Constrained and Ordered Level Planarity Parameterized by the Number of Levels

Václav Blažej

with Boris Klemz, Felix Klesen, Marie Diana Sieper, Alexander Wolff, and Johannes Zink

SoCG 2024

~~Constrained and~~ Ordered Level Planarity Parameterized by the Number of Levels

Václav Blažej

with Boris Klemz, Felix Klesen, Marie Diana Sieper, Alexander Wolff, and Johannes Zink

SoCG 2024

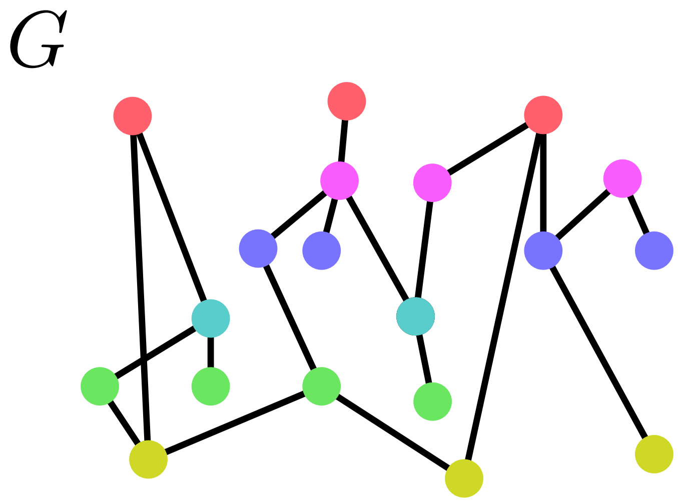
Engineering and Physical Sciences Research Council

WARWICK
THE UNIVERSITY OF WARWICK

Ordered Level Planarity

input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

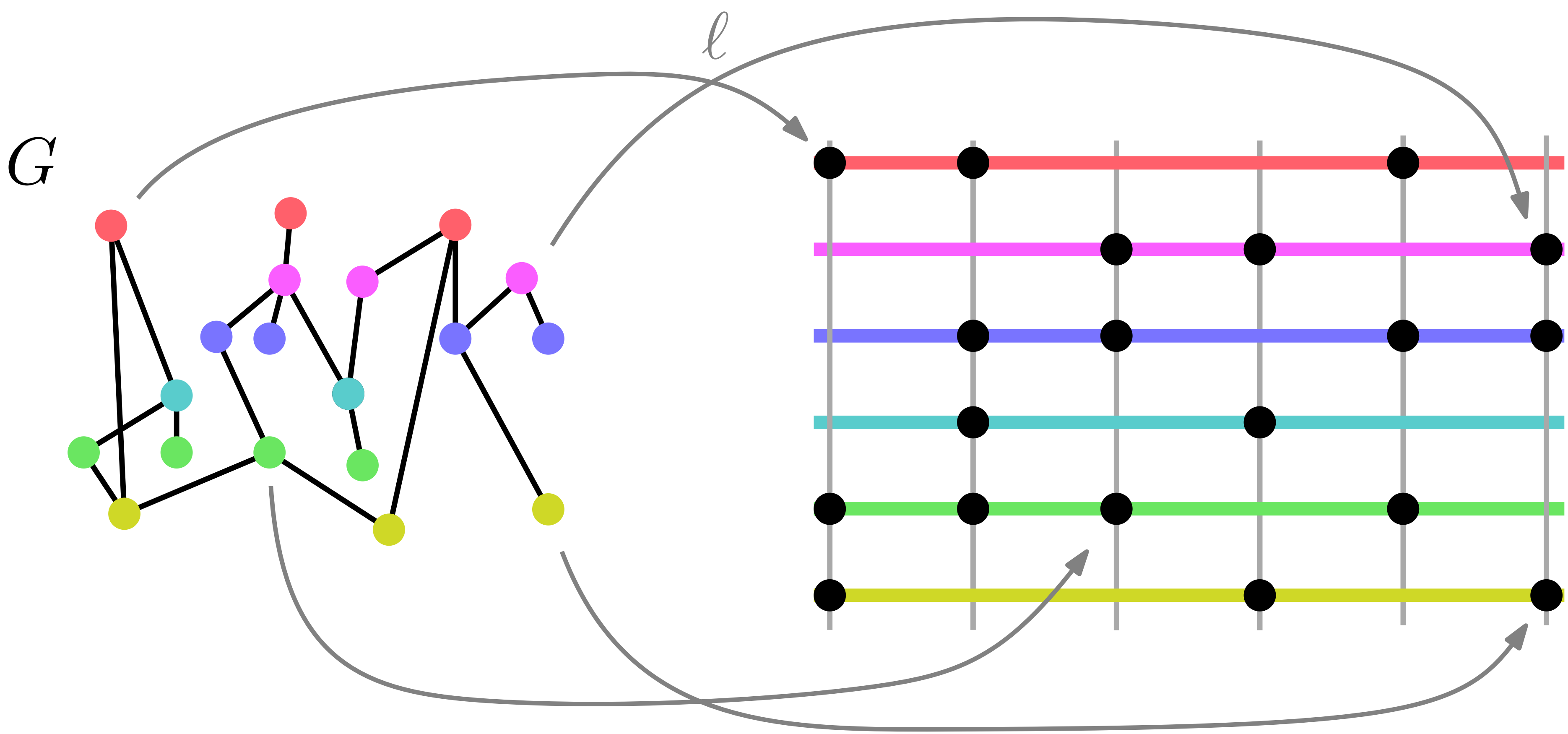
output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone



Ordered Level Planarity

input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

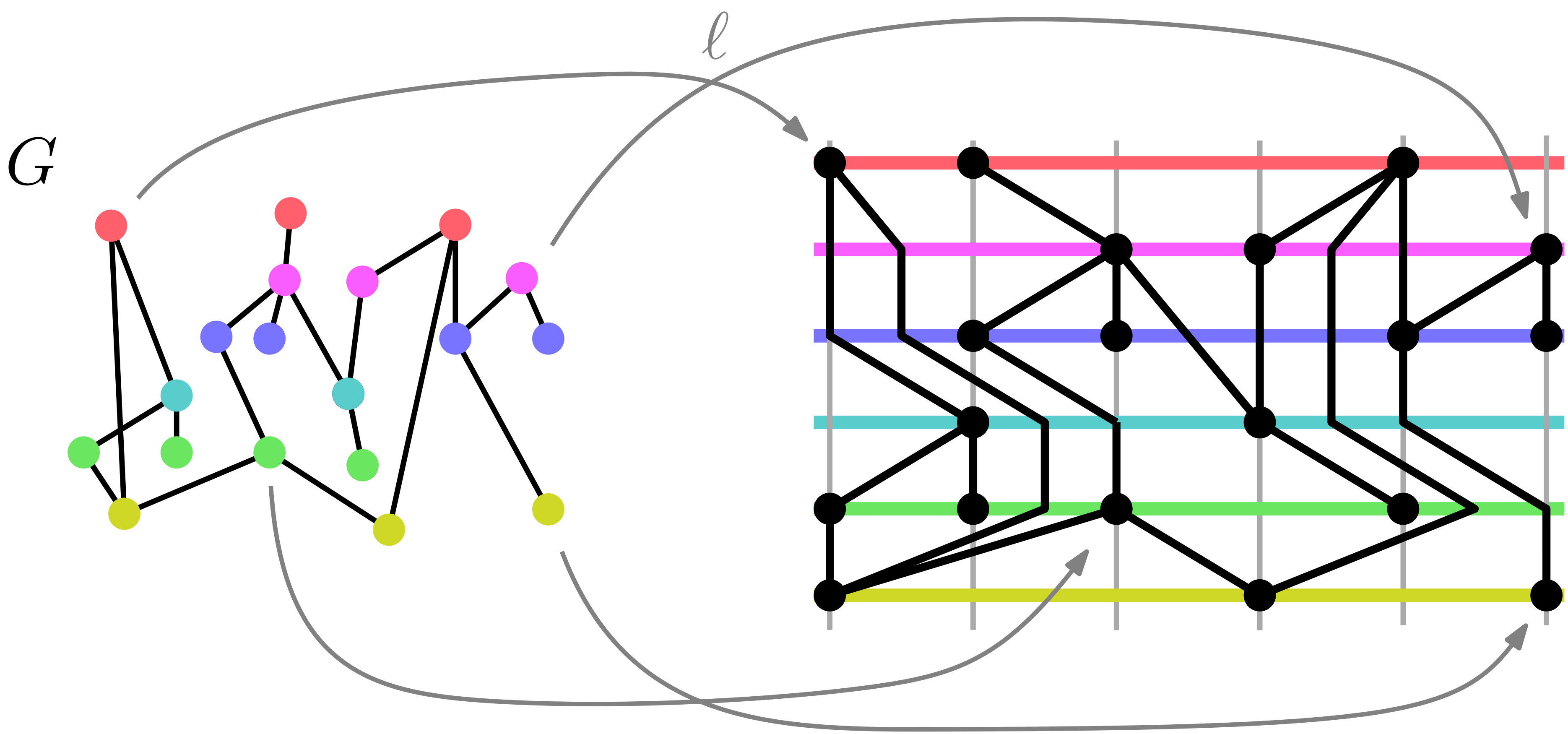
output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone



Ordered Level Planarity

input: a graph G with vertex coordinates $\ell \colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone



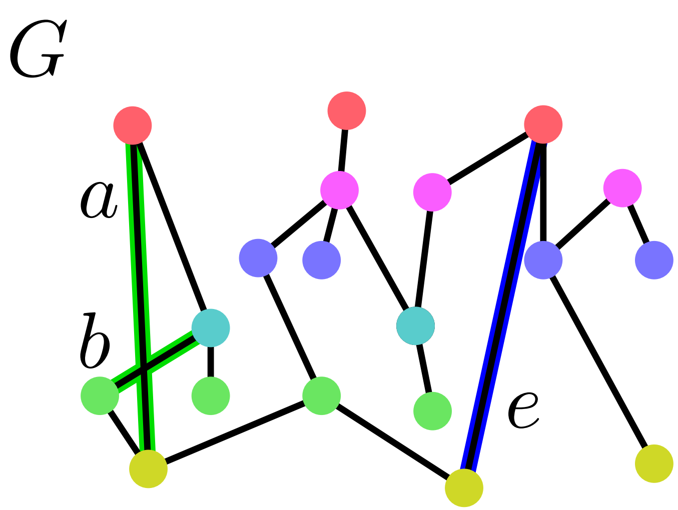
Ordered Level Planarity

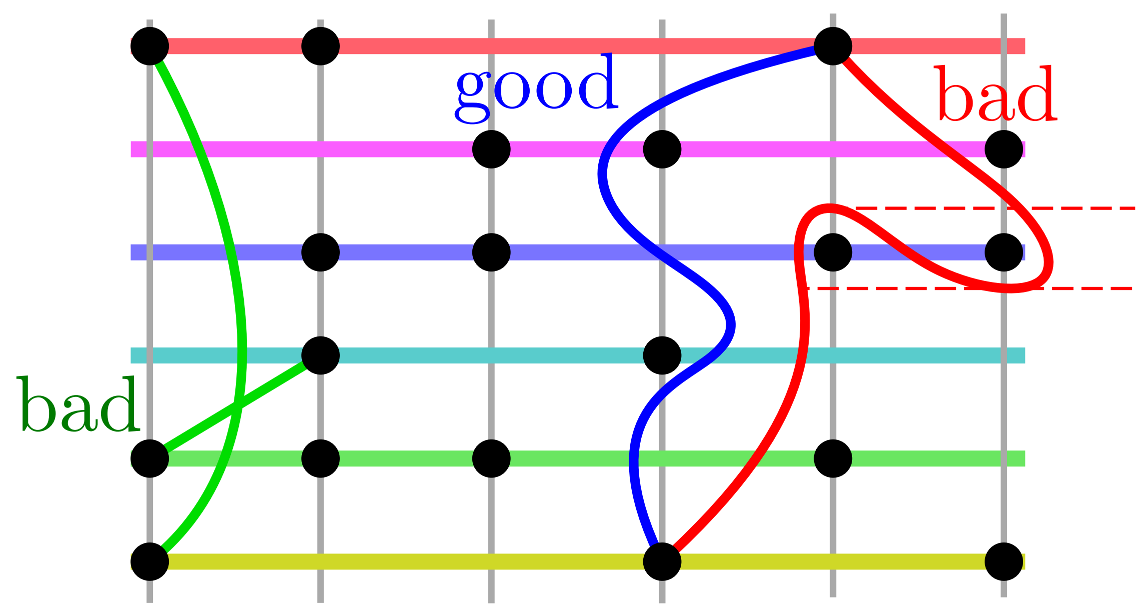
input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

no crossings

going only up

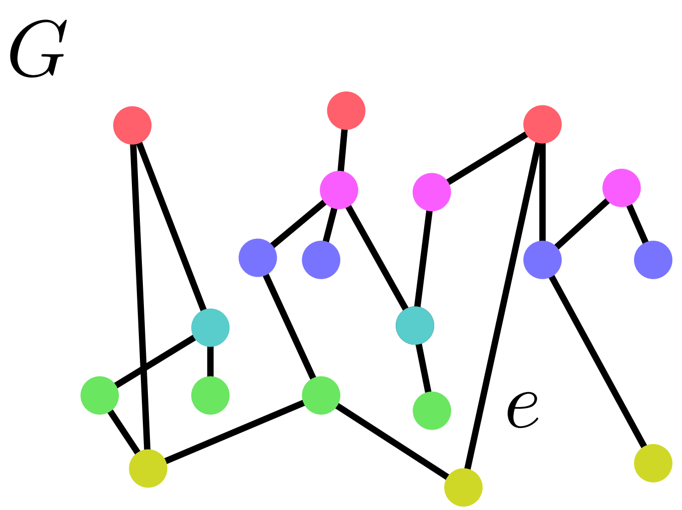


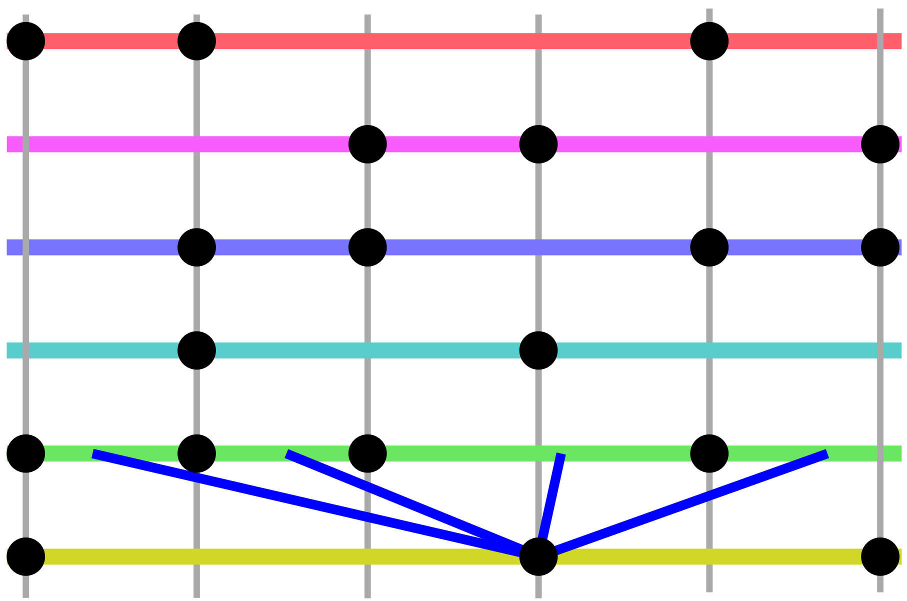


Ordered Level Planarity

input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

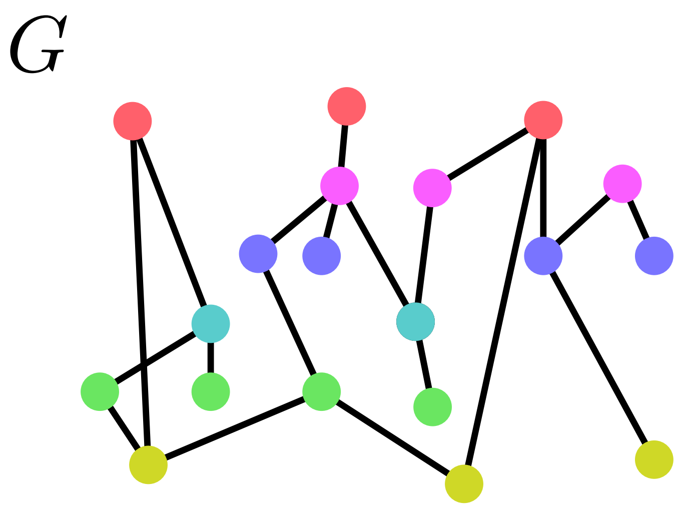


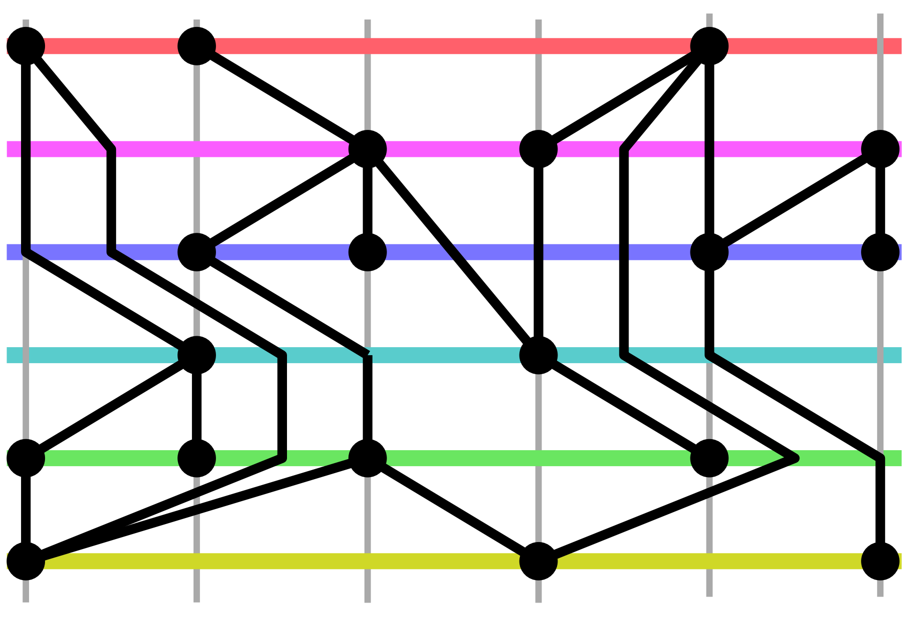


Ordered Level Planarity

input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

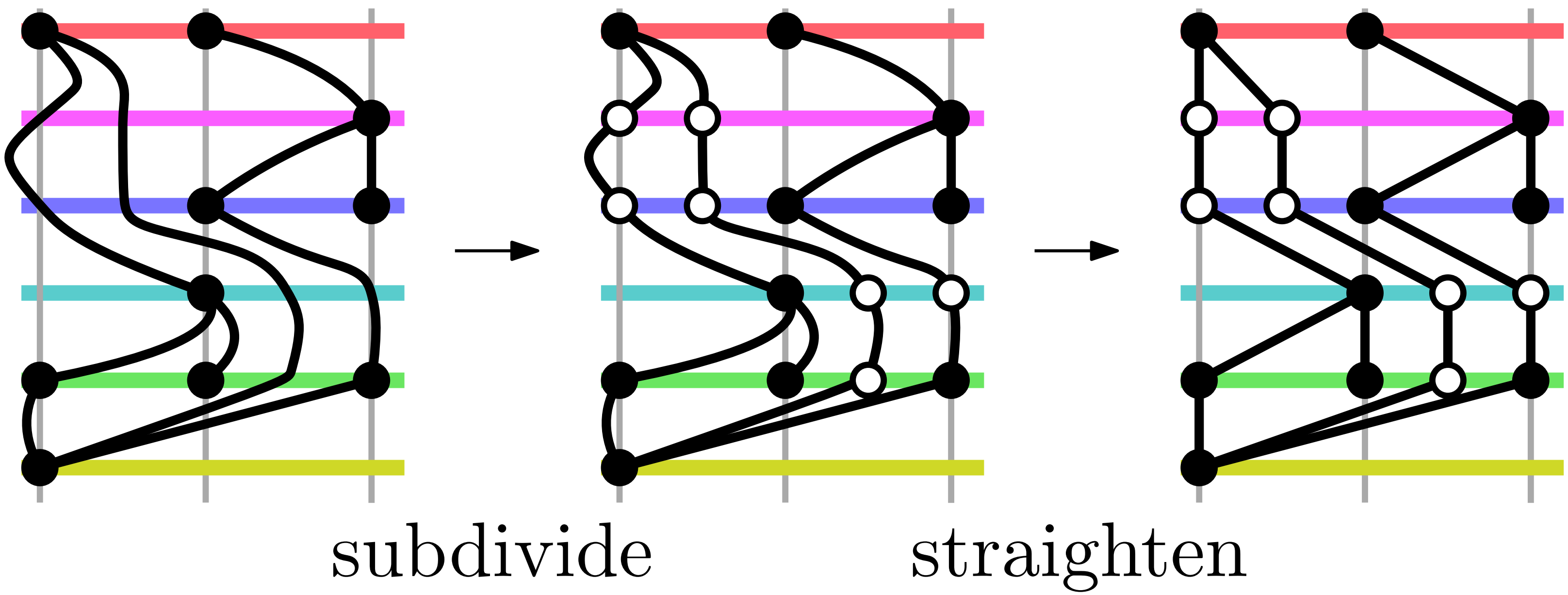
output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone





