

Parameterized Approaches to Orthogonal Compaction

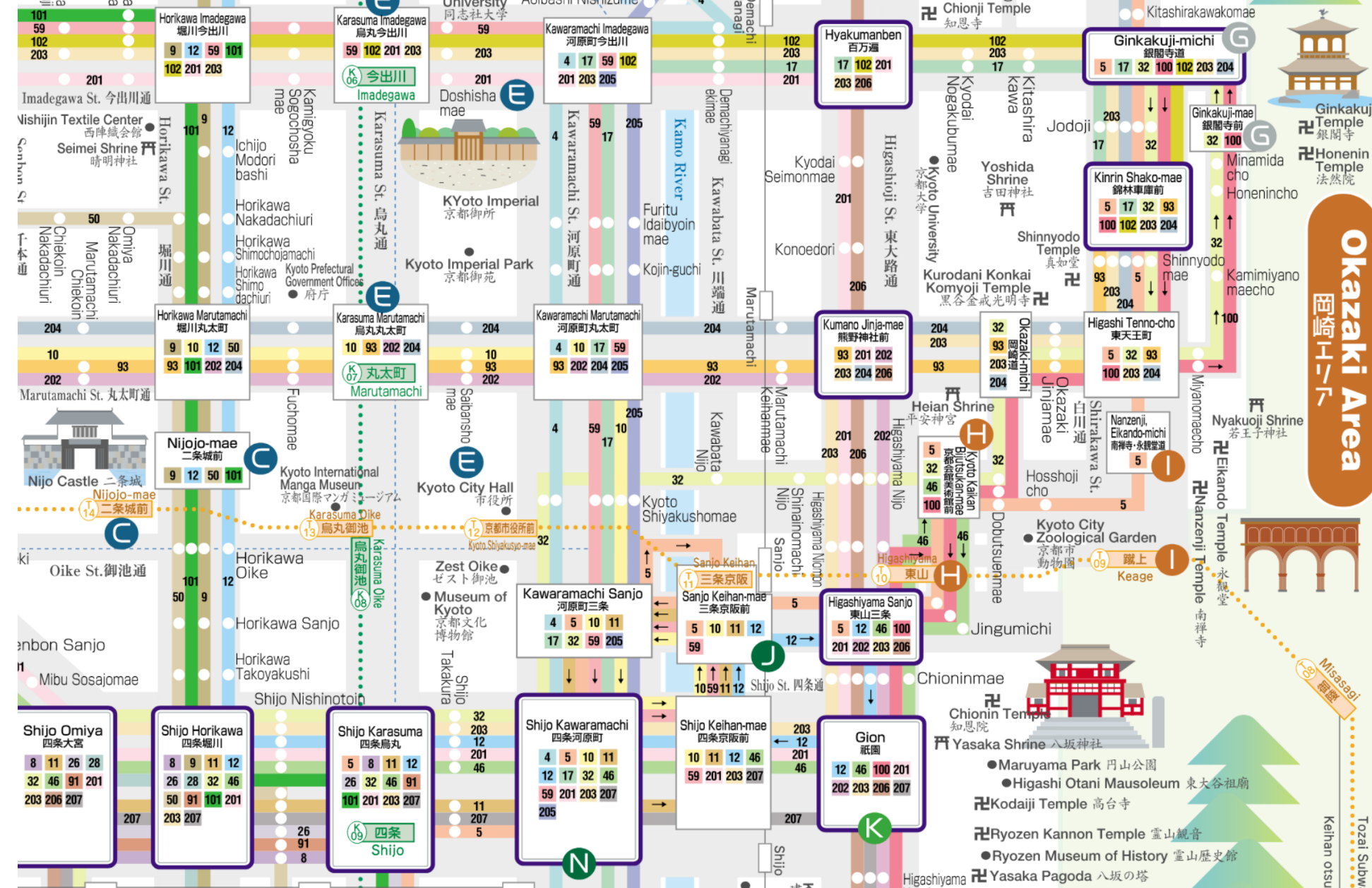
Walter Didimo
Giuseppe Liotta

Siddharth Gupta
Alexander Wolff

Philipp Kindermann
Meirav Zehavi

SOFSEM 2023

Orthogonal Graph Drawing



Kyoto Transportation Authority (c) 2008

The image is a composite of two parts: a UML class diagram on the left and a map of the Okazaki Area on the right.

UML Class Diagram:

- Customer** (Base Class):
 - Attributes: CRM_ID {id}, type: CustomerType, description: String [0..1]
 - Generalized by: **Company**
- Company** (Derived Class):
 - Attributes: name: String, phone: String [0..1], fax: String [0..1]
 - Generalized by: **Details**
- Details** (Derived Class):
 - Attributes: street: String [0..1], city: String [0..1], postalCode: String [0..1], state: String [0..1], country: String [0..1]
 - Relationships: **billing** (to Company), **shipping** (to Company)
- Contact** (Derived Class):
 - Attributes: firstName: String, lastName: String, middleName: String [0..1], email: String, locale: String [0..1] = "English"
- Product** (Derived Class):
 - Attributes: products: * {ordered, unique}
- «enumeration» CustomerType**:
 - Values: Individual, Company
- «enumeration» LockingType**:
 - Values: HL, SL-AdminMode, SL-UserMode

Map of the Okazaki Area:

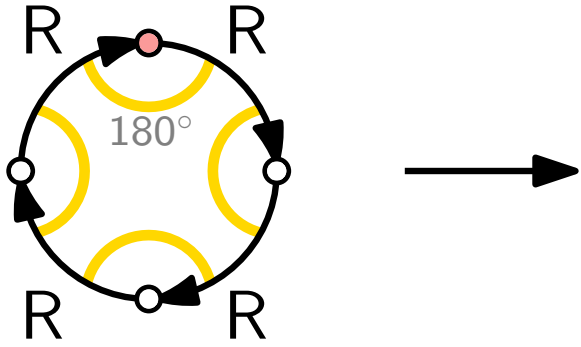
The map shows the city of Okazaki with various landmarks and streets. Key locations highlighted include:

- Ginkakuji-michi** (銀閣寺道): 5, 17, 32, 100, 102, 203, 204
- Kinrin Shako-mae** (錦林車庫前): 5, 17, 32, 93, 100, 102, 203, 204
- Higashi Tenno-cho** (東天王町): 5, 32, 93, 100, 203, 204
- Kumano Jinja-mae** (熊野神社前): 93, 201, 202, 203, 204, 206
- Higashiyama Sanjo** (東山三条): 5, 12, 46, 100, 201, 202, 203, 206
- Gion** (祇園): 12, 46, 100, 201, 202, 203, 206, 207
- Sanjo Keihan-mae** (三条京阪前): 5, 10, 11, 12, 59
- Shijo Keihan-mae** (四条京阪前): 10, 11, 12, 46, 59, 201, 203, 207

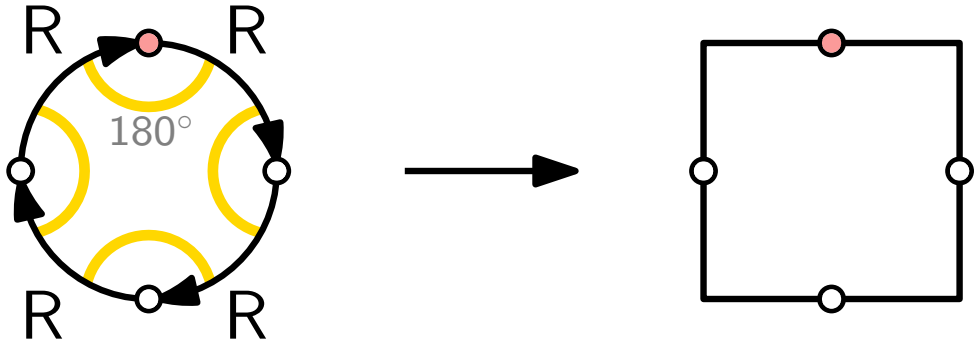
Other landmarks and streets shown include: Chionin Temple, Yasaka Shrine, Maruyama Park, Higashi Otani Mausoleum, Ryozen Kannon Temple, Ryozen Museum of History, Yasaka Pagoda, and various other temples and shrines.

Okazaki Area
岡崎エリア

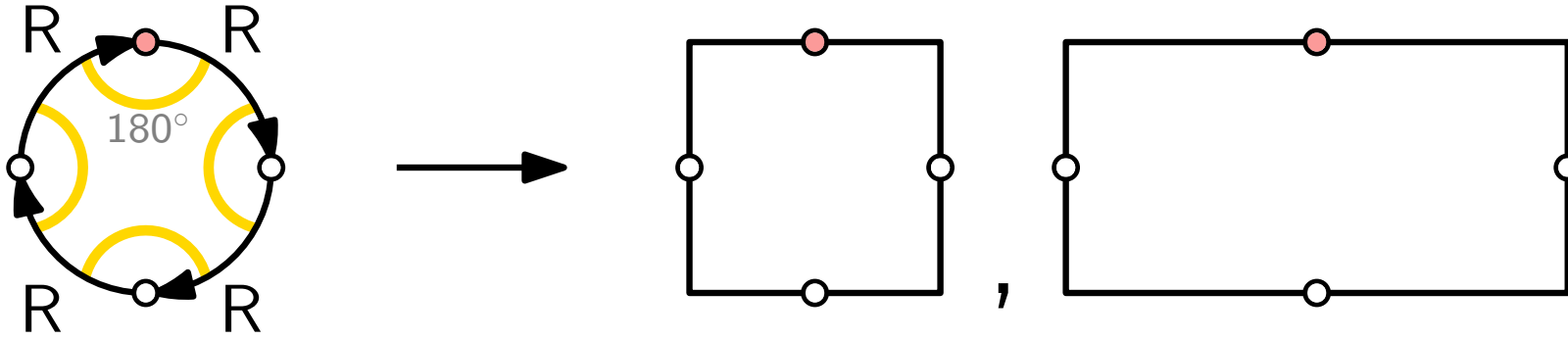
Orthogonal Representation vs. Orthogonal Drawing



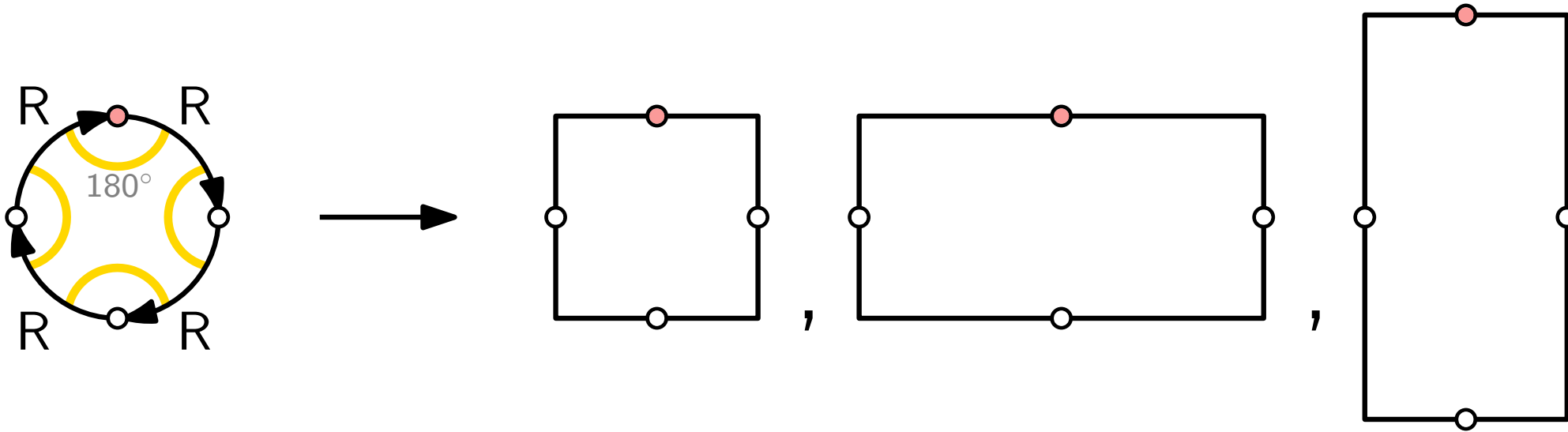
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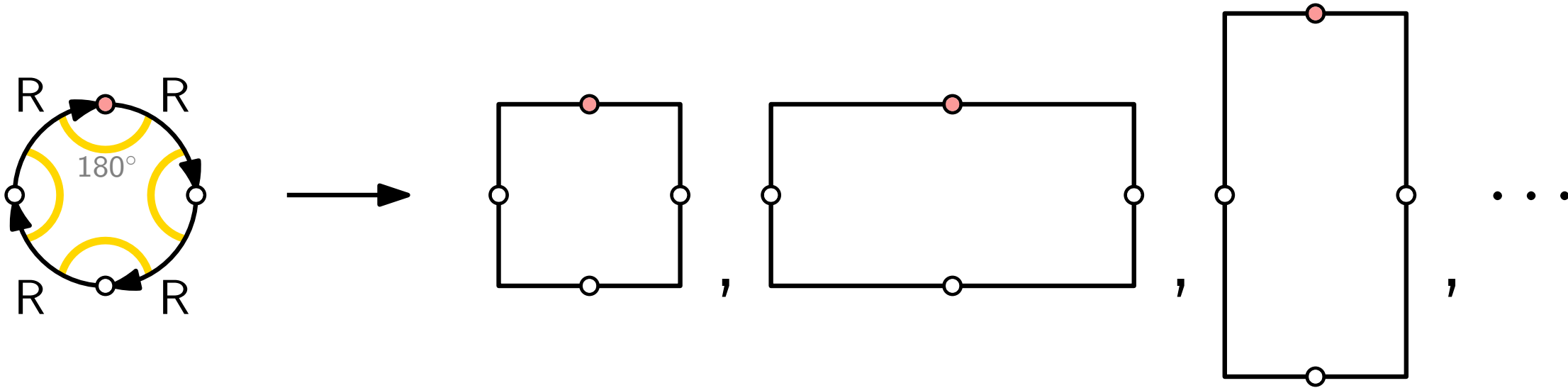
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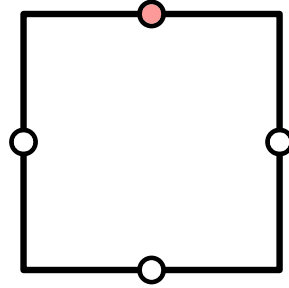
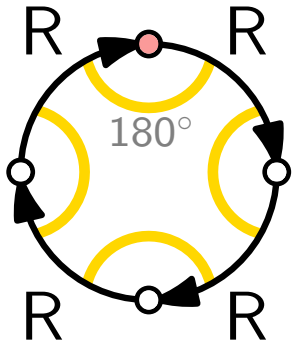
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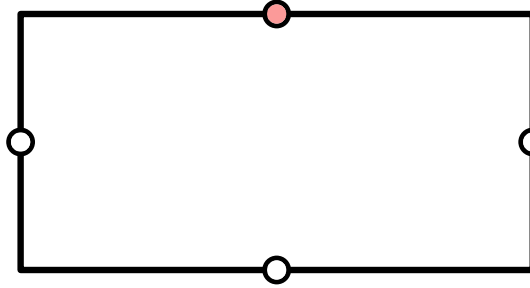
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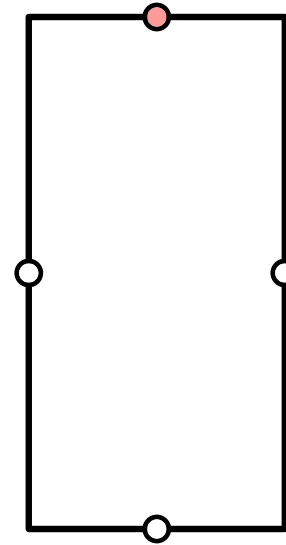
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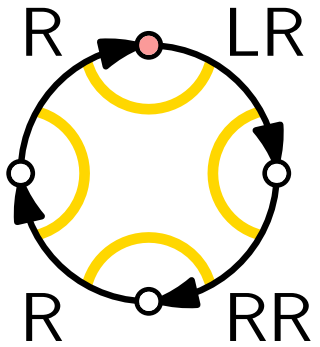


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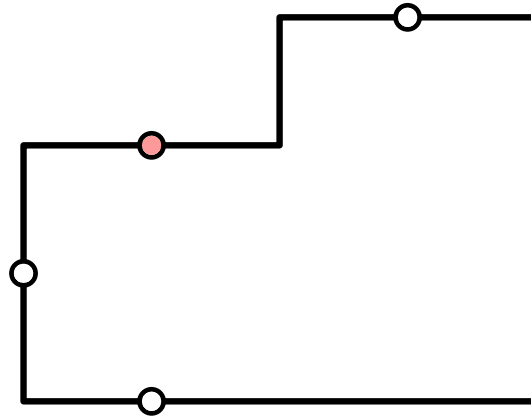
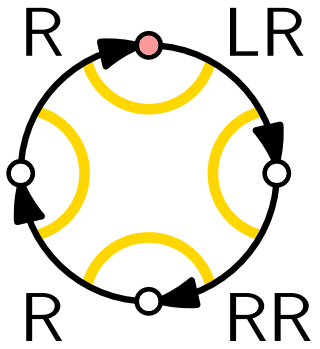
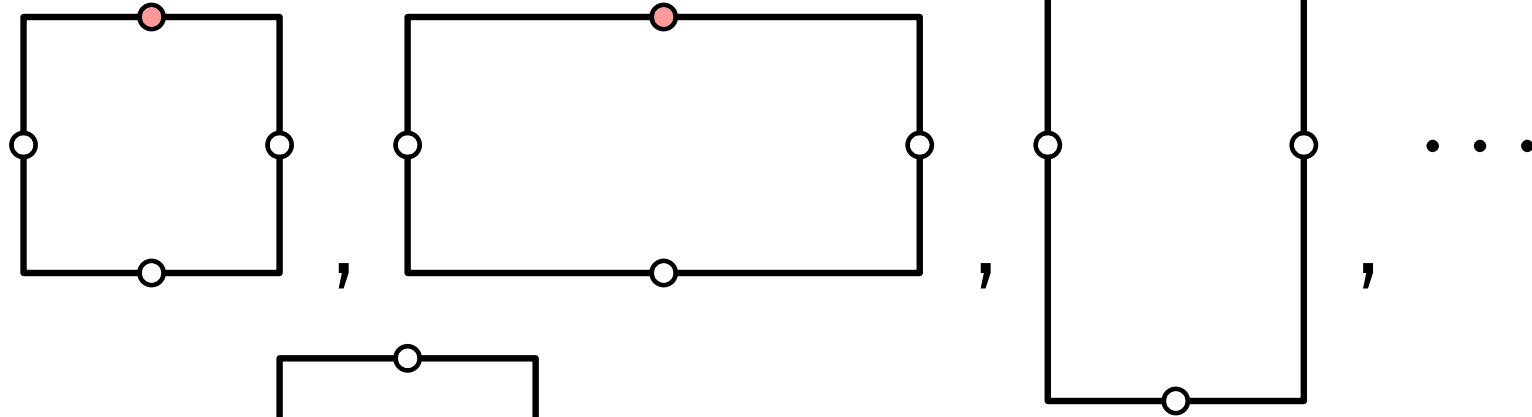
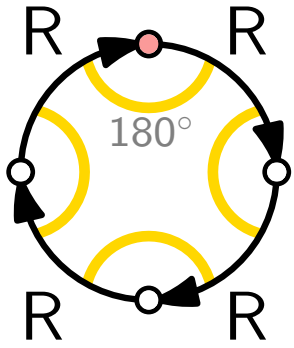


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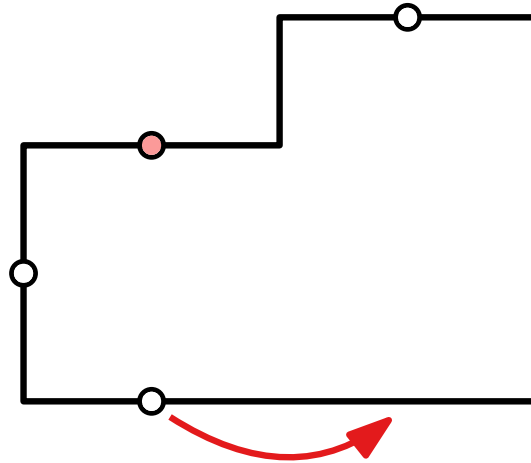
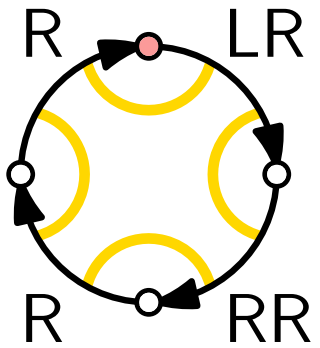
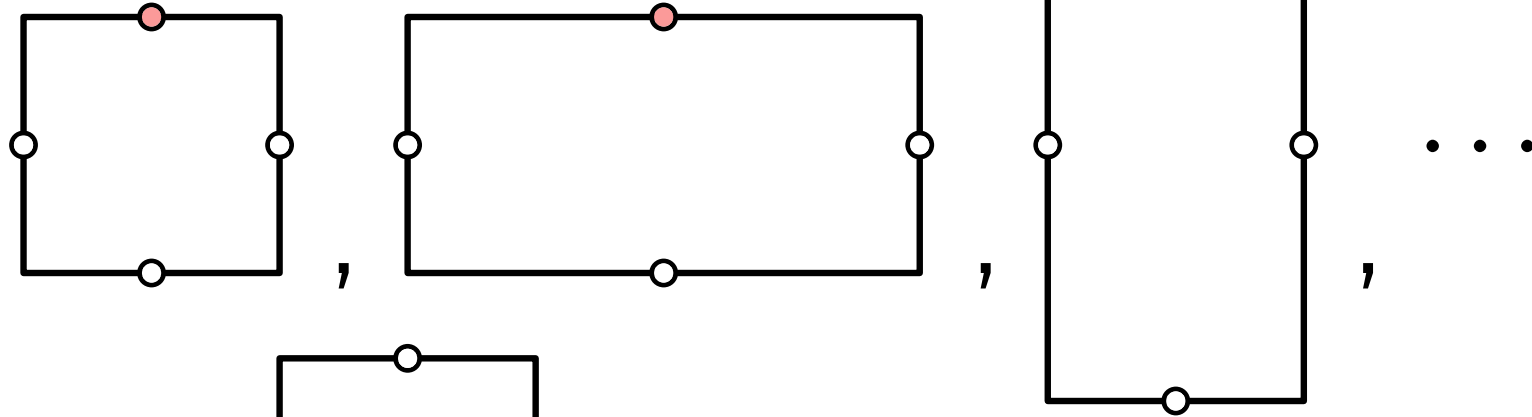
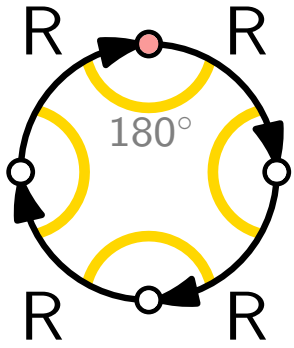
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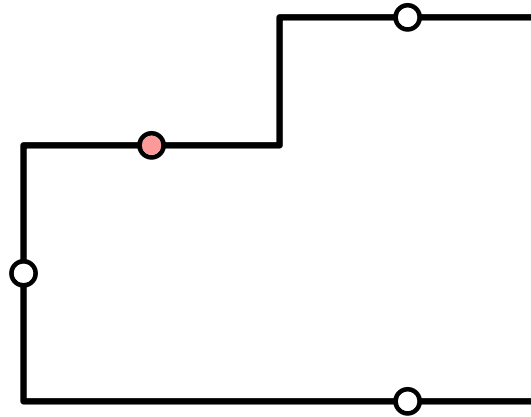
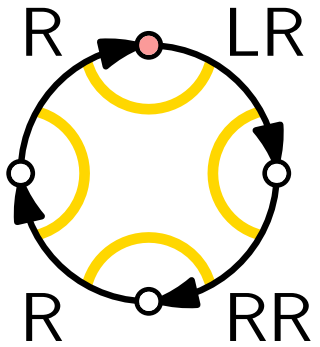
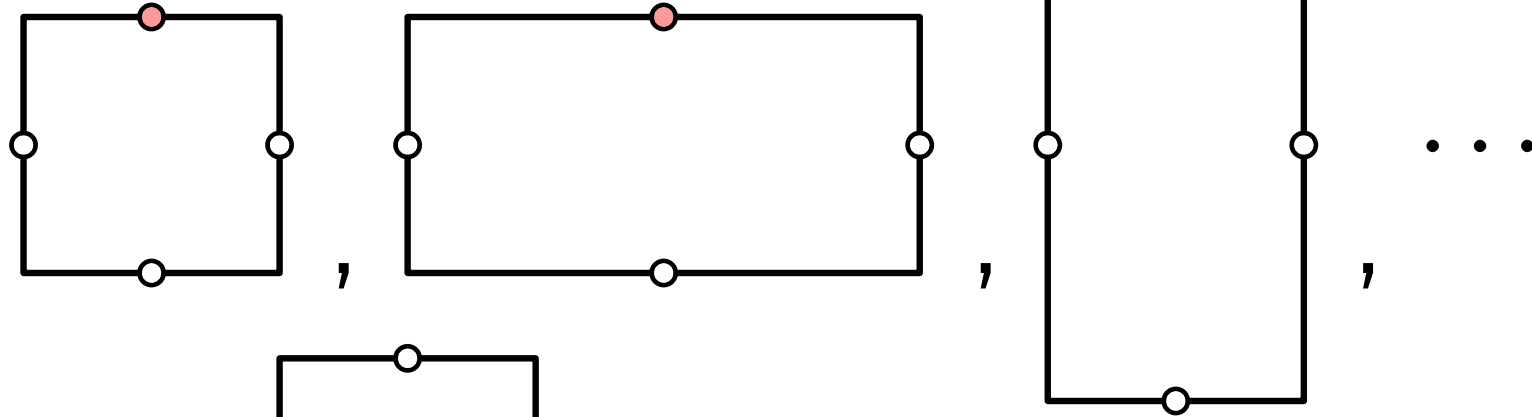
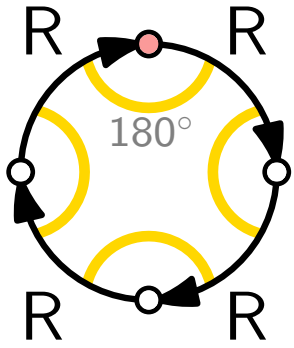
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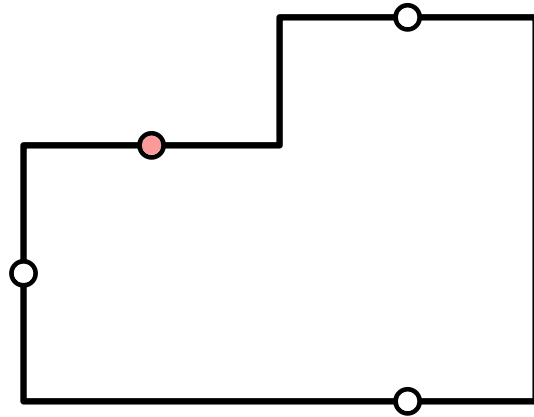
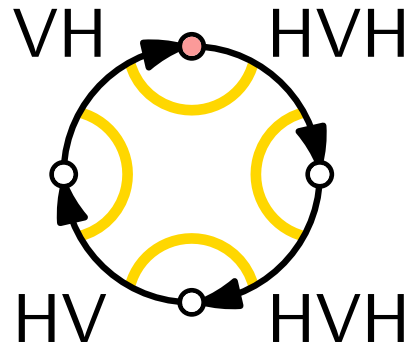
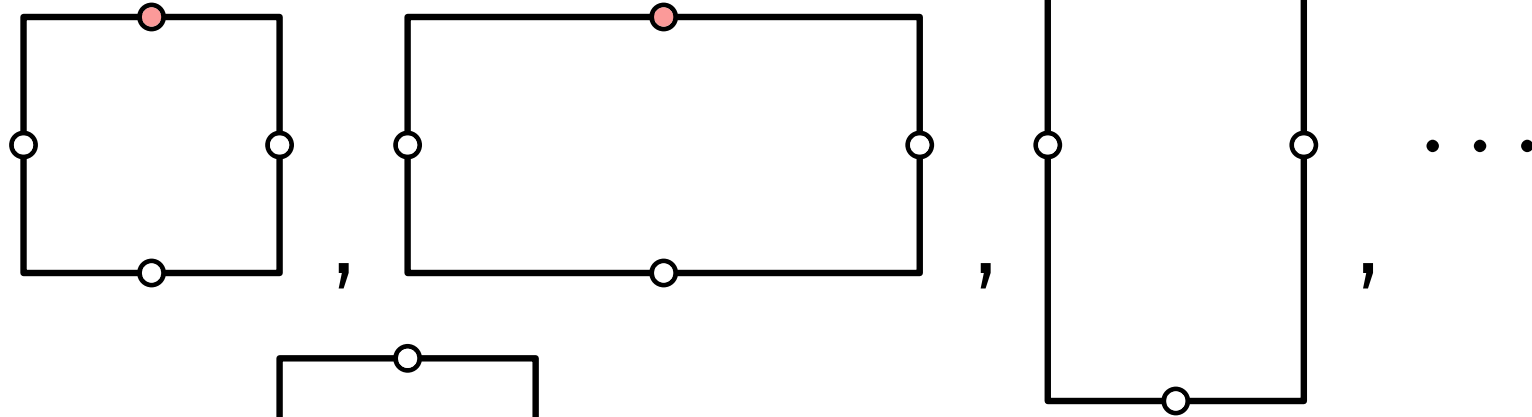
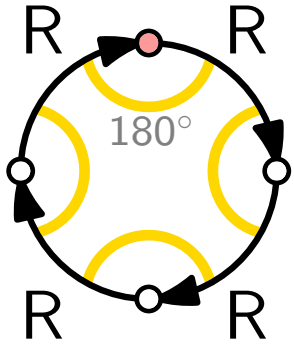
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- Even if the given graph is just a cycle :-(
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- Cycles have pathwidth (and treewidth) 2,
so cannot expect FPT algorithm for OC w.r.t. these parameters :-()

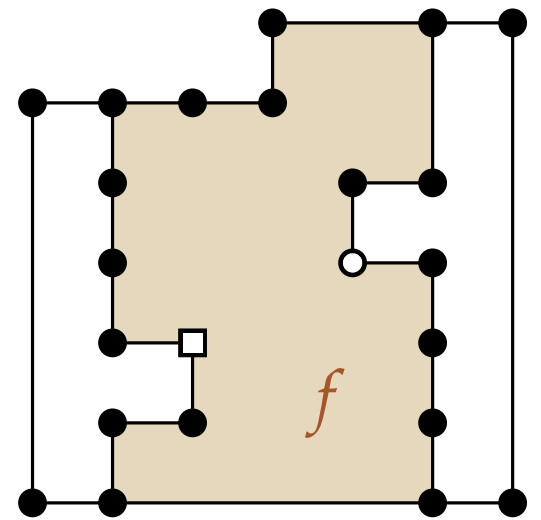
Kitty Corners

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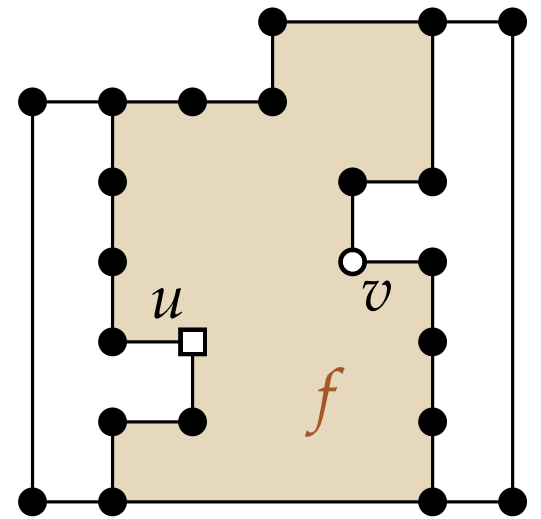
Let f be a face of G .



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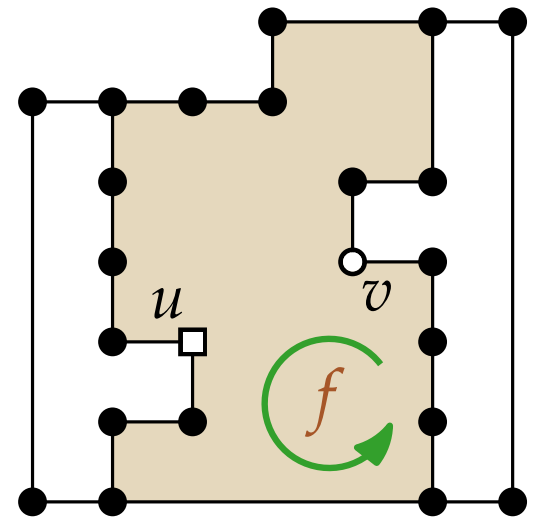
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Kitty Corners

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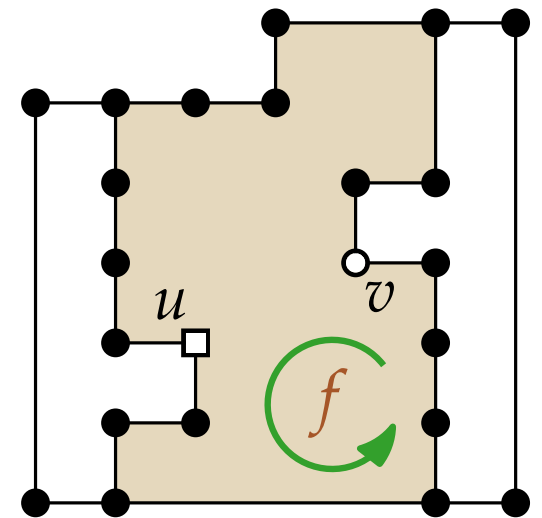


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$\text{rot}(u, v) := \# \text{ convex corners} - \# \text{ reflex corners on } \partial f \text{ from } u \text{ (included) to } v \text{ (excluded)};$
 a reflex vertex of degree 1 is counted like two reflex vertices.

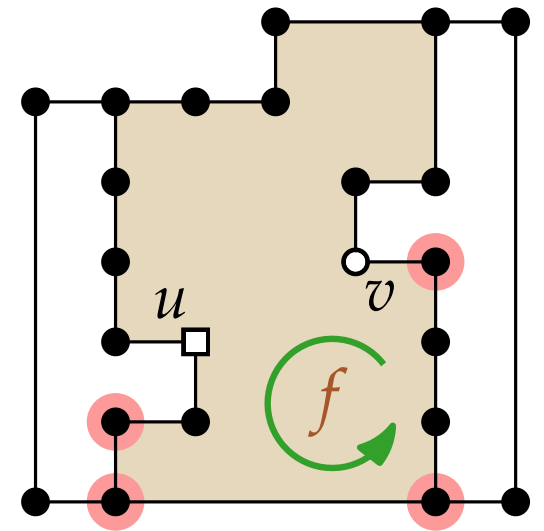


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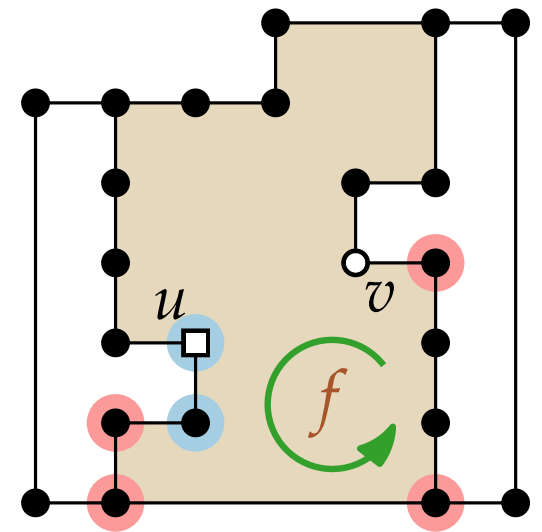


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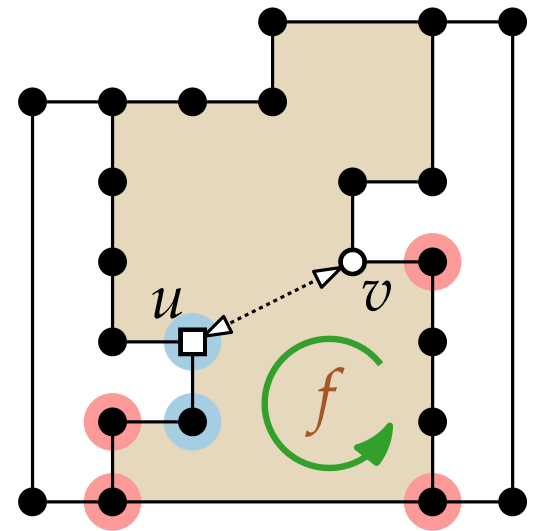
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We say that $\{u, v\}$ is a *pair of kitty corners* of f if $\text{rot}(u, v) = 2$ or $\text{rot}(v, u) = 2$.



