Julius-Maximilians-
UNIVERSITÄT WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and Knowledge-Based Systems

Institute for Informatics

Approximating the Generalized Minimum Manhattan Network Problem

ISAAC'13

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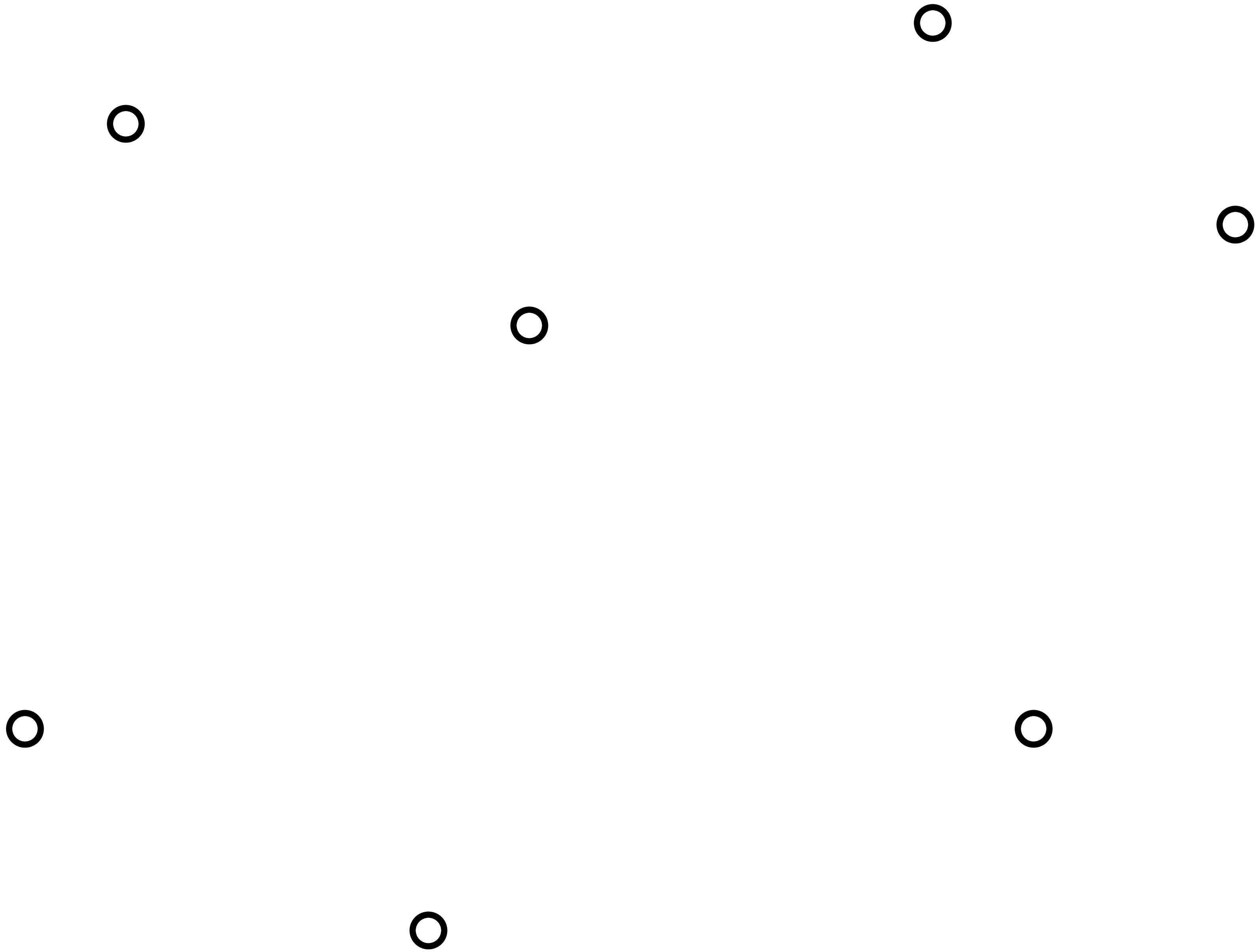
Joachim Spoerhase Sankar Veeramoni *Alexander Wolff*

Department of Computer Science
University of Arizona

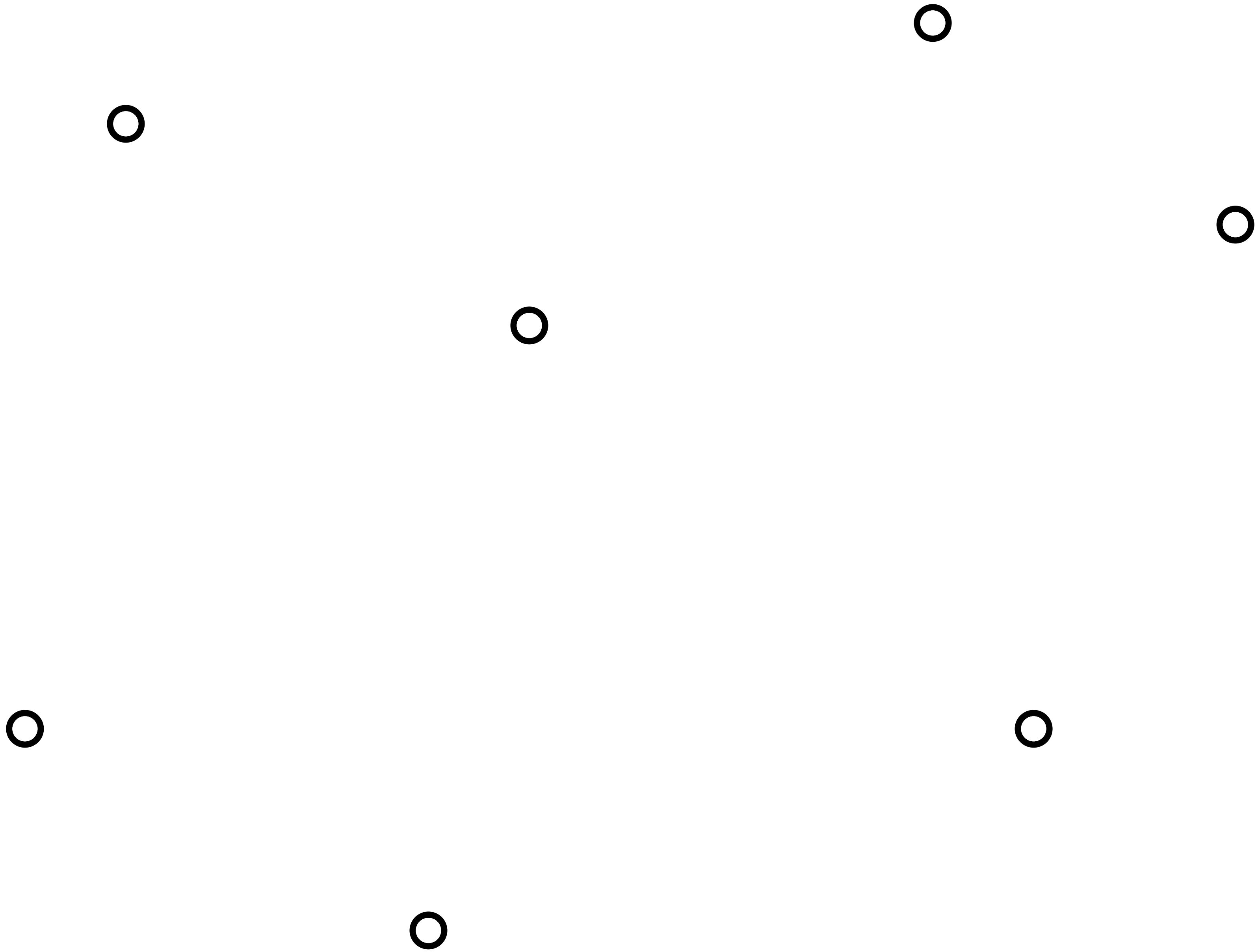
Institut für Informatik
Universität Würzburg

Definitions: Given a set of n points in the plane...



Definitions: Given a set of n points in the plane...

“terminals”



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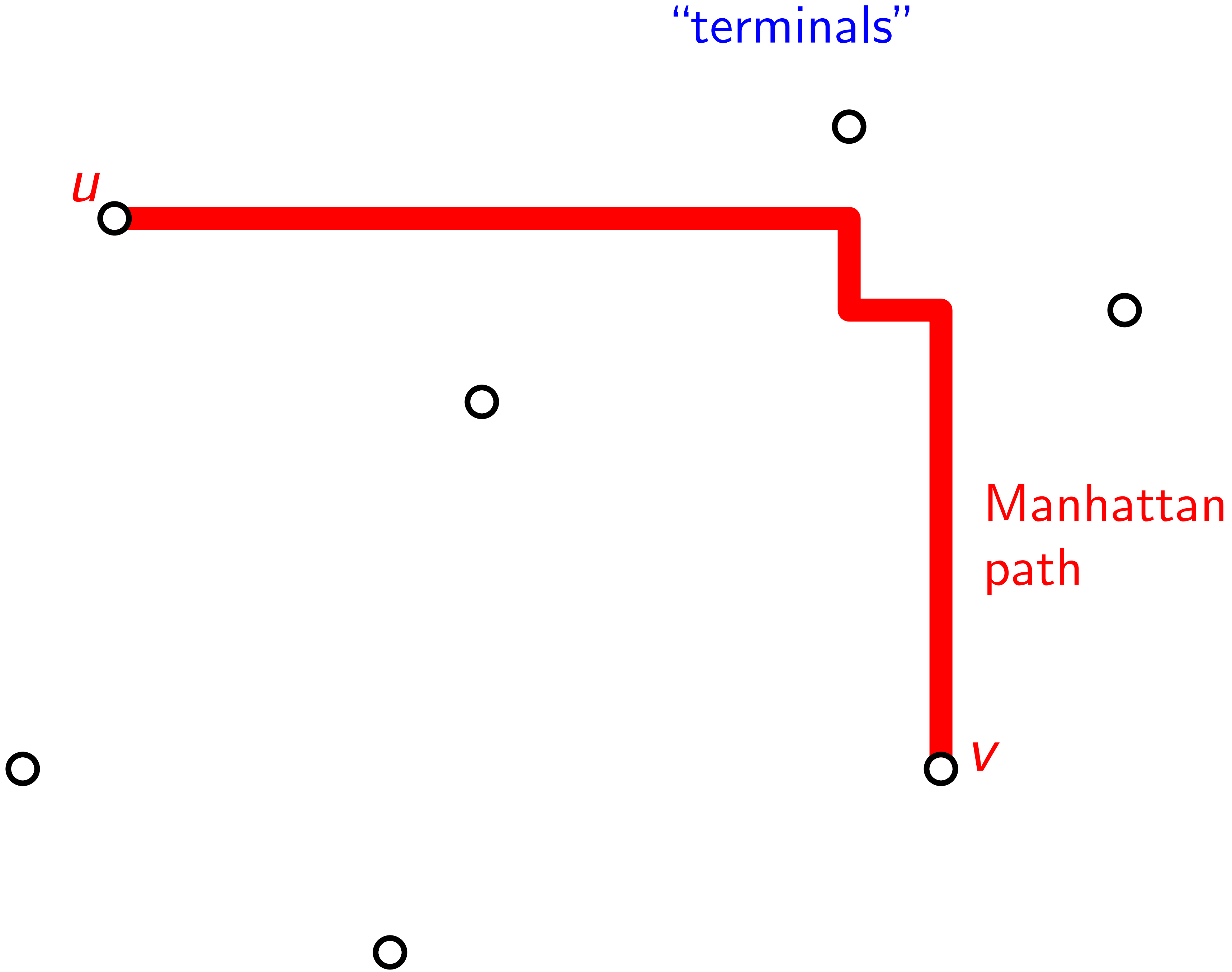

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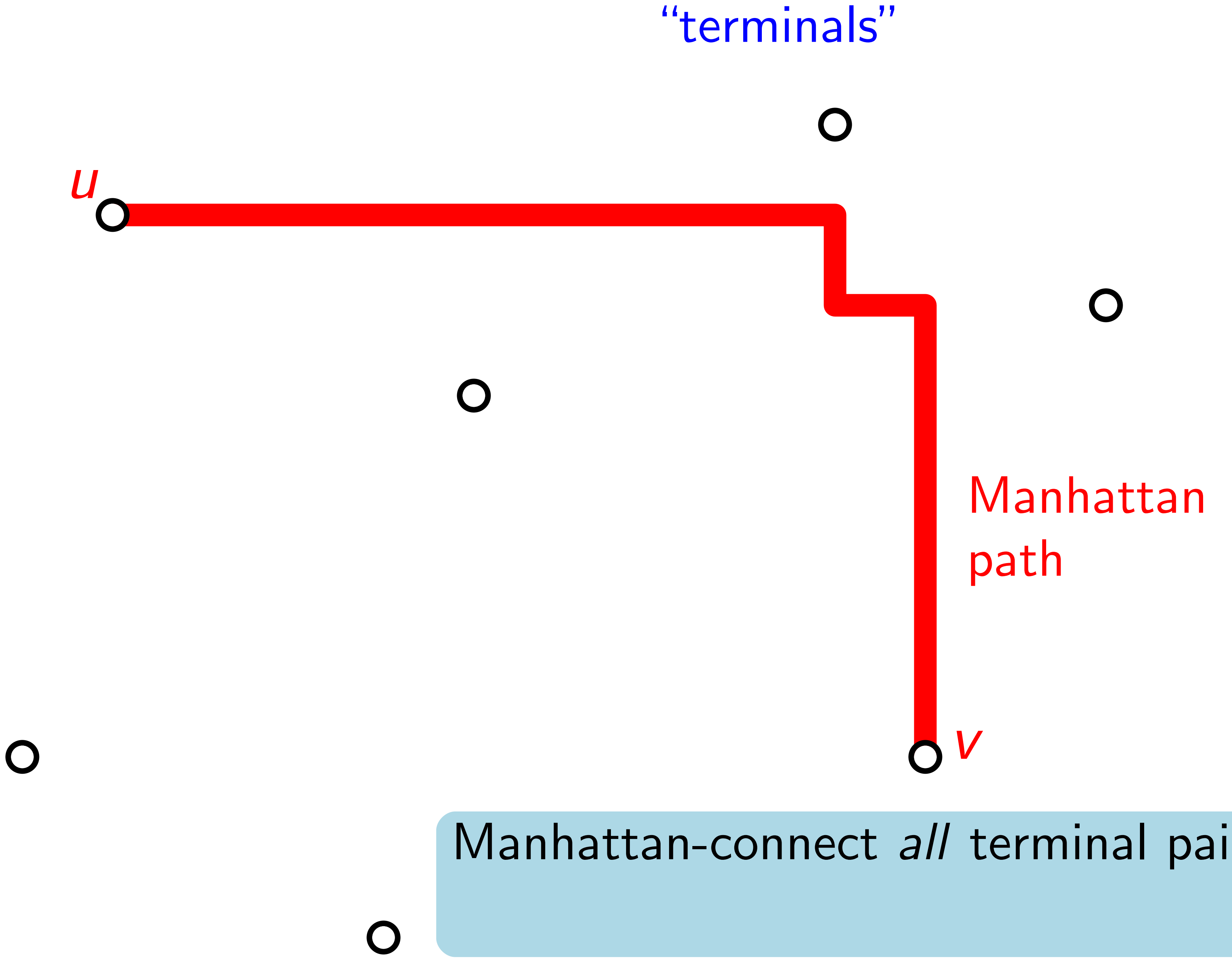
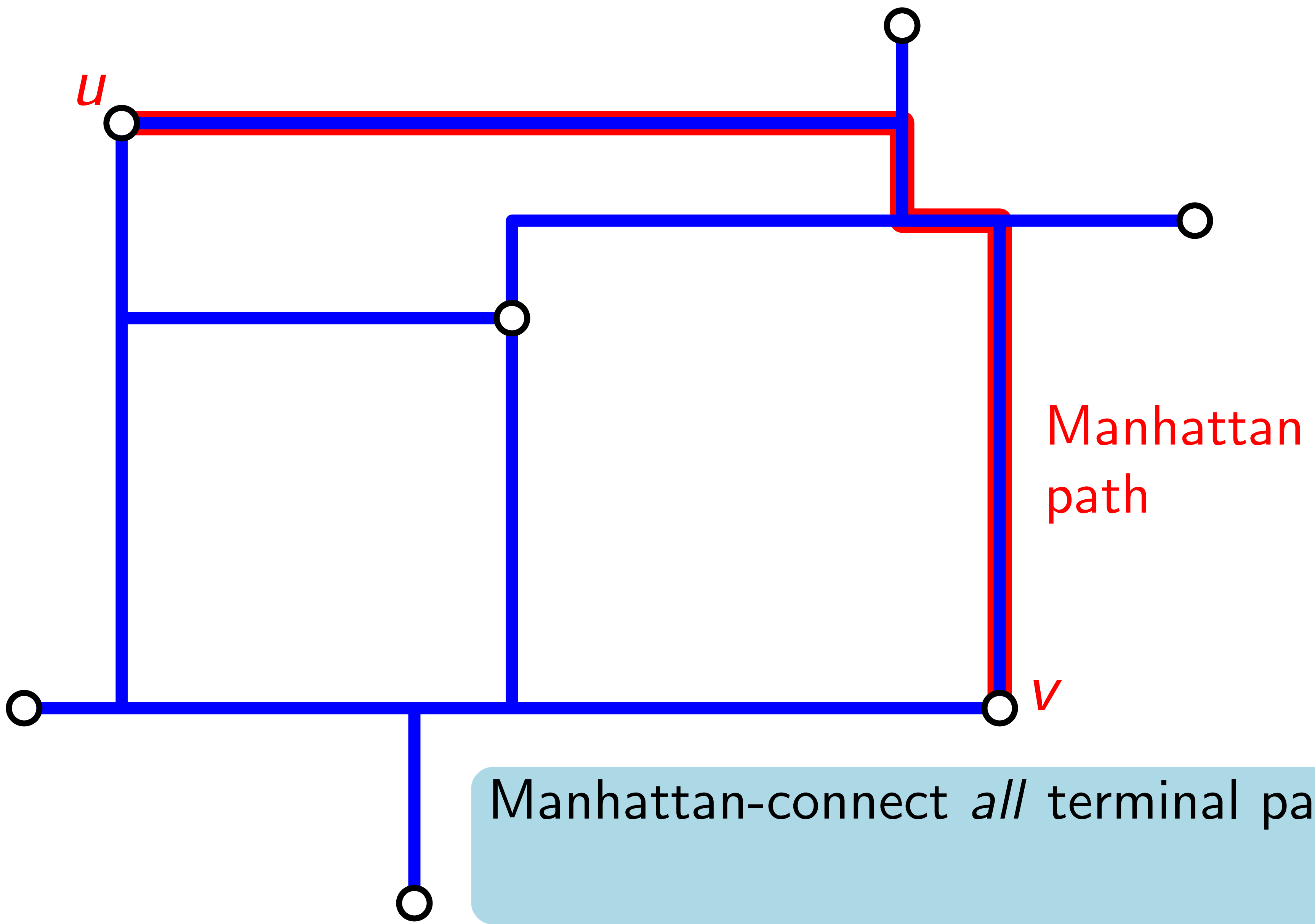


Manhattan-connect *all* terminal pairs.

Definitions: Given a set of *n* points in the plane...

“terminals”



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Definitions: Given a set of n points in the plane...

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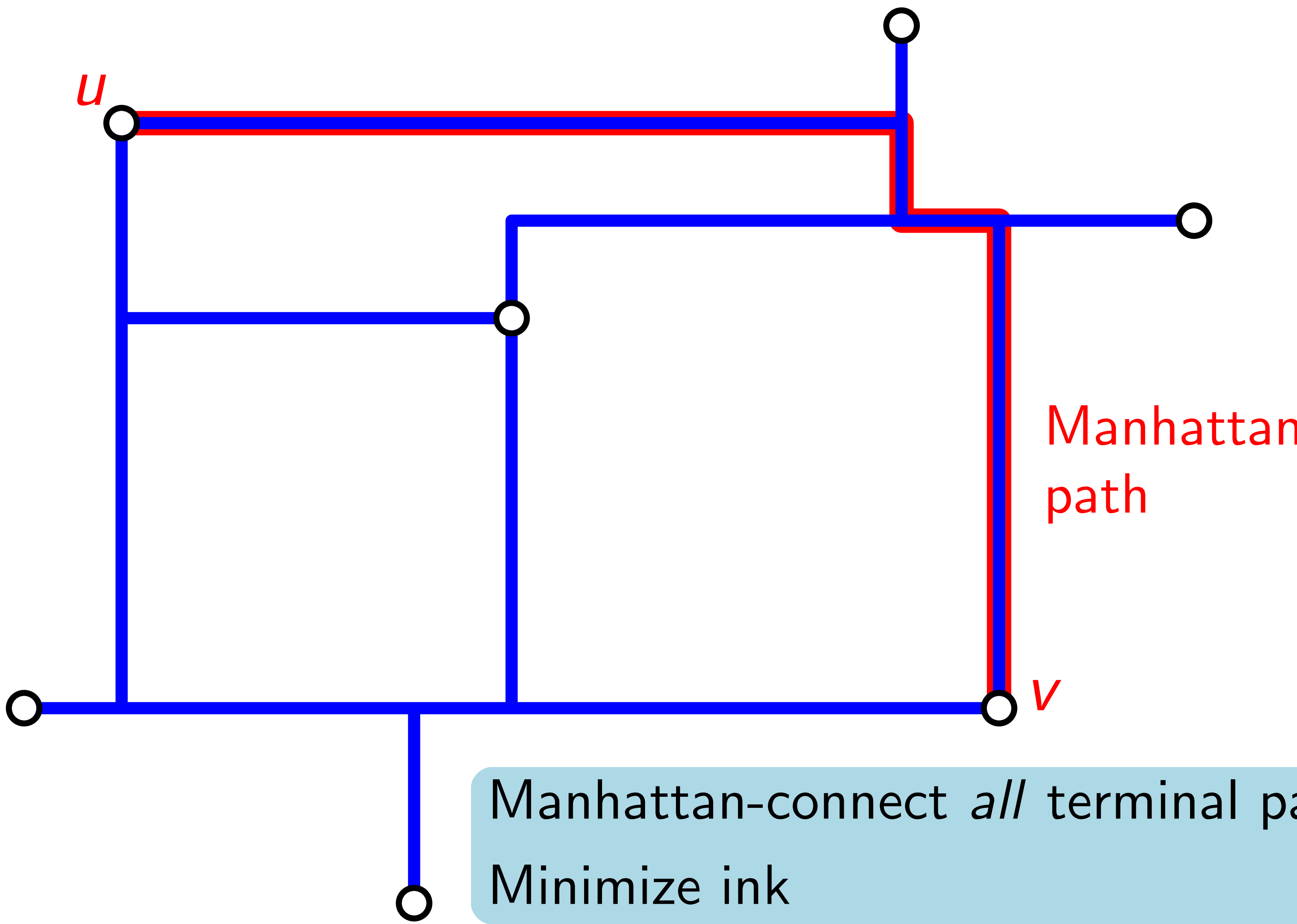
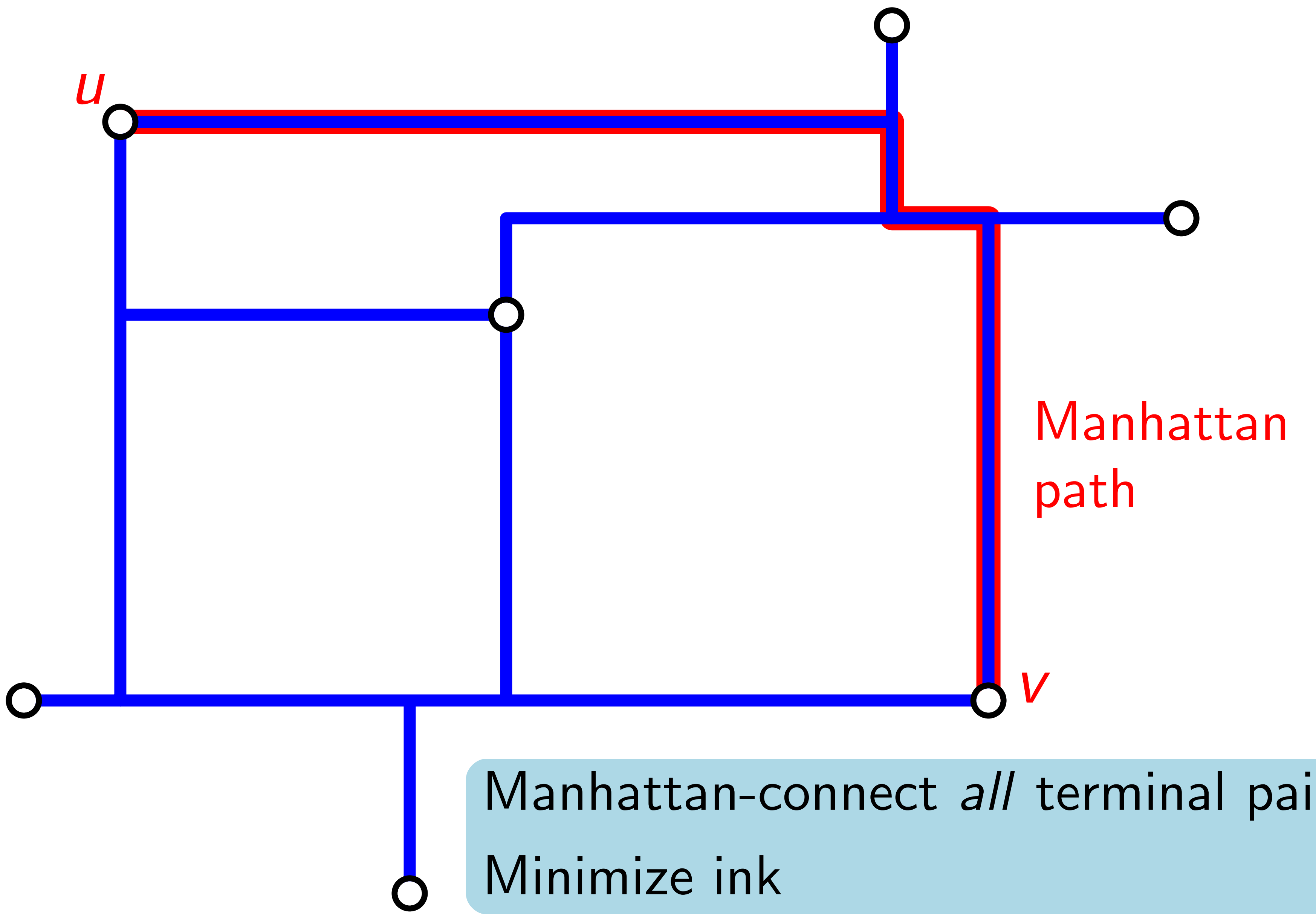


Manhattan-connect *all* terminal pairs.

Minimize ink

Definitions: Minimum Manhattan Network (MMN)



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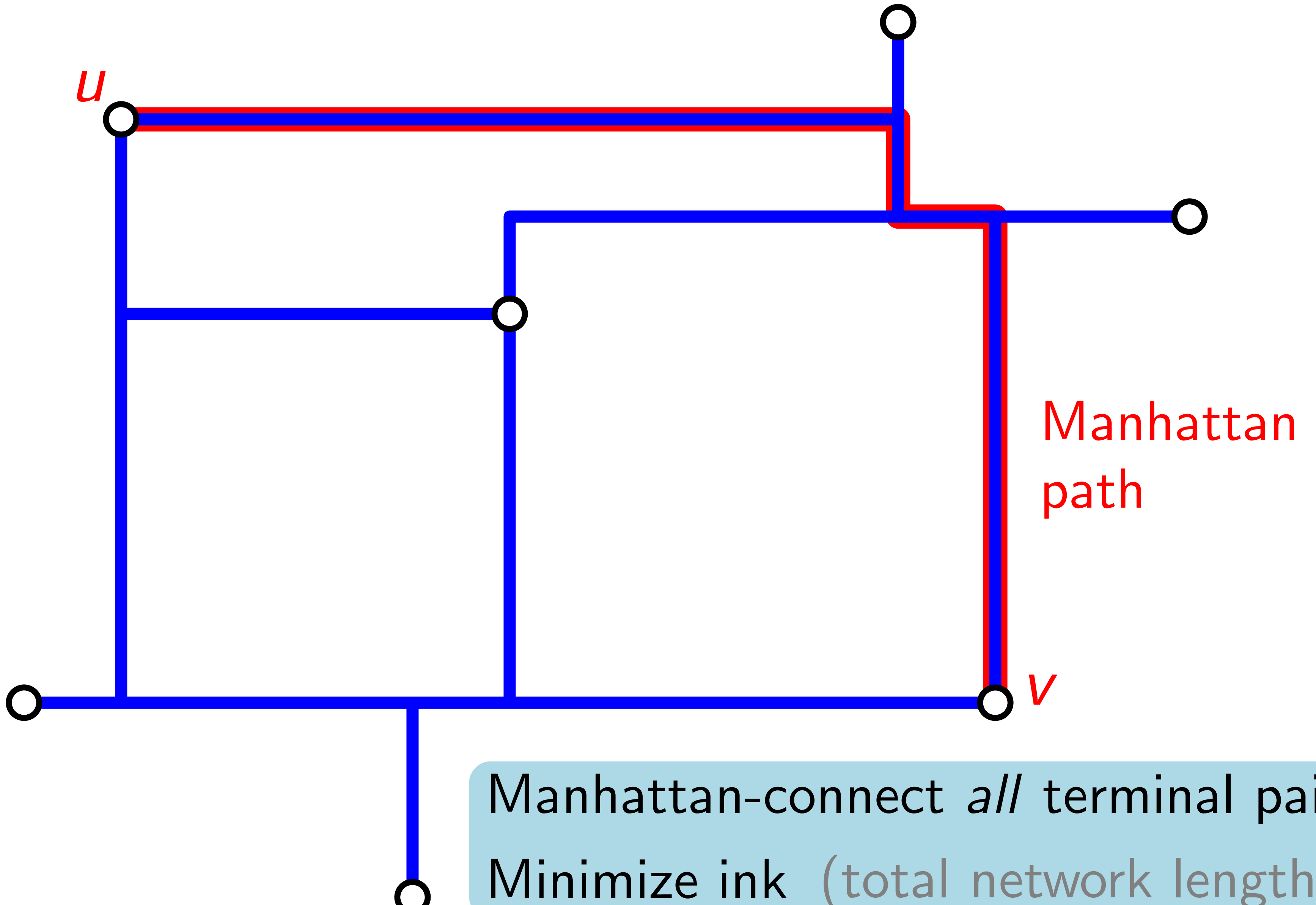
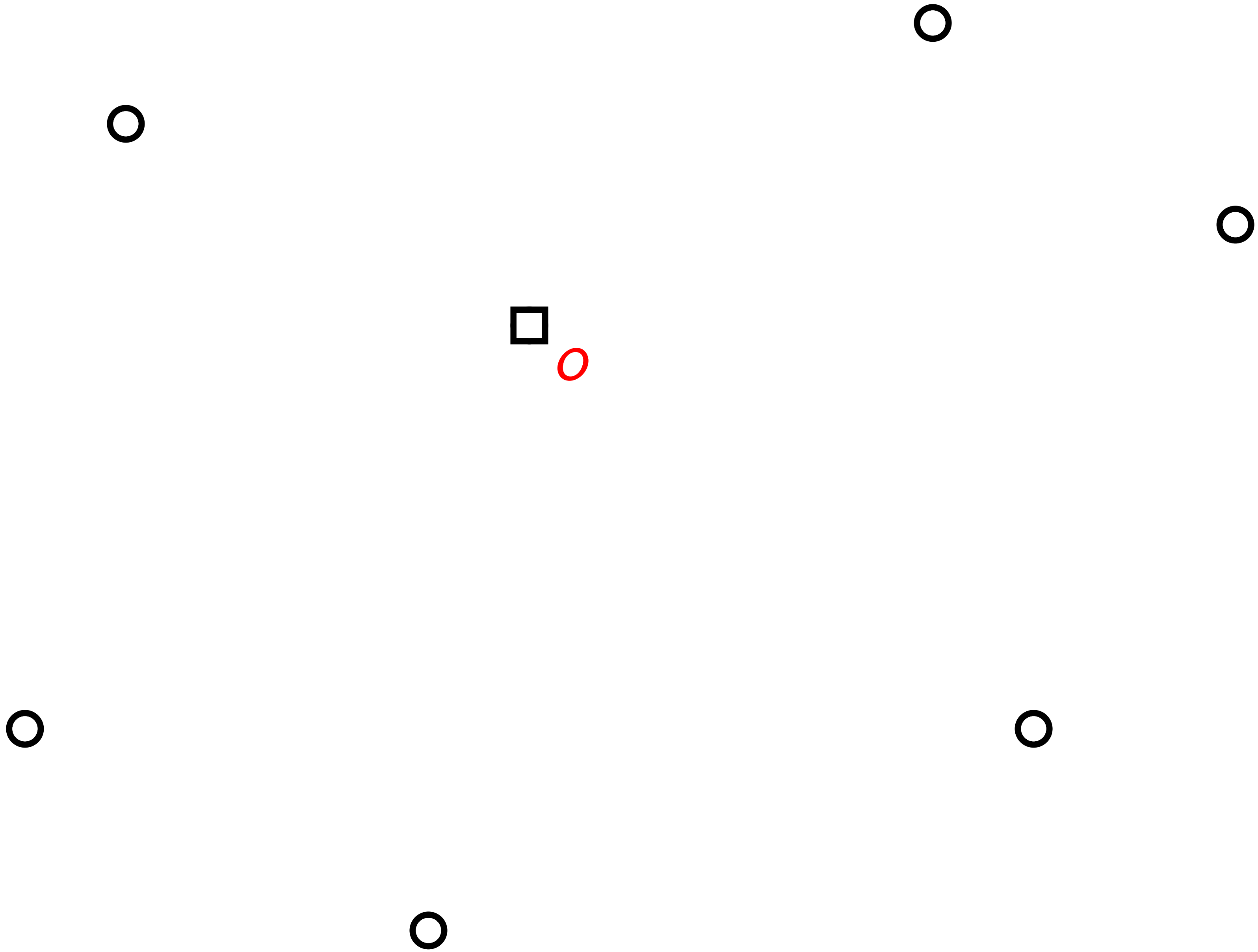


Manhattan-connect *all* terminal pairs.

Minimize ink (total network length)!

Definitions: Rectilinear Steiner Arborescence (RSA)



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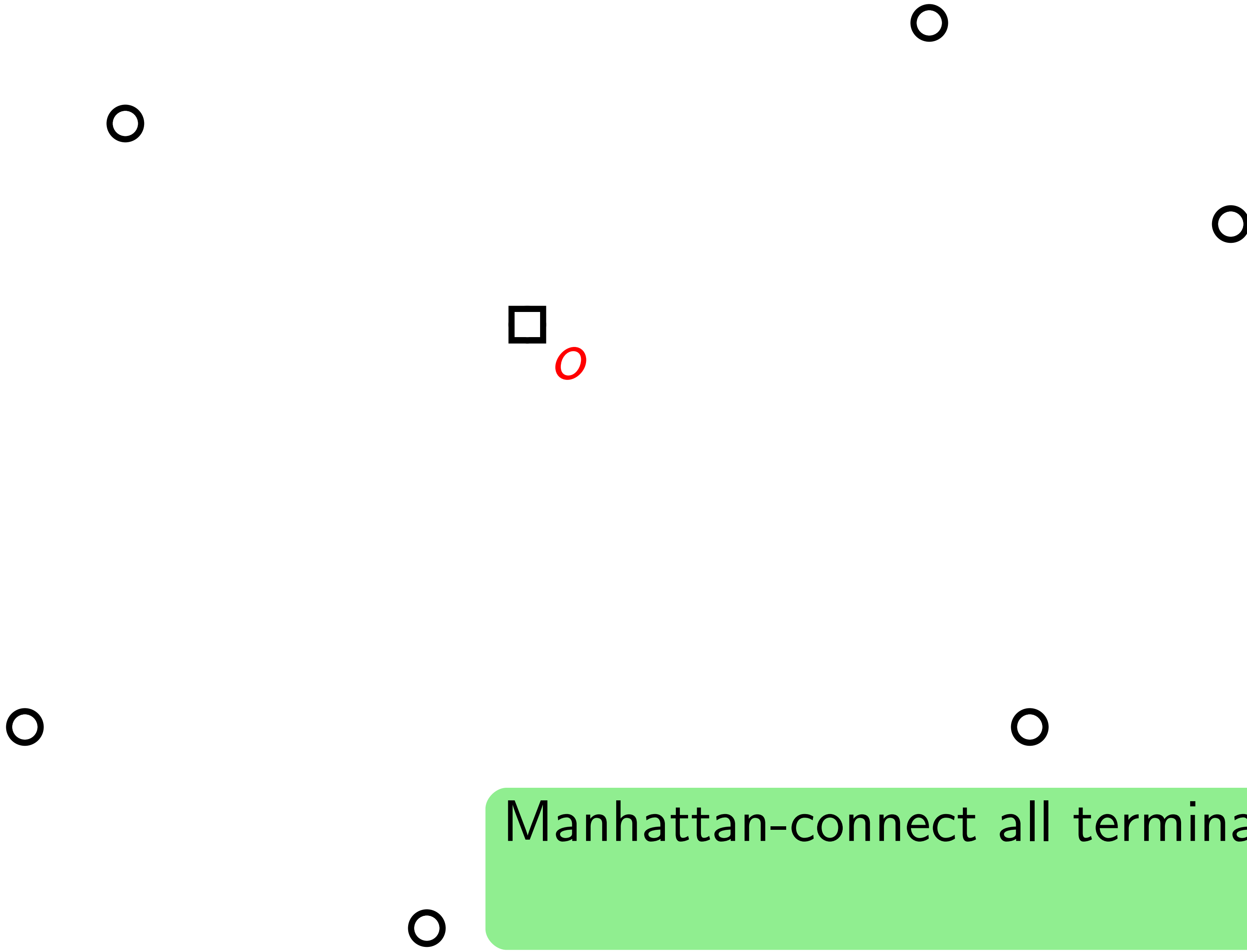
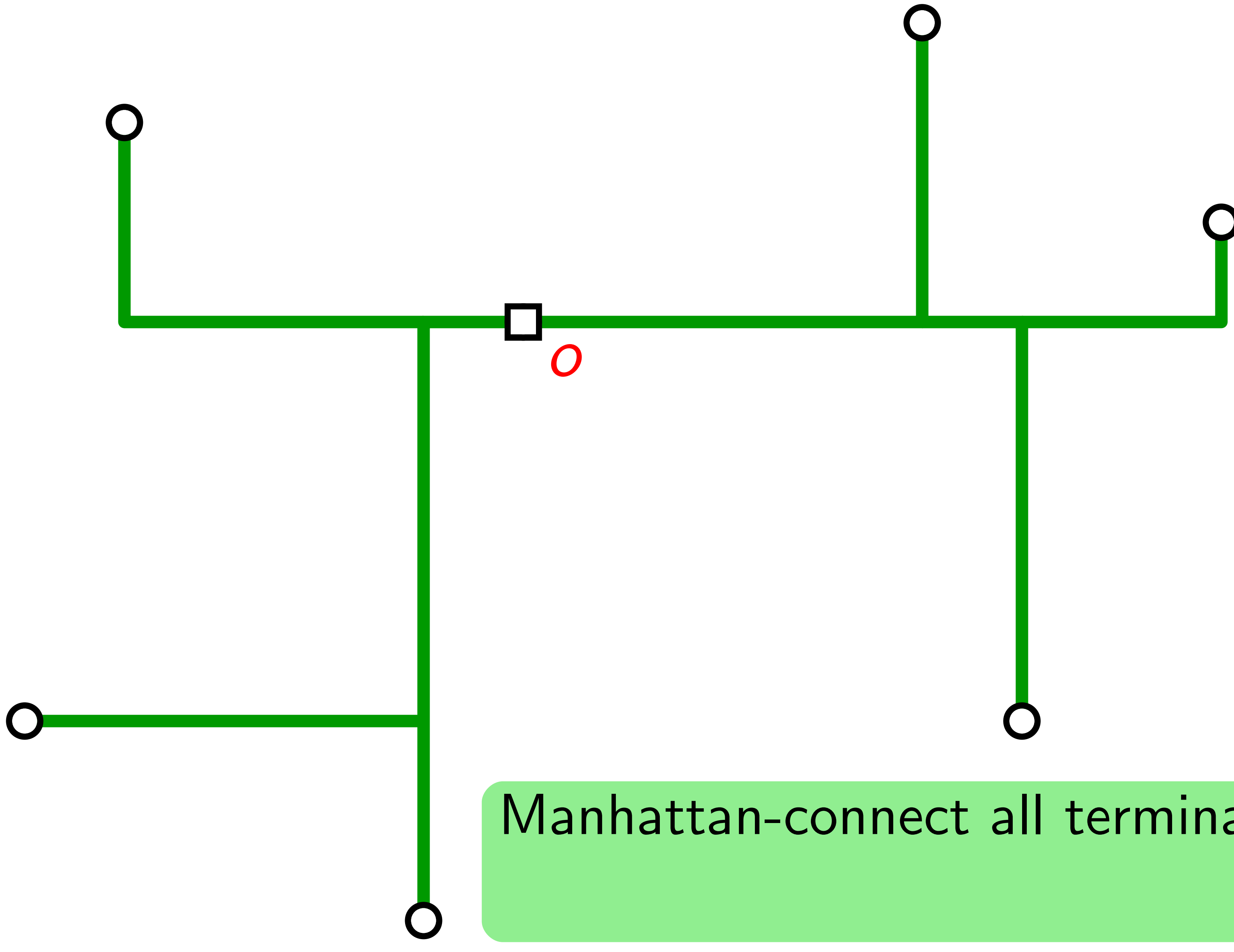


Manhattan-connect all terminals to o.

Definitions: Rectilinear Steiner Arborescence (RSA)



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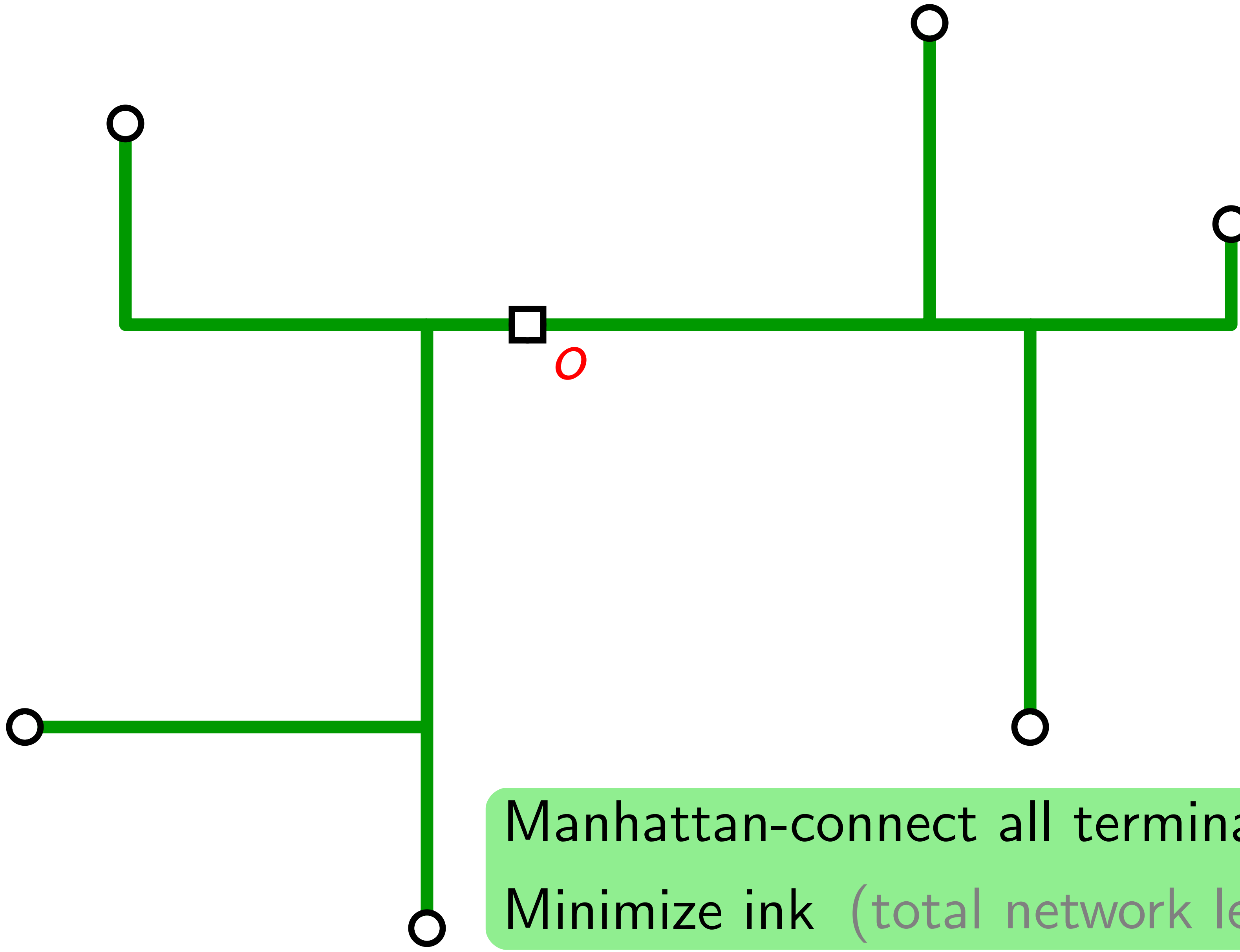
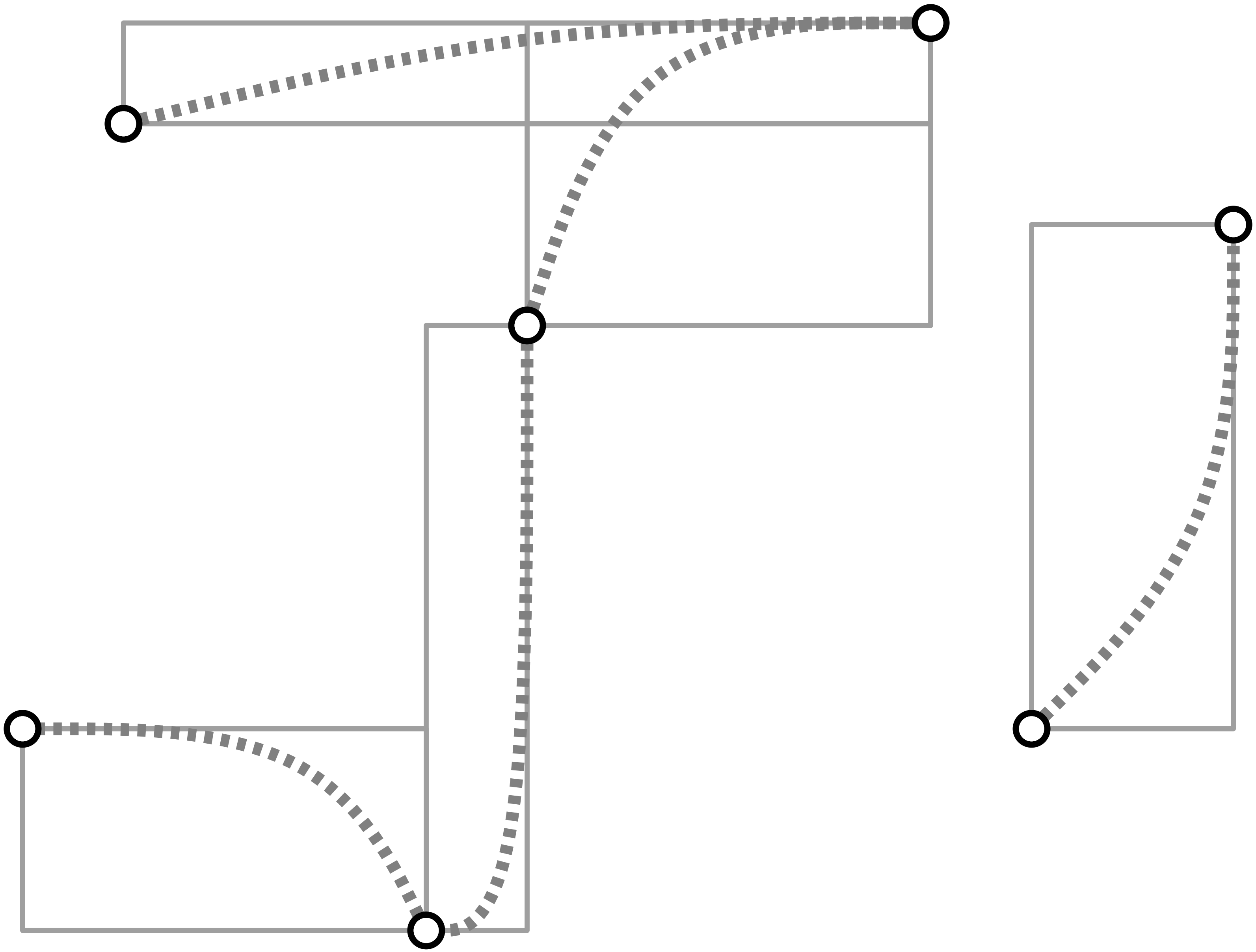


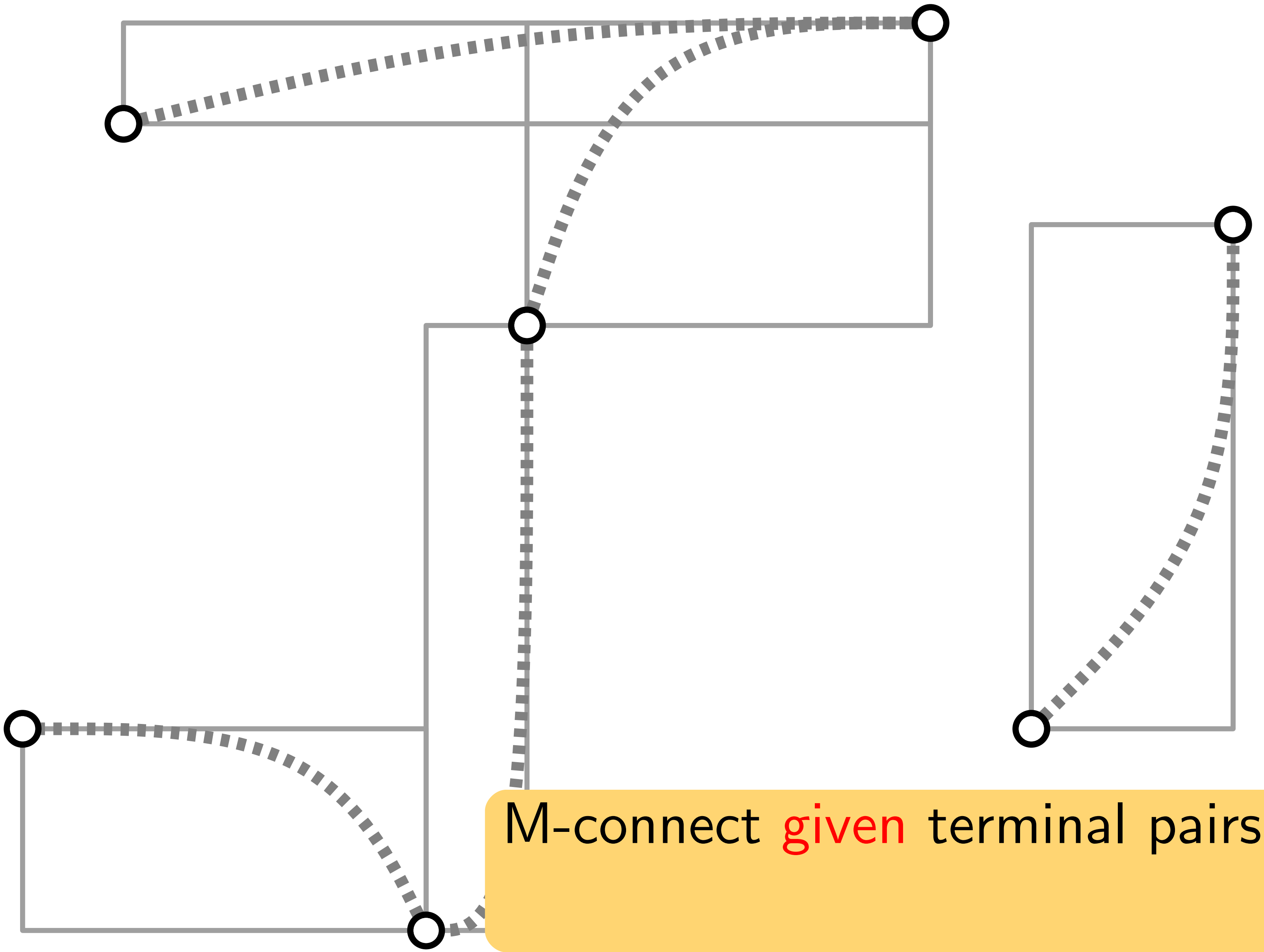
Manhattan-connect all terminals to *o*.

Minimize ink (total network length)!

Definitions: Generalized MMN (GMMN)

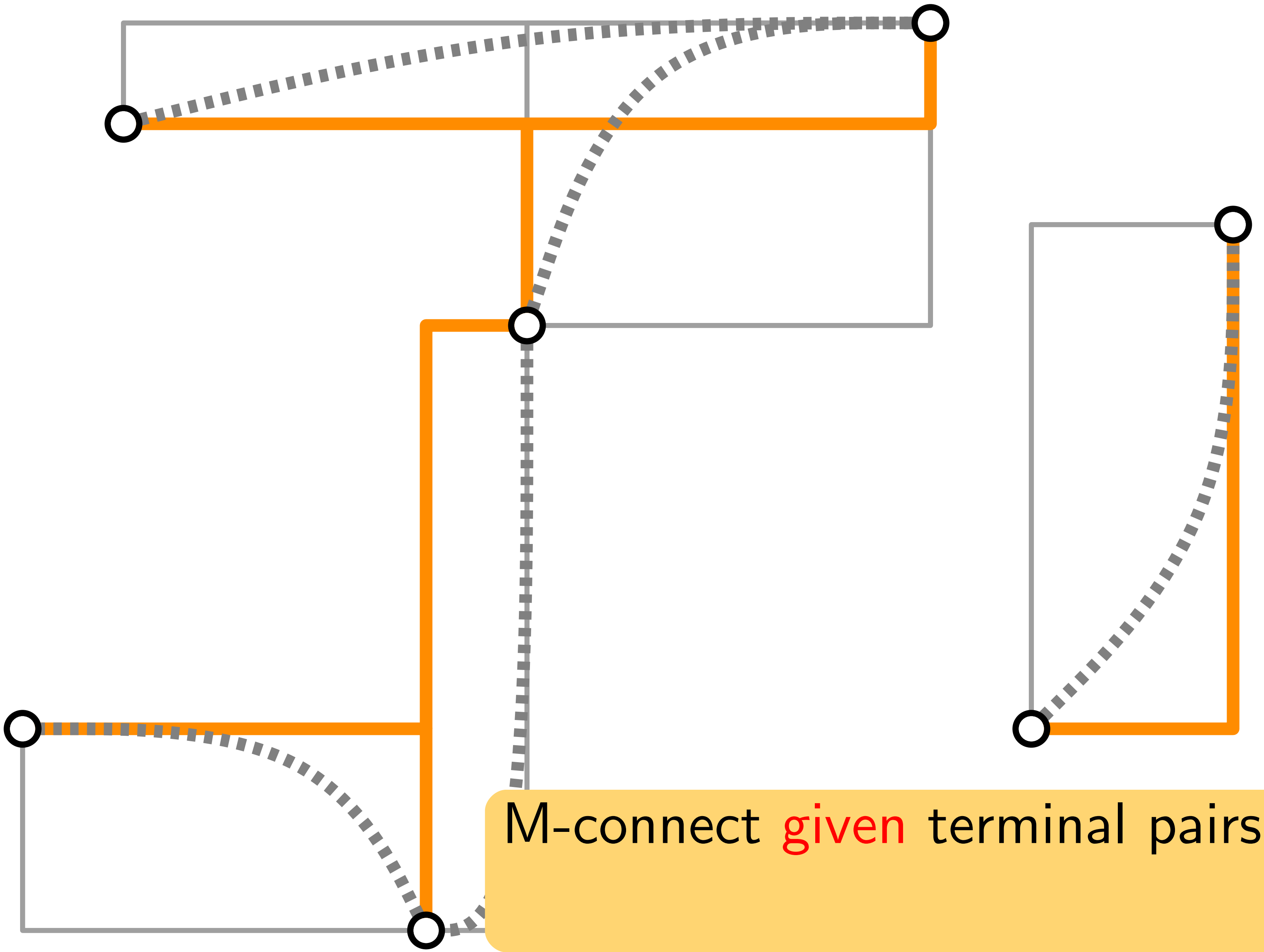


Definitions: Generalized MMN (GMMN)



M-connect given terminal pairs.

Definitions: Generalized MMN (GMMN)



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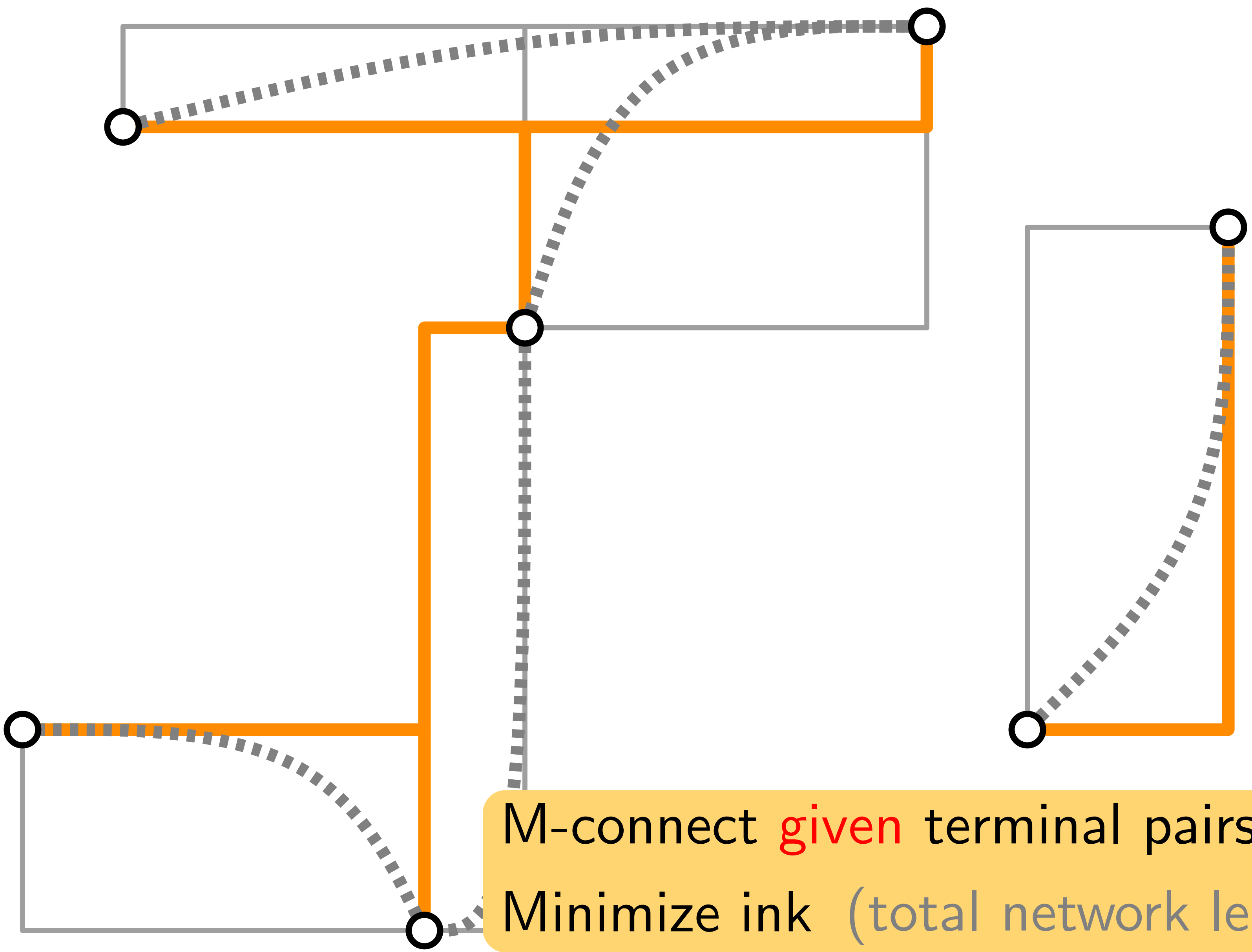
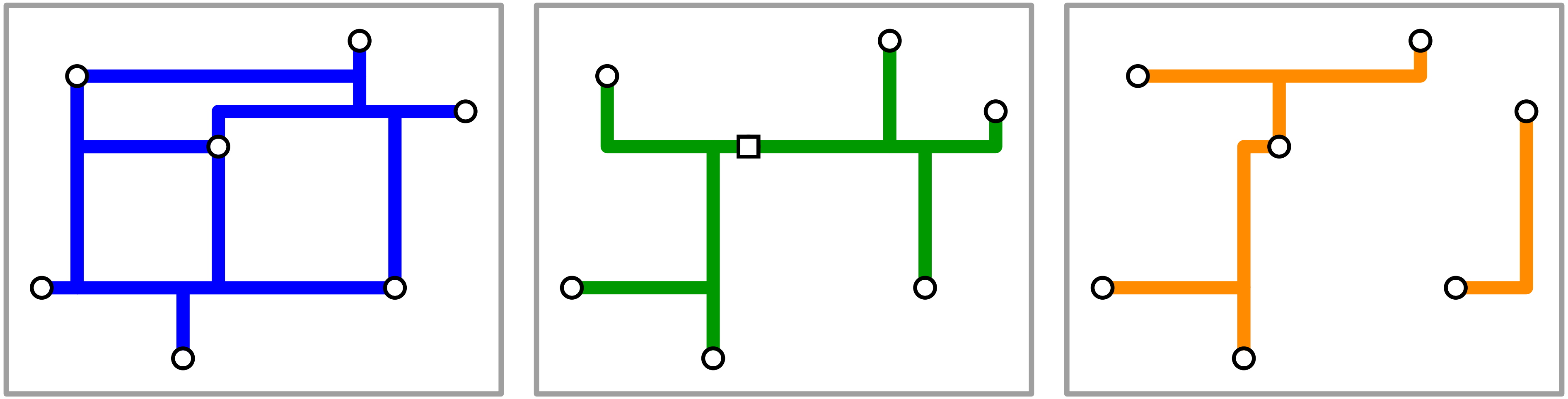


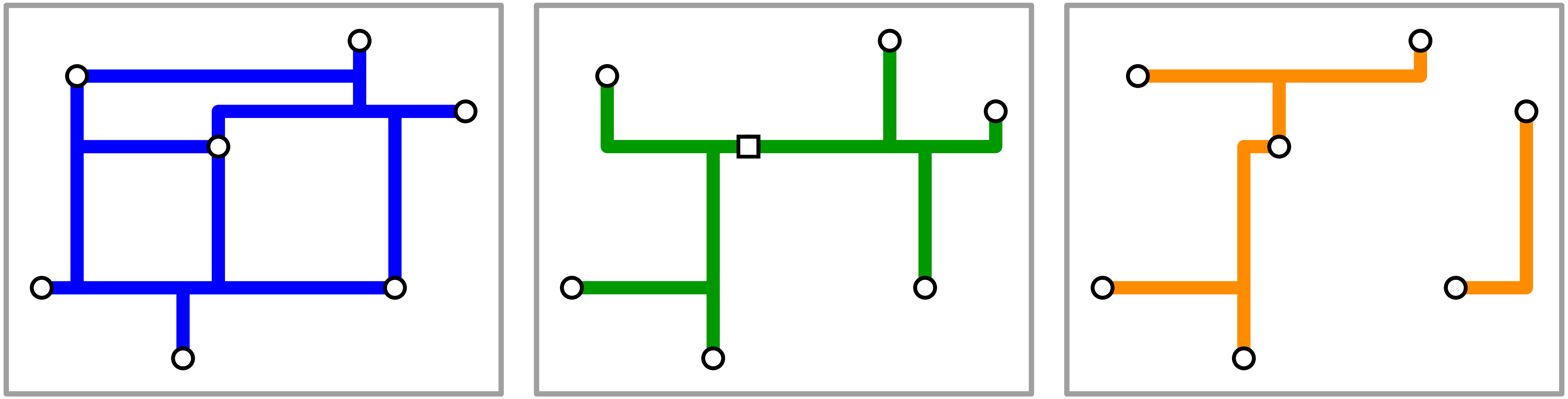
M-connect given terminal pairs.

Minimize ink (total network length)!

Applications

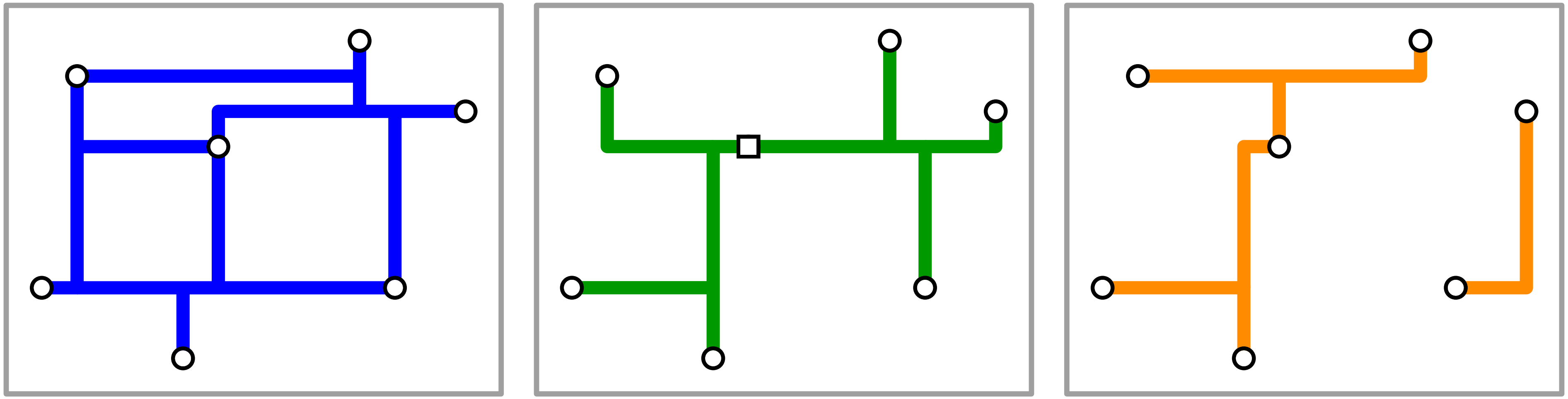


Applications



MMN

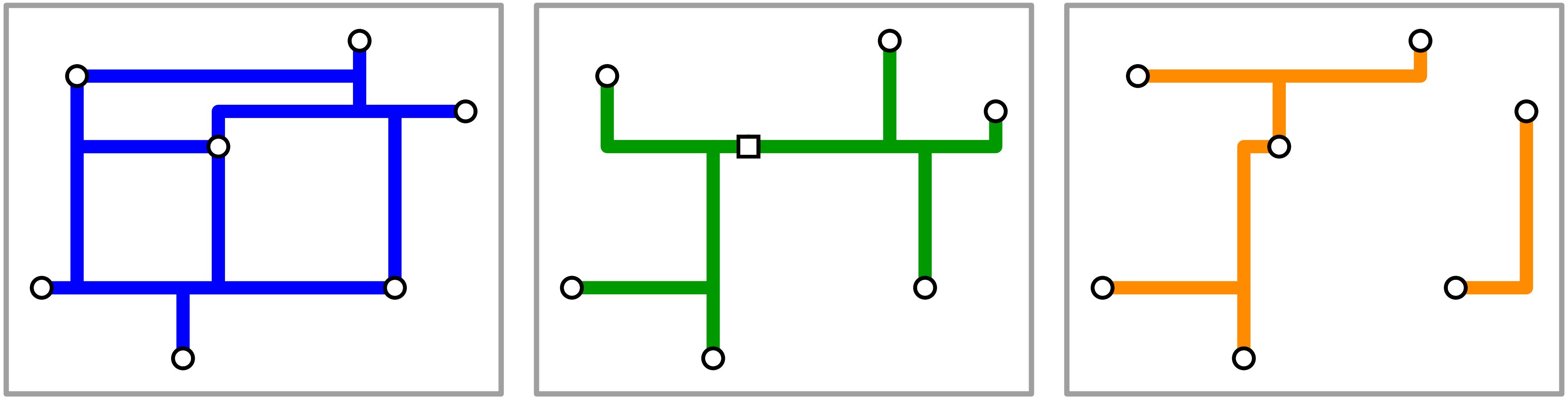
Applications



MMN

- point-set embedding:

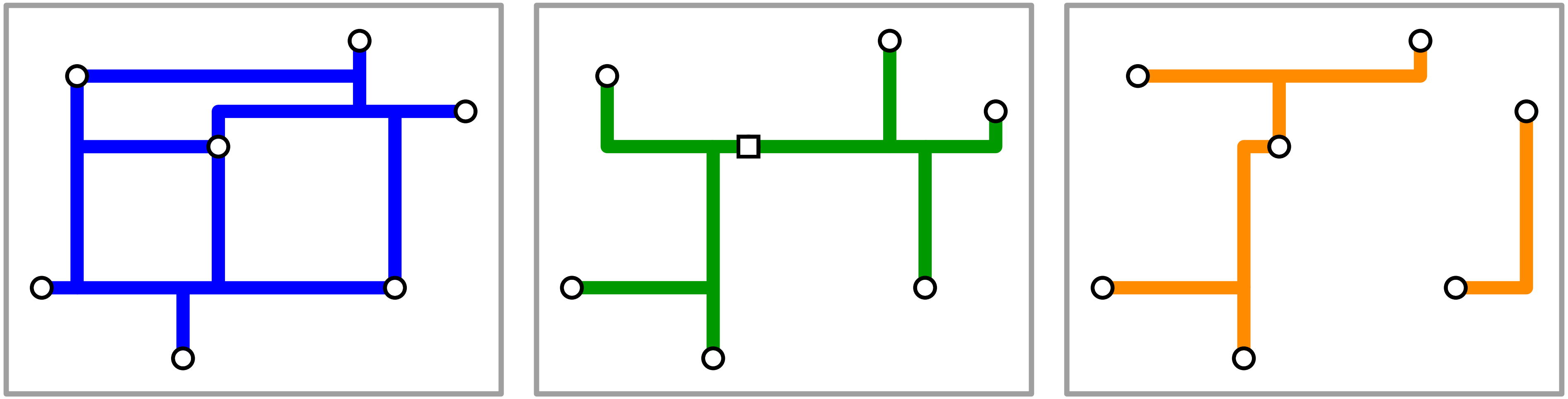
Applications



MMN

- point-set embedding:
  draw K_n with min. ink
  (using M-geodesics)

Applications



MMN

- point-set embedding:
  draw K_n with min. ink
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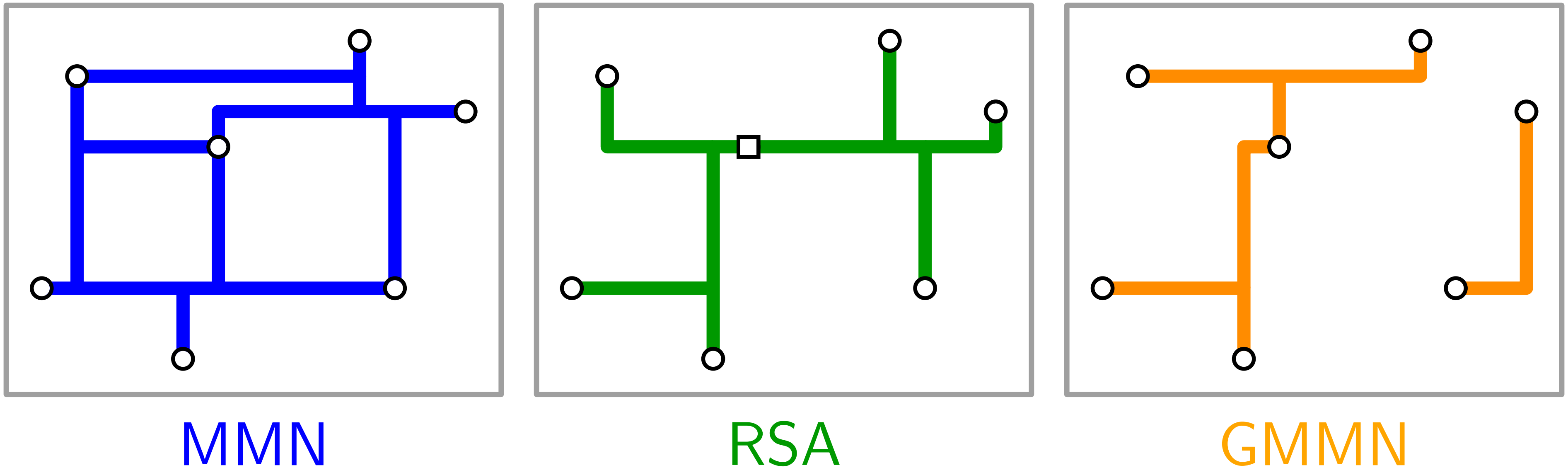
- visualization of
  split networks
  (also in higher dim.)

