Progress on Partial Edge Drawings

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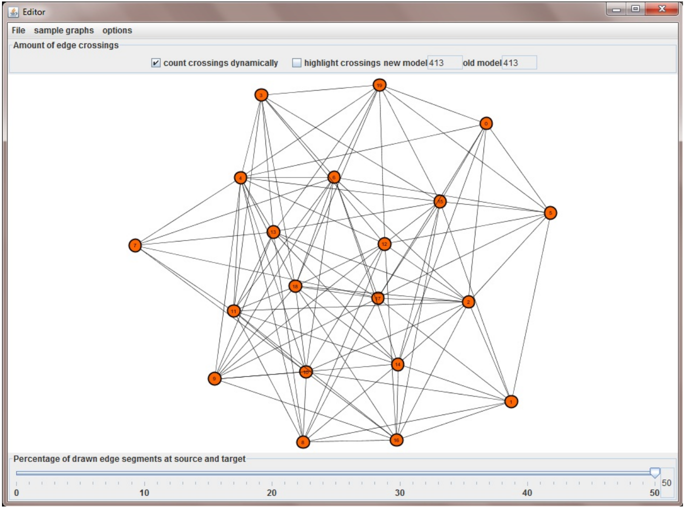
UNIVERSITÀ DEGLI STUDI DI PERUGIA

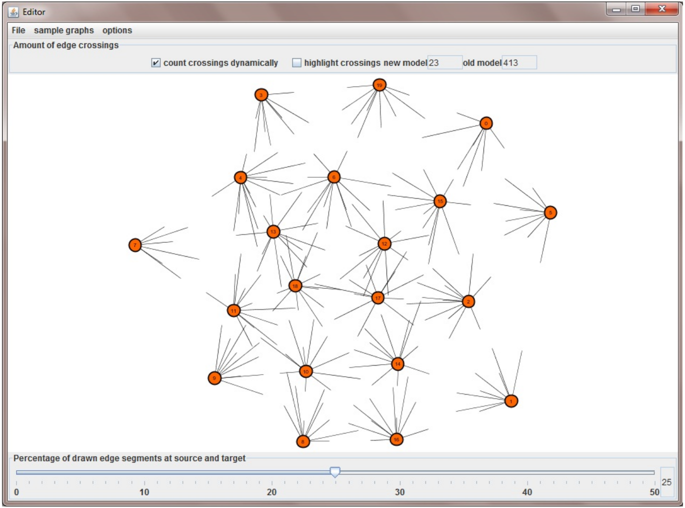
19-09-2012

gd12

Straight-line drawing of a graph with 20 vertices



Straight-line drawing of a graph with 20 vertices



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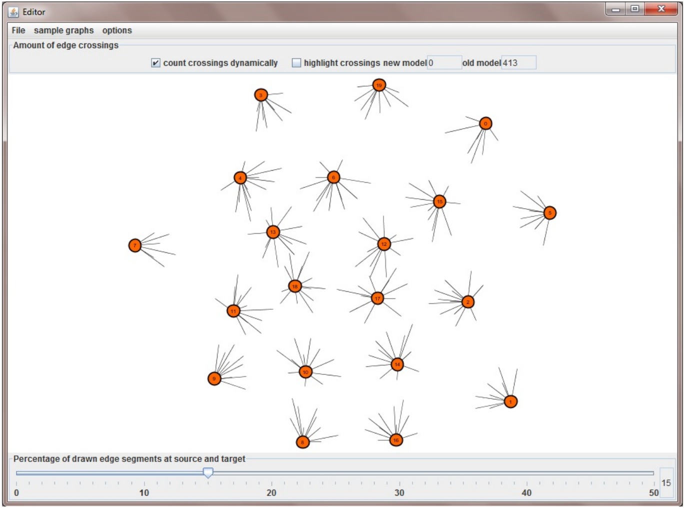
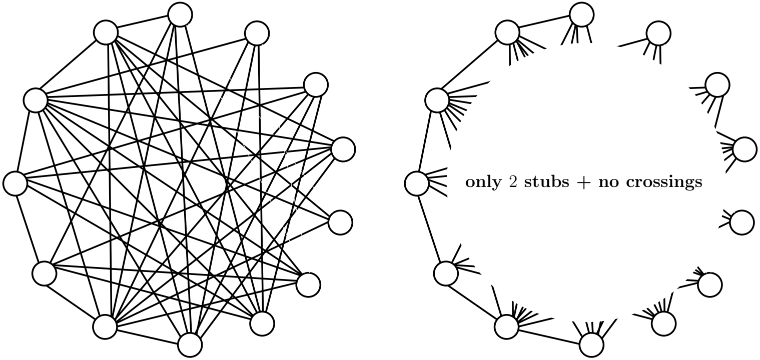
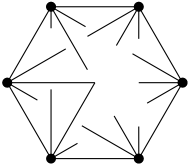


