Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG
Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems
Institute for Informatics

Maximum Matchings and Minimum Blocking Sets in Θ_6-Graphs

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Ahmad Biniaz
Kshitij Jain

Proximity Graphs

P: set of n points in the plane

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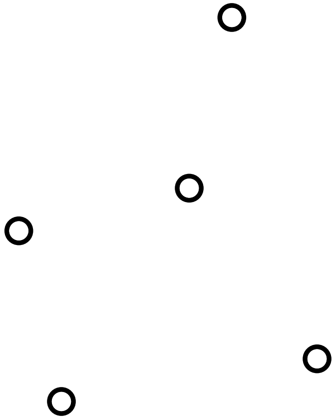

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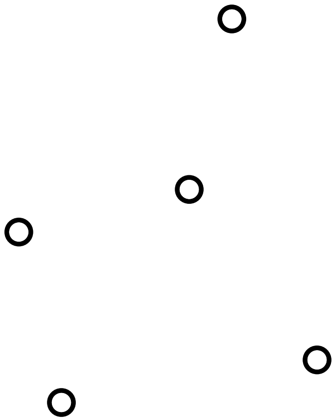

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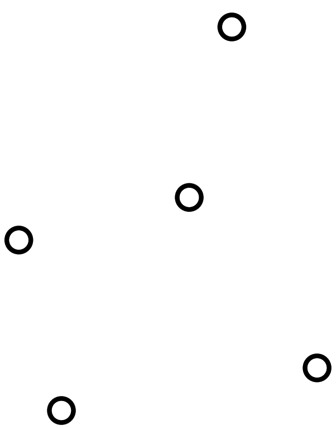

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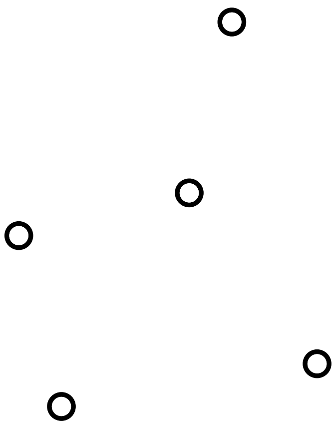

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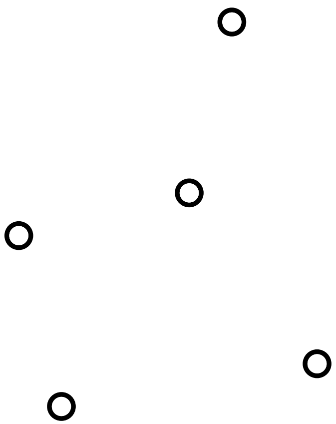

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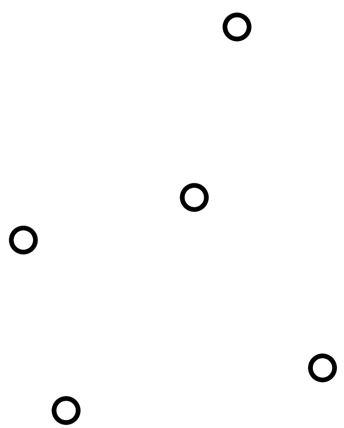

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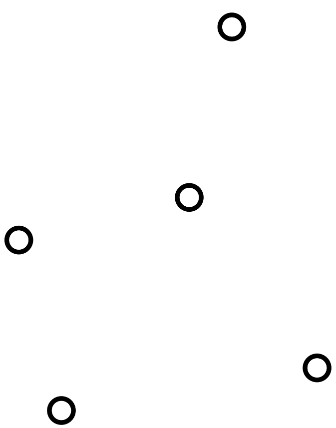

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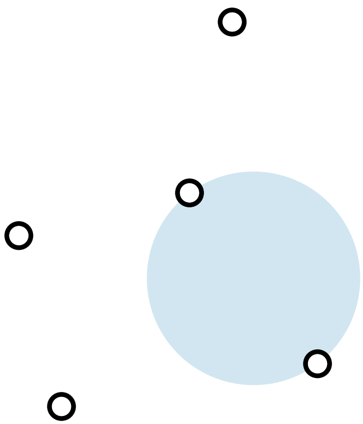

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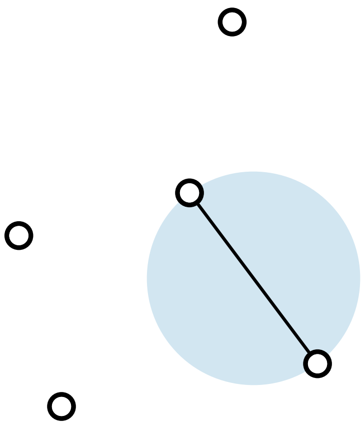

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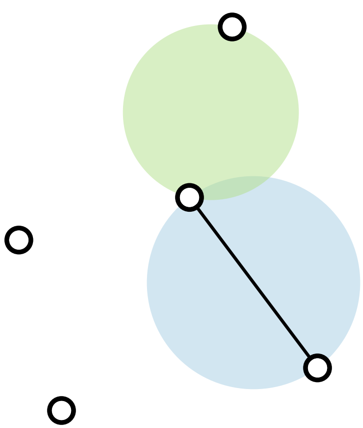

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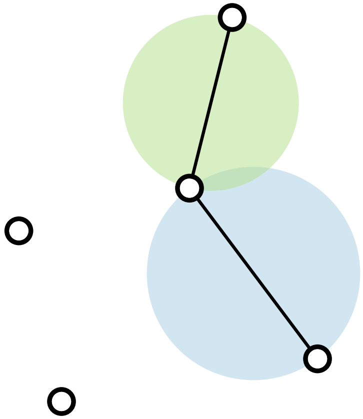

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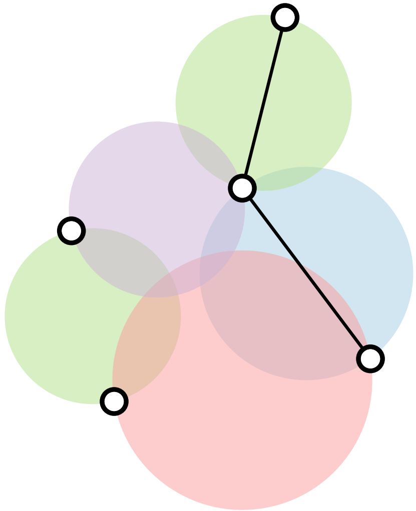

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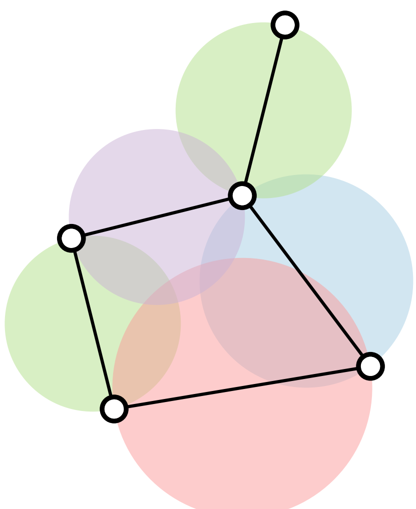

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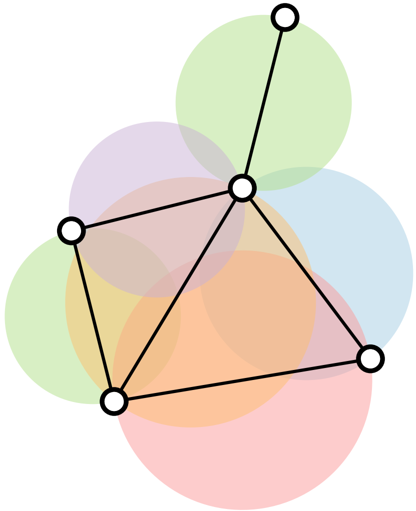

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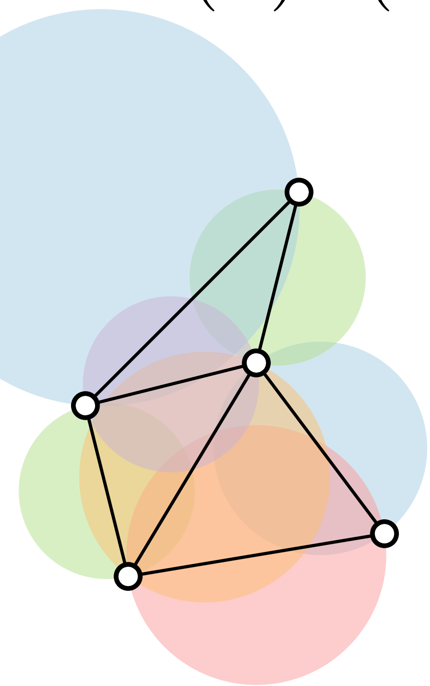

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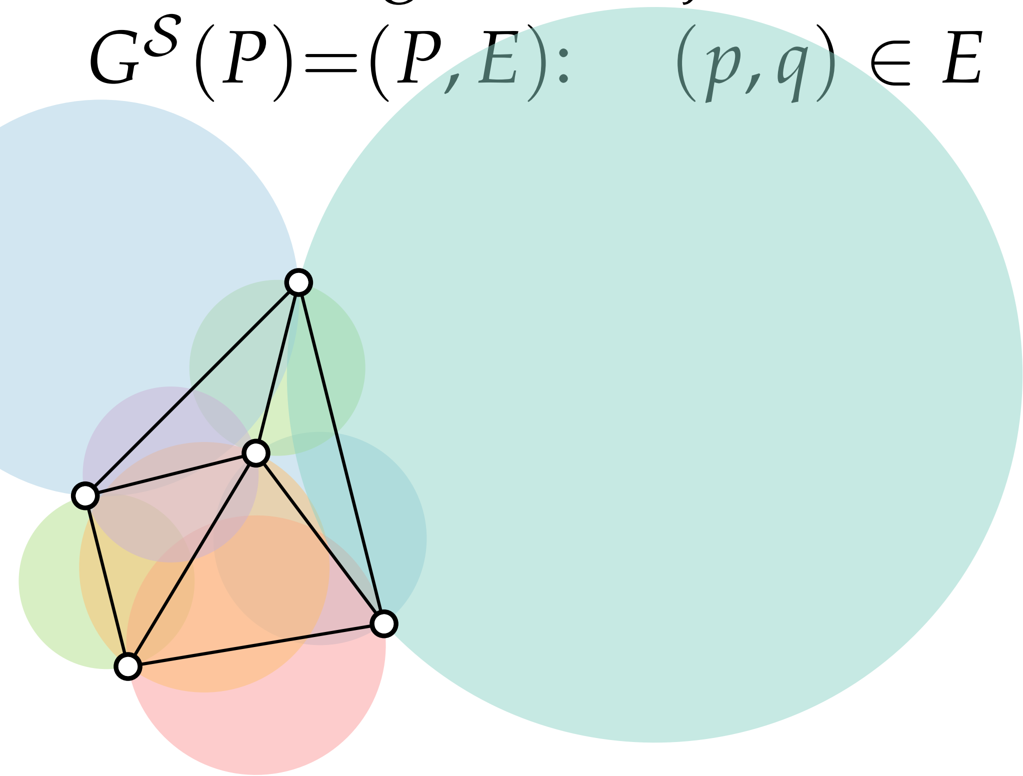

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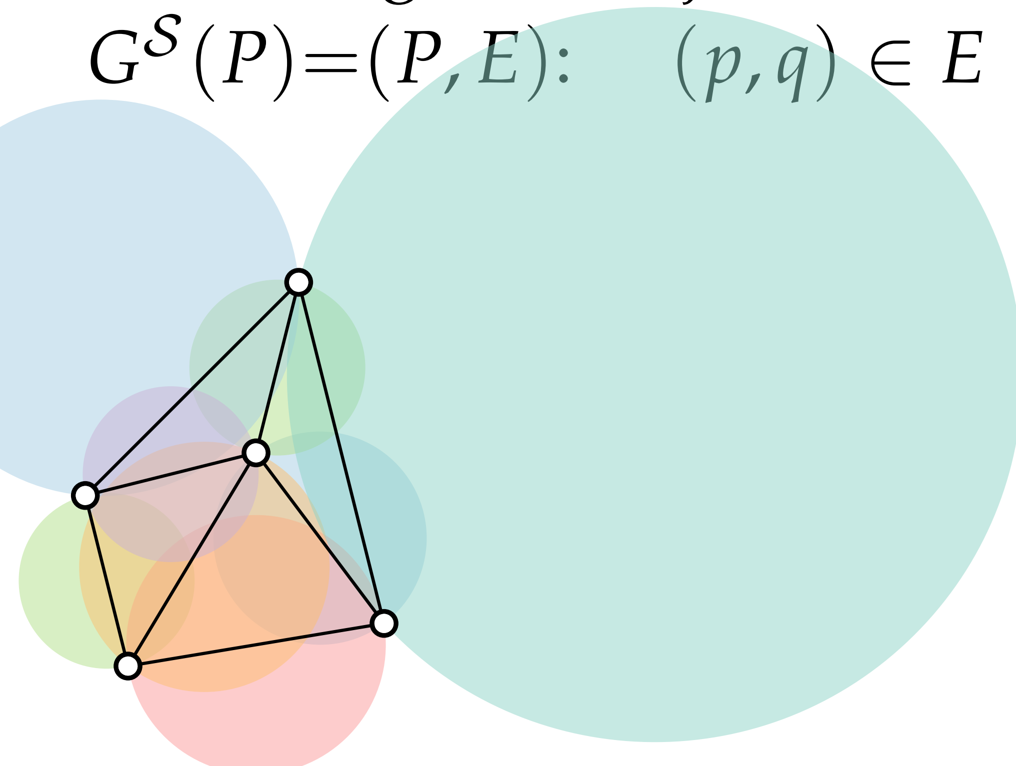

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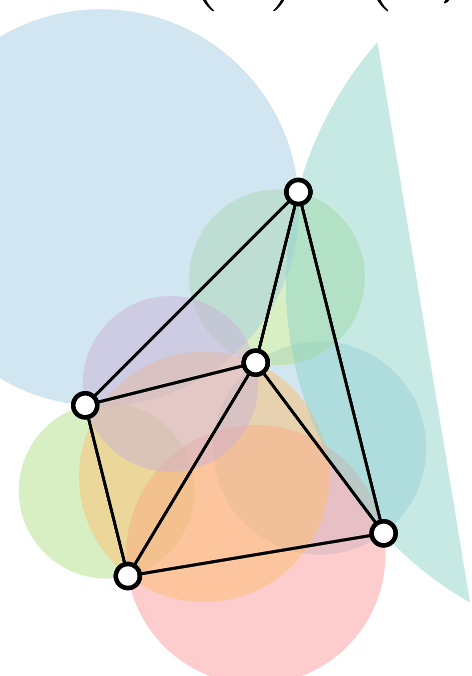

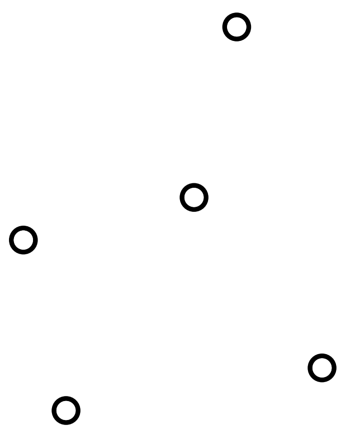

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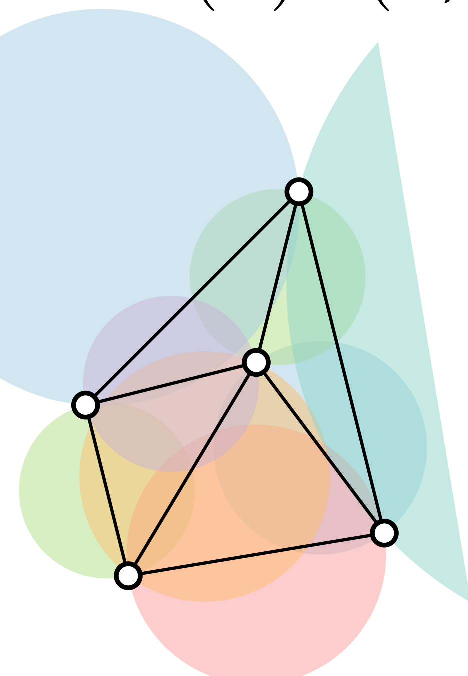

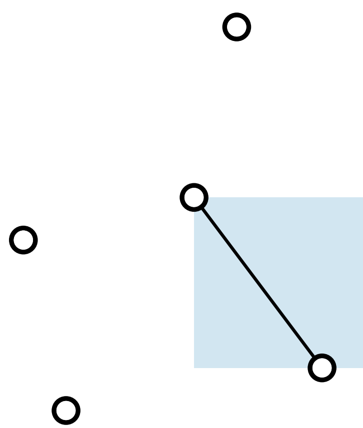

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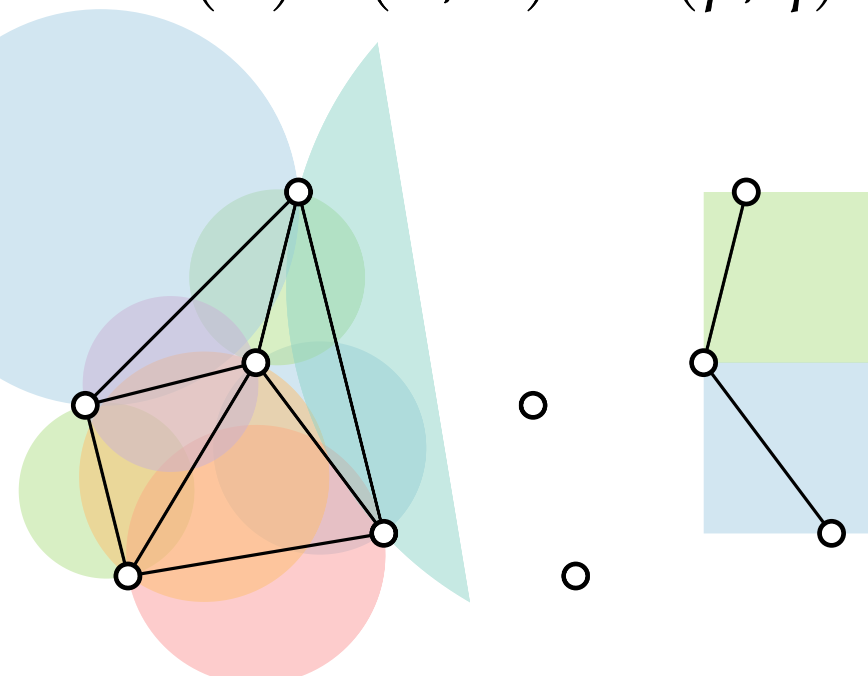

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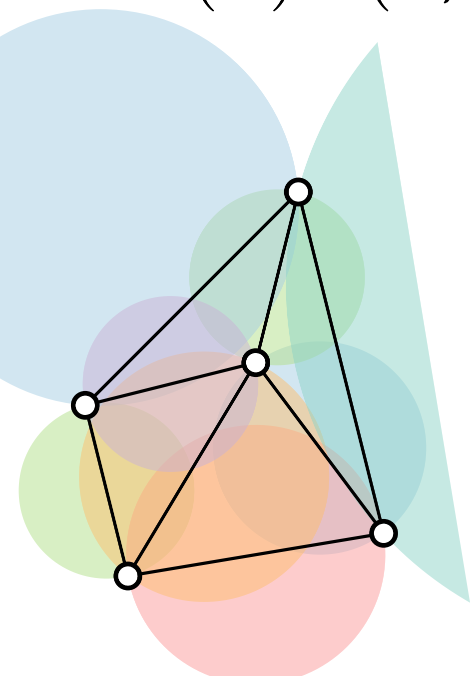

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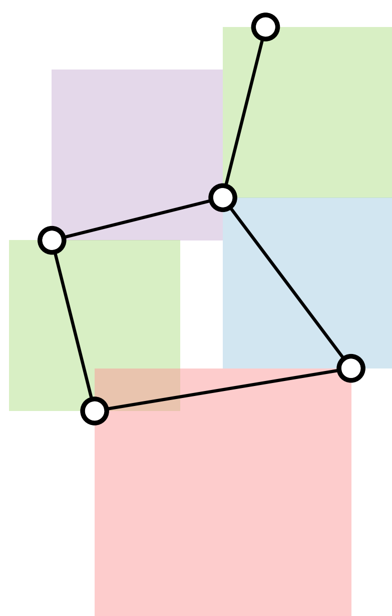

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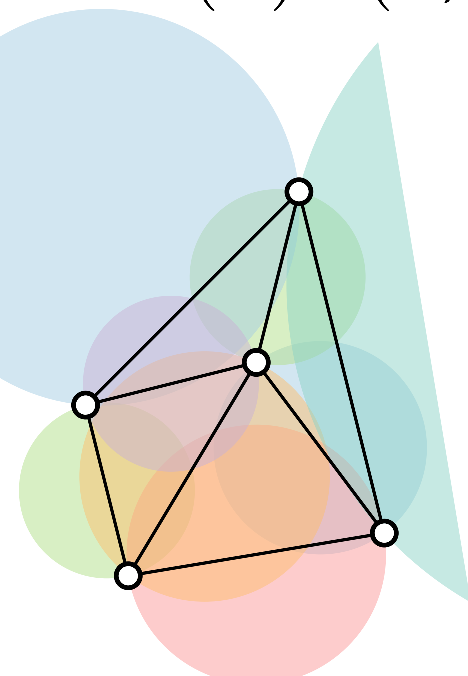

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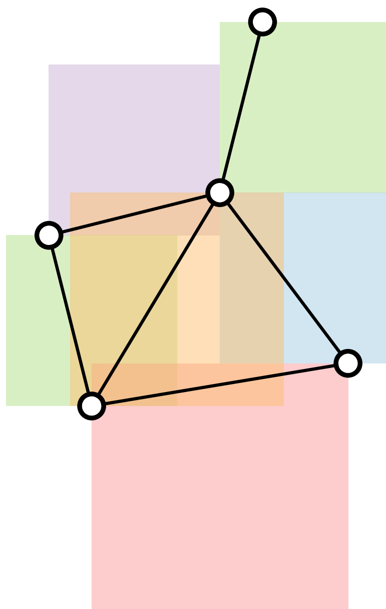

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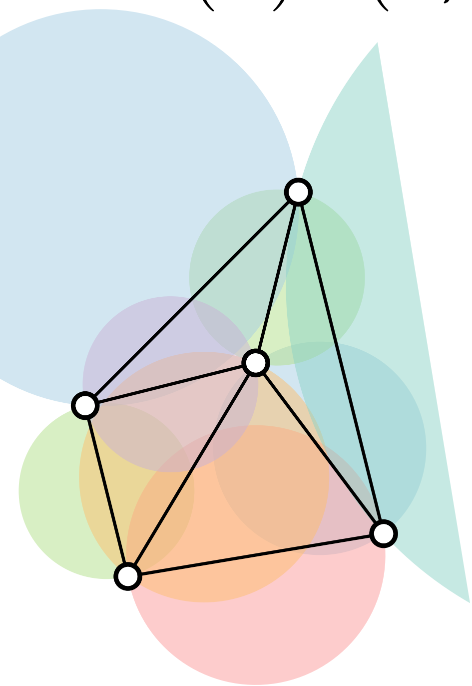

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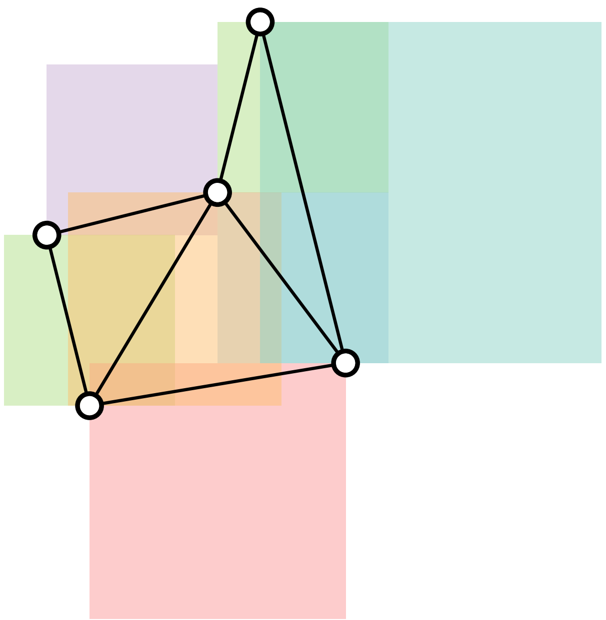

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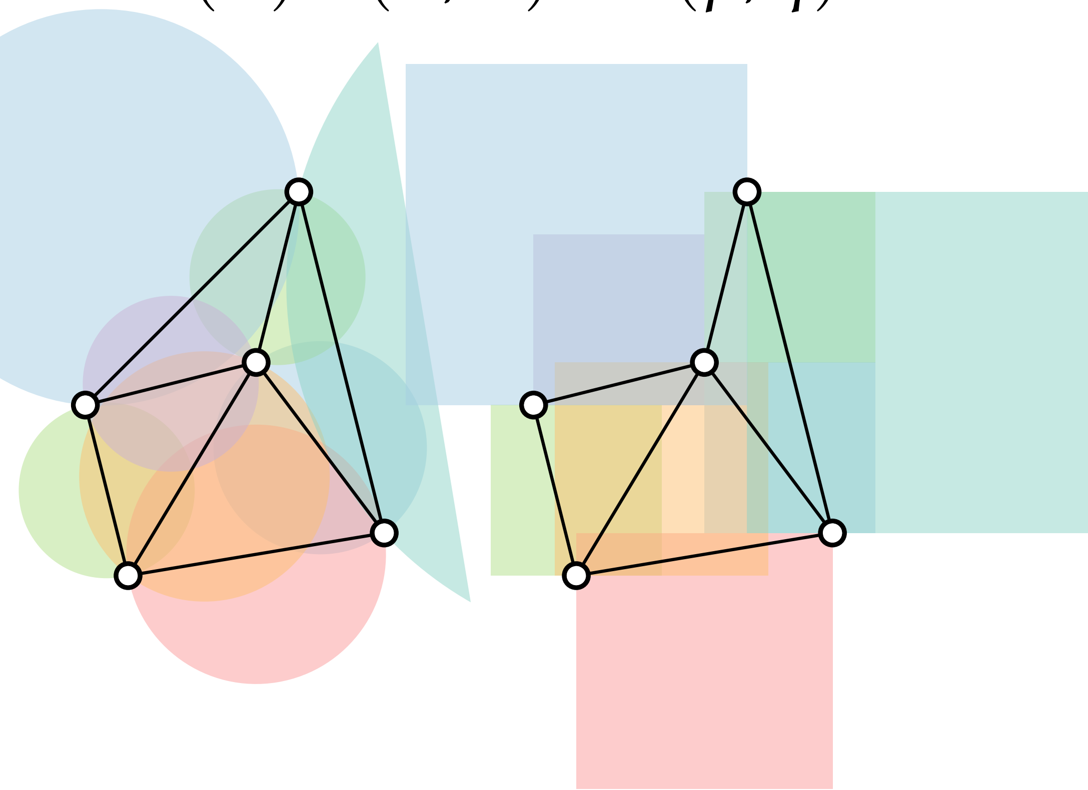

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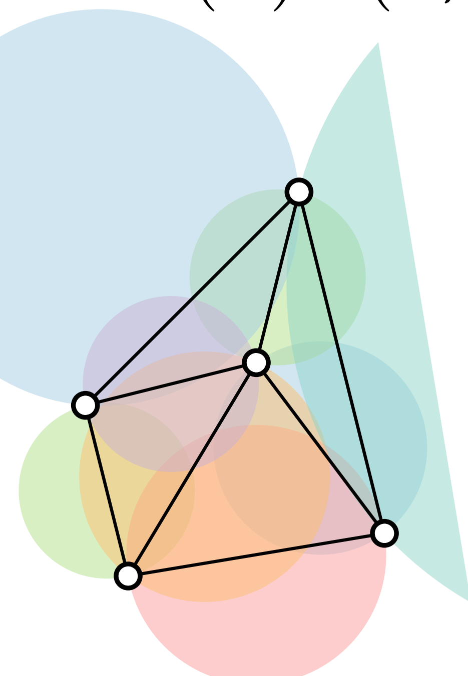

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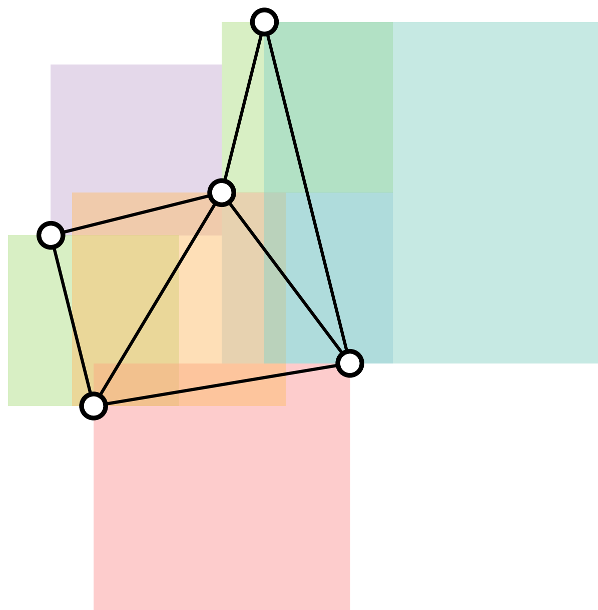

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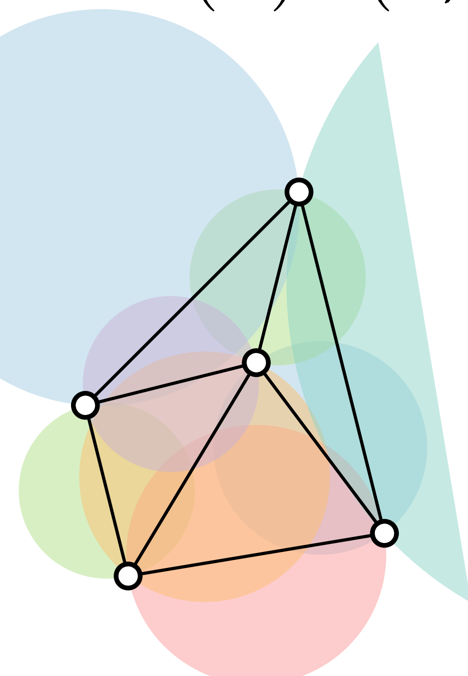
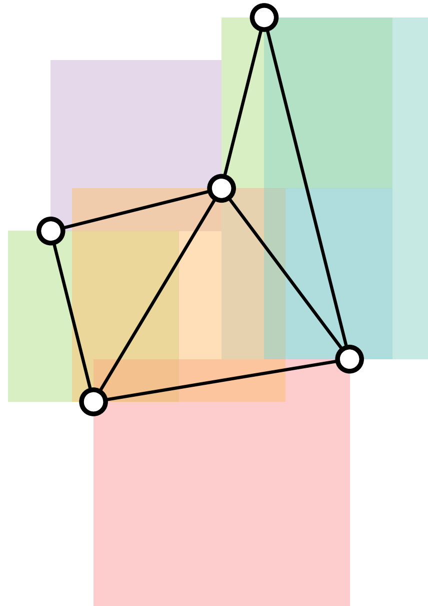
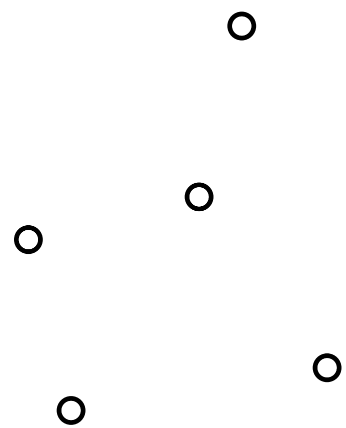

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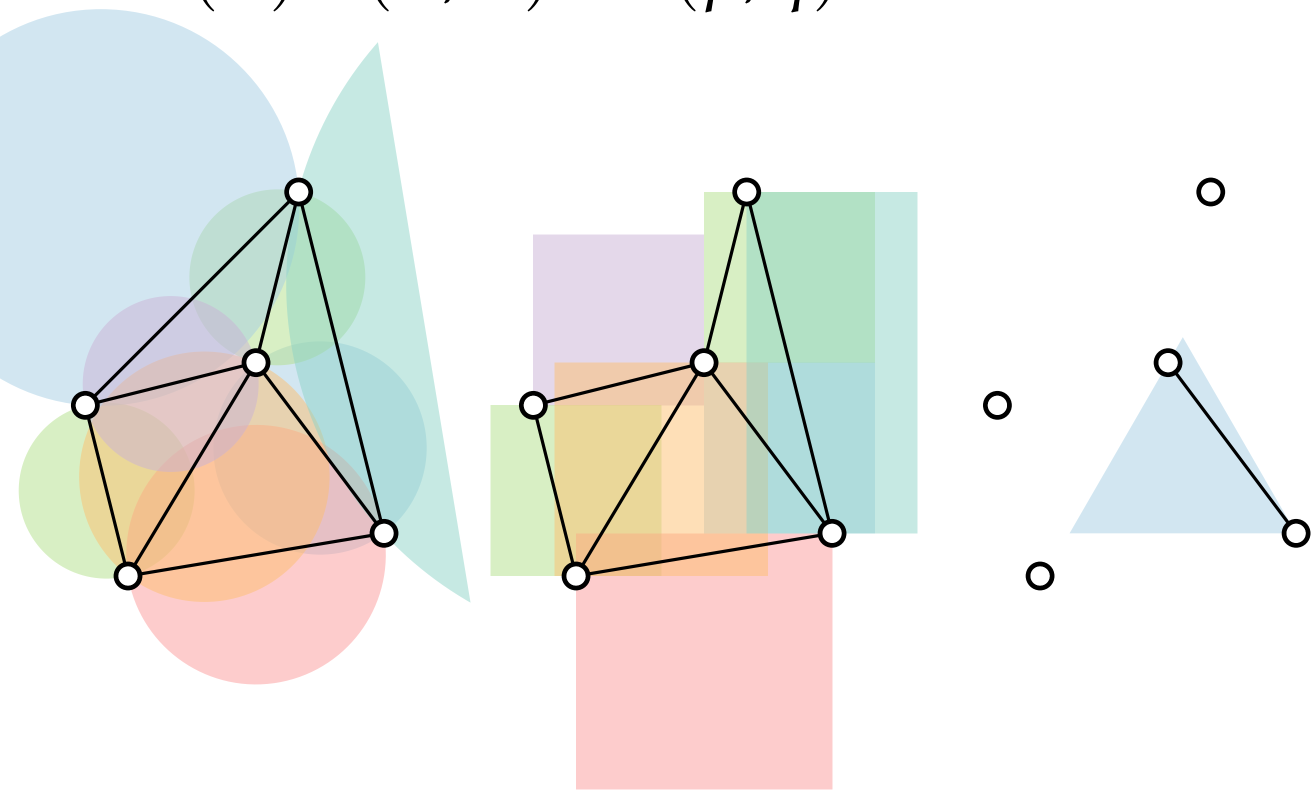

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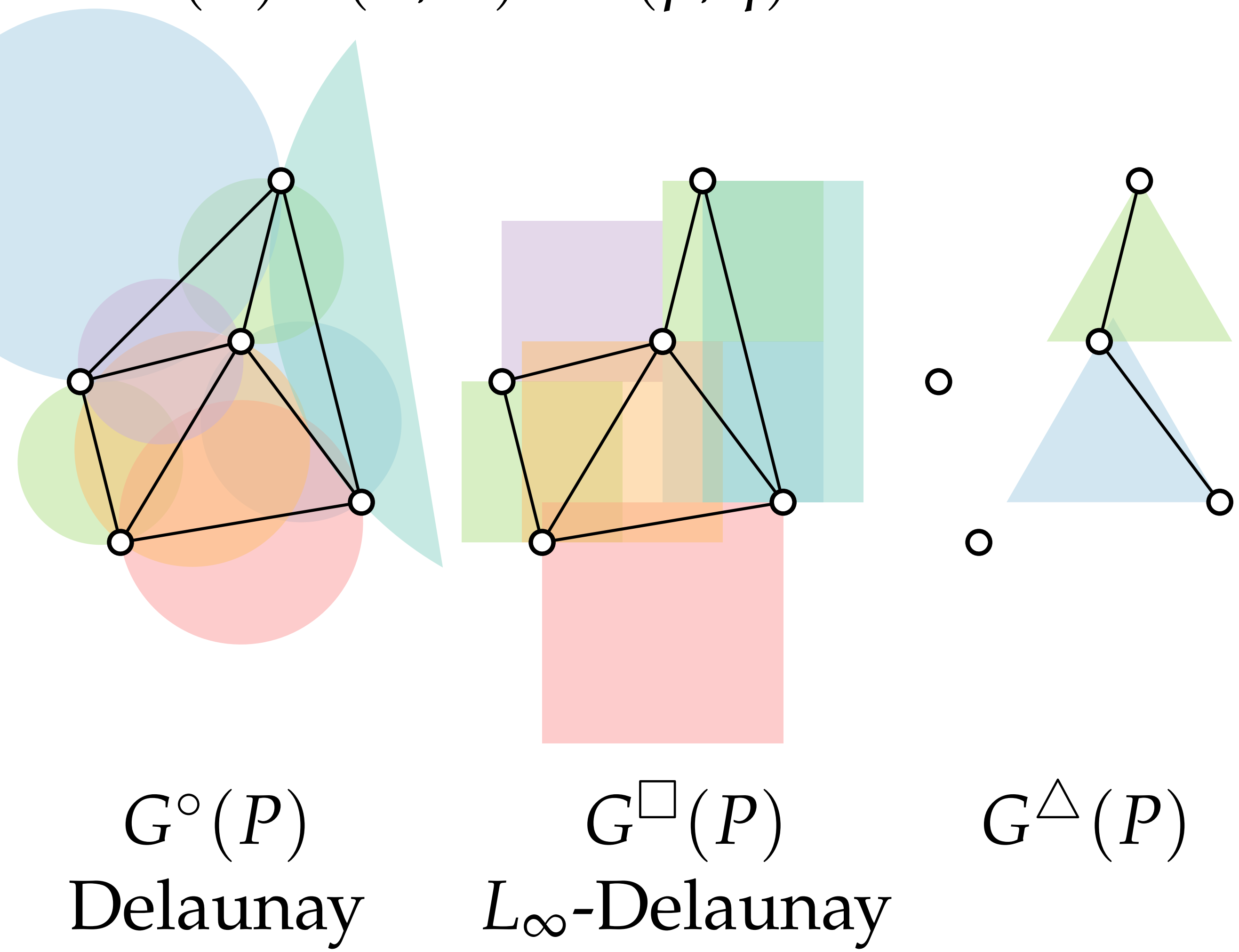

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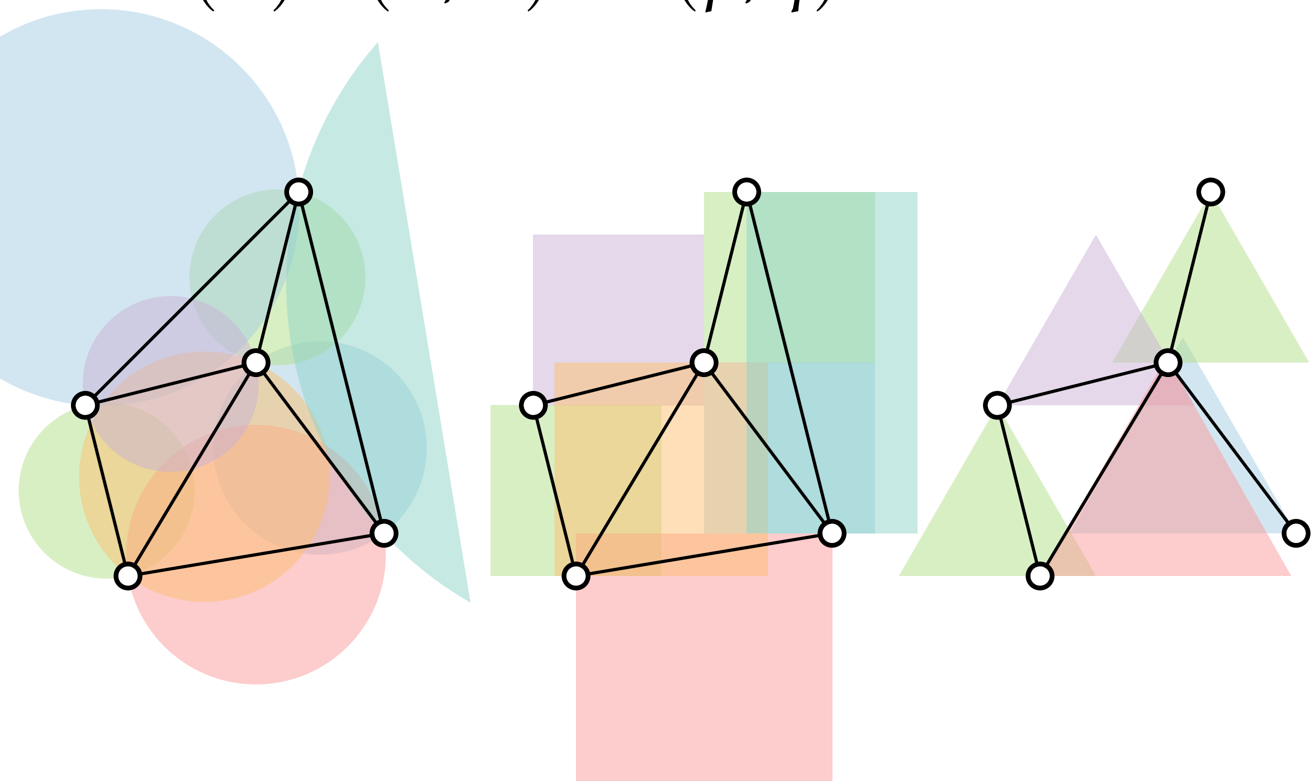

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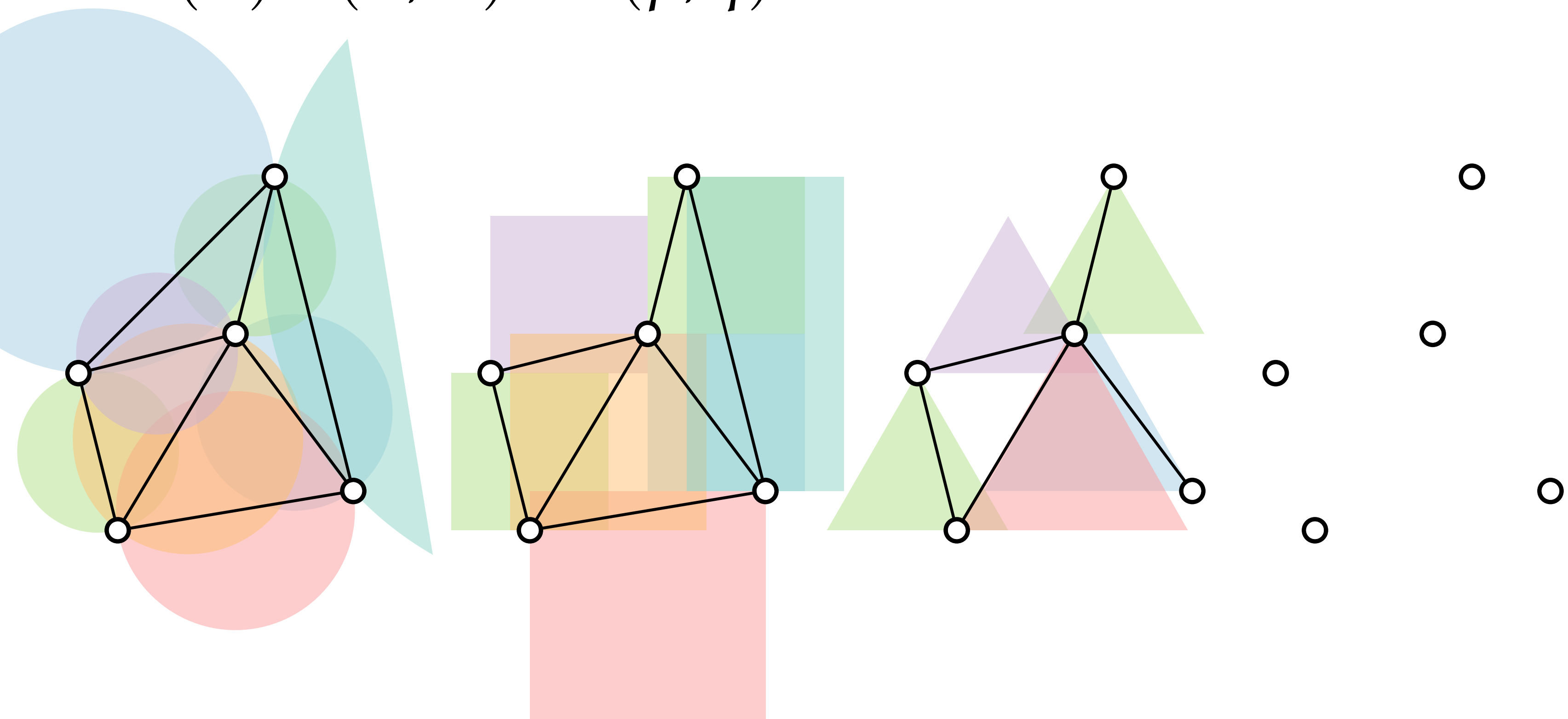