Applications



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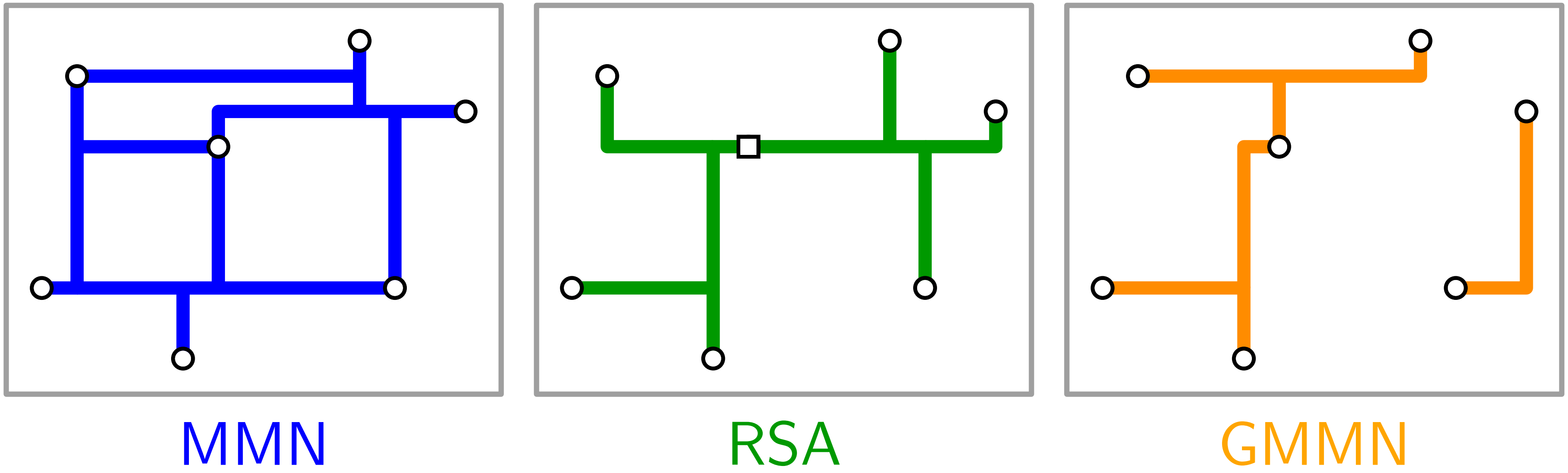

- point-set embedding:
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- visualization of
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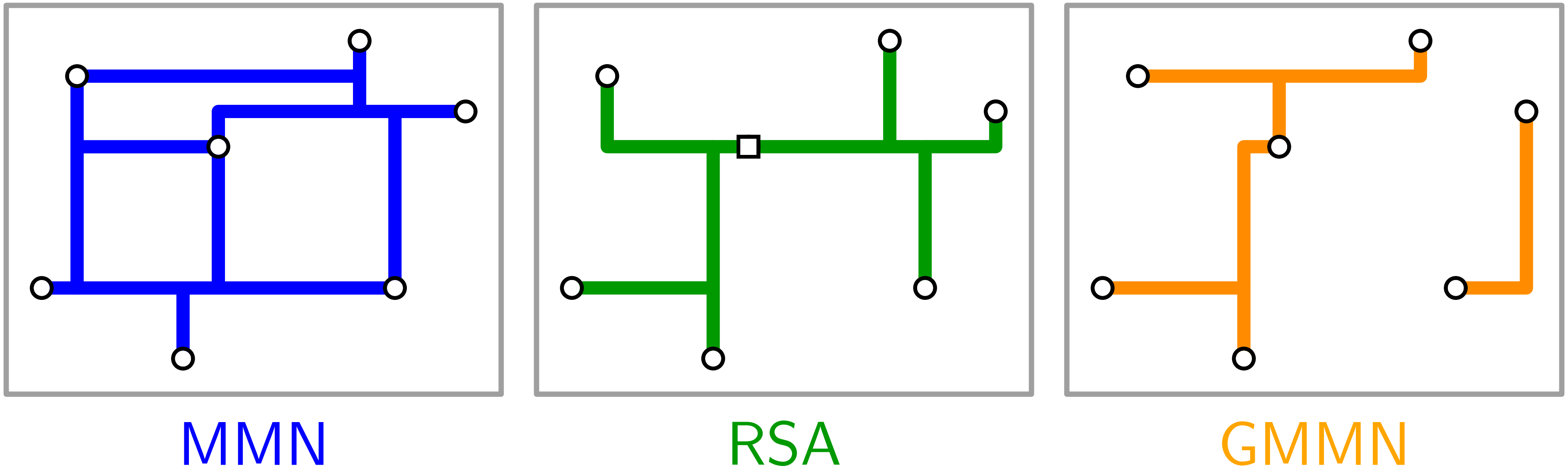
- VLSI layout:

Applications



- point-set embedding:
  draw K_n with min. ink
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- visualization of
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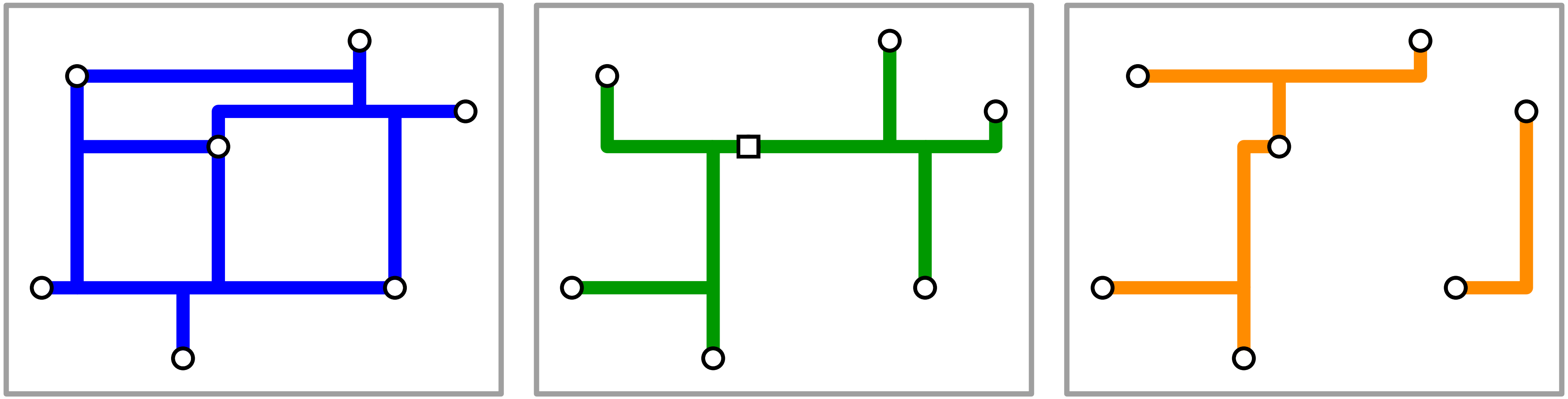
- VLSI layout:

minimize total wire length

Applications



MMN RSA GMMN

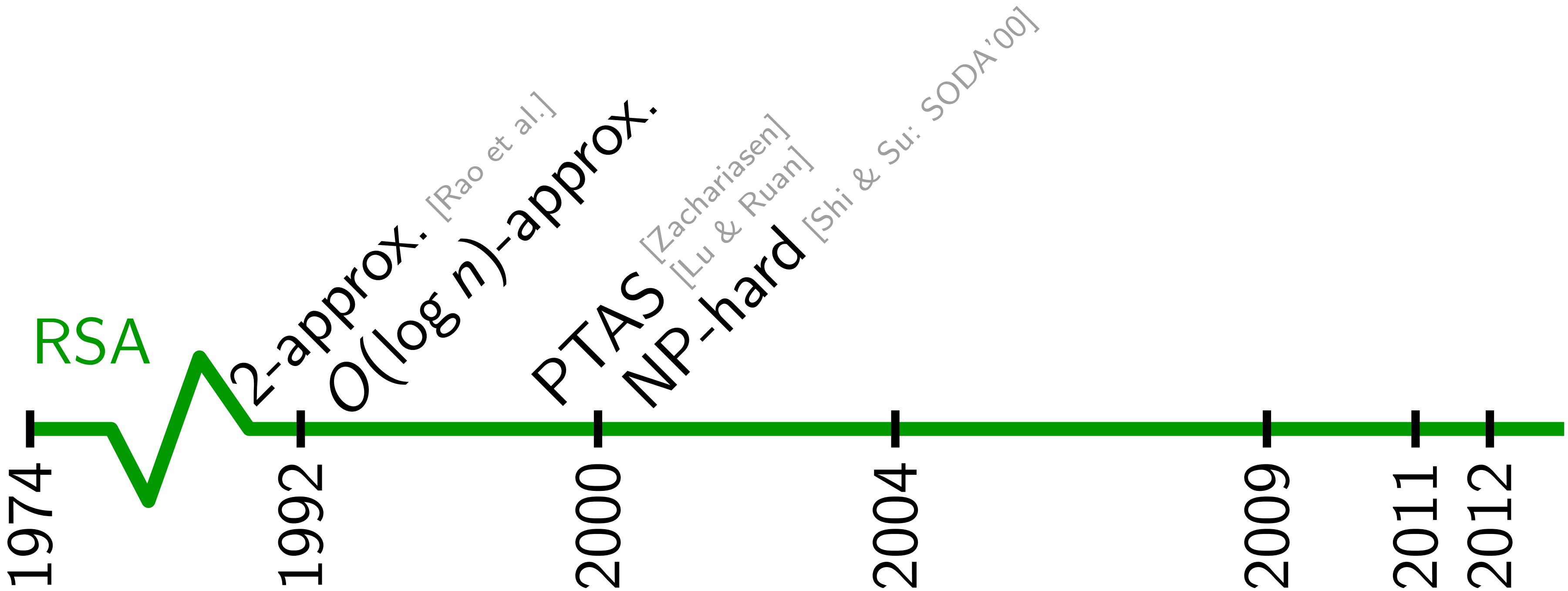
- point-set embedding:
  draw K_n with min. ink
  (using M-geodesics)
- visualization of
  split networks
  (also in higher dim.)

- VLSI layout:

minimize total wire length

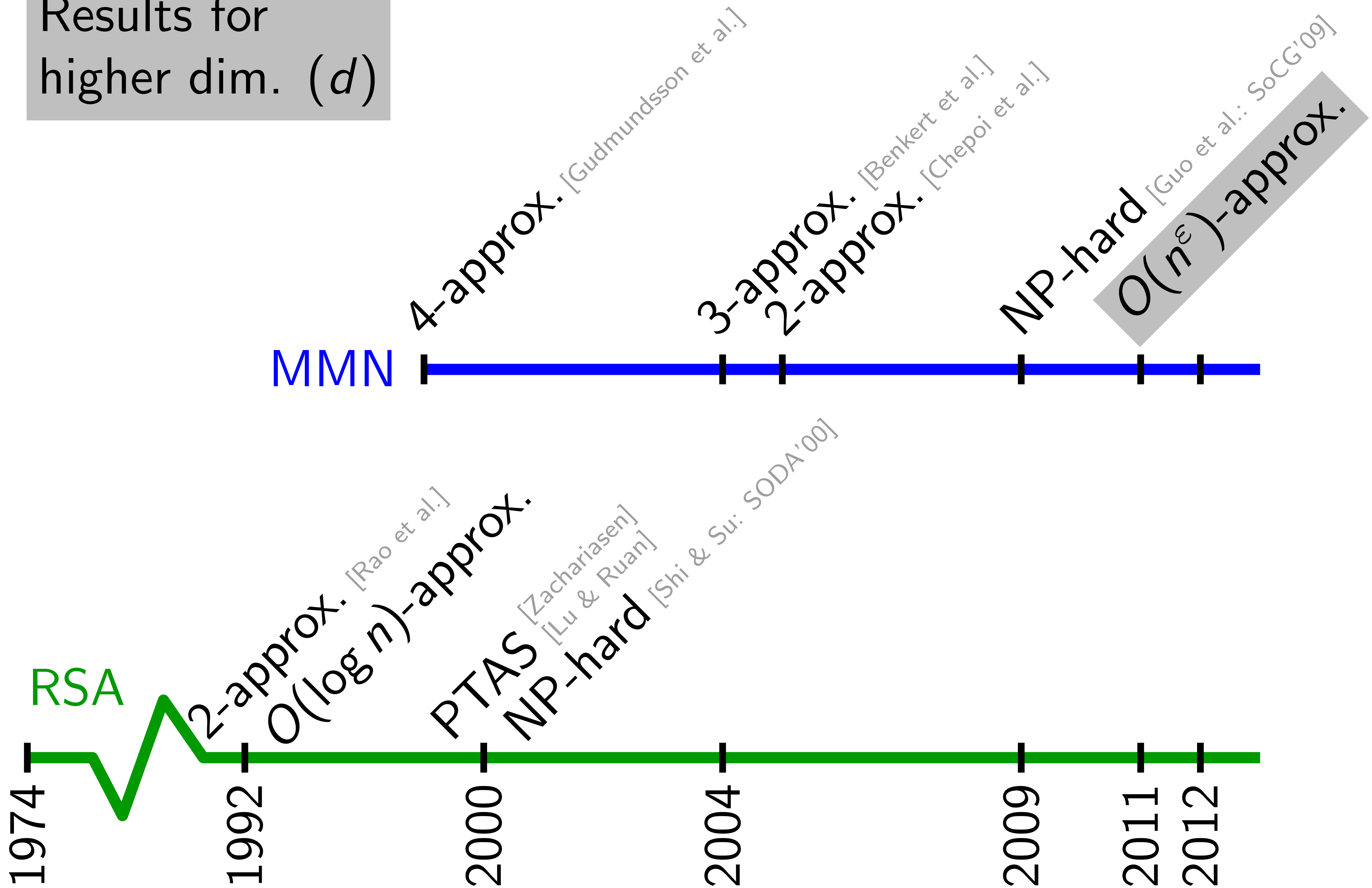
minimize signal travel time

Previous Work

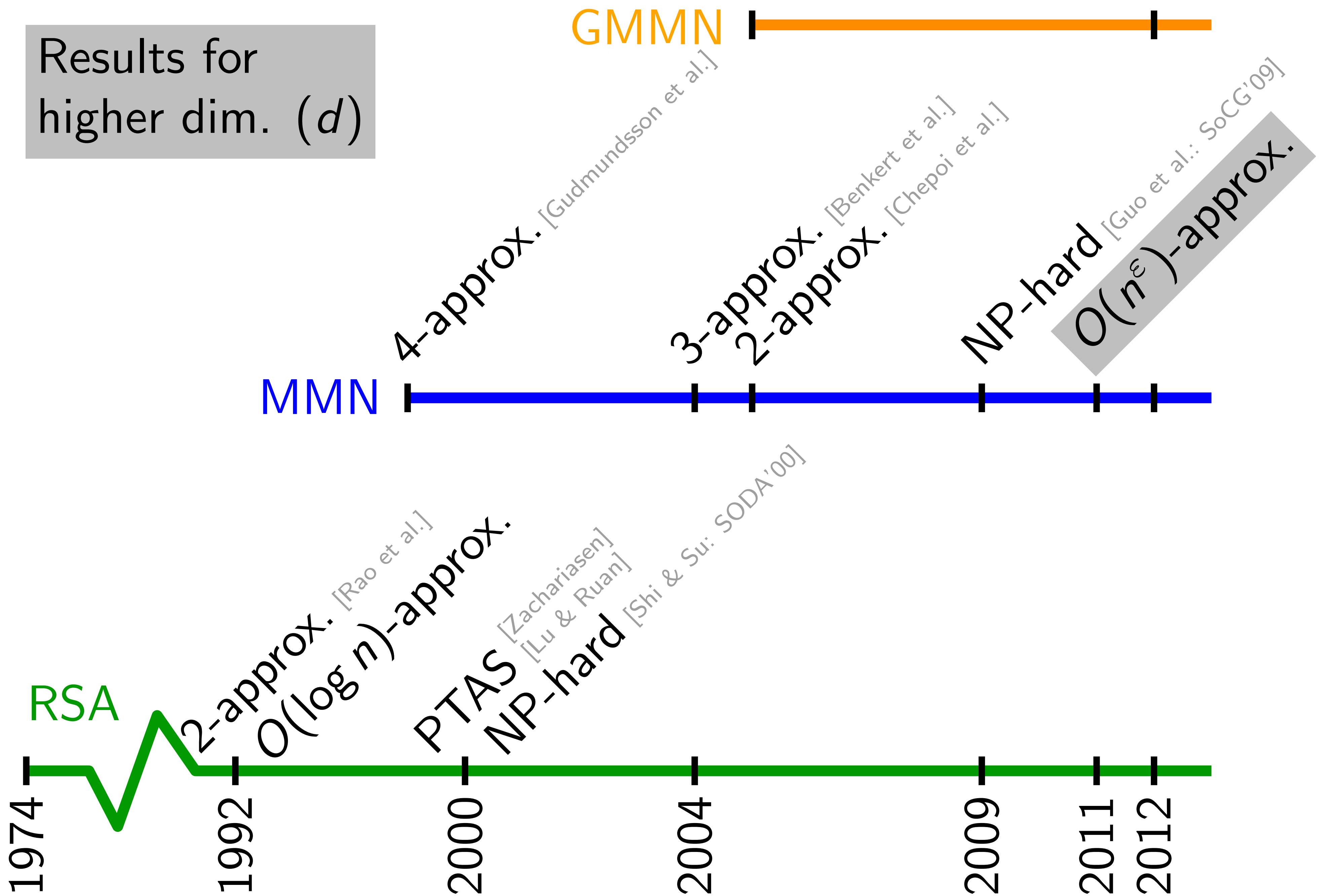


Previous Work

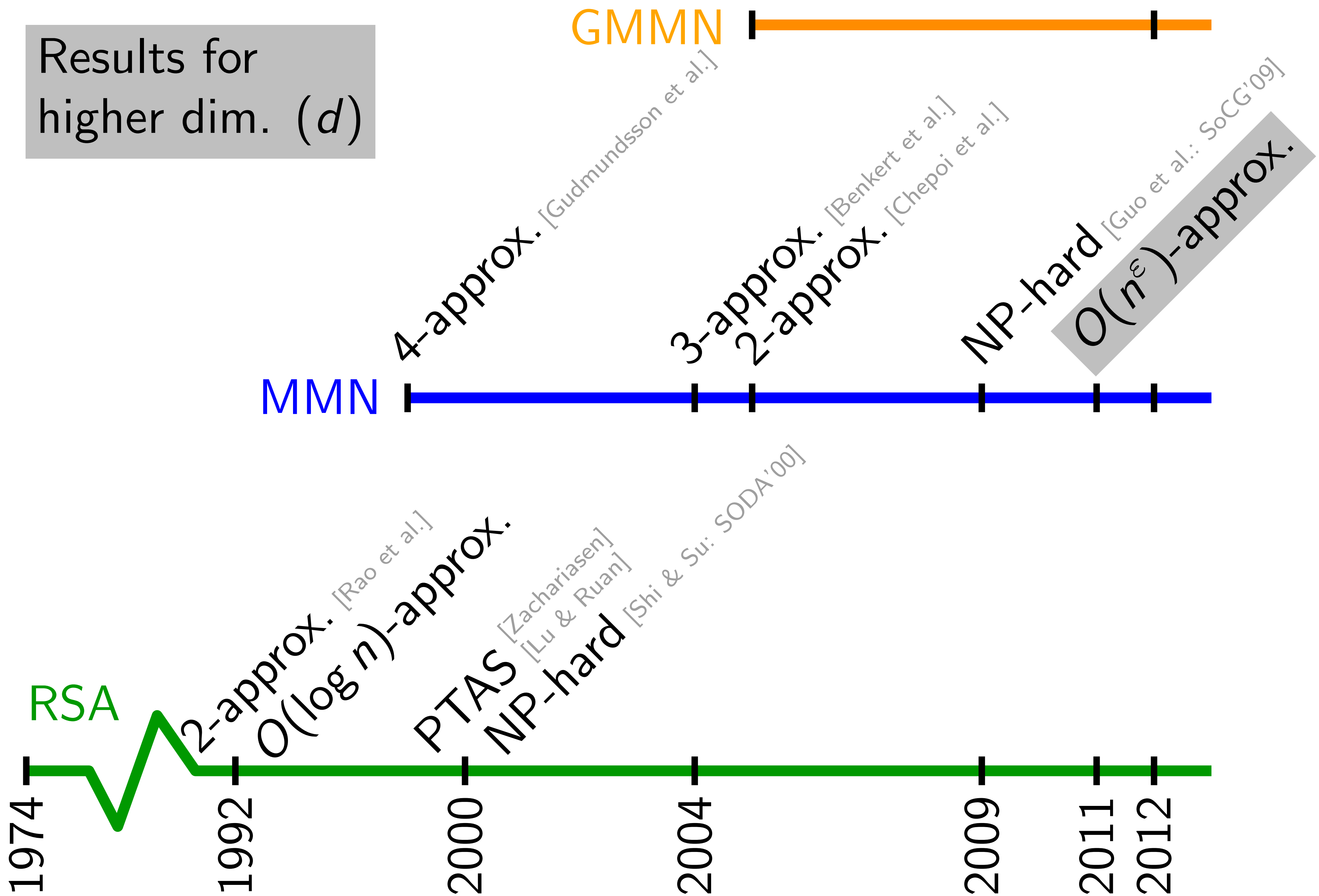
Results for
higher dim. (d)



Previous Work



Previous Work + Our Contributions



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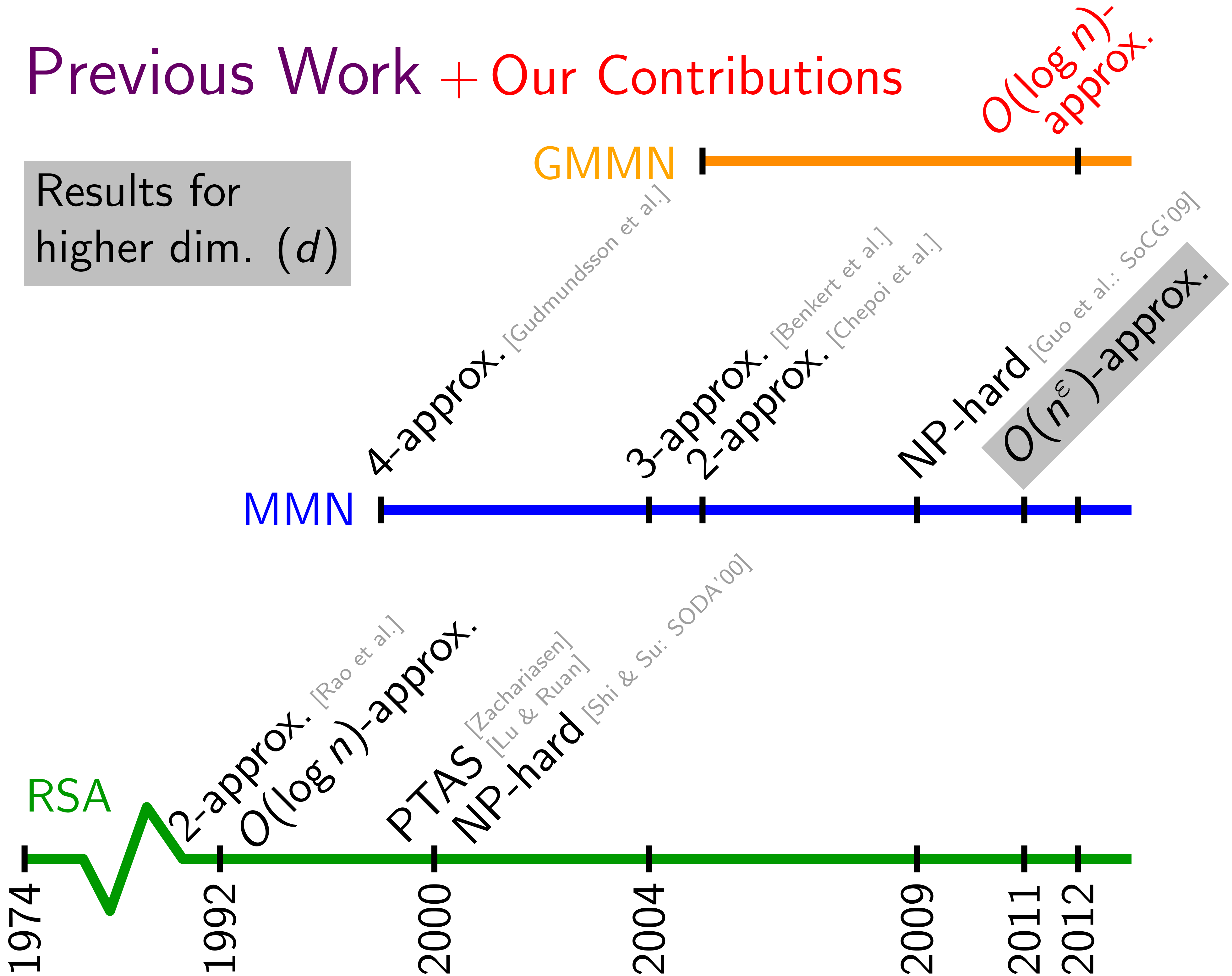

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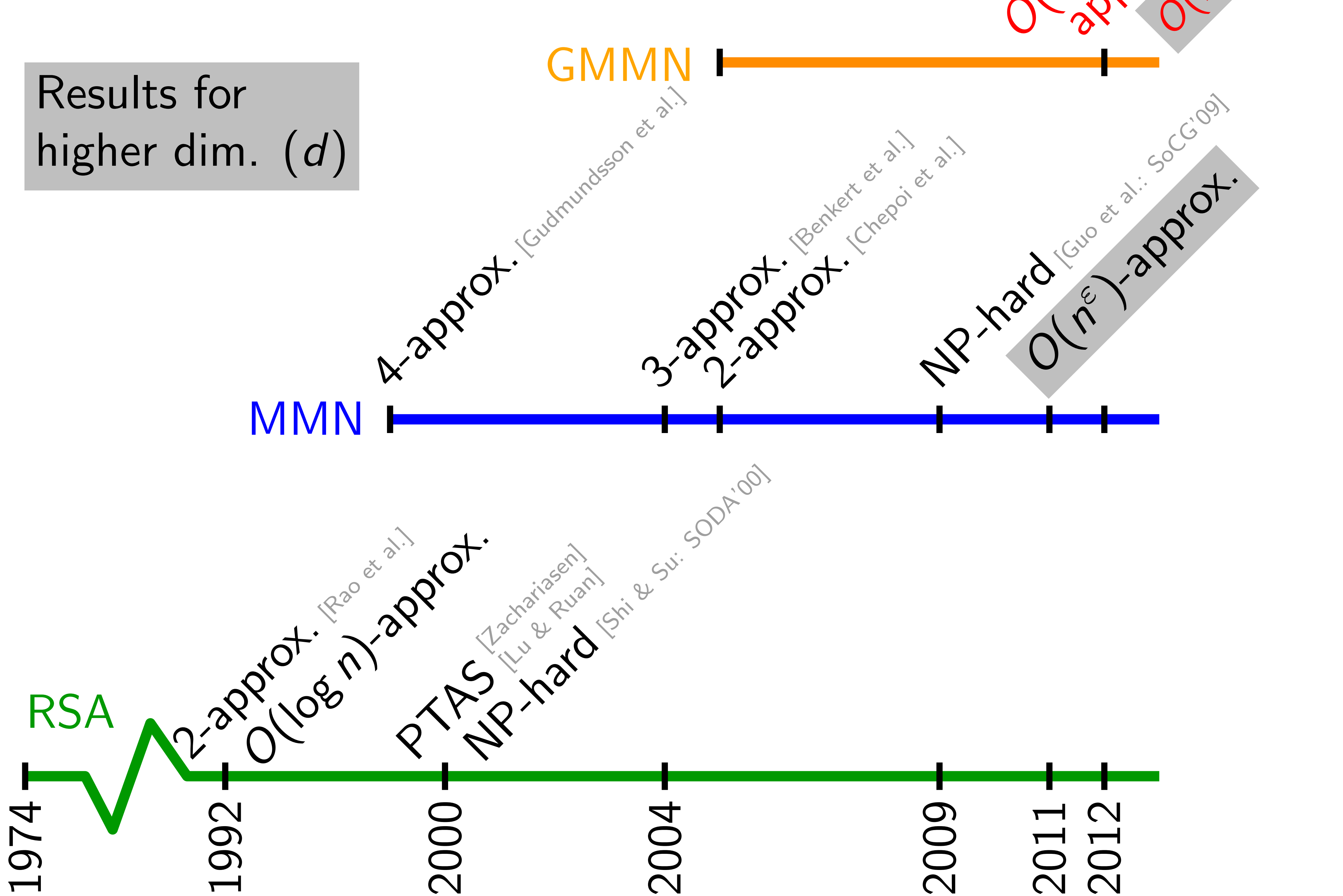

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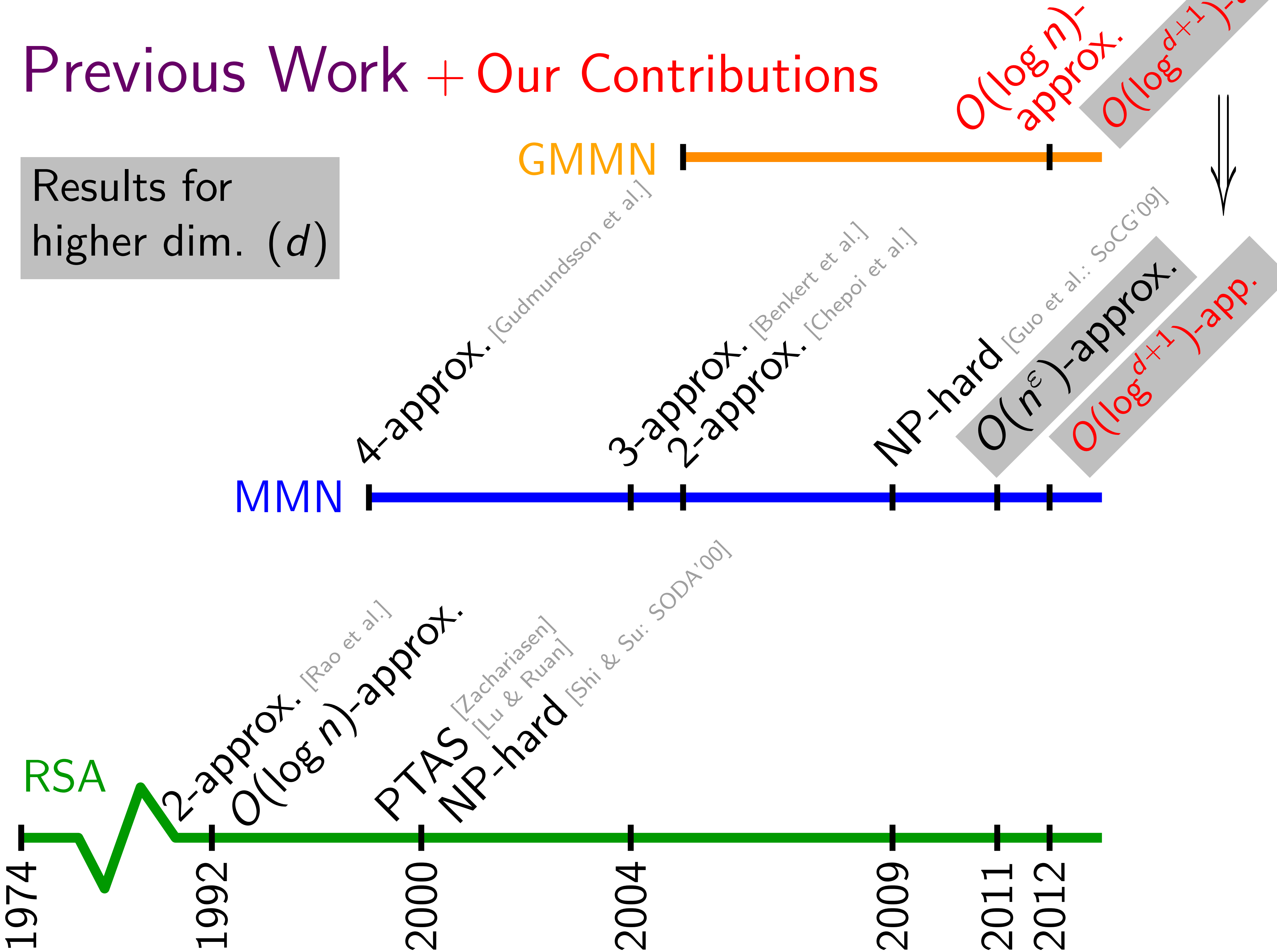

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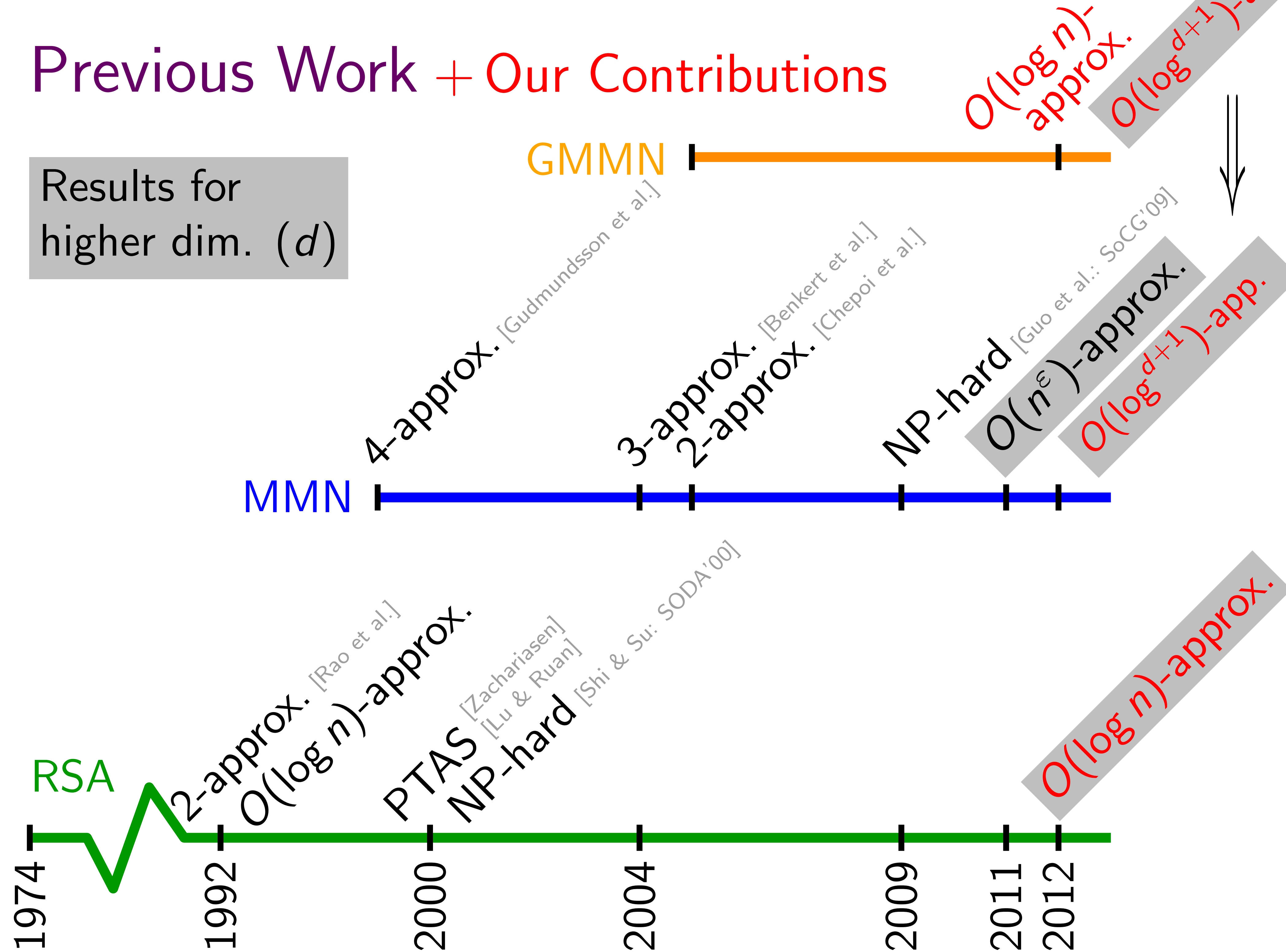

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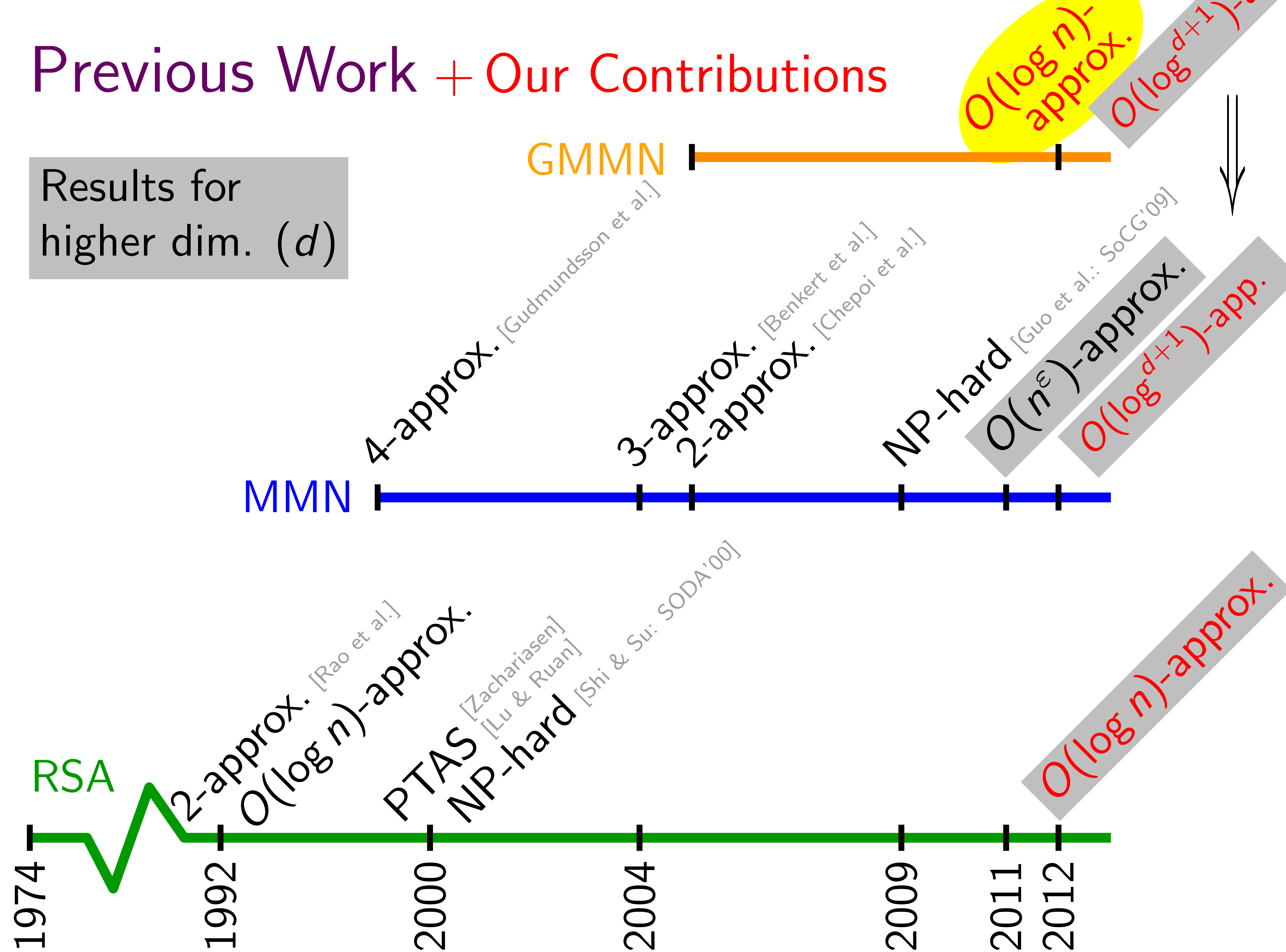
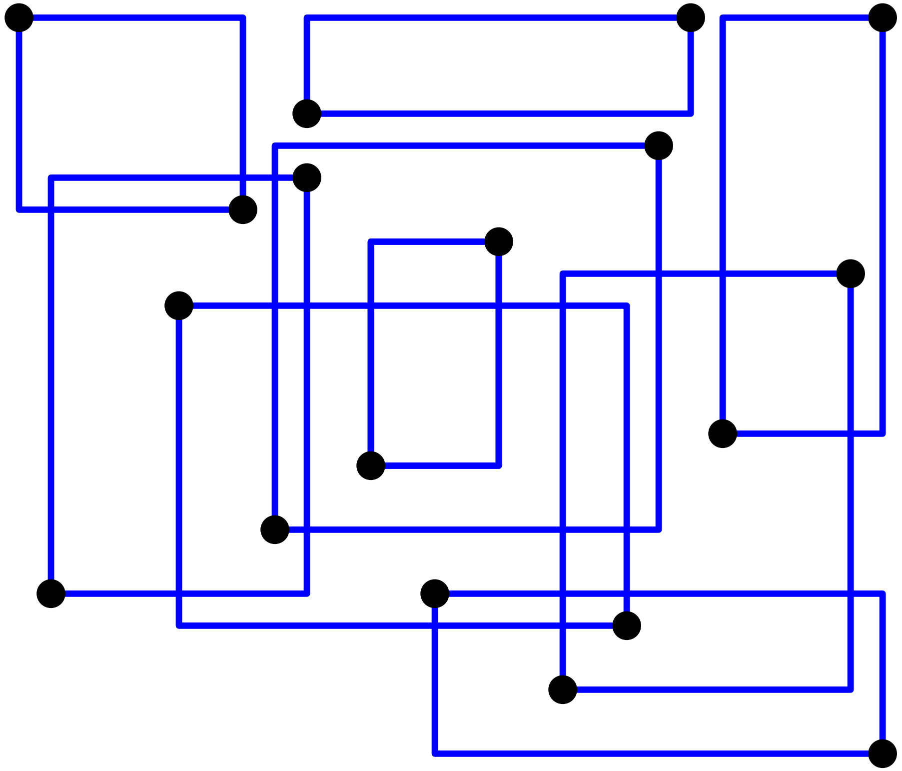
Results for higher dim. (d)



Part I

A Simple Recursive $O(\log^2 n)$-Approximation Algorithm for GMMN in the Plane

Main Algorithm for 2D-GMMN



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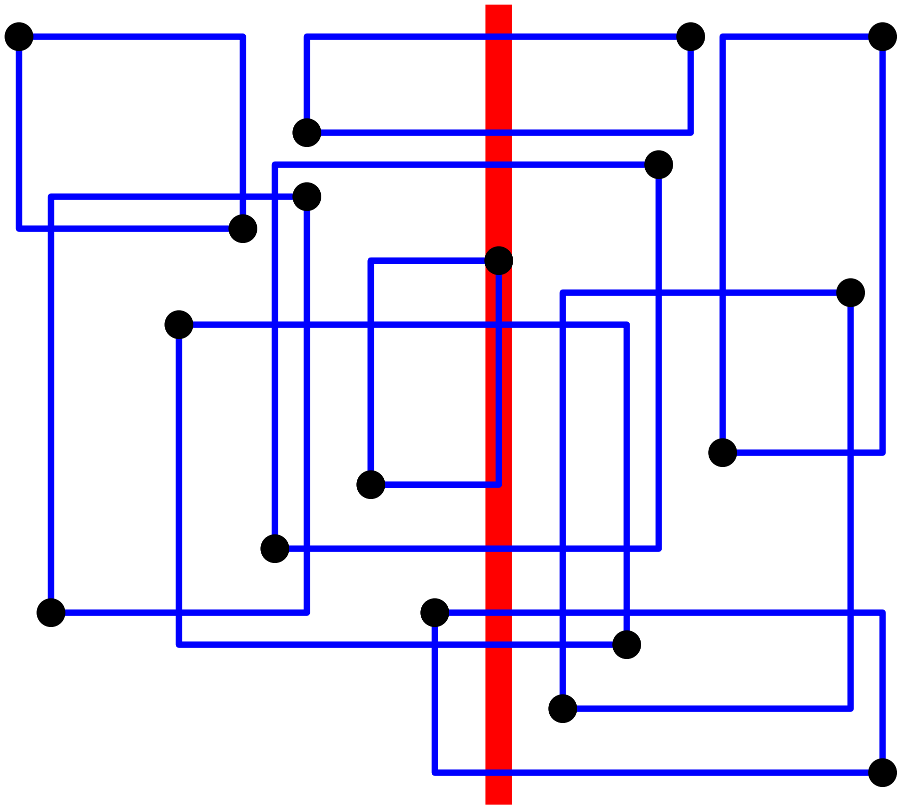

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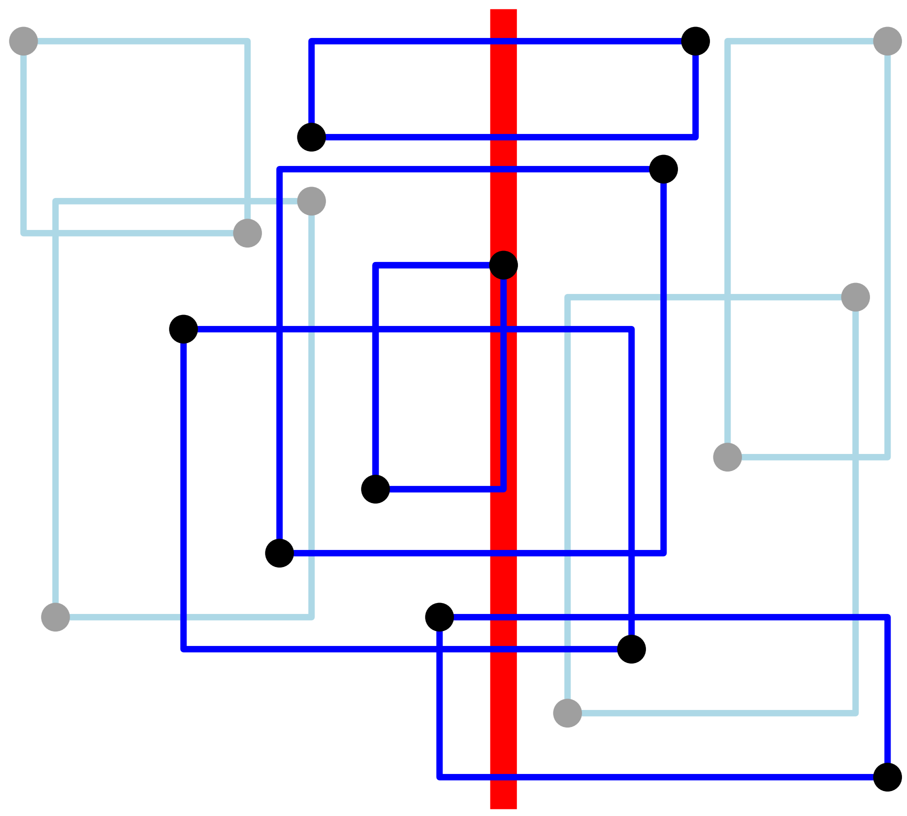

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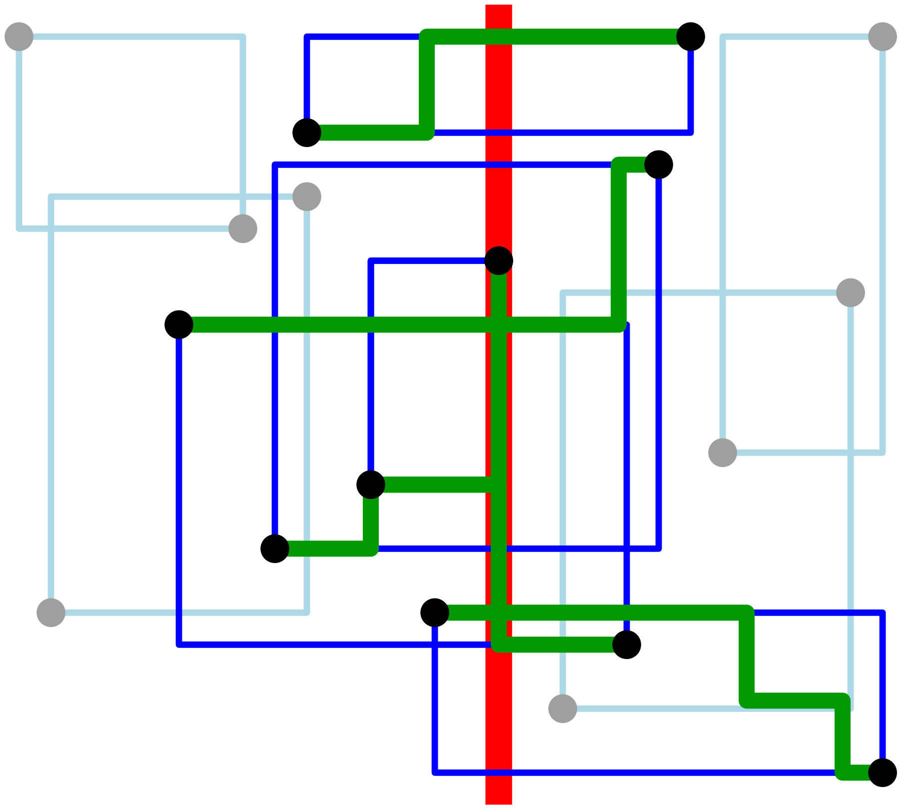

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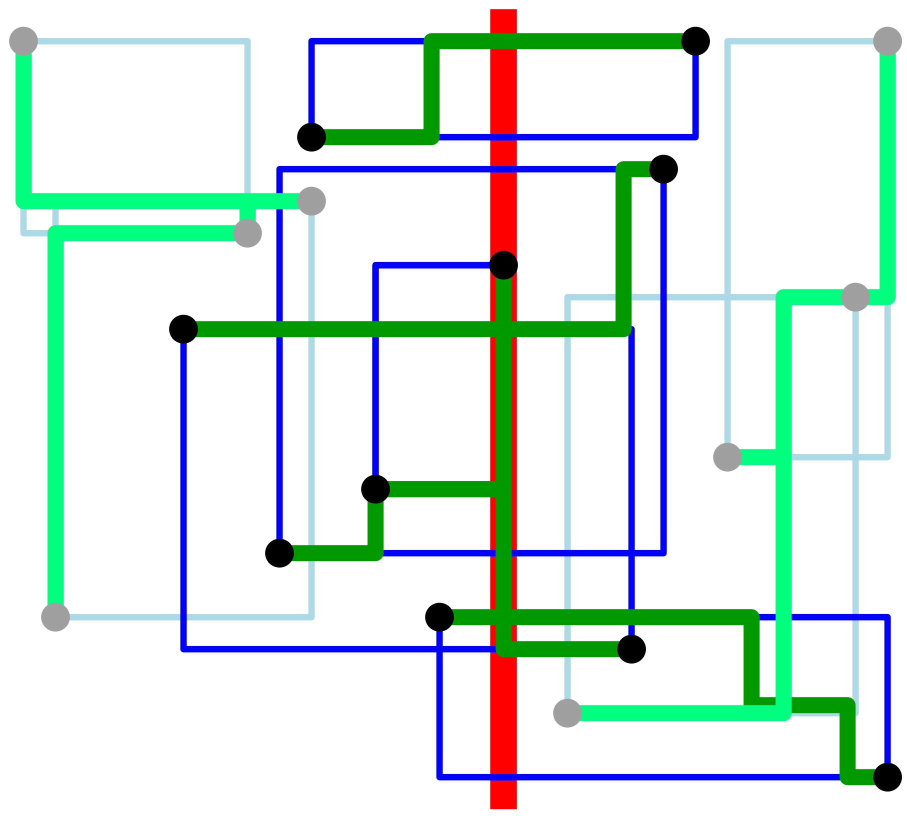
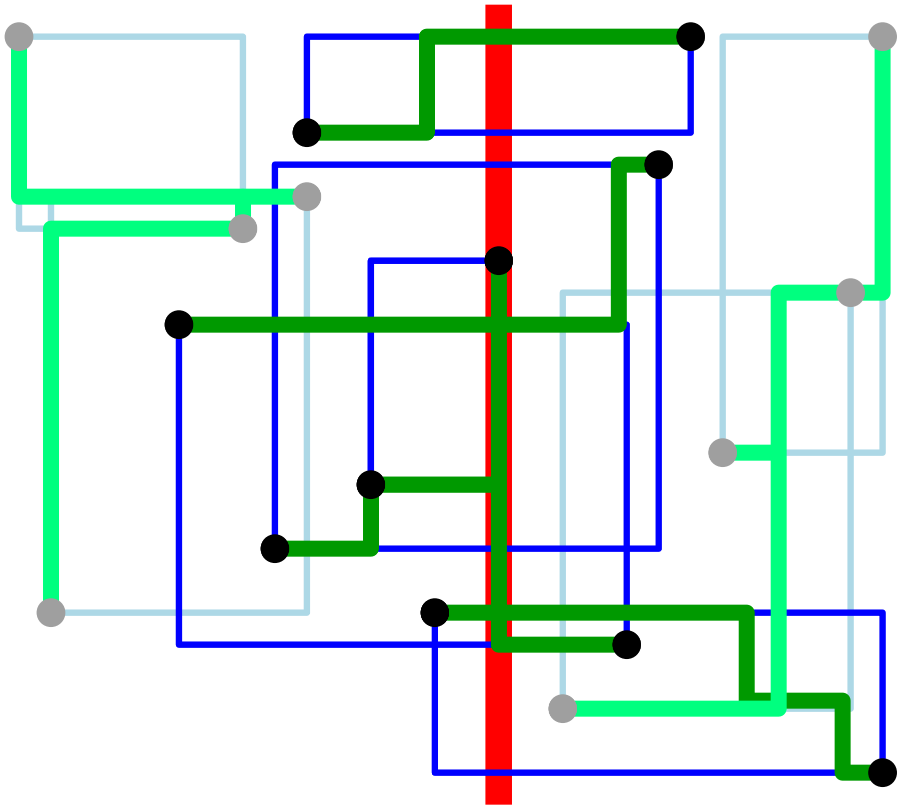
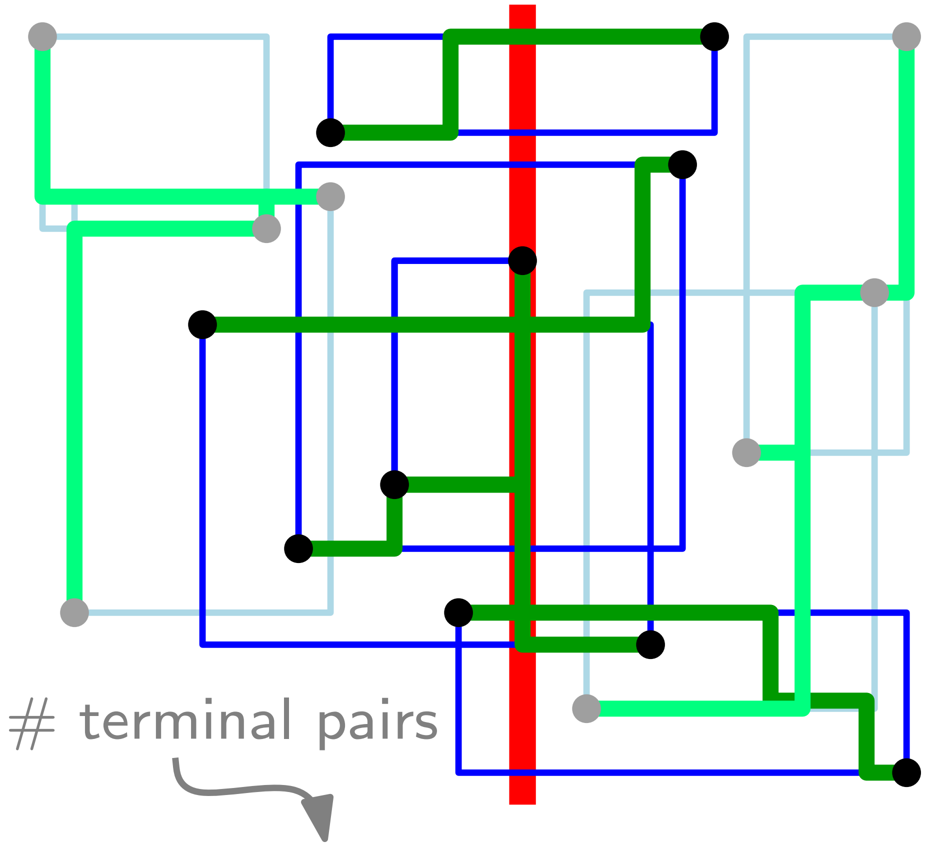


Main Algorithm for 2D-GMMN



Let $\rho_{\mathrm{2D}}(n) = \mathrm{cost}\,/\,\mathrm{OPT}$ be the performance ratio (in w-c).

Main Algorithm for 2D-GMMN



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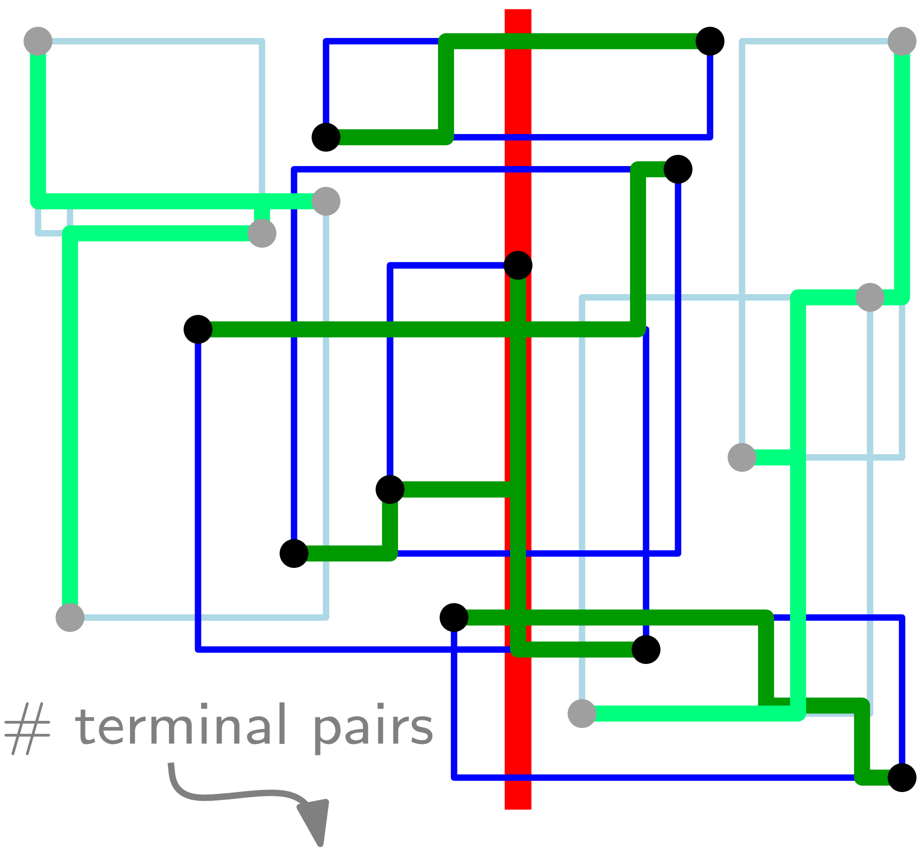
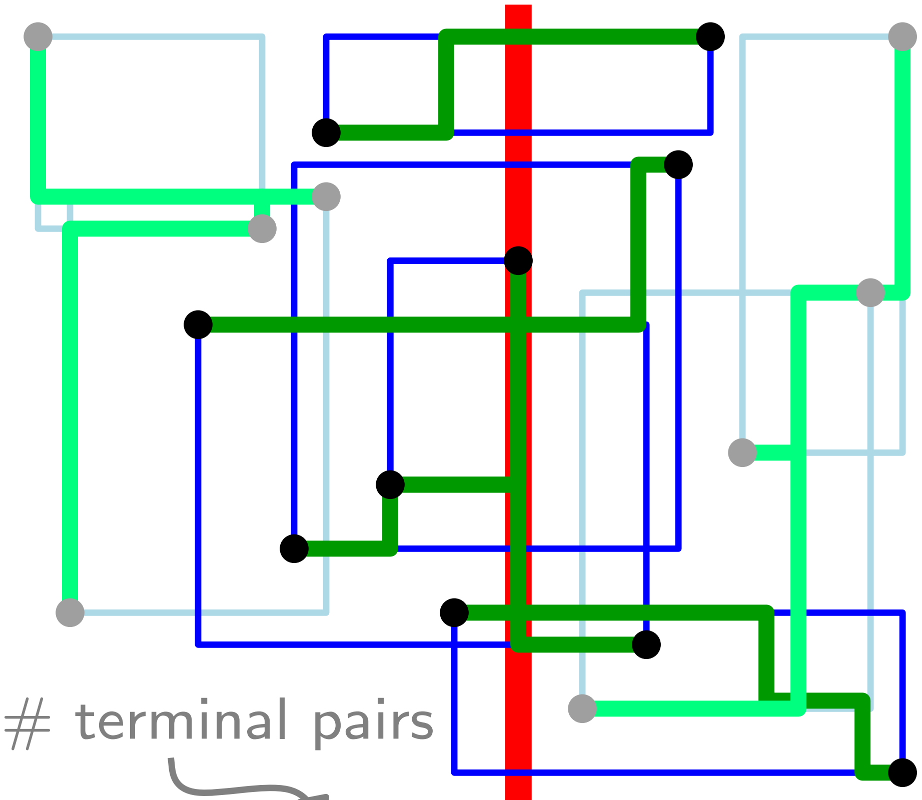


Let $\rho_{2\mathrm{D}}(n) = \mathrm{cost} \,/\, \mathrm{OPT}$ be the performance ratio (in w-c).

$\rho_{2\mathrm{D}}(n)\,\mathrm{OPT} = \mathrm{cost}$

Main Algorithm for 2D-GMMN

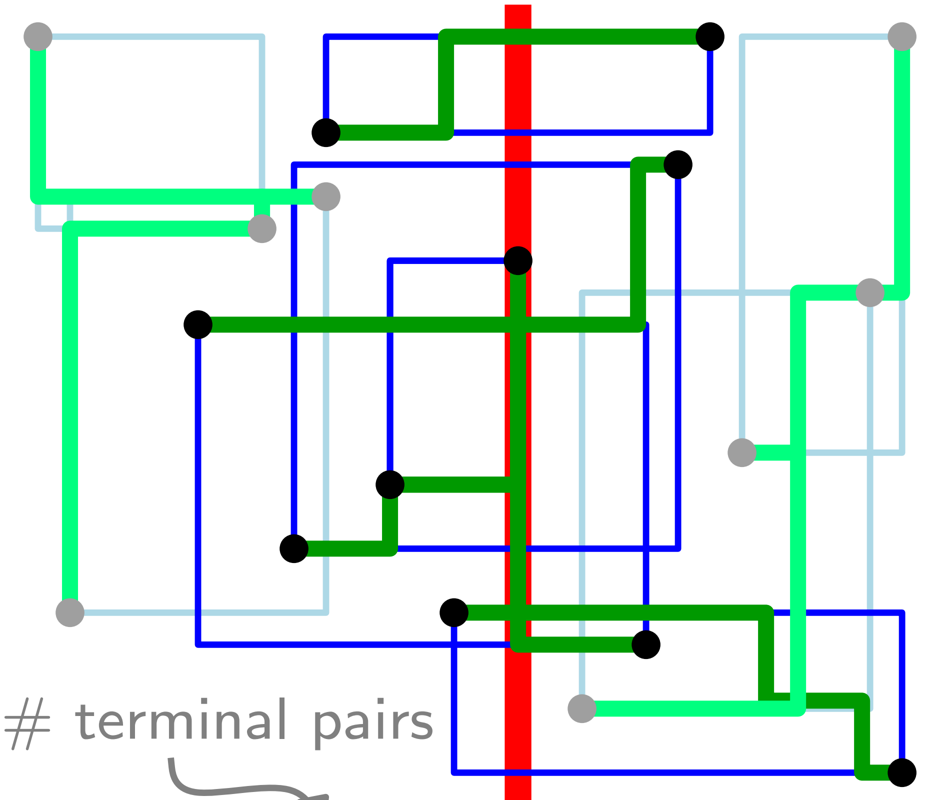


terminal pairs

Let $\rho_{2D}(n) = \text{cost} \,/\, \text{OPT}$ be the performance ratio (in w-c).

$\rho_{2D}(n)\,\text{OPT} = \text{cost} \;\le$

Main Algorithm for 2D-GMMN

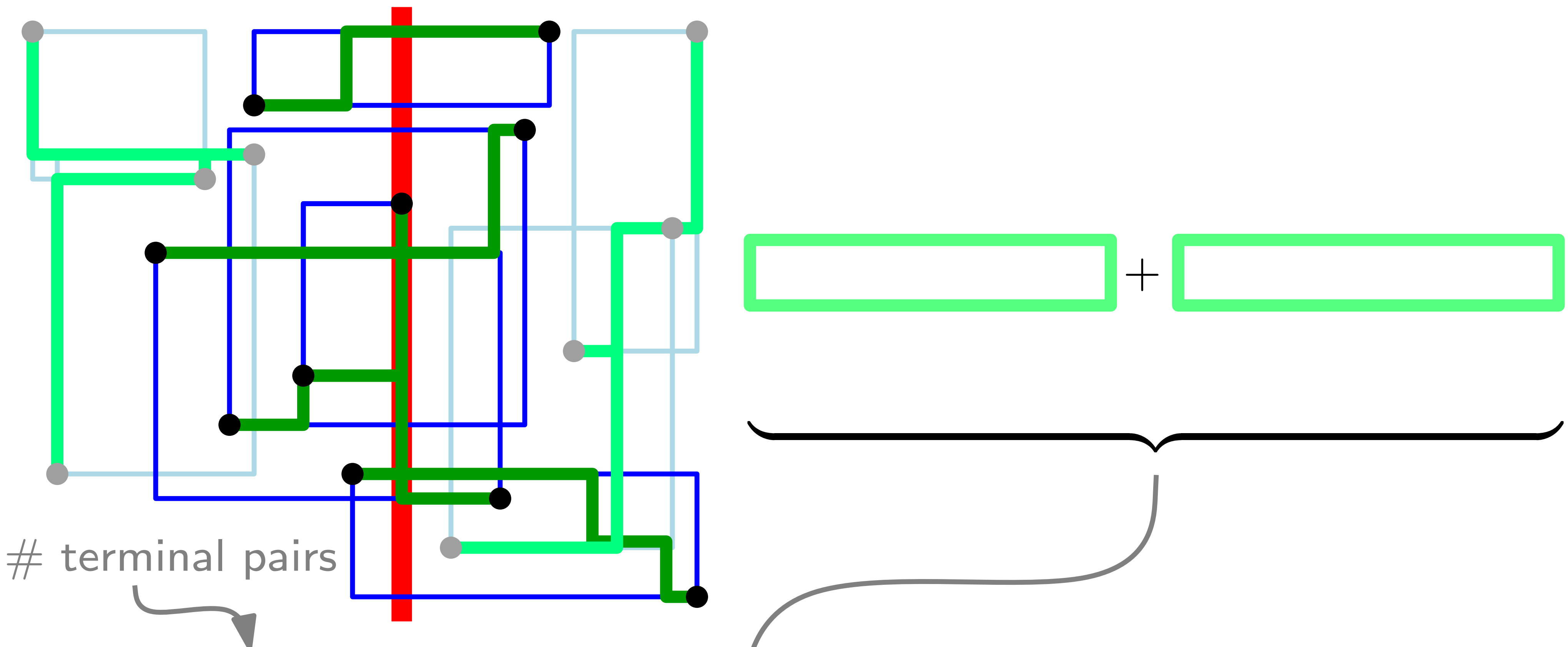


terminal pairs

Let $\rho_{2D}(n) = \text{cost} \,/\, \text{OPT}$ be the performance ratio (in w-c).

$\rho_{2D}(n)\,\text{OPT} = \text{cost} \leq \boxed{} + \boxed{}$

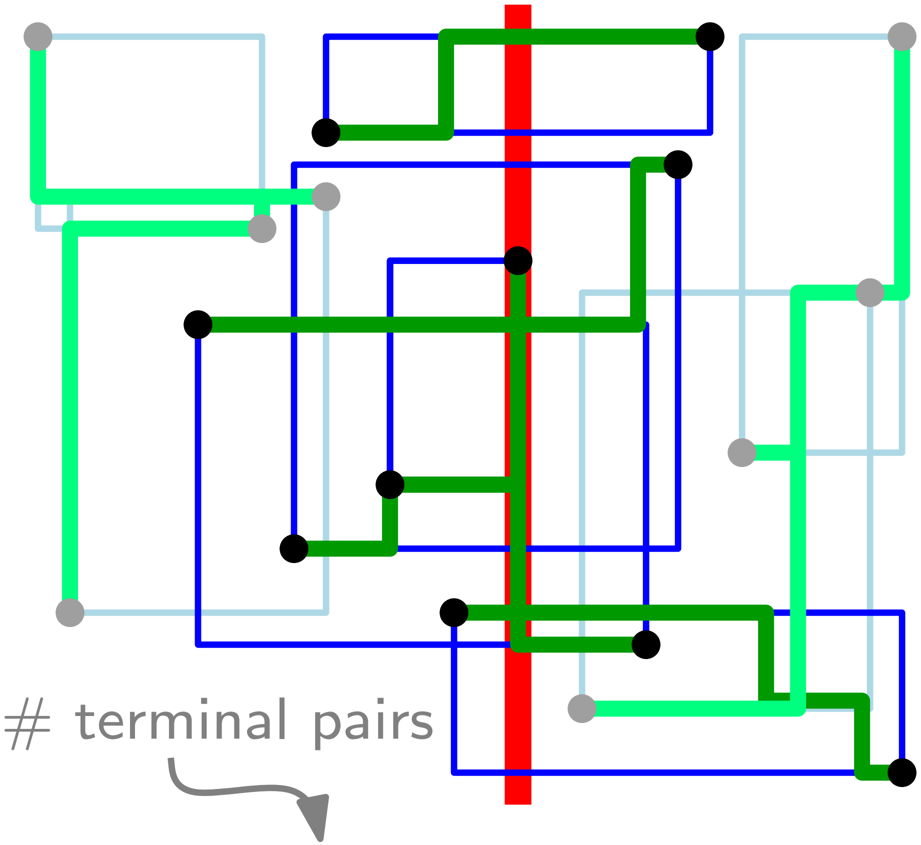
Main Algorithm for 2D-GMMN



Let $\rho_{2D}(n) = \text{cost} / \text{OPT}$ be the performance ratio (in w-c).

$\rho_{2D}(n)\,\text{OPT} = \text{cost} \leq \square + \square$

Main Algorithm for 2D-GMMN

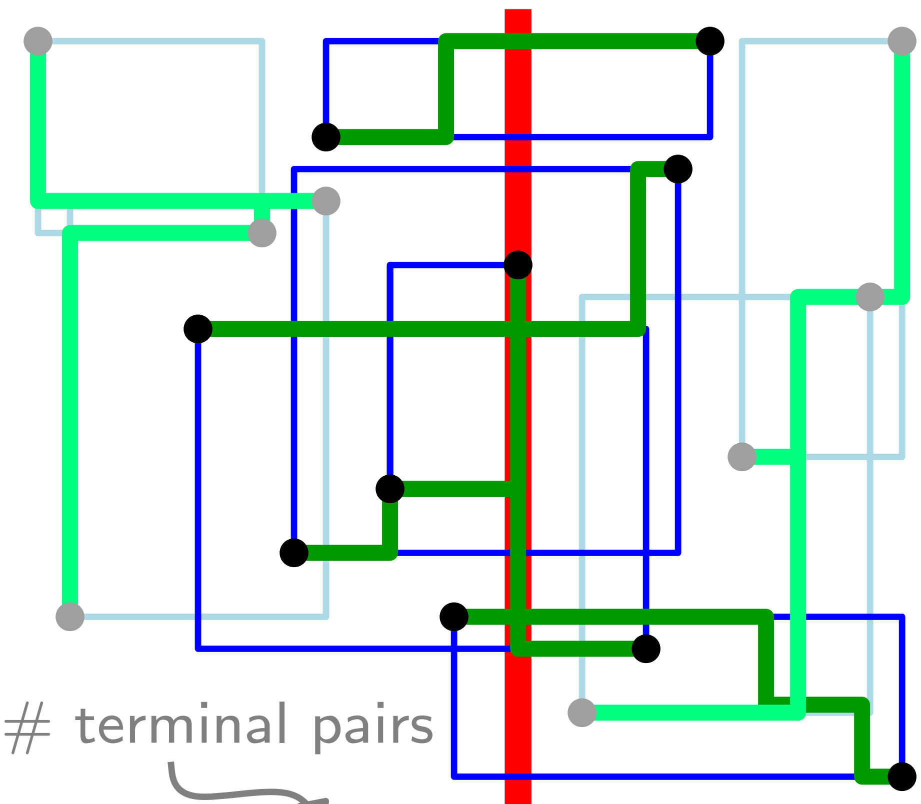


$\rho_{2D}(n/2)\,\mathrm{OPT}_{\text{left}} + \square$

Let $\rho_{2D}(n) = \text{cost} \,/\, \mathrm{OPT}$ be the performance ratio (in w-c).

$\rho_{2D}(n)\,\mathrm{OPT} = \text{cost} \leq \square + \square$

Main Algorithm for 2D-GMMN



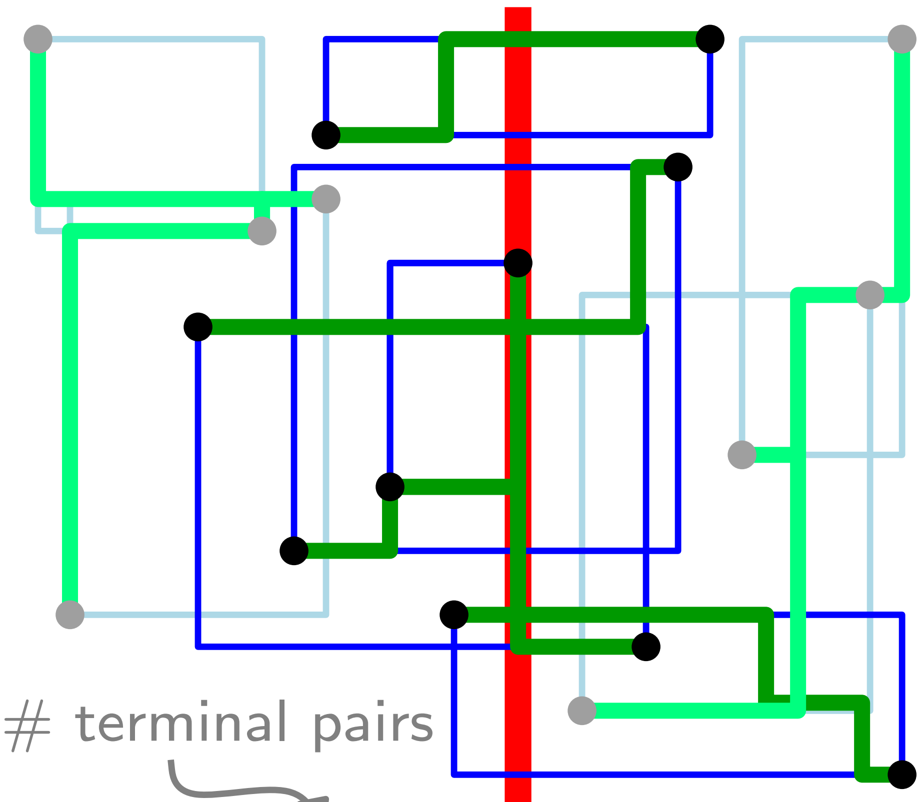
$\rho_{2D}(n/2)\,\mathrm{OPT}_{\text{left}} + \rho_{2D}(n/2)\,\mathrm{OPT}_{\text{right}}$

$\#$ terminal pairs

Let $\rho_{2D}(n) = \text{cost} \,/\, \mathrm{OPT}$ be the performance ratio (in w-c).

