The Complexity of Counting Turns in the Line-Based Dial-a-Ride Problem

SOFSEM 25

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WÜRZBURG

Dial-a-Ride-Problem (DARP)

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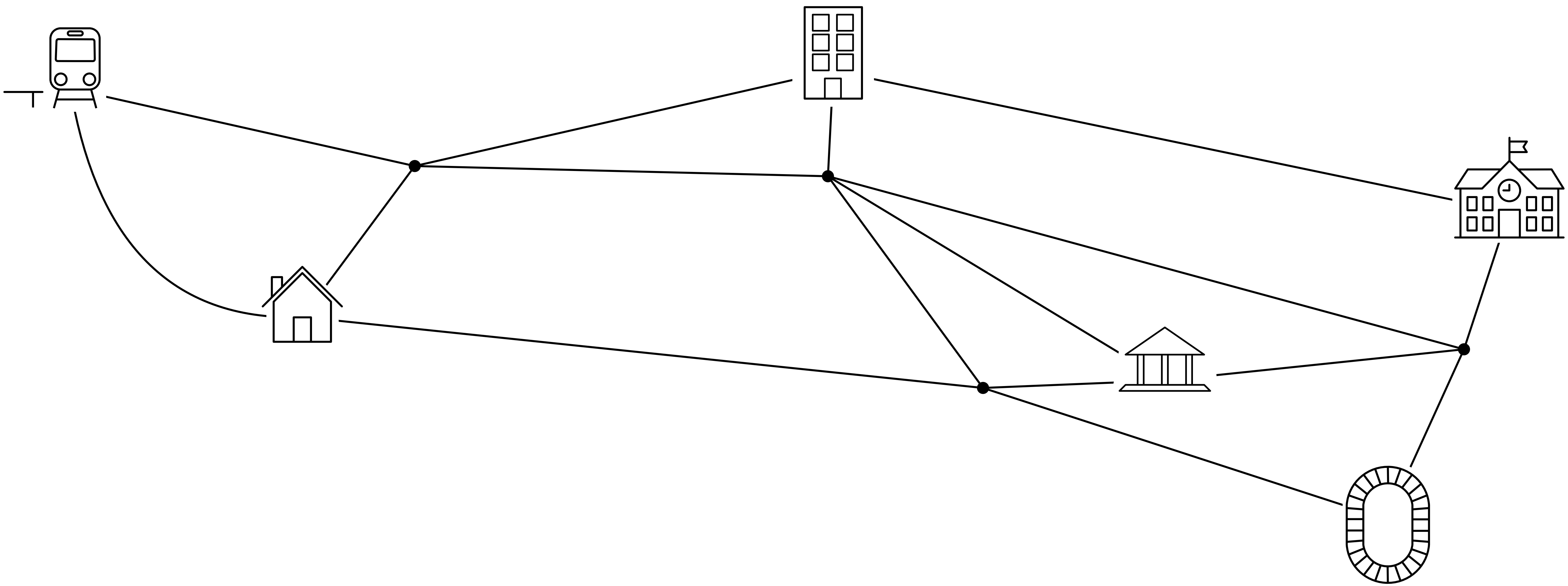
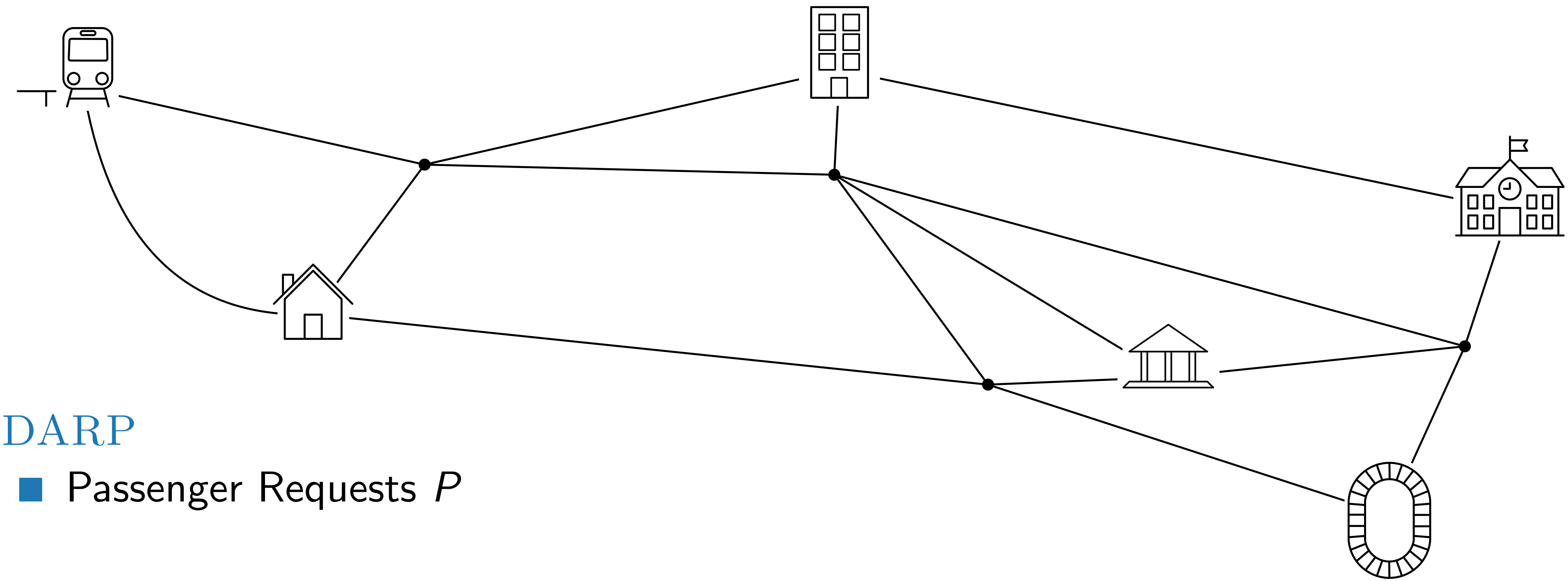


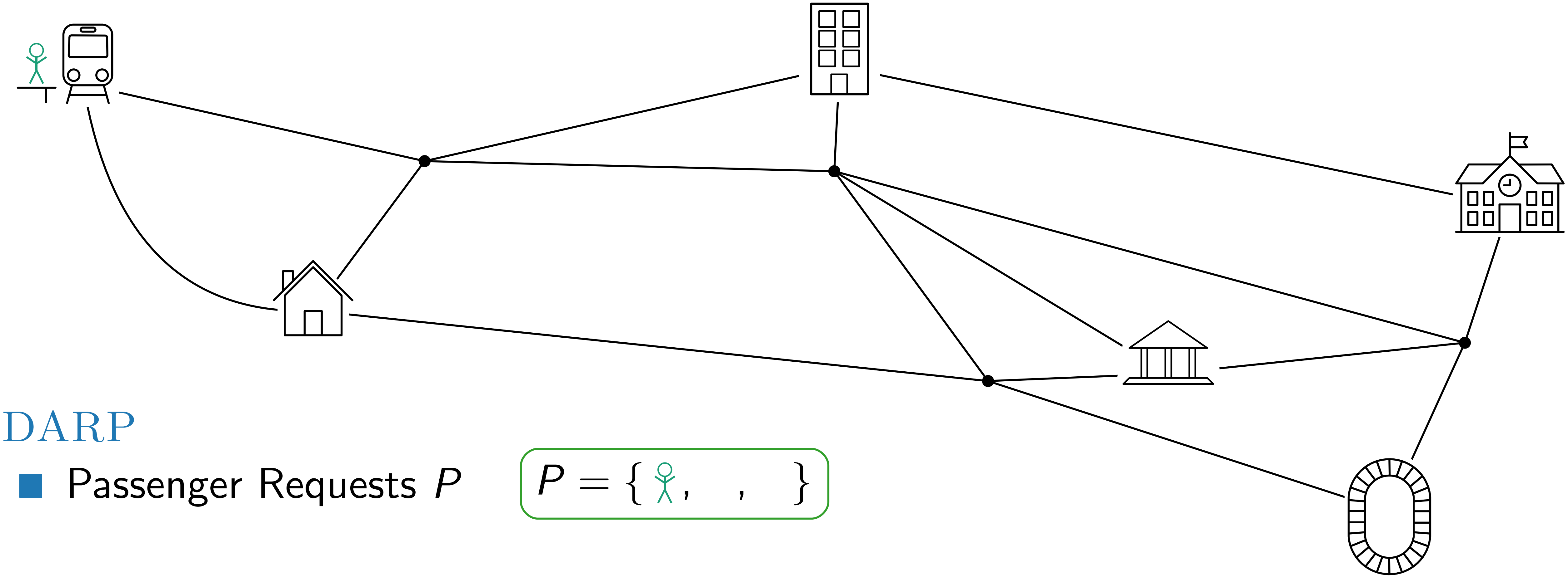
Dial-a-Ride-Problem (DARP)



DARP

- Passenger Requests P

Dial-a-Ride-Problem (DARP)

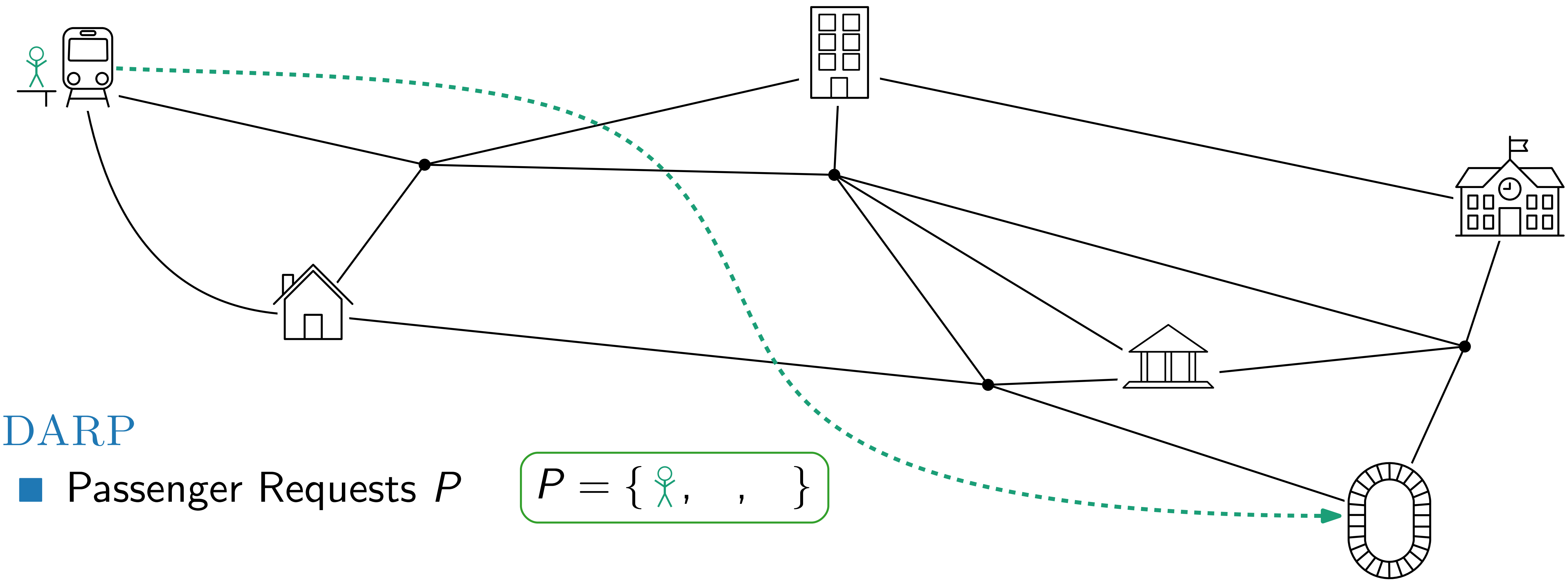


DARP

- Passenger Requests P

$P = \{\ \ ,\ \ ,\ \ \}$

Dial-a-Ride-Problem (DARP)

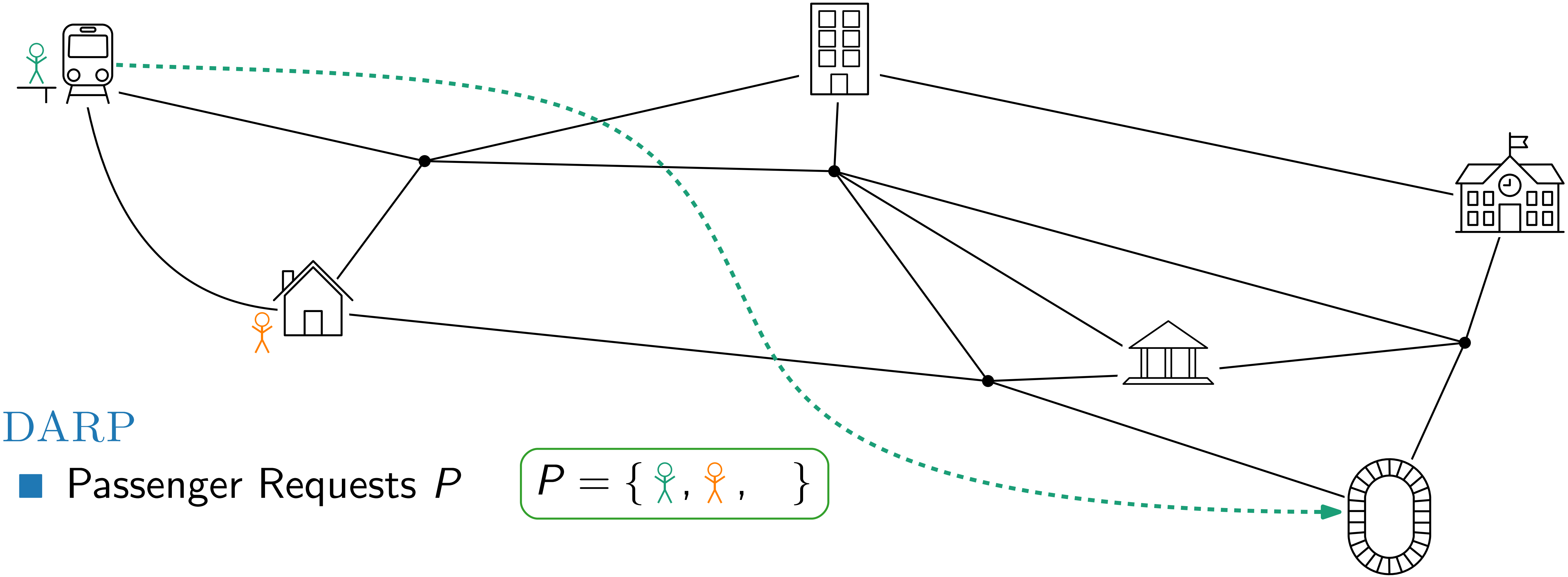


DARP

- Passenger Requests P

$P = \{\ \ ,\ \ ,\ \ \}$

Dial-a-Ride-Problem (DARP)

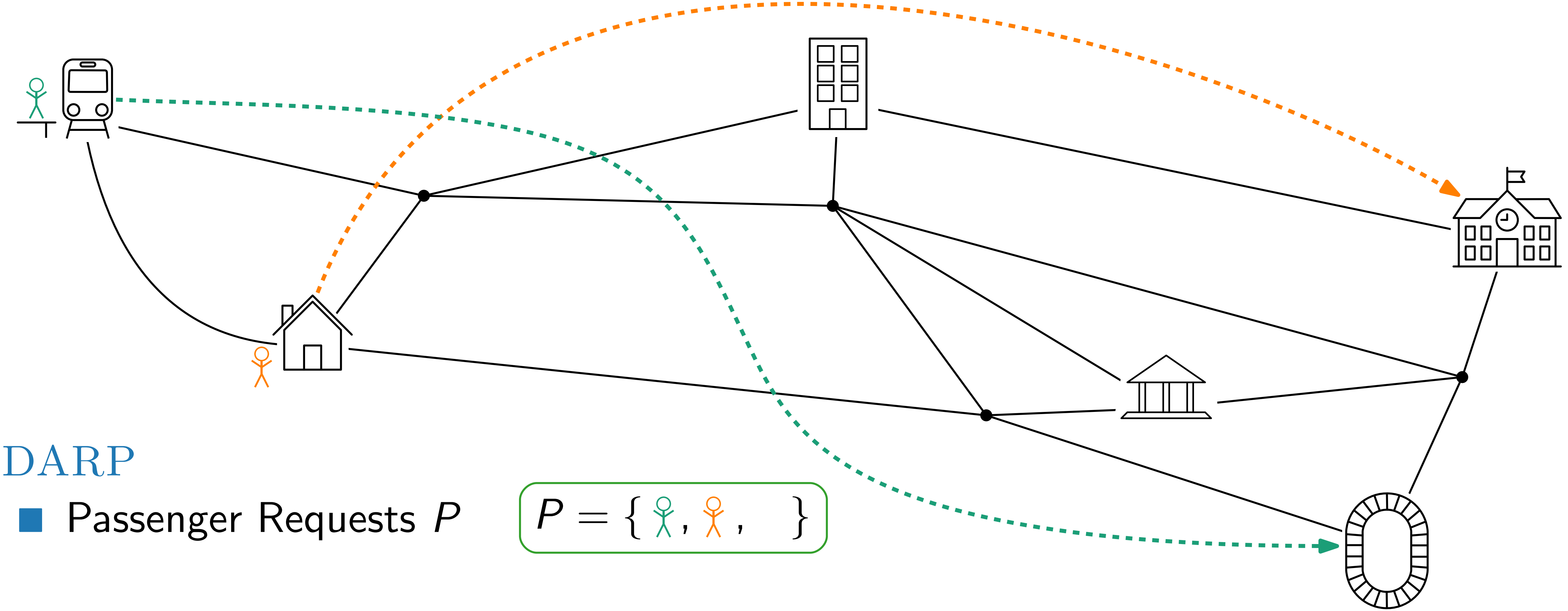


DARP

- Passenger Requests P

$$P = \{ \; , \; , \quad \}$$

Dial-a-Ride-Problem (DARP)

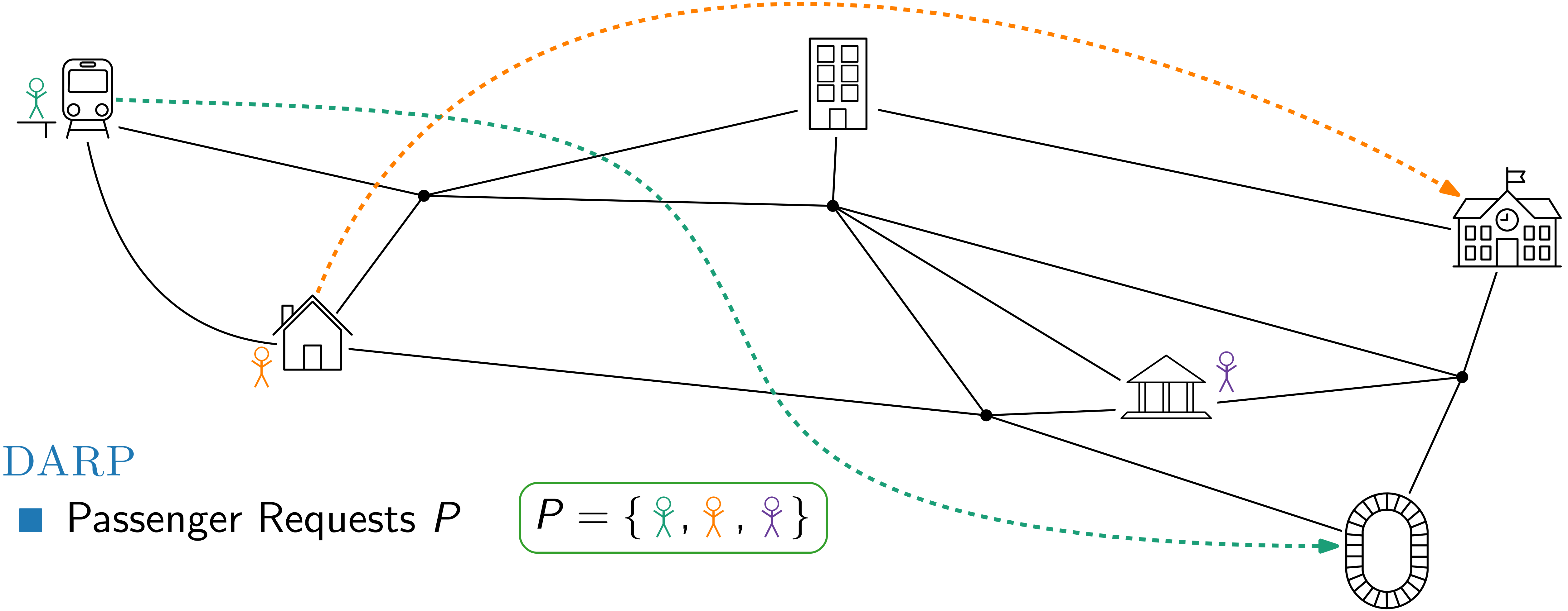


DARP

- Passenger Requests P

$P = \{ \quad , \quad , \quad \}$

Dial-a-Ride-Problem (DARP)

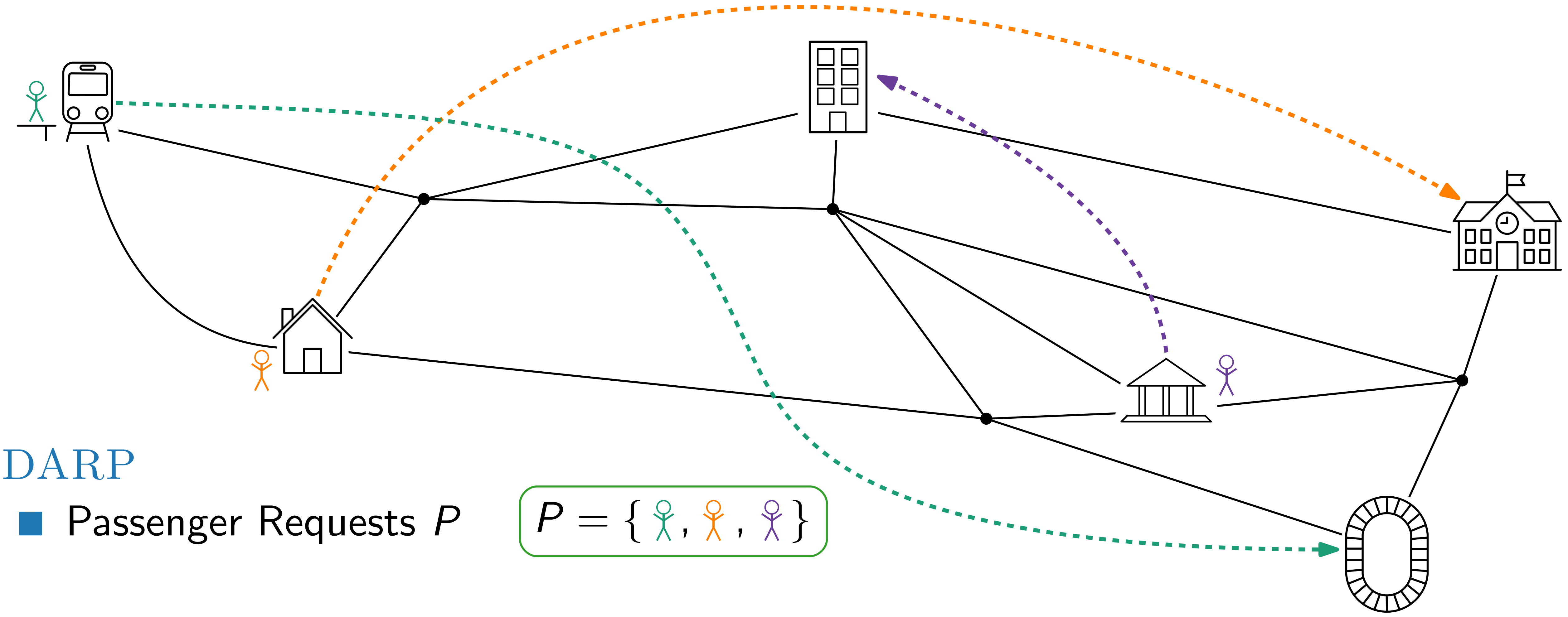


DARP

- Passenger Requests P

$$P = \{\ ,\ ,\ \}$$

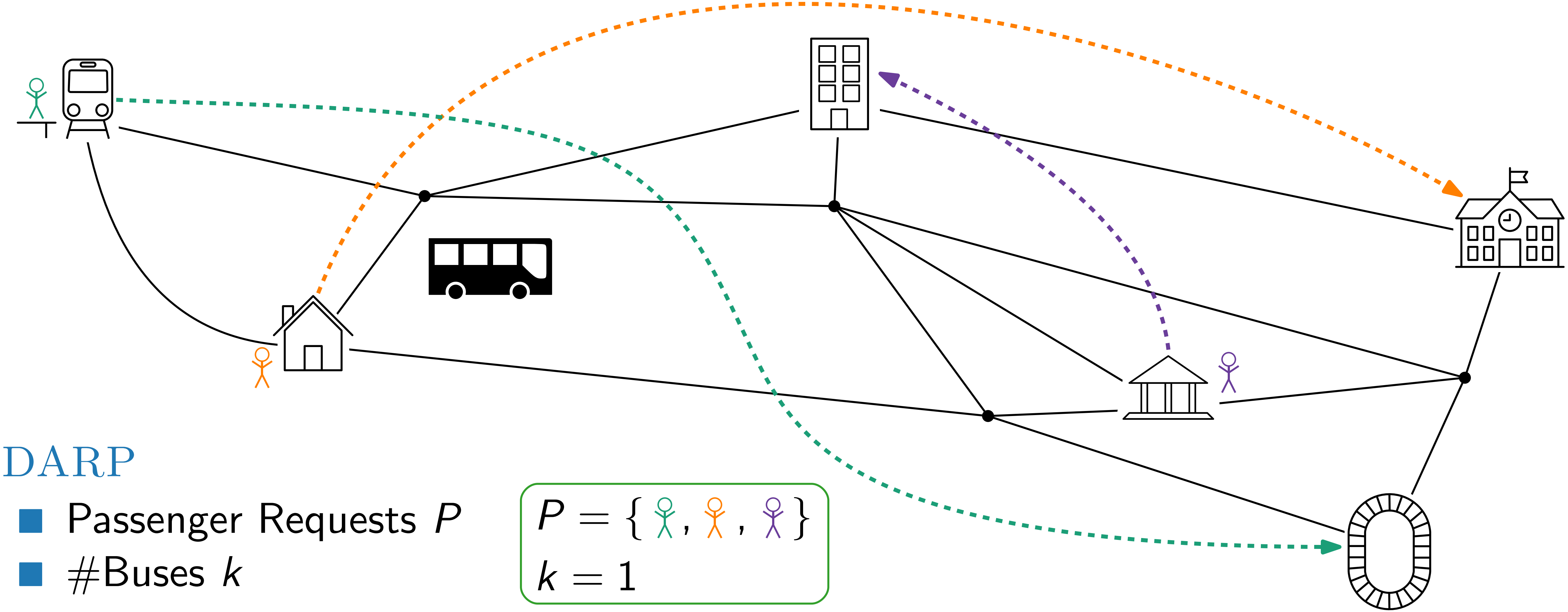
Dial-a-Ride-Problem (DARP)



DARP

- Passenger Requests P

Dial-a-Ride-Problem (DARP)

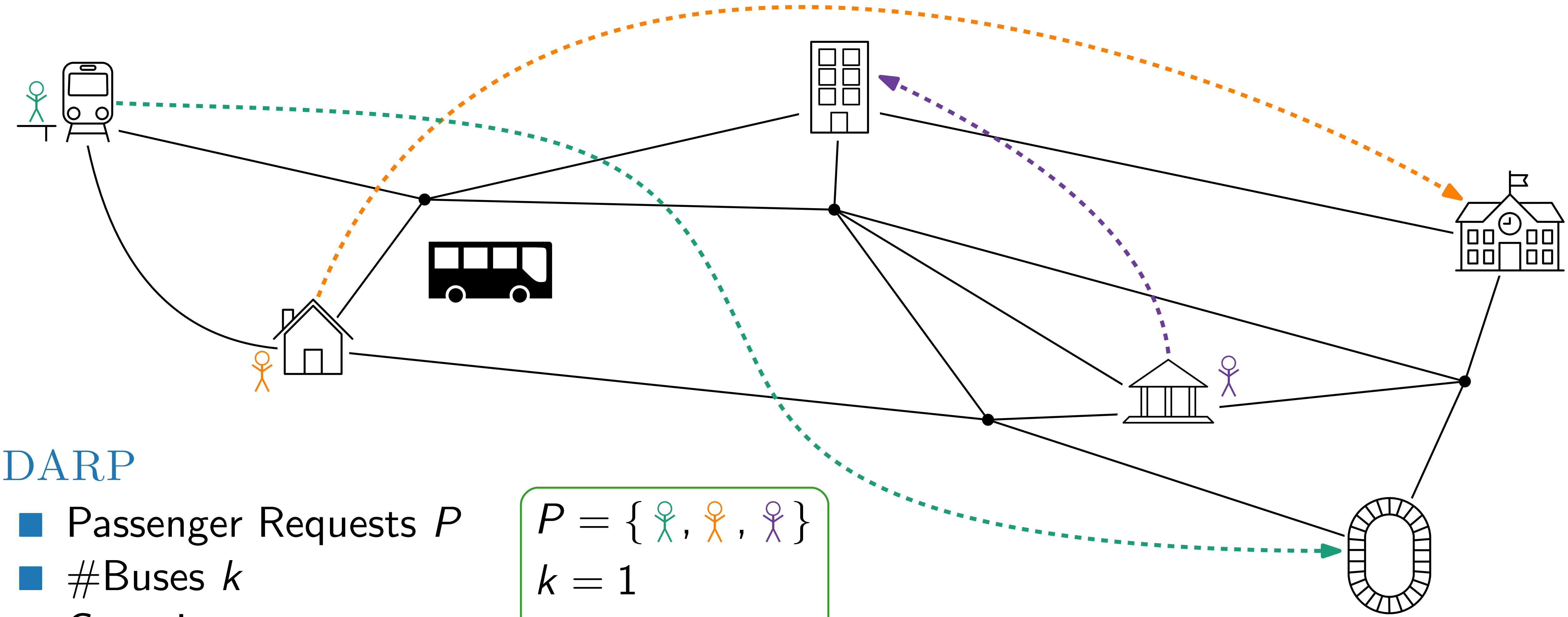


DARP

- Passenger Requests P
- #Buses k

$P = \{$ 🧍, 🧍, 🧍 $\}$
$k = 1$

Dial-a-Ride-Problem (DARP)



DARP

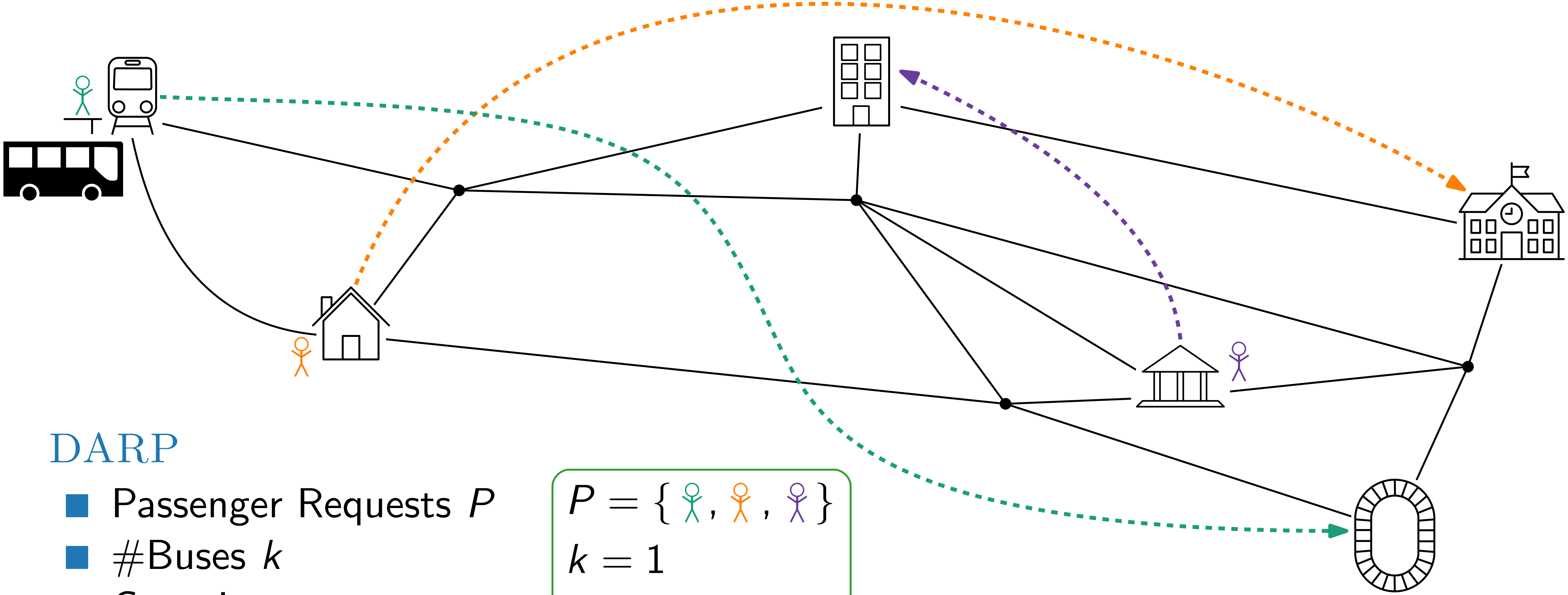
- Passenger Requests P
- #Buses k
- Capacity c

$P = \{\ ,\ ,\ \}$

$k = 1$

$c = 3$

Dial-a-Ride-Problem (DARP)



DARP

- Passenger Requests P
- #Buses k
- Capacity c

$P = \{$, , $\}$

$k = 1$

$c = 3$

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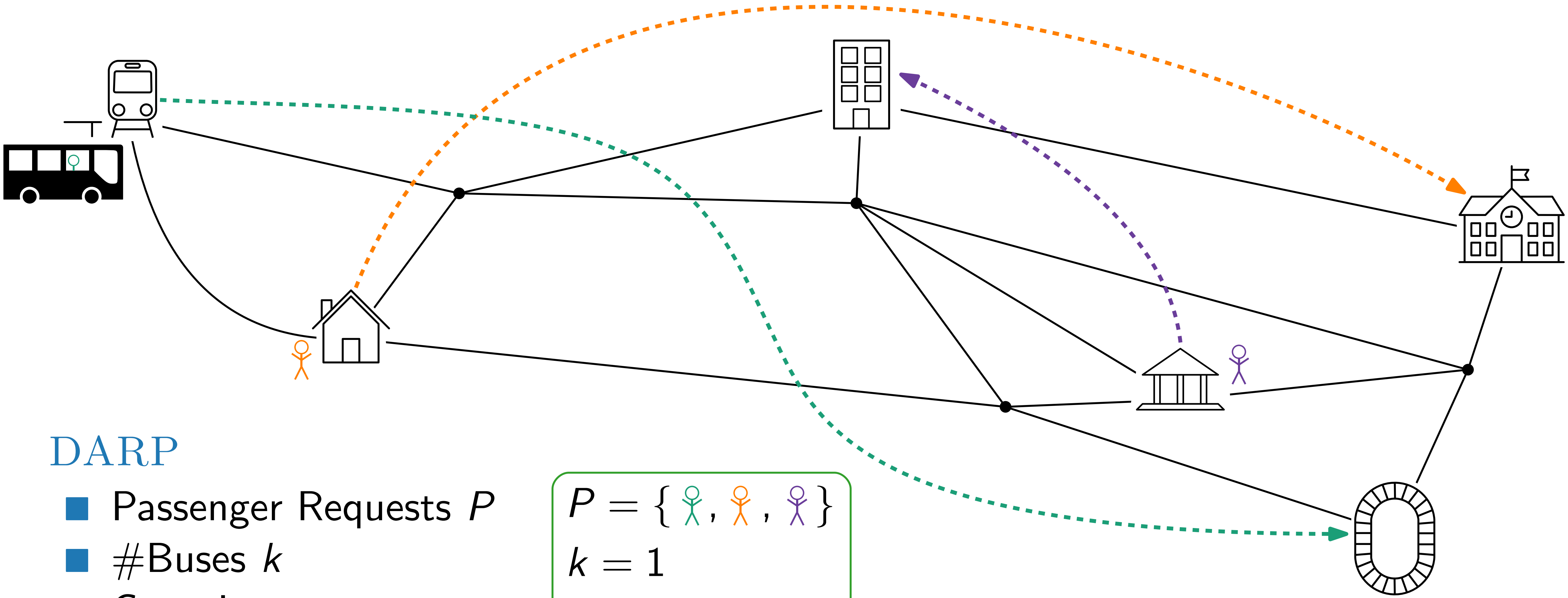


DARP

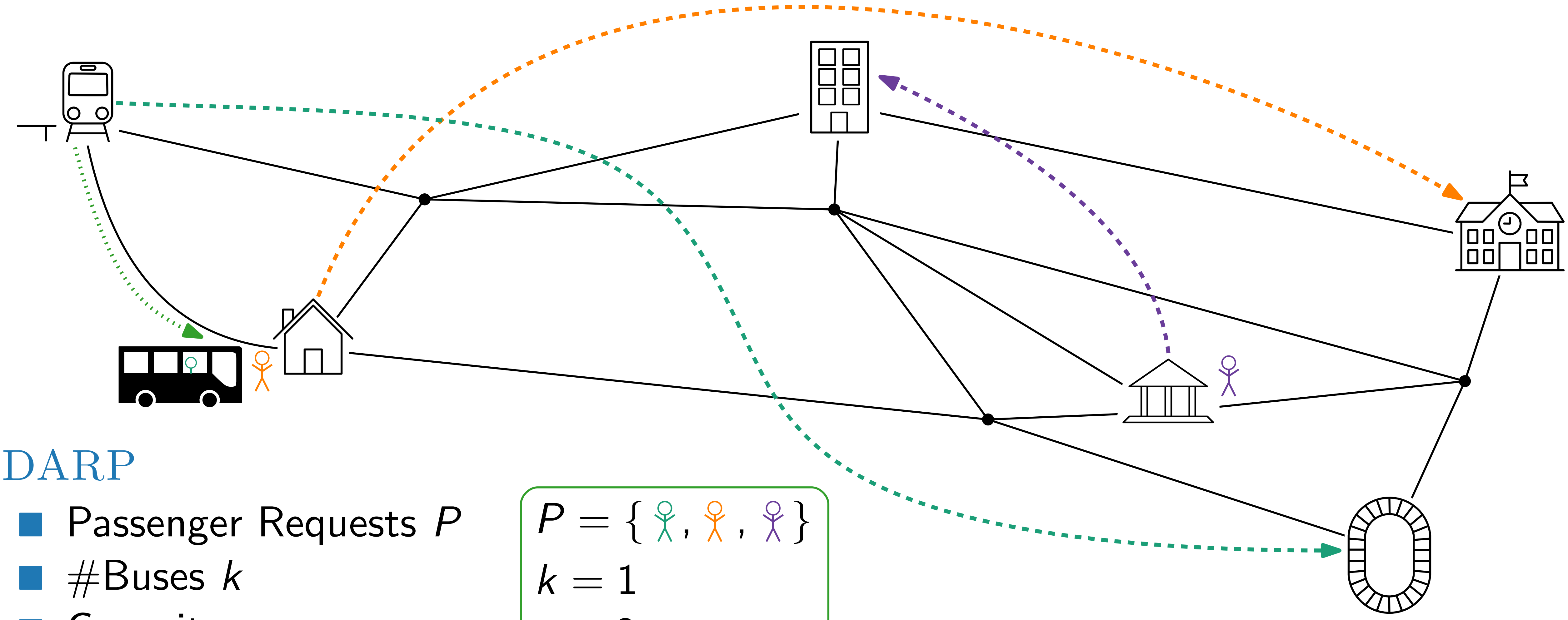
- Passenger Requests P
- #Buses k
- Capacity c

$P = \{$ 🧍, 🧍, 🧍 $\}$

$k = 1$

$c = 3$

Dial-a-Ride-Problem (DARP)



DARP

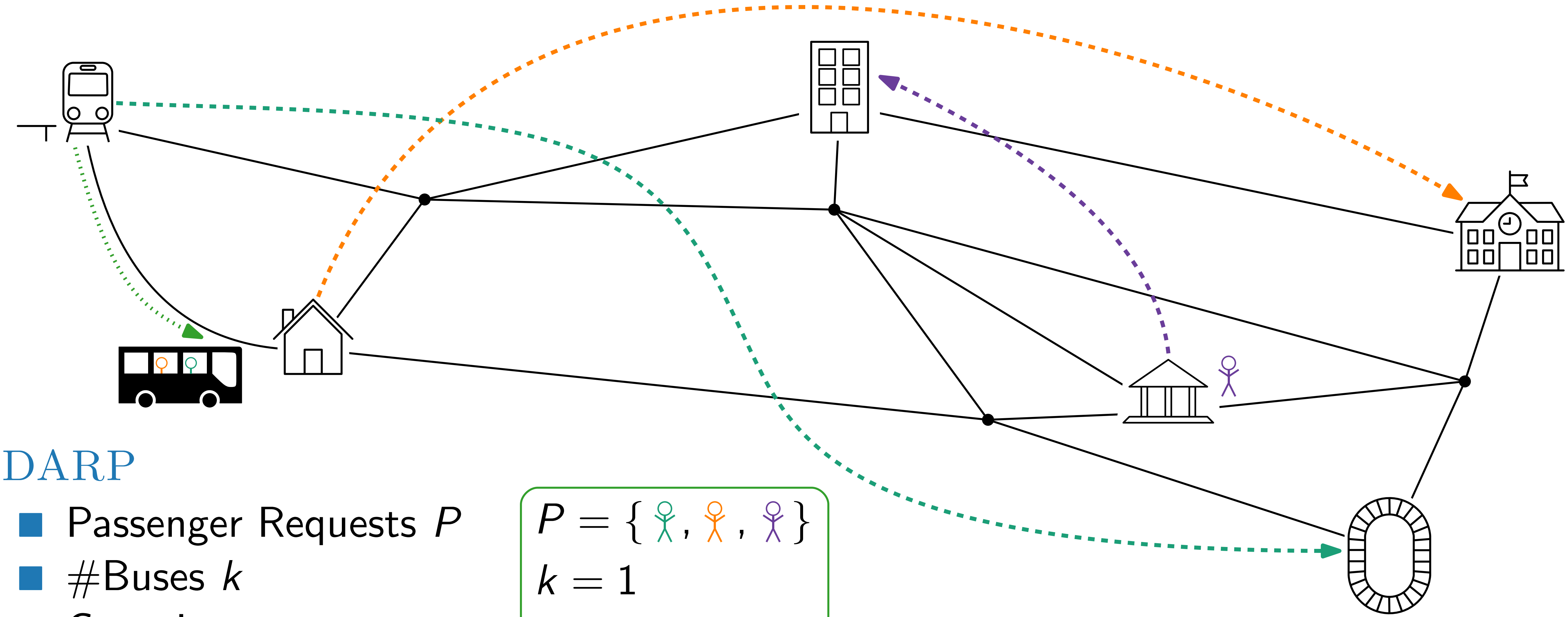
- Passenger Requests P
- #Buses k
- Capacity c

$P = \{ \text{🧍}, \text{🧍}, \text{🧍} \}$

$k = 1$

$c = 3$

Dial-a-Ride-Problem (DARP)



