The Parameterized Complexity of Coloring Mixed Graphs

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Warsaw University of Technology

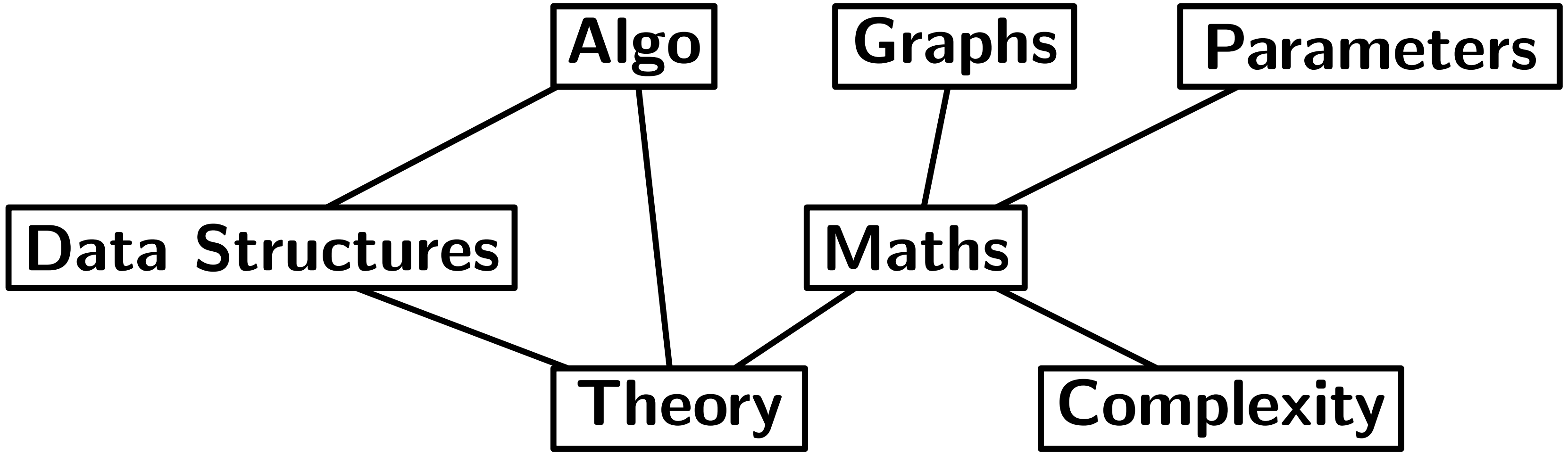
Introduction

Algo **Graphs** **Parameters**

Data Structures **Maths**

Theory **Complexity**

Introduction



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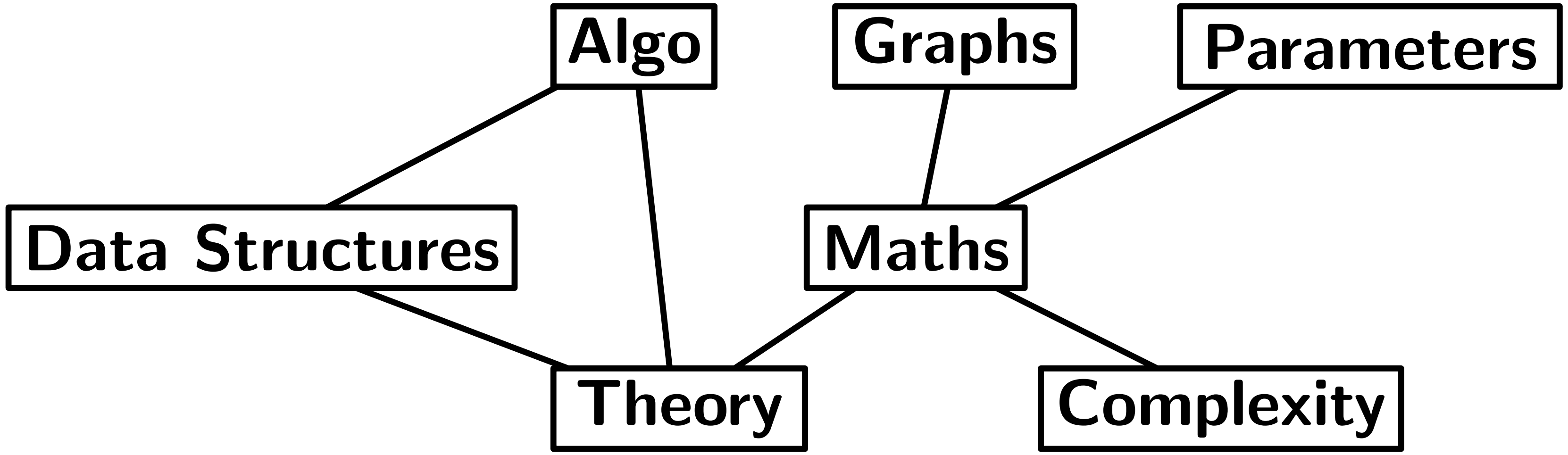
Undirected Graph

- vertices V
- edges E



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k-coloring $c\colon V \to [k]$ proper if

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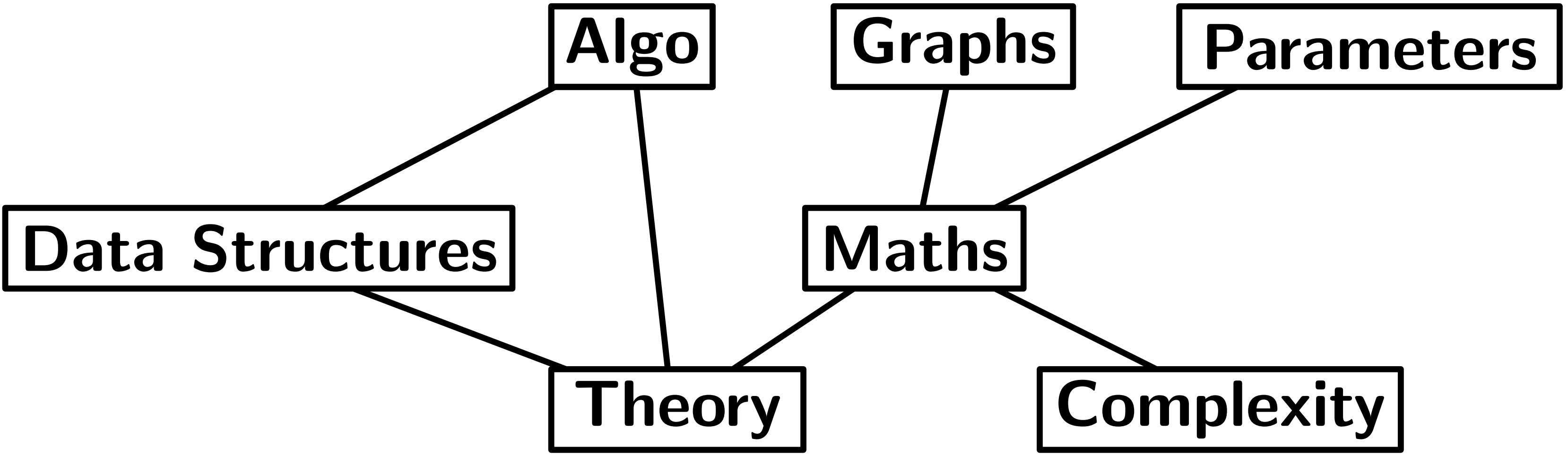

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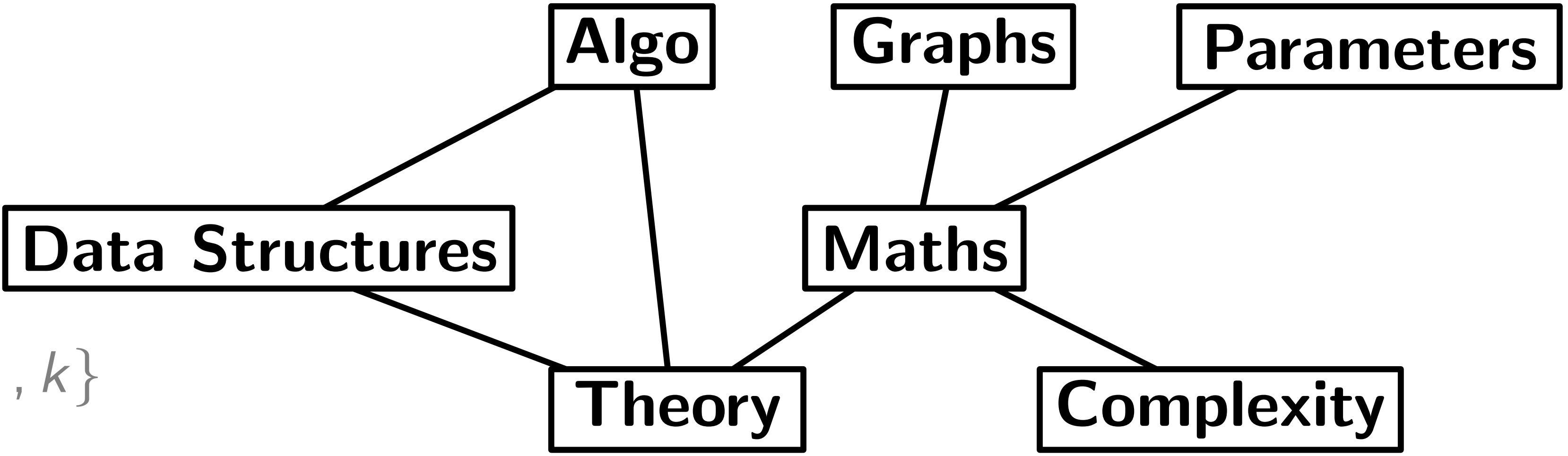

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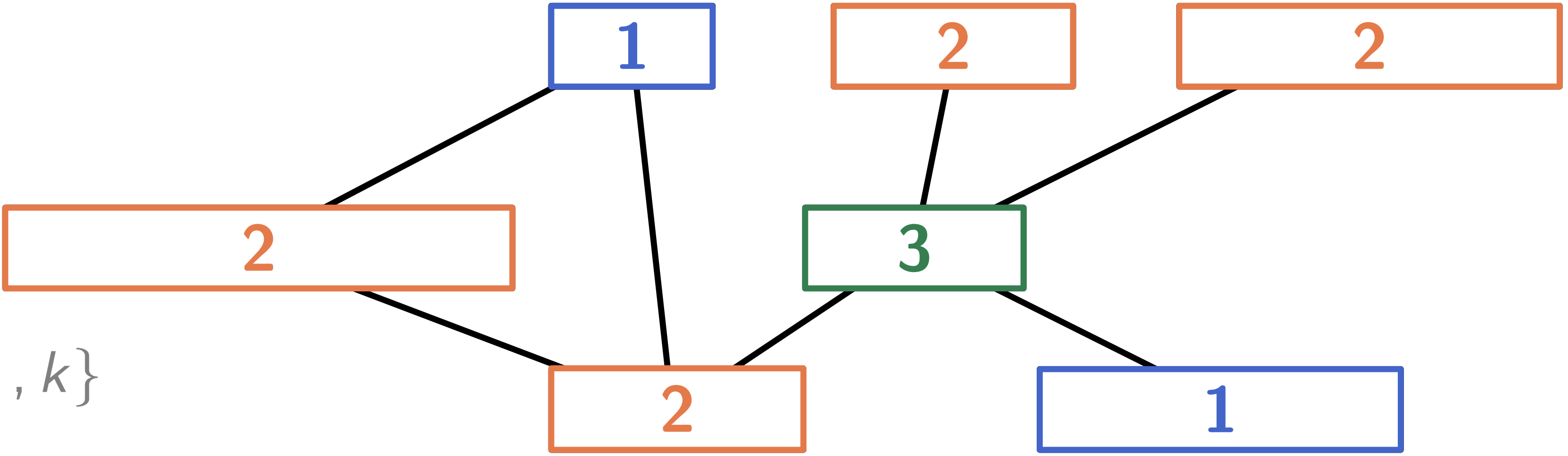

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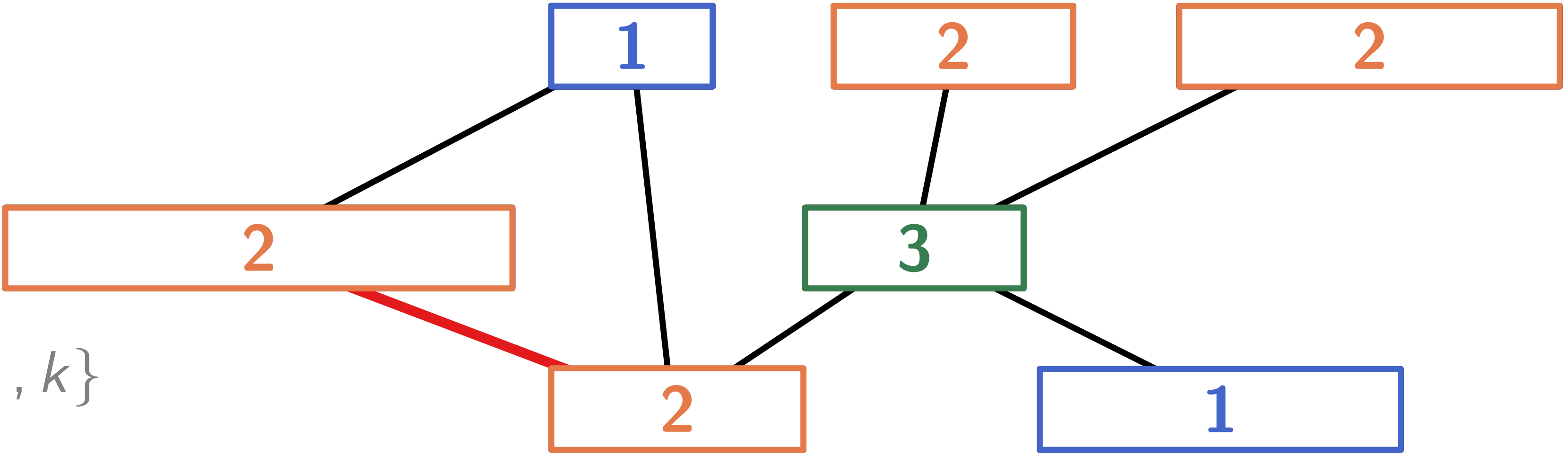

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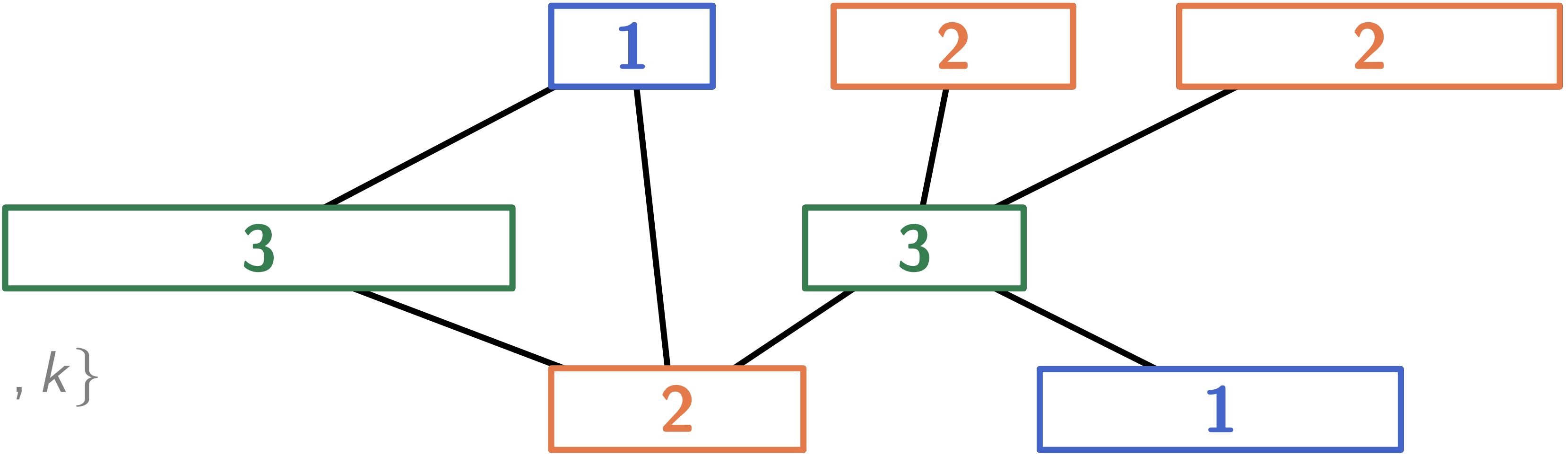

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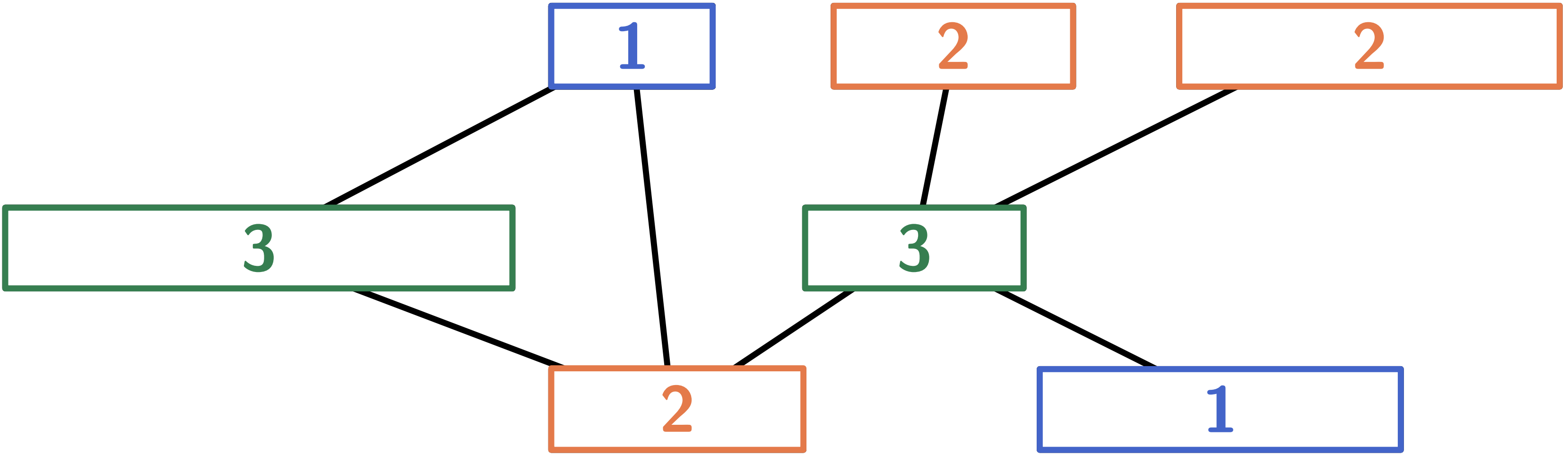

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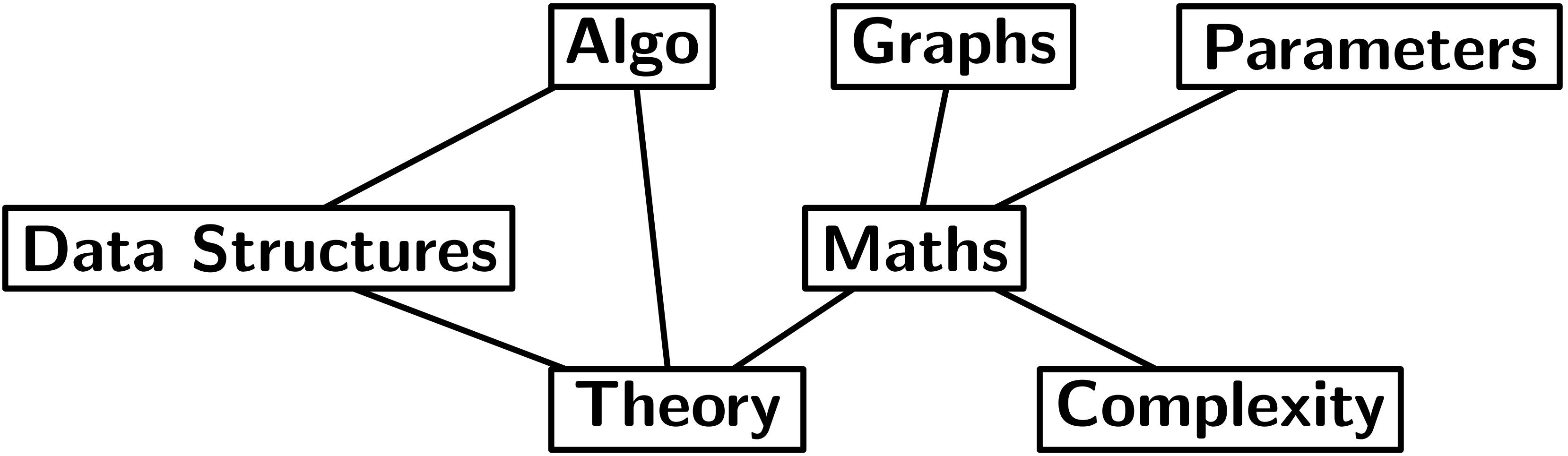

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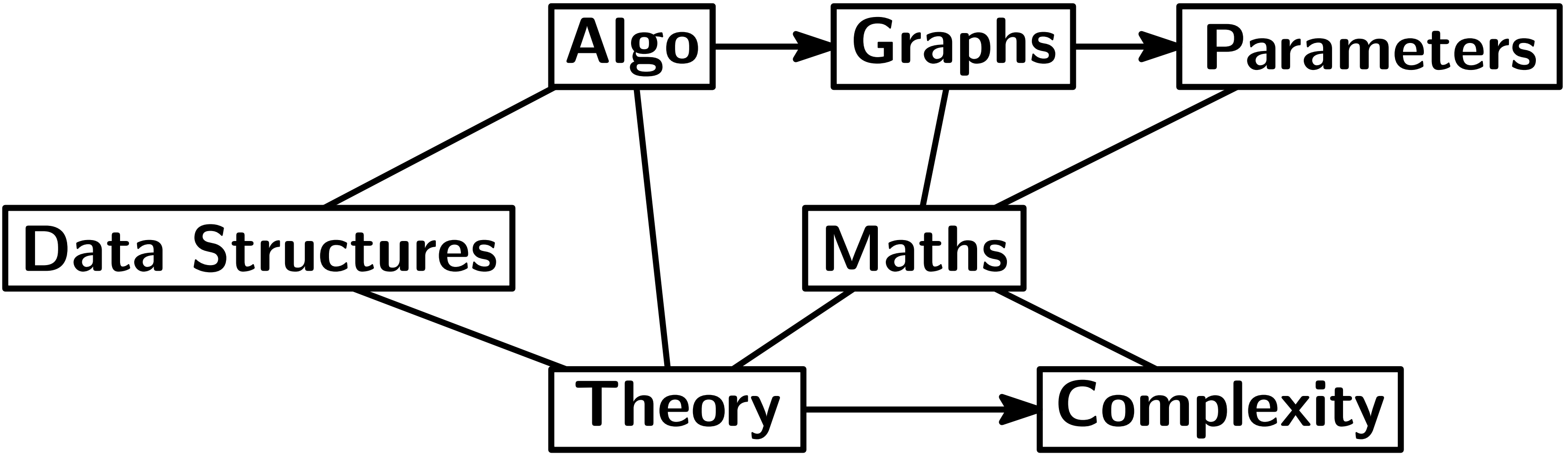

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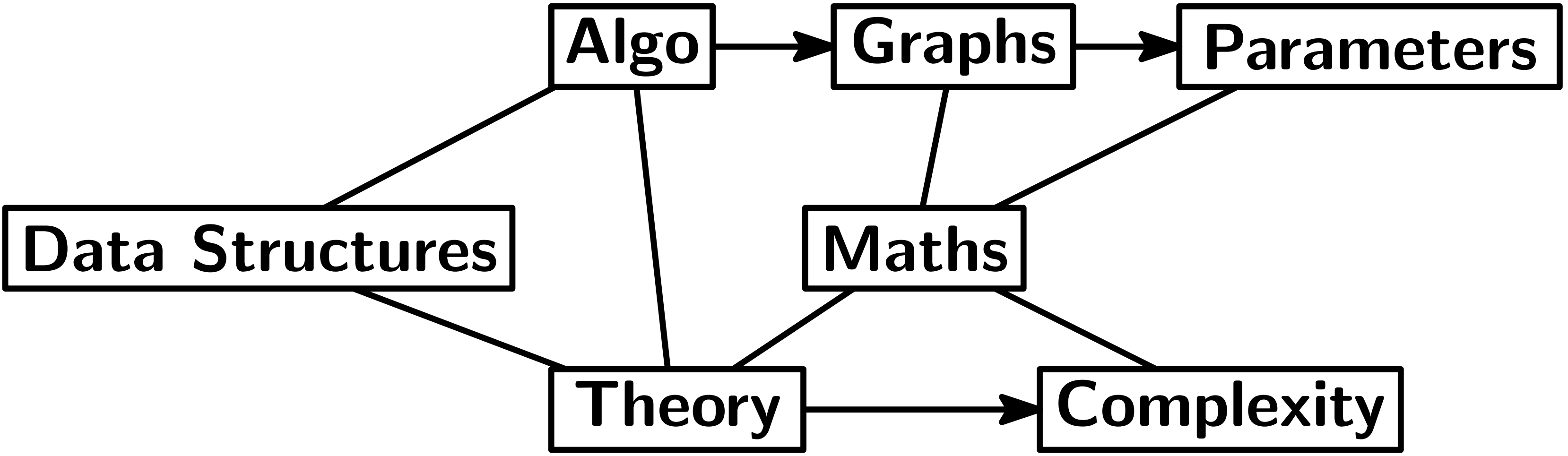

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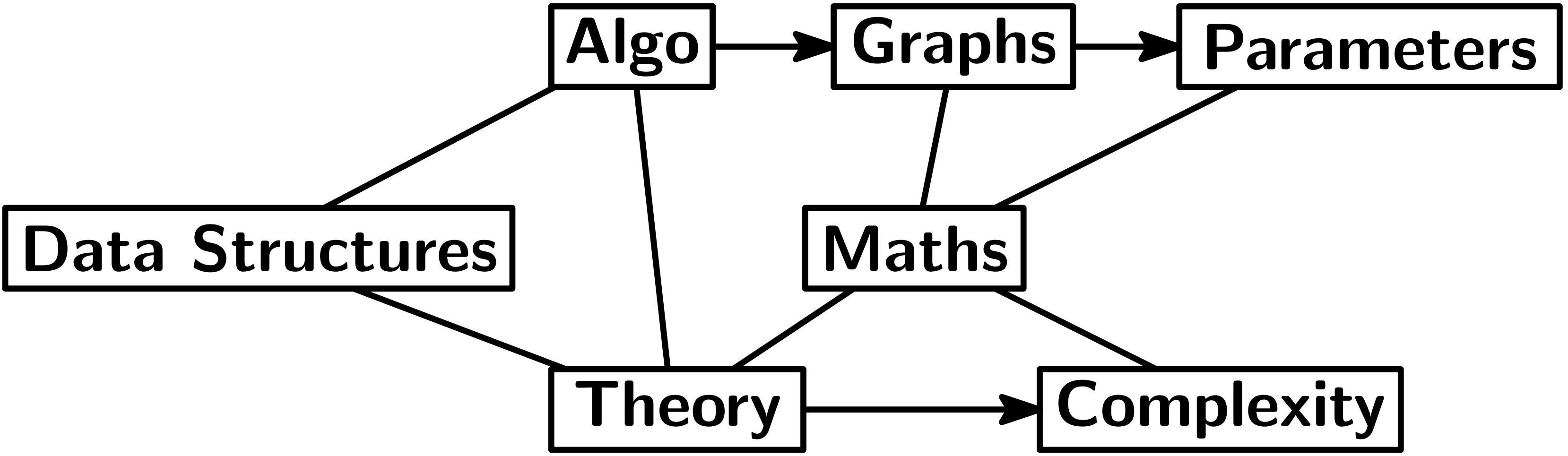

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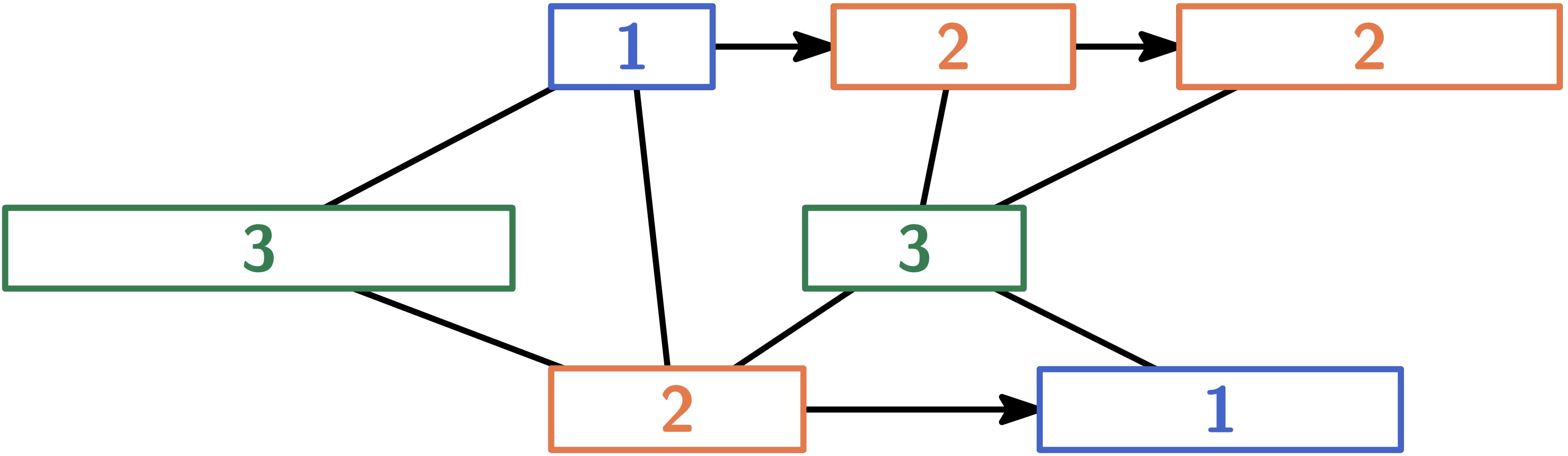

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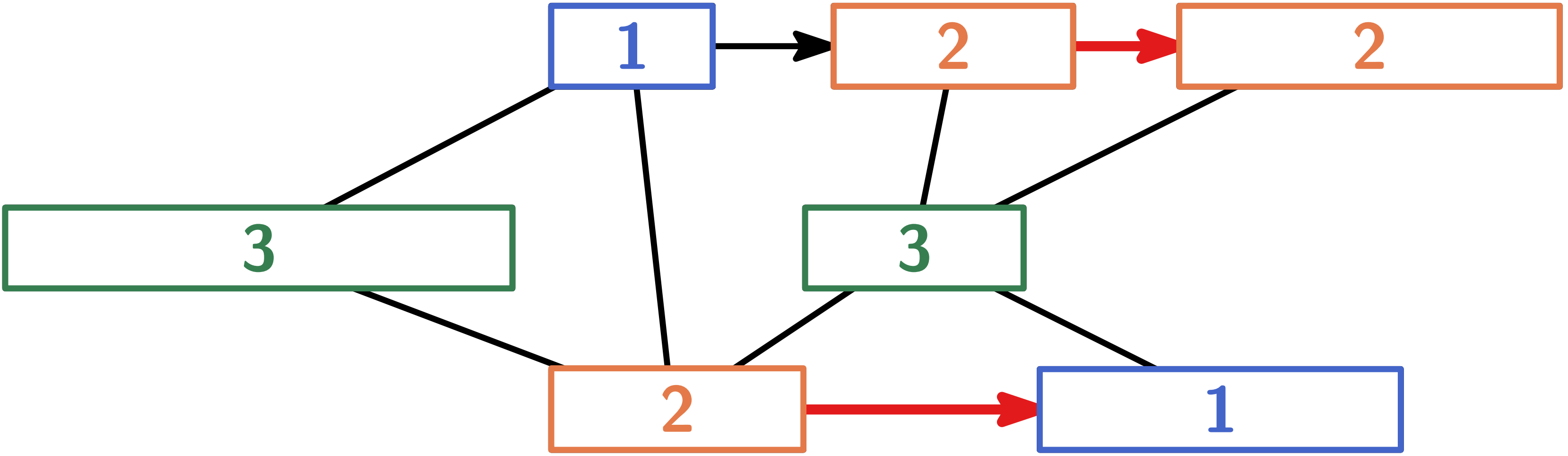

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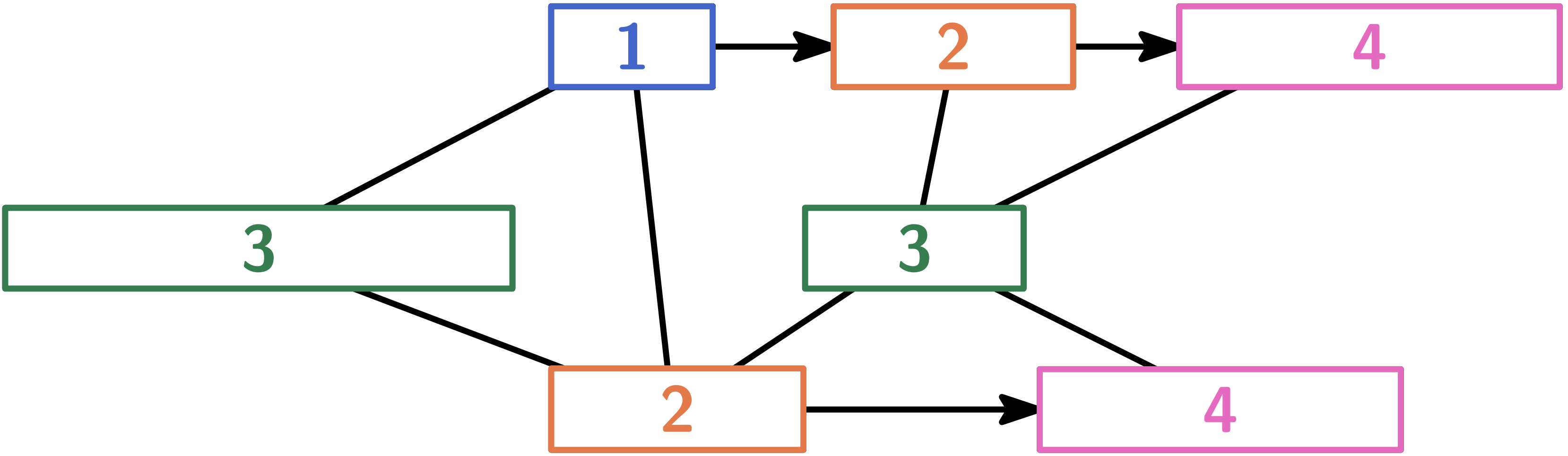

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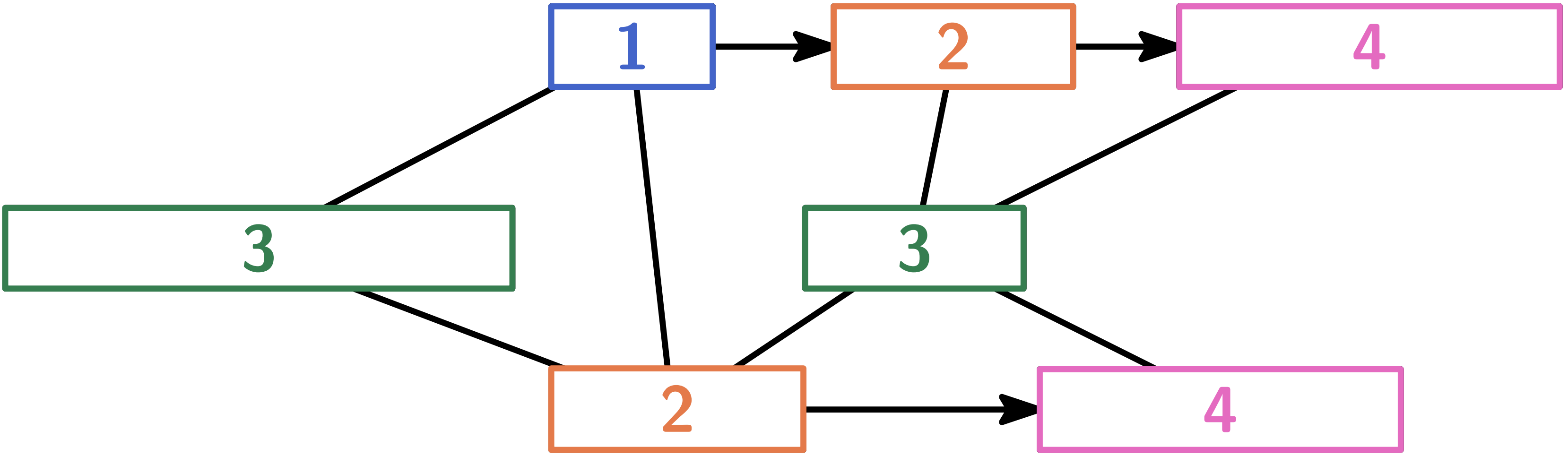