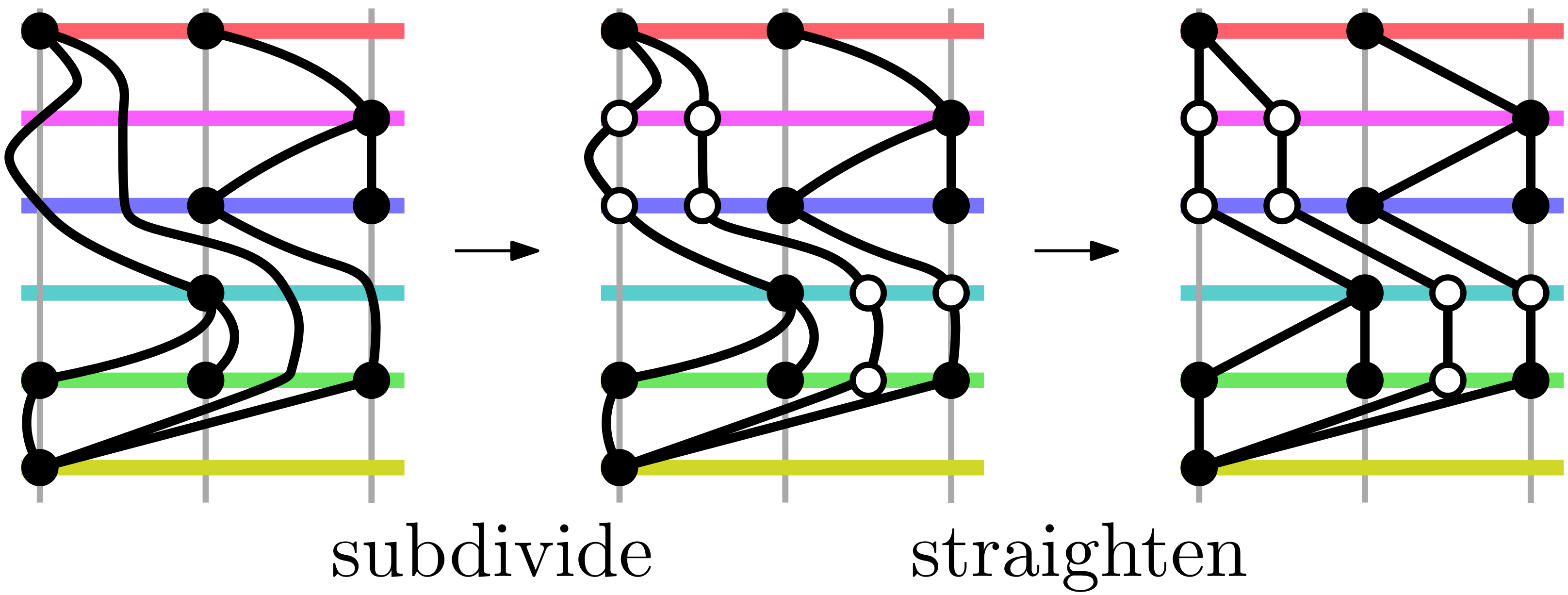
Ordered Level Planarity

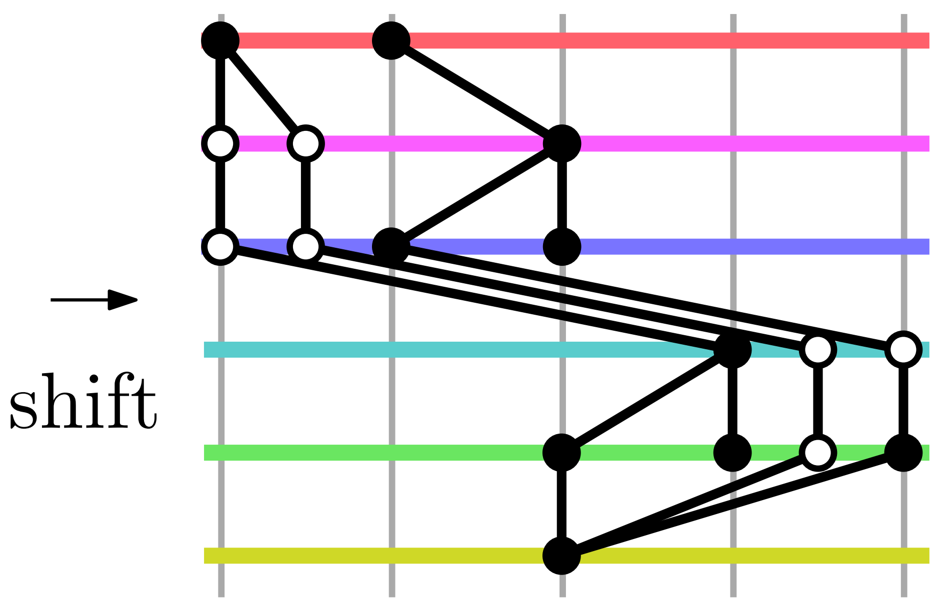
examples

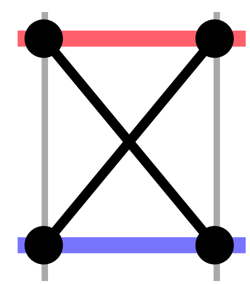


Ordered Level Planarity

examples



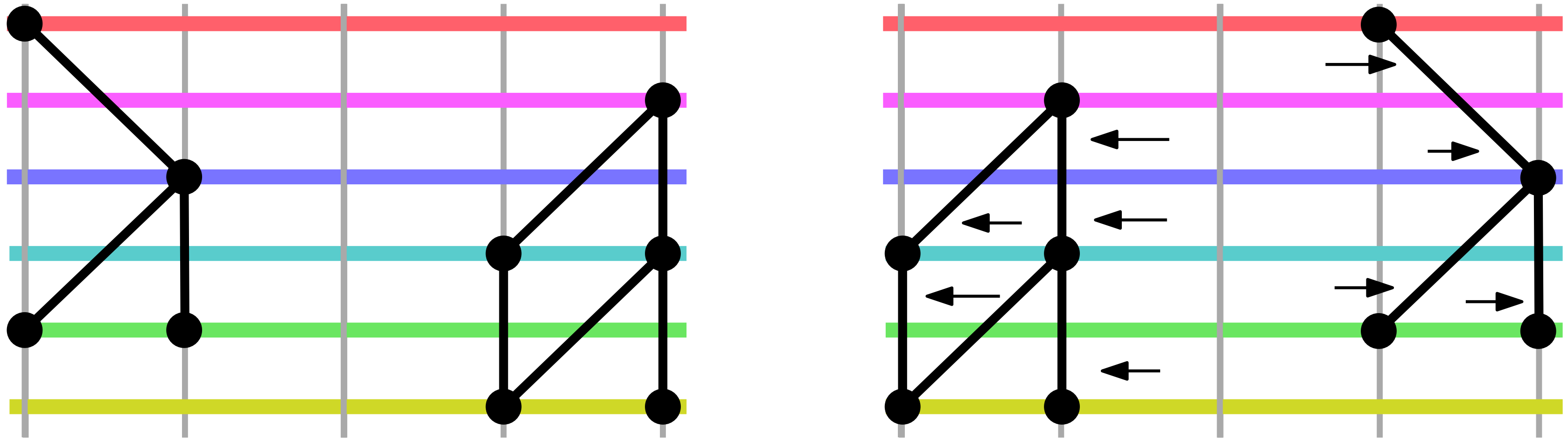




not realizable

Ordered Level Planarity

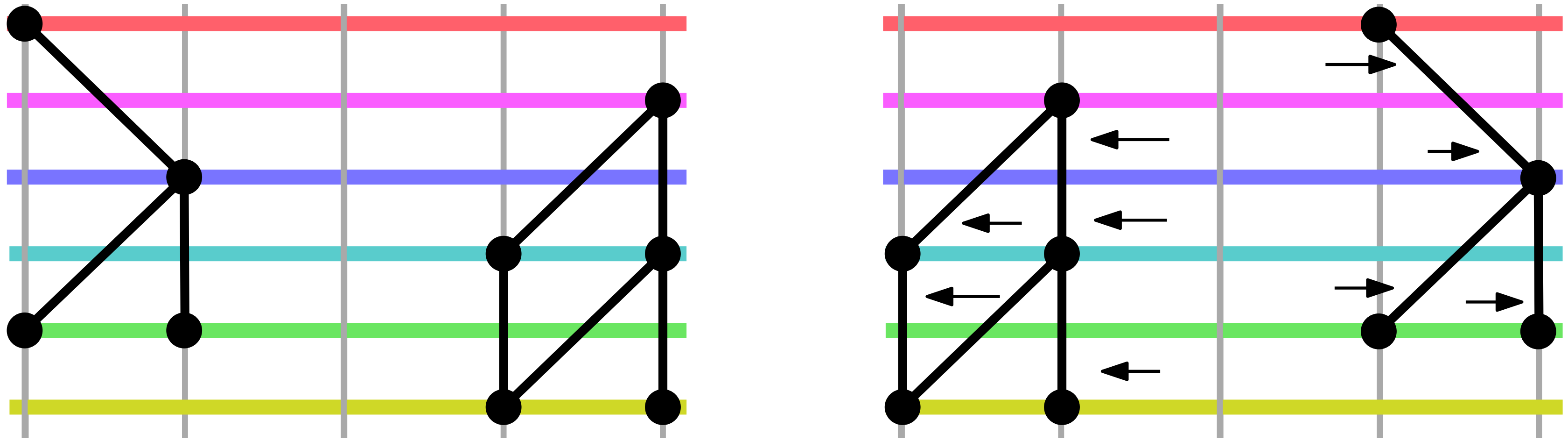
examples



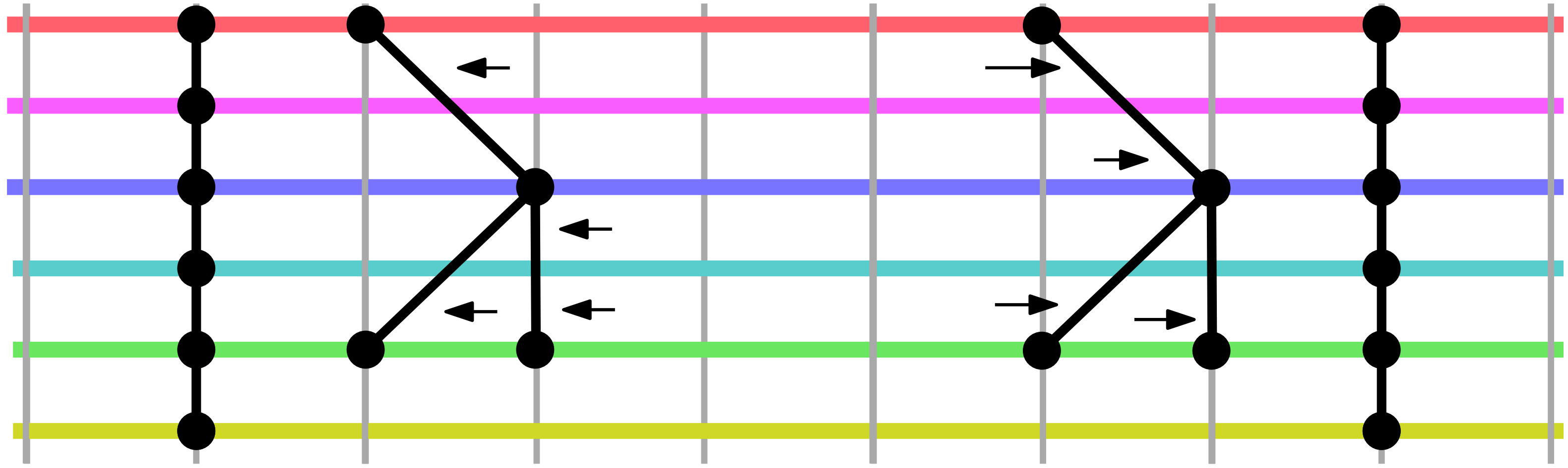
disjoint levels → independence

Ordered Level Planarity

examples

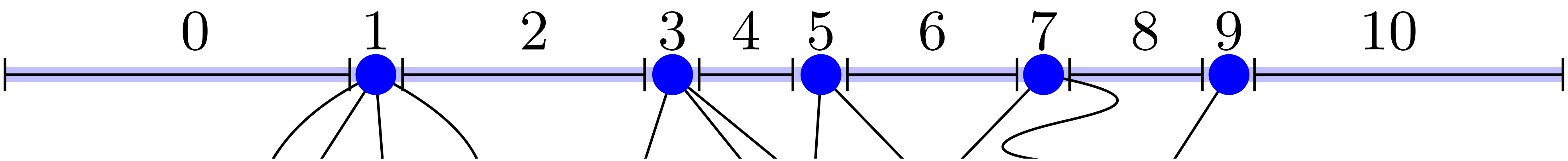


disjoint levels → independence



bounding walls

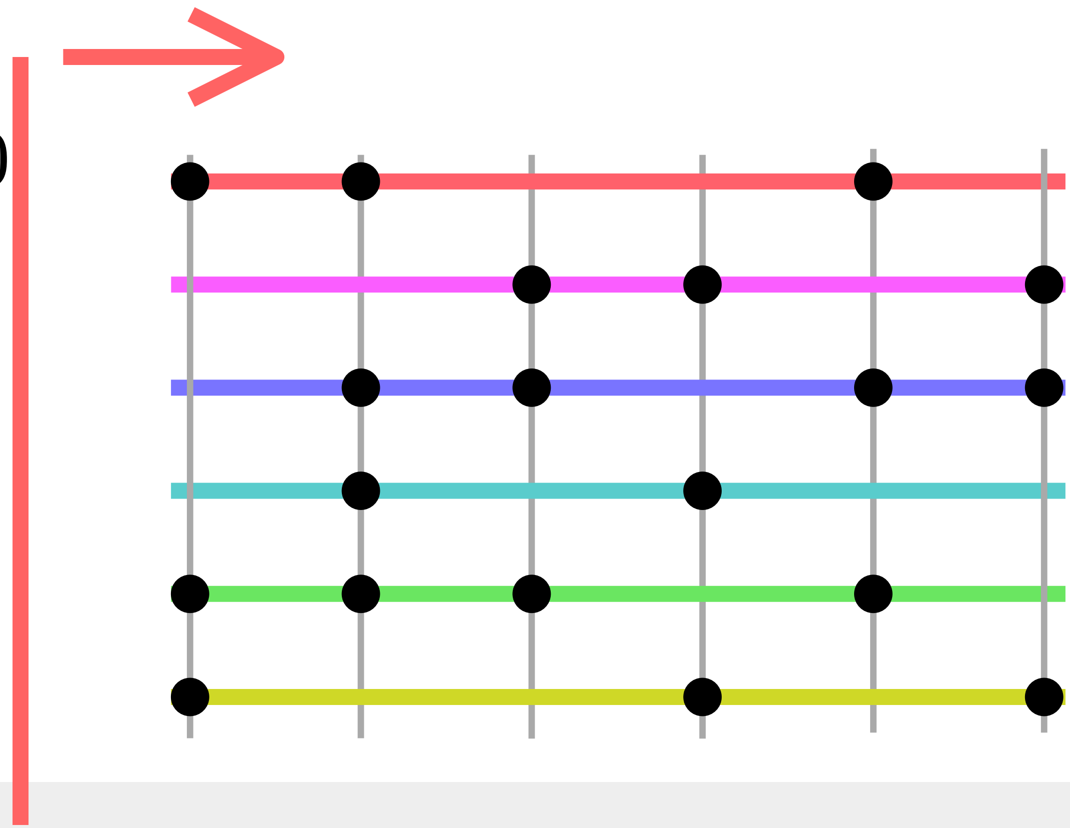
Ordered Level Planarity
(slow) algorithm