DARP

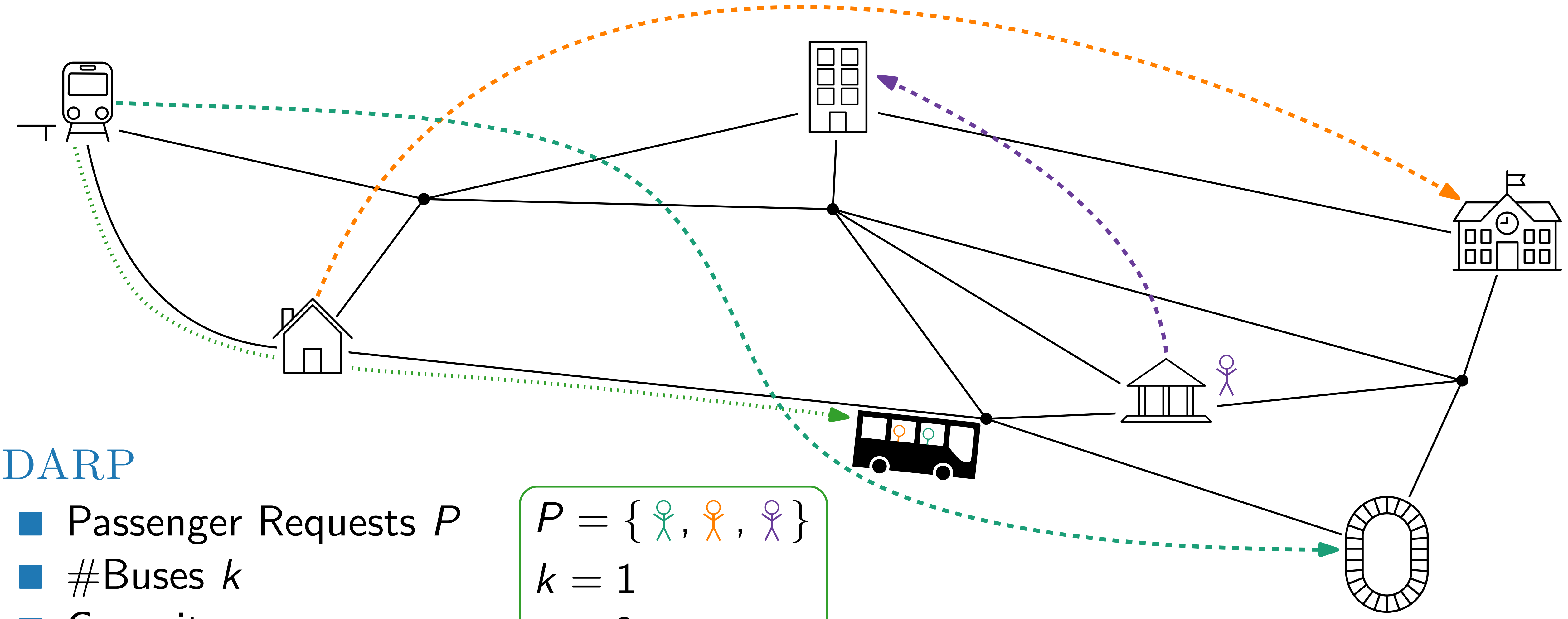
- Passenger Requests P
- #Buses k
- Capacity c

$P = \{ \text{(green passenger)}, \text{(orange passenger)}, \text{(purple passenger)} \}$

$k = 1$

$c = 3$

Dial-a-Ride-Problem (DARP)



DARP

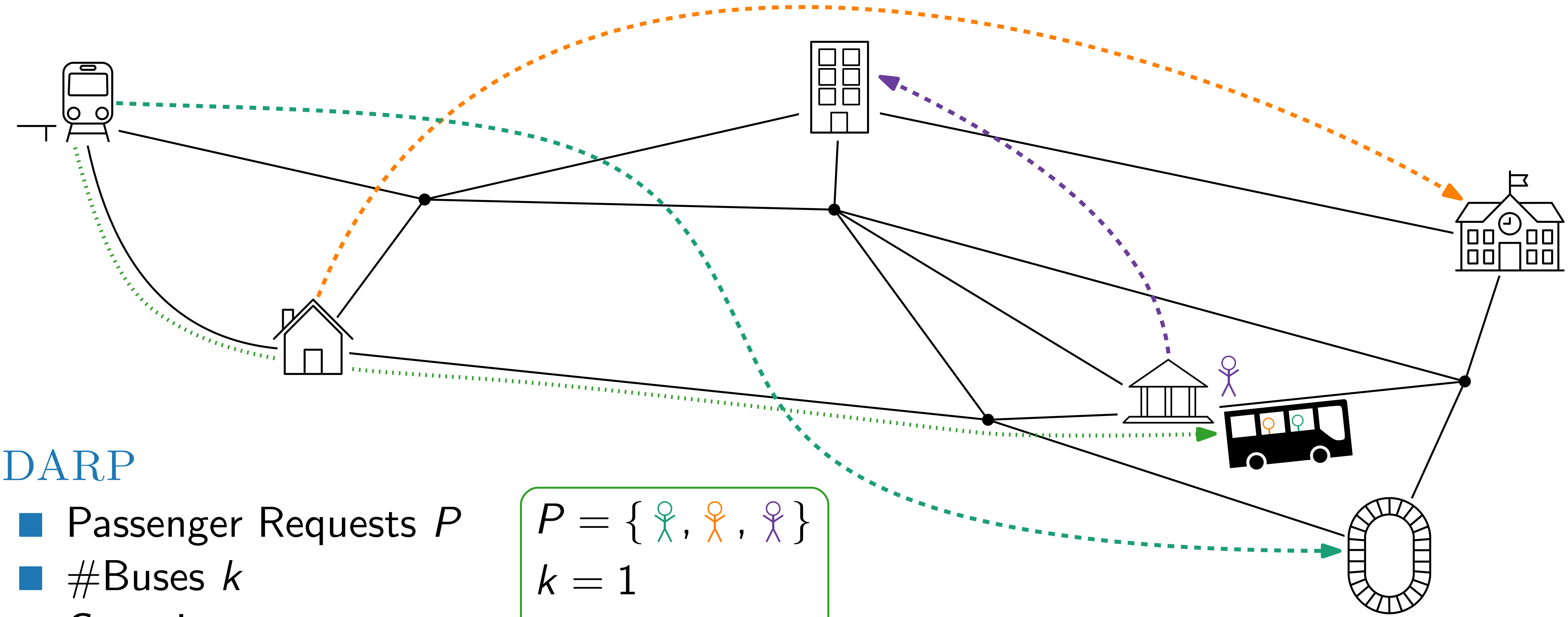
- Passenger Requests P
- #Buses k
- Capacity c

$P = \{ \text{🧍}, \text{🧍}, \text{🧍} \}$

$k = 1$

$c = 3$

Dial-a-Ride-Problem (DARP)



DARP

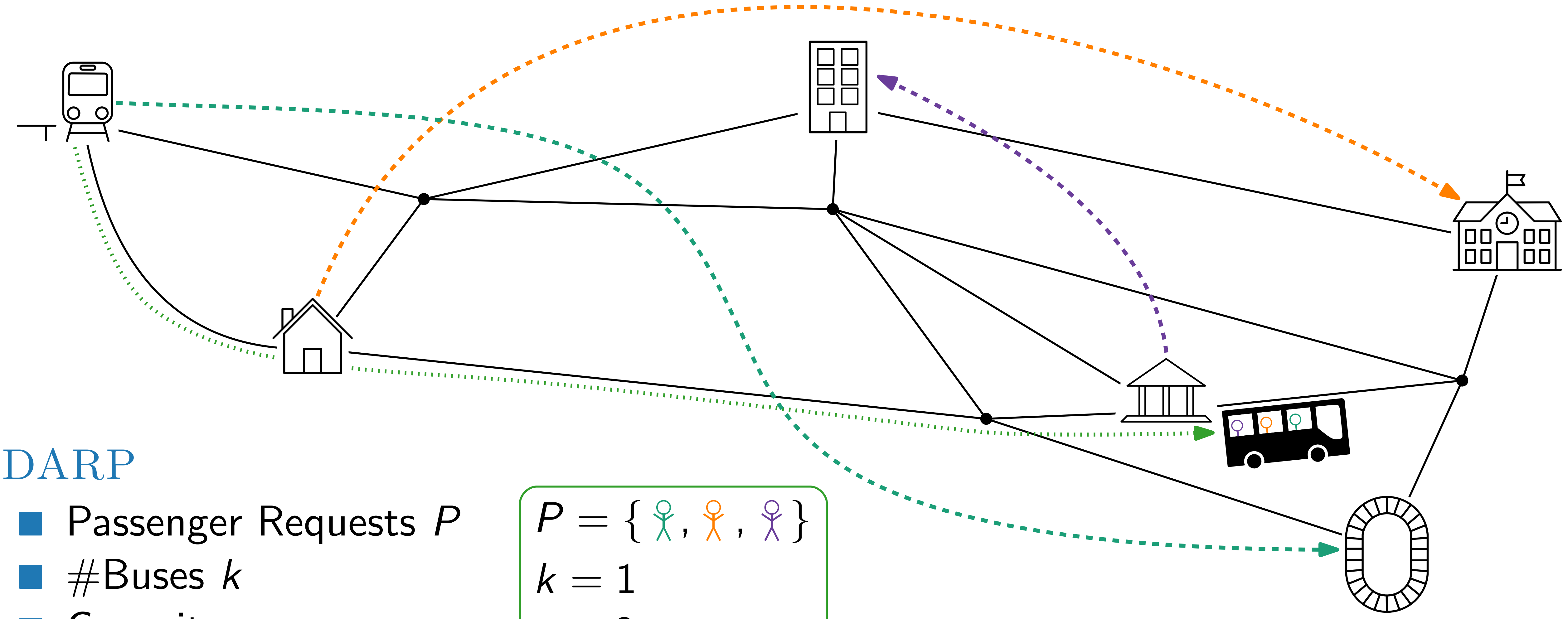
- Passenger Requests P
- #Buses k
- Capacity c

$P = \{ , , \}$

$k = 1$

$c = 3$

Dial-a-Ride-Problem (DARP)

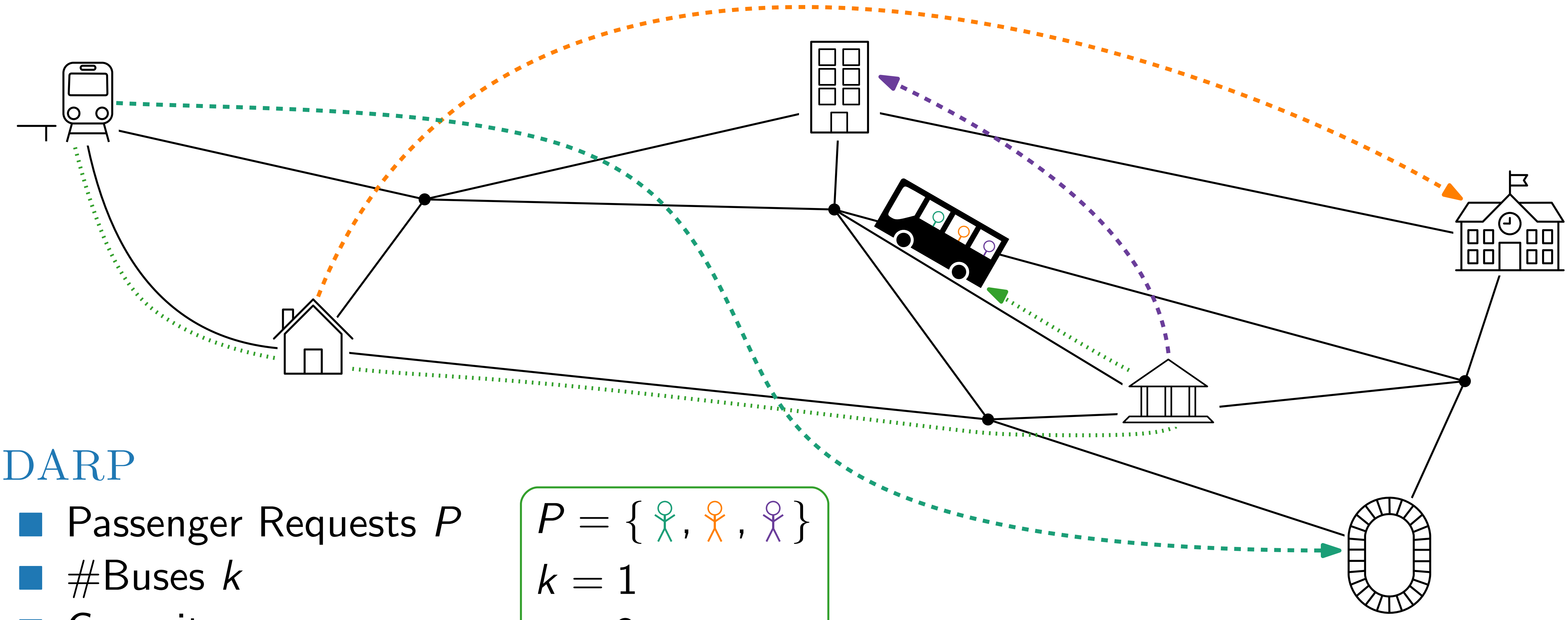


DARP

- Passenger Requests P
- #Buses k
- Capacity c

$P = \{$ 🯅 , 🯅 , 🯅 $\}$
$k = 1$
$c = 3$

Dial-a-Ride-Problem (DARP)



DARP

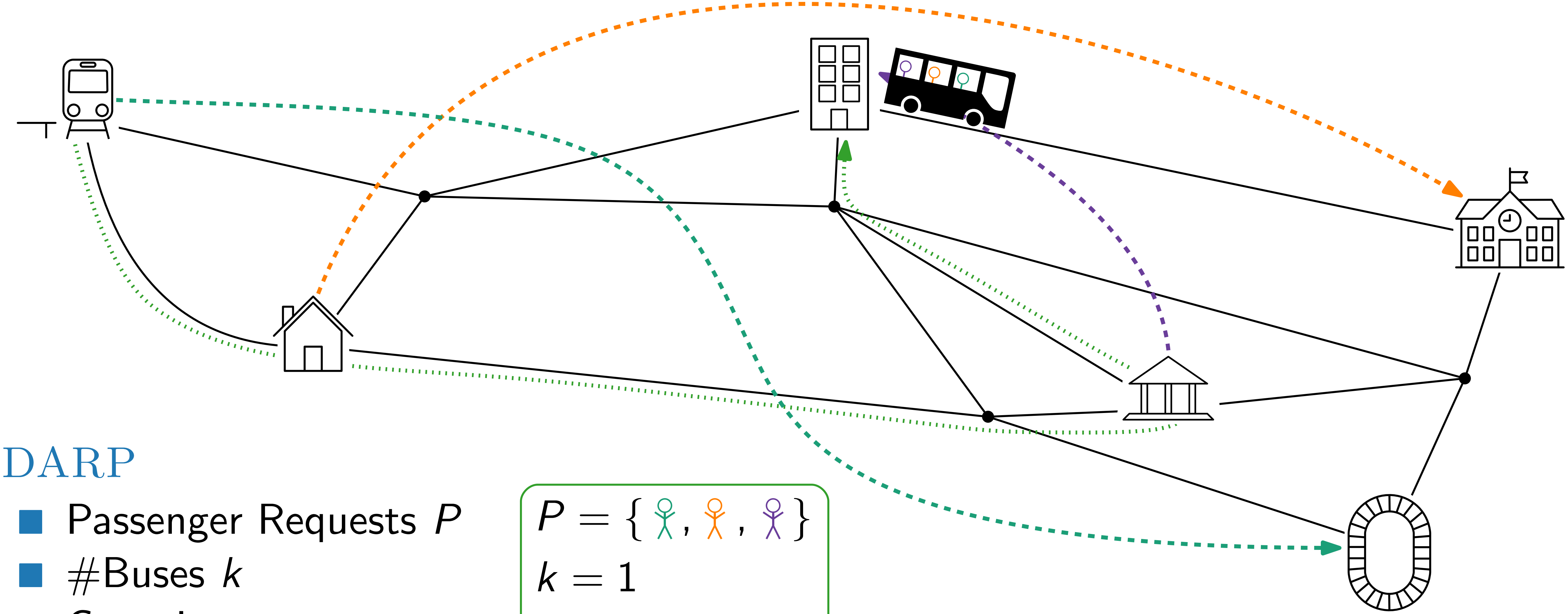
- Passenger Requests P
- #Buses k
- Capacity c

$P = \{ \text{passenger}, \text{passenger}, \text{passenger} \}$

$k = 1$

$c = 3$

Dial-a-Ride-Problem (DARP)



DARP

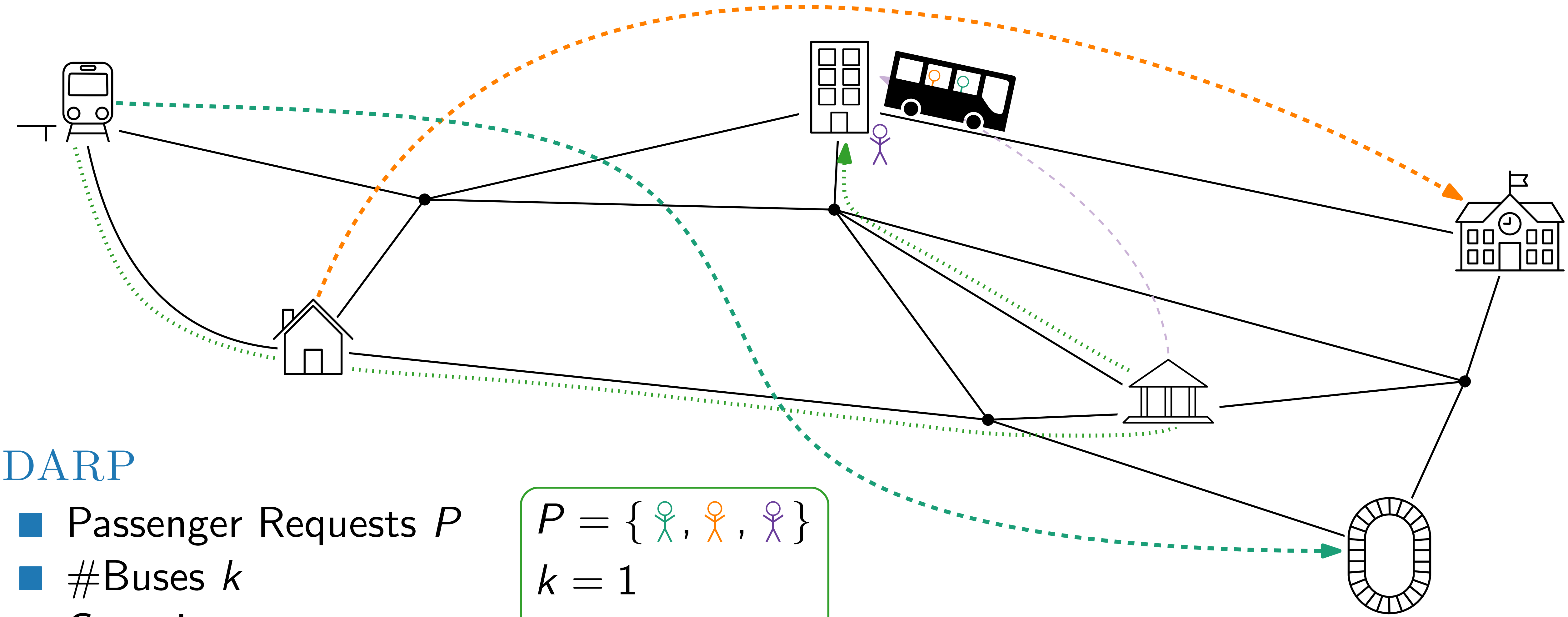
- Passenger Requests P
- #Buses k
- Capacity c

$P = \{ \; , \; , \; \}$

$k = 1$

$c = 3$

Dial-a-Ride-Problem (DARP)



DARP

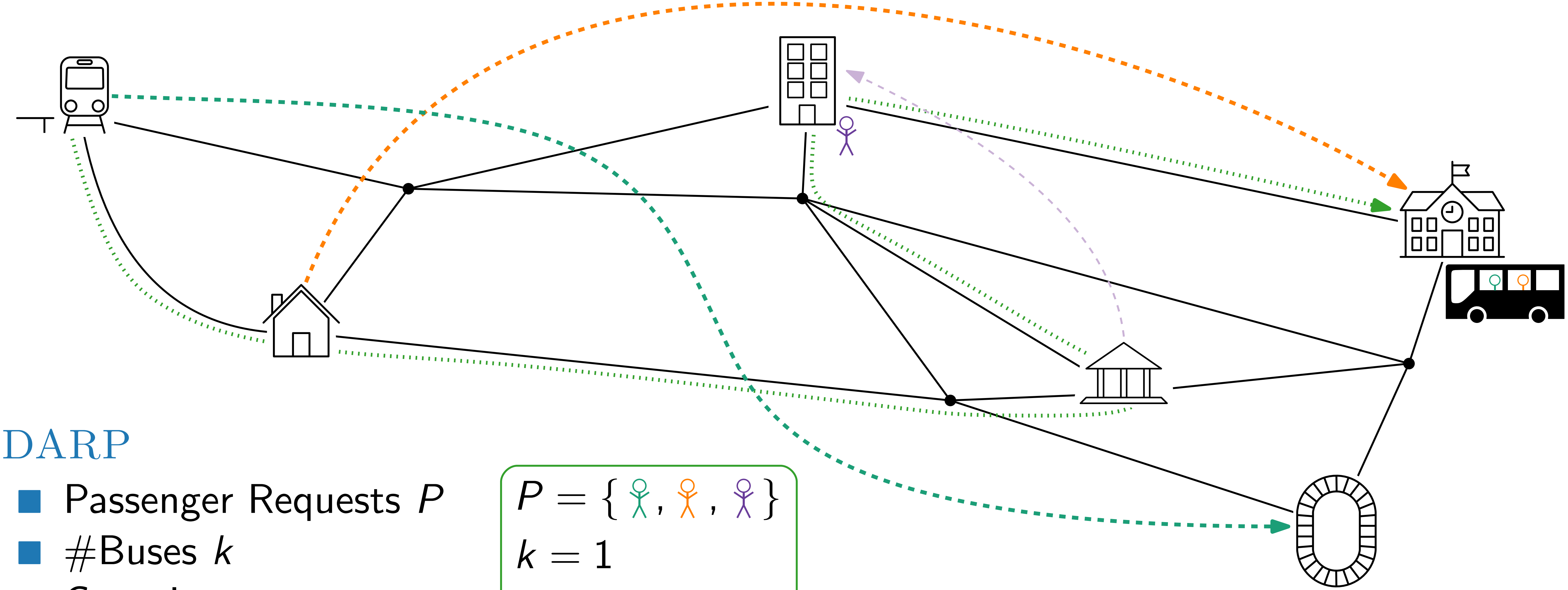
- Passenger Requests P
- #Buses k
- Capacity c

$P = \{ , , \}$

$k = 1$

$c = 3$

Dial-a-Ride-Problem (DARP)

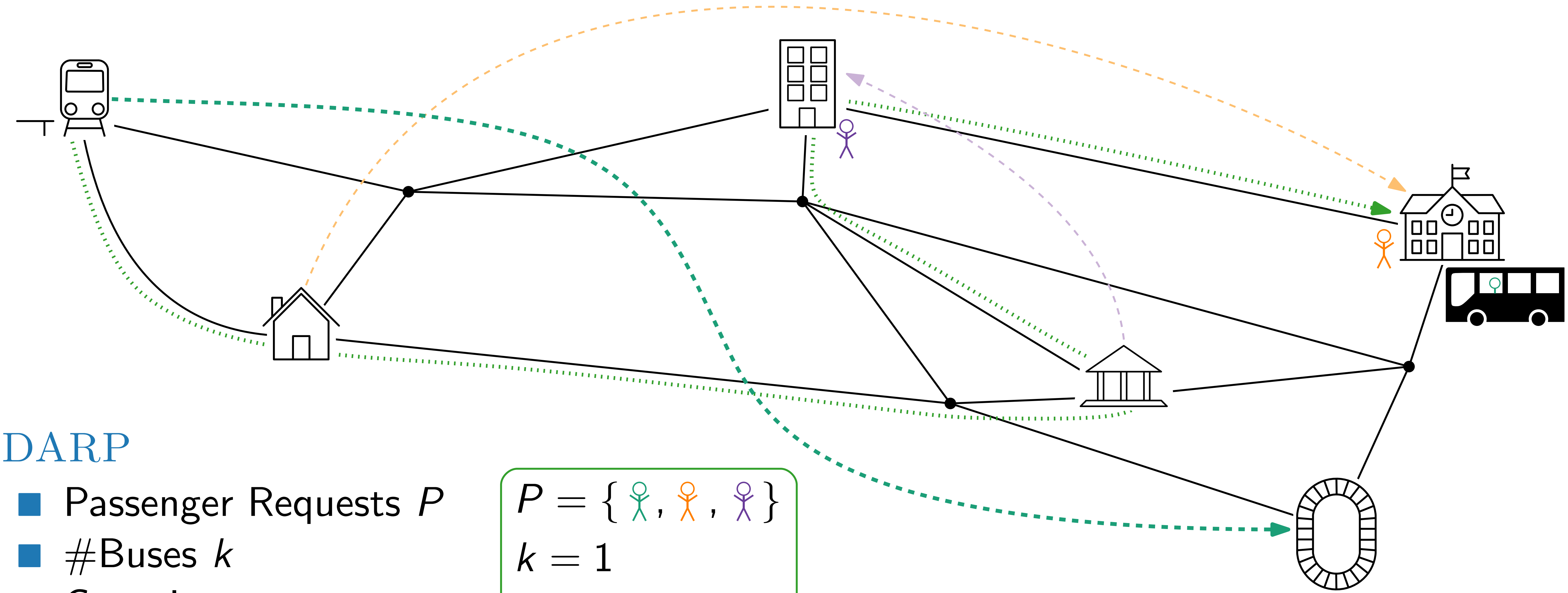


DARP

- Passenger Requests P
- #Buses k
- Capacity c

$P = \{$ ⚲, ⚲, ⚲ $\}$
$k = 1$
$c = 3$

Dial-a-Ride-Problem (DARP)



DARP

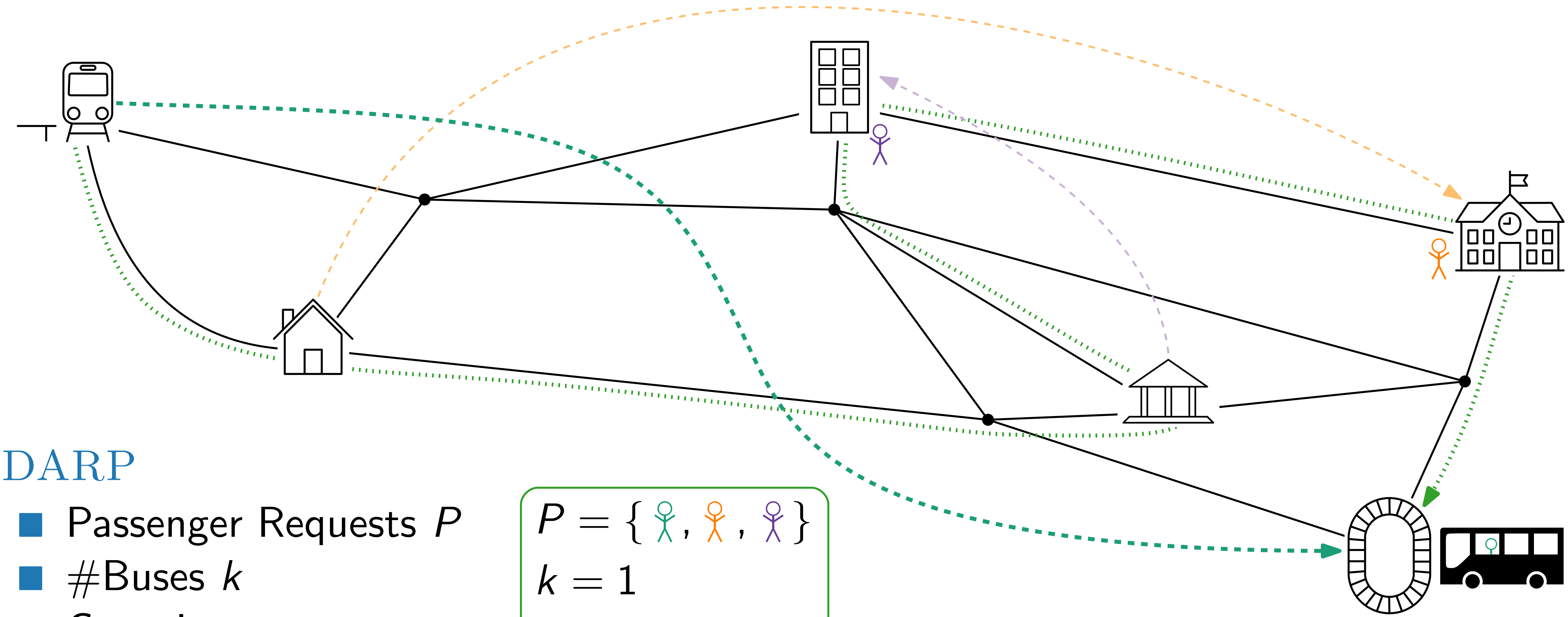
- Passenger Requests P
- #Buses k
- Capacity c

$P = \{ \text{ⓘ}, \text{ⓘ}, \text{ⓘ} \}$

$k = 1$

$c = 3$

Dial-a-Ride-Problem (DARP)

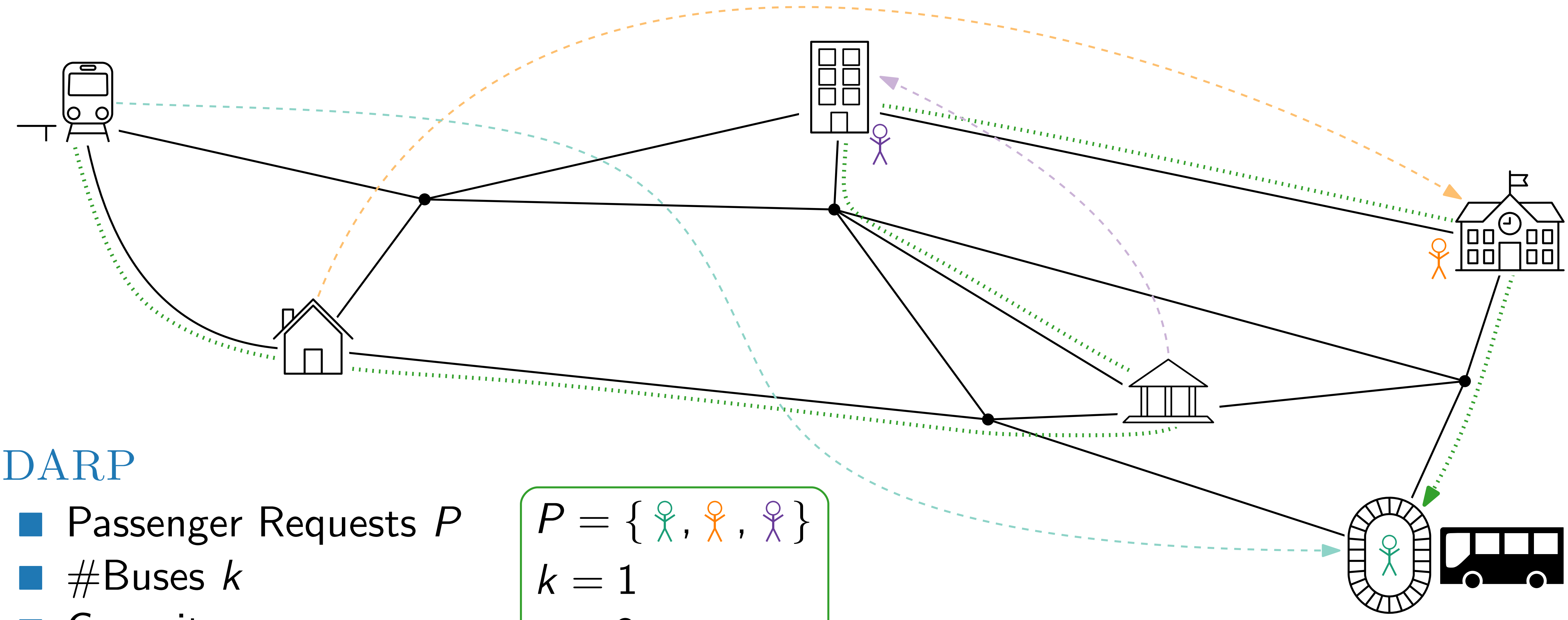


DARP

- Passenger Requests P
- #Buses k
- Capacity c

$P = \{ \text{☺}, \text{☺}, \text{☺} \}$
$k = 1$
$c = 3$

Dial-a-Ride-Problem (DARP)

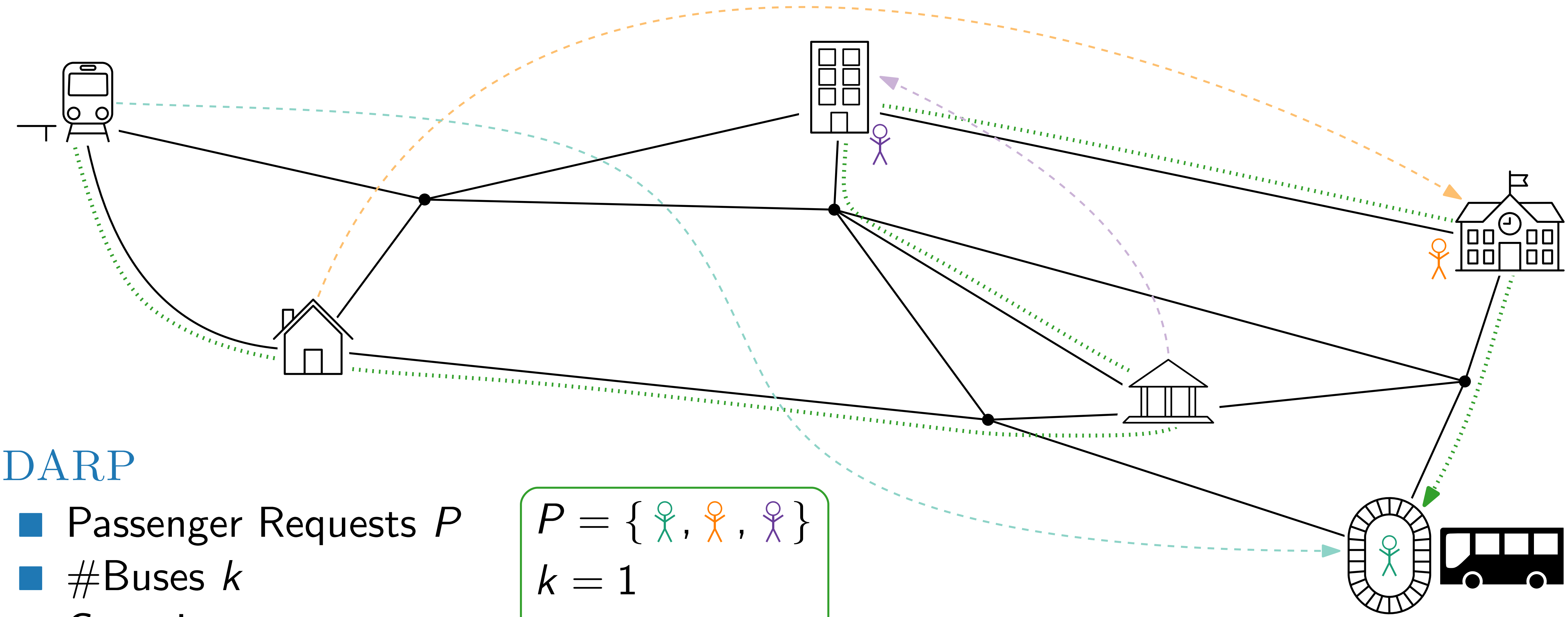


DARP

- Passenger Requests P
- #Buses k
- Capacity c

$P = \{ \ , \ , \ \}$
$k = 1$
$c = 3$

Dial-a-Ride-Problem (DARP)



DARP

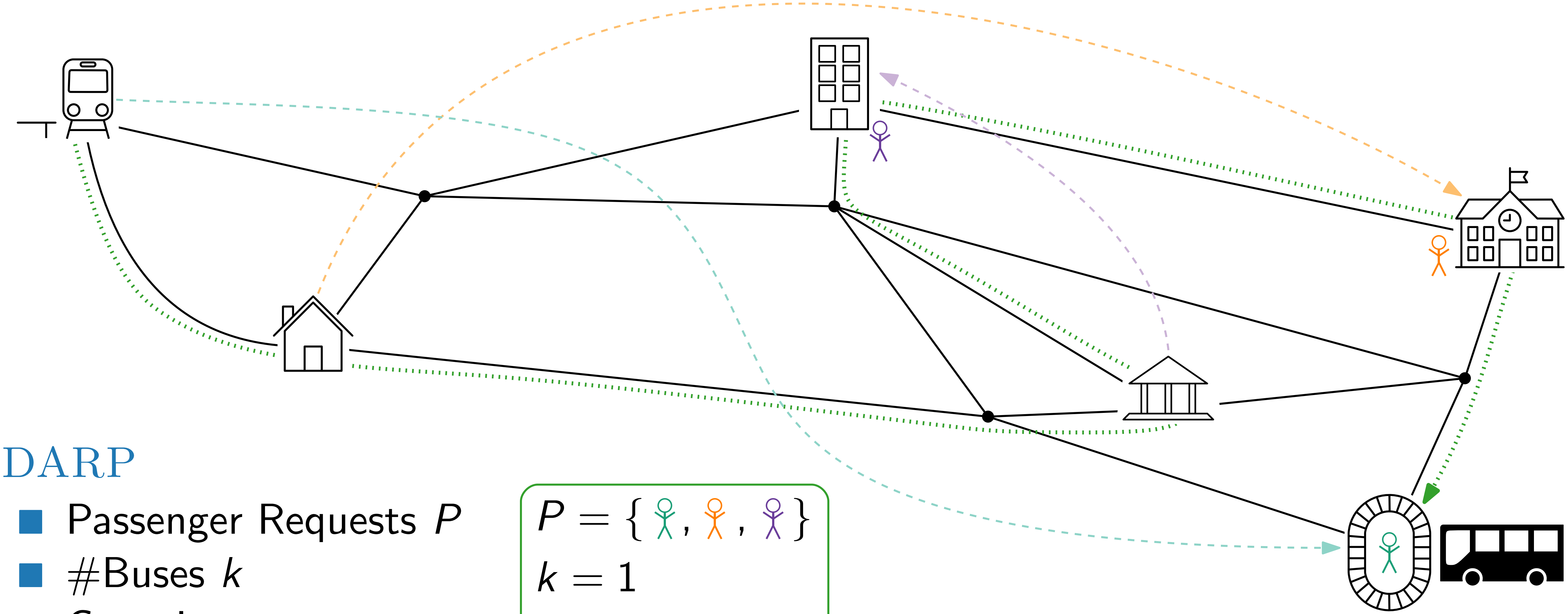
- Passenger Requests P
- #Buses k
- Capacity c
- Solution R

$P = \{$![green passenger], ![orange passenger], ![purple passenger] $\}$

$k = 1$

$c = 3$

Dial-a-Ride-Problem (DARP)



DARP

- Passenger Requests P
- #Buses k
- Capacity c
- Solution R

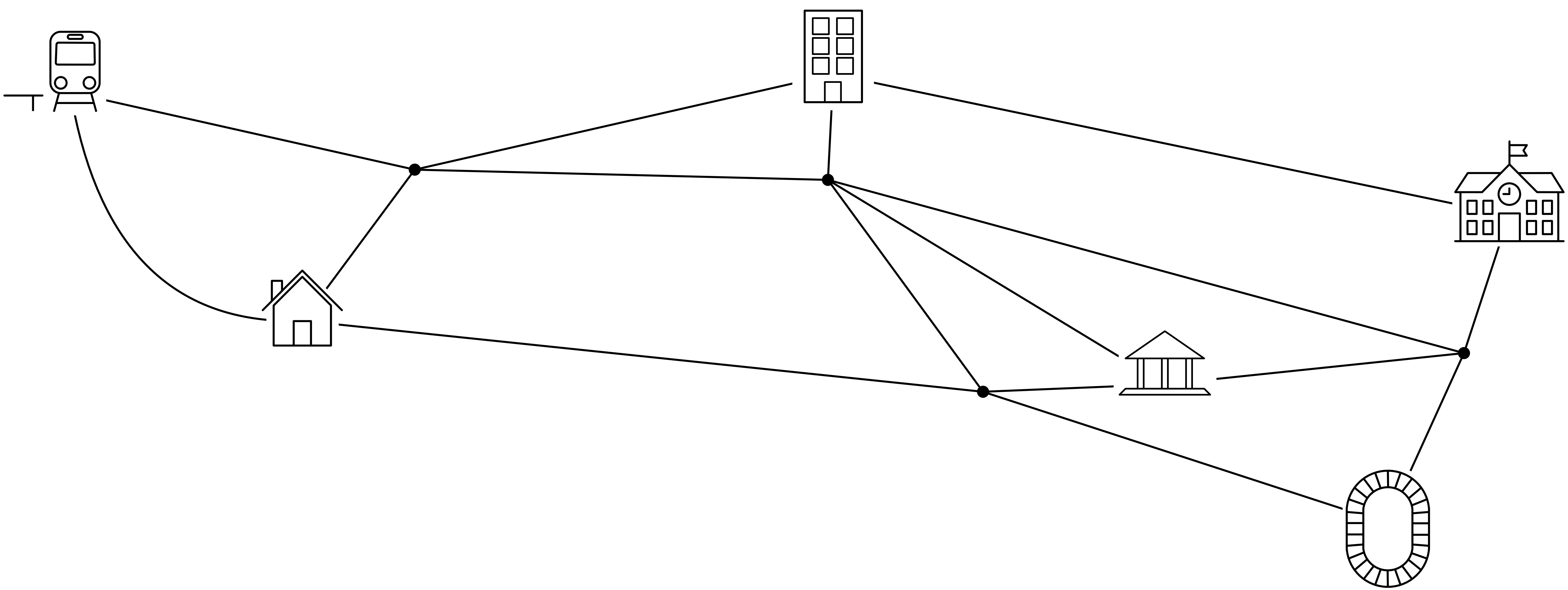
$P = \{\ ,\ ,\ \}$

$k = 1$

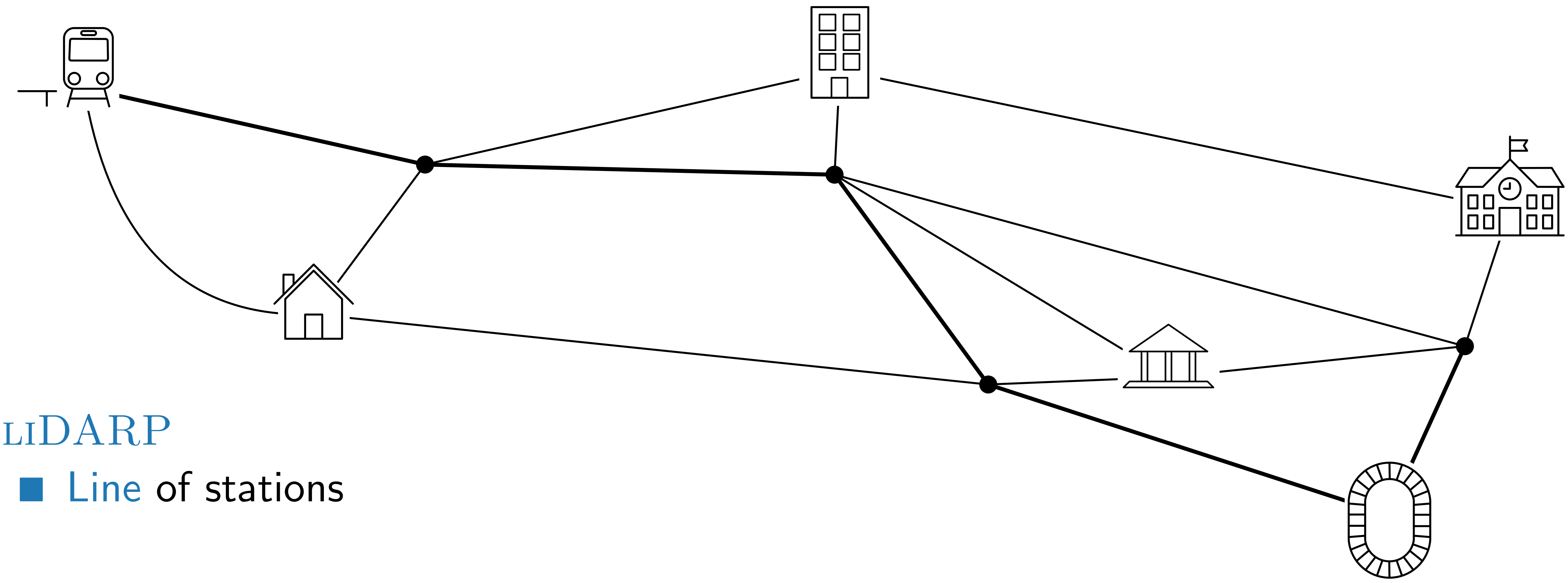
$c = 3$

$R = \{\ \}$

line-based Dial-a-Ride-Problem (liDARP)



