Navigating Posets with Few Maps

Stefan Felsner
TU Berlin

Jędrzej Hodor
UJ Kraków

Giacomo Ortali
U Telematica Int

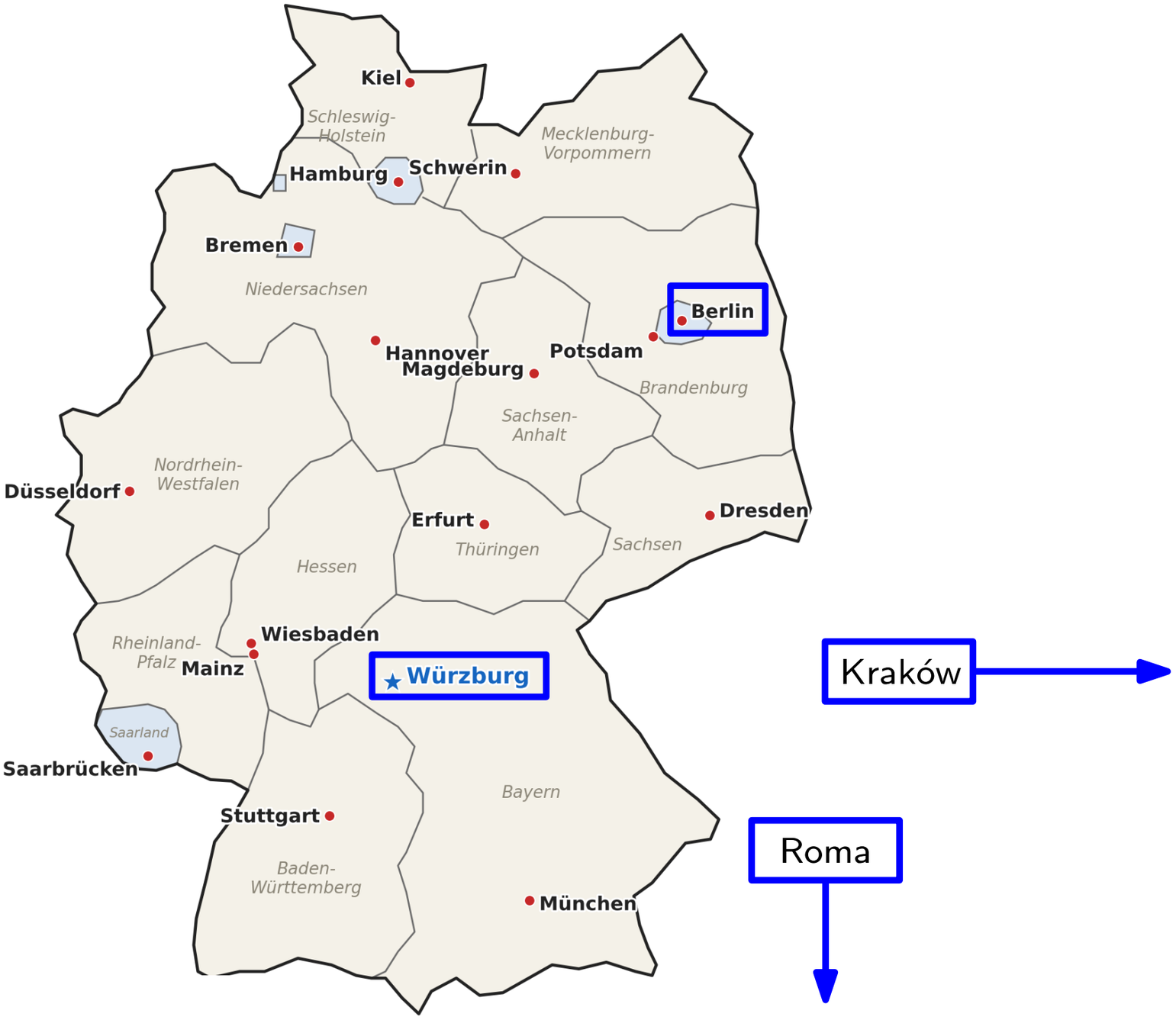
Alexander Wolff
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GD 2026

Germany – Bundesländer and their capitals



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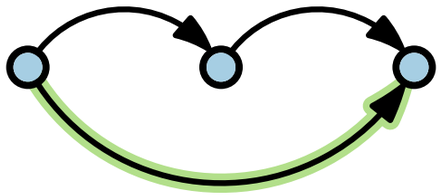
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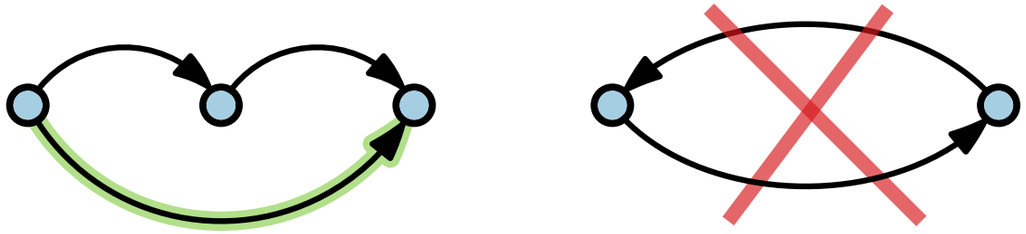
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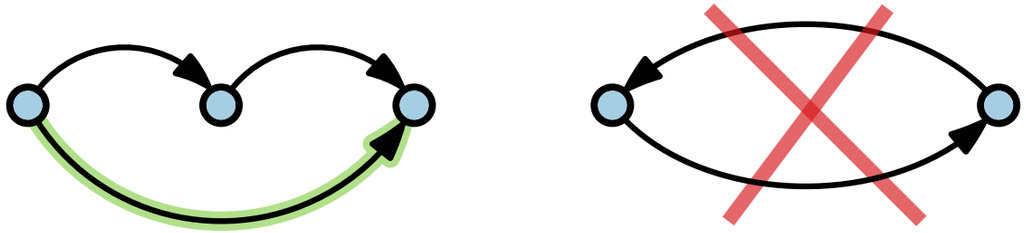
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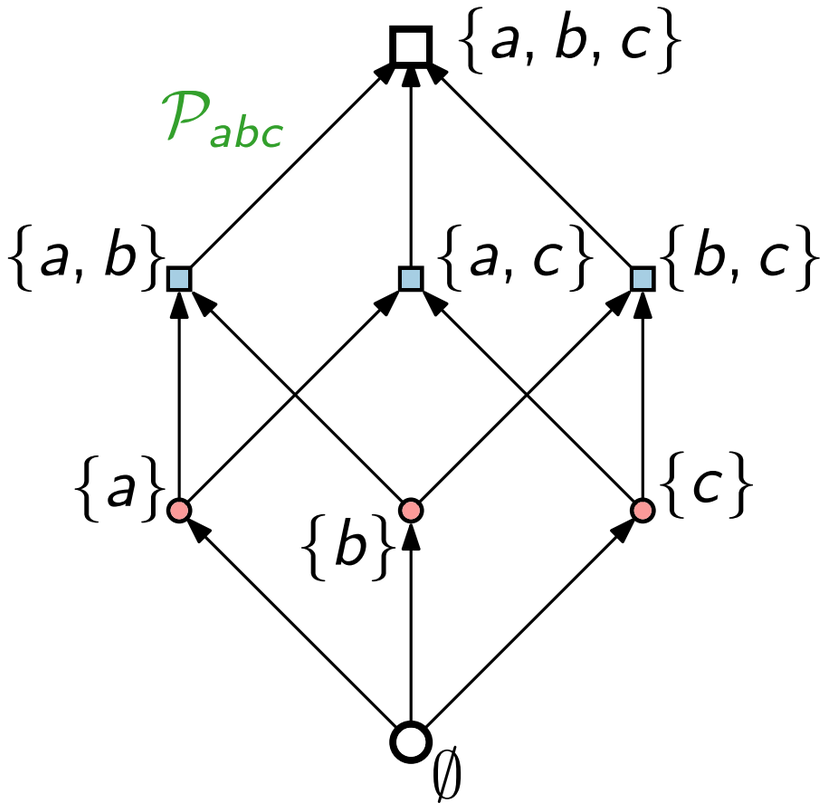
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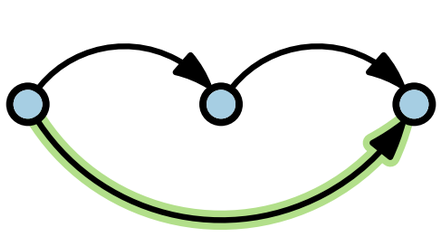
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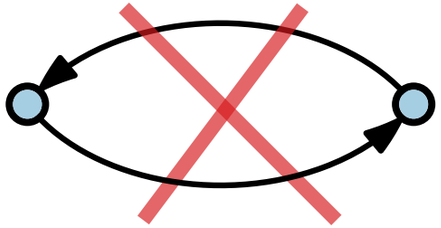


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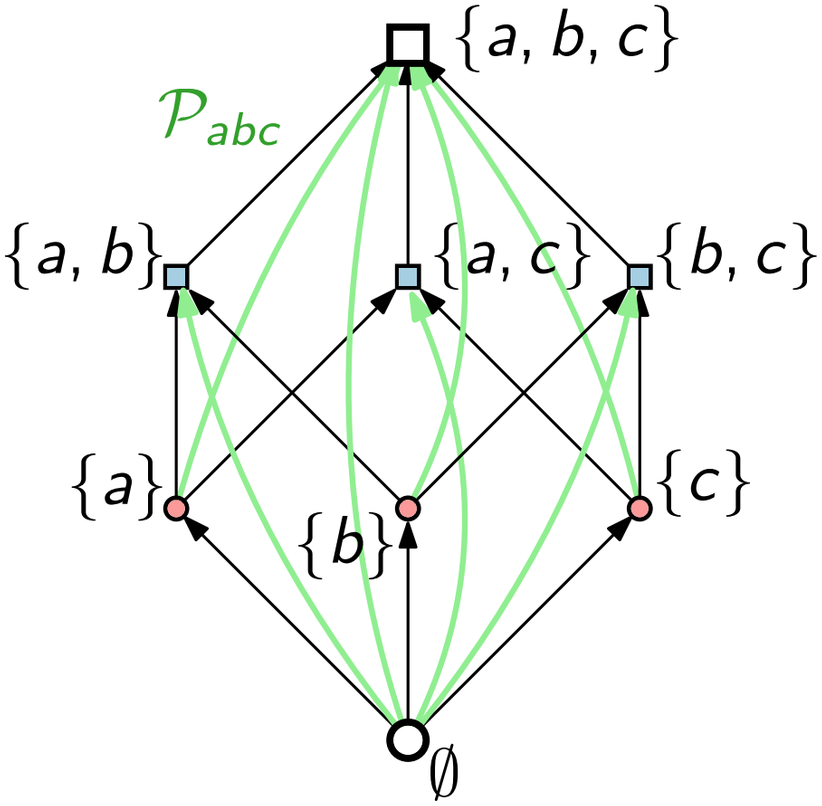
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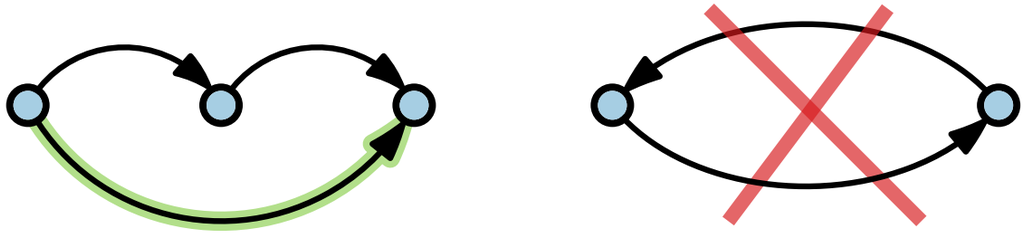
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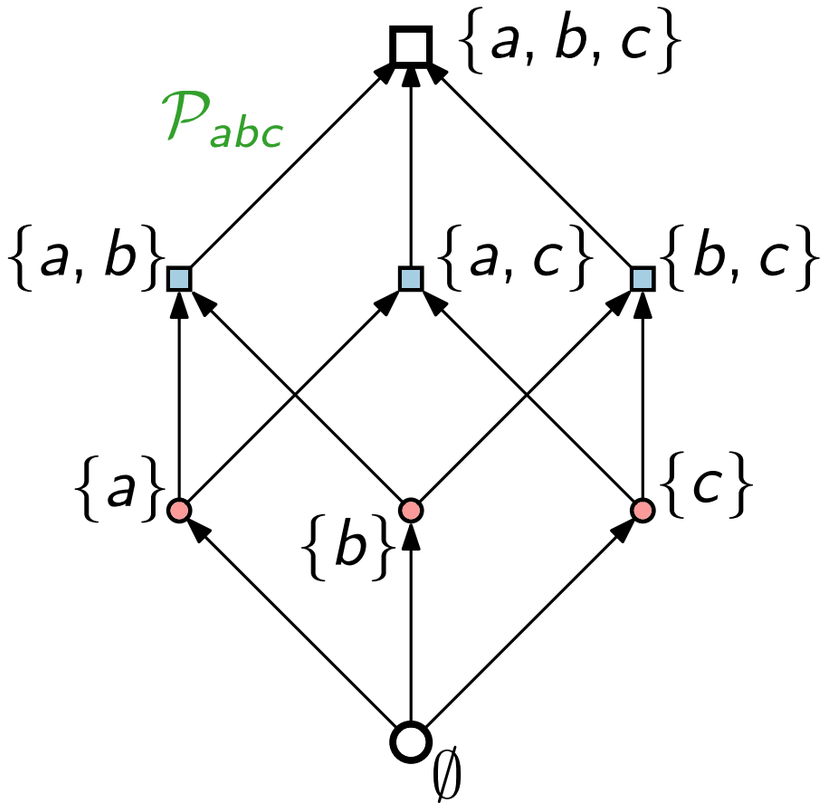
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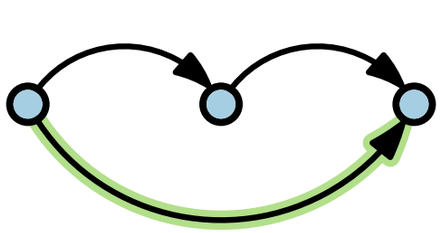
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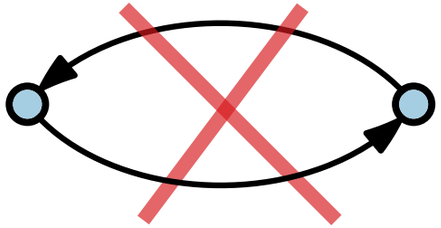


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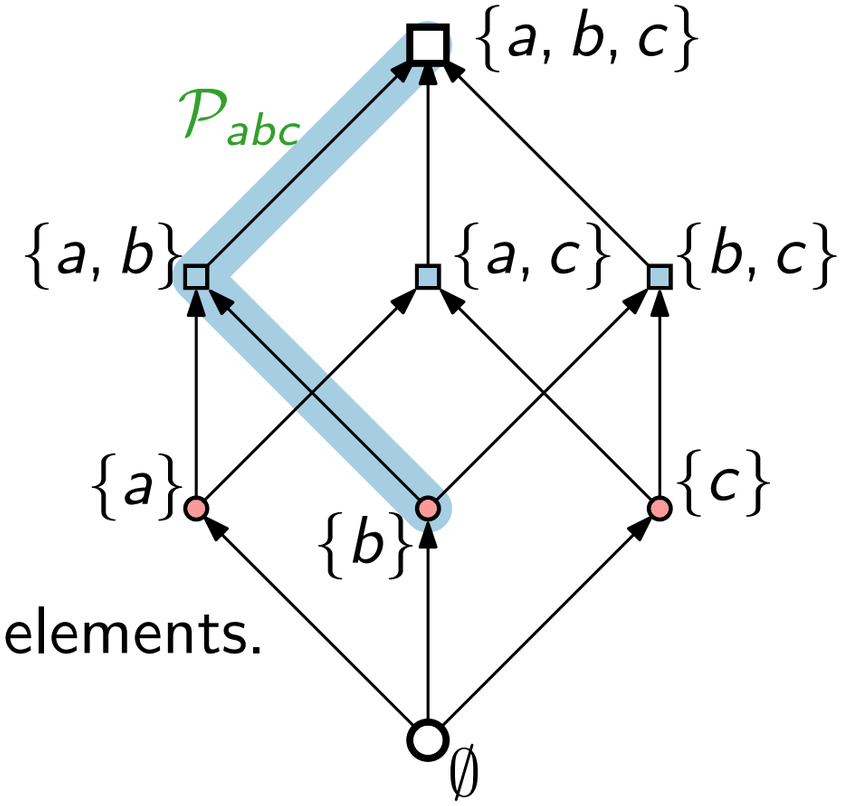
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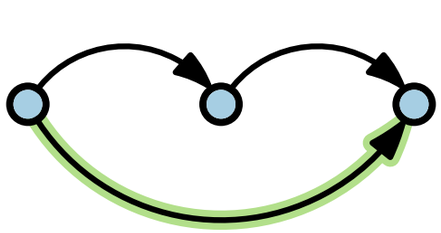


