Simultaneous Drawing of Planar Graphs with Right-Angle Crossings and Few Bends*

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1 Introduction

A simultaneous embedding of two planar graphs embeds each graph in a planar way—using the same vertex positions for both embeddings. Edges of one graph are allowed to intersect edges of the other graph. There are two versions of the problem: In the first version, called *Simultaneous Embedding with Fixed Edges* (SEFE), edges that occur in both graphs must be embedded in the same way in both graphs (and hence, cannot be crossed by any other edge). In the second version, these edges can be drawn differently for each of the graphs. Both versions of the problem have a geometric variant where edges must be drawn using straight-line segments.

When actually drawing these simultaneous embeddings, a natural choice is to use straight-line segments. Only very few graphs can be drawn in this way, however, and some existing results need exponential area. We suggest a new approach that overcomes these problems. We insist that crossings occur at right angles, thereby "taming" them. We do this while drawing on a grid of size $O(n) \times O(n)$ for n-vertex graphs, and we can still draw any pair of planar graphs simultaneously. We do not consider the problem of fixed edges. In a way, our results give a measure for the geometric complexity of simultaneous embeddability for various pairs of graph classes, some of which can be combined more easily (with fewer bends) and some not as easily (needing more bends).

More formally, in this paper we study the *RAC simultaneous drawing problem* (RACSIM *drawing problem*). Let $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ be two planar graphs on the same vertex set. We say that G_1 and G_2 admit a RACSIM drawing if we can place the vertices on the plane such that (i) each edge is drawn as a polyline, (ii) each graph is drawn planar, (iii) there are no edge overlaps and (iv) crossings between edges in E_1 and E_2 occur at right angles.

Argyriou et al. [1] introduced and studied the geometric version of RACSIM drawing. In particular, they proved that it is always possible to construct a geometric RACSIM drawing of a cycle and a matching in quadratic area, while there exists a pair of graphs (a wheel and a cycle) which which do not admit a geometric RACSIM drawing. The problem that we study was left as an open problem.

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Table 1. A short summary of our results

Graph classes			Number of bends	Grid size
Planar	+	Planar	$6+6$	$(14n-26)\times(14n-26)$
2-page book embed.	+	2-page book embed.	$4+4$	$(11n-32)\times(11n-32)$
Outerplanar	+	Outerplanar	$3+3$	$(7n-10)\times(7n-10)$
Cycle	+	Cycle	$1+1$	$2n\times 2n$
Caterpillar	+	Cycle	$1+1$	$(2n-1)\times 2n$
Four Matchings			$1+1+1+1$	$2n\times 2n$
Tree	+	Matching	$1+0$	$n\times(n-1)$
Wheel	+	Matching	$2+0$	$(1.5n-1)\times(n+2)$
Outerpath	+	Matching	$2+1$	$(3n-2)\times(3n-2)$

2 Our contribution

First, we look at the most general version of the problem, where the input is a pair of planar graphs. (In a simultaneous drawing, certainly both graphs must—individually—be planar.) We give a linear-time algorithm for this case, which produces a drawing in quadratic area with at most six bends per edge. For 2-page book embeddable graphs and outerplanar graphs, we give algorithms that guarantee four and three bends respectively. Then we turn our attention to graph classes that are more restricted, but for which we can give algorithms that use very few bends. See Table 1 for a full list of results. The main approach in these algorithms is to find linear orders on the vertices of the two graphs and then to compute coordinates for the vertices based on these orders. All drawings can be computed in linear time on a grid whose size is quadratic in the number of vertices. The proofs of these claims can be found in the full version of the paper [2].

3 Open Problems

The results presented in this paper raise several questions that remain open, such as the following. (i) As a variant of the problem, it might be possible to reduce the required number of bends per edge by relaxing the strict constraint that intersections must be right-angle and instead ask for drawings that have close to optimal crossing resolution. (ii) The computational complexity of the general problem remains open: Given two or more planar graphs on the same set of vertices and an integer k, is there a RACSIM drawing in which each graph is drawn with at most k bends per edge and the crossing are at right angles?

References

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