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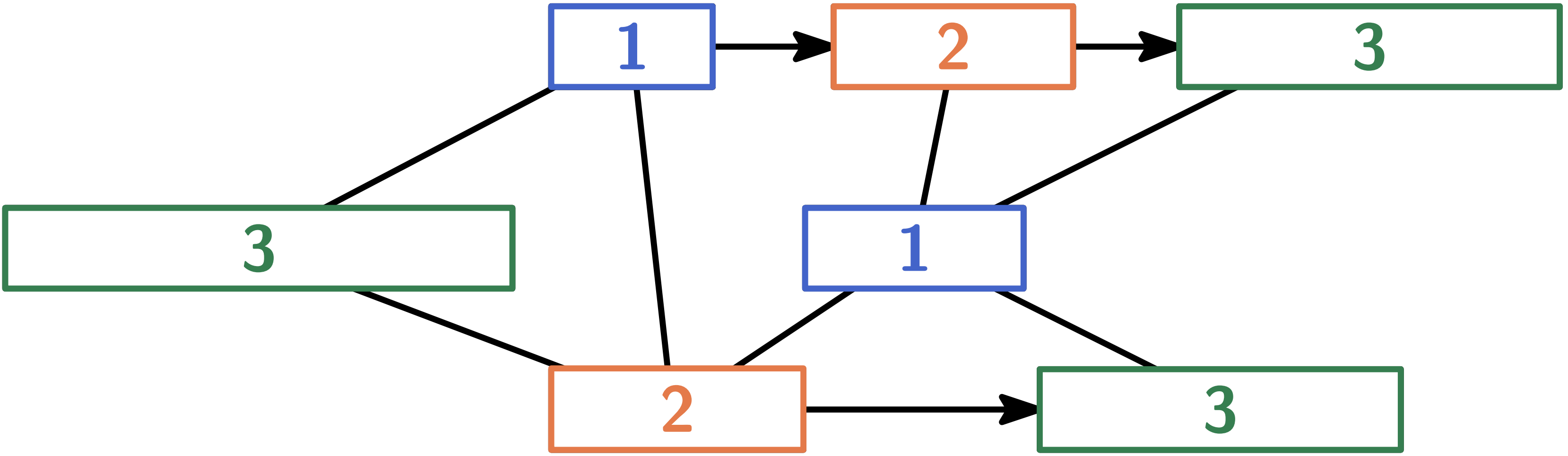

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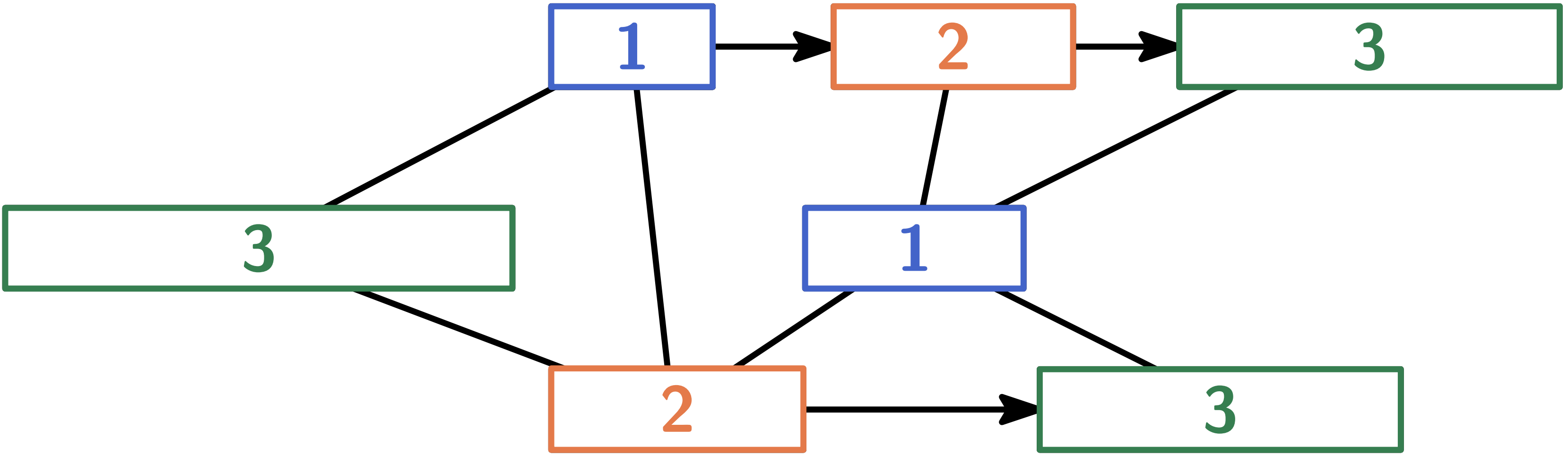

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MIXEDCOLORING

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Applications

scheduling, graph drawing

Related Work

MixedColoring

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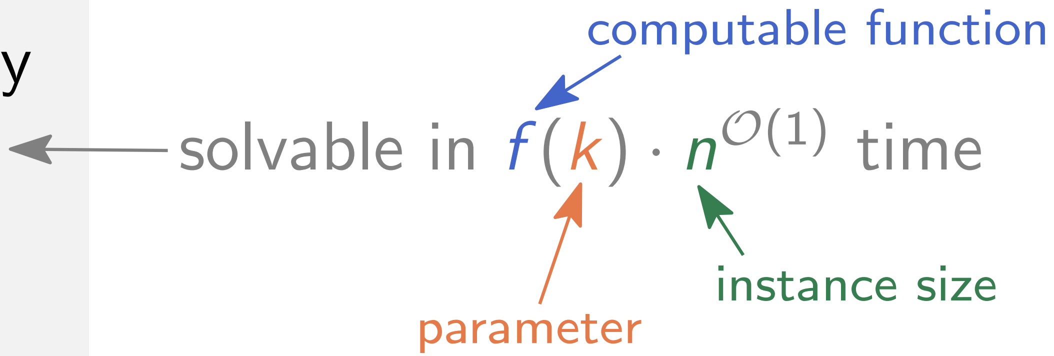

computable function

solvable in $f(k) \cdot n^{\mathcal{O}(1)}$ time

parameter

instance size

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treats arcs as edges

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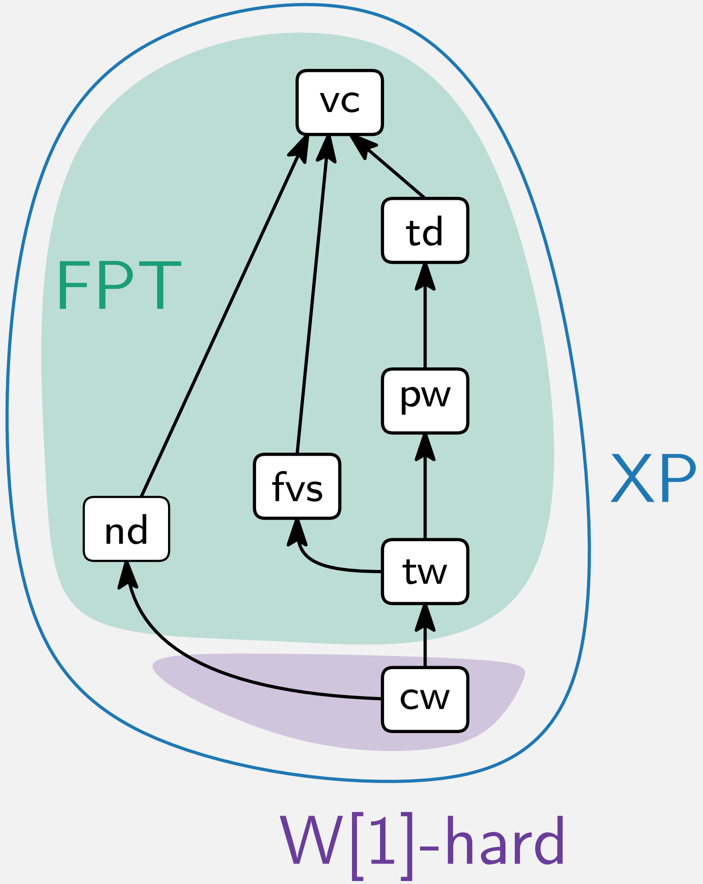

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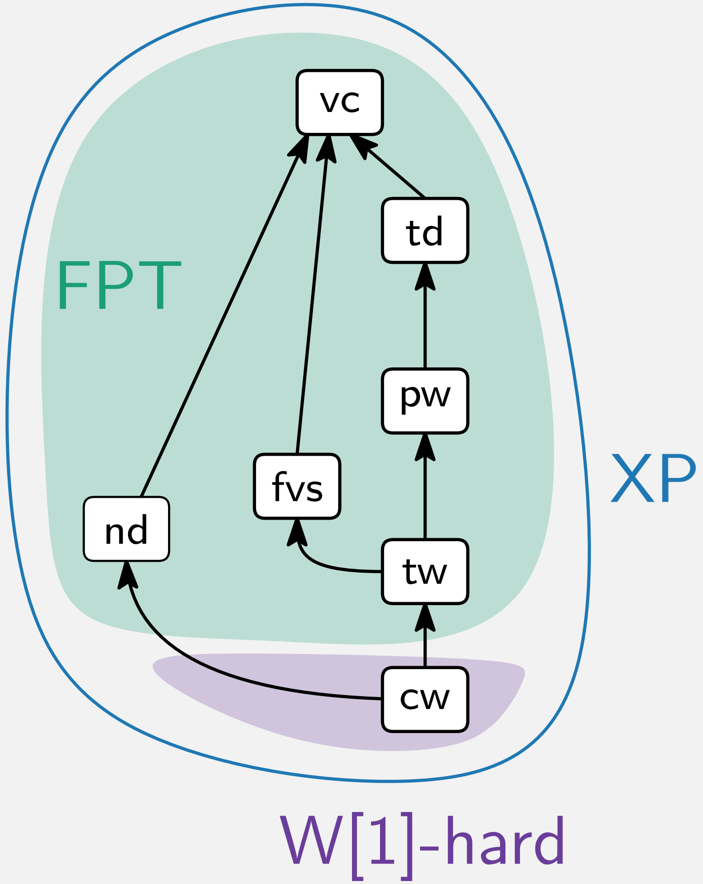

to FPT what NP-hardness is to P

Related Work

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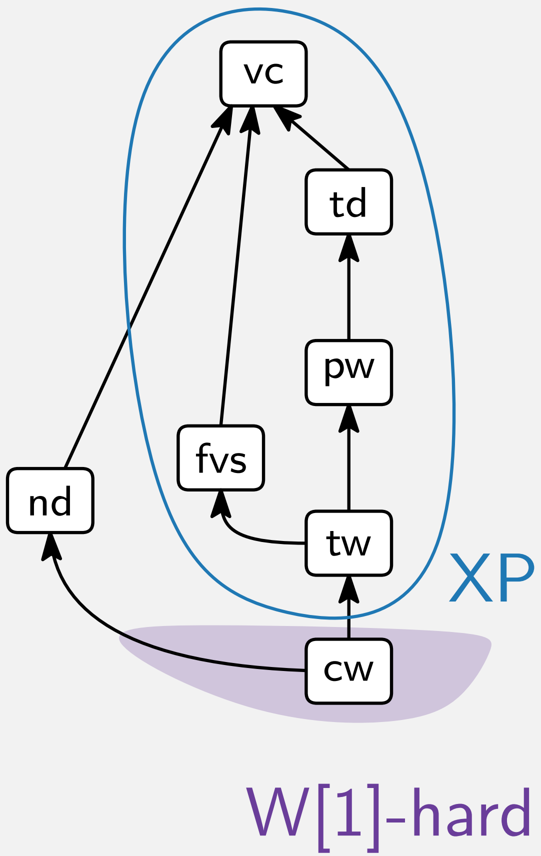

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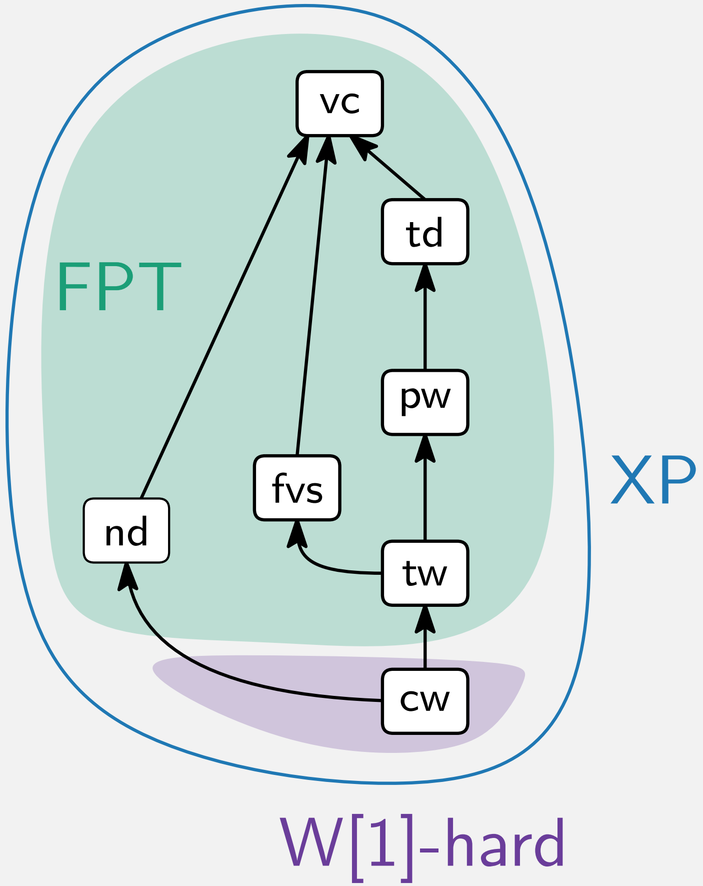

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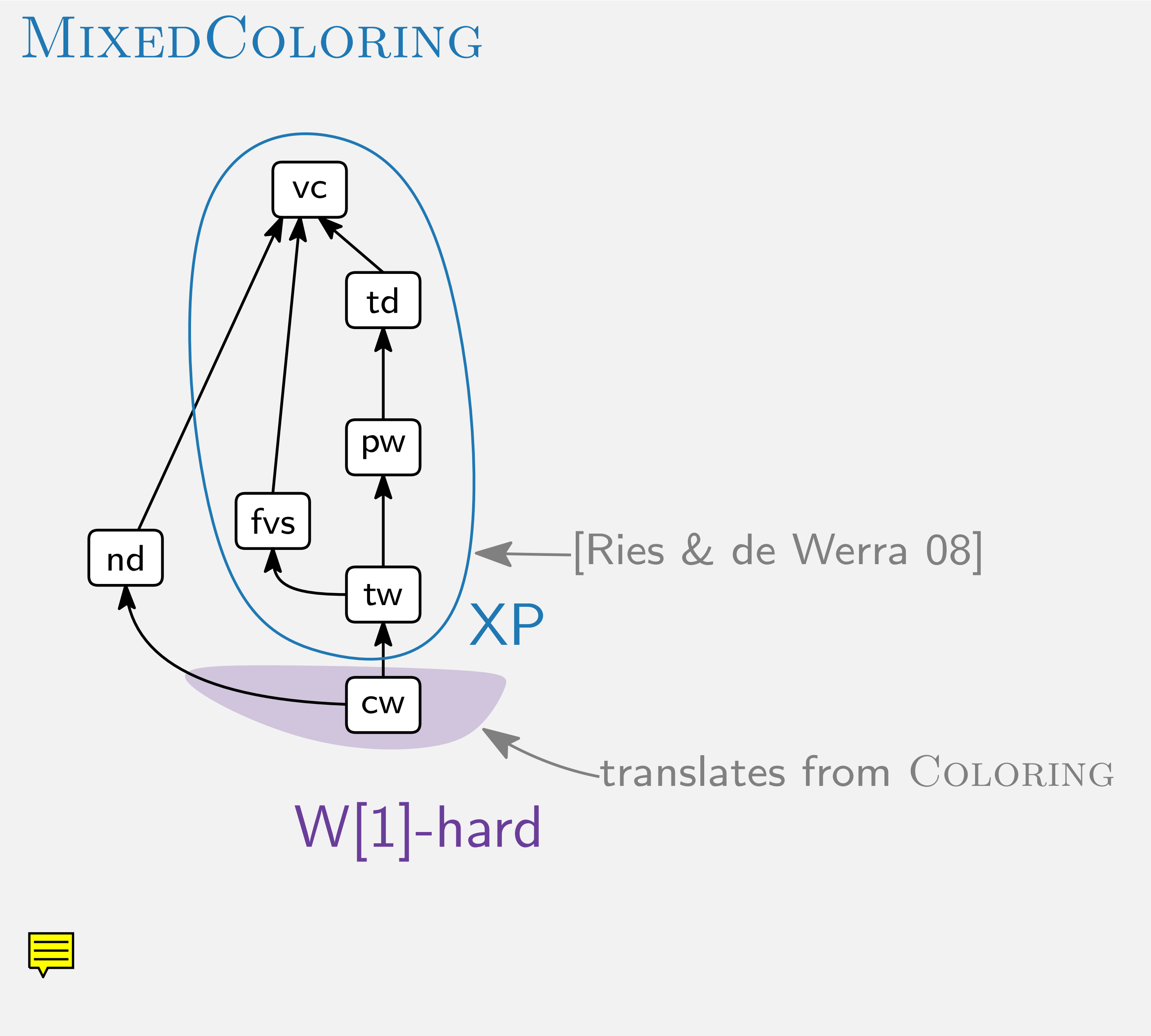
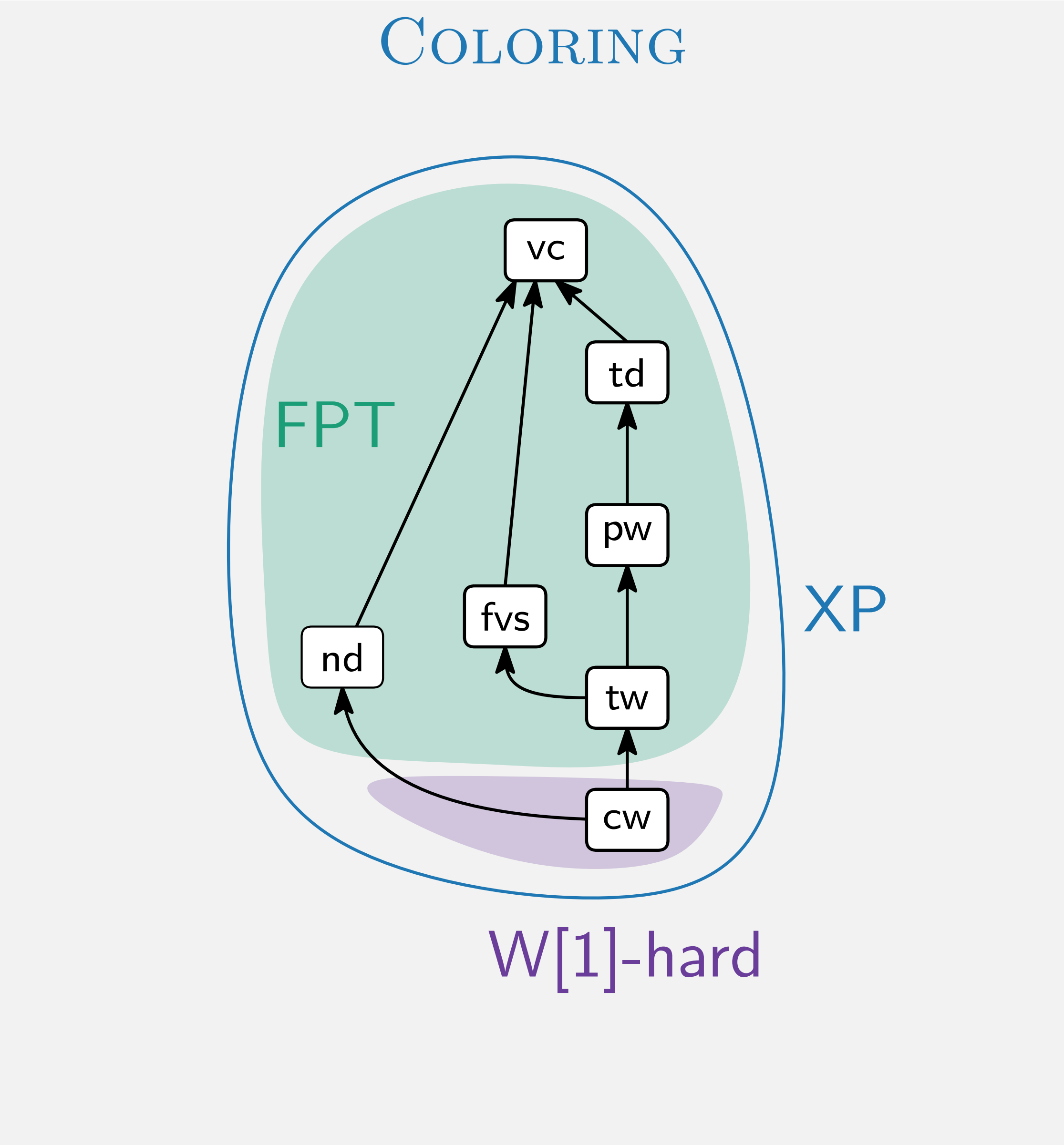
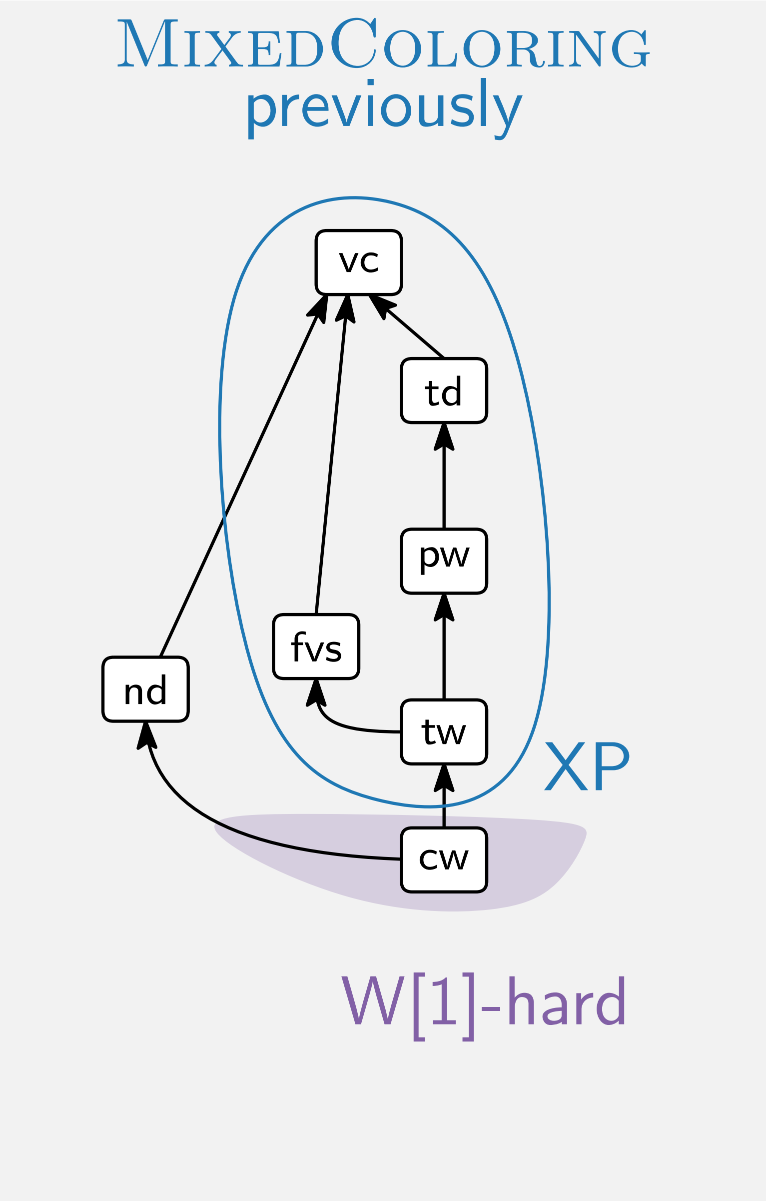




Parameterized Complexity



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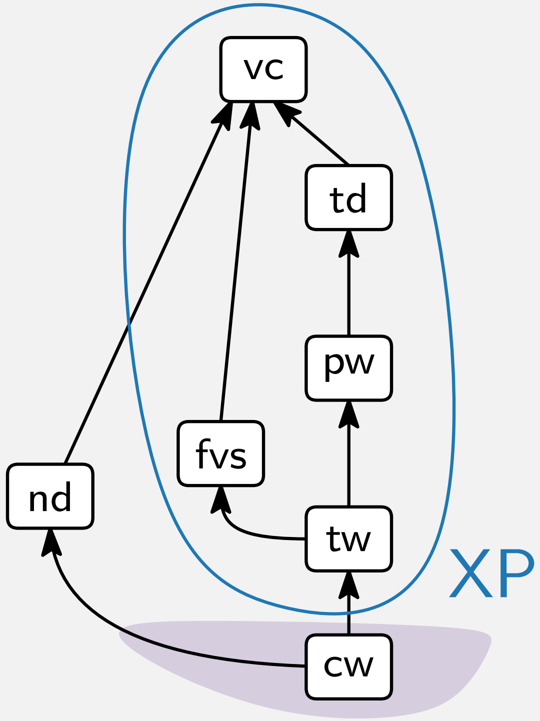
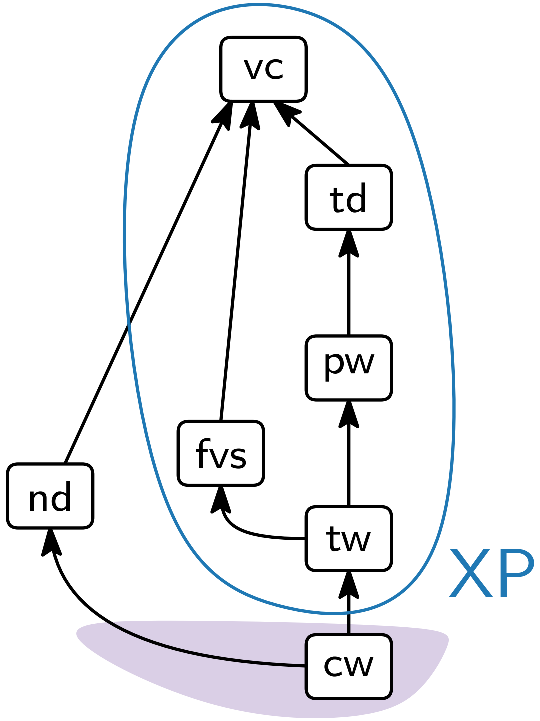
MixedColoring
previously