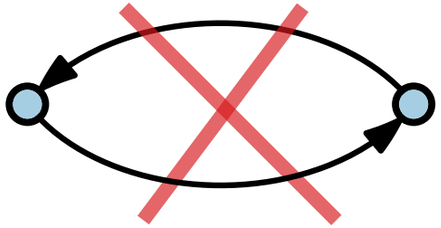
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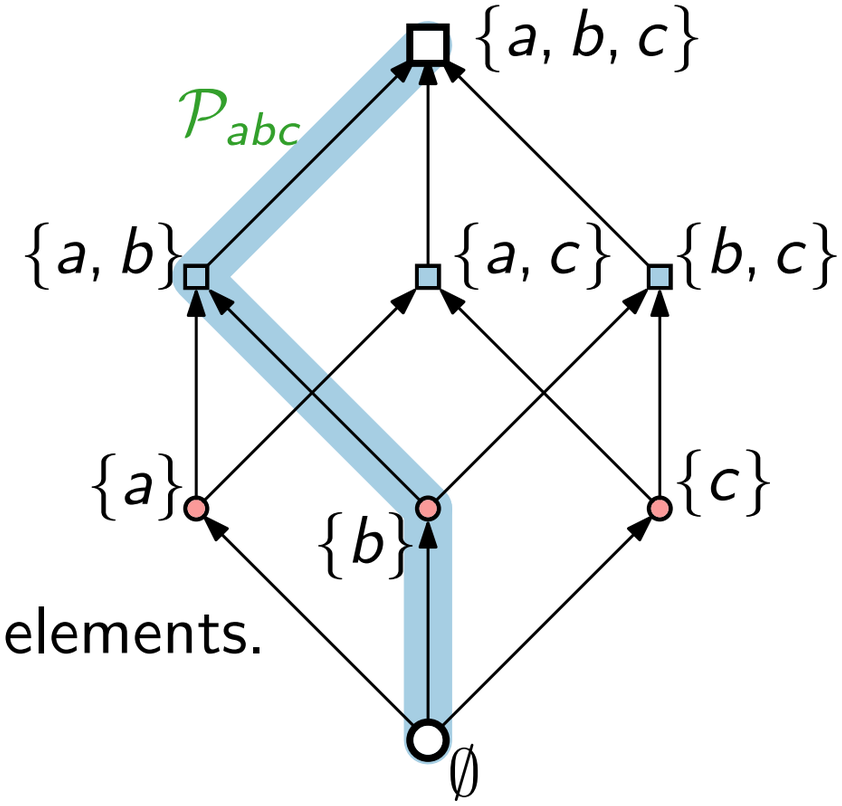
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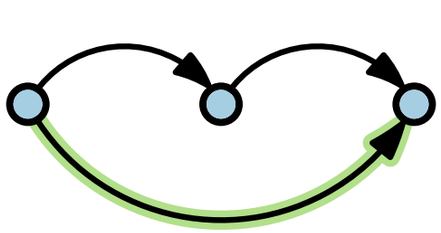


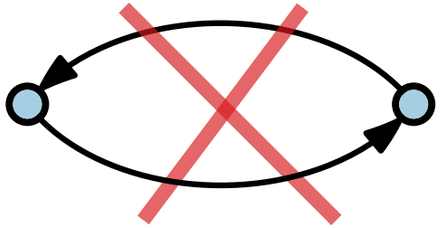
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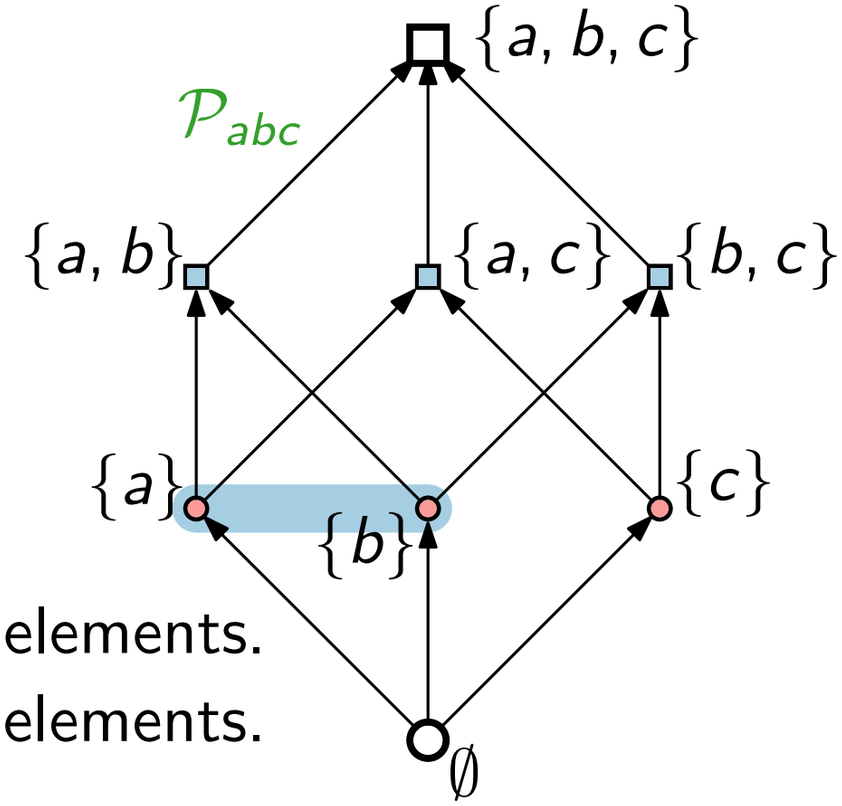
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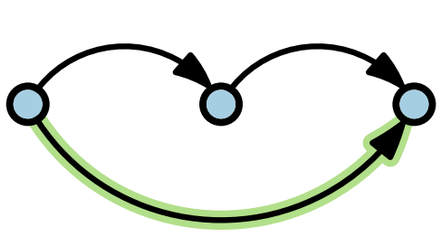
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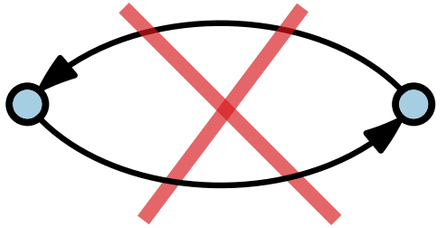
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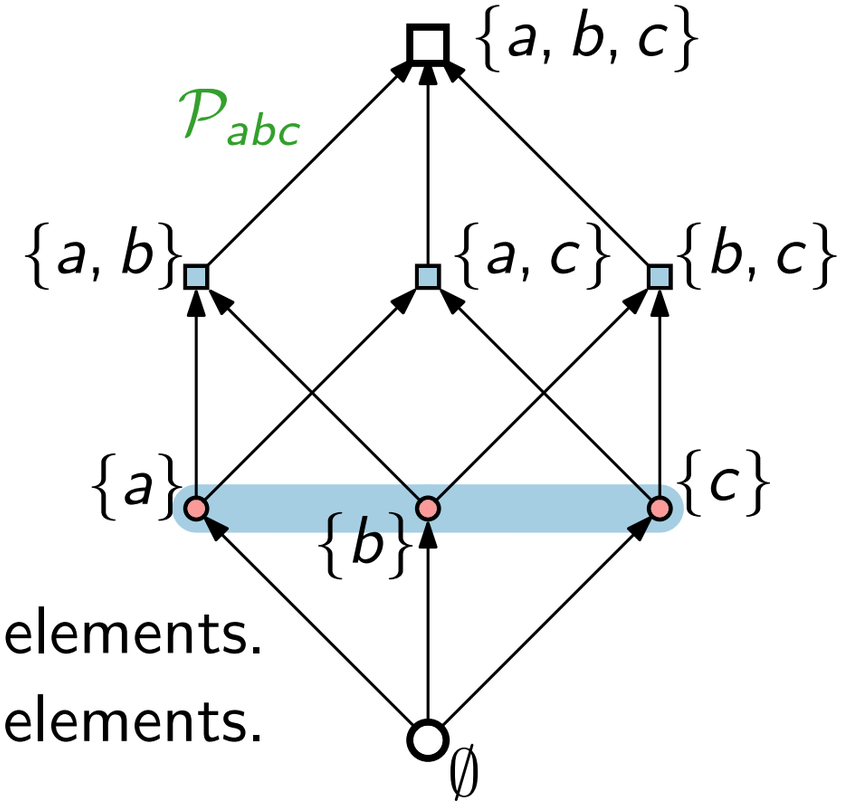
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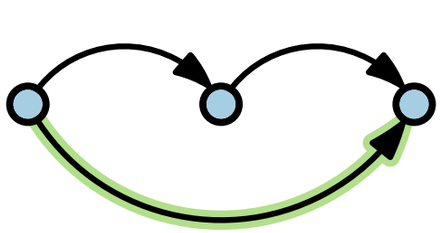
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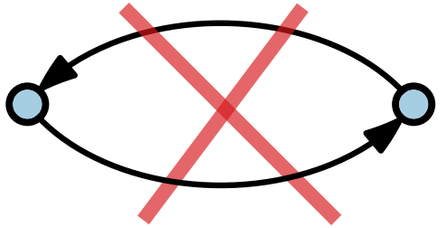
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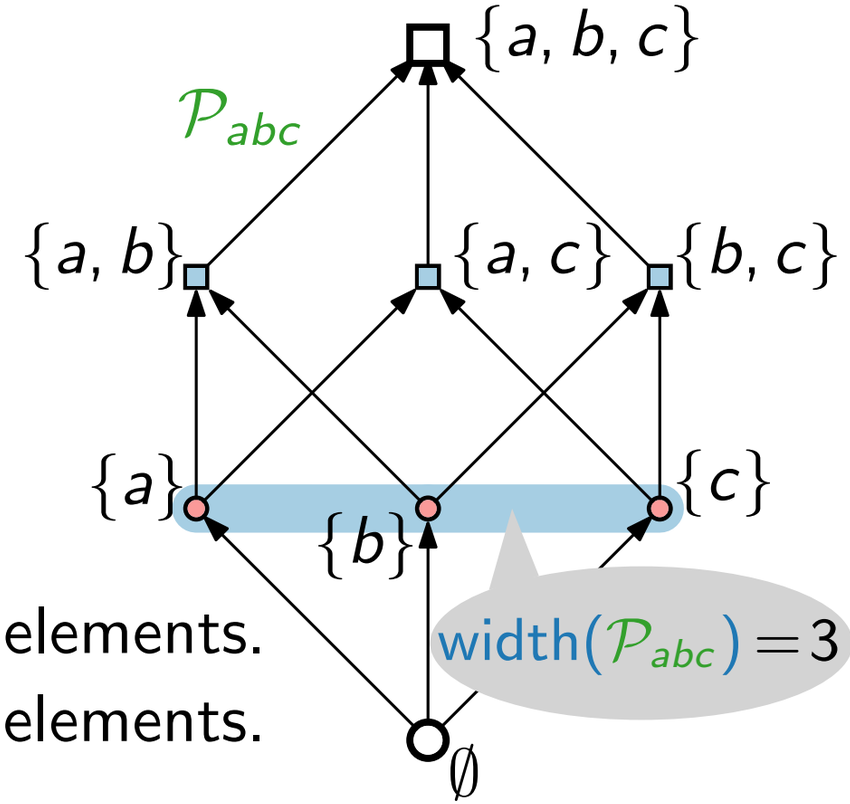
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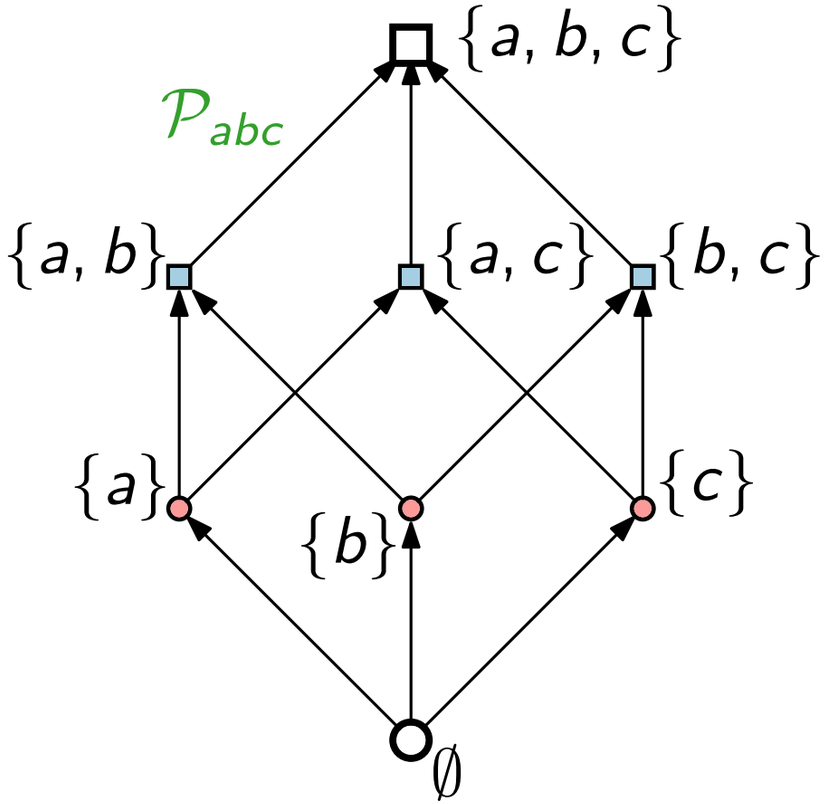
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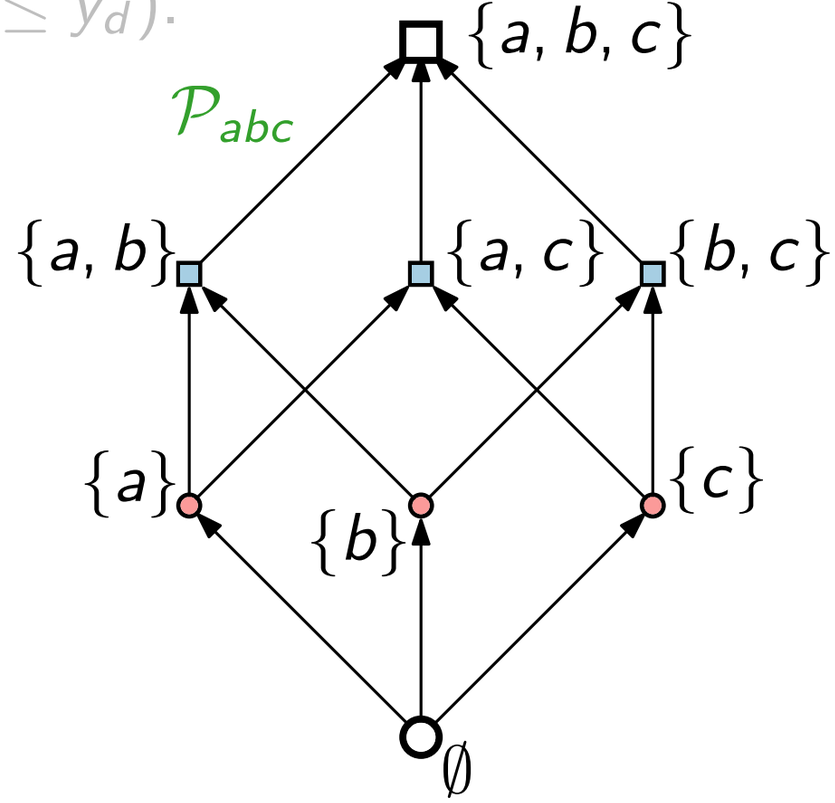


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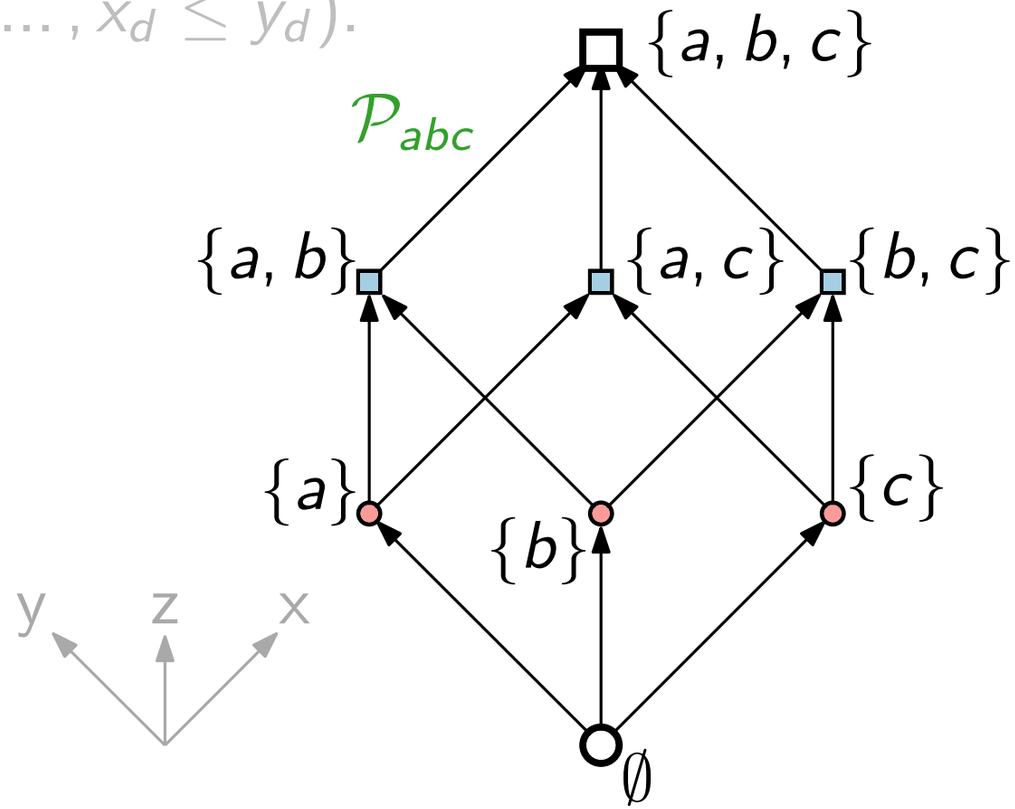