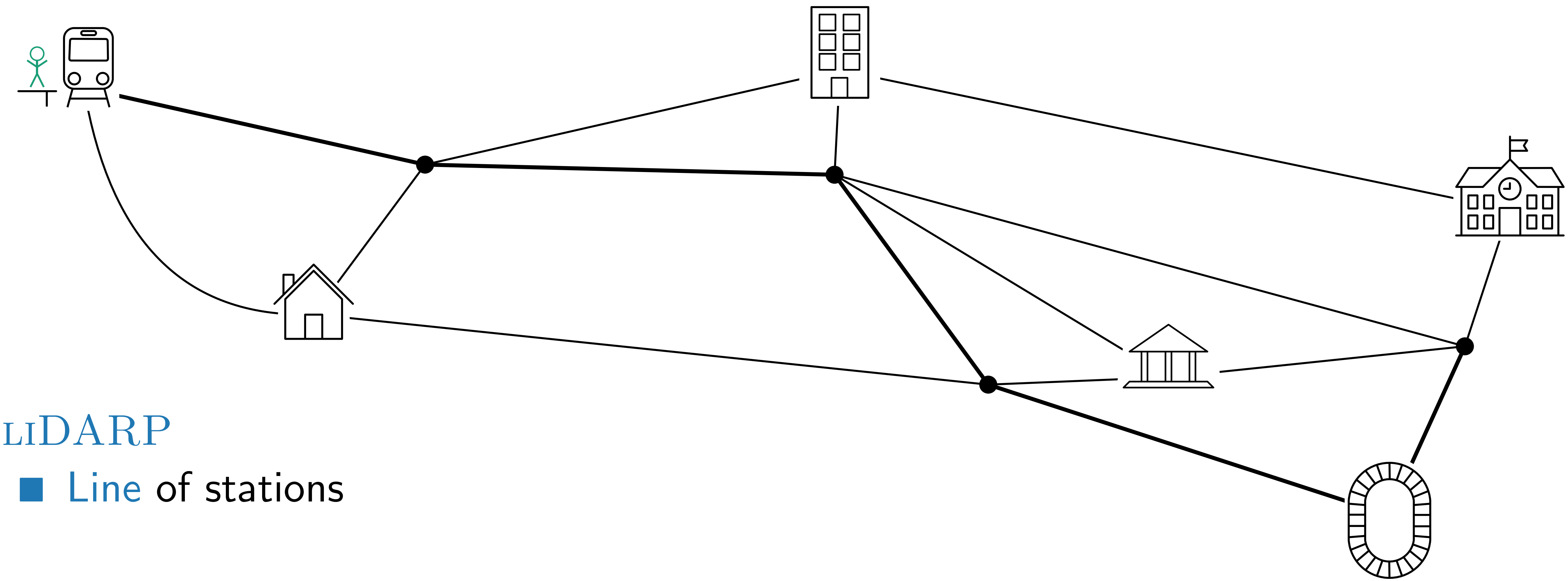
LINE-BASED DIAL-A-RIDE-PROBLEM (LIDARP)



LIDARP

- Line of stations

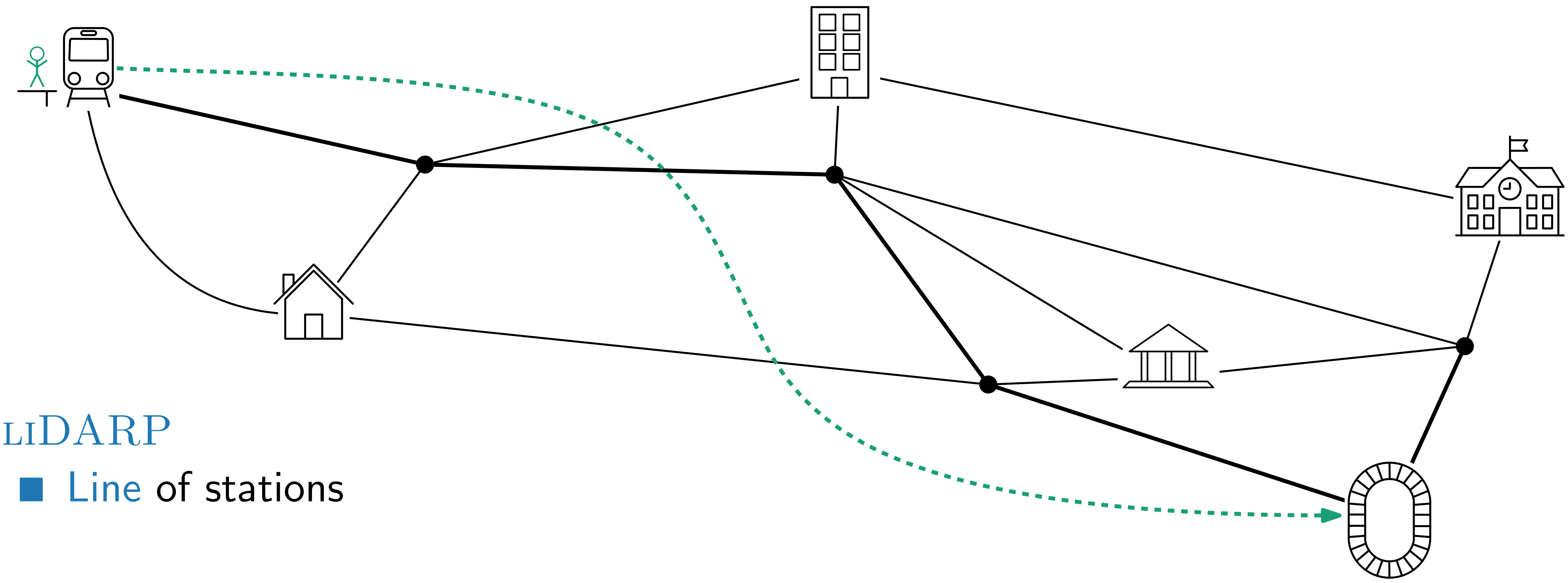
line-based Dial-a-Ride-Problem (liDARP)



liDARP

- Line of stations

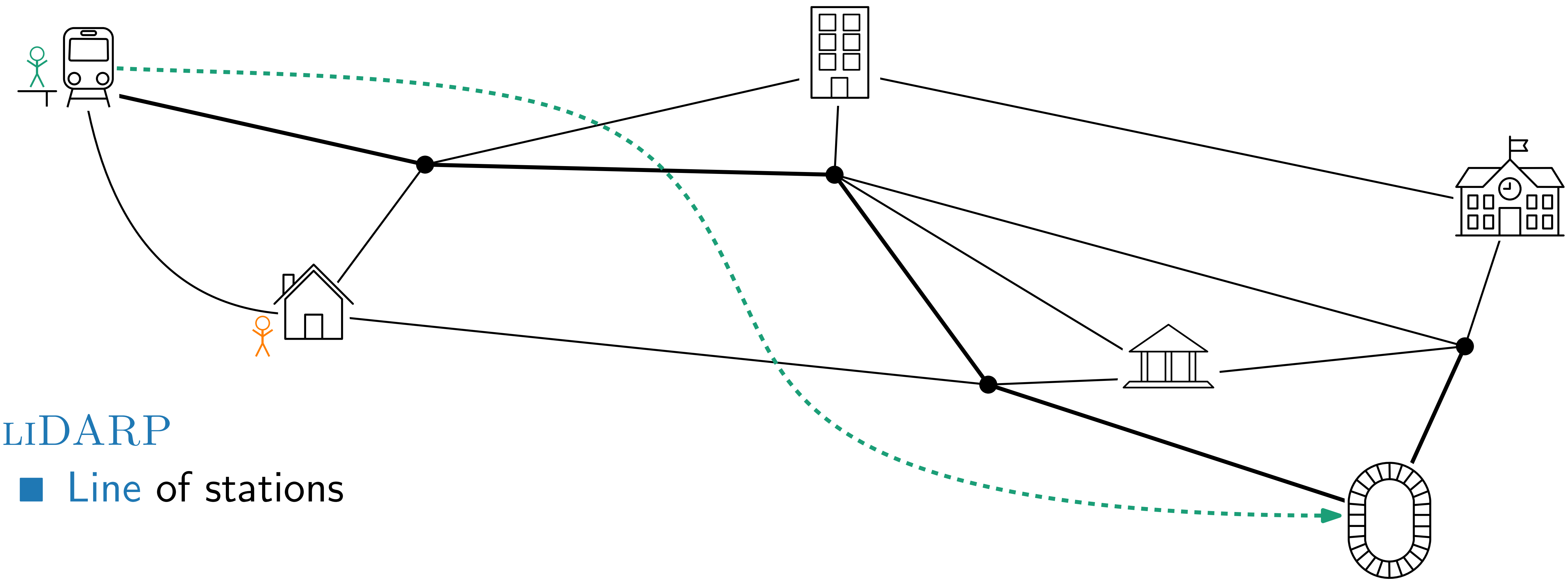
line-based Dial-a-Ride-Problem (liDARP)



liDARP

- Line of stations

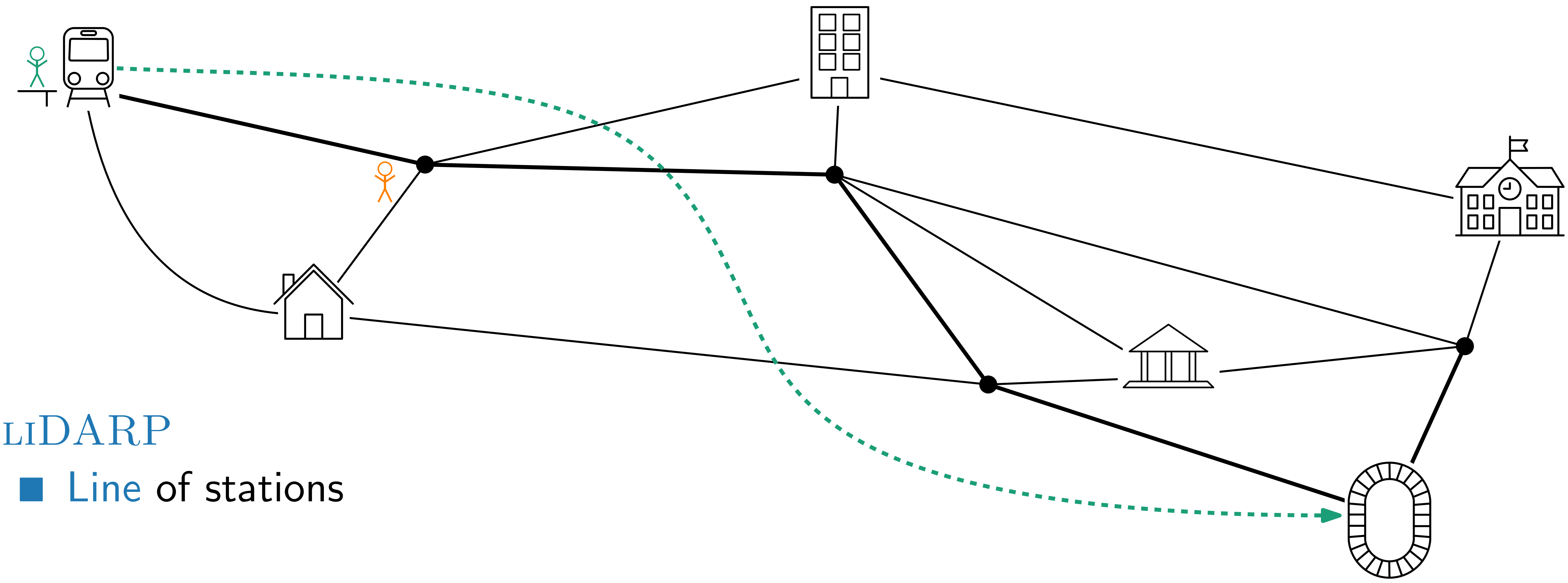
line-based Dial-a-Ride-Problem (liDARP)



liDARP

- Line of stations

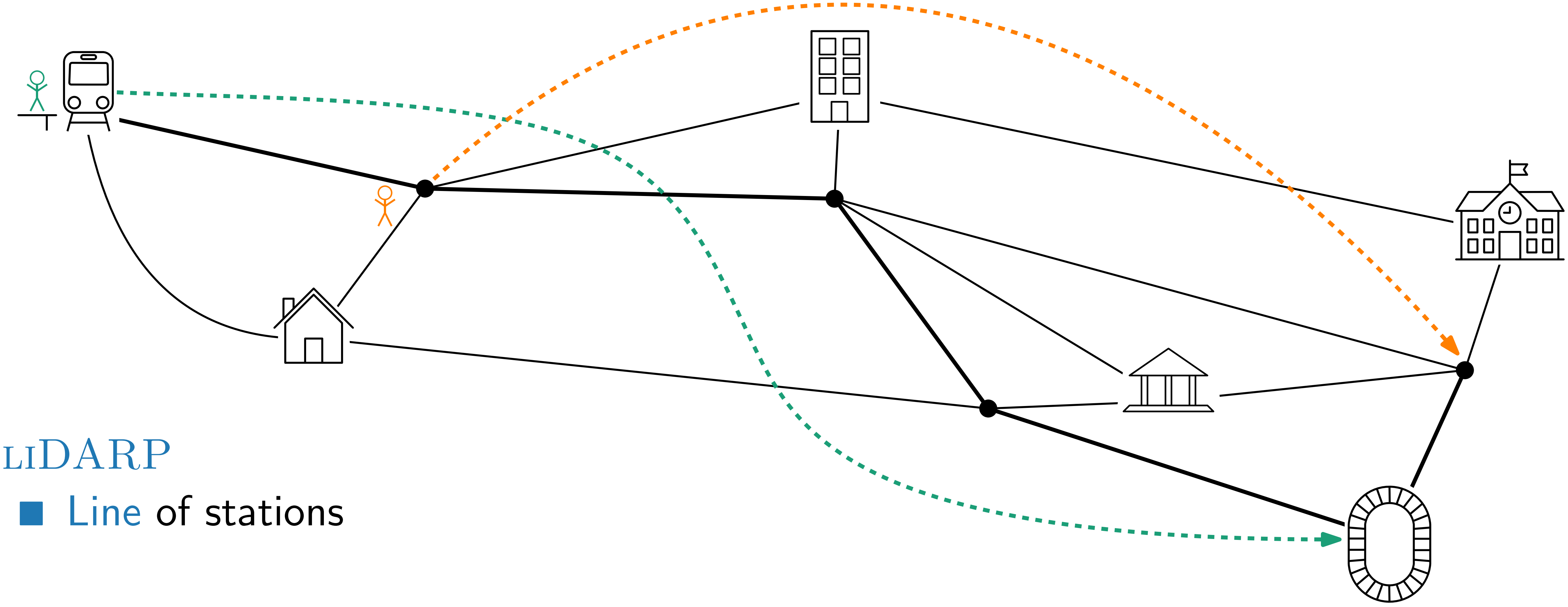
line-based Dial-a-Ride-Problem (liDARP)



liDARP

- **Line** of stations

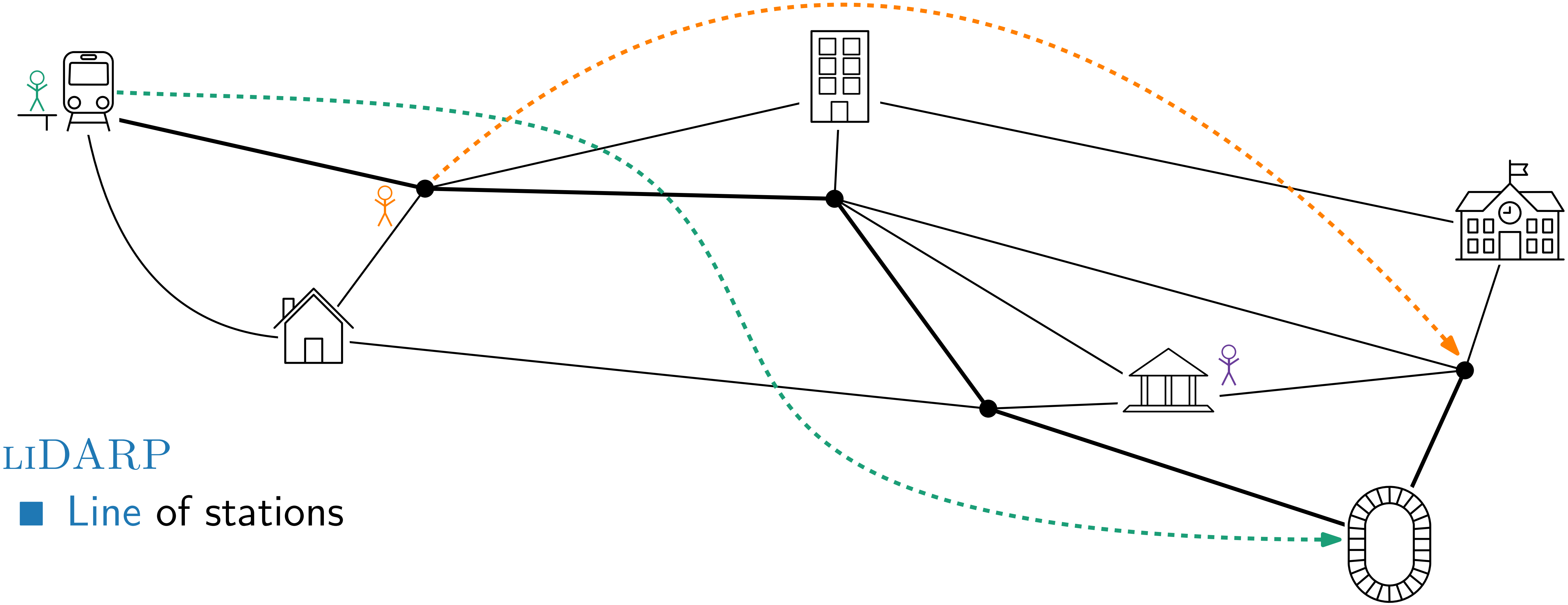
line-based Dial-a-Ride-Problem (liDARP)



liDARP

- Line of stations

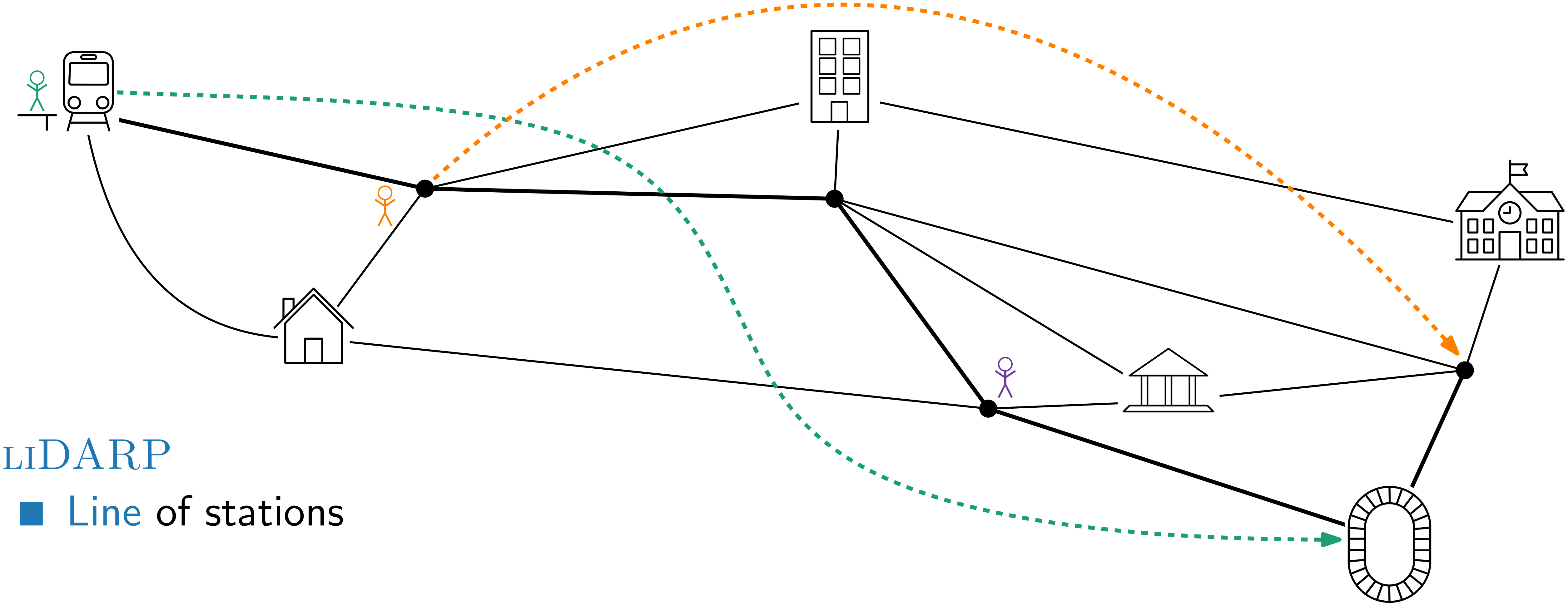
line-based Dial-a-Ride-Problem (liDARP)



liDARP

- Line of stations

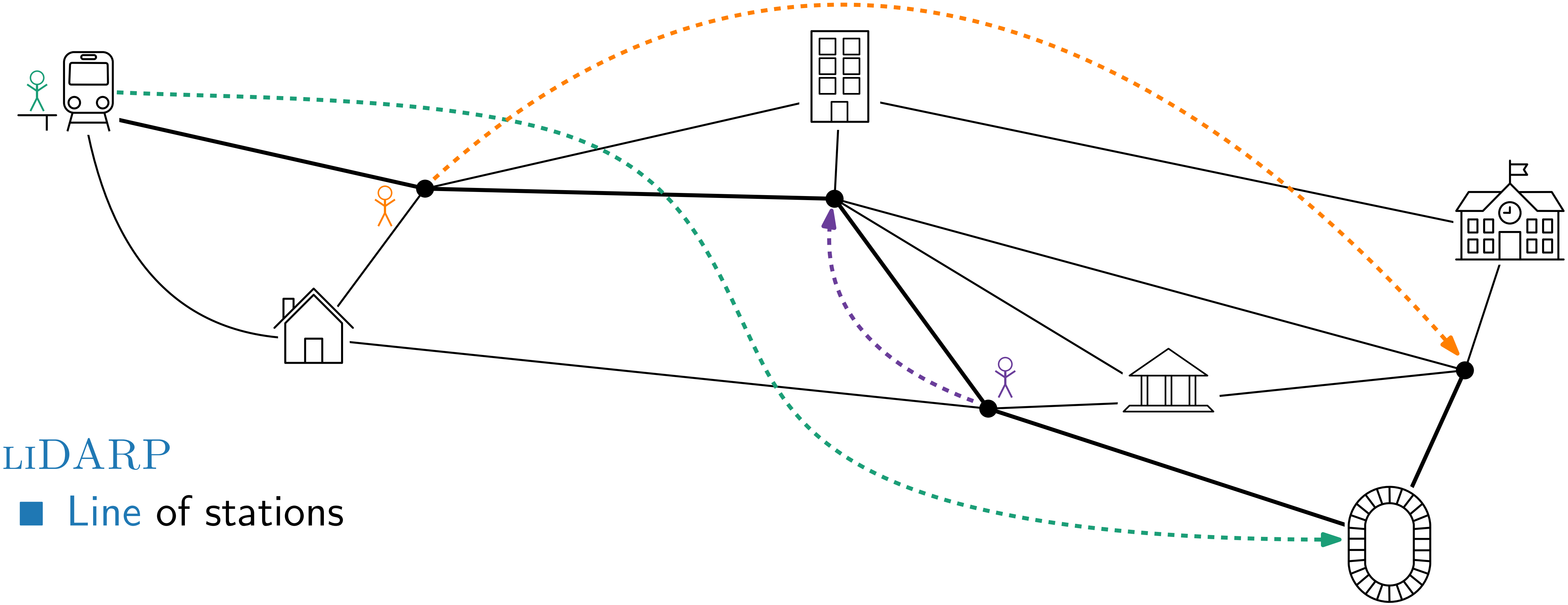
line-based Dial-a-Ride-Problem (liDARP)



liDARP

- Line of stations

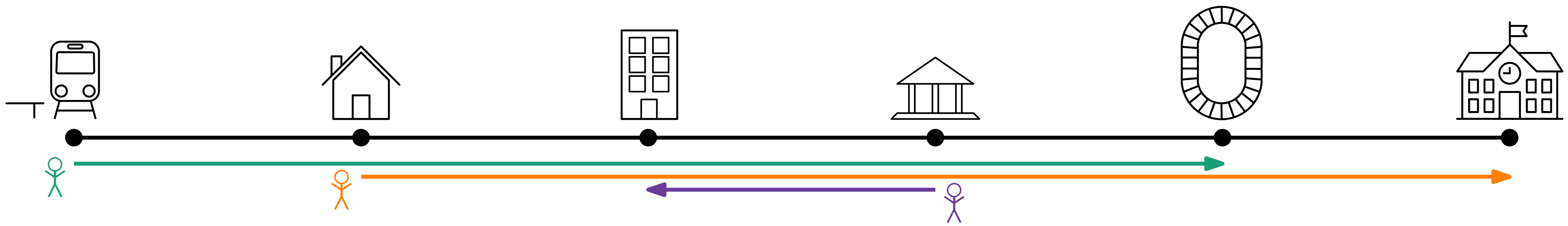
line-based Dial-a-Ride-Problem (liDARP)



liDARP

- Line of stations

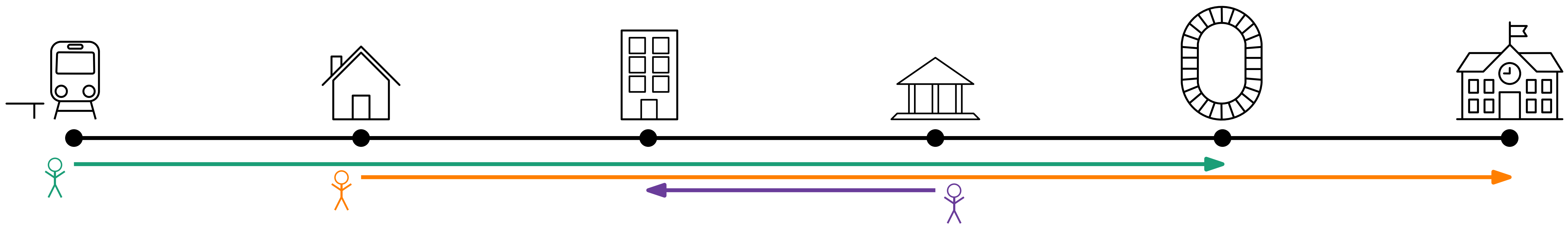
line-based Dial-a-Ride-Problem (liDARP)



liDARP

- Line of stations

line-based Dial-a-Ride-Problem (liDARP)



liDARP

- Line of stations
- Directionality: passengers always transported towards destination

line-based Dial-a-Ride-Problem (liDARP)

liDARP

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- Directionality: passengers always transported towards destination

line-based Dial-a-Ride-Problem (liDARP)

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line-based Dial-a-Ride-Problem (liDARP)

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line-based Dial-a-Ride-Problem (liDARP)

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line-based Dial-a-Ride-Problem (liDARP)

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line-based Dial-a-Ride-Problem (liDARP)

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- Directionality: passengers always transported towards destination

line-based Dial-a-Ride-Problem (liDARP)

liDARP

- Line of stations
- Directionality: passengers always transported towards destination

LINE-BASED DIAL-A-RIDE-PROBLEM (LIDARP)

LIDARP

- Line of stations
- Directionality: passengers always transported towards destination

line-based Dial-a-Ride-Problem (liDARP)

liDARP

- Line of stations
- Directionality: passengers always transported towards destination
  - bus empty when changing direction

line-based Dial-a-Ride-Problem (liDARP)

liDARP

- Line of stations
- Directionality: passengers always transported towards destination
  - bus empty when changing direction

line-based Dial-a-Ride-Problem (liDARP)

liDARP

- Line of stations
- Directionality: passengers always transported towards destination
  - bus empty when changing direction

LINE-BASED DIAL-A-RIDE-PROBLEM (LIDARP)

LIDARP

- Line of stations
- Directionality: passengers always transported towards destination
  - bus empty when changing direction

line-based Dial-a-Ride-Problem (liDARP)

liDARP

- Line of stations
- Directionality: passengers always transported towards destination
  - bus empty when changing direction
  - passengers only dropped-off at their destination

line-based Dial-a-Ride-Problem (liDARP)

liDARP

- Line of stations
- Directionality: passengers always transported towards destination
  - bus empty when changing direction
  - passengers only dropped-off at their destination

line-based Dial-a-Ride-Problem (liDARP)

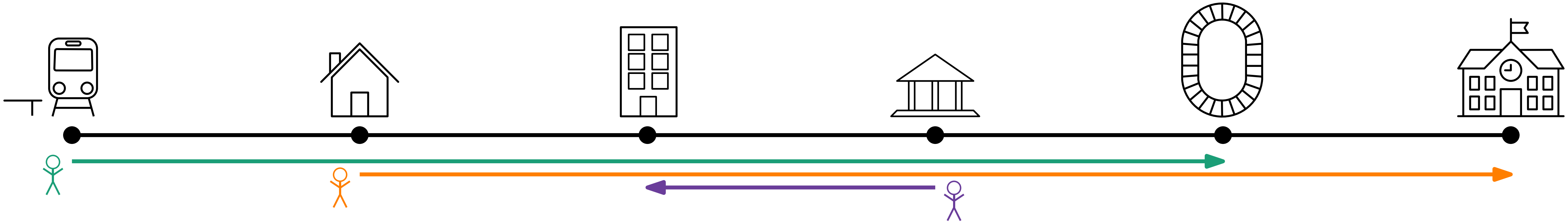
Variants

line-based Dial-a-Ride-Problem (liDARP)

Variants

- Time Windows (TW): earliest pick-up e_p and latest drop-off ℓ_p for request p

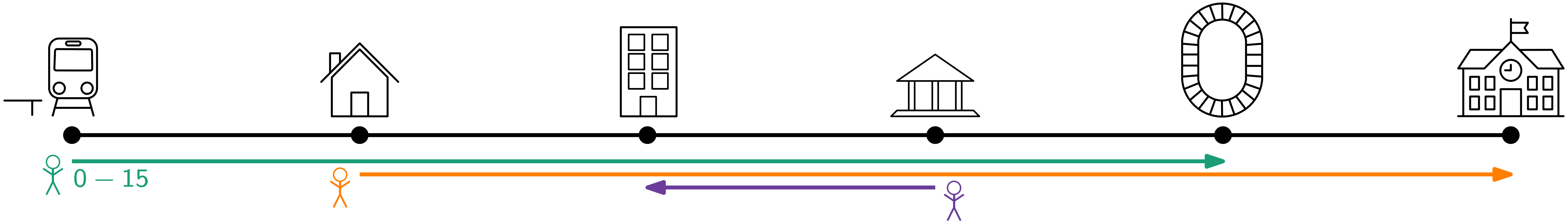
line-based Dial-a-Ride-Problem (liDARP)



Variants

- Time Windows (TW): earliest pick-up e_p and latest drop-off ℓ_p for request p

line-based Dial-a-Ride-Problem (liDARP)



Variants

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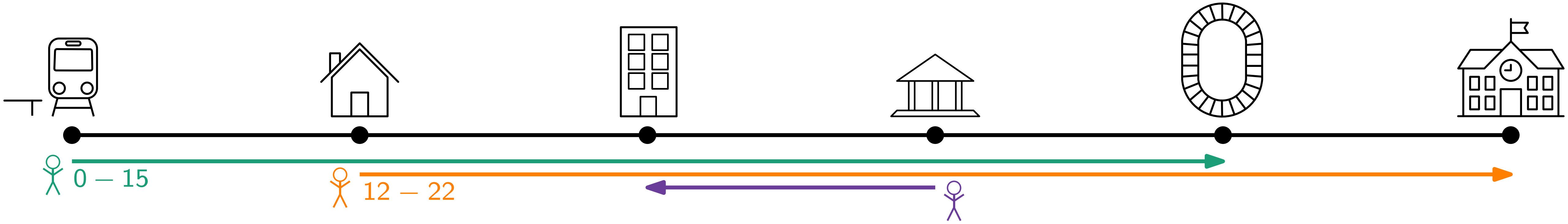
line-based Dial-a-Ride-Problem (liDARP)



Variants

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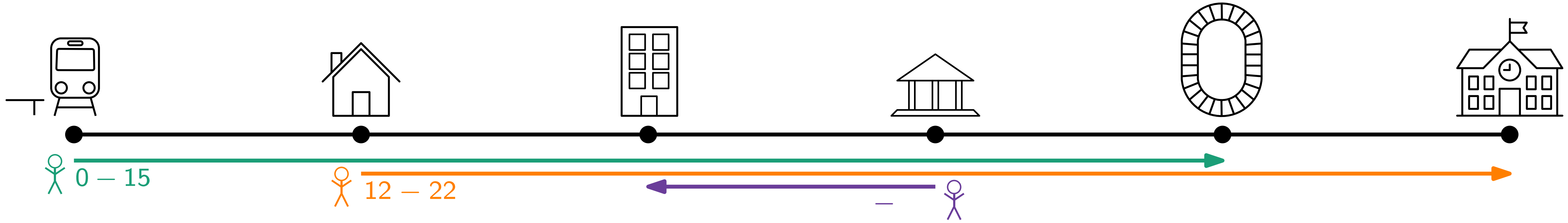
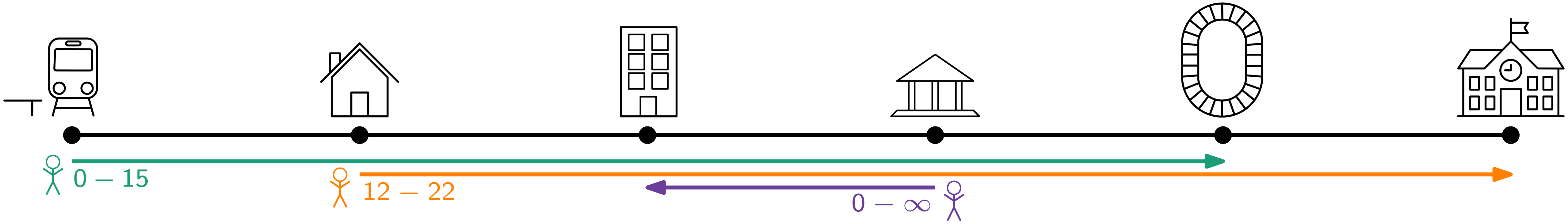


Variants

- Time Windows (TW): earliest pick-up e_p and latest drop-off ℓ_p for request p

LINE-BASED DIAL-A-RIDE-PROBLEM (LIDARP)



Variants

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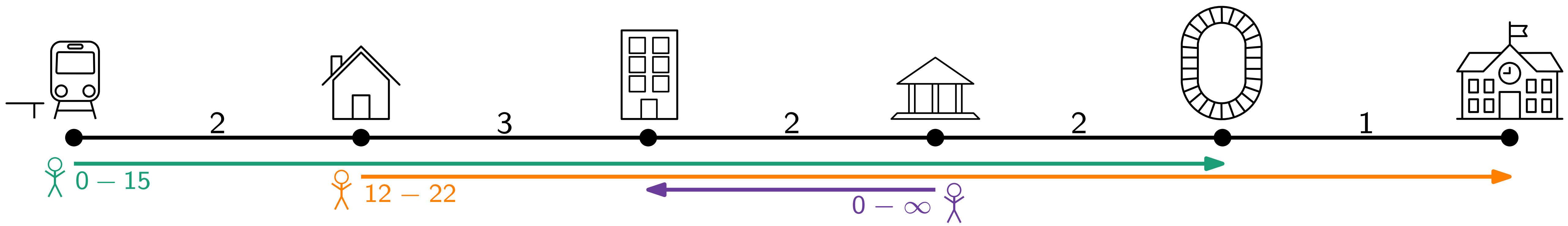
line-based Dial-a-Ride-Problem (liDARP)



Variants

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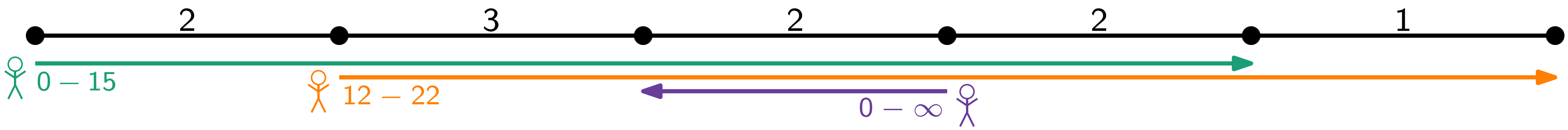
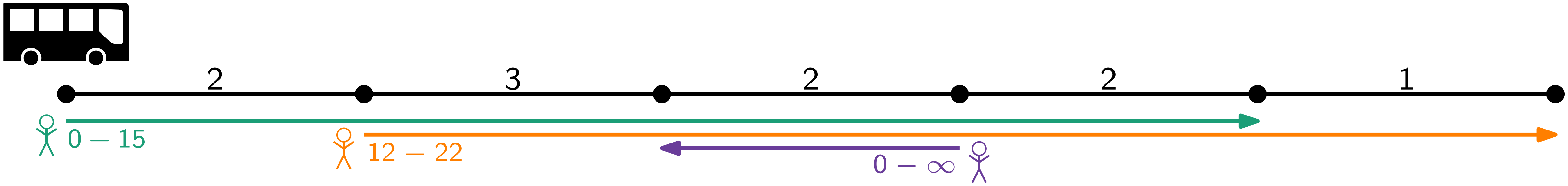


Variants

- Time Windows (TW): earliest pick-up e_p and latest drop-off ℓ_p for request p

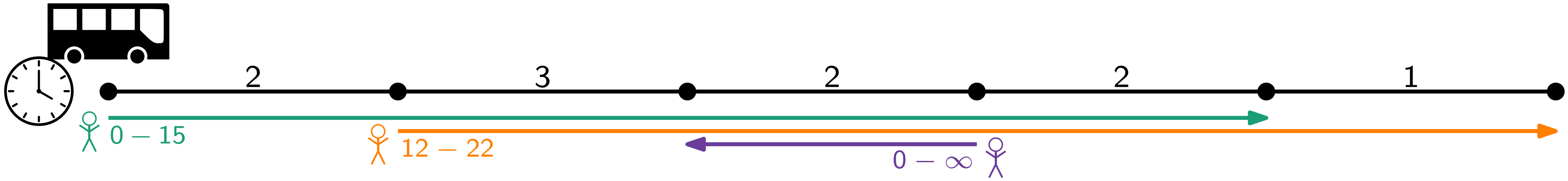
line-based Dial-a-Ride-Problem (liDARP)



Variants

- Time Windows (TW): earliest pick-up e_p and latest drop-off ℓ_p for request p

LINE-BASED DIAL-A-RIDE-PROBLEM (LIDARP)



Variants

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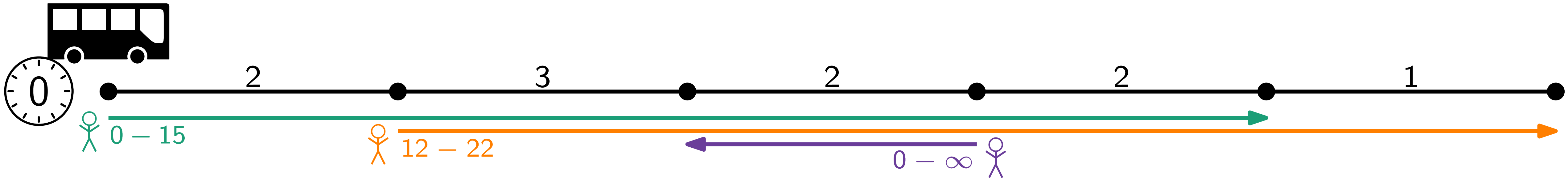
line-based Dial-a-Ride-Problem (liDARP)



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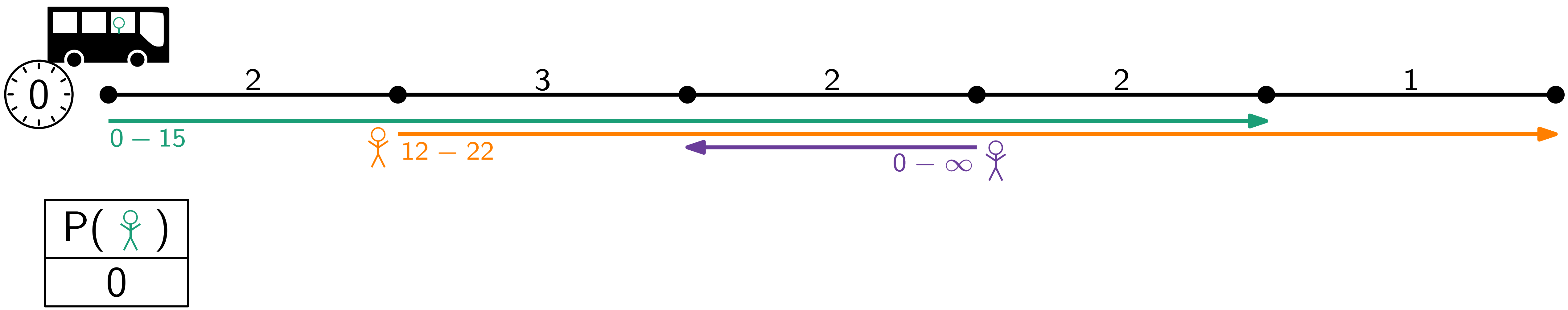

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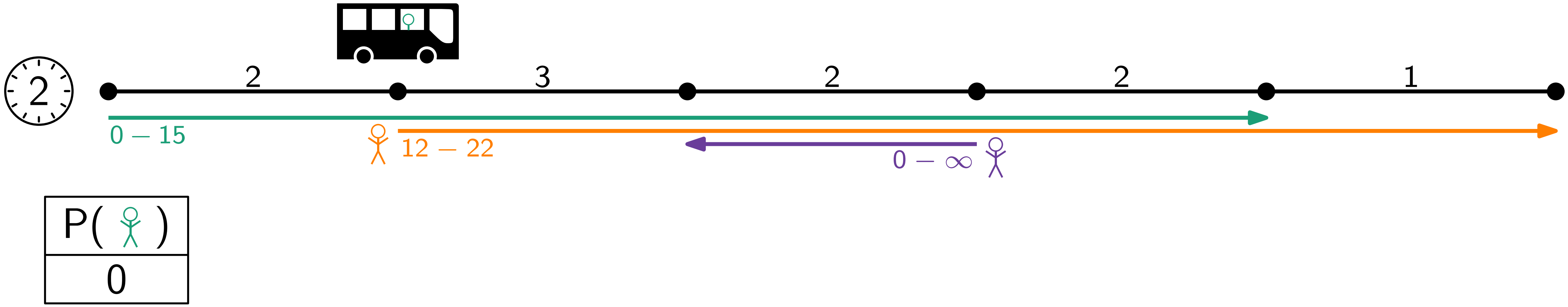

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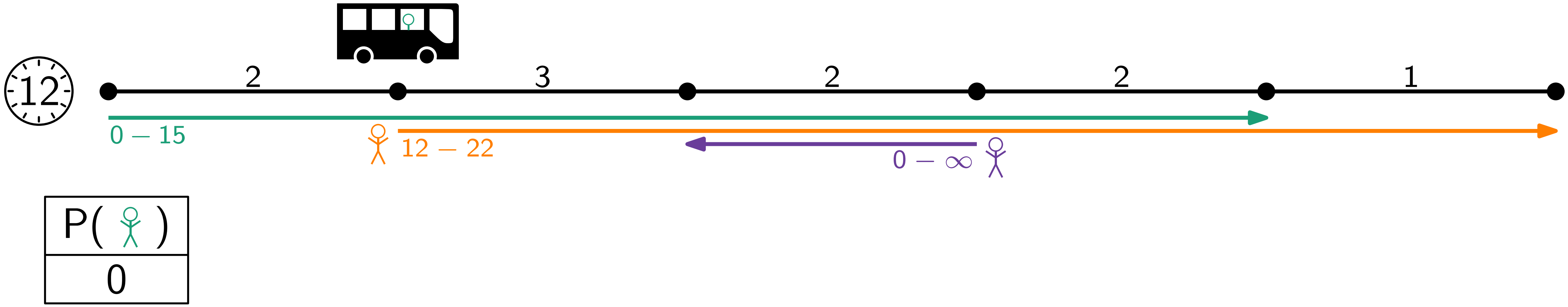

