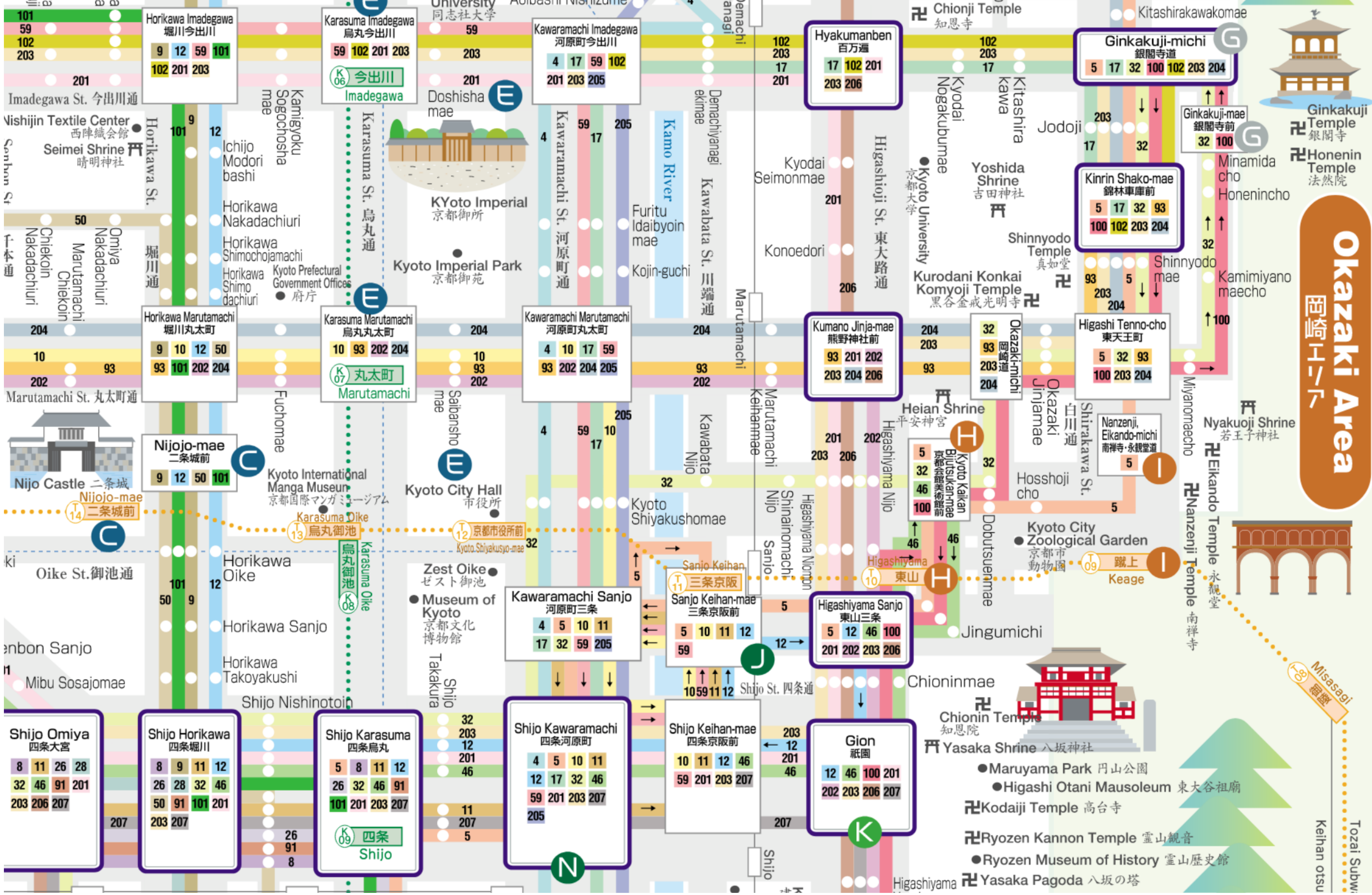
Parameterized Approaches to Orthogonal Compaction

Walter Didimo Siddharth Gupta Philipp Kindermann
Giuseppe Liotta *Alexander Wolff* Meirav Zehavi

SOFSEM 2023

Orthogonal Graph Drawing



Kyoto Transportation Authority (c) 2008

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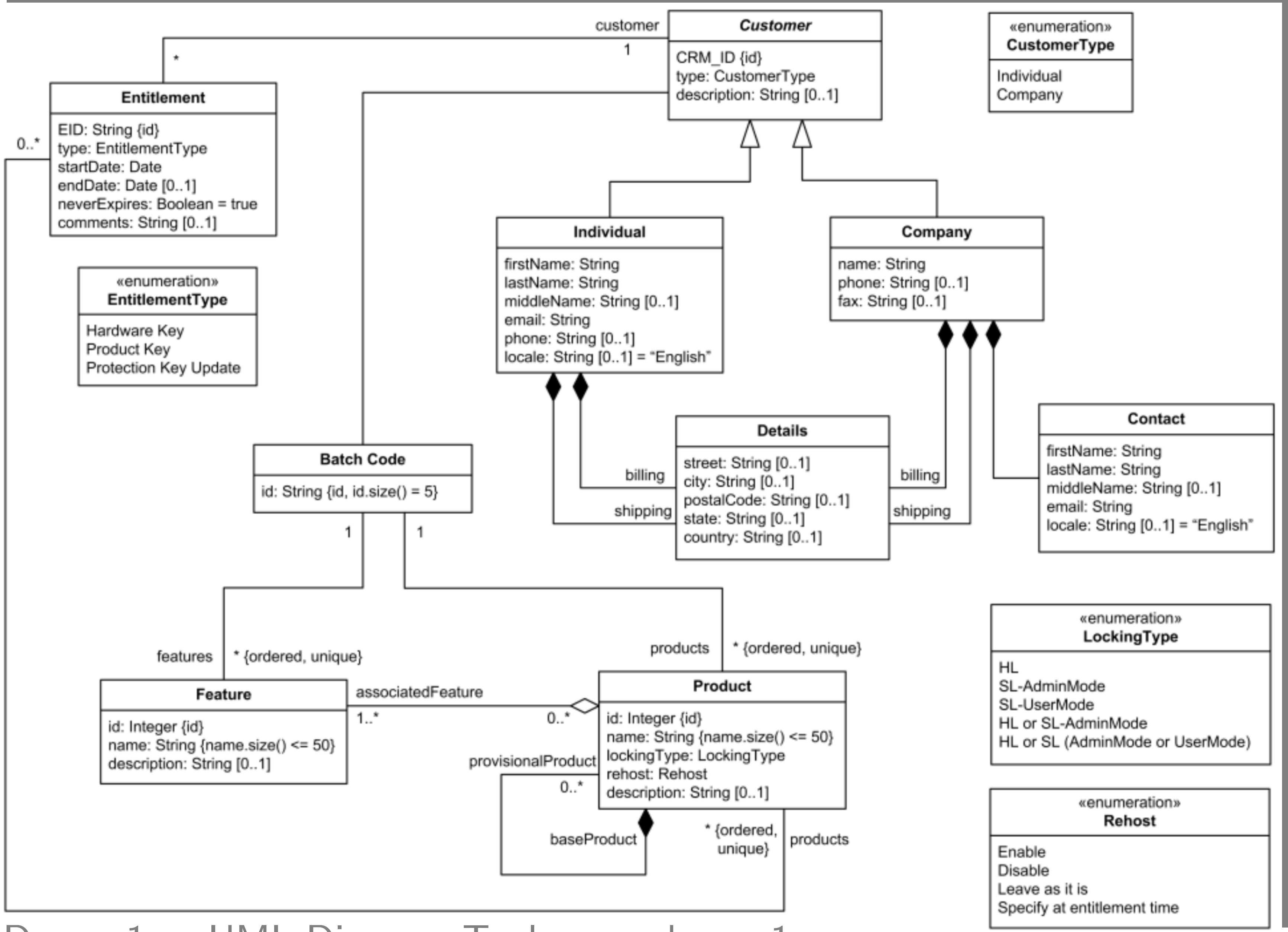
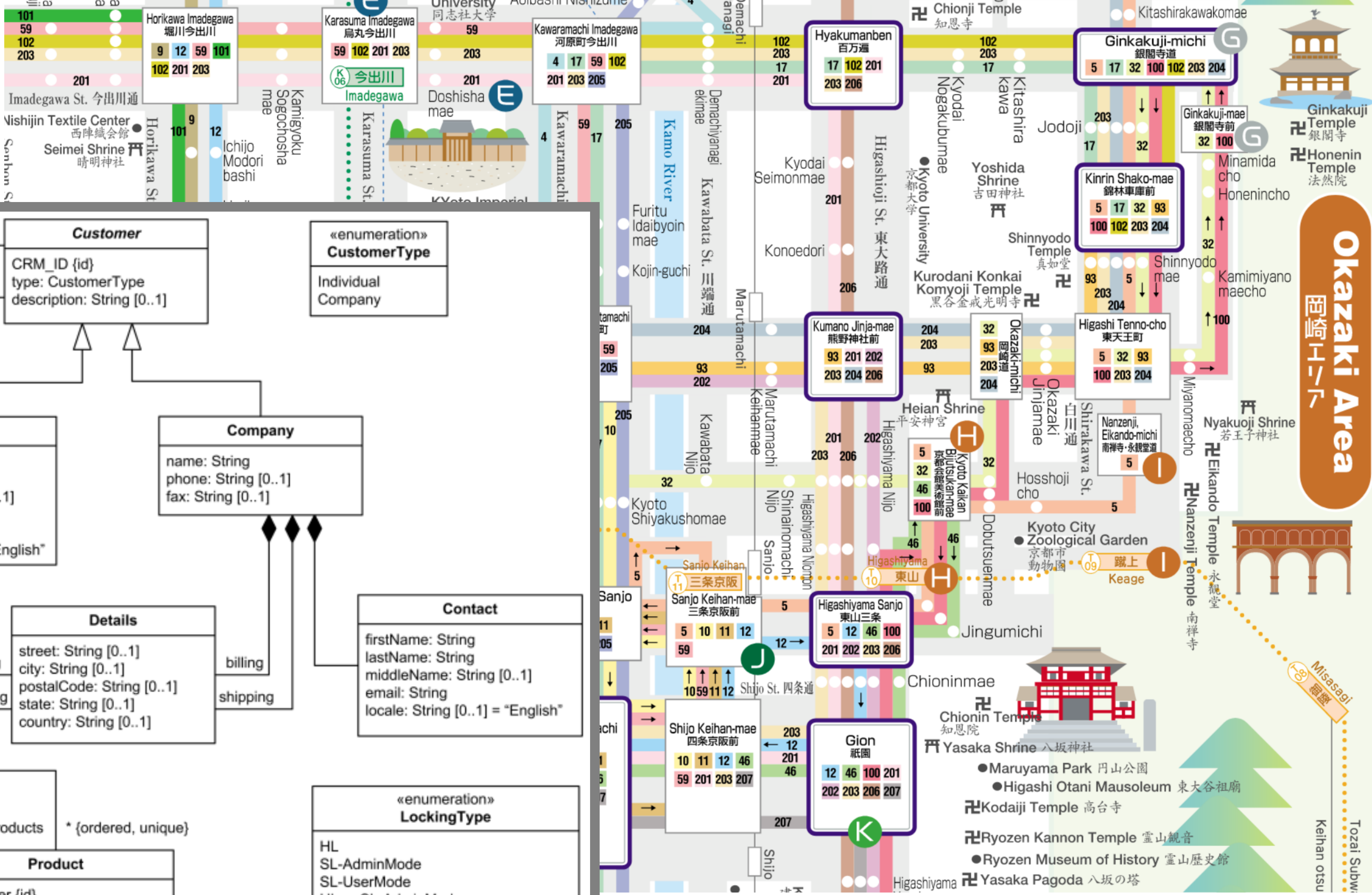


Dragon1 as UML Diagram Tool: www.dragon1.com



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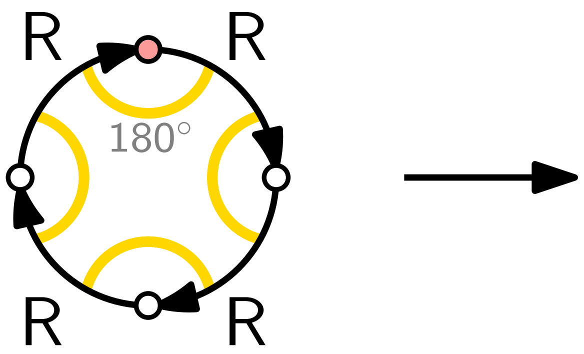
Orthogonal Representation vs. Orthogonal Drawing



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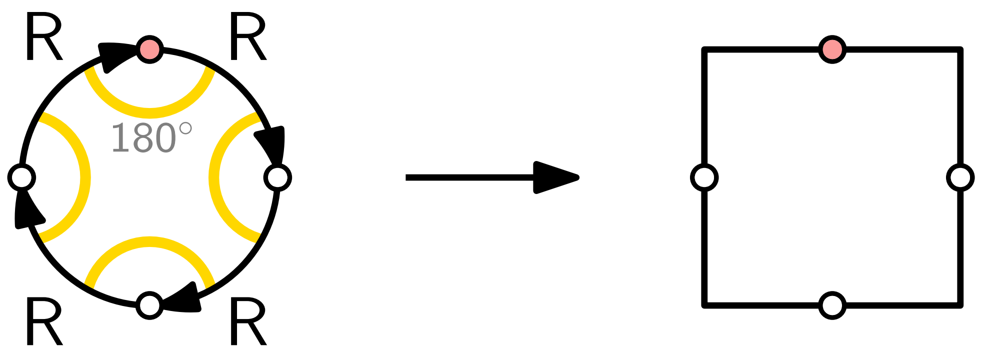

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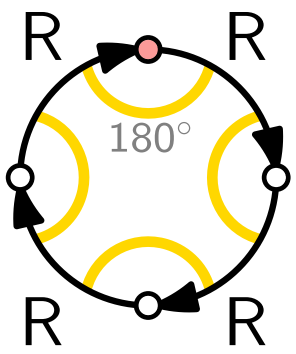

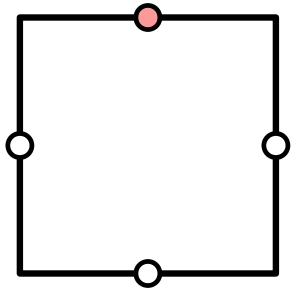

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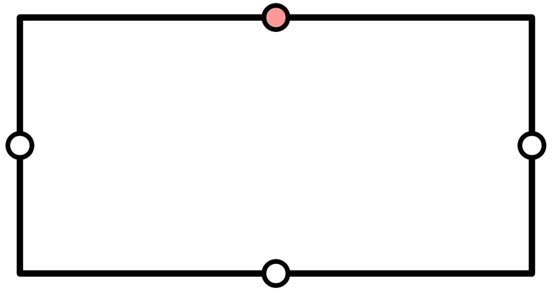

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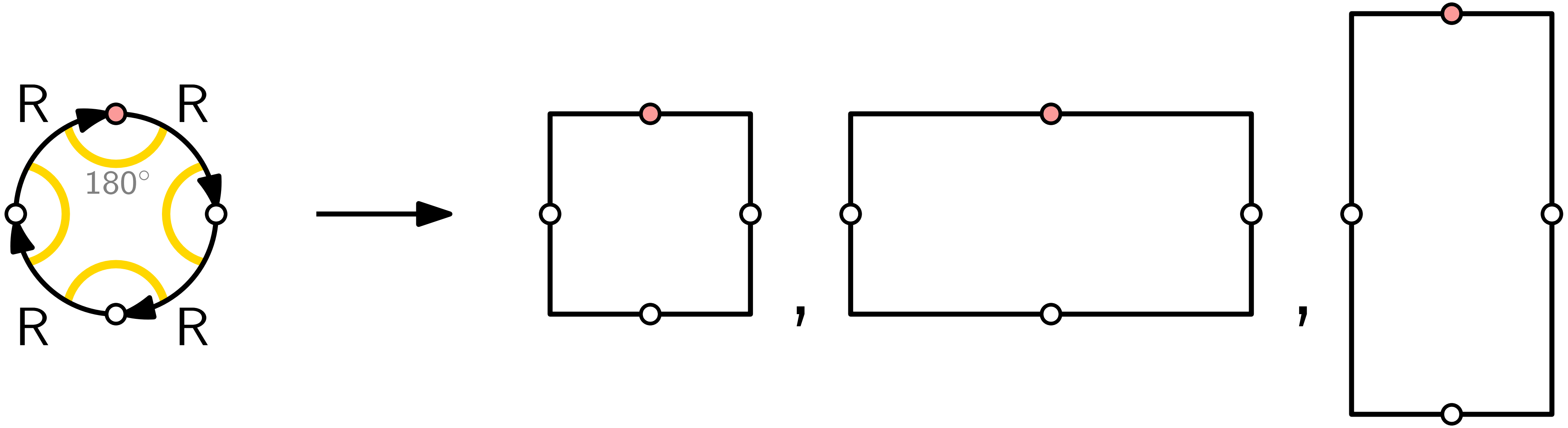

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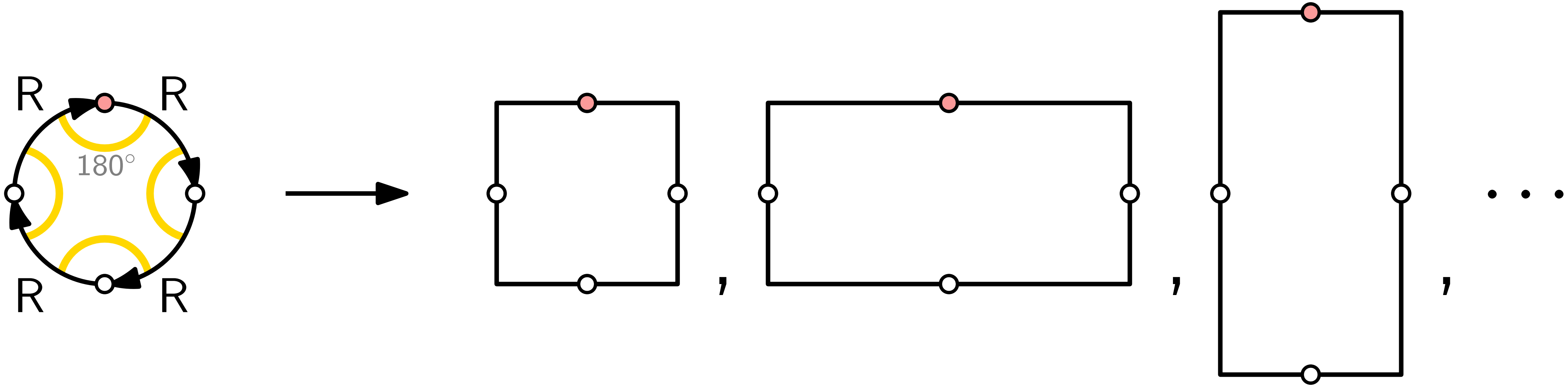

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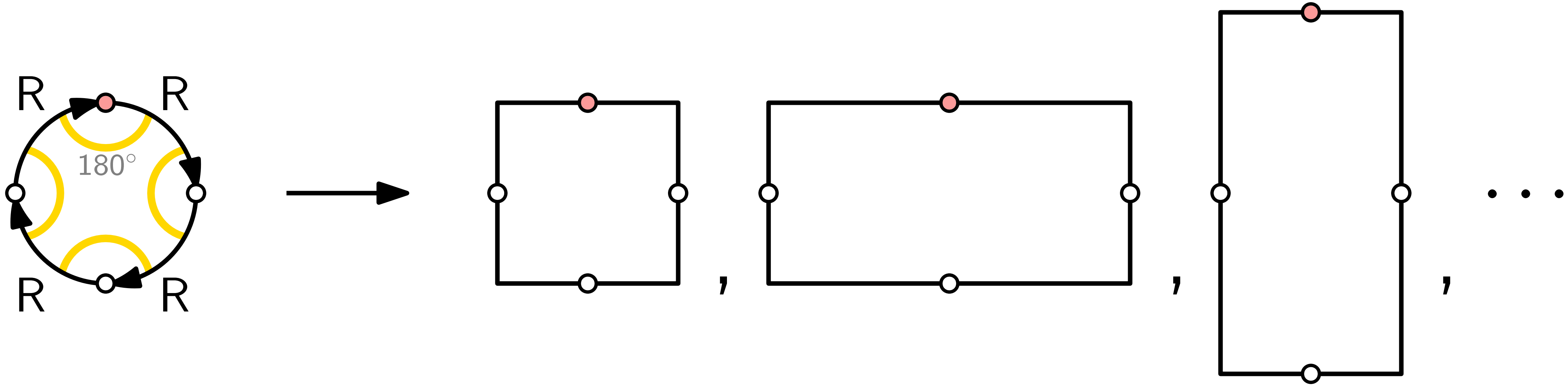

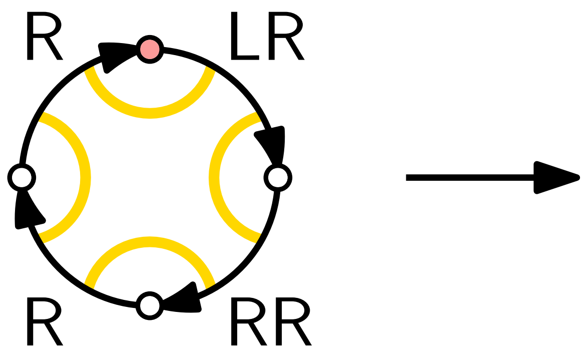

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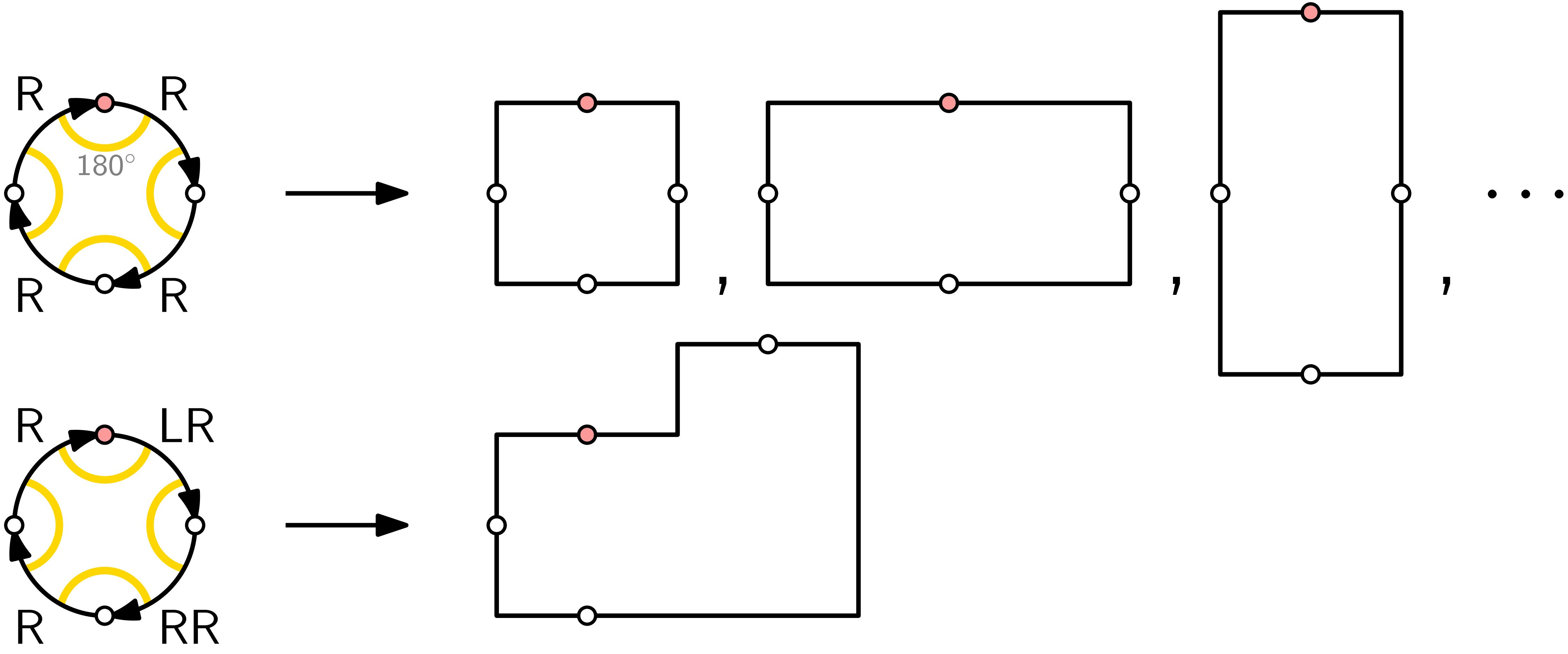

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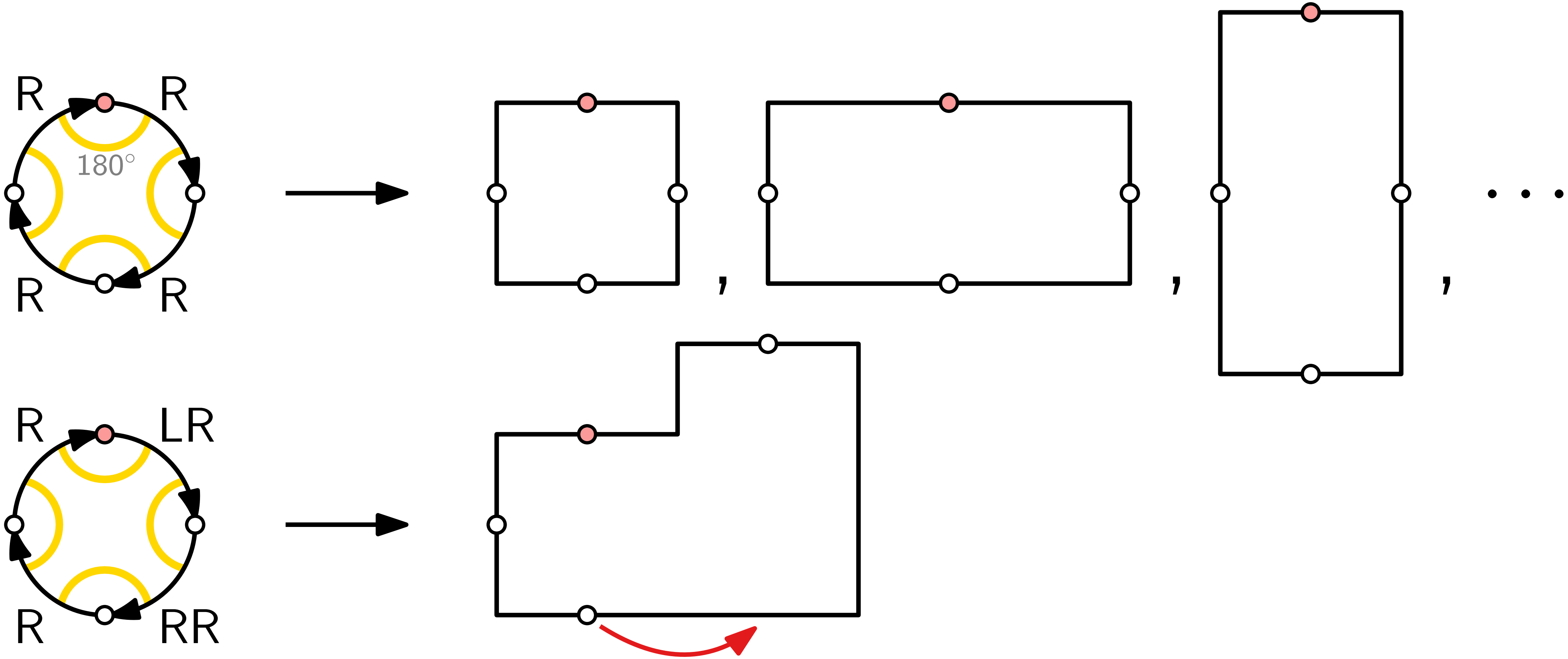

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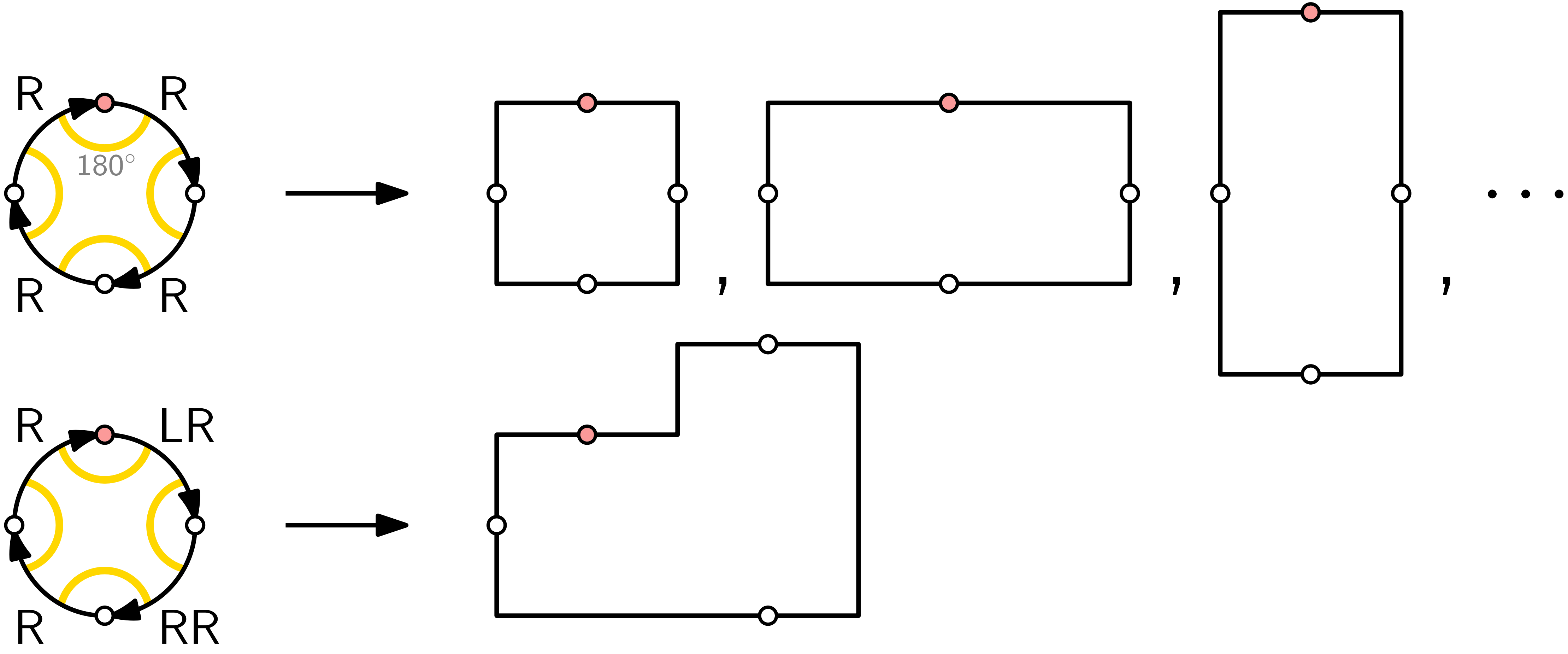

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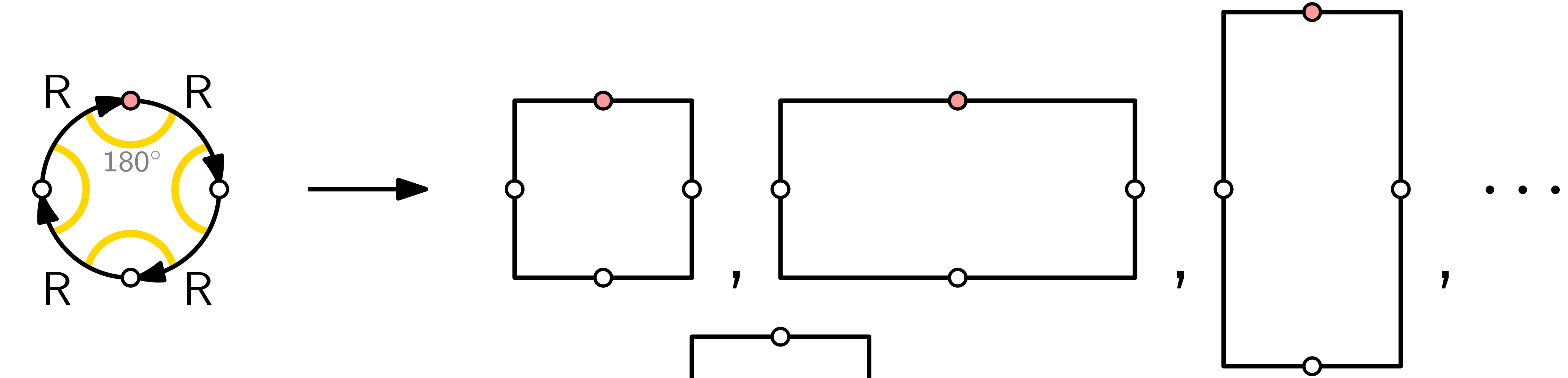

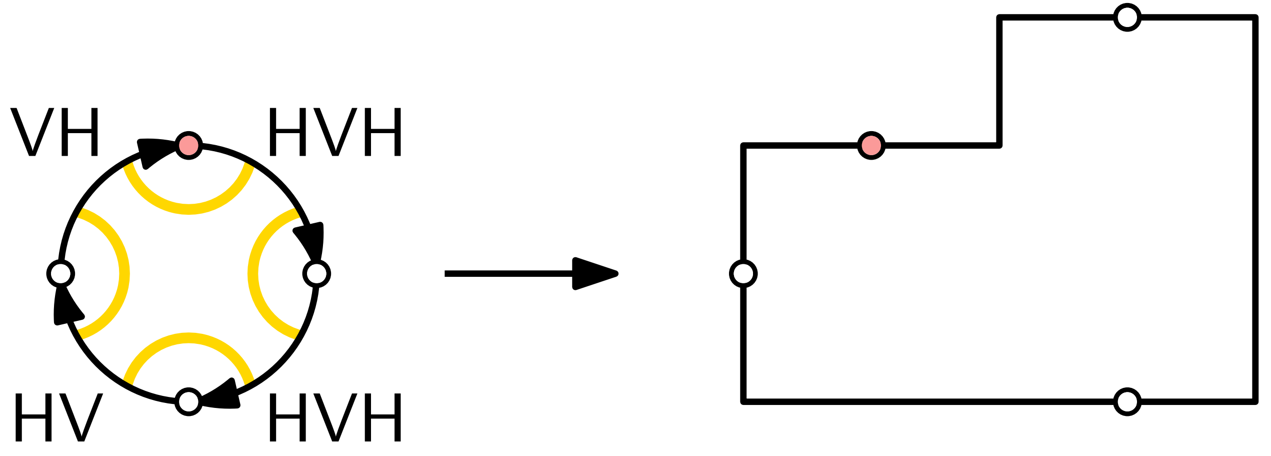

Previous Work

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- Cycles have pathwidth (and treewidth) 2, so cannot expect FPT algorithm for OC w.r.t. these parameters :-(

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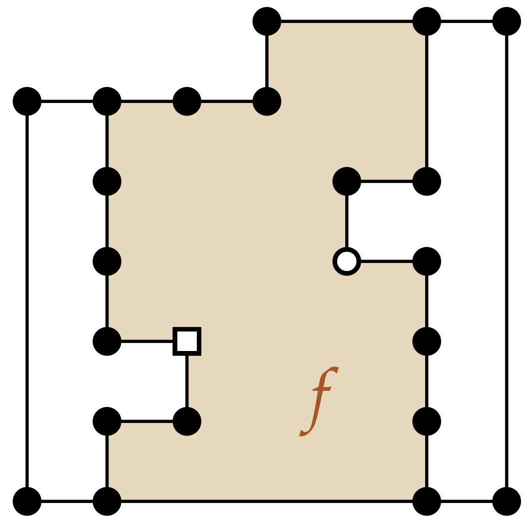

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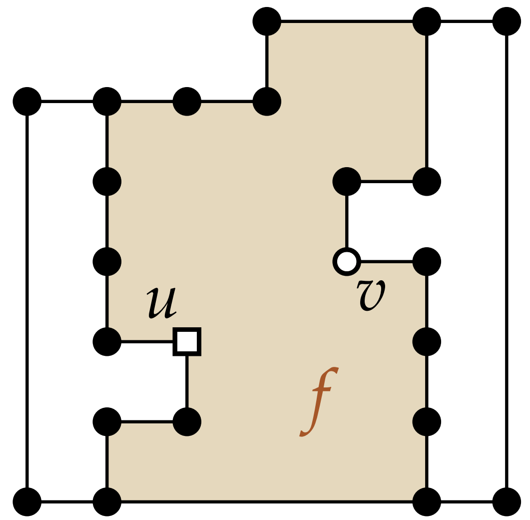

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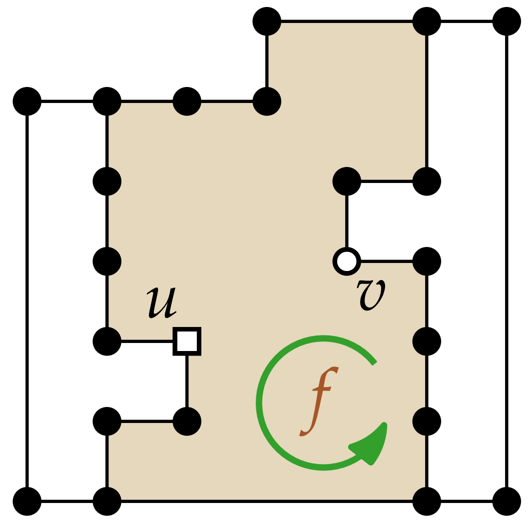

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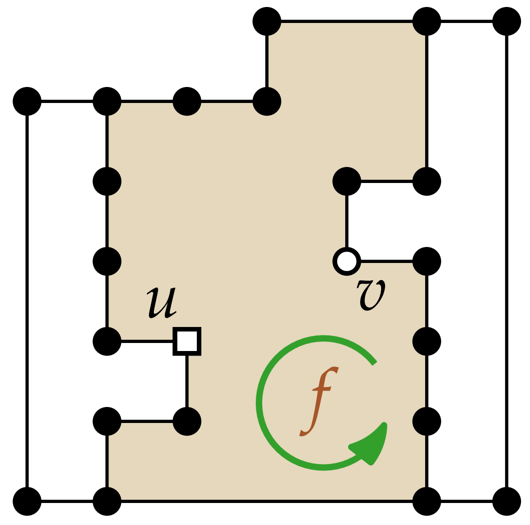

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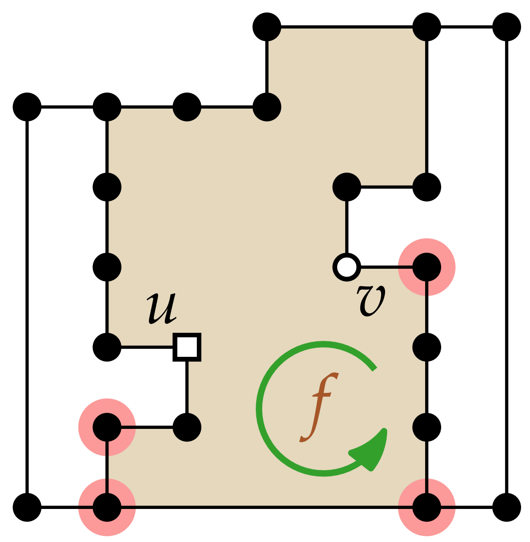

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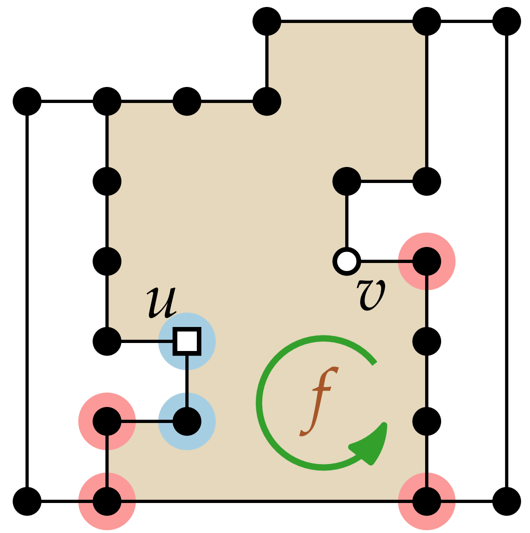

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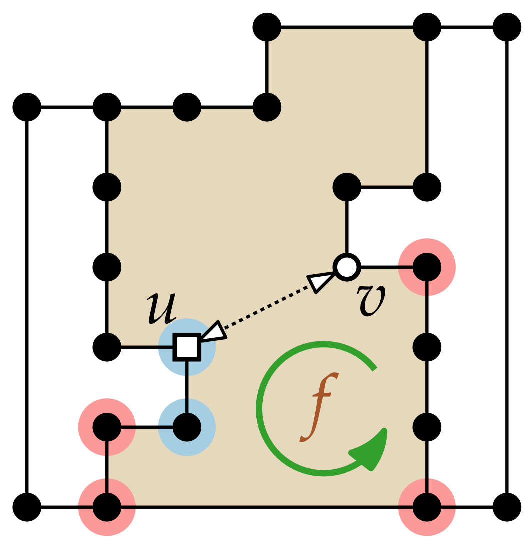
We say that $\{u, v\}$ is a *pair of kitty corners* of f
if $\text{rot}(u, v) = 2$ or $\text{rot}(v, u) = 2$.



