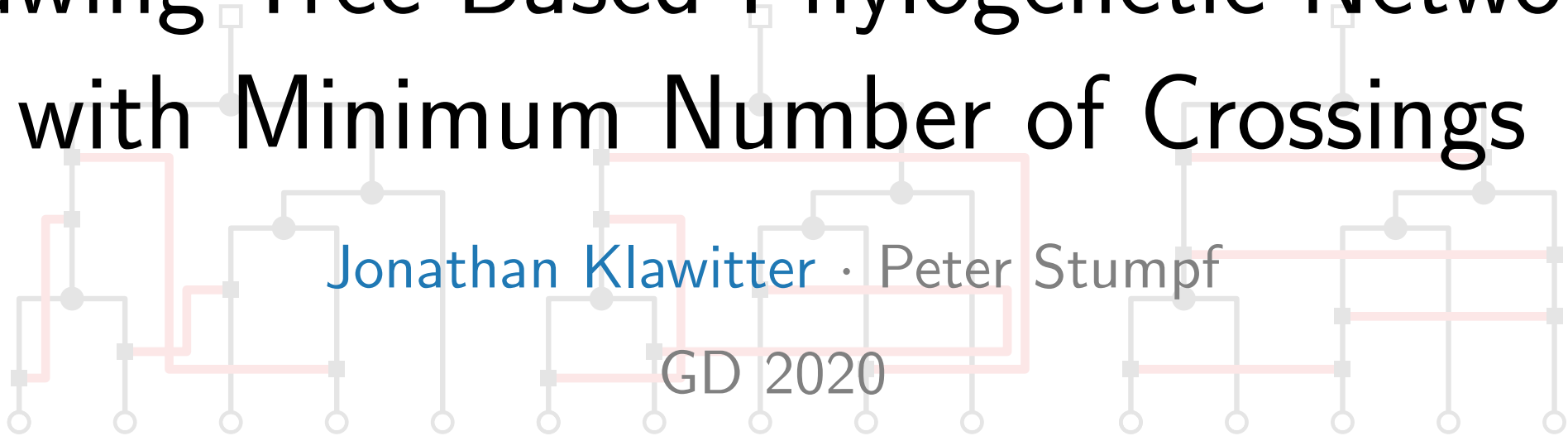


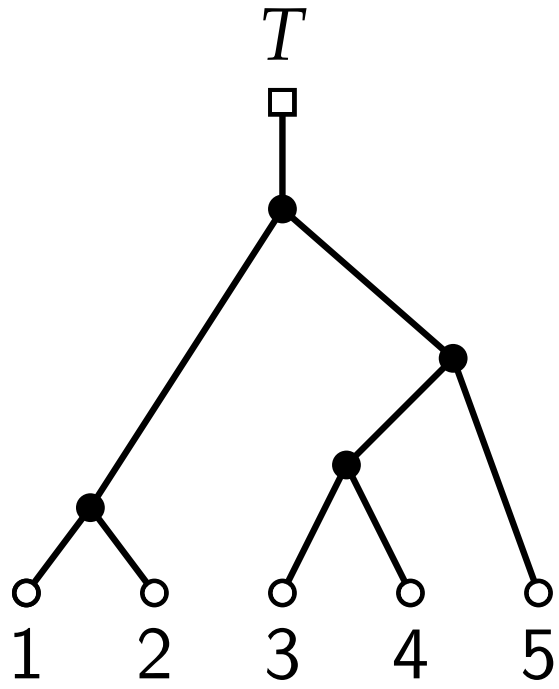
Drawing Tree-Based Phylogenetic Networks with Minimum Number of Crossings

Jonathan Klawitter · Peter Stumpf

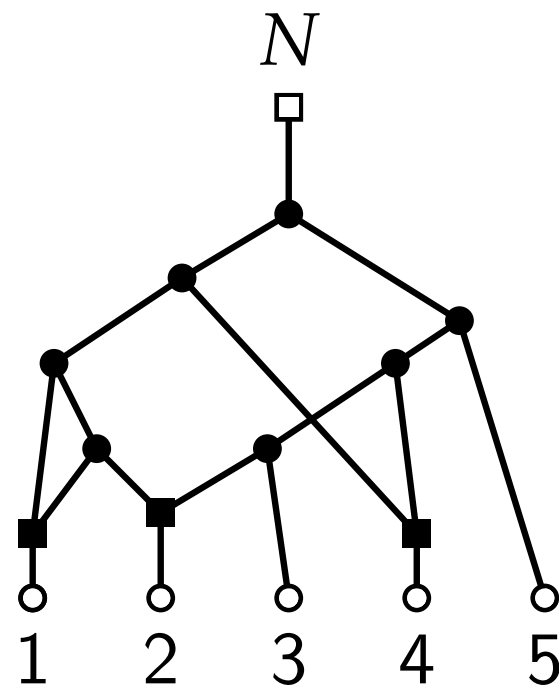
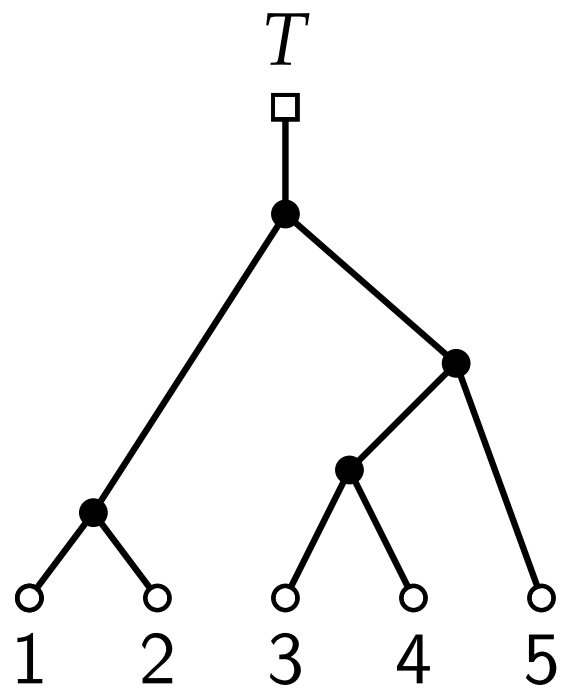
GD 2020



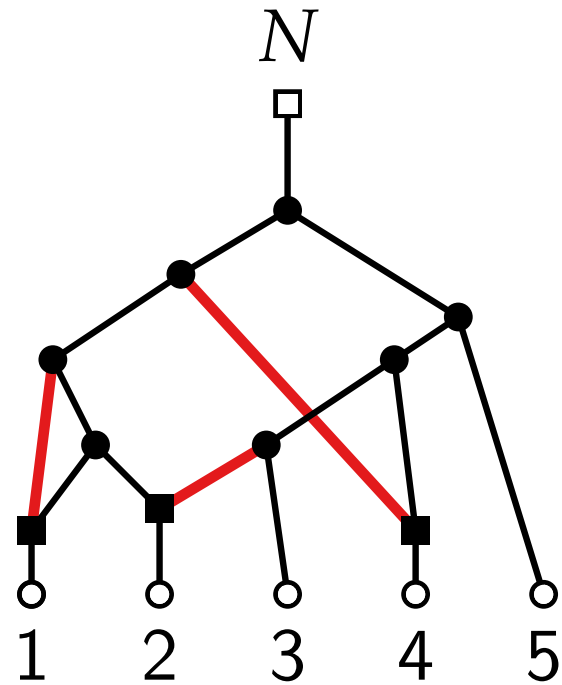
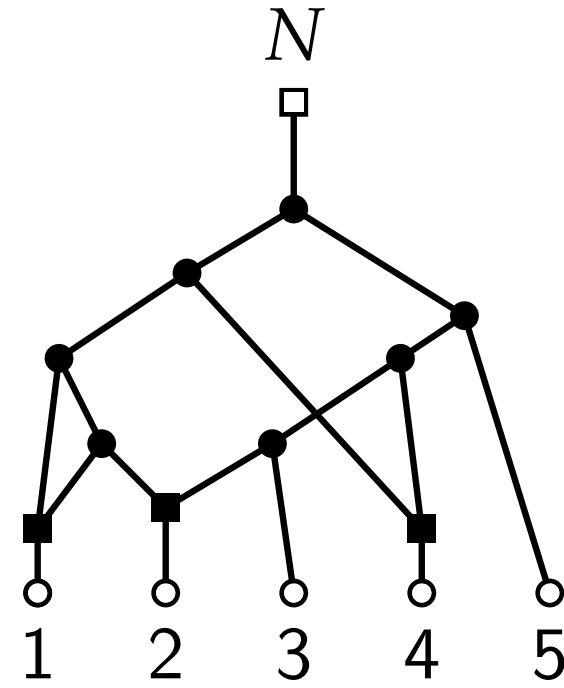
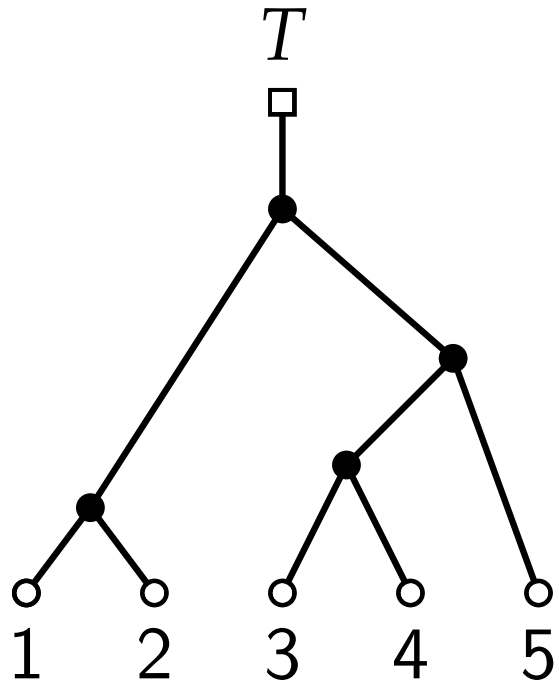
Phylogenetic networks



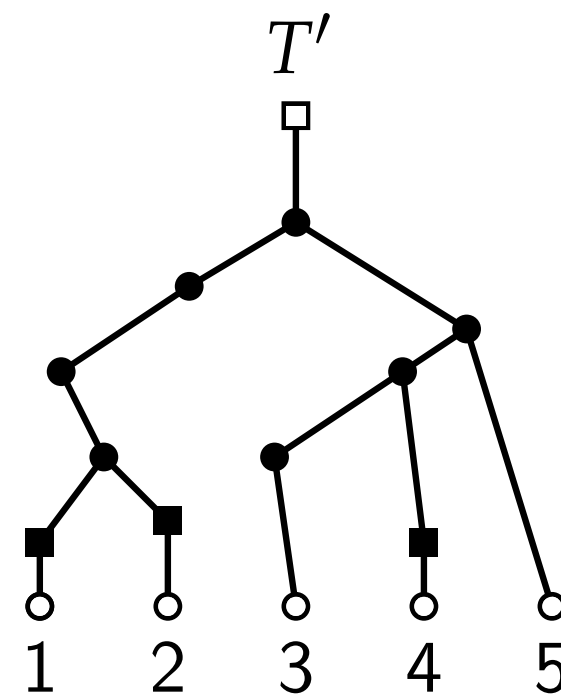
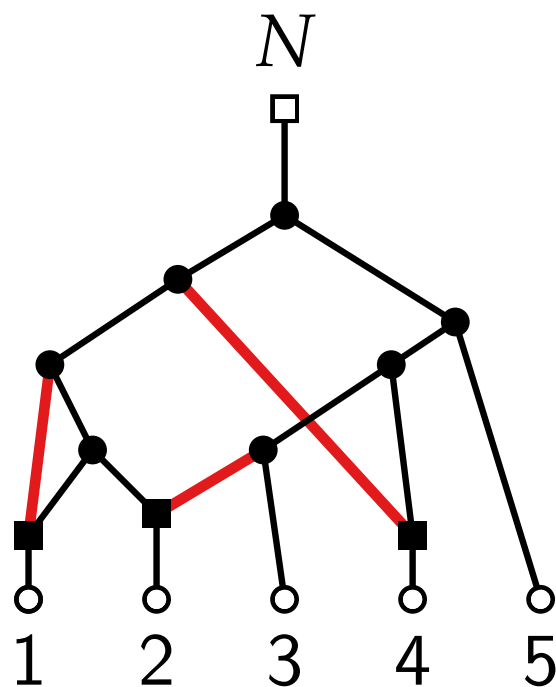
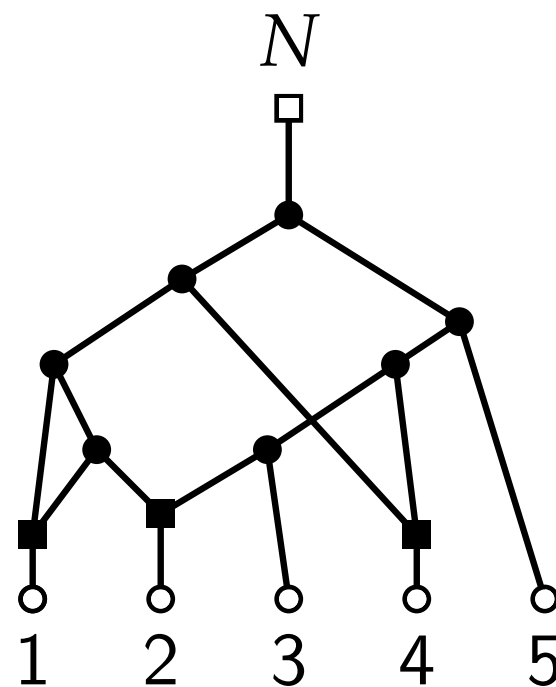
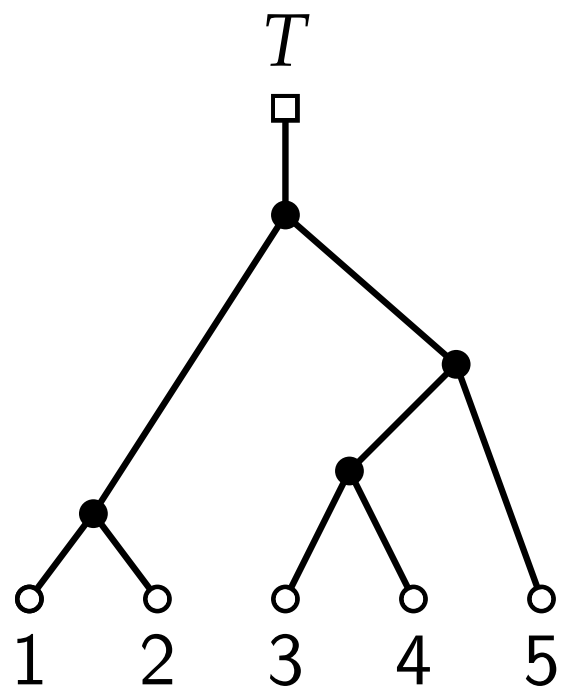
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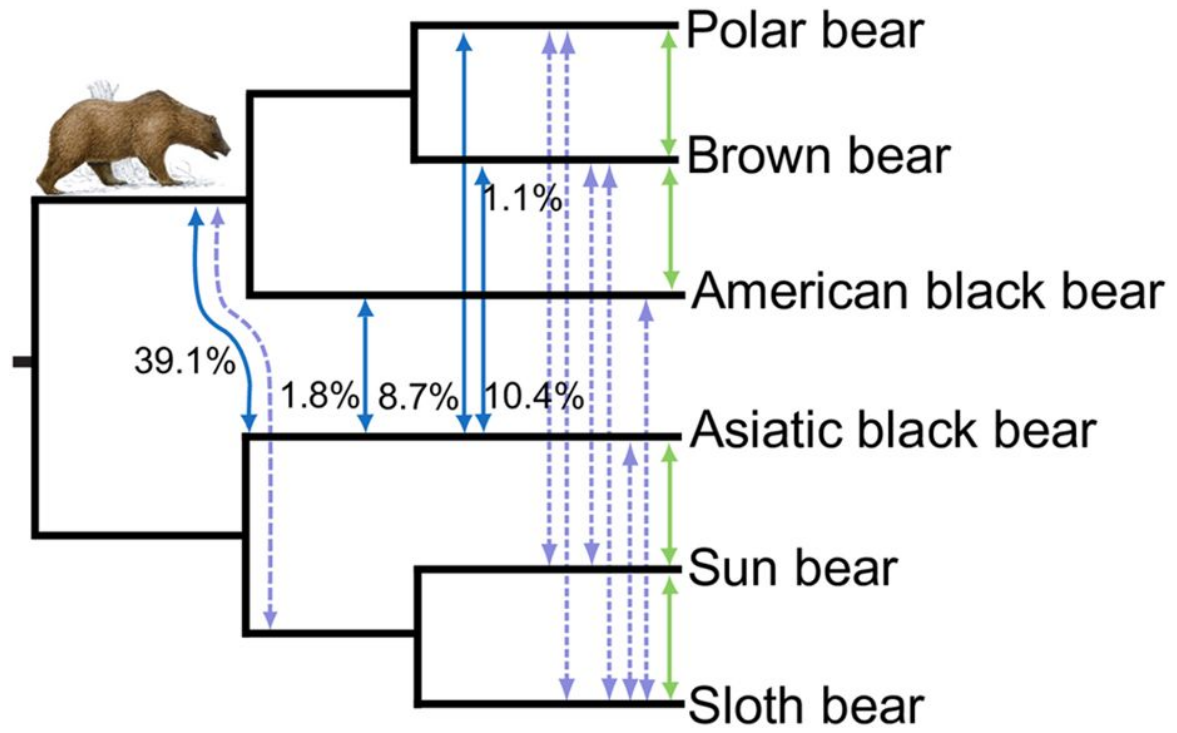


Phylogenetic networks

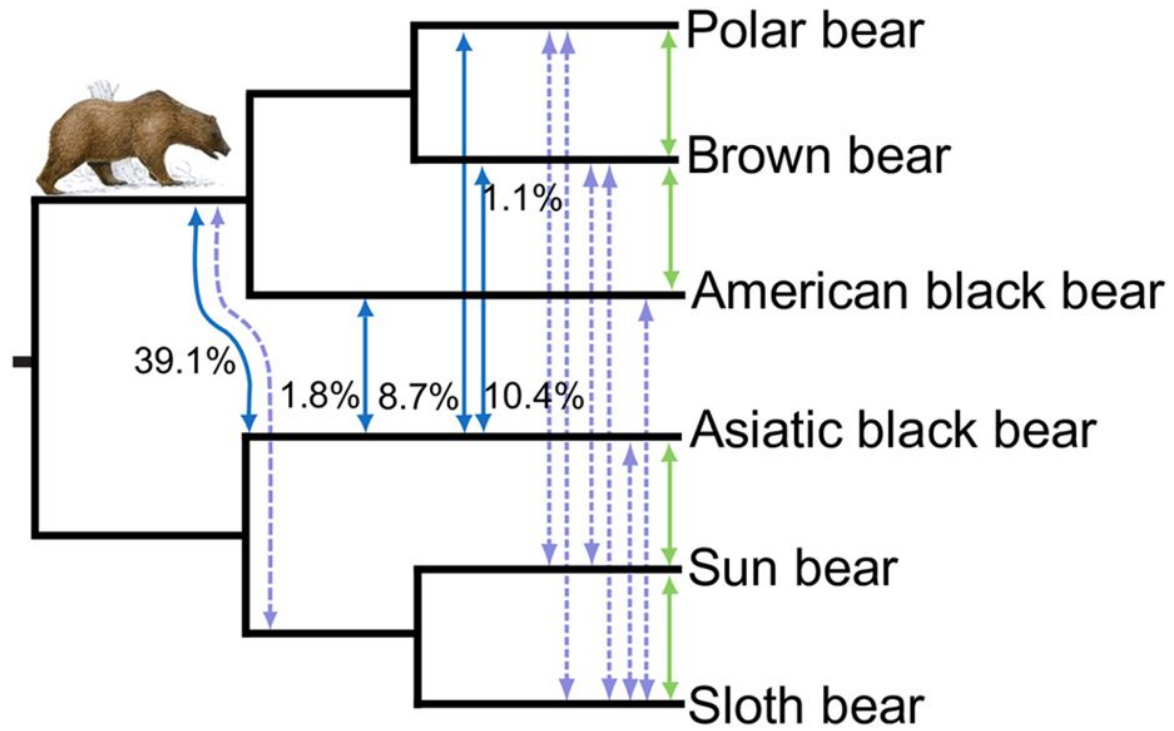


Phylogenetic networks

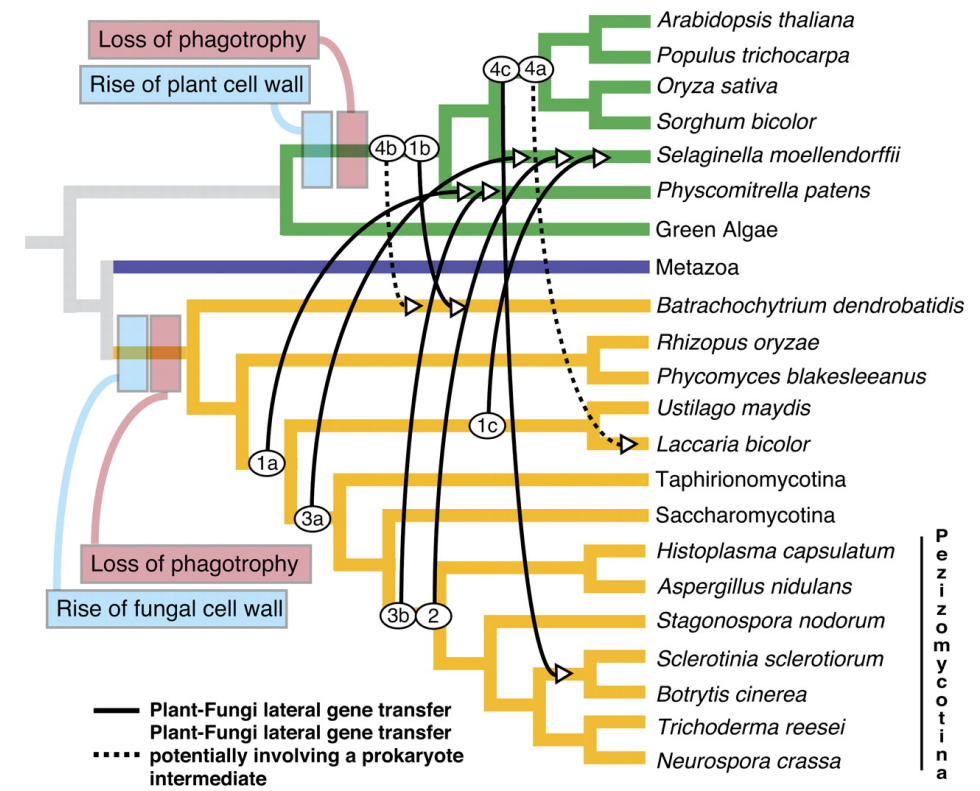




Kumar et al. "The evolutionary history of bears is characterized by gene flow across species"

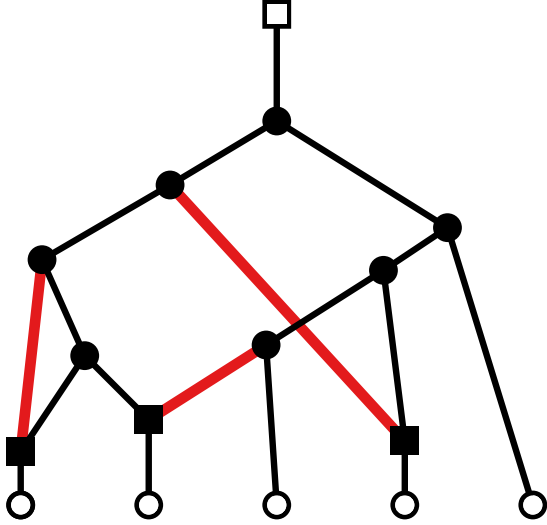


Kumar et al. "The evolutionary history of bears is characterized by gene flow across species"

