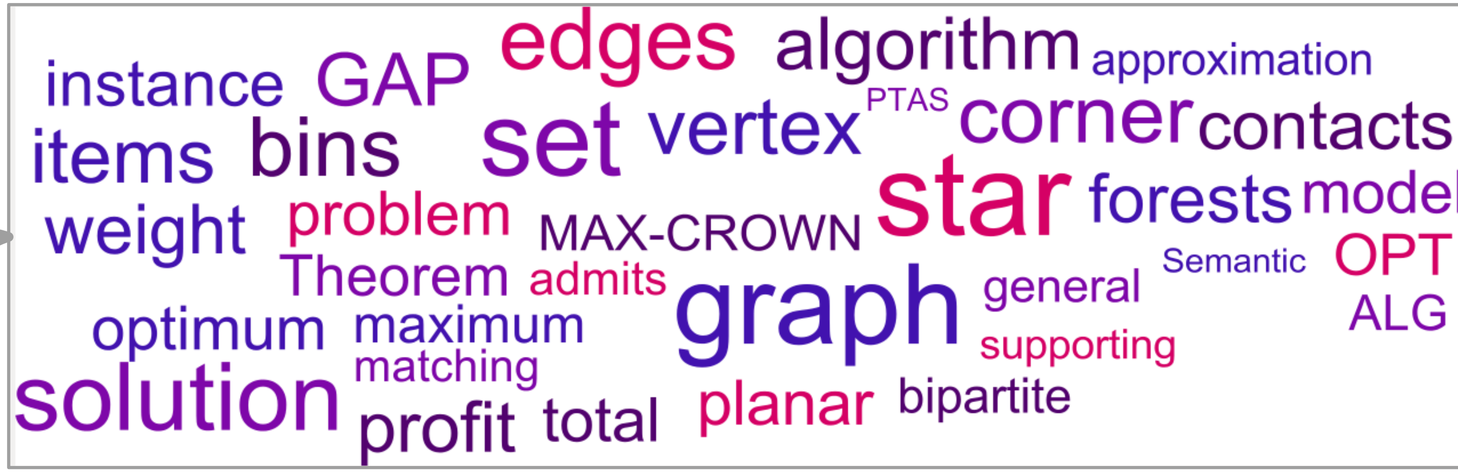

Approximation Algorithms for Contact Representations of Rectangles

Michalis Bekos Thomas van Dijk
Martin Fink Philipp Kindermann
Stephen Kobourov Sergey Pupyrev
Joachim Spoerhase *Alexander Wolff*

Universität Tübingen
University of Arizona
Universität Würzburg

2008 U.S. Presidential Elections

Heather Williams, June 2008

Coalition treaty 2013

Spiegel Online Nov. 2013

Coalition treaty 2013

Words that were more important in the 2013 than in the 2009 treaty

Spiegel Online Nov. 2013

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Coalition treaty 2013

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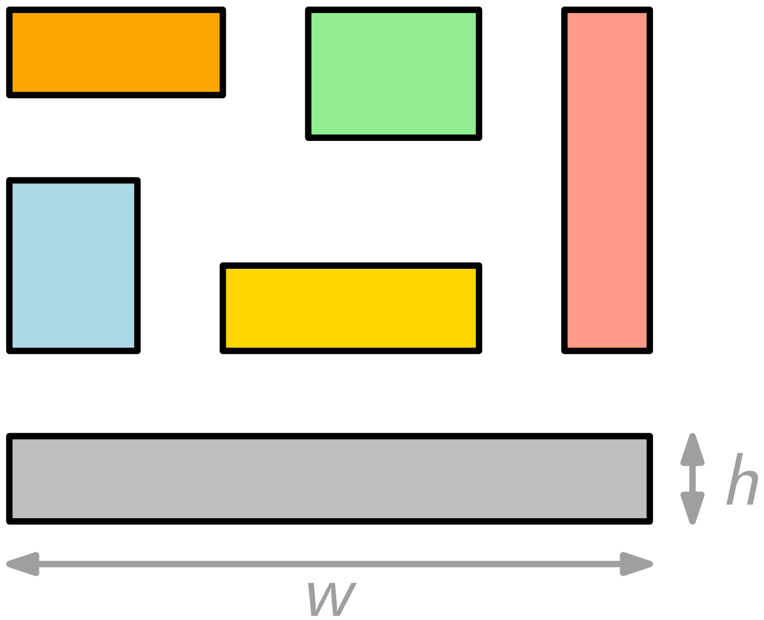
Spiegel Online Nov. 2013

*C*ontact *R*epresentation *O*f *W*ord *N*etworks

Input



– (integral) box dimensions

*C*ontact *R*epresentation *O*f *W*ord *N*etworks

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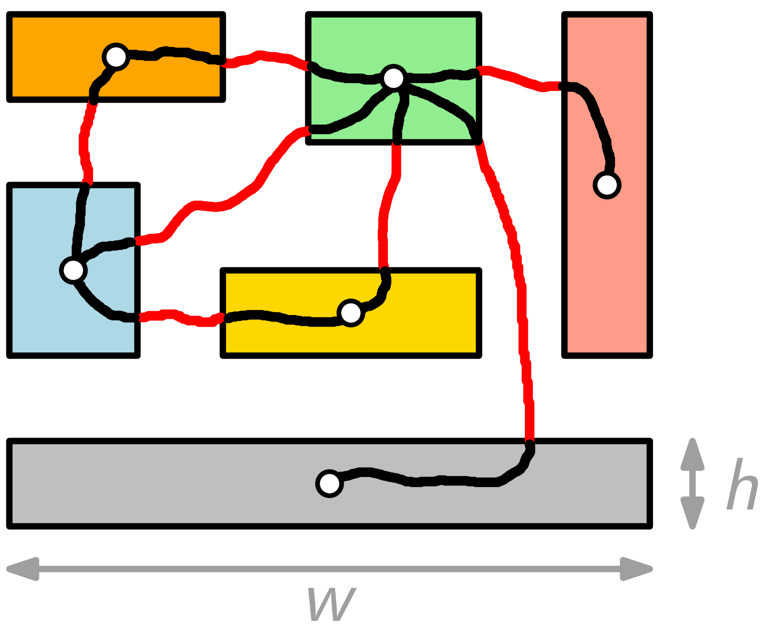

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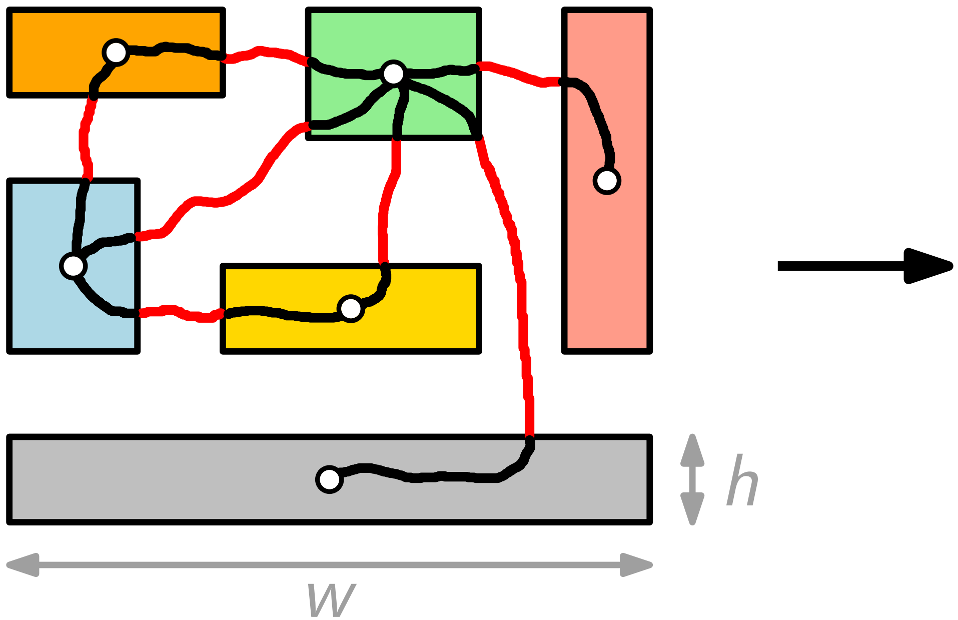
– desired contact graph

*C*ontact *R*epresentation *O*f *W*ord *N*etworks

Input **Output**



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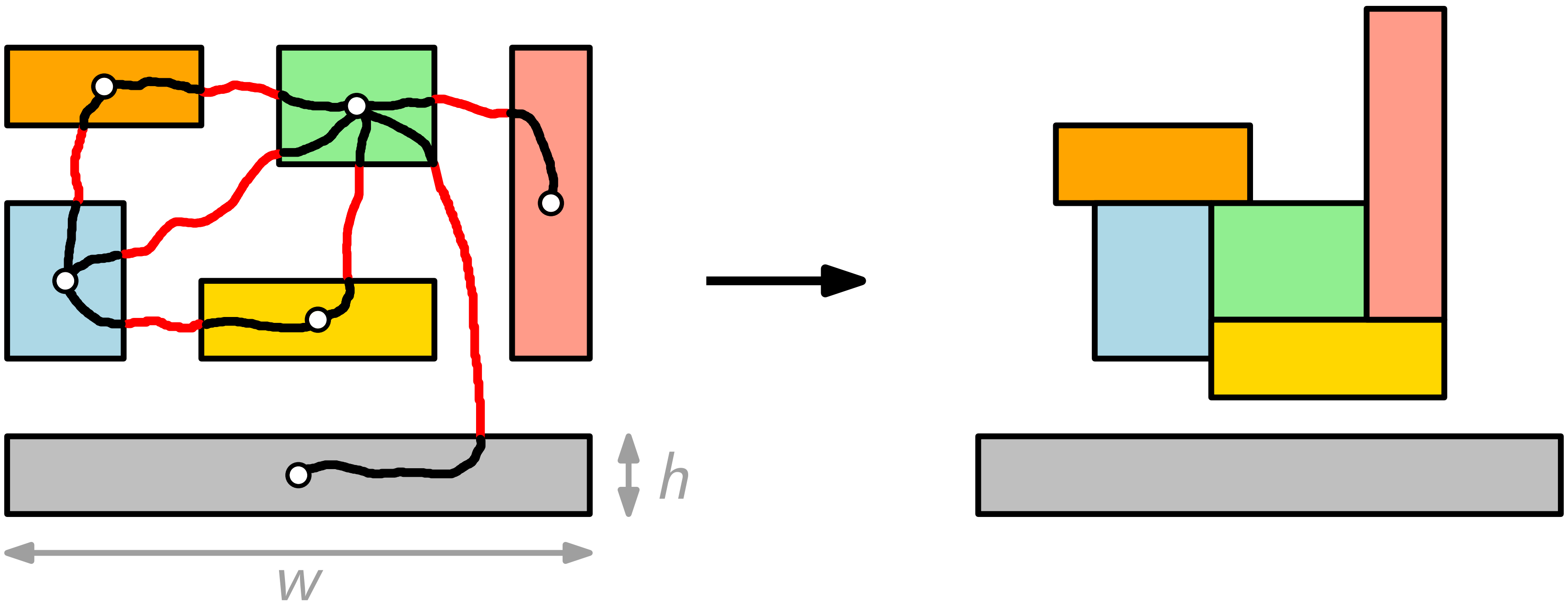

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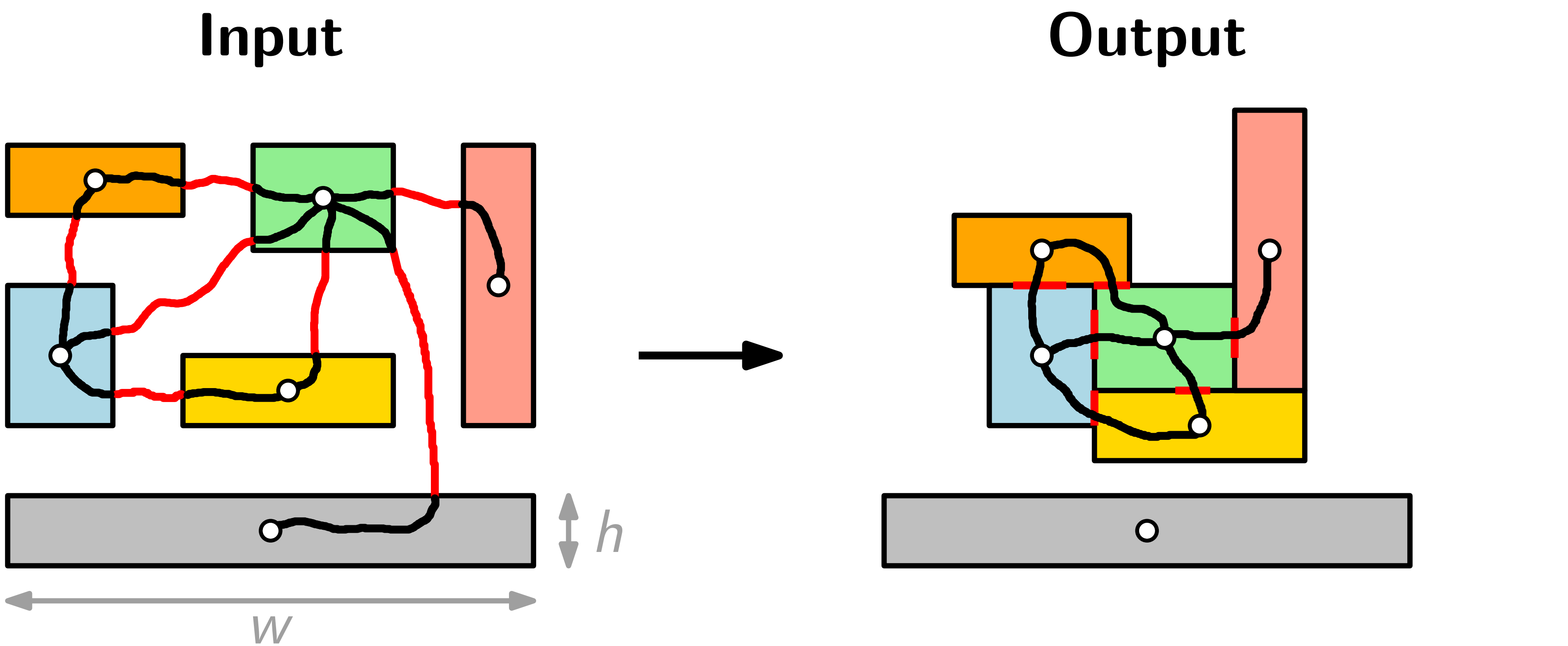
– placement of boxes

