Bounding and computing obstacle numbers of graphs

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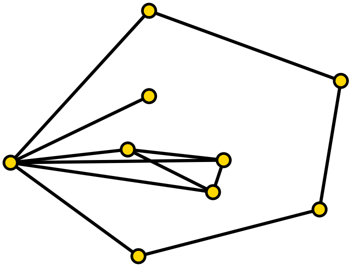

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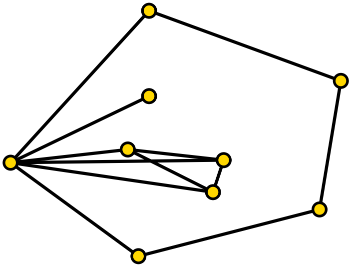

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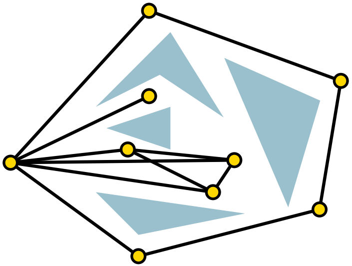

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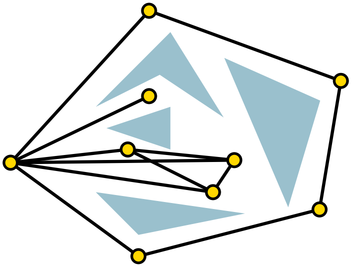

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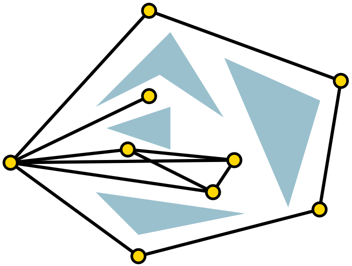

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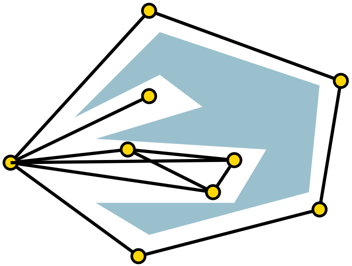

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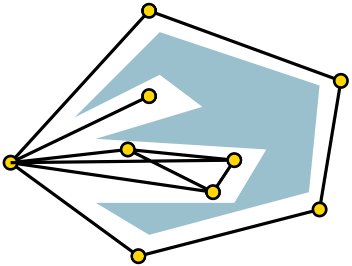

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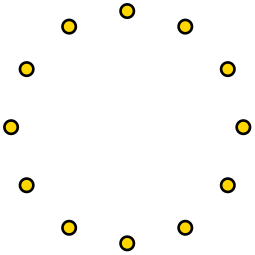

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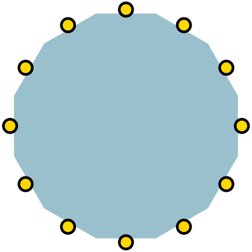

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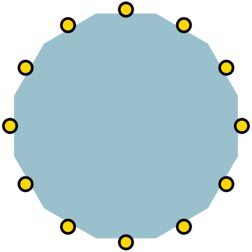

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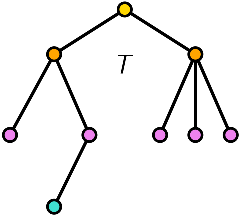

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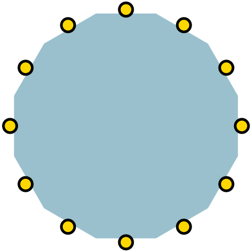

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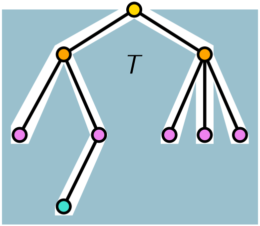

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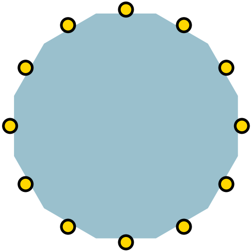