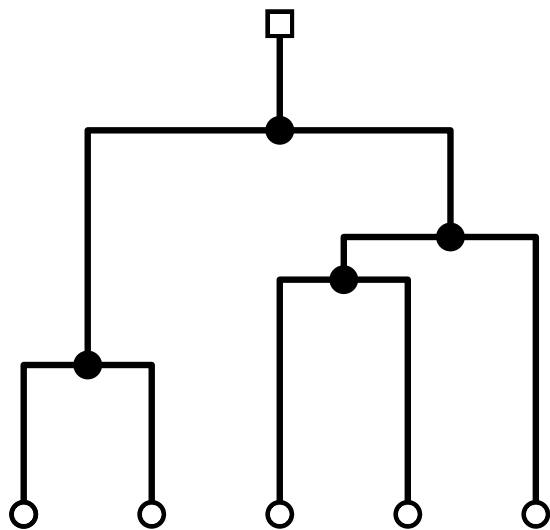
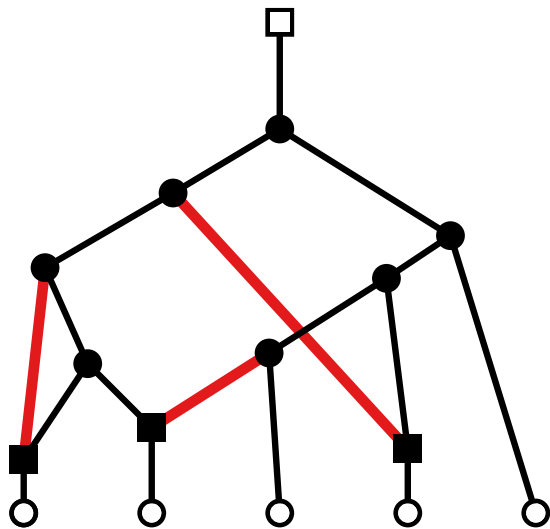


Richards et al. "Phylogenomic Analysis Demonstrates a Pattern of Rare and Ancient Horizontal Gene Transfer between Plants and Fungi"

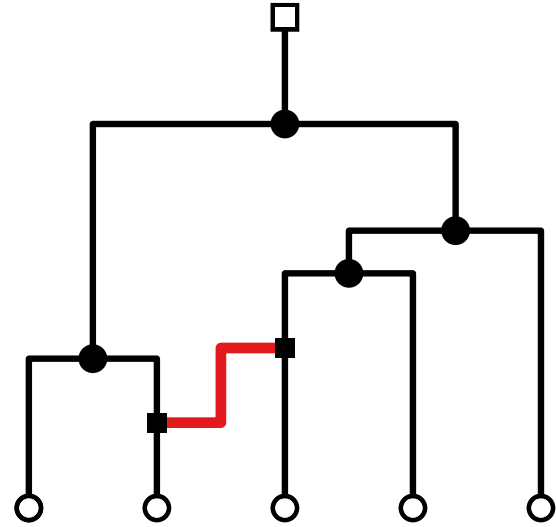
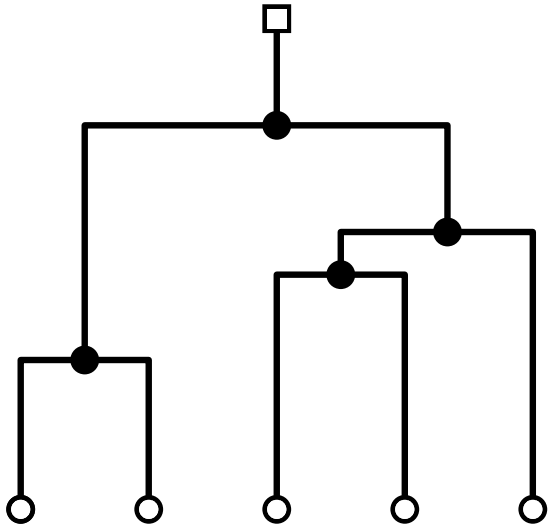
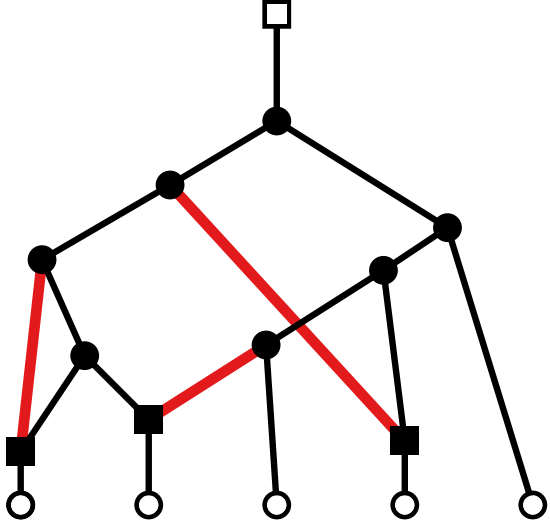
Drawing styles



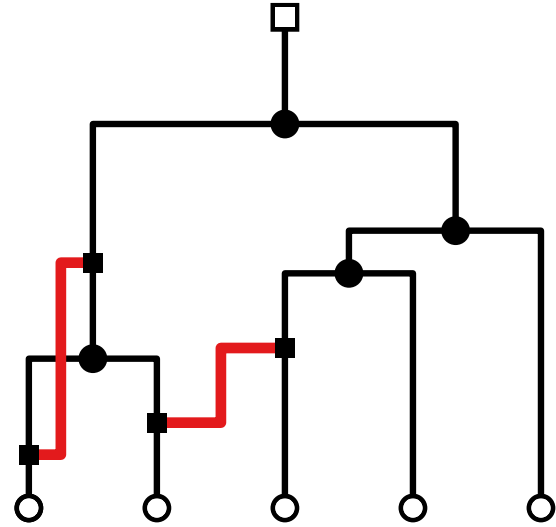
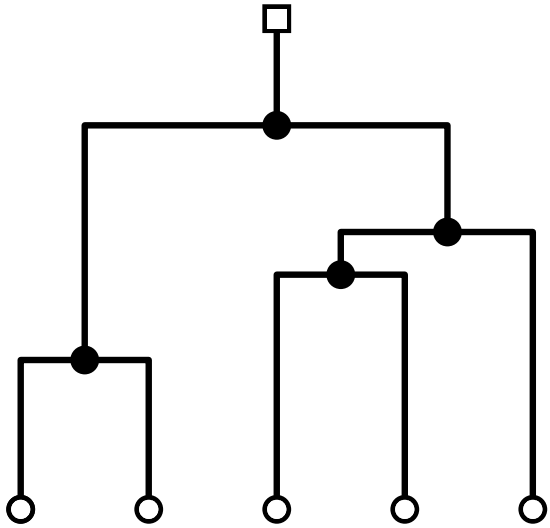
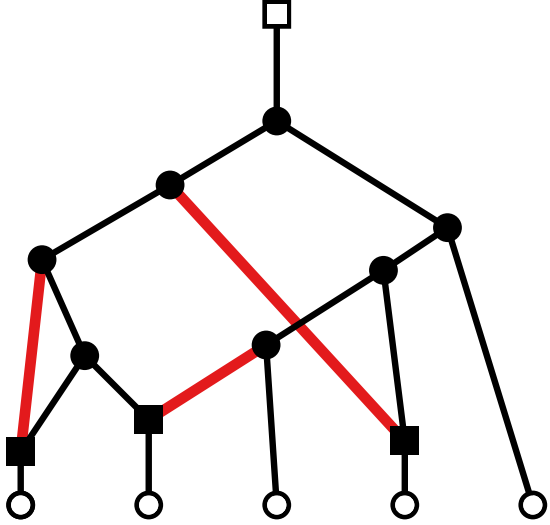
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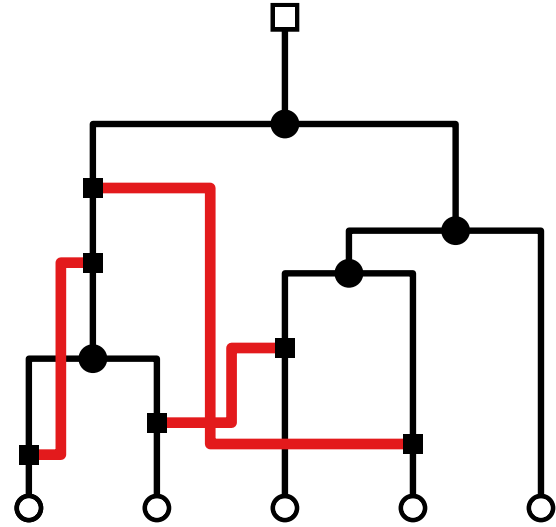
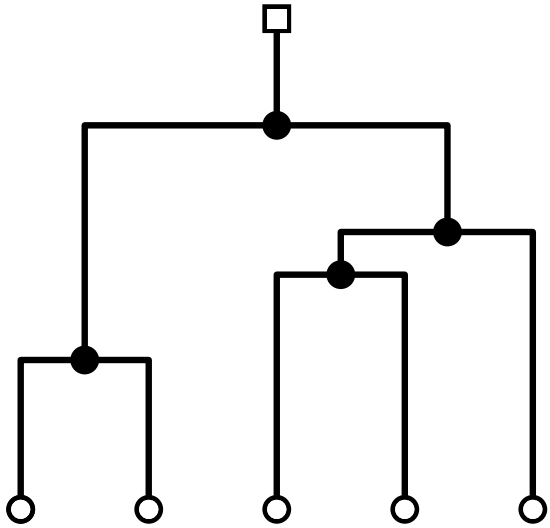
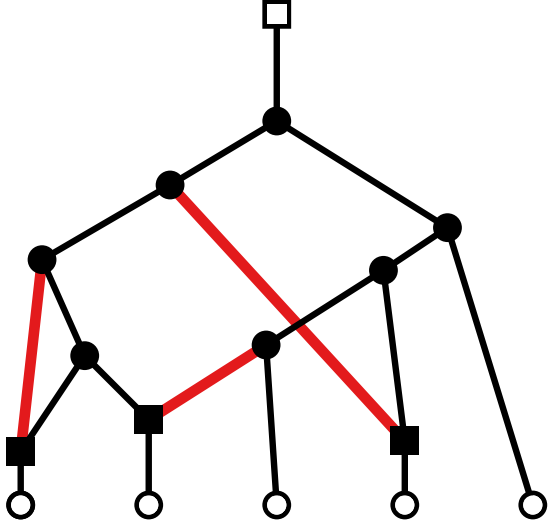
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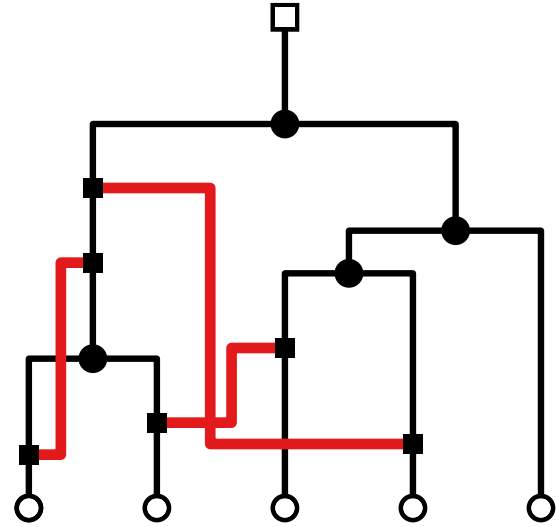
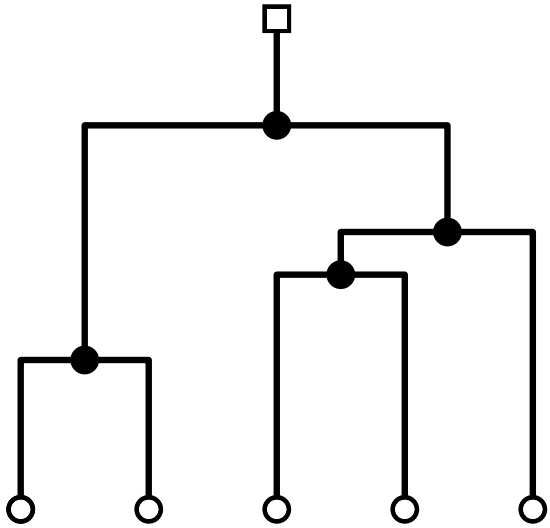
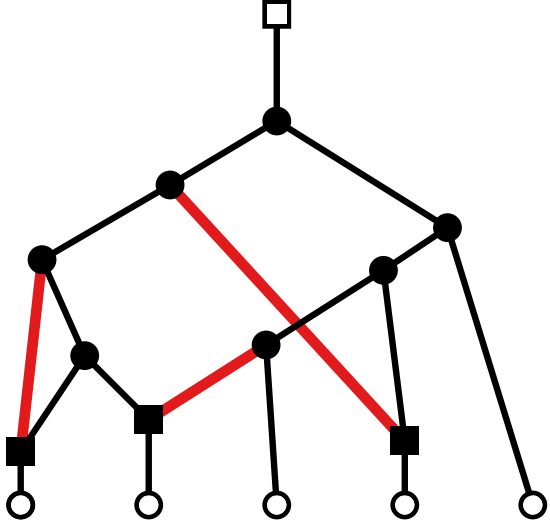
Drawing styles



Drawing styles

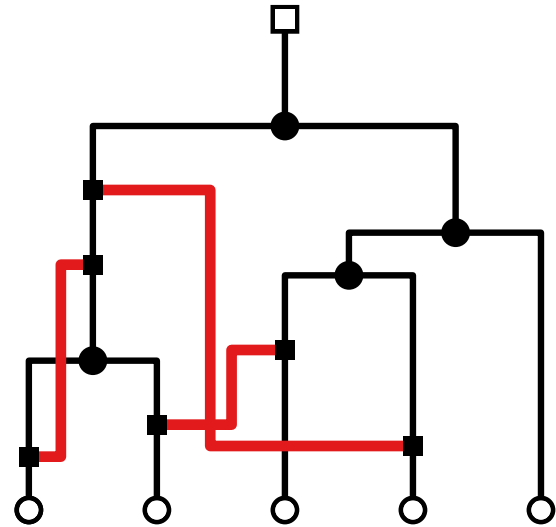
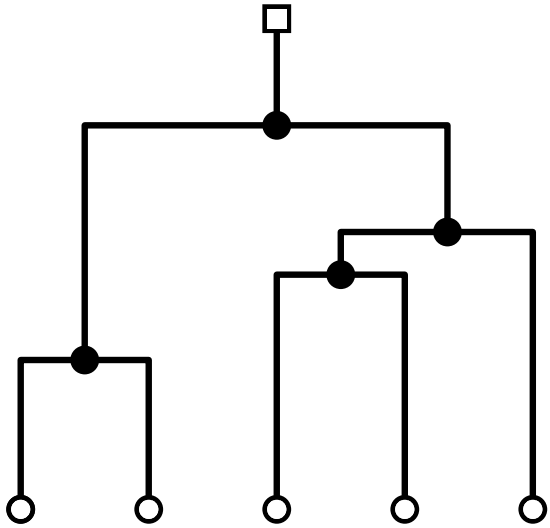
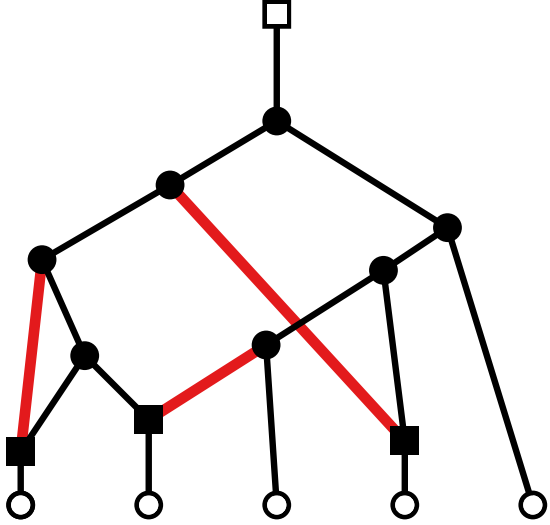


Drawing styles

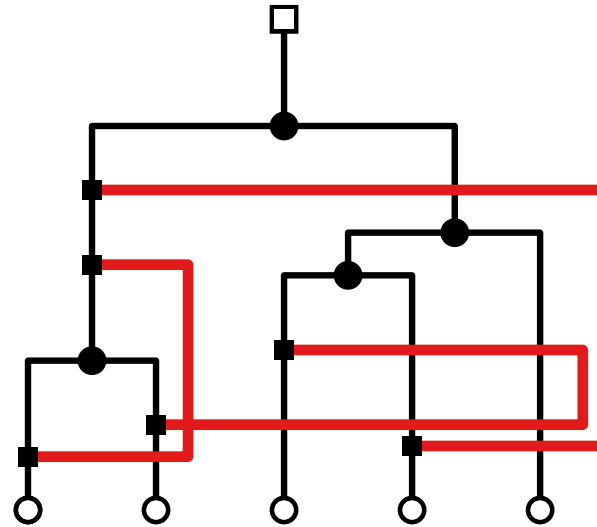


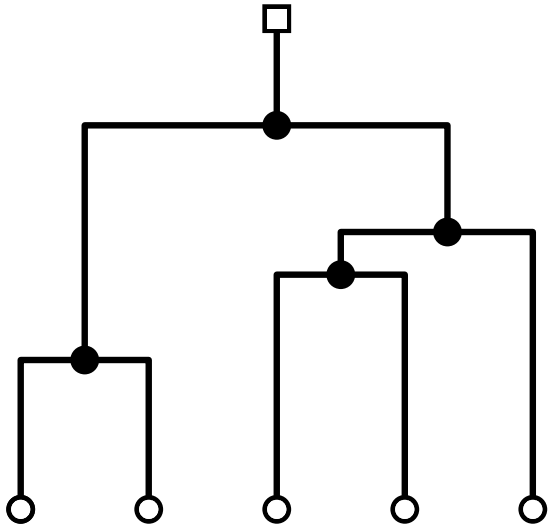
snake style

Drawing styles

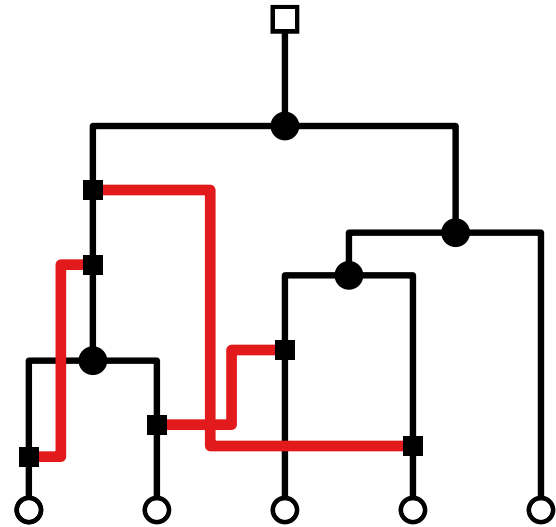
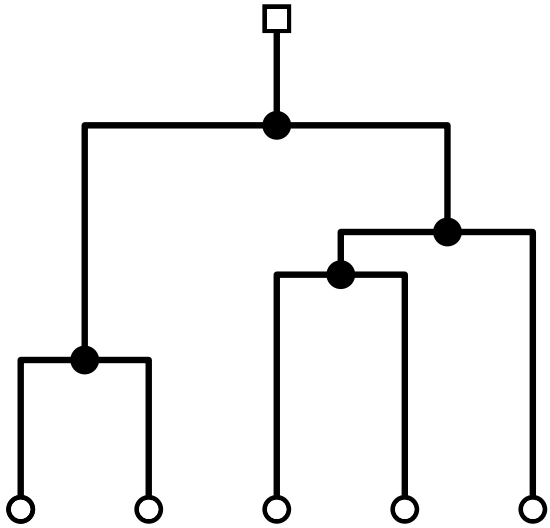
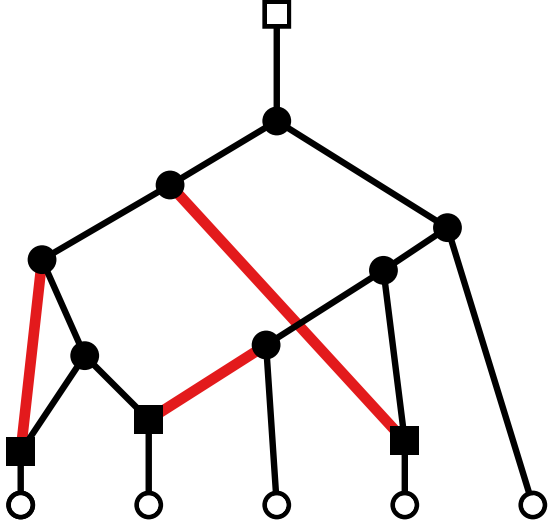


snake style

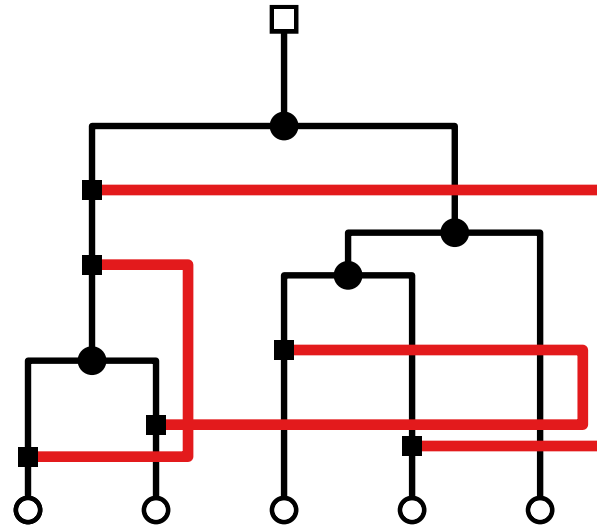




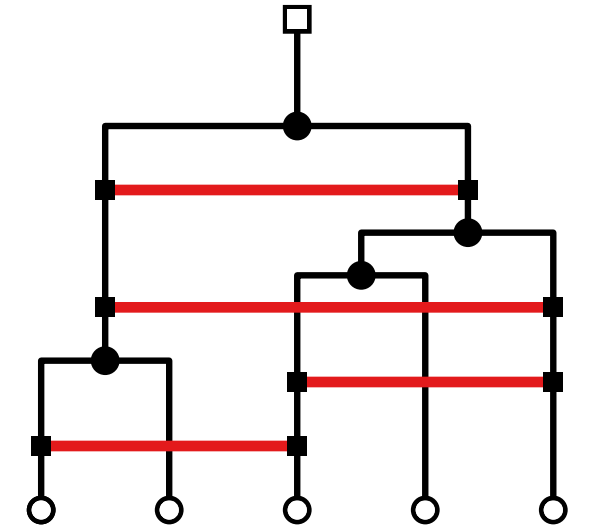
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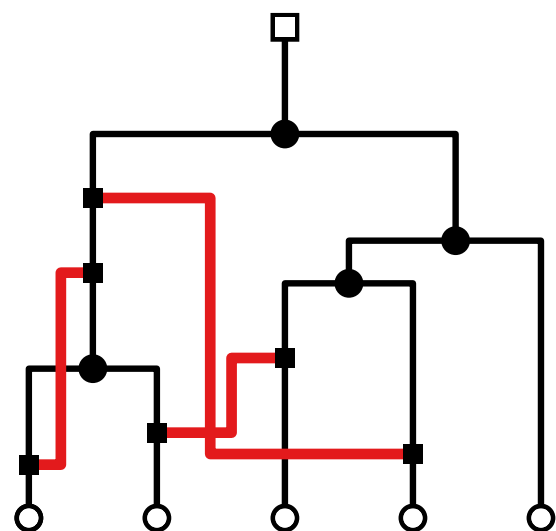
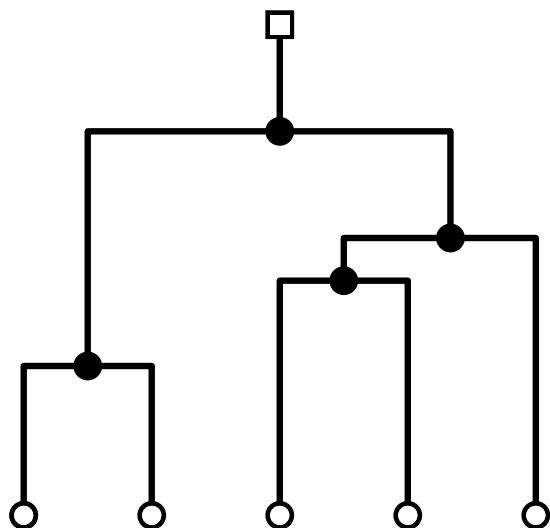
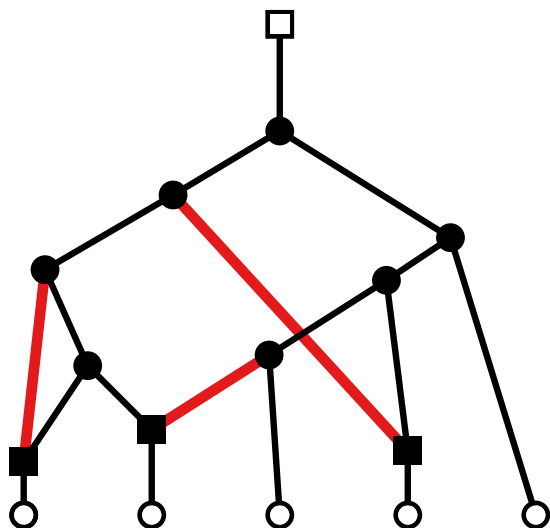
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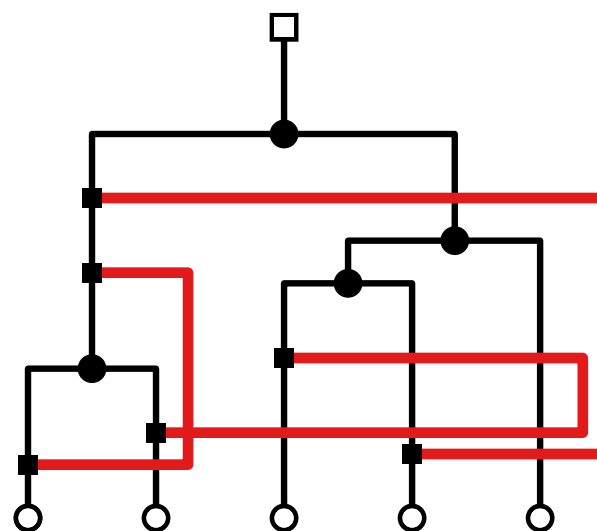
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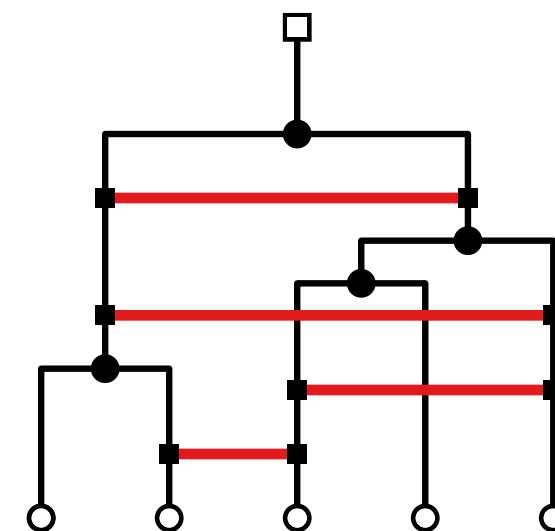
horizontal style