$\rho_{2D}(n)\,\mathrm{OPT} = \text{cost} \le \square + \square$

Main Algorithm for 2D-GMMN



$\rho_{2D}(n/2)\,\mathrm{OPT}_{\mathrm{left}} + \rho_{2D}(n/2)\,\mathrm{OPT}_{\mathrm{right}}$

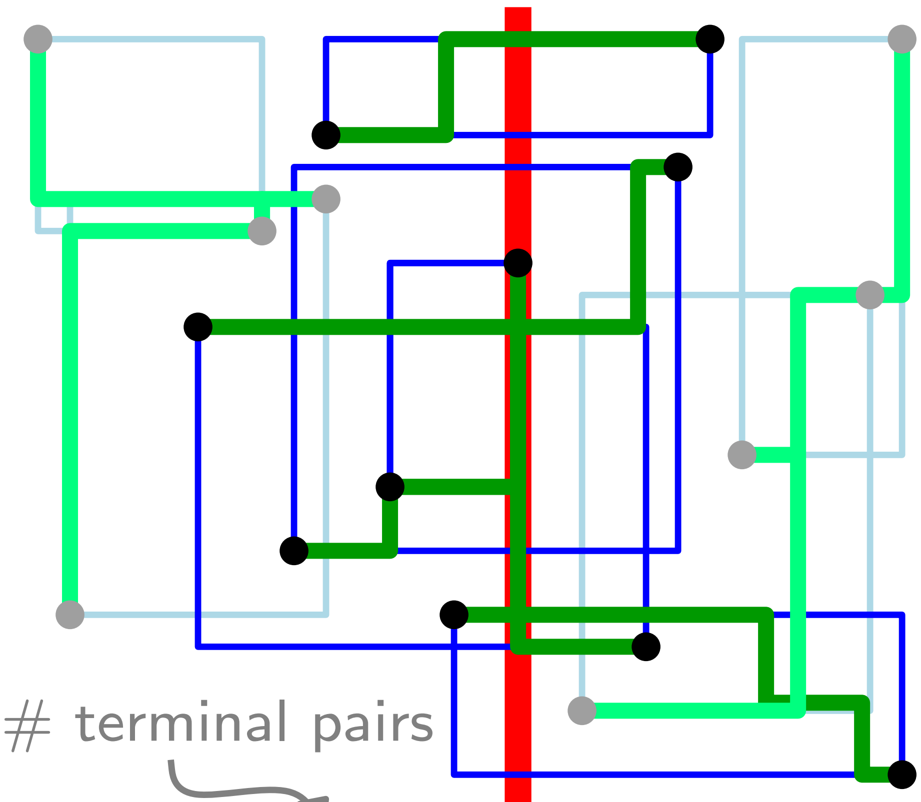
$\leq$

terminal pairs

Let $\rho_{2D}(n) = \mathrm{cost} / \mathrm{OPT}$ be the performance ratio (in w-c).

$\rho_{2D}(n)\,\mathrm{OPT} = \mathrm{cost} \leq \square + \square$

Main Algorithm for 2D-GMMN



$$\rho_{2D}(n/2)\,\mathrm{OPT}_{\text{left}} + \rho_{2D}(n/2)\,\mathrm{OPT}_{\text{right}}$$

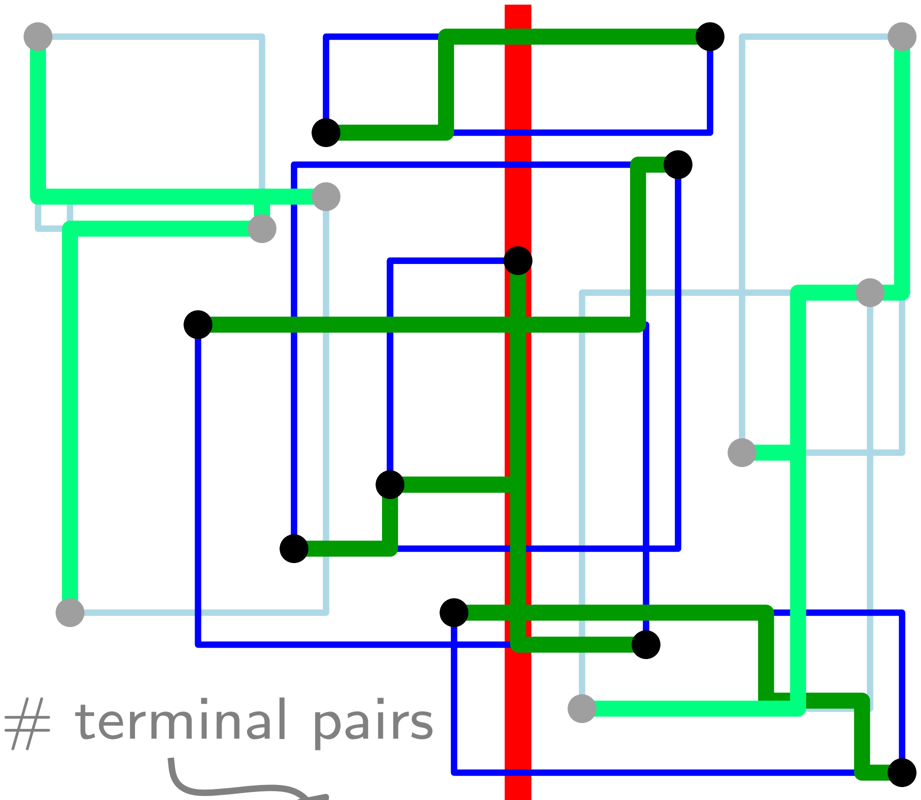
$$\leq \rho_{2D}(n/2)\,\mathrm{OPT}$$

terminal pairs

Let $\rho_{2D}(n) = \text{cost} \,/\, \mathrm{OPT}$ be the performance ratio (in w-c).

$$\rho_{2D}(n)\,\mathrm{OPT} = \text{cost} \leq \boxed{} + \boxed{}$$

Main Algorithm for 2D-GMMN



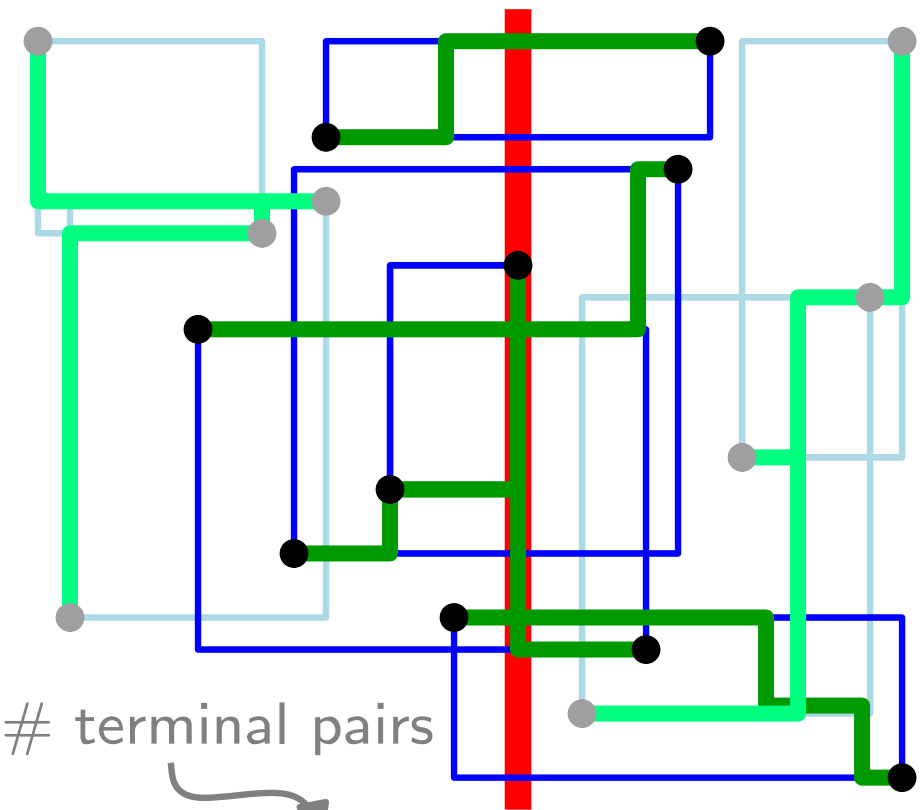
$\rho_{2D}(n/2)\,\mathrm{OPT}_{\mathrm{left}} + \rho_{2D}(n/2)\,\mathrm{OPT}_{\mathrm{right}}$

$\leq \rho_{2D}(n/2)\,\mathrm{OPT}$

Let $\rho_{2D}(n) = \mathrm{cost} / \mathrm{OPT}$ be the performance ratio (in w-c).

$\rho_{2D}(n)\,\mathrm{OPT} = \mathrm{cost} \leq \rho_{2D}(n/2)\,\mathrm{OPT} + \square$

Main Algorithm for 2D-GMMN



$$\rho_{2D}(n/2)\,\mathrm{OPT}_{\text{left}} + \rho_{2D}(n/2)\,\mathrm{OPT}_{\text{right}}$$

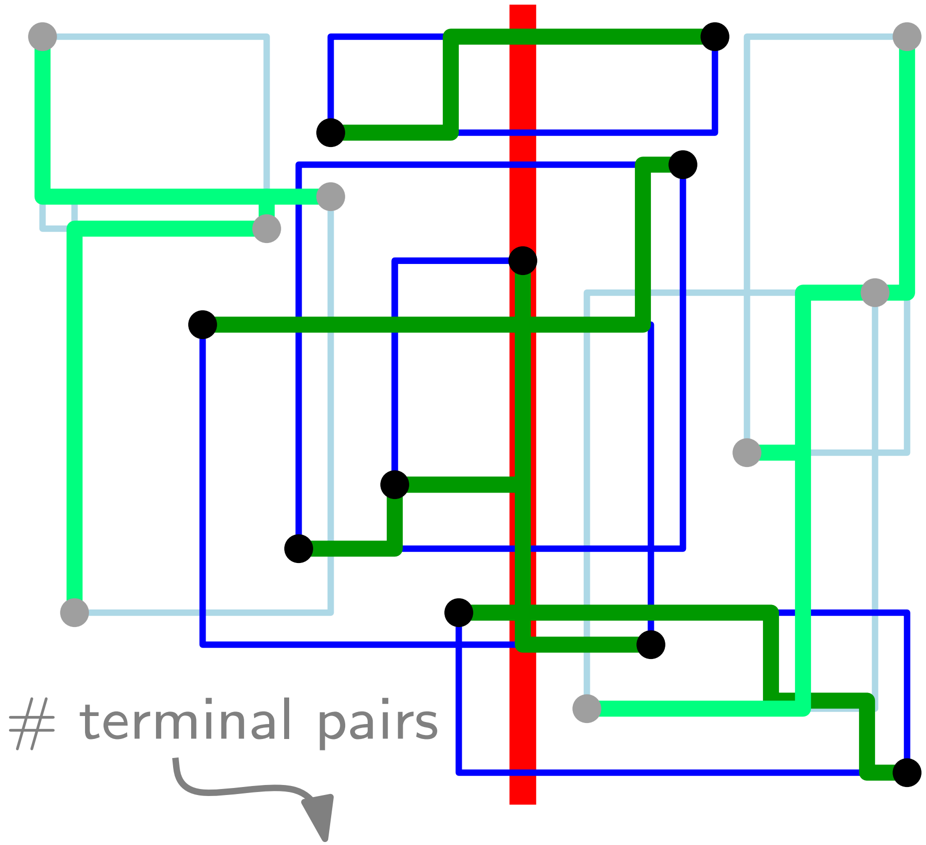
$$\le \rho_{2D}(n/2)\,\mathrm{OPT}$$

$\#$ terminal pairs

Let $\rho_{2D}(n) = \text{cost} / \mathrm{OPT}$ be the performance ratio (in w-c).

$$\rho_{2D}(n)\,\mathrm{OPT} = \text{cost} \le \rho_{2D}(n/2)\,\mathrm{OPT} + \rho_{\text{x-sep}}(n)\,\mathrm{OPT}$$

Main Algorithm for 2D-GMMN

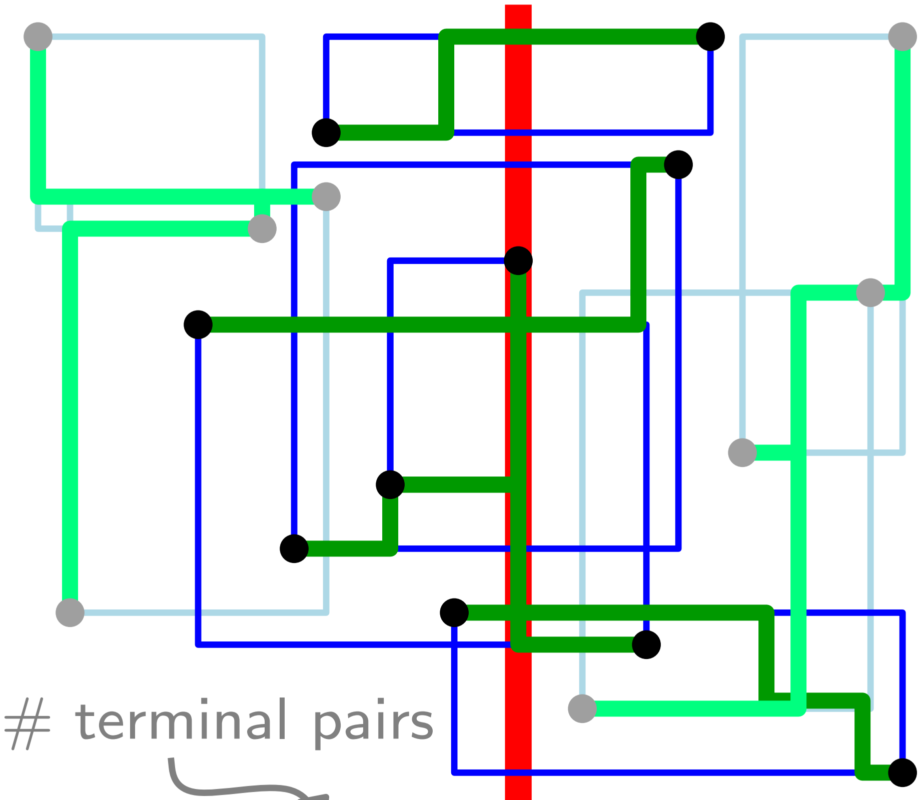


Let $\rho_{2D}(n) = \text{cost} \,/\, \text{OPT}$ be the performance ratio (in w-c).

$$\rho_{2D}(n)\,\text{OPT} = \text{cost} \;\le\; \boxed{\rho_{2D}(n/2)\,\text{OPT}} + \boxed{\rho_{\text{x-sep}}(n)\,\text{OPT}}$$

$$\Rightarrow \rho_{2D}(n) \le \rho_{2D}(n/2) + \rho_{\text{x-sep}}(n)$$

Main Algorithm for 2D-GMMN



TO DO:
Show that
$\rho_{\text{x-sep}}(n) \in O(\log n)$.

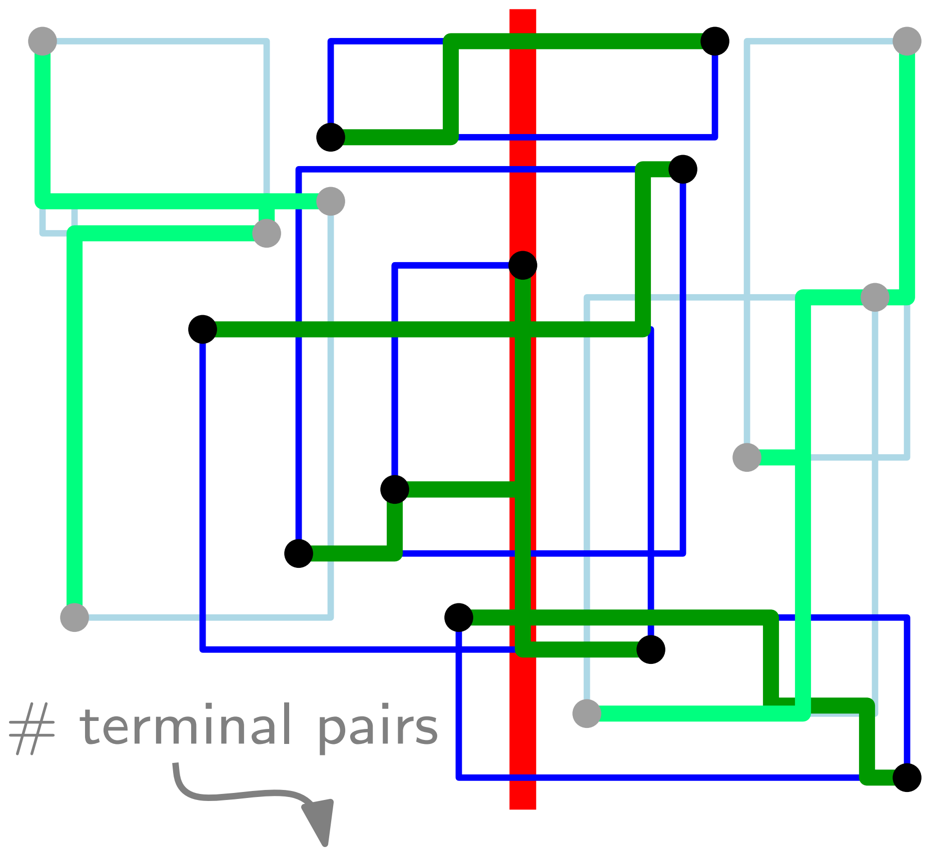
\# terminal pairs

Let $\rho_{2D}(n) = \text{cost} \,/\, \text{OPT}$ be the performance ratio (in w-c).

$\rho_{2D}(n)\,\text{OPT} = \text{cost} \leq \rho_{2D}(n/2)\,\text{OPT} + \rho_{\text{x-sep}}(n)\,\text{OPT}$

$\Rightarrow \rho_{2D}(n) \leq \rho_{2D}(n/2) + \rho_{\text{x-sep}}(n)$

Main Algorithm for 2D-GMMN



TO DO:

Show that

$\rho_{\text{x-sep}}(n) \in O(\log n)$.

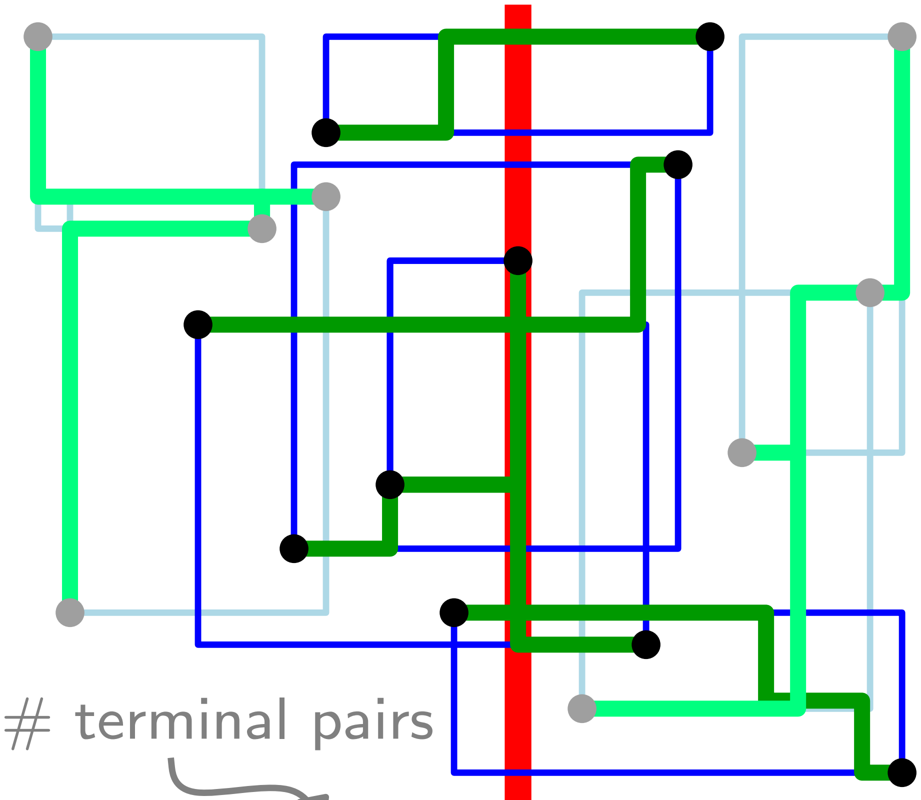
Let $\rho_{2D}(n) = \text{cost} / \text{OPT}$ be the performance ratio (in w-c).

$\rho_{2D}(n)\,\text{OPT} = \text{cost} \leq \rho_{2D}(n/2)\,\text{OPT} + \rho_{\text{x-sep}}(n)\,\text{OPT}$

$\Rightarrow \rho_{2D}(n) \leq \rho_{2D}(n/2) + \rho_{\text{x-sep}}(n)$

$\Rightarrow$

Main Algorithm for 2D-GMMN



terminal pairs

TO DO:

Show that

$\rho_{\text{x-sep}}(n) \in O(\log n)$.

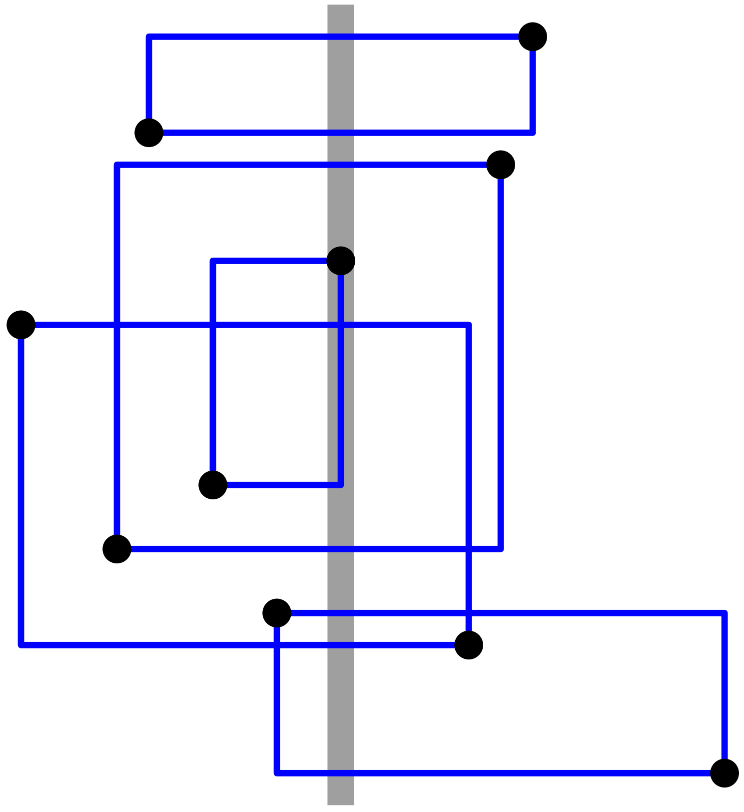
Let $\rho_{2D}(n) = \text{cost} / \text{OPT}$ be the performance ratio (in w-c).

$\rho_{2D}(n)\,\text{OPT} = \text{cost} \leq \rho_{2D}(n/2)\,\text{OPT} + \rho_{\text{x-sep}}(n)\,\text{OPT}$

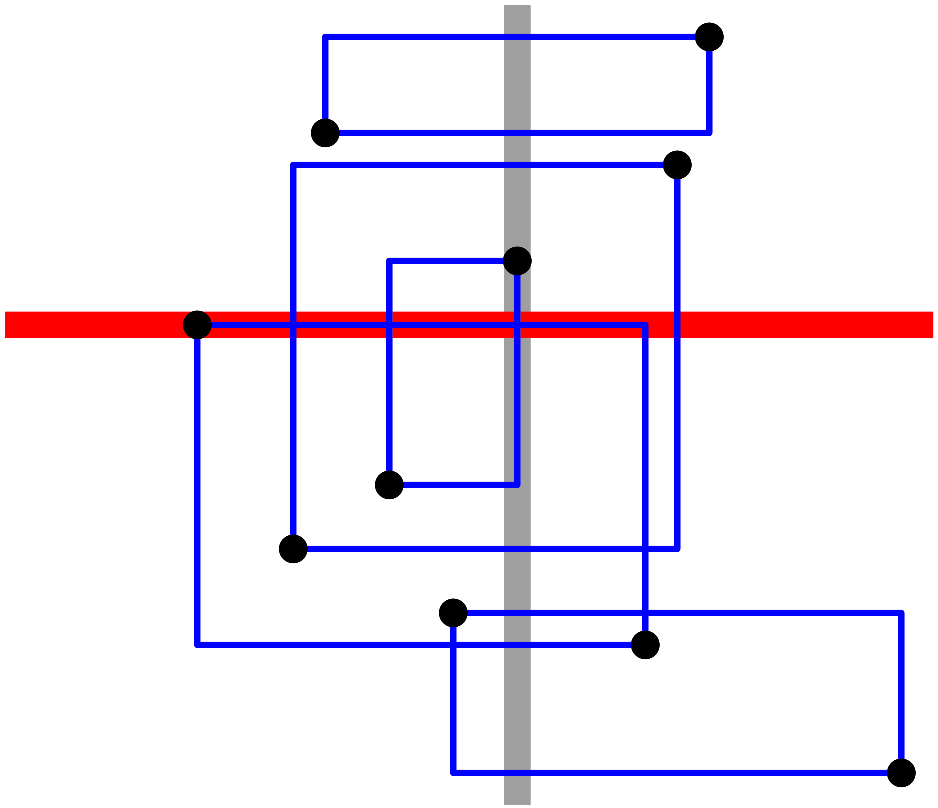
$\Rightarrow \rho_{2D}(n) \leq \rho_{2D}(n/2) + \rho_{\text{x-sep}}(n)$

$\Rightarrow \qquad \in O(\log^2 n)$ by Master theorem. $\square$

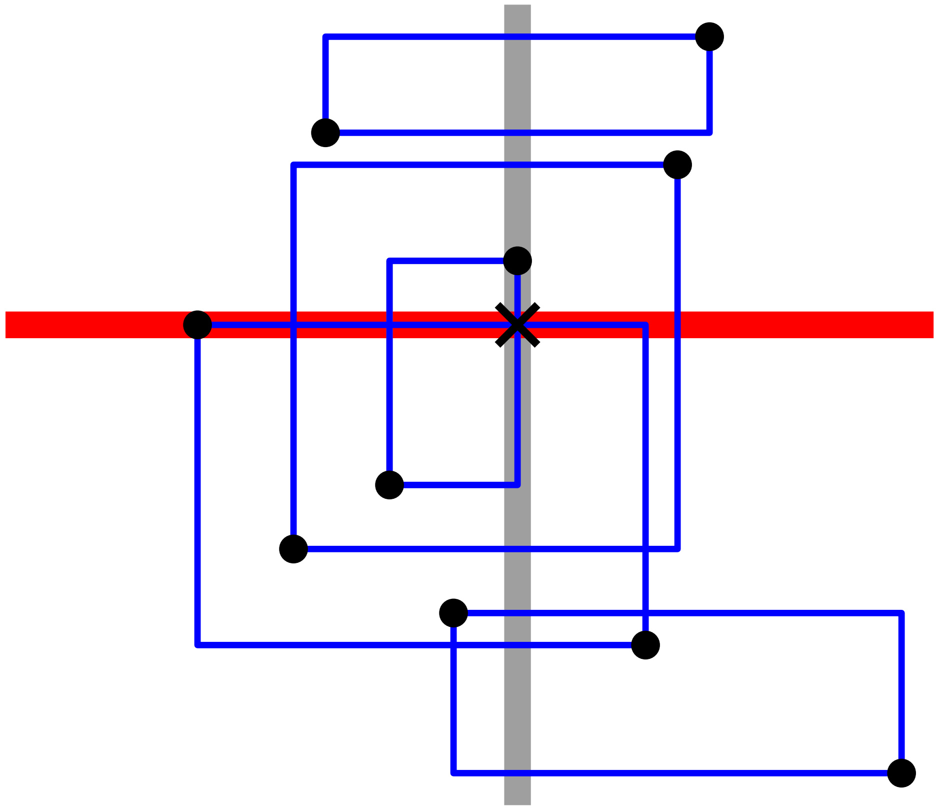
Algorithm for x-separated GMMN



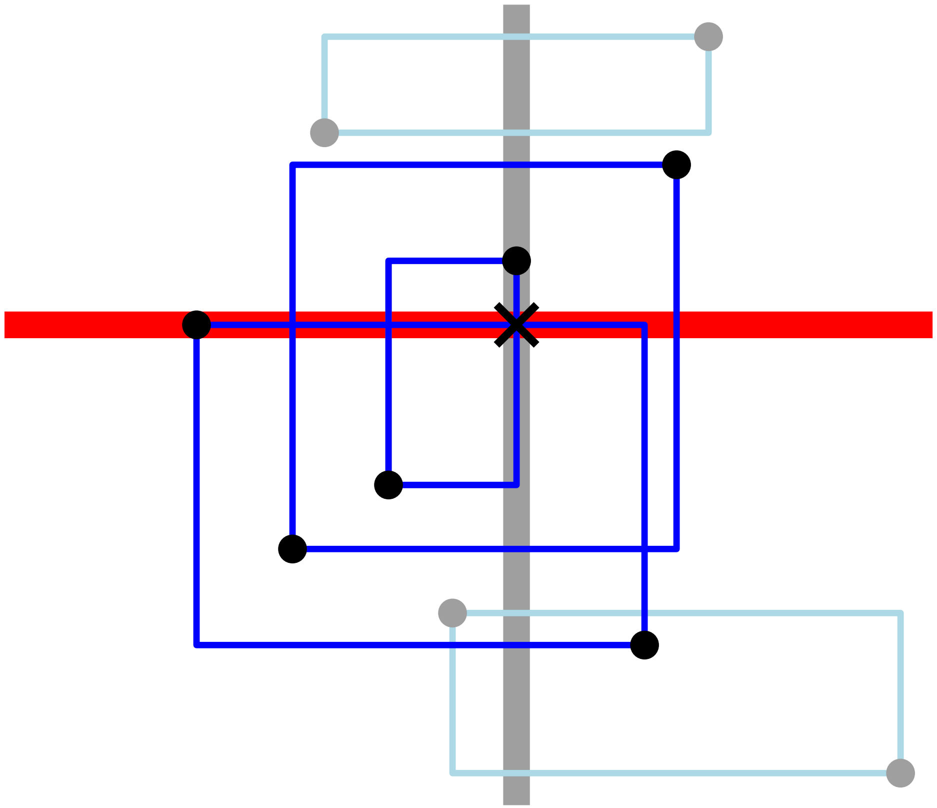
Algorithm for x-separated GMMN



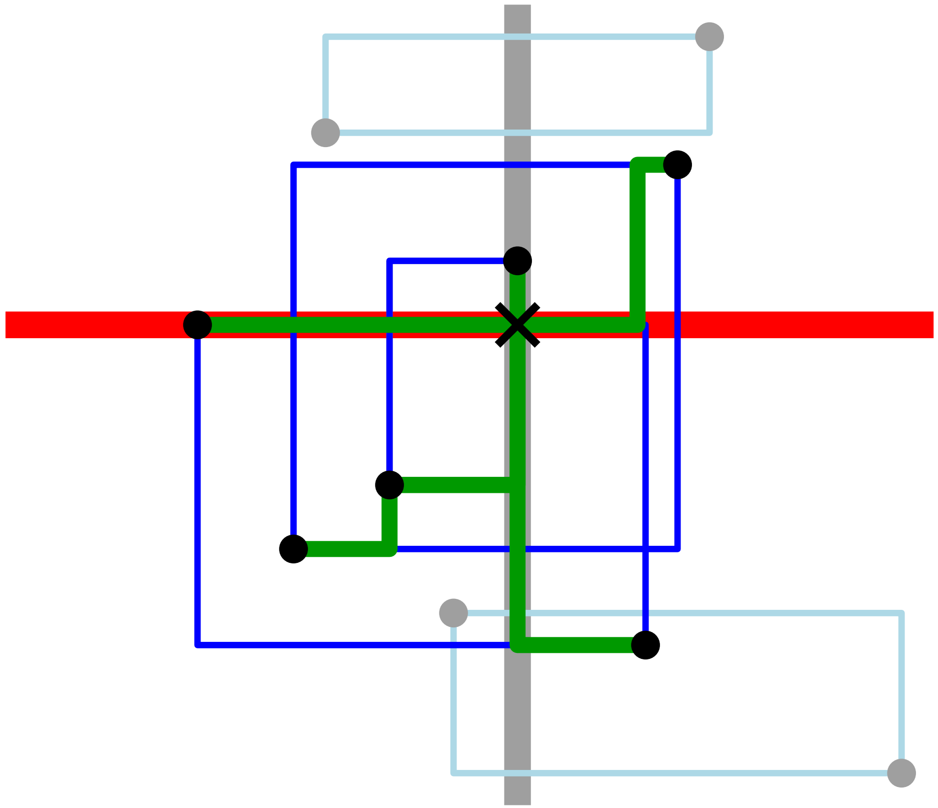
Algorithm for x-separated GMMN



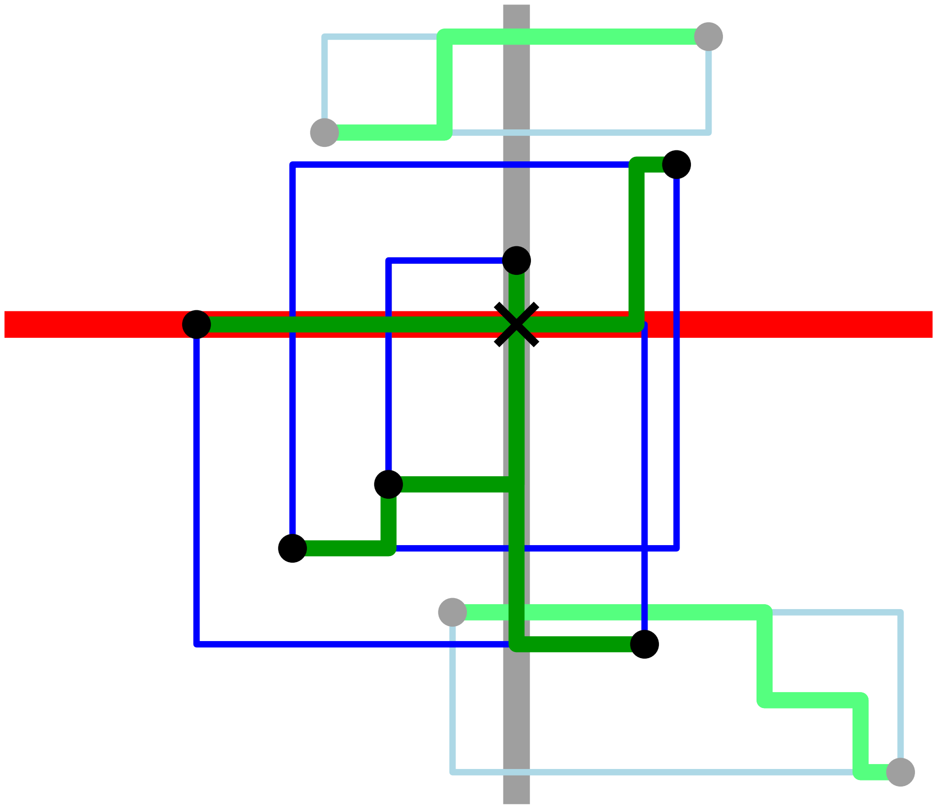
Algorithm for x-separated GMMN



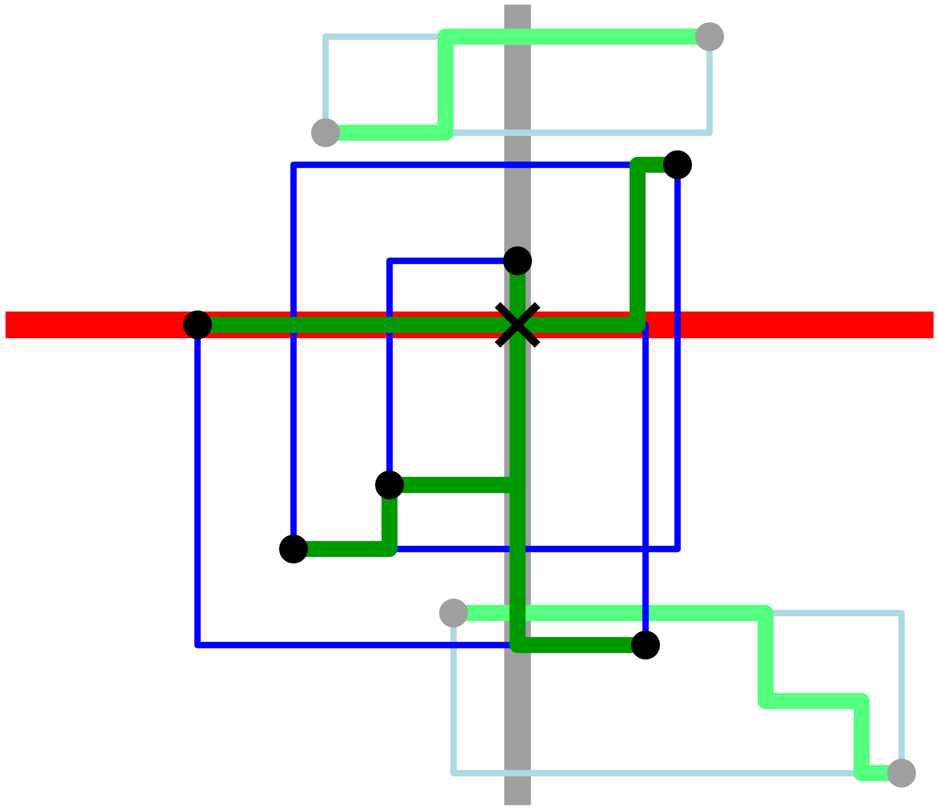
Algorithm for x-separated GMMN



Algorithm for x-separated GMMN

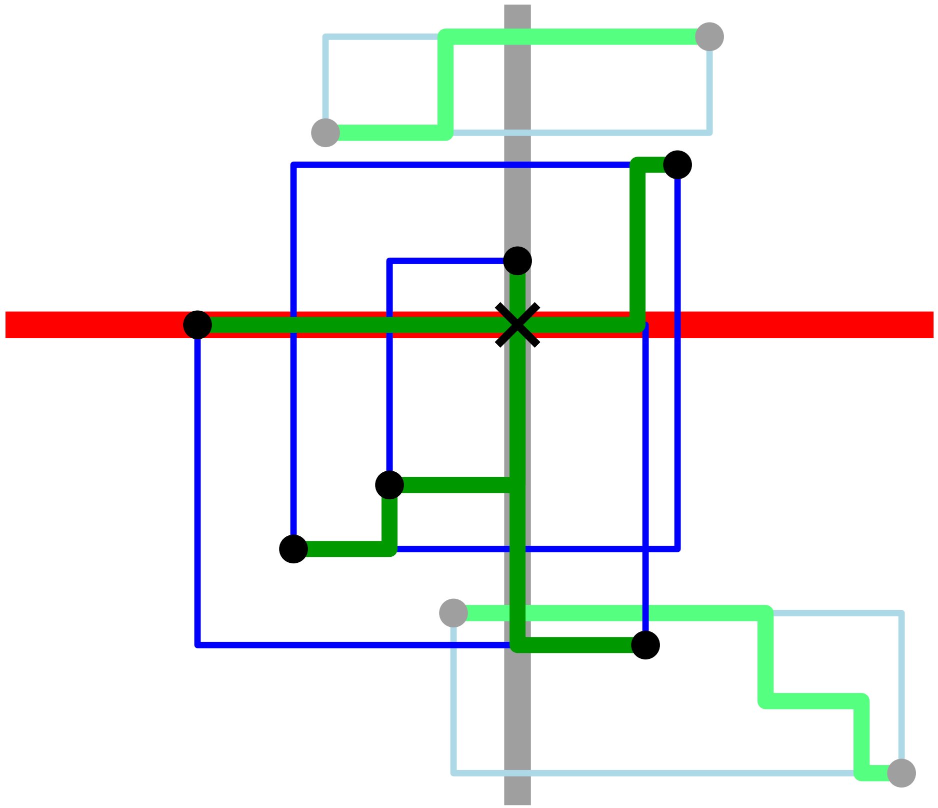


Algorithm for x-separated GMMN



Let $\rho_{\text{x-sep}}(n) = \text{cost} \,/\, \text{OPT}$ be the performance ratio (in w-c).

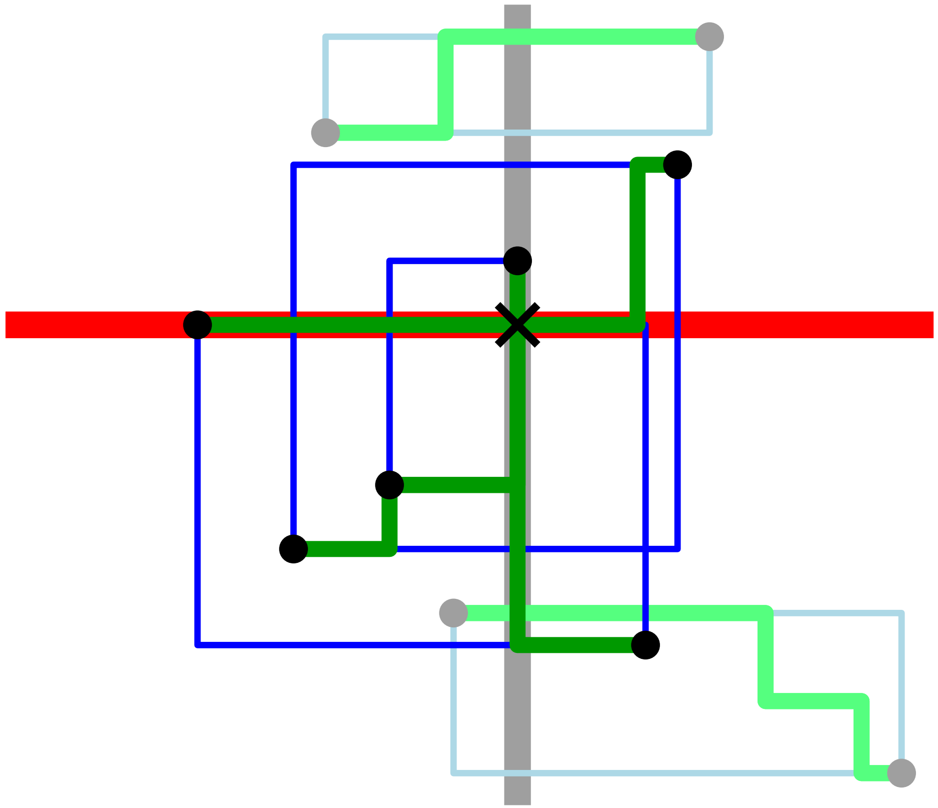
Algorithm for x-separated GMMN



Let $\rho_{\text{x-sep}}(n) = \text{cost} \,/\, \text{OPT}$ be the performance ratio (in w-c).

$\rho_{\text{x-sep}}(n)\,\text{OPT} = \text{cost}$

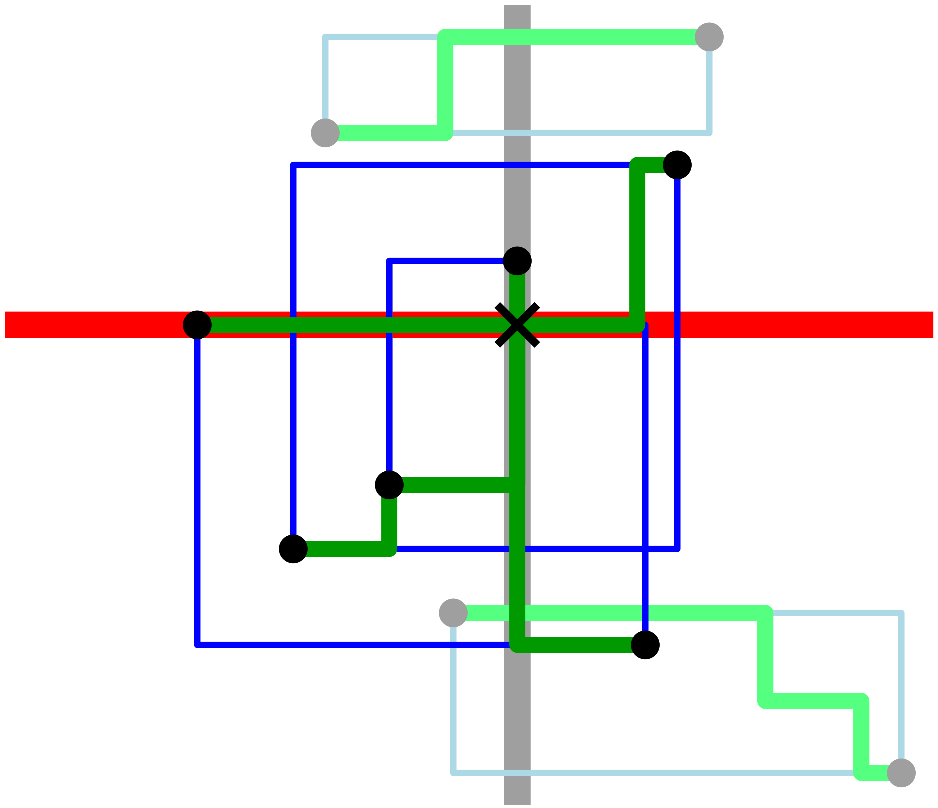
Algorithm for x-separated GMMN



Let $\rho_{\text{x-sep}}(n) = \text{cost} / \text{OPT}$ be the performance ratio (in w-c).

$$\rho_{\text{x-sep}}(n)\,\text{OPT} = \text{cost} \leq \boxed{\rho_{\text{x-sep}}(n/2)\,\text{OPT}} + \boxed{\rho_{\text{xy-sep}}(n)\,\text{OPT}}$$

Algorithm for x-separated GMMN



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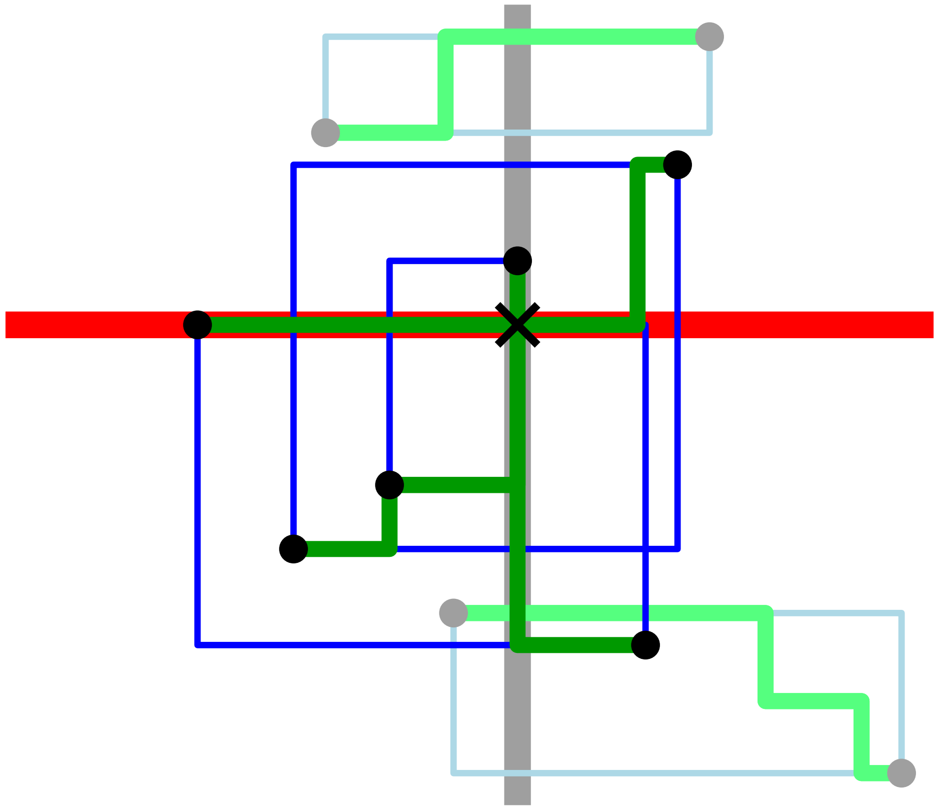
$$\rho_{\text{x-sep}}(n)\,\text{OPT} = \text{cost} \leq \boxed{\rho_{\text{x-sep}}(n/2)\,\text{OPT}} + \boxed{\rho_{\text{xy-sep}}(n)\,\text{OPT}}$$

$$\Rightarrow \rho_{\text{x-sep}}(n) \leq \rho_{\text{x-sep}}(n/2) + \rho_{\text{xy-sep}}(n)$$

Algorithm for x-separated GMMN



TO DO:

Show that

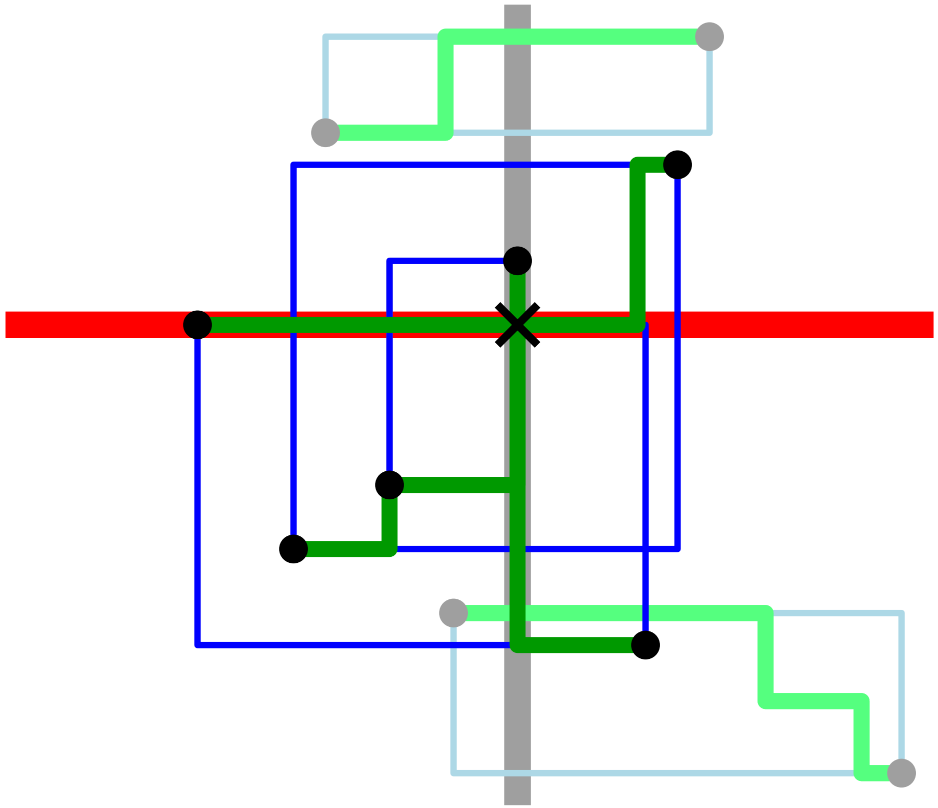
$\rho_{\text{xy-sep}}(n) \in O(1)$.

Let $\rho_{\text{x-sep}}(n) = \text{cost} / \text{OPT}$ be the performance ratio (in w-c).

$\rho_{\text{x-sep}}(n)\,\text{OPT} = \text{cost} \leq \rho_{\text{x-sep}}(n/2)\,\text{OPT} + \rho_{\text{xy-sep}}(n)\,\text{OPT}$

$\Rightarrow \rho_{\text{x-sep}}(n) \leq \rho_{\text{x-sep}}(n/2) + \rho_{\text{xy-sep}}(n)$

Algorithm for x-separated GMMN



TO DO:

Show that

$\rho_{\text{xy-sep}}(n) \in O(1)$.

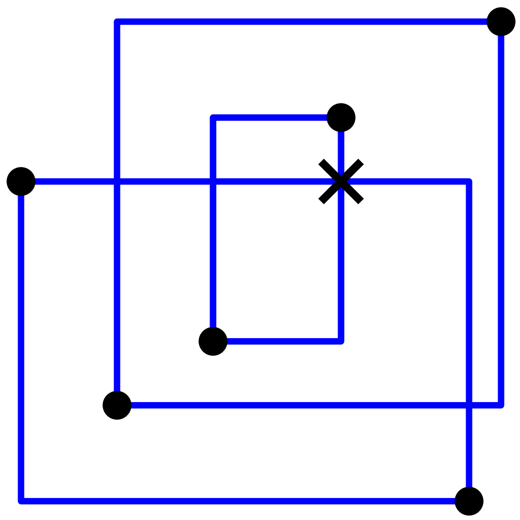
Let $\rho_{\text{x-sep}}(n) = \text{cost} \,/\, \text{OPT}$ be the performance ratio (in w-c).

$$\rho_{\text{x-sep}}(n)\,\text{OPT} = \text{cost} \le \rho_{\text{x-sep}}(n/2)\,\text{OPT} + \rho_{\text{xy-sep}}(n)\,\text{OPT}$$

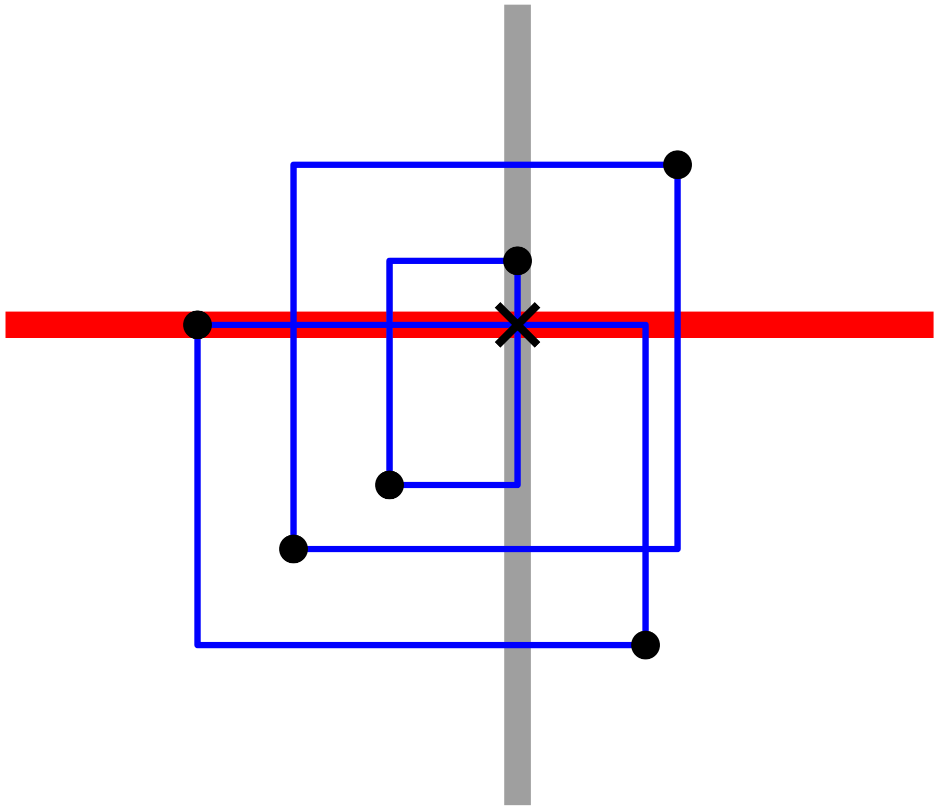
$$\Rightarrow \rho_{\text{x-sep}}(n) \le \rho_{\text{x-sep}}(n/2) + \rho_{\text{xy-sep}}(n)$$

$$\Rightarrow \in O(\log n)$$ by Master theorem. □

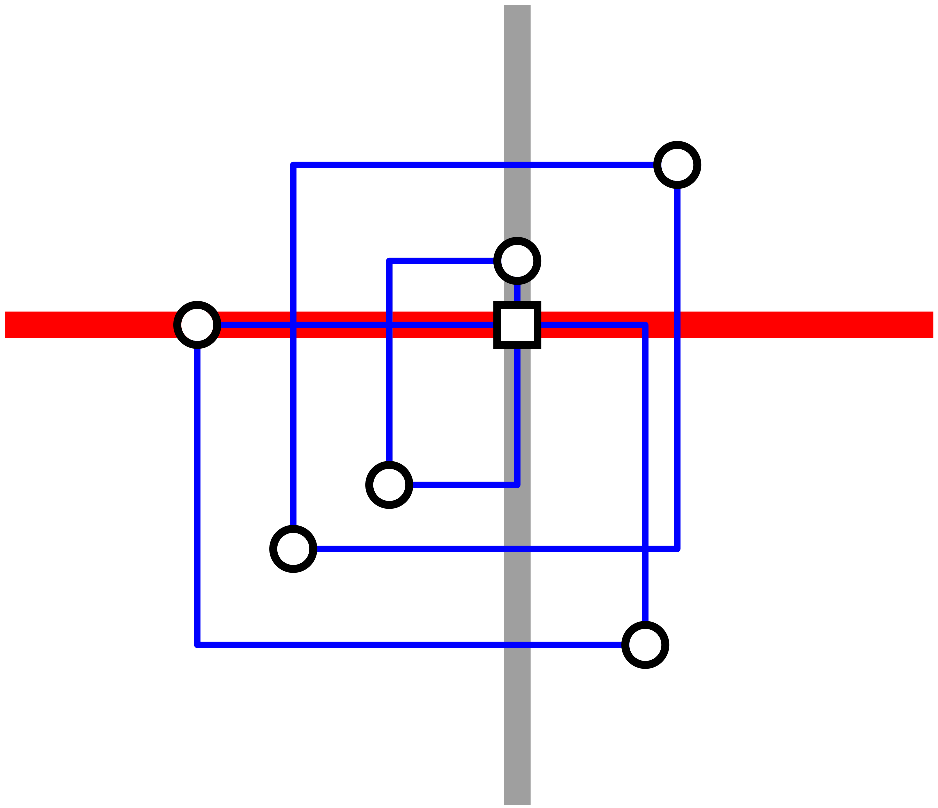
Algorithm for xy-separated GMMN



Algorithm for xy-separated GMMN

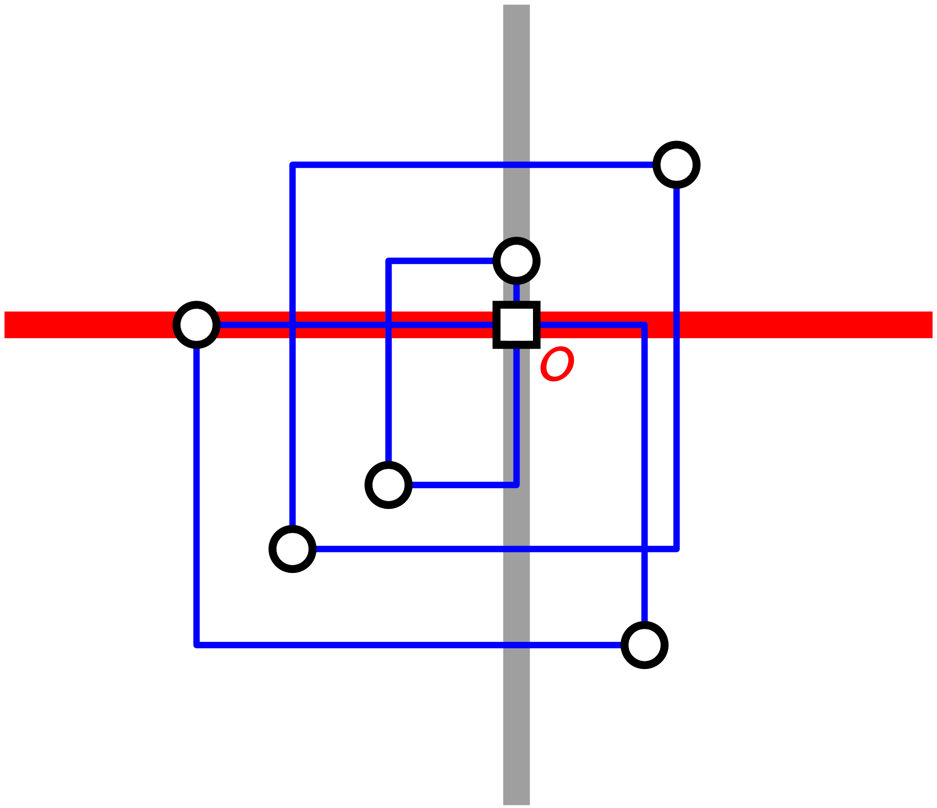


Algorithm for xy-separated GMMN



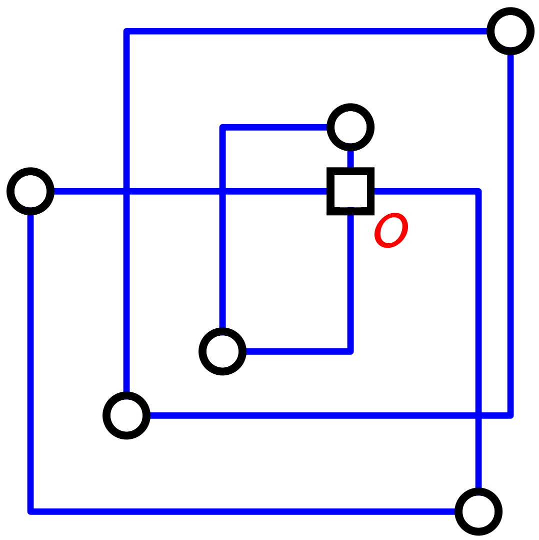
Idea: Use algorithm for RSA!

Algorithm for xy-separated GMMN



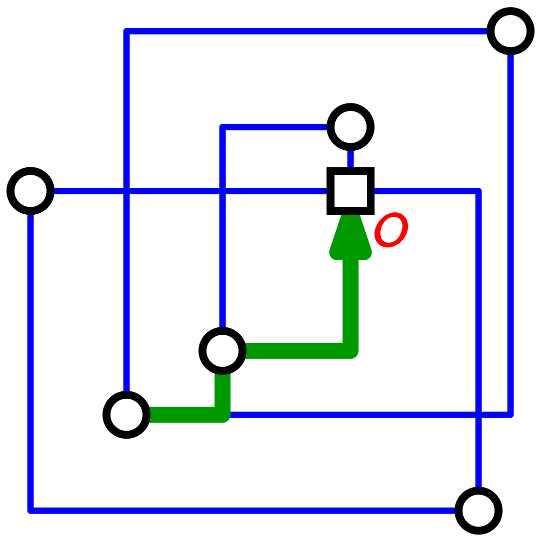
Idea: Use algorithm for RSA!

Algorithm for xy-separated GMMN



Idea: Use algorithm for RSA!

Algorithm for xy-separated GMMN



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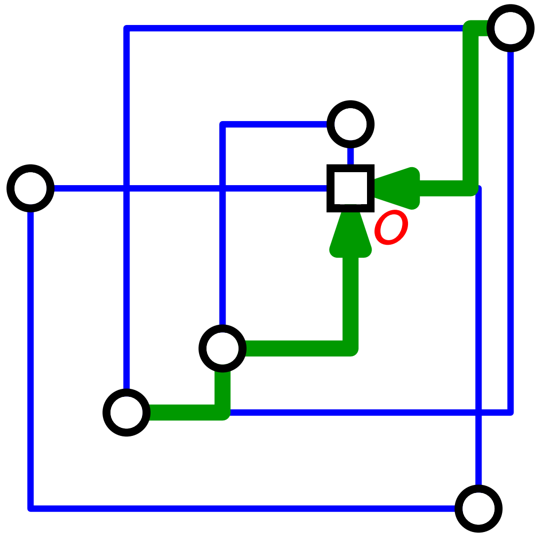
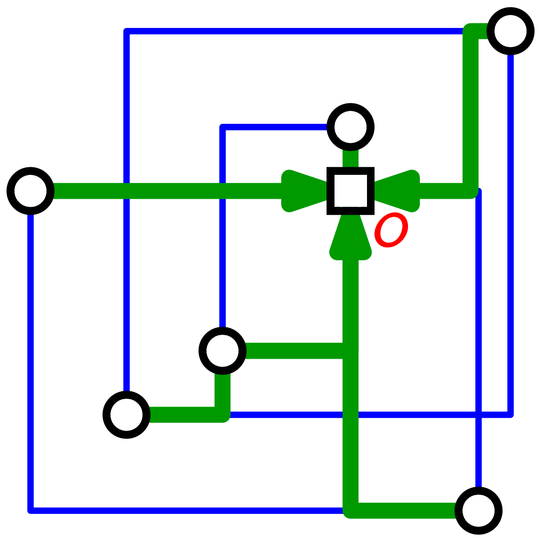
Algorithm for xy-separated GMMN



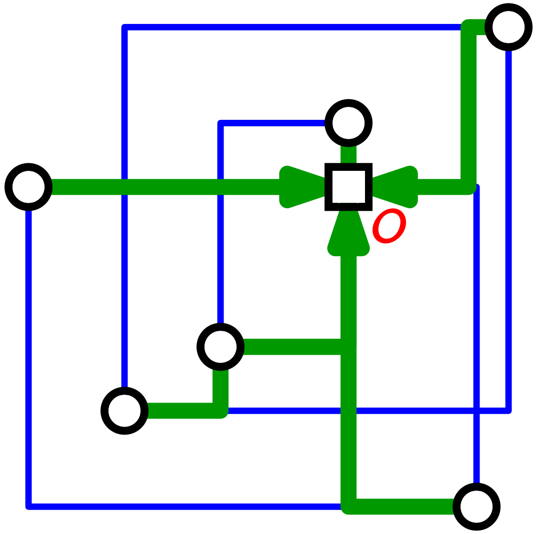
Idea: Use algorithm for RSA!

Algorithm for xy-separated GMMN



Idea: Use algorithm for RSA! Resulting network is. . .

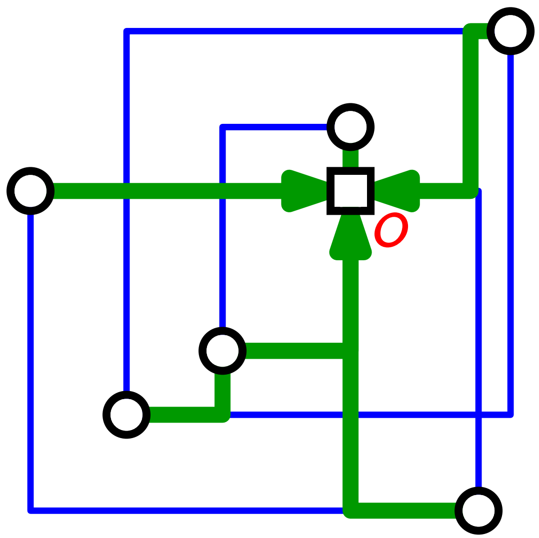
Algorithm for xy-separated GMMN



Idea: Use algorithm for RSA! Resulting network is. . .

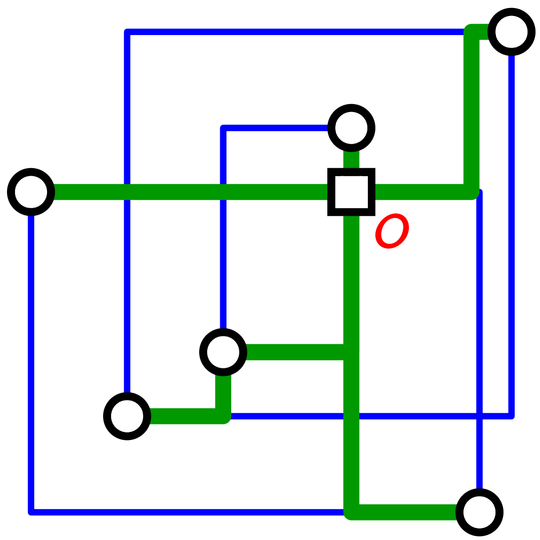
– feasible ✓

Algorithm for xy-separated GMMN



Idea: Use algorithm for RSA! Resulting network is. . .
– feasible ✓
– near-optimal:

Algorithm for xy-separated GMMN

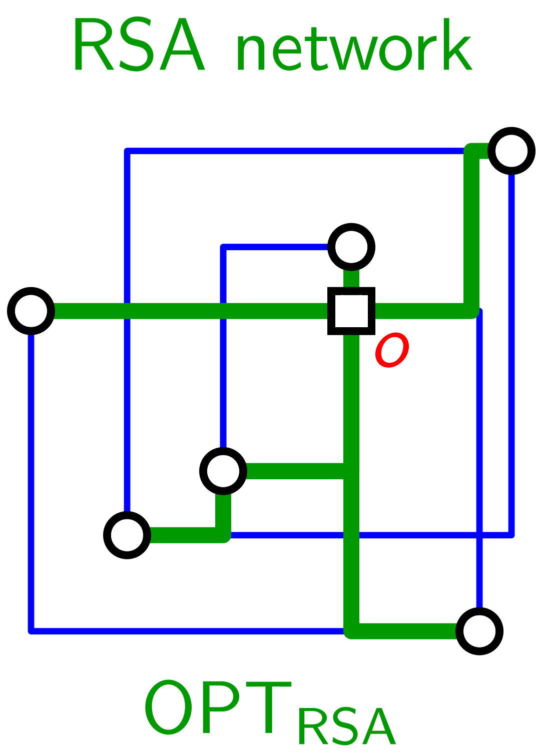


Idea: Use algorithm for RSA! Resulting network is. . .

– feasible ✓

– near-optimal:

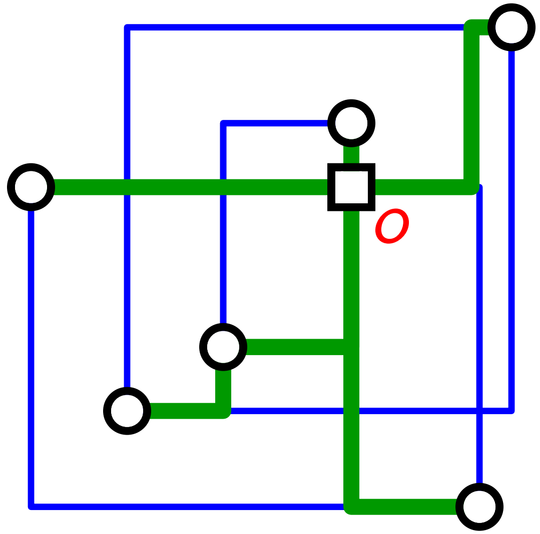
Algorithm for xy-separated GMMN



Idea: Use algorithm for RSA! Resulting network is...
– feasible ✓
– near-optimal:

Algorithm for xy-separated GMMN

RSA network

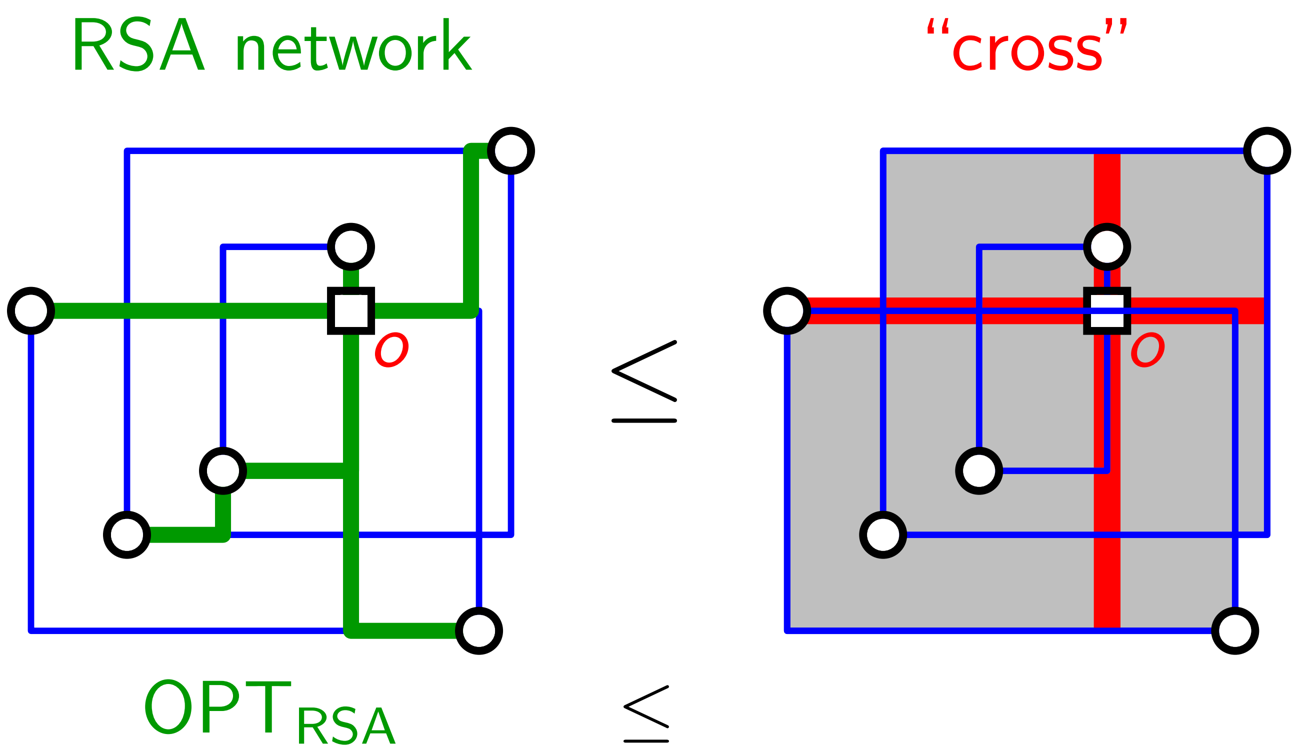


$\leq$

OPT_{RSA} $\leq$

Idea: Use algorithm for RSA! Resulting network is...
– feasible ✓
– near-optimal:

Algorithm for xy-separated GMMN



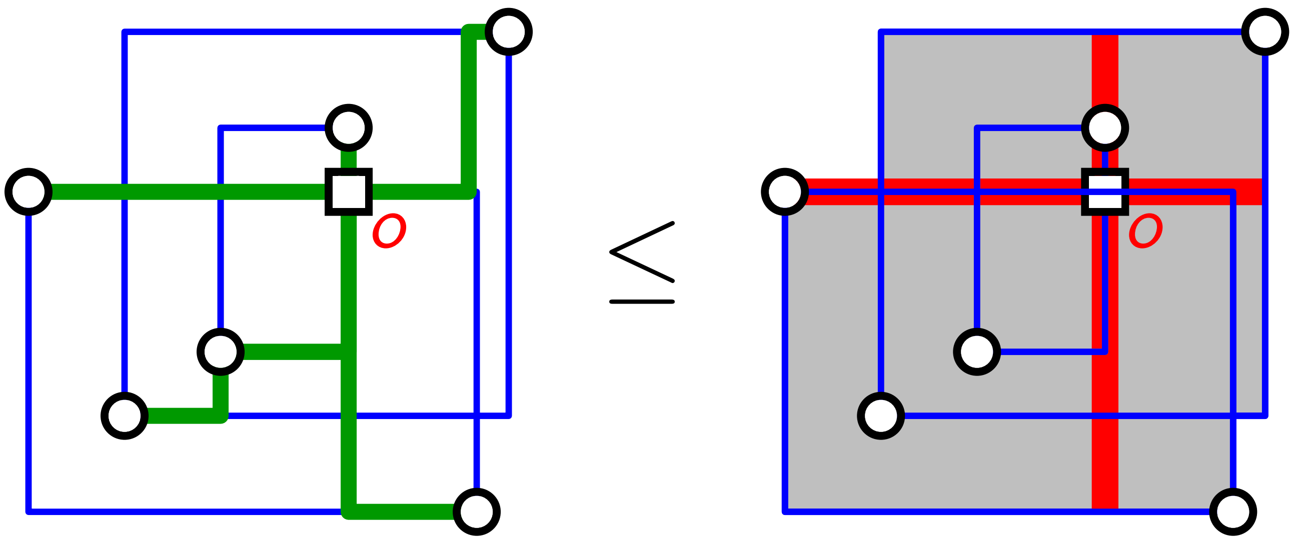
Idea: Use algorithm for RSA! Resulting network is. . .

