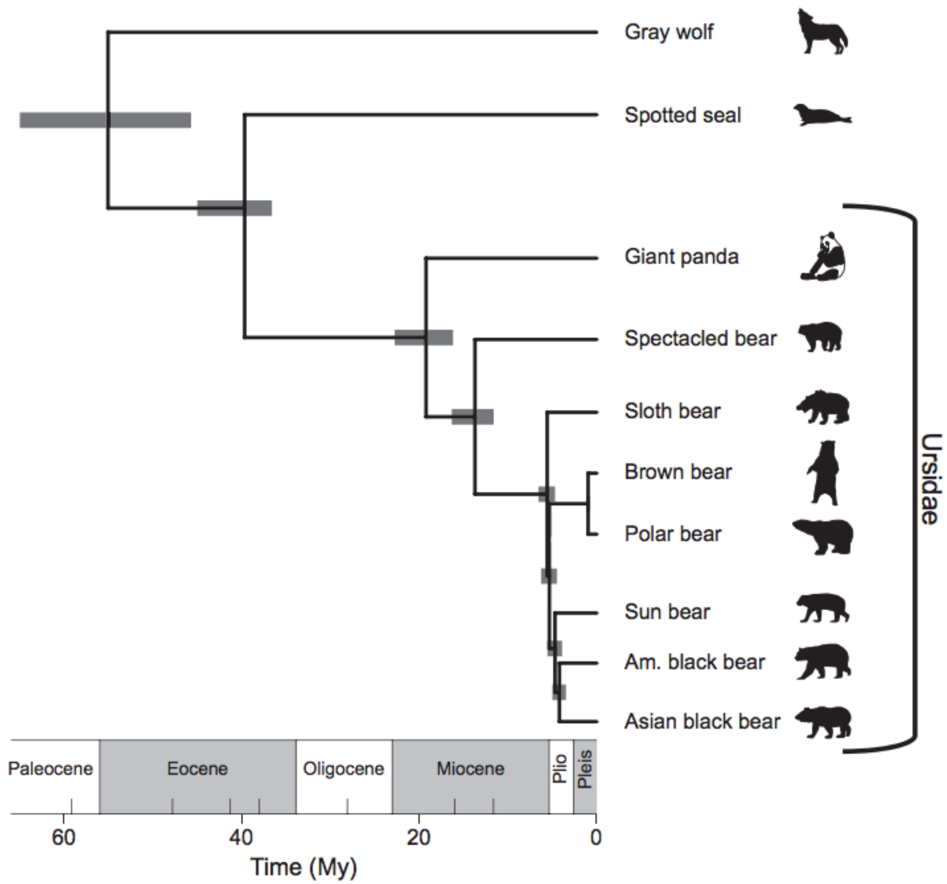
Spaces of phylogenetic networks

Jonathan Klawitter

PhD Exam · 5th March, 2020

Phylogenetic trees & networks



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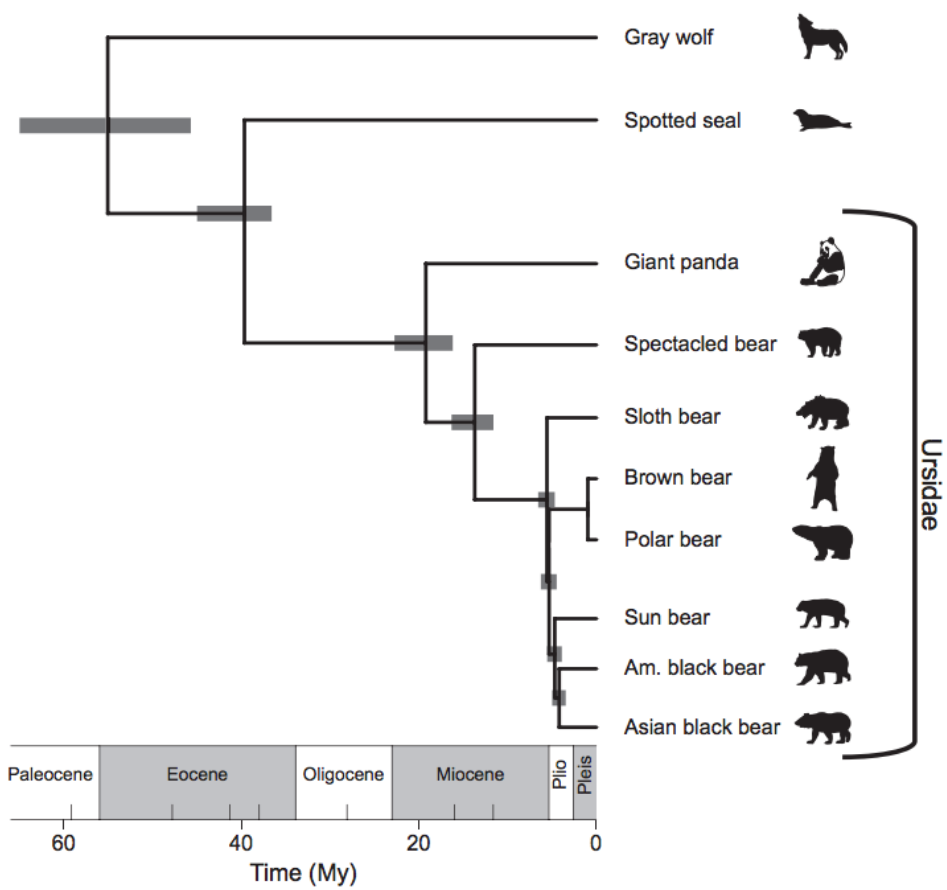

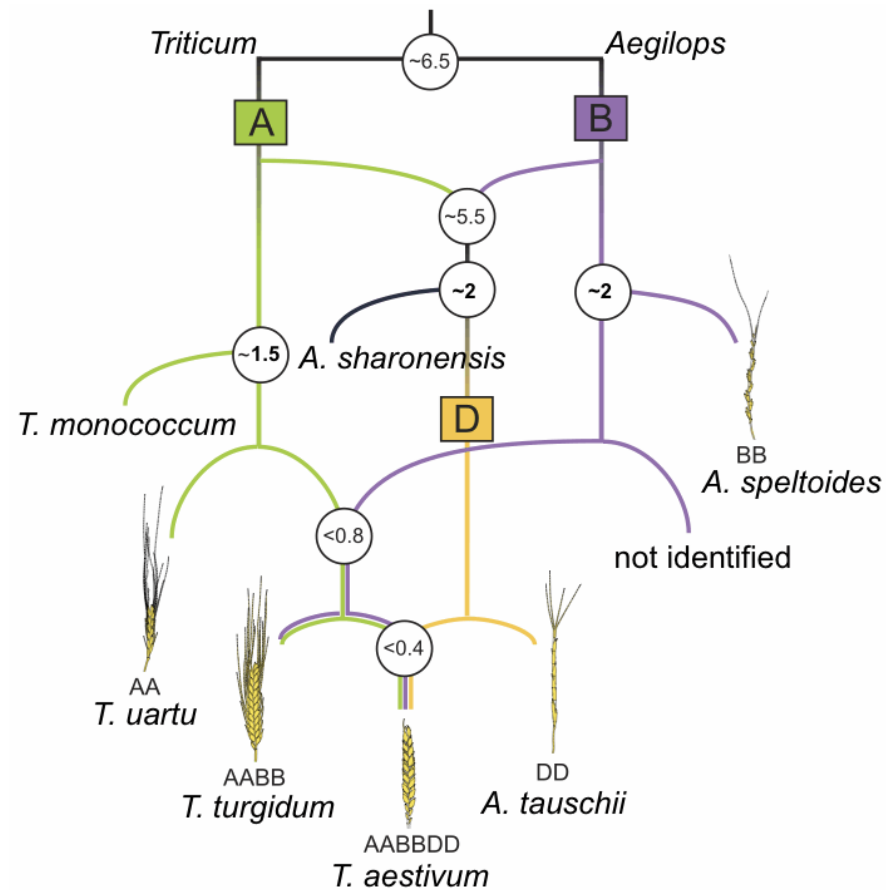

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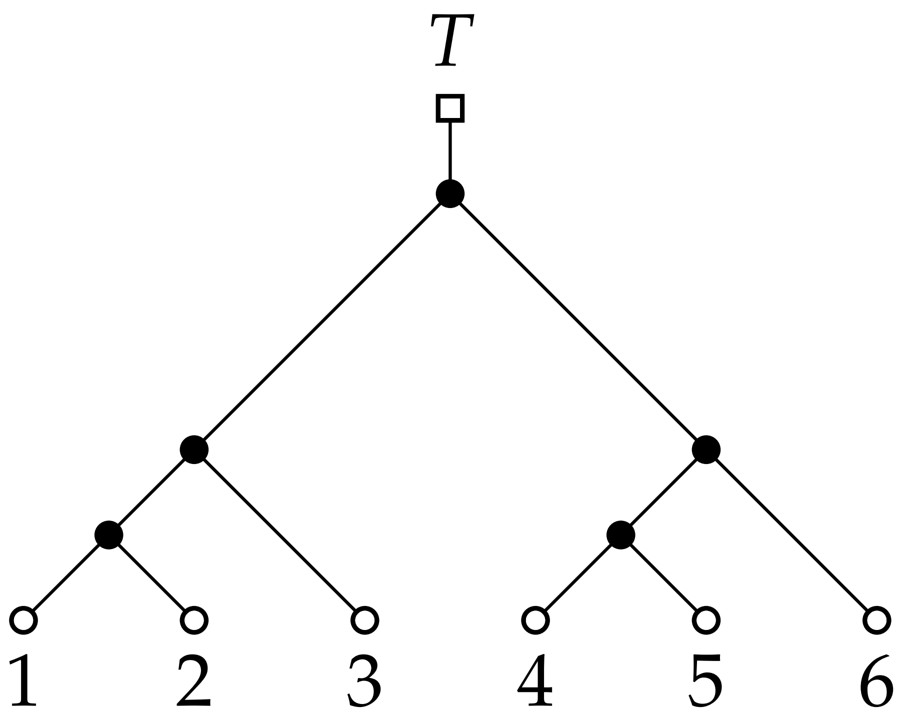