Partial edge drawing (PED)



PED for K_6 with additional properties



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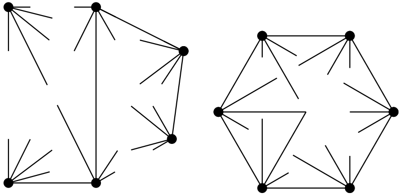
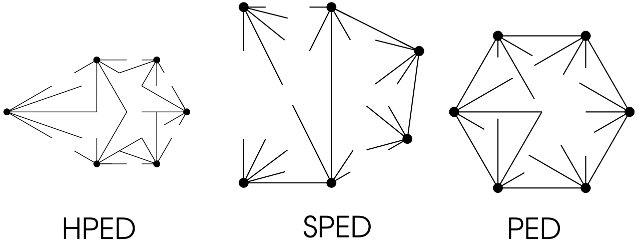


SPED PED

S = symmetric,

PED for K_6 with additional properties



S = symmetric, H = homogeneous,

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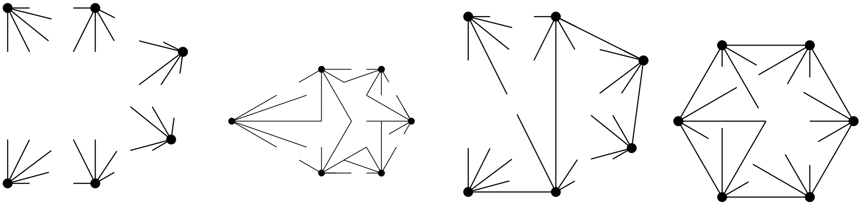

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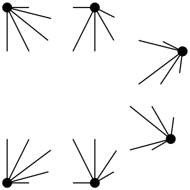
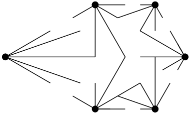
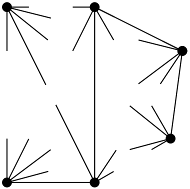


δ-SHPED
here $\delta = 1/4$



HPED



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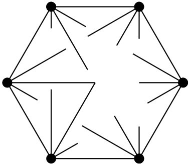
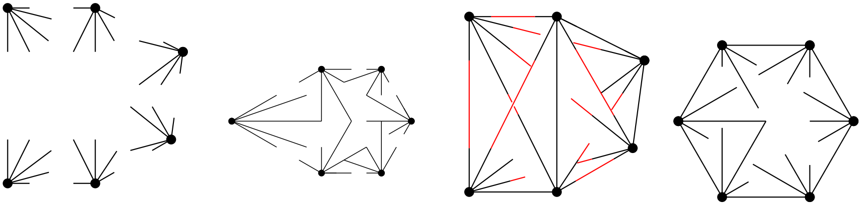


PED

S = symmetric, H = homogeneous, δ = stub-edge ratio

PED for K_6 with additional properties



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maxSPED
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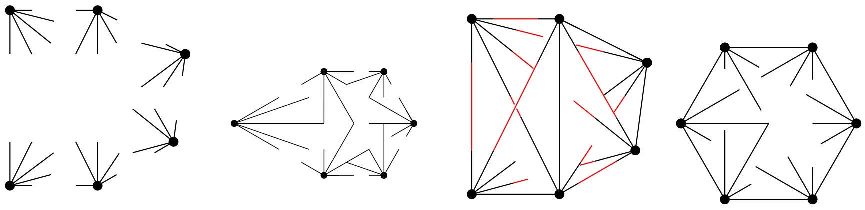

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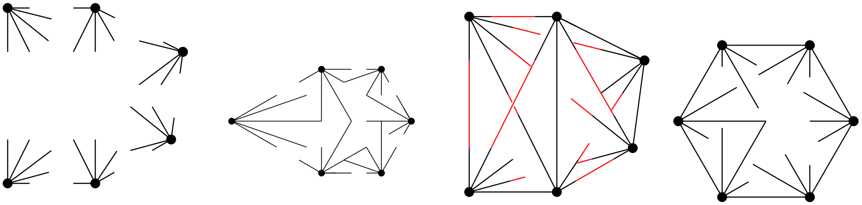