input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

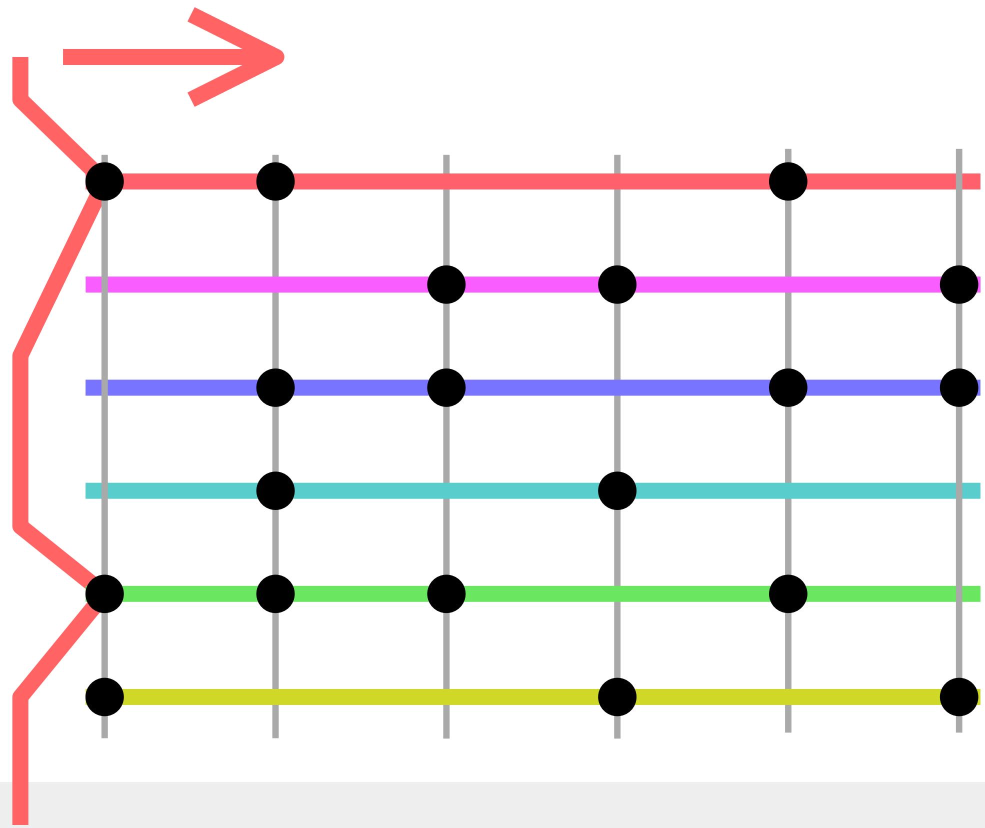


input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

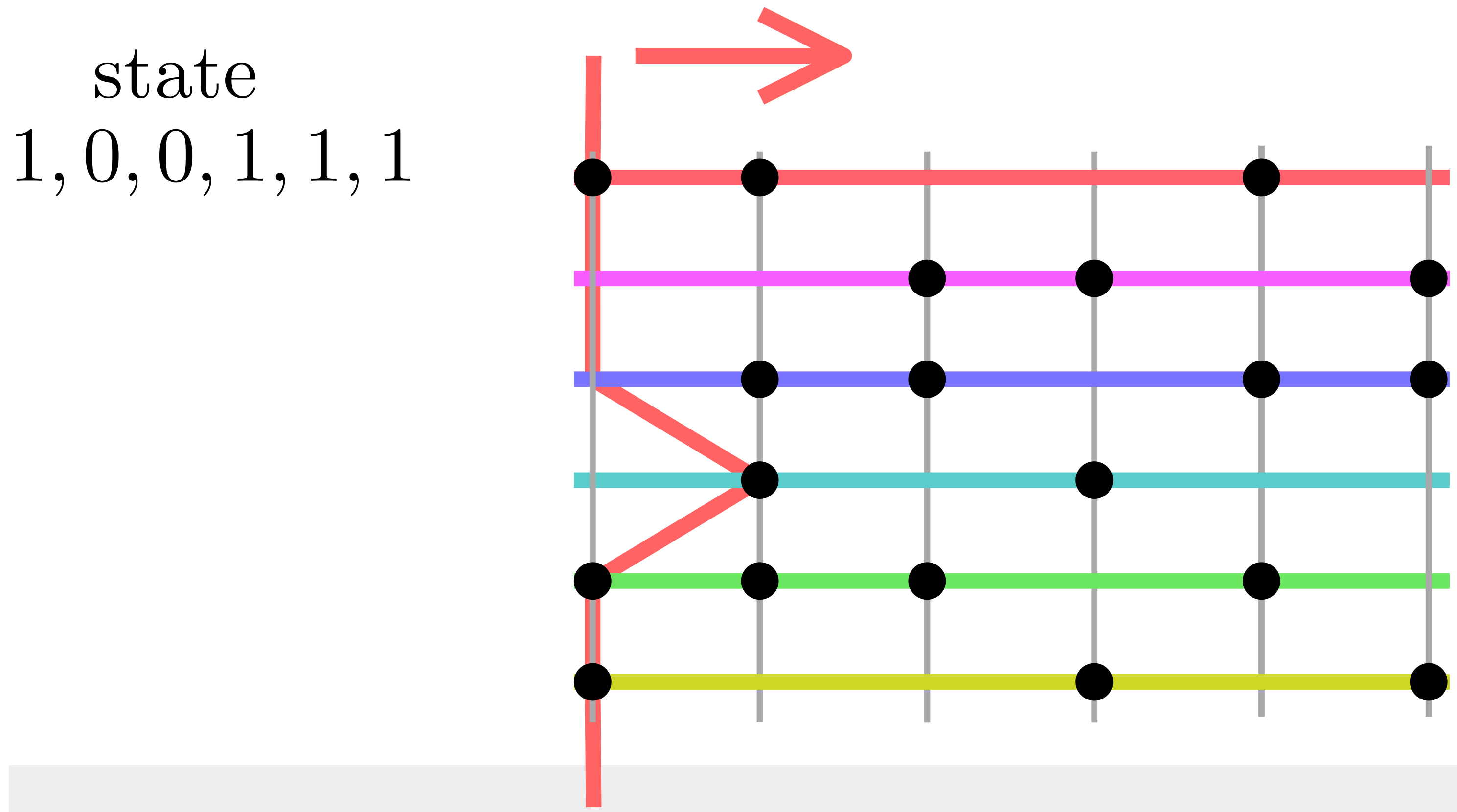
state
$1, 0, 0, 0, 1, 0$



input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

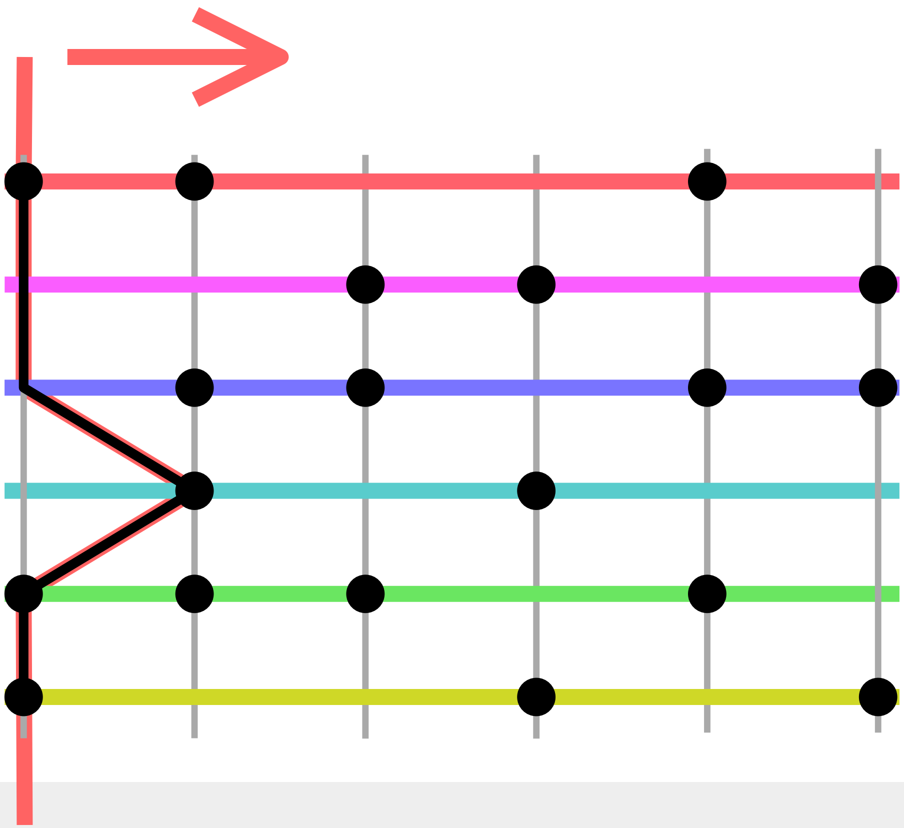


input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

state
1, 0, 0, 1, 1, 1

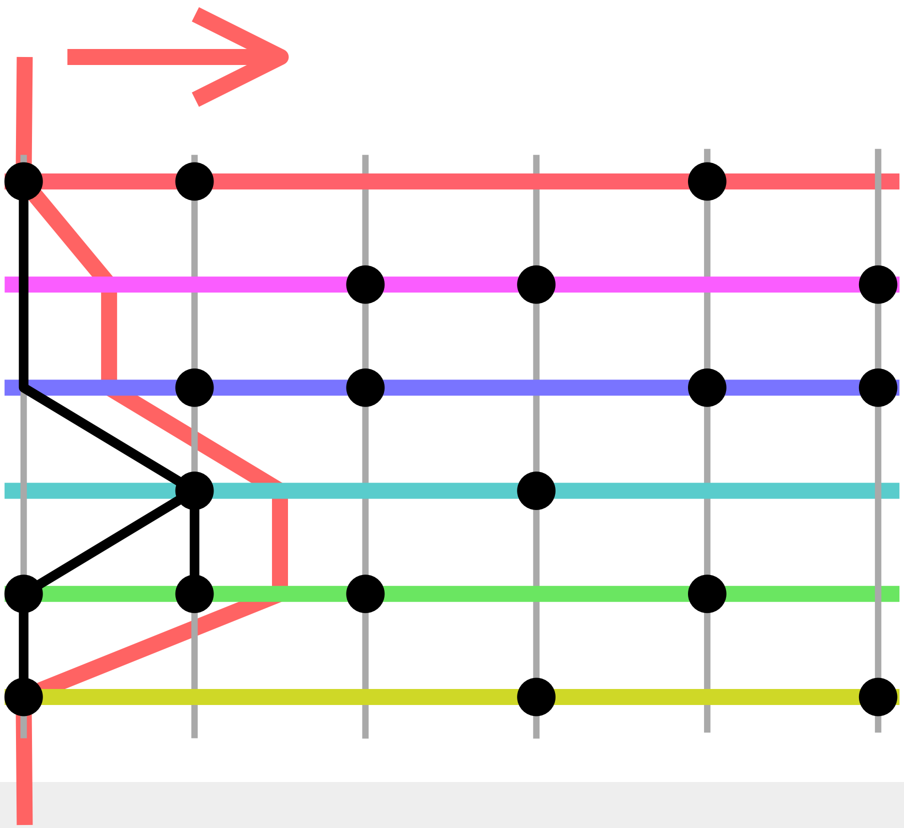


input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

state
1, 0, 0, 2, 4, 1

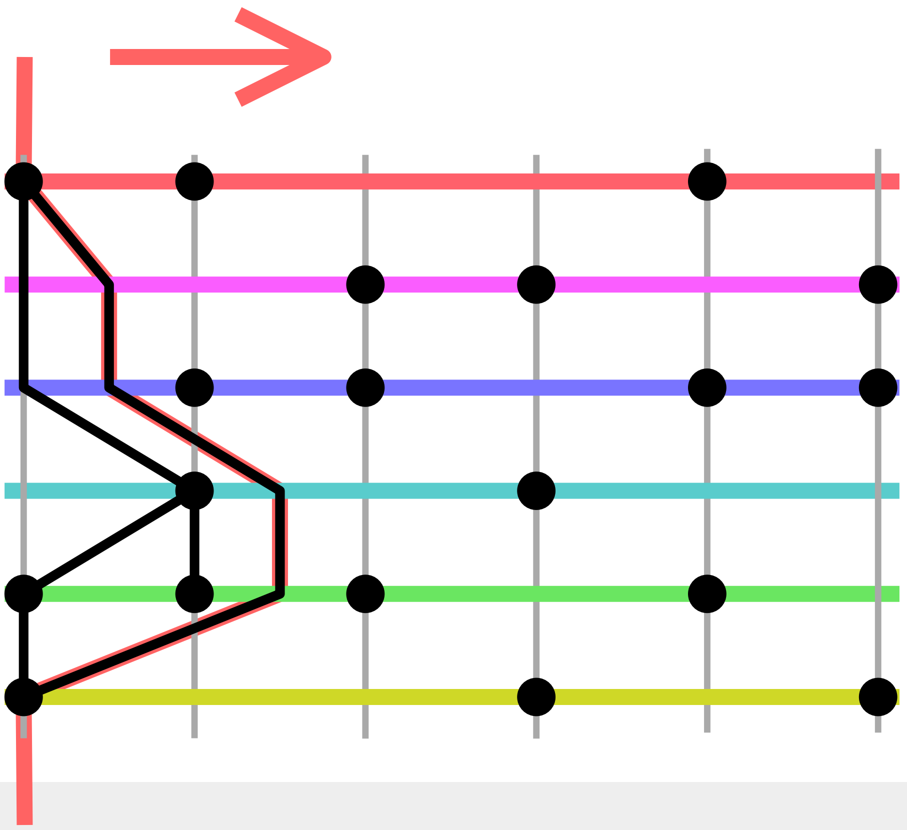


input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

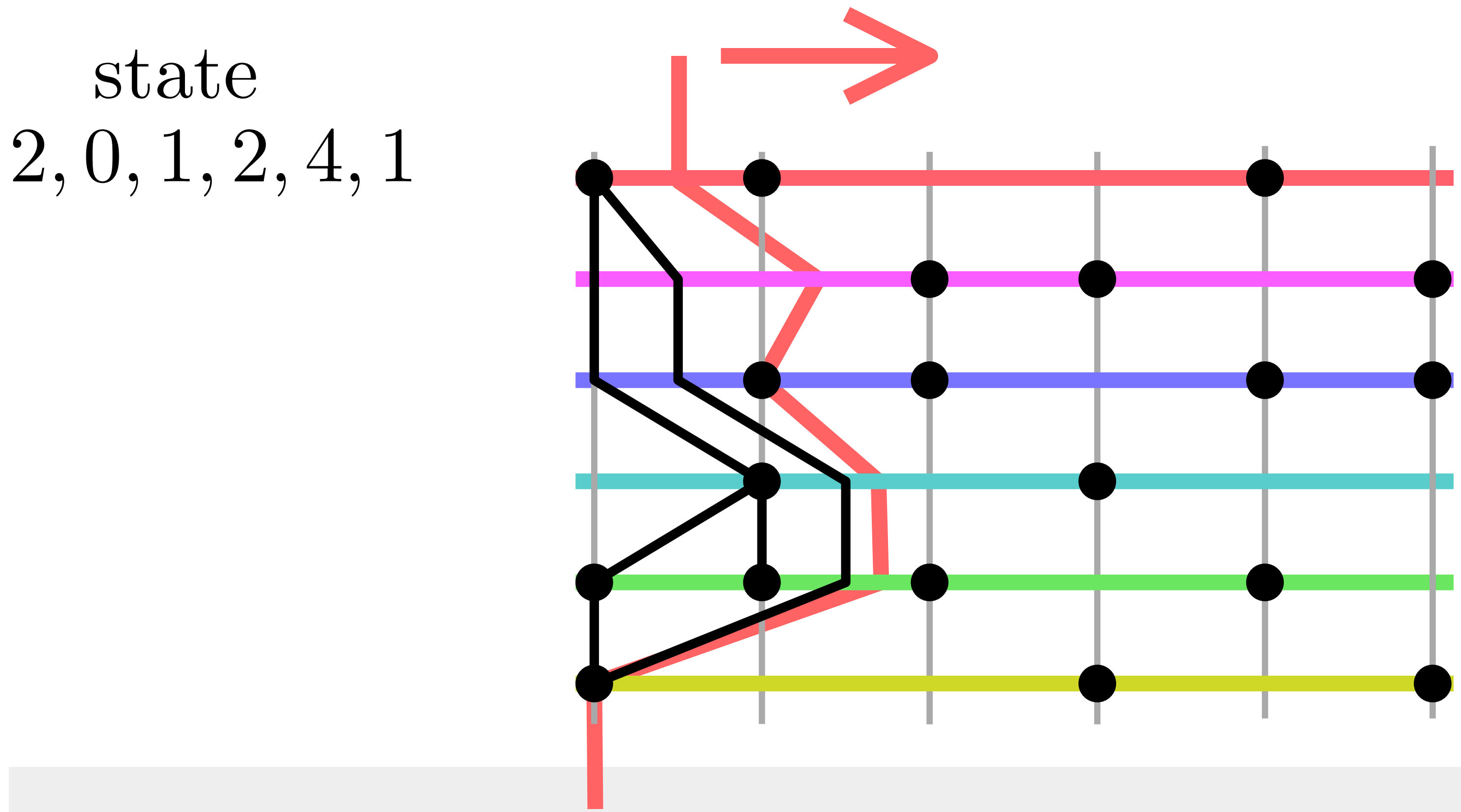
state
1, 0, 0, 2, 4, 1



input: a graph G with vertex coordinates $\ell \colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