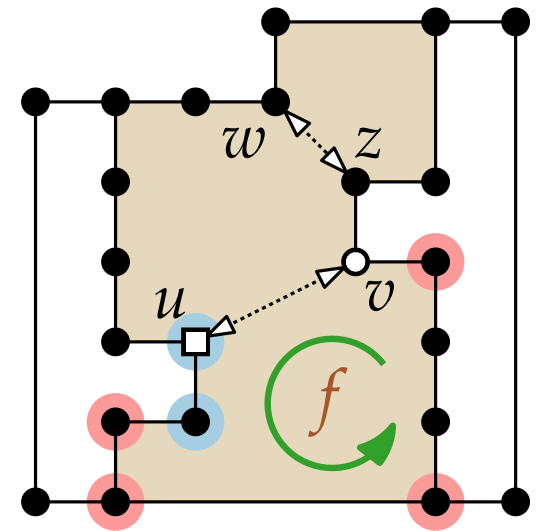
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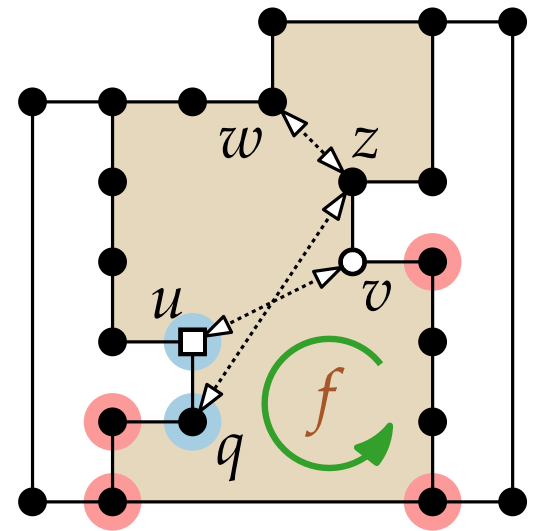
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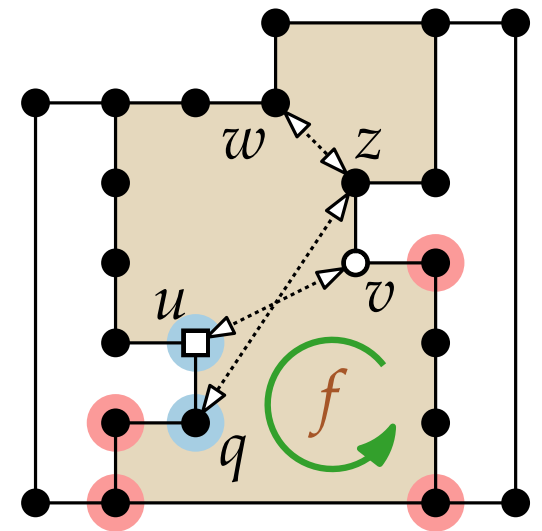
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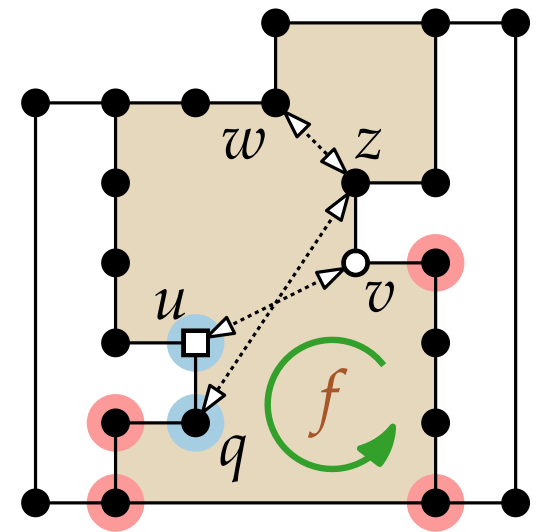
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A vertex is a **kitty corner** if it is part of a pair of kitty corners.

An orthogonal representation R is **turn-regular** if R has no kitty corners.



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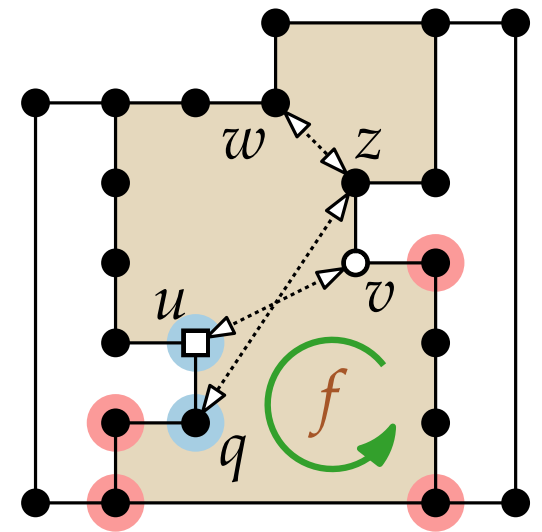
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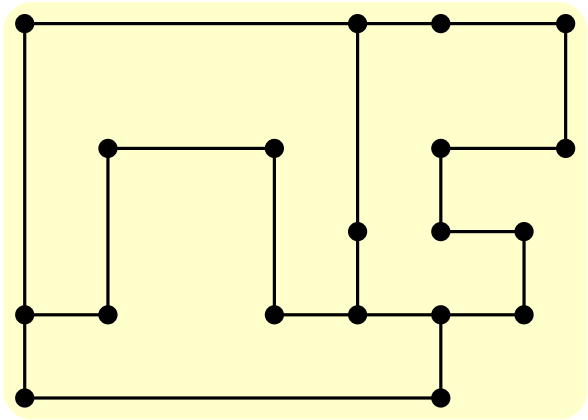
An orthogonal representation R is **turn-regular** if R has no kitty corners.



Theorem. If R is turn-regular, a min-area drawing of R can be computed in linear time.

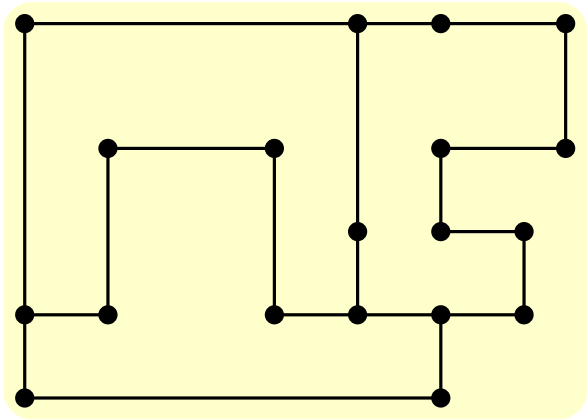
[Bridgeman, Di Battista, Didimo, Liotta, Tamassia, Vismara: CGTA 2000]

Drawing Turn-Regular Representations Optimally

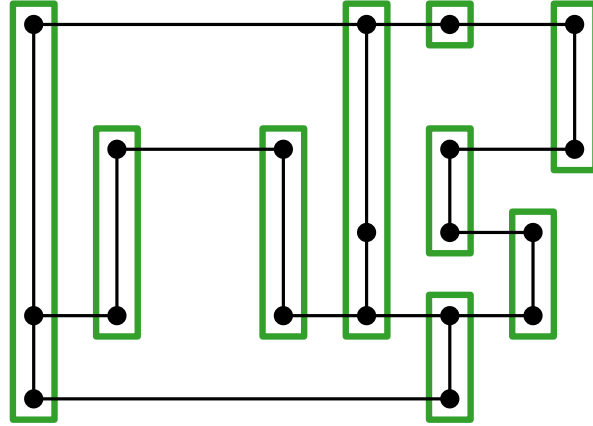


orthogonal representation

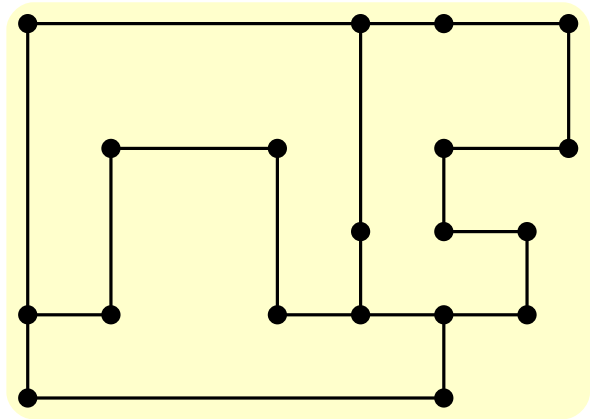
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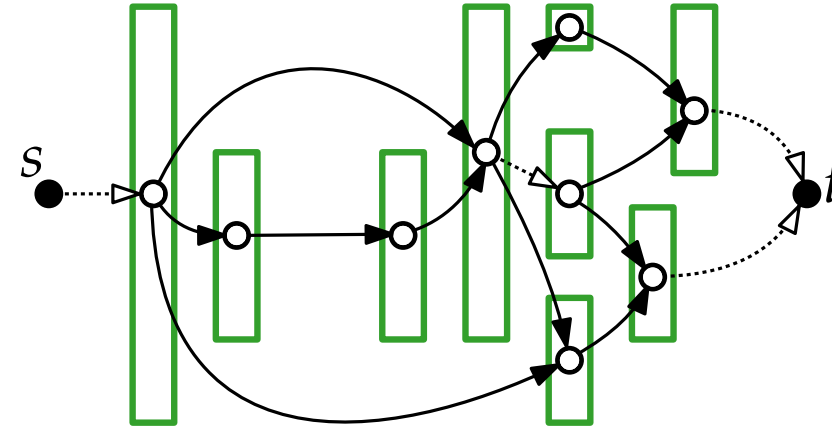
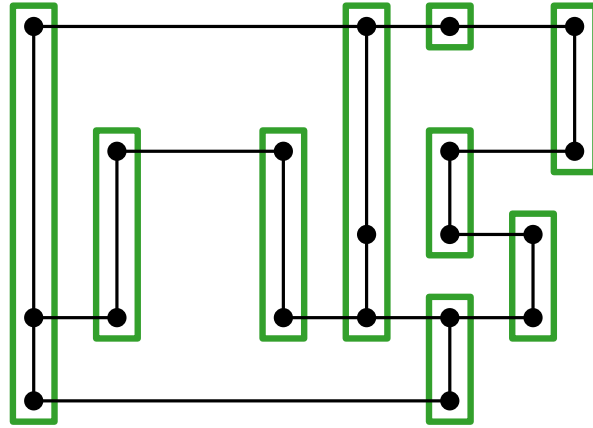
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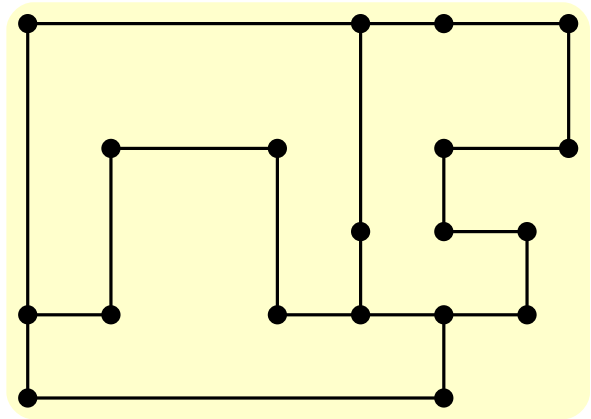


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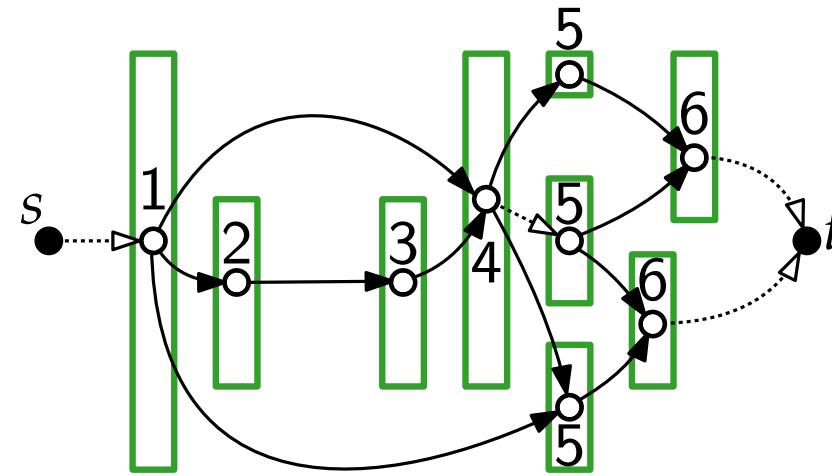
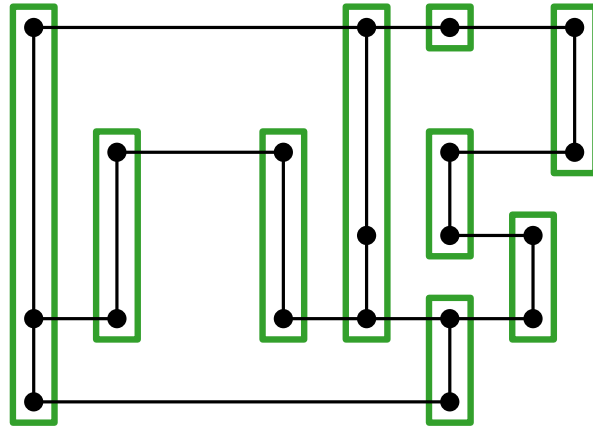


auxiliary graph (DAG) D_x

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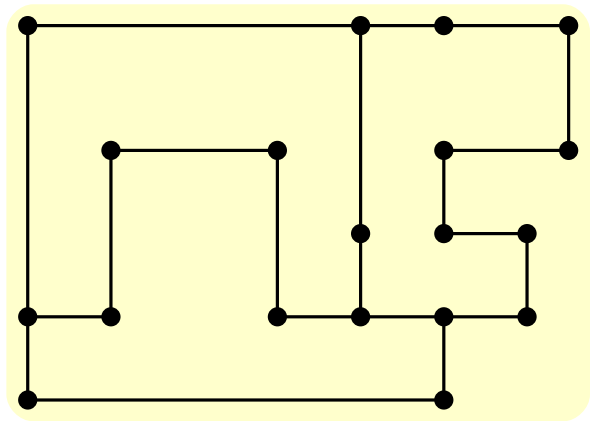


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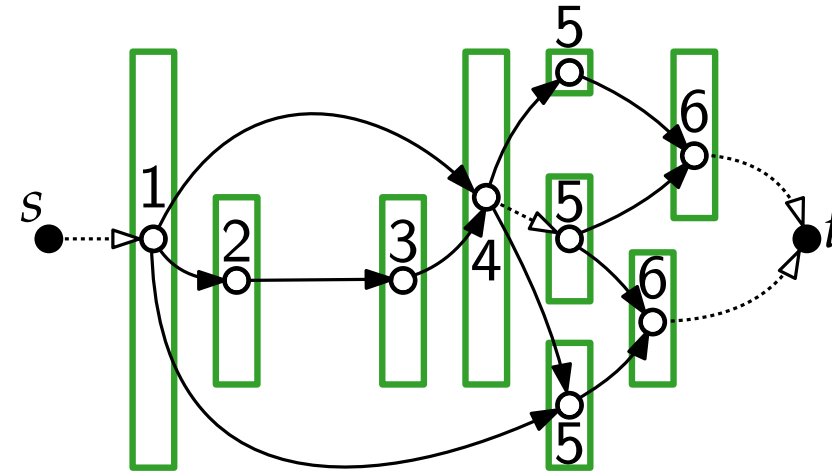
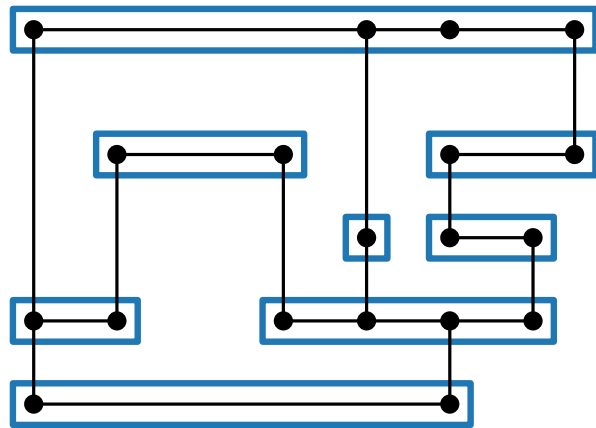
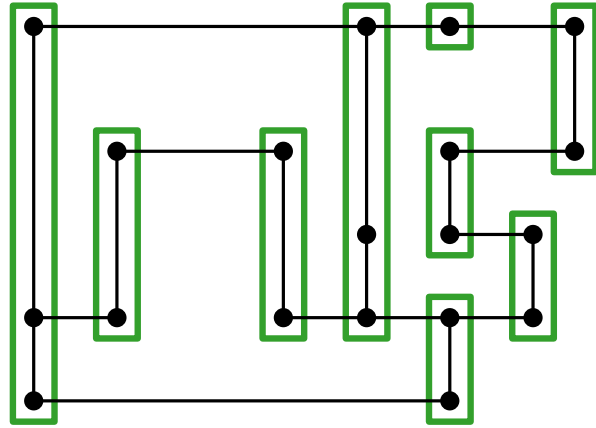


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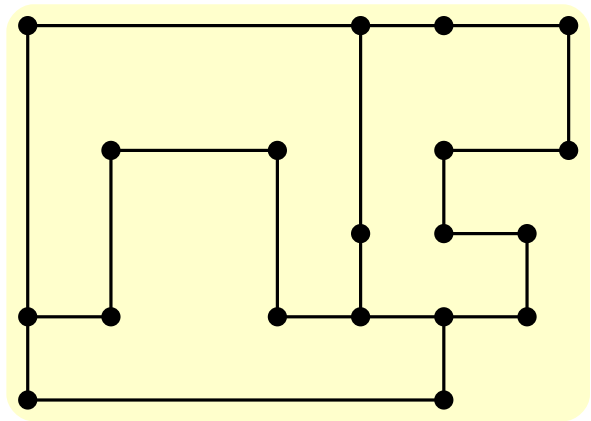


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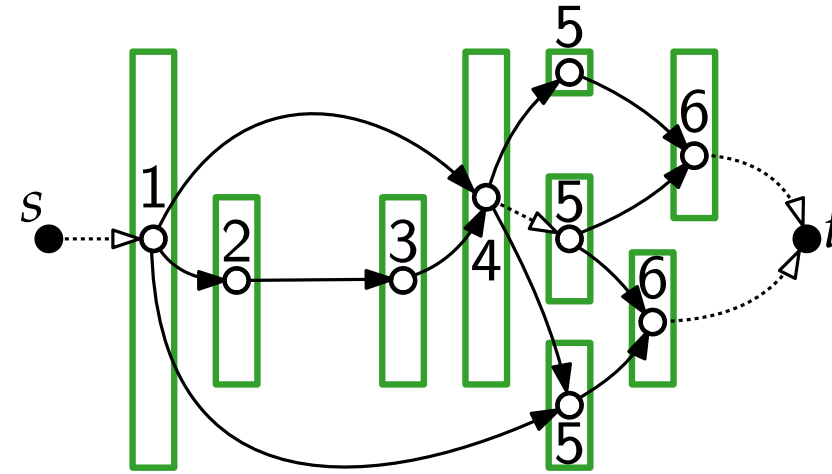
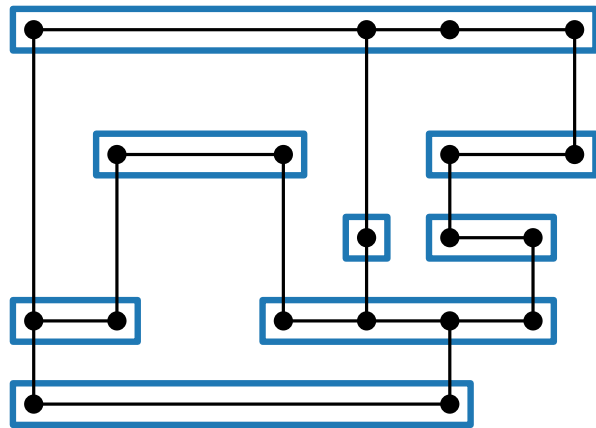
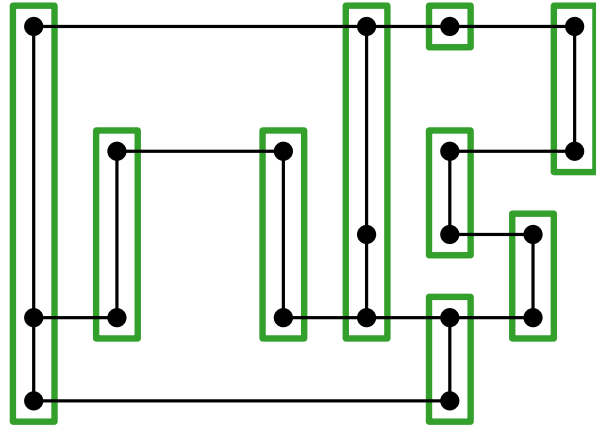


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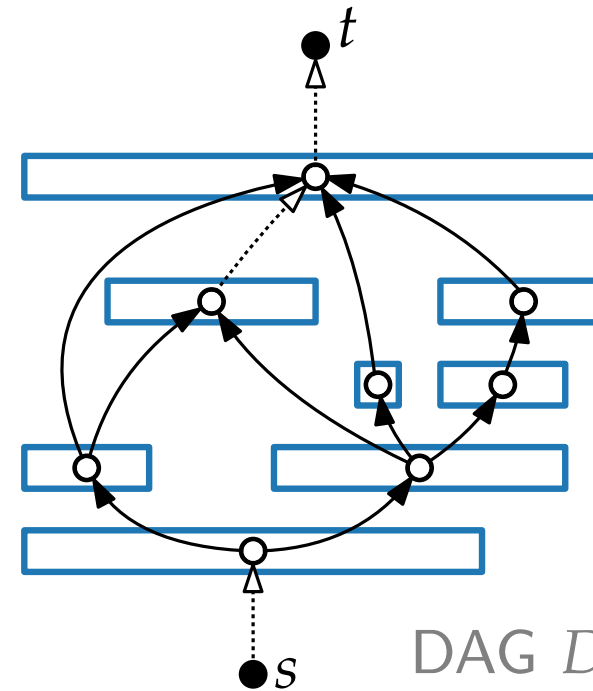
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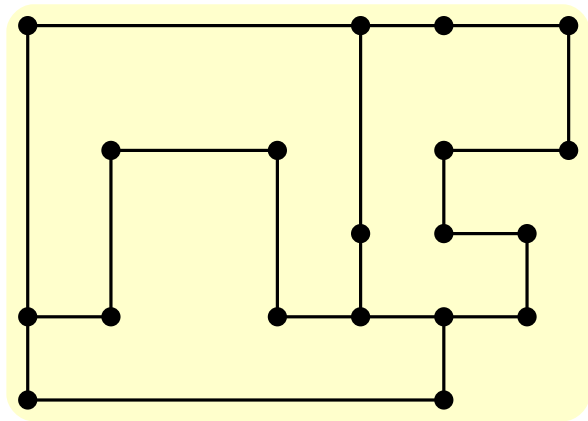


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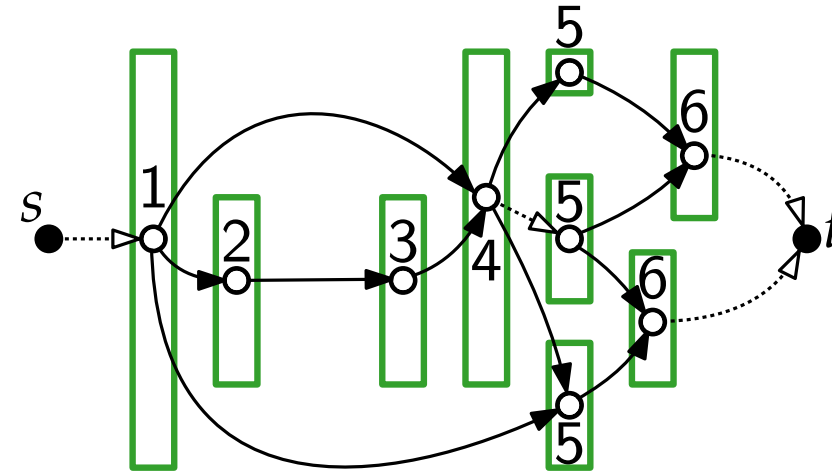
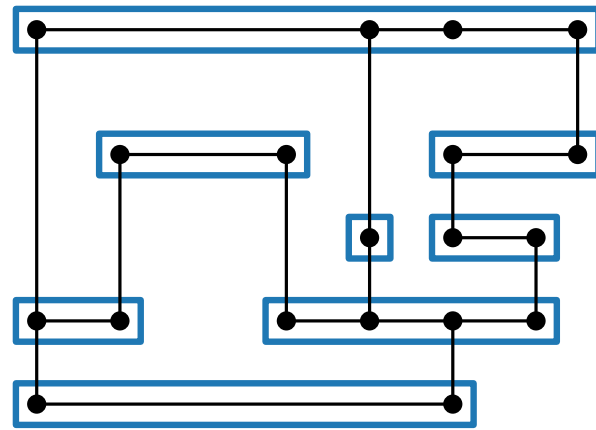
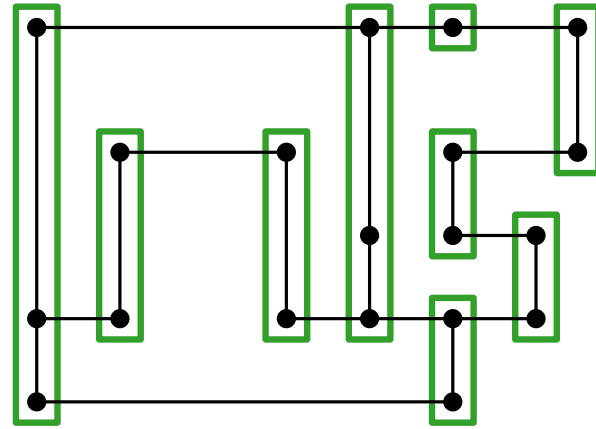


DAG D_y

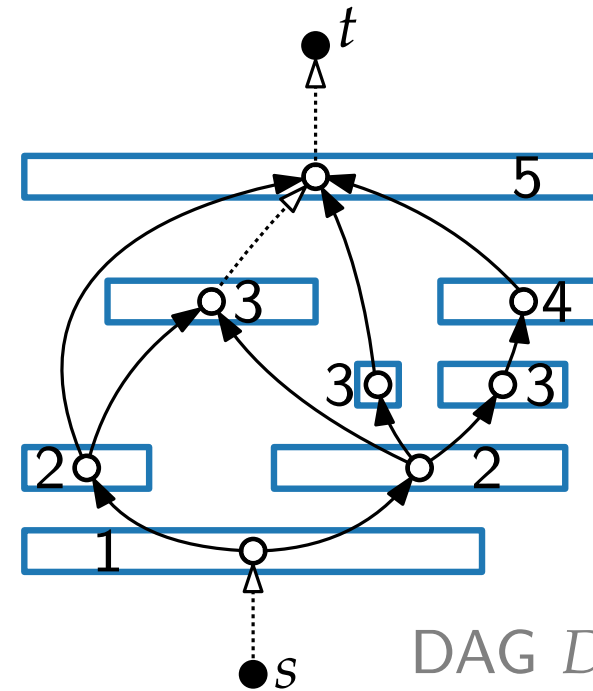
Drawing Turn-Regular Representations Optimally



orthogonal representation

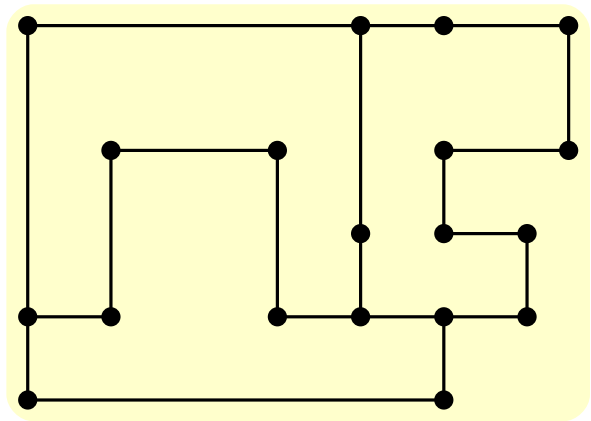


auxiliary graph (DAG) D_x

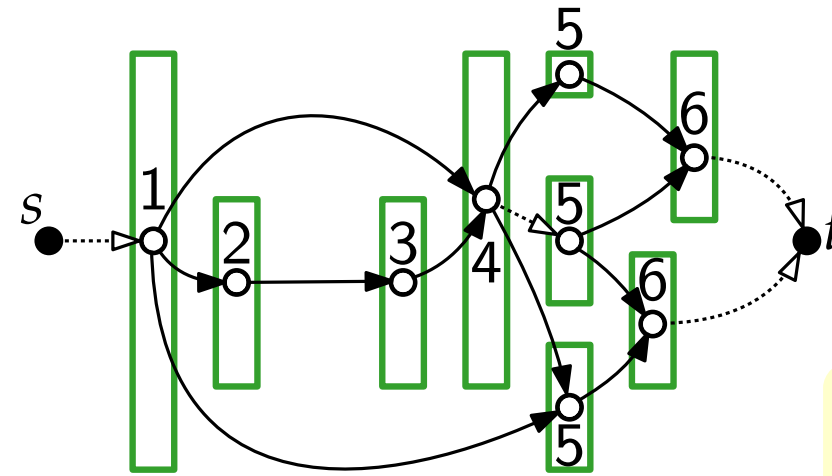
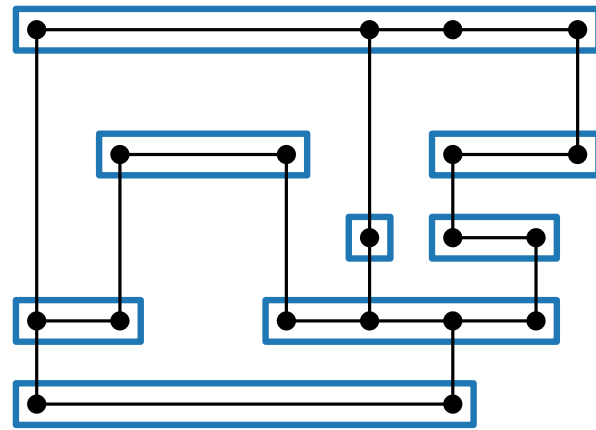
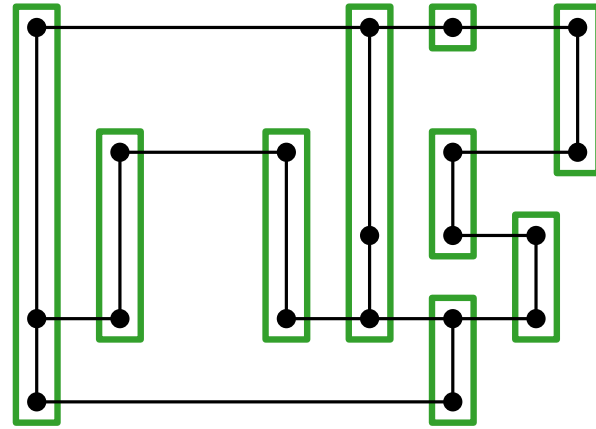


DAG D_y

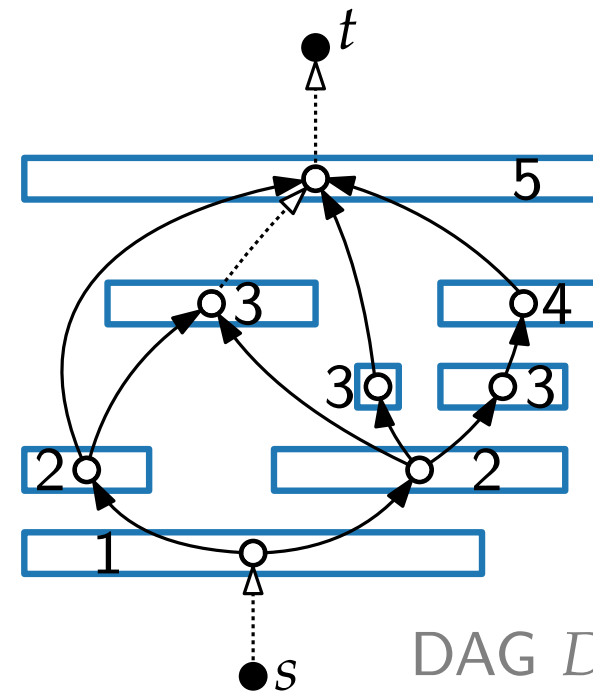
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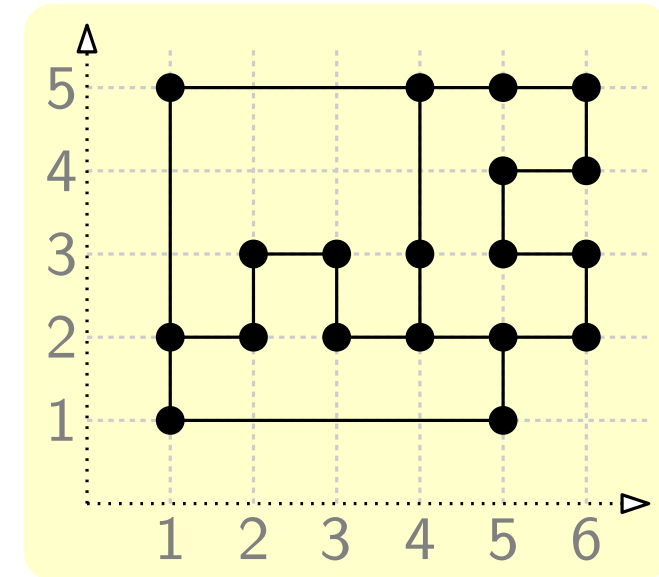


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topological numbering



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Our Contribution: A Parametrized View of OC

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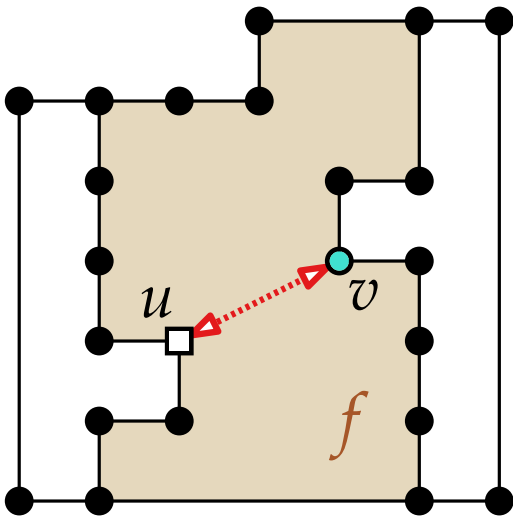
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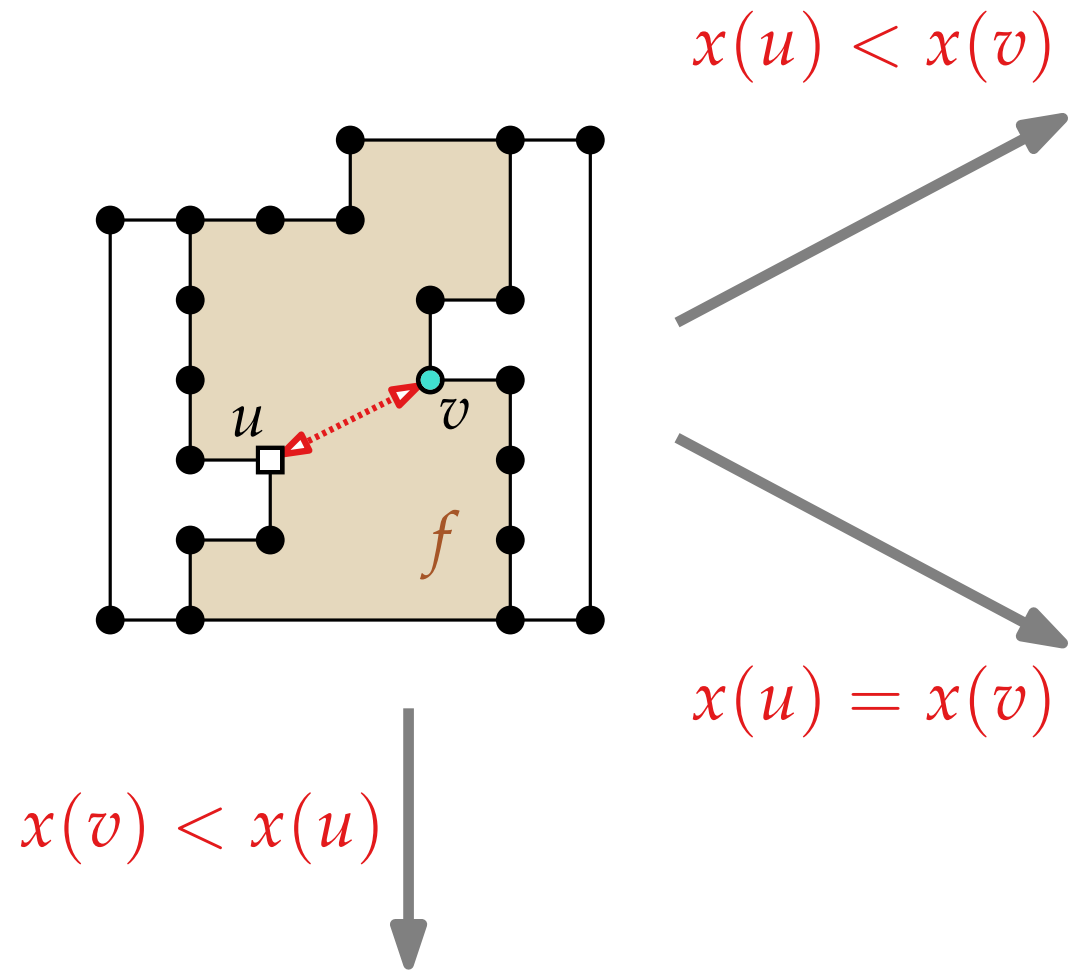
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Idea: For each (?) pair $\{u, v\}$ of kitty corners, guess whether $x(u) \leq x(v)$ and $y(u) \leq y(v)$.