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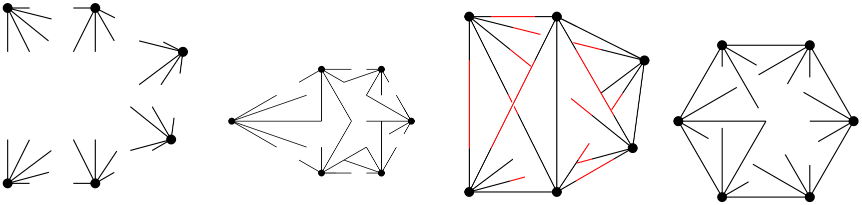

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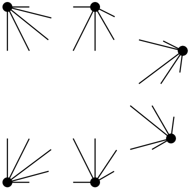
Symmetric homogeneous partial edge drawing



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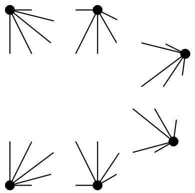

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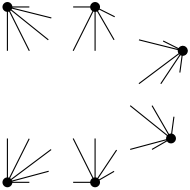

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Theorem (B., Kaufmann 2012)

K_n has a $1/4$-SHPED for all $n \leq 16$.

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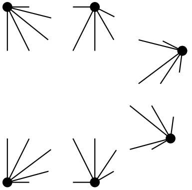

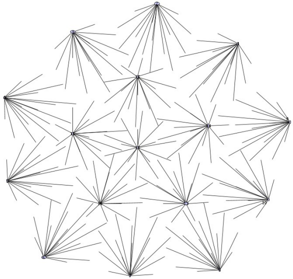

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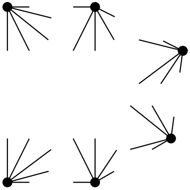

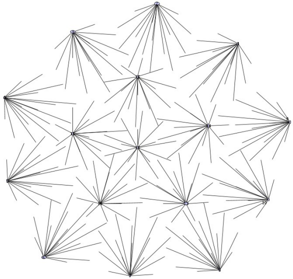

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Theorem (new)

For any $n > 182$, the graph K_n does not admit a $1/4$-SHPED.

Symmetric homogeneous partial edge drawing

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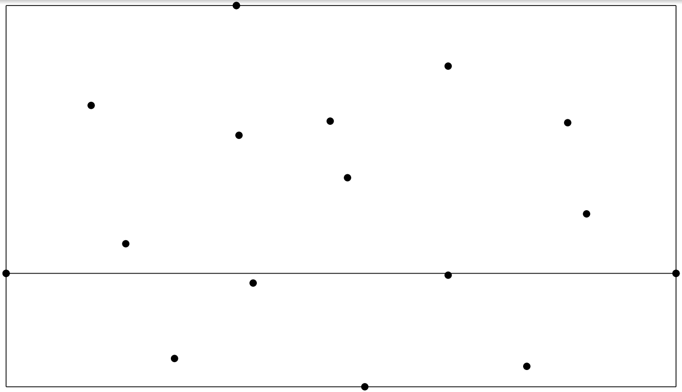

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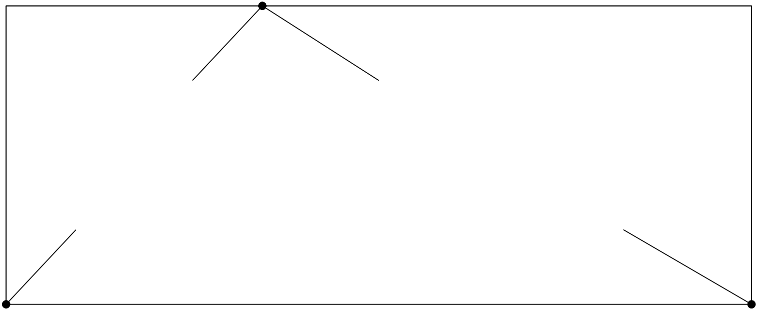

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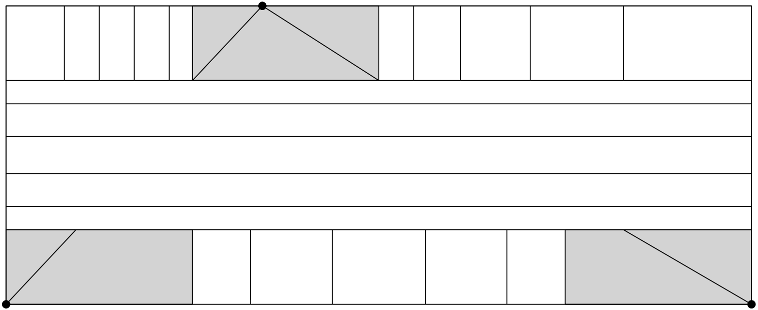