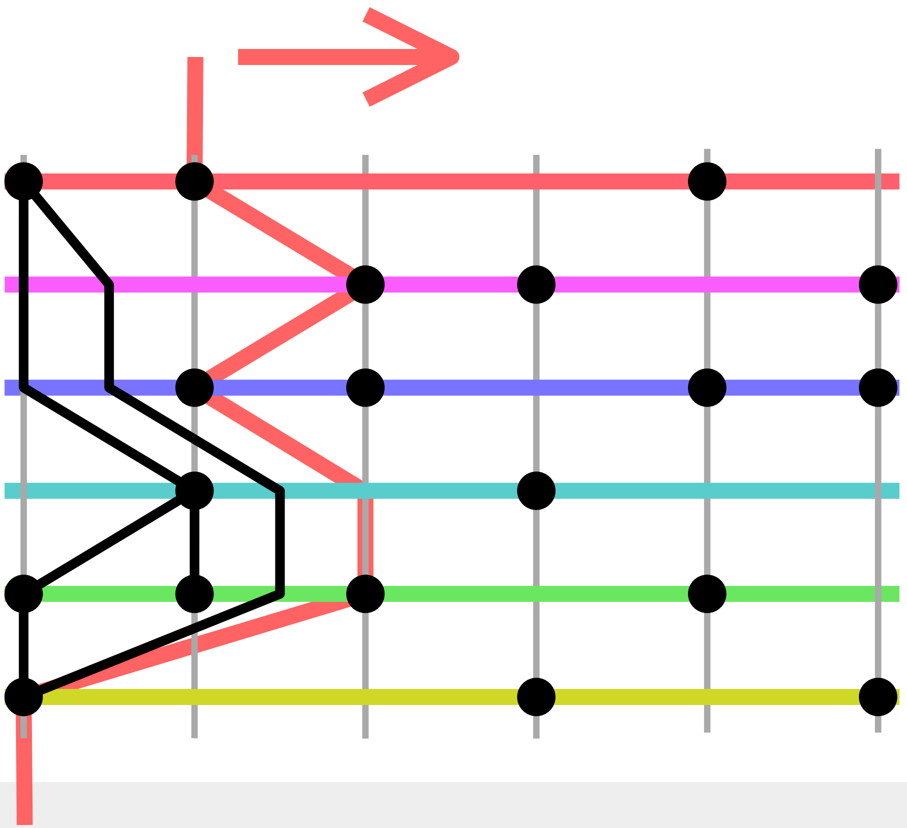


input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

state
3, 1, 1, 2, 5, 1

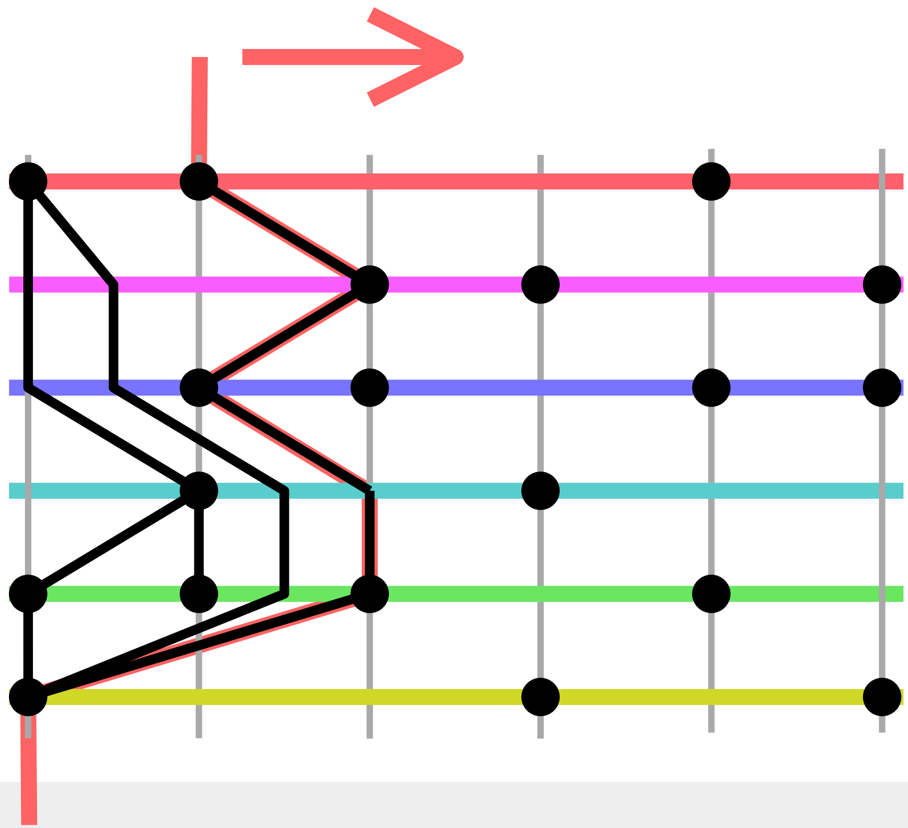


input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

state
3, 1, 1, 2, 5, 1

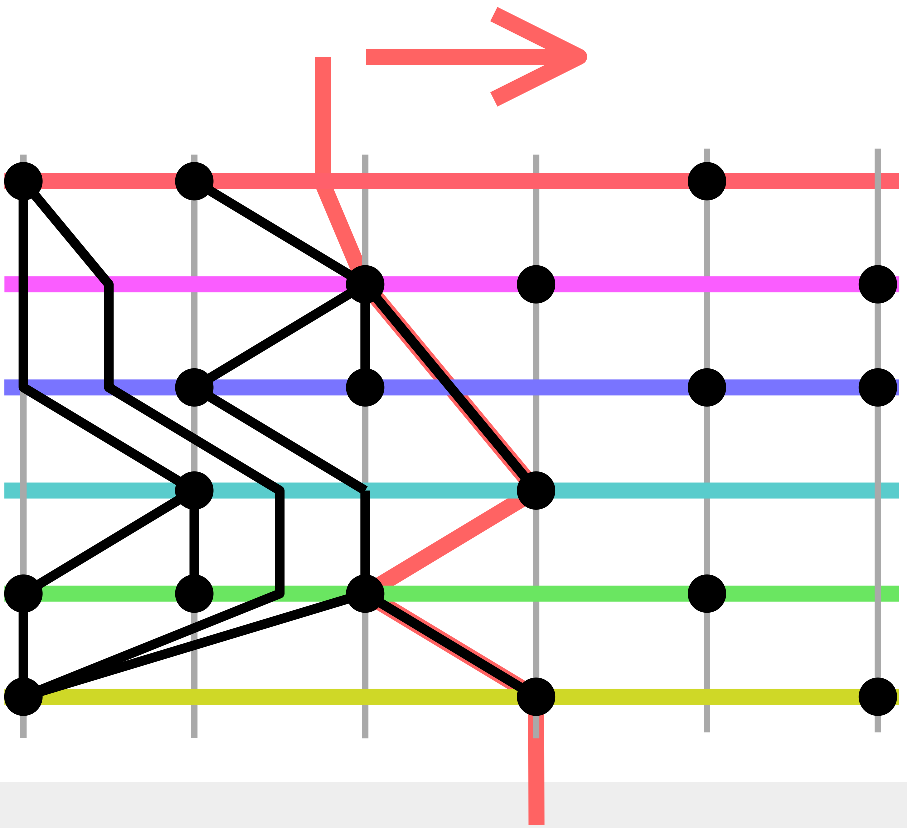


input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

state
4, 1, 4, 3, 5, 3

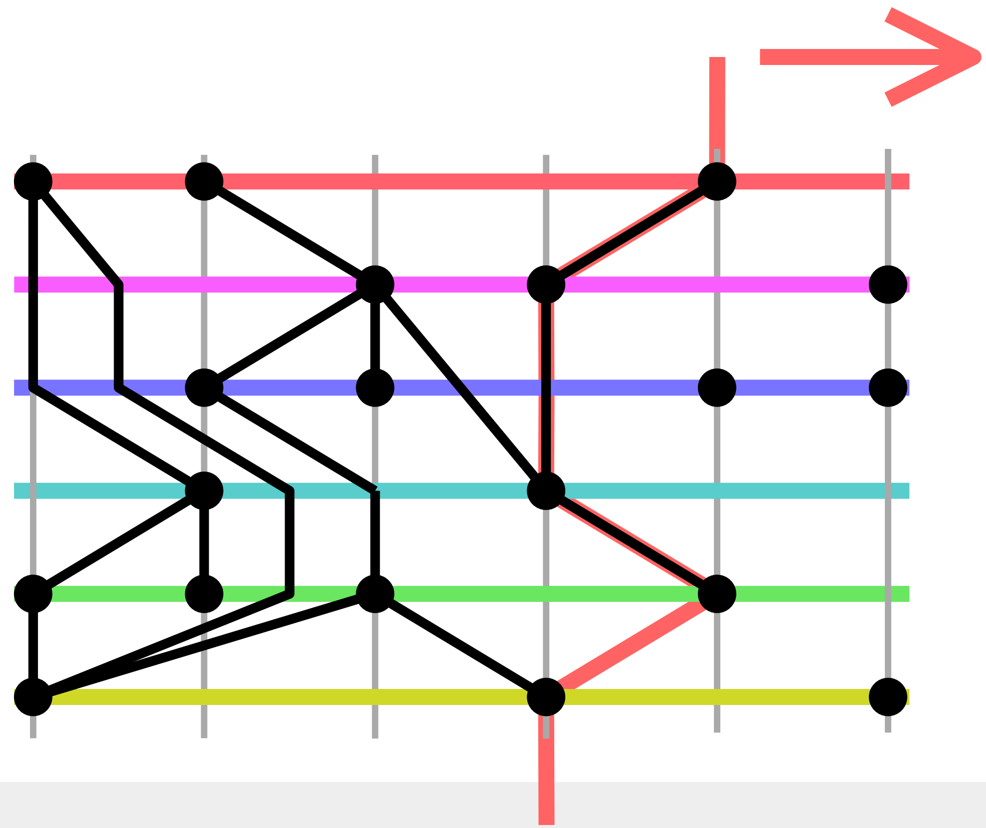


input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

state
$5, 3, 4, 3, 7, 3$

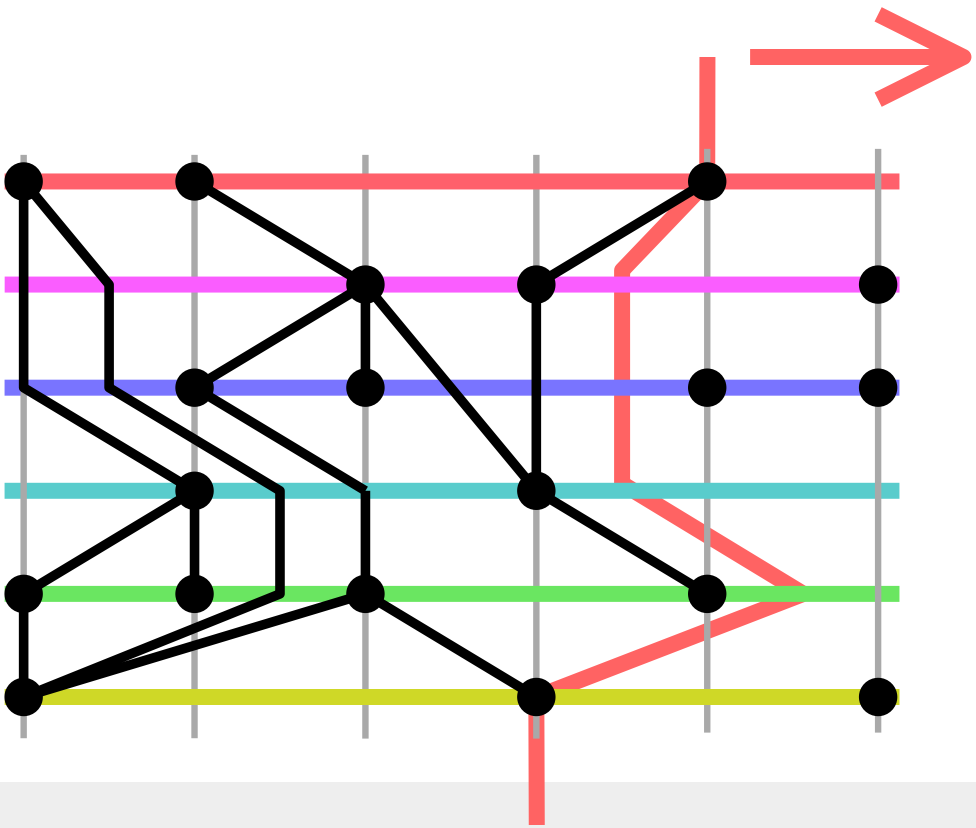


input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

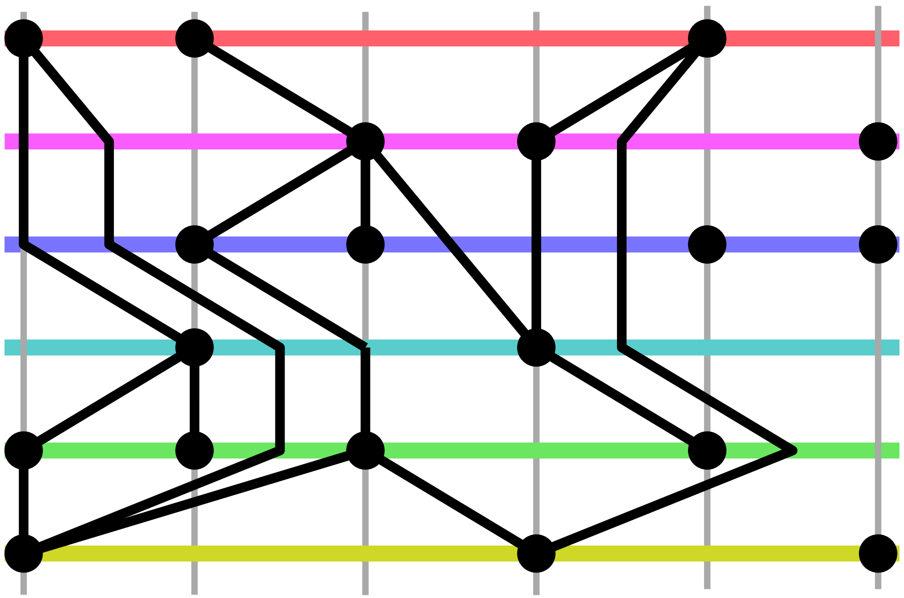
state
$5, 4, 4, 4, 8, 3$



input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

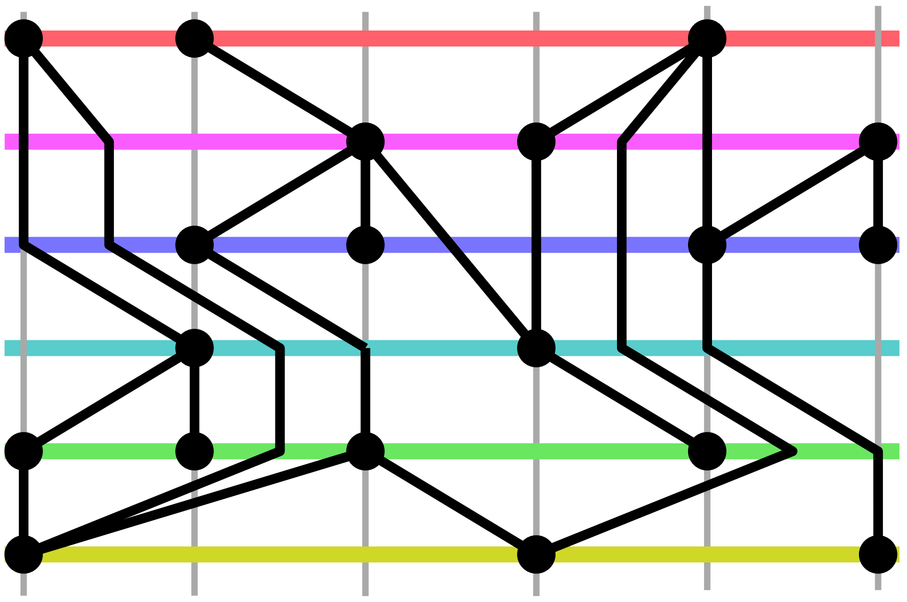
Ordered Level Planarity
(slow) algorithm



input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

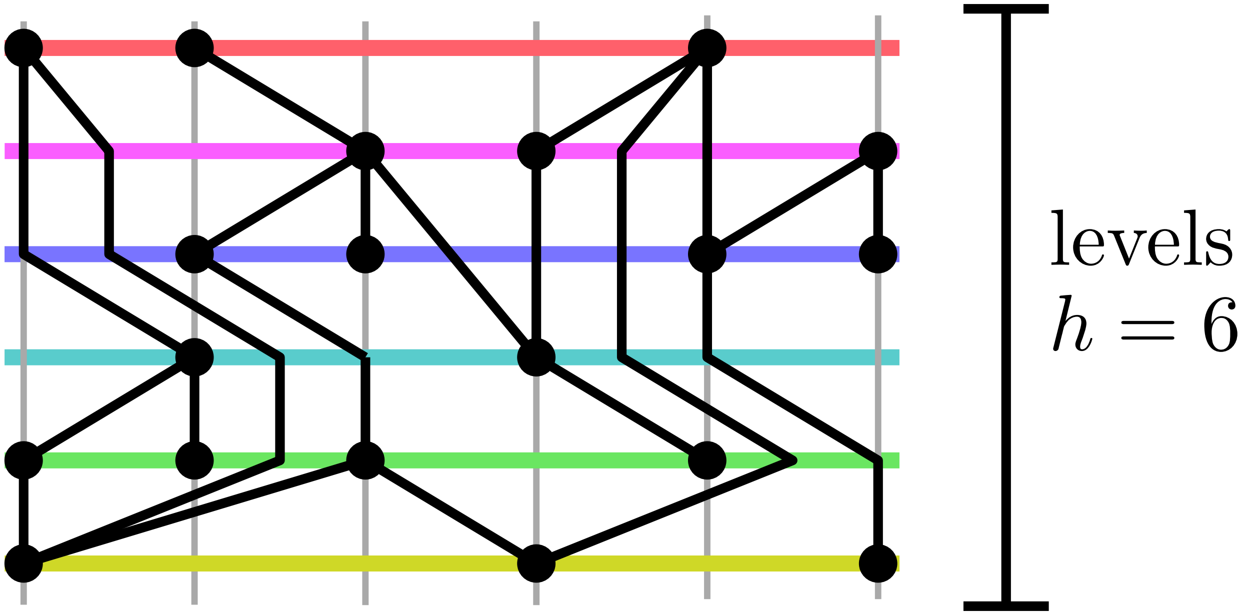


input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

Ordered Level Planarity
(slow) algorithm

using dynamic programming, runs in $n^{\mathcal{O}(h)}$



input: a graph G with vertex coordinates $\ell\colon V \to \mathbb{N}^2$

output: a planar drawing of G where vertices are on prescribed coordinates, and edges are y-monotone

NP-hardness

polynomial-time reduction (runs in $n^{O(1)}$)

of NP-hard problem to our problem

NP-hardness

polynomial-time reduction (runs in $n^{O(1)}$)
of NP-hard problem to our problem

- Satisfiability (SAT)
- 3-SAT
- 3-SAT-(2,2)
- Planar monotone 3-SAT
- 0-1 Integer programming
- Independent set

NP-hardness

polynomial-time reduction (runs in $n^{O(1)}$)
of NP-hard problem to our problem

Satisfiability (SAT)

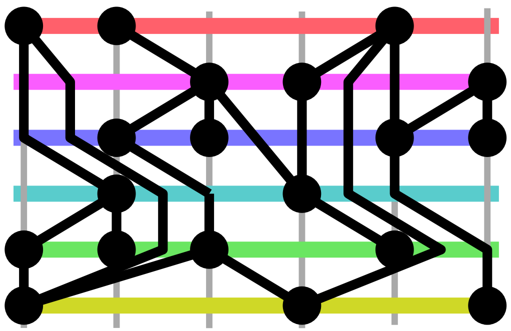
3-SAT

3-SAT-(2,2)

Planar monotone 3-SAT

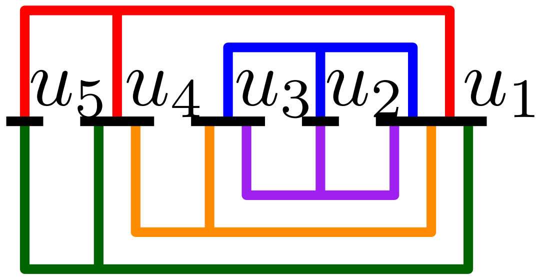
0-1 Integer programming

Independent set



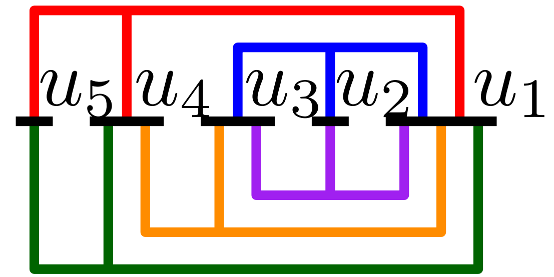
Ordered Level Planarity

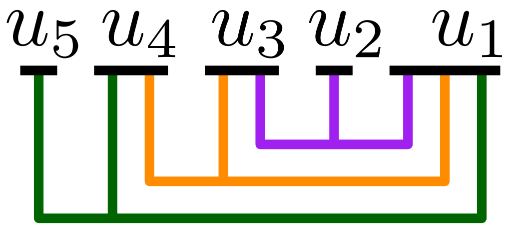
NP-hard from Planar Monotone 3-SAT

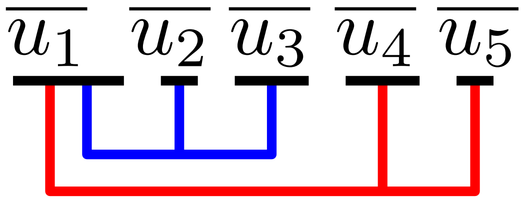


images of B. Klemz, G. Rote, Ordered Level Planarity ...

NP-hard from Planar Monotone 3-SAT

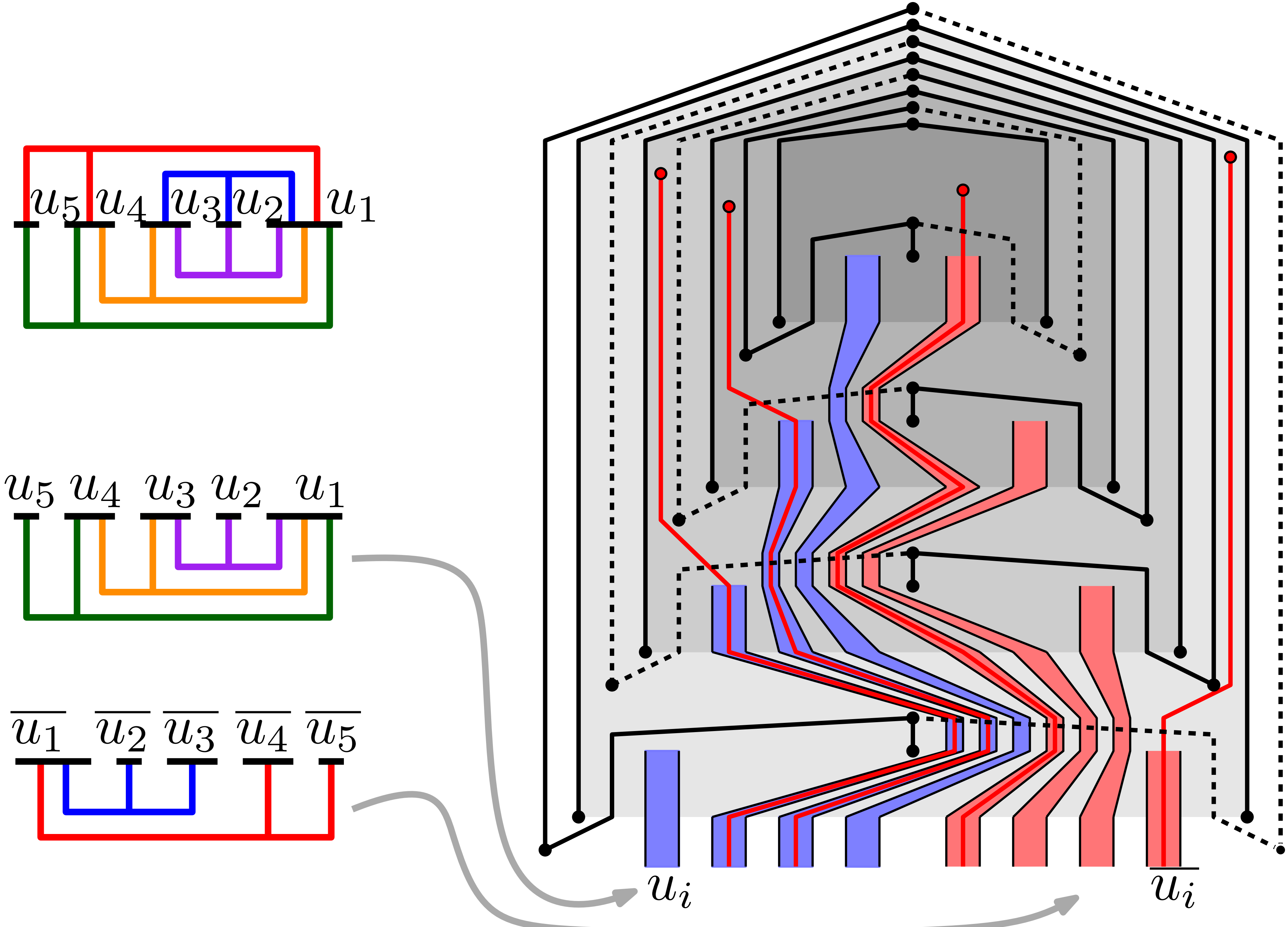






images of B. Klemz, G. Rote, Ordered Level Planarity . . .

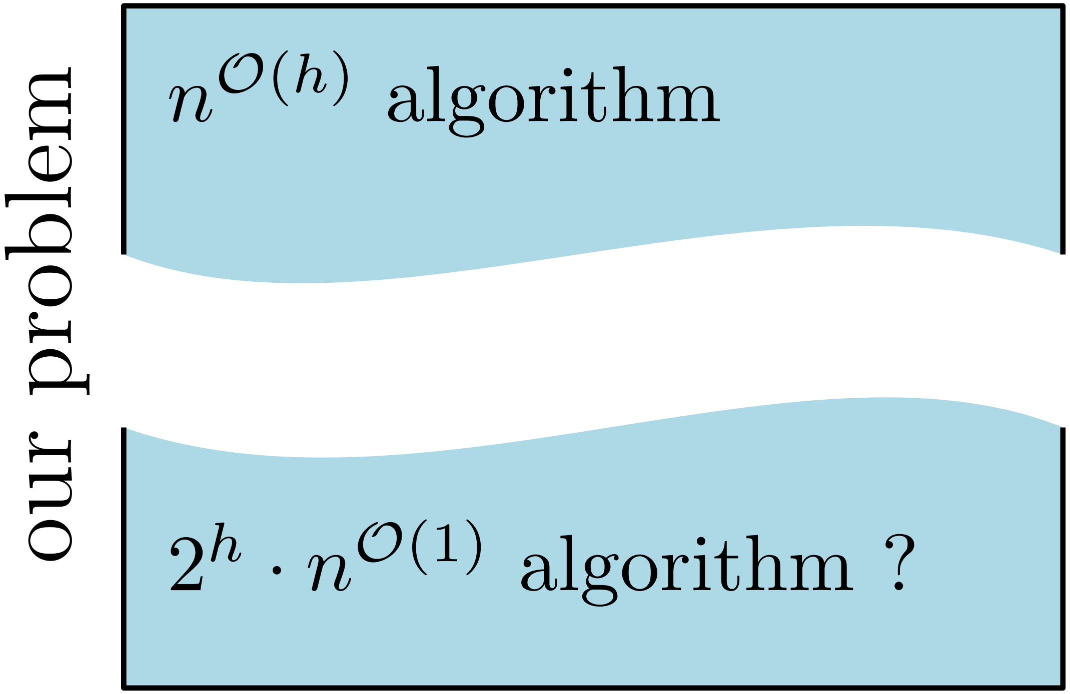
NP-hard from Planar Monotone 3-SAT



images of B. Klemz, G. Rote, Ordered Level Planarity . . .

h = height, number of distinct y-coordinates

NP-hard (for big h) $\rightarrow$ no $n^{\mathcal{O}(1)}$ algorithm

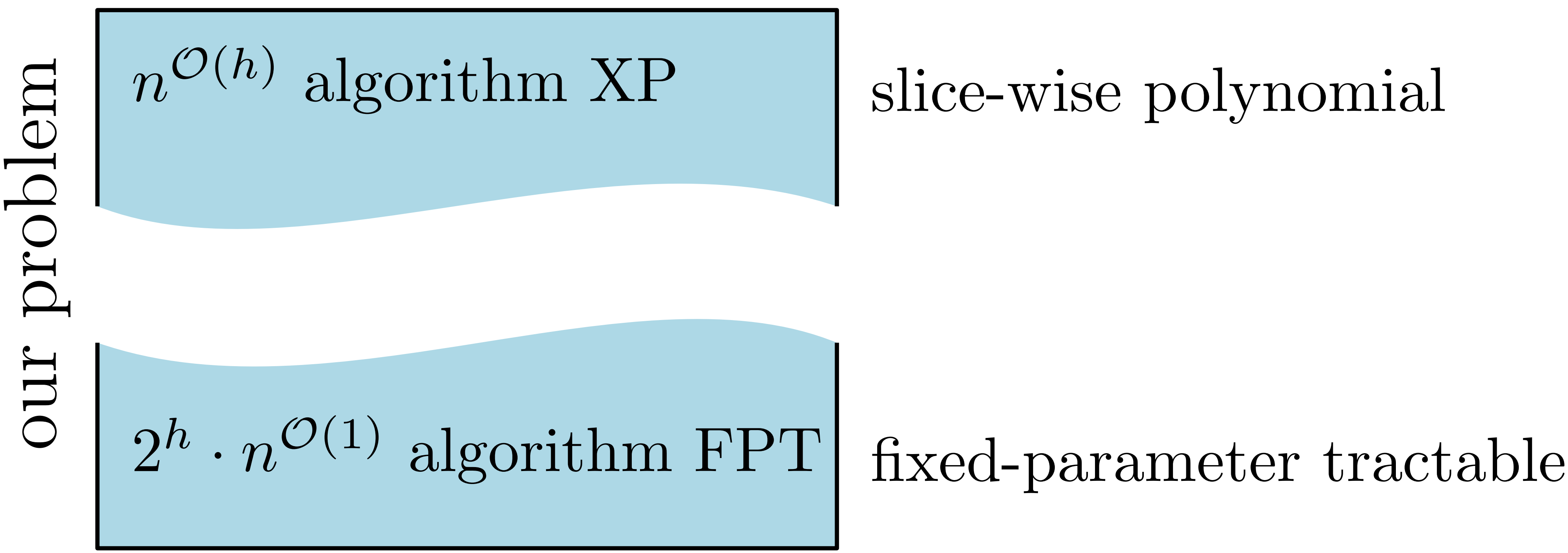


$n^{\mathcal{O}(1)}$ algorithm

Parameterized complexity

h = height, number of distinct y-coordinates

NP-hard (for big h) $\rightarrow$ no $n^{\mathcal{O}(1)}$ algorithm



$n^{\mathcal{O}(1)}$ algorithm

Parameterized complexity

h = height, number of distinct y-coordinates

NP-hard (for big h) $\rightarrow$ no $n^{\mathcal{O}(1)}$ algorithm

our problem

$n^{\mathcal{O}(h)}$ algorithm XP	slice-wise polynomial
W[1]-hardness	$\rightarrow$ no FPT algorithm
$2^h \cdot n^{\mathcal{O}(1)}$ algorithm FPT	fixed-parameter tractable

