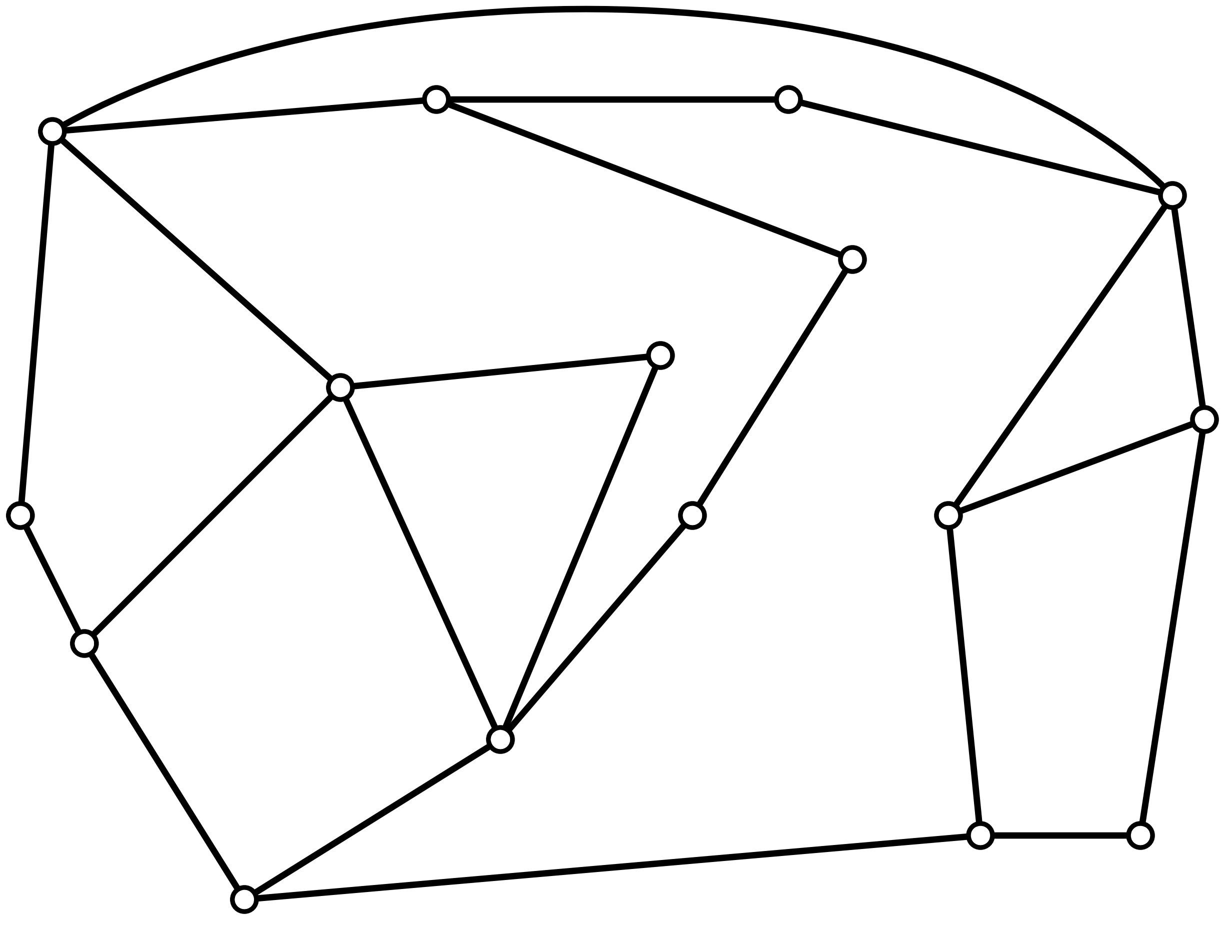
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG
Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems
Institute for Informatics

Finding Tutte Paths in Linear Time

Philipp Kindermann
Universität Würzburg

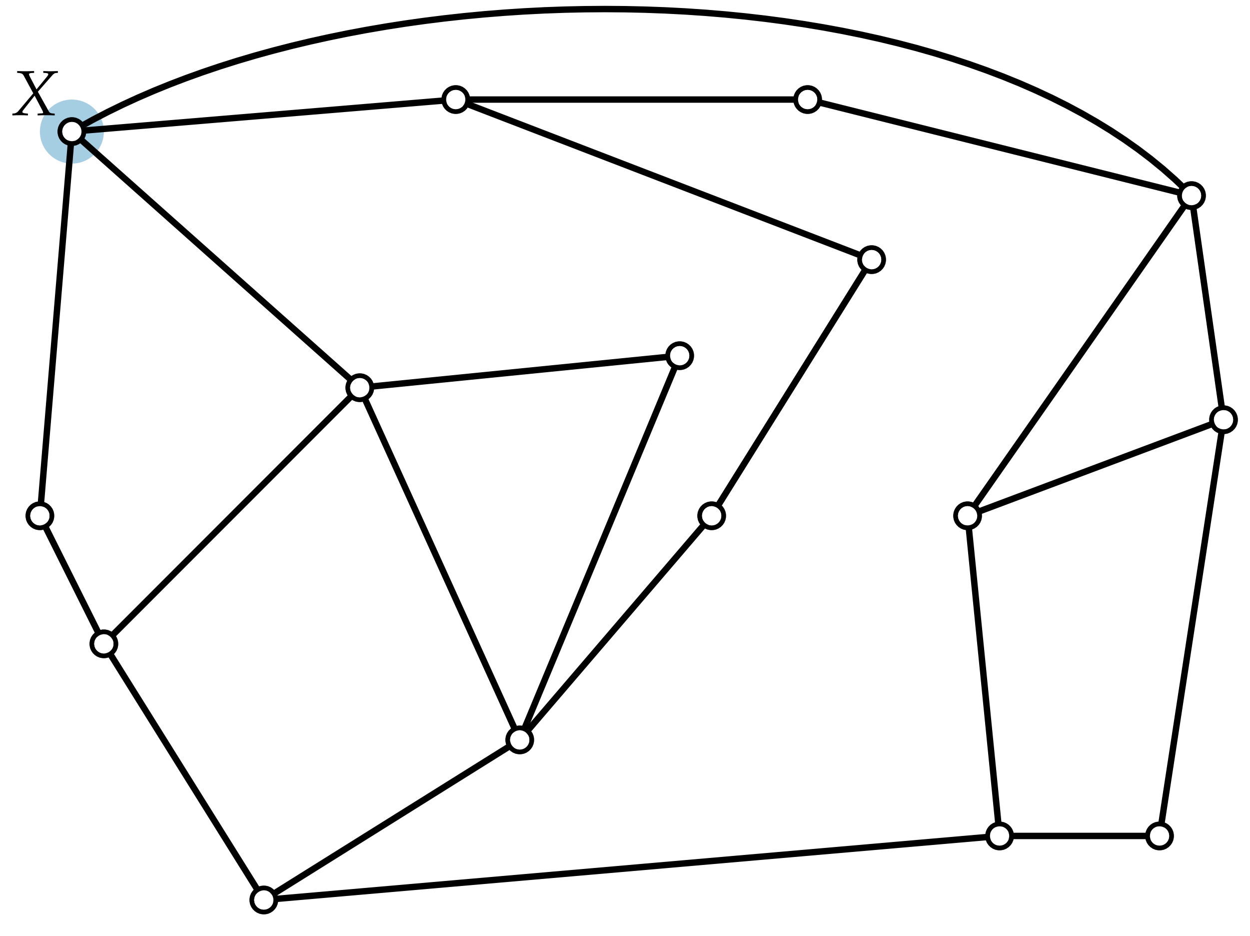
joint work with Therese Biedl
University of Waterloo

Tutte Paths



Planar graph G

Tutte Paths



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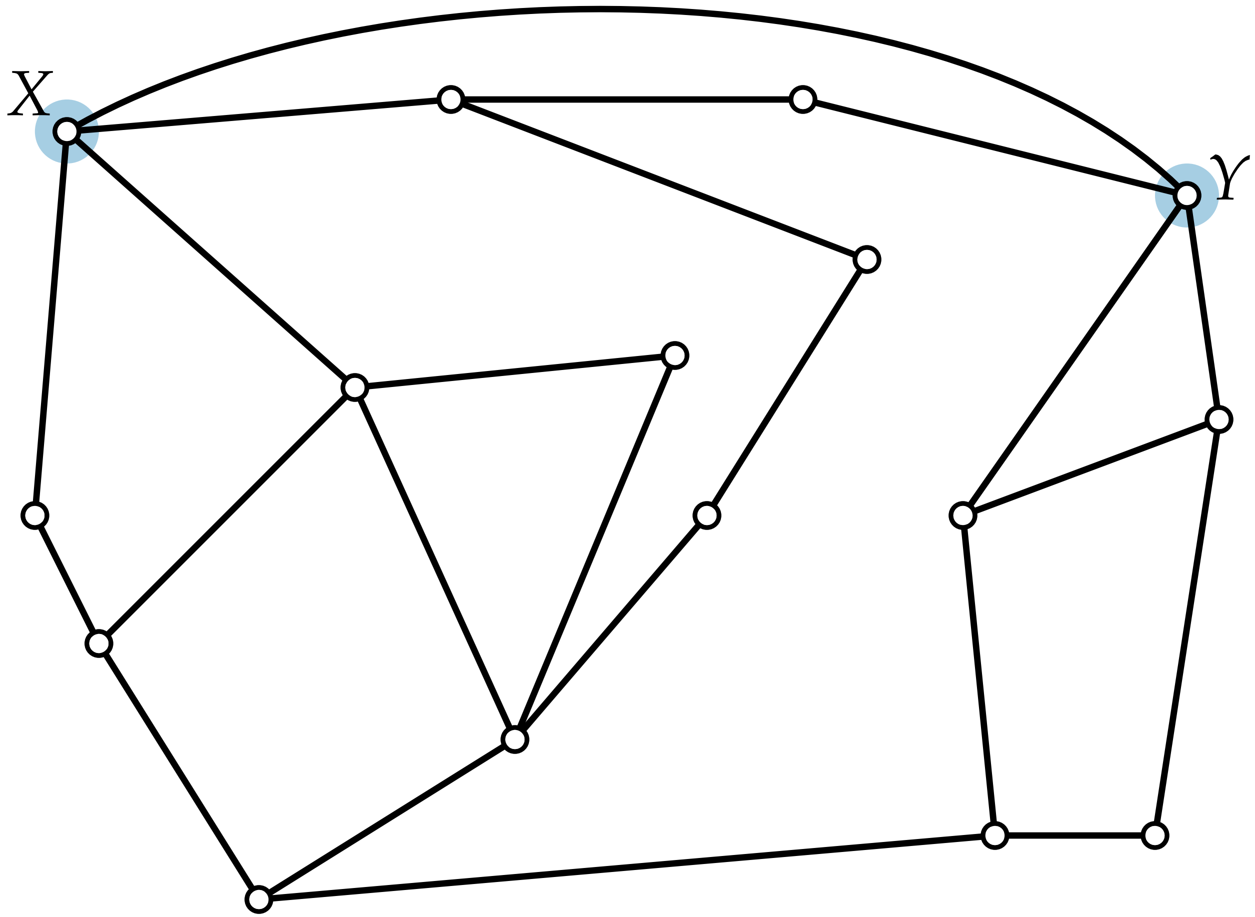

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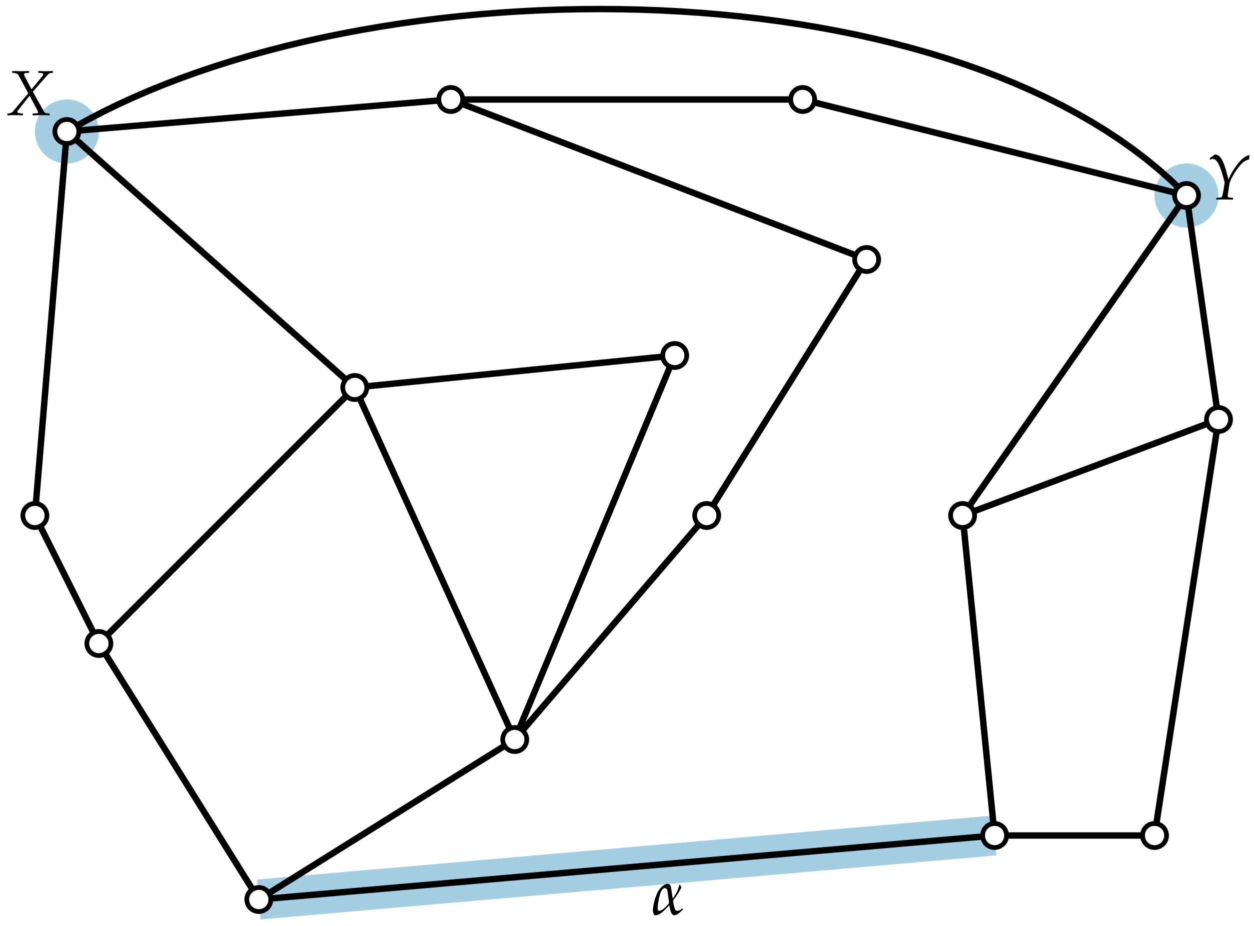

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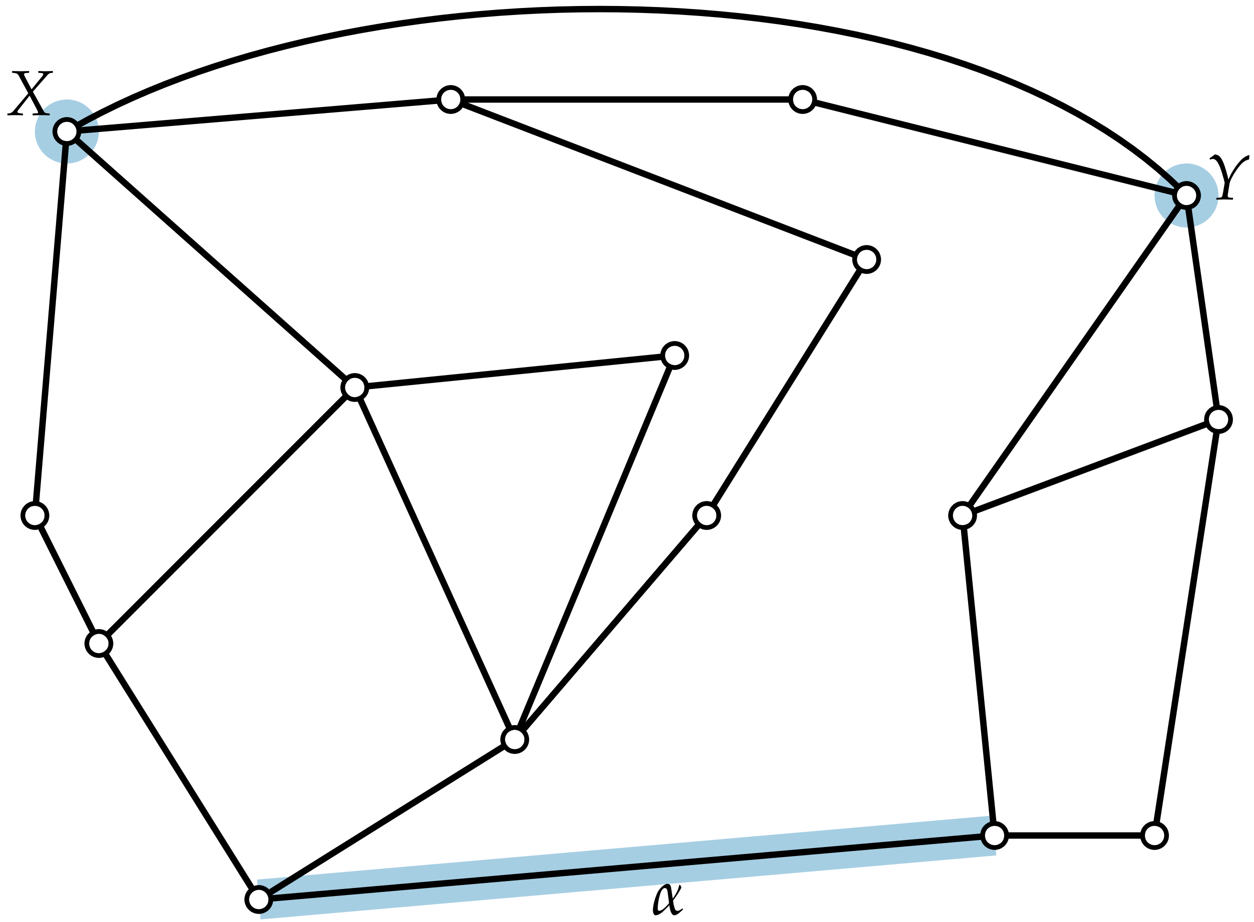

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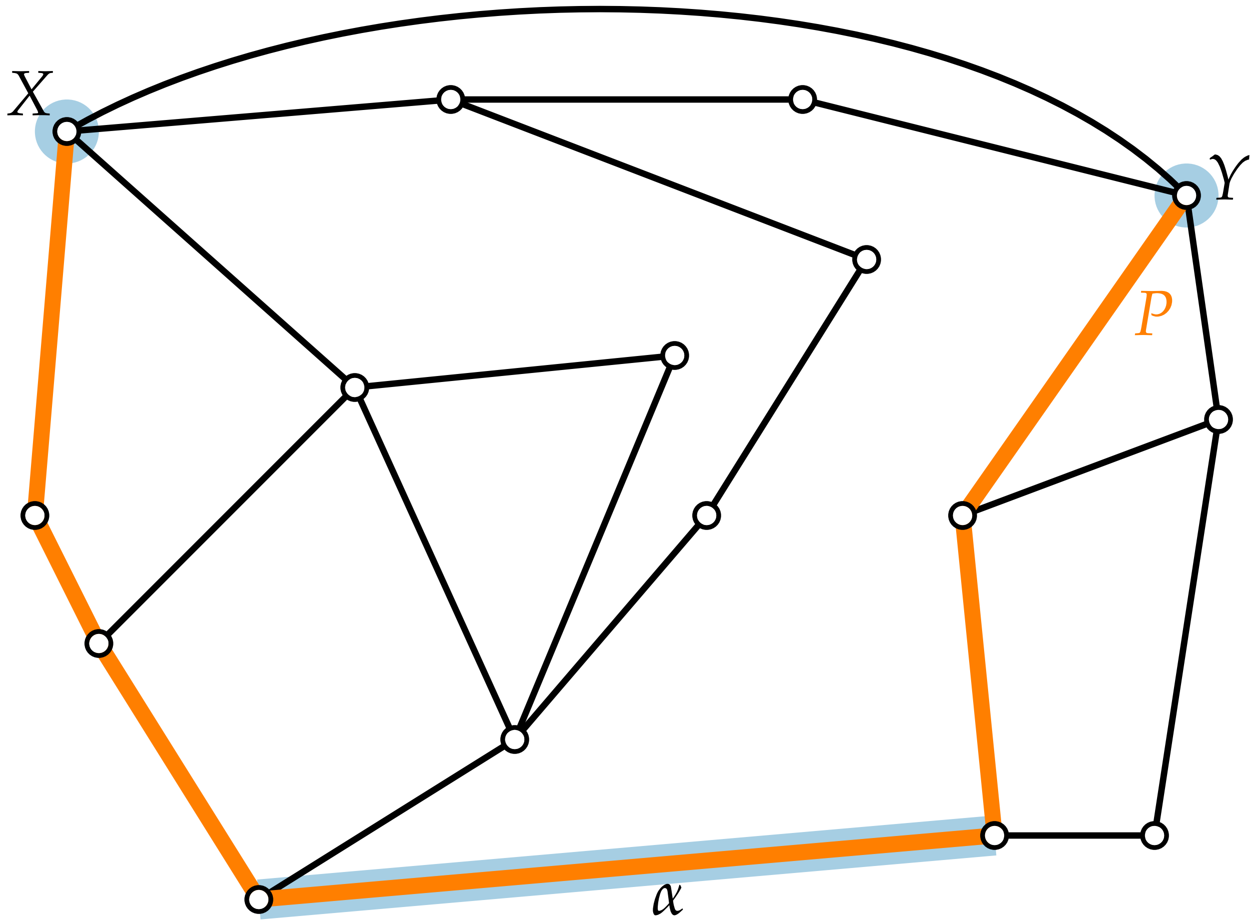
Tutte path: Path from X to Y via α

Tutte Paths



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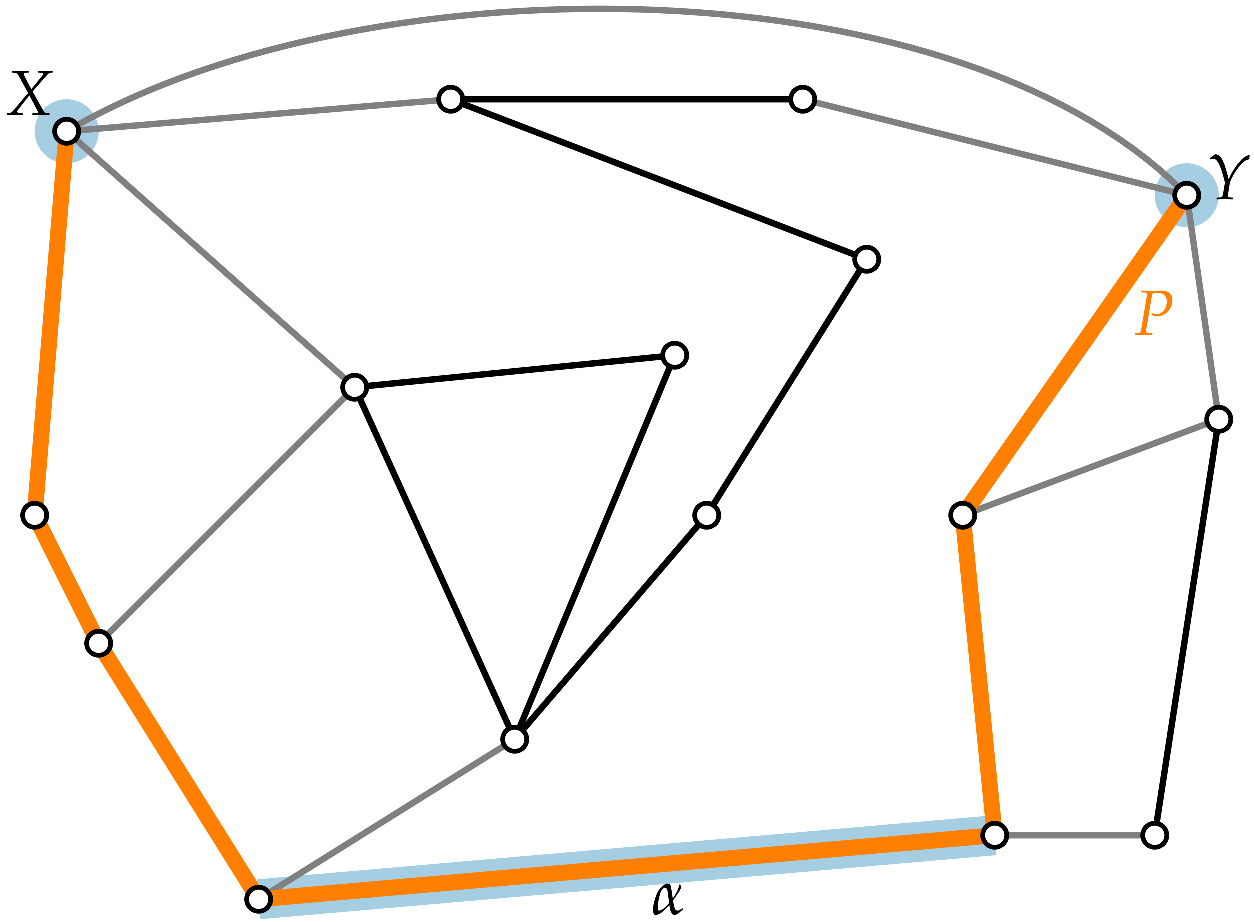

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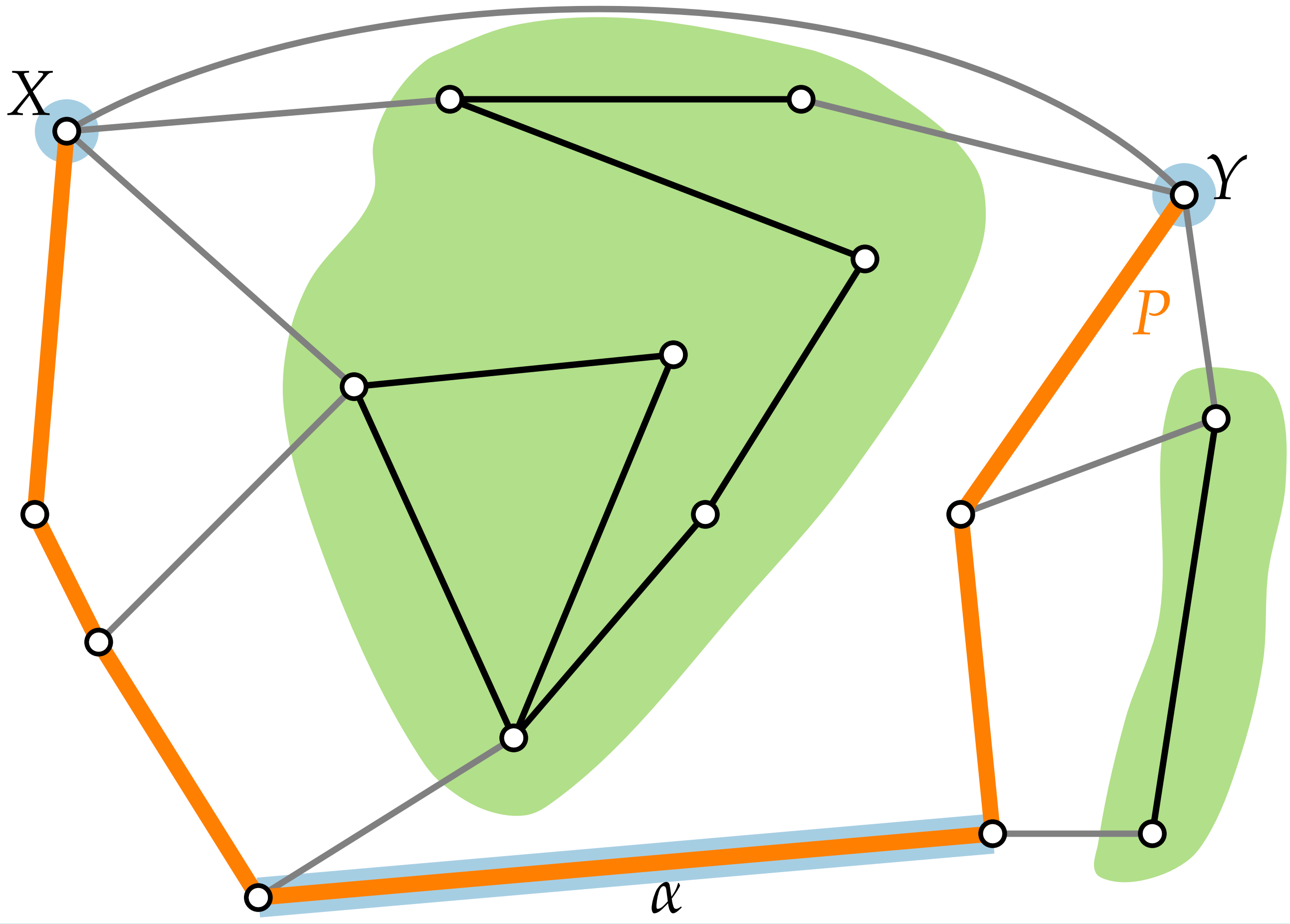

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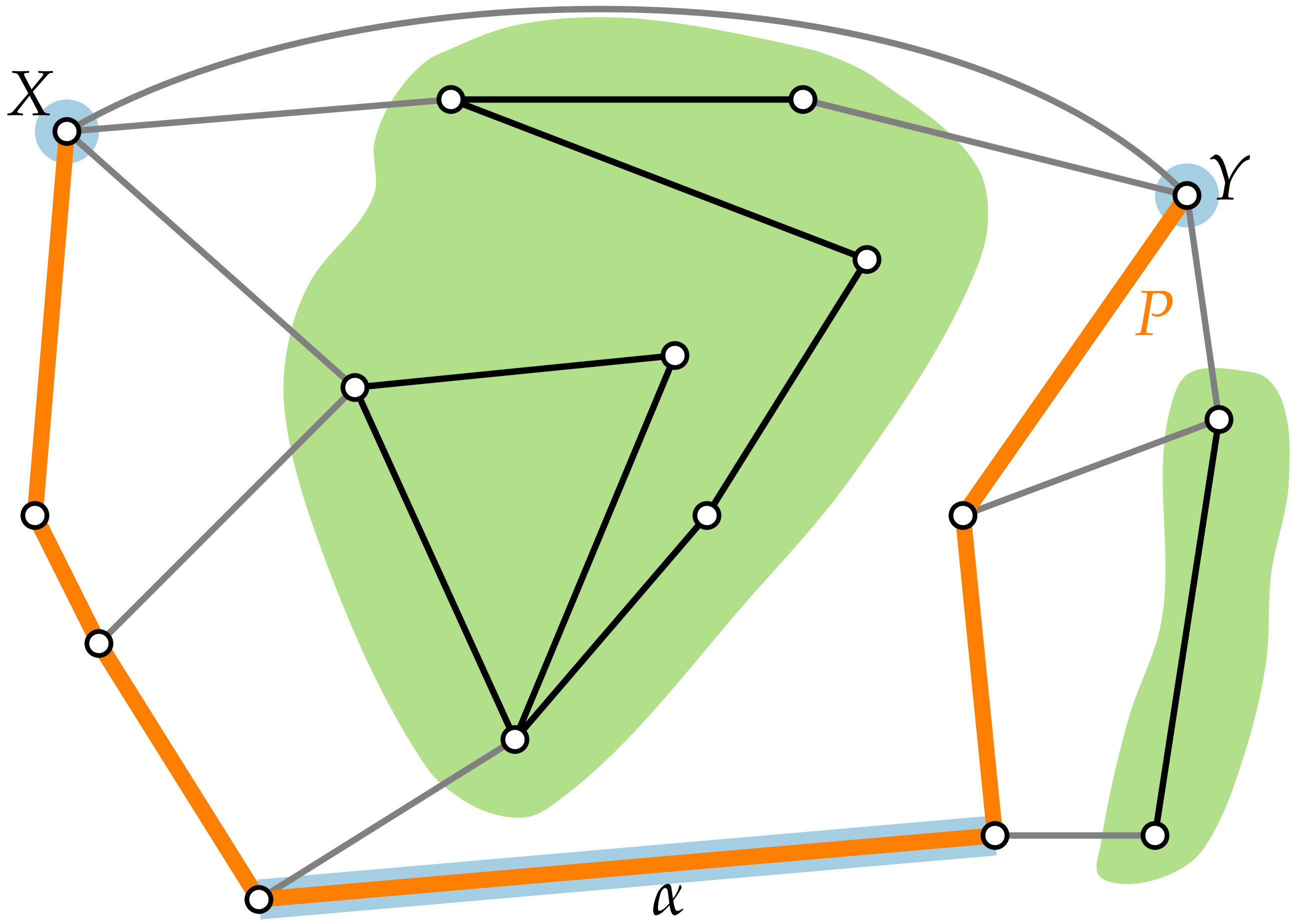

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Tutte path: Path from X to Y via α

Every comp. attached to ≤ 3 vtcs of P

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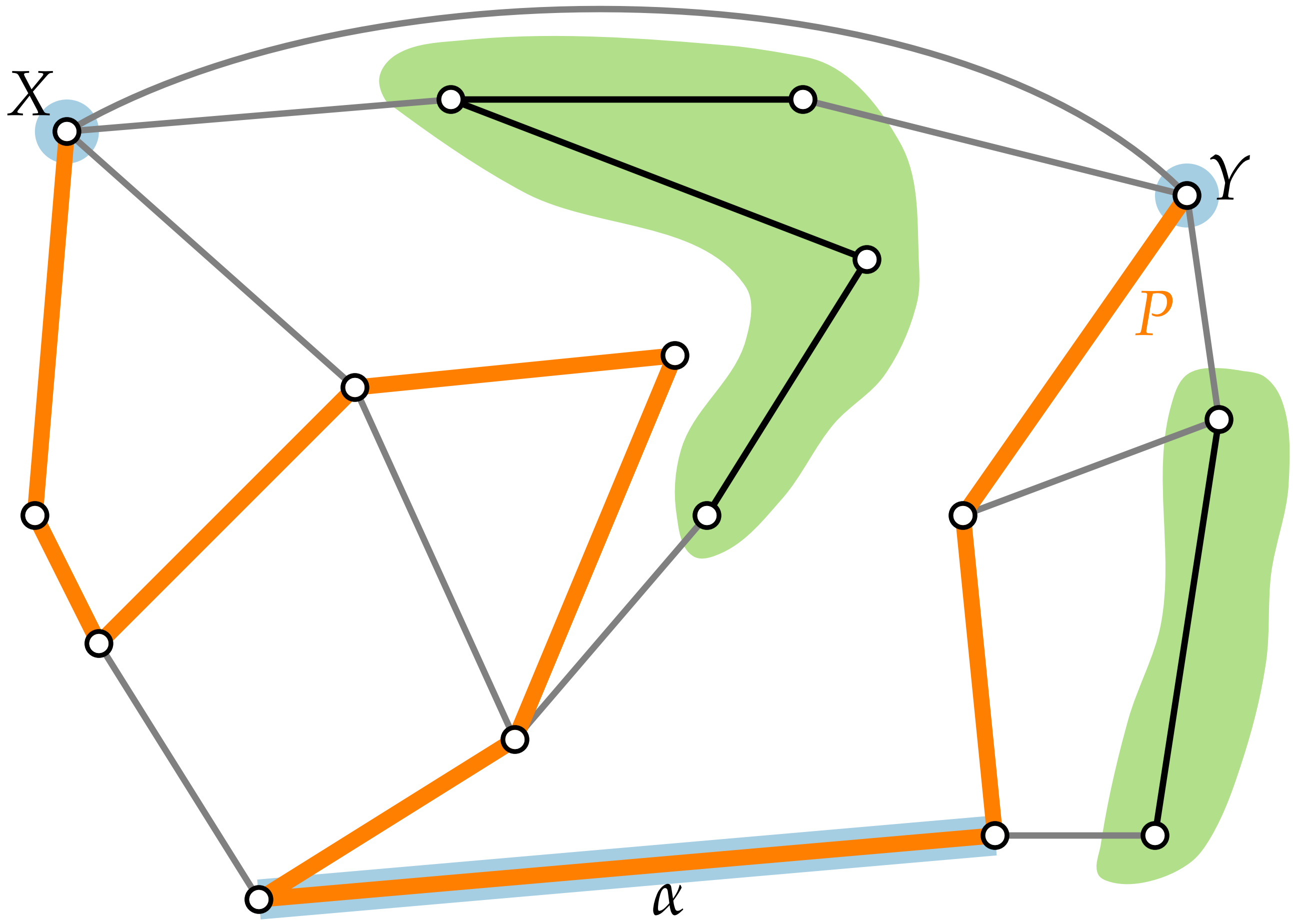

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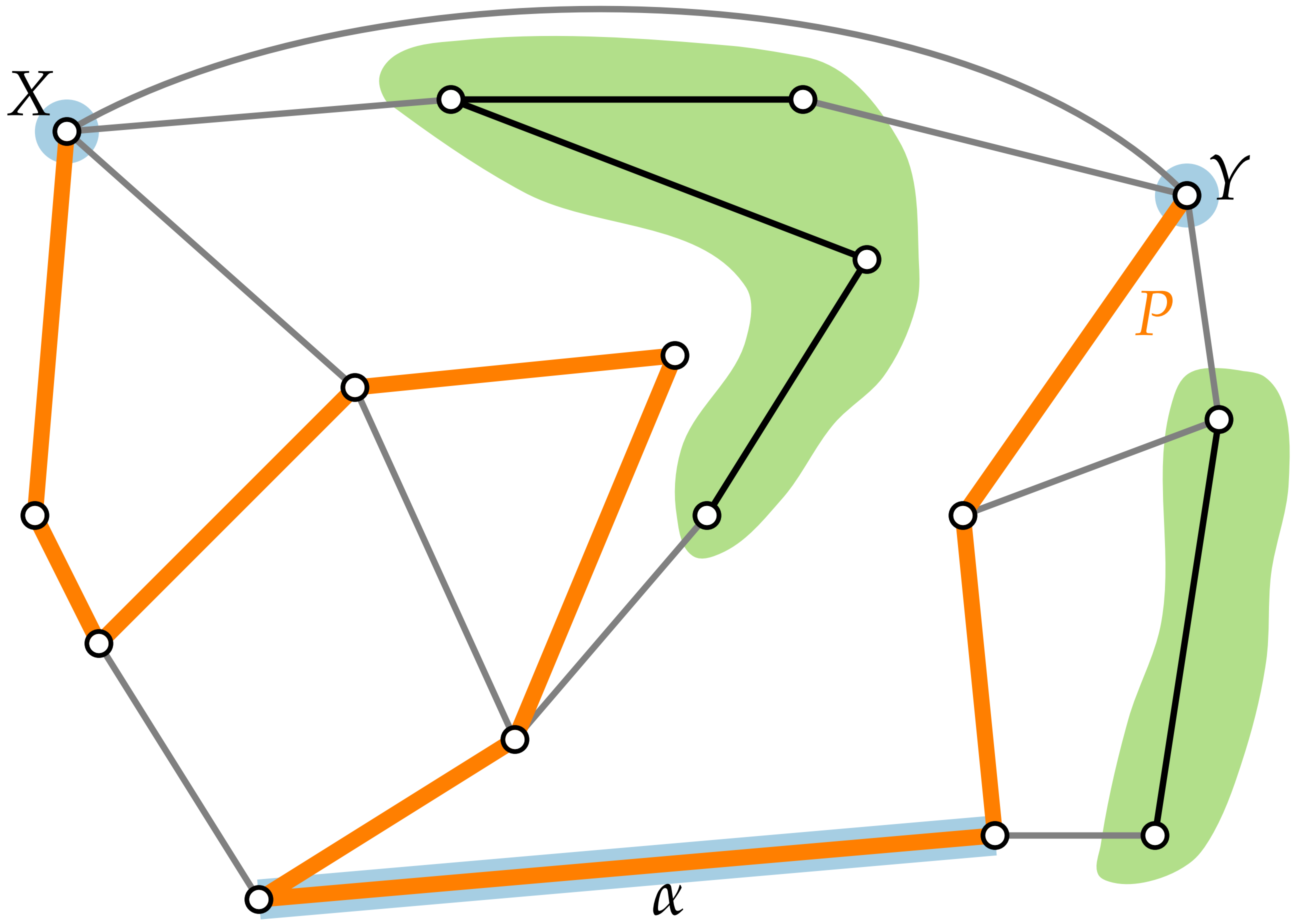