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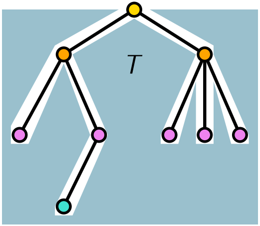

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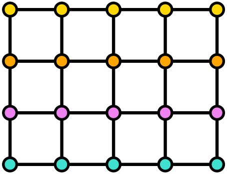

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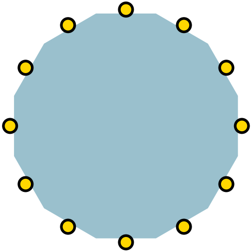

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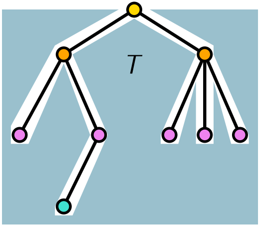

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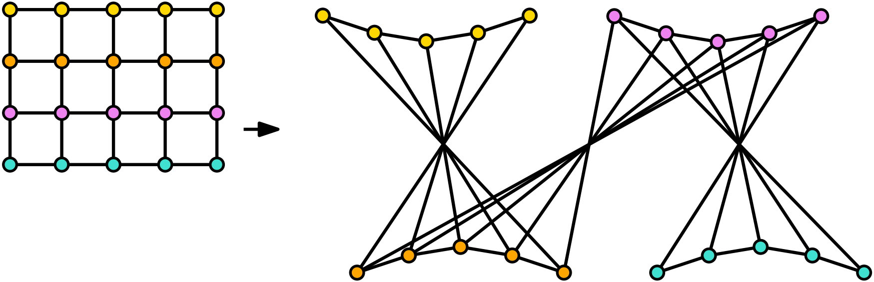

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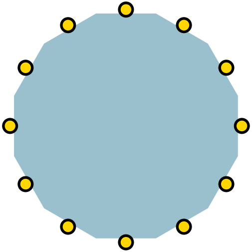

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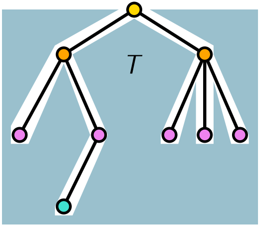

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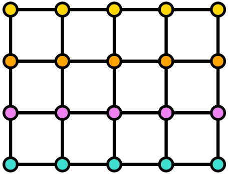
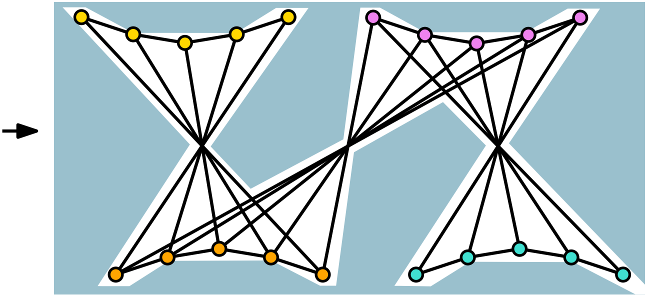


$\text{obs}(P_m \times P_n) = 1$

(Fabrizio Frati)



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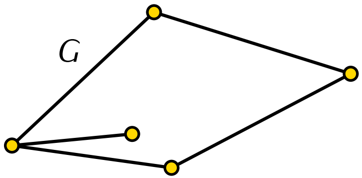

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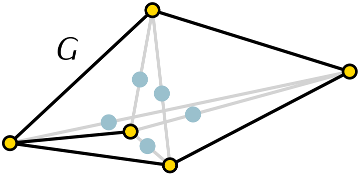

$$\text{obs}(G) \leq \binom{n}{2} - |E(G)|$$

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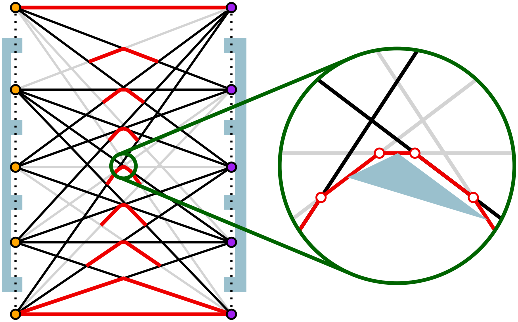