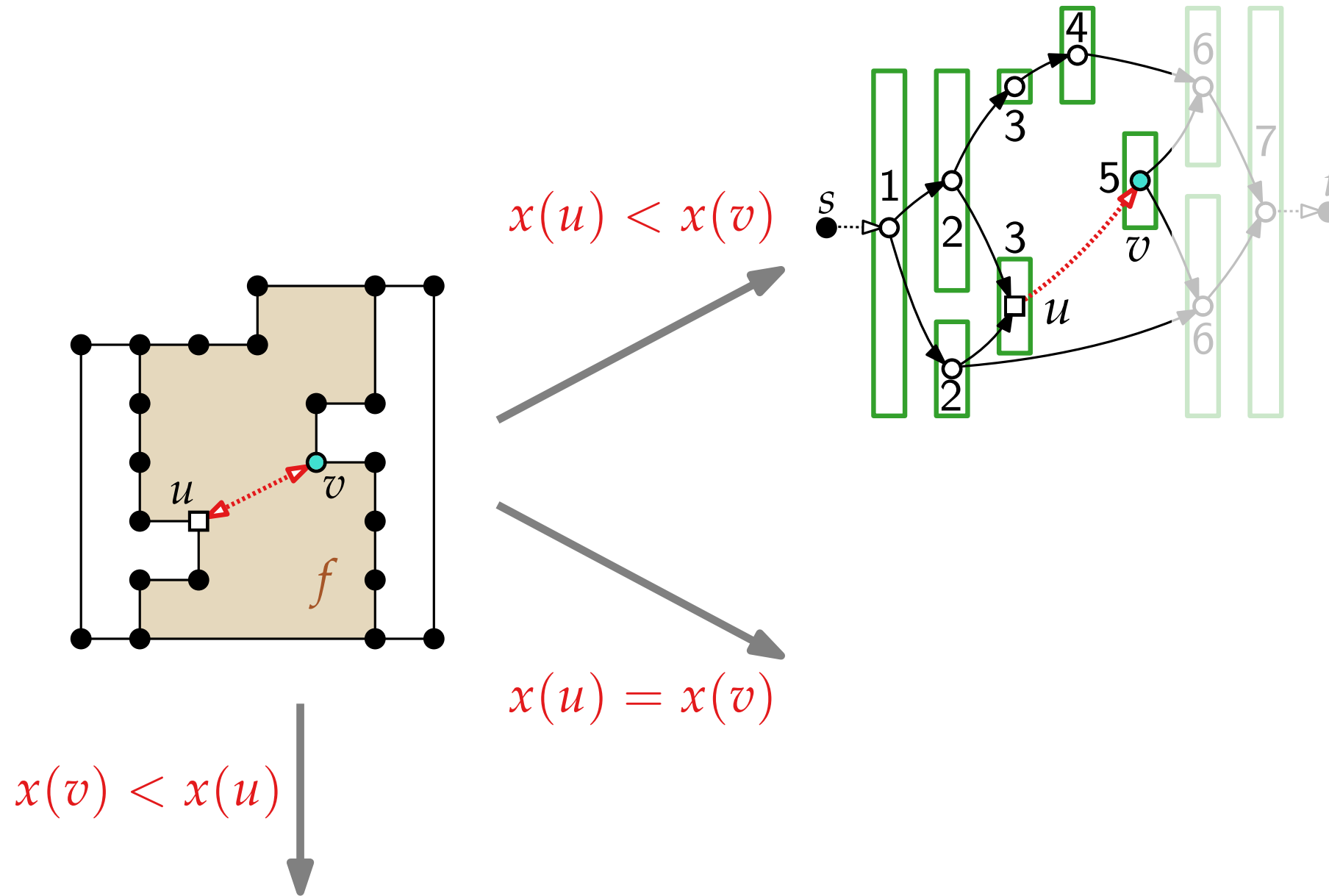
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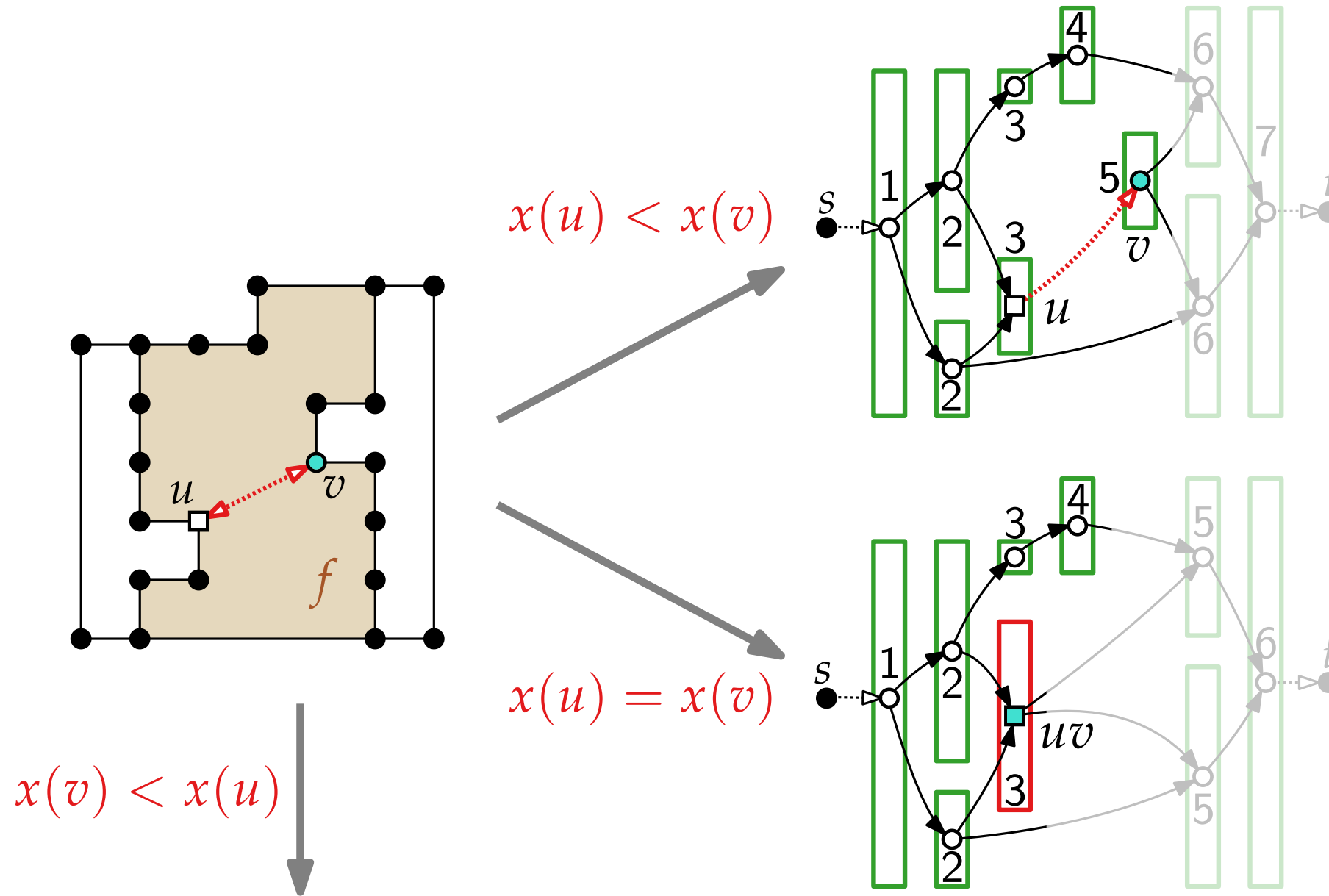
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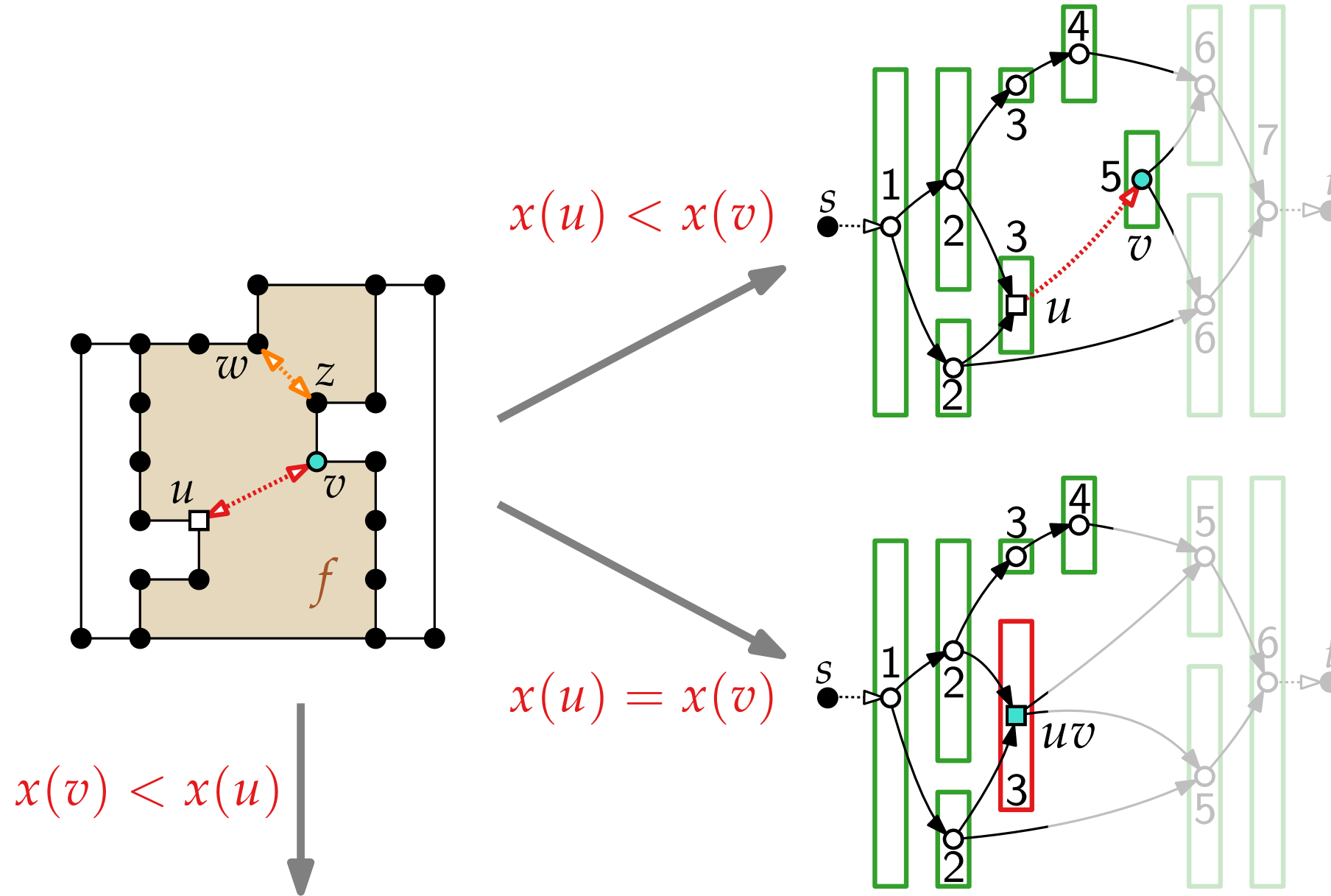
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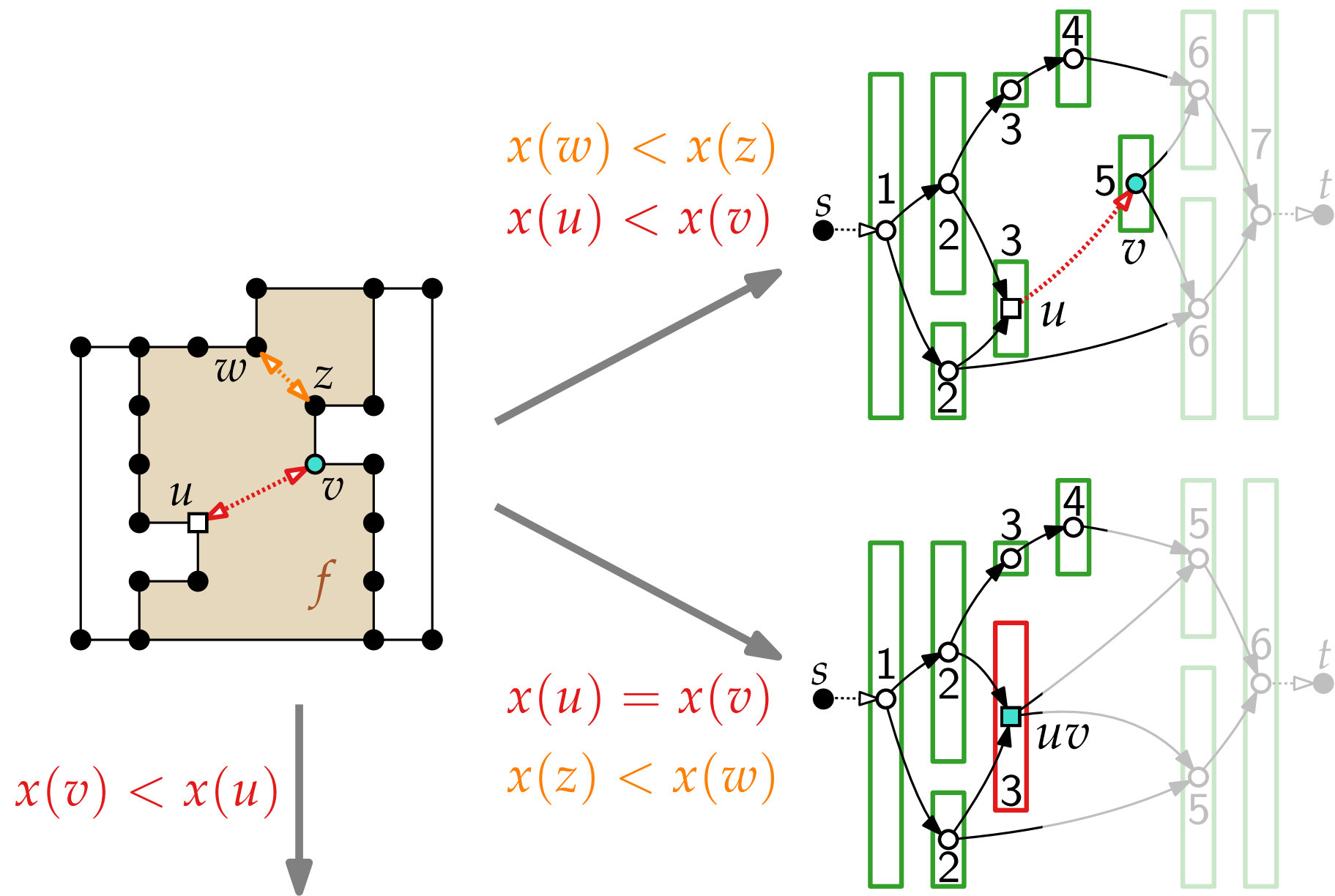
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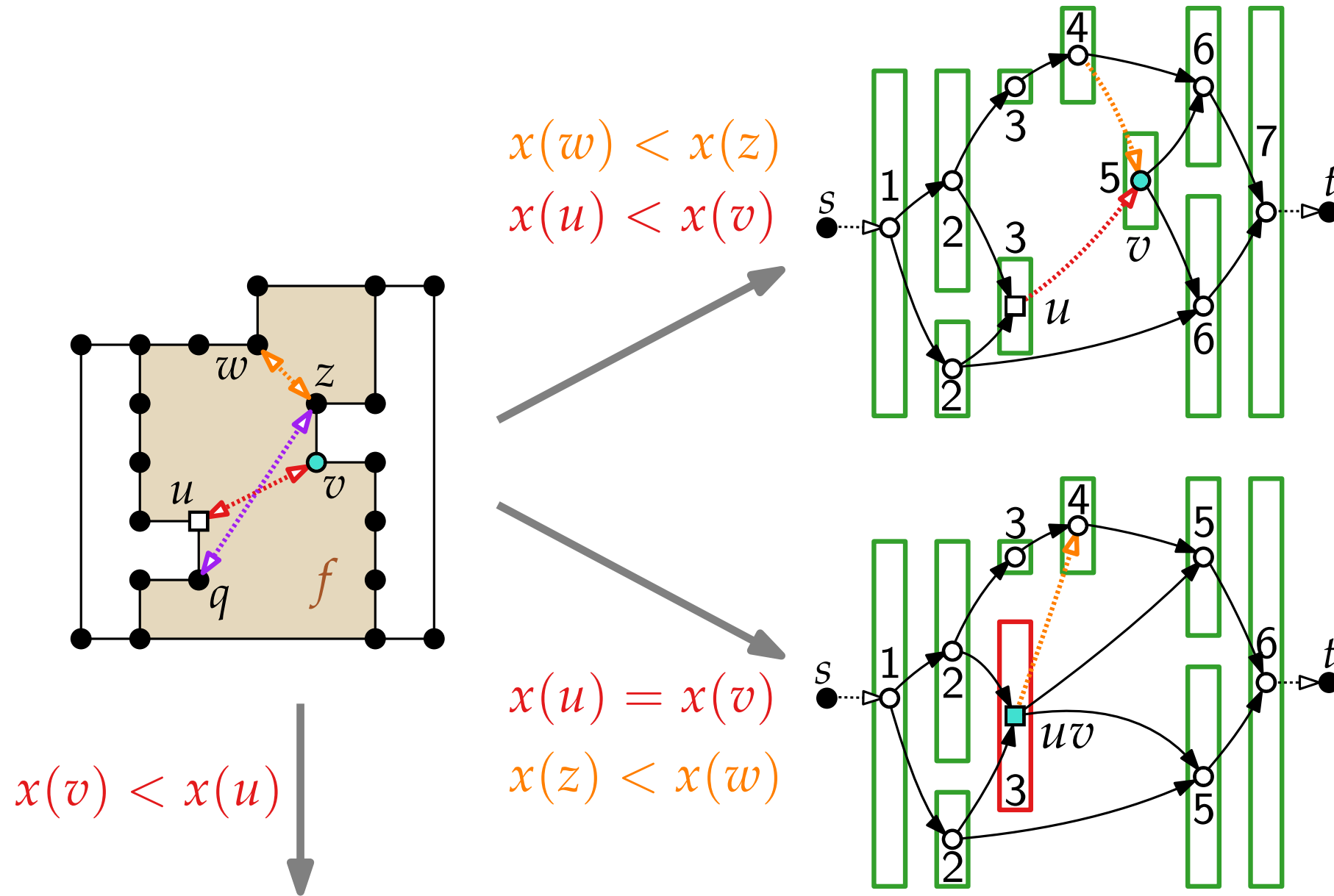
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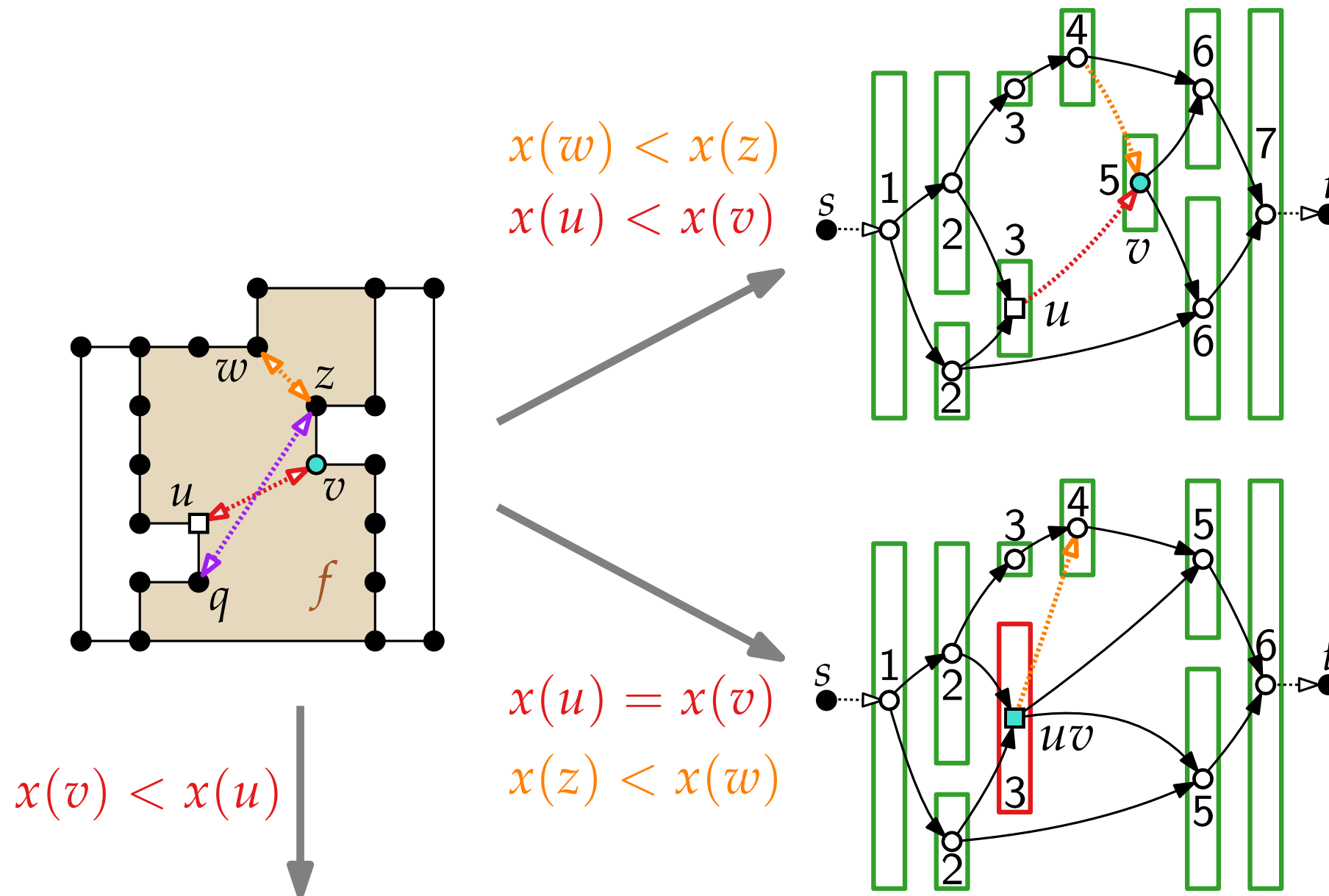
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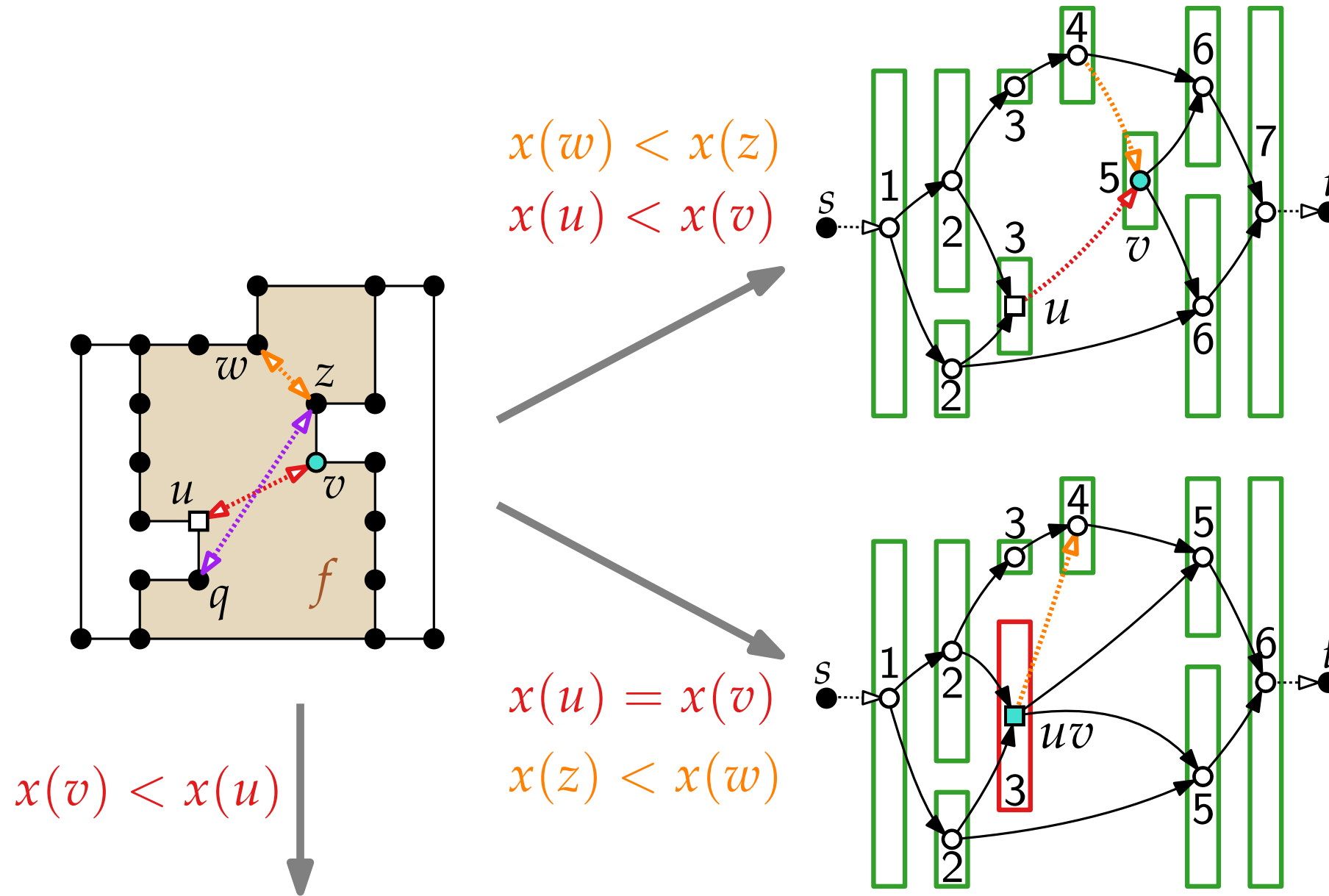


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Do we need to deal with $\{q, z\}$ analogously?

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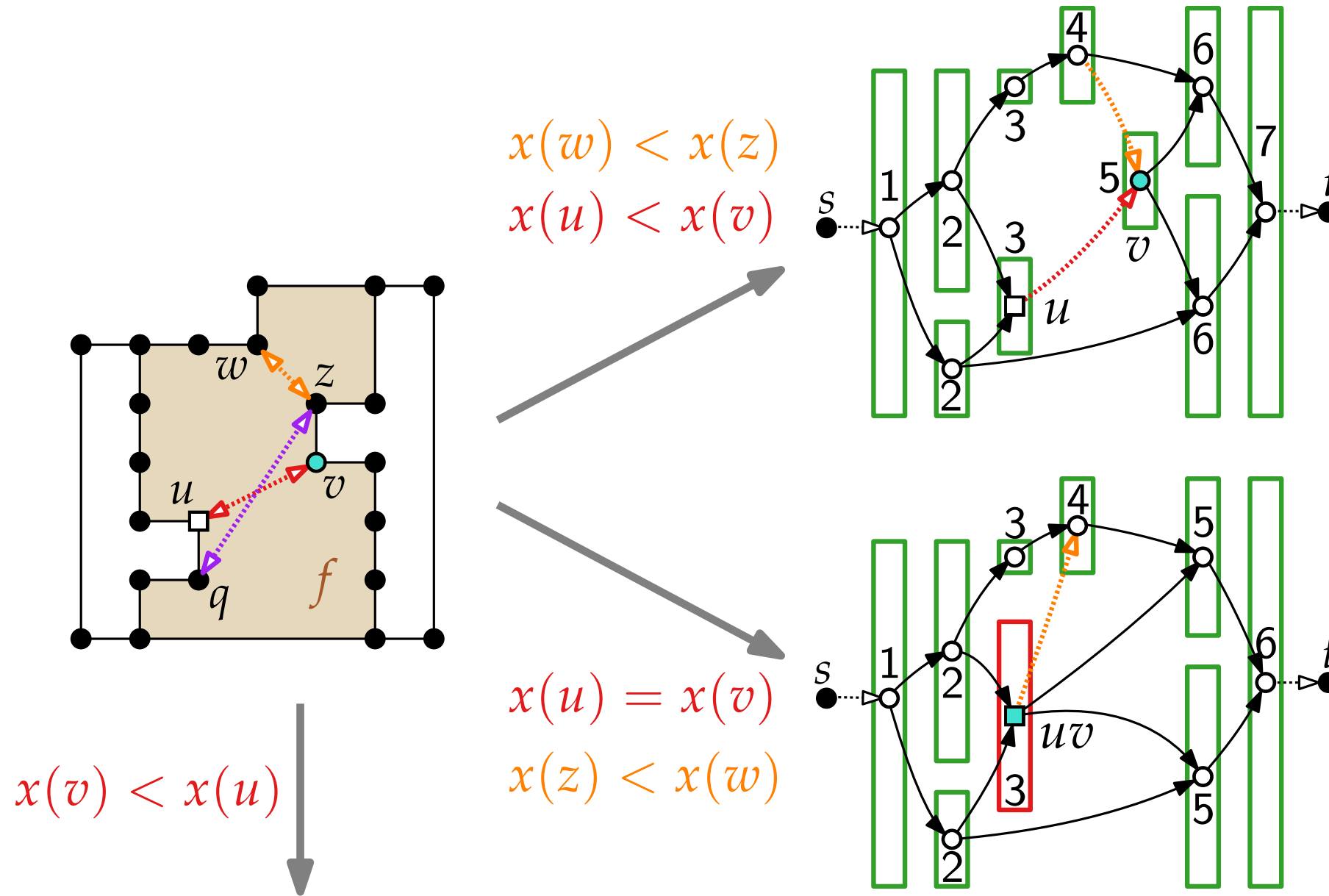
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Answer:

No, doesn't add new information.

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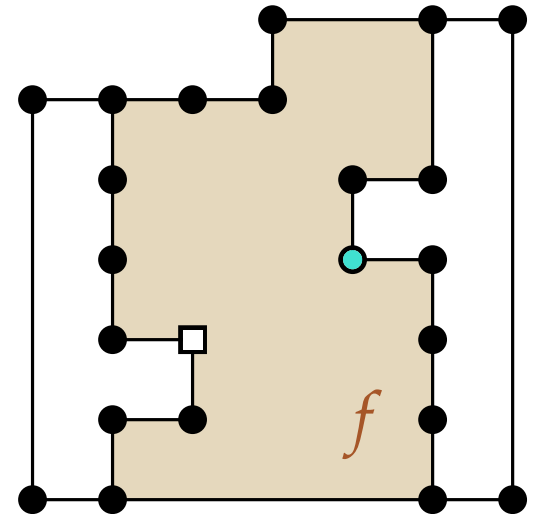
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Thus:

Suffices to consider a *maximal planar subset* of the set K_f of "kitty corner edges" of f .