W[1]-hard

MixedColoring
our contribution



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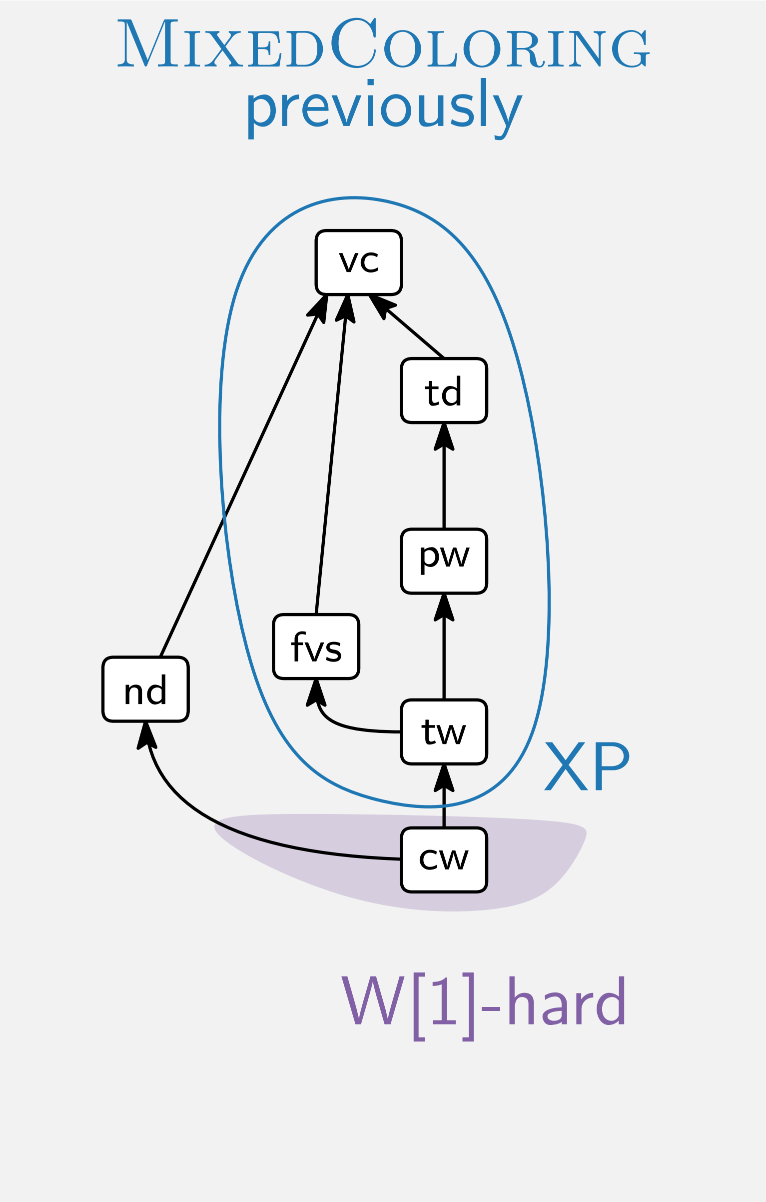
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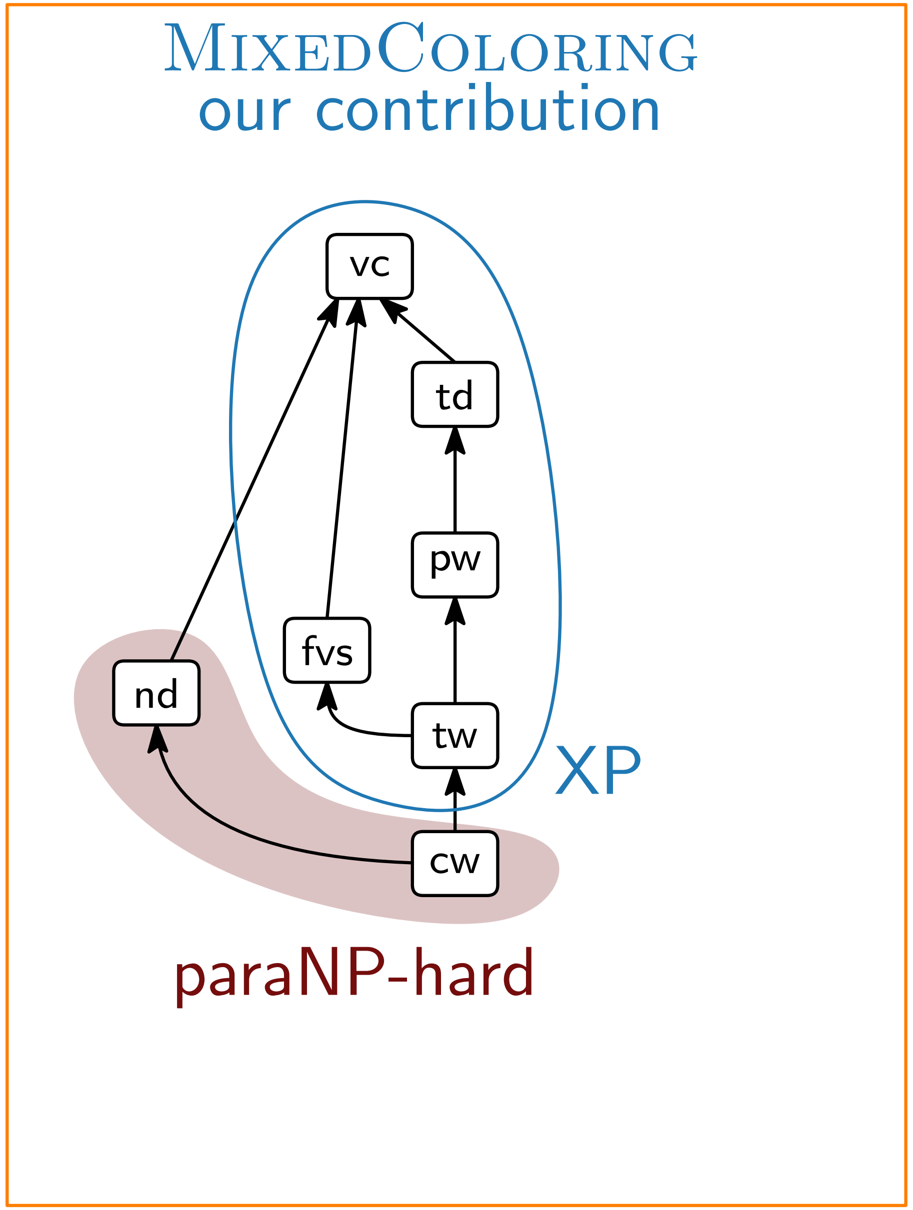
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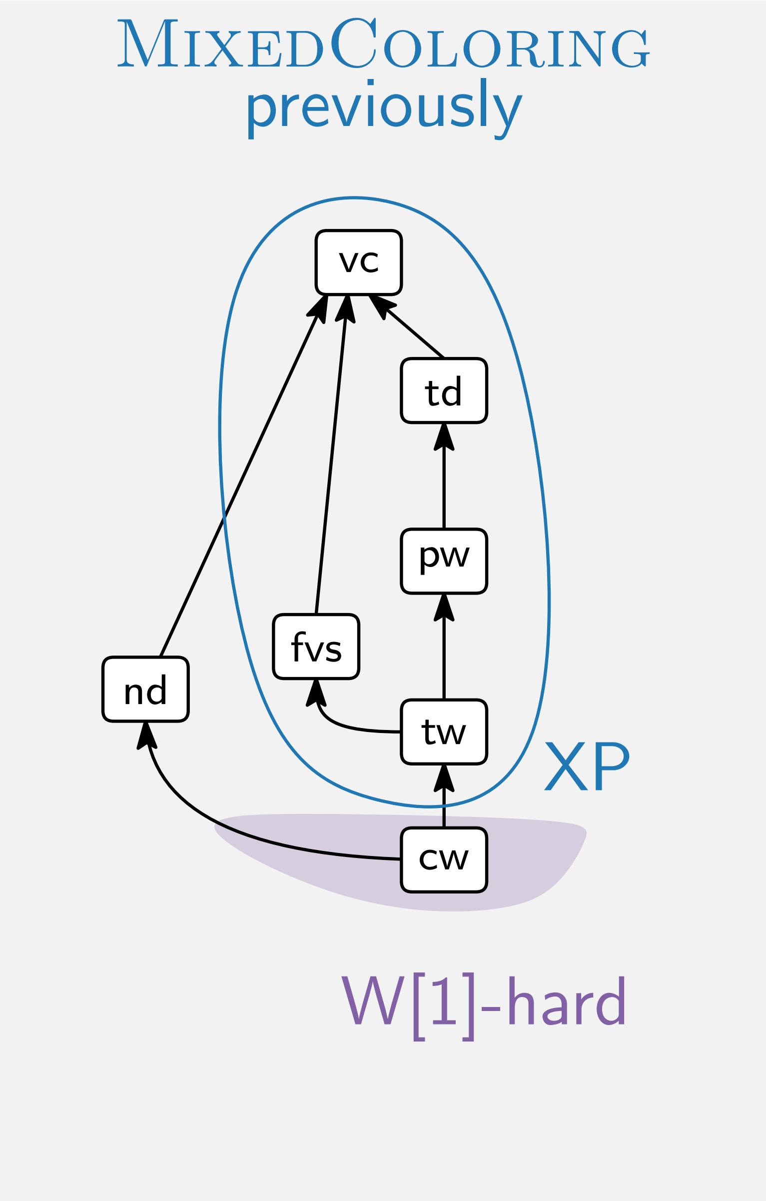
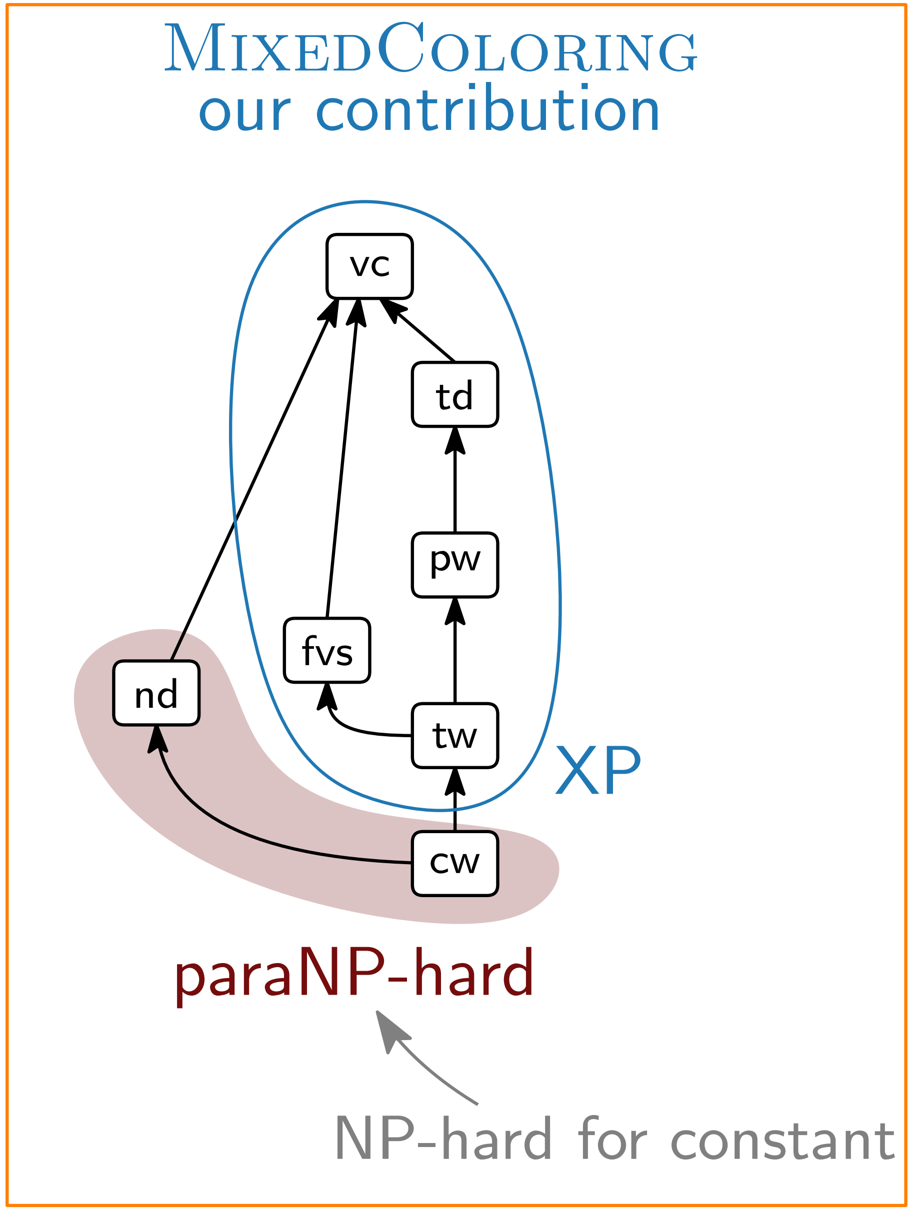
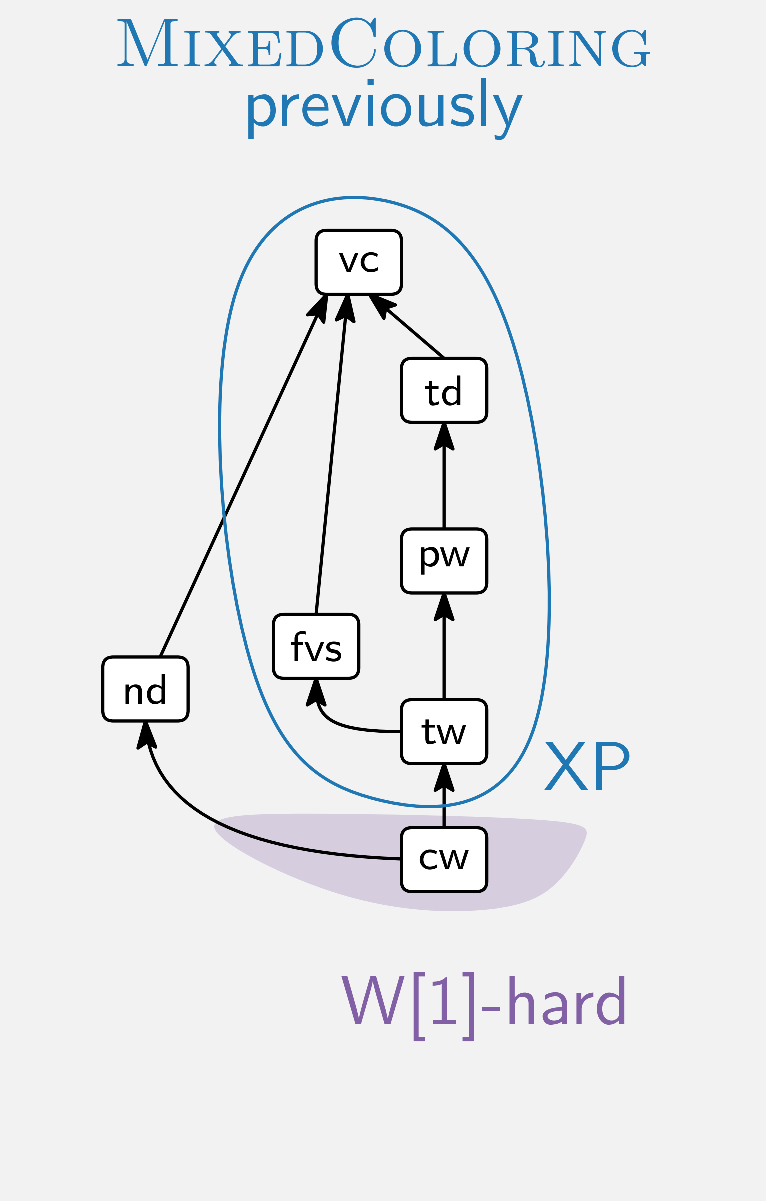
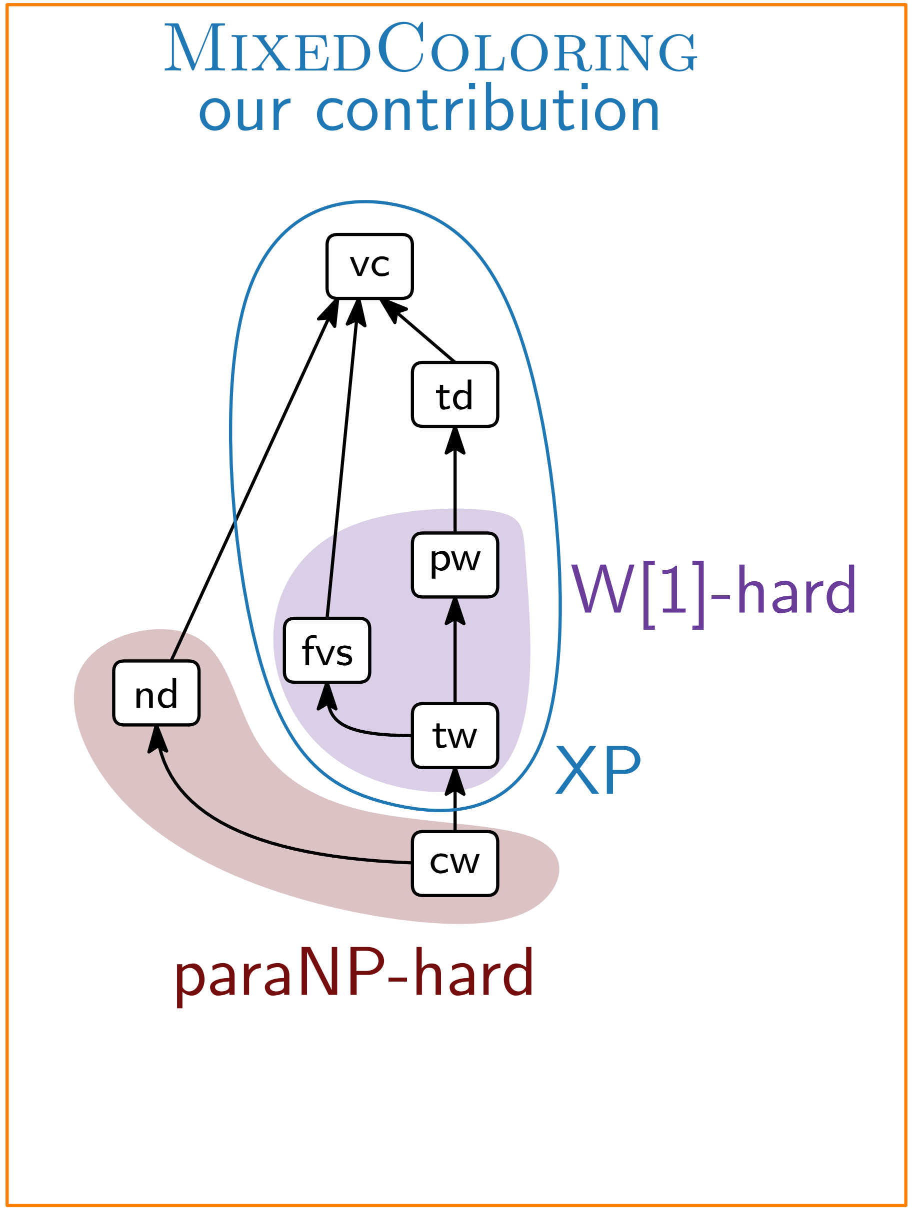




NP-hard for constant values of parameter

Parameterized Complexity





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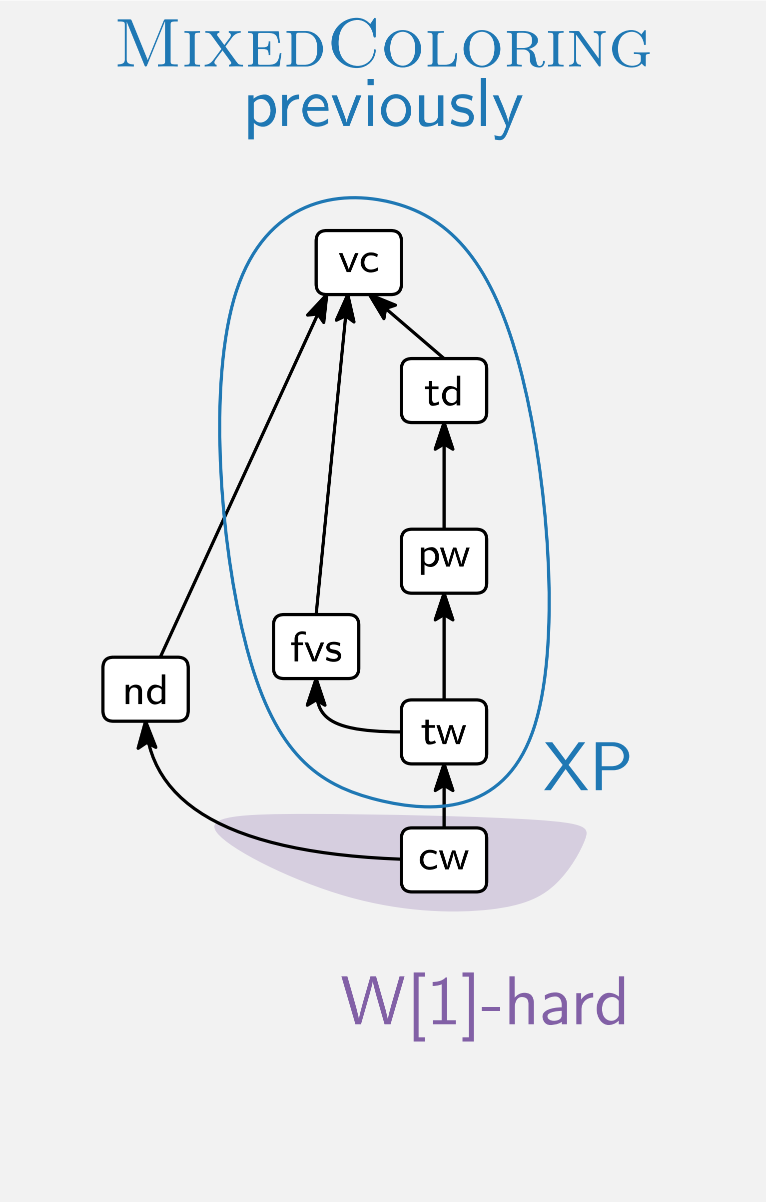

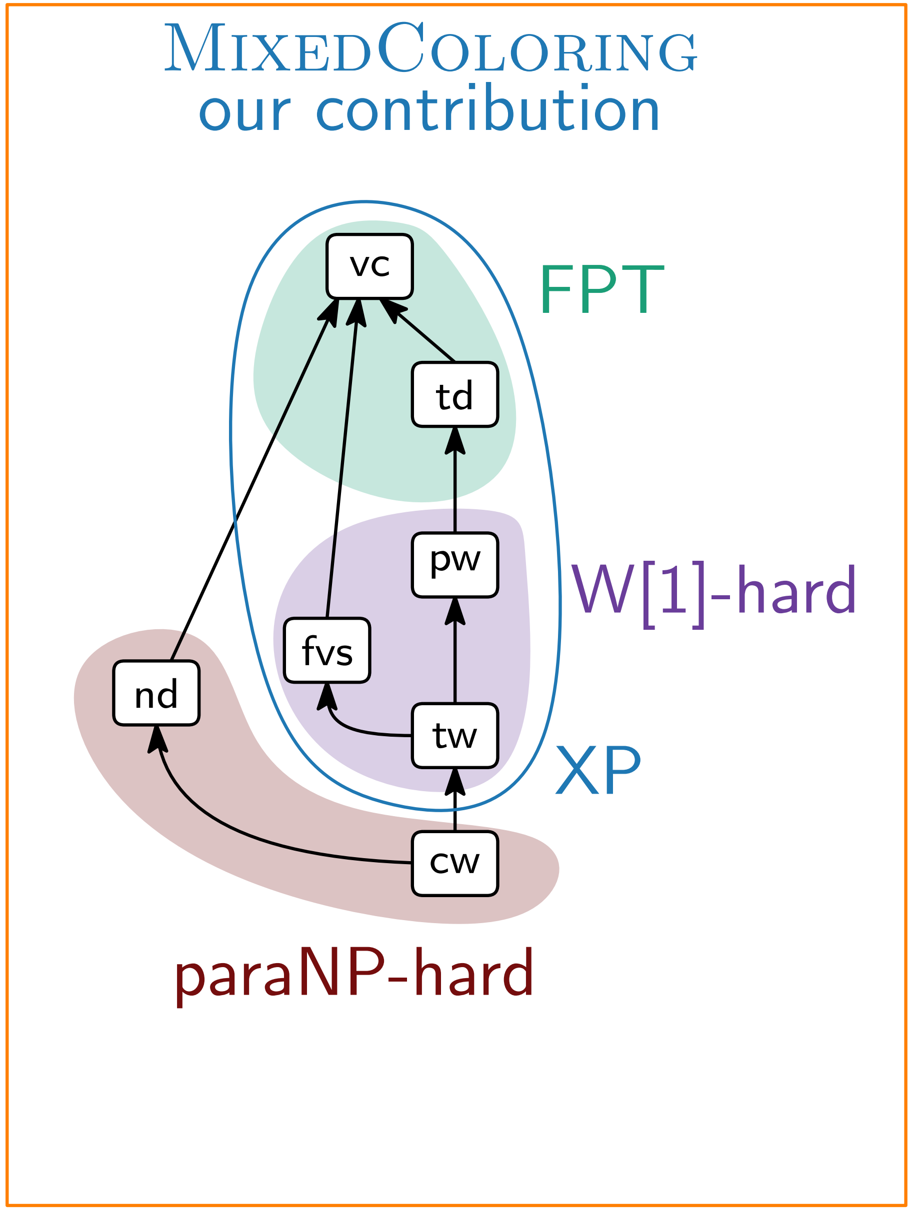

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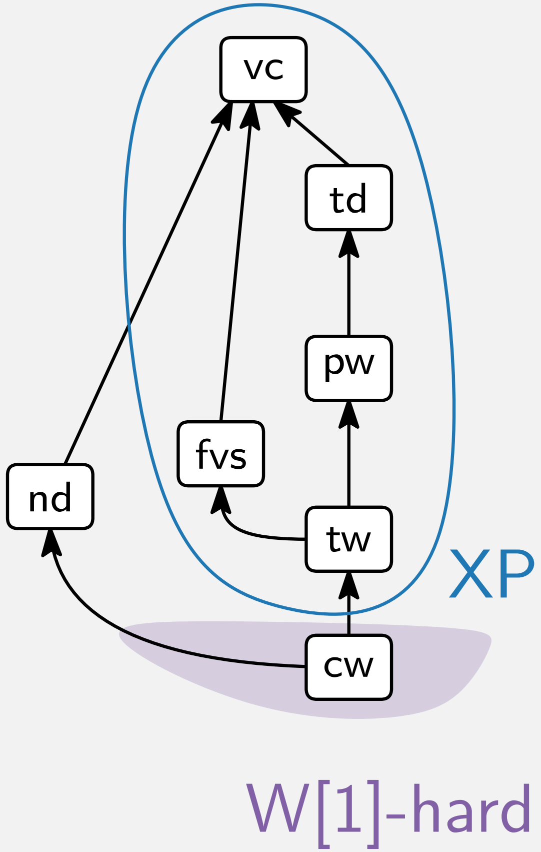

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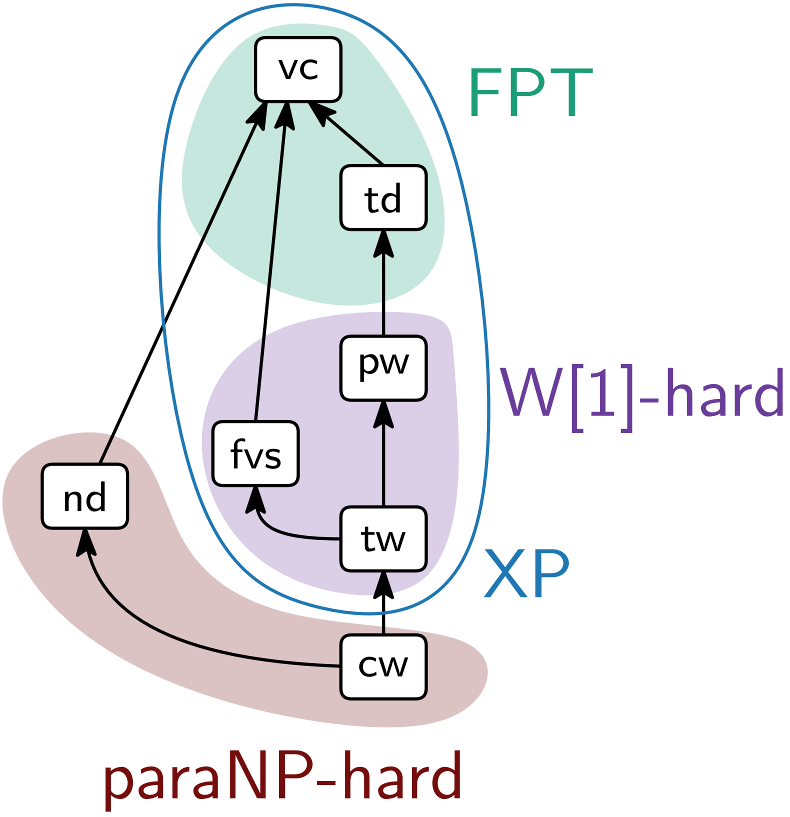

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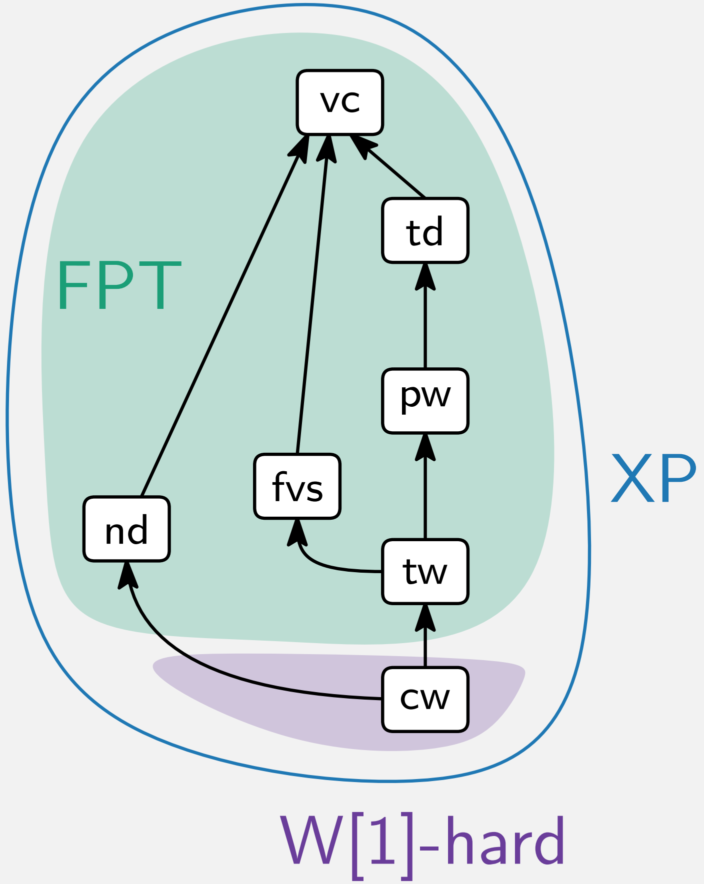

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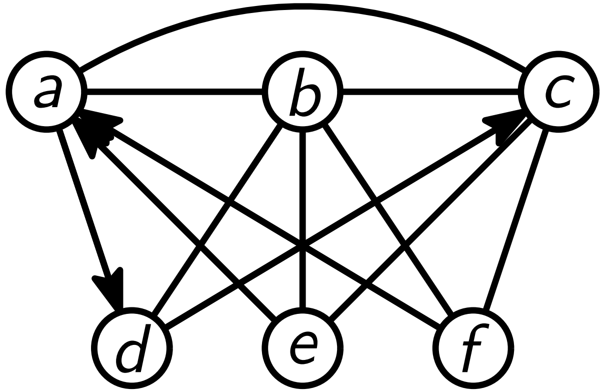

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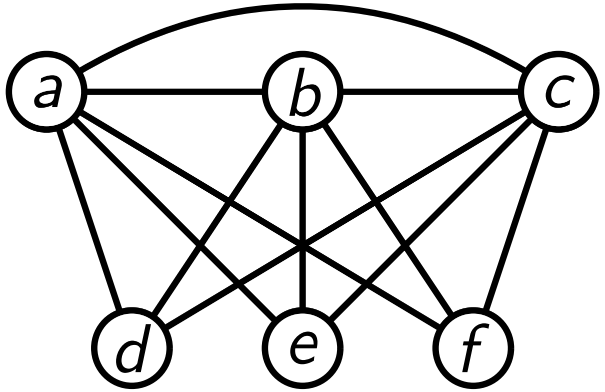

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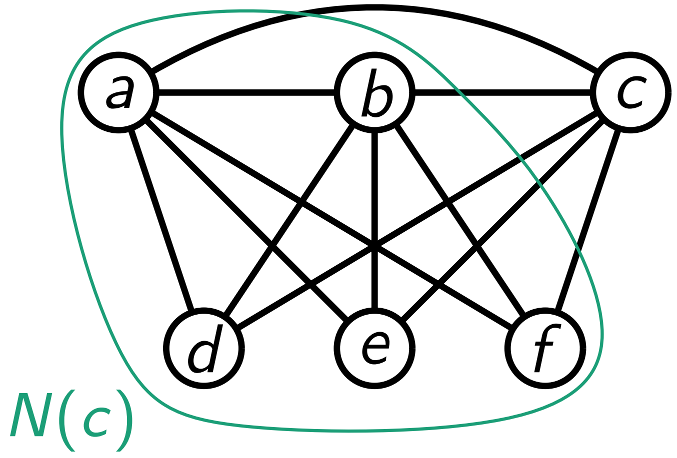

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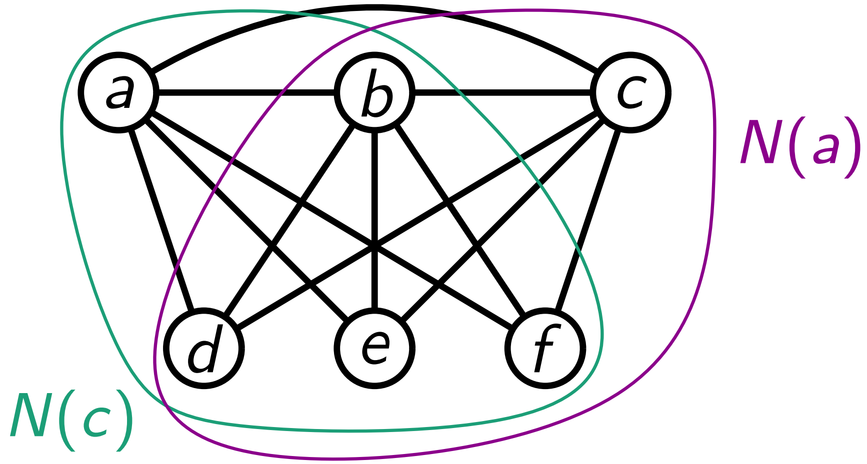

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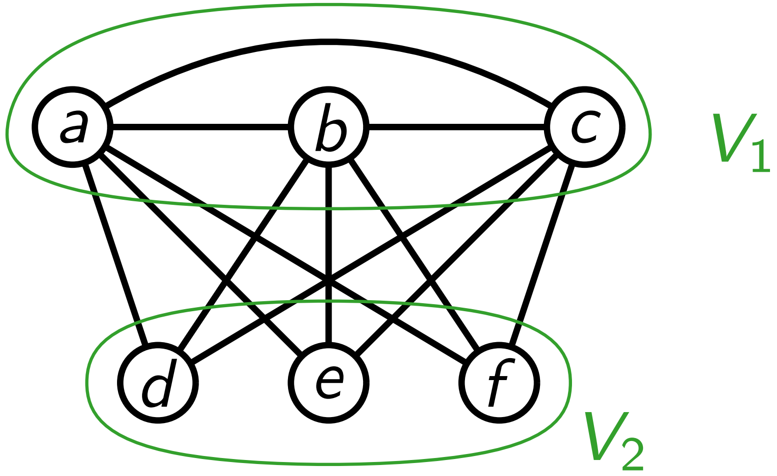

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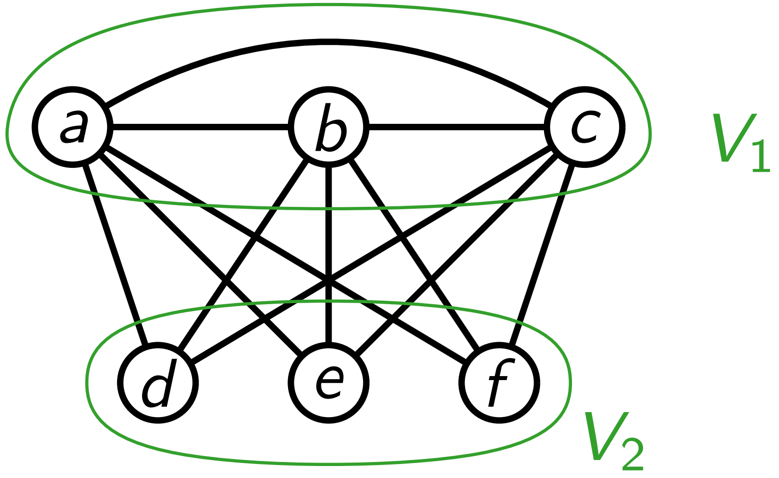
Observation



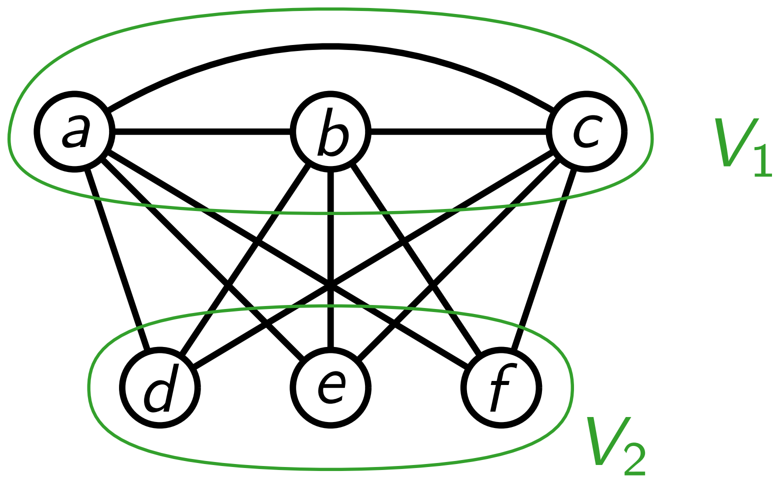
Mixed Parameters

Neighborhood Diversity

- counts types of vertices
- vertices u, v of same type iff $N(u) \setminus \{v\} = N(v) \setminus \{u\}$

Observation

- subgraphs induced by types form clique or independent set



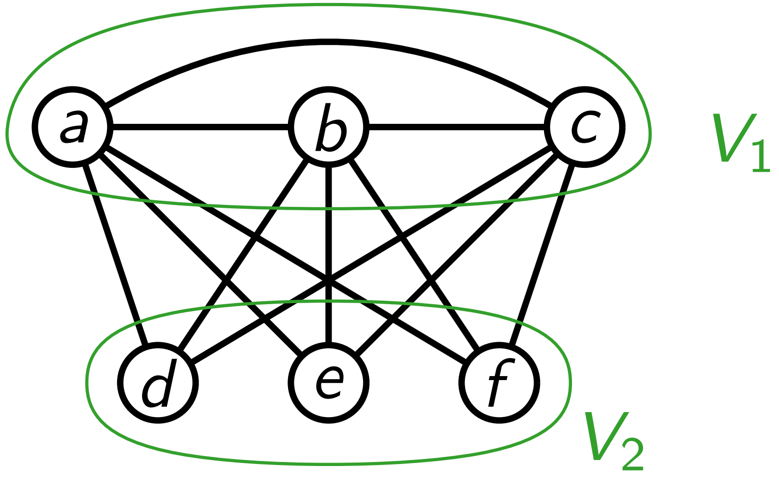
Mixed Parameters

Neighborhood Diversity

- counts types of vertices
- vertices u, v of same type iff $N(u) \setminus \{v\} = N(v) \setminus \{u\}$

Observation

- subgraphs induced by types form clique or independent set
- vertices in type inducing an independent set can receive same color



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Neighborhood Diversity

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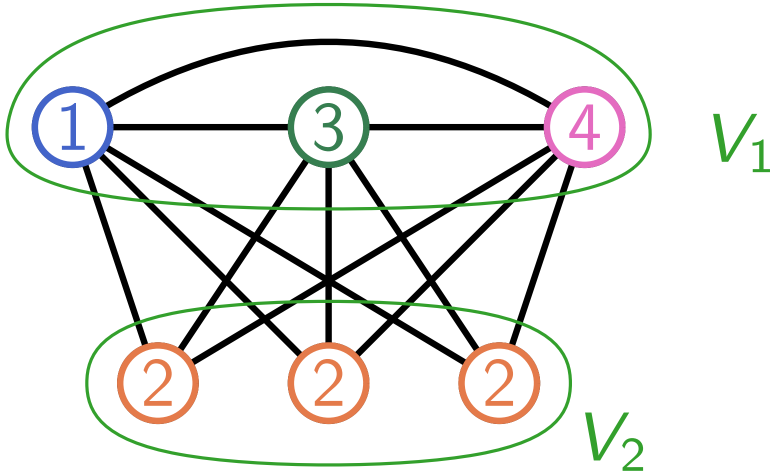

Mixed Parameters

Neighborhood Diversity

- counts types of vertices
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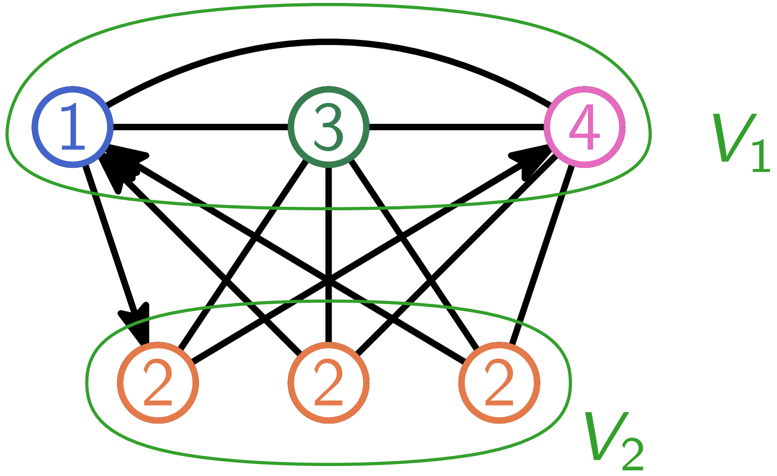
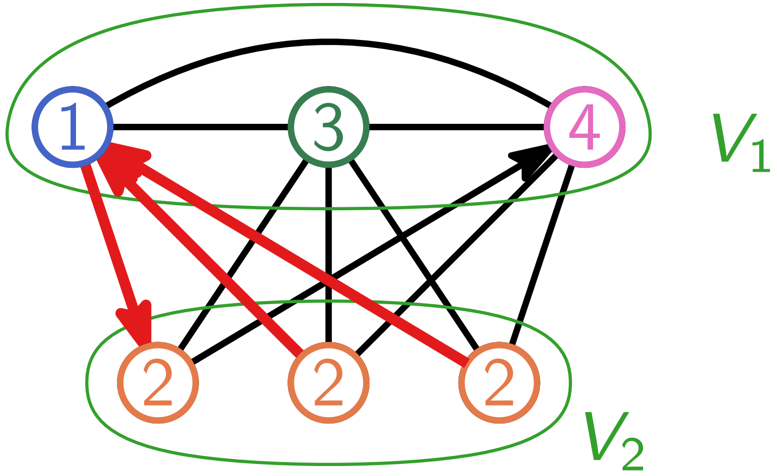


Mixed Parameters



Neighborhood Diversity

- counts types of vertices
- vertices u, v of same type iff $N(u) \setminus \{v\} = N(v) \setminus \{u\}$

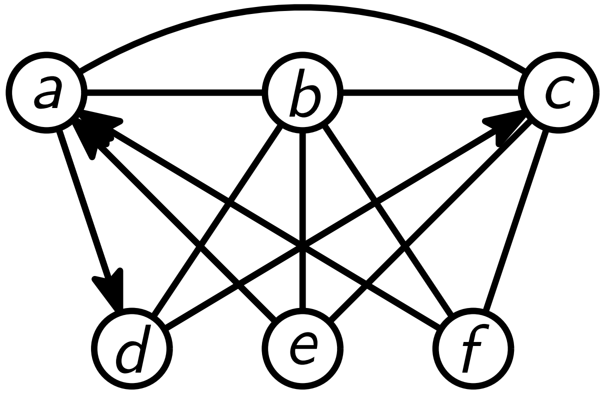
Observation

- subgraphs induced by types form clique or independent set
- vertices in type inducing an independent set can receive same color

doesn't work with arcs

Mixed Parameters

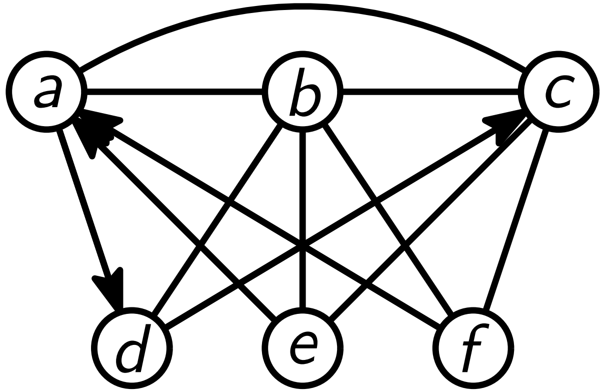
Mixed Neighborhood Diversity



Mixed Parameters

Mixed Neighborhood Diversity

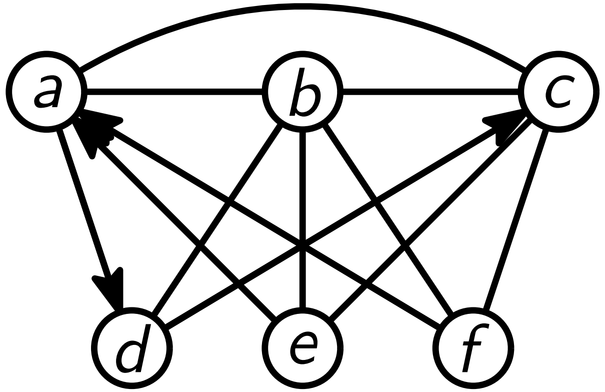
- counts types of vertices



Mixed Parameters

Mixed Neighborhood Diversity

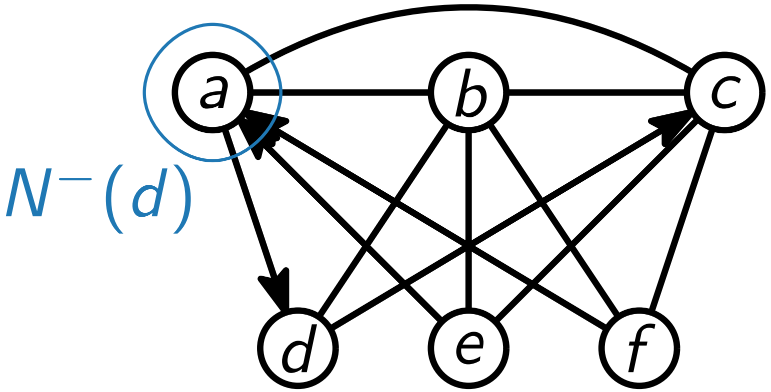
- counts types of vertices
- vertices u, v of same type iff