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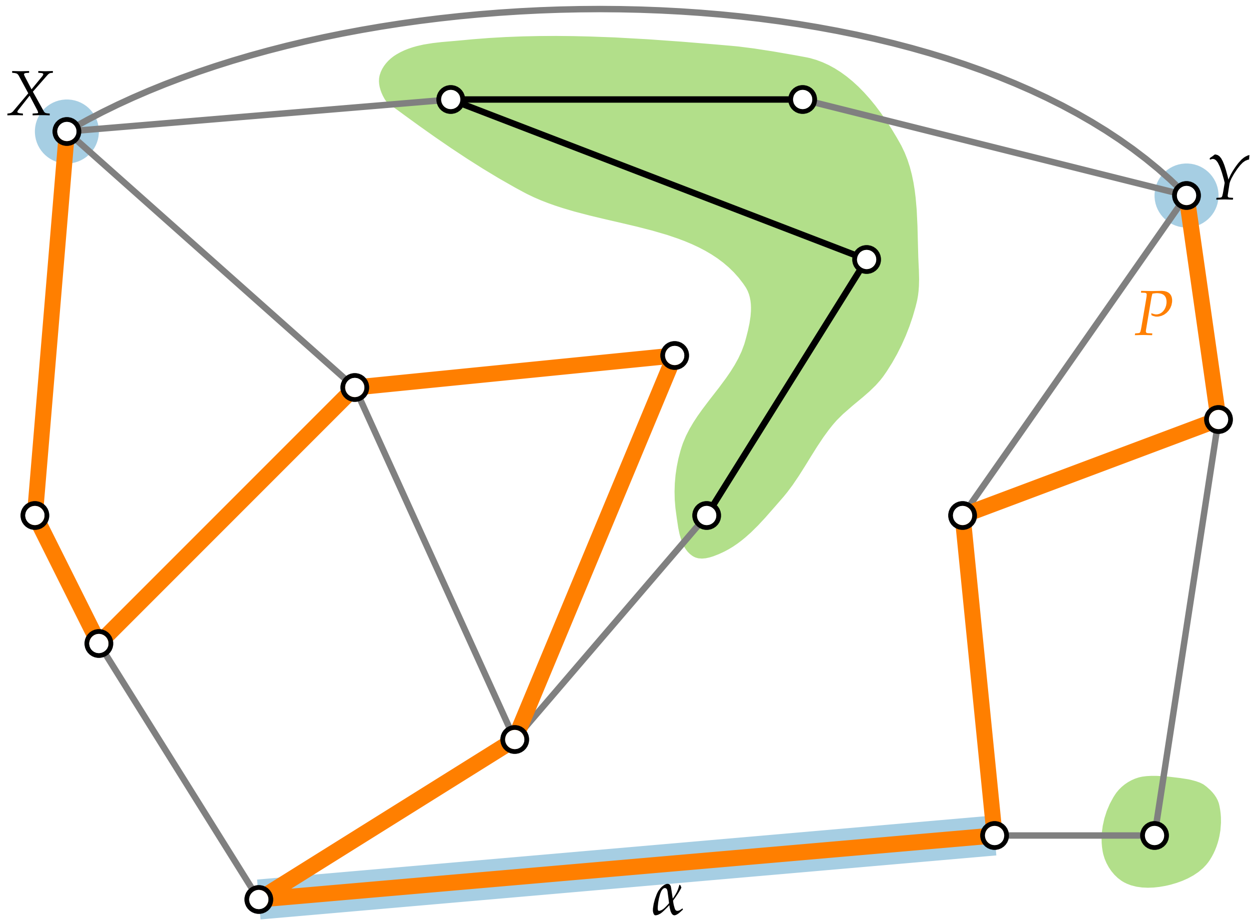

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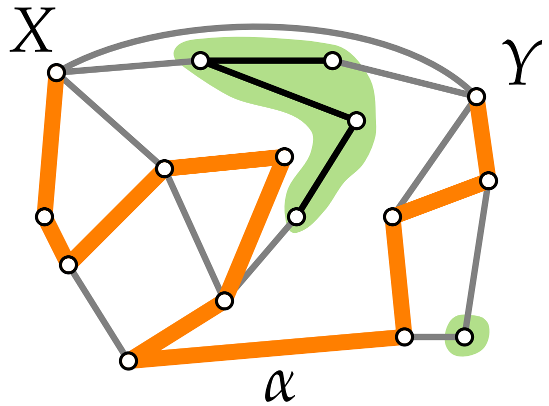

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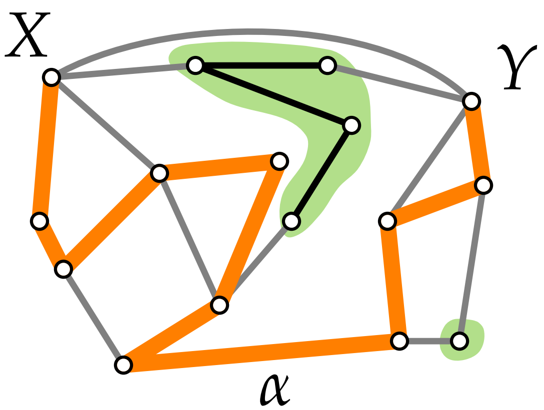

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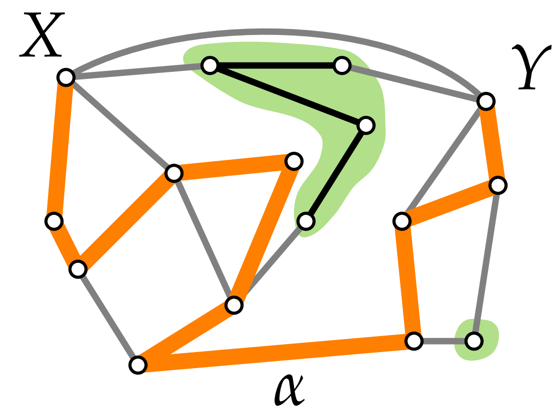

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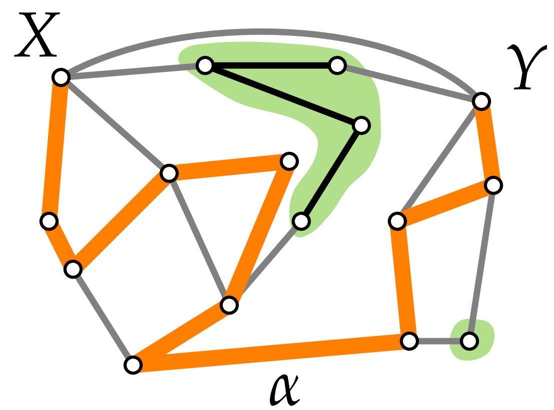

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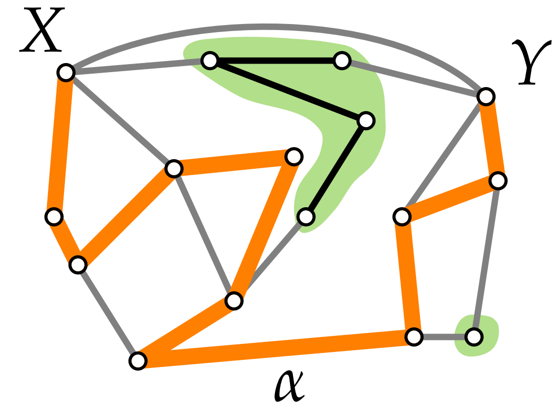

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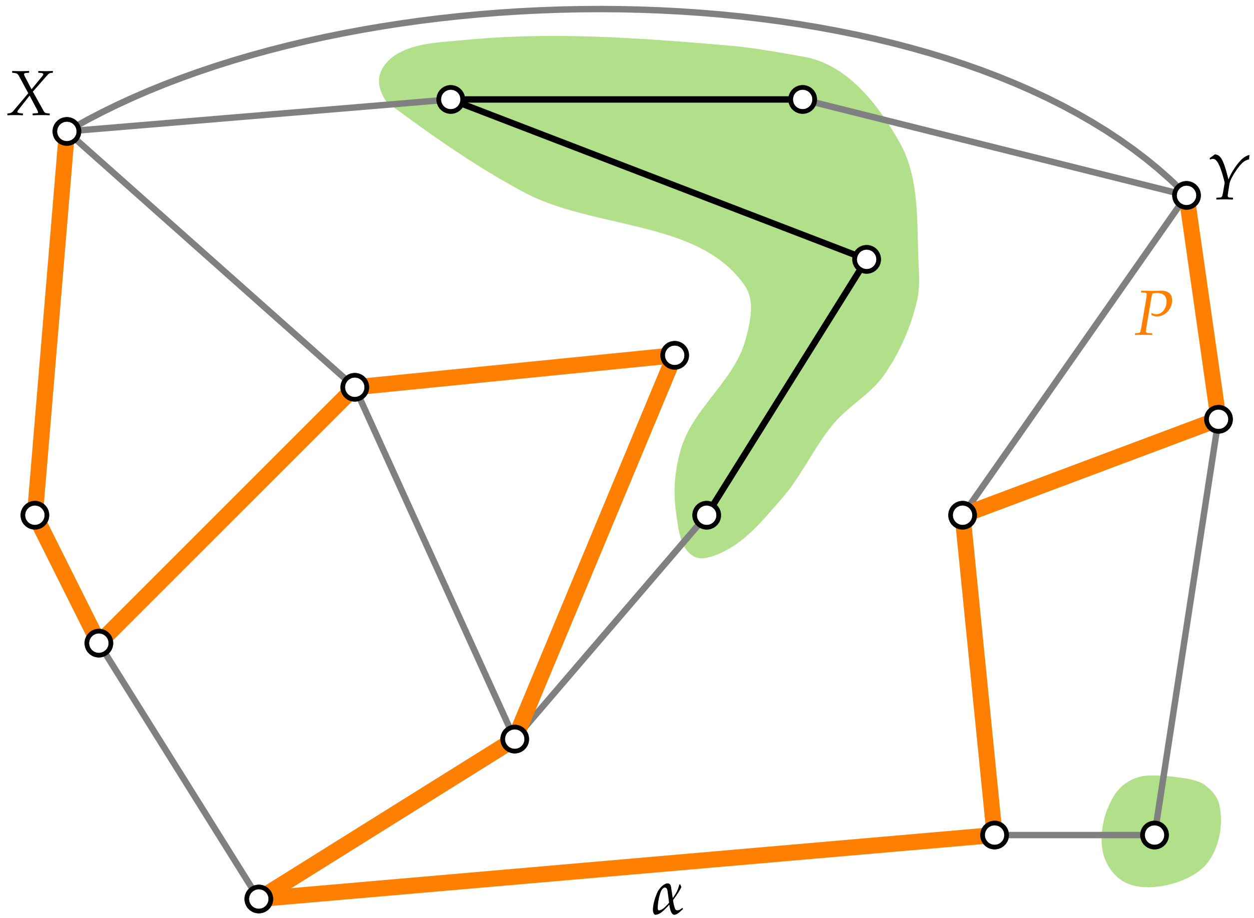

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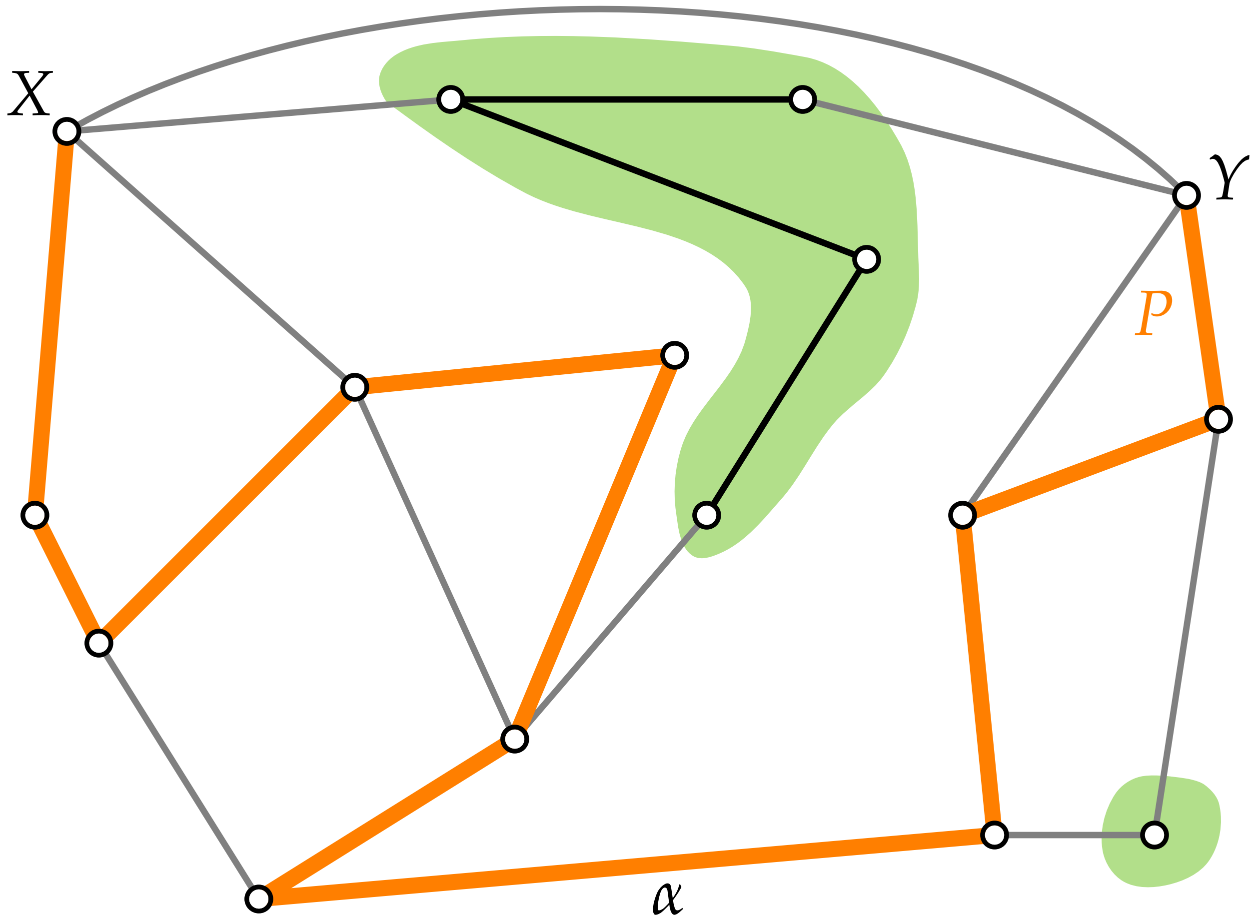

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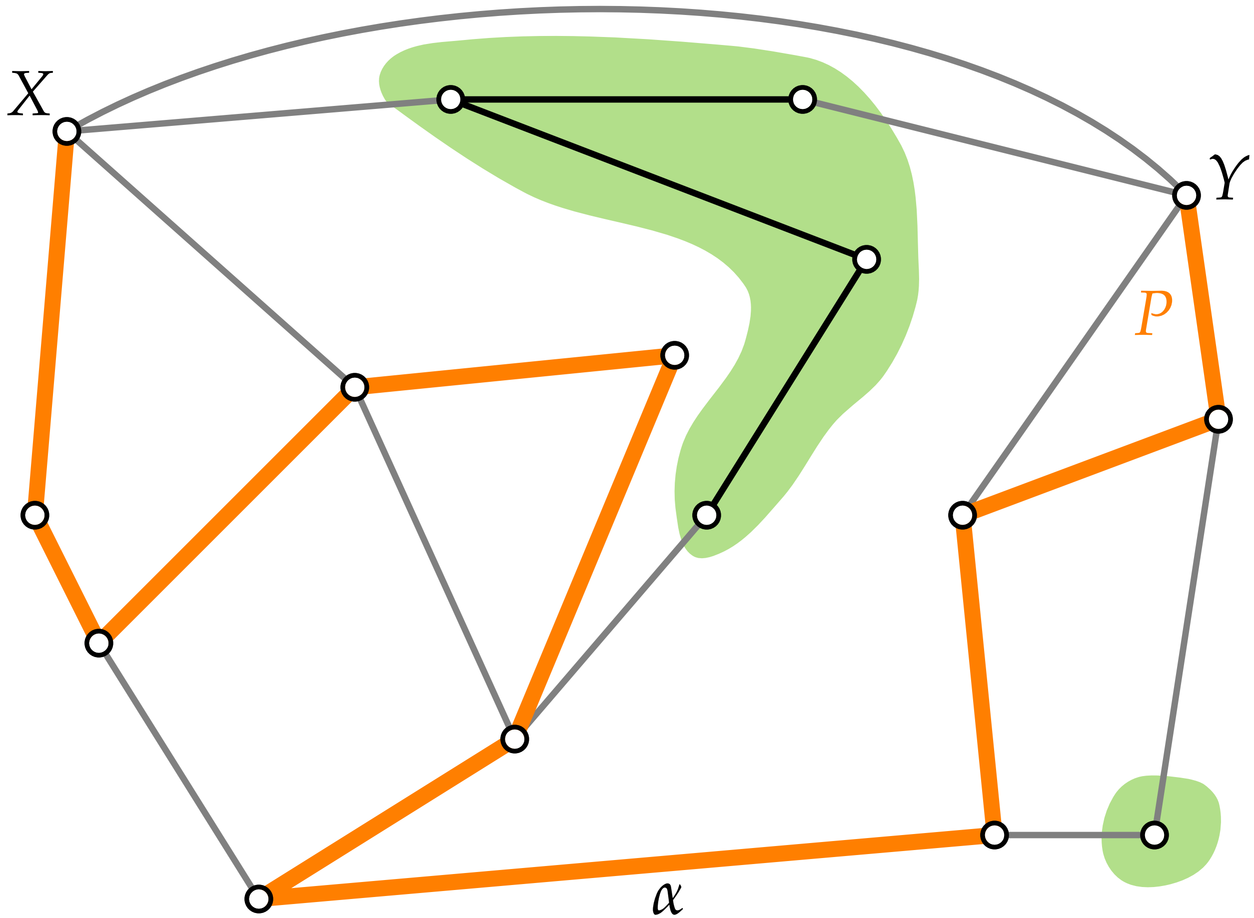

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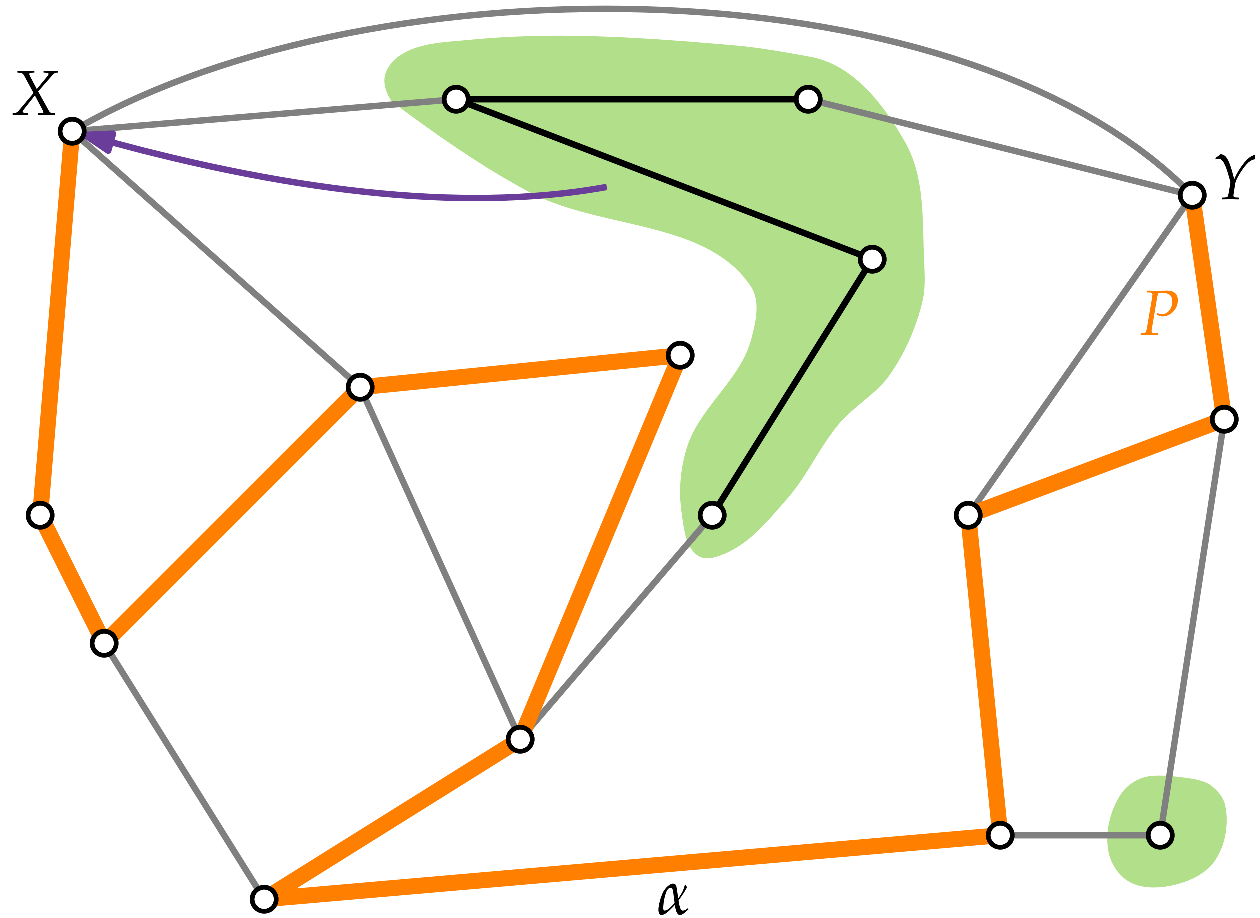

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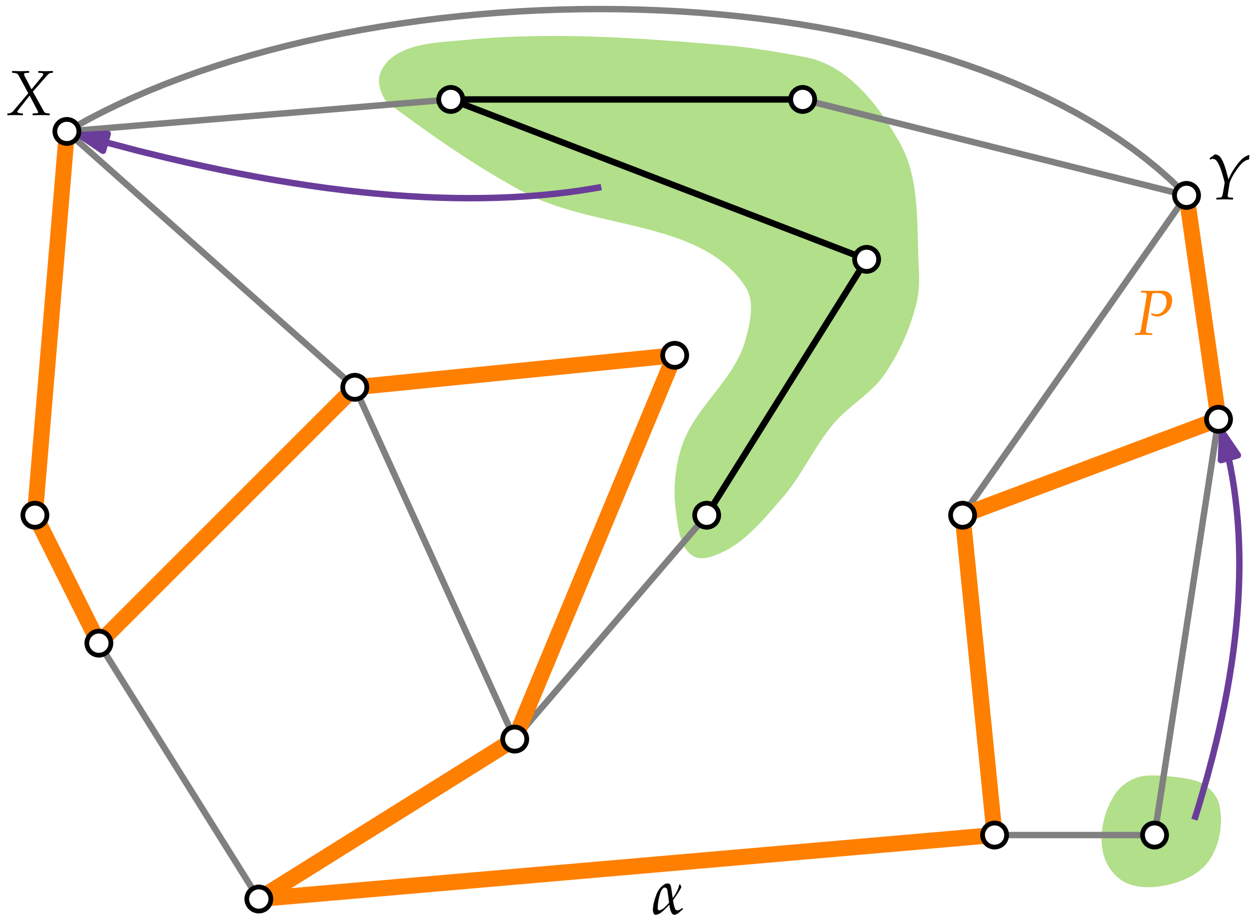

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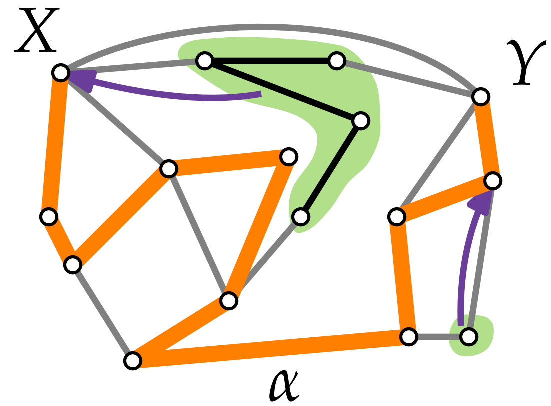

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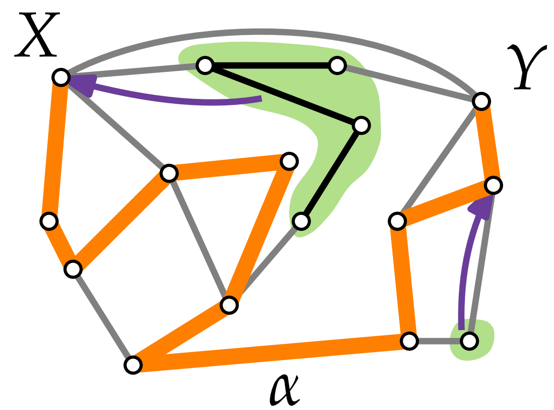

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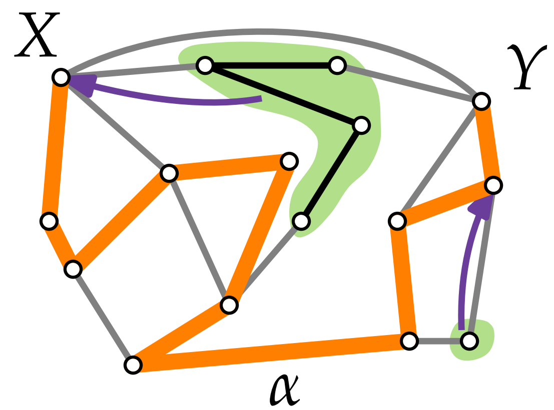

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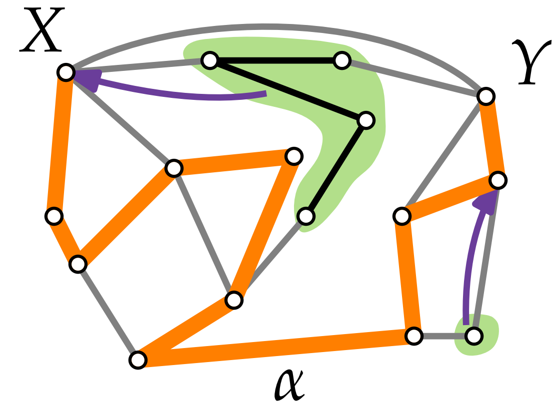

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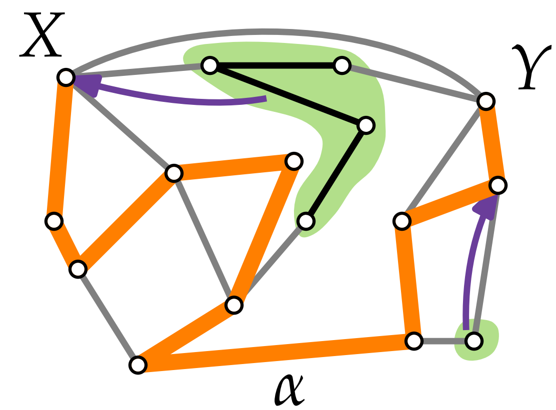

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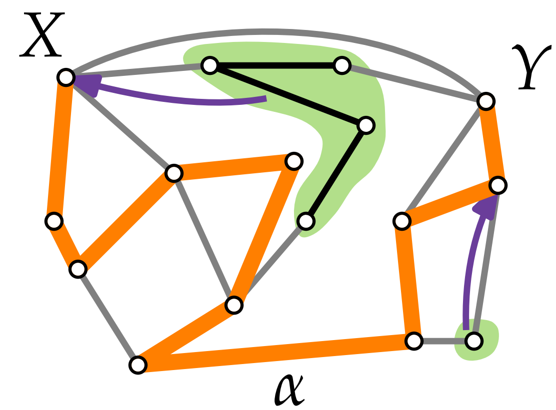

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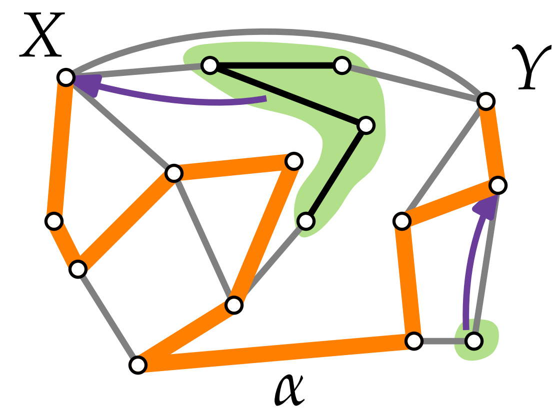

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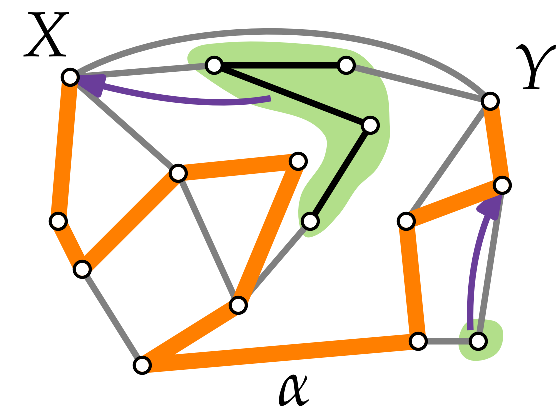

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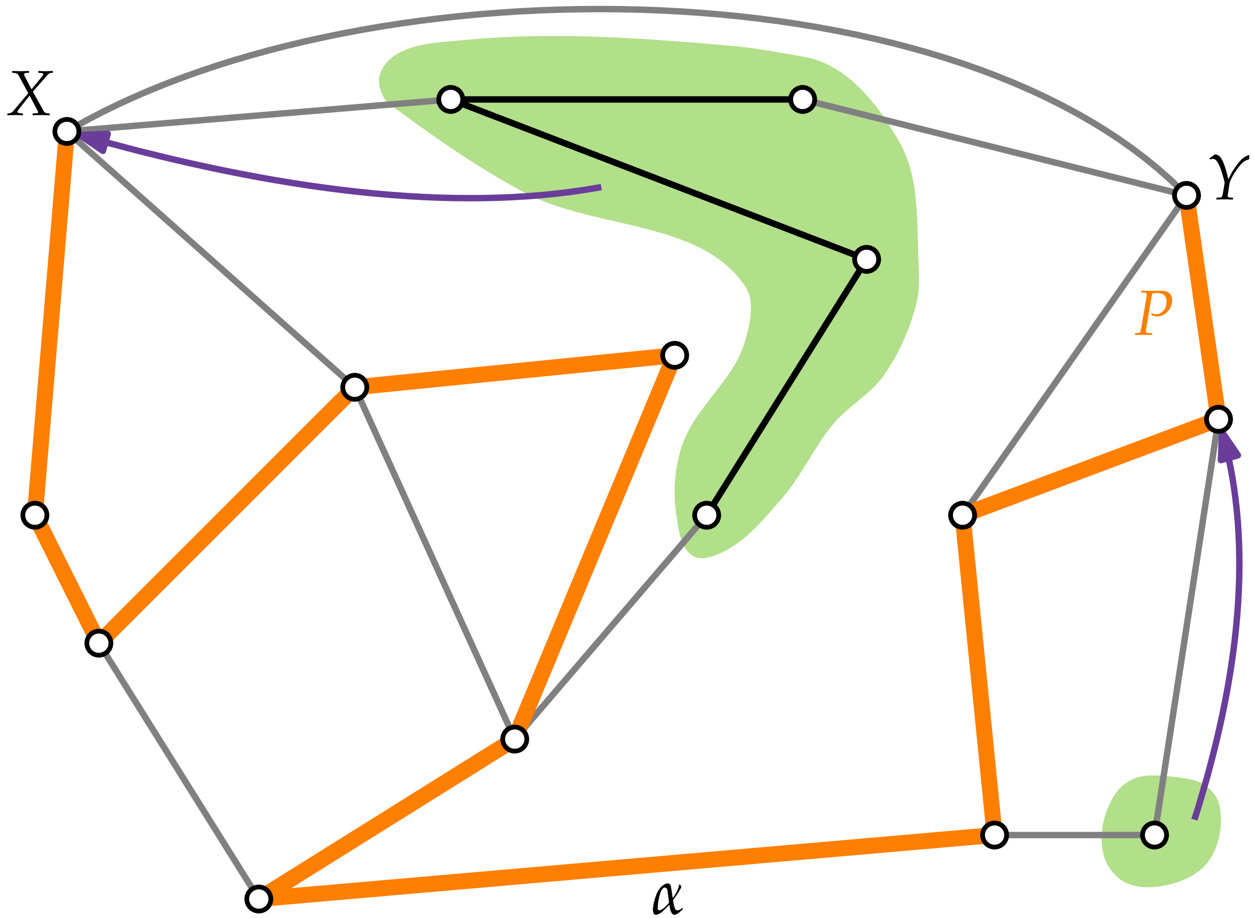
Tutte paths



Planar graph G

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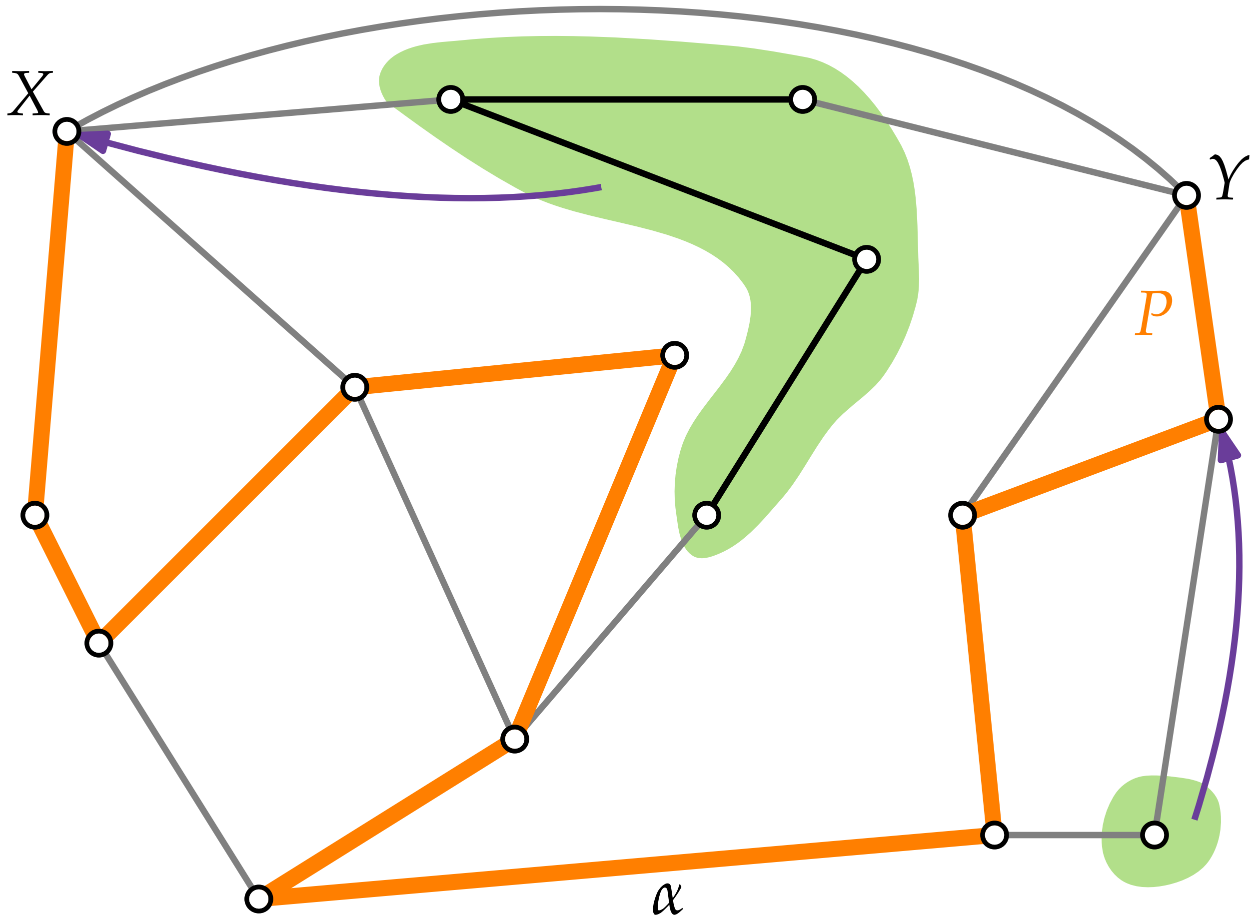

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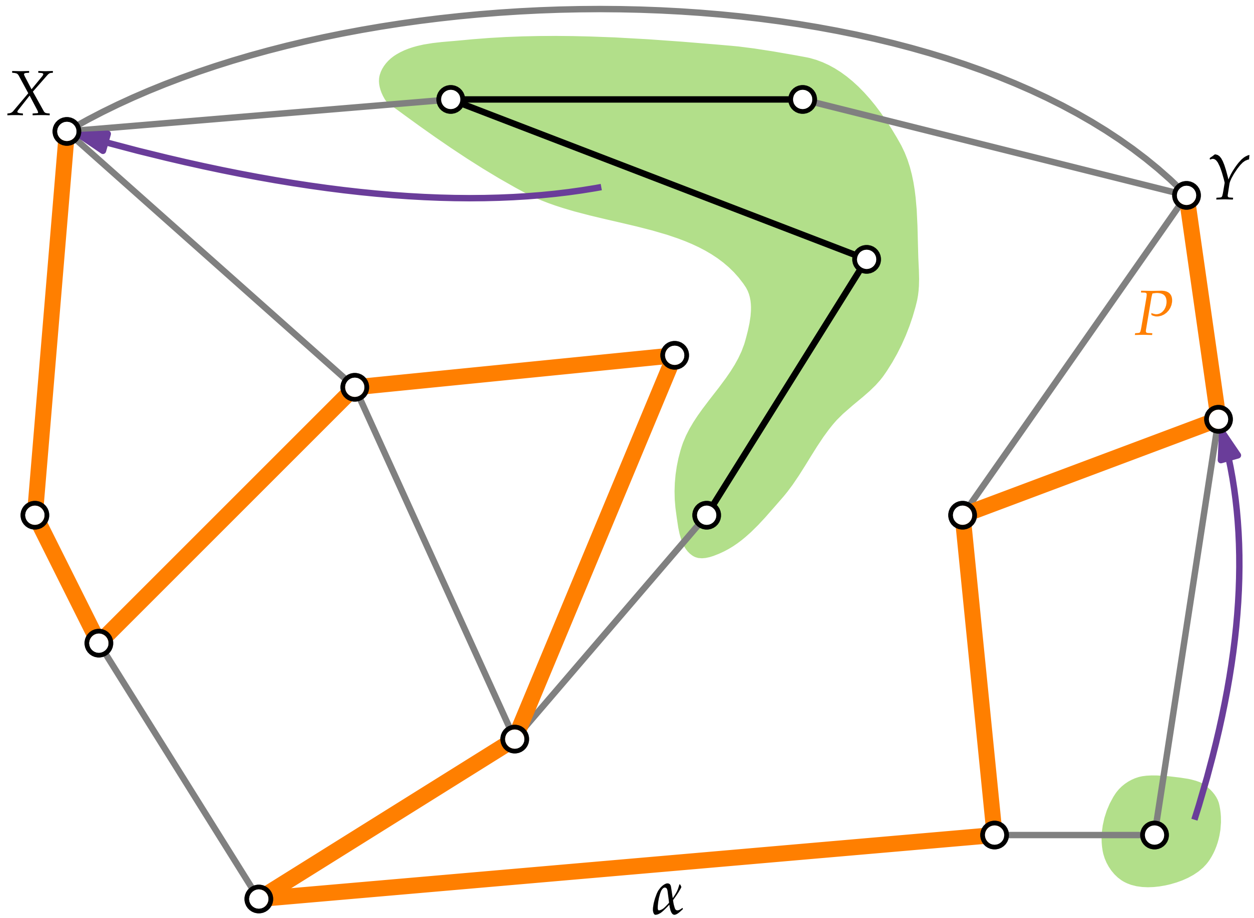