$n^{\mathcal{O}(1)}$ algorithm

W[1]-hardness

parameterized reduction (runs in $f(k) \cdot n^{O(1)}$)
of W[1]-hard problem to our problem

W[1]-hardness

parameterized reduction (runs in $f(k) \cdot n^{O(1)}$)
of W[1]-hard problem to our problem

parameter

Independent Set (its size)

Multicolored Ind. Set (size)

List Coloring (treewidth)

Odd Set (size)

Grid tiling (grid size)

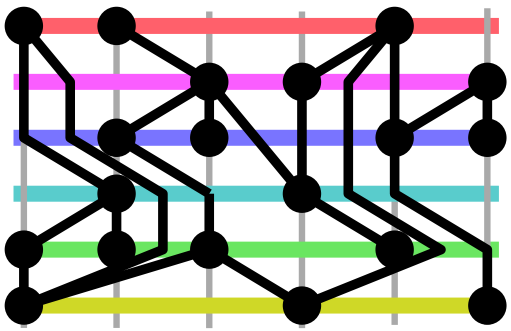
Partial Vertex Cover (size)

W[1]-hardness

parameterized reduction (runs in $f(k) \cdot n^{O(1)}$)
of W[1]-hard problem to our problem

parameter

Independent Set (its size)
Multicolored Ind. Set (size)
List Coloring (treewidth)
Odd Set (size)
Grid tiling (grid size)
Partial Vertex Cover (size)



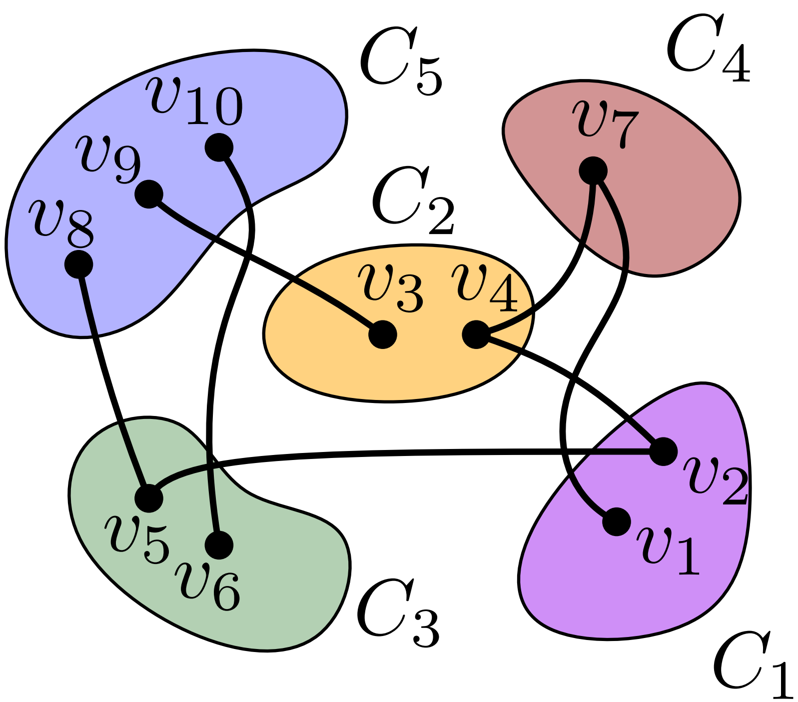
Ordered Level Planarity

W[1]-hard from MULTICOLORED INDEPENDENT SET

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?

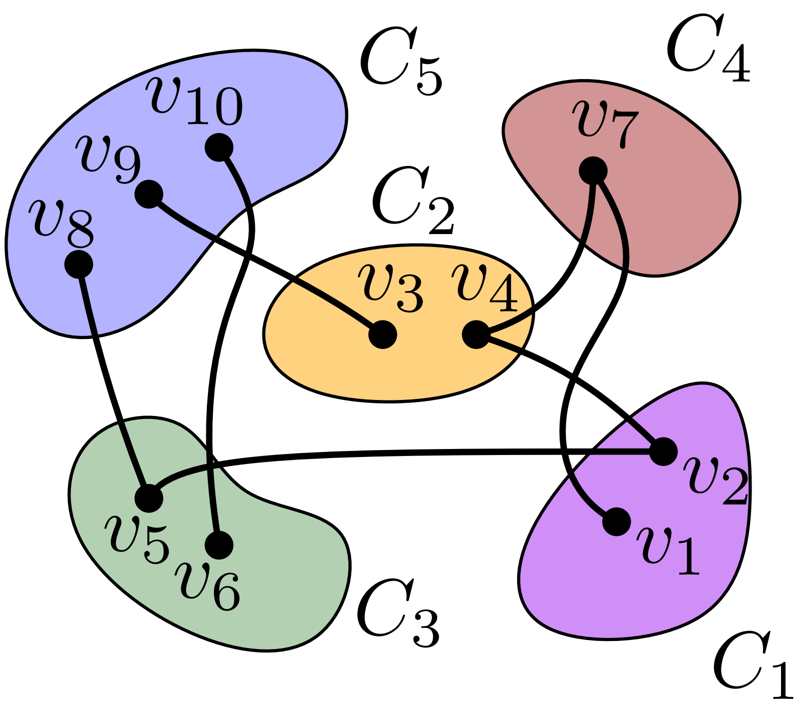


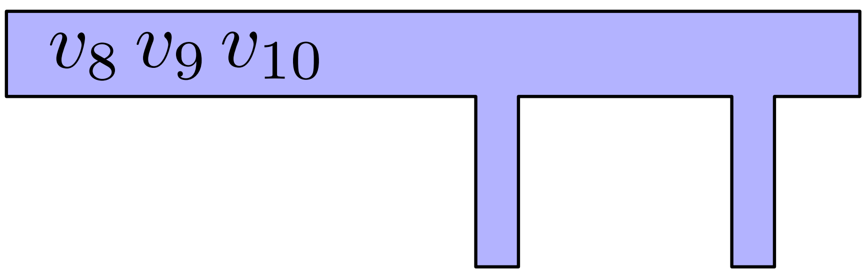
W[1]-hard from Multicolored Independent Set

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?



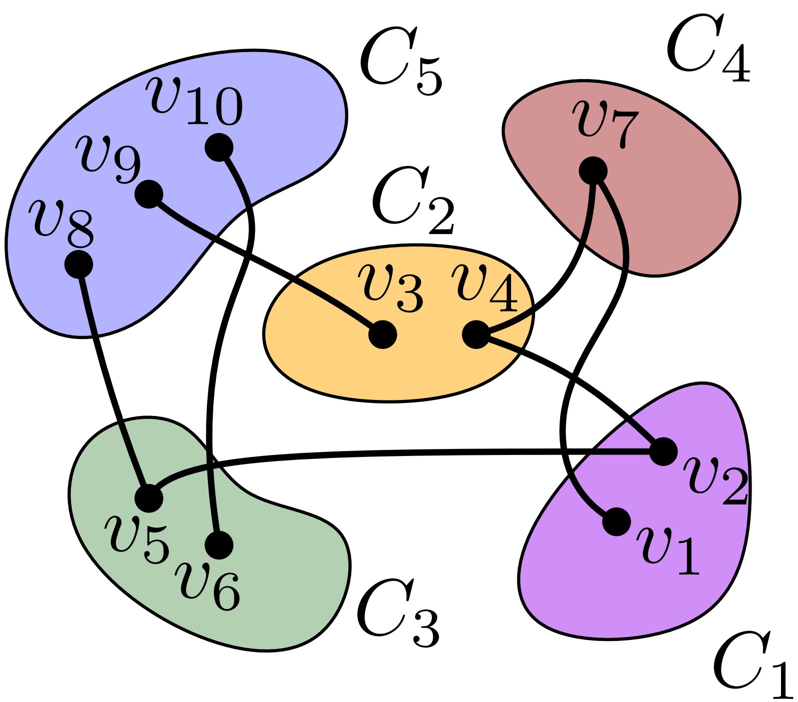


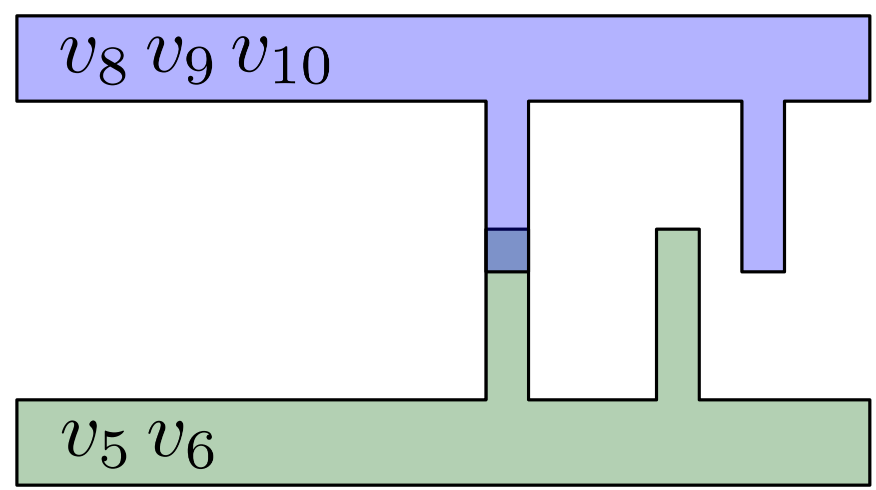
W[1]-hard from Multicolored Independent Set

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?



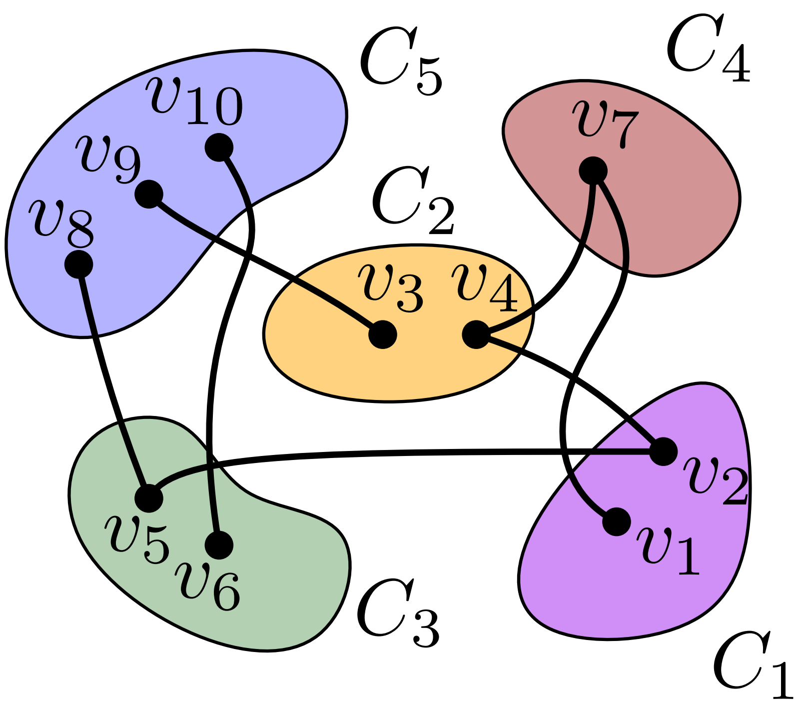


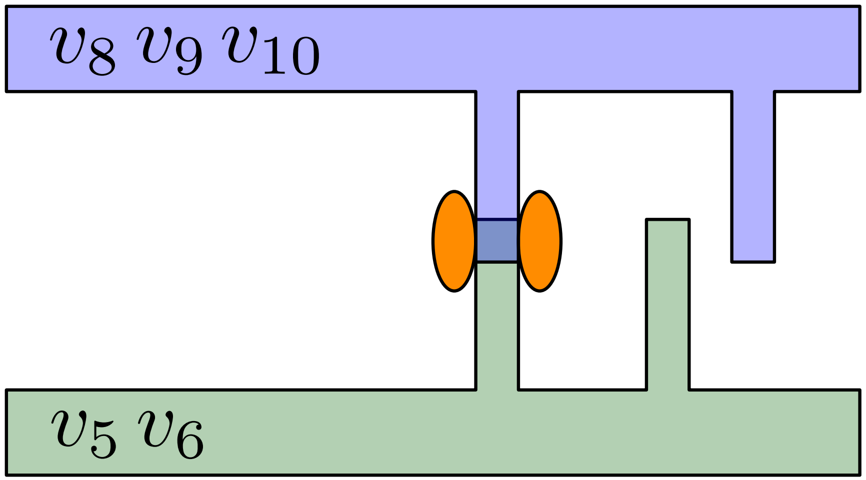
W[1]-hard from Multicolored Independent Set

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?



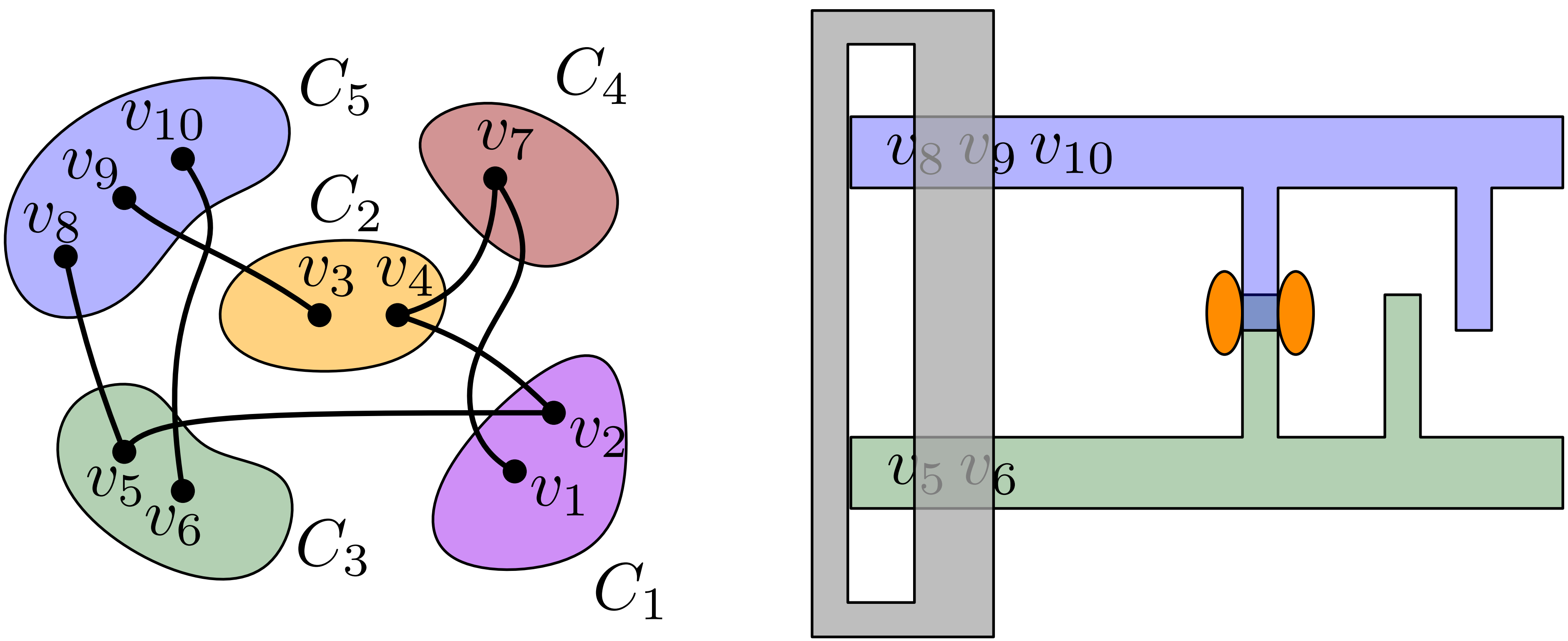


W[1]-hard from Multicolored Independent Set

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?

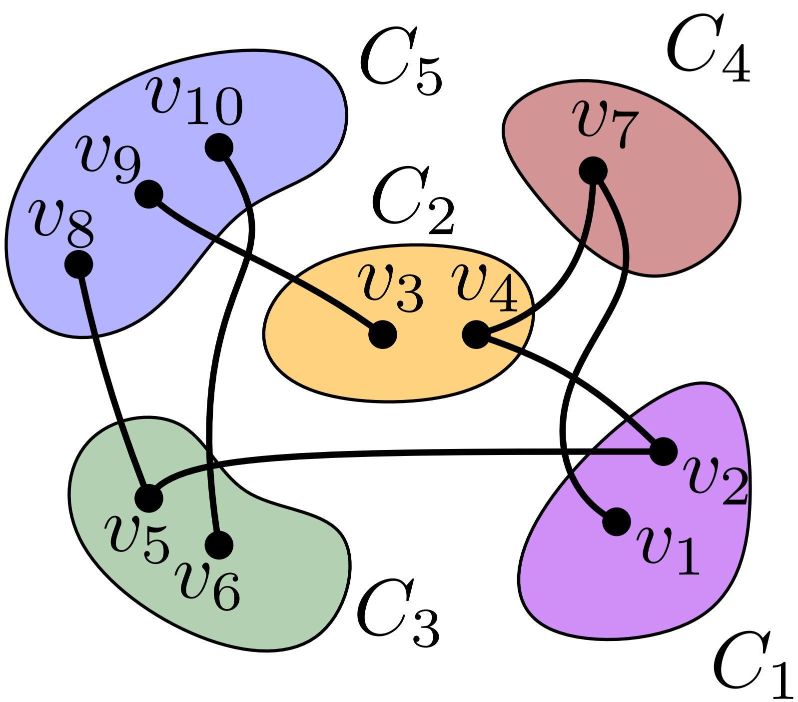


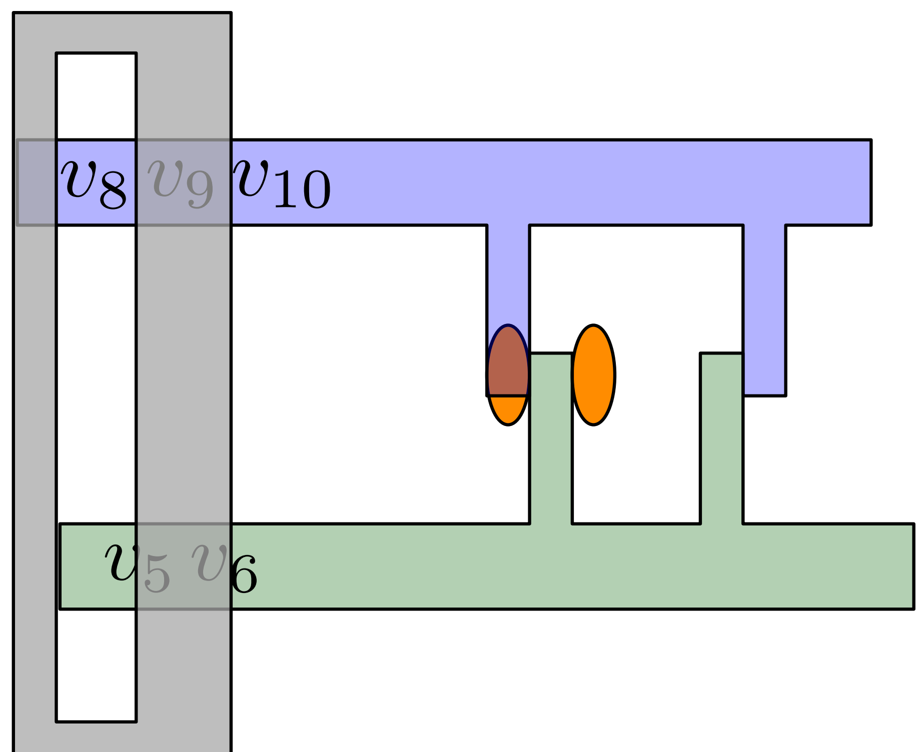
W[1]-hard from MULTICOLORED INDEPENDENT SET

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?