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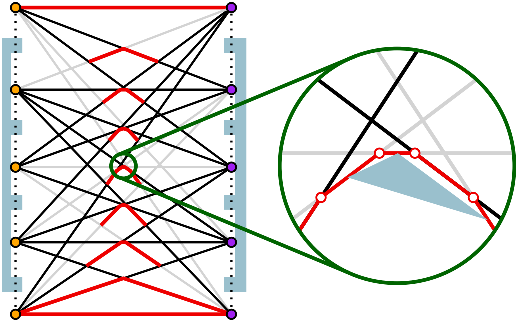

- The bounds apply even if the obstacles are required to be convex.

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- The proofs are not constructive and follow from a counting argument.

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- This is asymptotically tight for $h < n$ as Balko, Cibulka, and Valtr showed $f_c(h, n) \in 2^{\Omega(hn)}$ for $0 < h < n$ and $f_c(1, n) \in 2^{\Omega(n \log n)}$.

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- The complexity of deciding whether a given graph has obstacle number 1 is still open.

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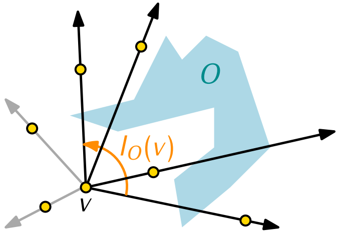

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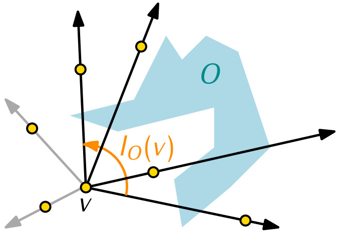

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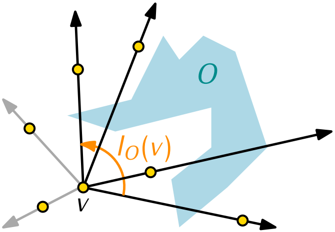

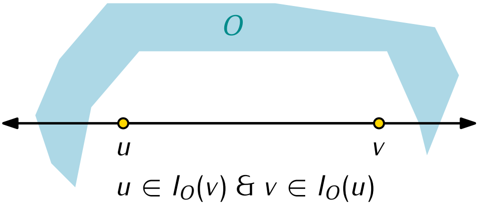

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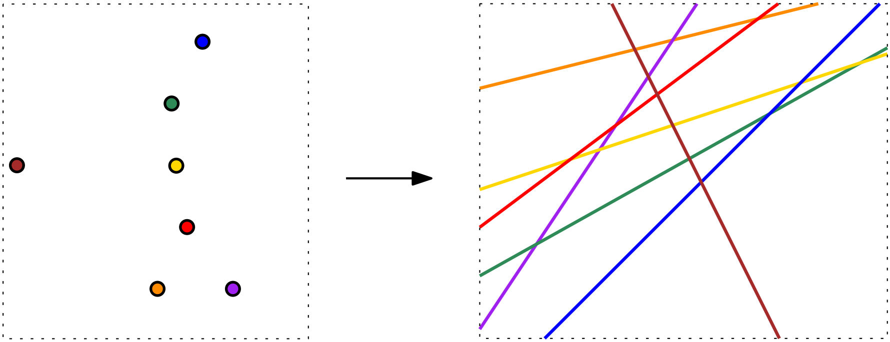