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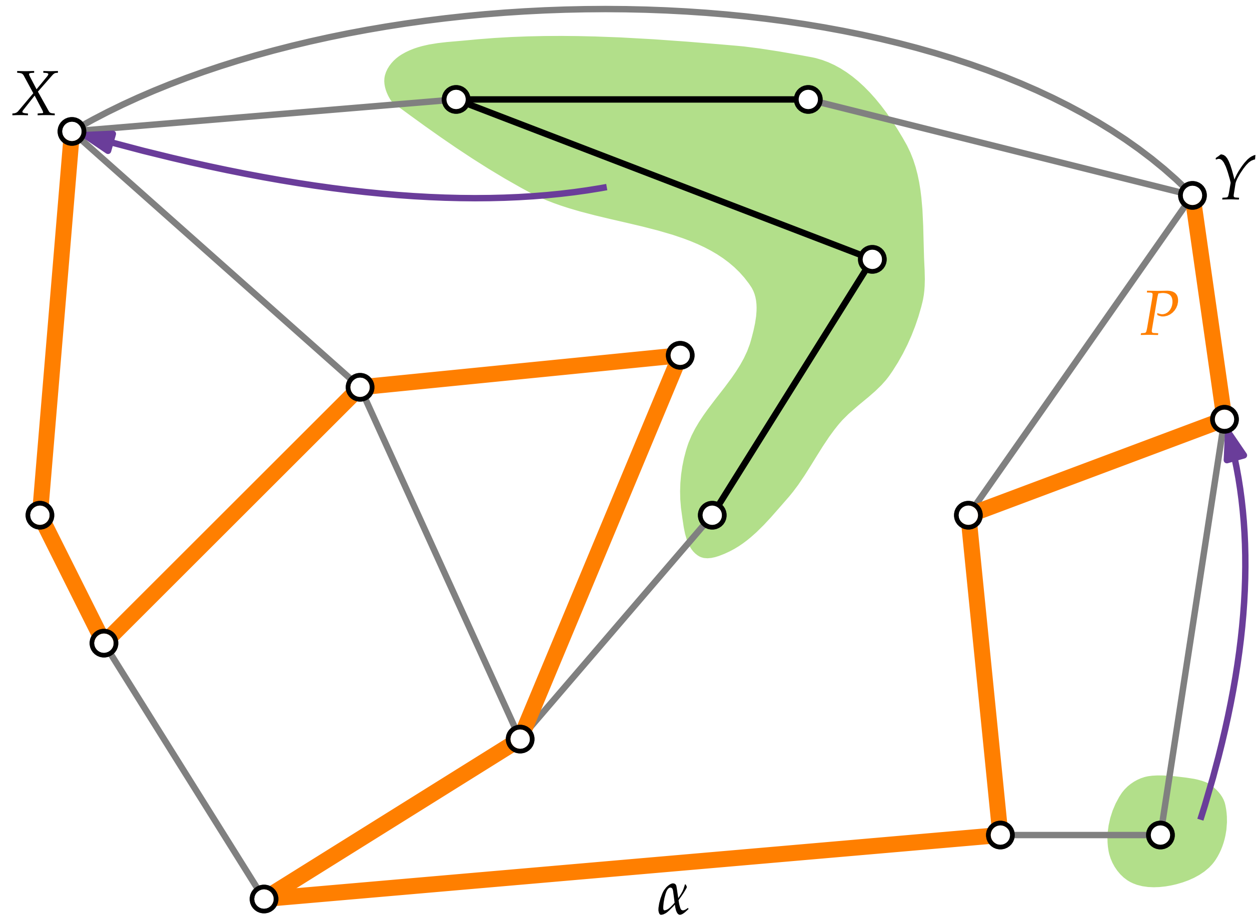

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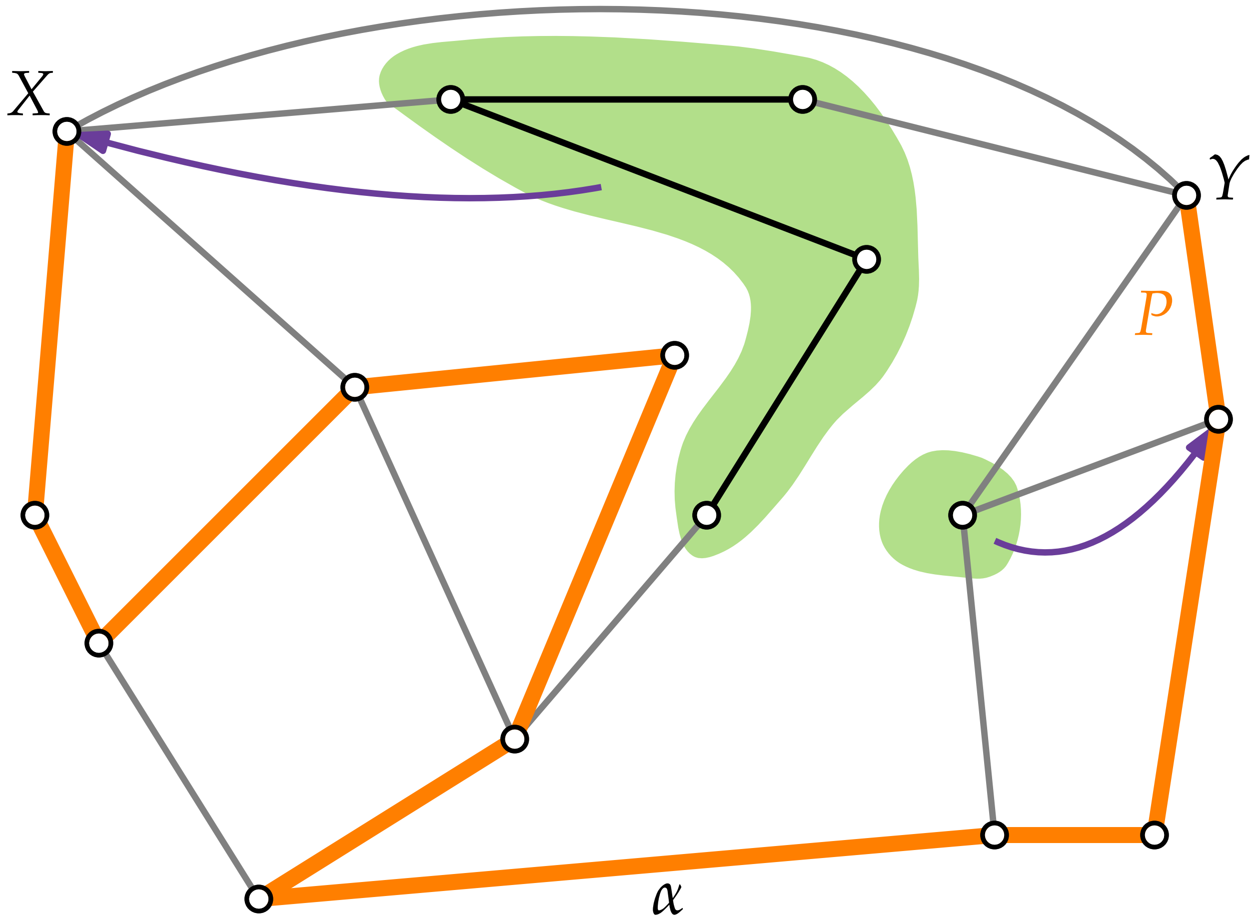

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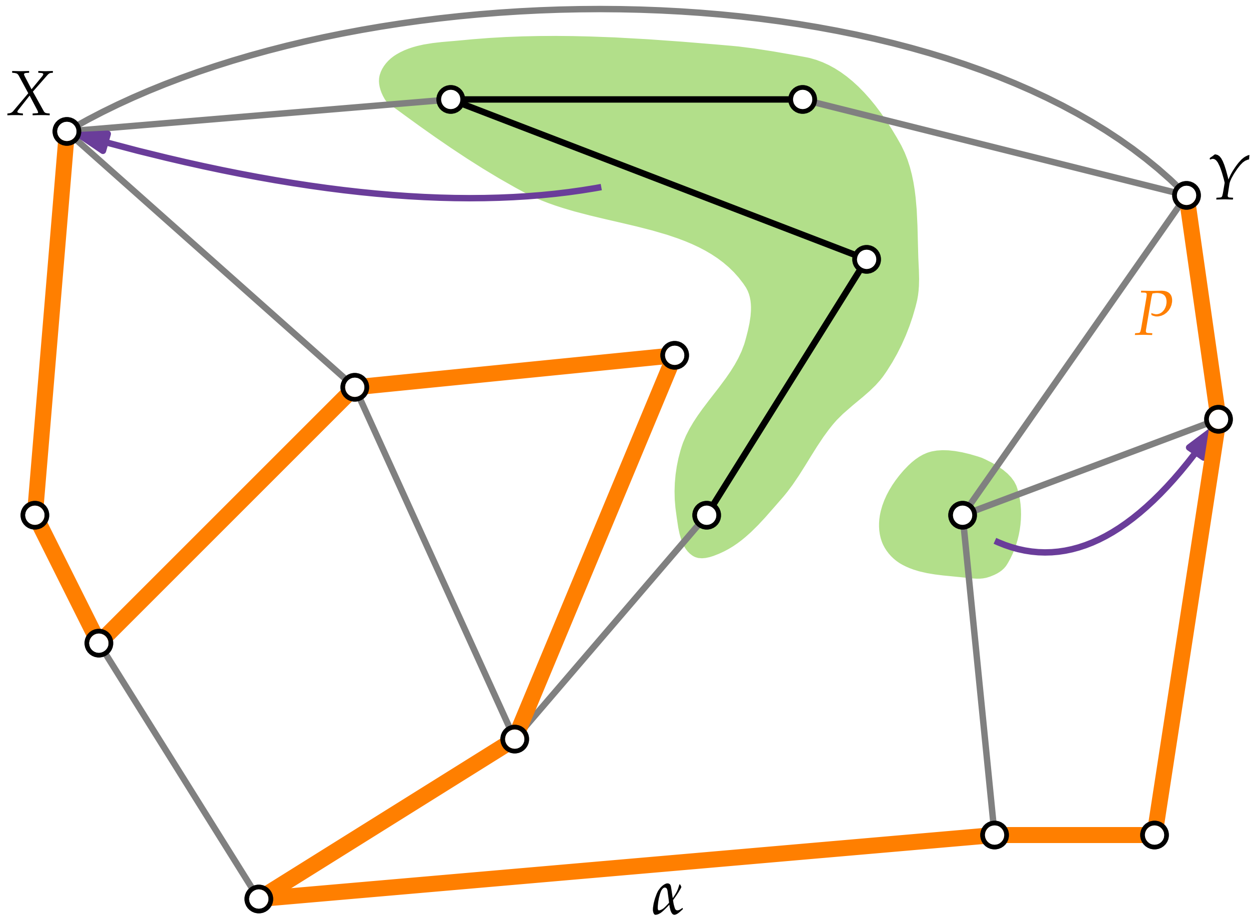

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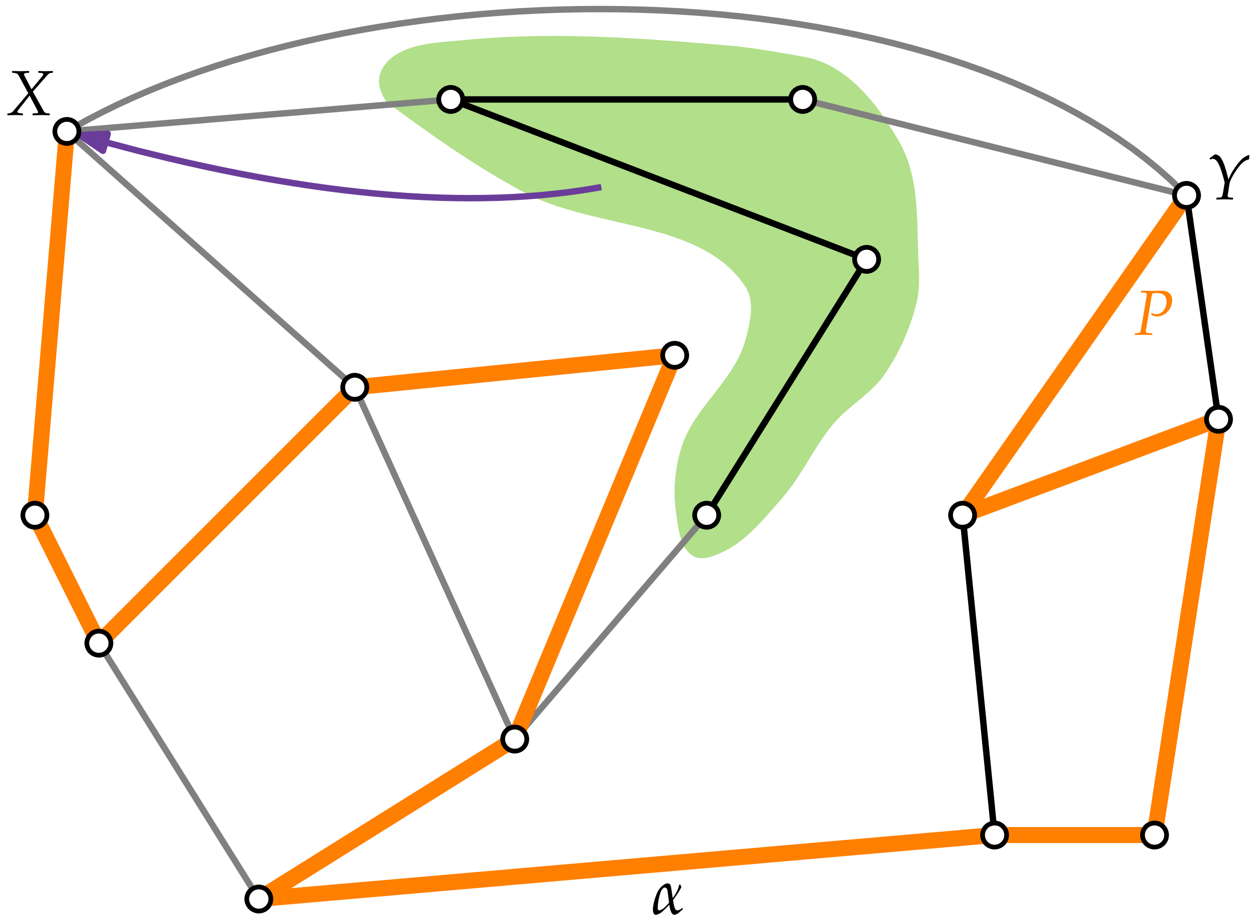

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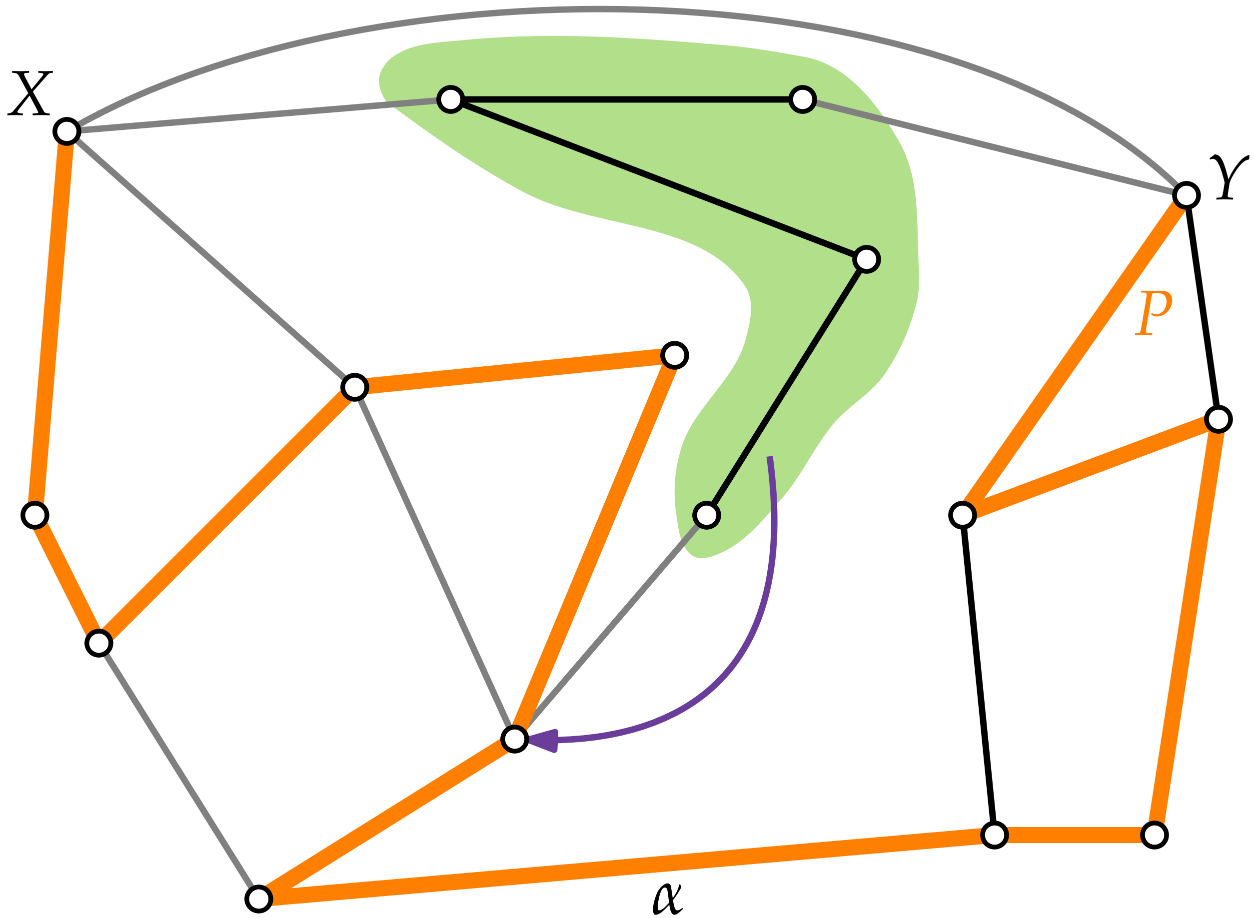

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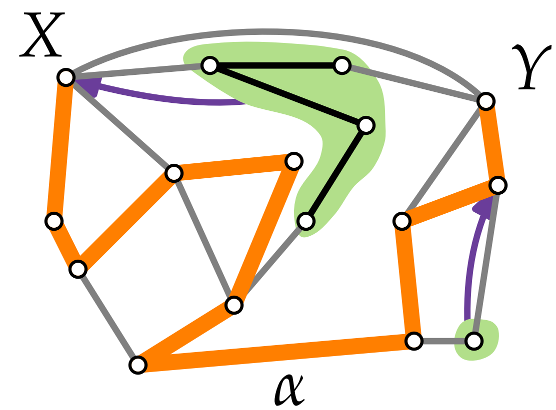

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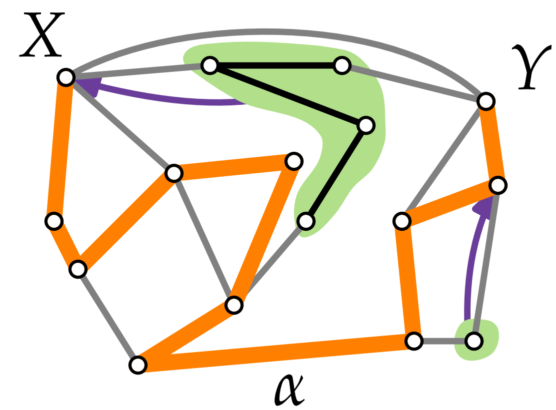

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G 2-conn., X, ~~Y~~, α on outer face $\Rightarrow$ Tutte path

[Sanders ’96]
G 2-conn., ~~X~~, ~~Y~~, α on outer face $\Rightarrow$ Tutte path

[Gao, Richter & Yu ’95, ’06]
int. T_{int}
G 3-conn., X, Y, α on outer face $\Rightarrow$ ~~T_{SDR}~~-path

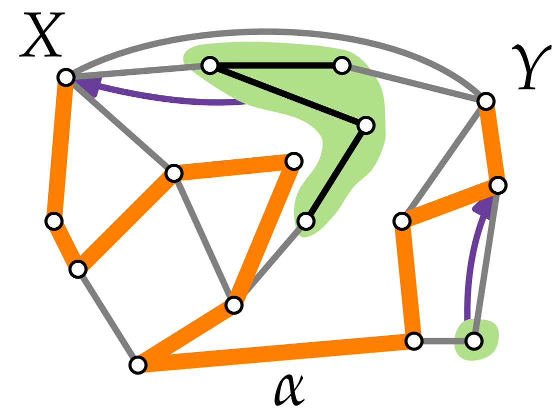
…in $O(n)$ time

[Chiba & Nishizeki ’89] (= Hamil. path)
G 4-conn. $\Rightarrow$ Tutte path in $O(n)$ time

[Schmid & Schmidt ’15]
…in $O(n^2)$ time

[Schmid & Schmidt ’18]
…in $O(n^2)$ time

What is known?



[Tutte '77]
G 2-conn., X, Y, α on outer face $\Rightarrow$ Tutte path

[Thomassen '83]
G 2-conn., X, ~~Y~~, α on outer face $\Rightarrow$ Tutte path

[Sanders '96]
G 2-conn., ~~X~~, ~~Y~~, α on outer face $\Rightarrow$ Tutte path

[Gao, Richter & Yu '95, '06]
G (int.) 3-conn., X, Y, α on outer face $\Rightarrow$ ~~T_{SDR}~~ T_{int}-path

. . . in $O(n)$ time

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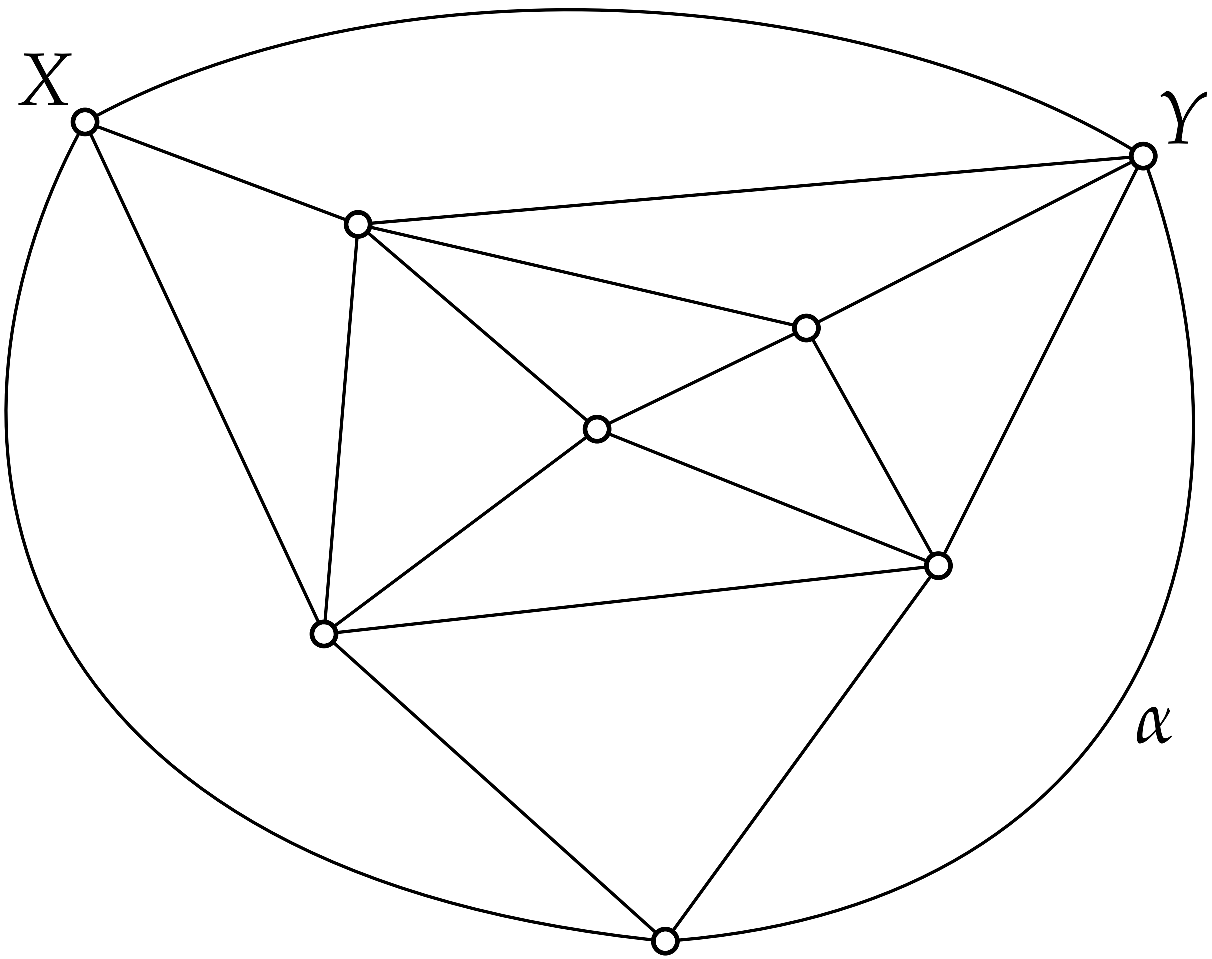
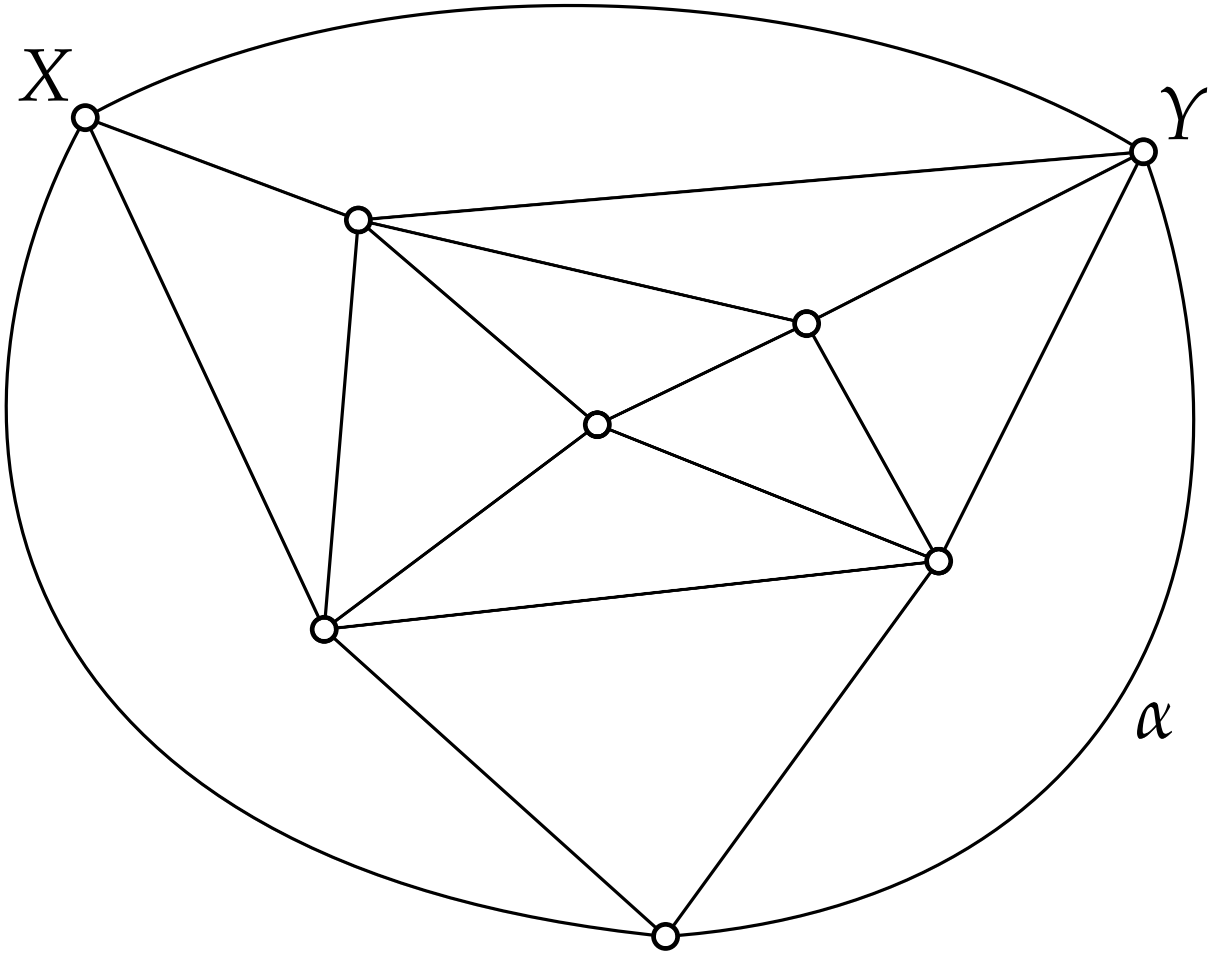
Triangulated Graphs

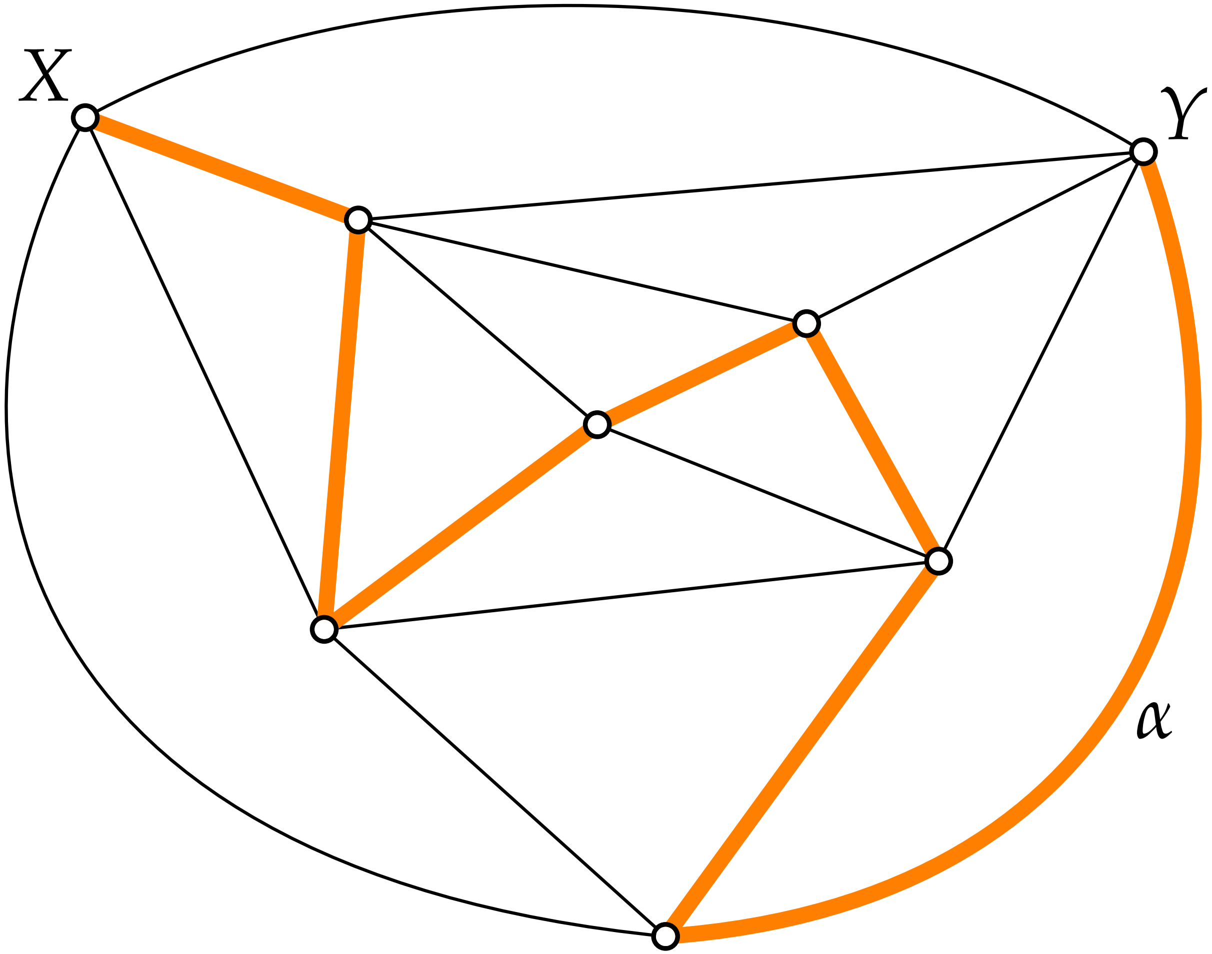


Triangulated Graphs



[Asano, Kikuchi & Saito '85]
4-conn. triangulation ⇒ Hamiltonian path in $O(n)$ time.

Triangulated Graphs



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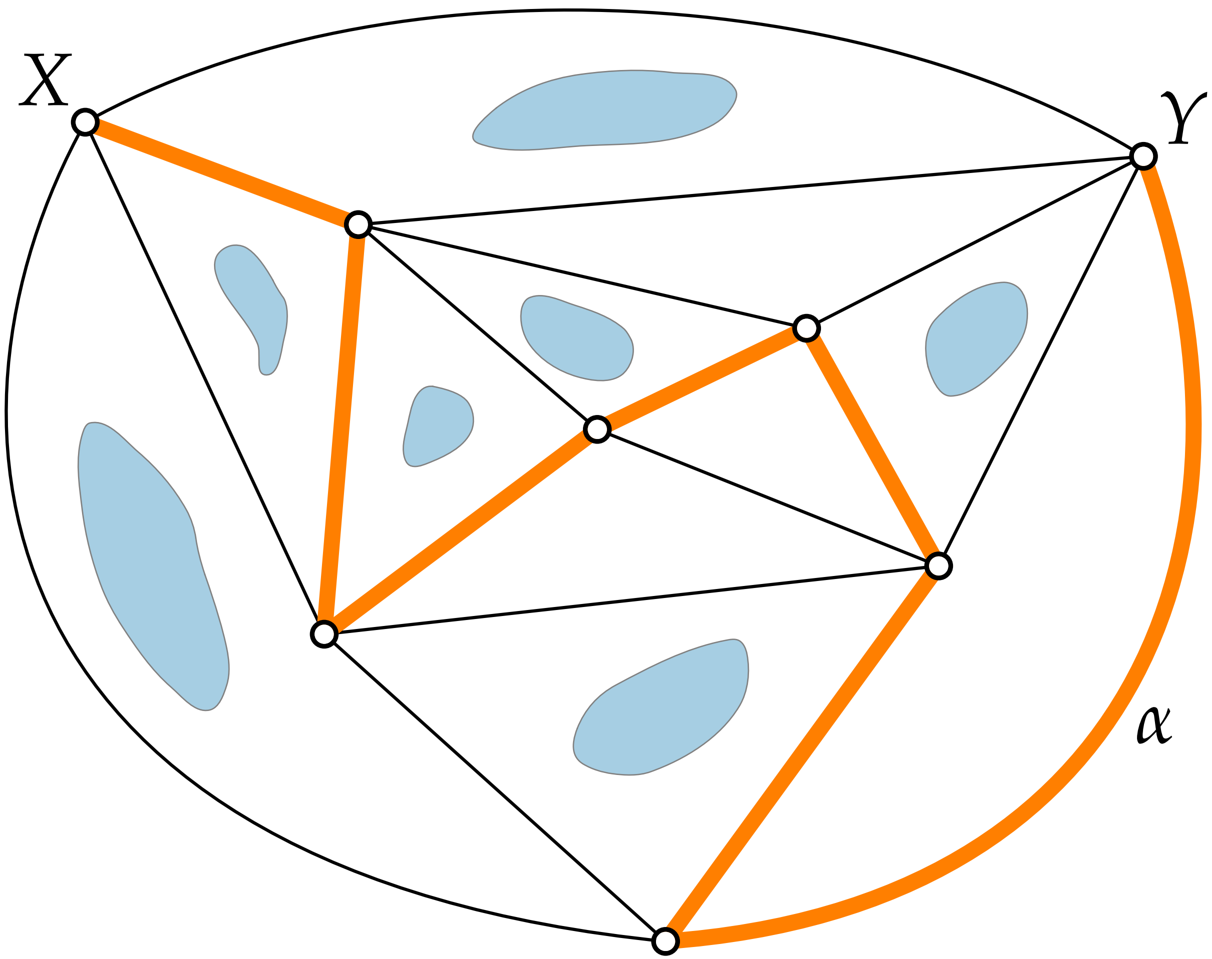

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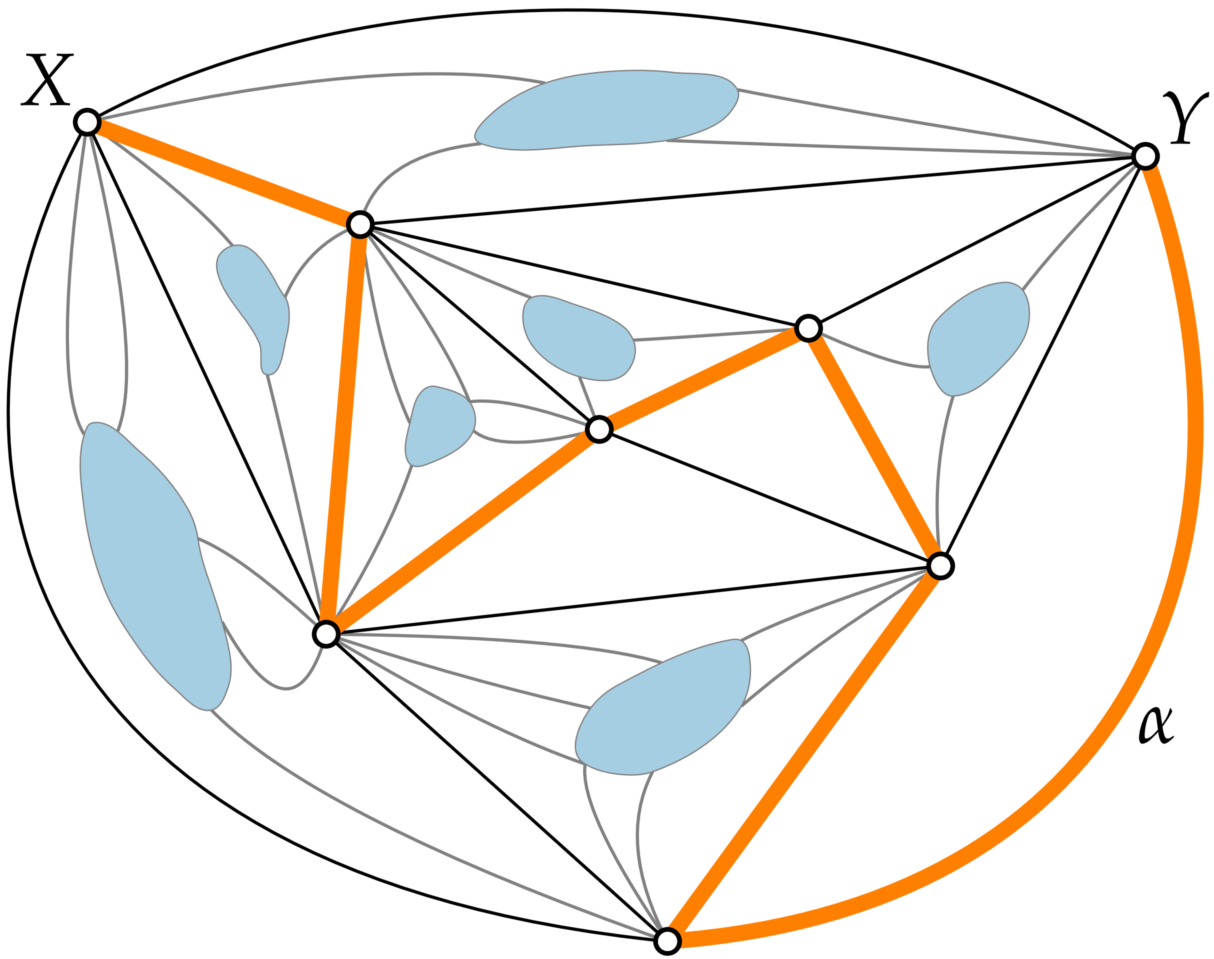

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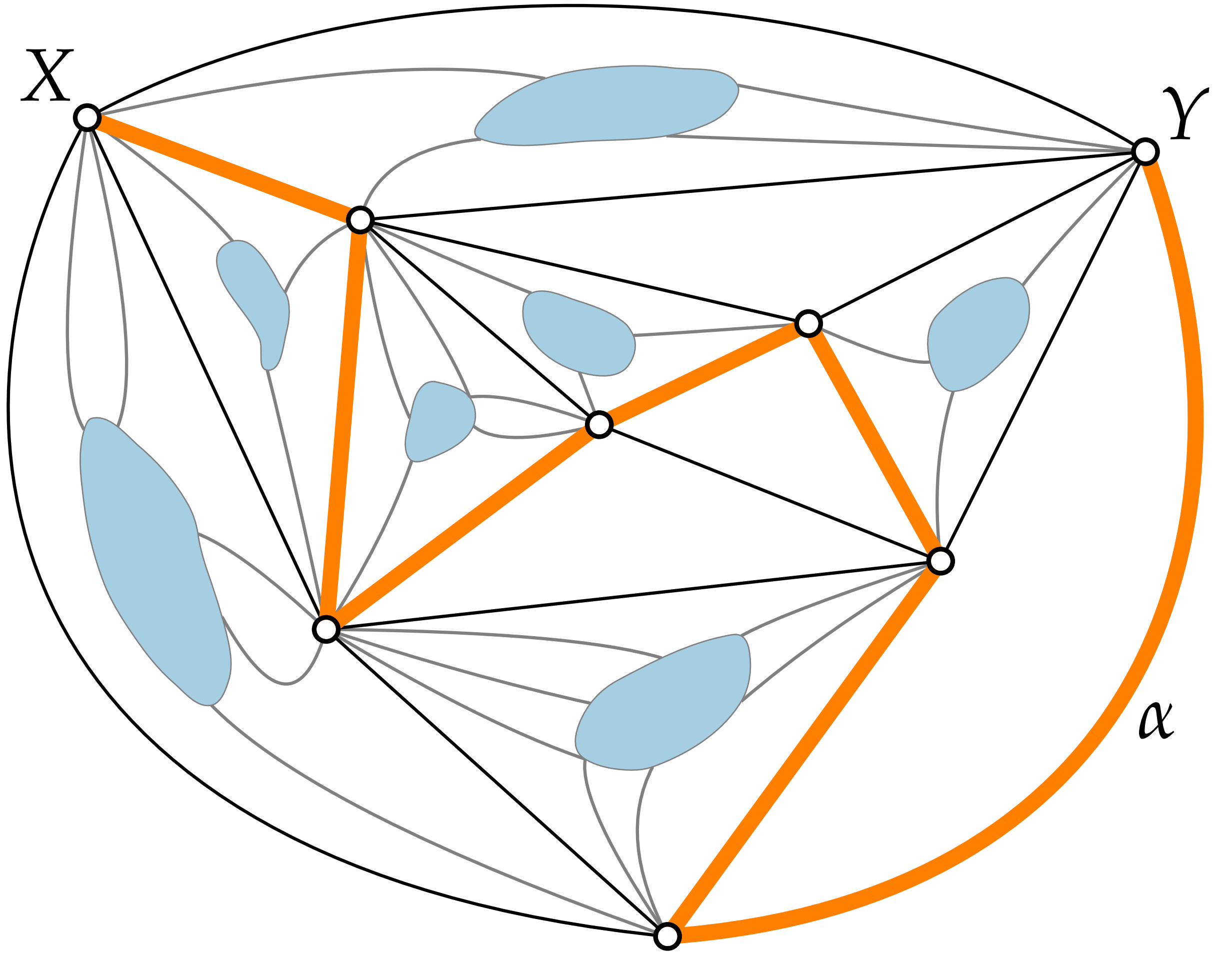