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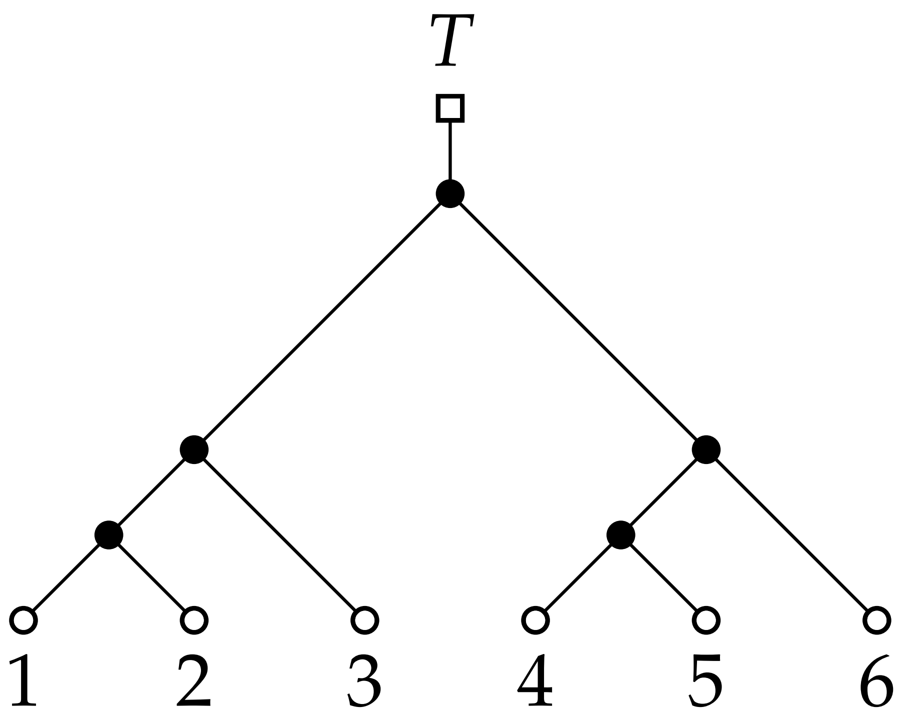
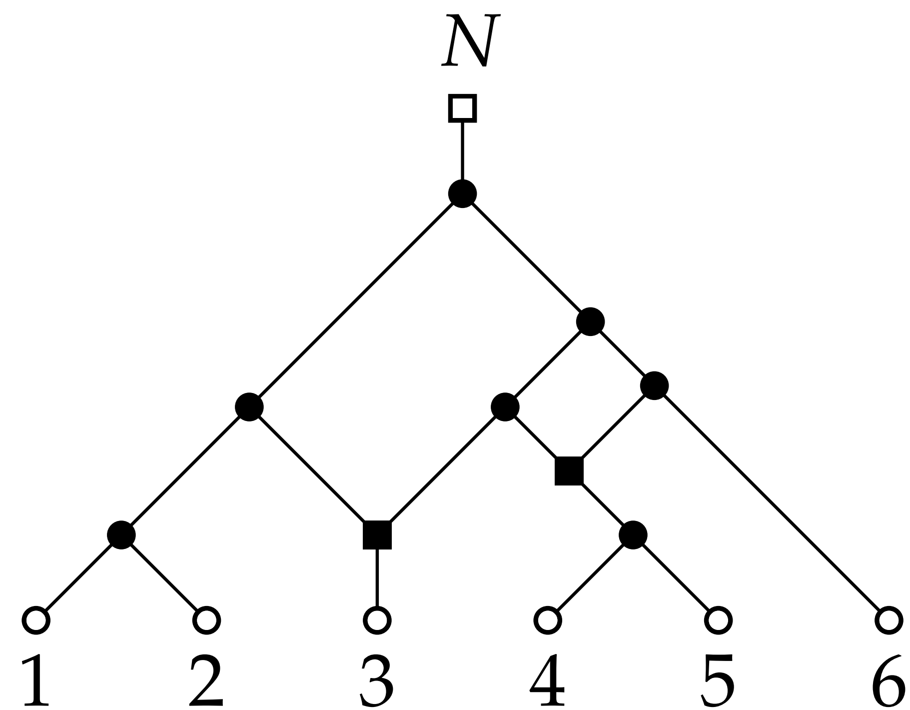
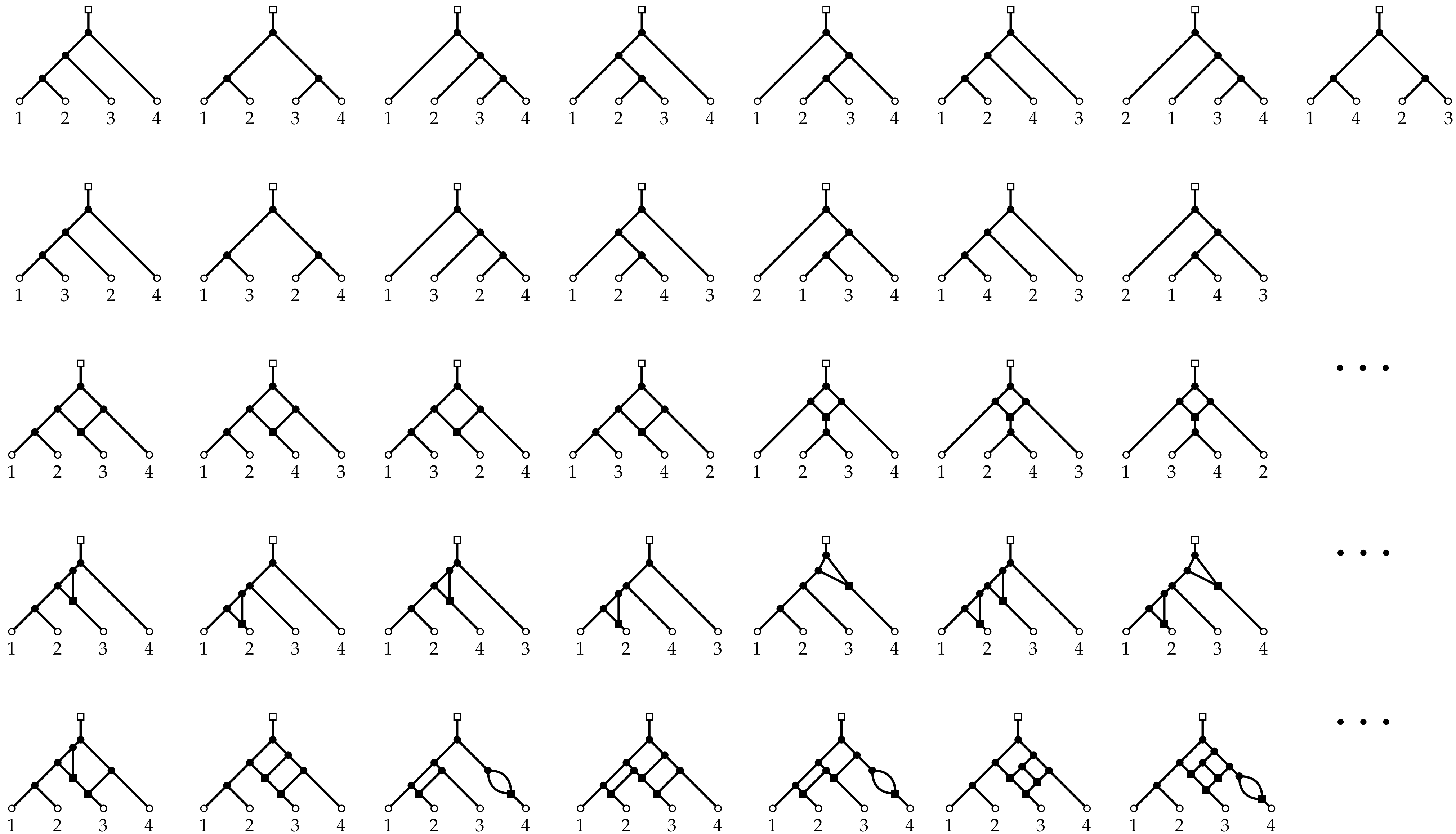




Set of networks



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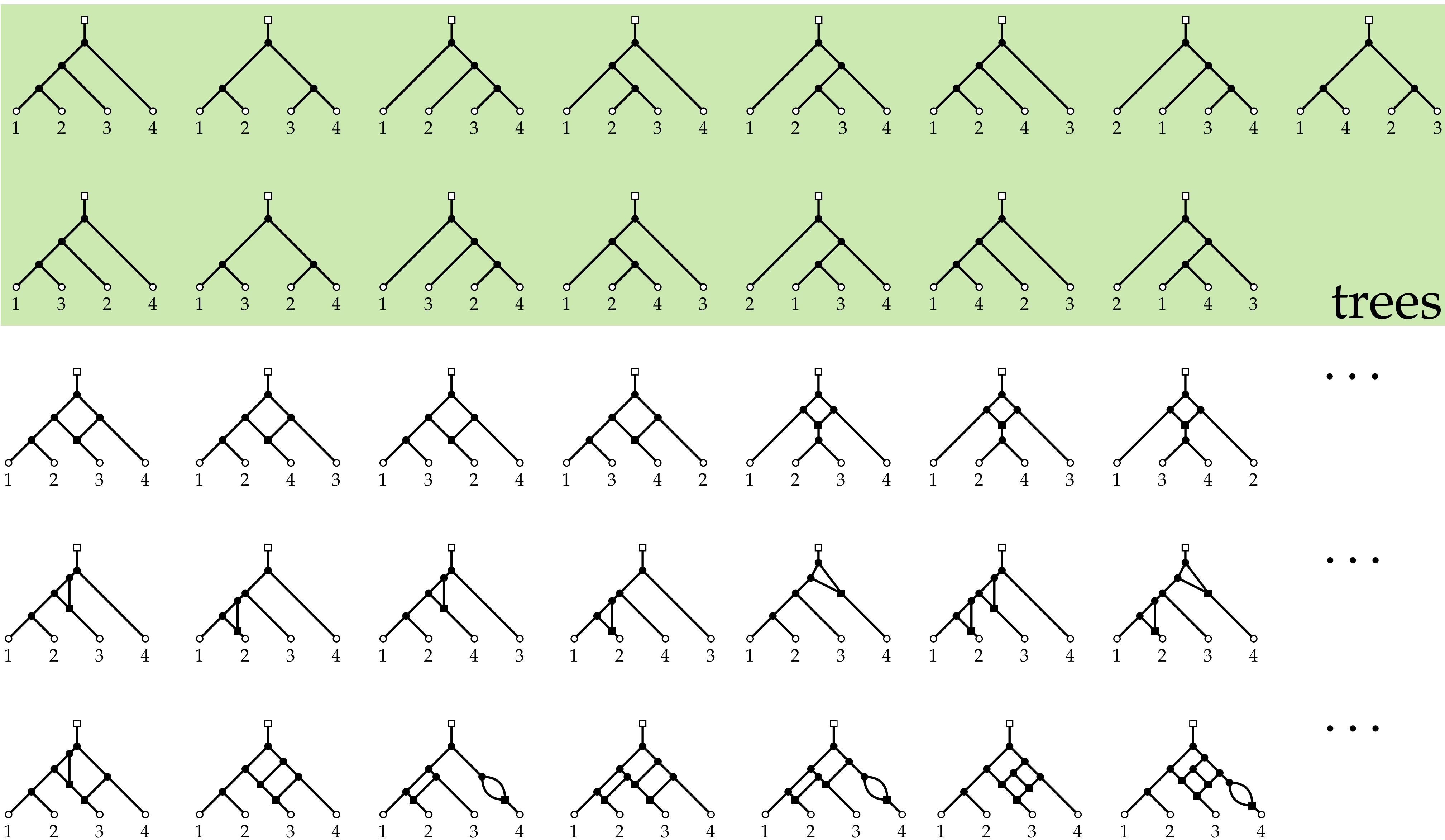

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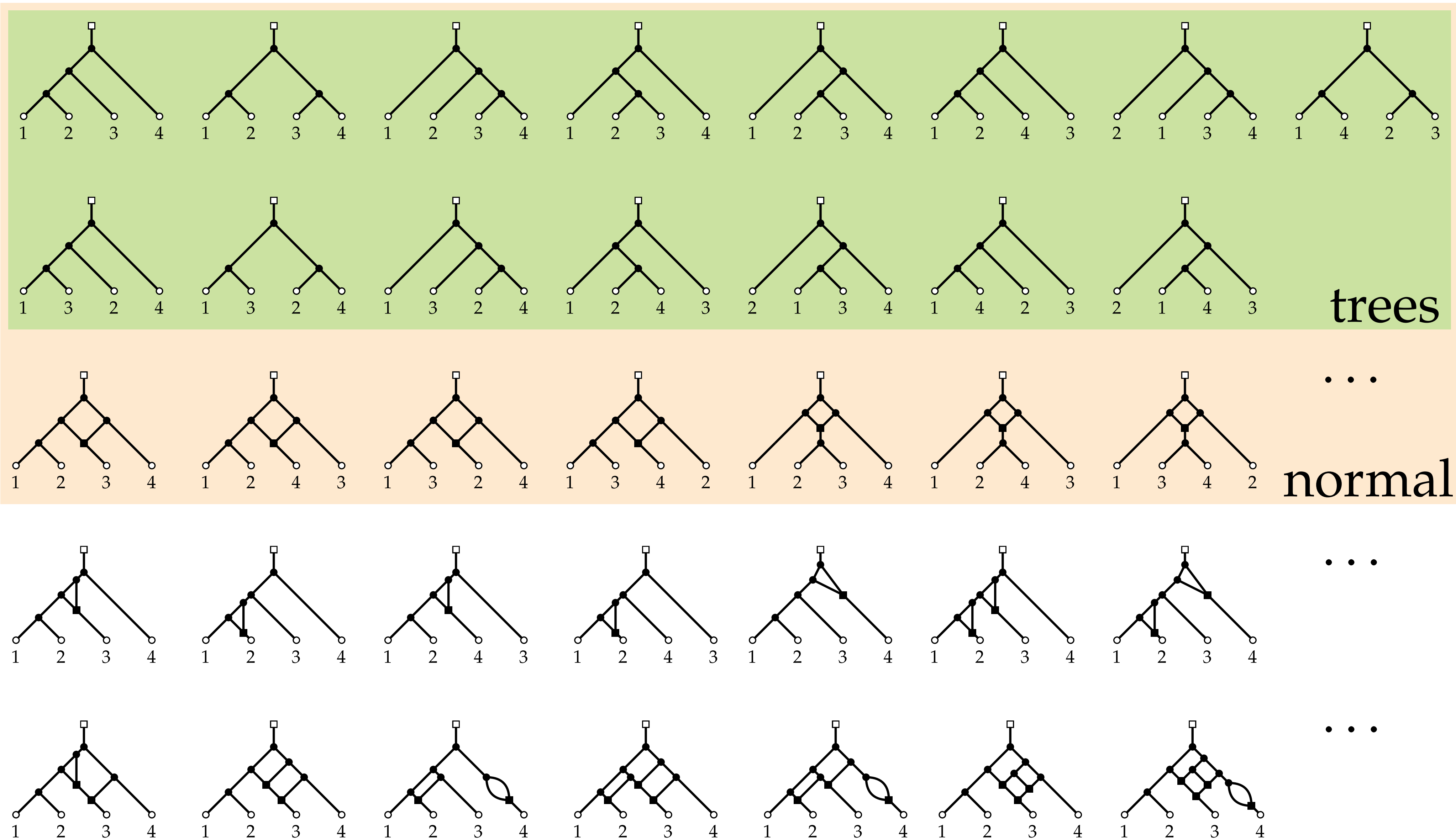