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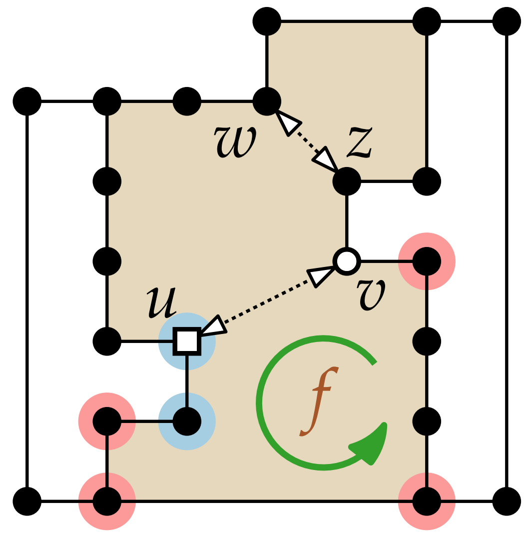

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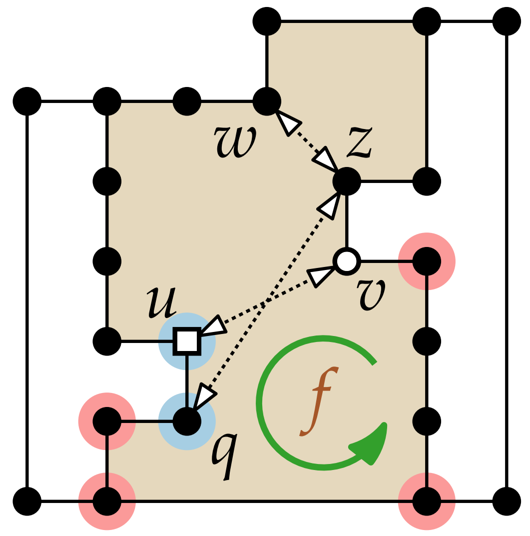

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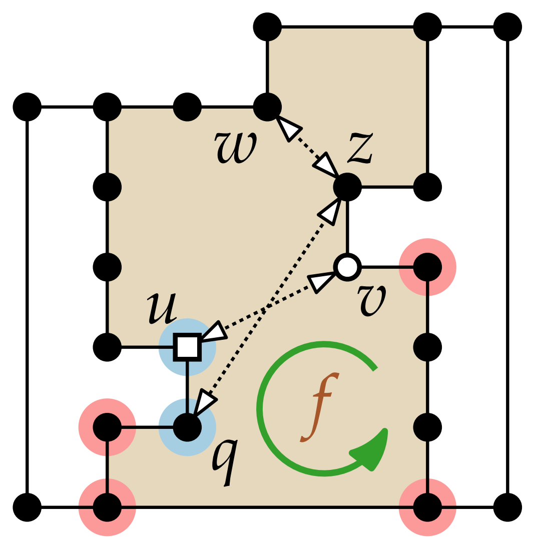

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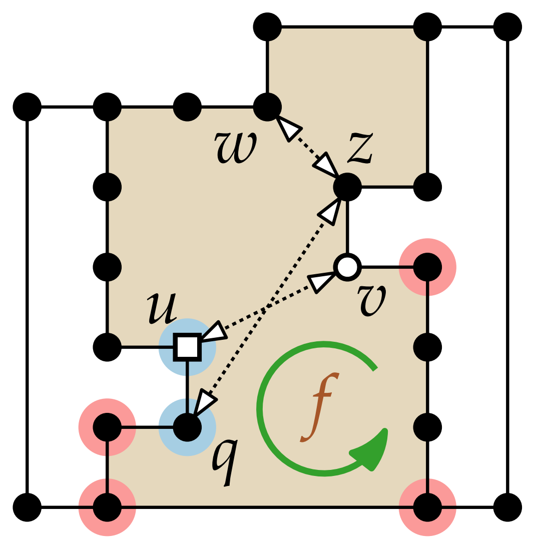

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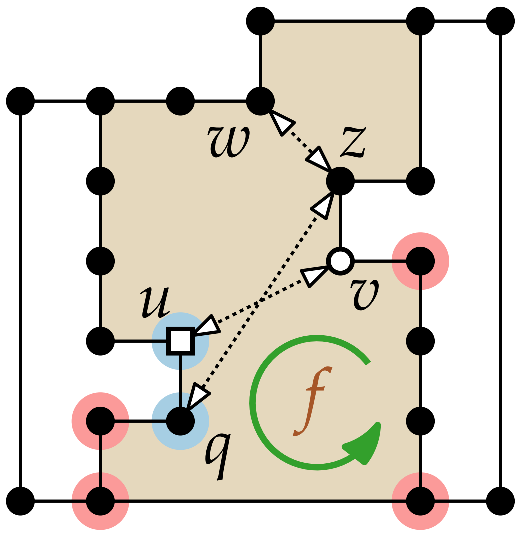

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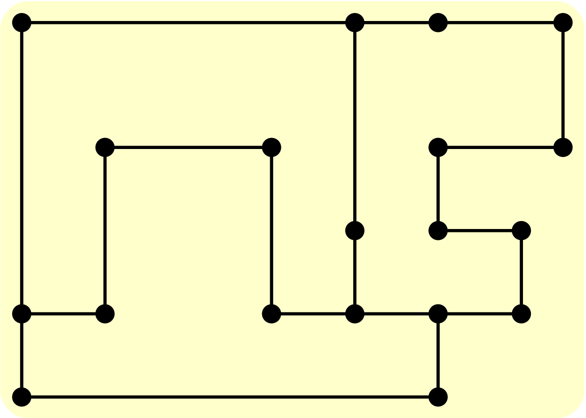
Theorem. If R is turn-regular, a min-area drawing of R can be computed in linear time.
[Bridgeman, Di Battista, Didimo, Liotta, Tamassia, Vismara: CGTA 2000]

Drawing Turn-Regular Representations Optimally



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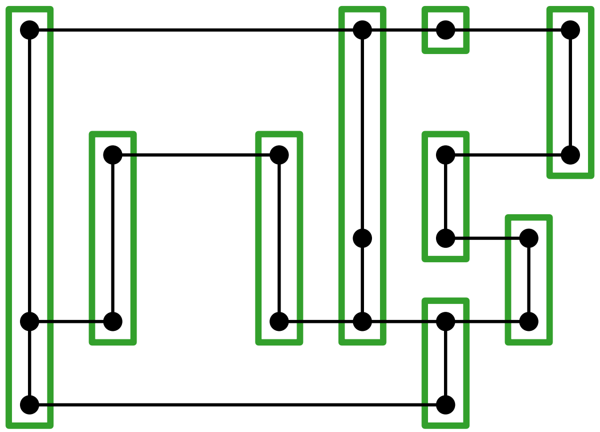

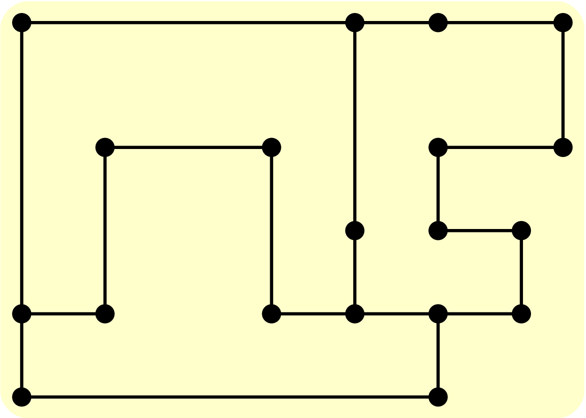

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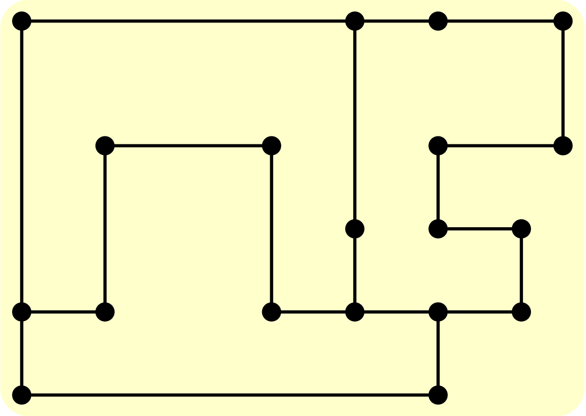

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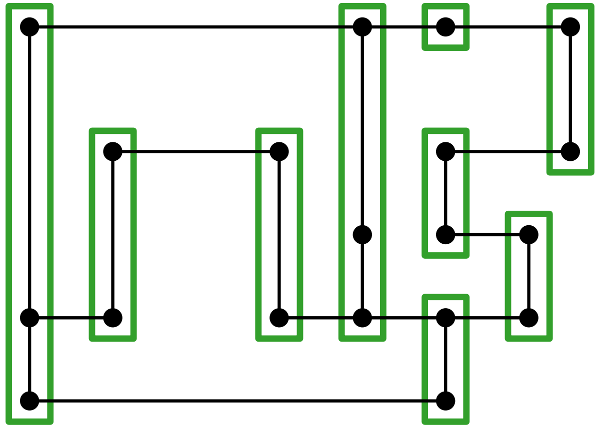

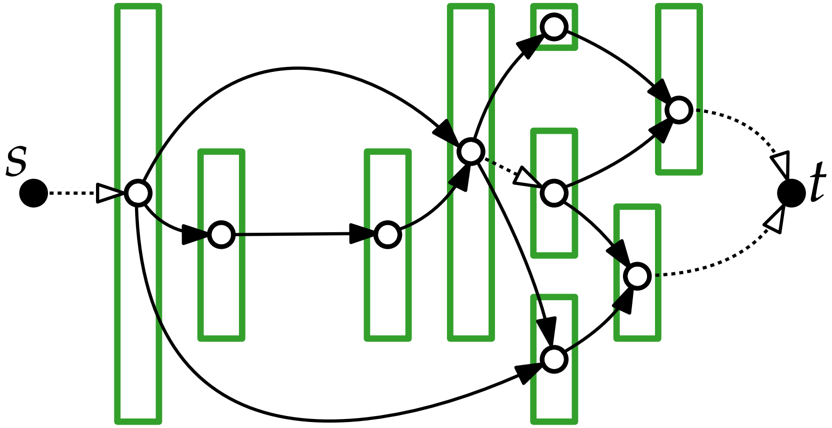

auxiliary graph (DAG) D_x

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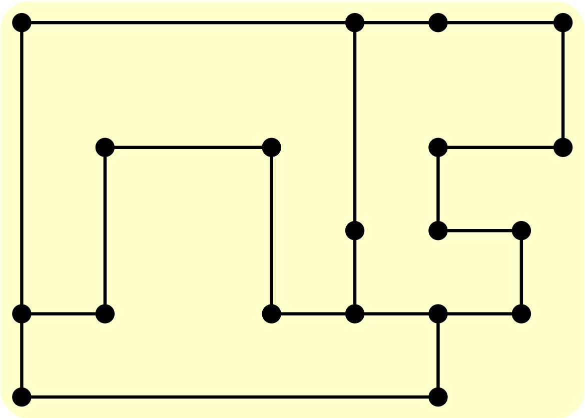

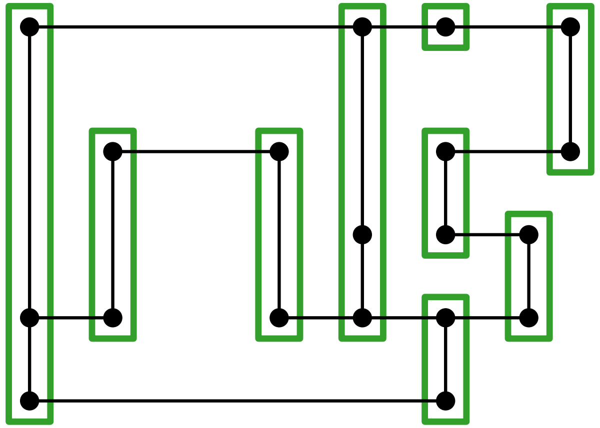

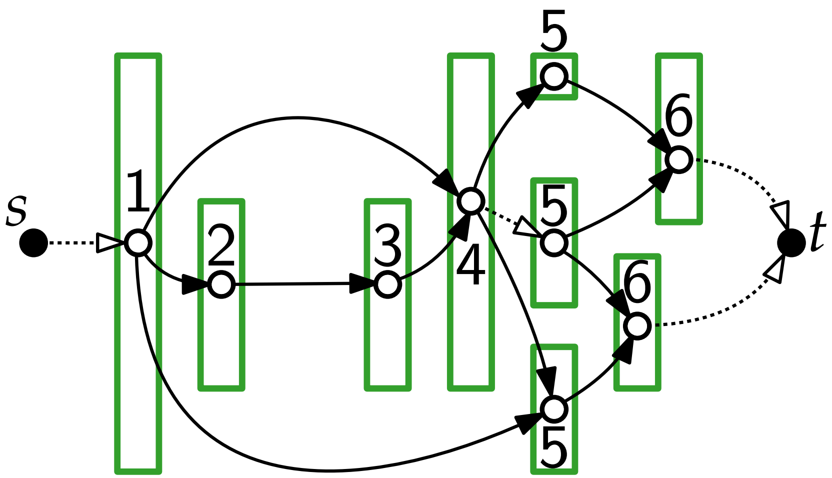

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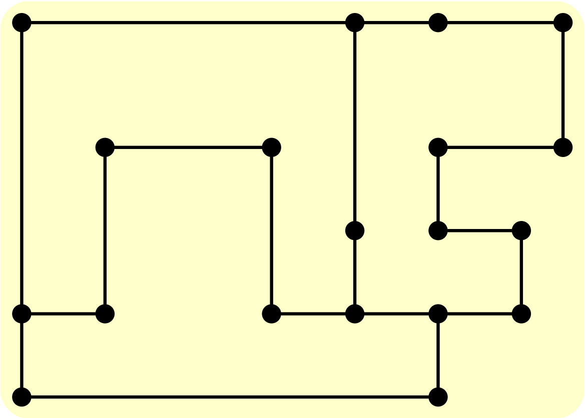

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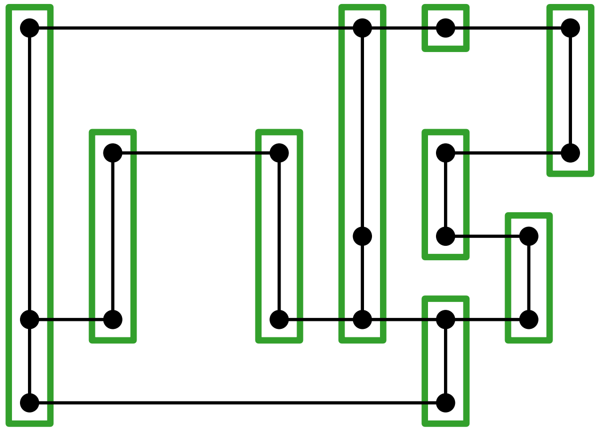

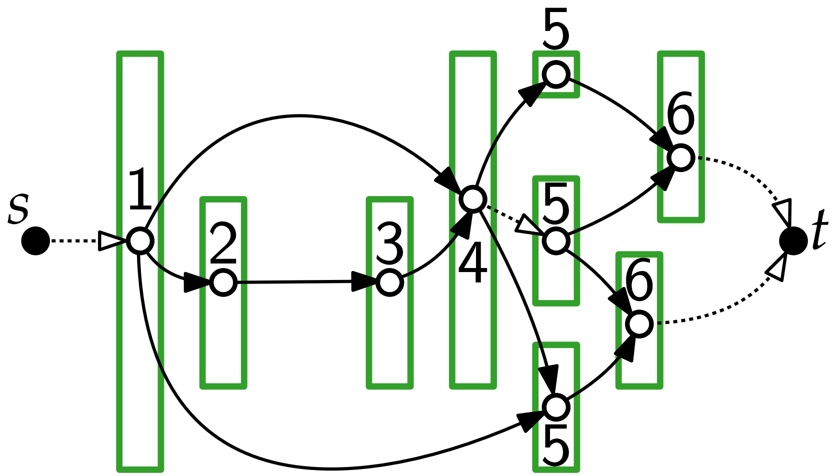

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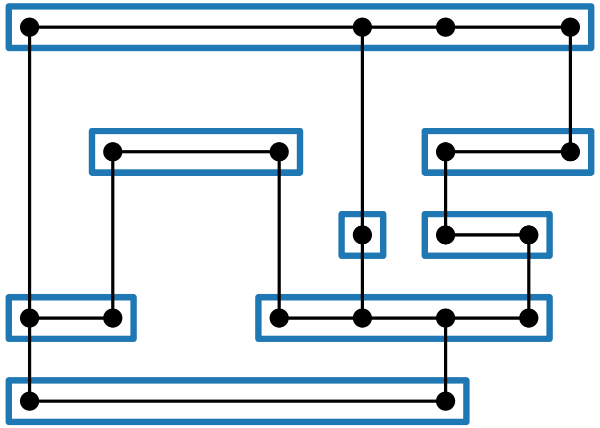

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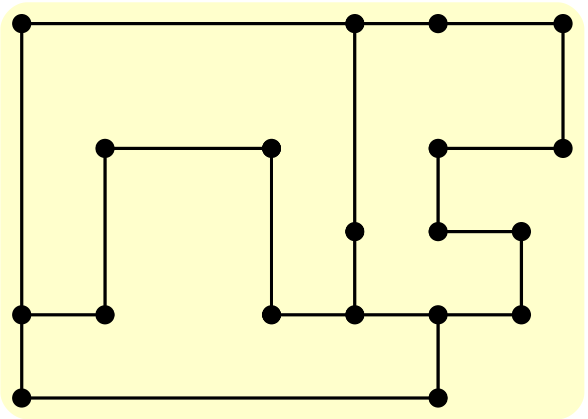

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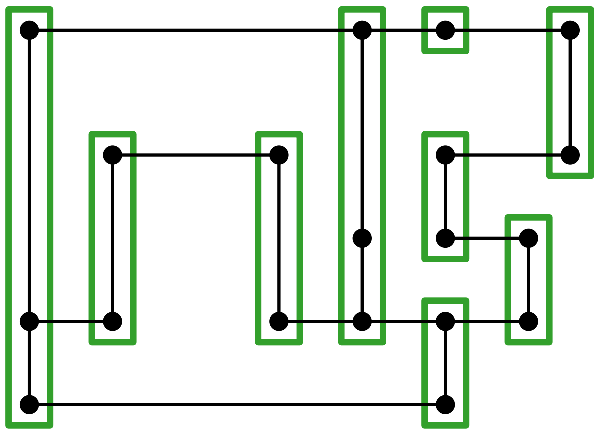

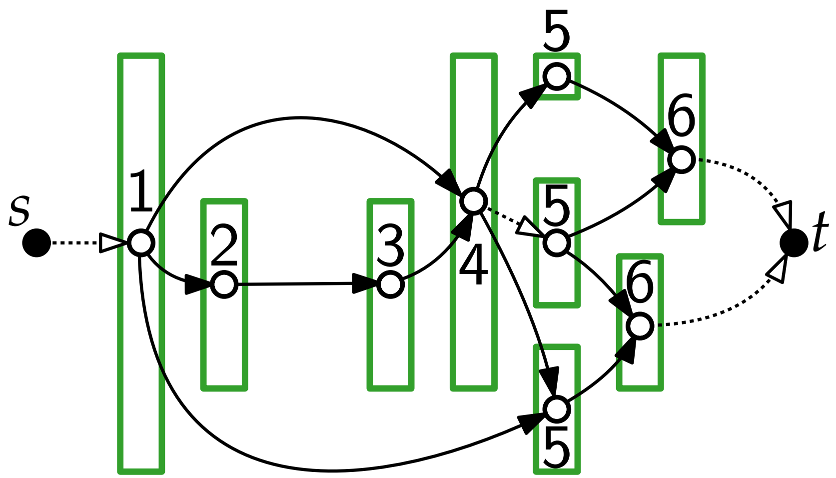

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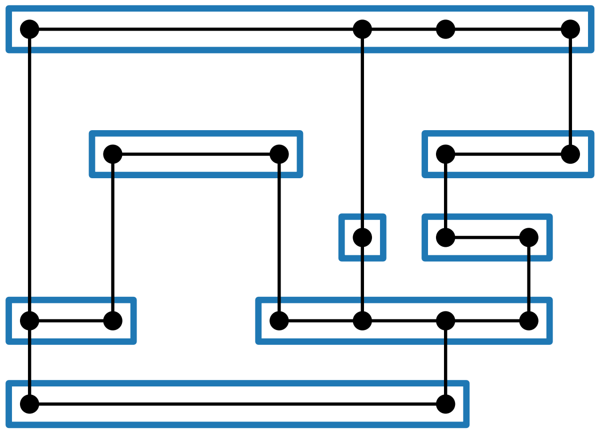

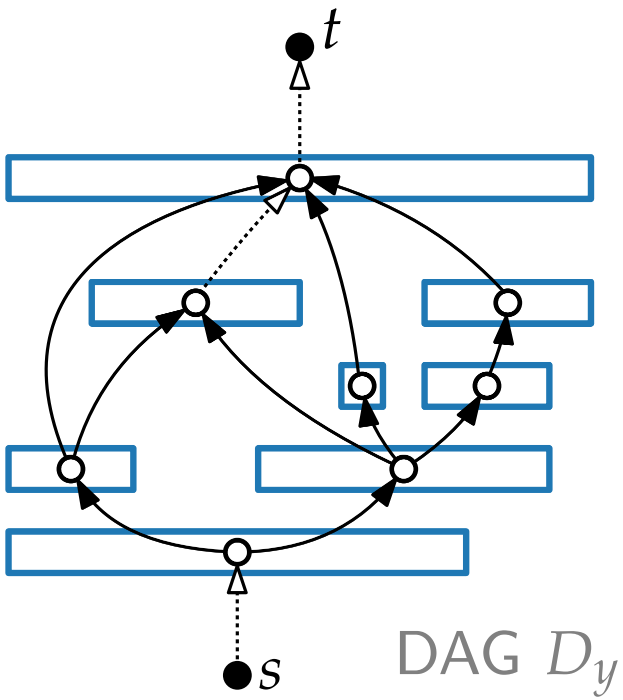

DAG D_y

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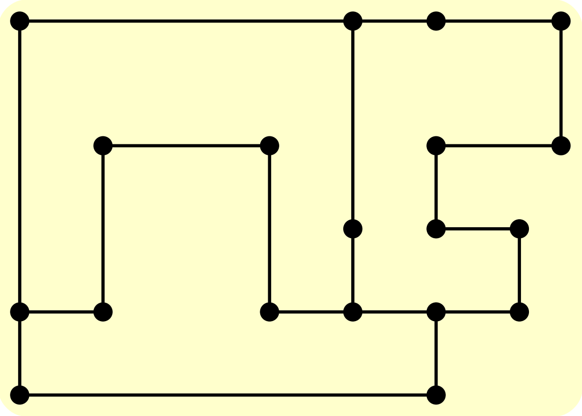

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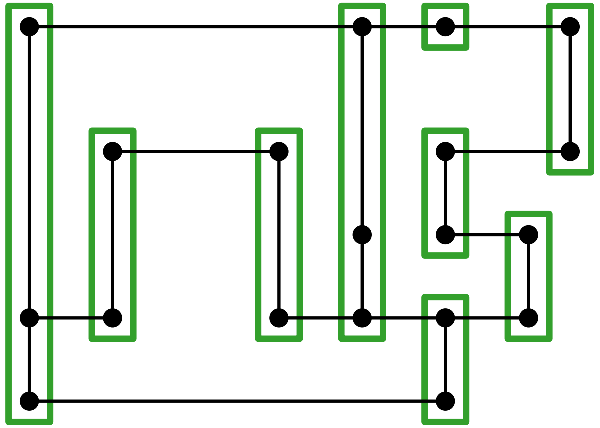

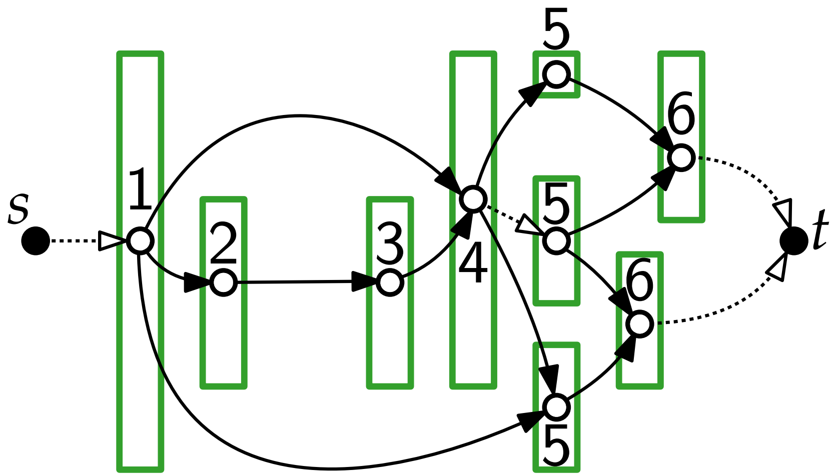

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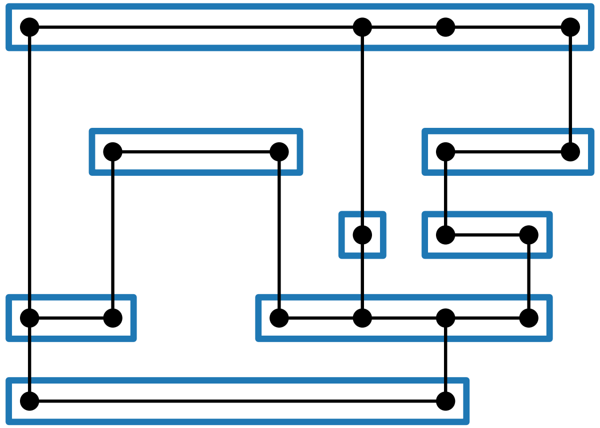

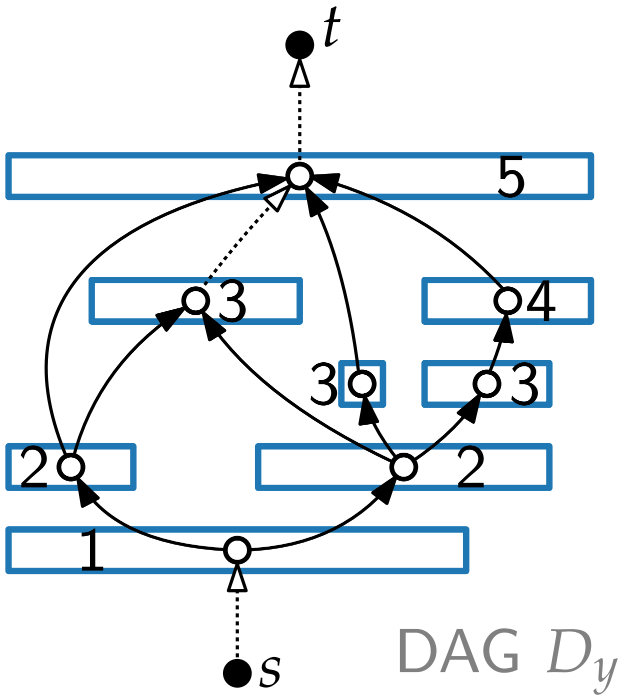

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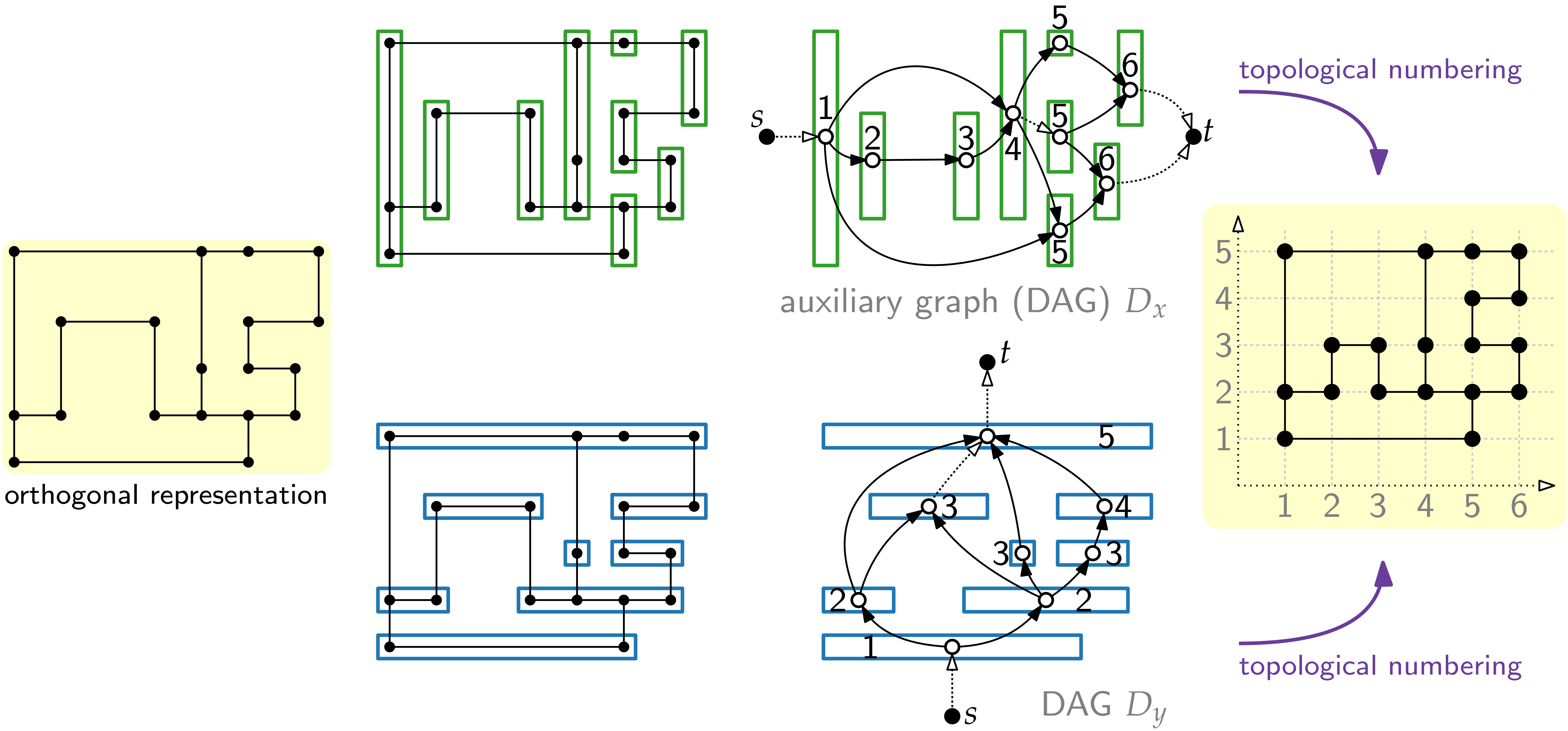

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- **Number of faces:** OC is para-NP-hard when parametrized by the number of faces.

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  We show: *OC admits an FPT algorithm parametrized by the number of kitty corners.*

- **Number of faces:** OC is para-NP-hard when parametrized by the number of faces.
  For *one* face (cycle), we show the *existence of a polynomial kernel for OC*.
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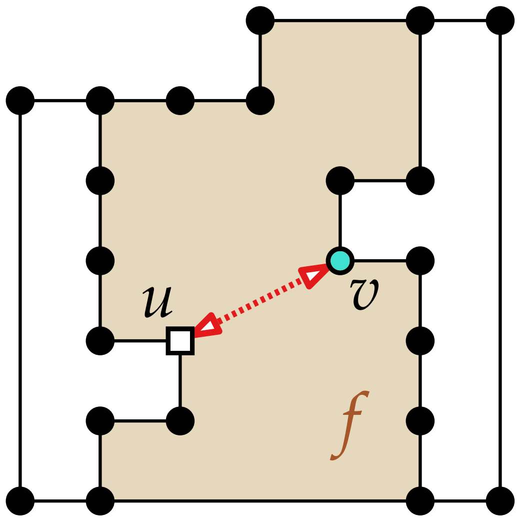
Idea: For each (?) pair $\{u, v\}$ of kitty corners, guess whether $x(u) \lesseqqgtr x(v)$ and $y(u) \lesseqqgtr y(v)$.