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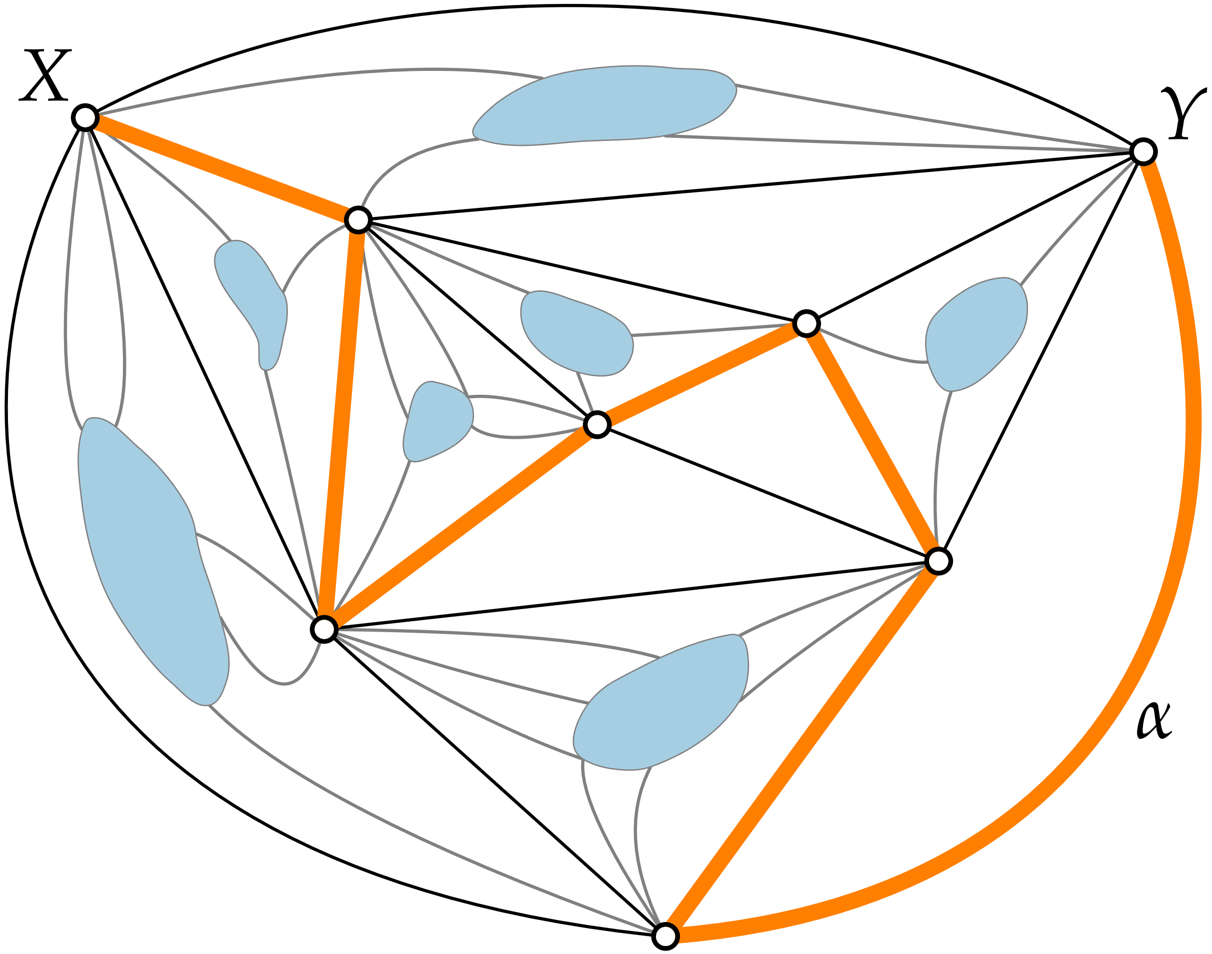

k vertices

[Asano, Kikuchi & Saito '85]

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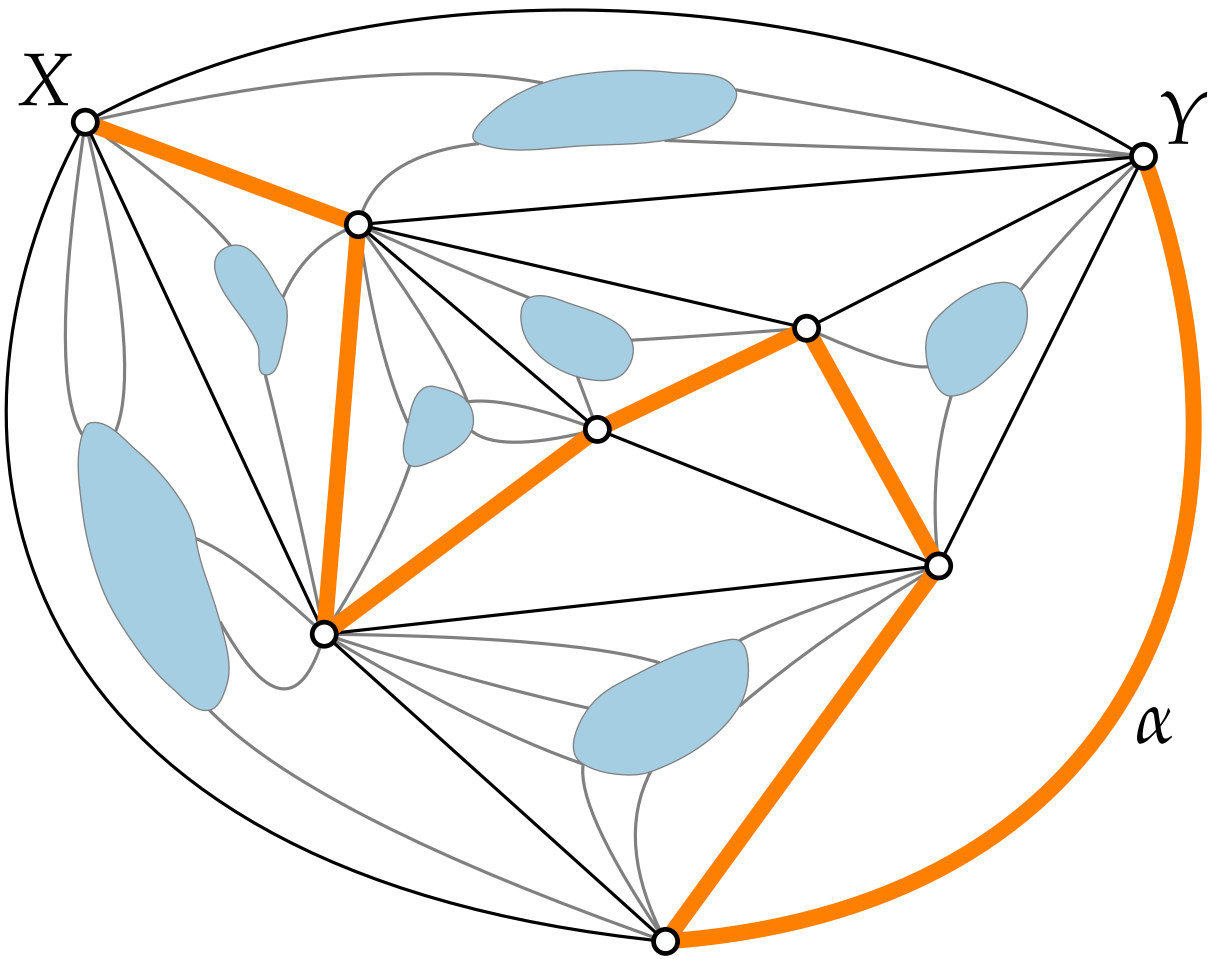

k vertices
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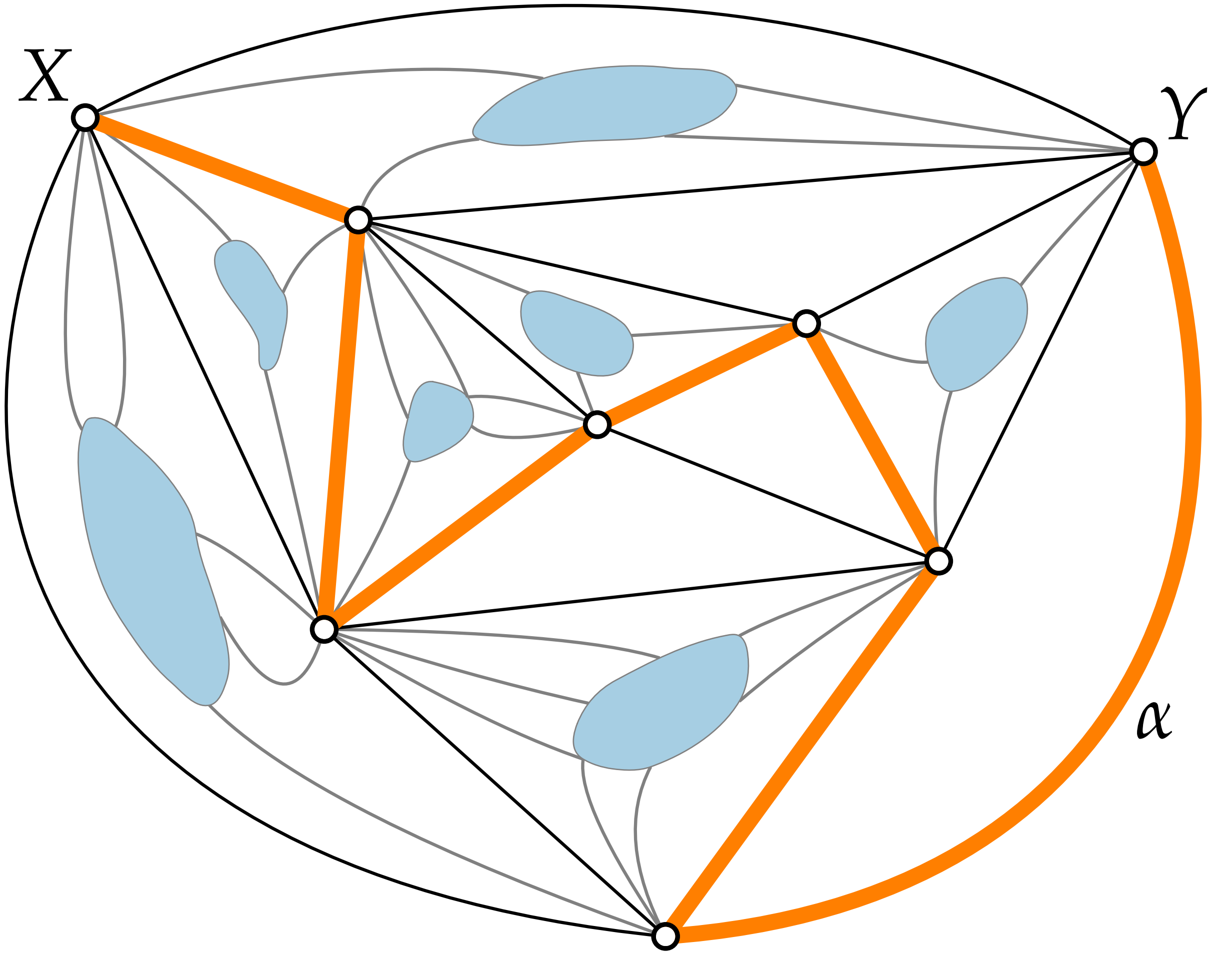

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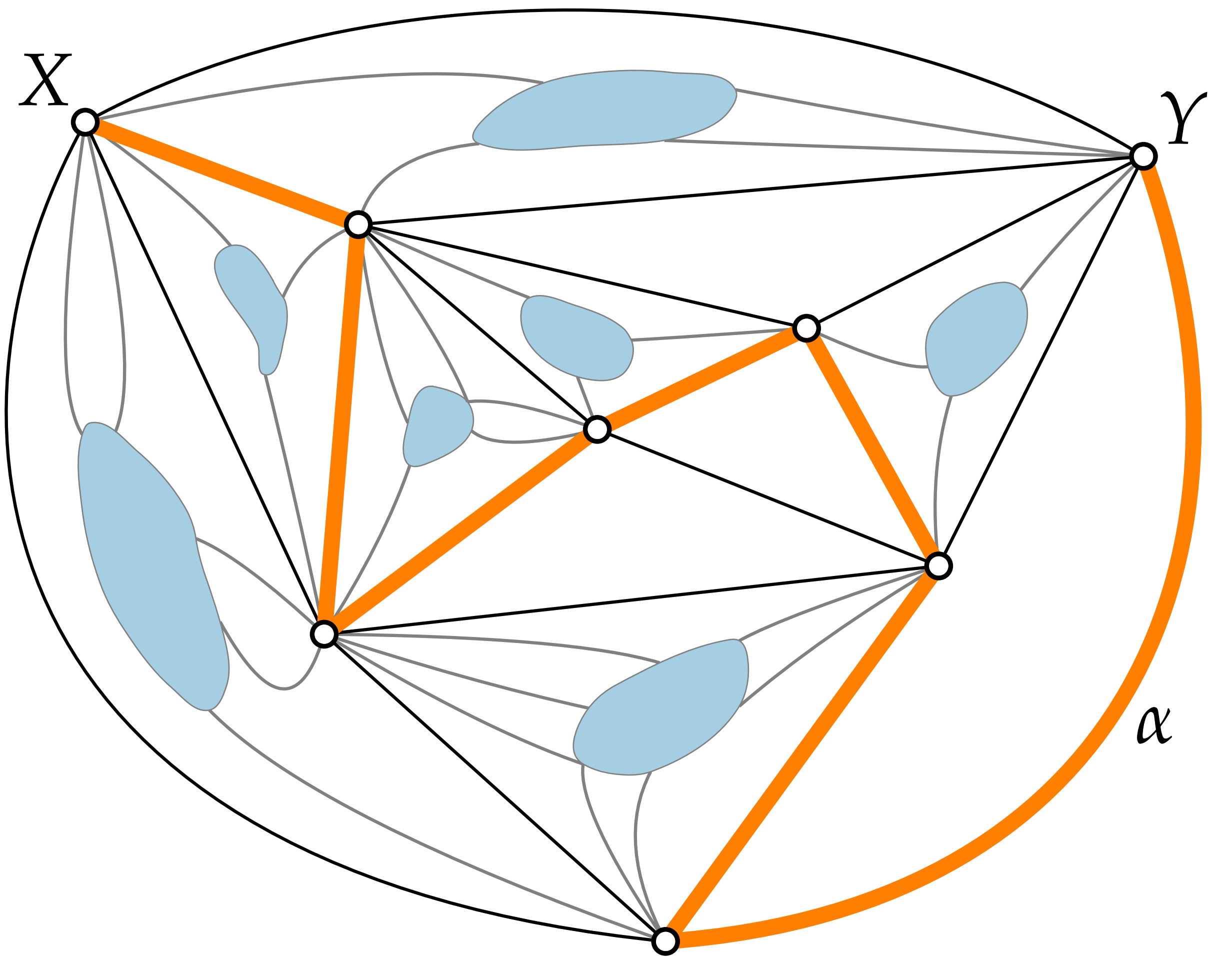

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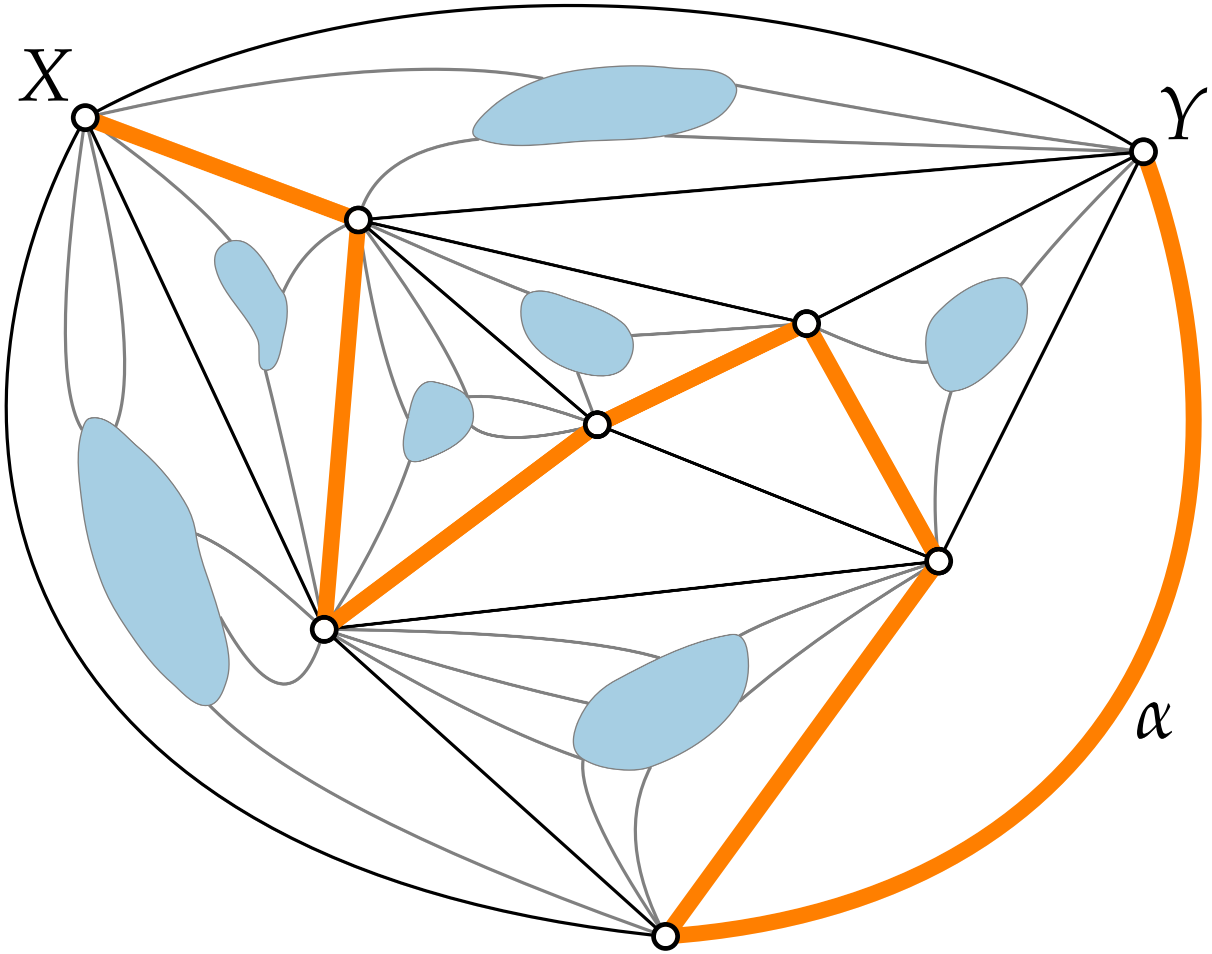

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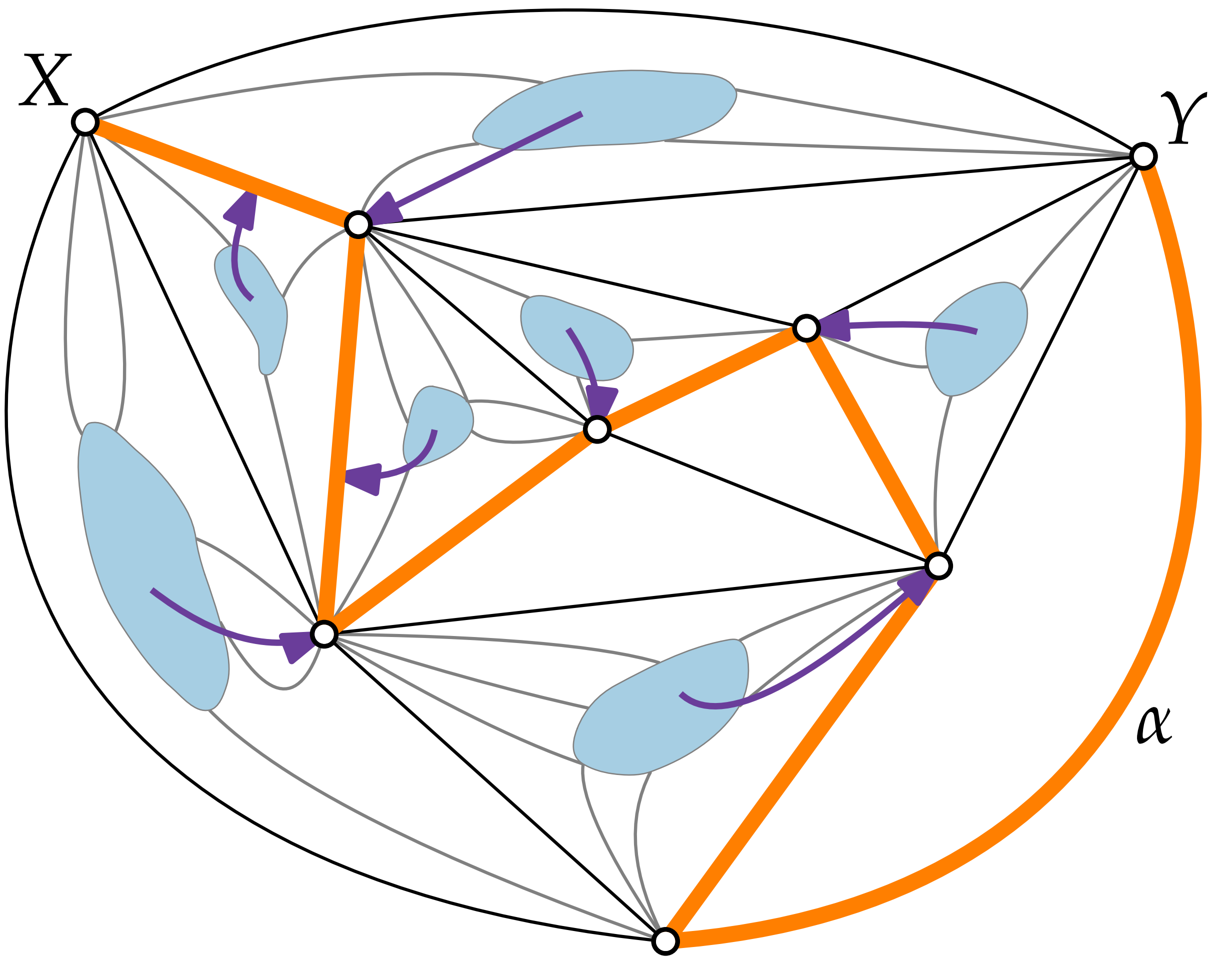

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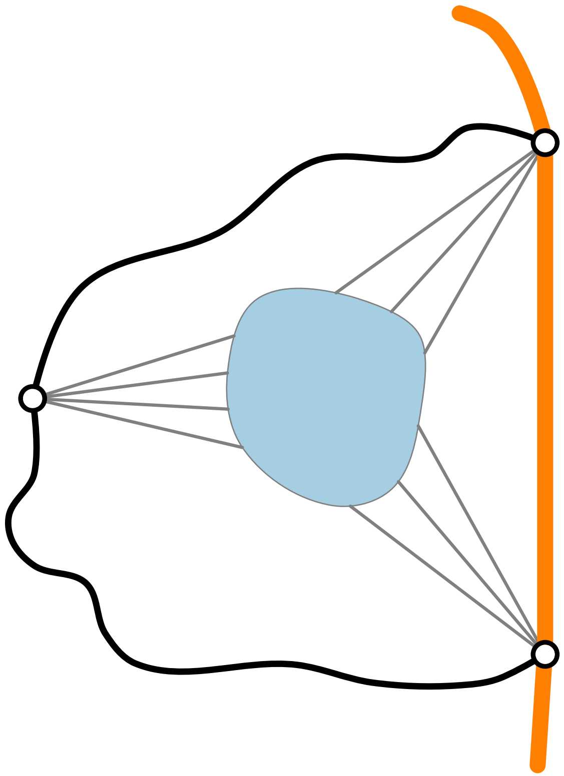
Substitution Trick



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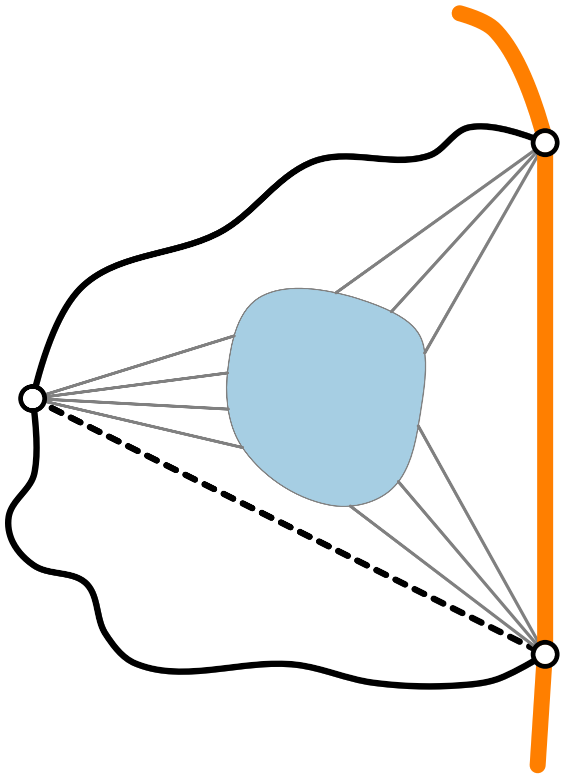

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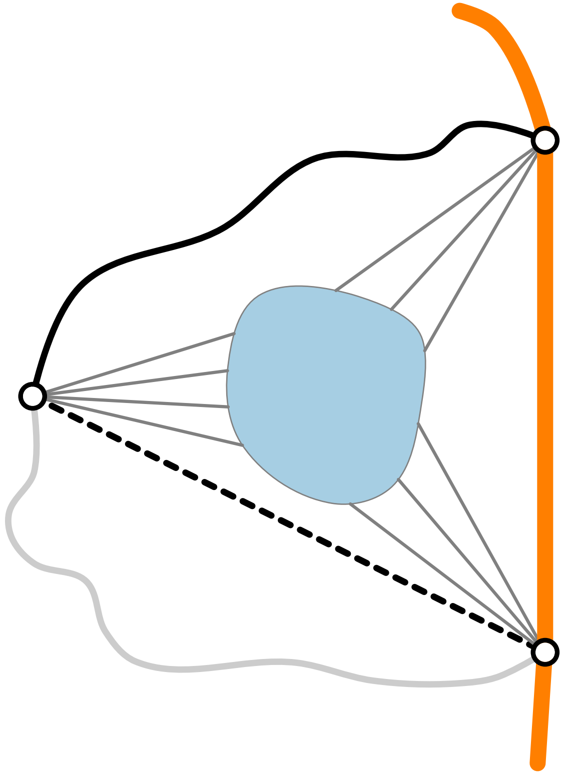

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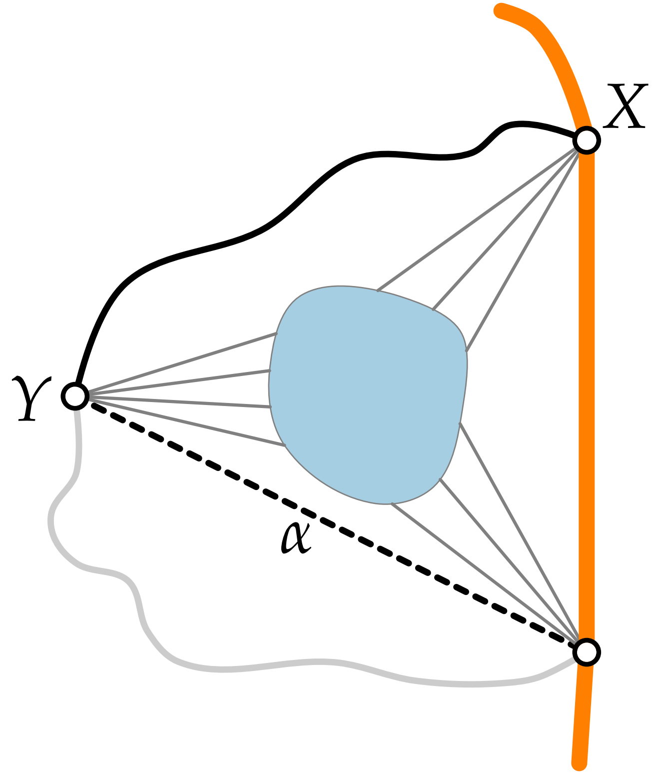

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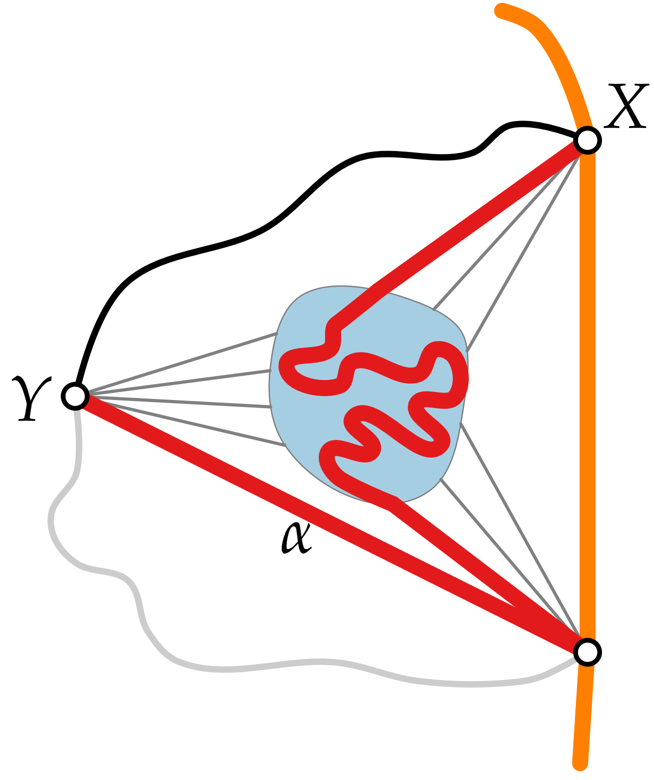

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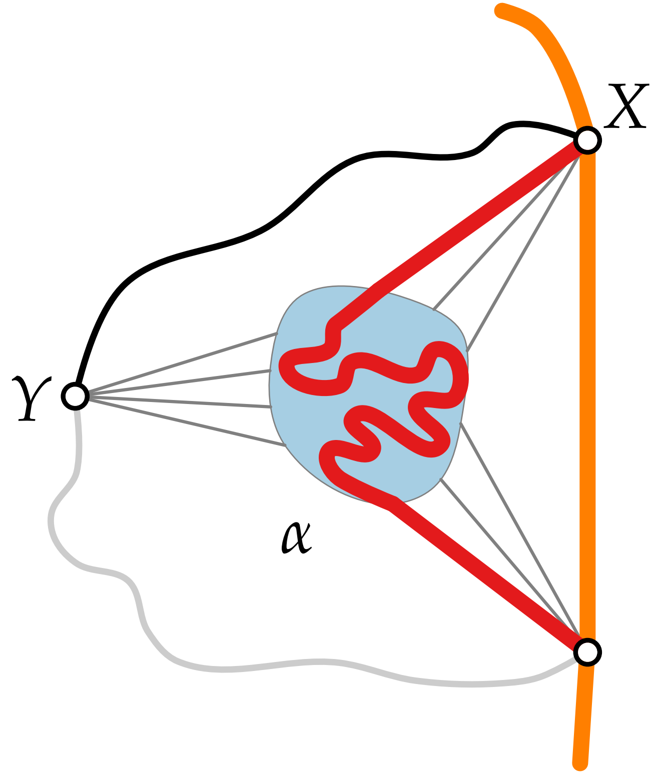

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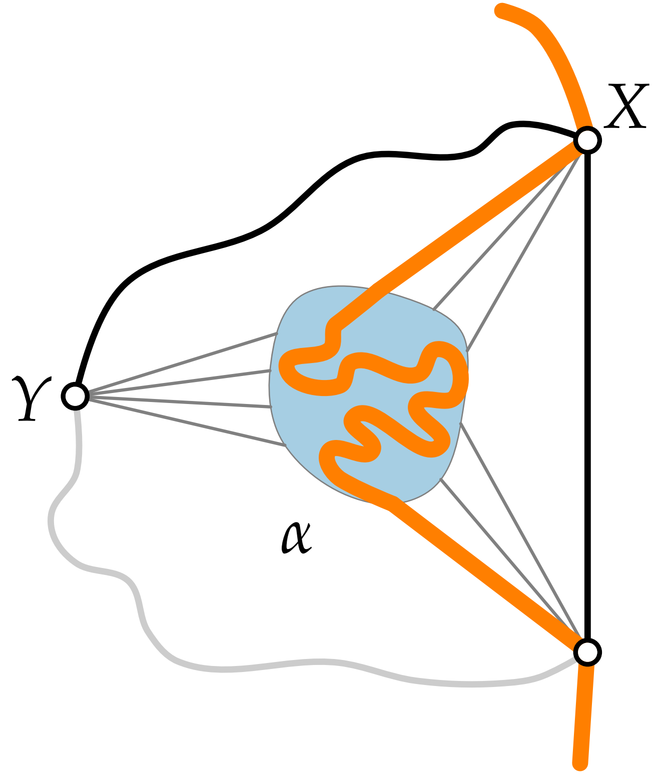

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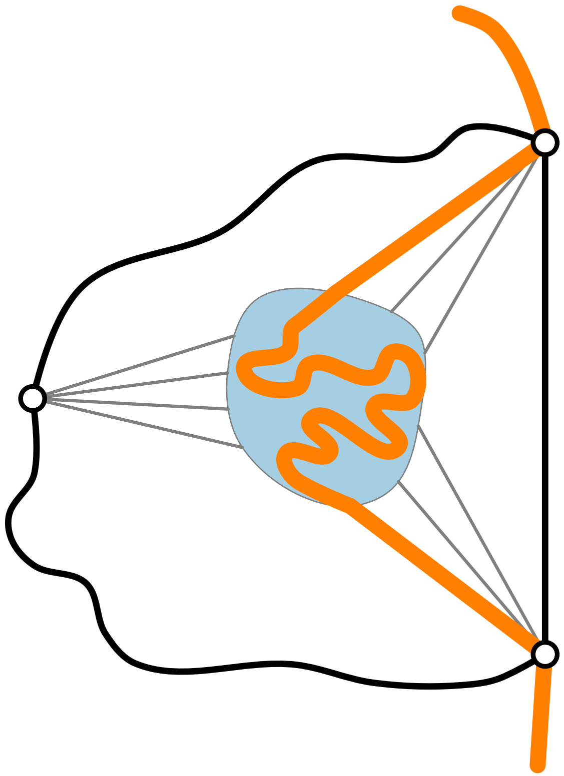
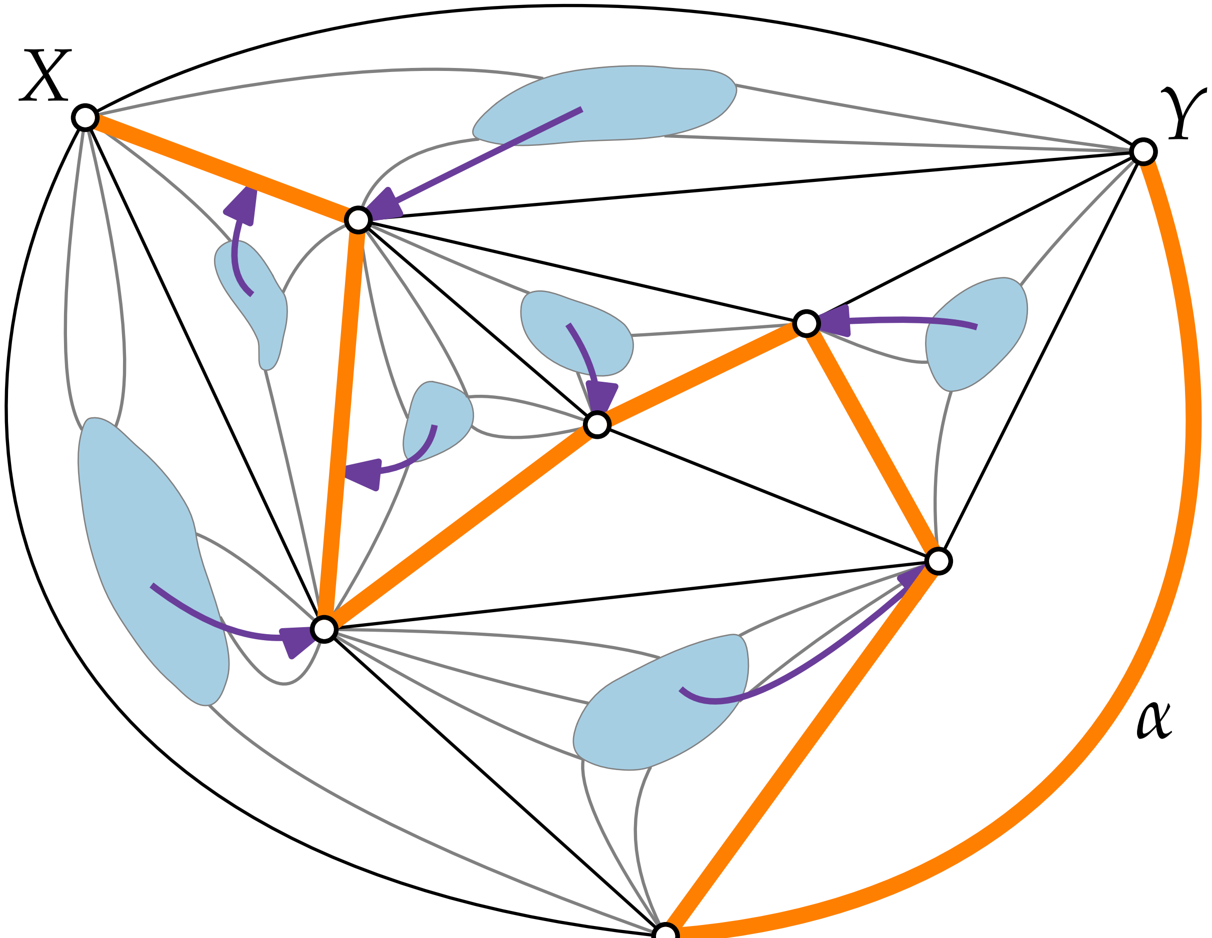


Triangulated graphs



k vertices
$2k - 5$ faces
$k - 2$ edges in $P - \alpha$
$k - 3$ int. vtcs

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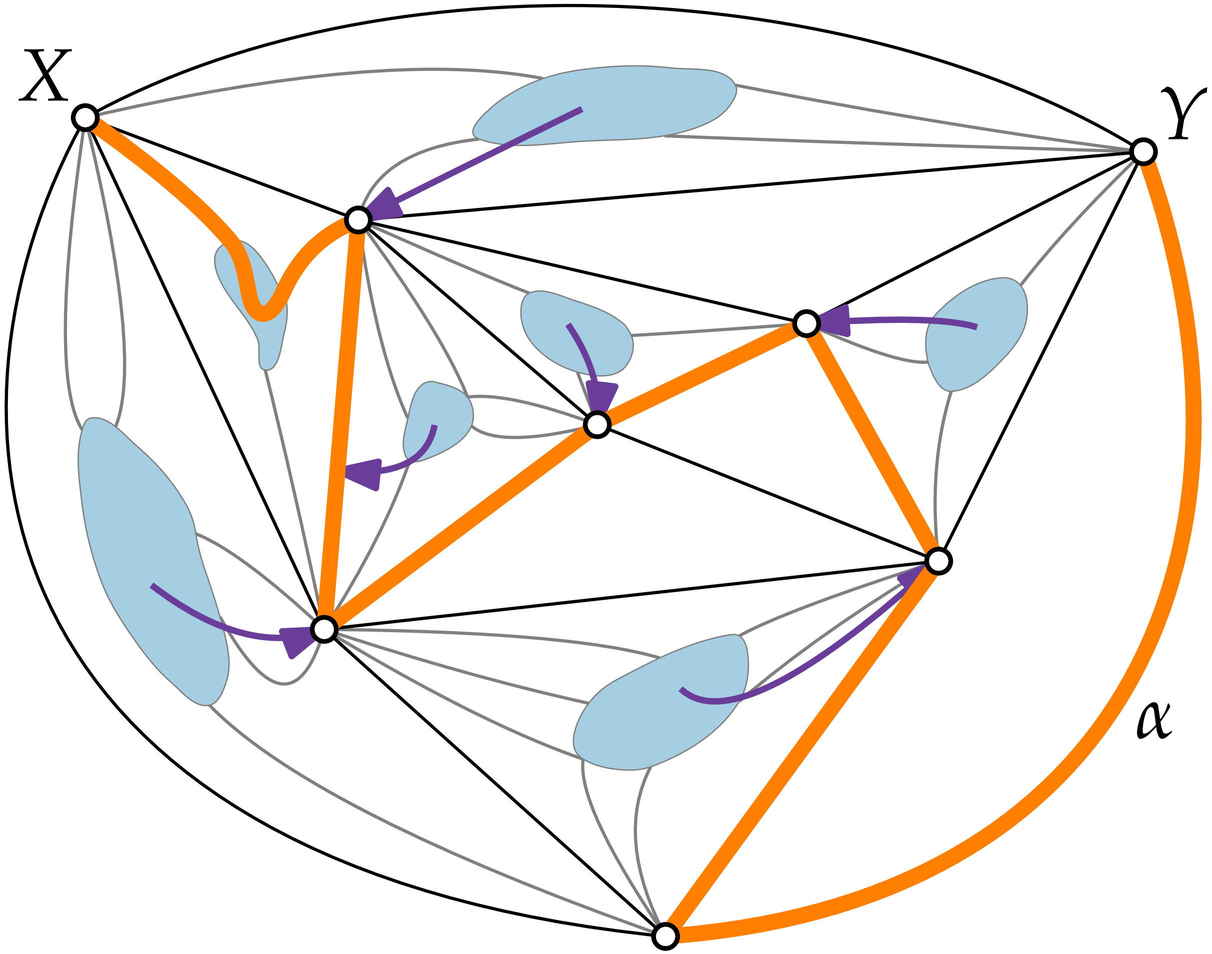