snake style

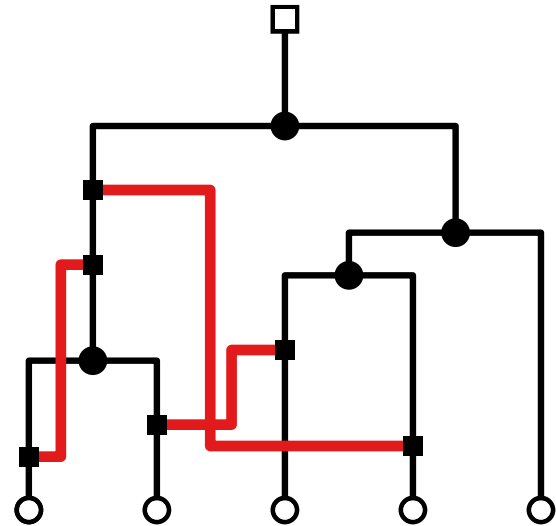
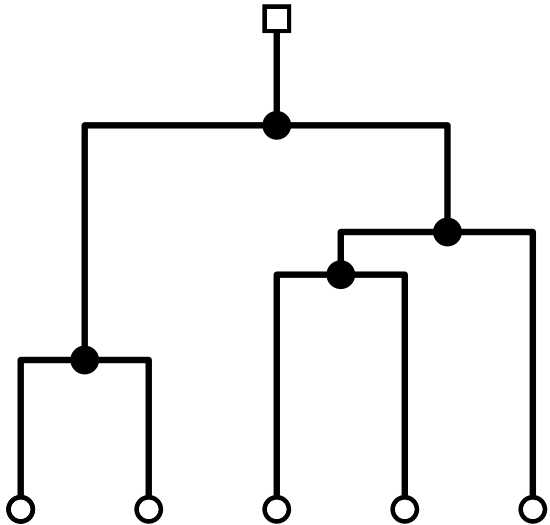
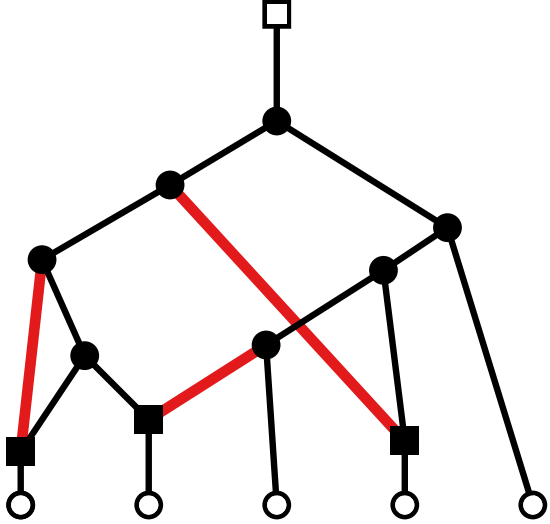


ear style

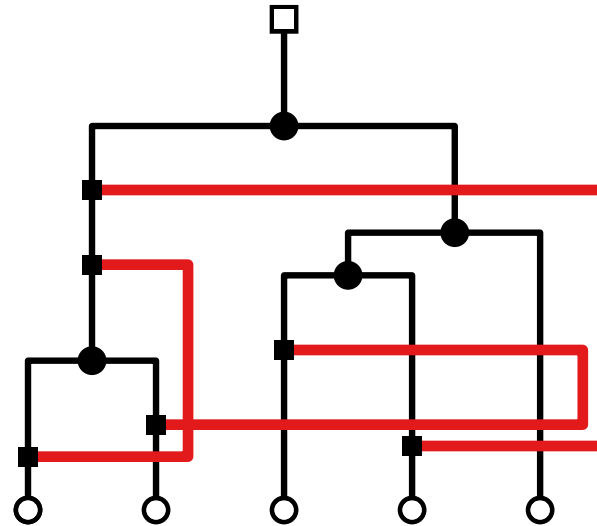


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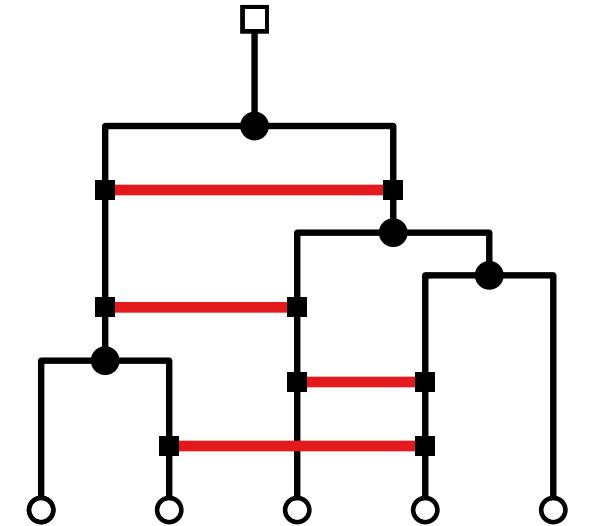
Drawing styles



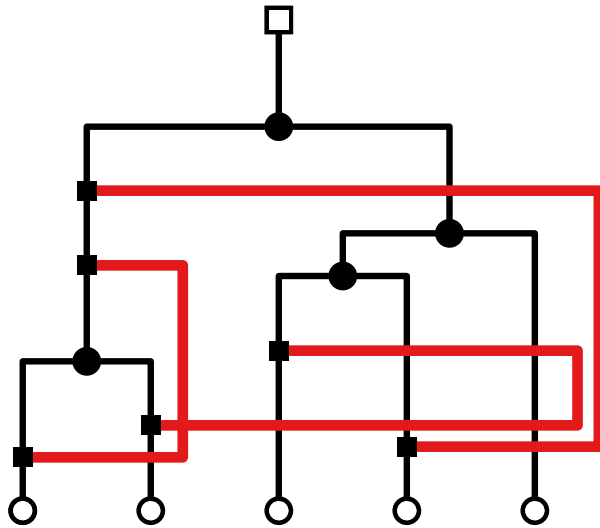
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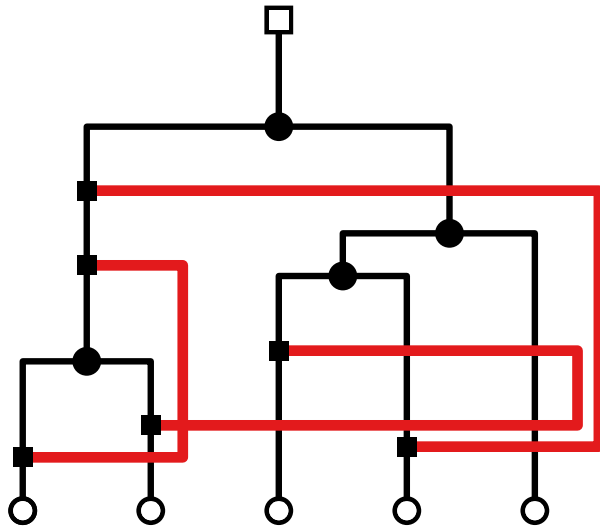
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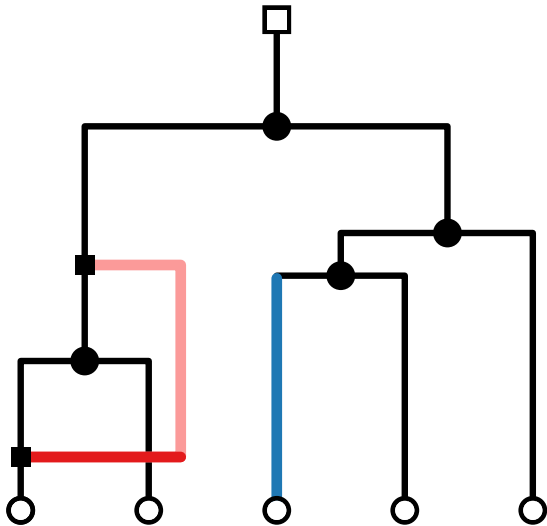
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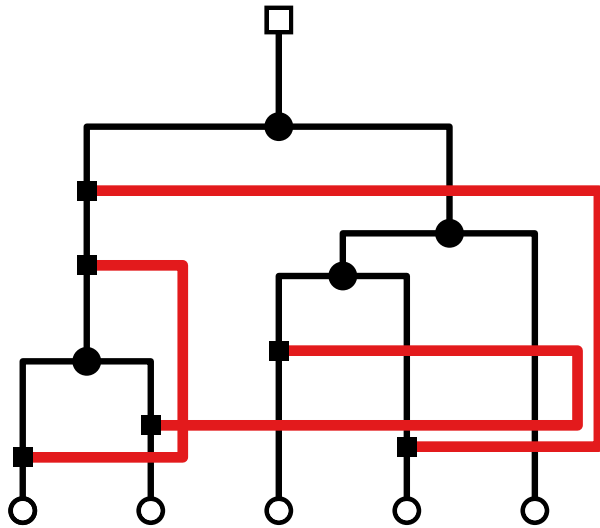


Let N be a tree-based network on n leaves and k **reticulation edges**.
 A **crossing-minimal ear-style** drawing of N can be computed in $\mathcal{O}(nk)$ time.

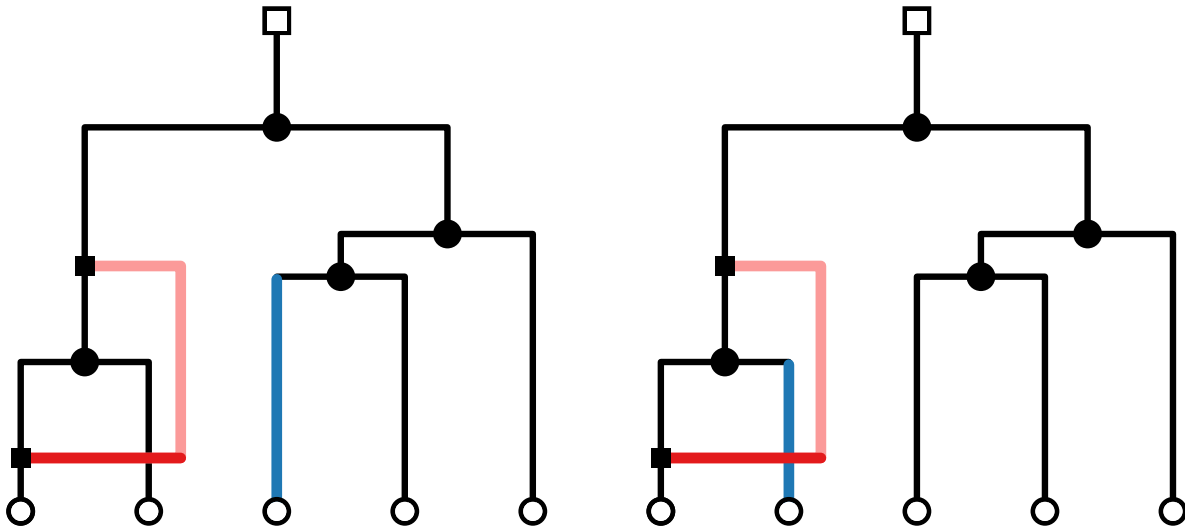


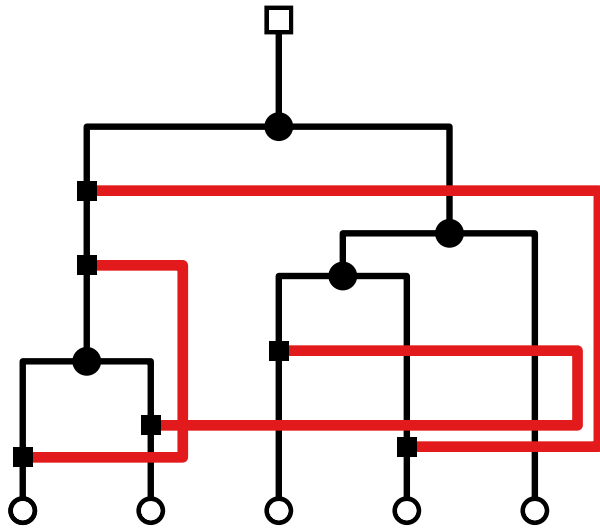
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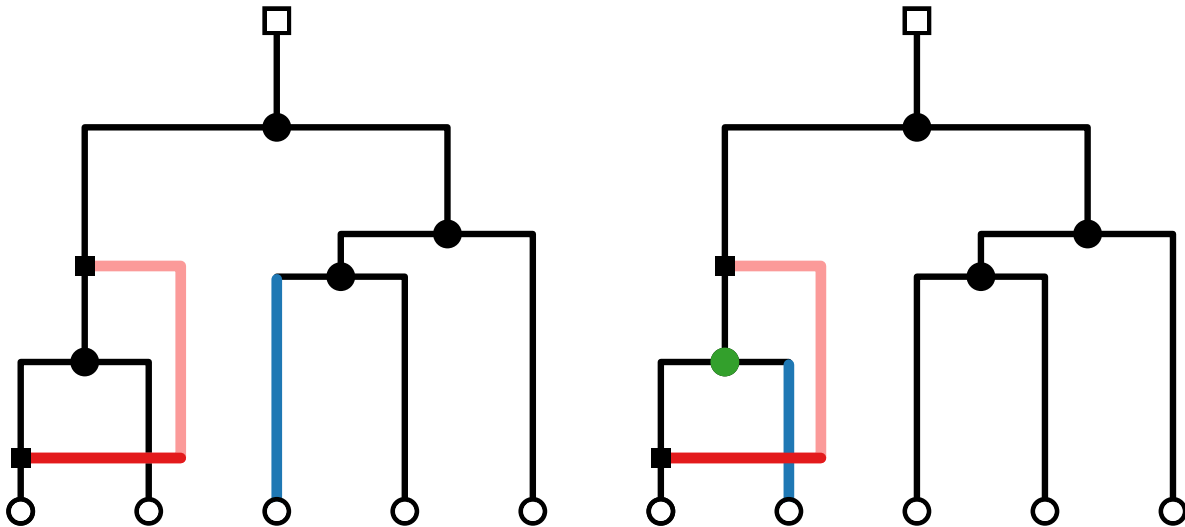


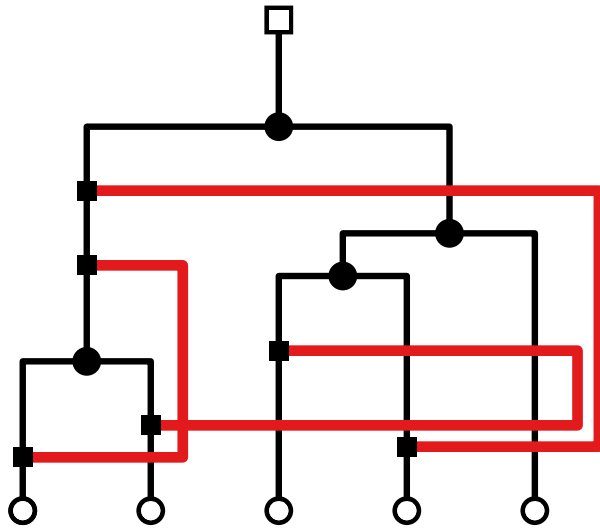
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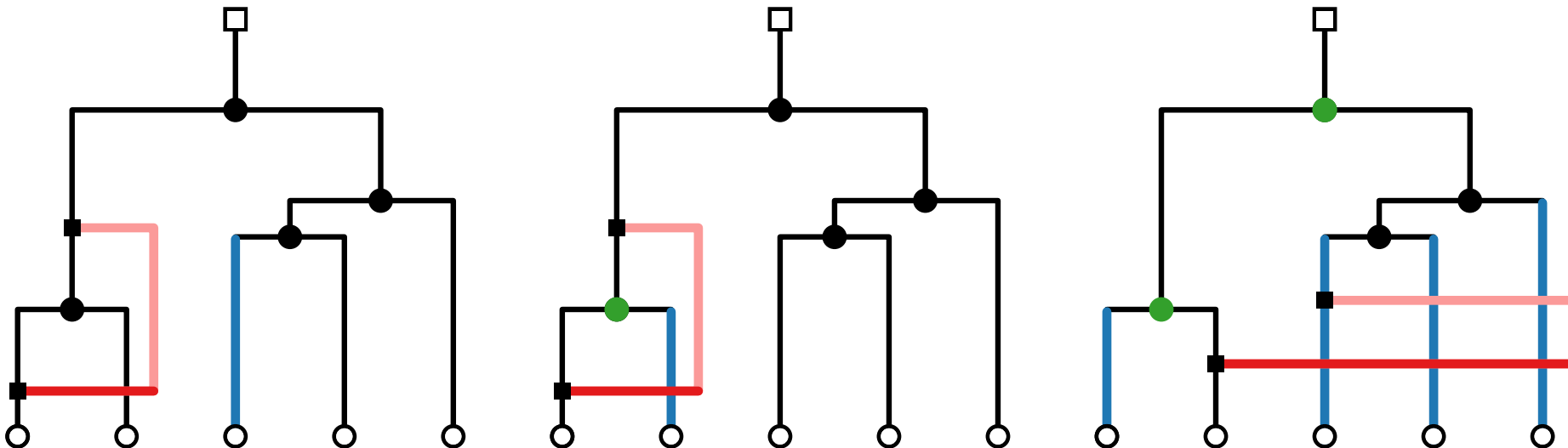


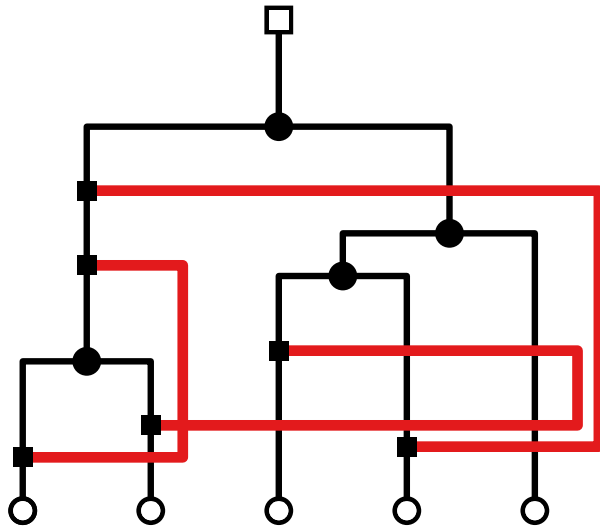
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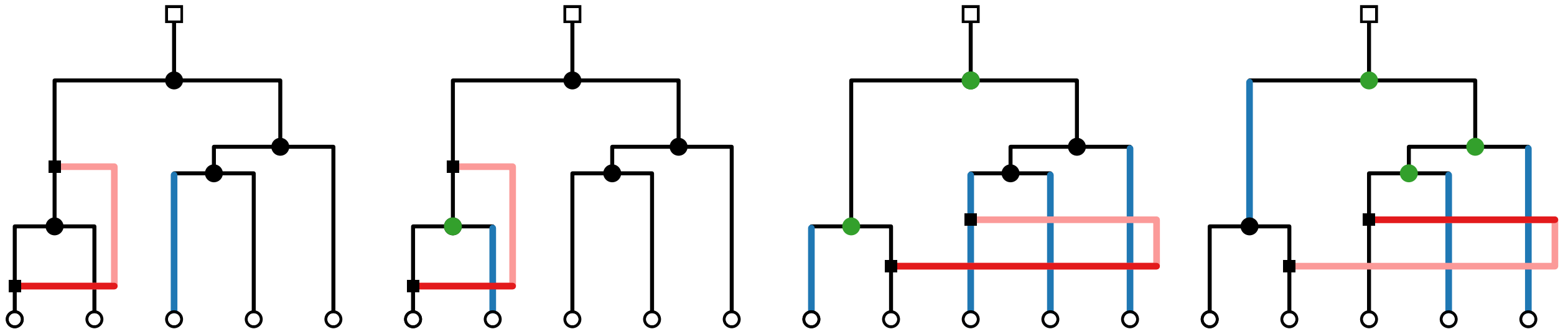


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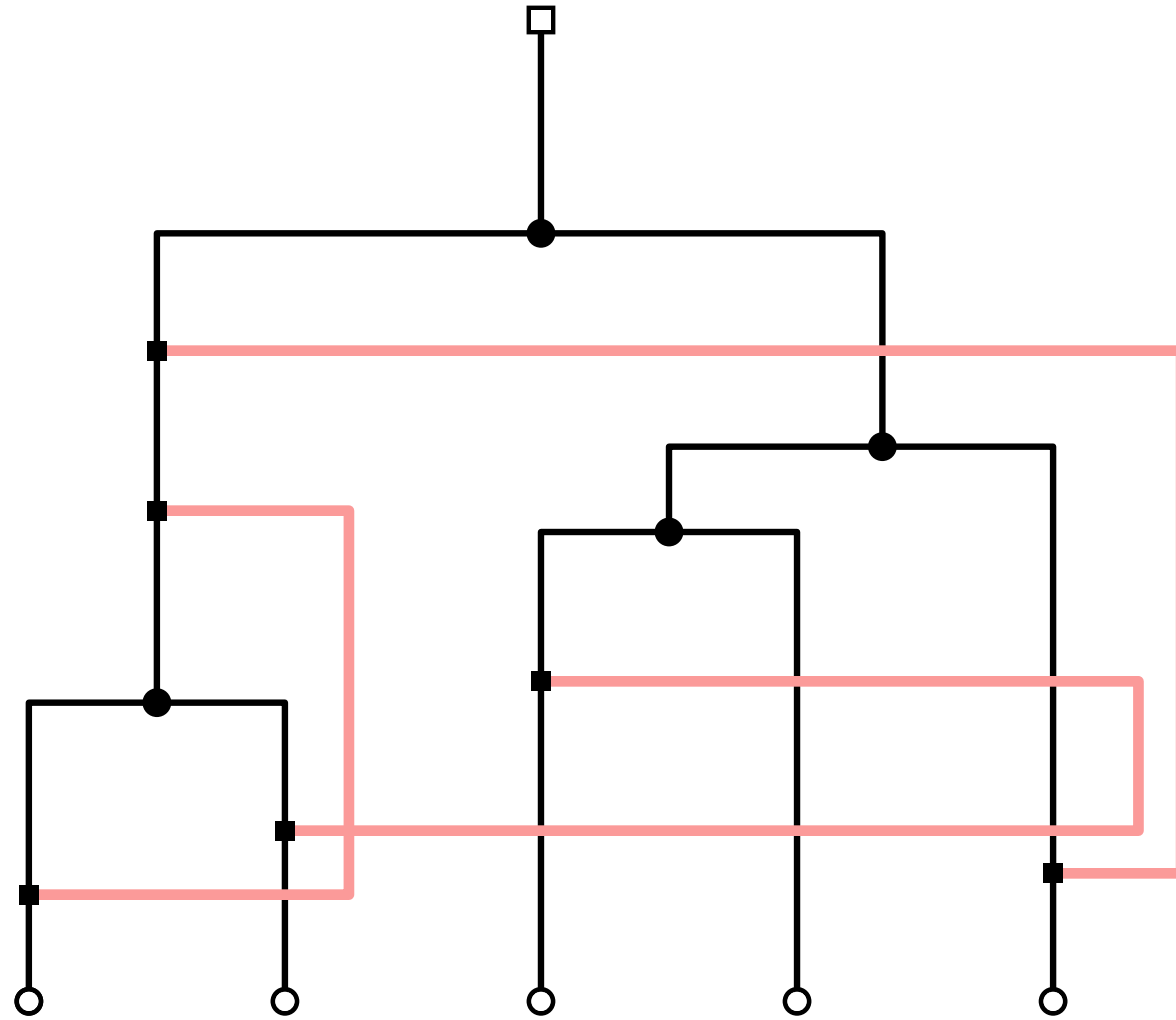




