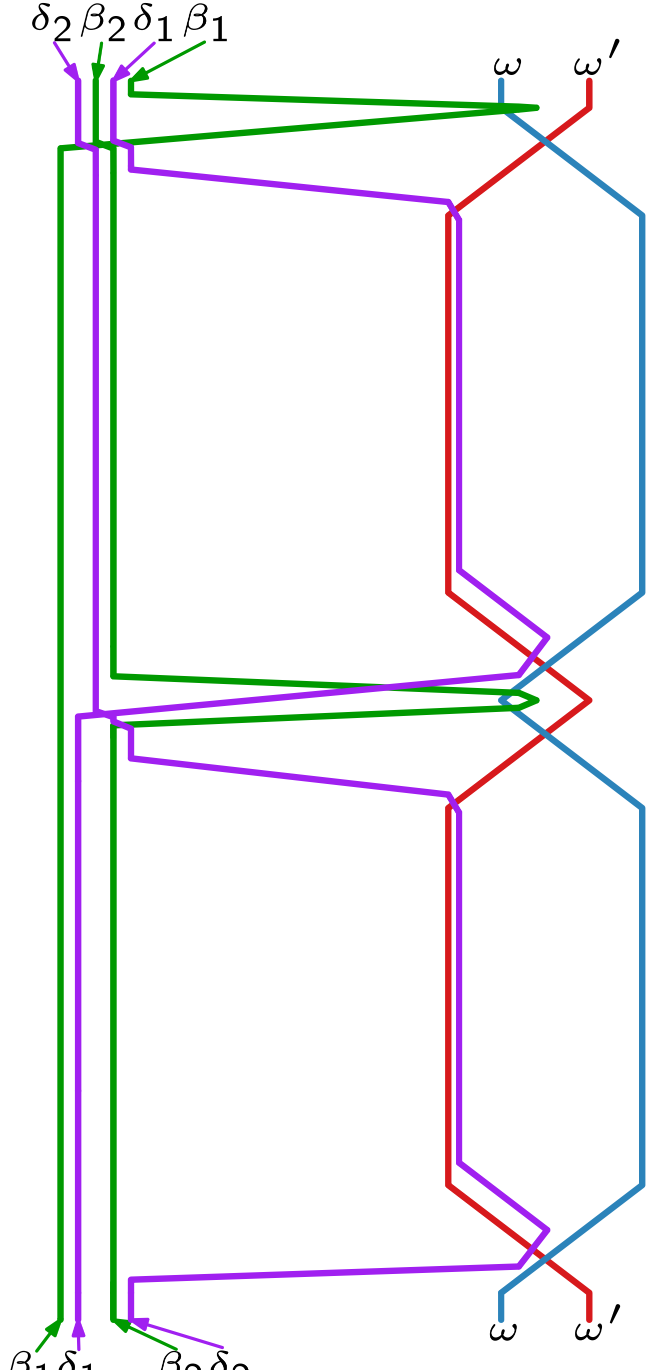

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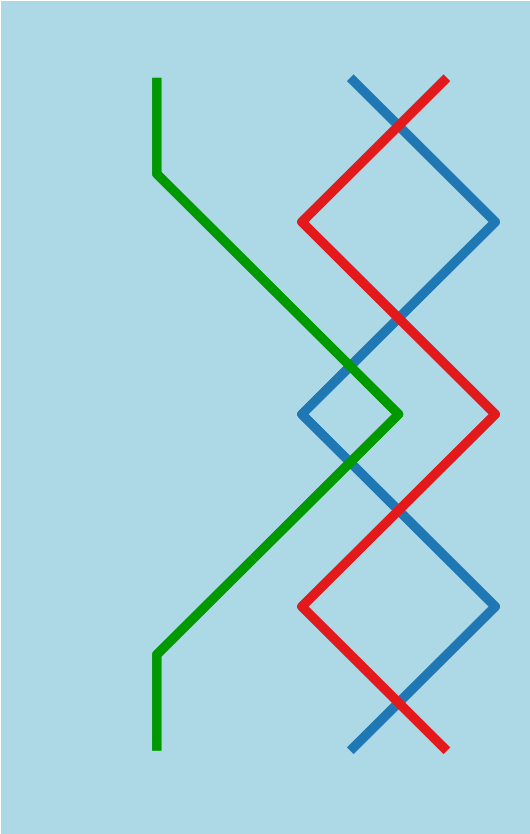

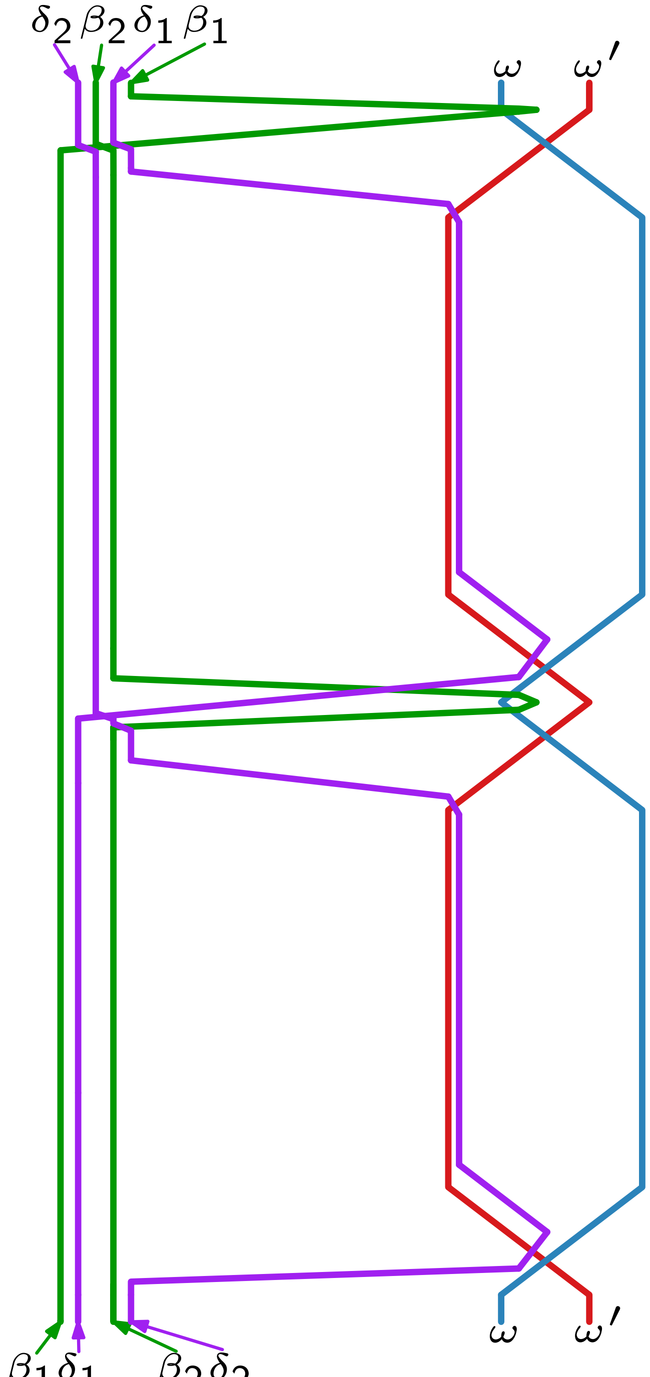

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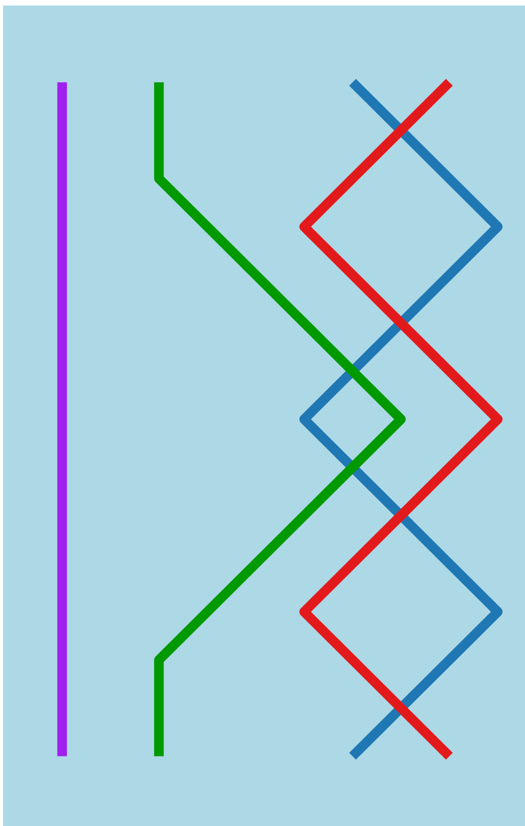

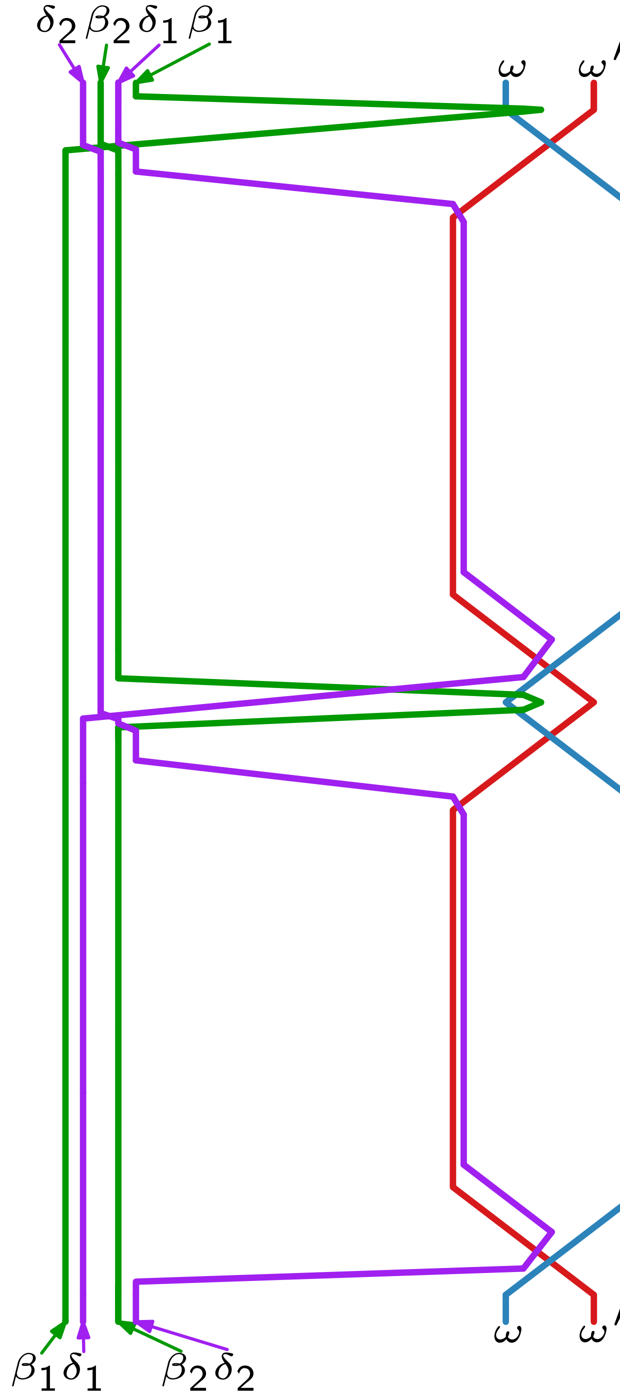

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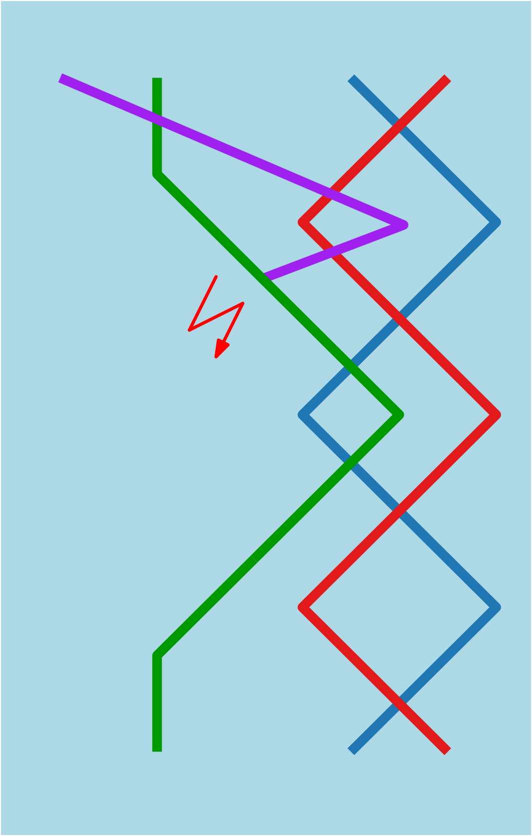