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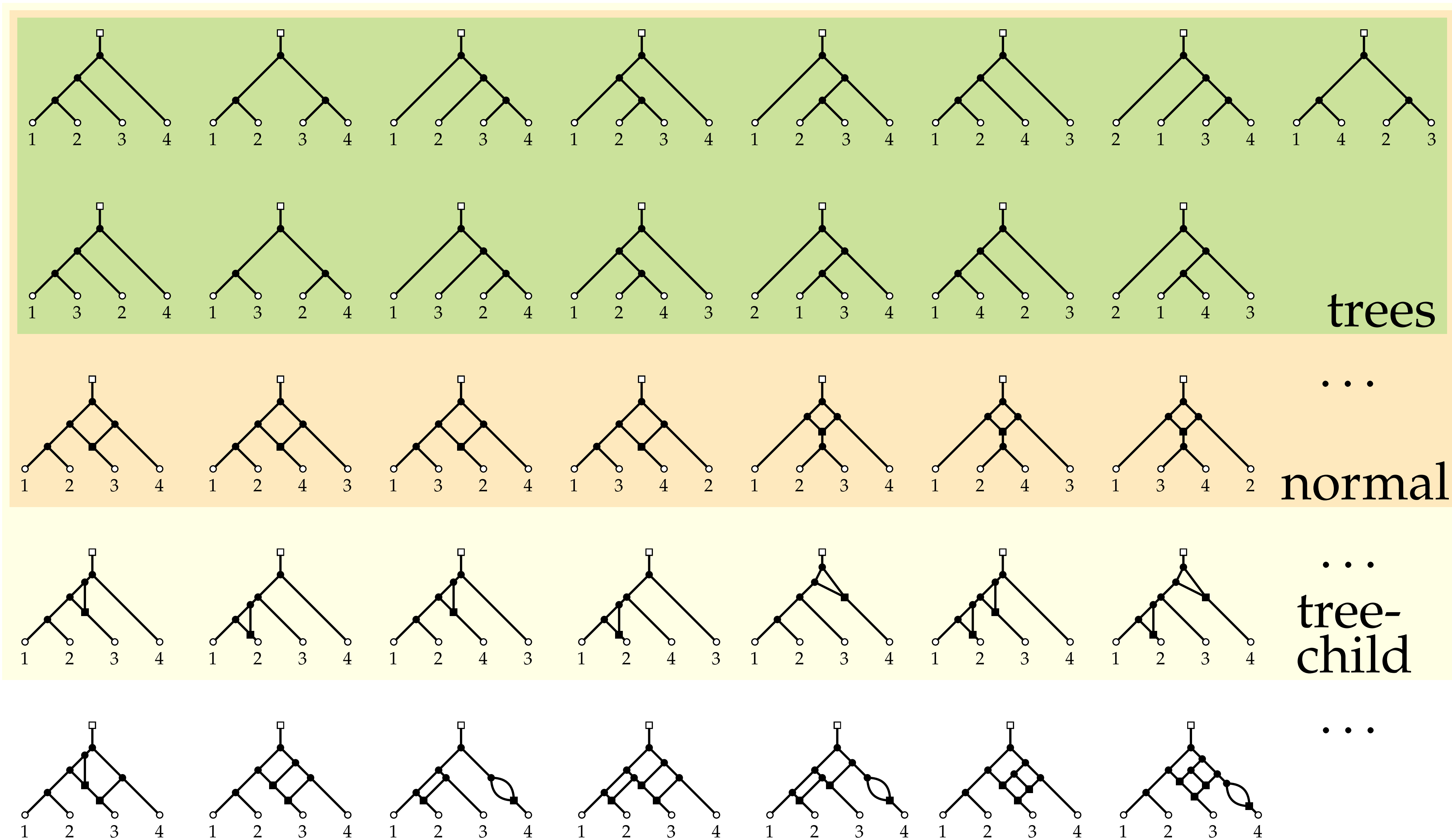

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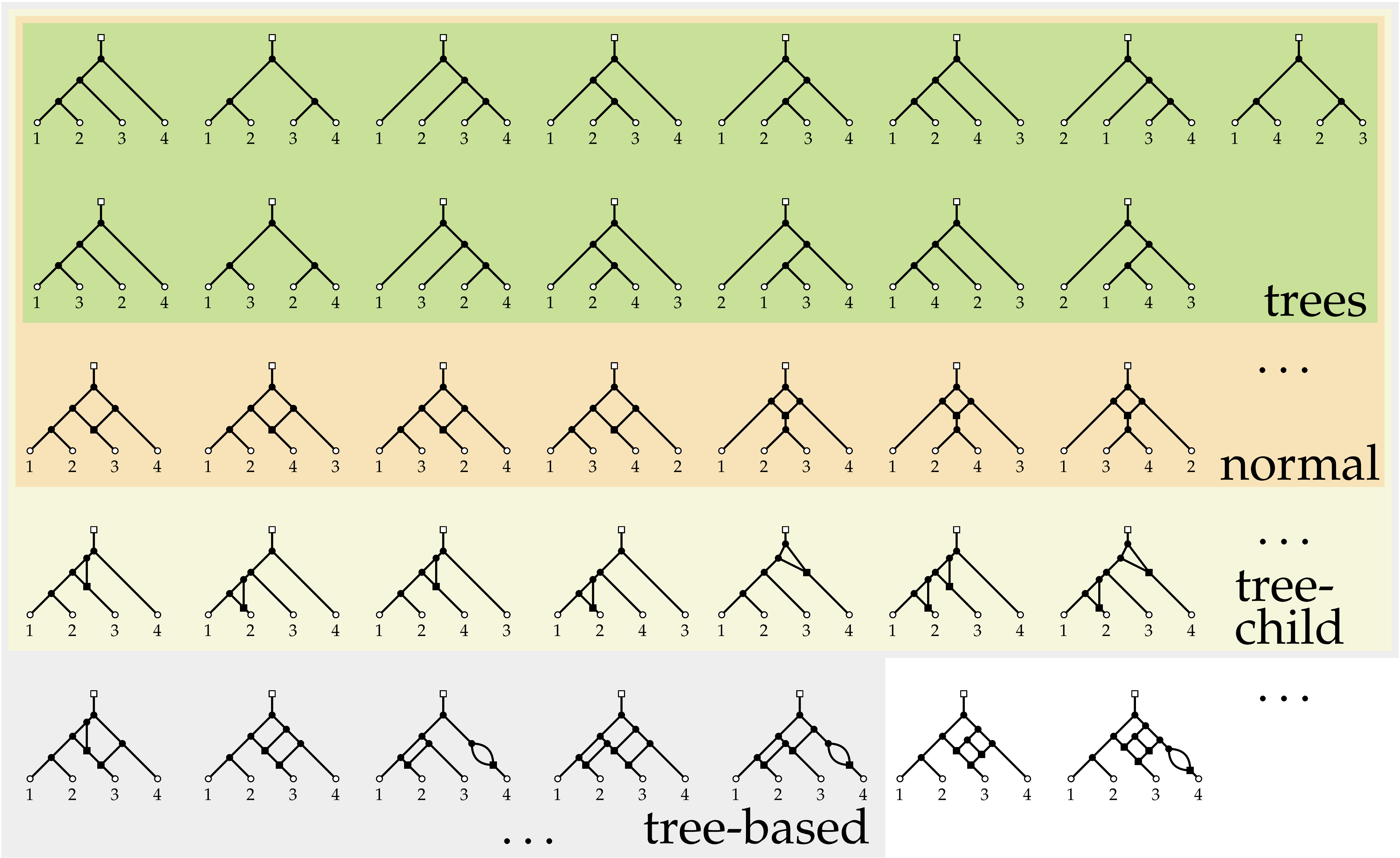
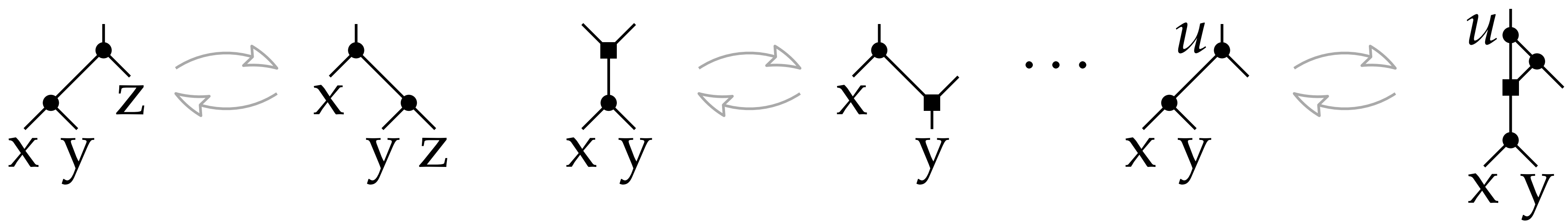


Rearrangement operations

NNI



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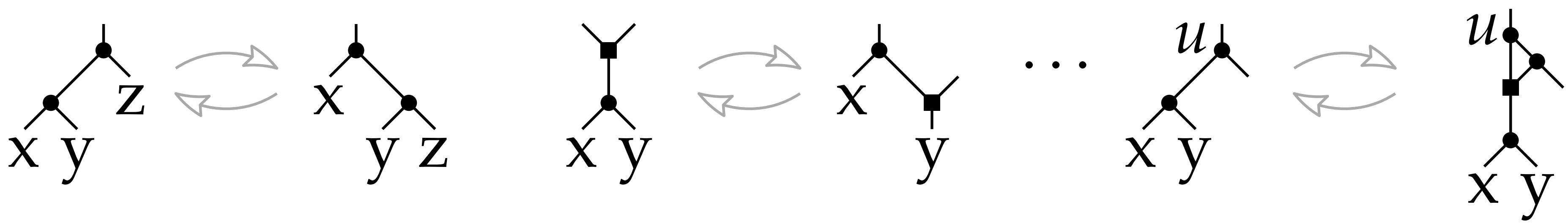
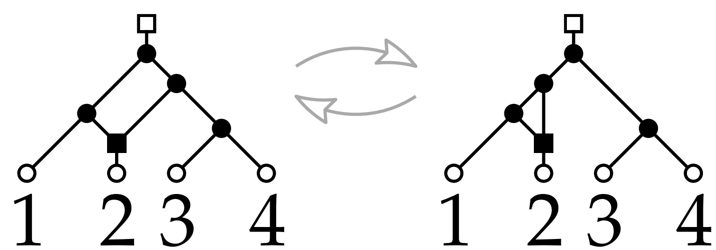


