Obstructing Visibilities with One Obstacle

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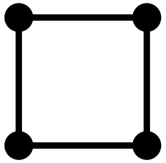
Steven Chaplick, Fabian Lipp, Alexander Wolff
(Universität Würzburg)

Obstacle Number of a Graph



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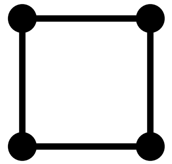

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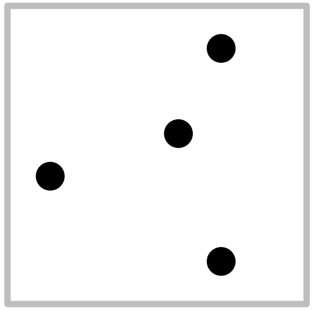

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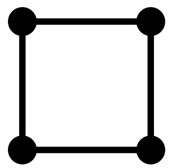

- G: a simple graph
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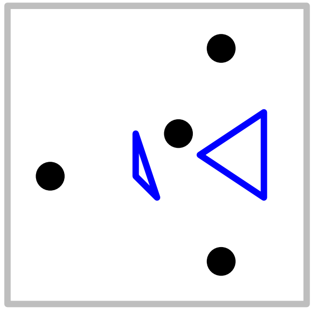
- Obstacles: (open) non-self-intersecting polygons



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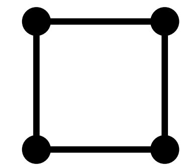

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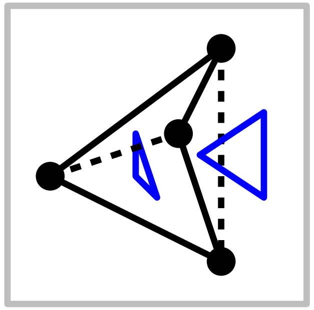

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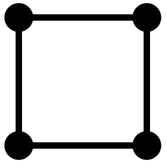

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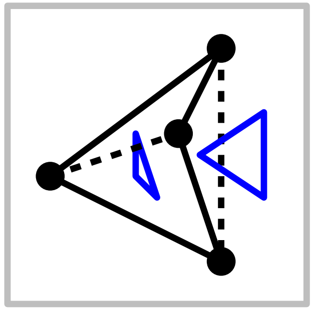

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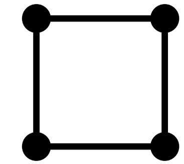

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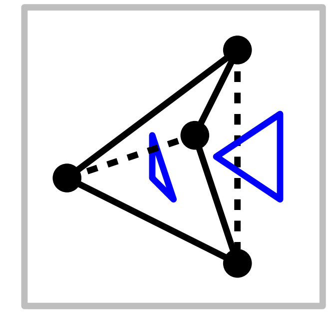

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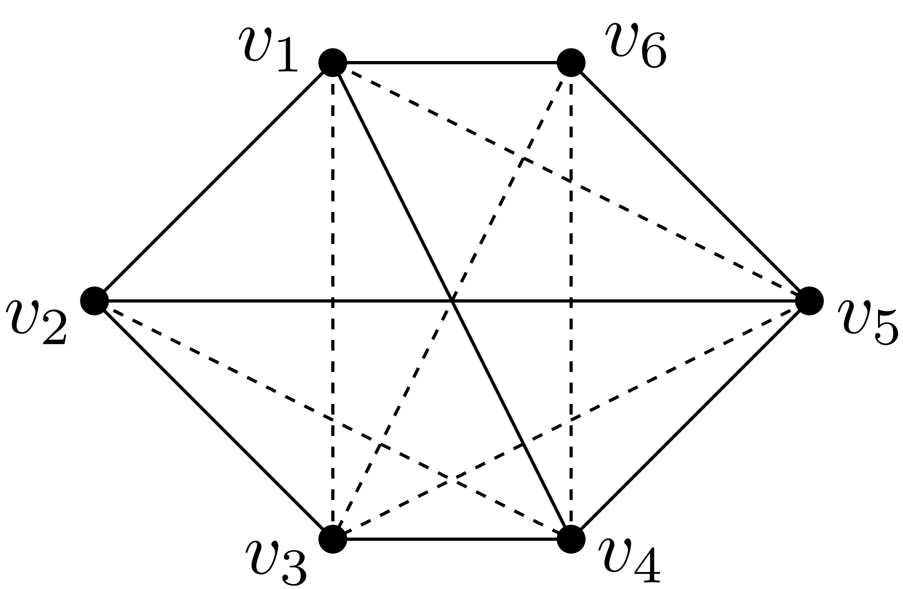

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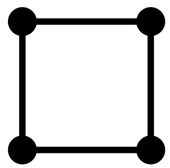

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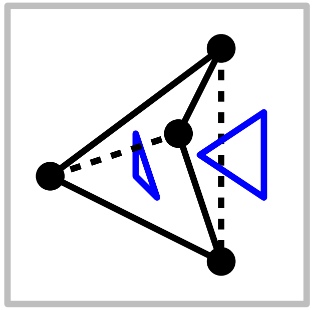

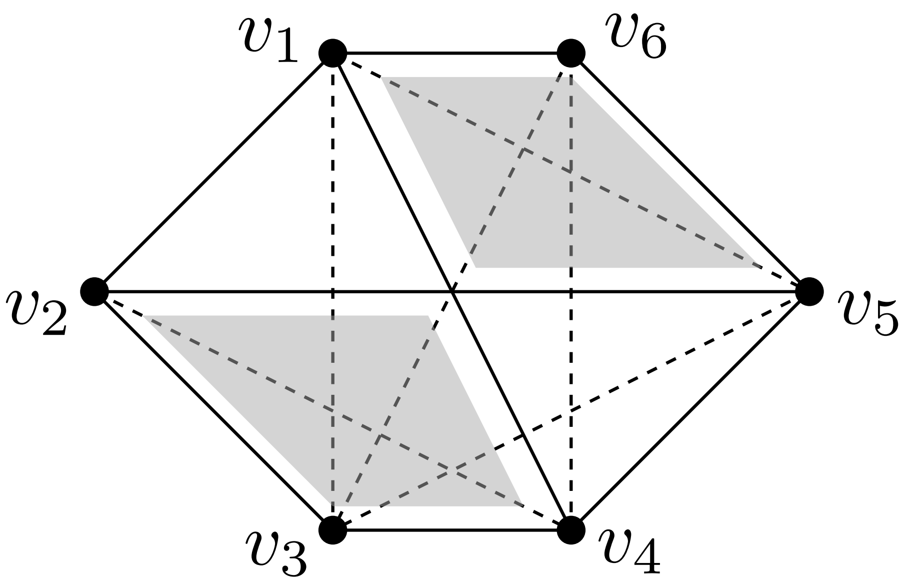

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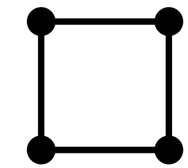

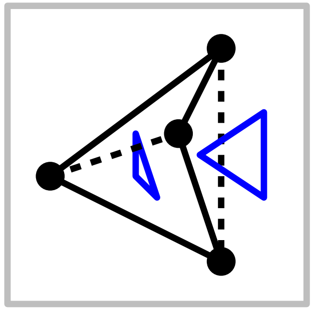

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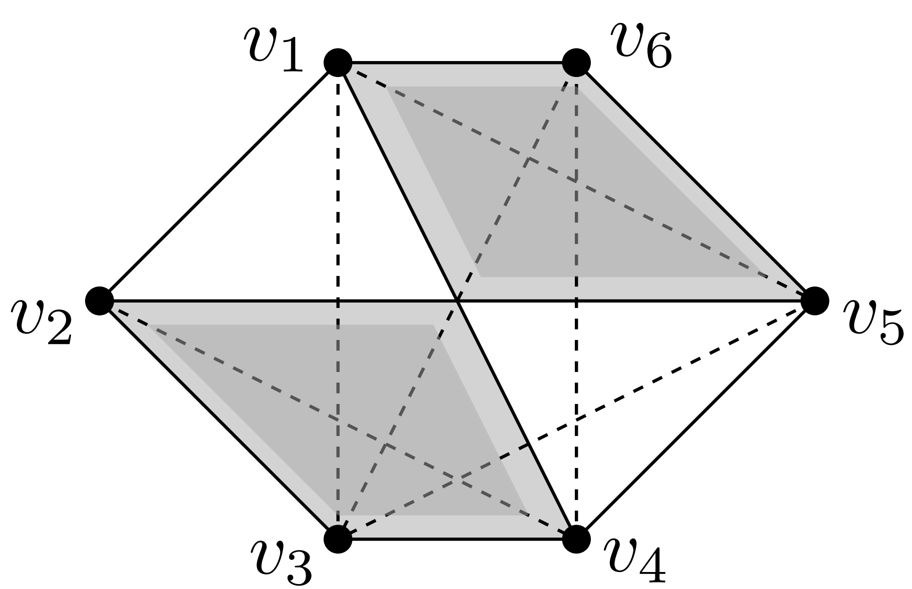

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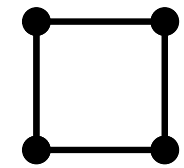

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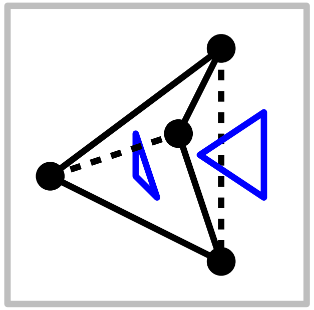

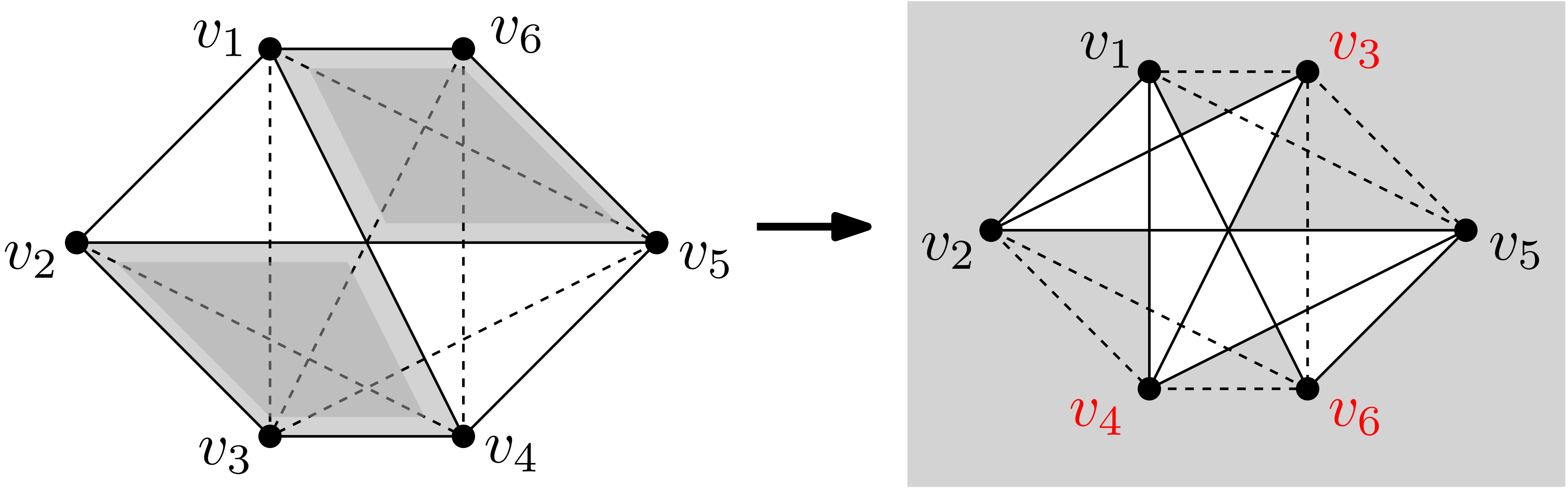

Obstacle Number of a Graph

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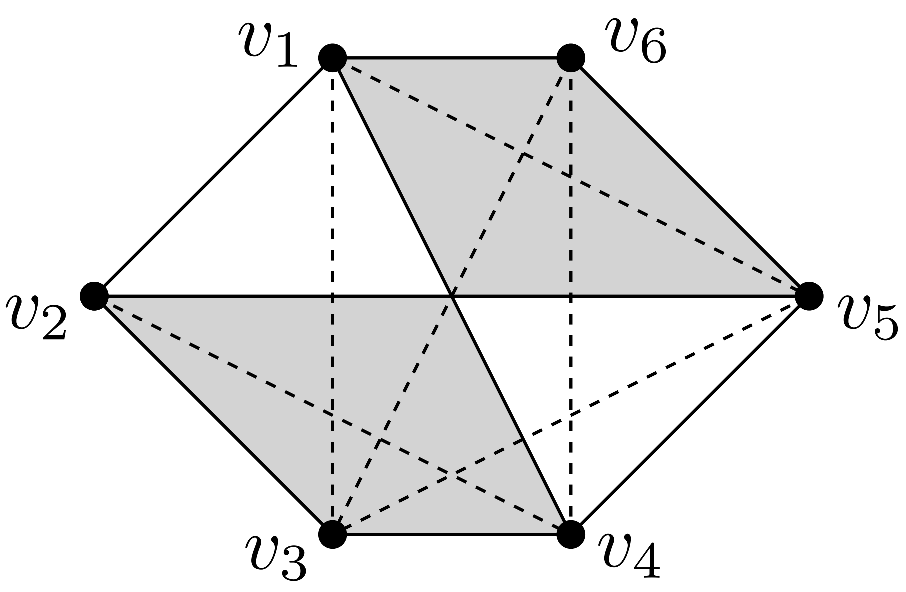
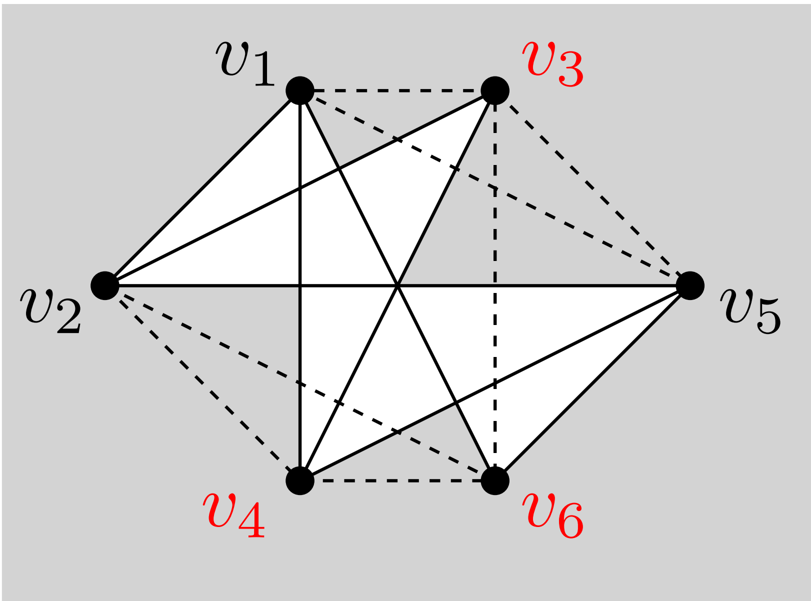
  drawn in the unbounded face