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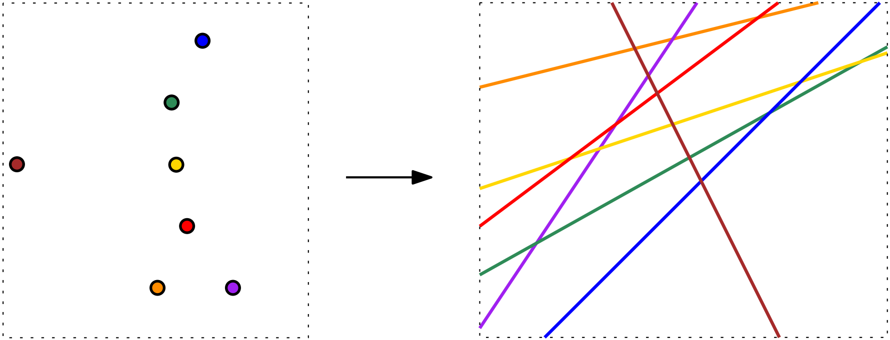

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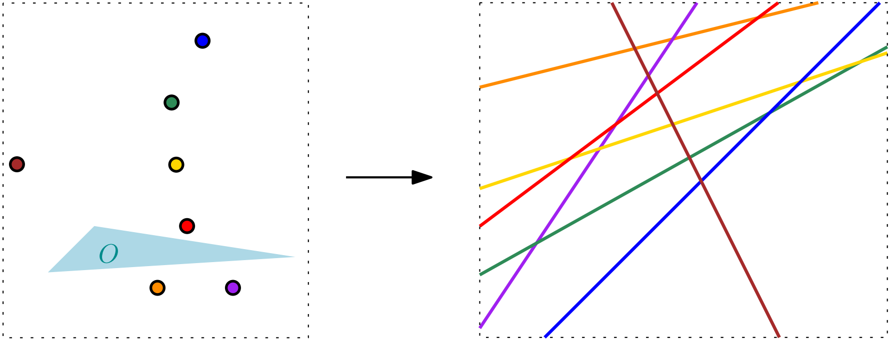

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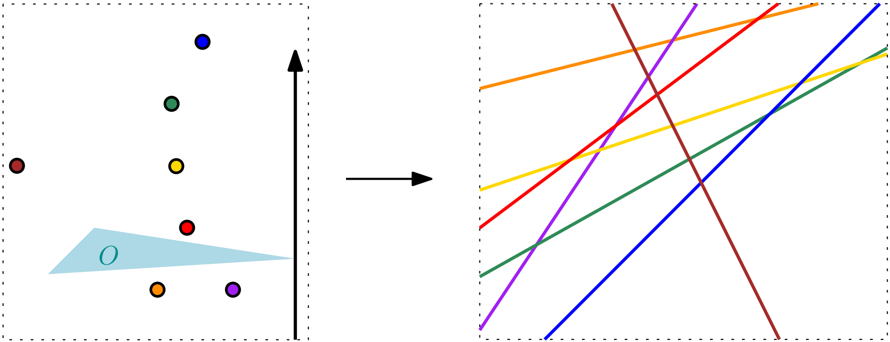

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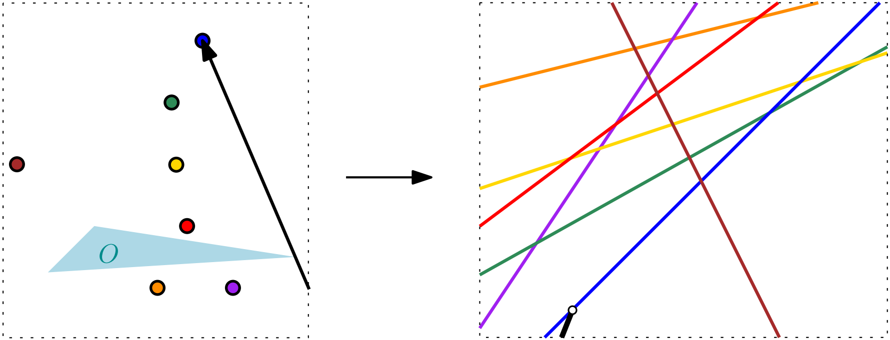