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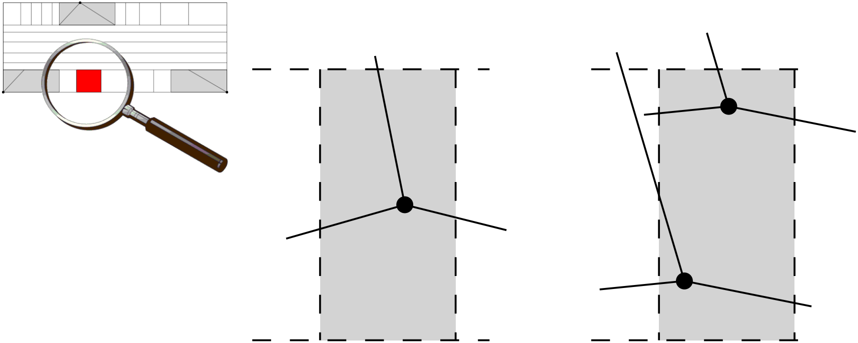

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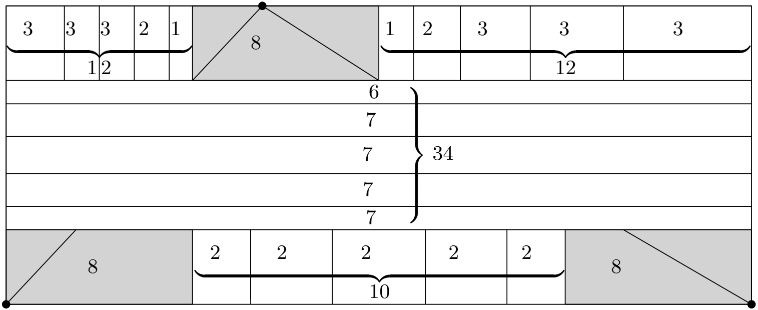

$$\sum = 92$$

Content

1. non-existence of $1/4$-SHPED for K_n with $n > 182$,
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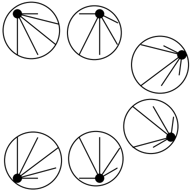
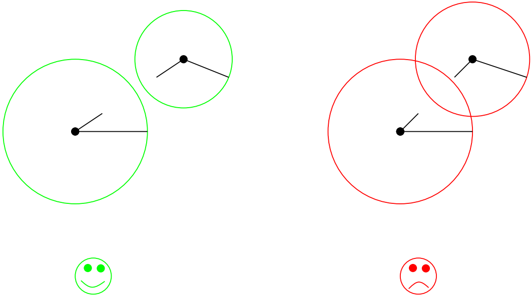
Key concept



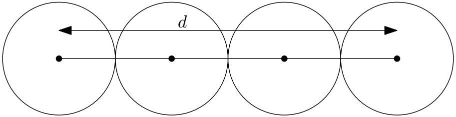
SHPED



Condition = no overlapping

Key concept

- cover all stubs of a vertex by a circle and ensure disjoint circles



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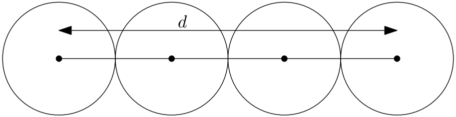

- distance $d = 3 \Rightarrow$ ratio $\delta = 1/6 = 1/(2d)$

Bandwidth k graphs and k-circulant graphs

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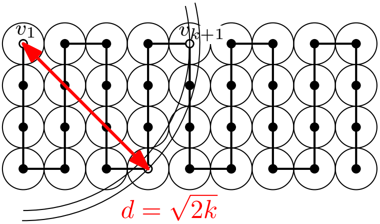

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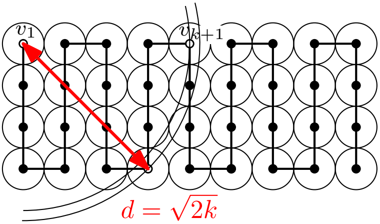

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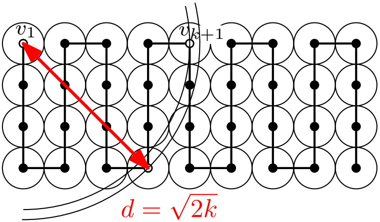