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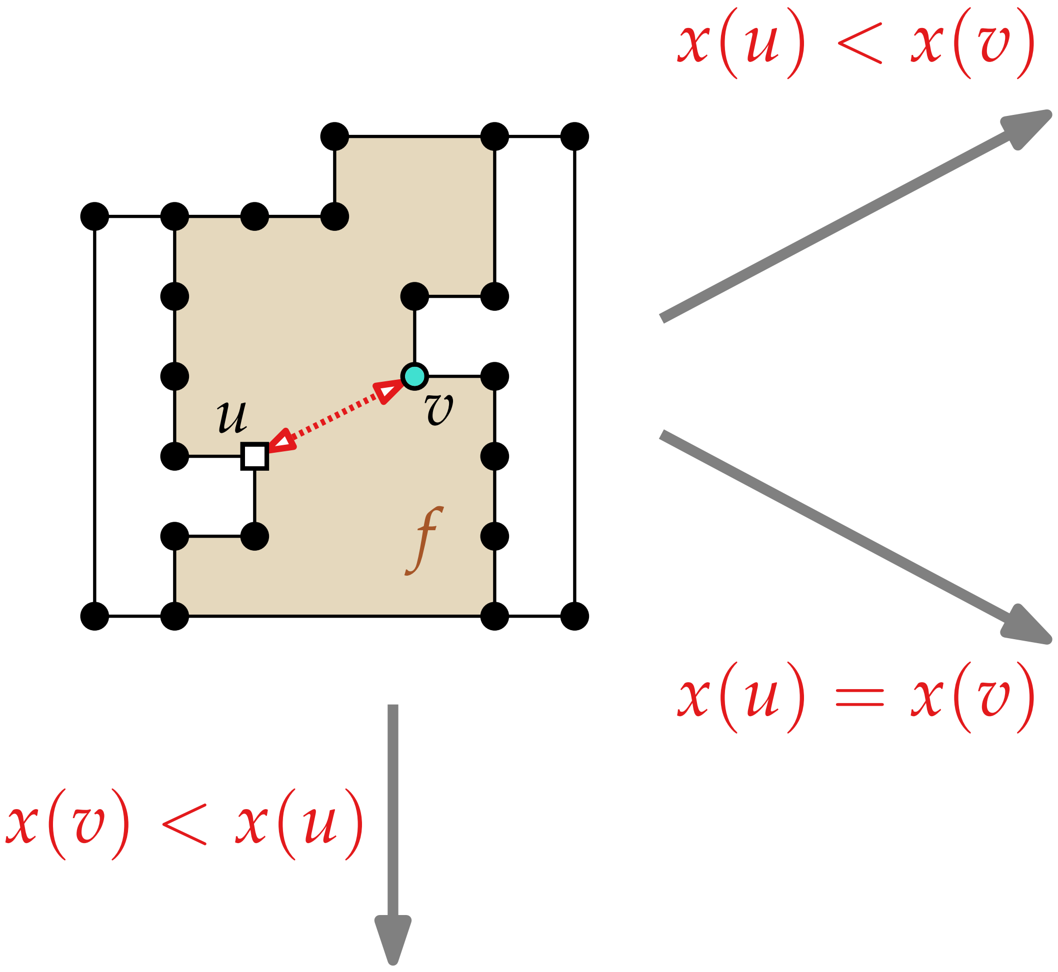

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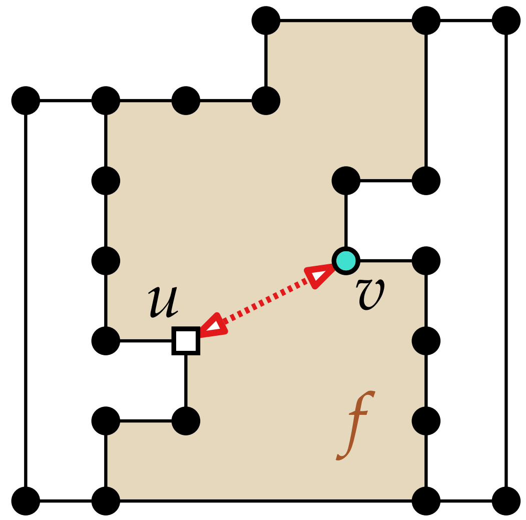
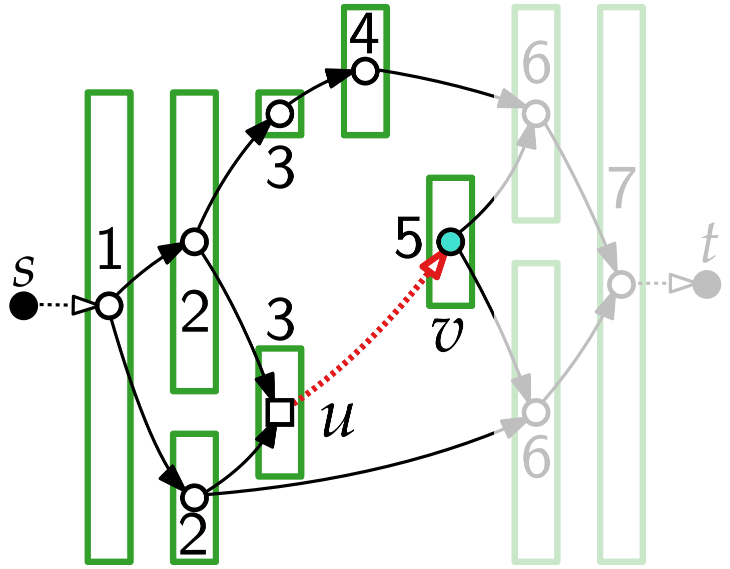


$x(u) < x(v)$



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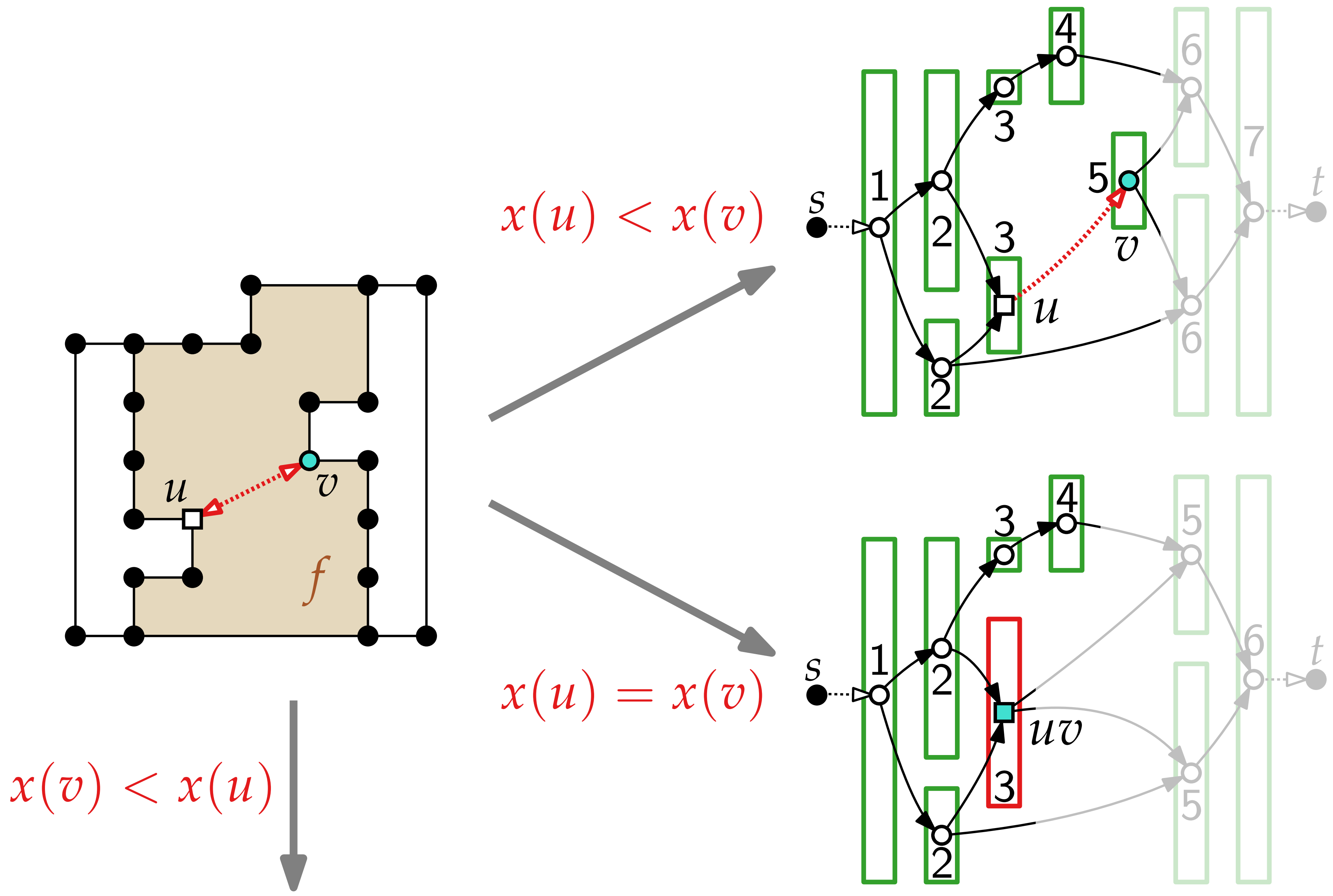

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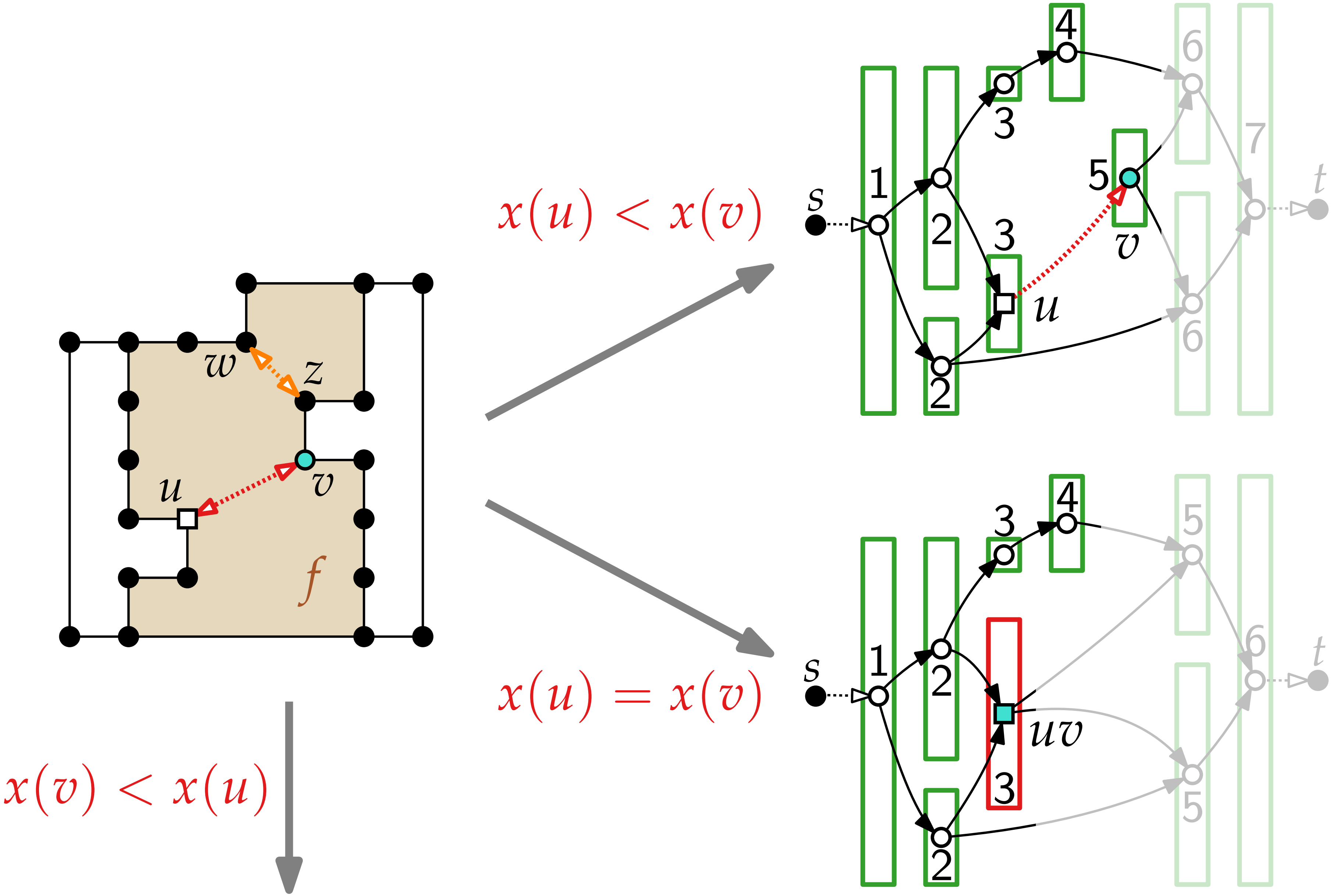

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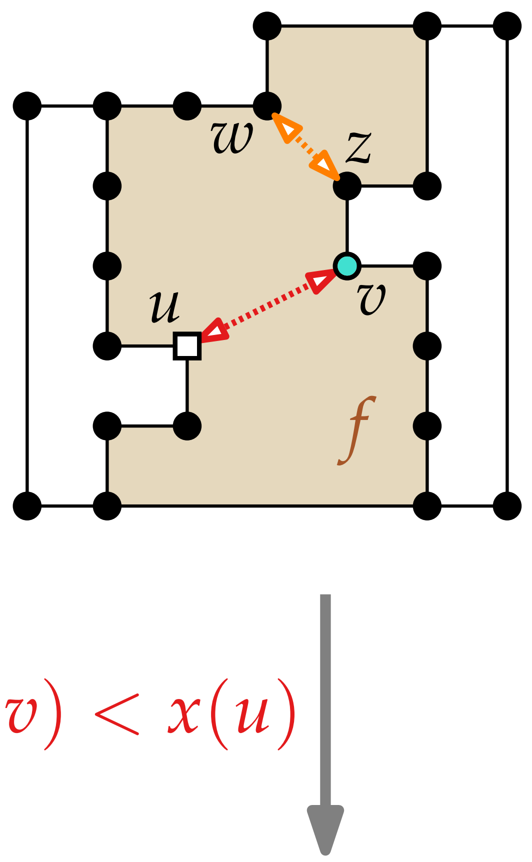

$x(v) < x(u)$

$x(w) < x(z)$

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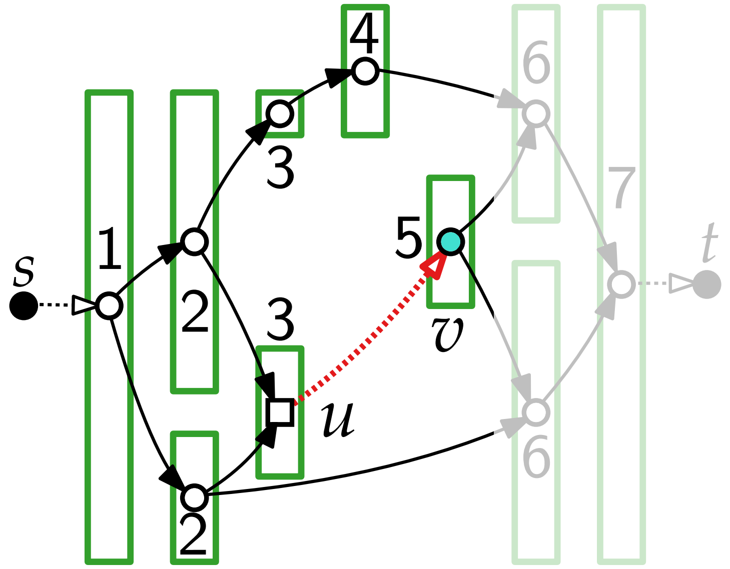

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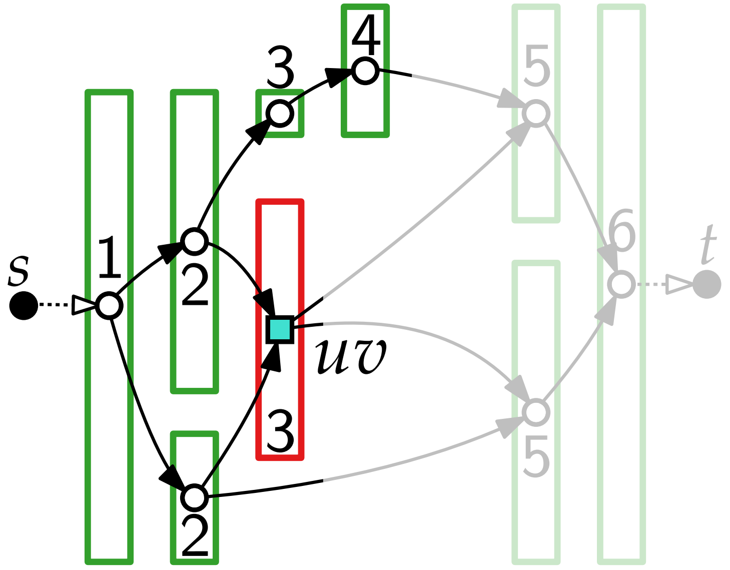

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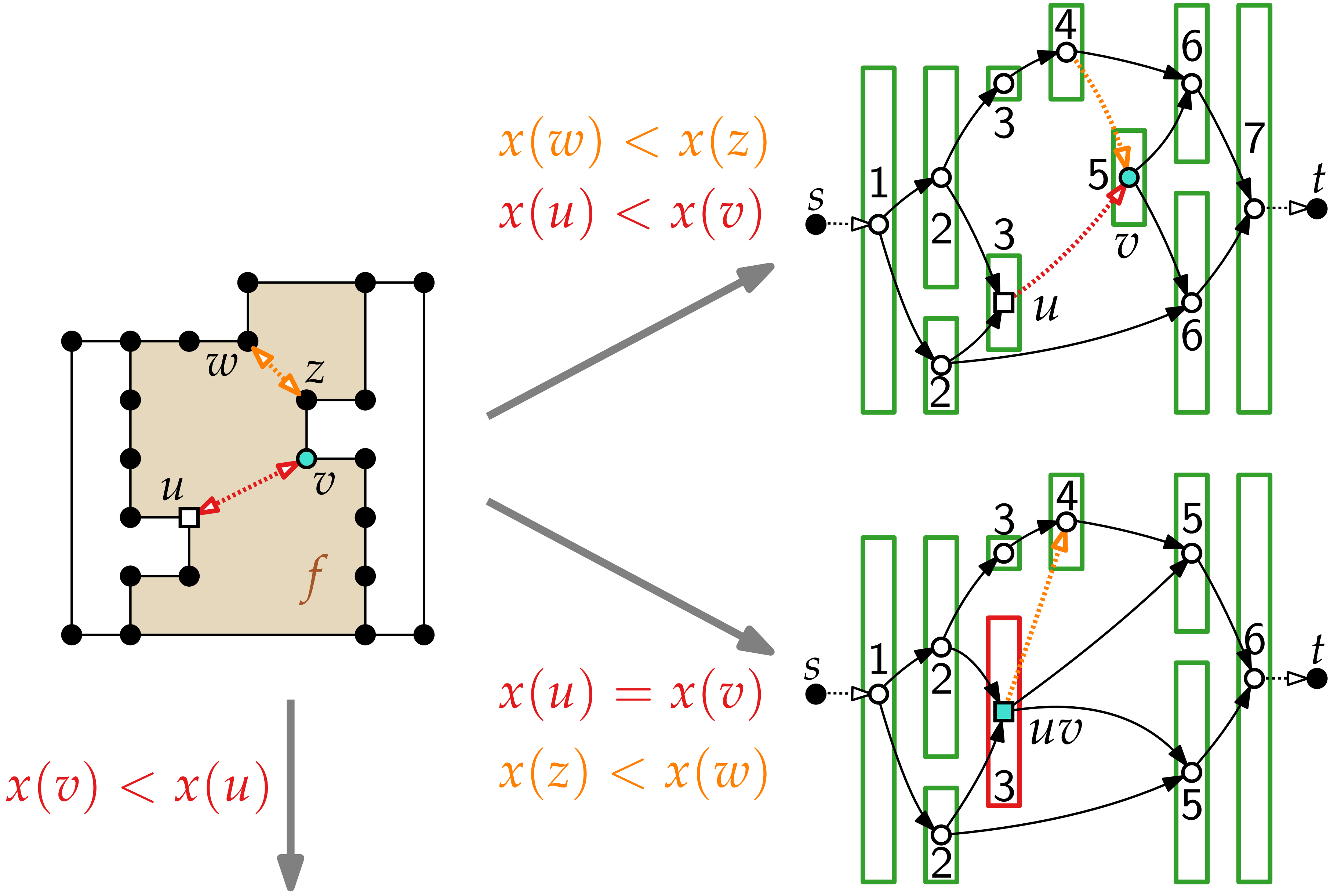

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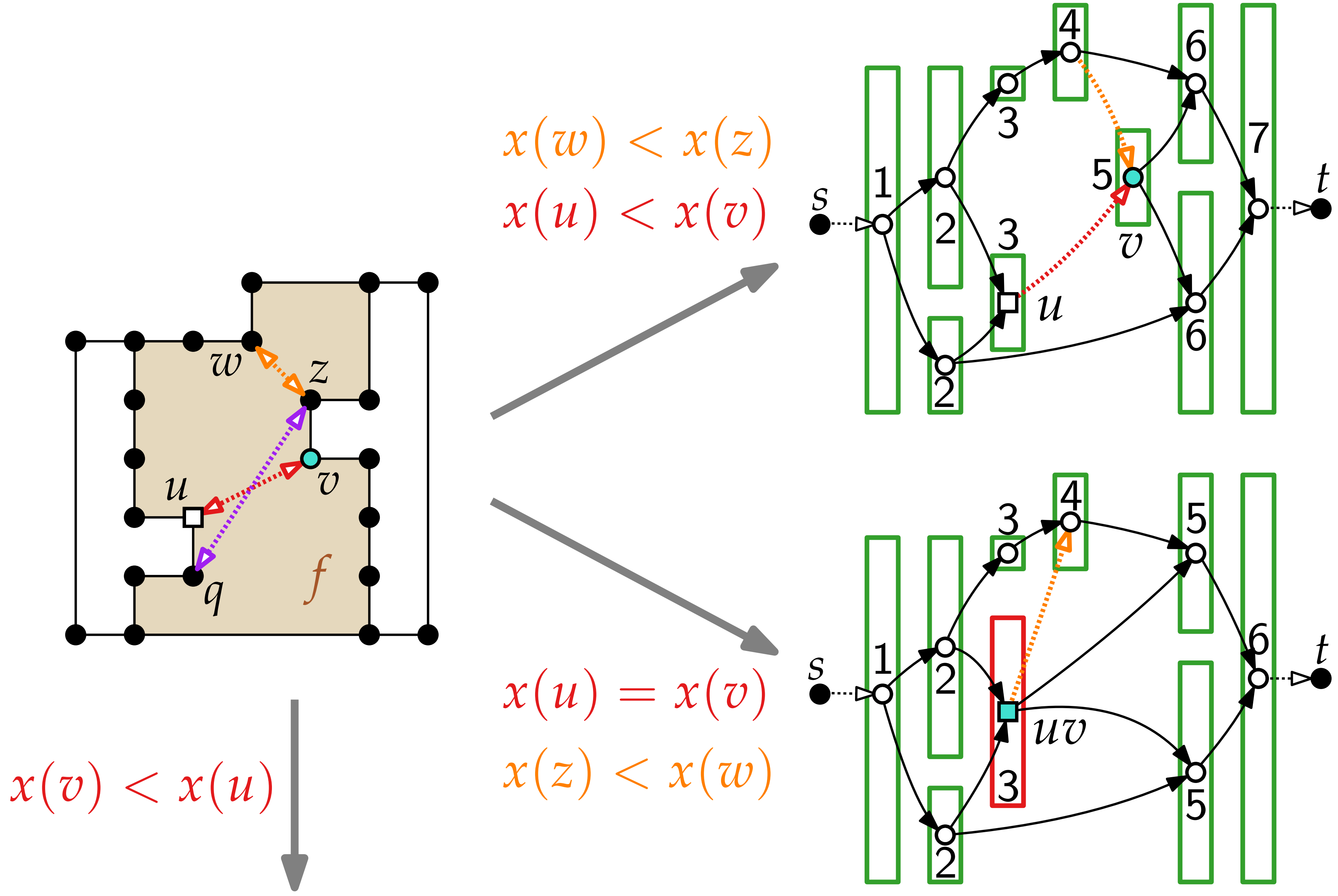

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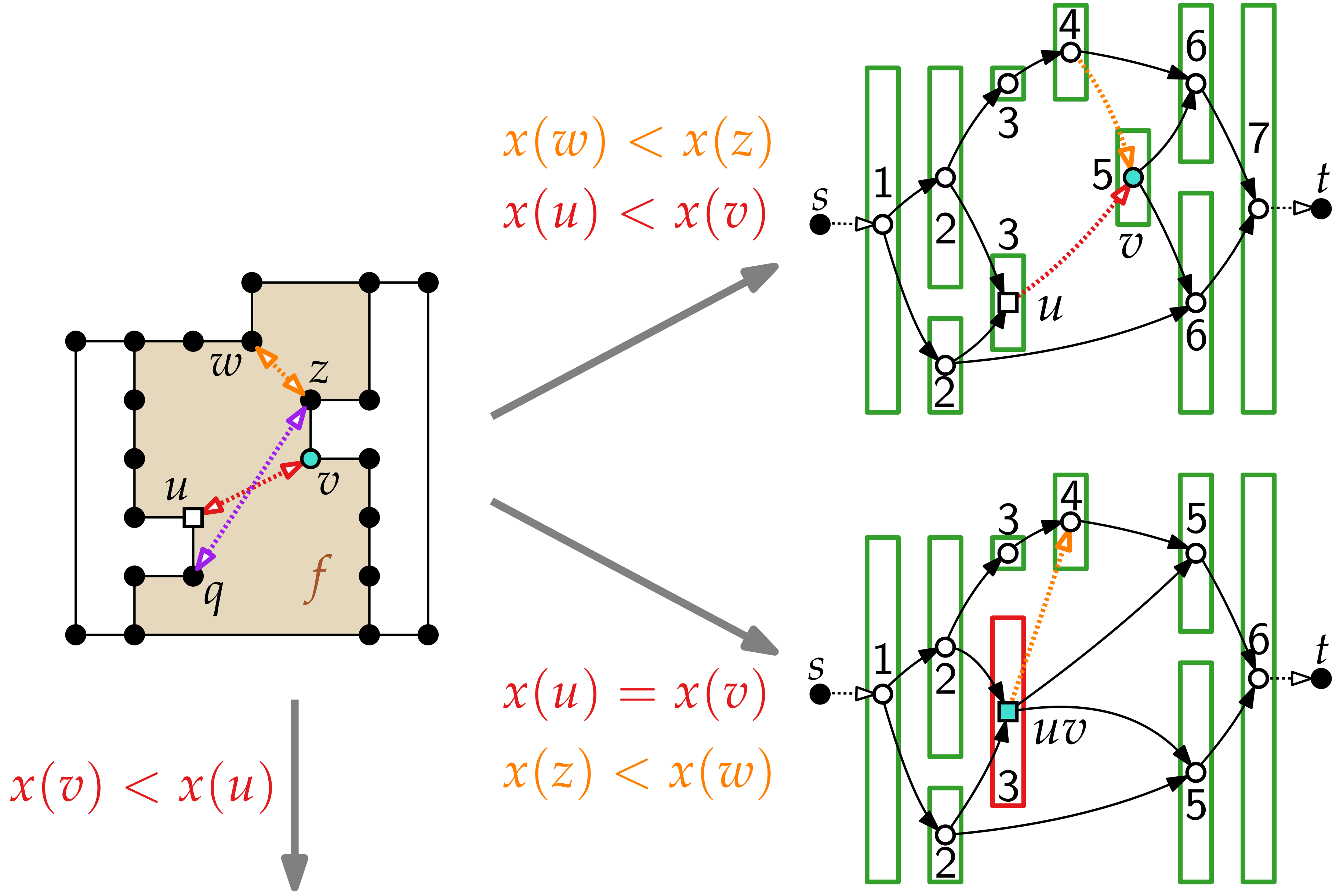

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Do we need to deal with $\{q, z\}$ analogously?

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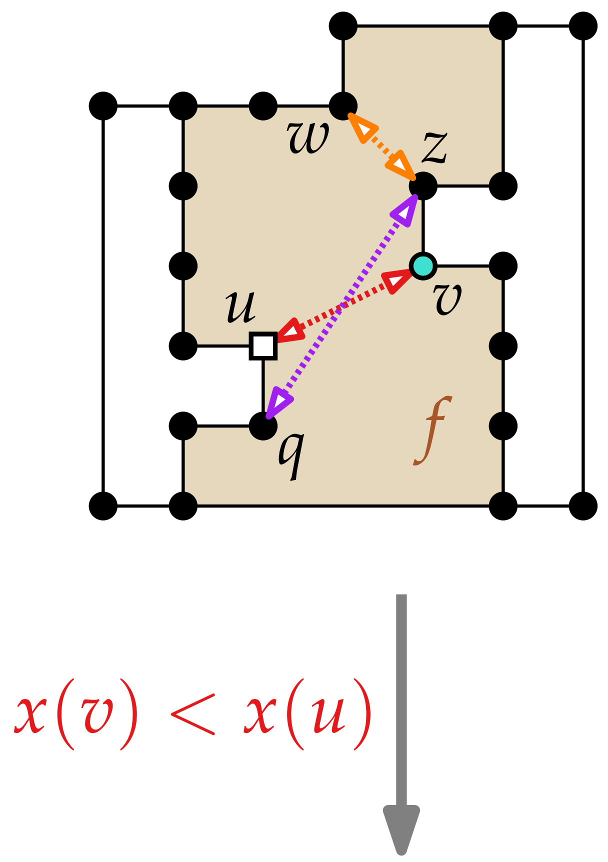

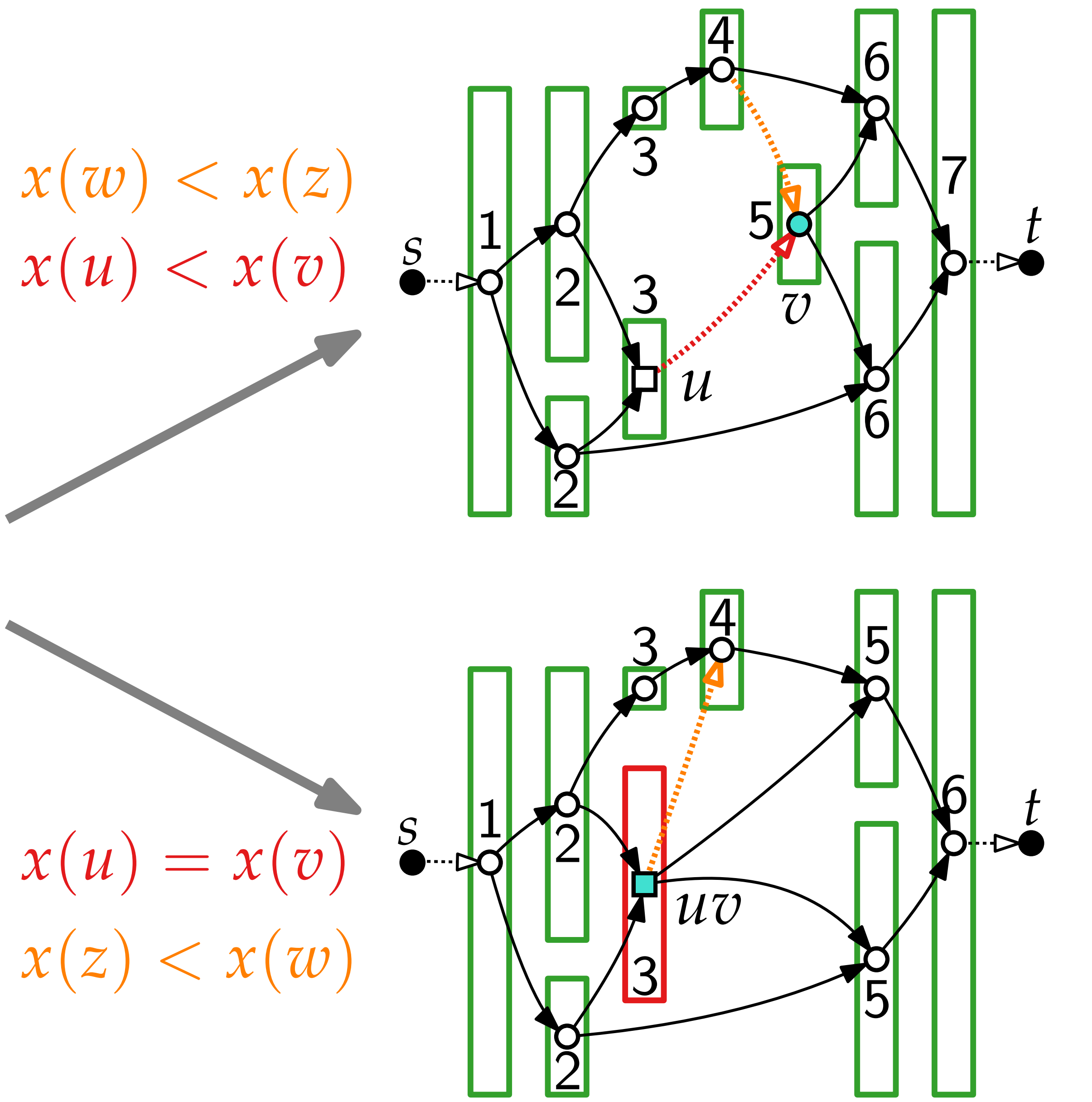

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No, doesn't add new information.

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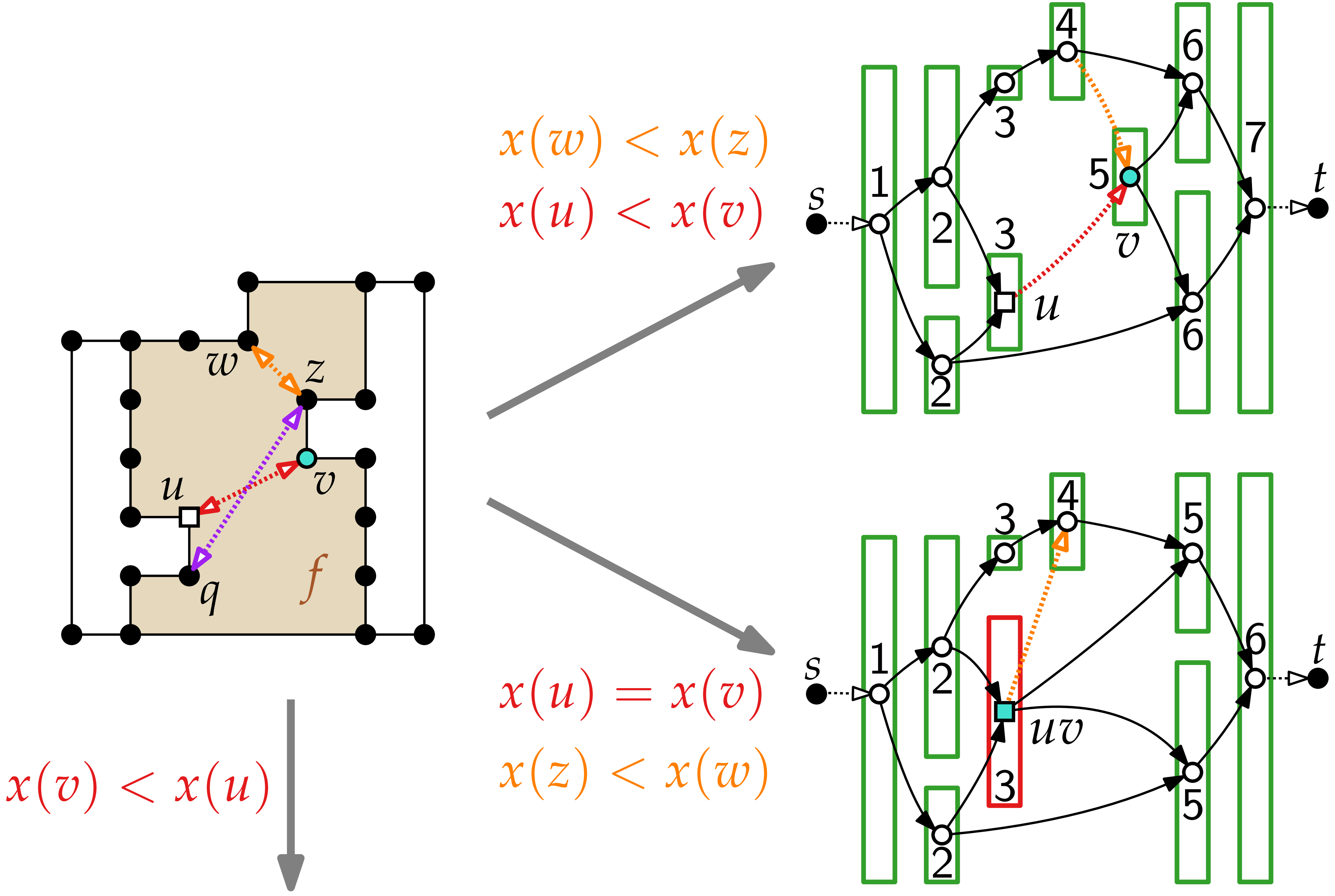

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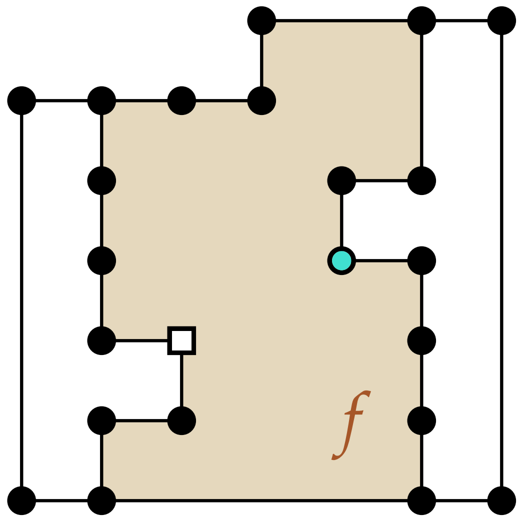
Thus:
Suffices to consider a *maximal planar subset* of the set K_f of "kitty corner edges" of f.