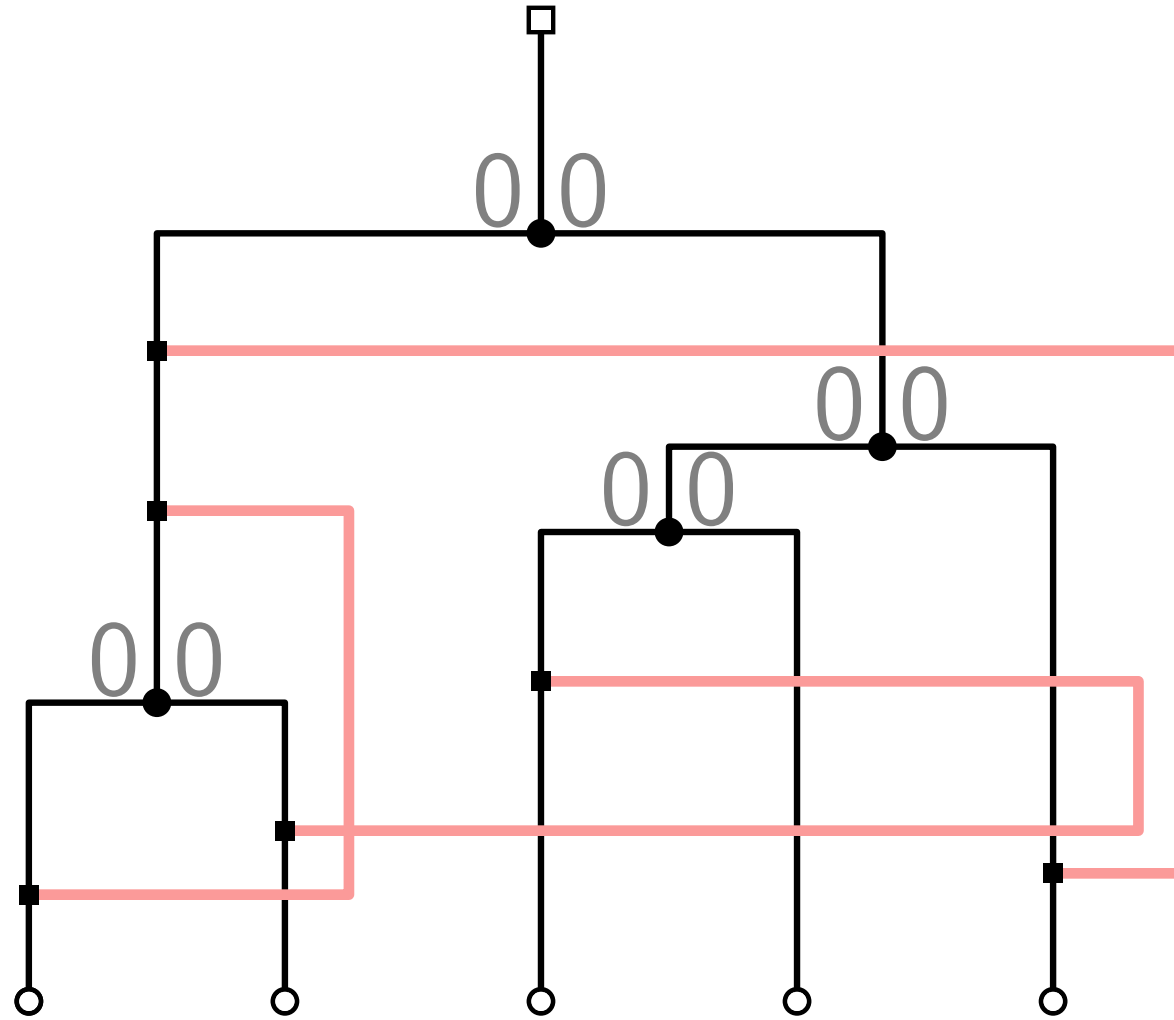
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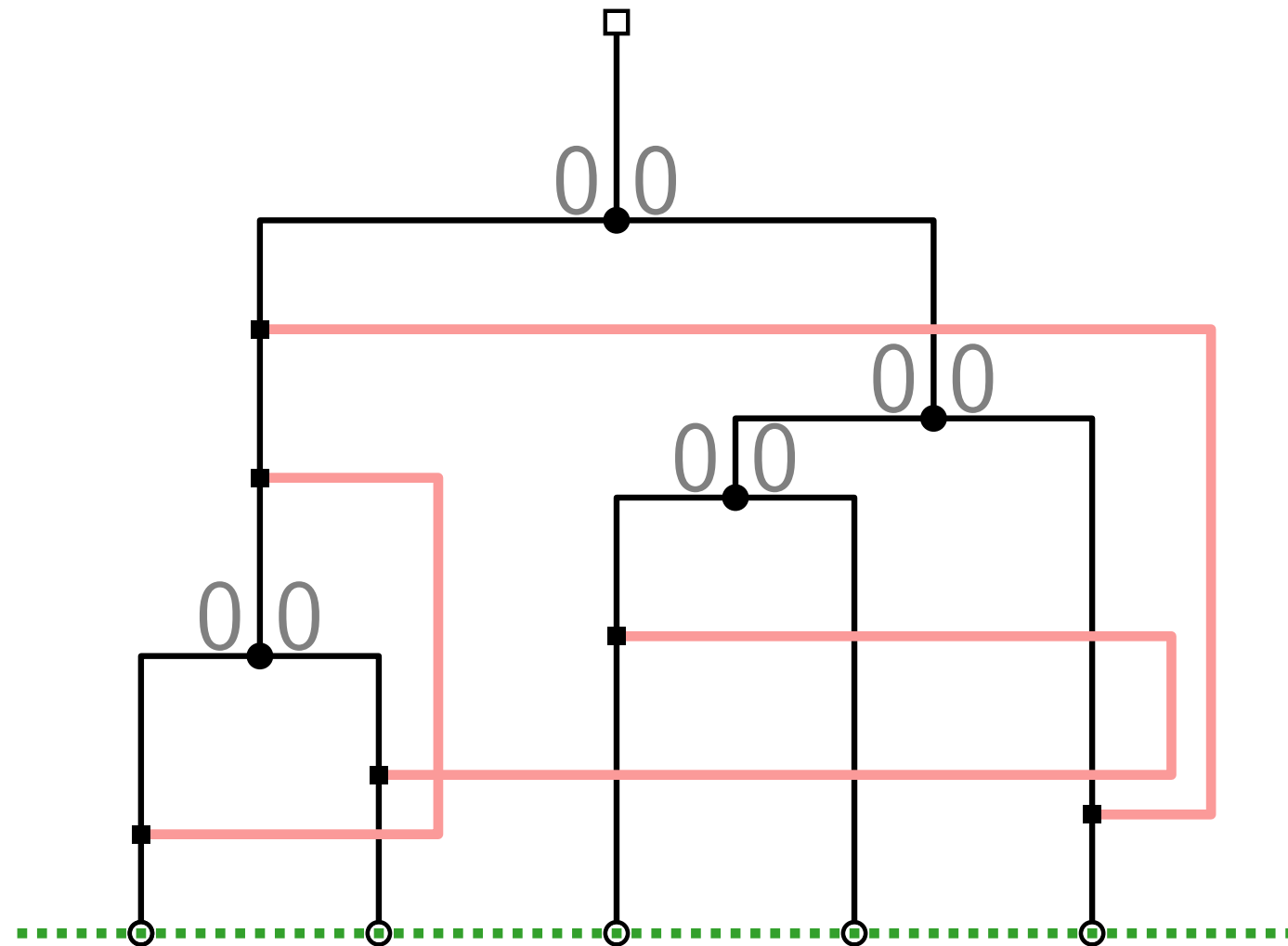
Ear-style algorithm