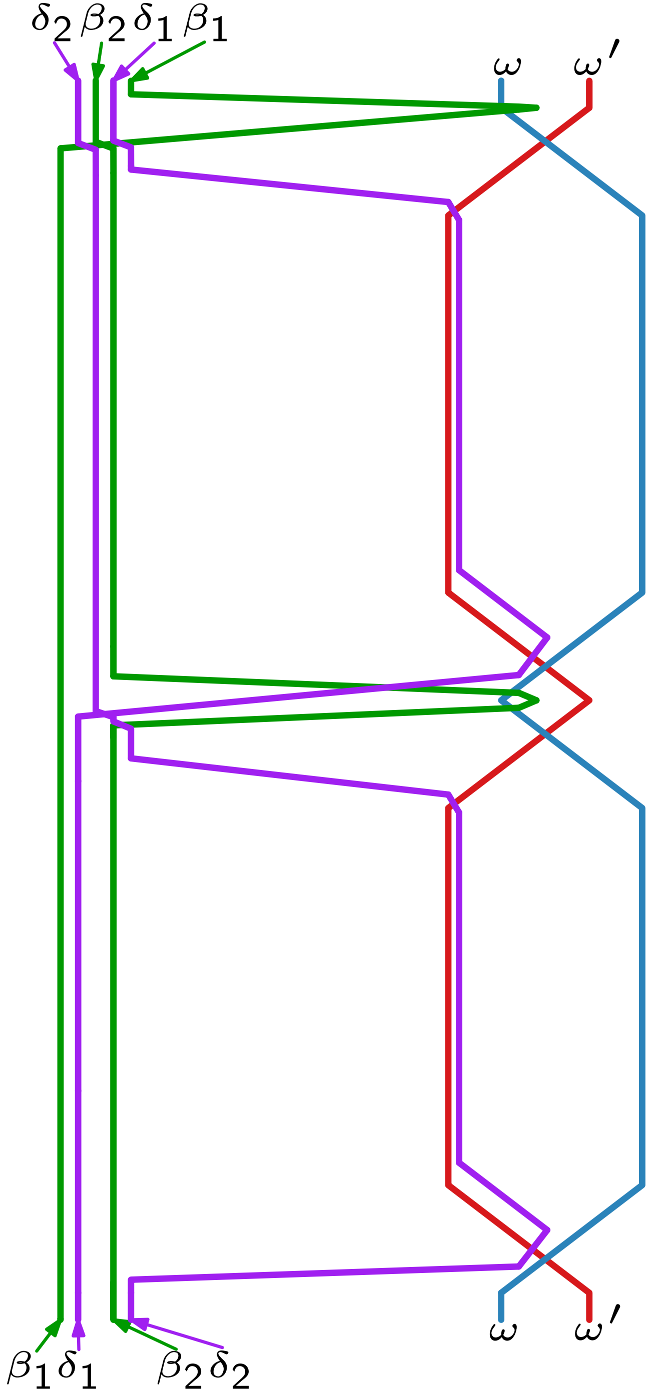

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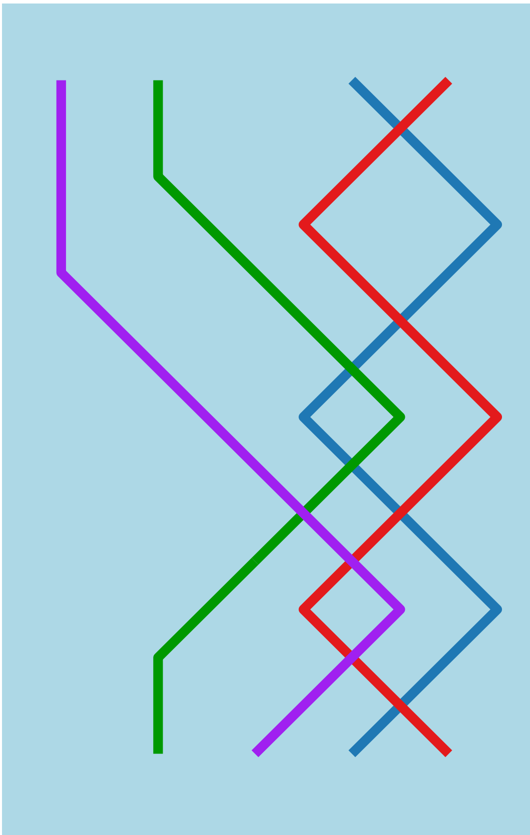