Mixed Parameters

Mixed Neighborhood Diversity

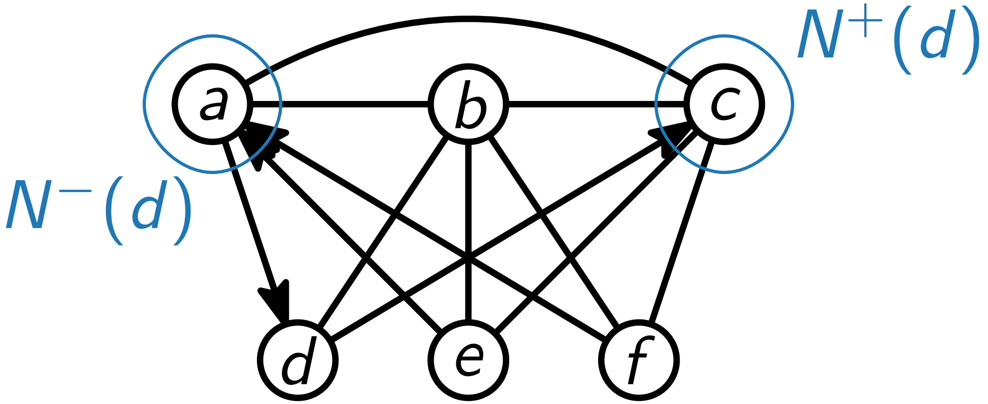
- counts types of vertices
- vertices u, v of same type iff
  - $N^{-}(u) = N^{-}(v)$



Mixed Parameters

Mixed Neighborhood Diversity

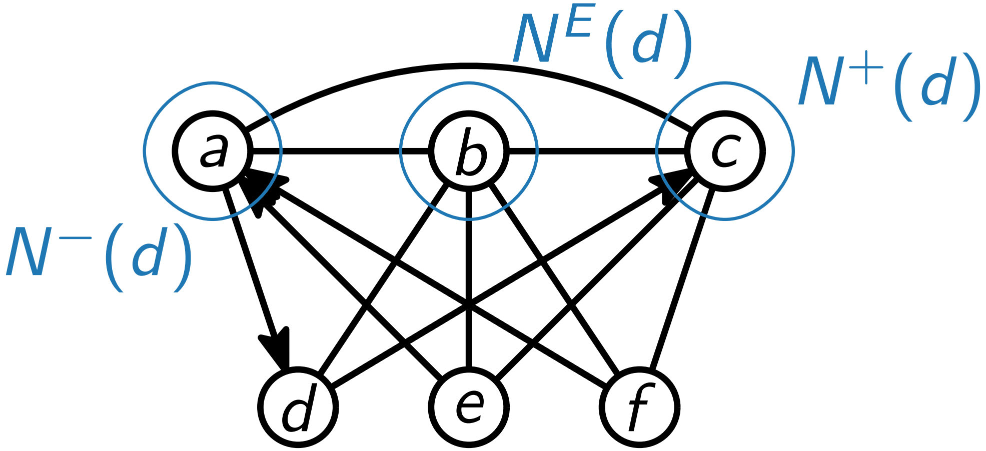
- counts types of vertices
- vertices u, v of same type iff
  - $N^-(u) = N^-(v)$
  - $N^+(u) = N^+(v)$



Mixed Parameters

Mixed Neighborhood Diversity

- counts types of vertices
- vertices u, v of same type iff
  - $N^-(u) = N^-(v)$
  - $N^+(u) = N^+(v)$
  - $N^E(u) \setminus \{v\} = N^E(v) \setminus \{u\}$



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Mixed Neighborhood Diversity

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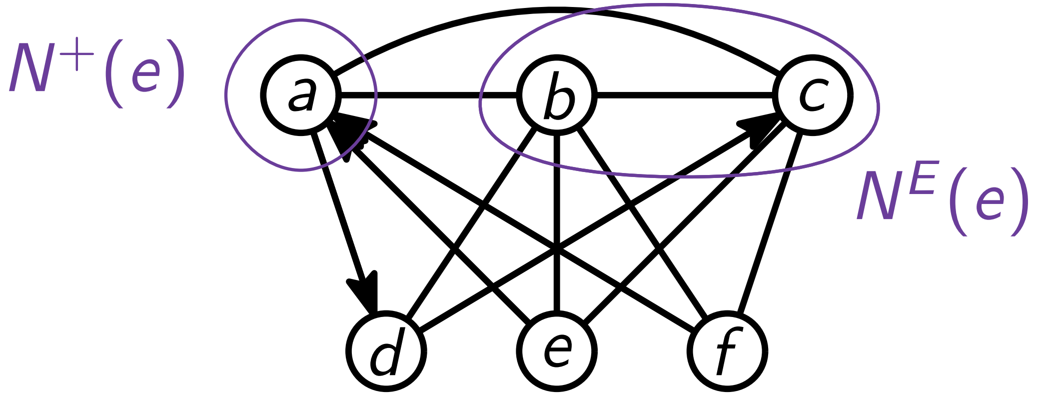

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Mixed Neighborhood Diversity

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  - $N^+(u) = N^+(v)$
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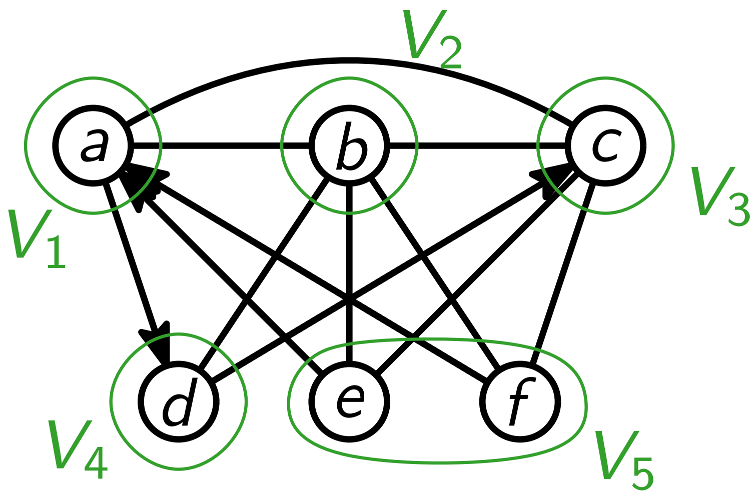

Mixed Parameters

Mixed Neighborhood Diversity

- counts types of vertices
- vertices u, v of same type iff
  - $N^-(u) = N^-(v)$
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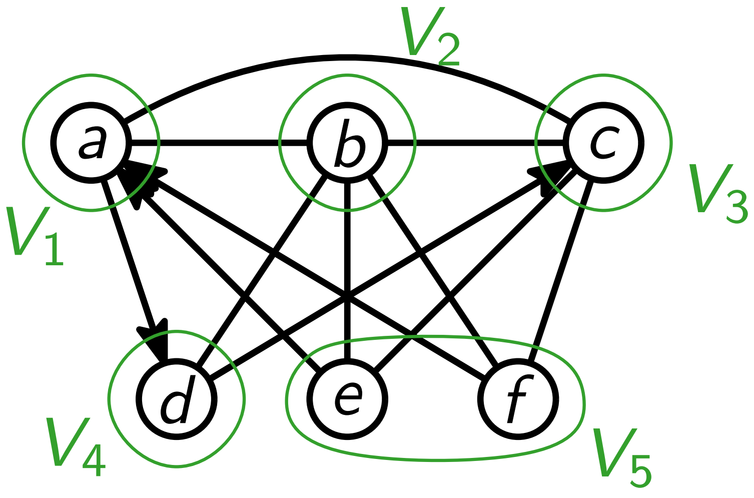
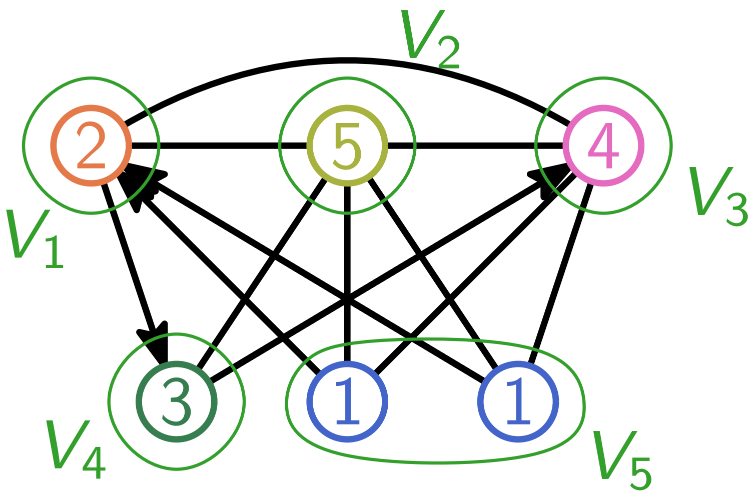
  - $N^E(u) \setminus \{v\} = N^E(v) \setminus \{u\}$

Observation

- subgraphs induced by types form clique or independent set
- vertices in type inducing IS can receive same color



Mixed Parameters



Mixed Neighborhood Diversity

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- vertices u, v of same type iff
  - $N^-(u) = N^-(v)$
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Observation

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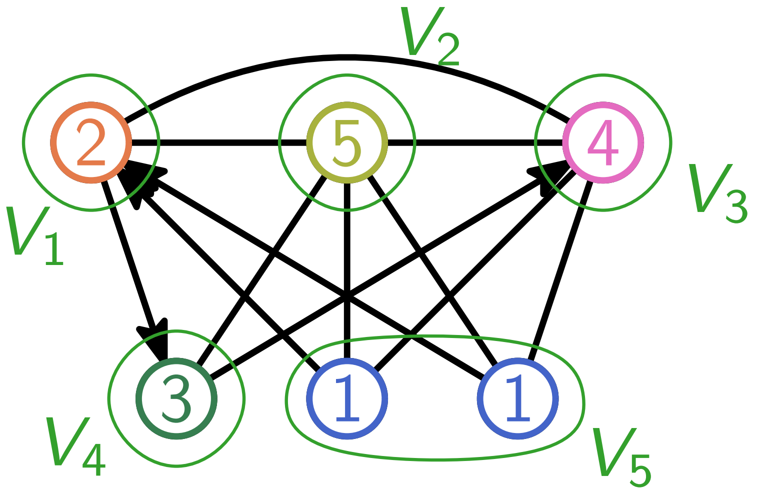
Mixed Parameters



Mixed Neighborhood Diversity

- counts types of vertices
- vertices u, v of same type iff
  - $N^-(u) = N^-(v)$
  - $N^+(u) = N^+(v)$
  - $N^E(u) \setminus \{v\} = N^E(v) \setminus \{u\}$

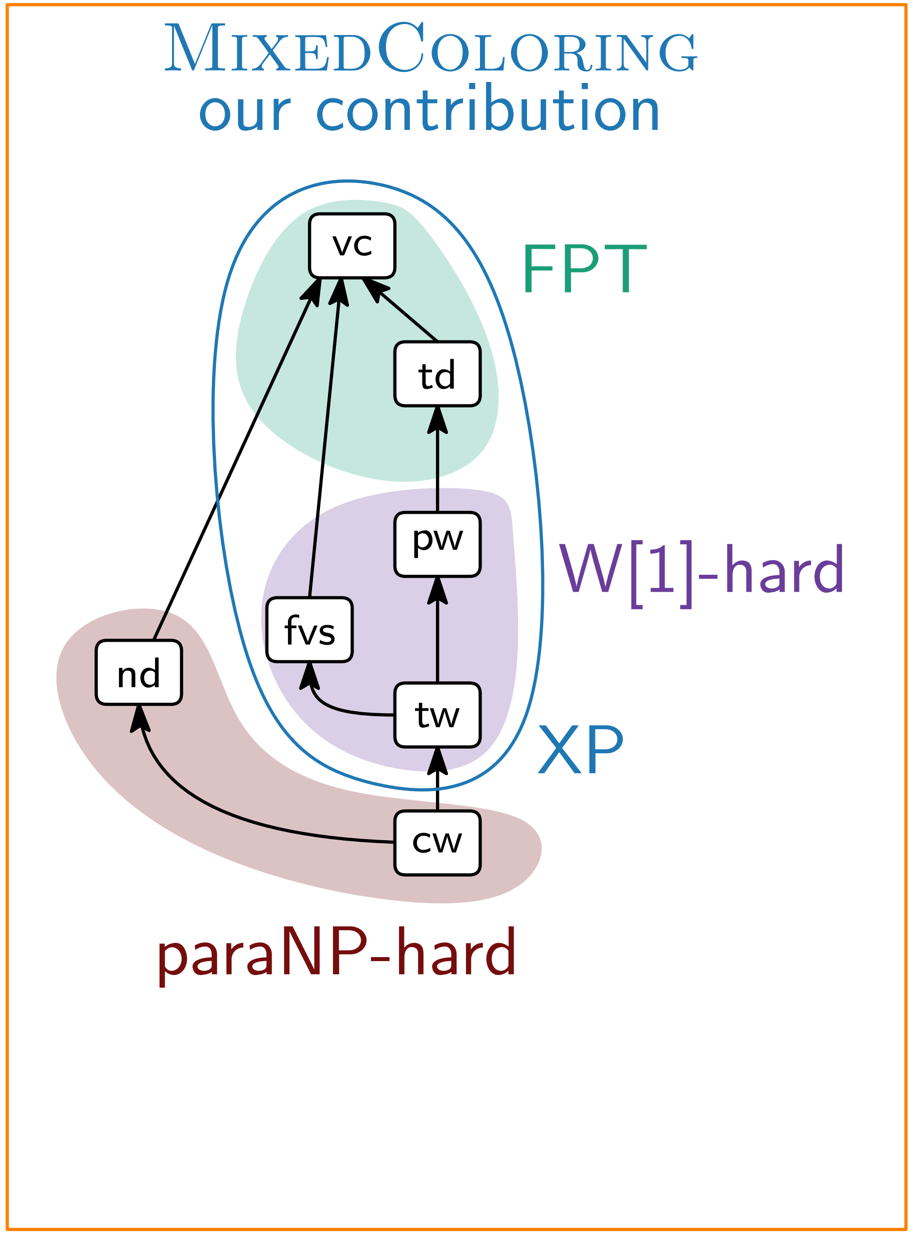
Observation

- subgraphs induced by types form clique or independent set
- vertices in type inducing IS can receive same color

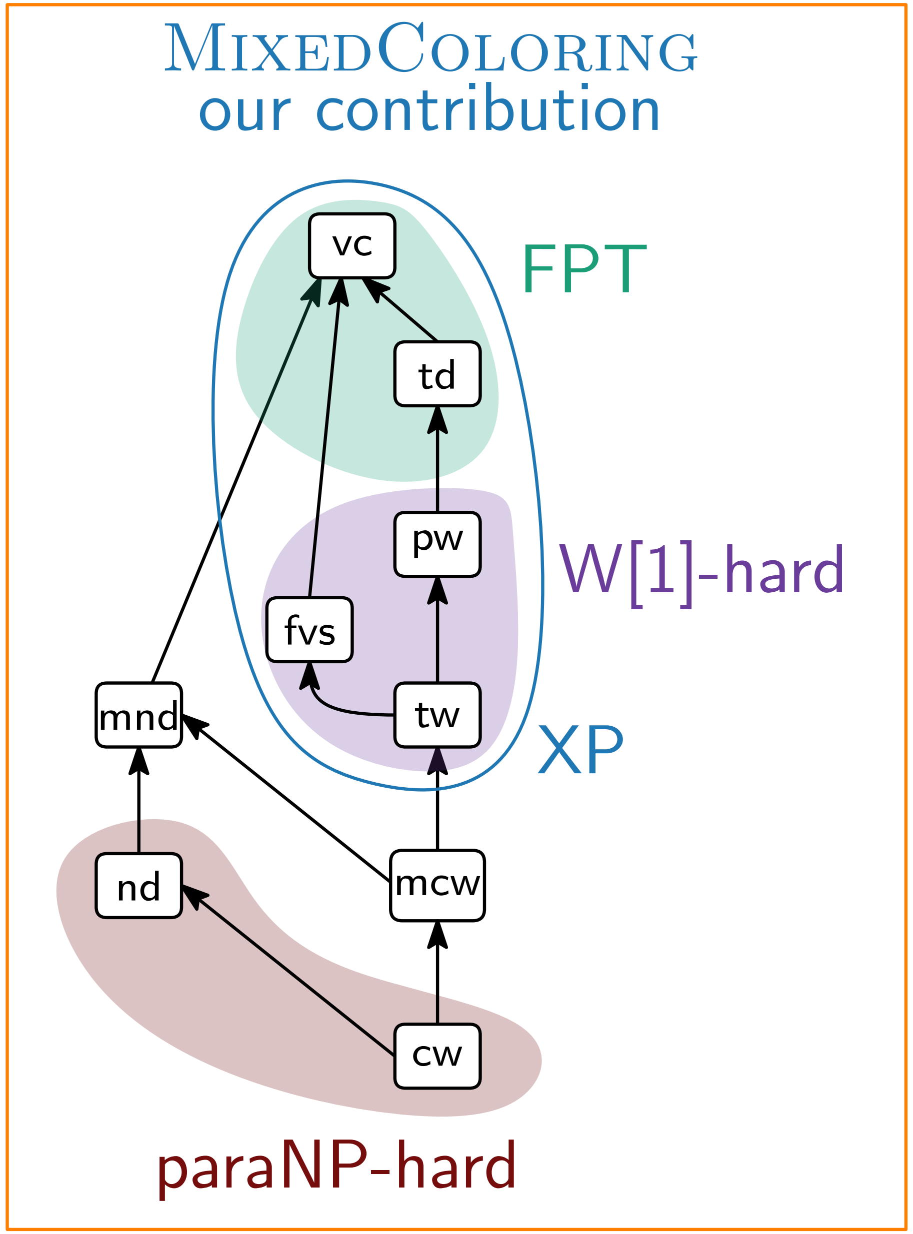
Mixed Cliquewidth

combines definitions of cliquewidth for undirected and directed graphs

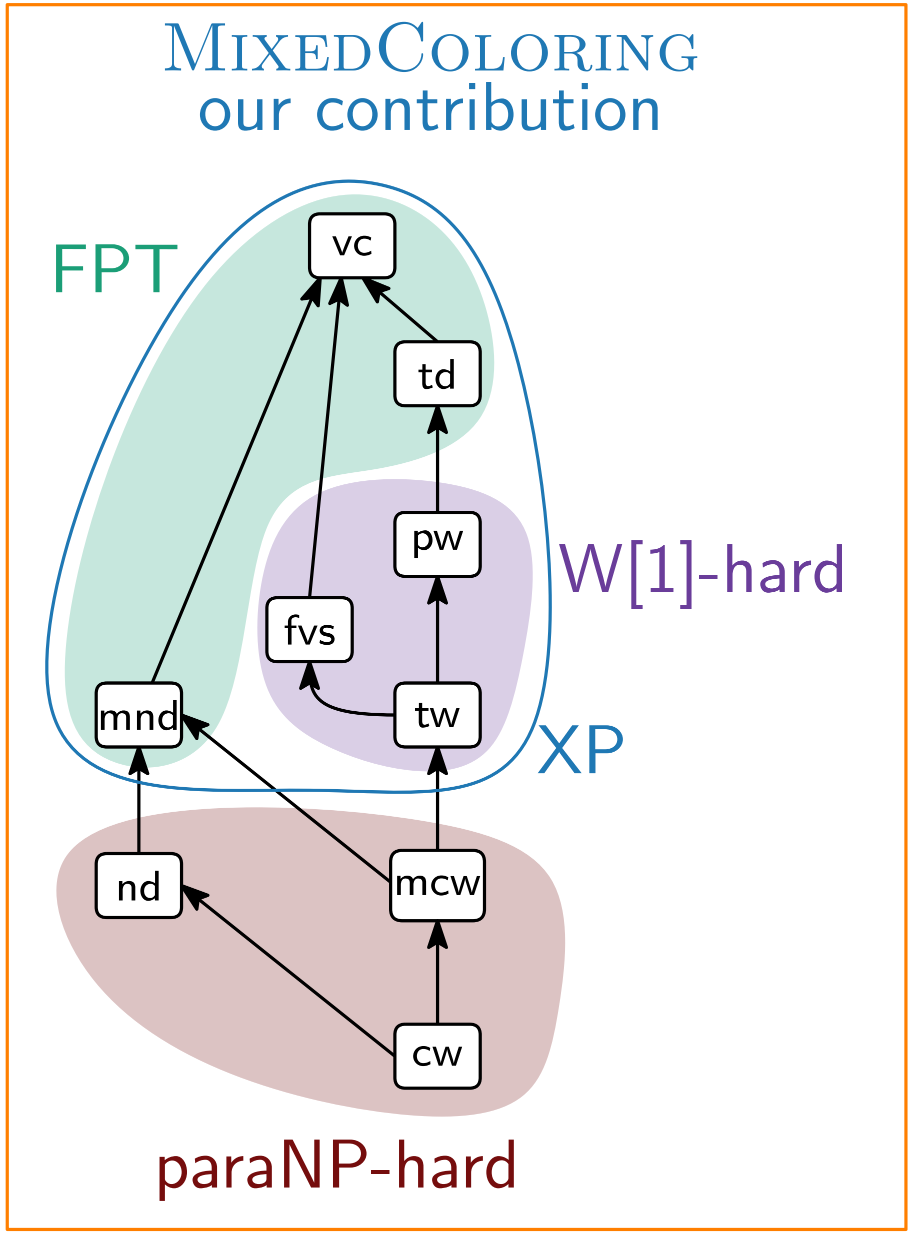
Parameterized Complexity



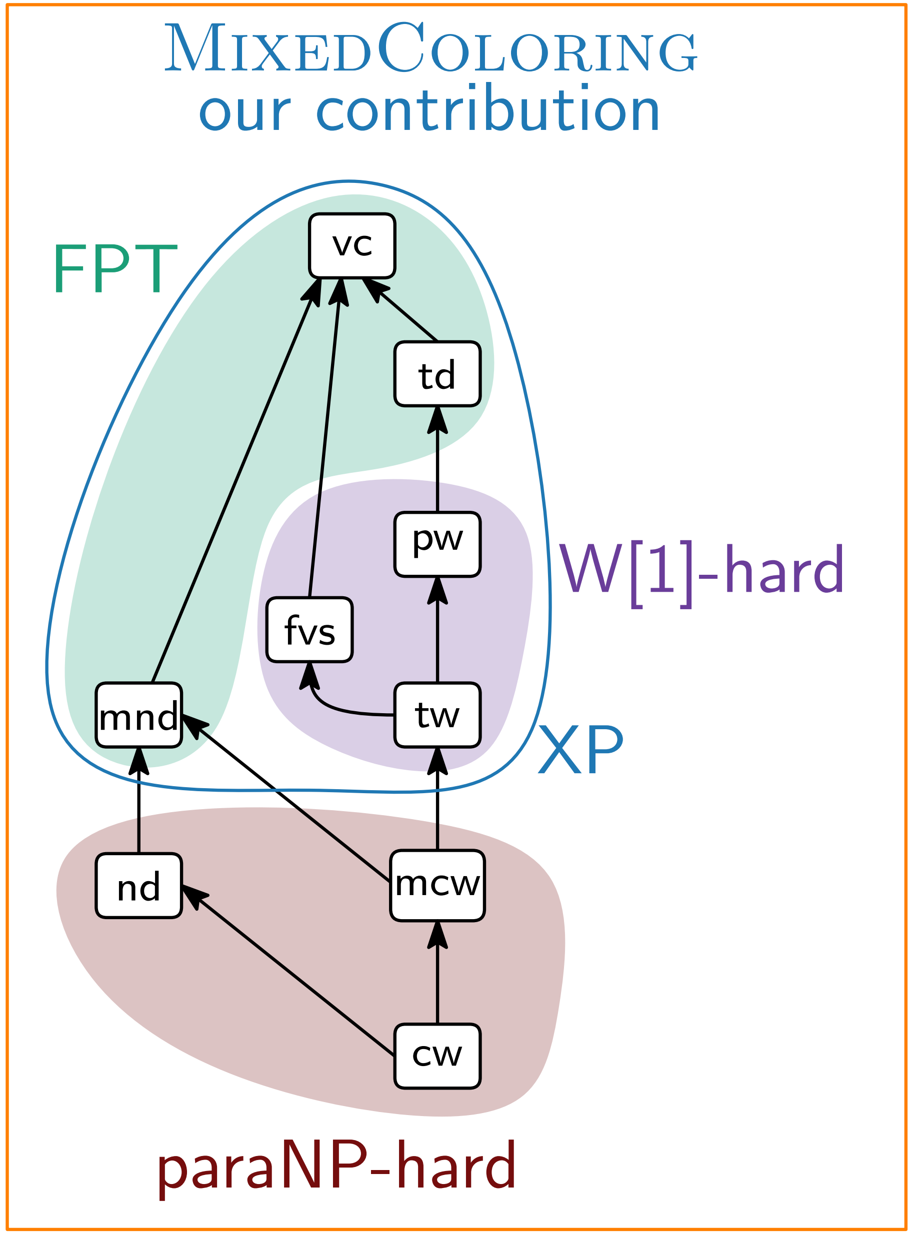
Parameterized Complexity

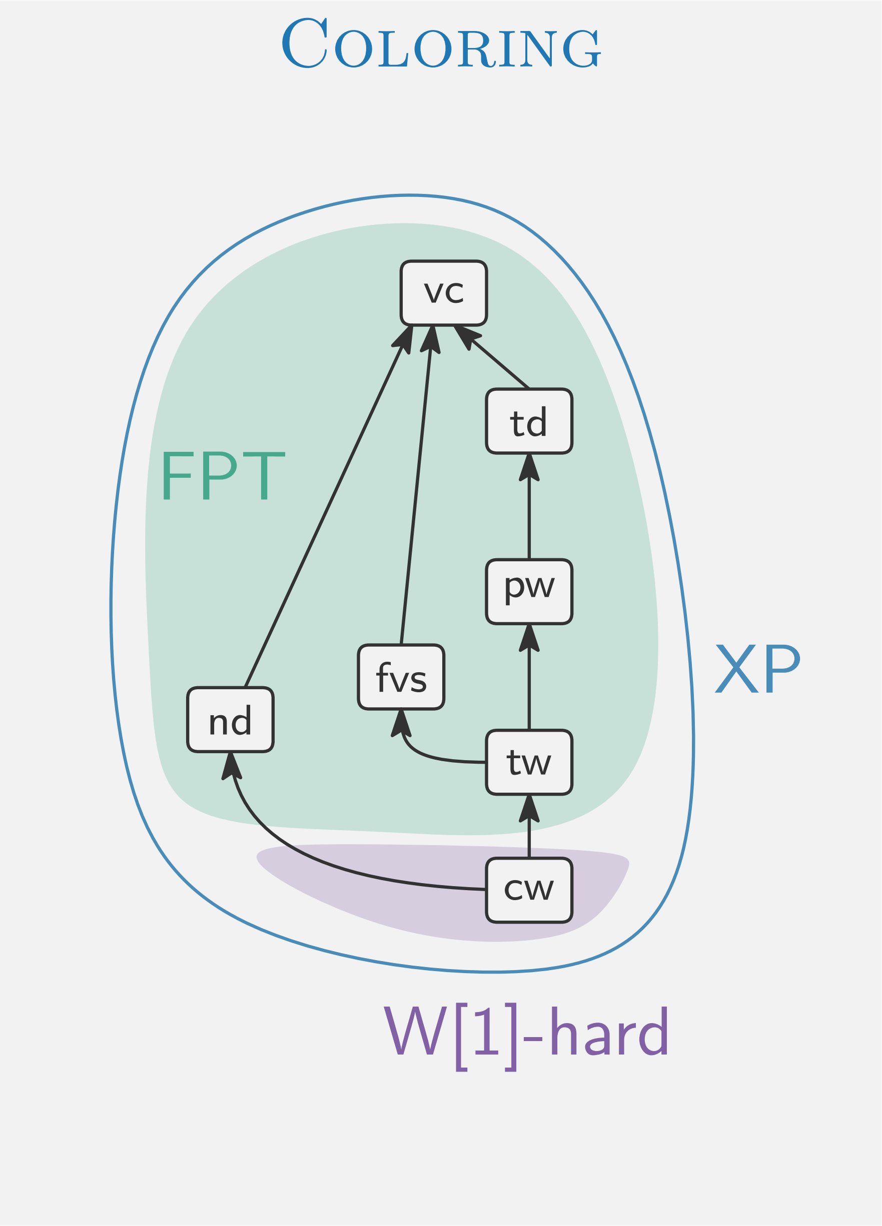


Parameterized Complexity

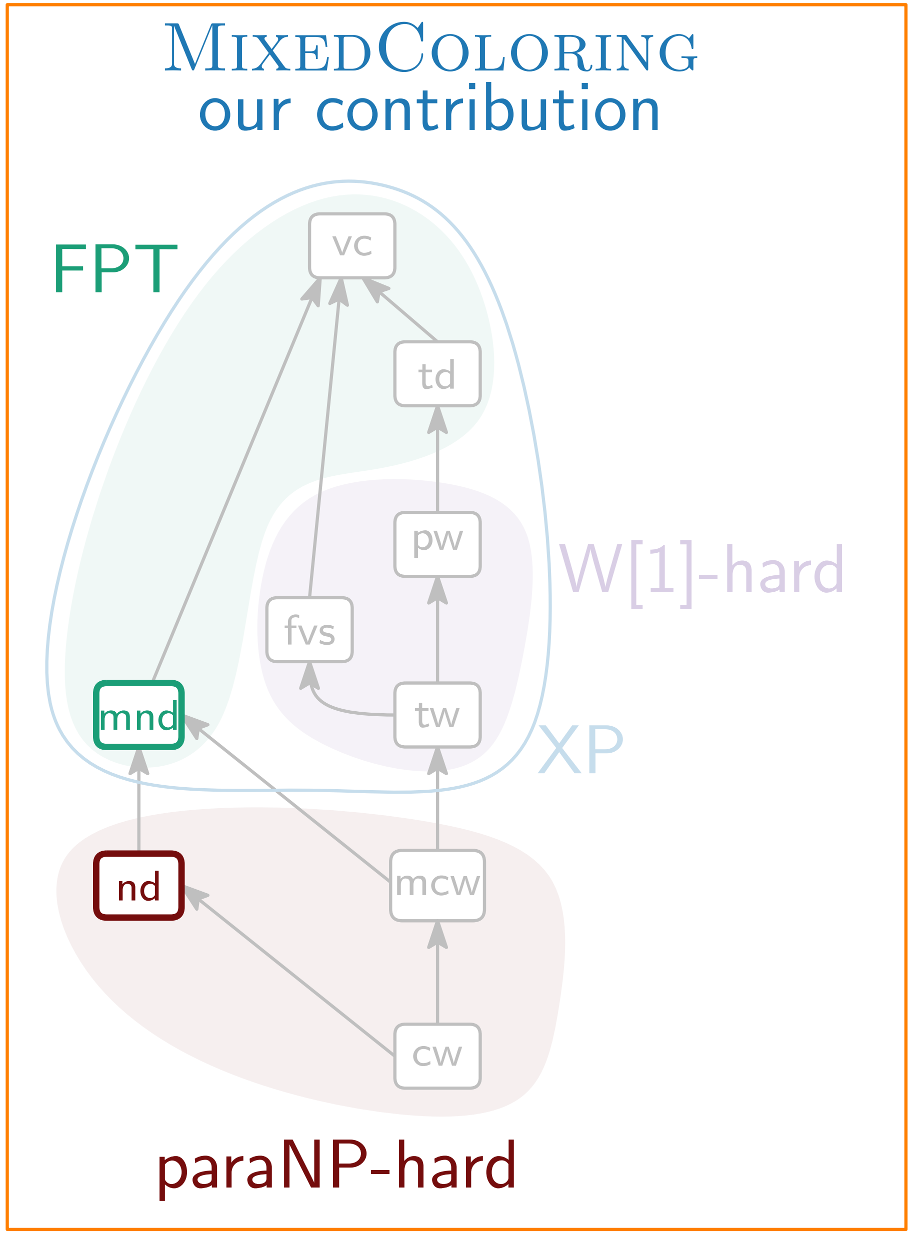


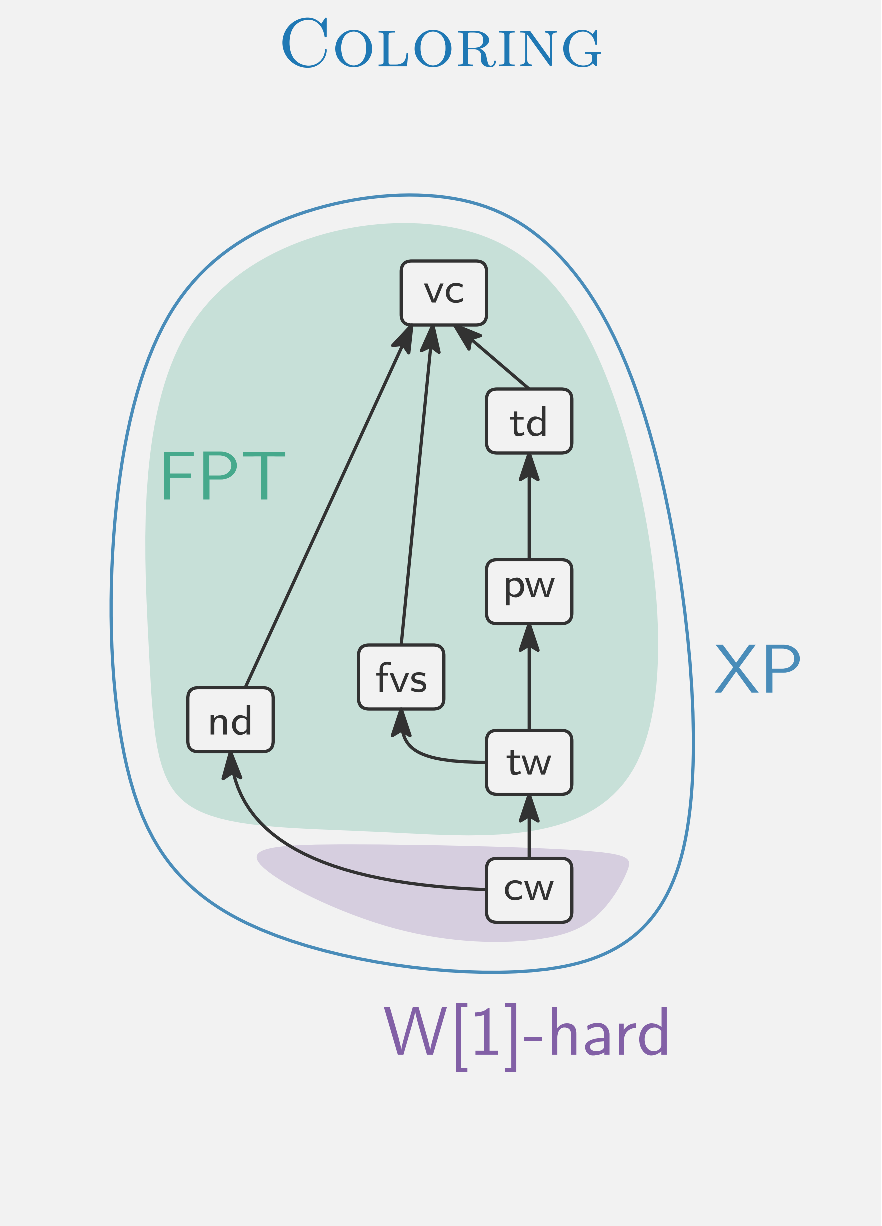
Parameterized Complexity





Parameterized Complexity





paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

paraNP-Hardness Neighborhood Diversity

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Reduction from

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from
PRECEDENCE CONSTRAINED SCHEDULING