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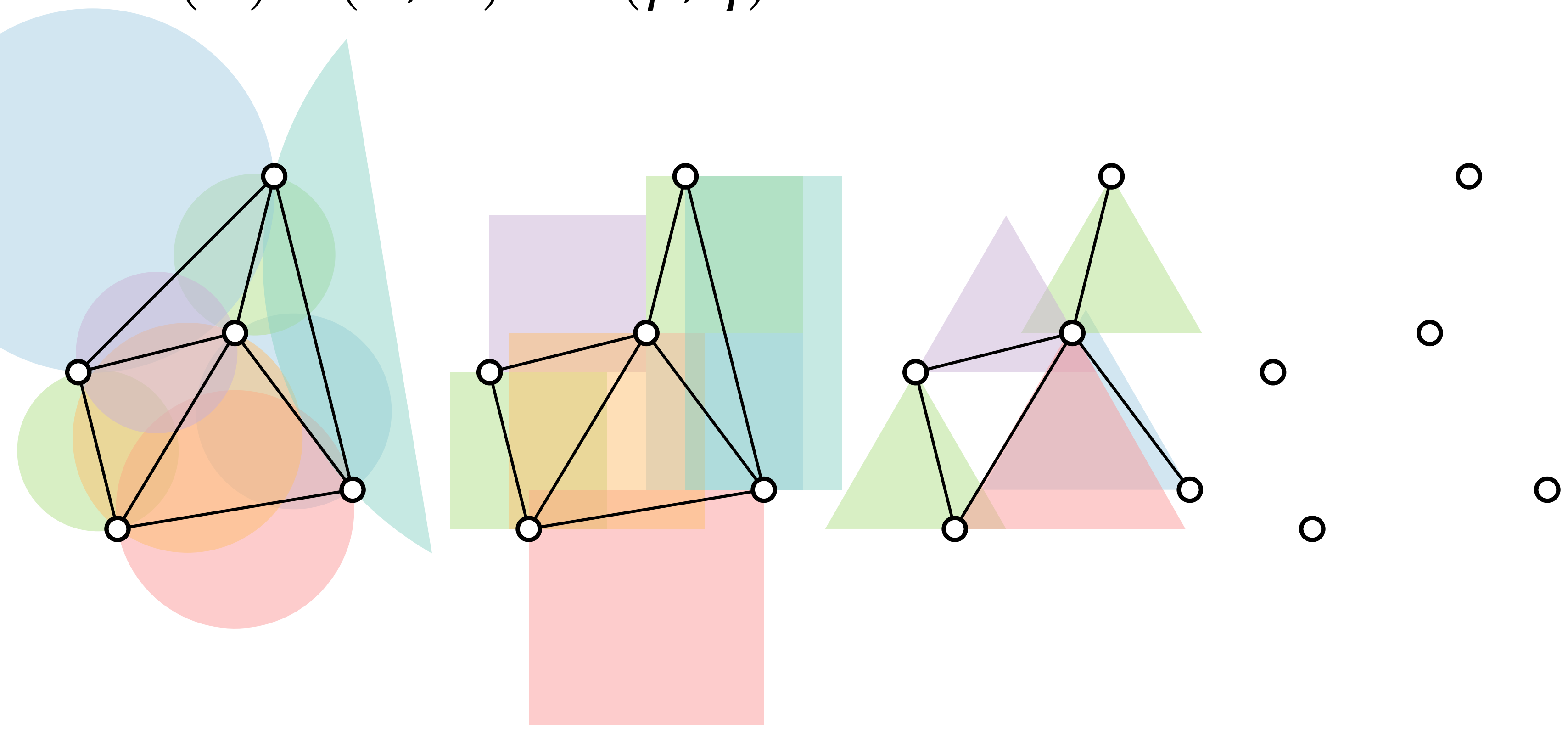

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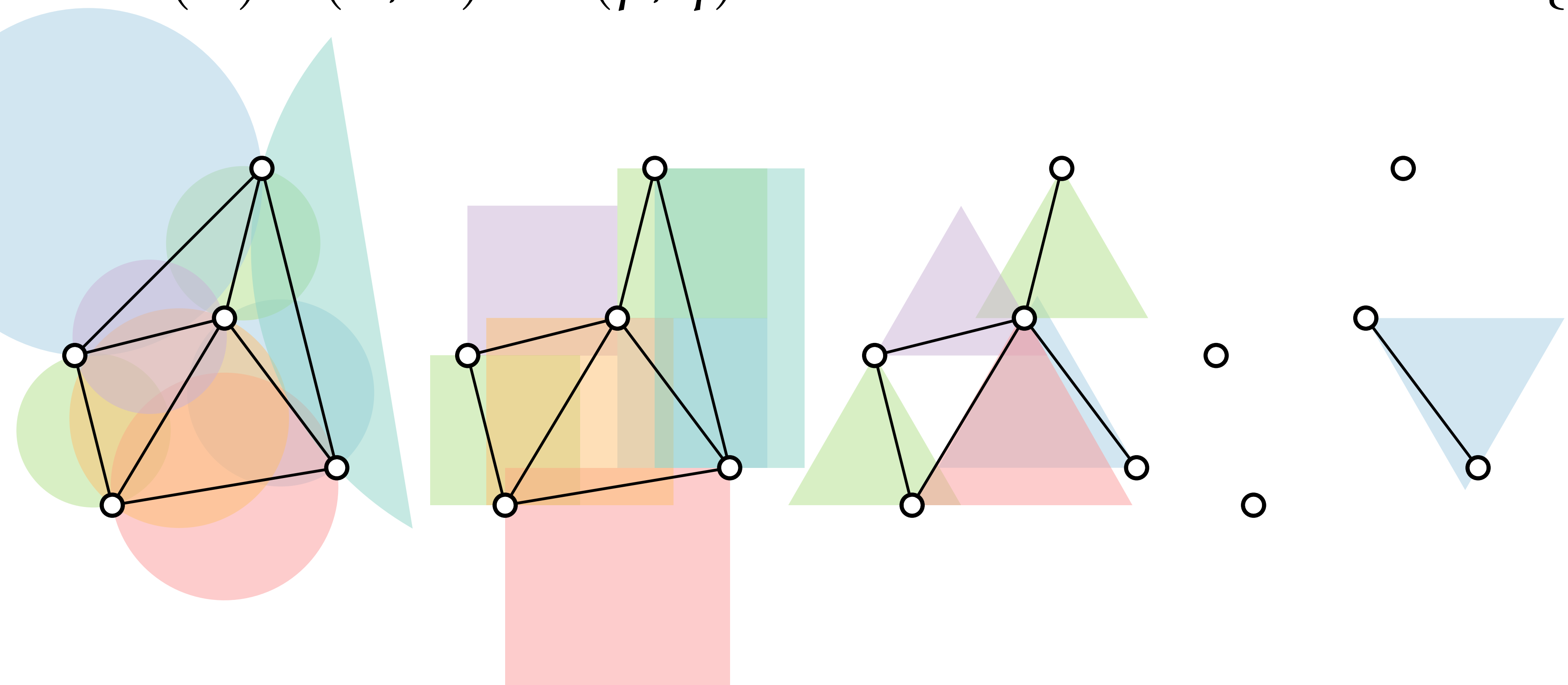

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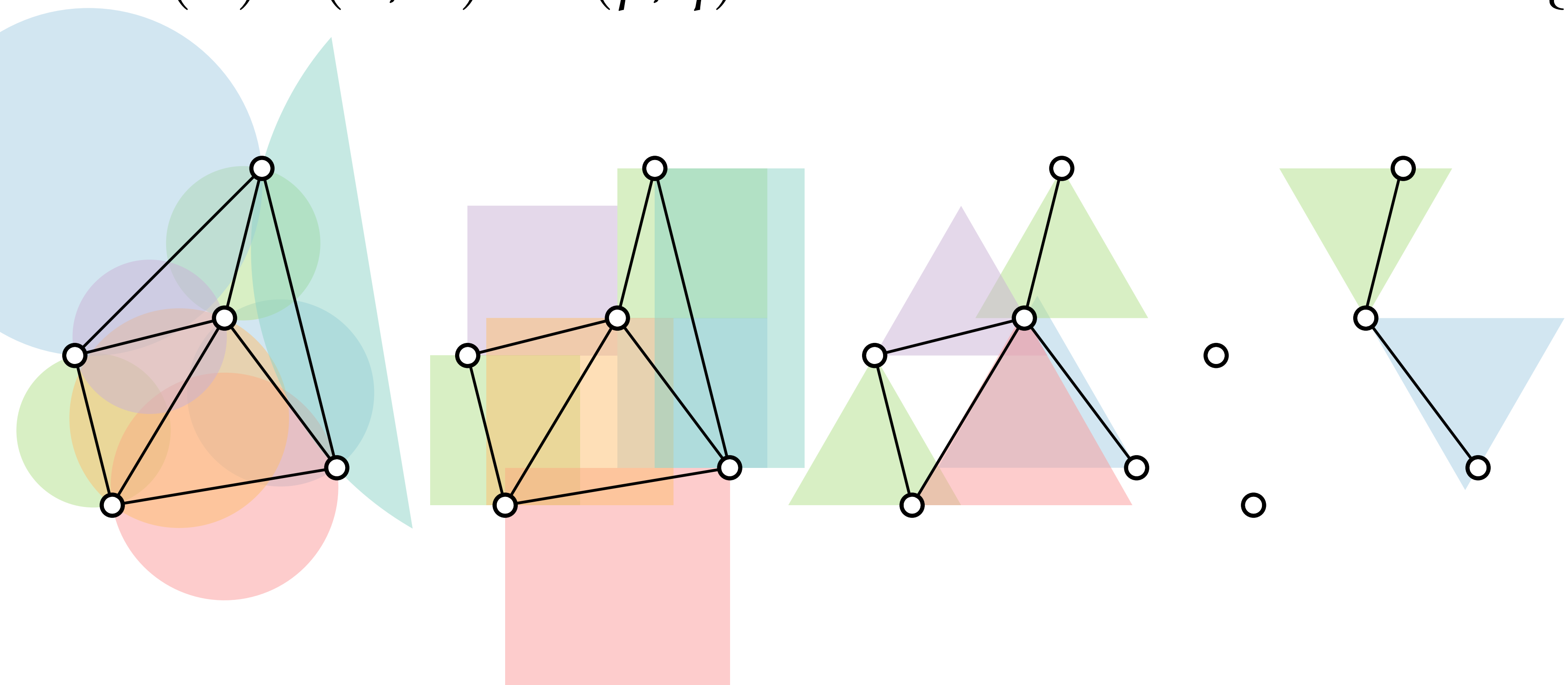

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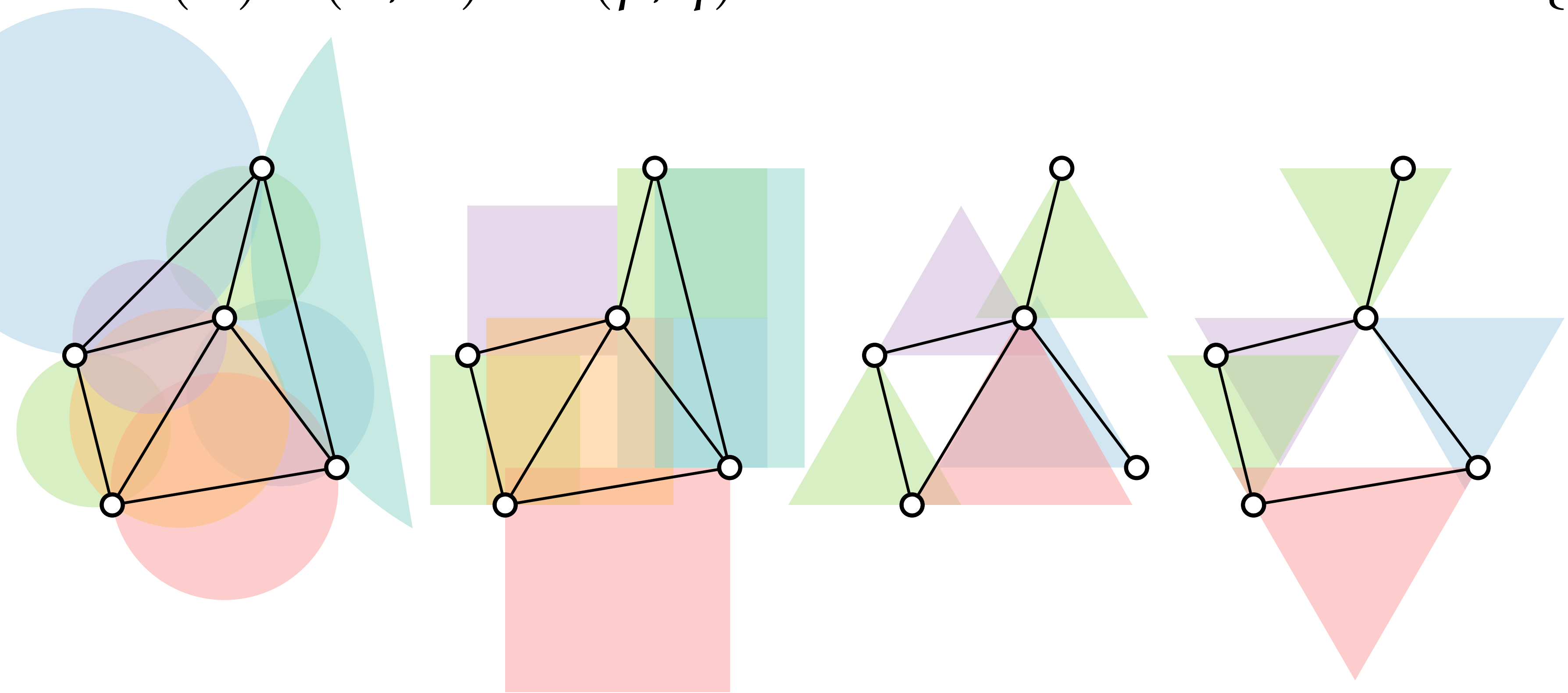

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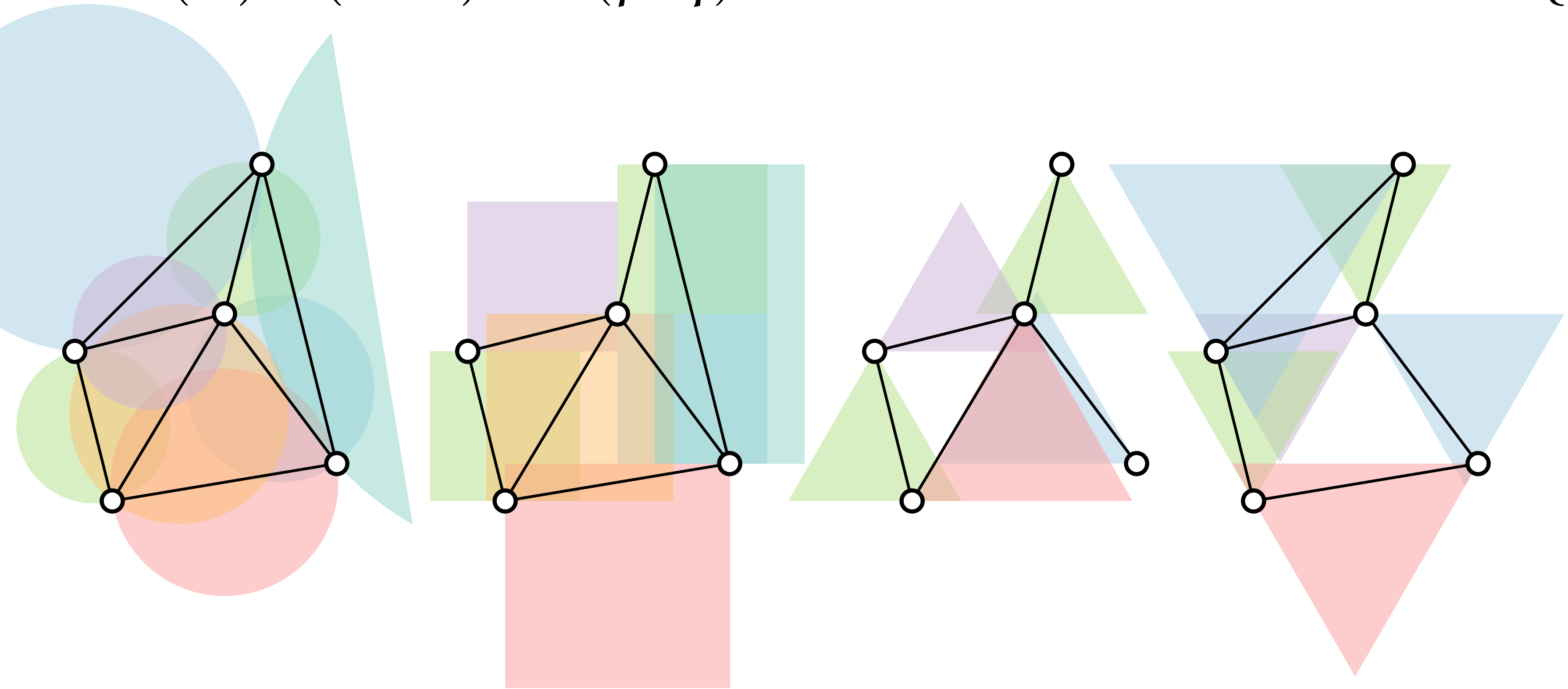

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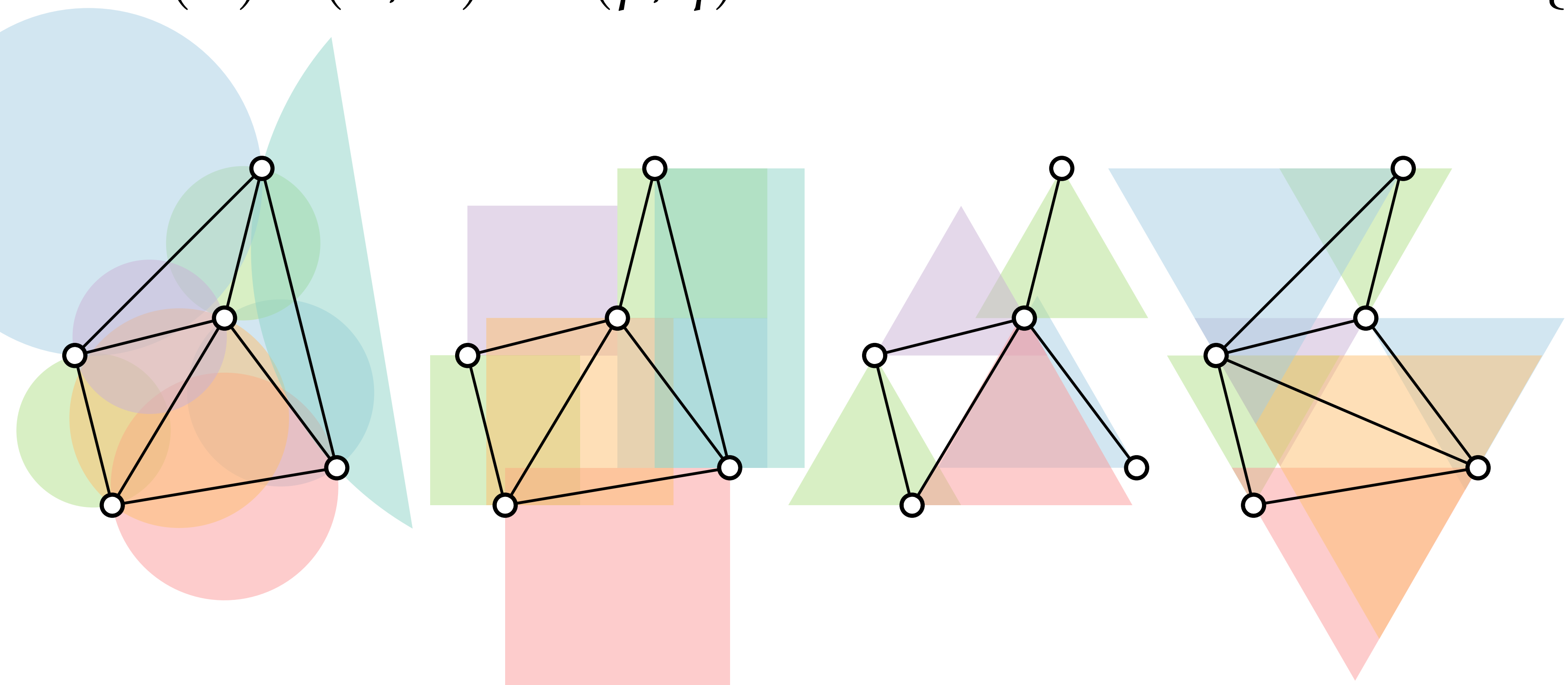
$G^{\mathcal{S}}(P)=(P,E)$: $\quad (p,q) \in E \;\Leftrightarrow\; \exists S \in \mathcal{S} : \; S \cap P = \{p,q\}$



$G^{\circ}(P)$ Delaunay

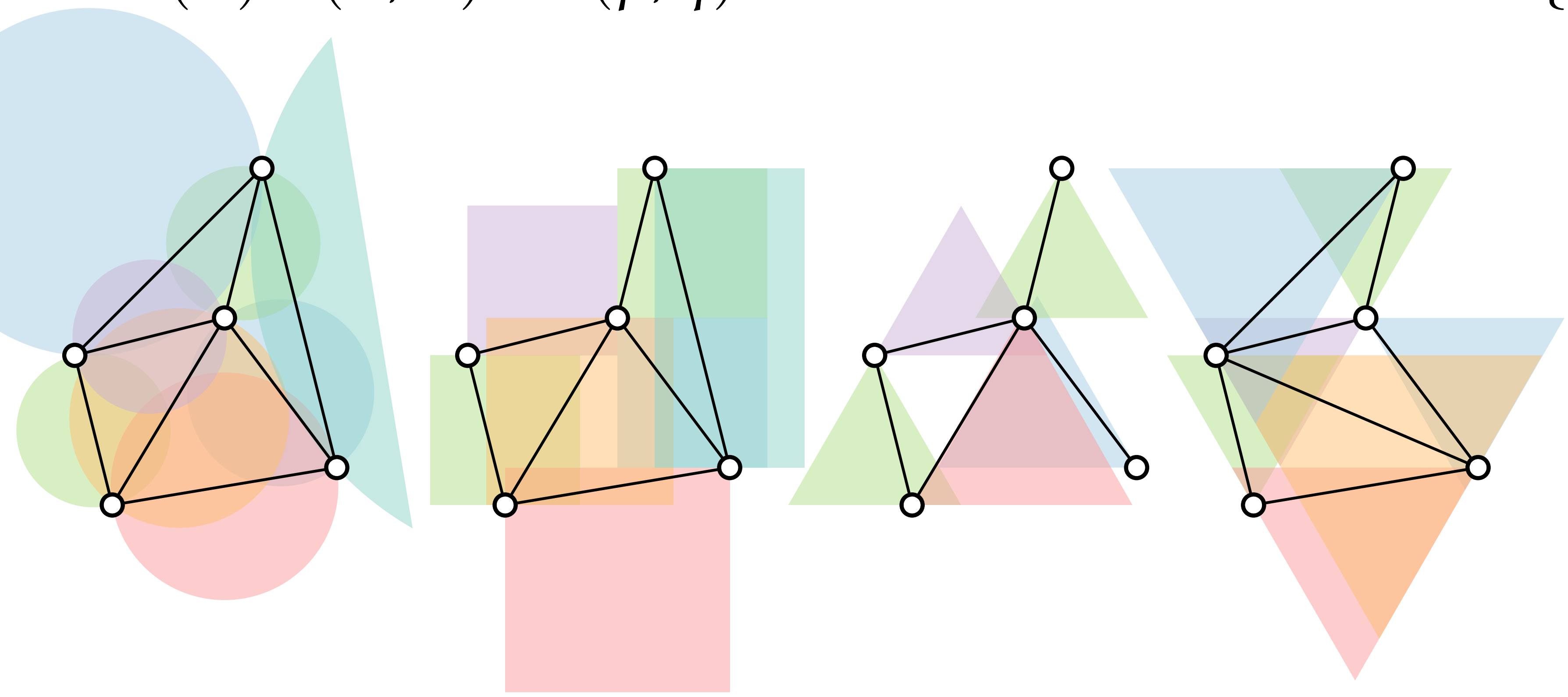
$G^{\square}(P)$ L_∞-Delaunay

$G^{\triangle}(P)$

$G^{\triangledown}(P)$

Proximity Graphs

P: set of n points in the plane
$\mathcal{S}$: set of geom. objects in the plane
$G^{\mathcal{S}}(P) = (P, E)$: $\quad (p,q) \in E \;\Leftrightarrow\; \exists S \in \mathcal{S} : \; S \cap P = \{p,q\}$



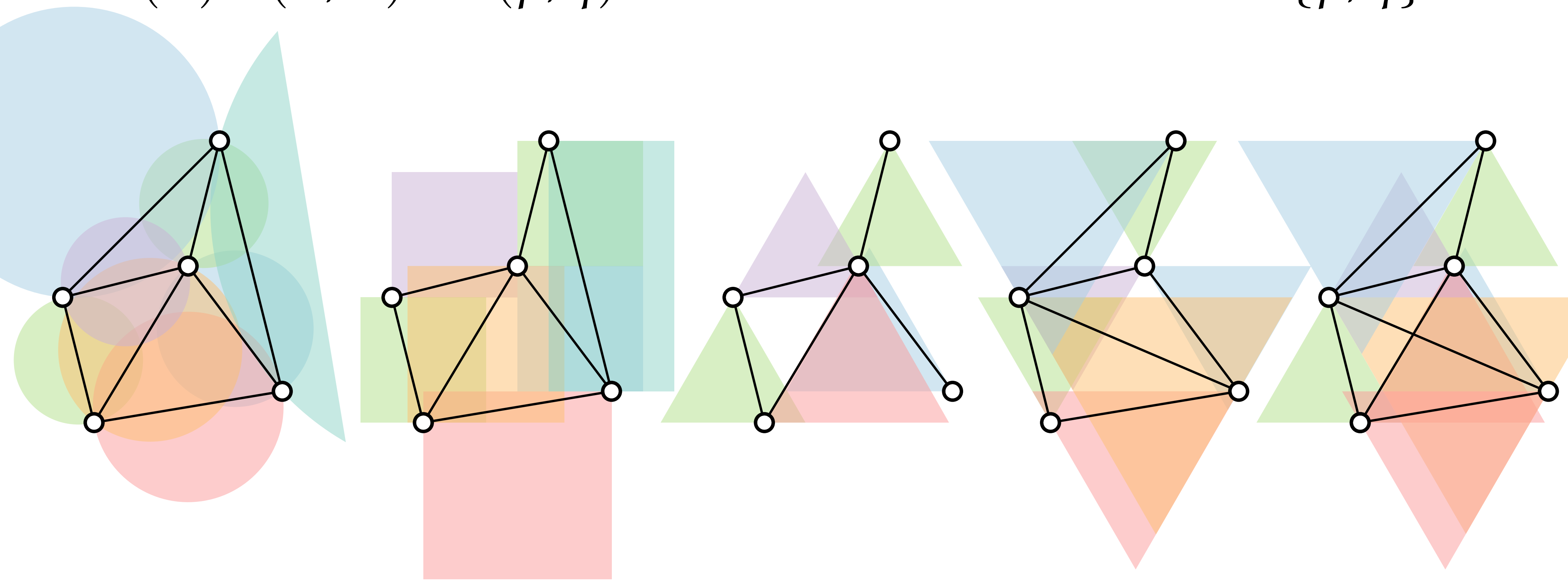
$G^{\circ}(P)$ Delaunay

$G^{\square}(P)$ L_∞-Delaunay

$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs

Proximity Graphs

P: set of n points in the plane
$\mathcal{S}$: set of geom. objects in the plane
$G^{\mathcal{S}}(P)=(P,E)$: $\quad (p,q) \in E \Leftrightarrow \exists S \in \mathcal{S} : S \cap P = \{p,q\}$



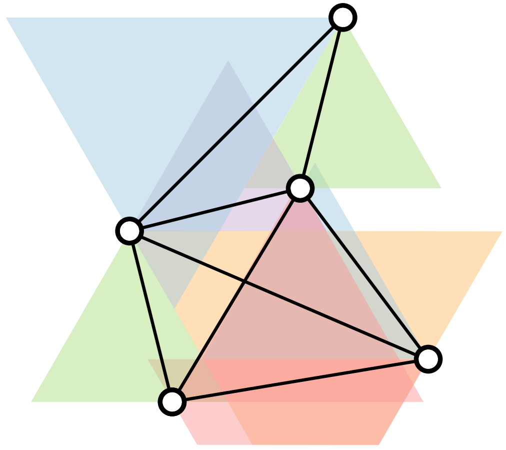
$G^{\circ}(P)$ Delaunay

$G^{\square}(P)$ L_∞-Delaunay

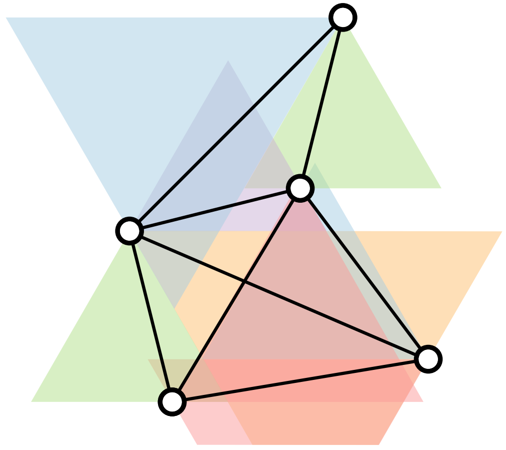
$G^{\triangle}(P) \cup G^{\triangledown}(P) = G^{\text{✡}}(P)$

Half-Θ_6-Graphs $\quad$ Θ_6-Graph

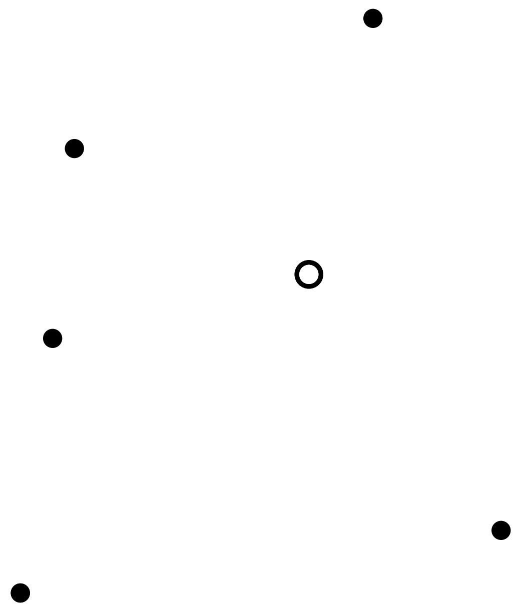
Alt. Definition of Θ_6-Graphs



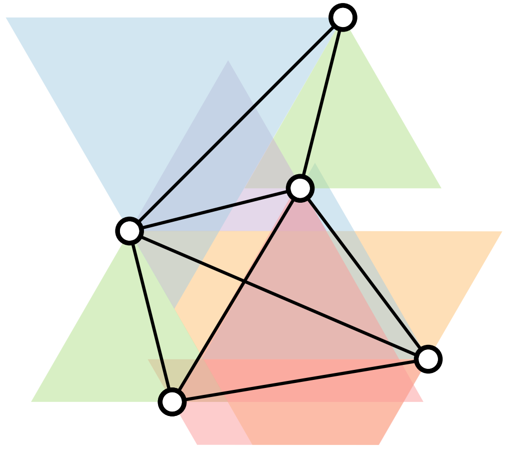
Alt. Definition of Θ_6-Graphs



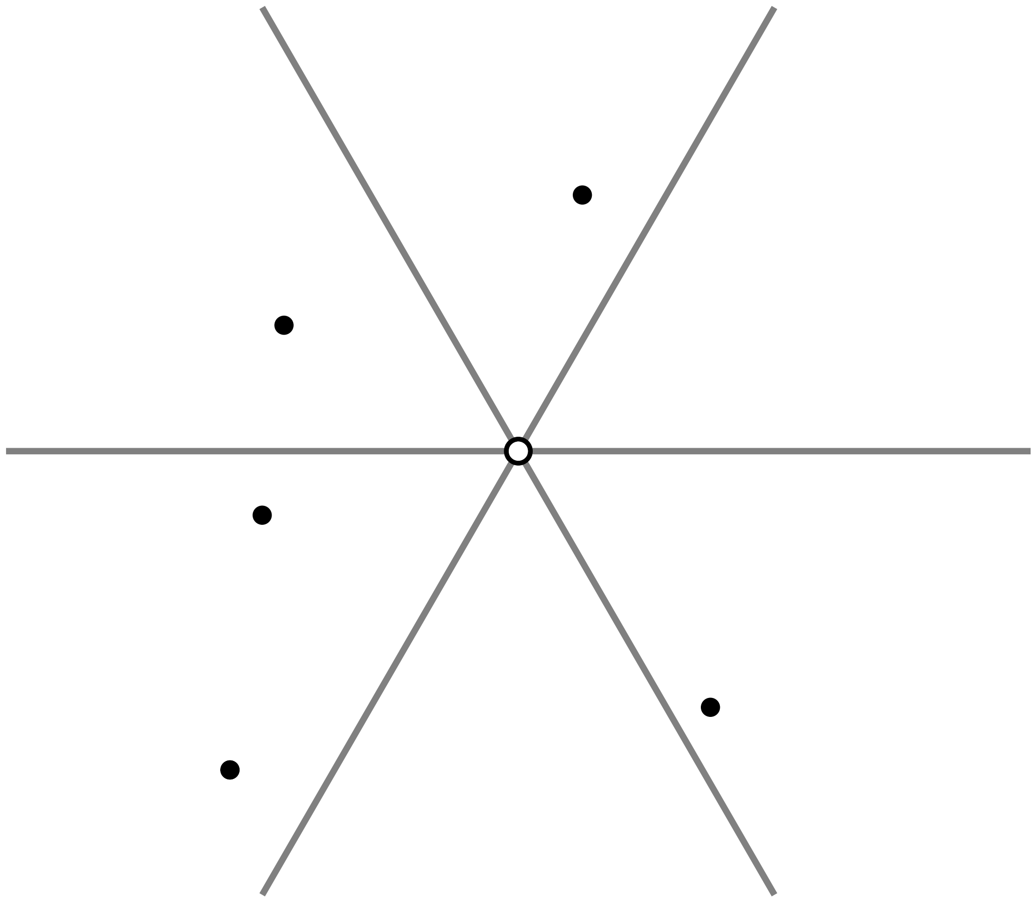
$G^{\triangle}(P)$



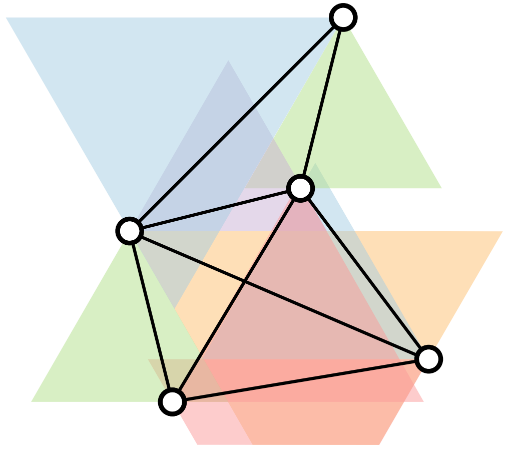
Alt. Definition of Θ_6-Graphs



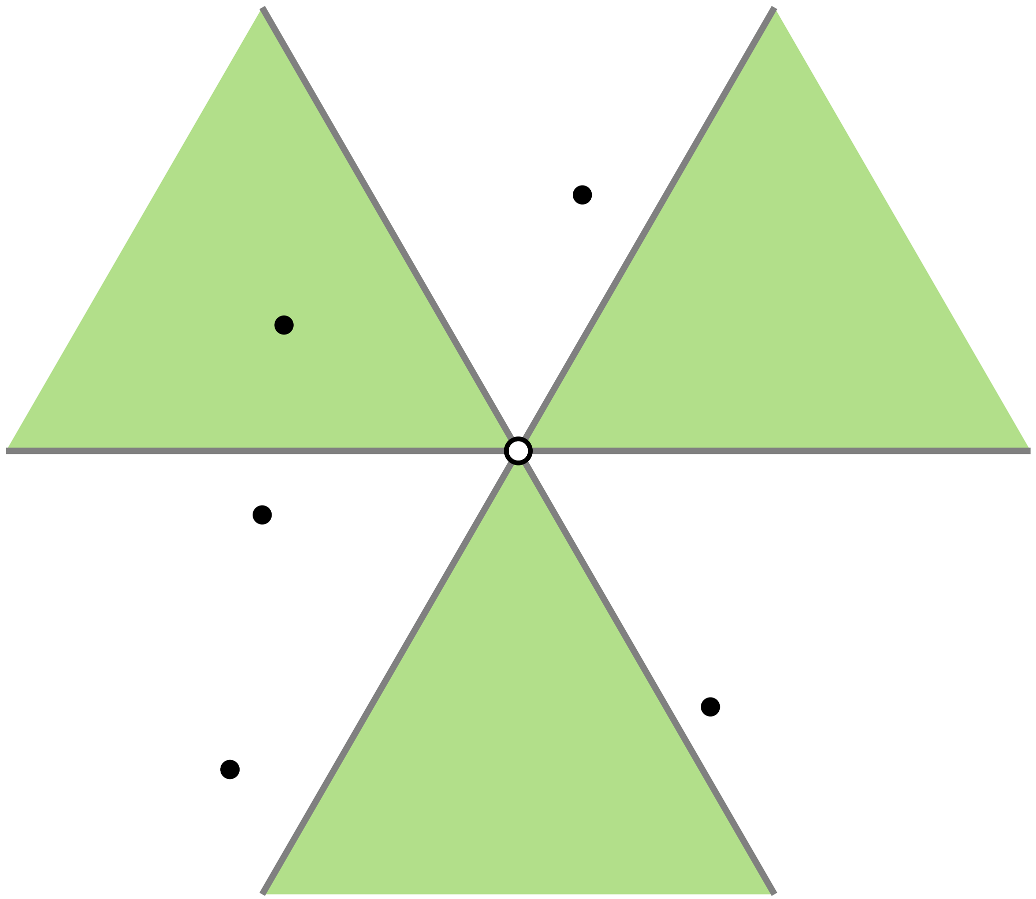
$G^{\triangle}(P)$



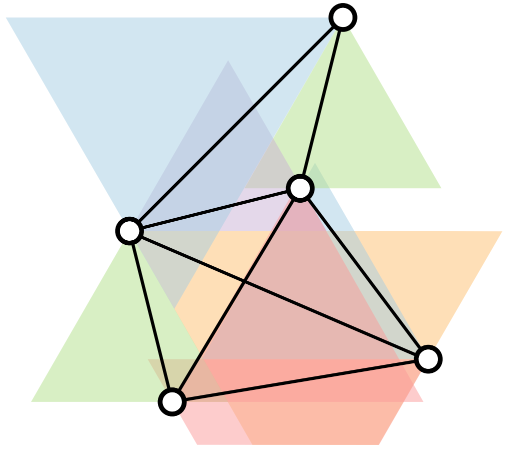
Alt. Definition of Θ_6-Graphs



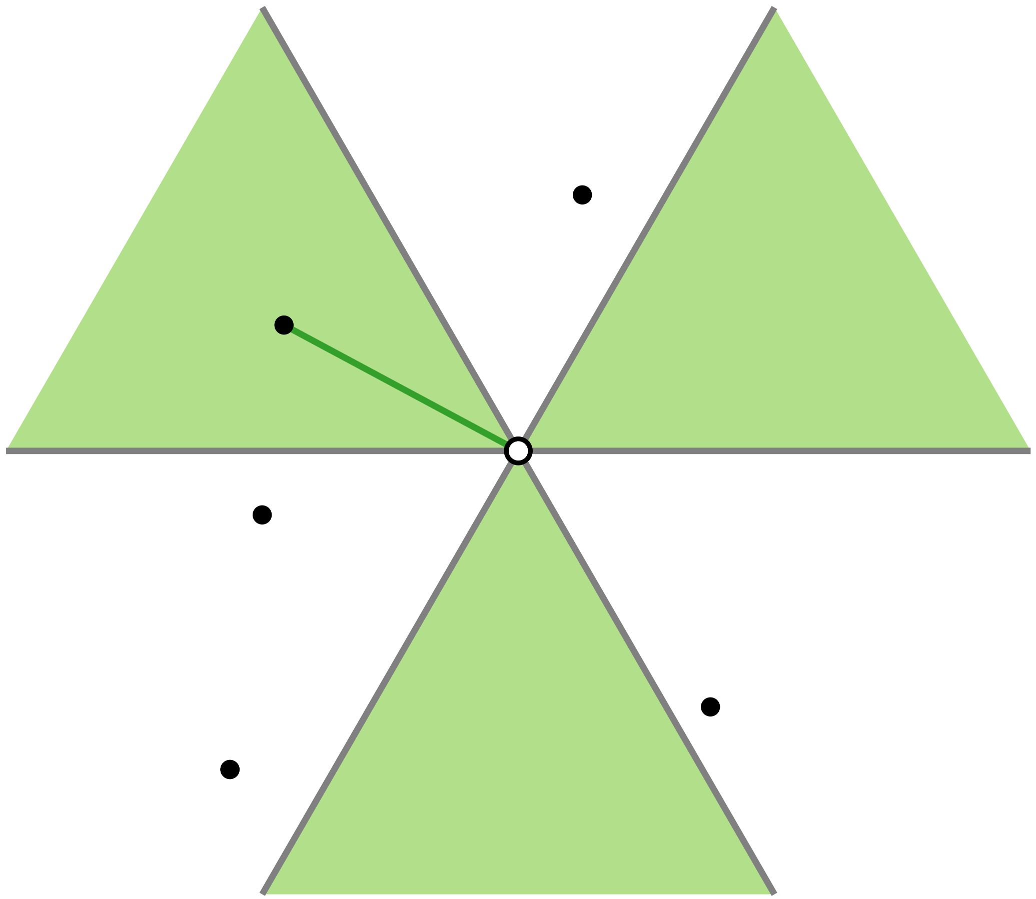
$G^{\triangle}(P)$



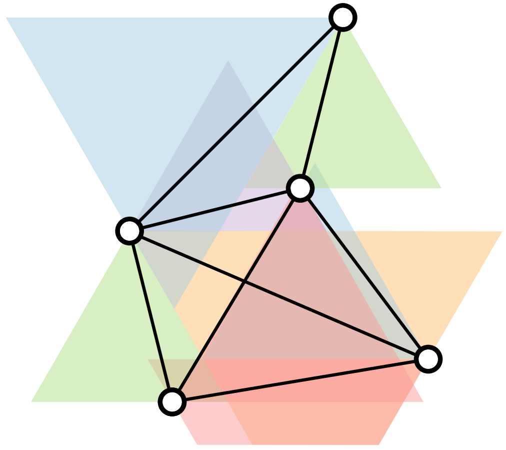
Alt. Definition of Θ_6-Graphs



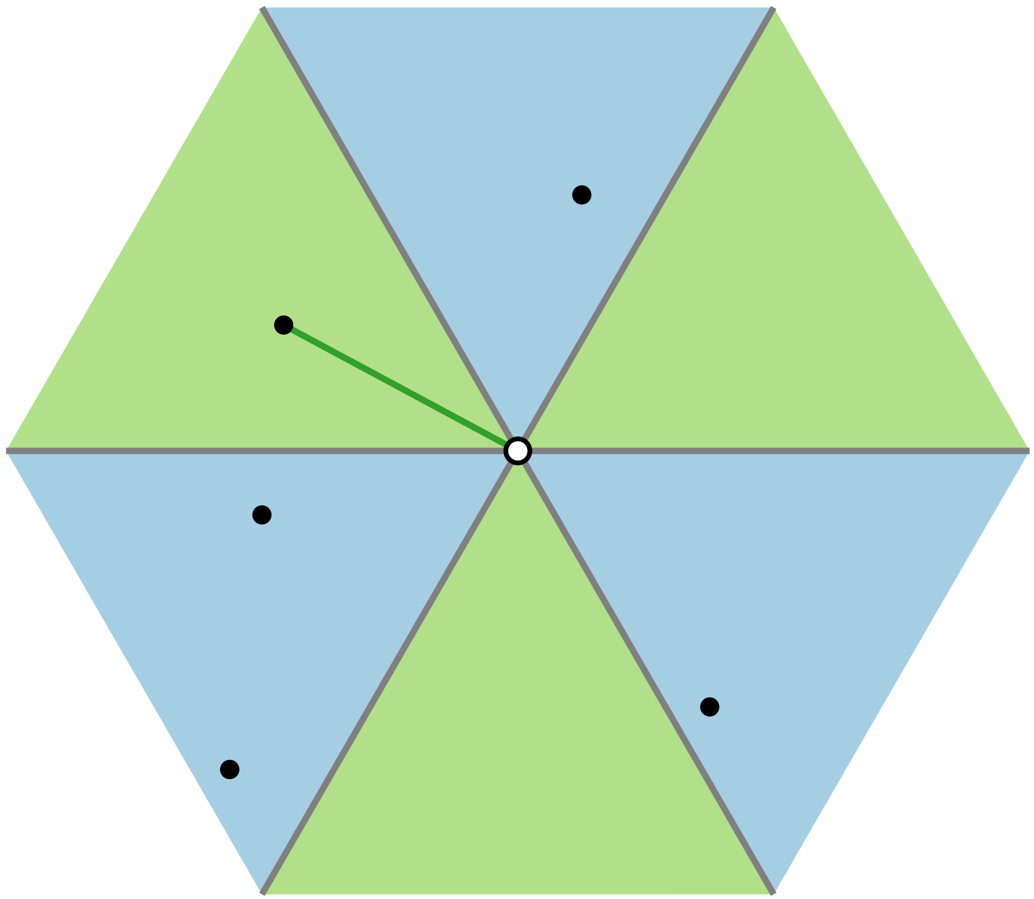
$G^{\triangle}(P)$



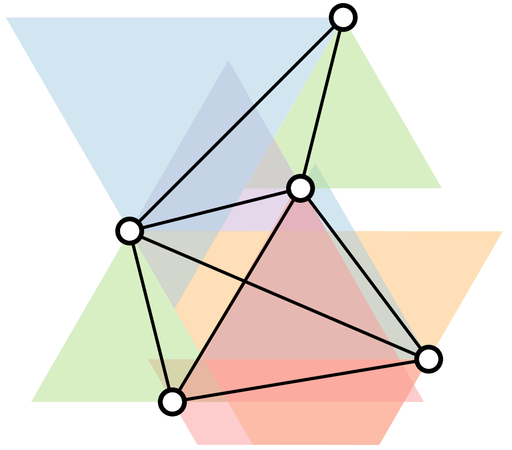
Alt. Definition of Θ_6-Graphs



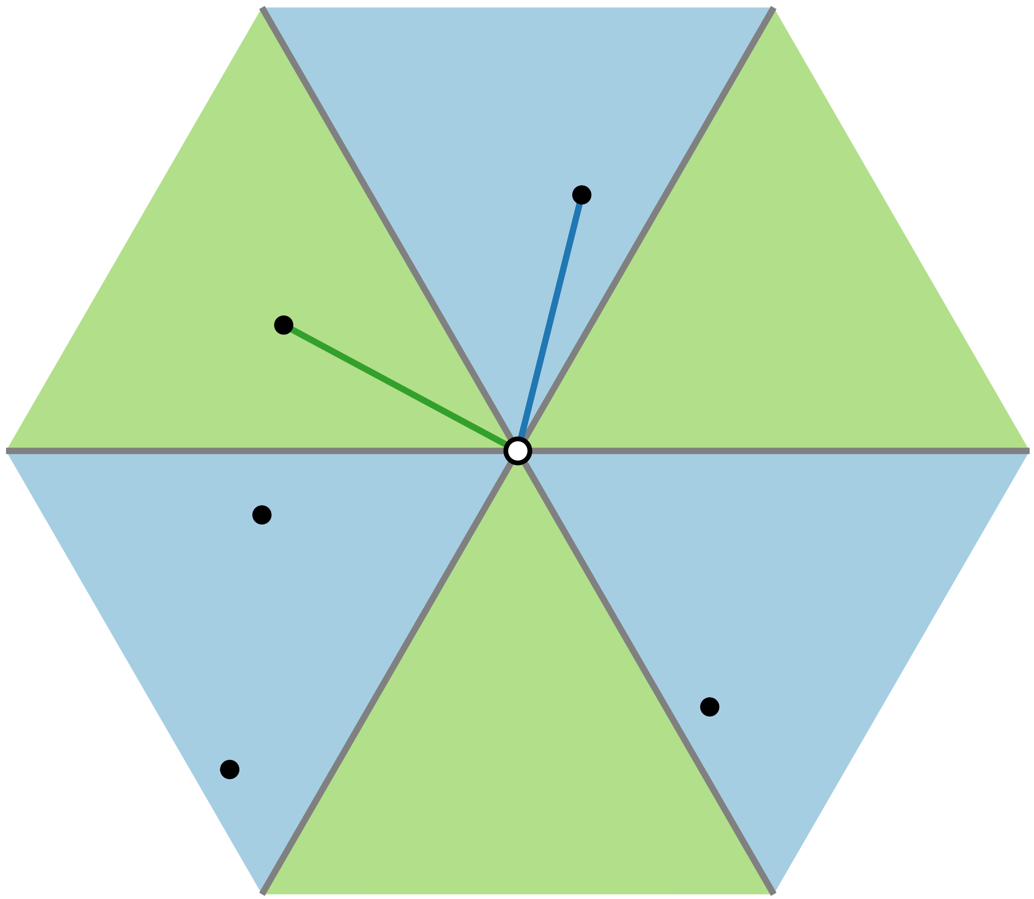
$G^{\triangle}(P)$ $G^{\triangledown}(P)$