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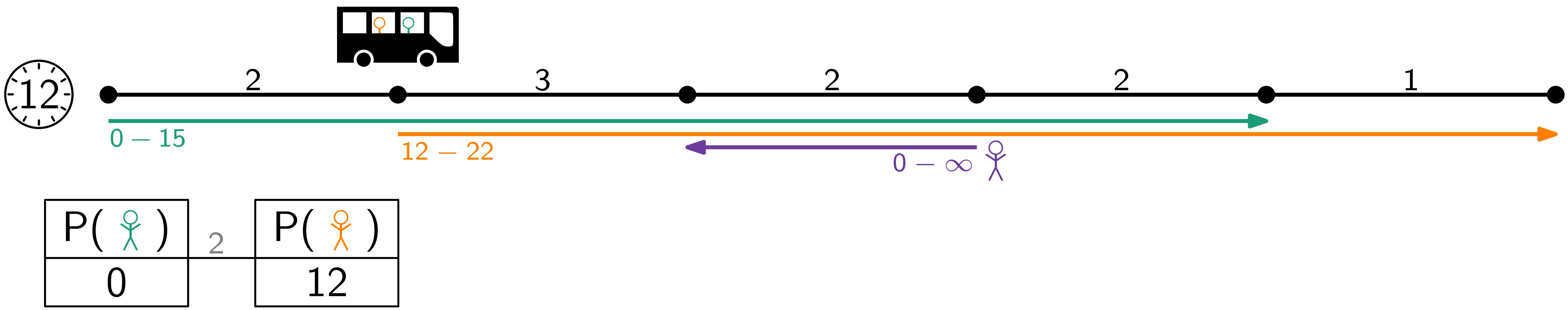

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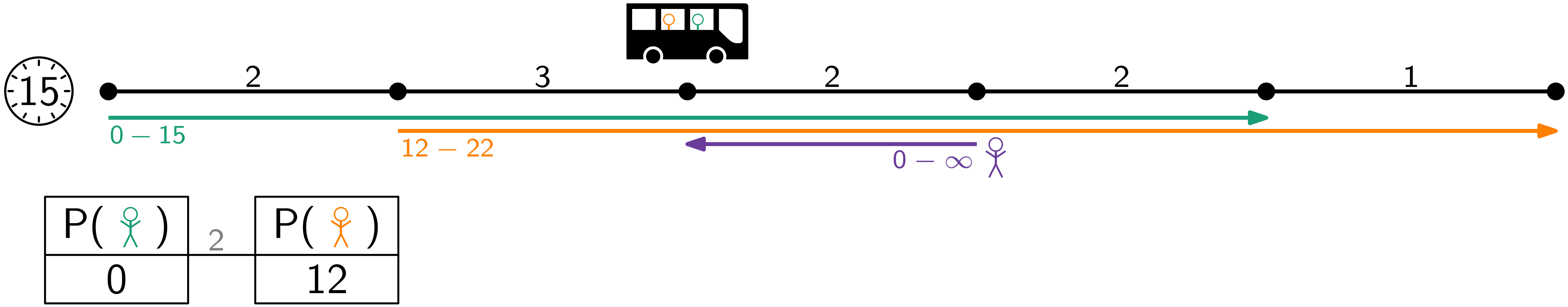

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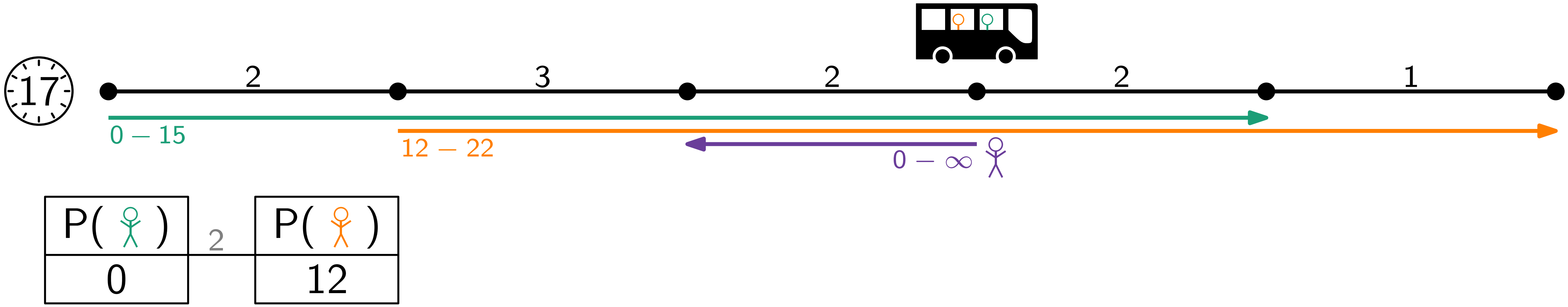

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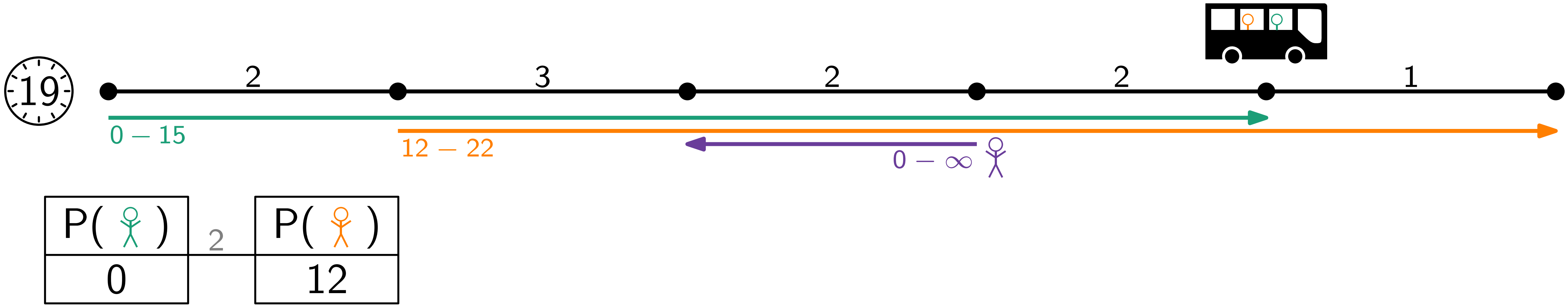

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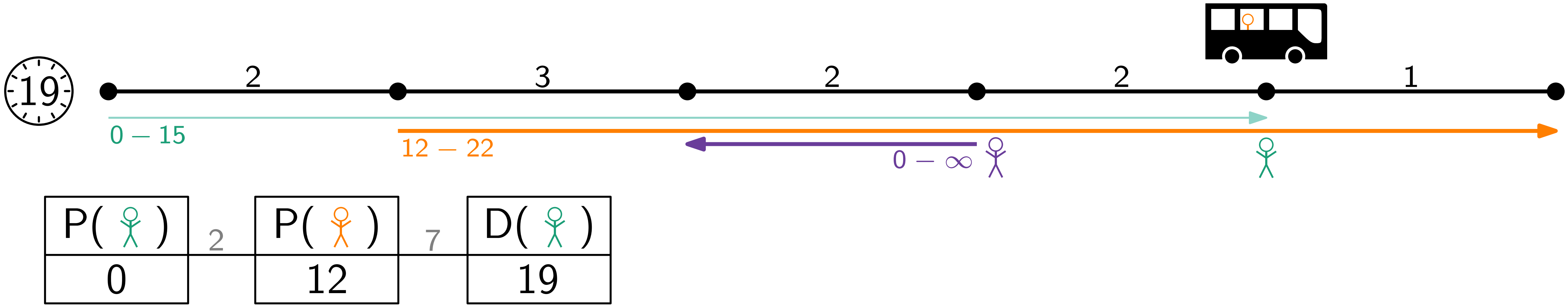

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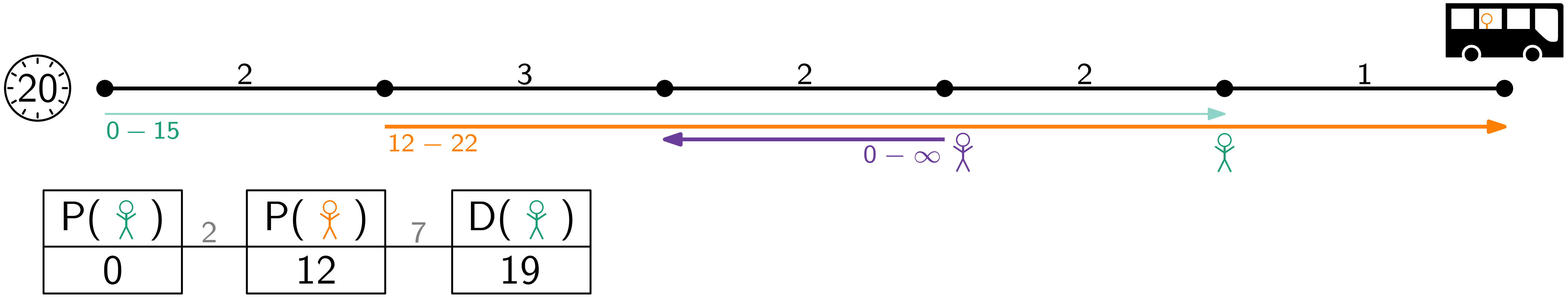

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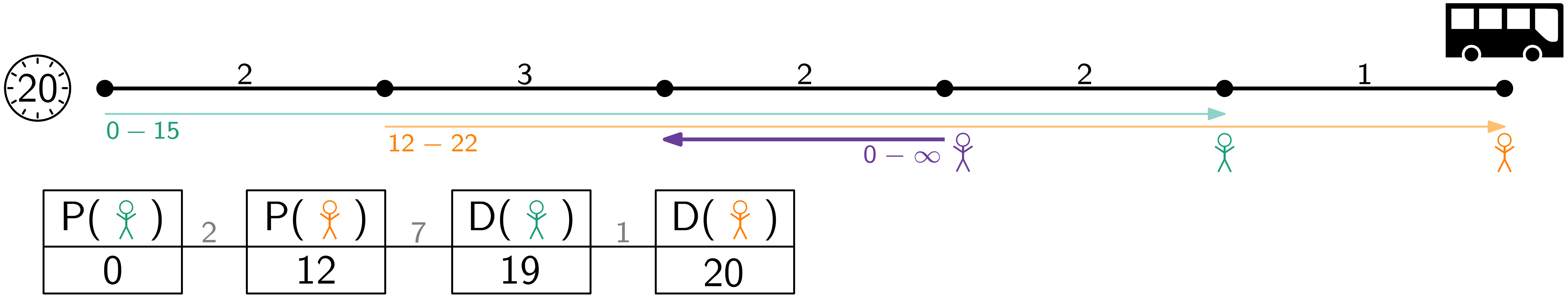

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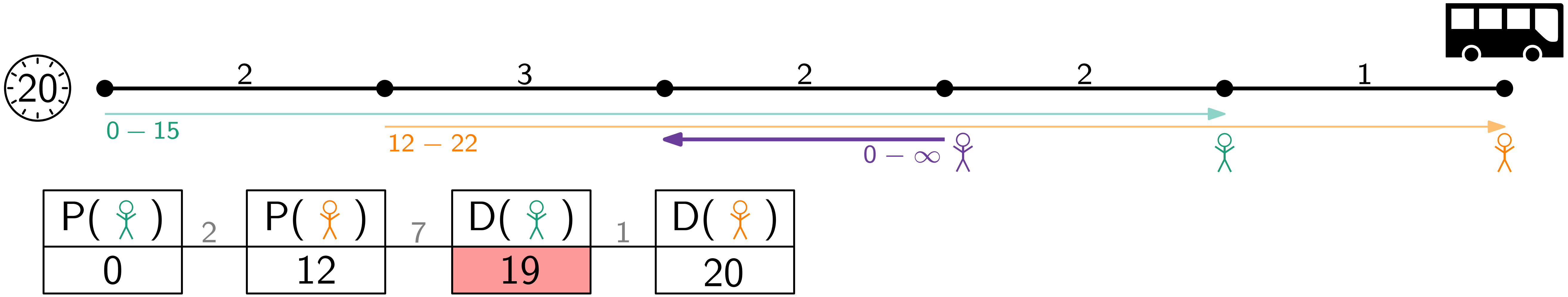

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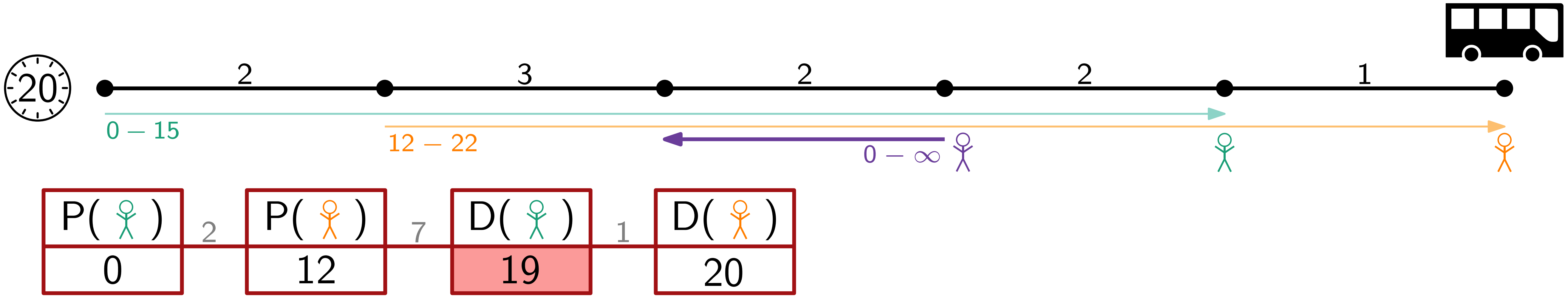

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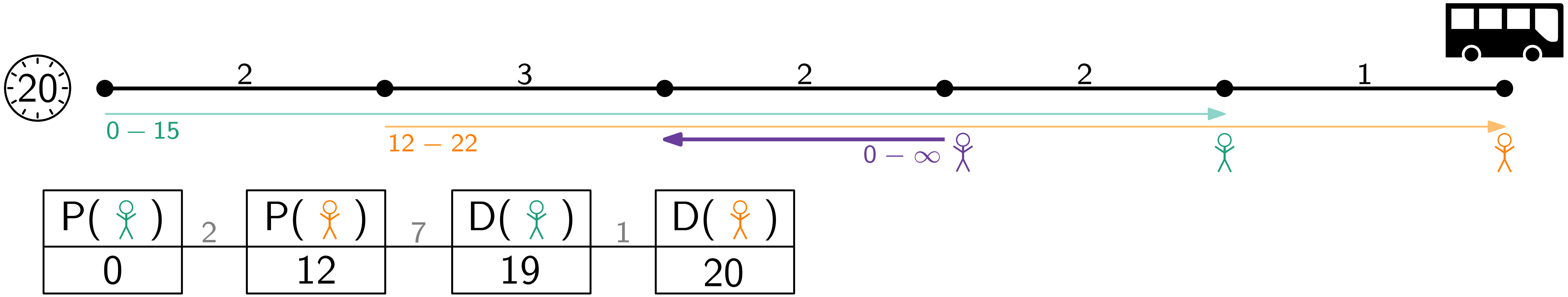

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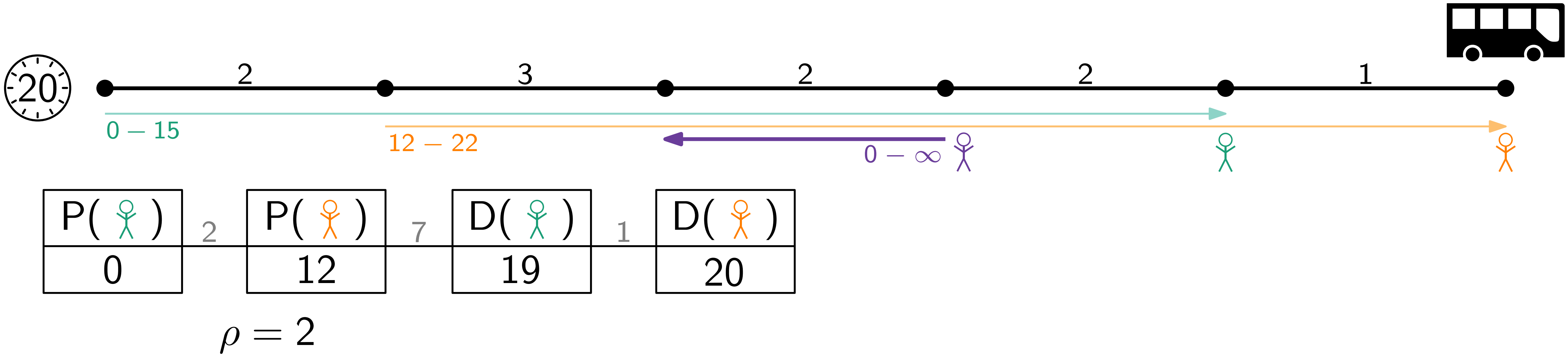

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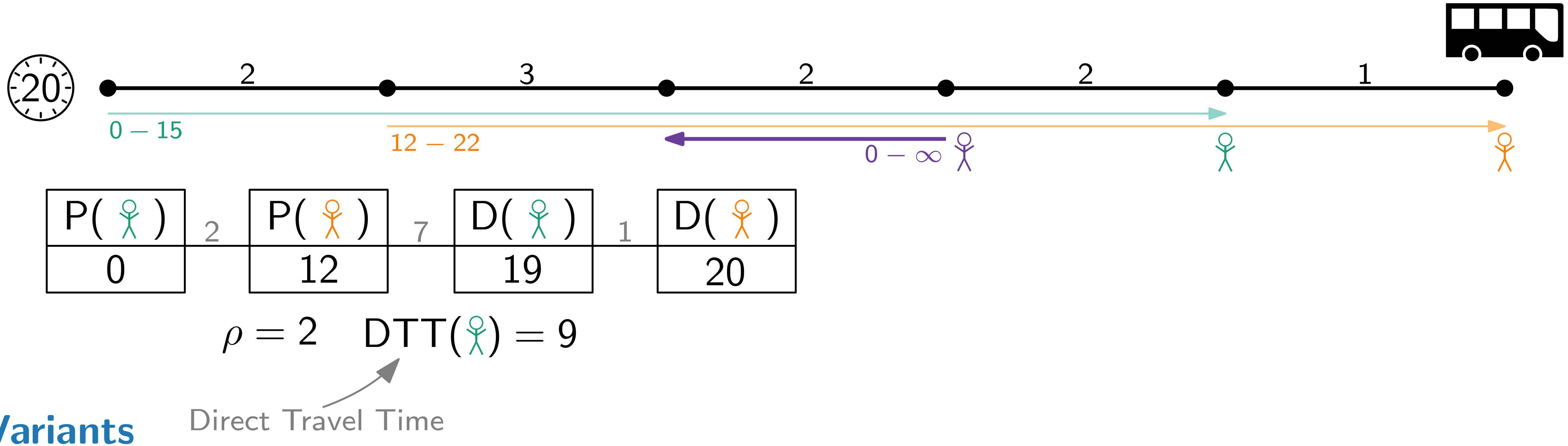

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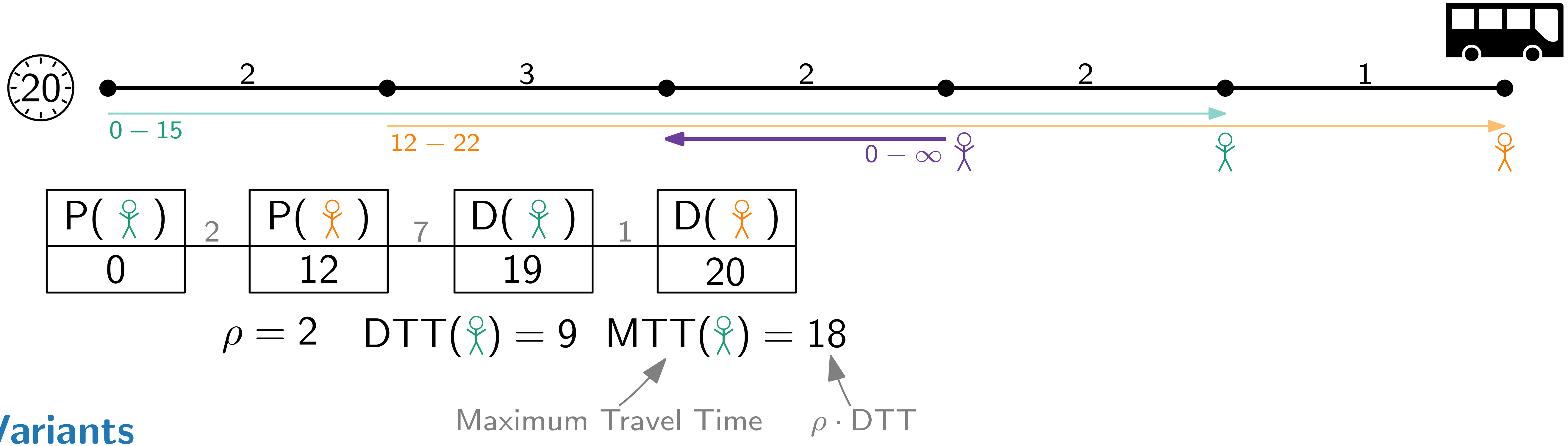

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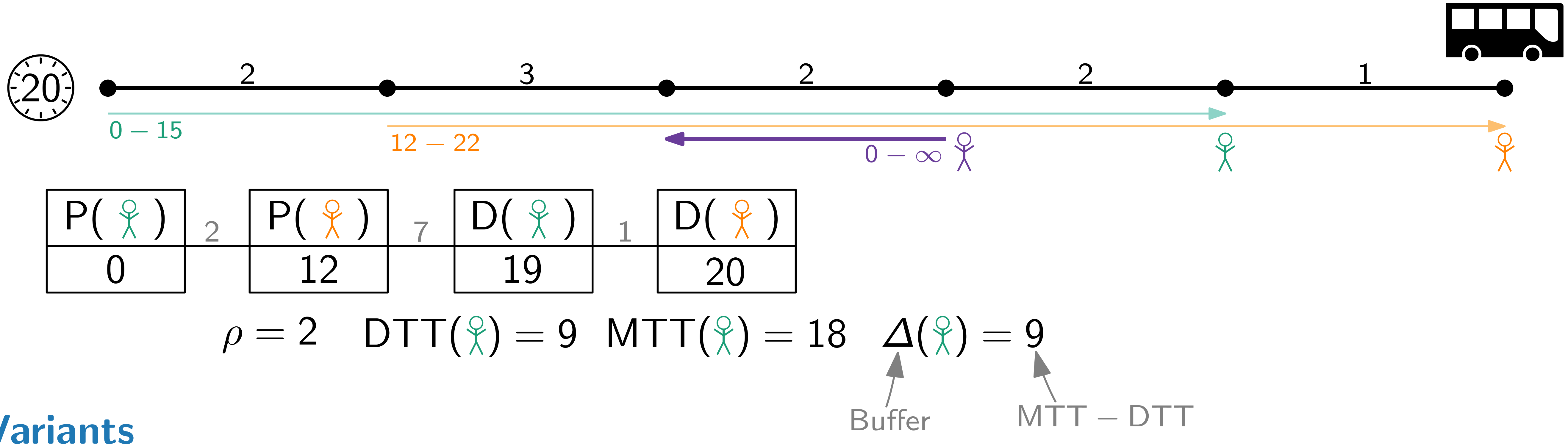

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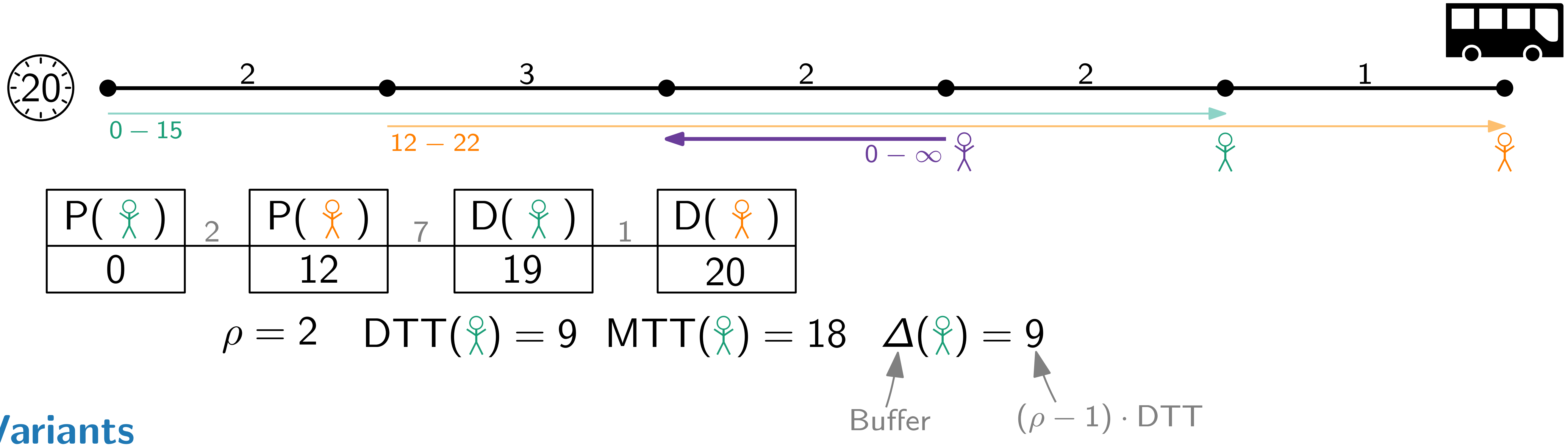

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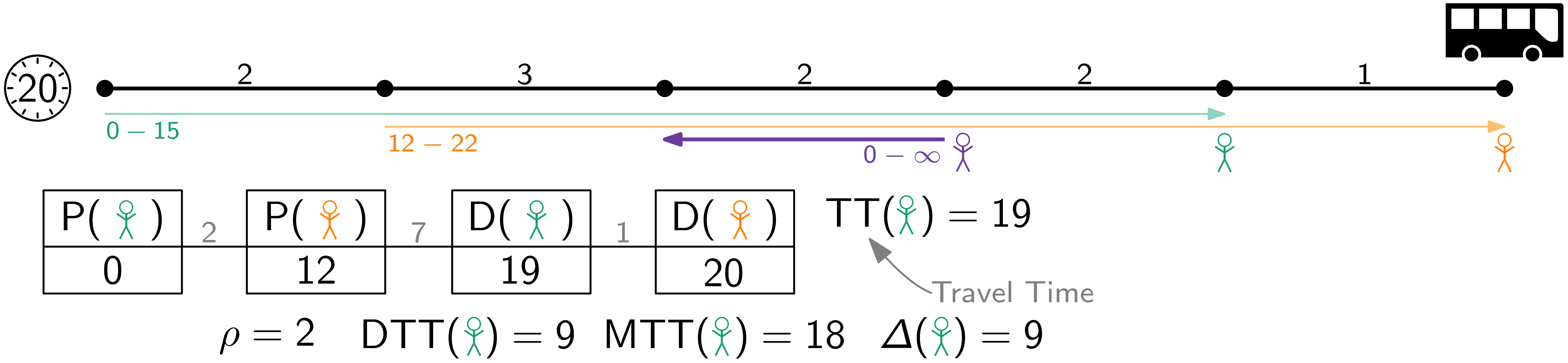

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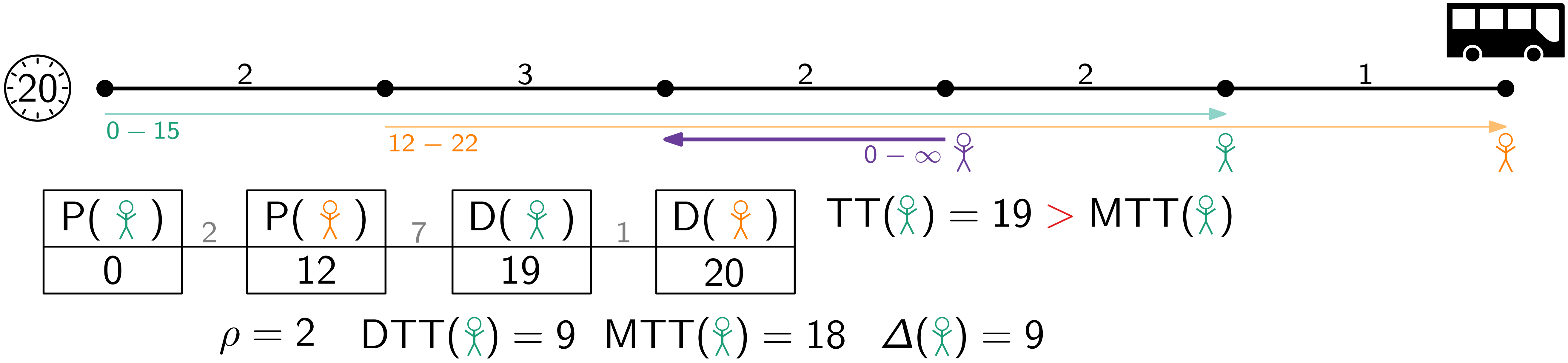

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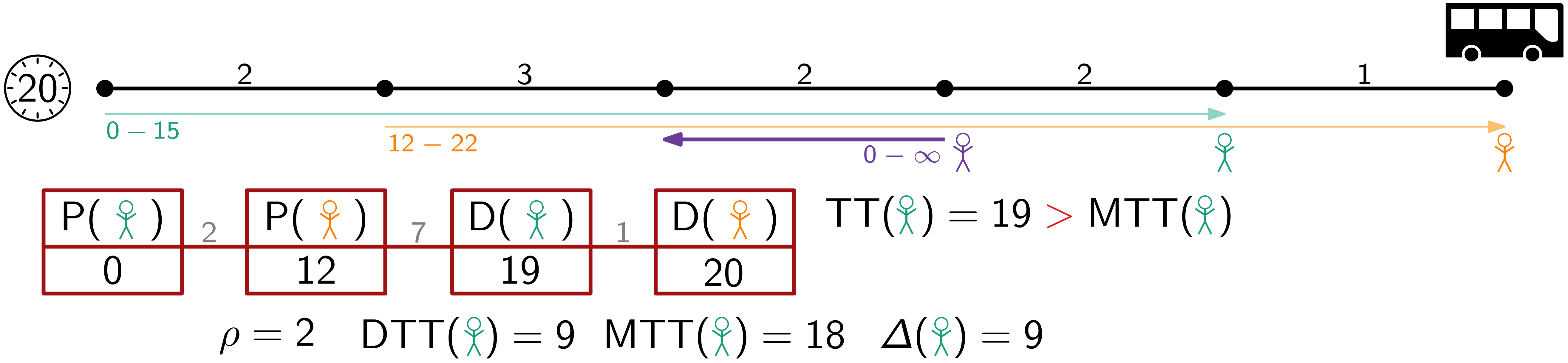

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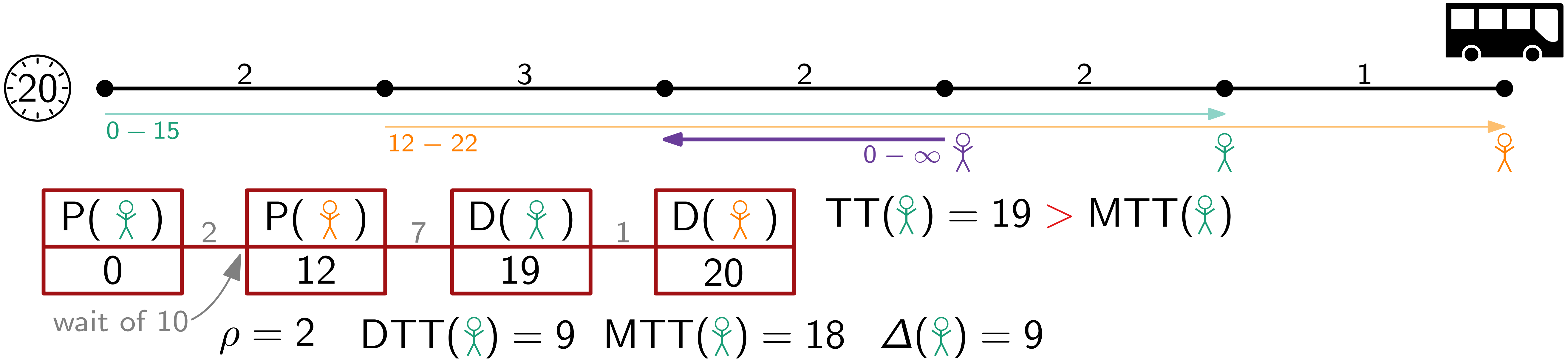

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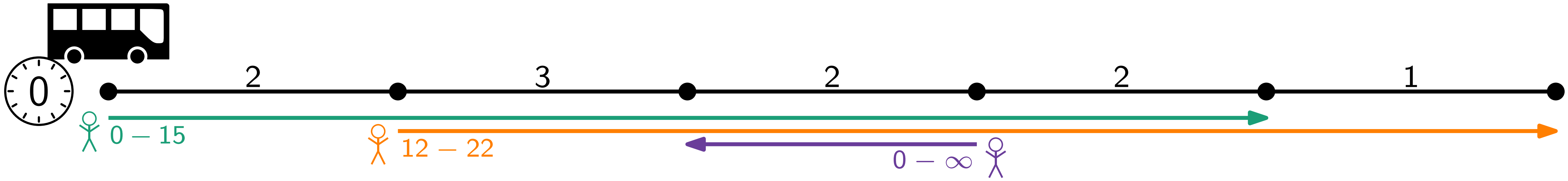

$\rho = 2 \quad \mathrm{DTT}(\;) = 9 \quad \mathrm{MTT}(\;) = 18 \quad \Delta(\;) = 9$

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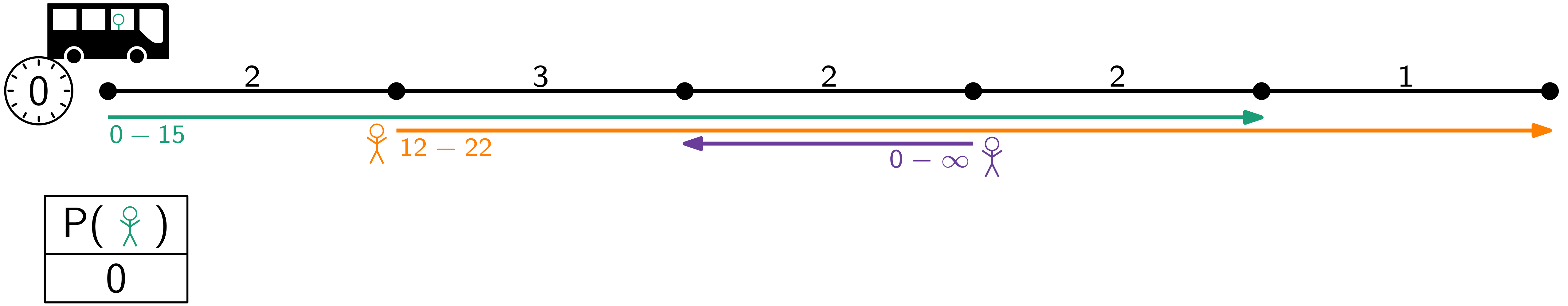

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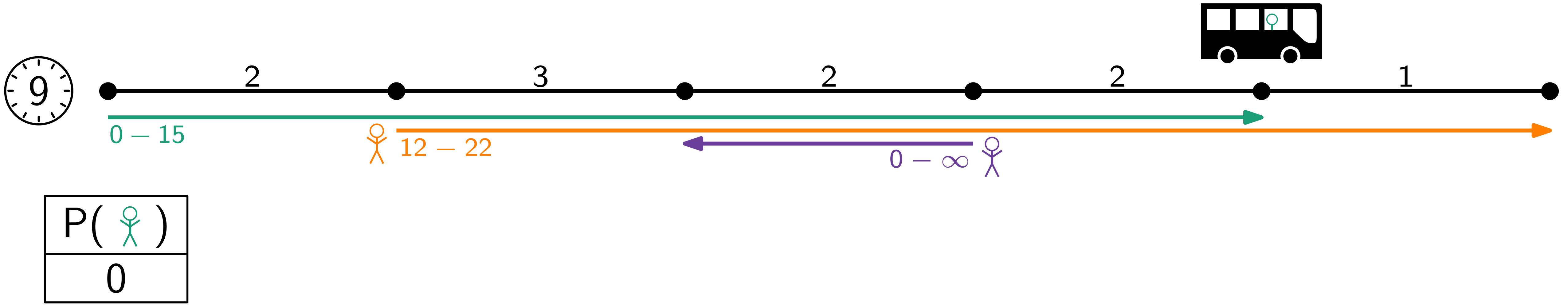

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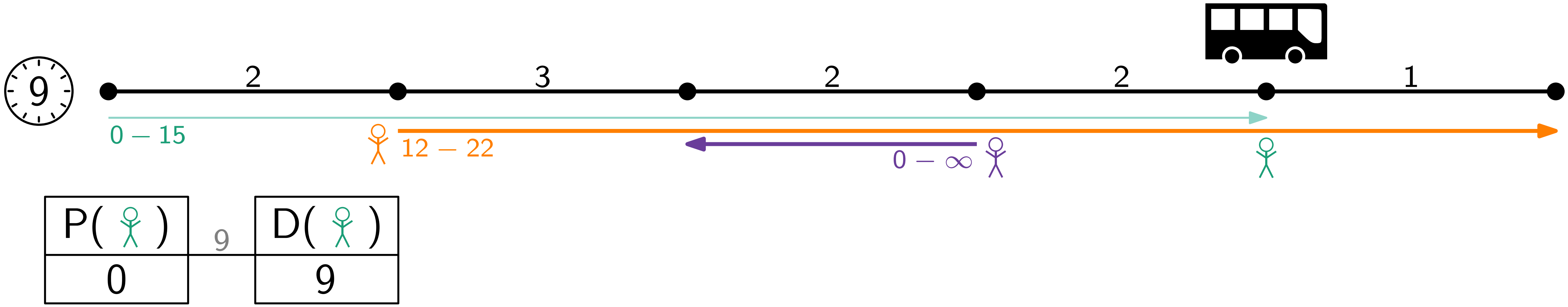

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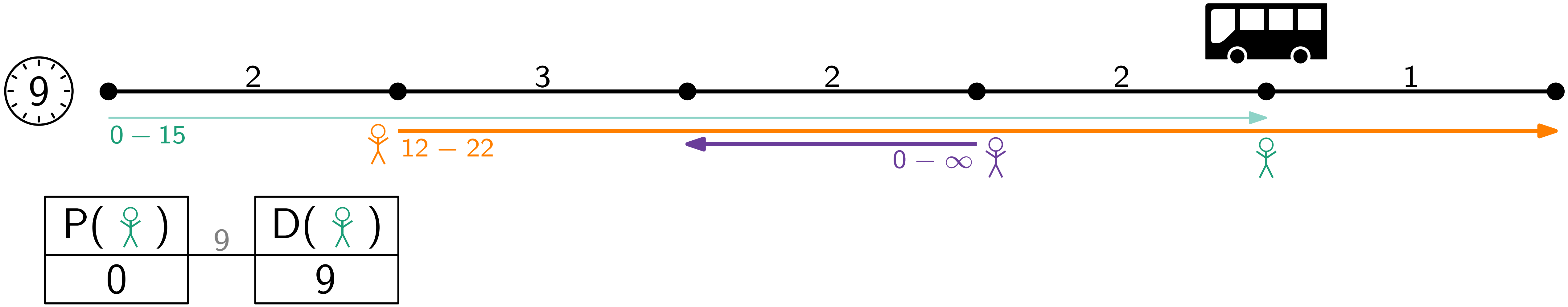

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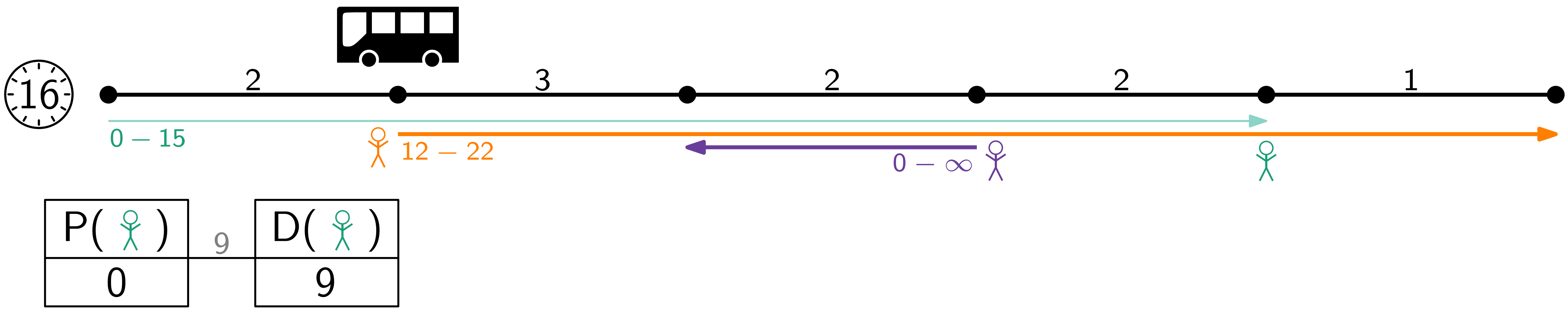

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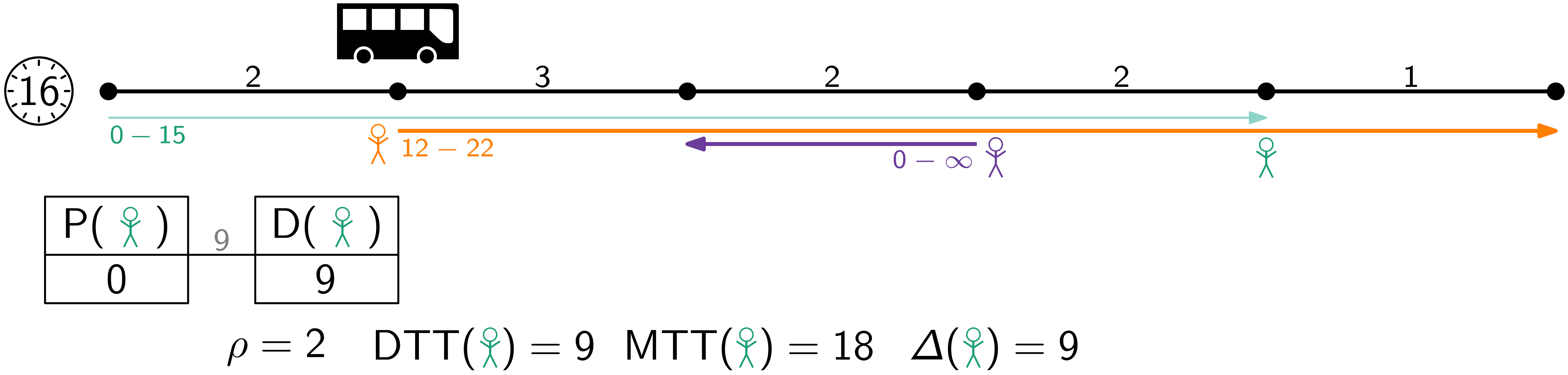
LINE-BASED DIAL-A-RIDE-PROBLEM (LIDARP)



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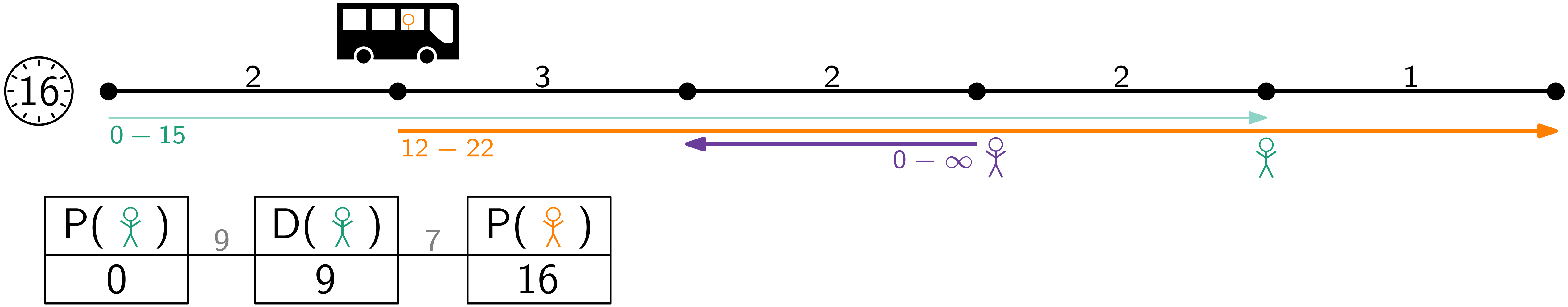