– feasible ✓

– near-optimal:

Algorithm for xy-separated GMMN

RSA network ≤ "cross"



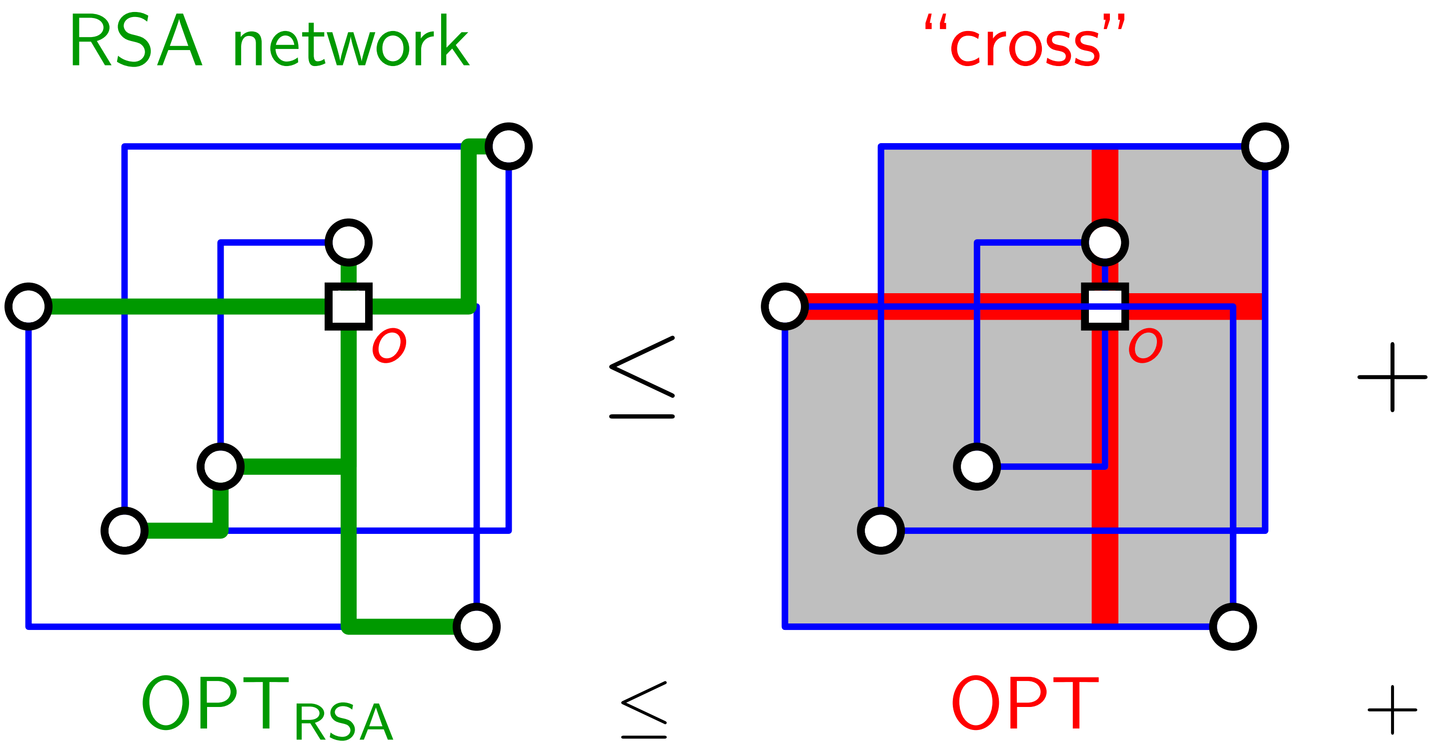
$\mathrm{OPT}_{\mathrm{RSA}} \leq \mathrm{OPT}$

Idea: Use algorithm for RSA! Resulting network is. . .

– feasible ✓

– near-optimal:

Algorithm for xy-separated GMMN

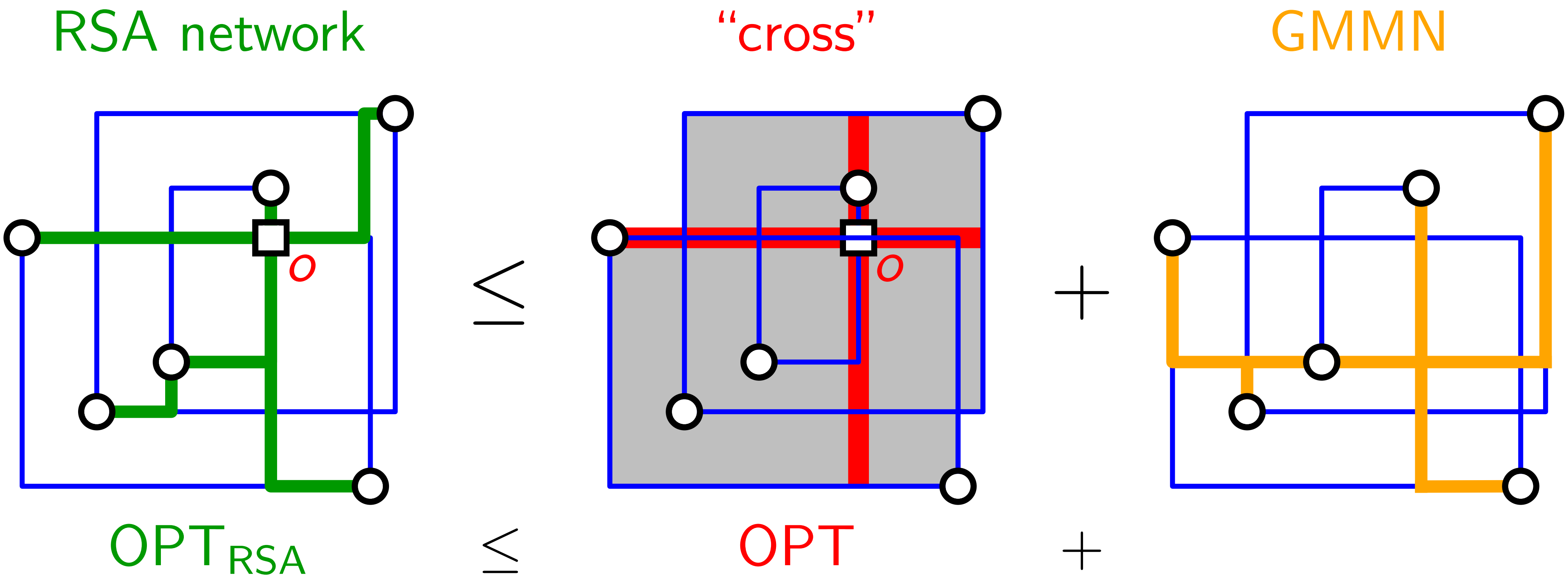


Idea: Use algorithm for RSA! Resulting network is...

– feasible ✓

– near-optimal:

Algorithm for xy-separated GMMN

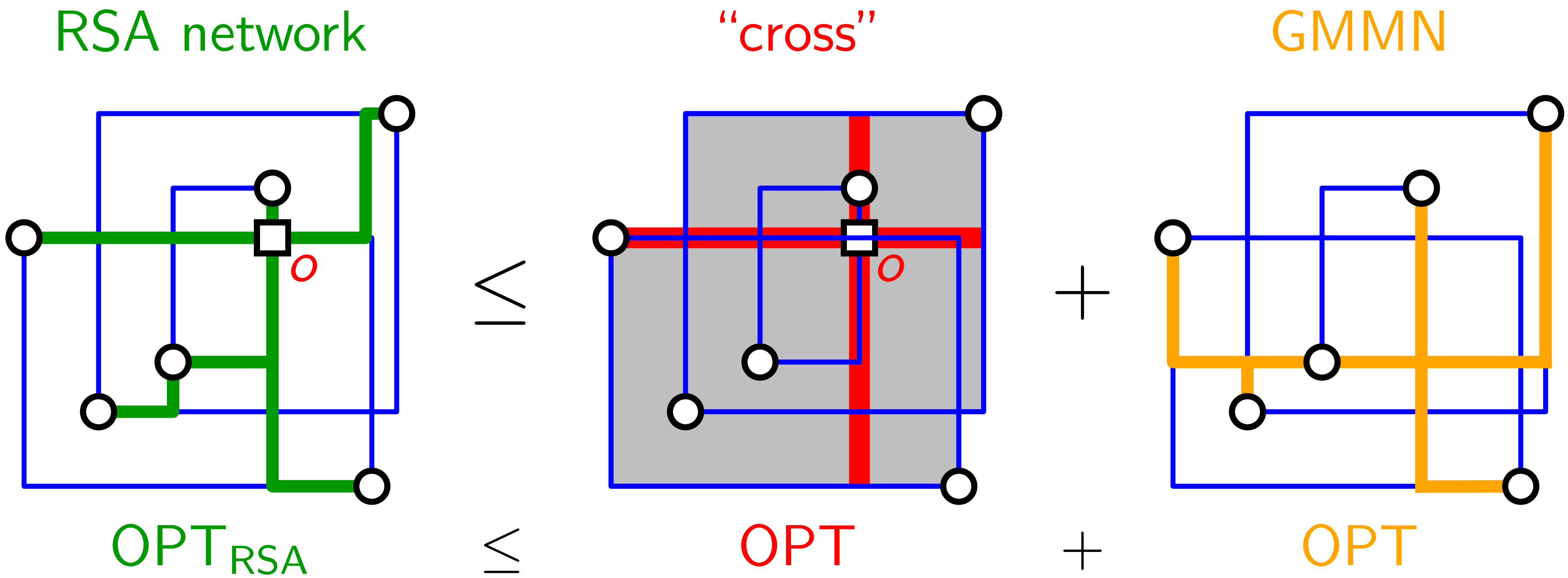


Idea: Use algorithm for RSA! Resulting network is. . .

– feasible ✓

– near-optimal:

Algorithm for xy-separated GMMN

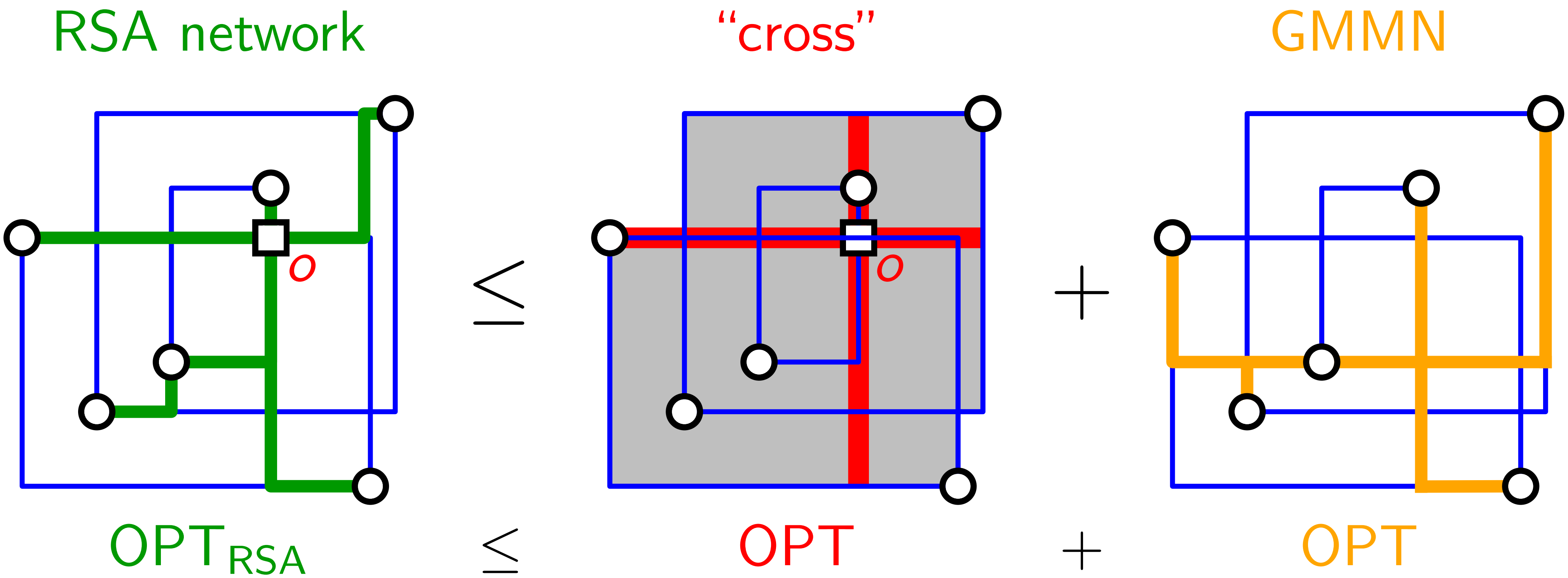


Idea: Use algorithm for RSA! Resulting network is. . .

– feasible ✓

– near-optimal:

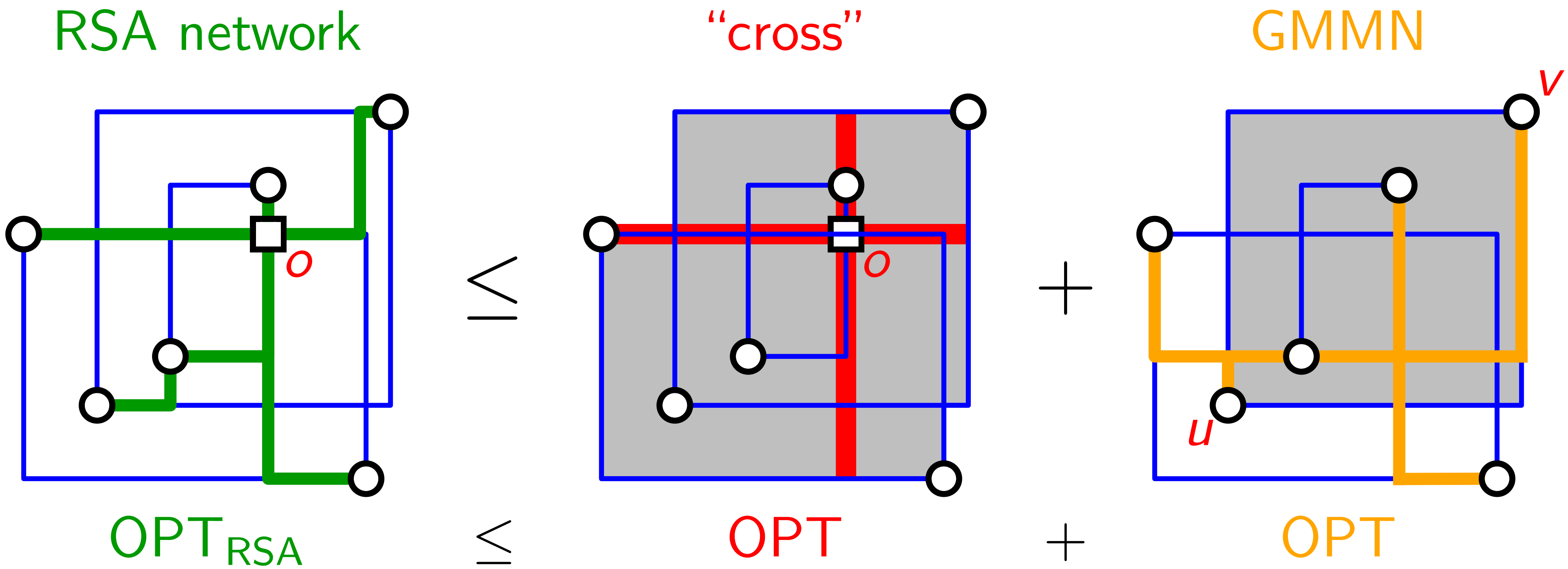
Algorithm for xy-separated GMMN



Idea: Use algorithm for RSA! Resulting network is. . .

- feasible ✓
- near-optimal: cross + GMMN network *is* RSA network

Algorithm for xy-separated GMMN



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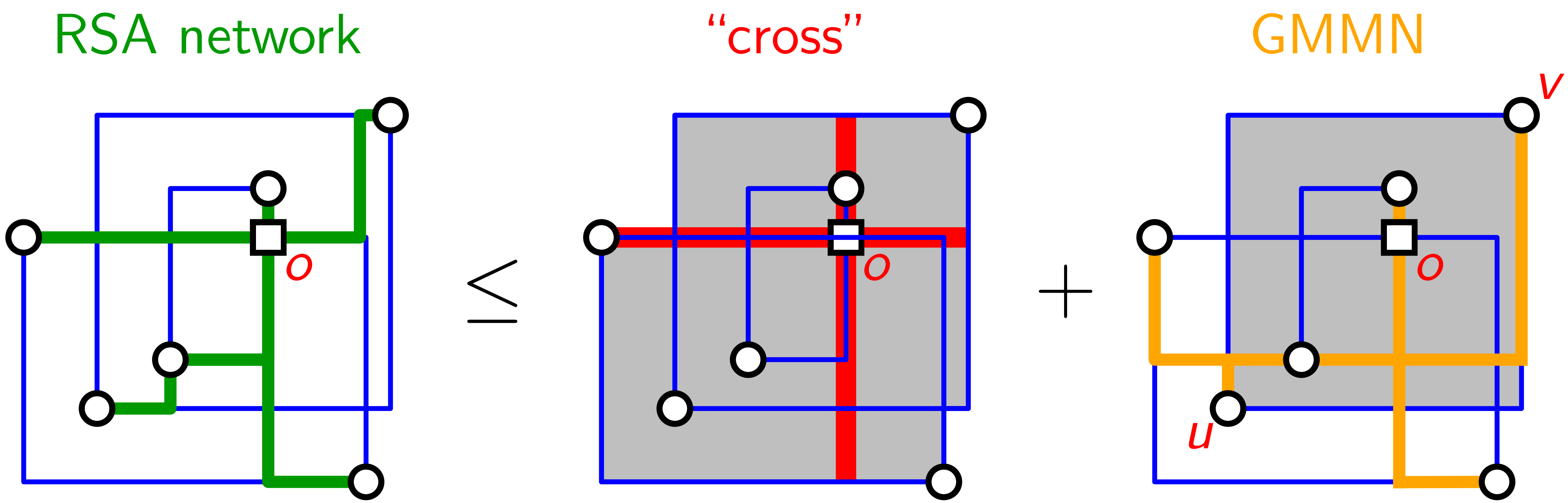
– near-optimal: cross + GMMN network *is* RSA network

Algorithm for xy-separated GMMN



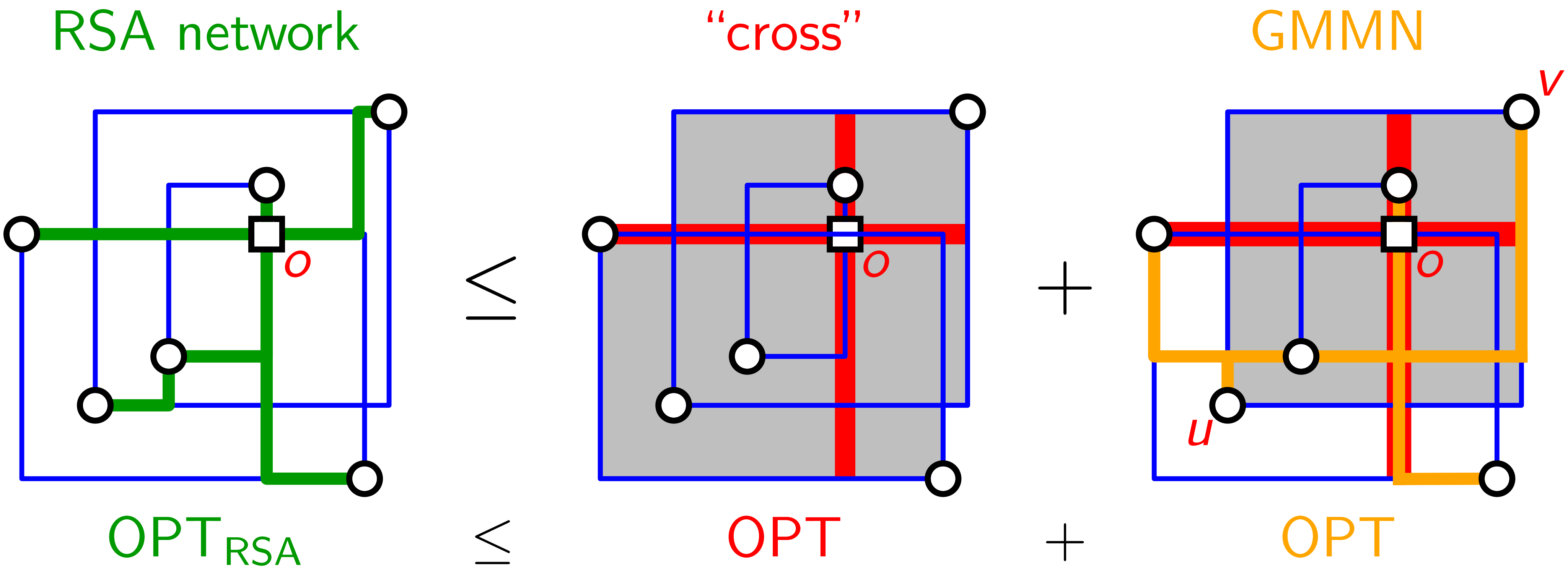
$\mathrm{OPT}_{\mathrm{RSA}} \leq \mathrm{OPT} + \mathrm{OPT}$

Idea: Use algorithm for RSA! Resulting network is. . .

– feasible ✓

– near-optimal: cross $+$ GMMN network *is* RSA network

Algorithm for xy-separated GMMN



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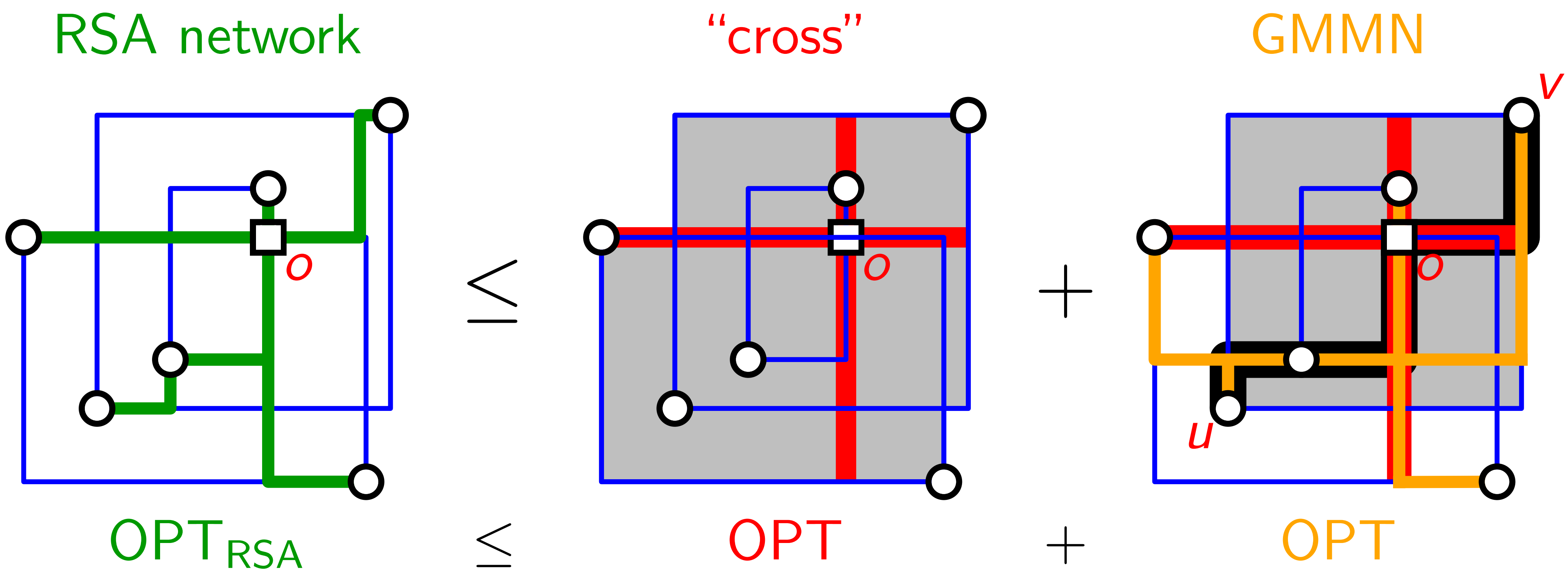
Algorithm for xy-separated GMMN



Idea: Use algorithm for RSA! Resulting network is. . .

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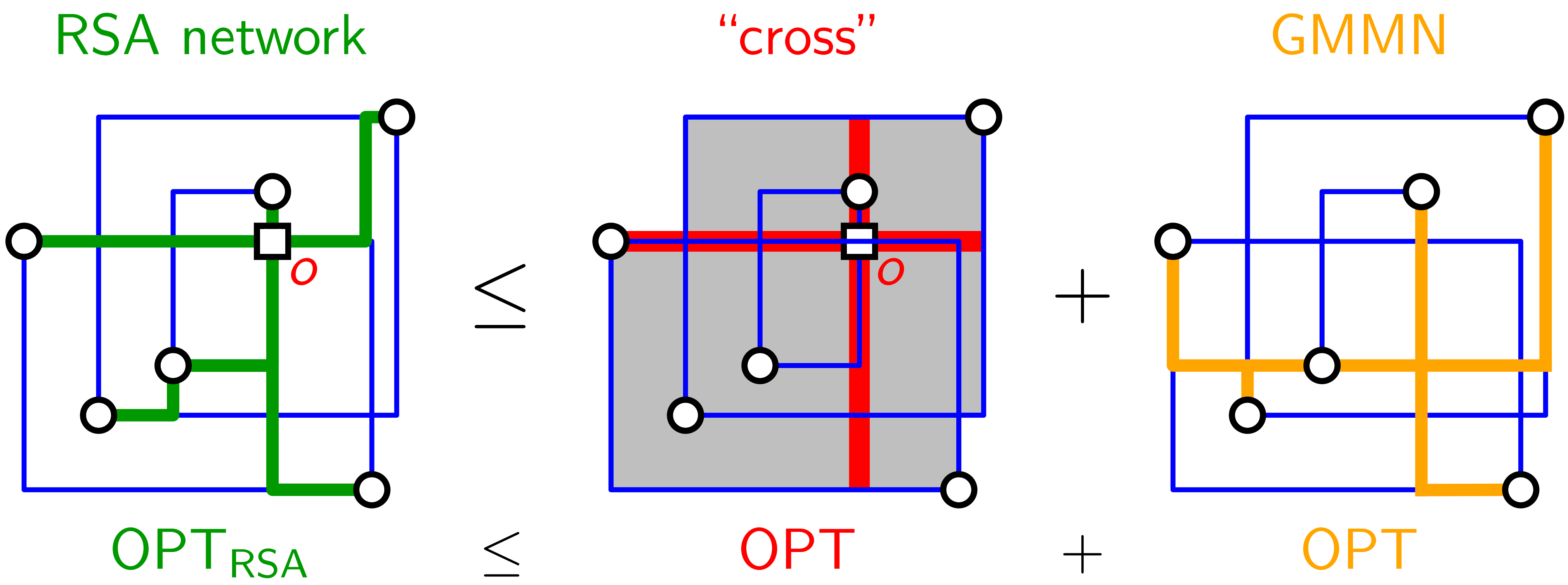
– near-optimal: cross + GMMN network *is* RSA network

Algorithm for xy-separated GMMN



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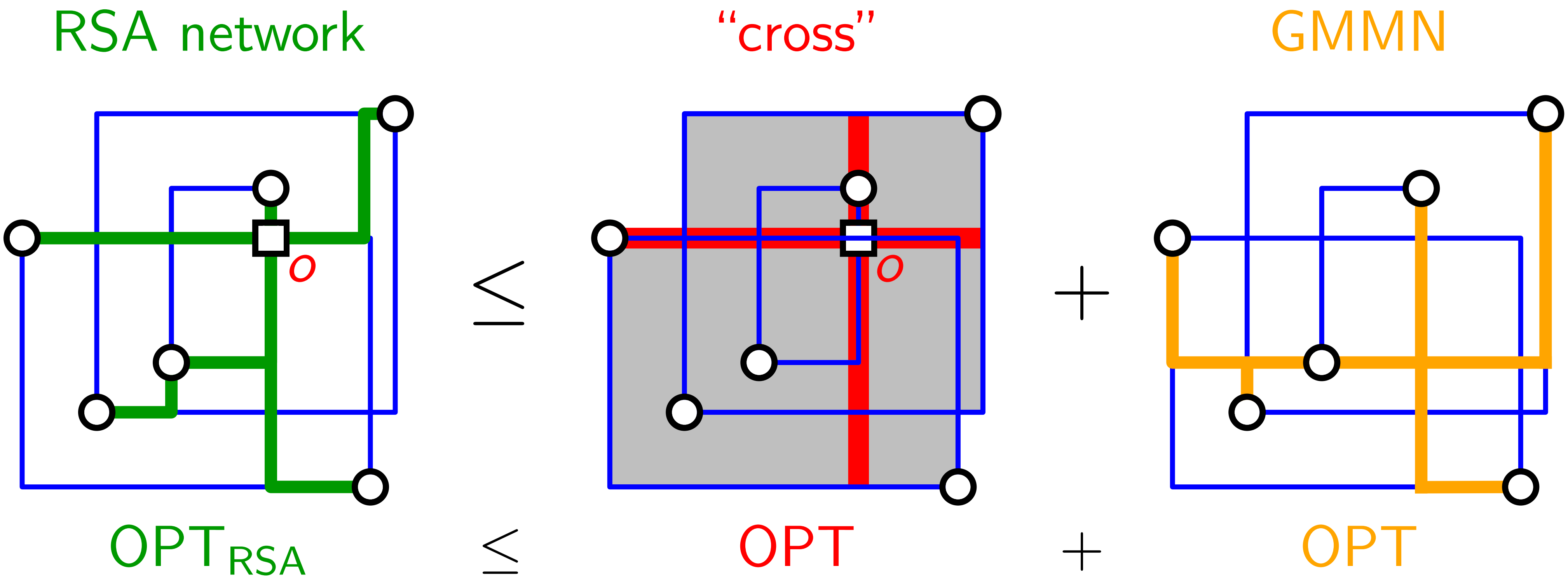
– feasible ✓

– near-optimal: cross + GMMN network *is* RSA network ✓

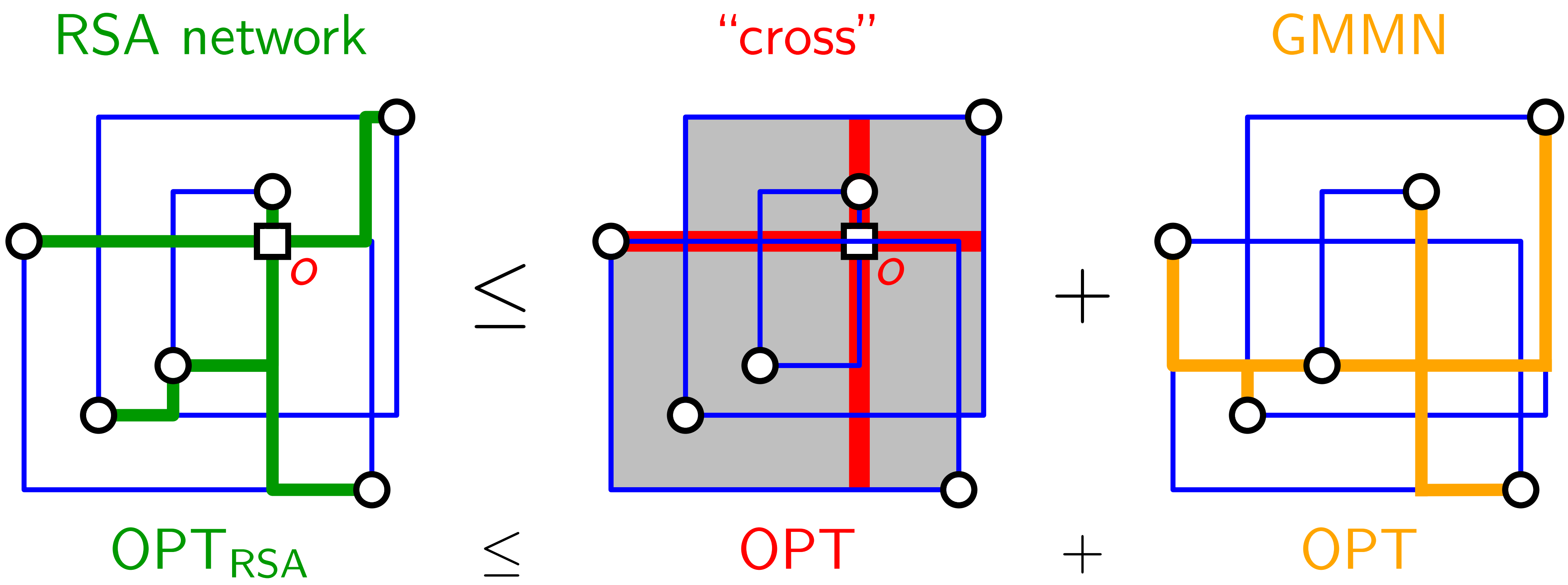
Algorithm for xy-separated GMMN



Idea: Use algorithm for RSA! Resulting network is...

– feasible ✓
– near-optimal: cross + GMMN network *is* RSA network ✓
– efficiently constructable: RSA admits PTAS in 2D.

Algorithm for xy-separated GMMN



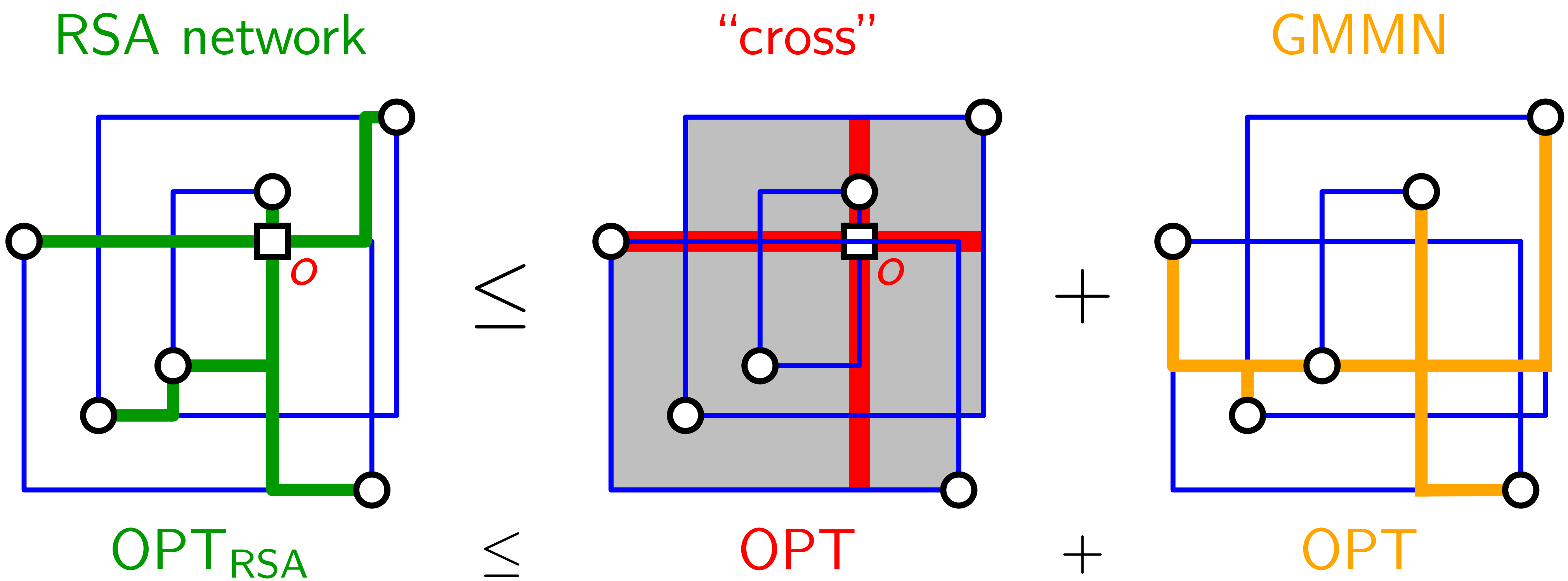
Idea: Use algorithm for RSA! Resulting network is. . .

– feasible ✓

– near-optimal: cross + GMMN network *is* RSA network ✓

– efficiently constructable: RSA admits PTAS in 2D. ✓

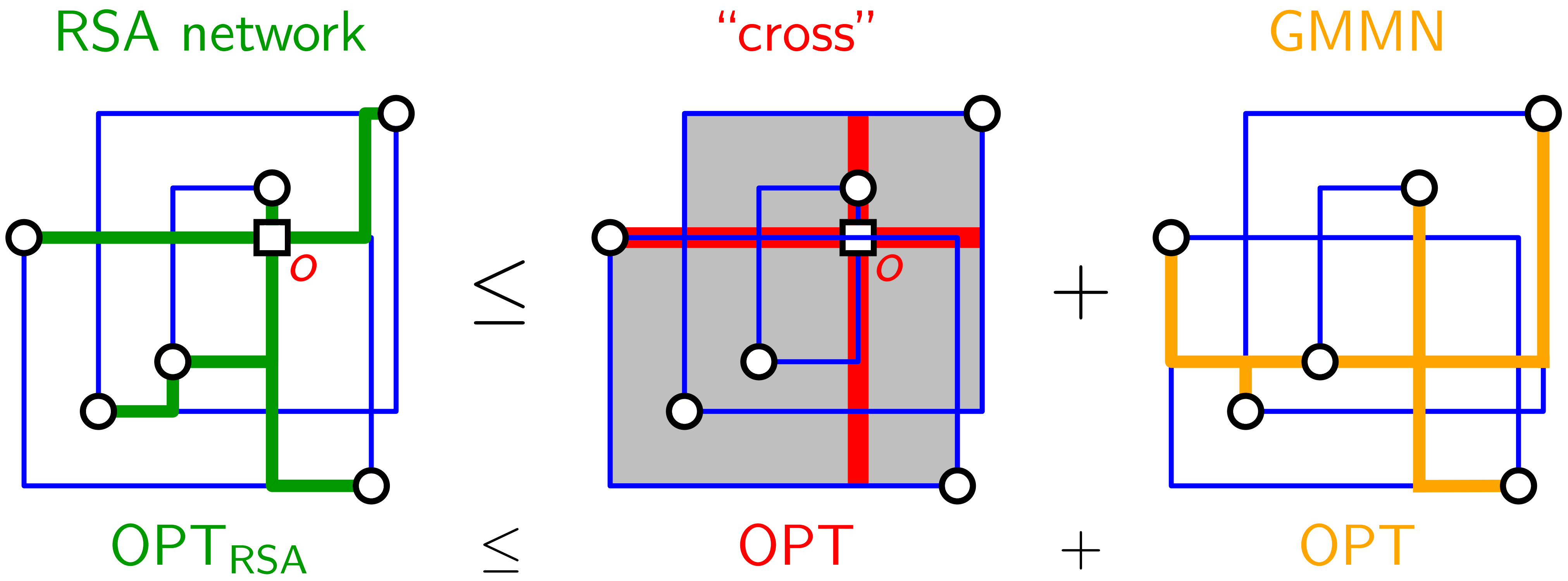
Algorithm for xy-separated GMMN



Idea: Use algorithm for RSA! Resulting network is. . .
– feasible ✓
– near-optimal: cross + GMMN network *is* RSA network ✓
– efficiently constructable: RSA admits PTAS in 2D. ✓

$\Rightarrow \rho_{\text{xy-sep}} \leq$

Algorithm for xy-separated GMMN



Idea: Use algorithm for RSA! Resulting network is. . .

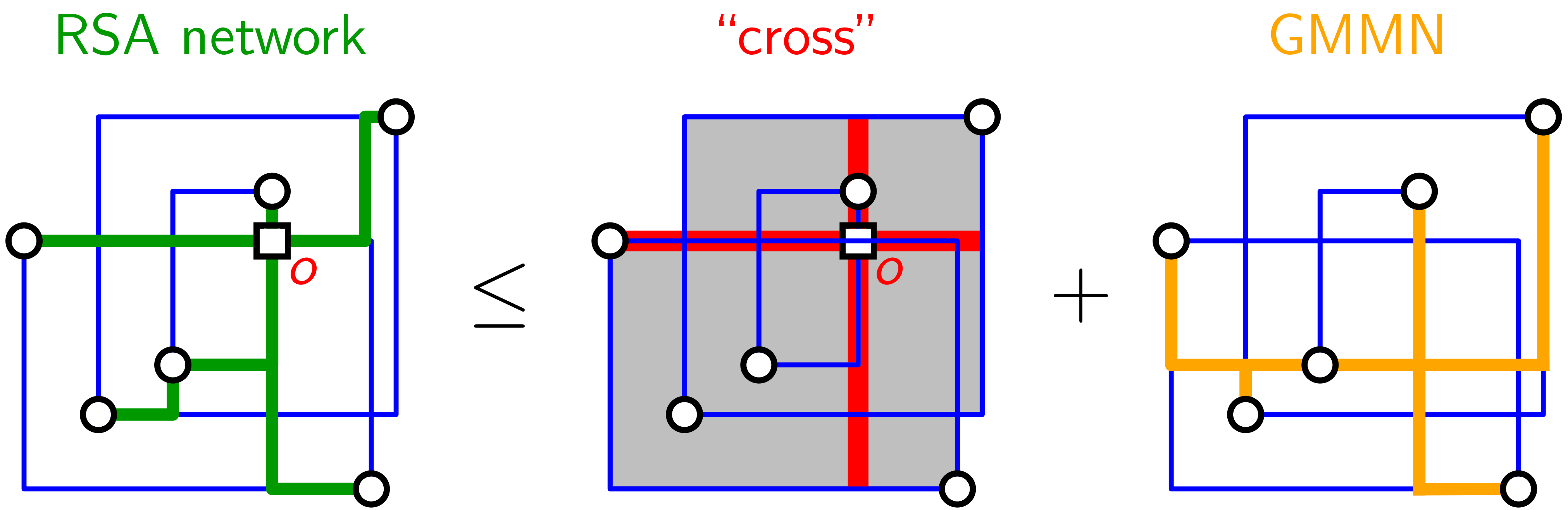
– feasible ✓

– near-optimal: cross + GMMN network *is* RSA network ✓

– efficiently constructable: RSA admits PTAS in 2D. ✓

$\Rightarrow \ \rho_{\text{xy-sep}} \leq \ 2(1+\varepsilon)$

Algorithm for xy-separated GMMN



$OPT_{RSA} \leq OPT + OPT$

Idea: Use algorithm for RSA! Resulting network is...

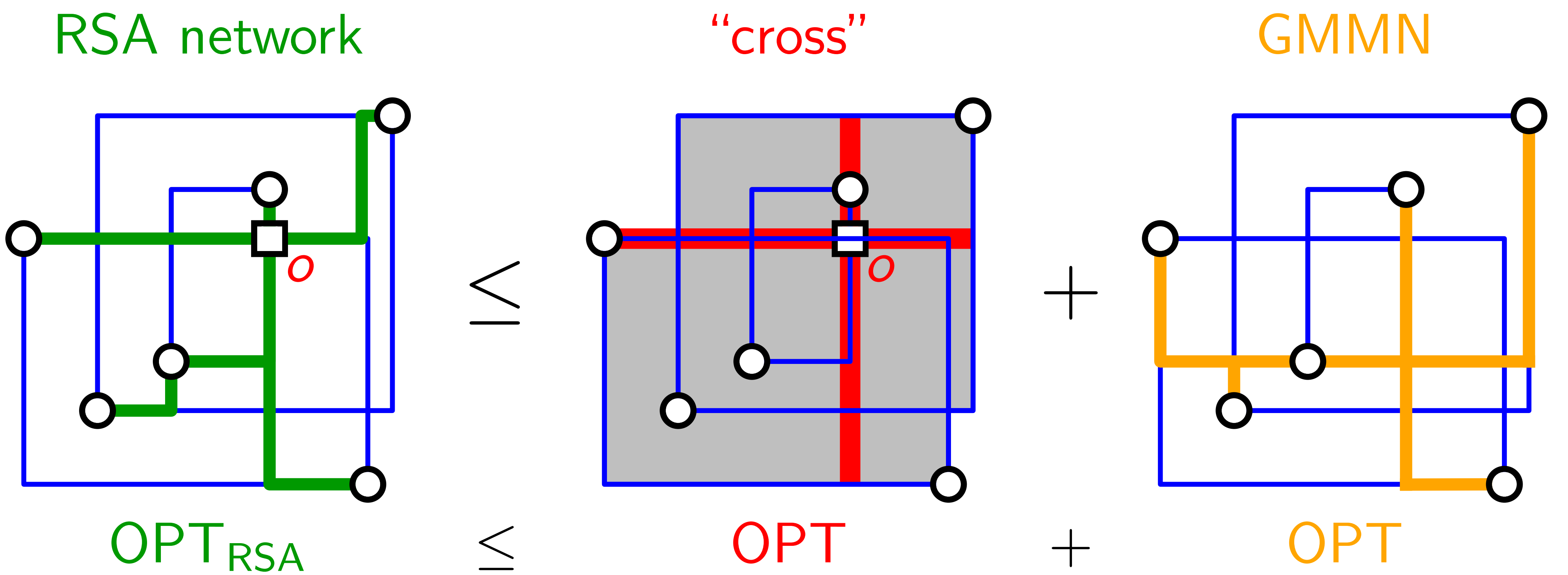
– feasible ✓

– near-optimal: cross + GMMN network *is* RSA network ✓

– efficiently constructable: RSA admits PTAS in 2D. ✓

$\Rightarrow \rho_{\text{xy-sep}} \leq 2(1+\varepsilon)$, $\rho_{\text{x-sep}} \in O(\log n)$

Algorithm for xy-separated GMMN



Idea: Use algorithm for RSA! Resulting network is. . .

- feasible ✓
- near-optimal: cross + GMMN network *is* RSA network ✓
- efficiently constructable: RSA admits PTAS in 2D. ✓

$\Rightarrow \rho_{\text{xy-sep}} \leq 2(1+\varepsilon)$, $\rho_{\text{x-sep}} \in O(\log n)$, $\rho_{\text{2D}} \in O(\log^2 n)$ □

Main Result for Higher Dimensions

	Approximation Factors		
Dimension	Step 1: Partition	Step 2: RSA	Result
2			
$d > 2$			

Main Result for Higher Dimensions

	Approximation Factors		
Dimension	Step 1: Partition	Step 2: RSA	Result
2	$O(\log^2 n)$	$O(1)$	$O(\log^2 n)$
$d > 2$			

Main Result for Higher Dimensions

	Approximation Factors		
Dimension	Step 1: Partition	Step 2: RSA	Result
2	$O(\log^2 n)$	$O(1)$	$O(\log^2 n)$
$d > 2$	$O(\log^d n)$	$O(\log n)$	$O(\log^{d+1} n)$

Main Result for Higher Dimensions

Dimension	Approximation Factors		
	Step 1: Partition	Step 2: RSA	Result
2	$O(\log^{\cancel{2}} n)$	$O(1)$	$O(\log^{\cancel{2}} n)$
$d > 2$	$O(\log^{d} n)$	$O(\log n)$	$O(\log^{d+1} n)$

Main Result for Higher Dimensions

Dimension	Approximation Factors		
	Step 1: Partition	Step 2: RSA	Result
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$d > 2$	$O(\log^{d} n)$	$O(\log n)$	$O(\log^{d+1} n)$

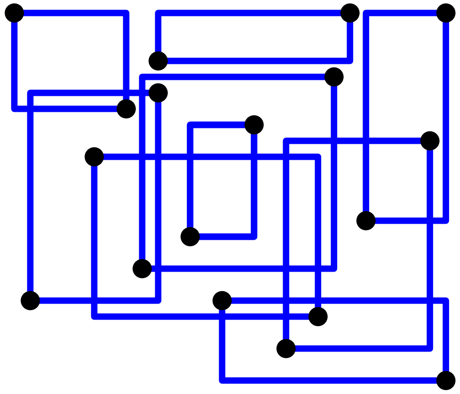
TO DO:

In 2D, remove one level of recursion!

Part II

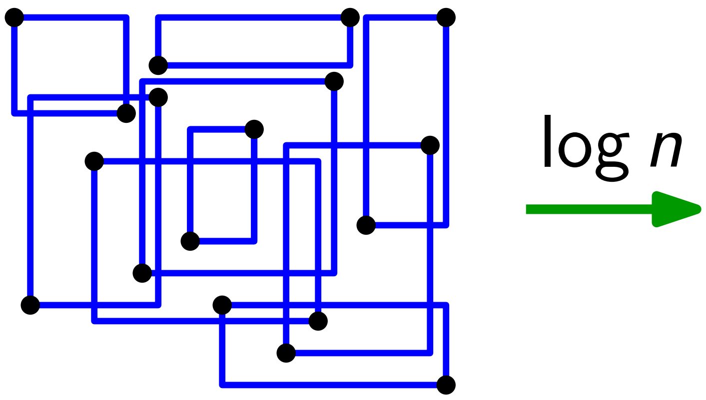
An Improved $O(\log n)$-Approximation Algorithm for GMMN in the Plane

Simple and Improved Approach in 2D



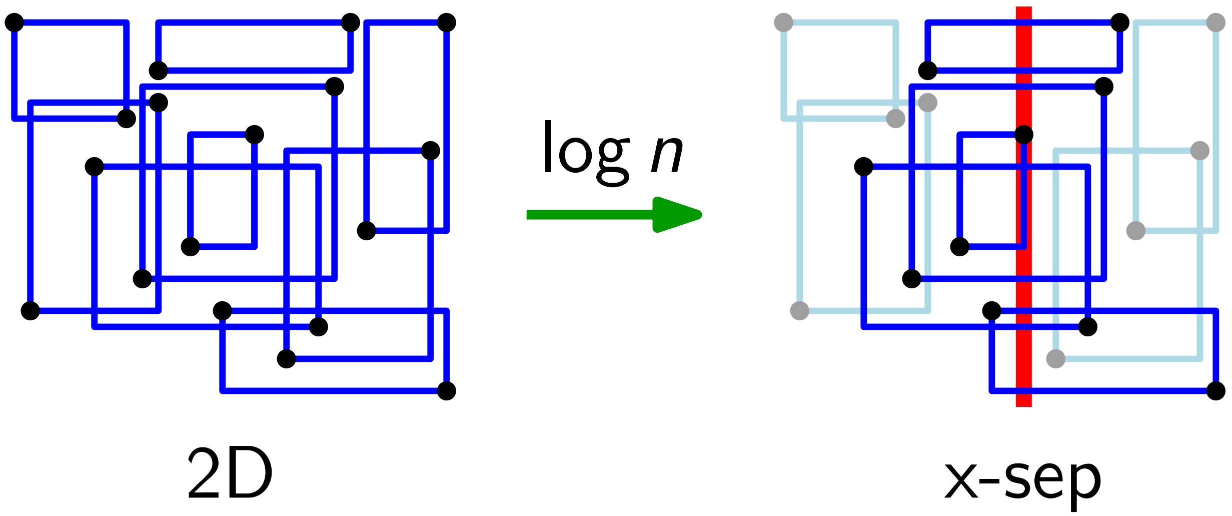
2D

Simple and Improved Approach in 2D

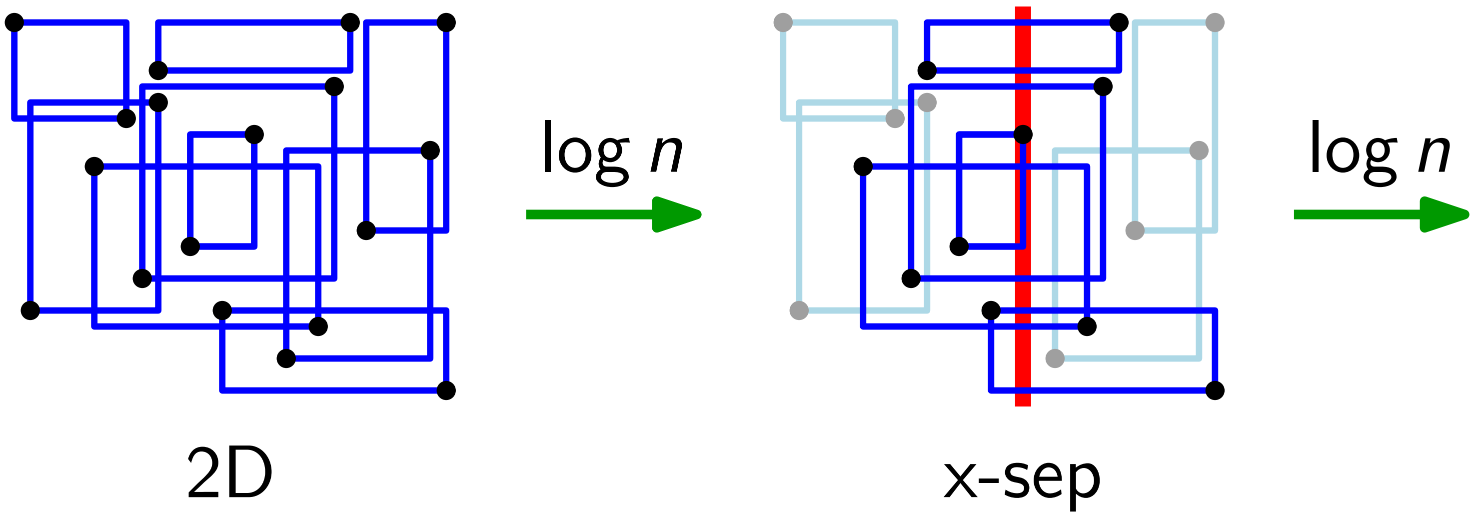


2D

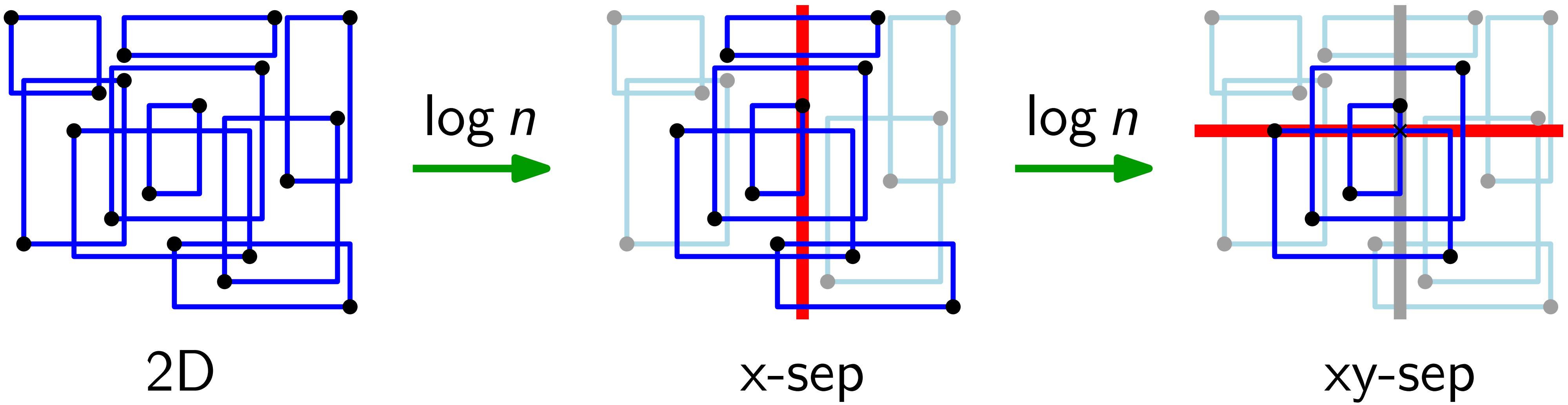
Simple and Improved Approach in 2D



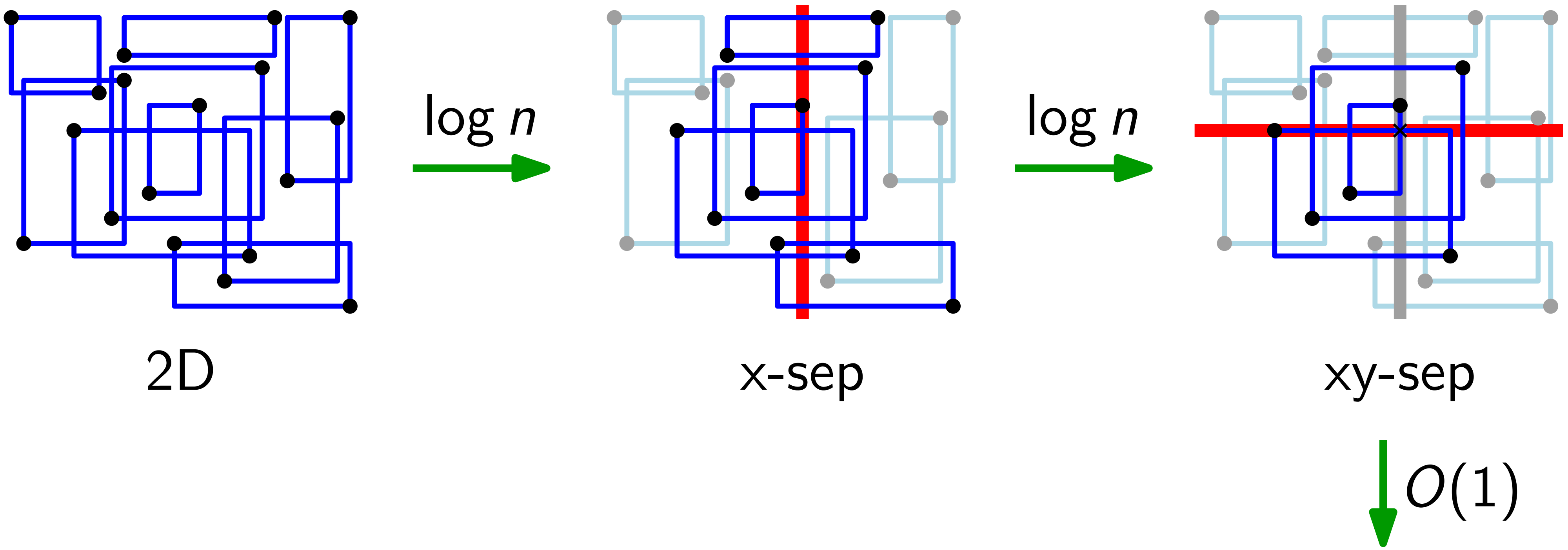
Simple and Improved Approach in 2D



Simple and Improved Approach in 2D



Simple and Improved Approach in 2D



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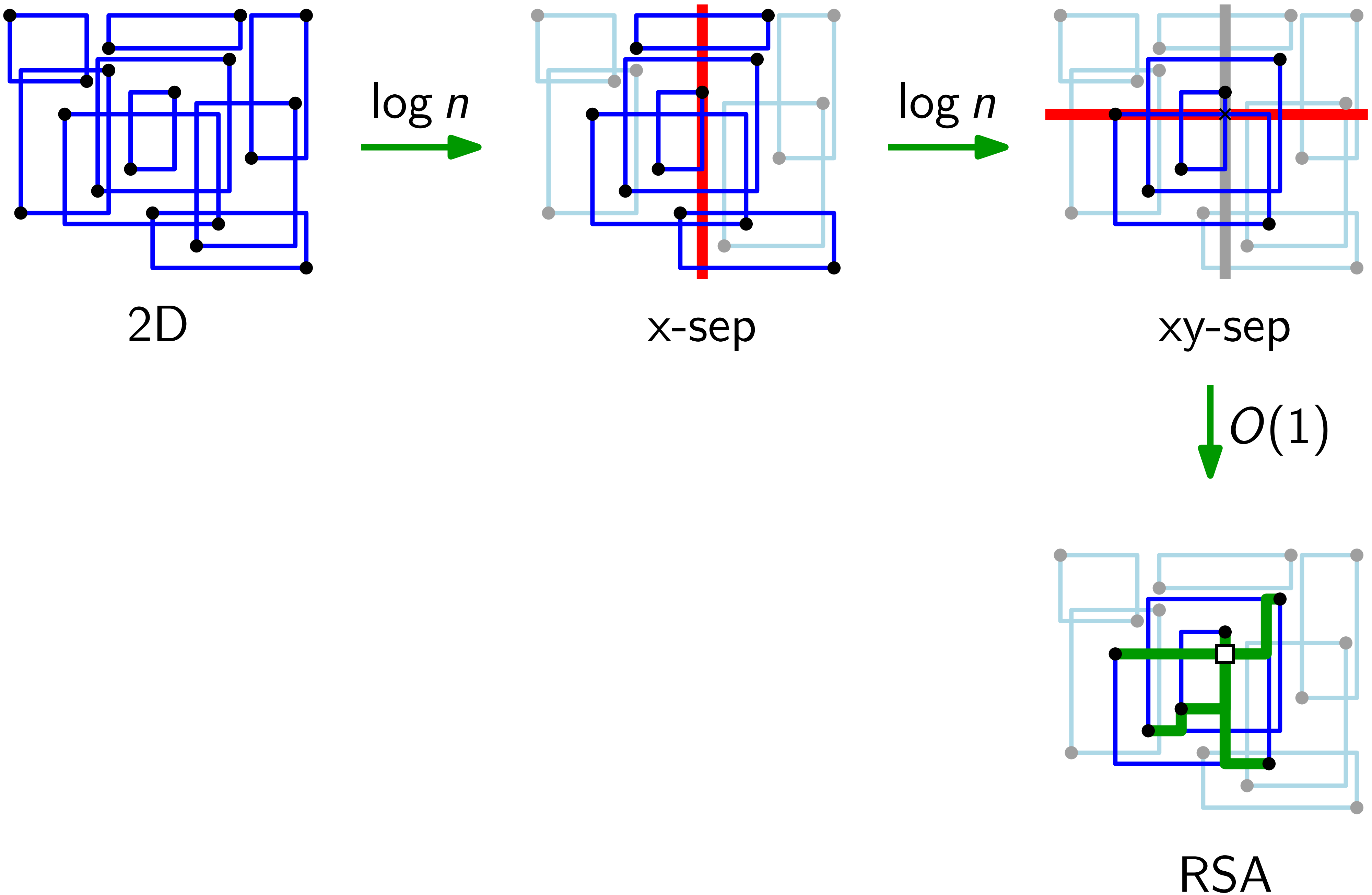

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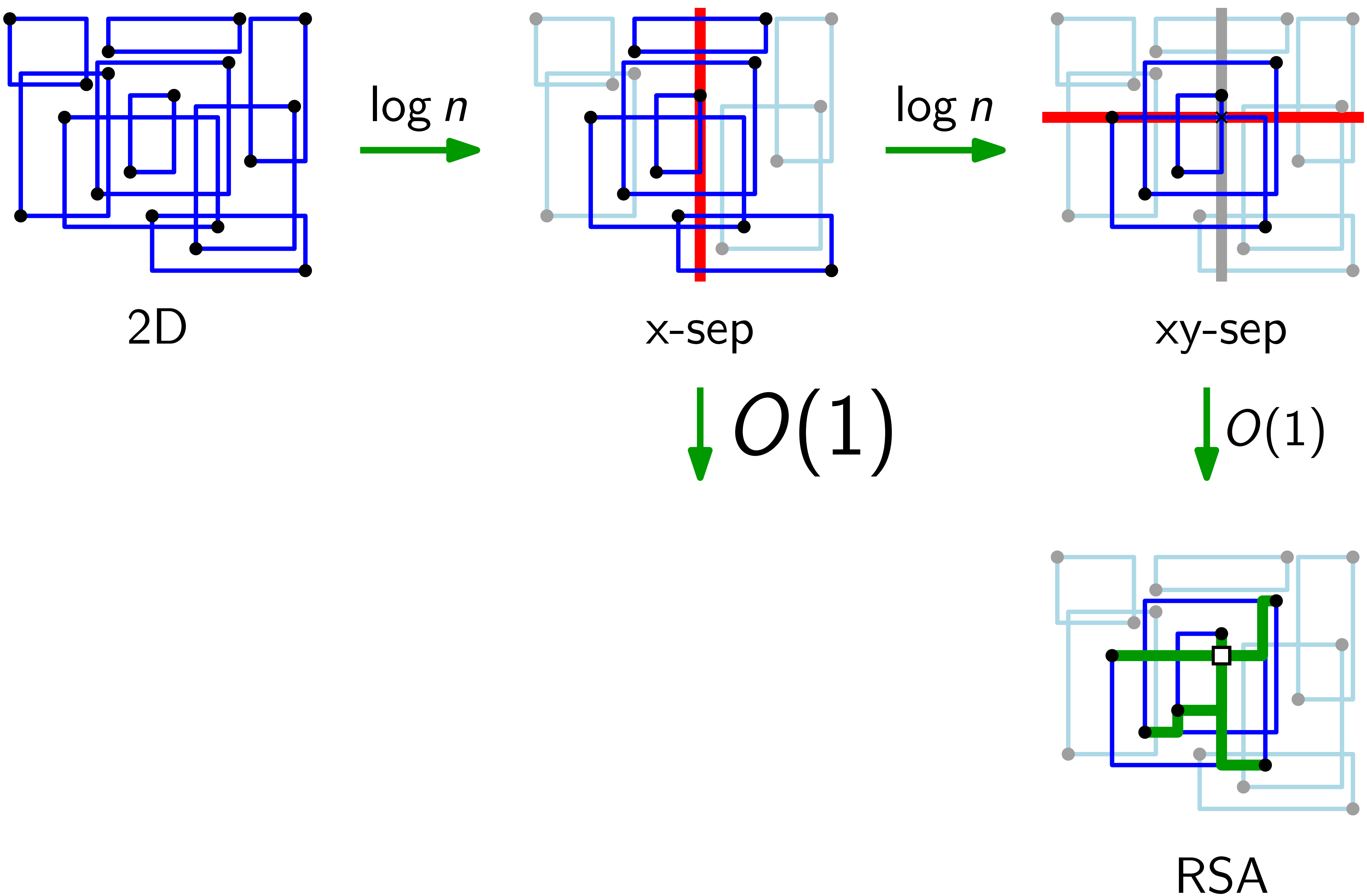
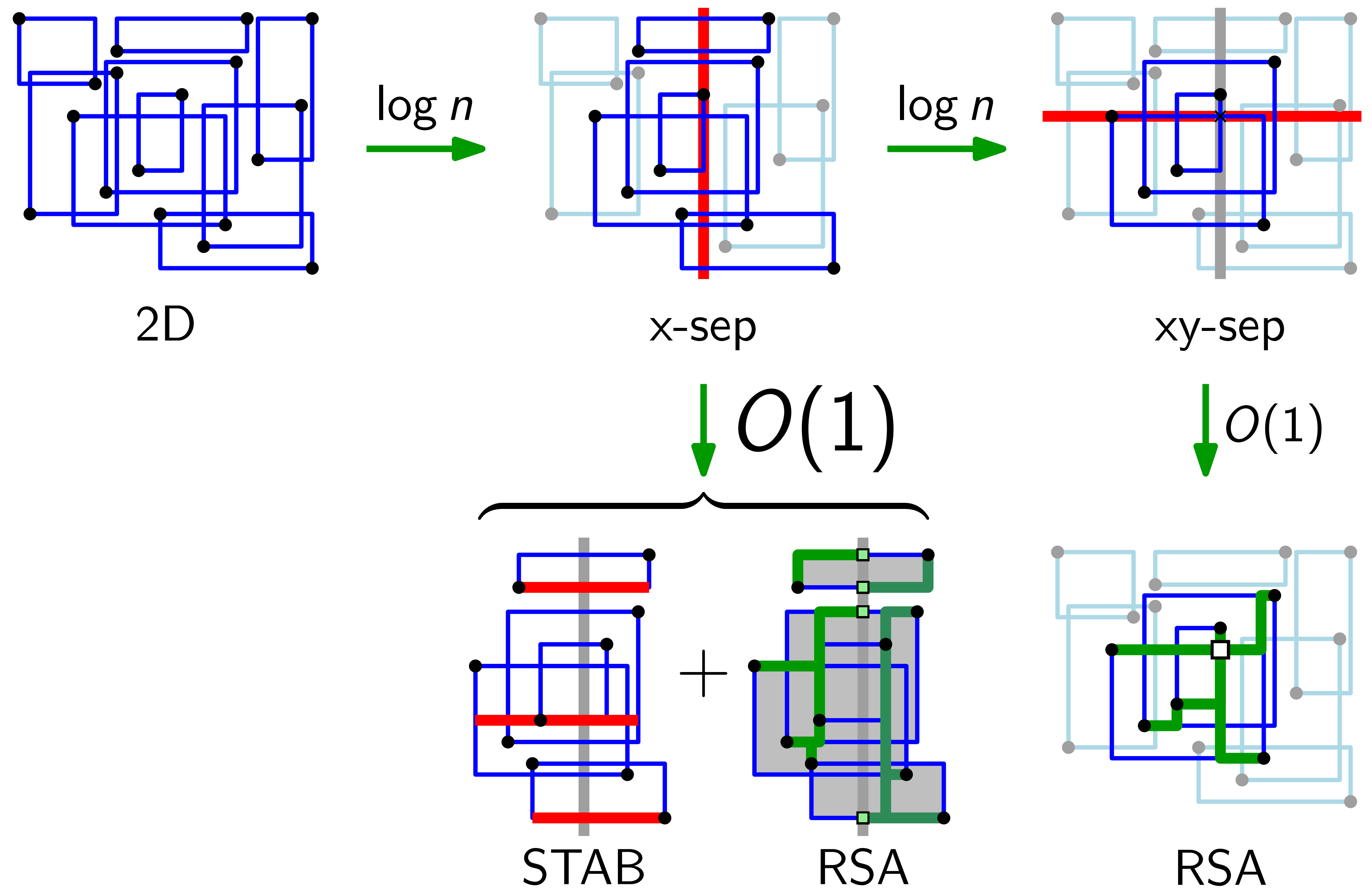
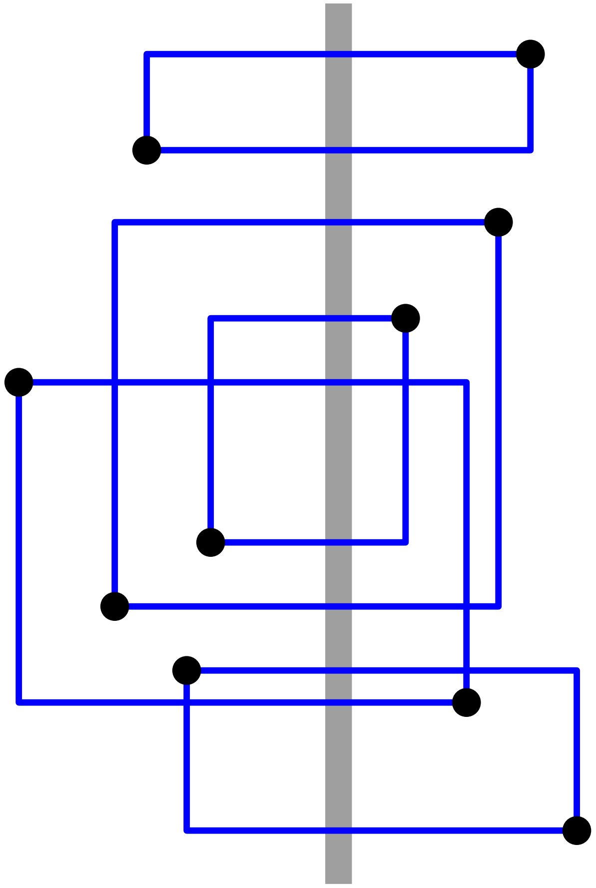


Simple and Improved Approach in 2D



Improved Approach: Idea



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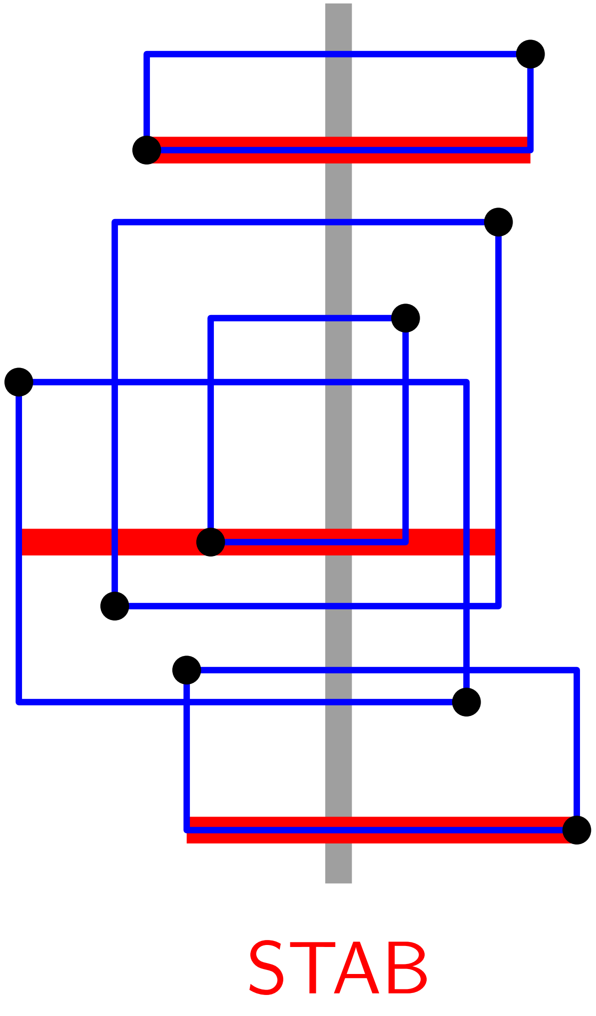