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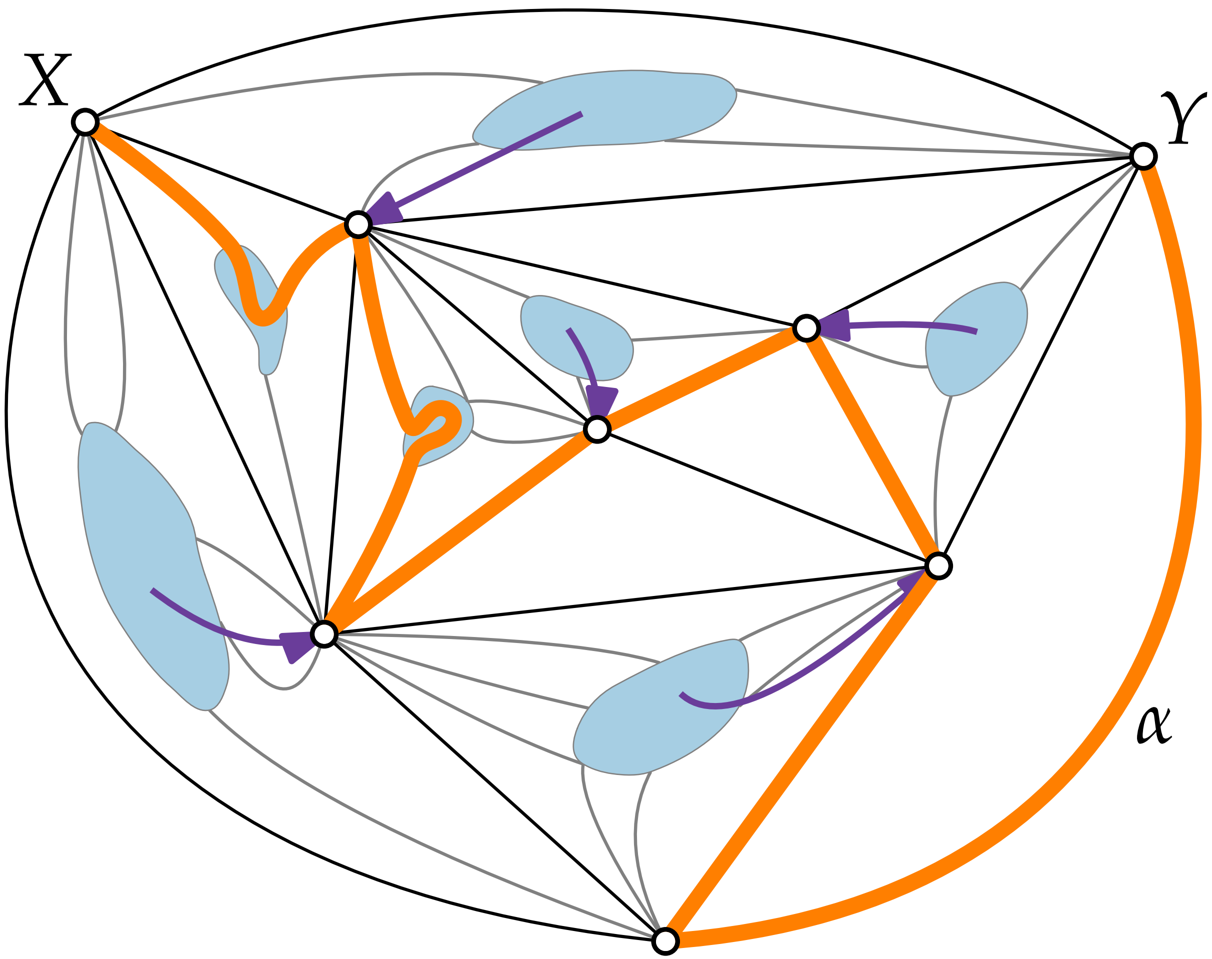

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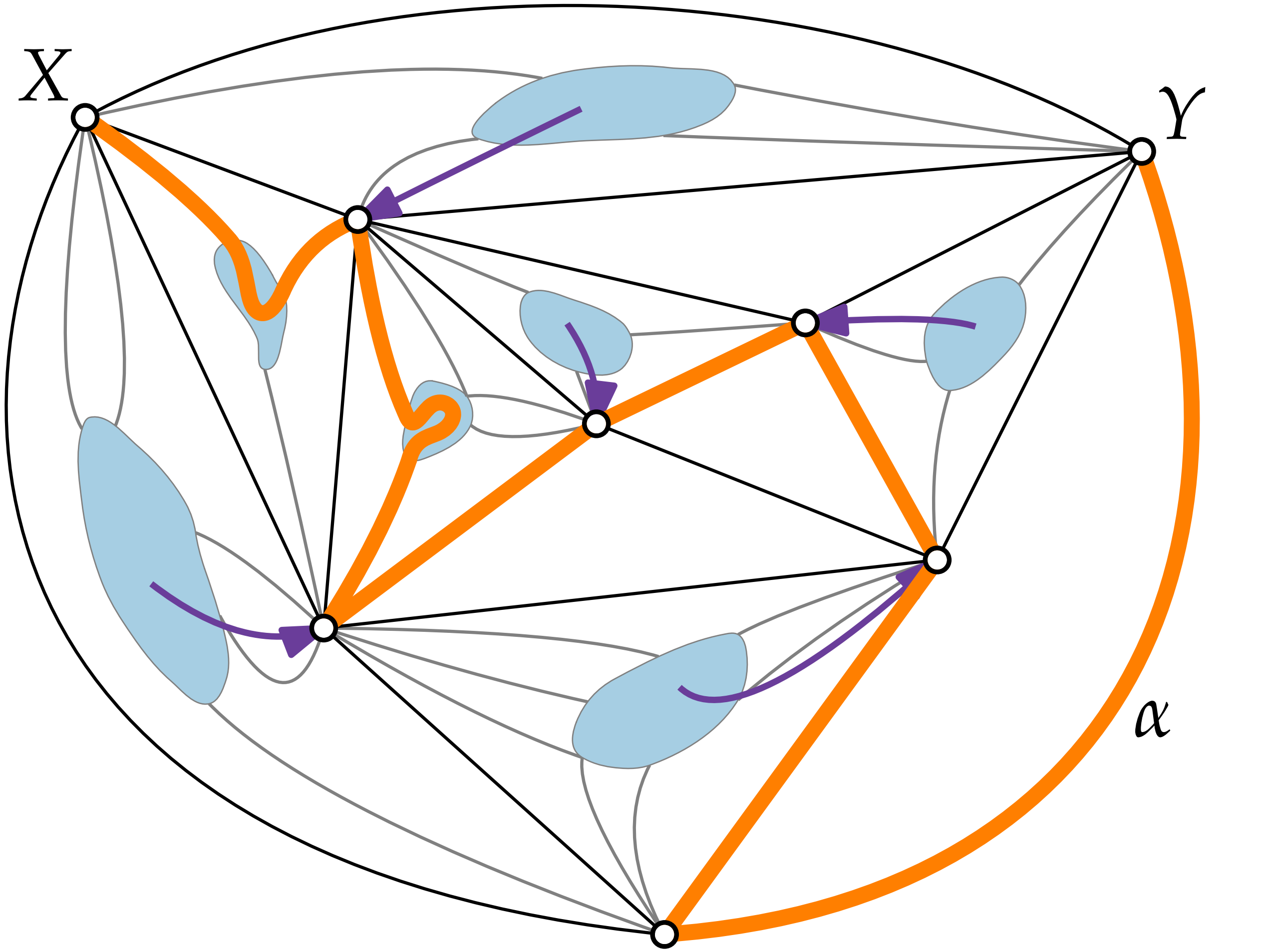

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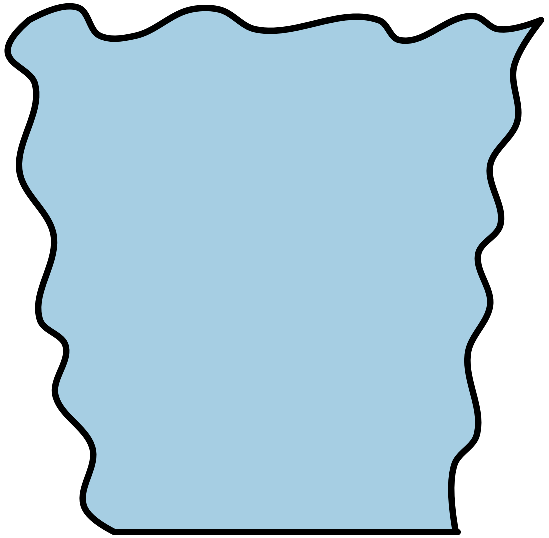
Corner-3-connectivity

int. 3-conn.



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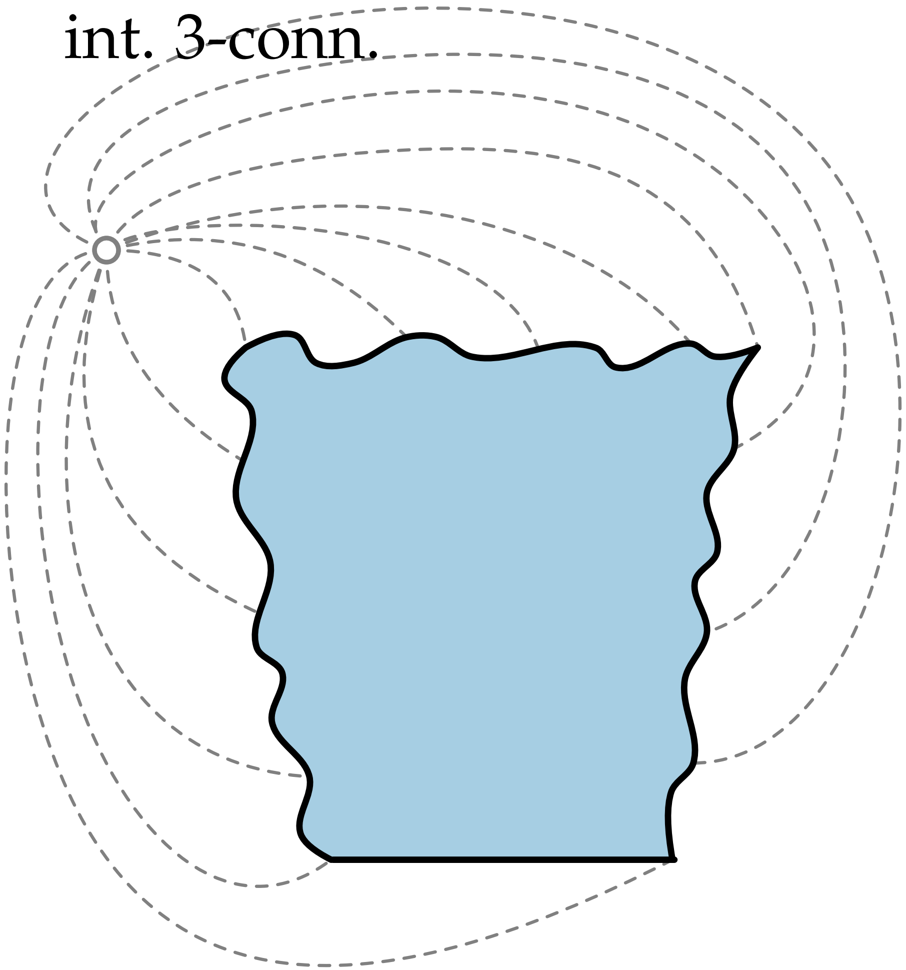

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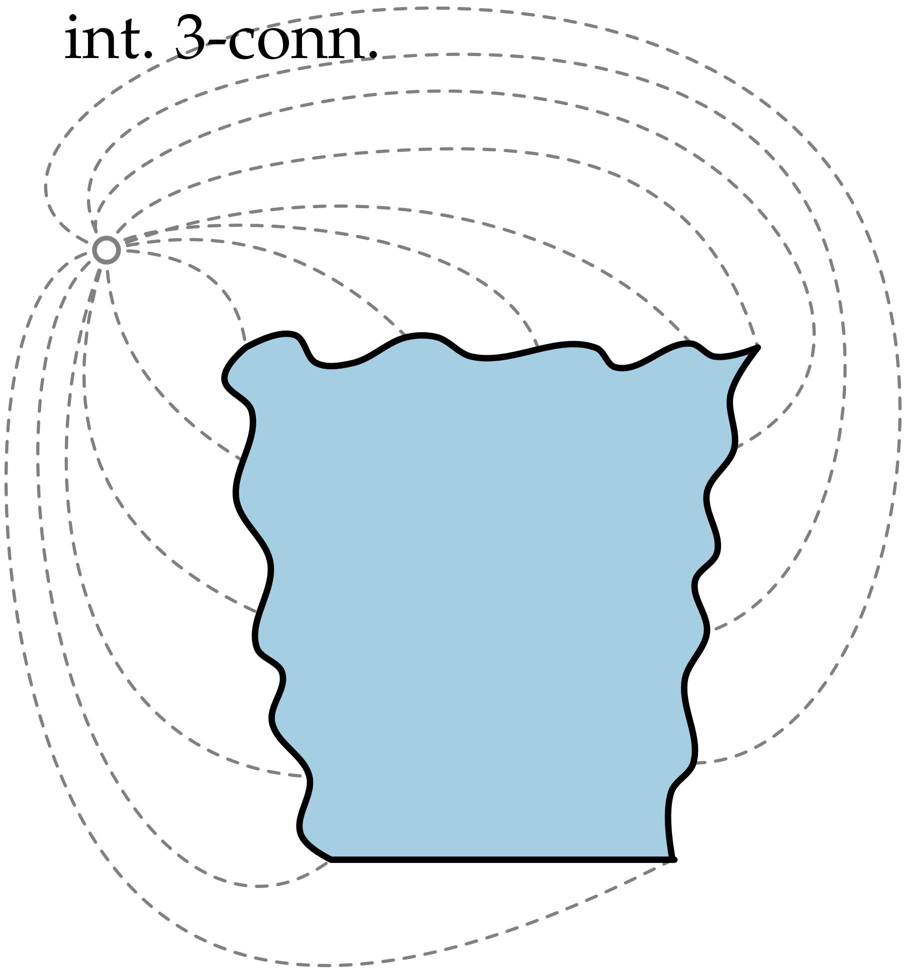

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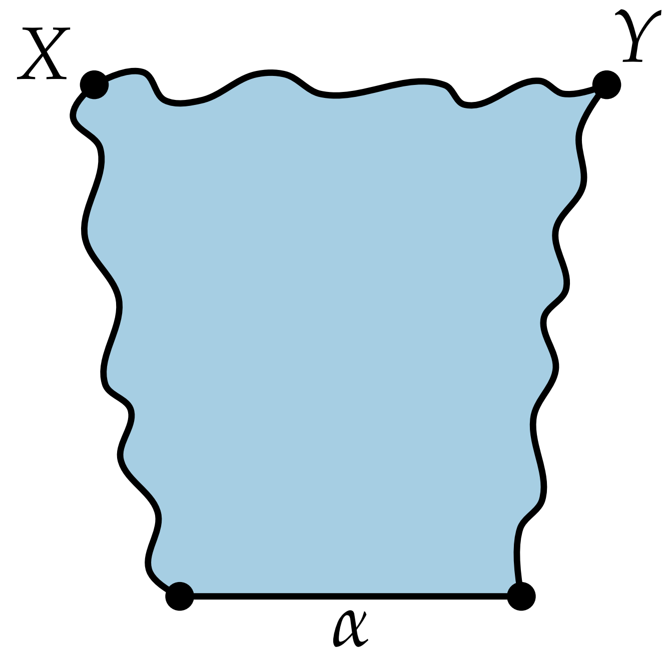

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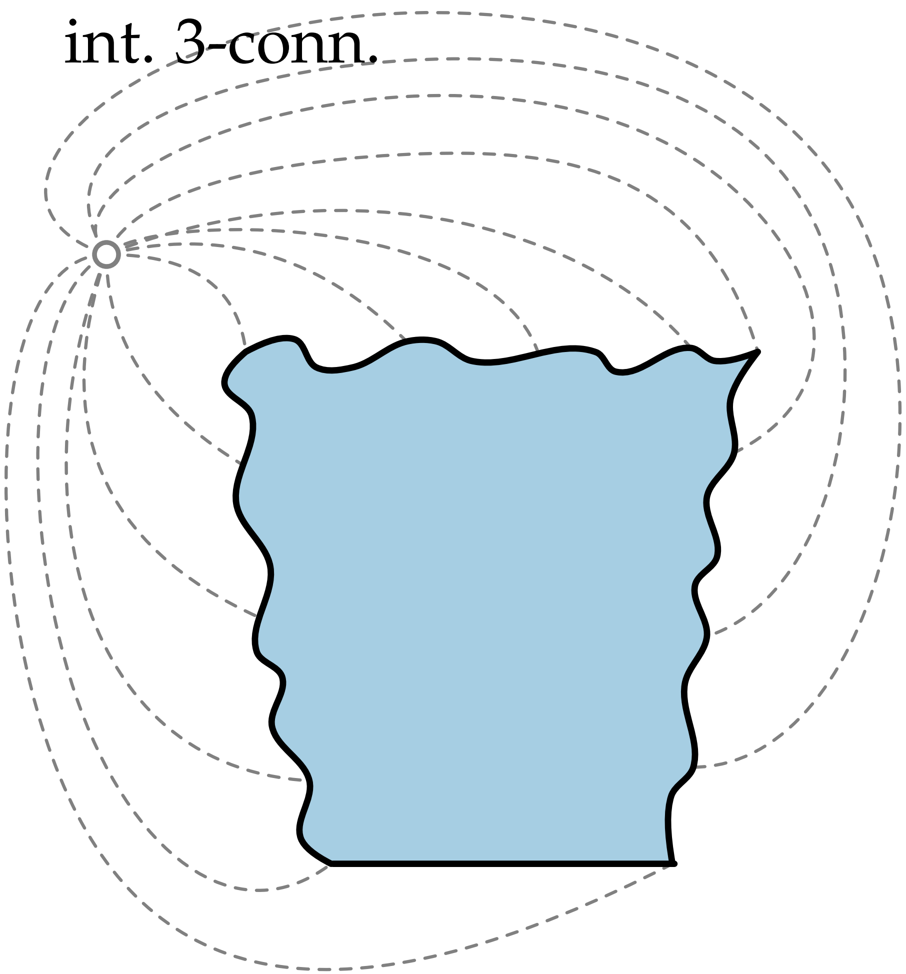

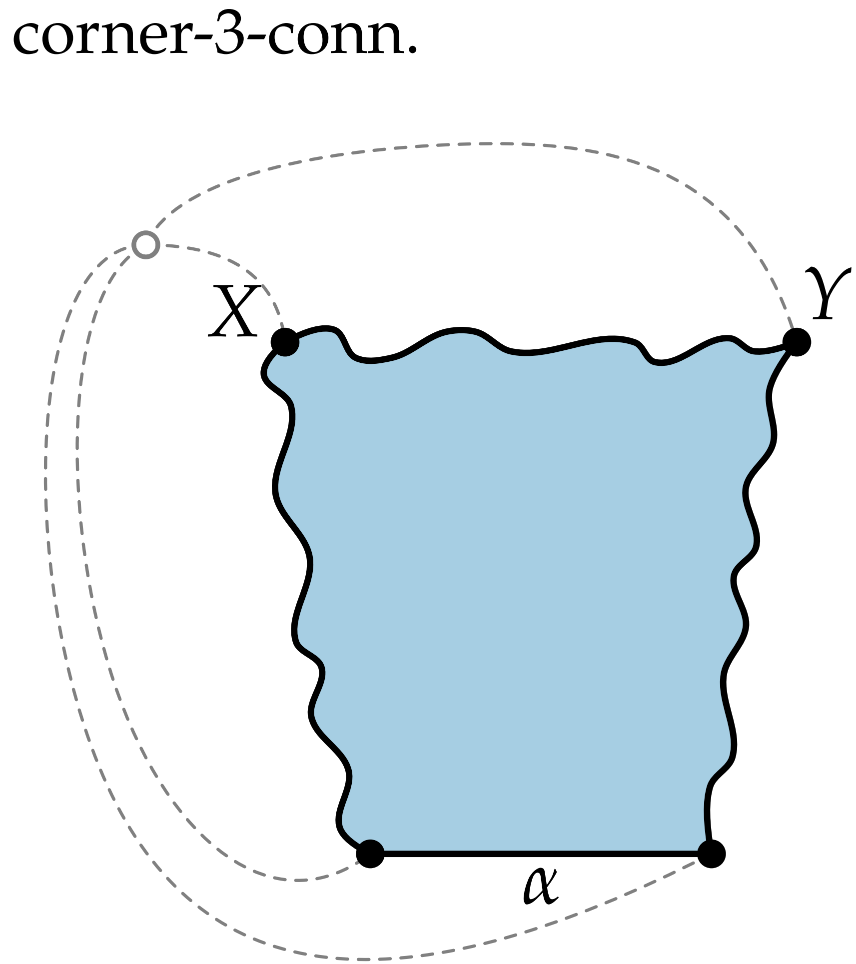

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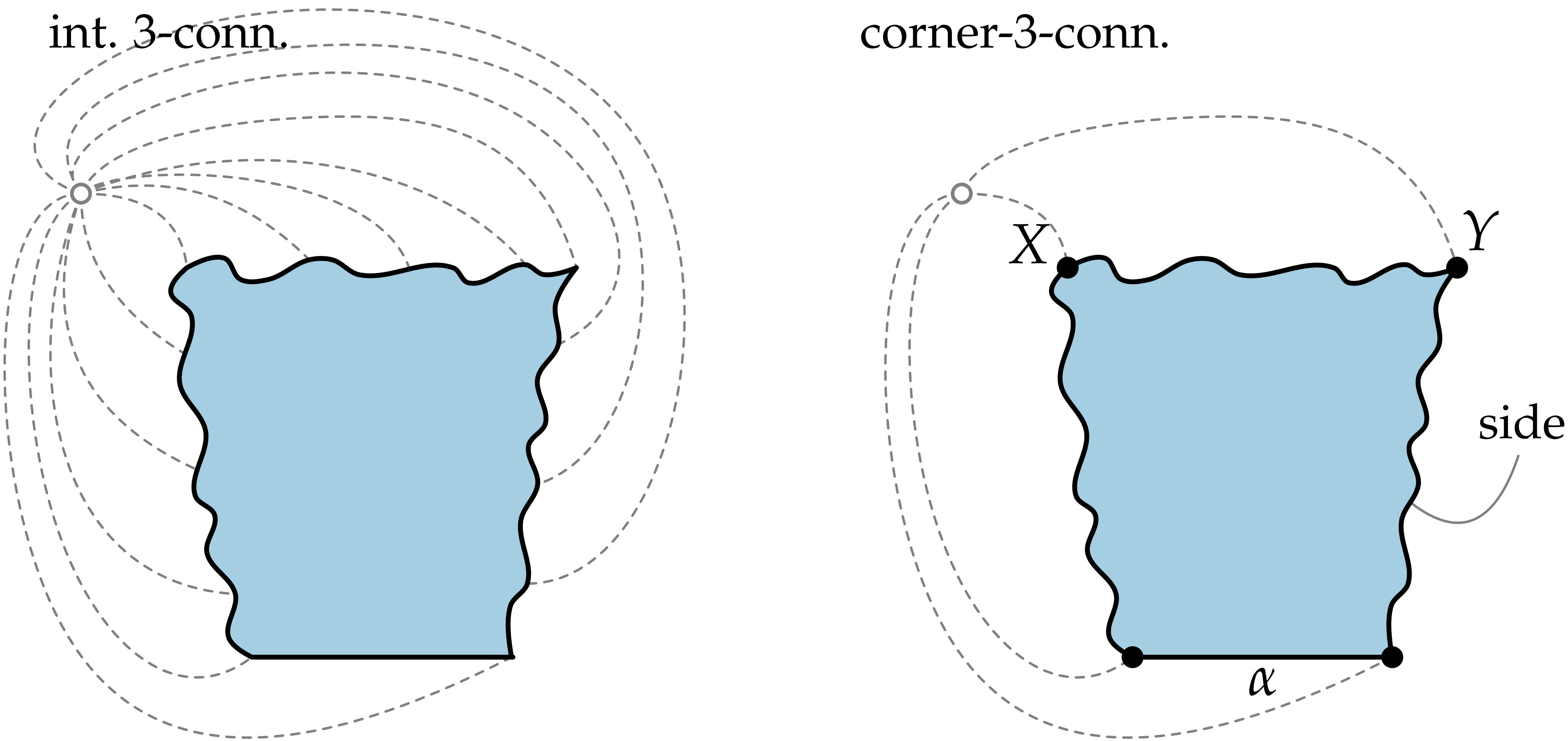

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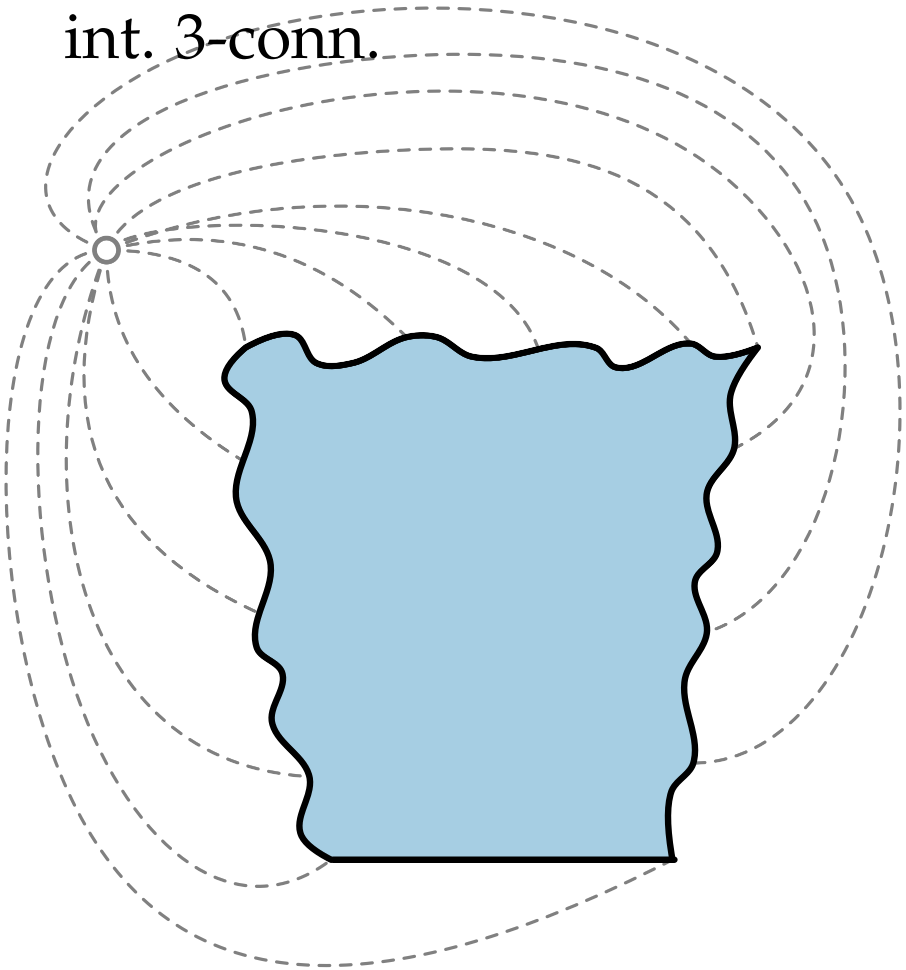

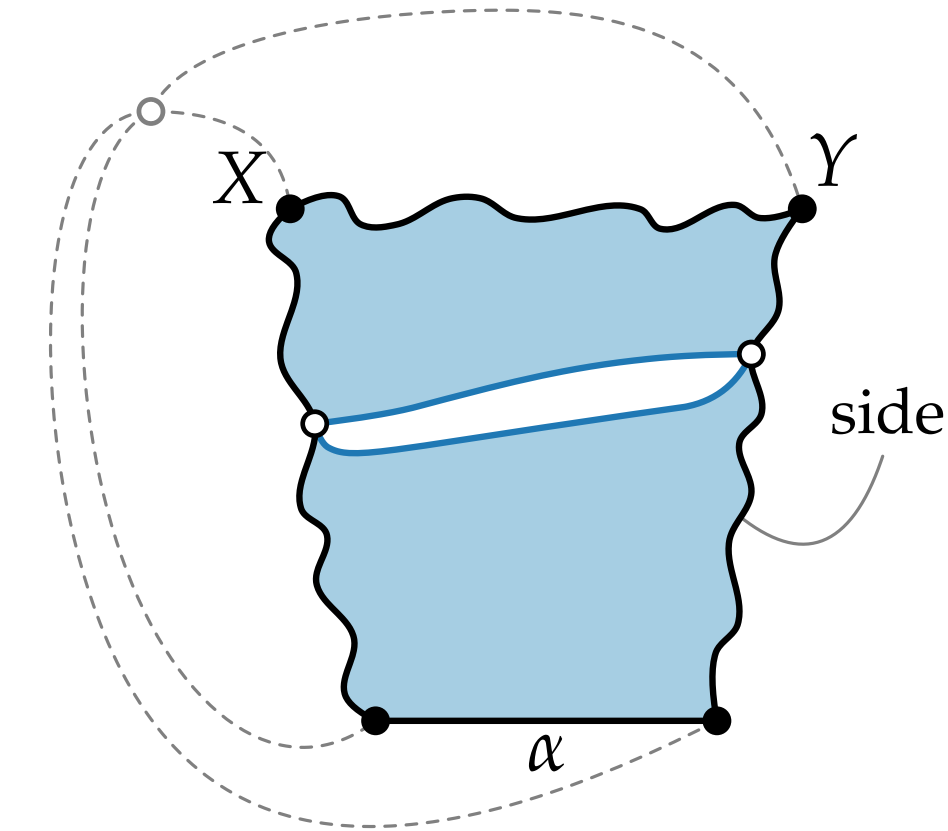

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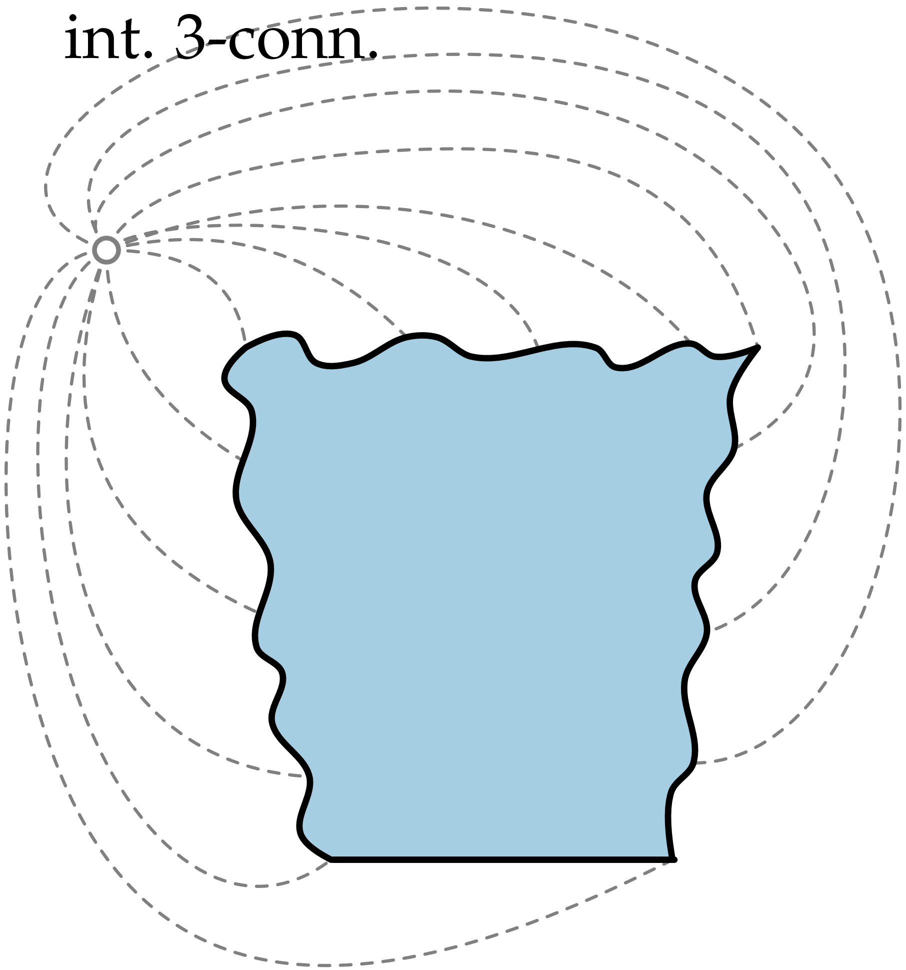

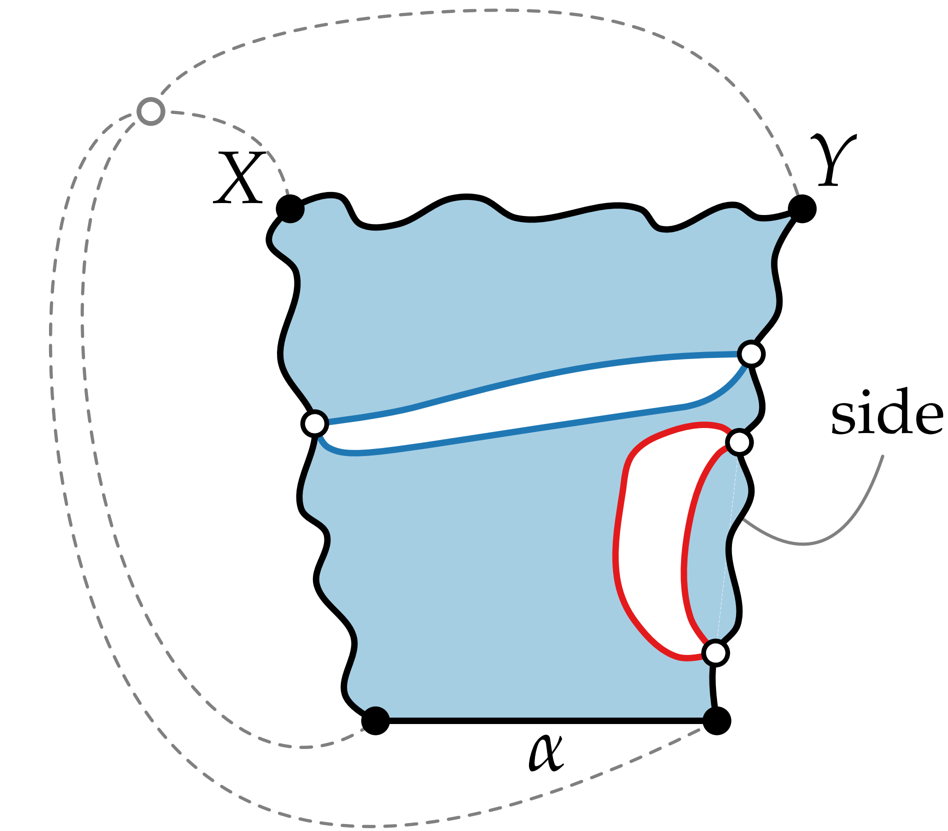

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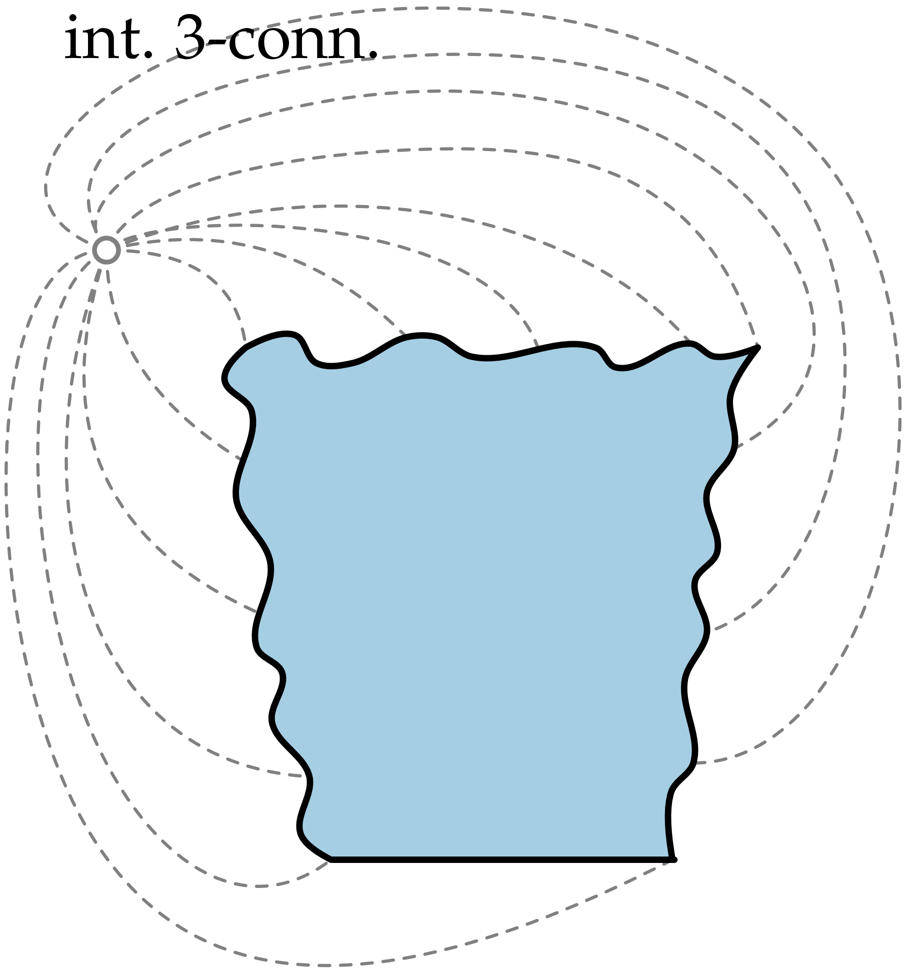

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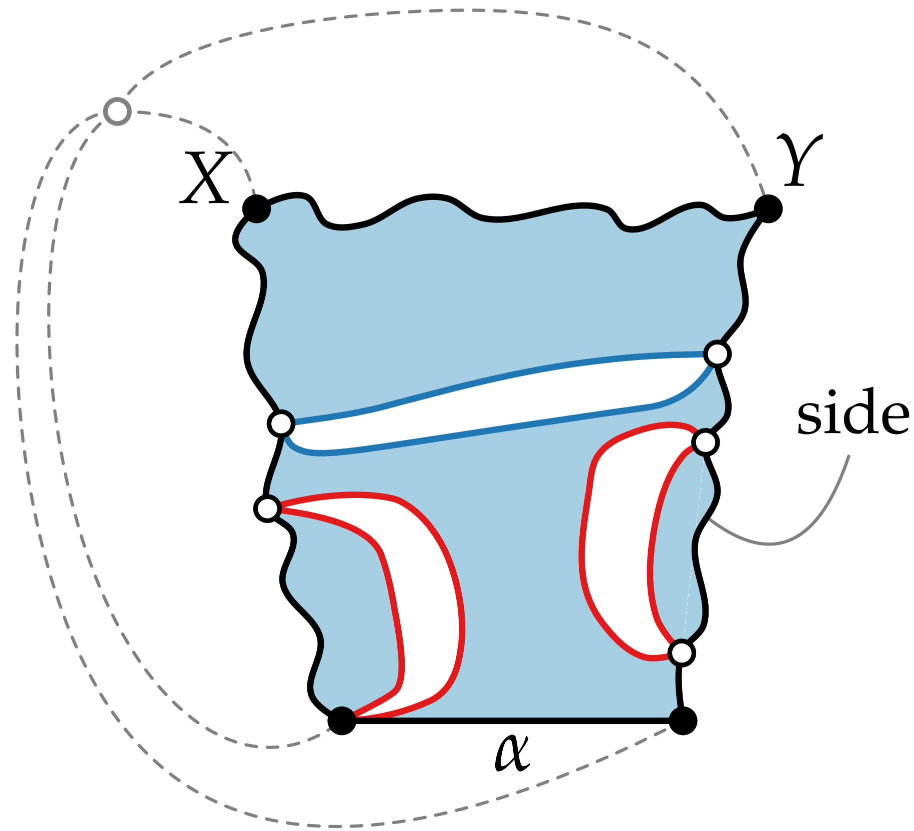