NP-hard [Garey & Johnson, SS9]

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

Input:

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

Input: ◆ sets $T_{\hexagon}$, $T_{\pentagon}$ of unit-length tasks

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

Input: ♦ sets $T_{\hexagon}$, $T_{\pentagon}$ of unit-length tasks

$T_{\hexagon} = \{t_1, t_5\}$

$T_{\pentagon} = \{t_2, t_3, t_4\}$

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

Input:
- sets $T_{\hexagon}$, $T_{\pentagon}$ of unit-length tasks
- partial order $\prec$ of tasks

$T_{\hexagon} = \{t_1, t_5\}$

$T_{\pentagon} = \{t_2, t_3, t_4\}$

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

Input:
- sets $T_{\hexagon}$, $T_{\pentagon}$ of unit-length tasks
- partial order $\prec$ of tasks

$T_{\hexagon} = \{t_1, t_5\}$

$T_{\pentagon} = \{t_2, t_3, t_4\}$

$t_1 \prec t_3 \prec t_4$

$t_2 \prec t_5$

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

Input:
- sets $T_{\hexagon}, T_{\pentagon}$ of unit-length tasks
- partial order $\prec$ of tasks
- deadline D

$T_{\hexagon} = \{t_1, t_5\}$

$T_{\pentagon} = \{t_2, t_3, t_4\}$

$t_1 \prec t_3 \prec t_4$

$t_2 \prec t_5$

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

Precedence Constrained Scheduling

Input:
- sets $T_{⬡}$, $T_{⬠}$ of unit-length tasks
- partial order $\prec$ of tasks
- deadline D

$T_{⬡} = \{t_1, t_5\}$

$T_{⬠} = \{t_2, t_3, t_4\}$

$t_1 \prec t_3 \prec t_4$

$t_2 \prec t_5$

$D = 3$

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

Precedence Constrained Scheduling

Input:
- sets $T_{\hexagon}$, $T_{\pentagon}$ of unit-length tasks
- partial order $\prec$ of tasks
- deadline D

Output:

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$t_1 \prec t_3 \prec t_4$

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$D = 3$

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

Input:
- sets $T_{\hexagon}$, $T_{\pentagon}$ of unit-length tasks
- partial order $\prec$ of tasks
- deadline D

Output: schedule on two machines $M_{\hexagon}$, $M_{\pentagon}$
respecting partial order $\prec$ and deadline D
$(T_{\hexagon}$ on $M_{\hexagon}$, $T_{\pentagon}$ on $M_{\pentagon})$

$T_{\hexagon} = \{t_1, t_5\}$
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$D = 3$

Reduction (Idea):

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

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Input:
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- partial order $\prec$ of tasks
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Output: schedule on two machines $M_{\hexagon}$, $M_{\pentagon}$
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$t_1 \prec t_3 \prec t_4$
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$D = 3$

Reduction (Idea):

- tasks as vertices

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

Input:
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- partial order $\prec$ of tasks
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respecting partial order $\prec$ and deadline D
($T_{\hexagon}$ on $M_{\hexagon}$, $T_{\pentagon}$ on $M_{\pentagon}$)

$T_{\hexagon} = \{t_1, t_5\}$

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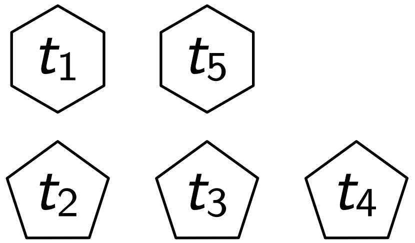
$D = 3$

Reduction (Idea):

- tasks as vertices



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

Input:
- ♦ sets $T_{\hexagon}$, $T_{\pentagon}$ of unit-length tasks
- ♦ partial order $\prec$ of tasks
- ♦ deadline D

Output: schedule on two machines $M_{\hexagon}$, $M_{\pentagon}$ respecting partial order $\prec$ and deadline D ($T_{\hexagon}$ on $M_{\hexagon}$, $T_{\pentagon}$ on $M_{\pentagon}$)

$T_{\hexagon} = \{t_1, t_5\}$

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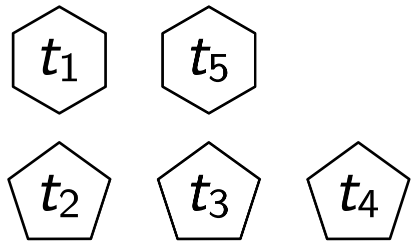
$t_1 \prec t_3 \prec t_4$

$t_2 \prec t_5$

$D = 3$

Reduction (Idea):

- tasks as vertices
- timeslots as colors



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

Input:
- ♦ sets $T_{\hexagon}$, $T_{\pentagon}$ of unit-length tasks
- ♦ partial order $\prec$ of tasks
- ♦ deadline D

Output: schedule on two machines $M_{\hexagon}$, $M_{\pentagon}$
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($T_{\hexagon}$ on $M_{\hexagon}$, $T_{\pentagon}$ on $M_{\pentagon}$)

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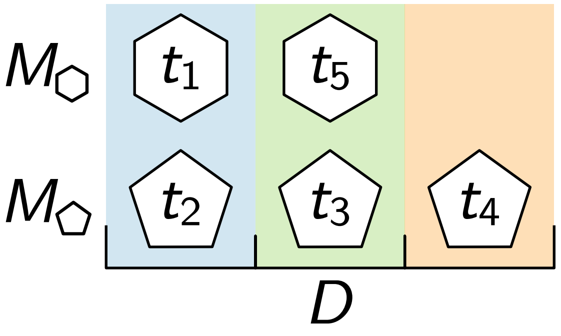
Reduction (Idea):

- tasks as vertices
- timeslots as colors



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

PRECEDENCE CONSTRAINED SCHEDULING

Input:
- sets $T_{\text{⬡}}$, $T_{\text{⬠}}$ of unit-length tasks
- partial order $\prec$ of tasks
- deadline D

Output: schedule on two machines $M_{\text{⬡}}$, $M_{\text{⬠}}$
respecting partial order $\prec$ and deadline D
($T_{\text{⬡}}$ on $M_{\text{⬡}}$, $T_{\text{⬠}}$ on $M_{\text{⬠}}$)

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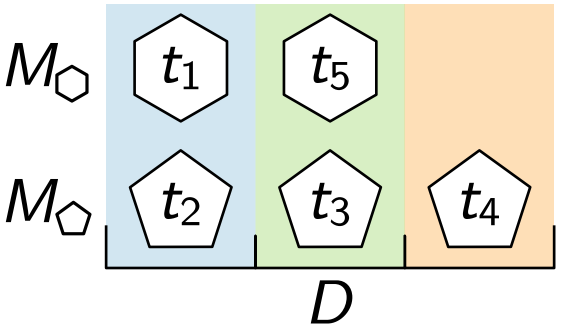
$t_1 \prec t_3 \prec t_4$

$t_2 \prec t_5$

$D = 3$

Reduction (Idea):

- tasks as vertices
- timeslots as colors
- arcs enforce partial order



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction from

Precedence Constrained Scheduling

Input:
- sets $T_{\hexagon}$, $T_{\pentagon}$ of unit-length tasks
- partial order $\prec$ of tasks
- deadline D

Output: schedule on two machines $M_{\hexagon}$, $M_{\pentagon}$ respecting partial order $\prec$ and deadline D ($T_{\hexagon}$ on $M_{\hexagon}$, $T_{\pentagon}$ on $M_{\pentagon}$)

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$t_1 \prec t_3 \prec t_4$

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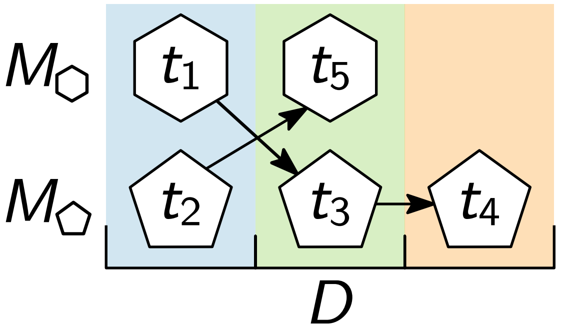
$D = 3$

Reduction (Idea):

- tasks as vertices
- timeslots as colors
- arcs enforce partial order



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

$T_{\hexagon} = \{t_1, t_5\}$

$T_{\pentagon} = \{t_2, t_3, t_4\}$

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Reduction:

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$t_1 \prec t_3 \prec t_4$

$t_2 \prec t_5$

$D = 3$

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
    - start t^- and end vertex t^+ for each task t

$T_{\text{⬡}} = \{t_1, t_5\}$

$T_{\text{⬠}} = \{t_2, t_3, t_4\}$

$t_1 \prec t_3 \prec t_4$

$t_2 \prec t_5$

$D = 3$

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
  - start t^- and end vertex t^+ for each task t

$T_{\hexagon} = \{t_1, t_5\}$

$T_{\pentagon} = \{t_2, t_3, t_4\}$

$t_1 \prec t_3 \prec t_4$

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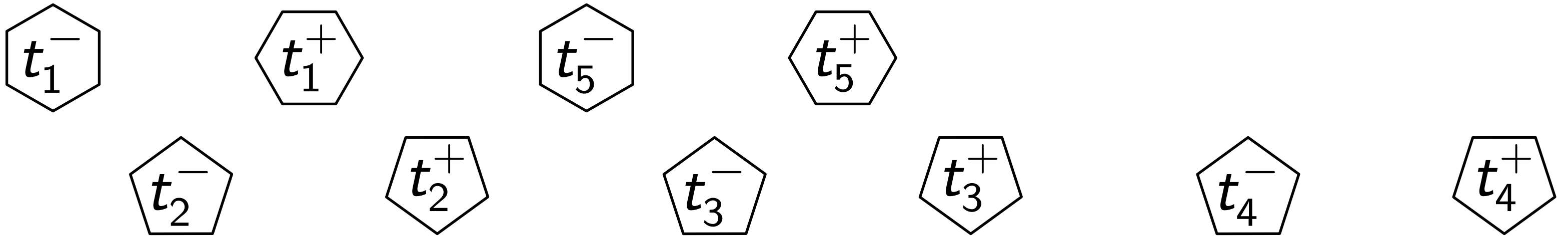
$D = 3$



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
    - start t^- and end vertex t^+ for each task t
    - arc (t^-, t^+) for each task t

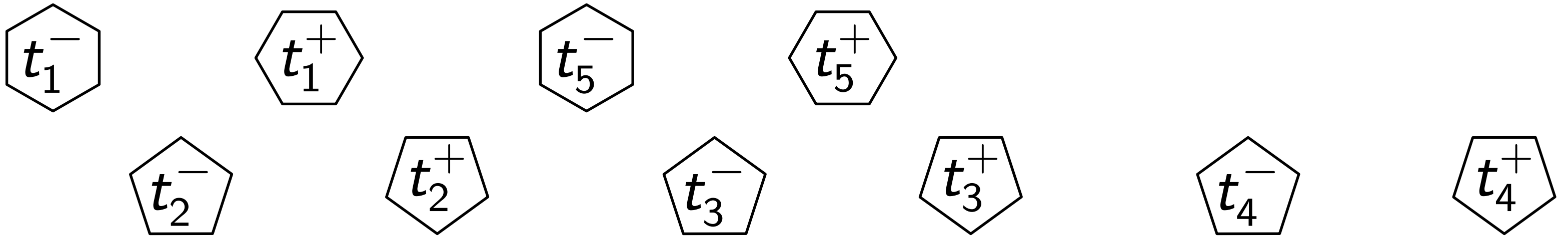
$T_{⬡} = \{t_1, t_5\}$

$T_{⬠} = \{t_2, t_3, t_4\}$

$t_1 \prec t_3 \prec t_4$

$t_2 \prec t_5$

$D = 3$



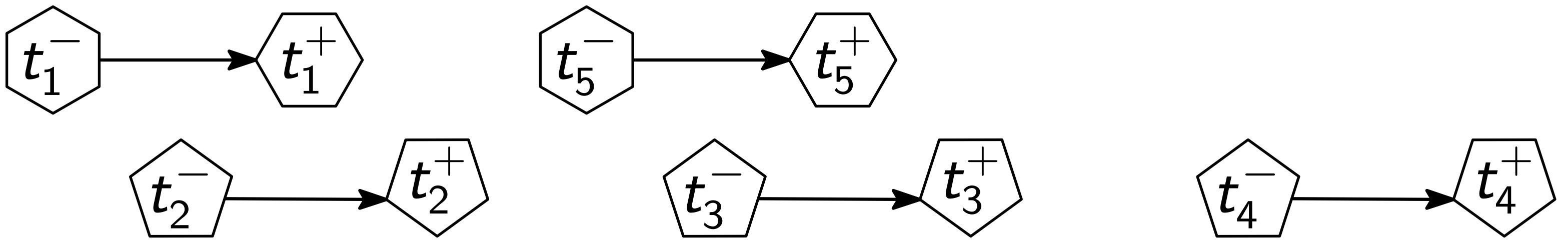
paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
  - start t^- and end vertex t^+ for each task t
  - arc (t^-, t^+) for each task t

$T_{\hexagon} = \{t_1, t_5\}$
$T_{\pentagon} = \{t_2, t_3, t_4\}$
$t_1 \prec t_3 \prec t_4$
$t_2 \prec t_5$
$D = 3$



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
  - start t^- and end vertex t^+ for each task t
  - arc (t^-, t^+) for each task t
  - arc (t_i^+, t_j^-) for $t_i \prec t_j$

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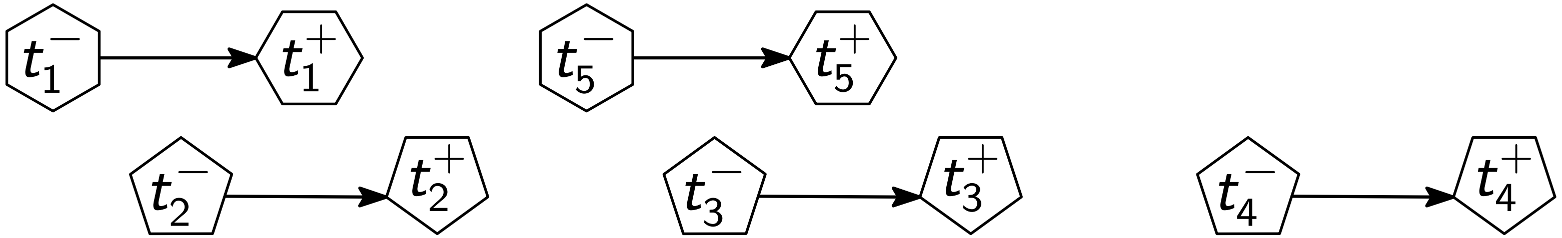
$t_2 \prec t_5$

$D = 3$



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
  - start t^- and end vertex t^+ for each task t
  - arc (t^-, t^+) for each task t
  - arc (t_i^+, t_j^-) for $t_i \prec t_j$

$T_{\hexagon} = \{t_1, t_5\}$

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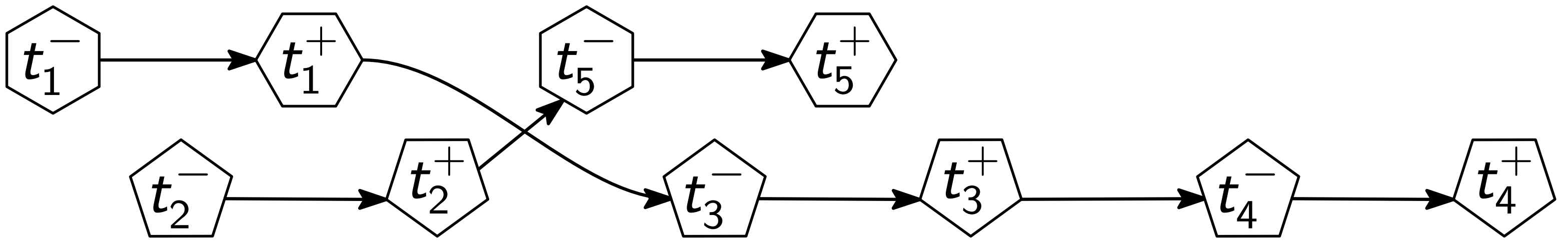
$D = 3$



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

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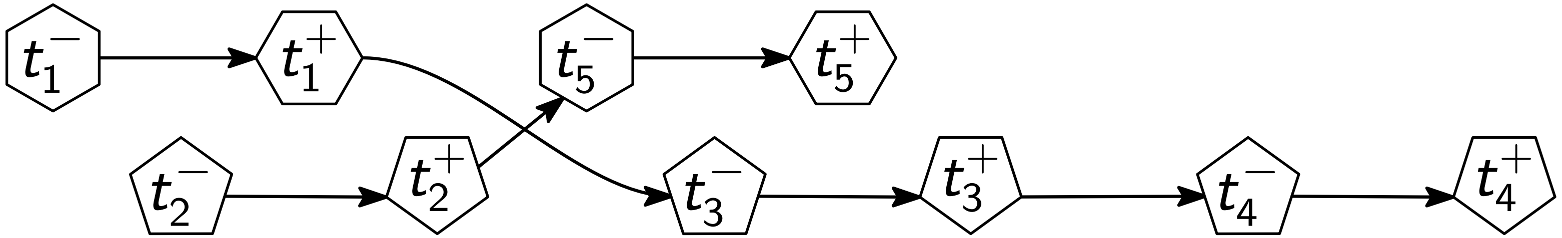

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors

$T_{\text{⬡}} = \{t_1, t_5\}$

$T_{\text{⬠}} = \{t_2, t_3, t_4\}$

$t_1 \prec t_3 \prec t_4$

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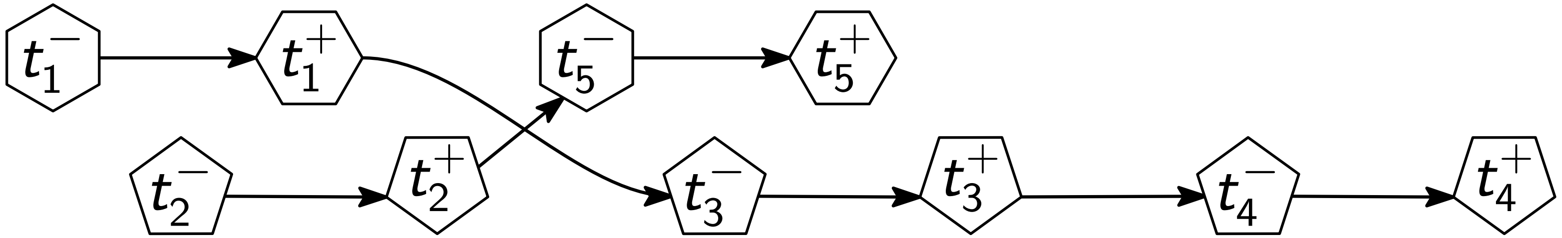
$D = 3$



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors
    - four colors per time unit

$T_{⬡} = \{t_1, t_5\}$

$T_{⬠} = \{t_2, t_3, t_4\}$

$t_1 \prec t_3 \prec t_4$

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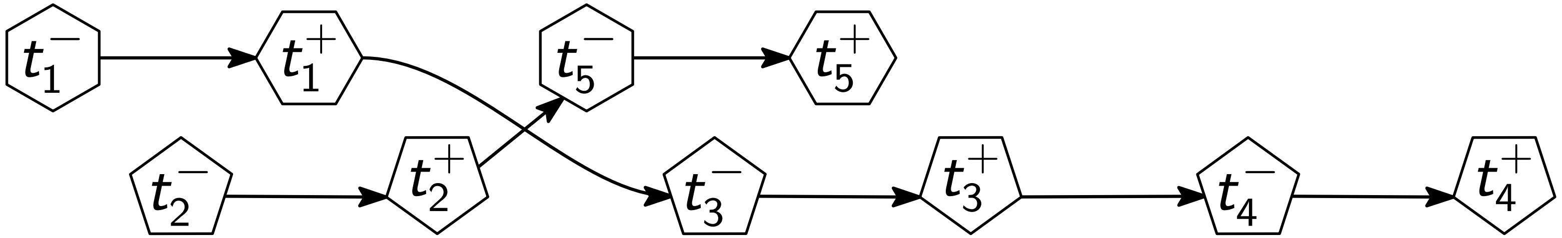
$D = 3$



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- tasks as vertices
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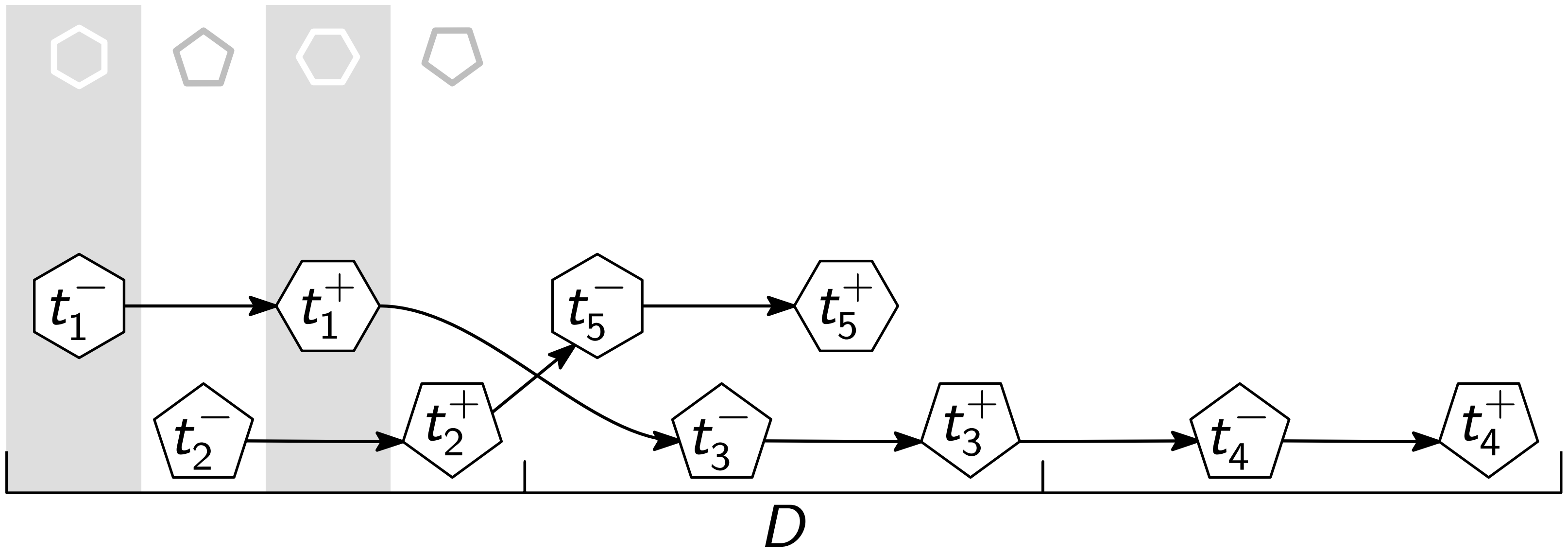

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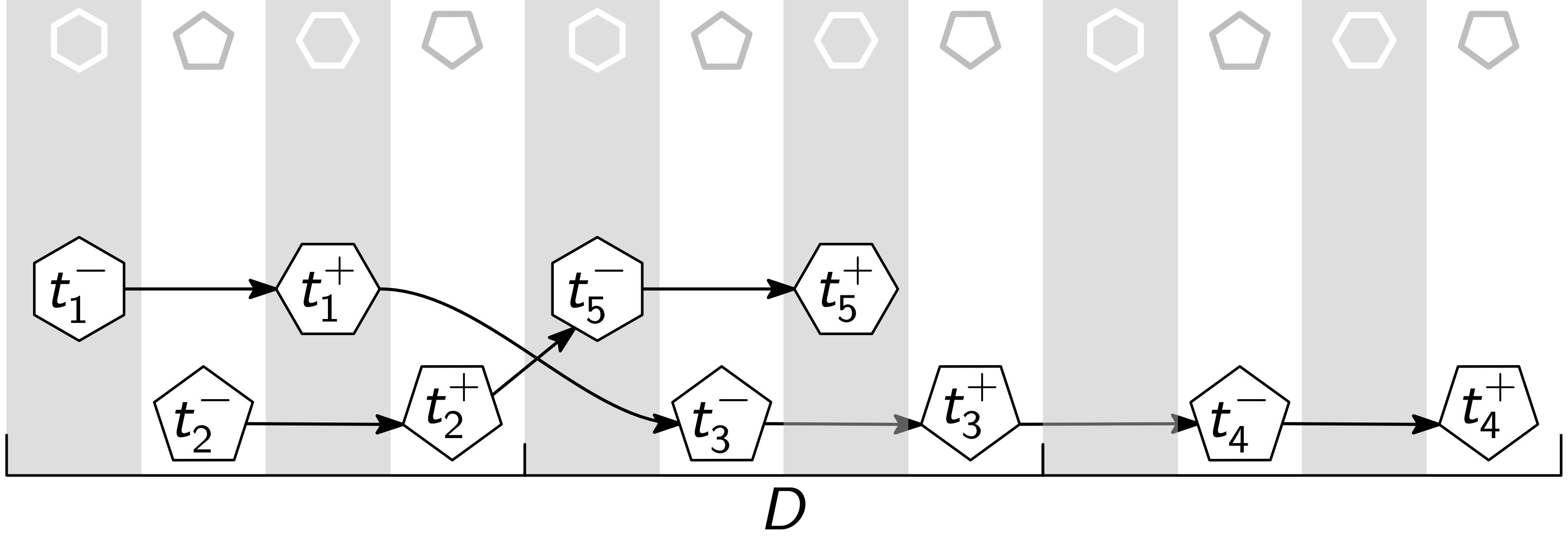

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors
  - four colors per time unit

$4D$-colors in total

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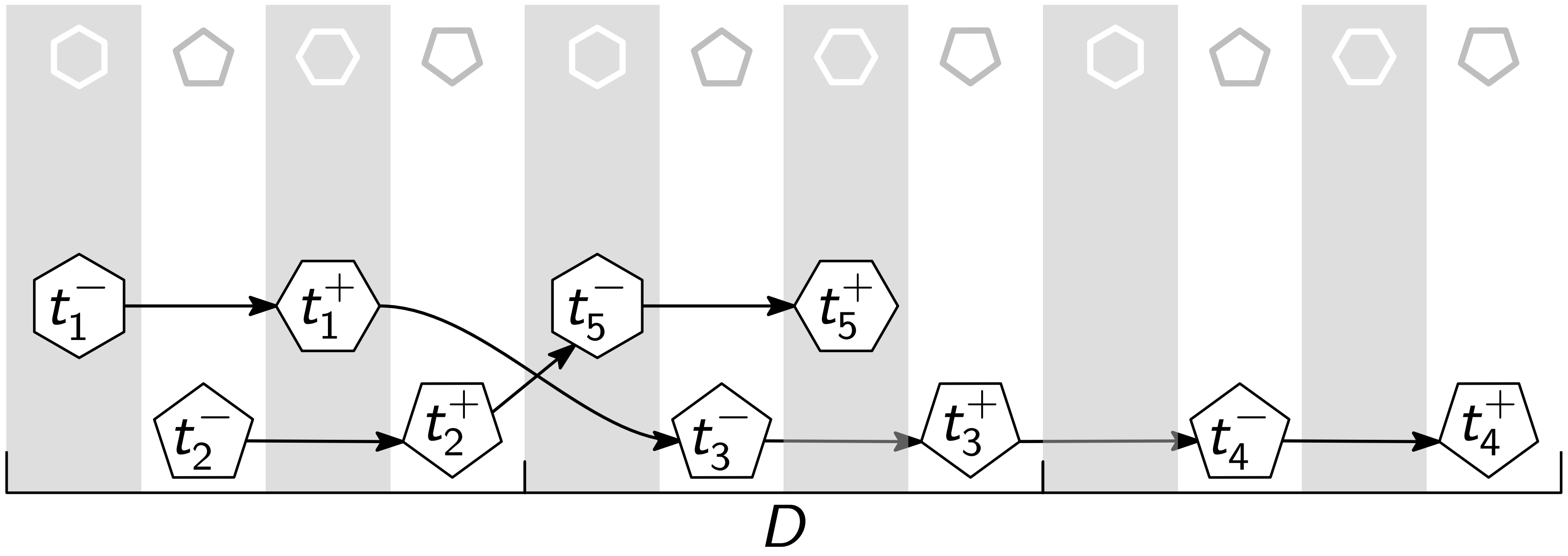
$D = 3$



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors
  - four colors per time unit
  - colors enforced by directed path of length $4D$

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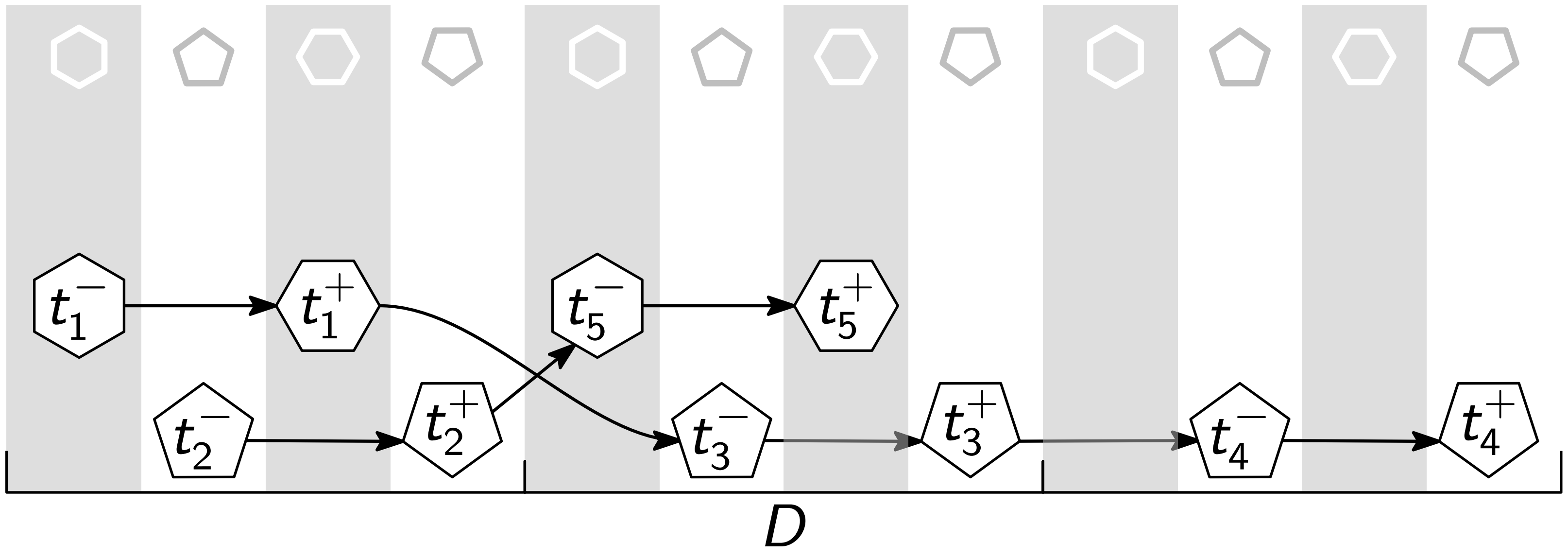

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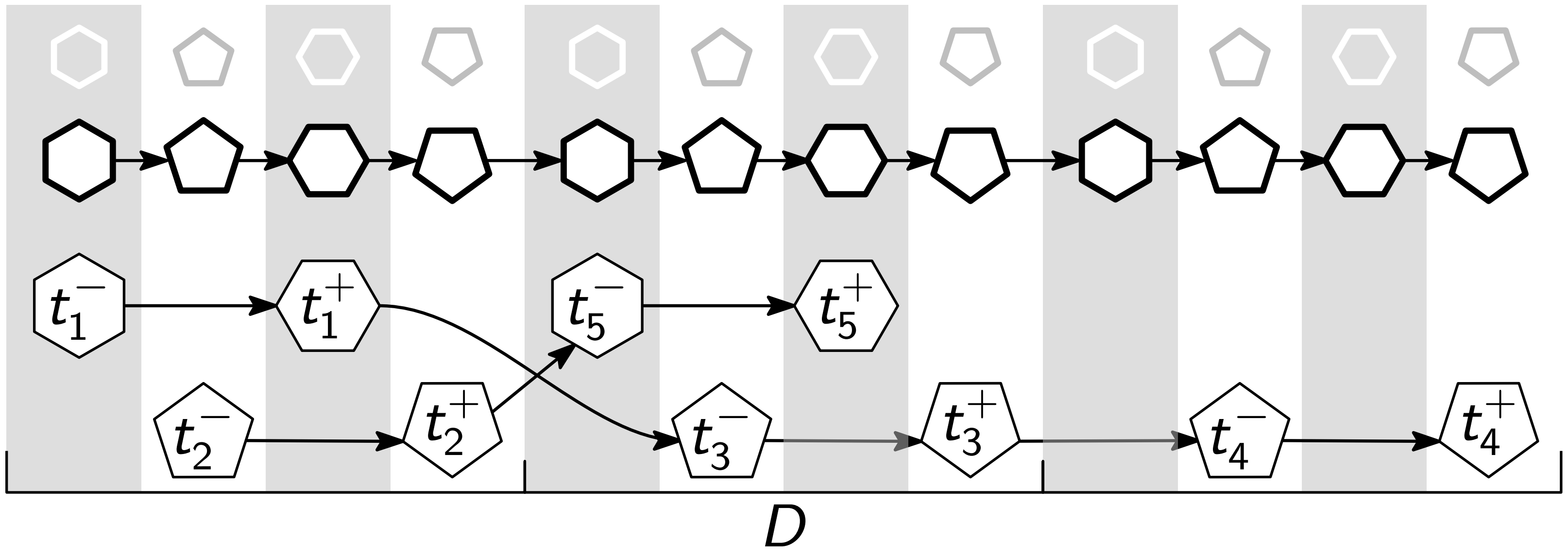

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors
  - four colors per time unit
  - colors enforced by directed path of length $4D$

unique $4D$-coloring

$T_{\hexagon} = \{t_1, t_5\}$

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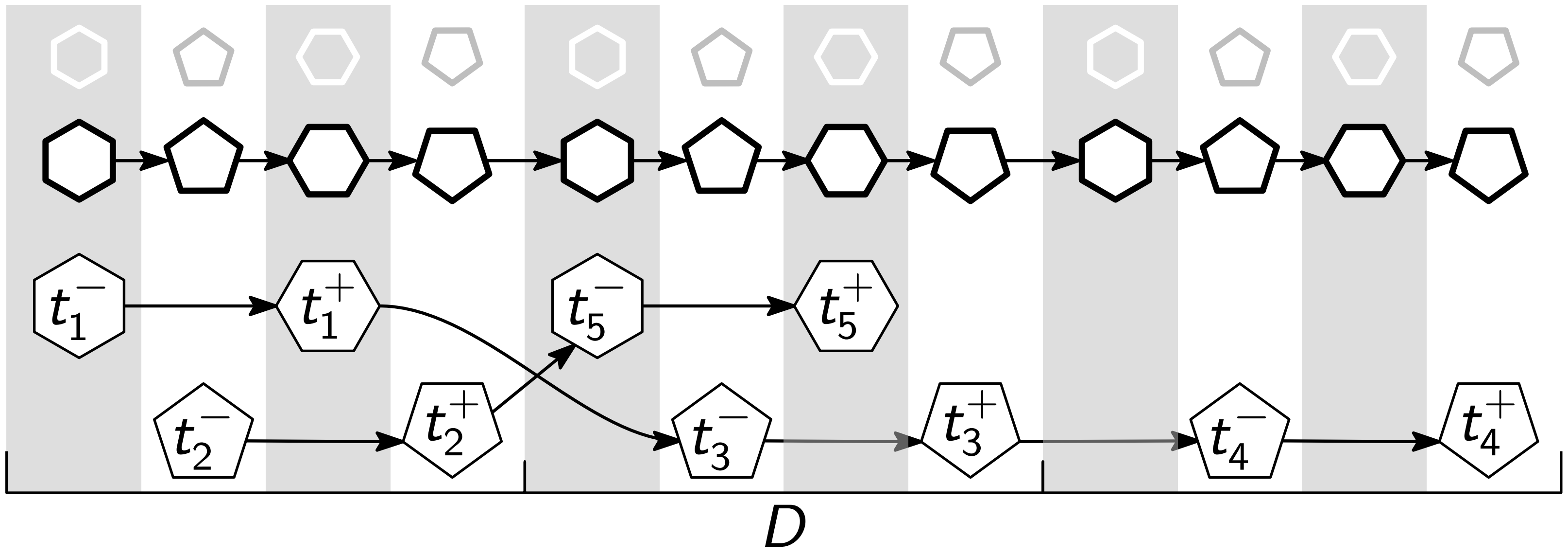