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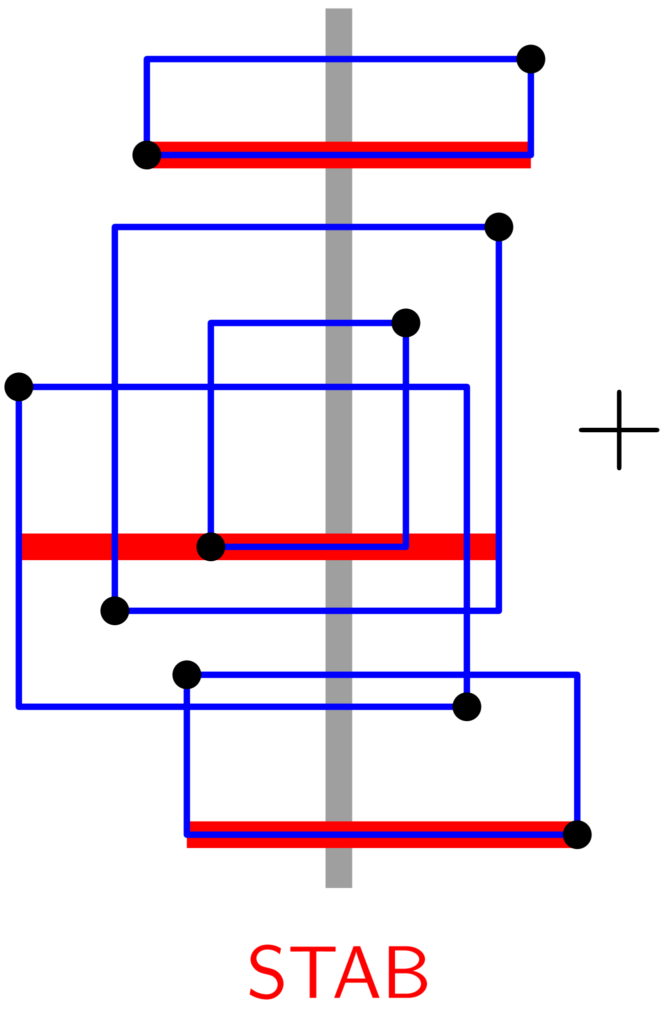

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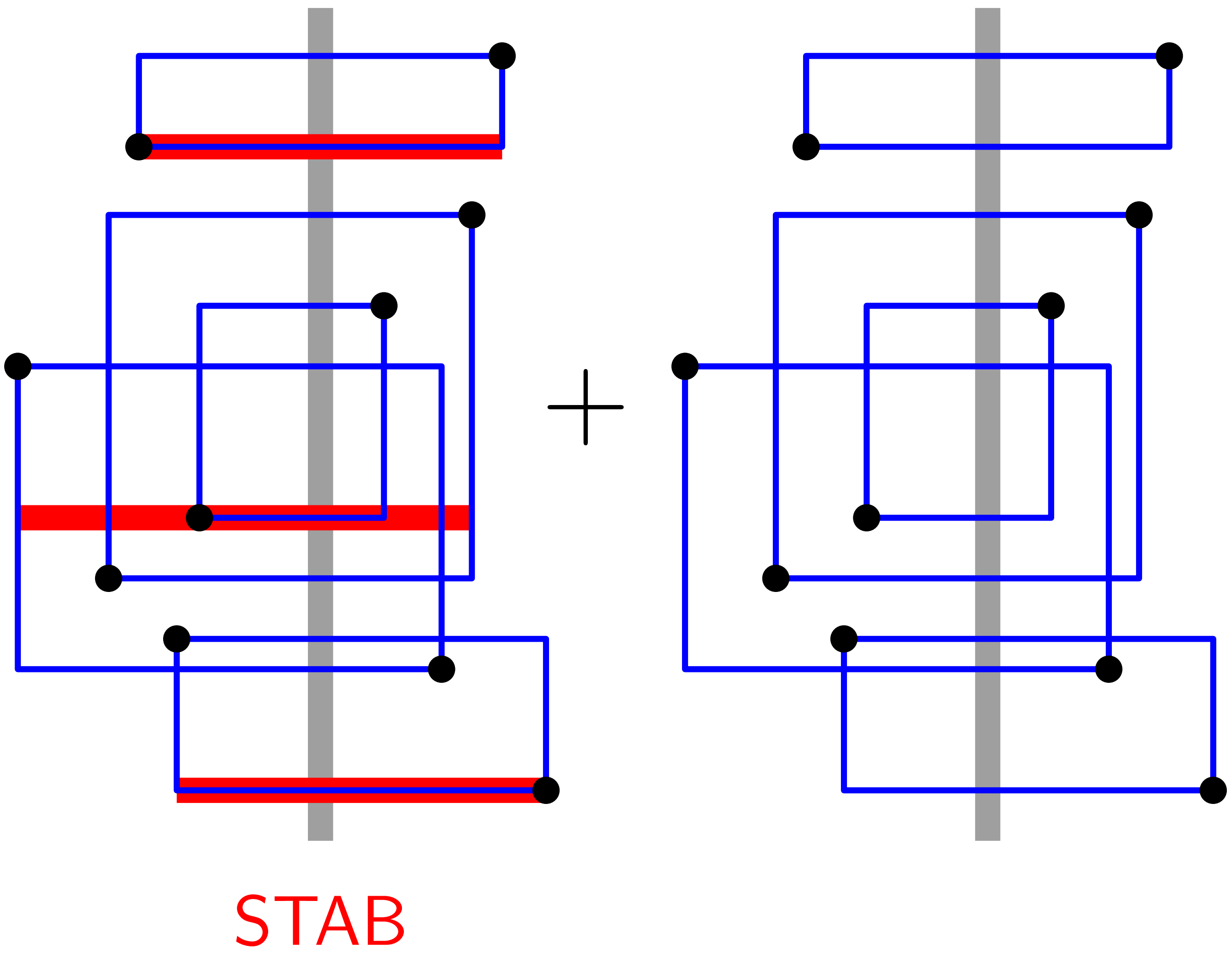

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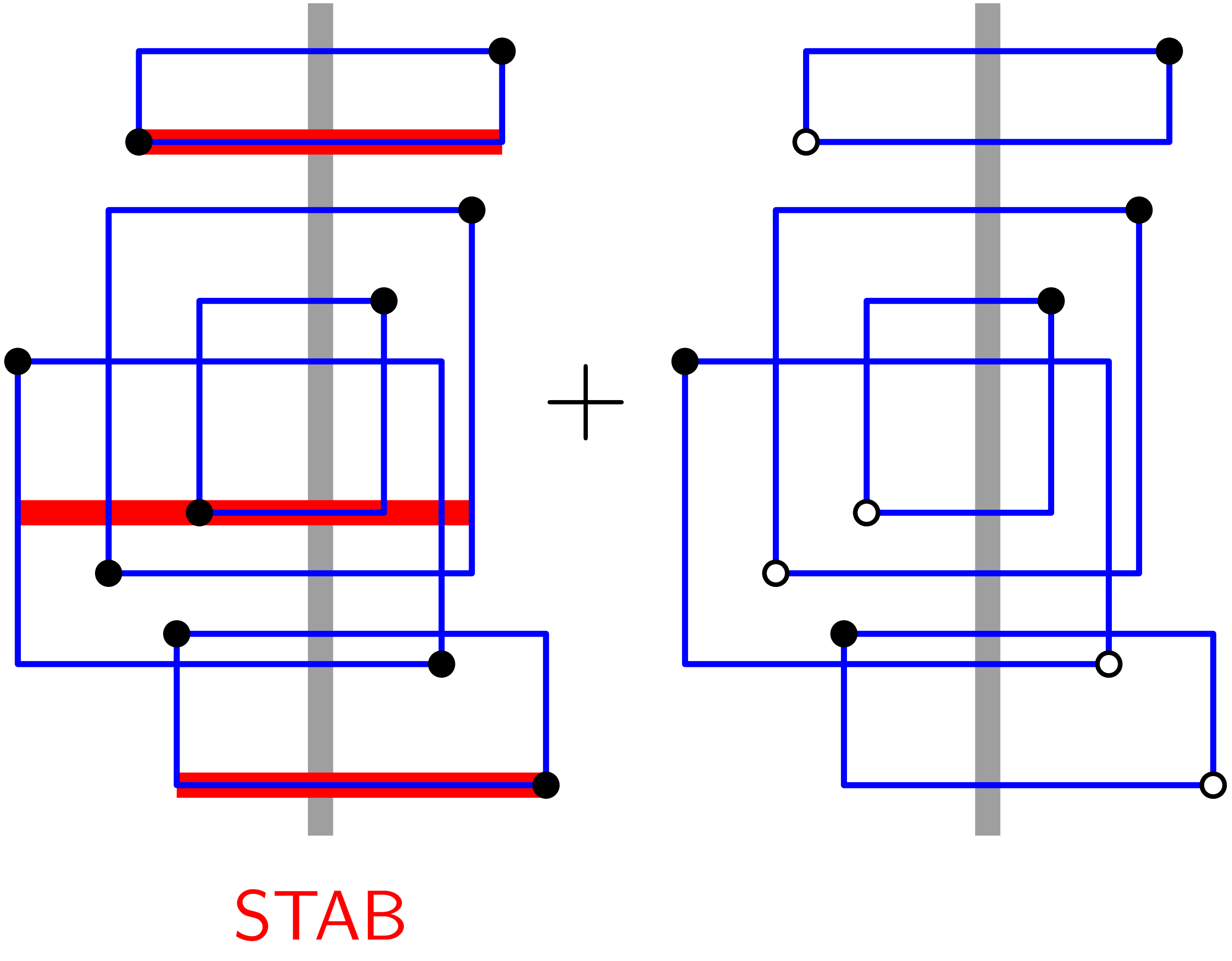

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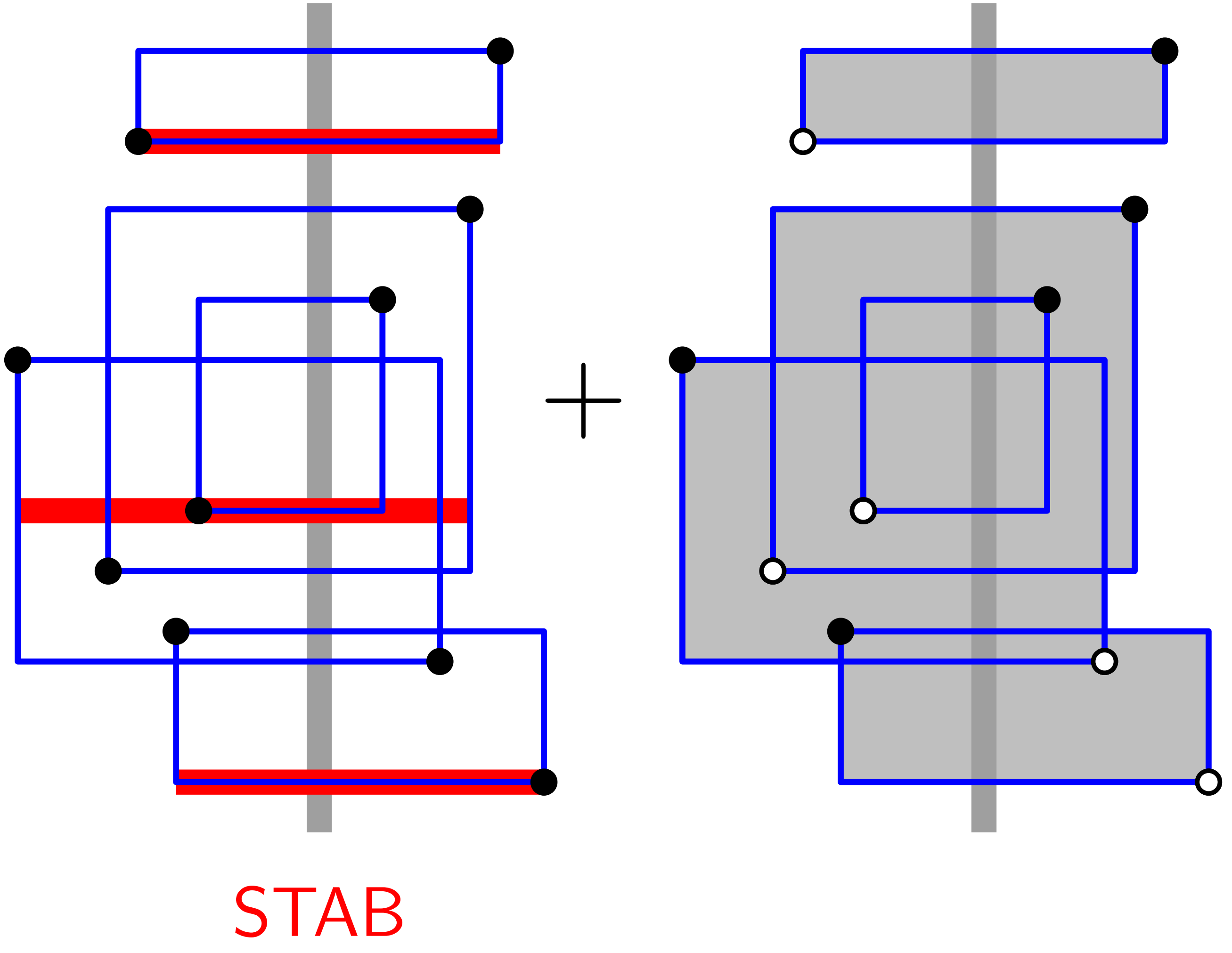

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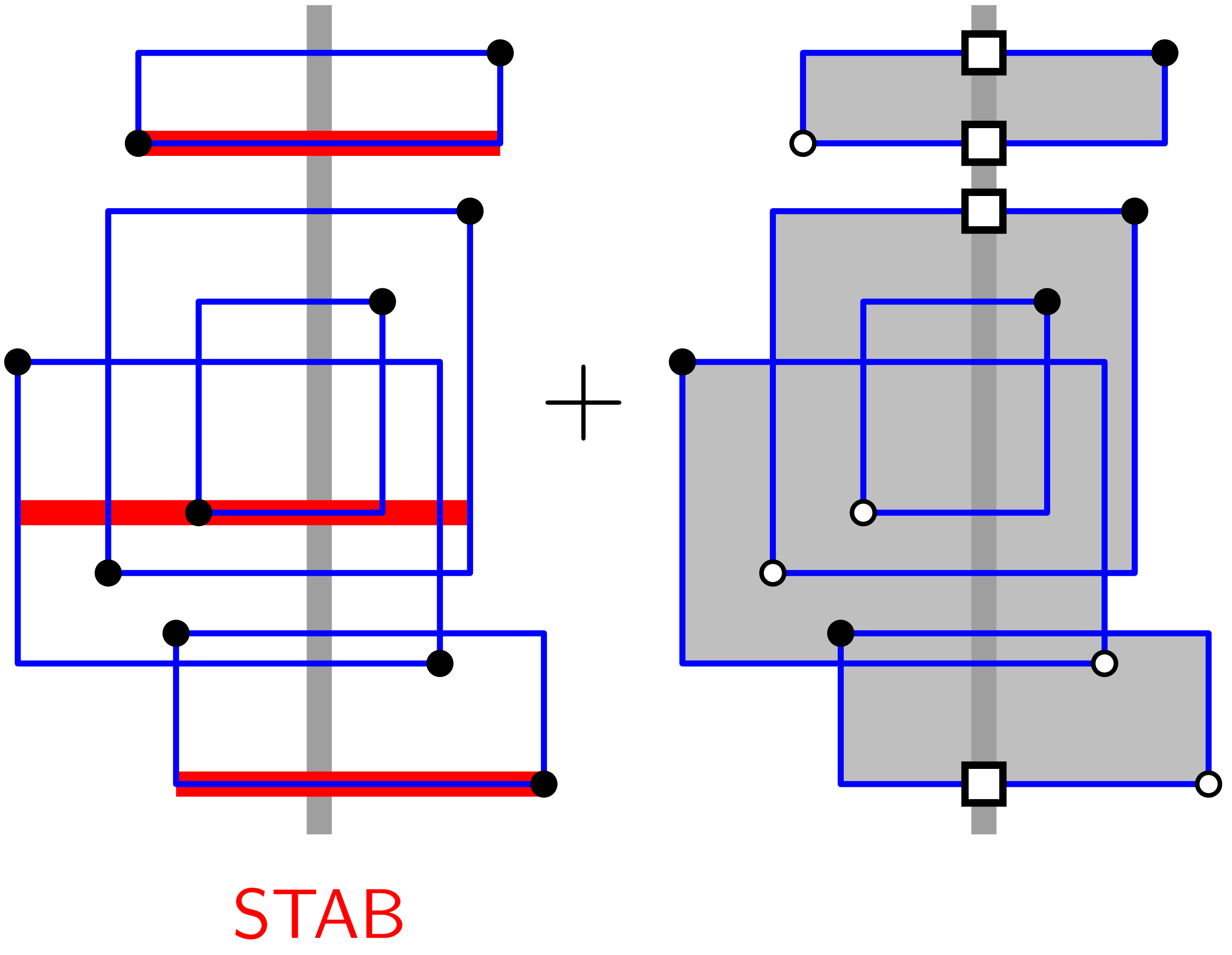

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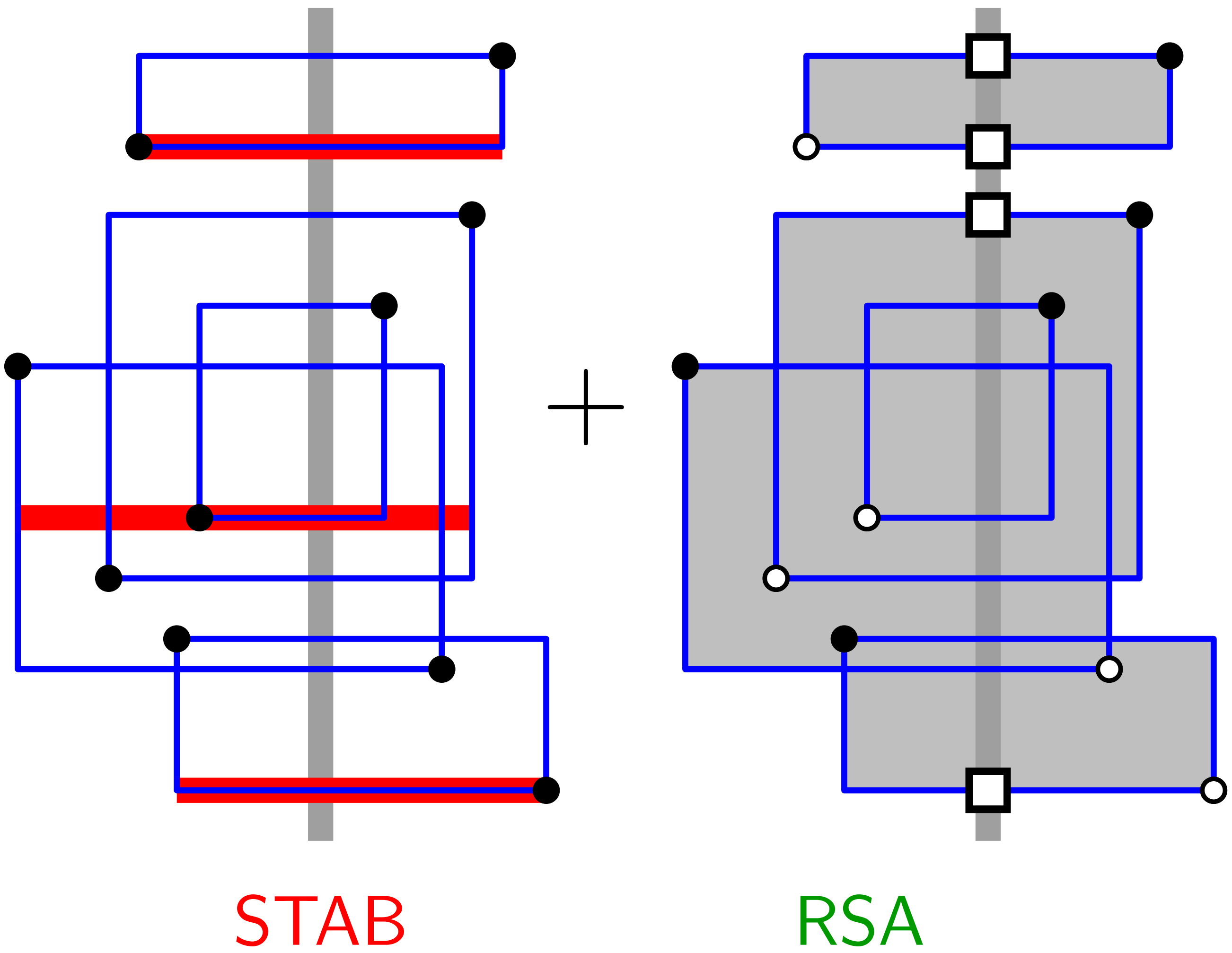

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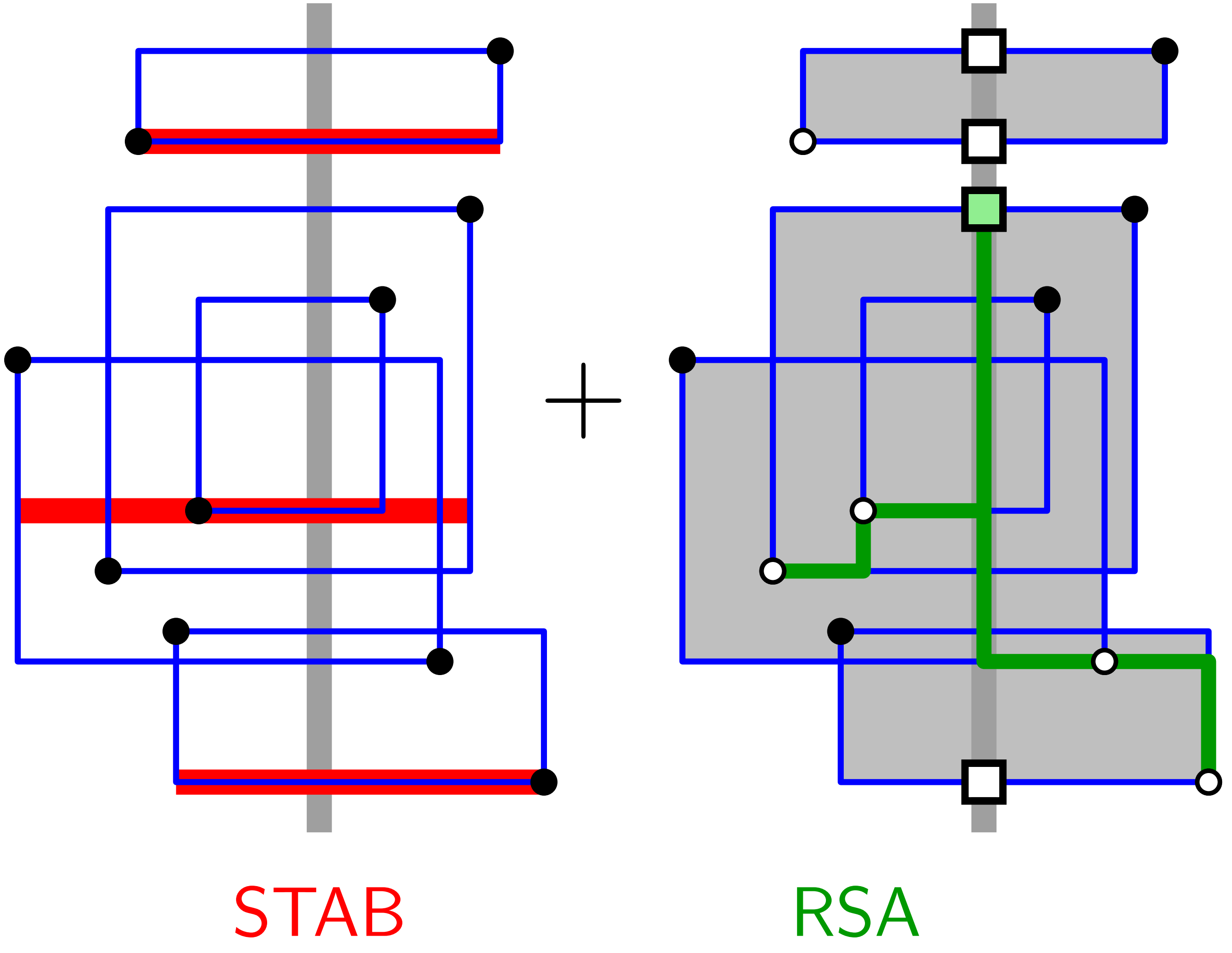

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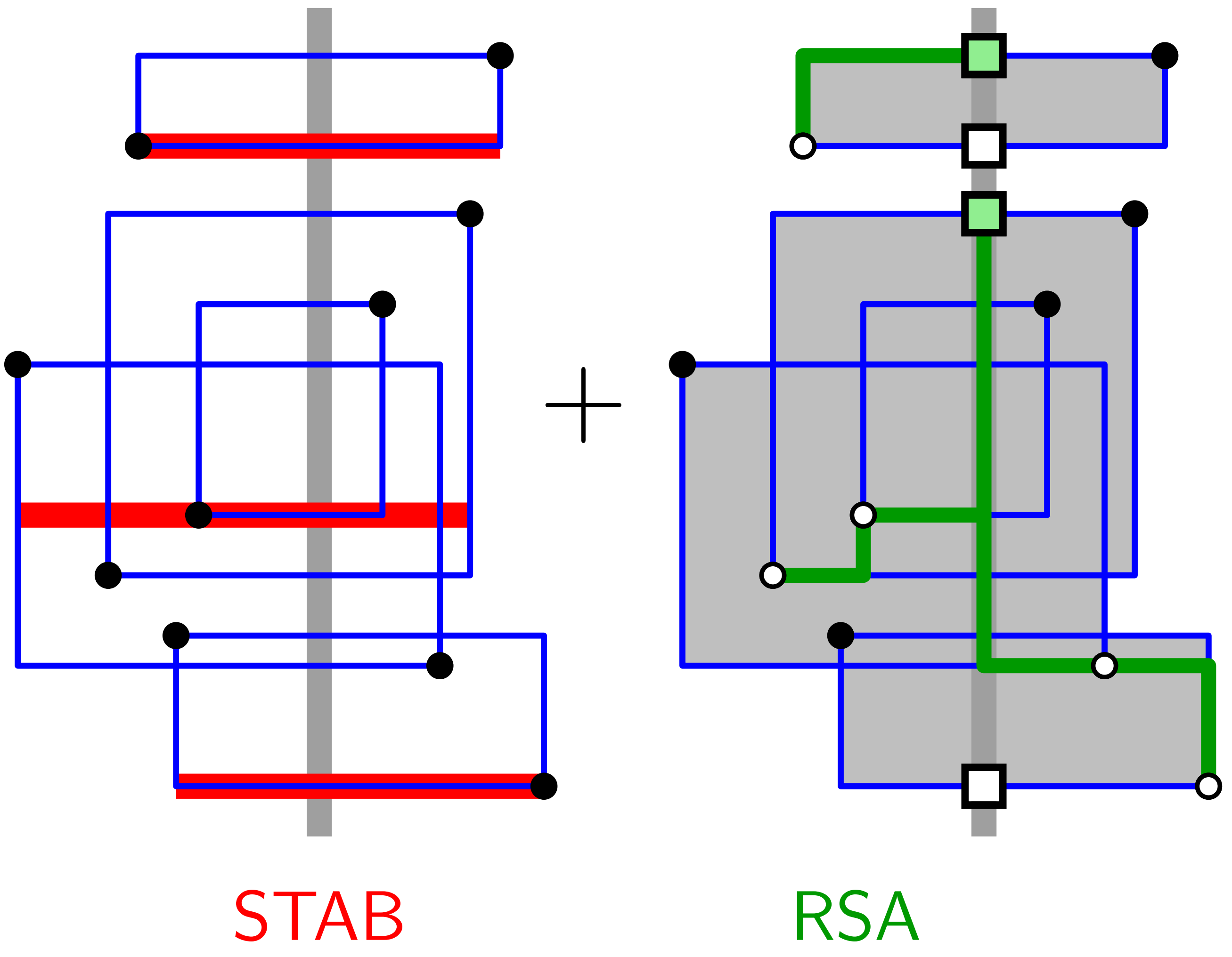

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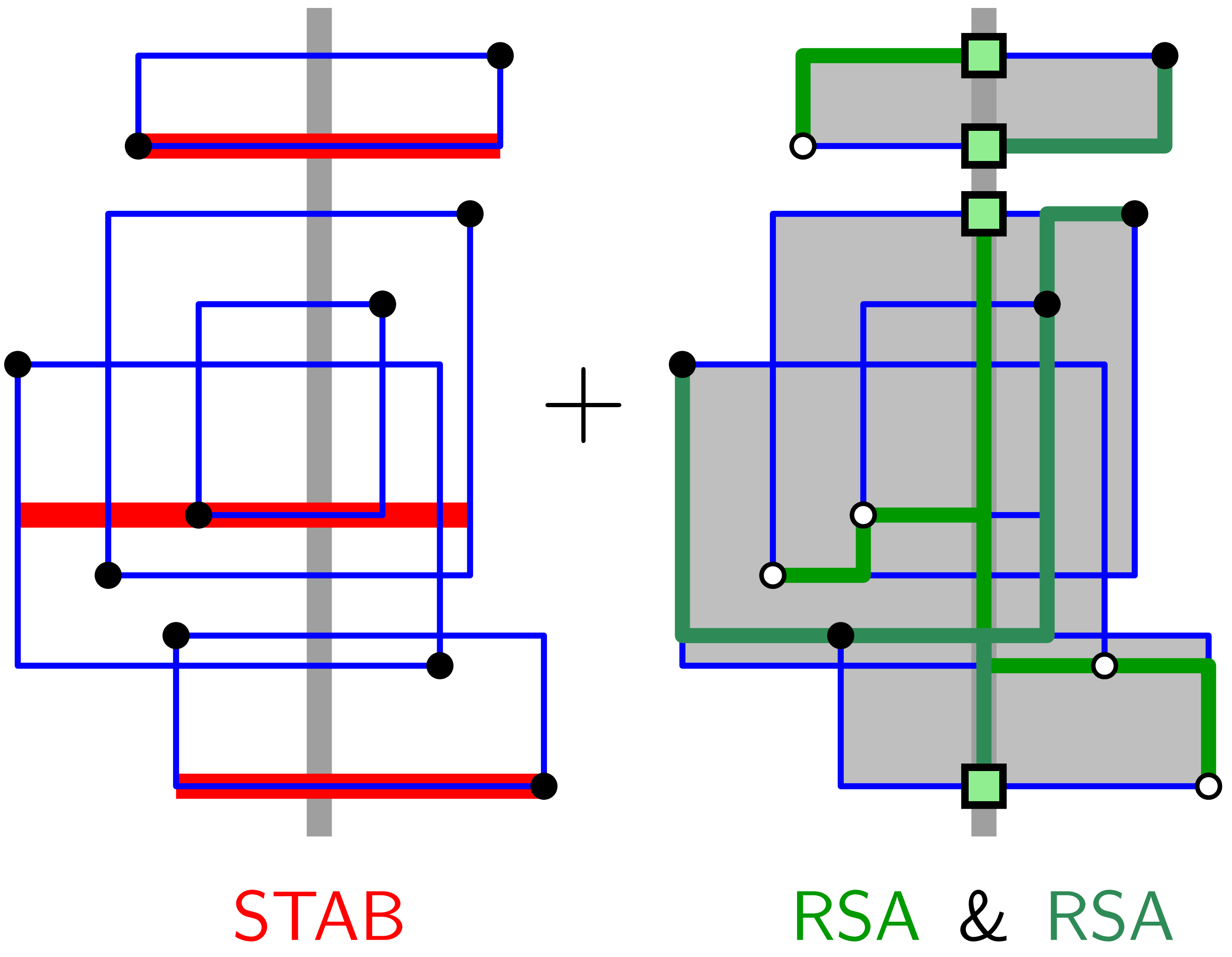

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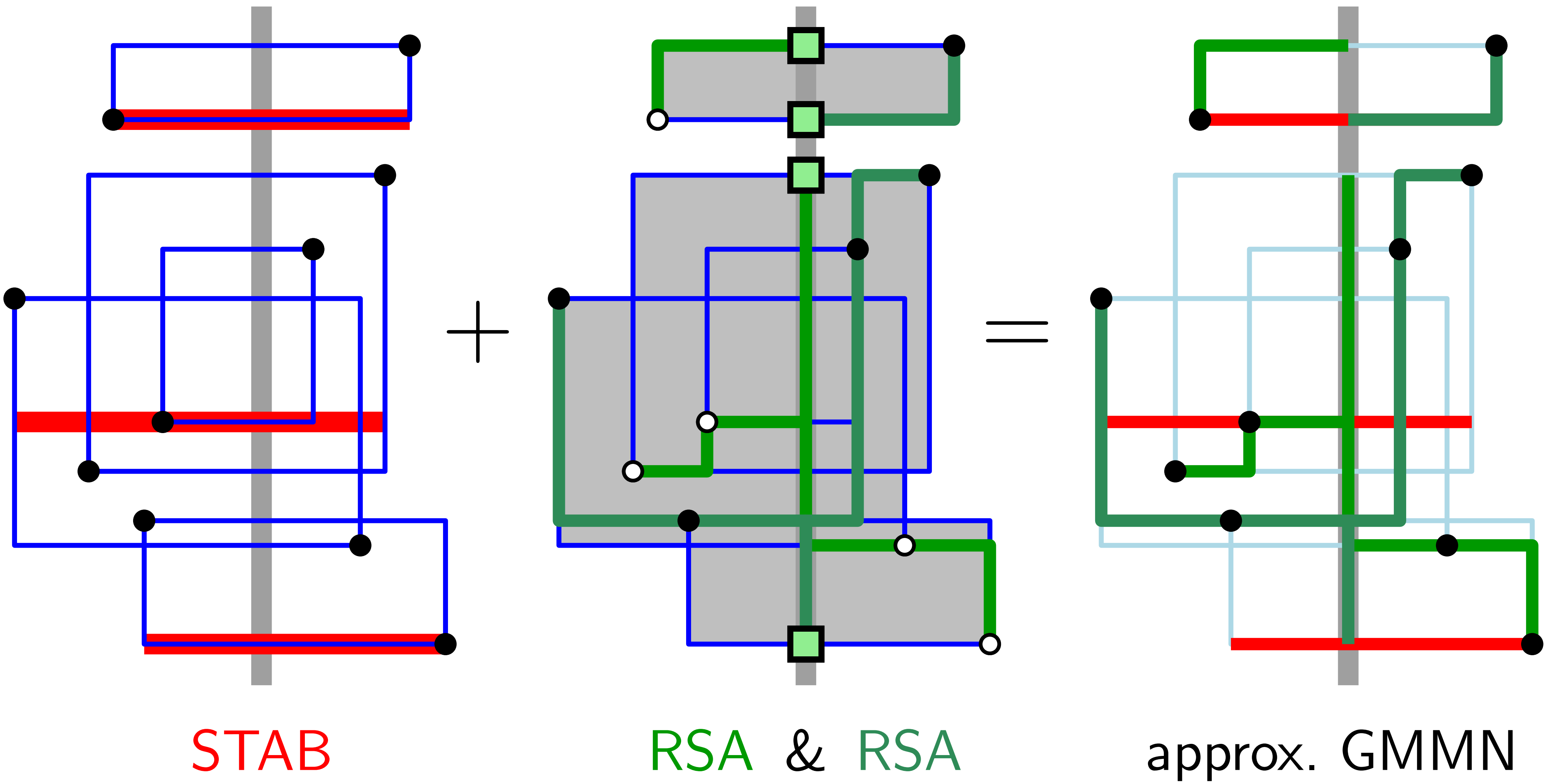

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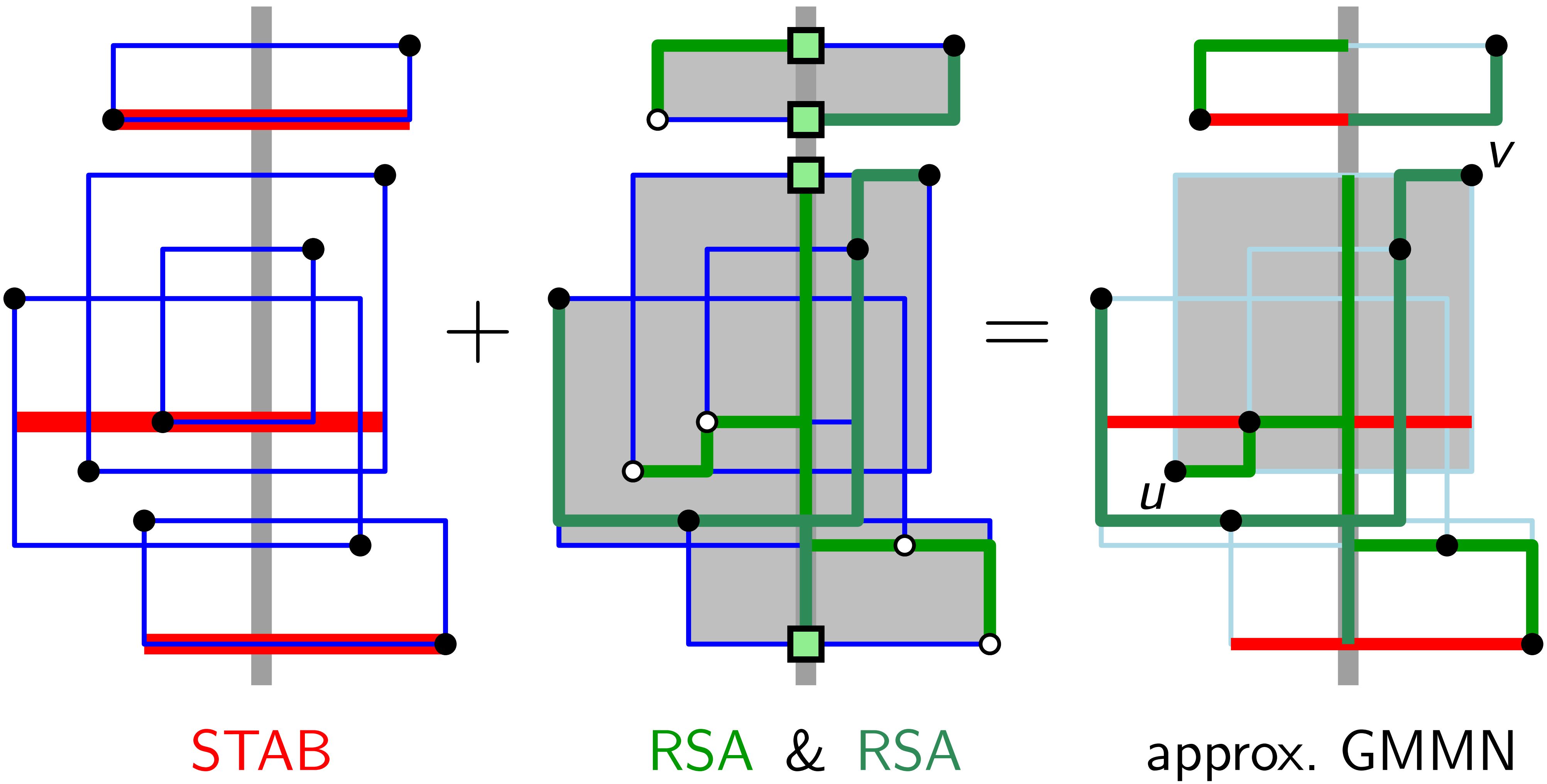

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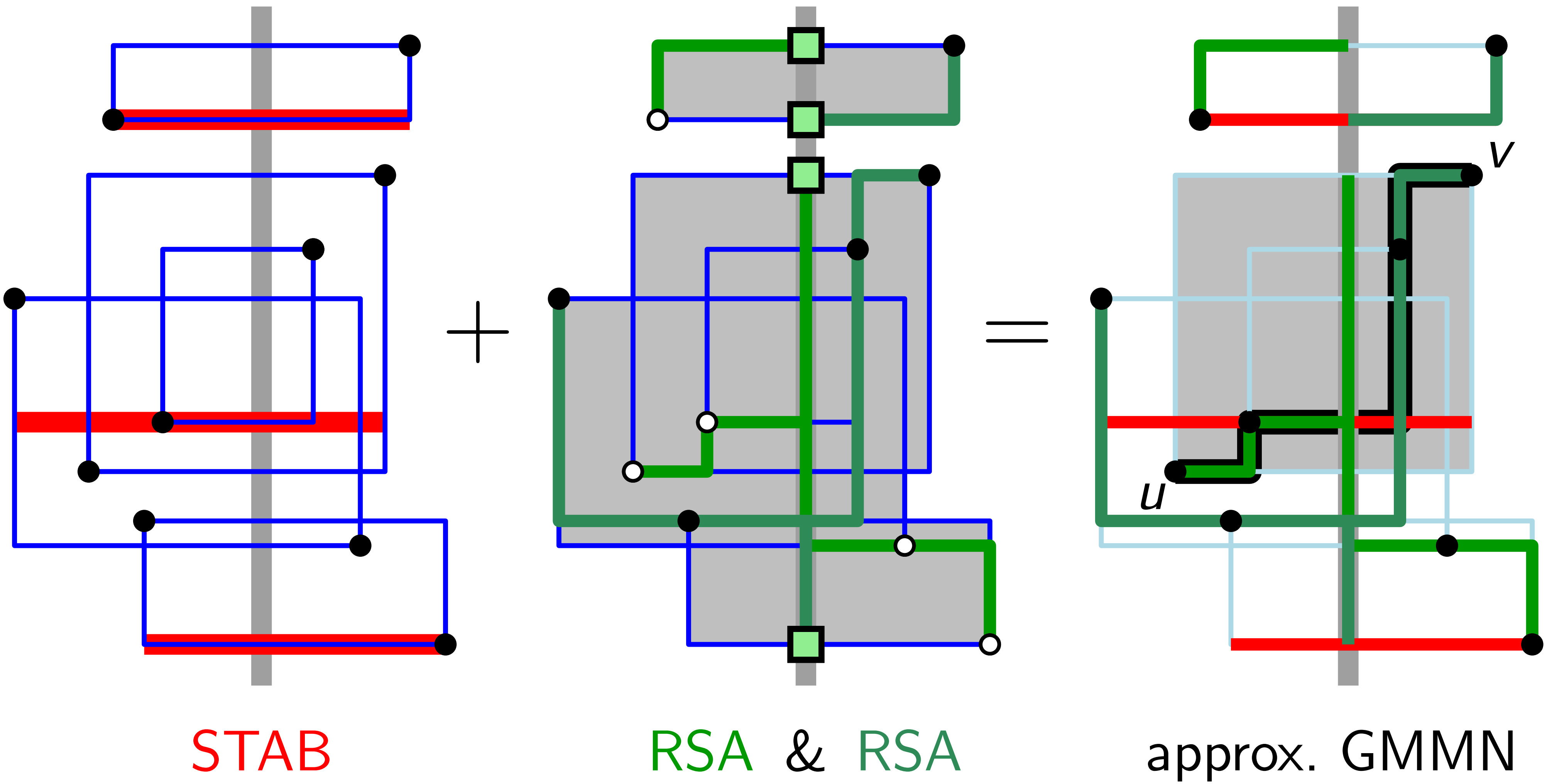

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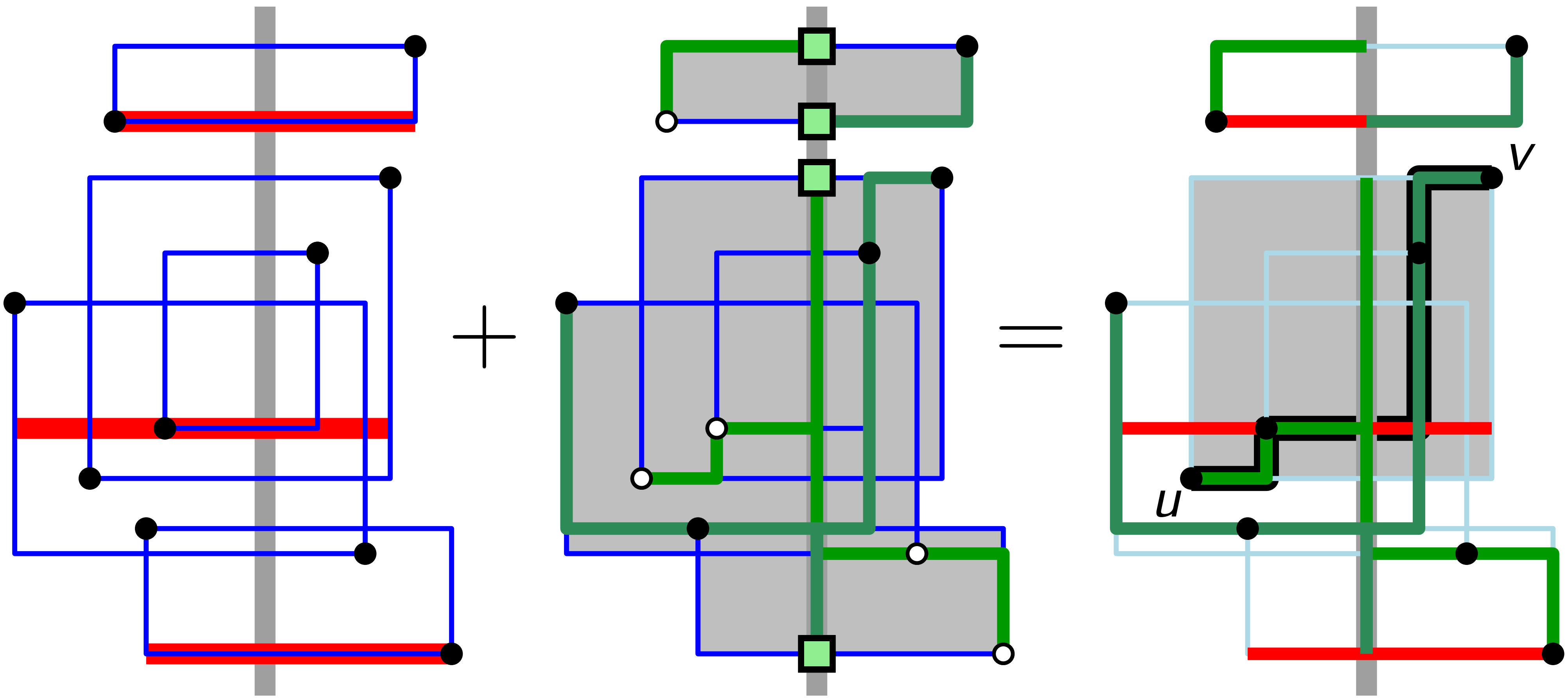


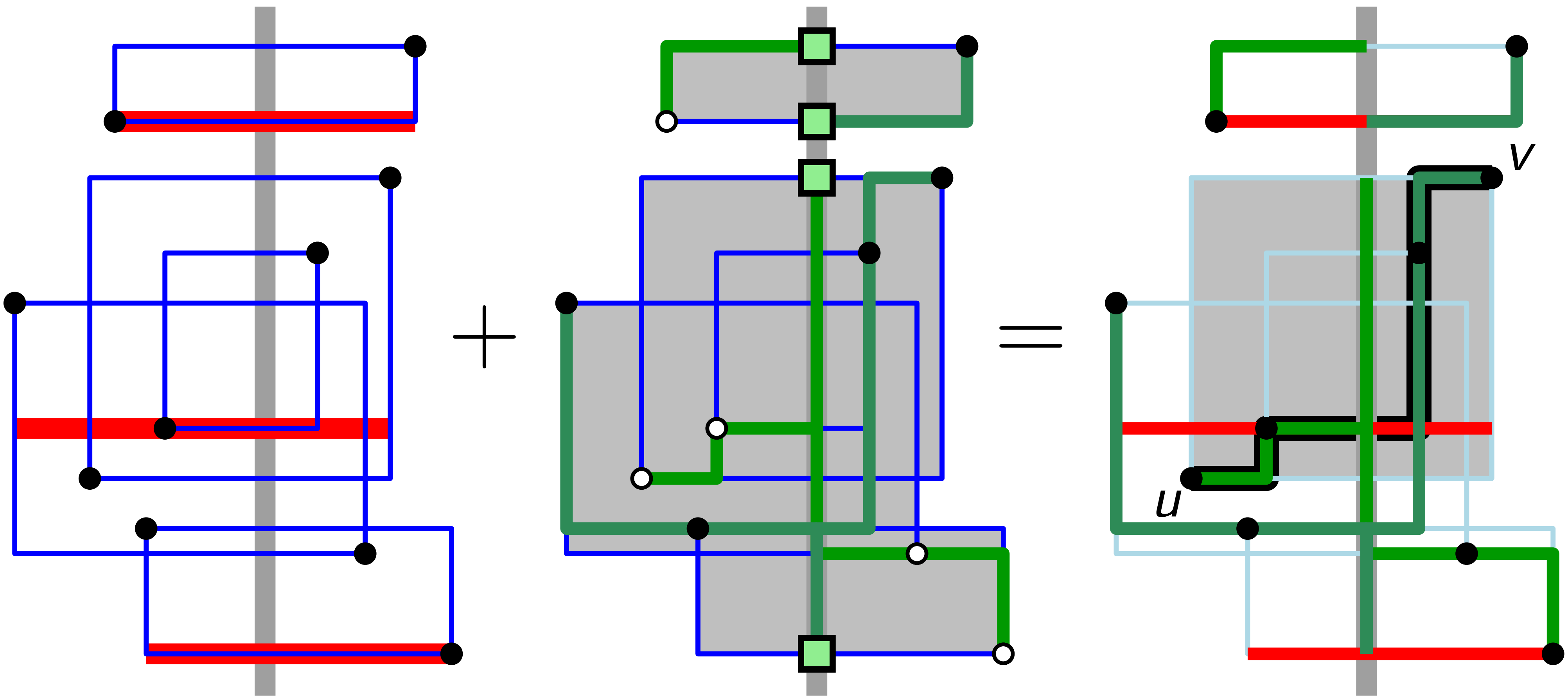
STAB

RSA & RSA

approx. GMMN

– feasible ✓

Improved Approach: Idea

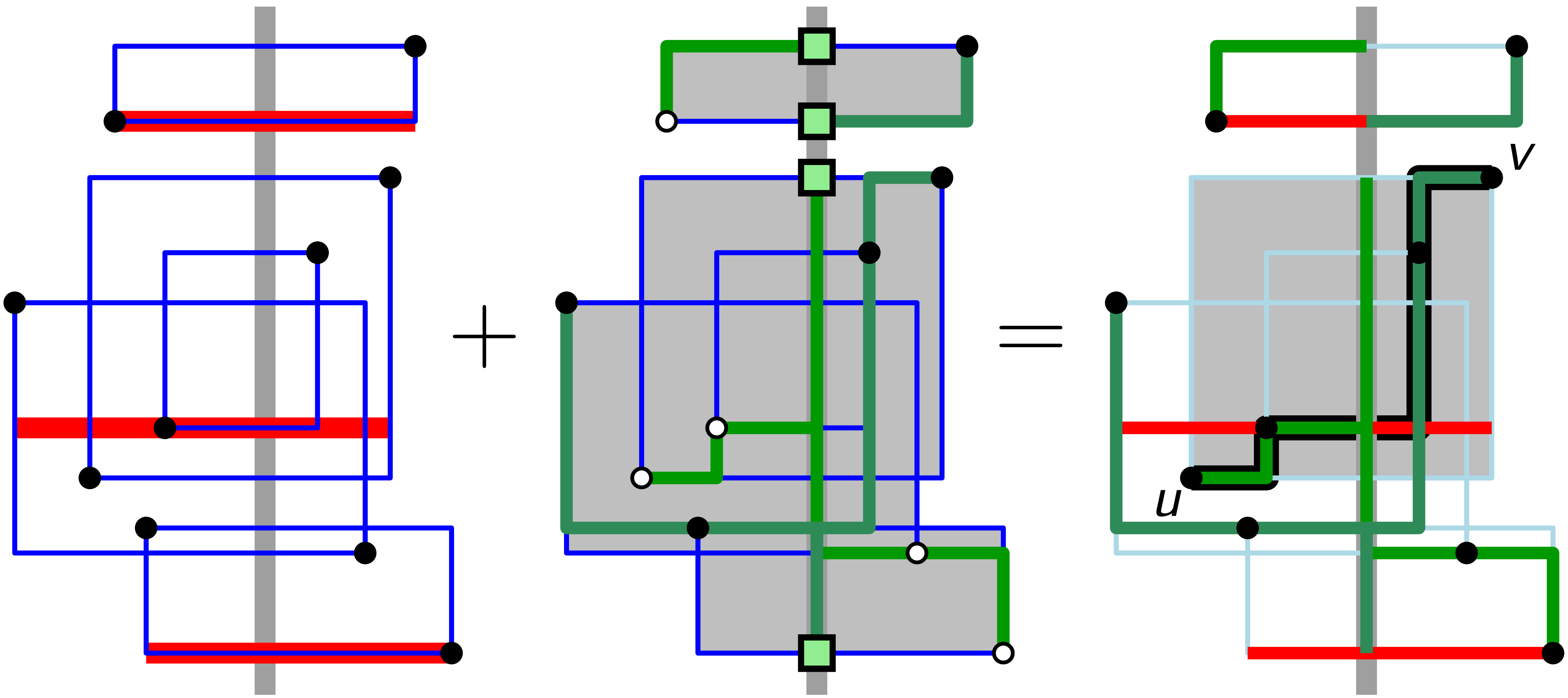


STAB + RSA & RSA = approx. GMMN

– feasible ✓

– near-optimal ?

Improved Approach: Idea



STAB

$\leq 4\,\mathrm{OPT}$

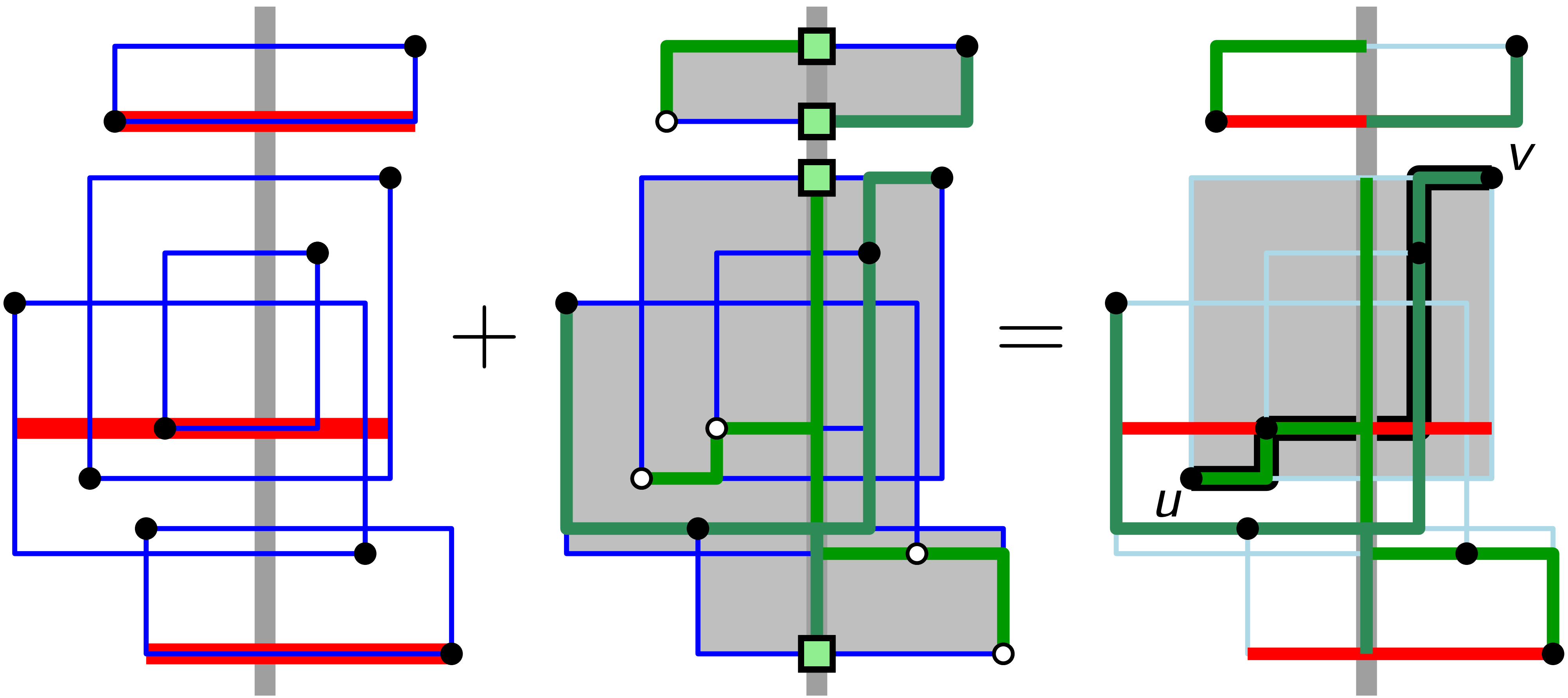
RSA & RSA

approx. GMMN

– feasible ✓

– near-optimal ?

Improved Approach: Idea



STAB $\leq 4\,\mathrm{OPT}$

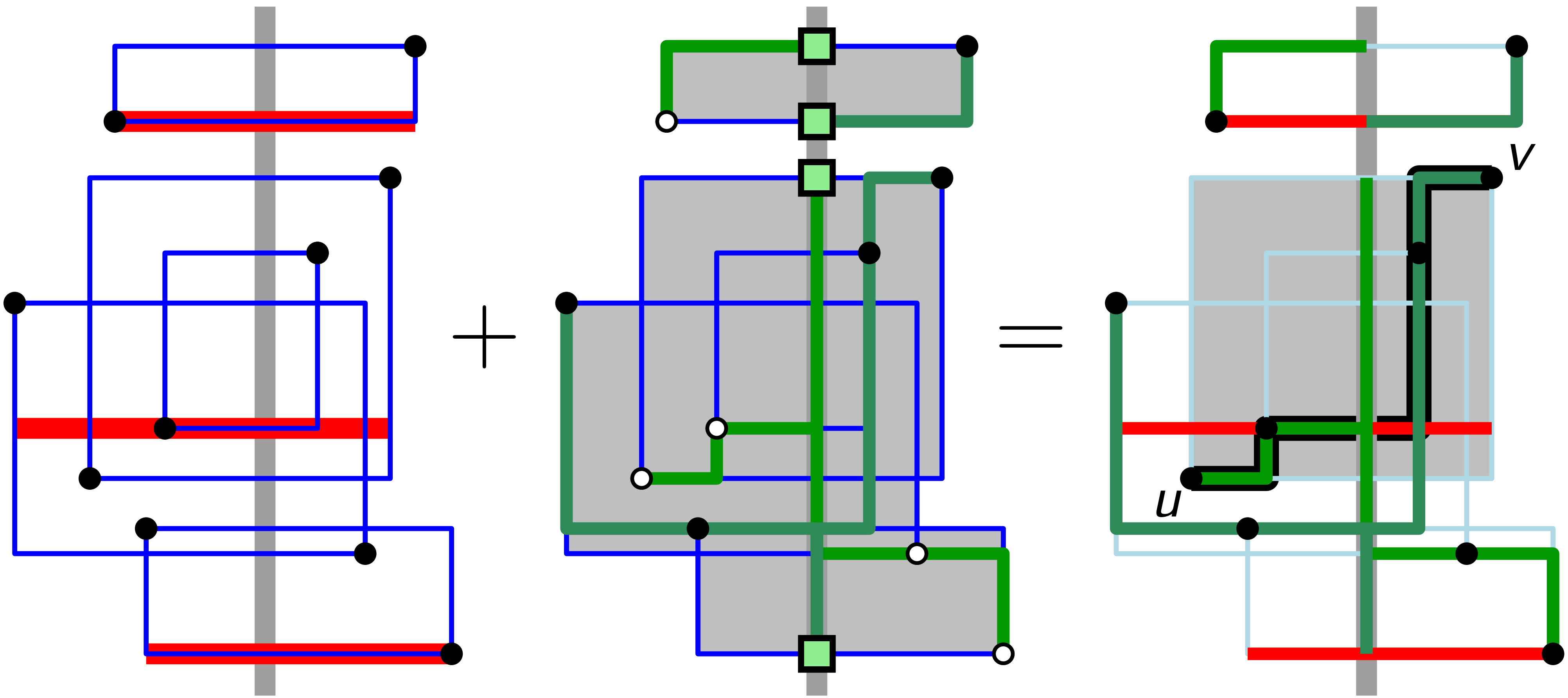
RSA & RSA $\leq 4(1+\varepsilon)\,\mathrm{OPT}$

approx. GMMN

– feasible ✓

– near-optimal ?

Improved Approach: Idea



STAB | RSA & RSA | approx. GMMN

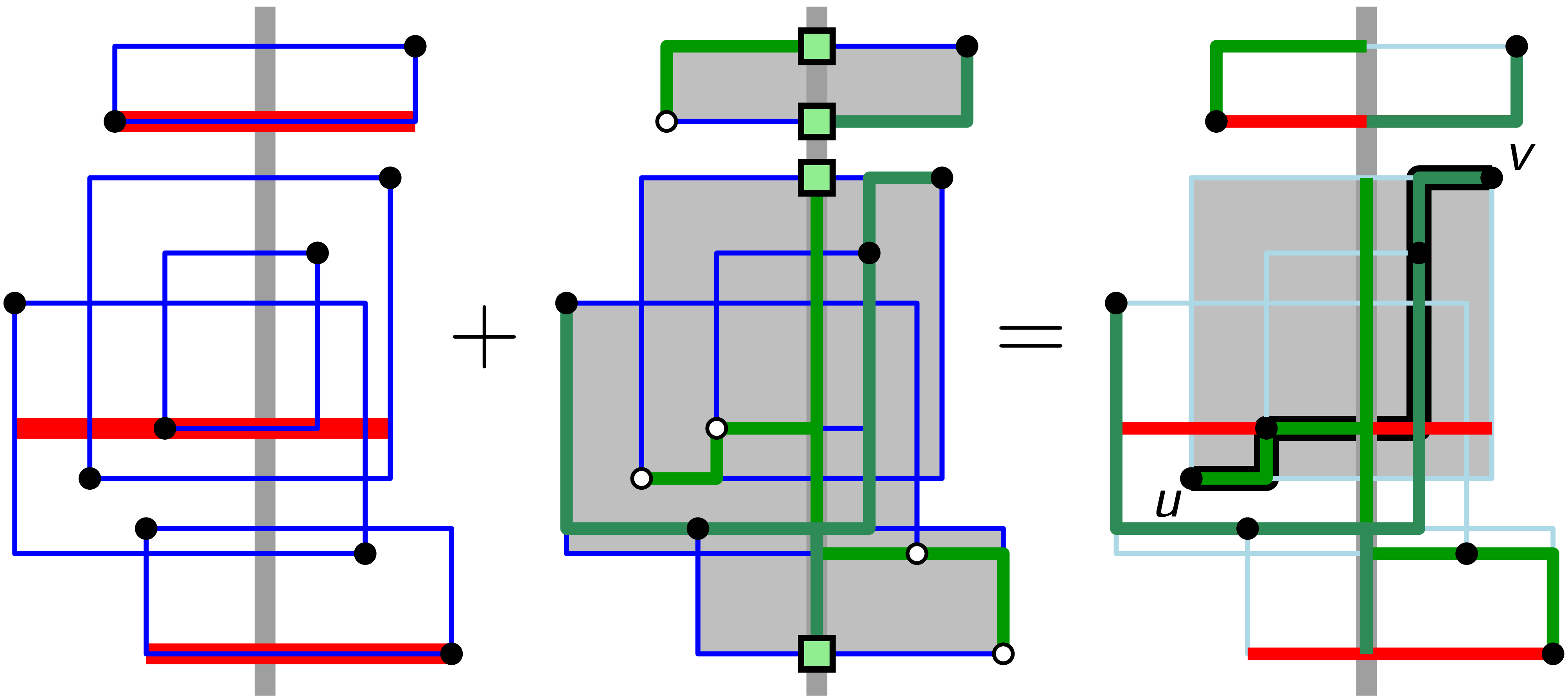
$\leq 4\,\mathrm{OPT}$ | $\leq 4(1+\varepsilon)\,\mathrm{OPT}$

$\leq 8(1+\varepsilon)\,\mathrm{OPT}$

– feasible ✓

– near-optimal ?

Improved Approach: Idea



STAB | RSA & RSA | approx. GMMN

$\leq 4\,\mathrm{OPT}$ $\leq 4(1+\varepsilon)\,\mathrm{OPT}$

$\underbrace{\qquad\qquad\qquad\qquad}_{\leq 8(1+\varepsilon)\,\mathrm{OPT}}$

– feasible ✓

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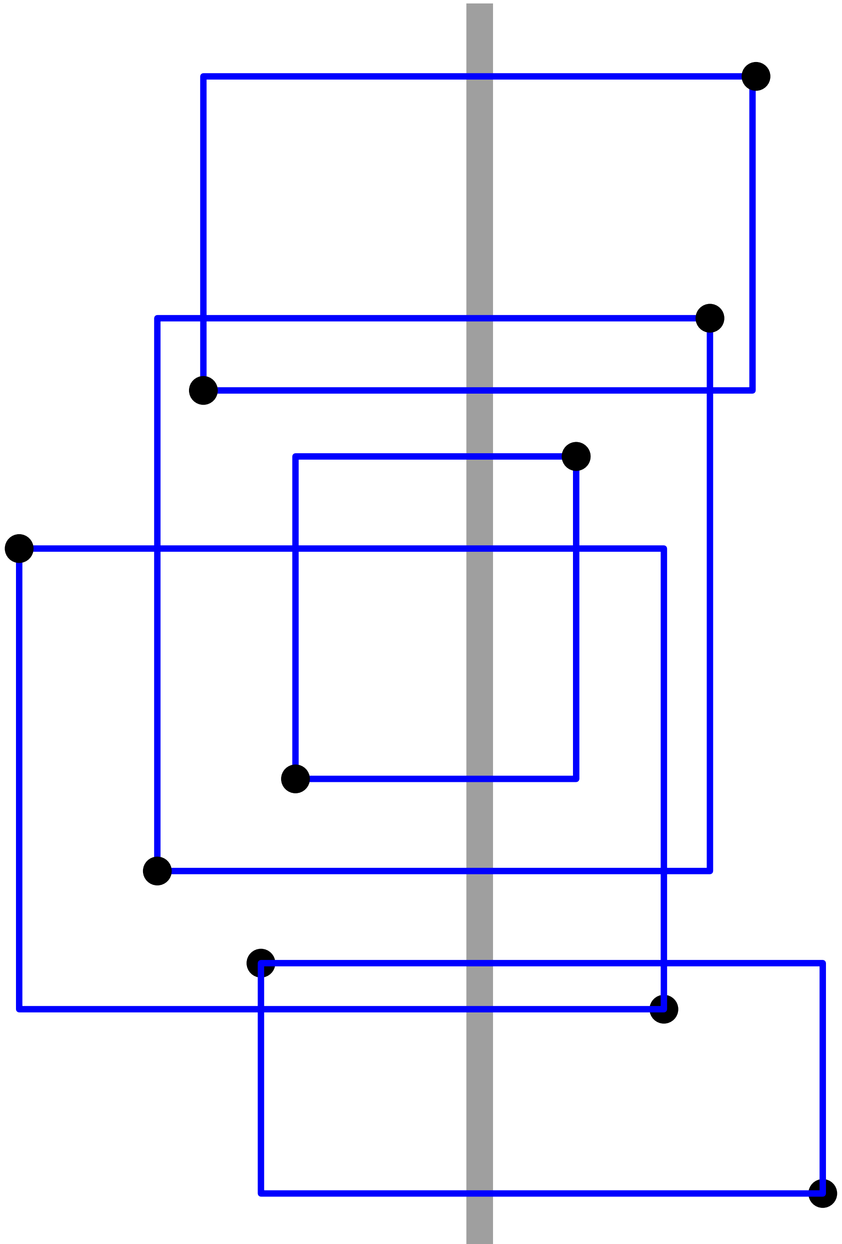
Piercing and Stabbing



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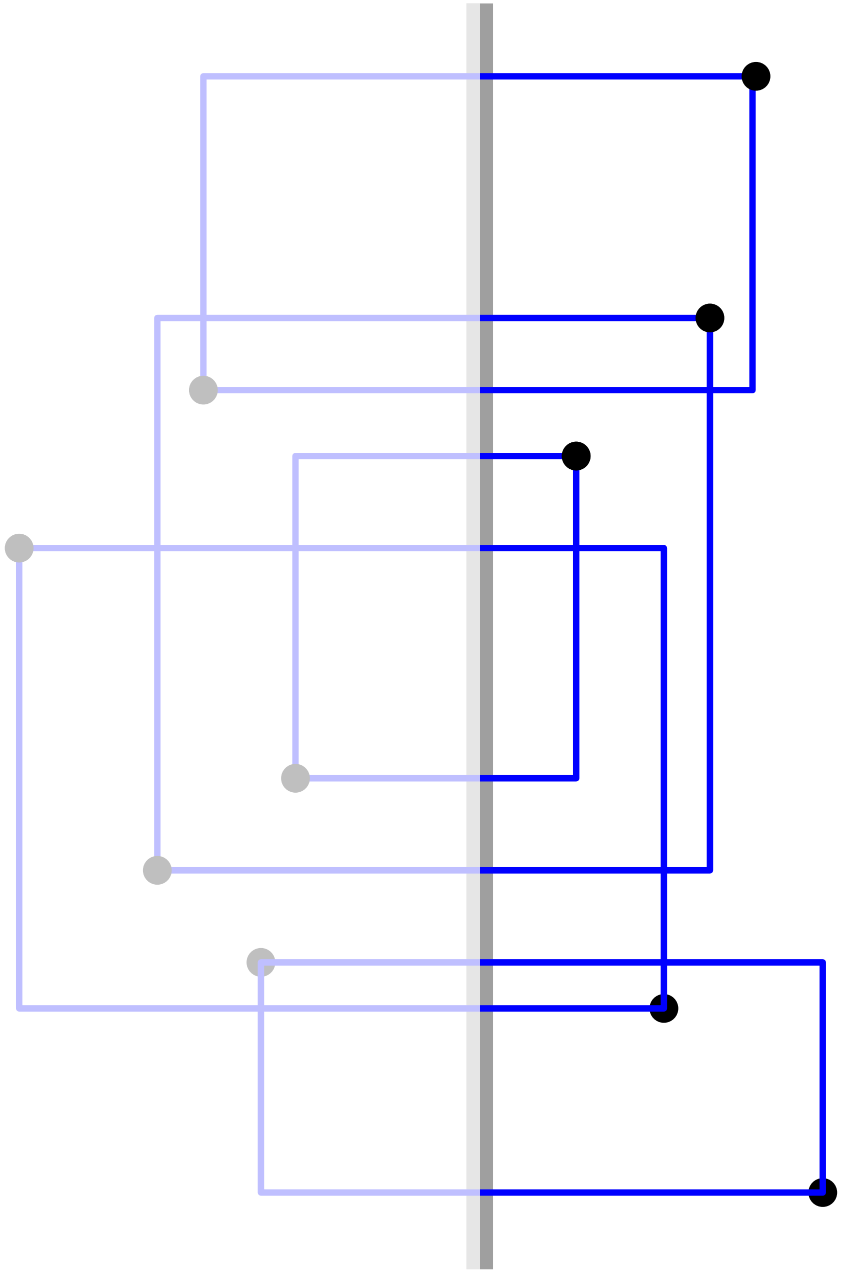

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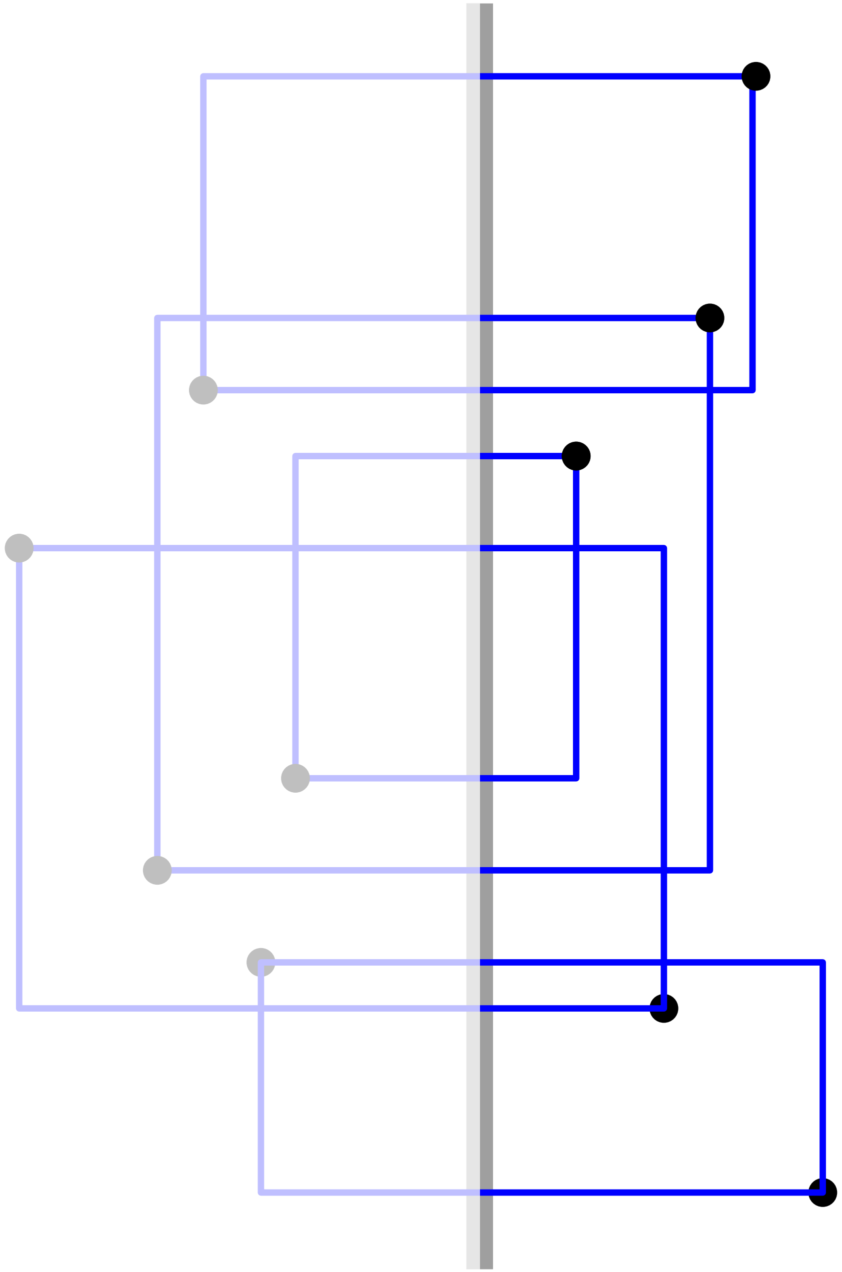
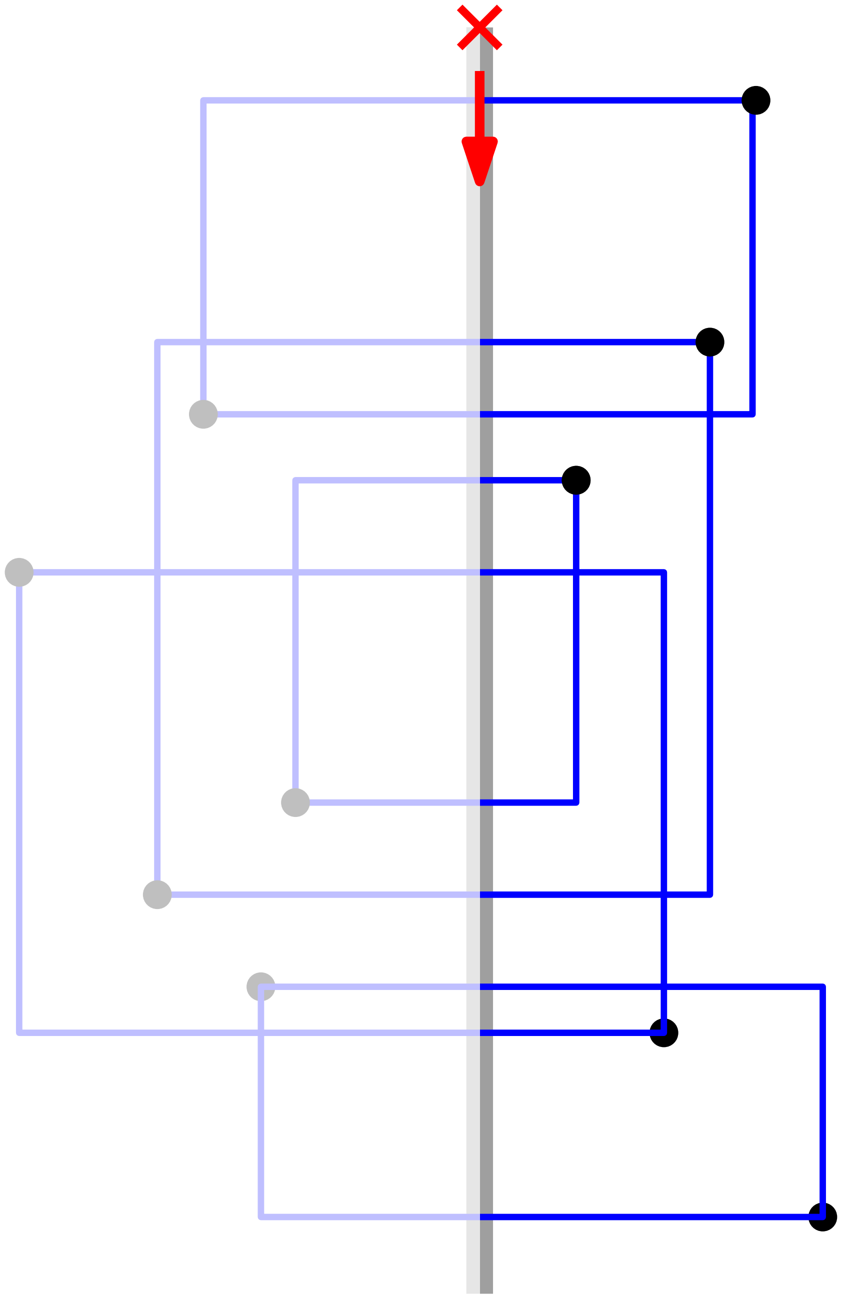


Compute and maintain inclusion-wise minimal piercing P_x for $x \geq 0$!