Contact Representation Of Word Networks



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Contact *R*epresentation *O*f *W*ord *N*etworks

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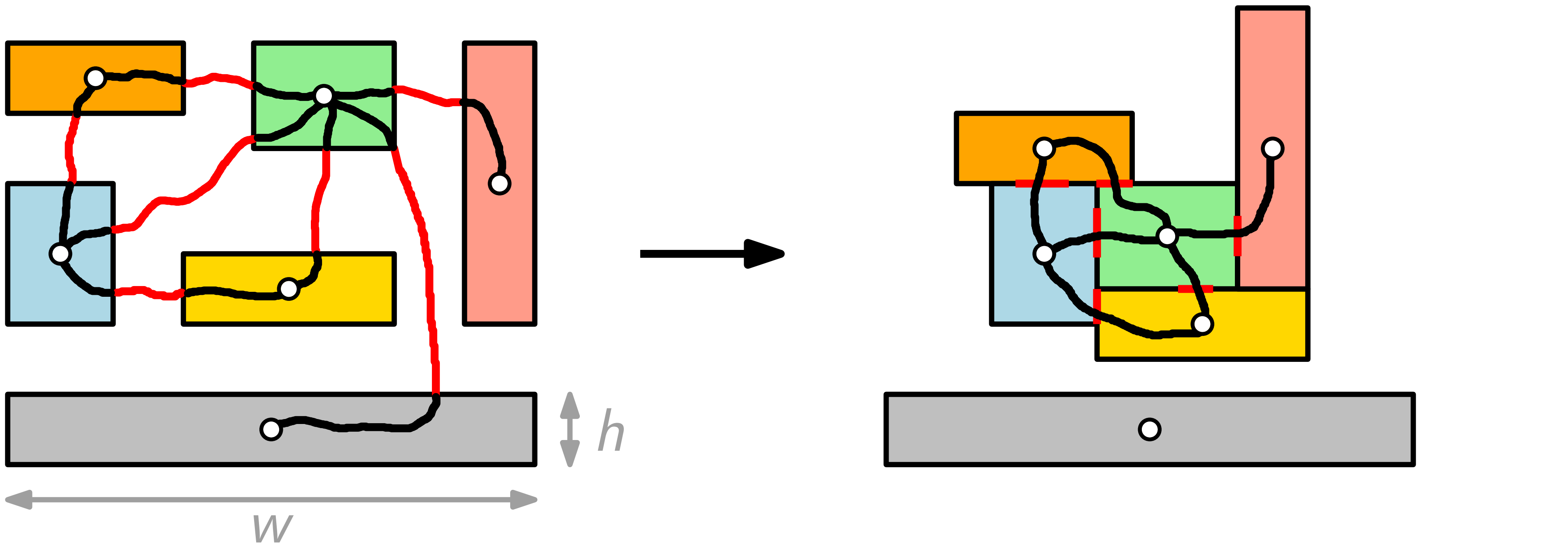

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– placement of boxes

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– profit: 1 unit / desired edge

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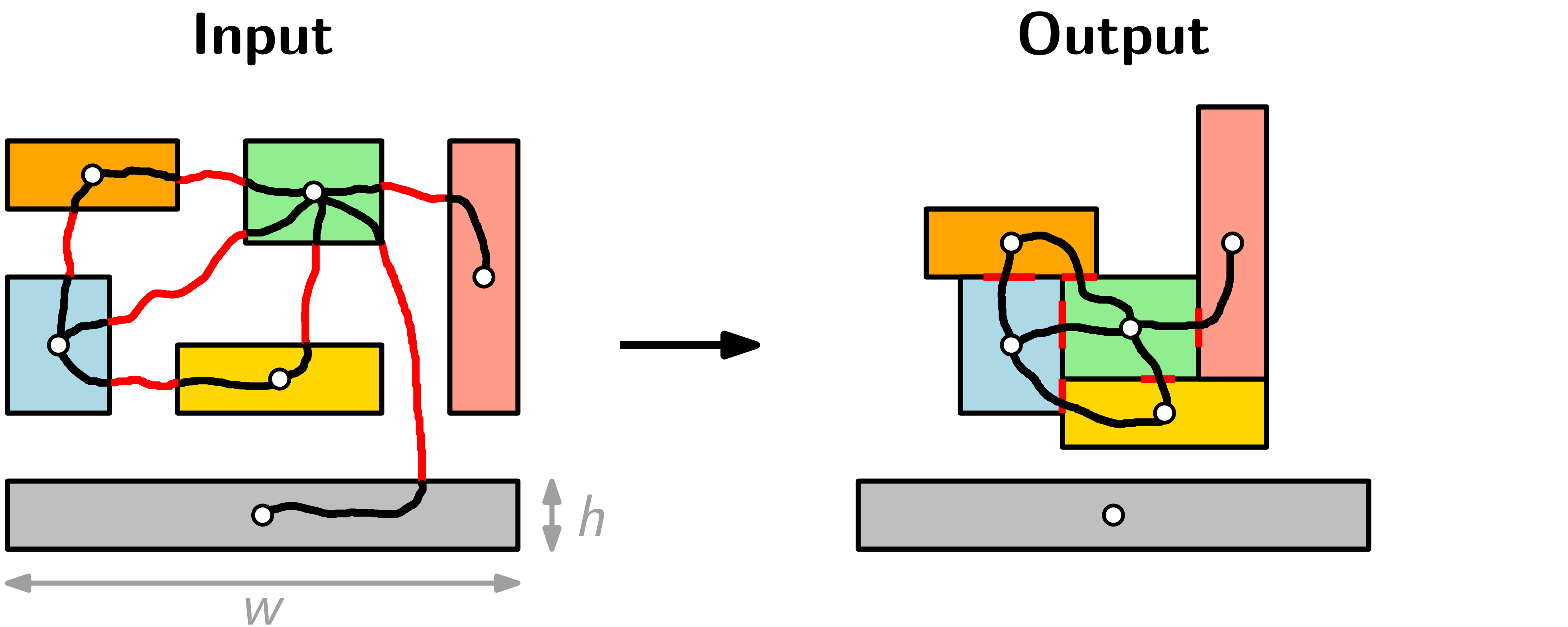

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MAX-CROWN:

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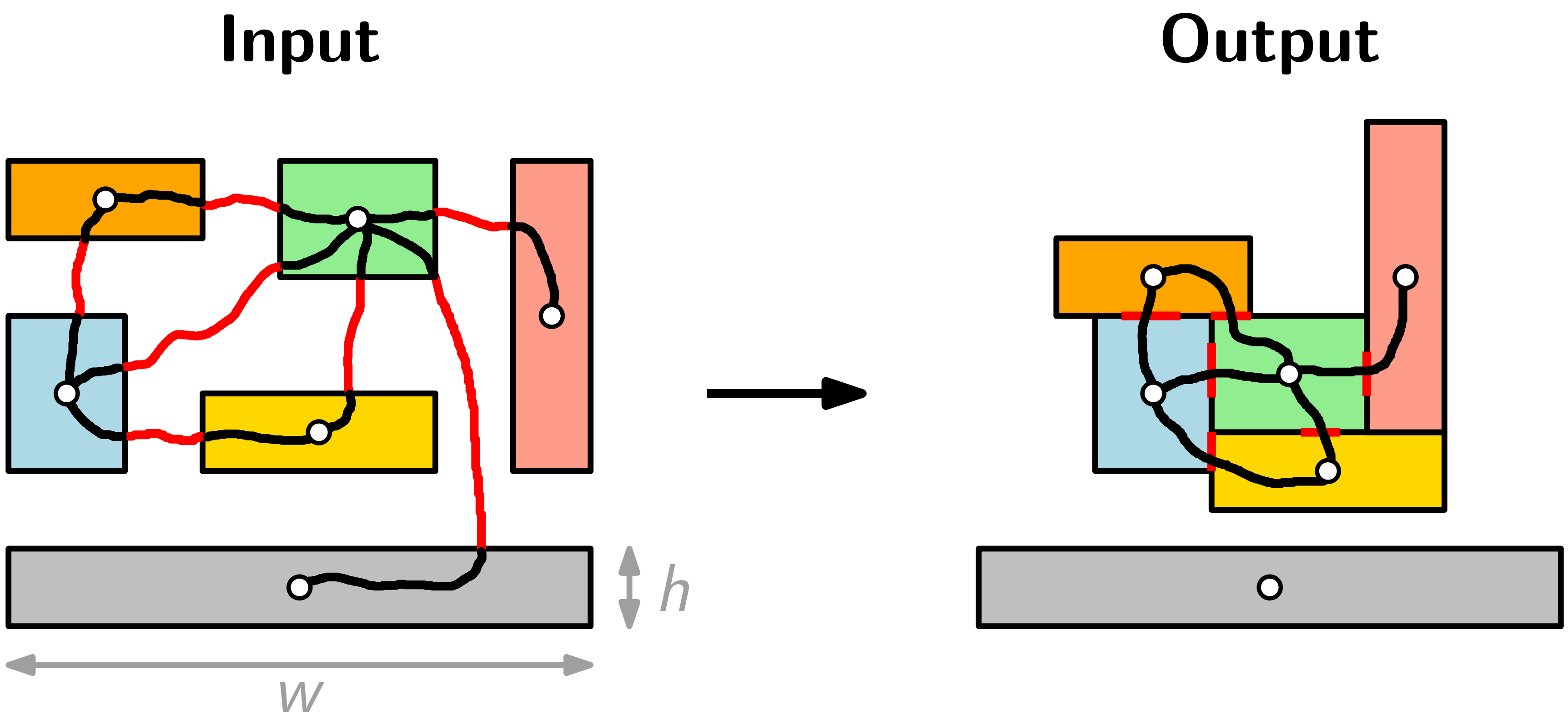

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– realized desired contacts
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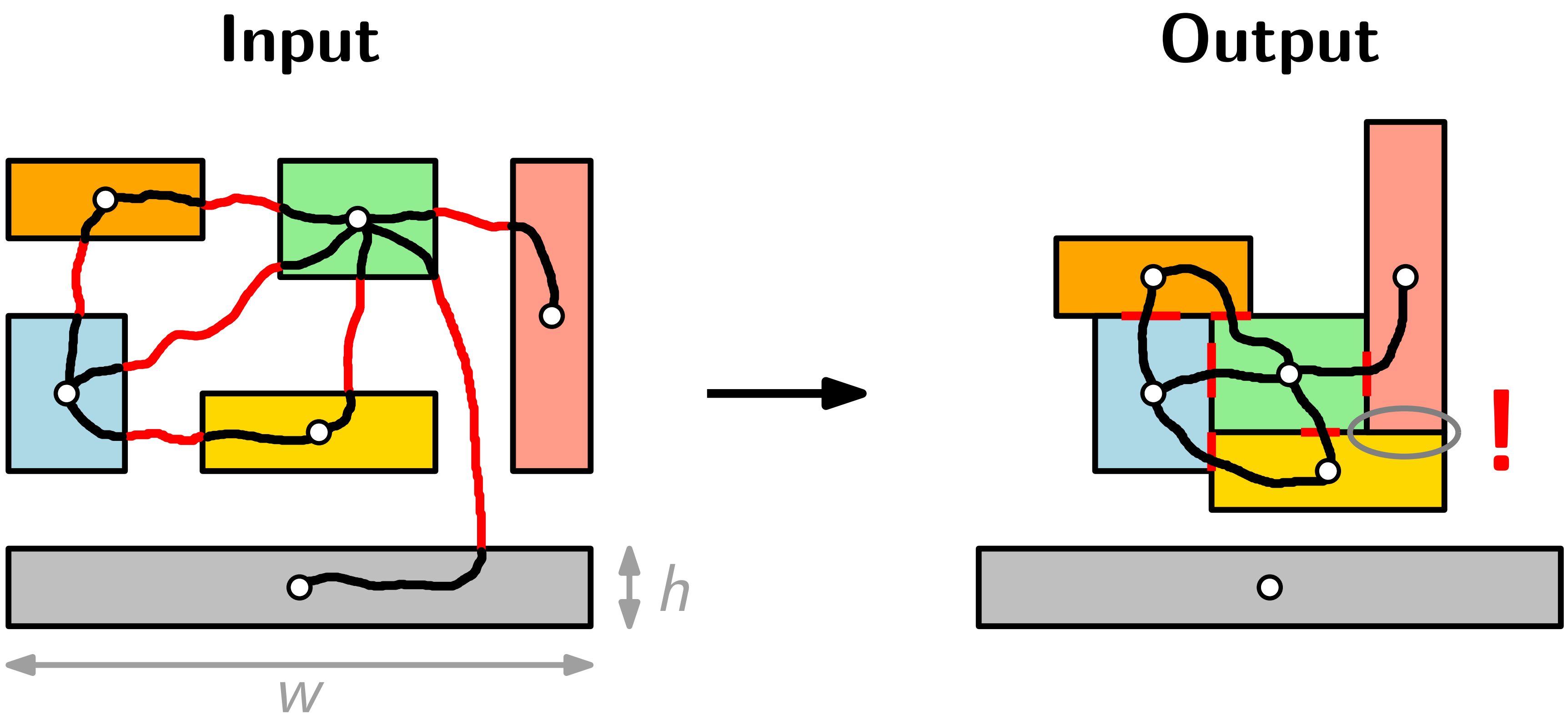
MAX-CROWN: Maximize profit!