An FPT Algorithm for OC, cont'd

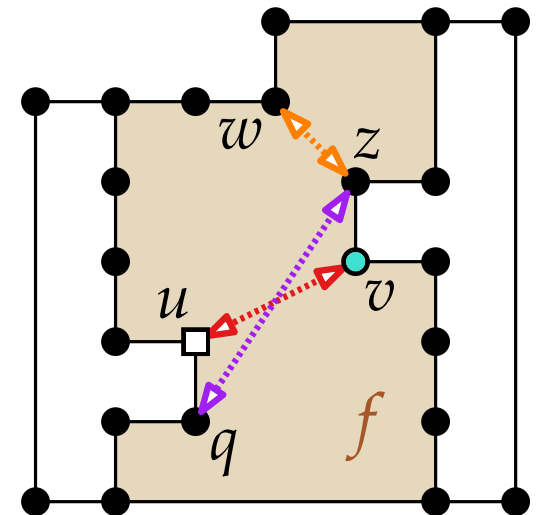
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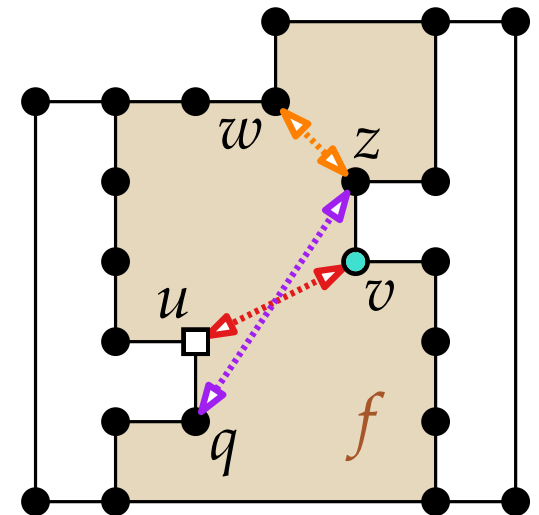
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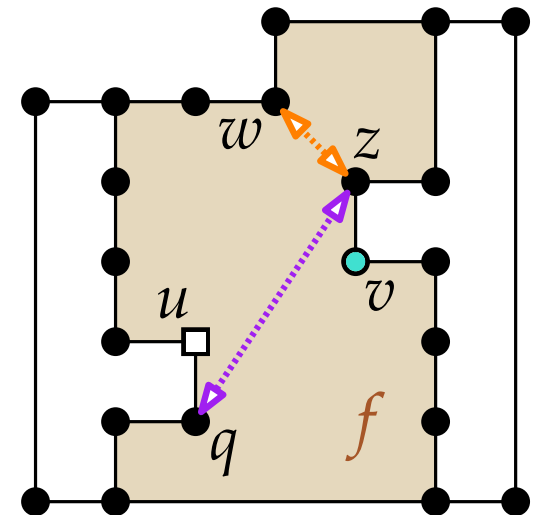
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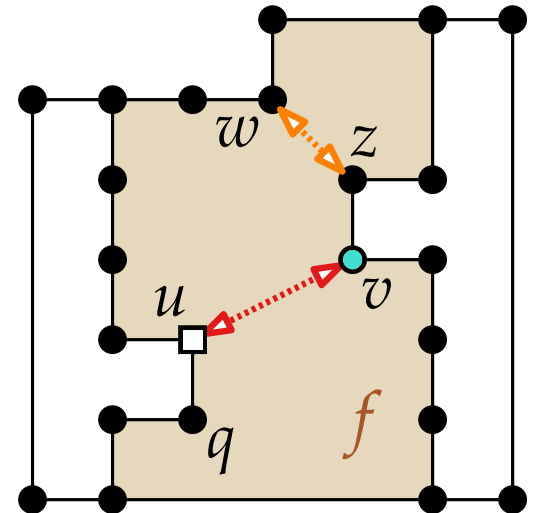
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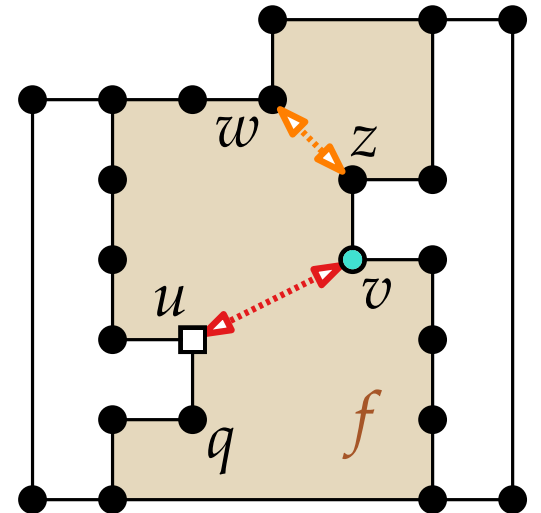
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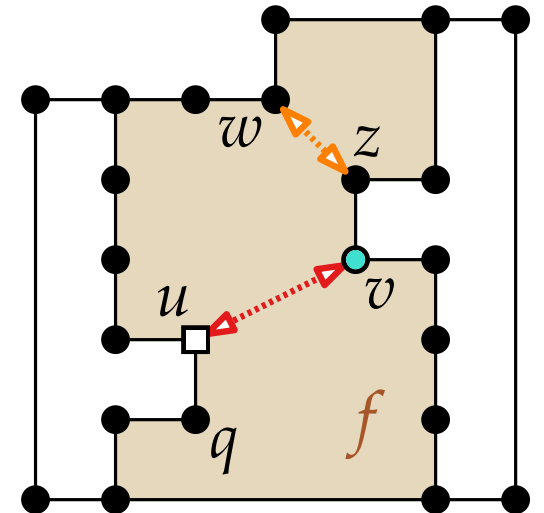
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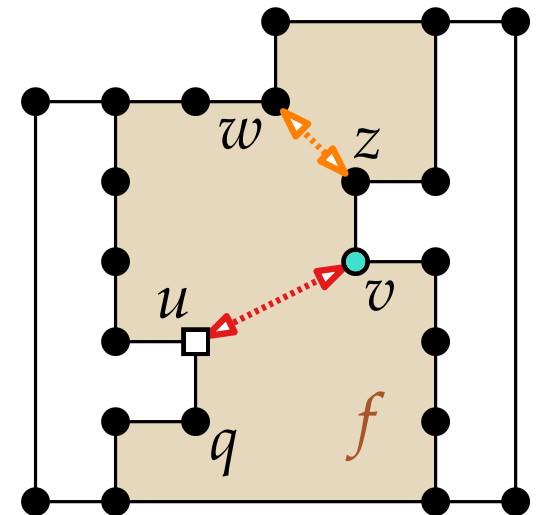
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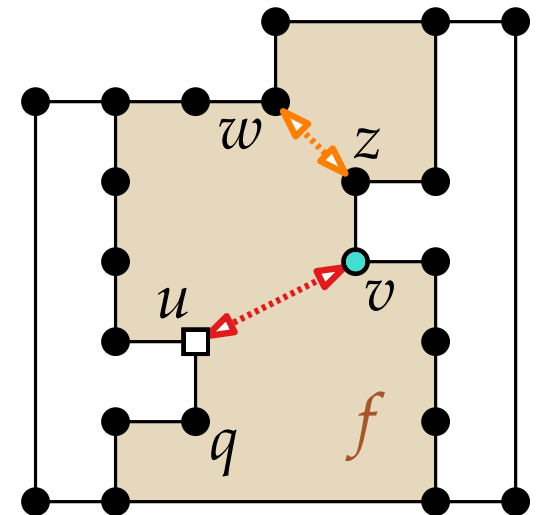
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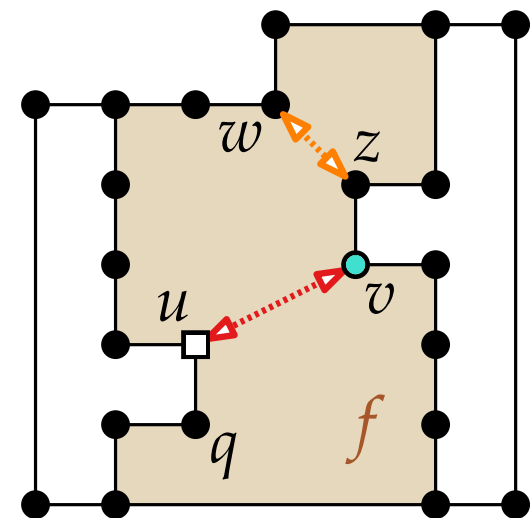
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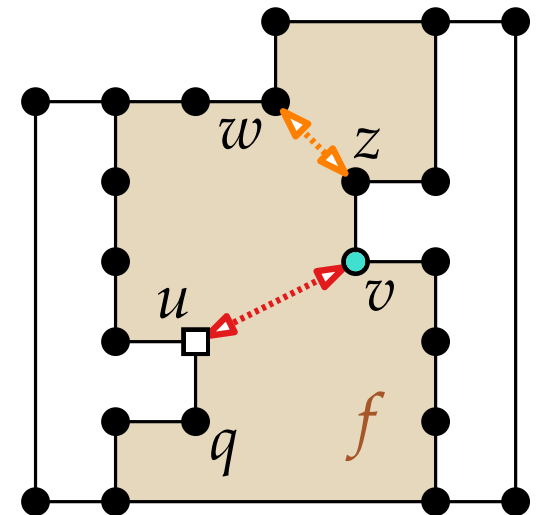
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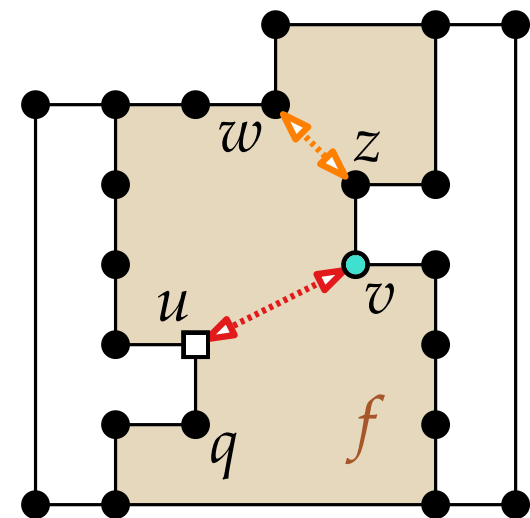


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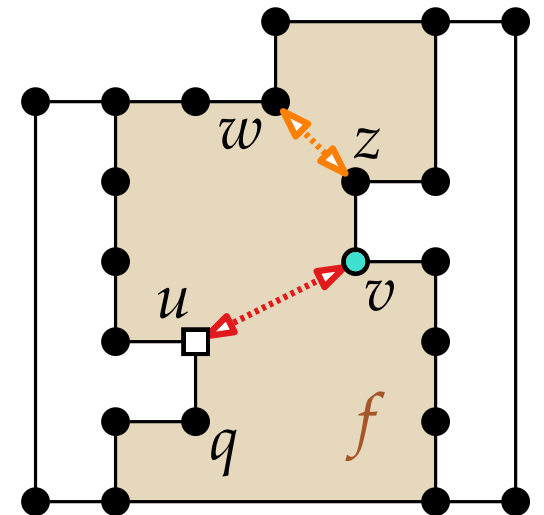
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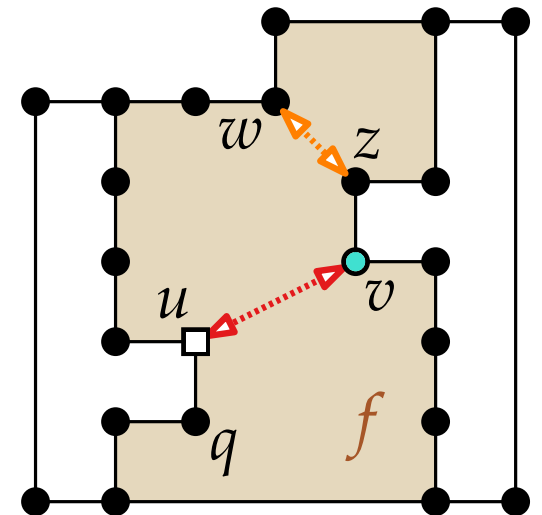
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$\#$ maximal planar subsets of $K_f = \#$ outerplanar graphs on k_f vertices



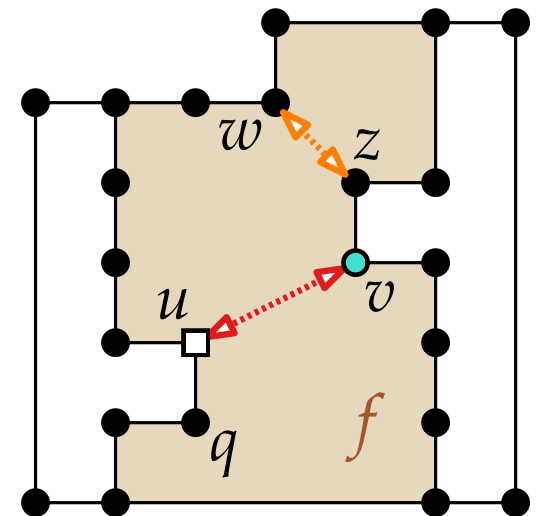
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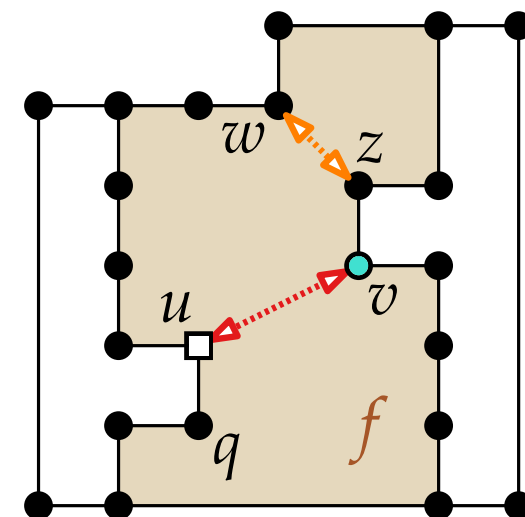
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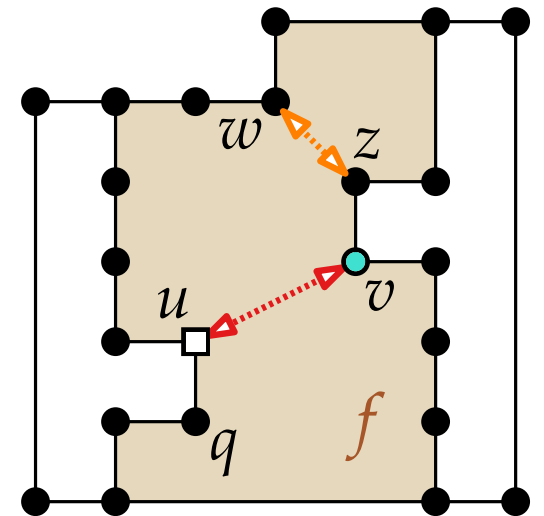
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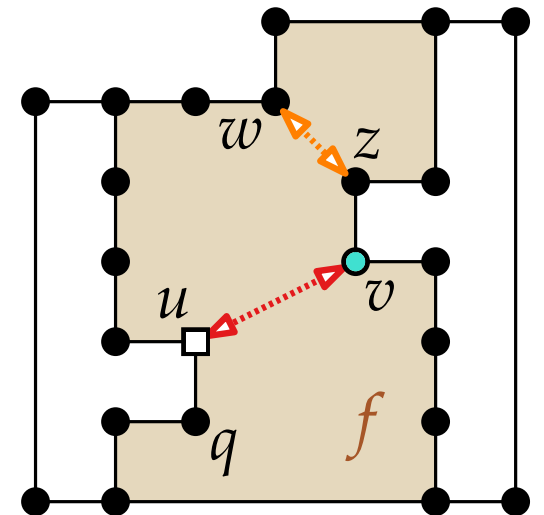
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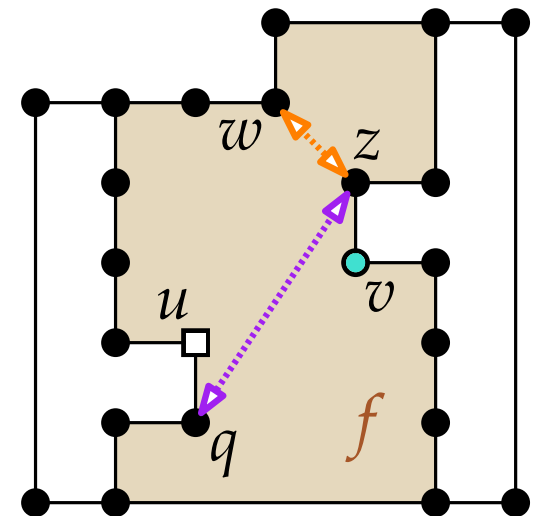
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$$\begin{aligned} \text{Total running time} &= \# \text{ augmentations} \cdot |\mathcal{D}_x \times \mathcal{D}_y| \cdot \text{self-intersection check} \\ &\leq 324^k \cdot O(n \log n) \end{aligned}$$

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Theorem. OC is FPT w.r.t. the number of kitty corners.

Table of Contents

- **Number of kitty corners:** (the number of corners involved in some pair of kitty corners)
We show: *OC admits an FPT algorithm parametrized by the number of kitty corners.*
- **Number of faces:** OC is para-NP-hard when parametrized by the number of faces.
For *one* face (cycle), we show the *existence of a polynomial kernel for OC.*
when parametrized by the number of kitty corners.
- **Maximum face-degree:** The reductions of Patrignani & Evans et al. use linear-size faces.
We show: *OC remains NP-hard when parametrized by maximum face degree.*
- **Height:** (minimum number of distinct y-coordinates required to draw the representation)
A $(w \times h)$ -grid has pathwidth at most h .
 \Rightarrow Graphs of bounded height have bounded pathwidth. But converse generally not true.
We show: *OC admits an XP algorithm parametrized by height.*

Boundary, Belt, and “Piston” Gadget

Theorem. OC is NP-hard.