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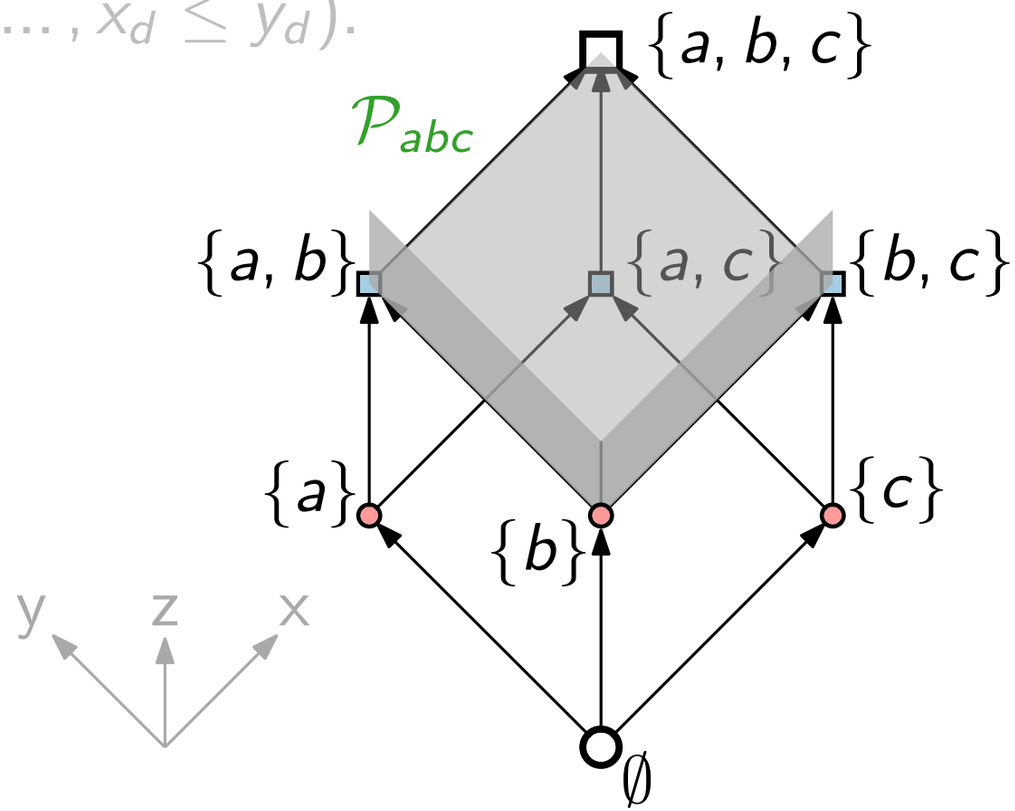


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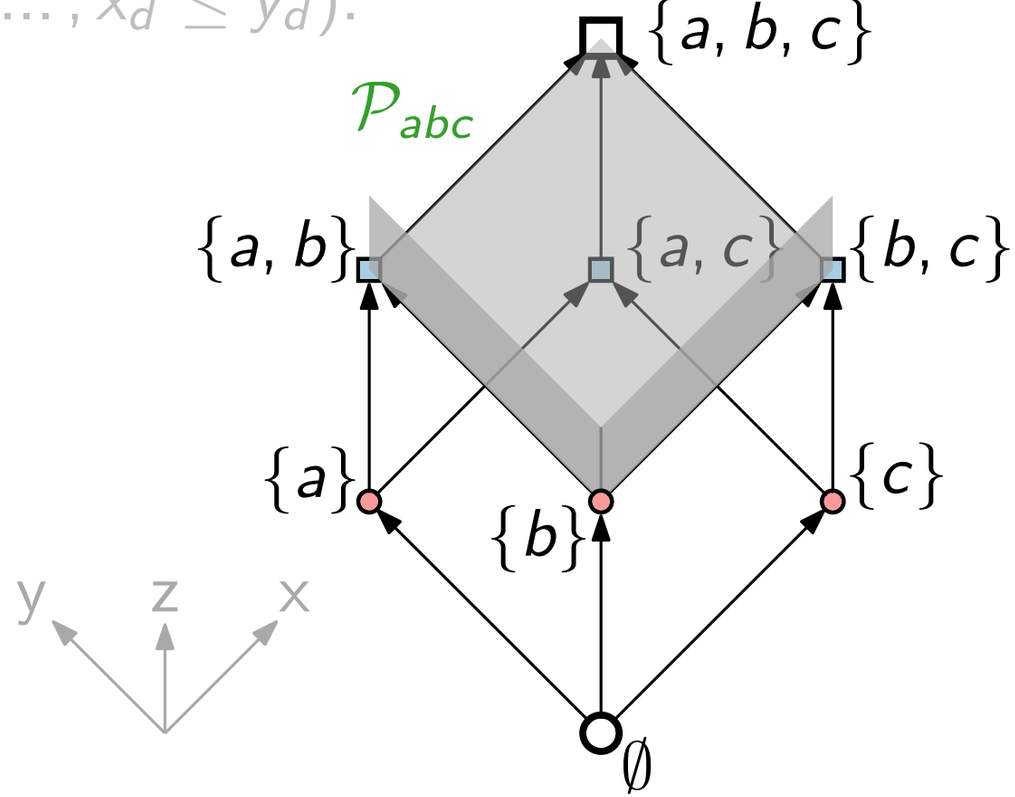
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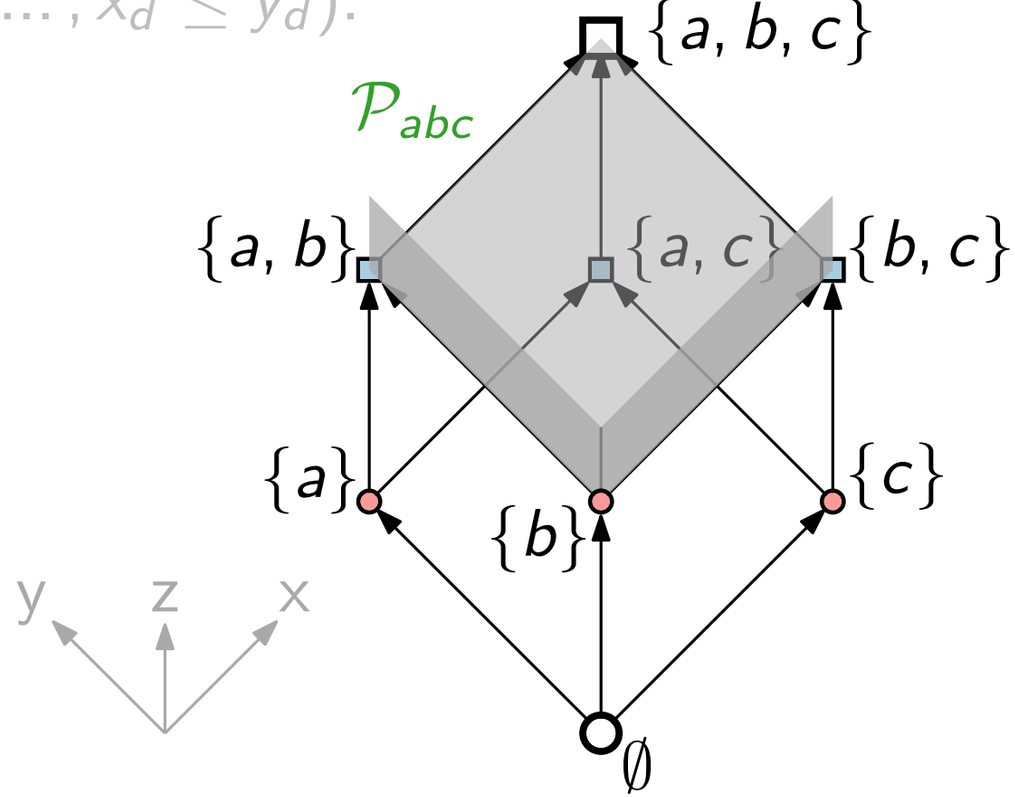
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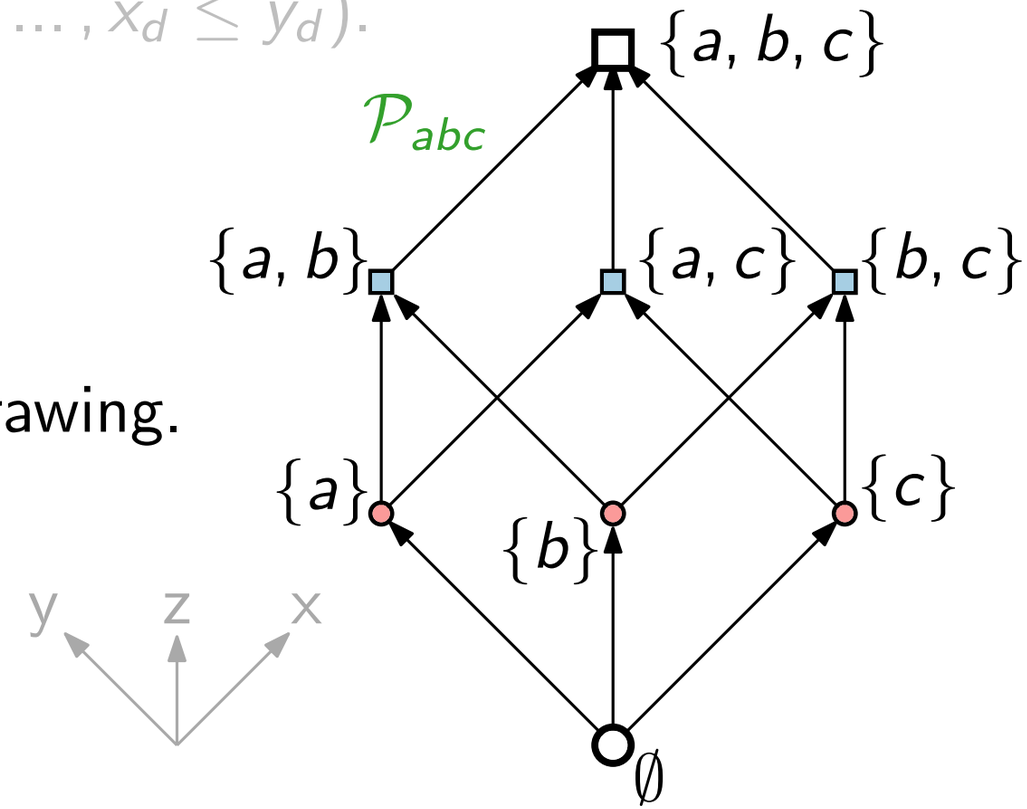
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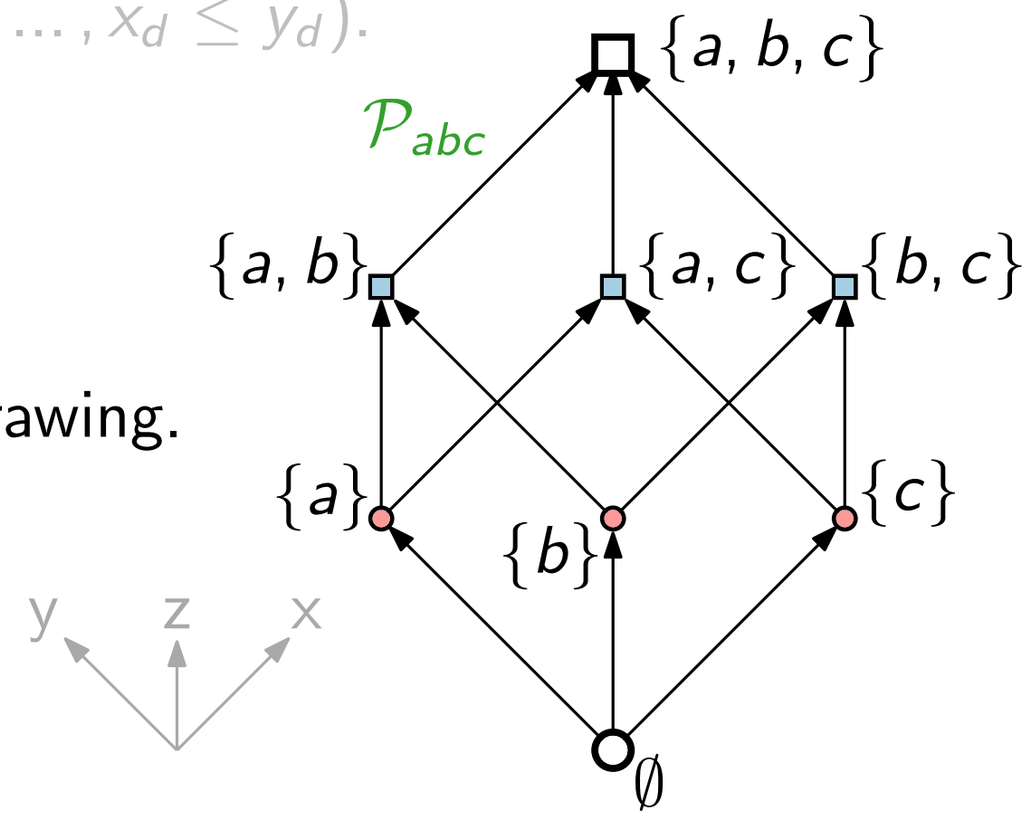
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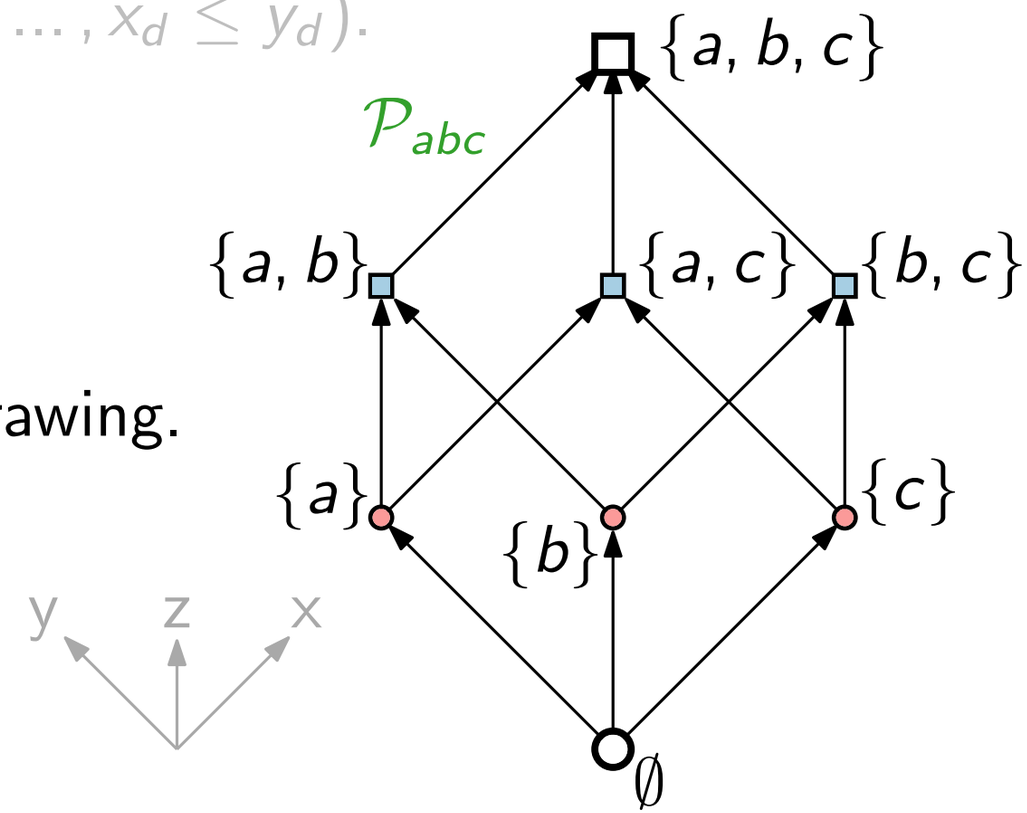
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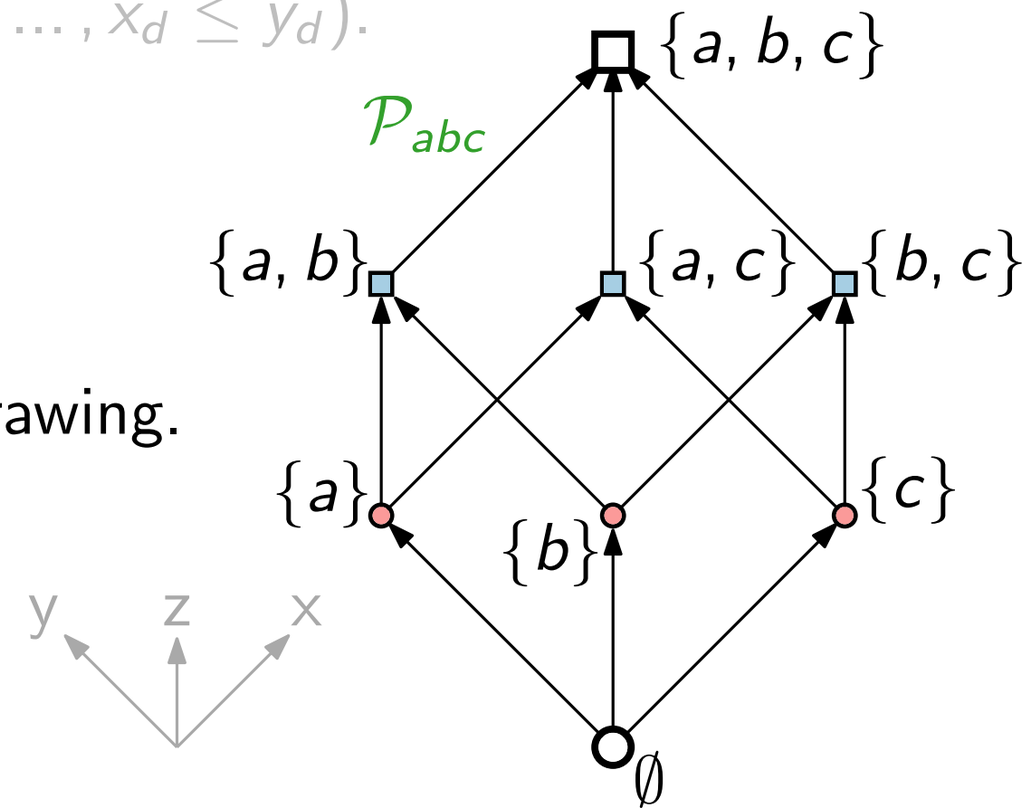
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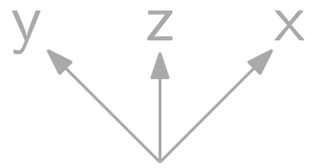
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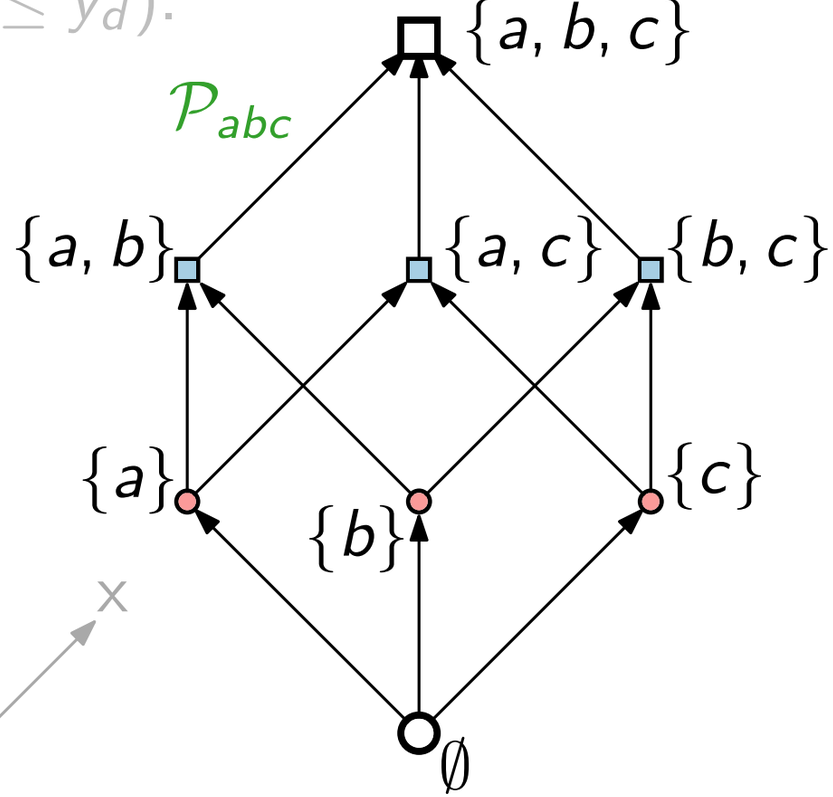
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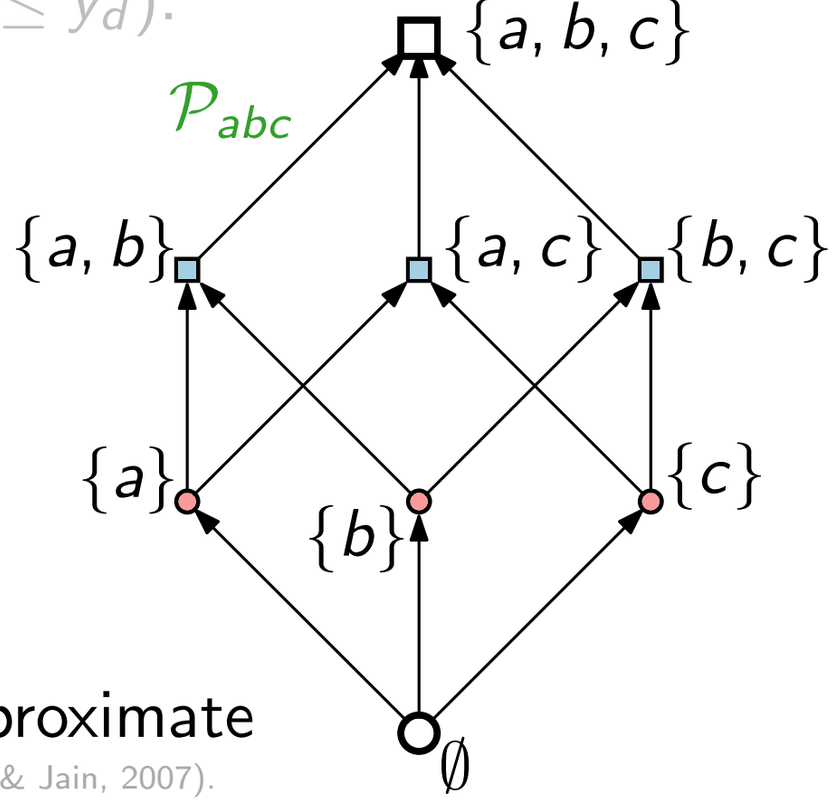
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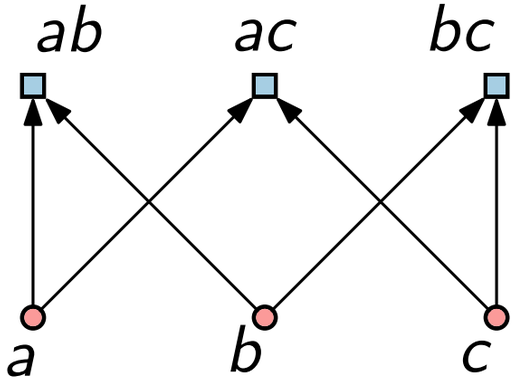
$\Rightarrow$ 3D dominance drawing $\Rightarrow \dim(\mathcal{P}_{abc}, \subseteq) \leq 3$.