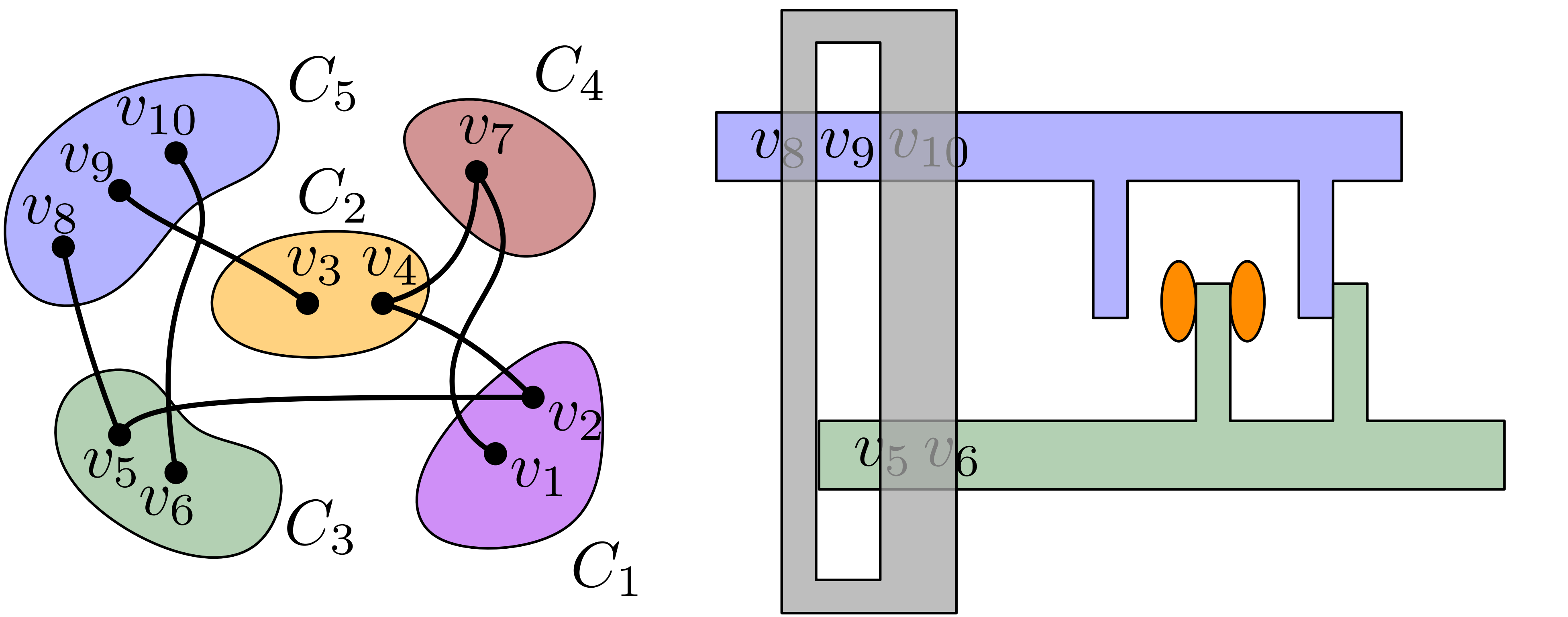


W[1]-hard from Multicolored Independent Set

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?

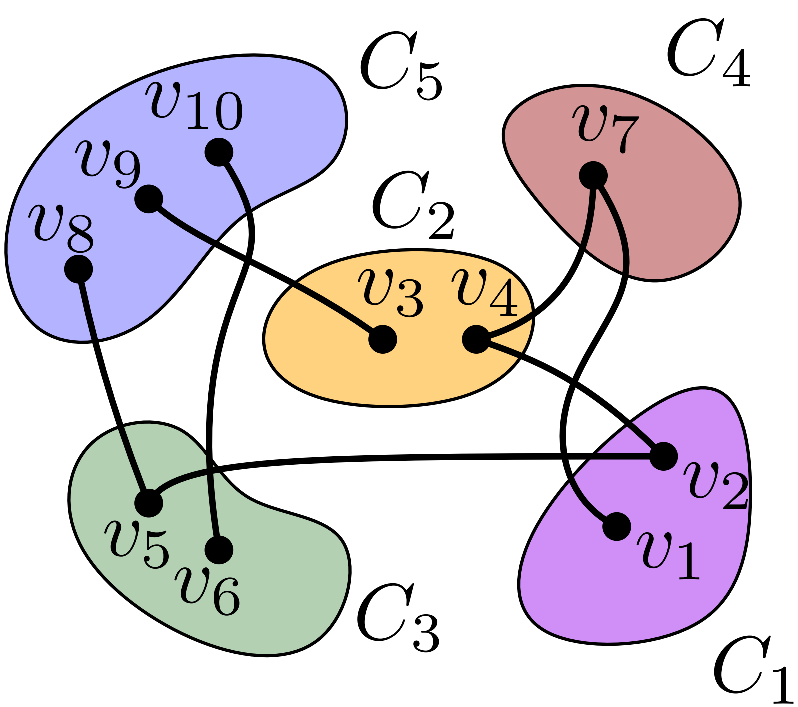


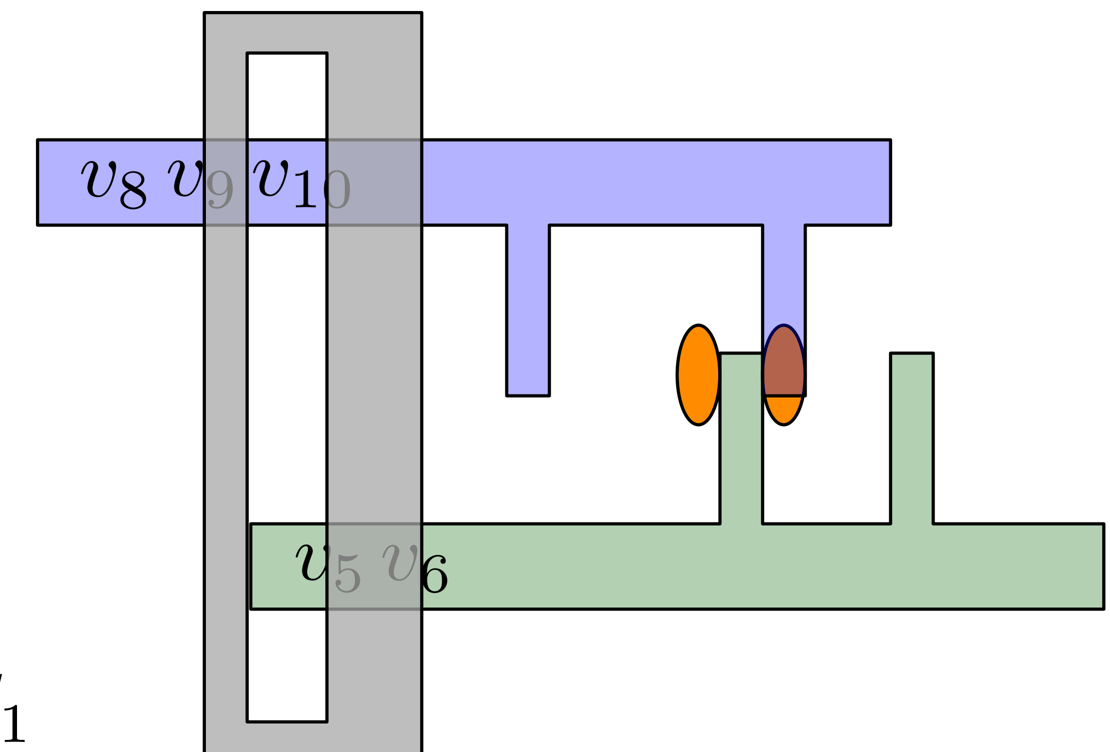
W[1]-hard from Multicolored Independent Set

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?



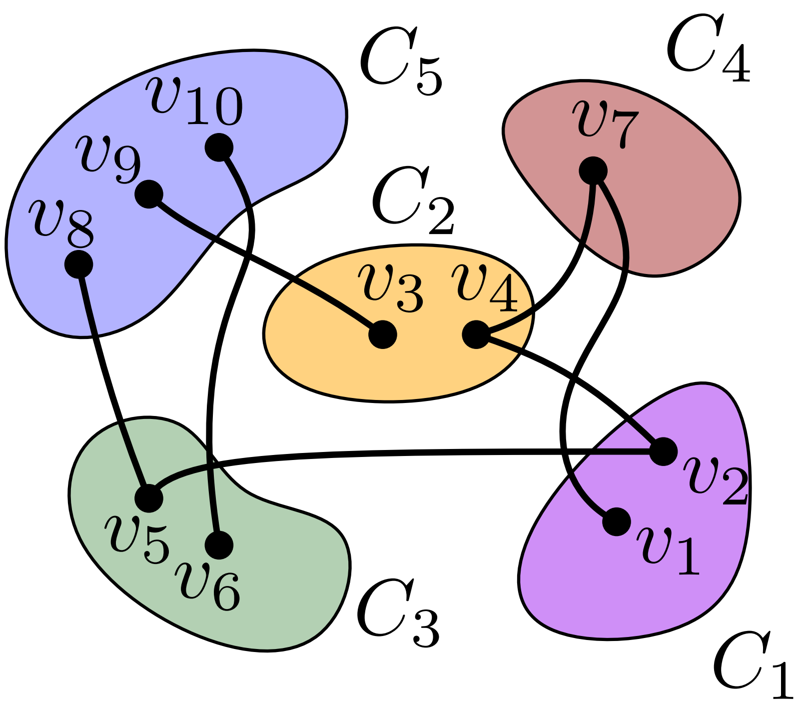


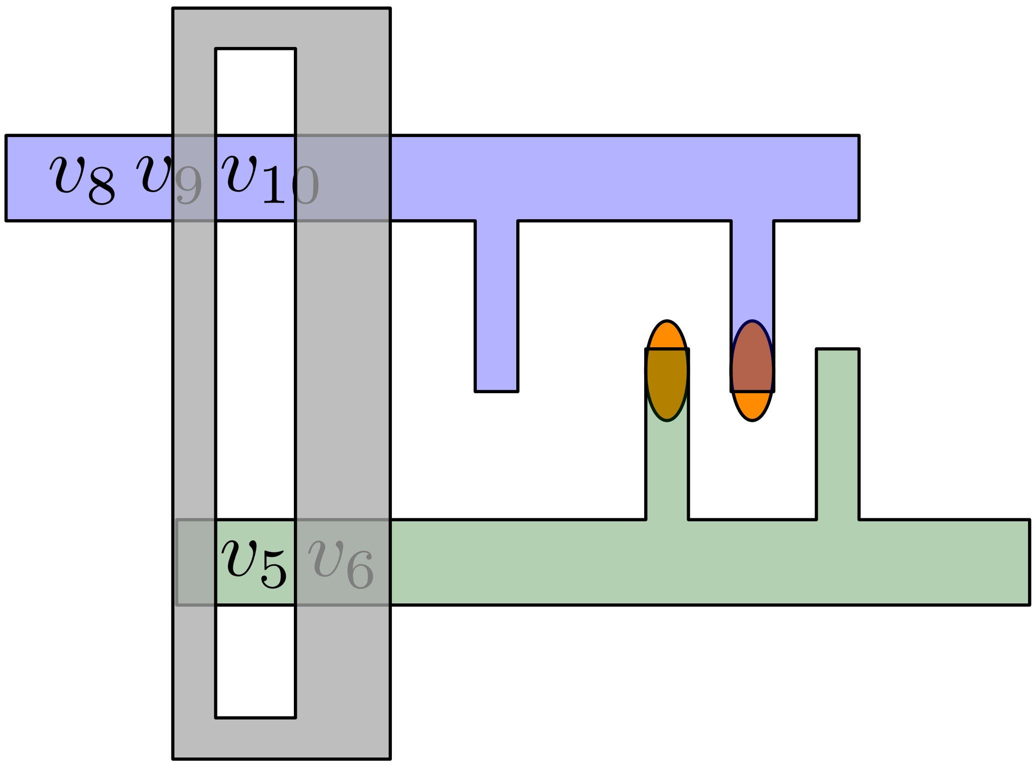
W[1]-hard from MULTICOLORED INDEPENDENT SET

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?



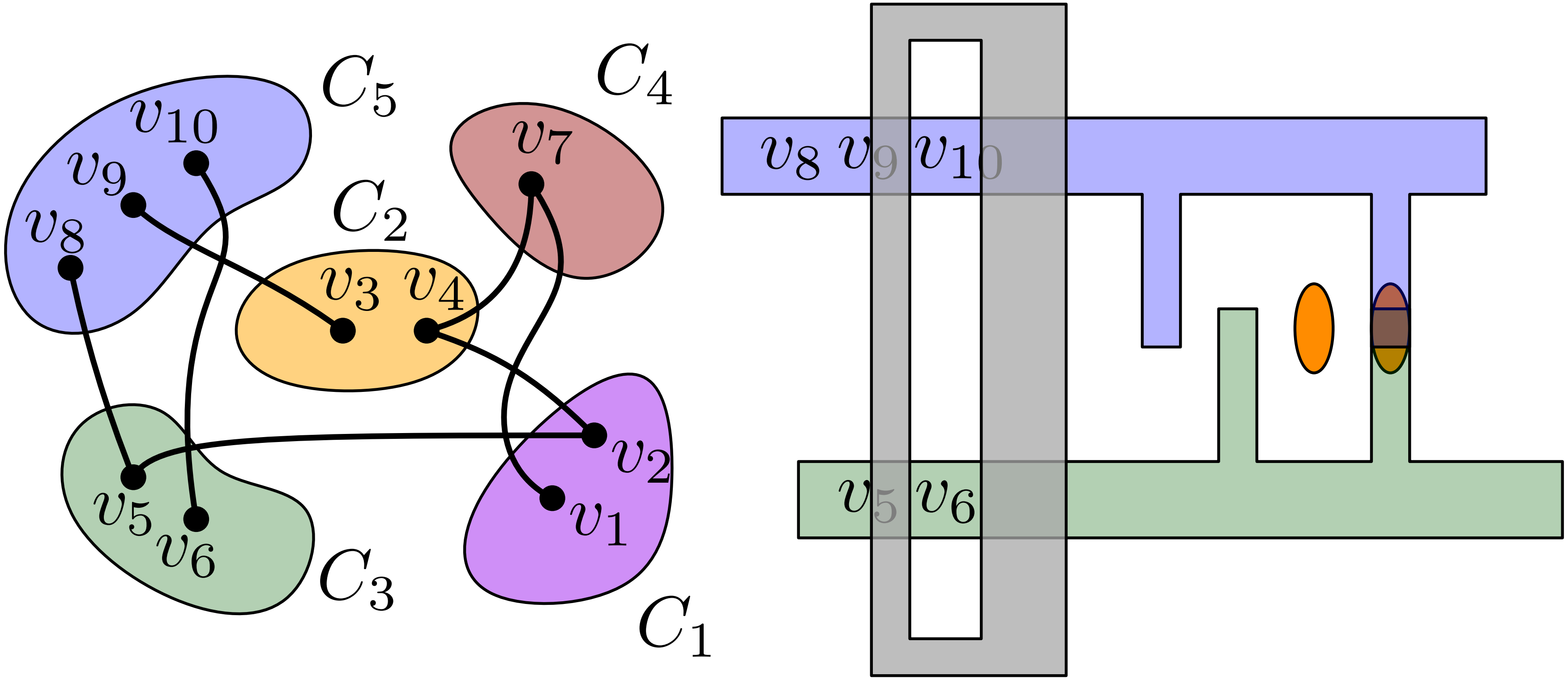


W[1]-hard from Multicolored Independent Set

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?

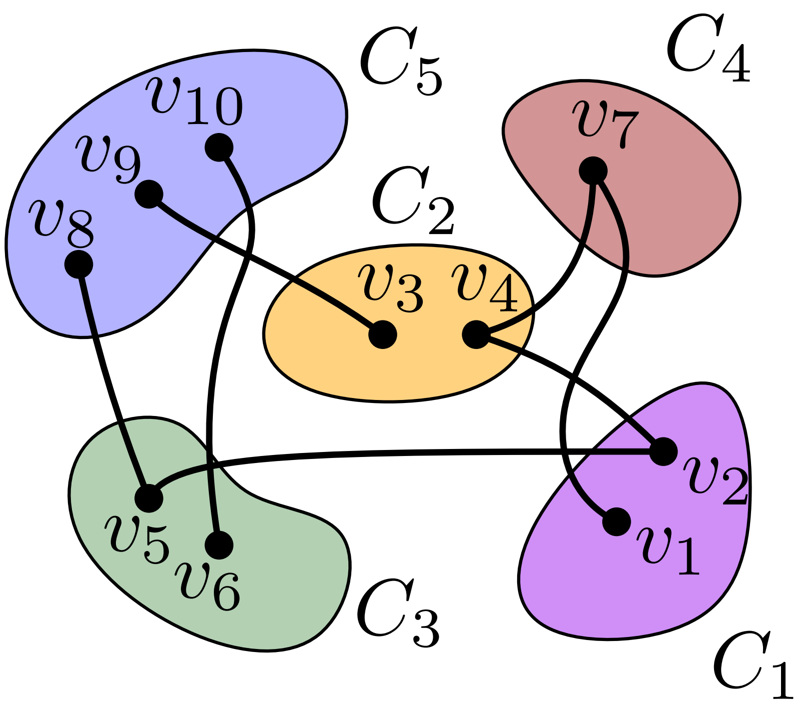


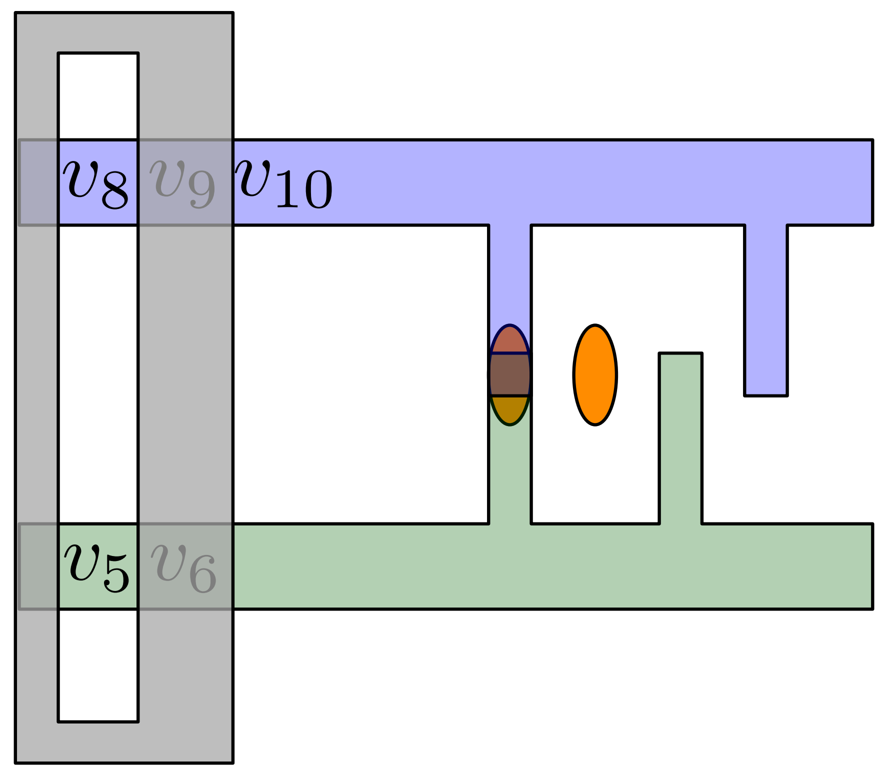
W[1]-hard from Multicolored Independent Set

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?