Contact *R*epresentation *O*f *W*ord *N*etworks



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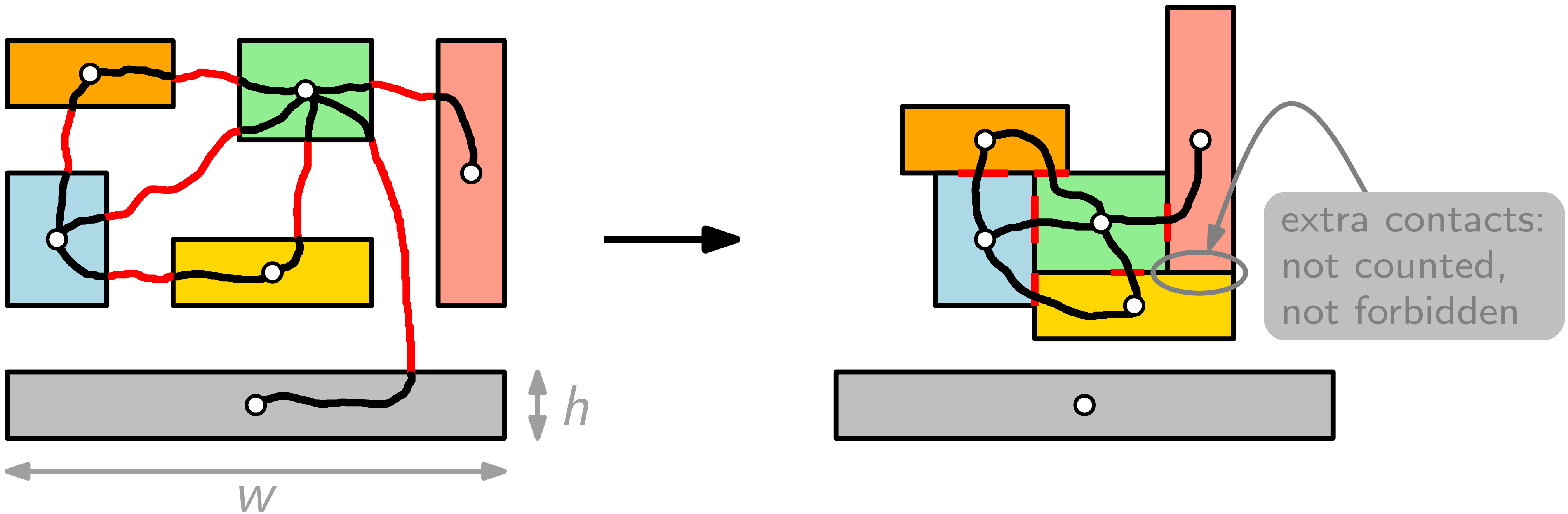

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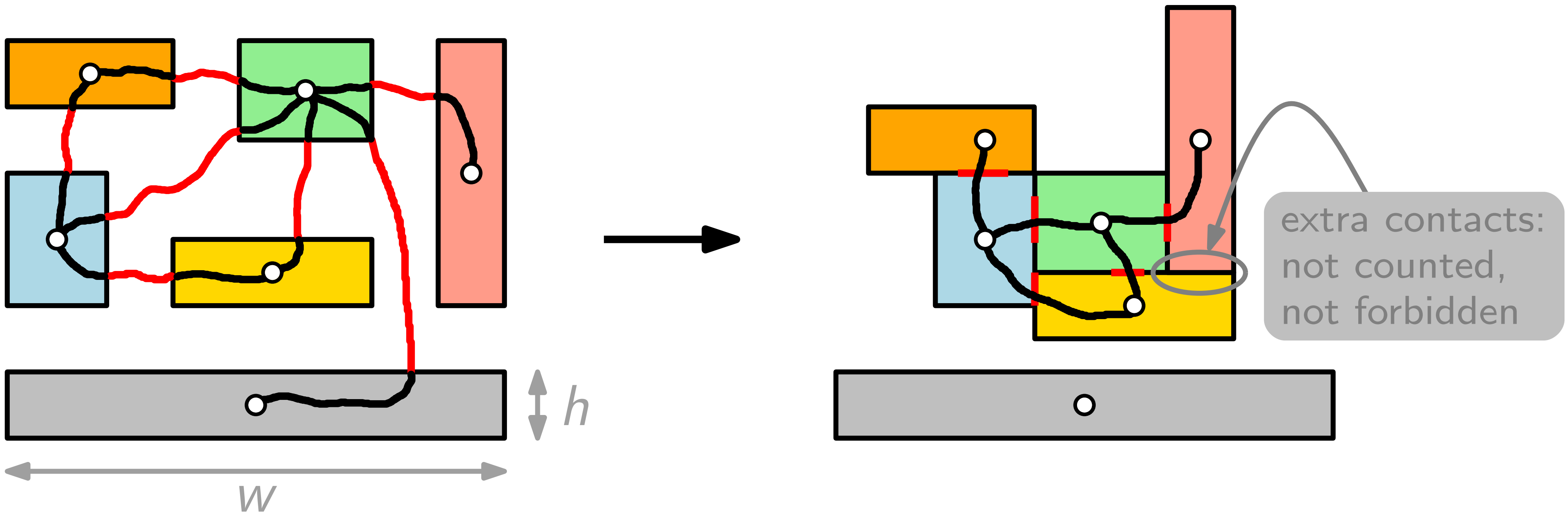

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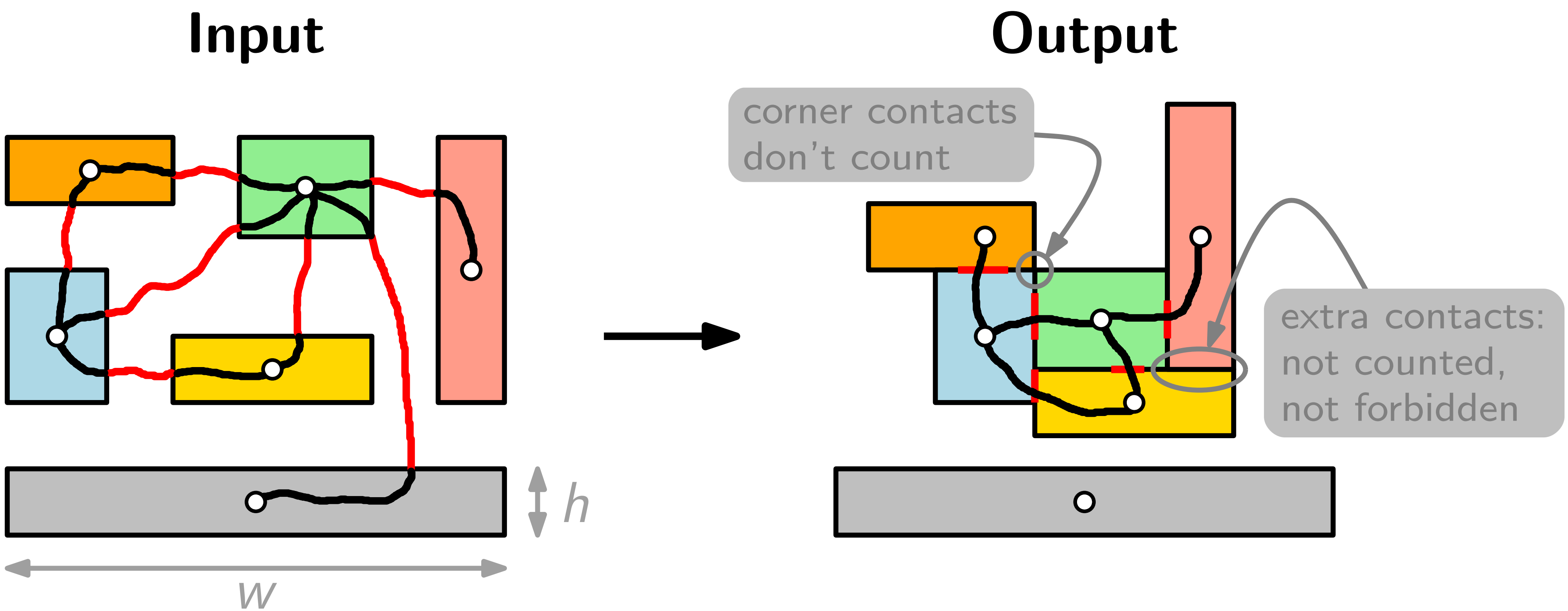

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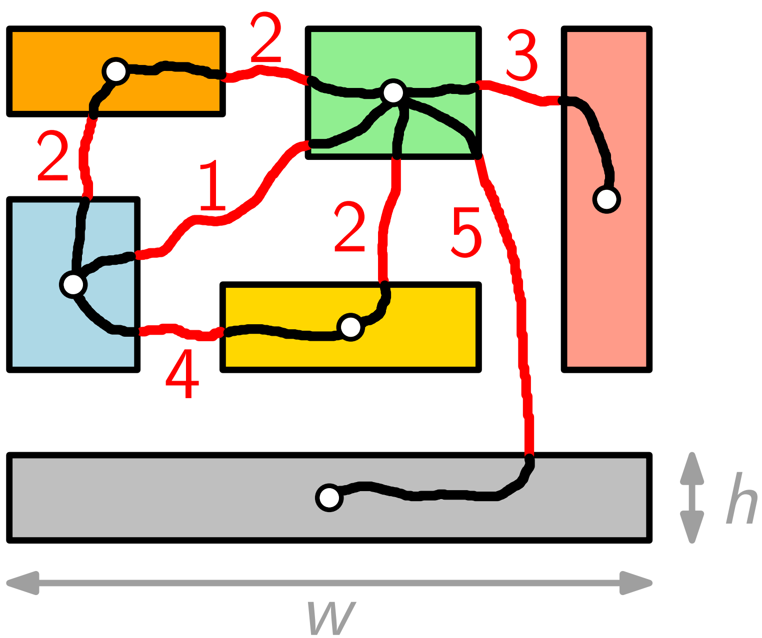
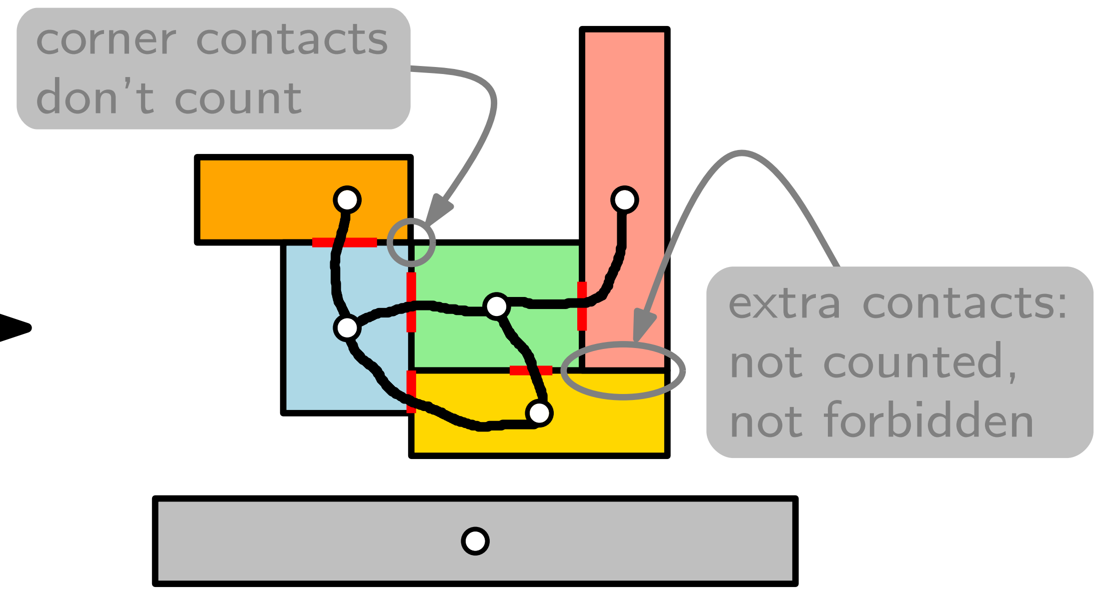


Output



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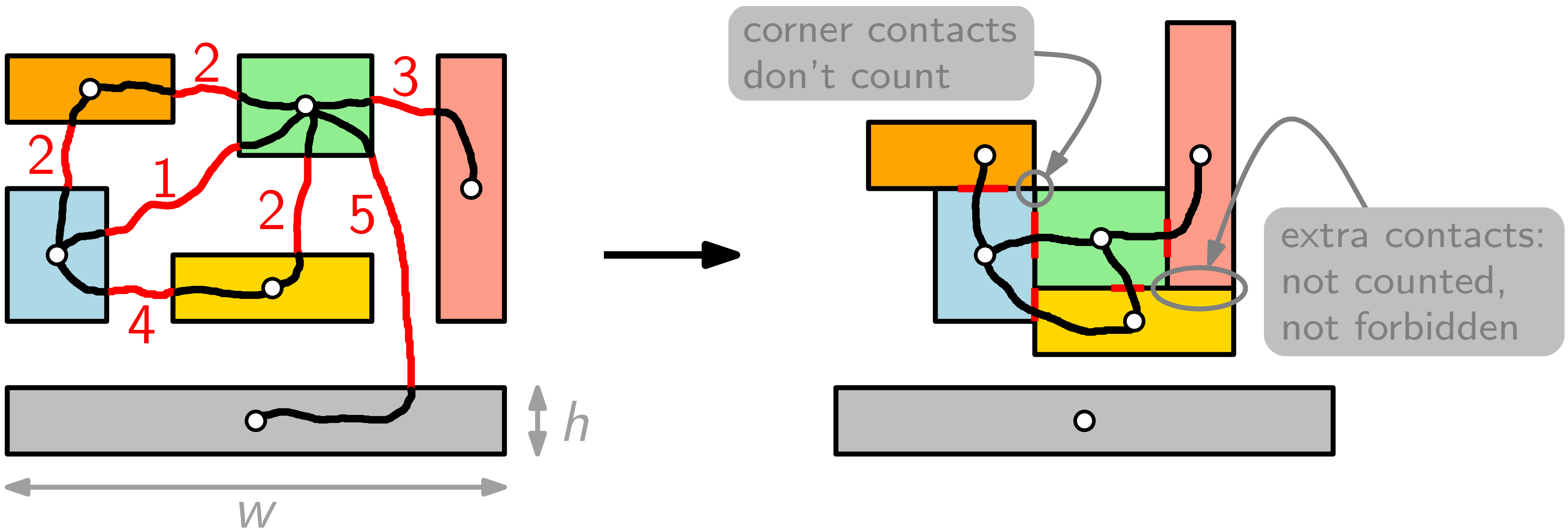
Input | **Output**



– (integral) box dimensions
– desired contact graph

– placement of boxes
– realized desired contacts
– profit: ~~1~~ $p(e)$ units / desired edge e

Max-Crown: Maximize profit!

