Drawing Graphs on Few Circles and Few Spheres

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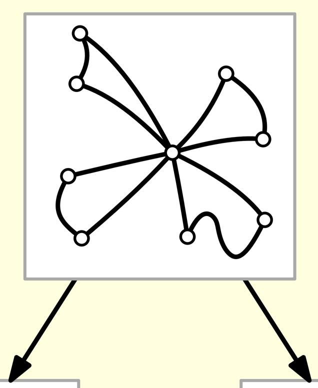
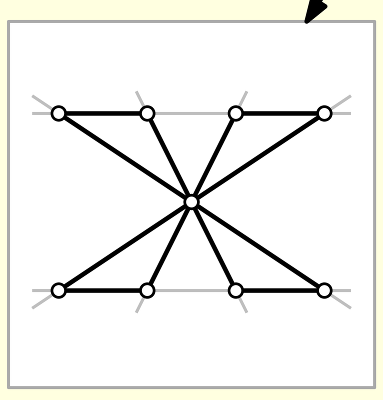
Problem

Given a planar graph G,

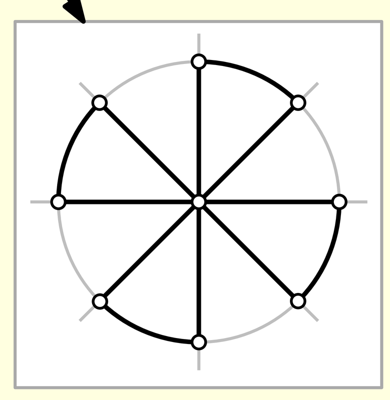
find a plane straight-line drawing of G that minimizes the number of lines that together cover the drawing.
This number is the *line cover number* of G.



find a plane circular-arc drawing of G that minimizes the number of circles that together cover the drawing.
This number is the *circle cover number* of G.

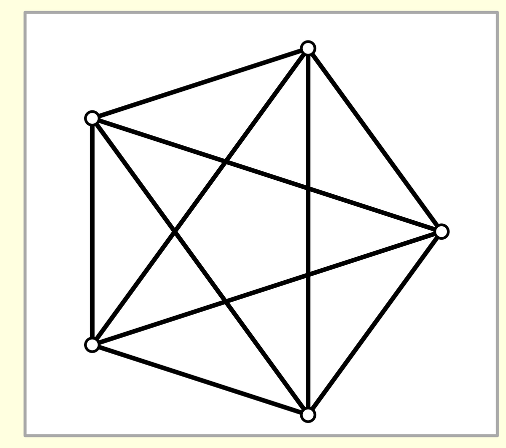


6 lines
8 segments



5 circles
8 arcs

Given a (non-planar) graph G,



find a circular-arc drawing without edge crossings on as few spheres as possible.

Results

Optimal Drawings of the Platonic Solids

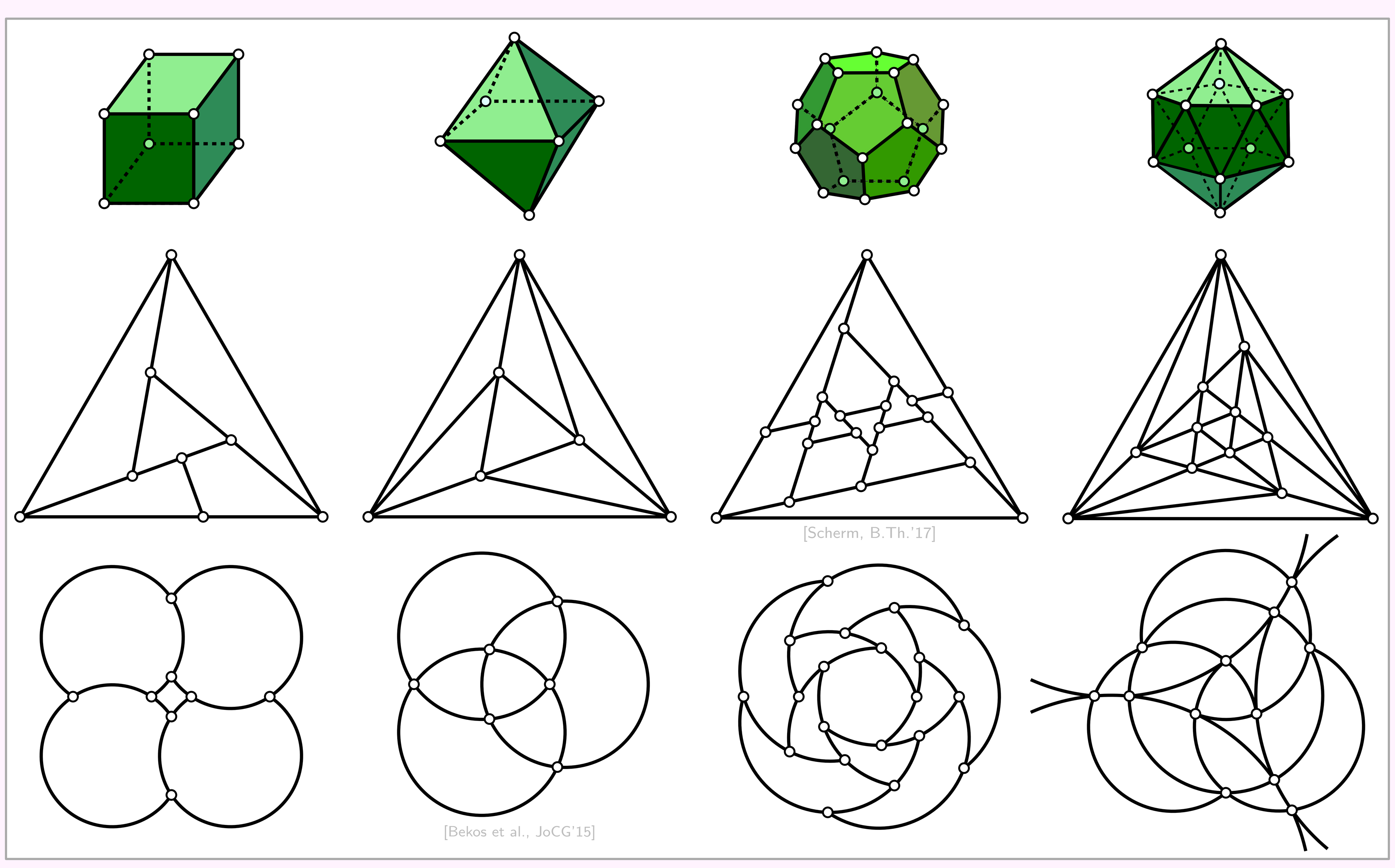
$G = (V, E)$	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert F\rvert$	segment number	line cover number	arc number	circle cover number
tetrahedron	4	6	4	6	6	3	3
cube	8	12	6	7	7	4	4
octahedron	6	12	8	9	9	3	3
dodecahedron	20	30	12	13	9...10	10	5
icosahedron	12	30	20	15	13...15	7	7