SNPR



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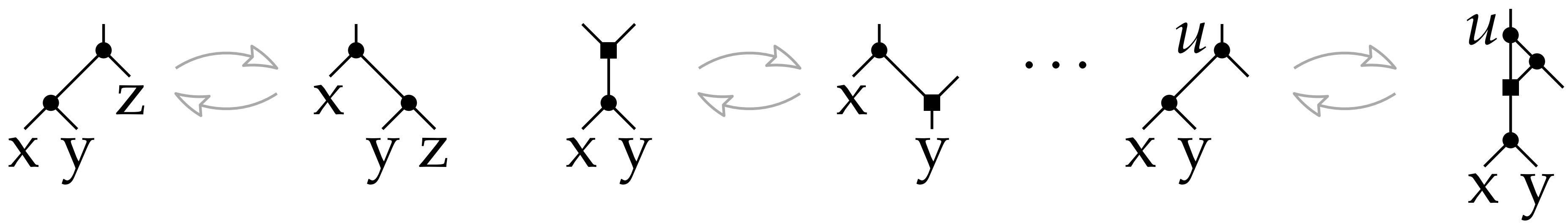

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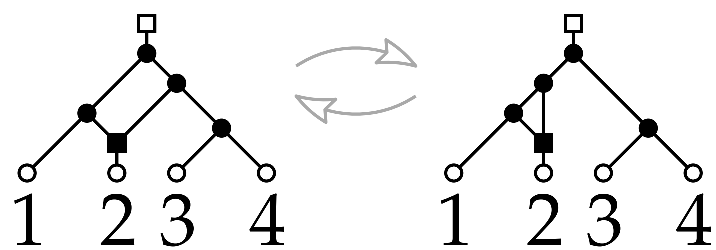

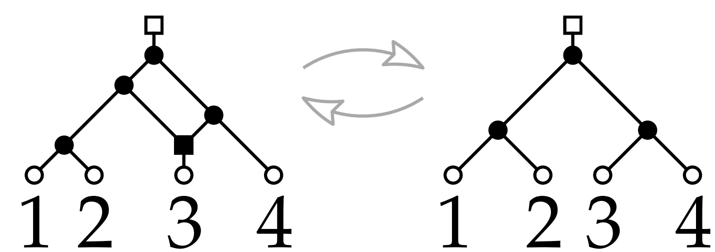

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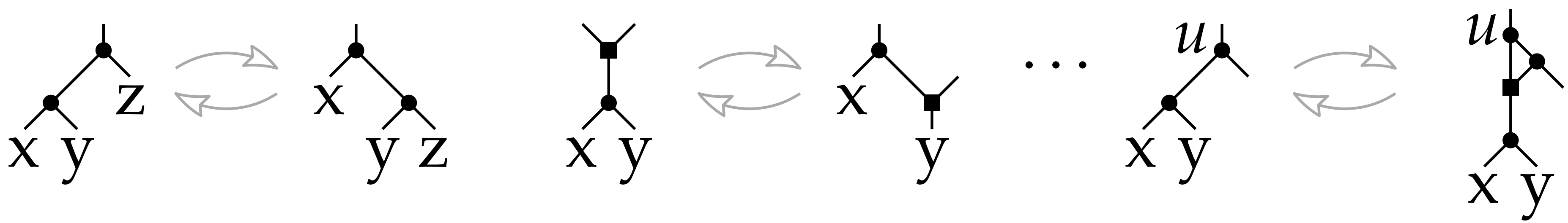

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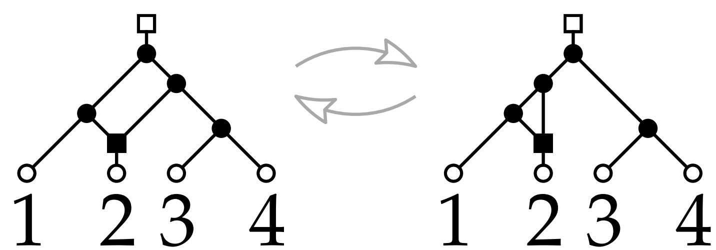
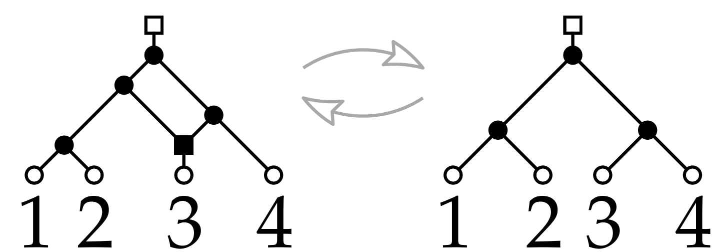
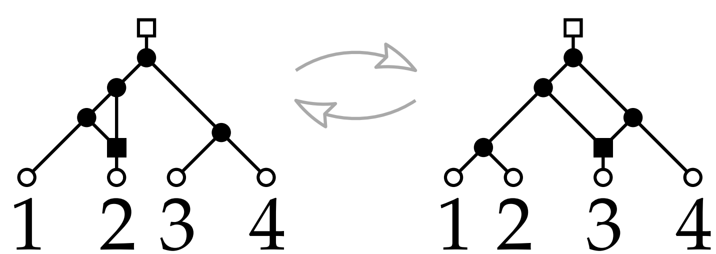
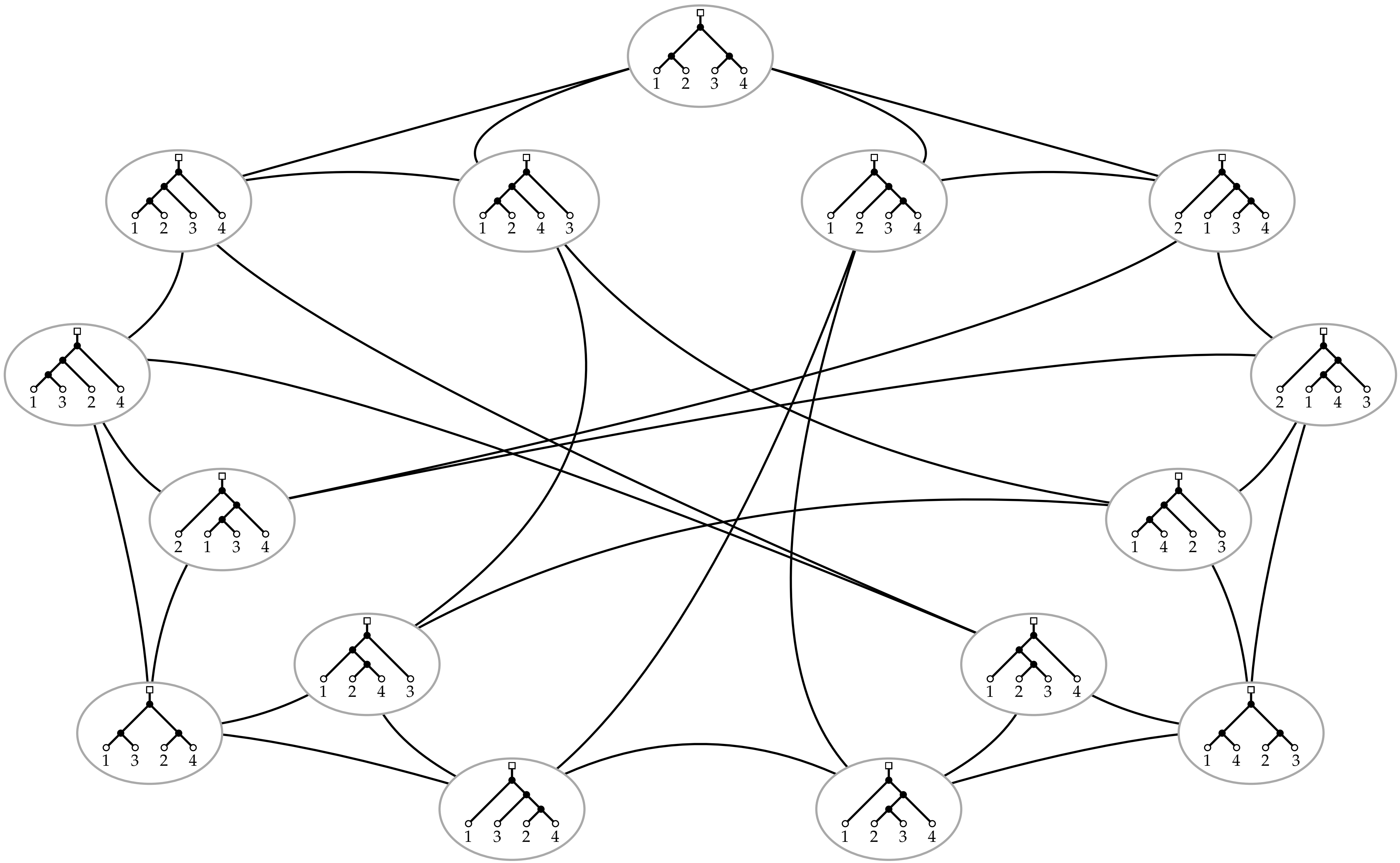




PR



NNI tree space



Chapters

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- Connectedness & diameter

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- Neighbourhood size

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- SNPR- and PR-distance

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- Maximum agreement graphs

Connectedness & diameter

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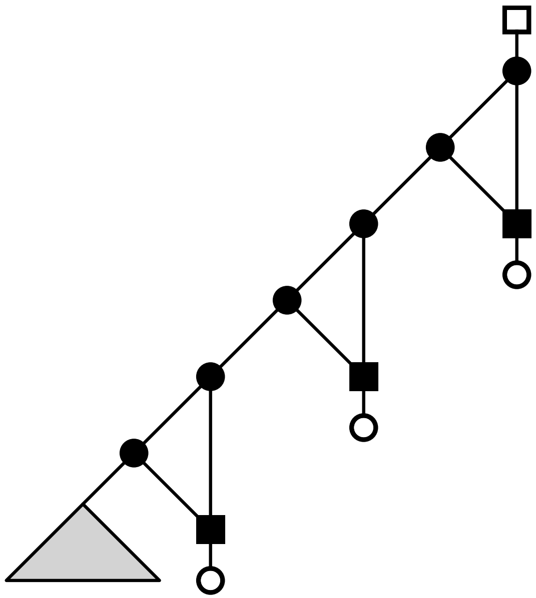
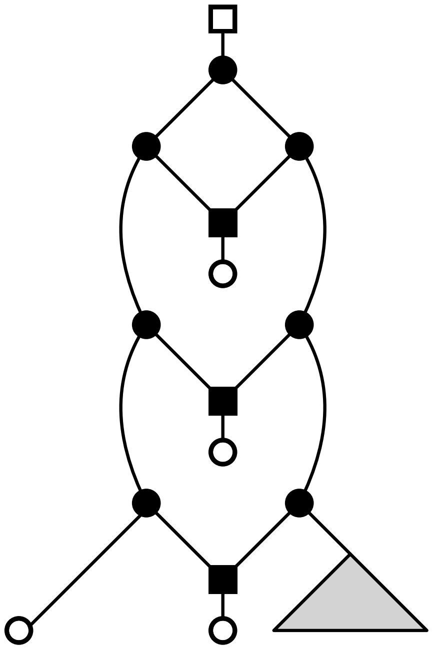
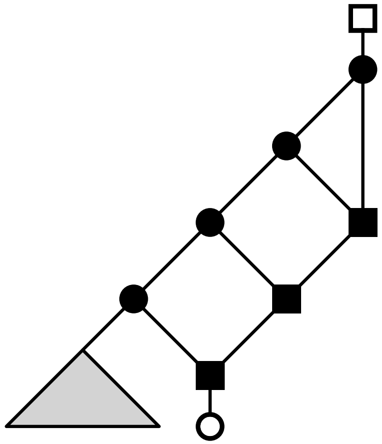