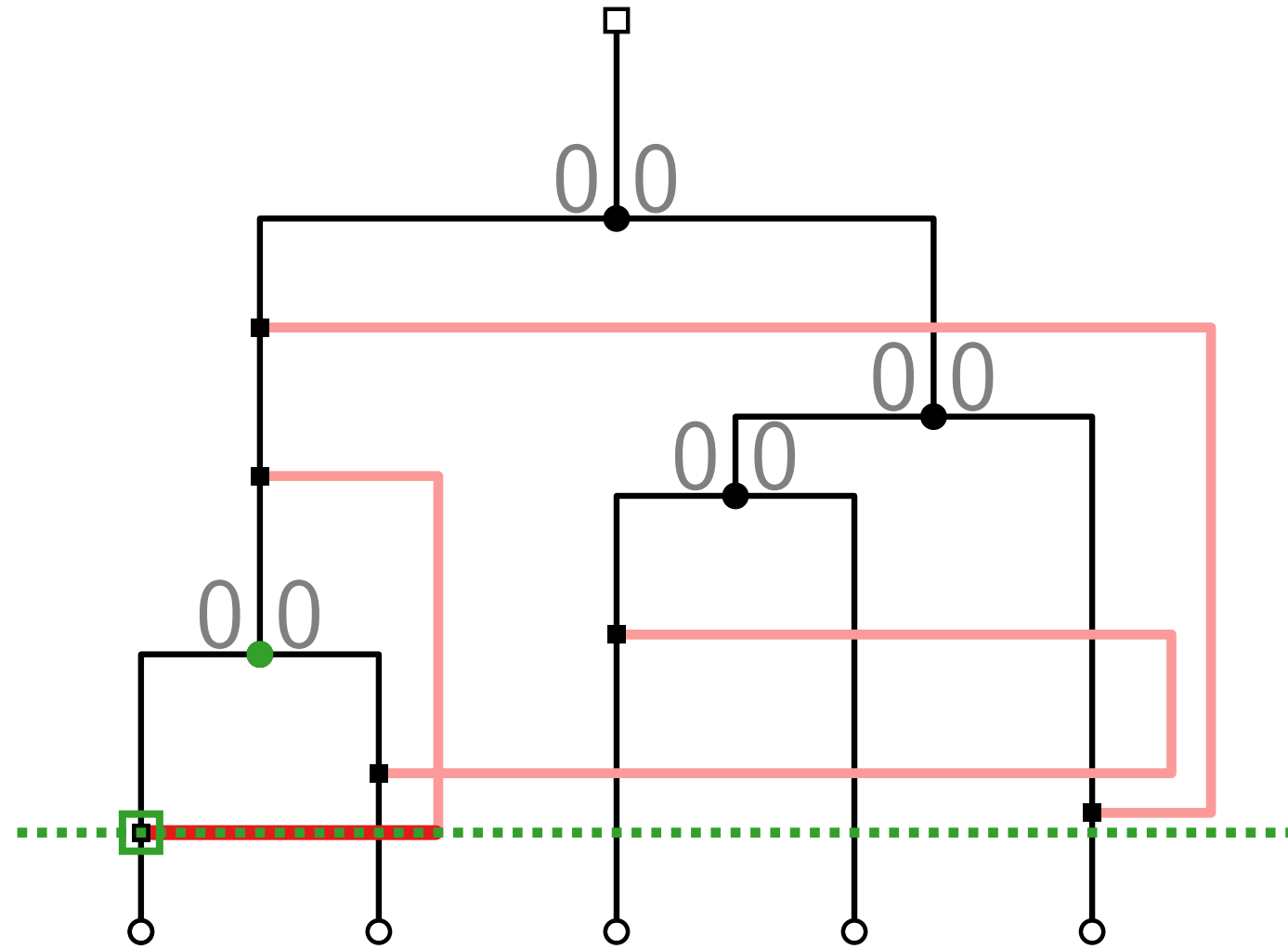
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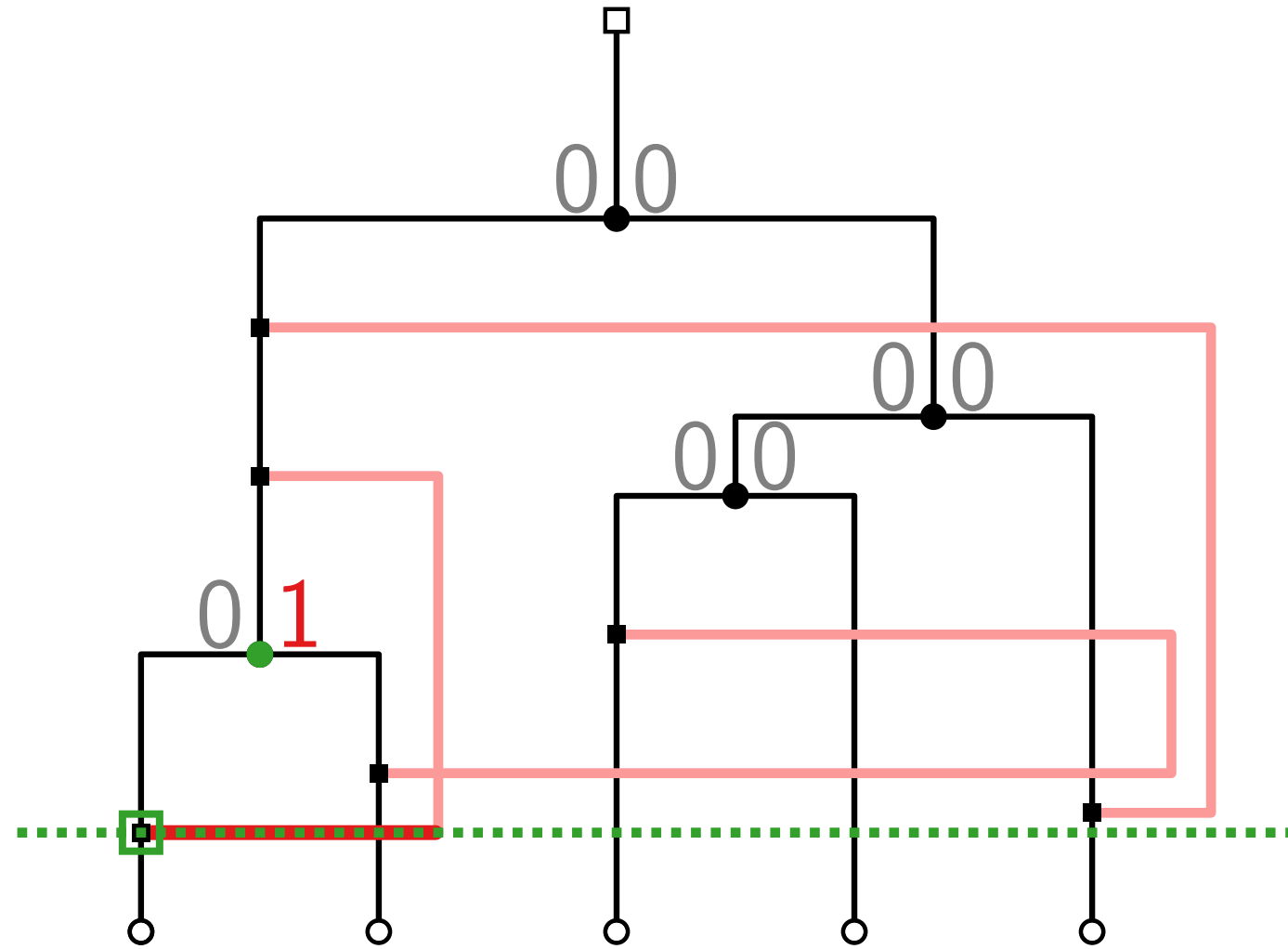
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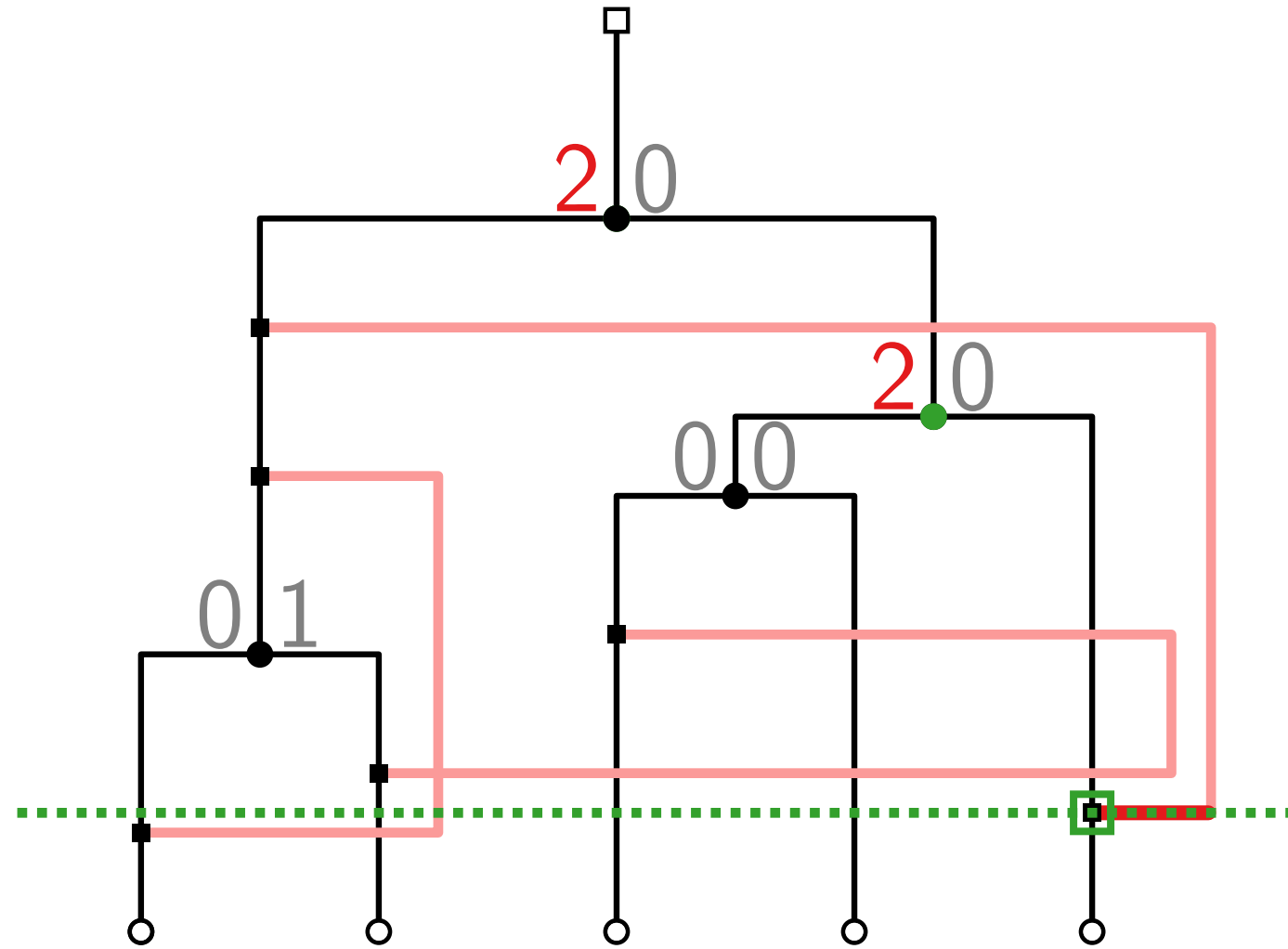
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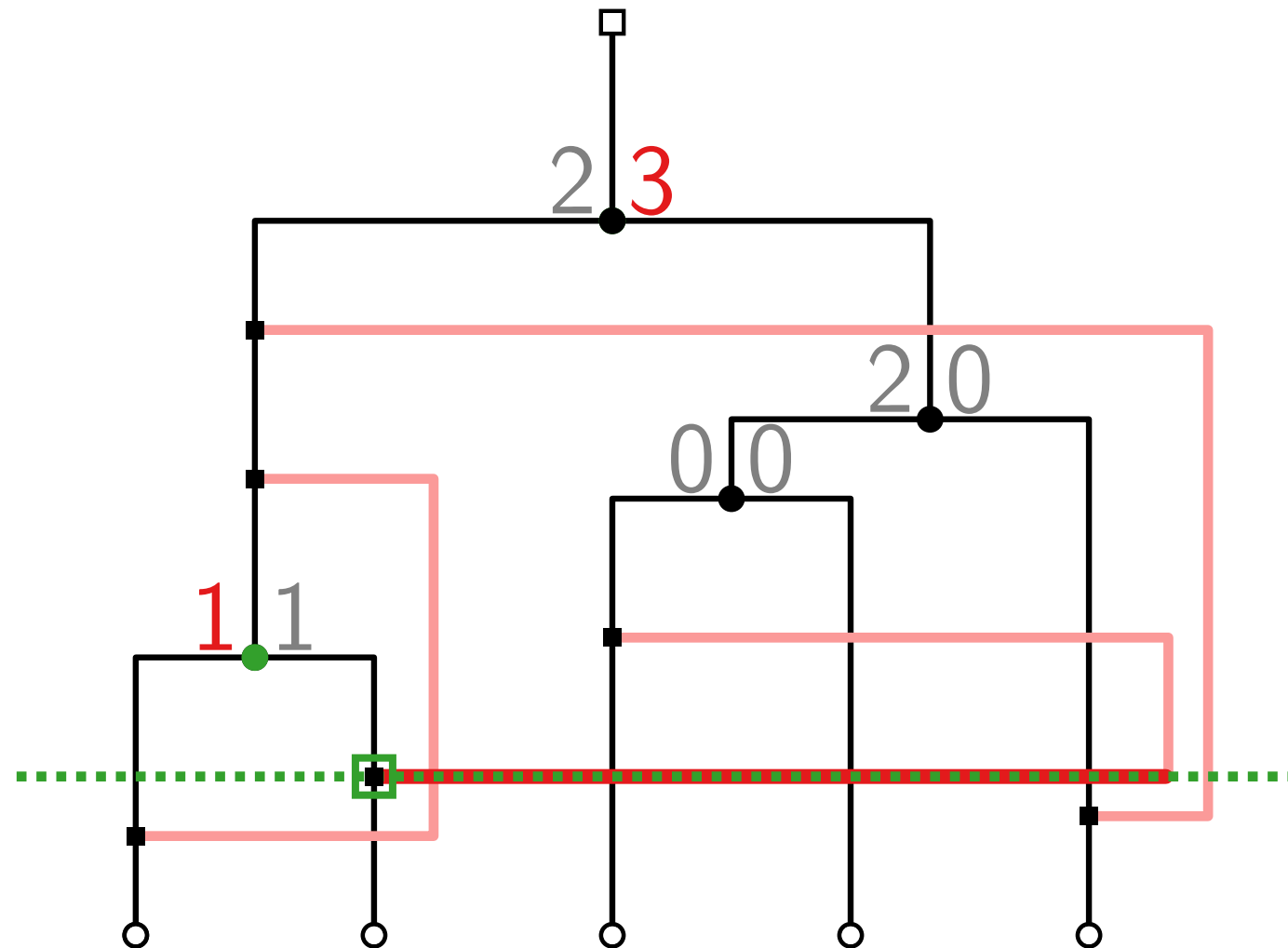
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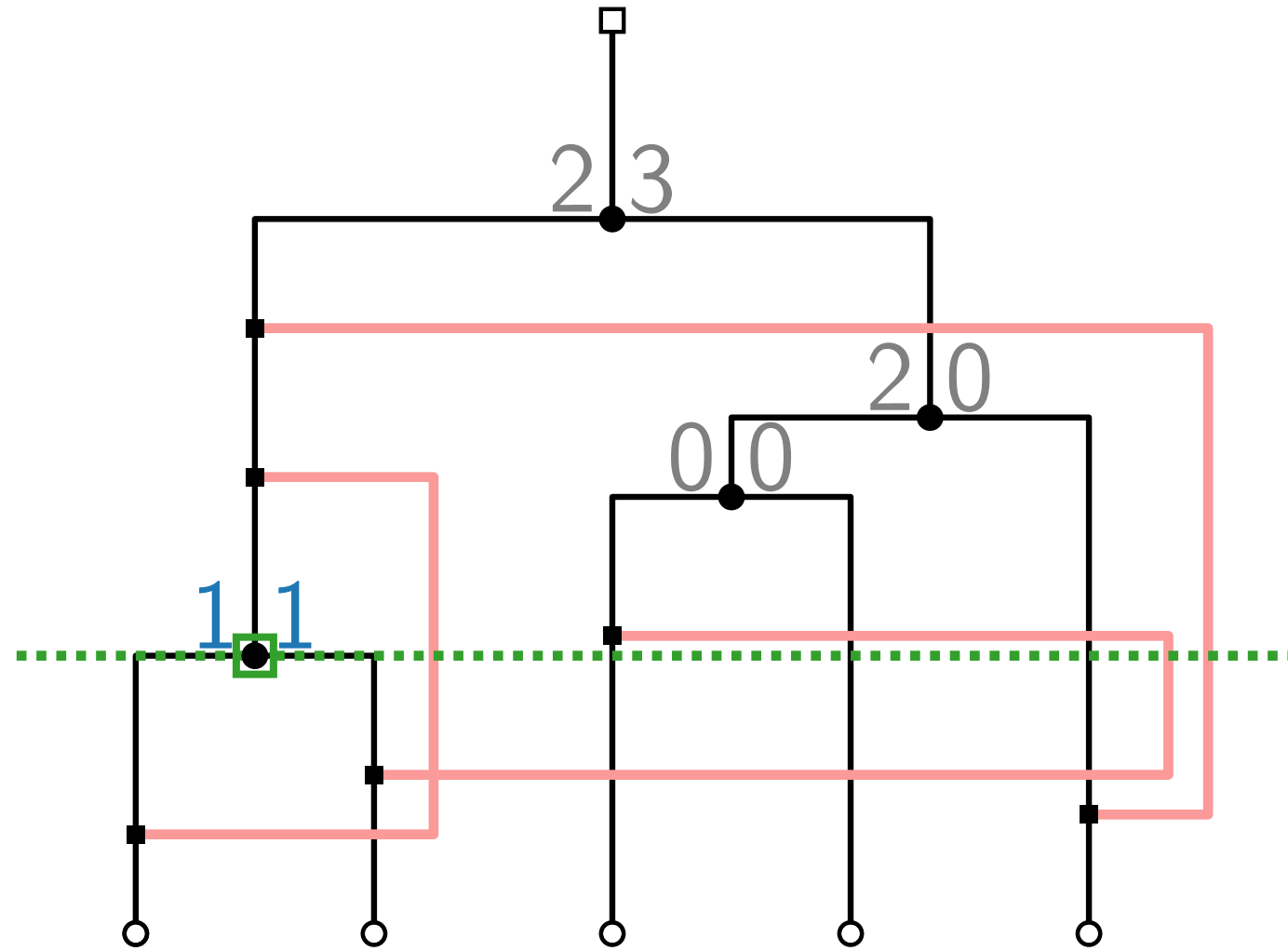
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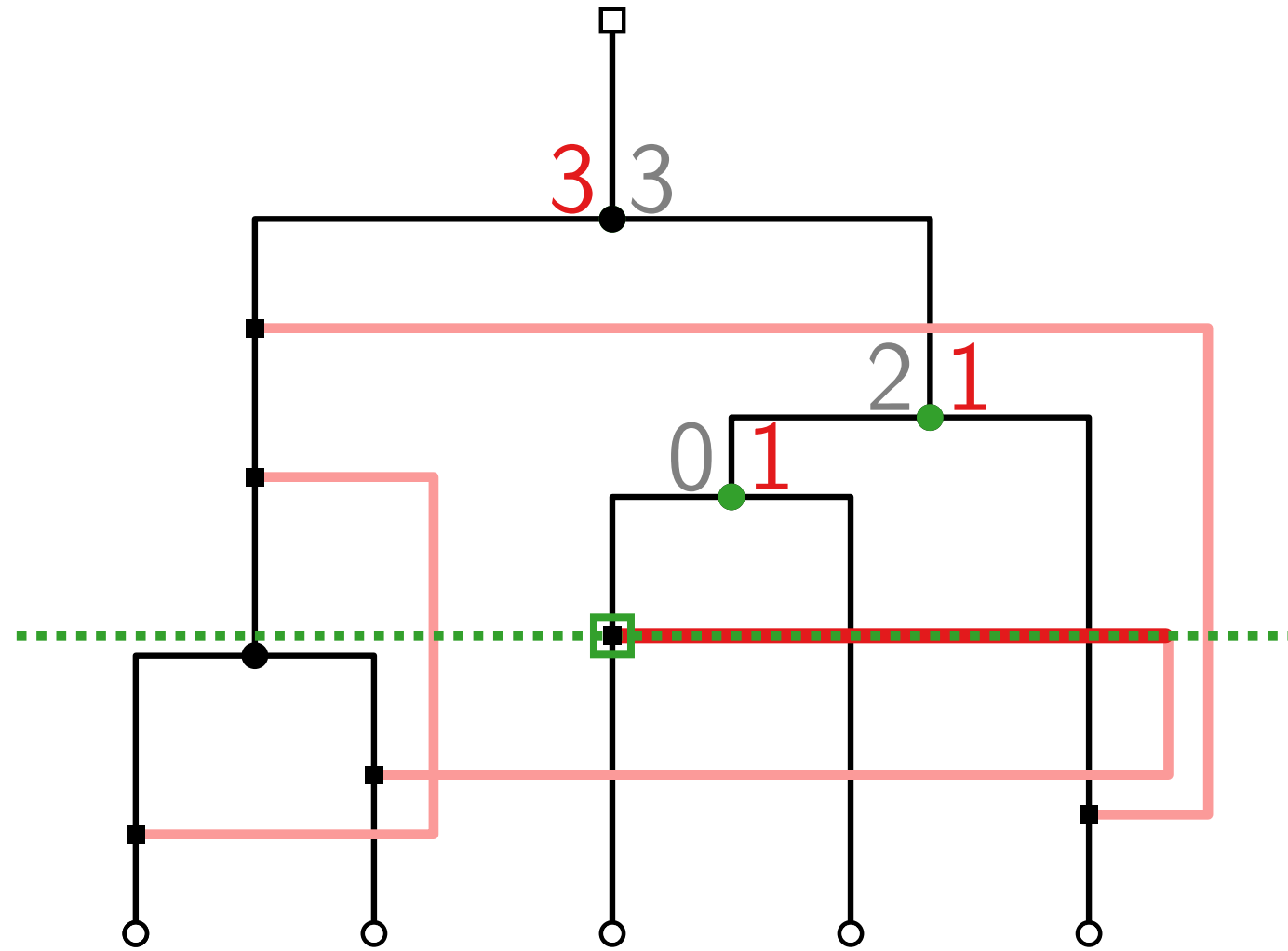
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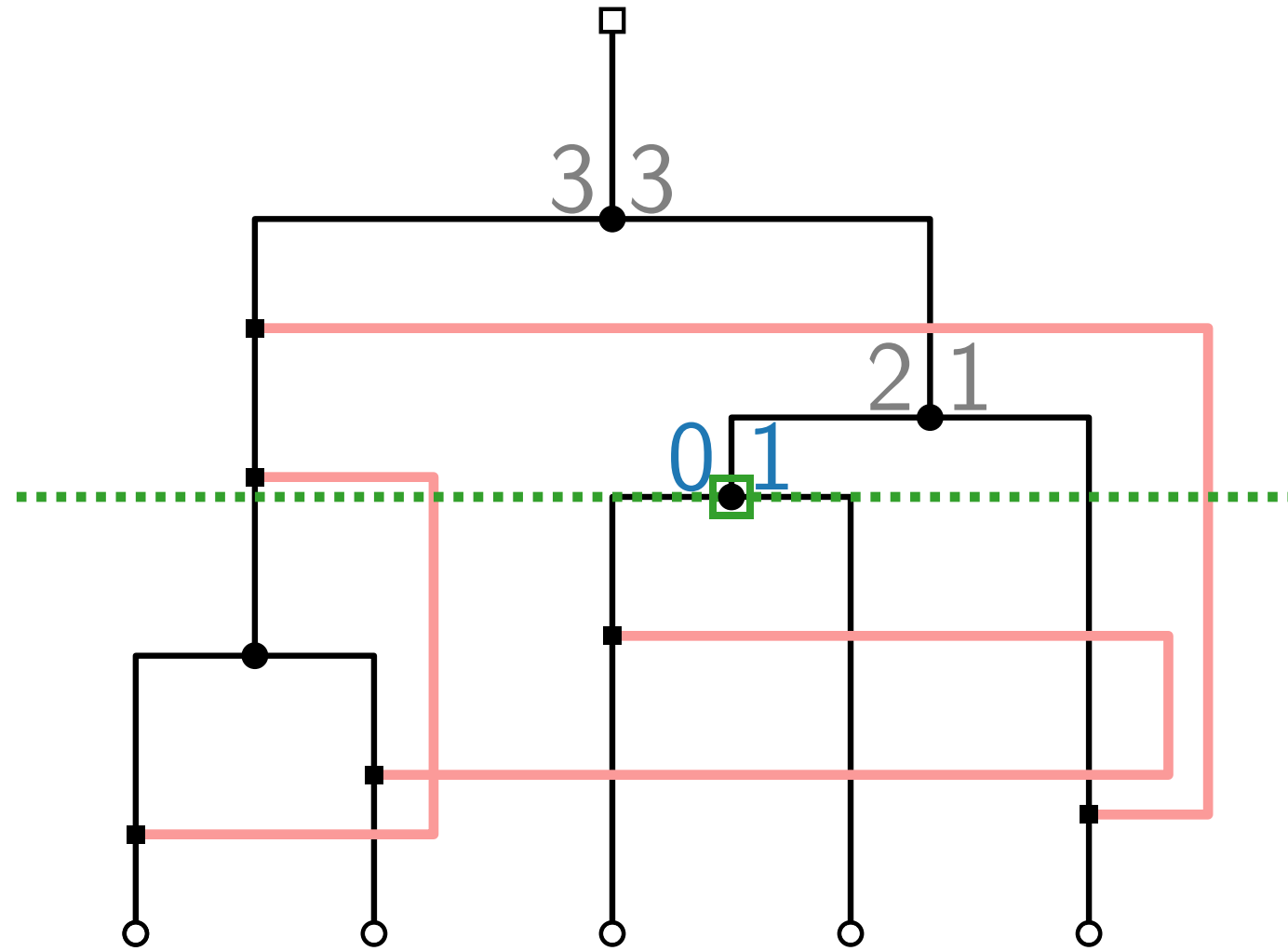
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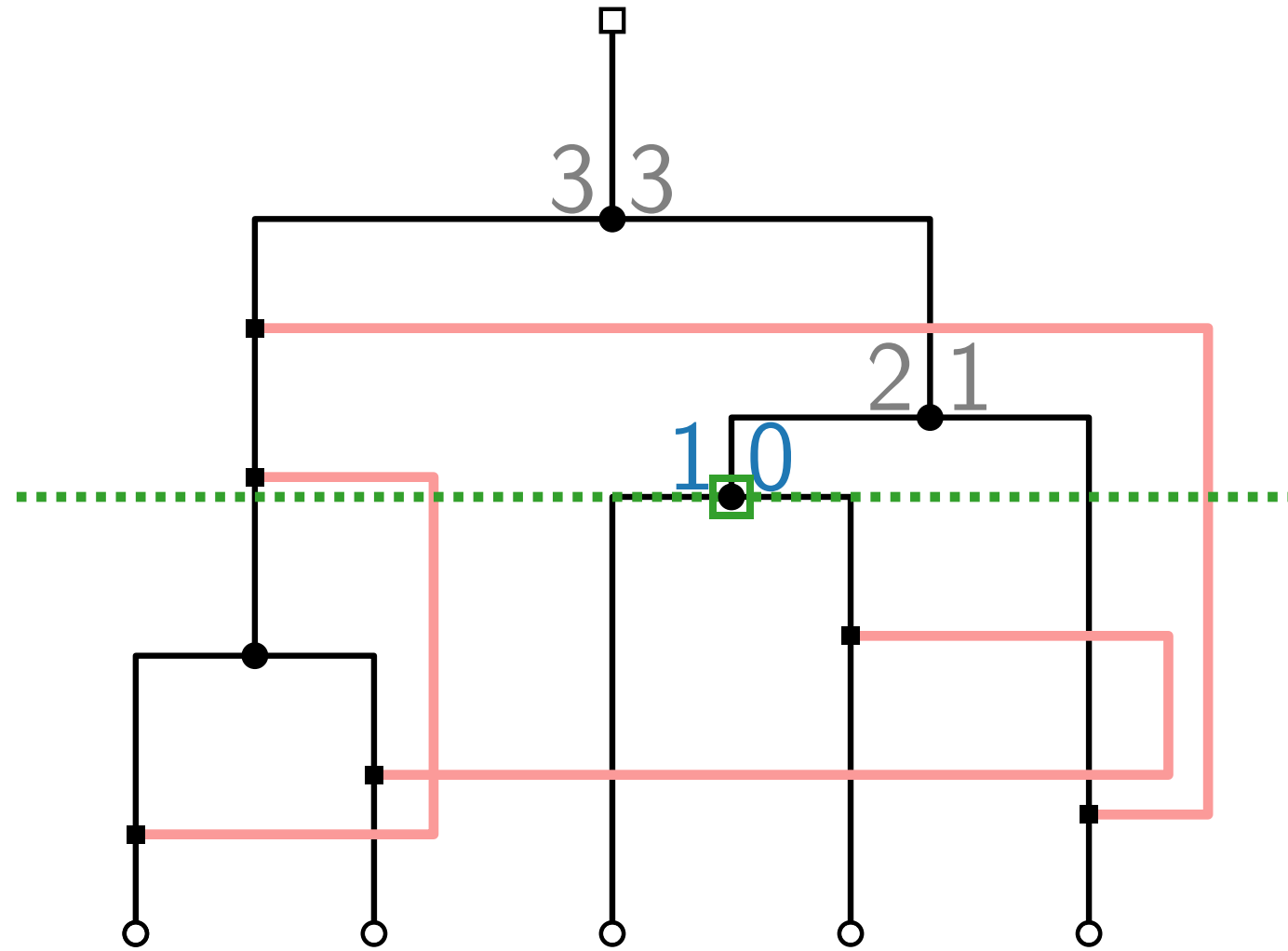
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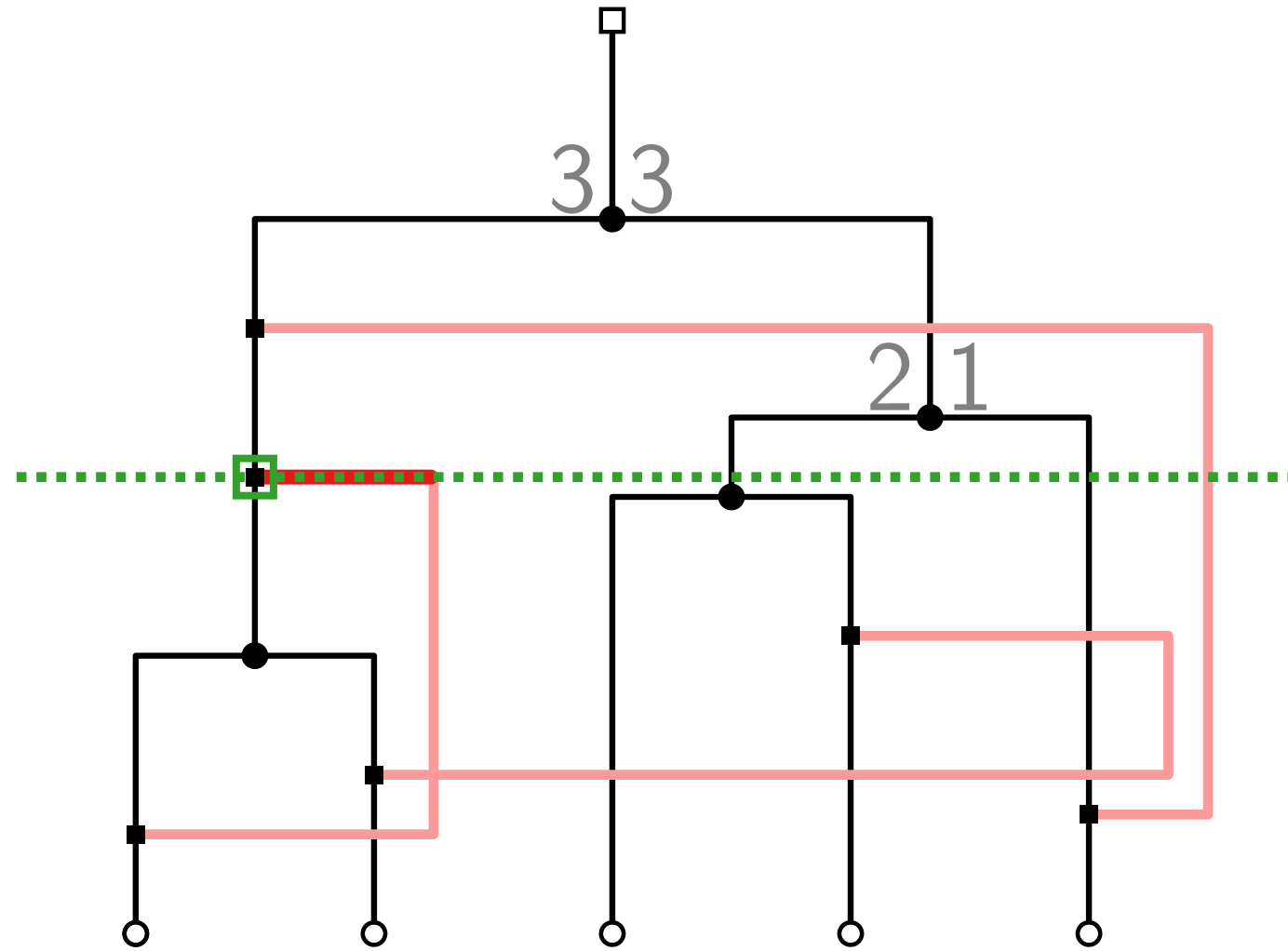
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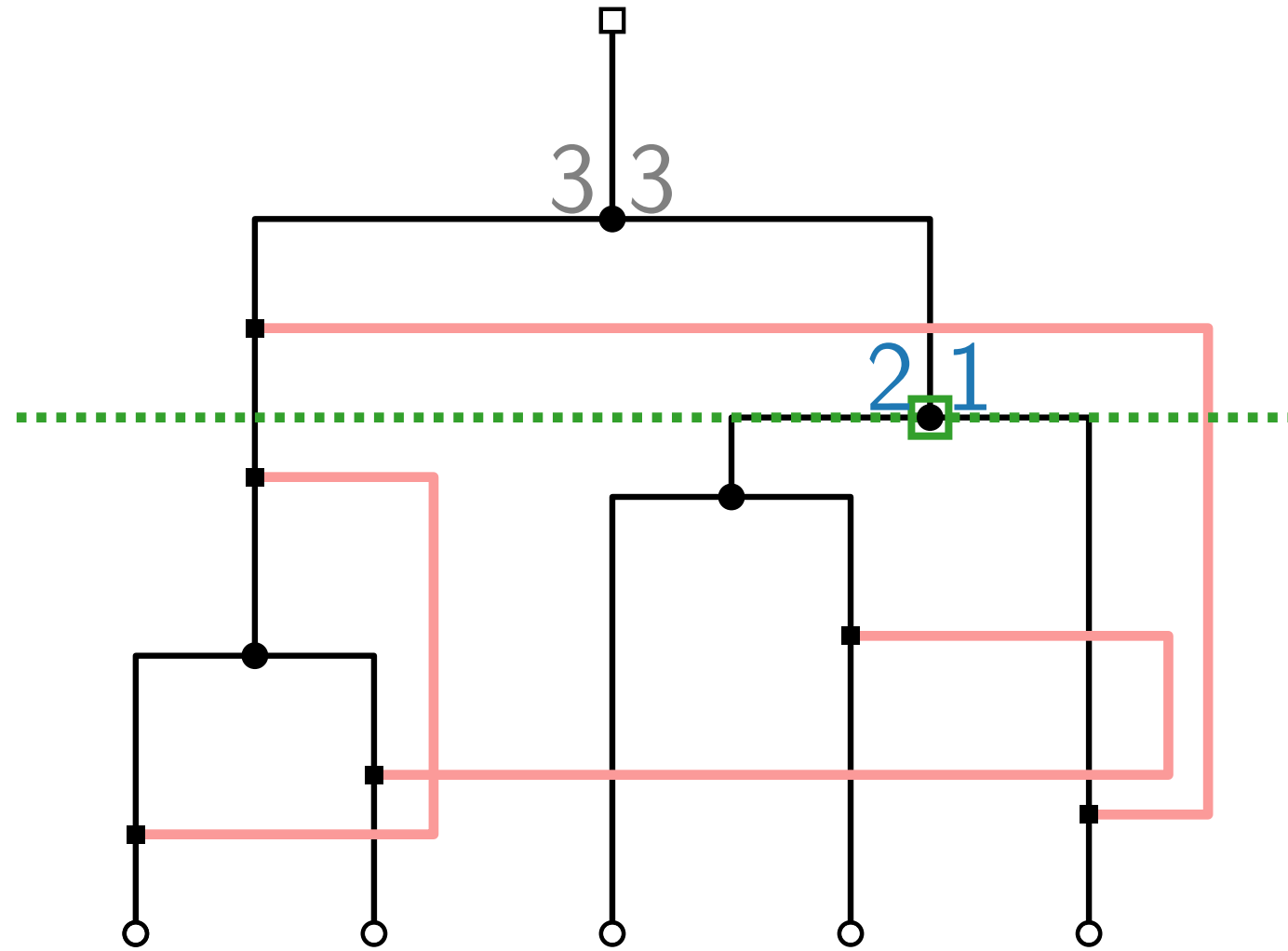
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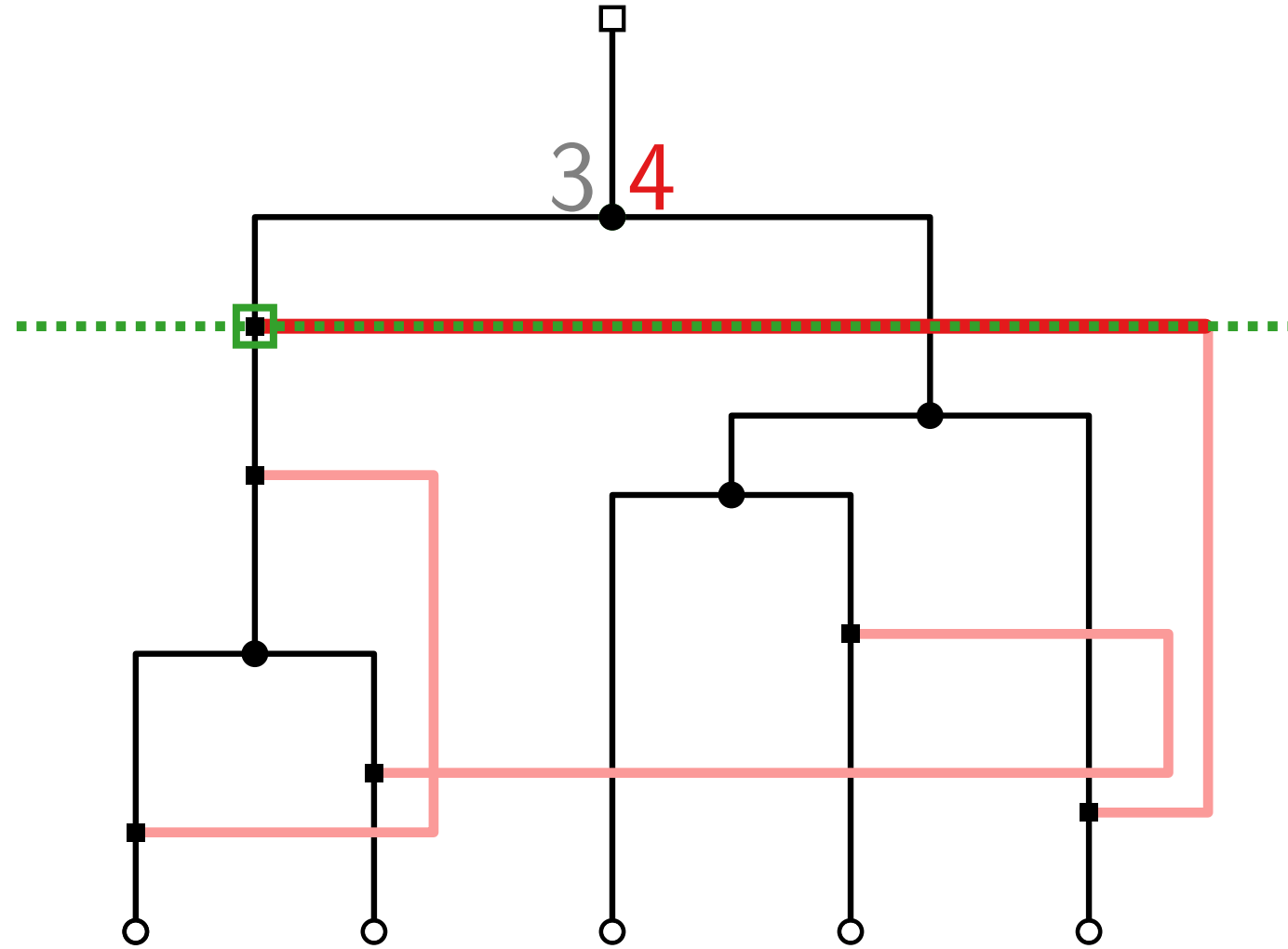
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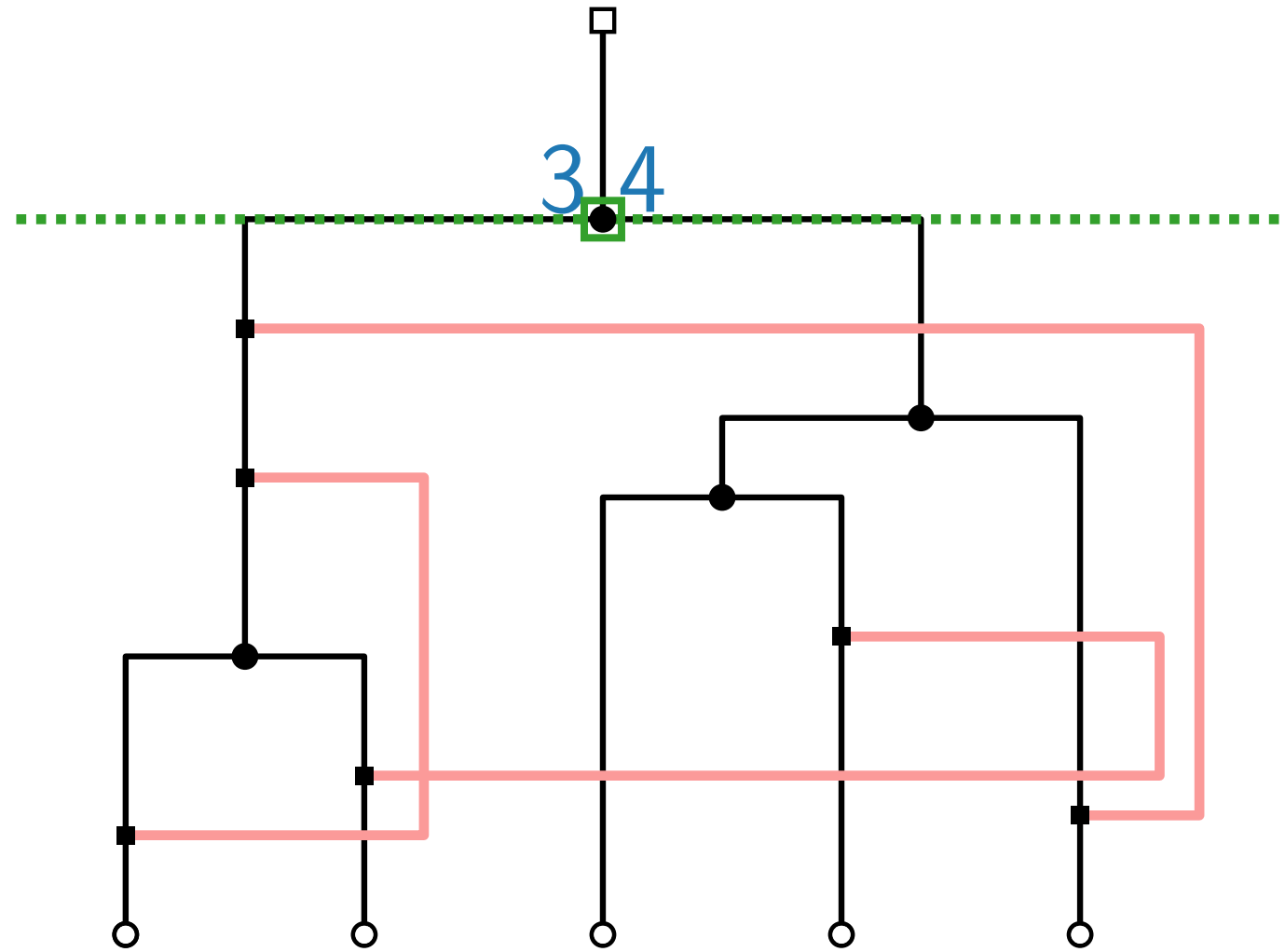
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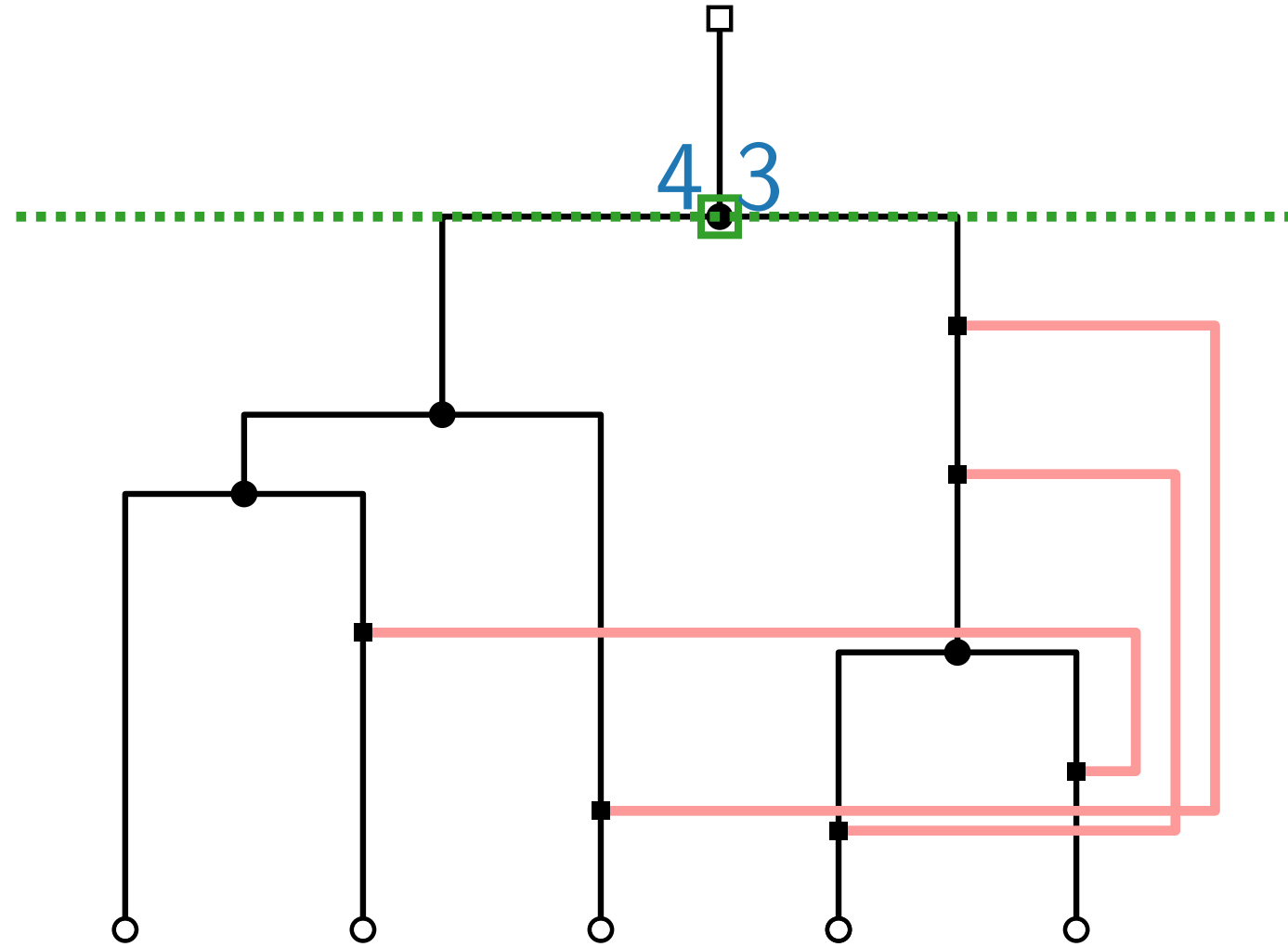
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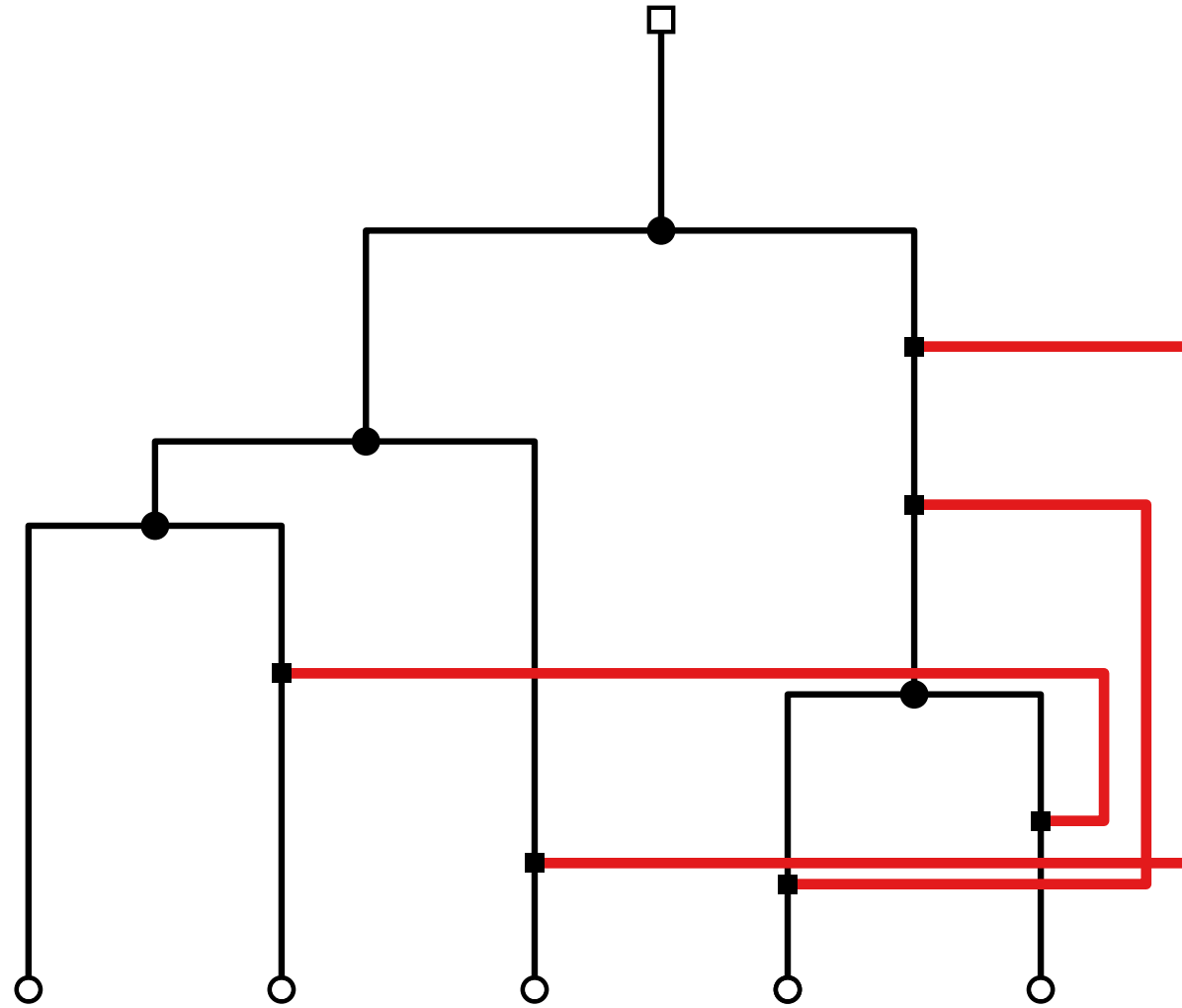
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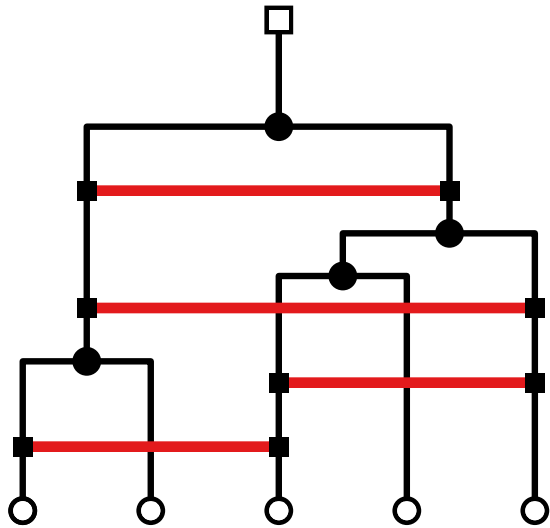