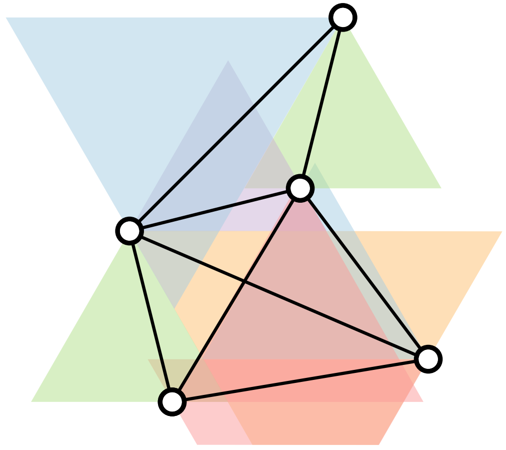
Alt. Definition of Θ_6-Graphs



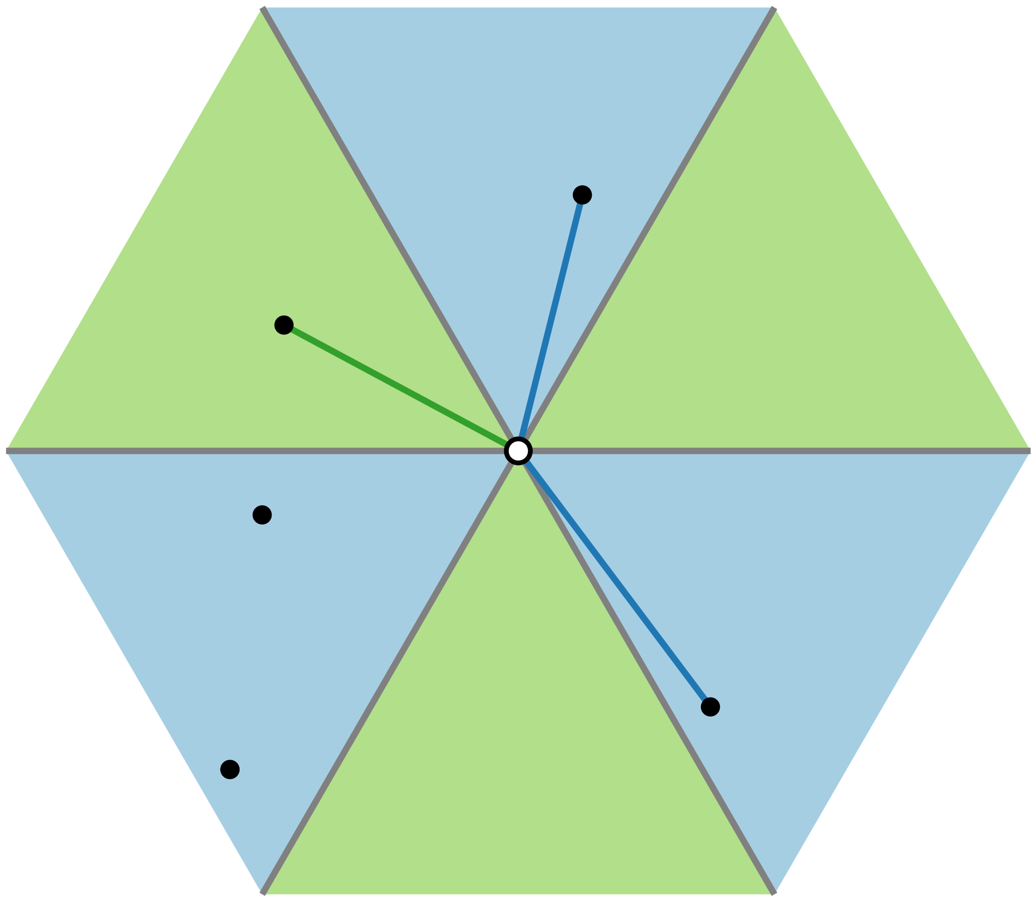
$G^{\triangle}(P)$ $\quad$ $G^{\nabla}(P)$



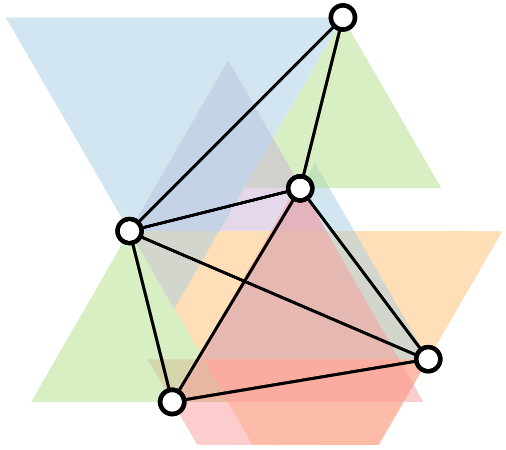
Alt. Definition of Θ_6-Graphs



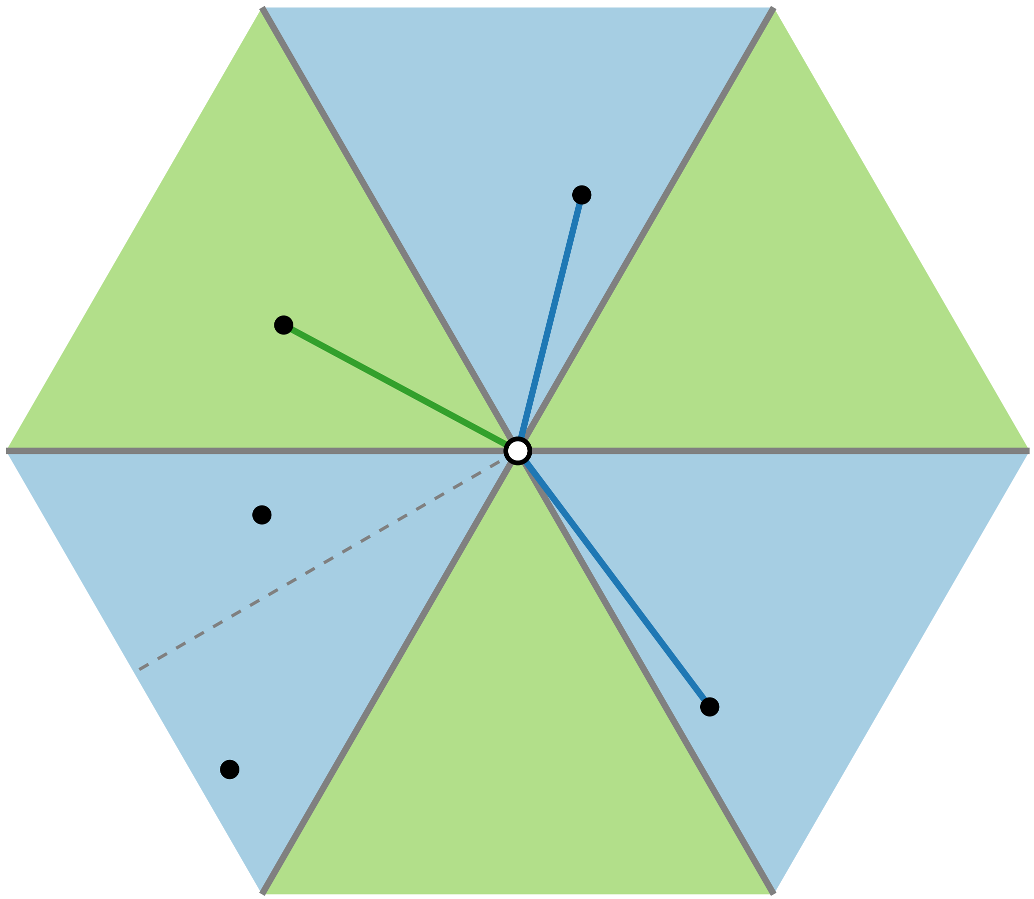
$G^{\triangle}(P)$ $\quad$ $G^{\triangledown}(P)$



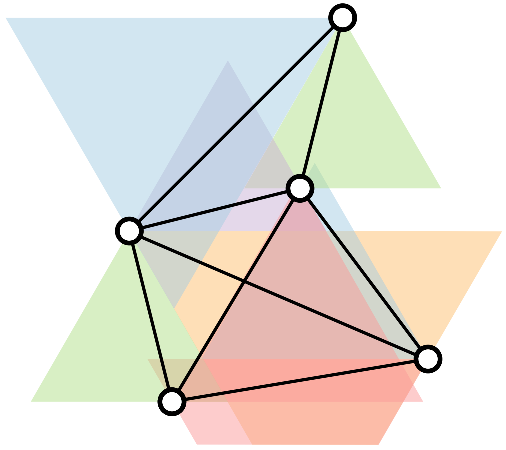
Alt. Definition of Θ_6-Graphs



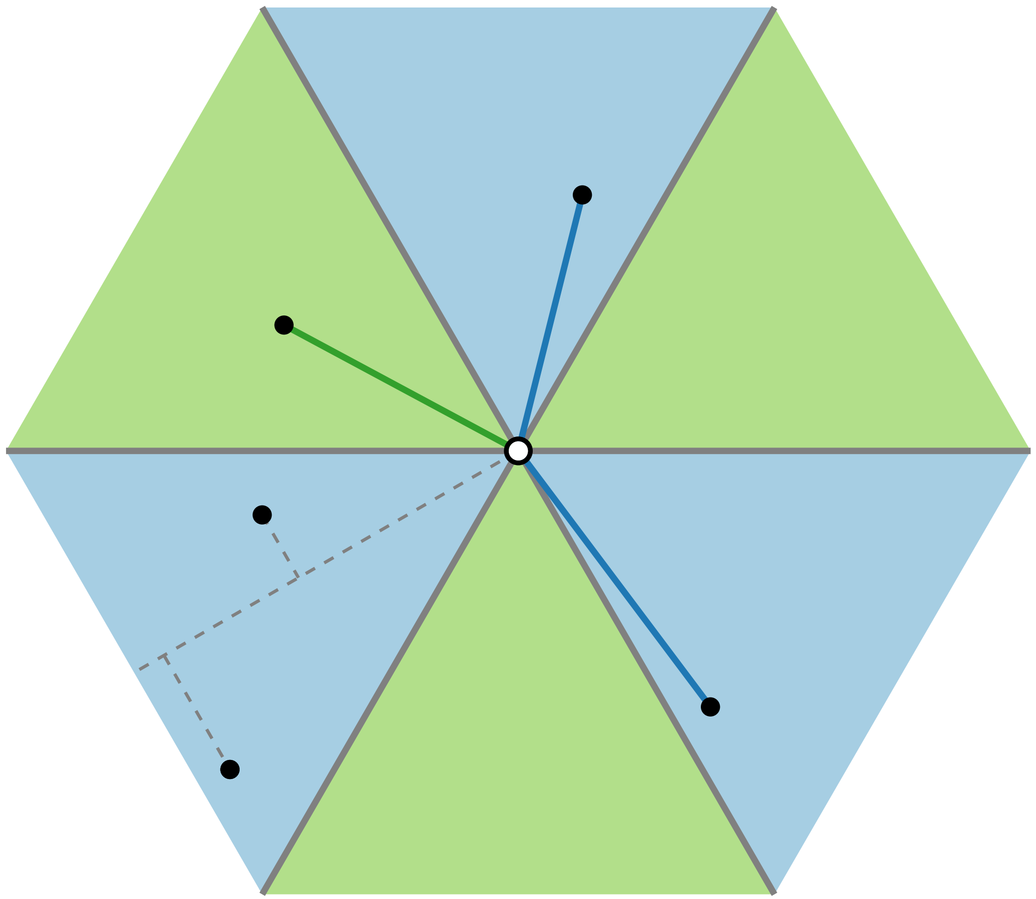
$G^{\triangle}(P)$ $G^{\bigtriangledown}(P)$



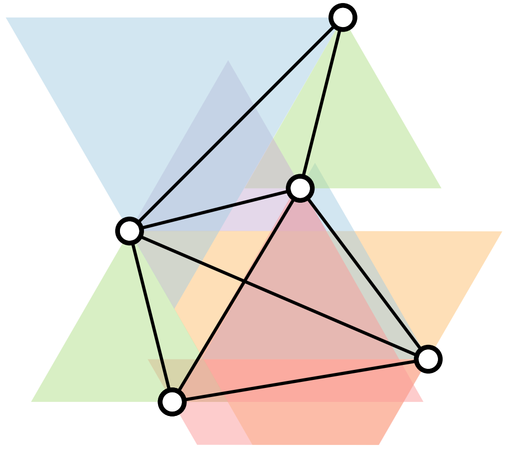
Alt. Definition of Θ_6-Graphs



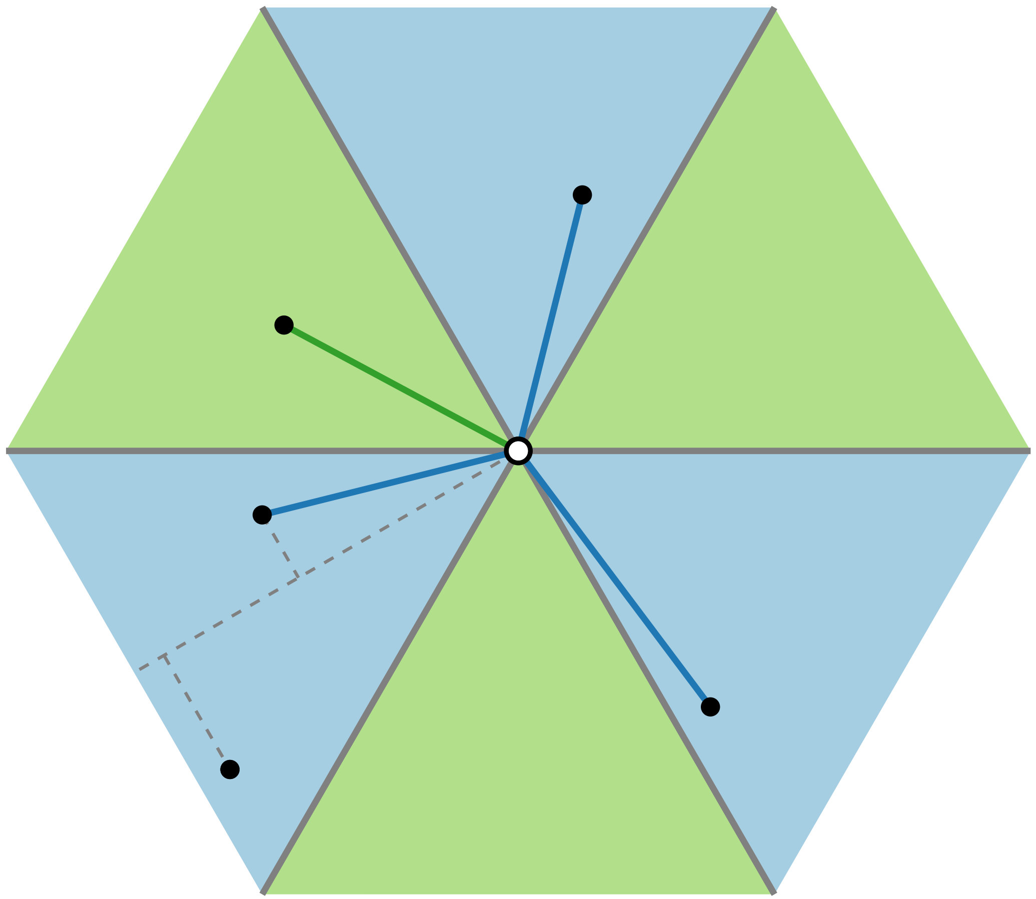
$G^{\triangle}(P)$ $\quad$ $G^{\triangledown}(P)$



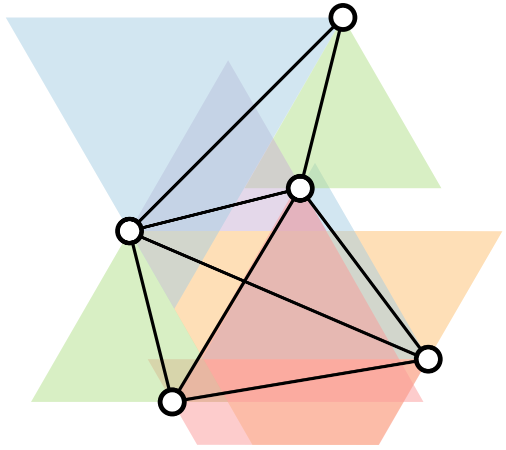
Alt. Definition of Θ_6-Graphs



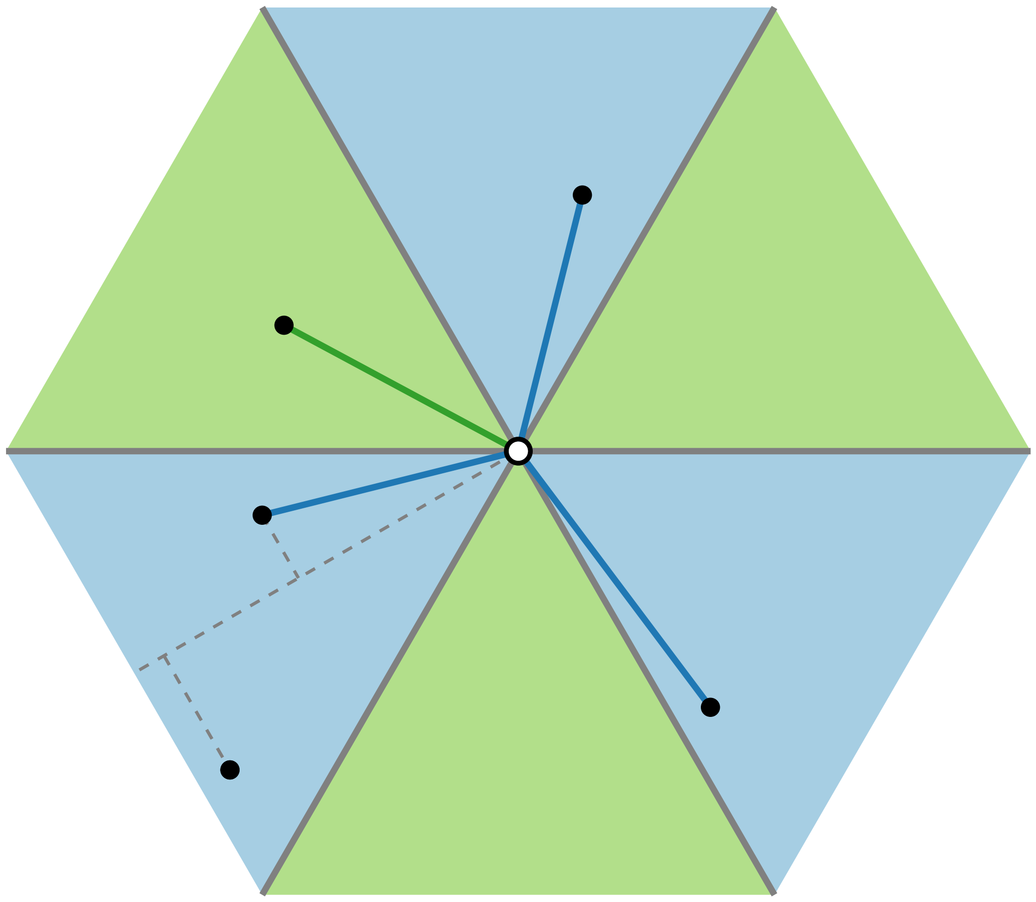
$G^{\triangle}(P)$ $G^{\triangledown}(P)$



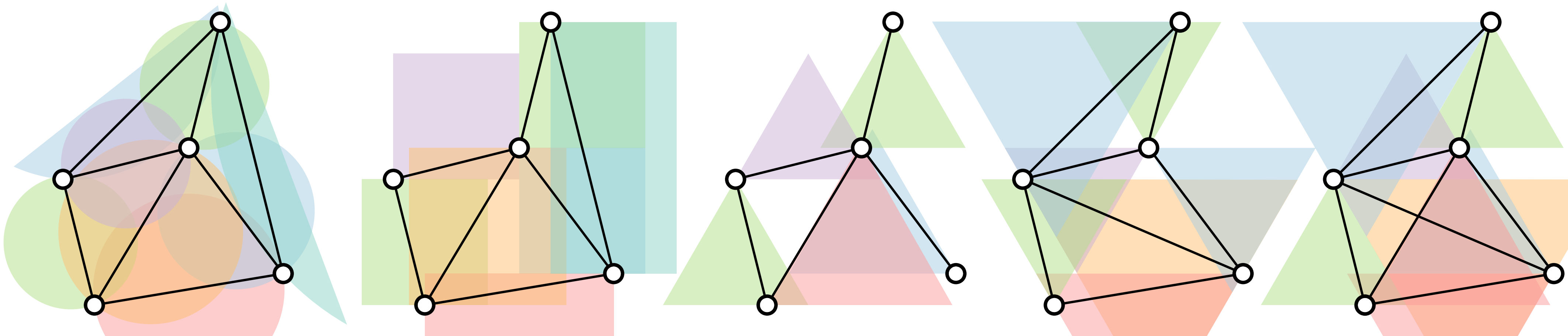
Alt. Definition of Θ_6-Graphs



$$G^{\triangle}(P) \cup G^{\triangledown}(P) = G^{\text{✡}}(P)$$



Known Results



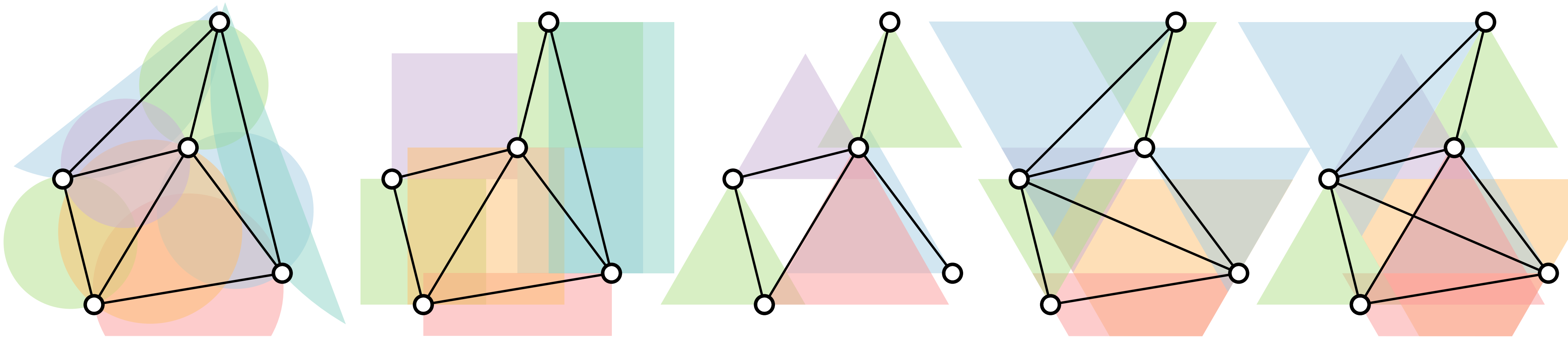
$G^{\circ}(P)$ Delaunay

$G^{\square}(P)$ L_∞-Delaunay

$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs

$G^{\text{✡}}(P)$ Θ_6-Graphs

Known Results



$G^{\circ}(P)$ Delaunay

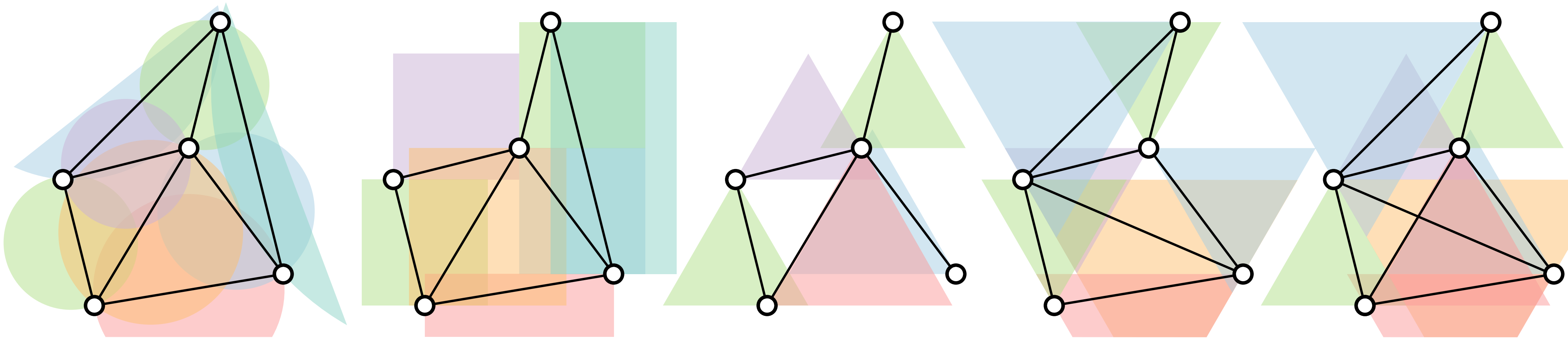
$G^{\square}(P)$ L_∞-Delaunay

$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs

$G^{\text{✡}}(P)$ Θ_6-Graphs

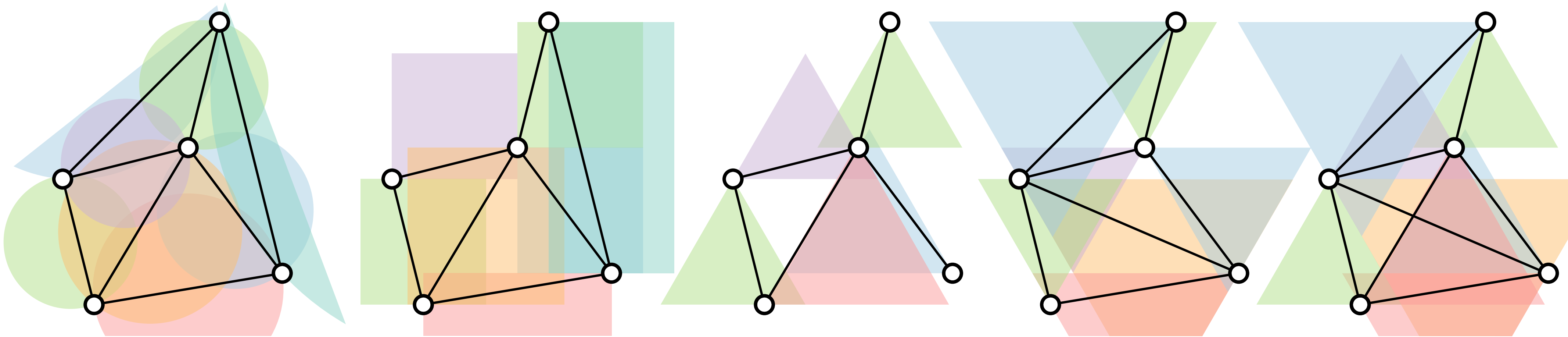
Spanning Ratio t

Known Results



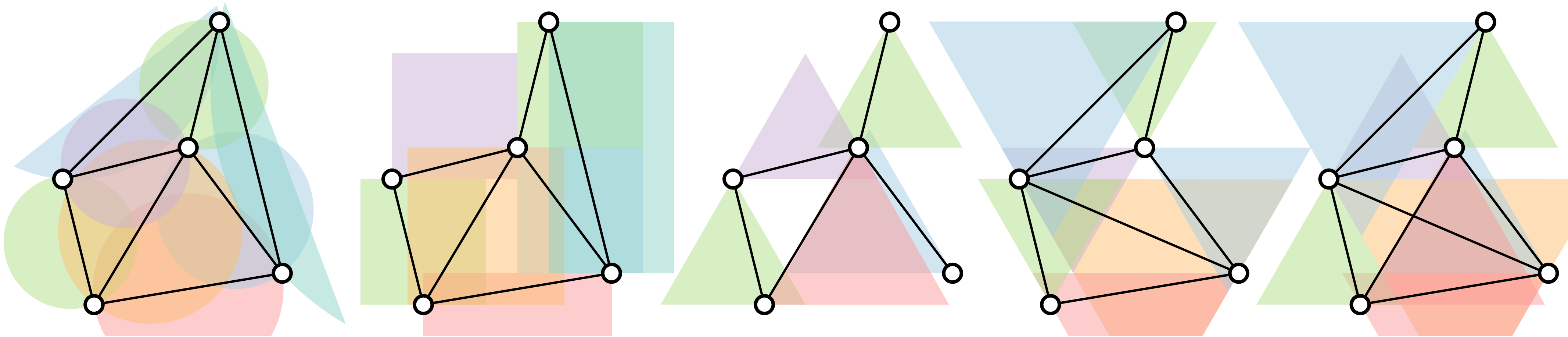
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t		$1.593 < t < 1.998$		

Known Results



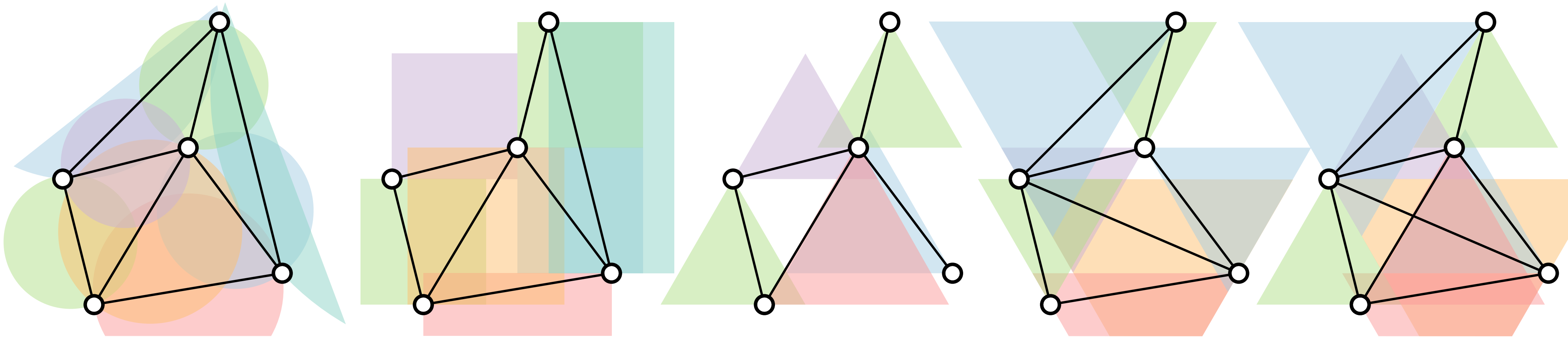
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$		

Known Results



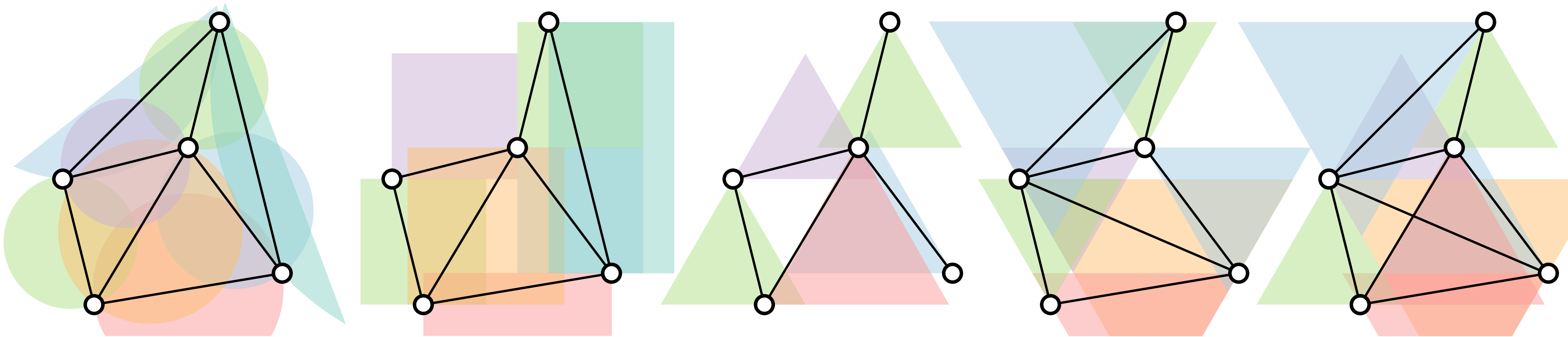
$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ Half-Θ_6-Graphs	$G^{\triangledown}(P)$	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	

Known Results



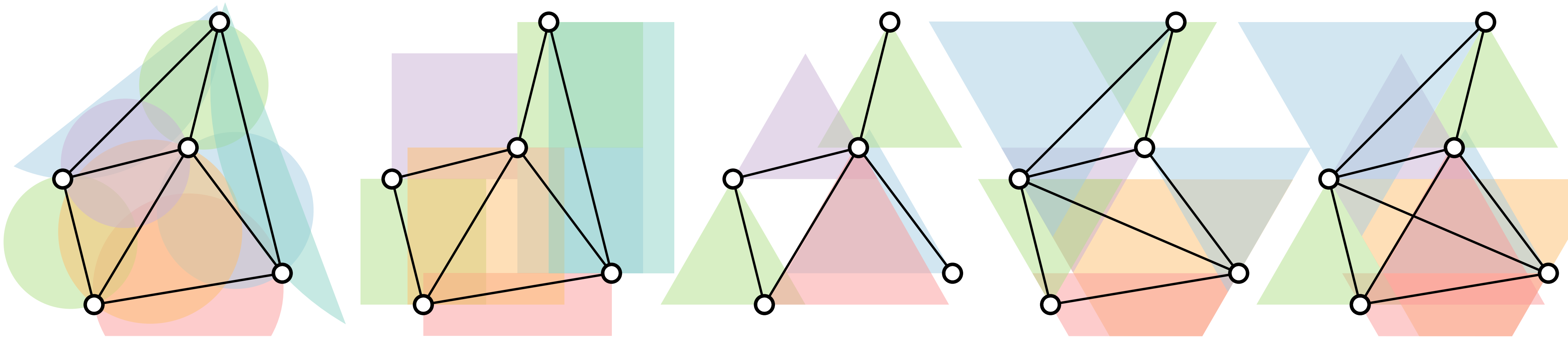
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ Half-Θ_6-Graphs	$G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t		$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$

Known Results



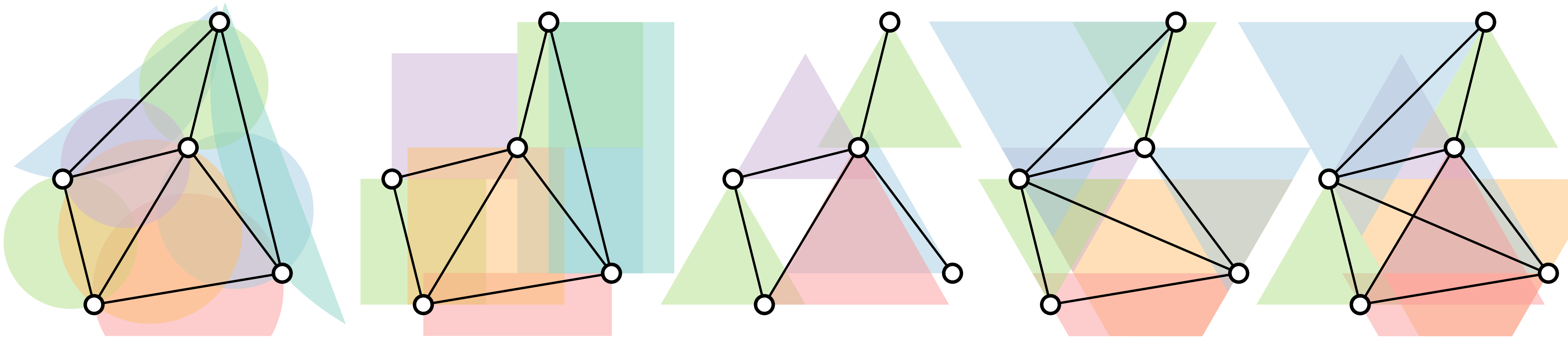
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$				

Known Results



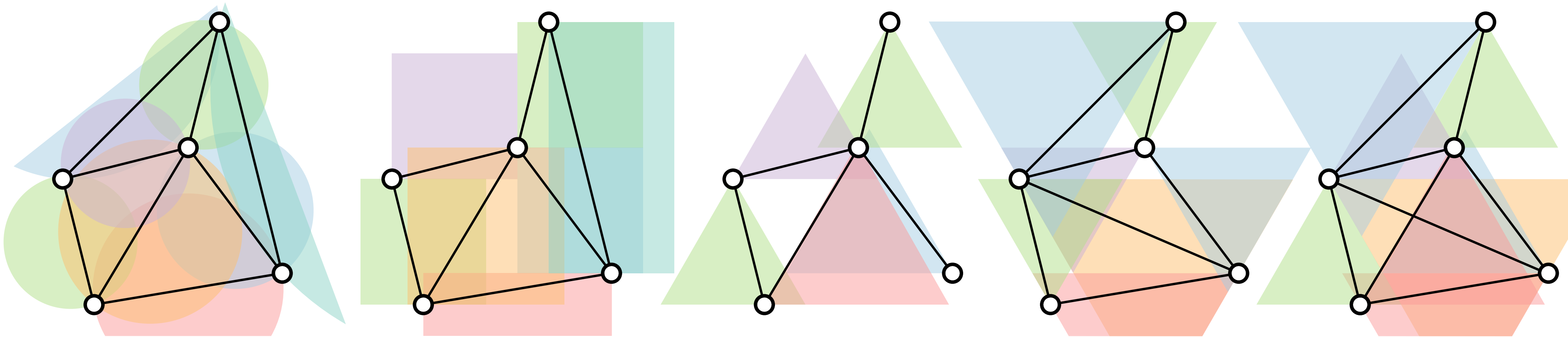
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ Half-Θ_6-Graphs	$G^{\triangledown}(P)$	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$		$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$				

Known Results



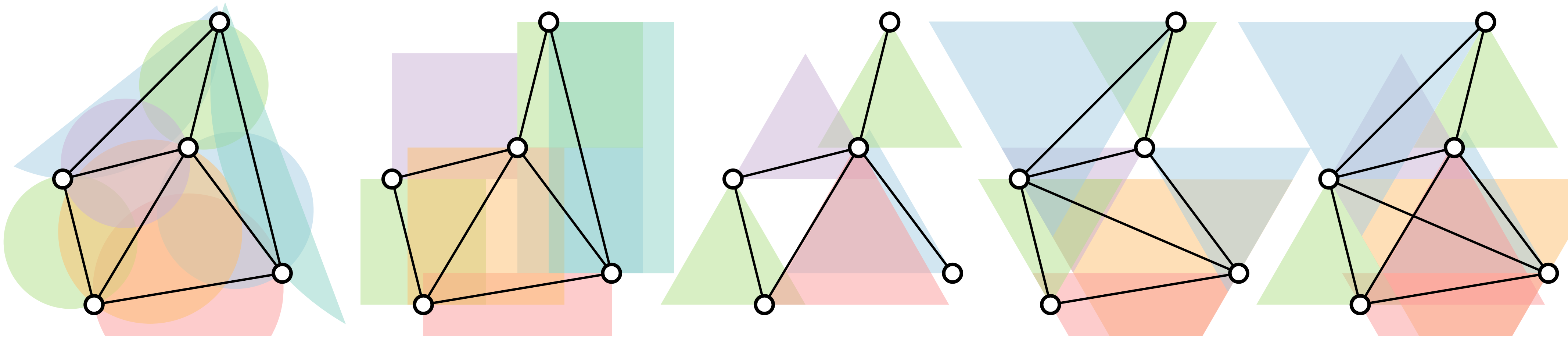
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$		

Known Results



	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)		

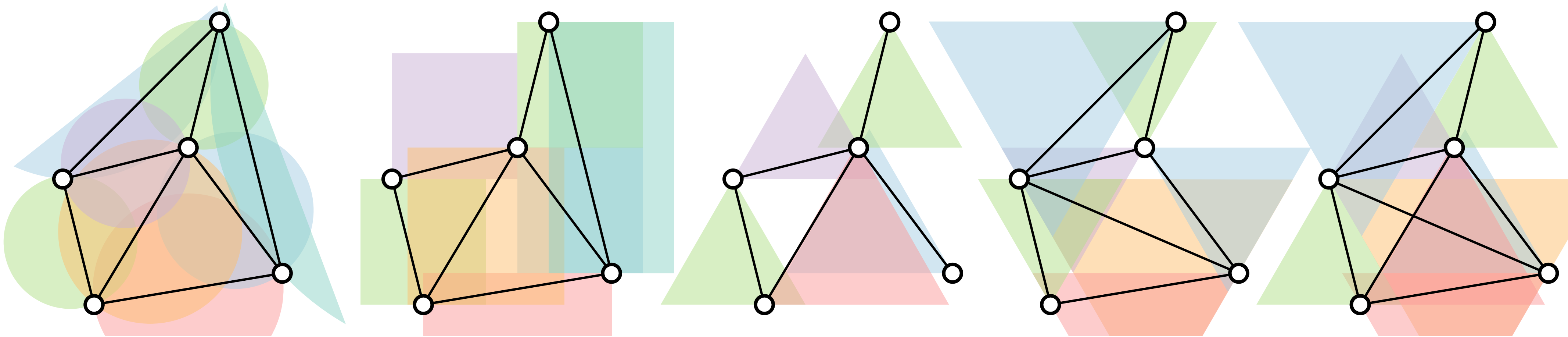
Known Results



$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$	$G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\hexstar}(P)$ Θ_6-Graphs

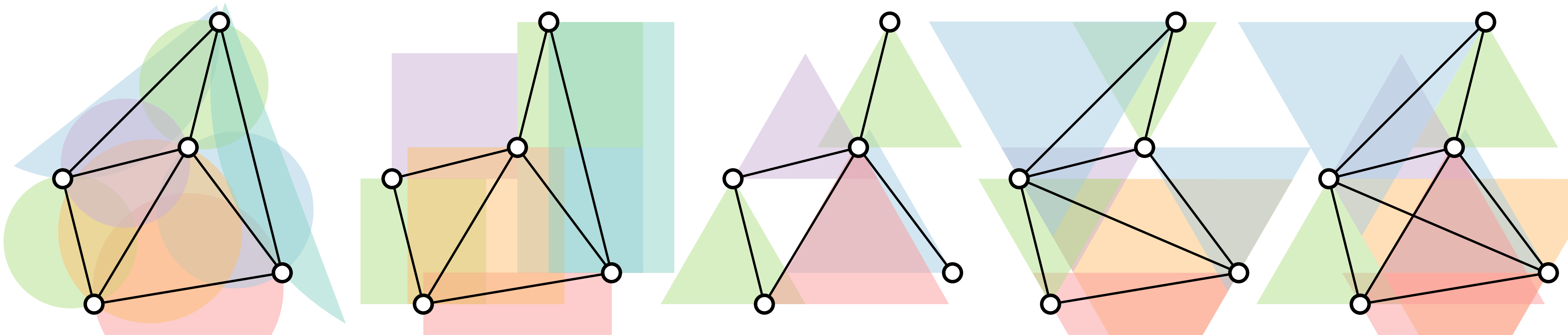
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	