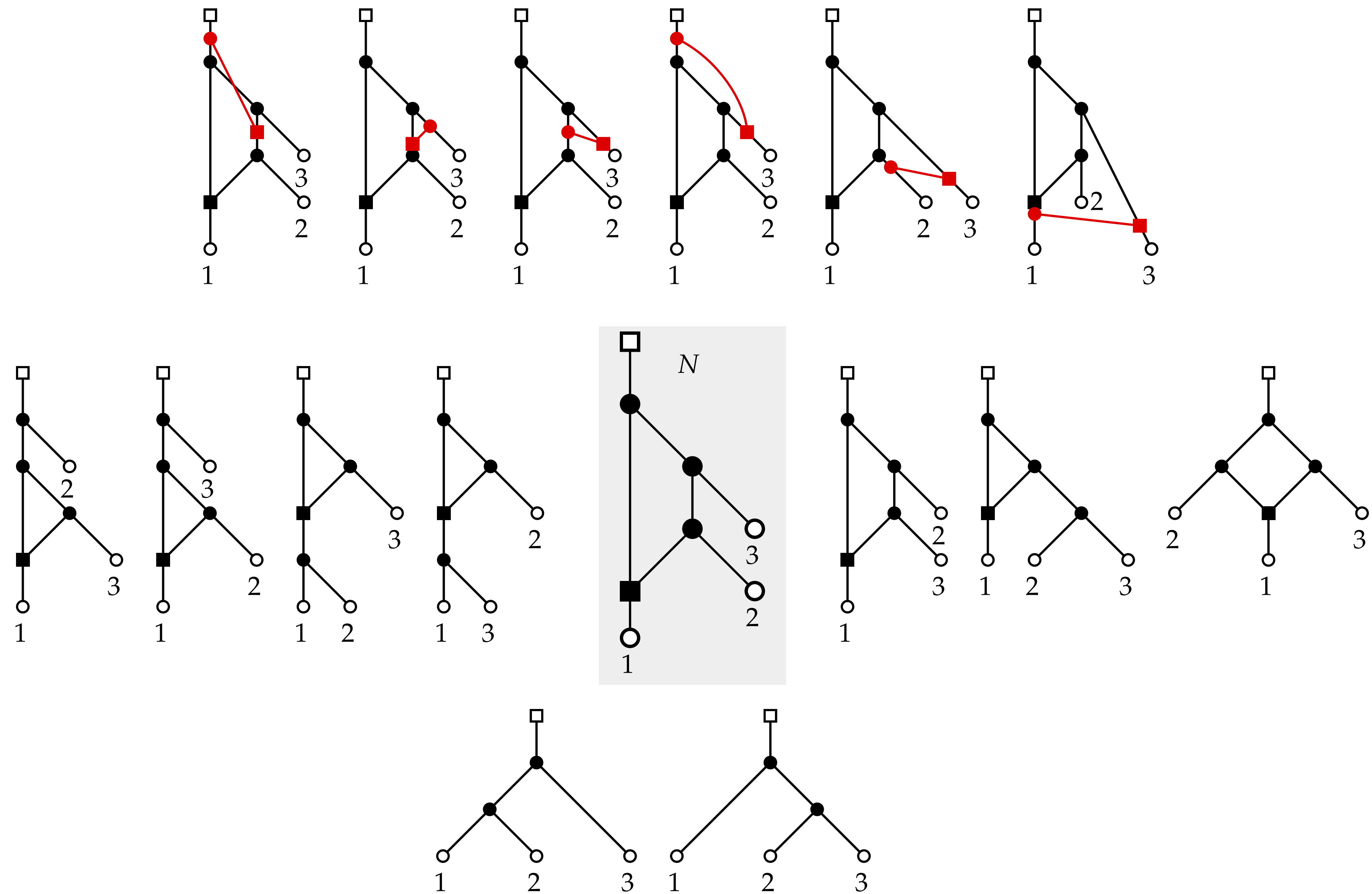




Connectedness & diameter

class	NNI	SNPR	PR
$\mathcal{T}_n$	$\Theta(n \log n)$ [LTZ96]	$\Theta(n)$ [Son03] (as SPR)	
$\mathcal{N}_{n,r}$	$\Omega(m \log m), \mathcal{O}(m^2)$ [JJE+18]	$\Theta(n+r)$ [JJE+18], L. 3.9	
$\mathcal{TC}_{n,r}$	$\mathcal{O}(n^2)$ T. 3.18	$r < n-1 : \Theta(n)$ [BLS17]	$\Theta(n)$ T. 3.17
		$r = n-1 : \times$ T. 3.16	
$\mathcal{NN}_{n,r}$		$r < n-2 : \Theta(n)$ T. 3.21	$\Theta(n)$ T. 3.23
		$r = n-2 : \times$ T. 3.22	
$\mathcal{TP}_{n,r}$		$r < n-2 : \Theta(n)$ T. 3.27	$\Theta(n)$ T. 3.28
		$r = n-2 : \times$ T. 3.26	
$\mathcal{TS}_{n,r}$		$\Theta(n+r)$ T. 3.31	
$^*\mathcal{RV}_{n,r}$		$\mathcal{O}(nr)$ [BLS17]	
$\mathcal{RV}_{n,r}$		$r \leq n-2$: $\mathcal{O}(n^2)$ T. 3.36	
	$r = 3n-3$: $\times$ L. 3.35		
$\mathcal{TB}_{n,r}(T)$	$\mathcal{O}(nr + r^2)$ T. 3.42	$\mathcal{O}(nr)$ [BLS17]	$\leq 2r$ L. 3.40
$\mathcal{TB}_{n,r}$	$\mathcal{O}(nr + r^2 + n \log n)$ C. 3.43	$\mathcal{O}(nr)$ [BLS17]	$\Theta(n+r)$ L. 3.41
$\mathcal{LV}_{k,n,r}$		$\mathcal{O}(n+r)$ T. 3.47	

Neighbourhood



Neighbourhood

- *Methods*
  - Count number of possible operations
  - Subtract number of trivial operations
  - Correct for double counting

Neighbourhood

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- *Results*

  Neighbourhood size for
  - trees under NNI and SNPR
  - tree-child networks under NNI and SNPR
  - normal networks under SNPR
  - general networks (bounds)

PR-distance

Theorem. Computing the SPR-distance of two trees is NP-hard.

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$$\mathrm{d}_{\mathrm{PR}}(T, N) = \min_{T' \in D(N)} \mathrm{d}_{\mathrm{PR}}(T, T') + r$$

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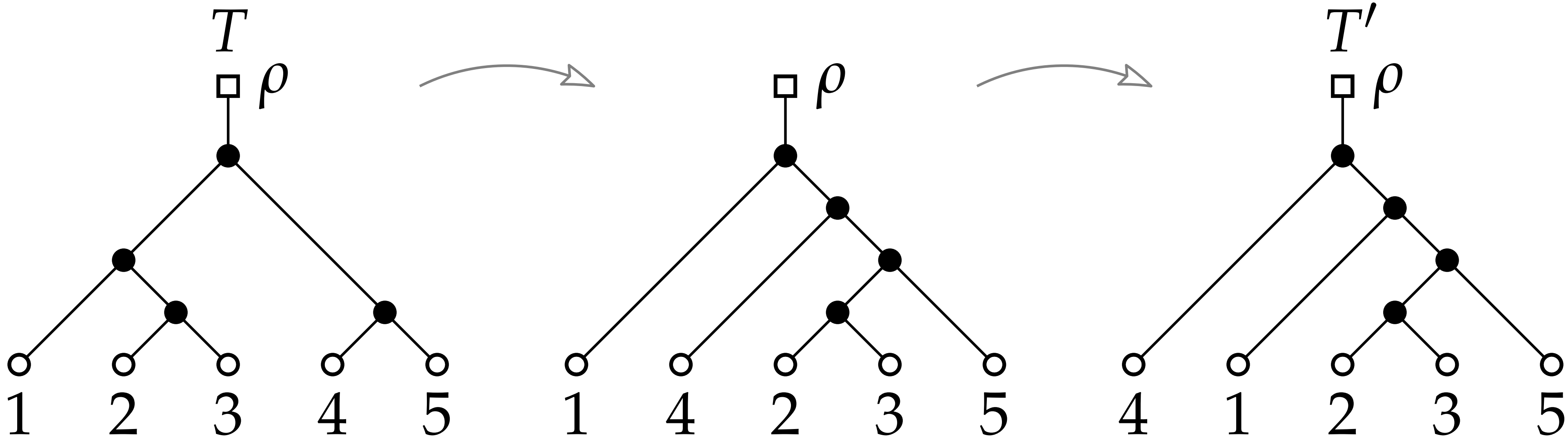
Corollary. Computing $\mathrm{d}_{\mathrm{PR}}(T, N)$ is fixed-parameter tractable.

Maximum agreement forests



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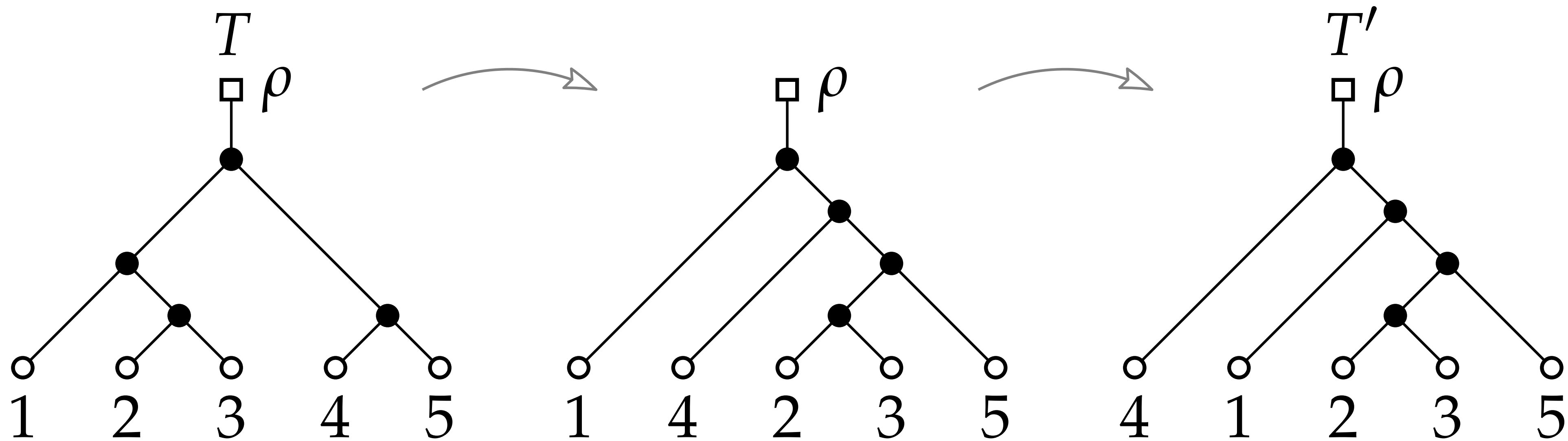