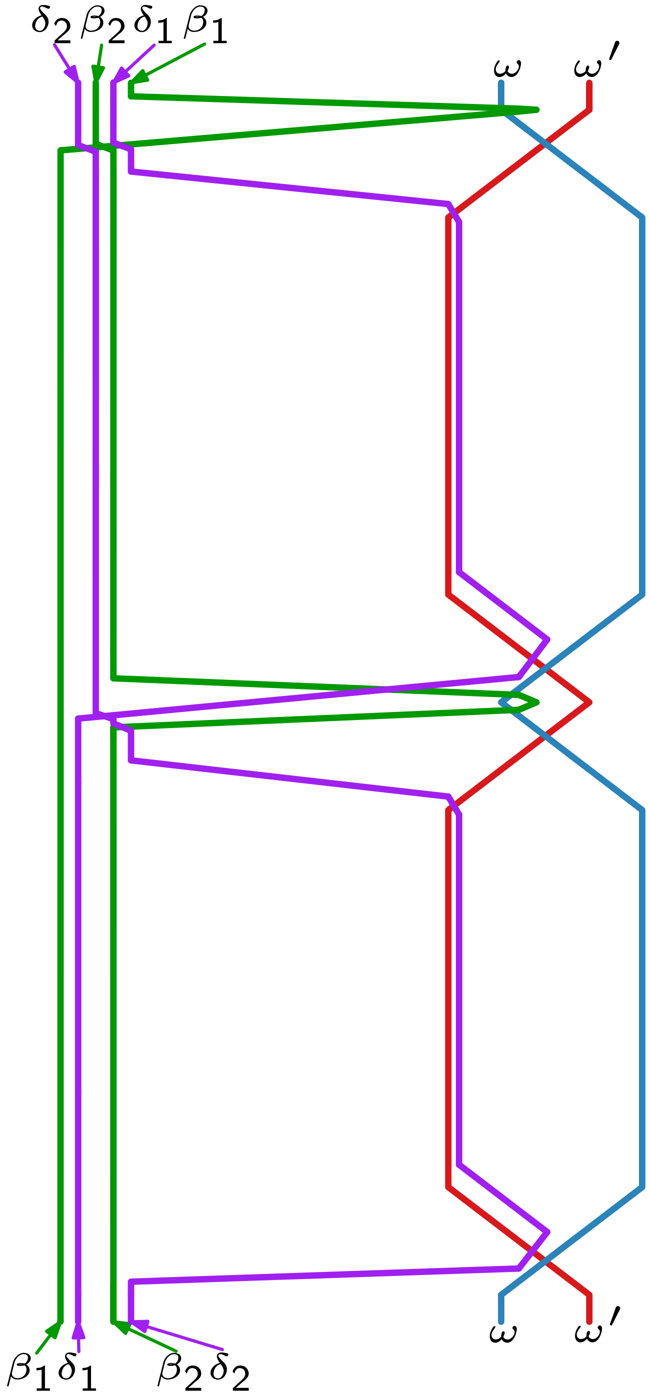

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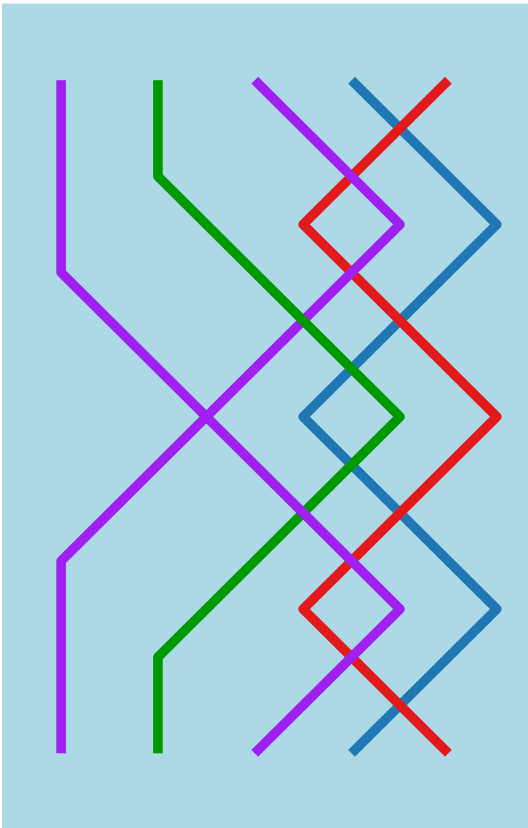
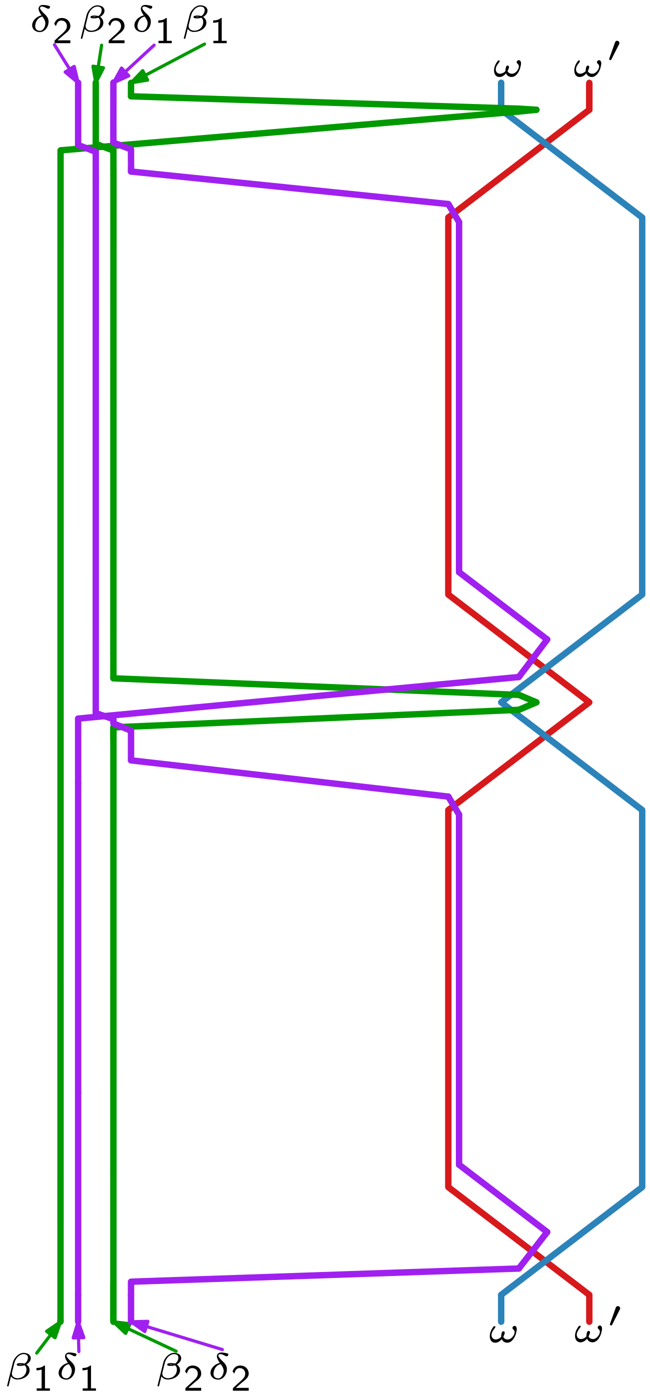
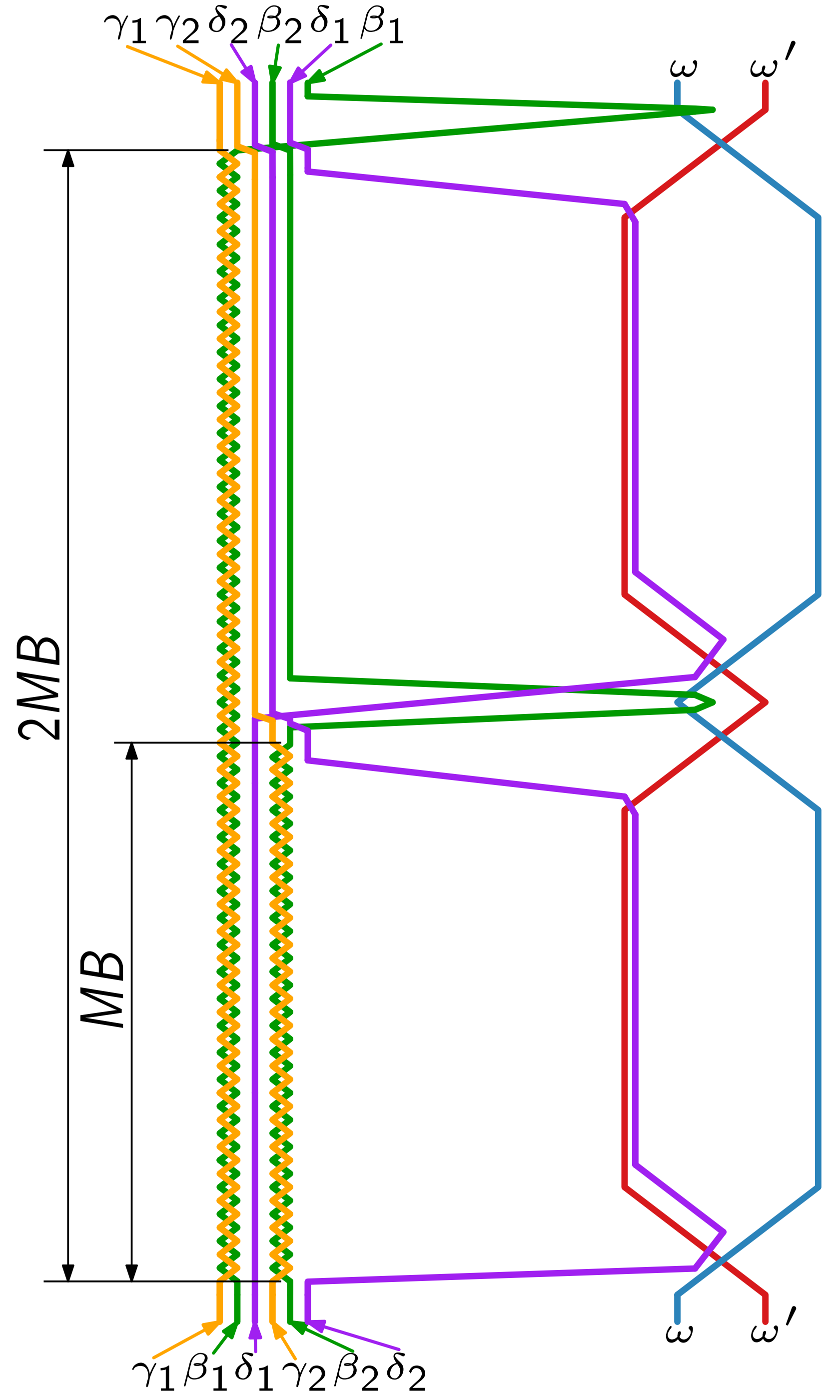




Making Sure That the “Pockets” Can’t Be Squeezed

$$M = 2m^3$$



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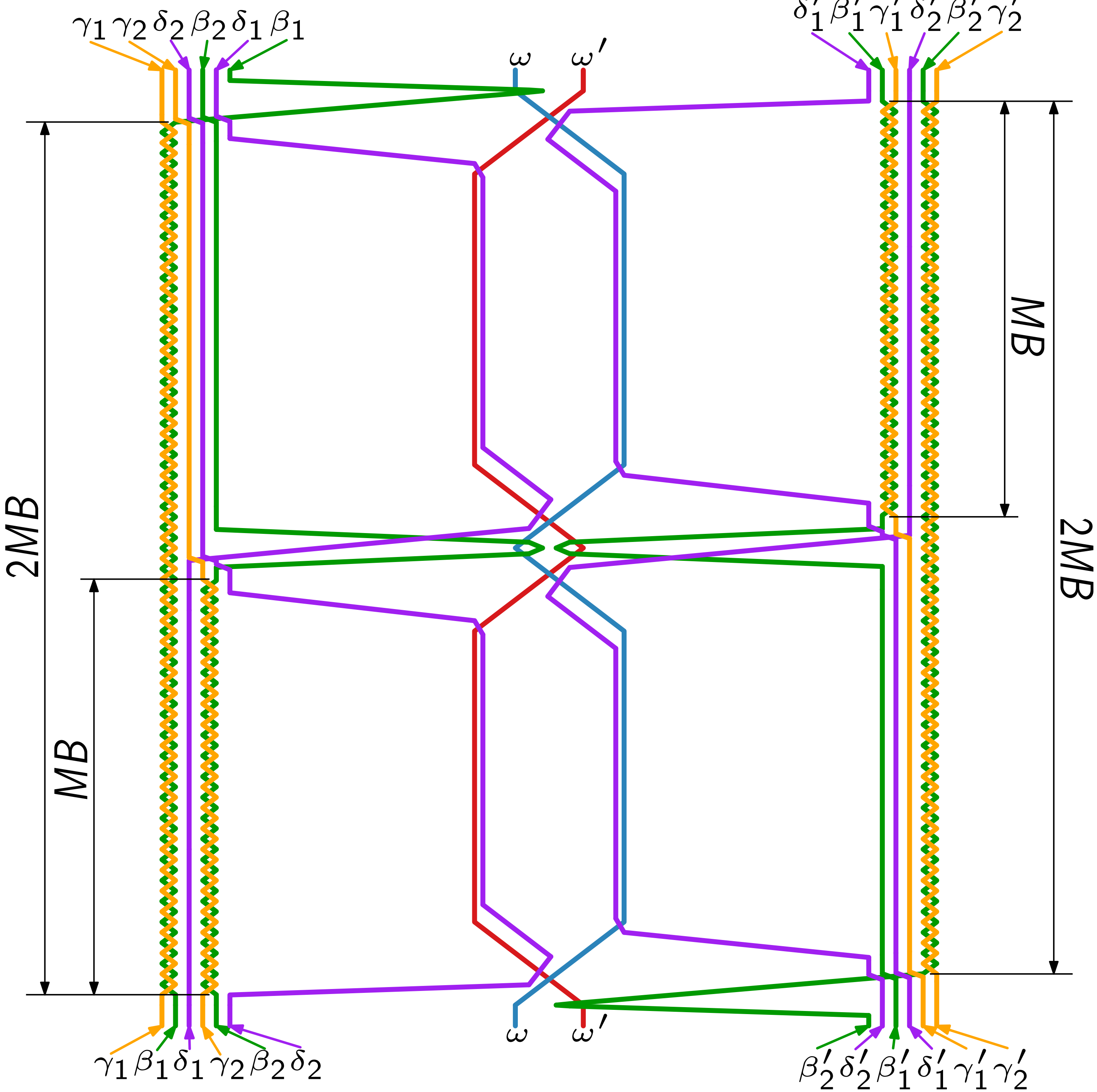