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- Inside obstacle:
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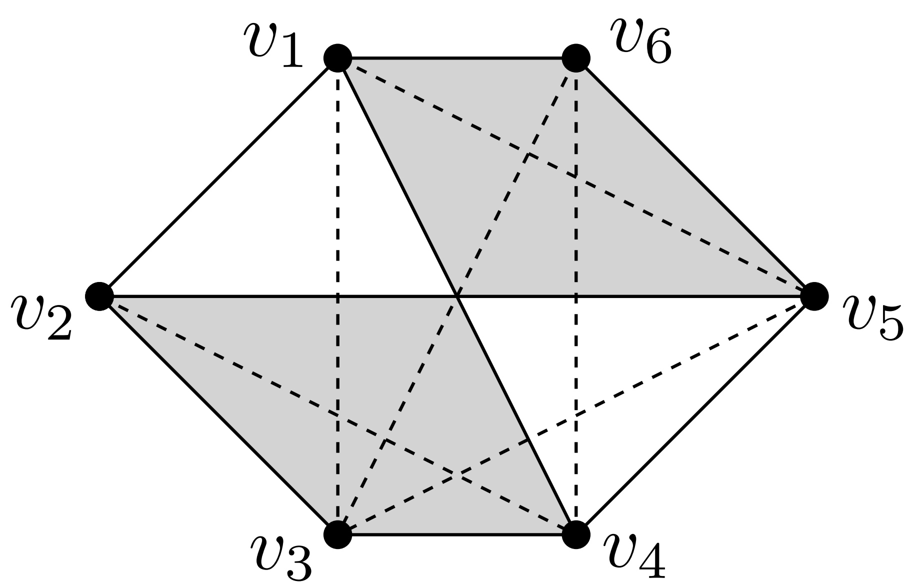
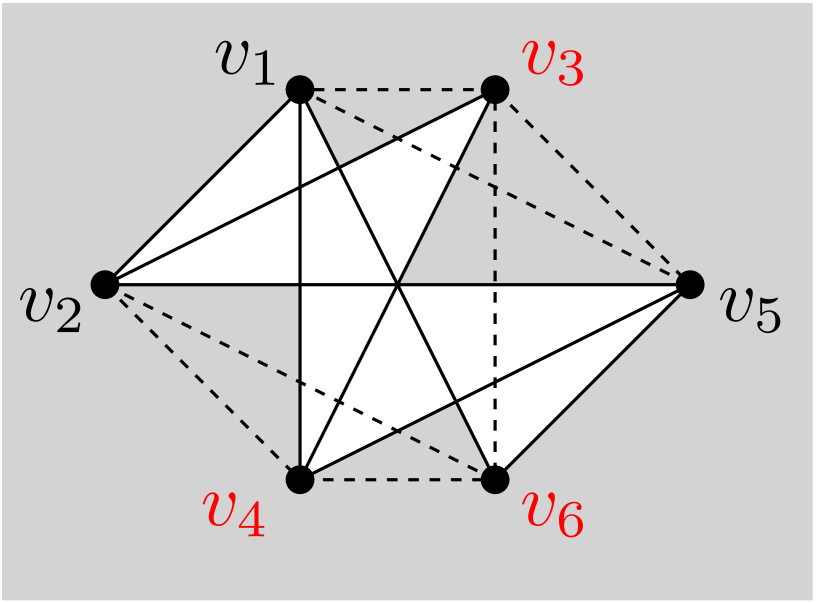
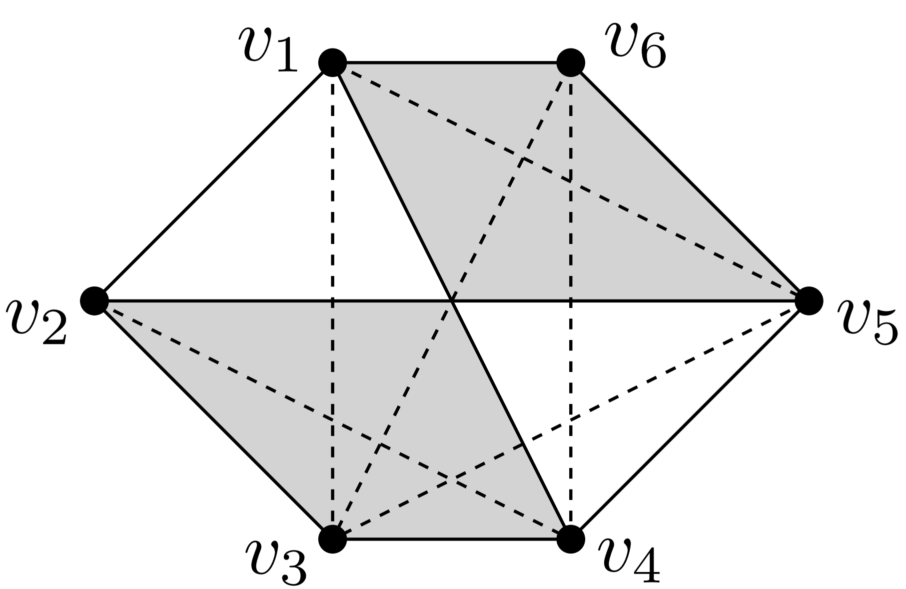
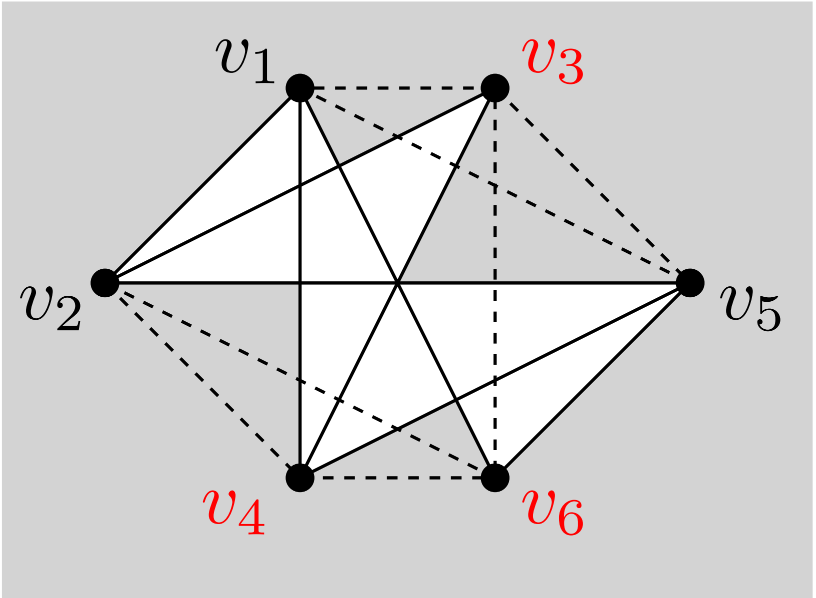




Obstacle Number of a Graph

- Outside obstacle:
  drawn in the unbounded face
- Inside obstacle:
  drawn in the *complement* of the unbounded face
- $\mathrm{obs}_{\mathrm{out}}(G) =$ Obstacle number using an outside obstacle
  $\mathrm{obs}_{\mathrm{in}}(G) =$ Obstacle number only using inside obstacles





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- There are graphs that require $\Omega(n/(\log\log n)^2)$ obstacles. [Dujmović and Morin '15]
- For each m, there exists a graph G s.t. $\text{obs}(G) = m$ [Mukkamala, Pach, Sariöz, WG'10]

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- What is the smallest graph of obstacle number 2?
- $\mathrm{obs}(K^*_{5,5}) = 2$ [Pach, Sariöz, '11]
- Can an outside obstacle and an inside obstacle do different jobs?
  i.e. $\{G : \mathrm{obs}_{\mathrm{out}}(G) = 1\}$ vs. $\{G : \mathrm{obs}_{\mathrm{in}}(G) = 1\}$

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  Given two graphs G and H, it is NP-hard to decide the existence of a graph K s.t. $G \subset K \subset H$ and $\mathrm{obs}(K) = 1$.

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- The following problems are all NP-hard:
  The outside-obstacle graph sandwich problem
  The inside-obstacle graph sandwich problem
  The simple-polygon visibility graph sandwich problem

Graphs of Obstacle Number 1

Thm. Every outerplanar graph has an outside-obstacle representation. [Alpert, Koch, Laison, '09]

Thm. Graphs represented by 1 convex polygon are non-double covering circular arc graphs. [Alpert, Koch, Laison, '09]

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Thm. Any graph whose longest cycle has length ≤ 6 has an outside-obstacle representation.

Thm. Any graph with at most 7 vertices has an outside-obstacle representation.

Co-bipartite Graphs

Let G be a co-bipartite graph with a co-bipartition Z, Z' with $\mathrm{obs}_{\mathrm{out}}(G) = 1$.
(A co-bipartite graph is the complement of a bipartite graph)

Obs. $\mathrm{CH}(Z)$ and $\mathrm{CH}(Z')$ cannot be pierced by the outside obstacle.

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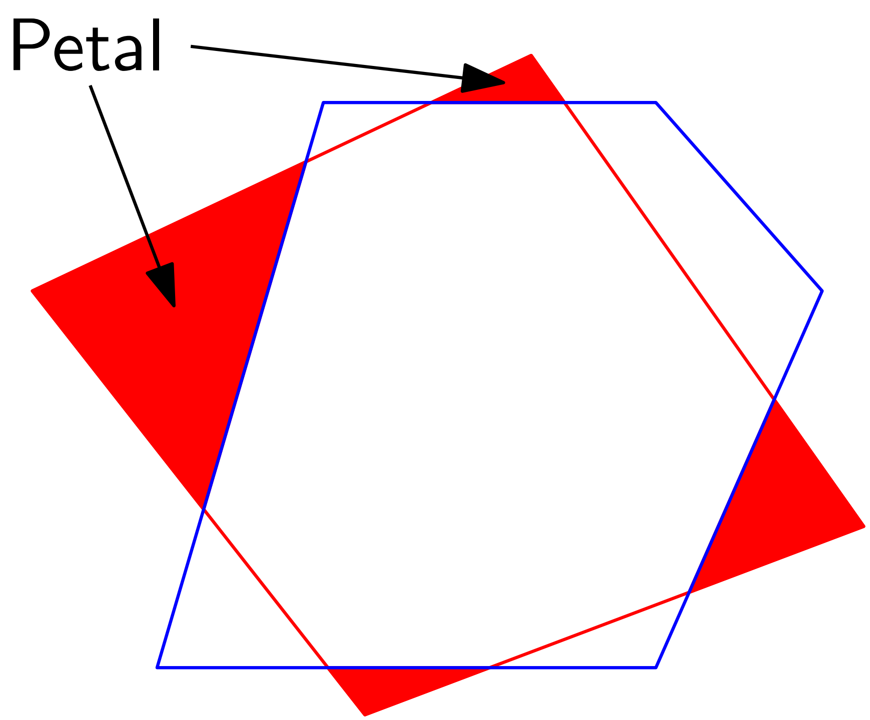
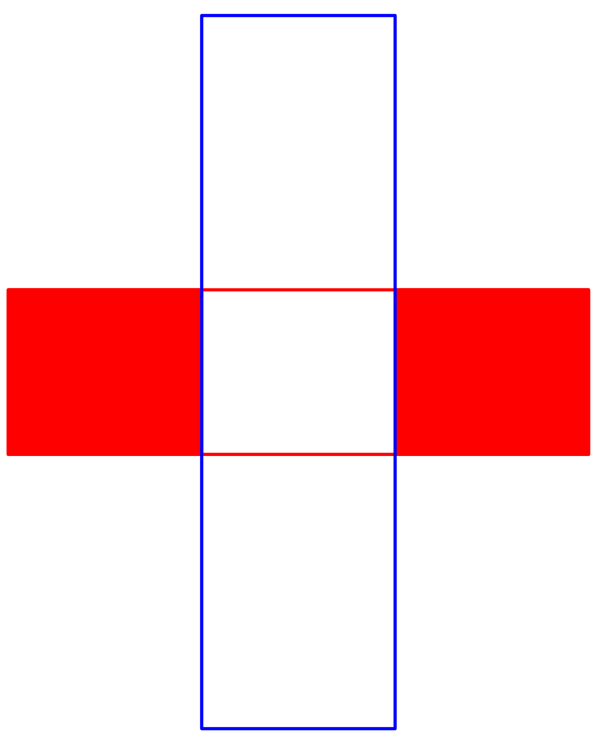
Let G be a co-bipartite graph with a co-bipartition Z, Z' with $\mathrm{obs}_{\mathrm{out}}(G) = 1$.

Def. $\mathrm{CH}(Z)$ and $\mathrm{CH}(Z')$ are k-crossing if $\mathrm{CH}(Z) \setminus \mathrm{CH}(Z')$ consists of $k+1$ disjoint regions.



3-crossing



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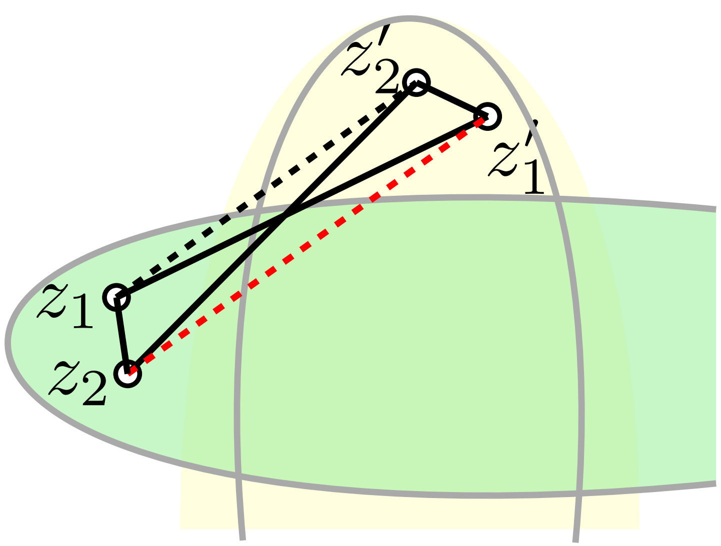
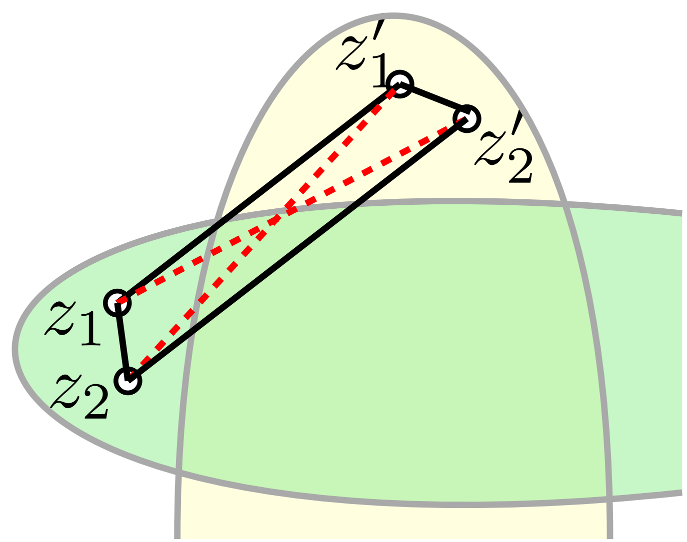
Lemma. Suppose $\mathrm{CH}(Z)$ and $\mathrm{CH}(Z')$ are 1-crossing. If G contains an induced 4-cycle $z_1 z'_1 z'_2 z_2$ where $\{z_1, z_2\} \subseteq Z$, $\{z'_1, z'_2\} \subseteq Z'$, then either z_1 and z_2 or z'_1 and z'_2 are in different petals.





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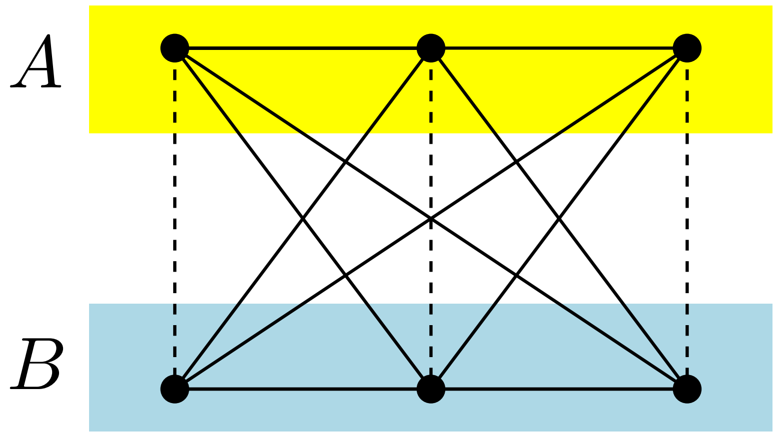
Lemma. Let A, B be a co-bipartition of K_6^*.
Then $\mathrm{CH}(A)$ and $\mathrm{CH}(B)$ are at least 1-crossing in any outside-obstacle representation.
Moreover, if G contains K_6^* as an induced subgraph, then $\mathrm{CH}(Z)$ and $\mathrm{CH}(Z')$ are at least 1-crossing.