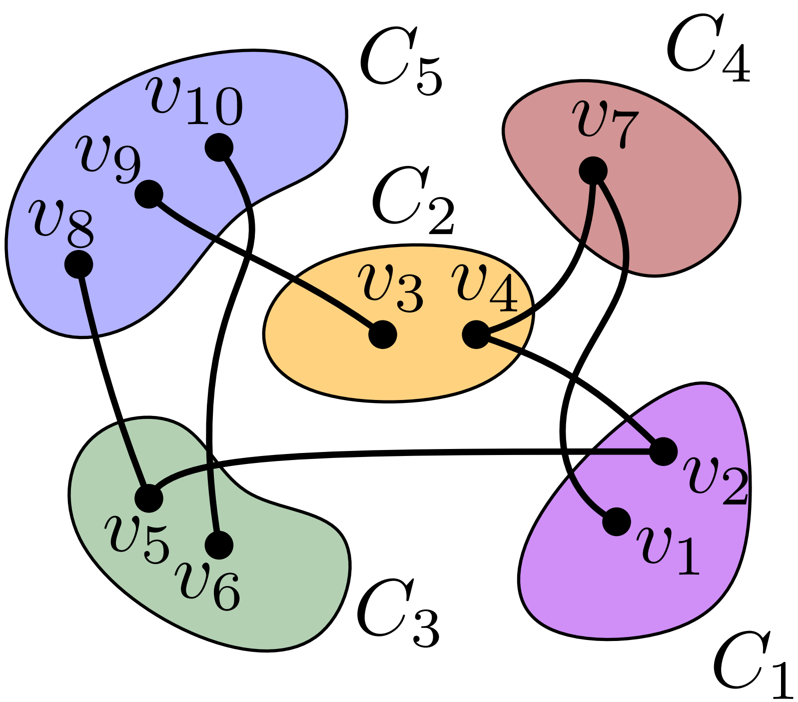


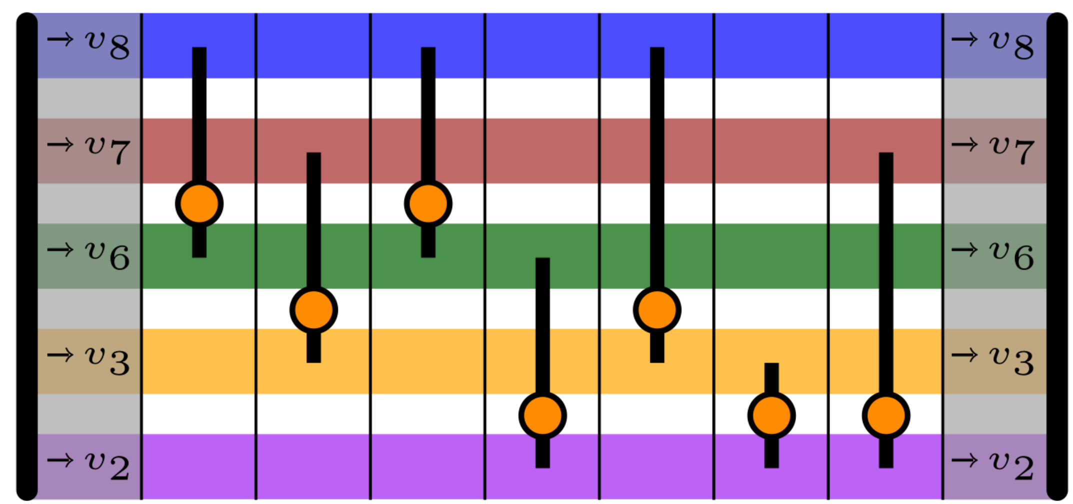
W[1]-hard from Multicolored Independent Set

input: a graph H, an integer k, and a k-coloring $C_1, C_2, \ldots, C_k$ of $V(H)$

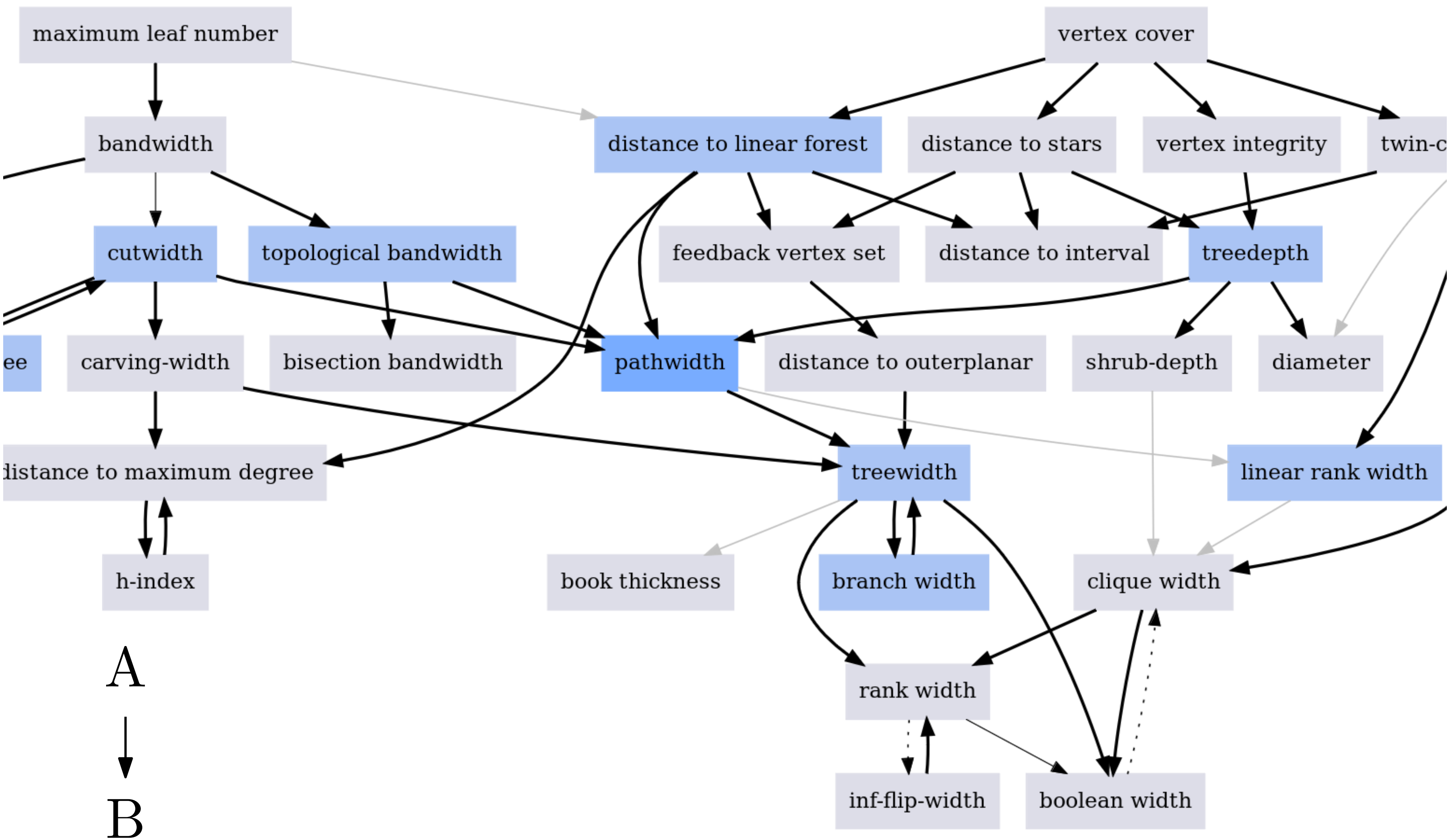
parameter: k

output: Does H have an independent set $X \subseteq V(H)$ that contains one vertex of C_j for every $j \in [k]$?