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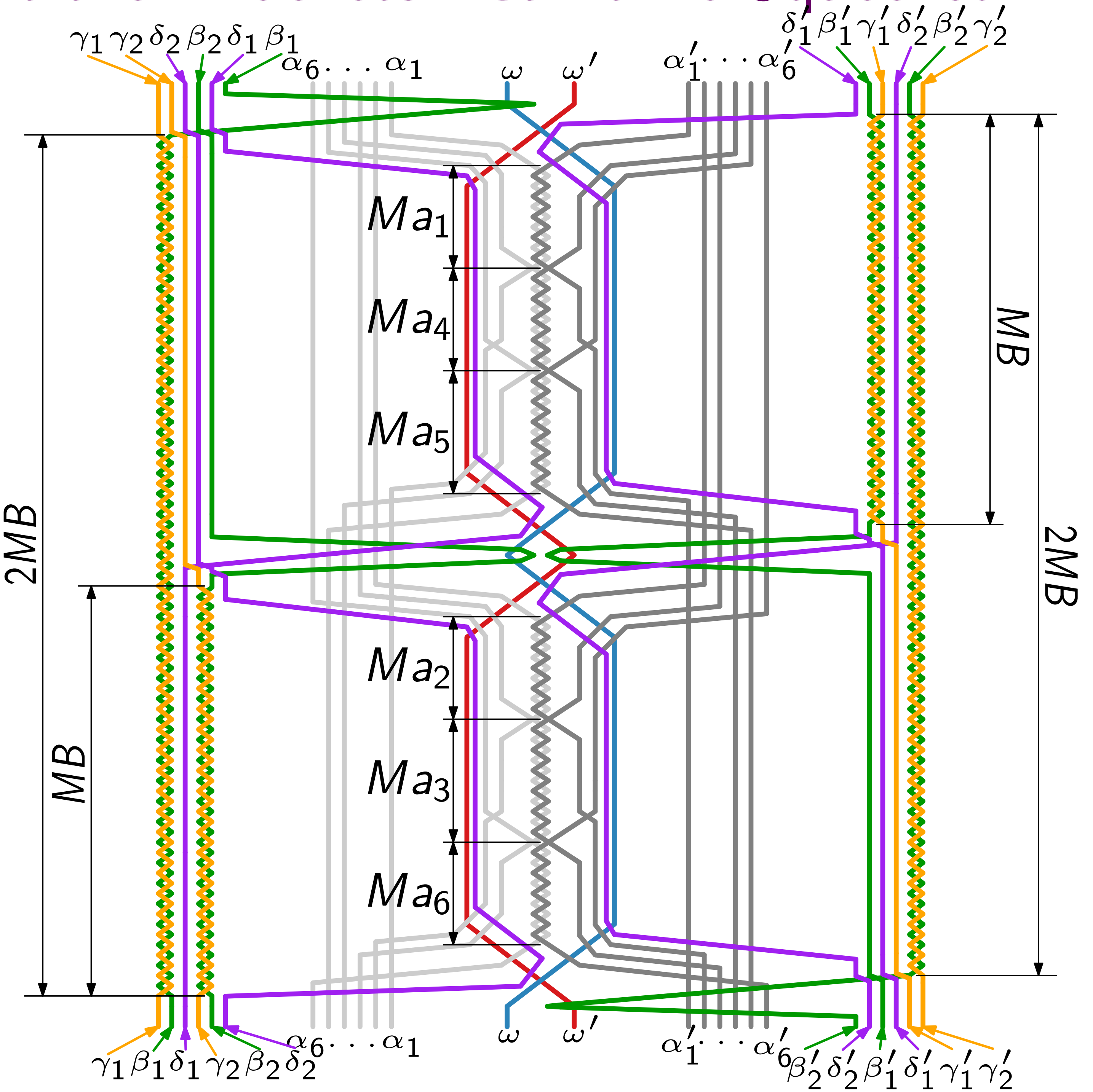
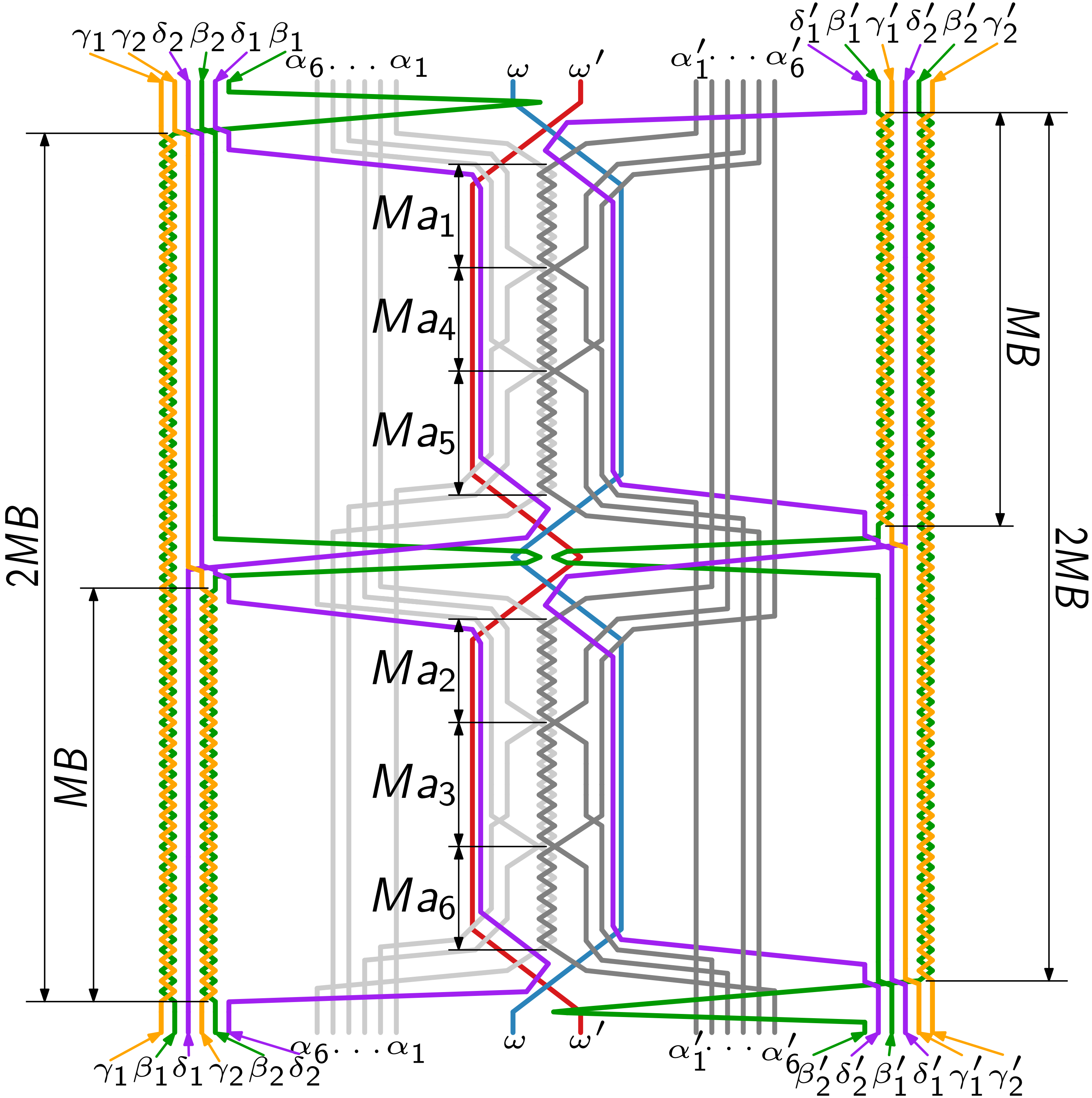


Proof of Correctness

$$M = 2m^3$$

A is a yes-instance



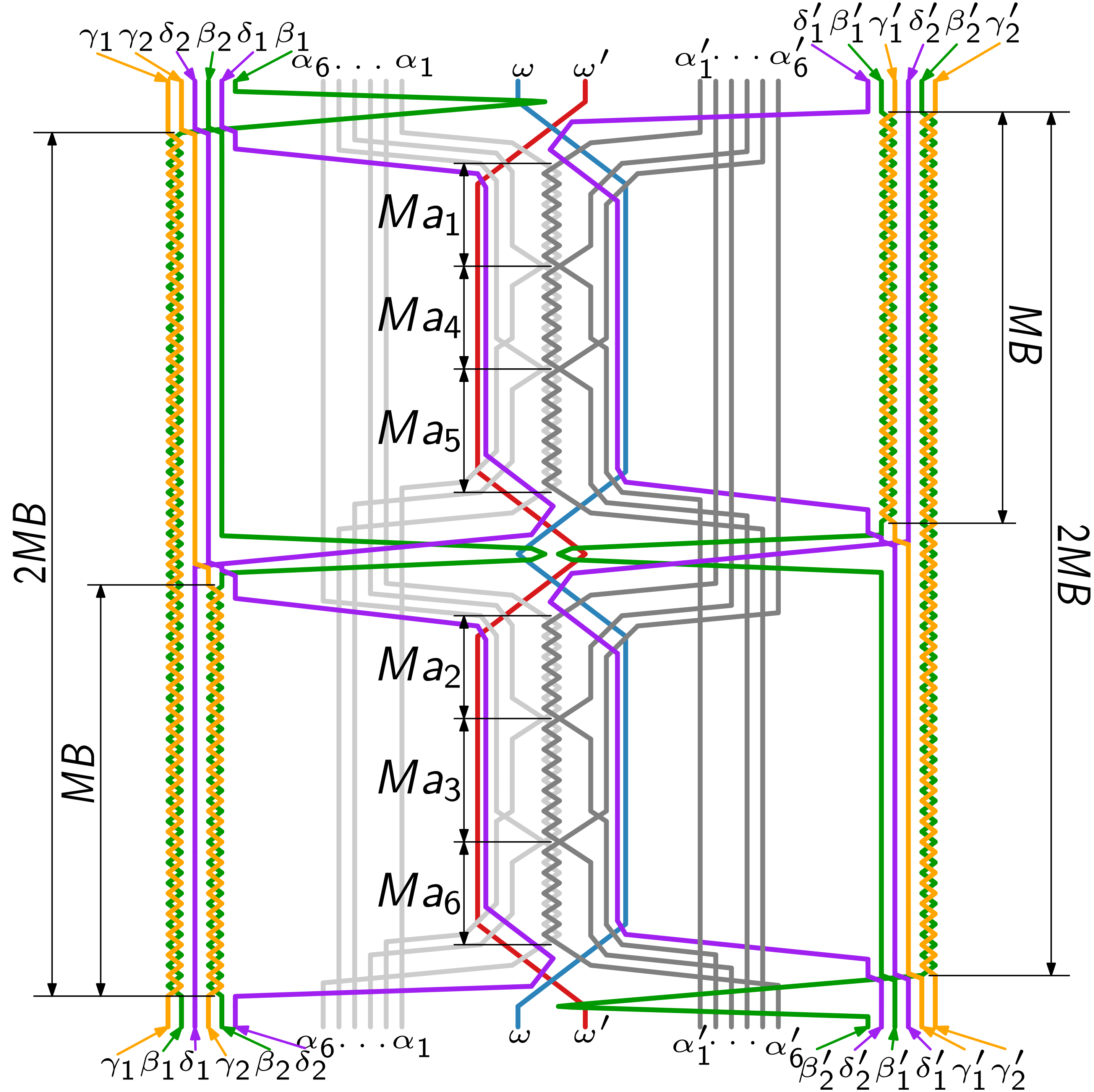
$H = 2m^3(\sum A) + 7m^2$ is the maximum allowed height for the reduction