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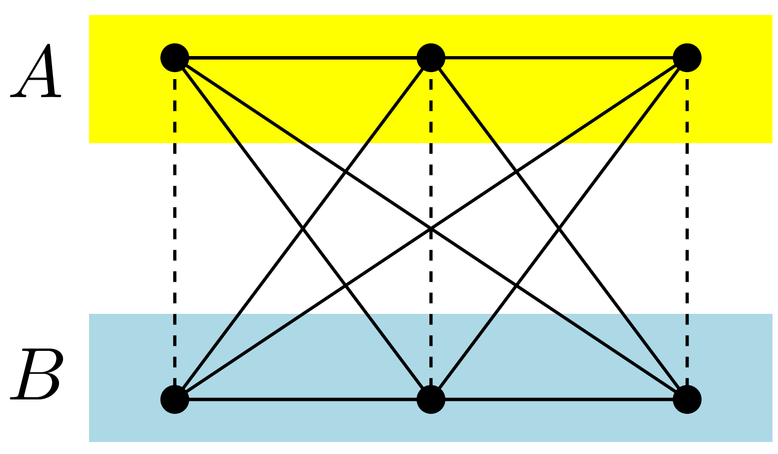

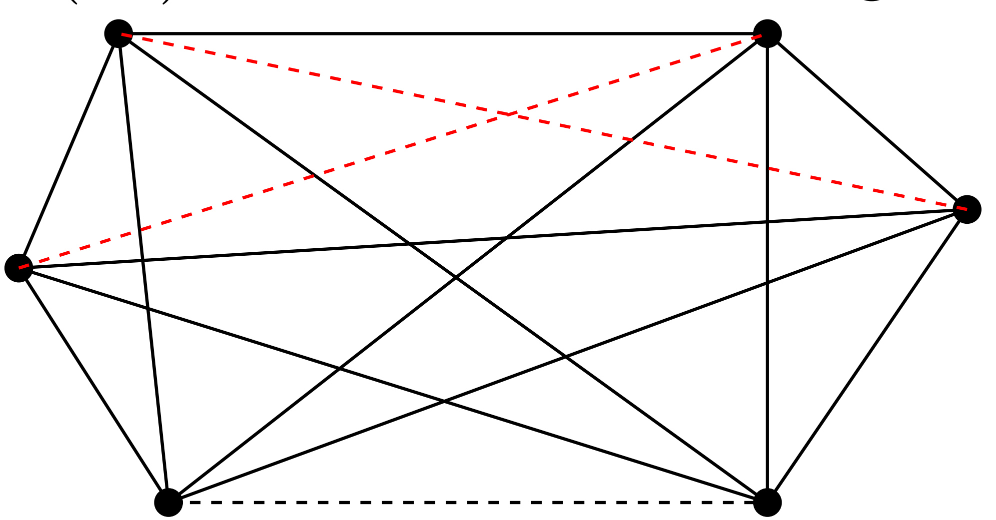

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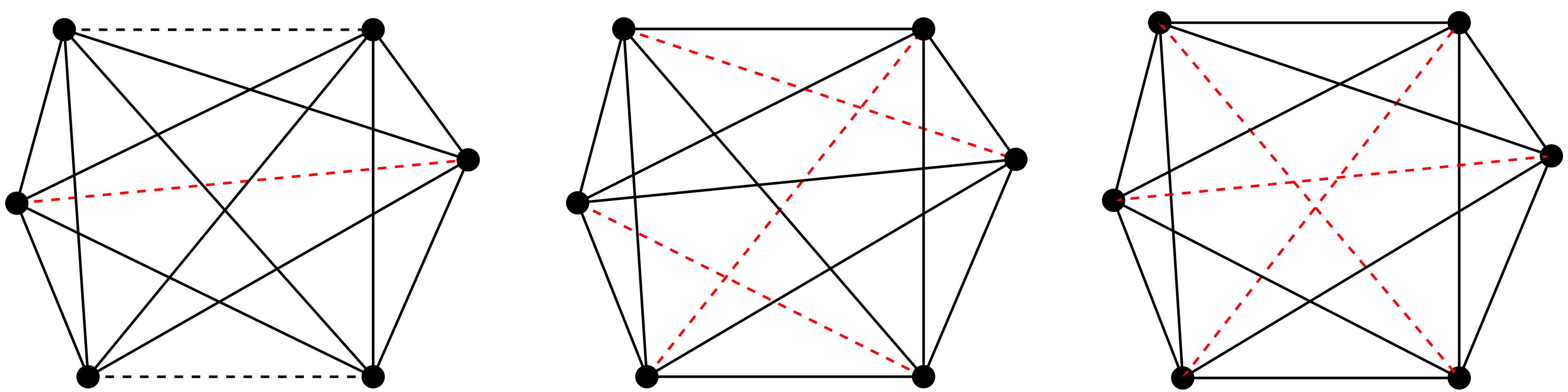
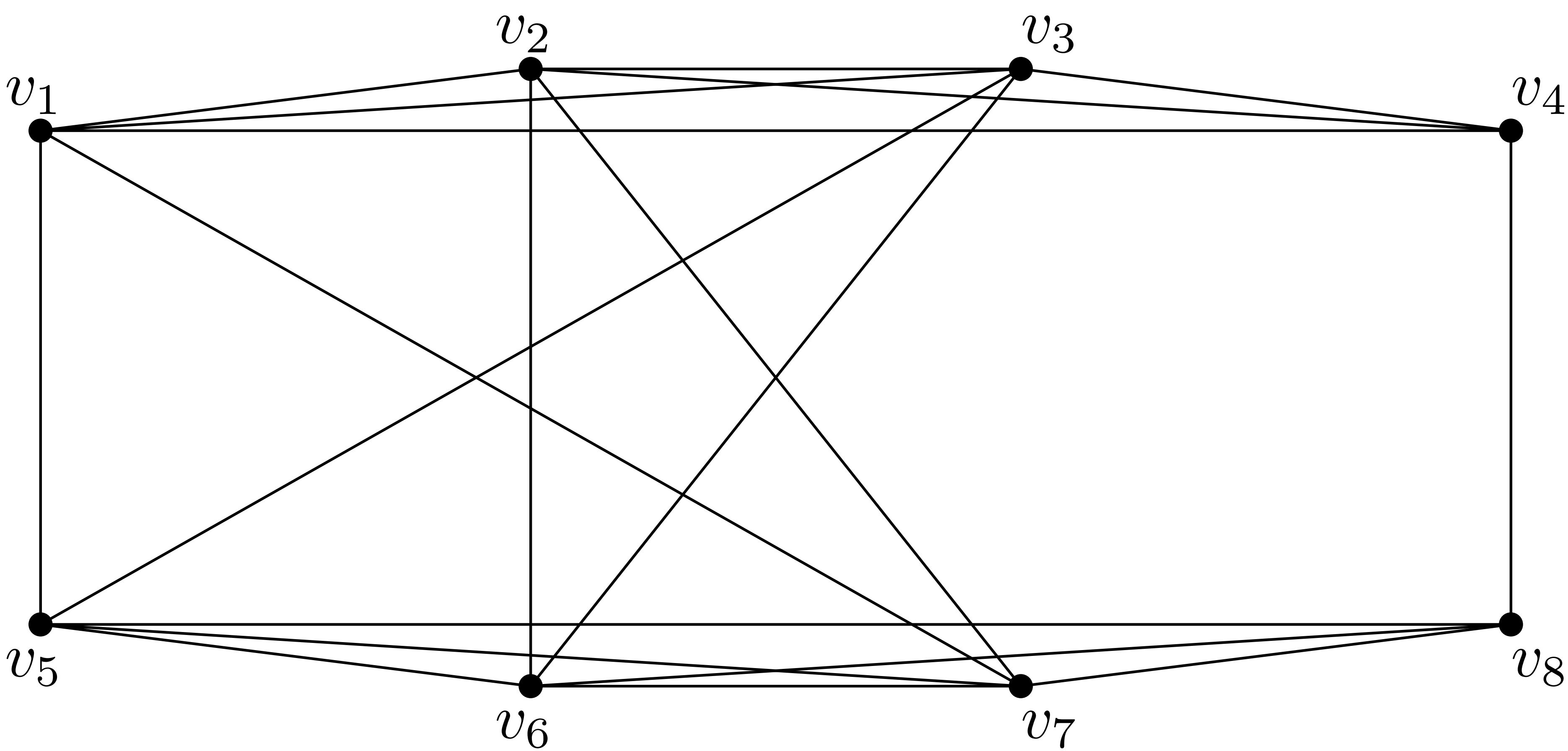


Smallest Graph of Obstacle Number 2

Thm. The smallest graph of obstacle number 2 has 8 vertices.



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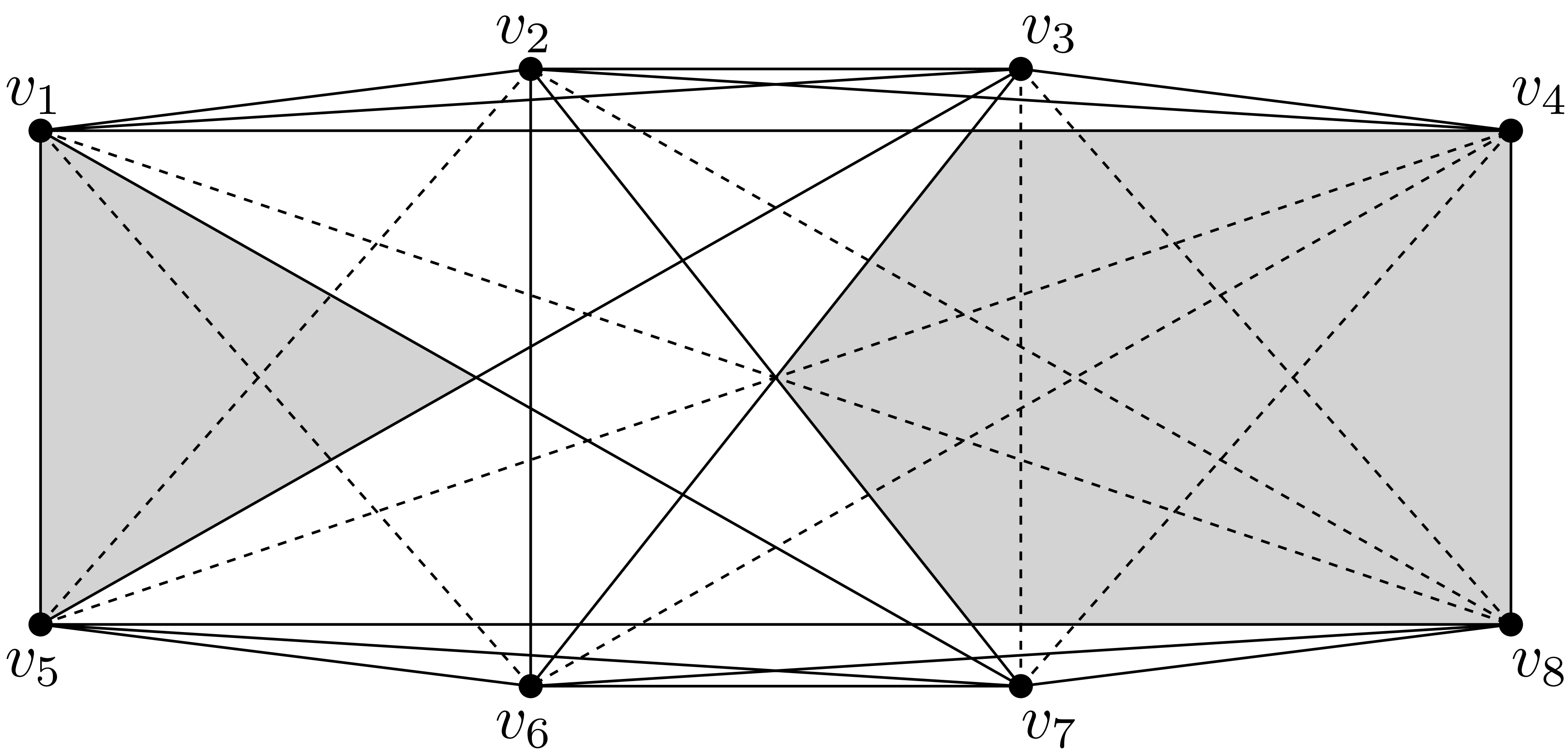

Proof. 1) $\mathrm{obs}(G) \leq 2$.

2) Every graph with at most 7 vertices has obstacle number 1.

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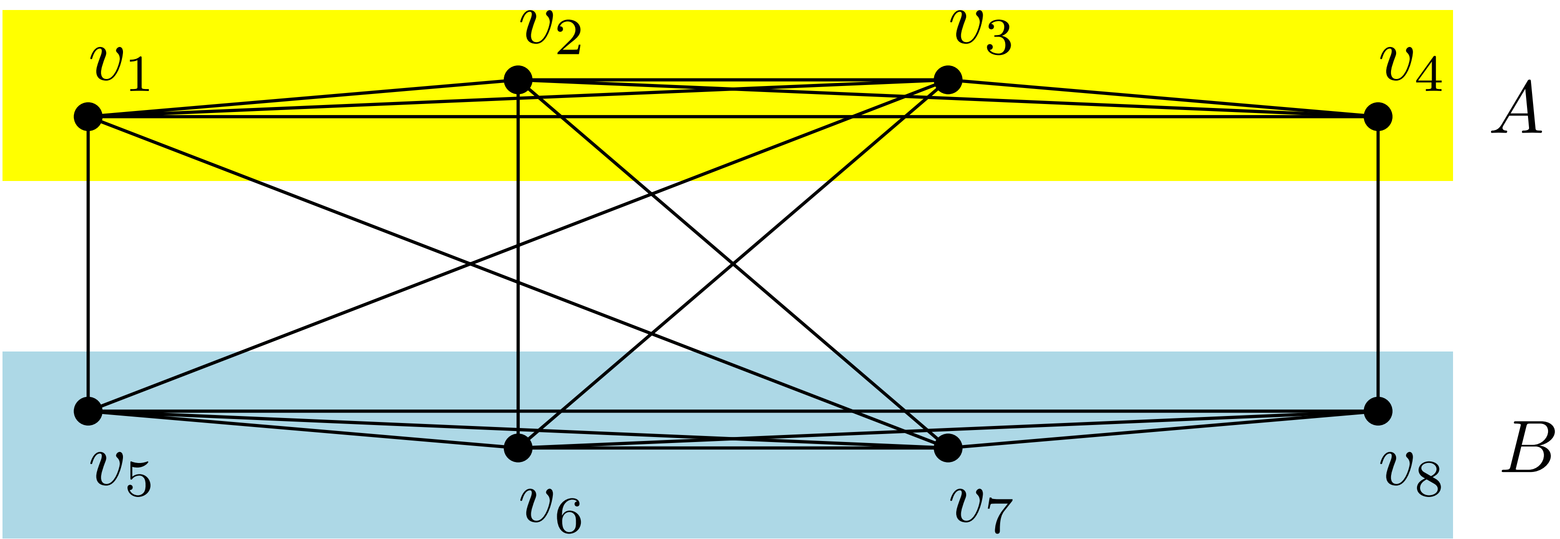

Proof. 3) $\mathrm{obs}_{\mathrm{out}}(G) > 1$

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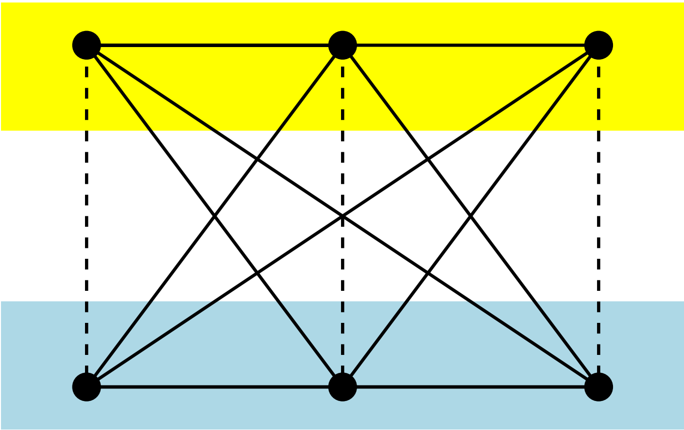