Known Results



	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \lceil \frac{n-1}{3} \rceil$

Known Results



$G^{\circ}(P)$ Delaunay

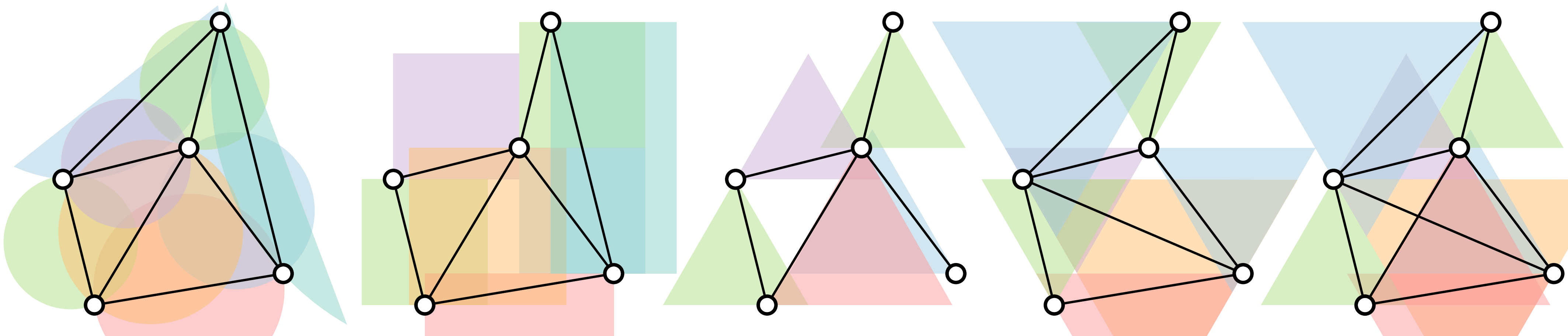
$G^{\square}(P)$ L_∞-Delaunay

$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs

$G^{\text{✡}}(P)$ Θ_6-Graphs

	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ / $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593<t<1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \lceil \frac{n-1}{3} \rceil$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$

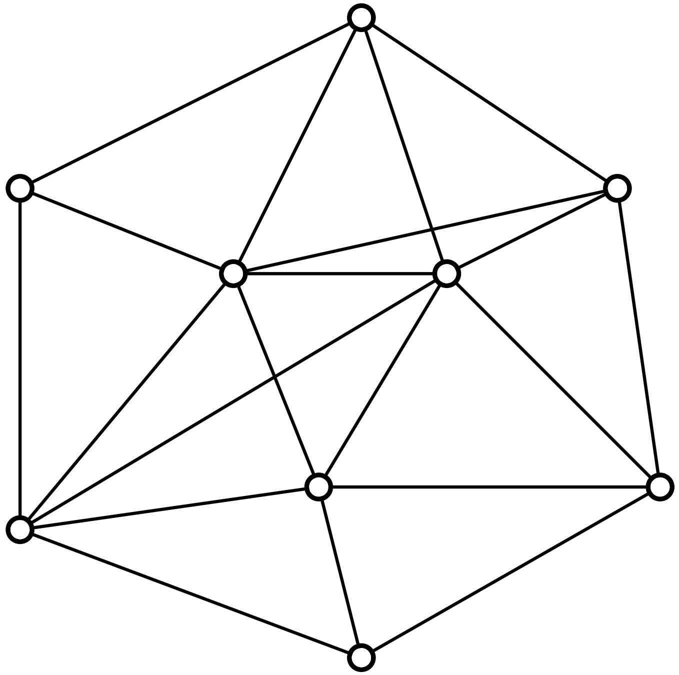
Known Results



	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \lceil \frac{n-1}{3} \rceil$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$

The Tutte-Berge Formula

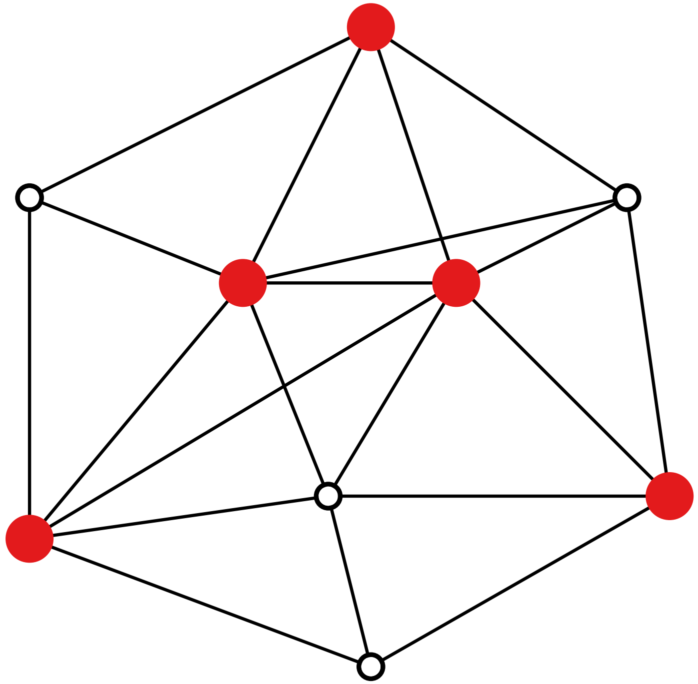
$G = (V, E)$



The Tutte-Berge Formula

$G = (V, E)$

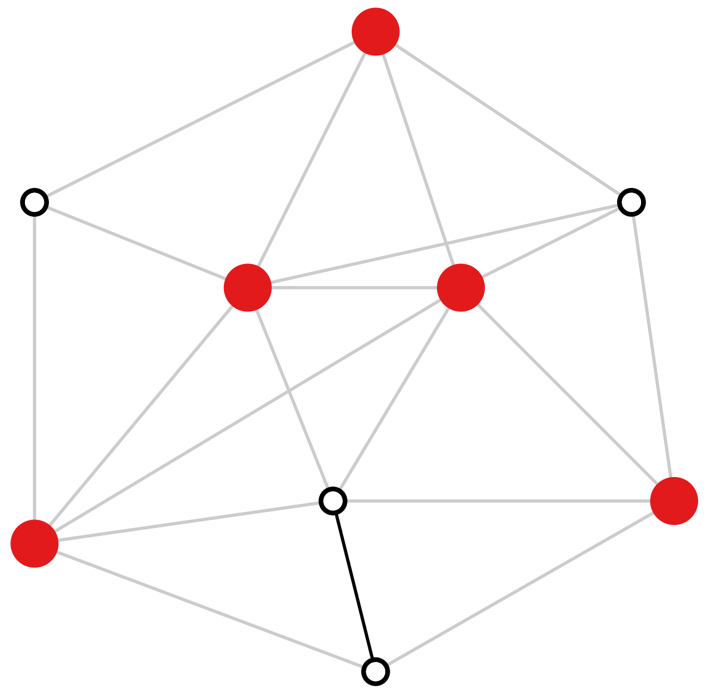
$S \subseteq V$



The Tutte-Berge Formula

$G = (V, E)$

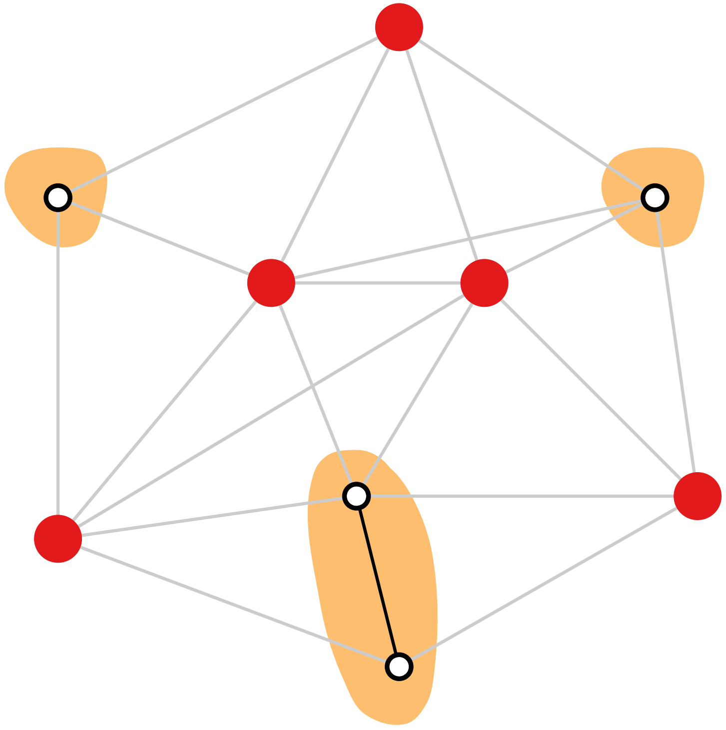
$S \subseteq V$



The Tutte-Berge Formula

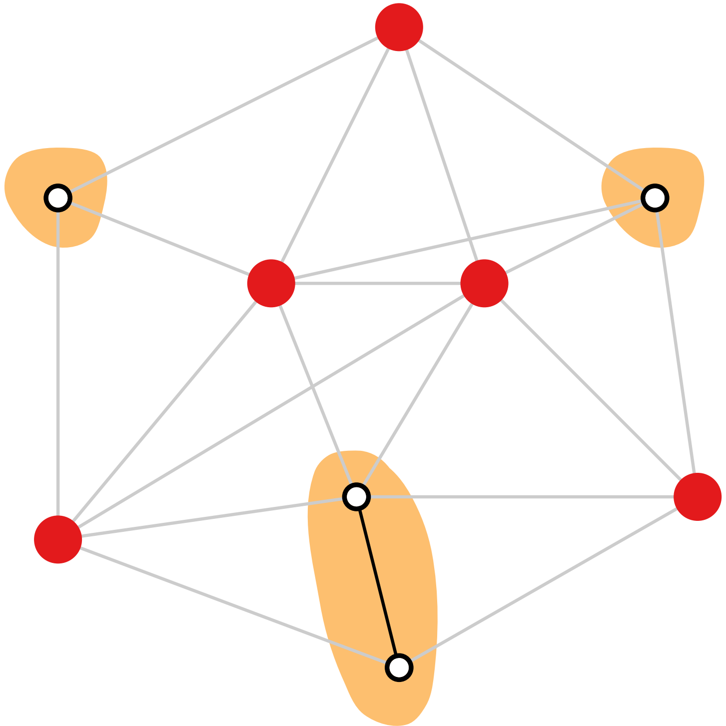
$G = (V, E)$

$S \subseteq V$



The Tutte-Berge Formula

$G = (V, E)$
$S \subseteq V$
$\mathrm{comp}(G \setminus S)$

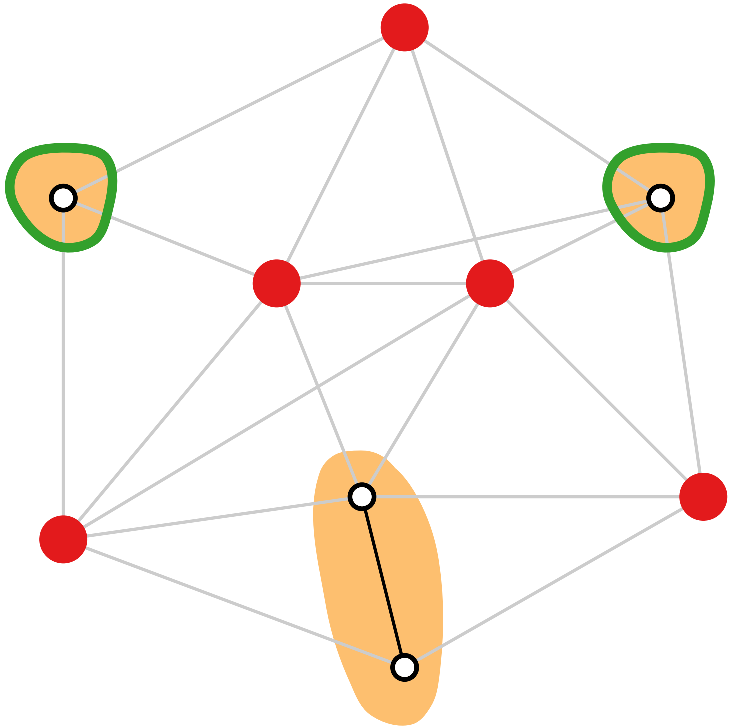


The Tutte-Berge Formula

$G = (V, E)$

$S \subseteq V$

$\text{comp}(G \setminus S)$



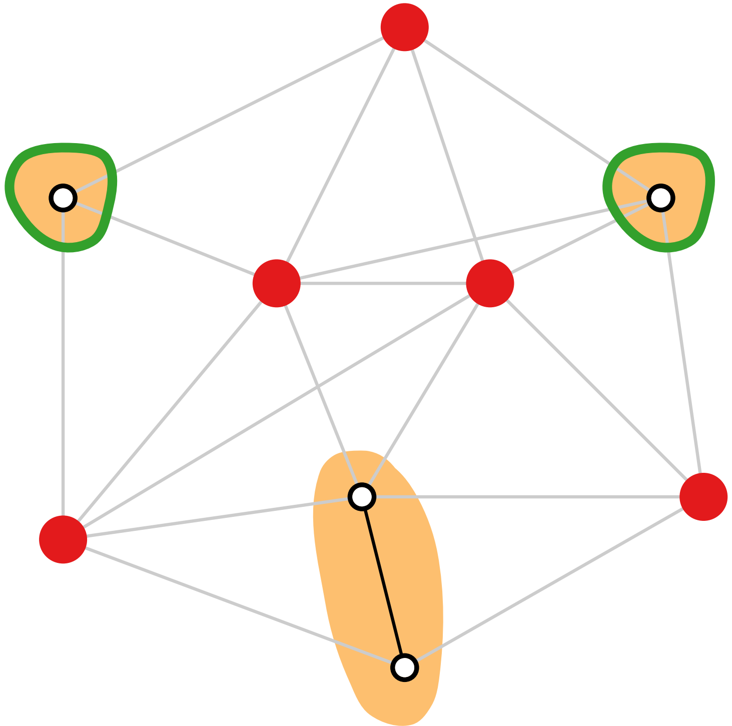
The Tutte-Berge Formula

$G = (V, E)$

$S \subseteq V$

$\mathrm{comp}(G \setminus S)$

$\mathrm{odd}(G \setminus S)$



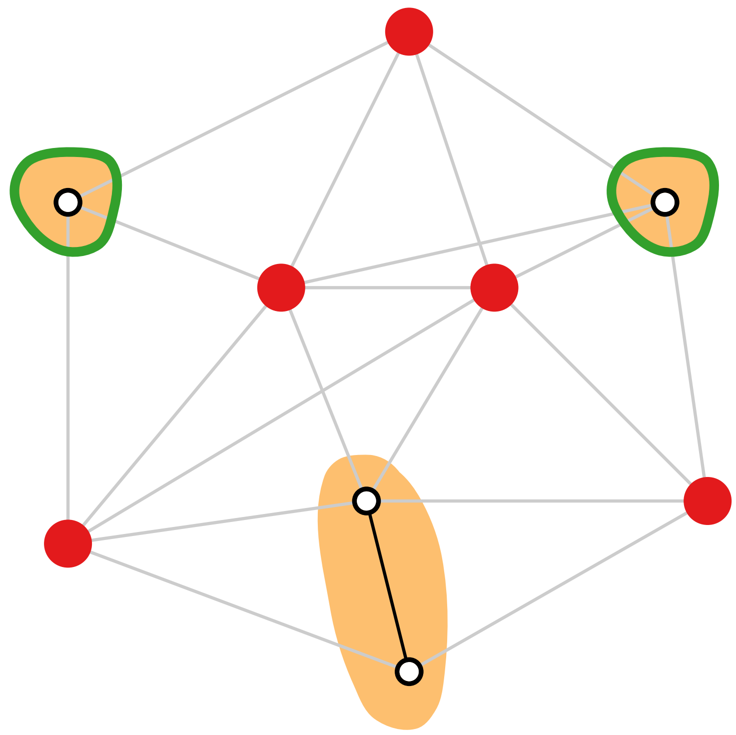
The Tutte-Berge Formula

$G = (V, E)$

$S \subseteq V$

$\text{comp}(G \setminus S)$

$\text{odd}(G \setminus S)$



[Tutte '47] G has a (near-)perfect matching

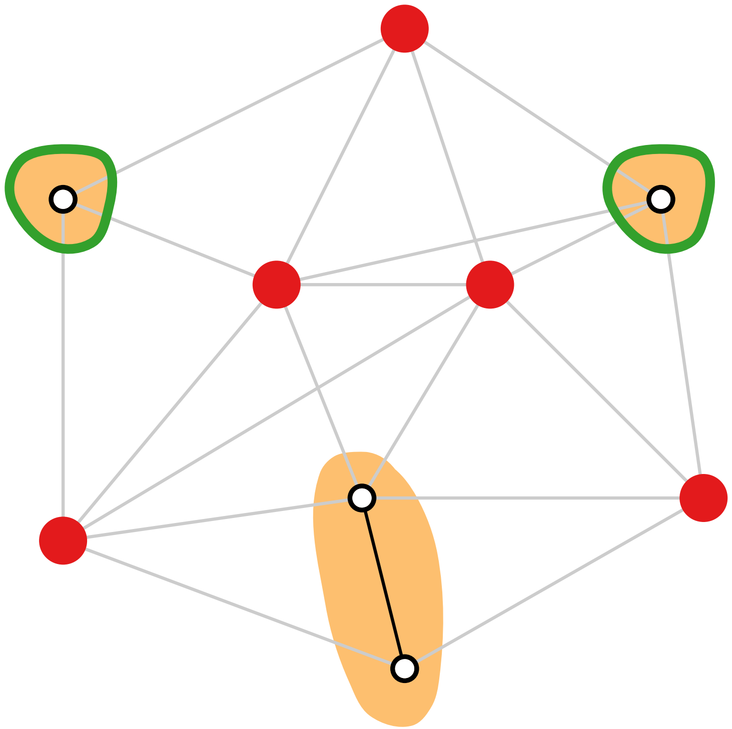
The Tutte-Berge Formula

$G = (V, E)$

$S \subseteq V$

$\mathrm{comp}(G \setminus S)$

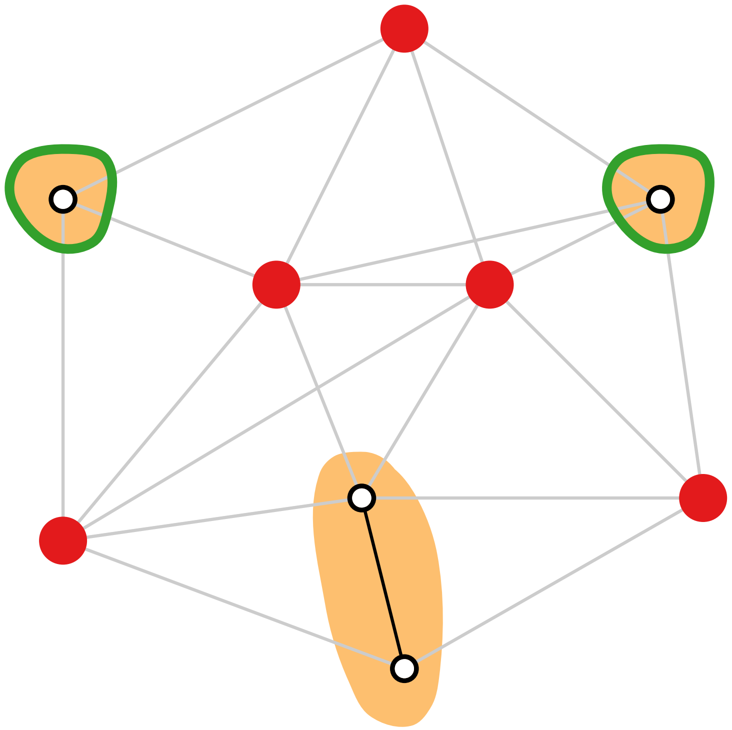
$\mathrm{odd}(G \setminus S)$



[Tutte '47] G has a (near-)perfect matching $\Leftrightarrow \mathrm{odd}(S) \leq |S|$

The Tutte-Berge Formula

$G = (V, E)$
$S \subseteq V$
$\mathrm{comp}(G \setminus S)$
$\mathrm{odd}(G \setminus S)$



[Tutte '47] G has a (near-)perfect matching
$\Leftrightarrow \mathrm{odd}(S) \leq |S|$ for every $S \subseteq V$

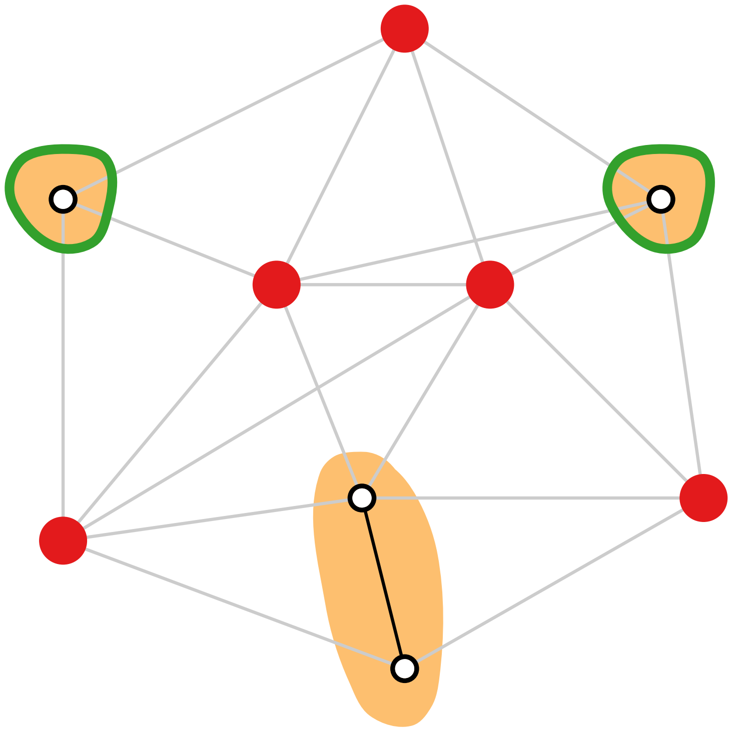
The Tutte-Berge Formula

$G = (V, E)$

$S \subseteq V$

$\mathrm{comp}(G \setminus S)$

$\mathrm{odd}(G \setminus S)$

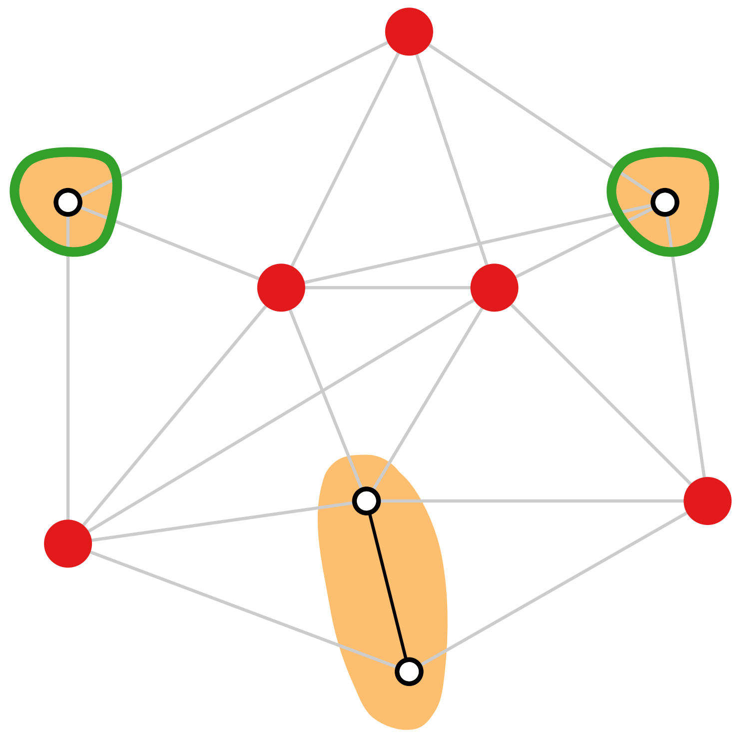


[Tutte '47] G has a (near-)perfect matching $\Leftrightarrow \mathrm{odd}(S) \leq |S|$ for every $S \subseteq V$

[Berge '57]

The Tutte-Berge Formula

$G = (V, E)$
$S \subseteq V$
$\text{comp}(G \setminus S)$
$\text{odd}(G \setminus S)$

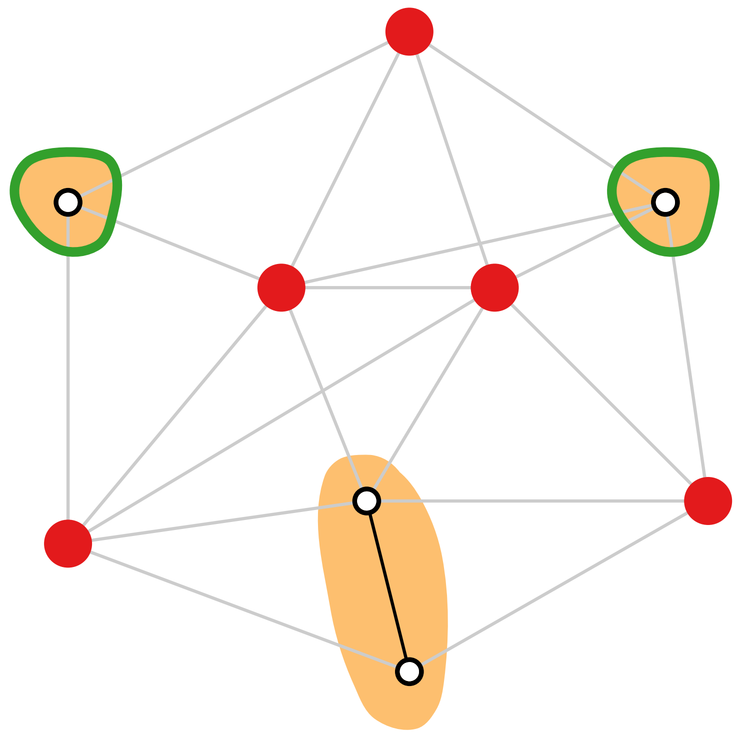


[Tutte '47] G has a (near-)perfect matching
$\Leftrightarrow \text{odd}(S) \leq |S|$ for every $S \subseteq V$

[Berge '57] A maximum matching of G has size

The Tutte-Berge Formula

$G = (V, E)$
$S \subseteq V$
$\mathrm{comp}(G \setminus S)$
$\mathrm{odd}(G \setminus S)$

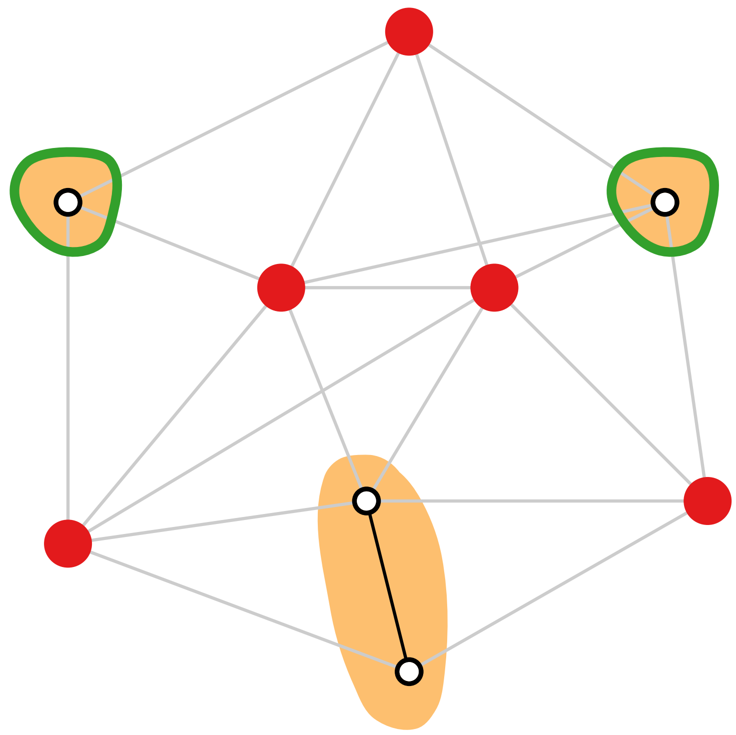


[Tutte '47] G has a (near-)perfect matching $\Leftrightarrow \mathrm{odd}(S) \leq |S|$ for every $S \subseteq V$

[Berge '57] A maximum matching of G has size $\frac{1}{2}(|V| - \qquad)$

The Tutte-Berge Formula

$G = (V, E)$
$S \subseteq V$
$\text{comp}(G \setminus S)$
$\text{odd}(G \setminus S)$

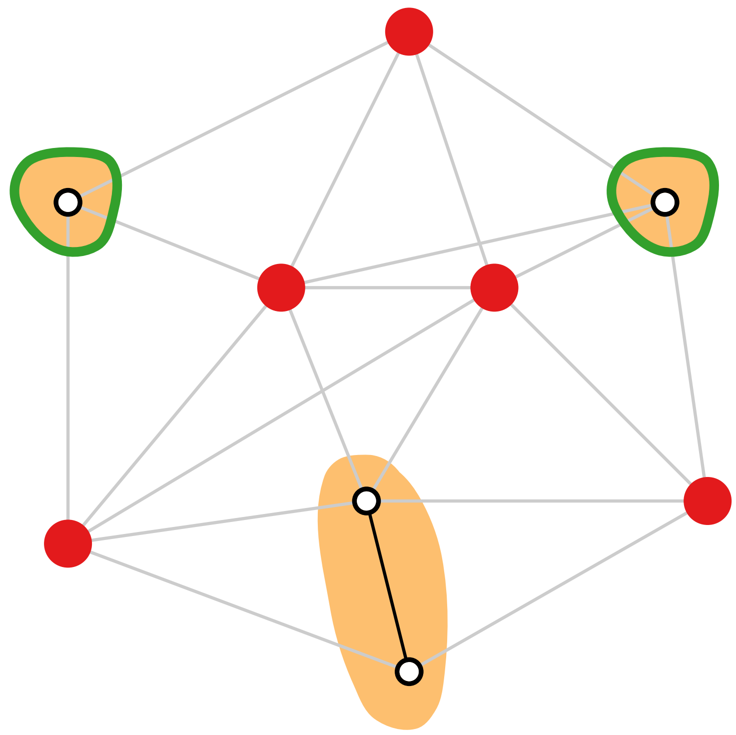


[Tutte '47] G has a (near-)perfect matching
$\Leftrightarrow \text{odd}(S) \leq |S|$ for every $S \subseteq V$

[Berge '57] A maximum matching of G has size
$\frac{1}{2}(|V| - \max_{S \subseteq V} (\qquad))$

The Tutte-Berge Formula

$G = (V, E)$
$S \subseteq V$
$\mathrm{comp}(G \setminus S)$
$\mathrm{odd}(G \setminus S)$

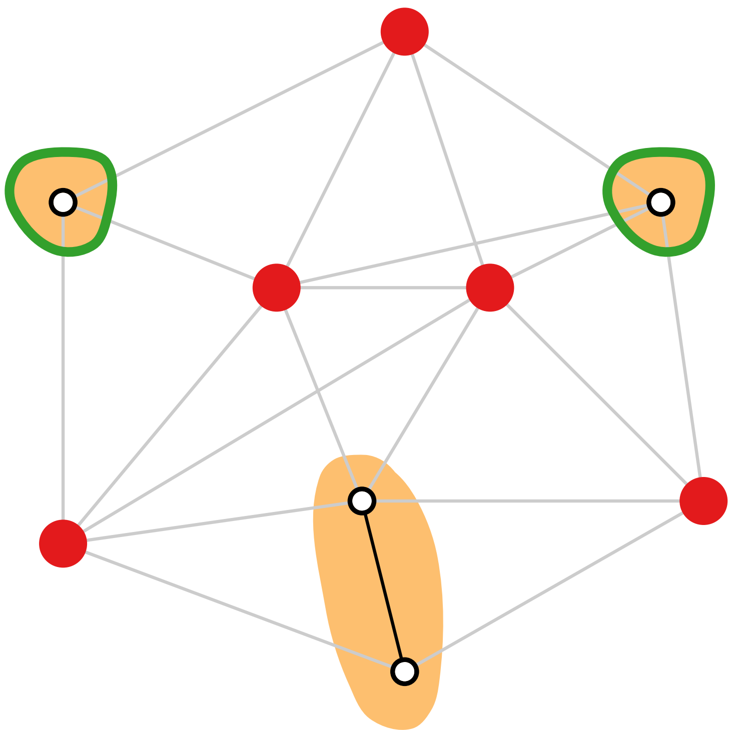


[Tutte '47] G has a (near-)perfect matching
$\Leftrightarrow \mathrm{odd}(S) \leq |S|$ for every $S \subseteq V$

[Berge '57] A maximum matching of G has size
$\frac{1}{2}(|V| - \max_{S \subseteq V}(\mathrm{odd}(G \setminus S) - |S|))$

The Tutte-Berge Formula

$G = (V, E)$
$S \subseteq V$
$\mathrm{comp}(G \setminus S)$
$\mathrm{odd}(G \setminus S)$



[Tutte '47] G has a (near-)perfect matching
$\Leftrightarrow \mathrm{odd}(S) \leq |S|$ for every $S \subseteq V$

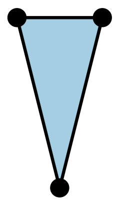
[Berge '57] A maximum matching of G has size
$\frac{1}{2}(|V| - \max_{S \subseteq V} (\mathrm{odd}(G \setminus S) - |S|))$
$\Rightarrow$ there are $\max_{S \subseteq V} (\mathrm{odd}(G \setminus S) - |S|)$ unmatched vtcs

Upper Bound on Maximum Matching

degree of a face

Upper Bound on Maximum Matching

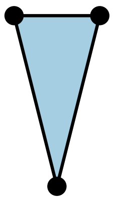
degree of a face



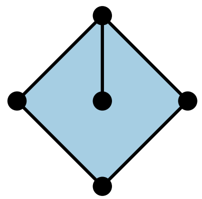
$d = 3$

Upper Bound on Maximum Matching

degree of a face

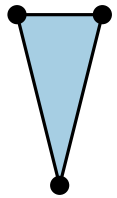


$d = 3$

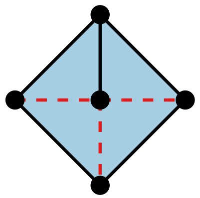


Upper Bound on Maximum Matching

degree of a face

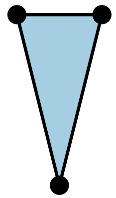


$d = 3$

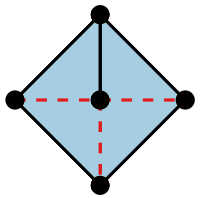


Upper Bound on Maximum Matching

degree of a face



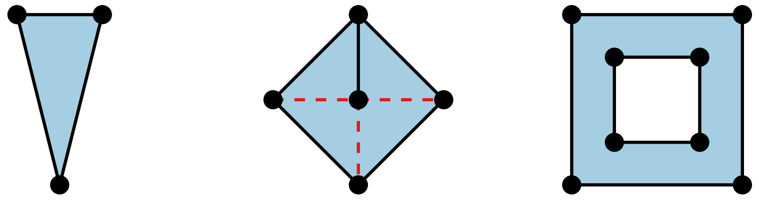
$d = 3$



$d = 6$

Upper Bound on Maximum Matching

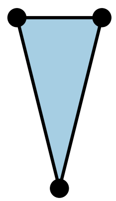
degree of a face



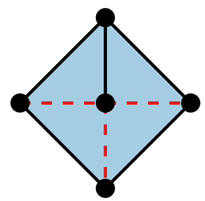
$d = 3$ $\quad$ $d = 6$

Upper Bound on Maximum Matching

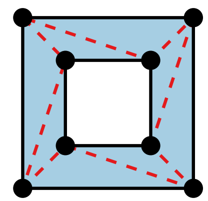
degree of a face



$d = 3$

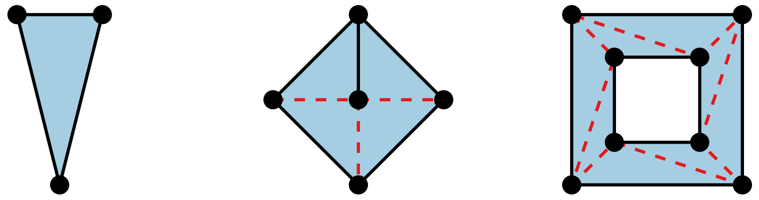


$d = 6$



Upper Bound on Maximum Matching

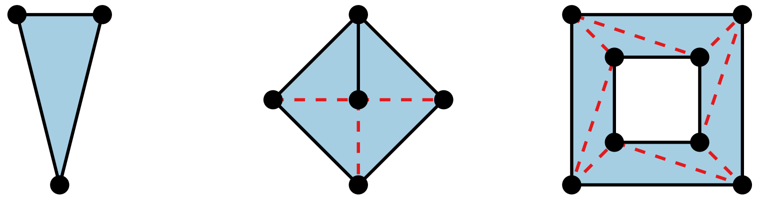
degree of a face



$d = 3$ $d = 6$ $d = 11$

Upper Bound on Maximum Matching

$$\sum_{d \geq 3} (d-2)|F|^{\triangle}_{\deg d} \leq 2n-4$$



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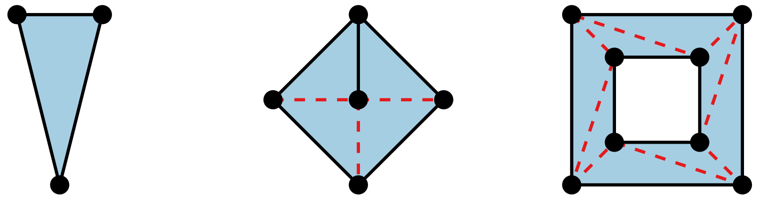
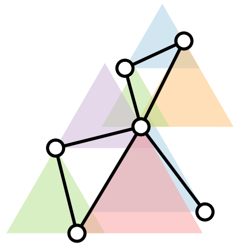
Upper Bound on Maximum Matching

$\sum_{d \geq 3} (d-2) |F|^{\triangle}_{\deg d} \leq 2n - 4$



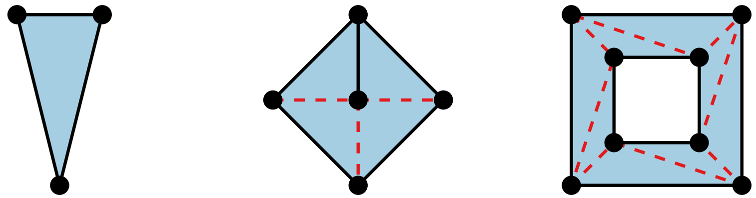
$d = 3$ $\quad$ $d = 6$ $\quad$ $d = 11$

$G^{\triangle}$



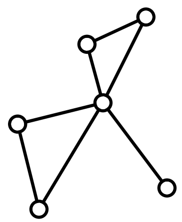
Upper Bound on Maximum Matching

$\sum_{d \geq 3} (d-2) |F|^{\triangle}_{\deg d} \leq 2n - 4$



$d = 3$ $d = 6$ $d = 11$

$G^{\triangle}$



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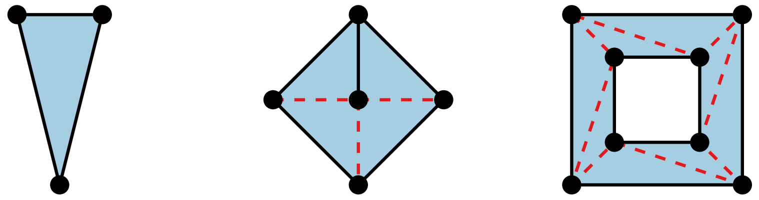

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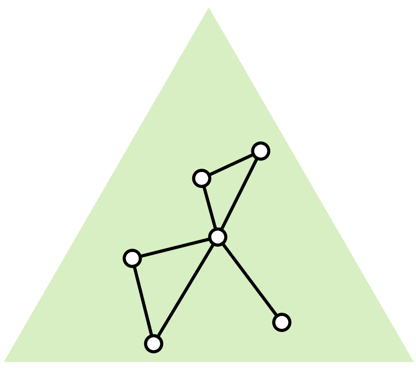
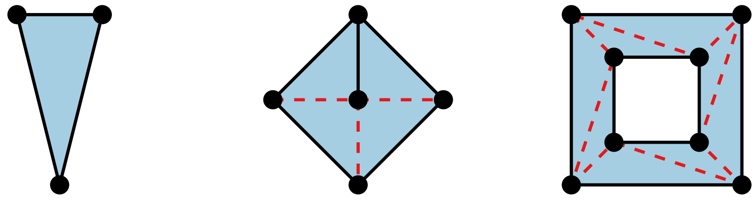


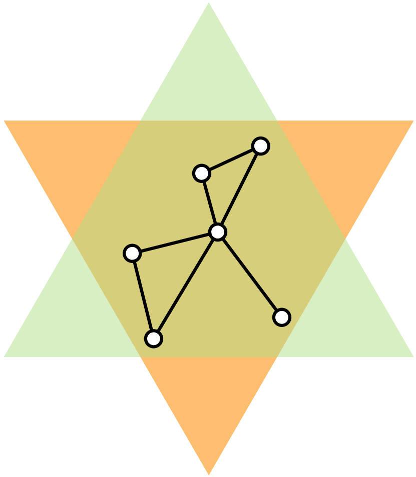
Upper Bound on Maximum Matching

$\sum_{d\geq 3}(d-2)|F|^{\triangle}_{\deg d} \leq 2n-4$

$G^{\triangle}$



$d=3$ $\quad d=6$ $\quad d=11$



Upper Bound on Maximum Matching

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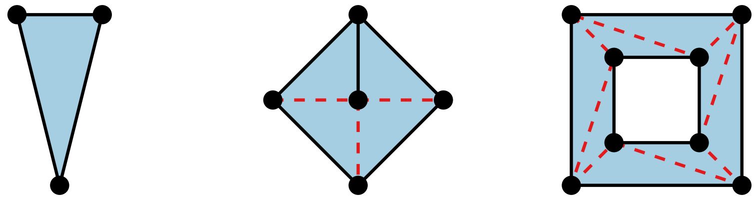

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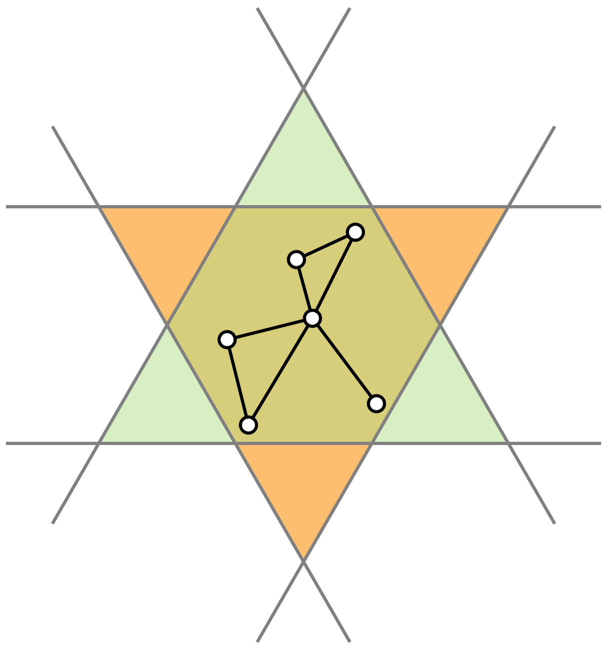

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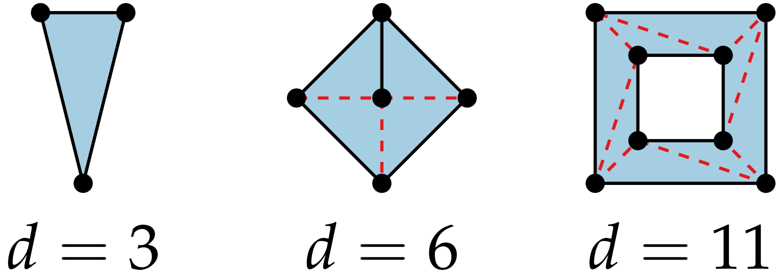
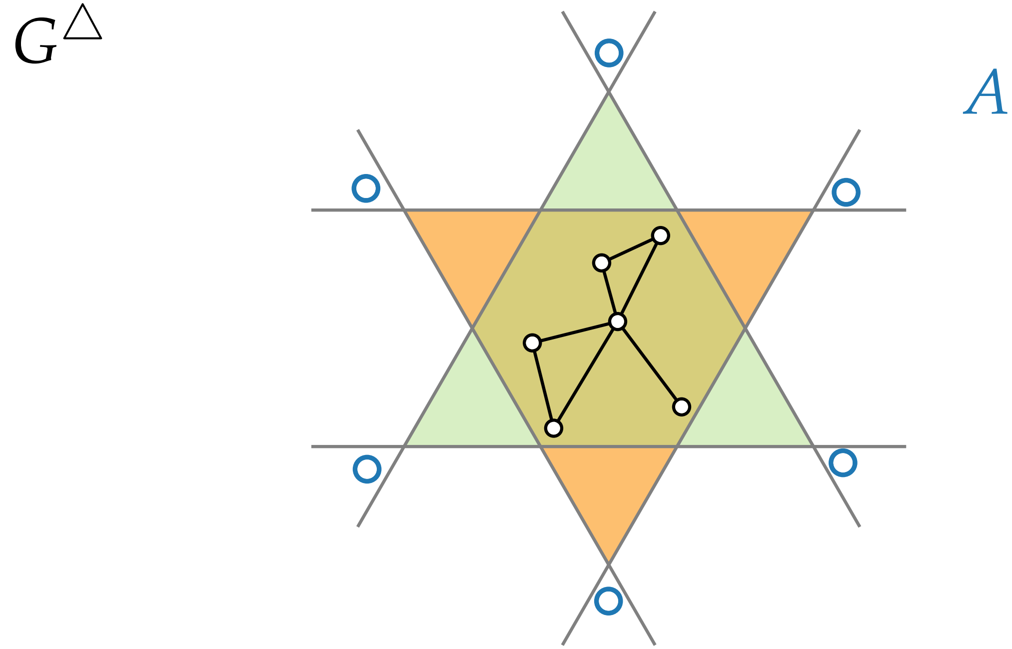
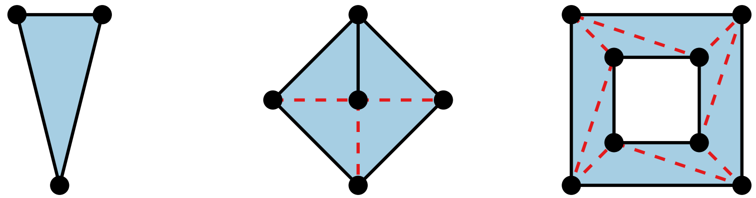