$\mathcal{P}_{abc}$ does not "fit" in $\mathbb{R}^2$.



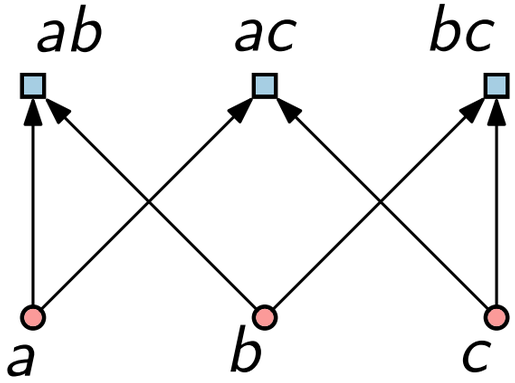
$\Rightarrow \dim(\mathcal{P}_{abc}, \subseteq) = 3$. Remark. dim is NP-hard to approximate (within a factor of $O(n^{0.5-\varepsilon})$ – Hedge & Jain, 2007).

Motivation

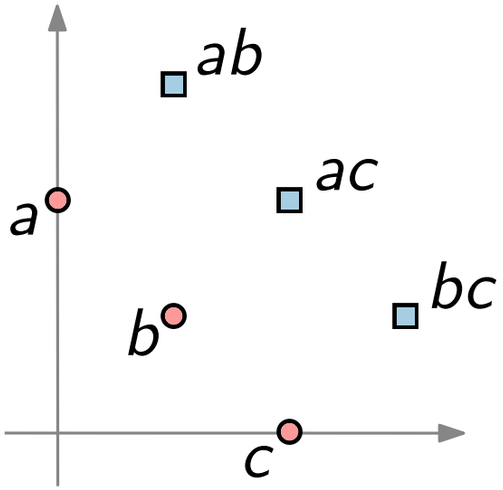


crown poset Q

Motivation

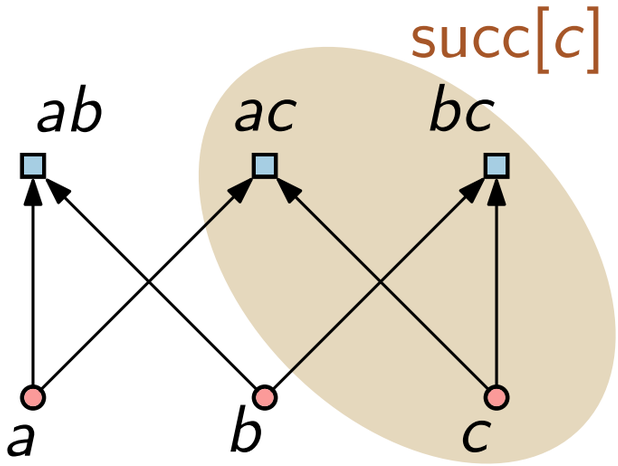


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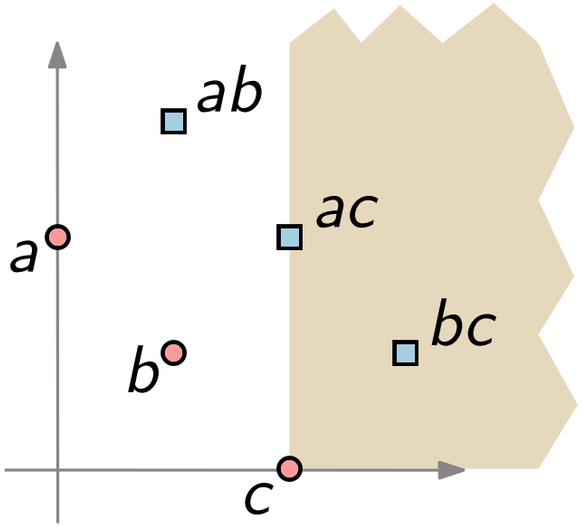


a plane map for Q

Motivation



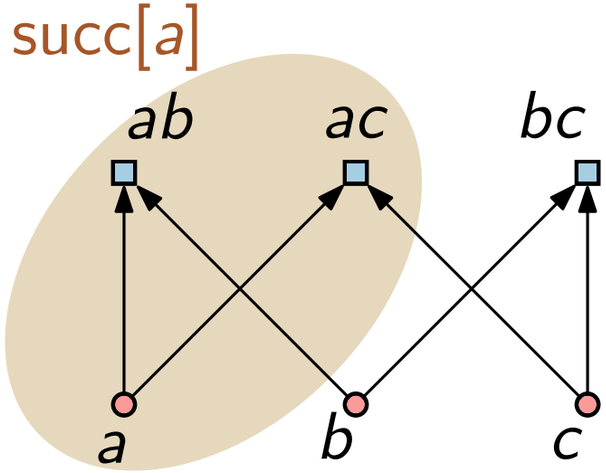
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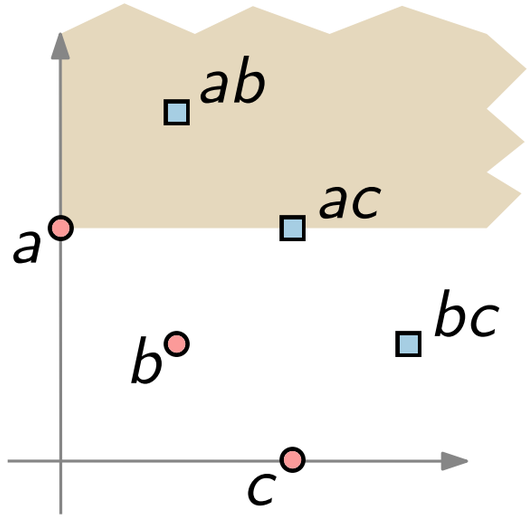
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– c is *tight*,

Motivation



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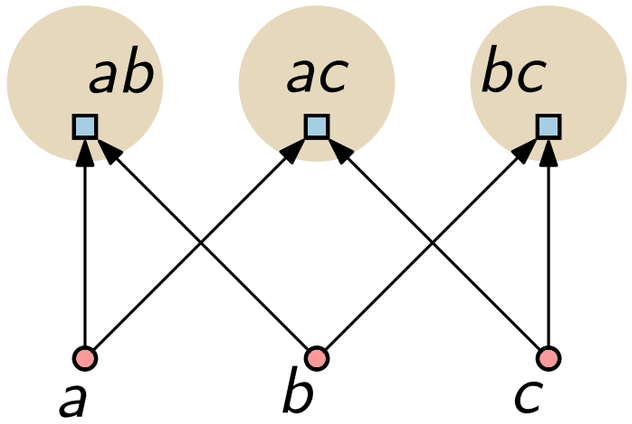


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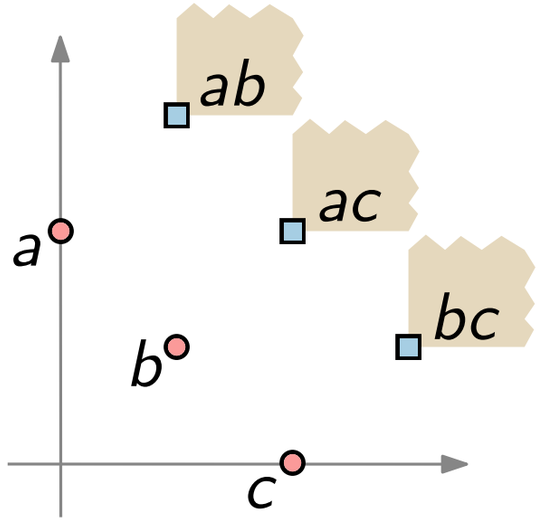
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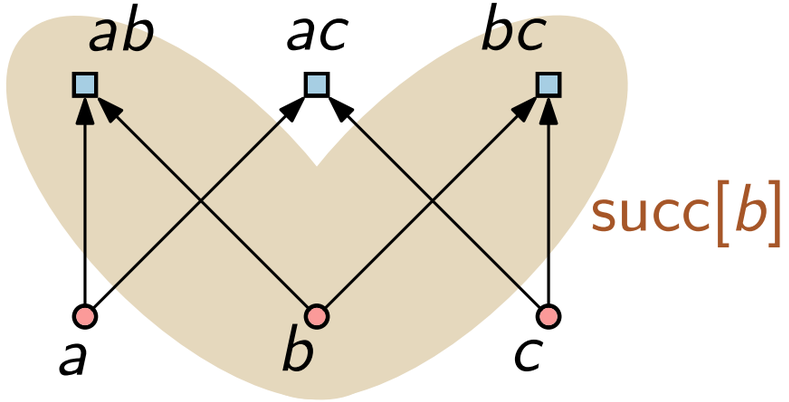
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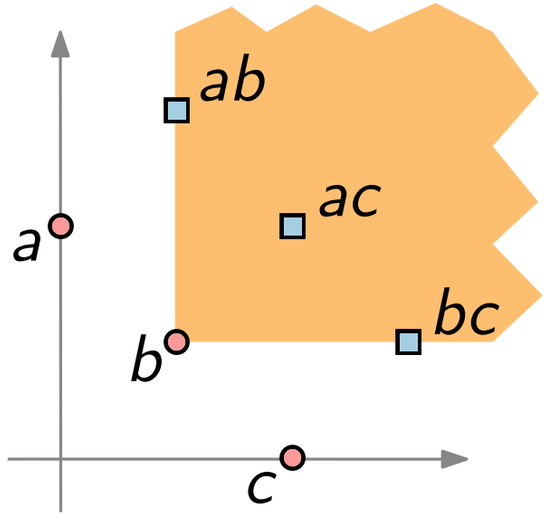
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Motivation



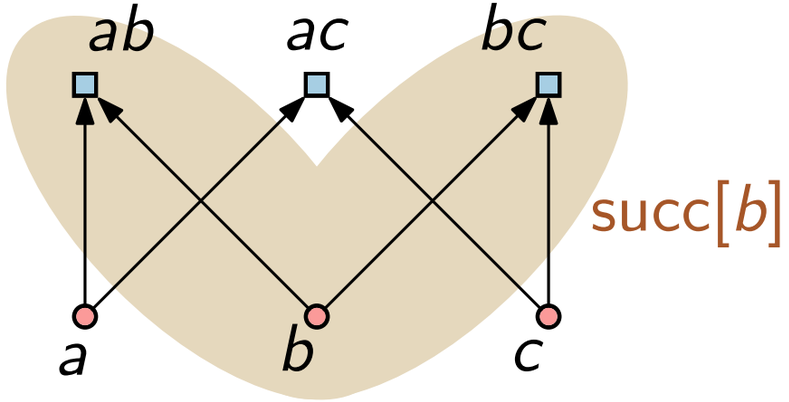
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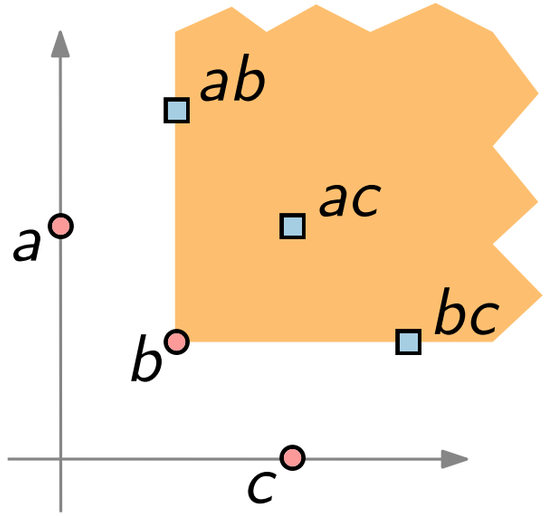
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Motivation



crown poset Q



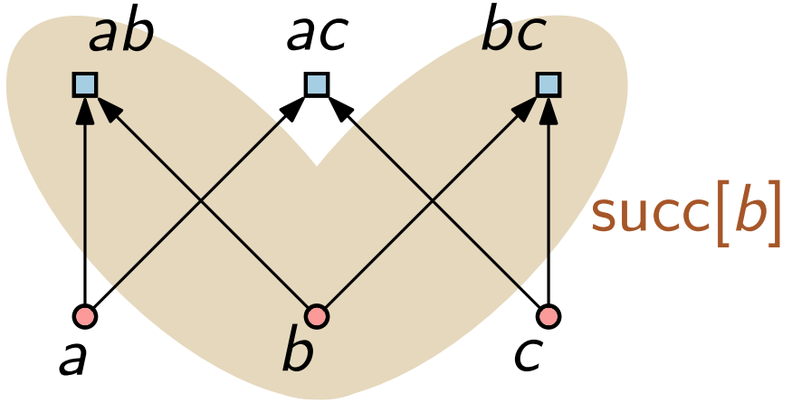
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The *(dominance) mapability* of Q, dmap(Q), is 5.