Piercing and Stabbing



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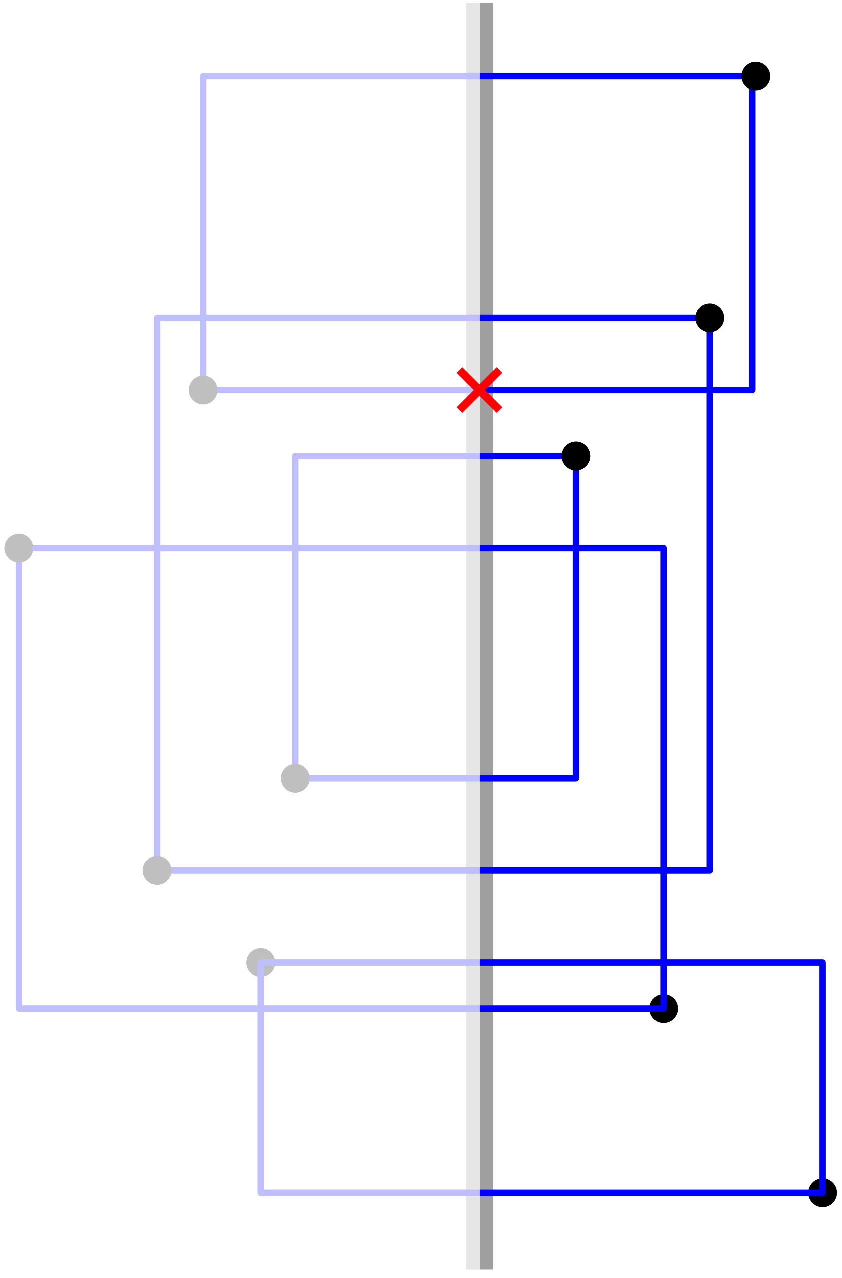

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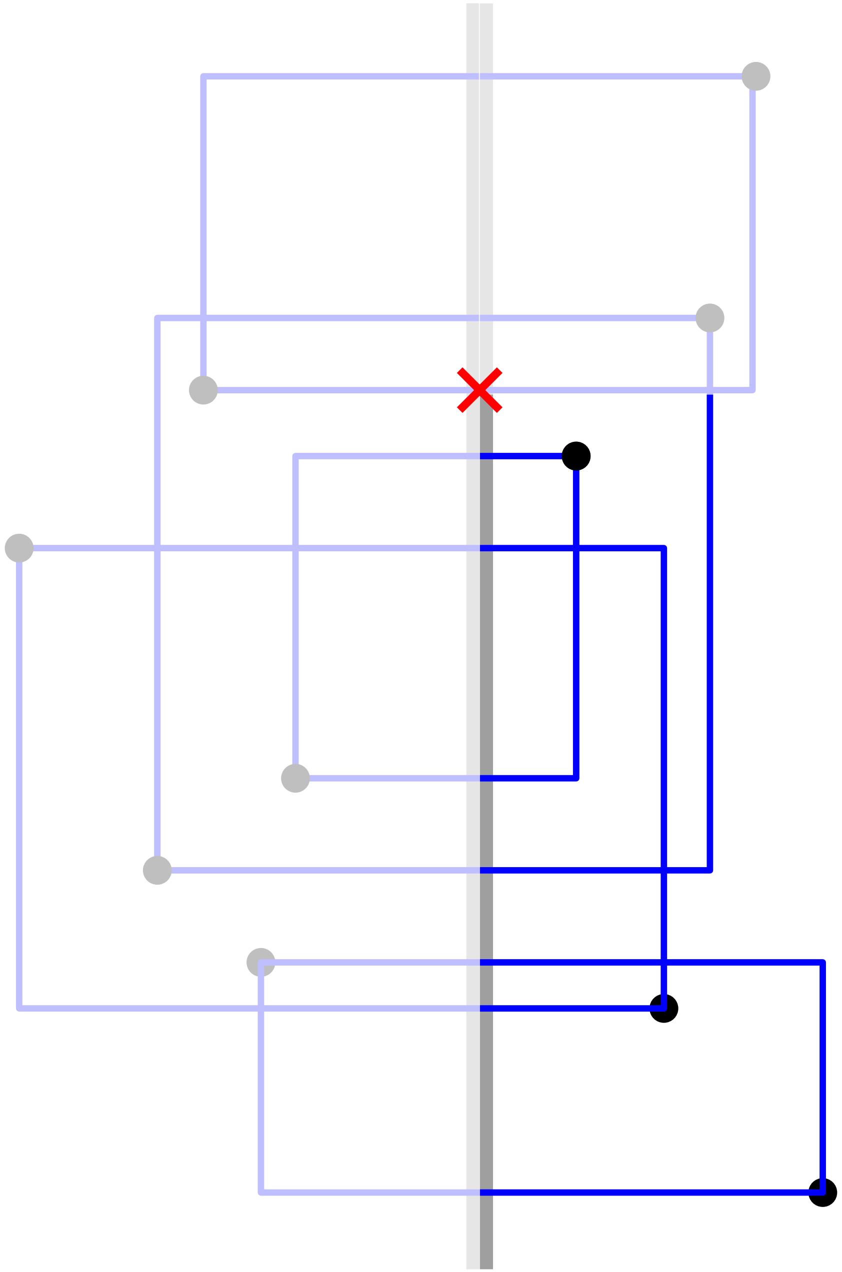

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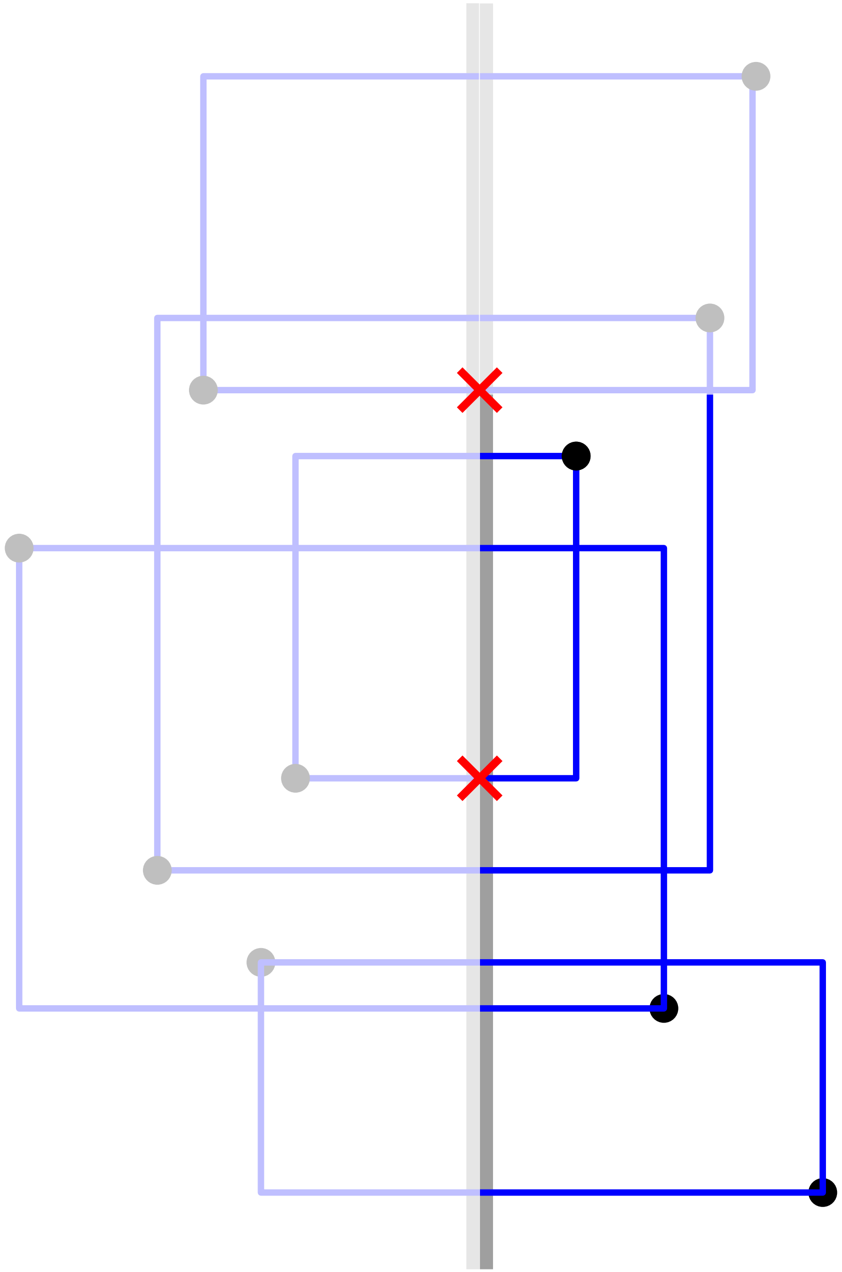

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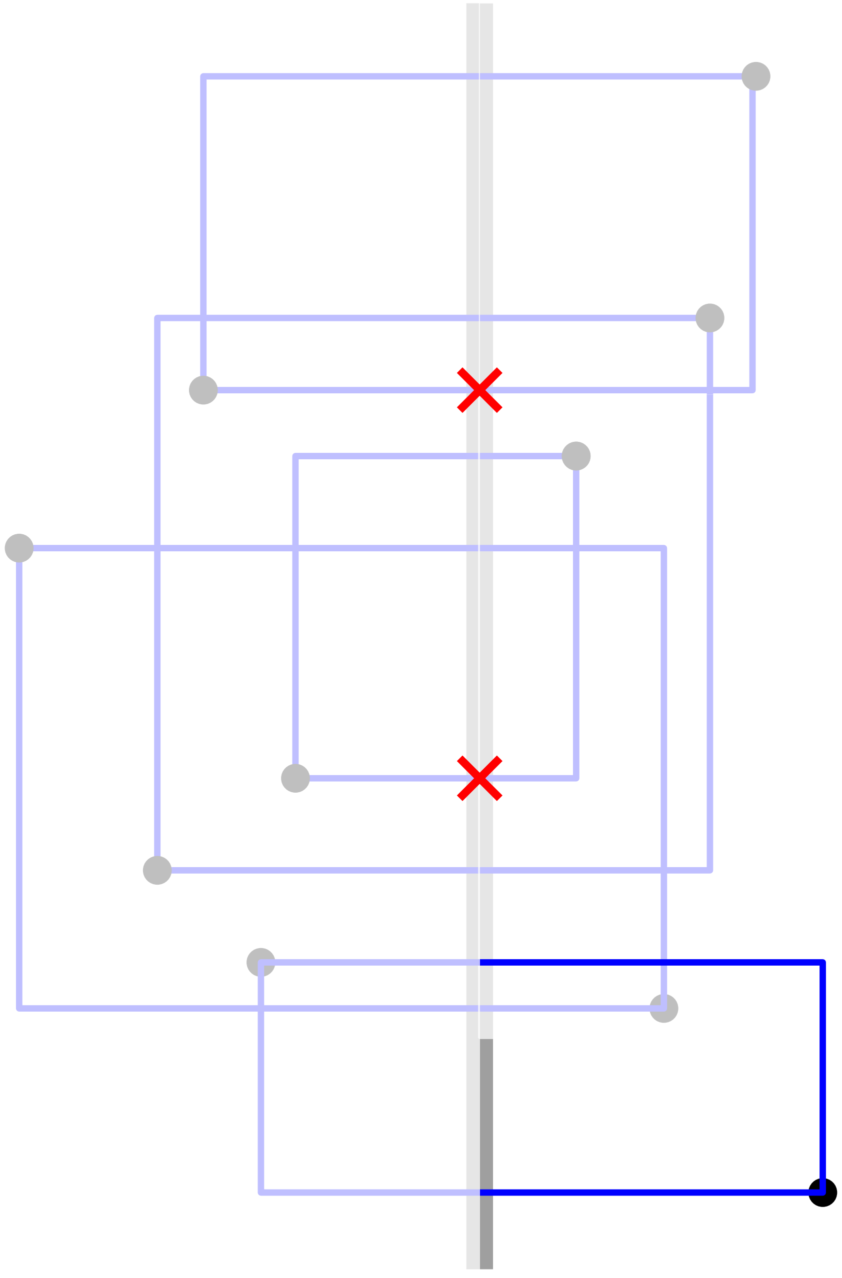

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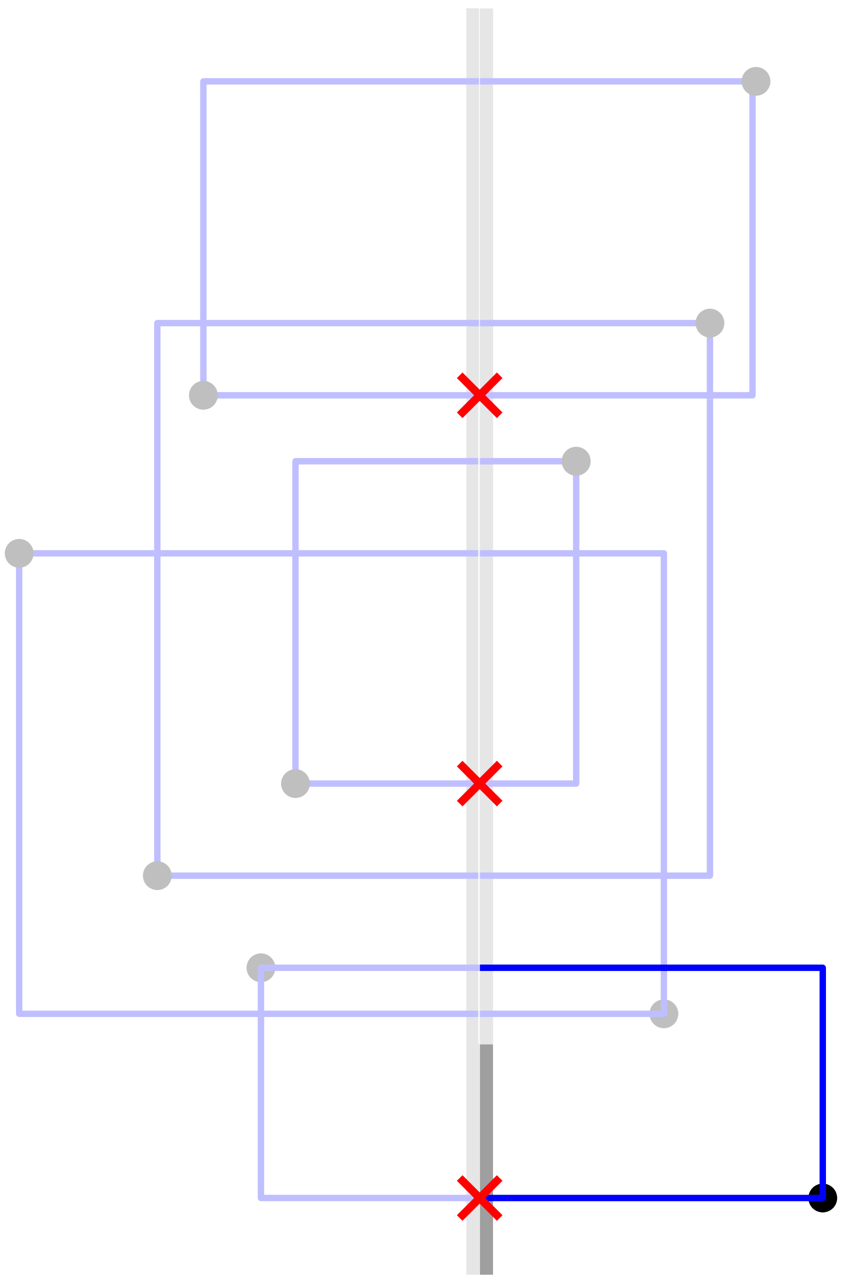

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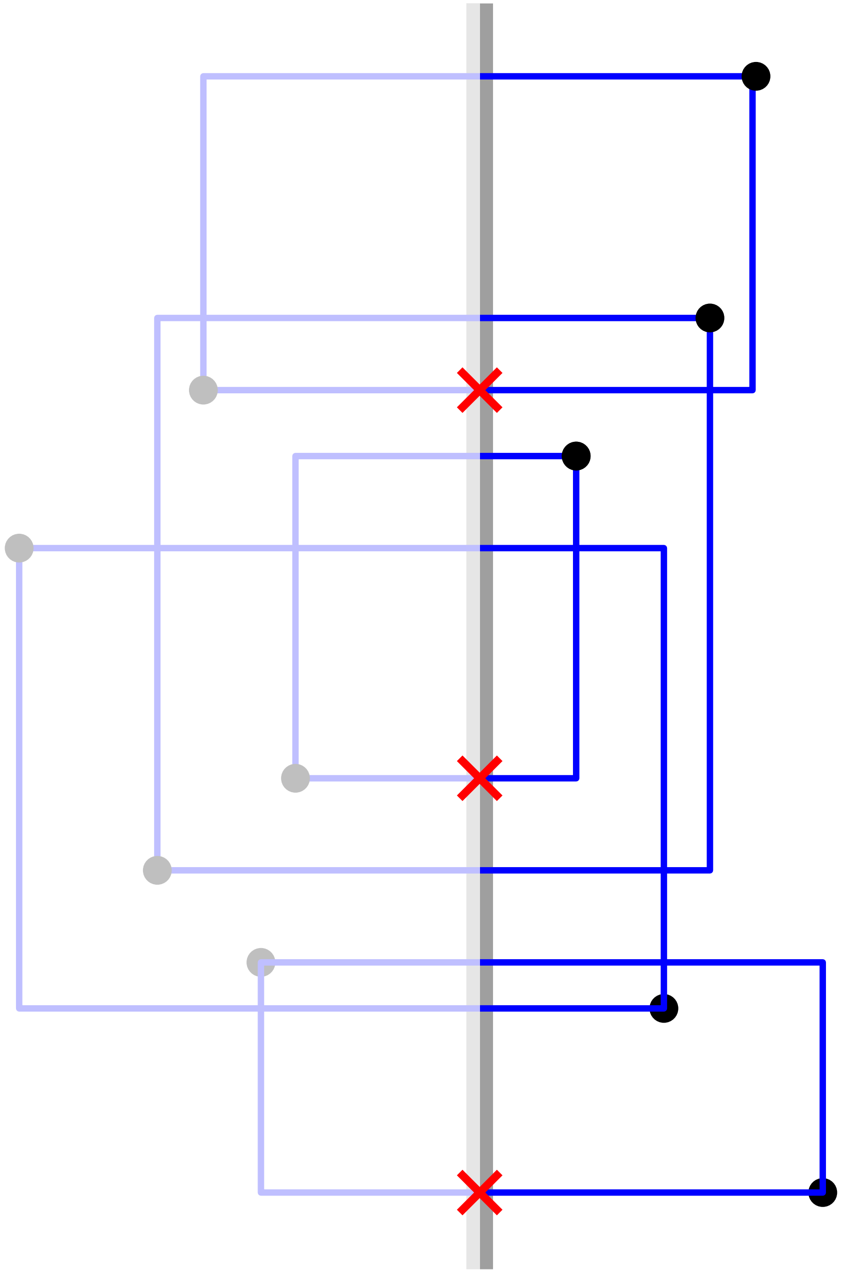

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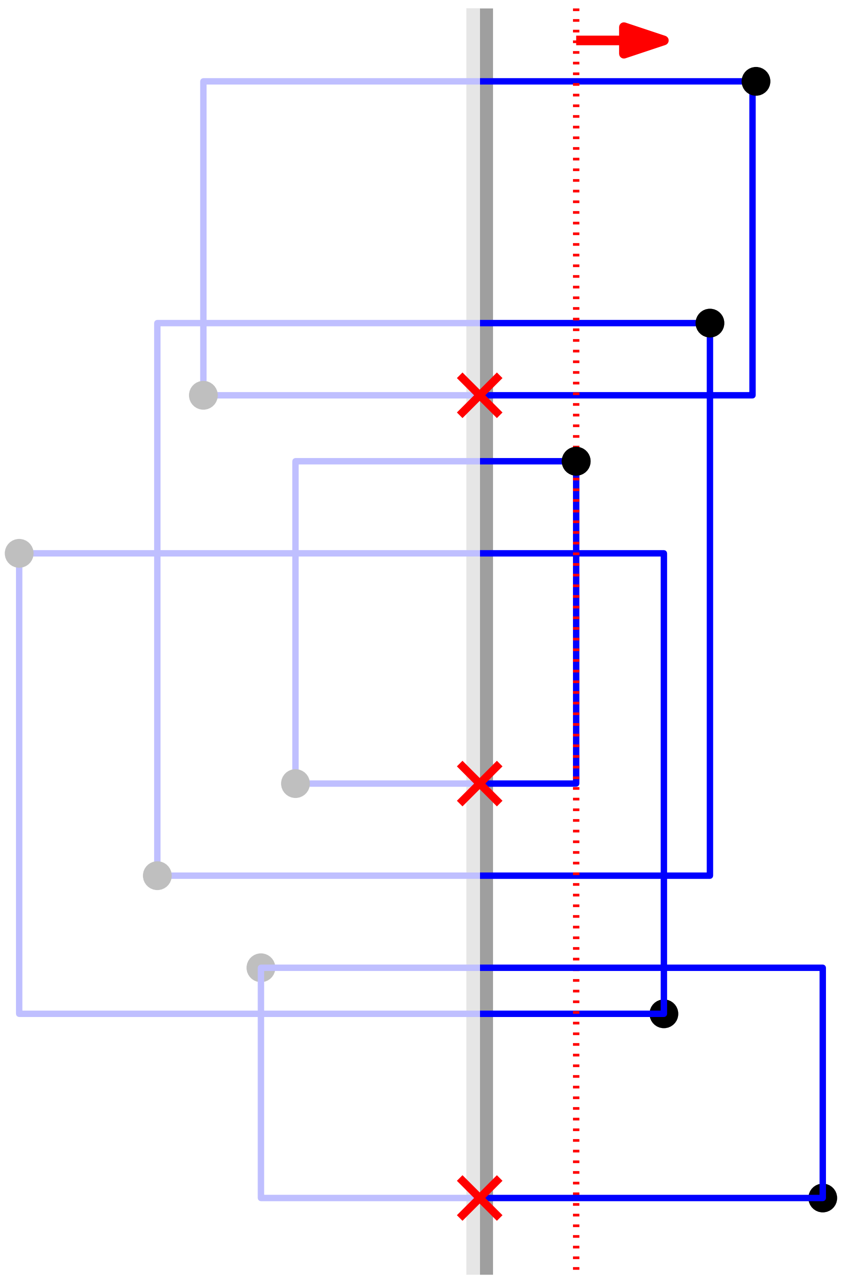

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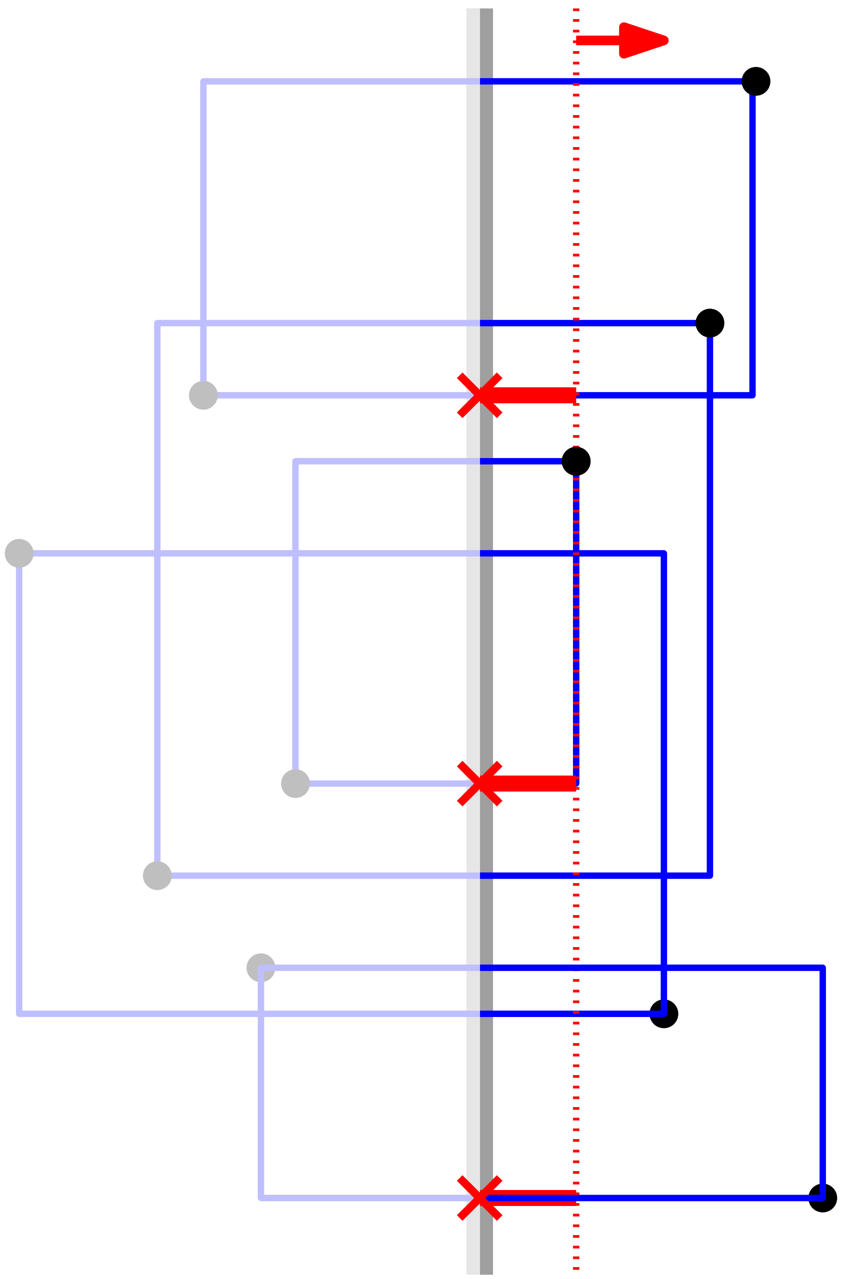

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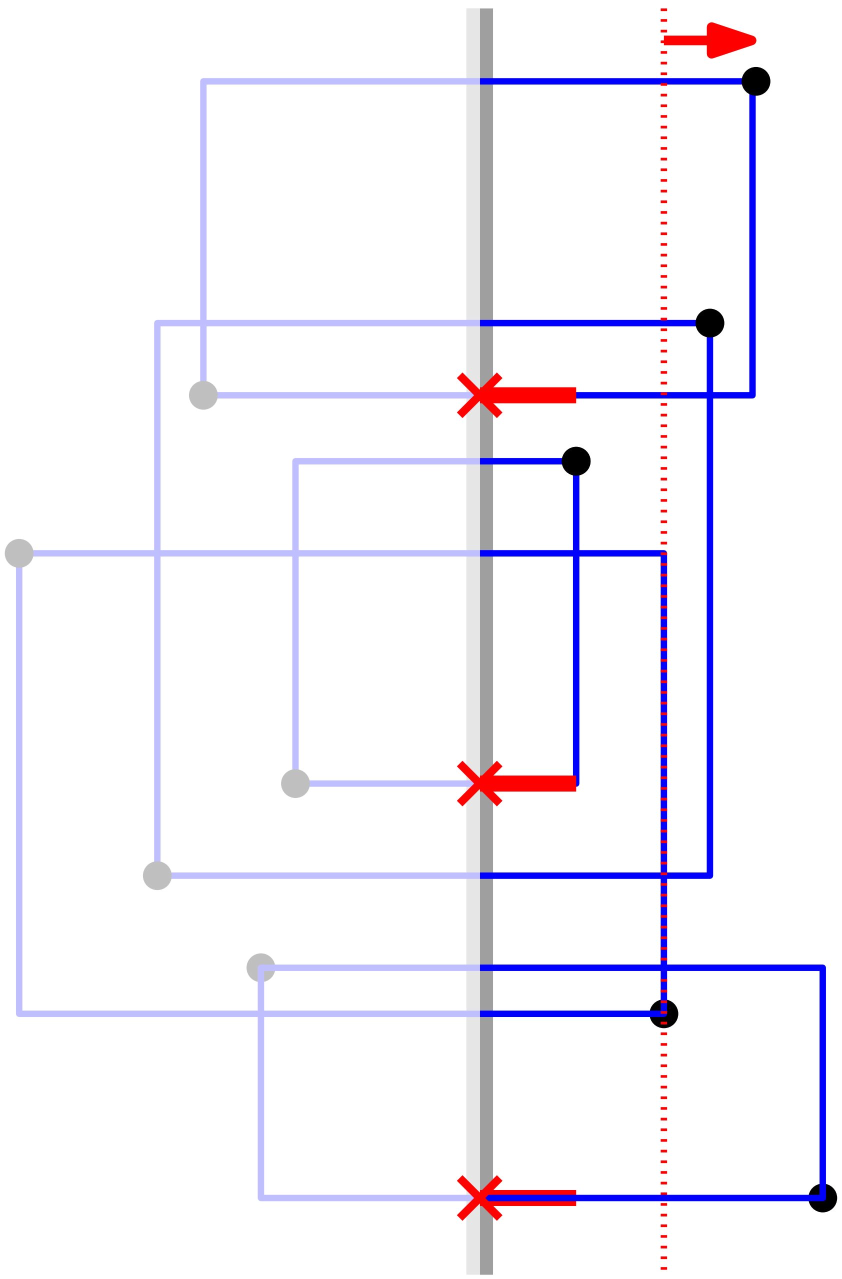

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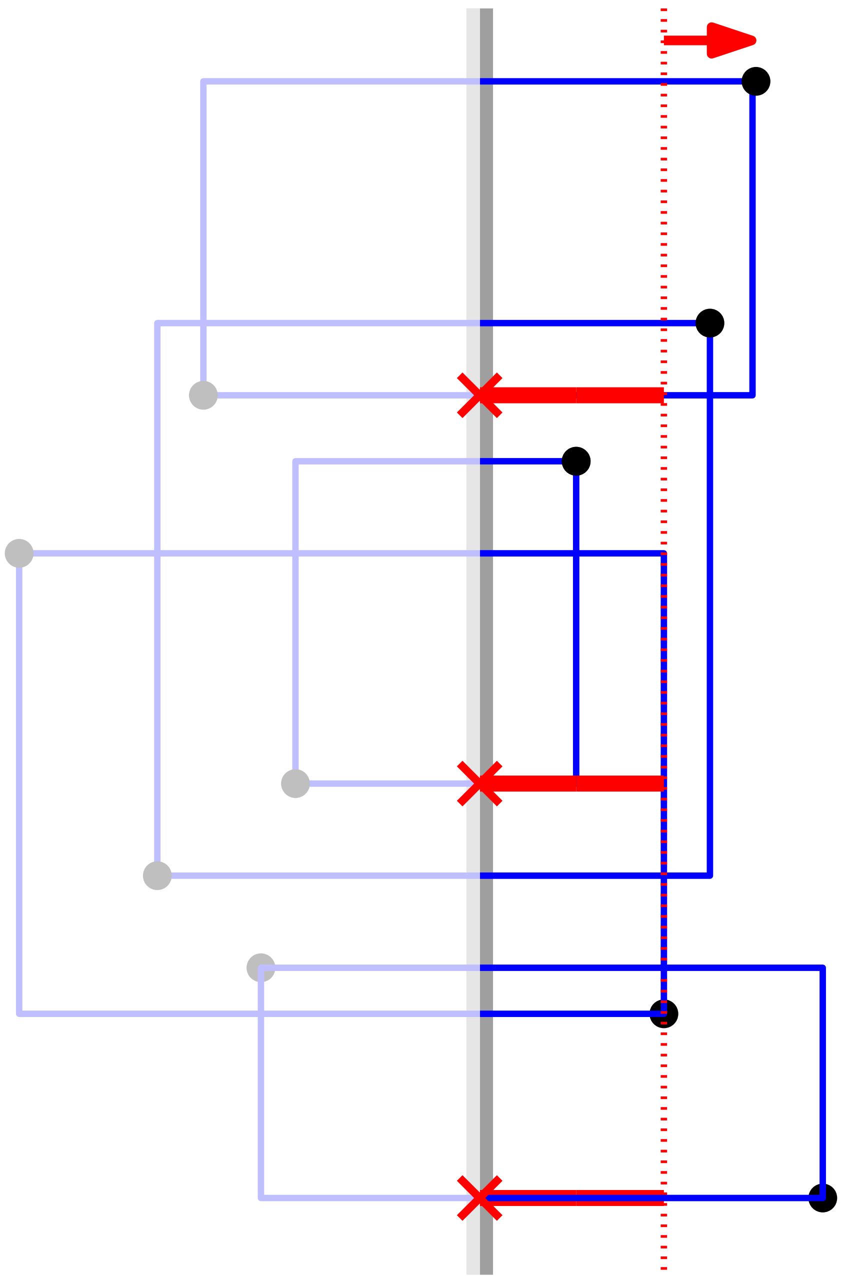

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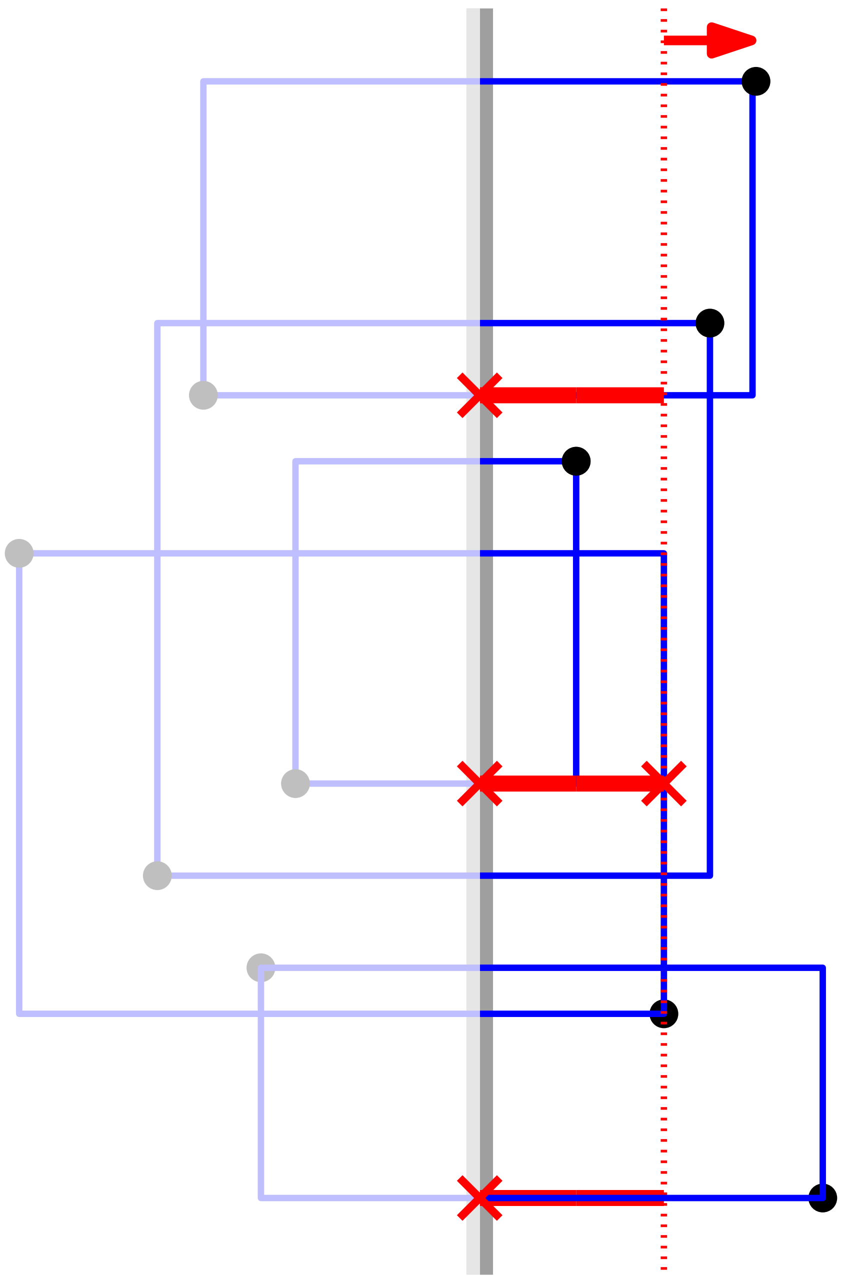

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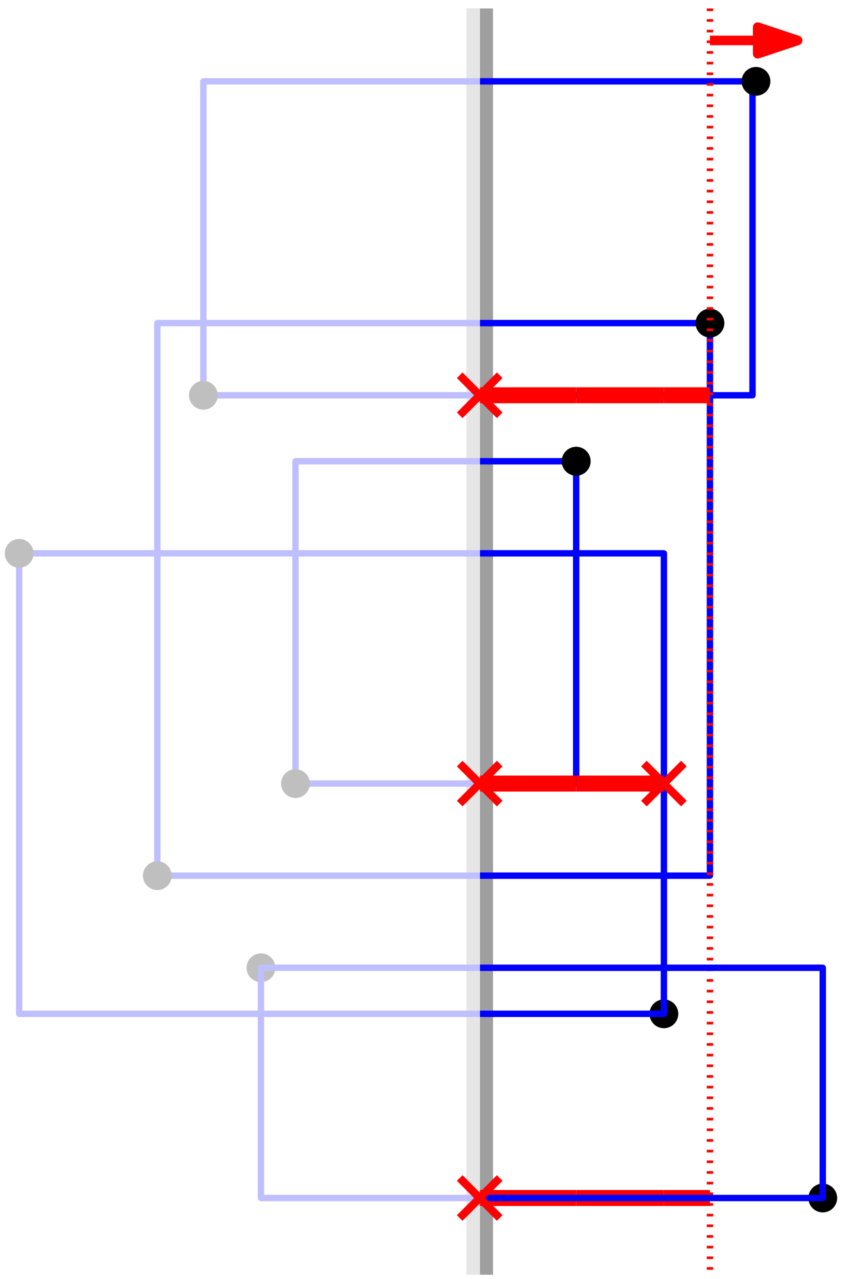

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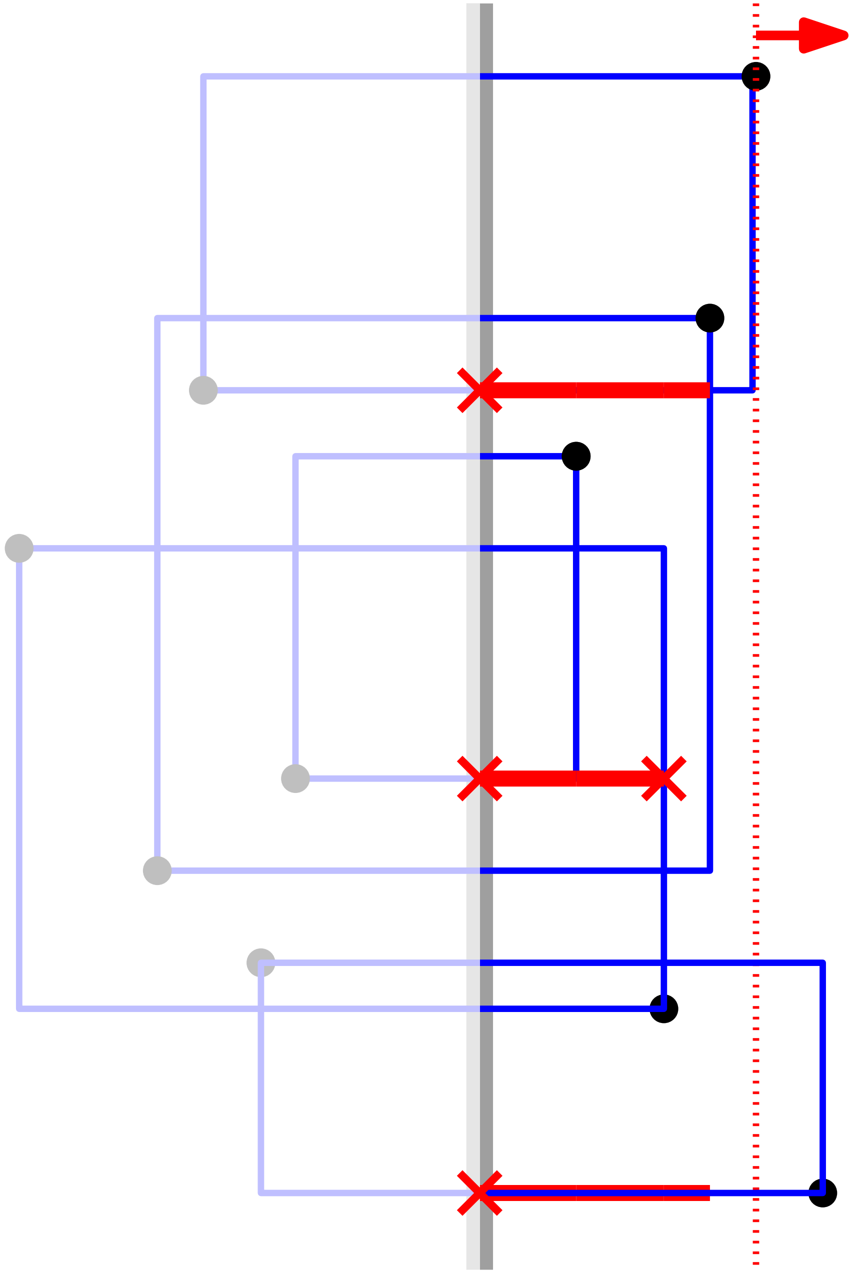

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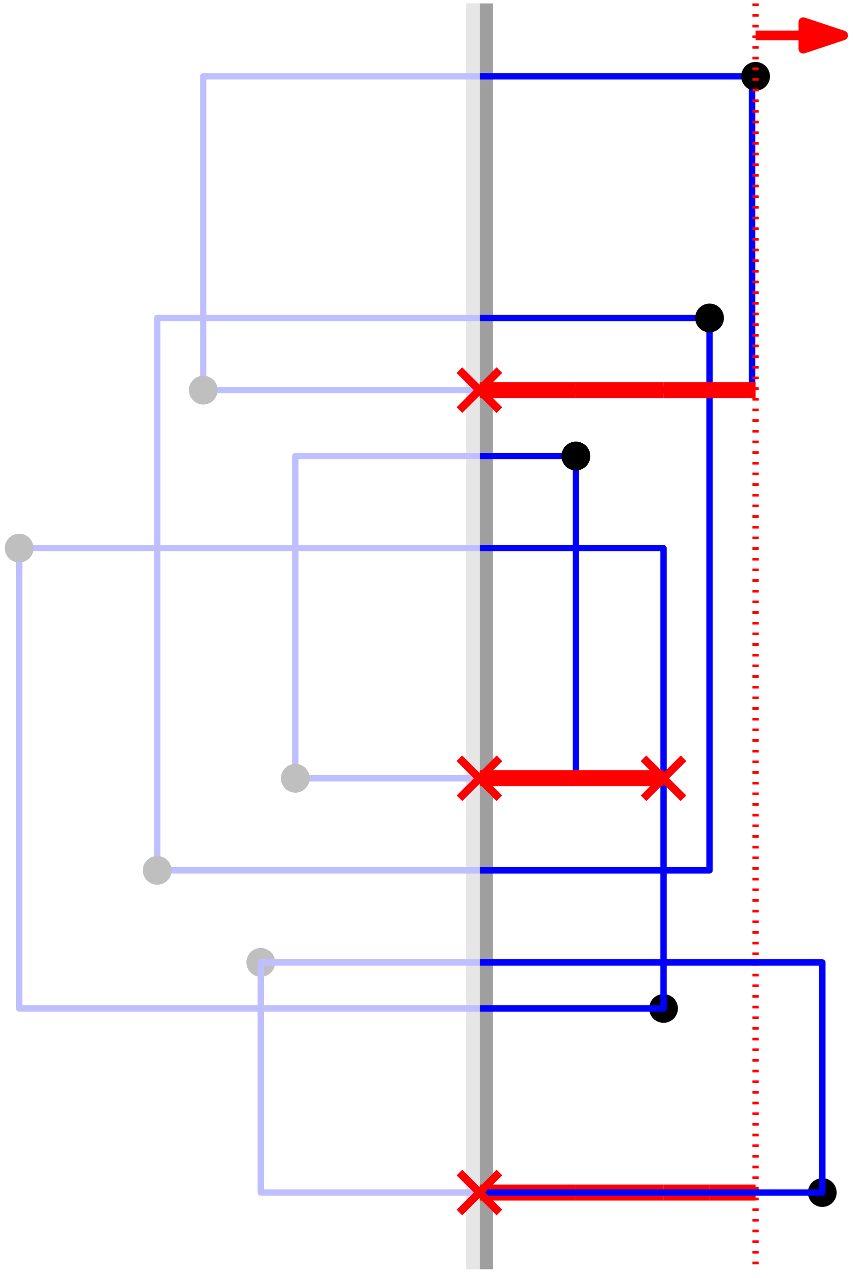

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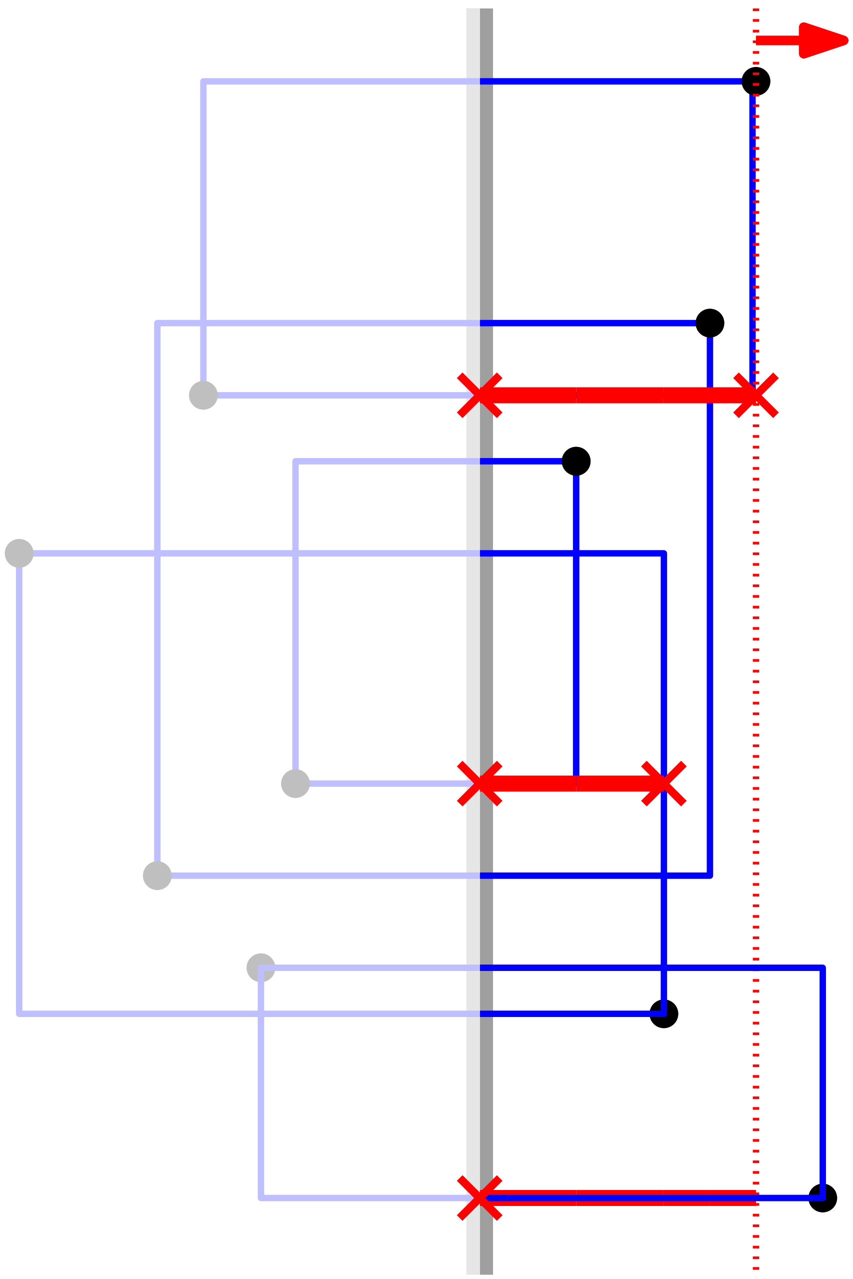

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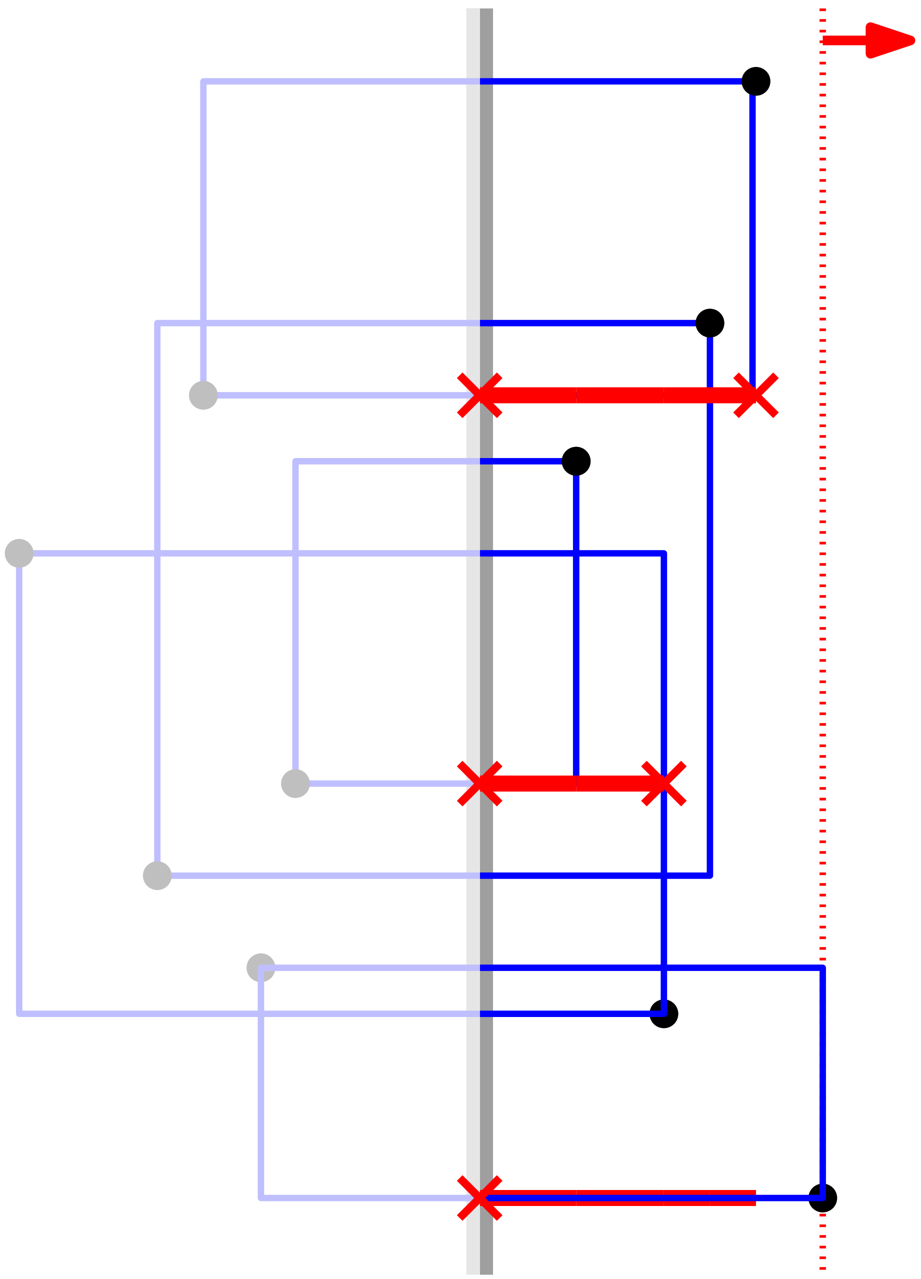

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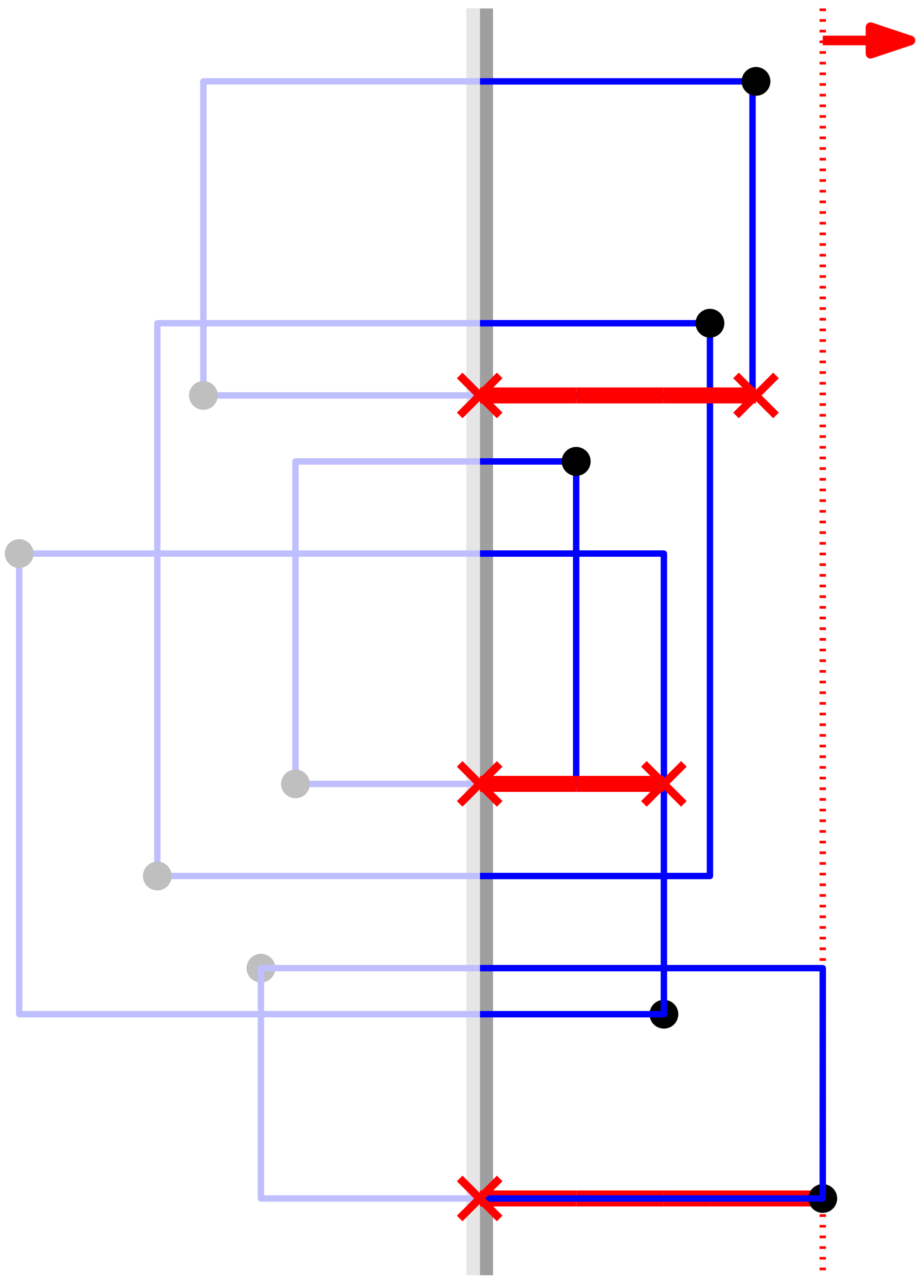

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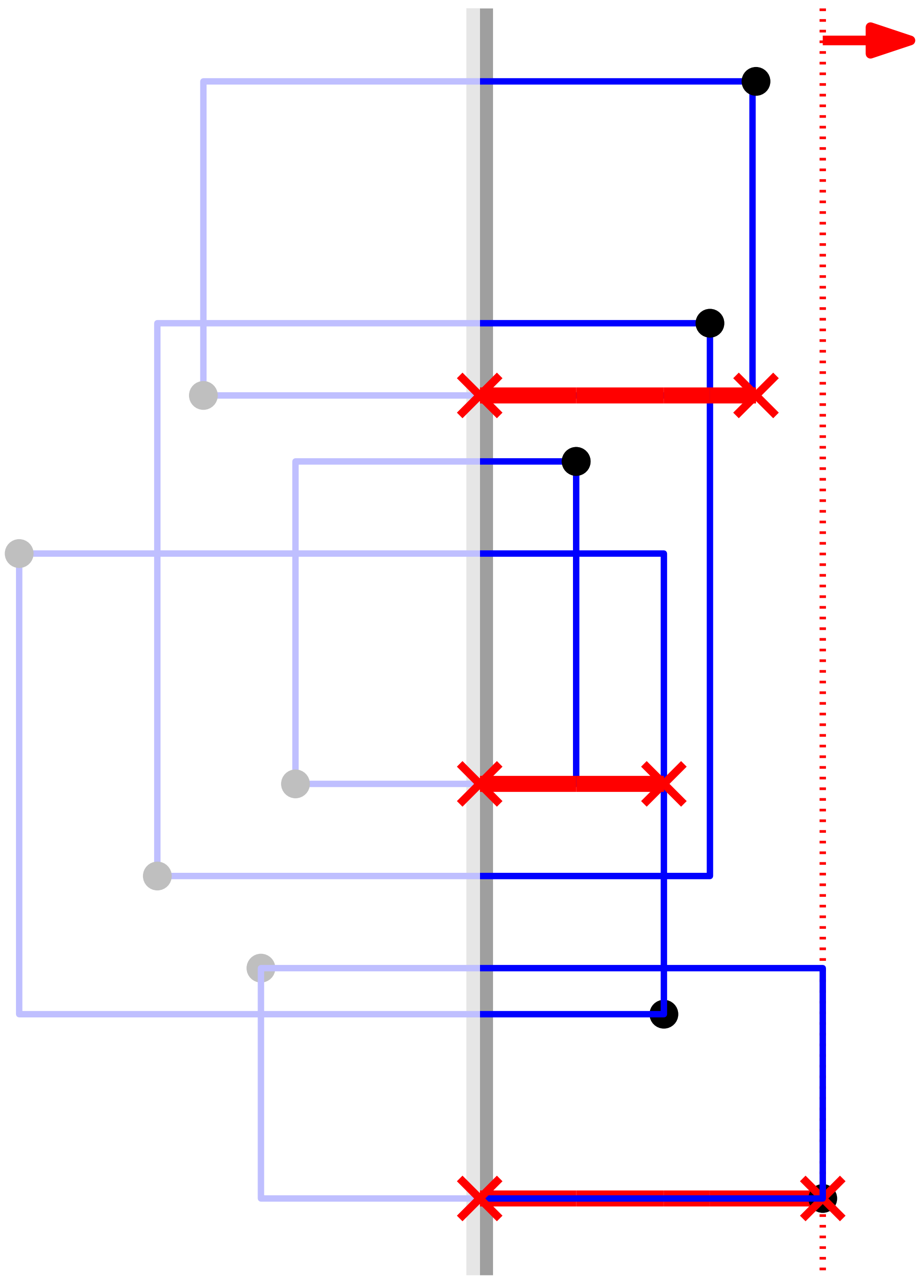

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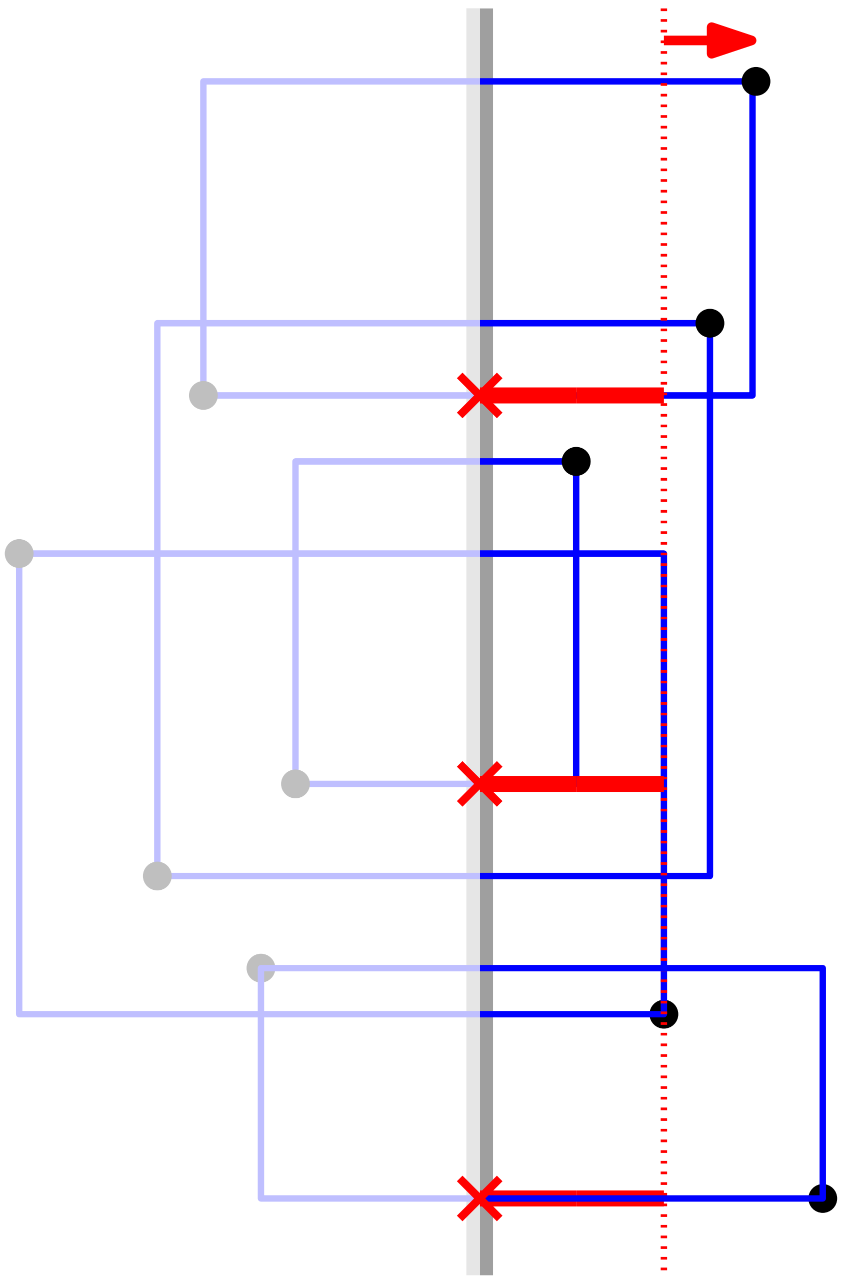

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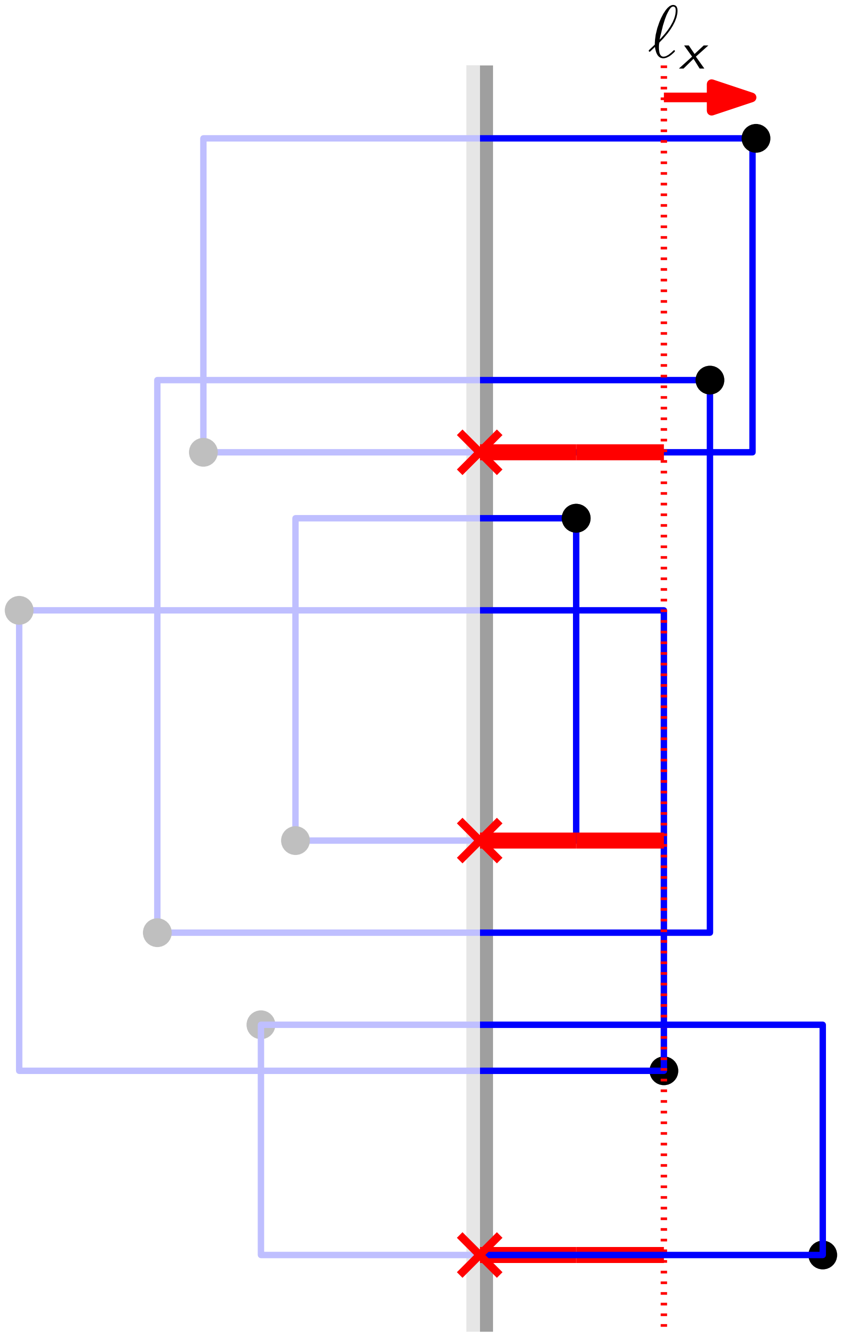
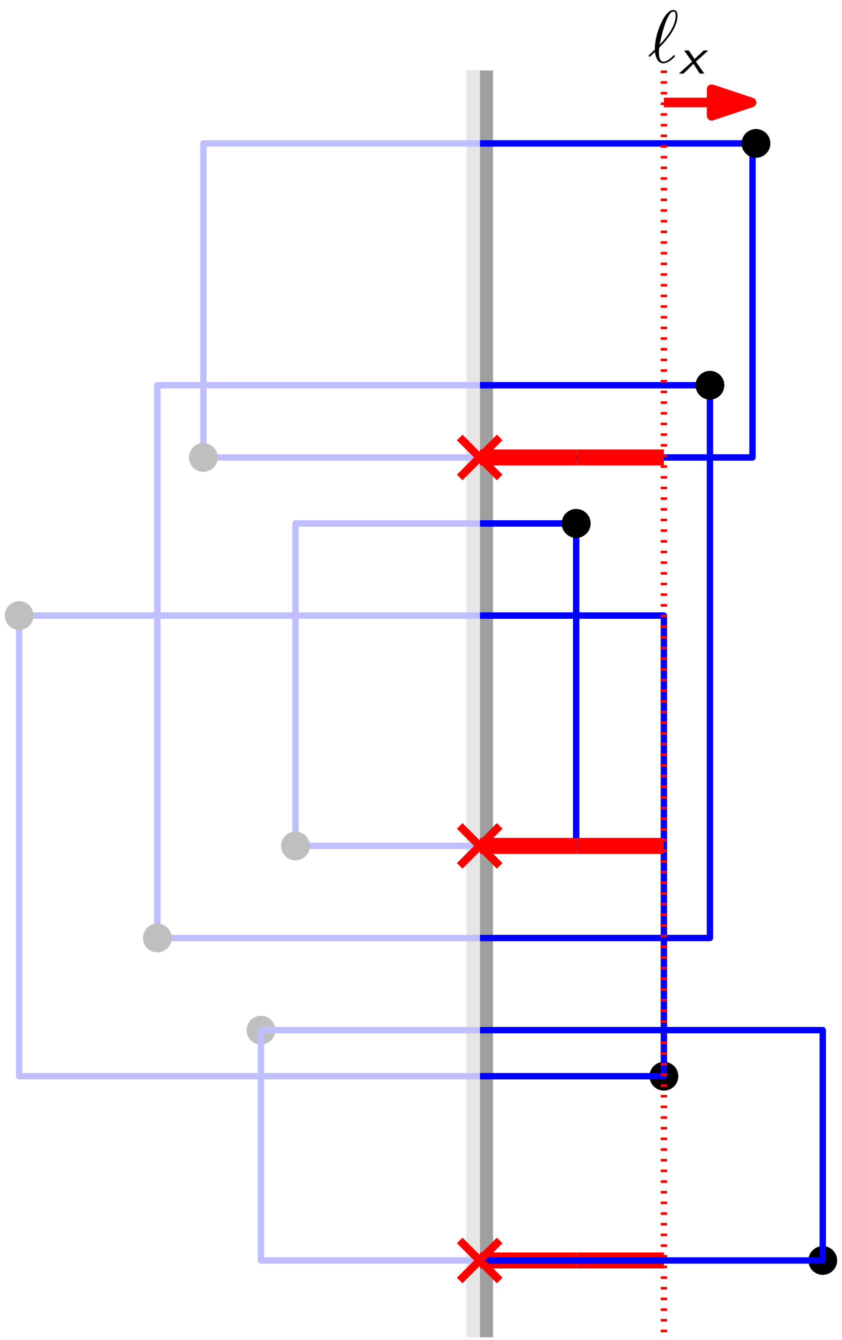


Piercing set

The right parts of the horizontal line segments in a fixed optimal solution.