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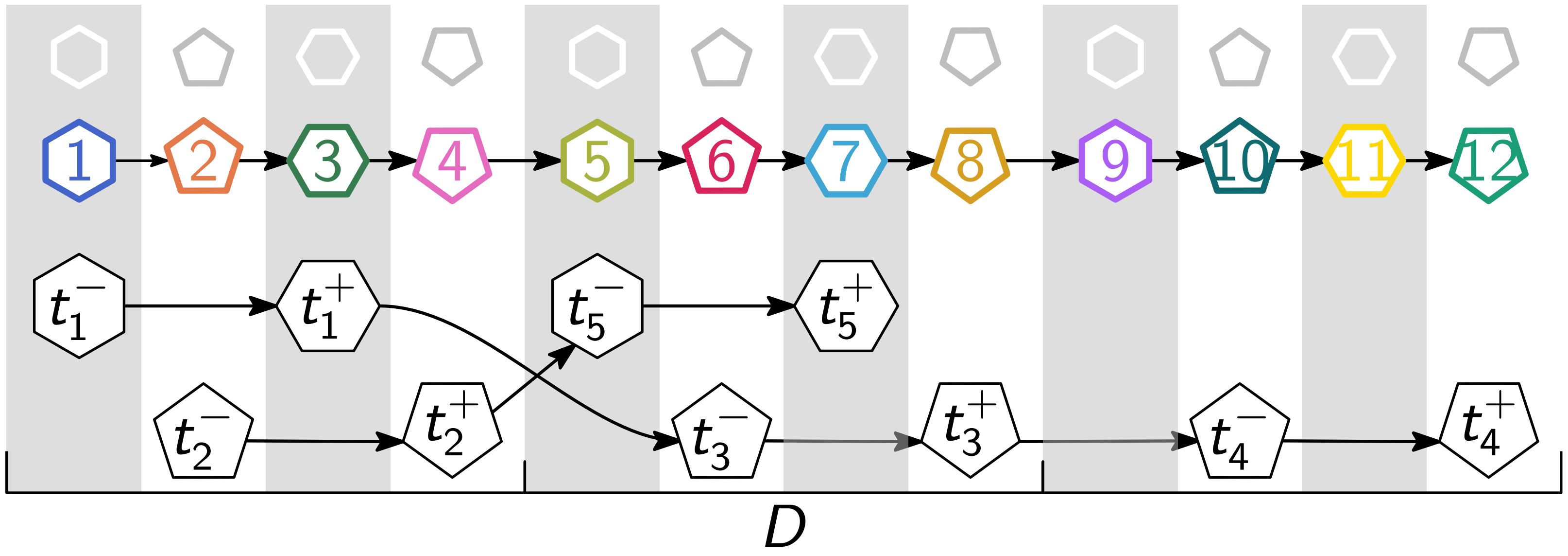

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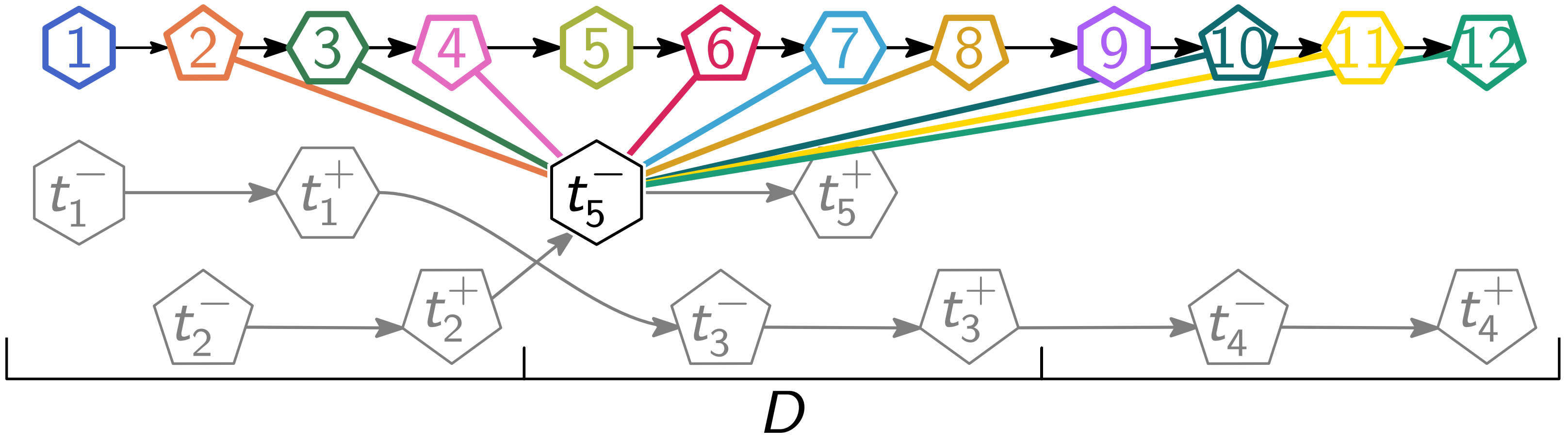

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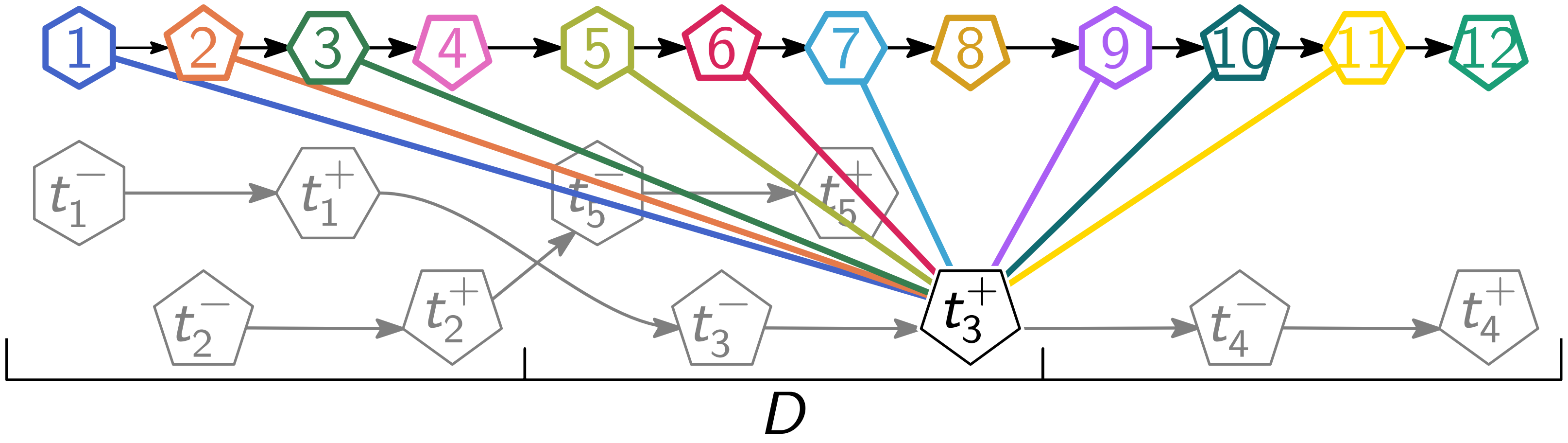

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

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- tasks as vertices
- timeslots as colors
    - four colors per time unit
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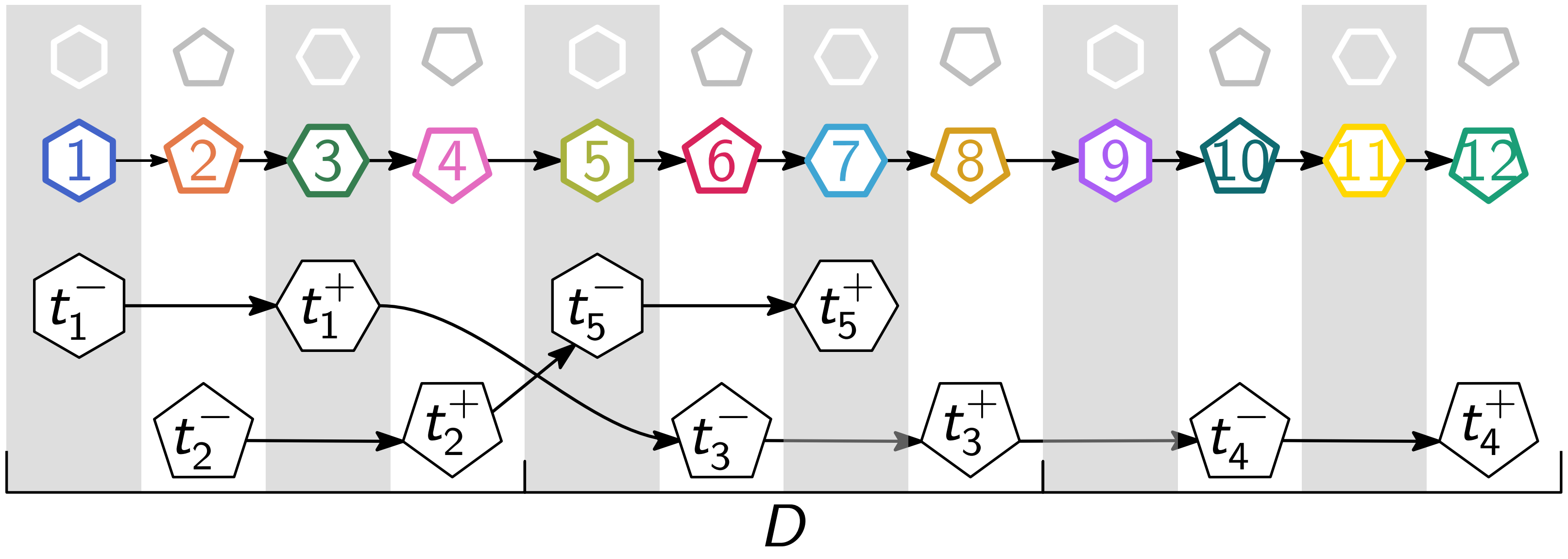

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors
    - four colors per time unit
    - colors enforced by directed path of length $4D$
    - task vertices pairwise connected

$T_{\text{⬡}} = \{t_1, t_5\}$

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$t_1 \prec t_3 \prec t_4$

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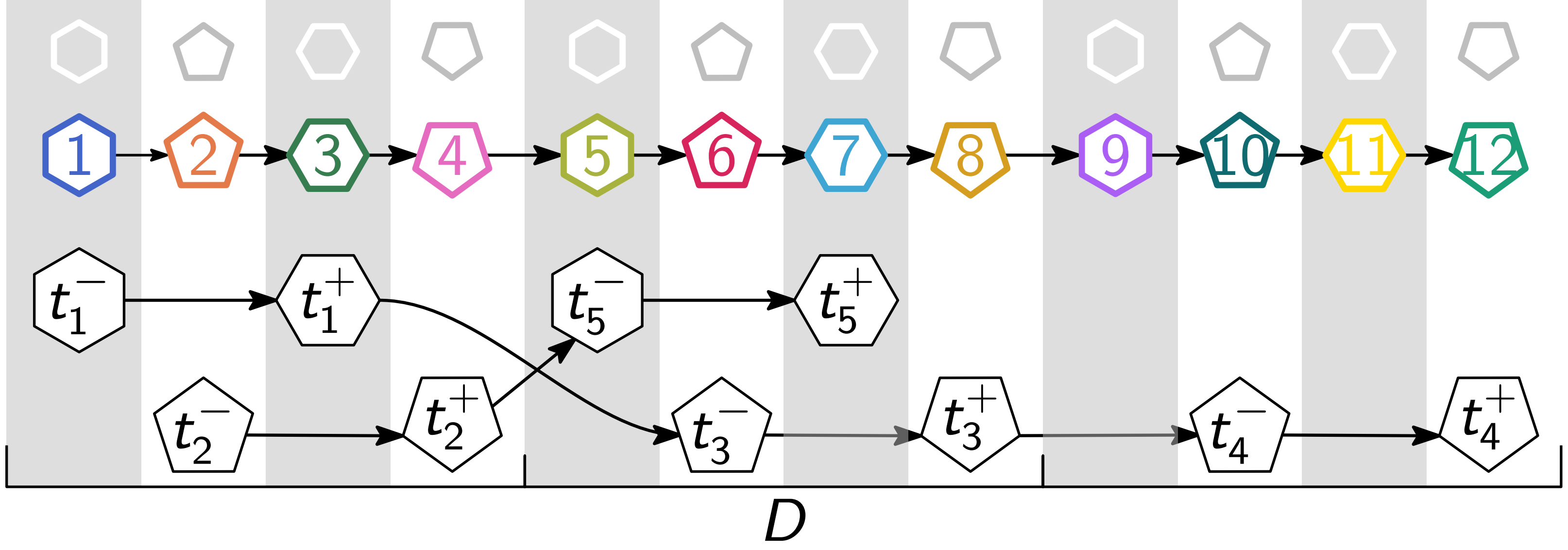
$D = 3$



paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors
  - four colors per time unit
  - colors enforced by directed path of length $4D$
  - task vertices pairwise connected
  - vertices of directed path pairwise connected

$T_{⬡} = \{t_1, t_5\}$

$T_{⬠} = \{t_2, t_3, t_4\}$

$t_1 \prec t_3 \prec t_4$

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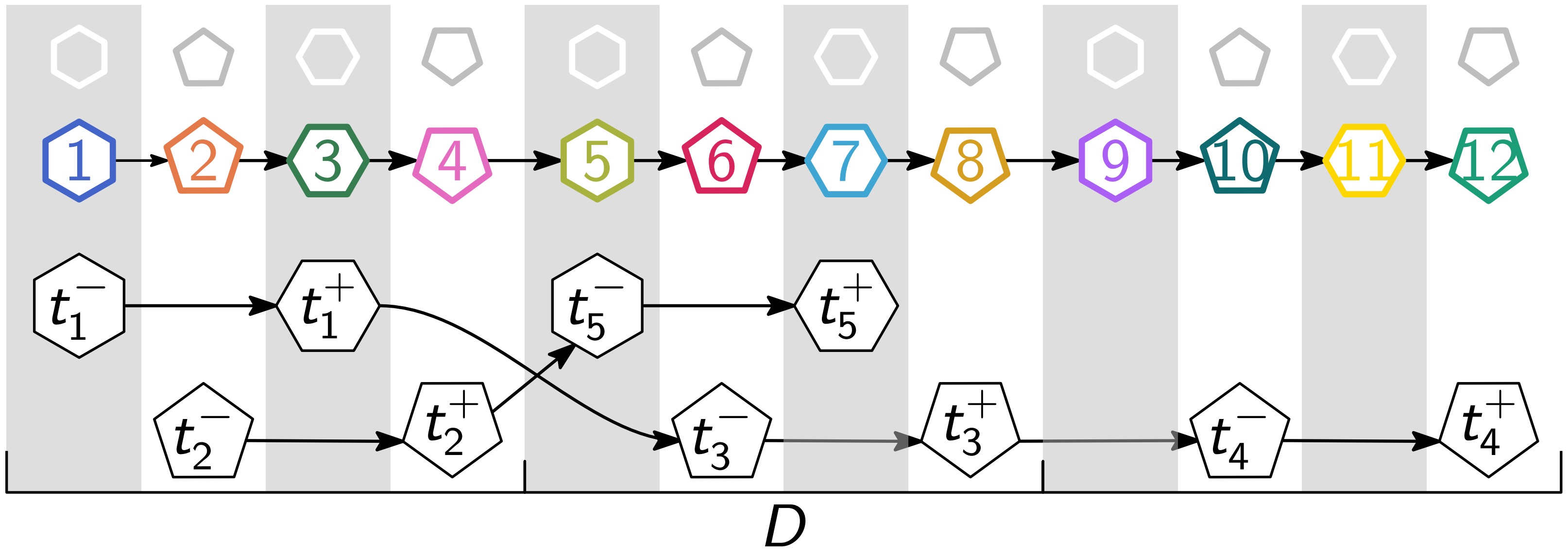

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors

$T_{\hexagon} = \{t_1, t_5\}$

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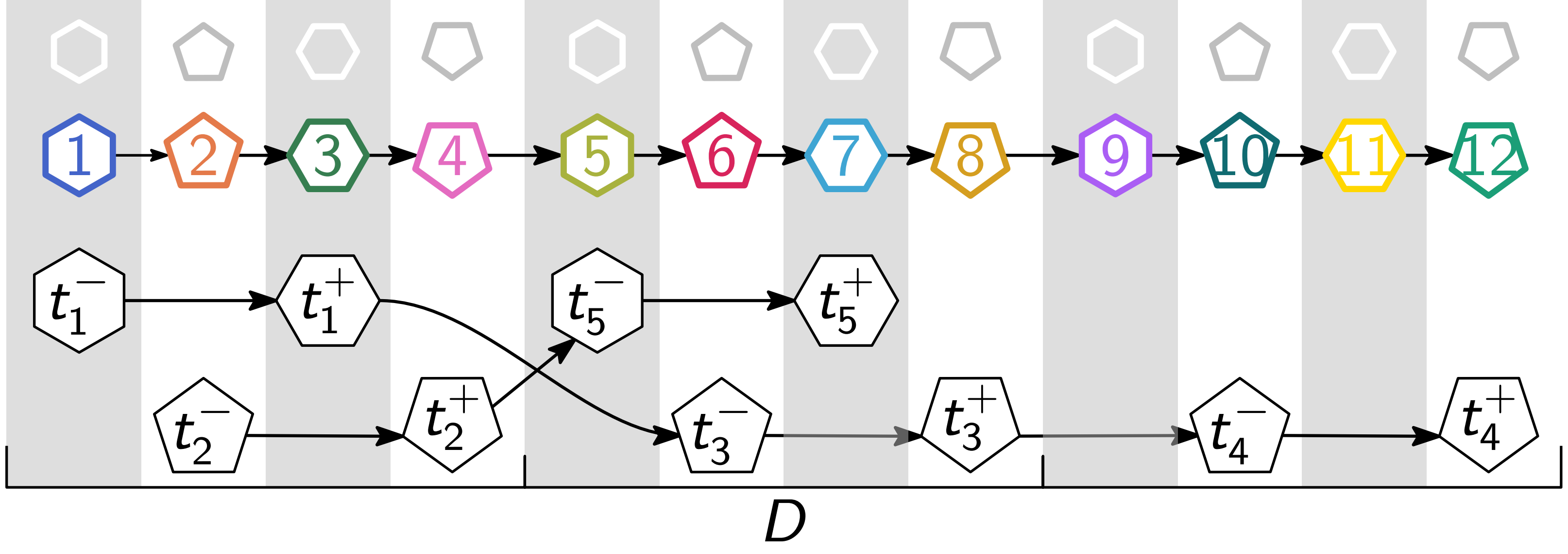

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors
- constant neighborhood diversity

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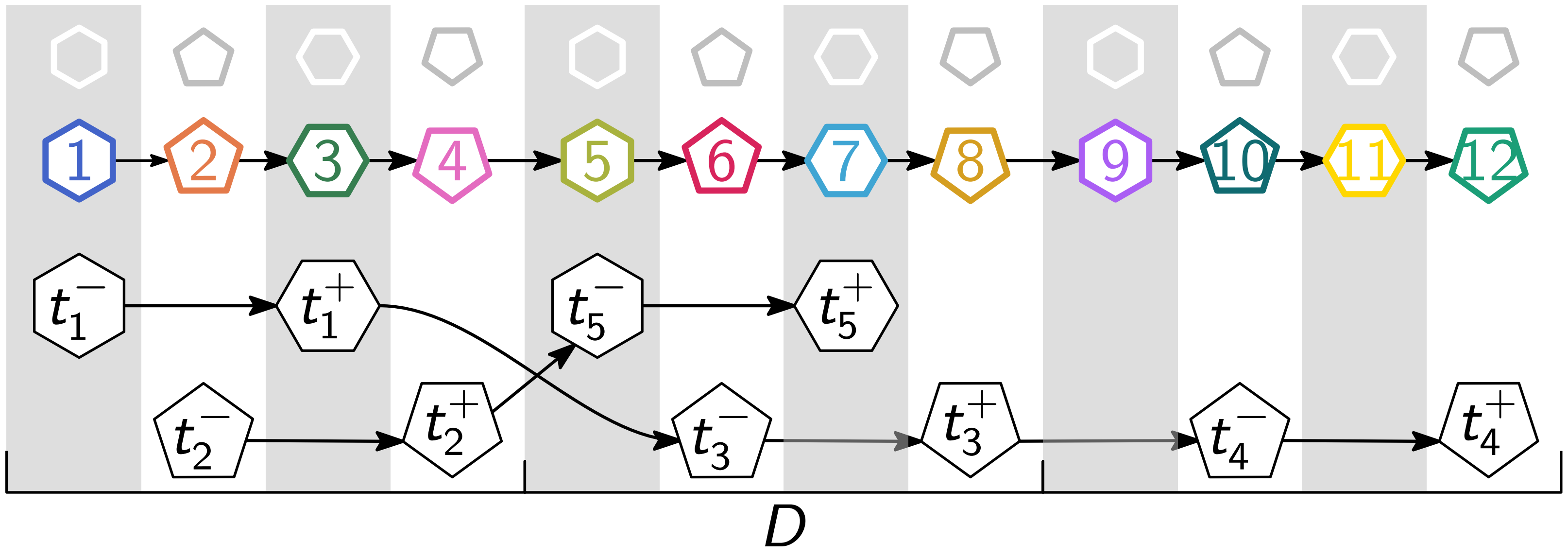

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors
- constant neighborhood diversity
  - 4 types of task vertices

$T_{\hexagon} = \{t_1, t_5\}$

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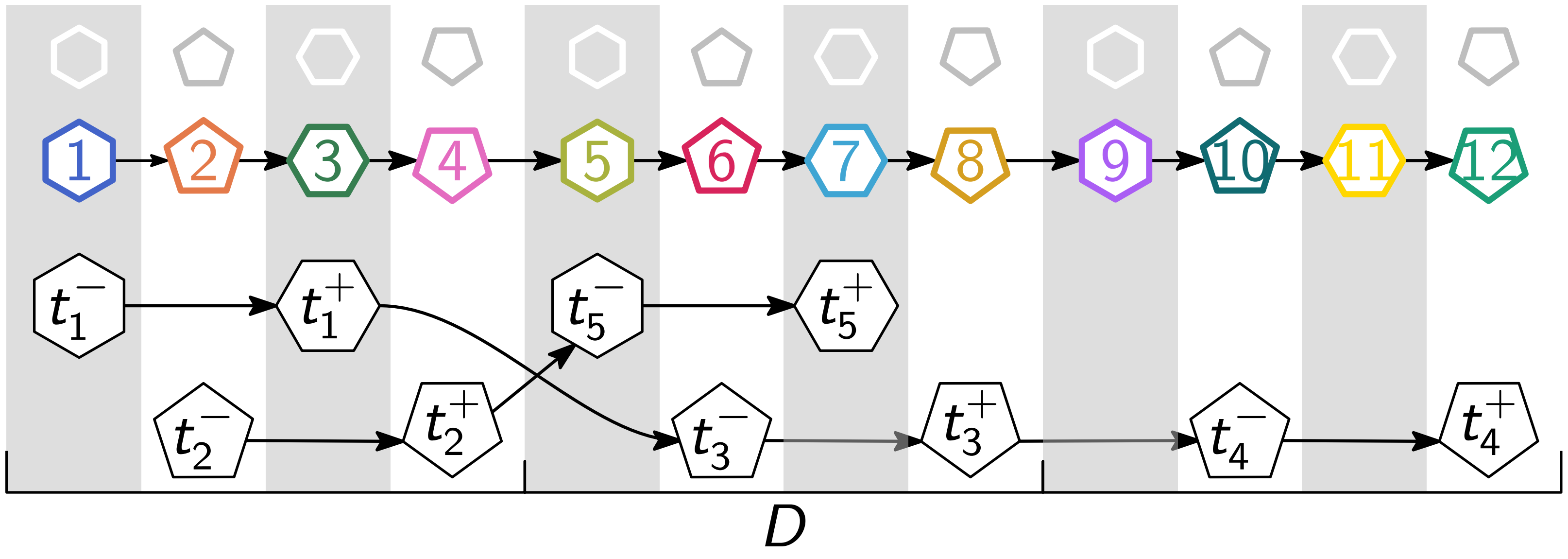

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors
- constant neighborhood diversity
    - 4 types of task vertices
    - 4 types of path vertices

$T_{\hexagon} = \{t_1, t_5\}$

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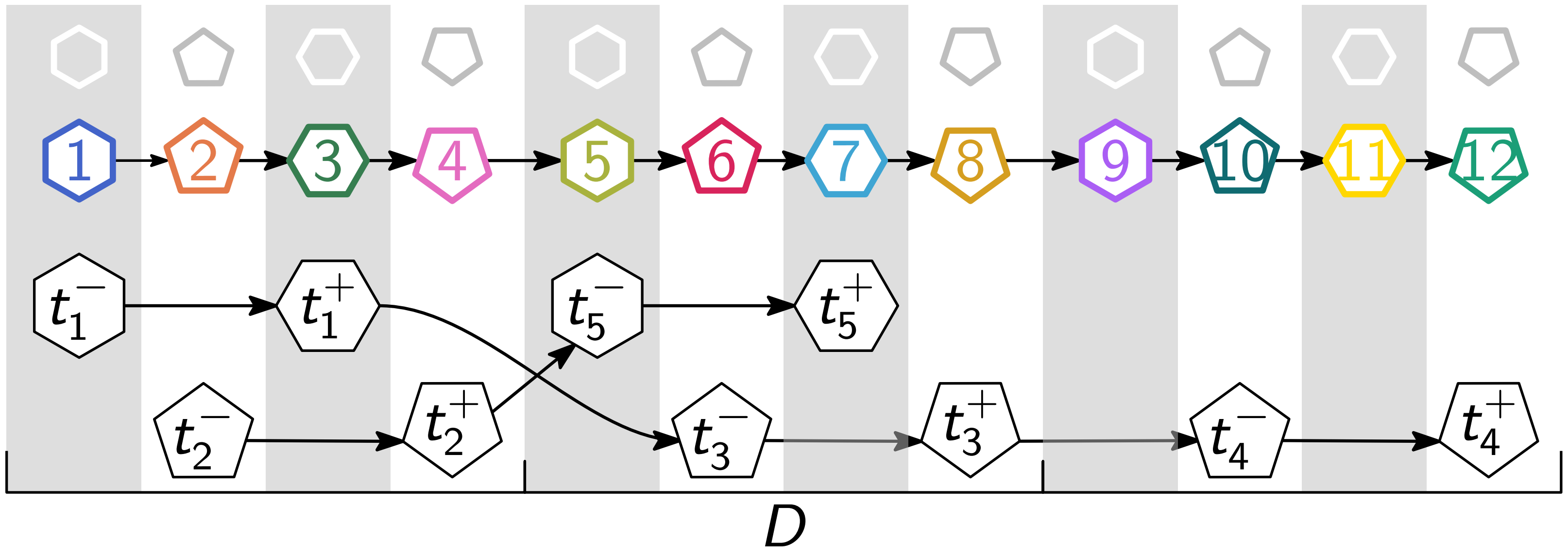

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors
- constant neighborhood diversity
- graph has proper $4D$-coloring ⇔ exists feasible schedule of length D

$T_{\hexagon} = \{t_1, t_5\}$

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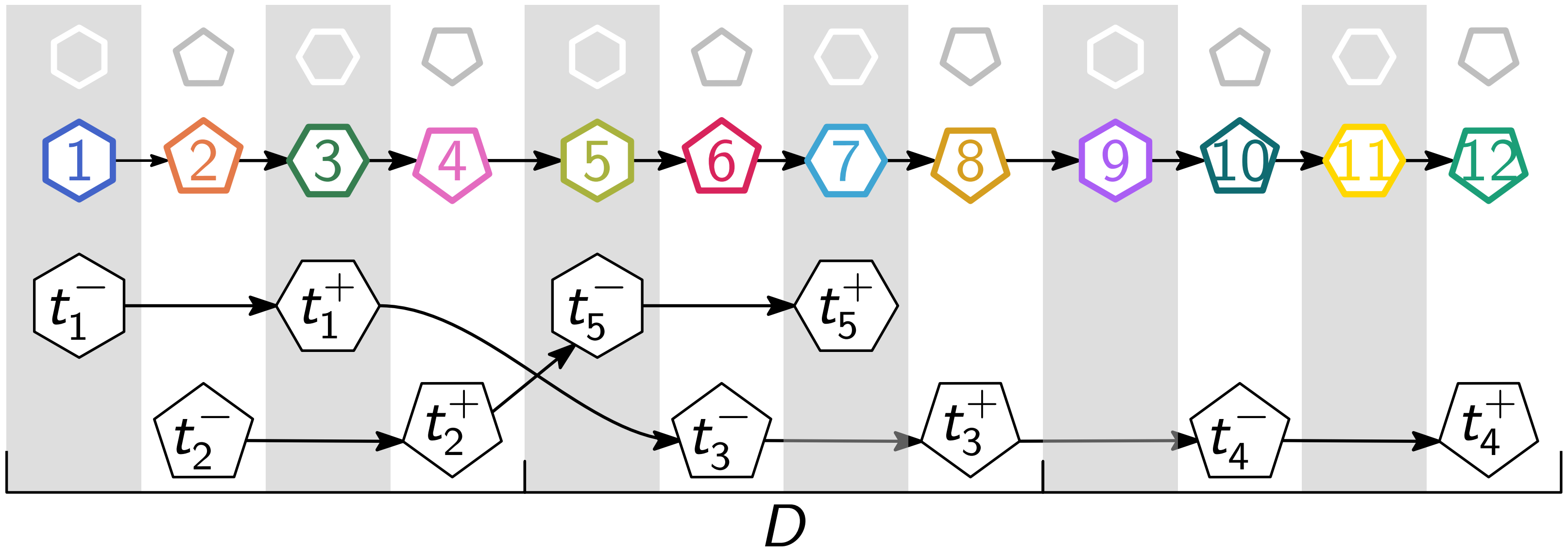

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Goal: show NP-hardness for constant neighborhood diversity

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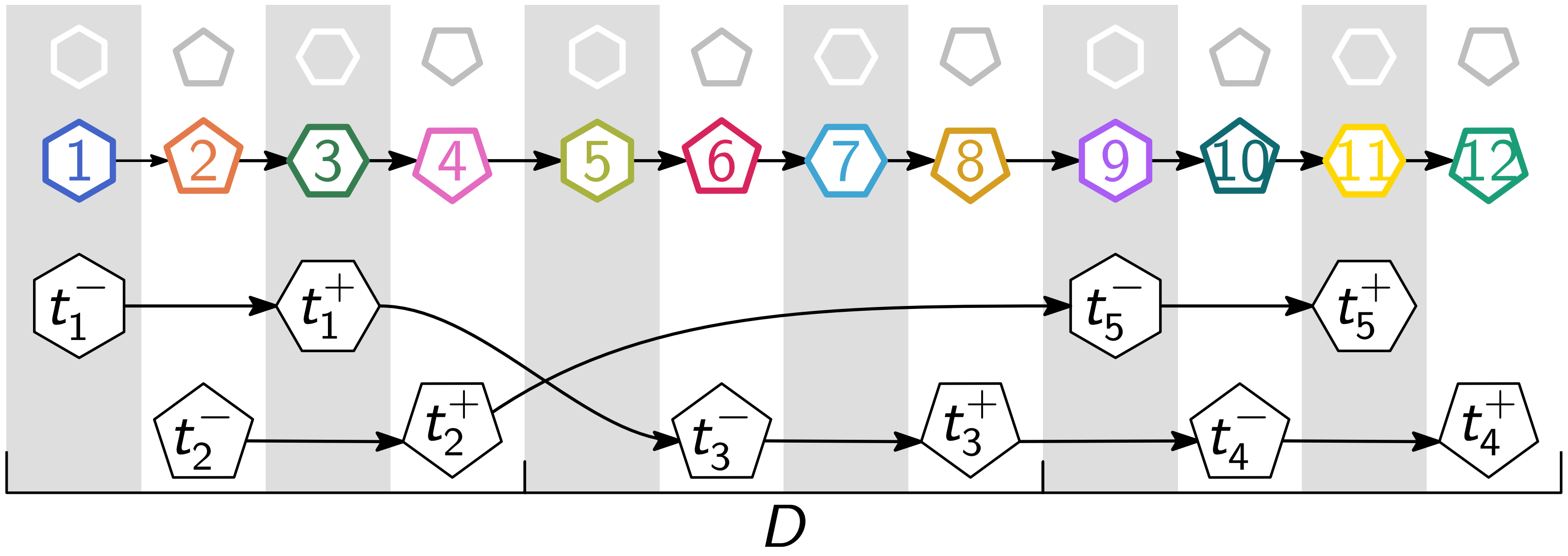

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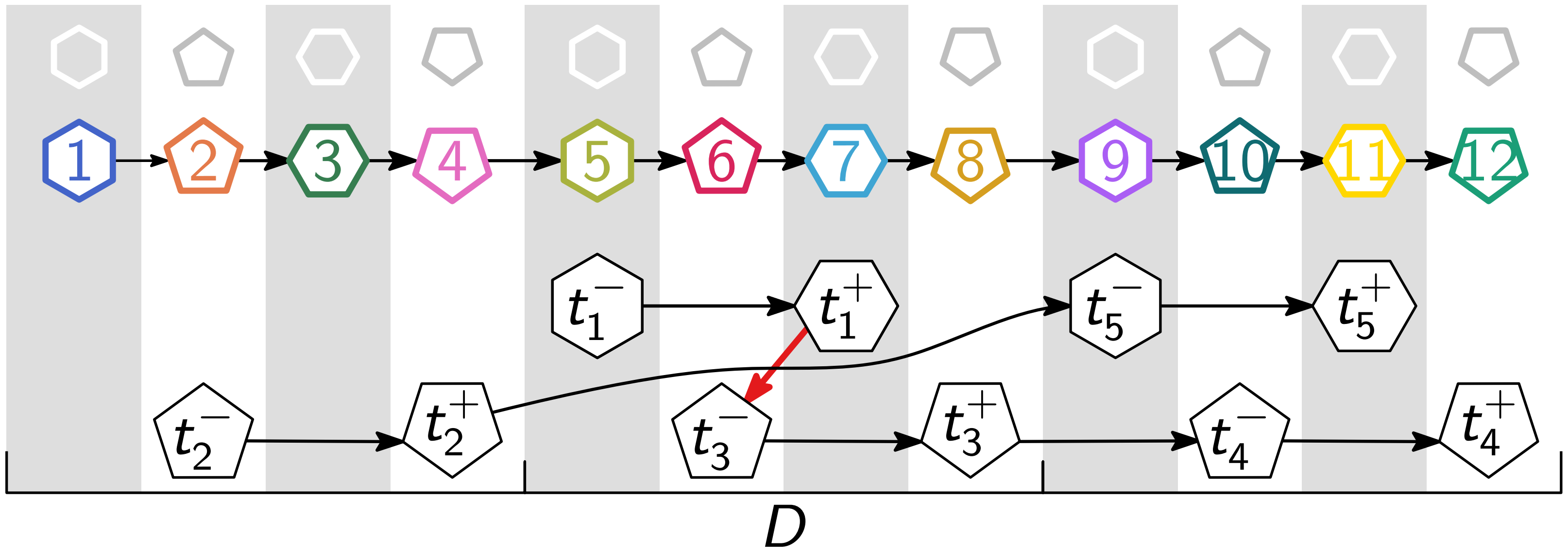