Related Work

- rectangle / cube representation of graphs

Related Work

- rectangle / cube representation of graphs
  – Every planar graph w/o sep. triangles has a touching rectangle representation

[Koźminński & Kinnen, Networks'85; He, SICOMP'93; He & Kant, TCS'97]

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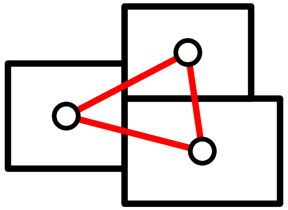

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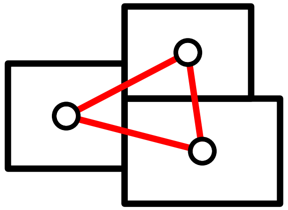

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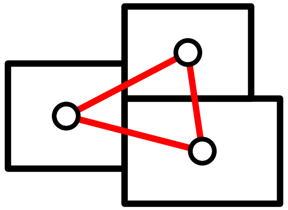

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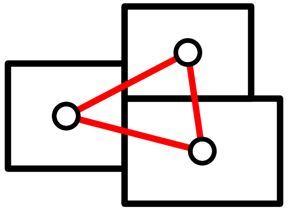

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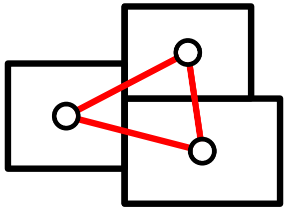

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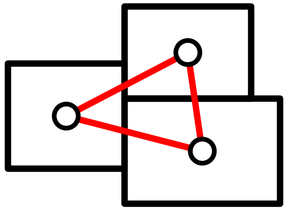

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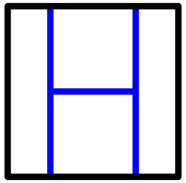

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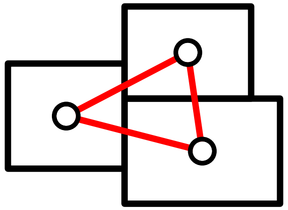

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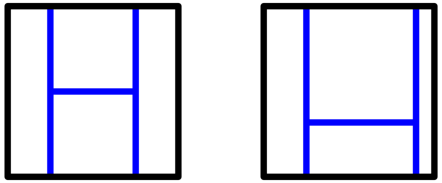

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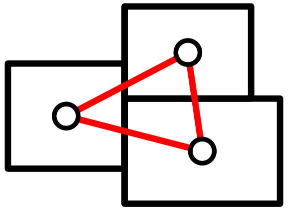

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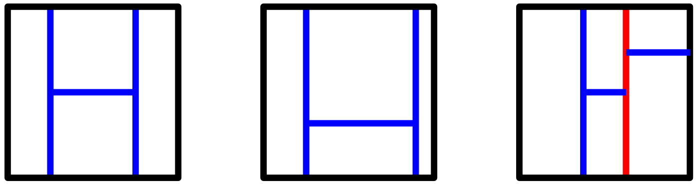

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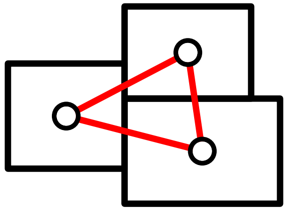

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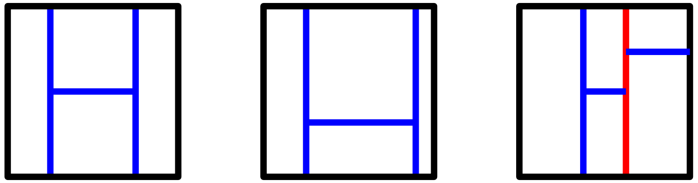

- rectangle representations with edge weights

Related Work

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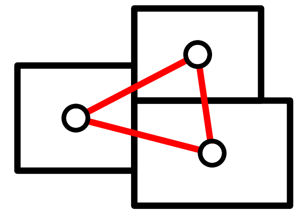

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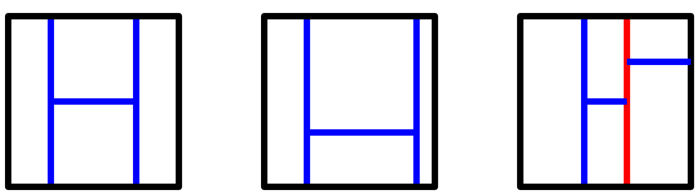

- rectangle representations with edge weights
  - edge weights prescribe length of contact [Nöllenburg et al., GD'12]

Our Results – Approximation Factors

Graph class	Weighted old*	Weighted new°
cycle, path	1	
star	α	$1+\varepsilon$
tree	2α, NP-hard	$2+\varepsilon$
max-degree Δ	$\lfloor(\Delta+1)/2\rfloor$	
planar max-deg. Δ		
outerplanar		$3+\varepsilon$
planar	5α	$5+\varepsilon$
bipartite		$16\alpha/3 \approx 8.4$ APX-hard
general		rand.: $32\alpha/3 \approx 16.9$ det.: $40\alpha/3 \approx 21.1$

*) [Barth, Fabrikant, Kobourov, Lubiw, Nöllenburg, Okamoto, Pupyrev, Squarcella, Ueckerdt, Wolff – LATIN'14]

°) [Bekos, van Dijk, Fink, Kindermann, Kobourov, Pupyrev, Spoerhase, Wolff – submitted]

$$\alpha = e/(e-1) \approx 1.58$$

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Tool #1: GAP

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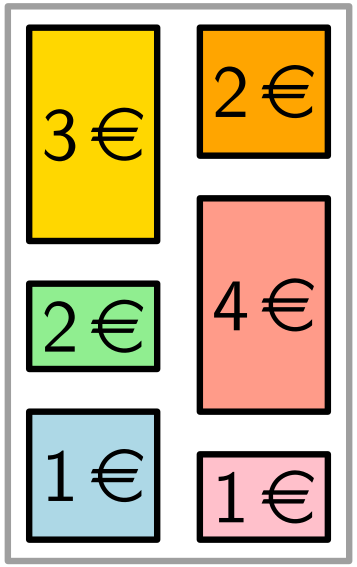
KNAPSACK

items

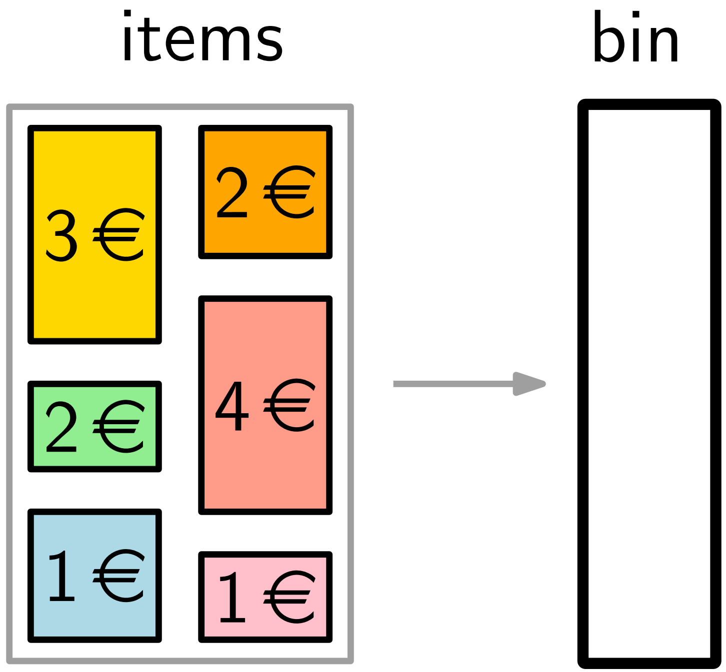


– size s_i

– value v_i

Tool #1: GAP

KNAPSACK



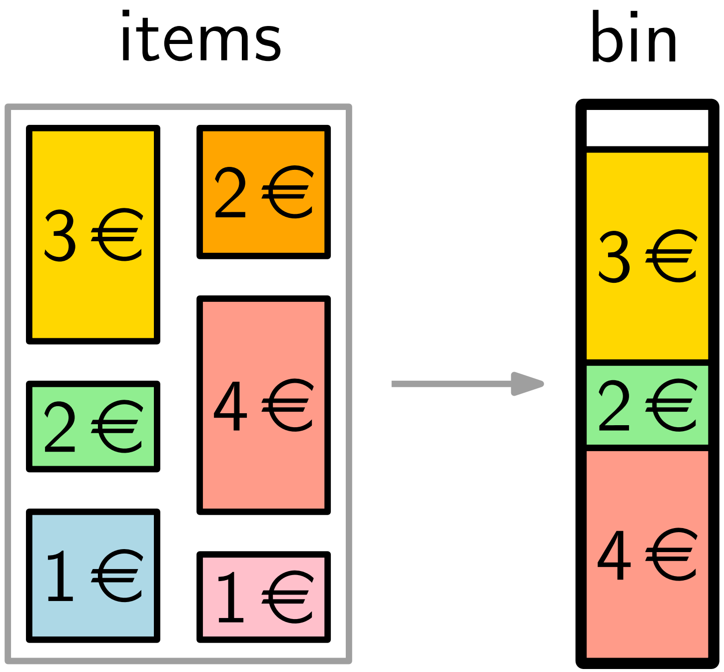
– size s_i

– value v_i

– bin has capacity c

Tool #1: GAP

KNAPSACK



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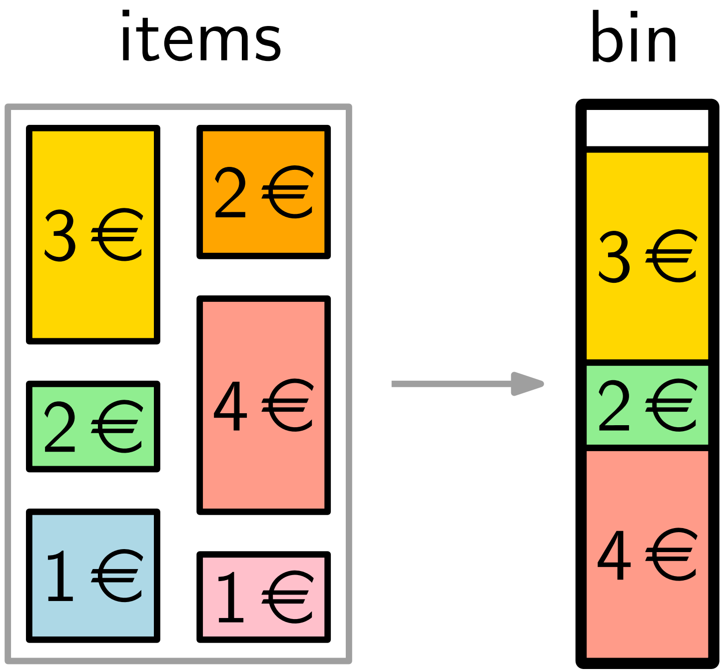
KNAPSACK



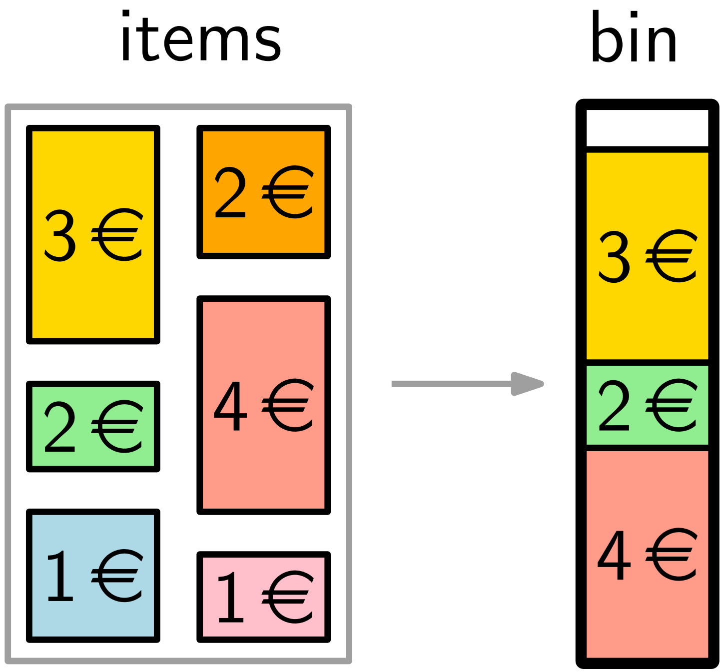
– size s_i – bin has capacity c

– value v_i – maximize total value packed

Tool #1: GAP

KNAPSACK

GENERALIZED ASSIGNMENT PROB.



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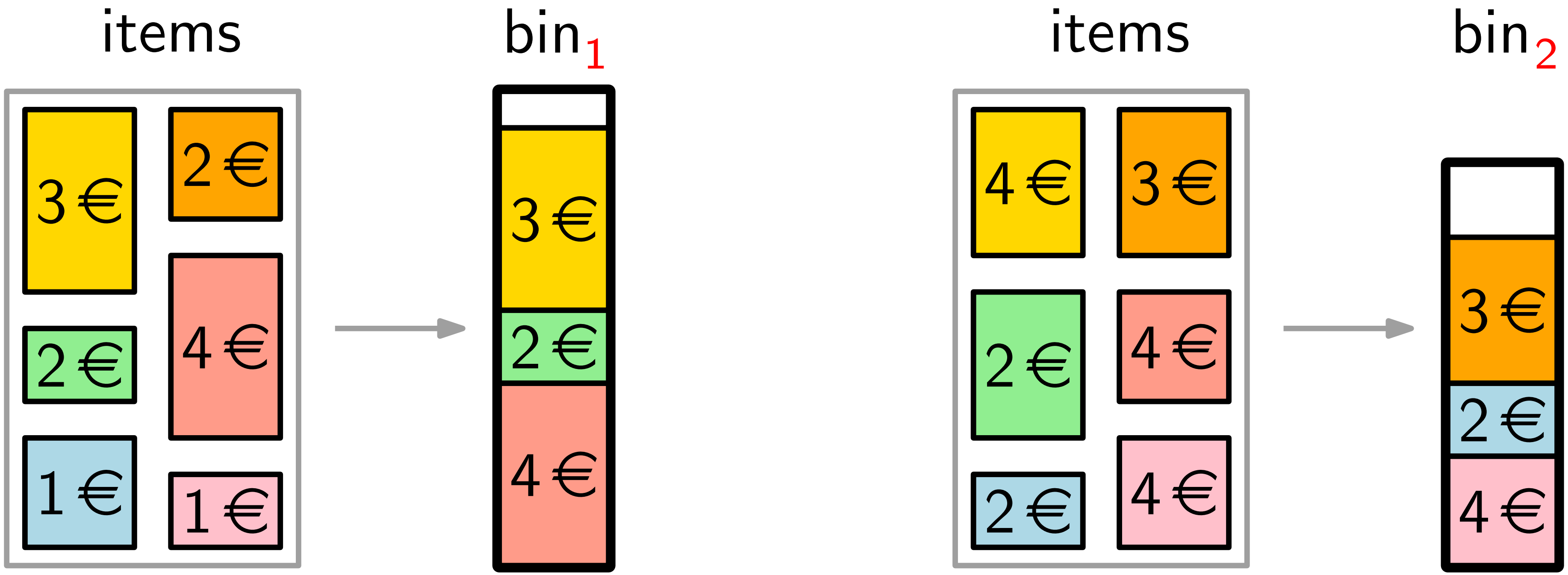