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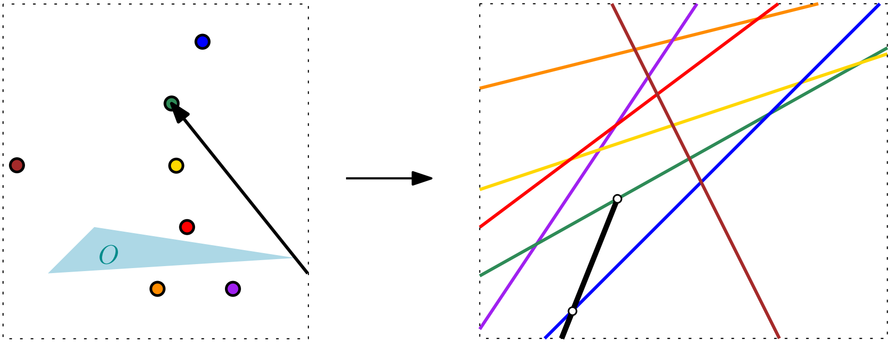

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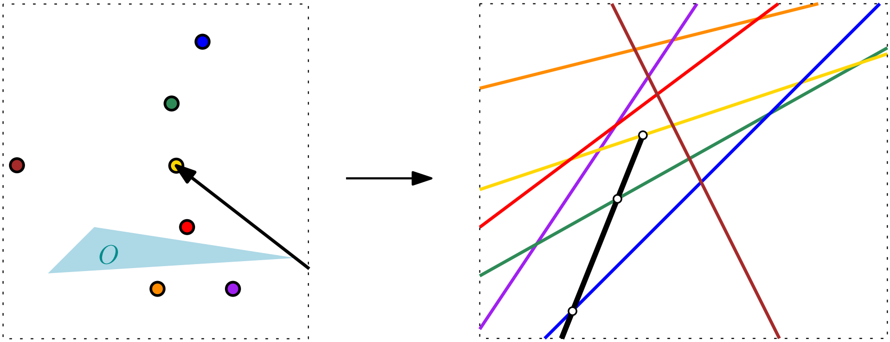

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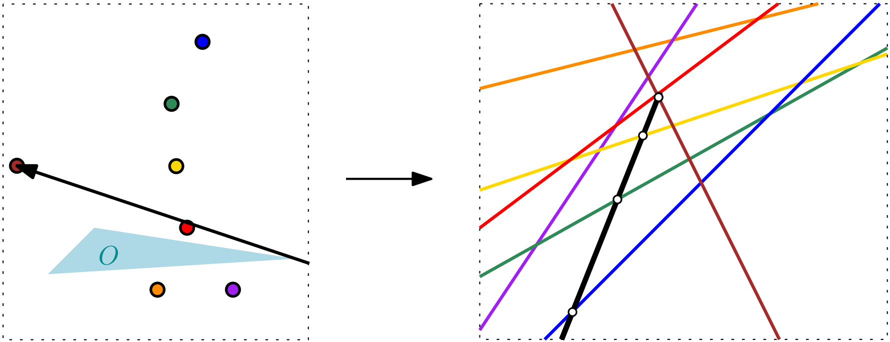

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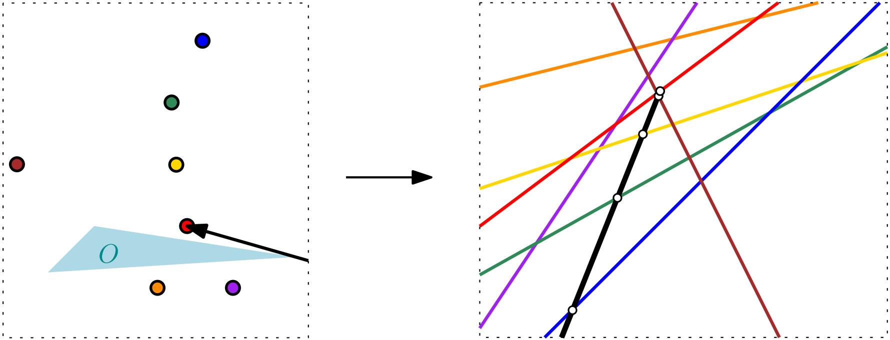