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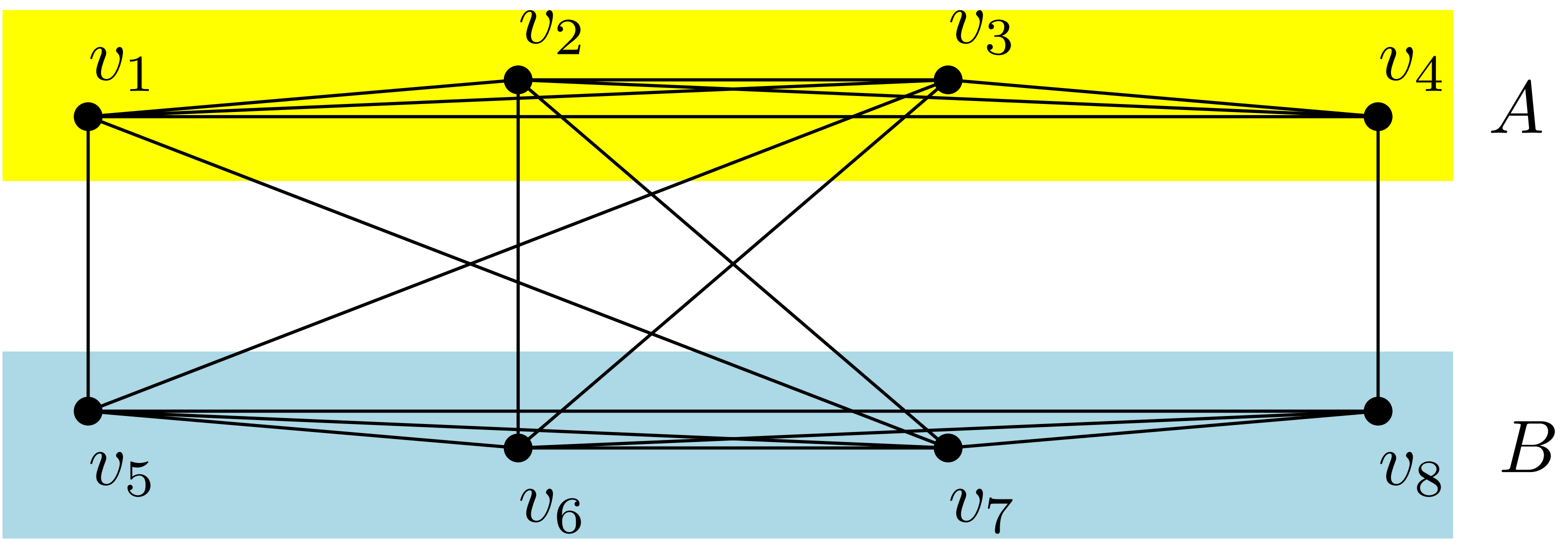

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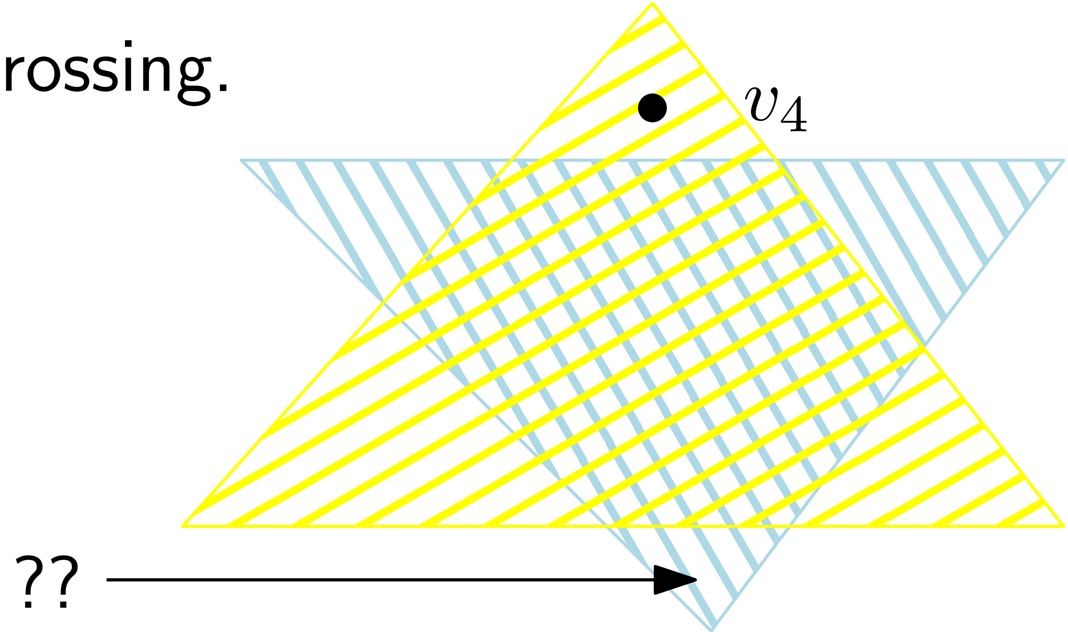

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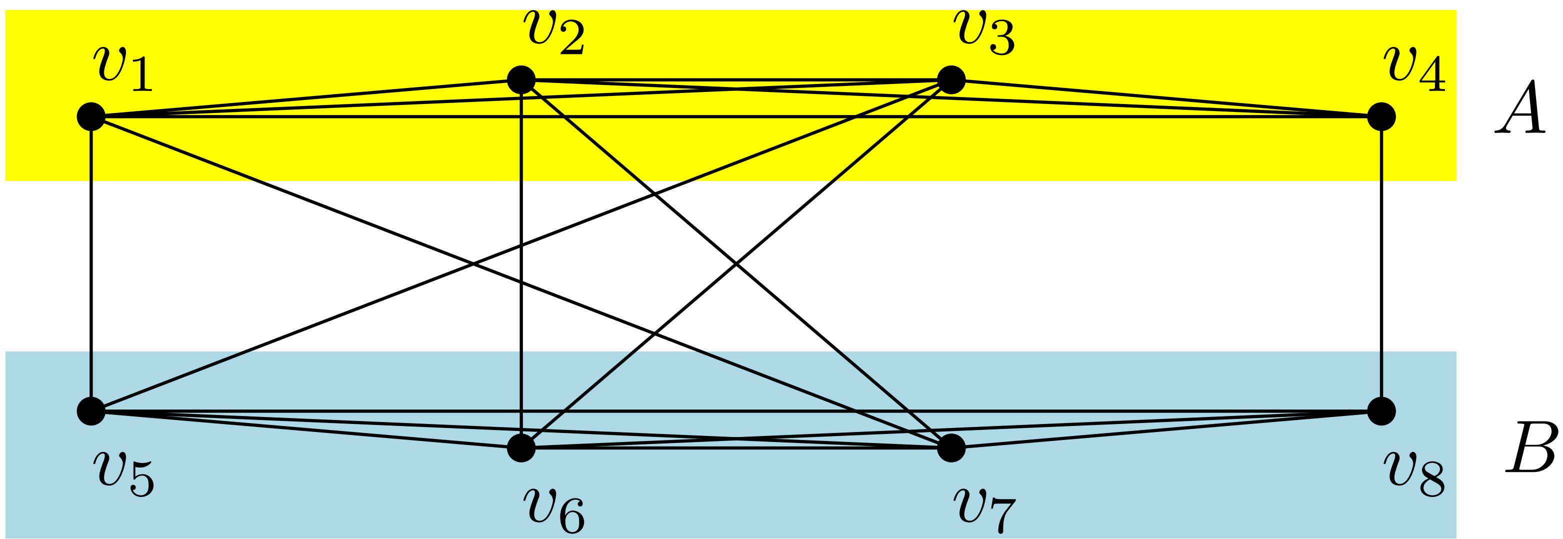

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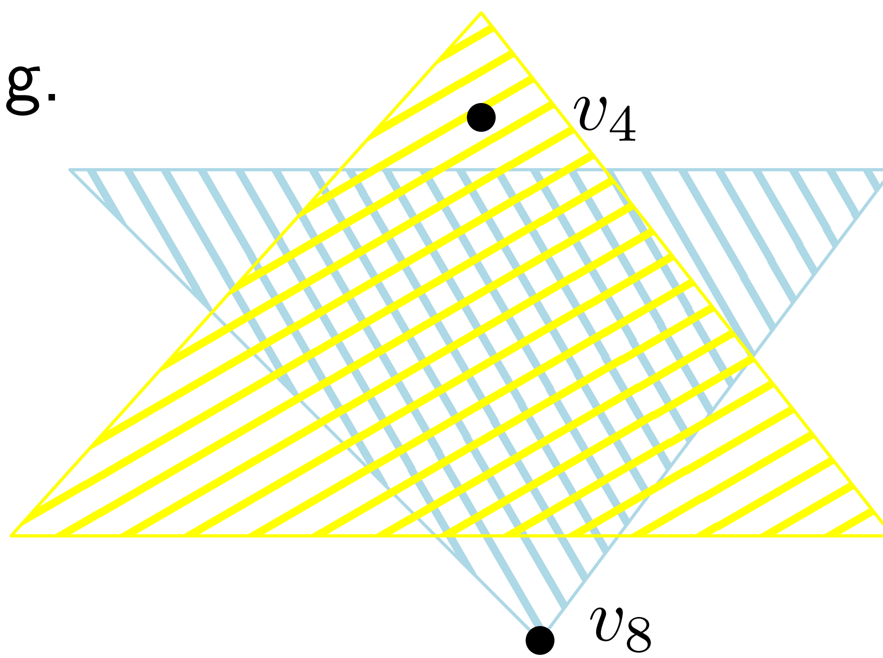

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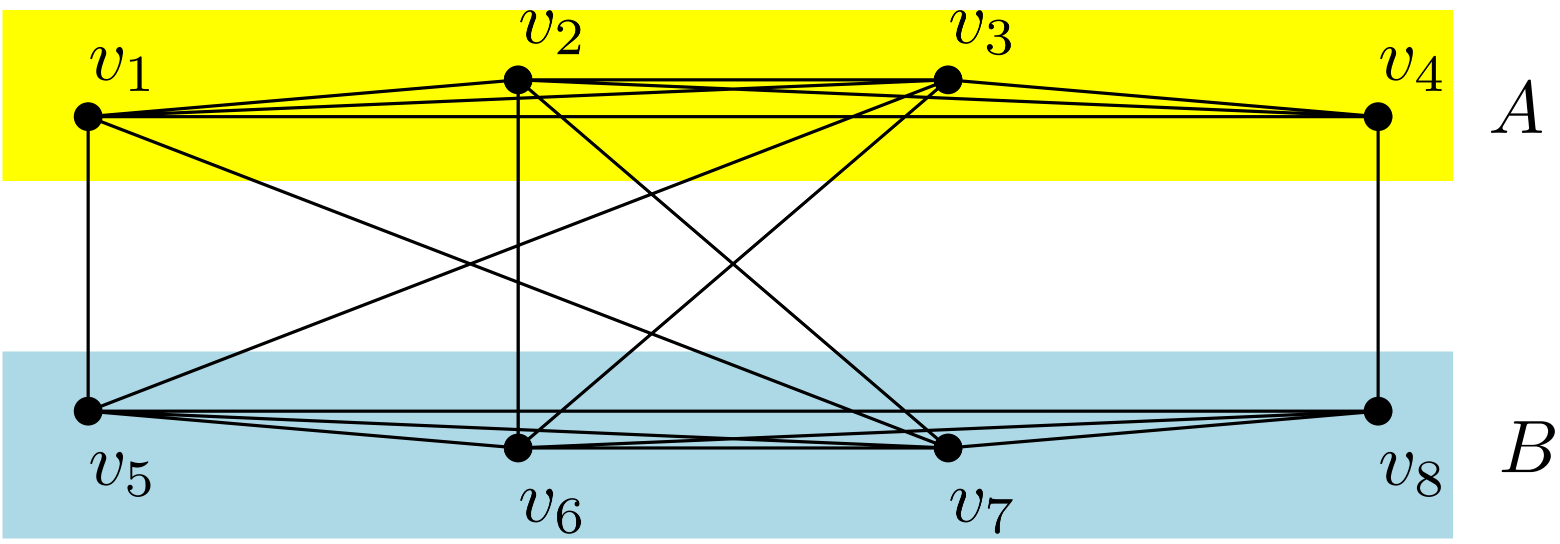

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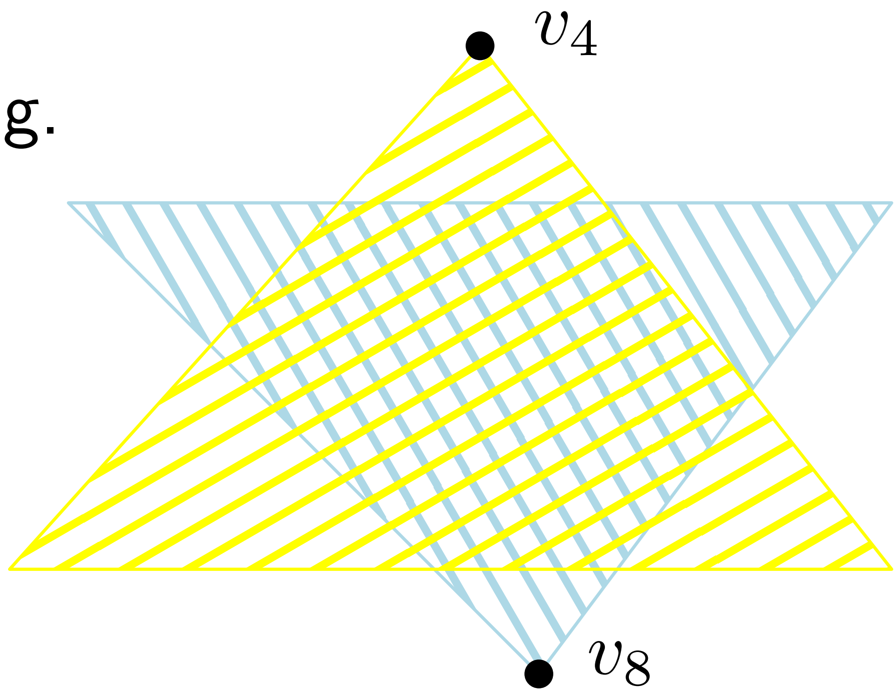

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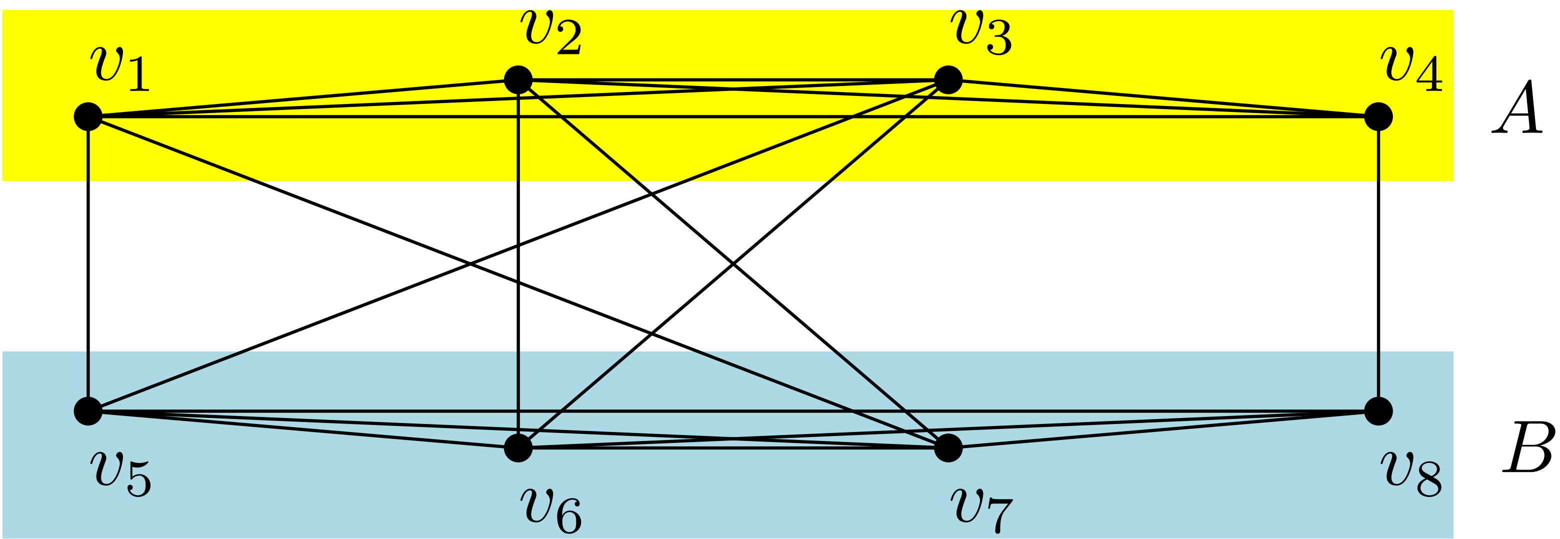

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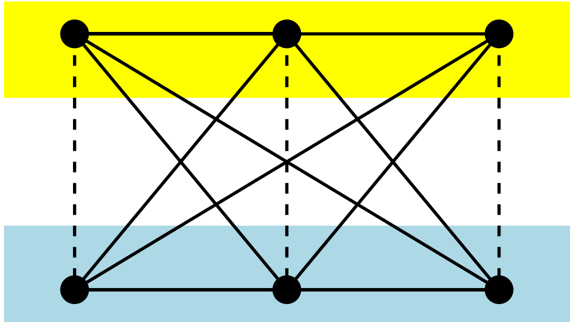
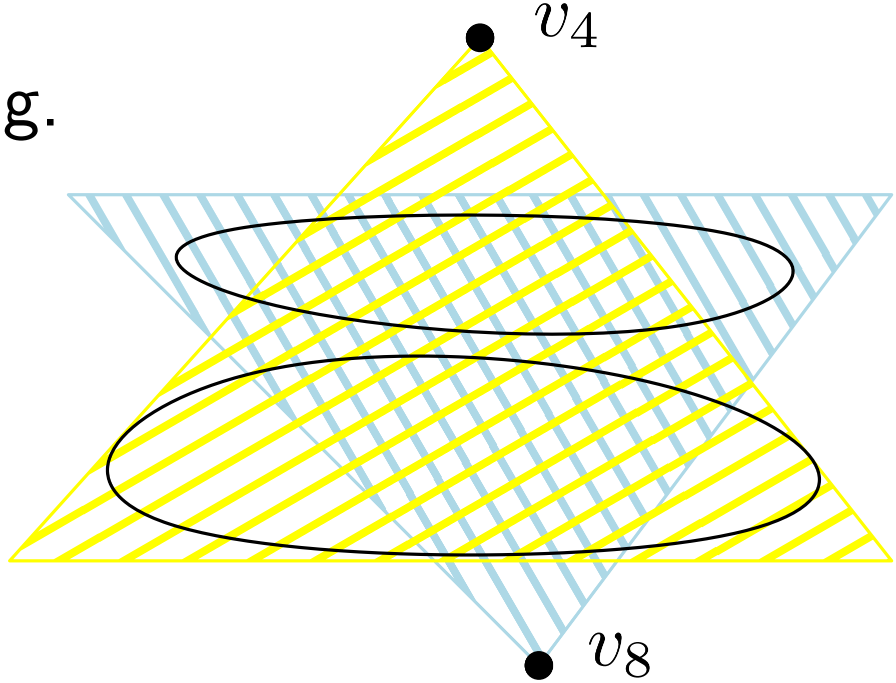
Consider $G - \{v_4, v_8\}$.