– size s_{ij}

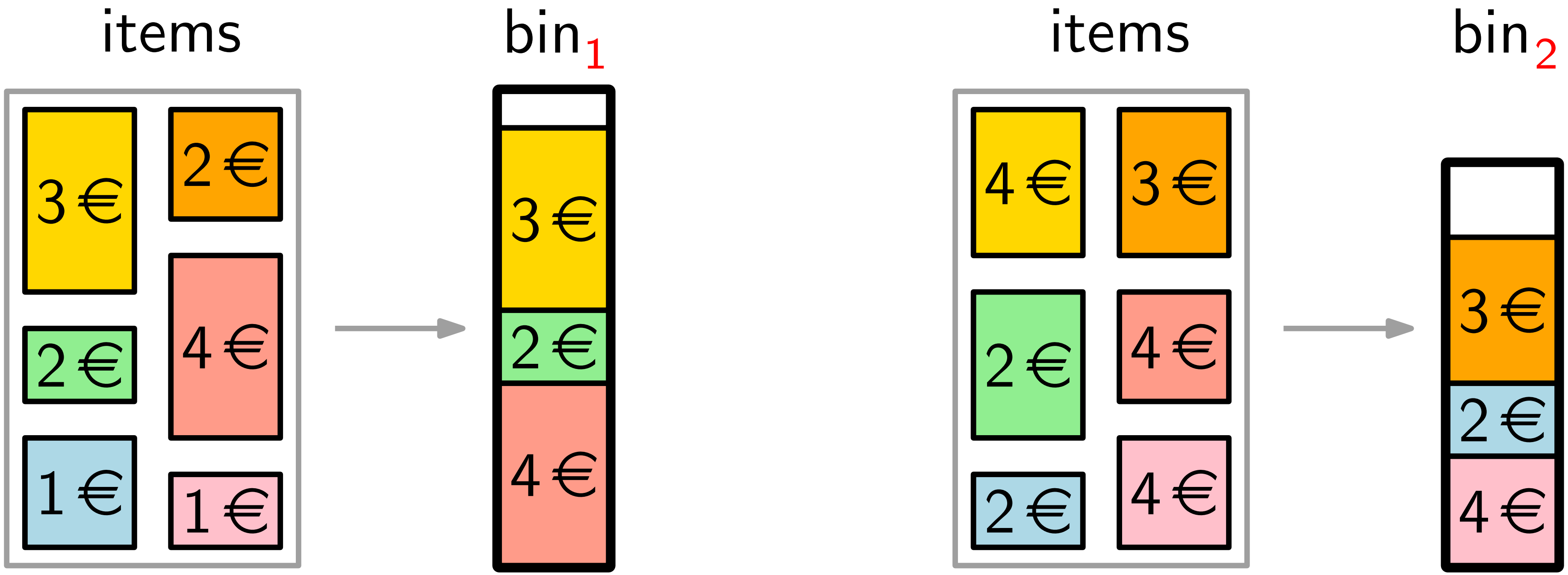
– value v_{ij}

– bin$_j$ has capacity c_j

– maximize total value packed

Tool #1: GAP

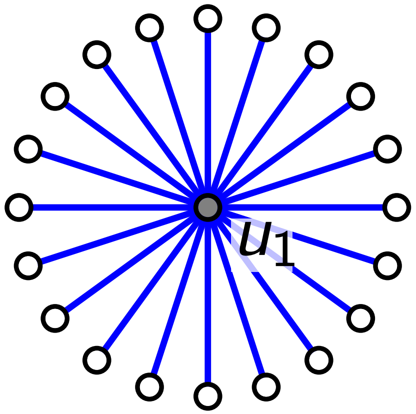
KNAPSACK → GENERALIZED ASSIGNMENT PROB.



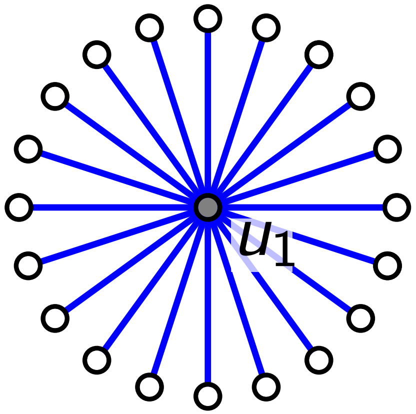
- size s_{i_j}
- value v_{i_j}
- bin$_j$ has capacity c_j
- maximize total value packed

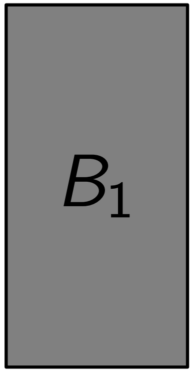
Theorem. GAP admits an approximation algorithm with ratio $\alpha = e/(e-1) \approx 1.58$. [Fleischer et al., MOR'11]

Max-Crown for Stars



Max-Crown for Stars





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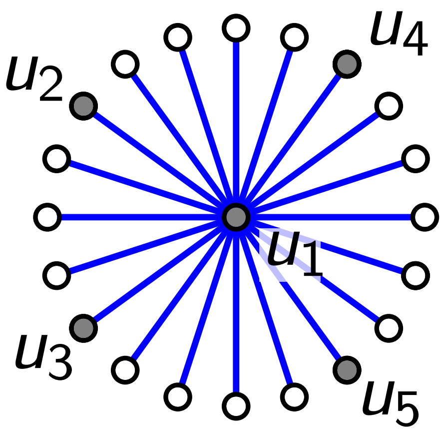

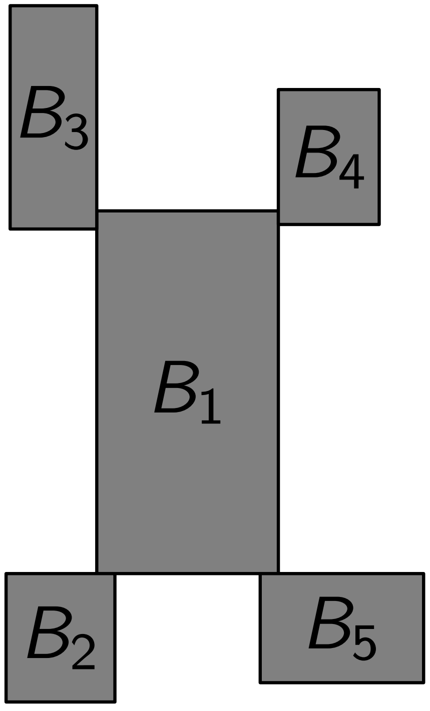

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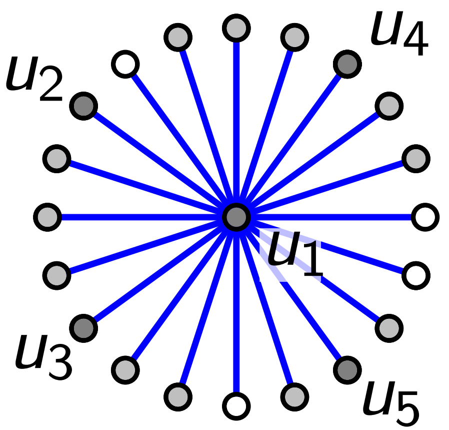

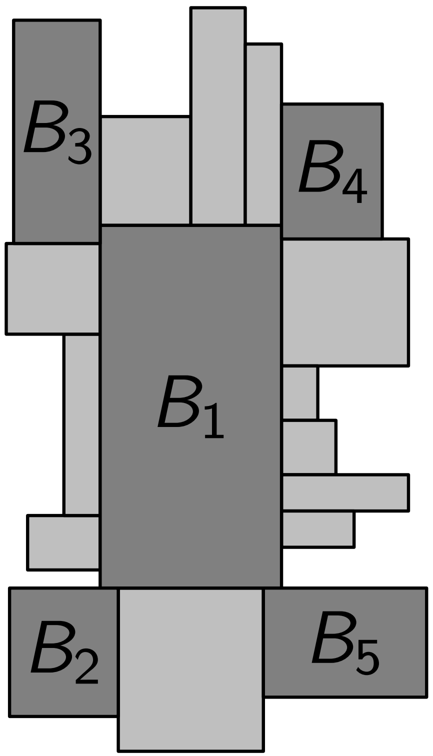

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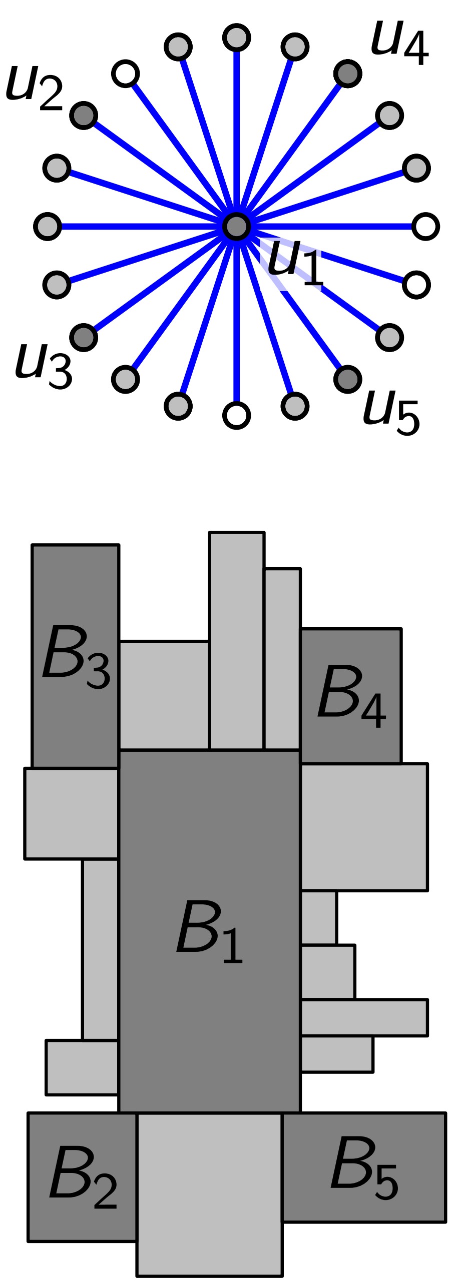

Set up Gap:

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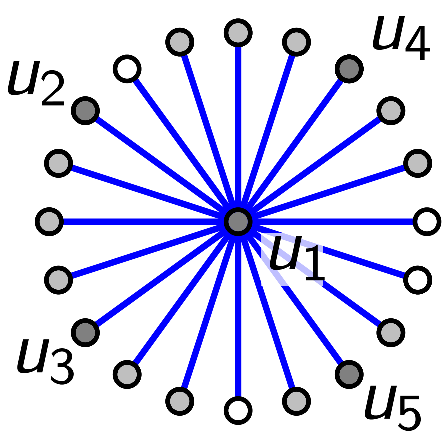
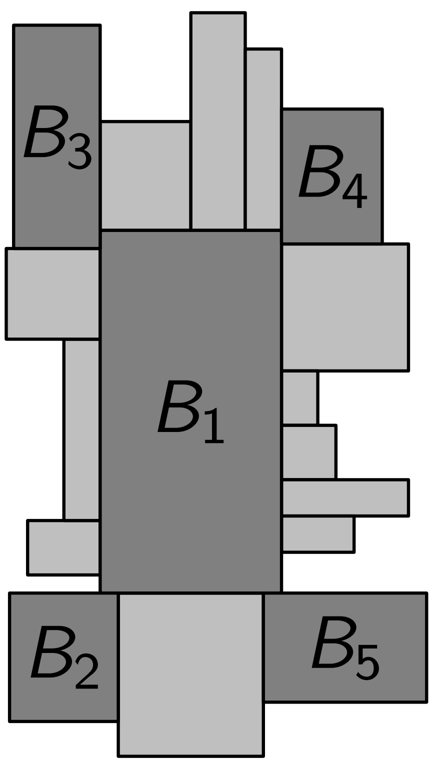


Set up GAP:

– eight bins (for the 4 sides and the 4 corners of B_1)



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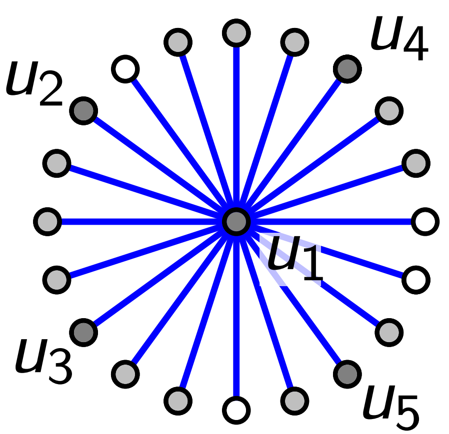
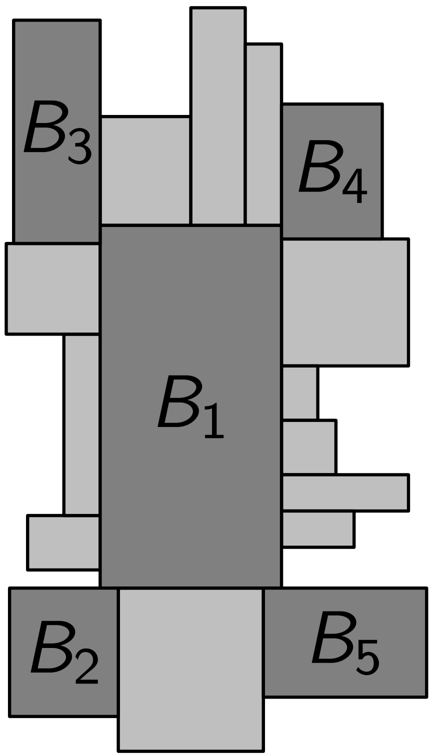
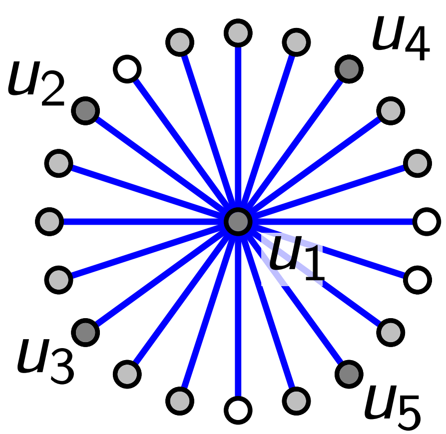