Proof of Correctness

$M = 2m^3$

A is a yes-instance

by construction



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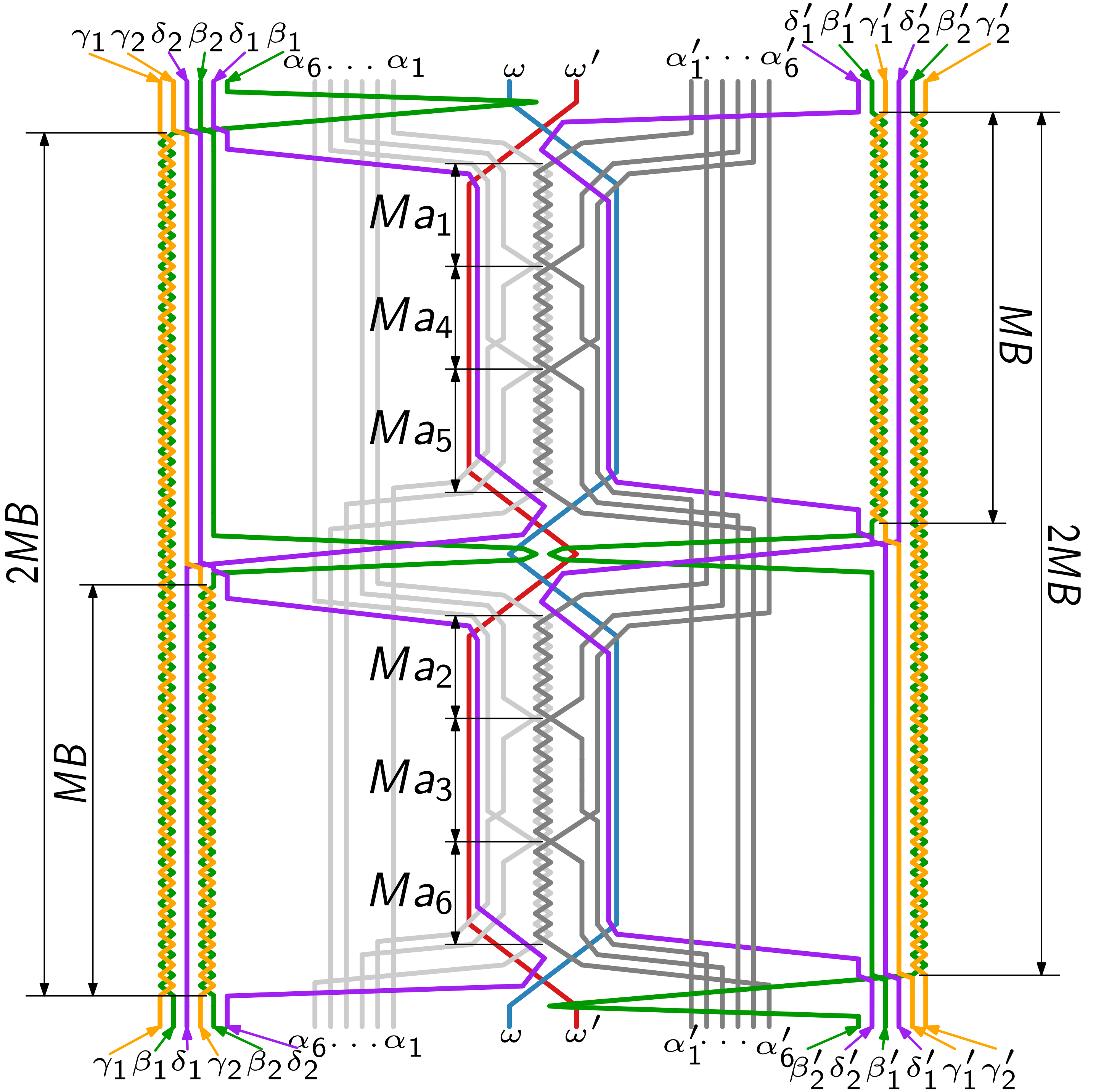
height $\leq H$



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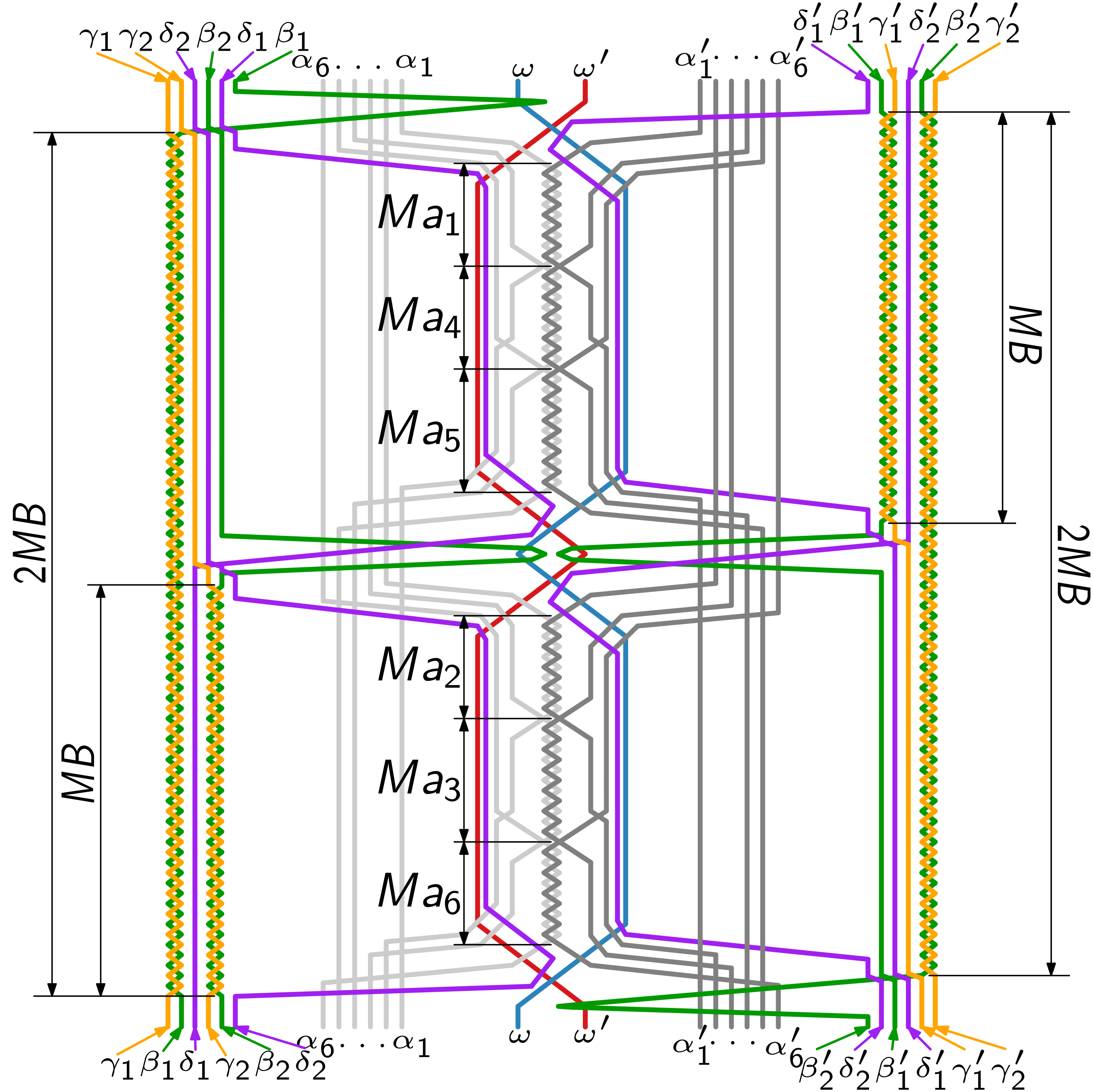
Proof of Correctness

$$M = 2m^3$$

A is a no-instance



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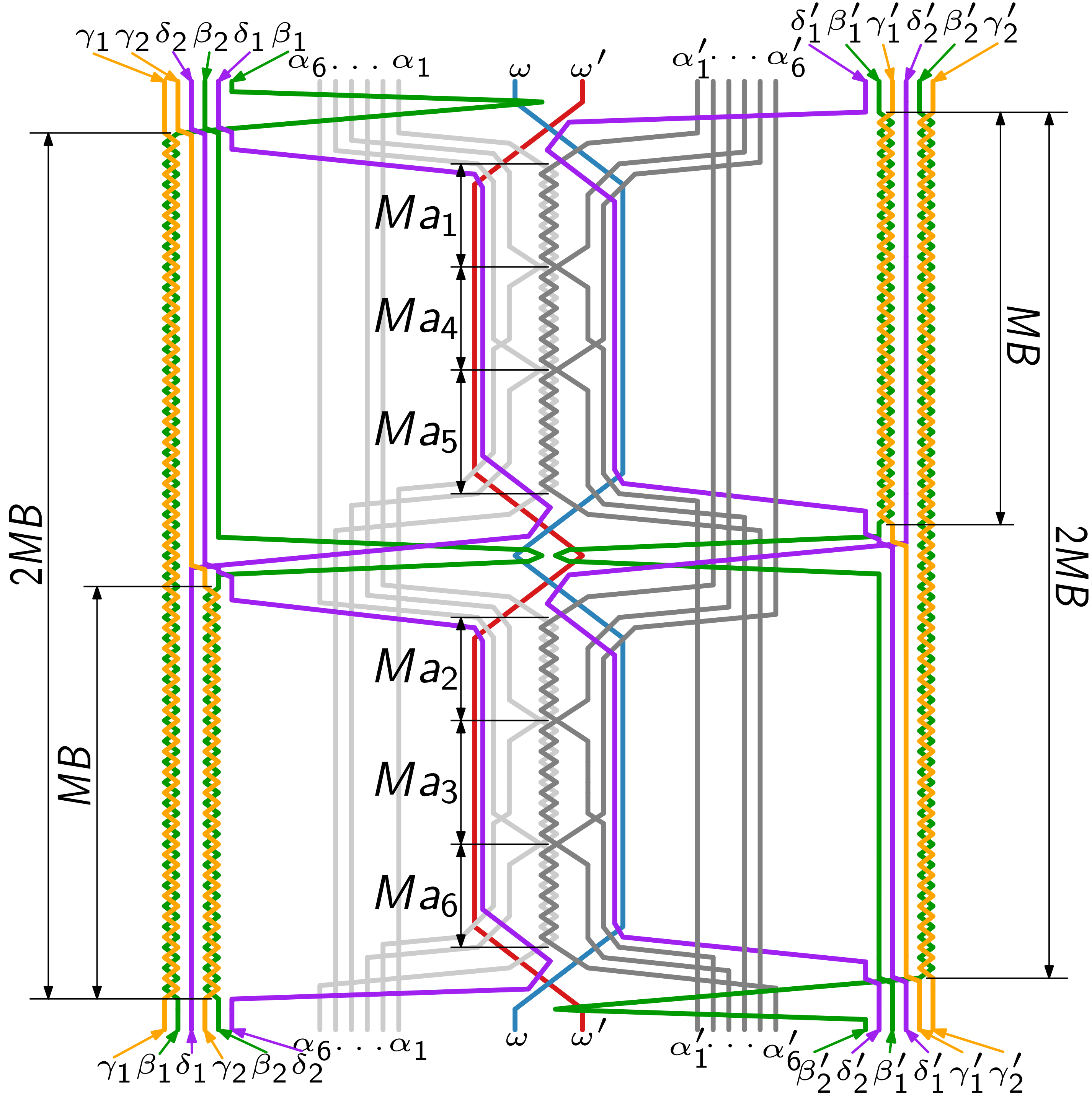
A is a no-instance