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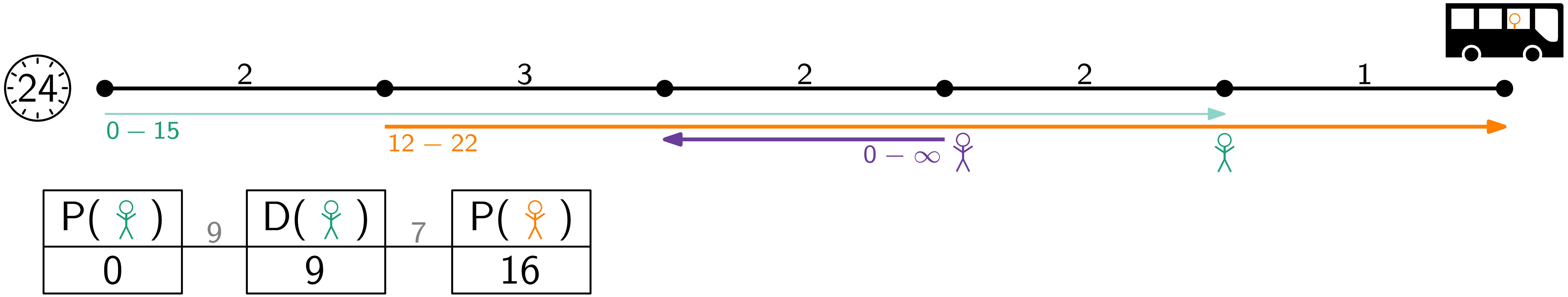

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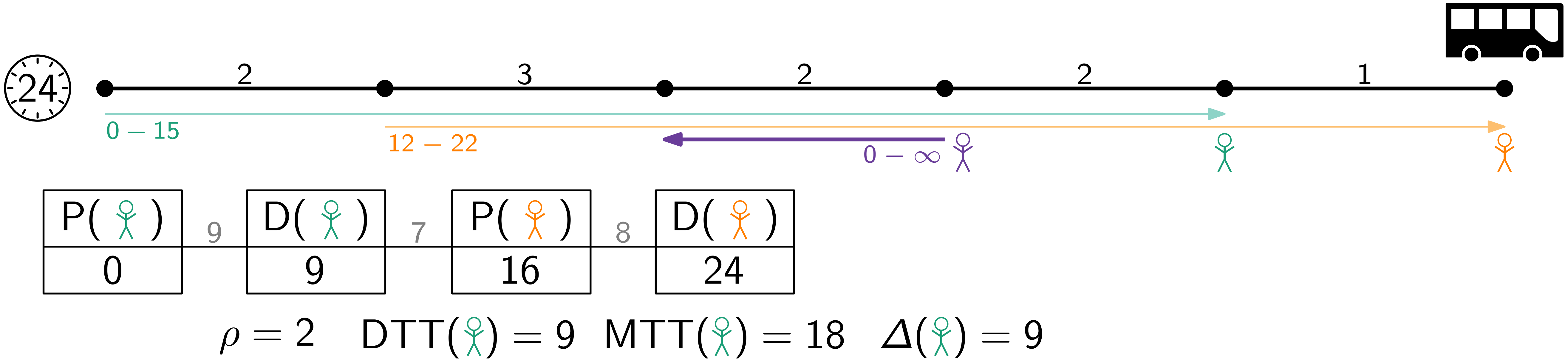

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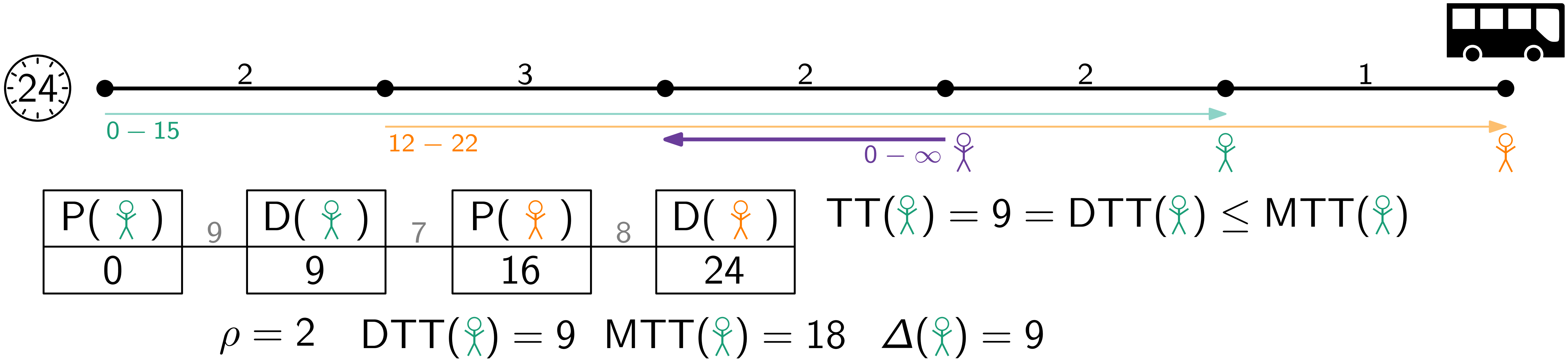

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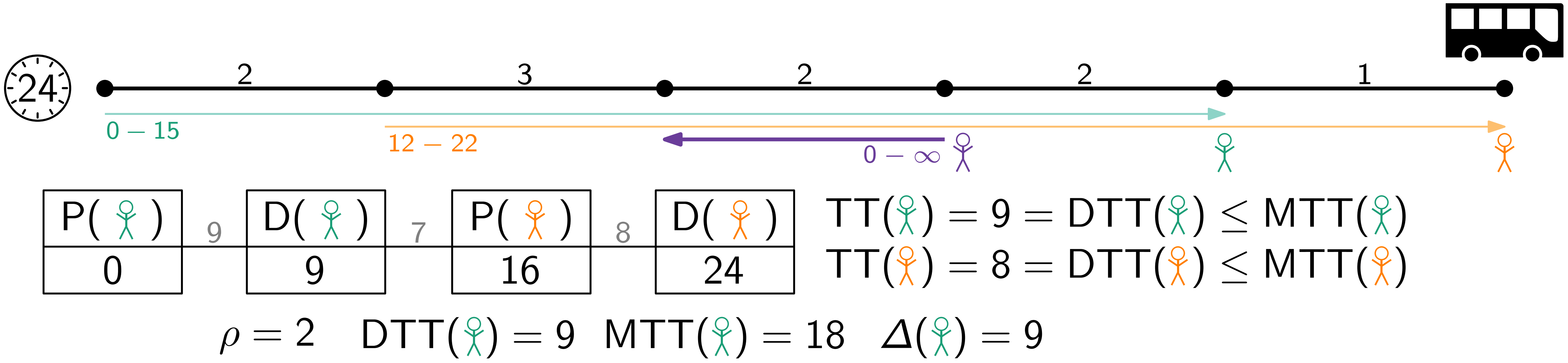

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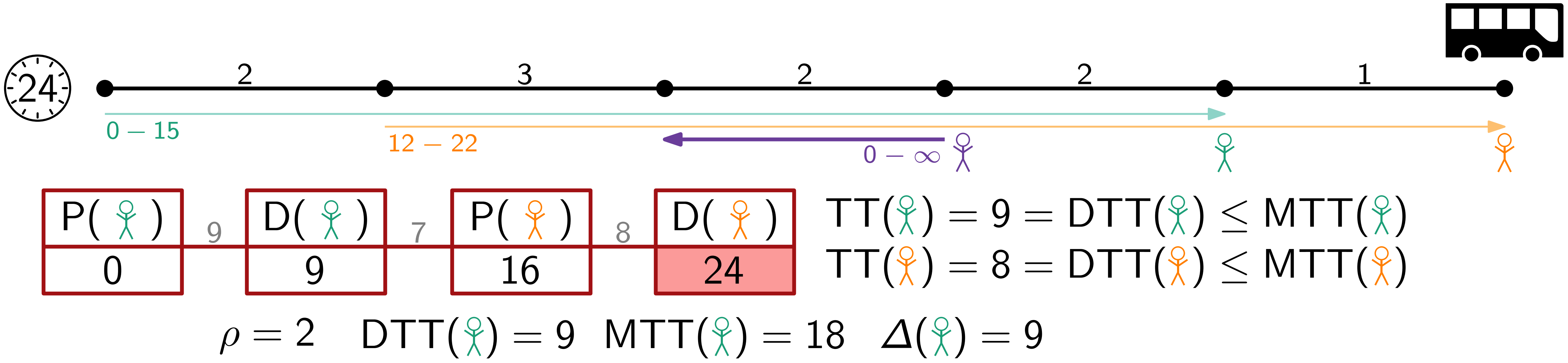

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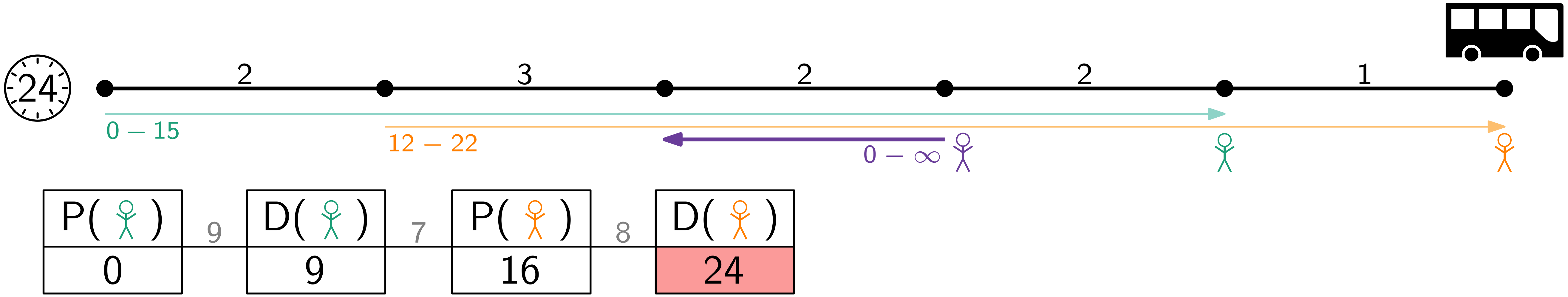

Variants

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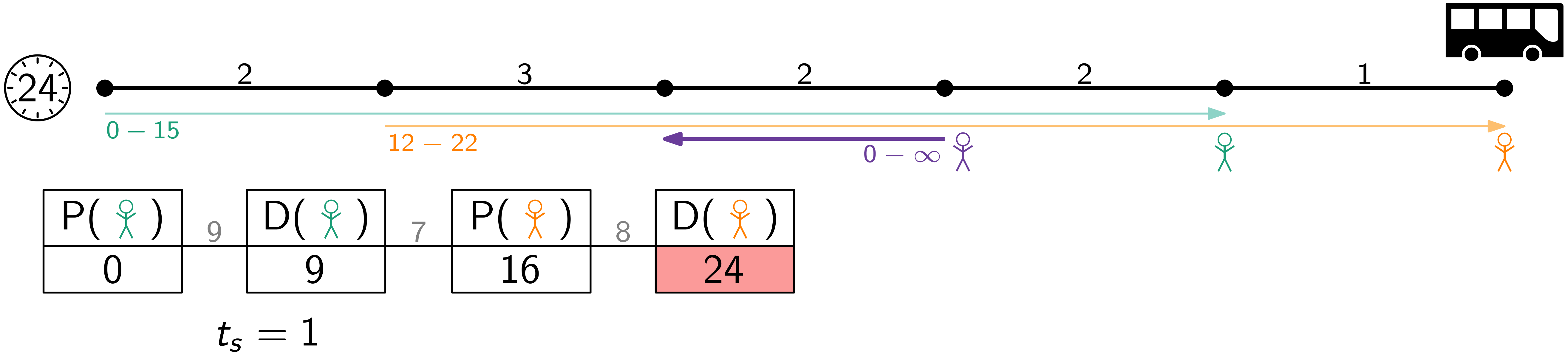
LINE-BASED DIAL-A-RIDE-PROBLEM (LIDARP)



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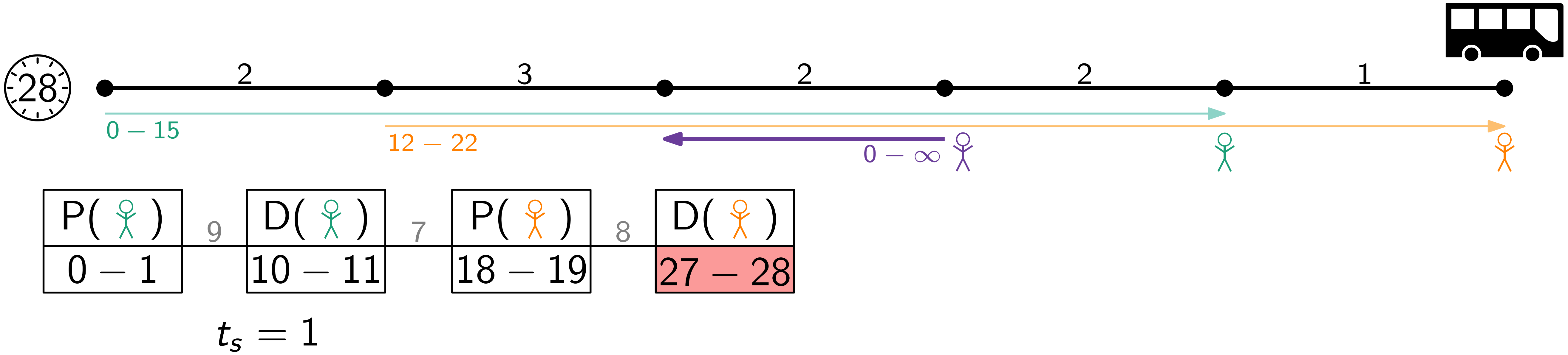

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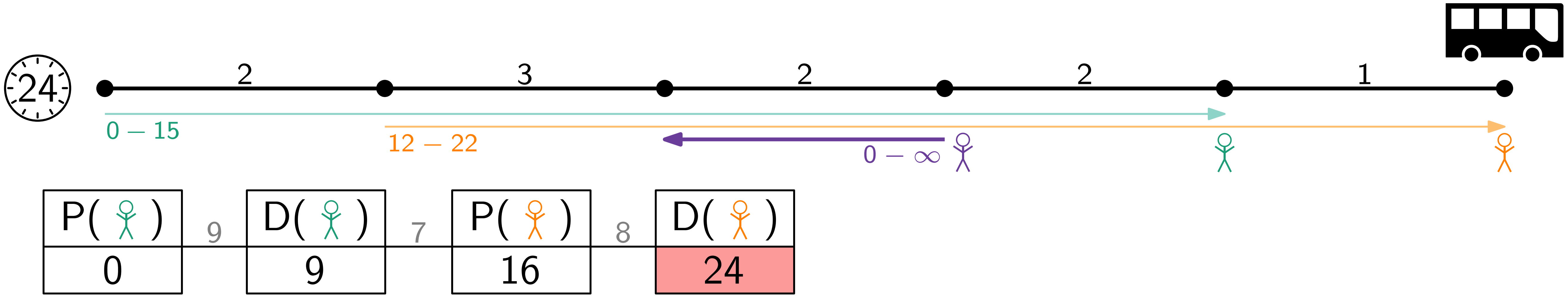

$t_s = 0$

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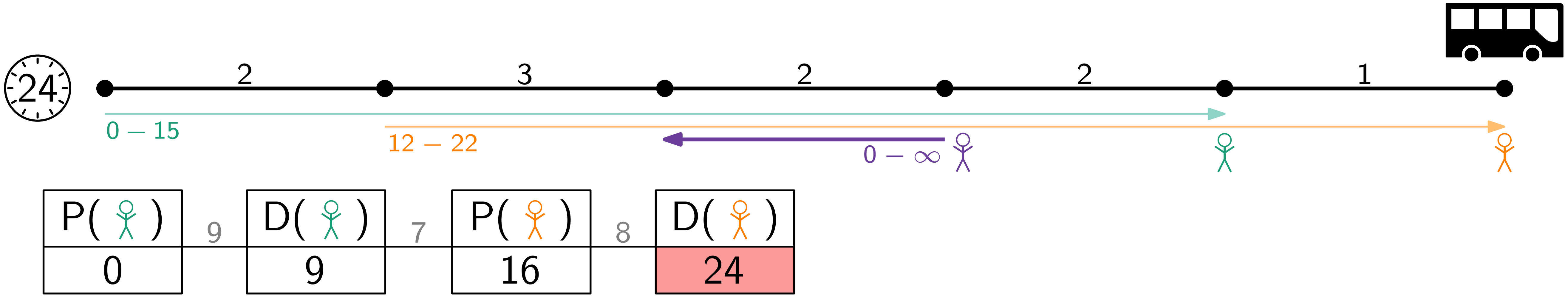

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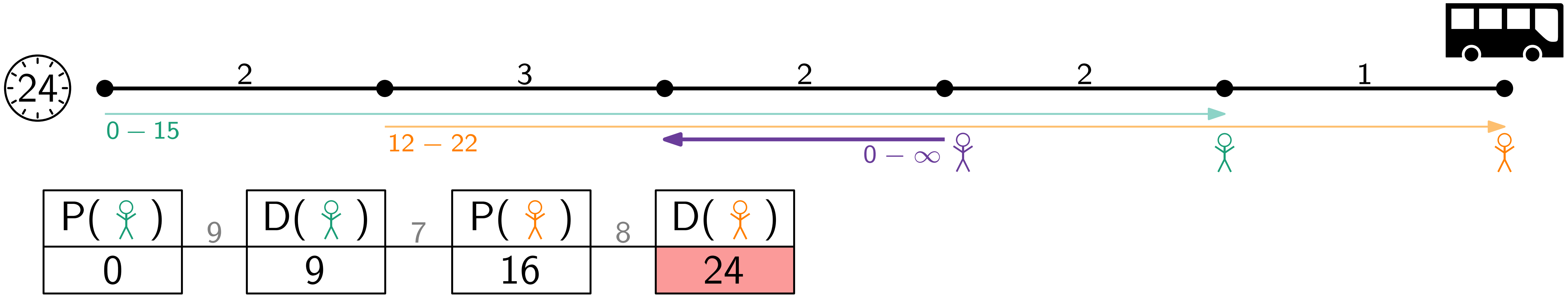
- Shortcuts (SC)

line-based Dial-a-Ride-Problem (liDARP)



Variants

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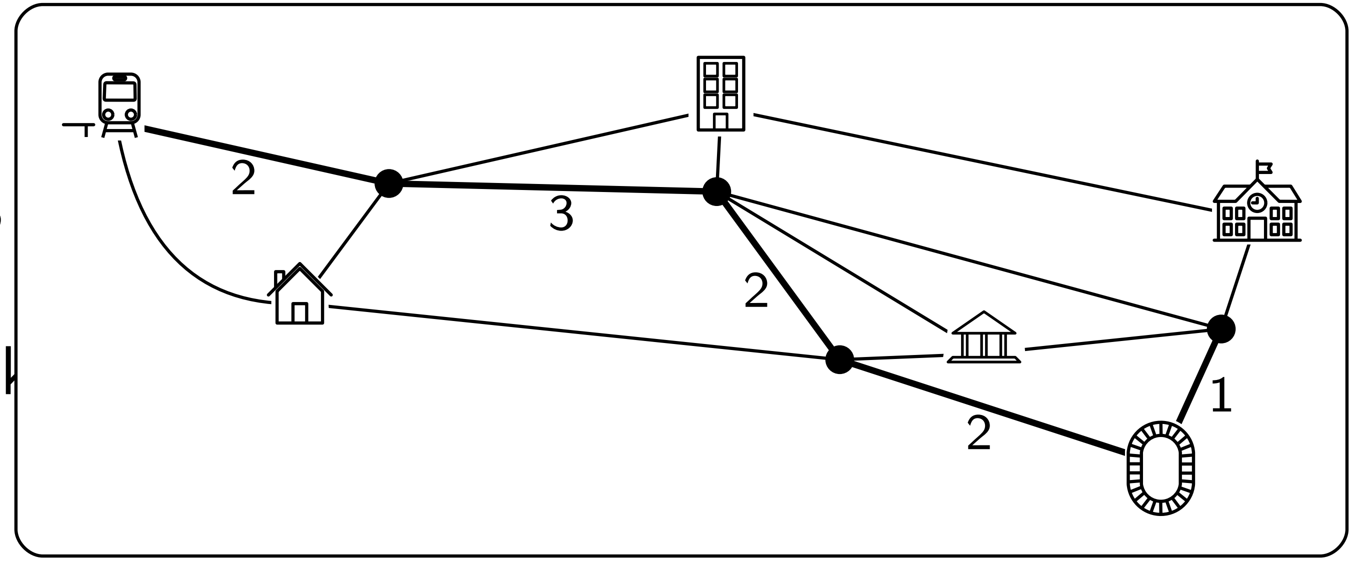

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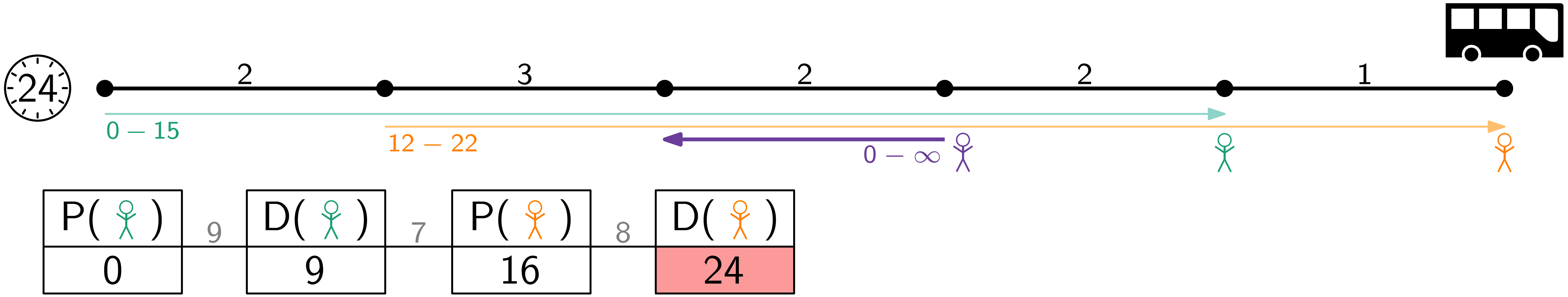

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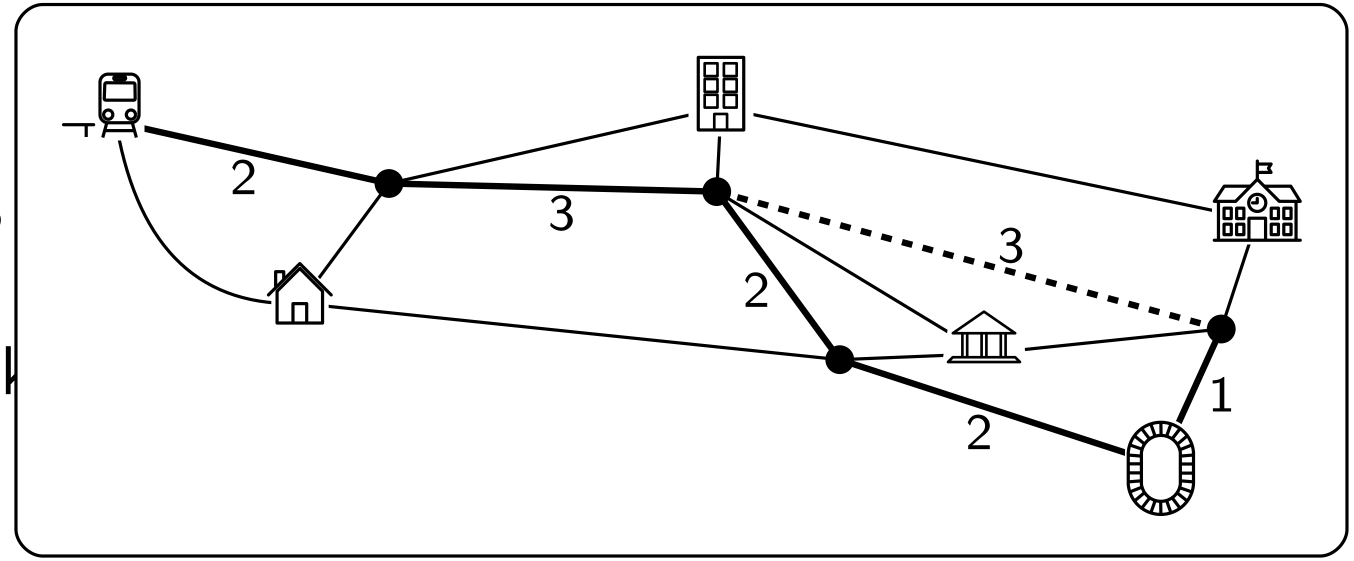
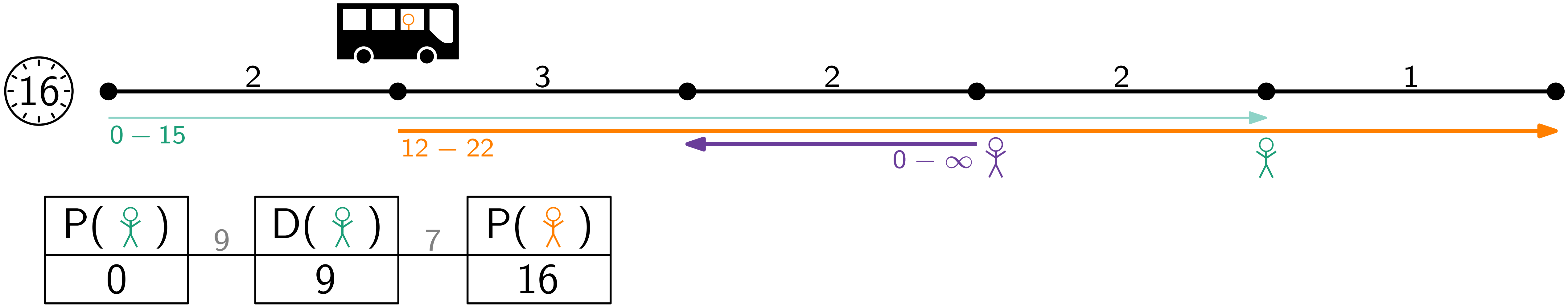


LINE-BASED DIAL-A-RIDE-PROBLEM (LIDARP)



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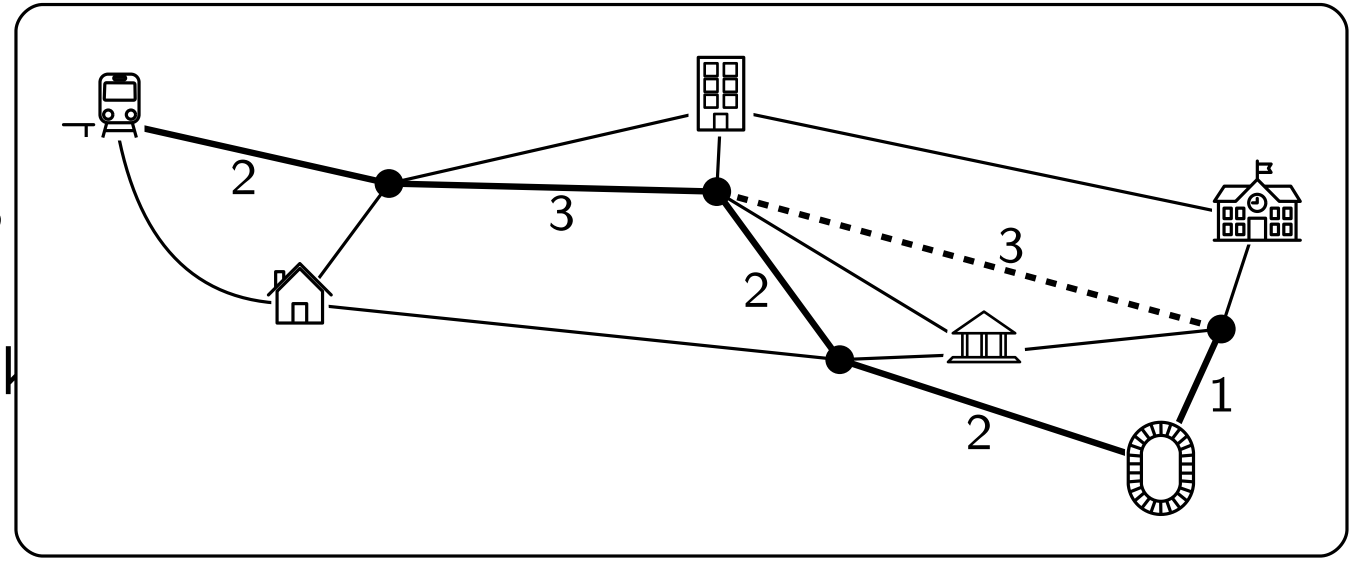
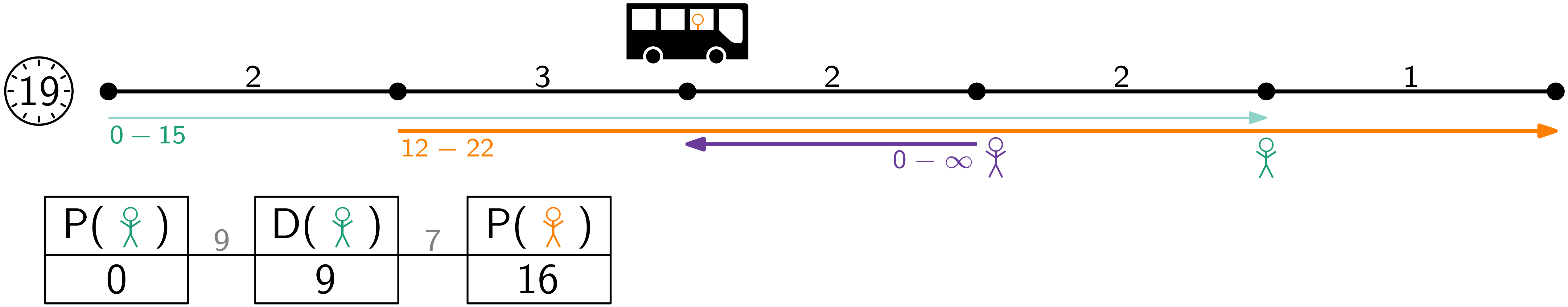
- Service Time (ST): pick-up / drop-off tak
- Shortcuts (SC)

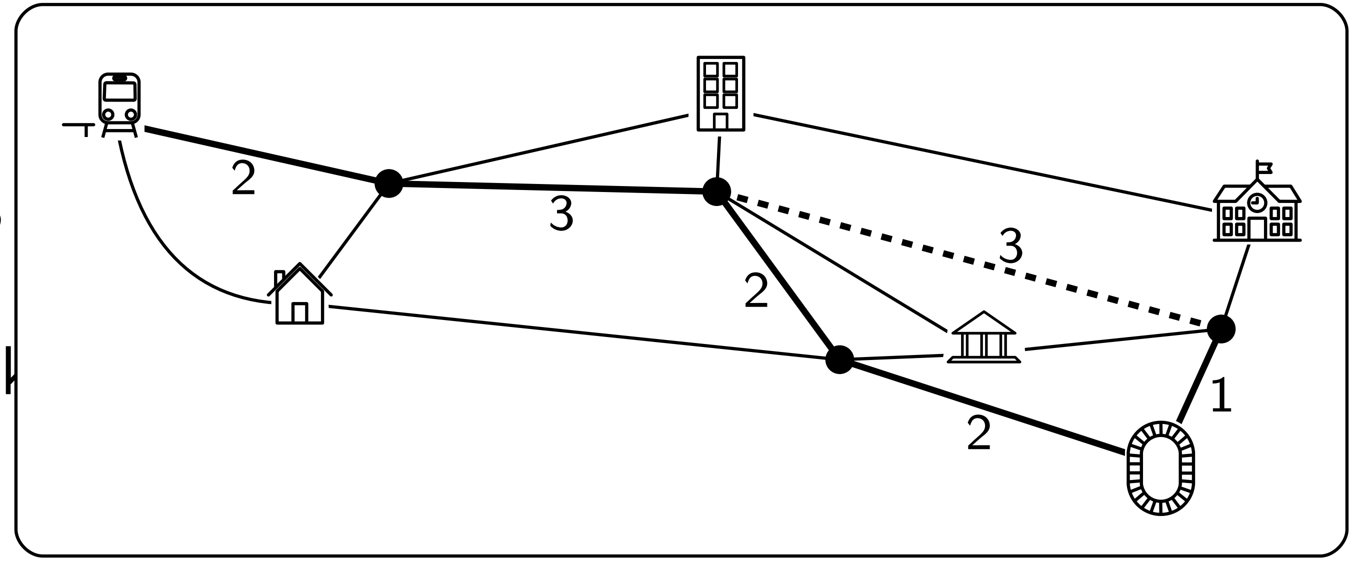


line-based Dial-a-Ride-Problem (liDARP)

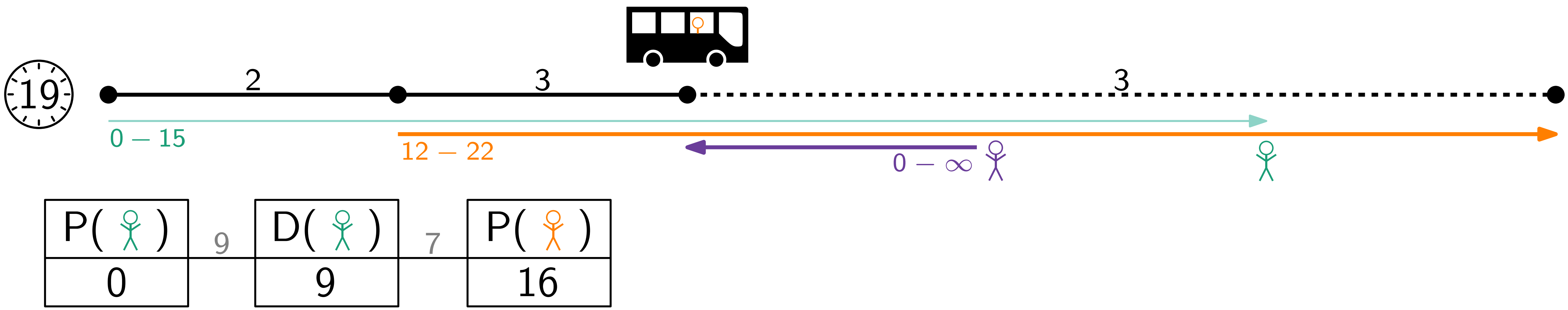


Variants

- Time Windows (TW): earliest pick-up e_p
- Service Promise (SP): passenger at most
- Service Time (ST): pick-up / drop-off tak
- Shortcuts (SC)

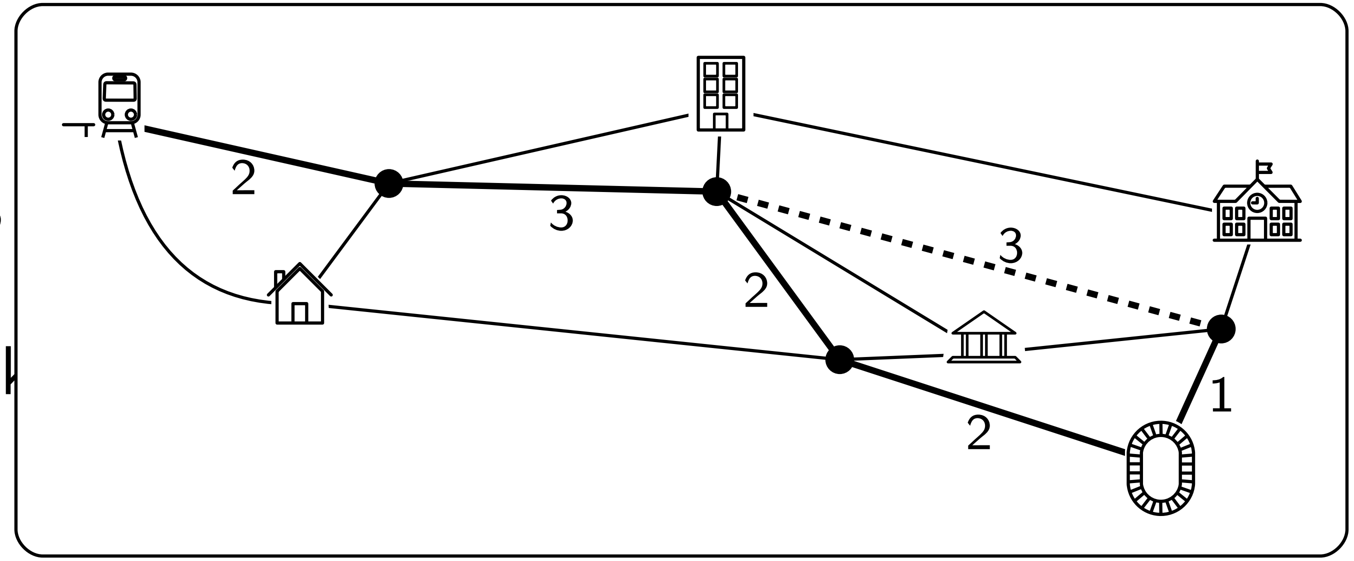


line-based Dial-a-Ride-Problem (liDARP)

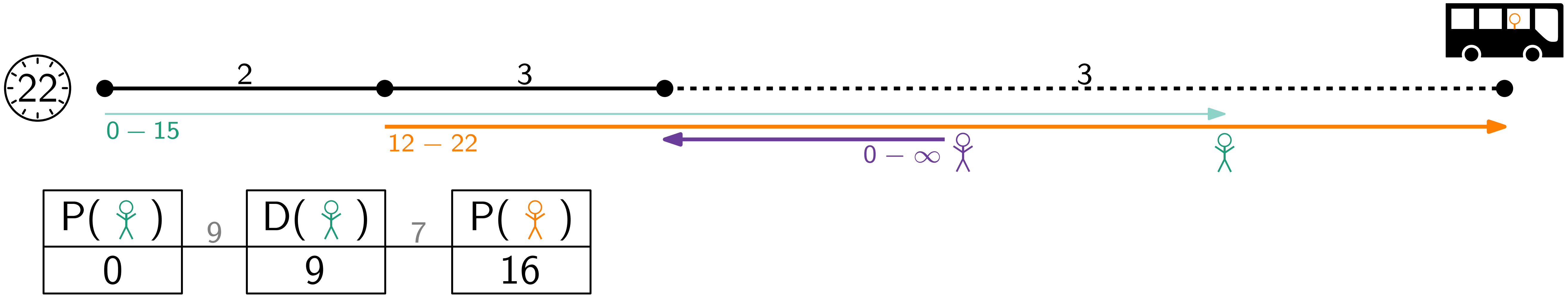


Variants

- Time Windows (TW): earliest pick-up e_p
- Service Promise (SP): passenger at most
- Service Time (ST): pick-up / drop-off tak
- Shortcuts (SC)

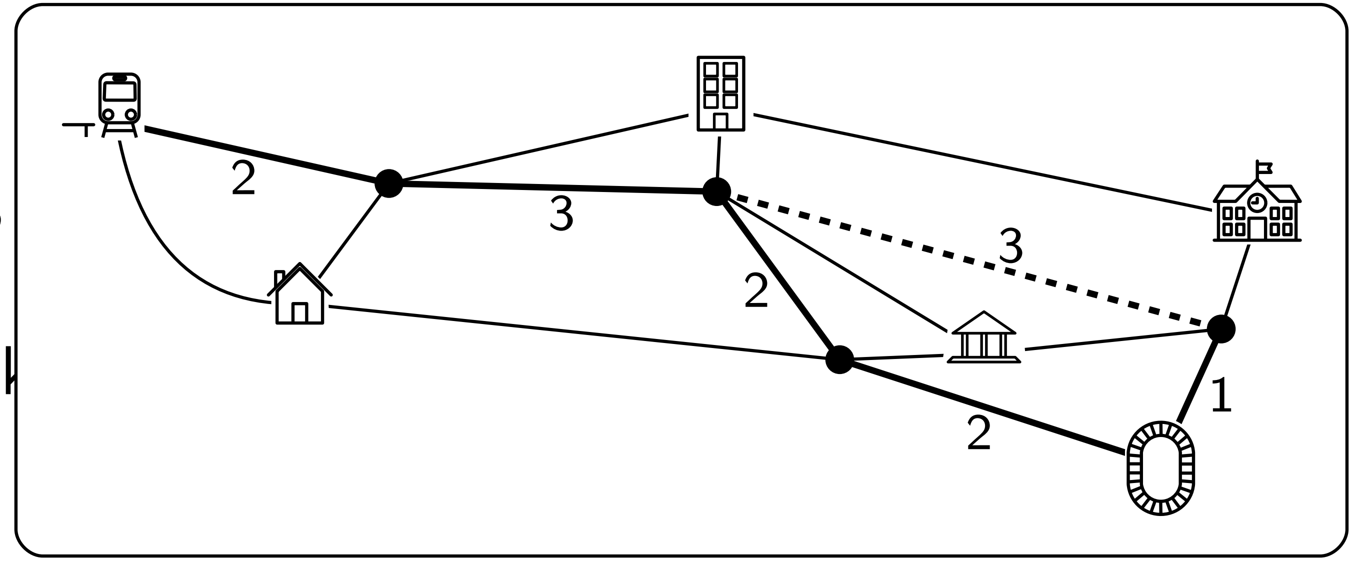


line-based Dial-a-Ride-Problem (liDARP)



Variants

- Time Windows (TW): earliest pick-up e_p
- Service Promise (SP): passenger at most
- Service Time (ST): pick-up / drop-off tak
- Shortcuts (SC)

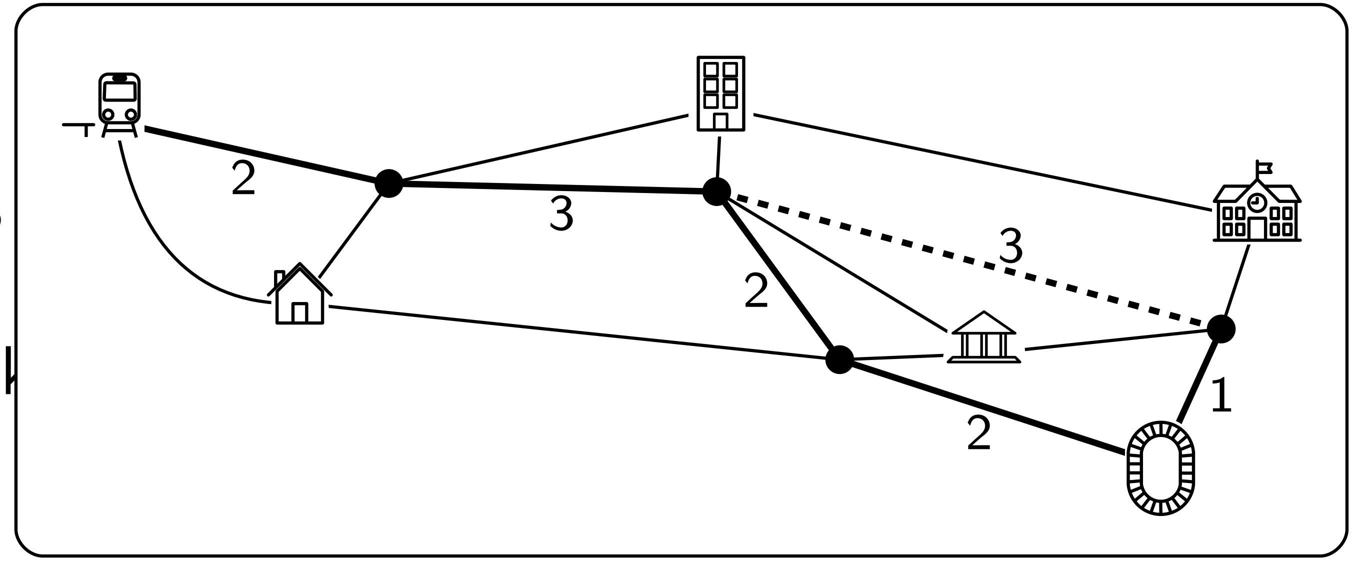


line-based Dial-a-Ride-Problem (liDARP)