Set up Gap:

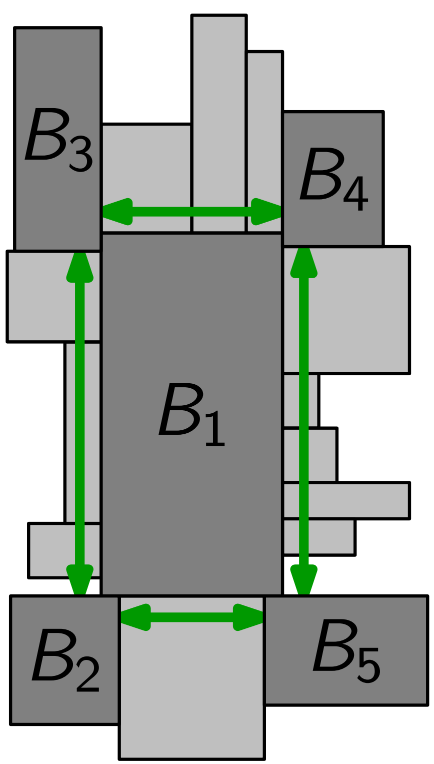
- eight bins (for the 4 sides and the 4 corners of B_1)
- corner bins have capacity $1/2$

MAX-CROWN for Stars



Set up GAP:

- eight bins (for the 4 sides and the 4 corners of B_1)
- corner bins have capacity $1/2$
- the capacity of side bins is their "free" length



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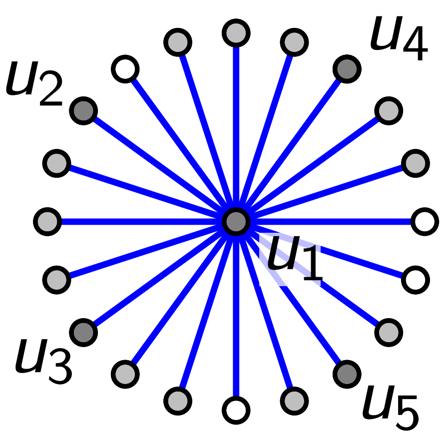

Set up Gap:

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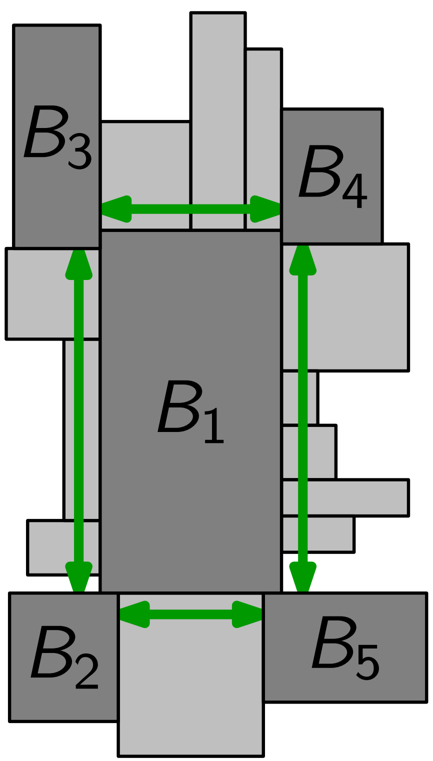

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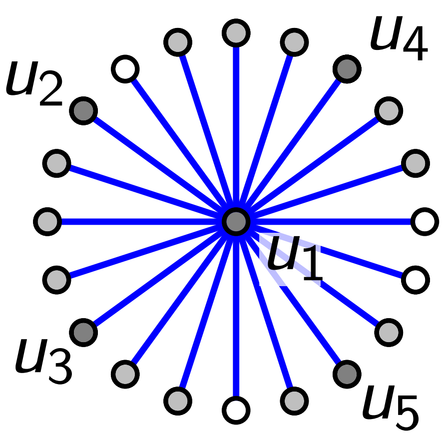

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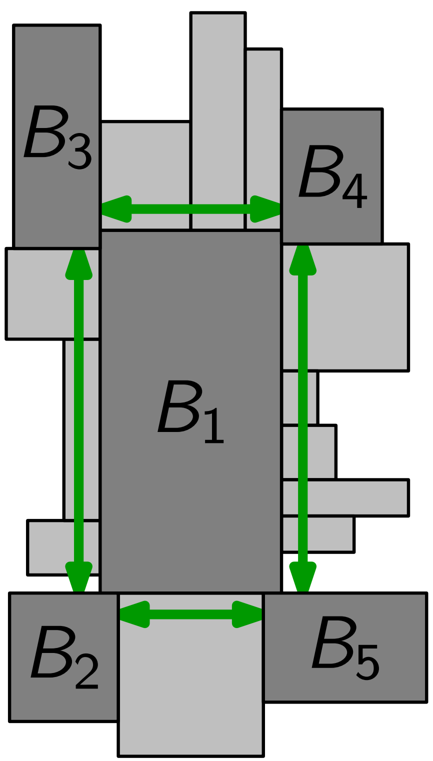

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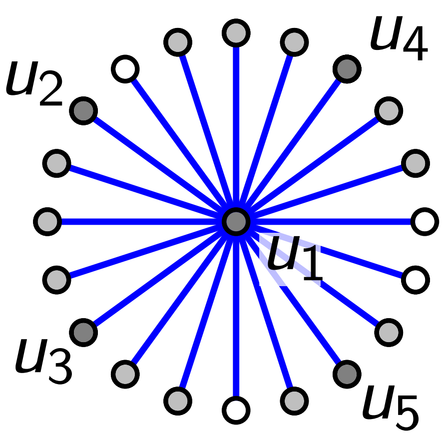

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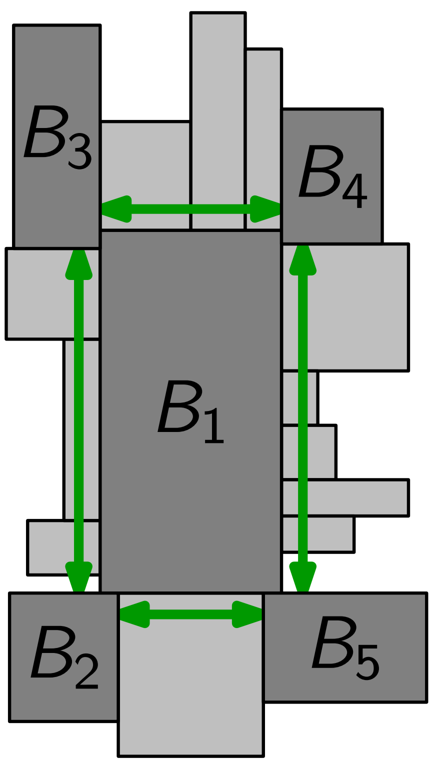

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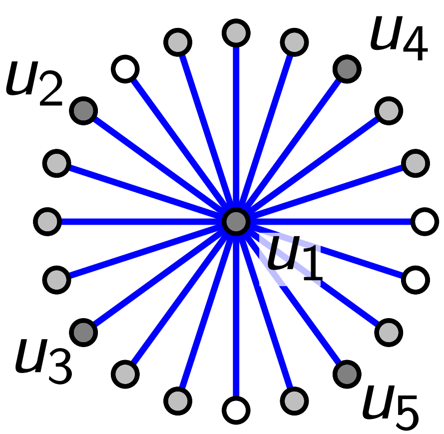

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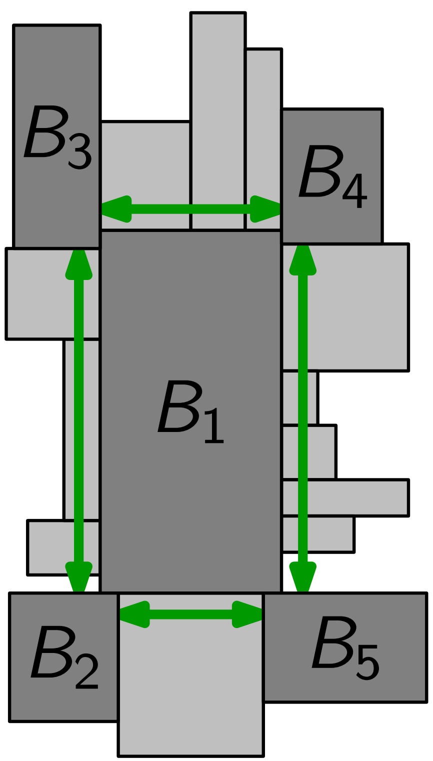
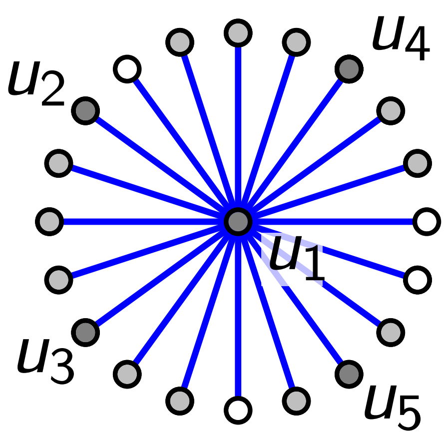


Algorithm:

Max-Crown for Stars



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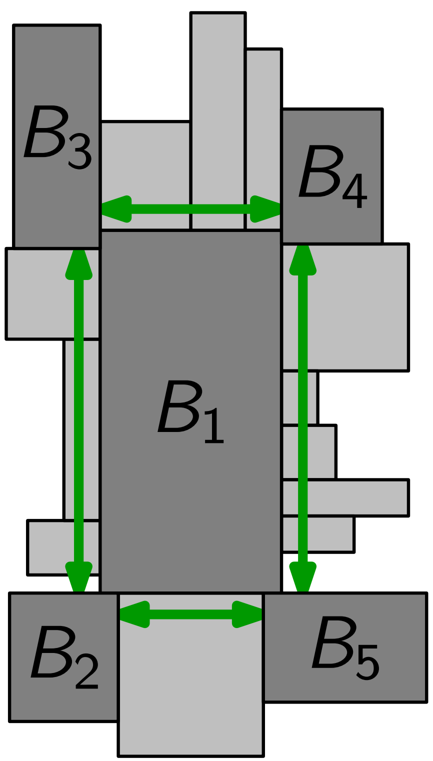

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- Assume that the 4 corner rectangles have contacts of length $\frac{1}{2}$ in a fixed optimal solution.

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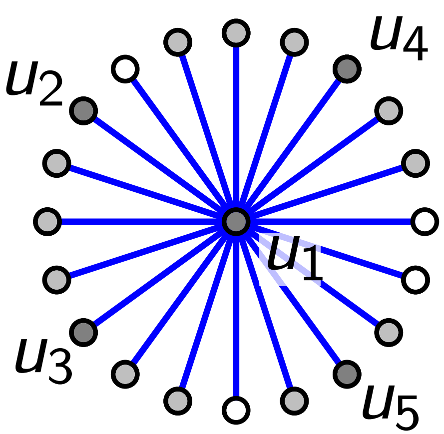

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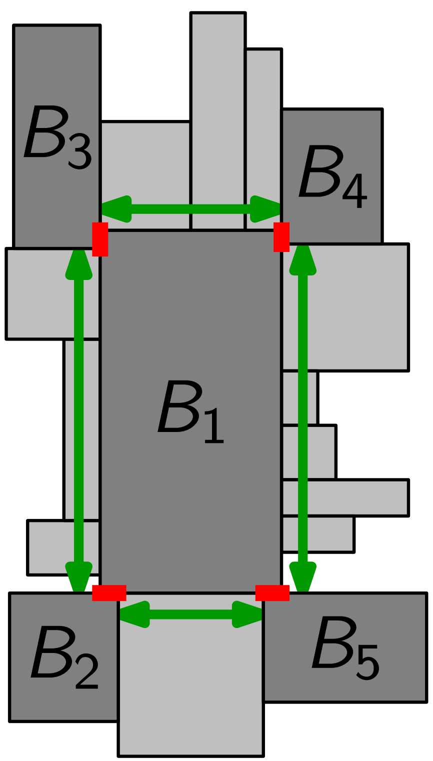

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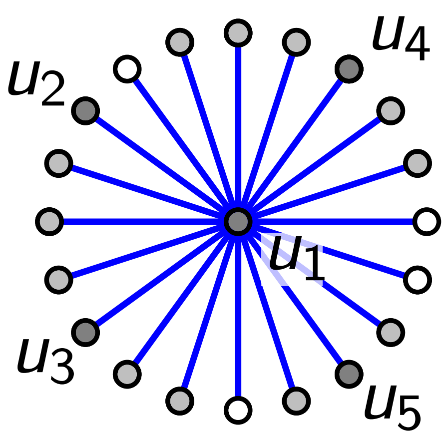

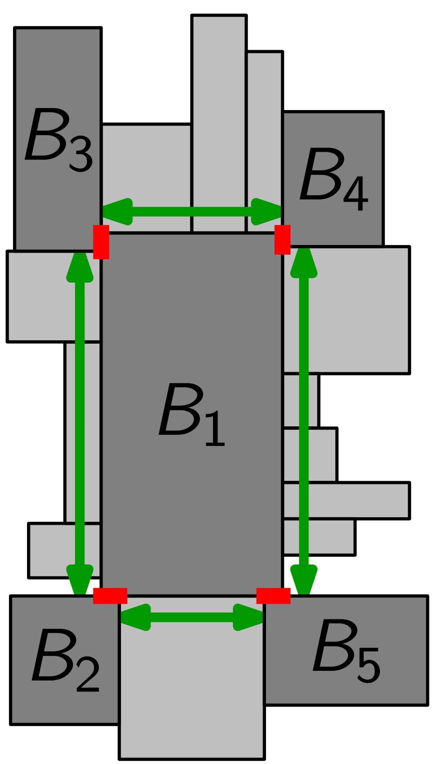

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- Each contact may be horizontal or vertical.