Upper bounds – follow from the drawings below.

Platonic solids:

(Near-) optimal line covers with min. number of segments:

Optimal circle covers with minimum number of arcs:



Lower bounds

Segment number:
Using an ILP, we find a locally consistent angle assignment with maximum number of 180°-angles.

Line cover number:
We use the number of nested cycles and the *internal degree* of the outer face.

Circle cover number:
We argue via the minimum number of circular arcs to cover the intersection points.

Sphere Covers

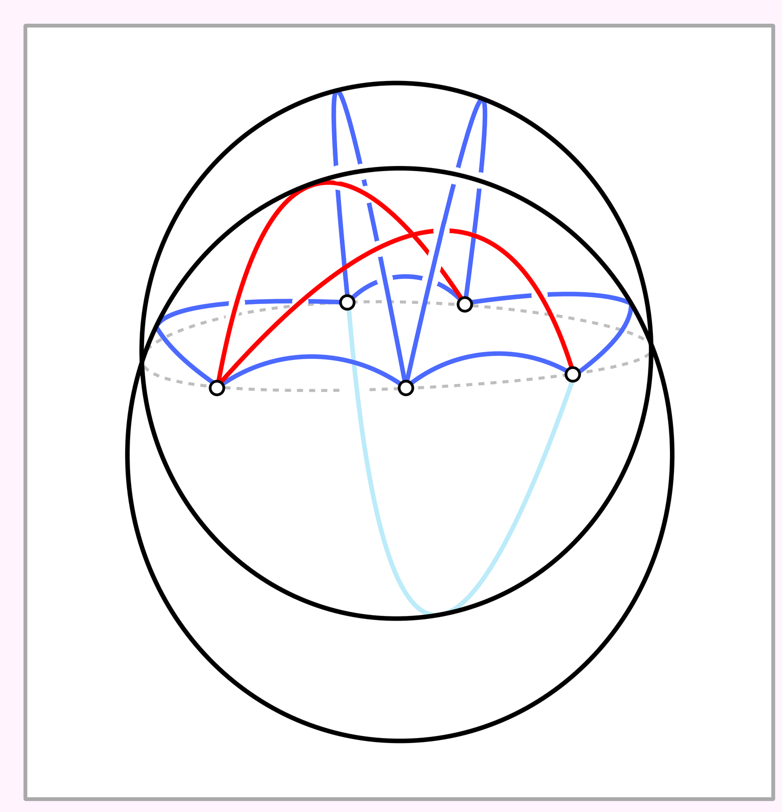
book-thickness(G)/2
$\leq$ sphere-cover-number(G)
$\leq$ thickness(G)

&

$$\text{thickness}(K_n) \approx \frac{n+7}{6}$$

$\Downarrow$

Proposition:
Any n-vertex graph G has sphere cover number $O(n)$.



Optimal sphere cover of K_5

https://arxiv.org/abs/1709.06965

Future work

Line cover vs. circle cover

Is there a family of planar graphs whose circle cover number grows asymptotically more slowly than their line cover number?

Size of circle cover vs. angular resolution