Lemma$_1$.
For any $x \geq 0$, it holds that
$$|P_x| \leq 2 \cdot |\ell_x \cap N^+_{\text{hor}}|.$$

Piercing and Stabbing



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Lemma$_1$**.**
For any $x \geq 0$, it holds that
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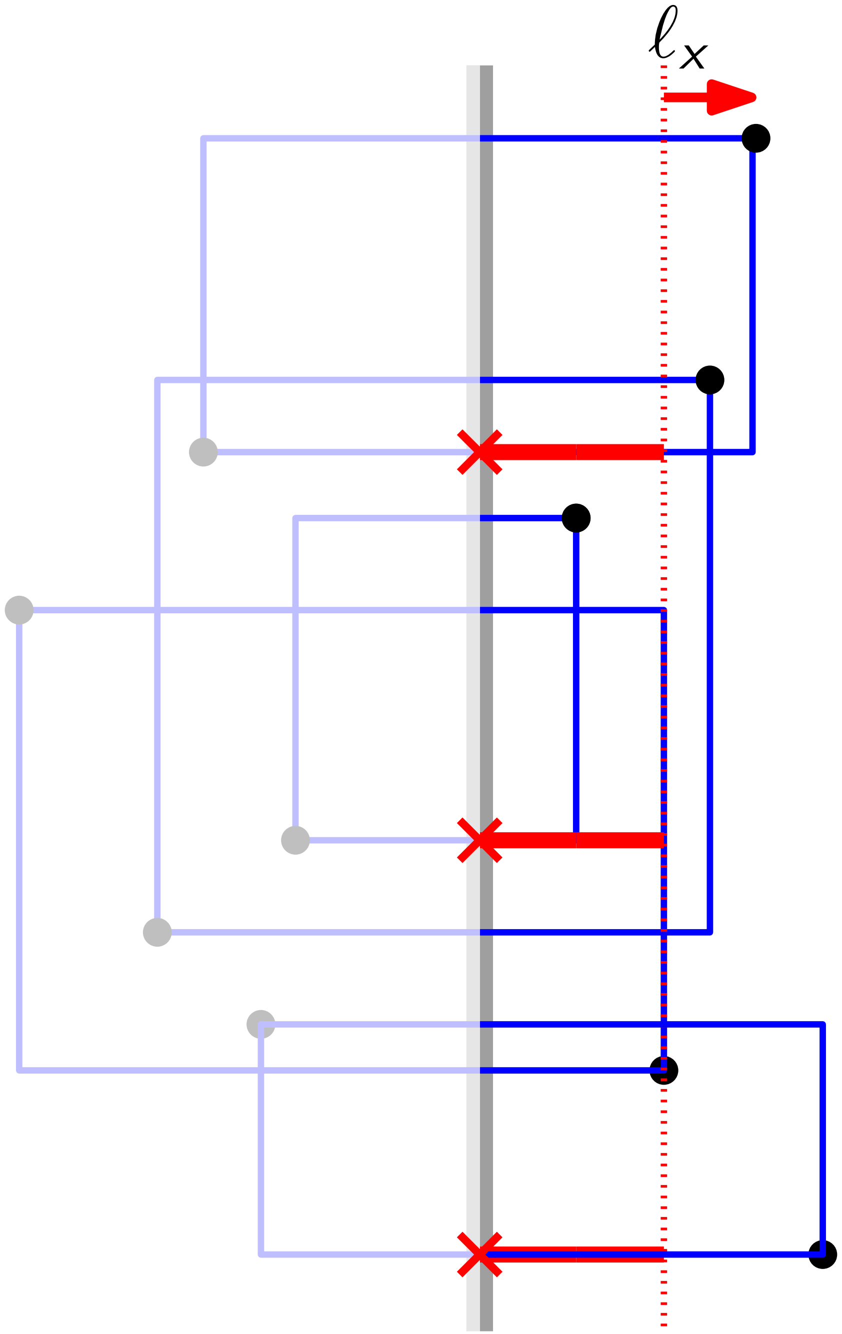
Proof.

Piercing and Stabbing



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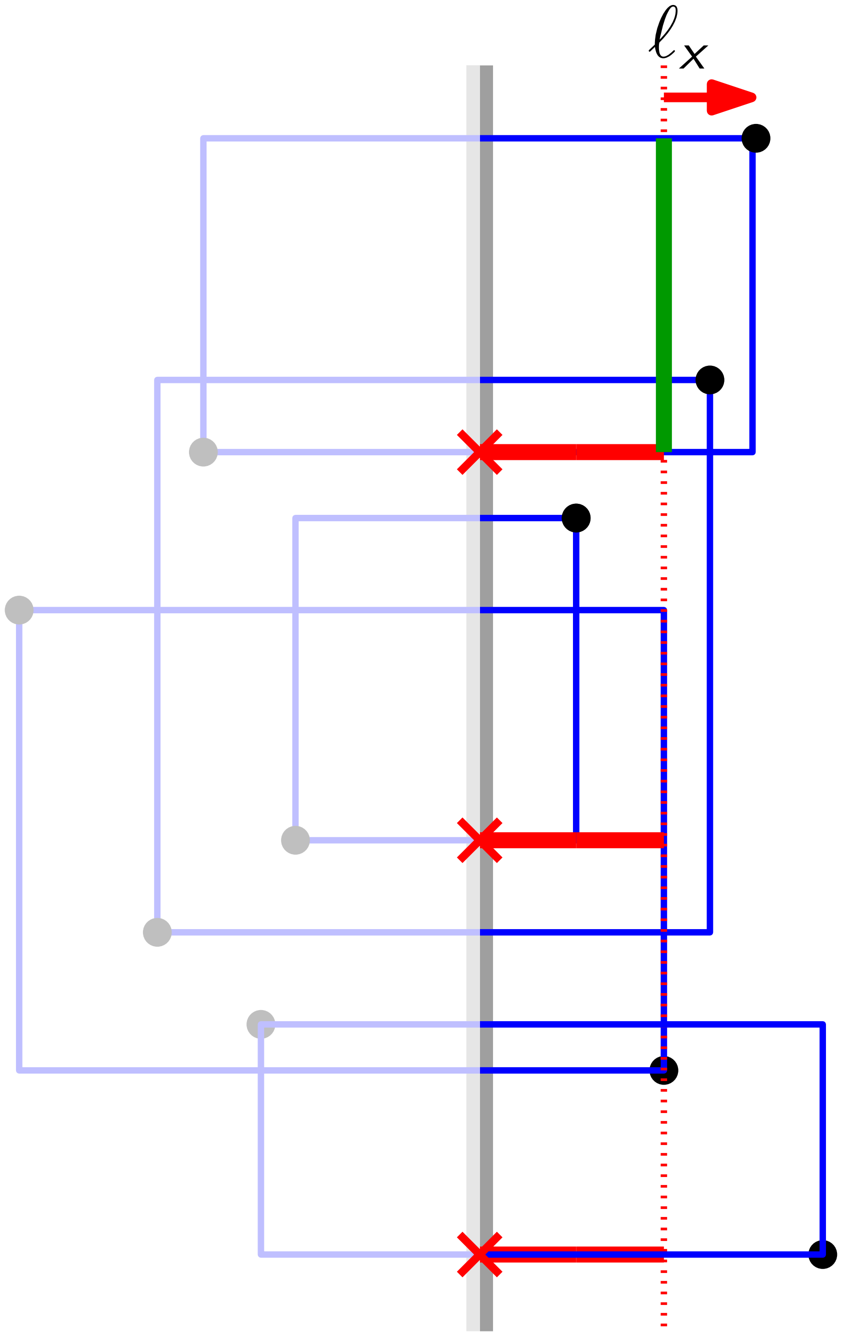
$\{r \cap \ell_x \mid r \in R^+\}$

Proof.
For every $p \in P_x$, let $I_p \in \mathcal{I}_x$ be a *witness* if it is pierced by p but not by $P_x \setminus \{p\}$.

Piercing and Stabbing



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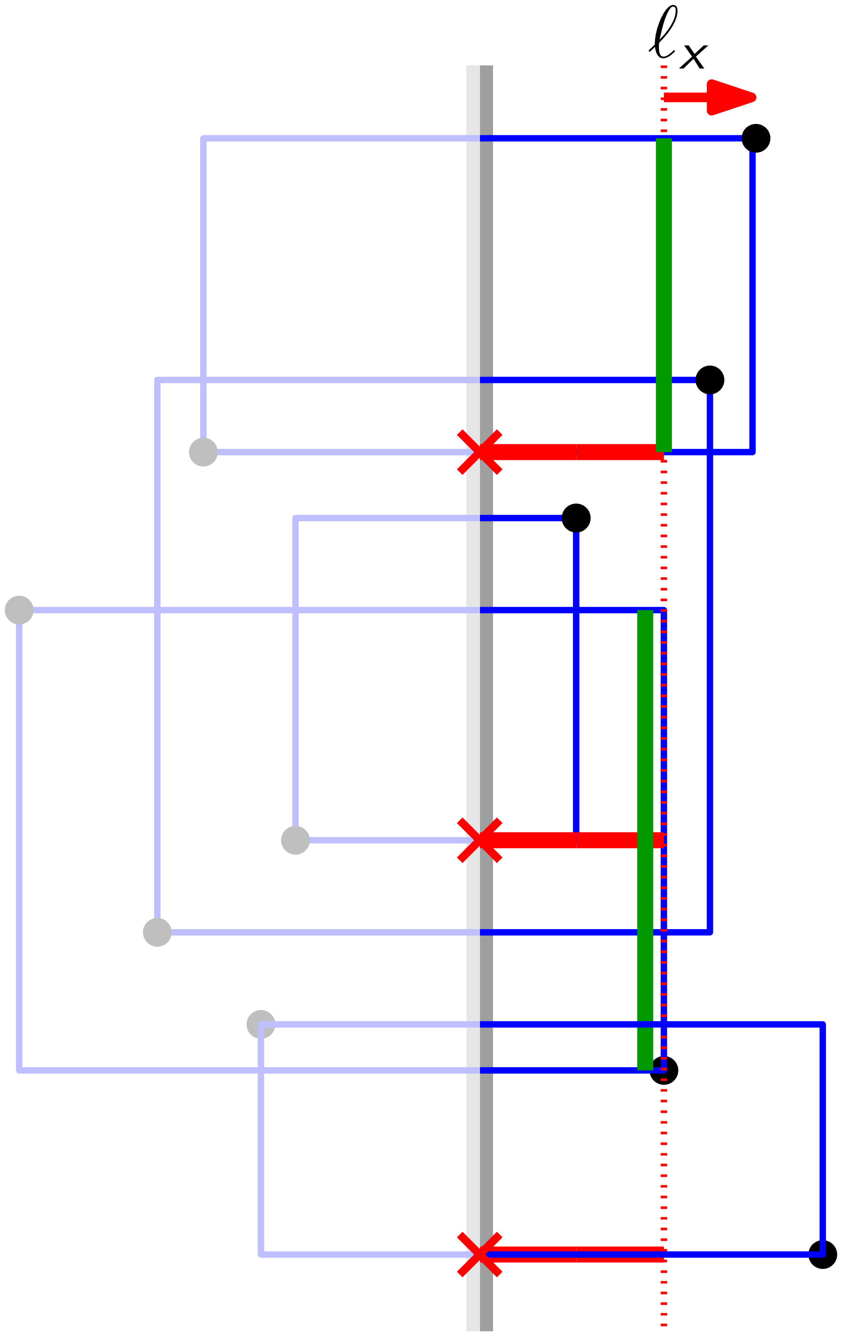

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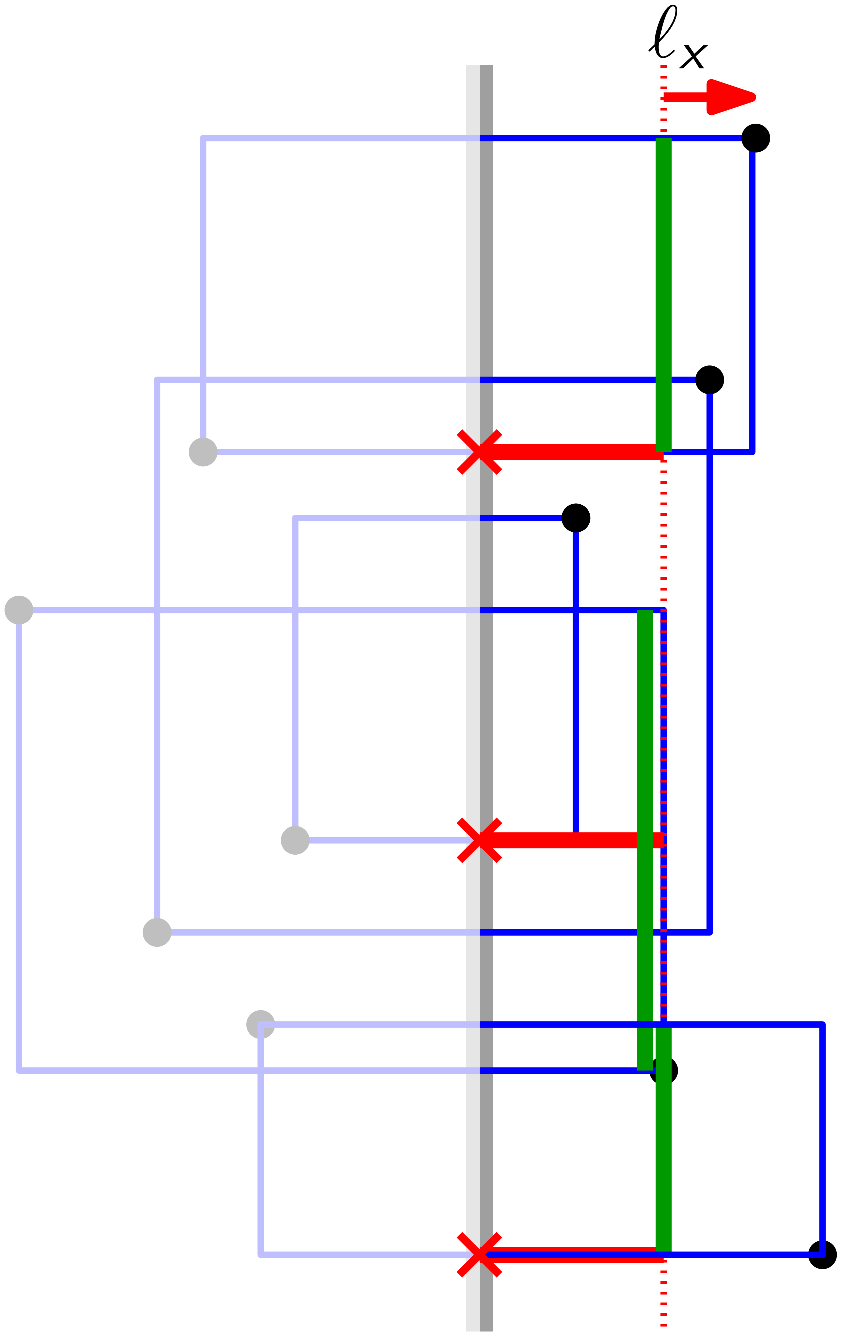

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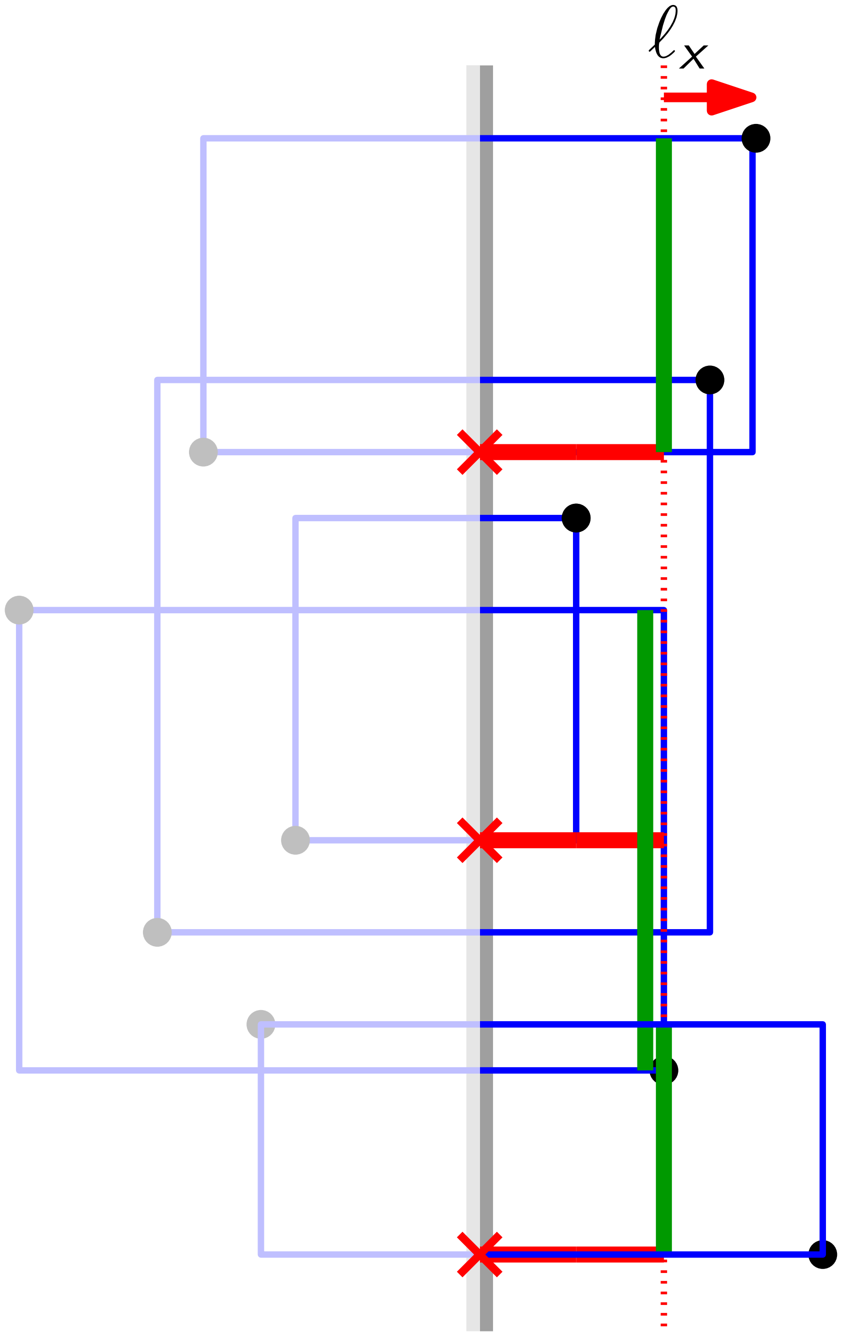
Piercing and Stabbing



Piercing set

The right parts of the horizontal line segments in a fixed optimal solution.

Lemma$_1$.
For any $x \geq 0$, it holds that
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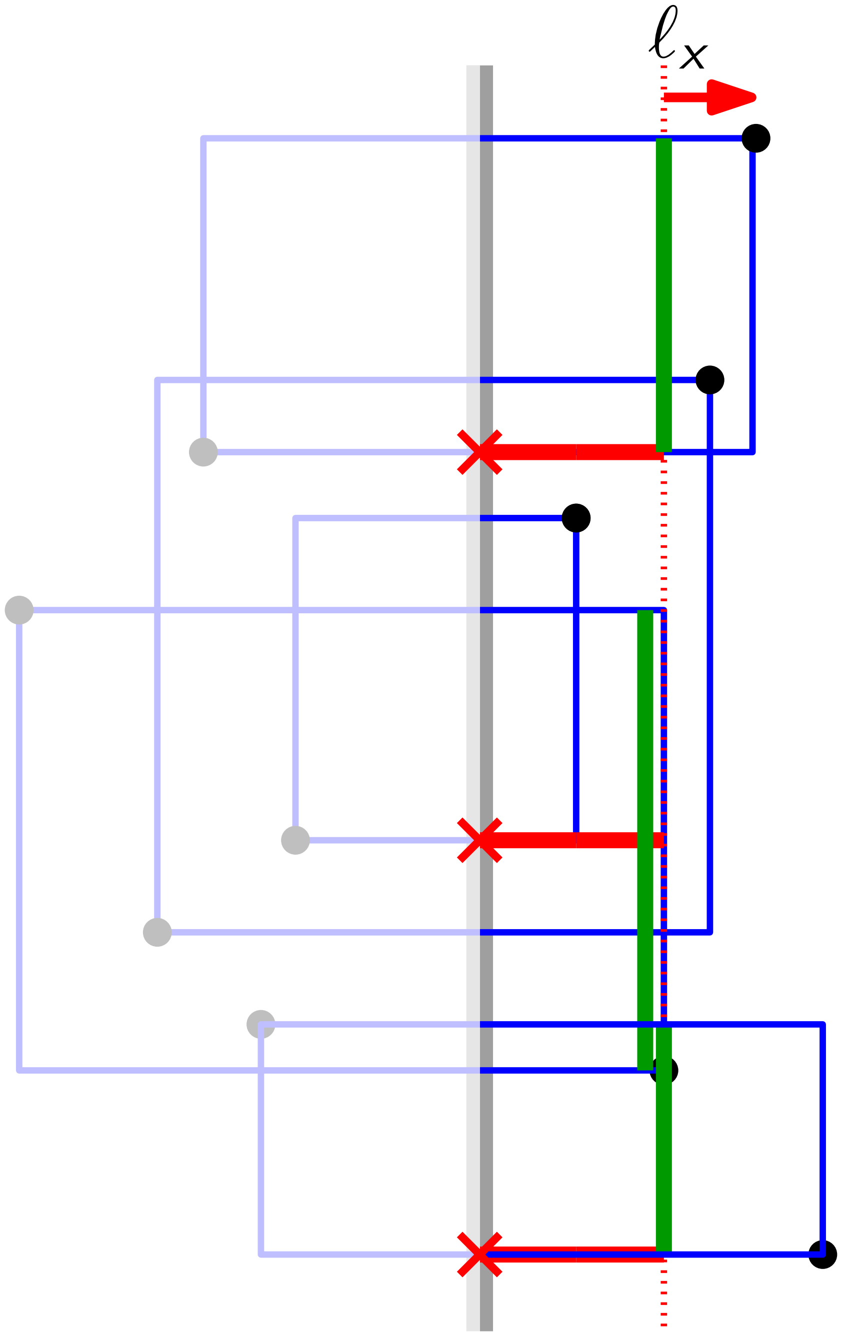
$\{r \cap \ell_x \mid r \in R^+\}$

Proof.
For every $p \in P_x$, let $I_p \in \mathcal{I}_x$ be a *witness* if it is pierced by p but not by $P_x \setminus \{p\}$.

- Any point q on ℓ_x pierces ≤ 2 witnesses

Piercing and Stabbing



Piercing set

The right parts of the horizontal line segments in a fixed optimal solution.

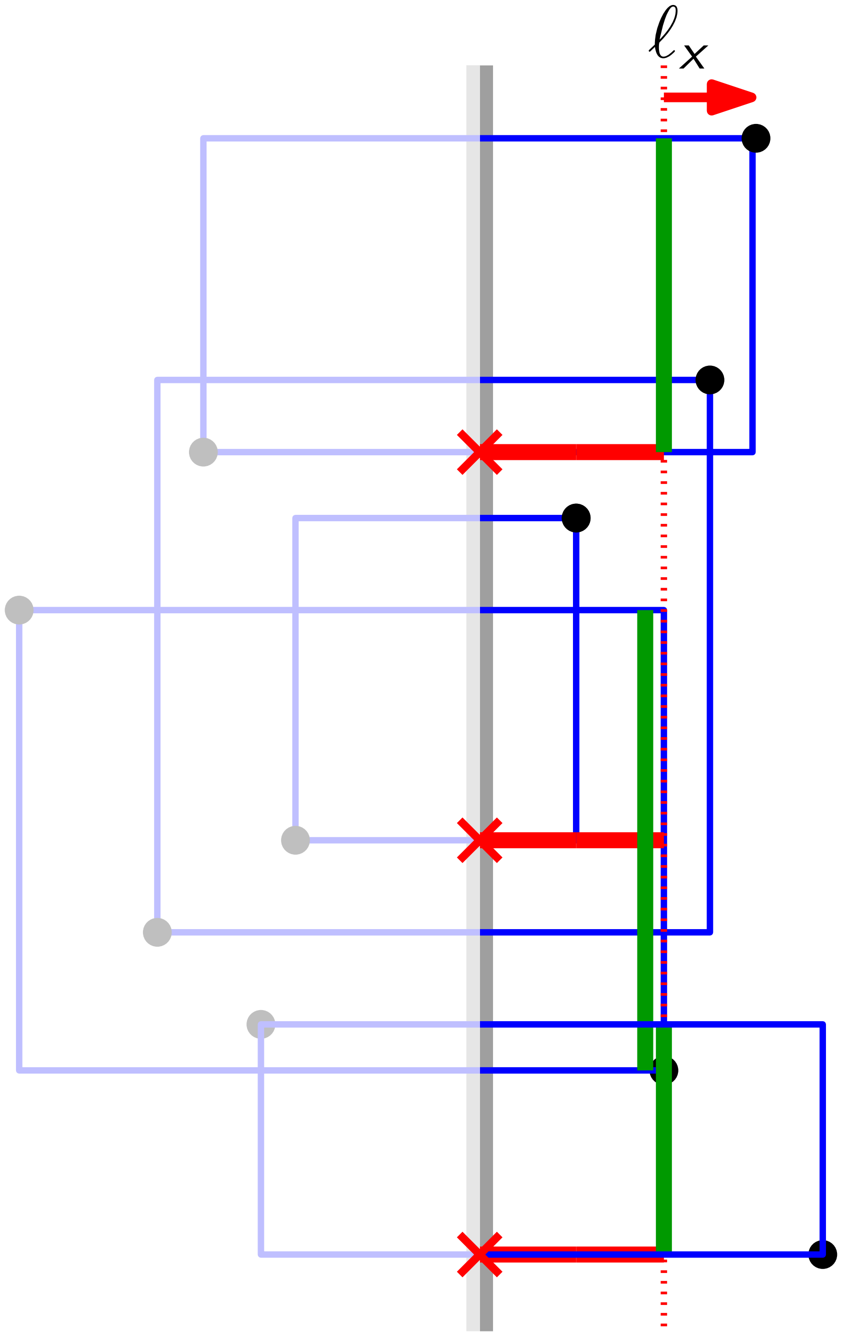
Lemma₁.
For any $x \geq 0$, it holds that
$$|P_x| \leq 2 \cdot |\ell_x \cap N^+_{hor}|.$$

$\{r \cap \ell_x \mid r \in R^+\}$

Proof.
For every $p \in P_x$, let $I_p \in \mathcal{I}_x$ be a *witness* if it is pierced by p but not by $P_x \setminus \{p\}$.

- Any point q on ℓ_x pierces ≤ 2 witnesses (by contradiction).

Piercing and Stabbing



Piercing set

The right parts of the horizontal line segments in a fixed optimal solution.

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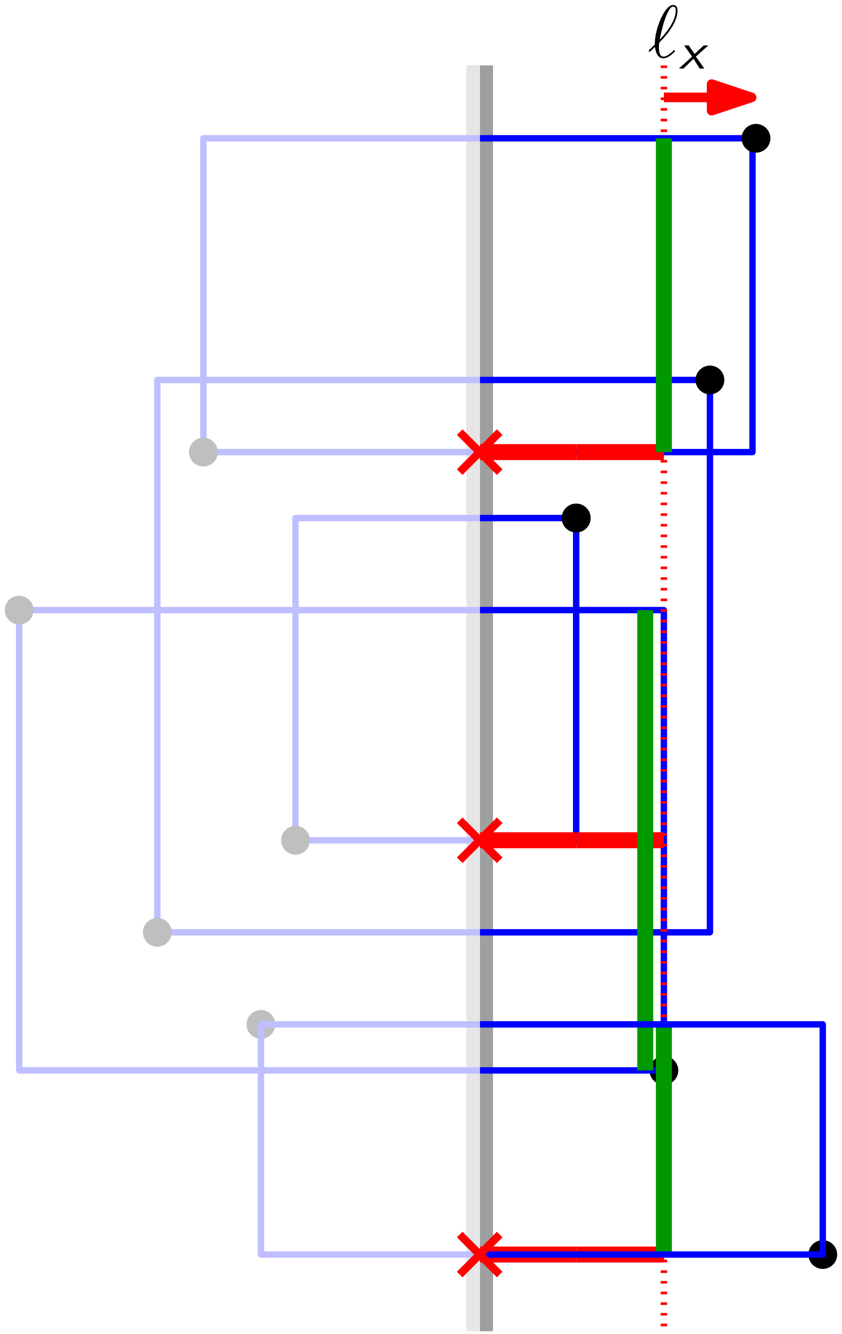
Piercing and Stabbing



Piercing set

The right parts of the horizontal line segments in a fixed optimal solution.

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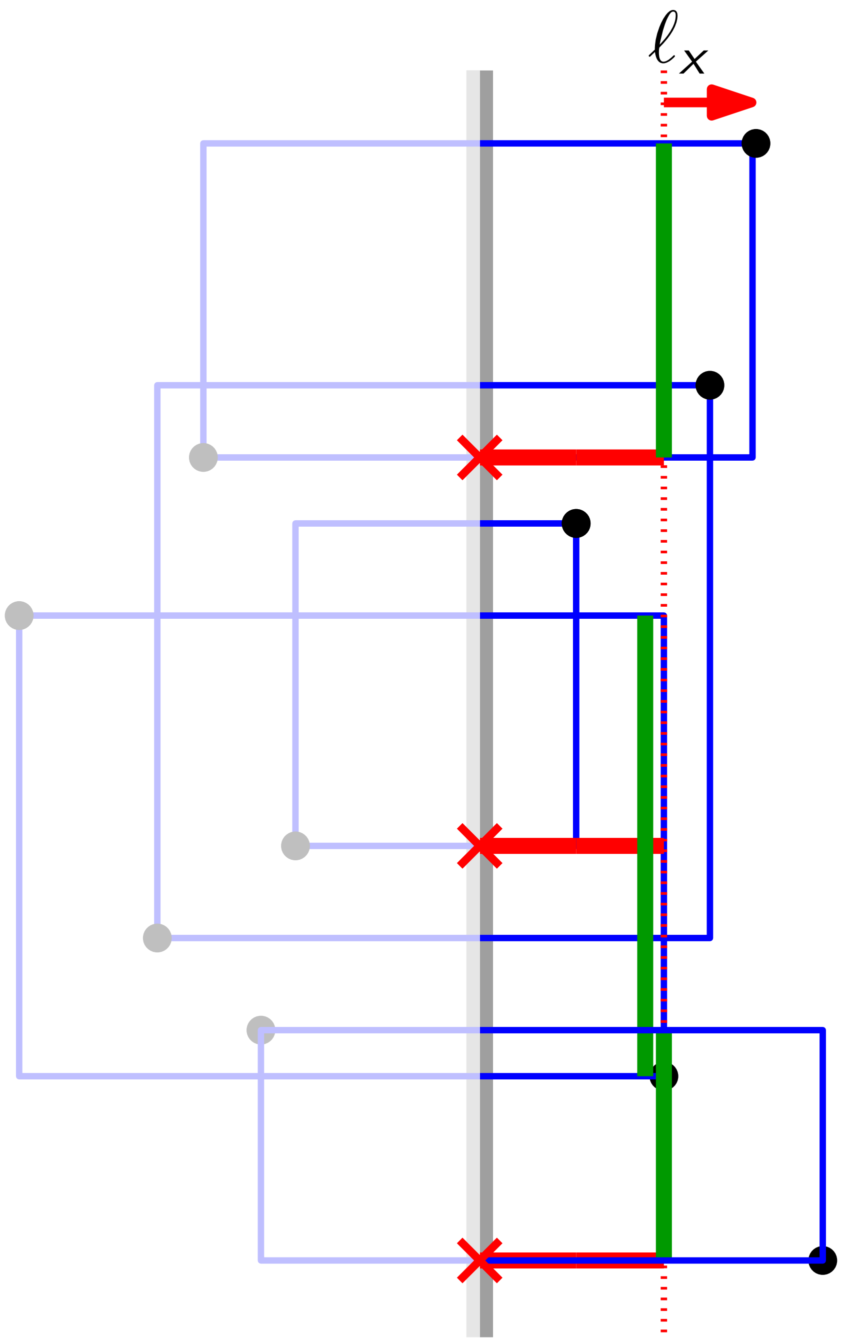

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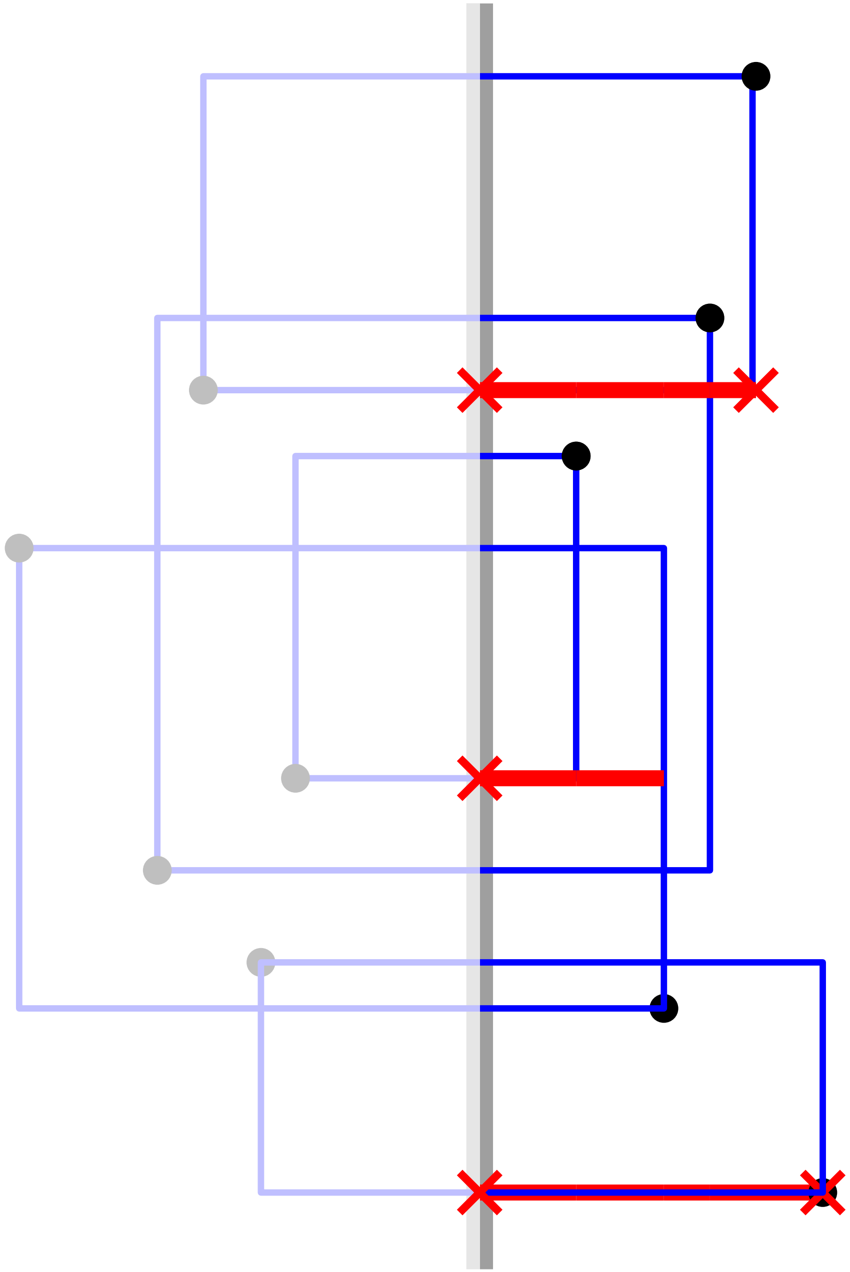

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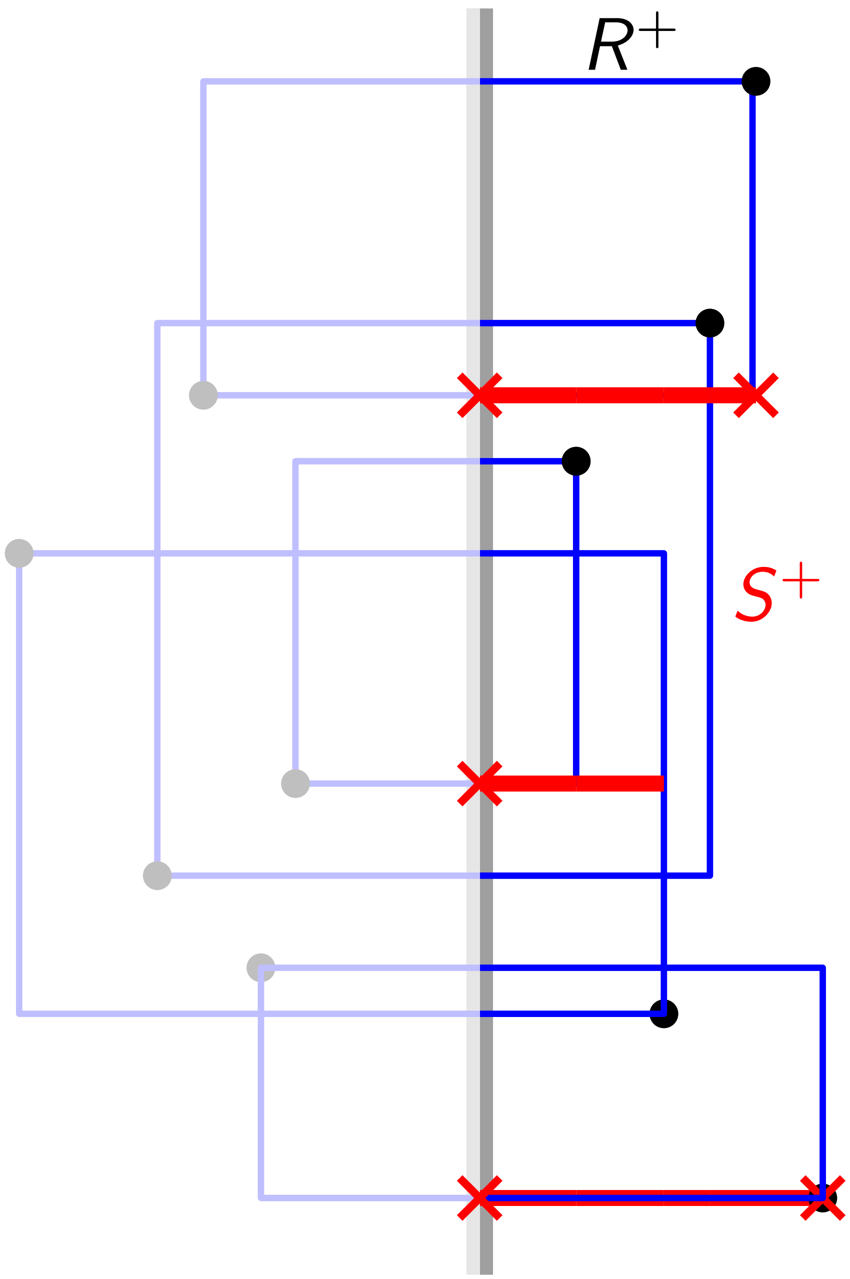

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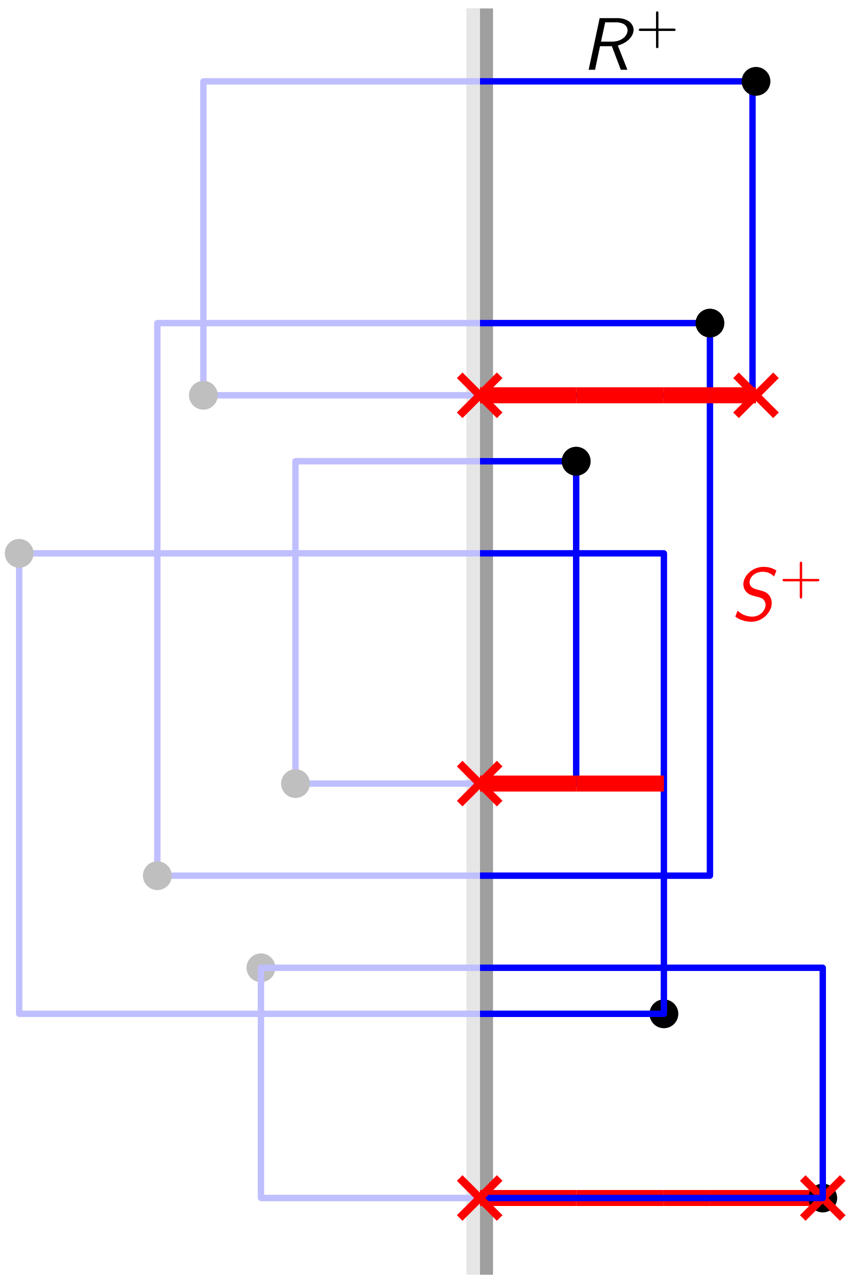

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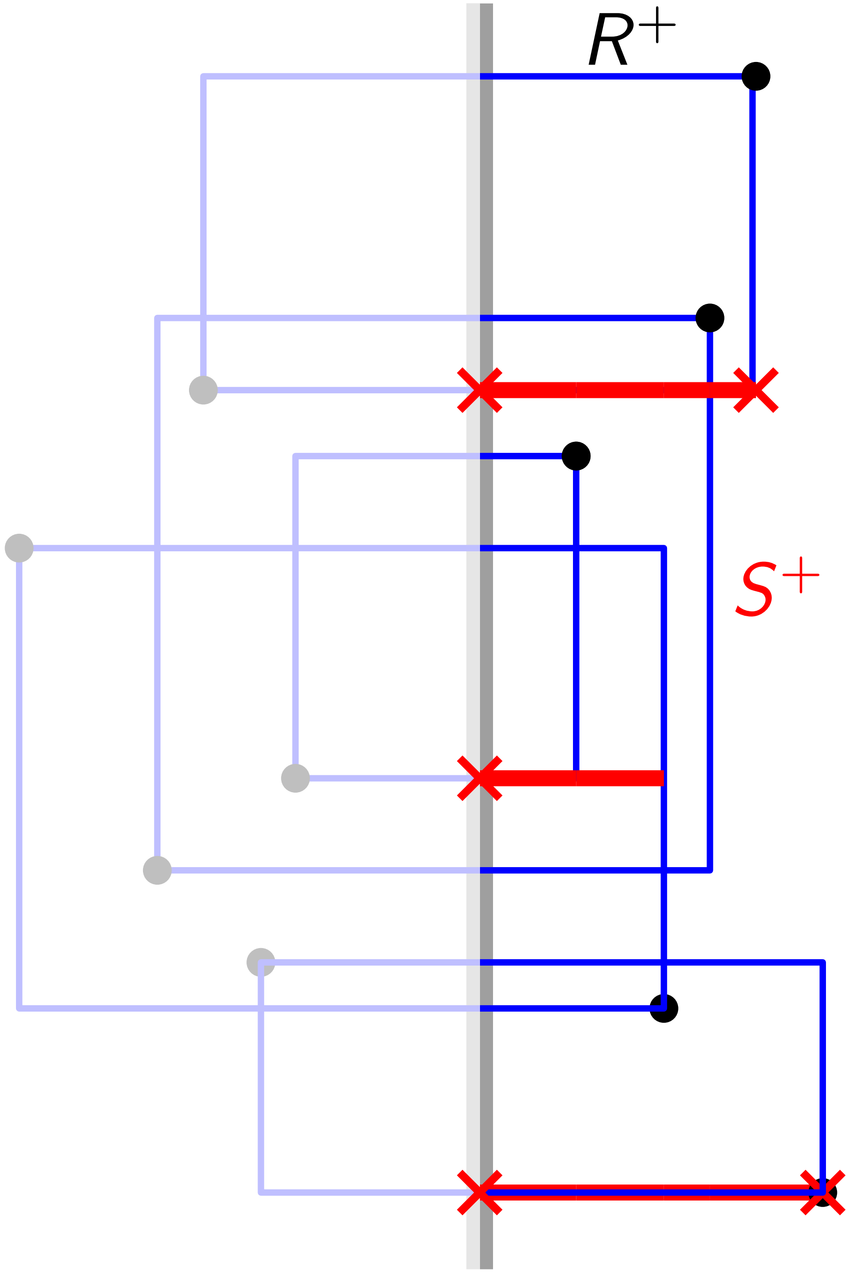

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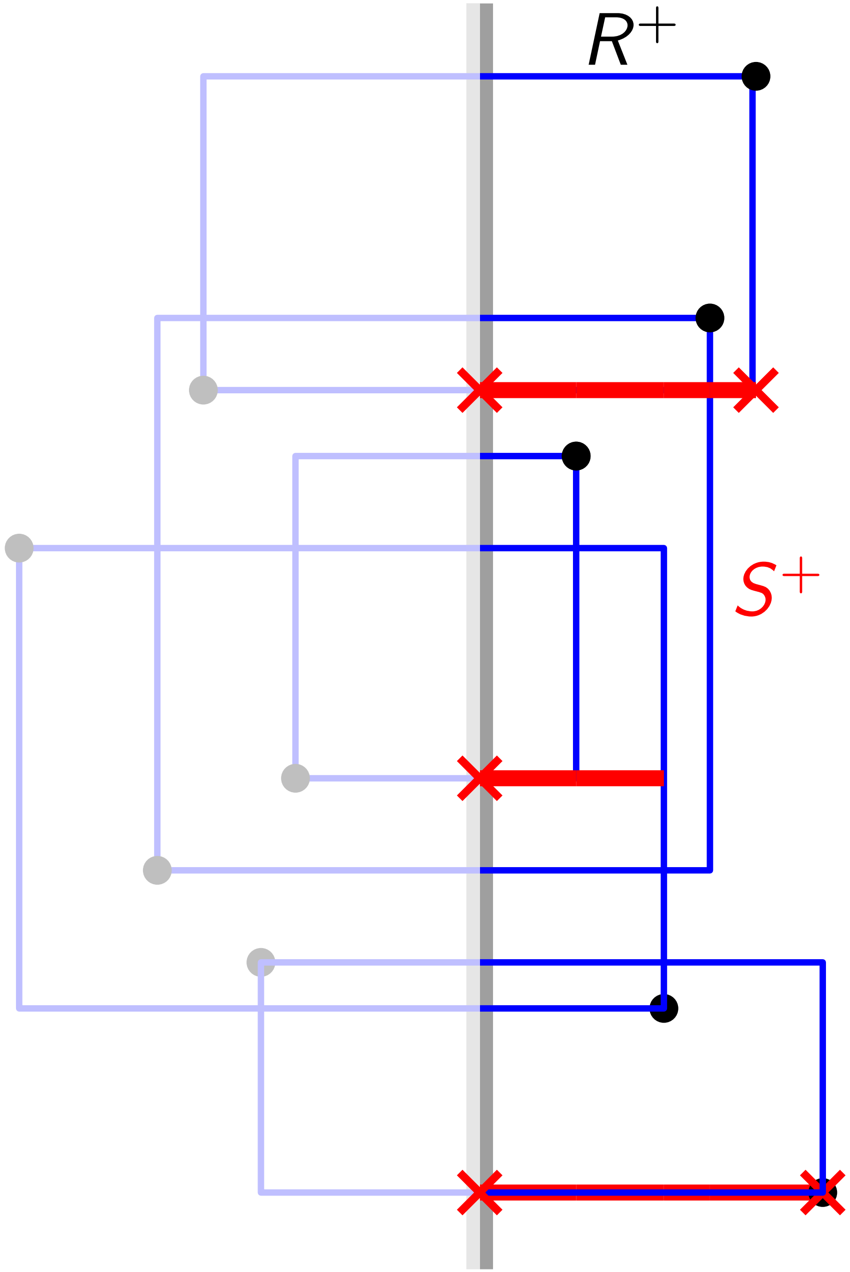

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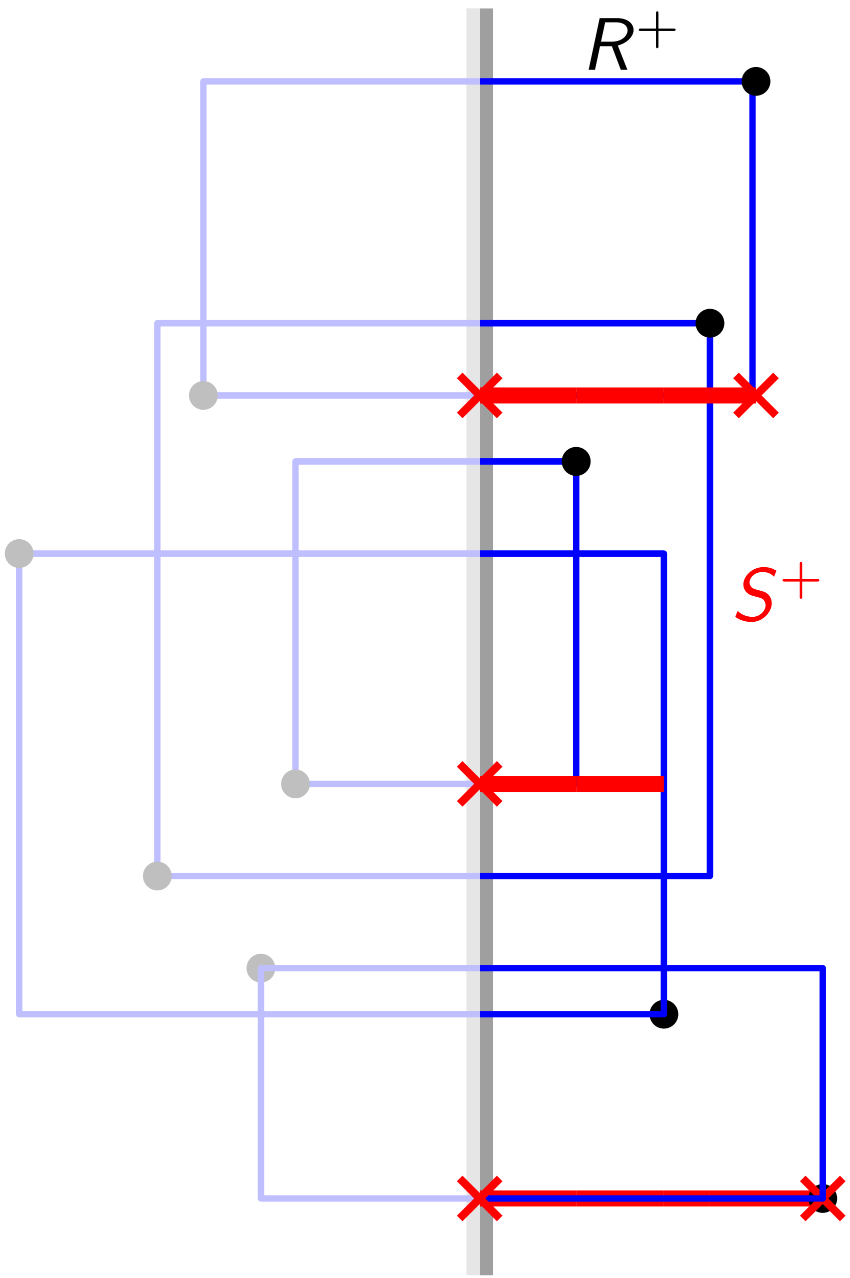

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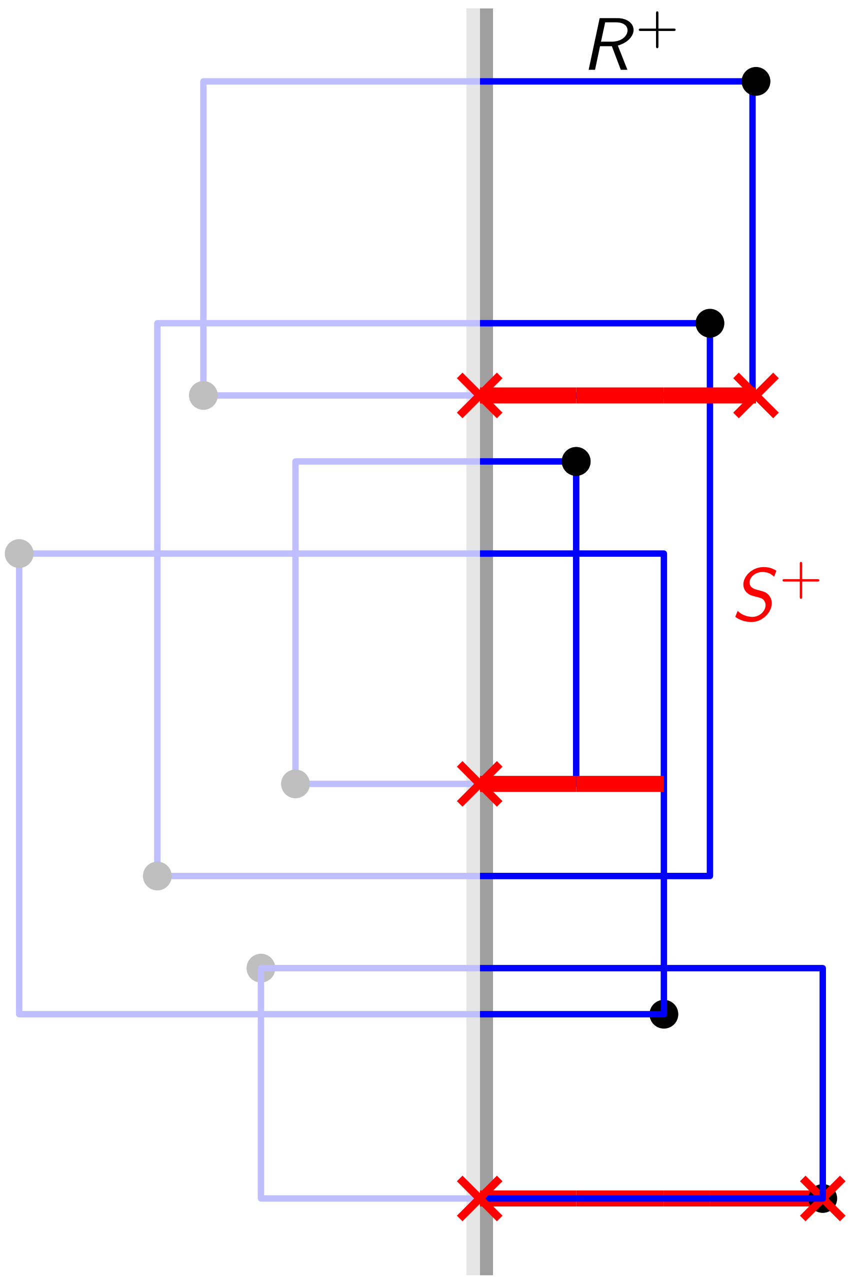

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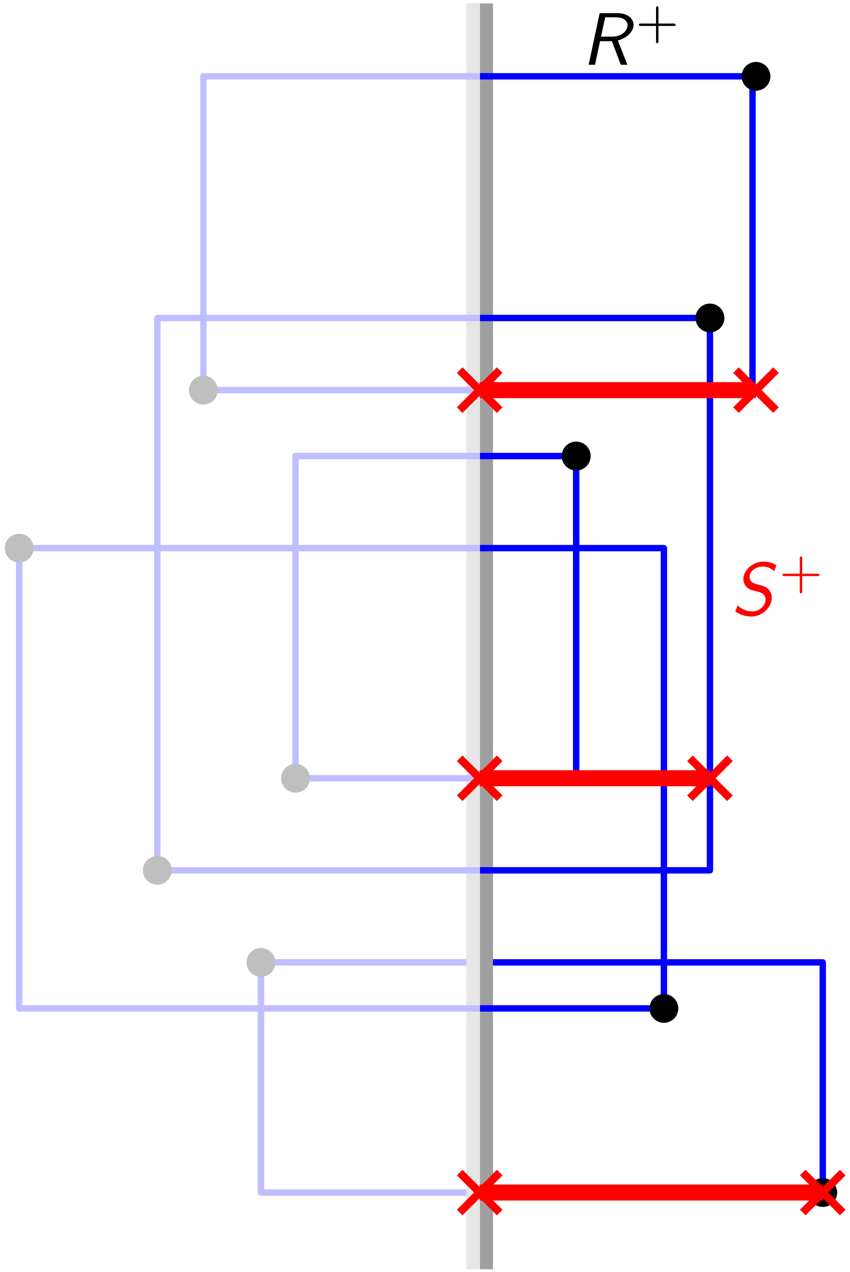
Stabbing Both Parts



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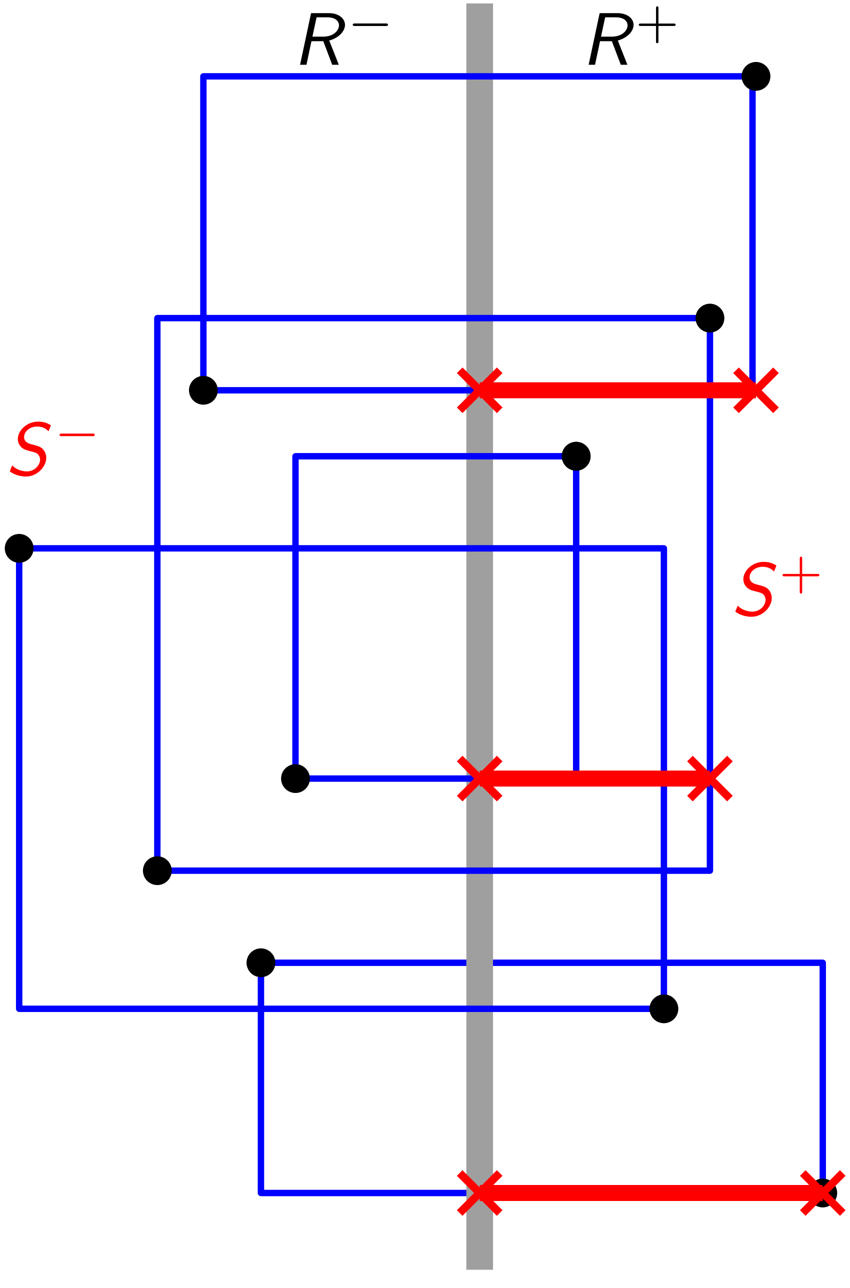

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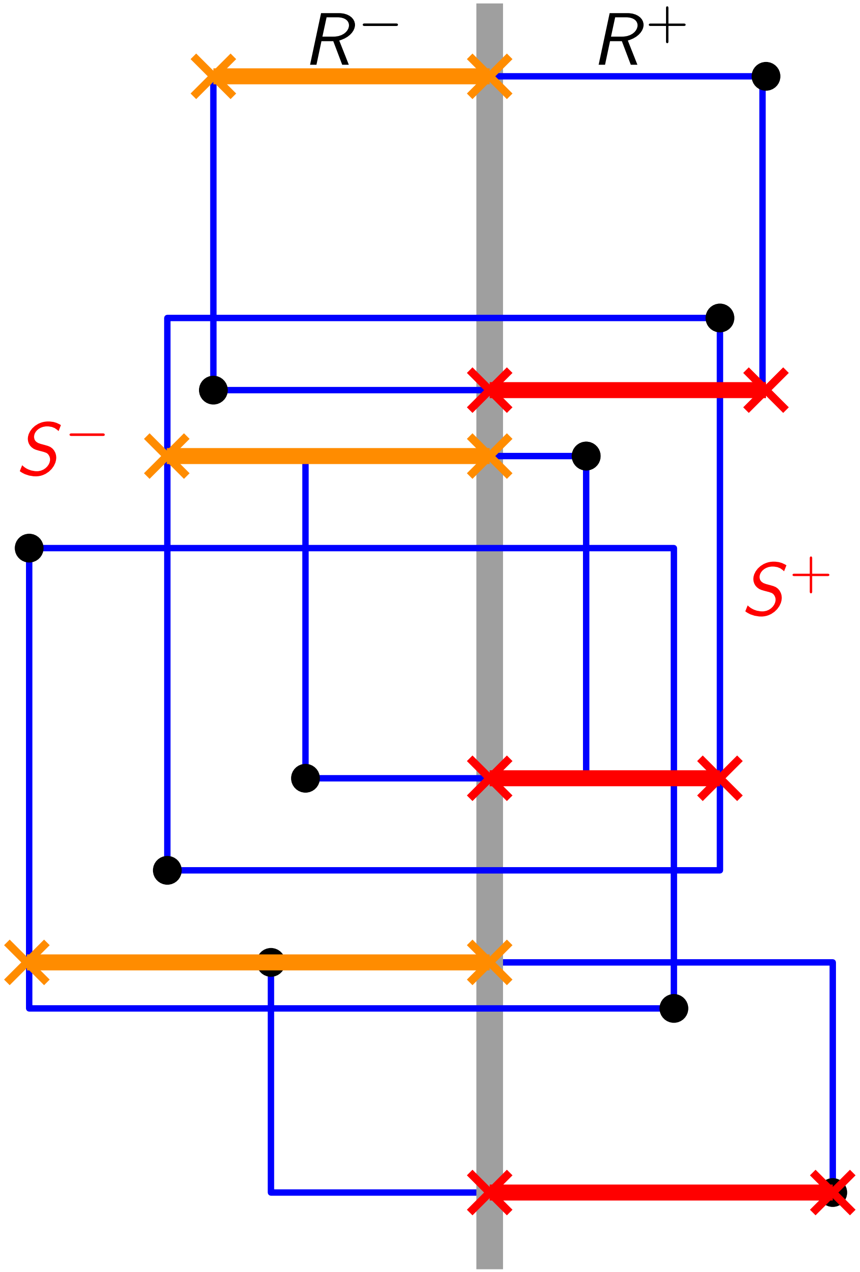

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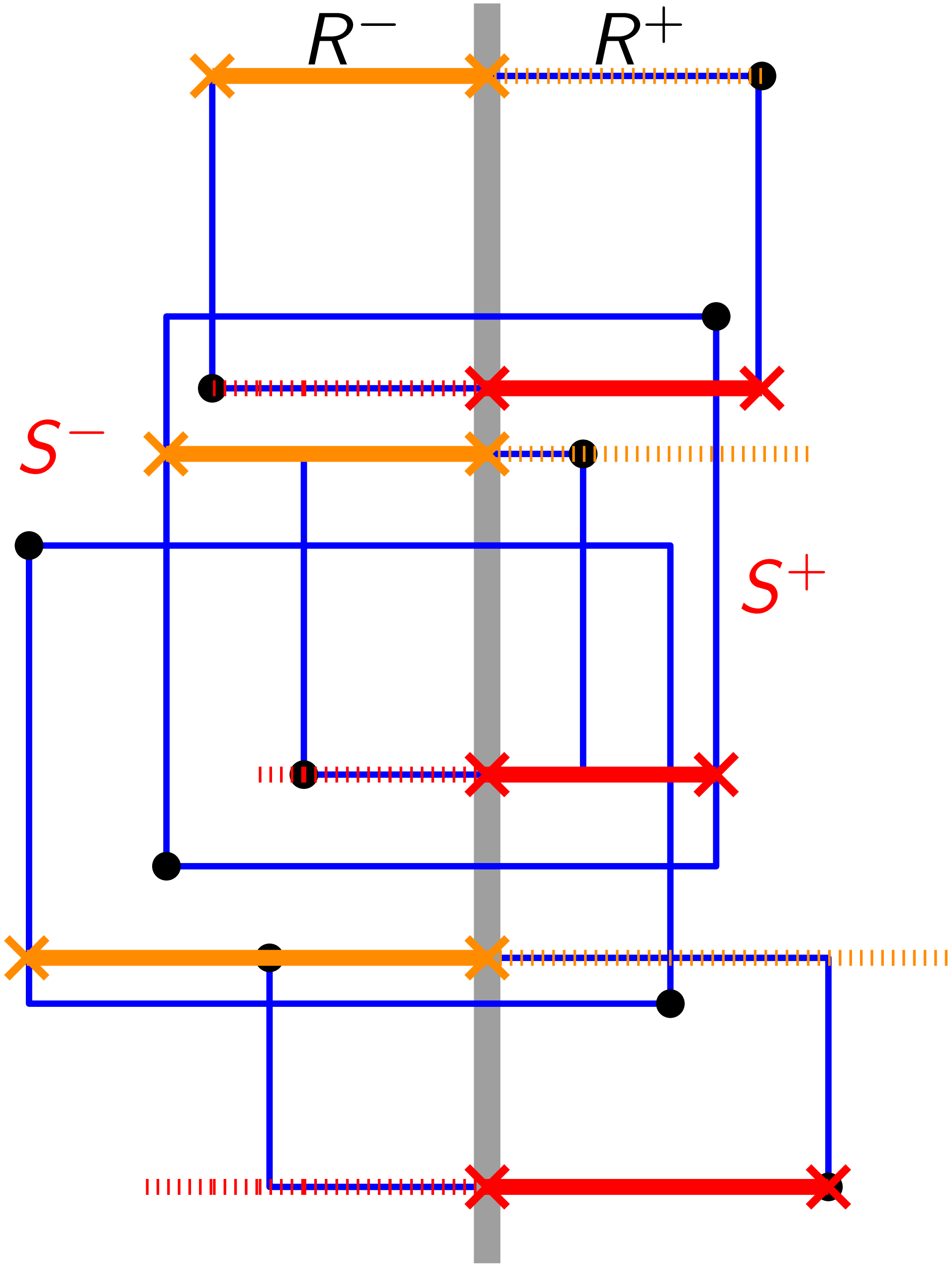