An FPT Algorithm for OC, cont'd

Let f be a face of the given orthogonal representation of G.



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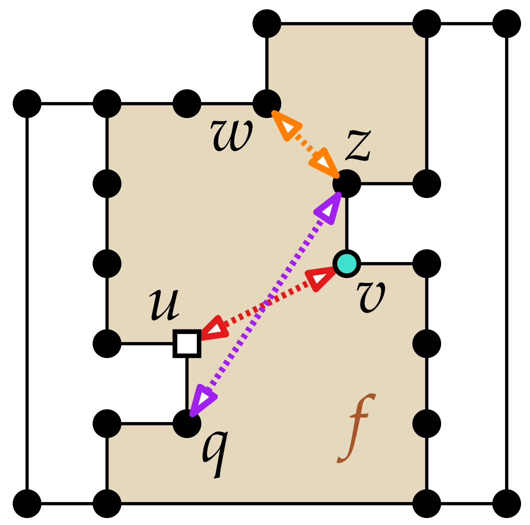

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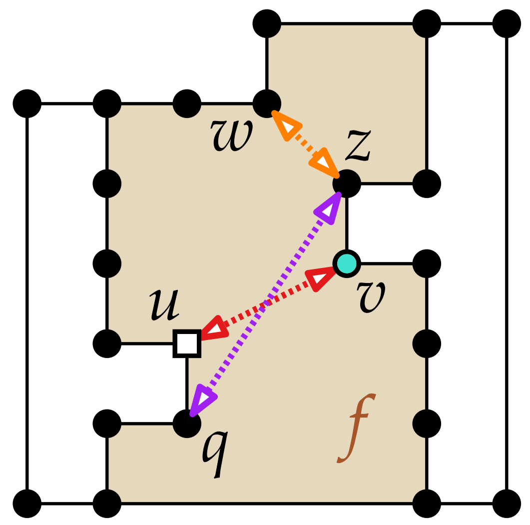

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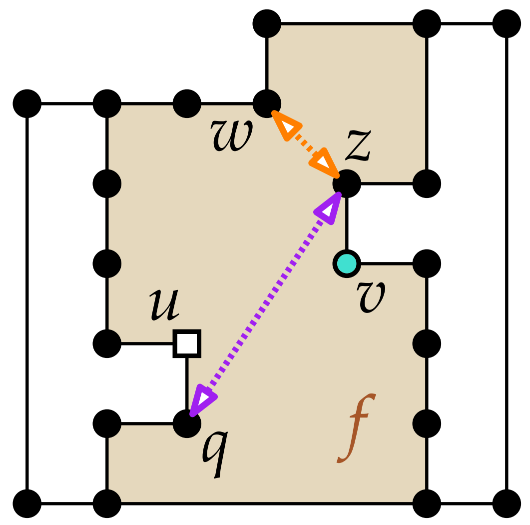

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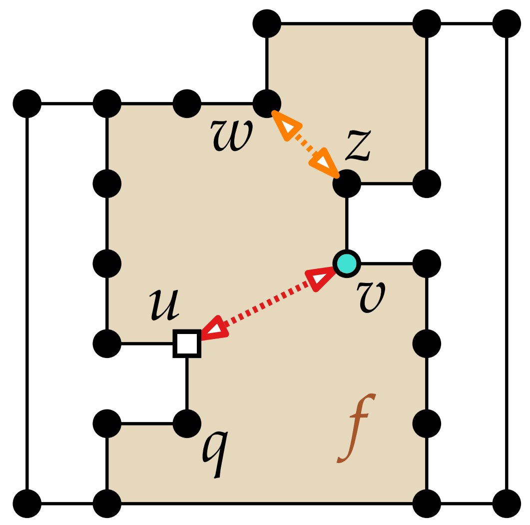

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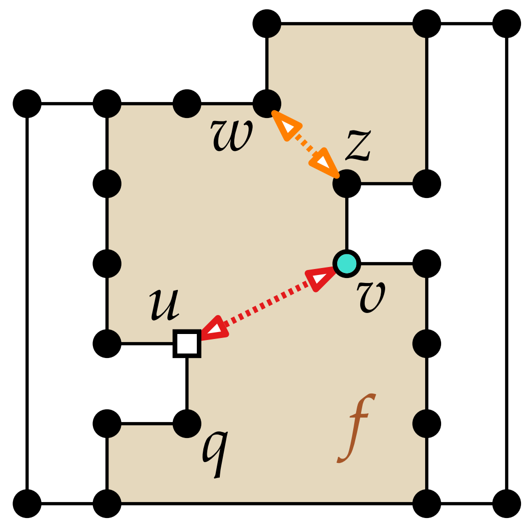

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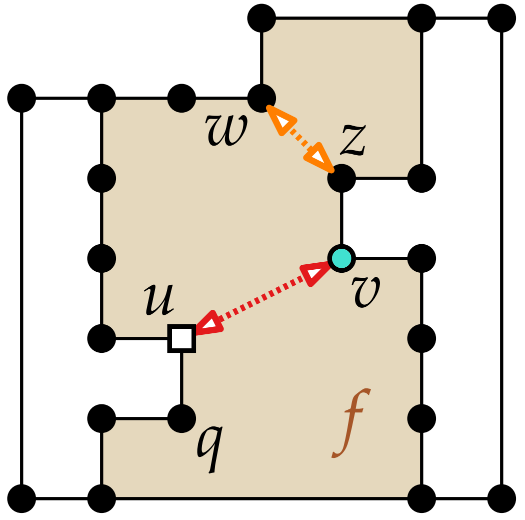

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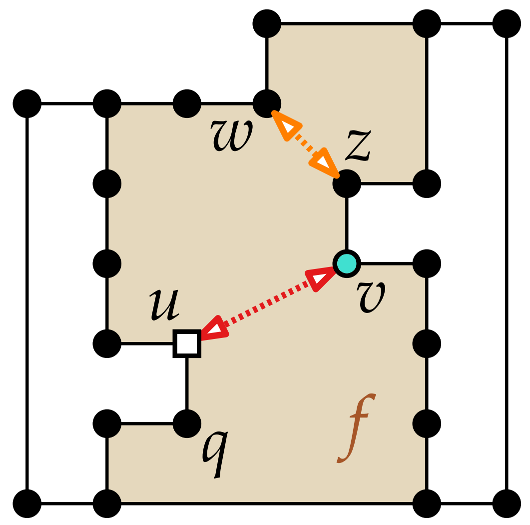

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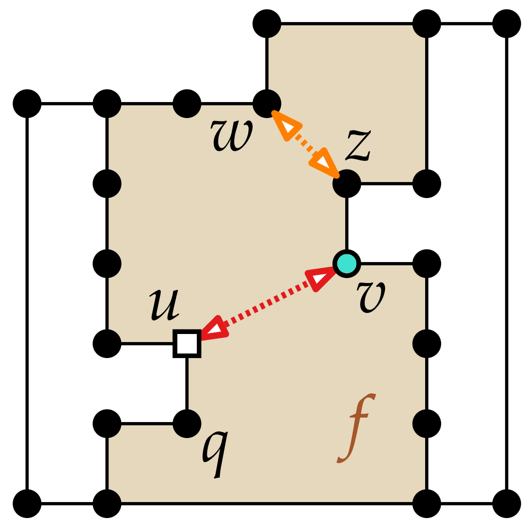
    - Run the algorithm for the turn-regular case.



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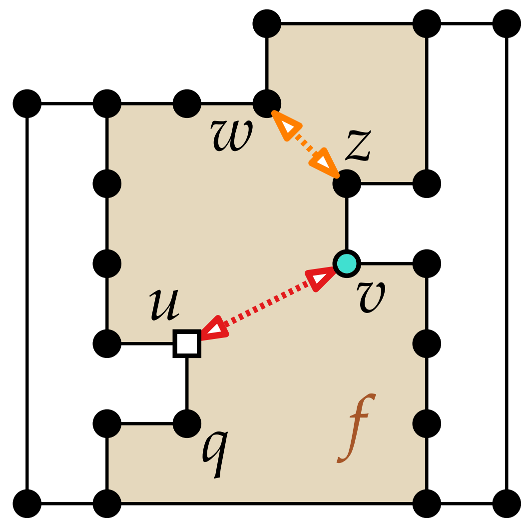
    - If the resulting drawing does not self-intersect, measure its area.



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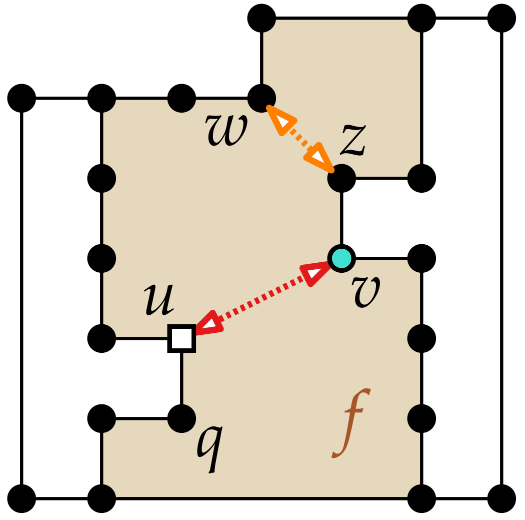
    - If the resulting drawing does not self-intersect, measure its area.
- Return the drawing of minimum area.



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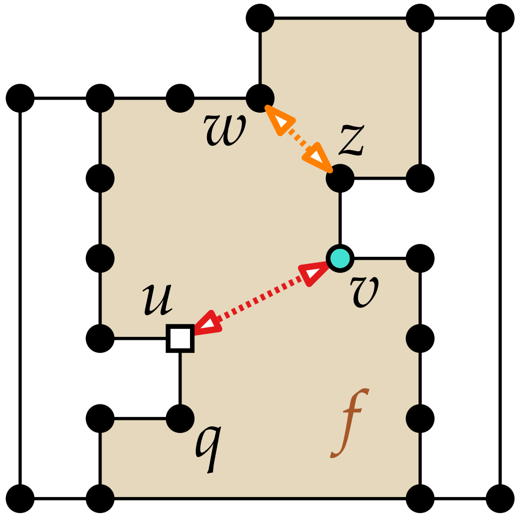

Runtime: Let $k_f = \#$kitty corners in face f.

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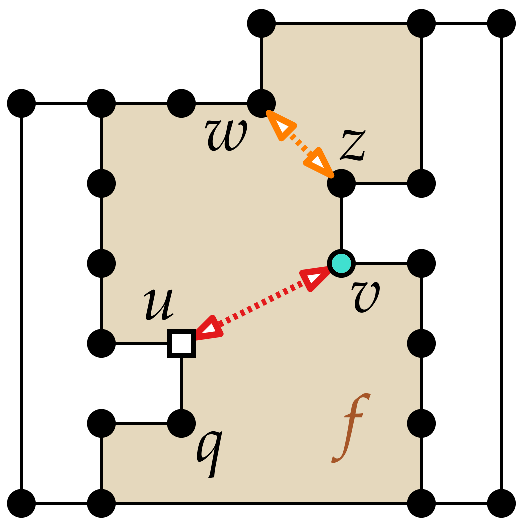

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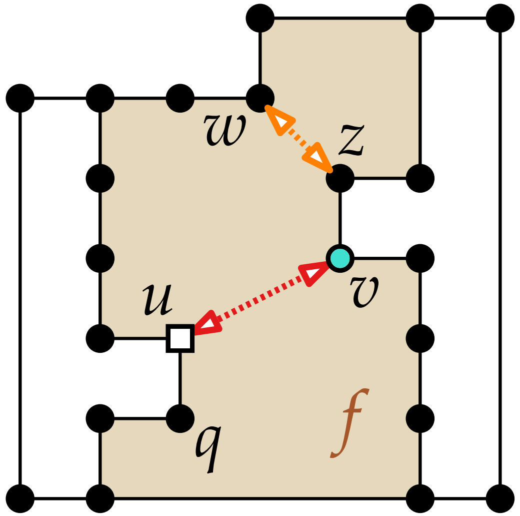

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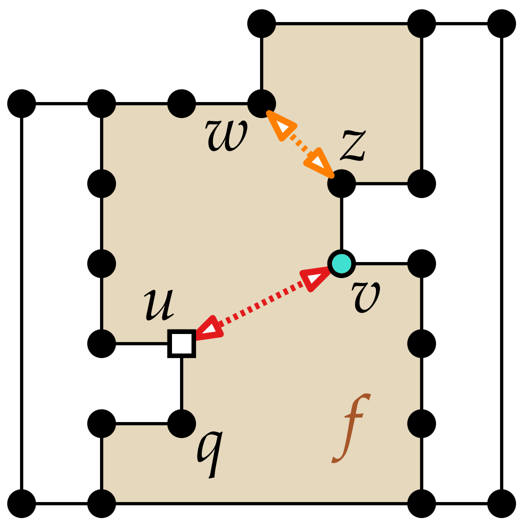

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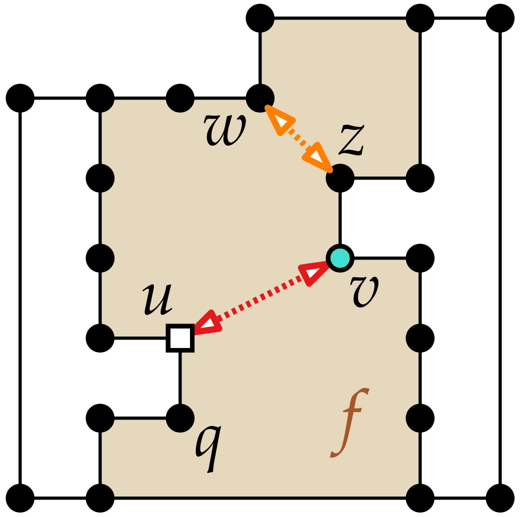

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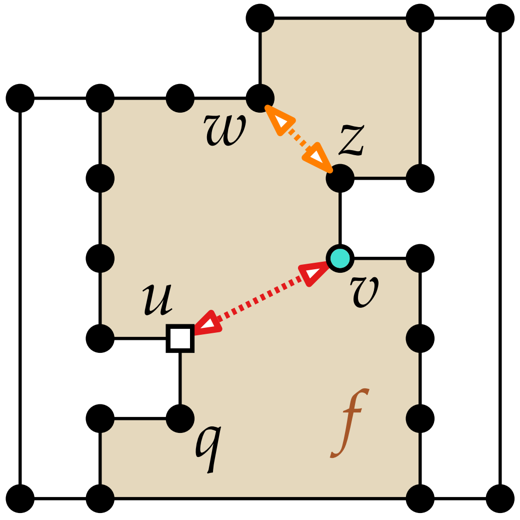

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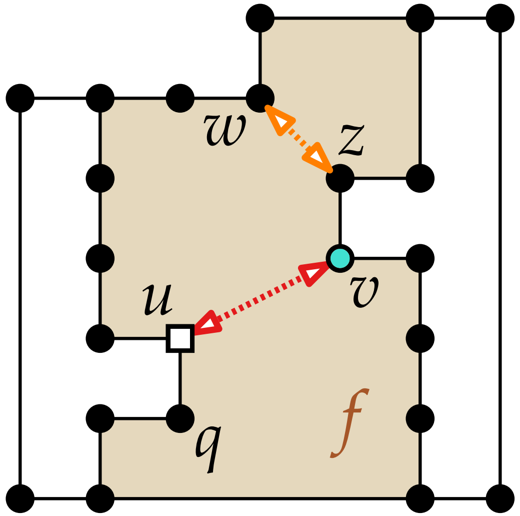

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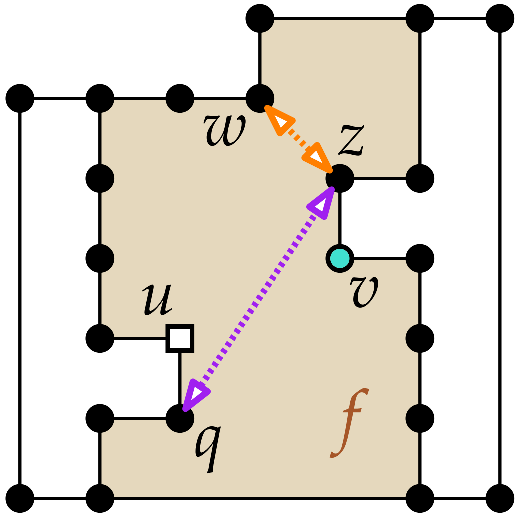

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Theorem. OC is FPT w.r.t. the number of kitty corners.

Table of Contents

- **Number of kitty corners:** (the number of corners involved in some pair of kitty corners)
  We show: *OC admits an FPT algorithm parametrized by the number of kitty corners.*

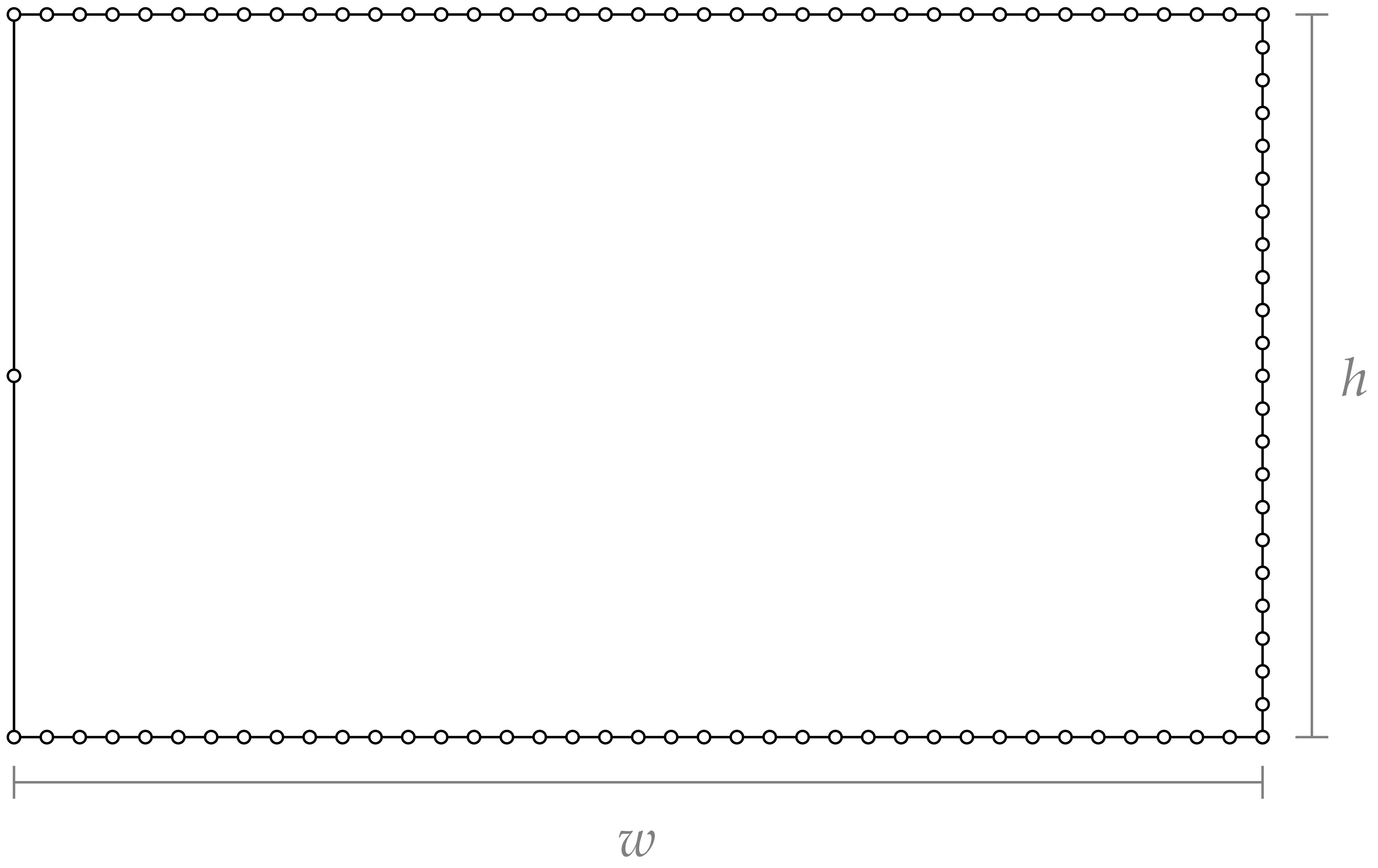
- **Number of faces:** OC is para-NP-hard when parametrized by the number of faces.
  For *one* face (cycle), we show the *existence of a polynomial kernel for OC*.
  when parametrized by the number of kitty corners.

- **Maximum face-degree:** The reductions of Patrignani & Evans et al. use linear-size faces.
  We show: *OC remains NP-hard when parametrized by maximum face degree.*

- **Height:** (minimum number of distinct y-coordinates required to draw the representation)
  A $(w \times h)$-grid has pathwidth at most h.
  $\Rightarrow$ Graphs of bounded height have bounded pathwidth. But converse generally not true.
  We show: *OC admits an XP algorithm parametrized by height.*

Boundary, Belt, and “Piston” Gadget

Theorem. OC is NP-hard. [Patrignani, CGTA 2001]



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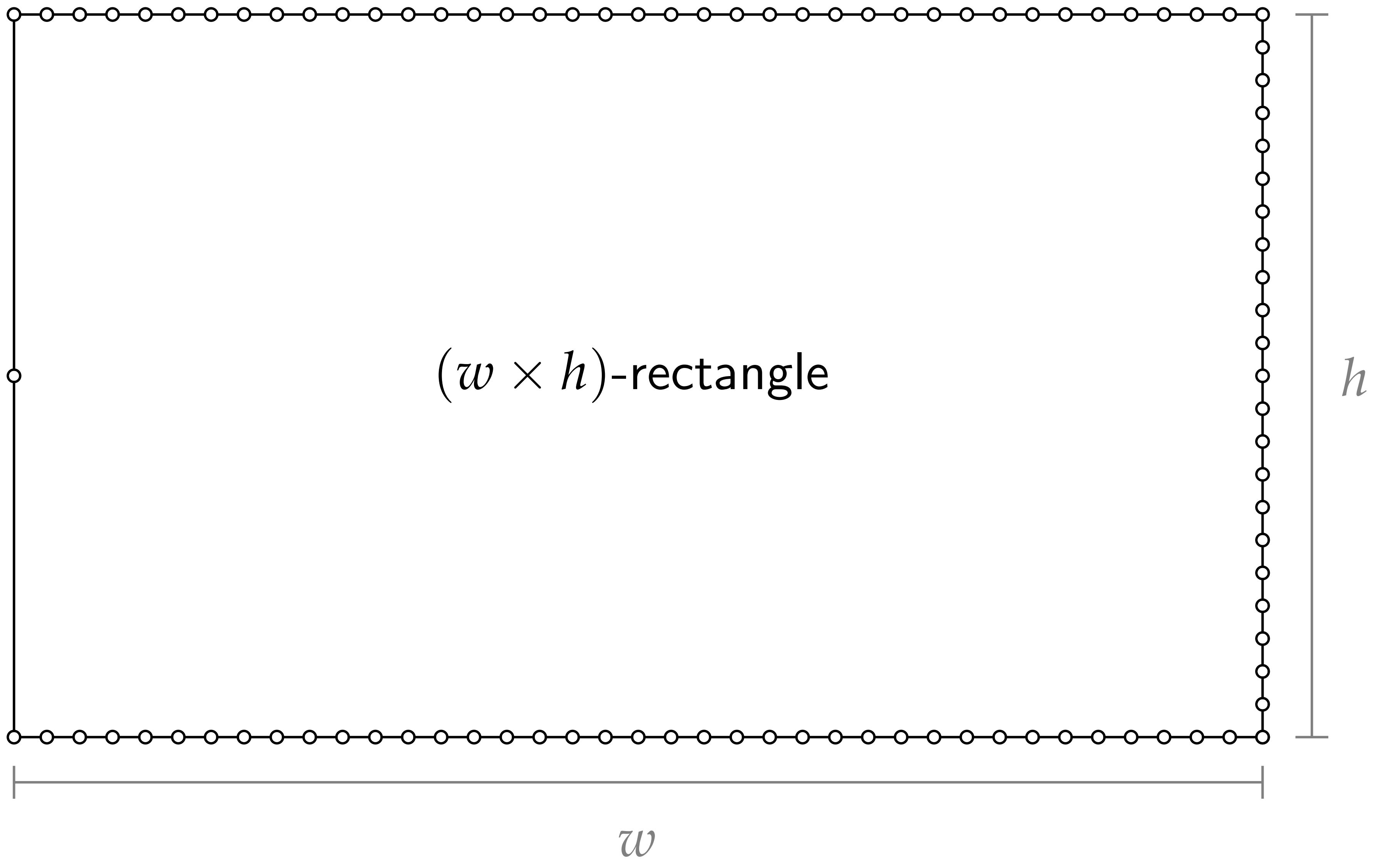

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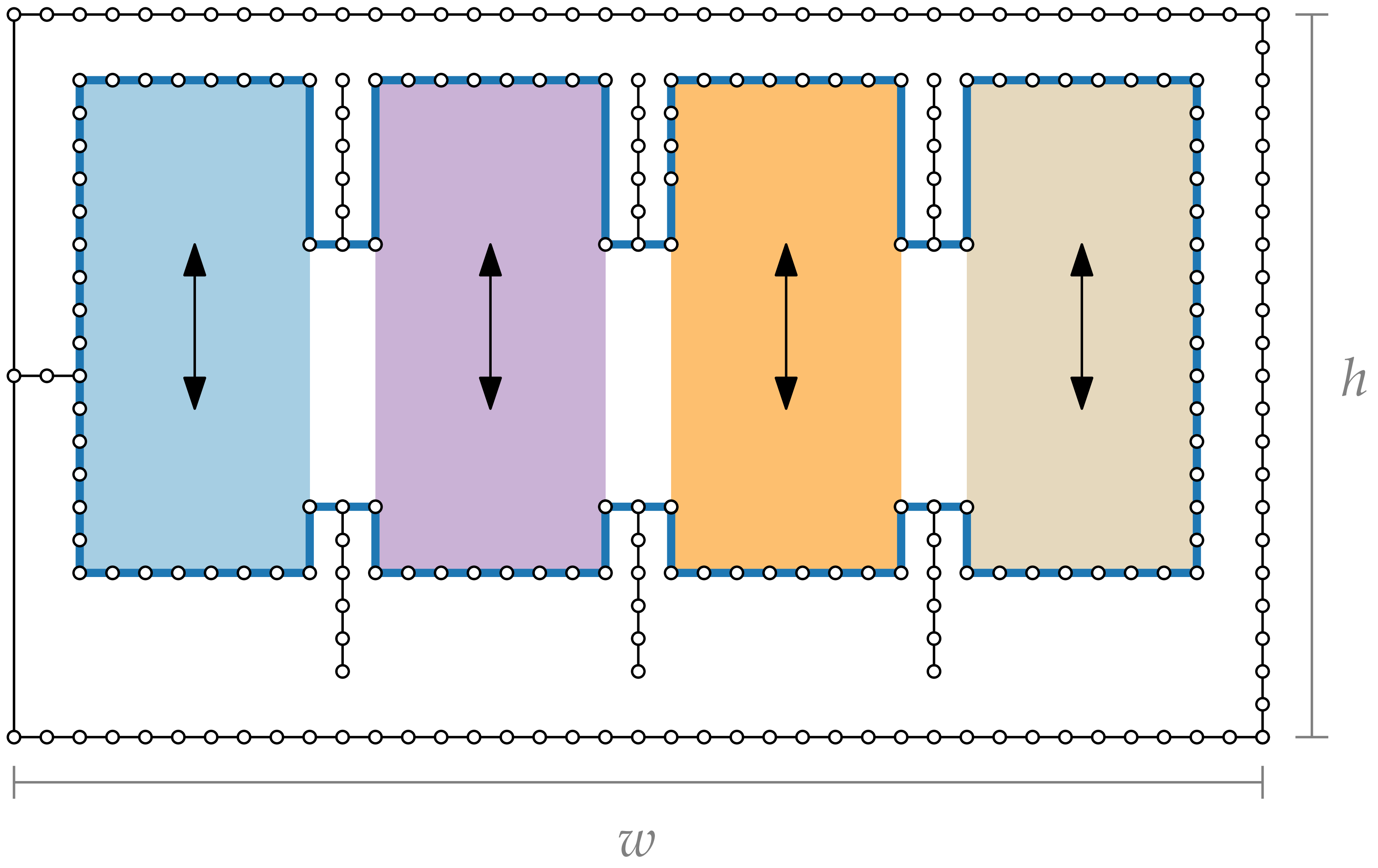

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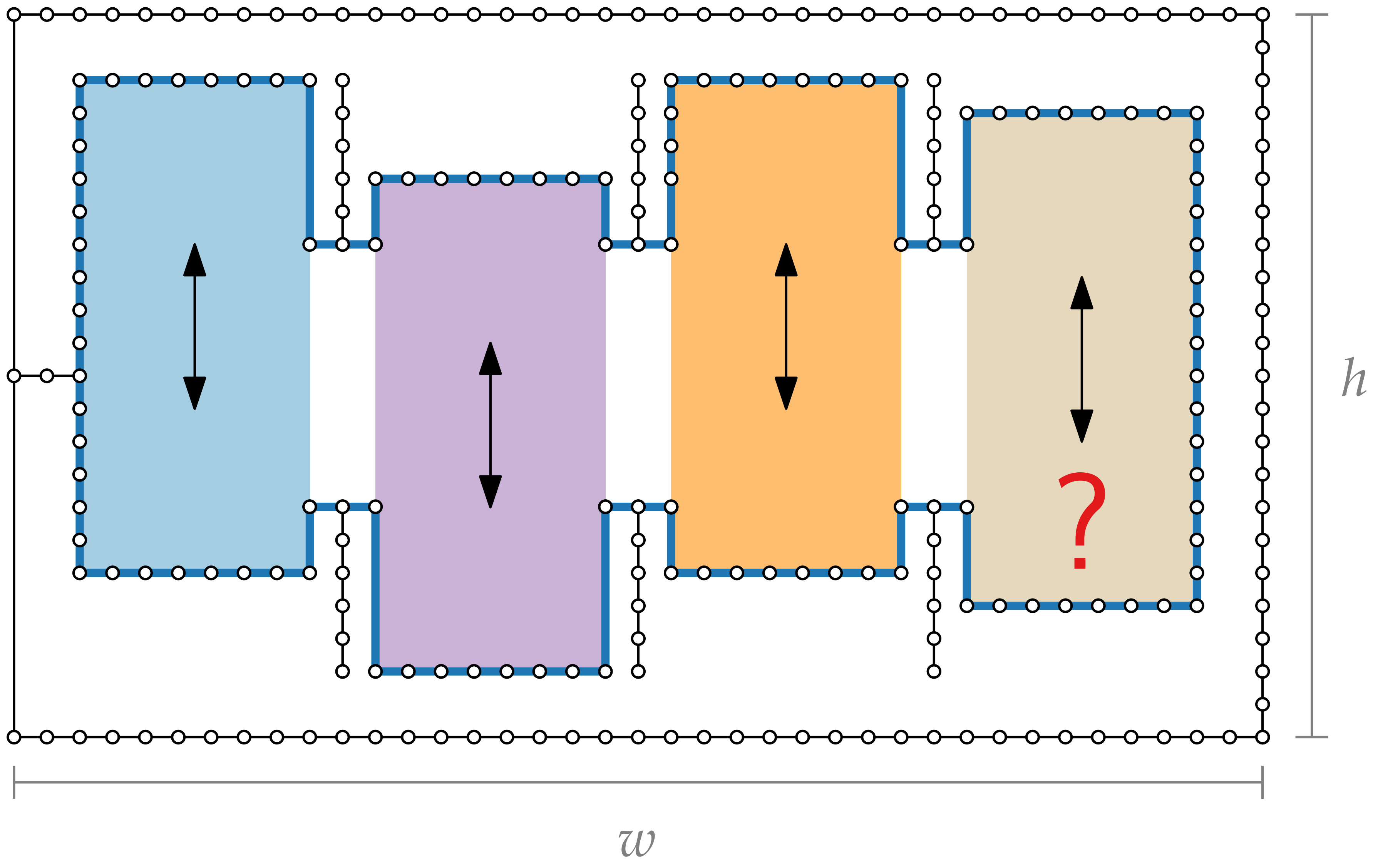

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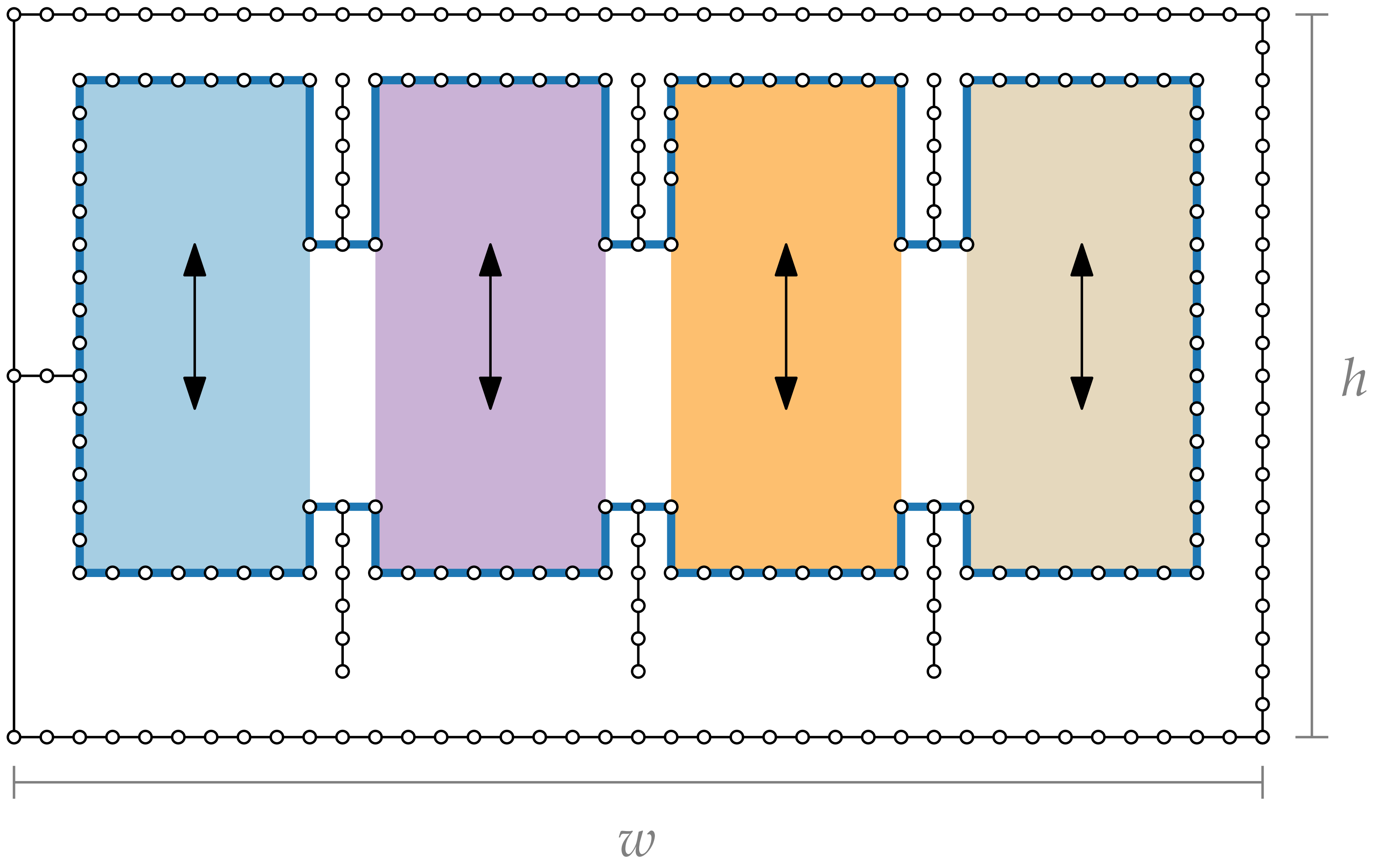