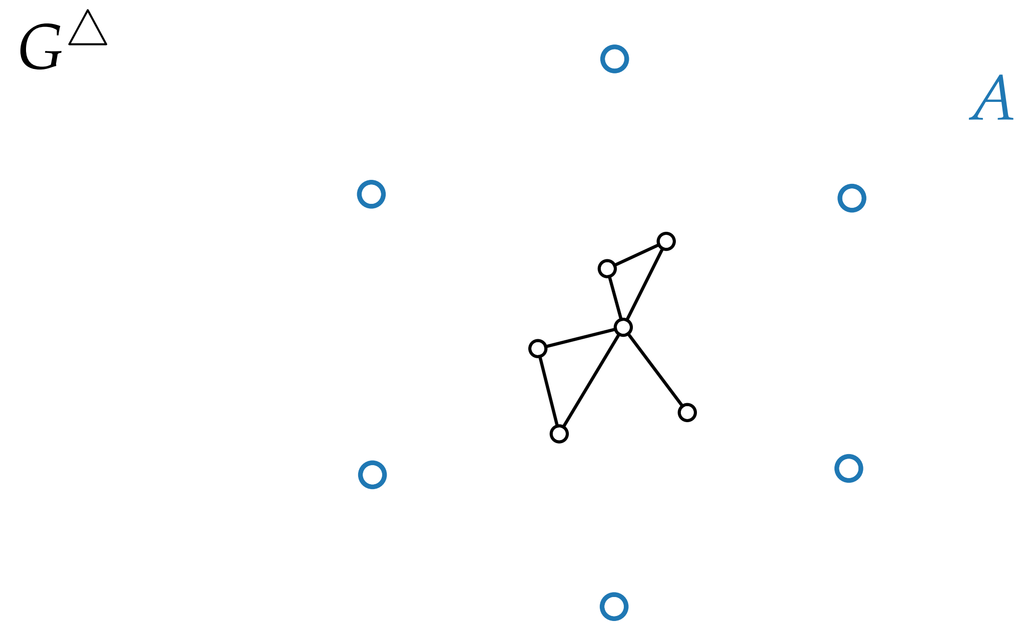


Upper Bound on Maximum Matching

$\sum_{d \geq 3} (d-2) |F|^{\triangle}_{\deg d} \leq 2n - 4$



$d = 3$ $\quad$ $d = 6$ $\quad$ $d = 11$



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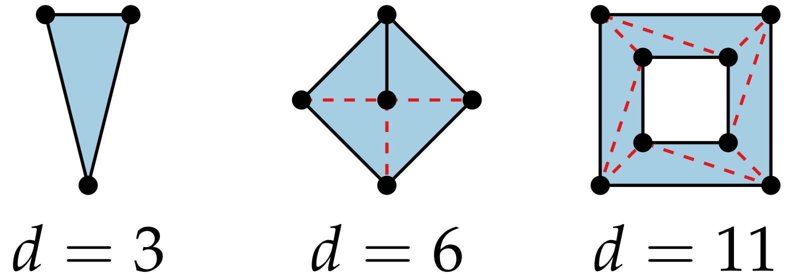
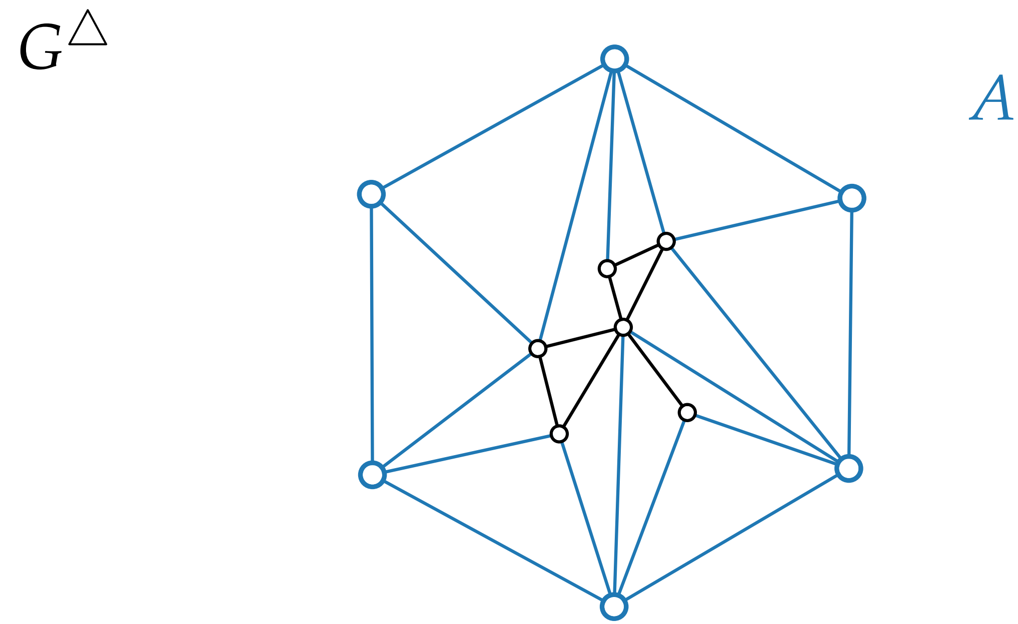
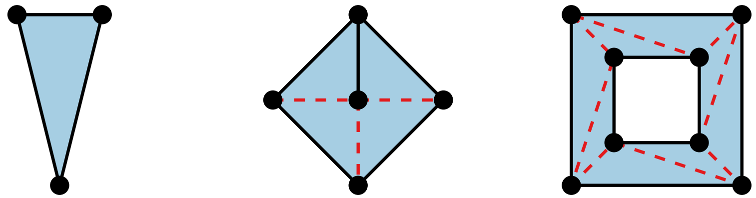




Upper Bound on Maximum Matching

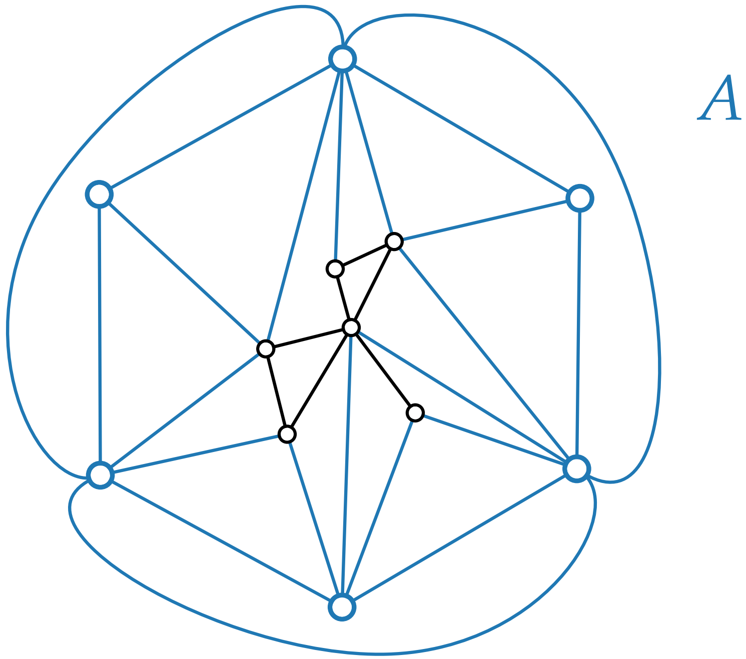
$\sum_{d\geq 3}(d-2)|F|^{\triangle}_{\deg d} \leq 2n-4$



$d = 3$ $d = 6$ $d = 11$

$G^{\triangle}$

$G^{\triangle}_{A}$



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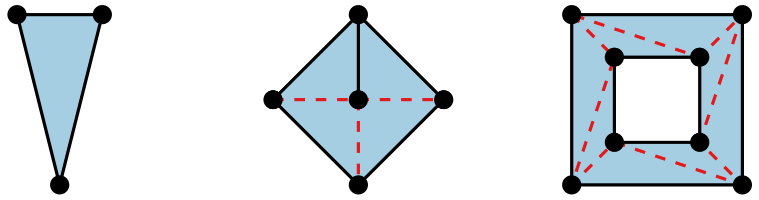


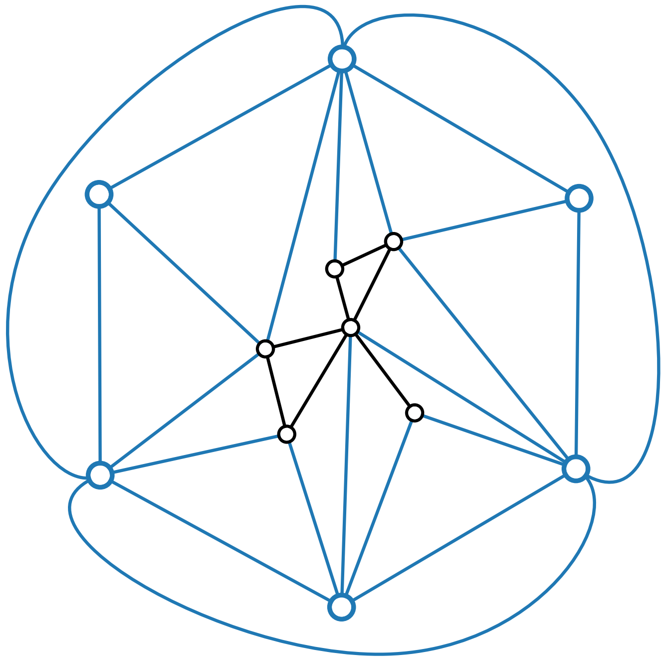
$d = 3$ $\quad d = 6$ $\quad d = 11$

[Dillencourt '90]

$G = (V, E)$ plane triangulation

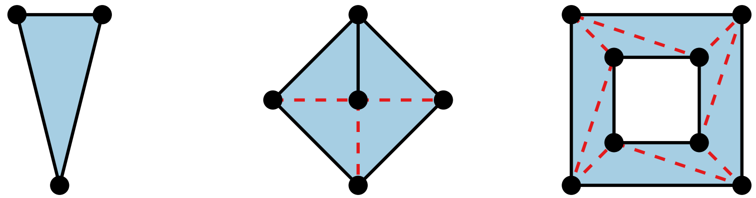
$G^{\triangle}$

$G^{\triangle}_{A}$



Upper Bound on Maximum Matching

$\sum_{d \geq 3} (d-2) |F|^{\triangle}_{\deg d} \leq 2n - 4$



$d = 3$ $d = 6$ $d = 11$

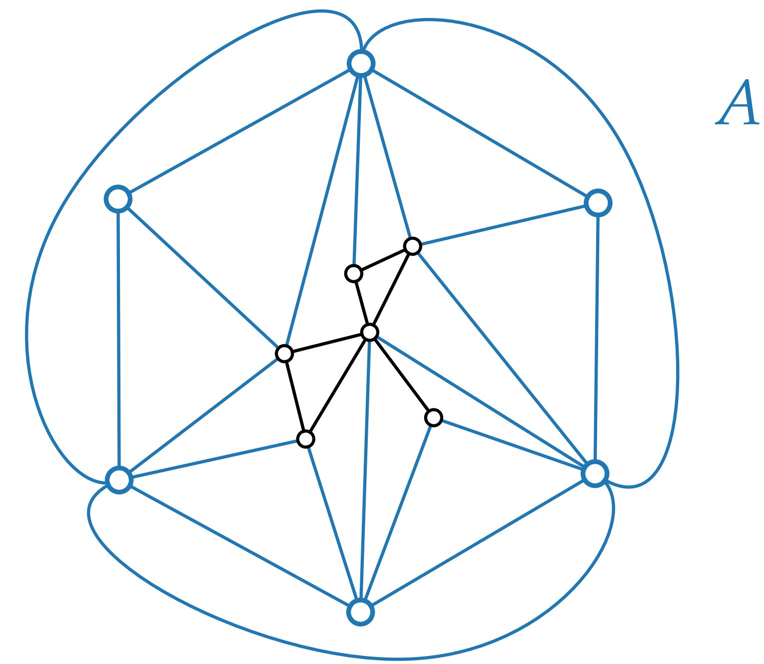
[Dillencourt ’90]

$G = (V, E)$ plane triangulation

$\Rightarrow \forall S \subseteq V :$

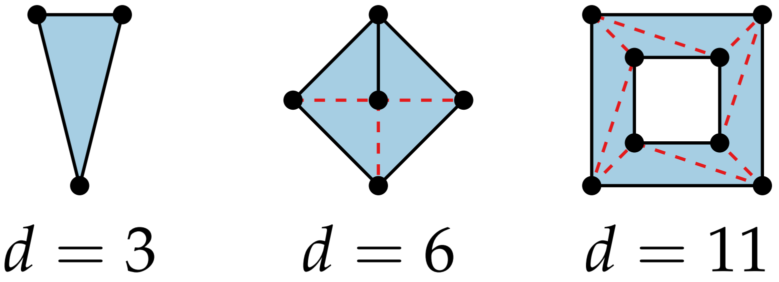
$G^{\triangle}$

$G^{\triangle}_{A}$



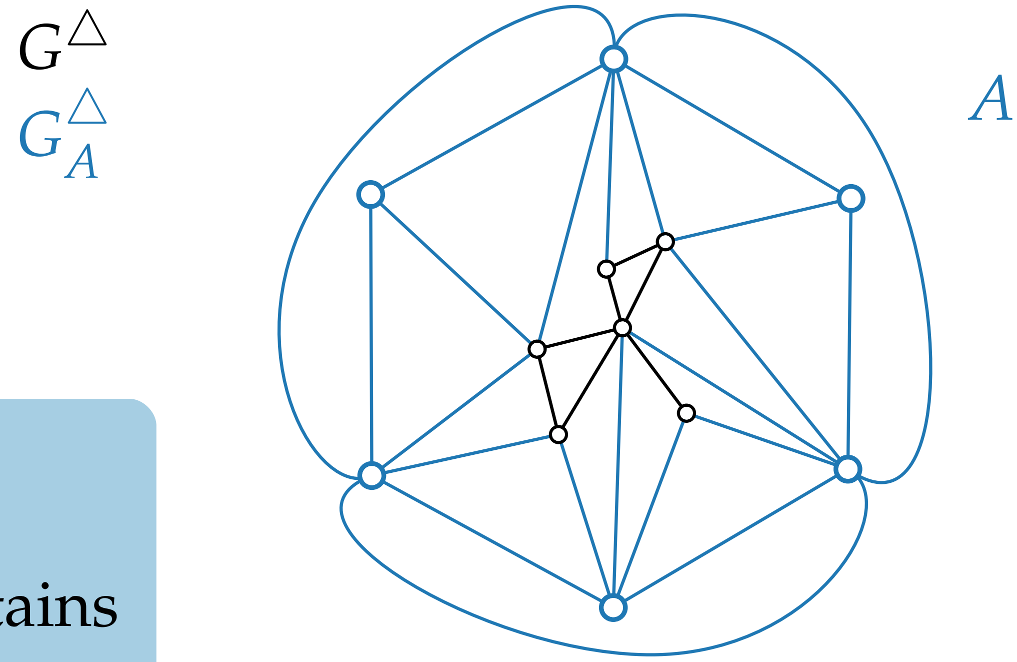
Upper Bound on Maximum Matching

$\sum_{d \geq 3} (d-2) |F|^{\triangle}_{\deg d} \leq 2n - 4$



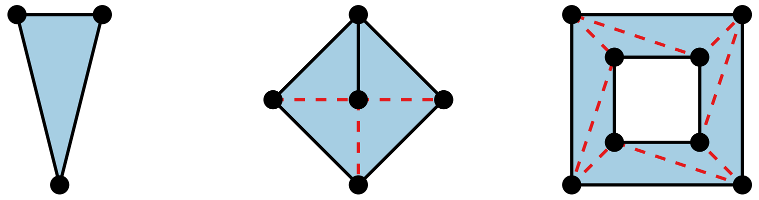
[Dillencourt '90]

$G = (V, E)$ plane triangulation
$\Rightarrow \forall S \subseteq V$: every face of $G[S]$ contains
≤ 1 comp. of $G \setminus S$



Upper Bound on Maximum Matching

$\sum_{d\geq 3}(d-2)|F|^{\triangle}_{\deg d} \leq 2n-4$



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[Dillencourt '90]

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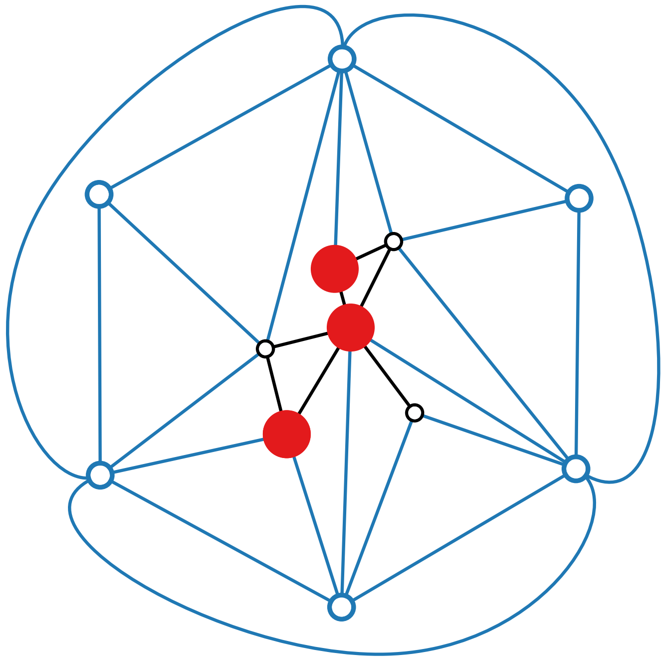
$G^{\triangle}$

$G^{\triangle}_{A}$

S



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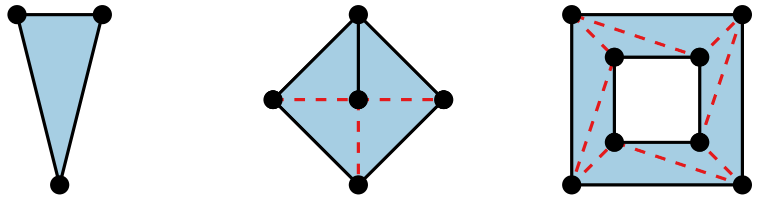

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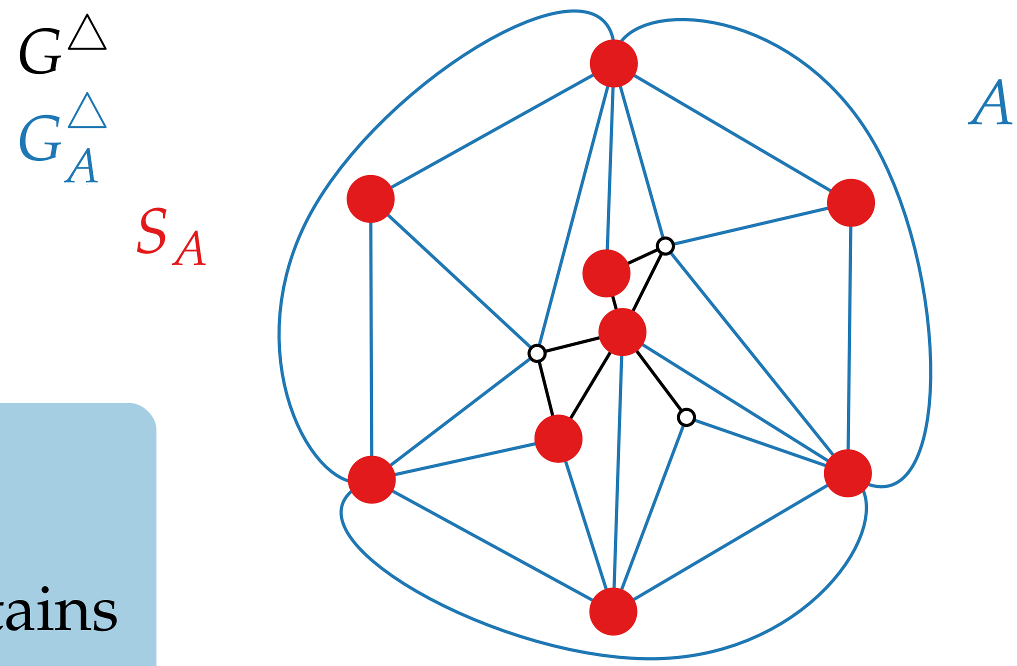

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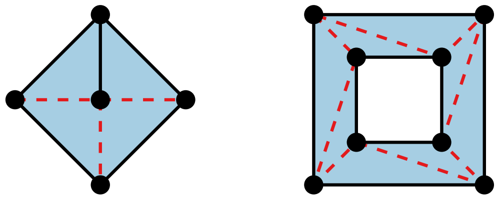

$d=3$ $d=6$ $d=11$

[Dillencourt '90]

$G=(V,E)$ plane triangulation

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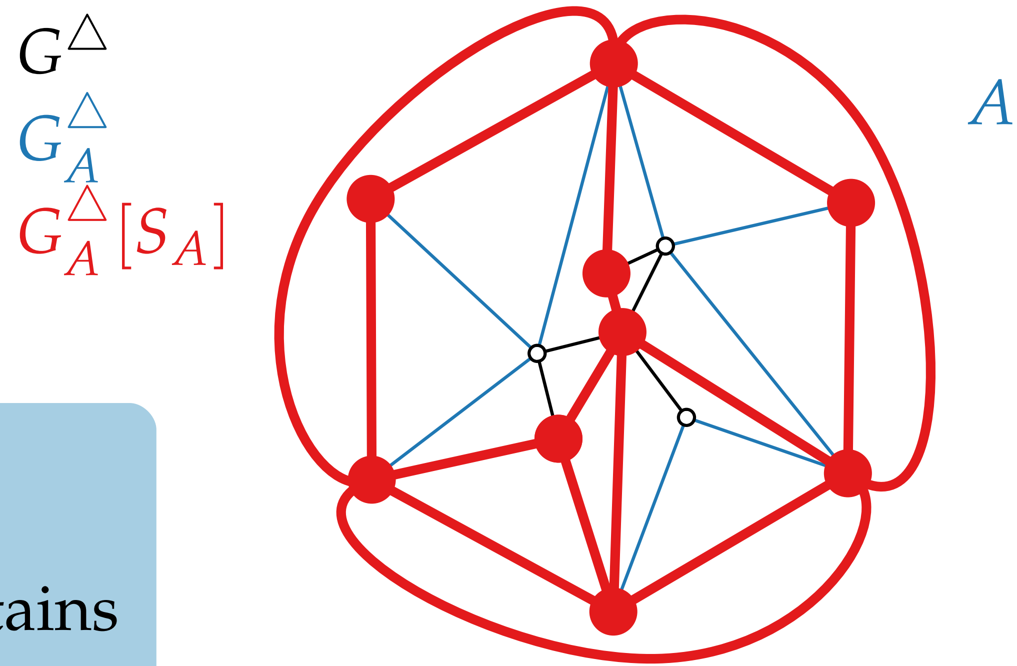

Upper Bound on Maximum Matching

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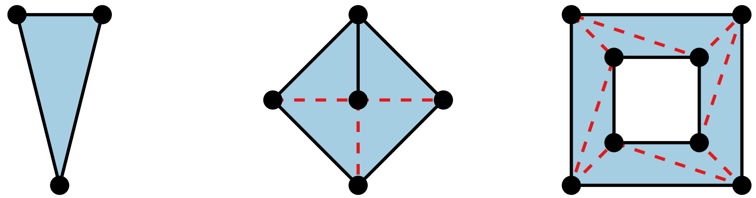

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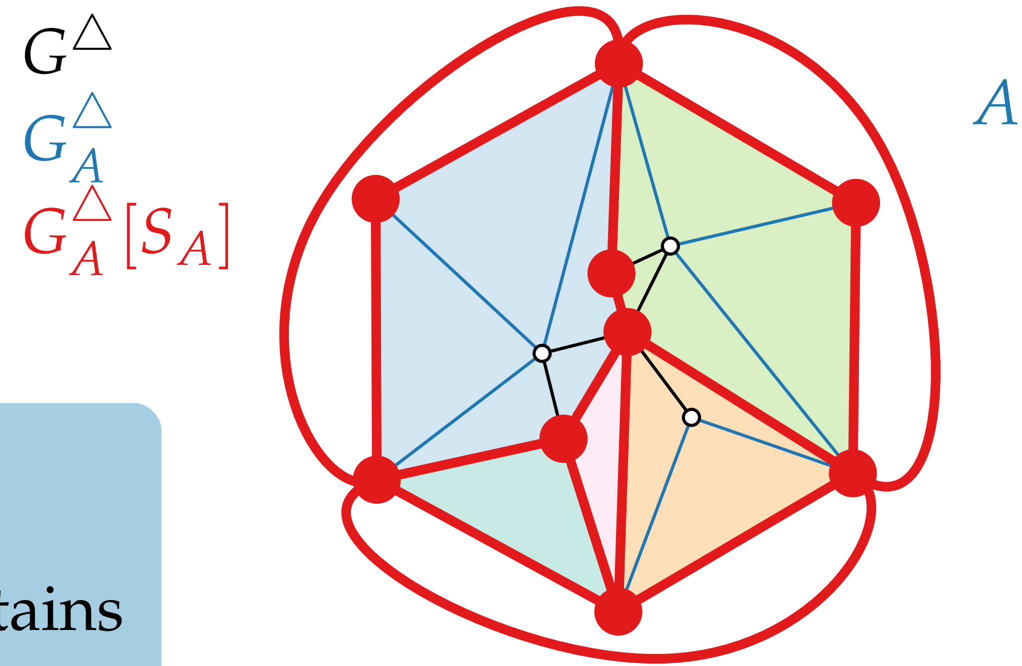
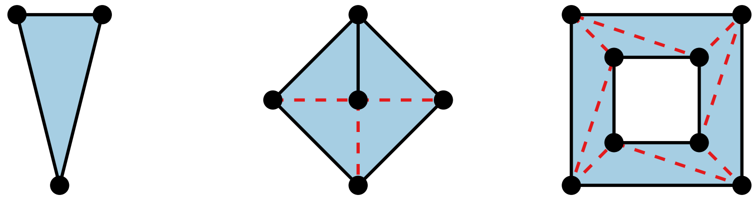


Upper Bound on Maximum Matching

$\sum_{d\geq 3}(d-2)|F|^{\triangle}_{\deg d} \leq 2n-4$



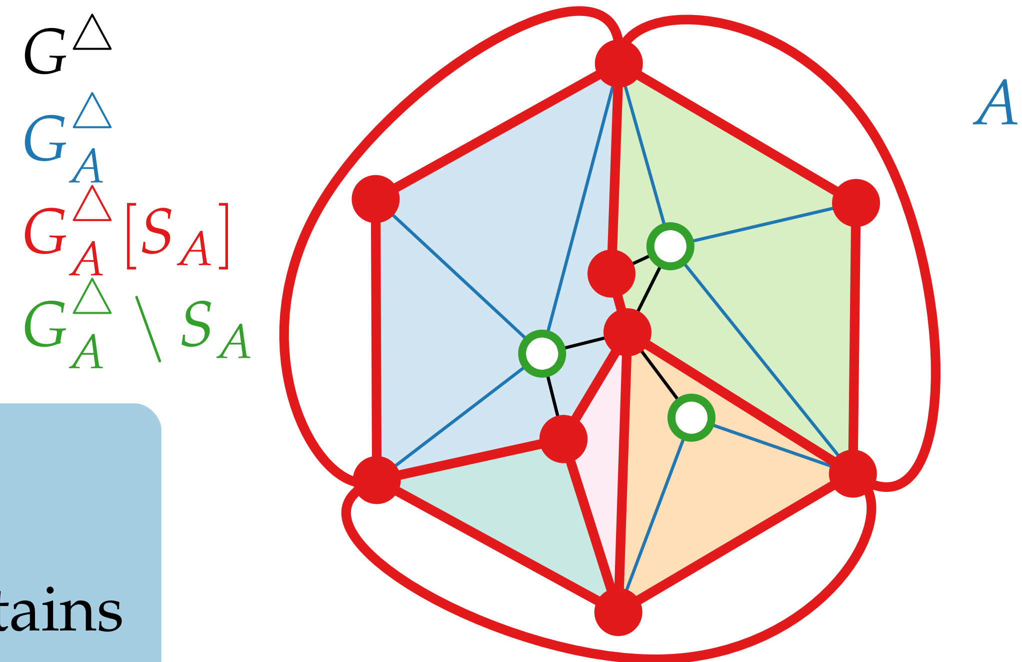
$d=3$ $\quad d=6$ $\quad d=11$

[Dillencourt '90]

$G=(V,E)$ plane triangulation

$\Rightarrow \forall S\subseteq V$: every face of $G[S]$ contains

≤ 1 comp. of $G\setminus S$



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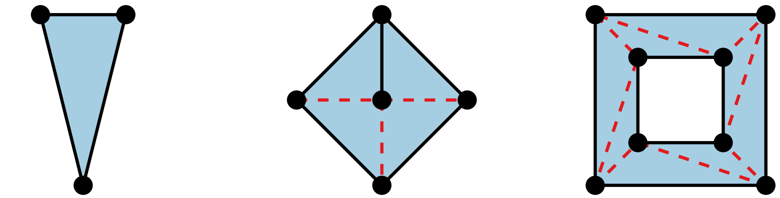

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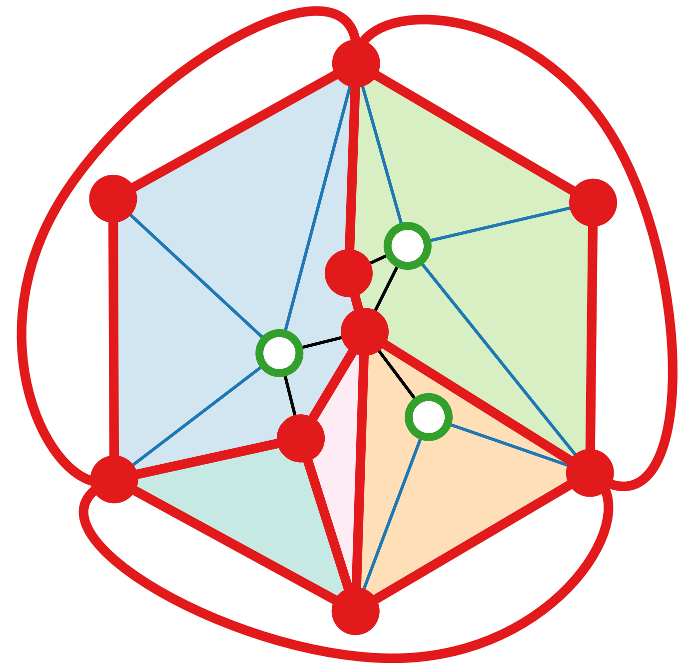
$G^{\triangle}$

$G^{\triangle}_{A}$

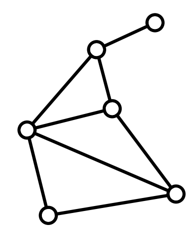
$G^{\triangle}_{A}[S_A]$

$G^{\triangle}_{A} \setminus S_A$



A

G^{∇}



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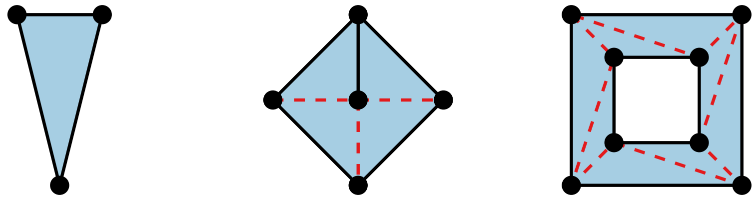

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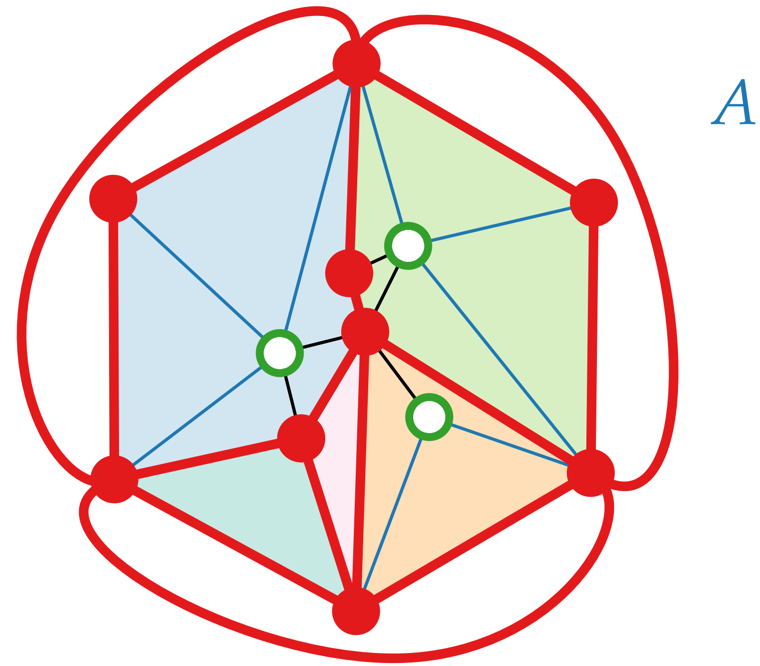
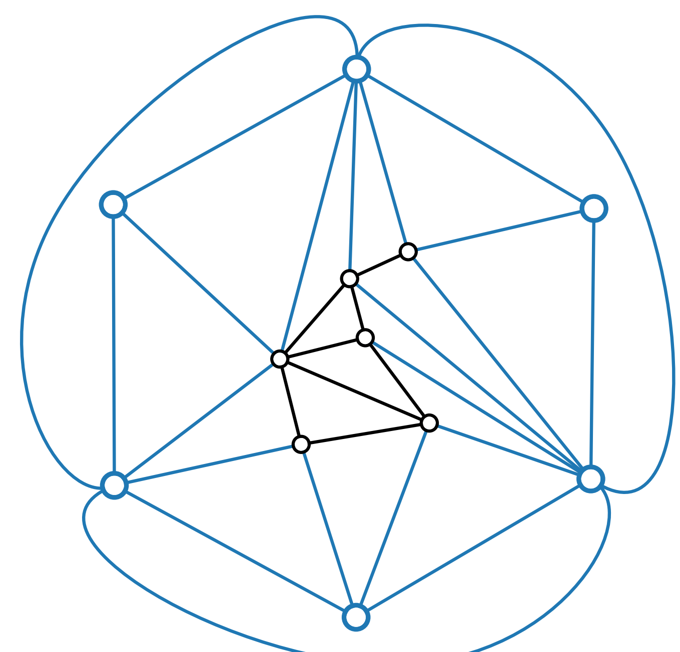
$G^{\triangle}_A$
$G^{\triangle}_A[S_A]$
$G^{\triangle}_A \setminus S_A$



$G^{\triangledown}$
$G^{\triangledown}_A$



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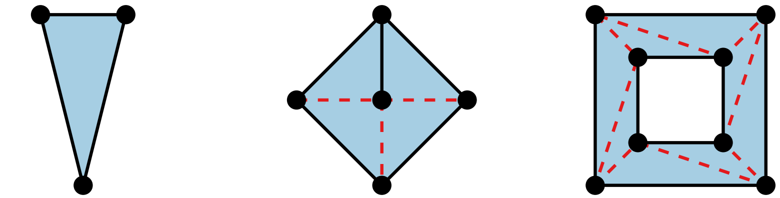

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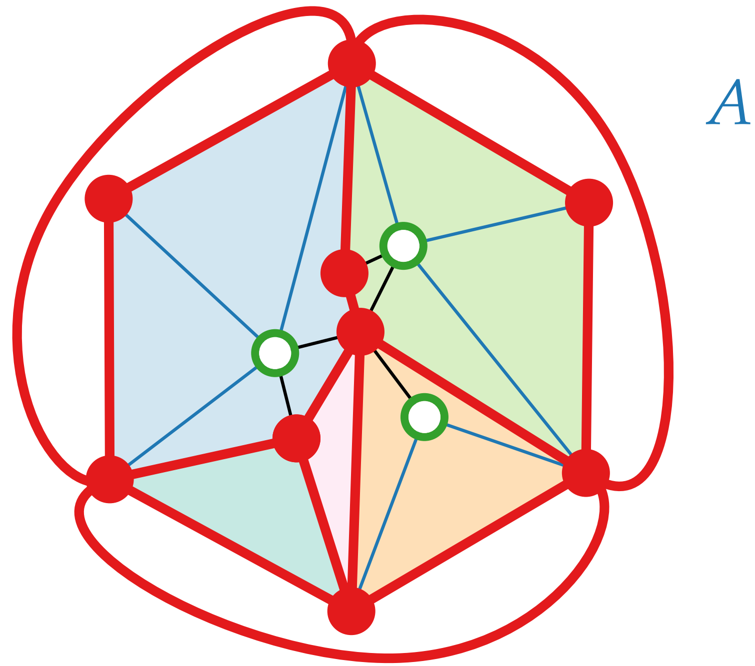
$G^{\triangle}_{A}$

$G^{\triangle}_{A}[S_A]$

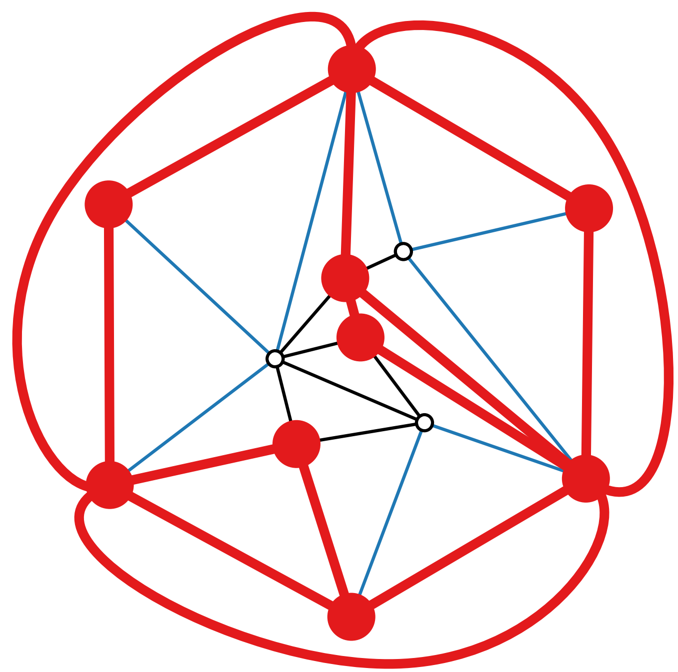
$G^{\triangle}_{A} \setminus S_A$



$G^{\triangledown}$

$G^{\triangledown}_{A}$

$G^{\triangledown}_{A}[S_A]$



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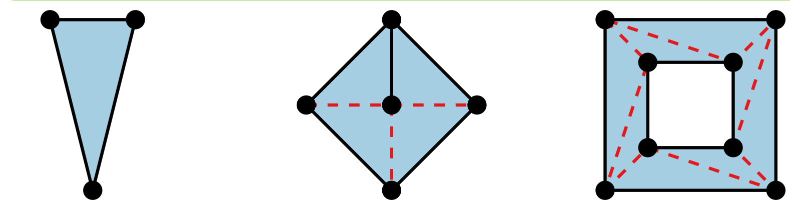

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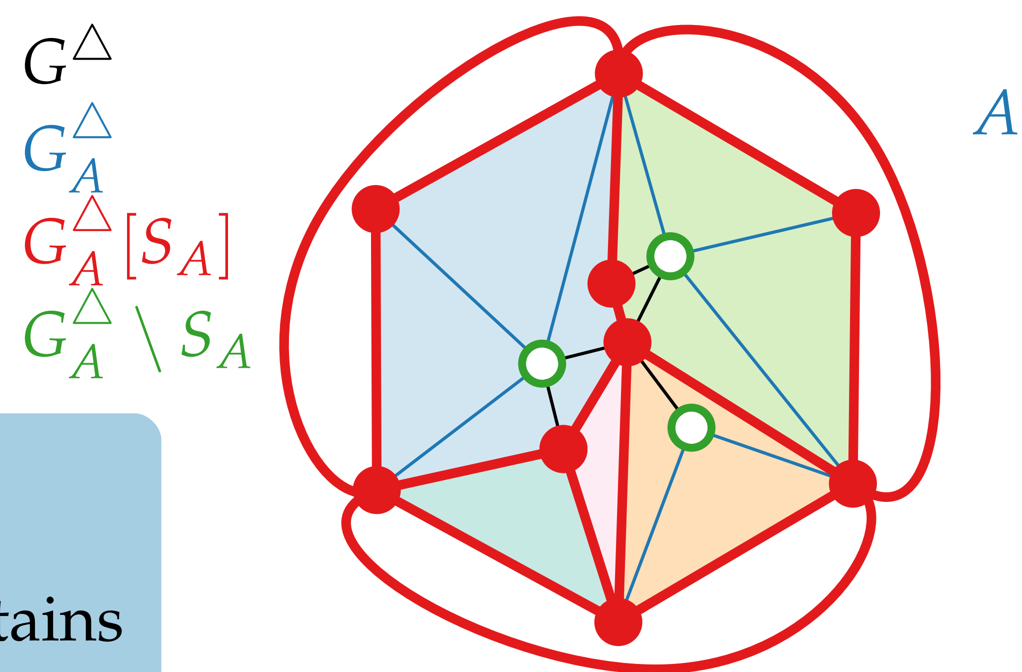

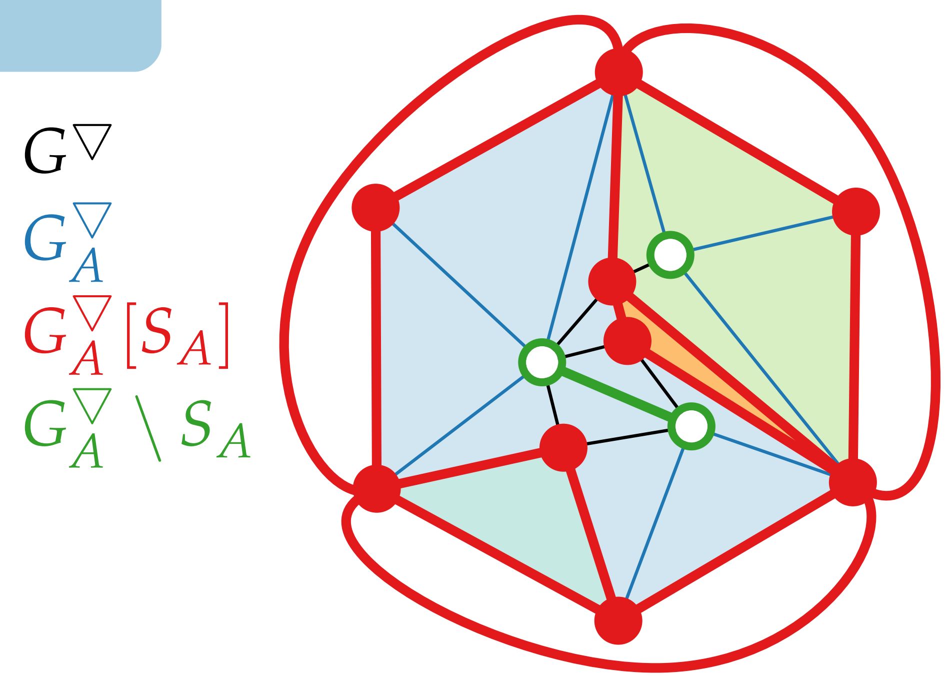

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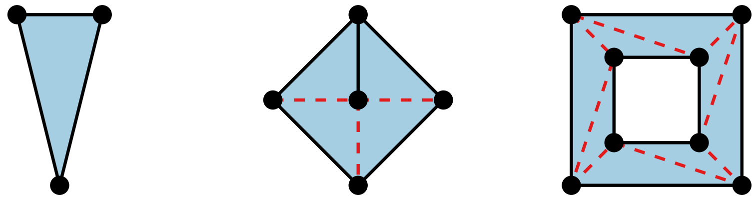

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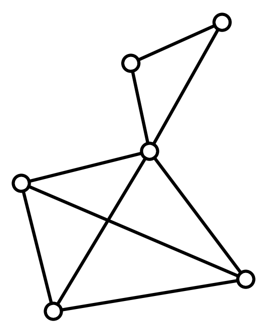
$G^{\text{✡}}$