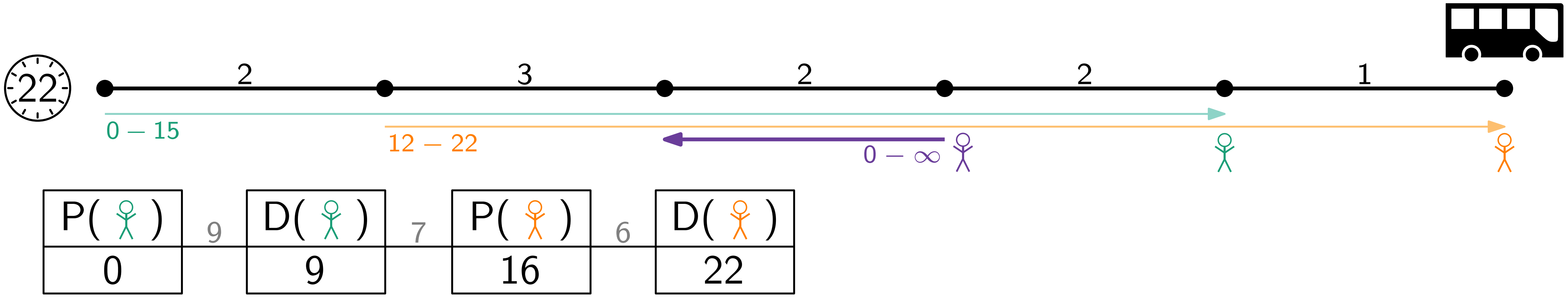
P( )	9	D( )	7	P( )	6	D( )
0		9		16		22

Variants

- Time Windows (TW): earliest pick-up e_p
- Service Promise (SP): passenger at most
- Service Time (ST): pick-up / drop-off tak
- Shortcuts (SC)



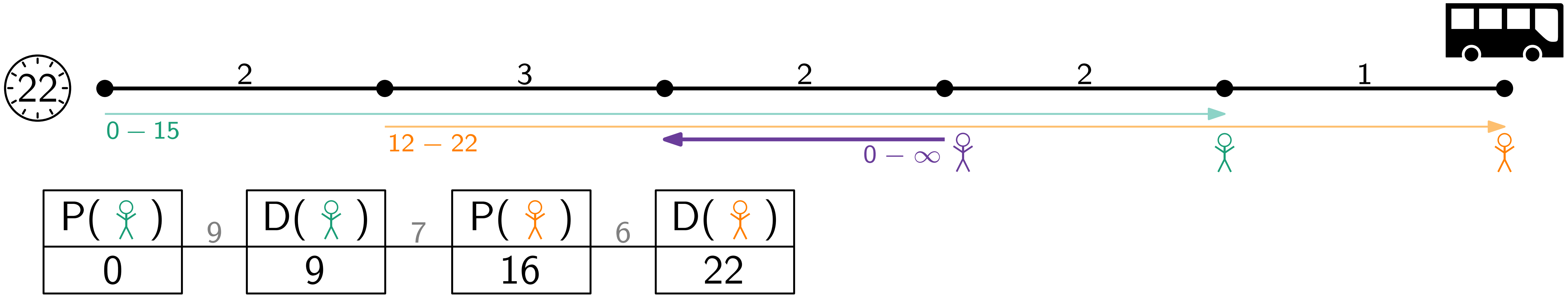
line-based Dial-a-Ride-Problem (liDARP)



Tour

- respects directionality and capacity
- timestamps adhere to distances and service time

line-based Dial-a-Ride-Problem (liDARP)



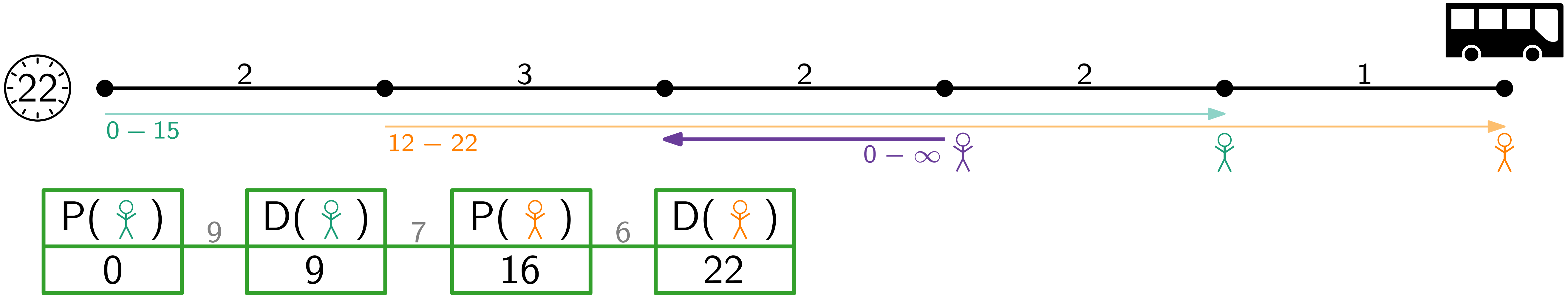
Tour

- respects directionality and capacity
- timestamps adhere to distances and service time

Feasible

- respects time windows and service promise

LINE-BASED DIAL-A-RIDE-PROBLEM (LIDARP)



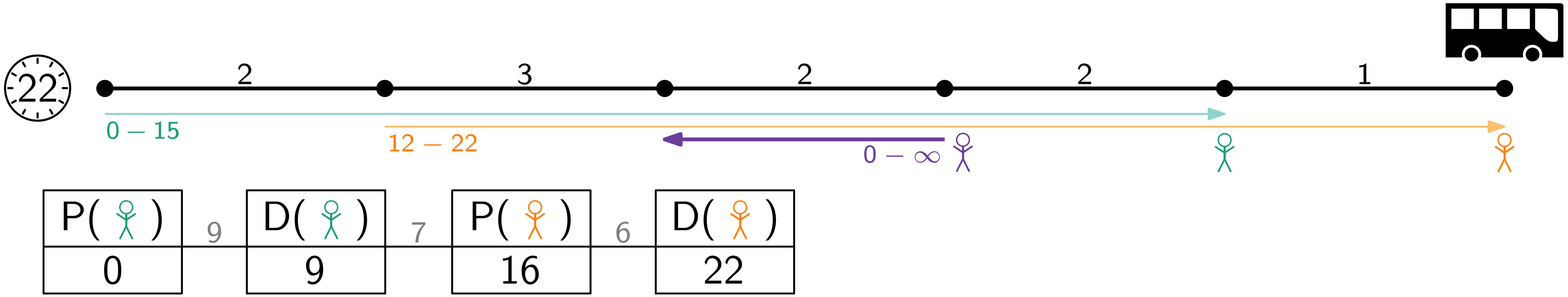
Tour

- respects directionality and capacity
- timestamps adhere to distances and service time

Feasible

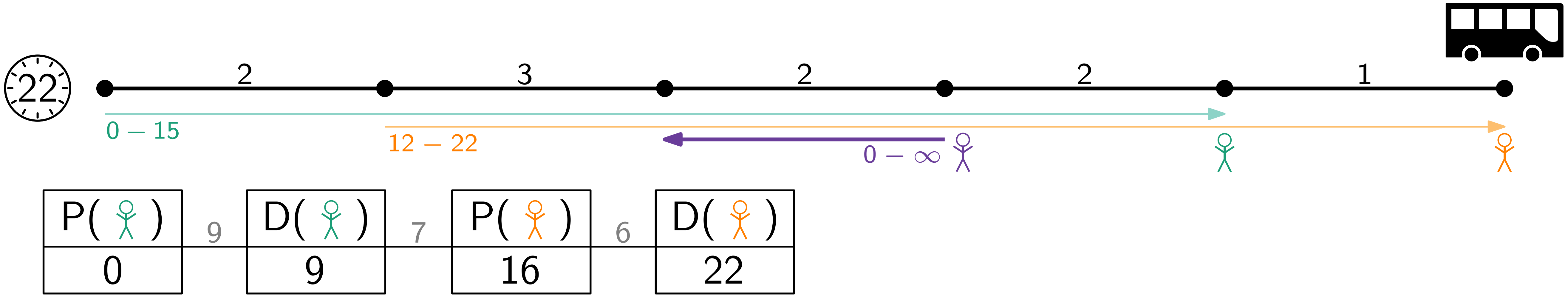
- respects time windows and service promise

line-based Dial-a-Ride-Problem (liDARP)



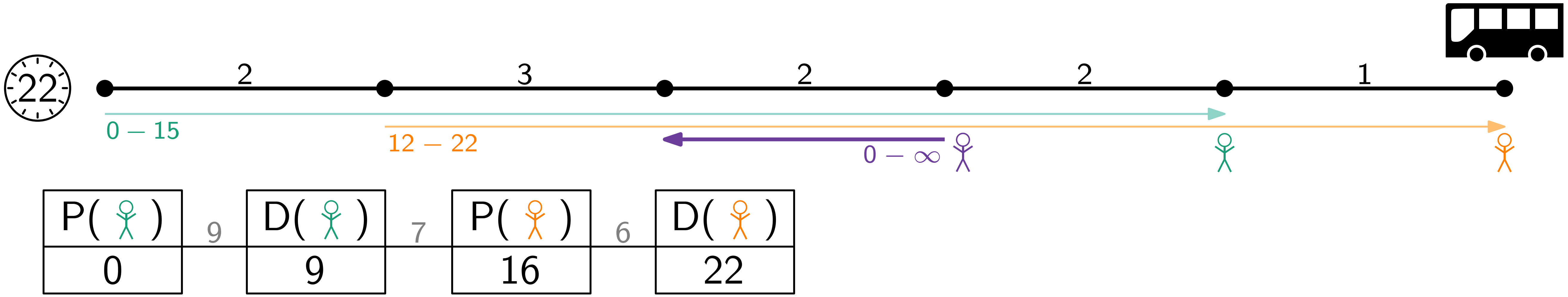
Objective:

line-based Dial-a-Ride-Problem (liDARP)



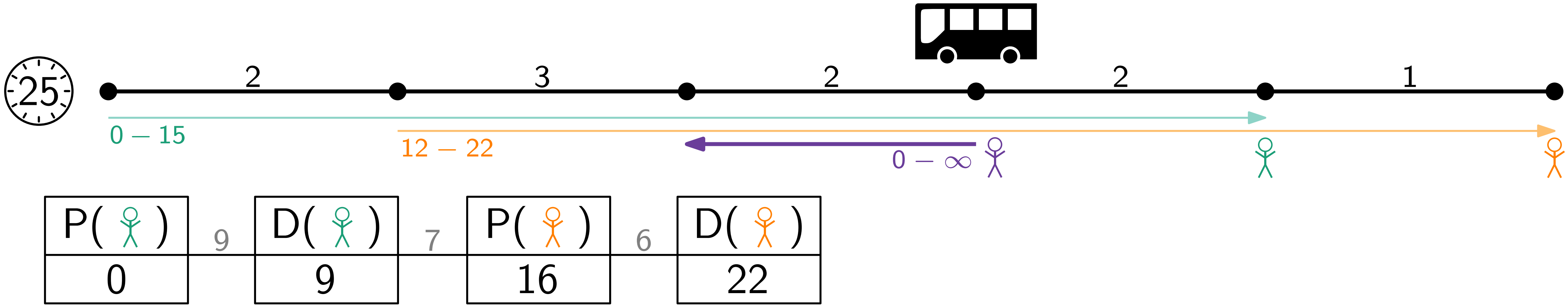
Objective: serve as many requests as possible

line-based Dial-a-Ride-Problem (liDARP)



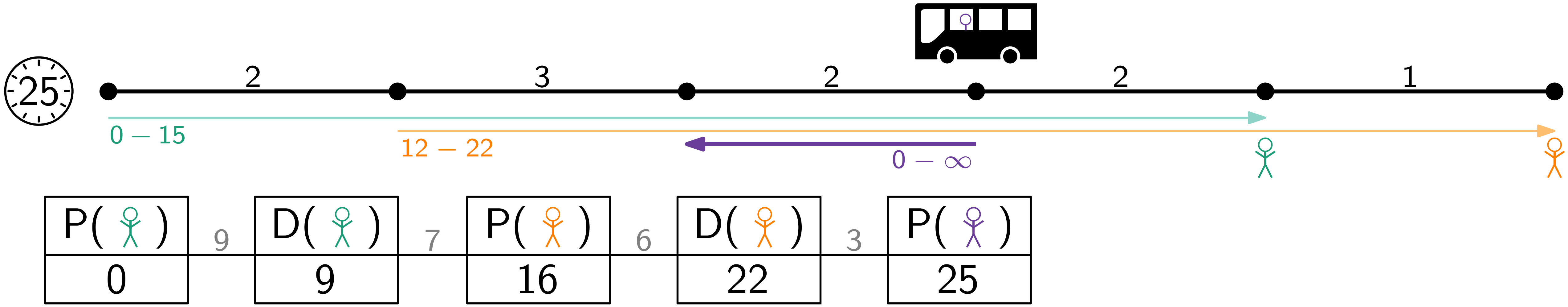
Objective: serve as many requests as possible

line-based Dial-a-Ride-Problem (liDARP)



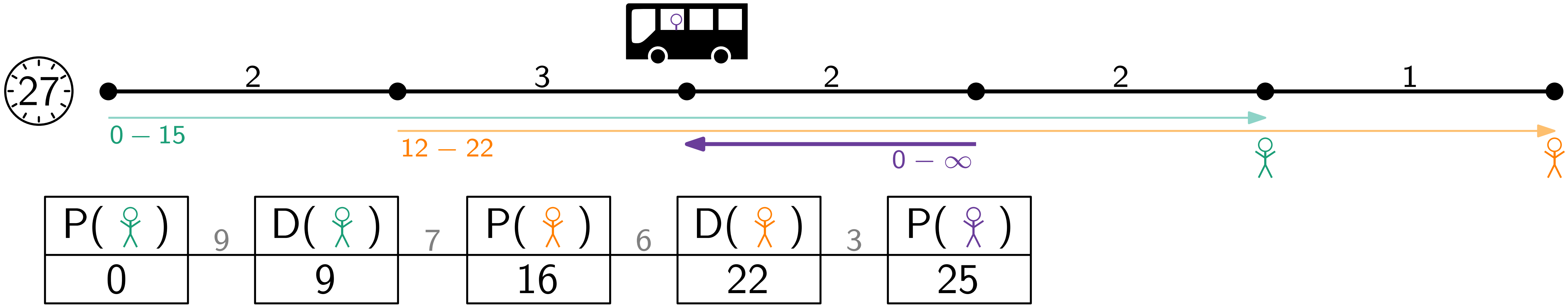
Objective: serve as many requests as possible

line-based Dial-a-Ride-Problem (liDARP)



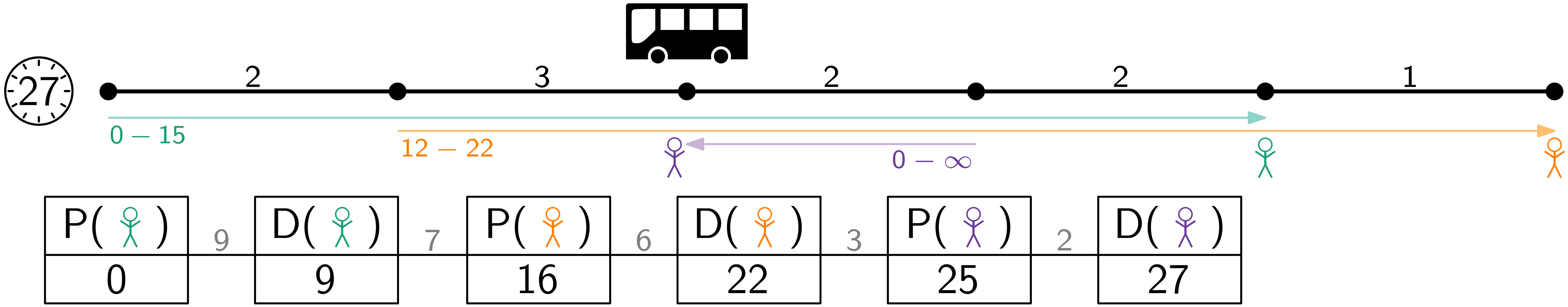
Objective: serve as many requests as possible

line-based Dial-a-Ride-Problem (liDARP)



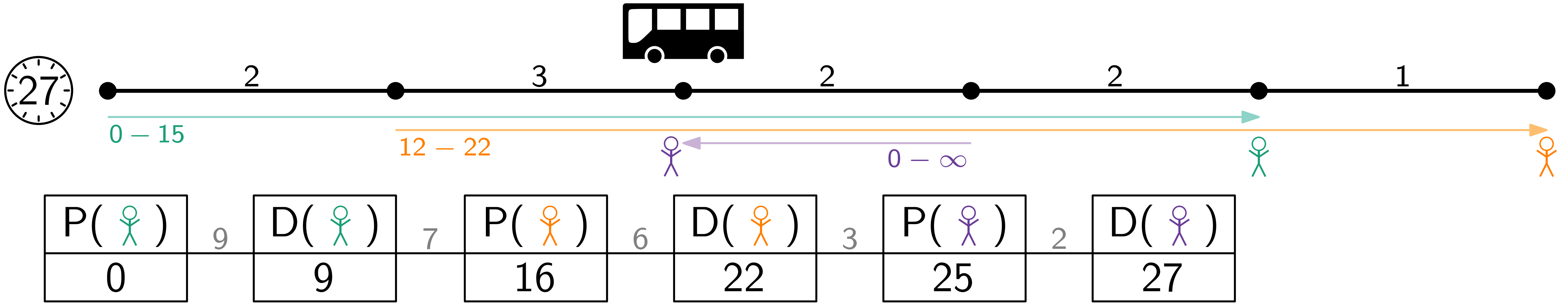
Objective: serve as many requests as possible

line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

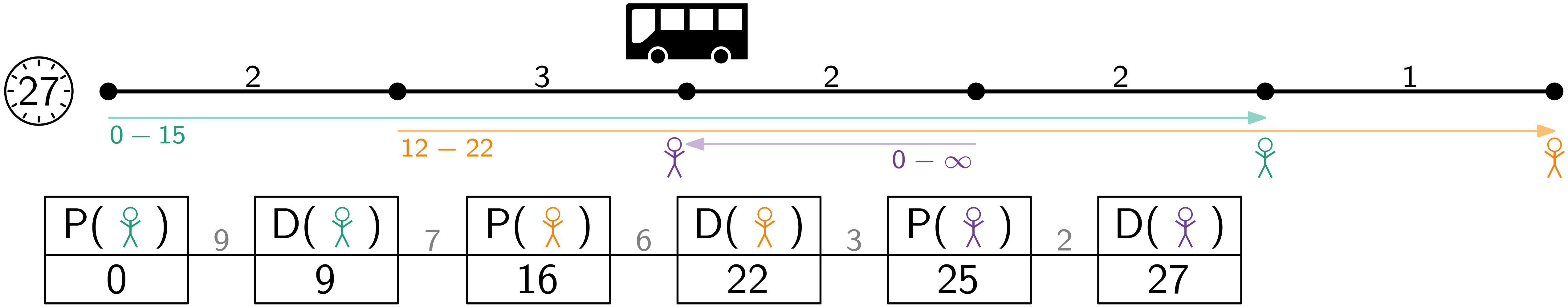
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✓

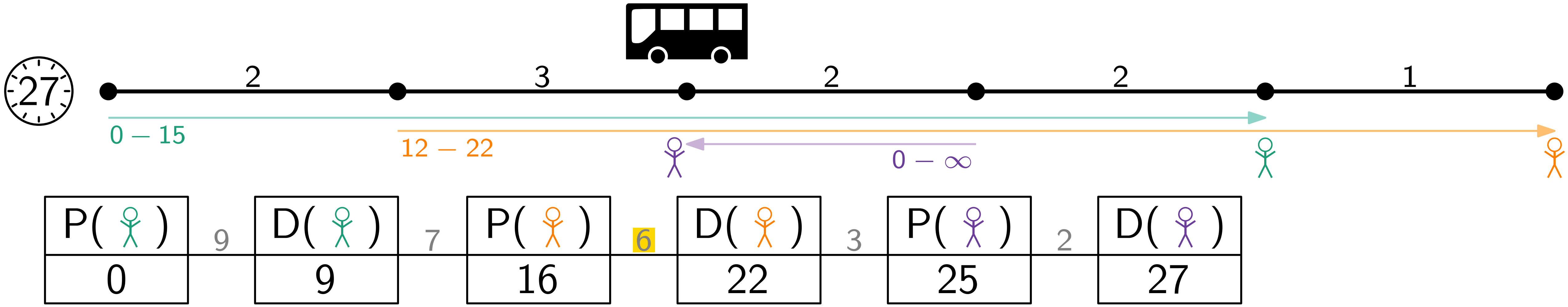
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

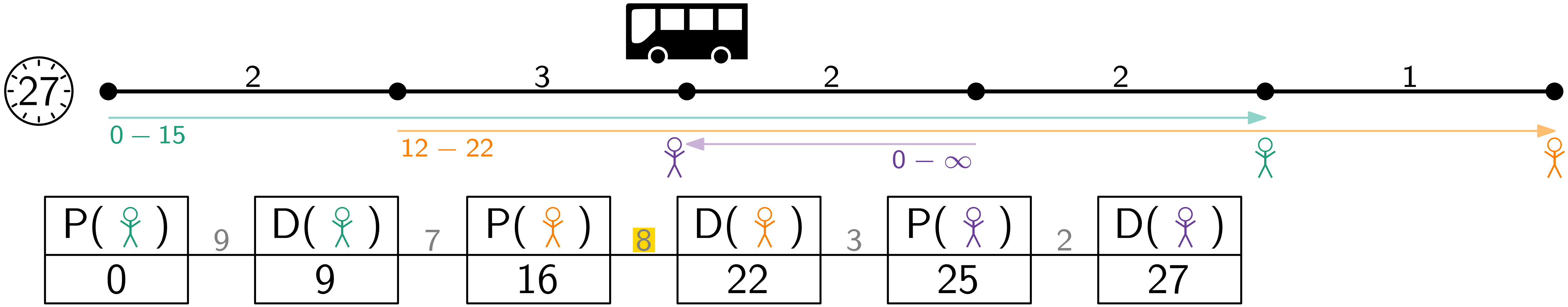
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

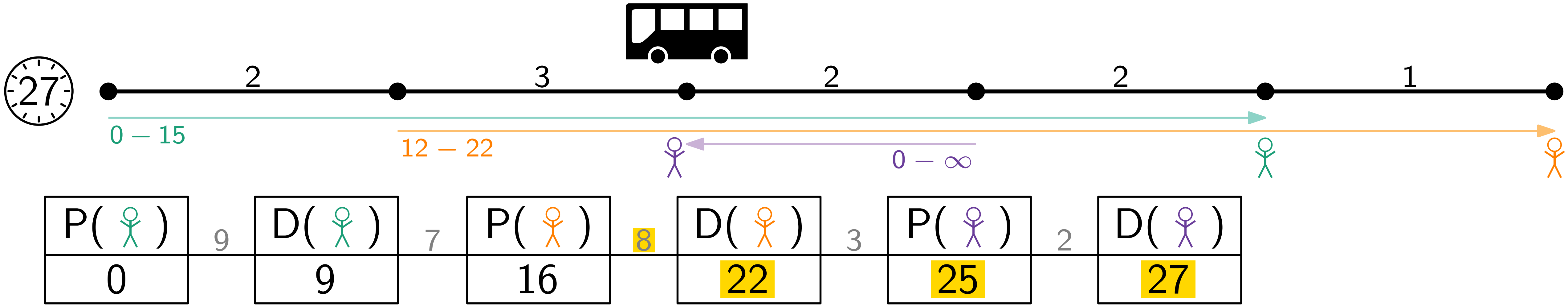
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

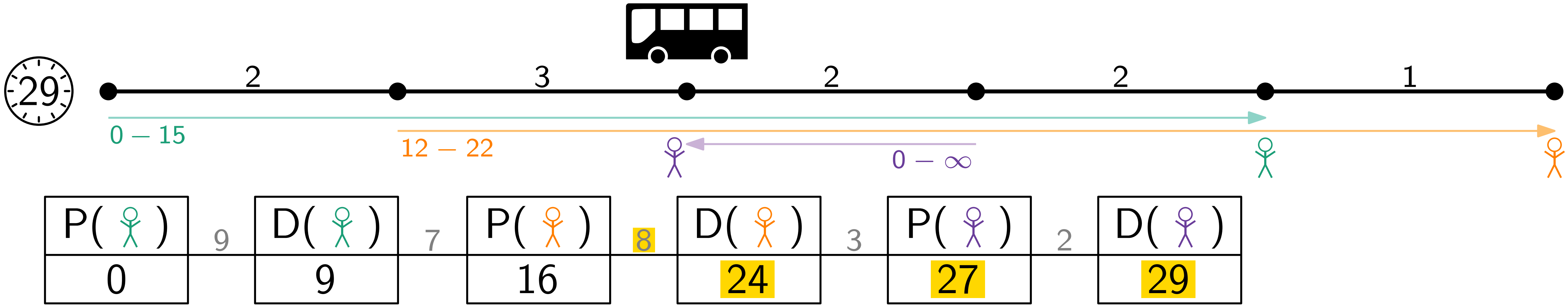
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

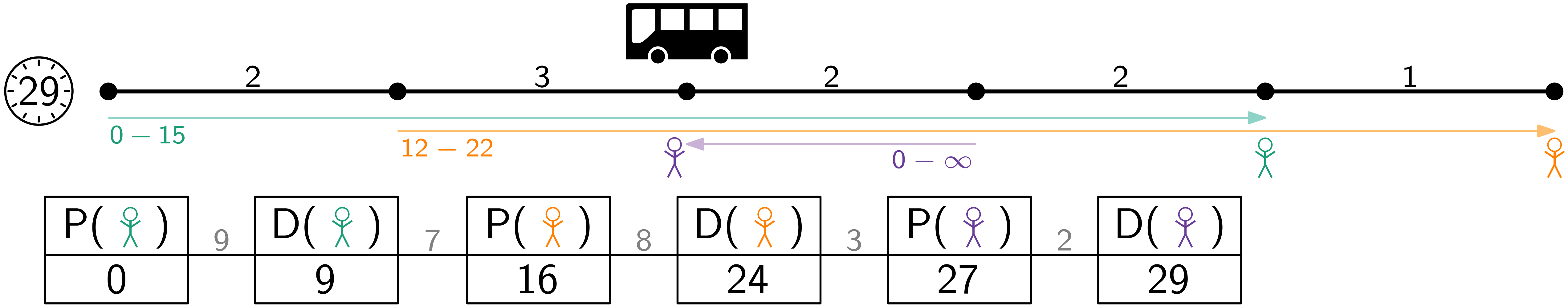
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

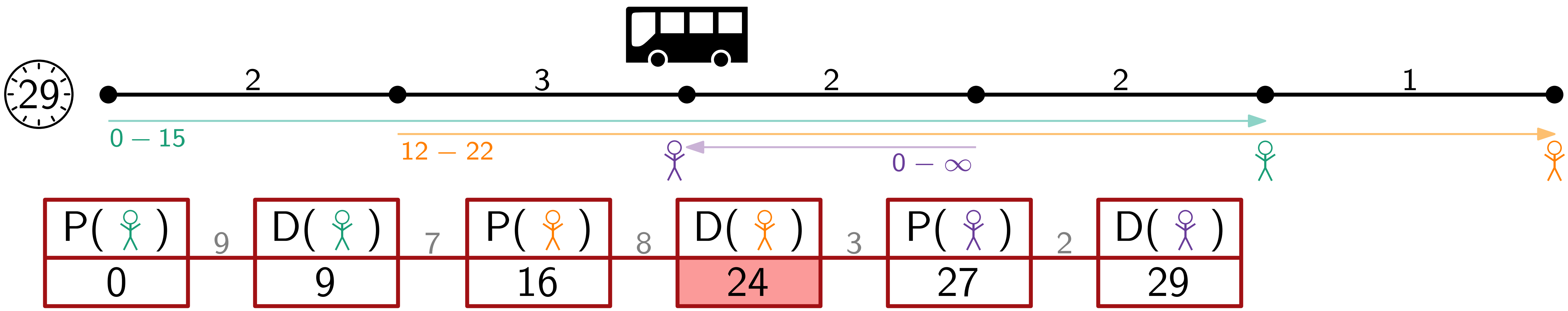
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

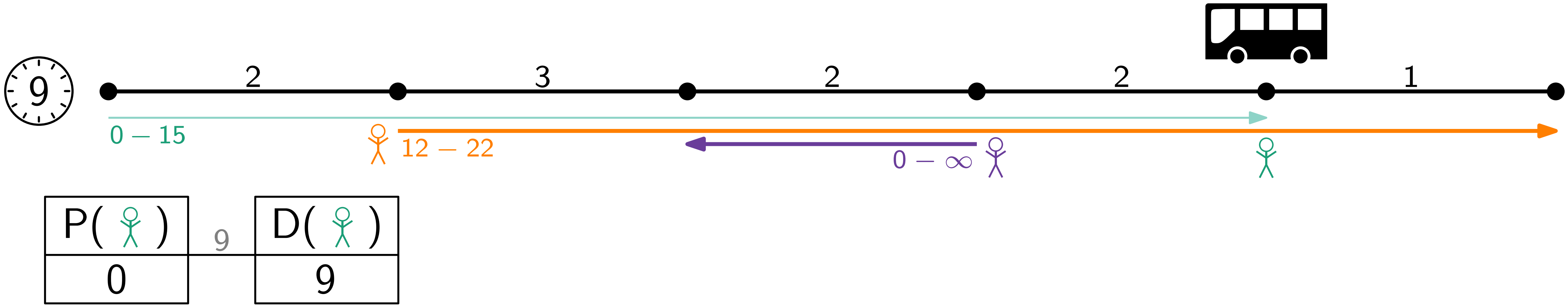
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

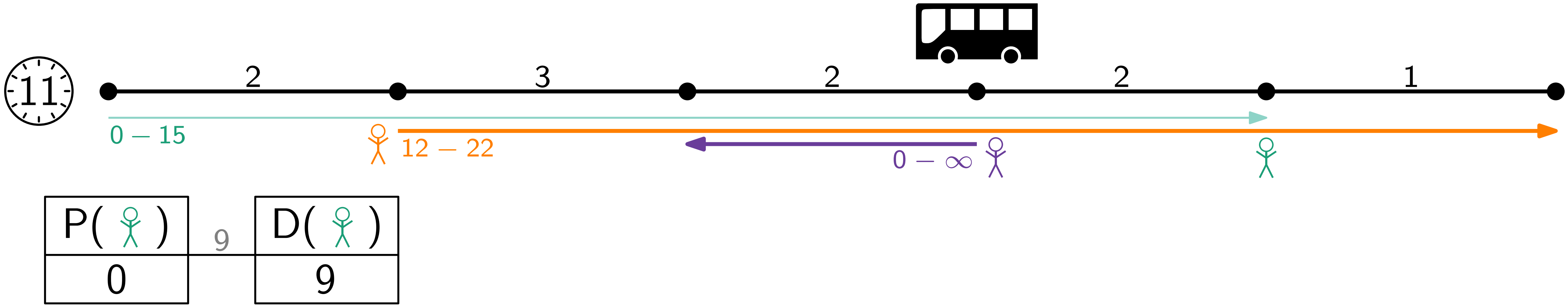
LINE-BASED DIAL-A-RIDE-PROBLEM (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

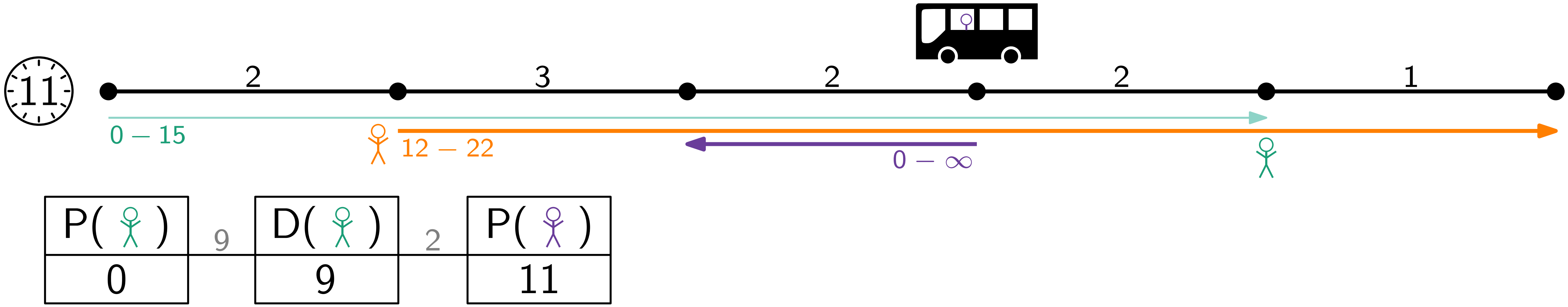
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

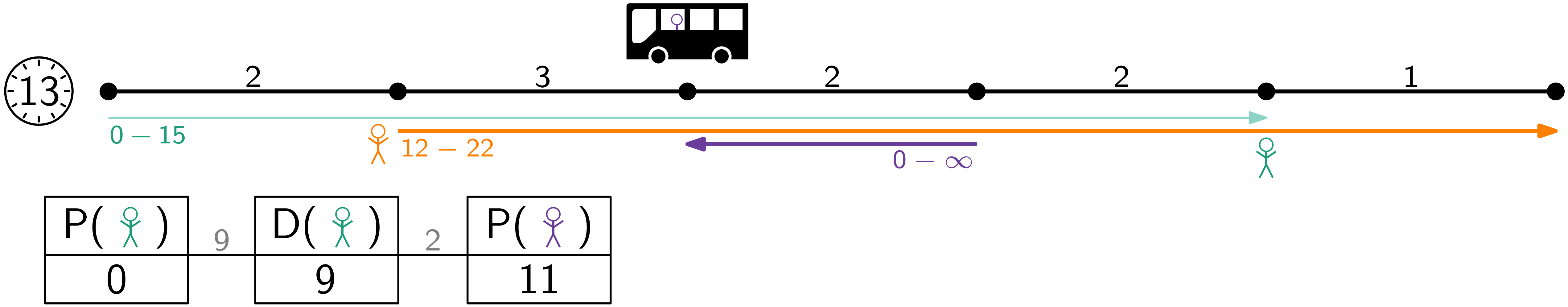
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

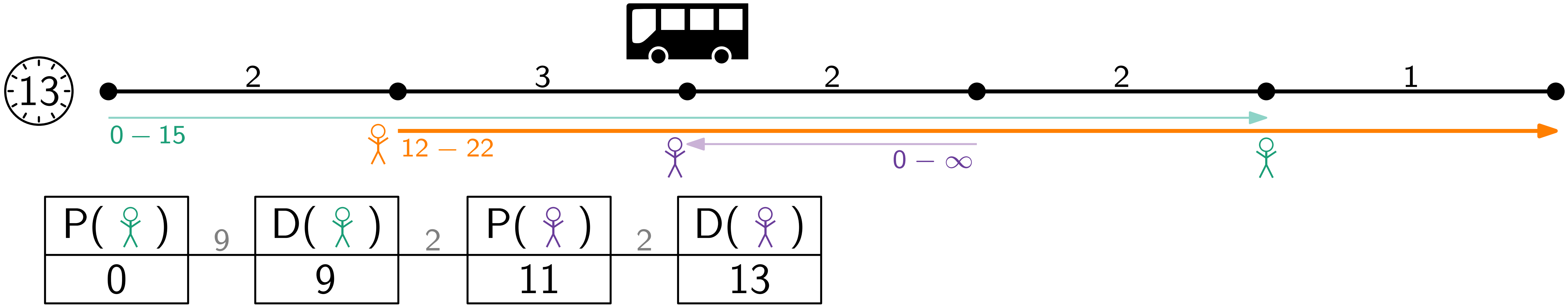
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

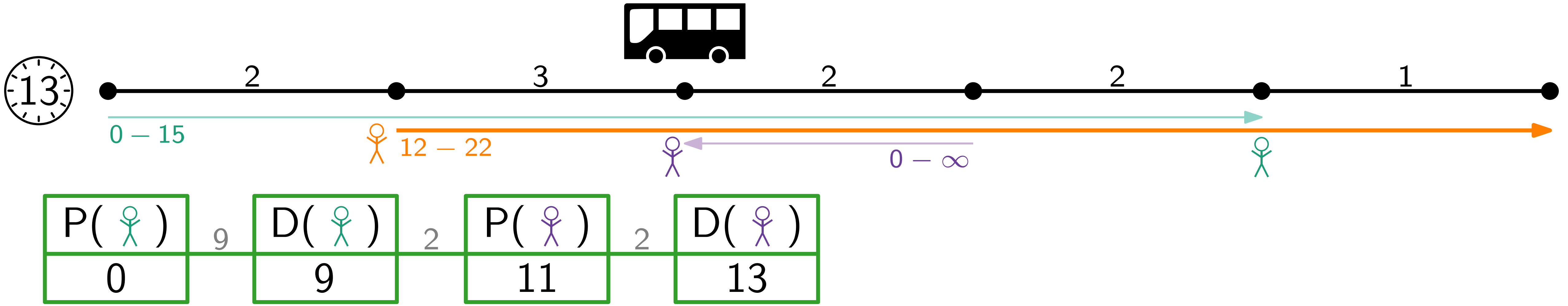
LINE-BASED DIAL-A-RIDE-PROBLEM (LIDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

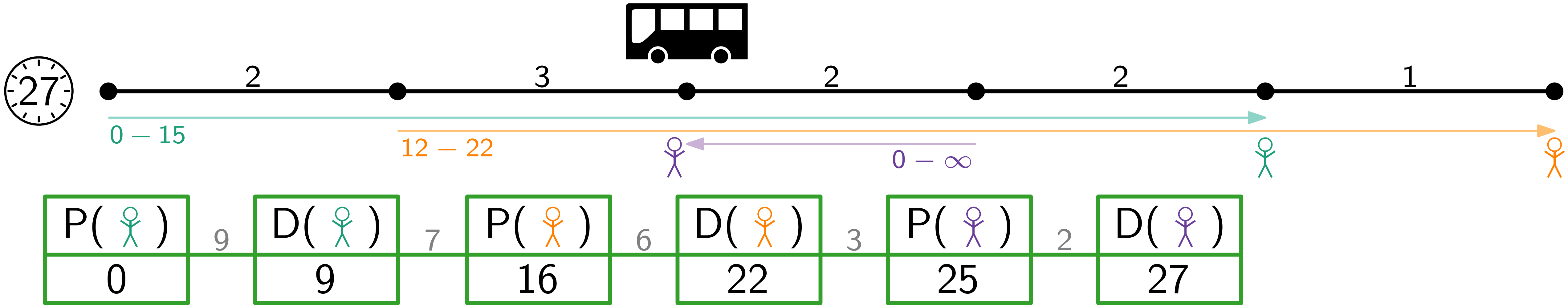
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

TW	✓
SP	✓
ST	✗
SC	✗

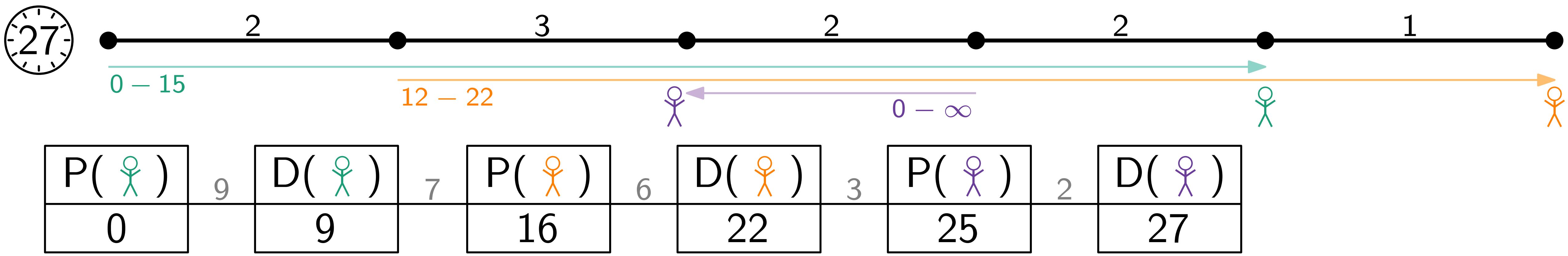
line-based Dial-a-Ride-Problem (liDARP)