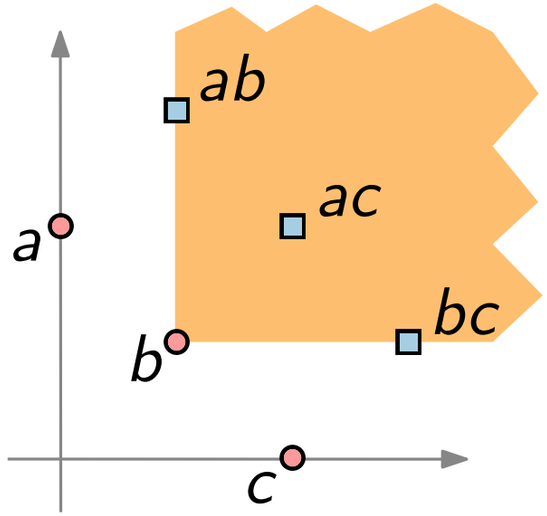
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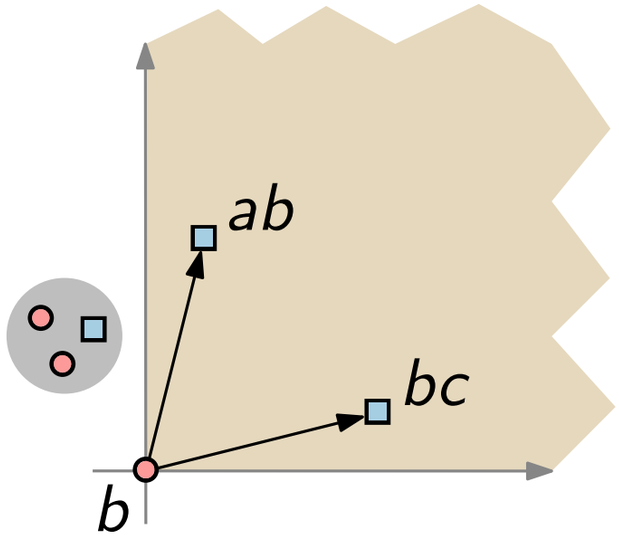
Motivation



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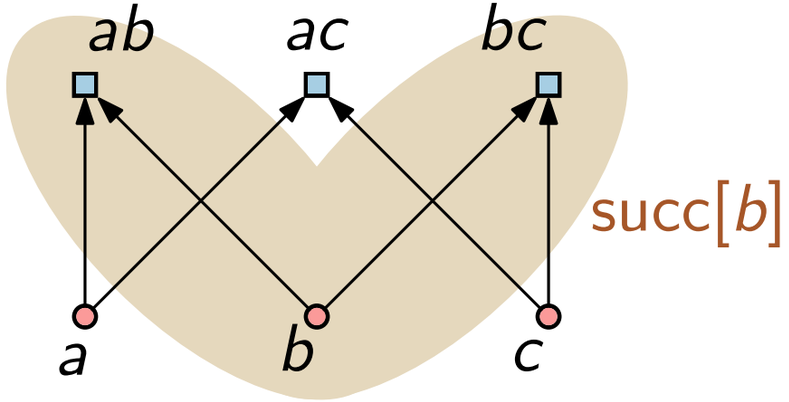


a 2nd map for Q

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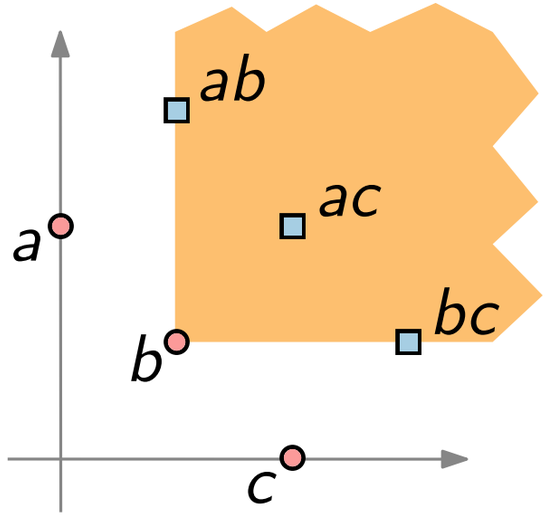
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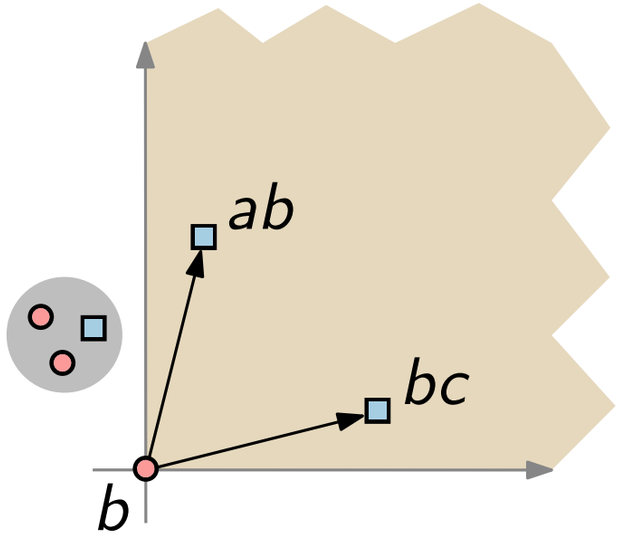
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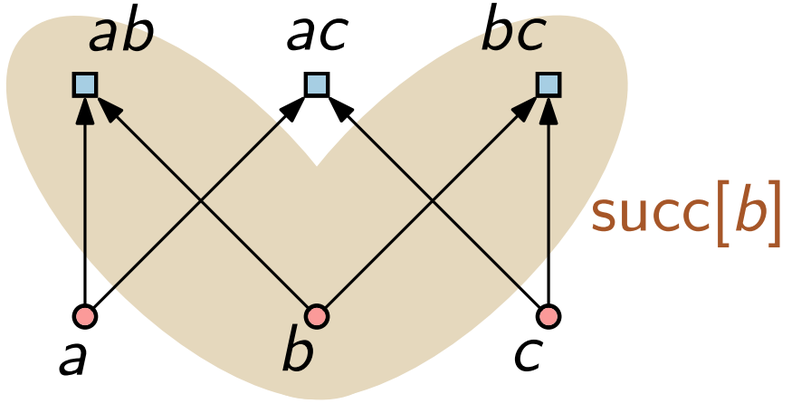
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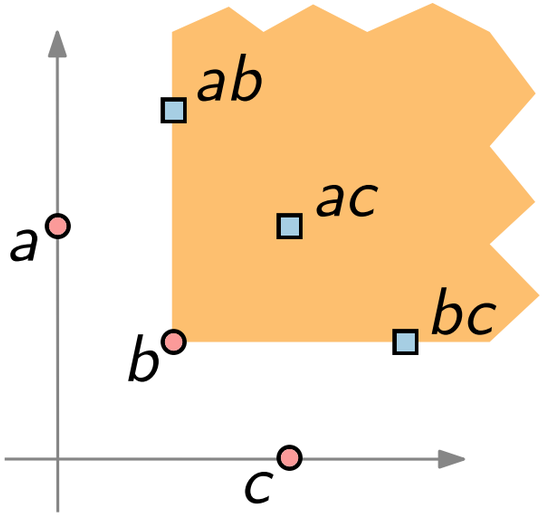
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Motivation



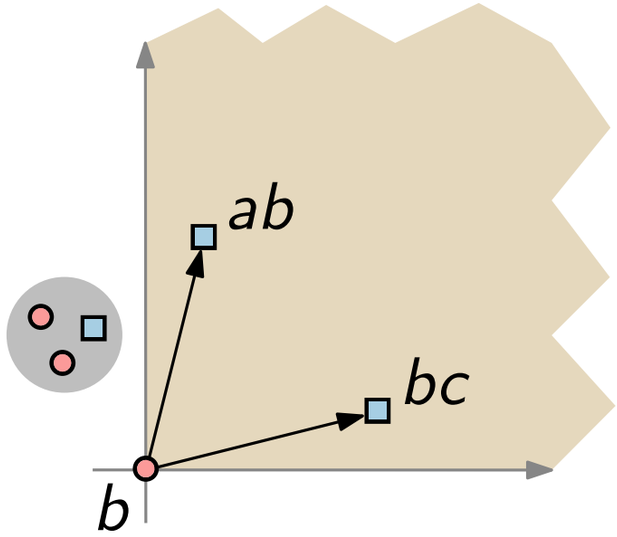
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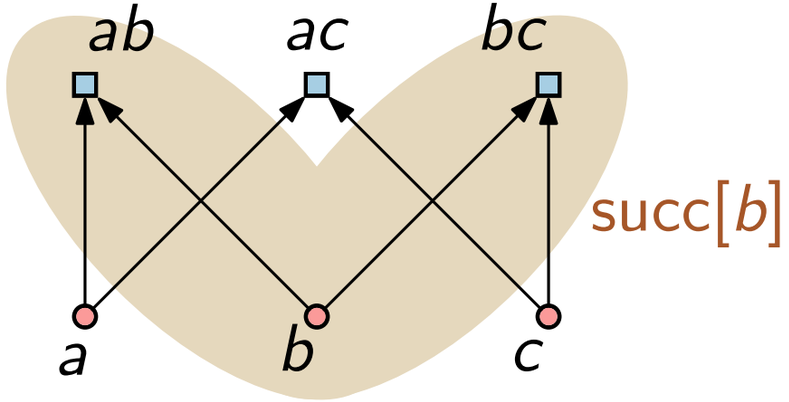


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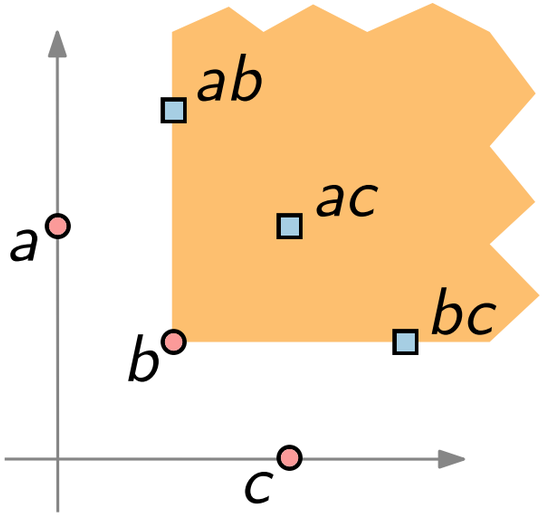
an *atlas* for Q

Motivation



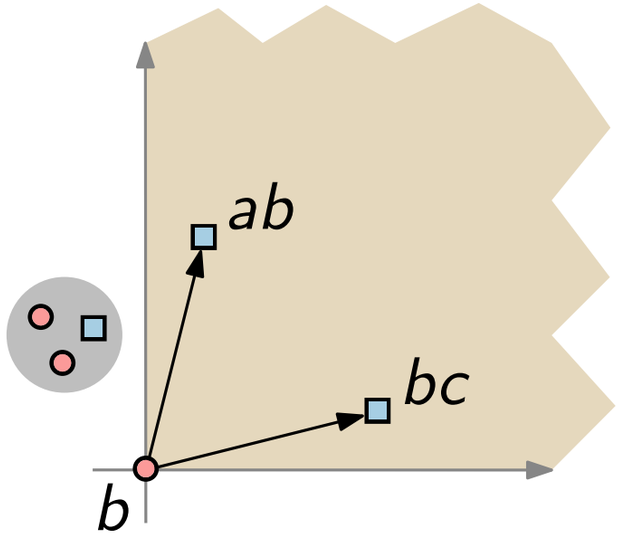
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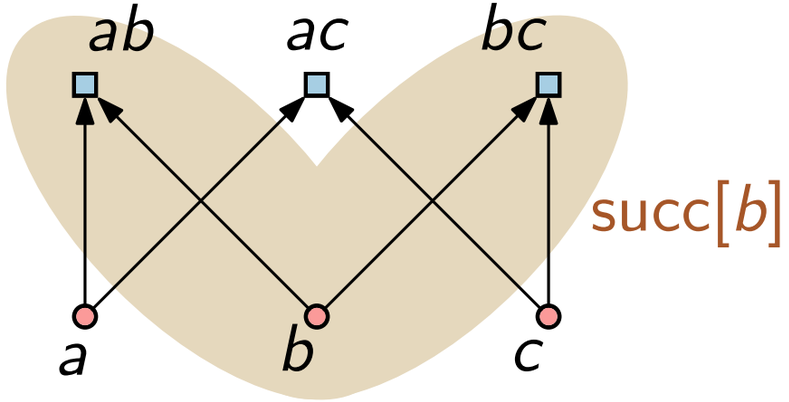


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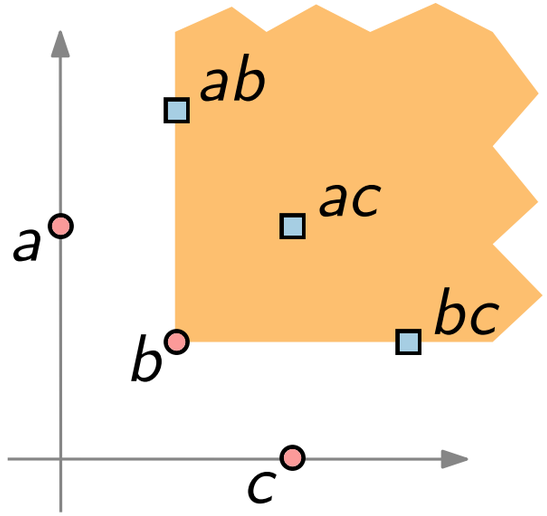
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an *atlas* for Q: a collection A of plane maps s.t. every $x \in Q$ is tight w.r.t. a map $\pi \in A$.

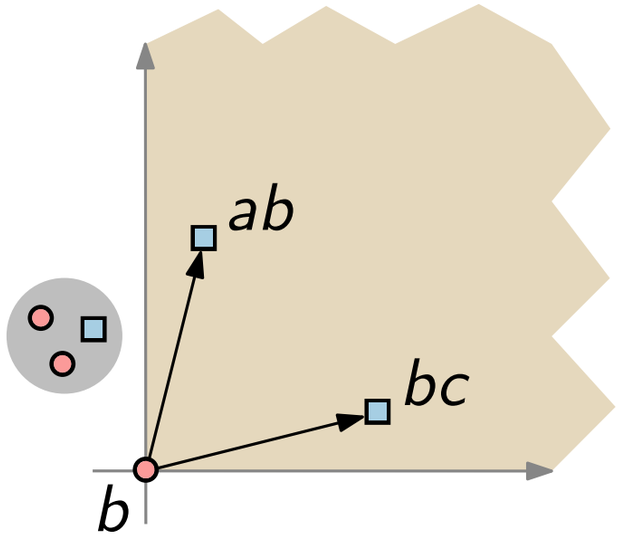
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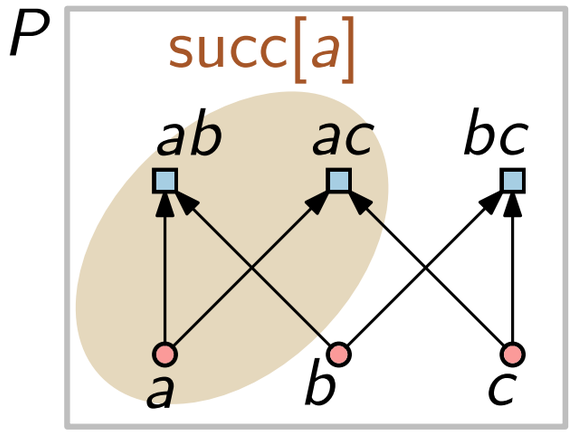
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The *atlas thickness* of Q, at(Q), is 2.

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Let's Recap...