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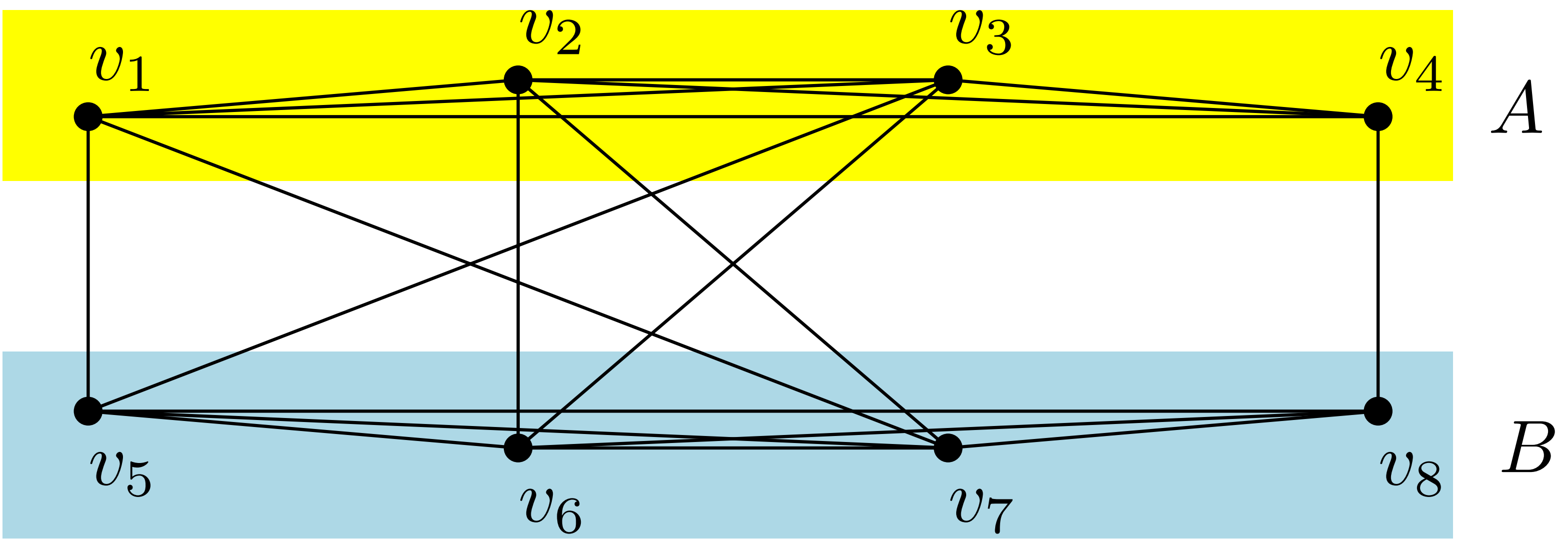

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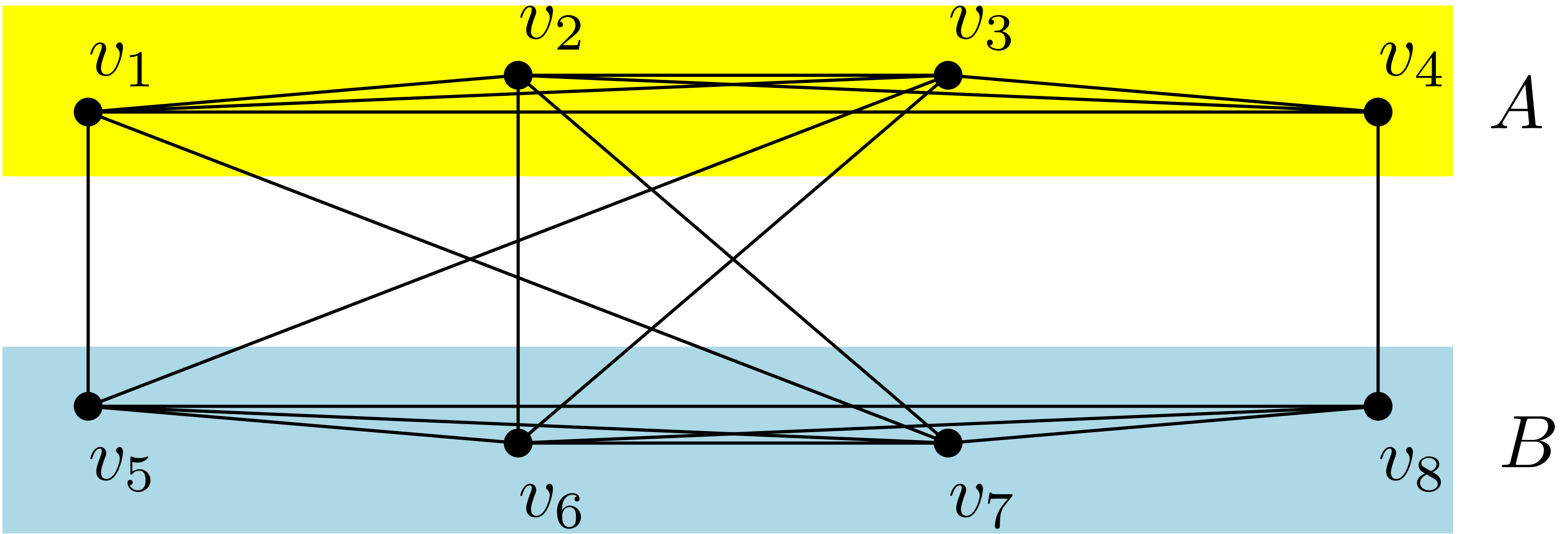

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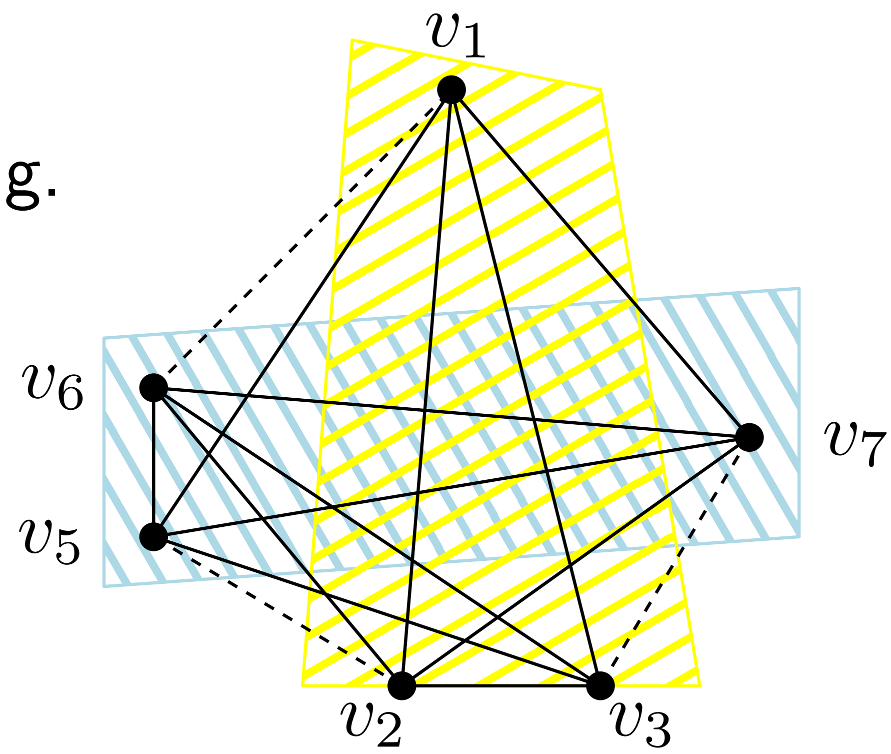
Cannot add v_4, v_8.



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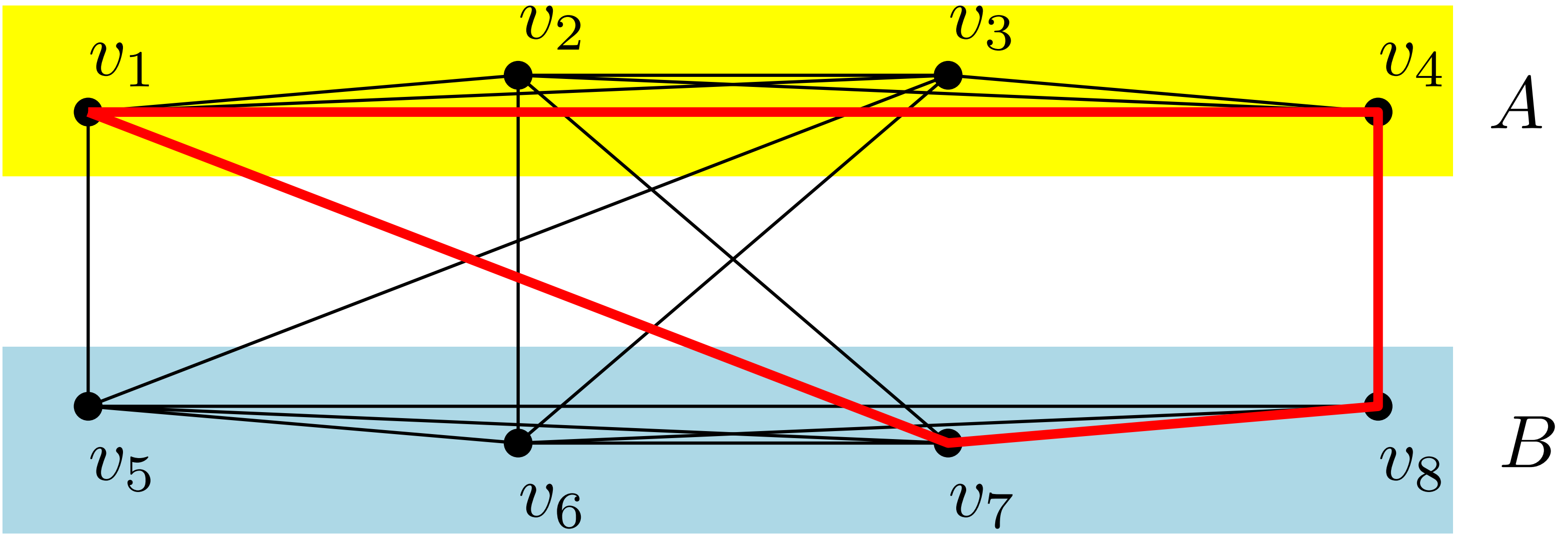

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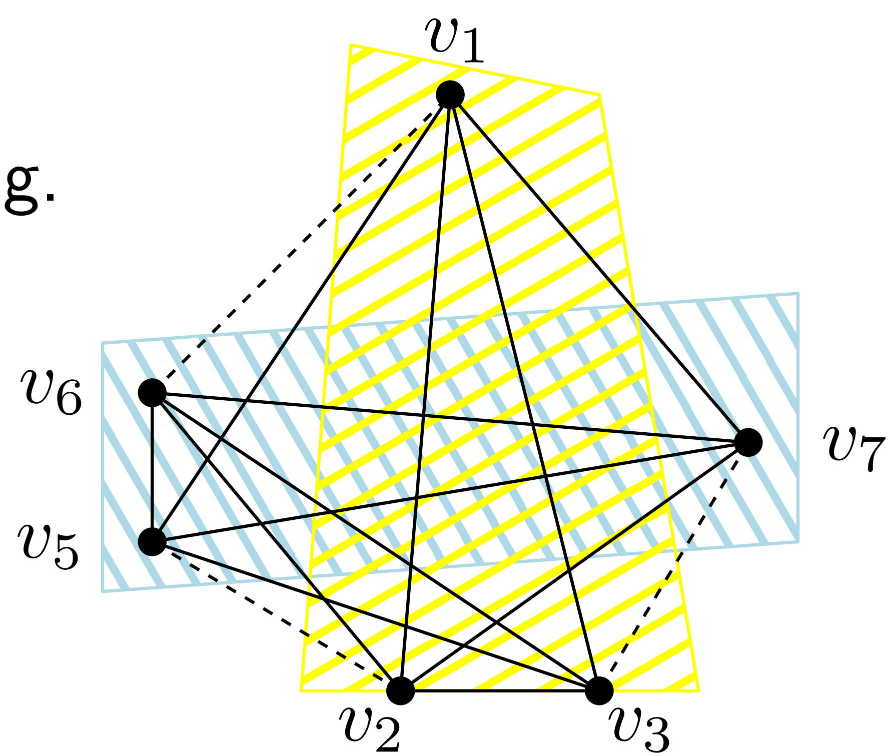

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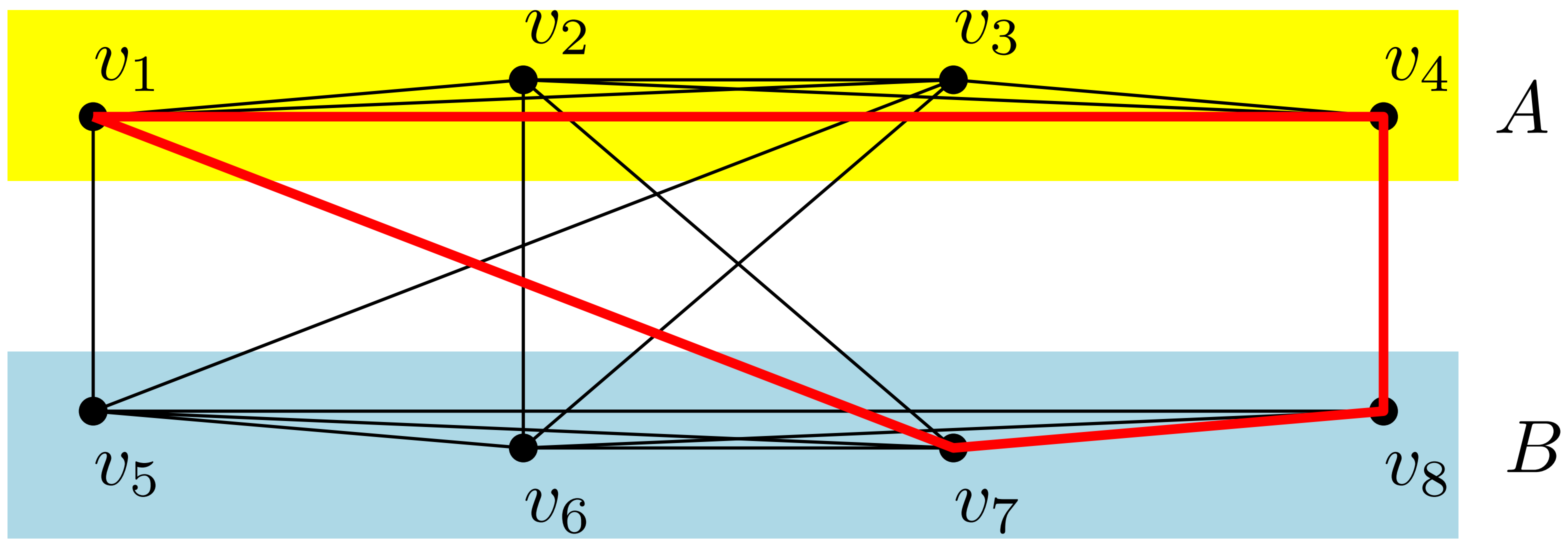