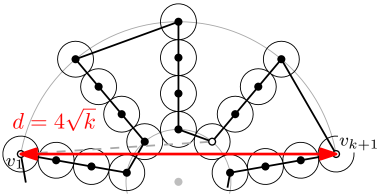

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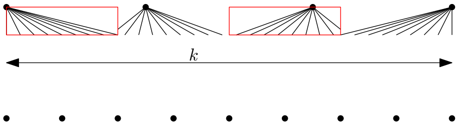

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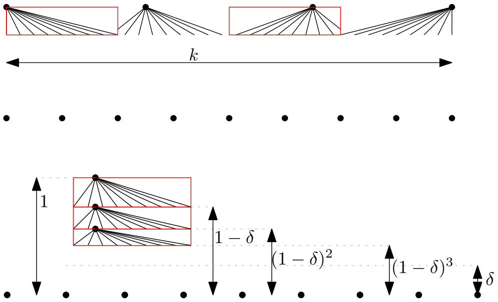

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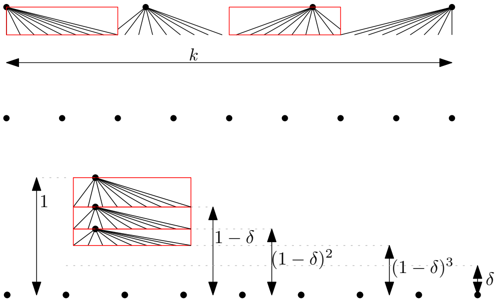

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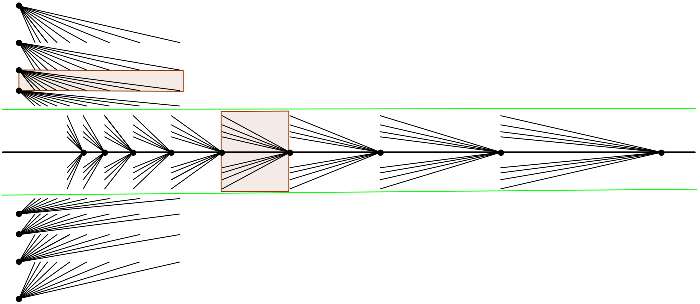

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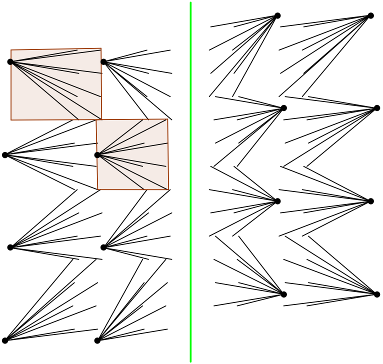

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- *can be 2-approximated in $O(m^4)$ time, for m edges.*

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  (Bar-Yehuda, Rawitz 2001)

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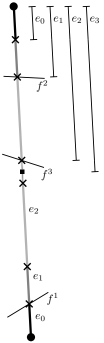

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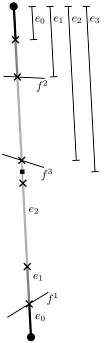

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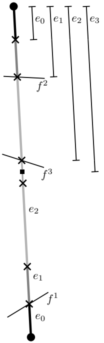

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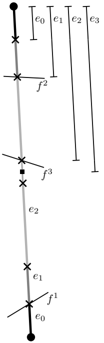

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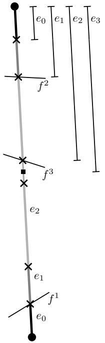

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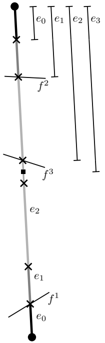
Theorem

A maxSPED can be 2-approximated in $O(m^4)$ time.



- pair of segments e_i create variable $\hat{e}_i$ in instance φ
- draw e_i not $\Leftrightarrow$ $\hat{e}_i$ = true
- introduce clauses $(\neg\hat{e}_{i+1} \Rightarrow \neg\hat{e}_i) \equiv (\hat{e}_{i+1} \vee \neg\hat{e}_i)$
- introduce clauses $(\hat{e}_i \vee \hat{f}_j)$, for e_i crosses f_j
- introduce weight $w_{e,i} = |e_i|$ for $\hat{e}_i$
- over all valid variable assignments Var(φ):

$$\text{minimize} \sum_{\hat{e}_i \in \text{Var}(\varphi)} w_{e,i}\hat{e}_i$$

Conclusion and future work

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- generalization: show $\nexists \delta$-SHPED for K_n with $n > n(\delta)$