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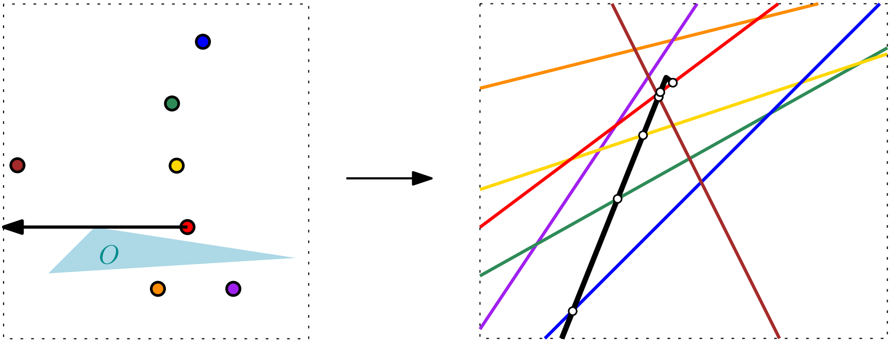

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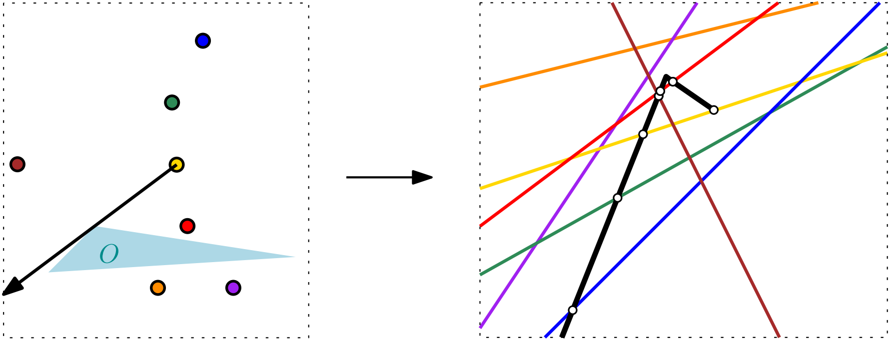

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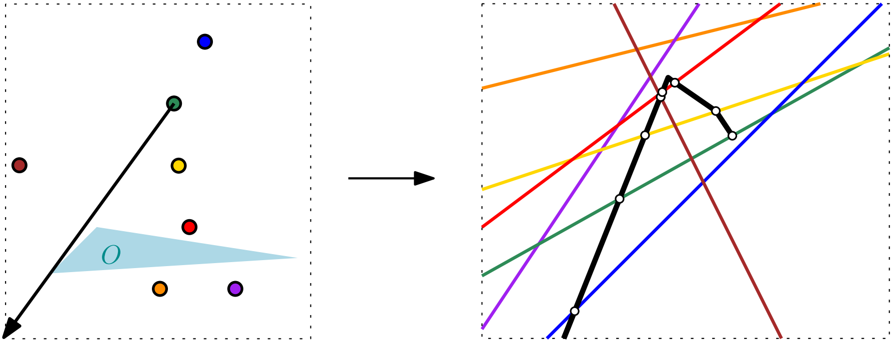

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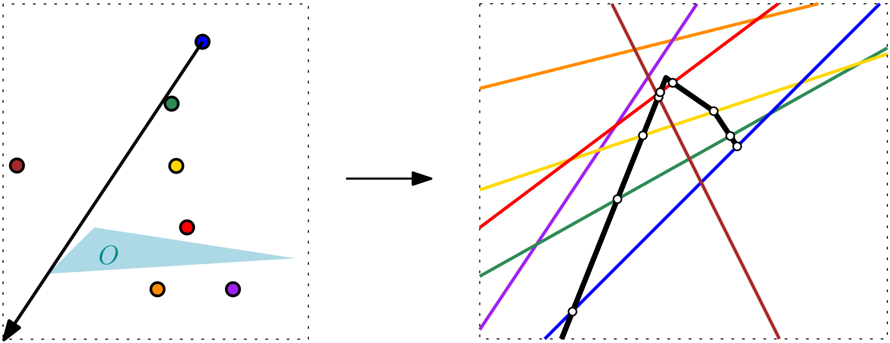