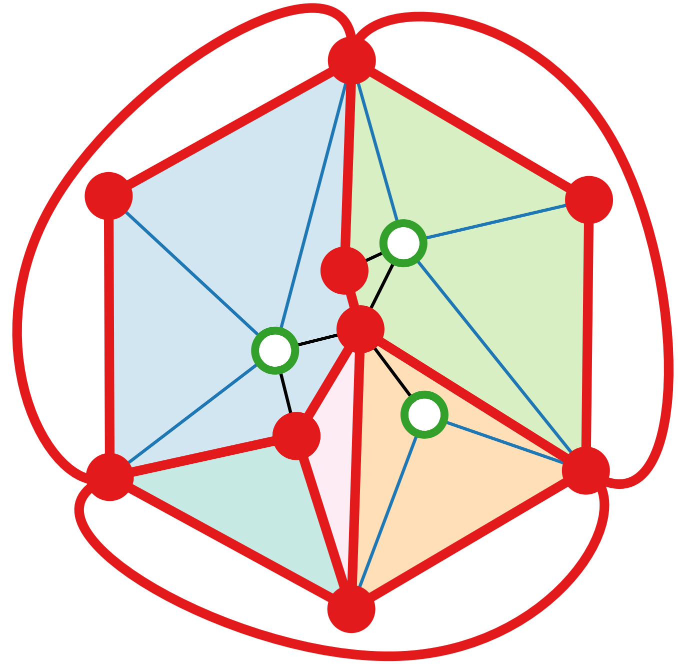


$G^{\triangle}$

$G^{\triangle}_{A}$

$G^{\triangle}_{A}[S_A]$

$G^{\triangle}_{A} \setminus S_A$



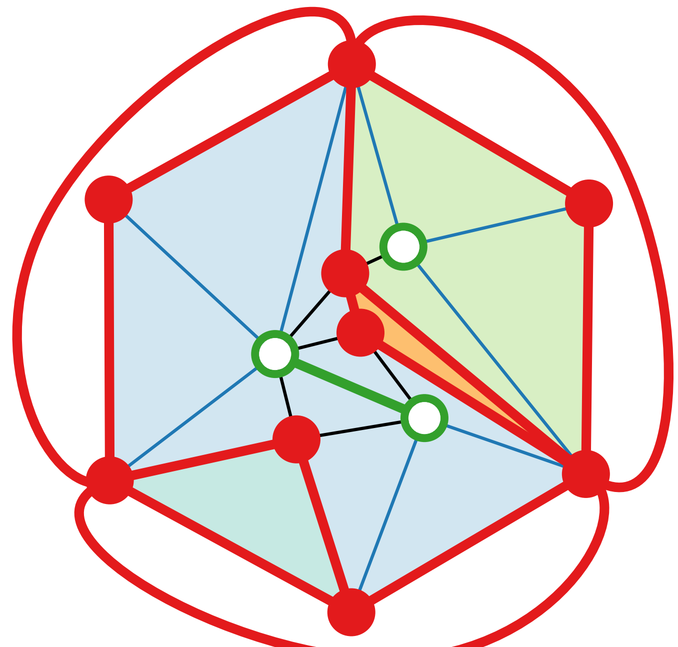
A

$G^{\triangledown}$

$G^{\triangledown}_{A}$

$G^{\triangledown}_{A}[S_A]$

$G^{\triangledown}_{A} \setminus S_A$



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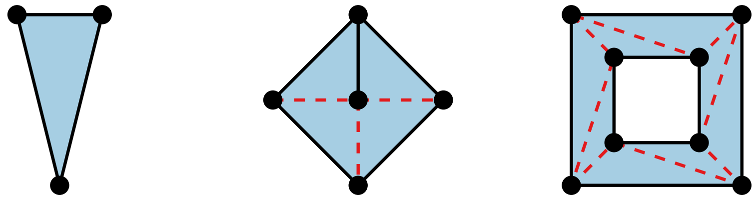

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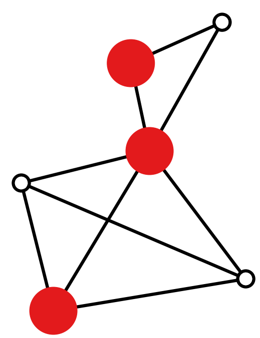
S

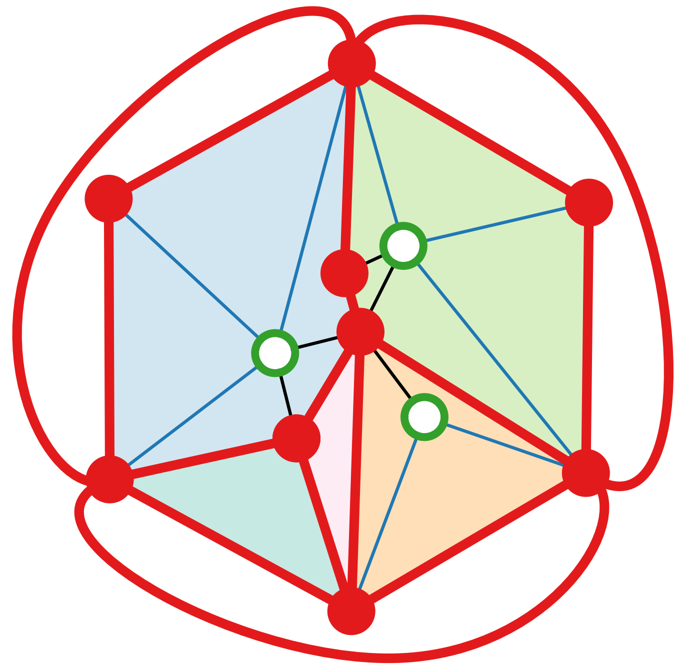


$G^{\triangle}$

$G^{\triangle}_{A}$

$G^{\triangle}_{A}[S_A]$

$G^{\triangle}_{A} \setminus S_A$



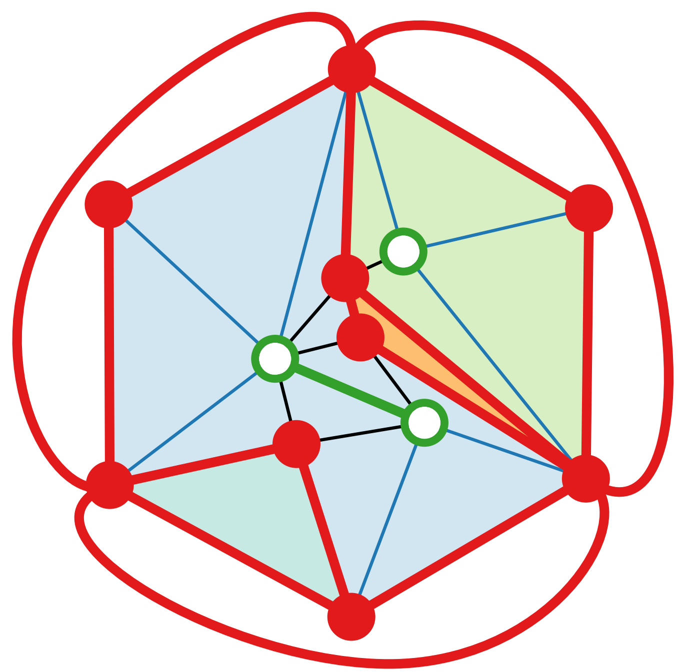
A

$G^{\triangledown}$

$G^{\triangledown}_{A}$

$G^{\triangledown}_{A}[S_A]$

$G^{\triangledown}_{A} \setminus S_A$



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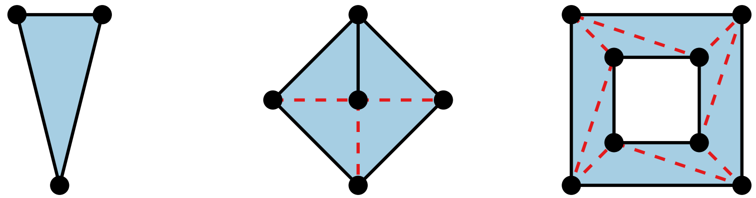

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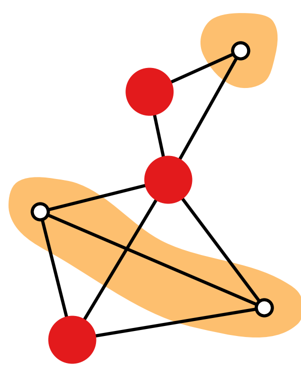
$G^{✡}$

S

$\text{comp}(G^{✡} \setminus S)$

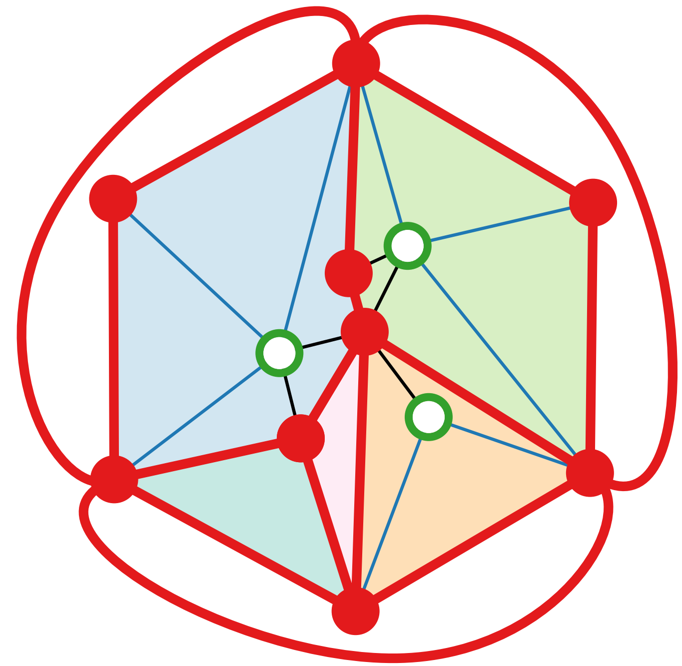


$G^{\triangle}$

$G^{\triangle}_A$

$G^{\triangle}_A[S_A]$

$G^{\triangle}_A \setminus S_A$



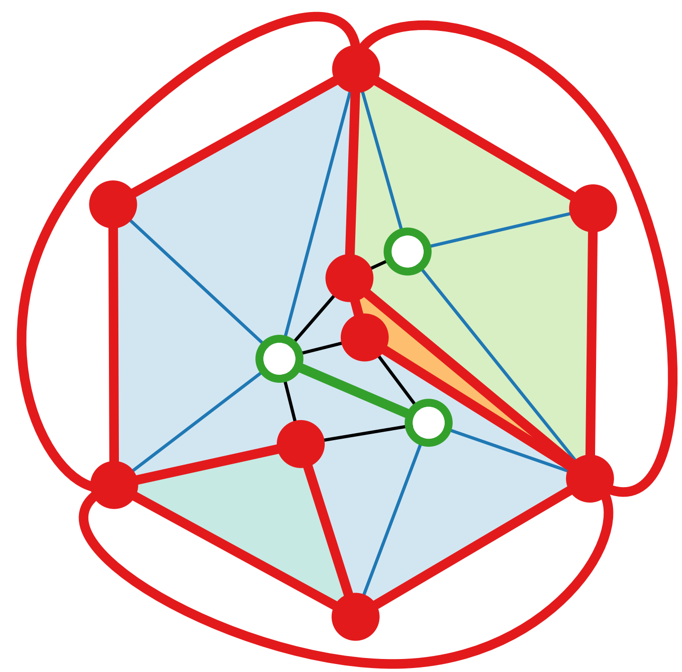
A

$G^{\triangledown}$

$G^{\triangledown}_A$

$G^{\triangledown}_A[S_A]$

$G^{\triangledown}_A \setminus S_A$



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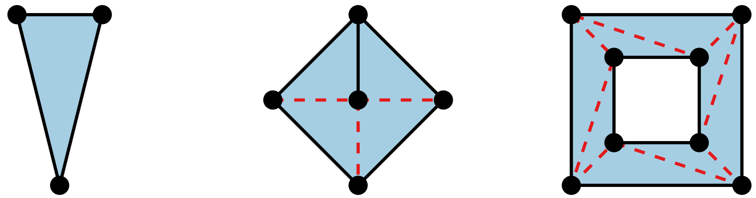

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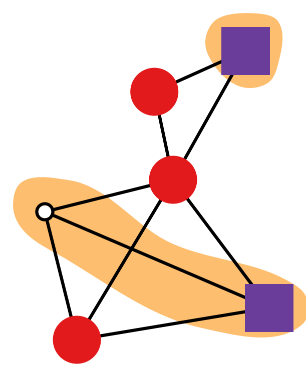
$G^{✡}$

S

$\mathrm{comp}(G^{✡}\setminus S)$

$|Q| = \mathrm{comp}(G^{✡}\setminus S)$

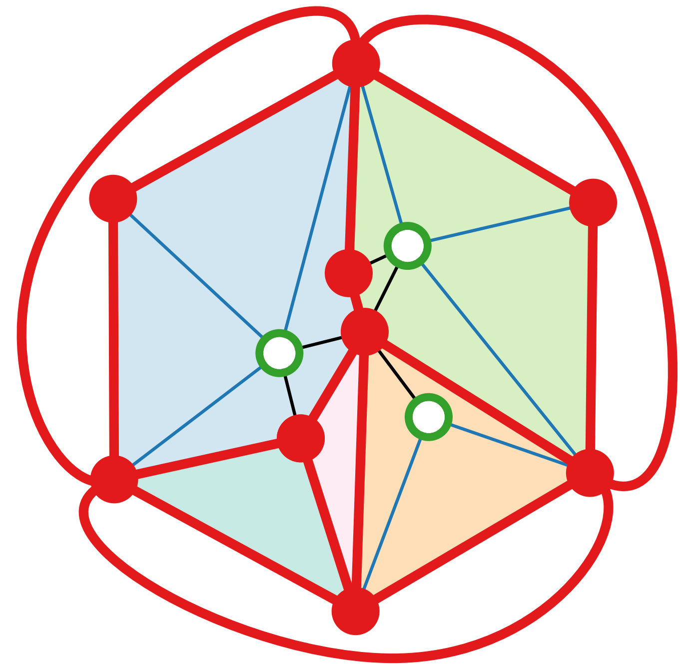


$G^{\triangle}$

$G^{\triangle}_A$

$G^{\triangle}_A[S_A]$

$G^{\triangle}_A\setminus S_A$

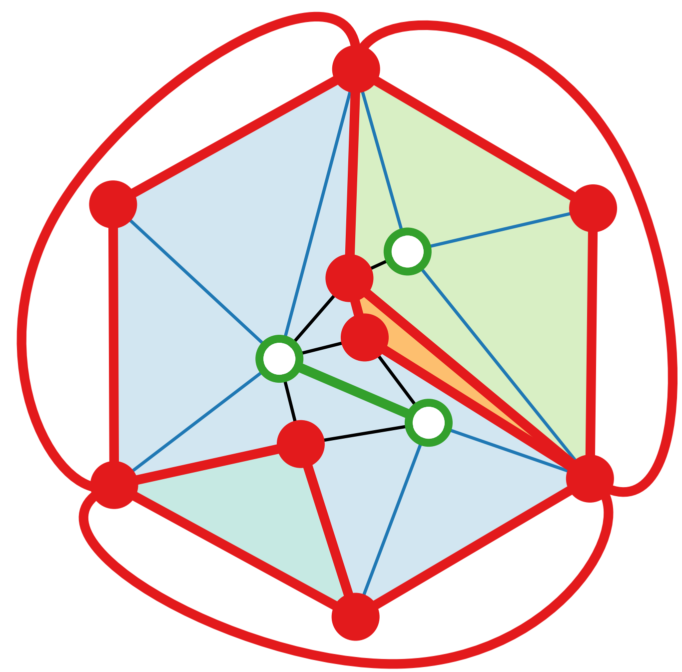


G^{∇}

G^{∇}_A

$G^{\nabla}_A[S_A]$

$G^{\nabla}_A\setminus S_A$



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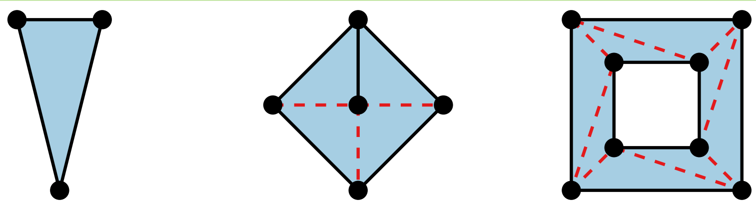

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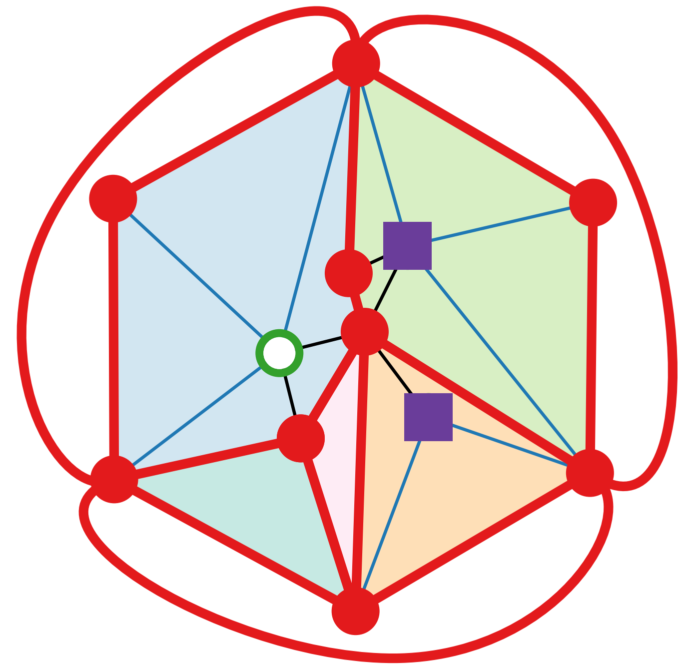
$G^{\triangle}$

$G^{\triangle}_{A}$

$G^{\triangle}_{A}[S_A]$

$G^{\triangle}_{A} \setminus S_A$



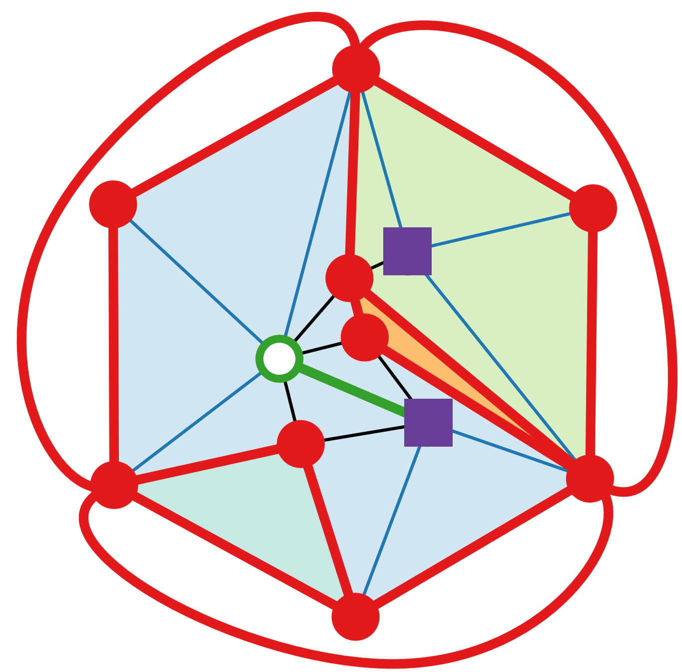
A

$G^{\triangledown}$

$G^{\triangledown}_{A}$

$G^{\triangledown}_{A}[S_A]$

$G^{\triangledown}_{A} \setminus S_A$

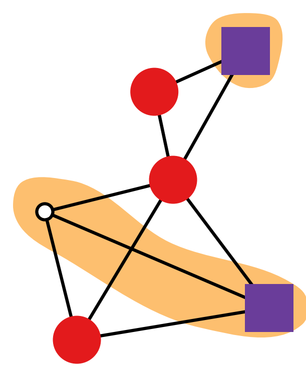


$G^{\text{✡}}$

S

$\mathrm{comp}(G^{\text{✡}} \setminus S)$

$|Q| = \mathrm{comp}(G^{\text{✡}} \setminus S)$



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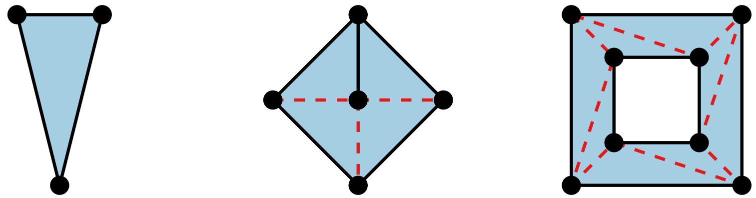

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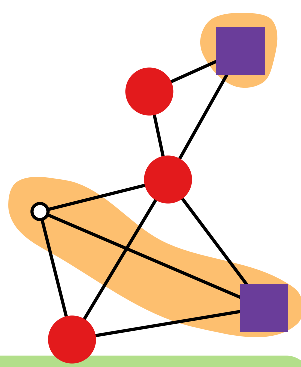
$G^{✡}$

S

$\mathrm{comp}(G^{✡} \setminus S)$

$|Q| = \mathrm{comp}(G^{✡} \setminus S)$



$q \in Q$ in deg-3 face of $G^{\triangle}_{A}[S_A]$

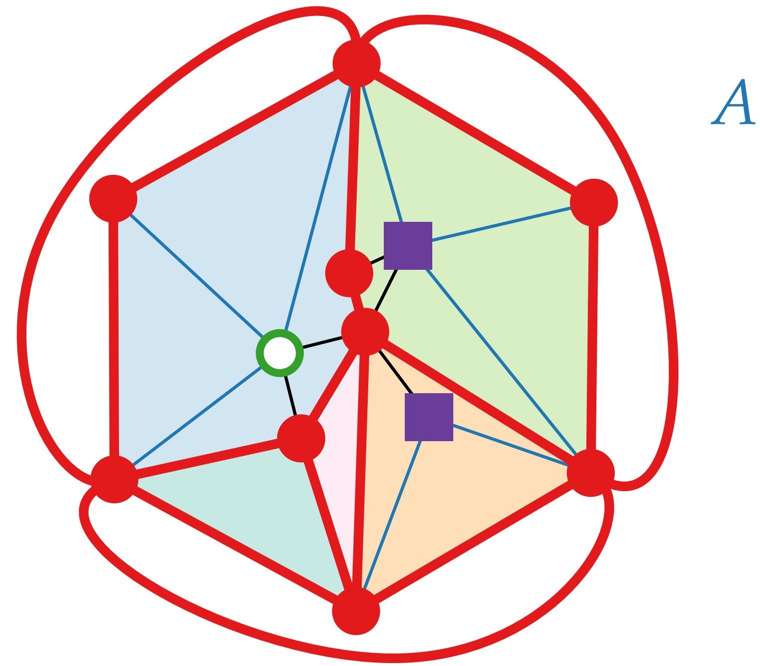
$\Rightarrow q$ in deg-4^{+} face of $G^{\triangledown}_{A}[S_A]$

$G^{\triangle}$

$G^{\triangle}_{A}$

$G^{\triangle}_{A}[S_A]$

$G^{\triangle}_{A} \setminus S_A$

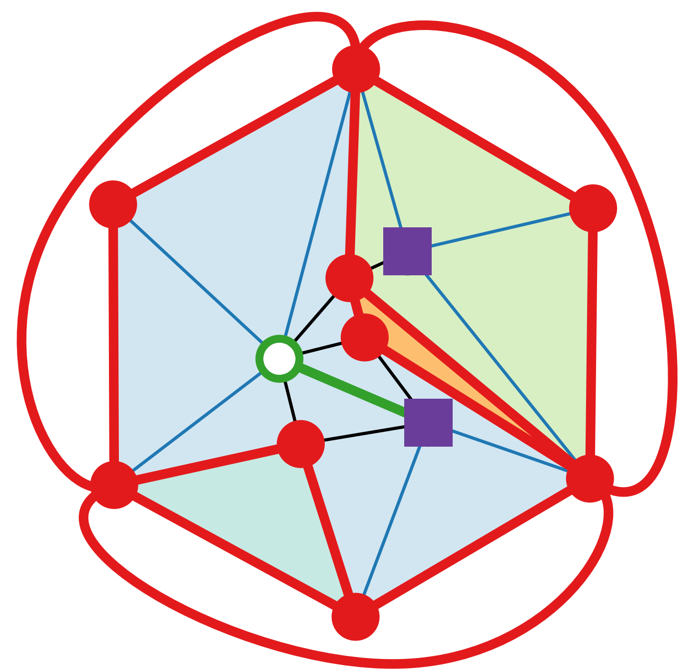


$G^{\triangledown}$

$G^{\triangledown}_{A}$

$G^{\triangledown}_{A}[S_A]$

$G^{\triangledown}_{A} \setminus S_A$



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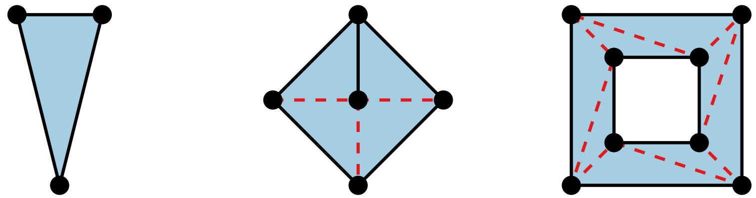

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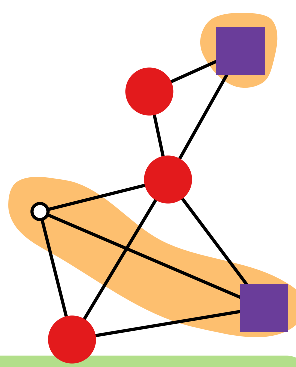
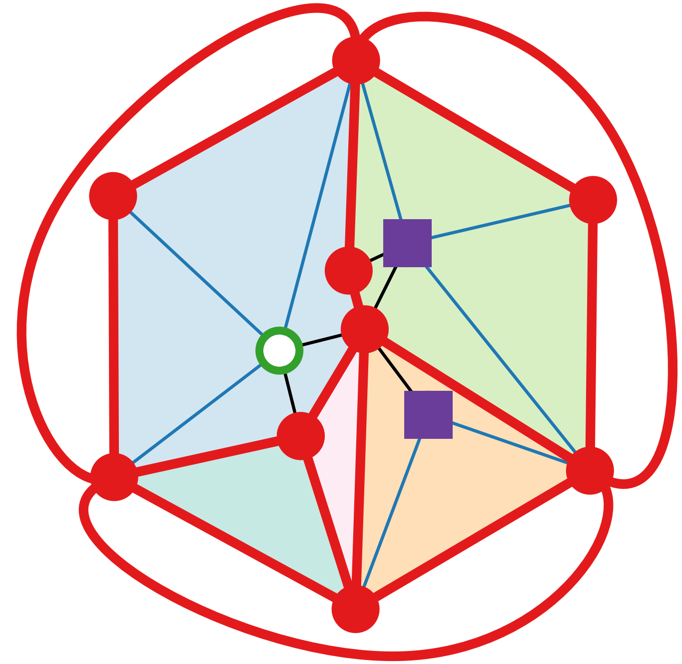
$G^{✡}$
S
$\mathrm{comp}(G^{✡} \setminus S)$
$|Q| = \mathrm{comp}(G^{✡} \setminus S)$



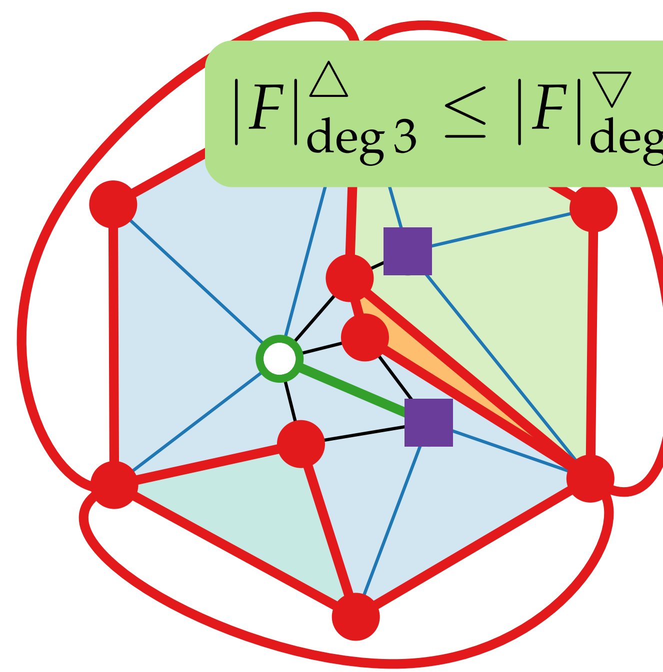
$q \in Q$ in deg-3 face of $G_A^{\triangle}[S_A]$
$\Rightarrow q$ in deg-4^+ face of $G_A^{\bigtriangledown}[S_A]$

$G^{\triangle}$
$G_A^{\triangle}$
$G_A^{\triangle}[S_A]$
$G_A^{\triangle} \setminus S_A$



A

$G^{\bigtriangledown}$
$G_A^{\bigtriangledown}$
$G_A^{\bigtriangledown}[S_A]$
$G_A^{\bigtriangledown} \setminus S_A$



$|F|^{\triangle}_{\deg 3} \leq |F|^{\bigtriangledown}_{\deg 4+}$

Upper Bound on Maximum Matching

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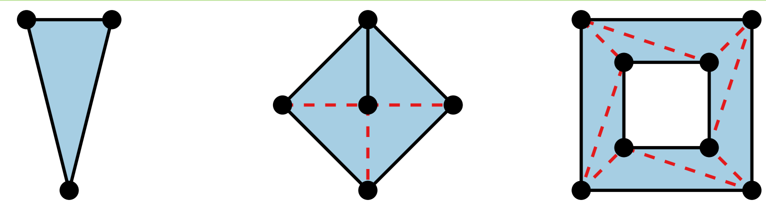

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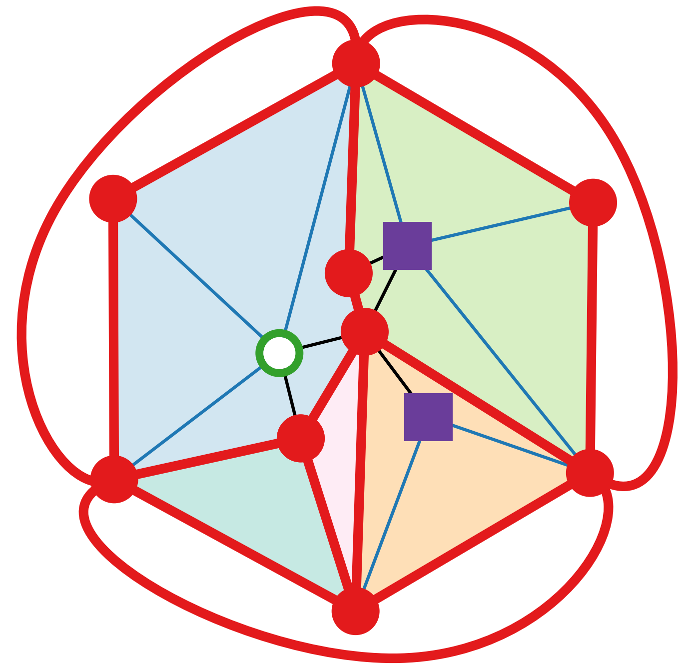
$G^{\triangle}$

$G^{\triangle}_A$

$G^{\triangle}_A[S_A]$

$G^{\triangle}_A \setminus S_A$

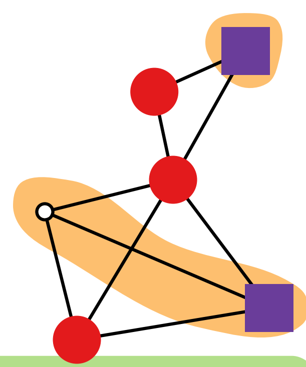


$G^{✡}$

S

$\mathrm{comp}(G^{✡} \setminus S)$

$|Q| = \mathrm{comp}(G^{✡} \setminus S)$



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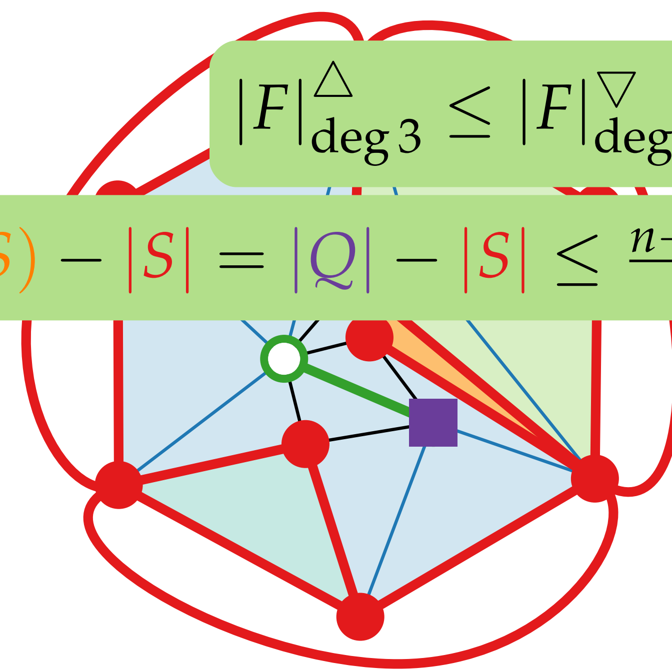
$\Rightarrow q$ in deg-4^+ face of $G^{\triangledown}_A[S_A]$

$G^{\triangledown}$

$G^{\triangledown}_A[S_A]$

$G^{\triangledown}_A \setminus S_A$



$|F|^{\triangle}_{\deg 3} \leq |F|^{\triangledown}_{\deg 4+}$

$\mathrm{comp}(G^{✡} \setminus S) - |S| = |Q| - |S| \leq \frac{n+16}{7}$

Upper Bound on Maximum Matching

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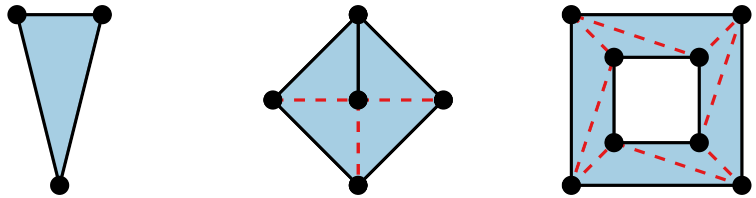

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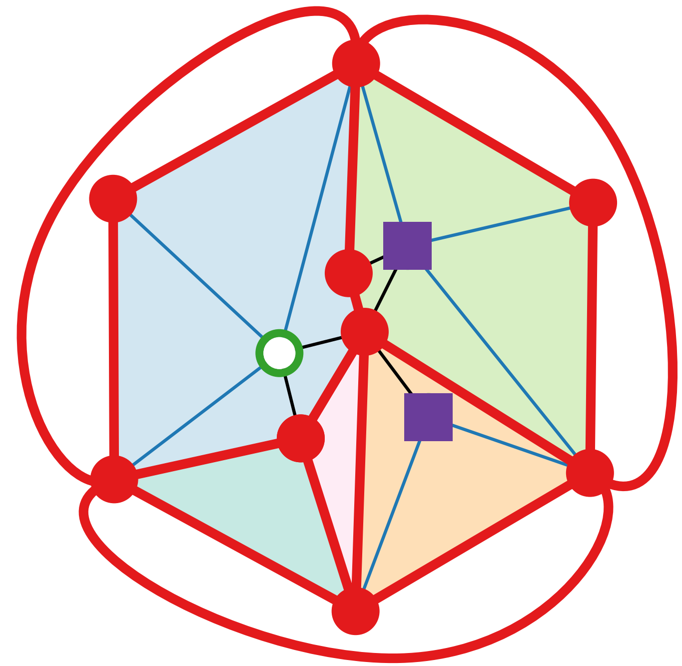
$G^{\triangle}_{A}$

$G^{\triangle}_{A}[S_A]$

$G^{\triangle}_{A} \setminus S_A$

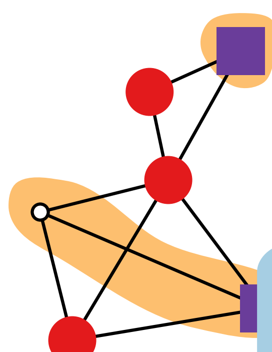


$G^{\text{✡}}$

S

$\mathrm{comp}(G^{\text{✡}} \setminus S)$

$|Q|=\mathrm{comp}(G^{\text{✡}} \setminus S)$



$G^{\triangledown}$

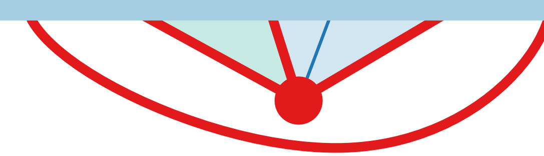
$|F|^{\triangle}_{\deg 3} \leq |F|^{\triangledown}_{\deg 4+}$

$\mathrm{comp}(G^{\text{✡}} \setminus S) - |S| = |Q| - |S| \leq \frac{n+16}{7}$

[Tutte-Berge]

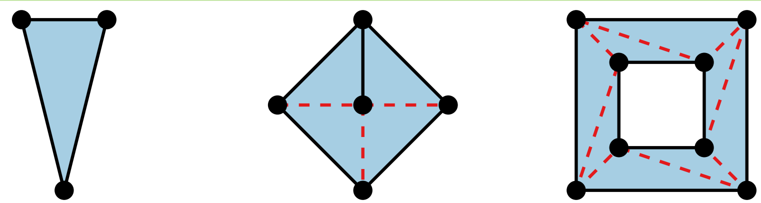
$\mu(G)=\frac{1}{2}(|V|-\max_{S\subseteq V}(\mathrm{odd}(G\setminus S)-|S|))$

$q \in Q$ in deg-3 face of $G^{\triangle}_{A}[S_A]$
$\Rightarrow q$ in deg-4^{+} face of $G^{\triangledown}_{A}[S_A]$

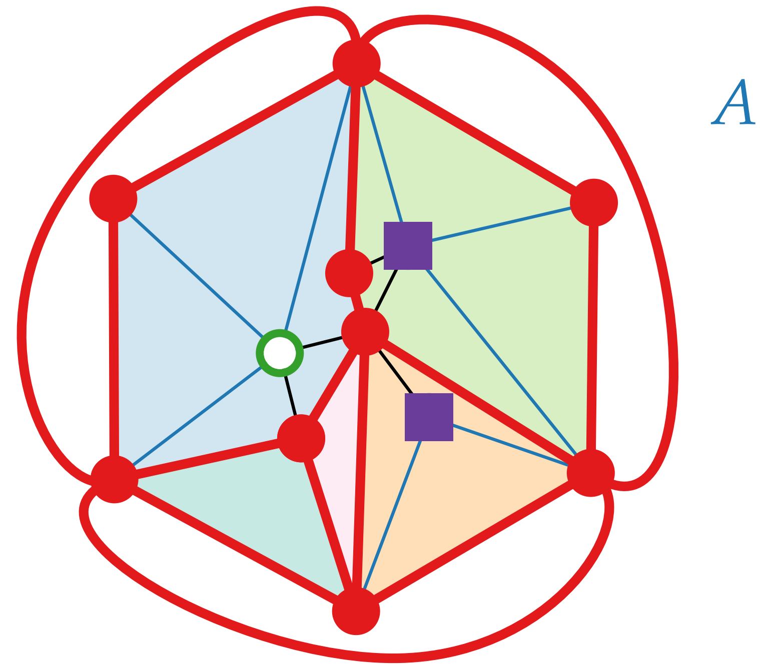


Upper Bound on Maximum Matching

$\sum_{d\geq 3}(d-2)|F|^{\triangle}_{\deg d} \leq 2n-4$



$G^{\triangle}$
$G^{\triangle}_{A}$
$G^{\triangle}_{A}[S_A]$
$G^{\triangle}_{A} \setminus S_A$

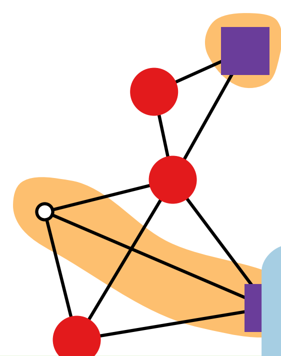


[Dillencourt '90]

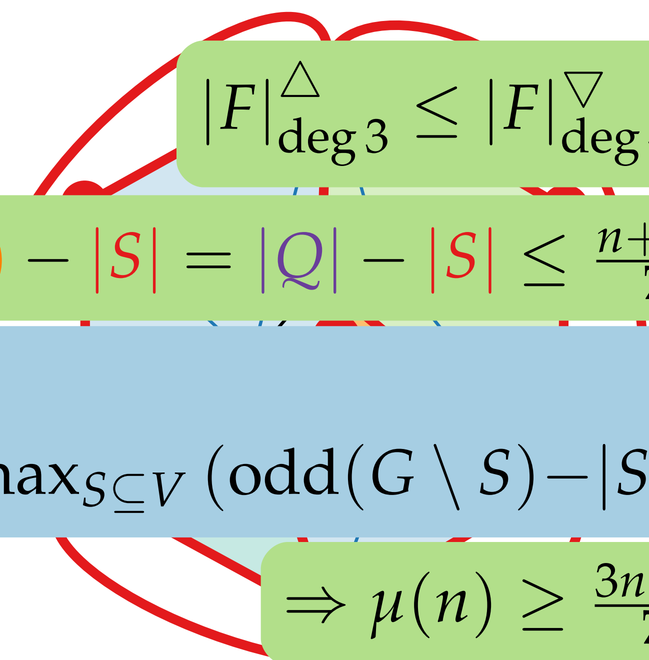
$G=(V,E)$ plane triangulation

$\Rightarrow \forall S \subseteq V$: every face of $G[S]$ contains ≤ 1 comp. of $G \setminus S$

$G^{✡}$
S
$\text{comp}(G^{✡} \setminus S)$
$|Q| = \text{comp}(G^{✡} \setminus S)$



$G^{\triangledown}$



$|F|^{\triangle}_{\deg 3} \leq |F|^{\triangledown}_{\deg 4+}$

$\text{comp}(G^{✡} \setminus S) - |S| = |Q| - |S| \leq \frac{n+16}{7}$

[Tutte-Berge]

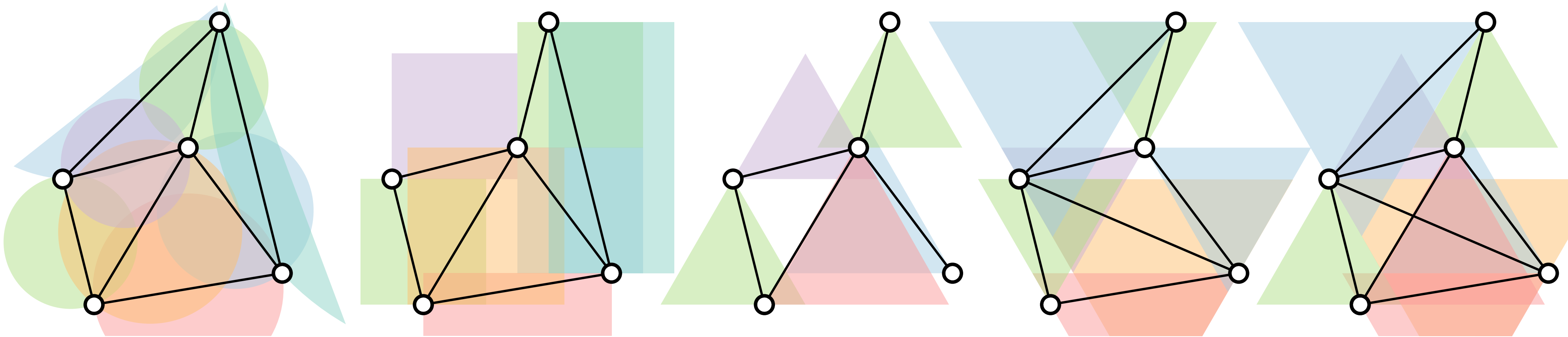
$\mu(G) = \frac{1}{2}(|V| - \max_{S\subseteq V}(\text{odd}(G \setminus S) - |S|))$

$q \in Q$ in deg-3 face of $G^{\triangle}_{A}[S_A]$

$\Rightarrow q$ in deg-4^{+} face of $G^{\triangledown}_{A}[S_A]$

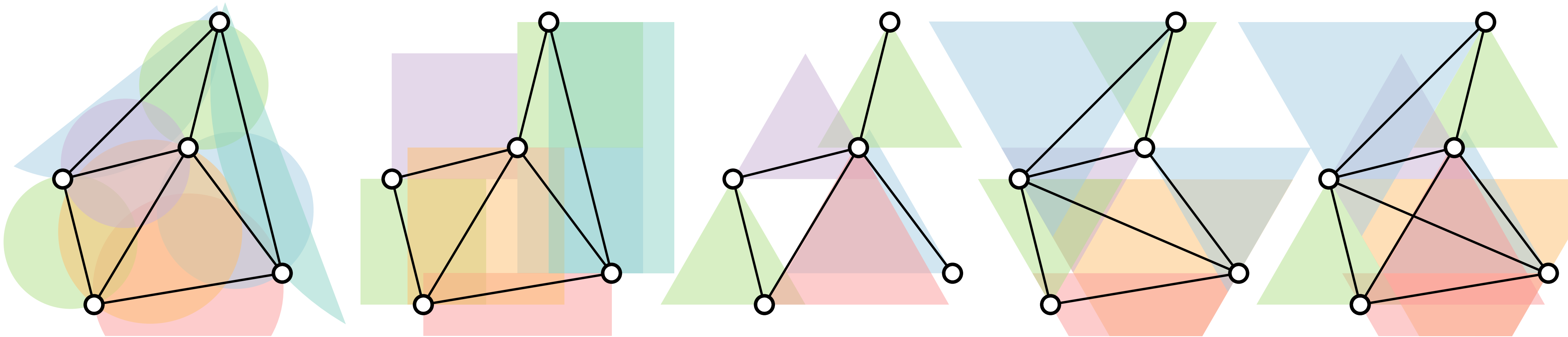
$\Rightarrow \mu(n) \geq \frac{3n-8}{7}$

Known Results



	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \lceil \frac{n-1}{3} \rceil$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$