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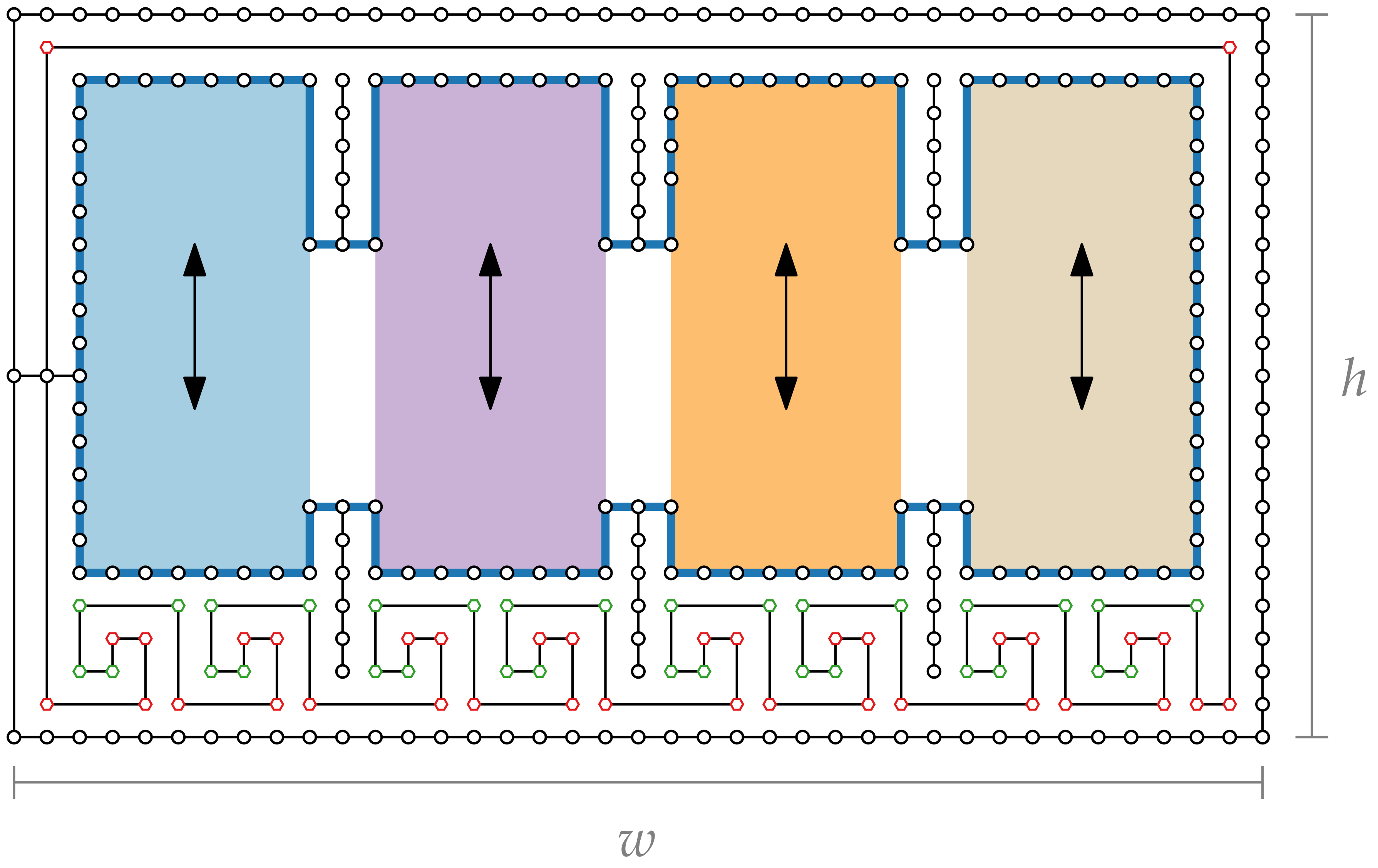

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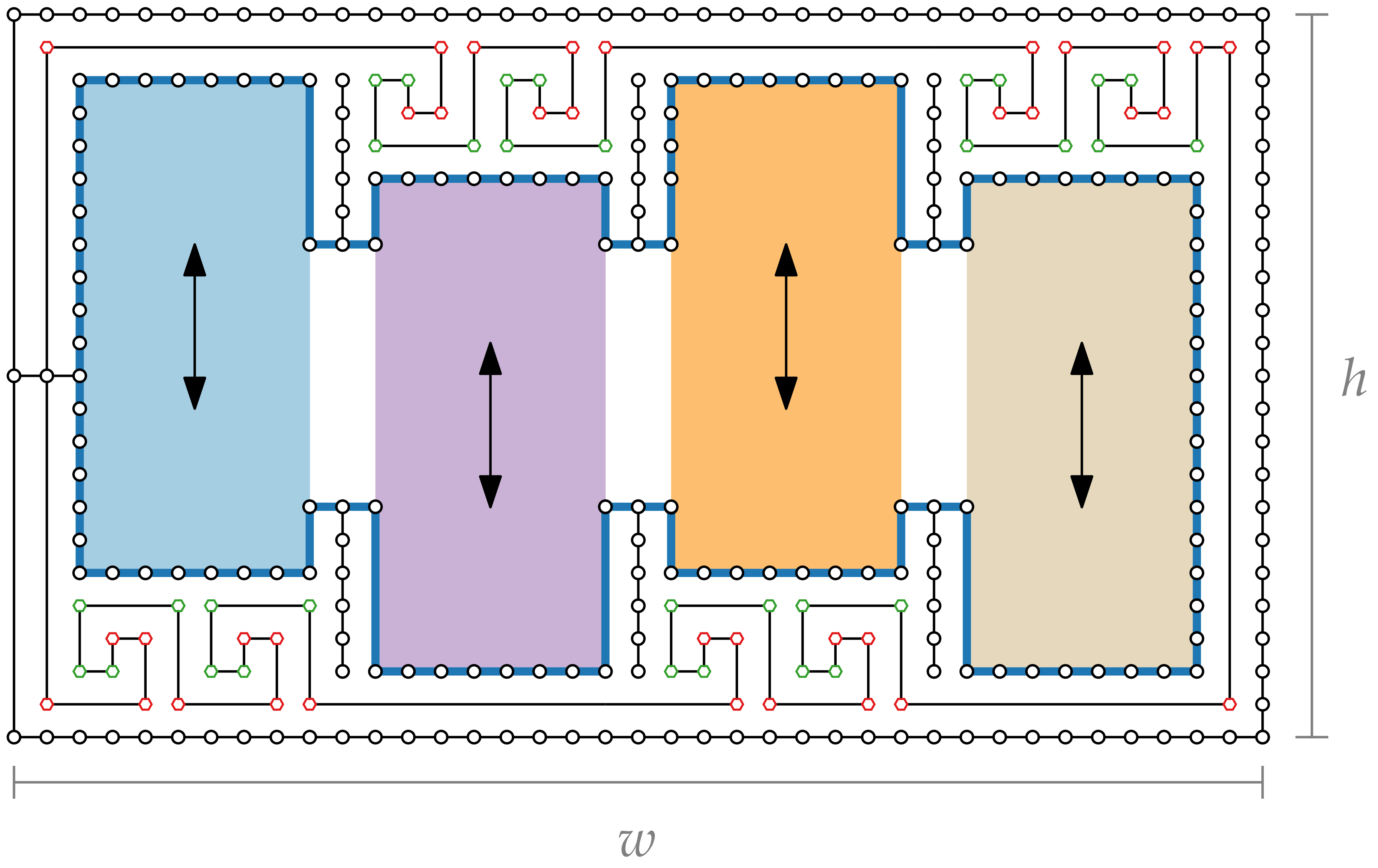

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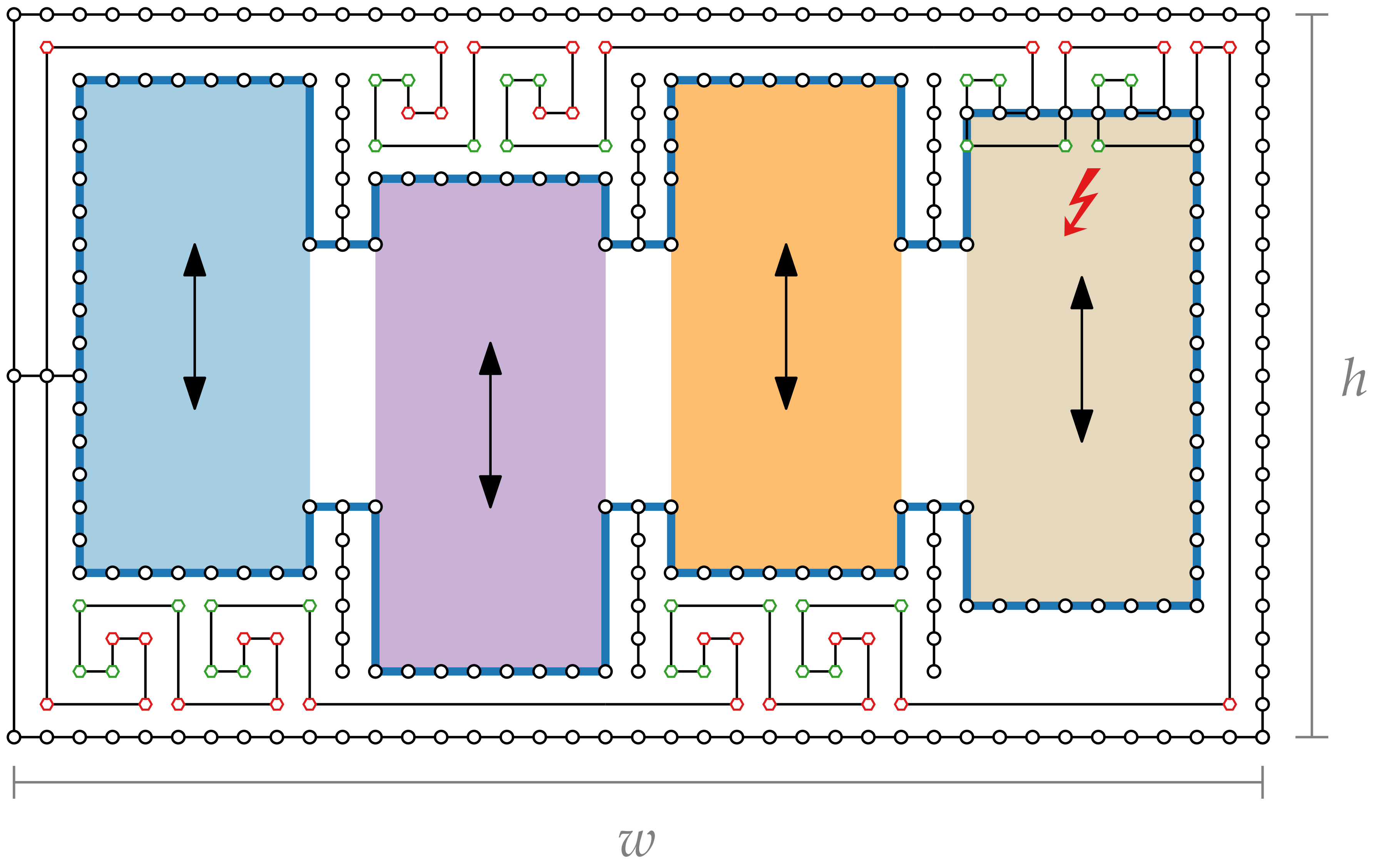

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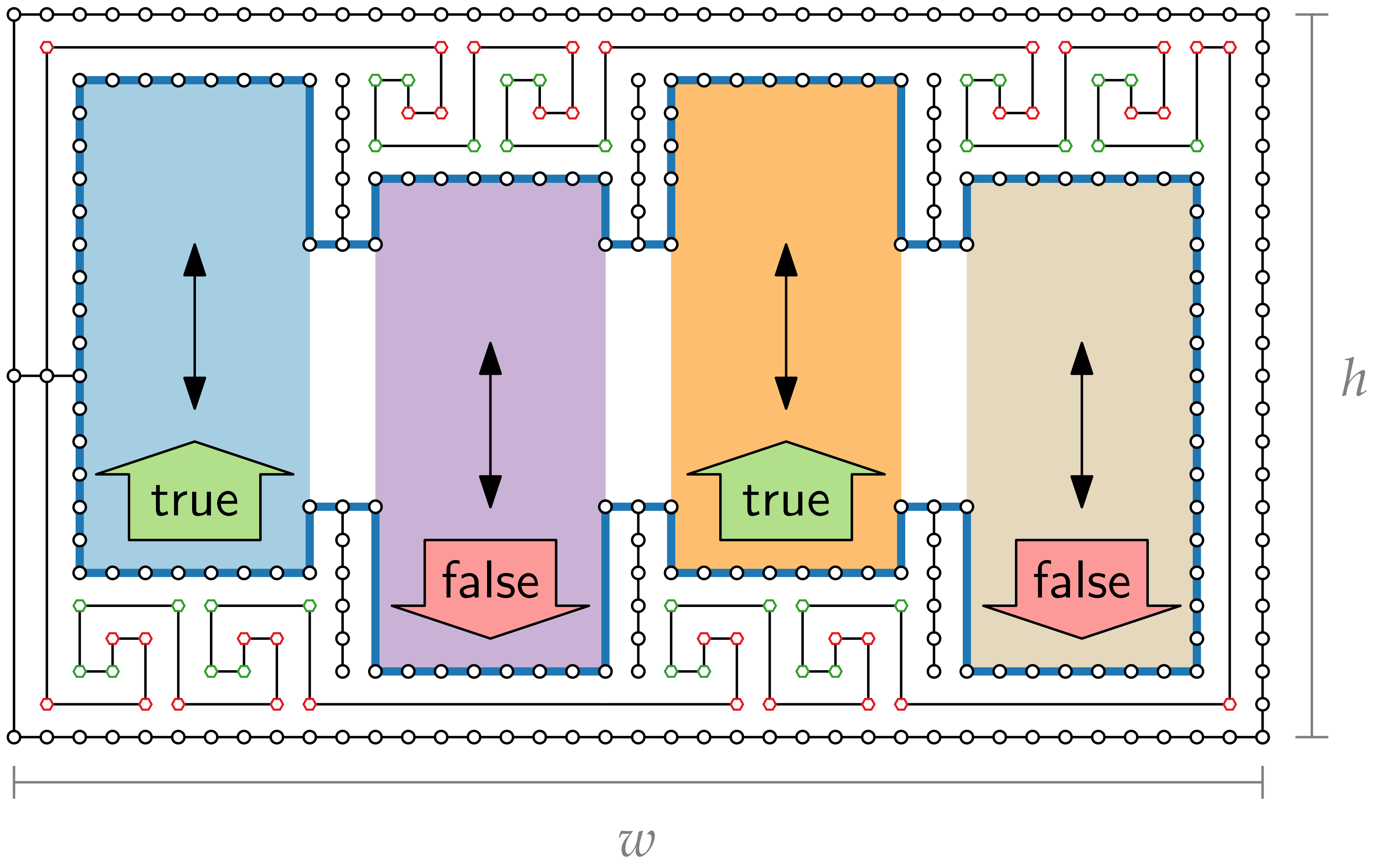

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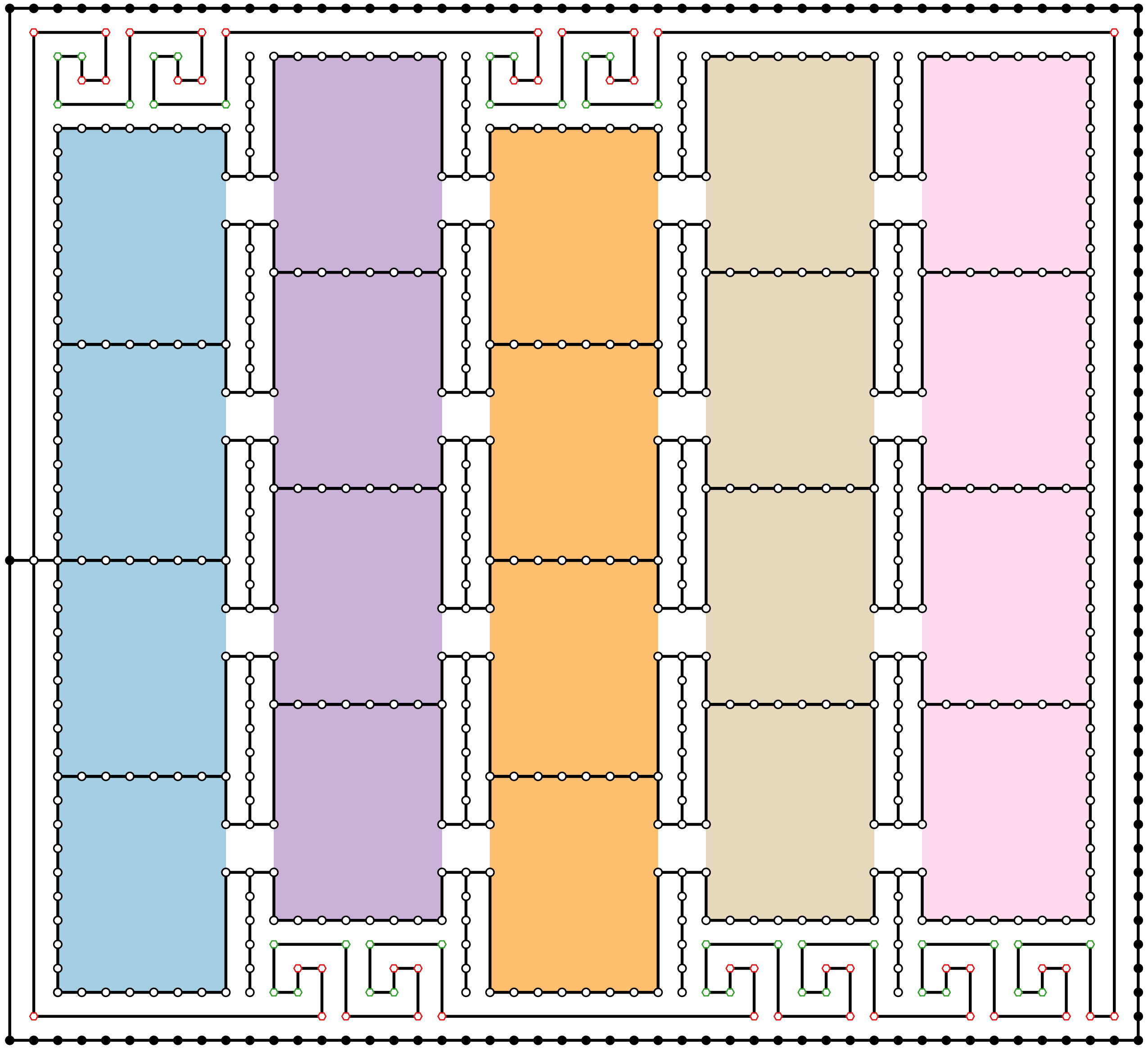

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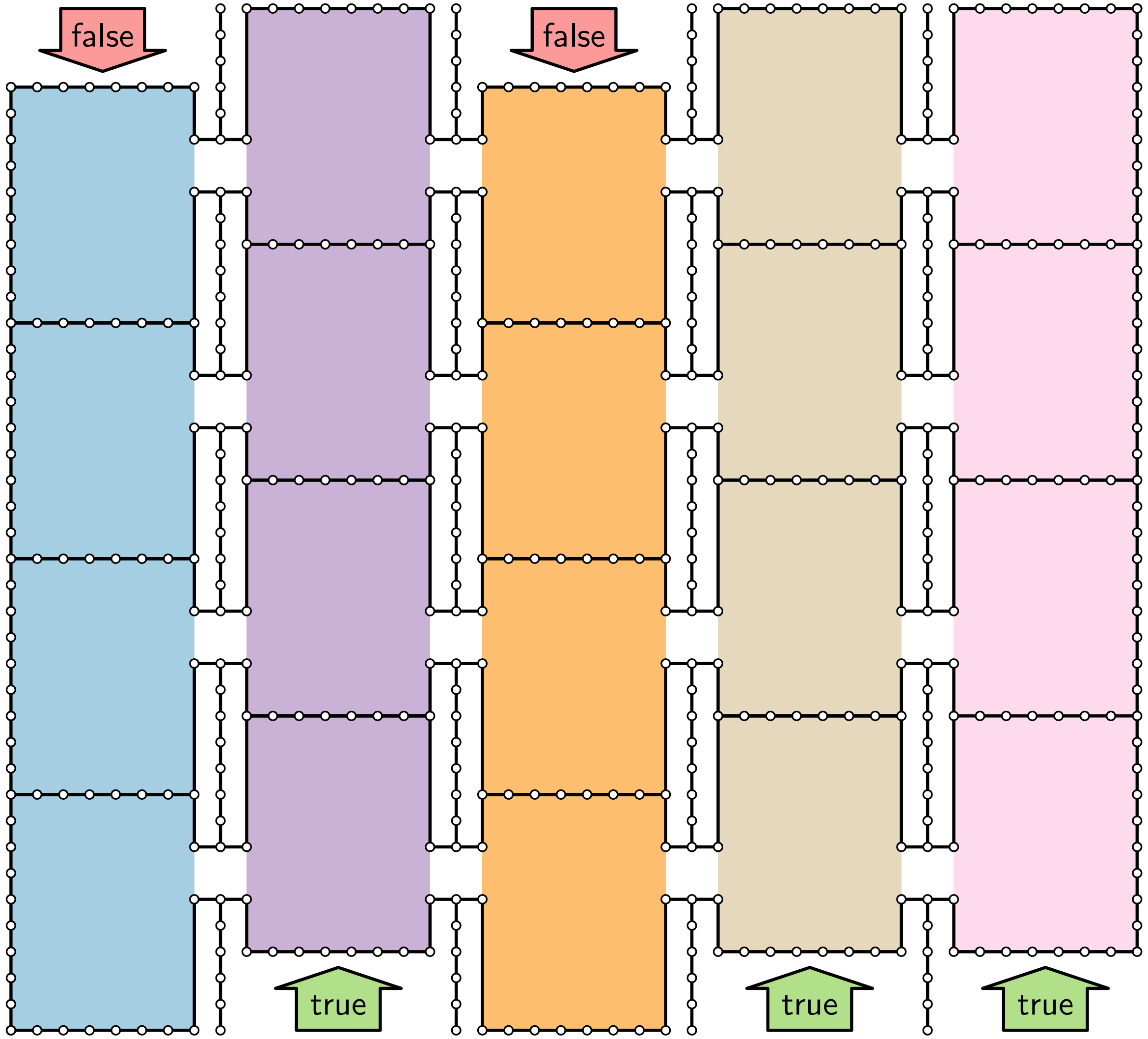

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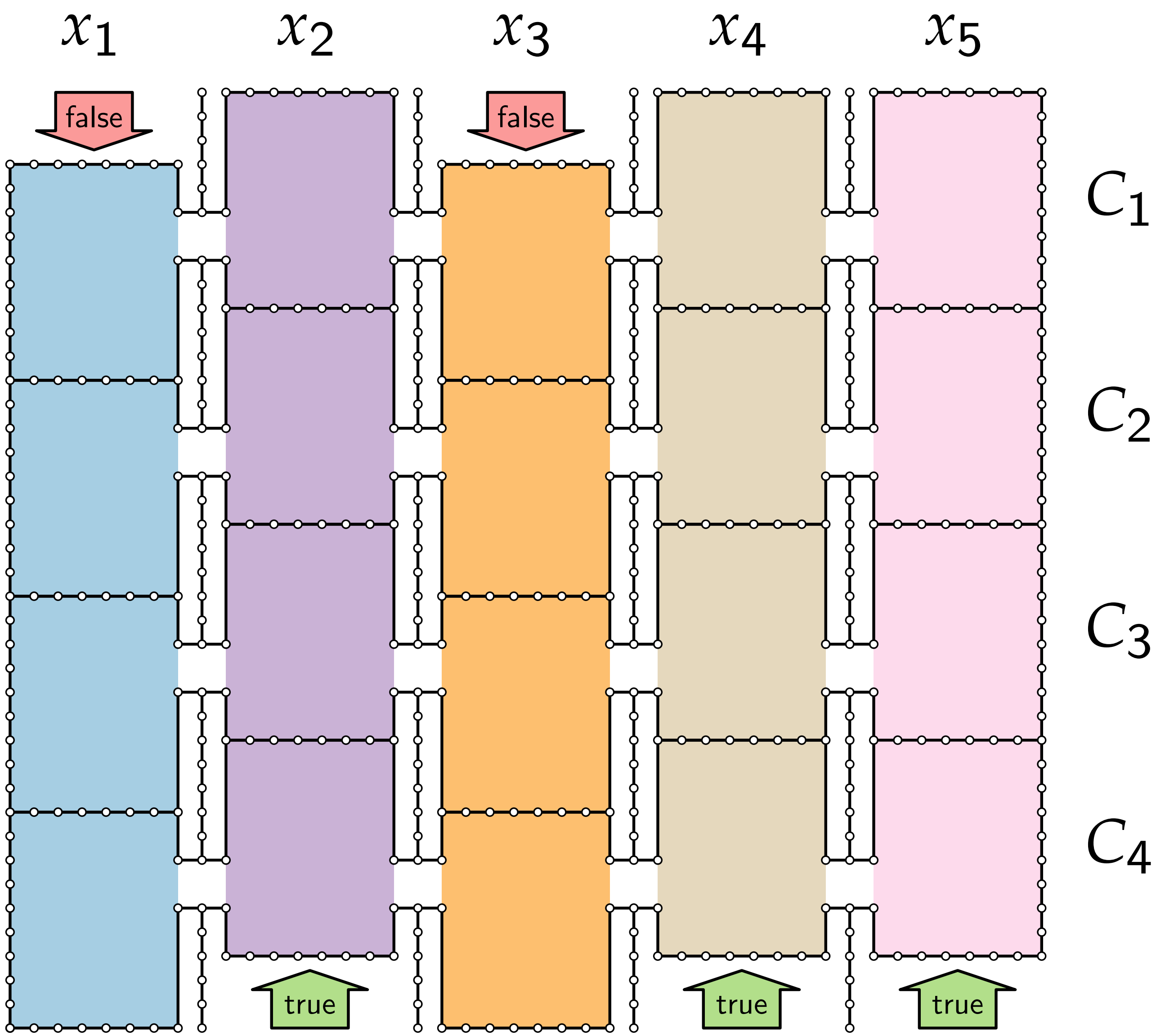

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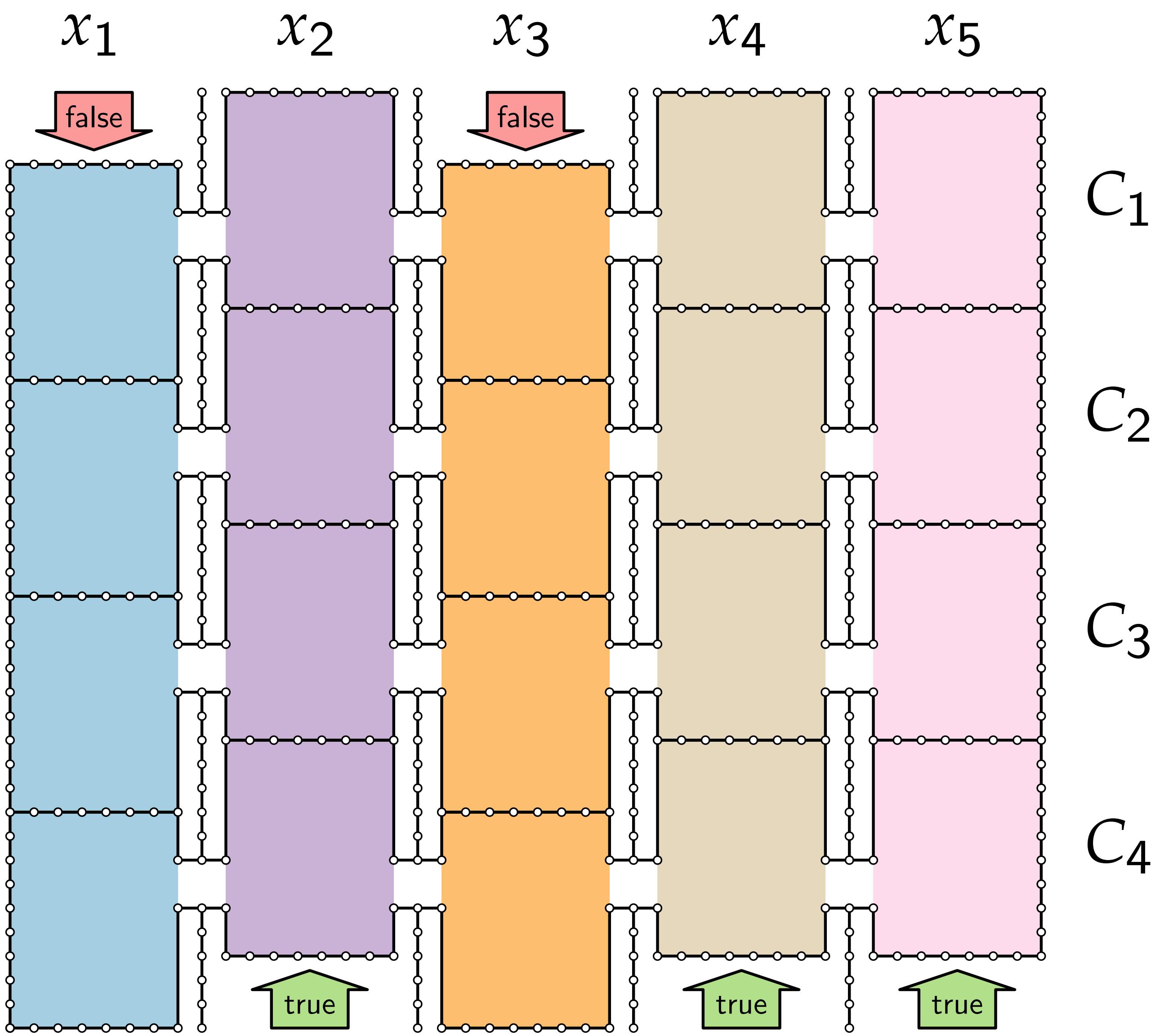

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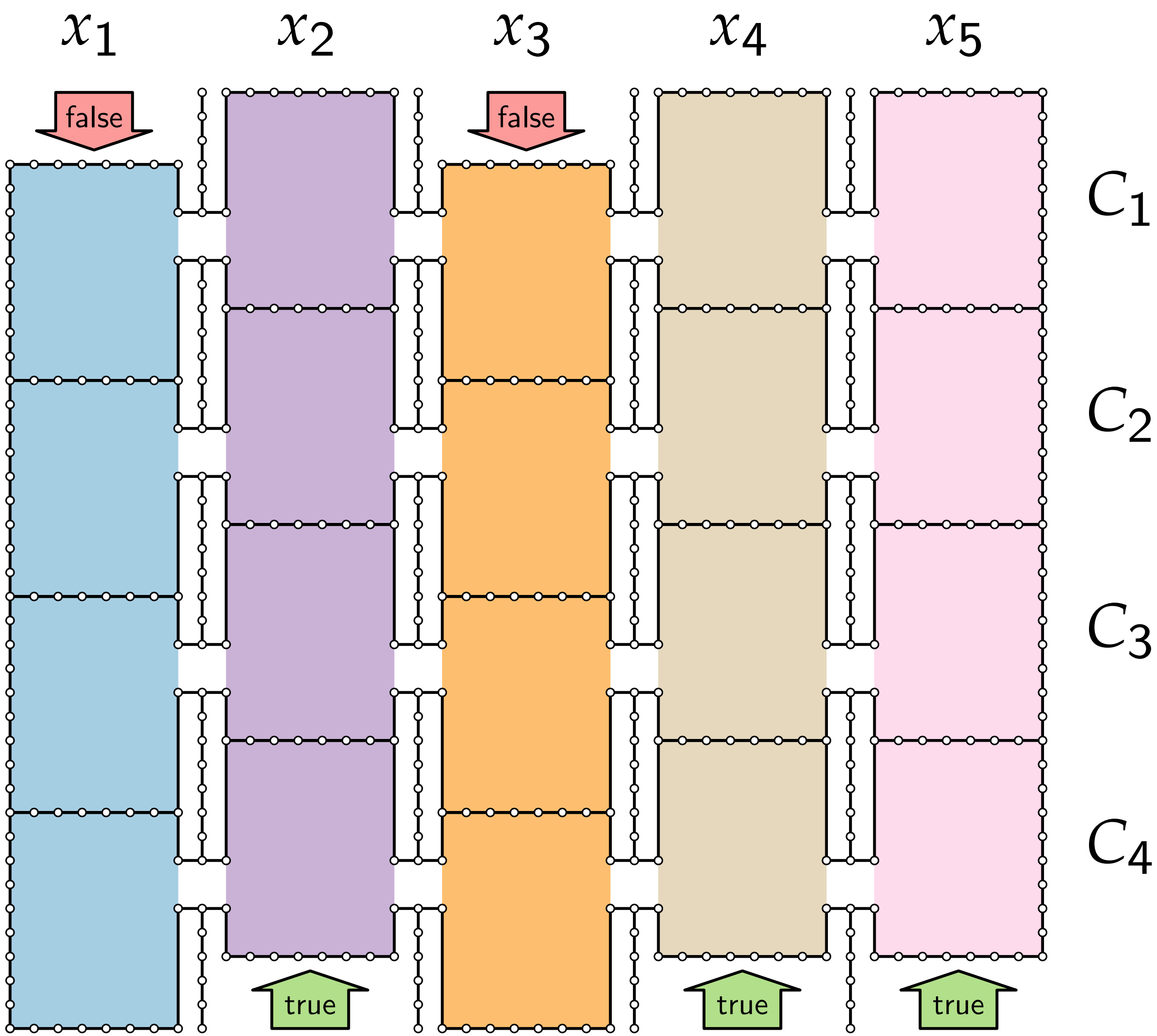

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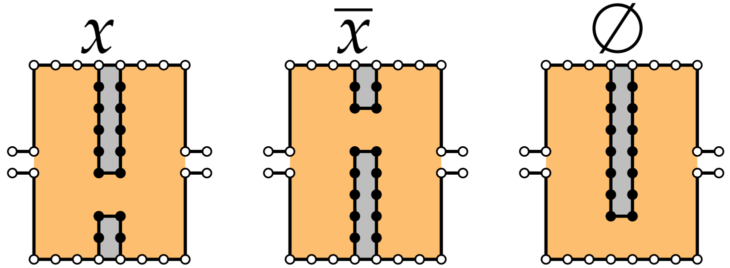

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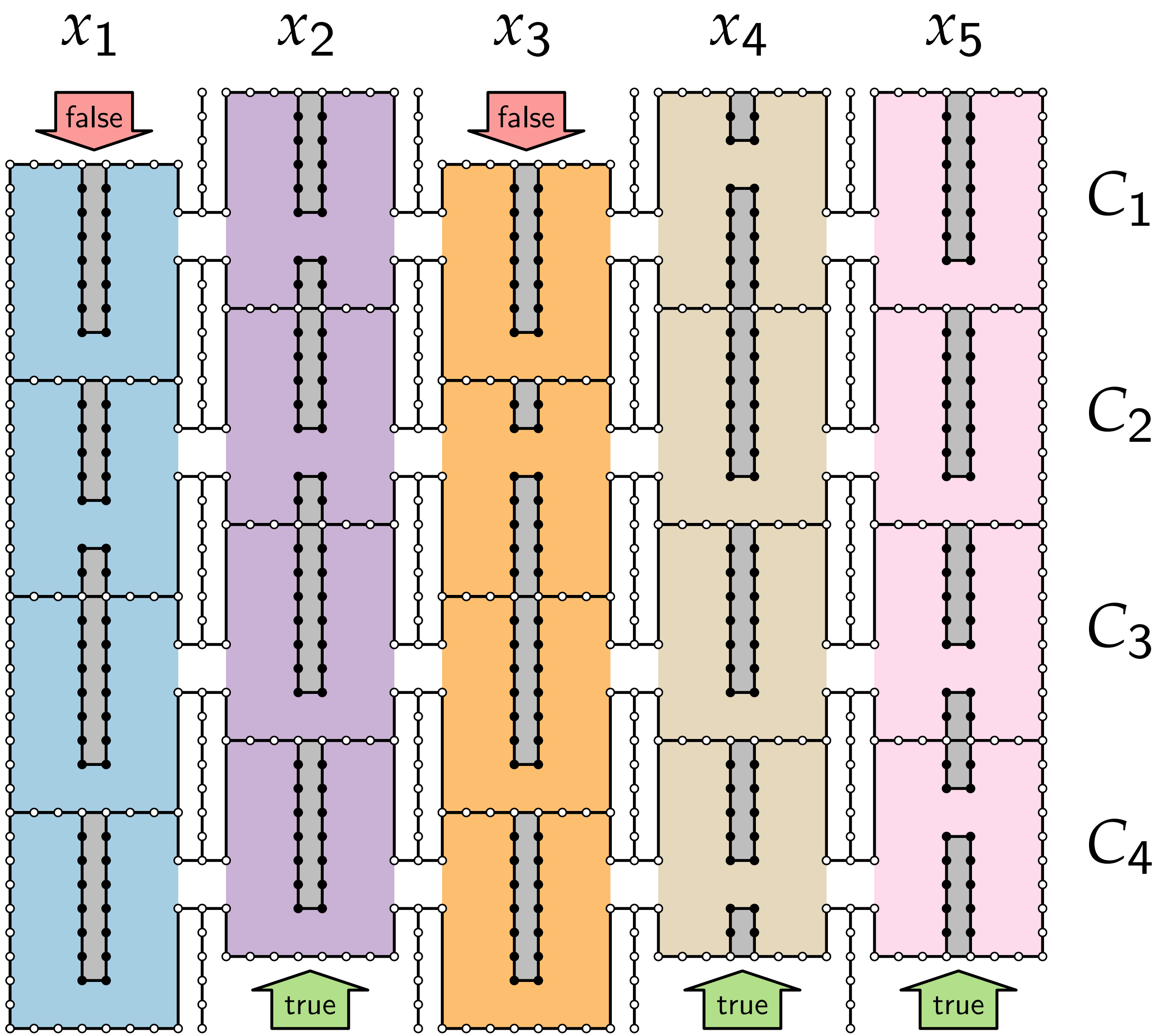