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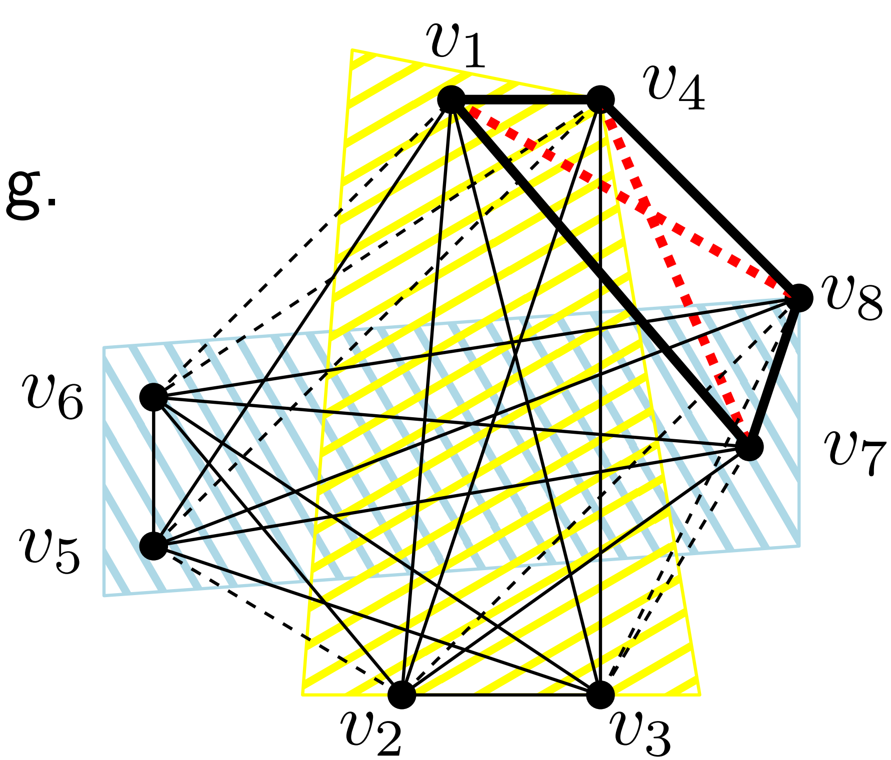

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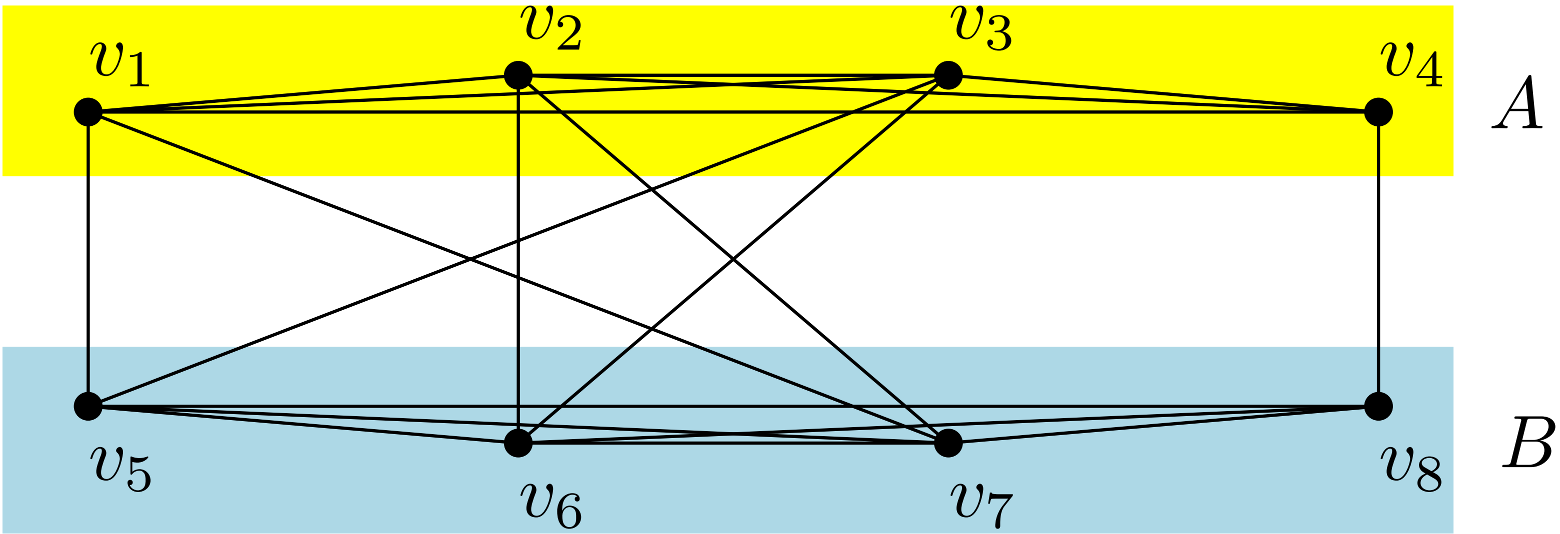

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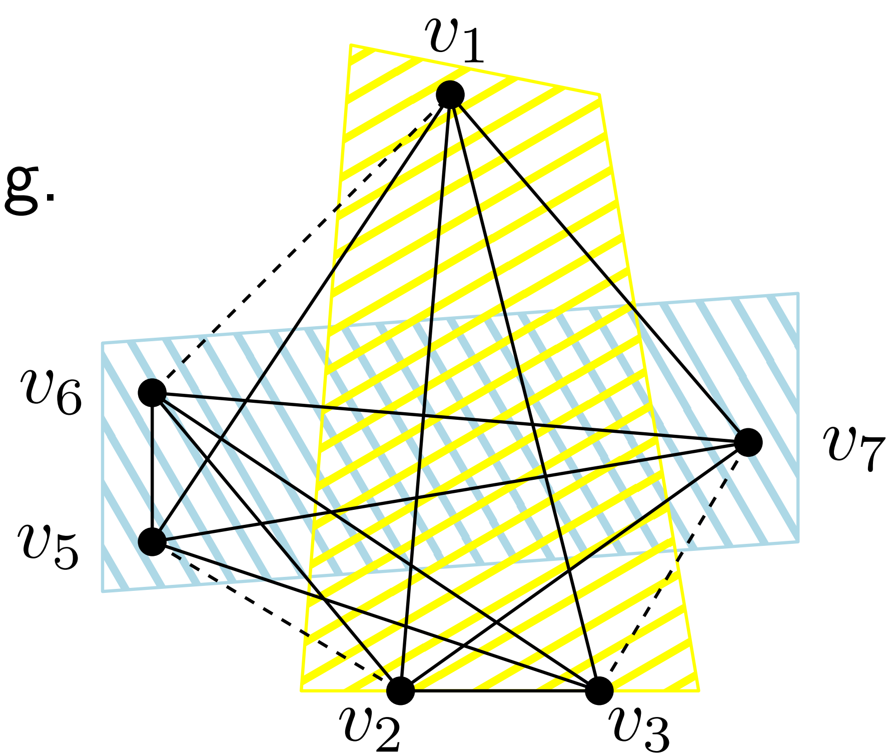
Induced 4-cycles $v_1v_4v_8v_7$, $v_1v_4v_8v_5$, $v_2v_4v_8v_6$, $v_2v_4v_8v_7$



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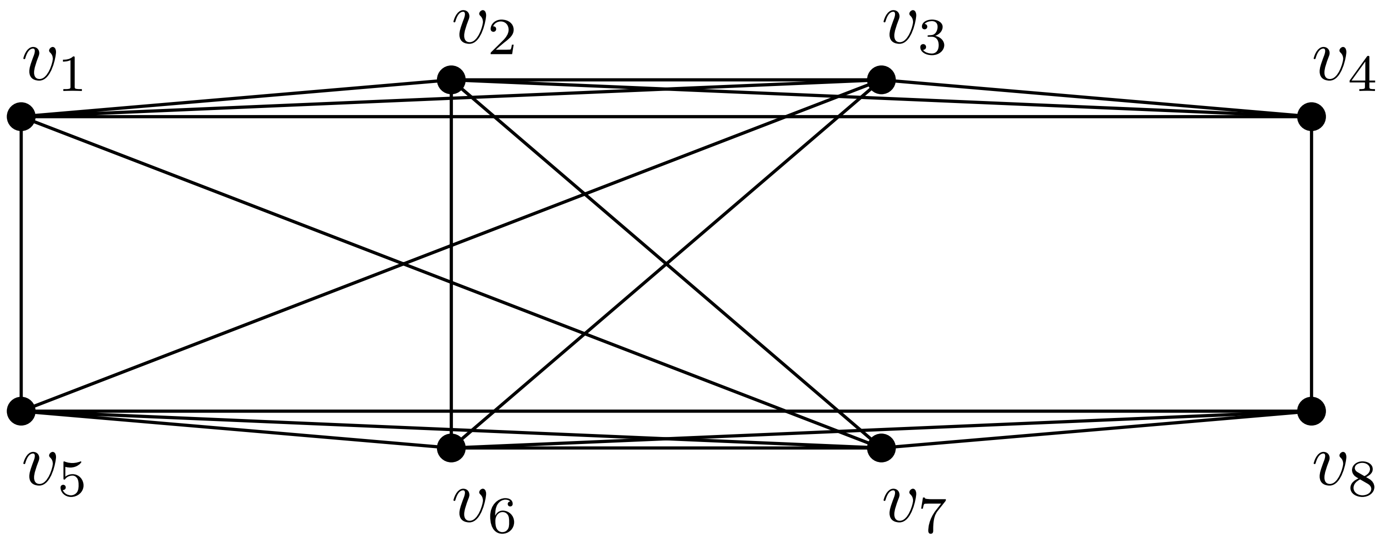

Proof. 4) $\mathrm{obs}_{\mathrm{in}}(G) > 1$

The convex hull of $V(G)$ forms a cycle.

Case analysis on vertices on CH

$$\{G : \mathrm{obs}_{\mathrm{out}}(G) = 1\} \not\subset \{G : \mathrm{obs}_{\mathrm{in}}(G) = 1\}$$

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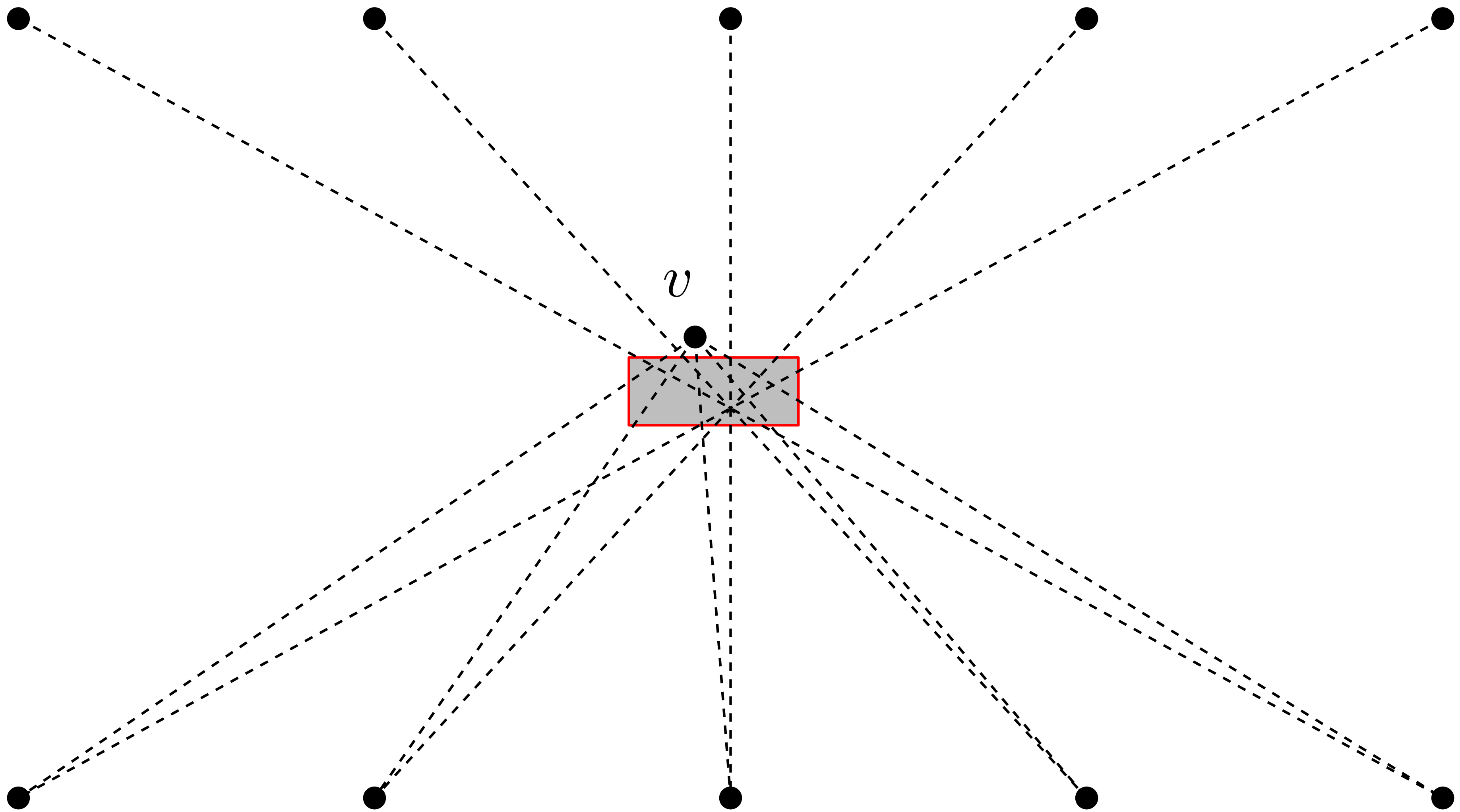
Thm. There is a graph G such that $\mathrm{obs}_{\mathrm{in}}(G) = 1$ but $\mathrm{obs}_{\mathrm{out}}(G) > 1$.