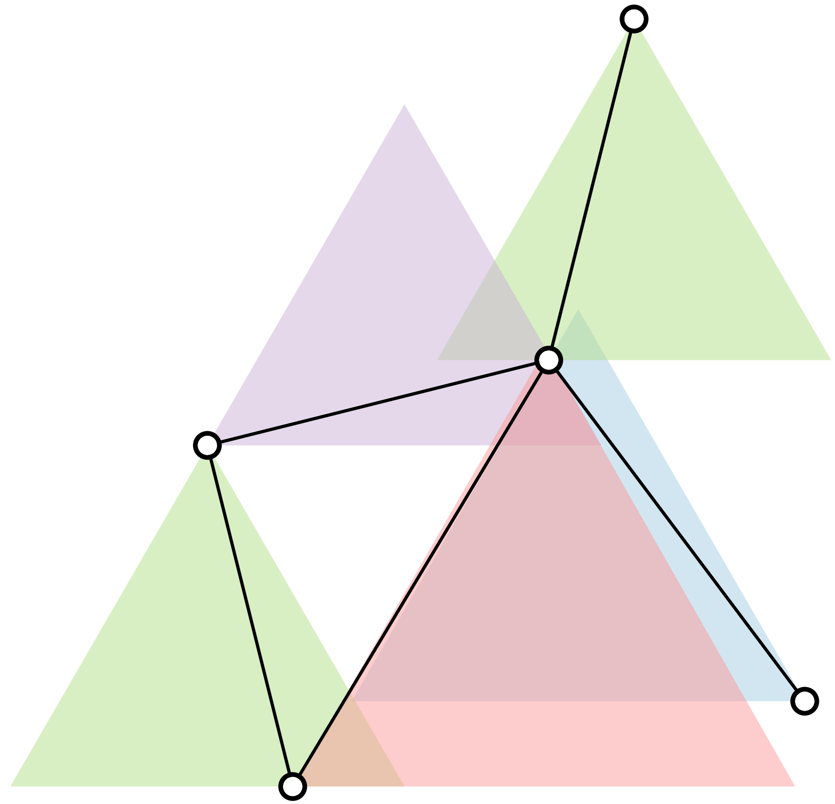
Known Results



	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593<t<1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$

Blocking Sets

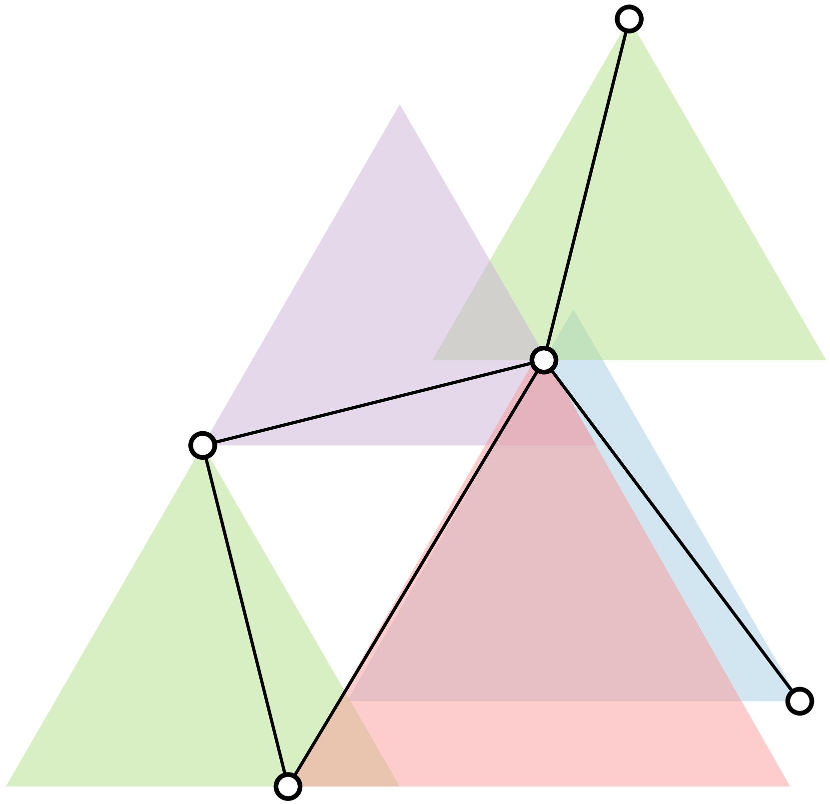
$G^{\triangle}(P)$



Blocking Sets

$$G^{\triangle}(P)$$

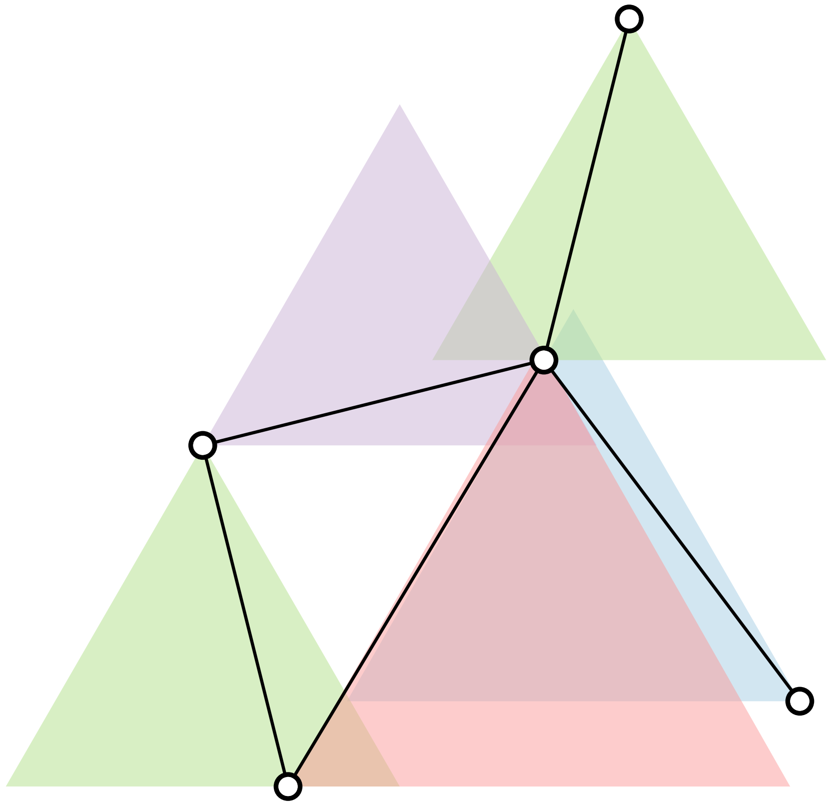
Blocking Set Q:



Blocking Sets

$G^{\triangle}(P)$

Blocking Set Q: $\forall (p,q) \in G^{\triangle}(P) : \exists r \in Q : r \in \triangle(p,q)$



Blocking Sets

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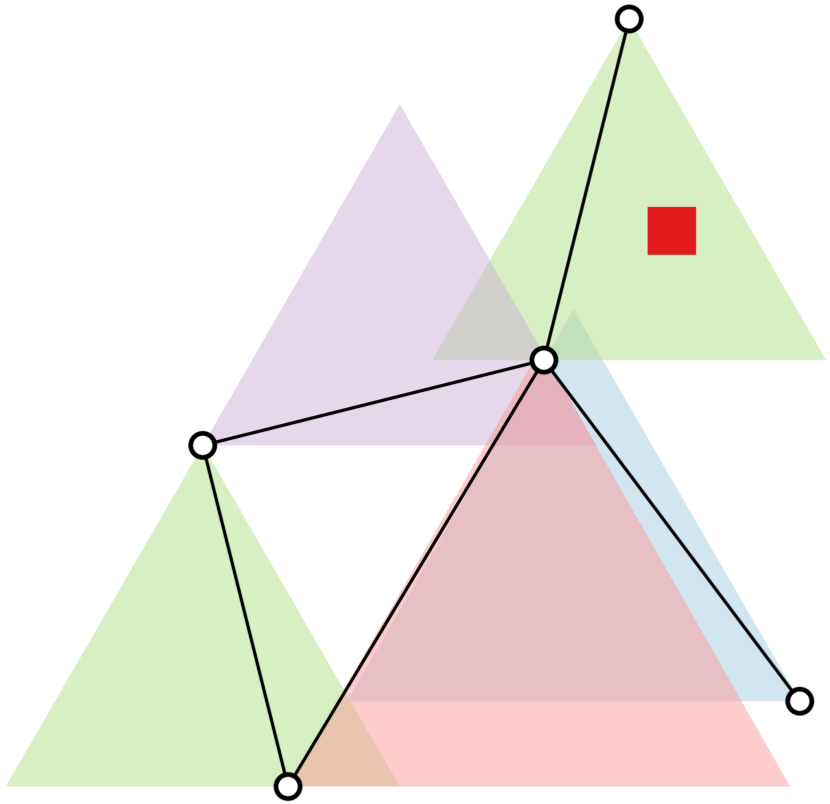
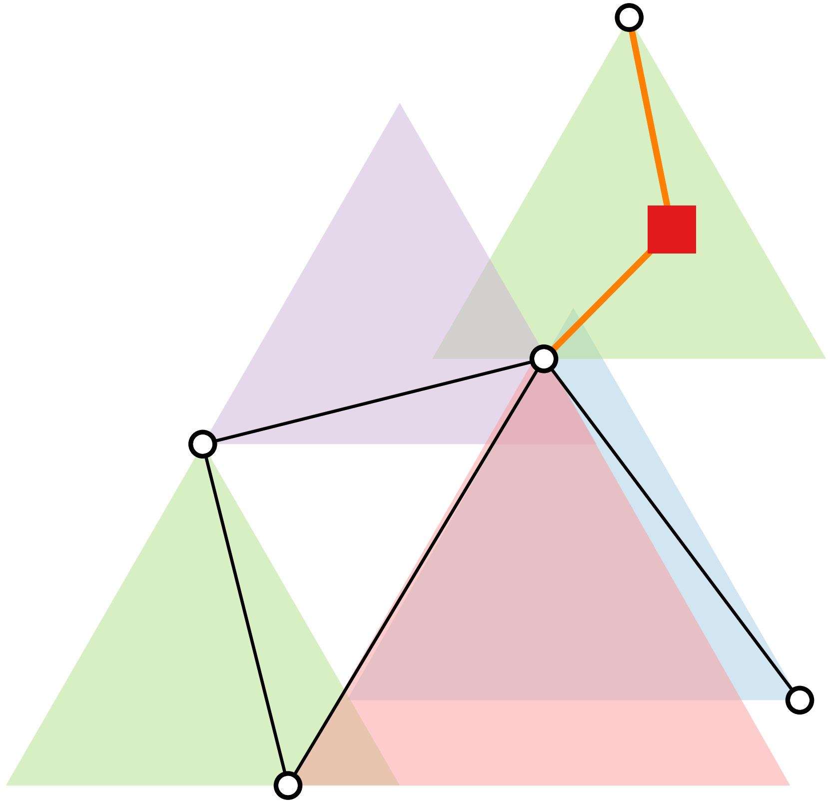
Blocking Set Q: $\forall (p,q) \in G^{\triangle}(P) : \exists r \in Q : r \in \triangle(p,q)$



Blocking Sets

$G^{\triangle}(P)$

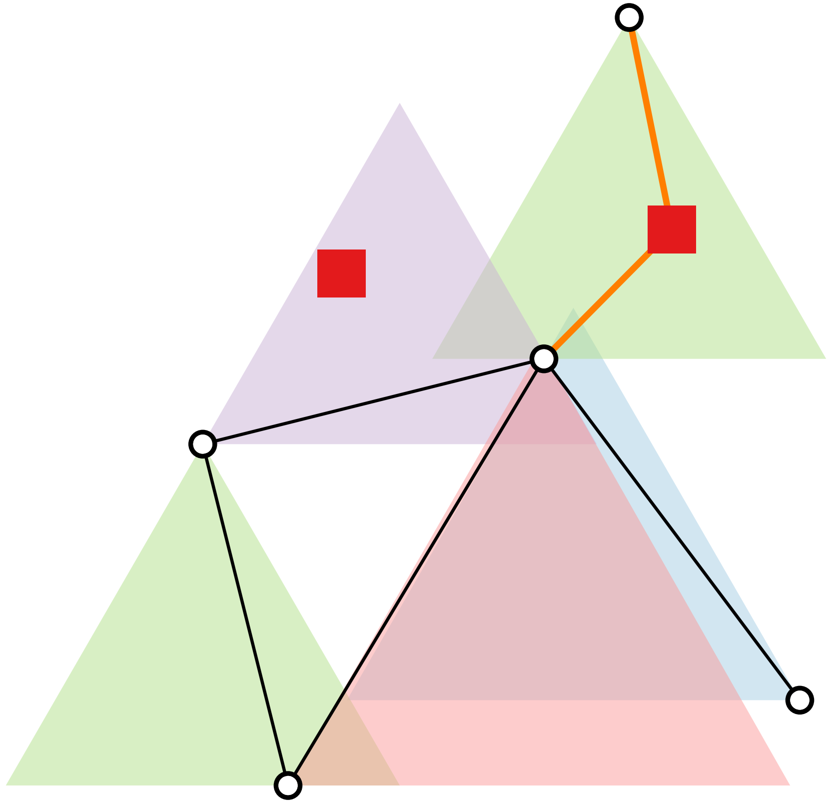
Blocking Set Q: $\forall(p,q) \in G^{\triangle}(P) : \exists r \in Q : r \in \triangle(p,q)$



Blocking Sets

$$G^{\triangle}(P)$$

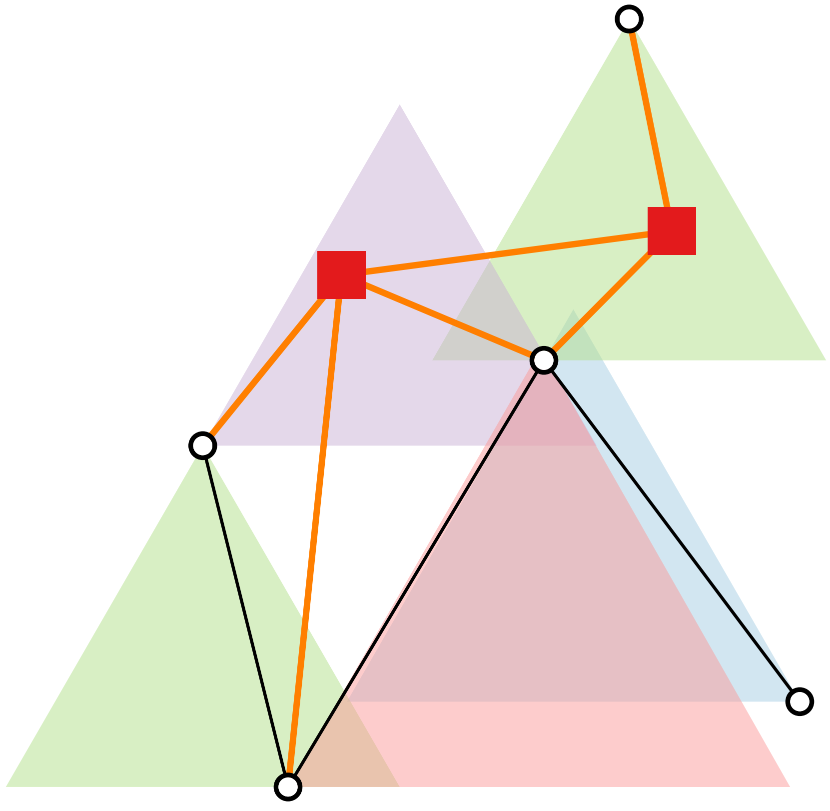
Blocking Set Q: $\forall (p,q) \in G^{\triangle}(P) : \exists r \in Q : r \in \triangle(p,q)$



Blocking Sets

$$G^{\triangle}(P)$$

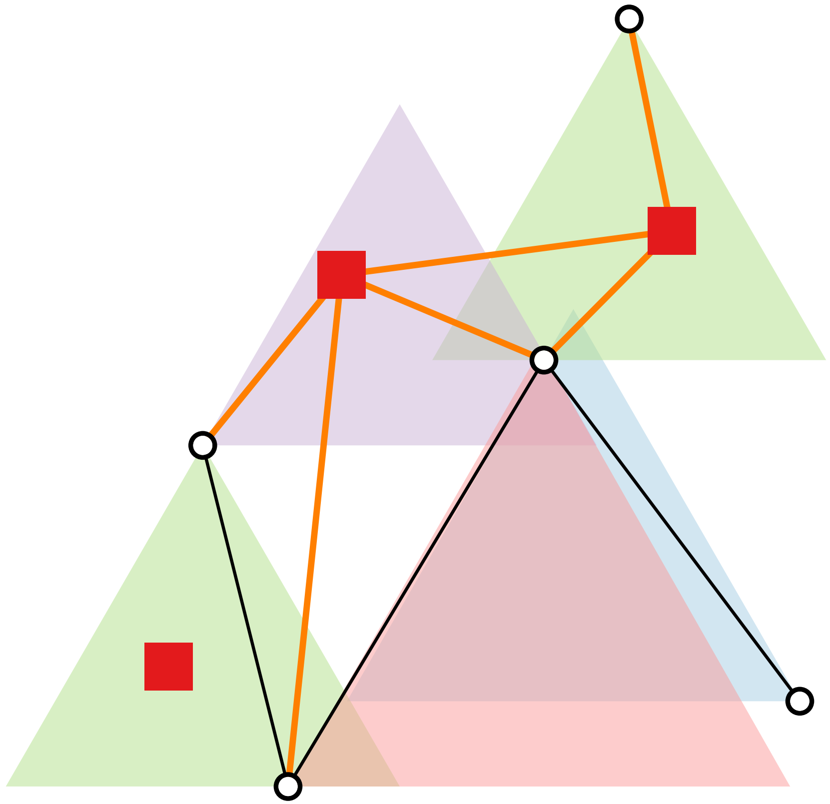
Blocking Set Q: $\forall (p,q) \in G^{\triangle}(P) : \exists r \in Q : r \in \triangle(p,q)$



Blocking Sets

$$G^{\triangle}(P)$$

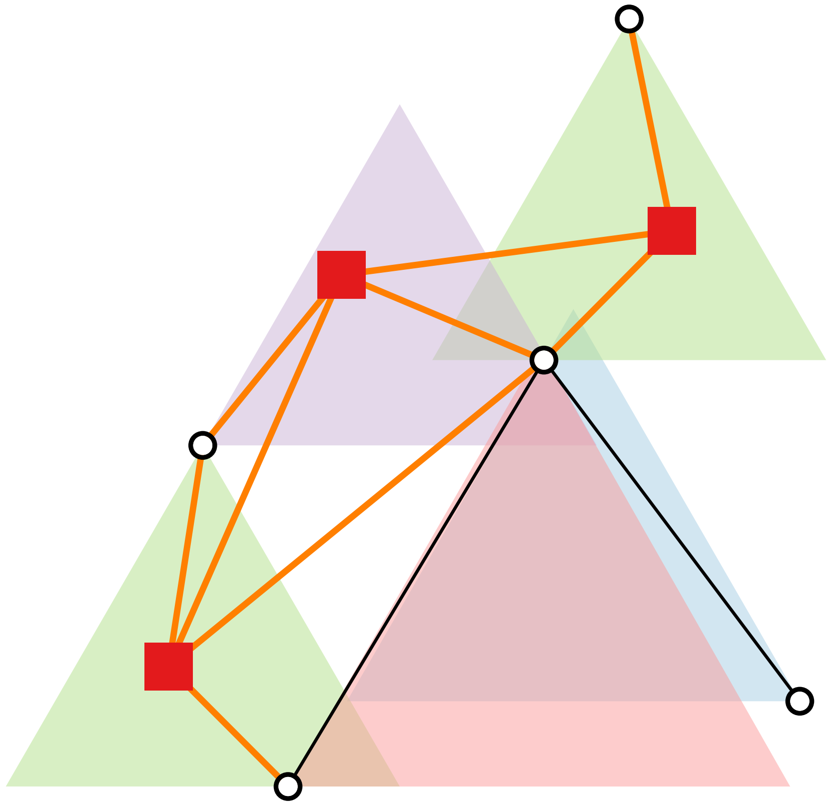
Blocking Set Q: $\forall(p,q) \in G^{\triangle}(P) : \exists r \in Q : r \in \triangle(p,q)$



Blocking Sets

$$G^{\triangle}(P)$$

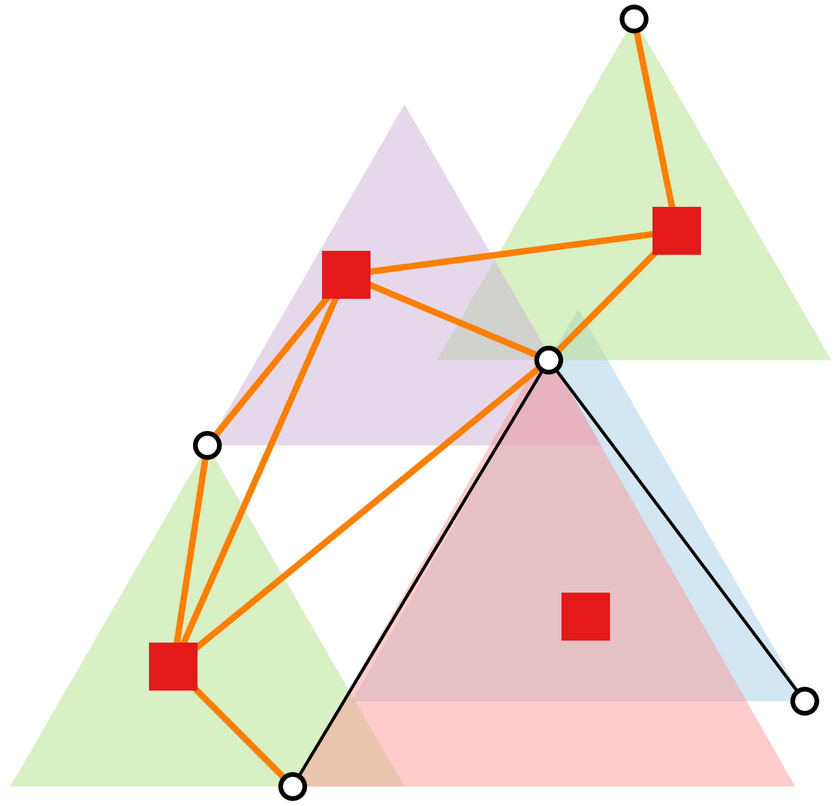
Blocking Set Q: $\forall (p,q) \in G^{\triangle}(P) : \exists r \in Q : r \in \triangle(p,q)$



Blocking Sets

$G^{\triangle}(P)$

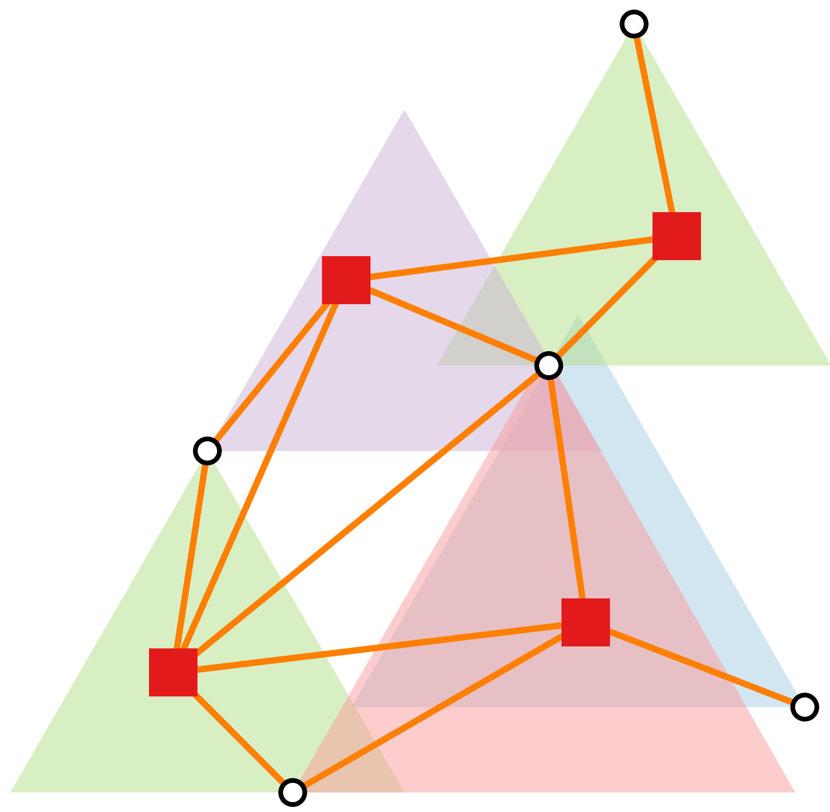
Blocking Set Q: $\forall (p,q) \in G^{\triangle}(P) : \exists r \in Q : r \in \triangle(p,q)$



Blocking Sets

$$G^{\triangle}(P)$$

Blocking Set Q: $\forall (p,q) \in G^{\triangle}(P) : \exists r \in Q : r \in \triangle(p,q)$

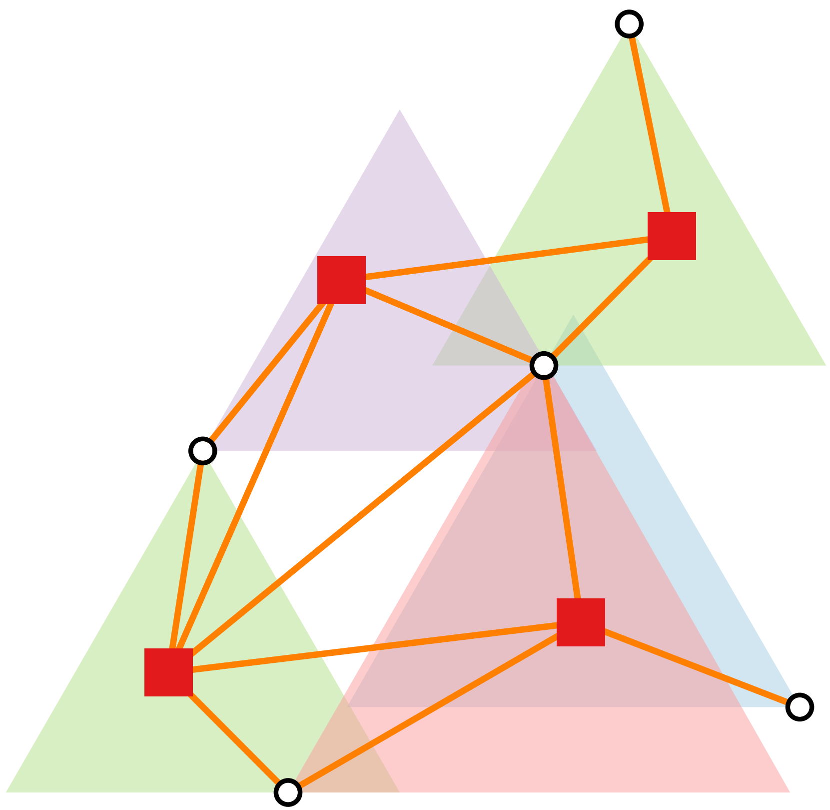


Blocking Sets

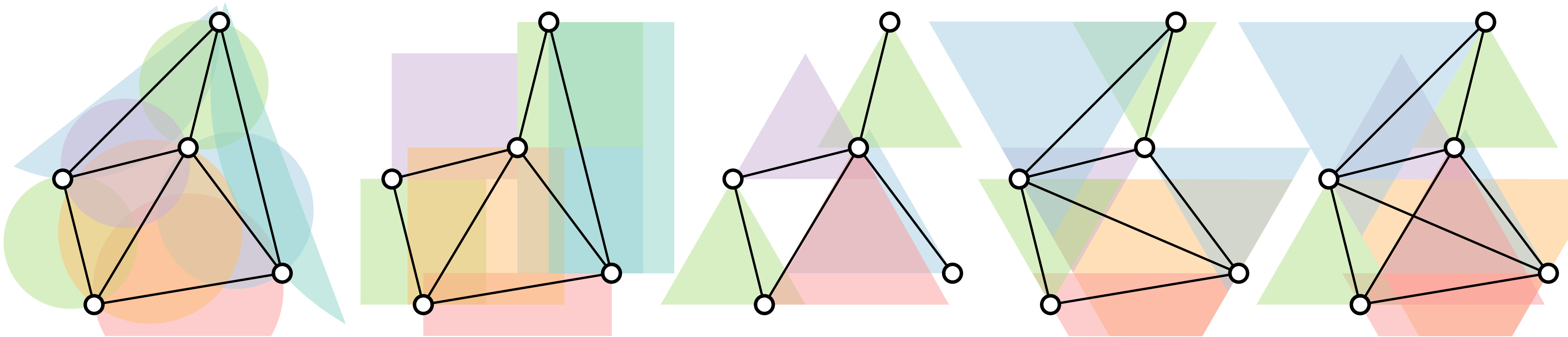
$$G^{\triangle}(P)$$

Blocking Set Q: $\forall (p,q) \in G^{\triangle}(P) : \exists r \in Q : r \in \triangle(p,q)$

or: $\forall (p,q) \in G^{\triangle}(P \cup Q) : q \in Q$

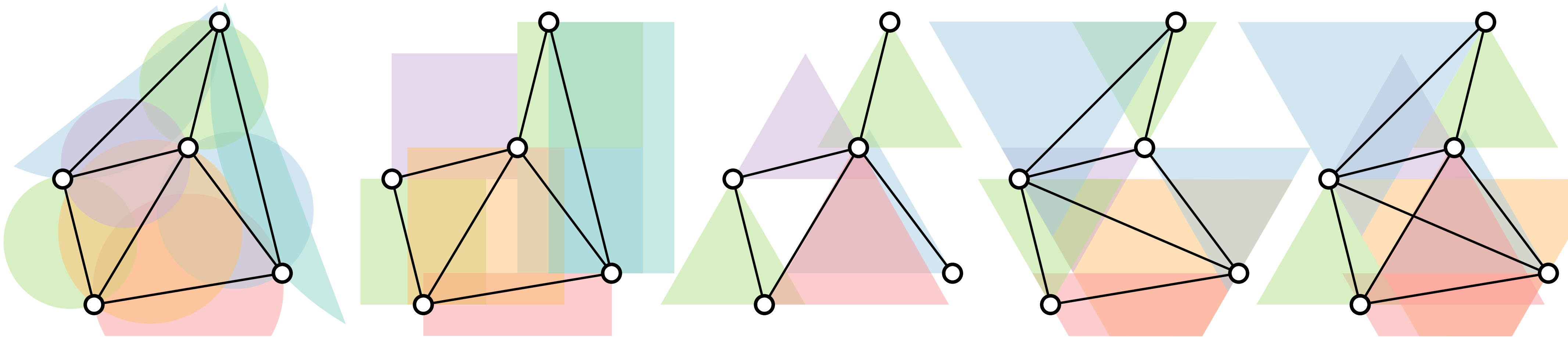


Known Results



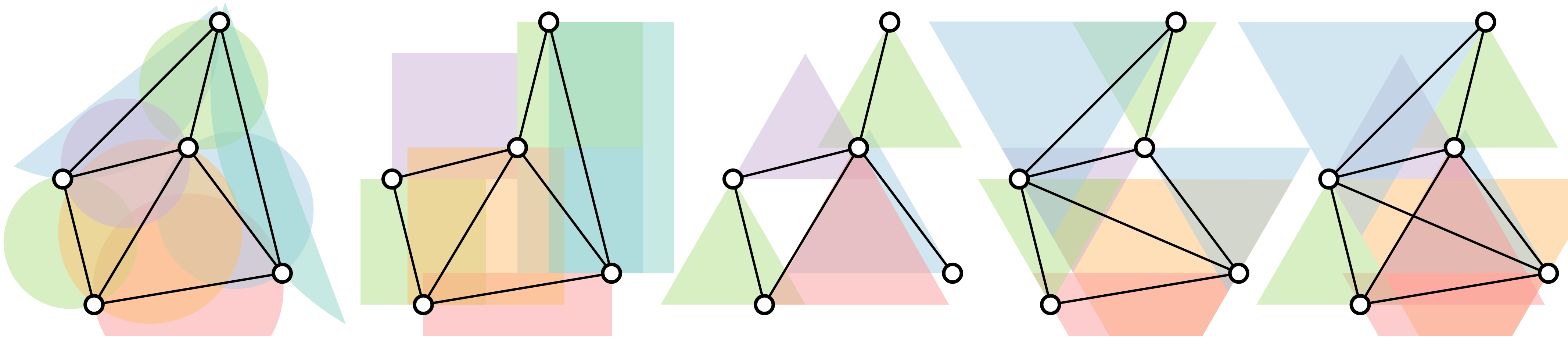
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$

Known Results



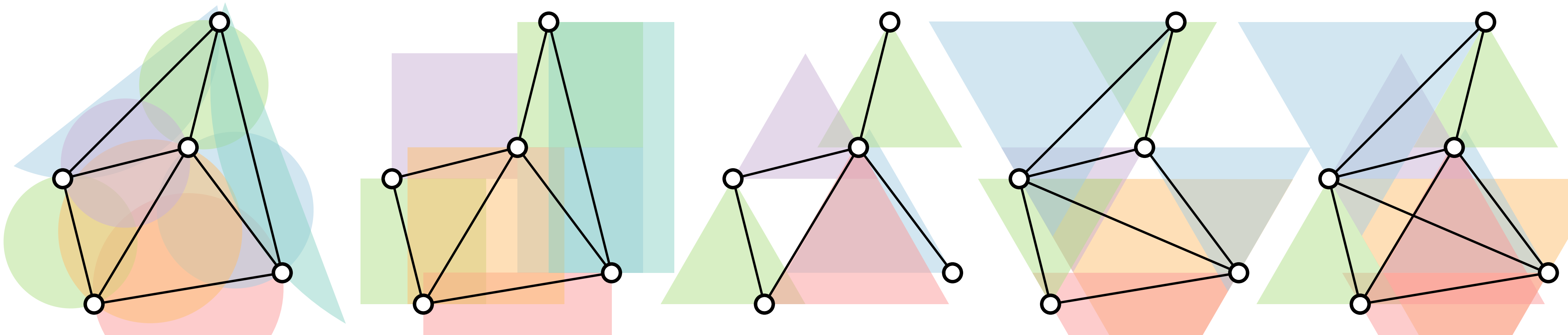
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\star}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$
Min. Blocking Set $\beta(n)$				

Known Results



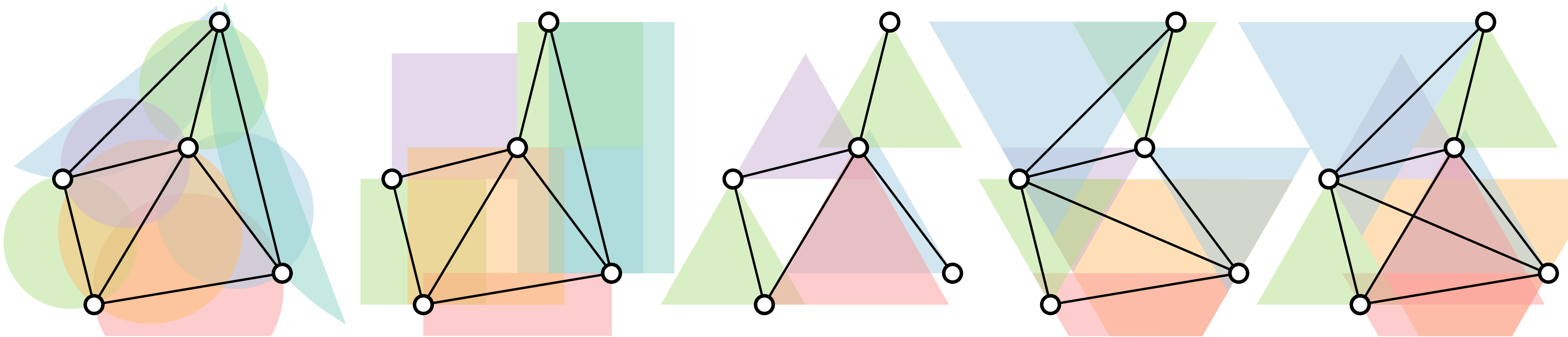
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$
Min. Blocking Set $\beta(n)$	$\beta(n) \geq n - 1$			

Known Results



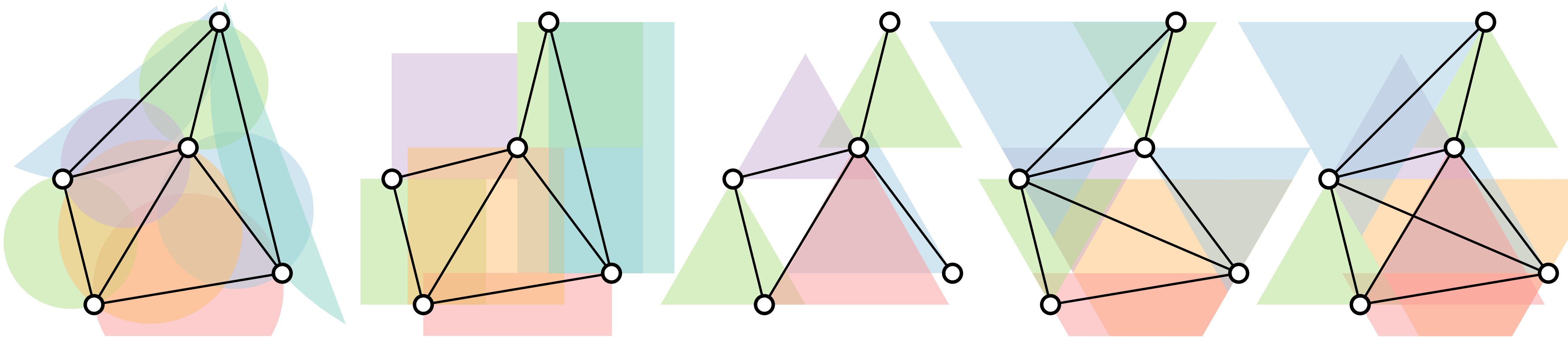
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$
Min. Blocking Set $\beta(n)$	$\beta(n) \geq n - 1$ $\beta(n) \leq \frac{3n}{2}$			

Known Results



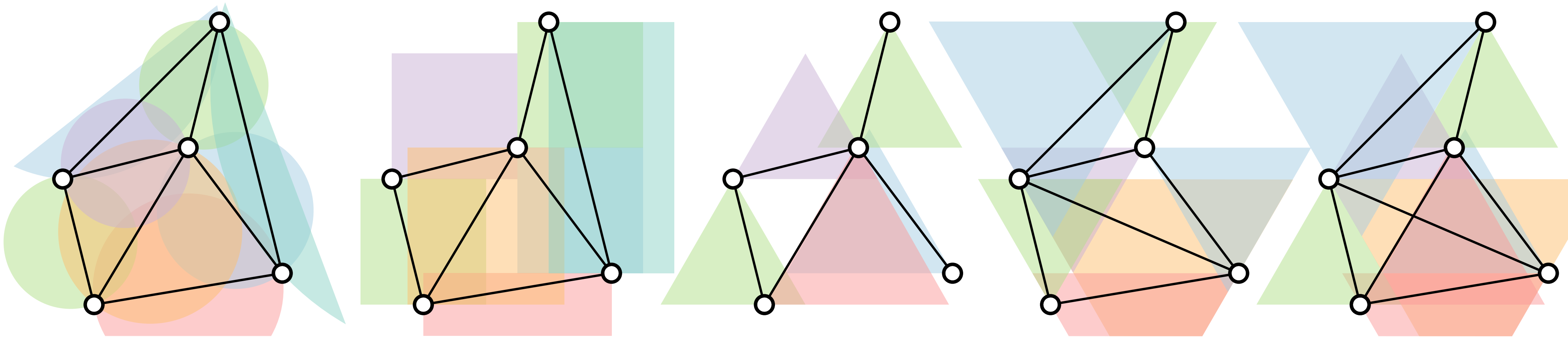
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$
Min. Blocking Set $\beta(n)$	$\beta(n) \geq n - 1$ $\beta(n) \leq \frac{3n}{2}$	$\beta(n) \geq \frac{5n}{4} - o(n)$		

Known Results



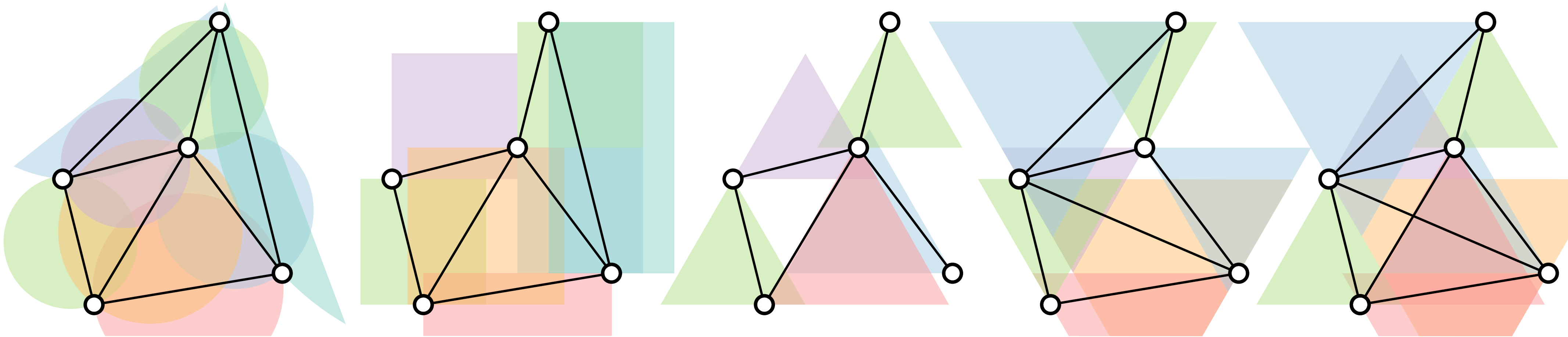
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$
Min. Blocking Set $\beta(n)$	$\beta(n) \geq n-1$ $\beta(n) \leq \frac{3n}{2}$	$\beta(n) \geq \frac{5n}{4} - o(n)$ $\beta(n) \leq 2n - o(n)$		

Known Results



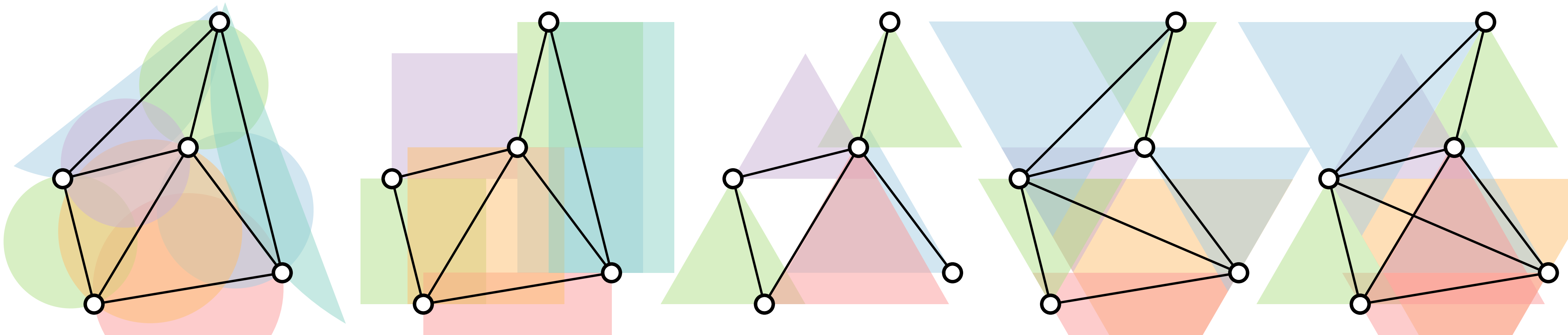
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ Half-Θ_6-Graphs	$G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$		$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$		$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$
Min. Blocking Set $\beta(n)$	$\beta(n) \geq n-1$ $\beta(n) \leq \frac{3n}{2}$	$\beta(n) \geq \frac{5n}{4} - o(n)$ $\beta(n) \leq 2n - o(n)$	$\beta(n) = \lceil \frac{n-1}{2} \rceil$		

Known Results



	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ Half-Θ_6-Graphs	$G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$		$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$		$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$
Min. Blocking Set $\beta(n)$		$\beta(n) \geq n - 1$ $\beta(n) \leq \frac{3n}{2}$	$\beta(n) \geq \frac{5n}{4} - o(n)$ $\beta(n) \leq 2n - o(n)$	$\beta(n) = \lceil \frac{n-1}{2} \rceil$	$\beta(n) \geq \lceil \frac{n-1}{2} \rceil$

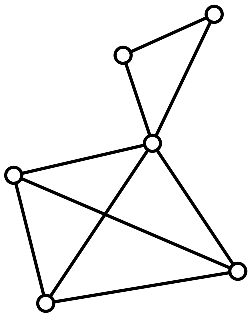
Known Results



	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{\text{✡}}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$
Min. Blocking Set $\beta(n)$	$\beta(n) \geq n-1$ $\beta(n) \leq \frac{3n}{2}$	$\beta(n) \geq \frac{5n}{4} - o(n)$ $\beta(n) \leq 2n - o(n)$	$\beta(n) = \lceil \frac{n-1}{2} \rceil$	$\beta(n) \geq \lceil \frac{n-1}{2} \rceil$ $\beta(n) \leq n-1$