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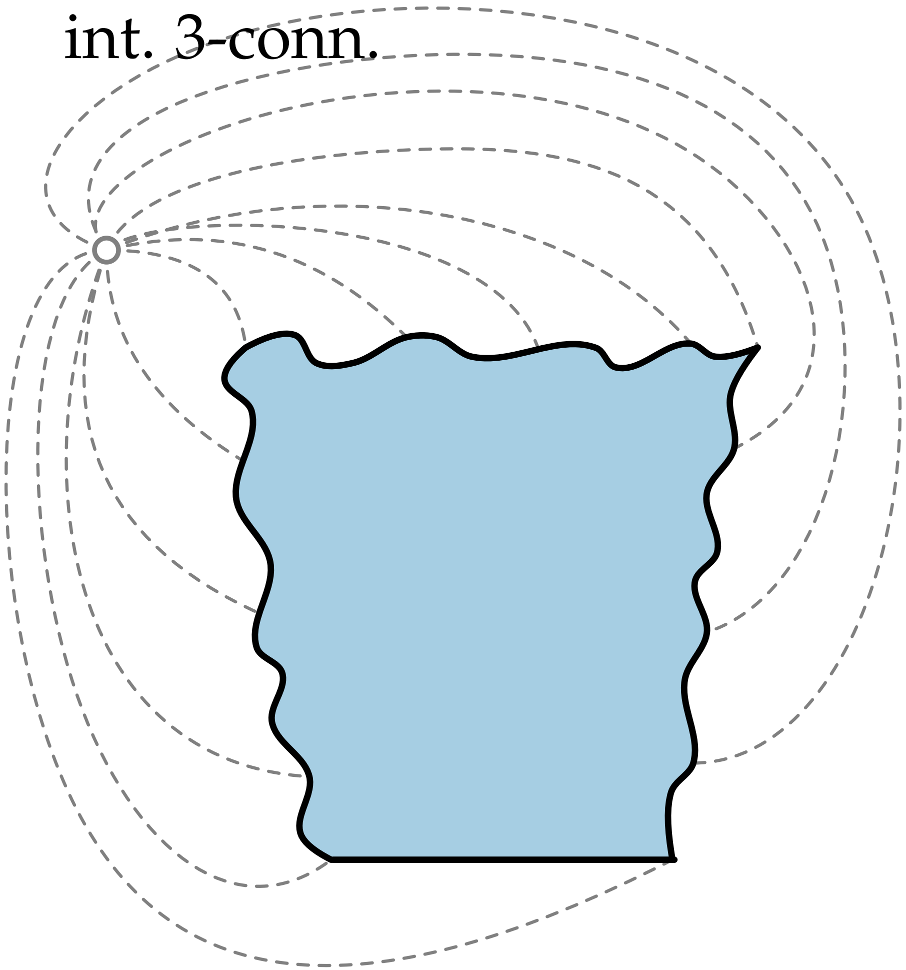

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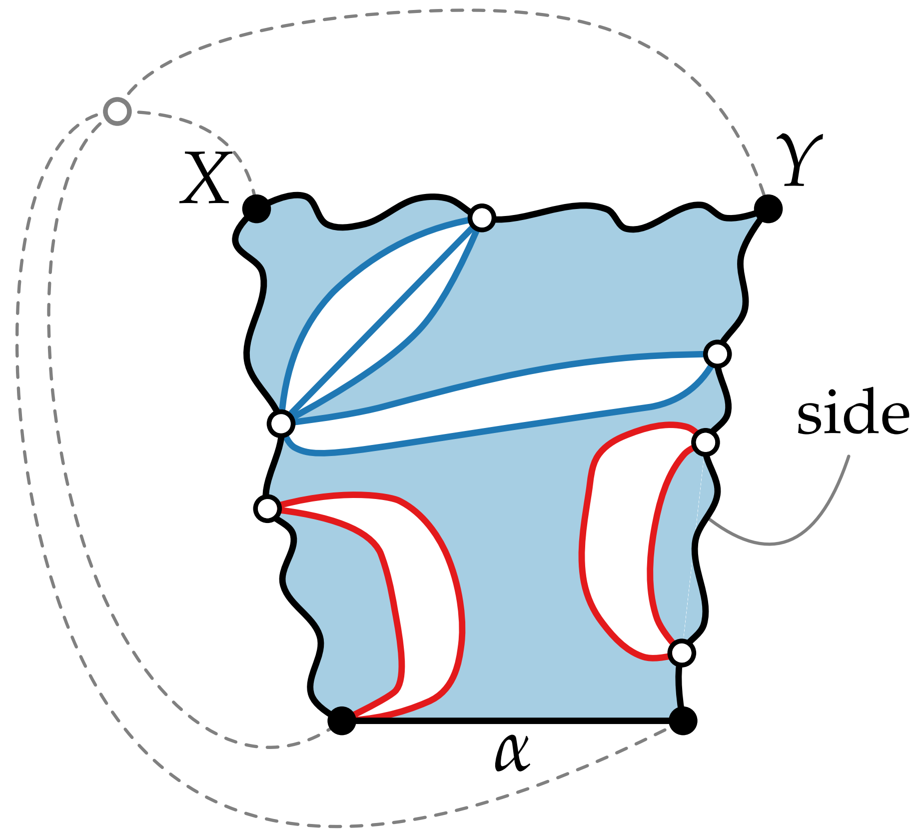

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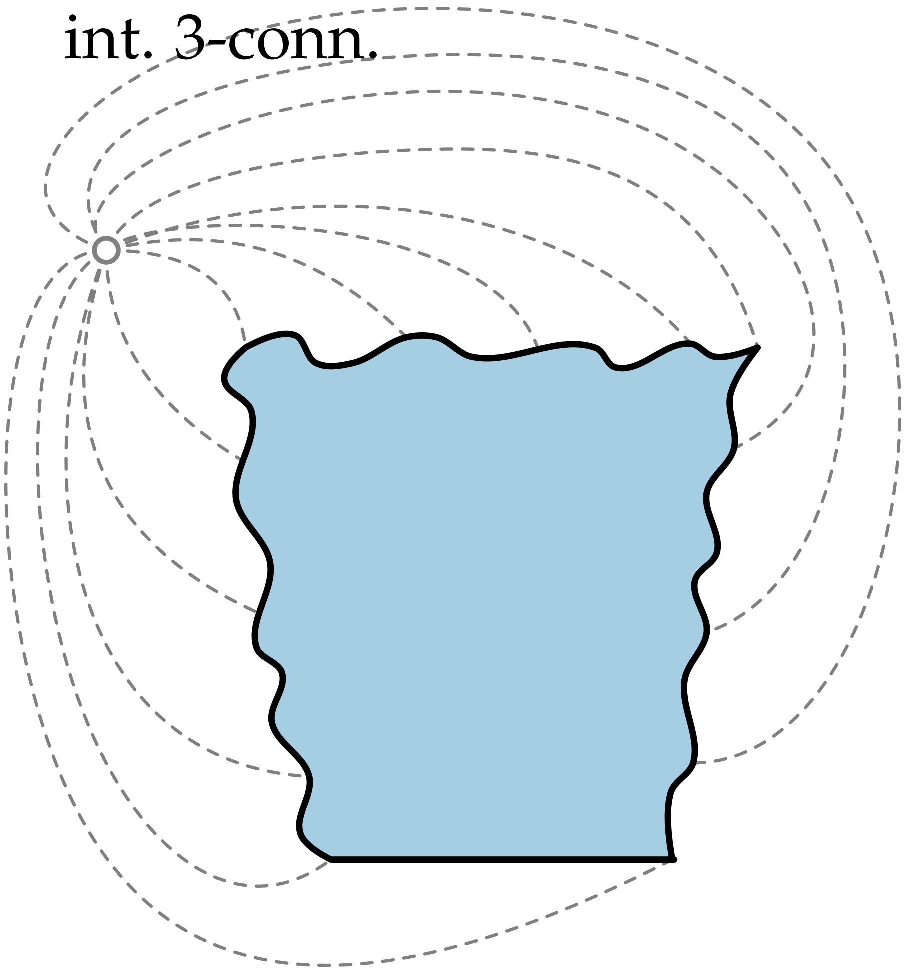

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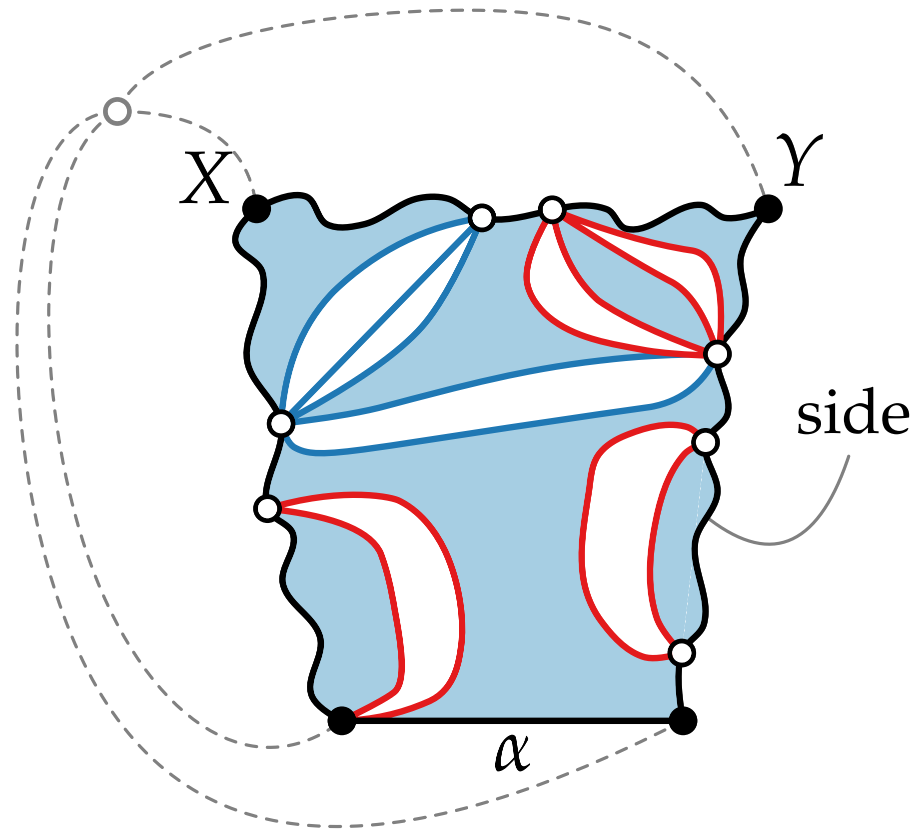

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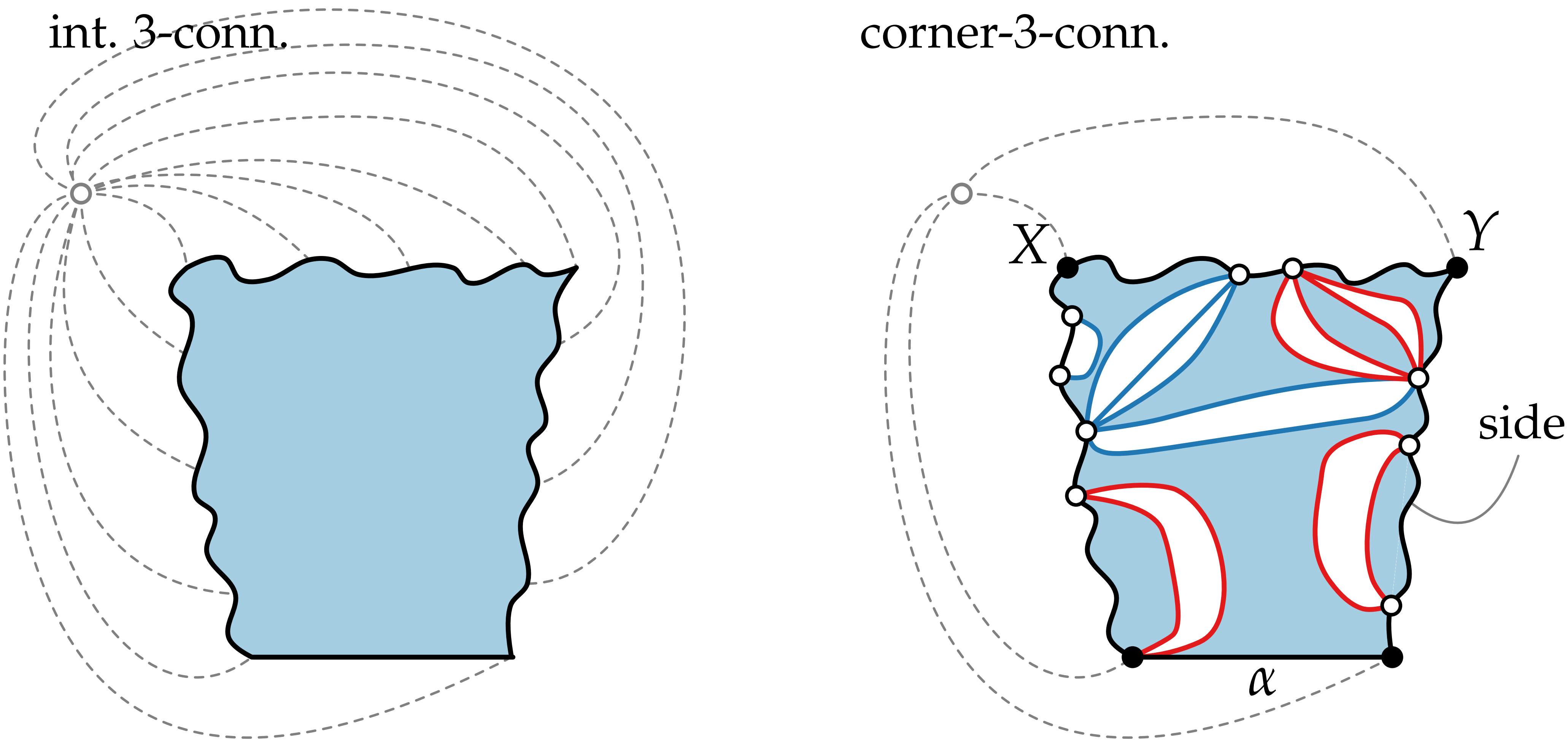

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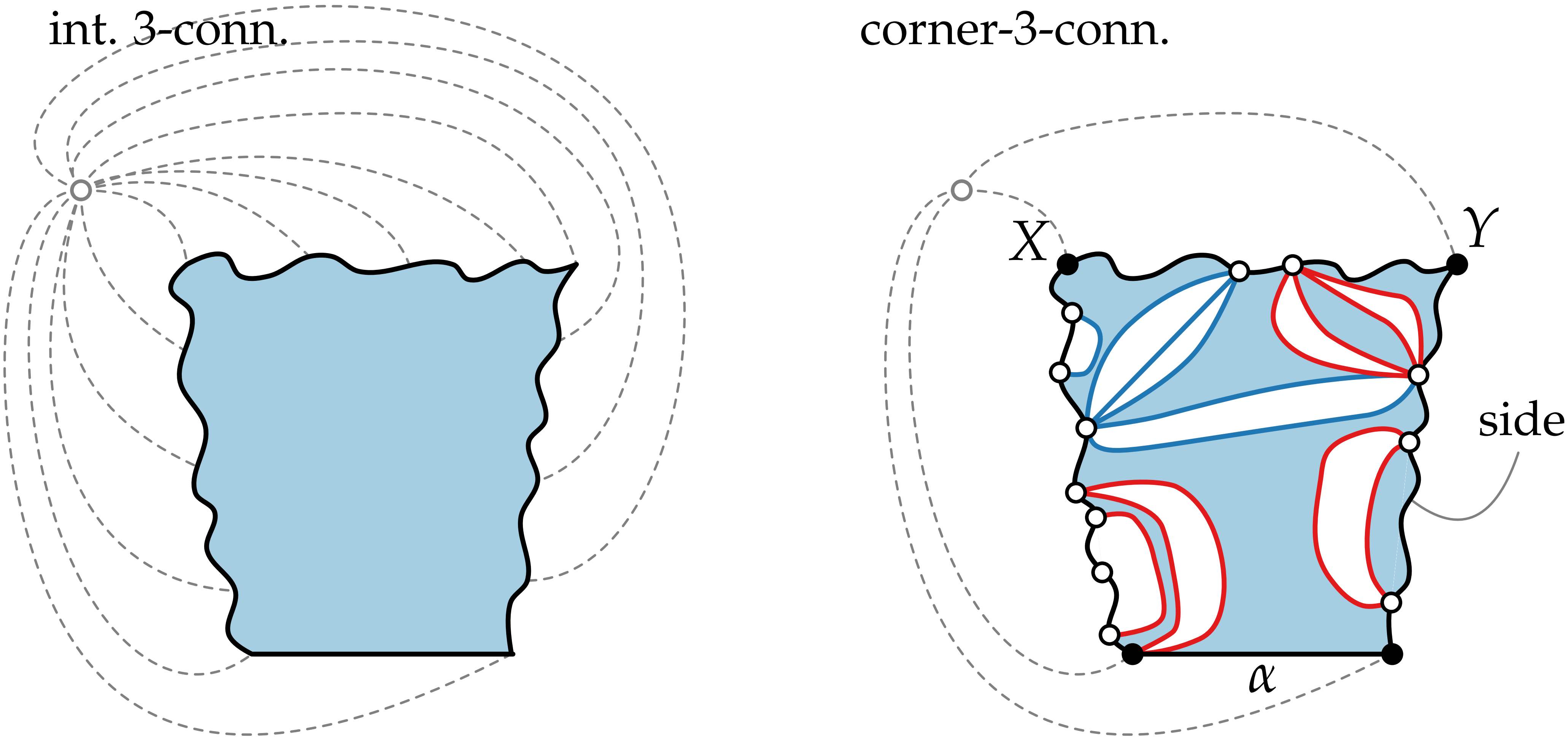

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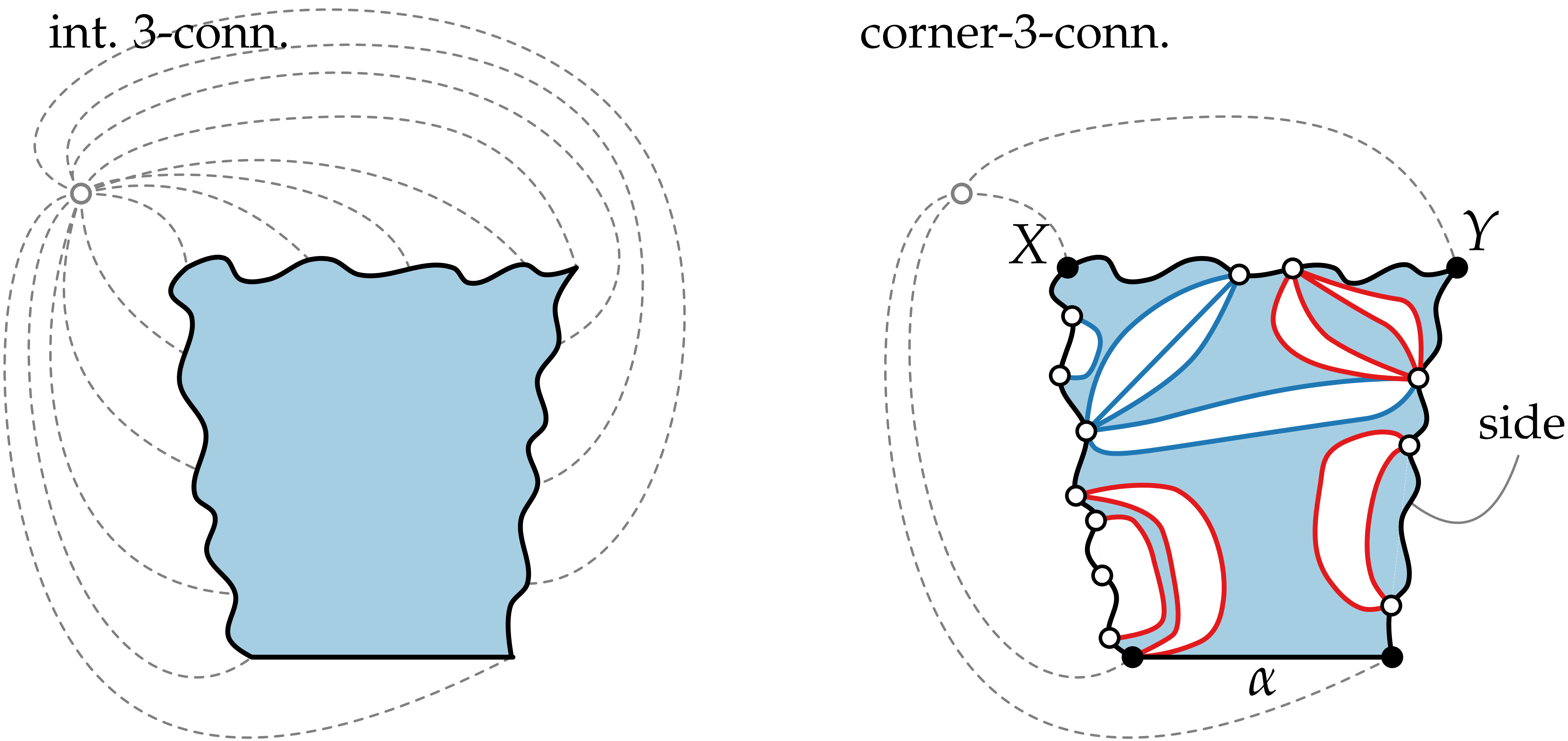

G is corner-3-conn., X, Y, α on outer face $\Rightarrow$ T_{int}-path

Corner-3-connectivity

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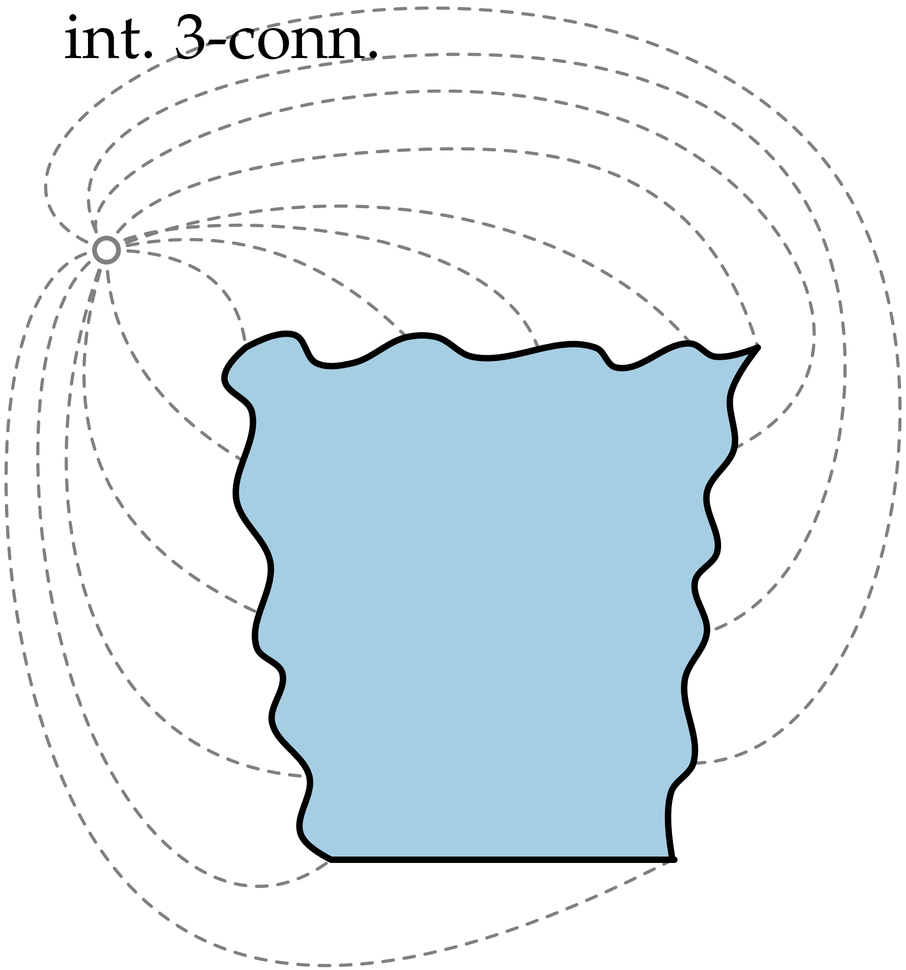

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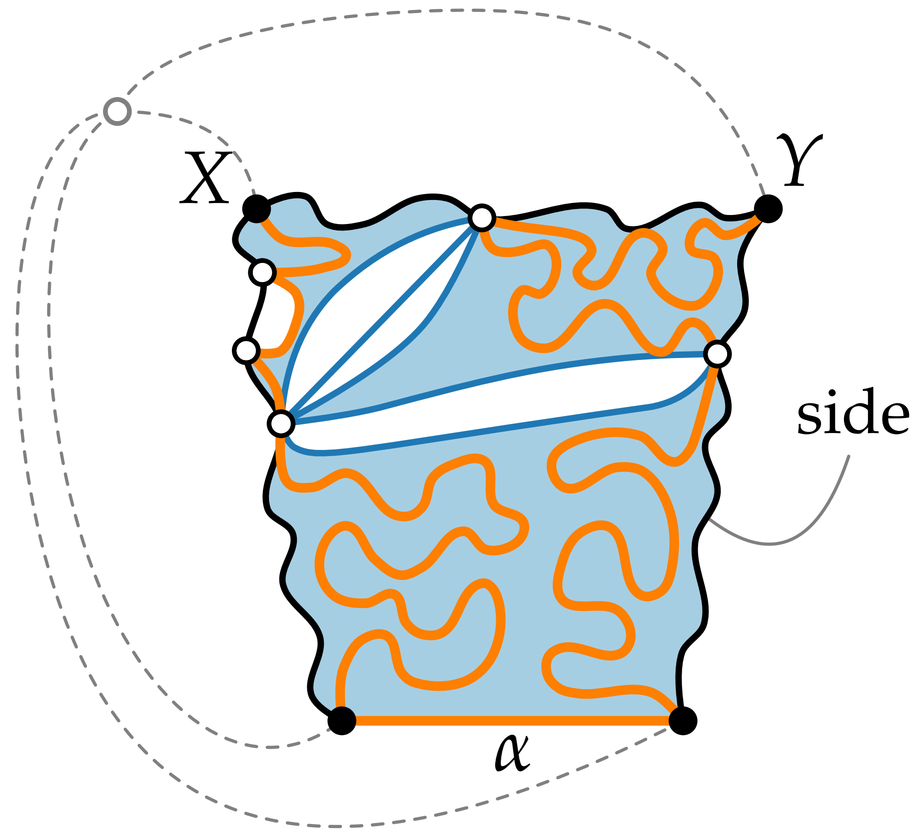

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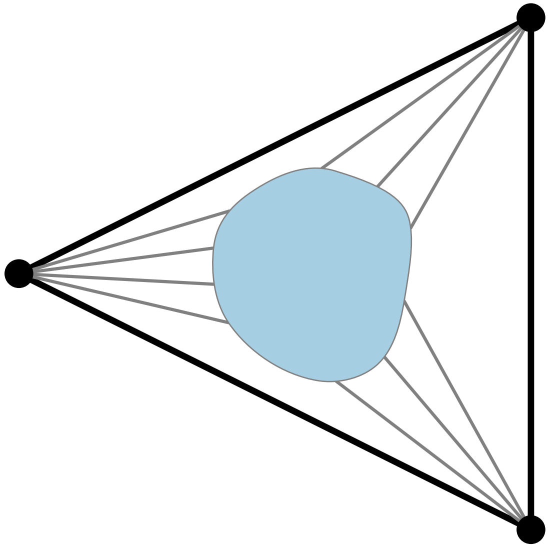
Case 1: Outer Face is Triangle



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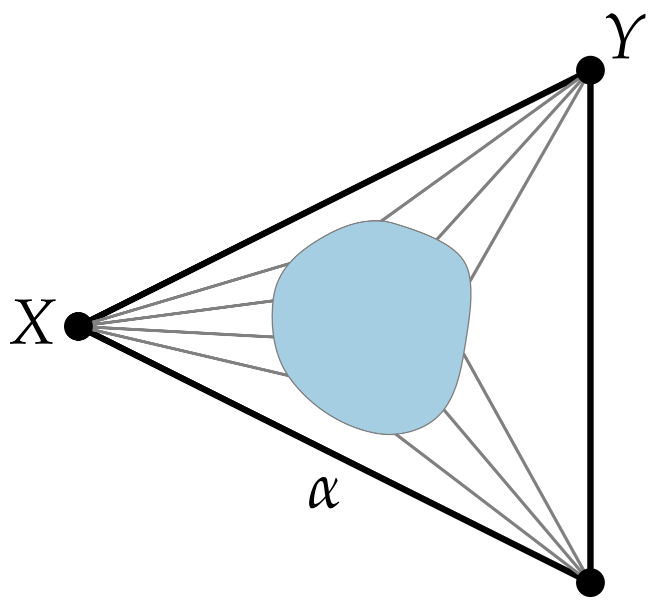

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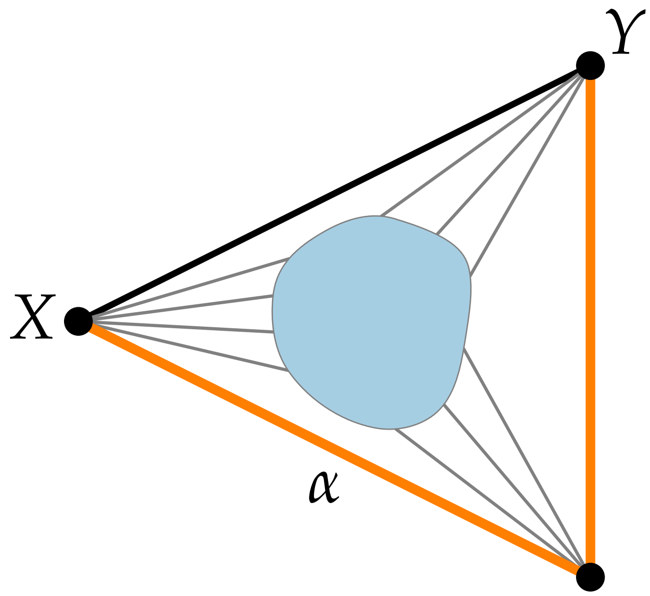

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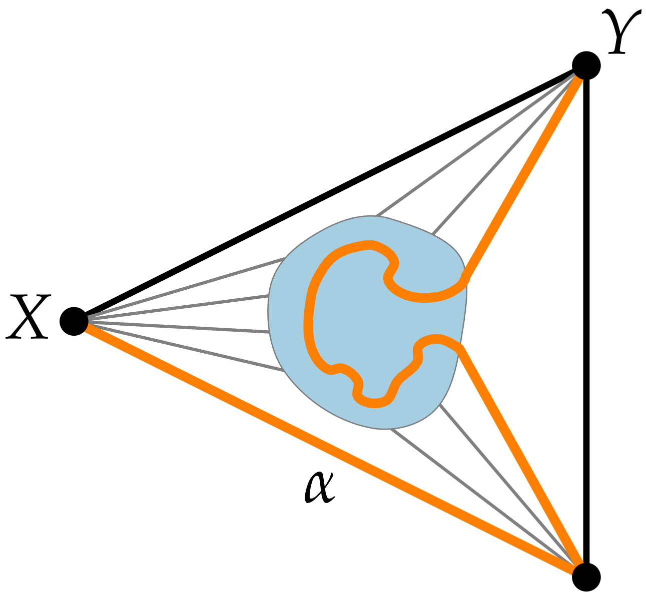
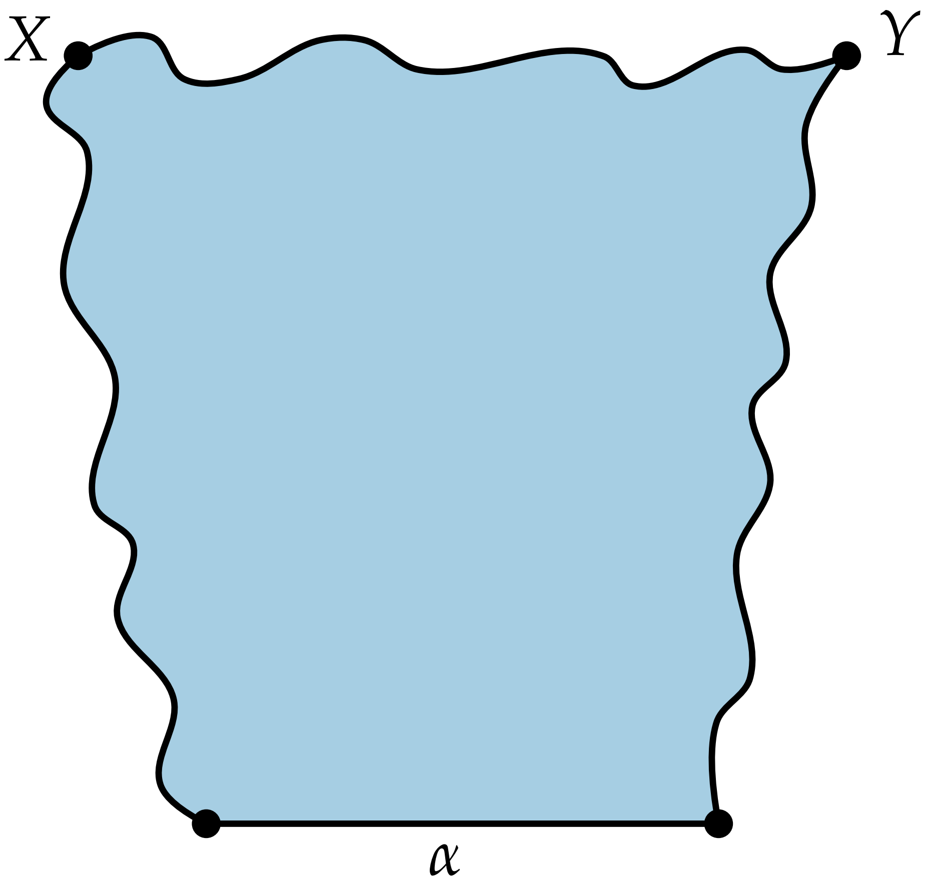


Case 2: left-right cutting pair



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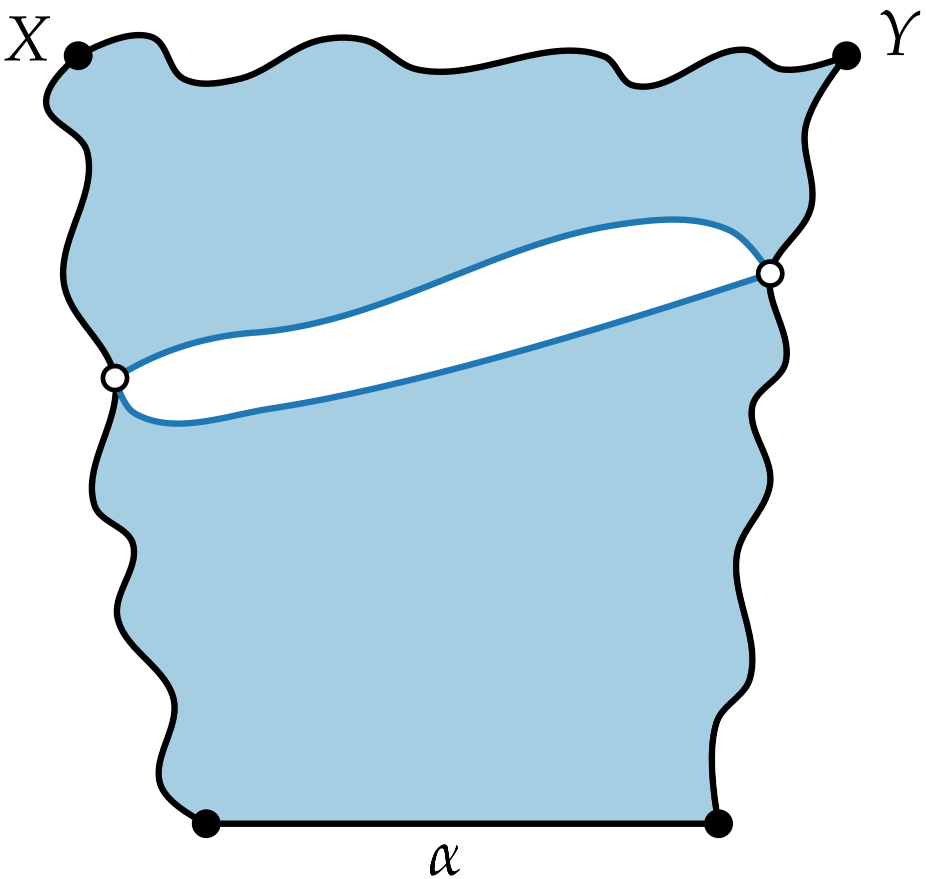

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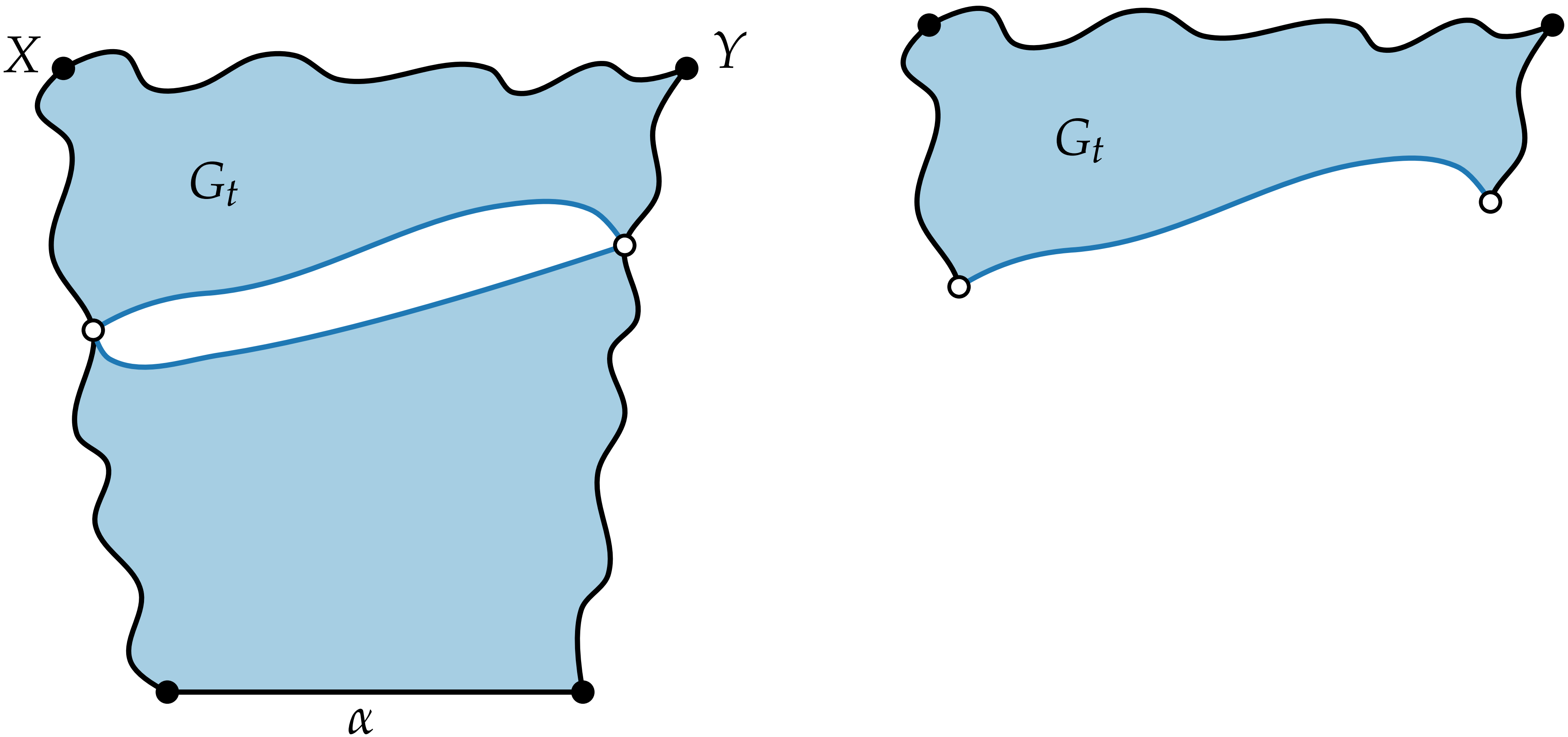

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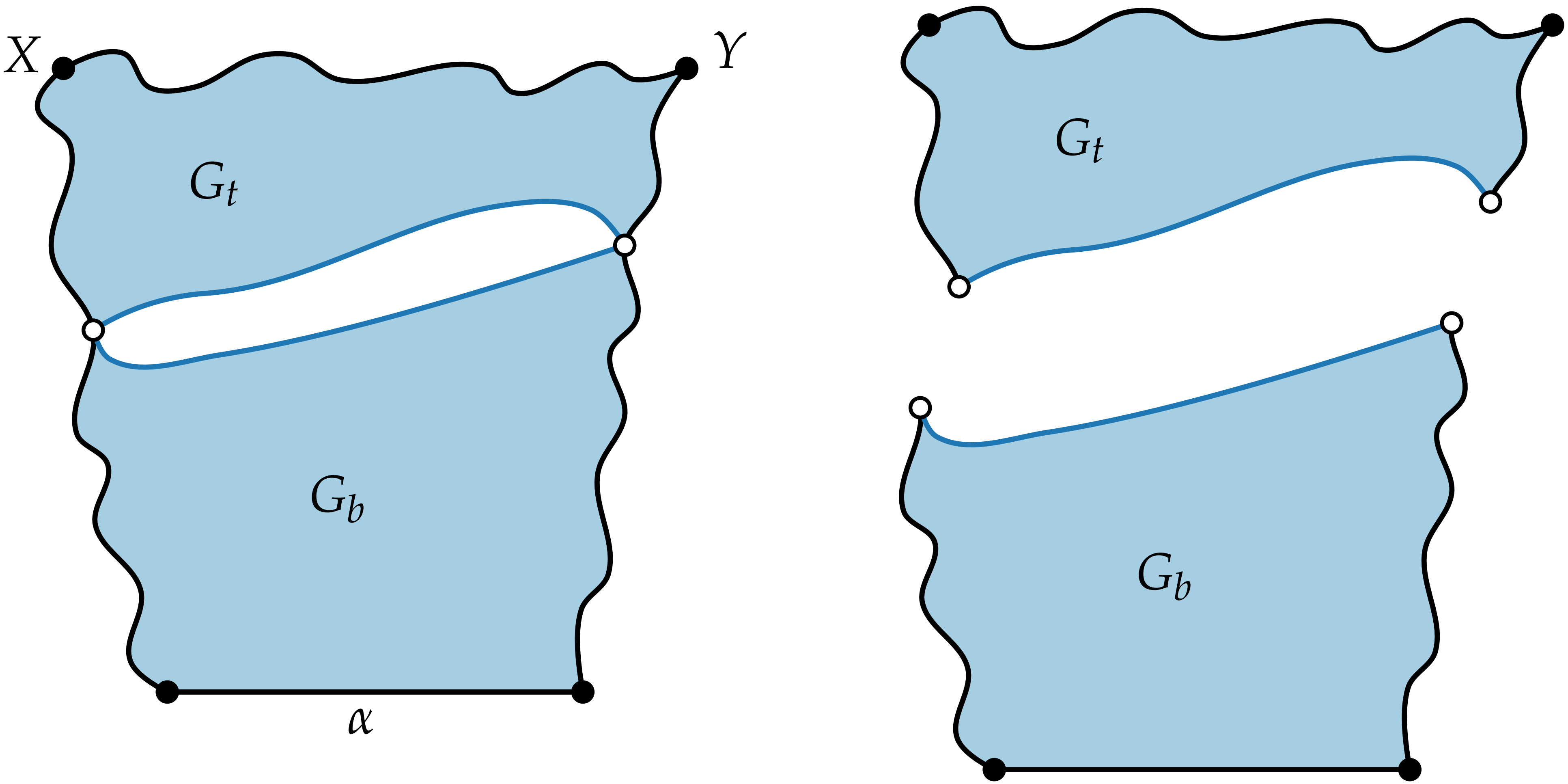