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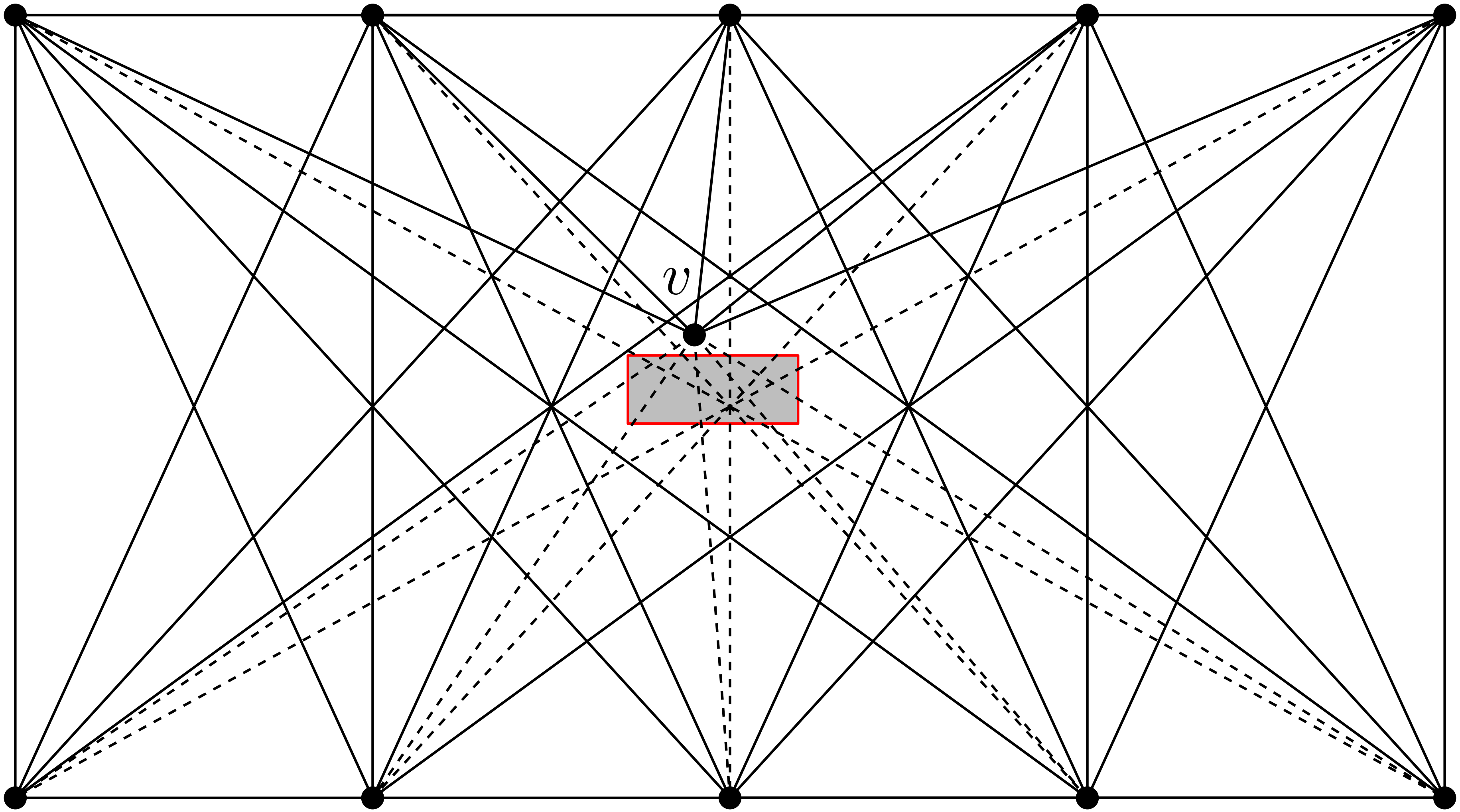

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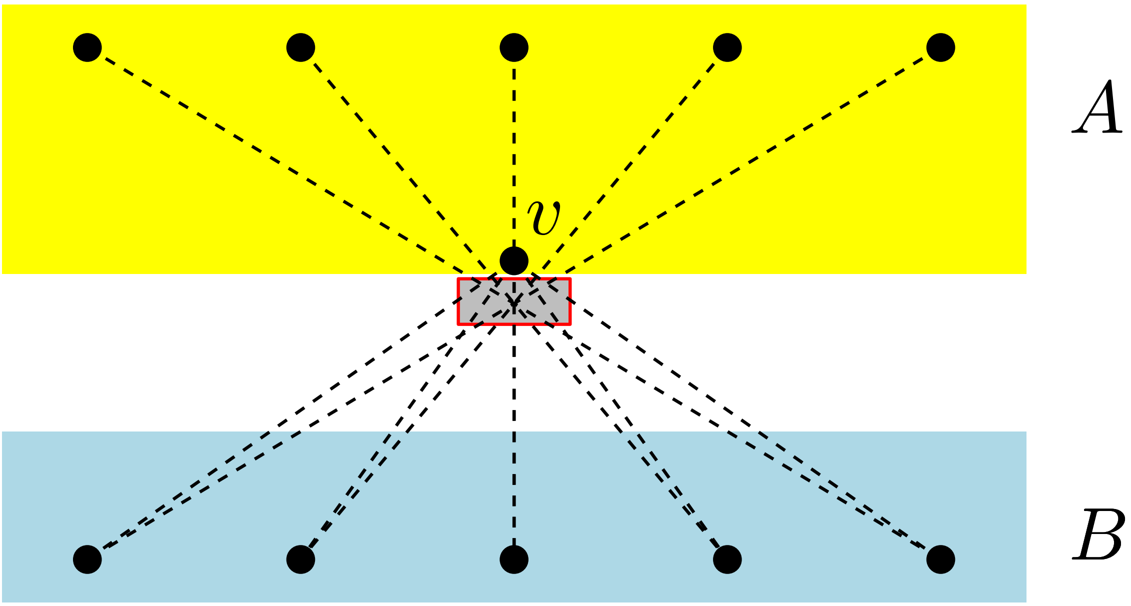
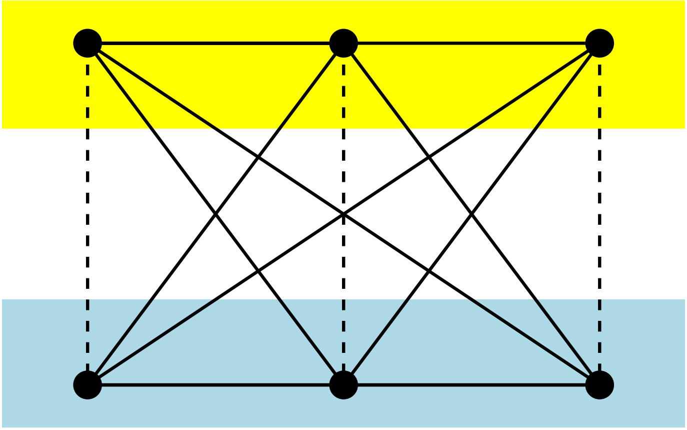


$\mathrm{CH}(A)$ and $\mathrm{CH}(B)$ are at least 1-crossing.



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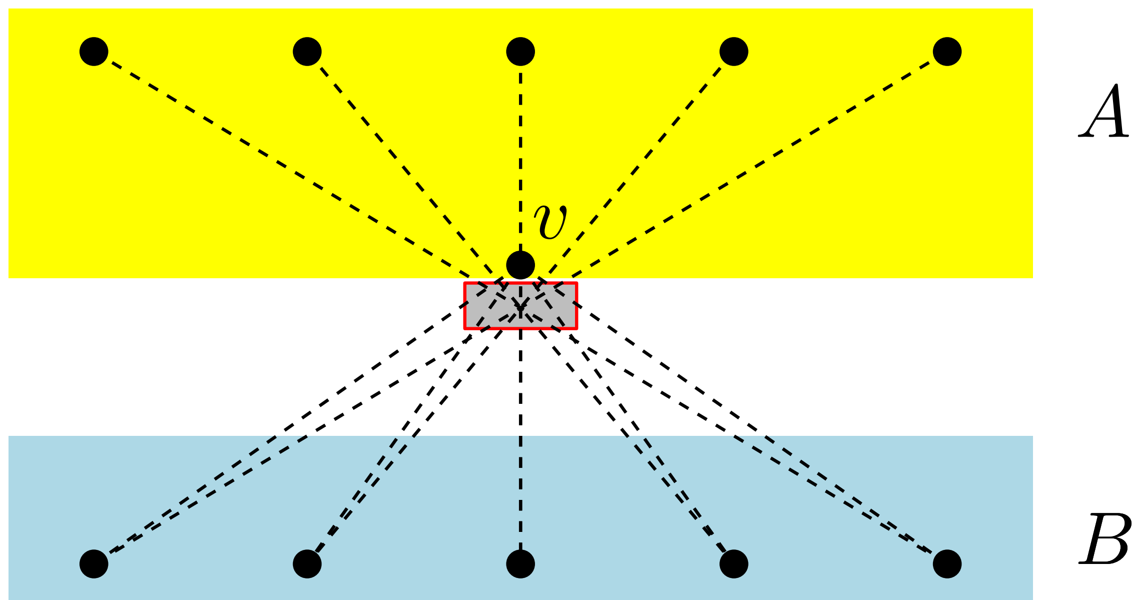
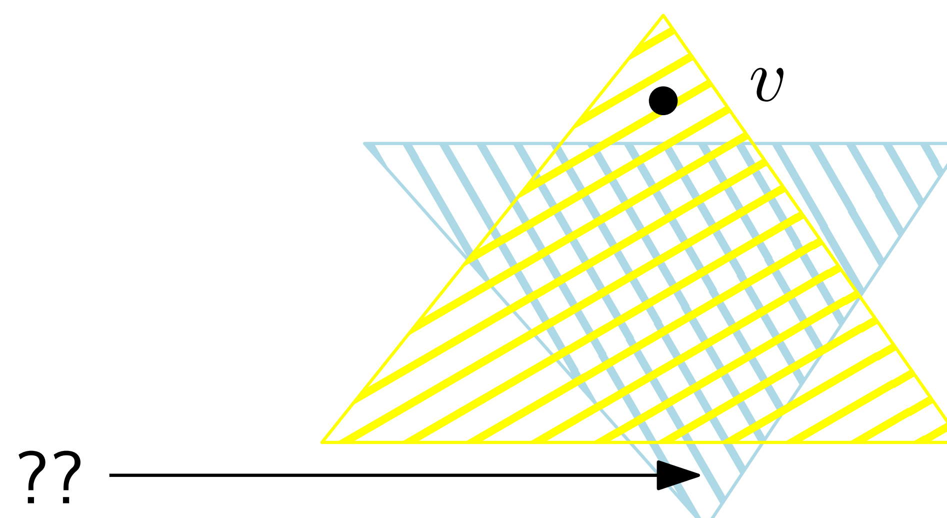


$\mathrm{CH}(A)$ and $\mathrm{CH}(B)$ are exactly 1-crossing.



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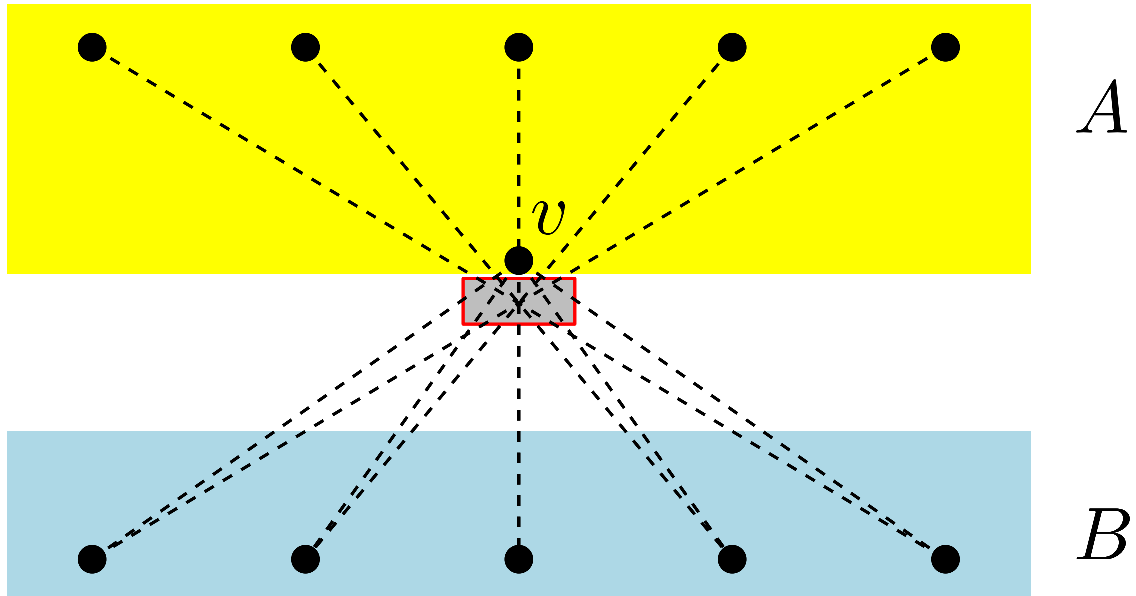

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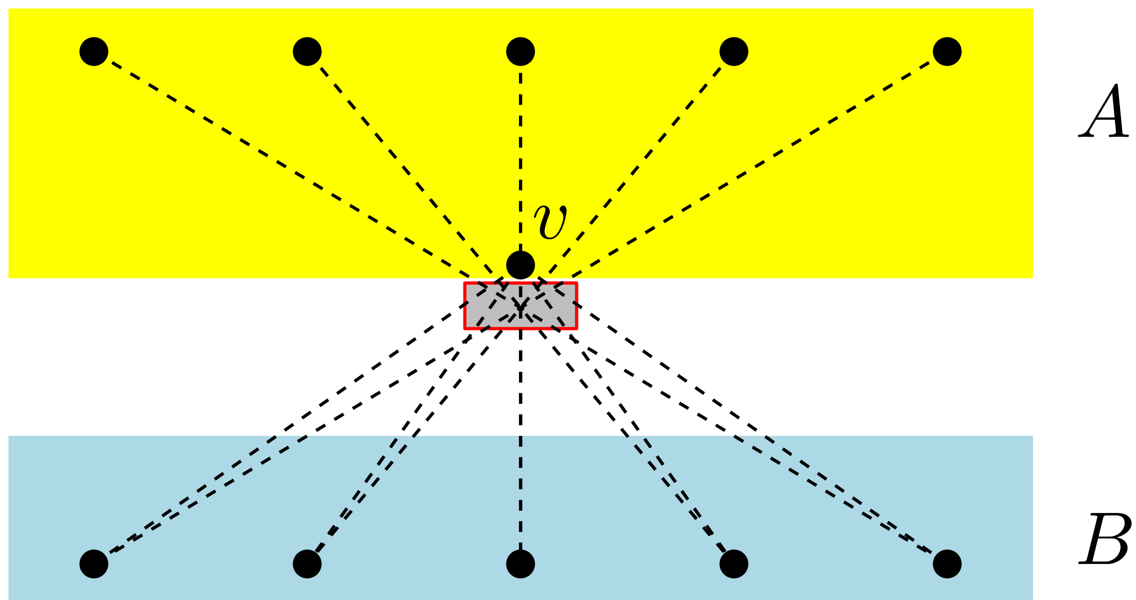

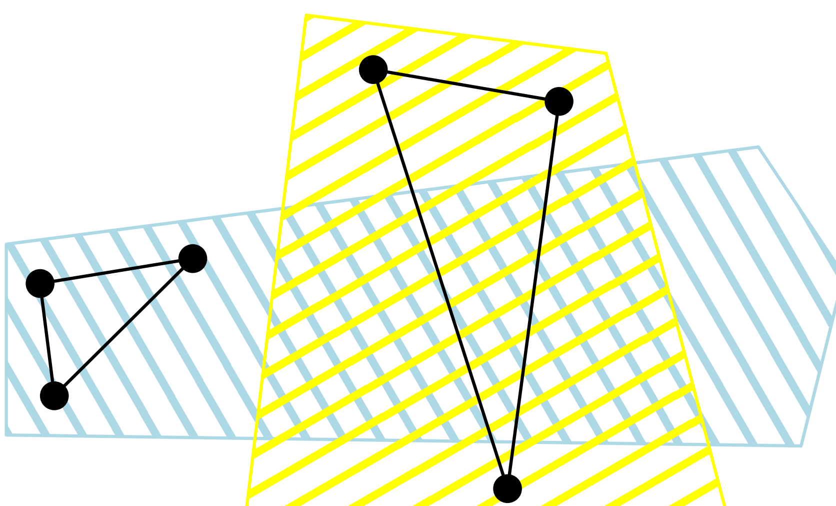

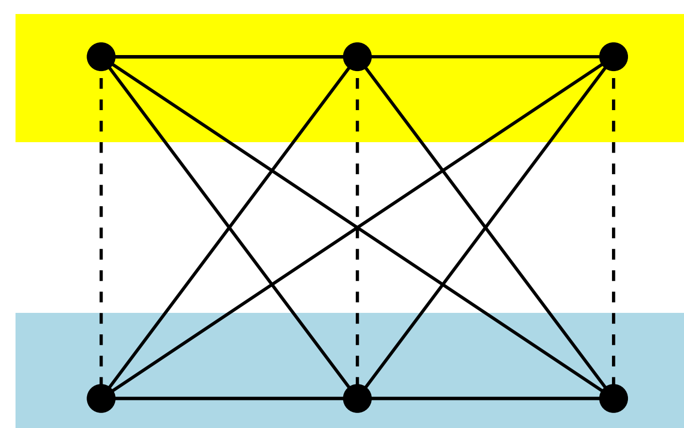

NP-hardness

Def. In a graph sandwich problem for a property Π, given two graphs $G \subseteq H$ with the same vertex set, we ask for a graph K s.t. $G \subseteq K \subseteq H$ and K has the property Π.