minimum height
$2m^3(\sum A + 1)$



$H = 2m^3(\sum A) + 7m^2$
is the maximum allowed
height for the reduction

Proof of Correctness

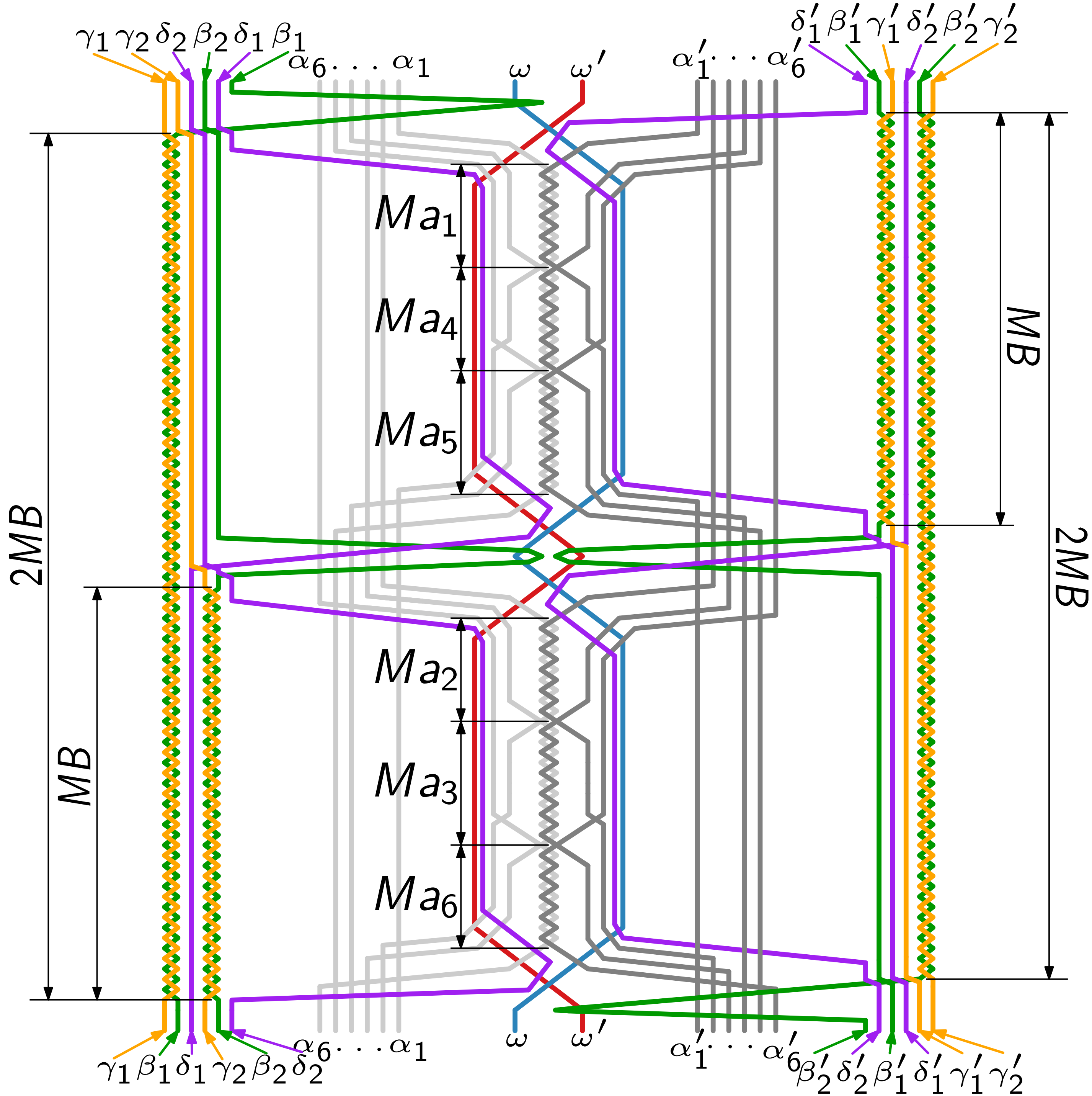
$$M = 2m^3$$

A is a no-instance

minimum height
$2m^3(\sum A + 1)$

height $> H$

$H = 2m^3(\sum A) + 7m^2$
is the maximum allowed height for the reduction



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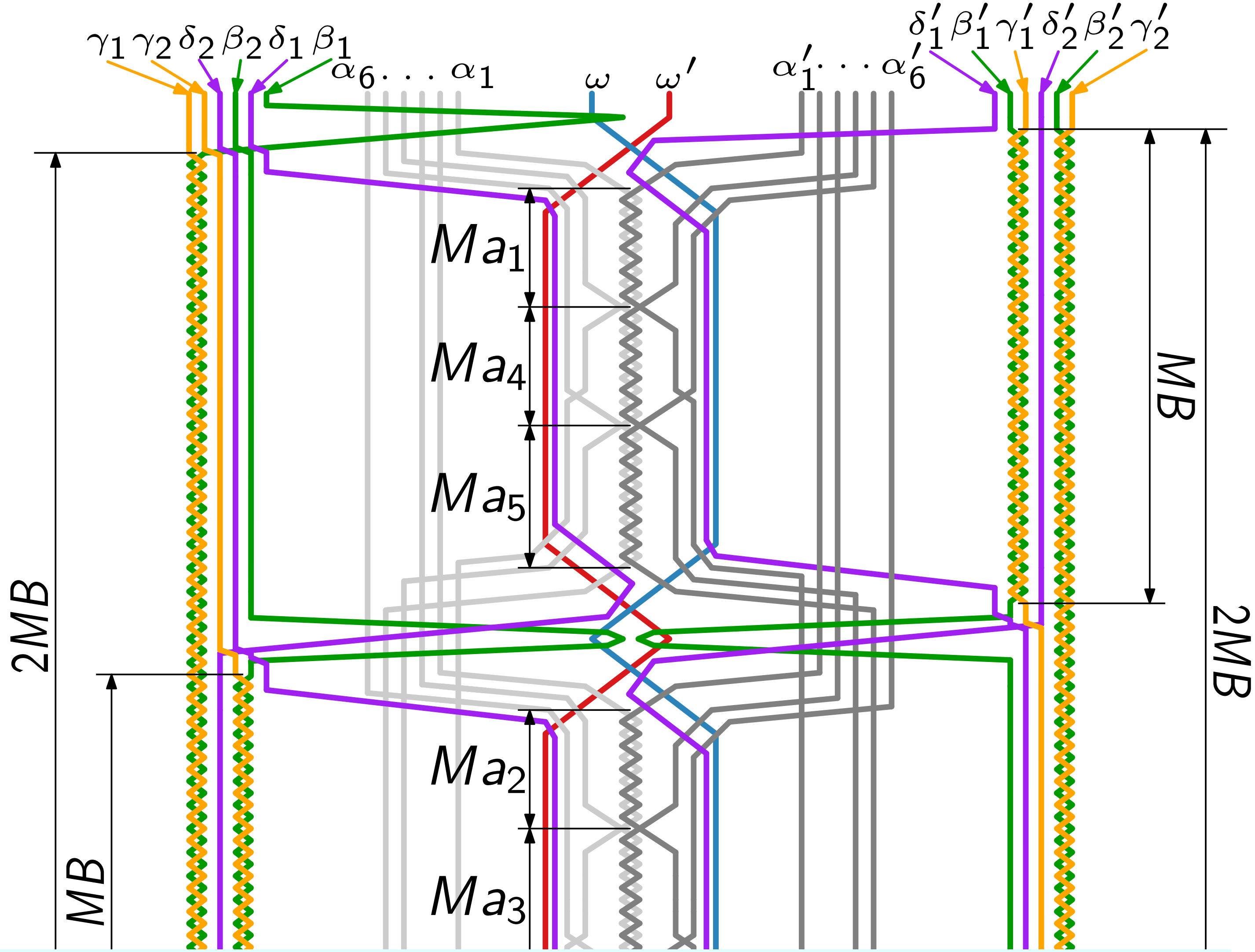
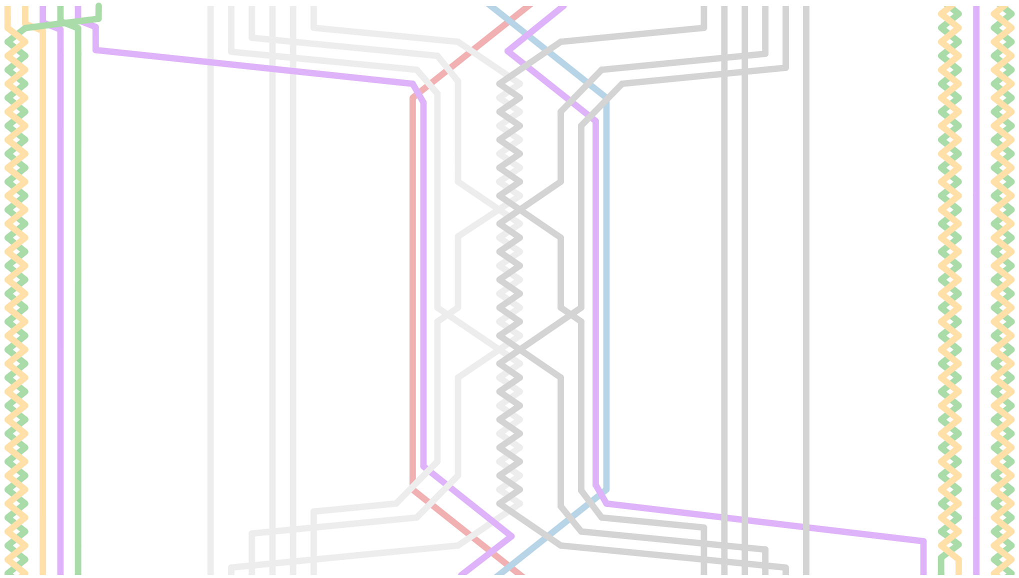


Theorem.
TANGLE-HEIGHT MINIMIZATION is NP-hard. ✓

Overview

- **Complexity:** NP-hardness by reduction from 3-Partition.



- **New algorithm:** using dynamic programming; asymptotically faster than [Olszewski et al., GD'18].

$$O\left(\frac{\varphi^{2|L|}}{5^{|L|/n}} n\right) \longrightarrow O\left(\left(\frac{2|L|}{n^2}+1\right)^{\frac{n^2}{2}} \varphi^n n\right)$$

- **Experiments:** comparison with [Olszewski et al., GD'18]

Improving Exact Algorithms

Tangle-Height Minimization can be solved in . . .

Simple lists

General lists

Improving Exact Algorithms

Tangle-Height Minimization can be solved in . . .

n – number of wires

Simple lists

[Olszewski et al., GD'18]

$2^{O(n^2)}$

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our runtime

$2^{O(n \log n)}$

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Improving Exact Algorithms

Tangle-Height Minimization can be solved in ...

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$|L|$ – length of the list L $(= \sum \ell_{ij})$
φ – golden ratio (≈ 1.618)

Simple lists

[Olszewski et al., GD'18]

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General lists

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$O\left(\frac{\varphi^{2|L|}}{5^{|L|/n}} n\right)$

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our runtime

$O\left(\left(\frac{2|L|}{n^2} + 1\right)^{\frac{n^2}{2}} \varphi^n n\right)$

polynomial in $|L|$
for fixed n

Dynamic Programming Algorithm

Let $L = (\ell_{ij})$ be the given list of swaps. $O\left(\left(\frac{2|L|}{n^2} + 1\right)^{n^2/2} \varphi^n n\right)$

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$$O\left(\left(\frac{2|L|}{n^2} + 1\right)^{n^2/2} \varphi^n n\right)$$

$\lambda = \#$ of **distinct sublists** of L.