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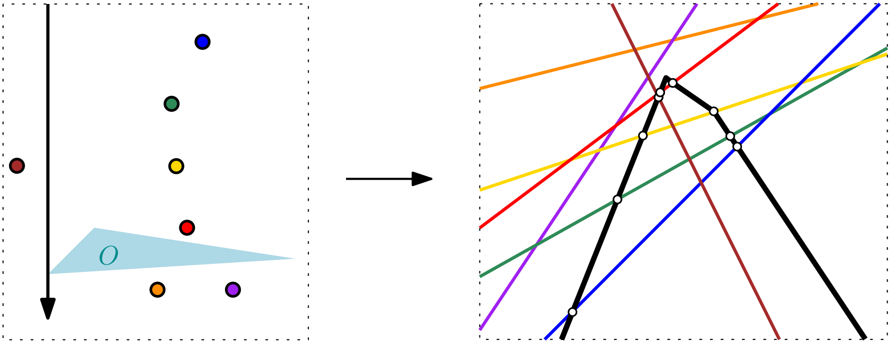

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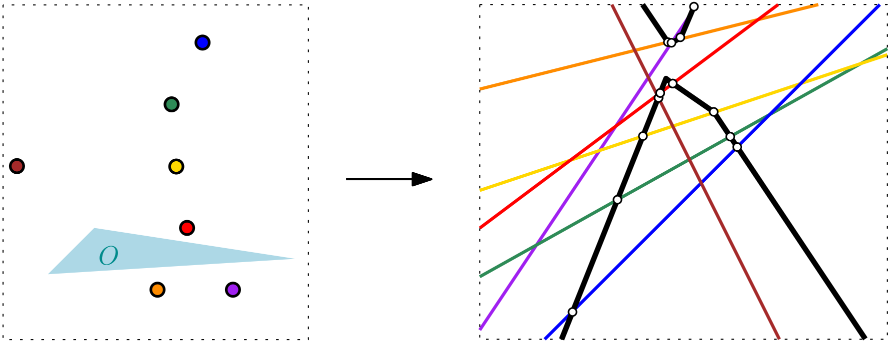
- Traversing around O with the tangent lines corresponds to two cutpaths in the dual line arrangement.



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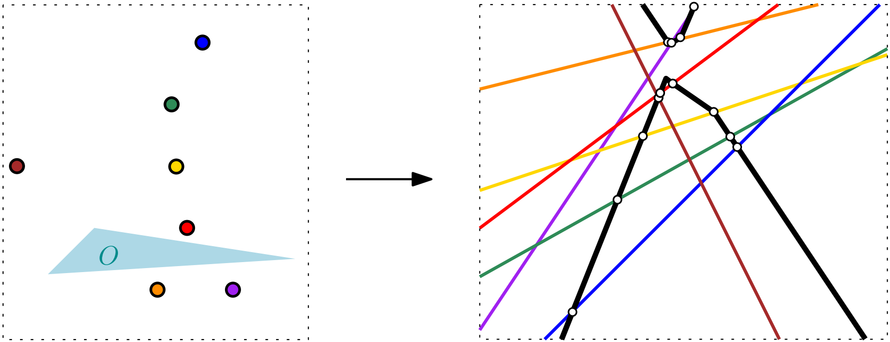