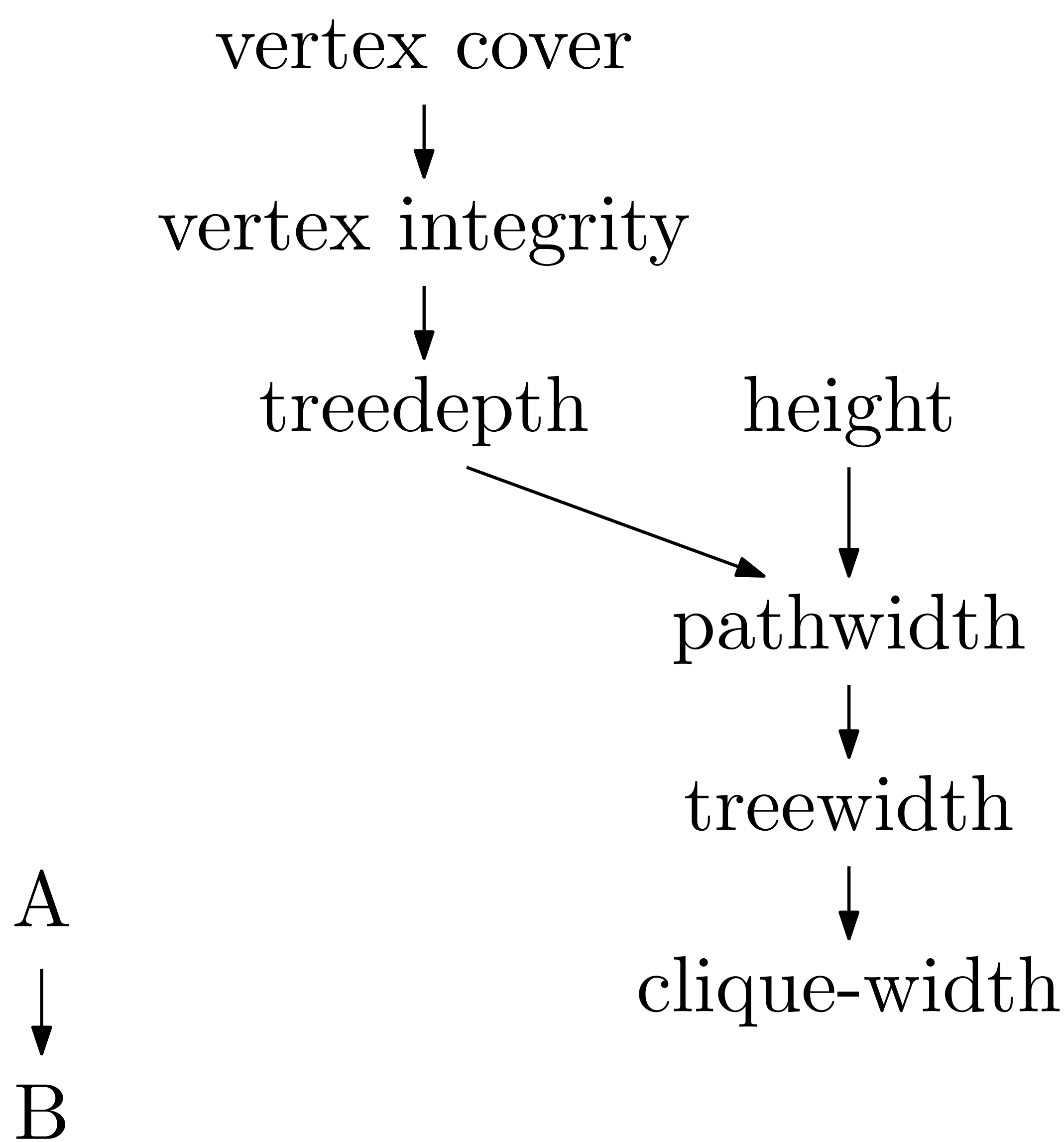


Complexity of Ordered Level Planarity



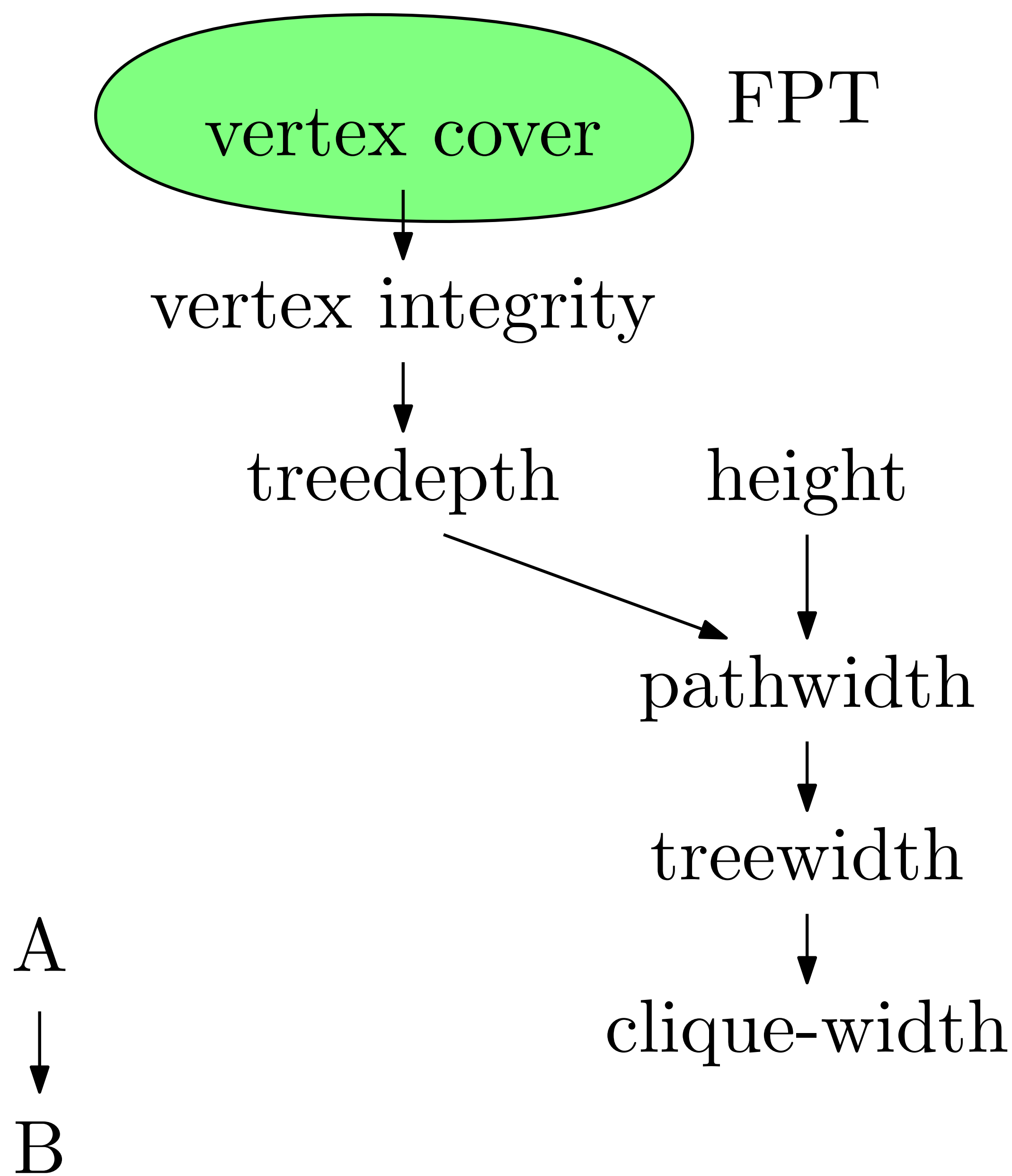
$A \leq f(B)$ for computable function f

Complexity of Ordered Level Planarity



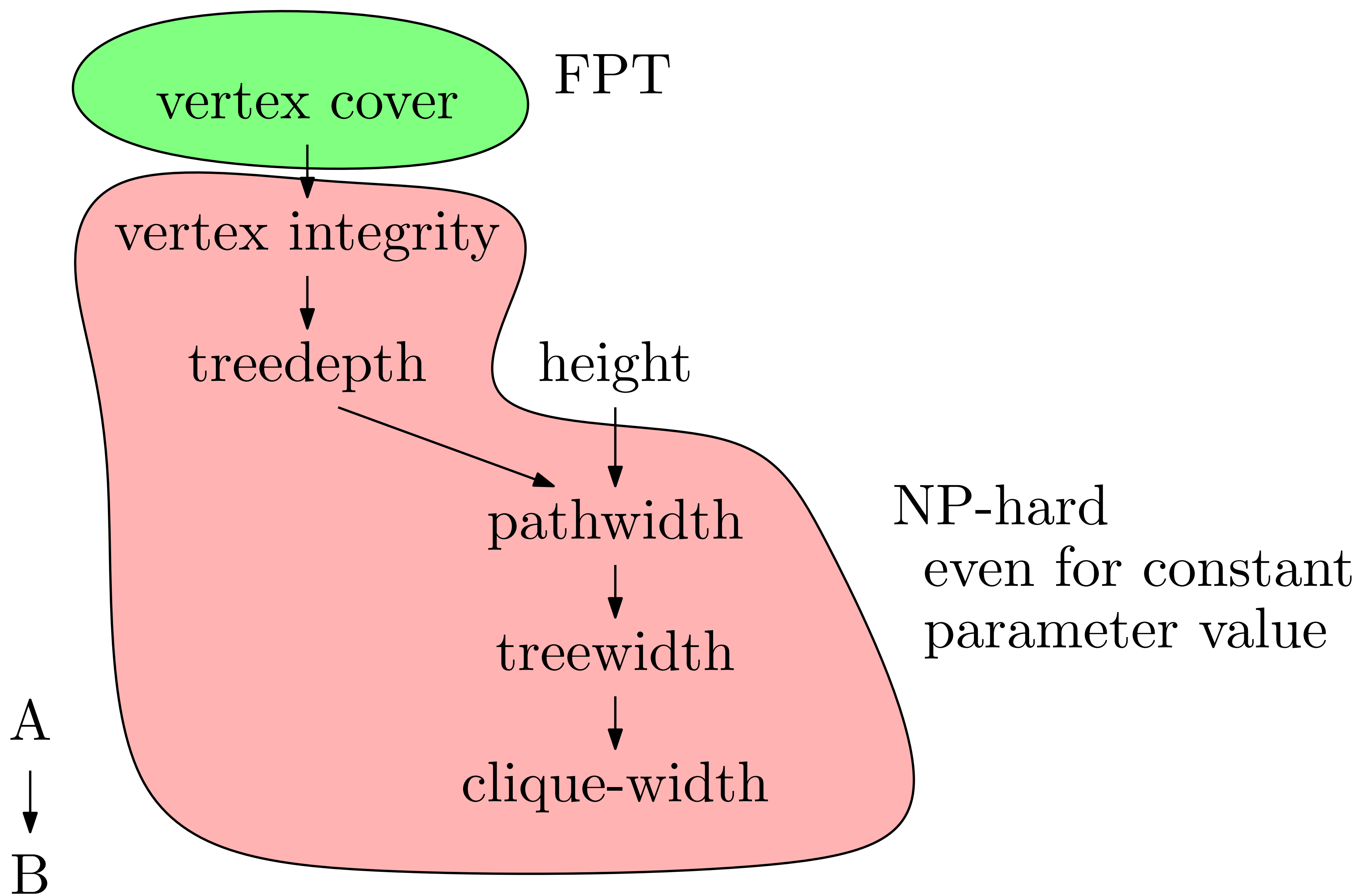
$A \leq f(B)$ for computable function f

Complexity of Ordered Level Planarity



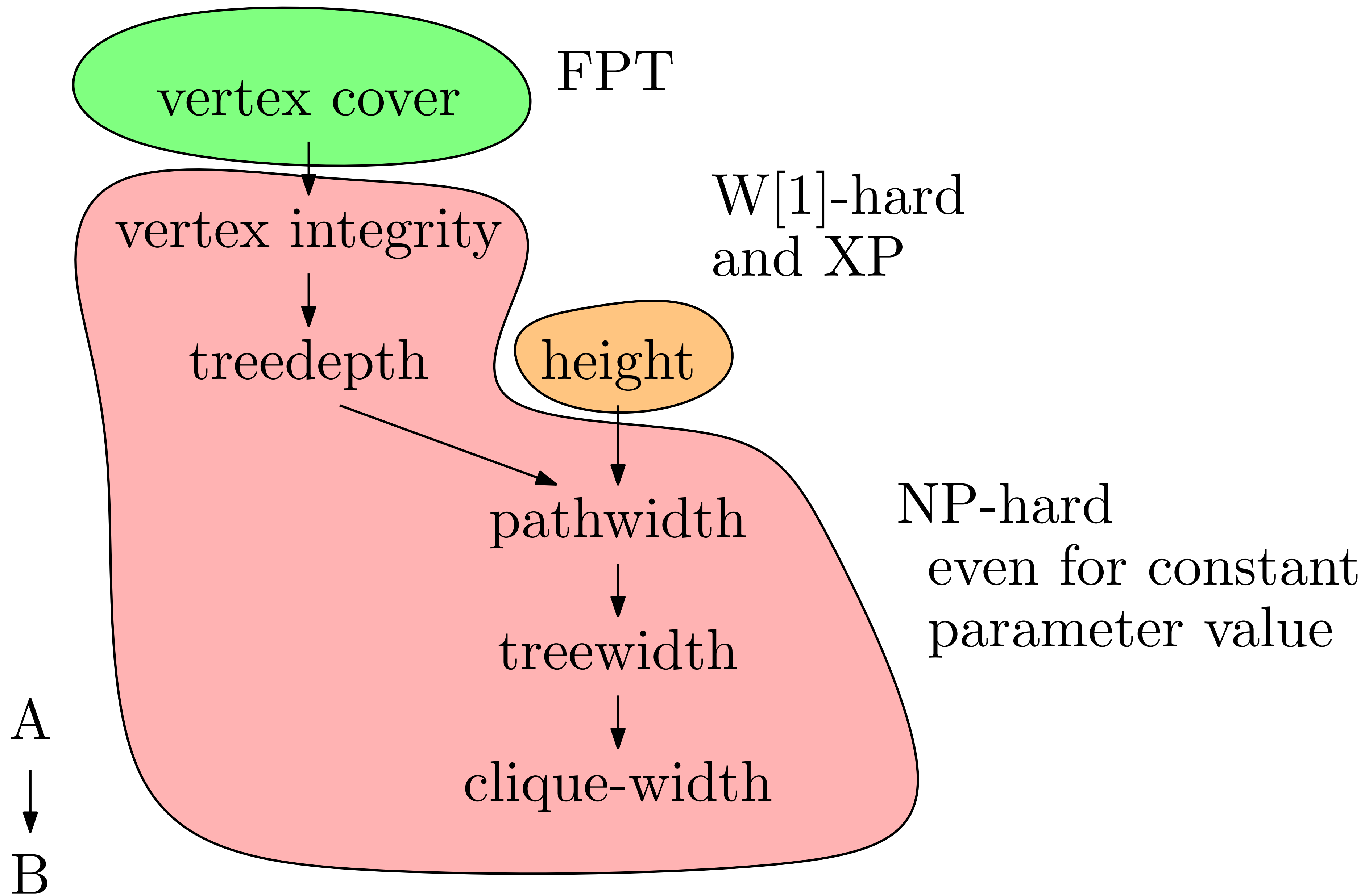
$A \leq f(B)$ for computable function f

Complexity of ORDERED LEVEL PLANARITY



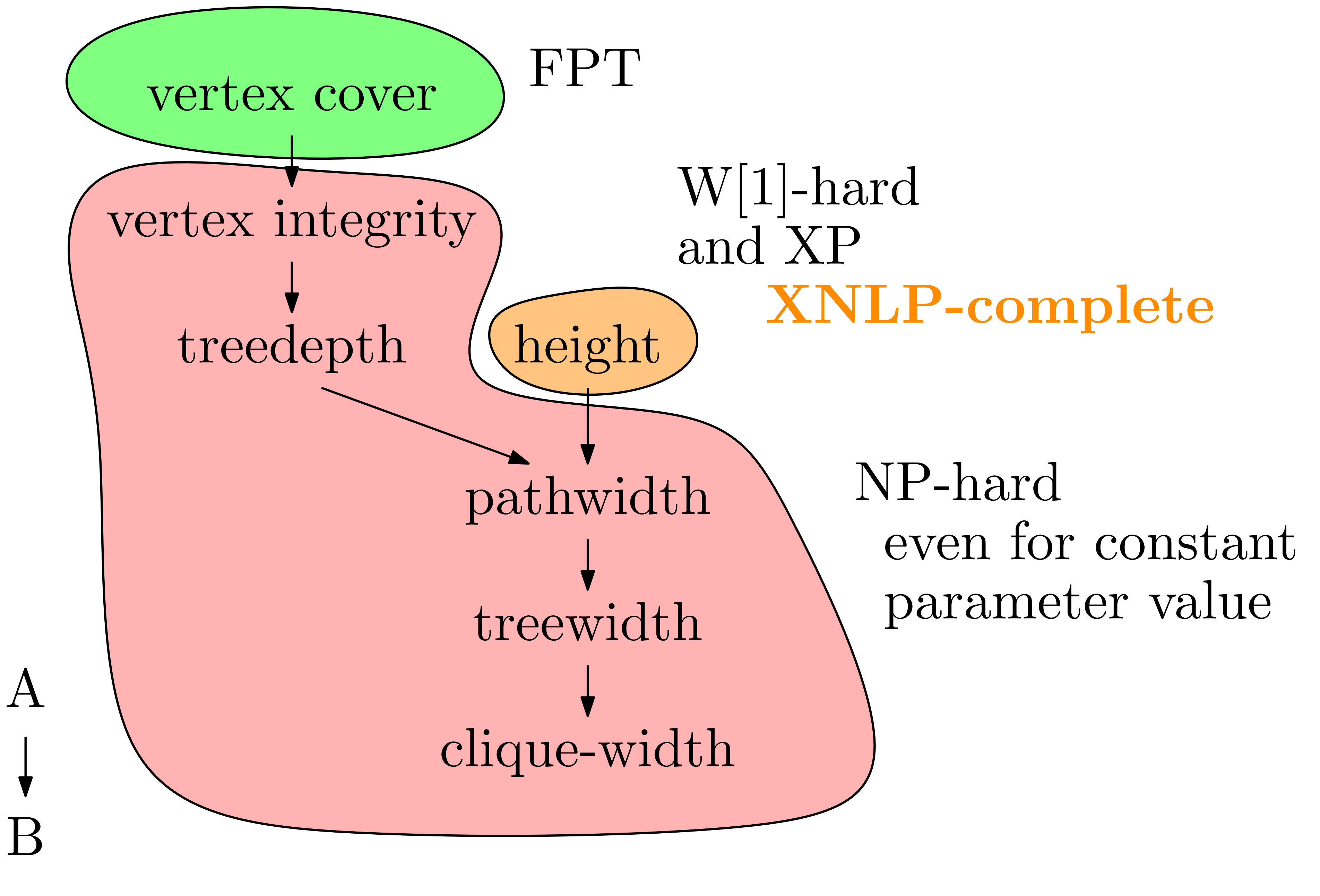
$A \leq f(B)$ for computable function f

Complexity of Ordered Level Planarity



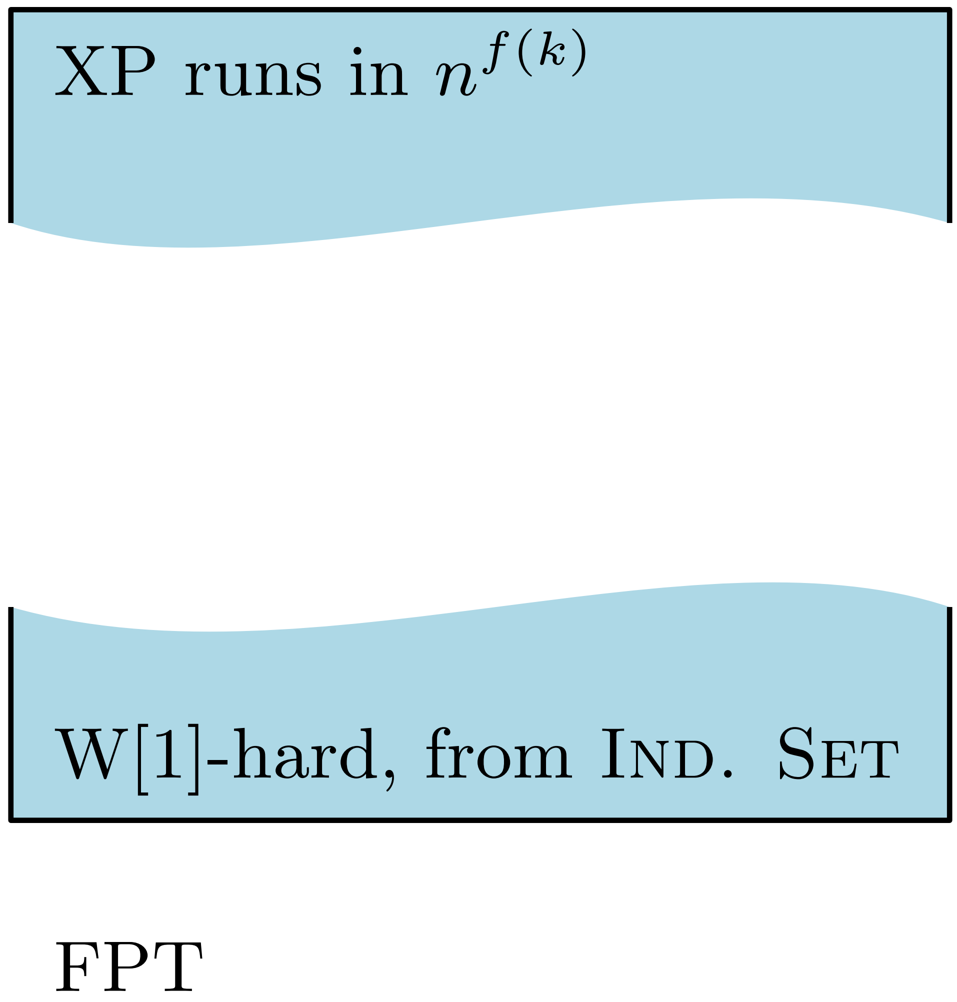
$A \leq f(B)$ for computable function f

Complexity of Ordered Level Planarity



$A \leq f(B)$ for computable function f

XP and W[1]-hard and linear structure → XNLP?



XP and W[1]-hard and linear structure $\rightarrow$ XNLP?

XP runs in $n^{f(k)}$
XNLP
XNLP-hard

W[t]-hard $\forall t$

W[3]-hard
W[2]-hard

W[1]-hard, from Ind. Set

FPT

XP and W[1]-hard and linear structure $\rightarrow$ XNLP?

XP

algorithm that runs in $n^{f(k)}$ time

XNLP membership

nondeterministic algorithm that runs in $f(k) \cdot n^c$ time and $g(k) \cdot \log n$ space

XP and W[1]-hard and linear structure $\to$ XNLP?

XP

algorithm that runs in $n^{f(k)}$ time

XNLP membership

nondeterministic algorithm that runs in $f(k) \cdot n^c$ time and $g(k) \cdot \log n$ space

W[1]-hard

parameterized reduction that runs in $\mathcal{O}(f(k) \cdot n^c)$ time

from Multicolored Independent Set

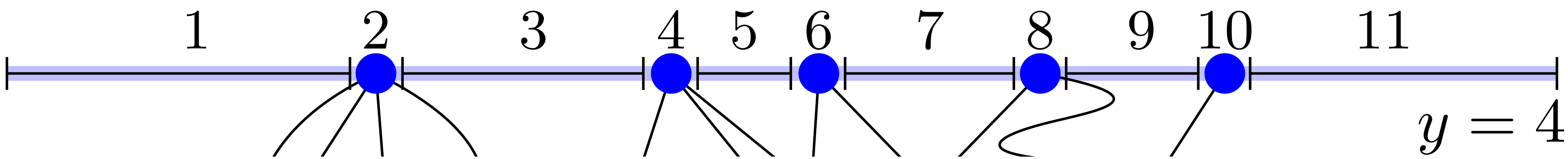
XNLP-hard

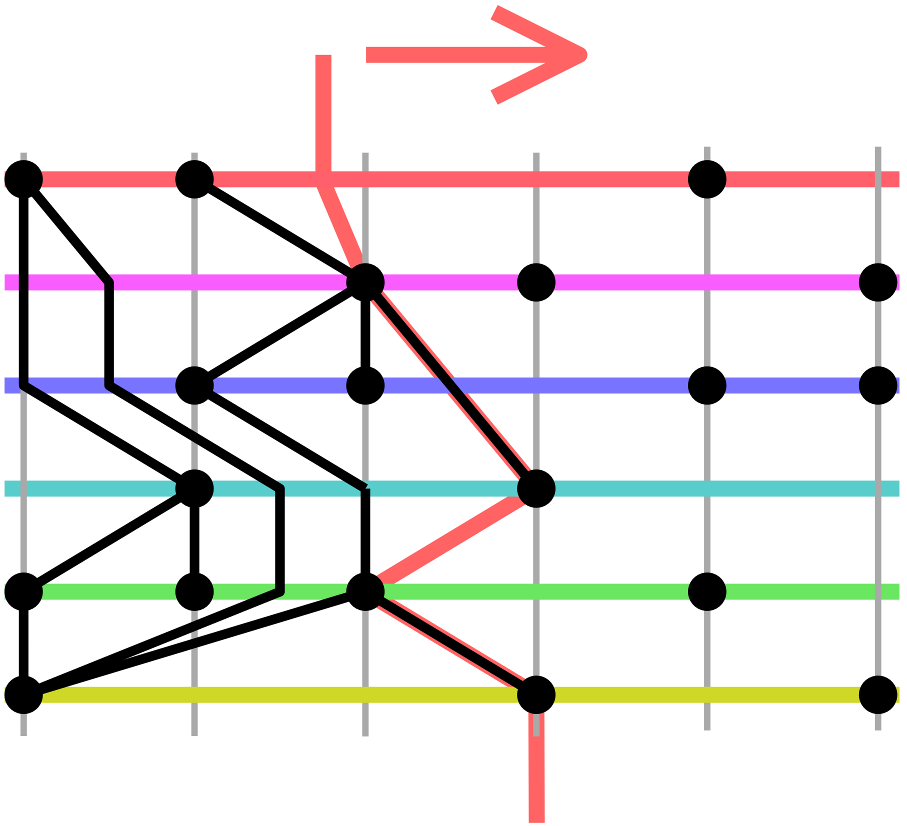
parameterized reduction that runs in $\mathcal{O}(f(k) \cdot n^c)$ time and uses only $\mathcal{O}(g(k) \cdot \log n)$ space

from Chained Multicolored Independent Set

XNLP membership

nondeterministic algorithm that runs in $f(k) \cdot n^c$ time and $g(k) \cdot \log n$ space





XNLP-hardness

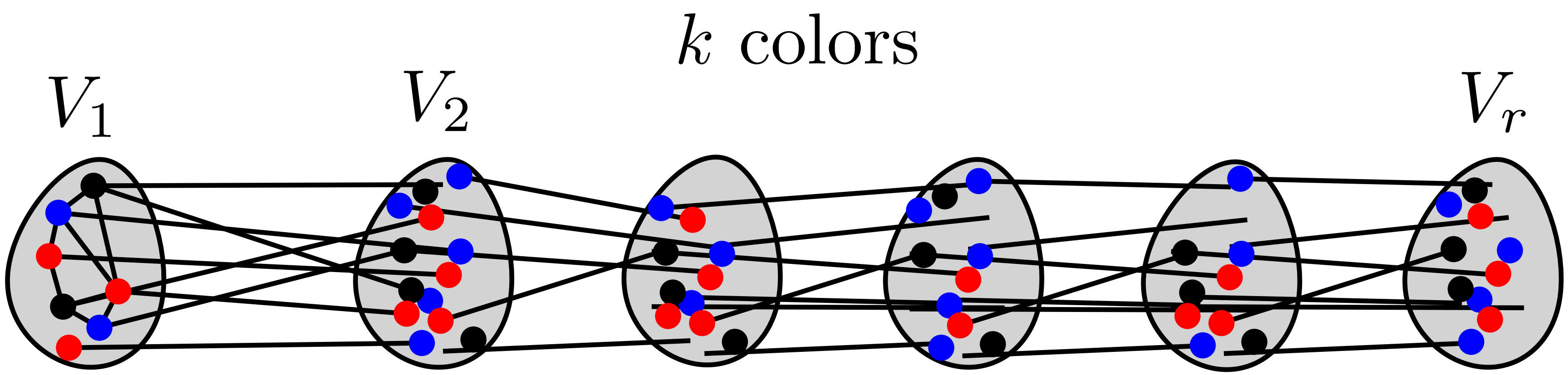
parameterized reduction that runs in $\mathcal{O}(f(k) \cdot n^c)$ time and uses only $\mathcal{O}(g(k) \cdot \log n)$ space

CHAINED MULTICOLORED INDEPENDENT SET

input: a graph H, an integer k, a k-coloring $C_1, C_2, \ldots, C_k$ and an r-partition $V_1, \ldots, V_r$ of $V(H)$ such that $\forall uv \in E(H), u \in V_i \Rightarrow v \in (V_{i-1} \cup V_i \cup V_{i+1})$.

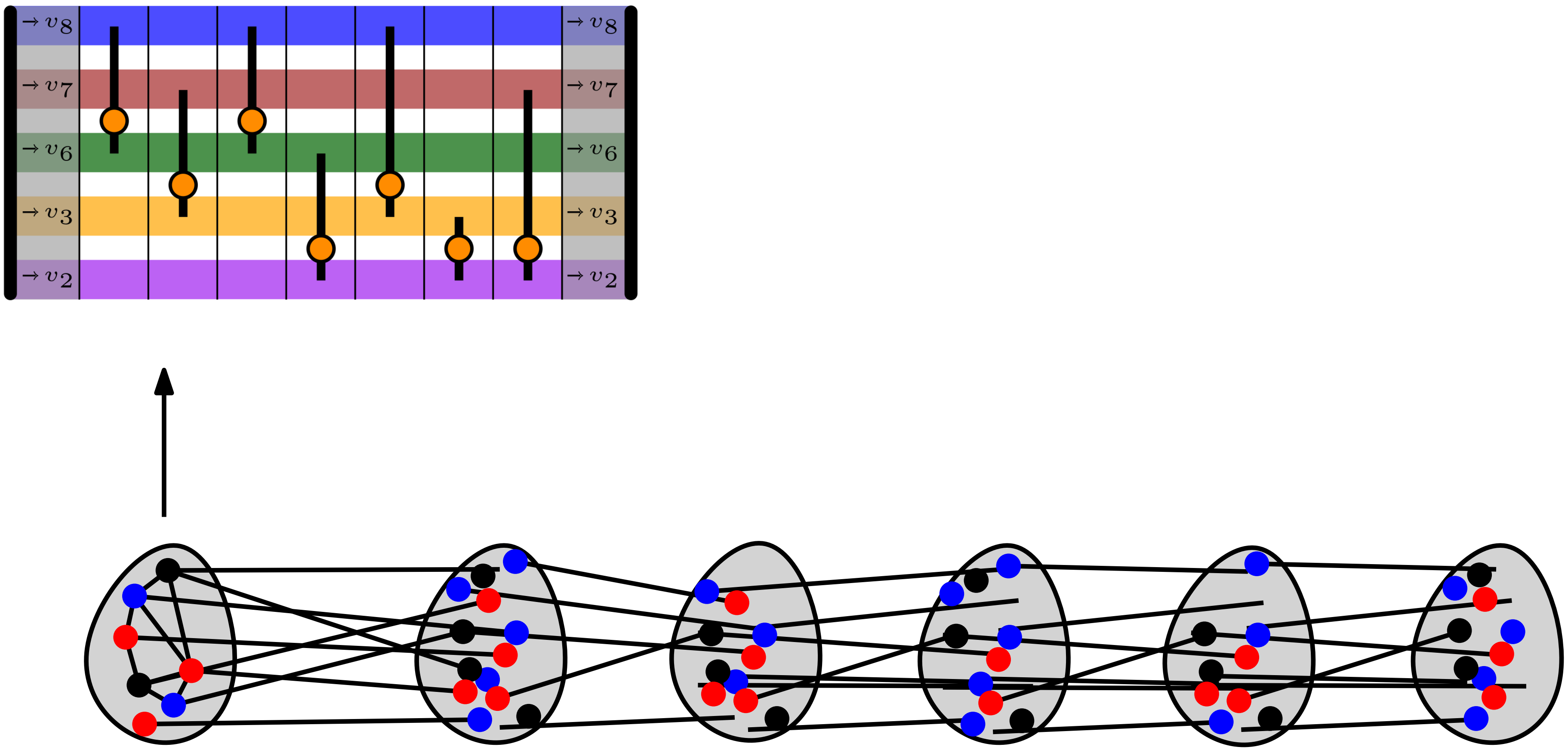
parameter: k

output: Does H have an indep. set $X \subseteq V(H)$ that contains one vertex of $C_j \cap V_i$ for every $j \in [k]$, $i \in [r]$?



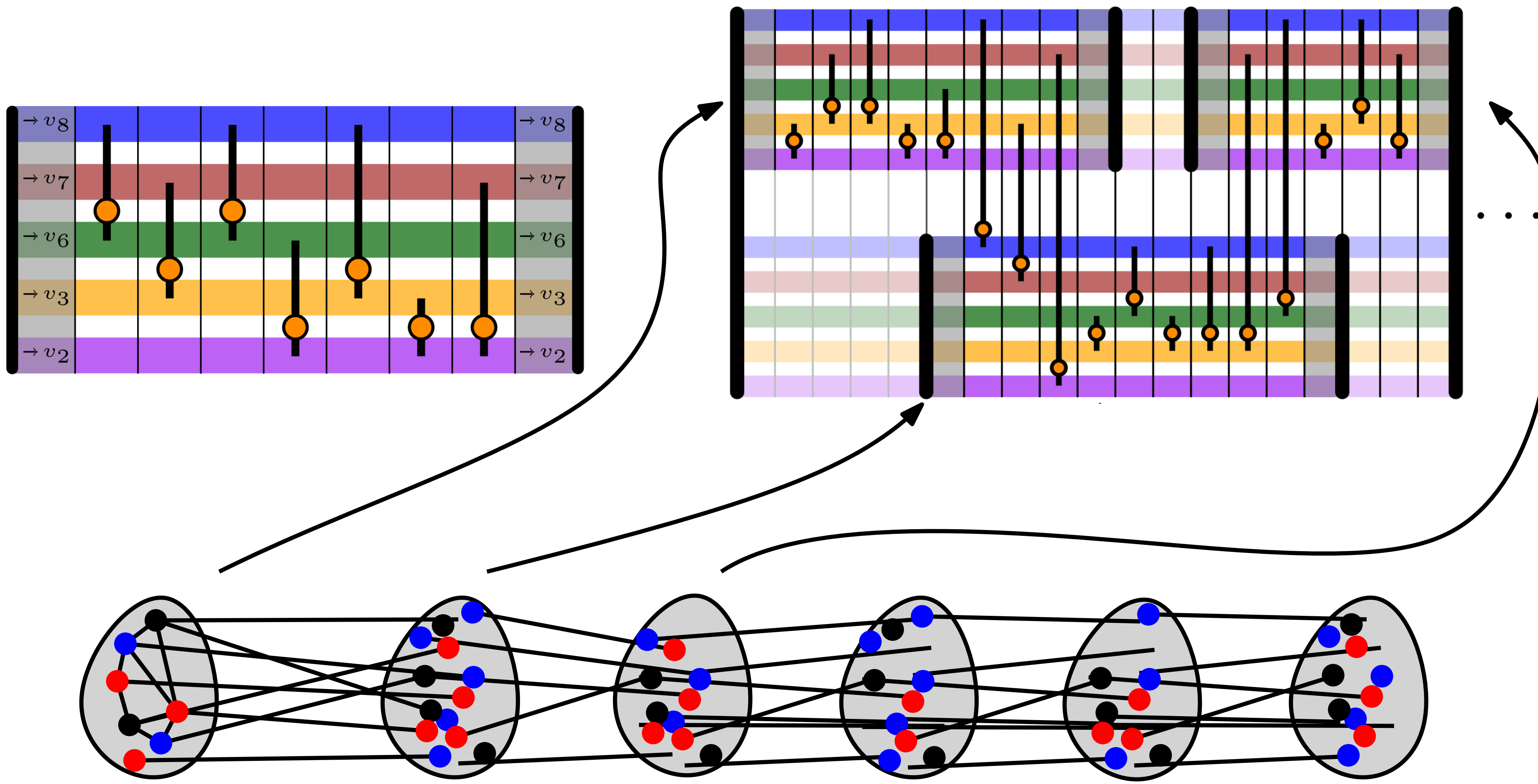
XNLP-hardness

parameterized reduction that runs in $\mathcal{O}(f(k) \cdot n^c)$ time and uses only $\mathcal{O}(g(k) \cdot \log n)$ space



XNLP-hardness

parameterized reduction that runs in $\mathcal{O}(f(k) \cdot n^c)$ time and uses only $\mathcal{O}(g(k) \cdot \log n)$ space



~~Constrained and~~ Ordered Level Planarity Parameterized by the Number of Levels is XNLP-complete

Václav Blažej

with Boris Klemz, Felix Klesen, Marie Diana Sieper, Alexander Wolff, and Johannes Zink

SoCG 2024

the end □