L' is a *sublist* of L if $\ell'_{ij} \leq \ell_{ij}$

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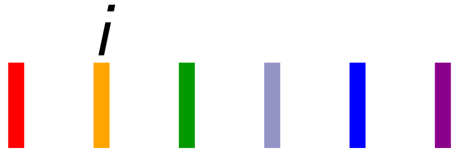

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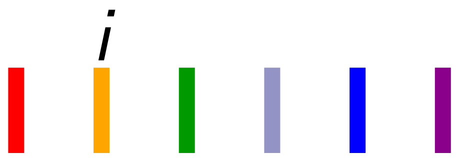

for each wire i:

    // find a position where it is after applying L'

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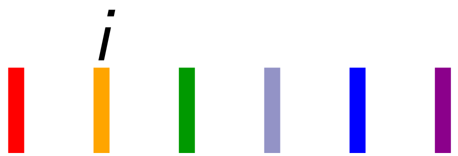

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    $i \mapsto i +$

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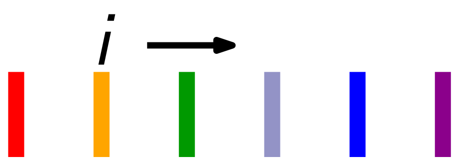

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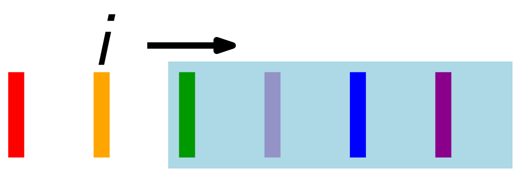

for each wire i:

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    $i \mapsto i + |\{j : j > i \text{ and } \ell'_{ij} \text{ is odd}\}|$

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check whether the map is indeed a permutation

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Compute the **final permutation** $\mathrm{id}_n L'$.

$\mathrm{id}_n L'$

Dynamic Programming Algorithm

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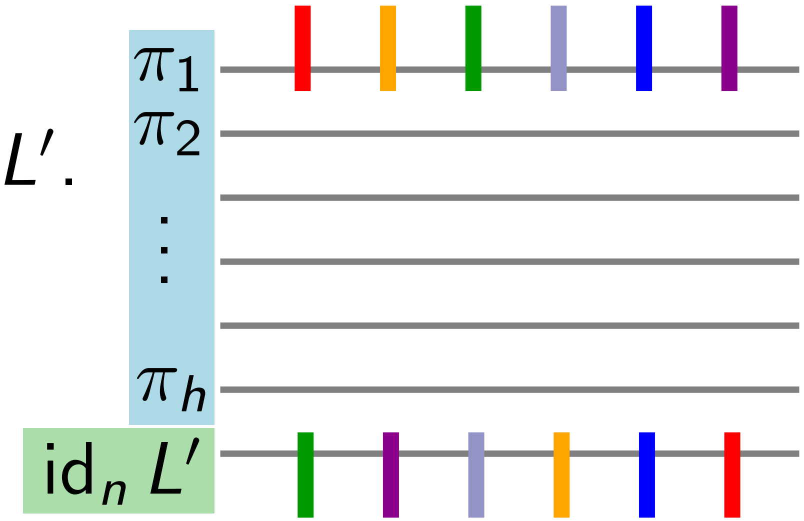
Choose the **shortest tangle** $T(L'')$.



π_h and $\mathrm{id}_n\, L'$ are adjacent

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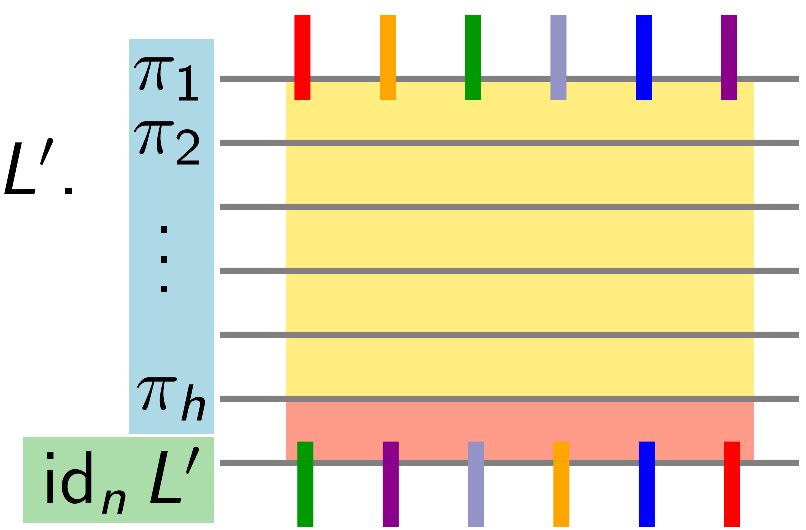

π_h and $\mathrm{id}_n L'$ are adjacent

$L'' \cup$ add. swaps $= L'$

Dynamic Programming Algorithm

$O\left(\left(\frac{2|L|}{n^2} + 1\right)^{n^2/2} \varphi^n n \right)$

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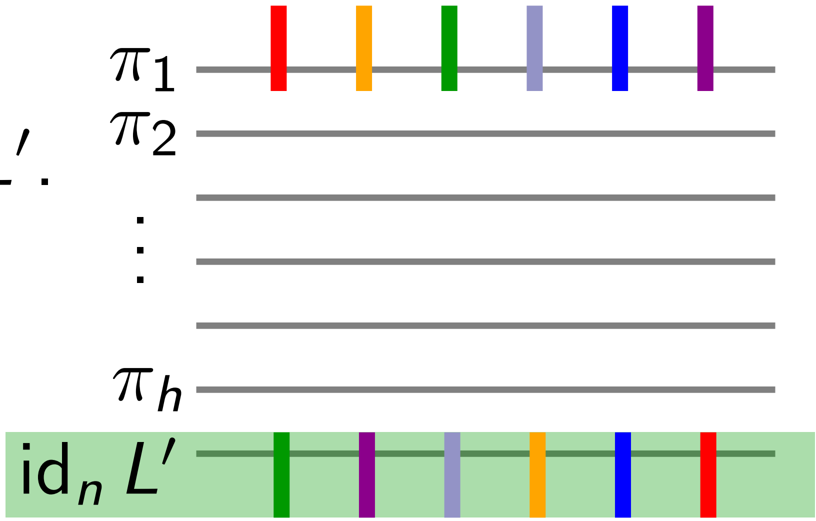

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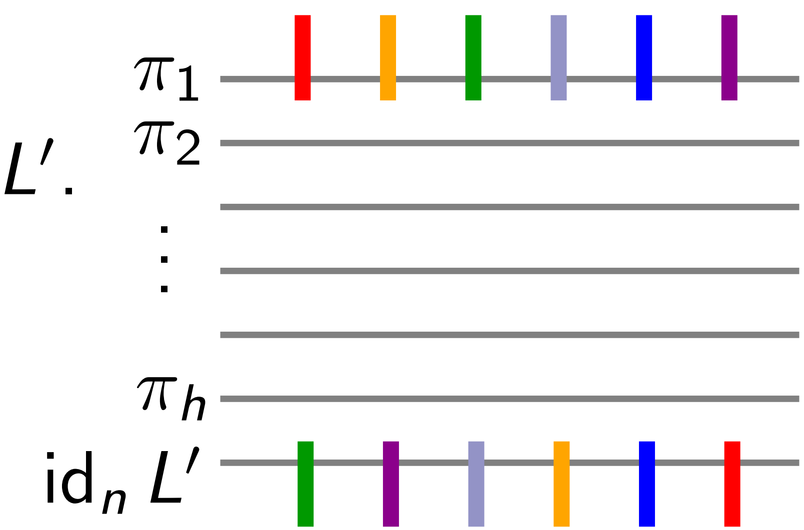

Running time

$O(\qquad\qquad\qquad)$

Dynamic Programming Algorithm

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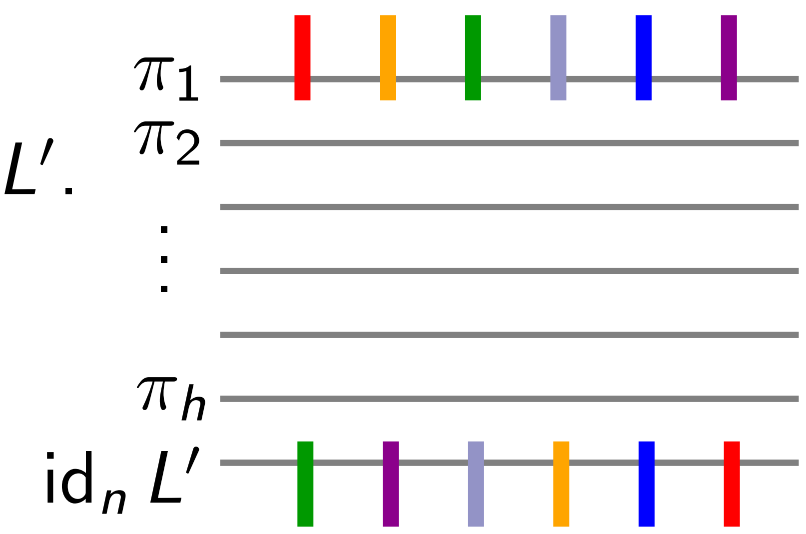

Running time

$O(\lambda \cdot \qquad\qquad\qquad)$

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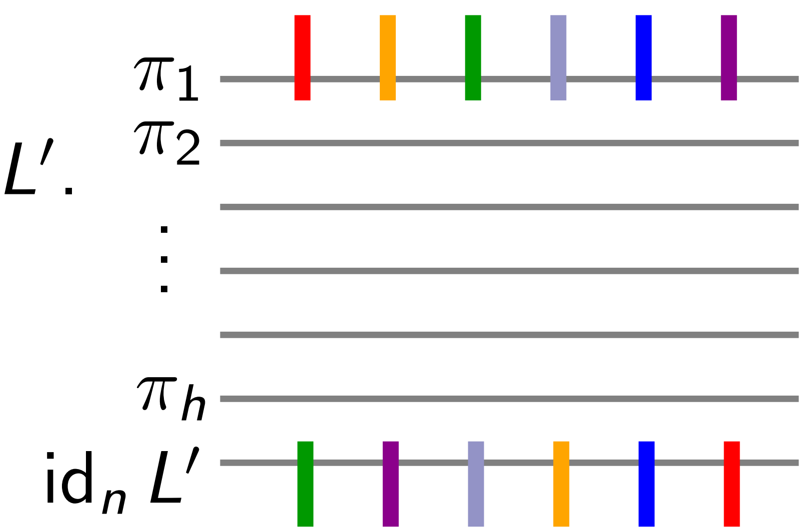

Running time

$O(\lambda \cdot (F_{n+1} - 1) \cdot n)$

F_n is the n-th Fibonacci number

Dynamic Programming Algorithm

$O\left(\left(\frac{2|L|}{n^2}+1\right)^{n^2/2}\varphi^n n\right)$

Let $L = (\ell_{ij})$ be the given list of swaps.

$\lambda = \#$ of **distinct sublists** of L.

L' is a *sublist* of L if $\ell'_{ij} \leq \ell_{ij}$

Consider them in order of **increasing length**.