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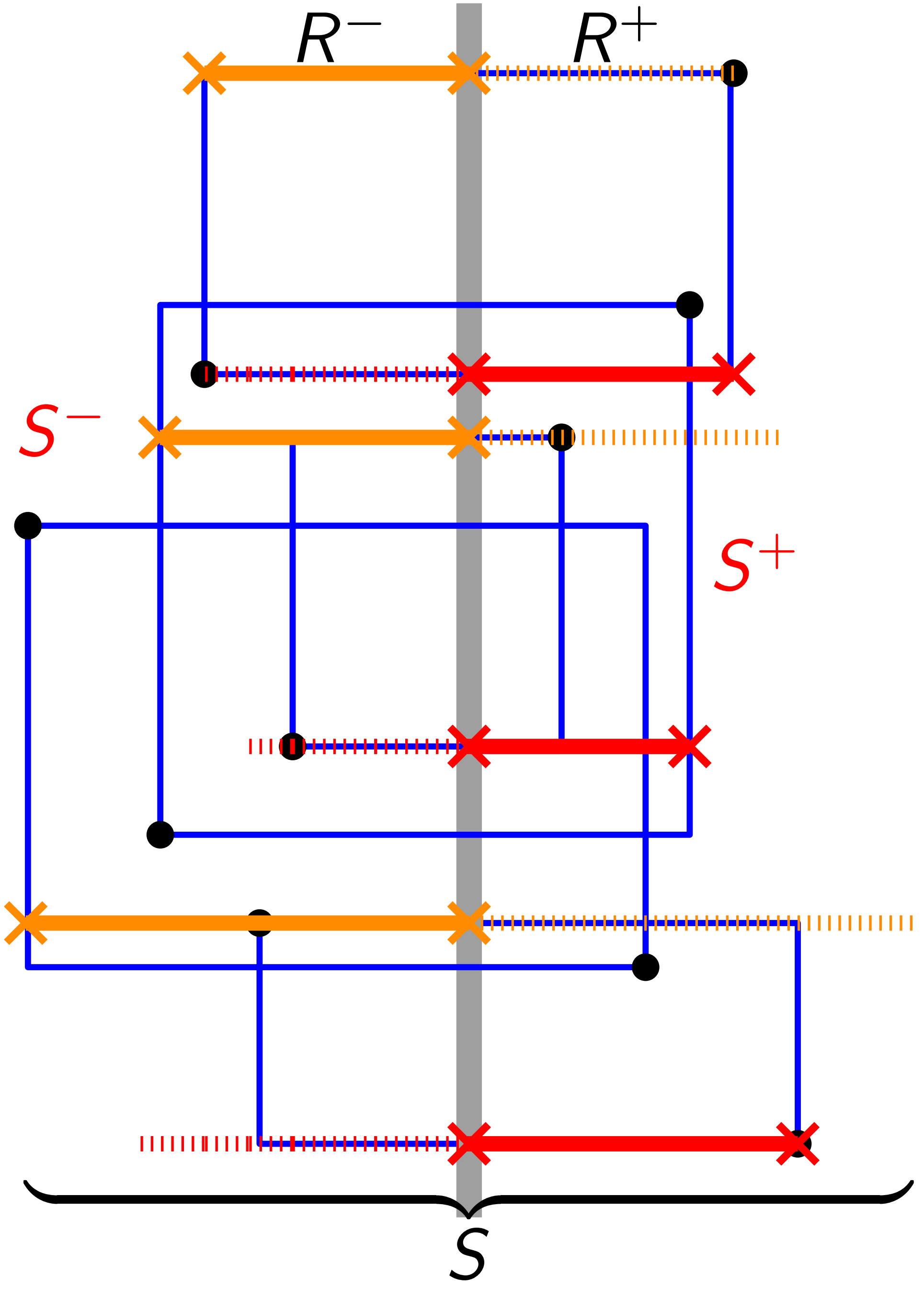

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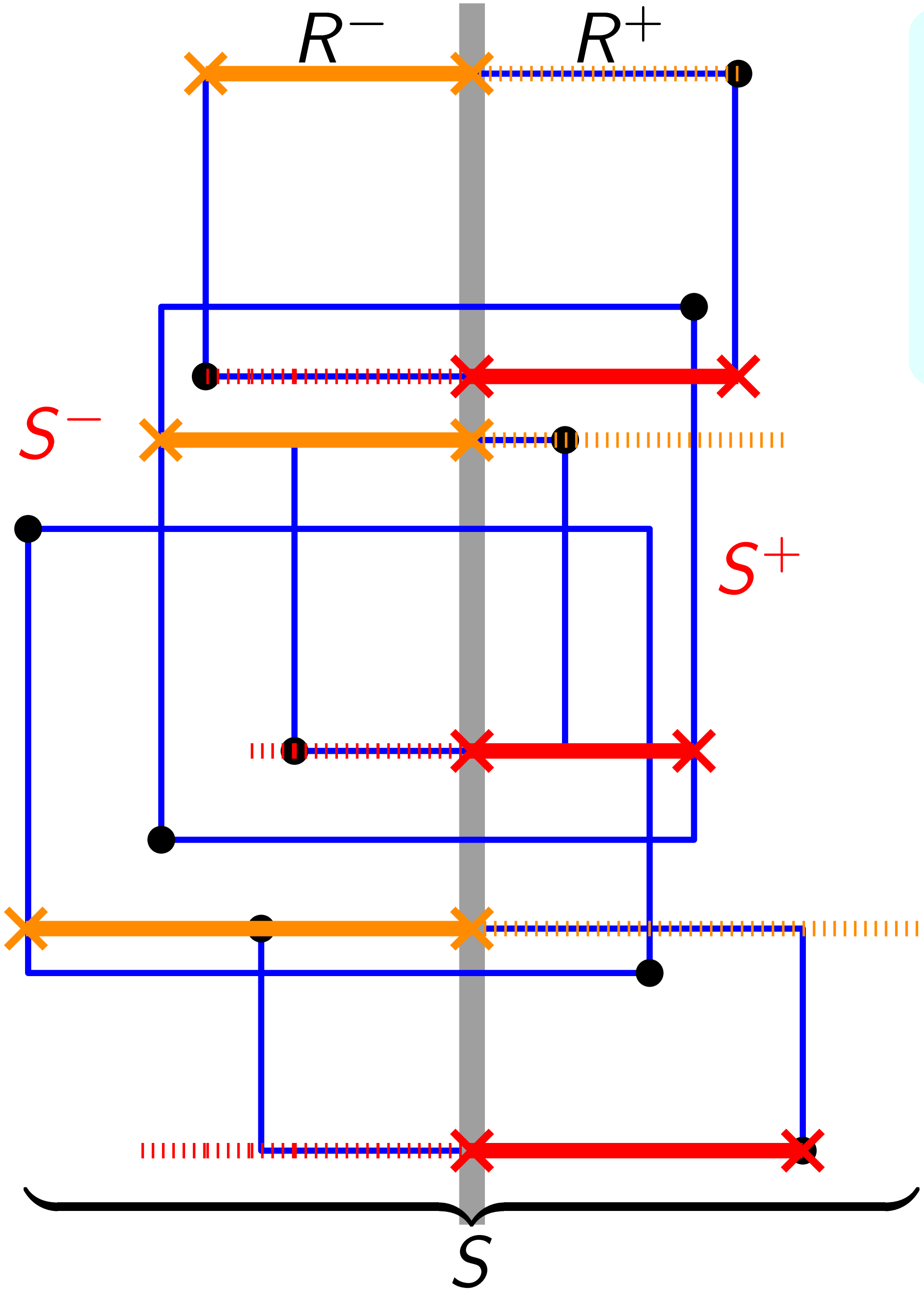

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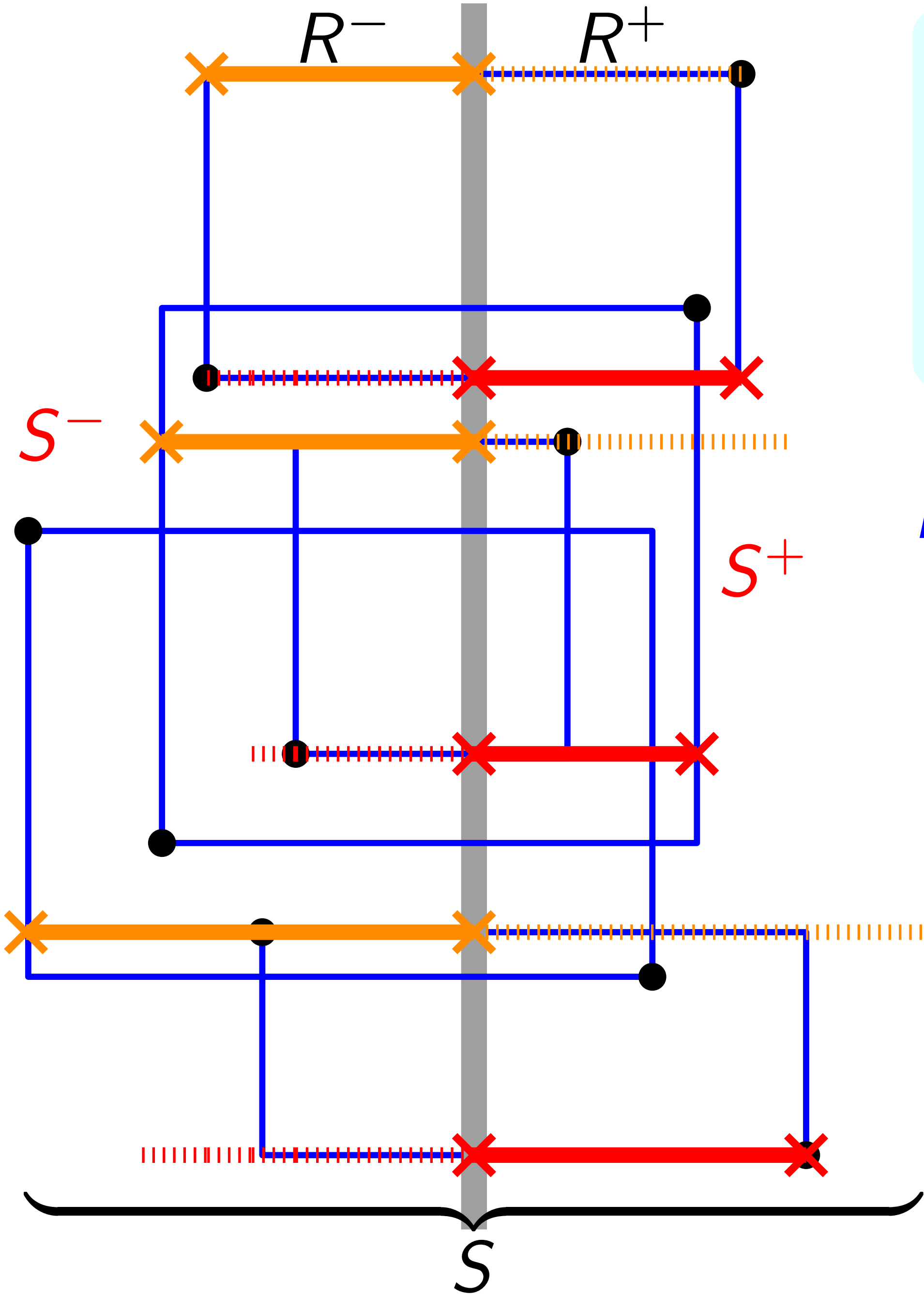

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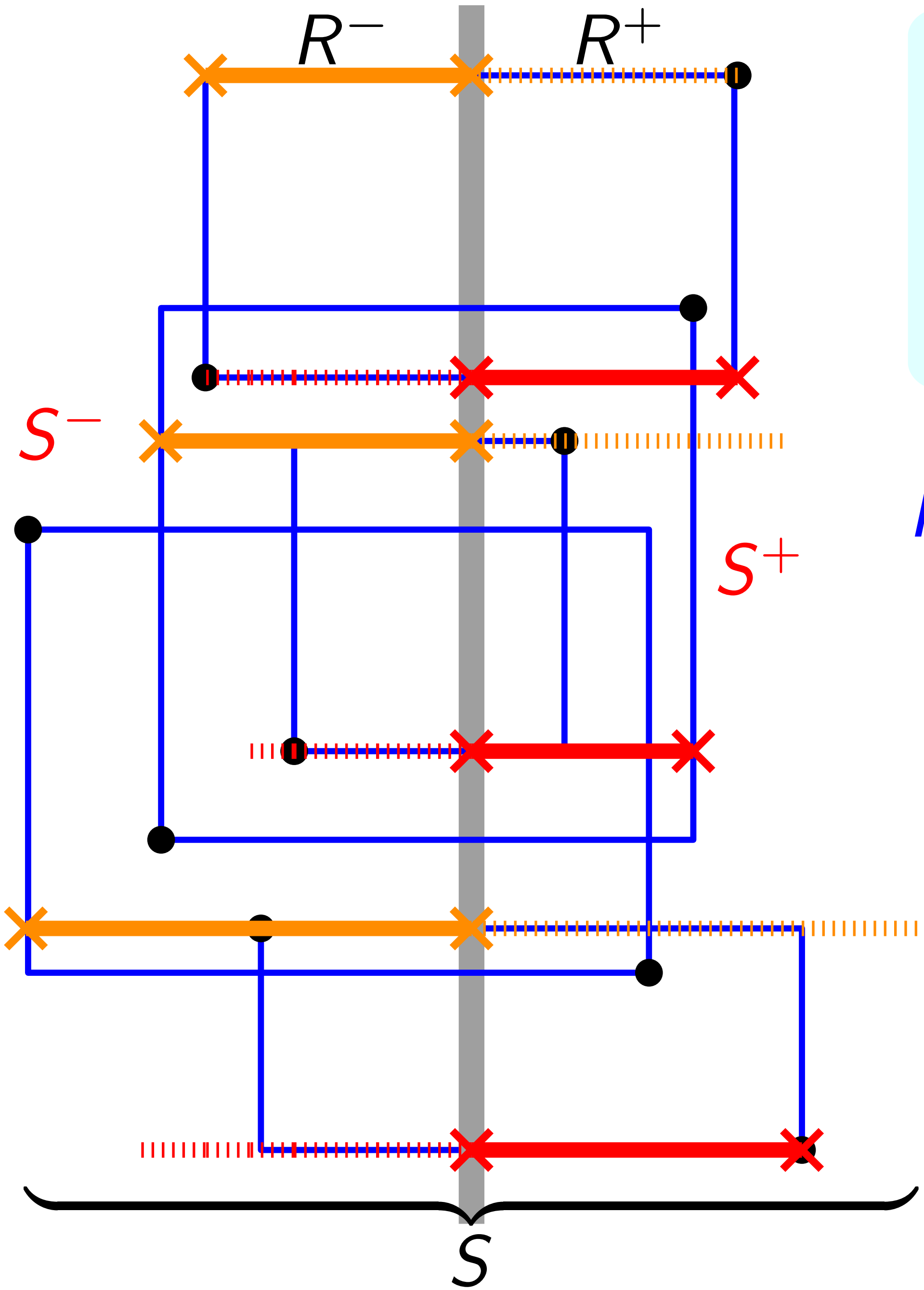

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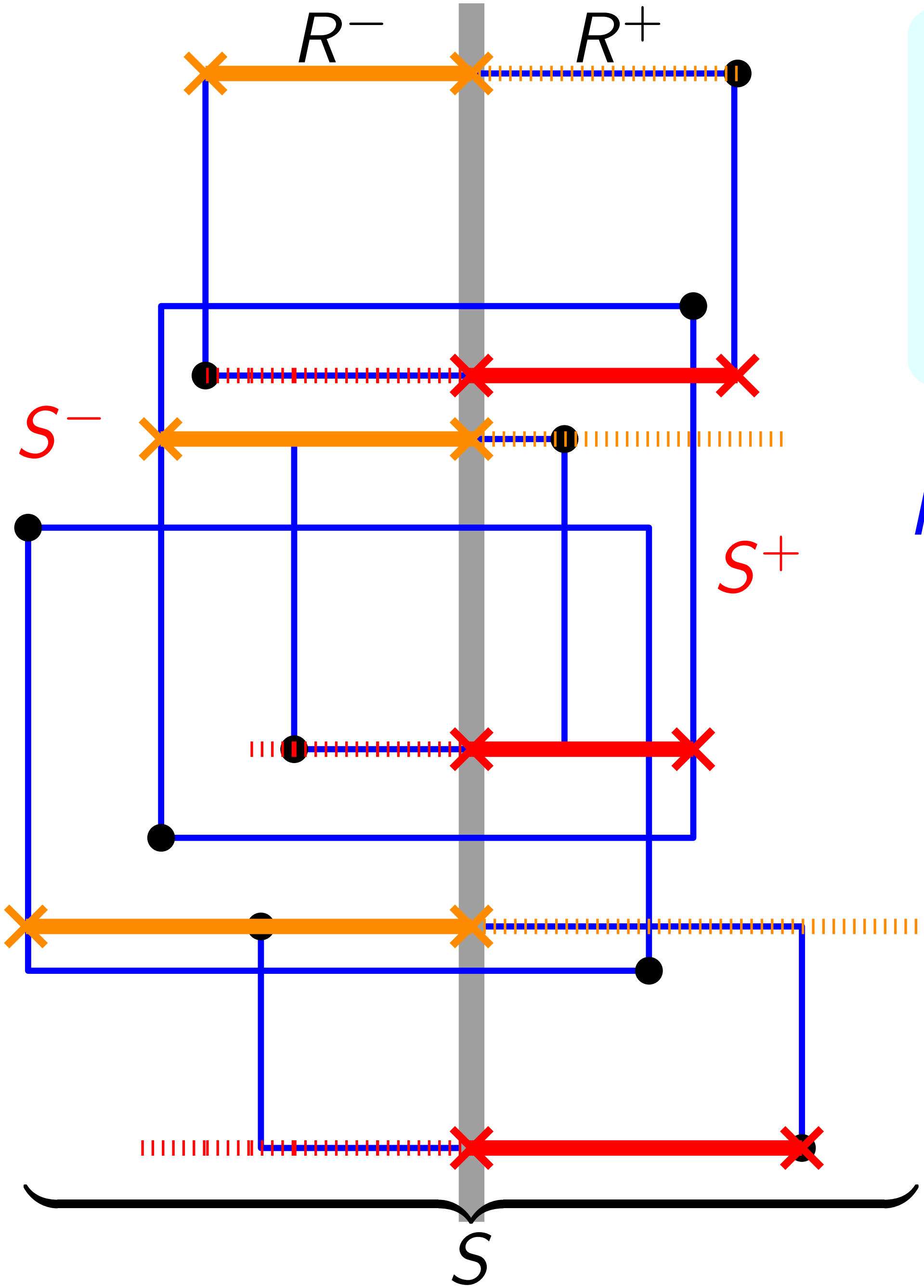

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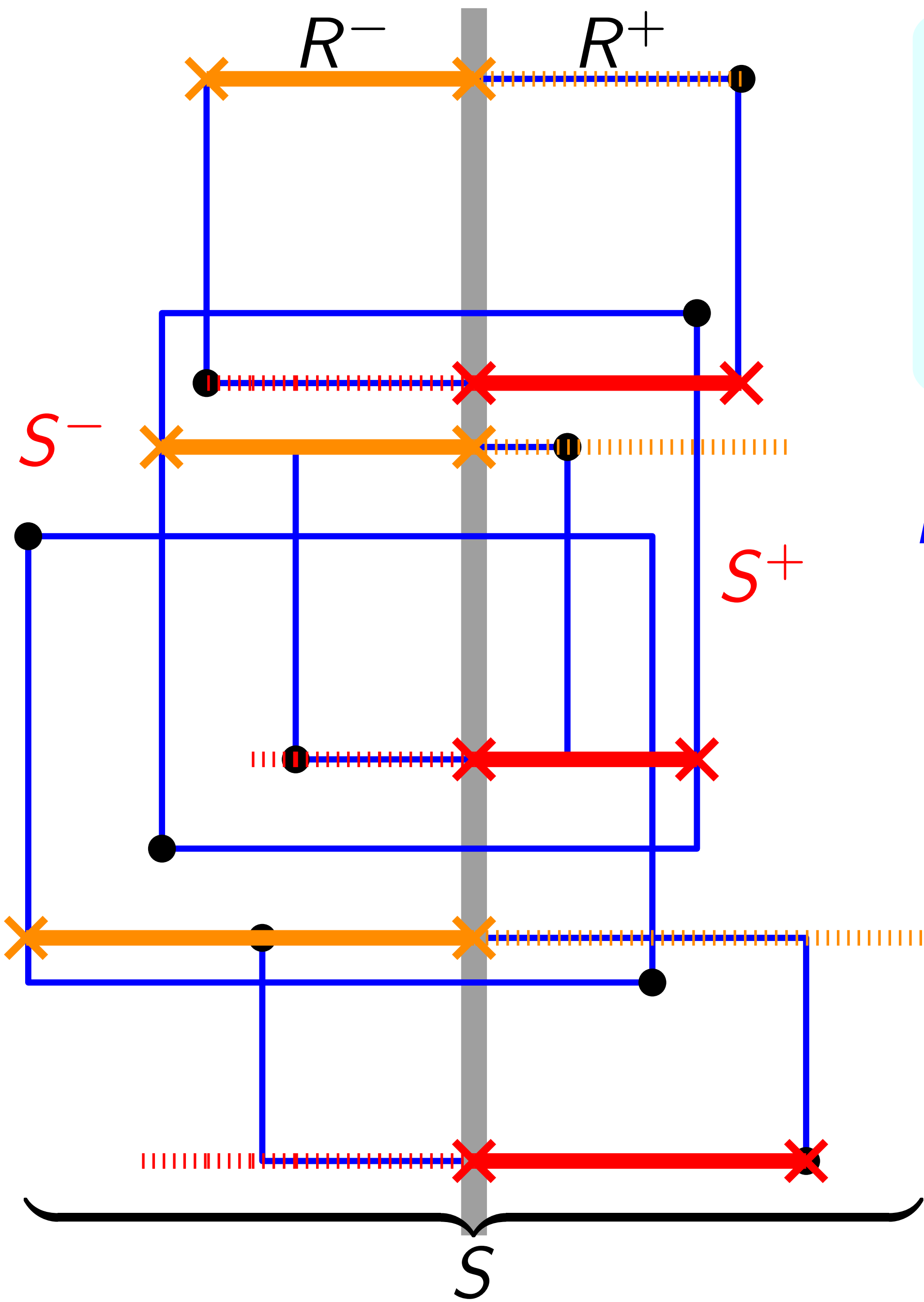

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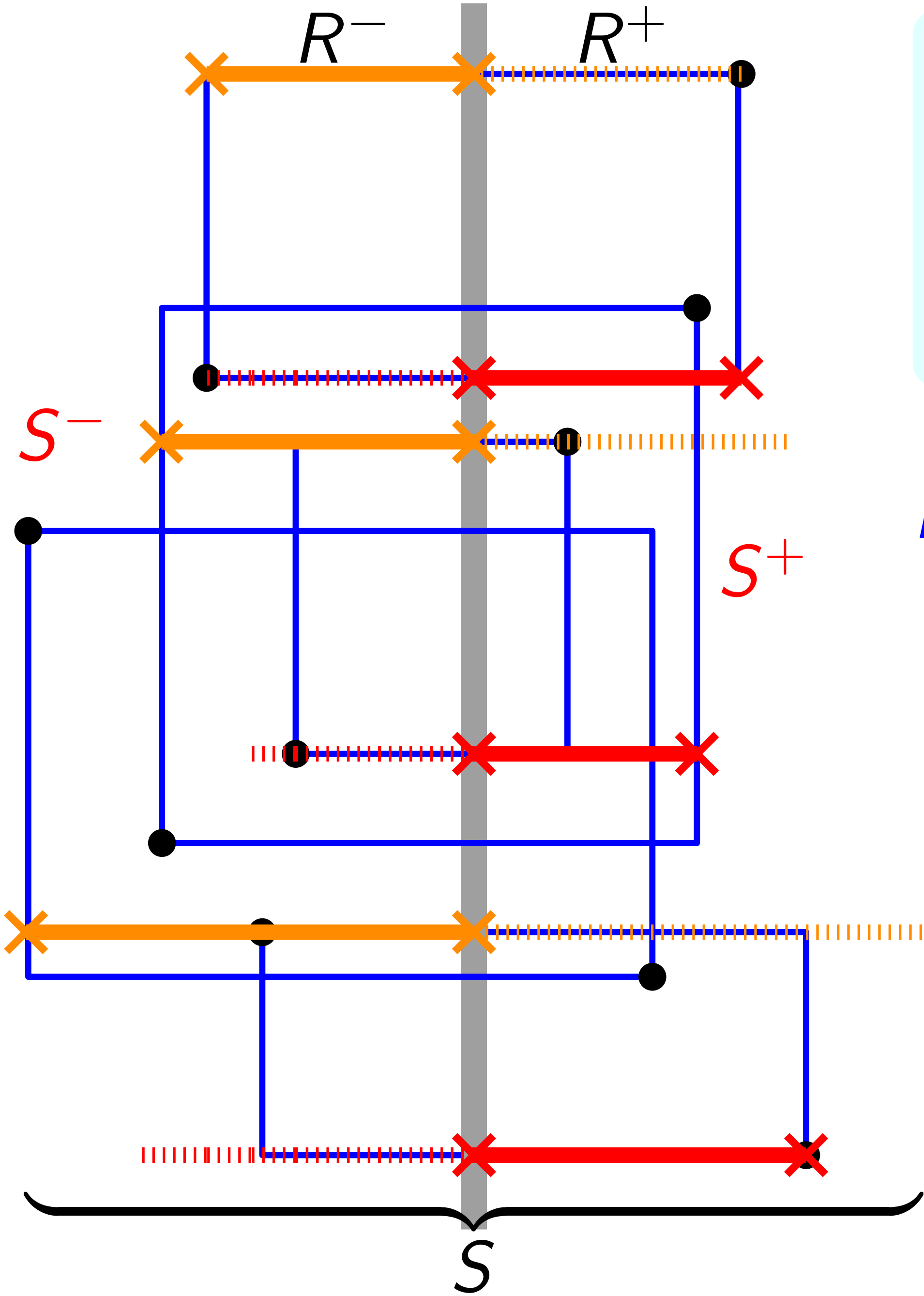

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Theorem. x-separated 2D-GMMN admits, for any $\varepsilon > 0$, a $(6 + \varepsilon)$-approximation.

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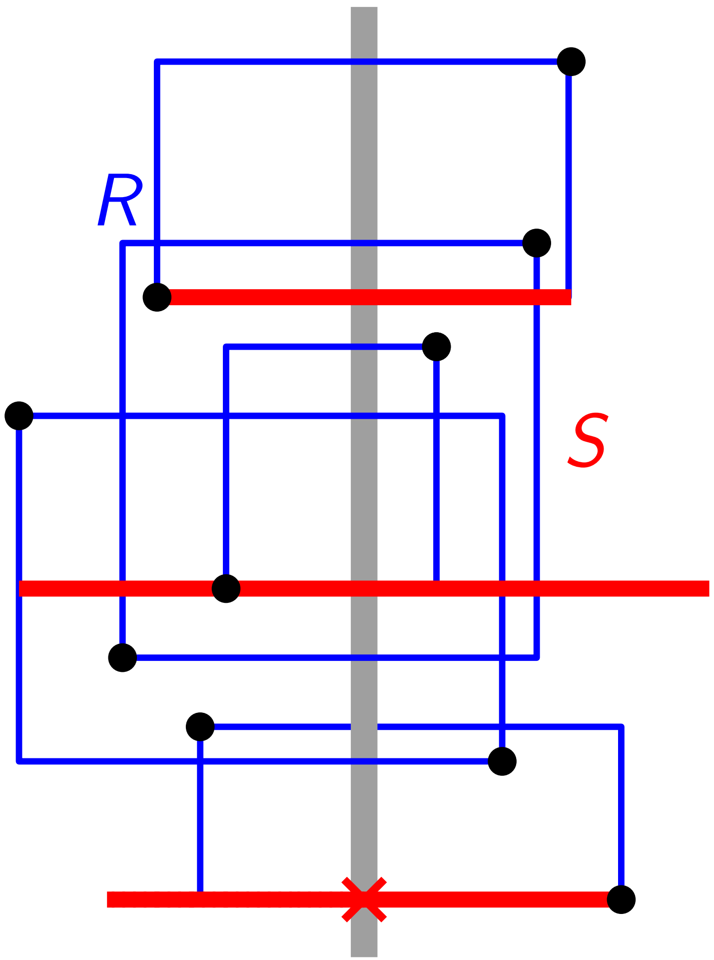

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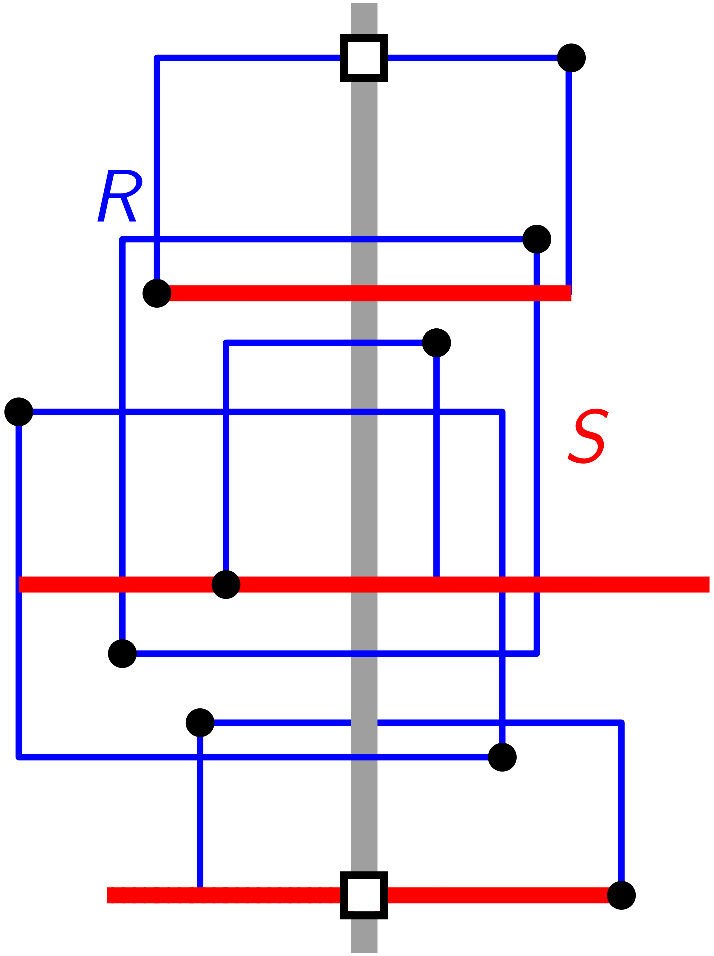

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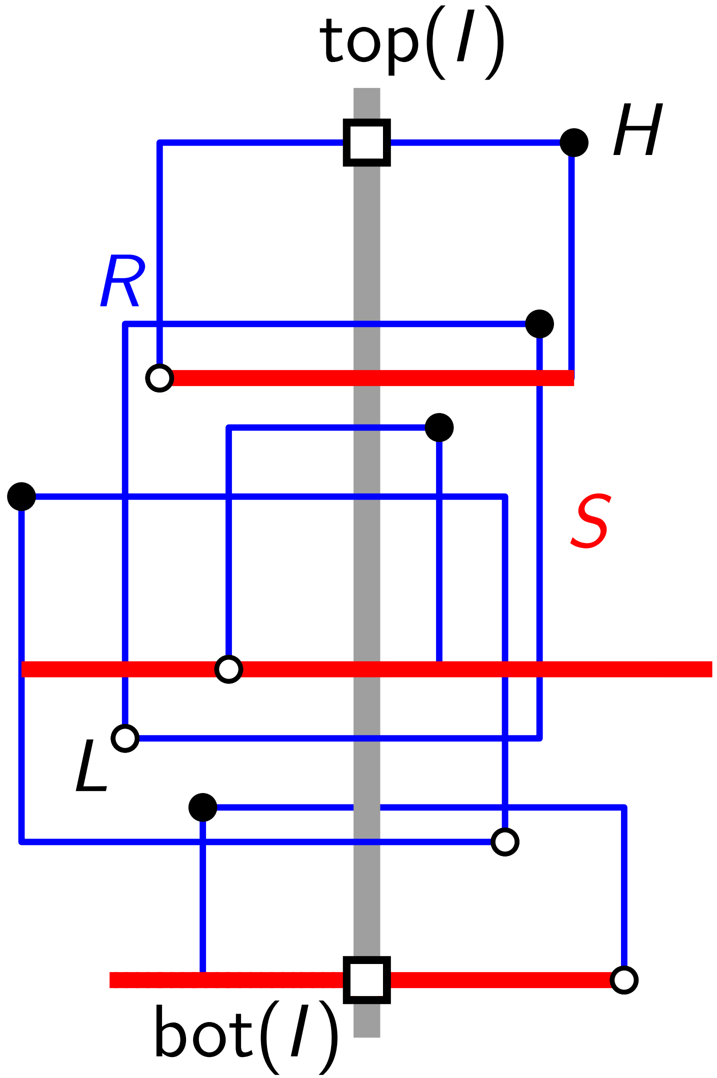

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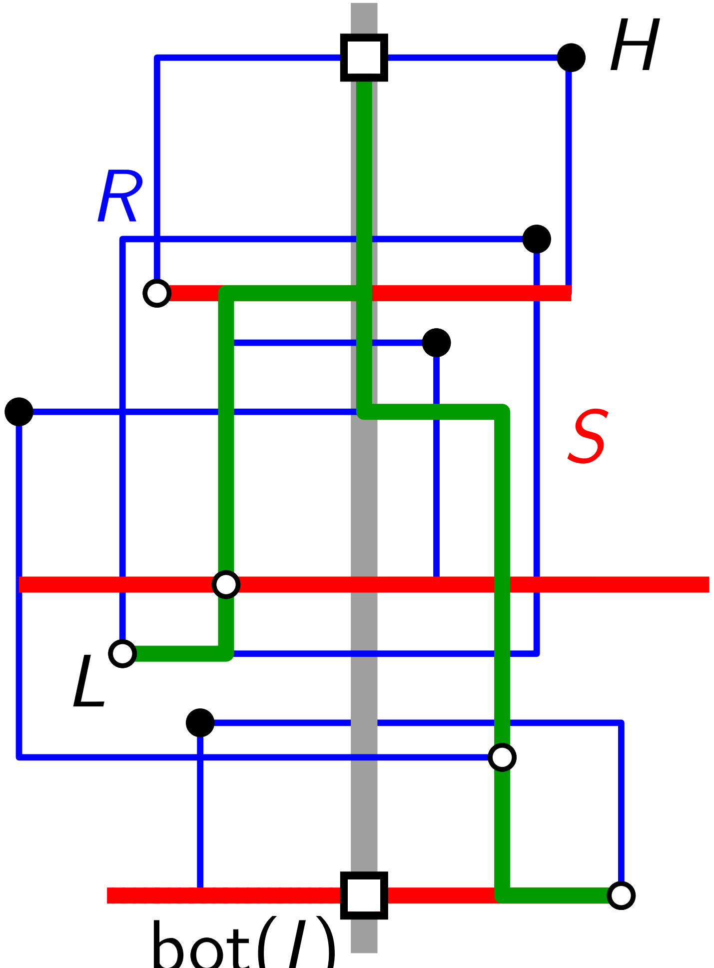

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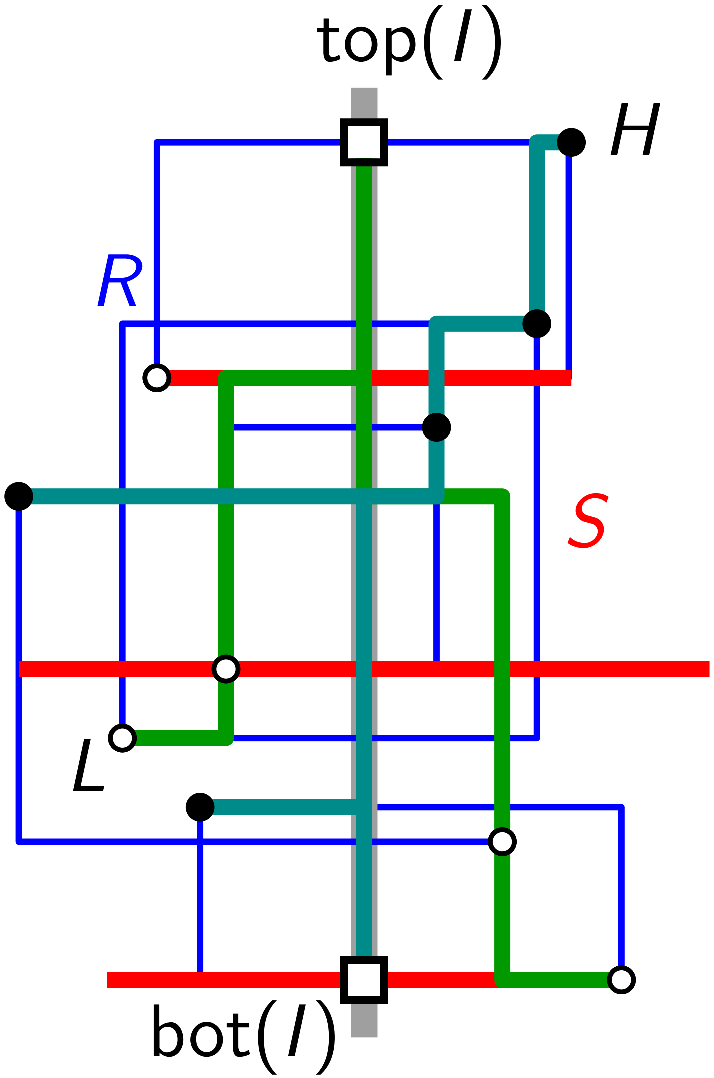

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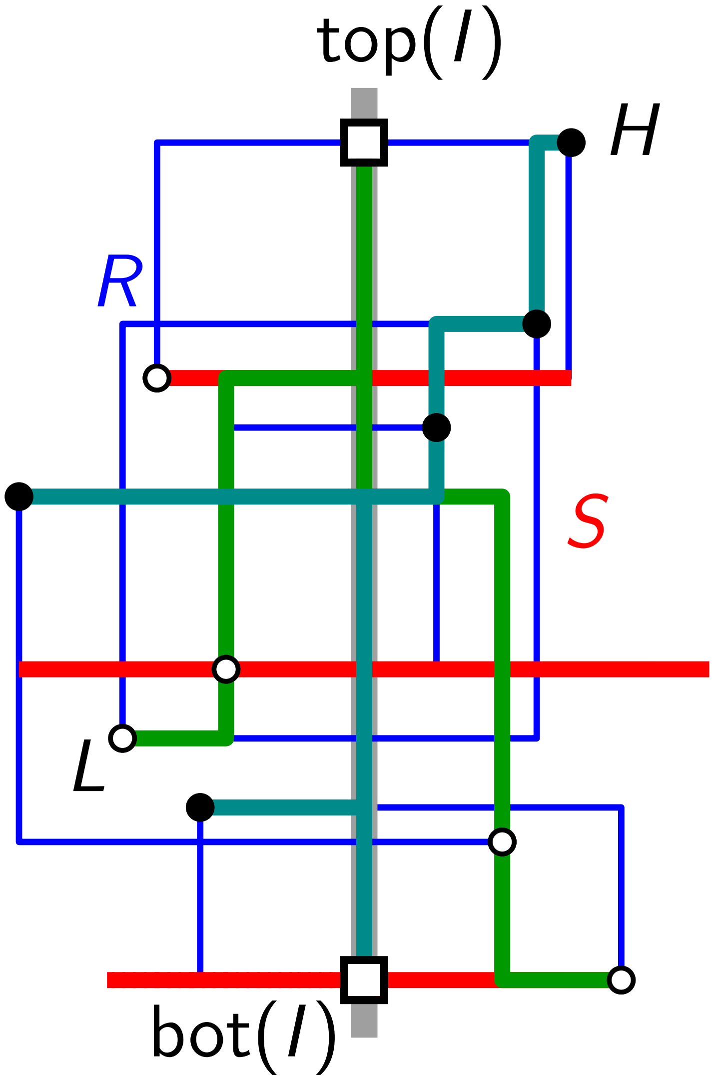

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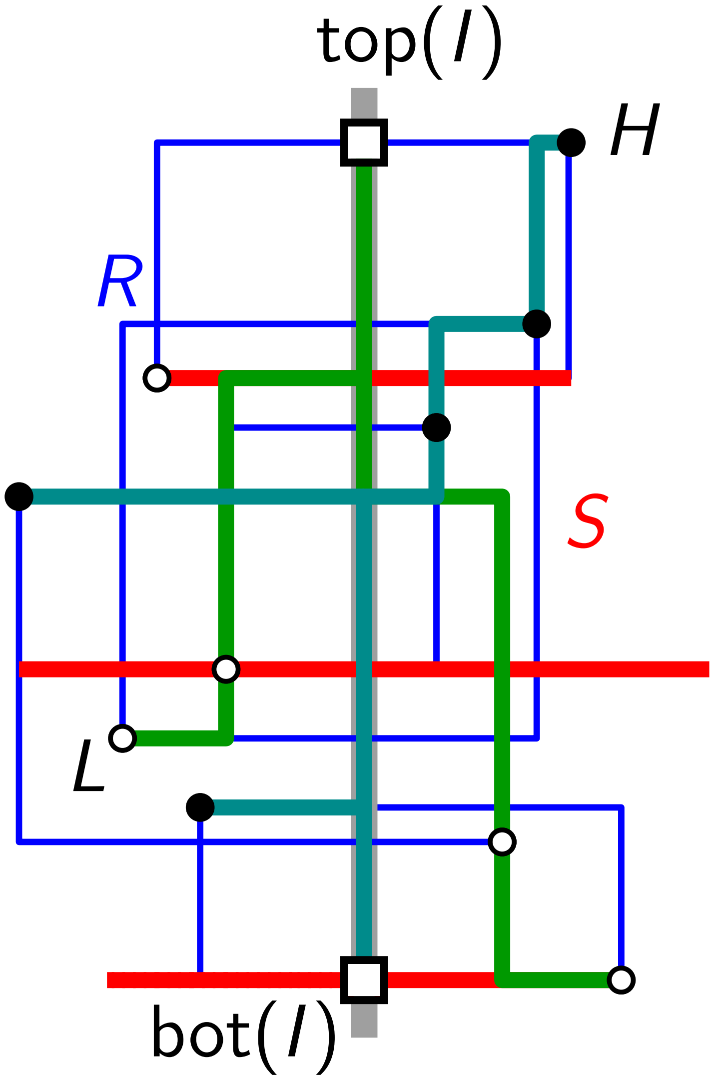

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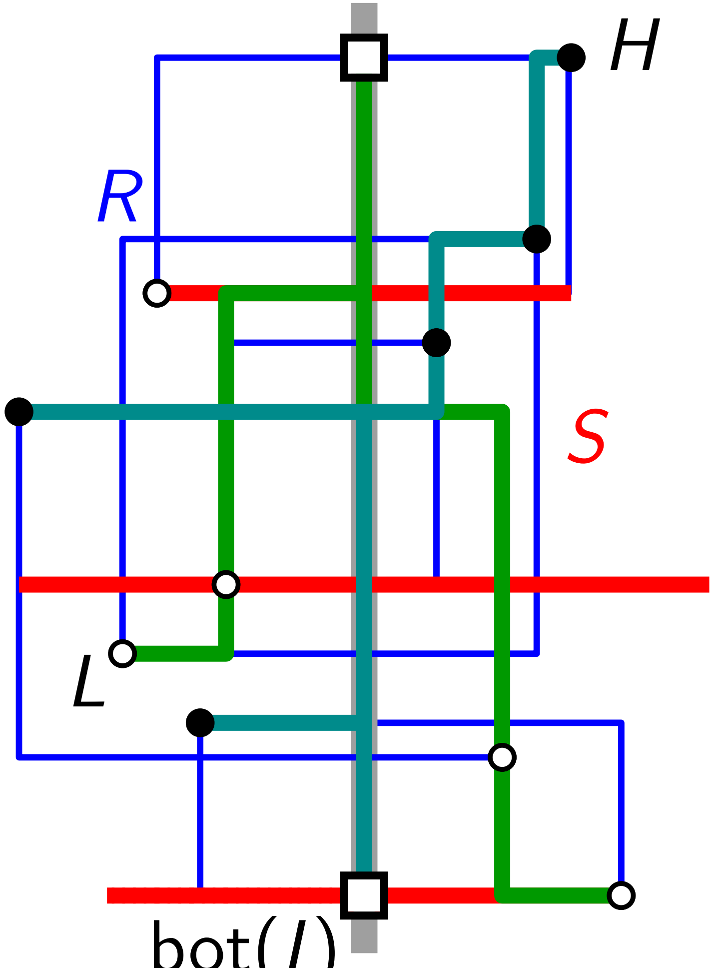

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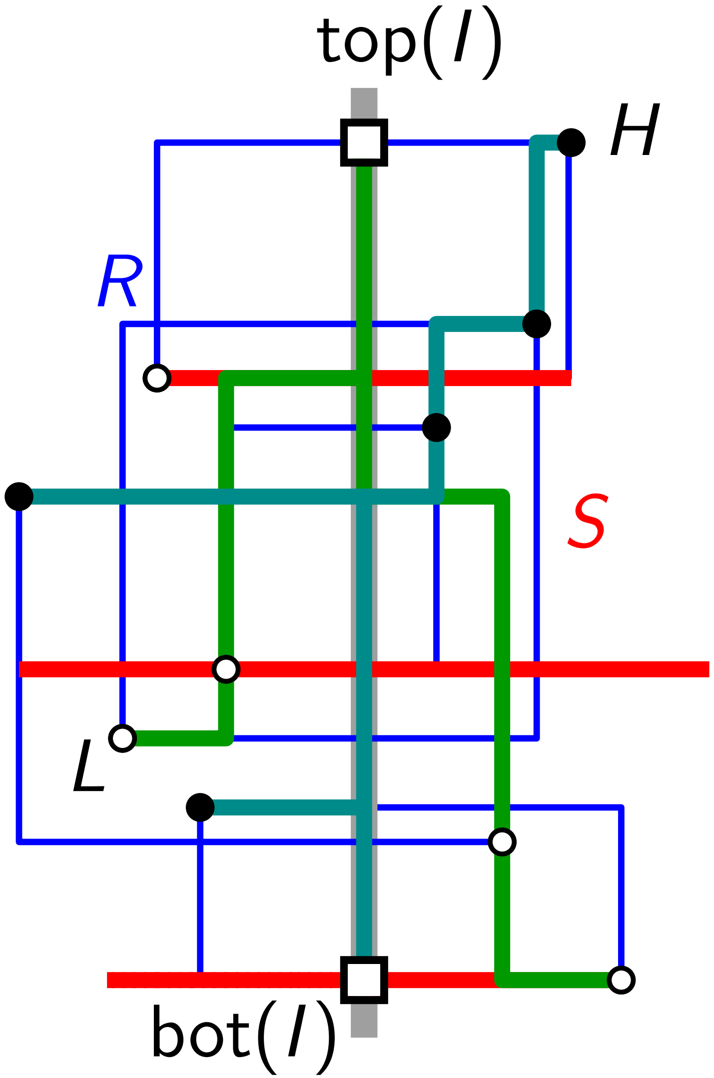

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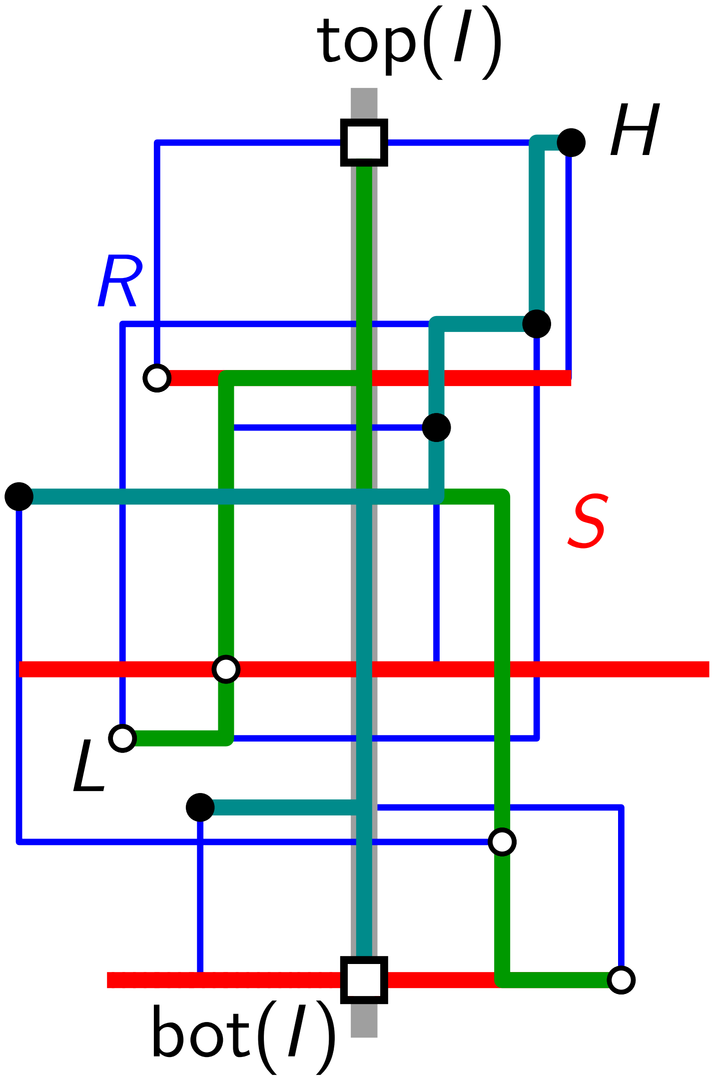

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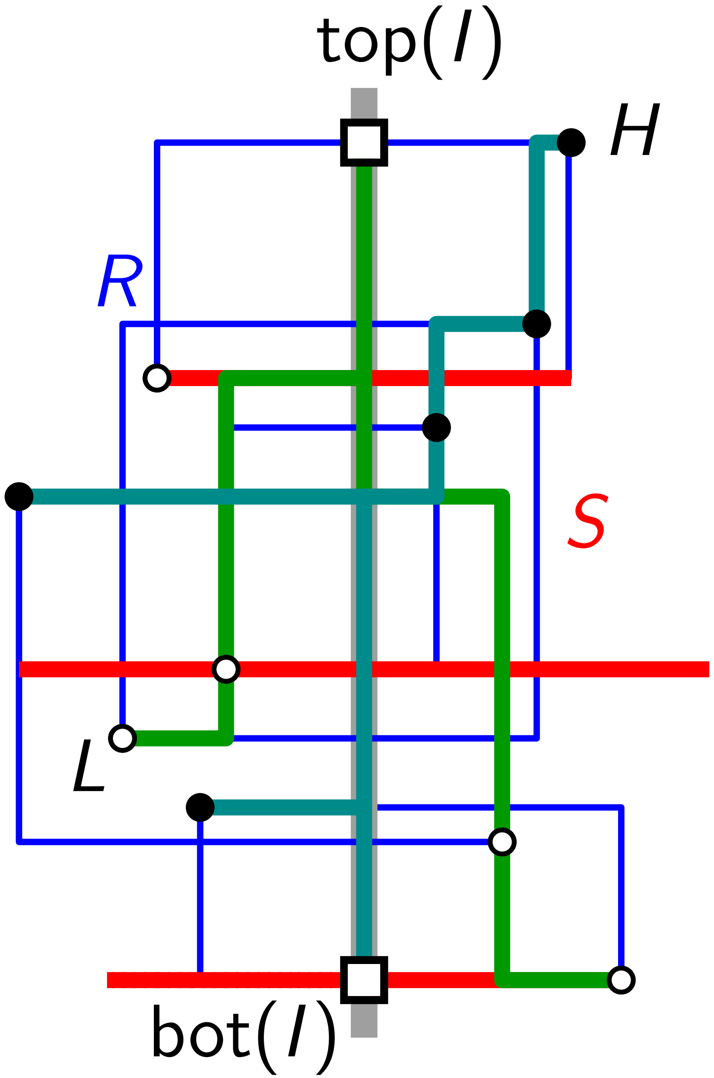

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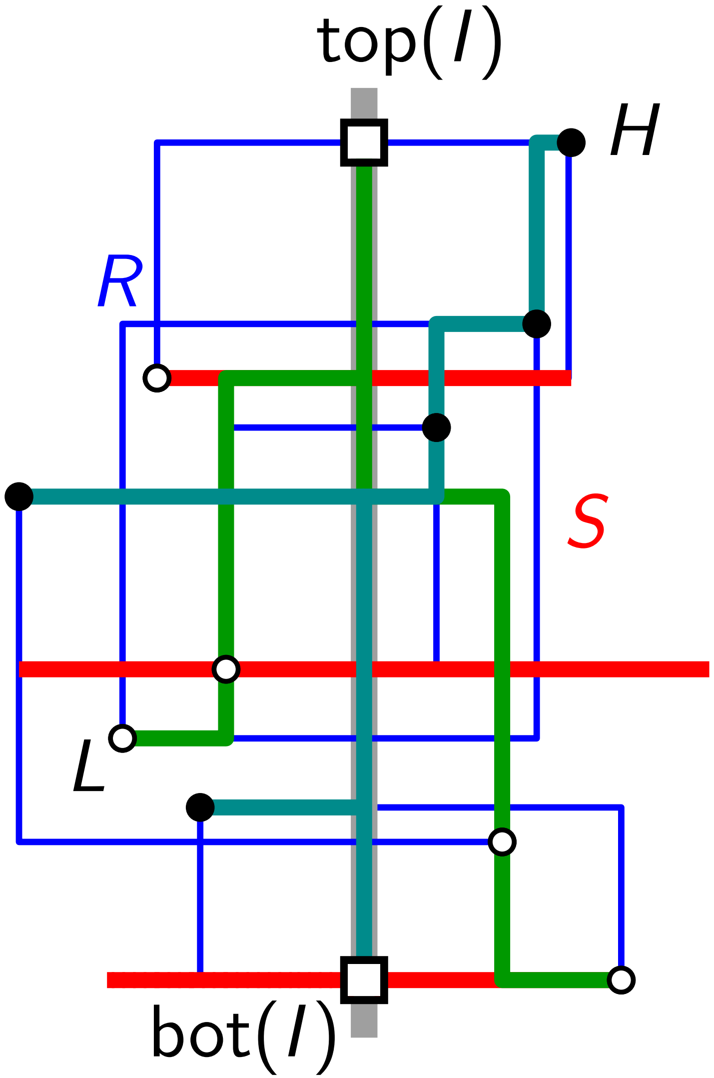

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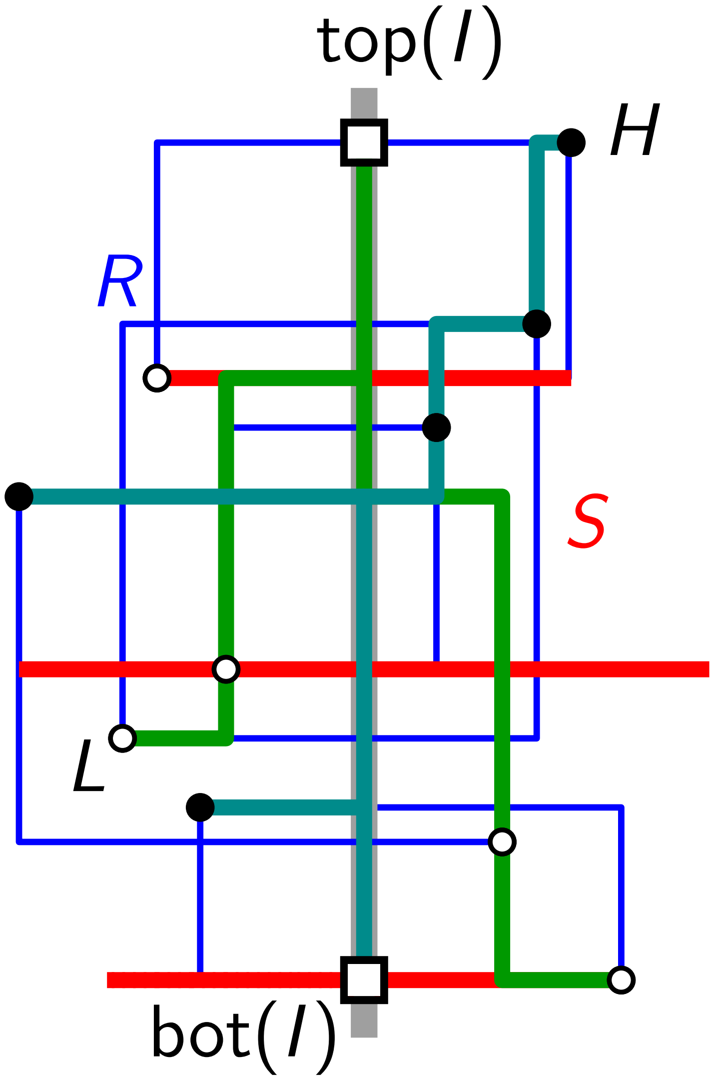

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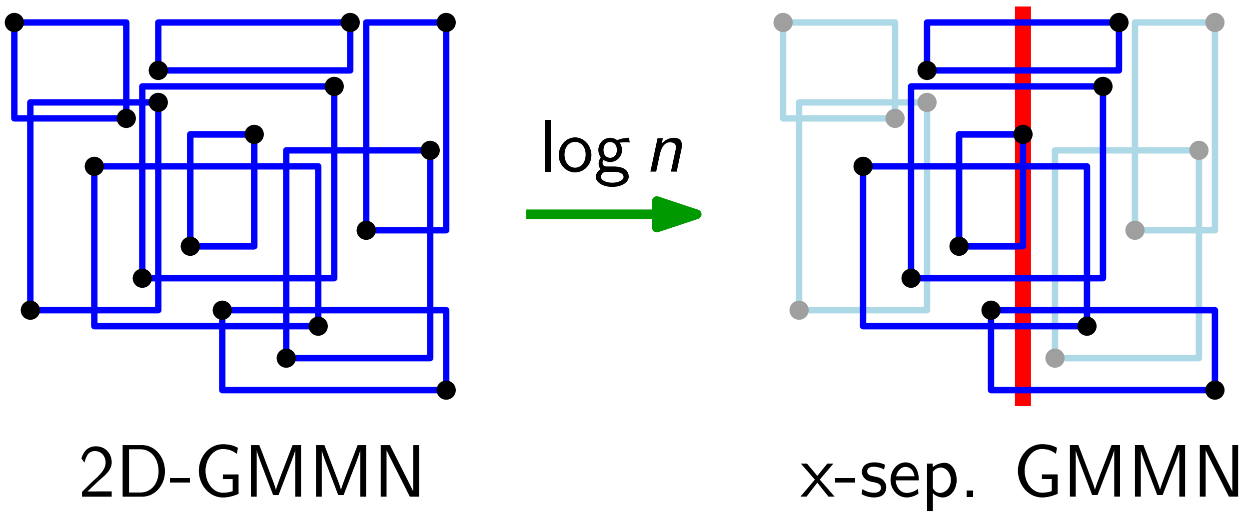
Conclusion & Open Problems in $\mathbb{R}^2$



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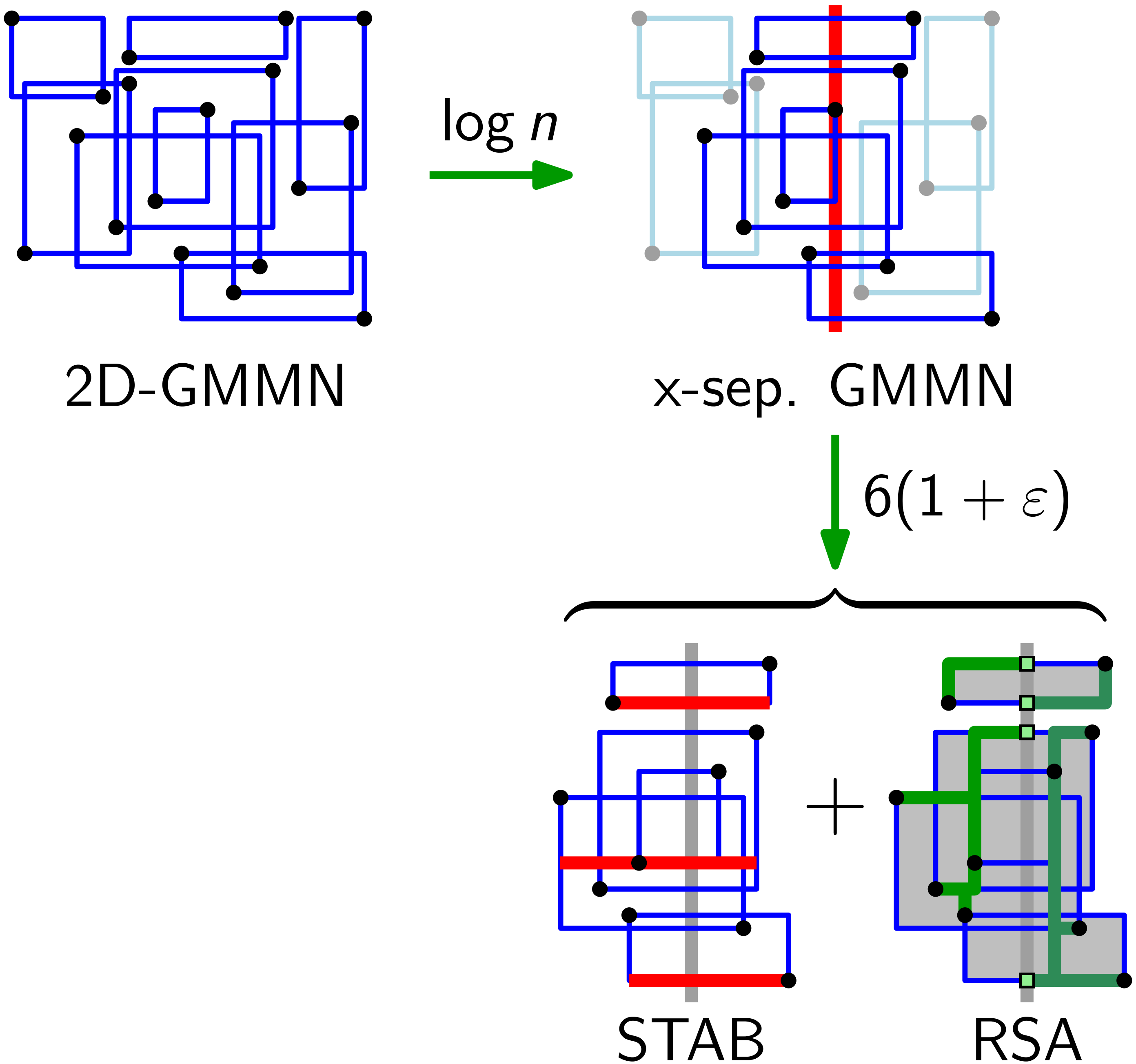

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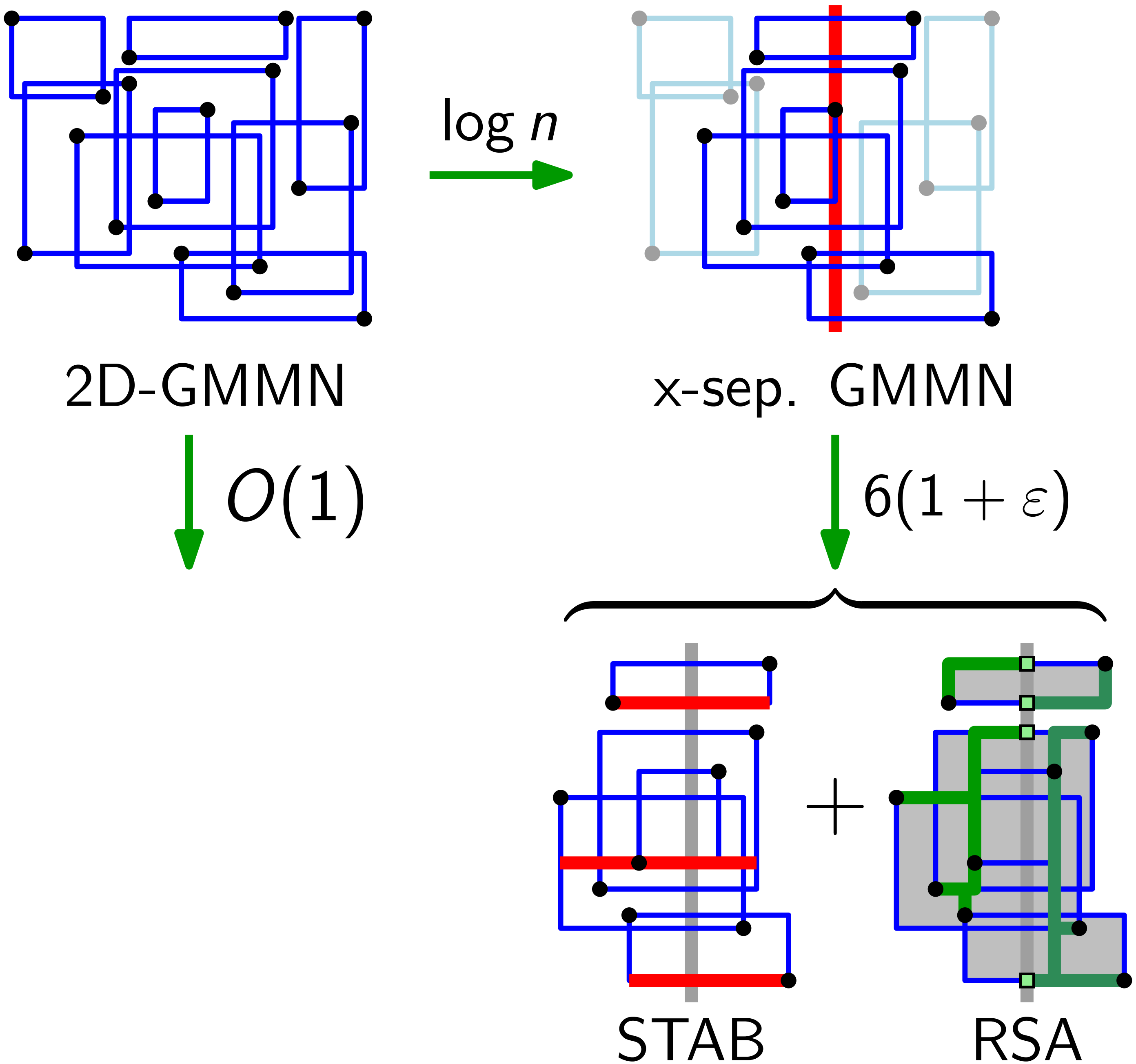

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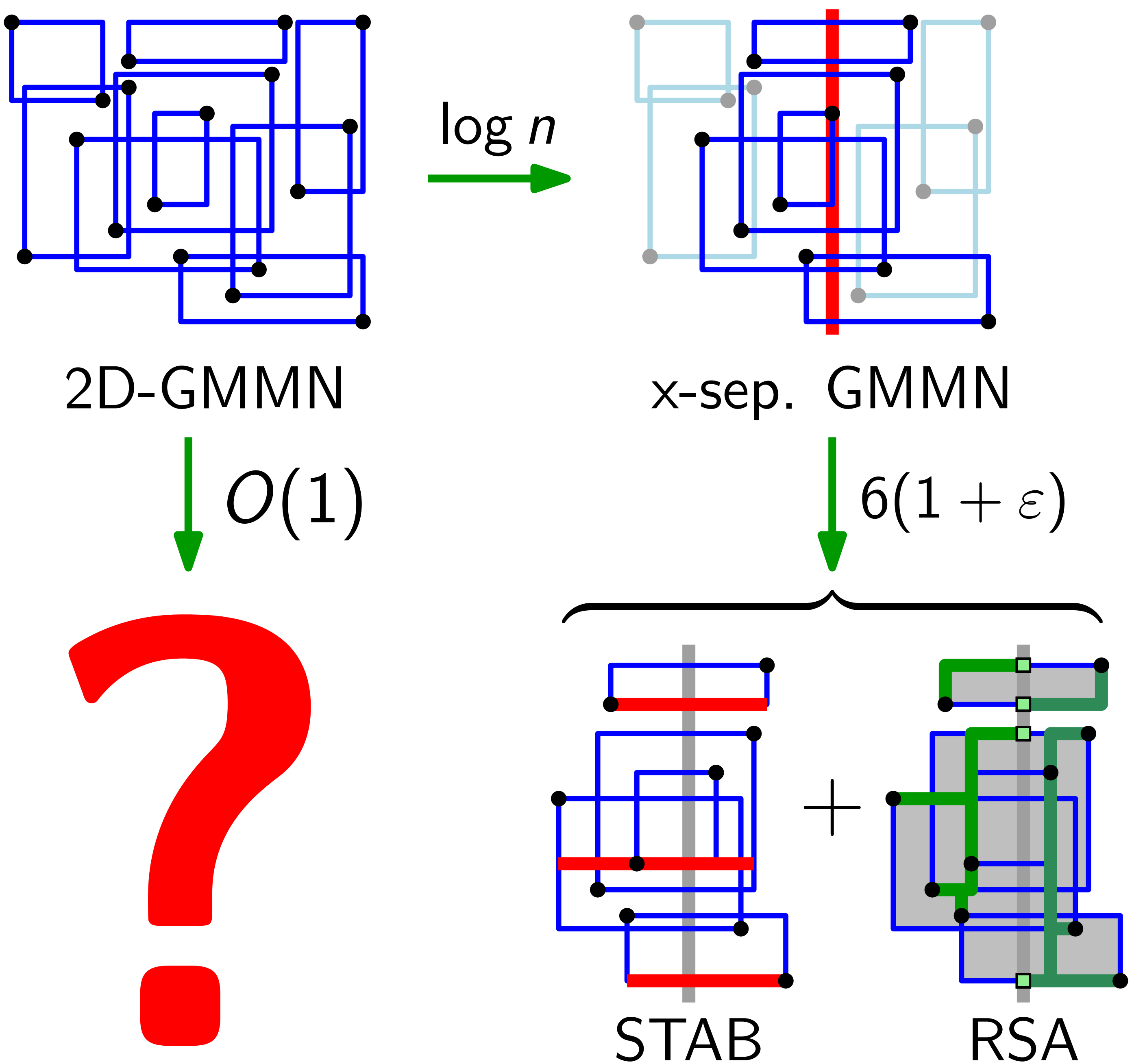
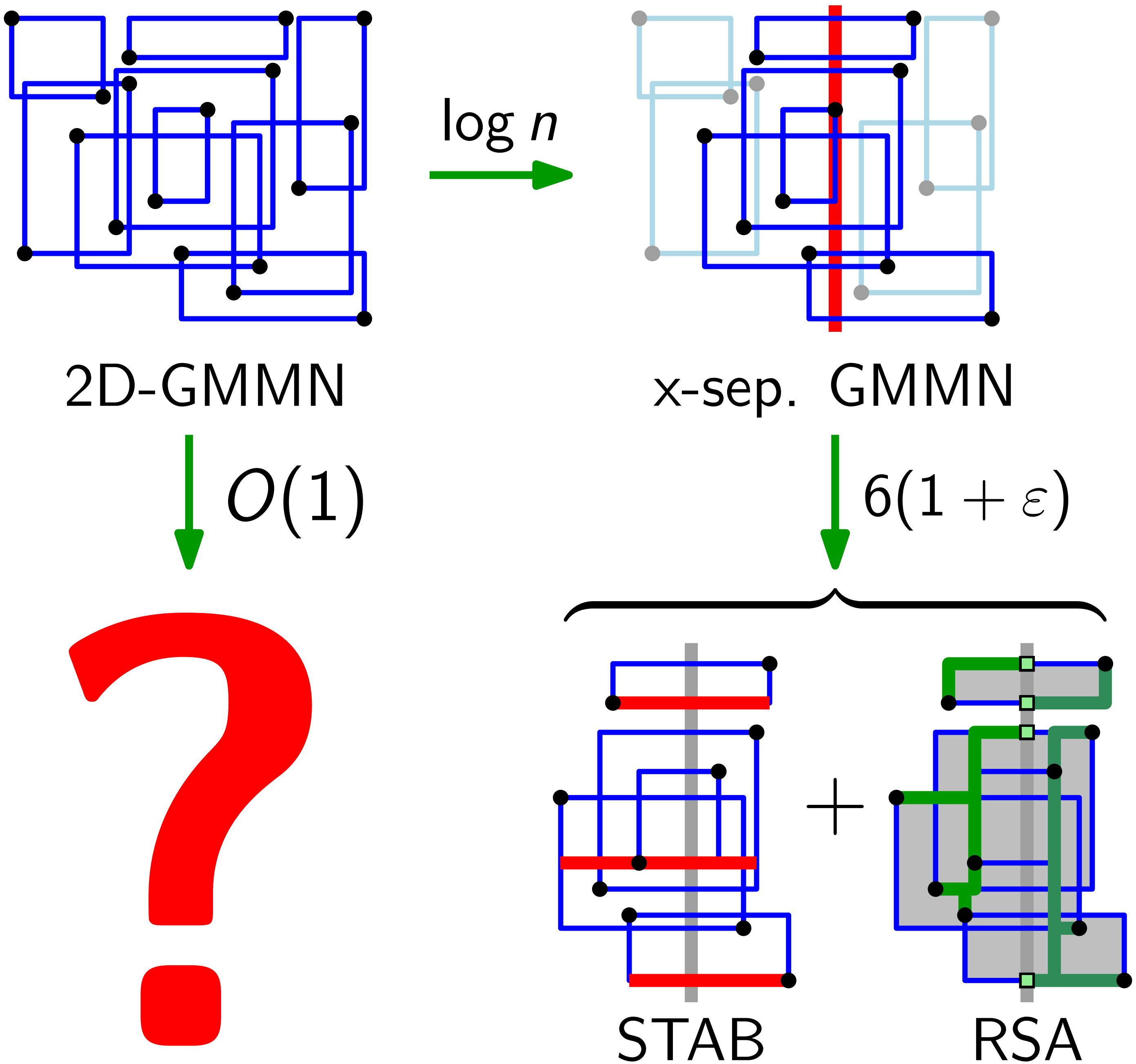


Conclusion & Open Problems in $\mathbb{R}^2$...



...and in $\mathbb{R}^d$

- $O(1)$-approx. for RSA?
- $O(\log^{\text{const}} n)$-approx. for GMMN?