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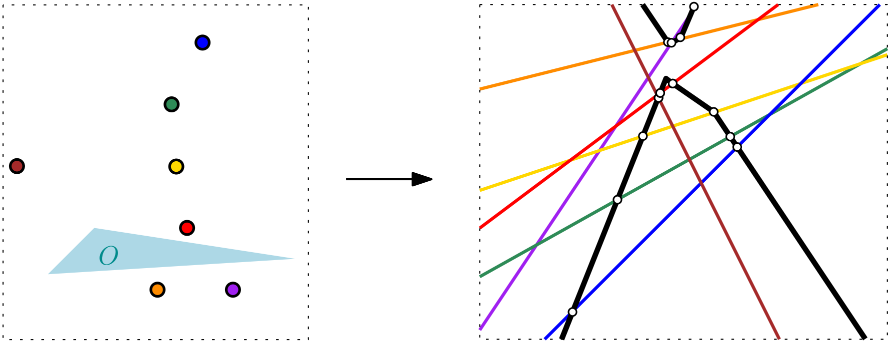

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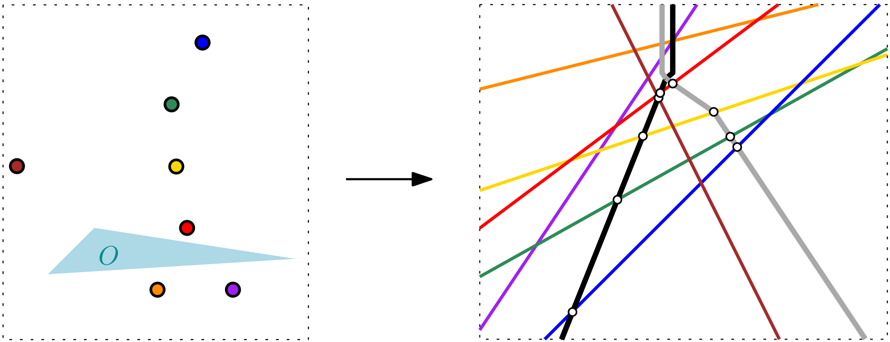

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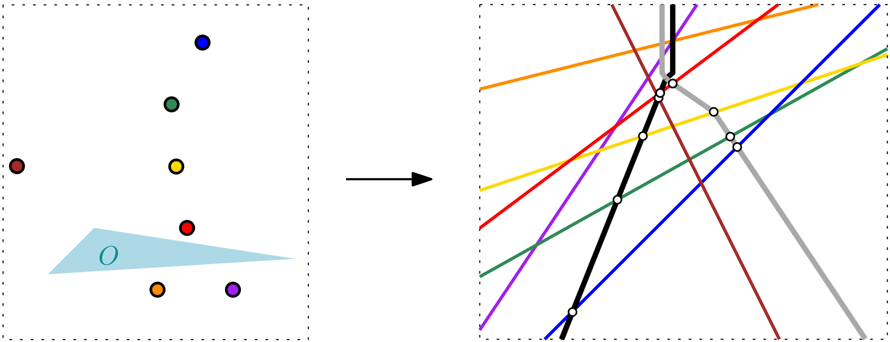

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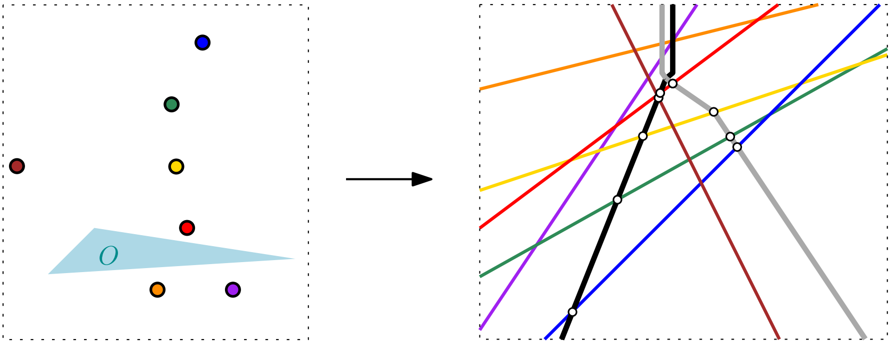

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