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
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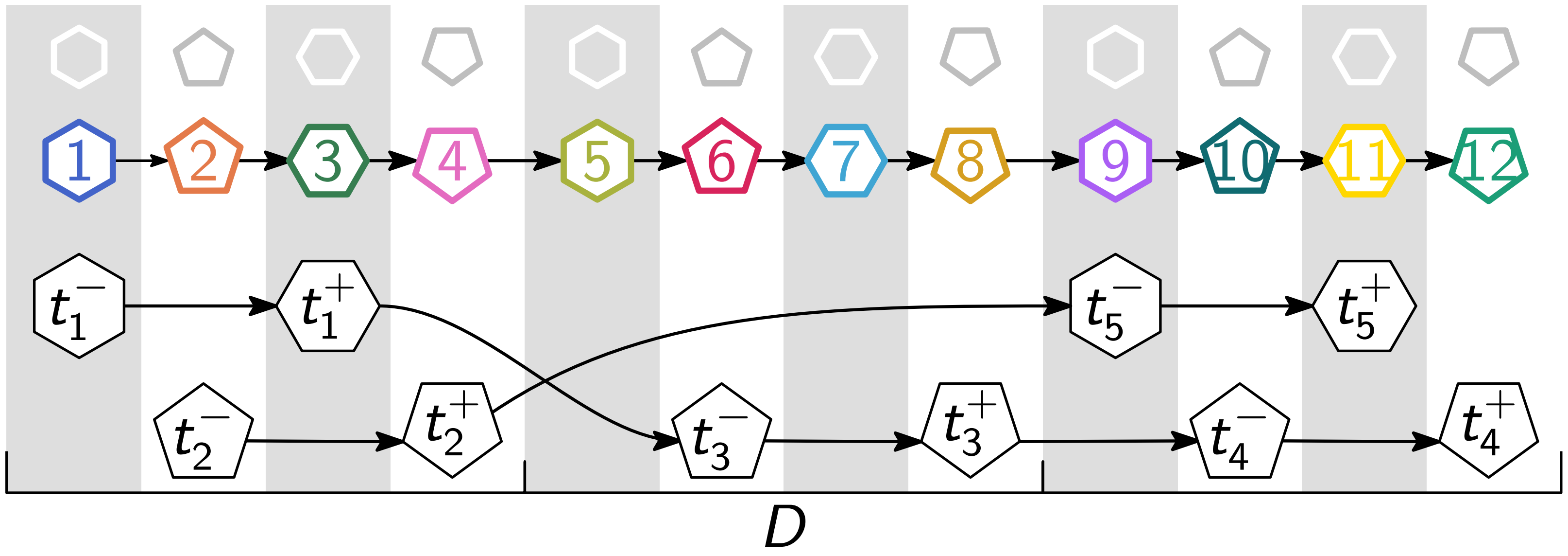

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
- timeslots as colors
- constant neighborhood diversity
- graph has proper $4D$-coloring $\Leftrightarrow$ exists feasible schedule of length D

$\implies$ MixedColoring NP-hard for constant neighborhood diversity

$T_{\text{⬡}} = \{t_1, t_5\}$

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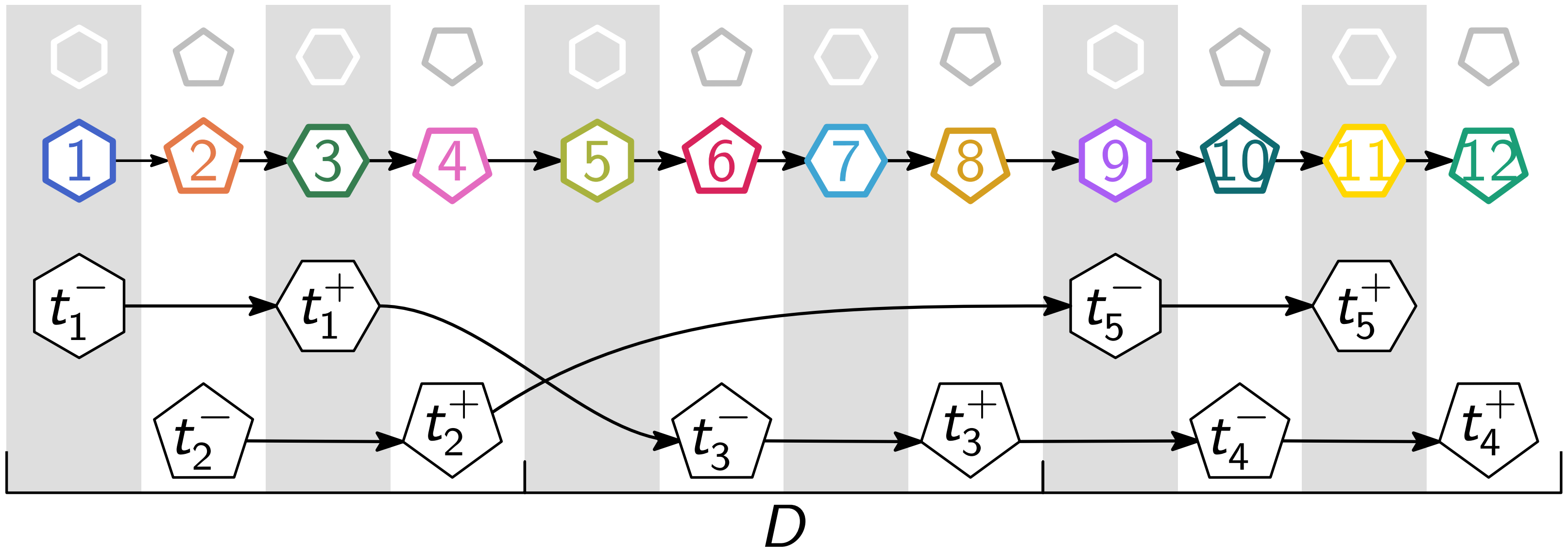

paraNP-Hardness Neighborhood Diversity

Goal: show NP-hardness for constant neighborhood diversity

Reduction:

- tasks as vertices
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- constant neighborhood diversity
- graph has proper $4D$-coloring $\Leftrightarrow$ exists feasible schedule of length D

$\Longrightarrow$ MixedColoring paraNP-hard w.r.t. neighborhood diversity

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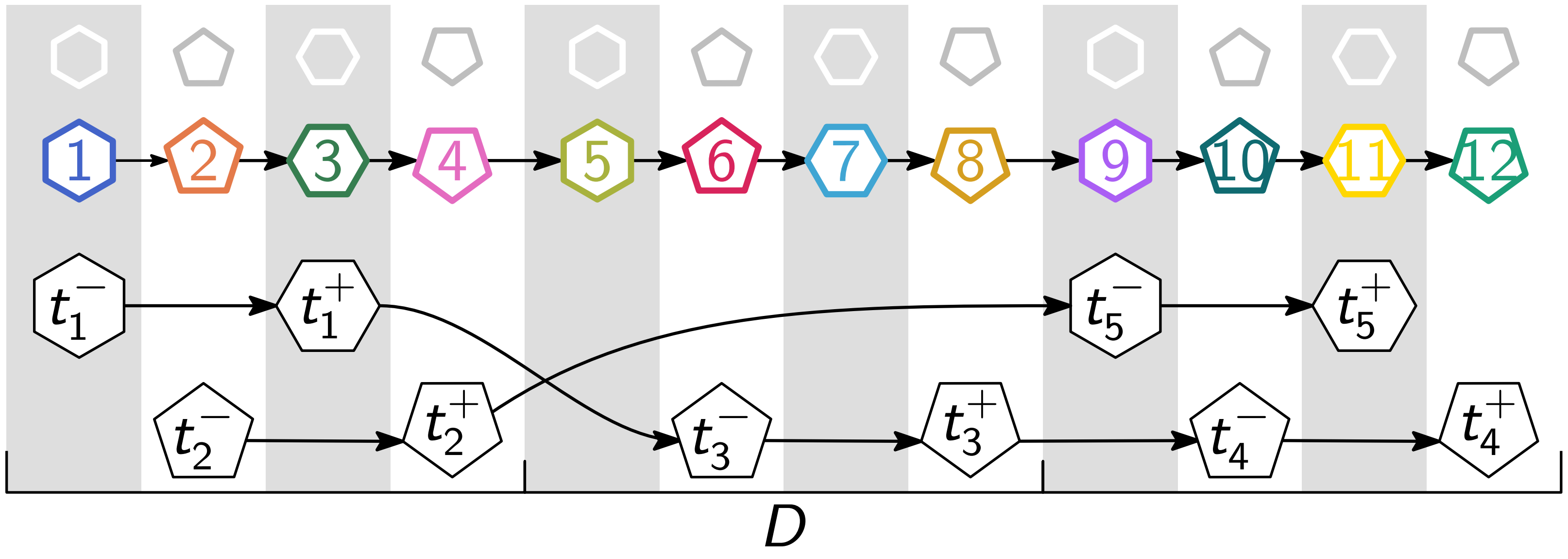


FPT Mixed Neighborhood Diversity

FPT Mixed Neighborhood Diversity

Theorem

MixedColoring is FPT w.r.t. mixed neighborhood diversity

FPT Mixed Neighborhood Diversity

Theorem

MIXEDCOLORING is FPT w.r.t. mixed neighborhood diversity

Recall:

- vertices of each type induce clique or independent set
- vertices in type inducing independent set can receive same color

FPT Mixed Neighborhood Diversity

Theorem

MixedColoring is FPT w.r.t. mixed neighborhood diversity

Recall:

- vertices of each type induce clique or independent set
- vertices in type inducing independent set can receive same color

Assumption: each type induces clique

FPT Mixed Neighborhood Diversity

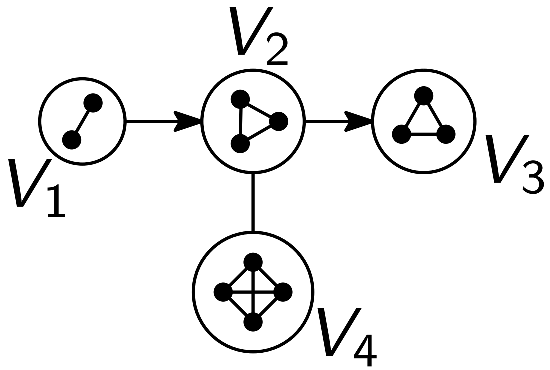
Theorem

MixedColoring is FPT w.r.t. mixed neighborhood diversity

Recall:

- vertices of each type induce clique or independent set
- vertices in type inducing independent set can receive same color

Assumption: each type induces clique



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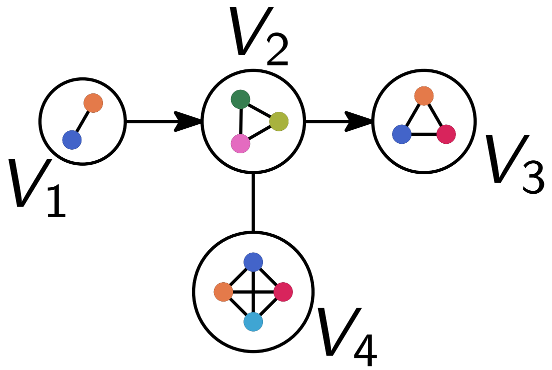

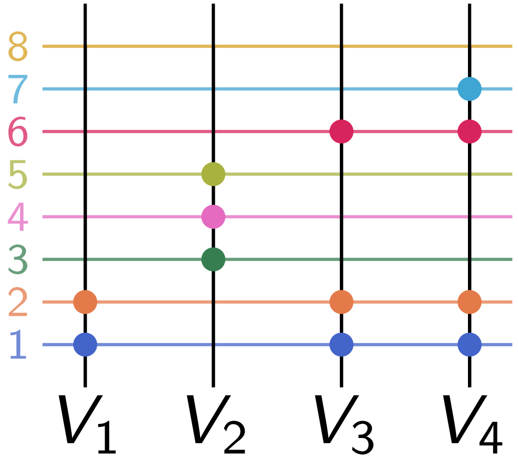

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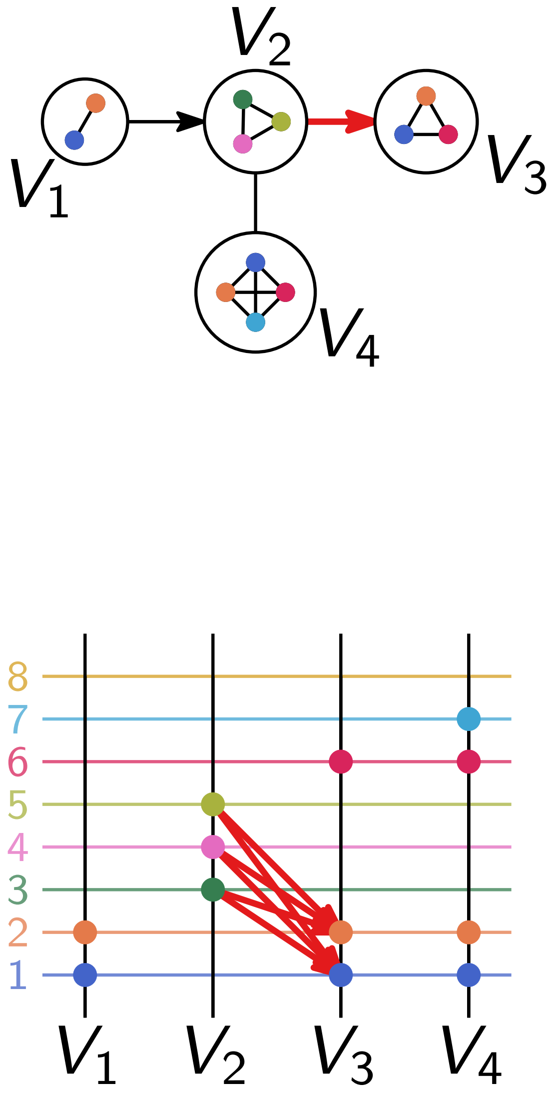


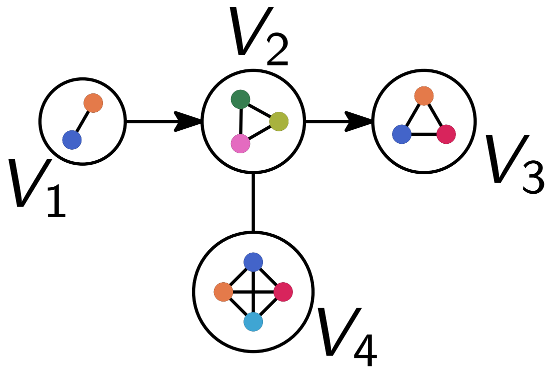
FPT Mixed Neighborhood Diversity

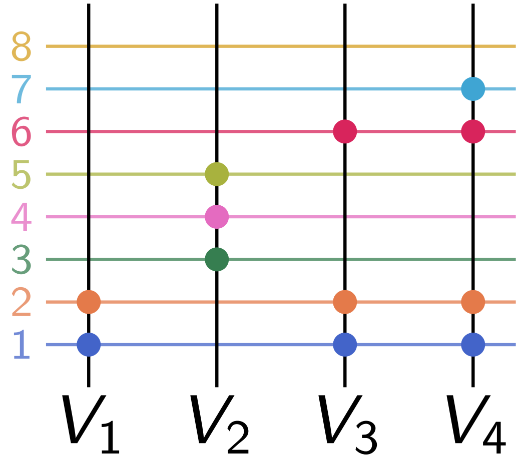
Theorem

MixedColoring is FPT w.r.t. mixed neighborhood diversity

Lemma

coloring c respects arcs if $\forall (V_i, V_j) \in A\colon c^{+}(V_i) \leq c^{-}(V_j)$





FPT Mixed Neighborhood Diversity

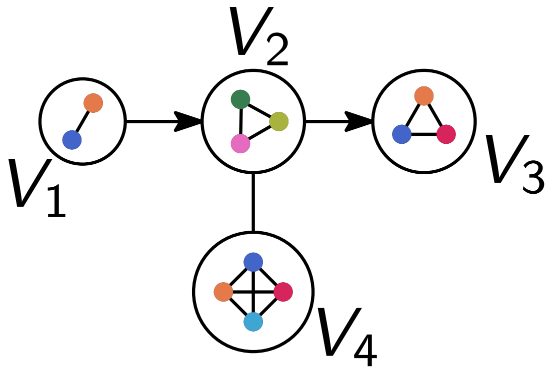
Theorem
MixedColoring is FPT w.r.t. mixed neighborhood diversity

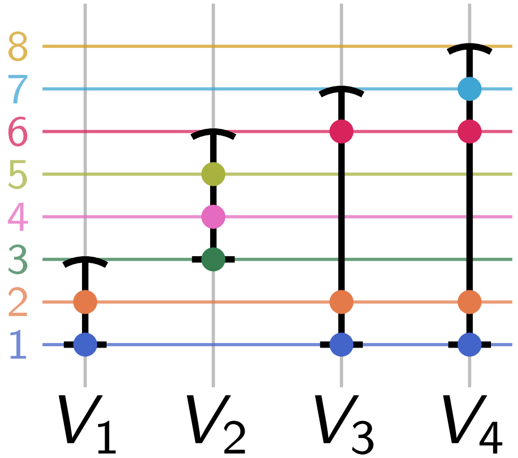
Lemma

coloring c respects arcs if $\forall (V_i, V_j) \in A\colon c^{+}(V_i) \leq c^{-}(V_j)$

largest color in $V_i + 1$

smallest color in V_j





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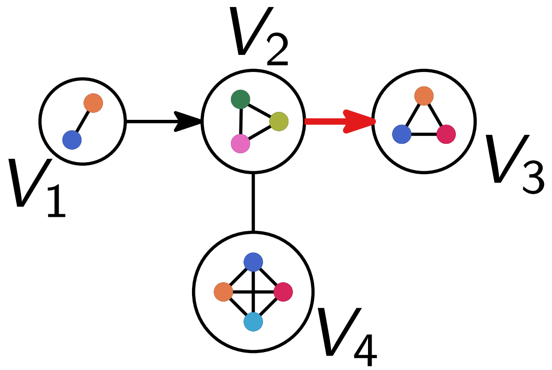

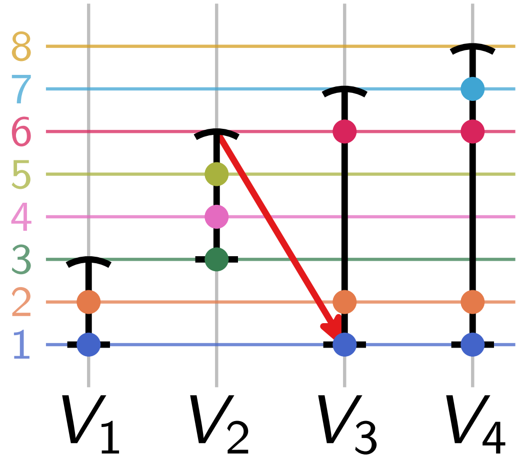

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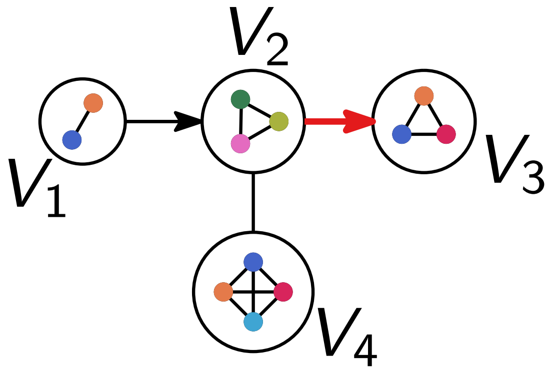

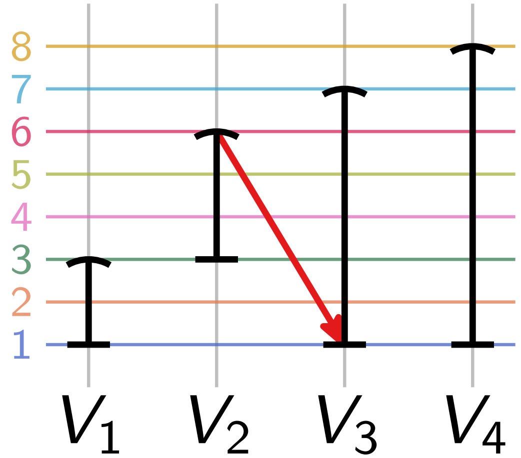

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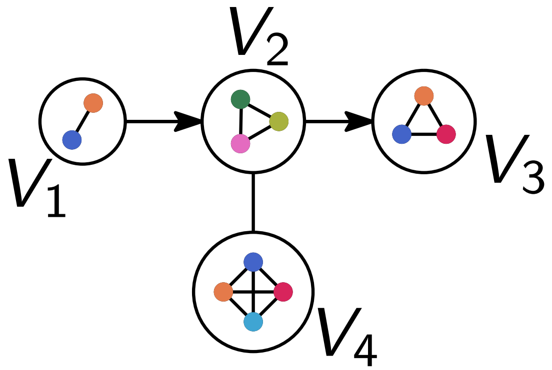

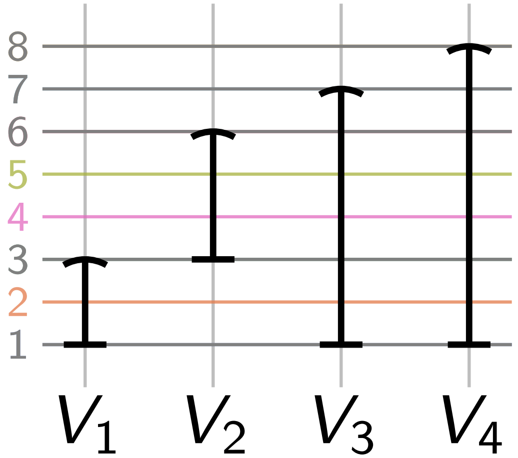

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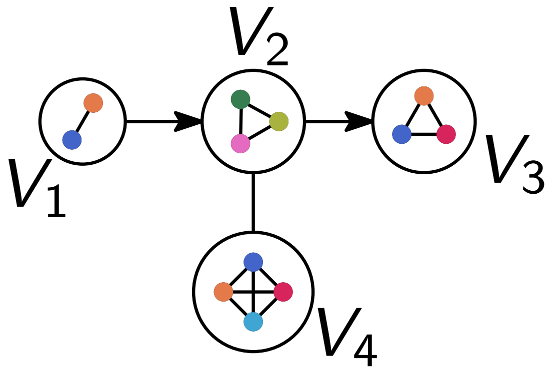

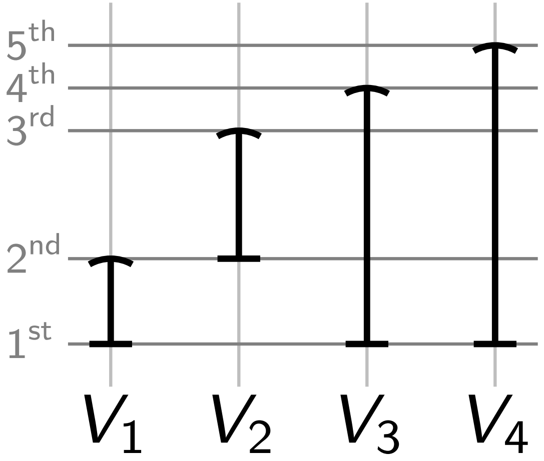

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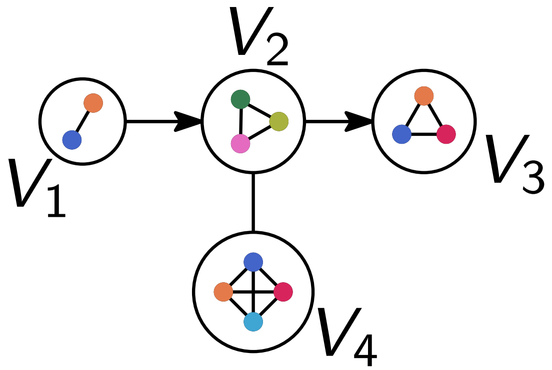

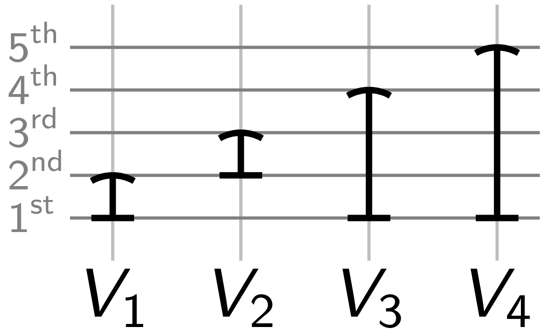

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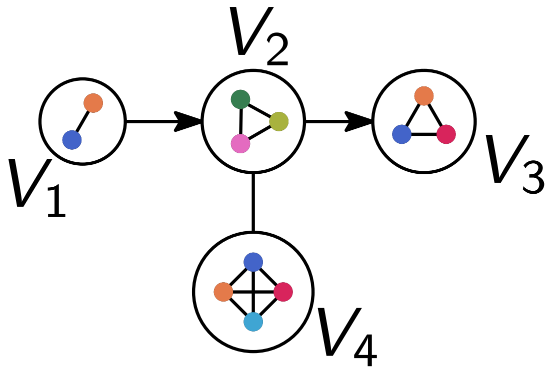

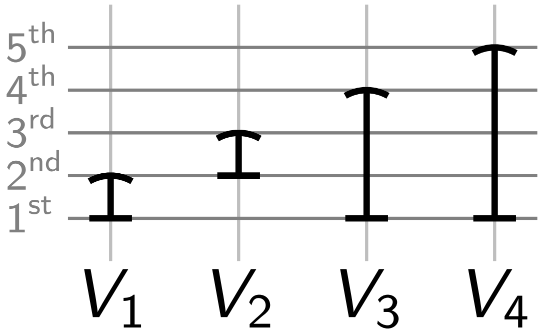

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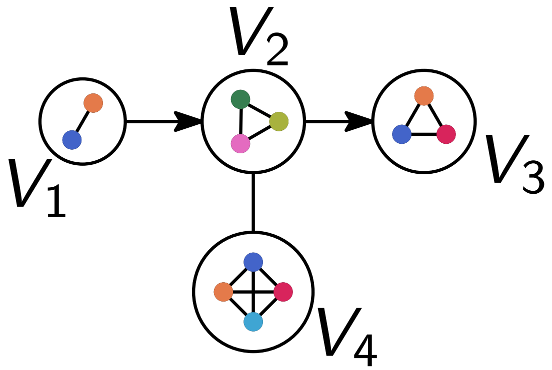

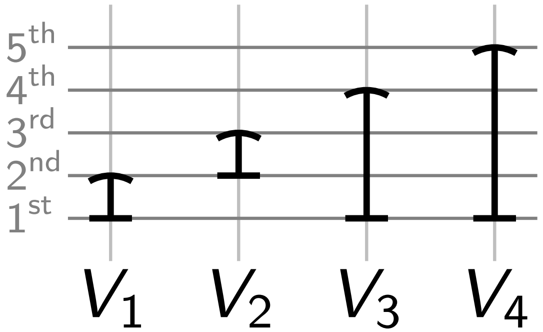

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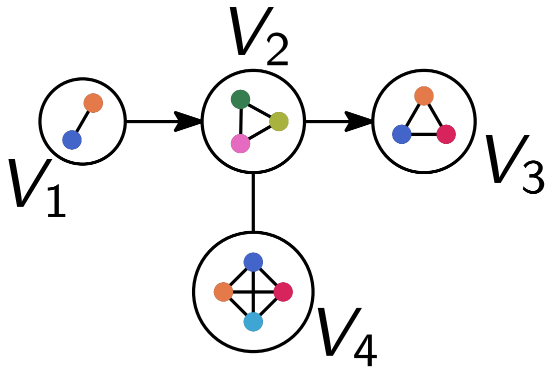

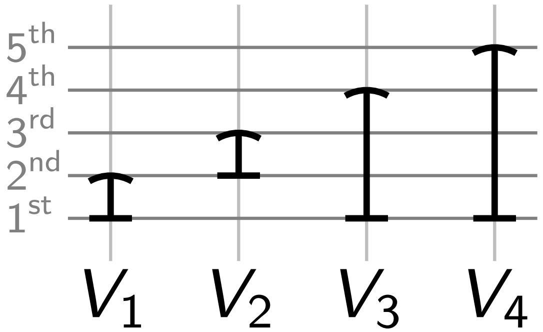

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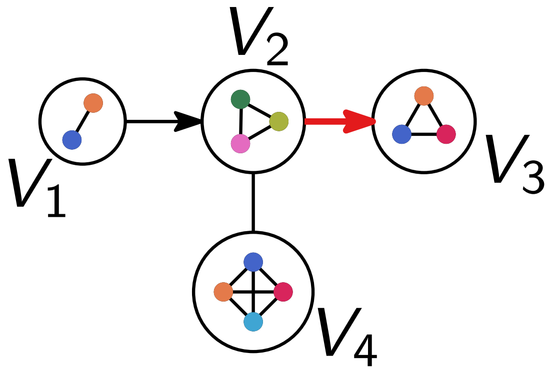

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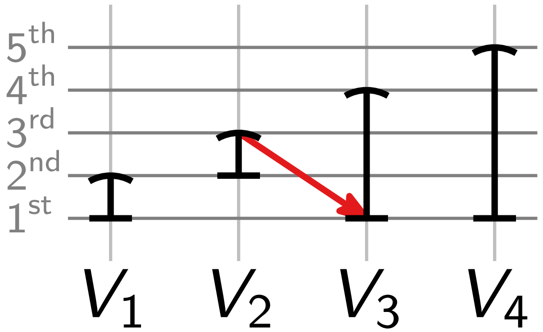

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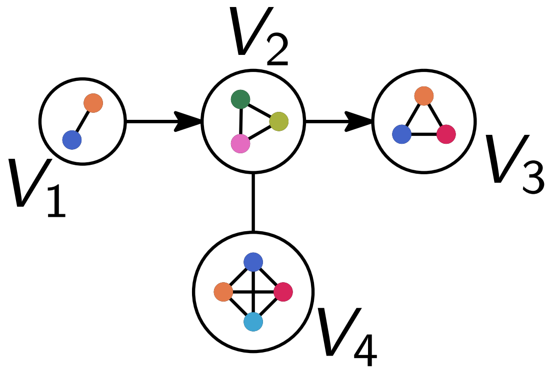

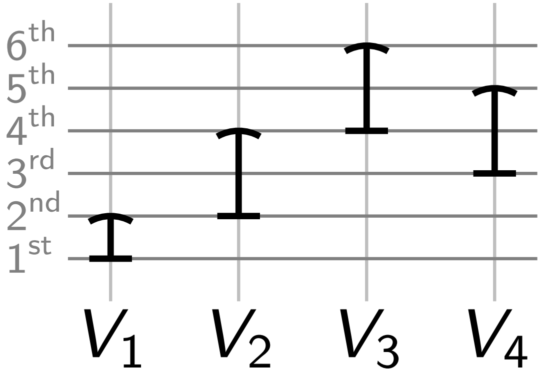

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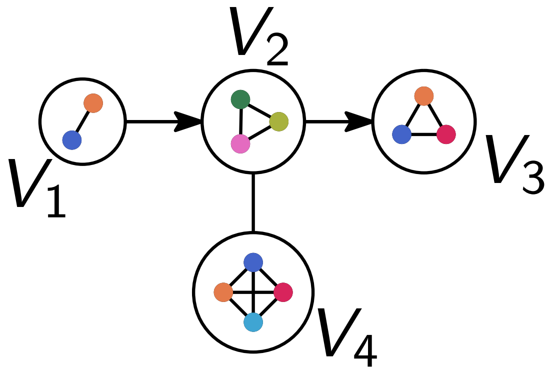

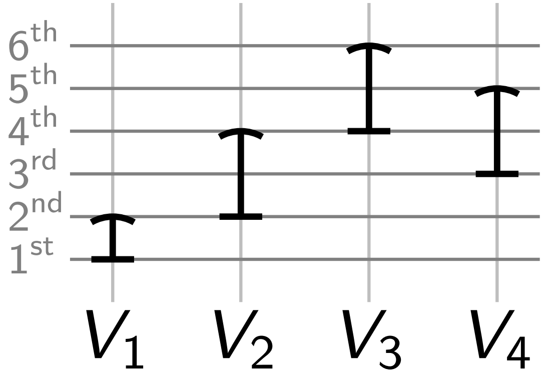

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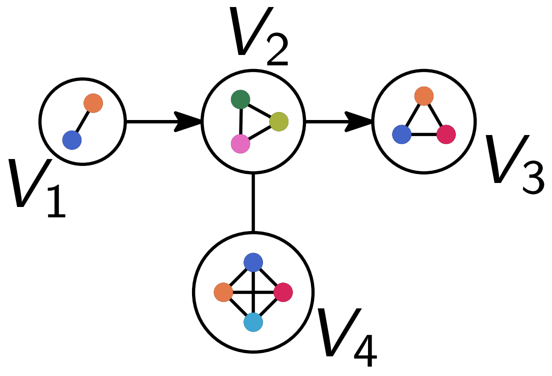

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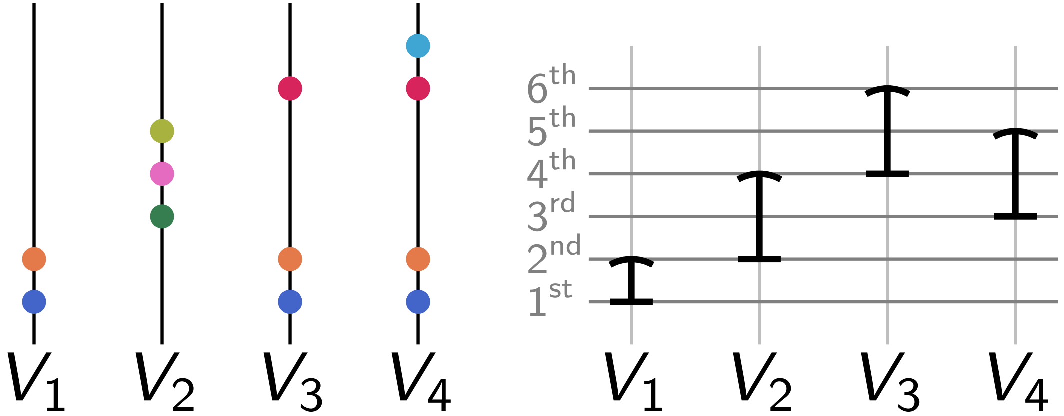

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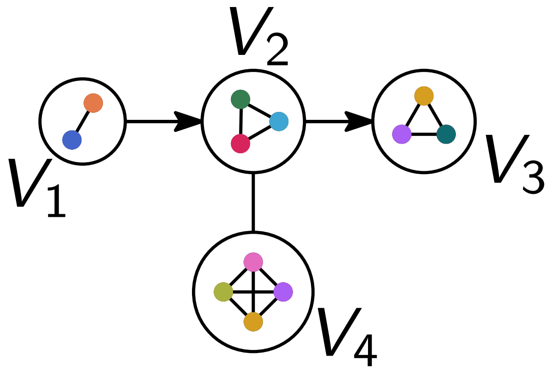

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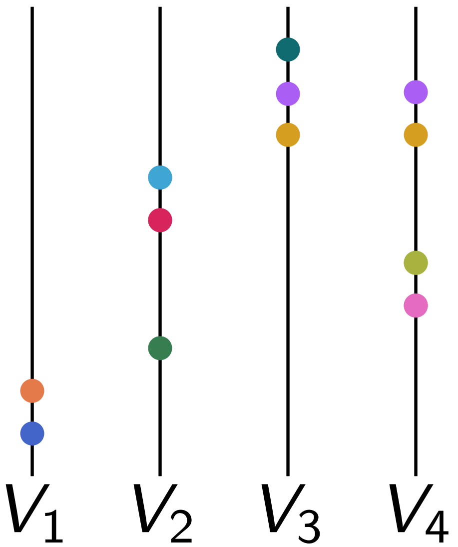

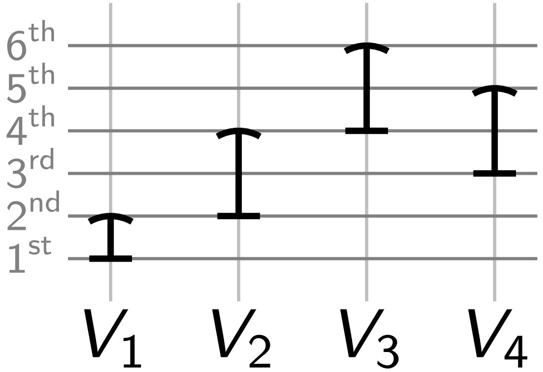

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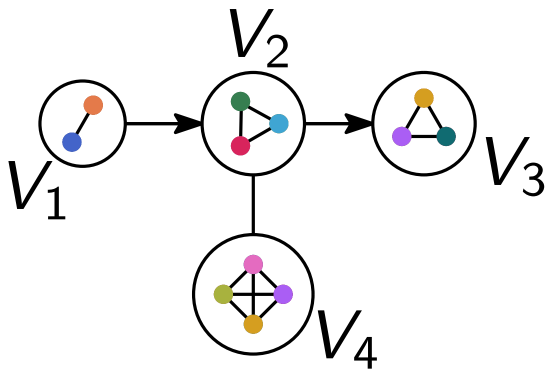