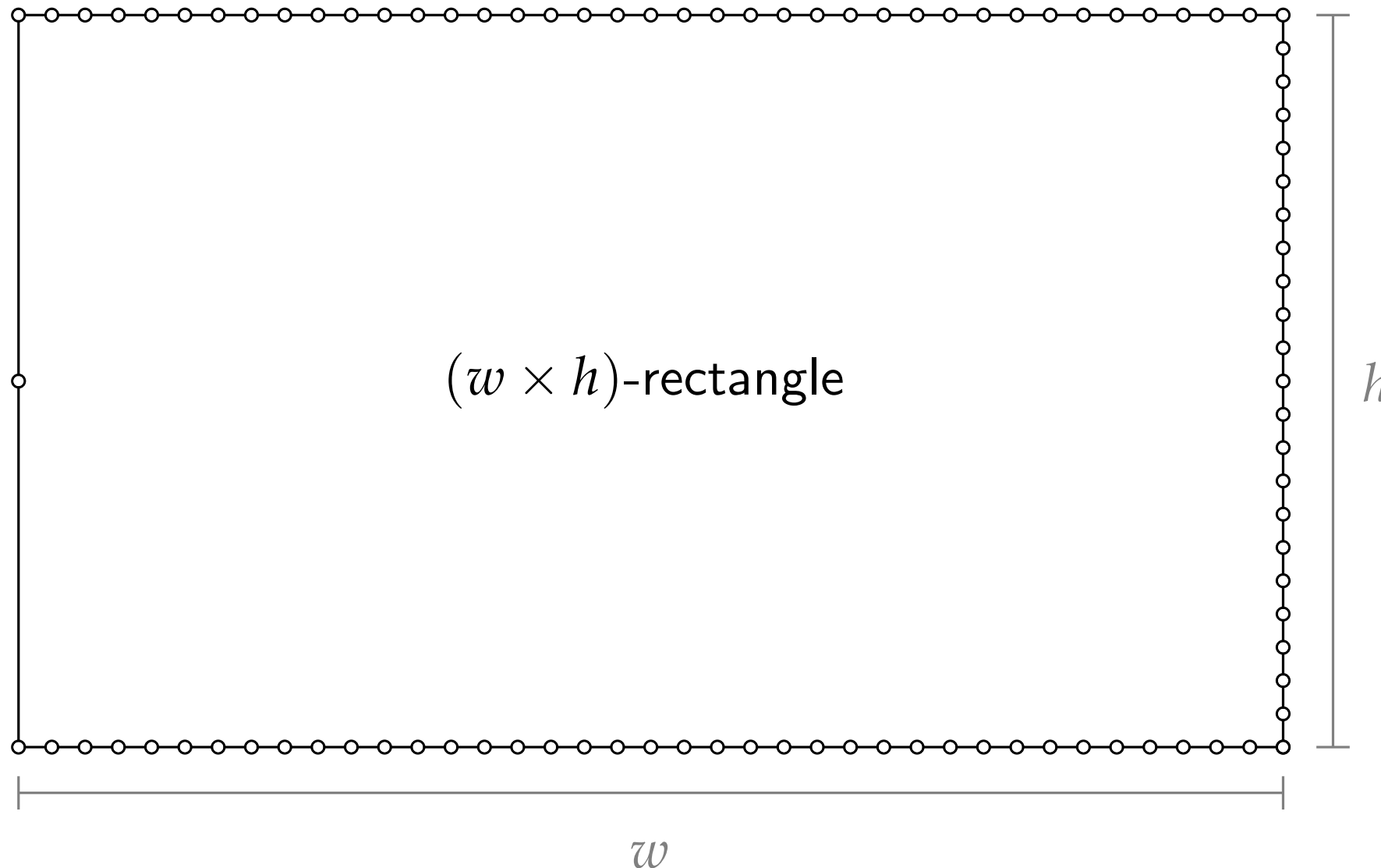
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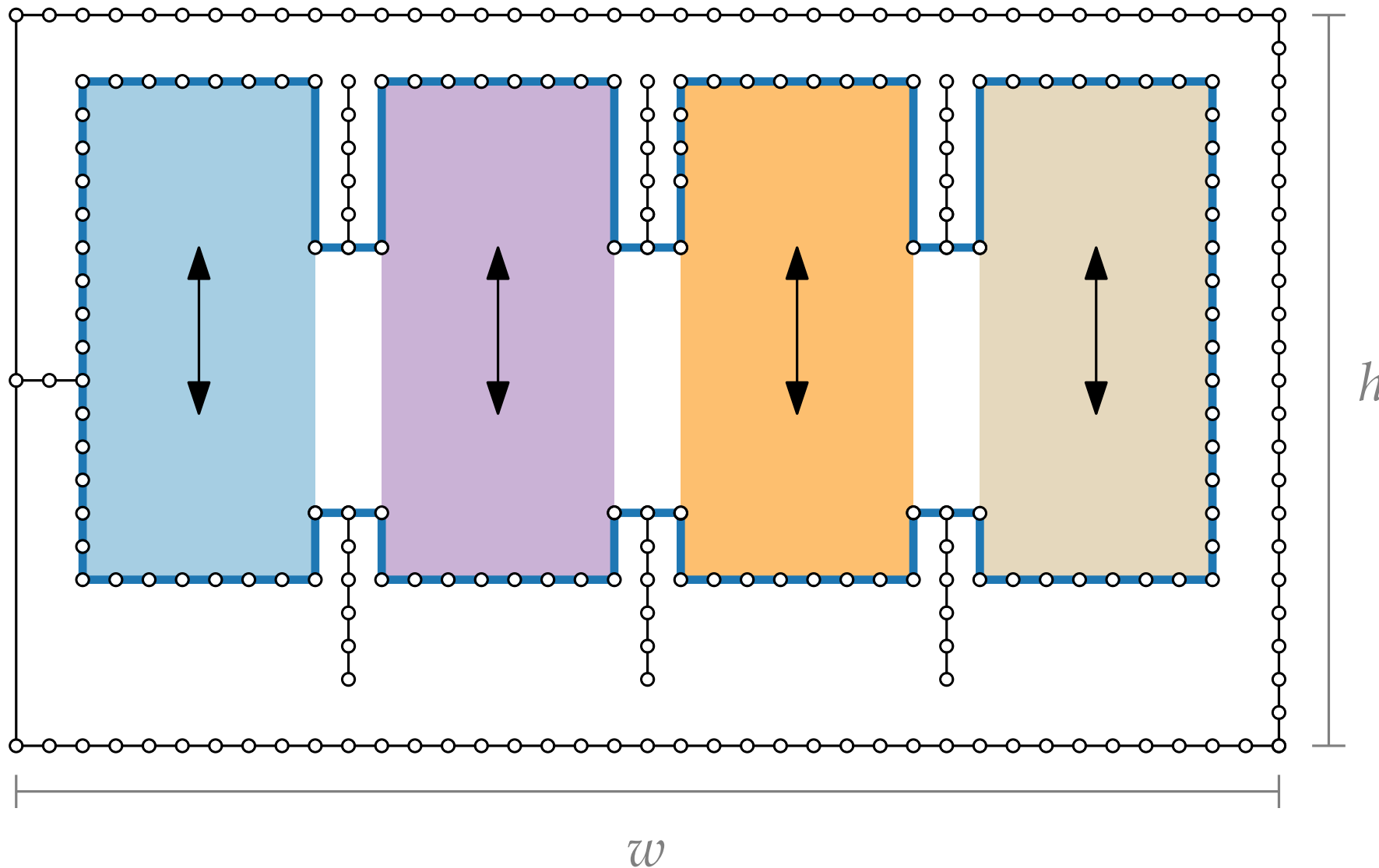
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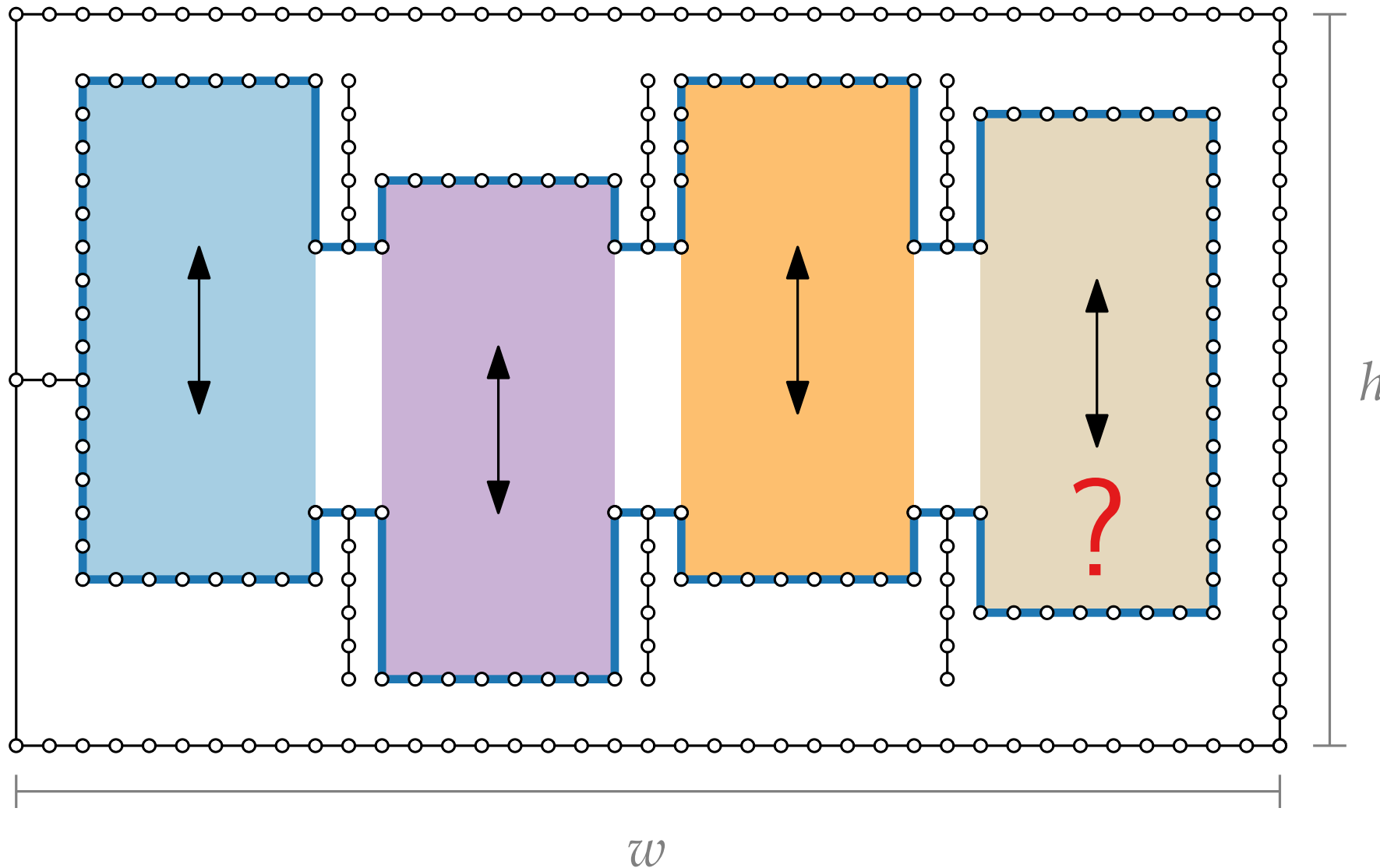
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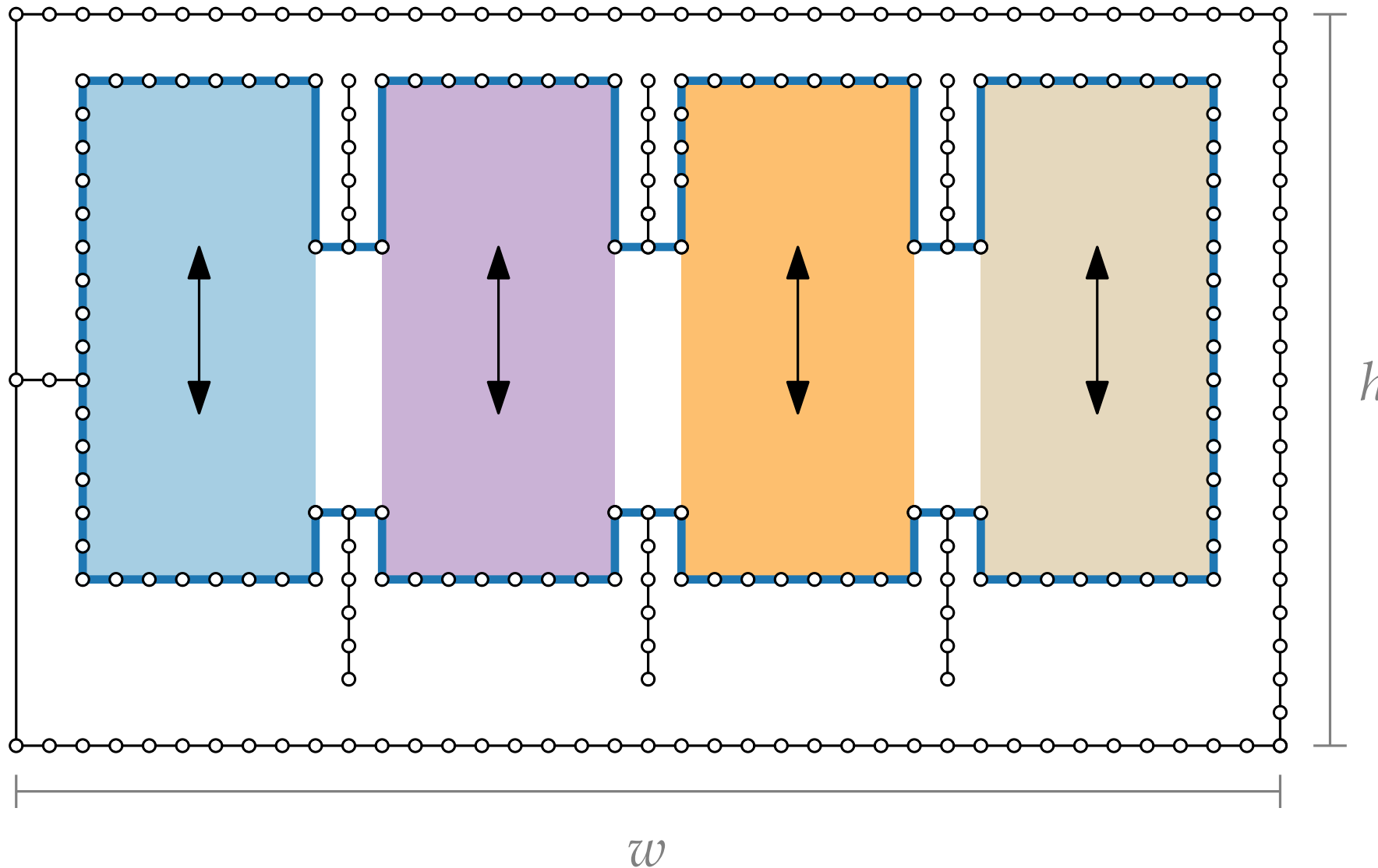
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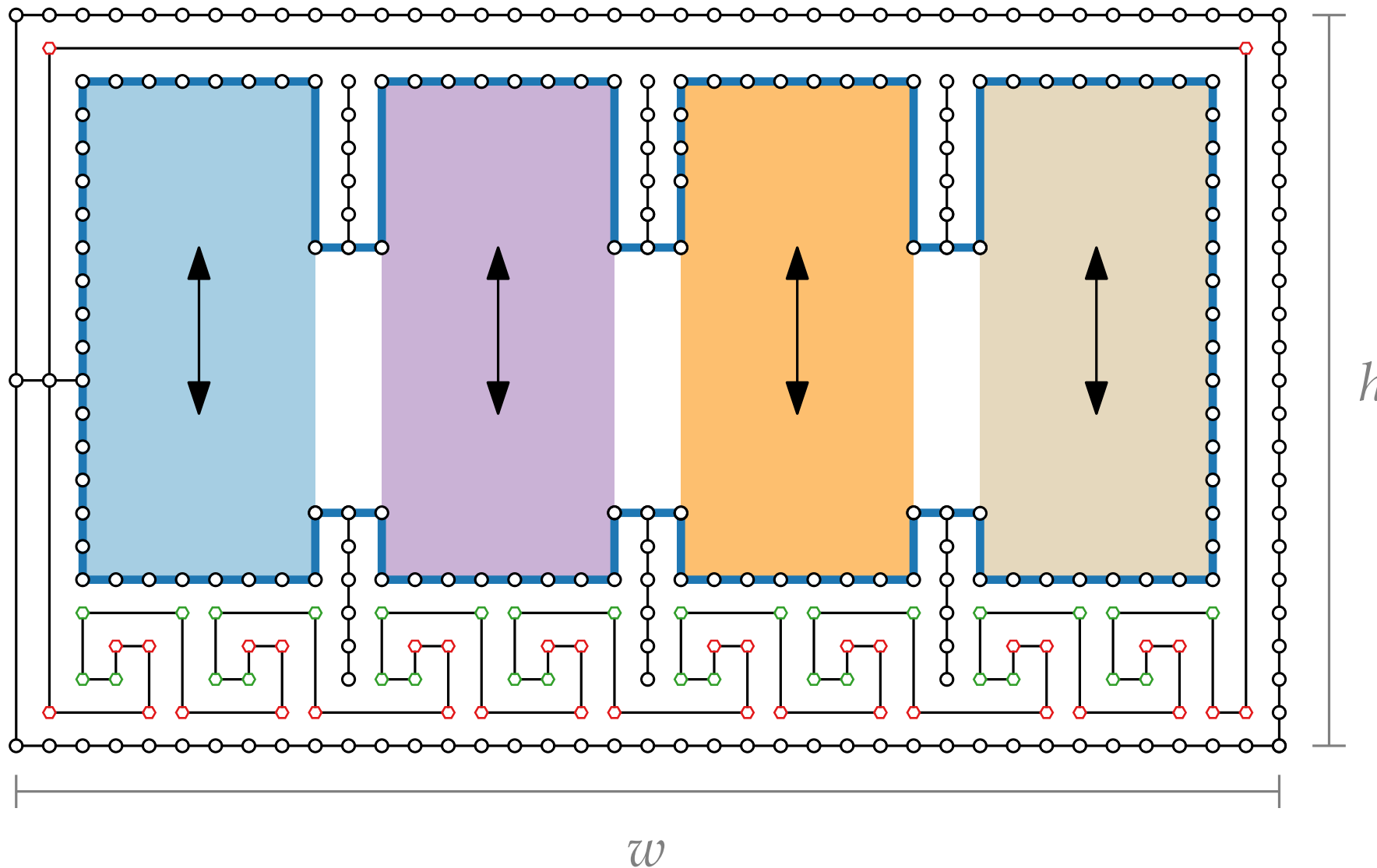
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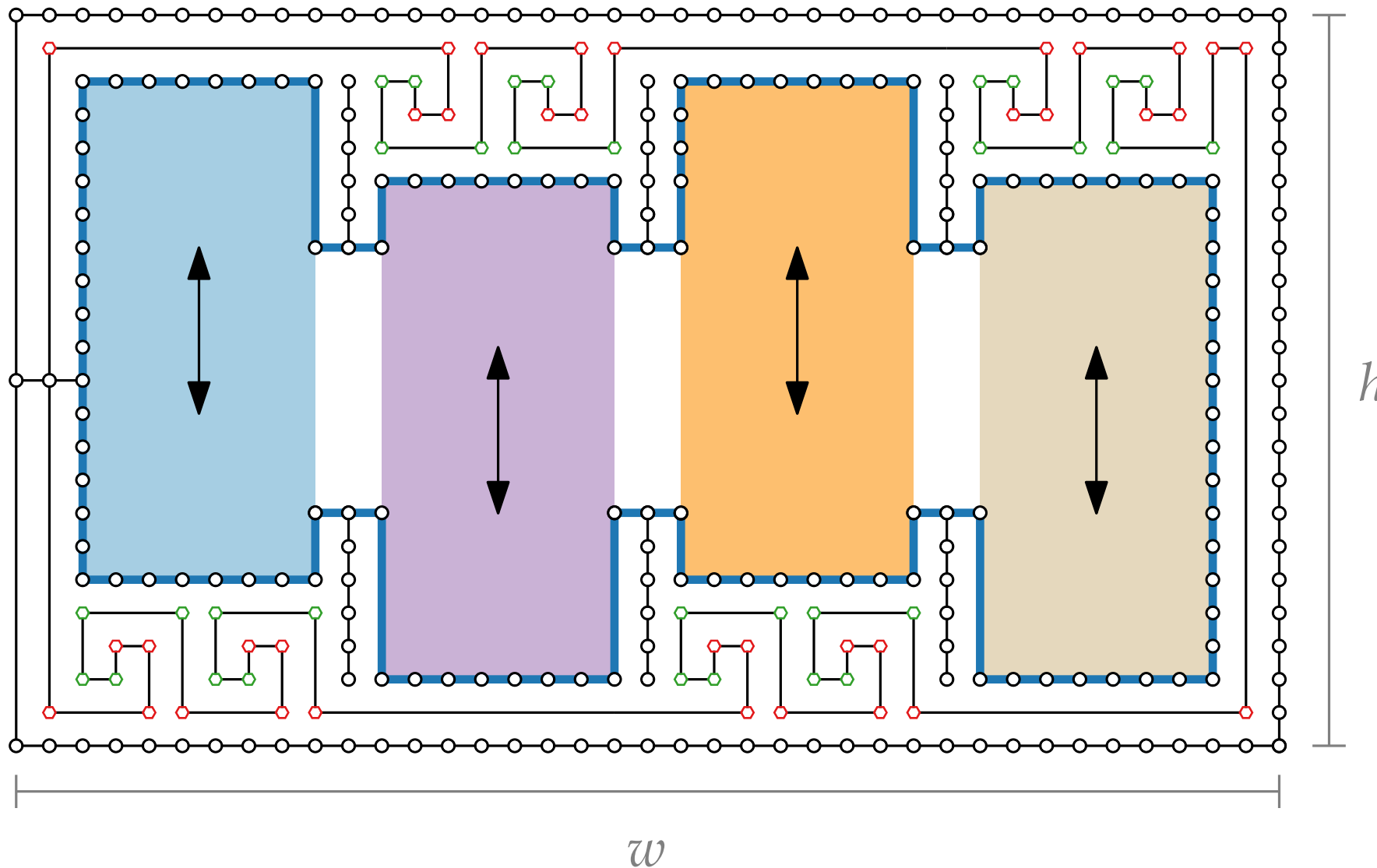
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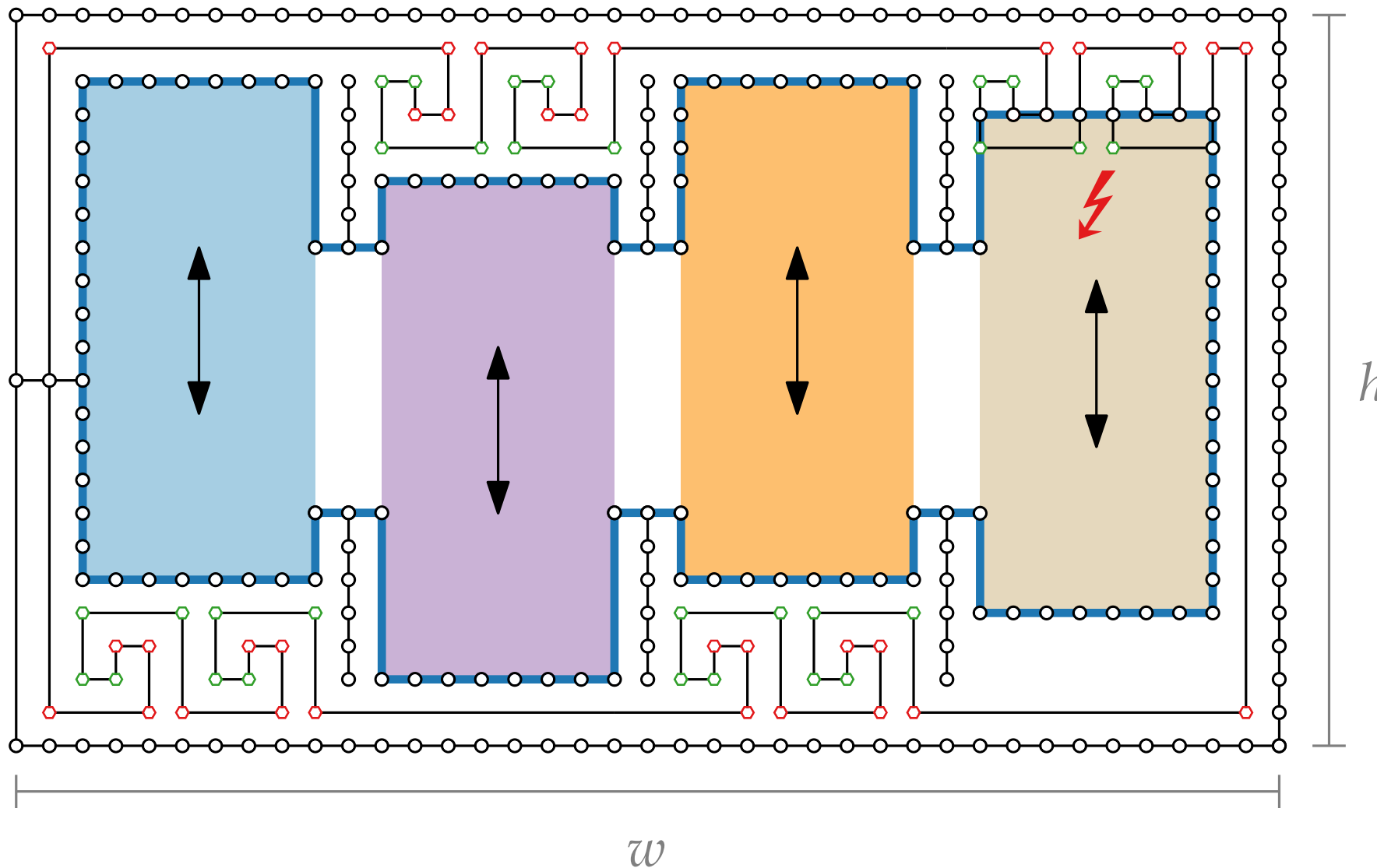
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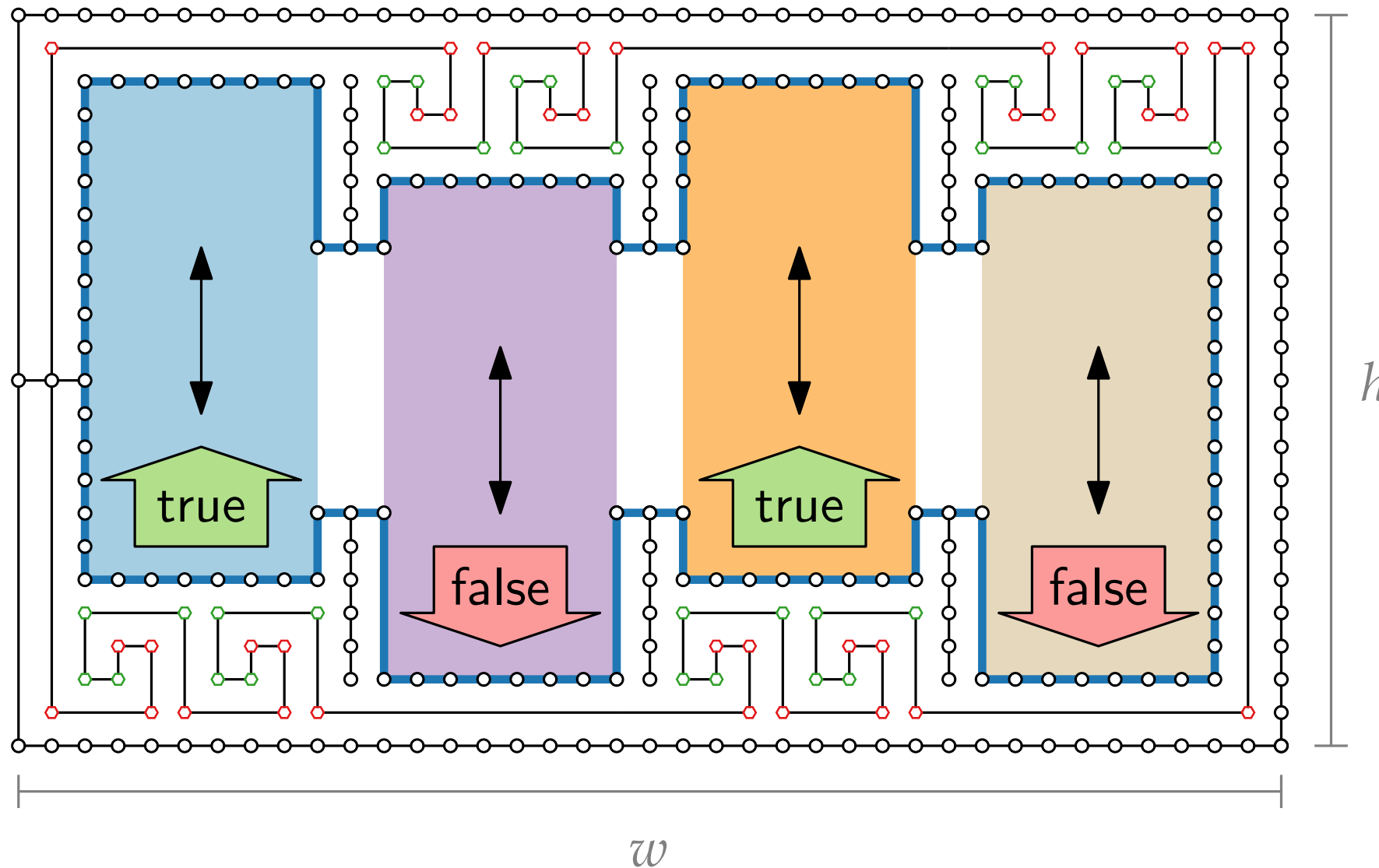
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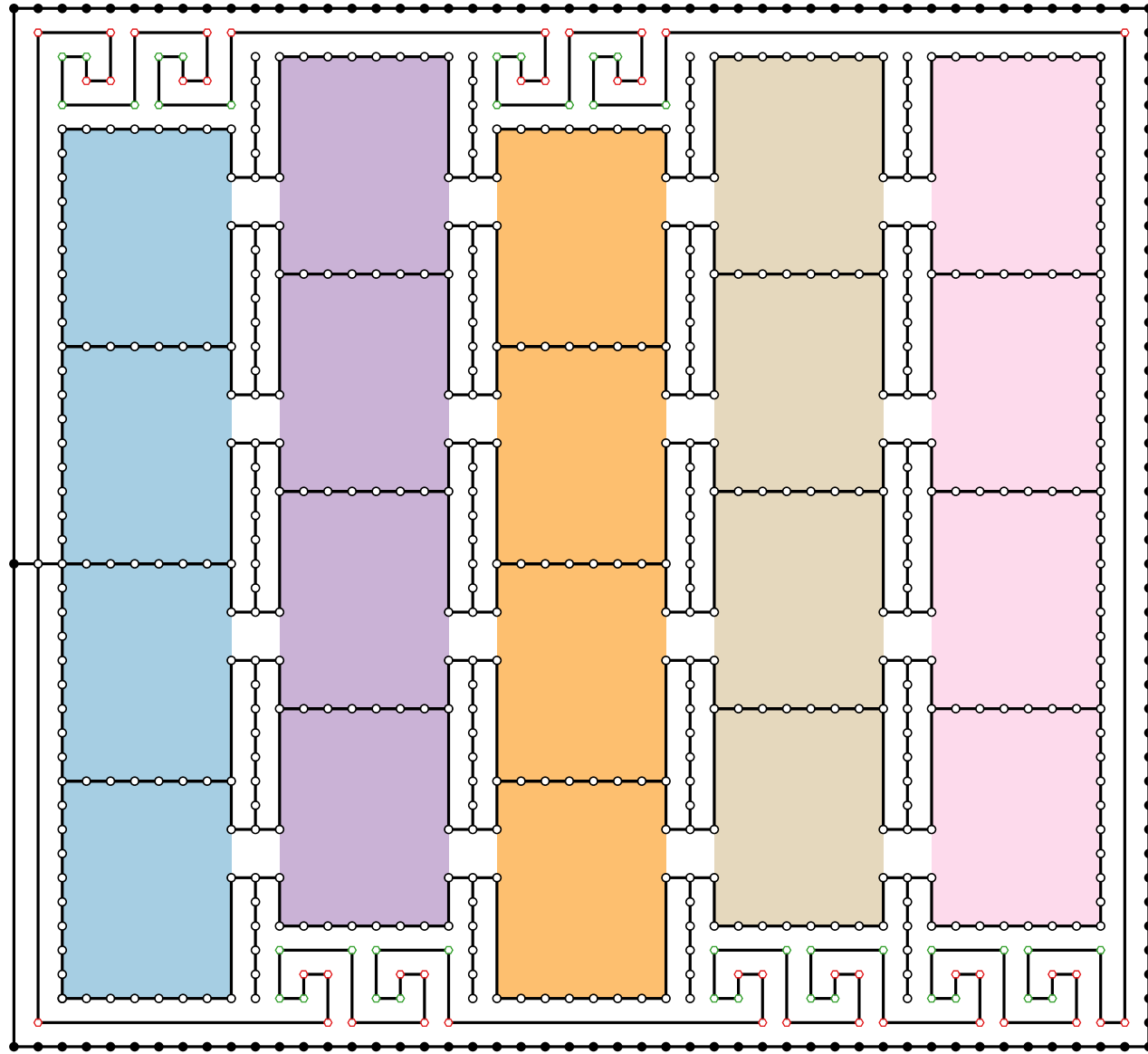
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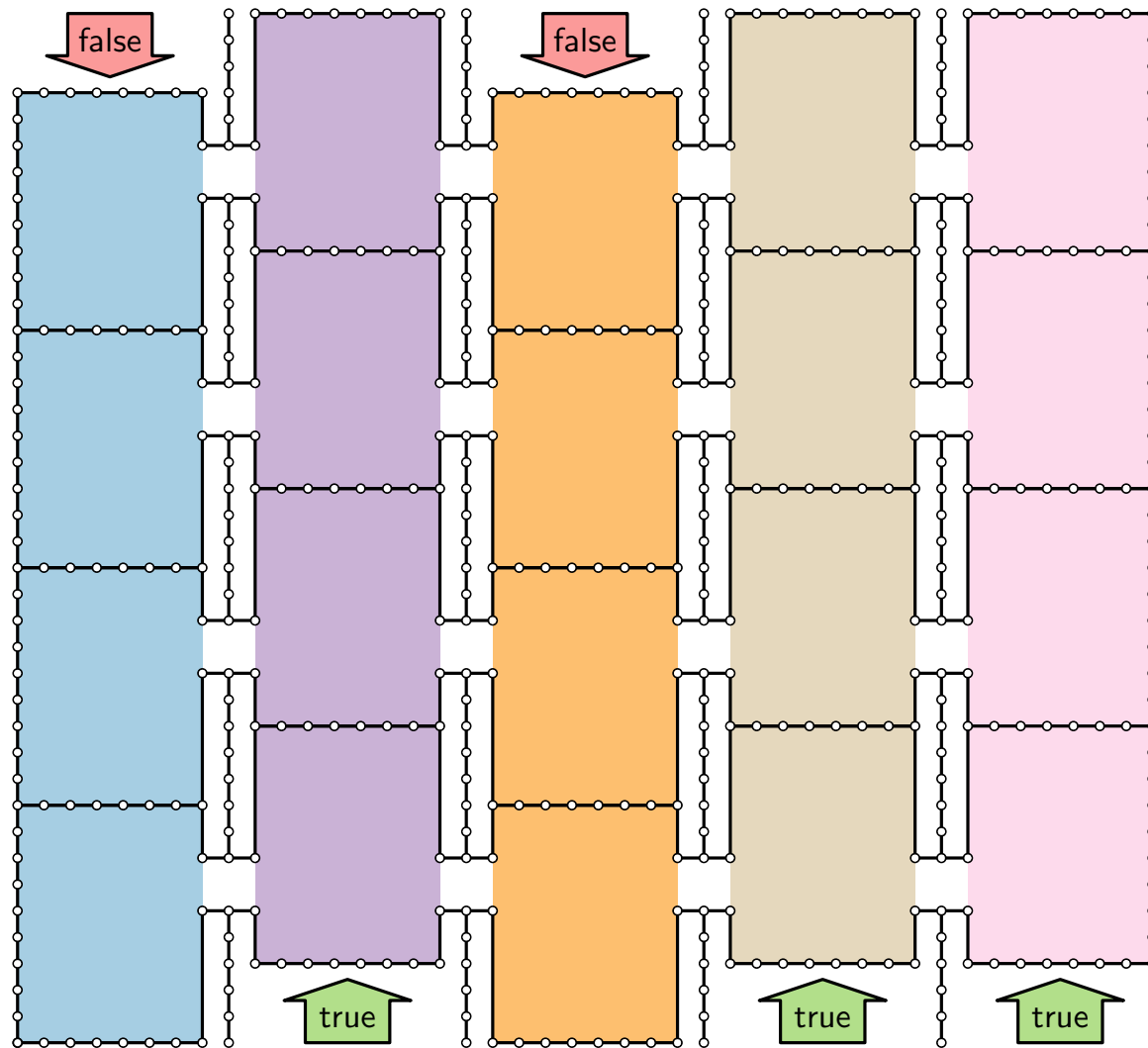
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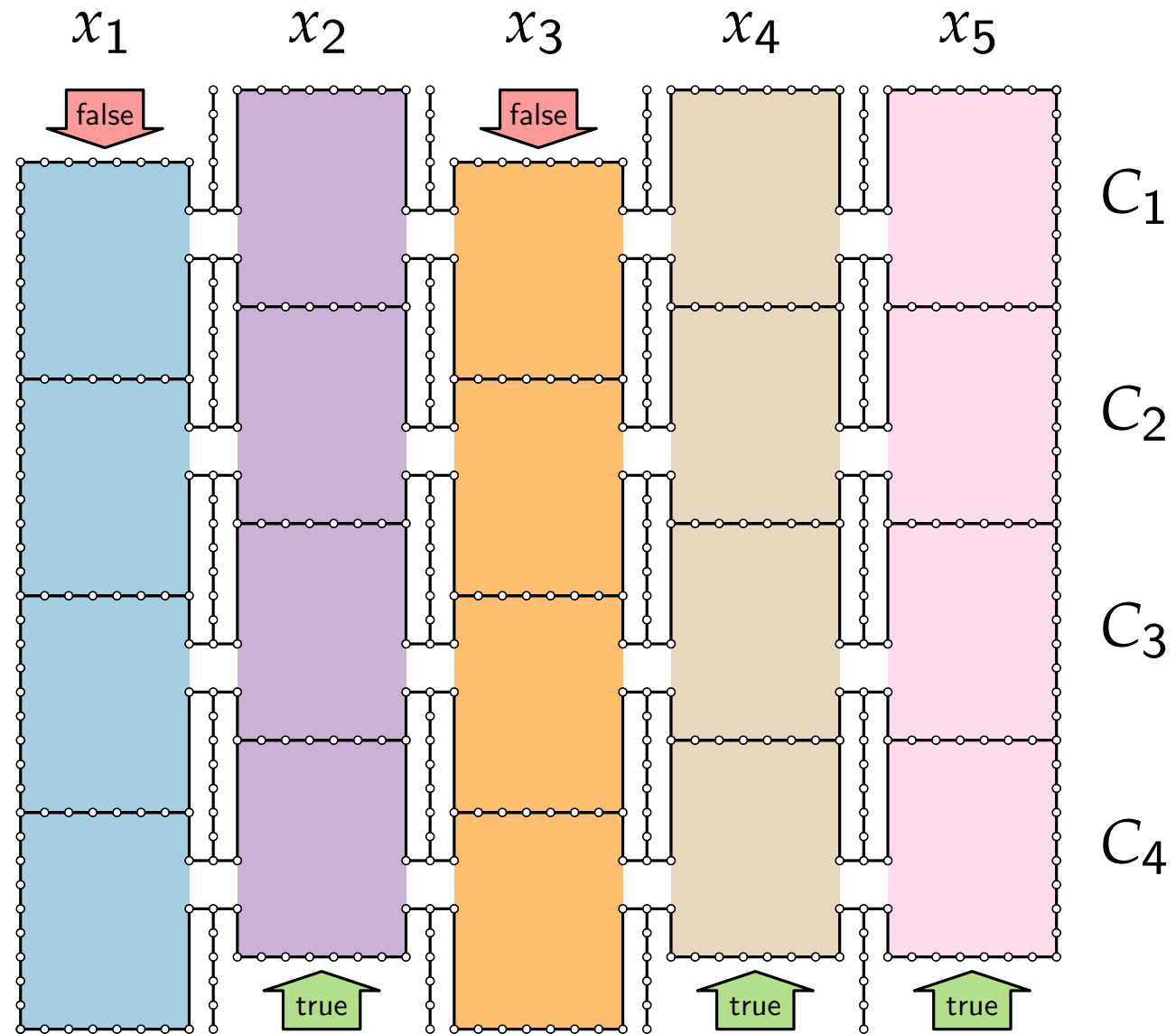
Clause Gadgets



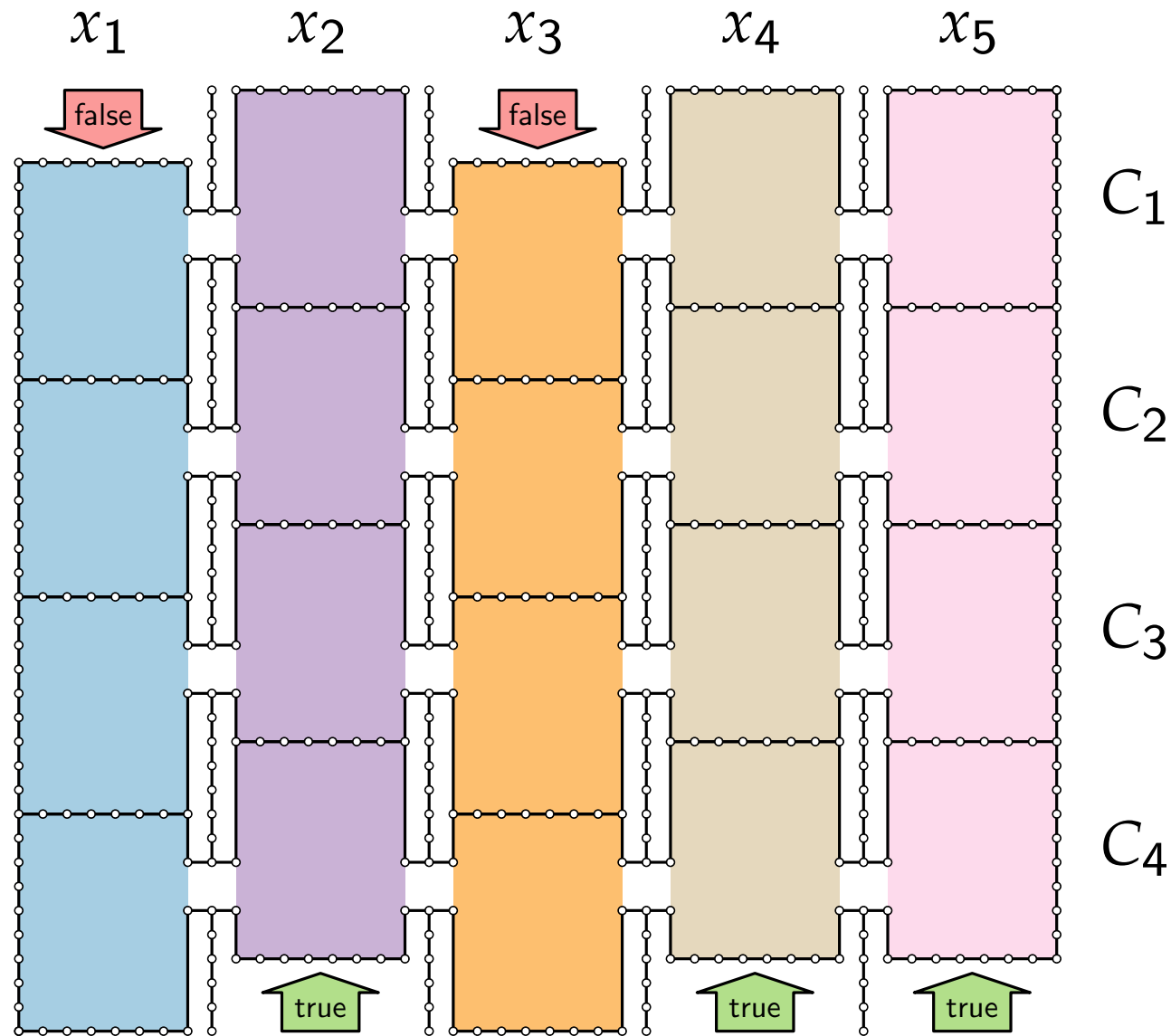
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Example:

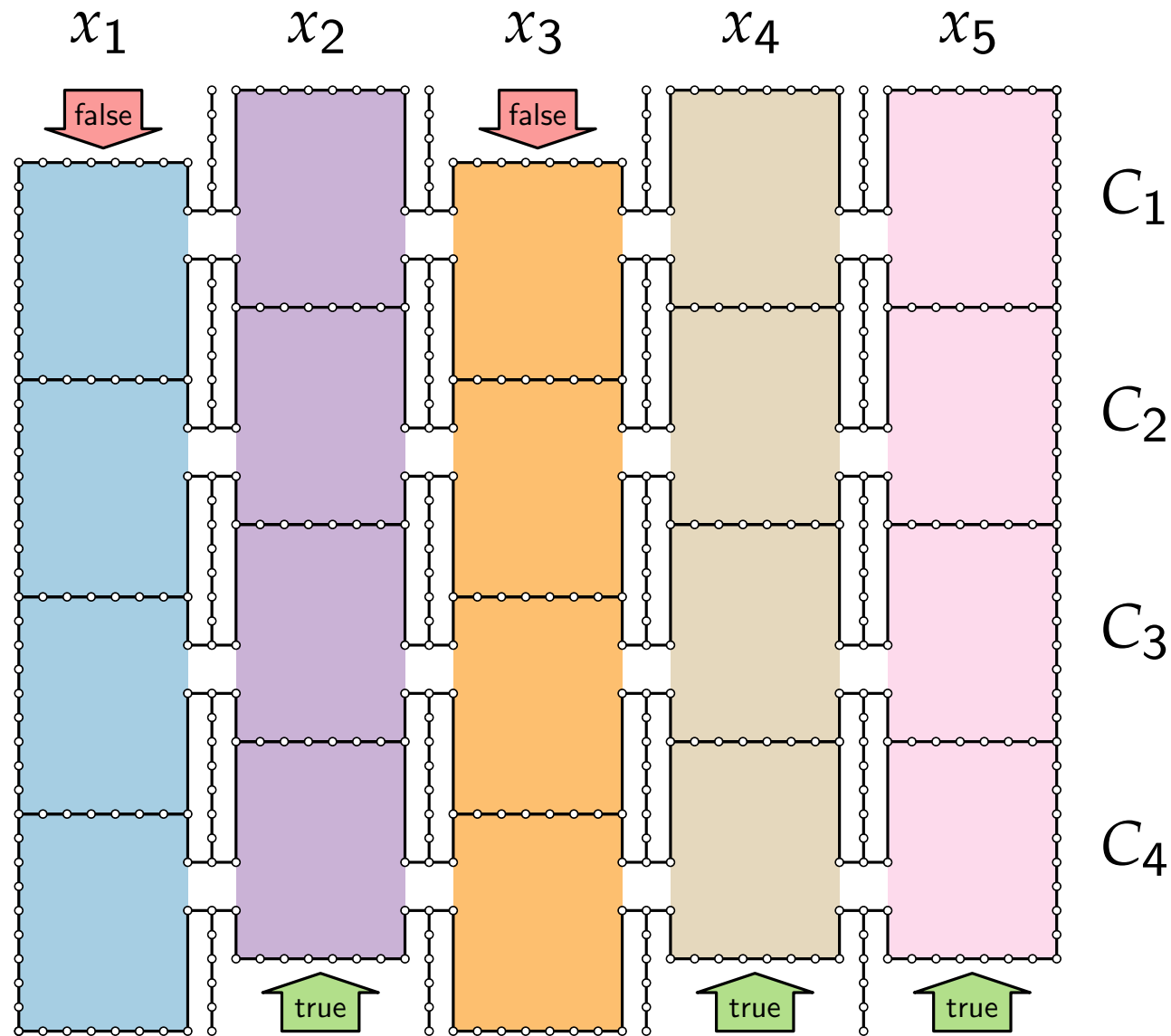
$$C_1 = x_2 \vee \overline{x_4}$$

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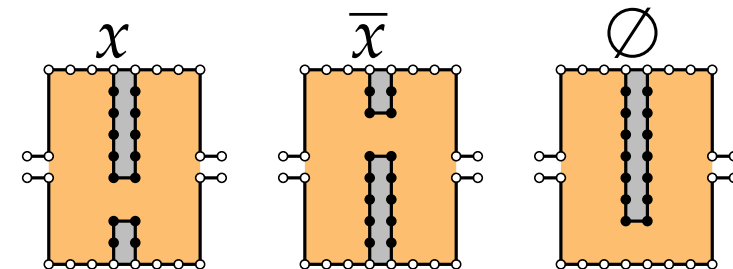
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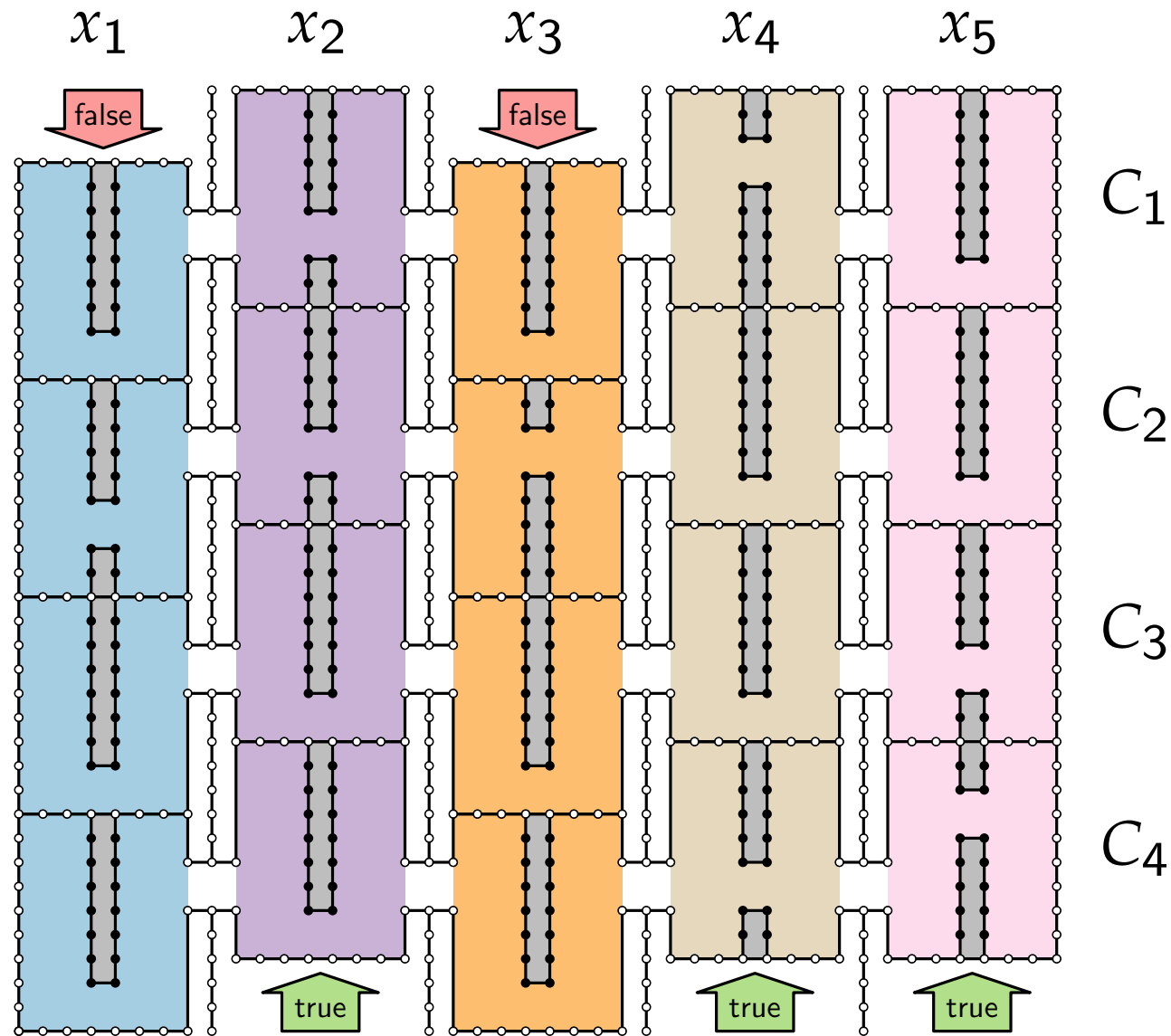
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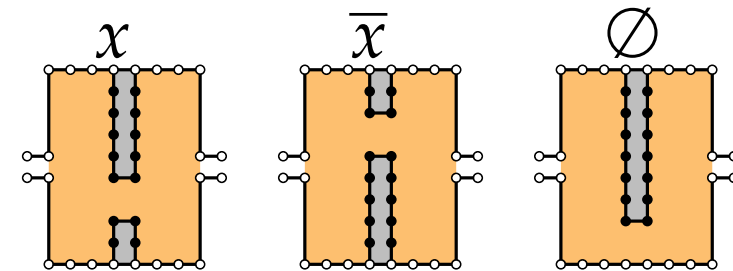
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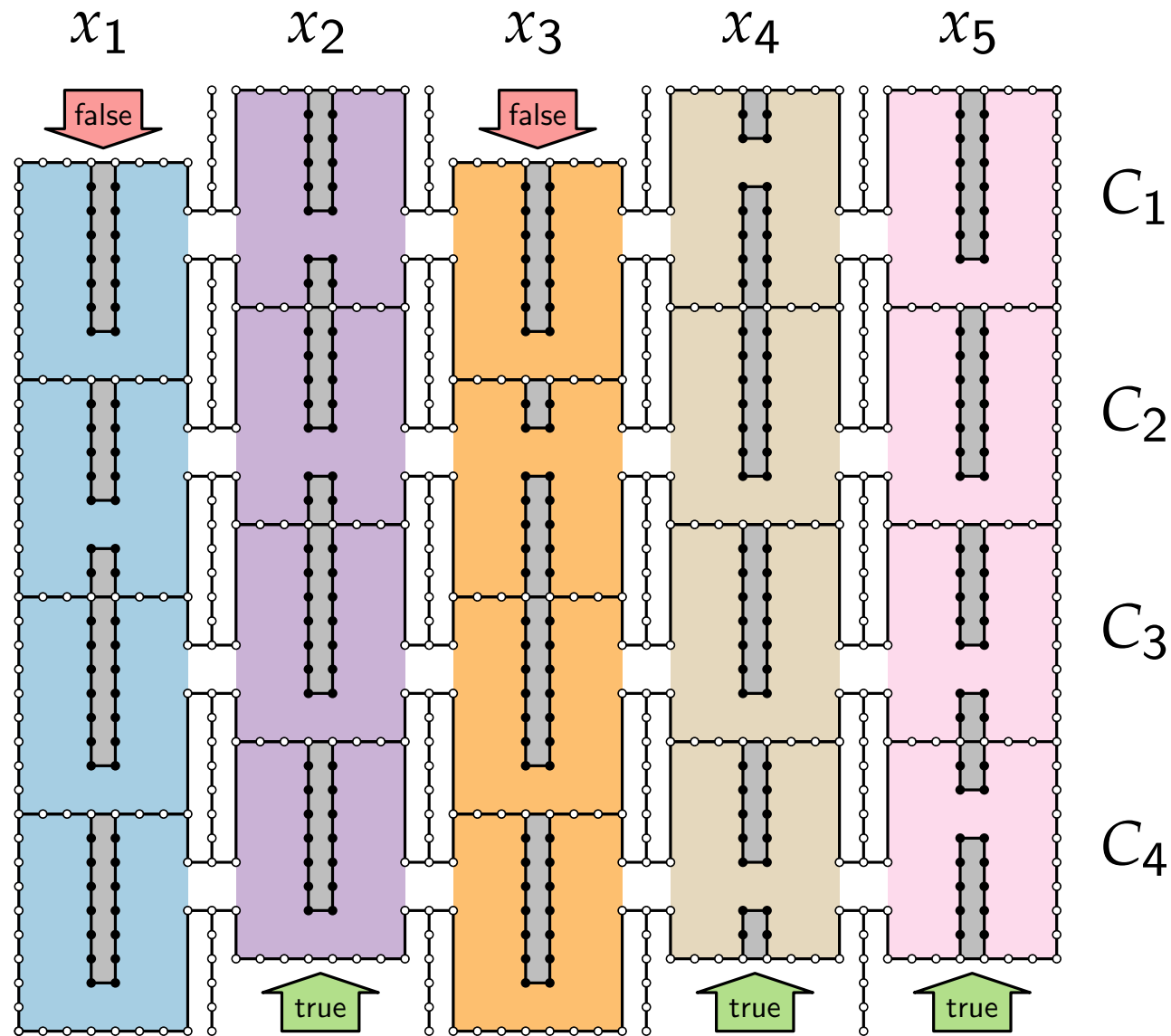
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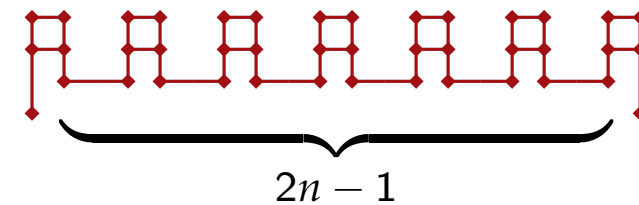
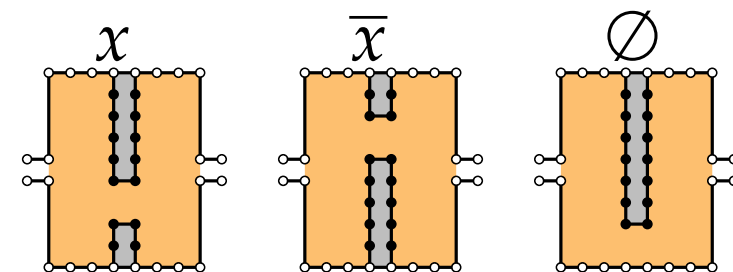
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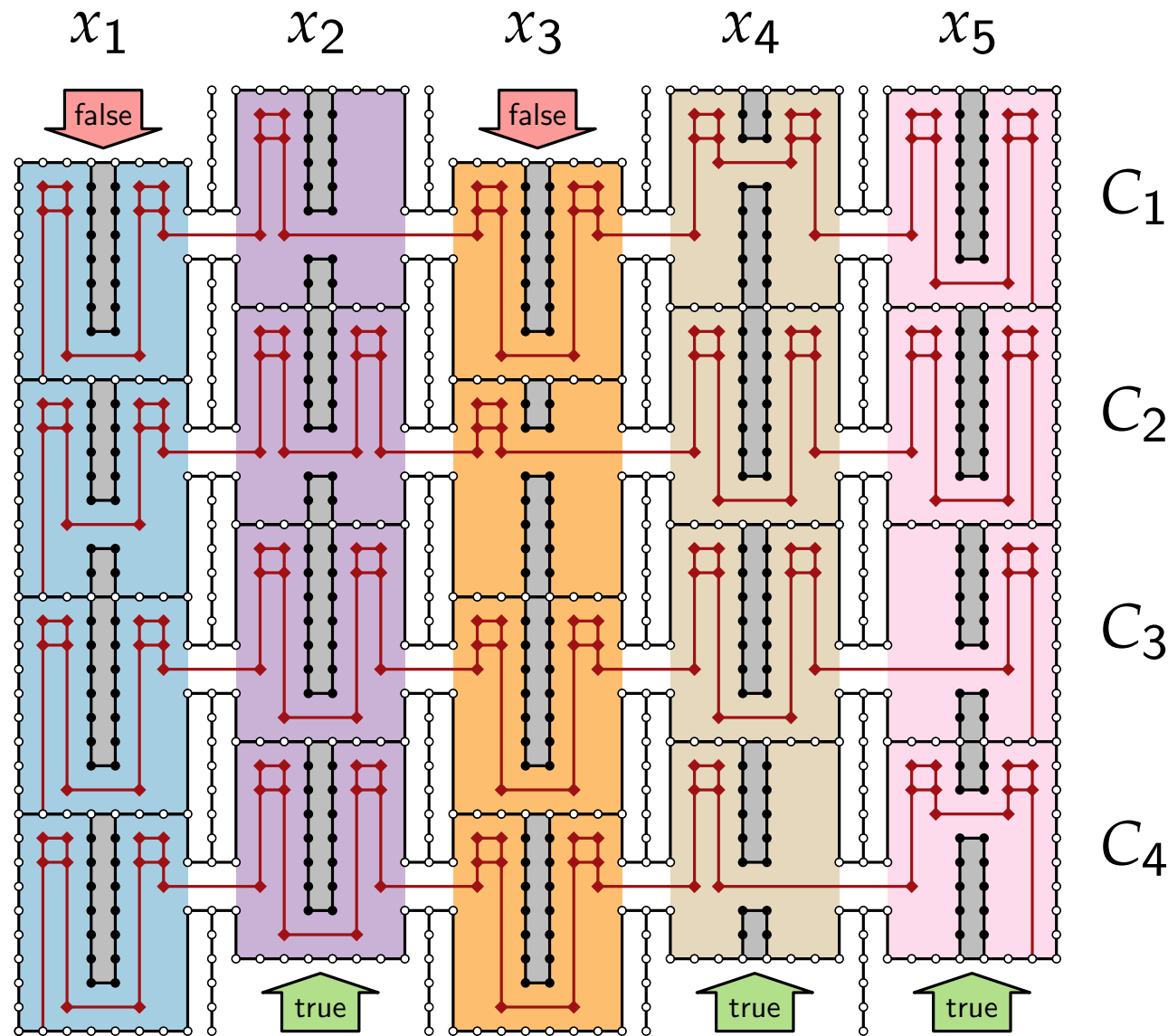
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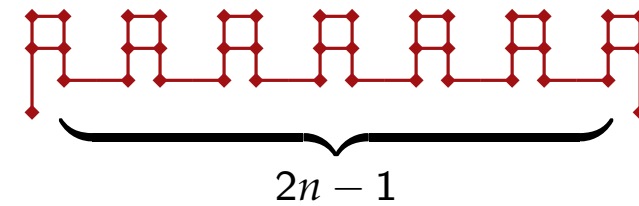
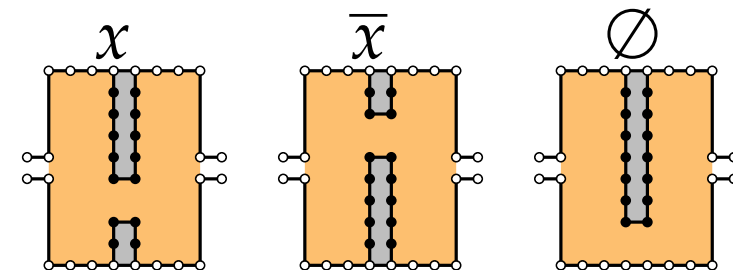
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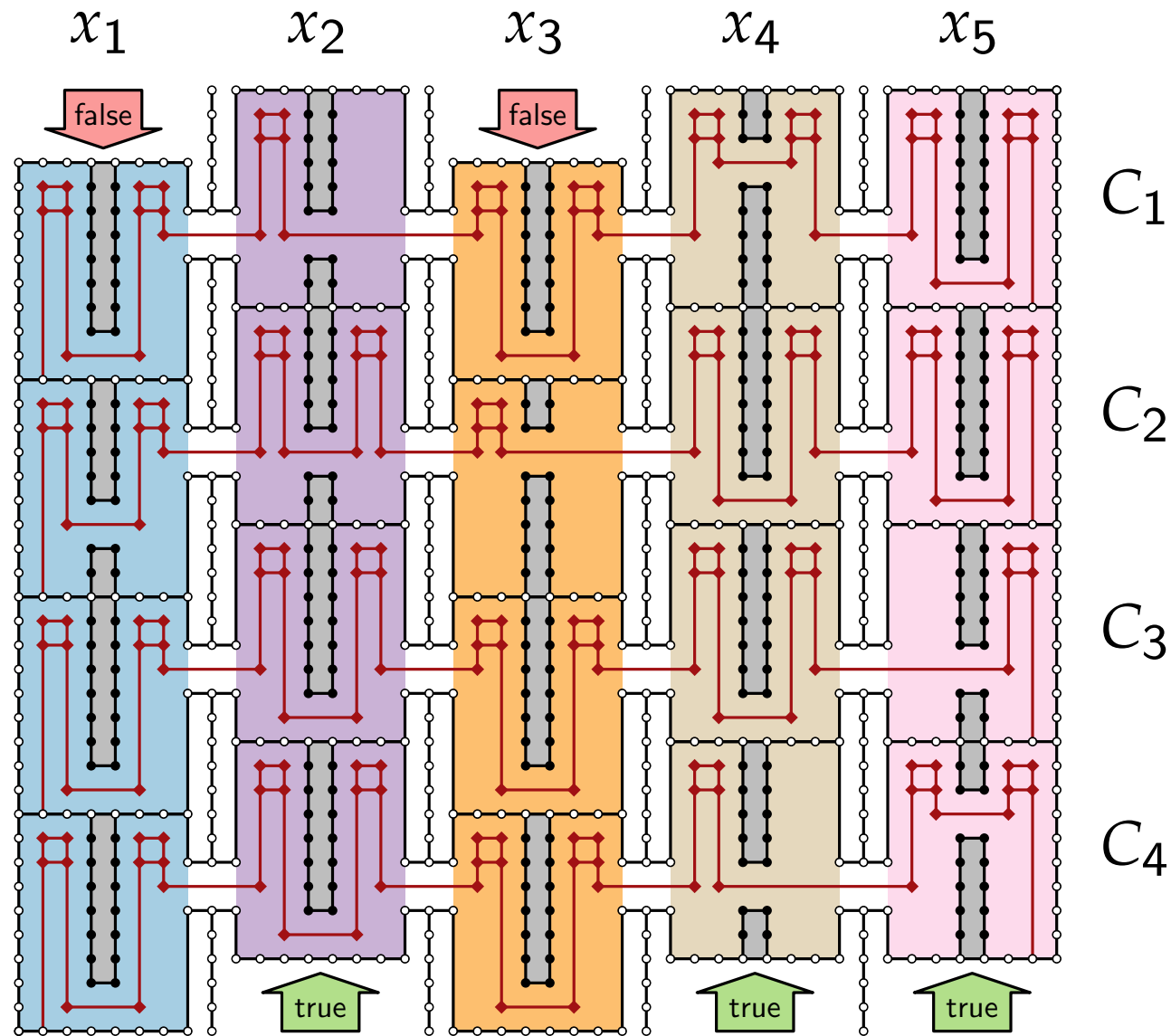
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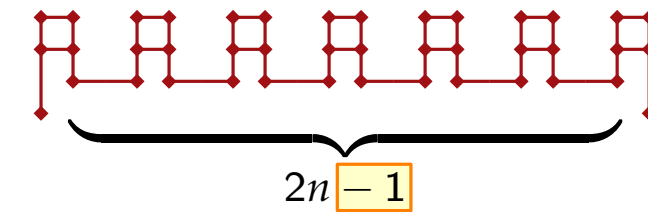
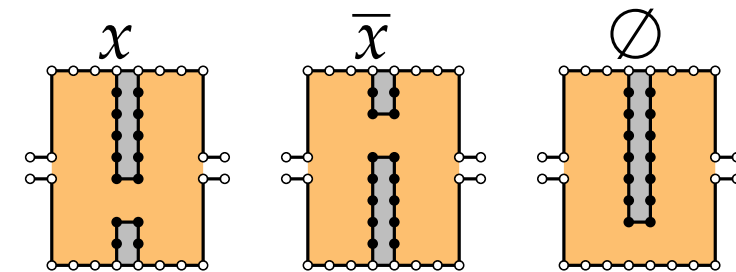
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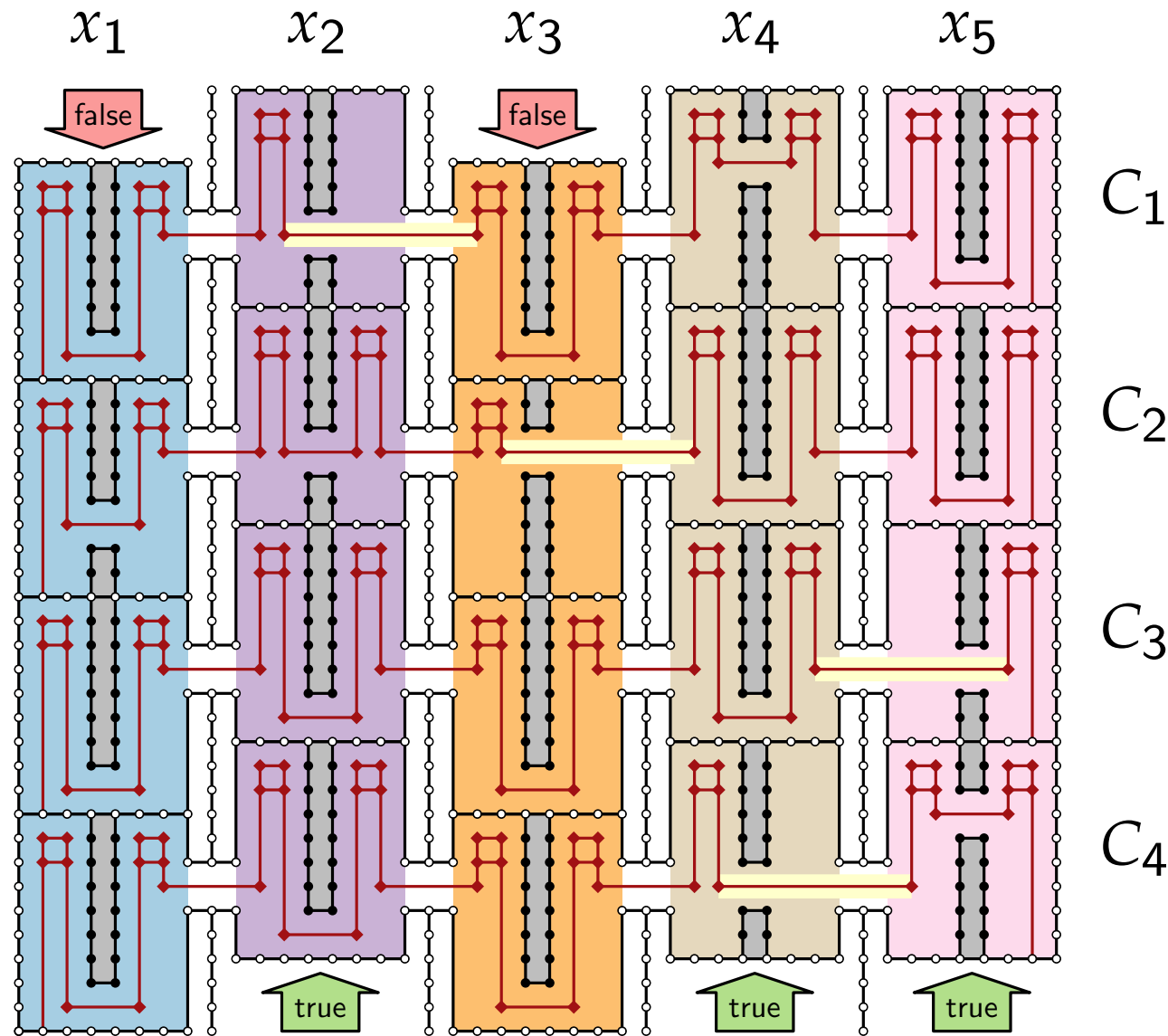
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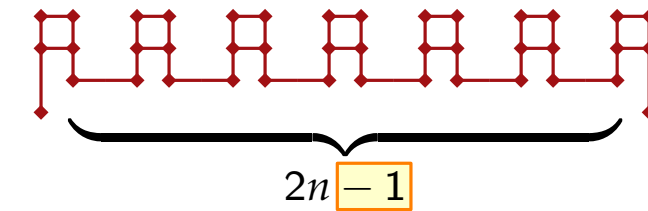
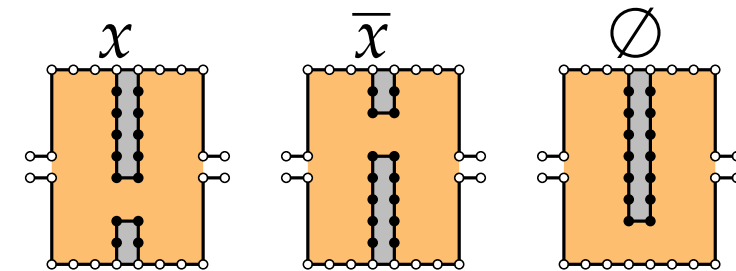
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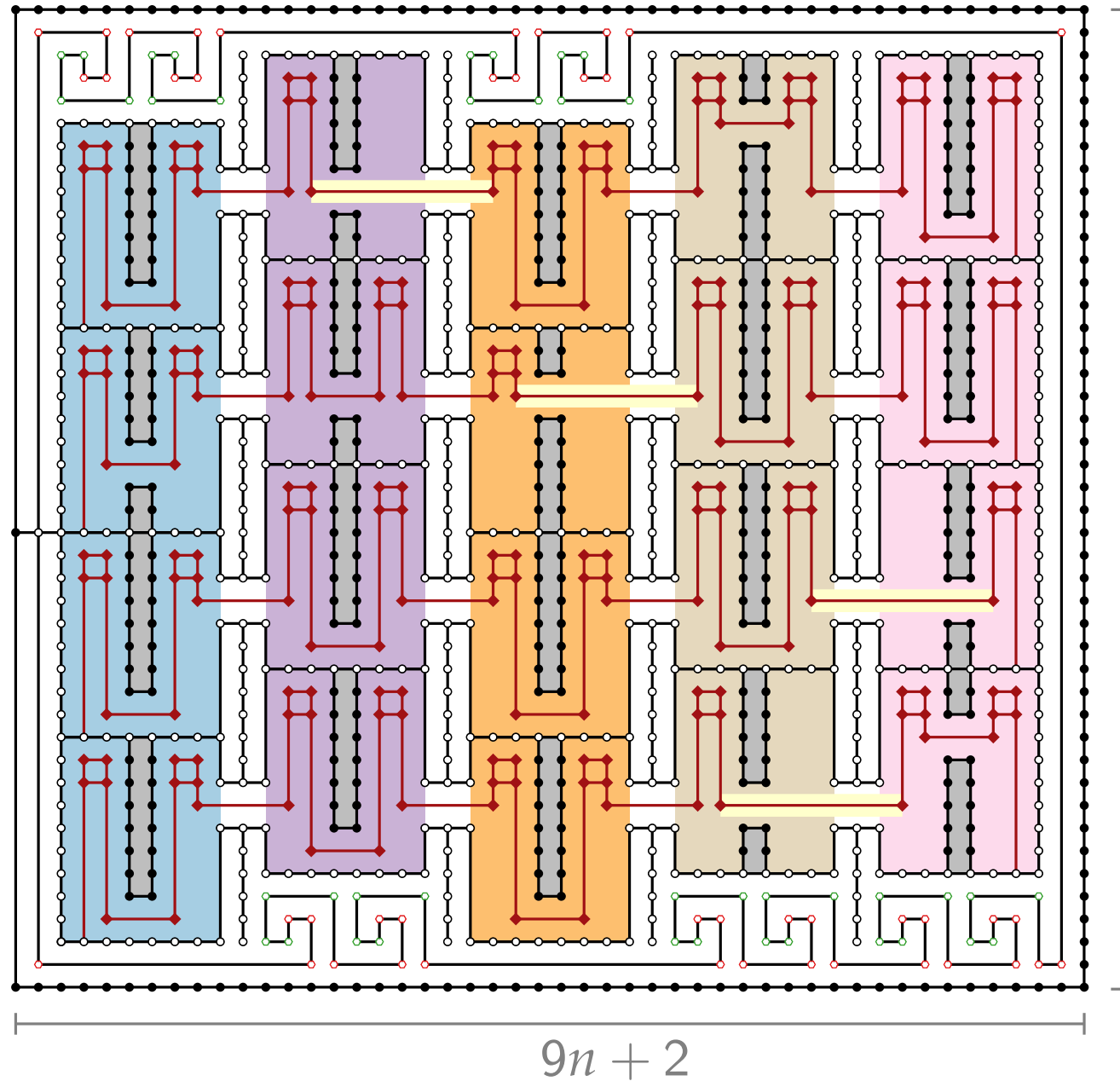
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Full reduction



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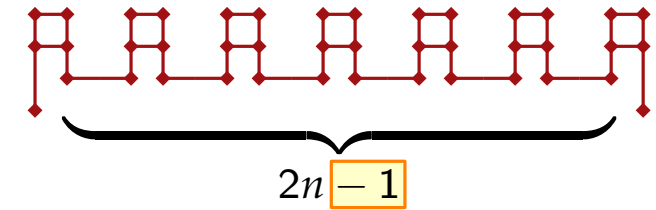
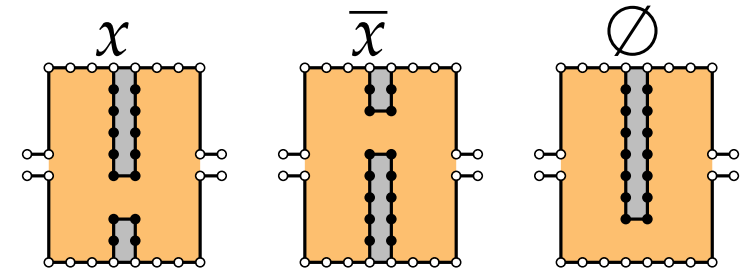
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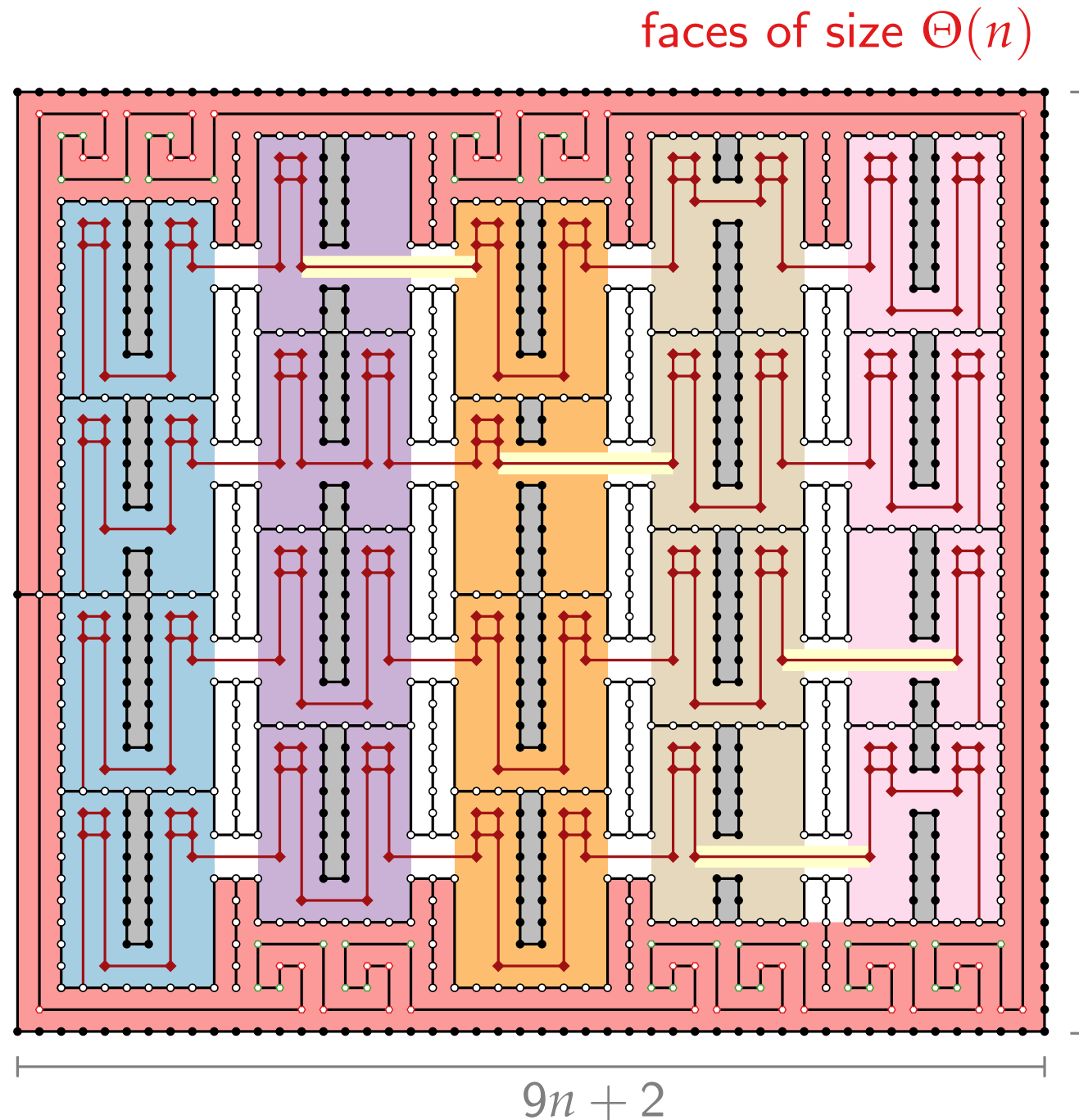
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$9m + 7$



Full reduction



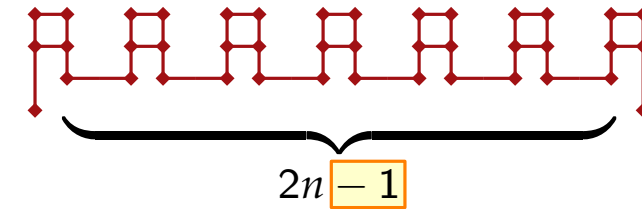
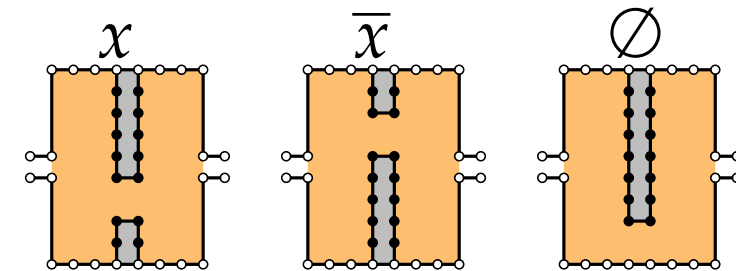
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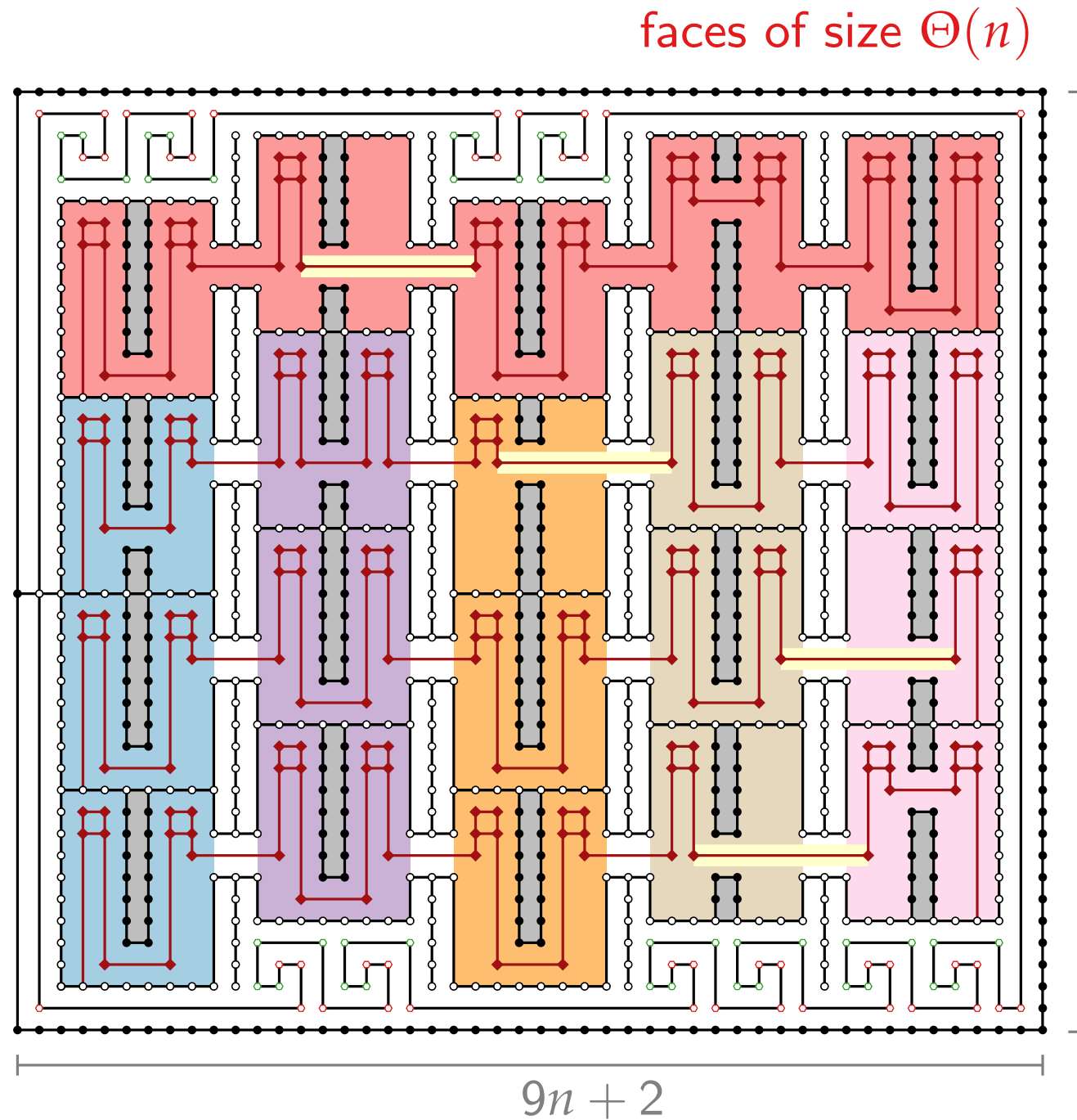
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Full reduction