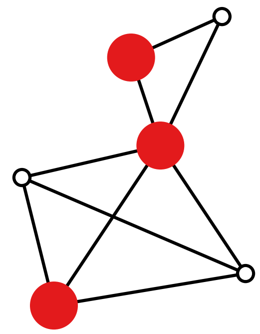
Relationship Blocking Set & Matching

$G^{\text{✡}}(P)$



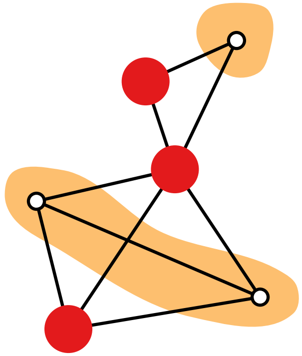
Relationship Blocking Set & Matching

$G^{\text{✡}}(P)$
S



Relationship Blocking Set & Matching

$G^{\text{✡}}(P)$
S
$\mathrm{comp}(G^{\text{✡}}(P) \setminus S)$



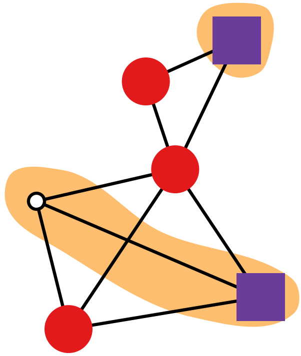
Relationship Blocking Set & Matching

$G^{\text{✡}}(P)$

S

$\mathrm{comp}(G^{\text{✡}}(P) \setminus S)$

$|Q| = \mathrm{comp}(G^{\text{✡}}(P) \setminus S)$



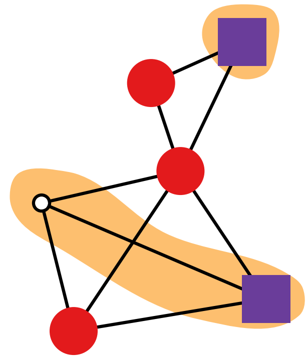
Relationship Blocking Set & Matching

$G^{✡}(P)$

S

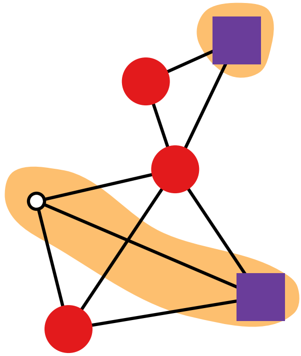
$\mathrm{comp}(G^{✡}(P) \setminus S)$

$|Q| = \mathrm{comp}(G^{✡}(P) \setminus S)$



Relationship Blocking Set & Matching

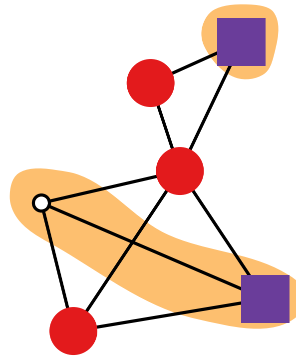
$G^{\text{✡}}(P)$
S
$\mathrm{comp}(G^{\text{✡}}(P) \setminus S)$
$|Q| = \mathrm{comp}(G^{\text{✡}}(P) \setminus S)$



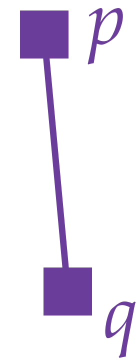
$G^{\text{✡}}(Q)$

Relationship Blocking Set & Matching

$G^{\text{✡}}(P)$
S
$\mathrm{comp}(G^{\text{✡}}(P) \setminus S)$
$|Q| = \mathrm{comp}(G^{\text{✡}}(P) \setminus S)$

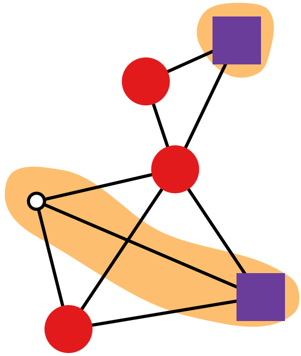


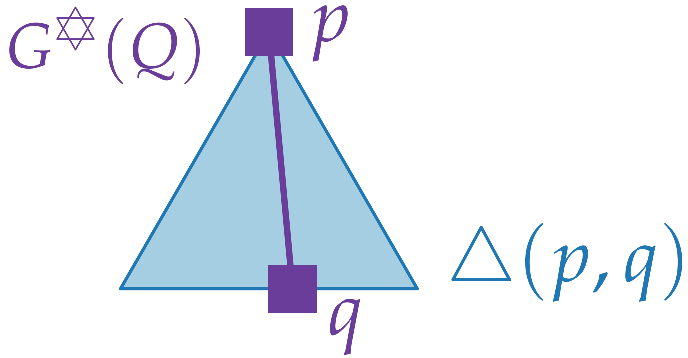
$G^{\text{✡}}(Q)$



Relationship Blocking Set & Matching

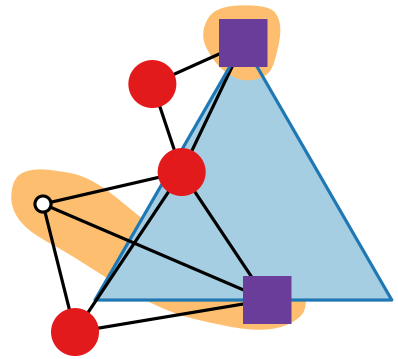
$G^{\hexstar}(P)$
S
$\mathrm{comp}(G^{\hexstar}(P) \setminus S)$
$|Q| = \mathrm{comp}(G^{\hexstar}(P) \setminus S)$

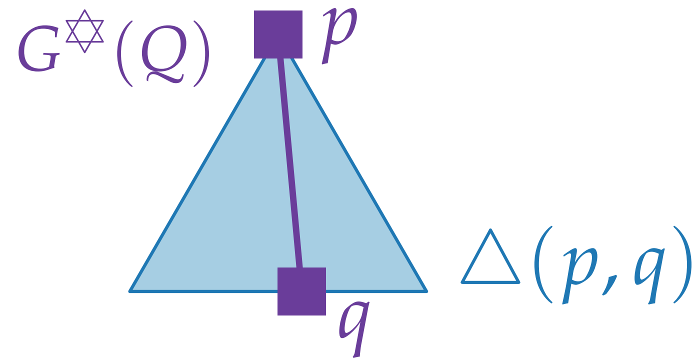




Relationship Blocking Set & Matching

$G^{✡}(P)$
S
$\mathrm{comp}(G^{✡}(P) \setminus S)$
$|Q| = \mathrm{comp}(G^{✡}(P) \setminus S)$





Relationship Blocking Set & Matching

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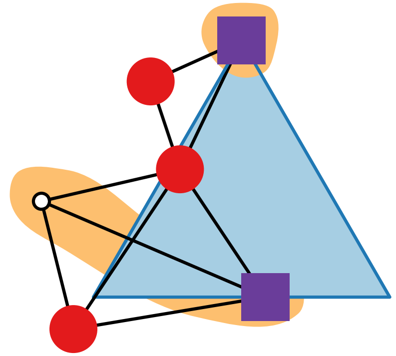
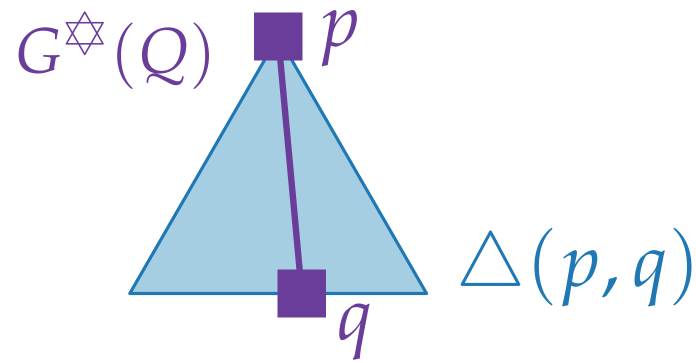




[Babu et al. ’14]
$\exists \langle p, \ldots, q \rangle$ s.t.
defining $\triangle$s lie
inside $\triangle(p, q)$

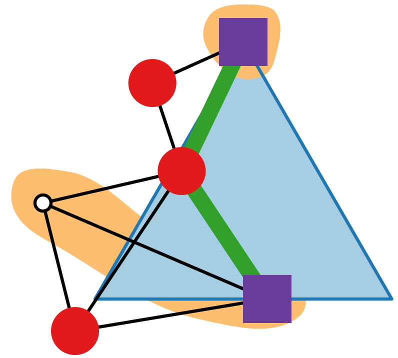
Relationship Blocking Set & Matching

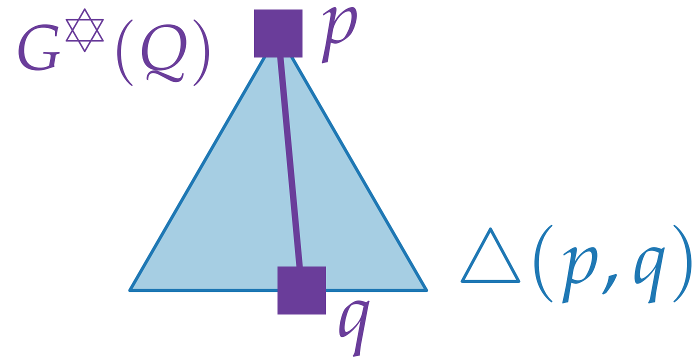
$G^{\text{✡}}(P)$

S

$\mathrm{comp}(G^{\text{✡}}(P) \setminus S)$

$|Q| = \mathrm{comp}(G^{\text{✡}}(P) \setminus S)$





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Relationship Blocking Set & Matching

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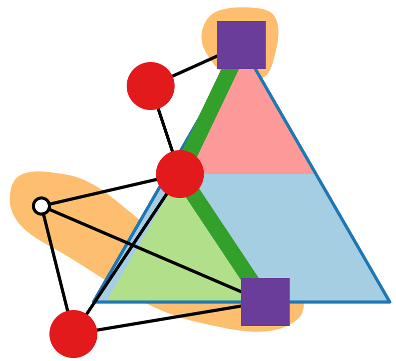

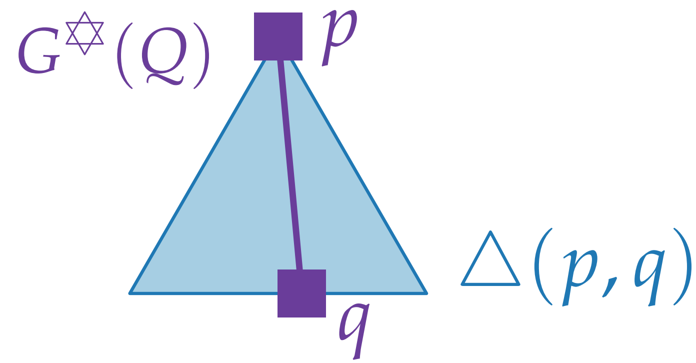

[Babu et al. '14]
$\exists \langle p, \ldots, q \rangle$ s.t.
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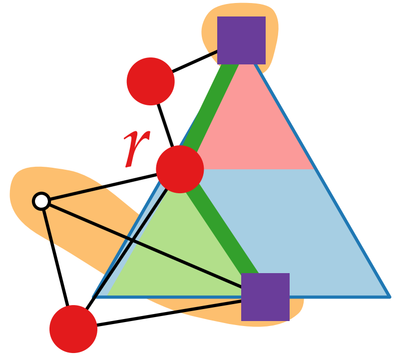
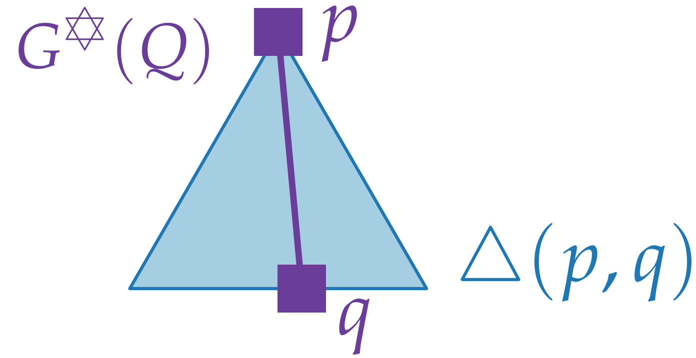
Relationship Blocking Set & Matching

$G^{\davidsstar}(P)$
S
$\mathrm{comp}(G^{\davidsstar}(P) \setminus S)$
$|Q| = \mathrm{comp}(G^{\davidsstar}(P) \setminus S)$





[Babu et al. '14]
$\exists \langle p, \ldots, q \rangle$ s.t.
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Relationship Blocking Set & Matching

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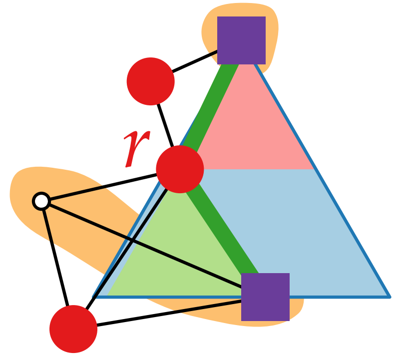

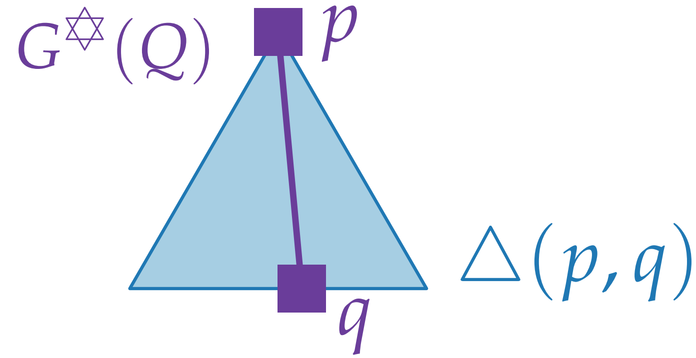

[Babu et al. '14]
$\exists \langle p, \ldots, q \rangle$ s.t. defining $\triangle$s lie inside $\triangle(p, q)$

p, q in different comp. of $G^{\text{✡}}(P) \setminus S \Rightarrow r \in S$

Relationship Blocking Set & Matching

$G^{\text{✡}}(P)$
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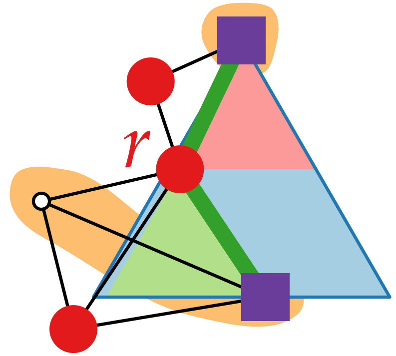
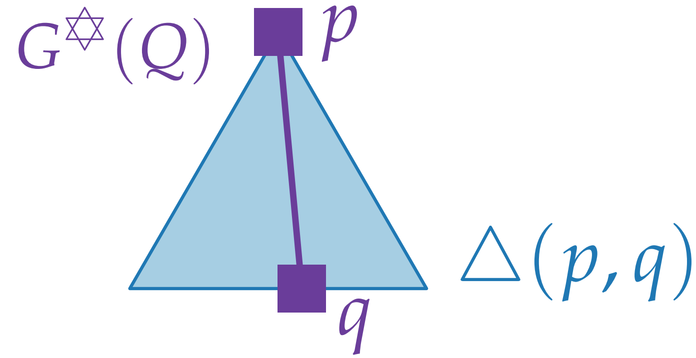
$\text{comp}(G^{\text{✡}}(P) \setminus S)$
$|Q| = \text{comp}(G^{\text{✡}}(P) \setminus S)$





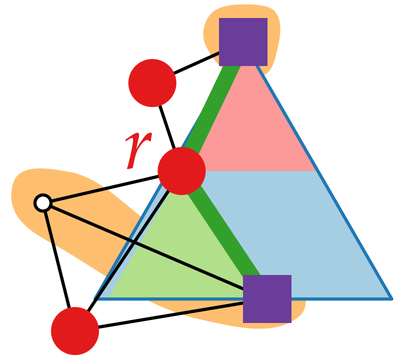
p, q in different comp. of $G^{\text{✡}}(P) \setminus S \Rightarrow r \in S$

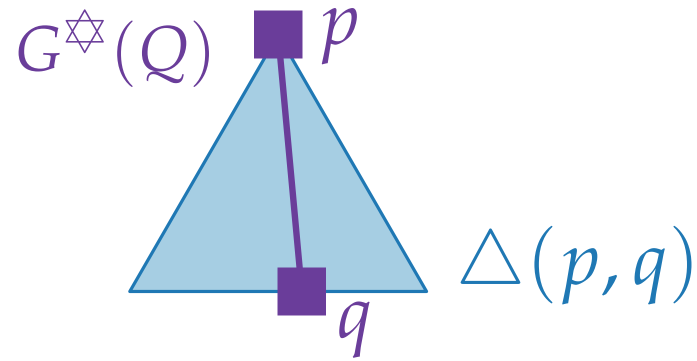
$\Rightarrow \forall (p,q) \in G^{\text{✡}}(Q) : \exists r \in S : r \in \triangle(p,q)$

[Babu et al. '14]
$\exists \langle p, \ldots, q \rangle$ s.t. defining $\triangle$s lie inside $\triangle(p,q)$

Relationship Blocking Set & Matching

$G^{\davidsstar}(P)$
S
$\text{comp}(G^{\davidsstar}(P) \setminus S)$
$|Q| = \text{comp}(G^{\davidsstar}(P) \setminus S)$





p, q in different comp. of $G^{\davidsstar}(P) \setminus S \Rightarrow r \in S$

$\Rightarrow \forall (p,q) \in G^{\davidsstar}(Q) : \exists r \in S : r \in \triangle(p,q)$

$\Rightarrow S$ is blocking set of Q

[Babu et al. ’14]
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Relationship Blocking Set & Matching

$G^{\text{✡}}(P)$
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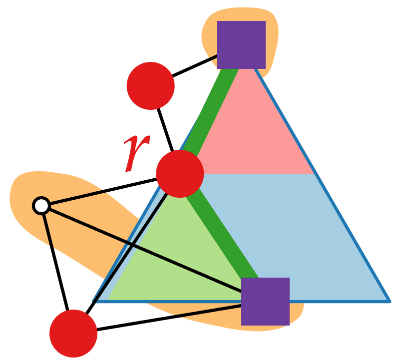
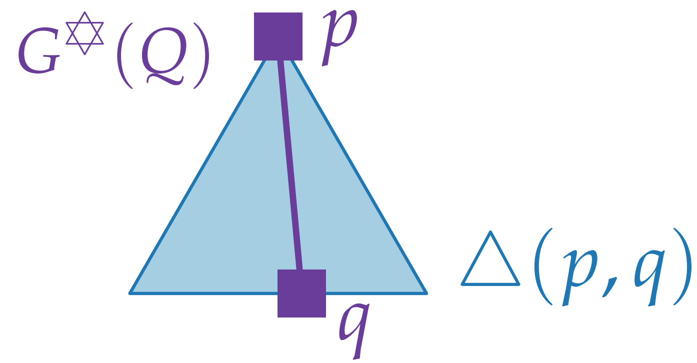




[Babu et al. '14]
$\exists\langle p,\ldots,q\rangle$ s.t. defining $\triangle$s lie inside $\triangle(p,q)$

p,q in different comp. of $G^{\text{✡}}(P) \setminus S \Rightarrow r \in S$

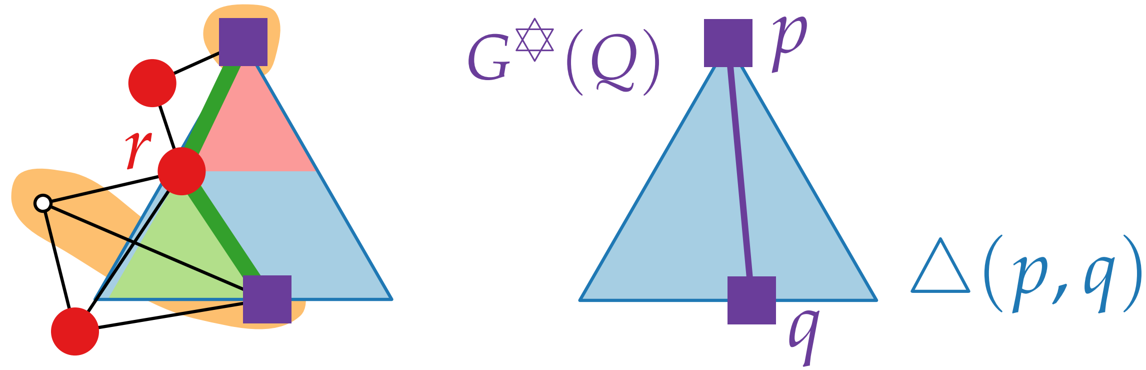
$\Rightarrow \forall (p,q) \in G^{\text{✡}}(Q) : \exists r \in S : r \in \triangle(p,q)$

$\Rightarrow$ S is blocking set of Q

$\Rightarrow |S| \geq \beta(|Q|) =$

Relationship Blocking Set & Matching

$G^{\davidsstar}(P)$
S
$\mathrm{comp}(G^{\davidsstar}(P) \setminus S)$
$|Q| = \mathrm{comp}(G^{\davidsstar}(P) \setminus S)$



[Babu et al. '14]
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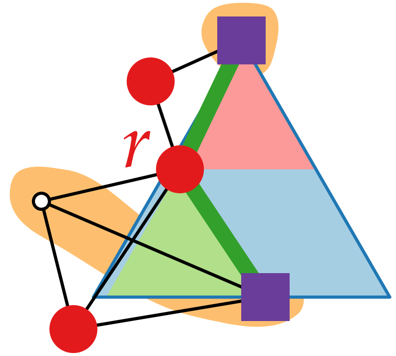
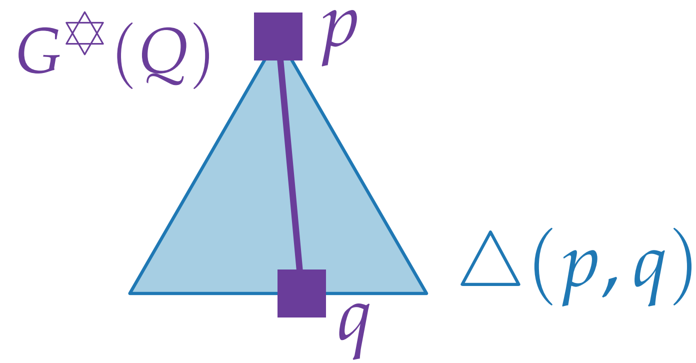
$\Rightarrow |S| \geq \beta(|Q|) = \beta(\mathrm{comp}(G^{\davidsstar}(P) \setminus S))$

Relationship Blocking Set & Matching

$G^{\text{✡}}(P)$
S
$\mathrm{comp}(G^{\text{✡}}(P) \setminus S)$
$|Q| = \mathrm{comp}(G^{\text{✡}}(P) \setminus S)$





[Babu et al. '14]
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p,q in different comp. of $G^{\text{✡}}(P) \setminus S \Rightarrow r \in S$

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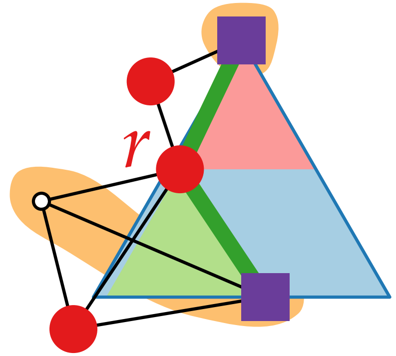
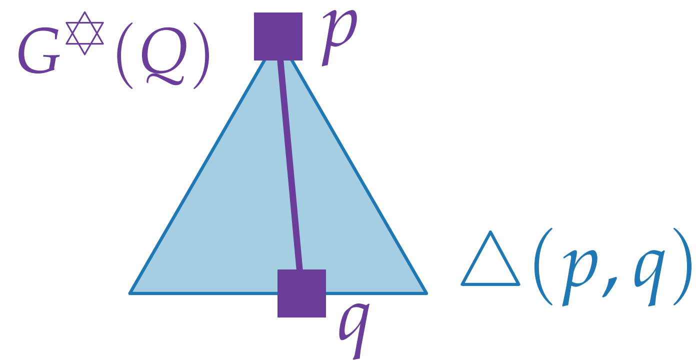
$\Rightarrow |S| \geq \beta(|Q|) = \beta(\mathrm{comp}(G^{\text{✡}}(P) \setminus S))$

[Tutte-Berge]
$\mu(G) = \frac{1}{2}(|V| - \max_{S \subseteq V}(\mathrm{odd}(G \setminus S) - |S|))$

Relationship Blocking Set & Matching

$G^{\davidsstar}(P)$
S
$\text{comp}(G^{\davidsstar}(P) \setminus S)$
$|Q| = \text{comp}(G^{\davidsstar}(P) \setminus S)$





[Babu et al. '14]
$\exists\langle p, \ldots, q\rangle$ s.t.
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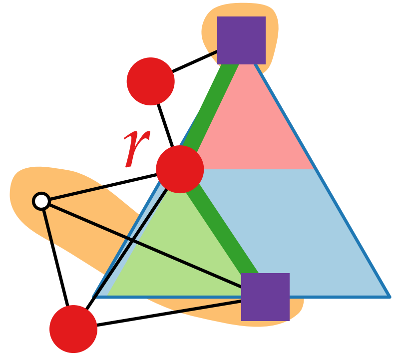
[Tutte-Berge]
$\mu(G) = \frac{1}{2}(|V| - \max_{S \subseteq V} (\text{odd}(G \setminus S) - |S|))$

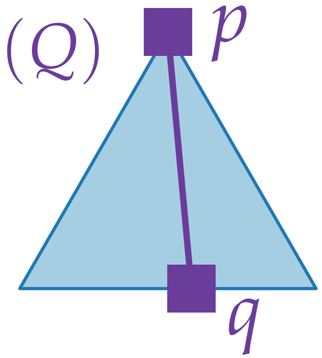
$\Rightarrow \mu(n)$

Relationship Blocking Set & Matching

$G^{\davidsstar}(P)$
S
$\text{comp}(G^{\davidsstar}(P) \setminus S)$
$|Q| = \text{comp}(G^{\davidsstar}(P) \setminus S)$



$G^{\davidsstar}(Q)$



$\triangle(p, q)$

[Babu et al. '14]
$\exists \langle p, \ldots, q \rangle$ s.t. defining $\triangle$s lie inside $\triangle(p, q)$

p, q in different comp. of $G^{\davidsstar}(P) \setminus S \Rightarrow r \in S$

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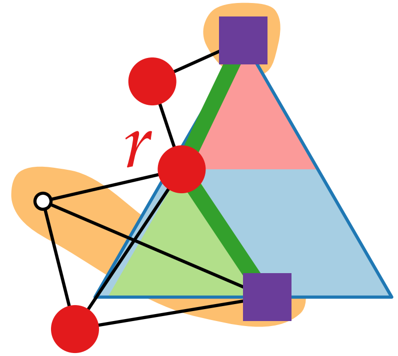
$\Rightarrow |S| \geq \beta(|Q|) = \beta(\text{comp}(G^{\davidsstar}(P) \setminus S))$

[Tutte-Berge]
$\mu(G) = \frac{1}{2}(|V| - \max_{S \subseteq V} (\text{odd}(G \setminus S) - |S|))$

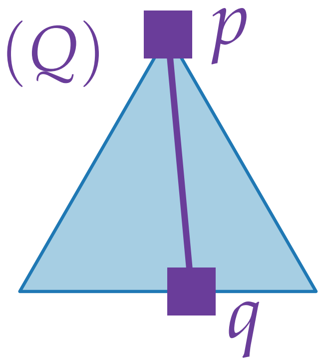
$\Rightarrow \mu(n) \geq \frac{n - (|Q| - \beta(|Q|))}{2}$

Relationship Blocking Set & Matching

$G^{\davidsstar}(P)$
S
$\mathrm{comp}(G^{\davidsstar}(P) \setminus S)$
$|Q| = \mathrm{comp}(G^{\davidsstar}(P) \setminus S)$



$G^{\davidsstar}(Q)$



$\triangle(p,q)$

[Babu et al. '14]
$\exists\langle p,\ldots,q\rangle$ s.t. defining $\triangle$s lie inside $\triangle(p,q)$

p,q in different comp. of $G^{\davidsstar}(P) \setminus S \Rightarrow r \in S$

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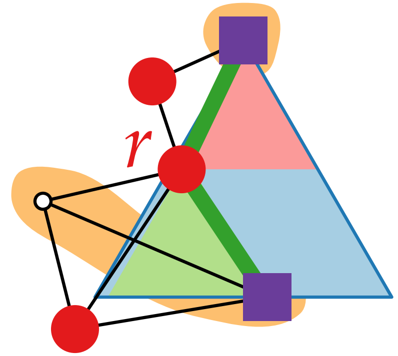
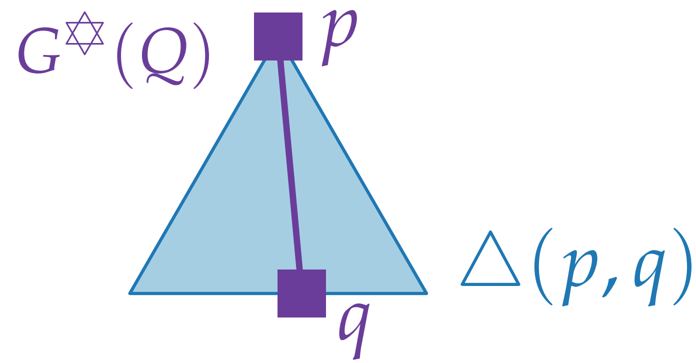
[Tutte-Berge]
$\mu(G) = \frac{1}{2}(|V| - \max_{S \subseteq V}(\mathrm{odd}(G \setminus S) - |S|))$

$\Rightarrow \mu(n) \geq \frac{n-(|Q|-\beta(|Q|))}{2} \geq \frac{n-(n-\beta(n))}{2}$

Relationship Blocking Set & Matching

$G^{✡}(P)$
S
$\mathrm{comp}(G^{✡}(P) \setminus S)$
$|Q| = \mathrm{comp}(G^{✡}(P) \setminus S)$





p, q in different comp. of $G^{✡}(P) \setminus S \Rightarrow r \in S$

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[Babu et al. '14]
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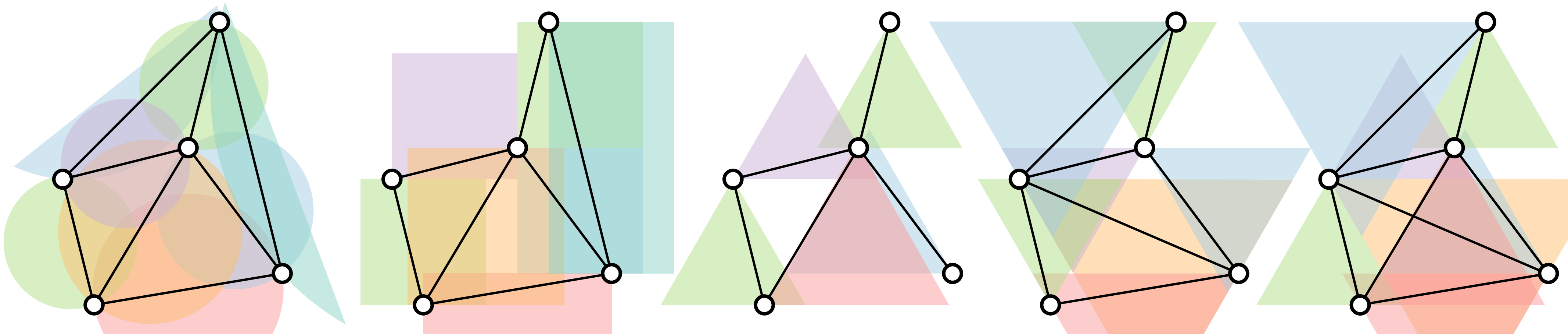
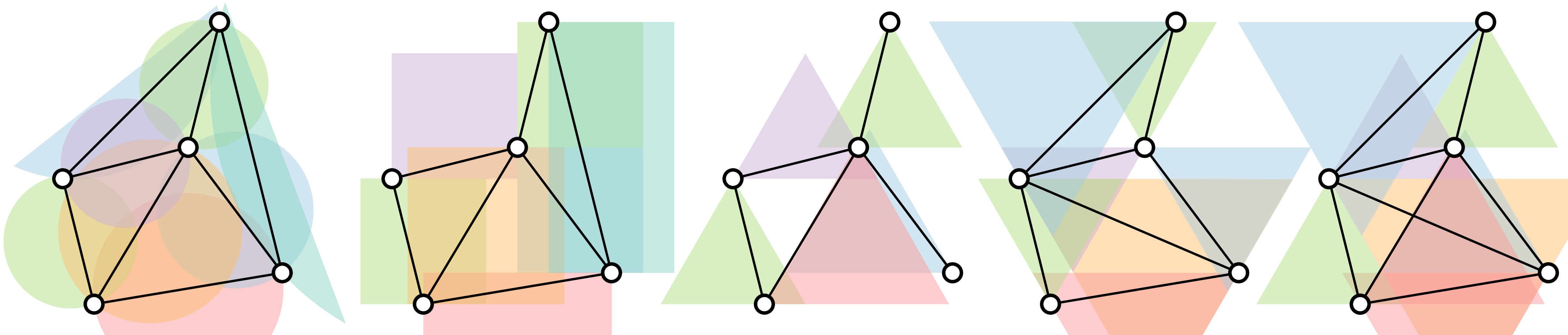
[Tutte-Berge]
$\mu(G) = \frac{1}{2}(|V| - \max_{S \subseteq V} (\mathrm{odd}(G \setminus S) - |S|))$

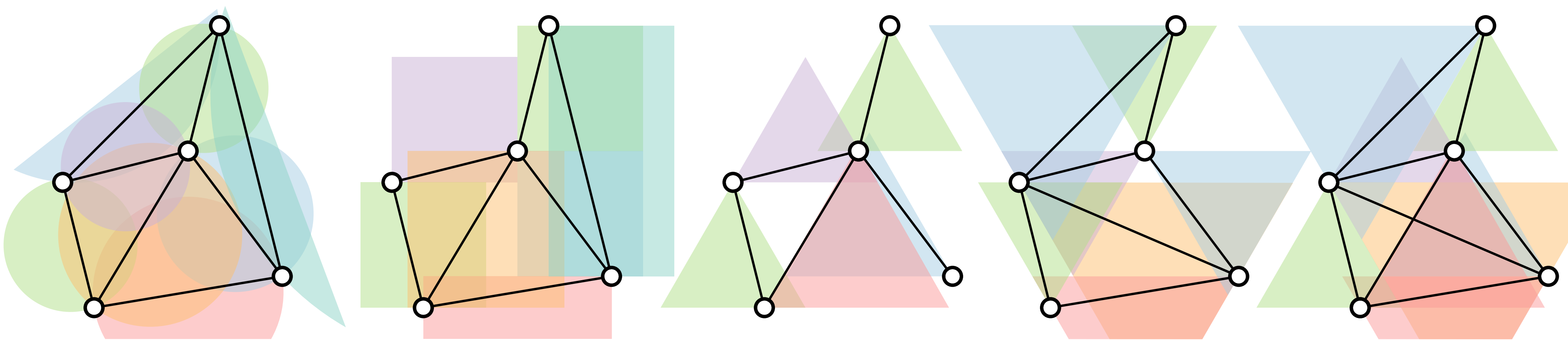
$$\Rightarrow \mu(n) \geq \frac{n - (|Q| - \beta(|Q|))}{2} \geq \frac{n - (n - \beta(n))}{2} = \frac{\beta(n)}{2}$$



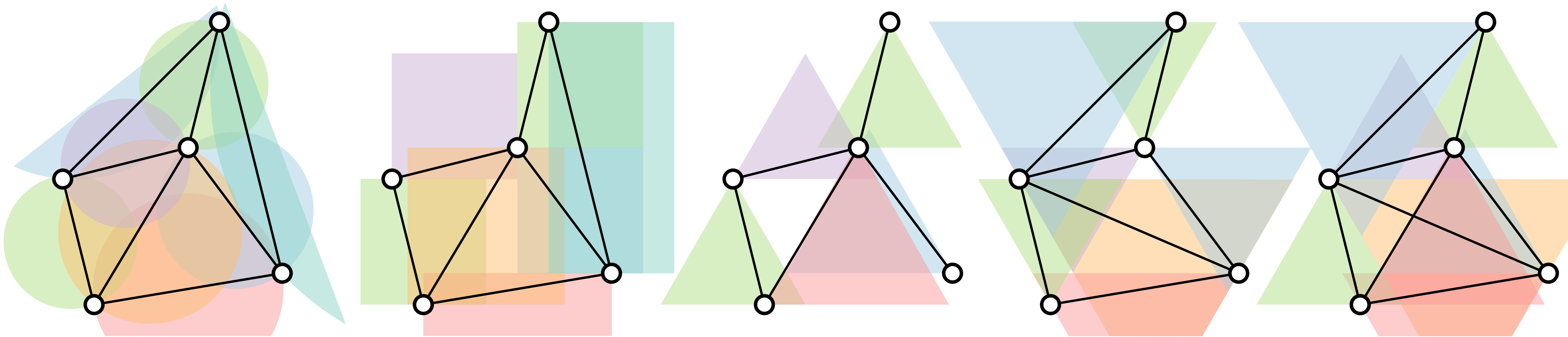
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593<t<1.998$	$t=2.61$	$t=2$	$t=2$
Max. Matching $\mu(n)$	$\mu(n)=\lfloor\frac{n}{2}\rfloor$	$\mu(n)=\lfloor\frac{n}{2}\rfloor$ (even Hamil. path)	$\mu(n)=\lceil\frac{n-1}{3}\rceil$	$\mu(n)\geq\frac{3n-8}{7}$ $\mu(n)\leq\lfloor\frac{n}{2}\rfloor$
Min. Blocking Set $\beta(n)$	$\beta(n)\geq n-1$ $\beta(n)\leq\frac{3n}{2}$	$\beta(n)\geq\frac{5n}{4}-o(n)$ $\beta(n)\leq 2n-o(n)$	$\beta(n)=\lceil\frac{n-1}{2}\rceil$	$\beta(n)\geq\lceil\frac{n-1}{2}\rceil$ $\beta(n)\leq n-1$



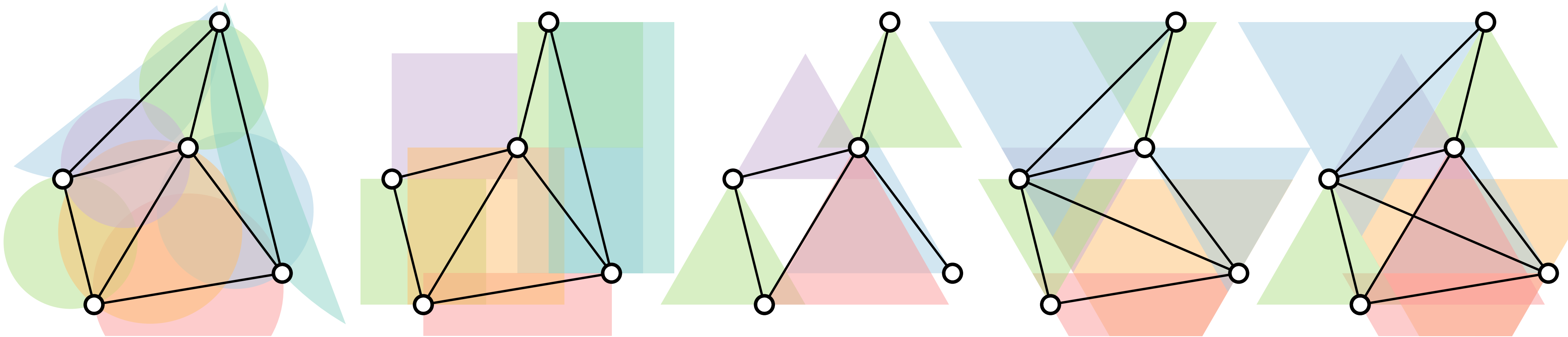
	$G^\circ(P)$ Delaunay / $G^\square(P)$ L_∞-Delaunay	$G^\triangle(P)$ Half-Θ_6-Graphs	$G^\triangledown(P)$ Half-Θ_6-Graphs	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593<t<1.998$	$t=2.61$	$t=2$	$t=2$
Max. Matching $\mu(n)$	$\mu(n)=\lfloor\frac{n}{2}\rfloor$	$\mu(n)=\lfloor\frac{n}{2}\rfloor$ (even Hamil. path)	$\mu(n)=\lceil\frac{n-1}{3}\rceil$	$\mu(n)\geq\frac{3n-8}{7}$ $\mu(n)\leq\lfloor\frac{n}{2}\rfloor$ $\mu(n)\geq\frac{\beta(n)}{2}$
Min. Blocking Set $\beta(n)$	$\beta(n)\geq n-1$ $\beta(n)\leq\frac{3n}{2}$	$\beta(n)\geq\frac{5n}{4}-o(n)$ $\beta(n)\leq 2n-o(n)$	$\beta(n)=\lceil\frac{n-1}{2}\rceil$	$\beta(n)\geq\lceil\frac{n-1}{2}\rceil$ $\beta(n)\leq n-1$



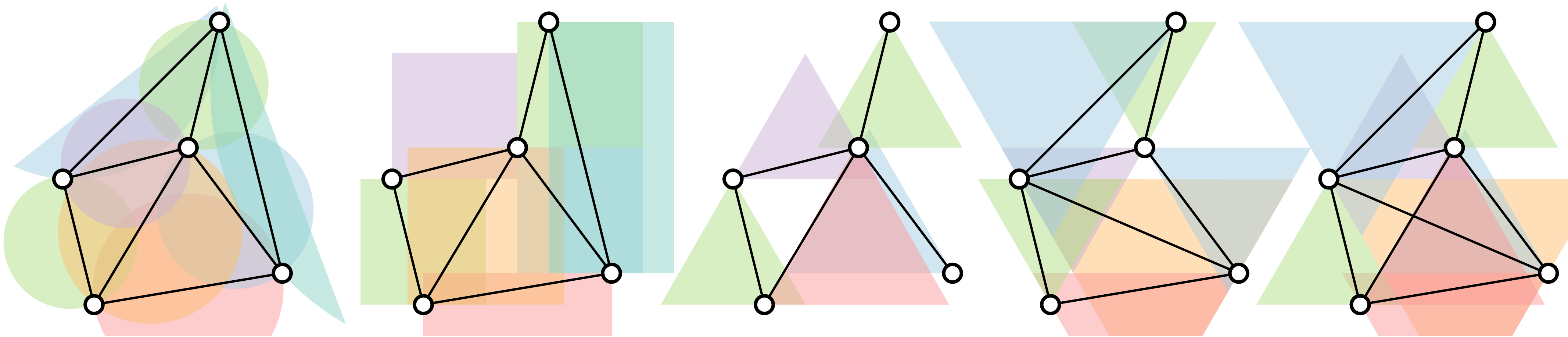
	$G^{\circ}(P)$ Delaunay	$G^{\square}(P)$ L_∞-Delaunay	$G^{\triangle}(P)$ $G^{\triangledown}(P)$ Half-Θ_6-Graphs	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$ $\mu(n) \geq \frac{\beta(n)}{2}$
Min. Blocking Set $\beta(n)$	$\beta(n) \geq n-1$ $\beta(n) \leq \frac{3n}{2}$	$\beta(n) \geq \frac{5n}{4} - o(n)$ $\beta(n) \leq 2n - o(n)$	$\beta(n) = \lceil \frac{n-1}{2} \rceil$	$\beta(n) \geq \frac{3n-8}{4}$ $\beta(n) \leq n-1$



	$G^\circ(P)$ Delaunay	$G^\square(P)$ L_∞-Delaunay	$G^\triangle(P)$ Half-Θ_6-Graphs	$G^\triangledown(P)$ Half-Θ_6-Graphs	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$		$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ $\mu(n) \geq \lfloor \frac{n}{2} \rfloor$?	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$		$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$ $\mu(n) \geq \frac{\beta(n)}{2}$
Min. Blocking Set $\beta(n)$	$\beta(n) \geq n - 1$ $\beta(n) \leq \frac{3n}{2}$	$\beta(n) \geq \frac{5n}{4} - o(n)$ $\beta(n) \leq 2n - o(n)$	$\beta(n) = \lceil \frac{n-1}{2} \rceil$		$\beta(n) \geq \frac{3n-8}{4}$ $\beta(n) \leq n - 1$



	$G^\circ(P)$ Delaunay	$G^\square(P)$ L_∞-Delaunay	$G^\triangle(P)$ $G^\triangledown(P)$ Half-Θ_6-Graphs	$G^{✡}(P)$ Θ_6-Graphs
Spanning Ratio t	$1.593<t<1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$ $\mu(n) \geq \frac{\beta(n)}{2}$
	$\mu(n) \geq \lfloor \frac{n}{2} \rfloor$?	$\beta(n) \geq n-1$?		
Min. Blocking Set $\beta(n)$	$\beta(n) \geq n-1$ $\beta(n) \leq \frac{3n}{2}$	$\beta(n) \geq \frac{5n}{4} - o(n)$ $\beta(n) \leq 2n - o(n)$	$\beta(n) = \lceil \frac{n-1}{2} \rceil$	$\beta(n) \geq \frac{3n-8}{4}$ $\beta(n) \leq n-1$



$G^\circ(P)$ Delaunay | $G^\square(P)$ L_∞-Delaunay | $G^\triangle(P)$ $G^\triangledown(P)$ Half-Θ_6-Graphs | $G^{✡}(P)$ Θ_6-Graphs

	Delaunay	L_∞-Delaunay	Half-Θ_6-Graphs	Θ_6-Graphs
Spanning Ratio t	$1.593 < t < 1.998$	$t = 2.61$	$t = 2$	$t = 2$
Max. Matching $\mu(n)$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$	$\mu(n) = \lfloor \frac{n}{2} \rfloor$ (even Hamil. path)	$\mu(n) = \lceil \frac{n-1}{3} \rceil$	$\mu(n) \geq \frac{3n-8}{7}$ $\mu(n) \leq \lfloor \frac{n}{2} \rfloor$ $\mu(n) \geq \frac{\beta(n)}{2}$
Min. Blocking Set $\beta(n)$	$\beta(n) \geq n-1$ $\beta(n) \leq \frac{3n}{2}$	$\beta(n) \geq \frac{5n}{4} - o(n)$ $\beta(n) \leq 2n - o(n)$	$\beta(n) = \lceil \frac{n-1}{2} \rceil$	$\beta(n) \geq \frac{3n-8}{4}$ $\beta(n) \leq n-1$

$\mu(n) \geq \lfloor \frac{n}{2} \rfloor$? $\Leftrightarrow$ $\beta(n) \geq n-1$?