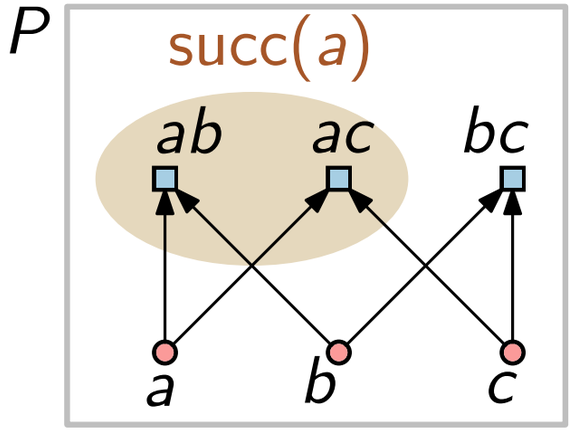
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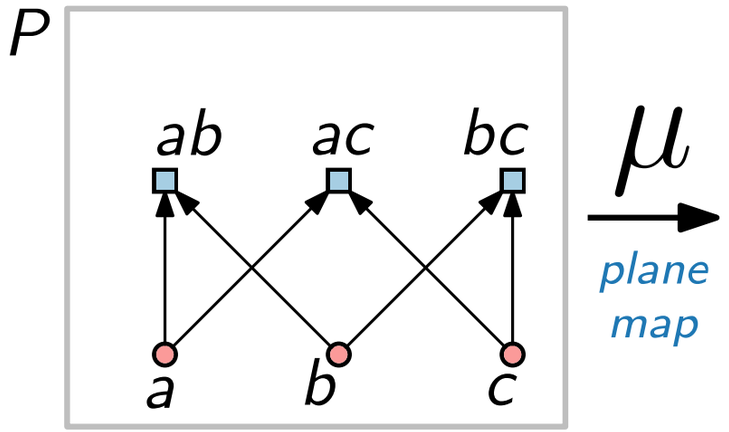
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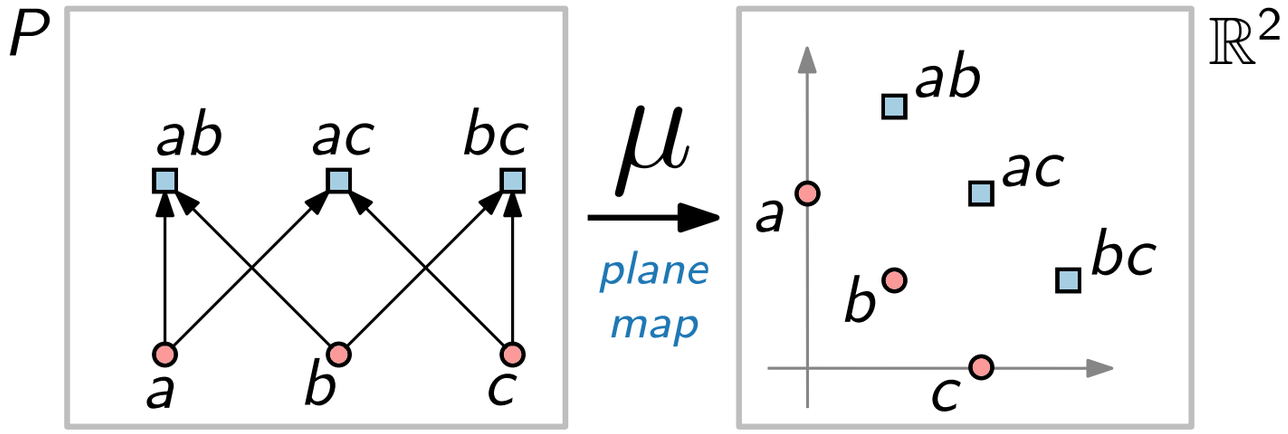
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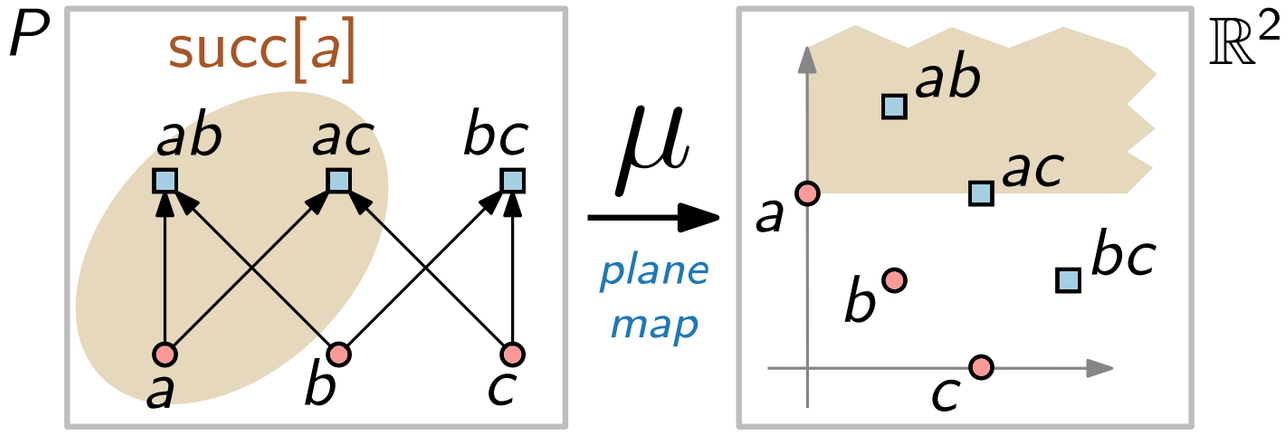


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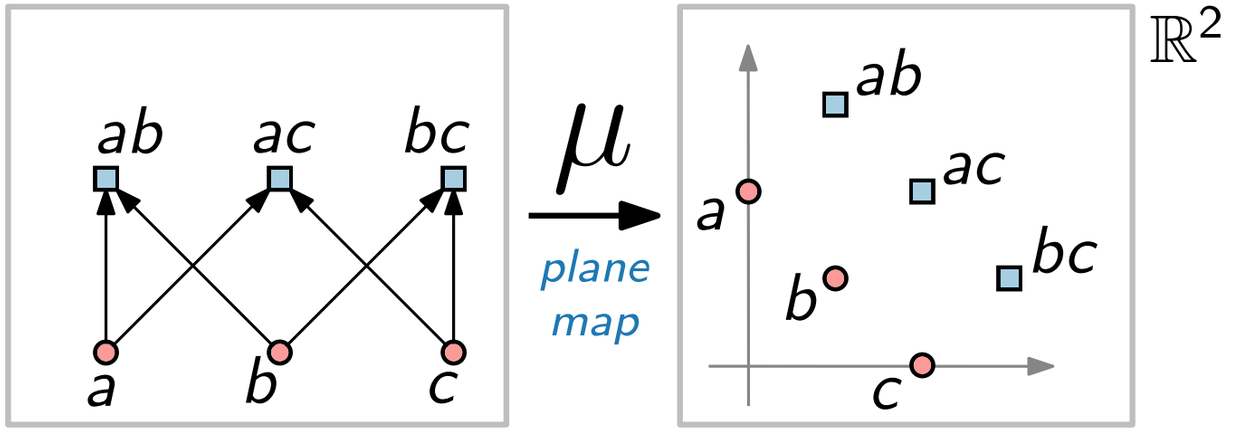
An element x of P is *tight* on a map μ if $\mathrm{succ}_P[x] = \{y \in P : \mu(x) \leq_{\mathrm{dom}} \mu(y)\}$.



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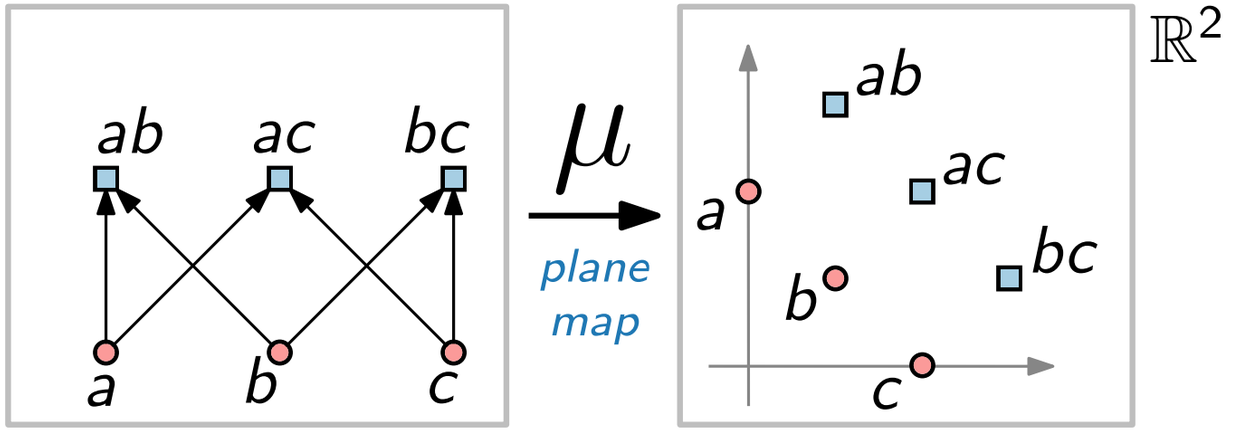
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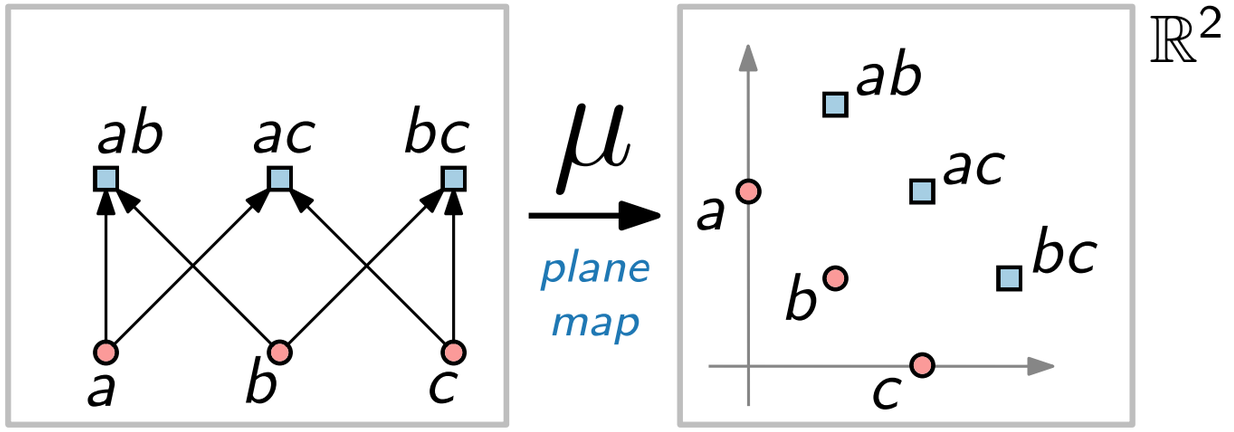
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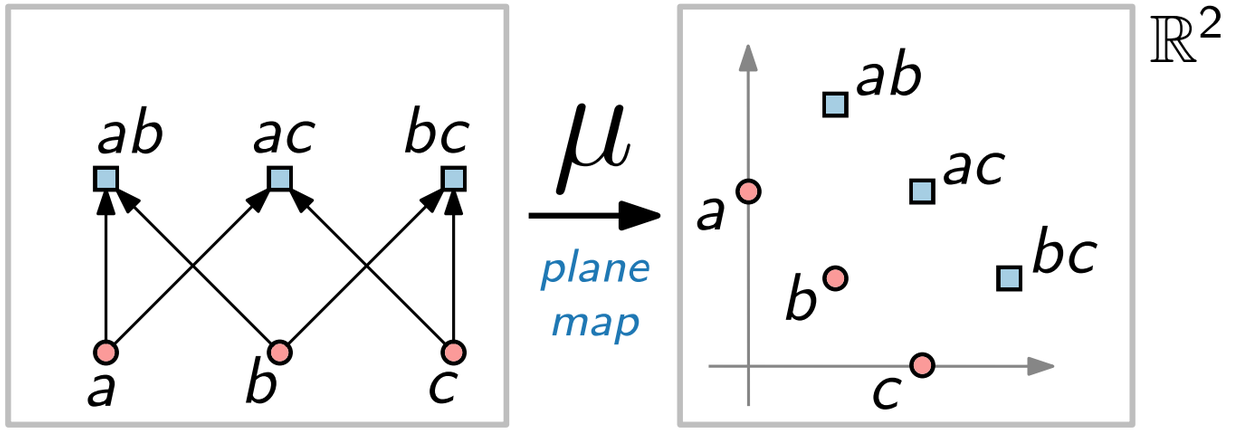
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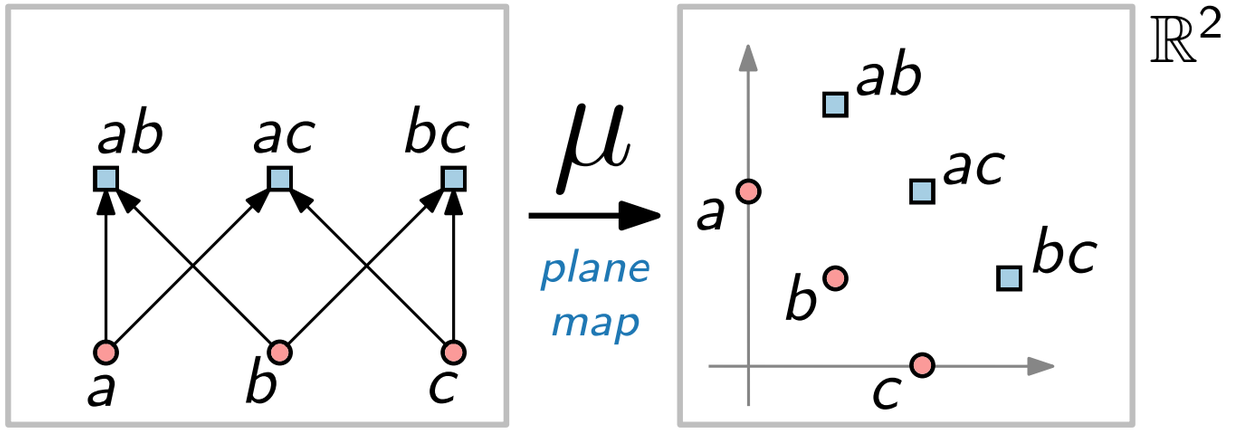
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Related Work

Trend (?): Visualize a complex combinatorial or geometric object (say, a graph) by means of a collection of drawings, each of which includes all parts (say, edges), but highlights only a subset of them.

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collection of straight-line drawings of a graph s.t. every edge of the graph is *uncrossed* in at least one drawing in the collection.

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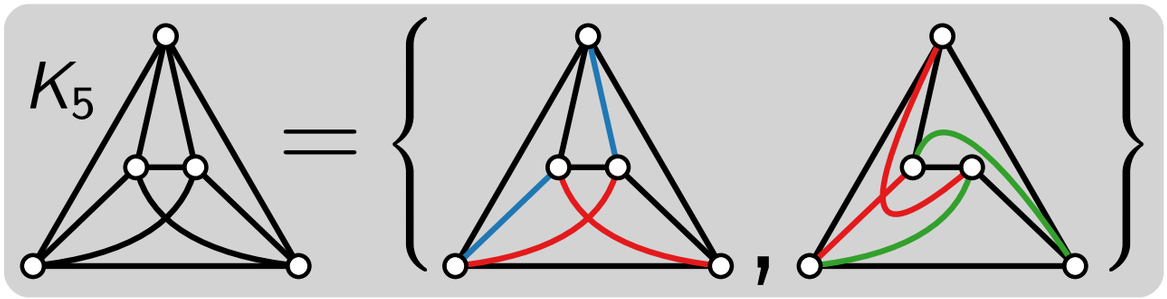


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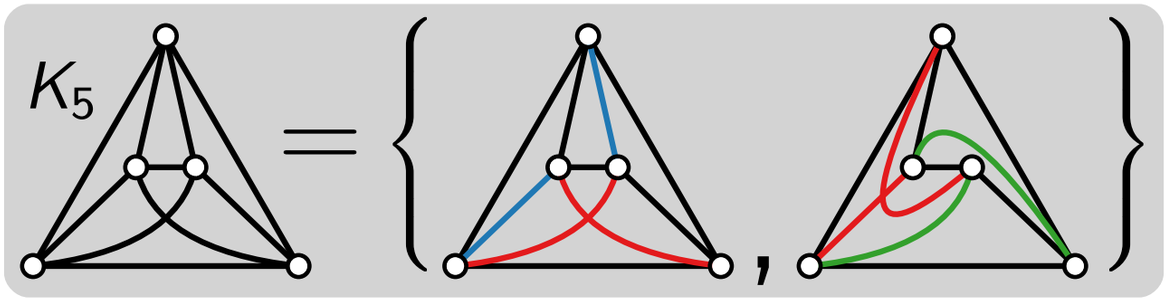