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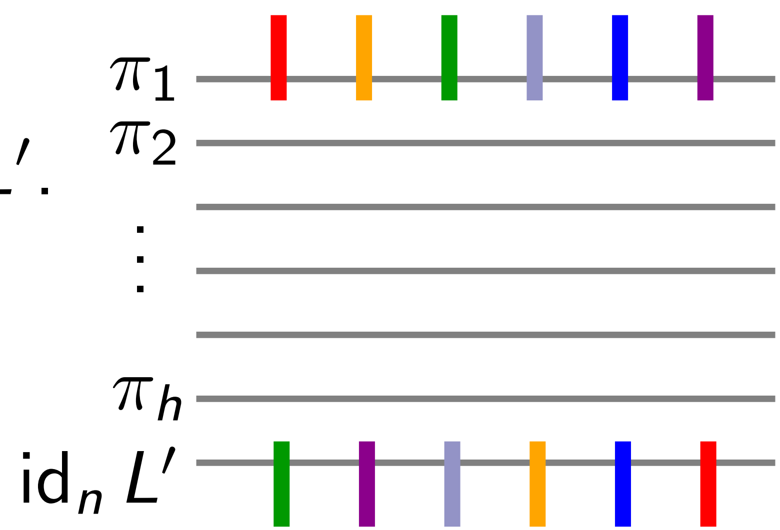

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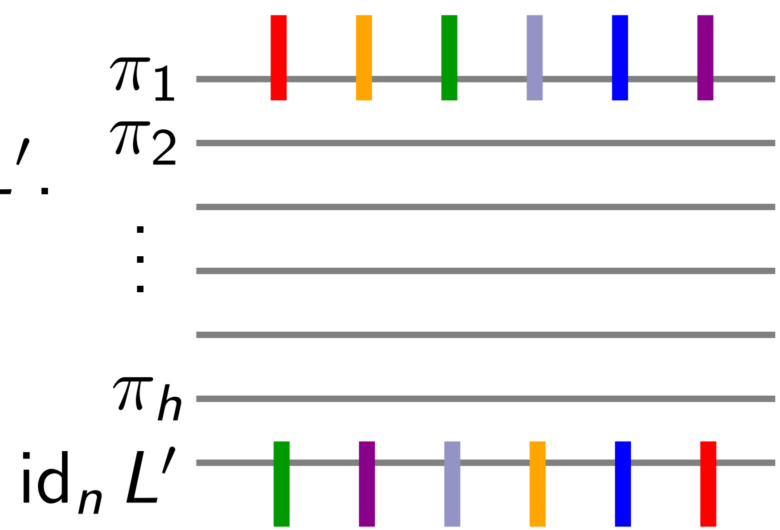

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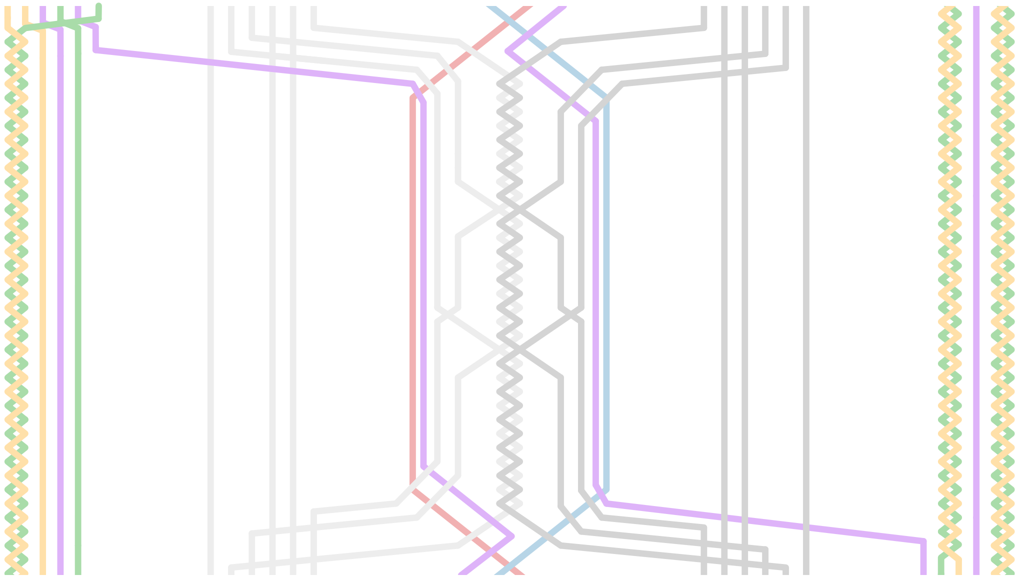
Overview

- **Complexity:** NP-hardness by reduction from 3-Partition.

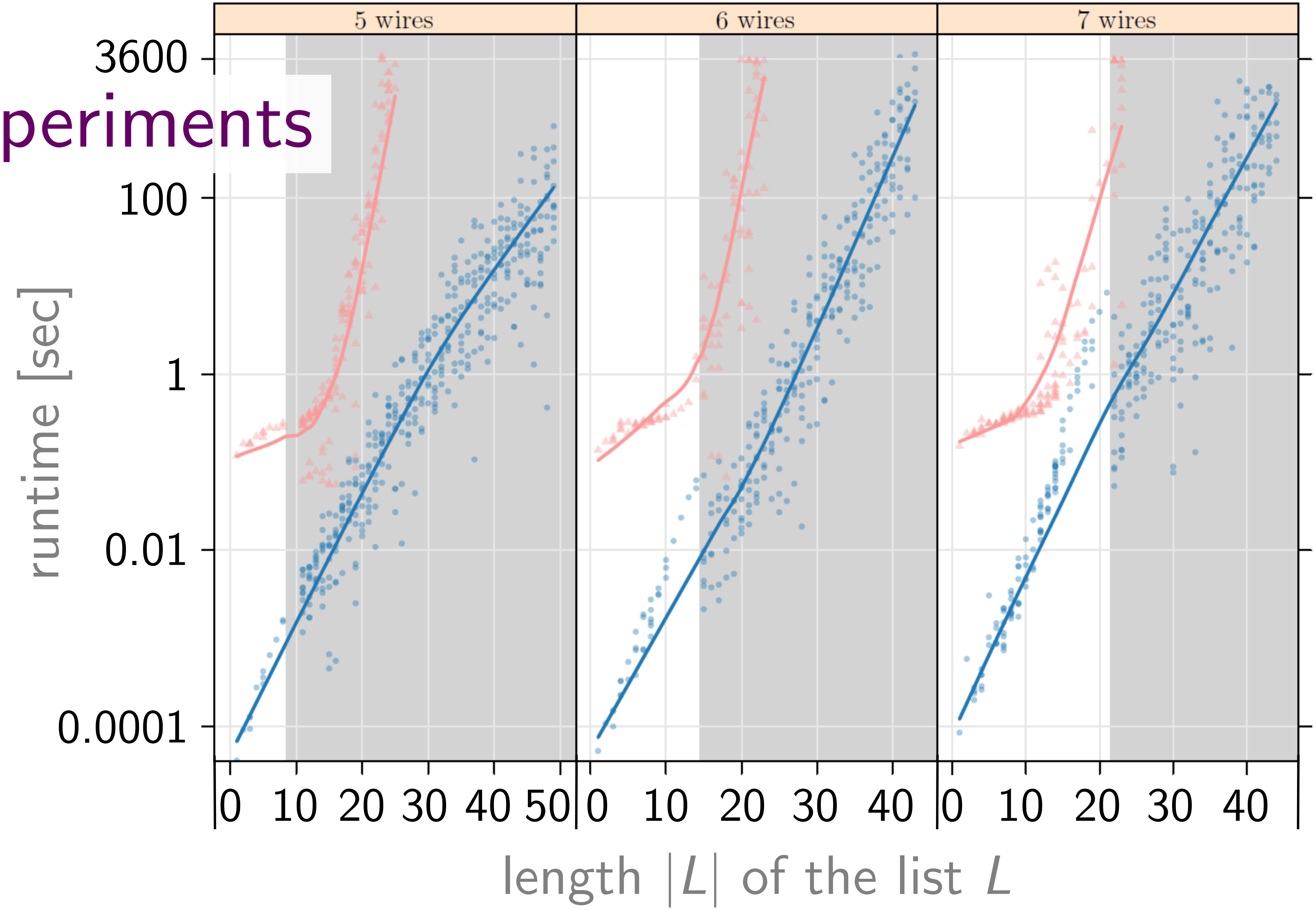


- **New algorithm:** using dynamic programming; asymptotically faster than [Olszewski et al., GD'18].

$$O\left(\frac{\varphi^{2|L|}}{5^{|L|/n}}n\right) \longrightarrow O\left(\left(\frac{2|L|}{n^2}+1\right)^{\frac{n^2}{2}}\varphi^n n\right)$$

- **Experiments:** comparison with [Olszewski et al., GD'18]

Experiments



[Olszewski et al., GD'18]

$O\left(\frac{\varphi^{2|L|}}{5^{|L|/n}} n\right)$

Our algorithm

$O\left(\left(\frac{2|L|}{n^2}+1\right)^{\frac{n^2}{2}} \varphi^n n\right)$

Open Problems

Problem 1

Is it NP-hard to test the feasibility of a given (non-simple) list?

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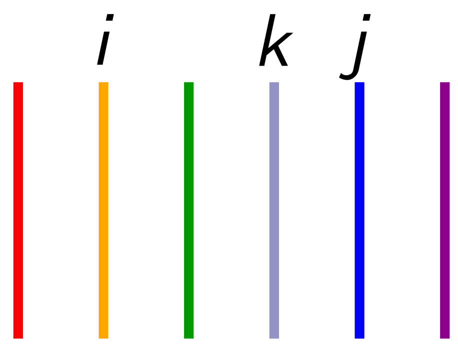

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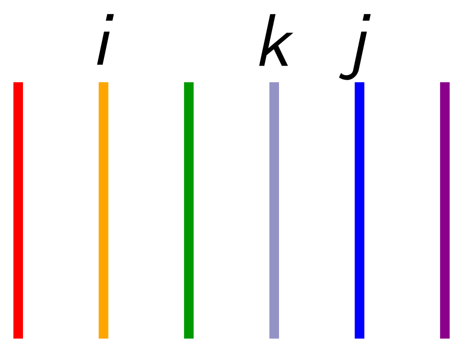

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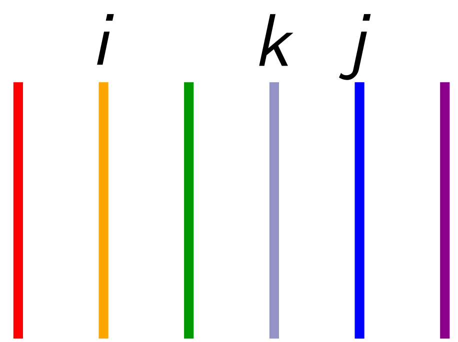

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Thank you!

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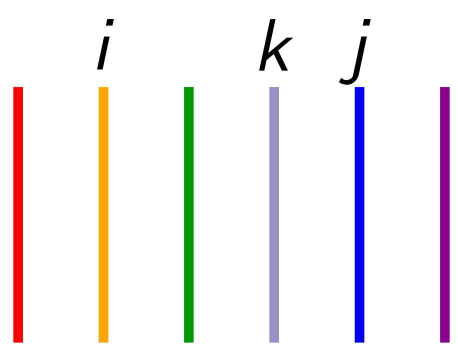

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