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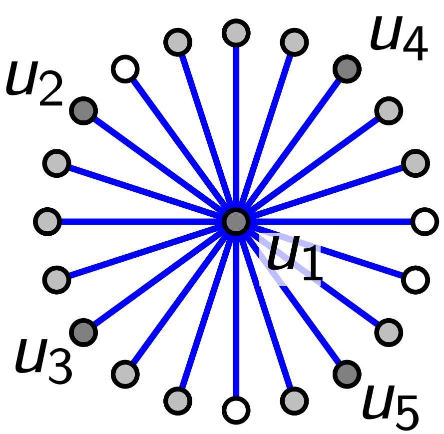

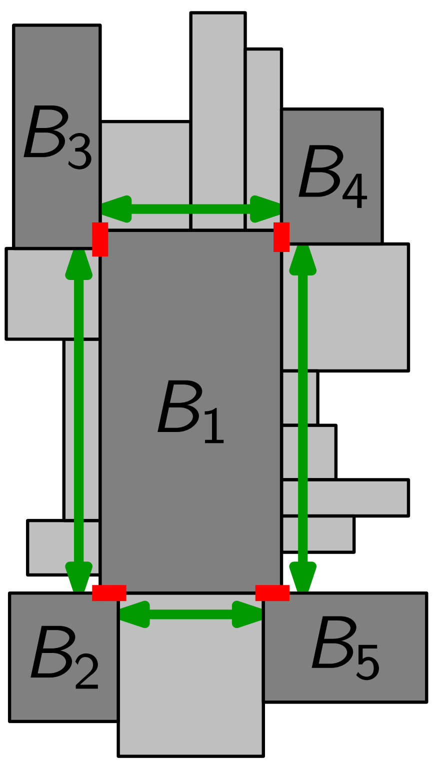

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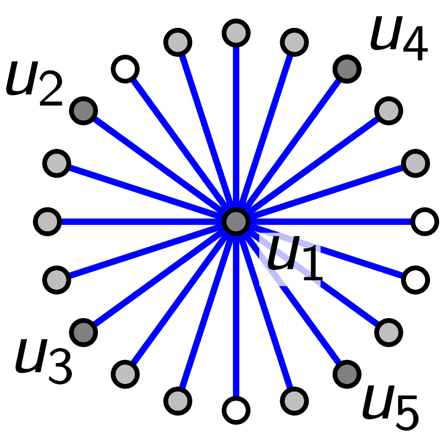
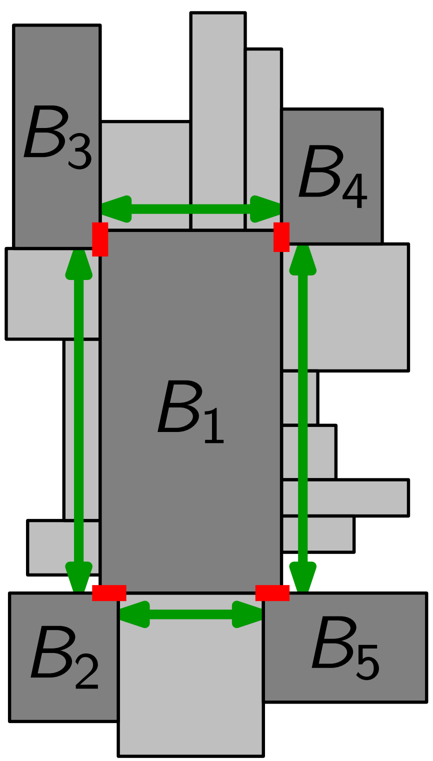
- Try all 2^4 possibilities

Max-Crown for Stars





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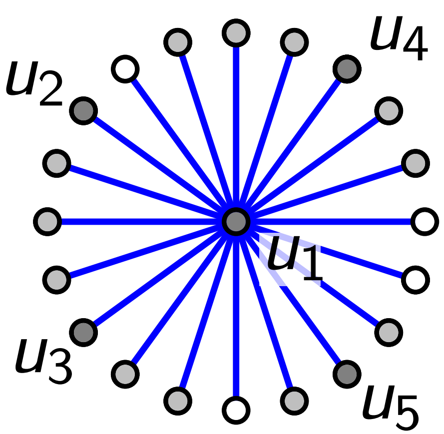
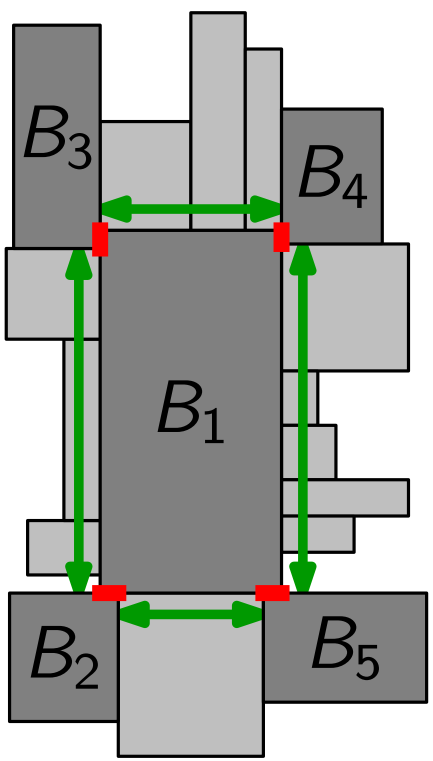
Max-Crown for Stars





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$\Rightarrow$ α-approx. algorithm for Max-Crown on stars $\square$

Overview

Graph class	Weighted old*	Weighted new°	Unweighted new°
cycle, path	1		
star	α	$1+\varepsilon$	
tree	2α, NP-hard	$2+\varepsilon$	2
max-degree Δ	$\lfloor(\Delta+1)/2\rfloor$		
planar max-deg. Δ			$1+\varepsilon$
outerplanar		$3+\varepsilon$	
planar	5α	$5+\varepsilon$	
bipartite		$16\alpha/3 \approx 8.4$ APX-hard	
general		rand.: $32\alpha/3 \approx 16.9$ det.: $40\alpha/3 \approx 21.1$	$5+16\alpha/3 \approx 13.4$

*) [Barth, Fabrikant, Kobourov, Lubiw, Nöllenburg, Okamoto, Pupyrev, Squarcella, Ueckerdt & Wolff, LATIN'14]

°) [Bekos, van Dijk, Fink, Kindermann, Kobourov, Pupyrev, Spoerhase, Wolff – submitted]

$$\alpha = e/(e-1) \approx 1.58$$

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star	α ✓	$1+\varepsilon$	
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max-degree Δ	$\lfloor(\Delta+1)/2\rfloor$		
planar max-deg. Δ			$1+\varepsilon$
outerplanar	3α	$3+\varepsilon$	
planar	5α	$5+\varepsilon$	
bipartite		$16\alpha/3 \approx 8.4$ APX-hard	
general		rand.: $32\alpha/3 \approx 16.9$ det.: $40\alpha/3 \approx 21.1$	$5+16\alpha/3$ ≈ 13.4

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$$\alpha = e/(e-1) \approx 1.58$$

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Lemma. Let $G_1 = (V, E_1)$, $G_2 = (V, E_2)$, $G = (V, E_1 \cup E_2)$.

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Proof. Algorithm.

Analysis.

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Return result with larger profit for G.

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By def., $\mathrm{ALG}_i > \mathrm{OPT}_i / \alpha_i$.

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Assume $\mathrm{OPT}_1 / \alpha_1 \ge \mathrm{OPT}_2 / \alpha_2$.

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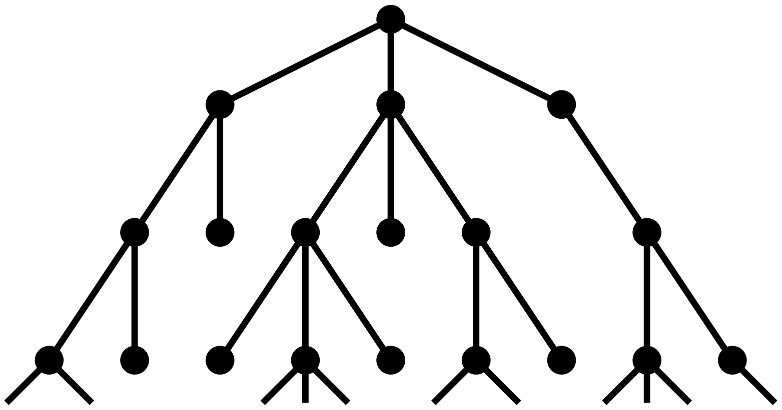

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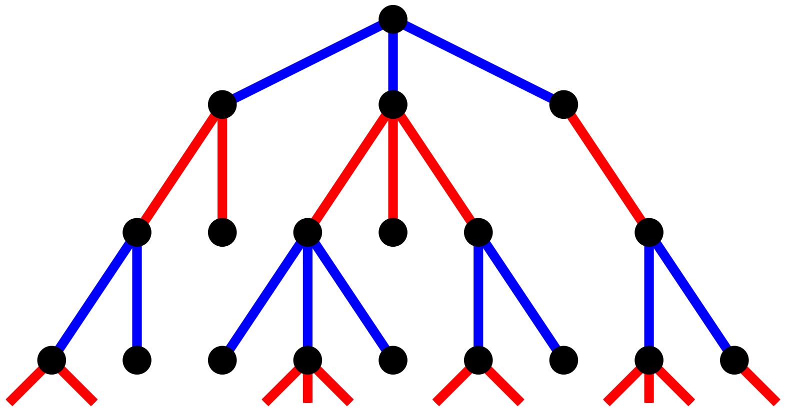

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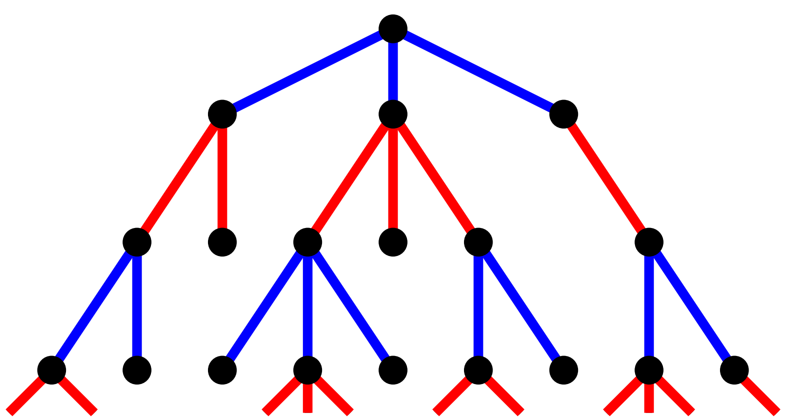

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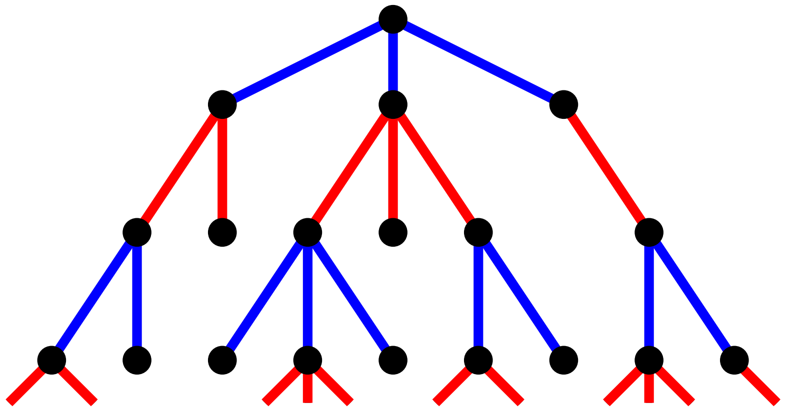

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Outerplanar | planar graphs have star arboricity 3|5. [Hakimi et al., DM'96]

□

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Graph class	Weighted old*	Weighted new°	Unweighted new°
cycle, path	1		
star	α ✓	$1+\varepsilon$	
tree	2α NP-hard	$2+\varepsilon$	2
max-degree Δ	$\lfloor(\Delta+1)/2\rfloor$		
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bipartite		$16\alpha/3 \approx 8.4$ APX-hard	
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[Chekuri & Khanna: SIAM J. Comput.'05]

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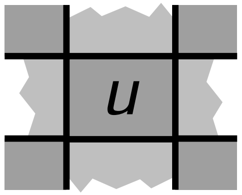

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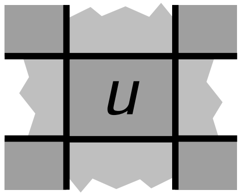

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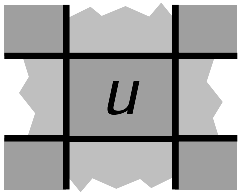

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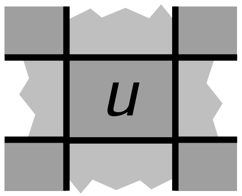

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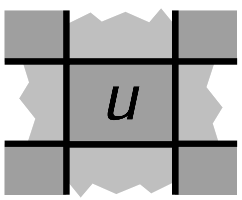

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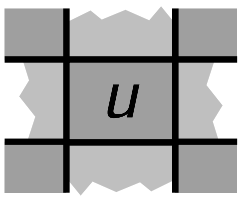

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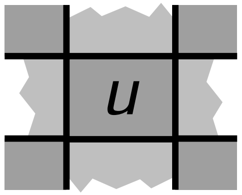

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No slack? $\Rightarrow$ Remove two cheapest items from
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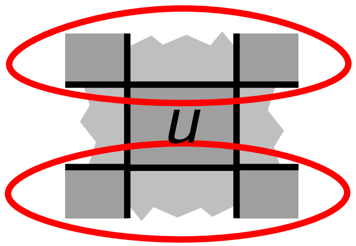

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Bipartite Graphs

Thm. Max-Crown admits a $16\alpha/3$-approx. on bip. graphs.