Thm. The outside-obstacle graph sandwich problem is NP-hard. In other words, given two graphs $G \subseteq H$ with the same vertex set, it is NP-hard to decide if there is a graph K s.t. $G \subseteq K \subseteq H$ and $\mathrm{obs}_{\mathrm{out}}(K) = 1$.

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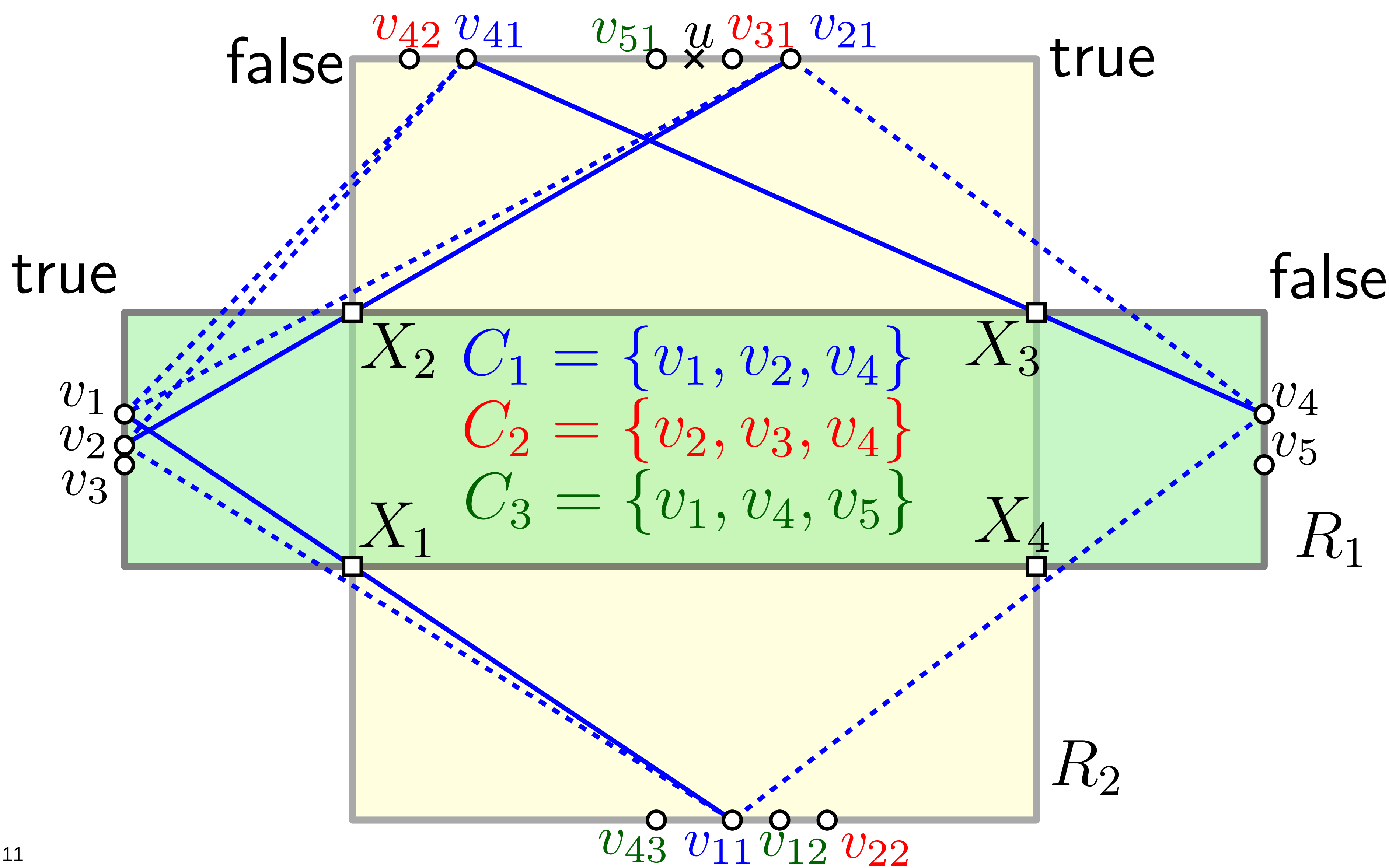
Thm. The inside-obstacle graph sandwich problem and the single-obstacle graph sandwich problem are NP-hard.

Def. The simple-polygon visibility graph problem asks to recognize the visibility graph of a simple polygon where the obstacle is the complement of the polygon.

Thm. The simple-polygon visibility graph sandwich problem is NP-hard.

NP-hardness

Reduction from MONOTONENOTALLEQUAL3SAT where each clause contains 3 variables, not all of which are equal.



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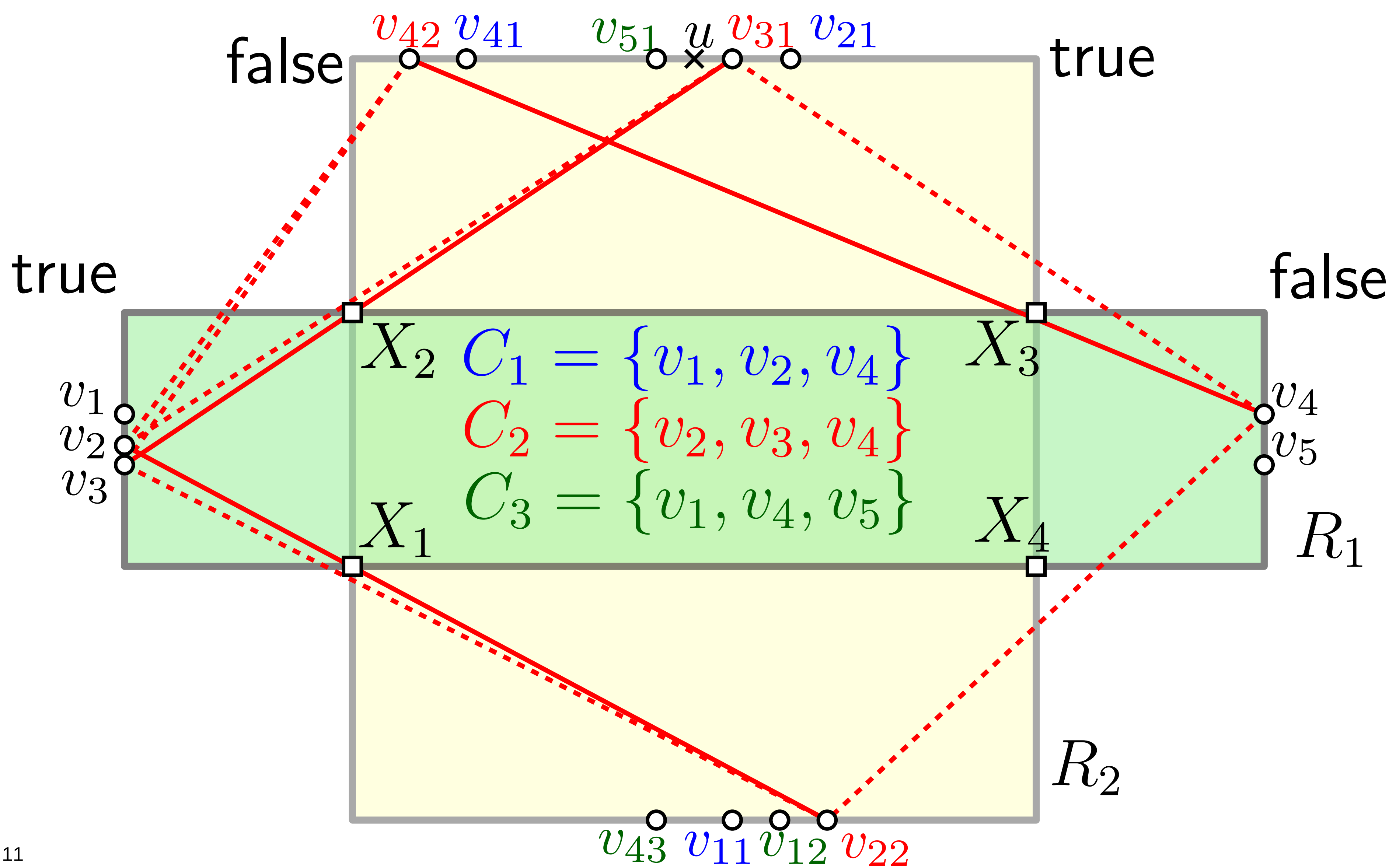

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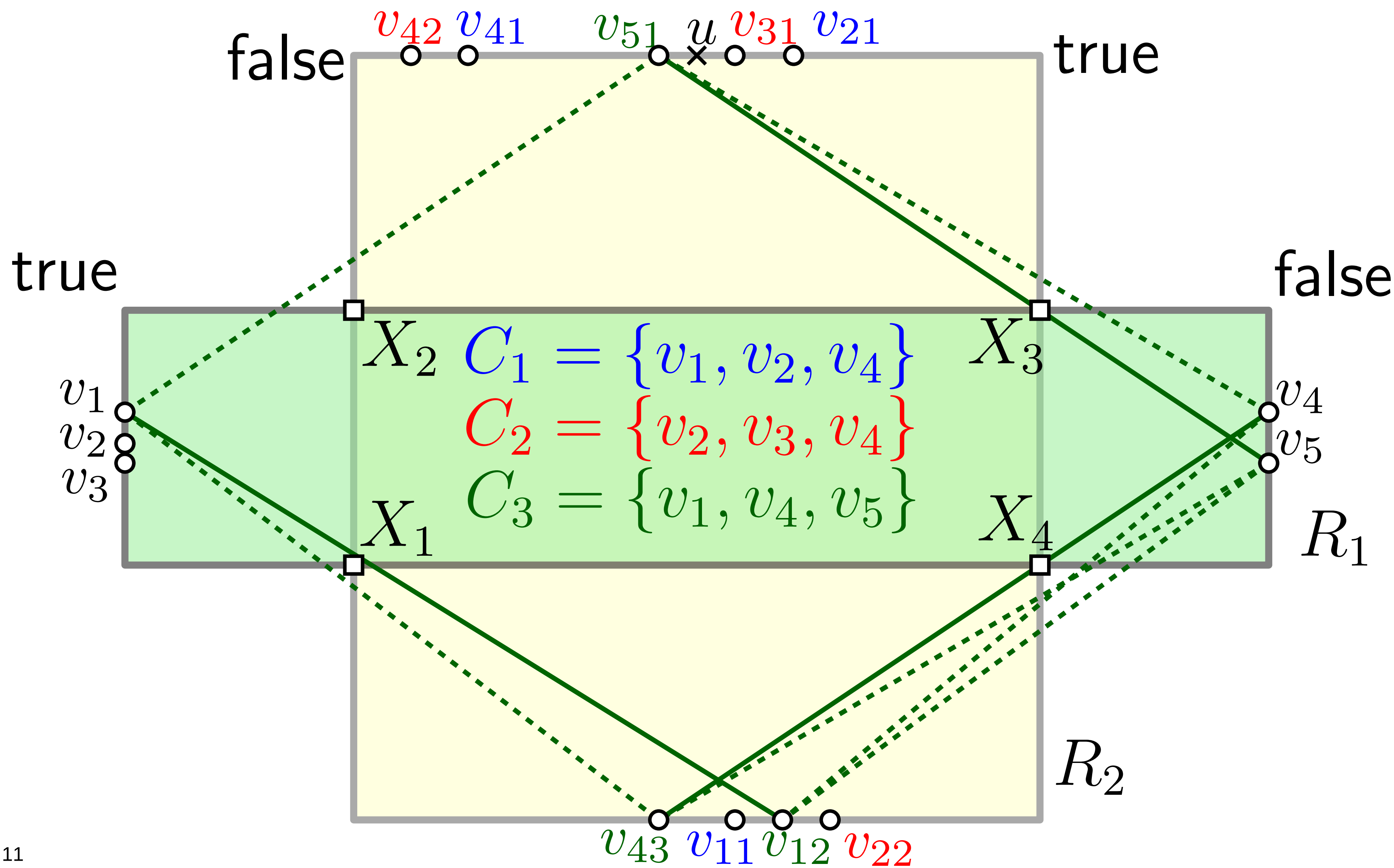

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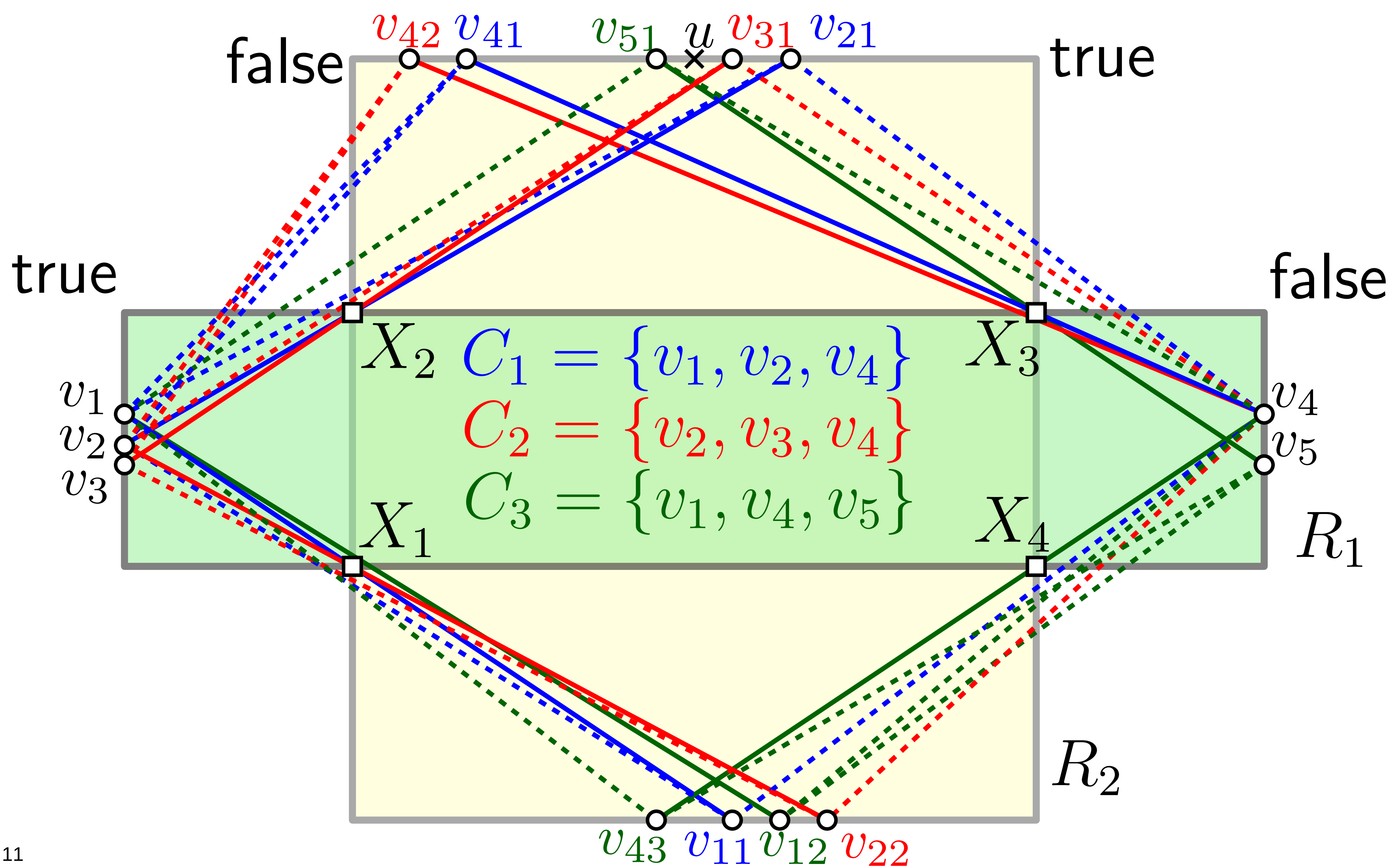

Summary and Open Problems

- Graphs of circumference at most 6 and graphs with at most 7 vertices have obstacle number 1.
- Smallest graph of obstacle number 2 has 8 vertices.
- $\{G : \mathrm{obs}_{\mathrm{out}}(G) = 1\}$ and $\{G : \mathrm{obs}_{\mathrm{in}}(G) = 1\}$ are incomparable.
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  Shown to be tight: $\mathrm{obs}(G) \leq \mathrm{obs}_{\mathrm{out}}(G) \leq \mathrm{obs}(G) + 1$

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