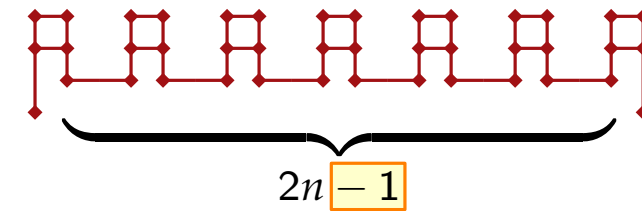
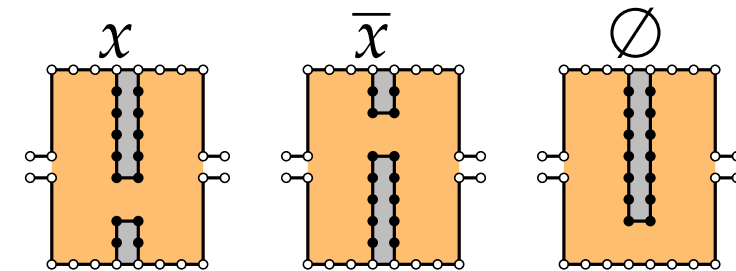
Example:

$$C_1 = x_2 \vee \overline{x_4}$$

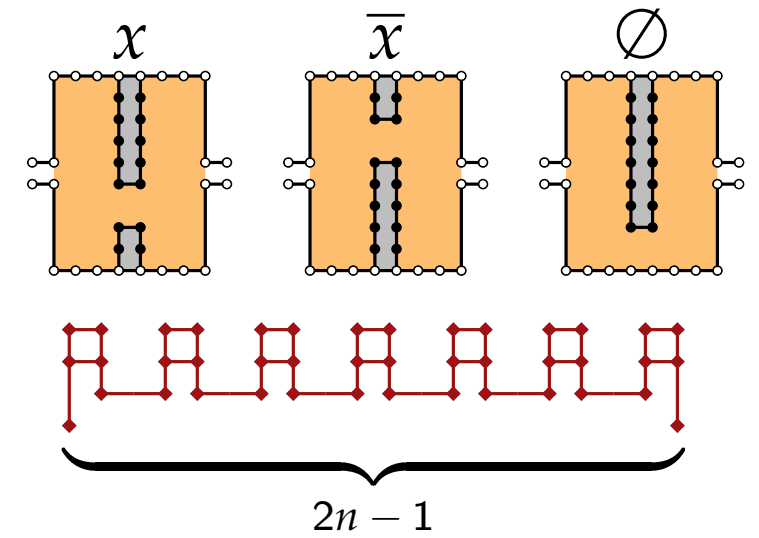
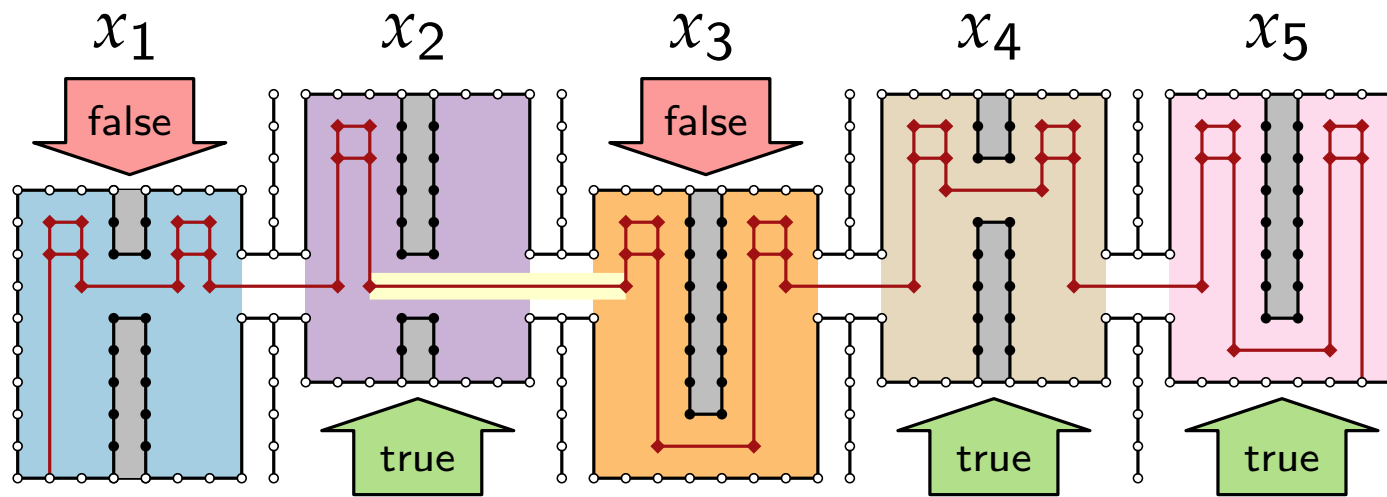
$$C_2 = x_1 \vee x_2 \vee \overline{x_3}$$

$$C_3 = x_5$$

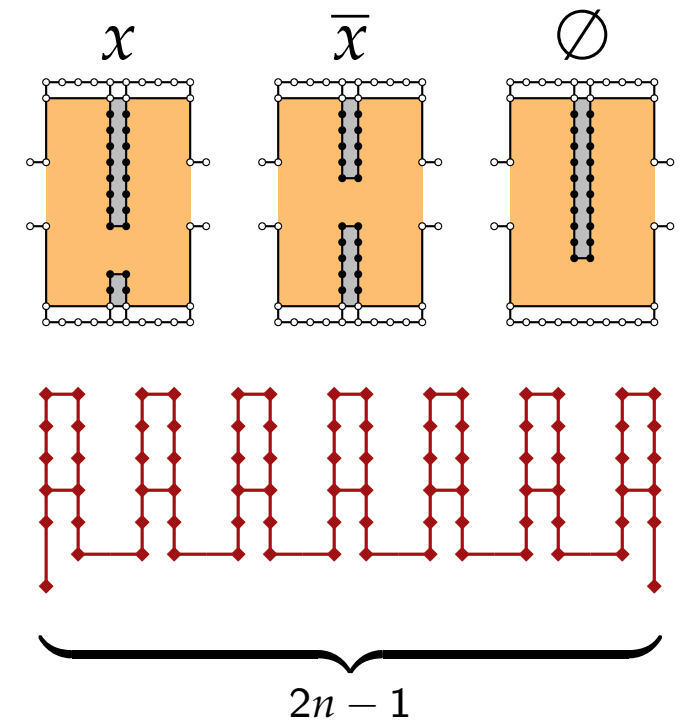
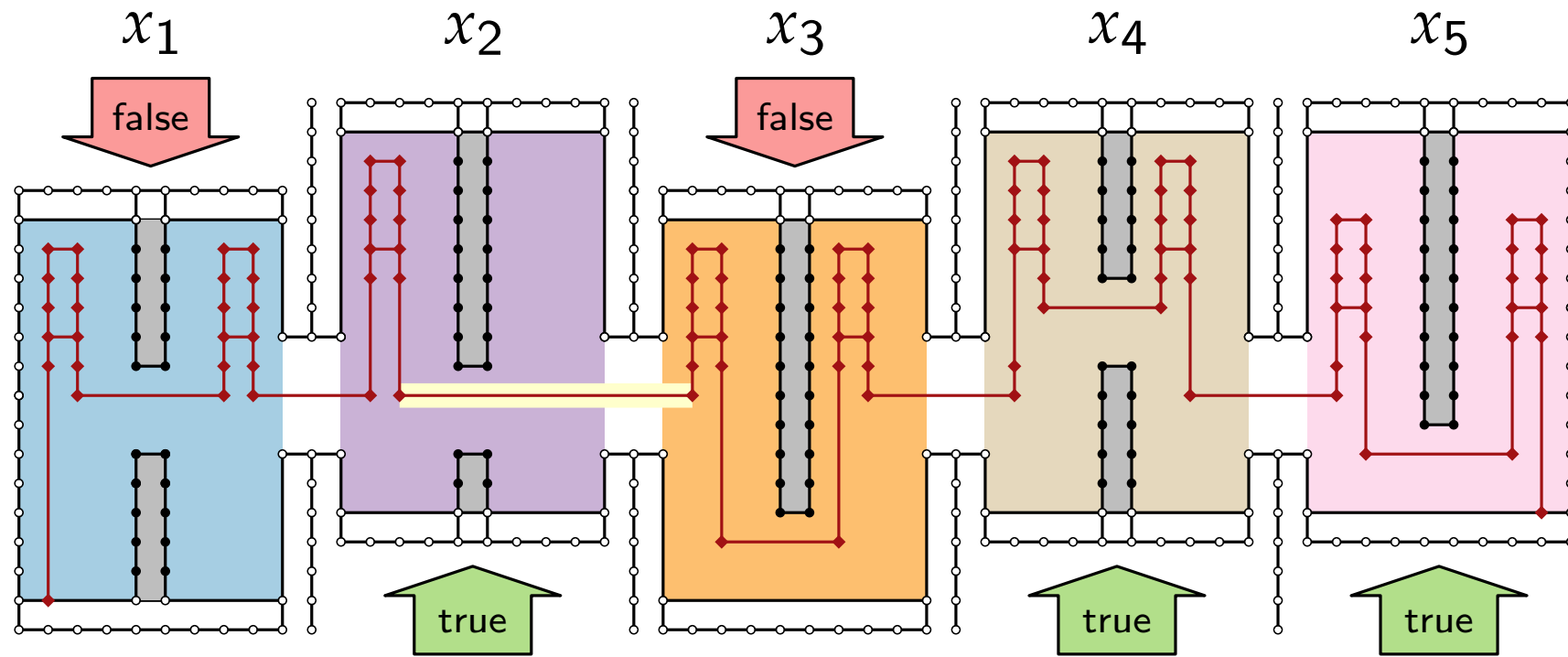
$$C_4 = x_4 \vee \overline{x_5}$$



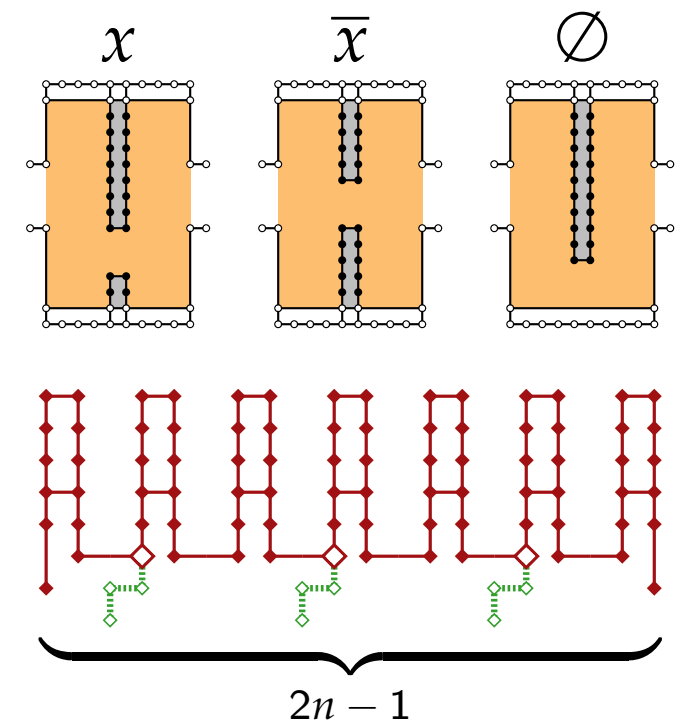
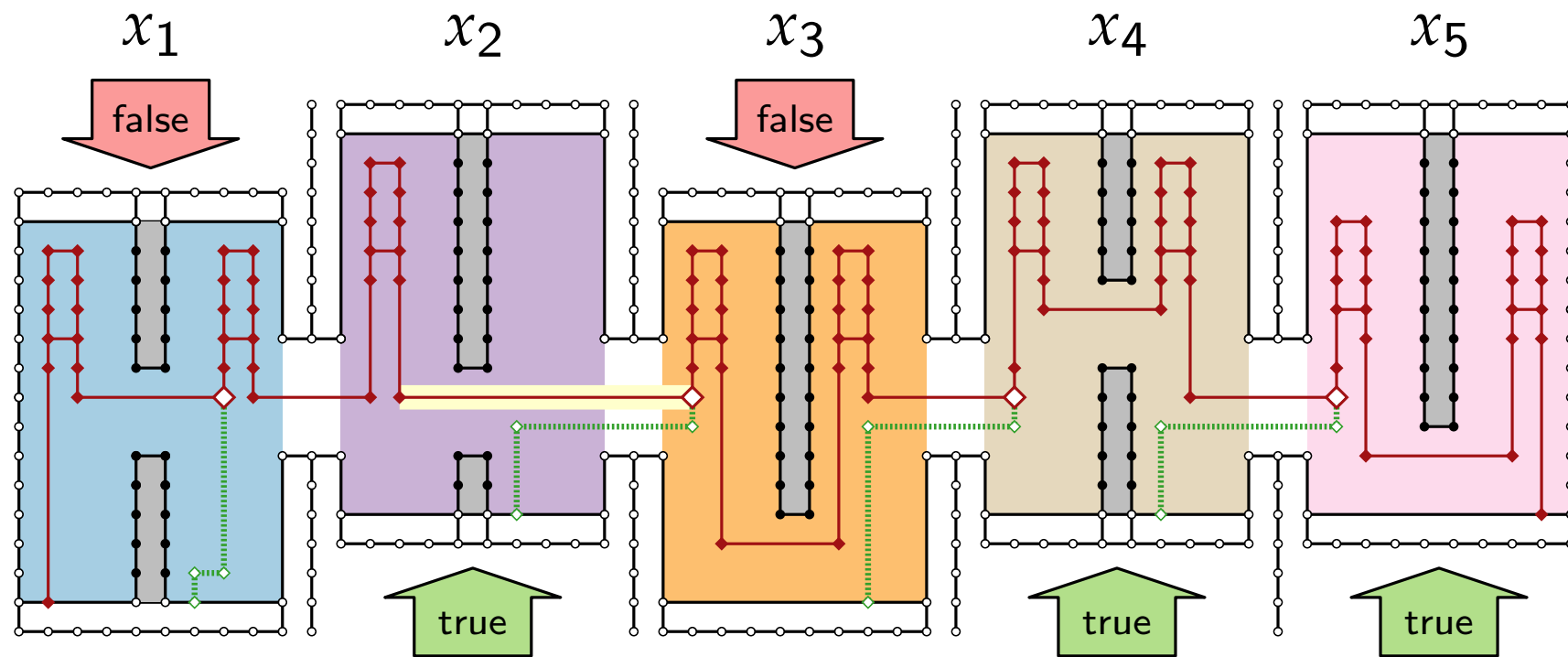
Adjusting Clause Gadgets



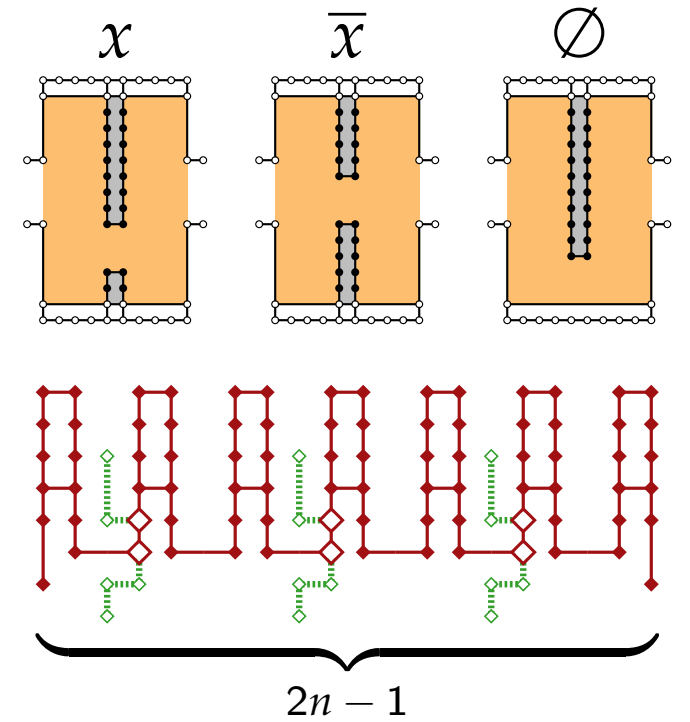
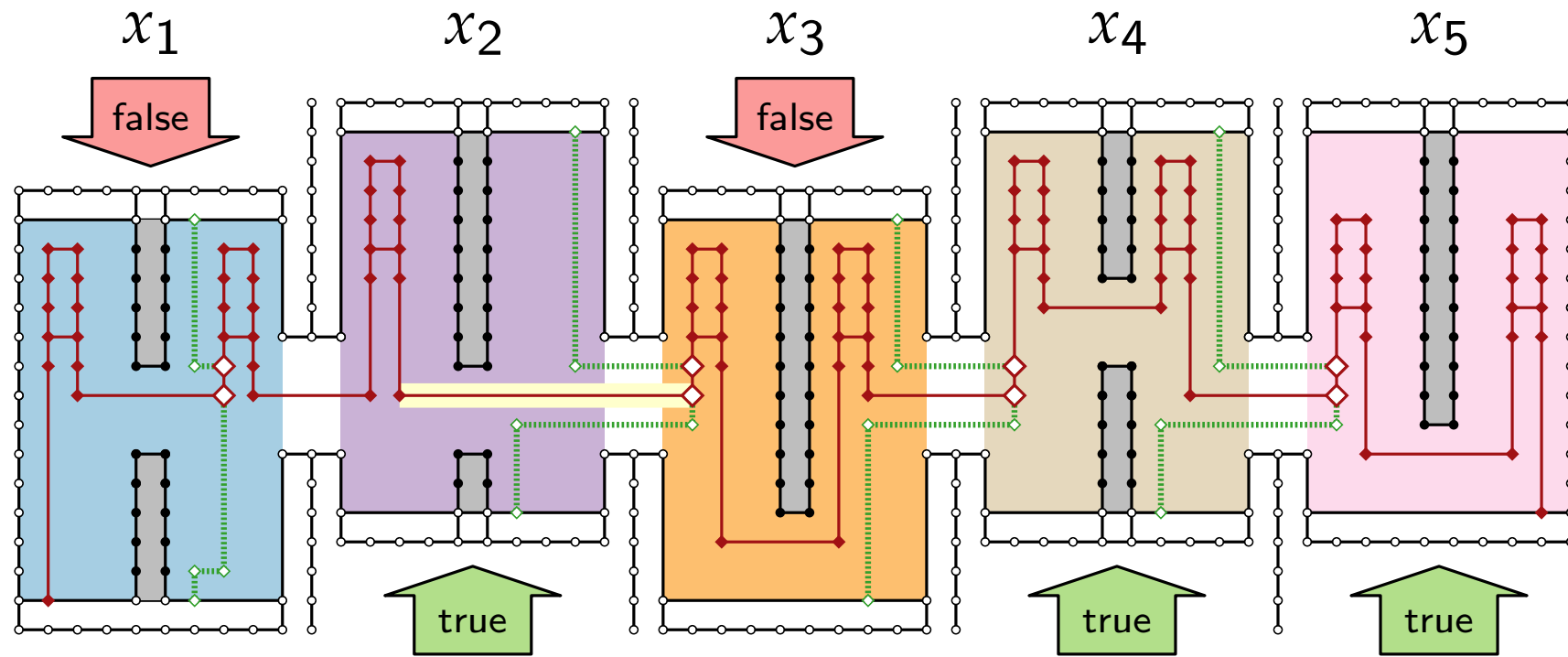
Adjusting Clause Gadgets



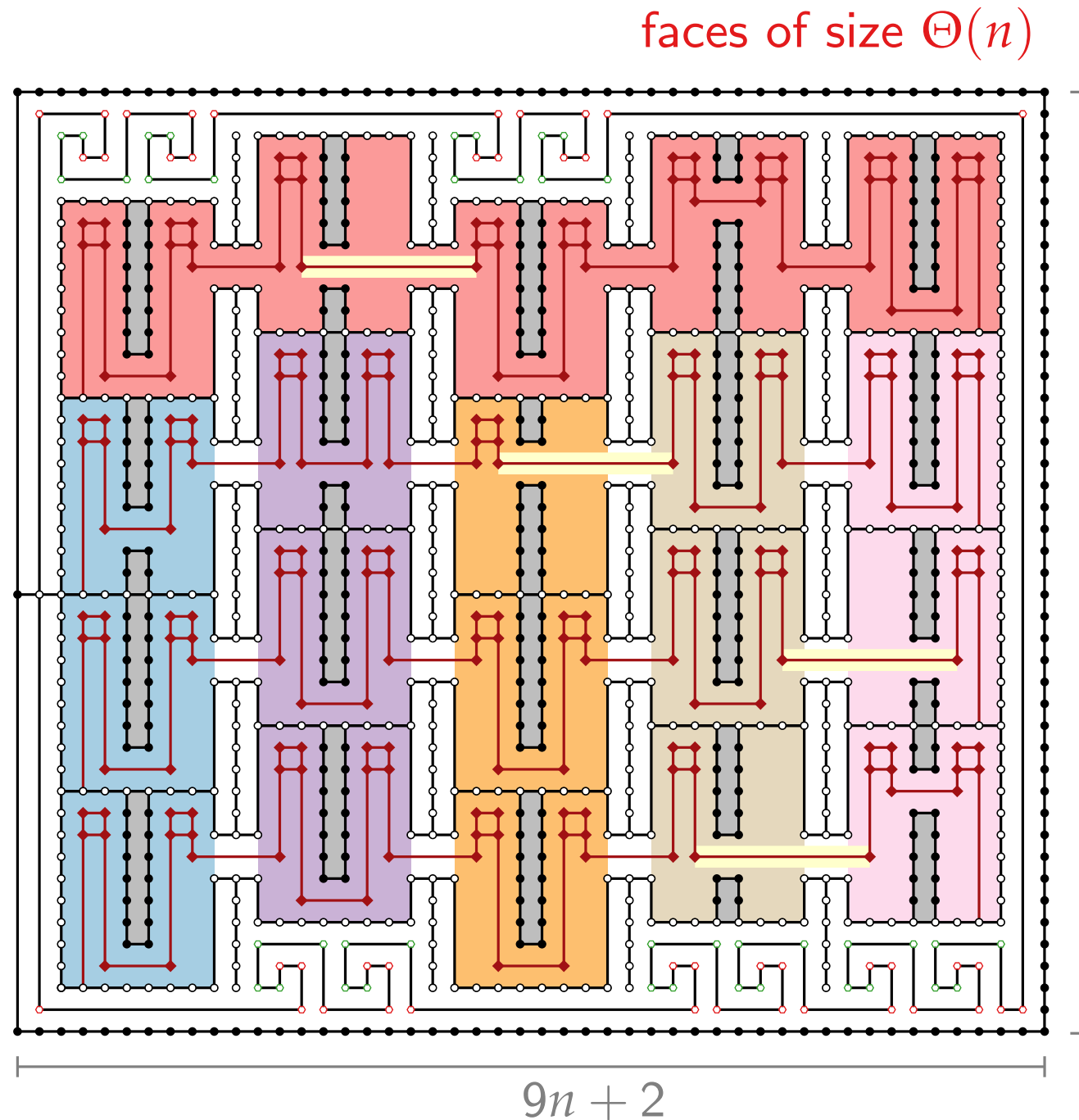
Adjusting Clause Gadgets



Adjusting Clause Gadgets



Full Reduction



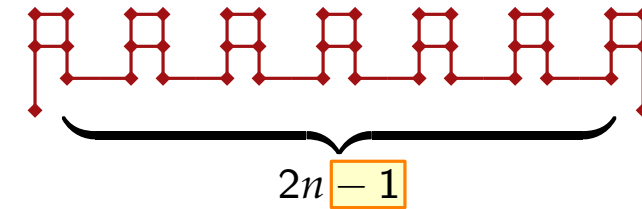
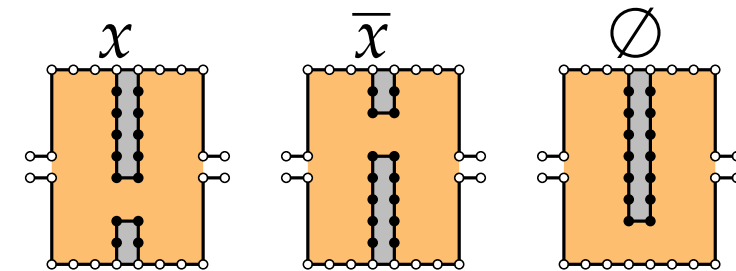
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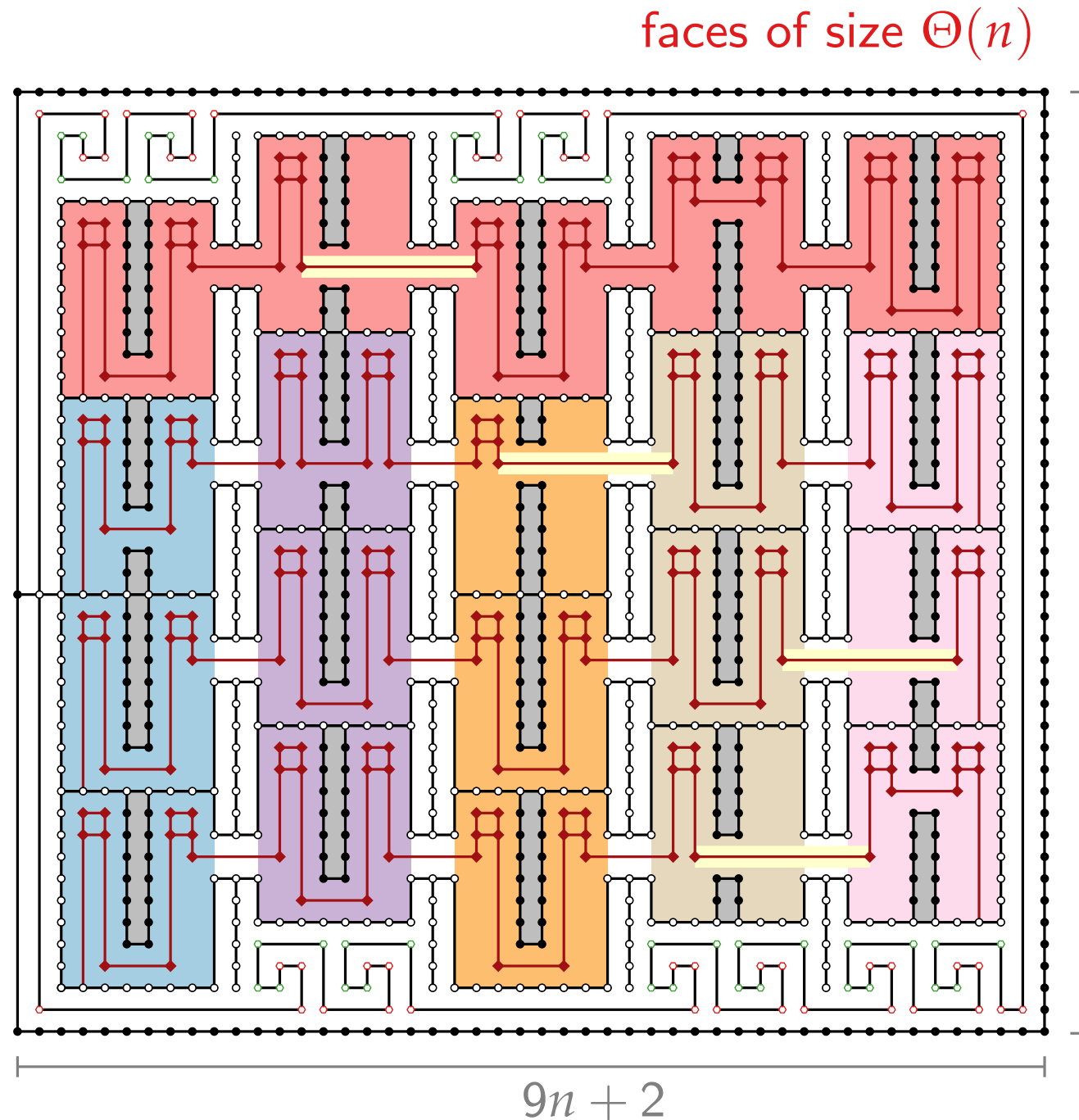
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Full Reduction



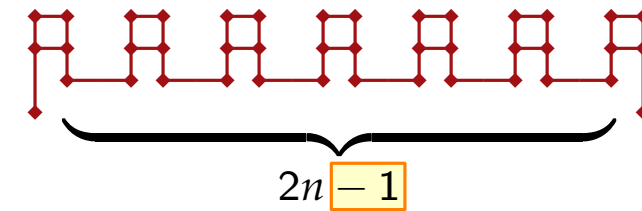
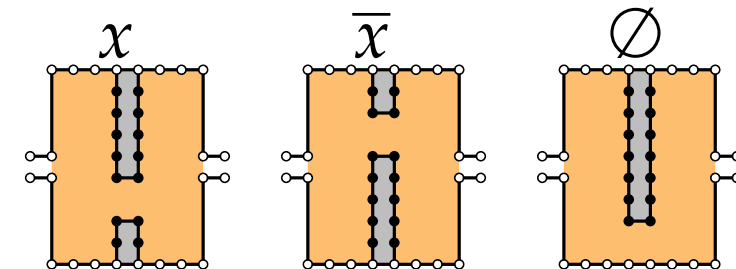
Example:

$$C_1 = x_2 \vee \overline{x_4}$$

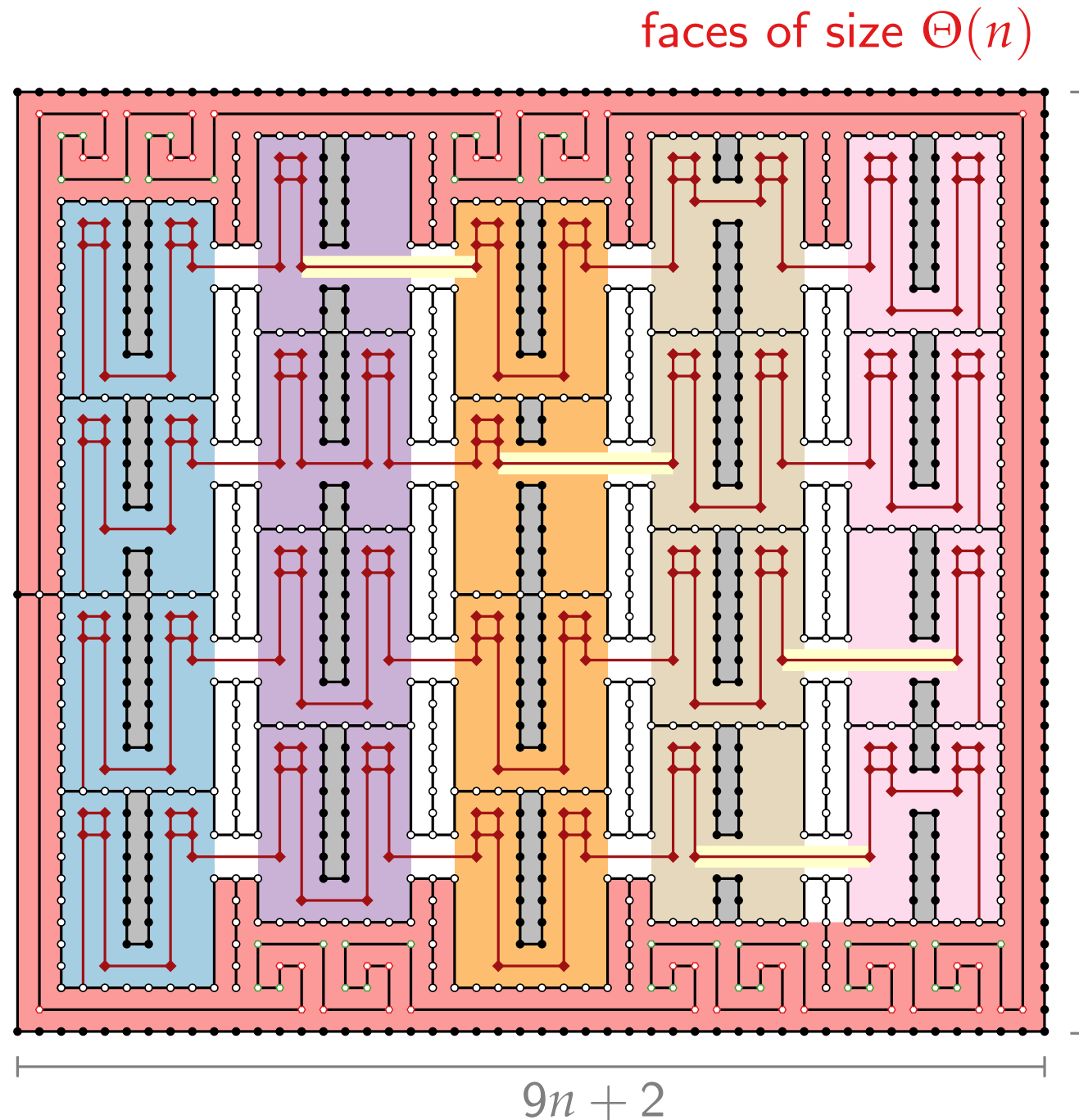
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Full Reduction



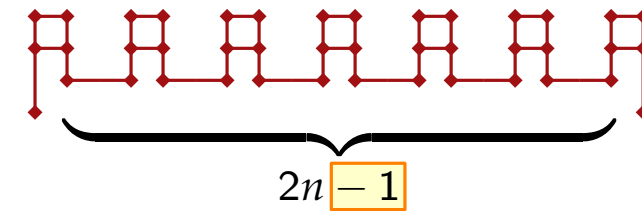
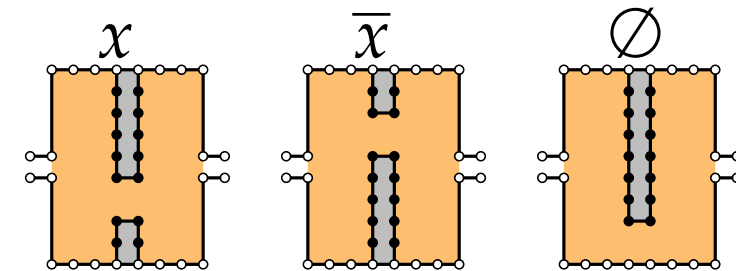
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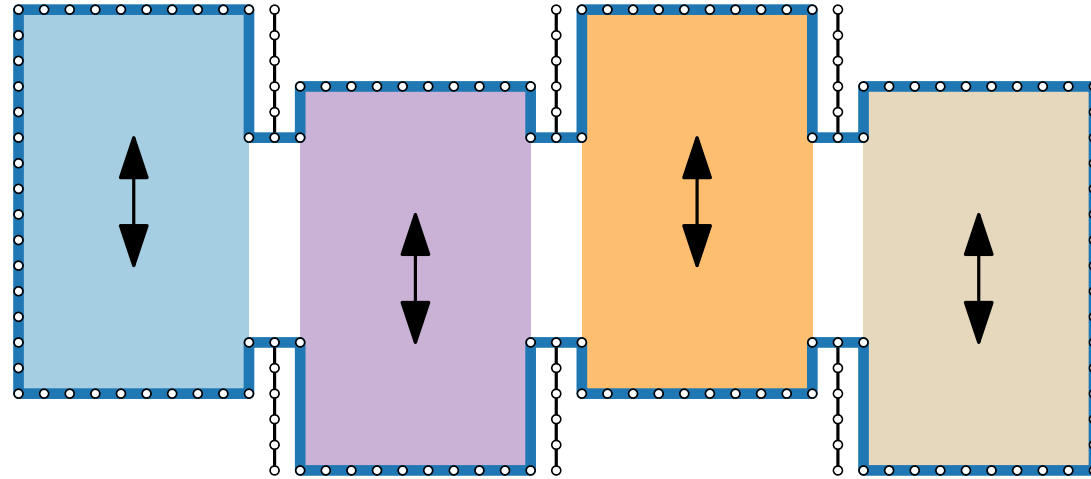
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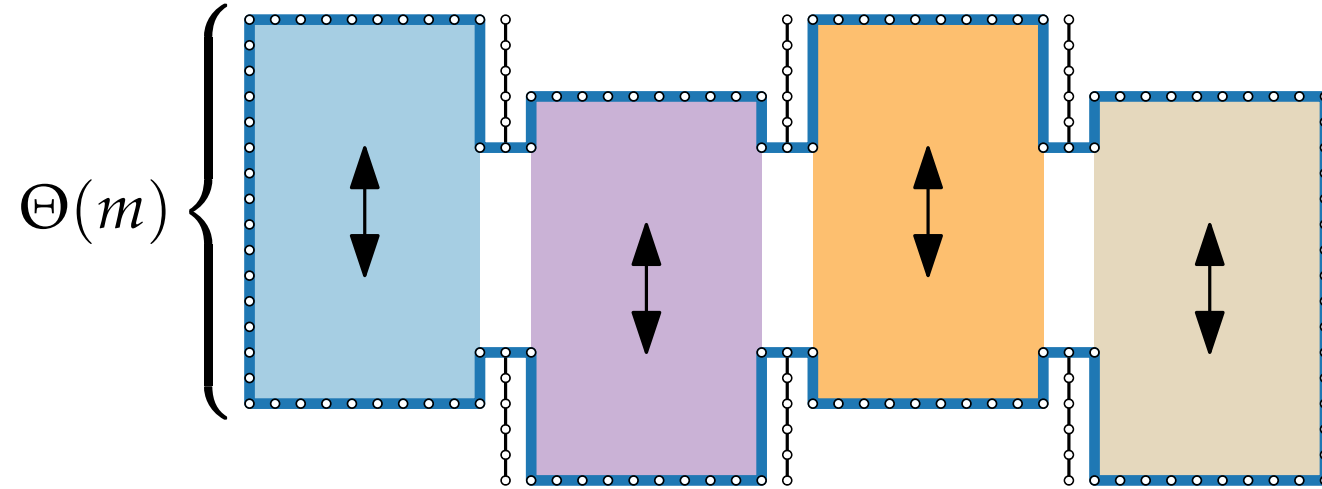
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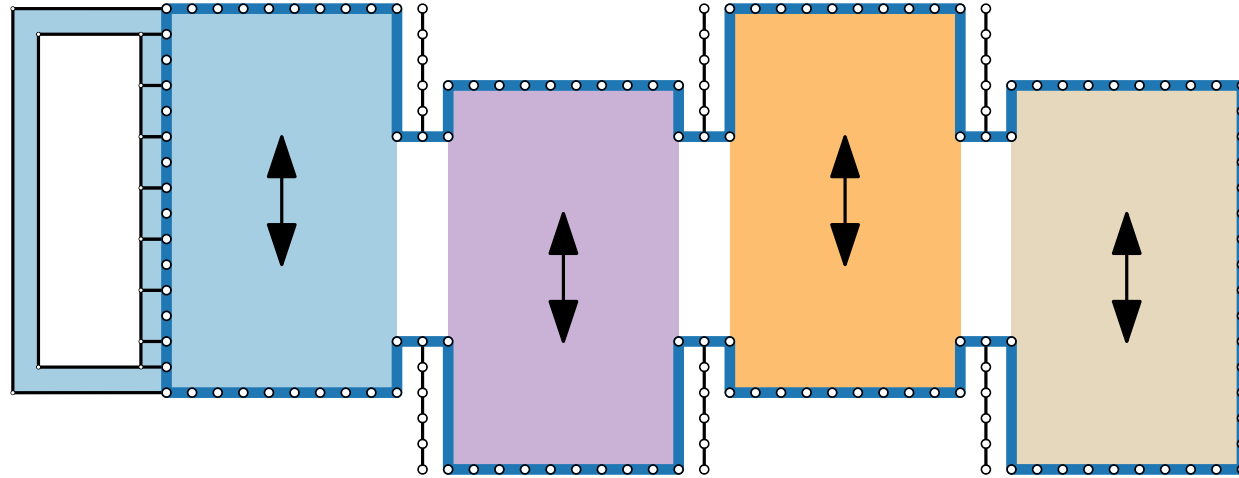
Adjusting Boundary and Belt