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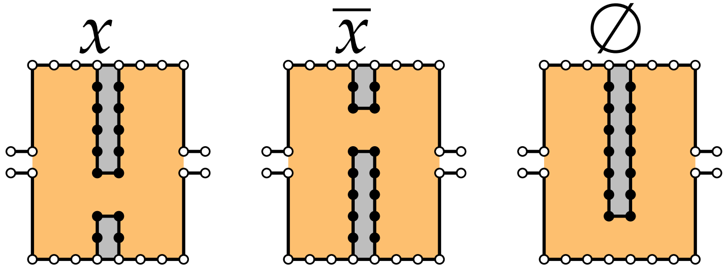

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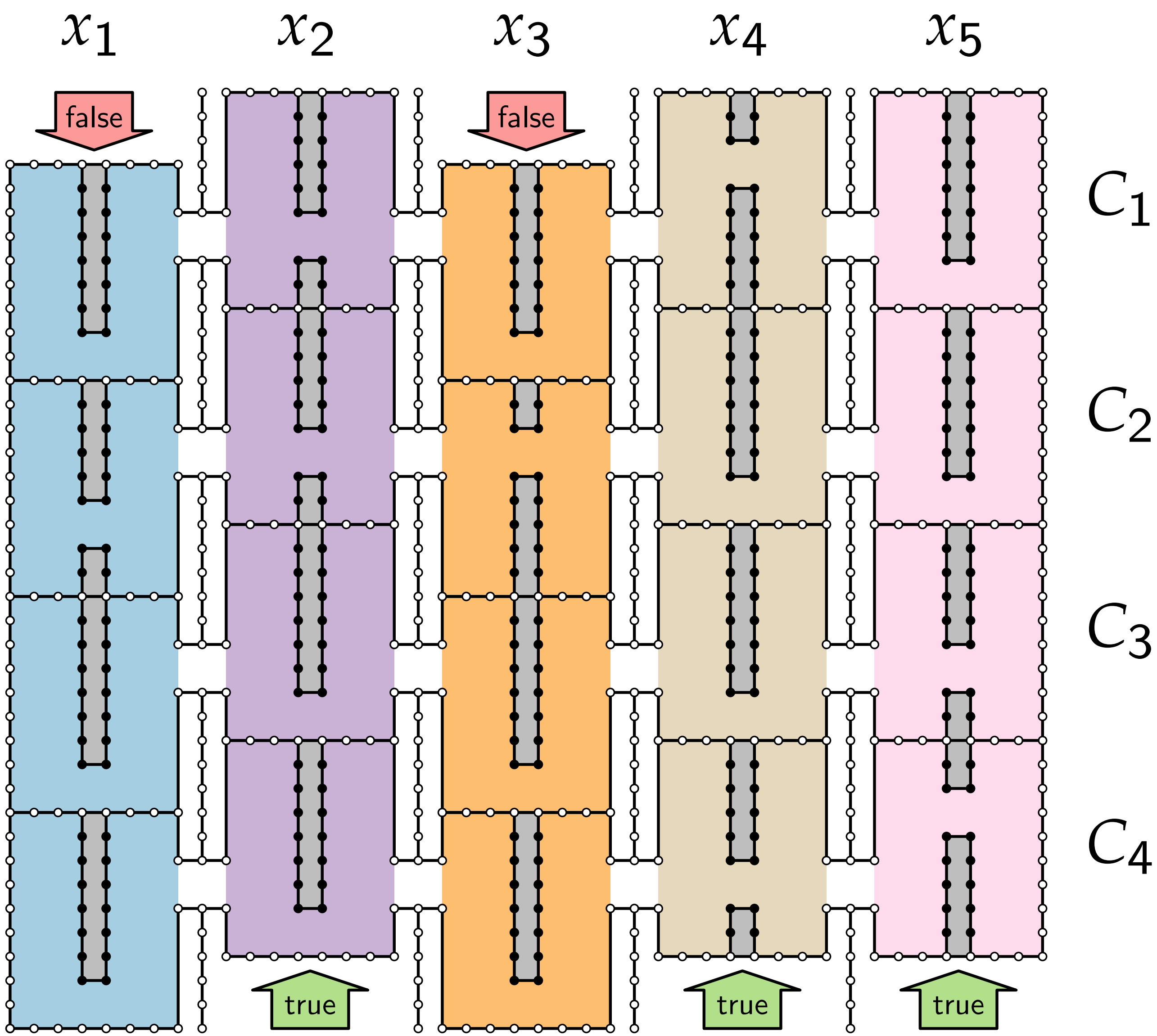

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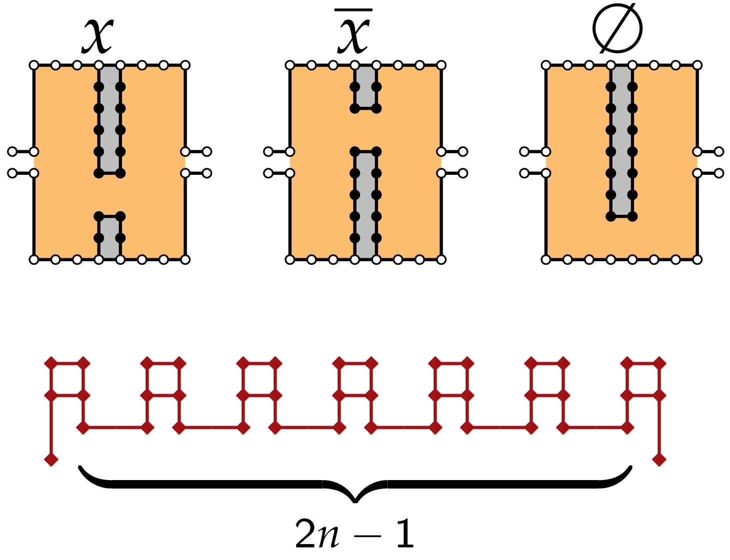

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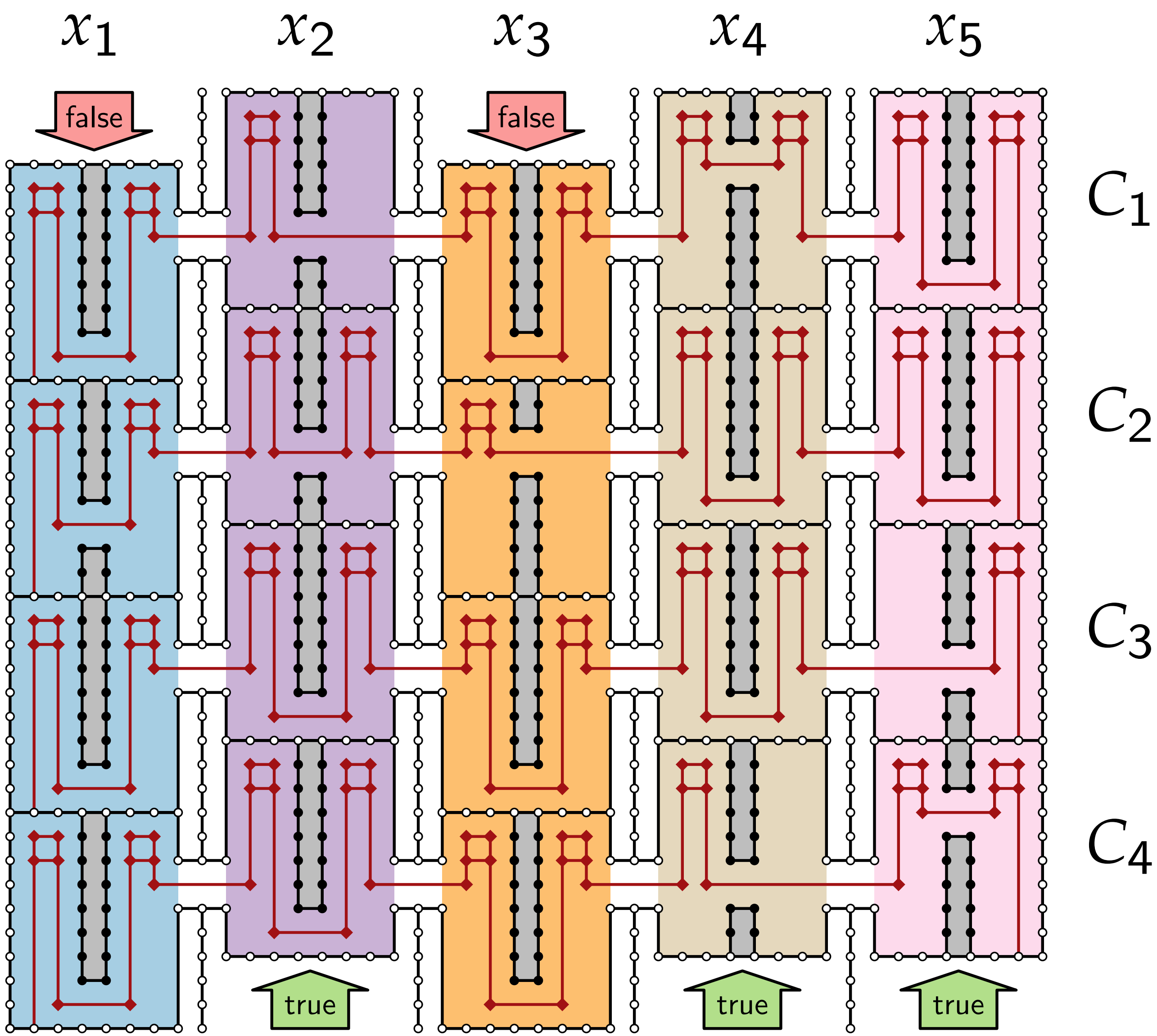

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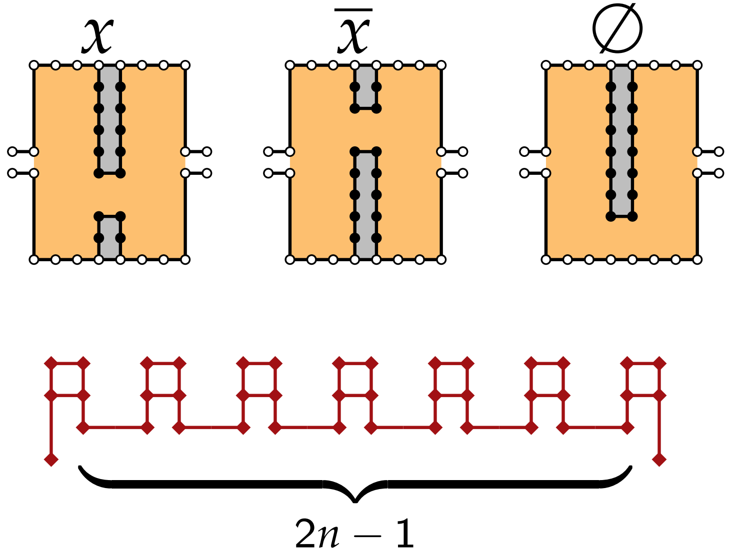

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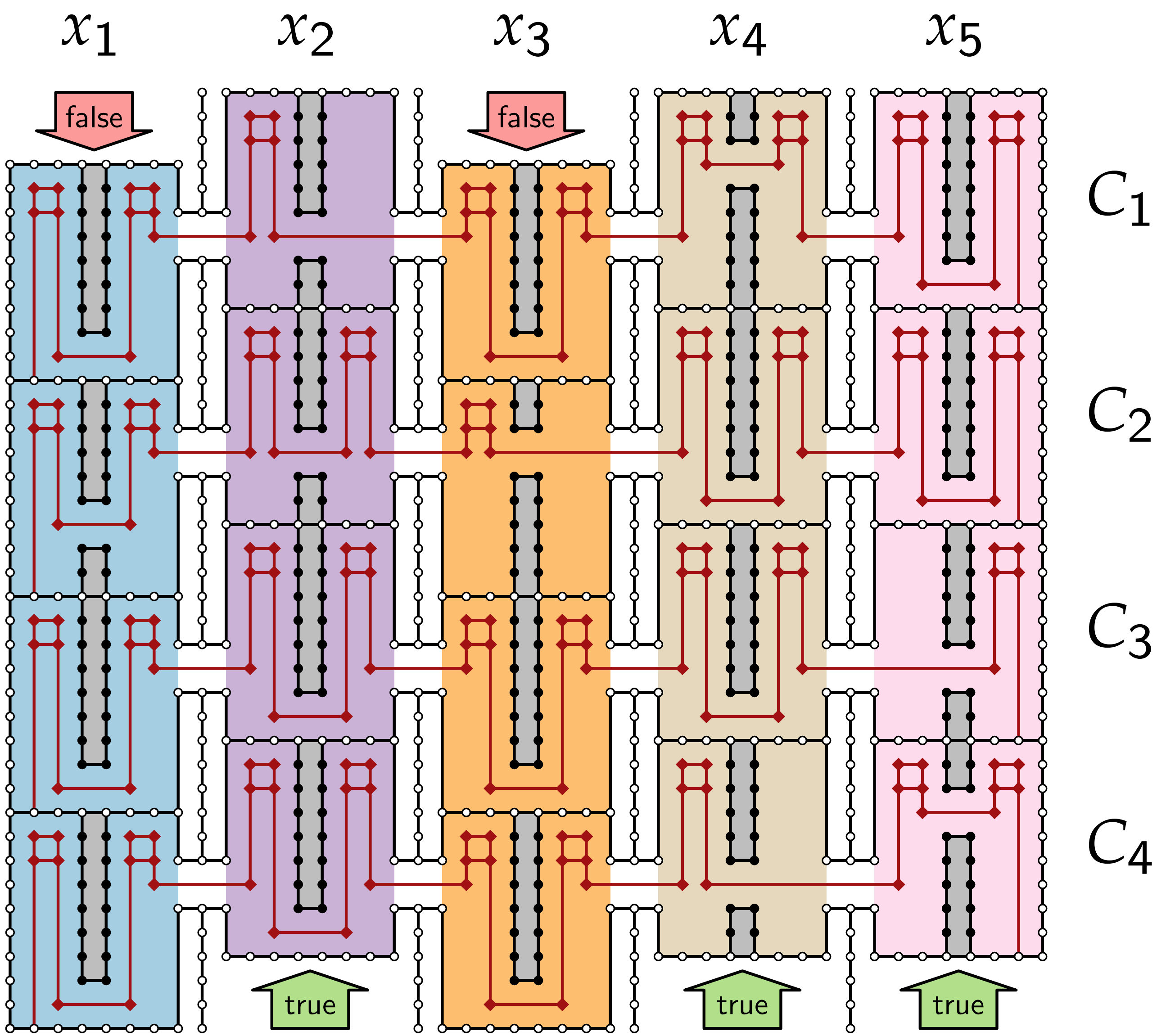

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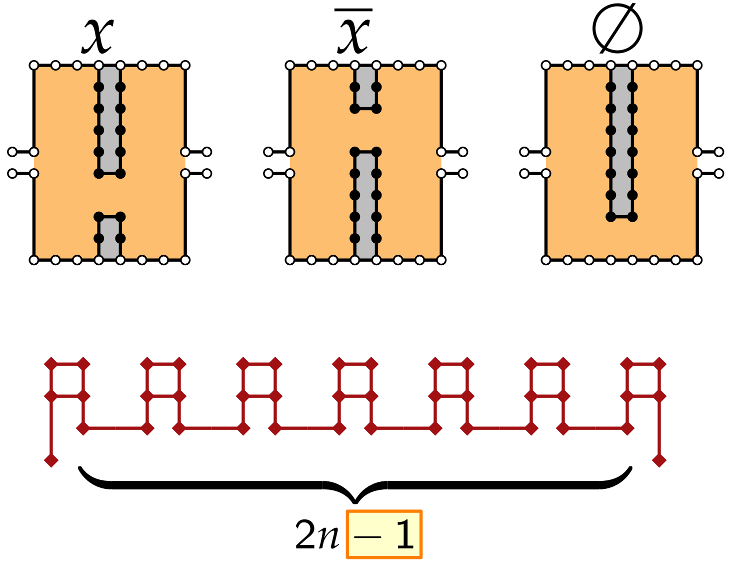
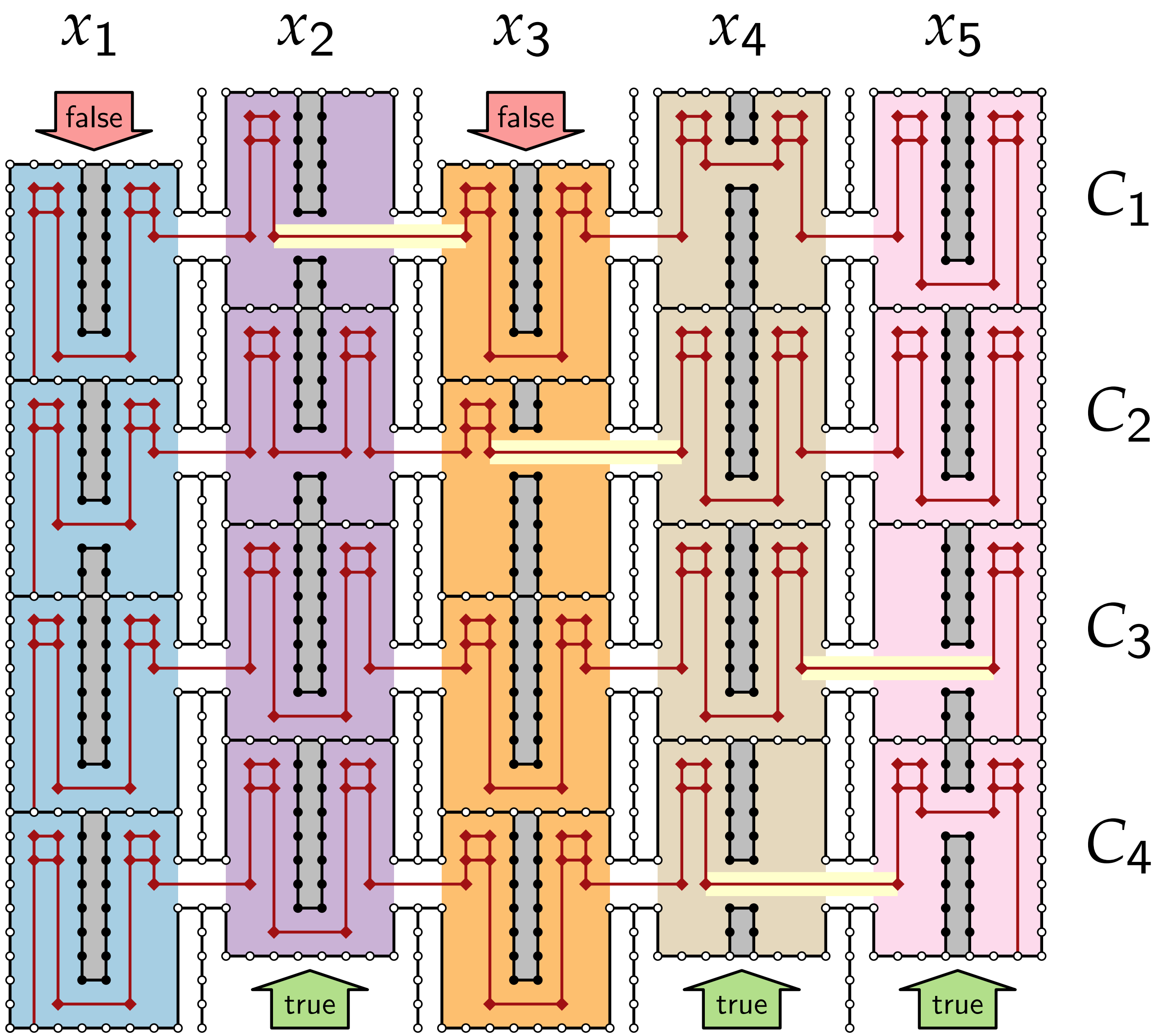


Clause Gadgets



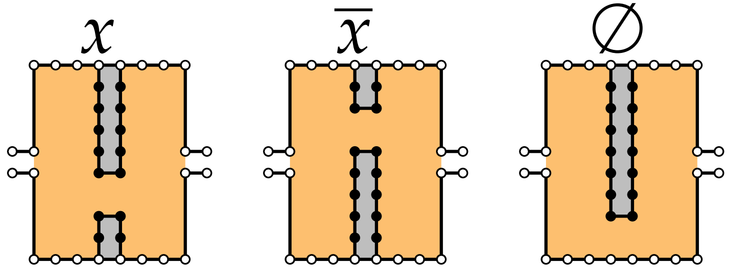
Example:

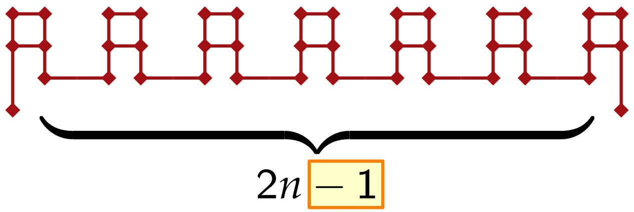
$C_1 = x_2 \vee \overline{x_4}$

$C_2 = x_1 \vee x_2 \vee \overline{x_3}$

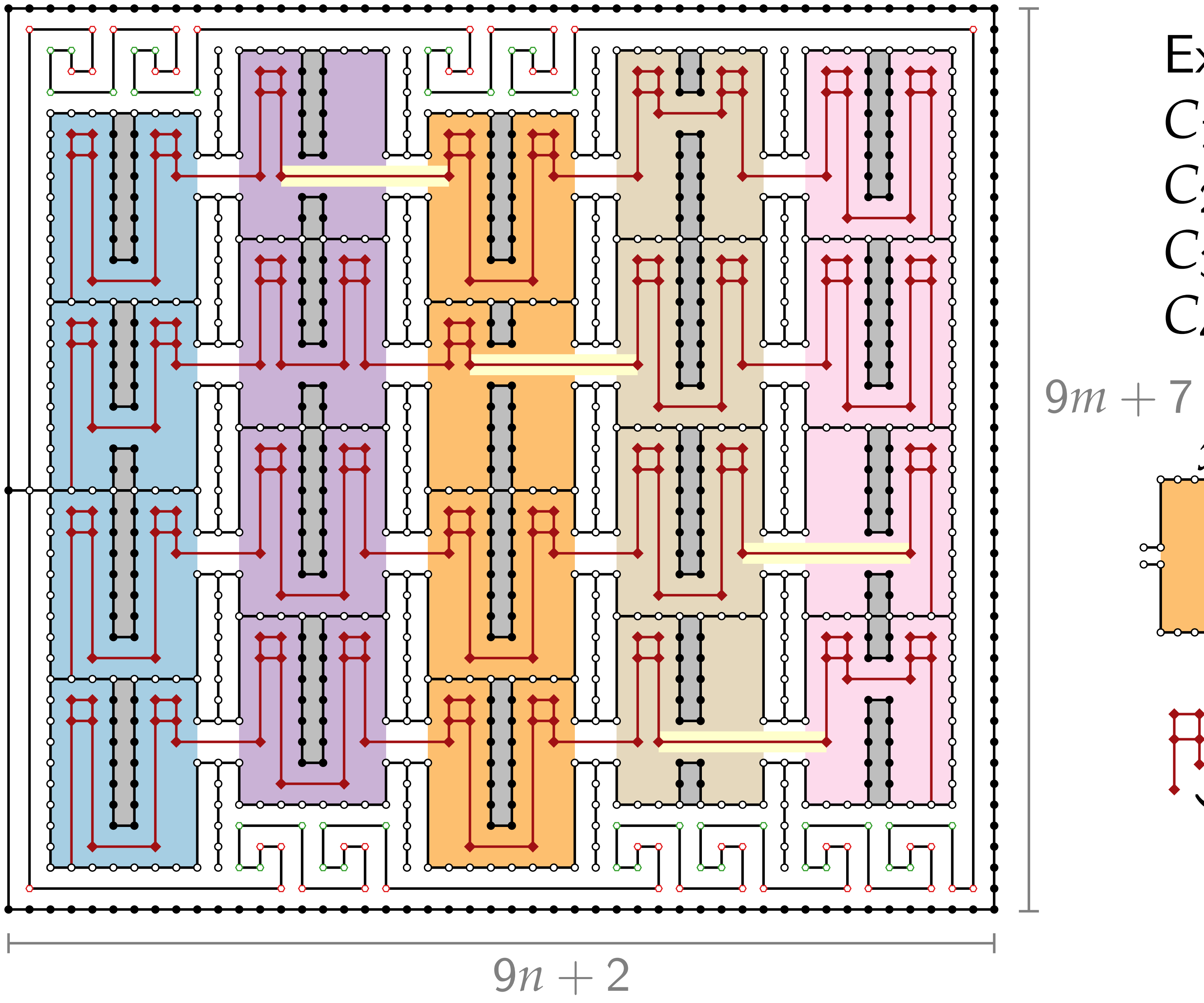
$C_3 = x_5$

$C_4 = x_4 \vee \overline{x_5}$





Full reduction



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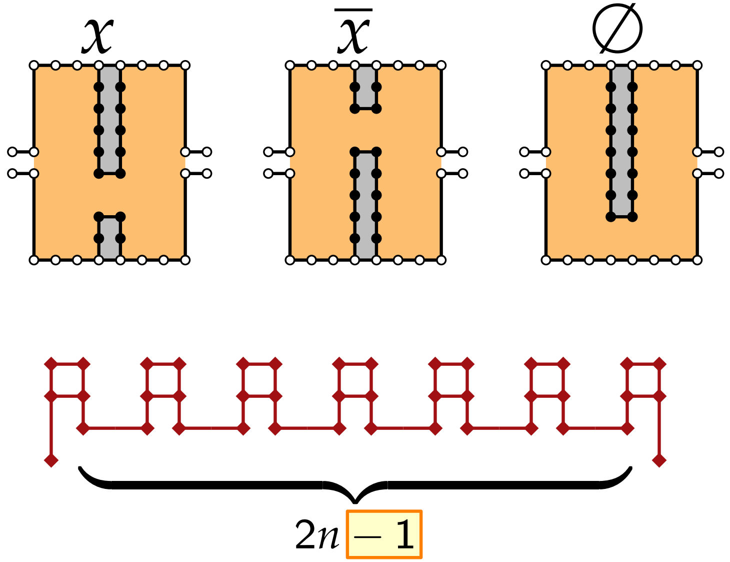
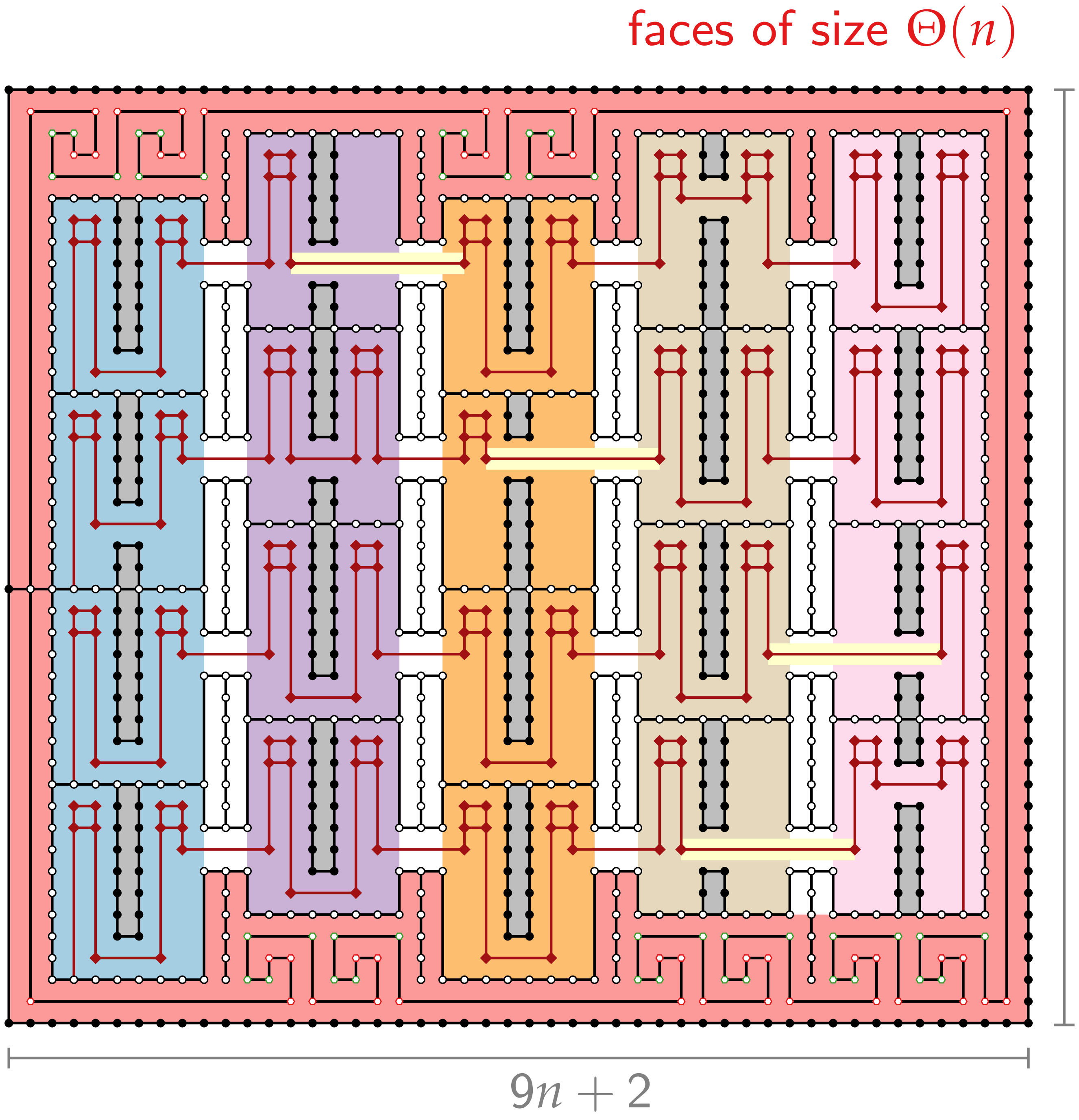


Full reduction



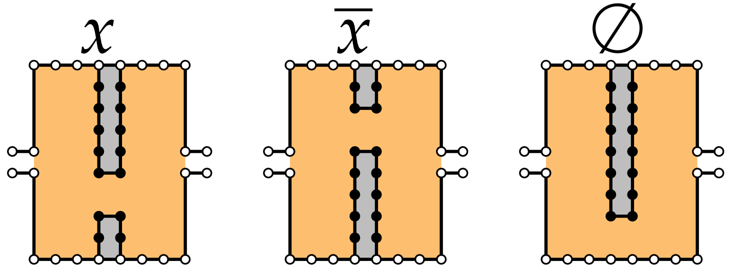
Example:

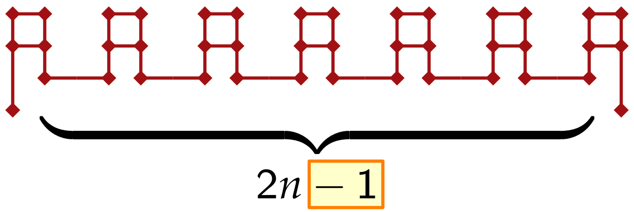
$C_1 = x_2 \lor \overline{x_4}$

$C_2 = x_1 \lor x_2 \lor \overline{x_3}$

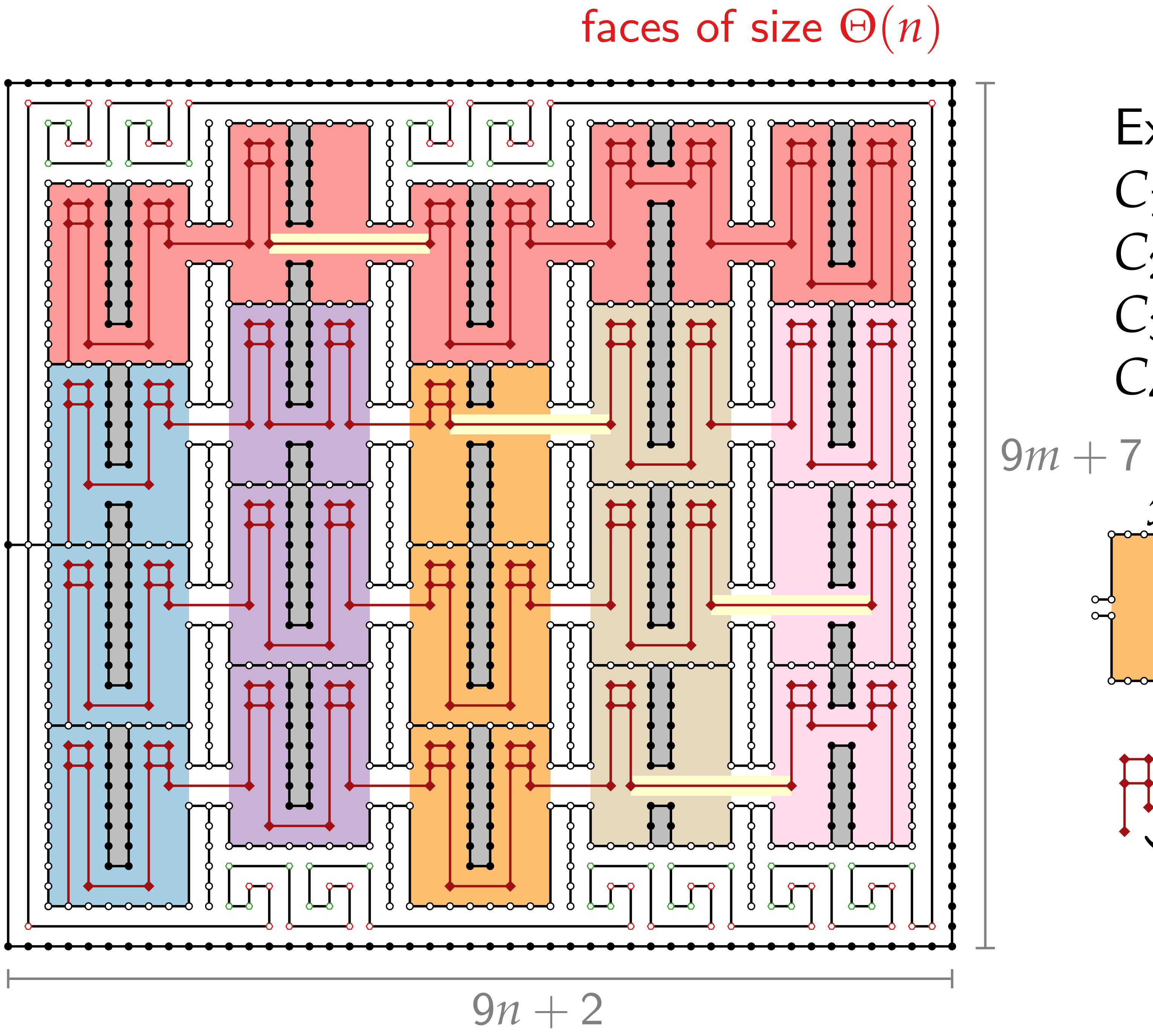
$C_3 = x_5$

$C_4 = x_4 \lor \overline{x_5}$





Full reduction



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$C_2 = x_1 \vee x_2 \vee \overline{x_3}$