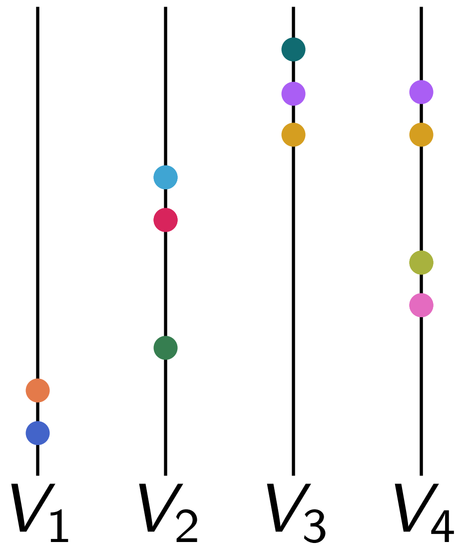

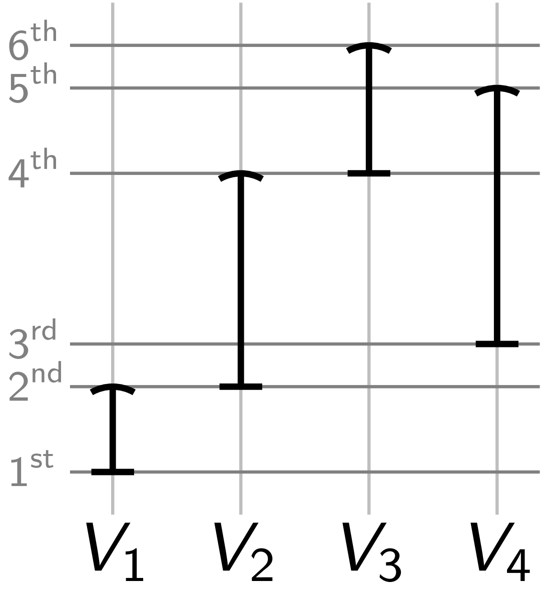

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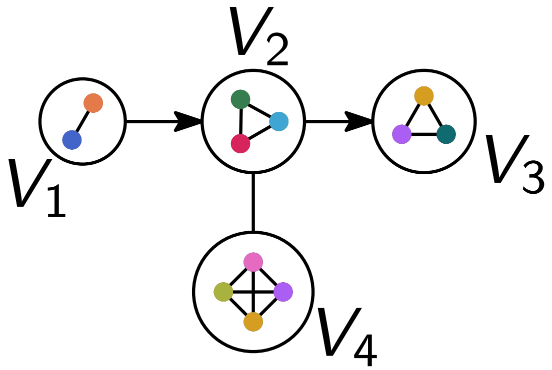

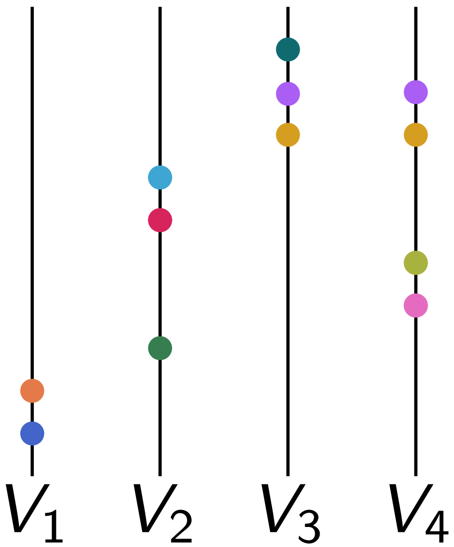

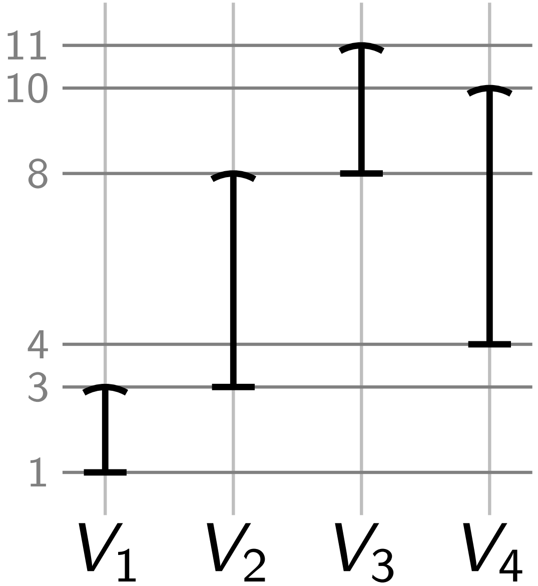

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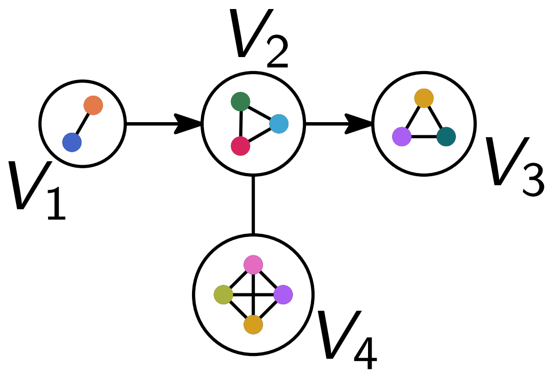

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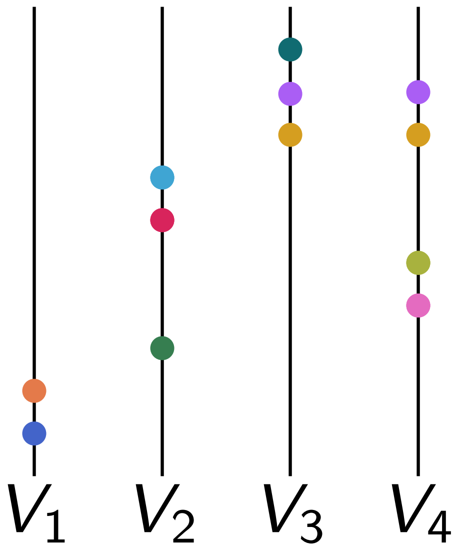

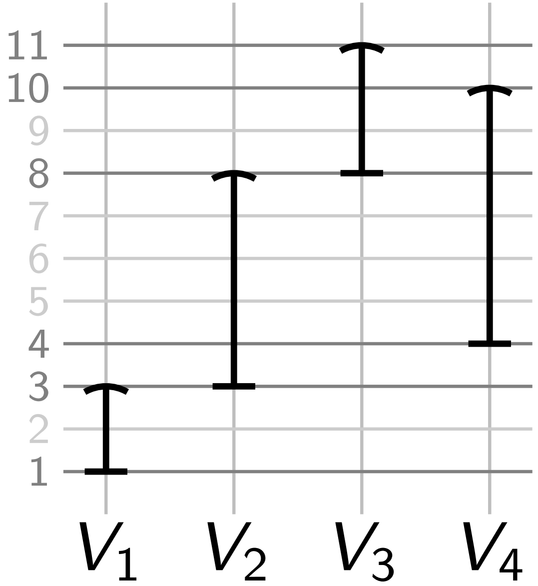

FPT Mixed Neighborhood Diversity

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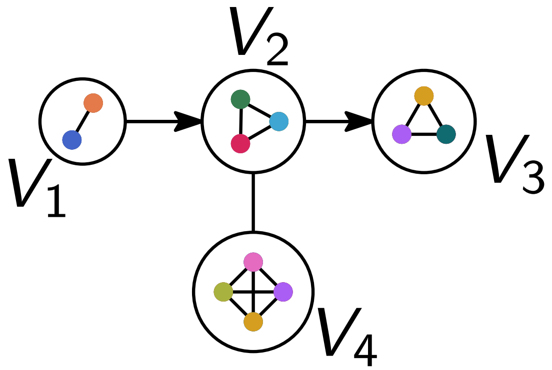

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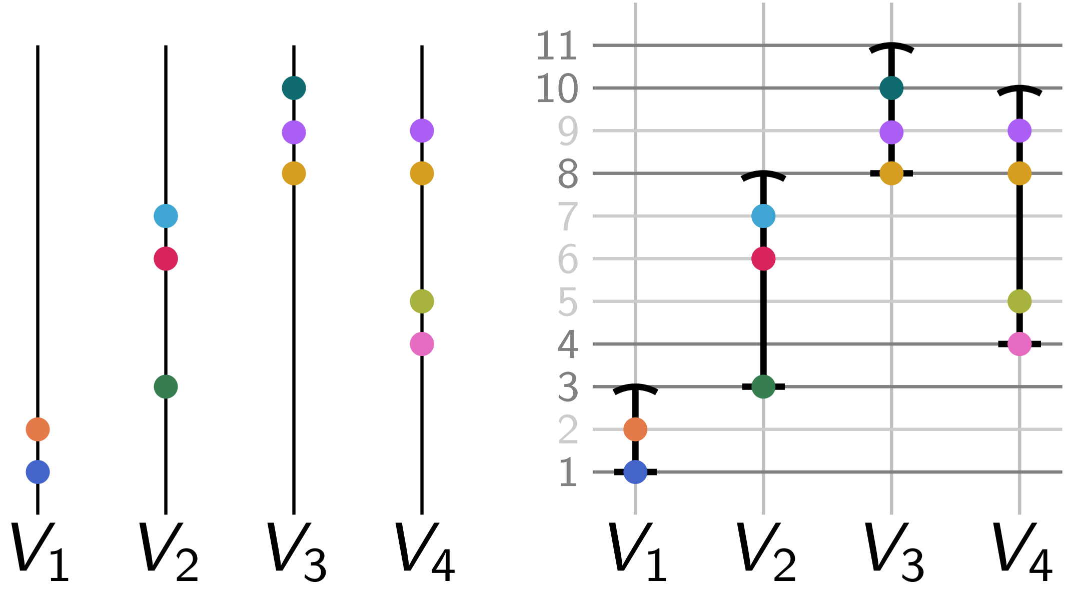

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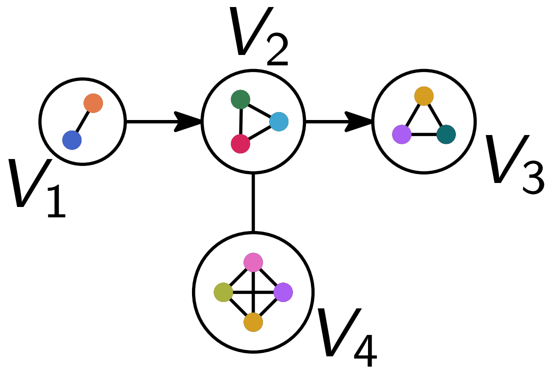

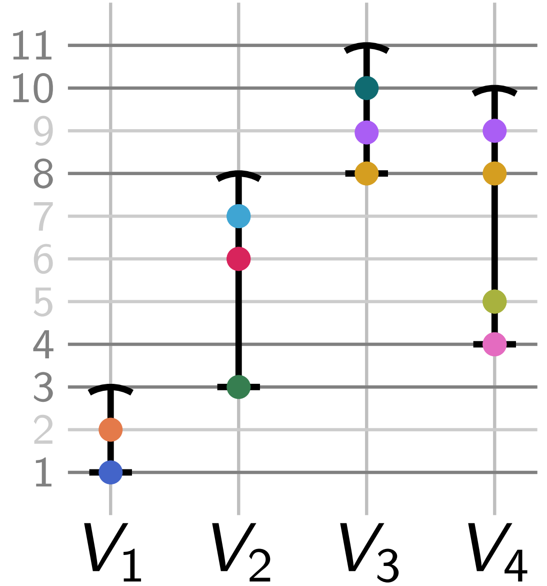

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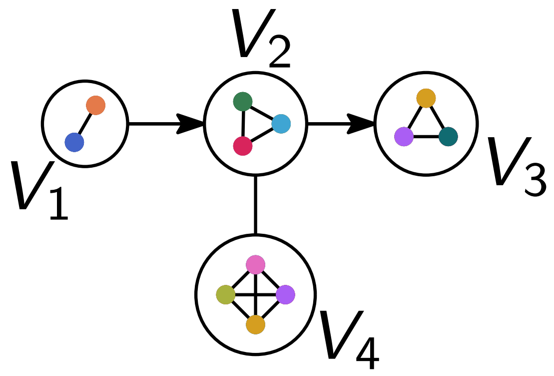
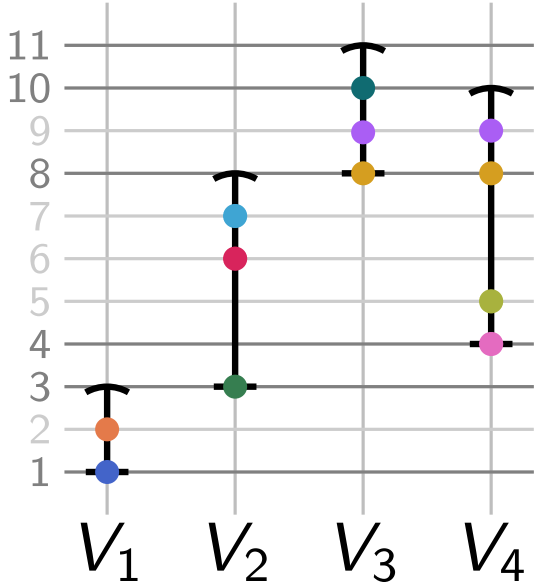
Lemma

- coloring corresponding to proper preorder respects arcs
- each proper coloring corresponds to at least one proper preorder





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foreach proper preorder do
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FPT Mixed Neighborhood Diversity

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$2^{\mathcal{O}(\text{mnd} \log \text{mnd})}$

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Lemma

finding a proper k-coloring corresponding to a proper preorder is FPT w.r.t. mixed neighborhood diversity

FPT Mixed Neighborhood Diversity

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$\implies$ FPT w.r.t. mixed neighborhood diversity

Lemma

finding a proper k-coloring corresponding to a proper preorder is FPT w.r.t. mixed neighborhood diversity

FPT Mixed Neighborhood Diversity

Theorem

MixedColoring is FPT w.r.t. mixed neighborhood diversity

Algorithm

$2^{\mathcal{O}(\text{mnd}\log\text{mnd})}$

```
foreach proper preorder do
    check if there exists a corresponding proper $k$-coloring
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$\implies$ FPT w.r.t. mixed neighborhood diversity

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Proof (Idea):

FPT Mixed Neighborhood Diversity

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$\implies$ FPT w.r.t. mixed neighborhood diversity

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finding a proper k-coloring corresponding to a proper preorder is FPT w.r.t. mixed neighborhood diversity

Proof (Idea):

- arcs no longer relevant

FPT Mixed Neighborhood Diversity

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MixedColoring is FPT w.r.t. mixed neighborhood diversity

Algorithm

$2^{\mathcal{O}(\text{mnd} \log \text{mnd})}$

```
foreach proper preorder do
    check if there exists a corresponding proper $k$-coloring
```

$\Longrightarrow$ FPT w.r.t. mixed neighborhood diversity

Lemma

finding a proper *k*-coloring corresponding to a proper preorder is FPT w.r.t. mixed neighborhood diversity

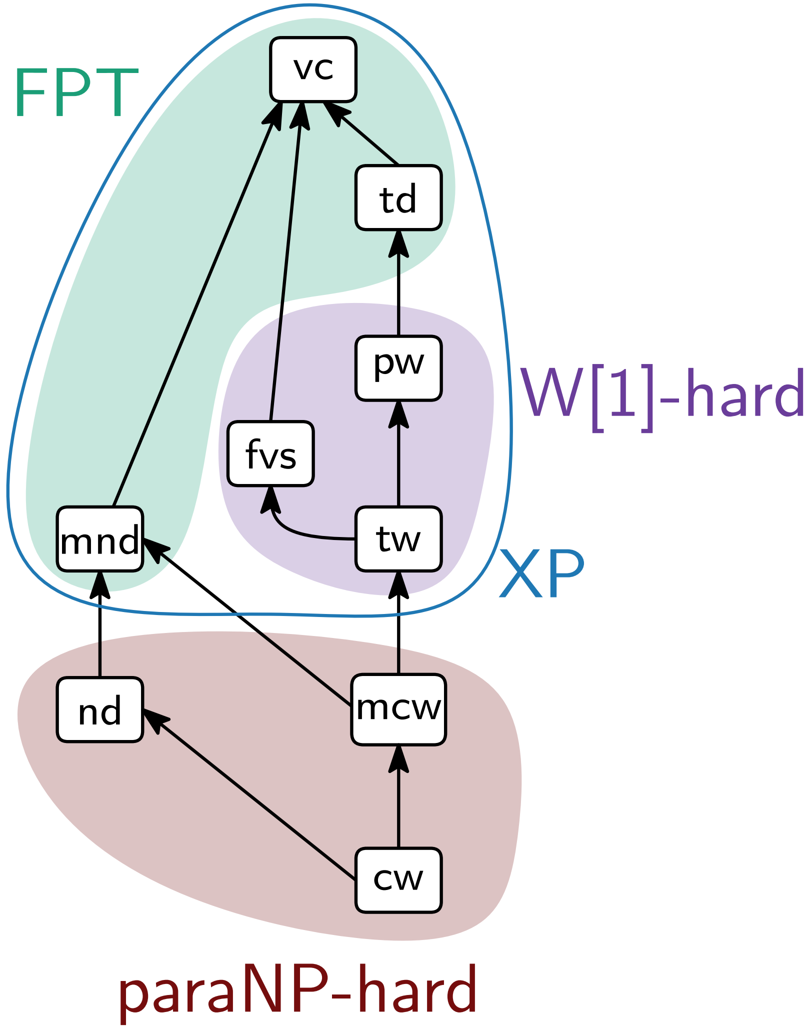
Proof (Idea):

- arcs no longer relevant
- similar approach as FPT-Algo w.r.t. neighborhood diversity for Coloring

Conclusion

Complexity

MixedColoring

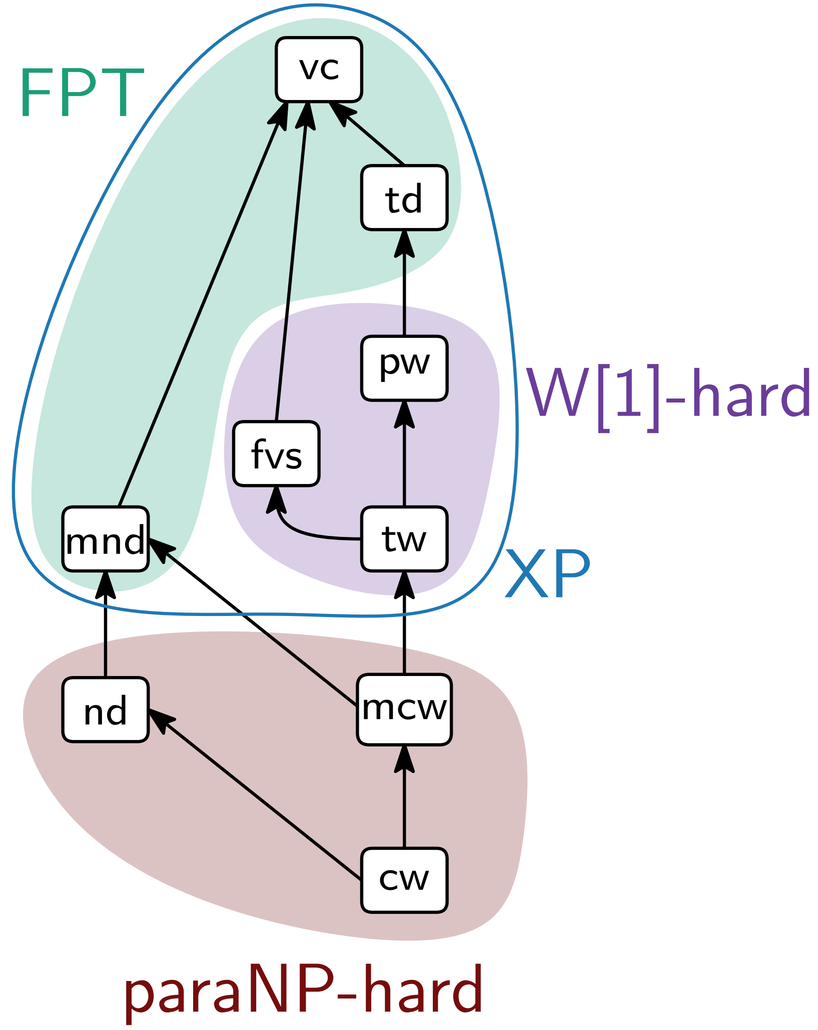


Conclusion

Open Problems

Complexity

MixedColoring



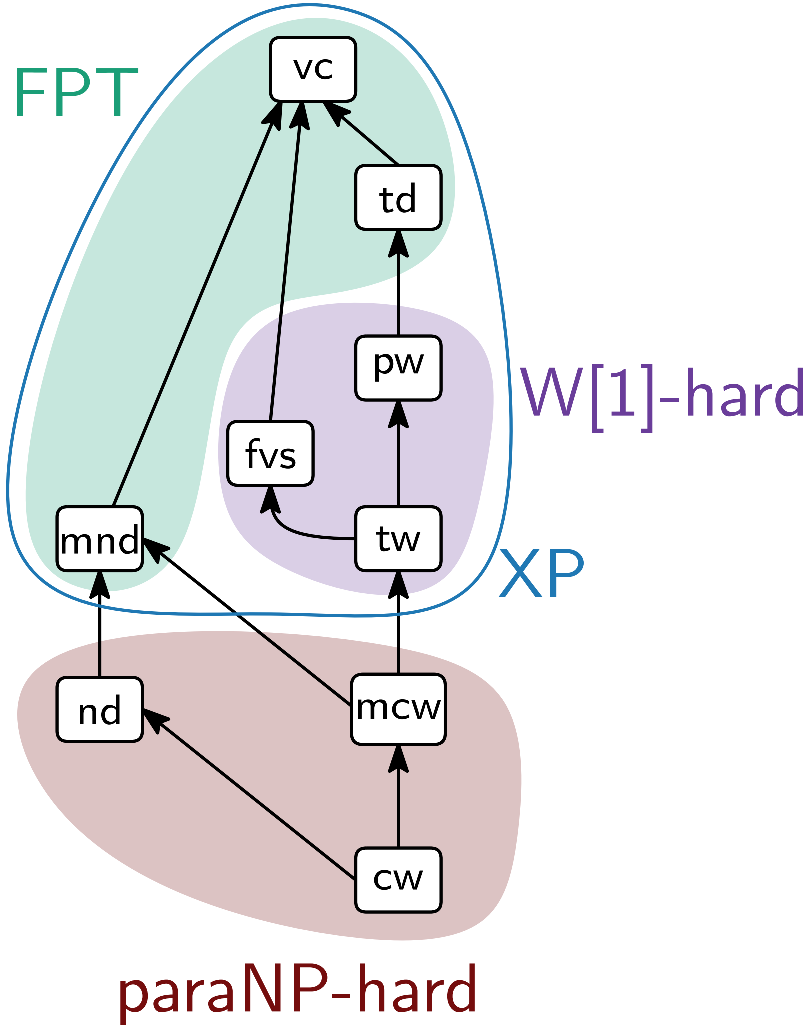
Conclusion

Open Problems

Further Parameters

Complexity

MixedColoring



Conclusion

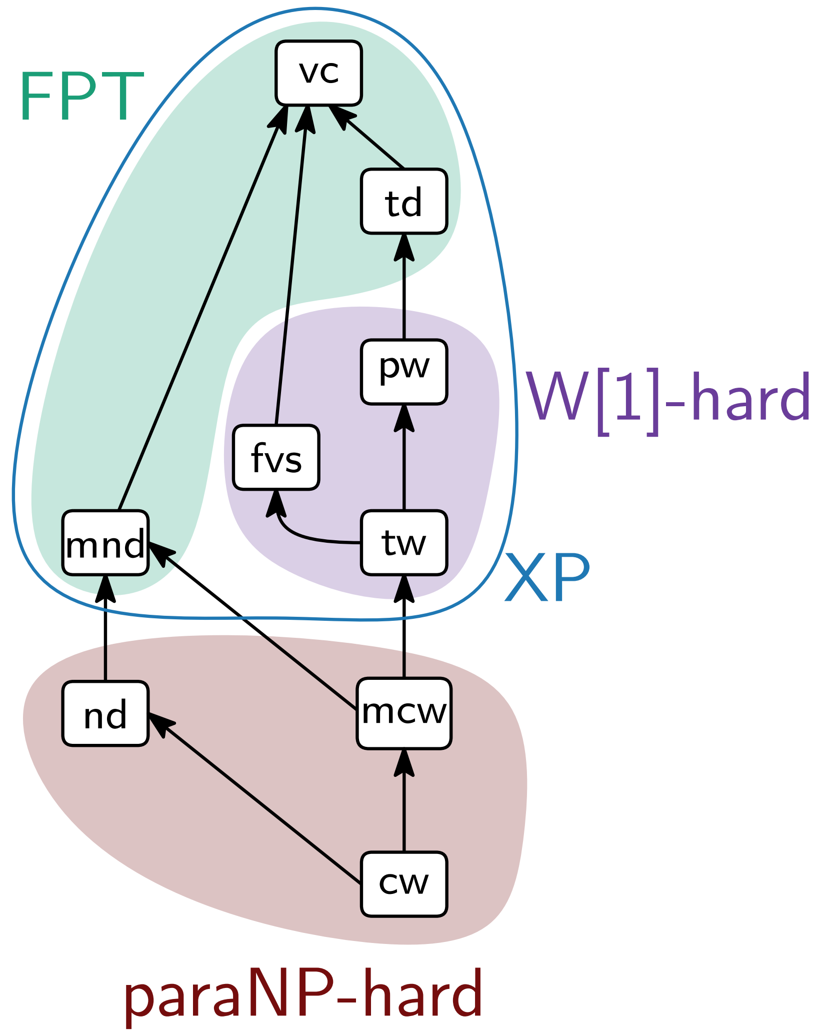
Open Problems

Further Parameters

- mixed variants

Complexity

MixedColoring



Conclusion

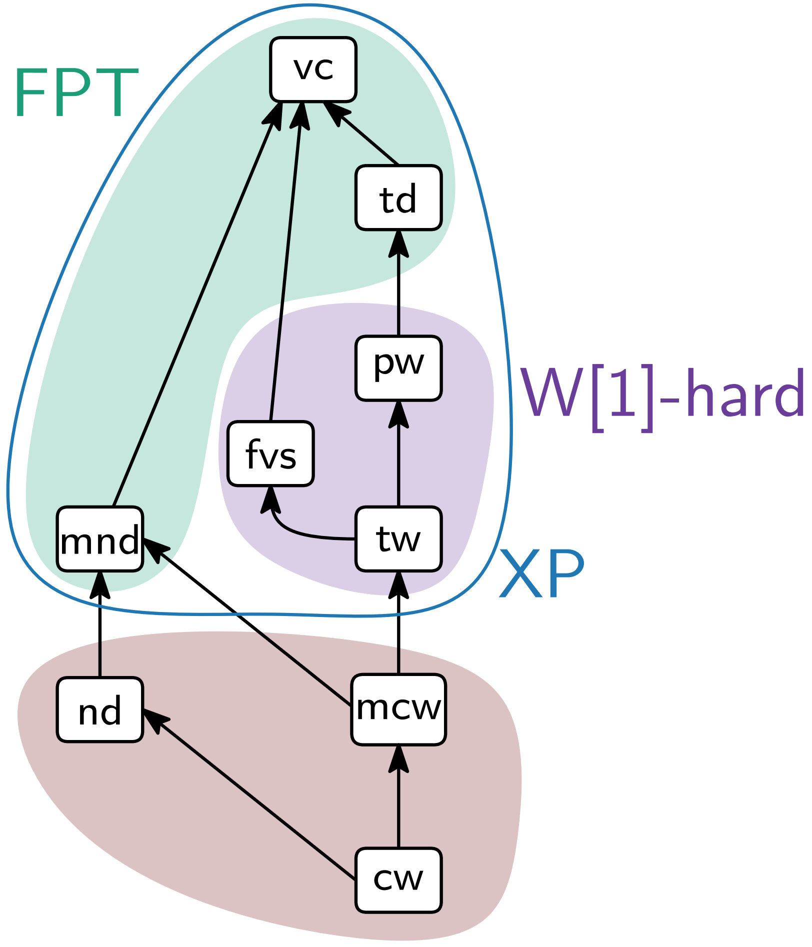
Open Problems

Further Parameters

- mixed variants
- complexity

Complexity

MixedColoring



paraNP-hard

Conclusion

Open Problems

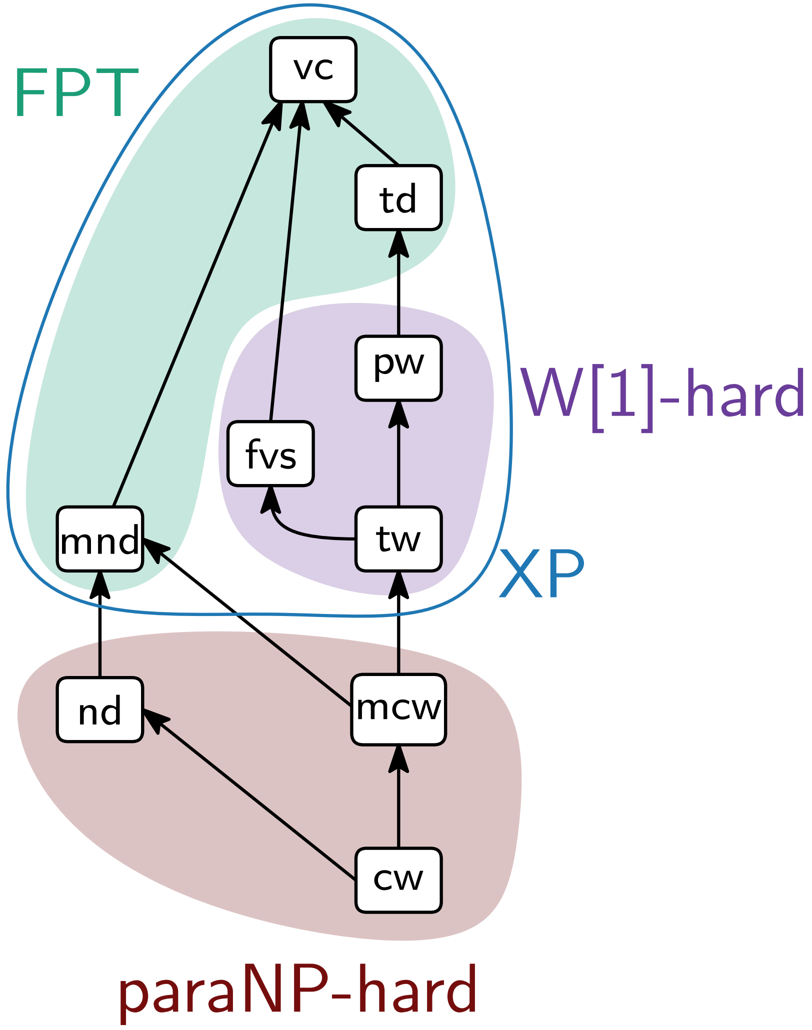
Further Parameters

- mixed variants
- complexity

Meta Theorems

Complexity

Mixed Coloring



Conclusion

Open Problems

Further Parameters

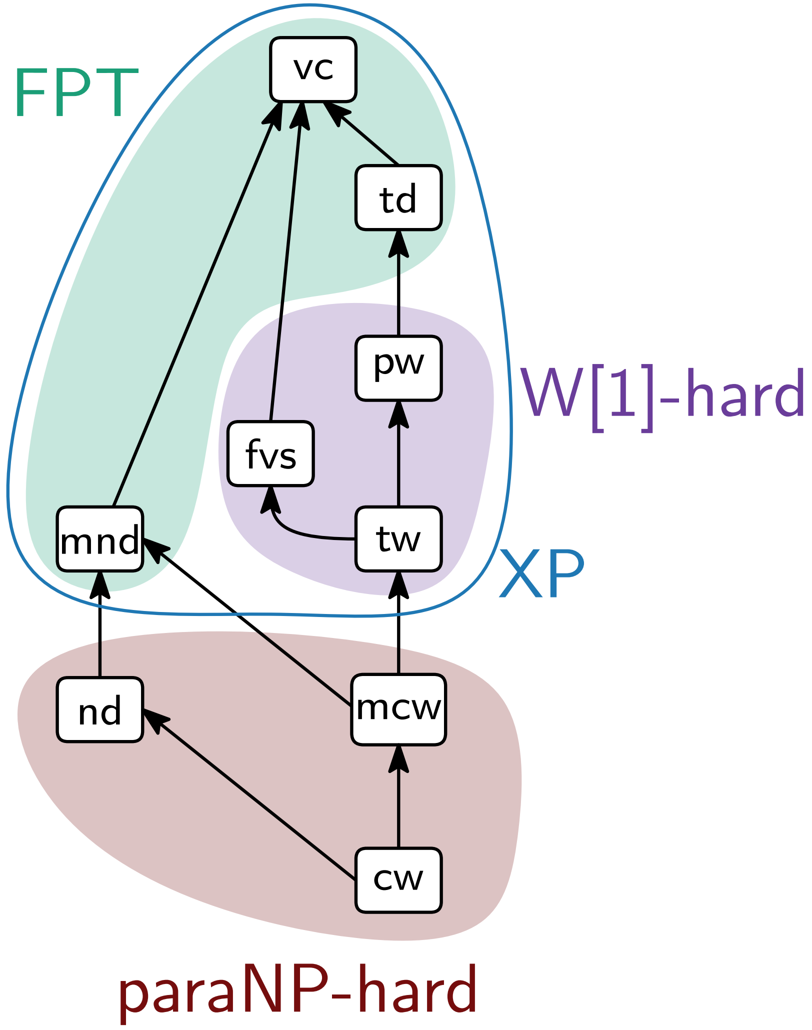
- mixed variants
- complexity

Meta Theorems

- Courcelle's Theorem

Complexity

MixedColoring



Conclusion

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Further Parameters

- mixed variants
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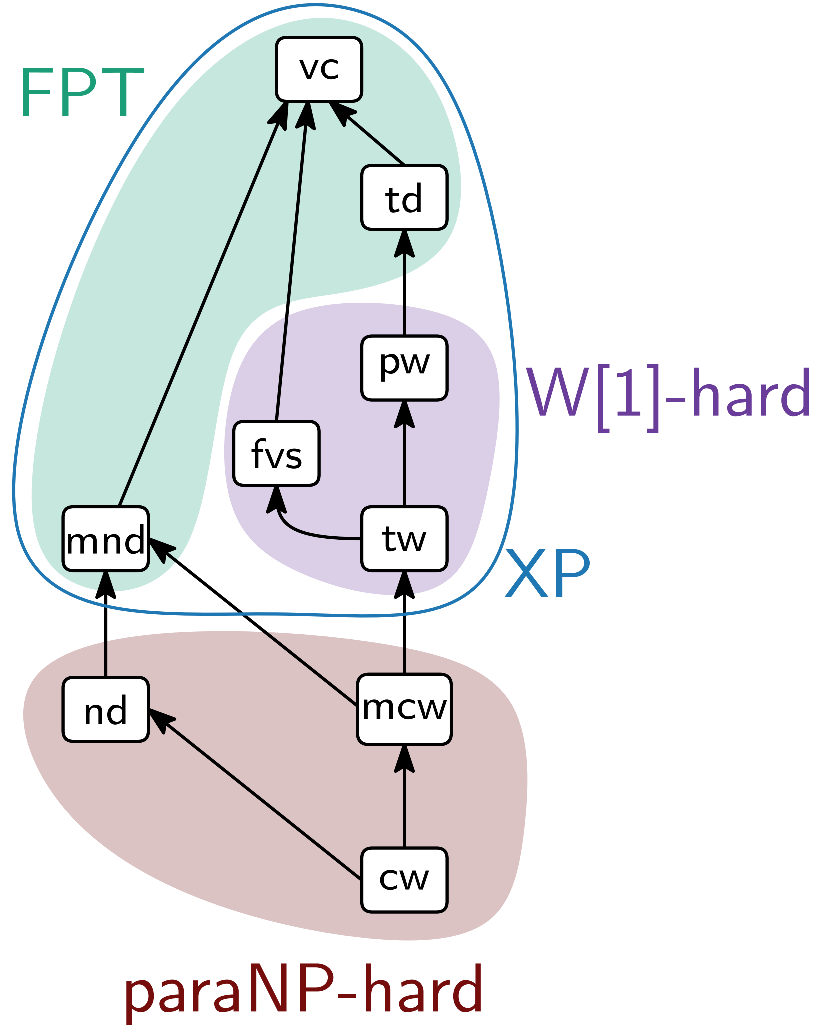
Variants of Coloring

Complexity

MixedColoring



Conclusion

Open Problems

Further Parameters

- mixed variants
- complexity

Meta Theorems

- Courcelle's Theorem

Variants of Coloring

- ListColoring

Complexity

MixedColoring