Objective: serve as many requests as possible

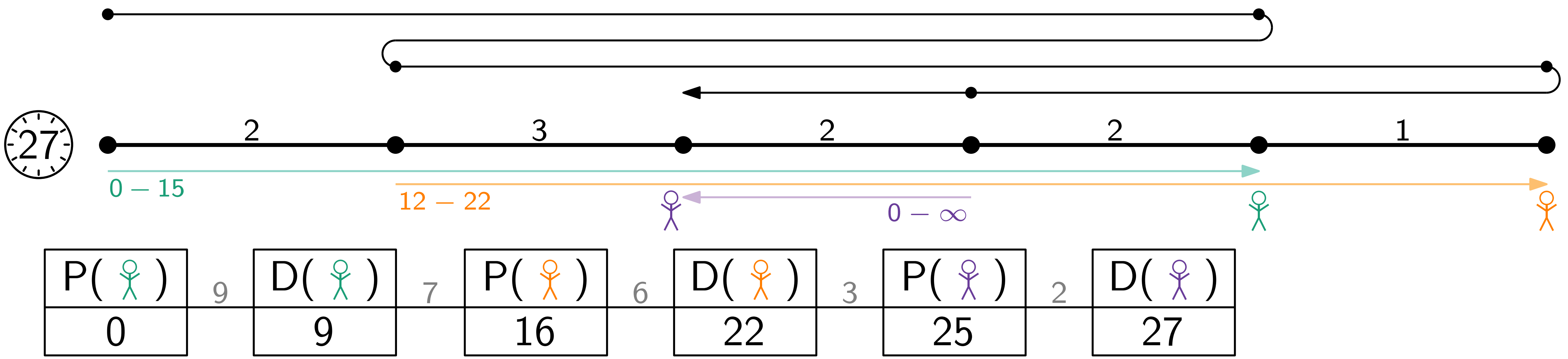
TW	✓
SP	✓
ST	✗
SC	✓

MinTurn



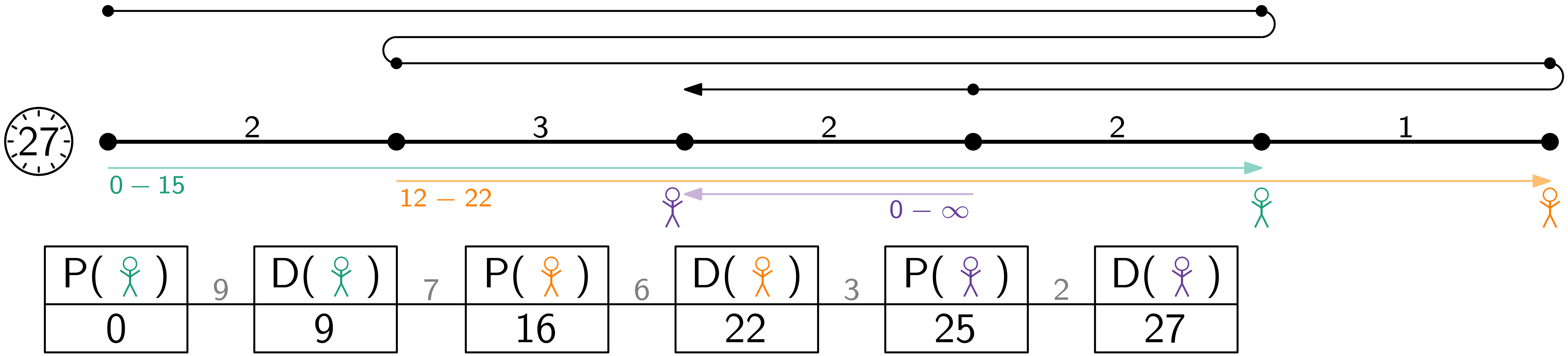
TW	✓
SP	✓
ST	✗
SC	✓

MinTurn



TW	✓
SP	✓
ST	✗
SC	✓

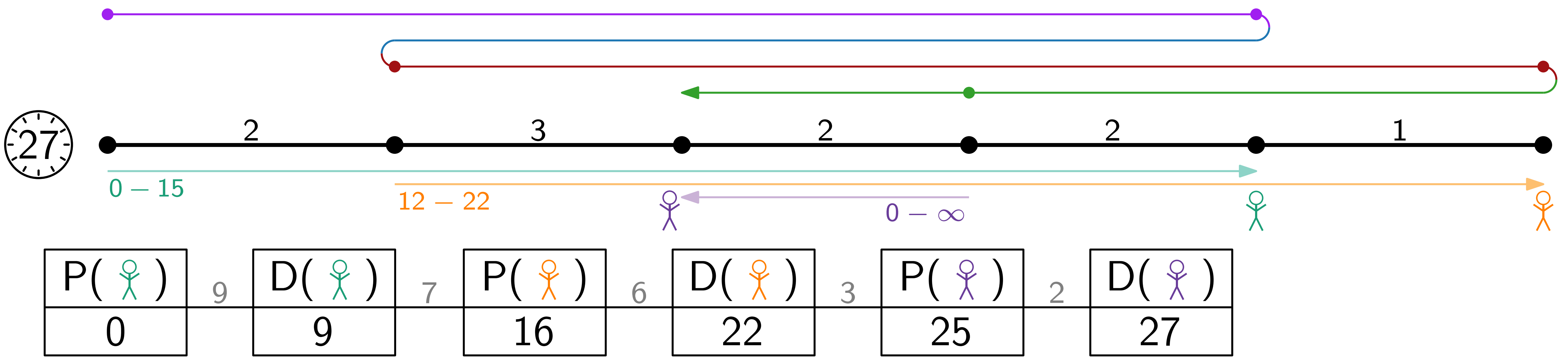
MinTurn



Subtour: tour segment where bus does not change direction

TW	✓
SP	✓
ST	✗
SC	✓

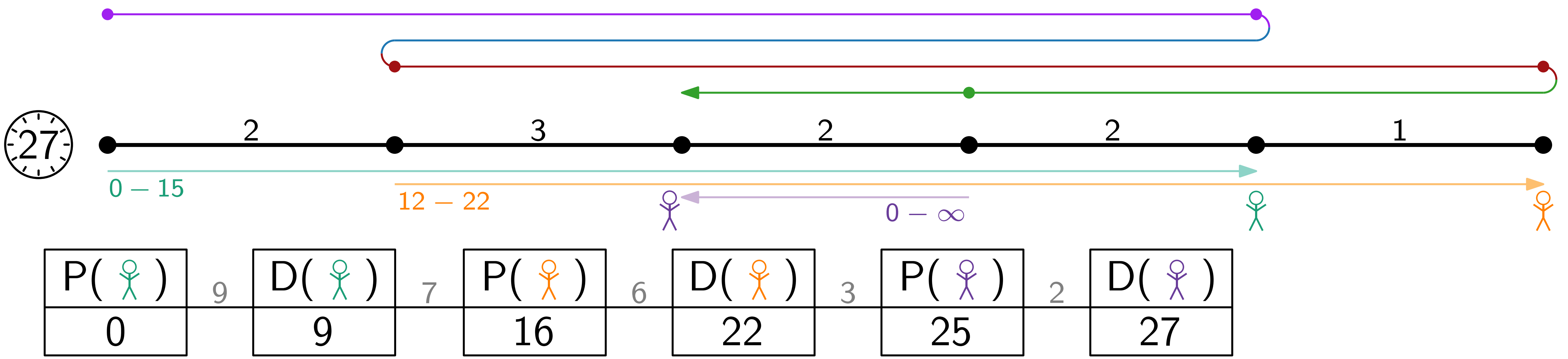
MinTurn



Subtour: tour segment where bus does not change direction

TW	✓
SP	✓
ST	✗
SC	✓

MinTurn

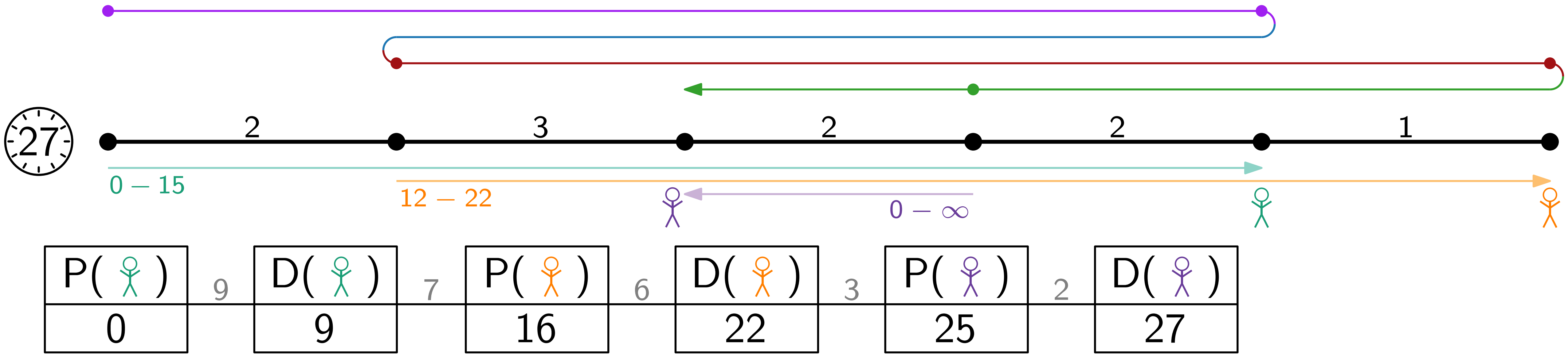


Subtour: tour segment where bus does not change direction

MinTurn

TW	✓
SP	✓
ST	✗
SC	✓

MinTurn



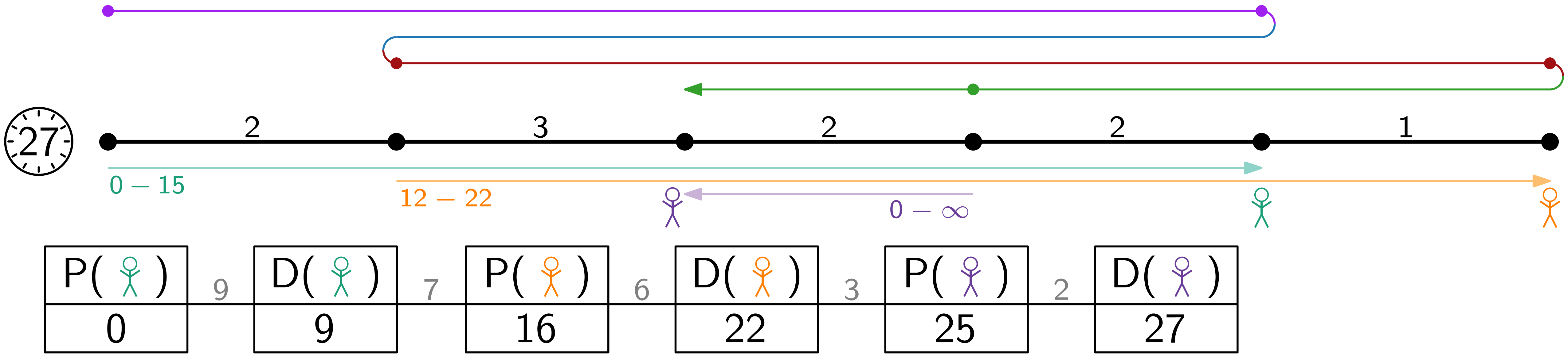
Subtour: tour segment where bus does not change direction

MinTurn

Input: liDARP-Instance

TW	✓
SP	✓
ST	✗
SC	✓

MinTurn



Subtour: tour segment where bus does not change direction

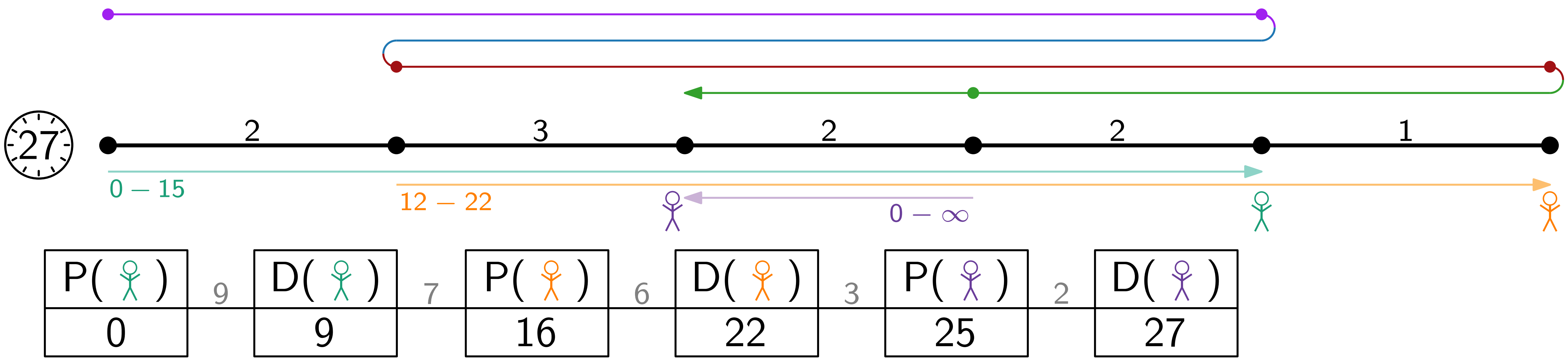
MinTurn

Input: liDARP-Instance

Output: minimal number of subtours of a tour in an optimal liDARP solution

TW	✓
SP	✓
ST	✗
SC	✓

MinTurn



Subtour: tour segment where bus does not change direction

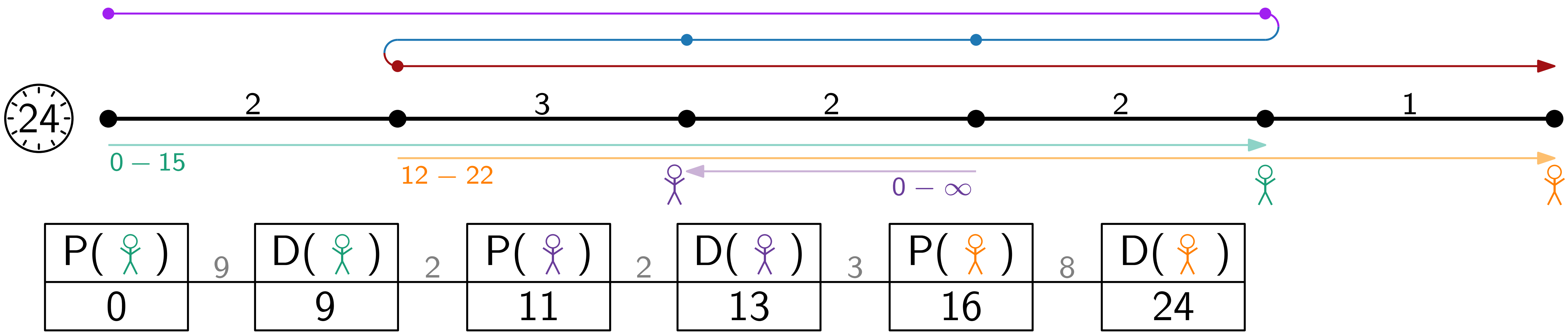
MinTurn

Input: liDARP-Instance

Output: minimal number of subtours of a tour in an optimal liDARP solution $=: \tau$

TW	✓
SP	✓
ST	✗
SC	✓

MinTurn



Subtour: tour segment where bus does not change direction

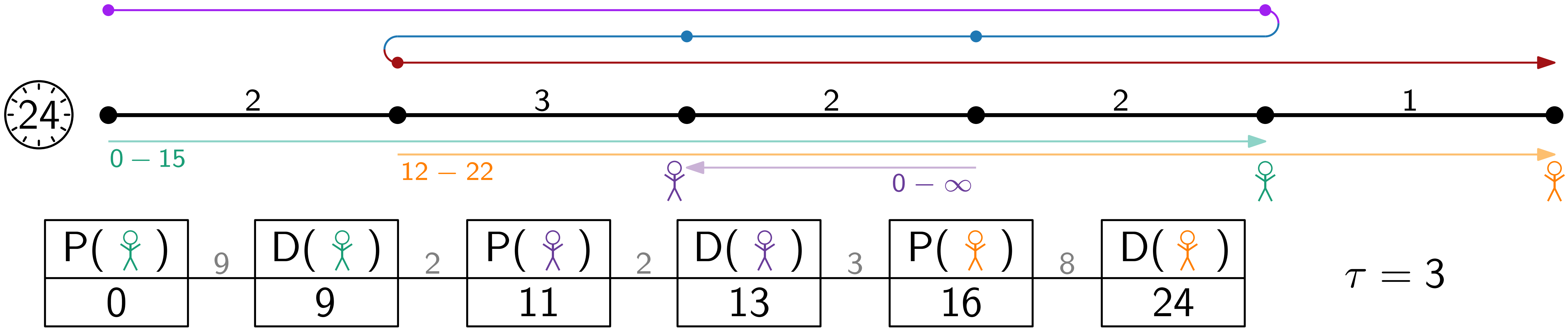
MinTurn

Input: LIDARP-Instance

Output: minimal number of subtours of a tour in an optimal LIDARP solution $=: \tau$

TW	✓
SP	✓
ST	✗
SC	✓

MinTurn



$\tau = 3$

Subtour: tour segment where bus does not change direction

MinTurn

Input: liDARP-Instance

Output: minimal number of subtours of a tour in an optimal liDARP solution $=: \tau$

TW	✓
SP	✓
ST	✗
SC	✓

Contribution

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

Complexity

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

#Vehicles	k
Capacity	c
Time Windows	TW
Service Promise	SP
Service Time	ST
Shortcuts	SC

Complexity

LIDARP

k	c	TW	SP	ST	SC

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

$\mathcal{P}$

#Vehicles	k
Capacity	c
Time Windows	TW
Service Promise	SP
Service Time	ST
Shortcuts	SC

Complexity

LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

$\mathcal{P}$	$\mathcal{NP}$-hard

#Vehicles	k
Capacity	c
Time Windows	TW
Service Promise	SP
Service Time	ST
Shortcuts	SC

Complexity

LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

$\mathcal{P}$	$\mathcal{NP}$-hard

#Vehicles	k
Capacity	c
Time Windows	TW
Service Promise	SP
Service Time	ST
Shortcuts	SC

Complexity

LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MINTURN

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

Parameterized Algorithms

Complexity

LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MinTurn

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

Parameterized Algorithms

Parameters

- $k :=$ #vehicles
- $c :=$ capacity

Complexity

LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MINTURN

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

Parameterized Algorithms

Parameters

- k := #vehicles
- c := capacity
- h := #stations
- t := max. time

determined by time windows

Complexity

LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MINTURN

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

Parameterized Algorithms

- FPT-algorithm for LIDARP and MINTURN

$$O^*((h^2 \cdot t^3 \cdot c \cdot k)^{2 \cdot t \cdot c \cdot k})$$

Parameters

- $k :=$ #vehicles
- $c :=$ capacity
- $h :=$ #stations
- $t :=$ max. time

Complexity

LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MINTURN

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

Parameterized Algorithms

- FPT-algorithm for LIDARP and MINTURN

$$O^*((h^2 \cdot t^3 \cdot c \cdot k)^{2 \cdot t \cdot c \cdot k})$$

Problem: without time windows $t = \infty$

Parameters

- $k :=$ #vehicles
- $c :=$ capacity
- $h :=$ #stations
- $t :=$ max. time

Complexity

LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MINTURN

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Contribution

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Parameterized Algorithms

- FPT-algorithm for LIDARP and MINTURN

$$O^*((h^2 \cdot t^3 \cdot c \cdot k)^{2 \cdot t \cdot c \cdot k})$$

- XP-algorithm for MINTURN without time windows

$$O^*(n^{h^2} \cdot h^{4 \cdot c \cdot h})$$

Parameters

- $k :=$ #vehicles
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- $h :=$ #stations
- $t :=$ max. time

Complexity

LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MINTURN

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Contribution

Related Work

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$$O^*(n^{h^2} \cdot h^{4 \cdot c \cdot h})$$

$n :=$ #requests

Parameters

- $k :=$ #vehicles
- $c :=$ capacity
- $h :=$ #stations
- $t :=$ max. time

Complexity

LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MINTURN

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Contribution

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≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MINTURN

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Contribution

Related Work

- study LIDARP with other objectives
- assume all requests can be served

Parameterized Algorithms

- FPT-algorithm for LIDARP and MINTURN

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Complexity

LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MINTURN

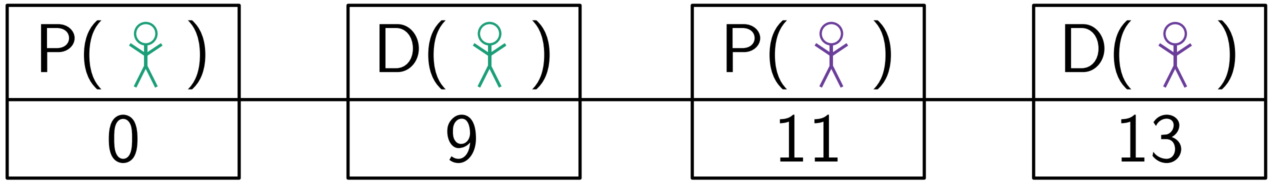
k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Easy liDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓

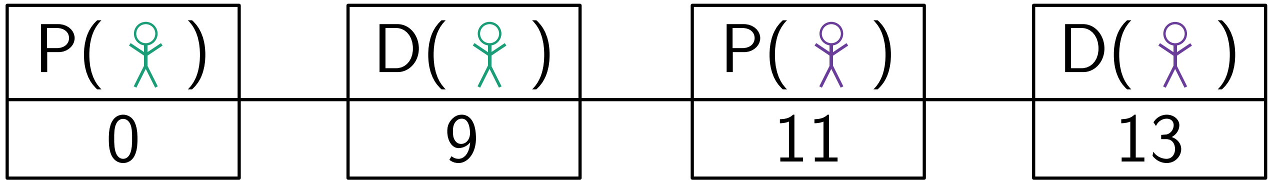
Easy LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓



Easy liDARP

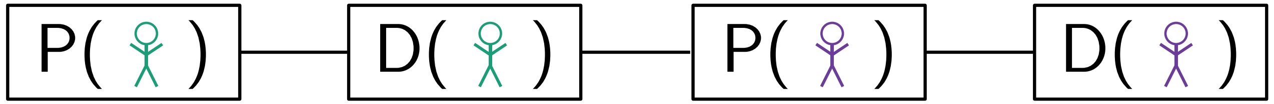
k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓



Route: tour without timestamps

Easy liDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓



Route: tour without timestamps

Easy liDARP

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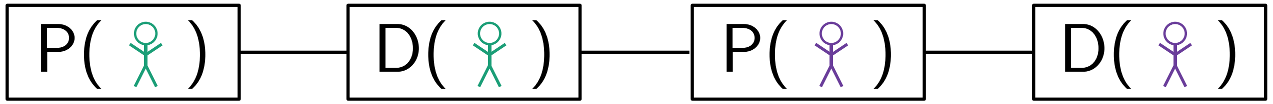
≥ 1	≥ 1	✗	✓	✓	✓



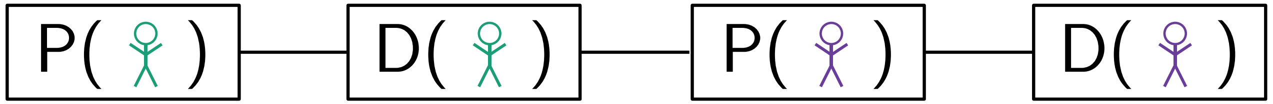
Route: tour without timestamps

- route feasible if there is feasible corresponding tour

Easy liDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓

Lemma [Haugland & Ho 2010]
Testing feasibility of routes (and constructing feasible tours) is possible in polynomial time



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Lemma
Joining two feasible routes yields feasible route if there are no time windows

Easy liDARP

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≥ 1	≥ 1	✗	✓	✓	✓

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Theorem
Without time windows a tour serving all request can be computed in polynomial time

Easy liDARP

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≥ 1	≥ 1	✗	✓	✓	✓

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Proof:

Easy LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓

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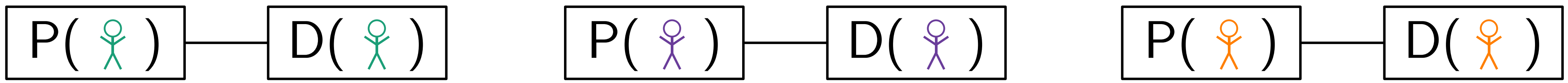

Easy LIDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓

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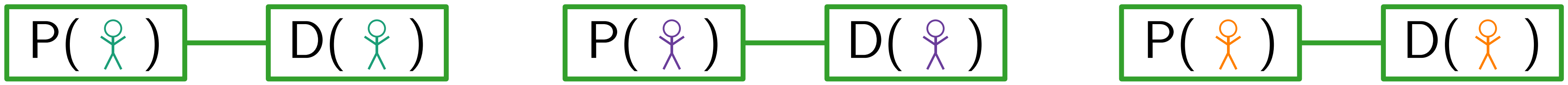

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Proof:

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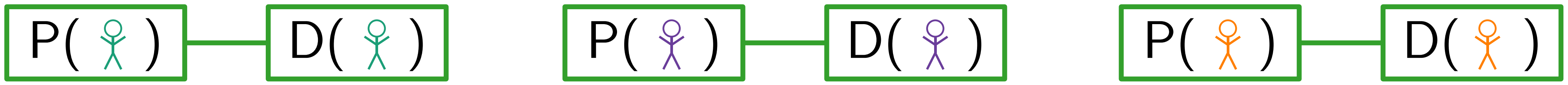
- join all routes into feasible route serving all requests



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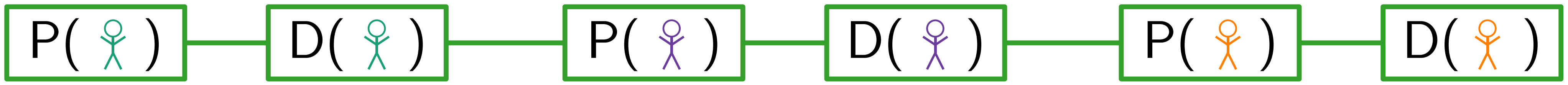

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k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓

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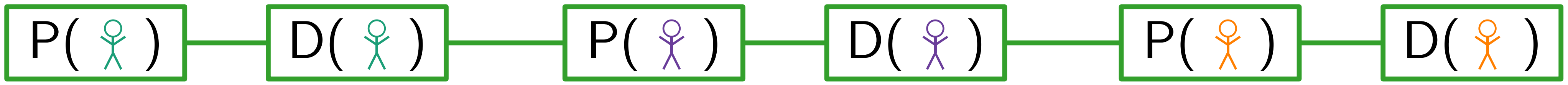
Proof:

- serve each request in separate route (all feasible)
- join all routes into feasible route serving all requests
- compute corresponding feasible tour



Easy liDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓

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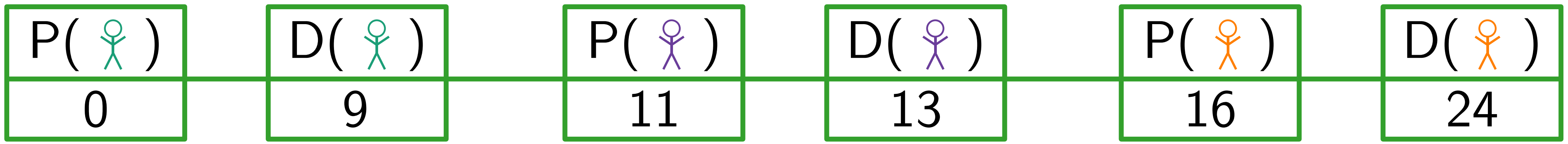

Hard MinTurn

liDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MinTurn

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Hard MinTurn

liDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MinTurn

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Hard MinTurn

liDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MinTurn

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
$= 1$	$= 2$	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗

General Idea

Reduction from 3-Partition

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗

General Idea

Reduction from 3-Partition

Definition (3-Partition):

Given: Multiset S of $3m$ positive integers with $\sum_{s\in S} = mT$

Question: Is there a partition of S into m disjoint subsets $S_1, \ldots, S_m$ s.t. each sums up to T?

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗

General Idea
Reduction from 3-Partition

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

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Reduction from 3-Partition

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$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

$\{1, 6, 8\}, \{3, 7, 5\}$

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗

General Idea

Reduction from 3-Partition

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Given: Multiset S of $3m$ positive integers with $\sum_{s \in S} = mT$

Question: Is there a partition of S into m disjoint subsets $S_1, \dots, S_m$ s.t. each sums up to T?

Theorem [Garey & Johnson 1979]

3-Partition is **strongly** $\mathcal{NP}$-hard

$S = \{1, 6, 8, 3, 7, 5\}$ $m = 2$, $T = 15$
$\{1, 6, 8\}, \{3, 7, 5\}$

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗

General Idea

Given 3-Partition-Instance S:

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2,\ T = 15$

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗

General Idea

Given 3-Partition-Instance S:

- for each value s_i there is a *value request* v_i

$S = \{1, 6, 8, 3, 7, 5\}$
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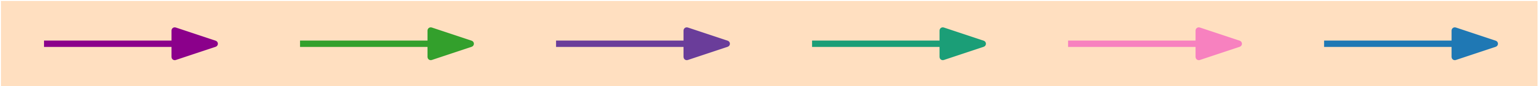
Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



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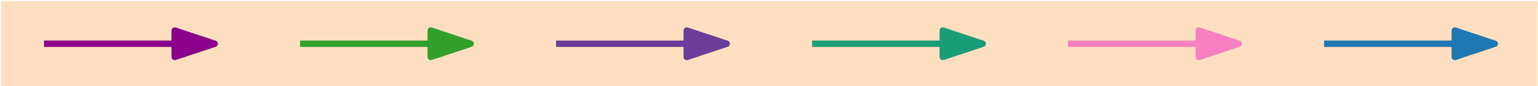
Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



General Idea

Given 3-Partition-Instance S:

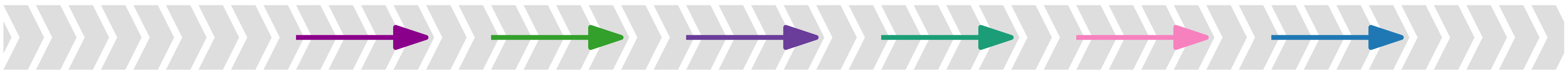
- for each value s_i there is a *value request* v_i

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Goal: partition of values corresponds to grouping of value requests into m *ascending* subroutes

Hard $\textsc{MinTurn}$

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



General Idea

Given 3-$\textsc{Partition}$-Instance S:

- for each value s_i there is a *value request* v_i

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

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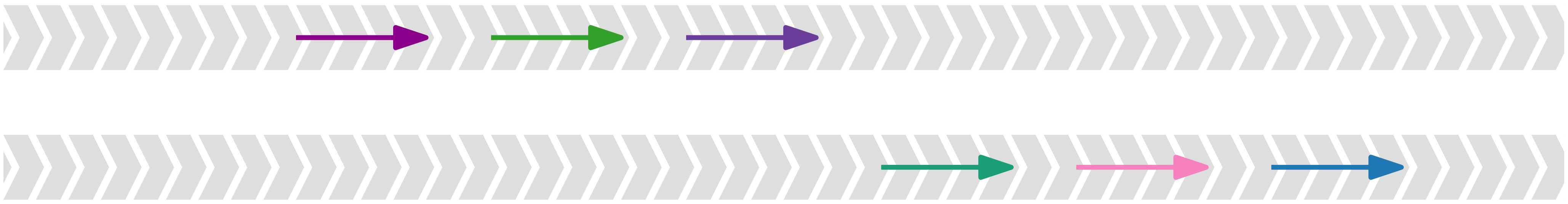
Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



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Given 3-Partition-Instance S:

- for each value s_i there is a *value request* v_i

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

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Hard MinTurn

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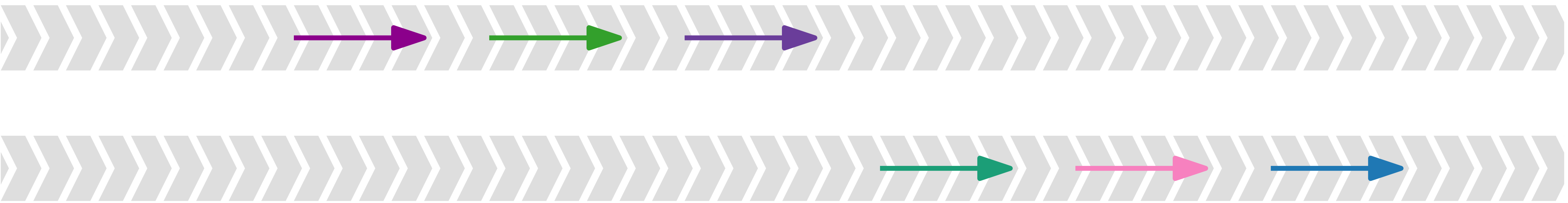
$= 1$	$= 2$	✗	✓	✓	✗



$\{1, 6, 8\}$

$\{3, 7, 5\}$

General Idea

Given 3-Partition-Instance S:

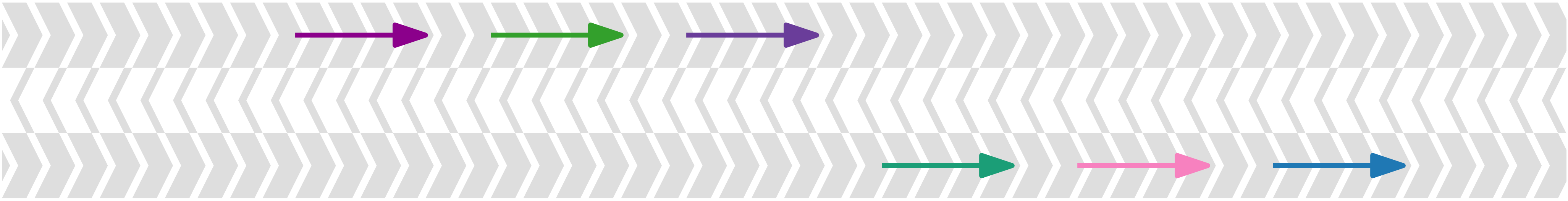
- for each value s_i there is a *value request* v_i

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Goal: partition of values corresponds to grouping of value requests into *m ascending* subroutes

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



$\{1, 6, 8\}$

$\{3, 7, 5\}$

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

General Idea

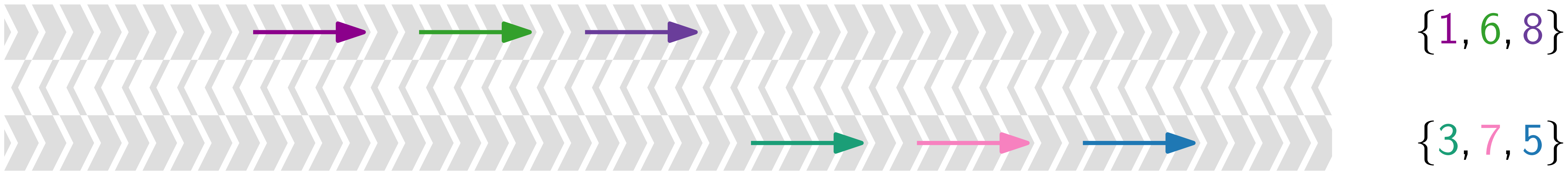
Given 3-Partition-Instance S:

- for each value s_i there is a *value request* v_i

Goal: partition of values corresponds to grouping of value requests into m *ascending* subroutes

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



General Idea

Given 3-Partition-Instance S:

- for each value s_i there is a *value request* v_i

$S = \{1, 6, 8, 3, 7, 5\}$
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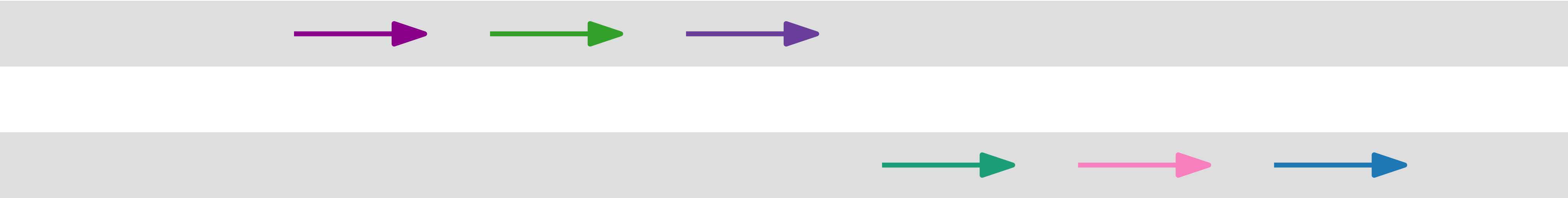
Goal: partition of values corresponds to grouping of value requests into m *ascending* subroutes
$\Leftrightarrow \tau = 2m - 1$

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

General Idea

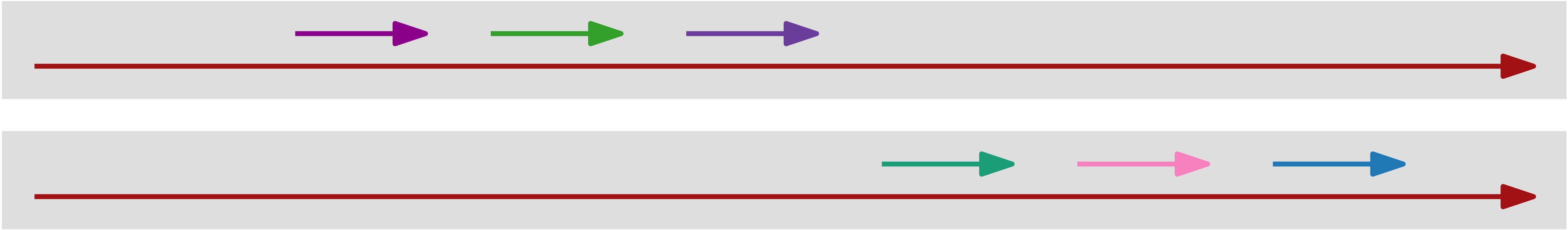
Given 3-Partition-Instance S:

- for each value s_i there is a *value request* v_i

Goal: partition of values corresponds to grouping of value requests into m *ascending* subroutes $\Leftrightarrow \tau = 2m - 1$

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

General Idea

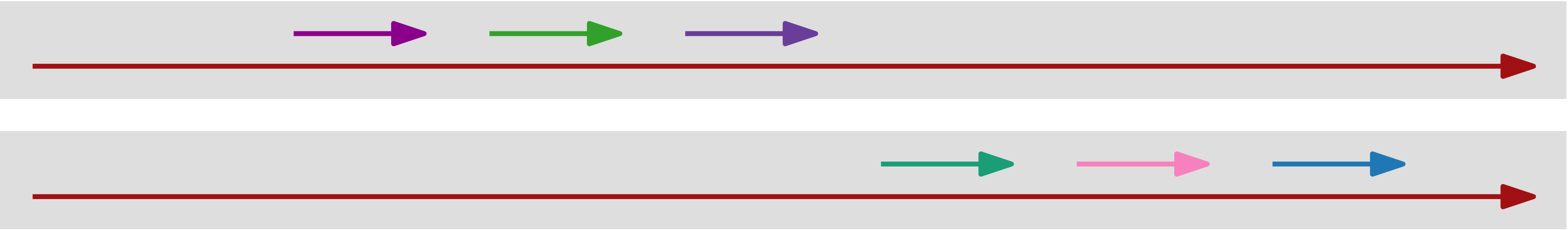
Given 3-Partition-Instance S:

- for each value s_i there is a *value request* v_i
- sum of values to T in each set is enforced by a *promise request* per set

Goal: partition of values corresponds to grouping of value requests into m *ascending* subroutes
$\Leftrightarrow \tau = 2m - 1$

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

General Idea

Given 3-Partition-Instance S:

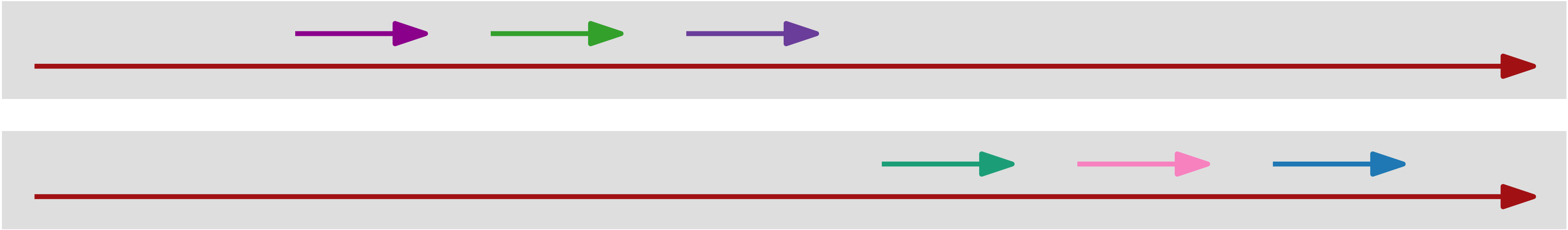
- for each value s_i there is a *value request* v_i
- sum of values to T in each set is enforced by a *promise request* per set

$\Delta = 2T$

Goal: partition of values corresponds to grouping of value requests into m *ascending* subroutes
$\Leftrightarrow \tau = 2m - 1$

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



General Idea

Given 3-Partition-Instance S:

delay of $2s_i$

- for each value s_i there is a *value request* v_i
- sum of values to T in each set is enforced by a *promise request* per set

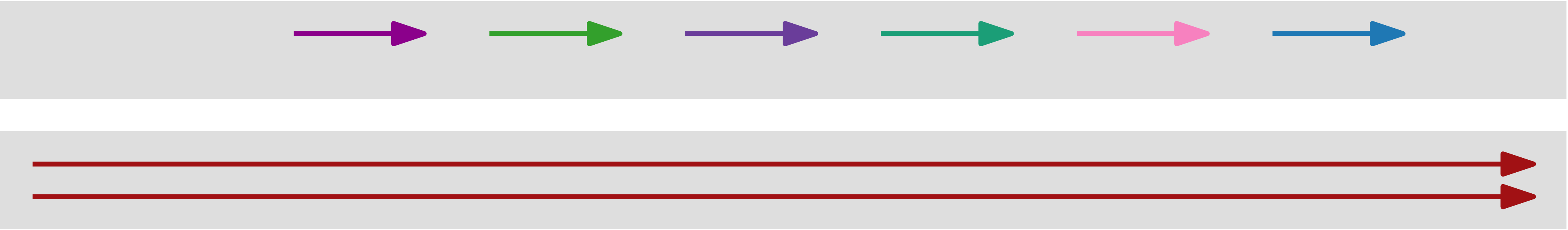
$\Delta = 2T$

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2,\ T = 15$

Goal: partition of values corresponds to grouping of value requests into m *ascending* subroutes $\Leftrightarrow \tau = 2m - 1$

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

General Idea

Given 3-Partition-Instance S:

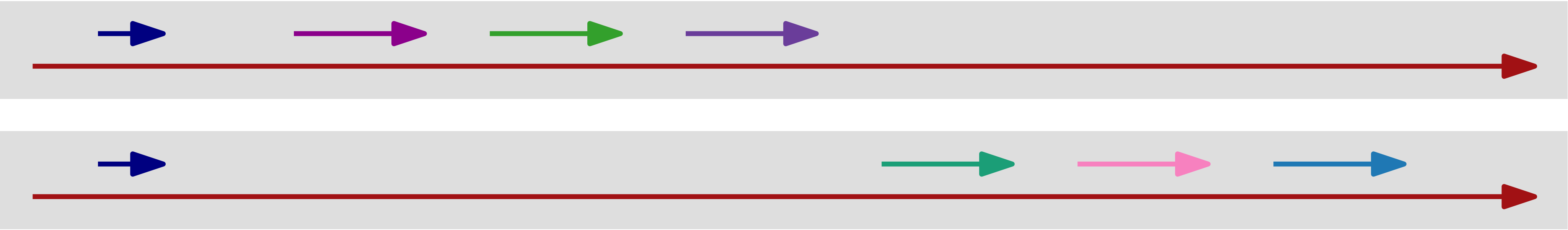
- for each value s_i there is a *value request* v_i
- sum of values to T in each set is enforced by a *promise request* per set

Problem: not always one promise request in each subroute

Goal: partition of values corresponds to
grouping of value requests into m *ascending* subroutes
$\Leftrightarrow \tau = 2m - 1$

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2, T = 15$

General Idea

Given 3-Partition-Instance S:

- for each value s_i there is a *value request* v_i
- sum of values to T in each set is enforced by a *promise request* per set
- one promise requests per subroute is enforced by m *filter requests*

Goal: partition of values corresponds to
grouping of value requests into m *ascending* subroutes
$\Leftrightarrow \tau = 2m - 1$

Hard MinTurn

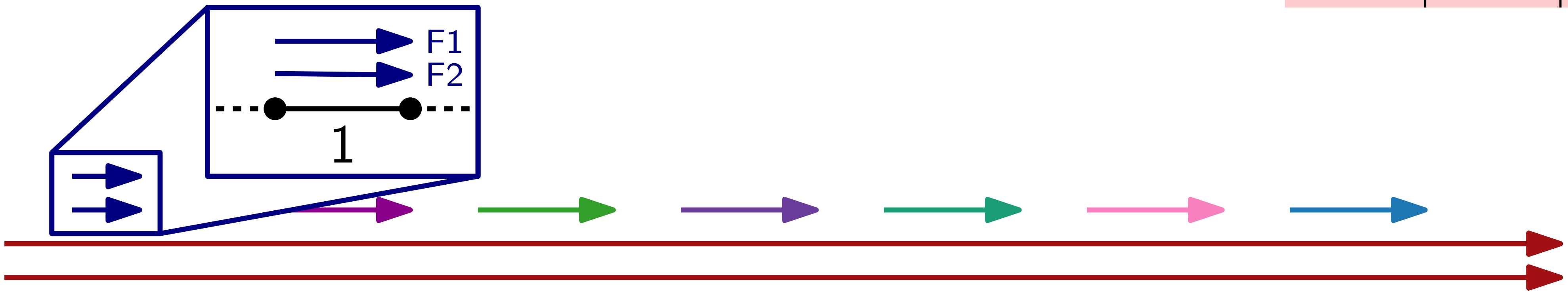
k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗

Set service time $t_s = 1$ and service promise $\rho < 2$

$$S = \{1, 6, 8, 3, 7, 5\}$$
$$m = 2,\ T = 15$$

Hard $\textsc{MinTurn}$

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



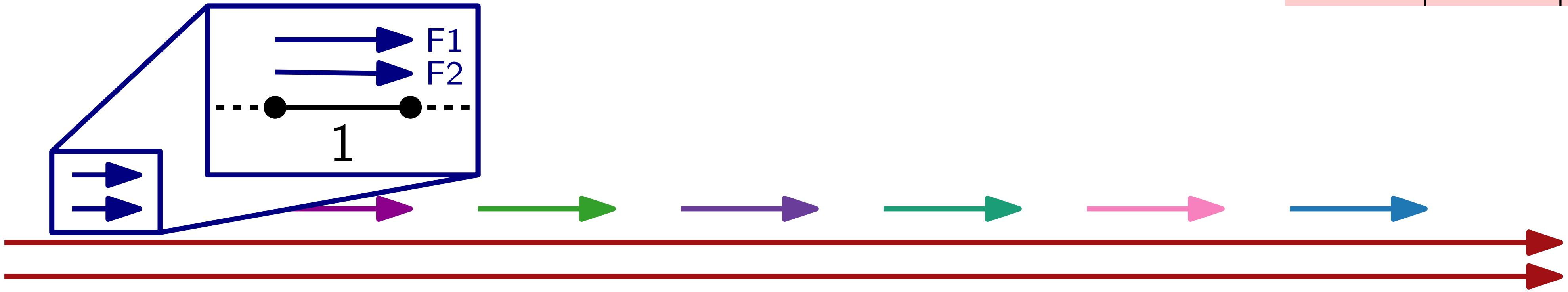
Set service time $t_s = 1$ and service promise $\rho < 2$

Filter Requests

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



$$S = \{1, 6, 8, 3, 7, 5\}$$
$$m = 2,\ T = 15$$

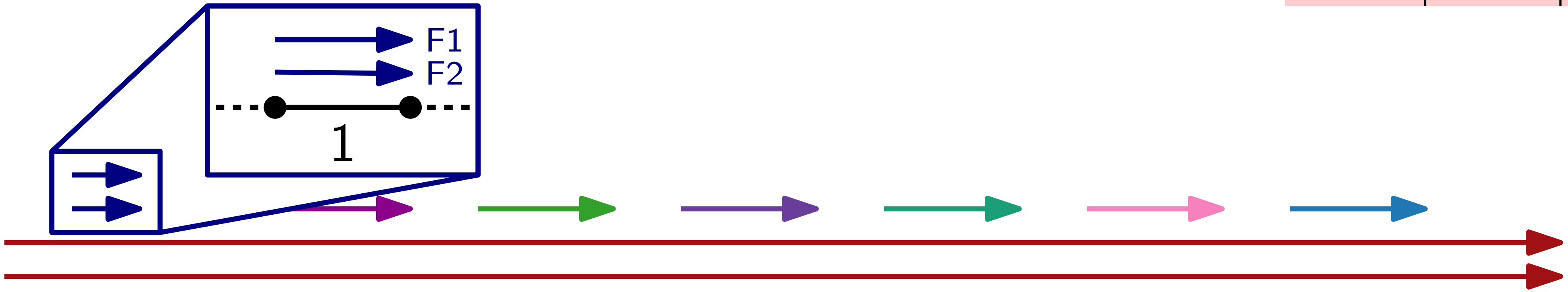
Set service time $t_s = 1$ and service promise $\rho < 2$