Ear-style algorithm

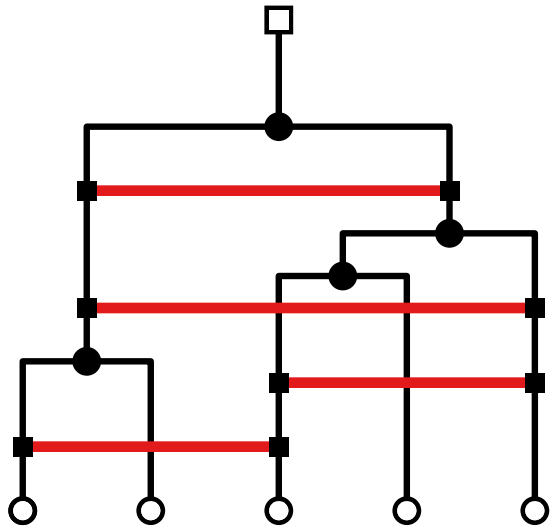


Ear-style algorithm



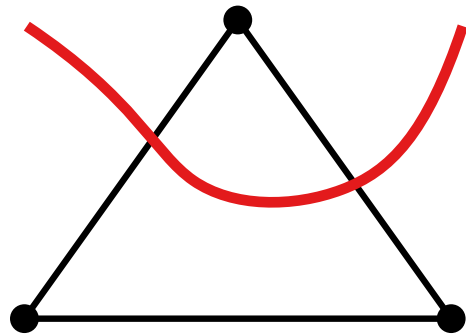


The CROSSING MINIMISATION problem for **horizontal-style** drawings of tree-based networks is NP-complete.



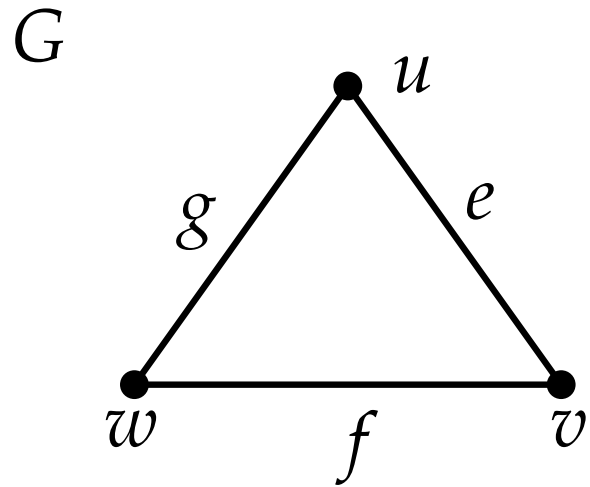
The CROSSING MINIMISATION problem for **horizontal-style** drawings of tree-based networks is NP-complete.

G

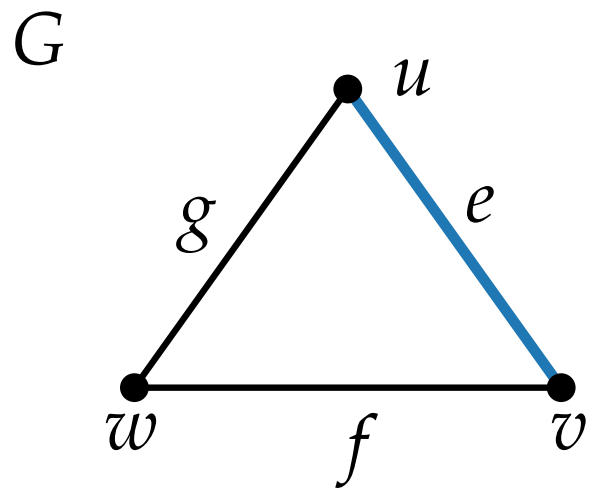
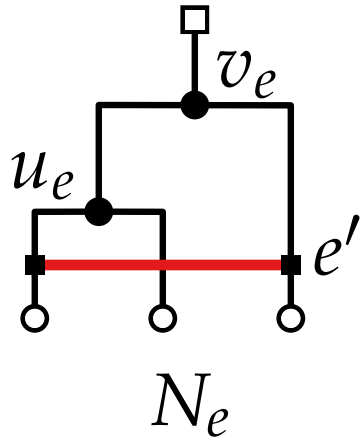


MAXCUT

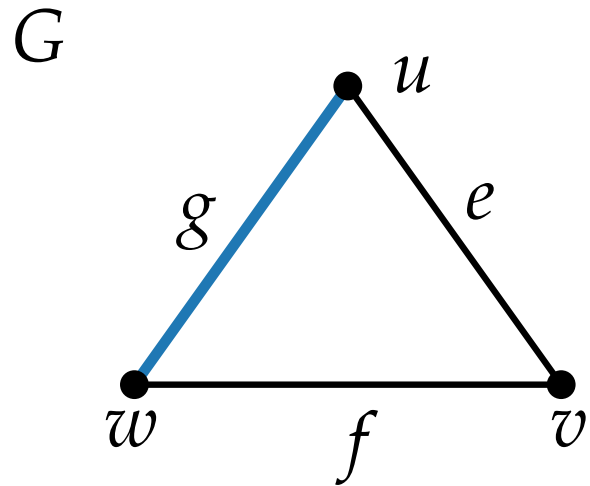
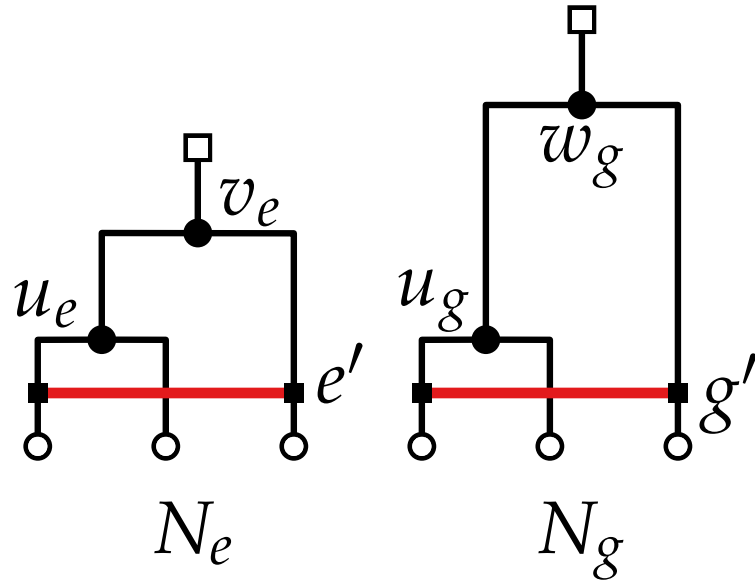
Horizontal style NP-reduction



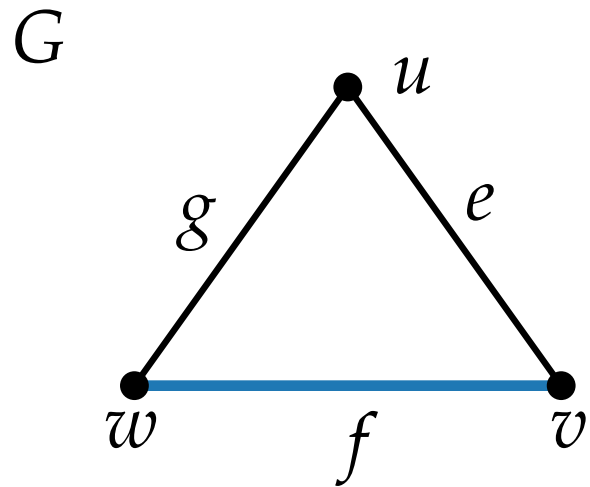
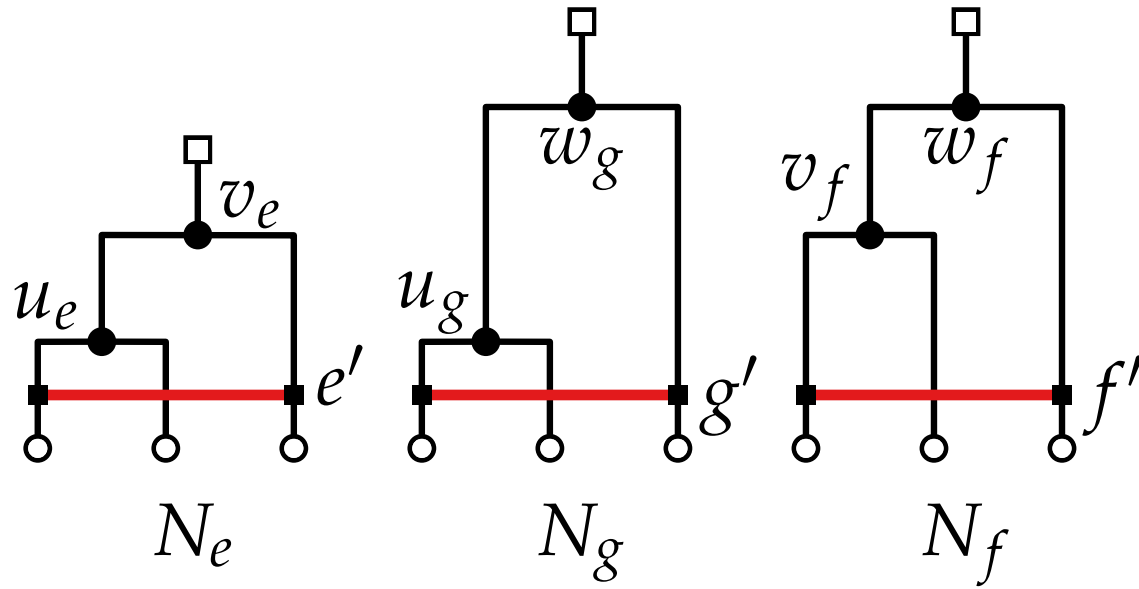
Horizontal style NP-reduction



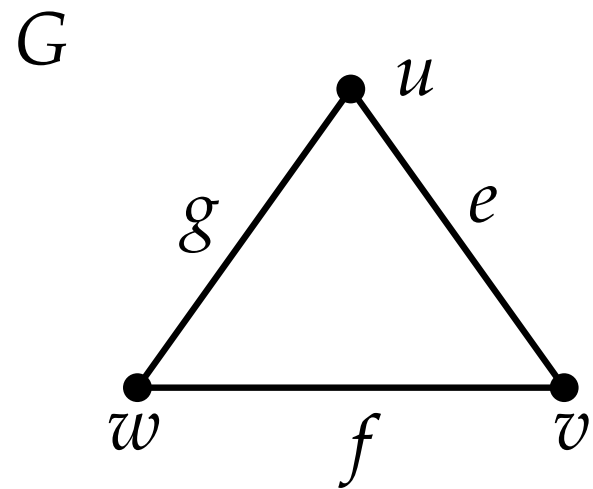
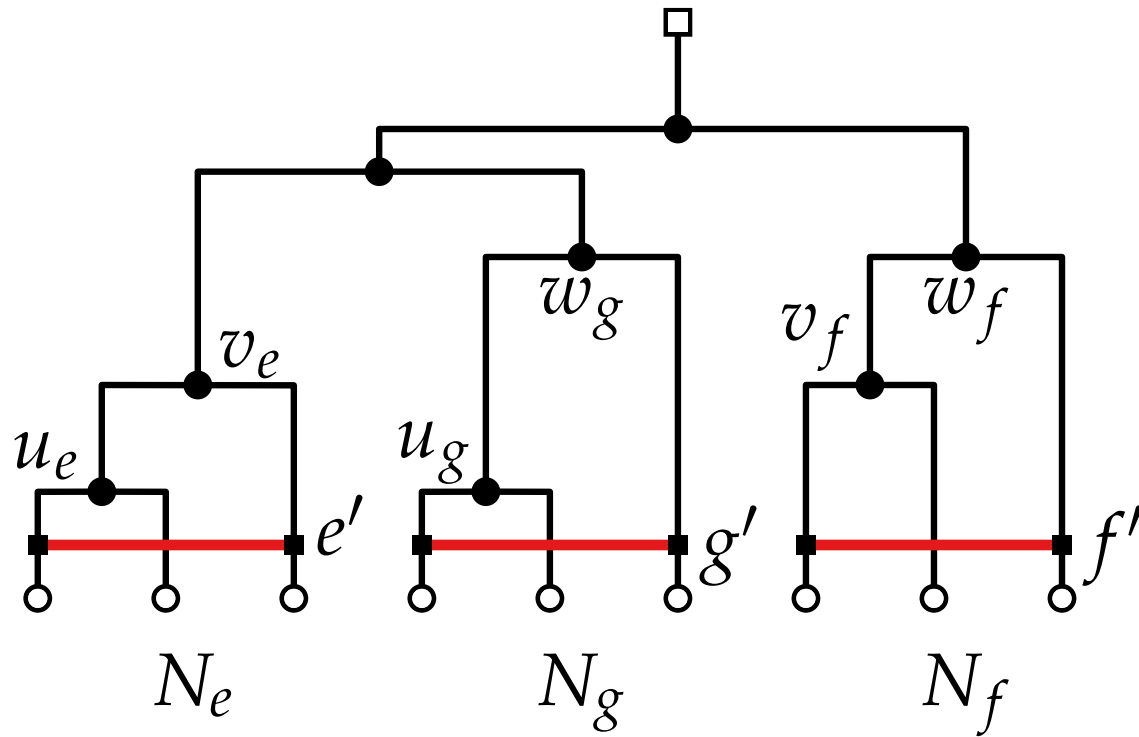
Horizontal style NP-reduction