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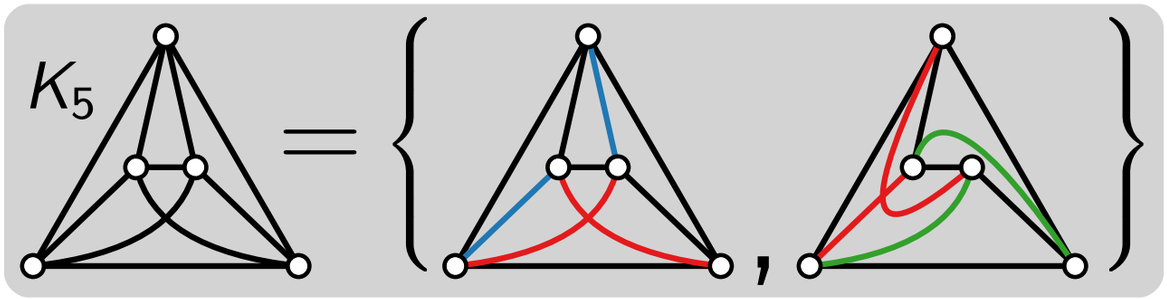
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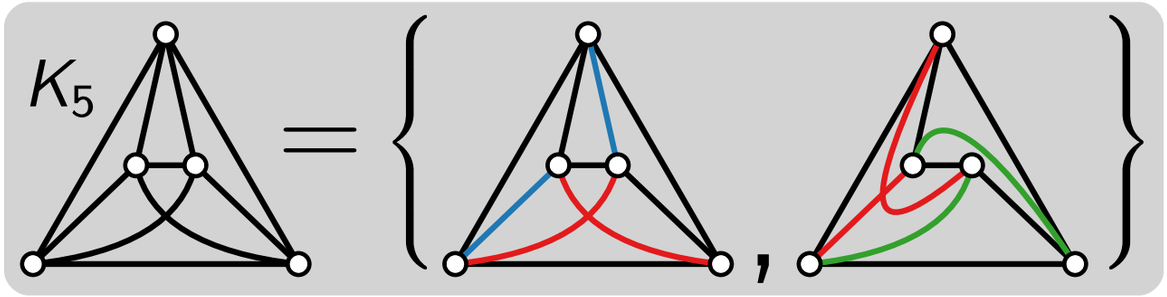
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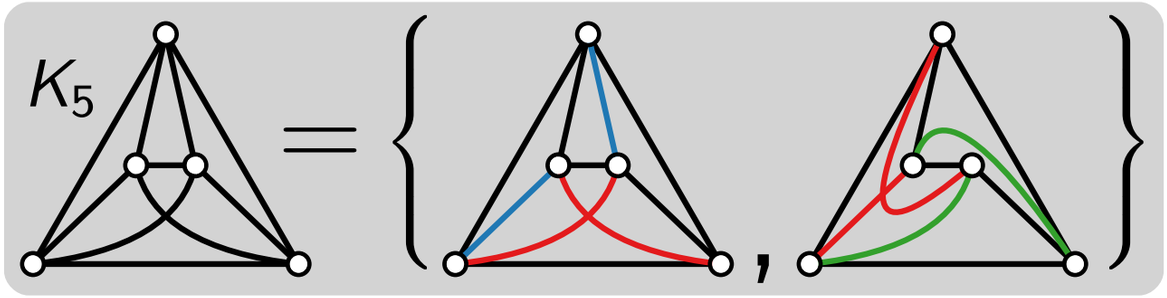
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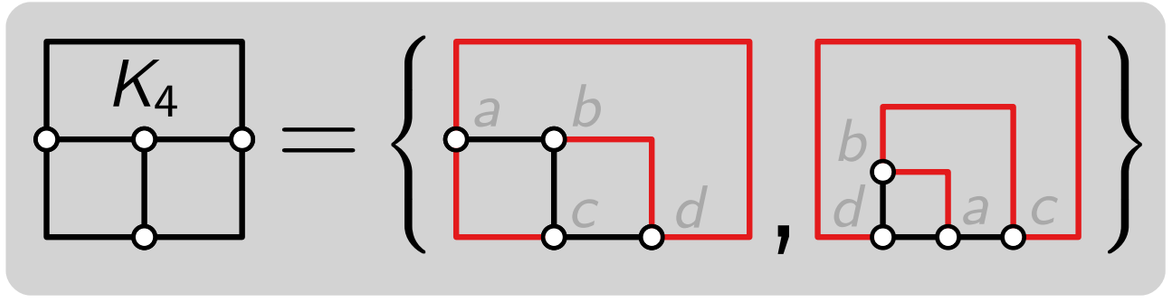
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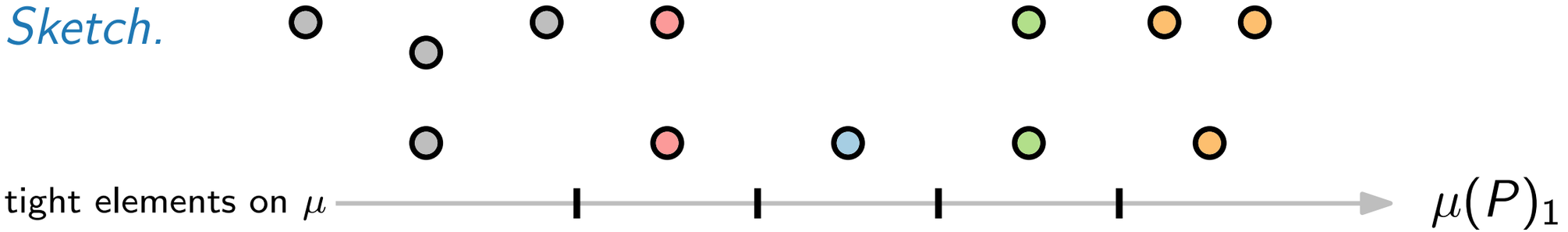
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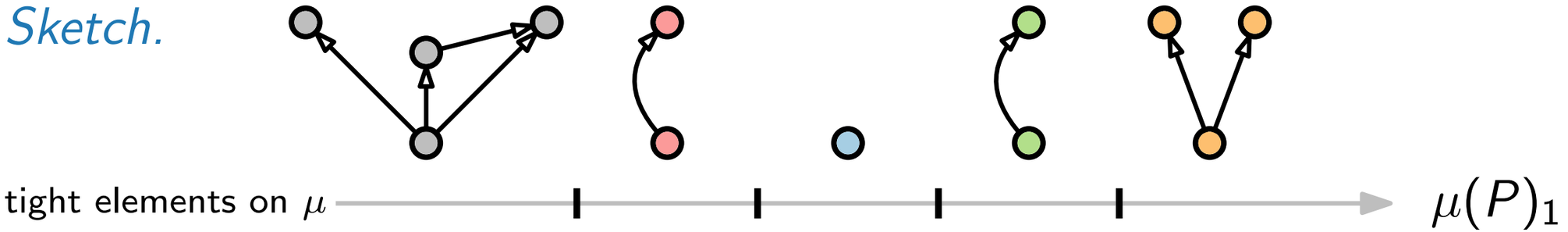
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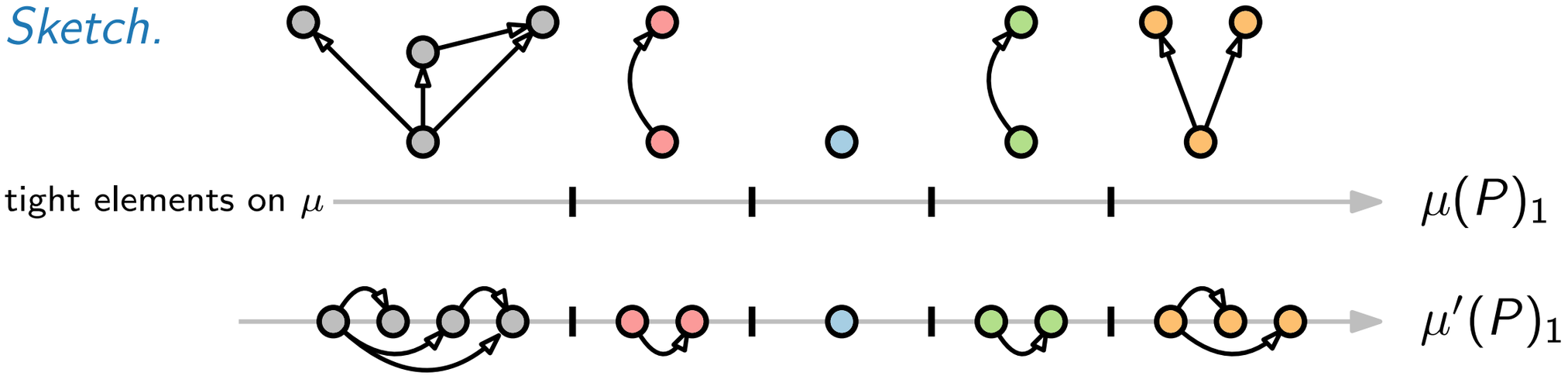
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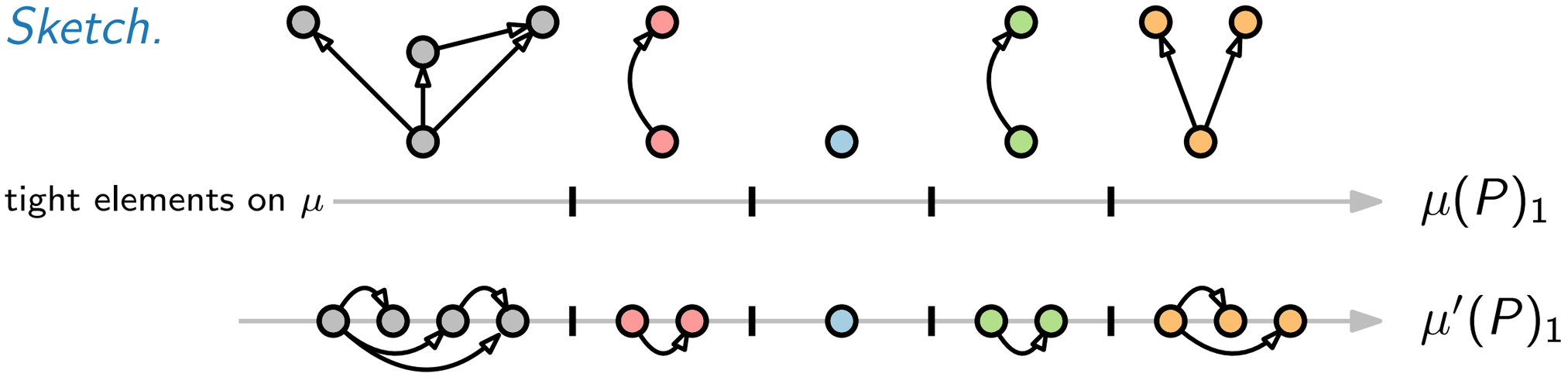
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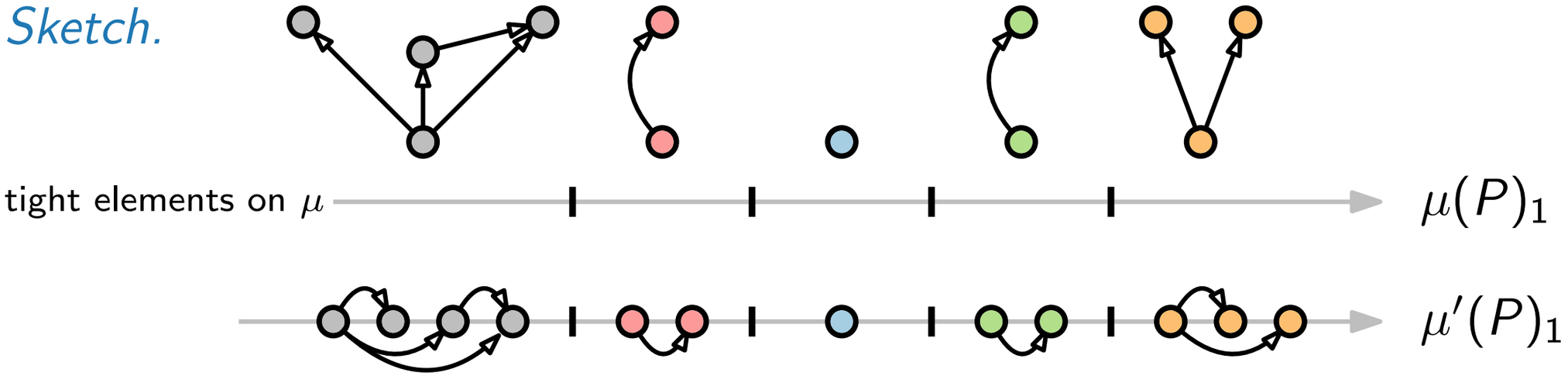


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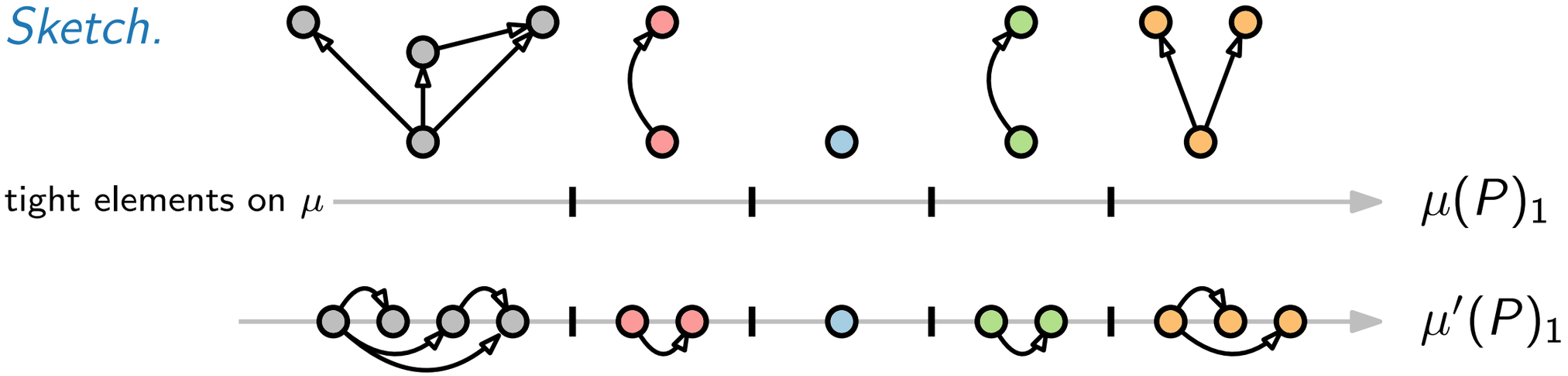
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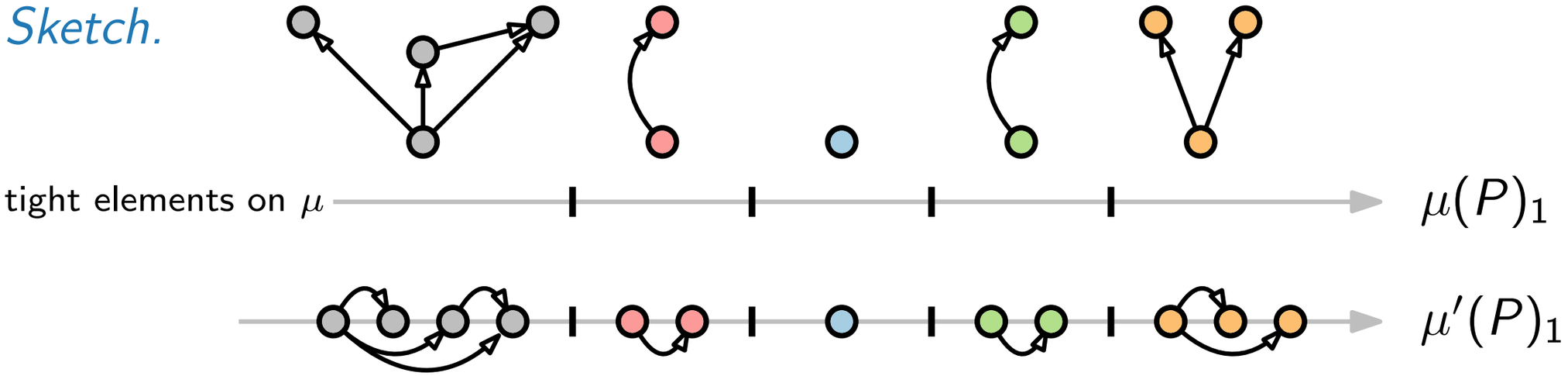
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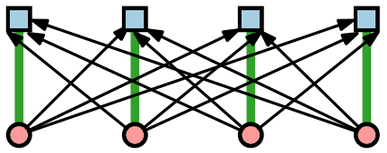
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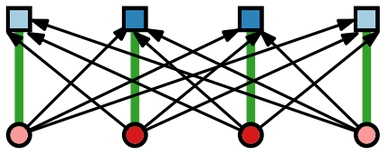
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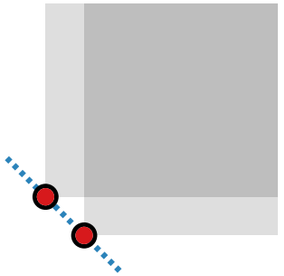
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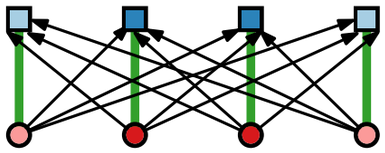
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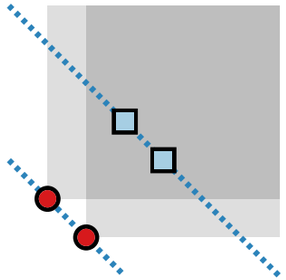
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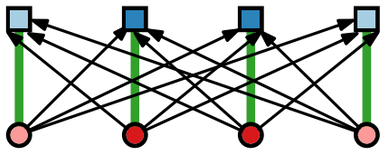
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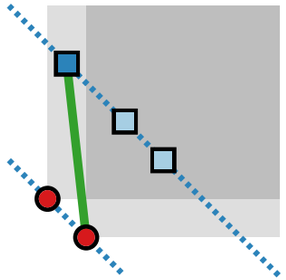
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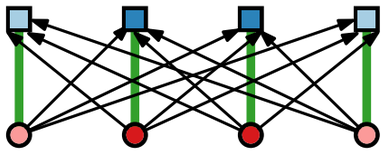
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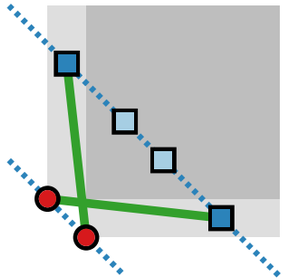
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Universal Bounds for the Atlas Thickness (II)

Lemma. $\forall$ poset P and $\forall$ two chains C and D in P,
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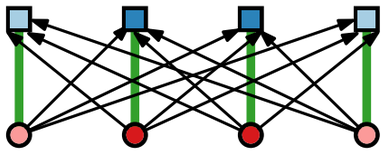
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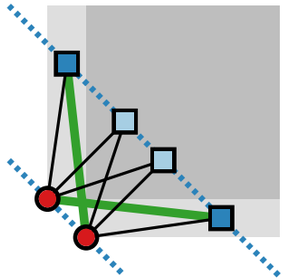
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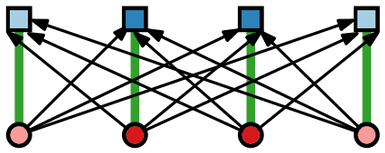
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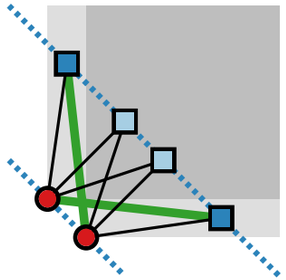
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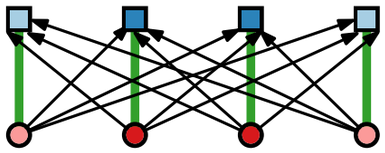
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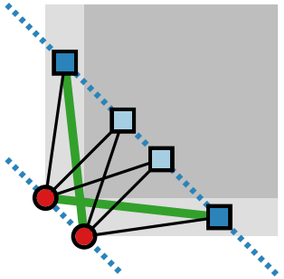
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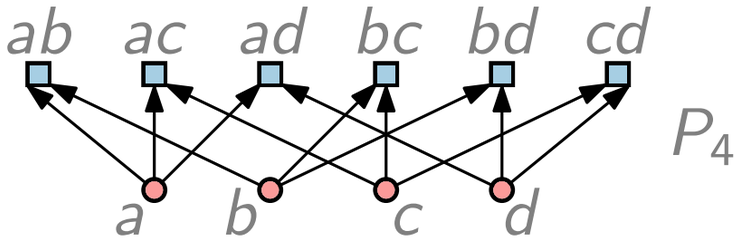
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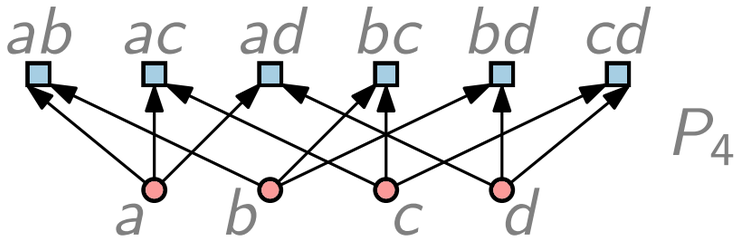