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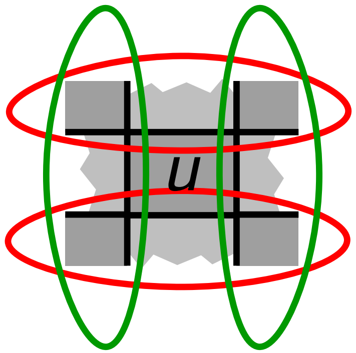

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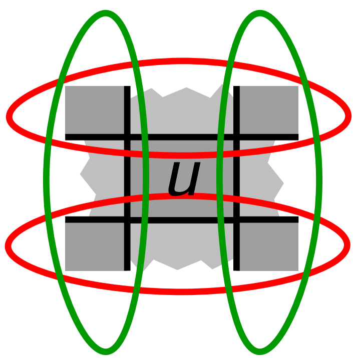

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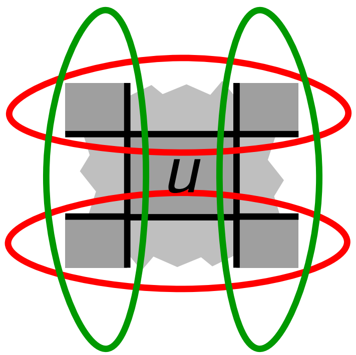

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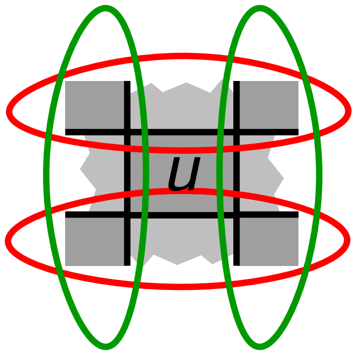

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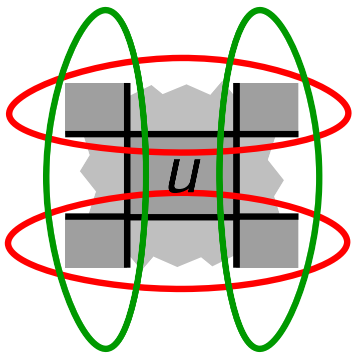

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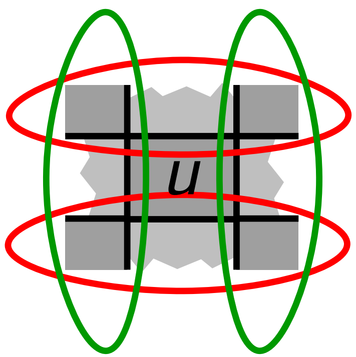

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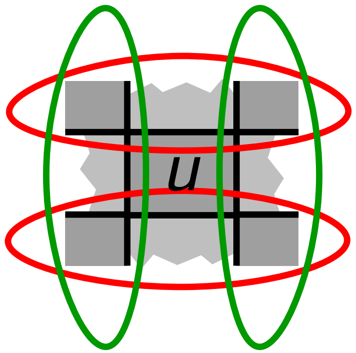

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Overview

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general		rand.: $32\alpha/3 \approx 16.9$ det.: $40\alpha/3 \approx 21.1$	$5+16\alpha/3$ ≈ 13.4

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Overview

Graph class	Weighted: old*	Weighted: new°	Unweighted: new°
cycle, path	1		
star	α ✓	$1+\varepsilon$ ✓	
tree	✓ 2α, NP-hard	$2+\varepsilon$ ✓	2
max-degree Δ	$\lfloor(\Delta+1)/2\rfloor$		
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outerplanar	3α ✓	$3+\varepsilon$ ✓	
planar	5α ✓	$5+\varepsilon$ ✓	
bipartite		$16\alpha/3$ ✓ ≈ 8.4 APX-hard	
general		rand.: $32\alpha/3 \approx 16.9$ det.: $40\alpha/3 \approx 21.1$	$5+16\alpha/3$ ≈ 13.4

*) [Barth, Fabrikant, Kobourov, Lubiw, Nöllenburg, Okamoto, Pupyrev, Squarcella, Ueckerdt & Wolff, LATIN'14]

°) [Bekos, van Dijk, Fink, Kindermann, Kobourov, Pupyrev, Spoerhase, Wolff – submitted]

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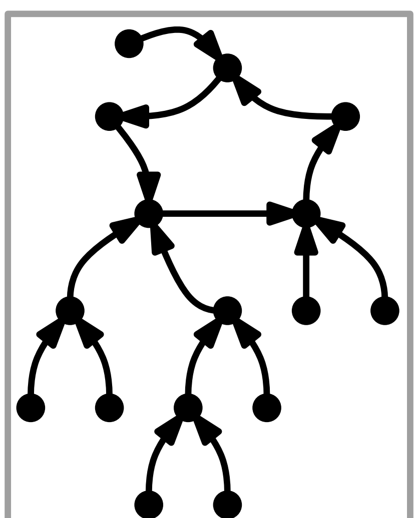

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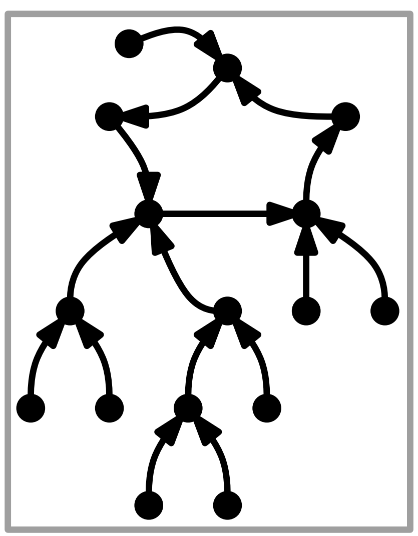

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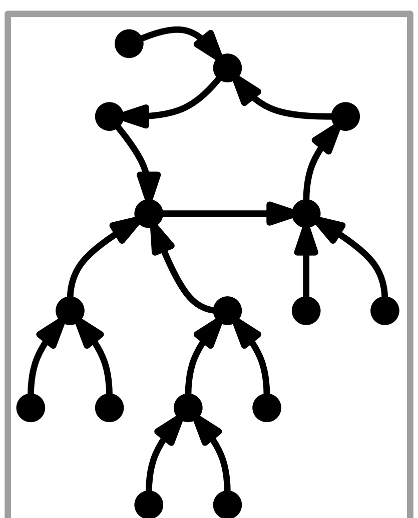

Def. G_{GAP} with edge uv iff item u is placed into a bin of v.
outdeg $\leq 1 \Rightarrow$ connected components of G_{GAP} are 1-trees.

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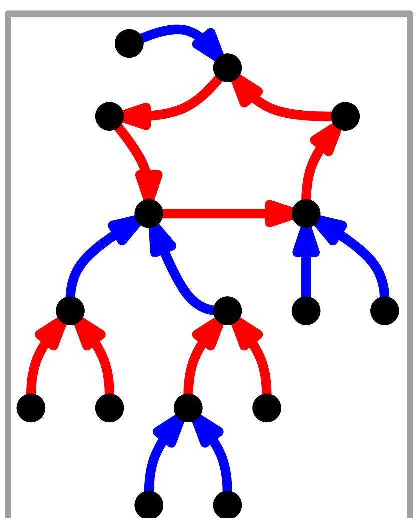

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Partition each into star forest S_1 and star forest + cycle S_2.

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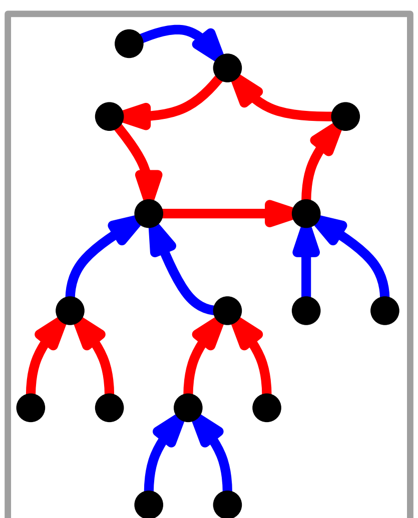

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All contacts in S_i can be realized – with corner contacts!

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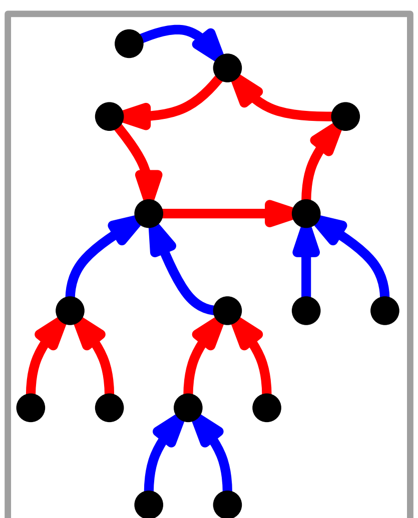

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Choose heavier of S_1 and S_2.

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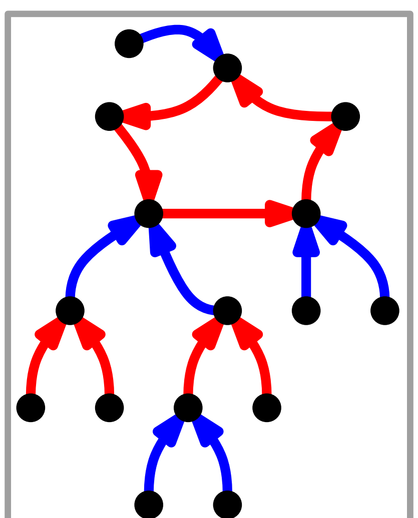

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Choose heavier of S_1 and S_2. Remove corner contacts.

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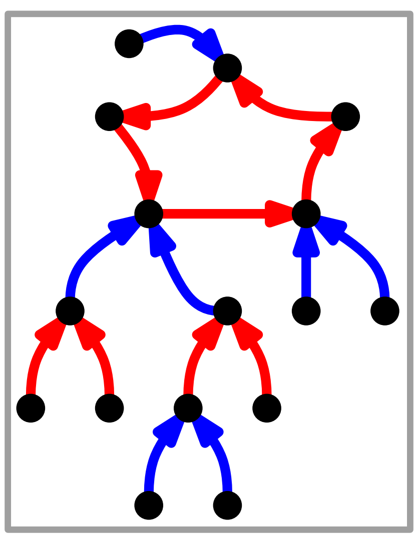
$\Rightarrow \mathrm{OPT}_{\mathrm{GAP}} \geq \mathrm{OPT}/5$. Use α-approx. alg. for Gap.

$\Rightarrow \mathrm{ALG}_{\mathrm{GAP}} \geq \mathrm{OPT}_{\mathrm{GAP}}/\alpha \geq \mathrm{OPT}/(5\alpha)$.



Def. G_{GAP} with edge uv iff item u is placed into a bin of v.

outdeg $\leq 1 \Rightarrow$ connected components of G_{GAP} are 1-trees.

Partition each into star forest S_1 and star forest + cycle S_2.

All contacts in S_i can be realized – with corner contacts!

Choose heavier of S_1 and S_2. Remove corner contacts.

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Tool #4: Let GAP Take the Decisions!

Thm. MAX-CROWN admits a deterministic $40\alpha/3$-approx.

Proof. Let $G = (V, E)$ be any graph. Use GAP – as in bipartite case!

New: for every vertex, we construct both 8 bins *and* 1 item.

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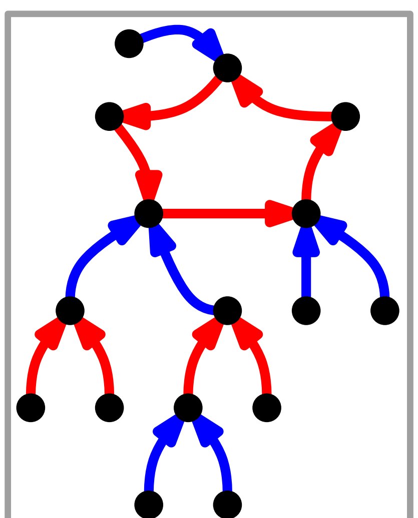

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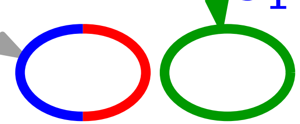
$\Rightarrow \mathrm{ALG} \geq$  $\mathrm{ALG}_{\mathrm{GAP}}$

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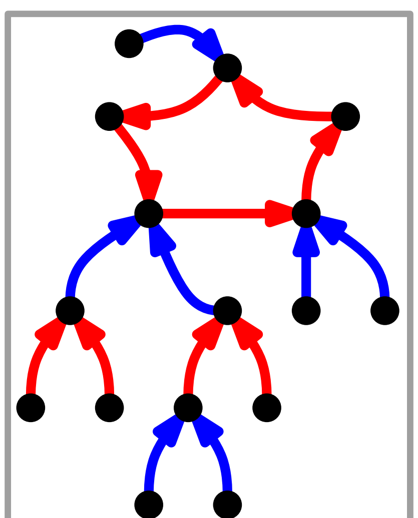

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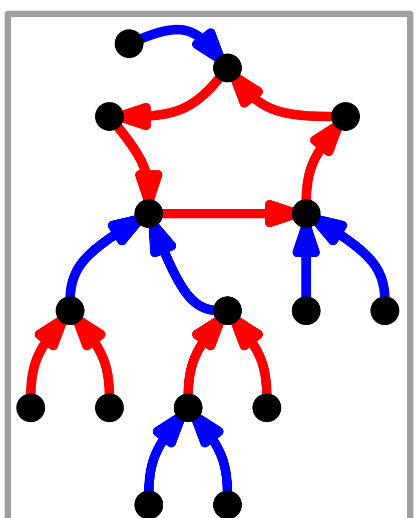

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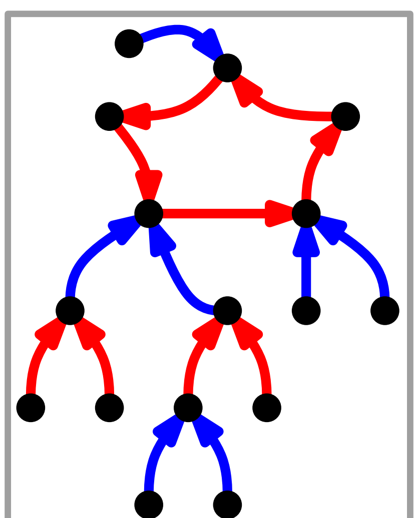

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Overview

Graph class	Weighted old*	Weighted new°	Unweighted new°
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star	α ✓	$1+\varepsilon$ ✓	
tree	✓ 2α, NP-hard	$2+\varepsilon$ ✓	2
max-degree Δ	$\lfloor(\Delta+1)/2\rfloor$		
planar max-deg. Δ			$1+\varepsilon$
outerplanar	3α ✓	$3+\varepsilon$ ✓	
planar	5α ✓	$5+\varepsilon$ ✓	
bipartite		$16\alpha/3$ ✓ ≈ 8.4 APX-hard	
general		rand.: $32\alpha/3$ ✓ ≈ 16.9 det.: $40\alpha/3$ ≈ 21.1	$5+16\alpha/3$ ≈ 13.4

*) [Barth, Fabrikant, Kobourov, Lubiw, Nöllenburg, Okamoto, Pupyrev, Squarcella, Ueckerdt & Wolff, LATIN'14]

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  What other problems have been solved combinatorially, but are interesting to optimize when we add more constraints?