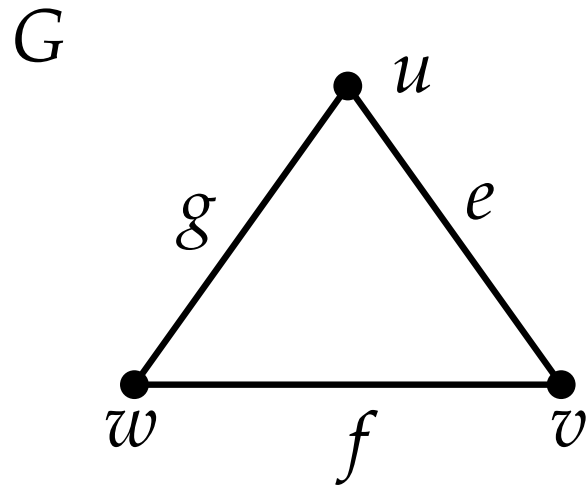
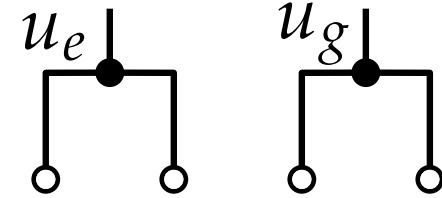
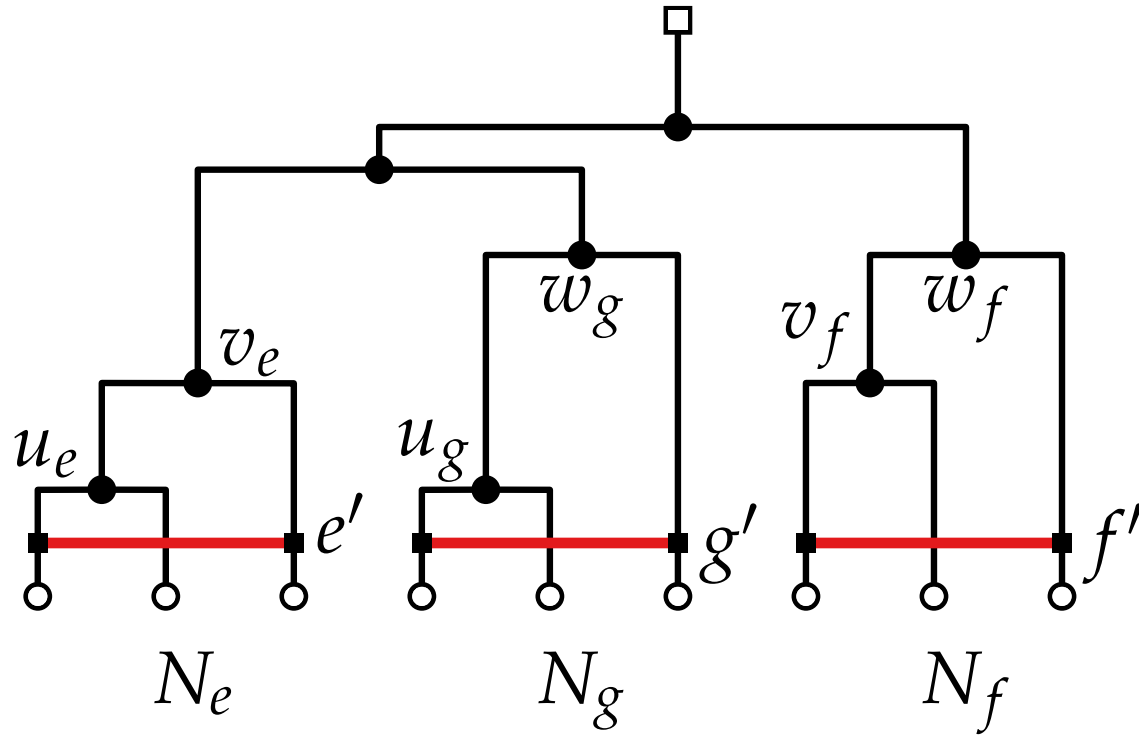
Horizontal style NP-reduction



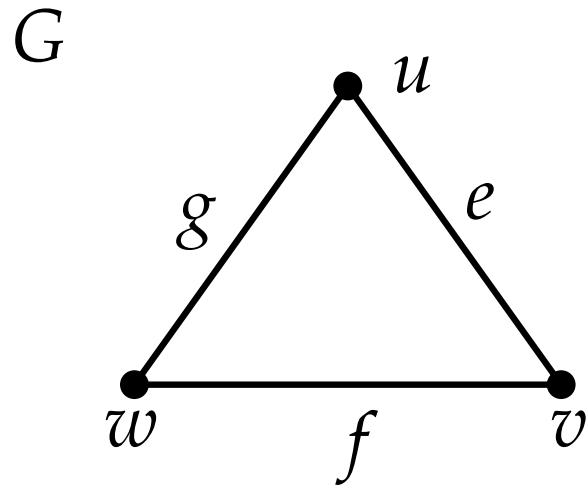
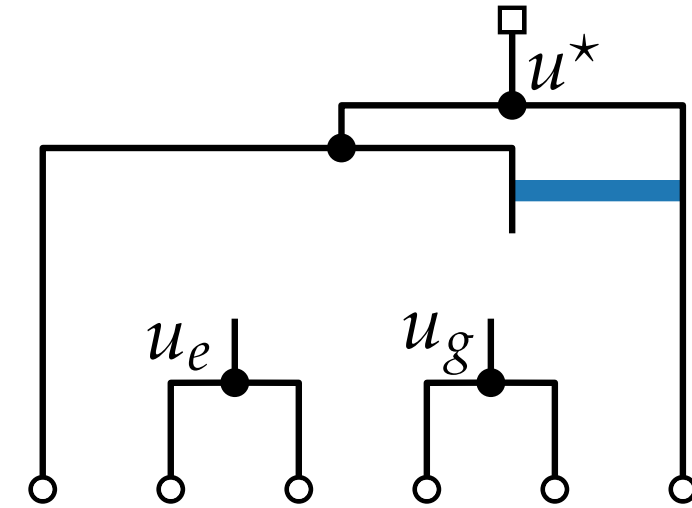
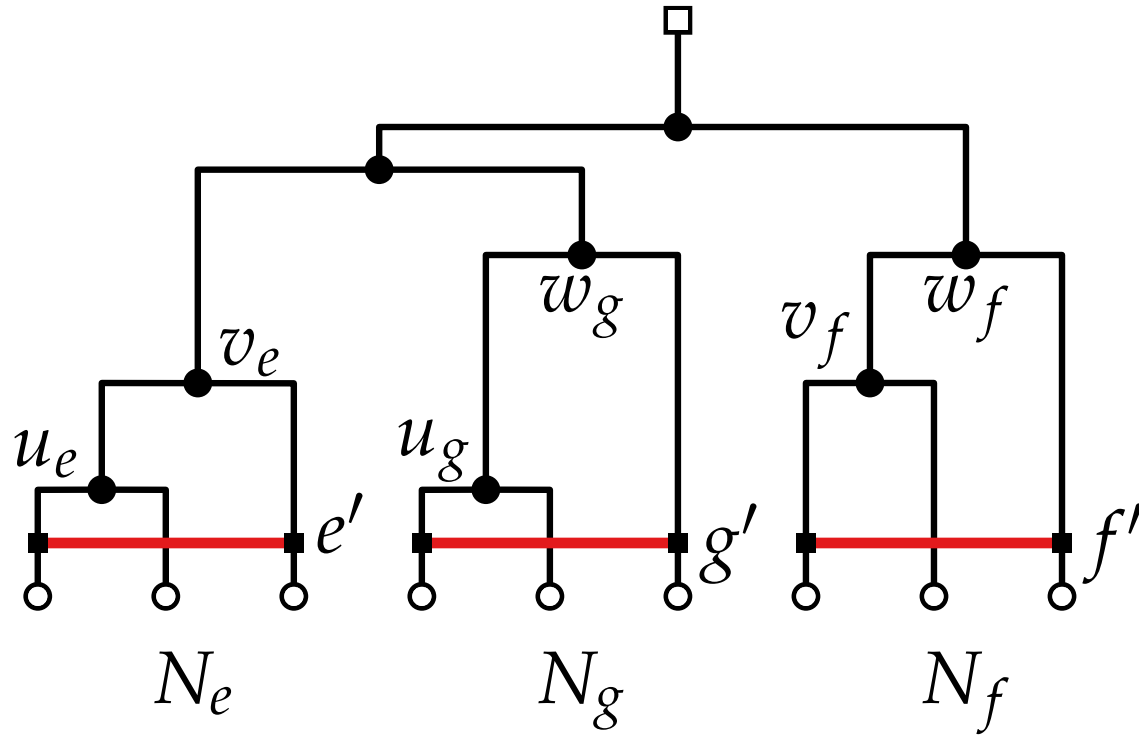
Horizontal style NP-reduction



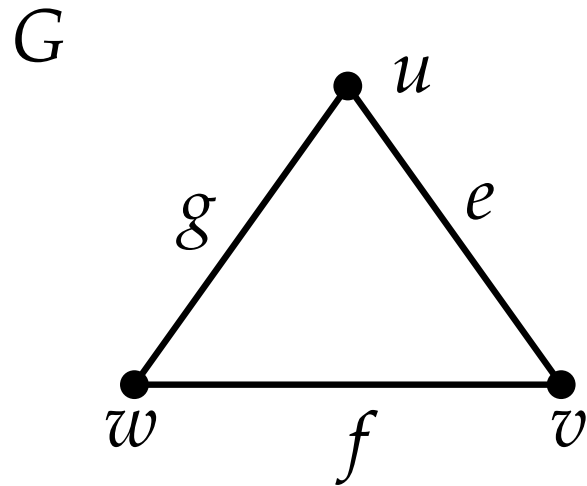
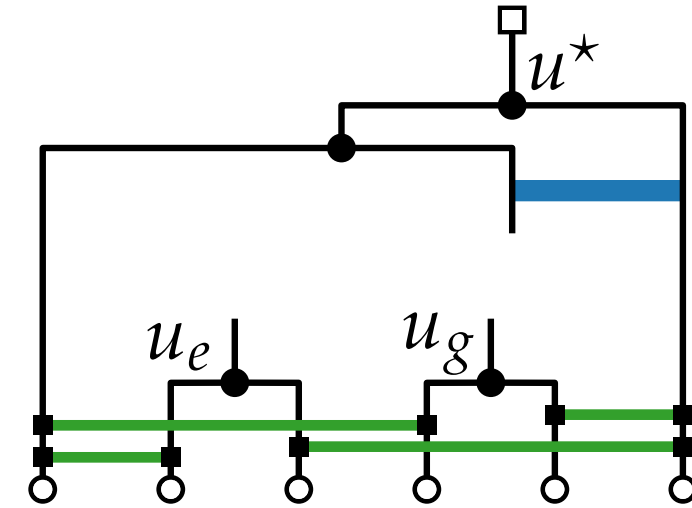
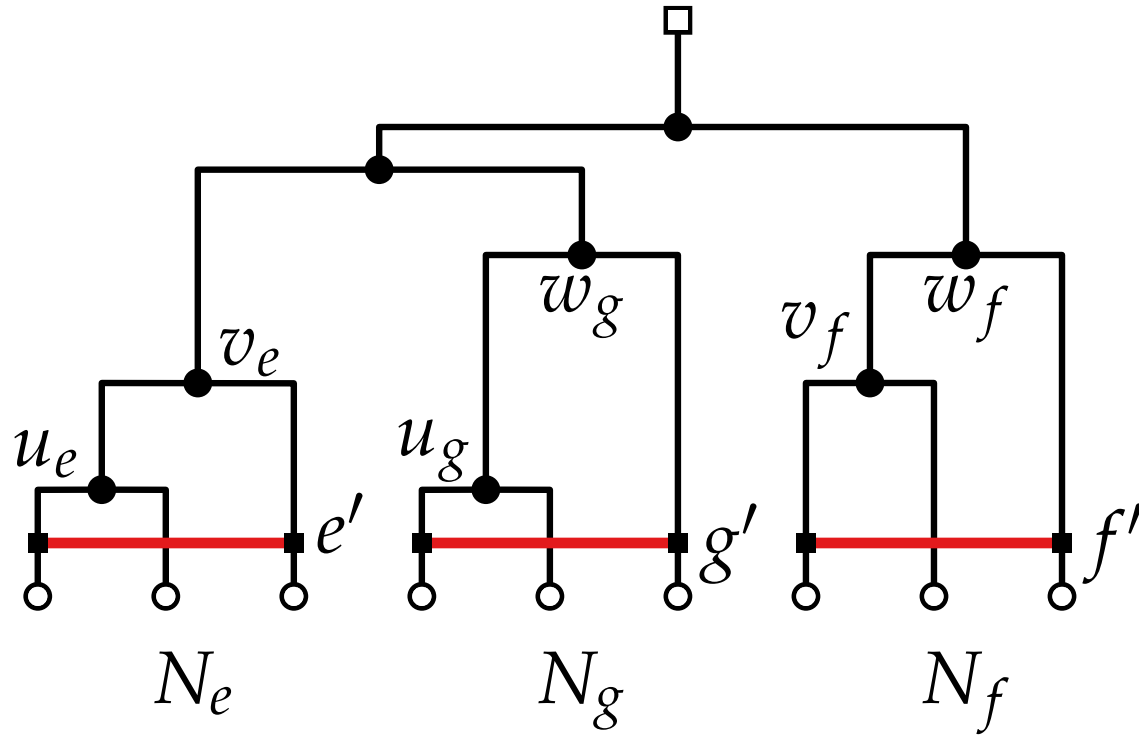
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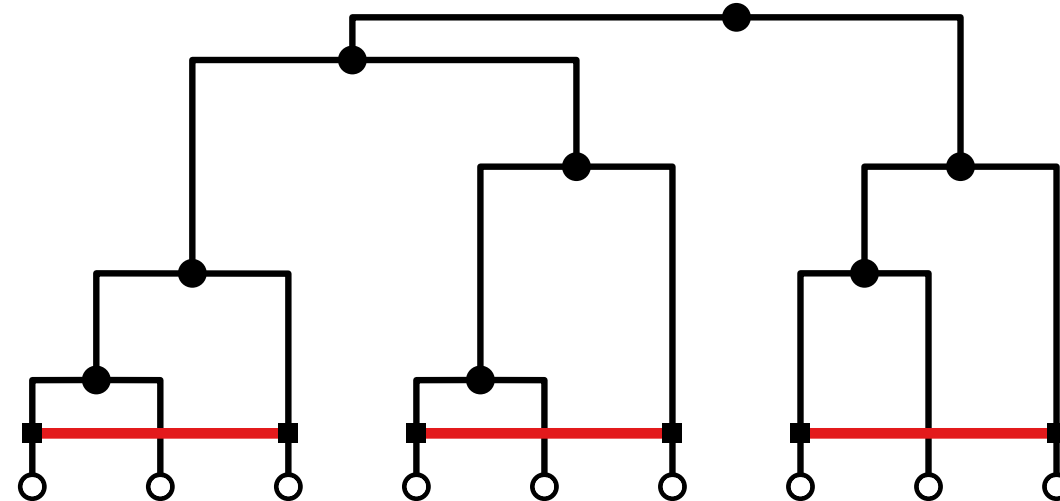
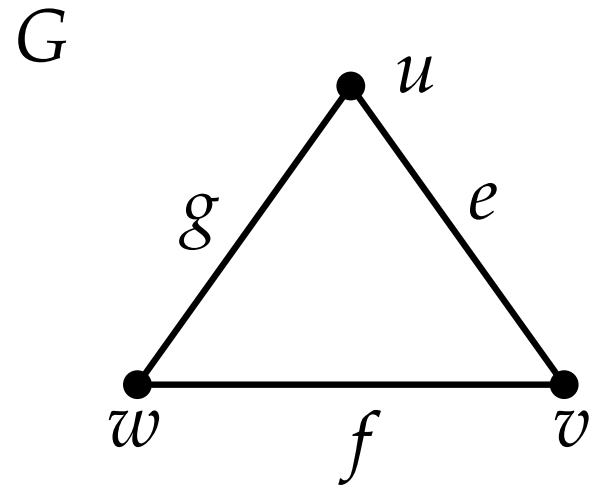
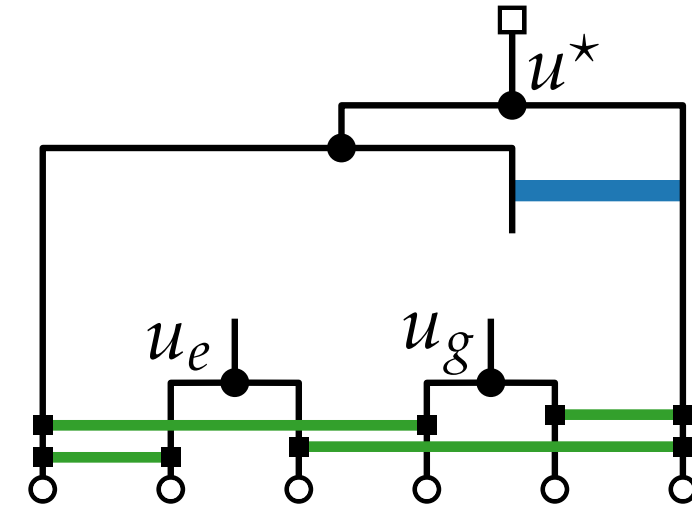
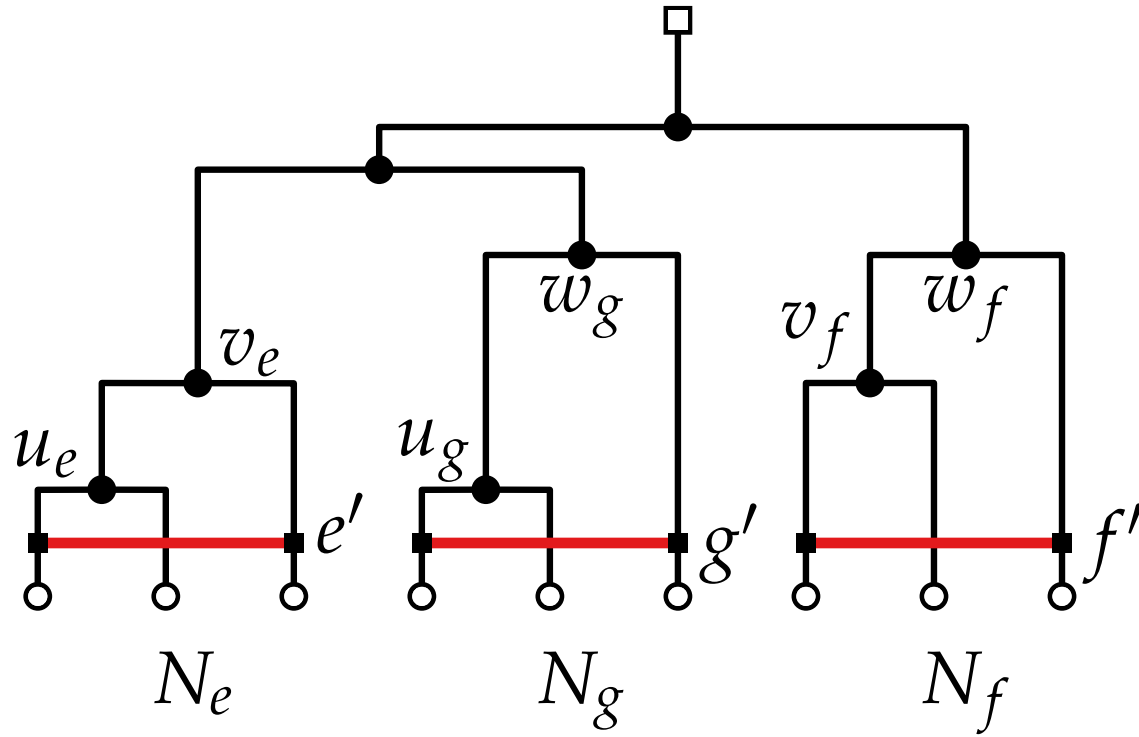
Horizontal style NP-reduction



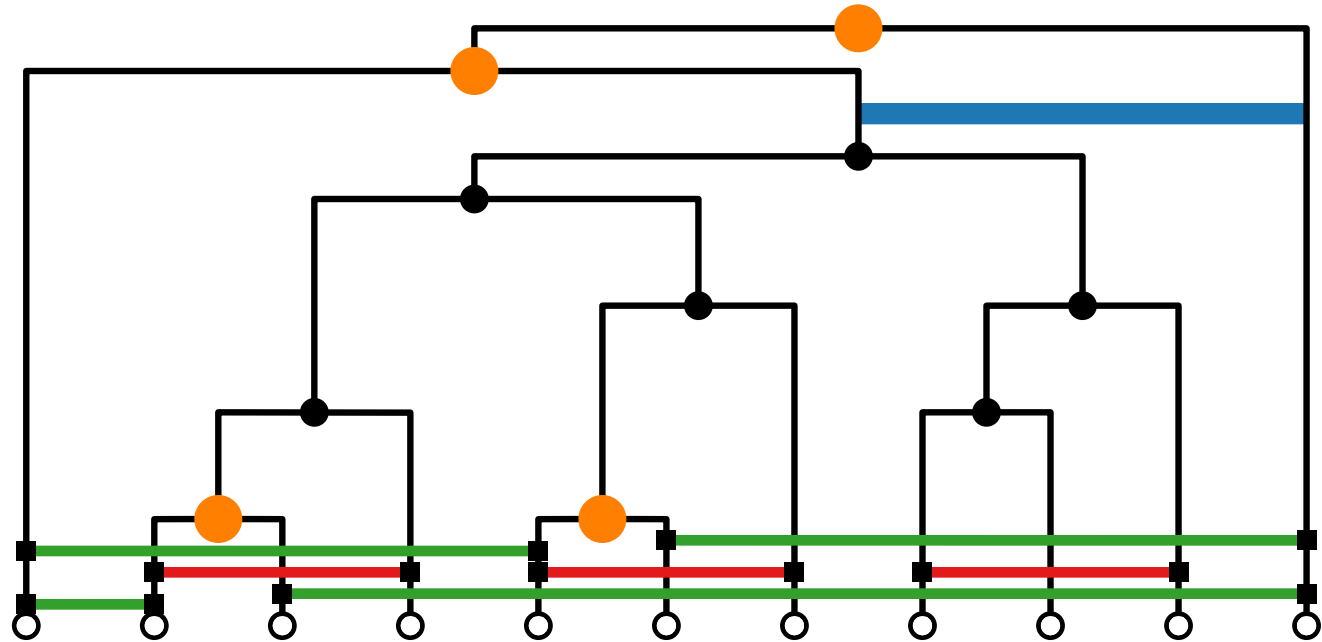
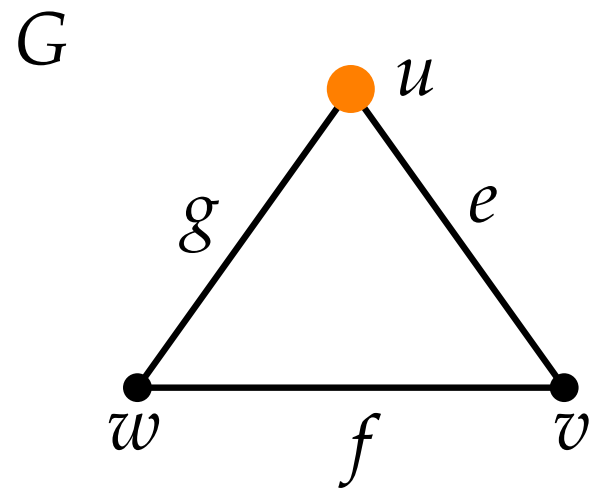
Horizontal style NP-reduction