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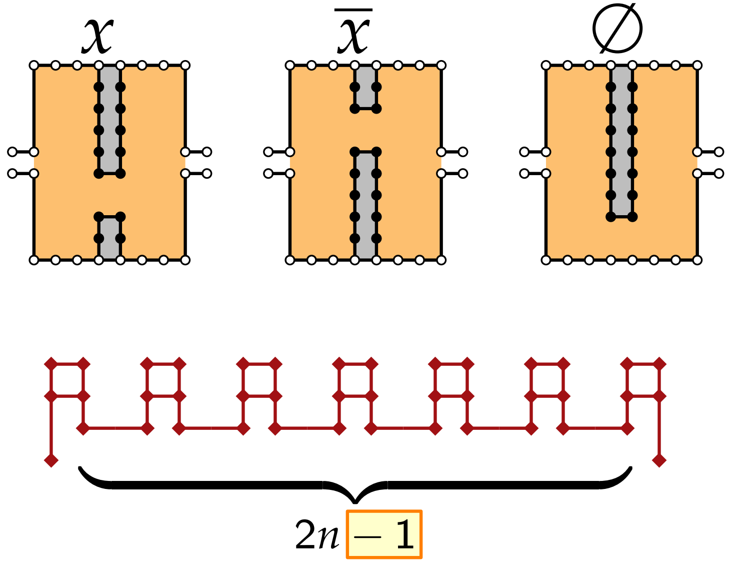
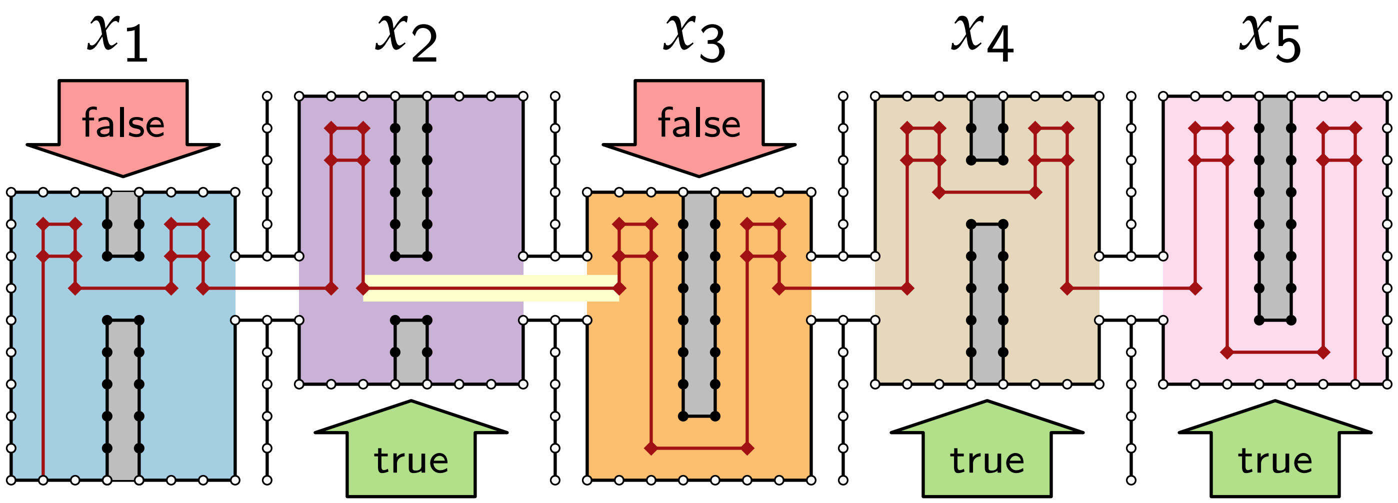
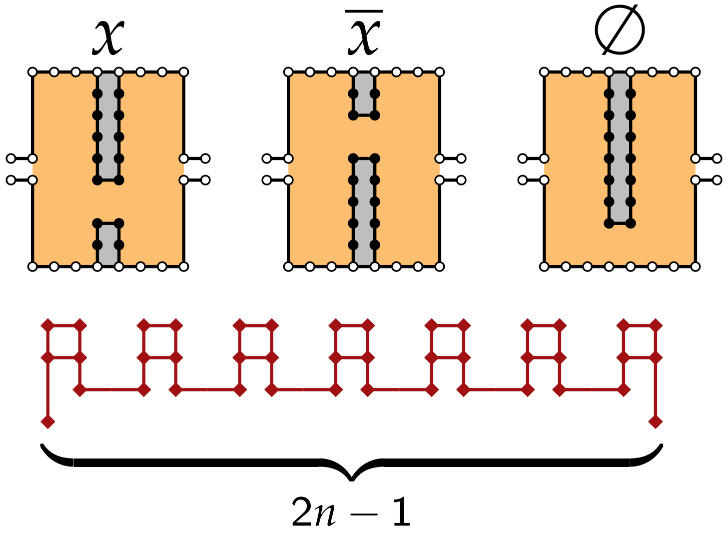


Adjusting Clause Gadgets





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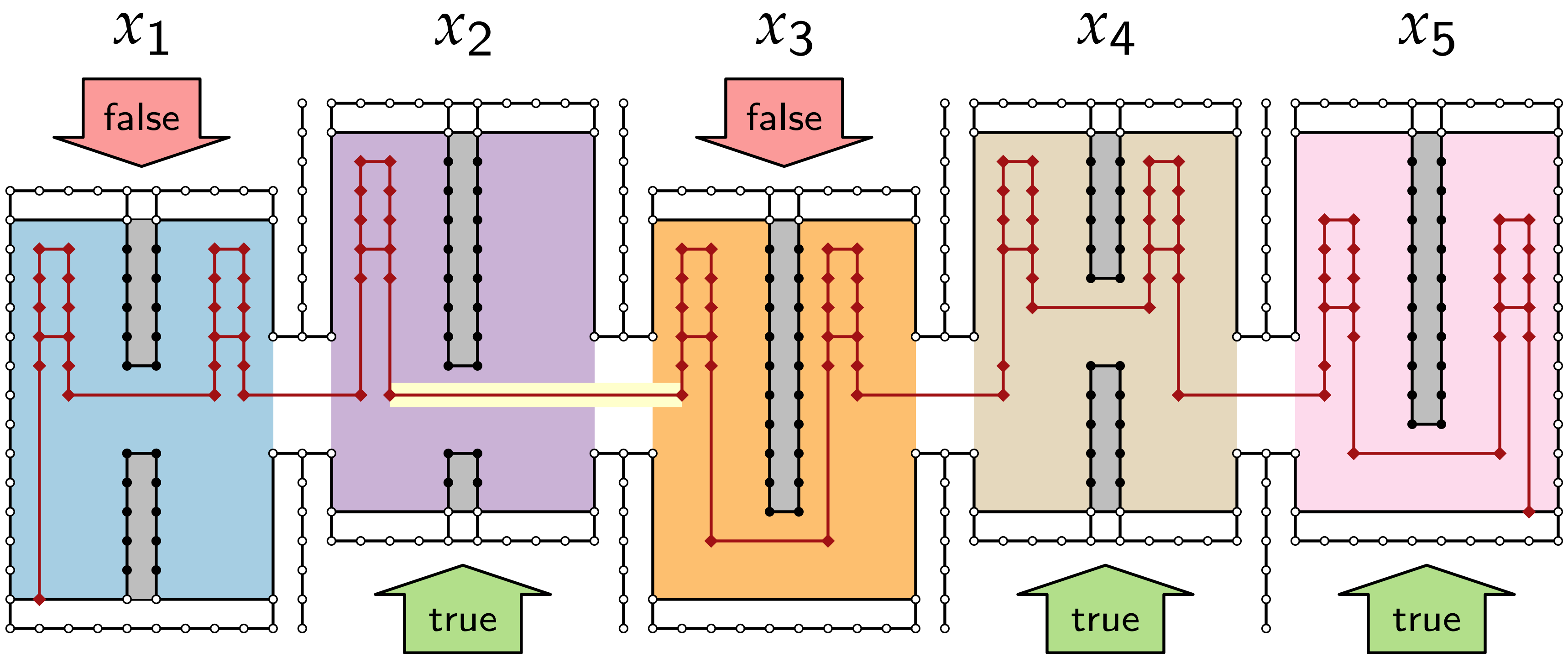

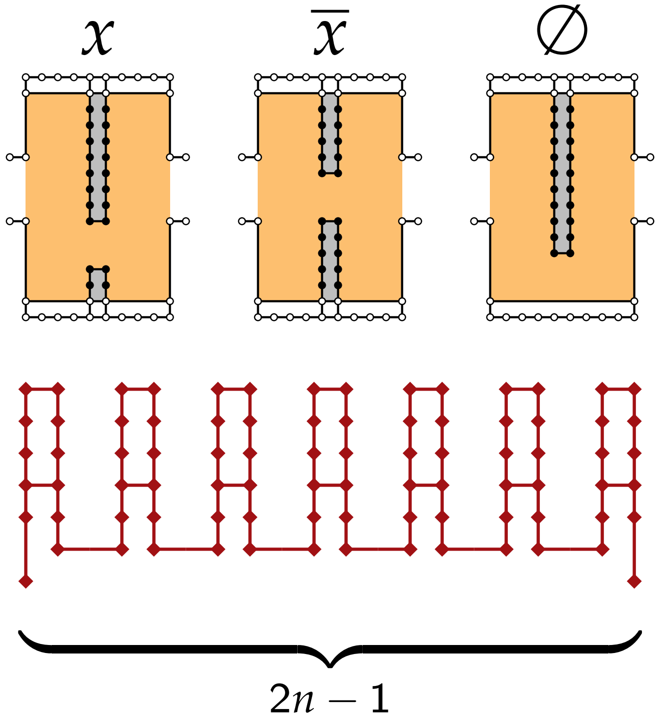

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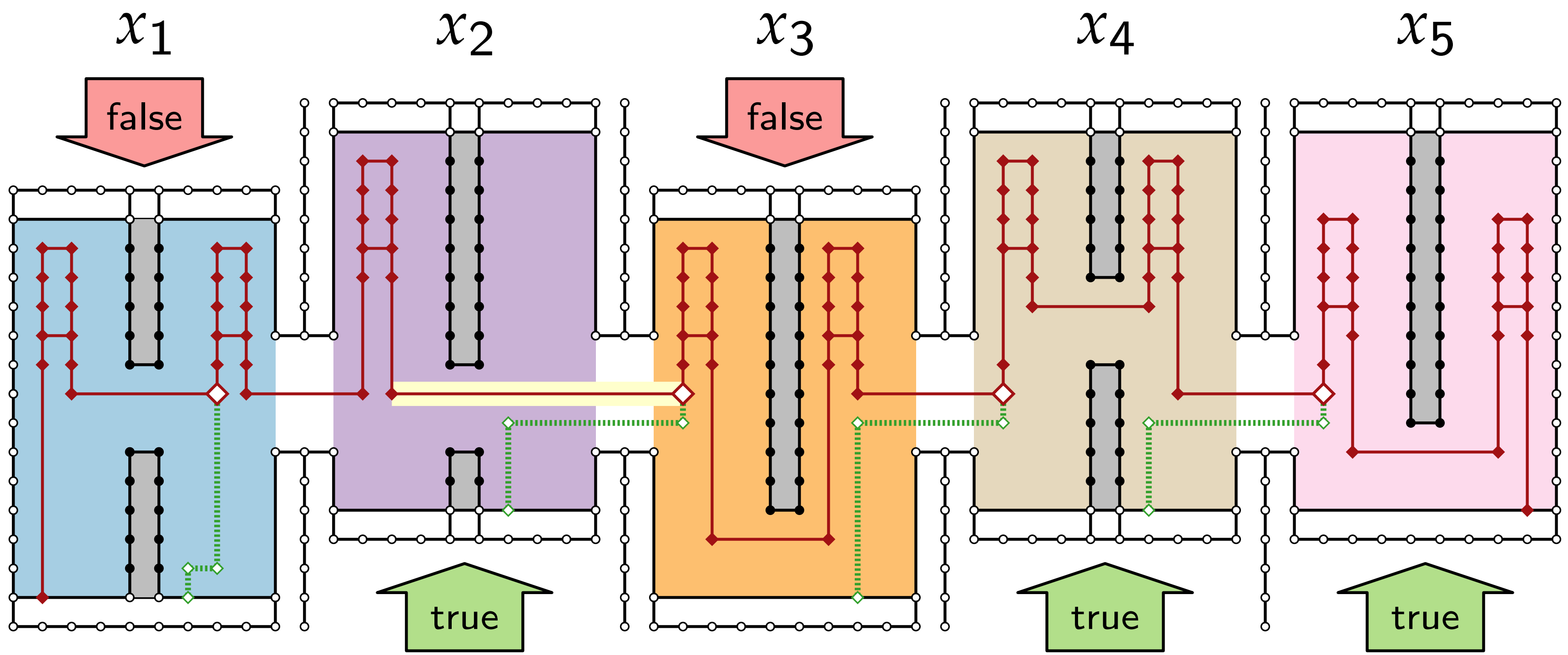

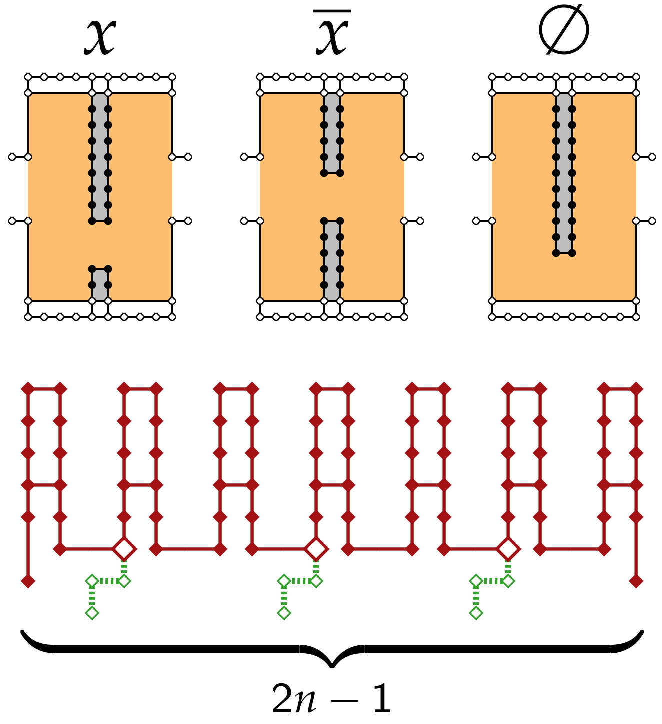

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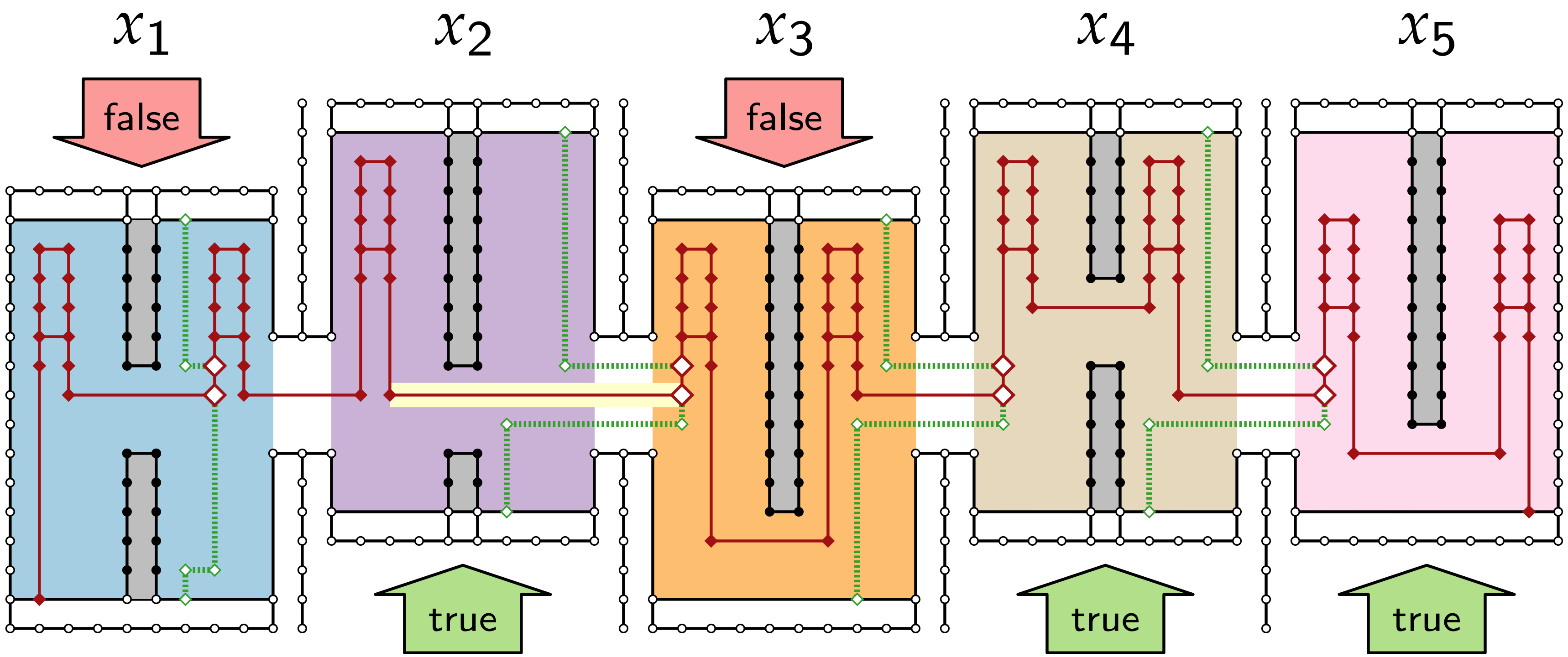
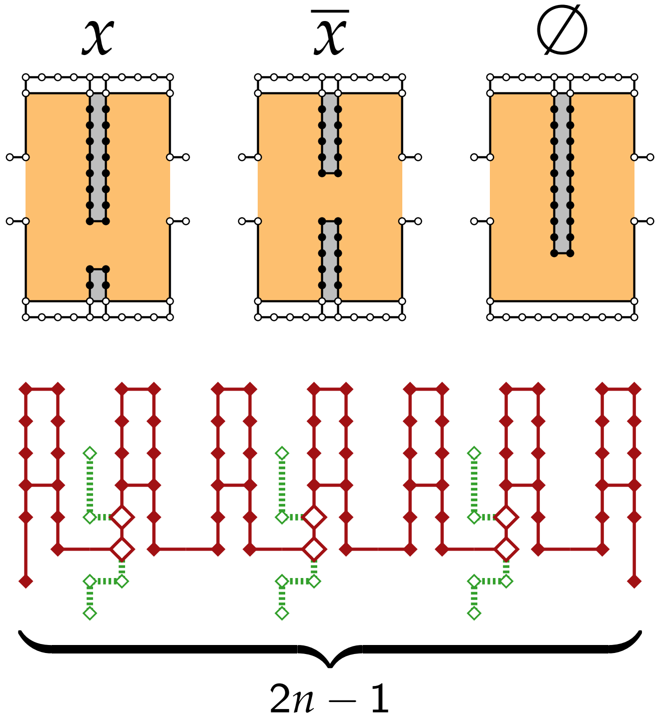
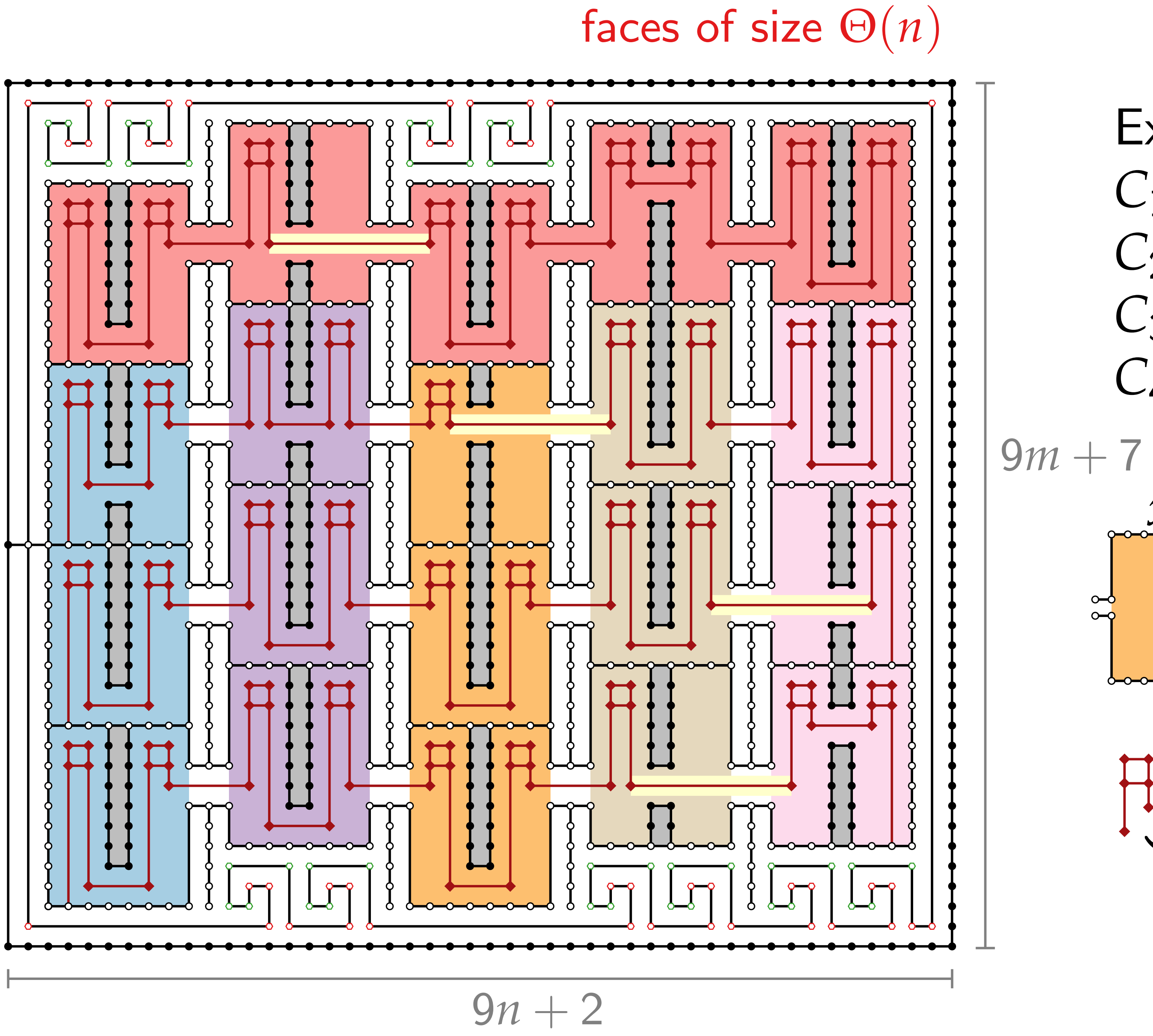




Full Reduction



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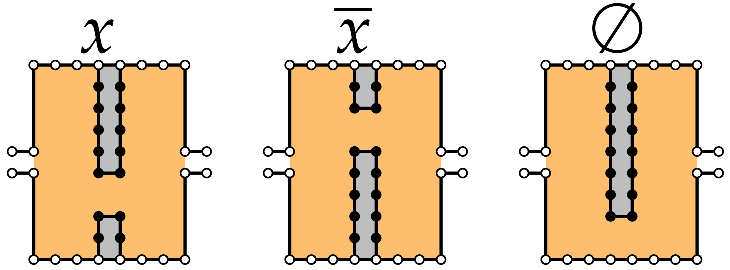
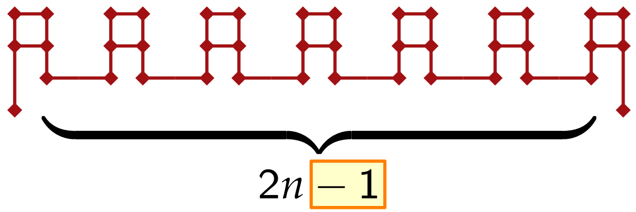
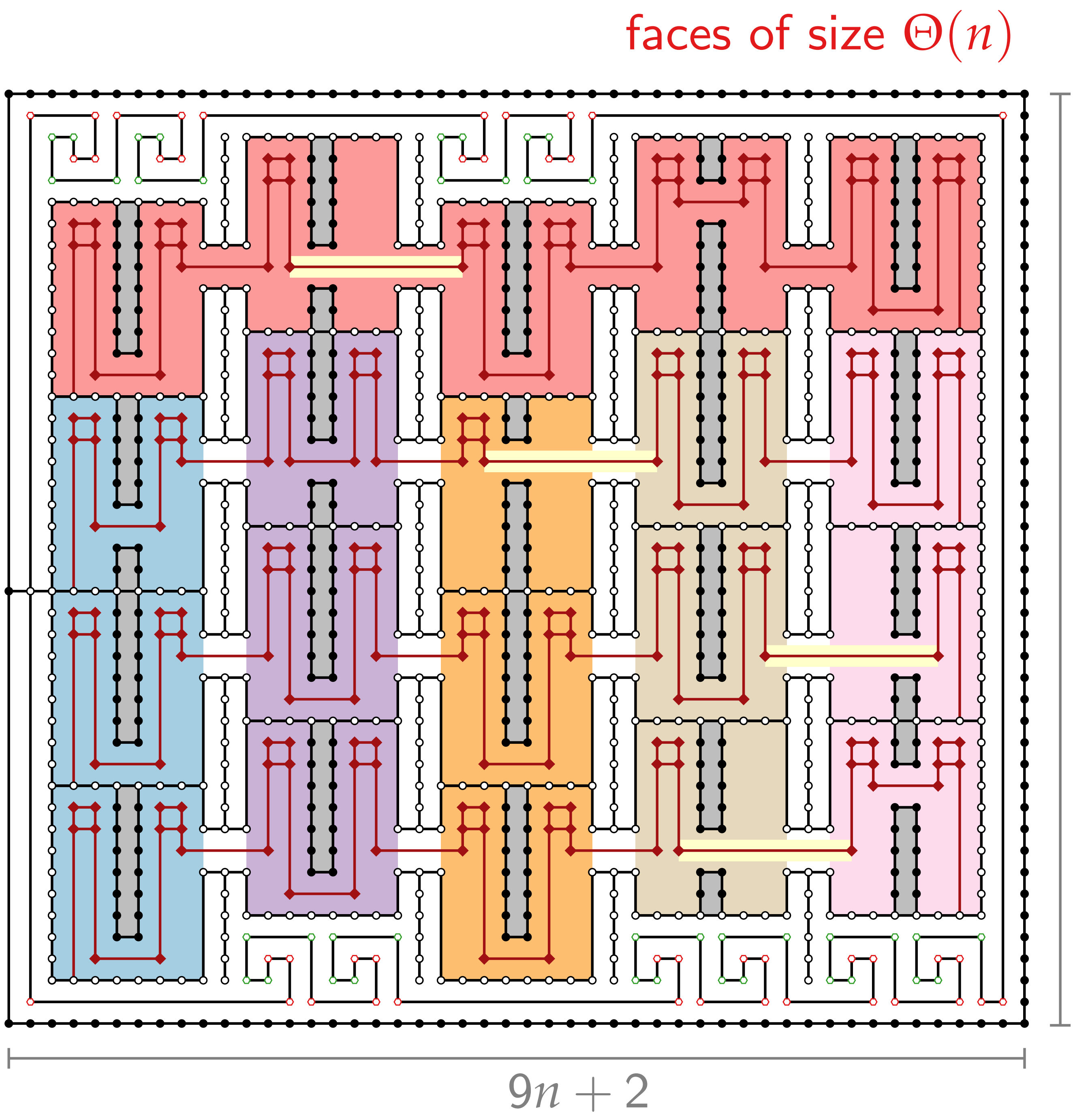




Full Reduction



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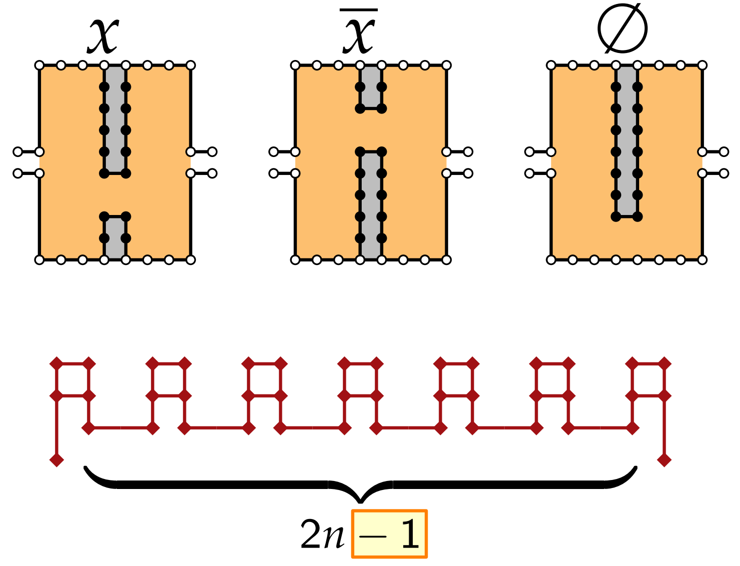

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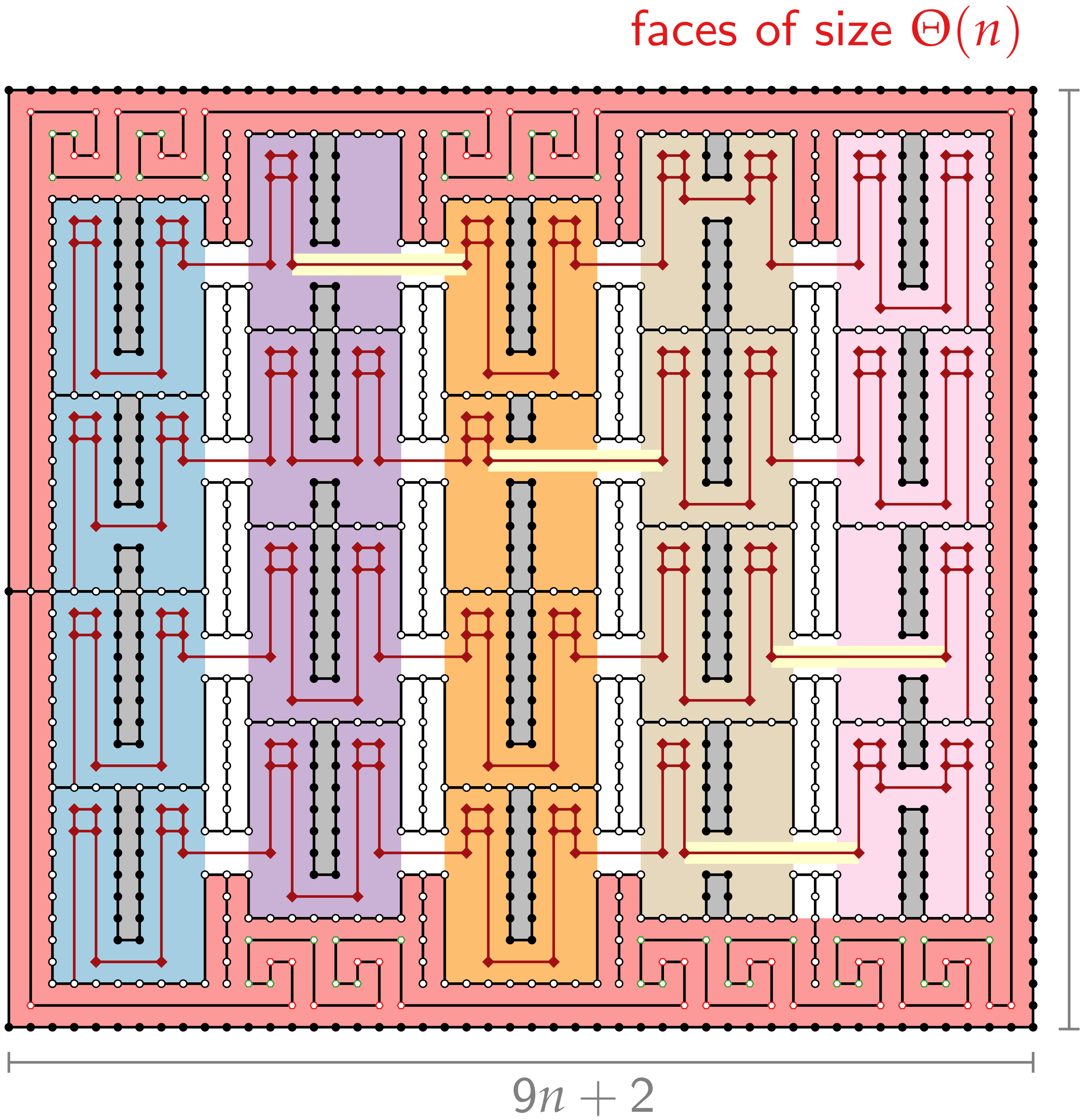

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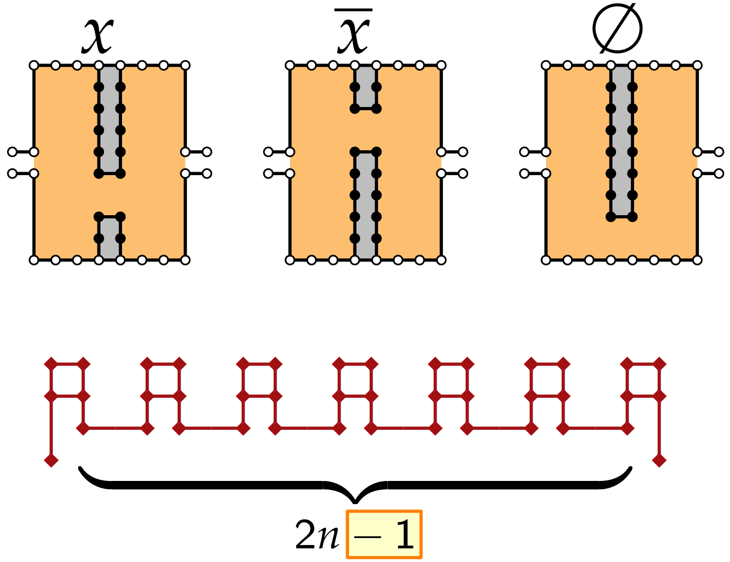
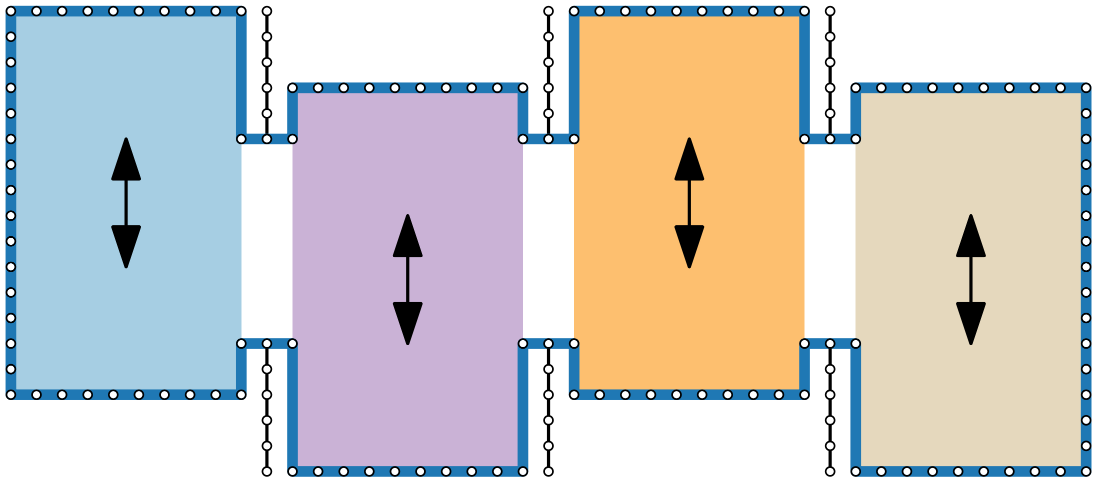


Adjusting Boundary and Belt



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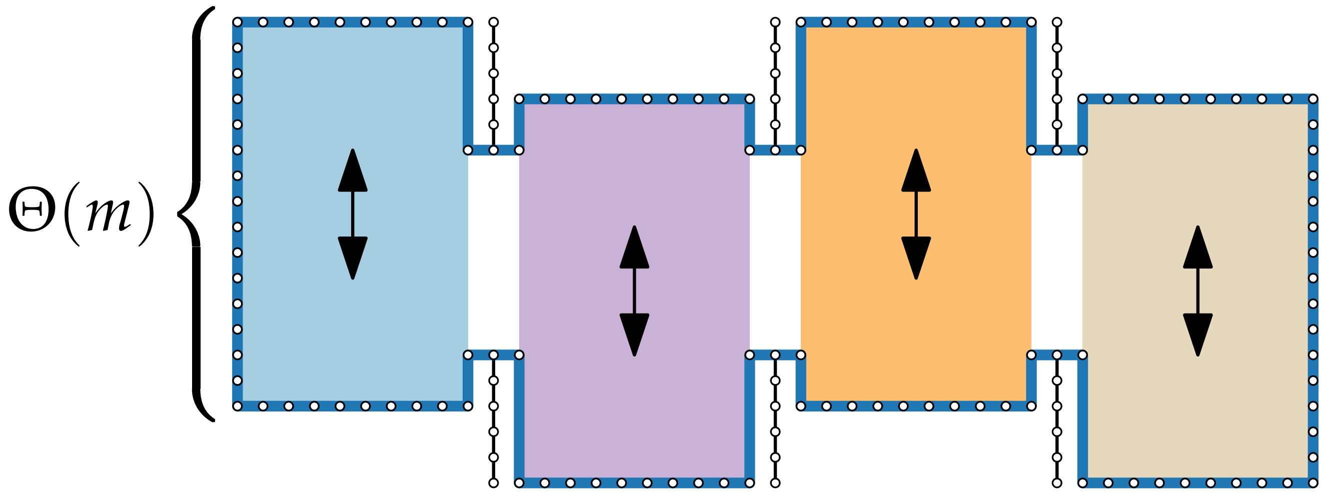