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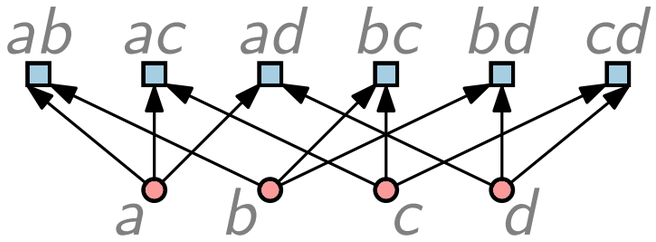
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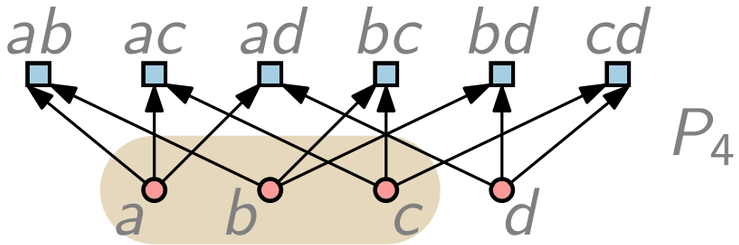


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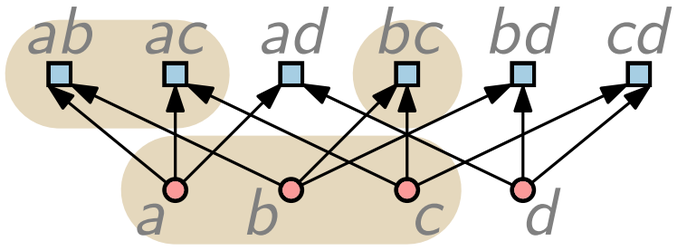
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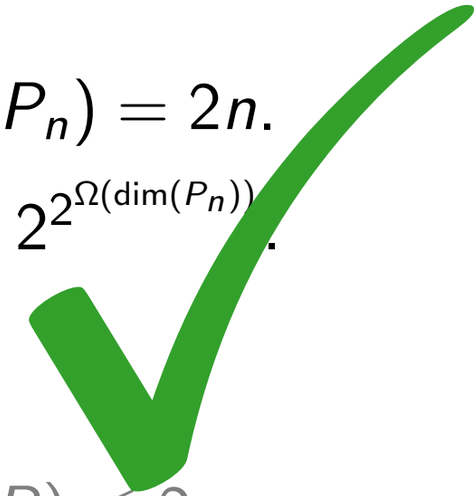
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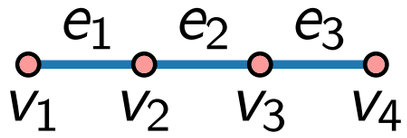
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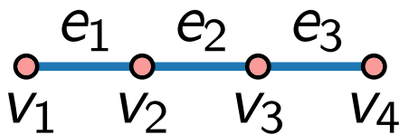
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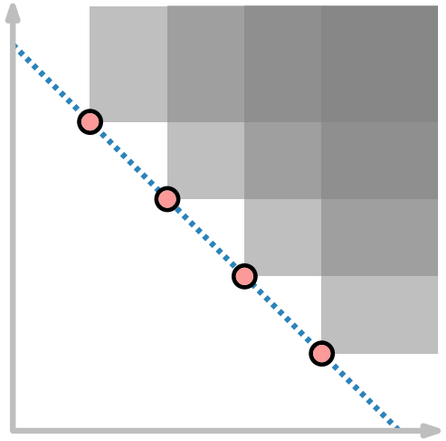


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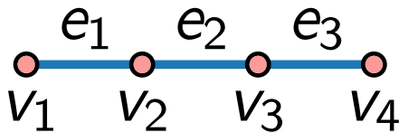


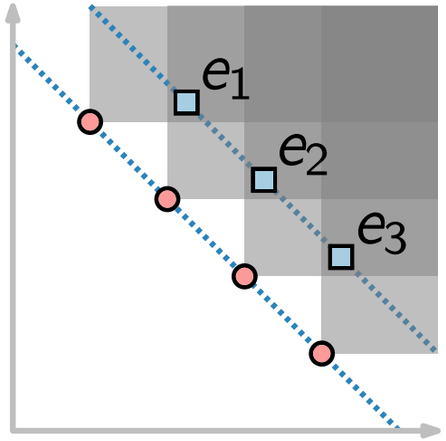


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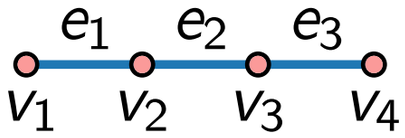


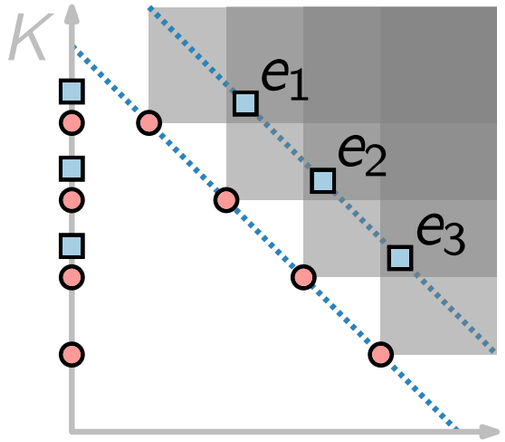


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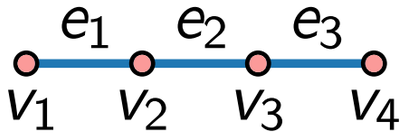


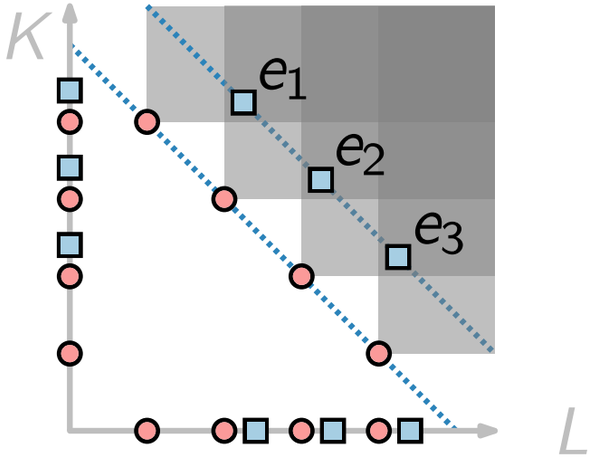


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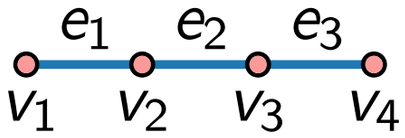


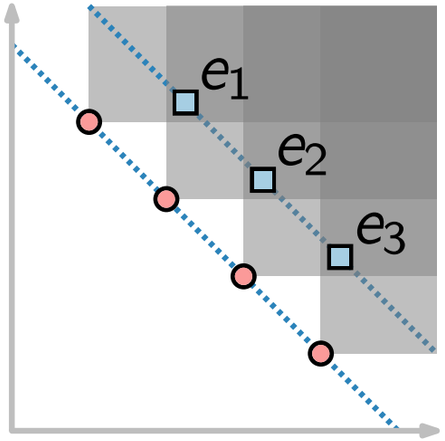


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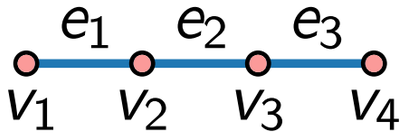


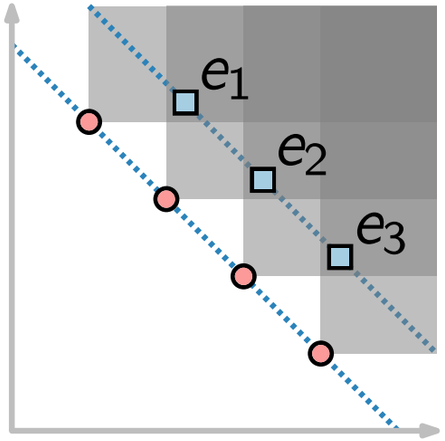
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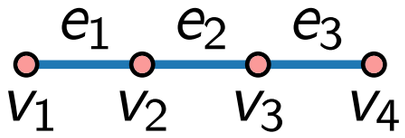
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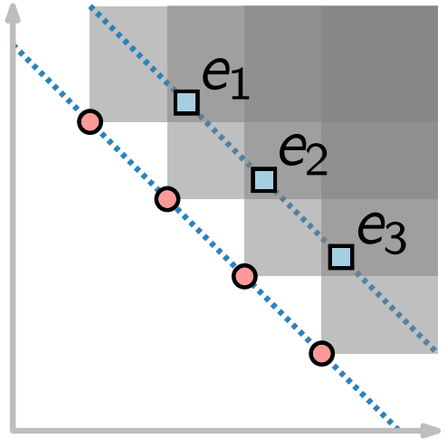
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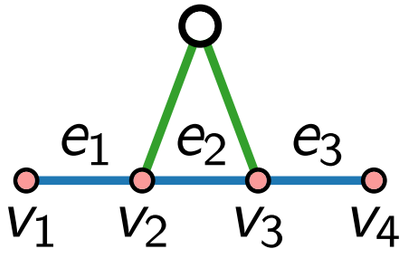
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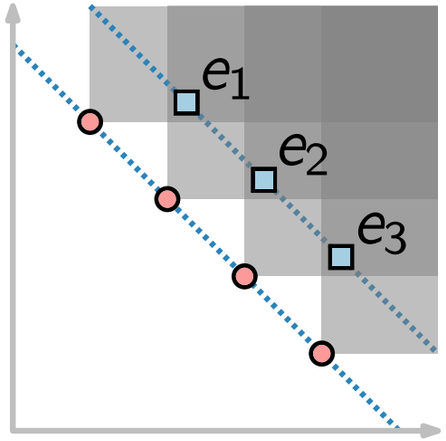
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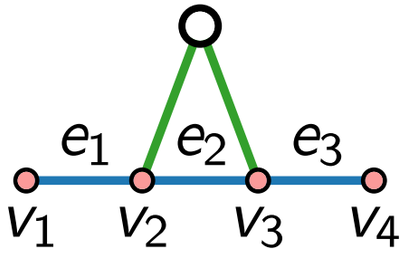
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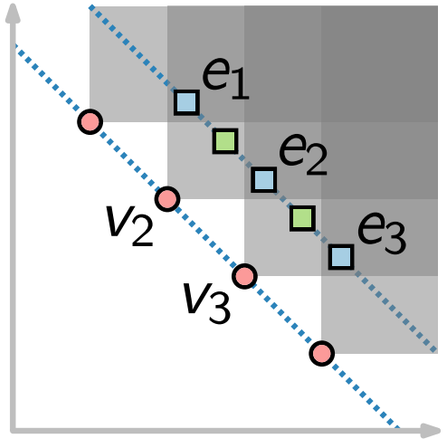
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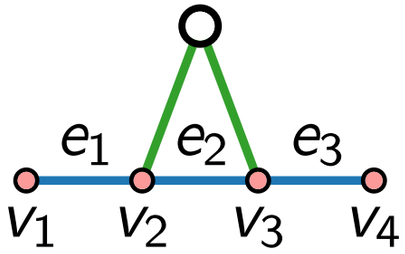
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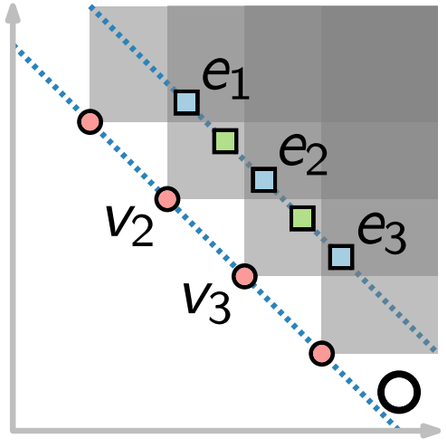
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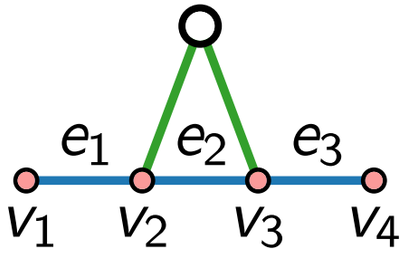
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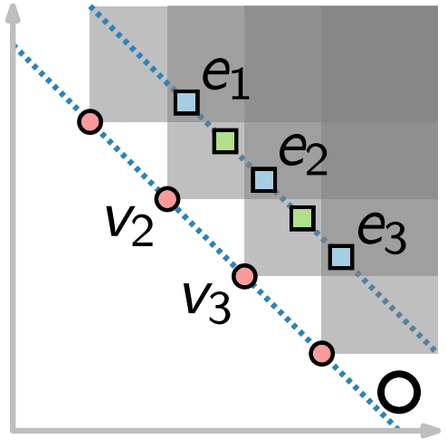
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$G[U]$ linear forest $\Rightarrow \exists$ map of P_G s.t. every $x \in U \cup E(G)$ is tight.

Complexity of Computing the Atlas Thickness

Prop. Let G be a graph, and let P_G be its incidence poset. Then:
$\dim(P_G) = 2 \Leftrightarrow G$ is a linear forest.

Proof. "$\Leftarrow$":





"$\Rightarrow$": Show that cycles and 3-stars cannot be done.

They have dim 3. □

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Thm. 4. It is NP-hard to decide, given a poset P, whether $\mathrm{at}(P) \leq 2$.

Testing if a graph can be partitioned into two linear forests is NP-hard.

Farrugia [EJC'04], Chaplick, Fleszar, Lipp, Ravsky, Verbitsky, Wolff [JGAA'23]

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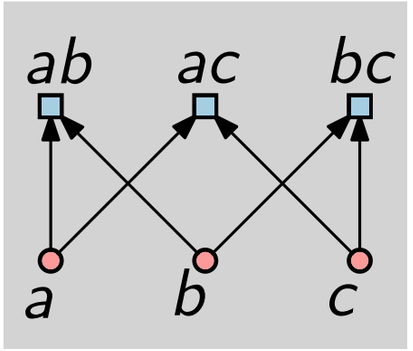
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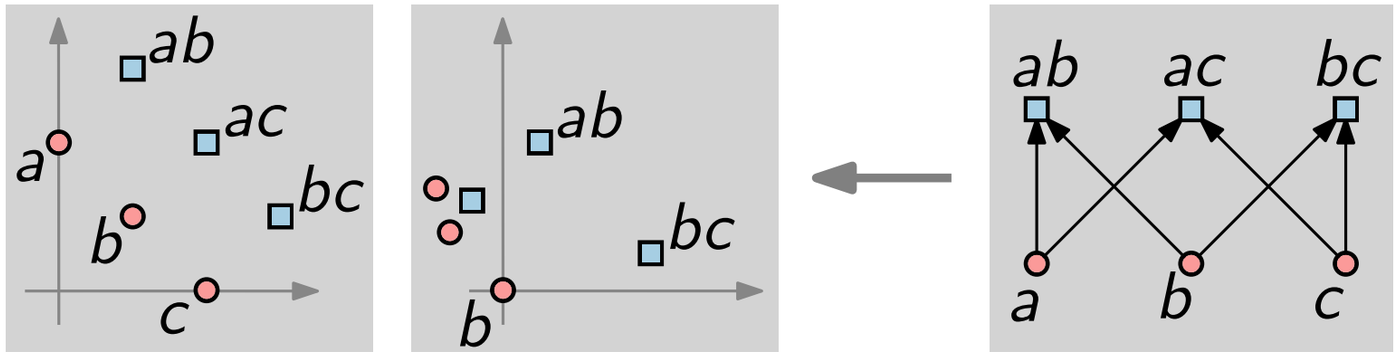
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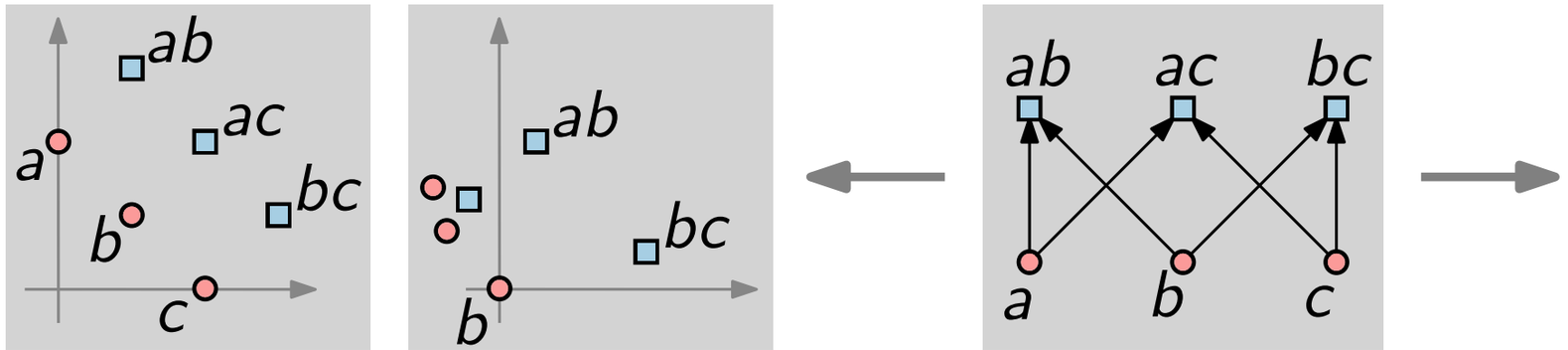
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