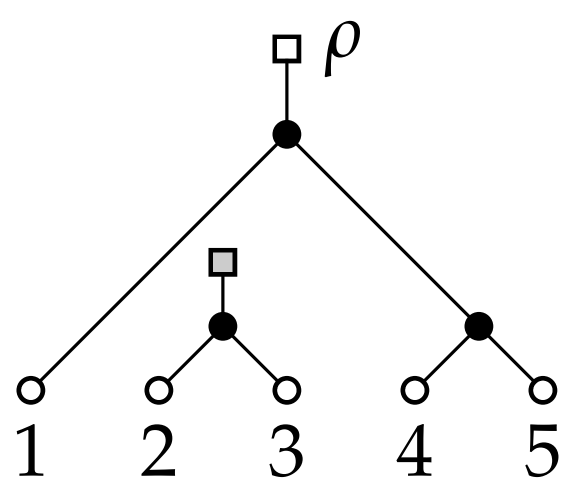

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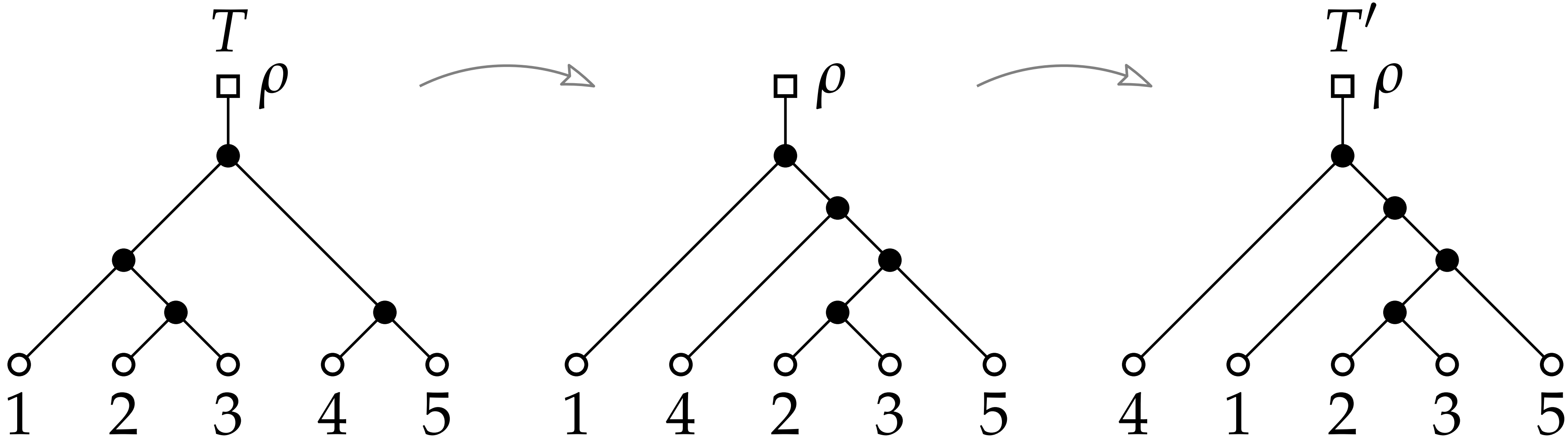

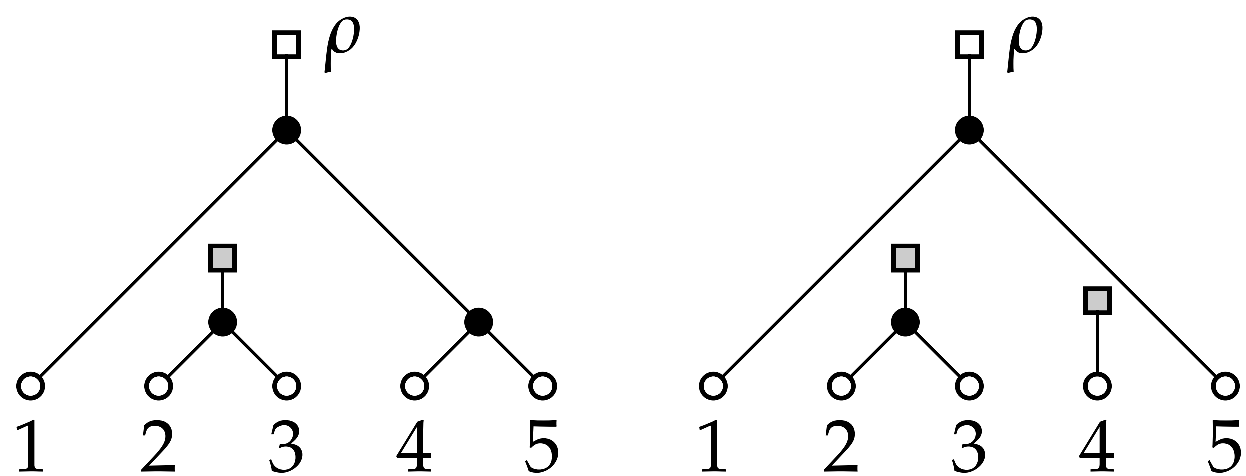

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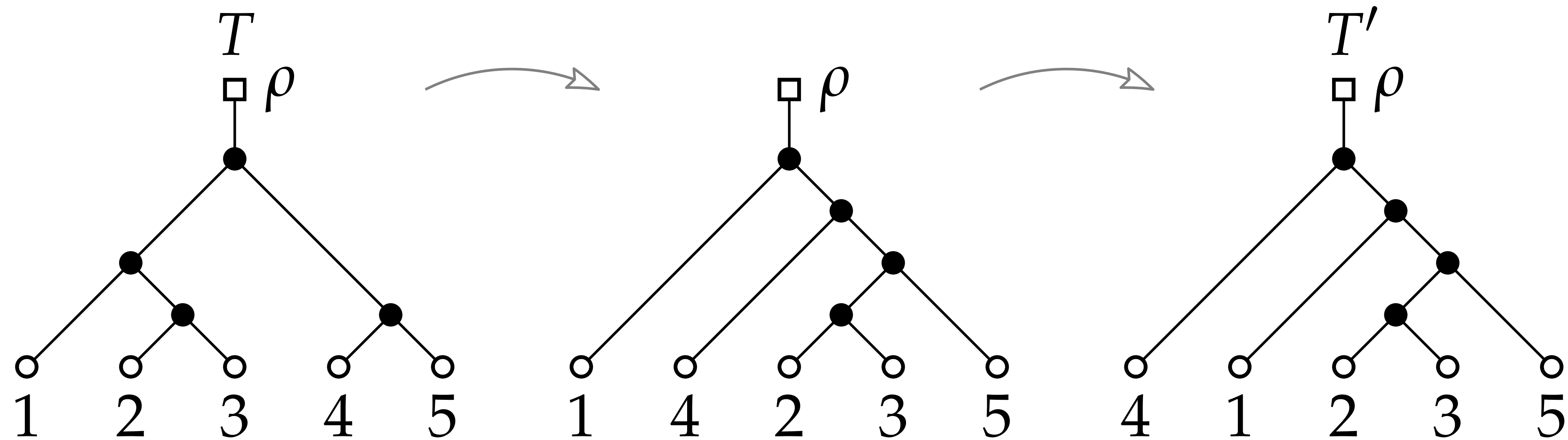

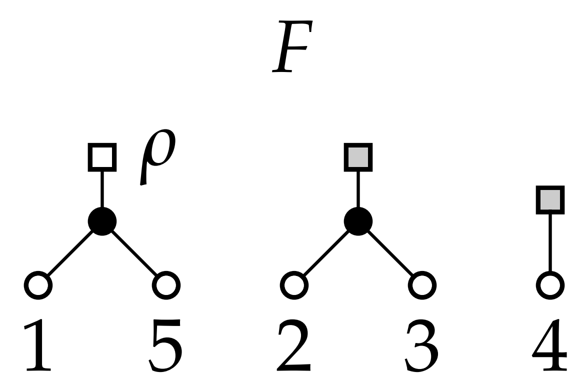

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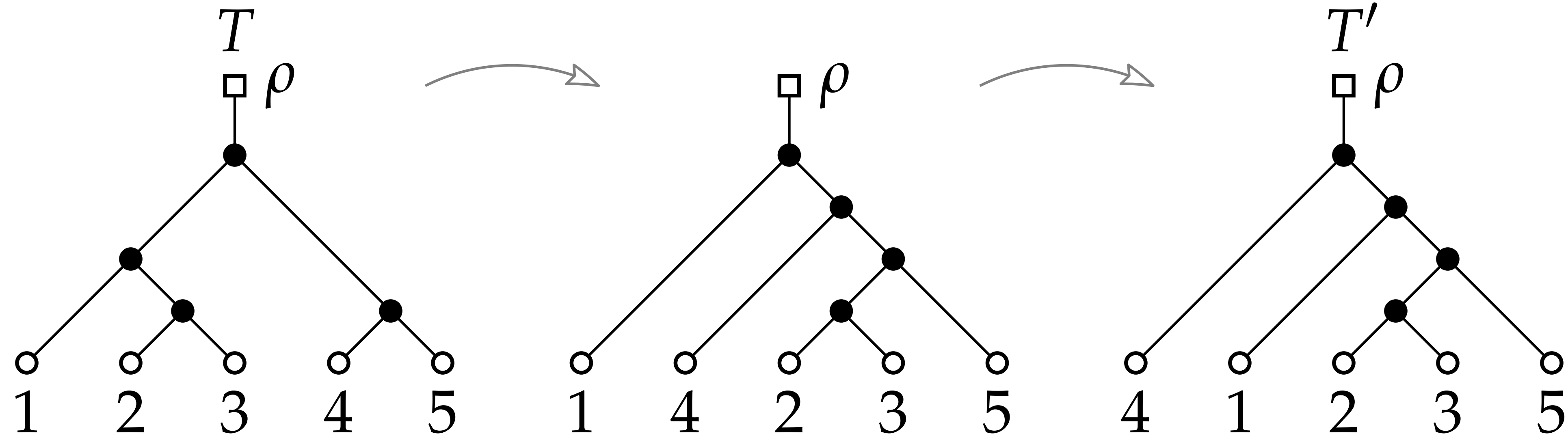

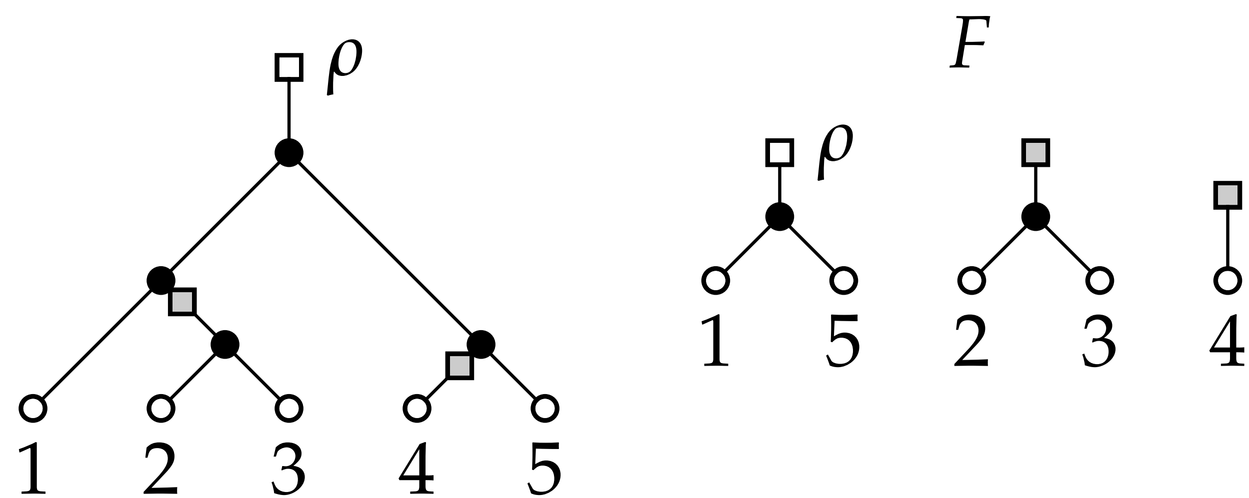

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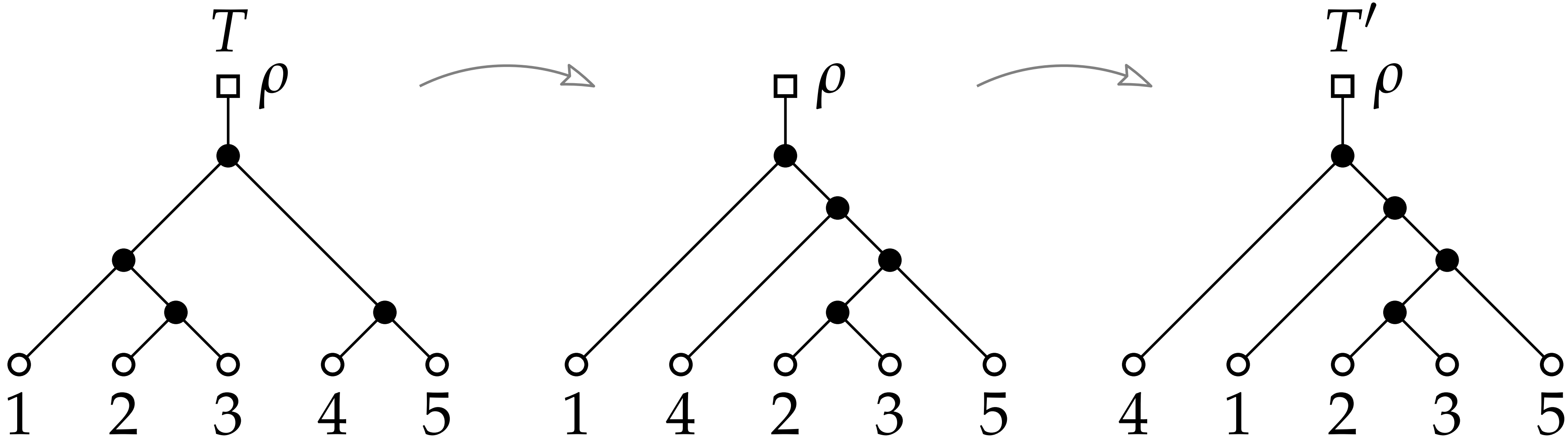