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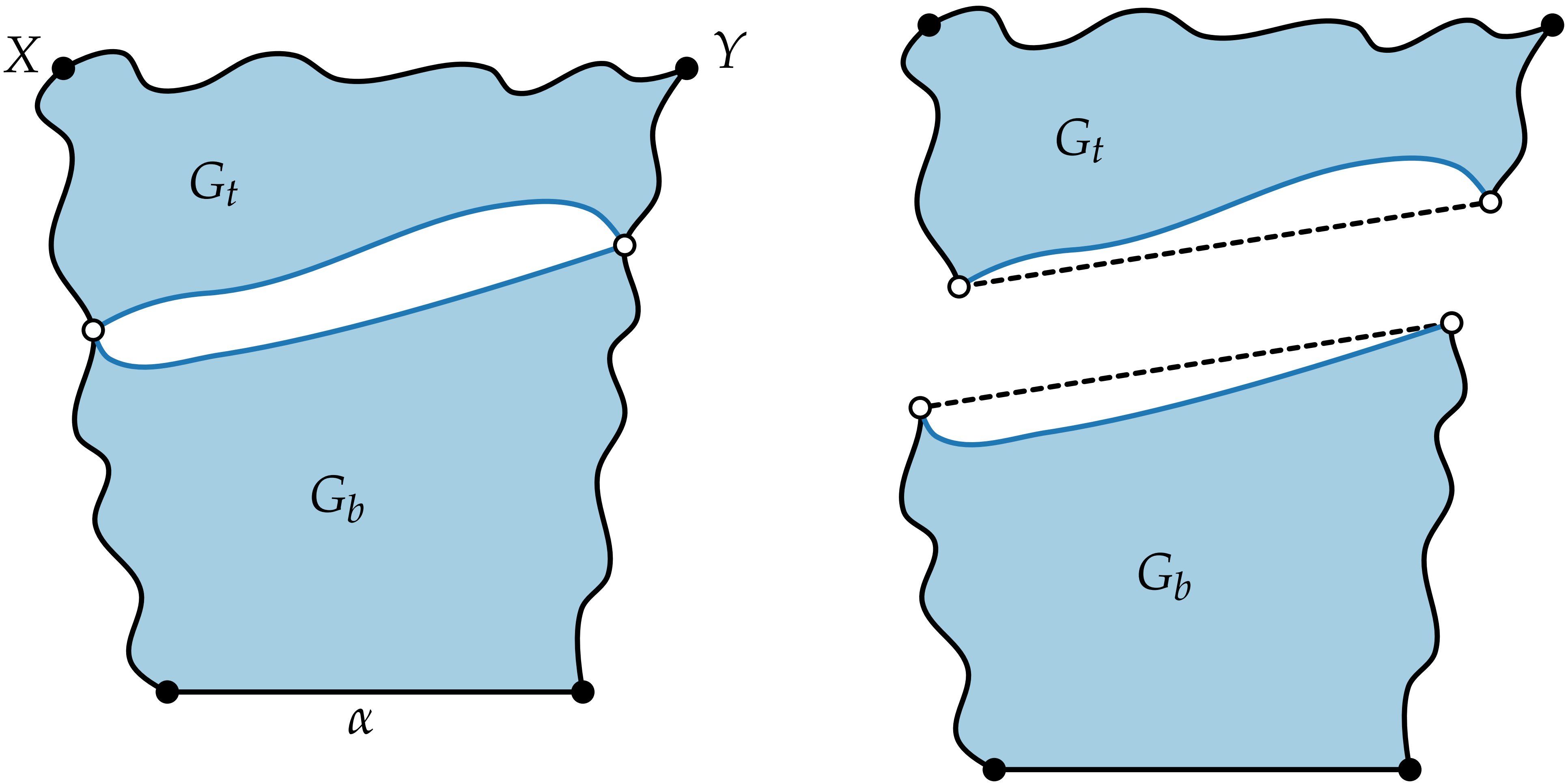

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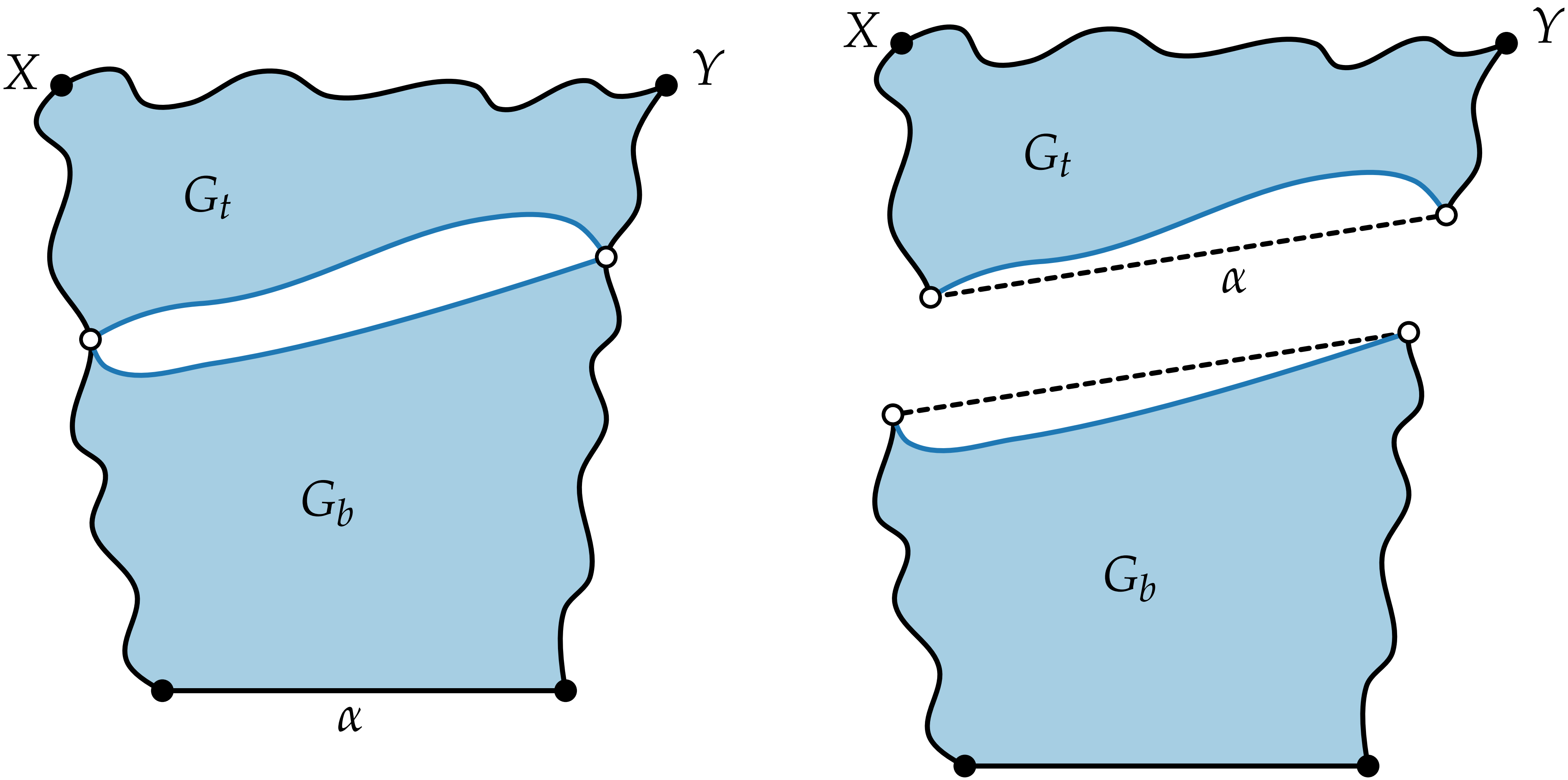

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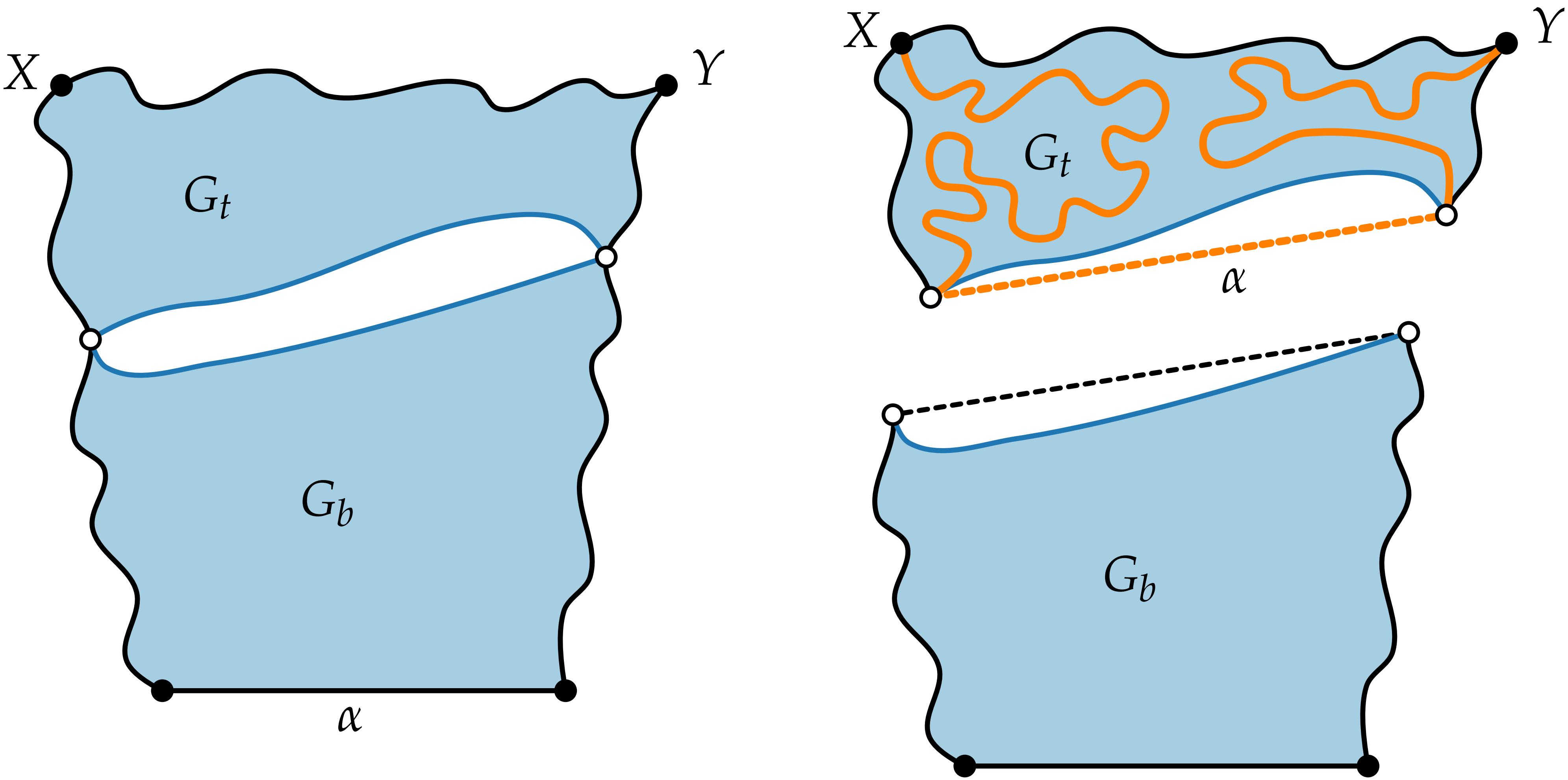
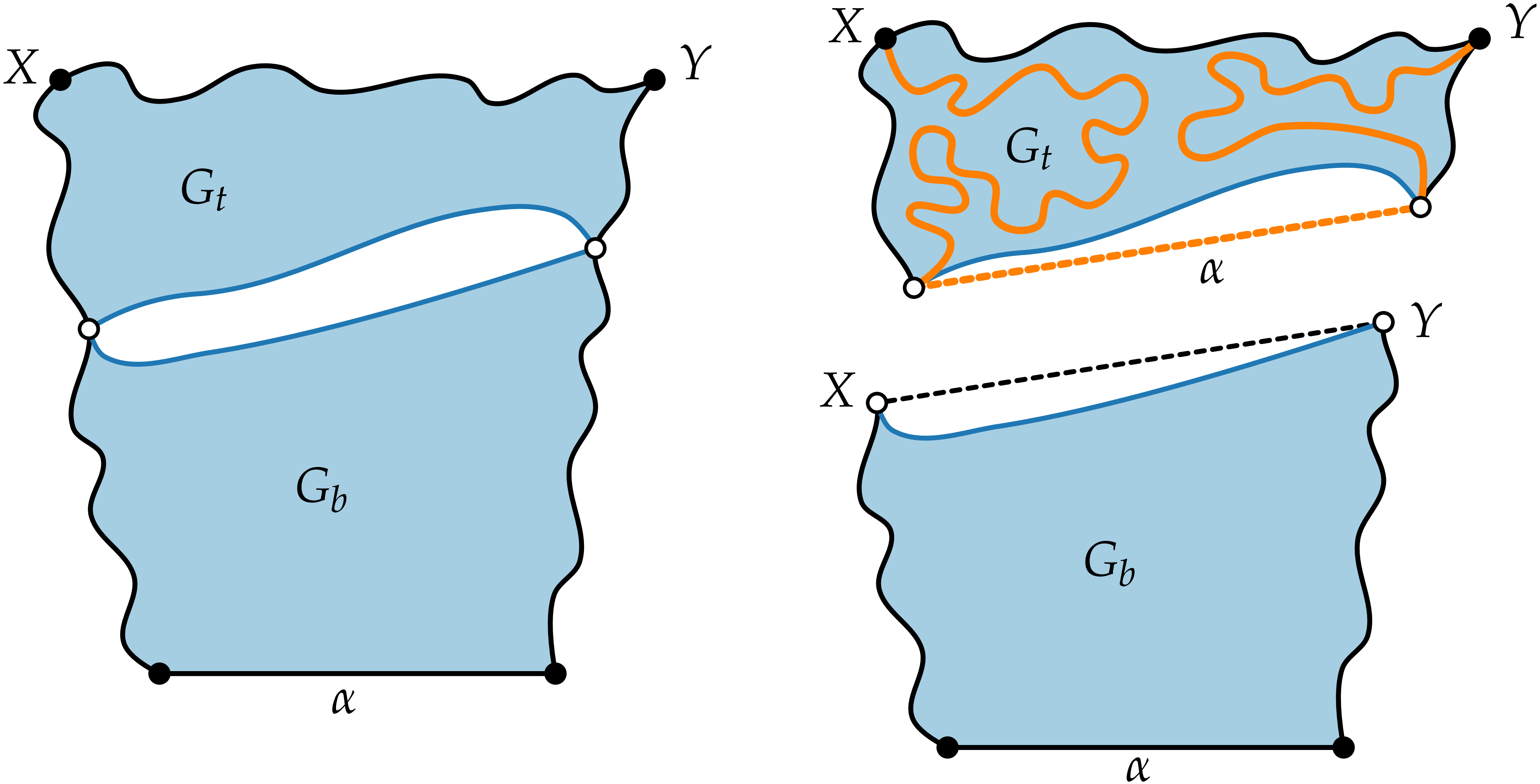
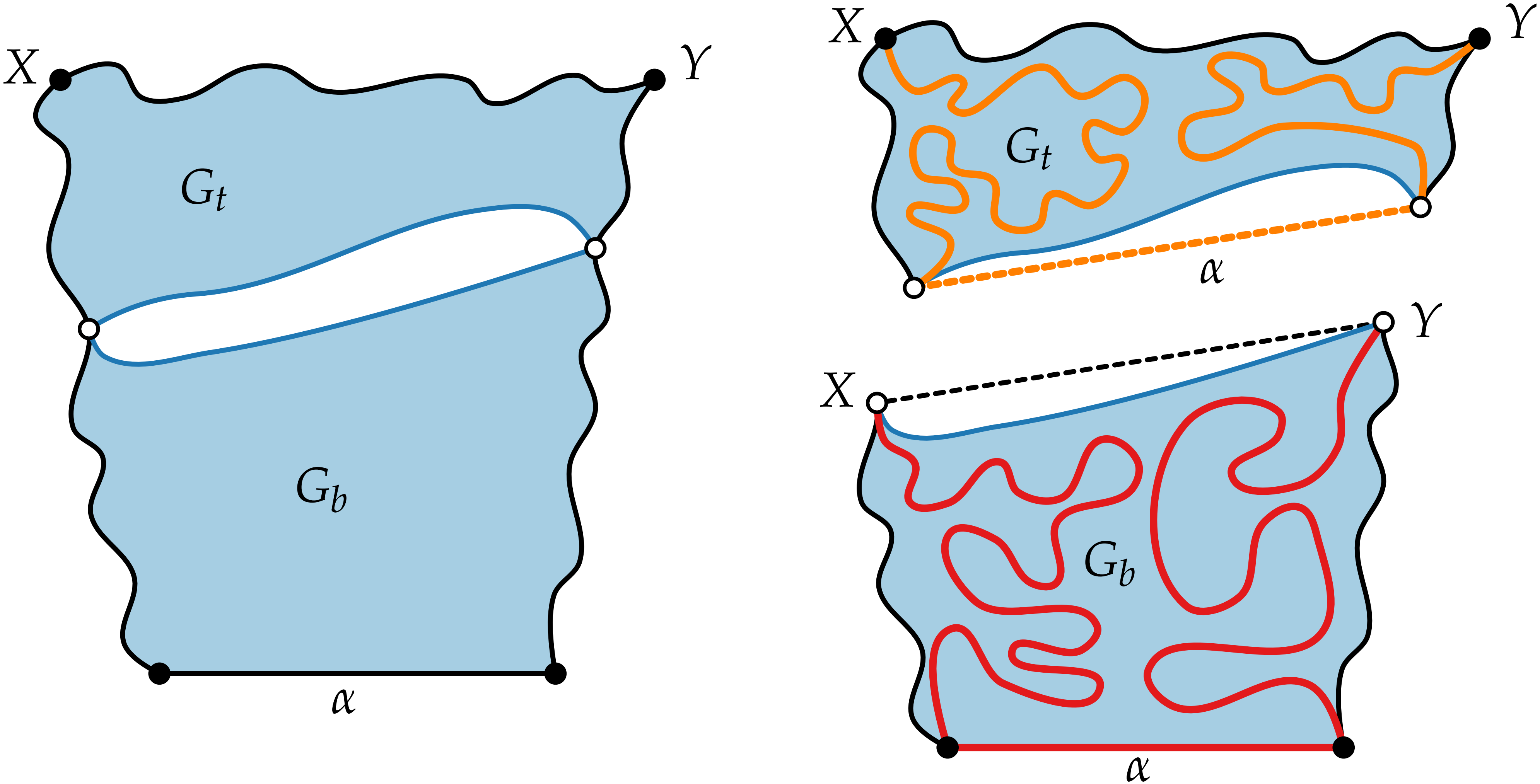