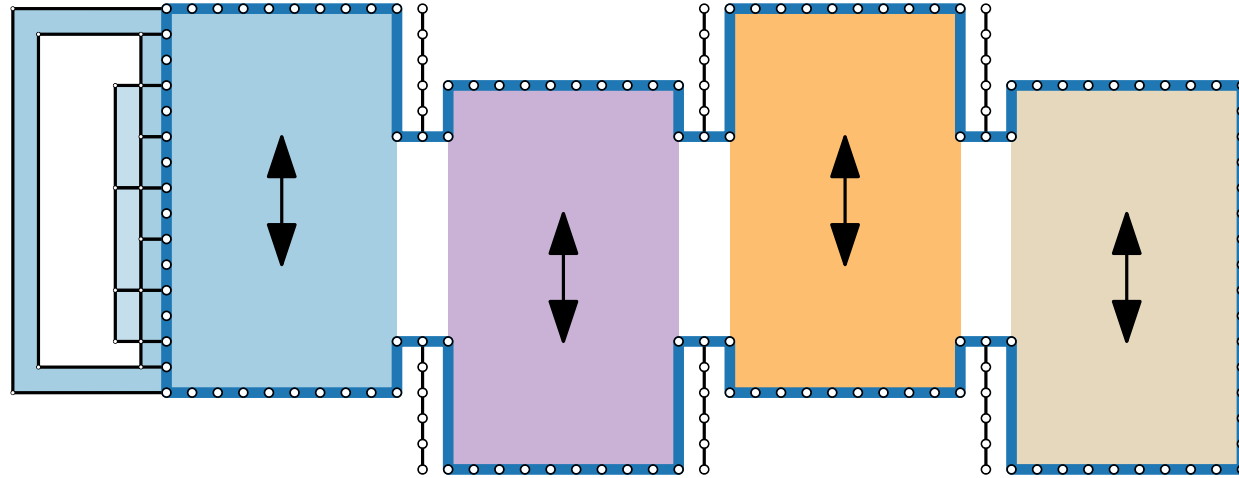
Adjusting Boundary and Belt



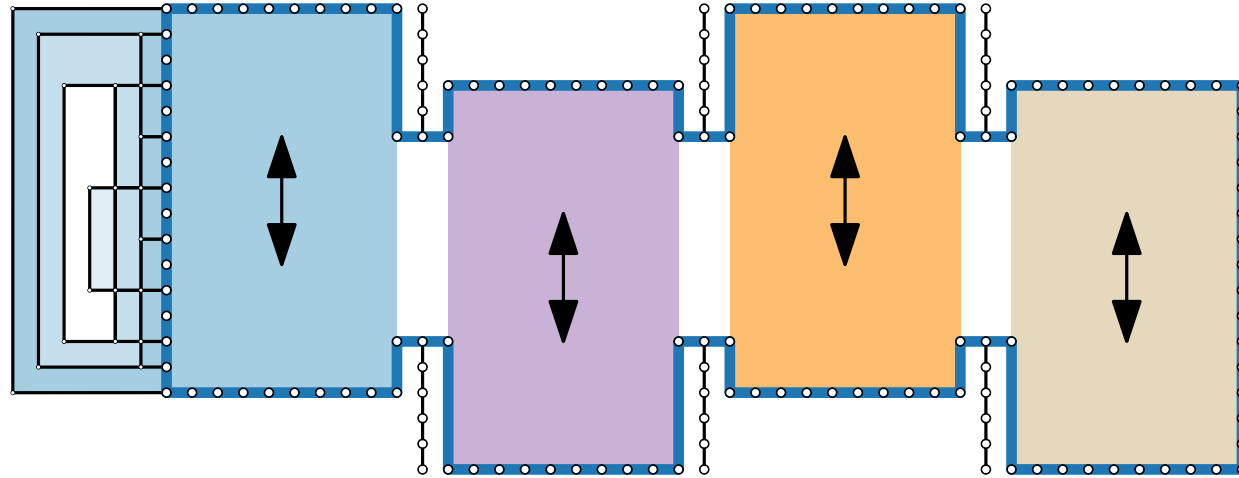
Adjusting Boundary and Belt



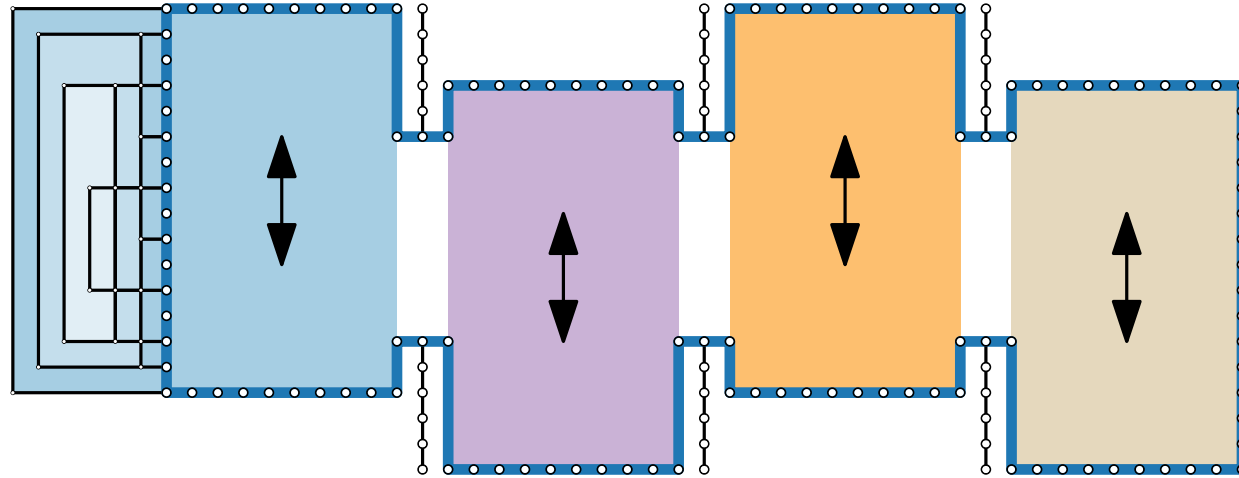
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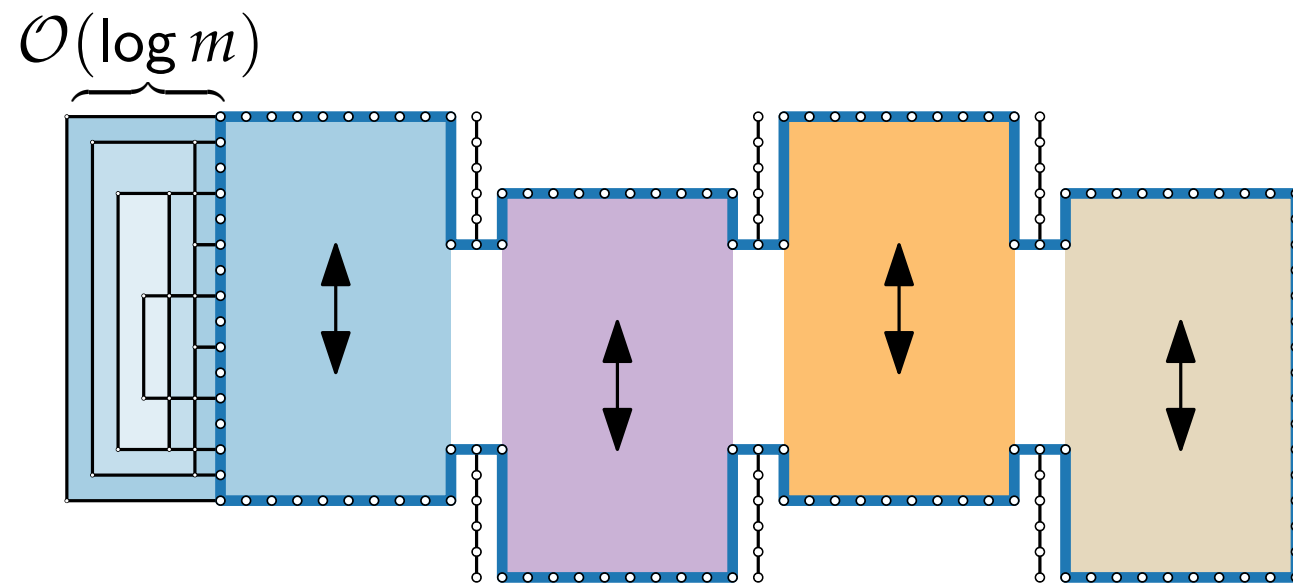
Adjusting Boundary and Belt



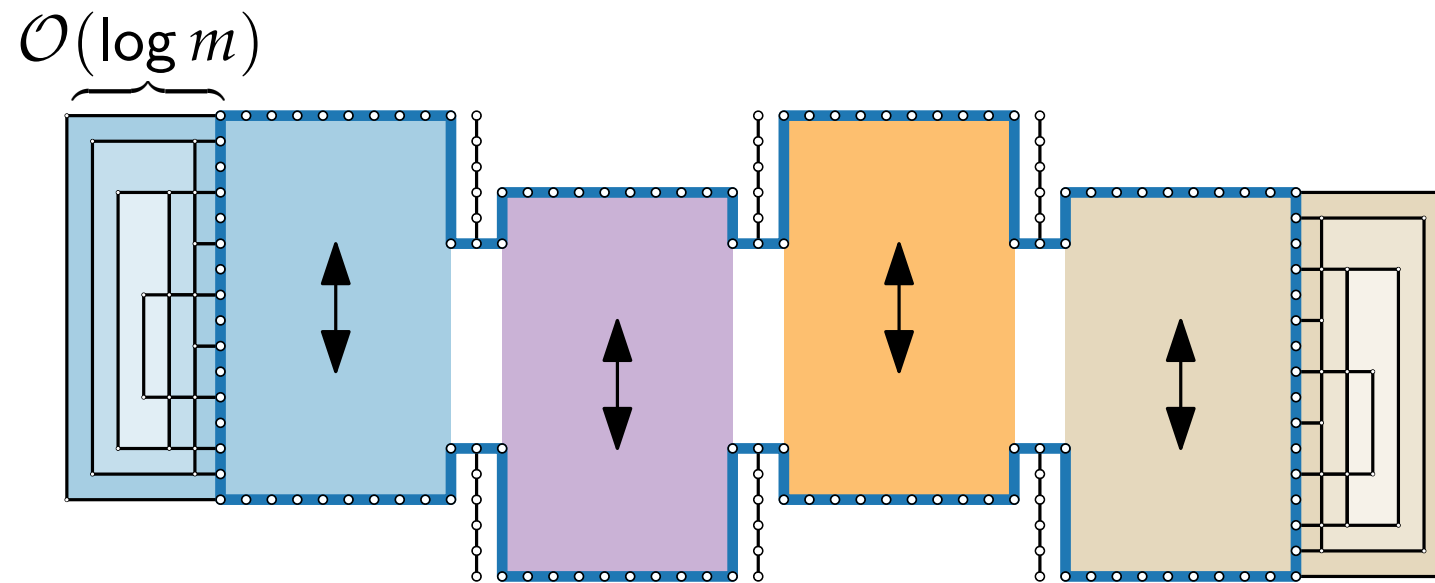
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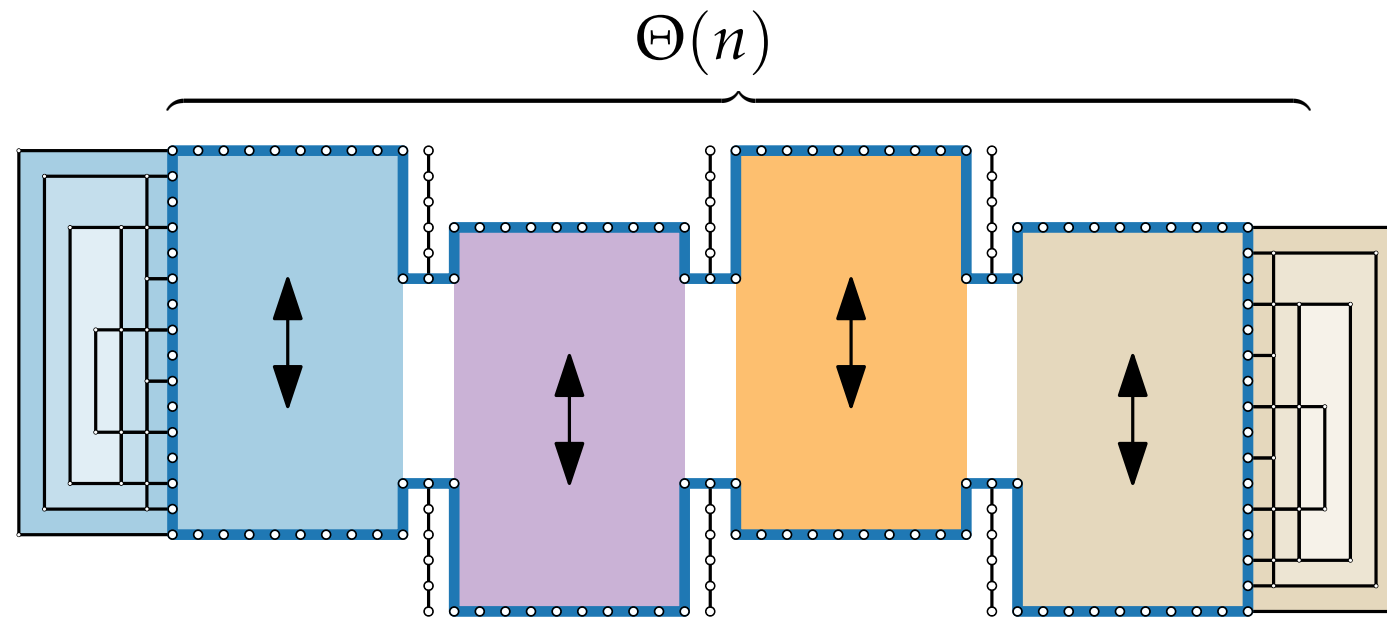
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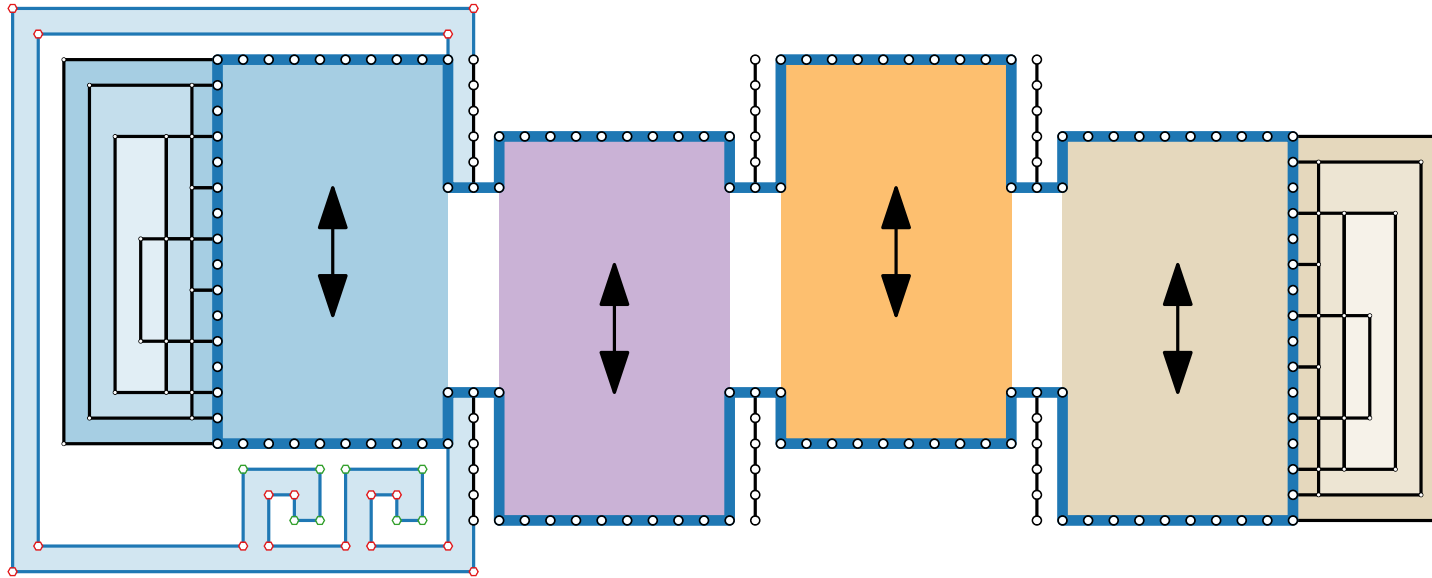
Adjusting Boundary and Belt



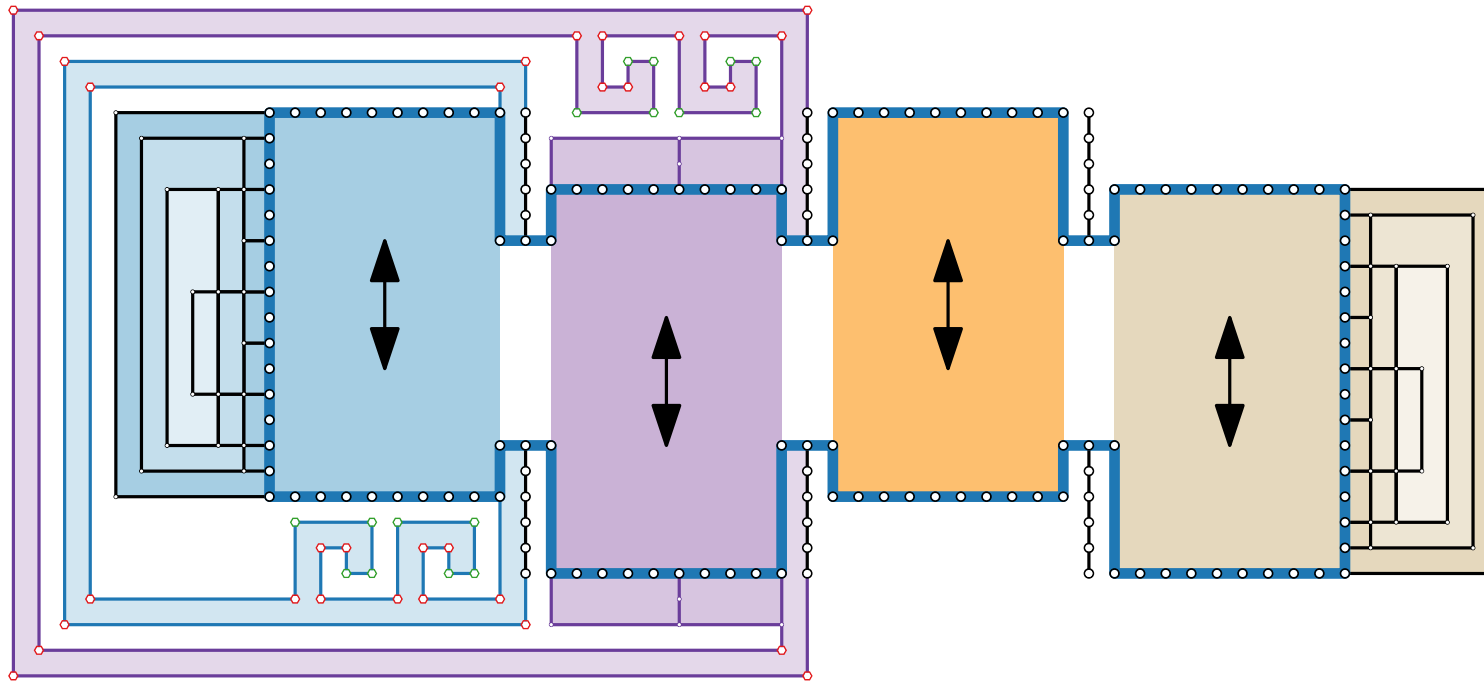
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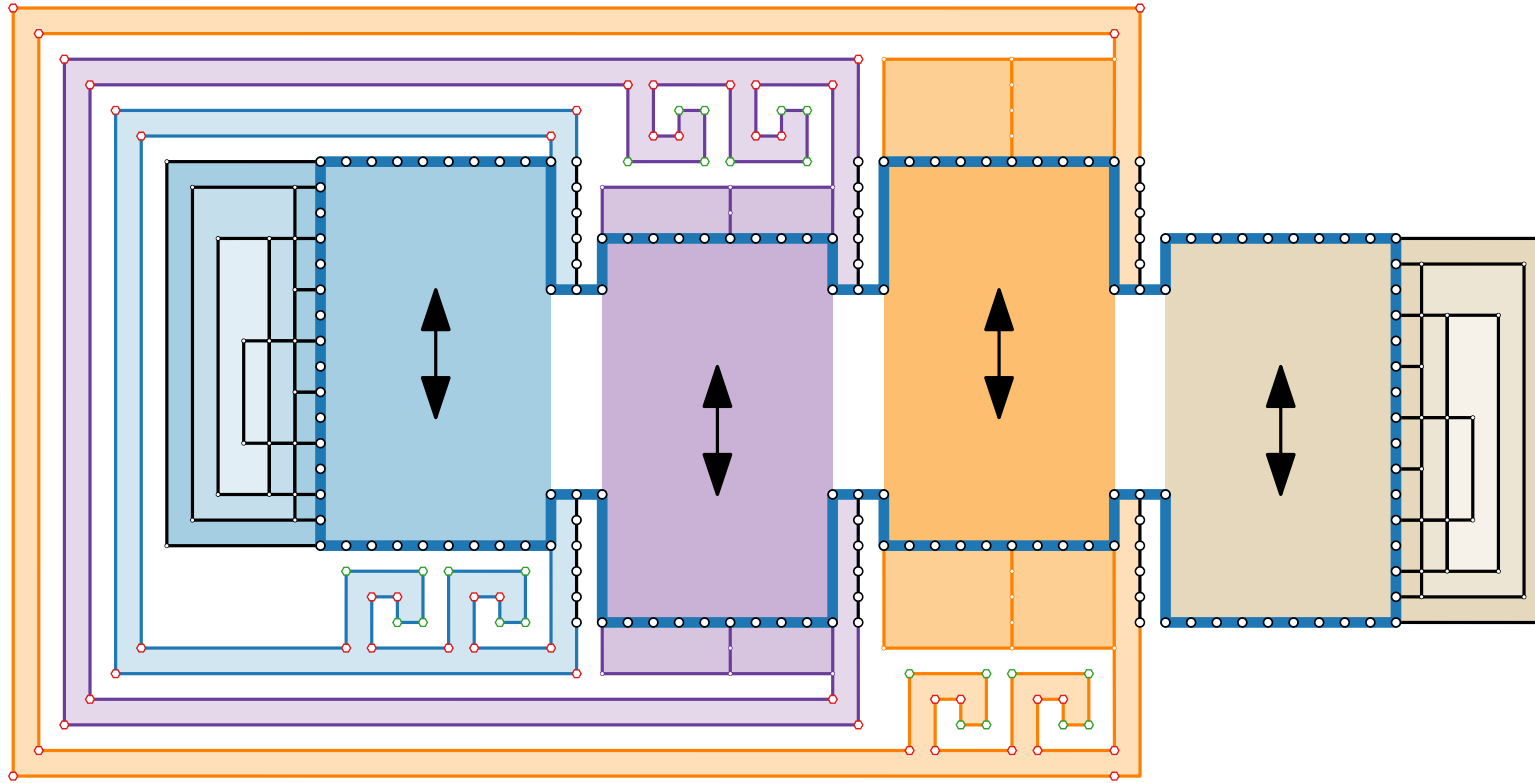
Adjusting Boundary and Belt



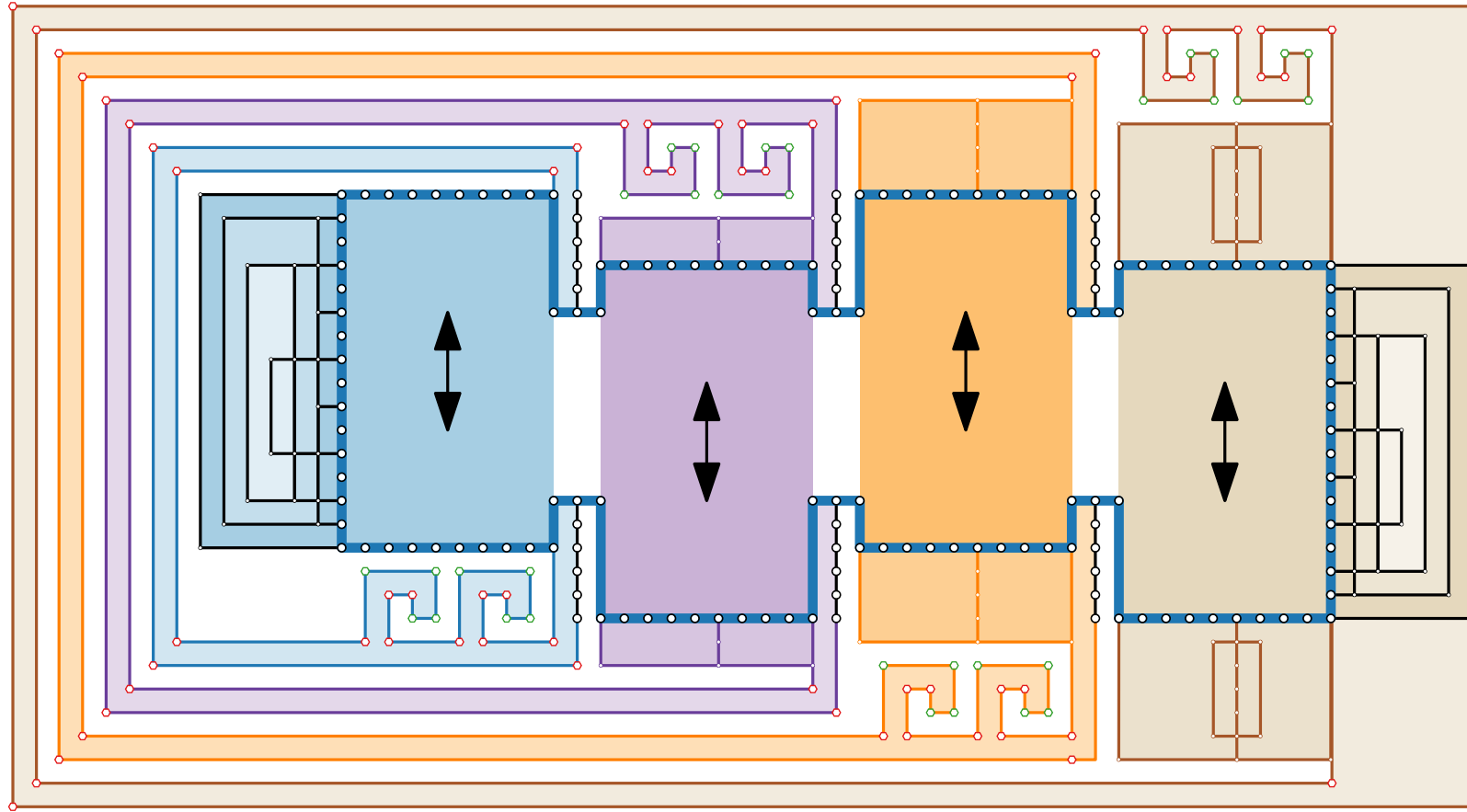
Adjusting Boundary and Belt



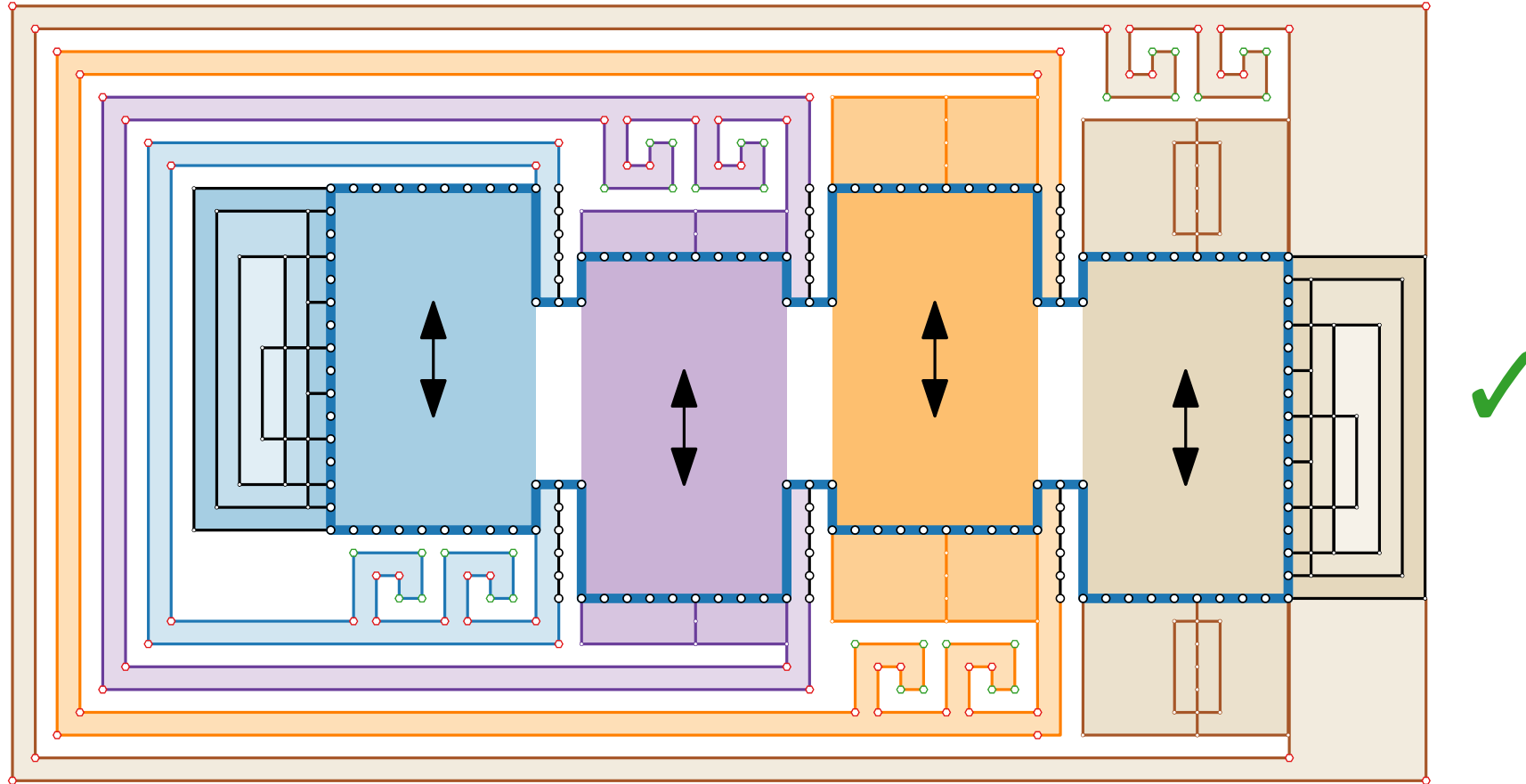
Adjusting Boundary and Belt



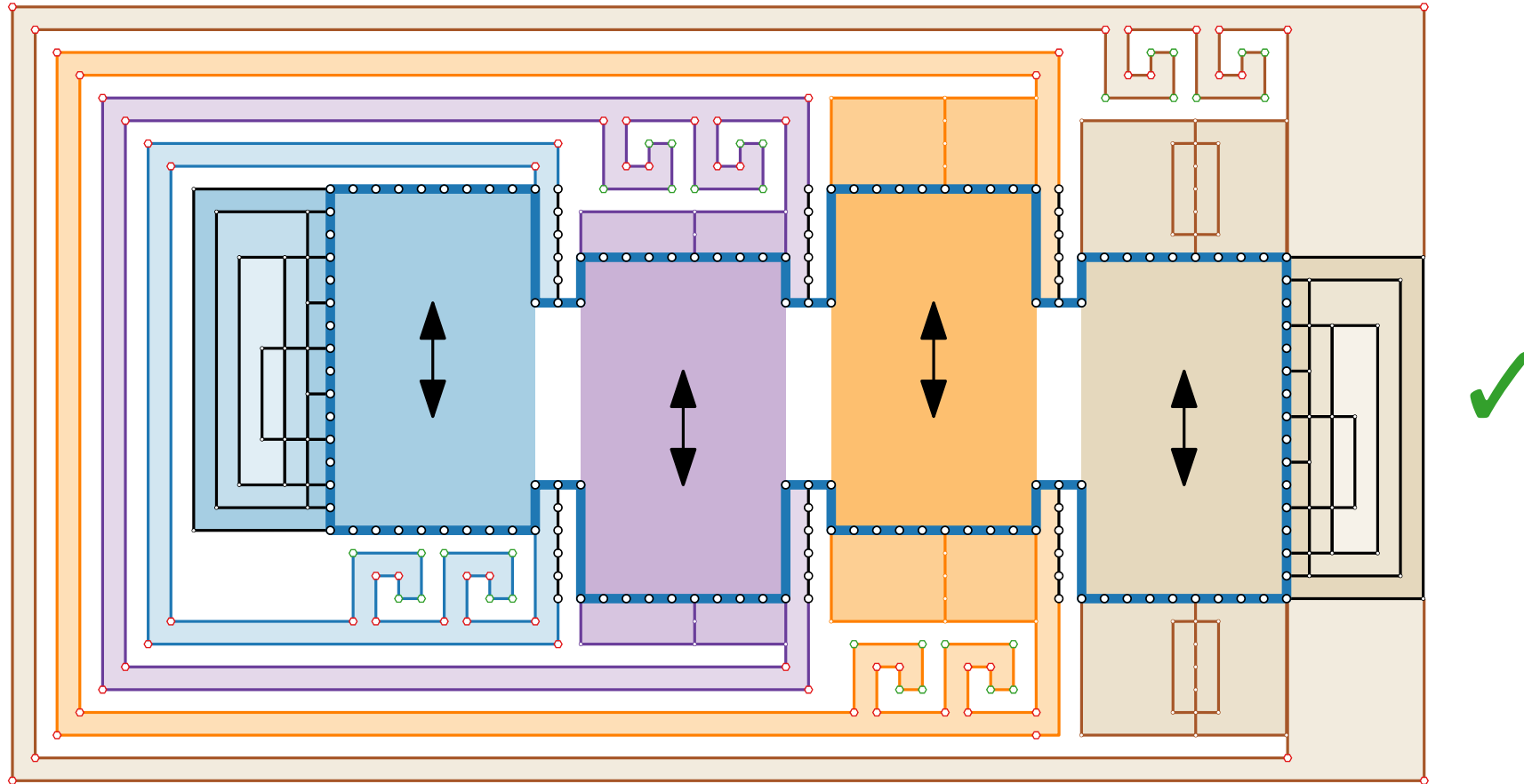
Adjusting Boundary and Belt



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Adjusting Boundary and Belt



Theorem. OC is para-NP-hard when parameterized by the maximum face degree.

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- Can we find a polynomial kernel for OC w.r.t. the number of kitty corners, or at least w.r.t. the number of kitty corners *plus the number of faces*, for general graphs?

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- If we parametrize by the number of *pairs* of kitty corners, can we achieve substantially better running times?