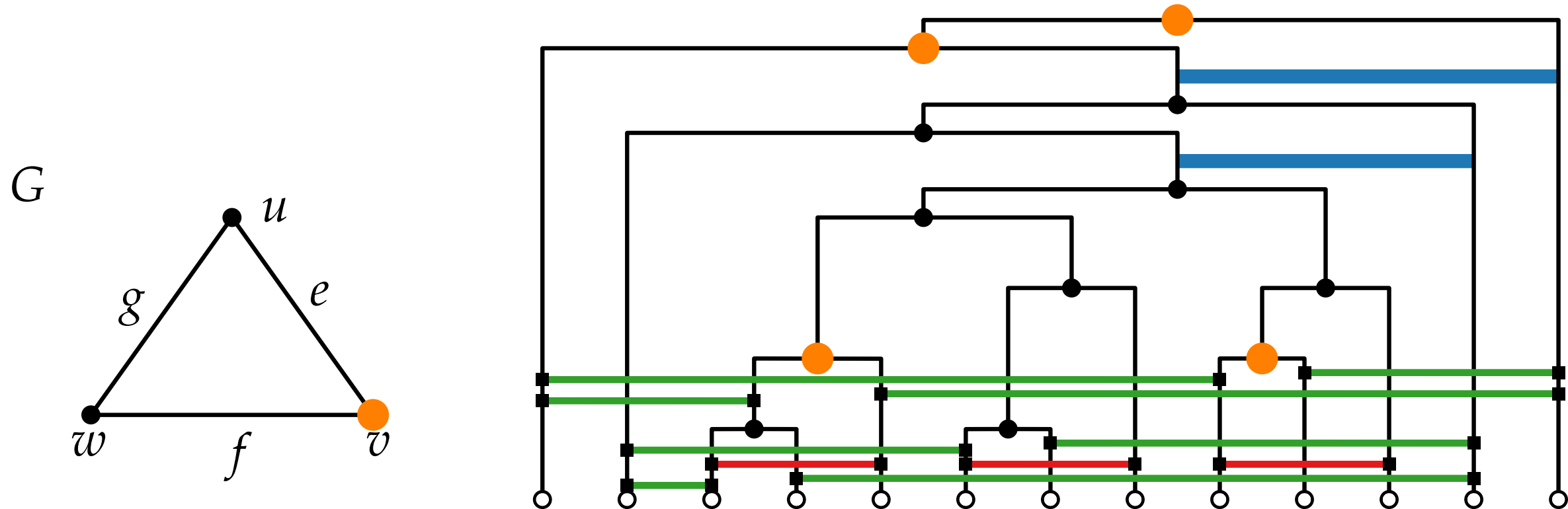
Horizontal style NP-reduction



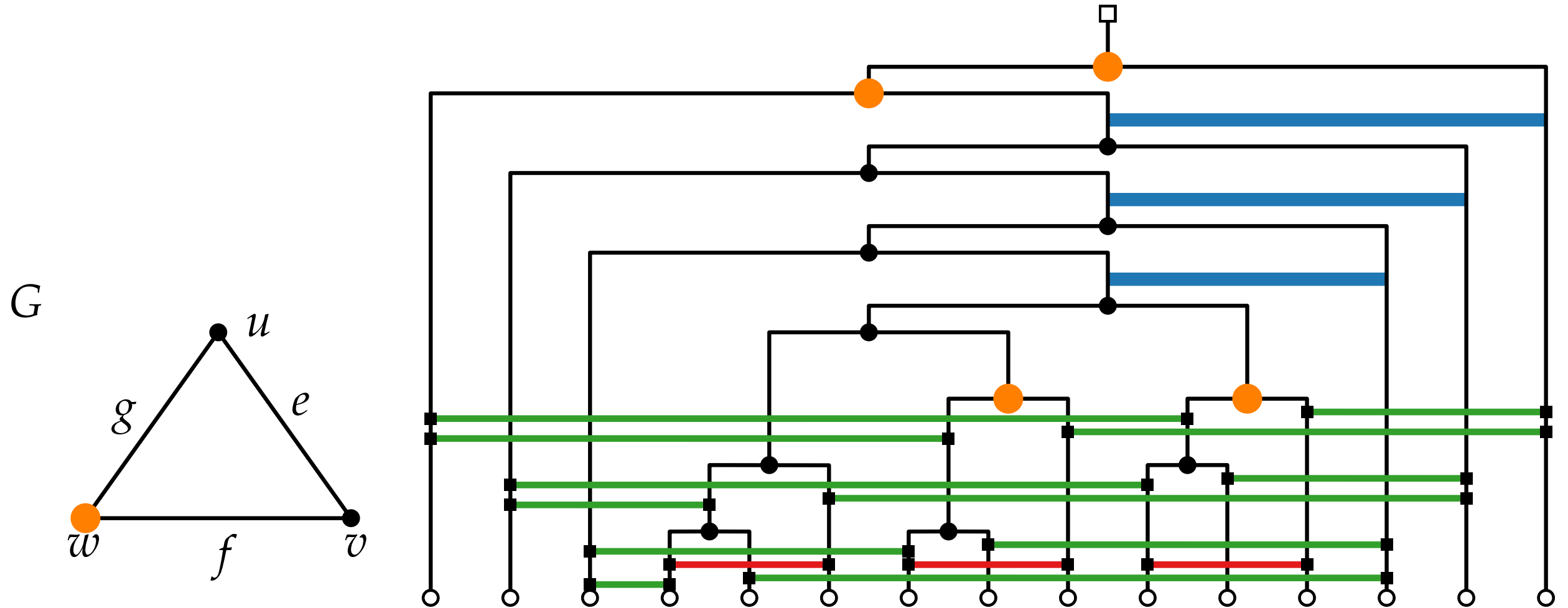
Horizontal style NP-reduction



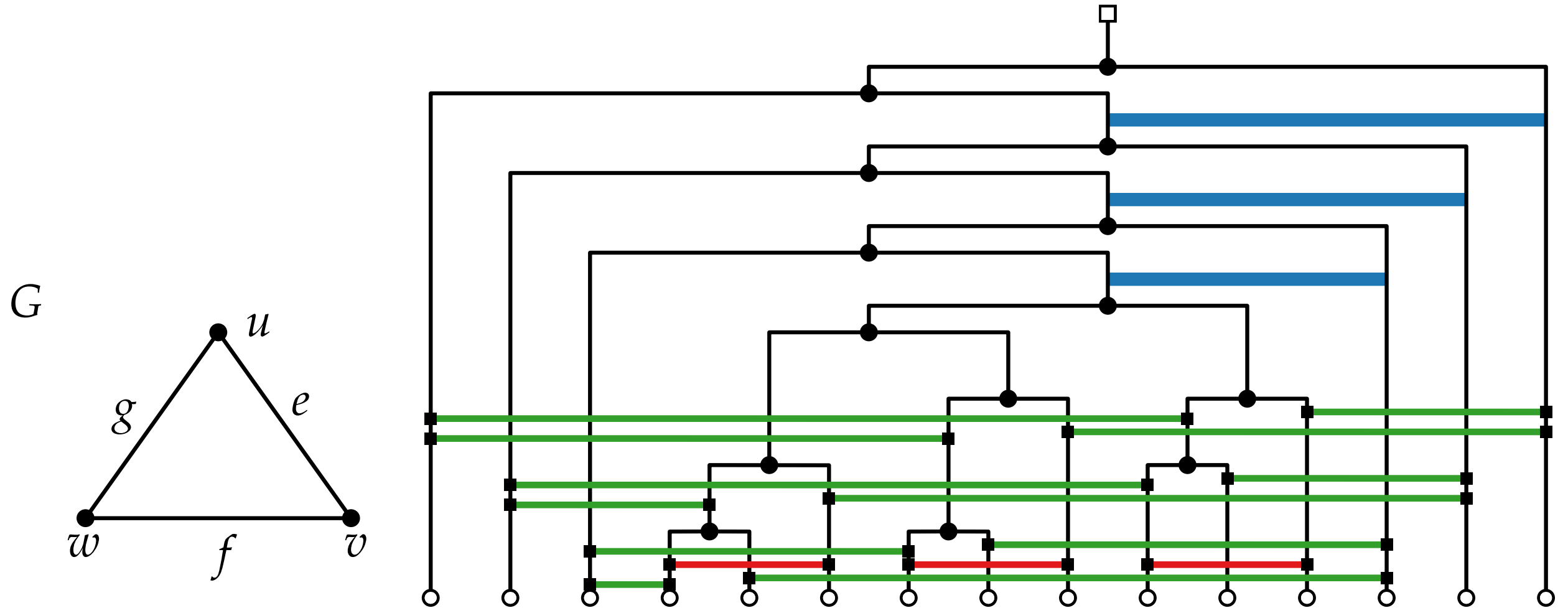
Horizontal style NP-reduction



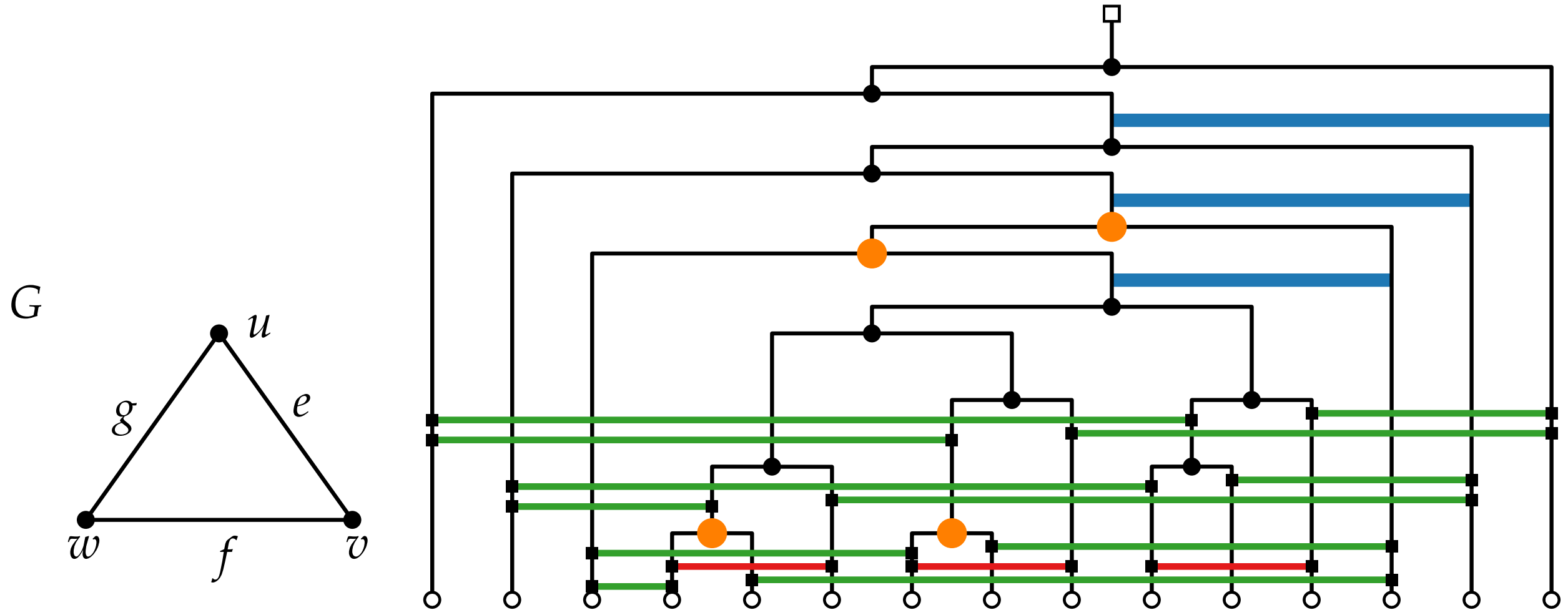
Horizontal style NP-reduction



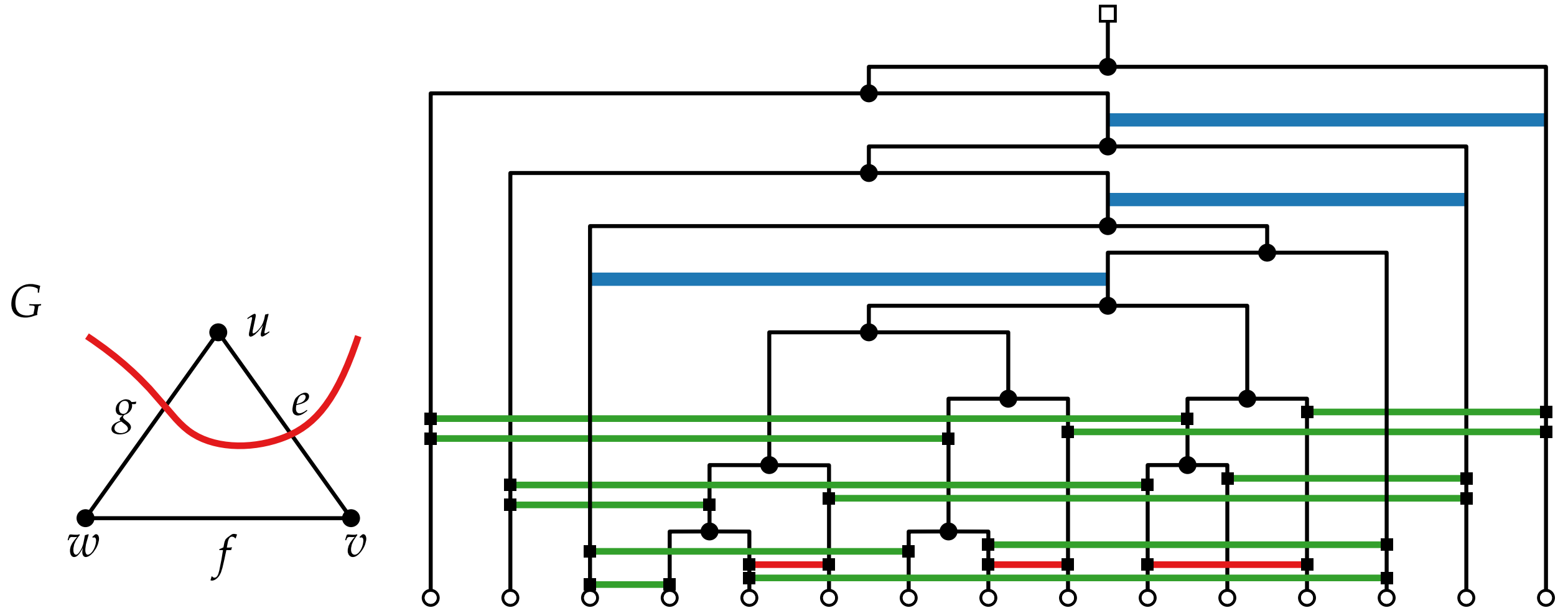
Horizontal style NP-reduction

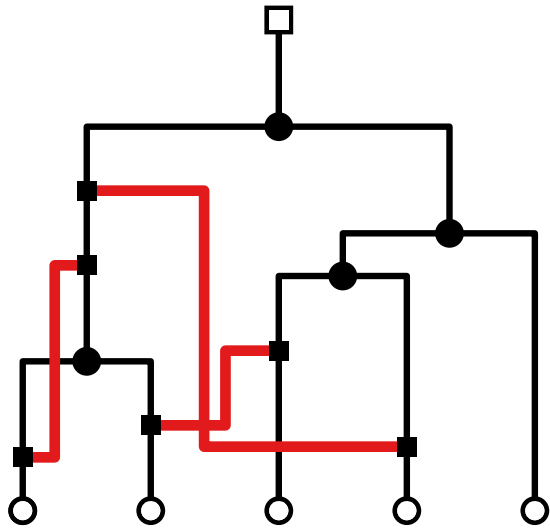


Horizontal style NP-reduction

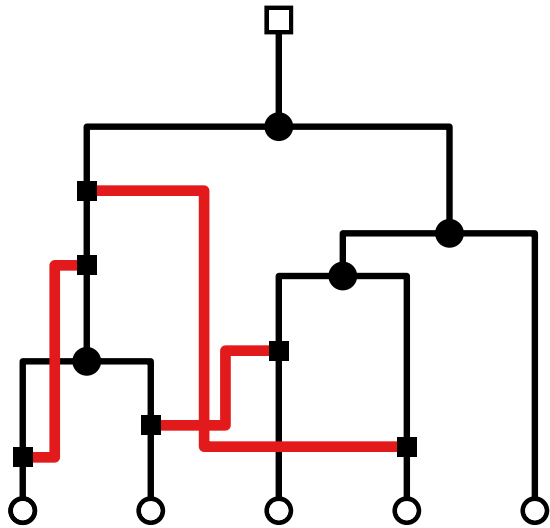


Horizontal style NP-reduction

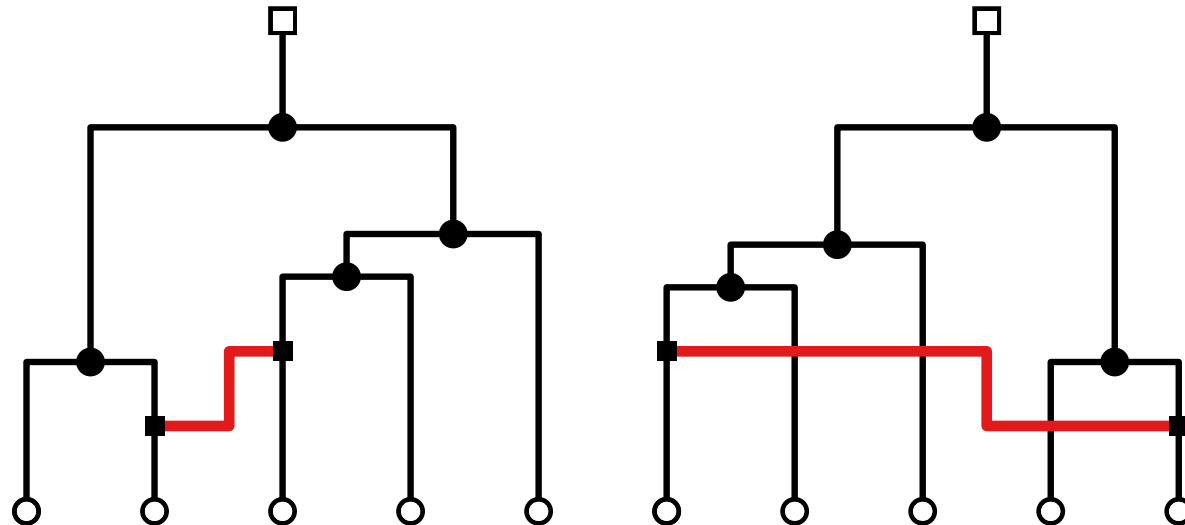


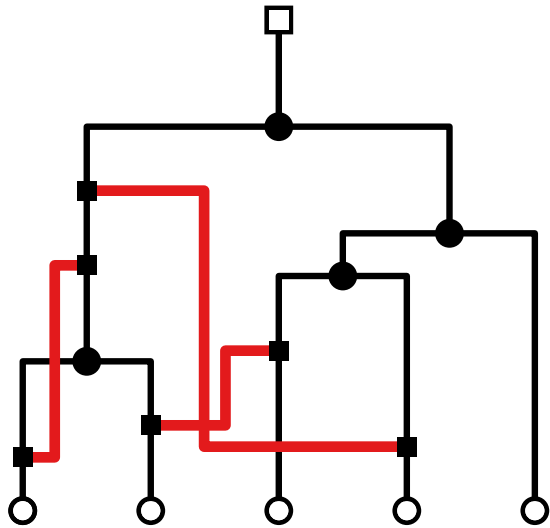


Let N be a tree-based network on n leaves and k **reticulation edges**.
 A **crossing-minimal snake-style** drawing of N can be computed in $\mathcal{O}(2^k nk)$ time.

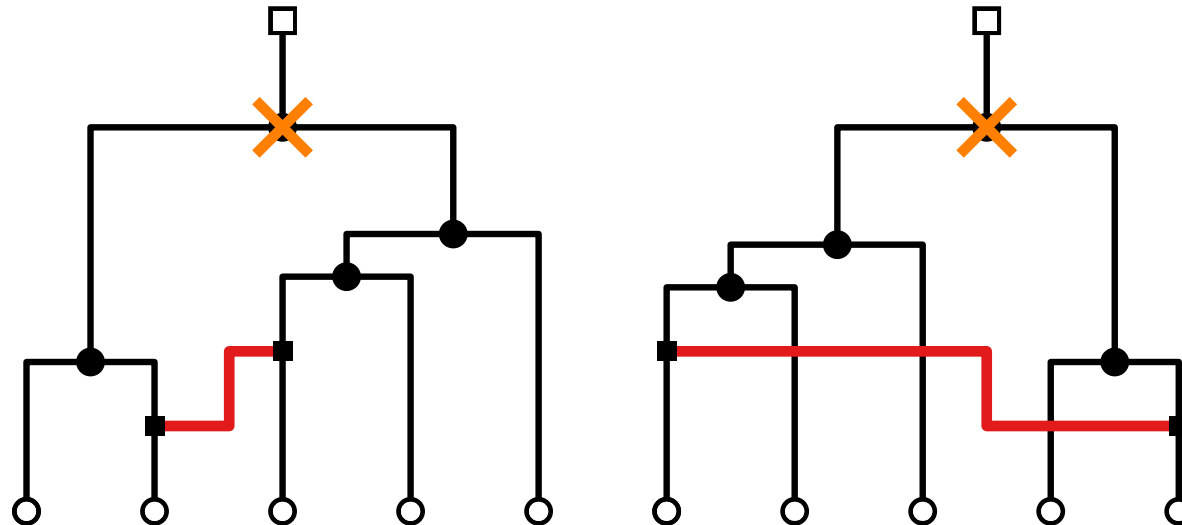


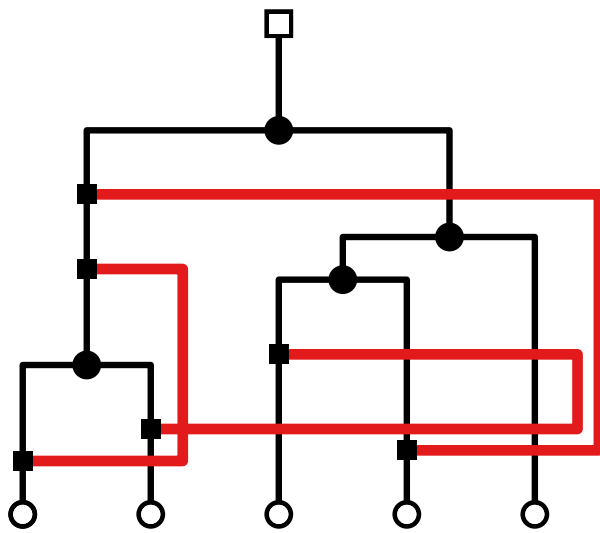
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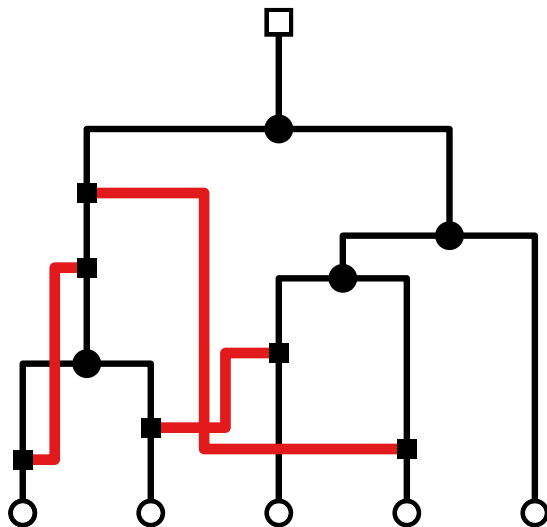
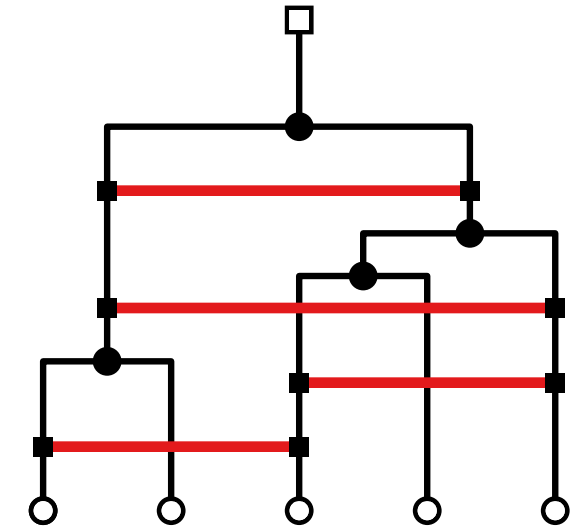
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A **crossing-minimal snake-style** drawing of N can be computed in $\mathcal{O}(2^k nk)$ time.





Let N be a tree-based network on n leaves and k **reticulation edges**.
A **crossing-minimal ear-style** drawing of N can be computed in $\mathcal{O}(nk)$ time.

The CROSSING MINIMISATION problem for **horizontal-style** drawings of tree-based networks is NP-complete.



Let N be a tree-based network on n leaves and k **reticulation edges**.
A **crossing-minimal snake-style** drawing of N can be computed in $\mathcal{O}(2^k nk)$ time.