Filter Requests

- DTT $= 1 \Rightarrow$ MTT < 2

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Set service time $t_s = 1$ and service promise $\rho < 2$

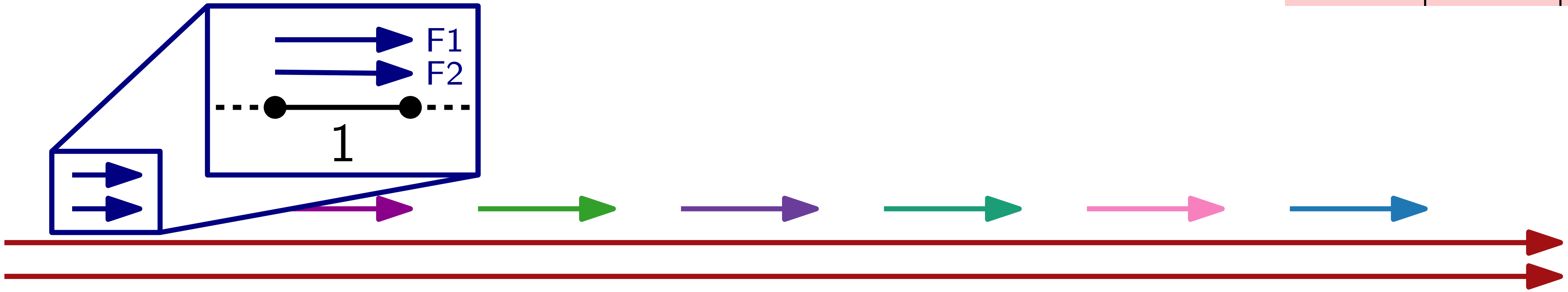
Filter Requests

- $\text{DTT} = 1 \Rightarrow \text{MTT} < 2$

P(F1)	P(F2)	——	D(F1)	D(F2)

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Set service time $t_s = 1$ and service promise $\rho < 2$

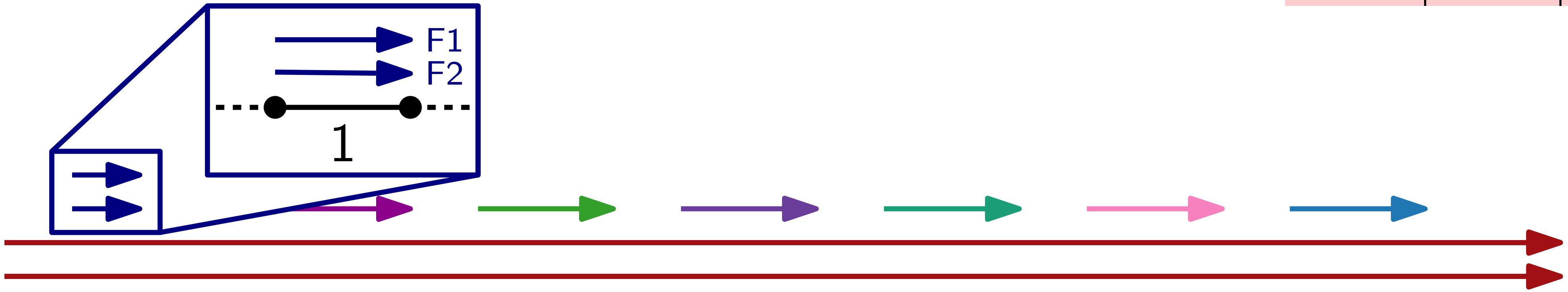
Filter Requests

- DTT $= 1 \Rightarrow$ MTT < 2

P(F1)	P(F2)	1	D(F1)	D(F2)
$2-3$	$3-4$		$5-6$	$6-7$

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗

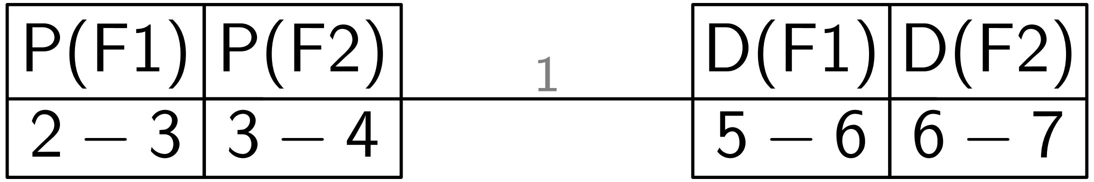


$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Set service time $t_s = 1$ and service promise $\rho < 2$

Filter Requests

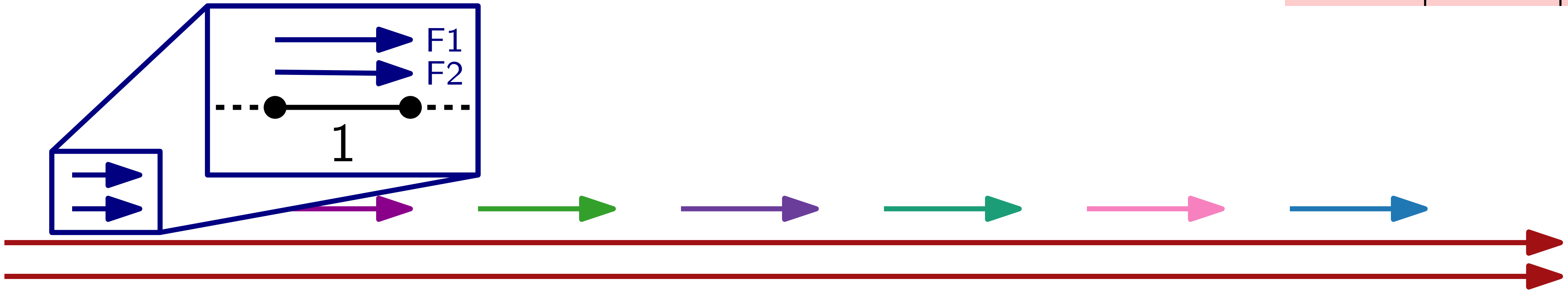
- DTT $= 1 \Rightarrow$ MTT < 2



TT(F1) $= 5 - 3 = 2 >$ MTT(F1)

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗

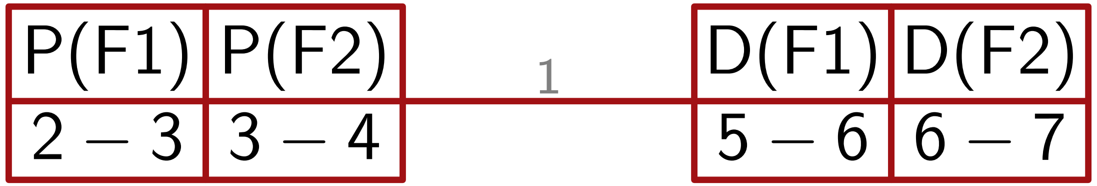


Set service time $t_s = 1$ and service promise $\rho < 2$

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Filter Requests

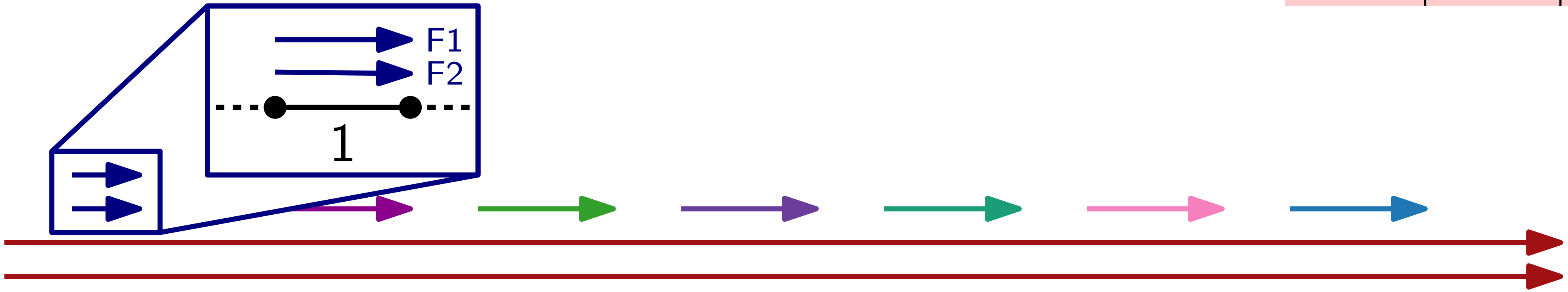
- $\text{DTT} = 1 \Rightarrow \text{MTT} < 2$



$\text{TT(F1)} = 5 - 3 = 2 > \text{MTT(F1)}$

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗

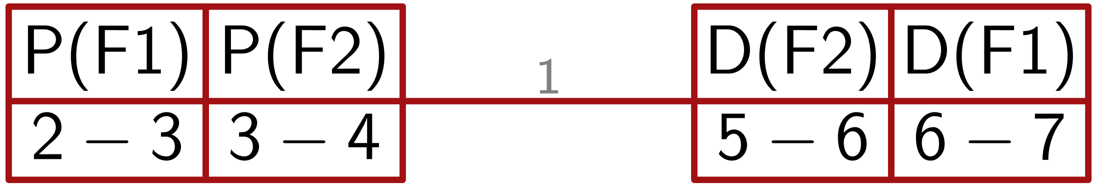


$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Set service time $t_s = 1$ and service promise $\rho < 2$

Filter Requests

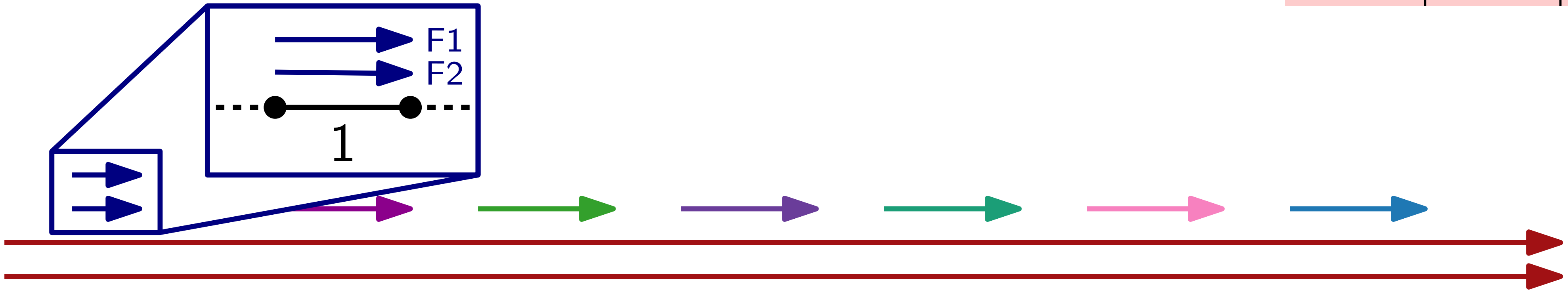
- DTT $= 1 \Rightarrow$ MTT < 2



TT(F1) $= 6 - 3 = 3 >$ MTT(F1)

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Set service time $t_s = 1$ and service promise $\rho < 2$

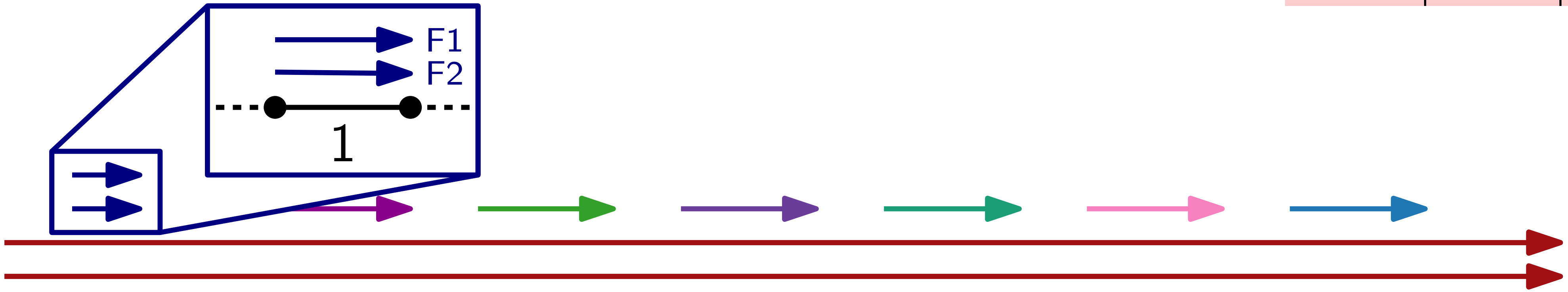
Filter Requests

- DTT $= 1 \Rightarrow$ MTT < 2

$\Rightarrow$ no two filter requests can be served in same subroute

Hard $\textsc{MinTurn}$

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Set service time $t_s = 1$ and service promise $\rho < 2$

Filter Requests

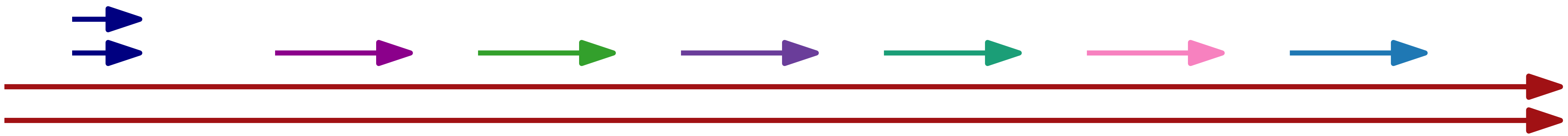
- DTT $= 1 \Rightarrow$ MTT < 2

$\Rightarrow$ no two filter requests can be served in same subroute

$\Rightarrow$ each of the m subroutes contains one filter and one promise request

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



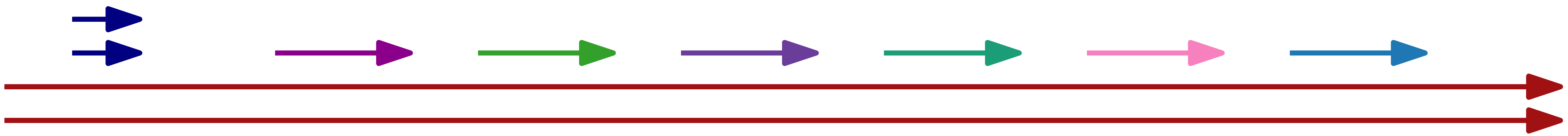
Set service time $t_s = 1$ and service promise $\rho < 2$

Value Requests

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2, T = 15$

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

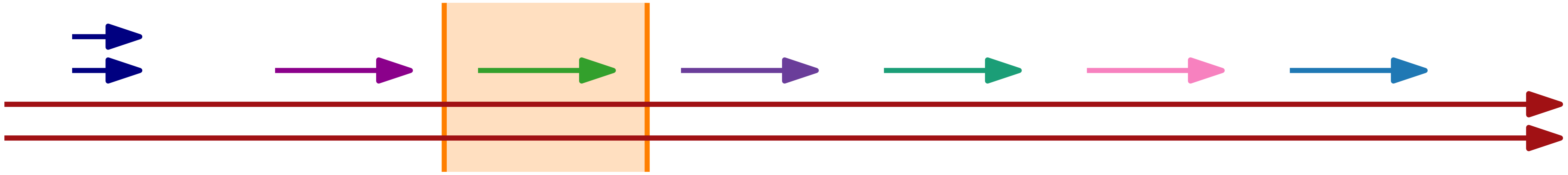
Set service time $t_s = 1$ and service promise $\rho < 2$

Value Requests

- value request v_i consists of s_i smaller requests

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

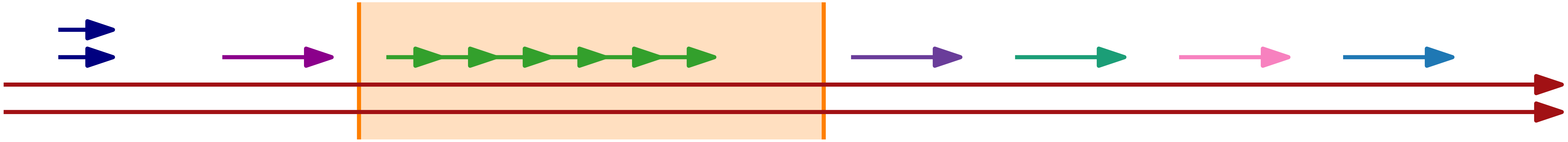
$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2, T = 15$

Value Requests

- value request v_i consists of s_i smaller requests

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

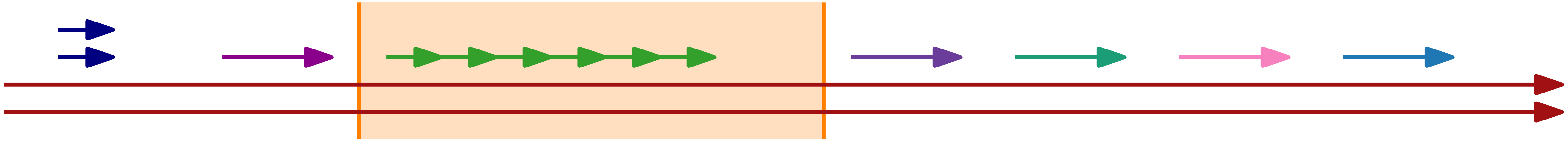
$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Value Requests

- value request v_i consists of s_i smaller requests

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

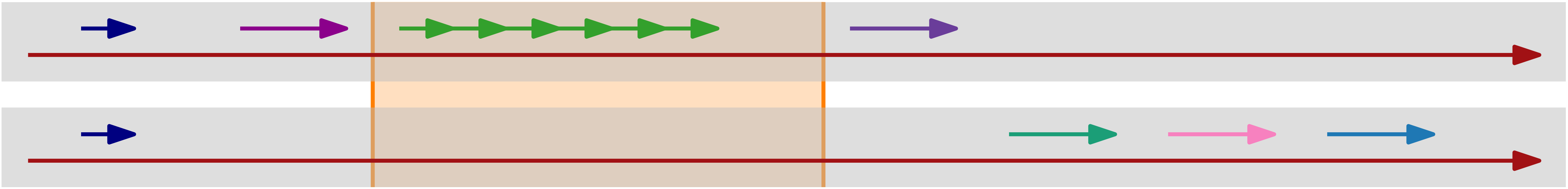
Value Requests

delay of $2s_i$

- value request v_i consists of s_i smaller requests

Hard $\textsc{MinTurn}$

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

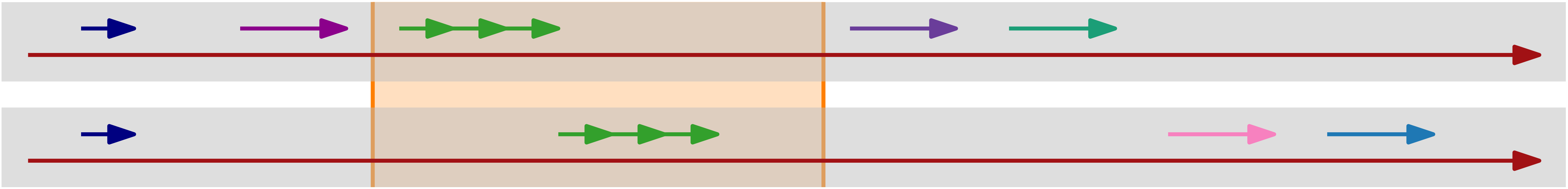
$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Value Requests

- value request v_i consists of s_i smaller requests

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

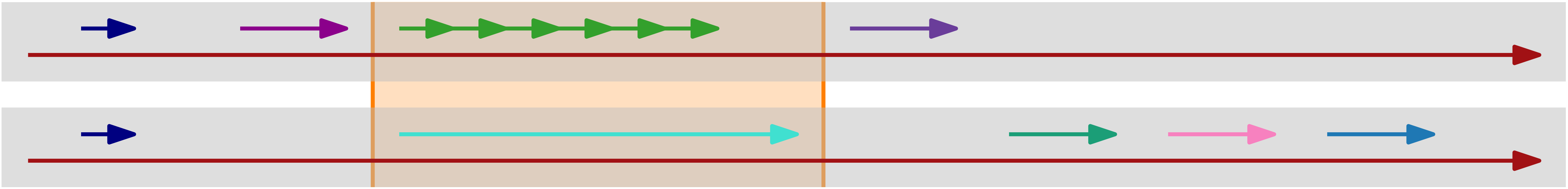
Value Requests

- value request v_i consists of s_i smaller requests

Problem: value request may be split between subroutes

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

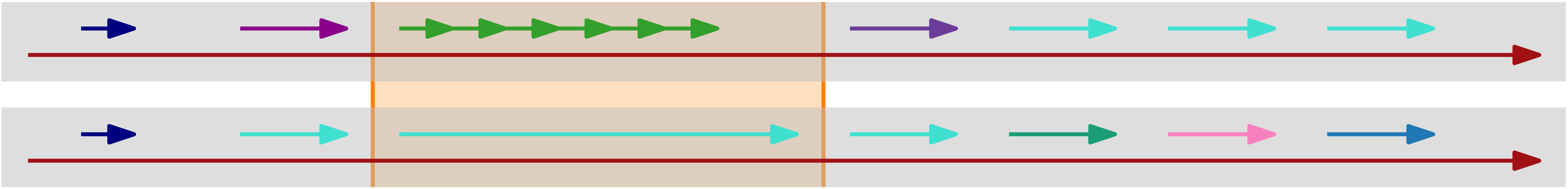
$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Value Requests

- value request v_i consists of s_i smaller requests
- splitting of value requests is prevented by a plug request per subroute

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

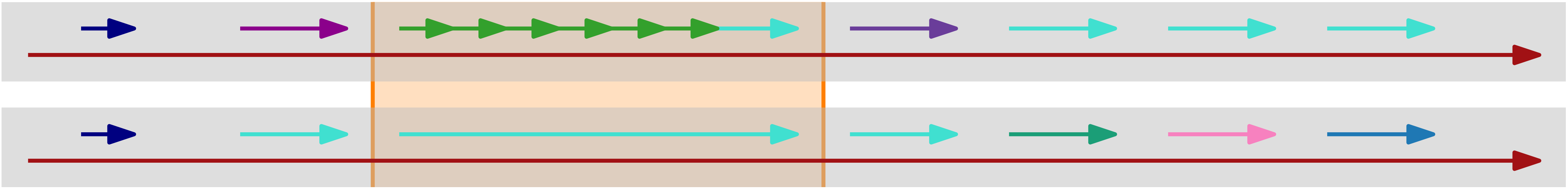
$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Value Requests

- value request v_i consists of s_i smaller requests
- splitting of value requests is prevented by a plug request per subroute

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

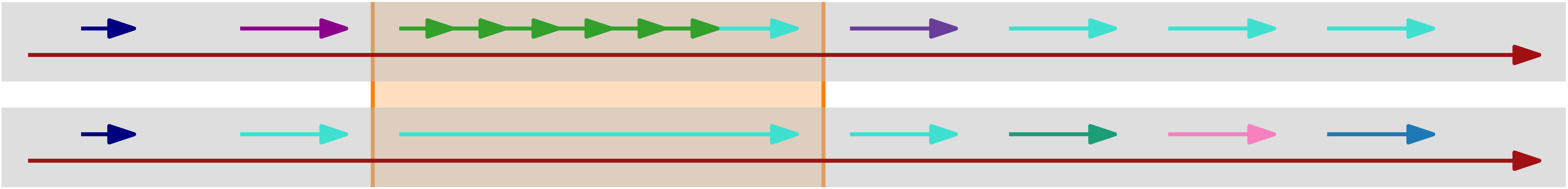
Set service time $t_s = 1$ and service promise $\rho < 2$

Value Requests

- value request v_i consists of s_i smaller requests
- splitting of value requests is prevented by a plug request per subroute

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

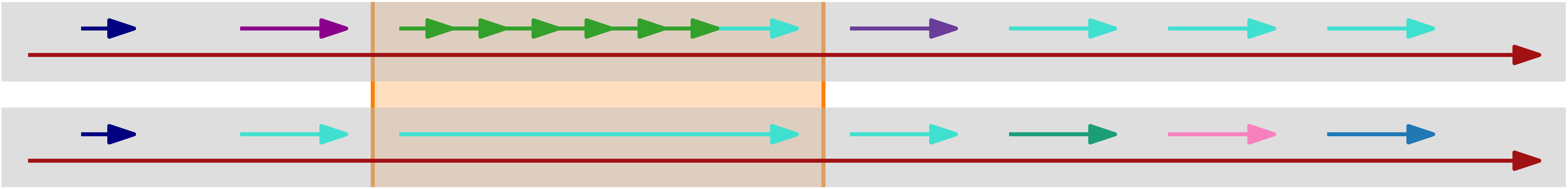
Each subroute should contain:

$S = \{1, 6, 8, 3, 7, 5\}$

$m = 2$, $T = 15$

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

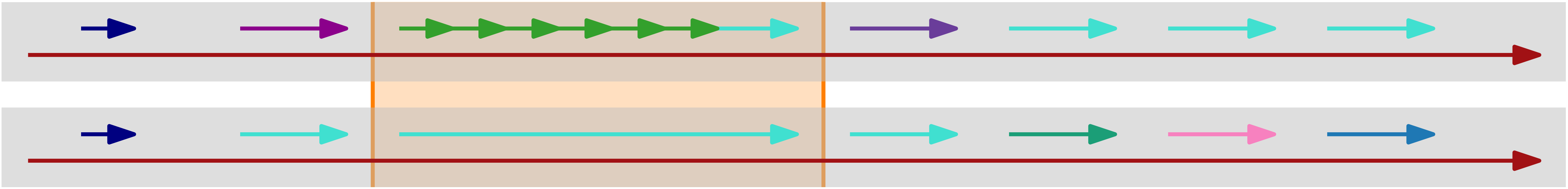
Each subroute should contain:

- 1 promise request

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2,\ T = 15$

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

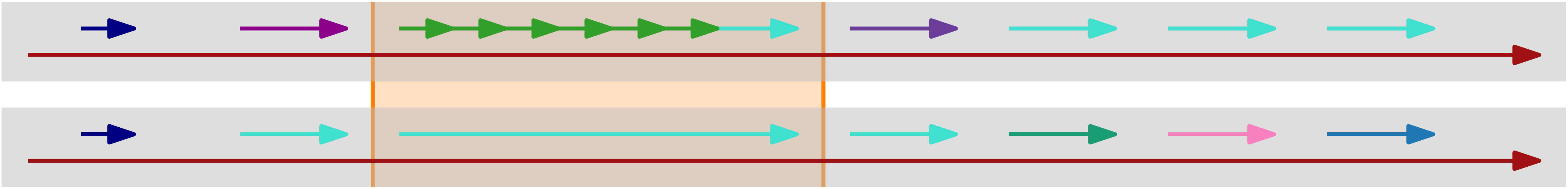
Set service time $t_s = 1$ and service promise $\rho < 2$

Each subroute should contain:

- 1 promise request
- 1 filter request

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

Each subroute should contain:

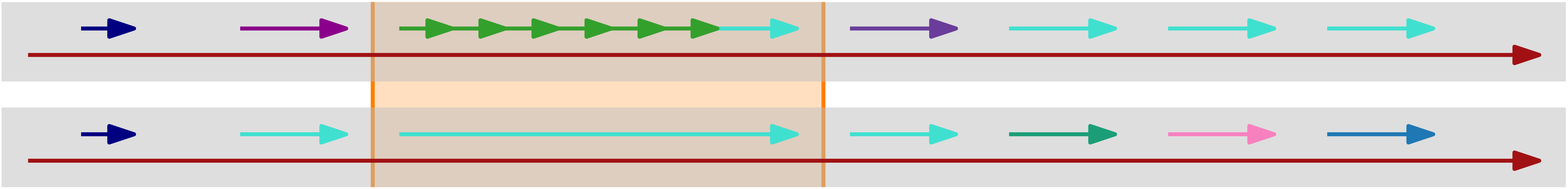
- 1 promise request
- 1 filter request
- T short value requests

$S = \{1, 6, 8, 3, 7, 5\}$

$m = 2, T = 15$

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

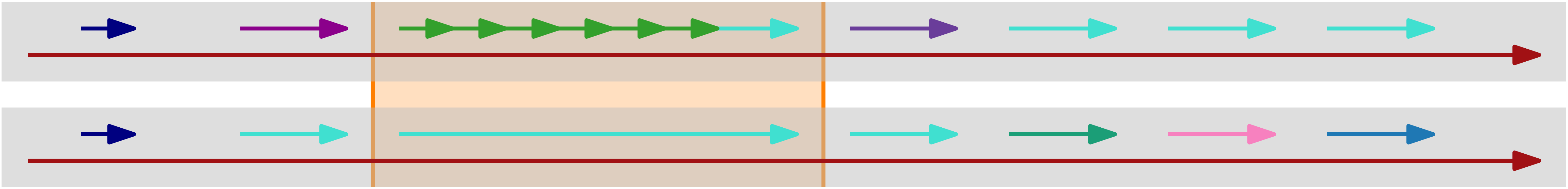
Set service time $t_s = 1$ and service promise $\rho < 2$

Each subroute should contain:

- 1 promise request
- 1 filter request
- T short value requests
- $3m$ plug requests

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

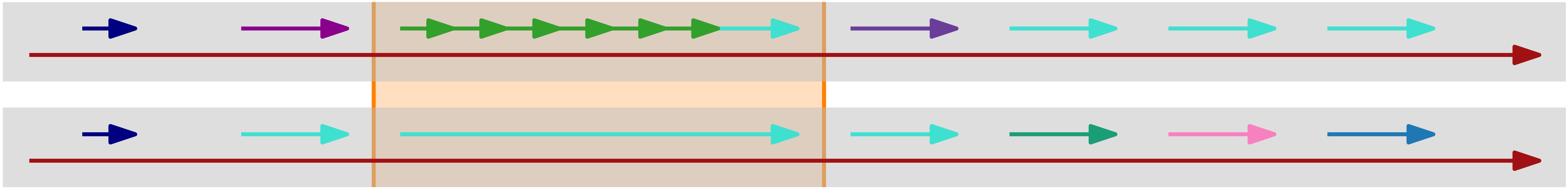
Each subroute should contain:

- 1 promise request ✓
- 1 filter request ✓
- T short value requests
- $3m$ plug requests

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2, T = 15$

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

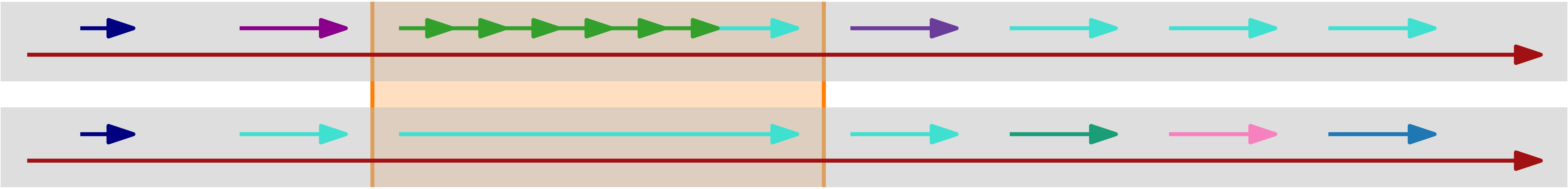
$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2, T = 15$

Each subroute should contain:

- 1 promise request ✓
- 1 filter request ✓
- T short value requests
- $3m$ plug requests ✓

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

Each subroute should contain:

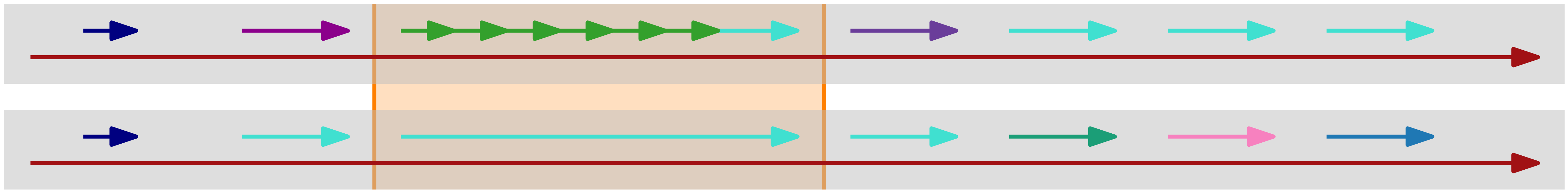
- 1 promise request ✓
- 1 filter request ✓
- T short value requests ⟵ enforced by service promise on promise requests
- $3m$ plug requests ✓

$S = \{1, 6, 8, 3, 7, 5\}$

$m = 2$, $T = 15$

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

Each subroute should contain:

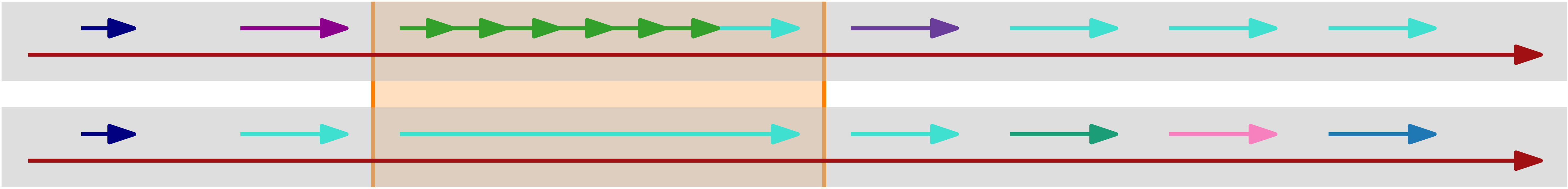
- 1 promise request
- 1 filter request
- T short value requests
- $3m$ plug requests

$S = \{1, 6, 8, 3, 7, 5\}$

$m = 2$, $T = 15$

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

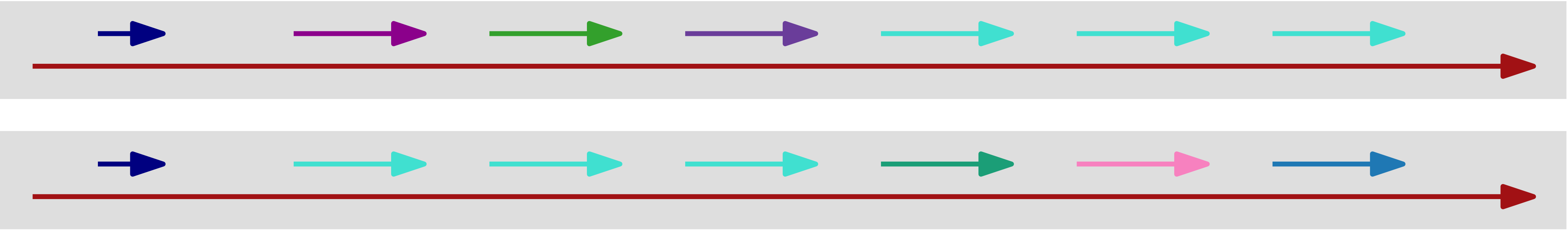
$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Each subroute should contain:

$$\left.\begin{array}{l} \text{1 promise request} \\ \text{1 filter request} \\ T \text{ short value requests} \\ 3m \text{ plug requests} \end{array}\right\} \text{delay: } 2(1 + T + 3m)$$

Hard $\textsc{MinTurn}$

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

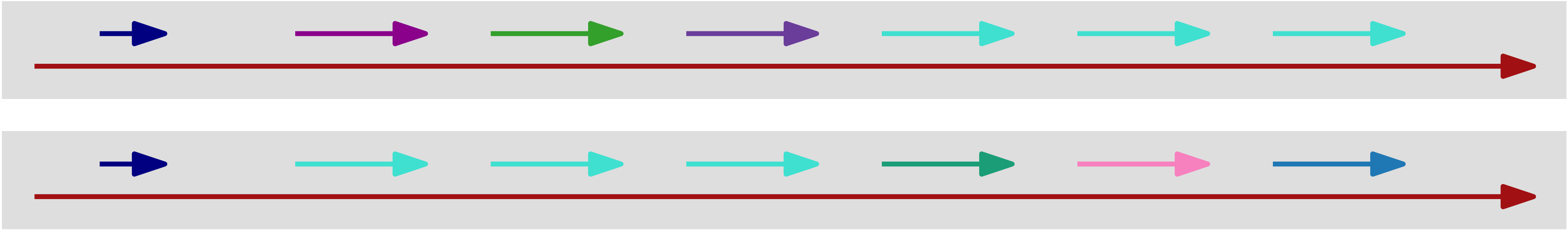
Set service time $t_s = 1$ and service promise $\rho < 2$

Promise Requests

- need $\Delta = 2(1 + T + 3m)$

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

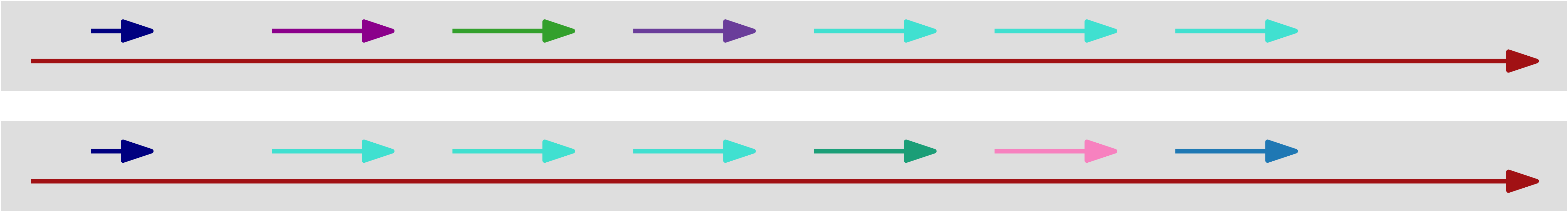
$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2,\ T = 15$

Promise Requests

- need $\Delta = 2(1 + T + 3m)$ $(\rho - 1) \cdot \mathrm{DTT}$

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

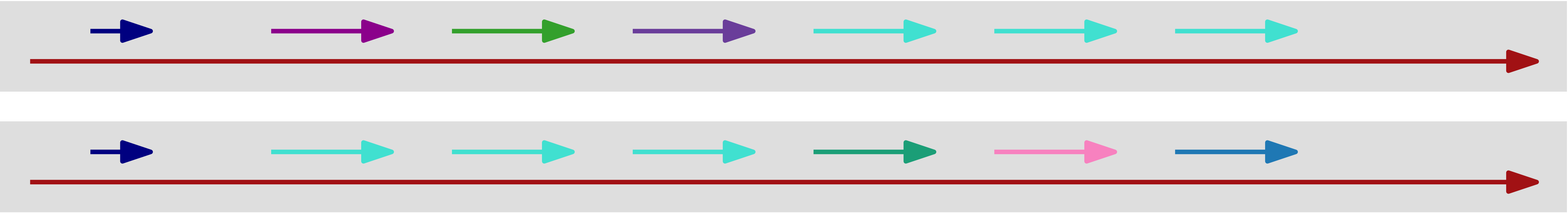
$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Promise Requests

- need $\Delta = 2(1 + T + 3m)$ $(\rho - 1) \cdot \mathrm{DTT}$
- make DTT large enough such that $\rho < 2$

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2,\ T = 15$

Promise Requests

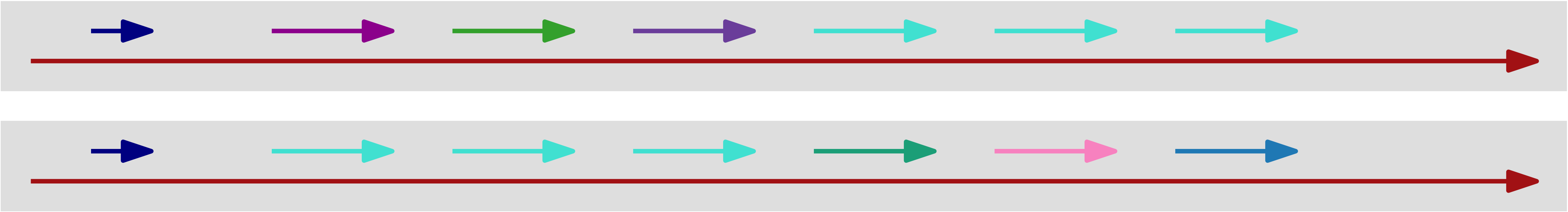
$(\rho - 1) \cdot \mathrm{DTT}$

- need $\Delta = 2(1 + T + 3m)$
- make DTT large enough such that $\rho < 2$

Goal: grouping of value requests into m subroutes corresponds to 3-partition of S ✓

Hard MinTurn

k	c	TW	SP	ST	SC
$= 1$	$= 2$	✗	✓	✓	✗



$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Set service time $t_s = 1$ and service promise $\rho < 2$

Promise Requests

$(\rho - 1) \cdot \text{DTT}$

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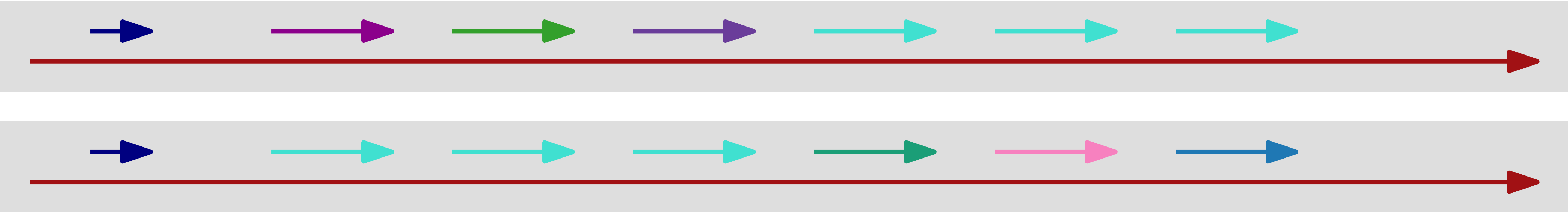
Goal: grouping of value requests into m subroutes corresponds to 3-partition of S ✓

$\tau = 2m - 1 \Leftrightarrow S$ has 3-partition

Hard MinTurn

k	c	TW	SP	ST	SC
$=1$	$=2$	✗	✓	✓	✗



Set service time $t_s = 1$ and service promise $\rho < 2$

$S = \{1, 6, 8, 3, 7, 5\}$
$m = 2$, $T = 15$

Promise Requests

$(\rho - 1) \cdot \mathrm{DTT}$

- need $\Delta = 2(1 + T + 3m)$
- make DTT large enough such that $\rho < 2$

Goal: grouping of value requests into m subroutes corresponds to 3-partition of S ✓

$\tau = 2m - 1 \Leftrightarrow S$ has 3-partition

Theorem
MinTurn with service promise & service time is $\mathcal{NP}$-hard

Conclusion

Parameterized Algorithms

- FPT-algorithm for liDARP and MinTurn

  $$O^*((h^2 \cdot t^3 \cdot c \cdot k)^{2 \cdot t \cdot c \cdot k})$$

- XP-algorithm for MinTurn without time windows

  $$O^*(n^{h^2} \cdot h^{4 \cdot c \cdot h})$$

Complexity

liDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MinTurn

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗

Conclusion

Parameterized Algorithms

- FPT-algorithm for liDARP and MinTurn

  $$O^*((h^2 \cdot t^3 \cdot c \cdot k)^{2 \cdot t \cdot c \cdot k})$$

- XP-algorithm for MinTurn without time windows

  $$O^*(n^{h^2} \cdot h^{4 \cdot c \cdot h})$$

Open Problems

- Can the parameterized algorithms be improved? Is there an FPT algorithm for MinTurn without time windows?
- What about heuristics / approximation algorithms for MinTurn?

Complexity

liDARP

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✓	✓
≥ 1	≥ 1	✓	✗	✗	✗

MinTurn

k	c	TW	SP	ST	SC
≥ 1	≥ 1	✗	✓	✗	✗
≥ 1	≥ 1	✗	✗	✓	✓
≥ 1	1	✗	✓	✓	✓
≥ 1	≥ 2	✗	✓	✗	✓
≥ 1	≥ 2	✗	✓	✓	✗
≥ 1	≥ 1	✓	✗	✗	✗