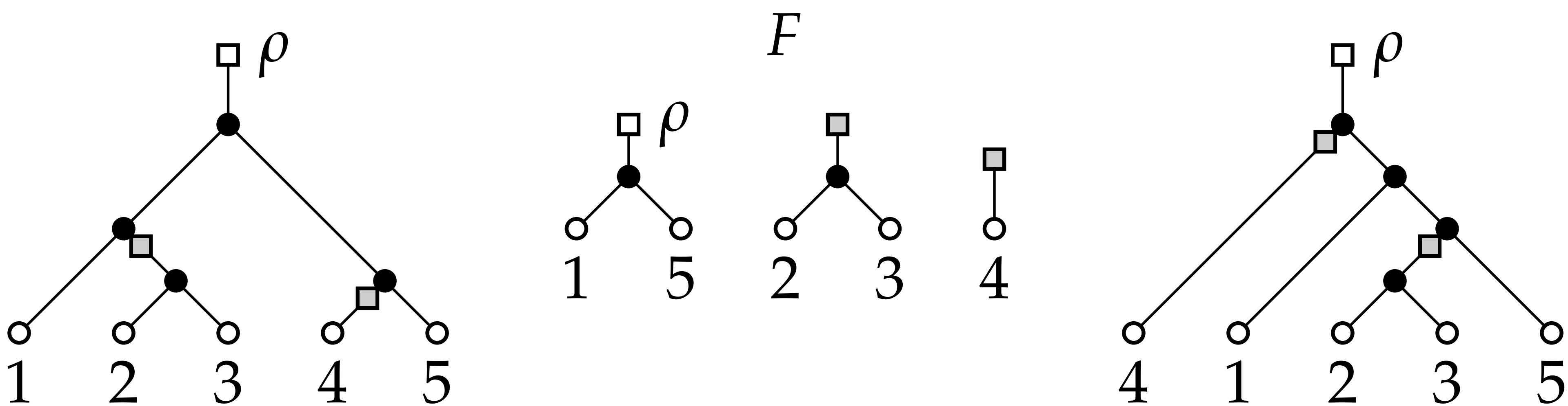

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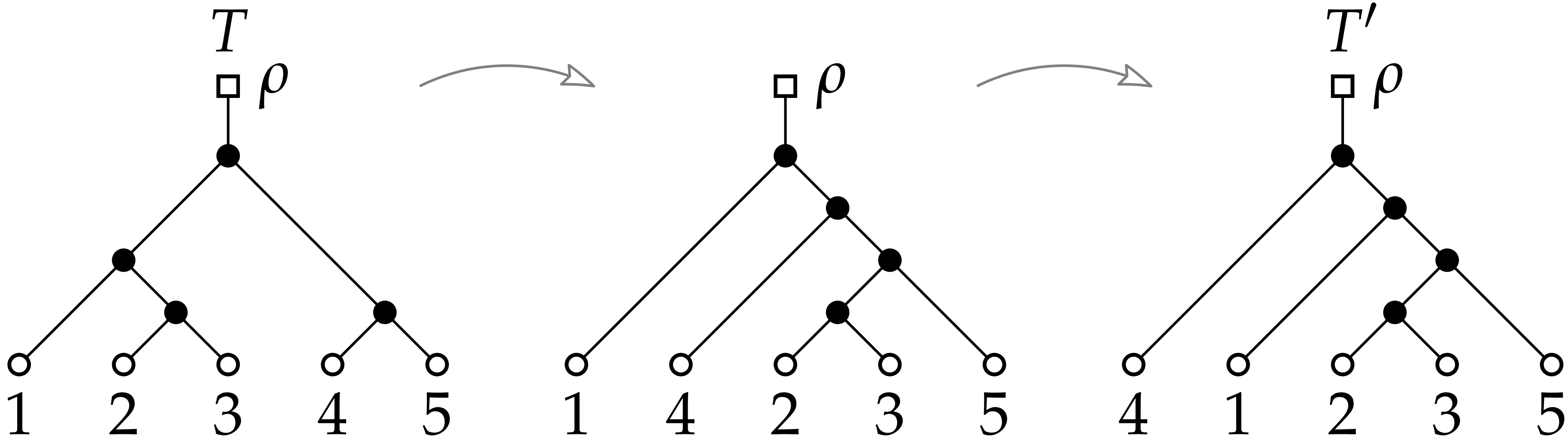
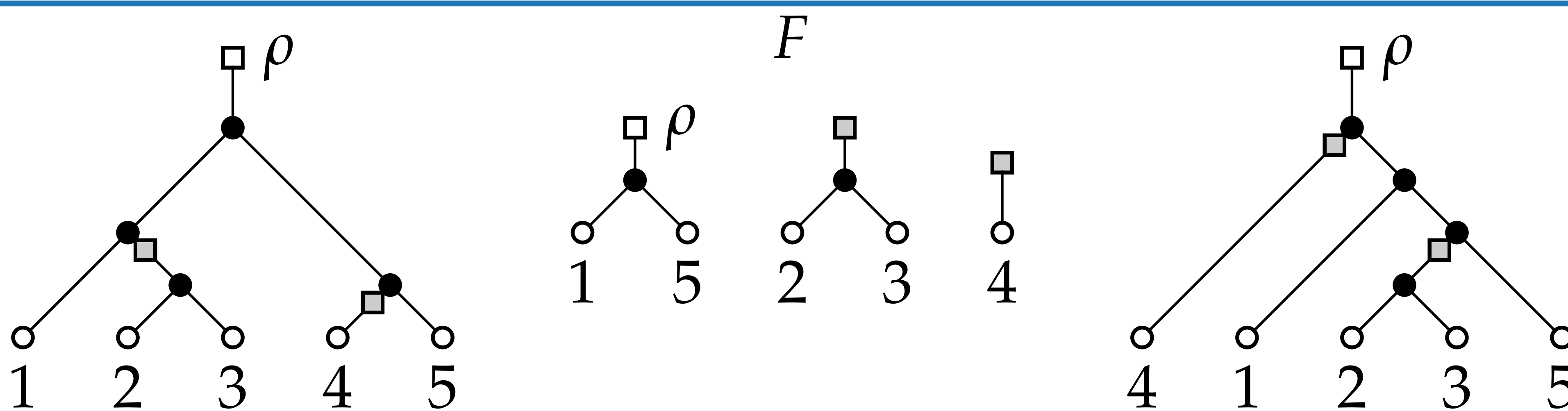
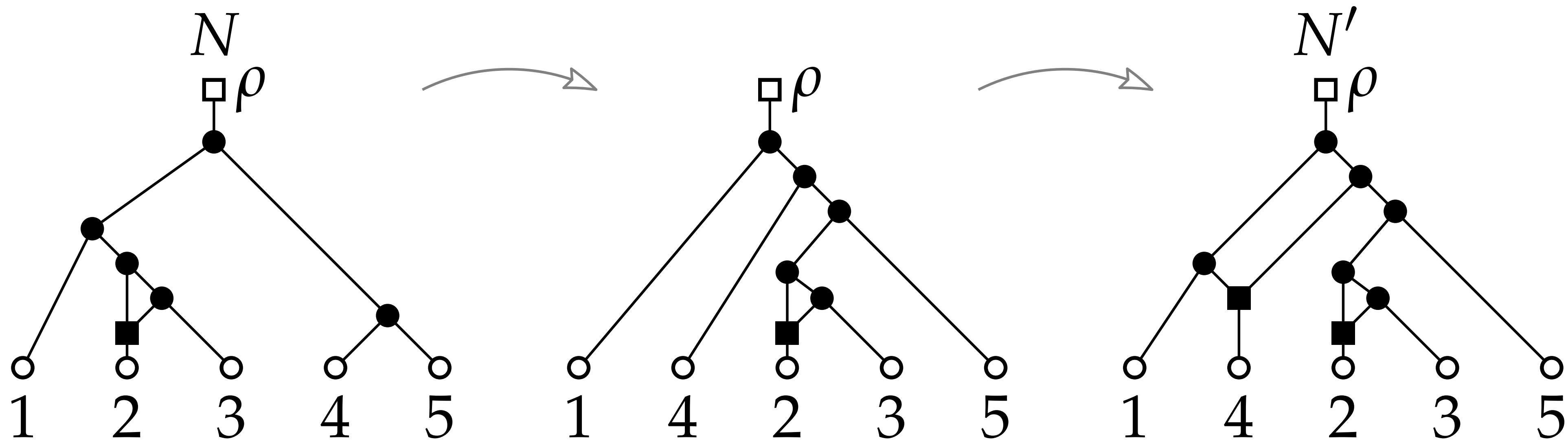


Theorem. The size of a maximum agreement forest for two trees characterises their SPR-distance.



Maximum agreement graphs



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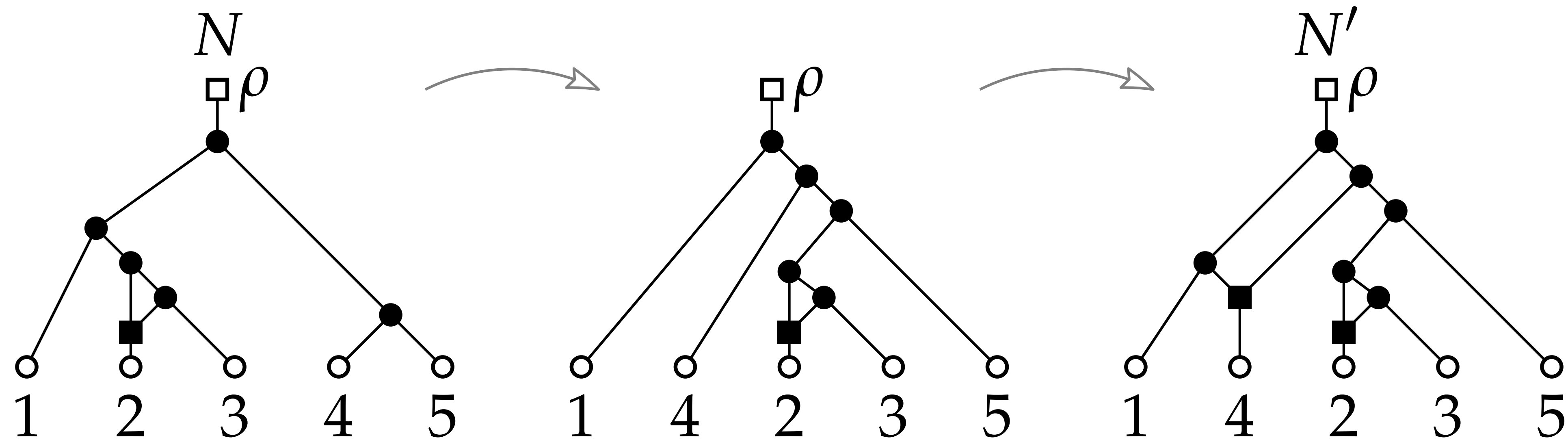