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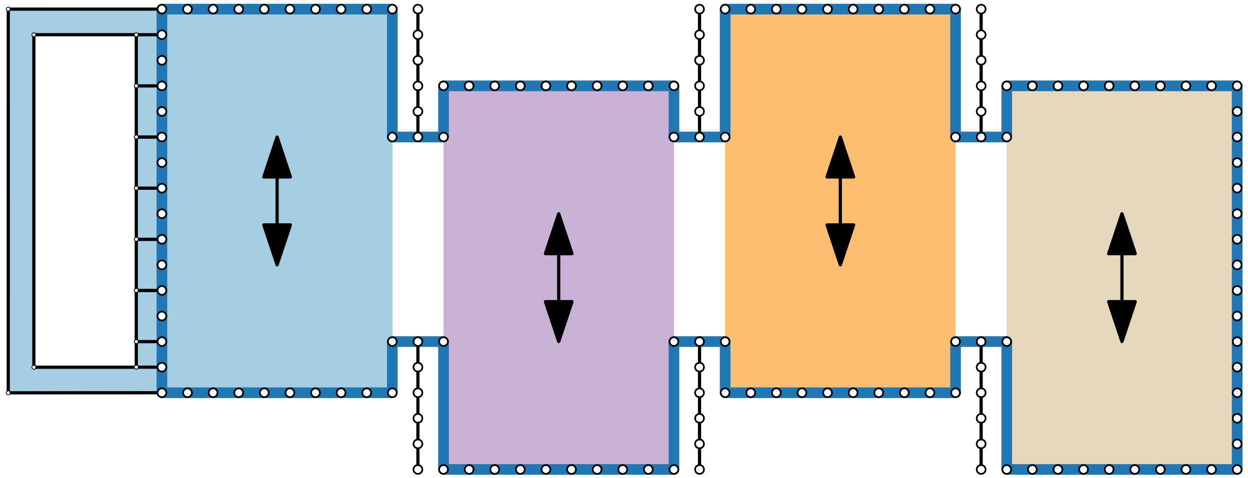

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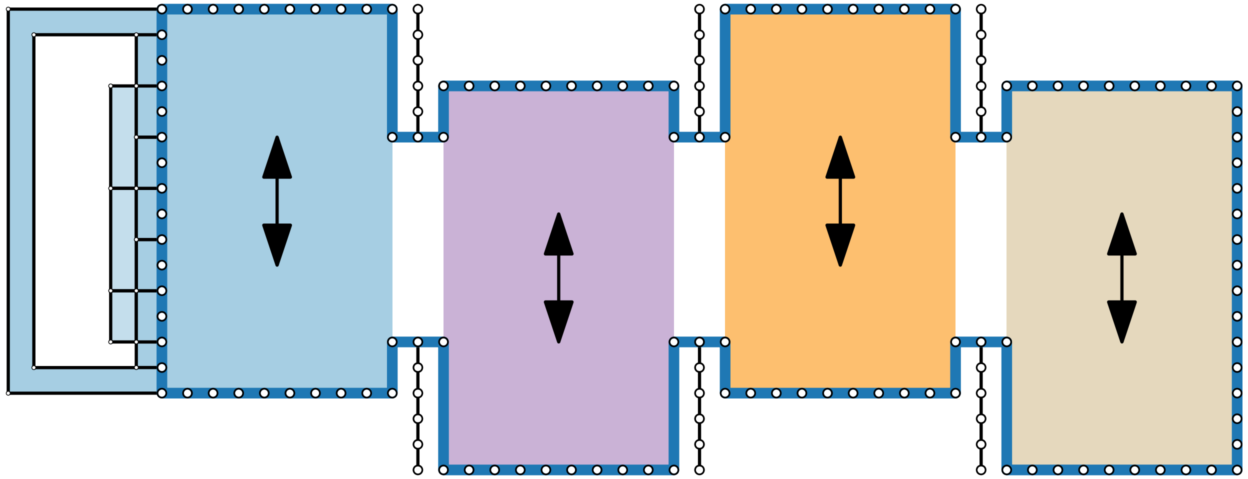

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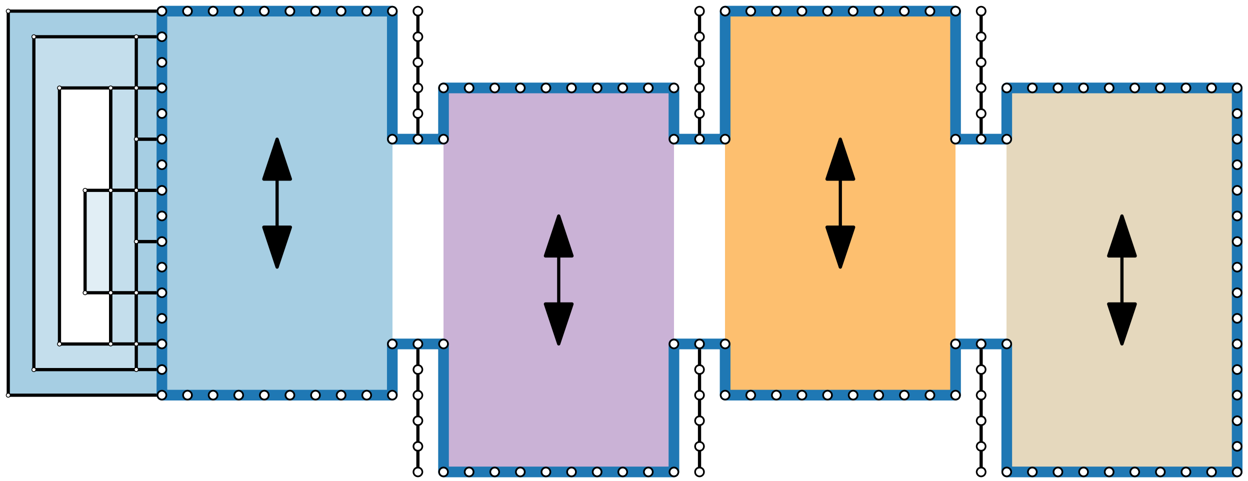

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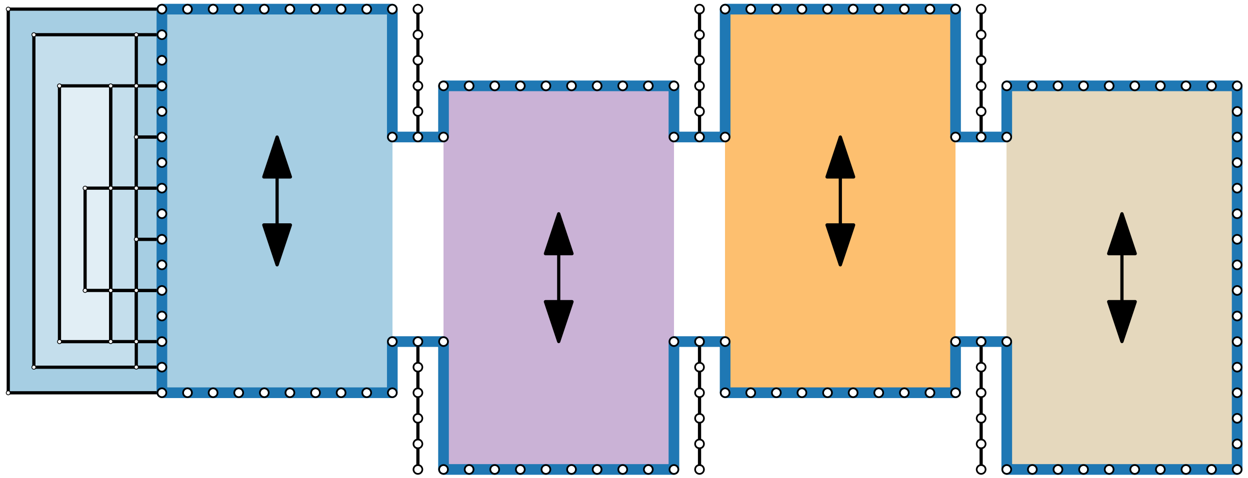

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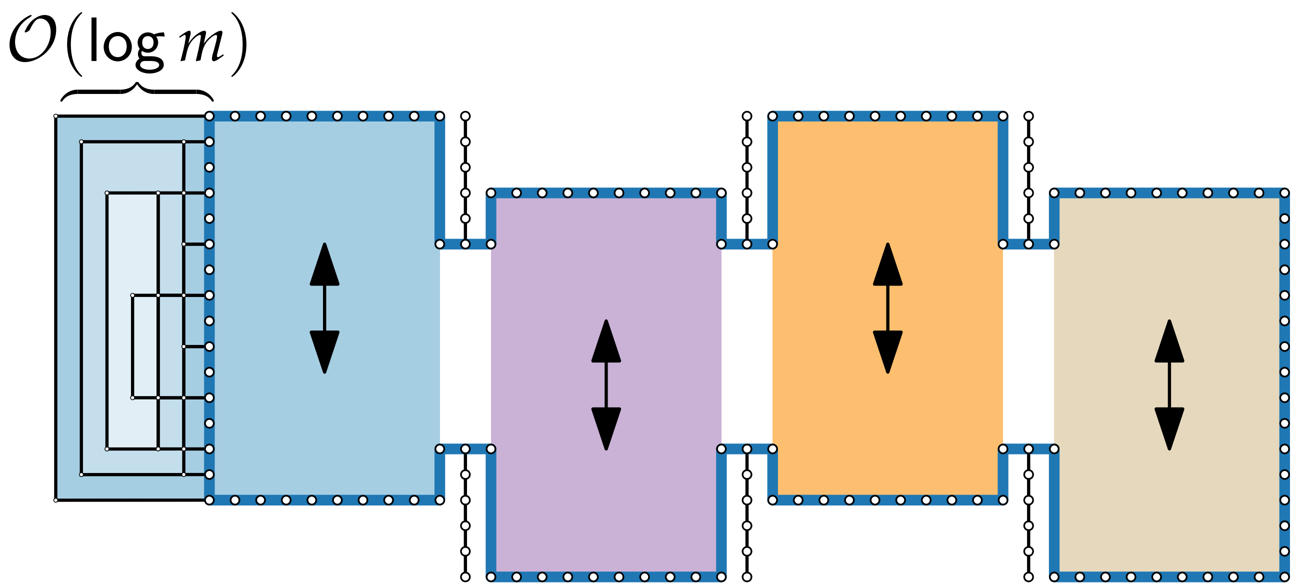

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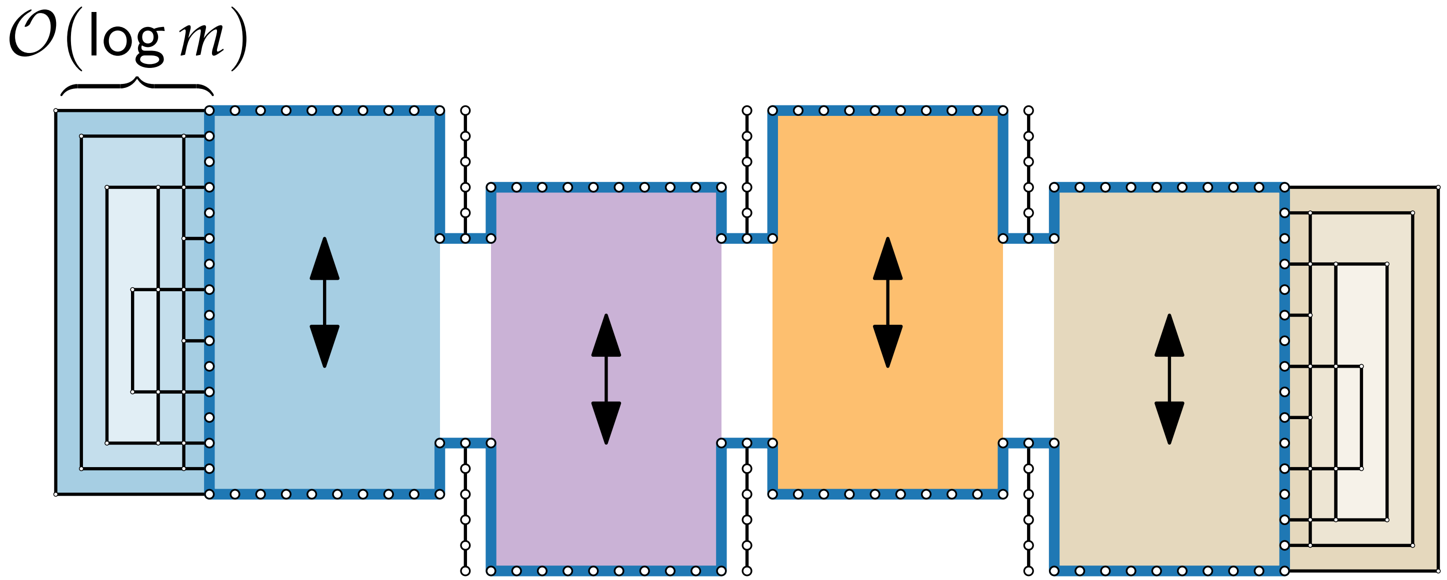

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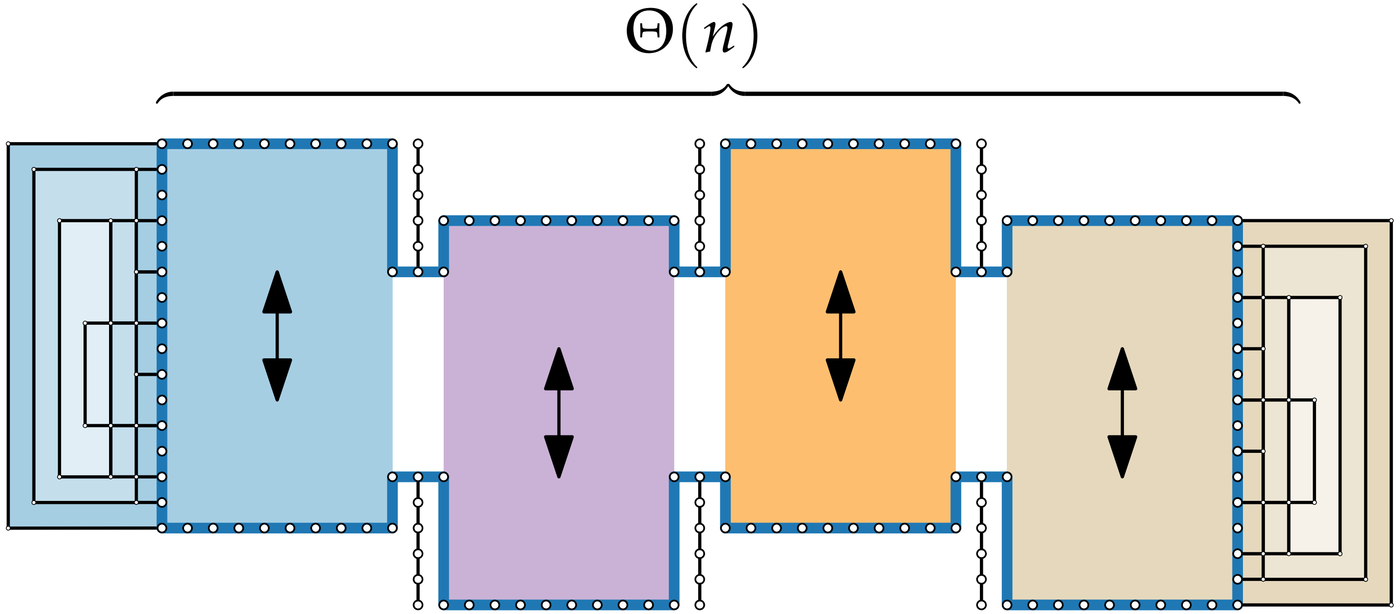

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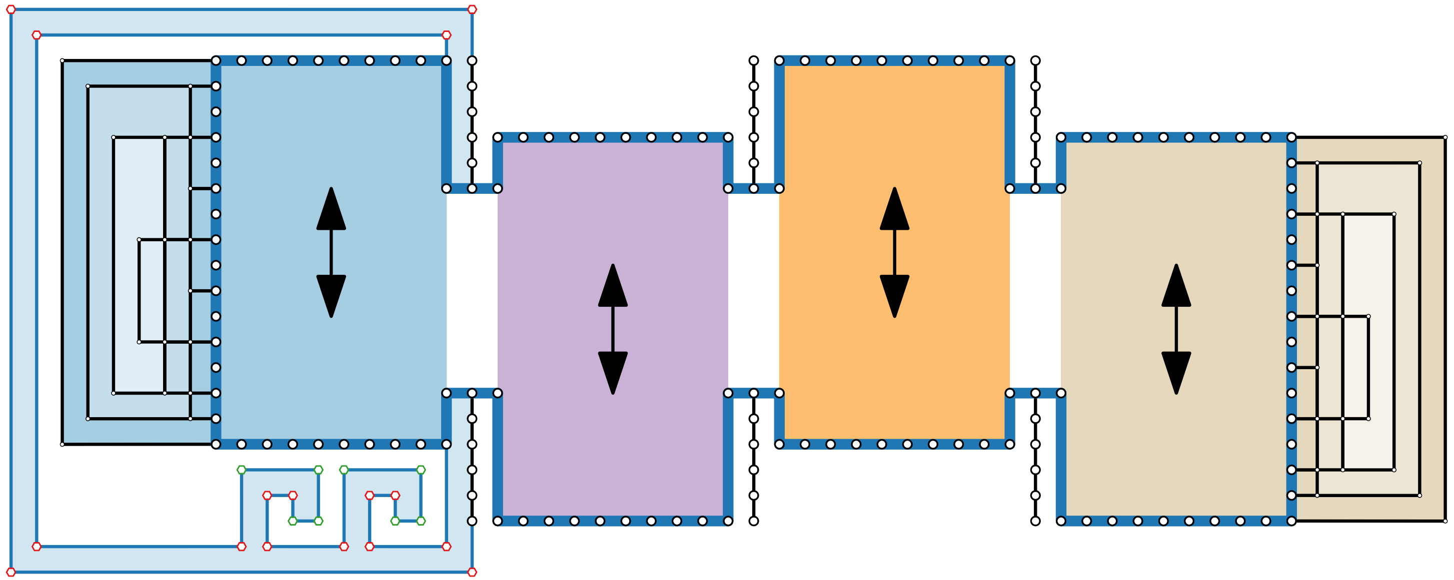
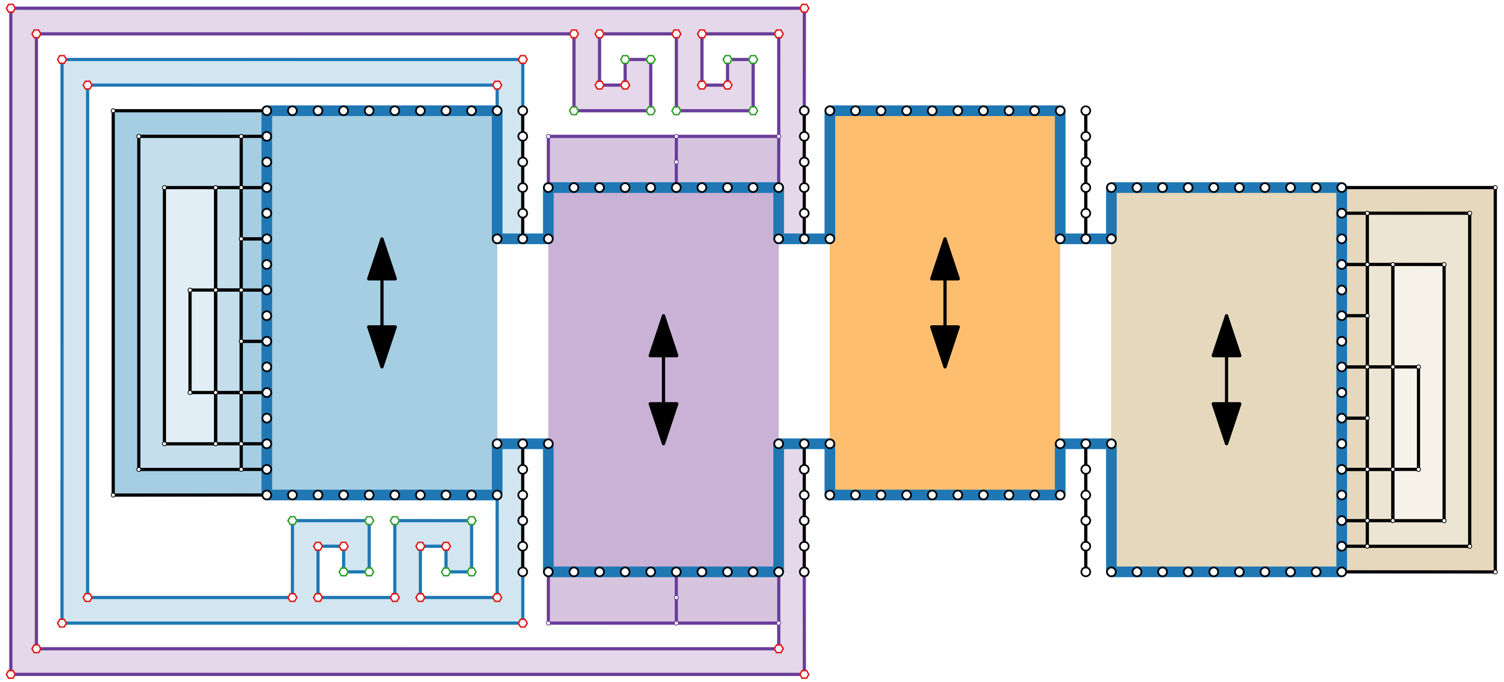


Adjusting Boundary and Belt



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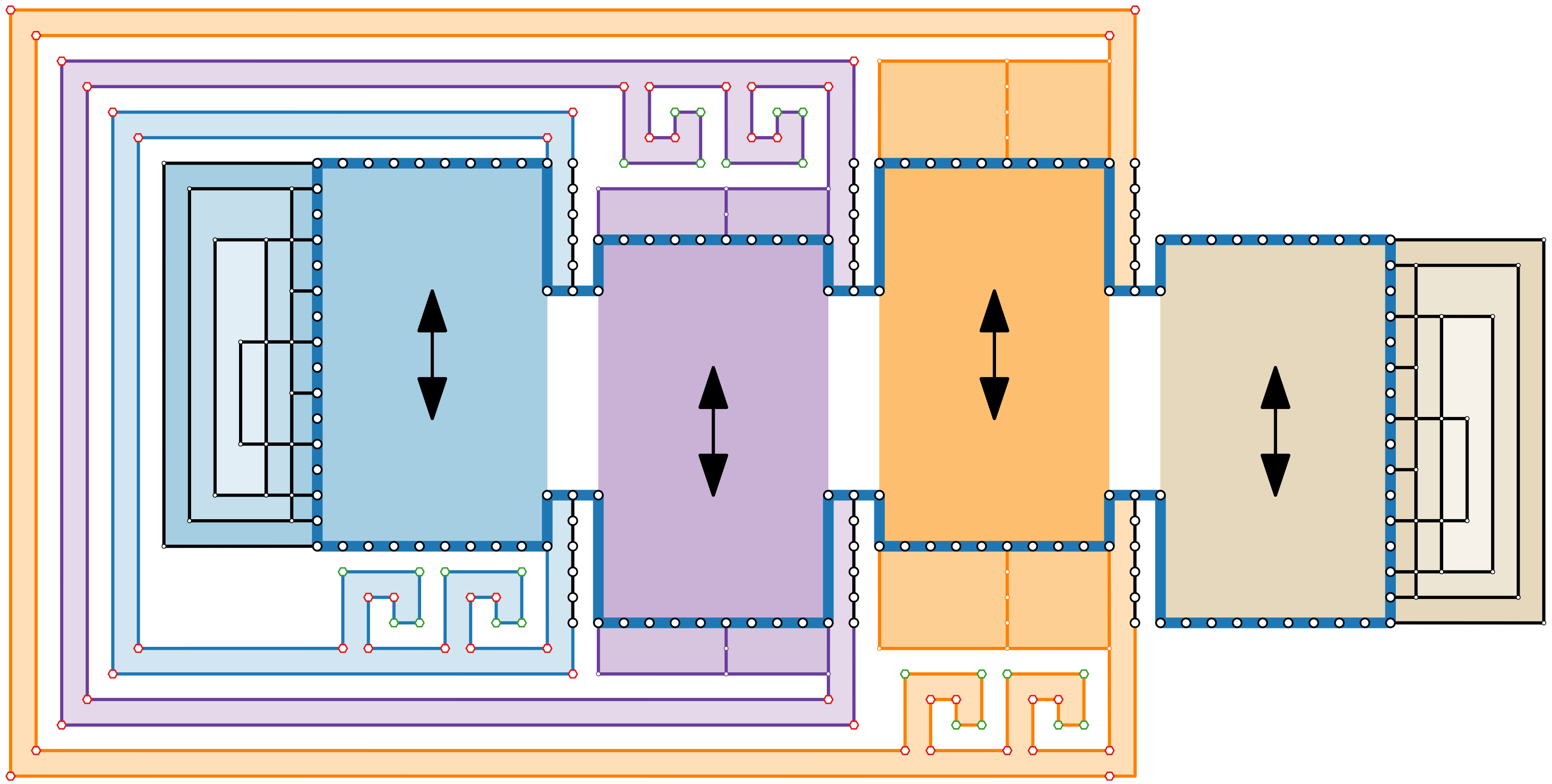

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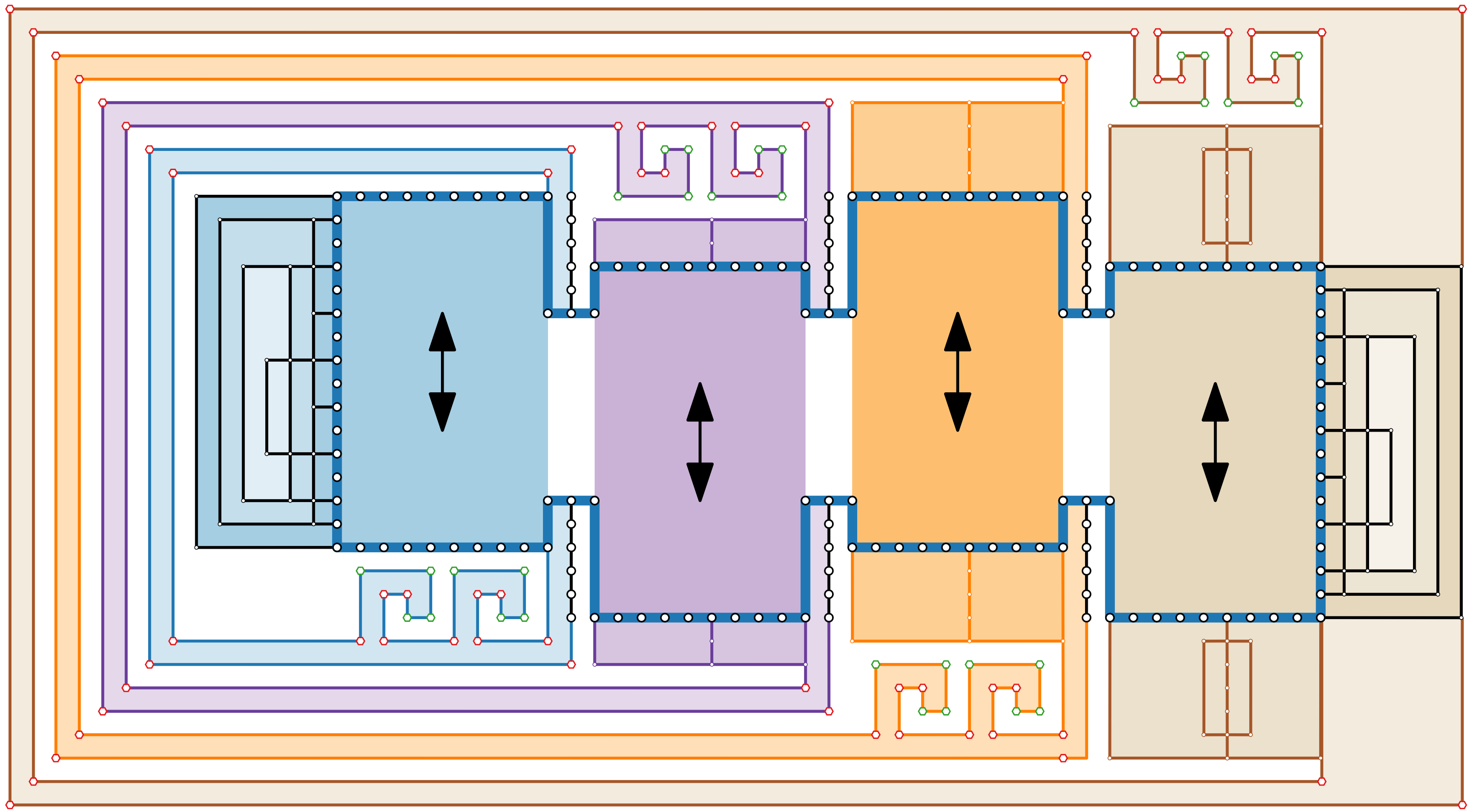
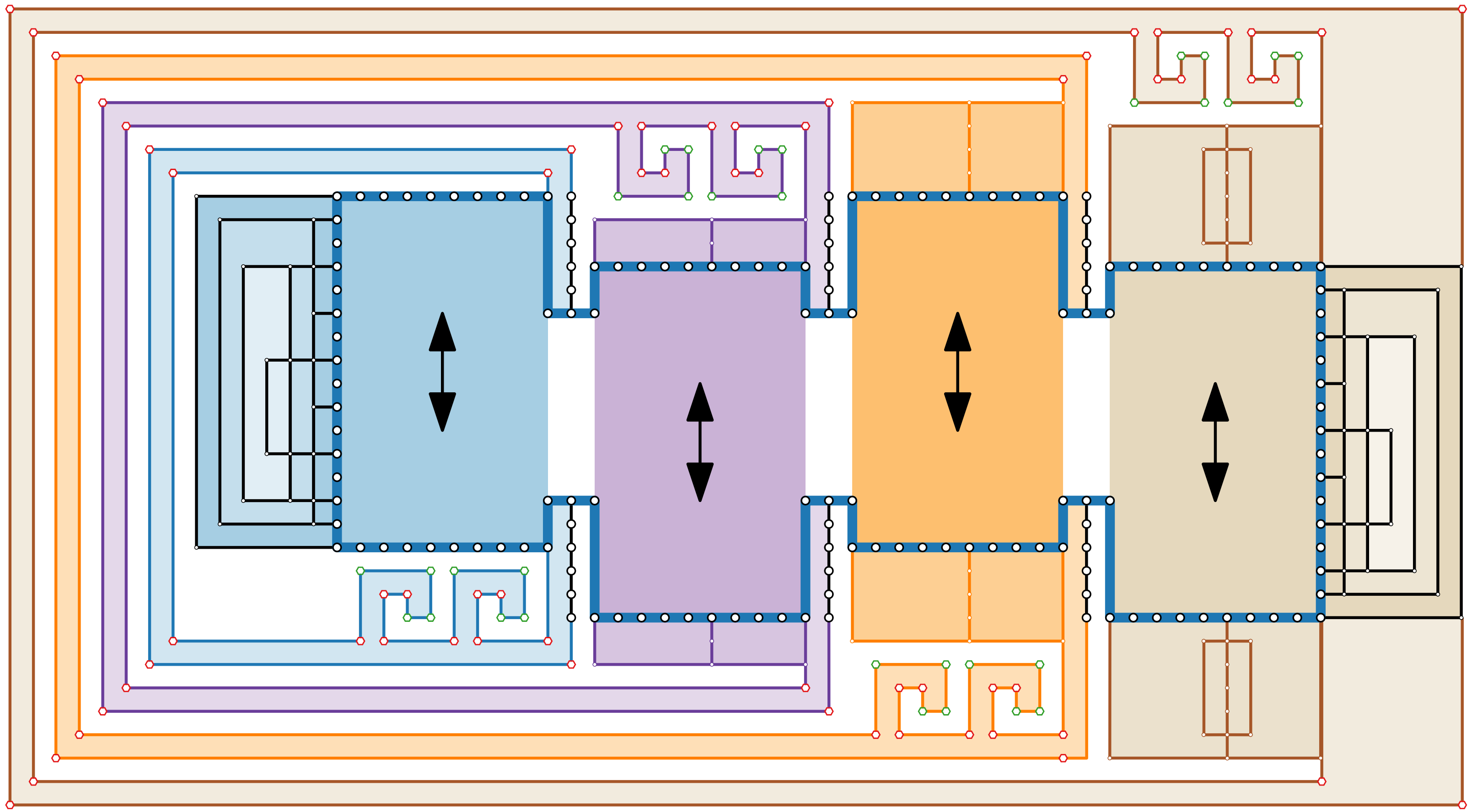
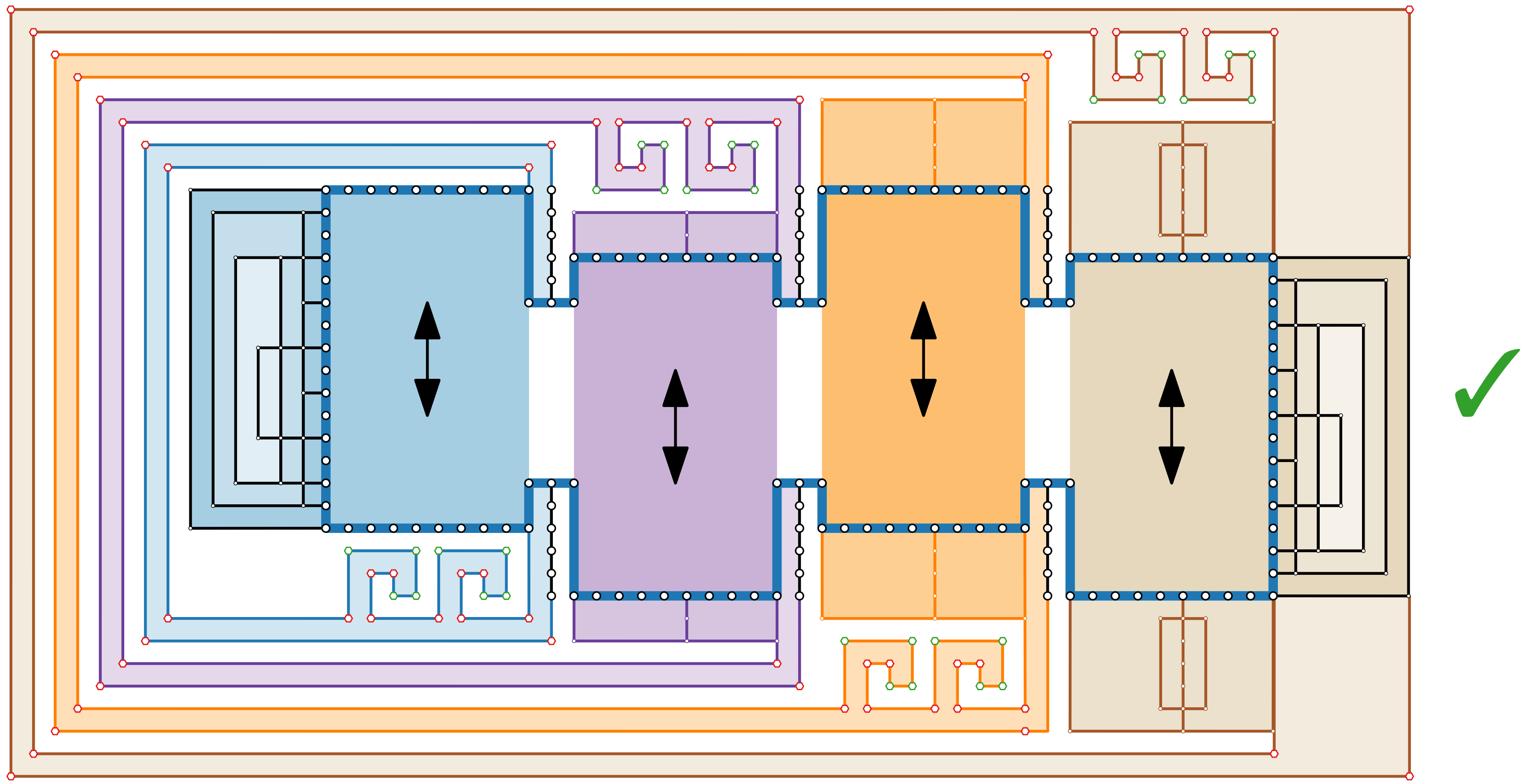


Adjusting Boundary and Belt



✓

Adjusting Boundary and Belt



Theorem. OC is para-NP-hard when parameterized by the maximum face degree.

Open Problems

- Can we find a polynomial kernel for OC w.r.t. the number of kitty corners, or at least w.r.t. the number of kitty corners *plus the number of faces*, for general graphs?

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- If we parametrize by the number of *pairs* of kitty corners, can we achieve substantially better running times?