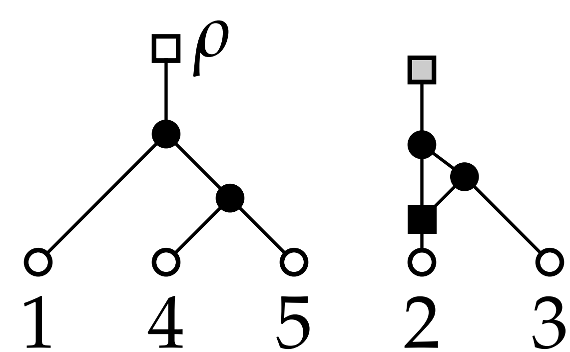

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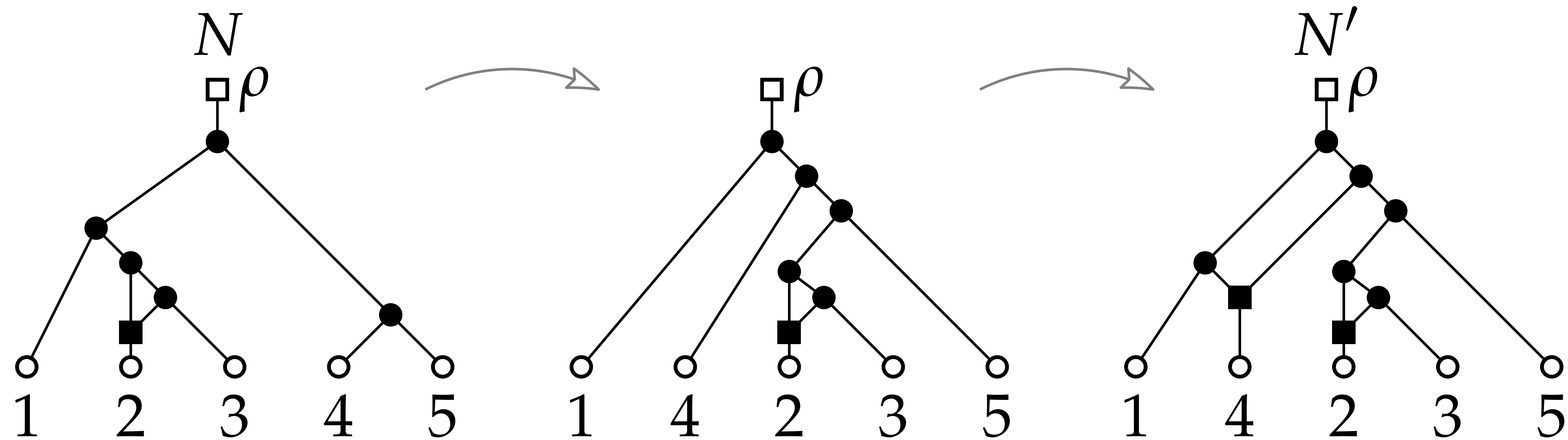

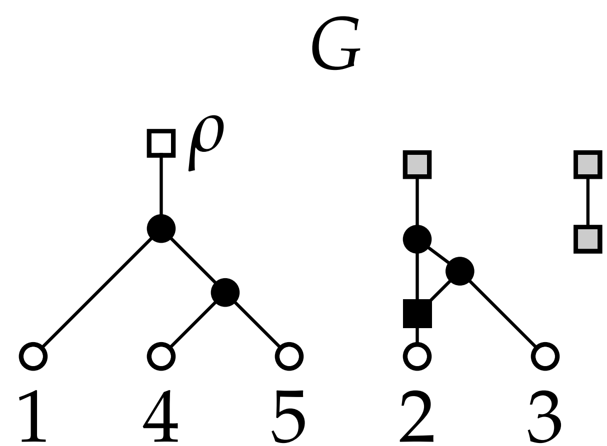

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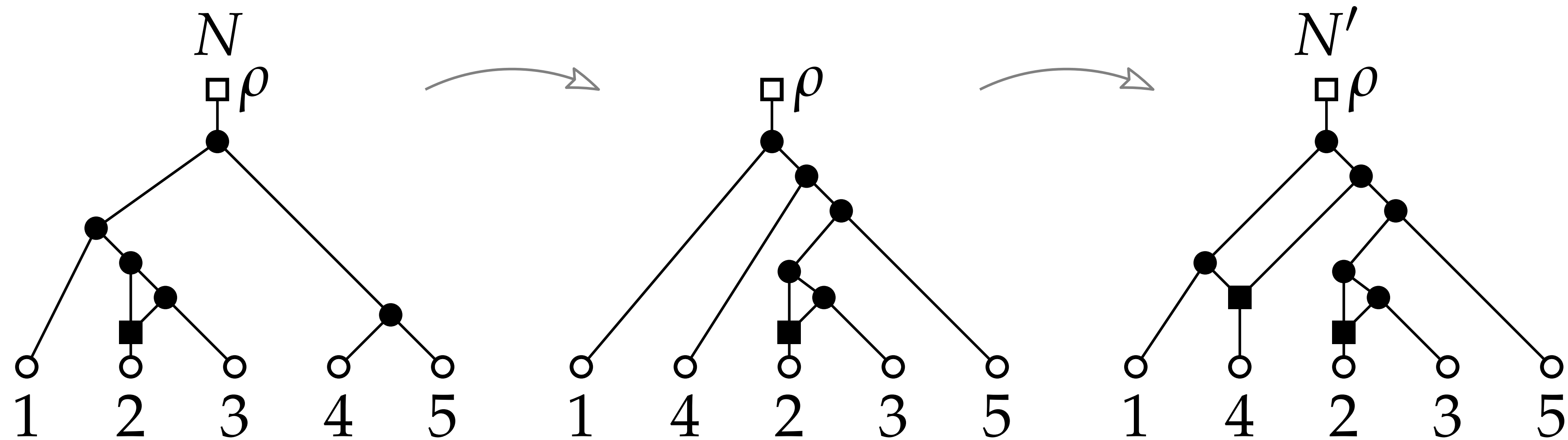

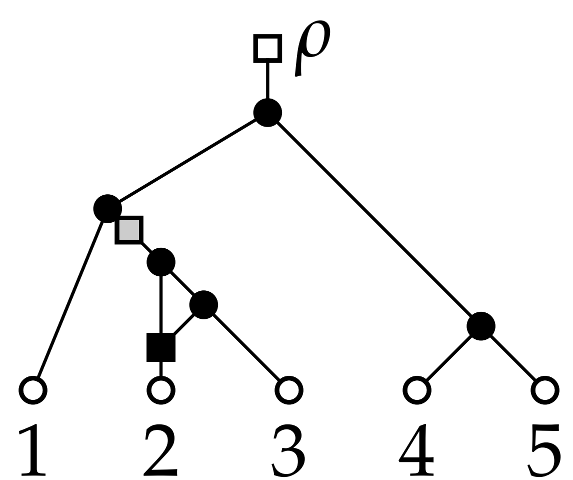

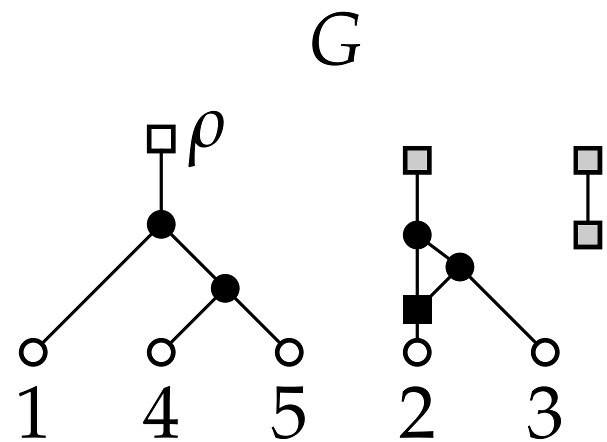

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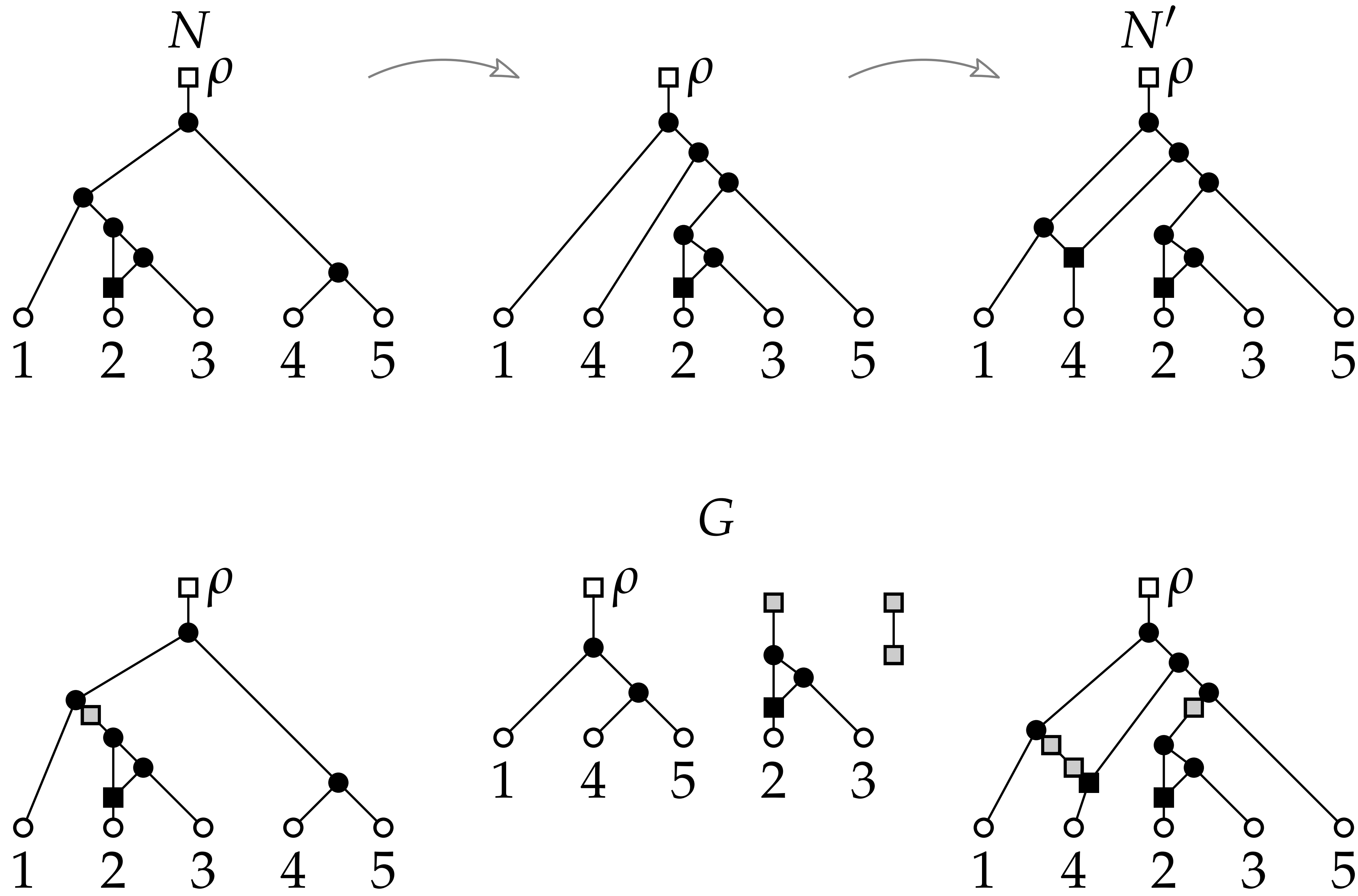

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Theorem. $\mathrm{d}_{\mathrm{AD}}(N, N') \leq \mathrm{d}_{\mathrm{PR}}(N, N') \leq 3\,\mathrm{d}_{\mathrm{AD}}(N, N')$

$$\mathrm{d}_{\mathrm{AD}}(N, N') \leq \mathrm{d}_{\mathrm{SNPR}}(N, N') \leq 6\,\mathrm{d}_{\mathrm{AD}}(N, N')$$

Publications

- K., “The SNPR-Neighbourhood of tree-child networks”, Journal of Graph Algorithms & Applications, 2018.
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- Janssen, K., “Rearrangement operations on unrooted phylogenetic networks ”, Theory and Applications of Graphs, 2019.
- K., “The agreement distance of unrooted phylogenetic networks ”, Discrete Mathematics & Theoretical Computer Science, 2020 (to appear).