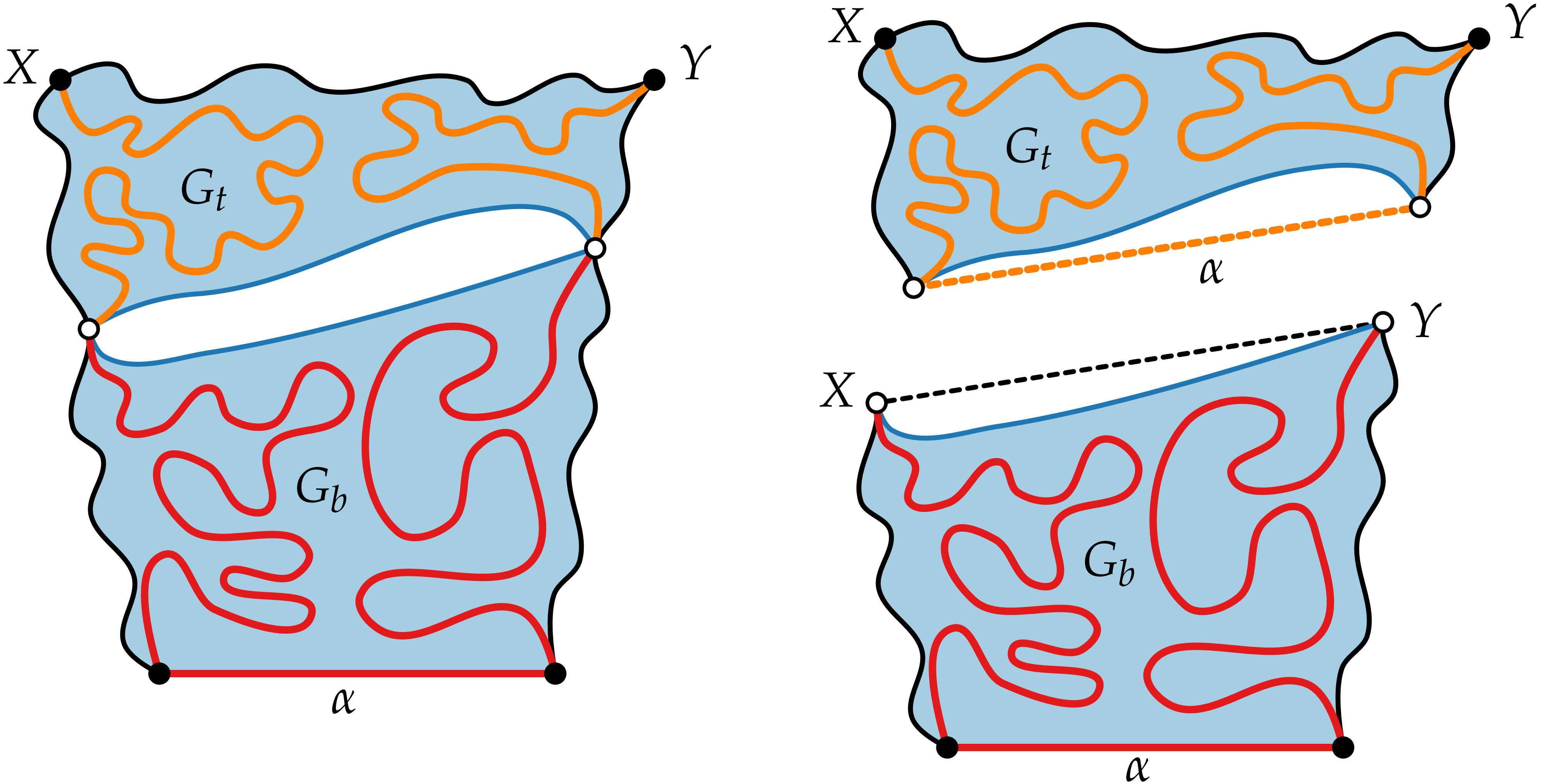
Case 2: left-right cutting pair



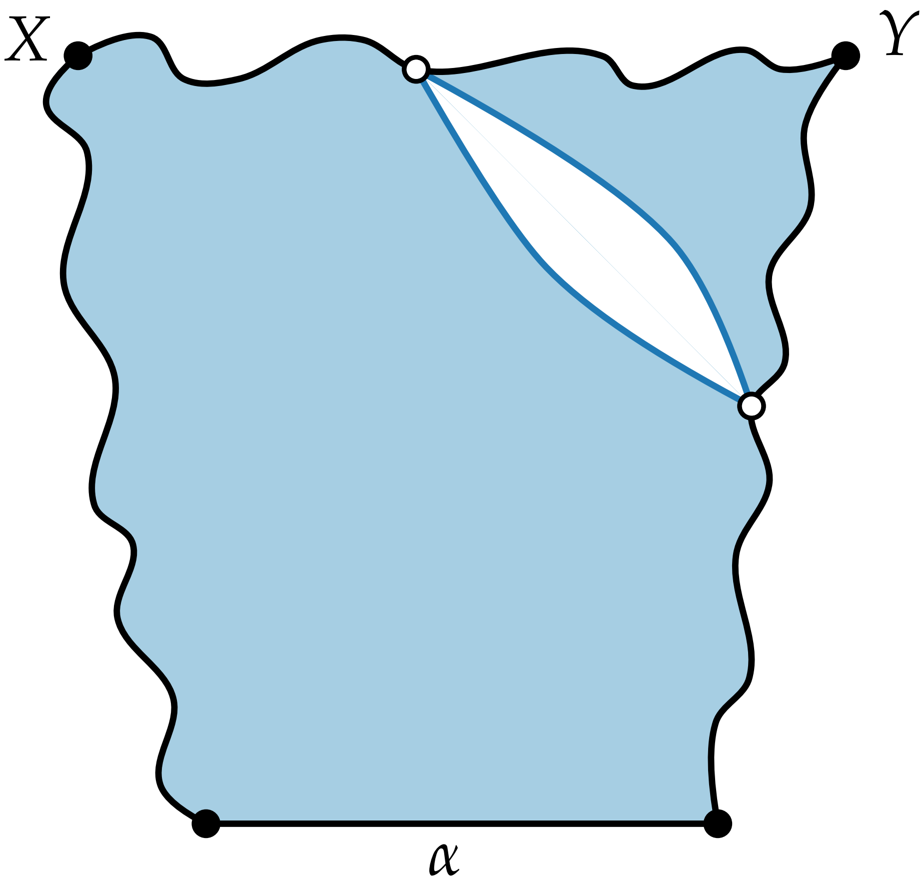
Case 2: left-right cutting pair



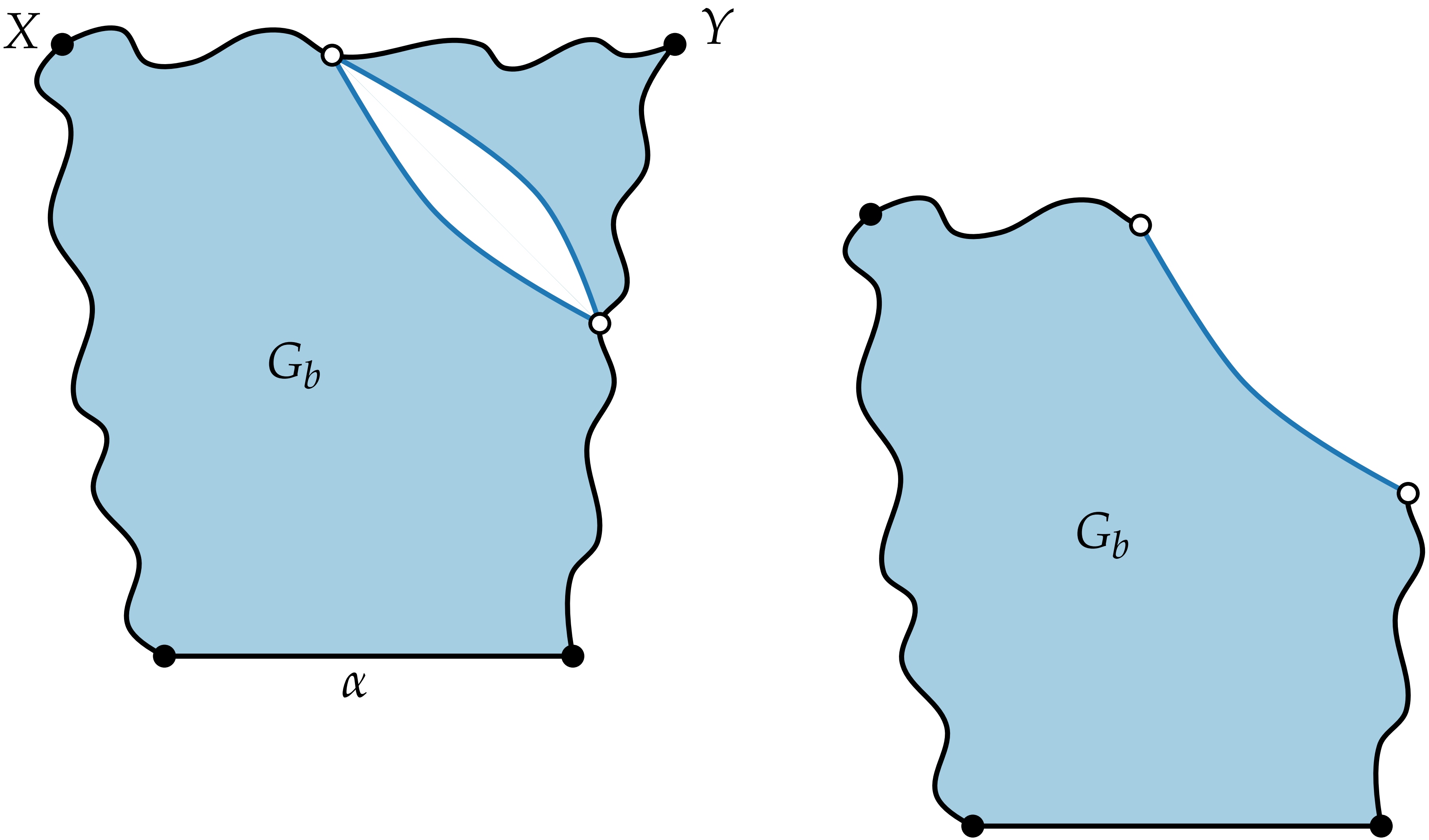
Case 2: left-right cutting pair



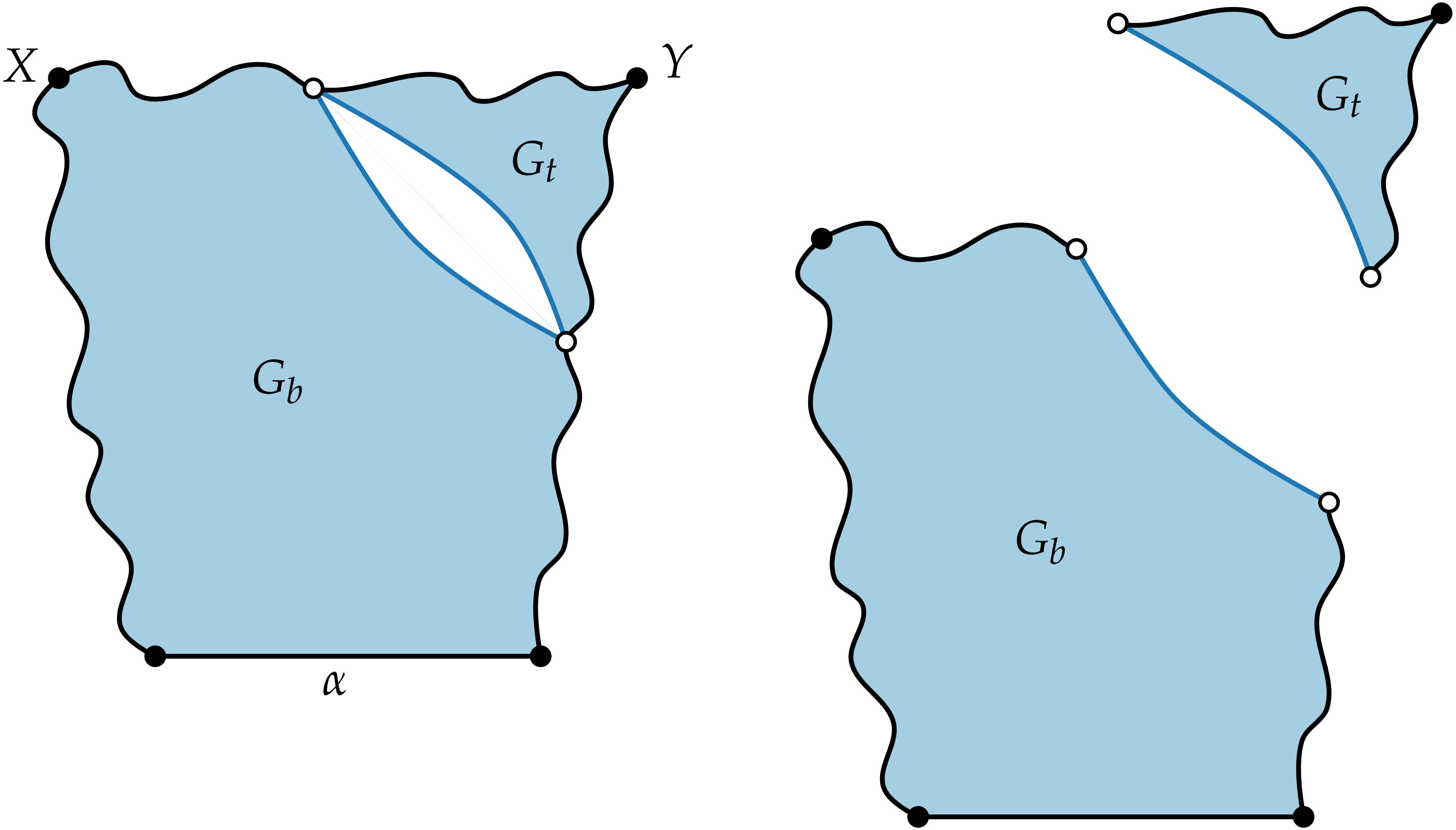
Case 3: top-right cutting pair



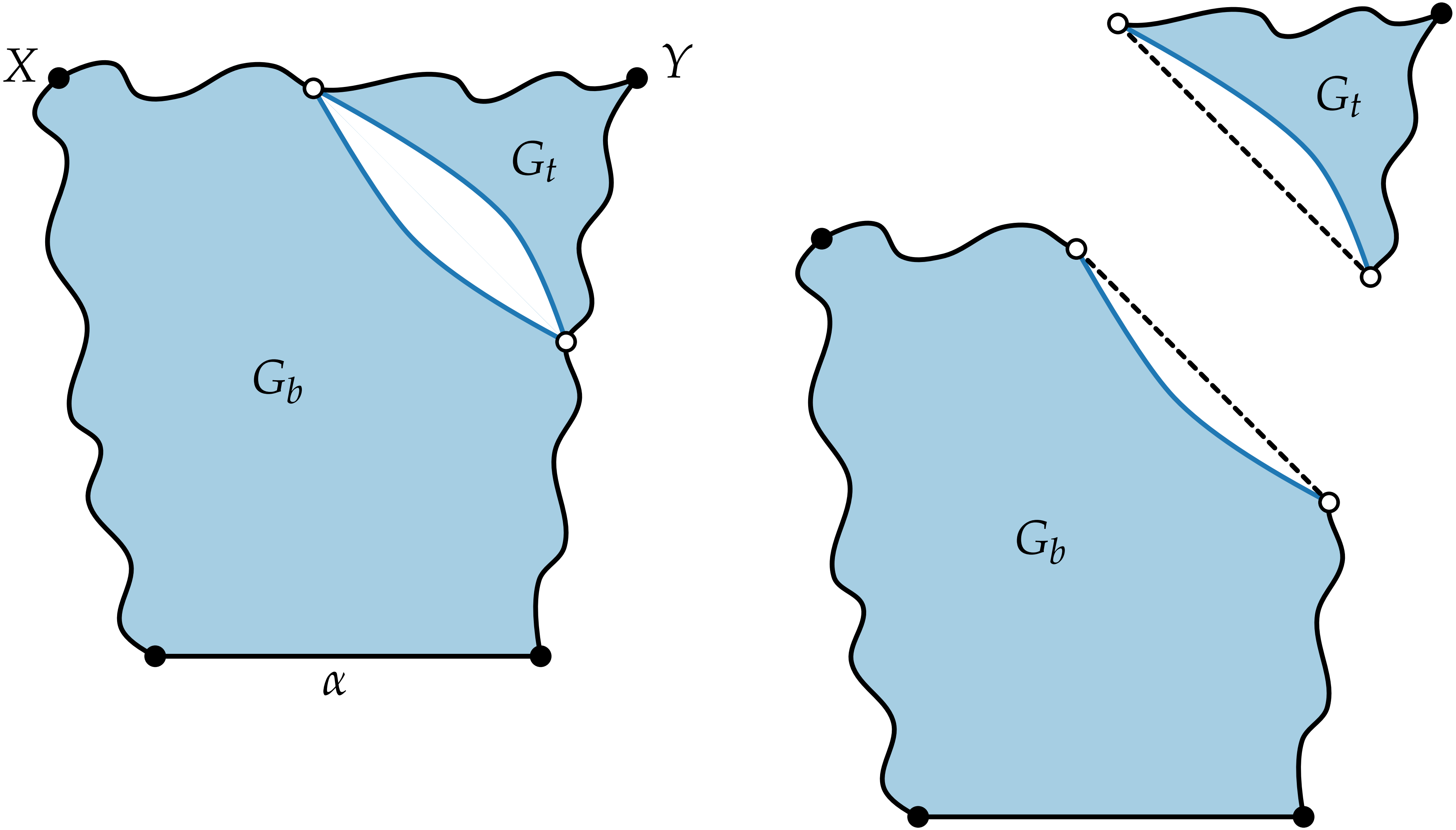
Case 3: top-right cutting pair



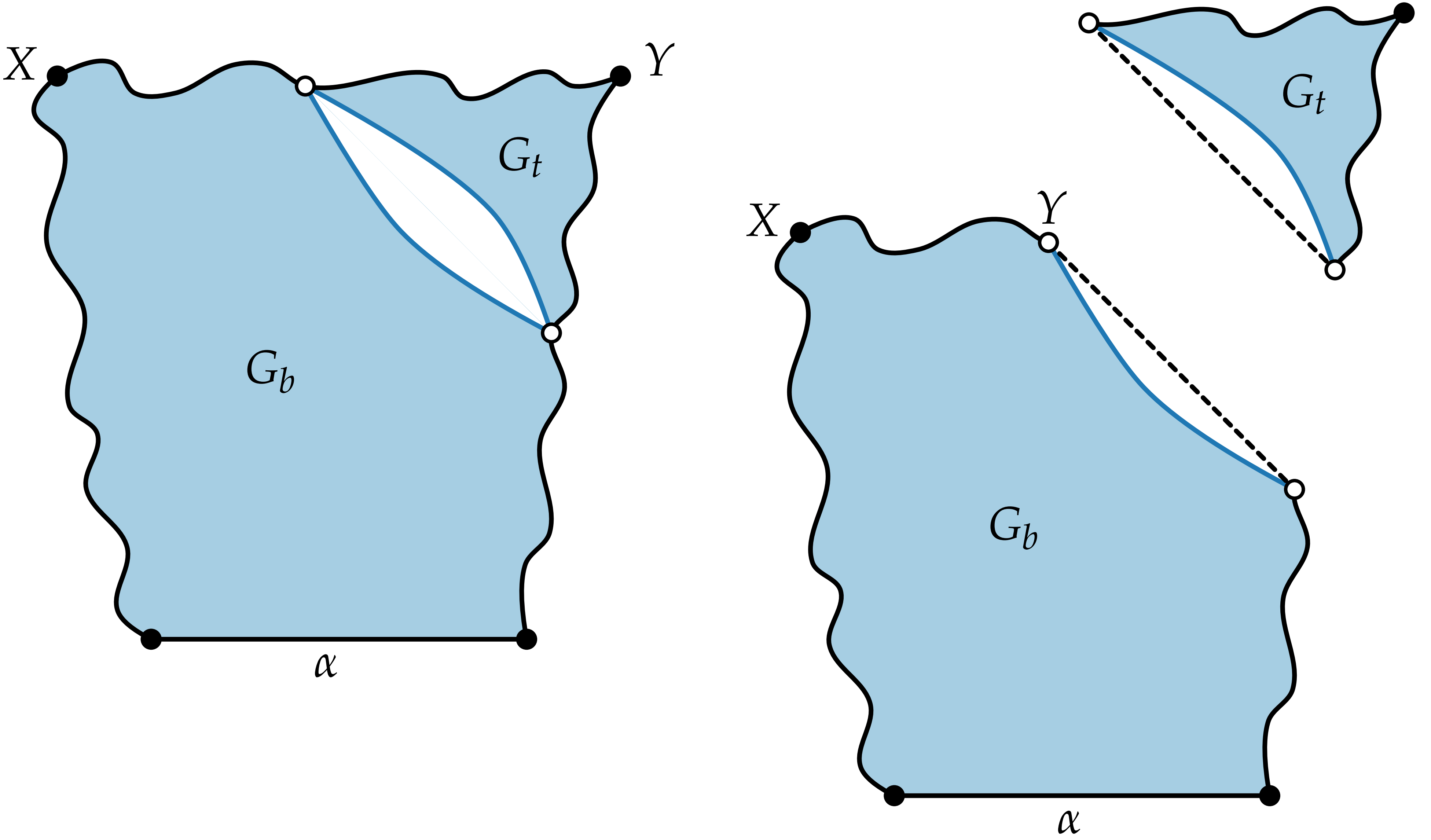
Case 3: top-right cutting pair



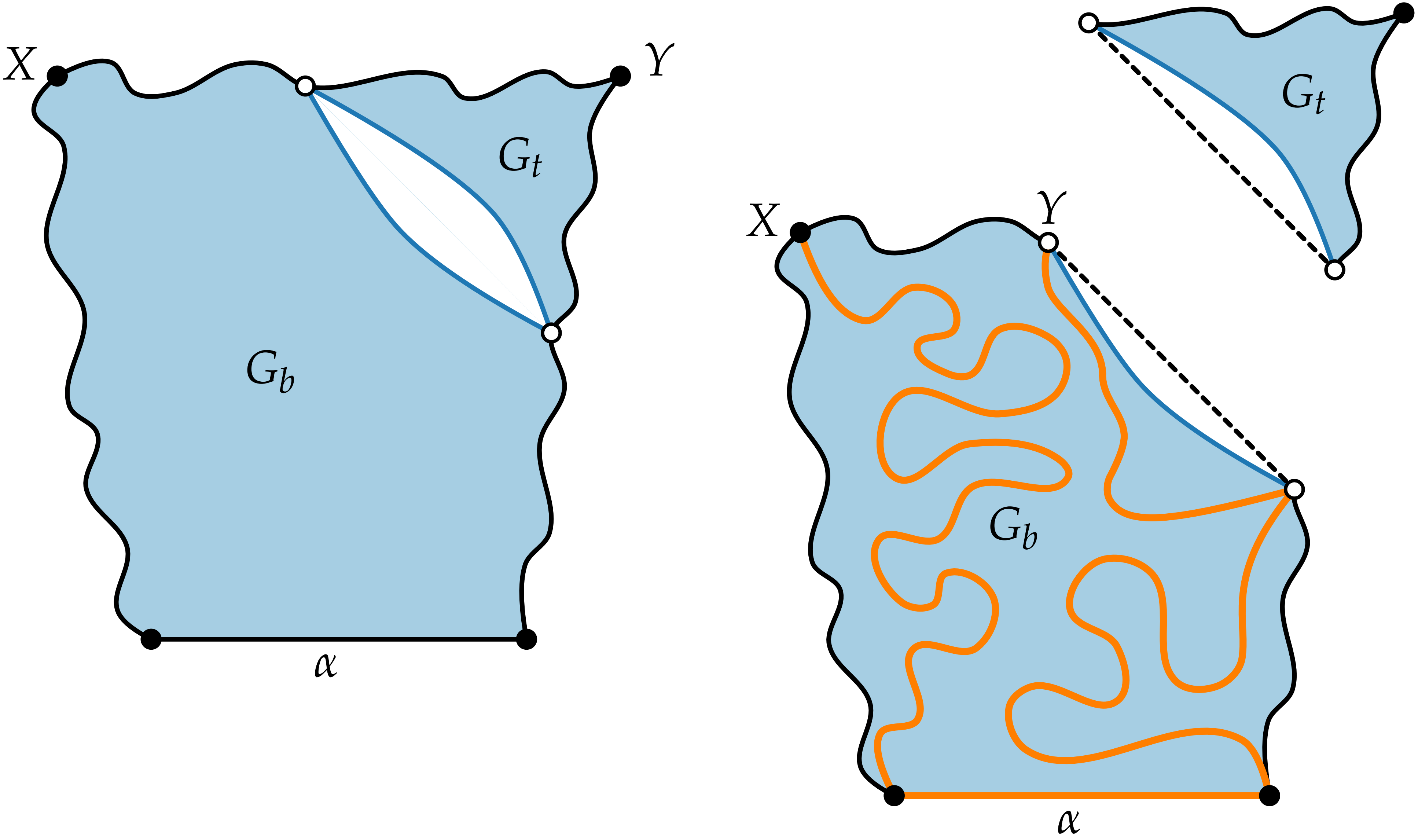
Case 3: top-right cutting pair



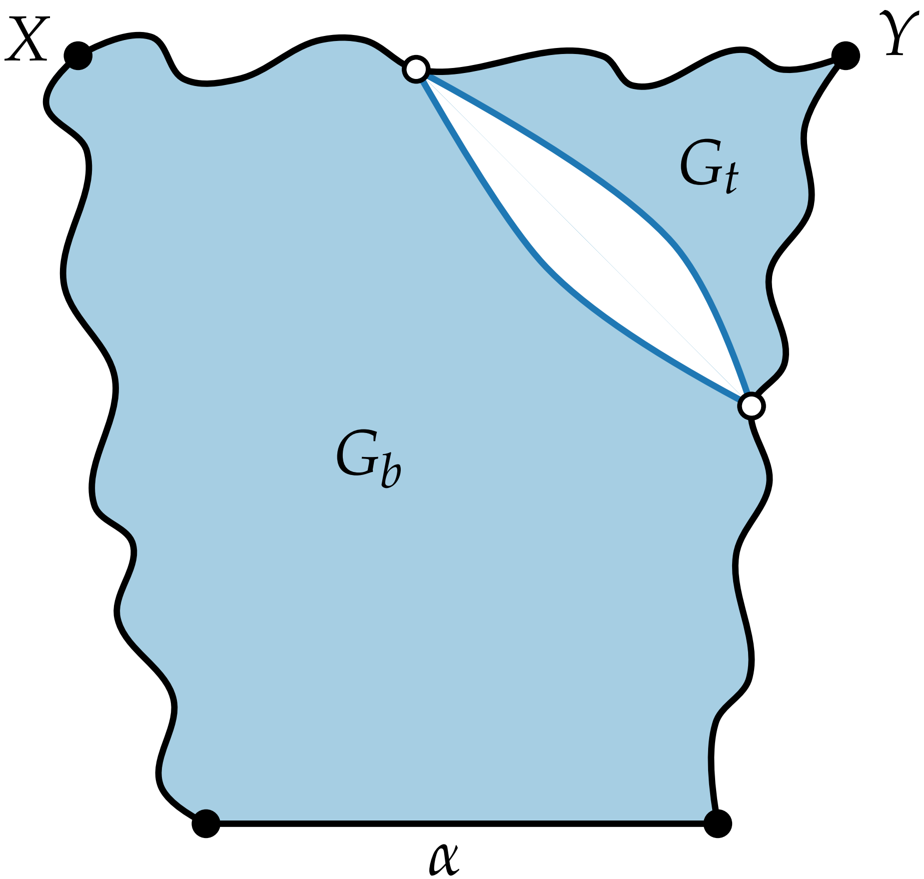
Case 3: top-right cutting pair

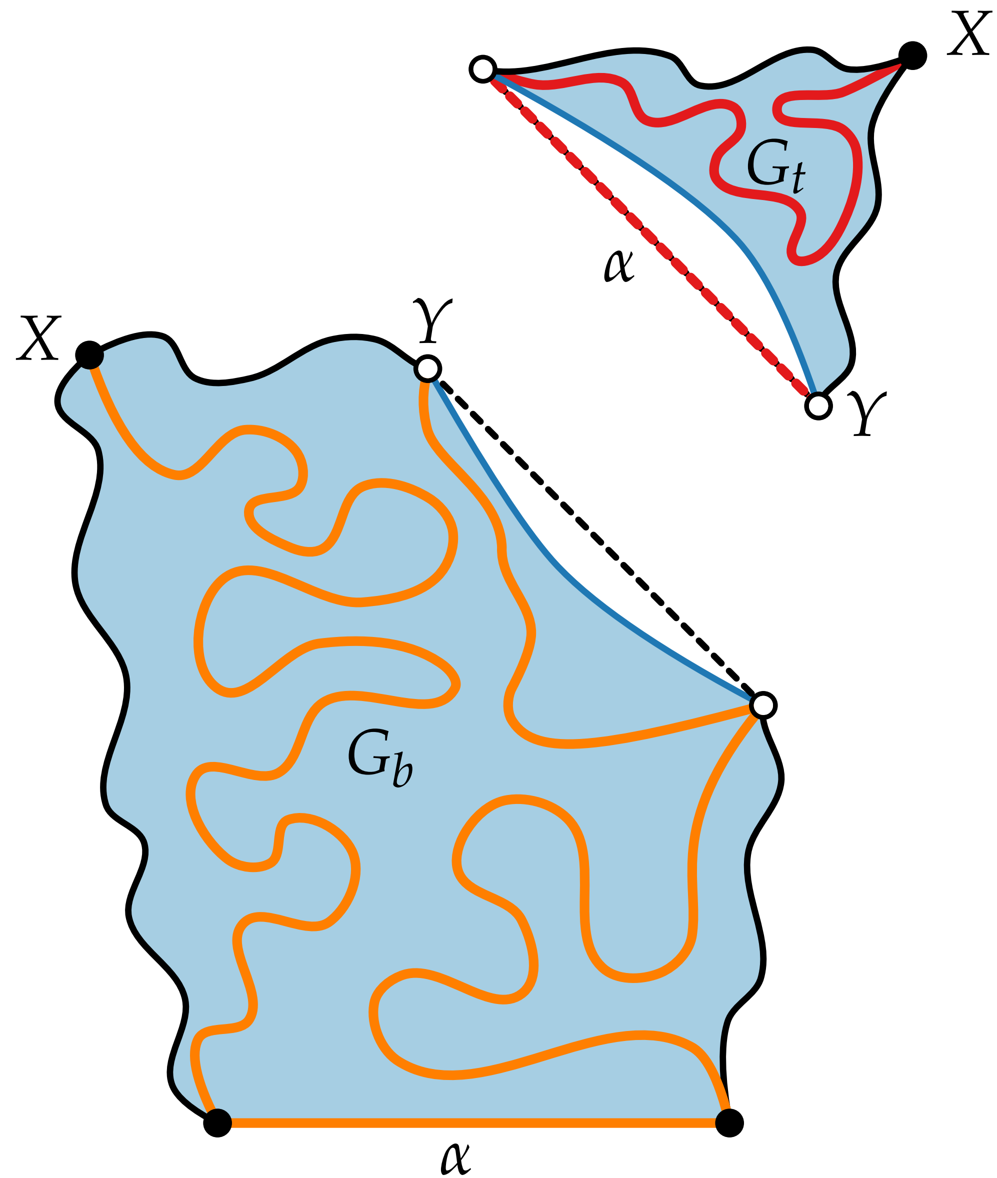


Case 3: top-right cutting pair

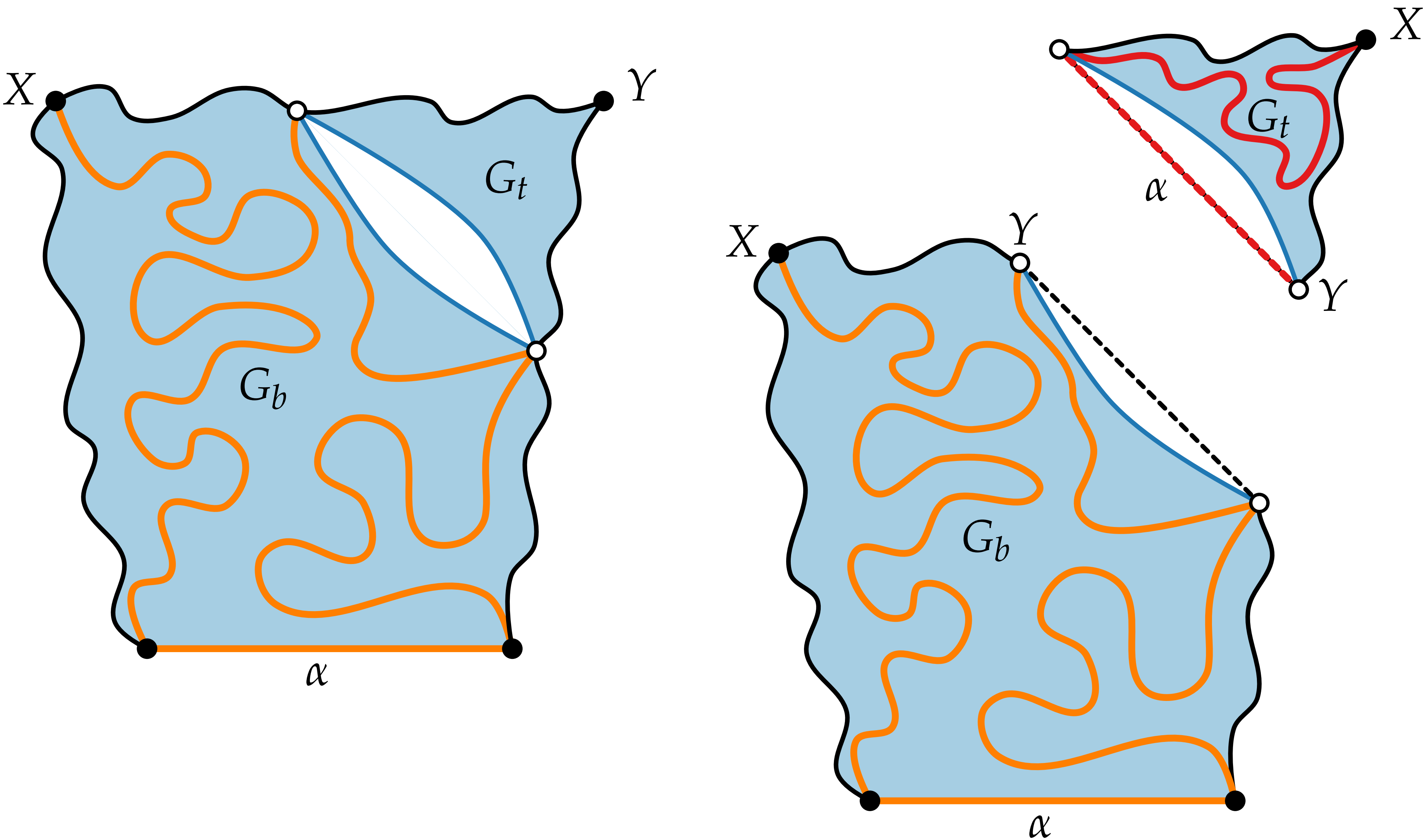


Case 3: top-right cutting pair

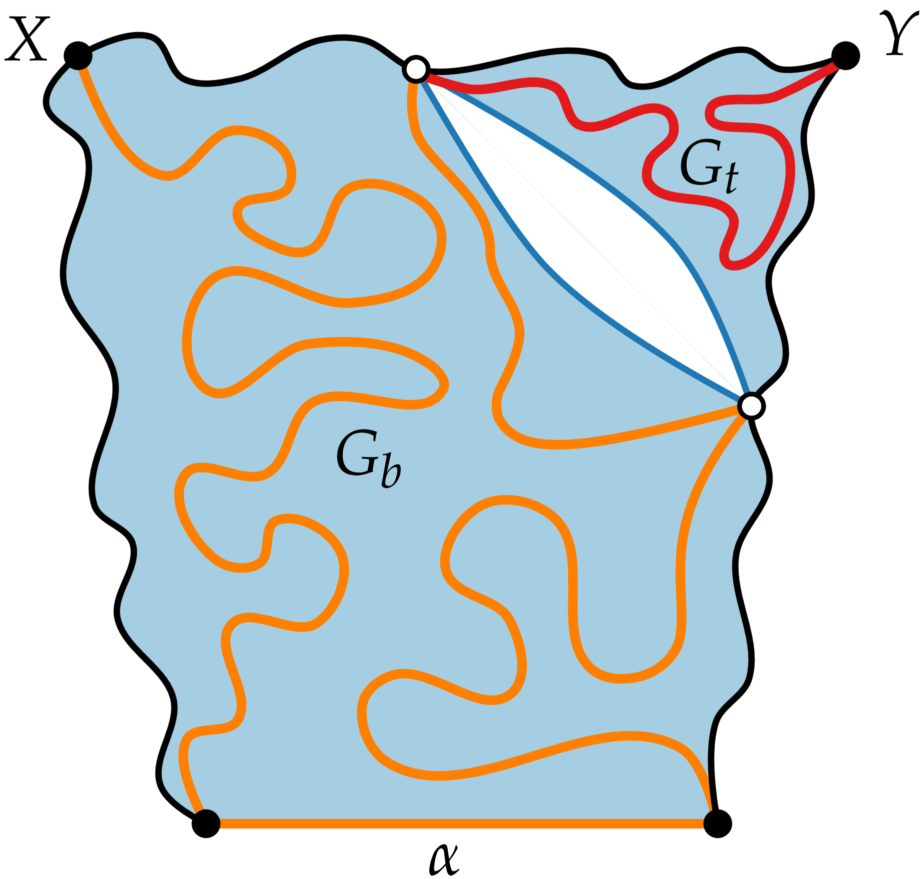


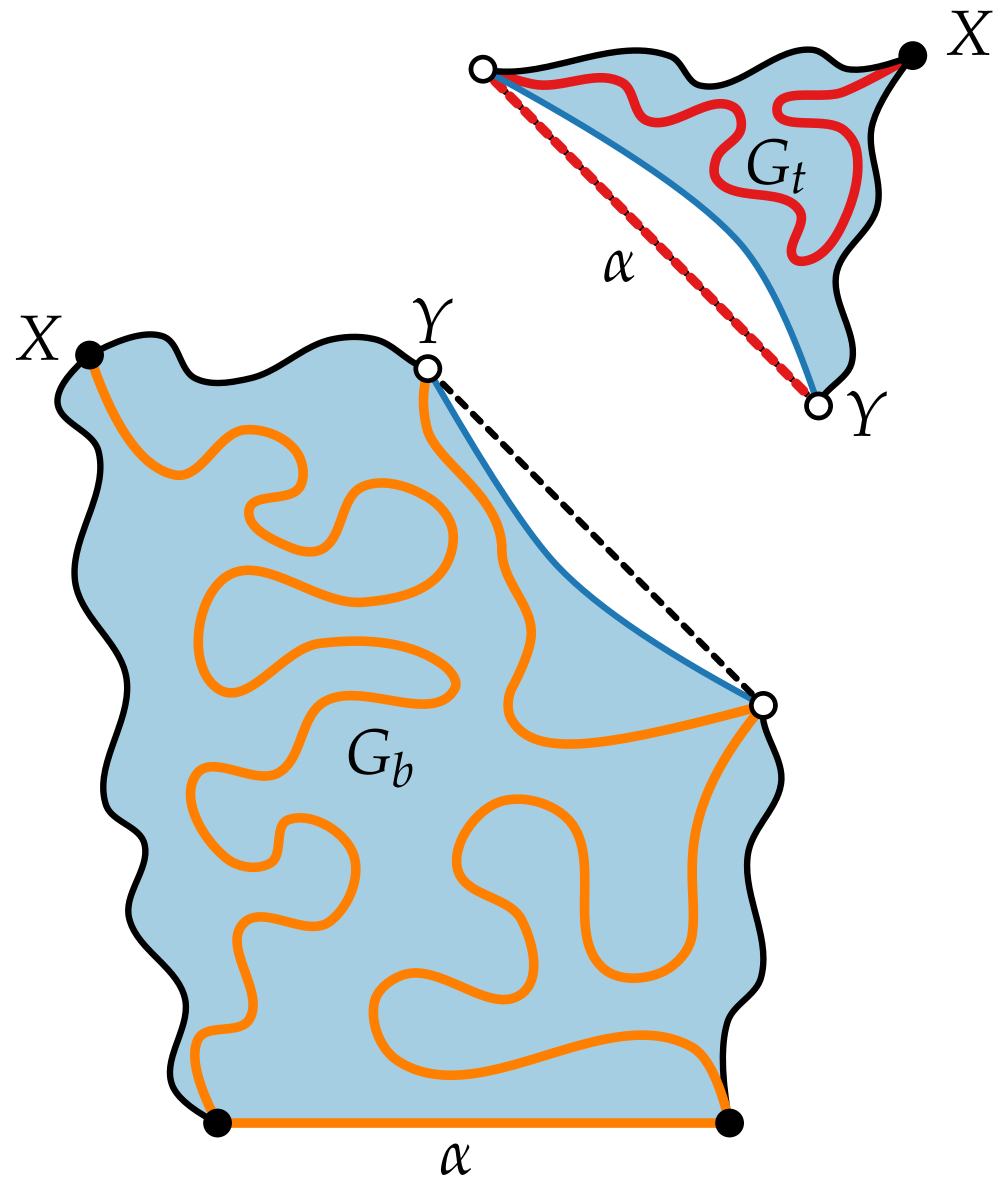


Case 3: top-right cutting pair

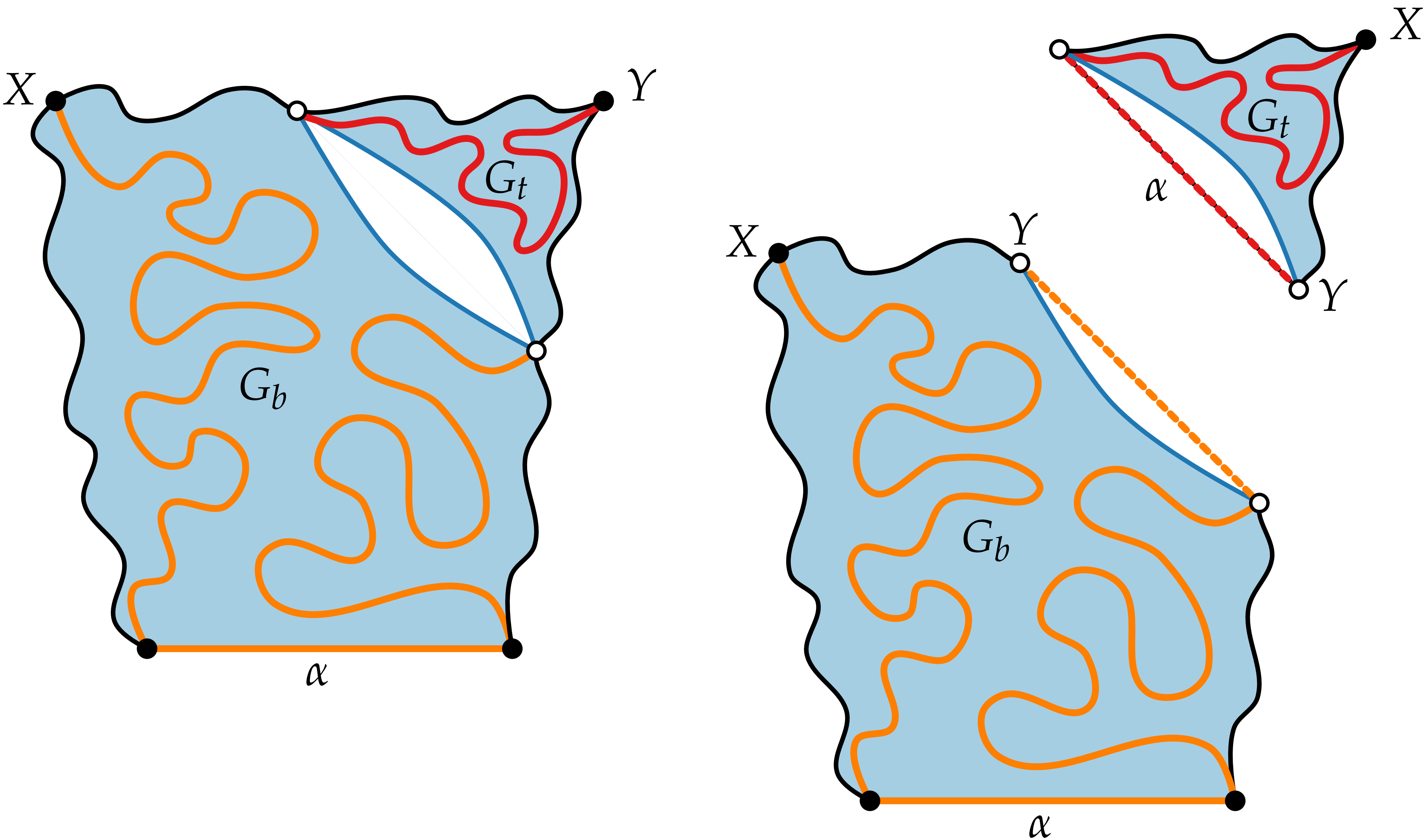


Case 3: top-right cutting pair

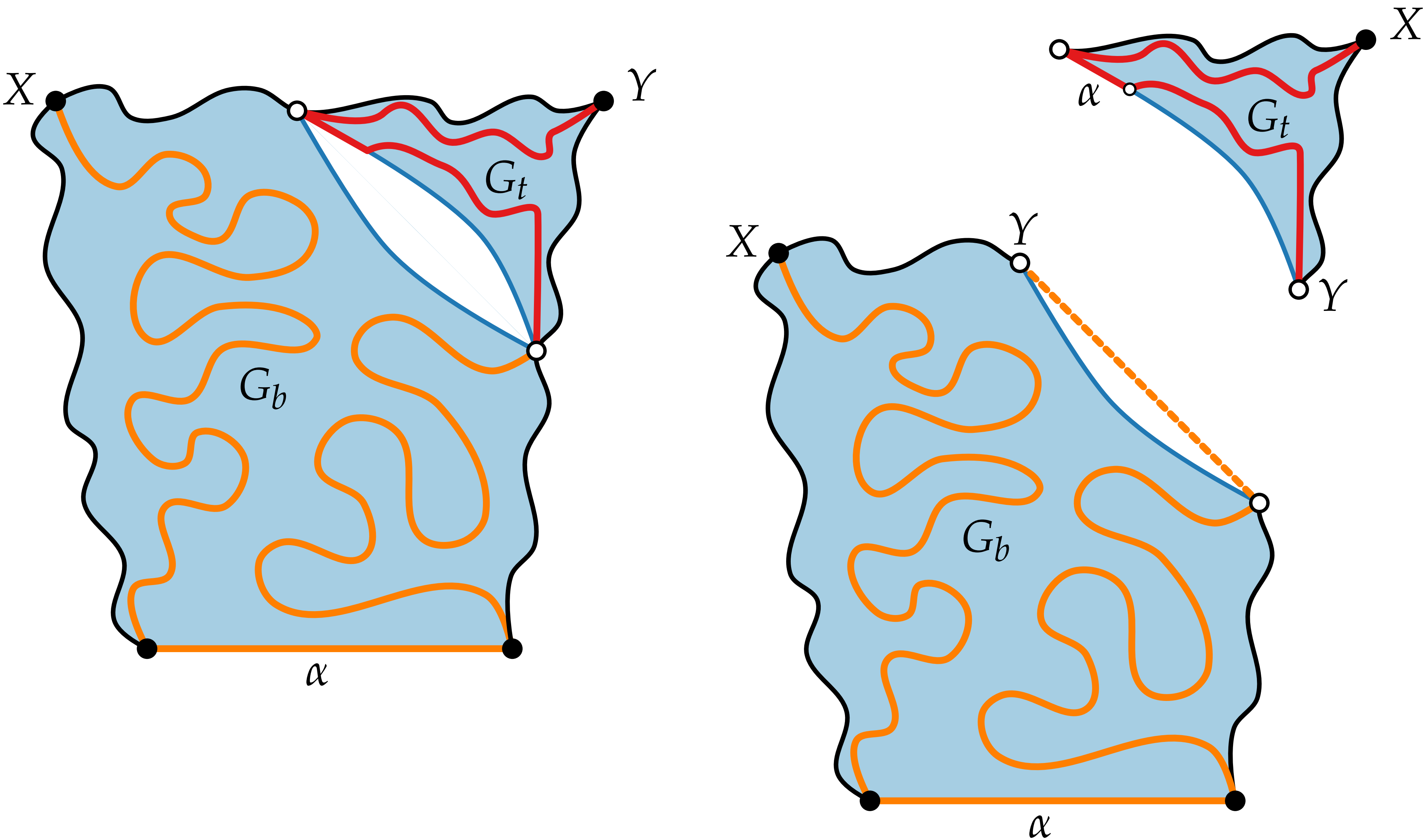




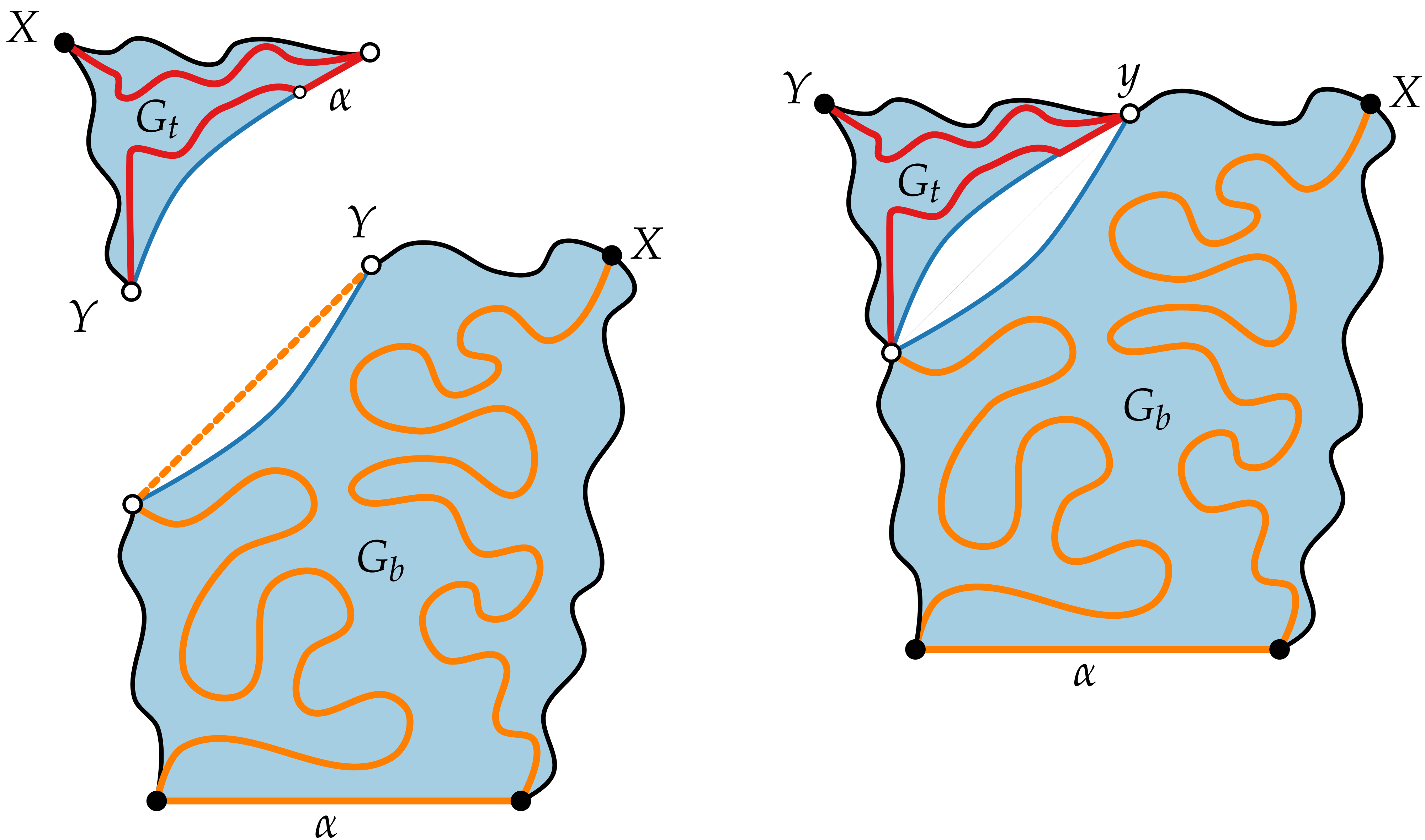
Case 3: top-right cutting pair



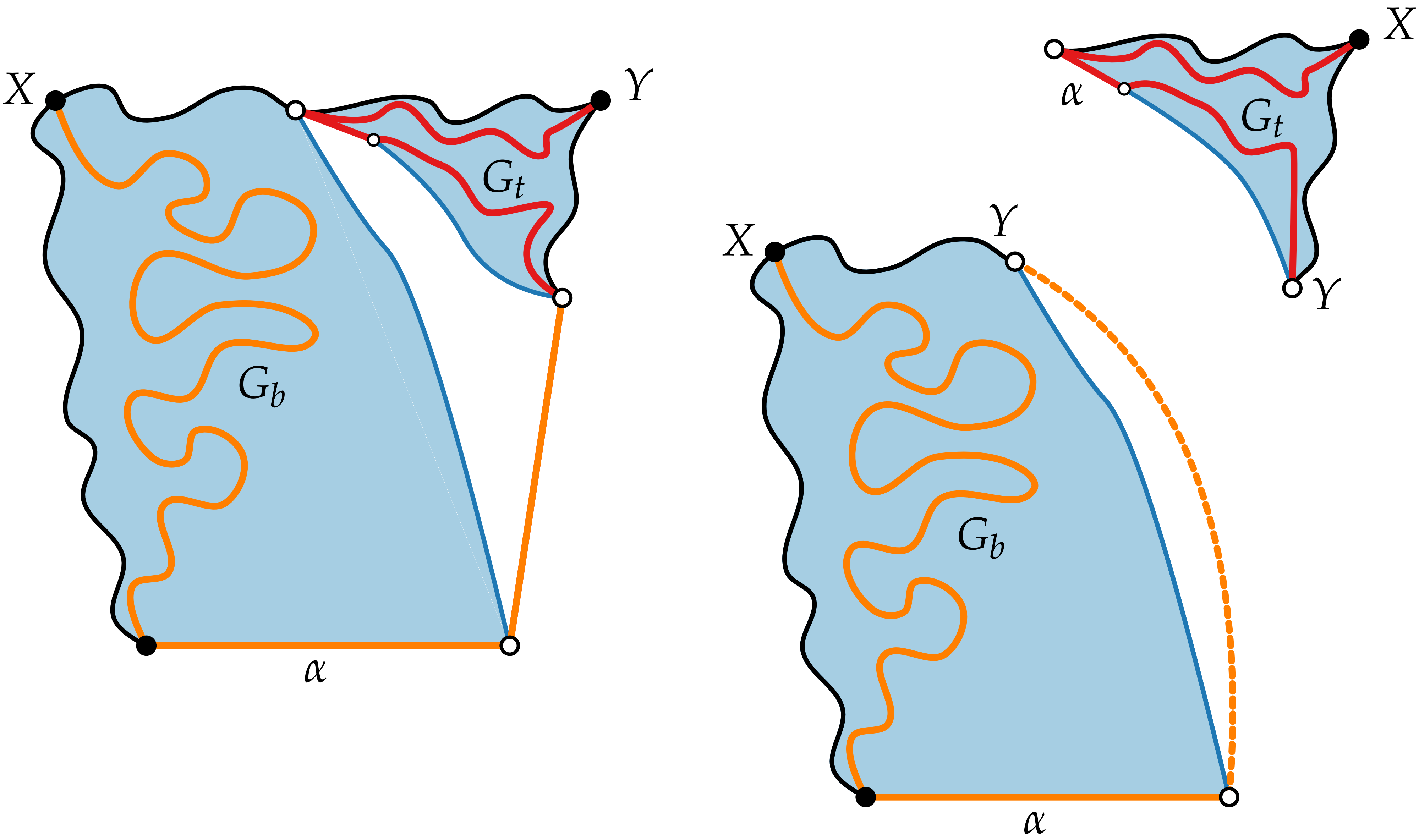
Case 3: top-right cutting pair



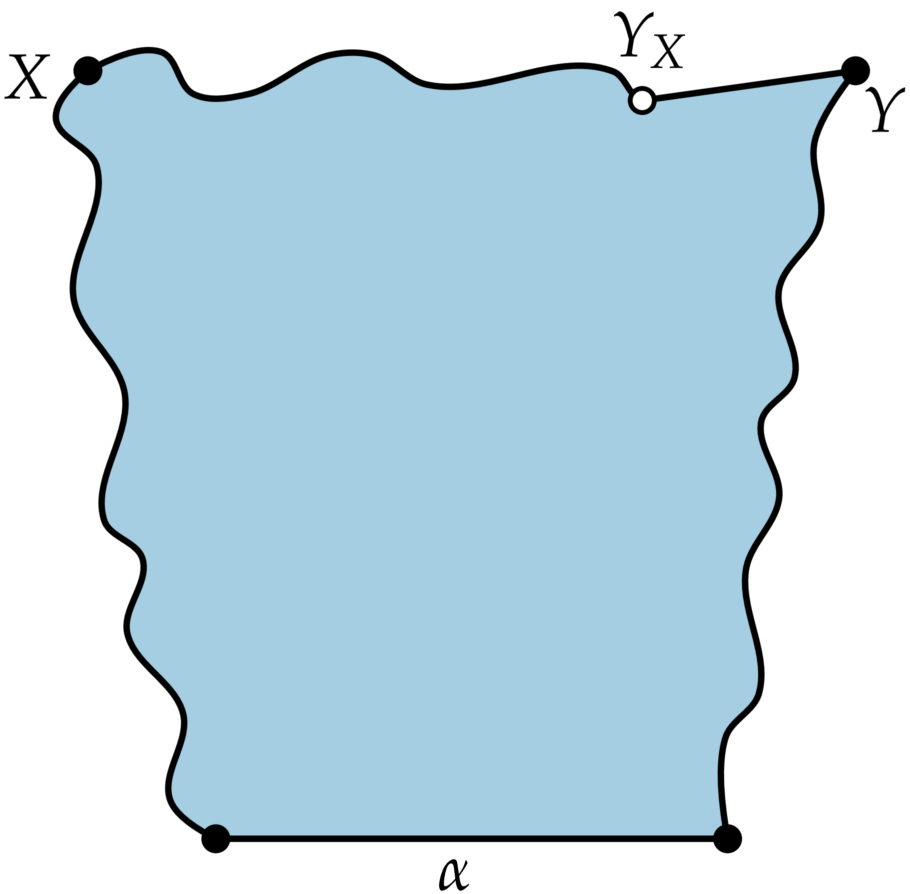
Case 3′: top-left cutting pair



Case 3″: top-bottom cutting pair

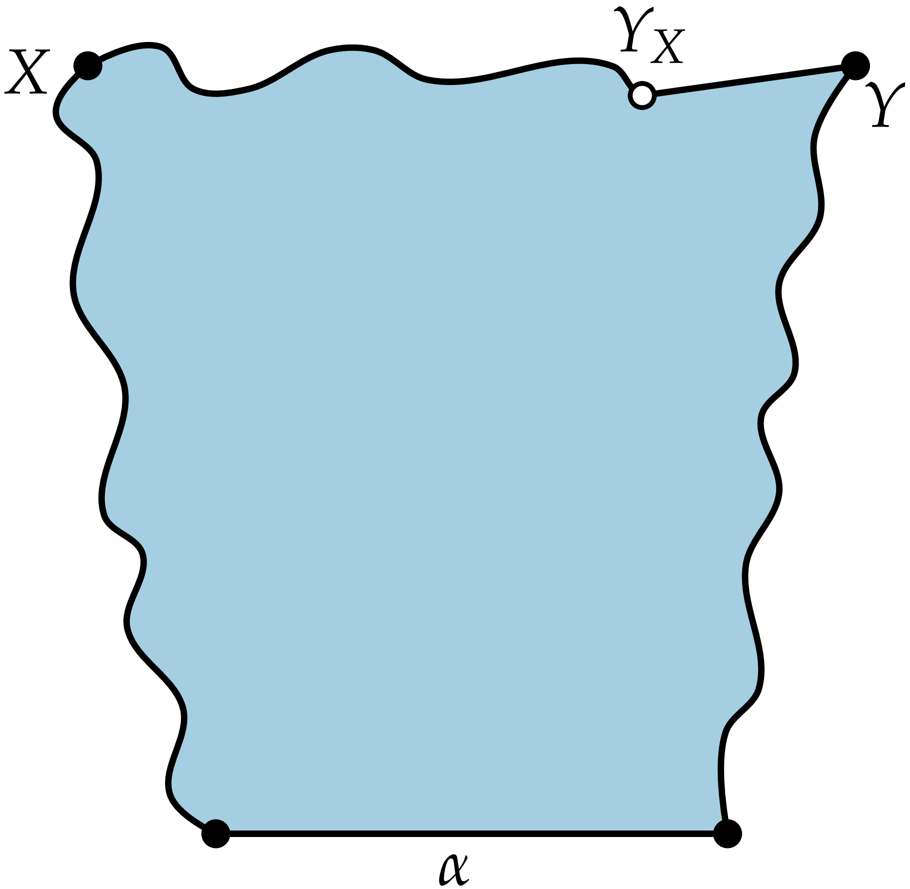


Case 4: No cutting pair



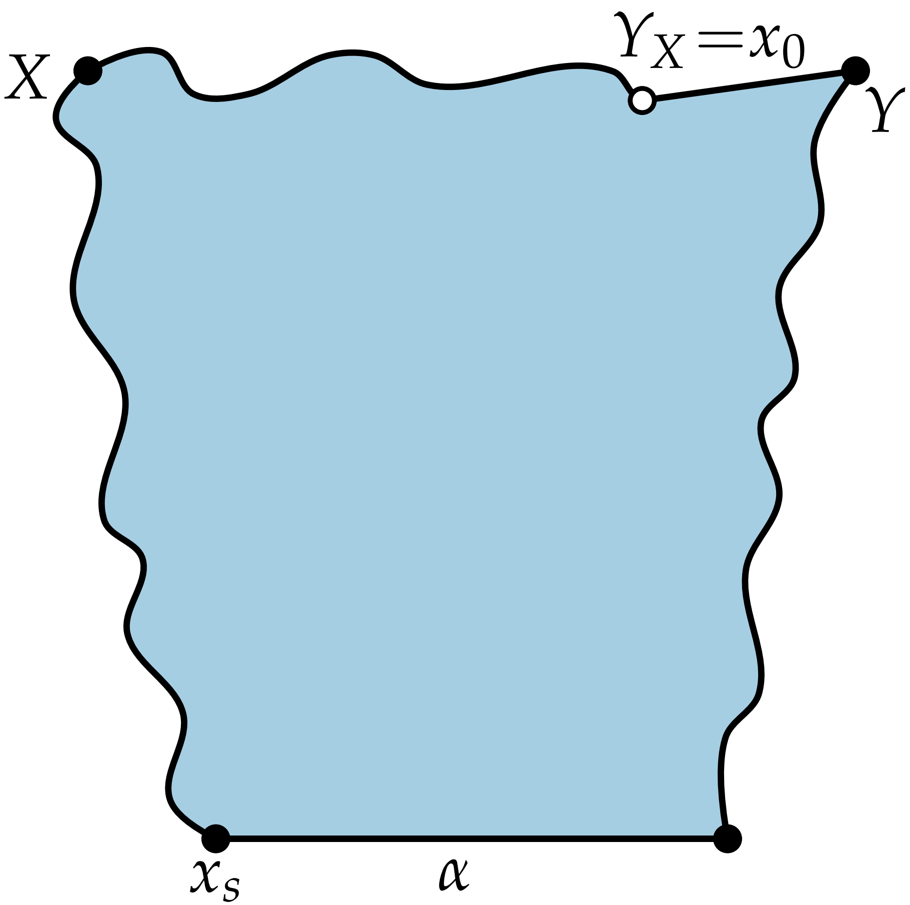
Case 4: No cutting pair

Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$



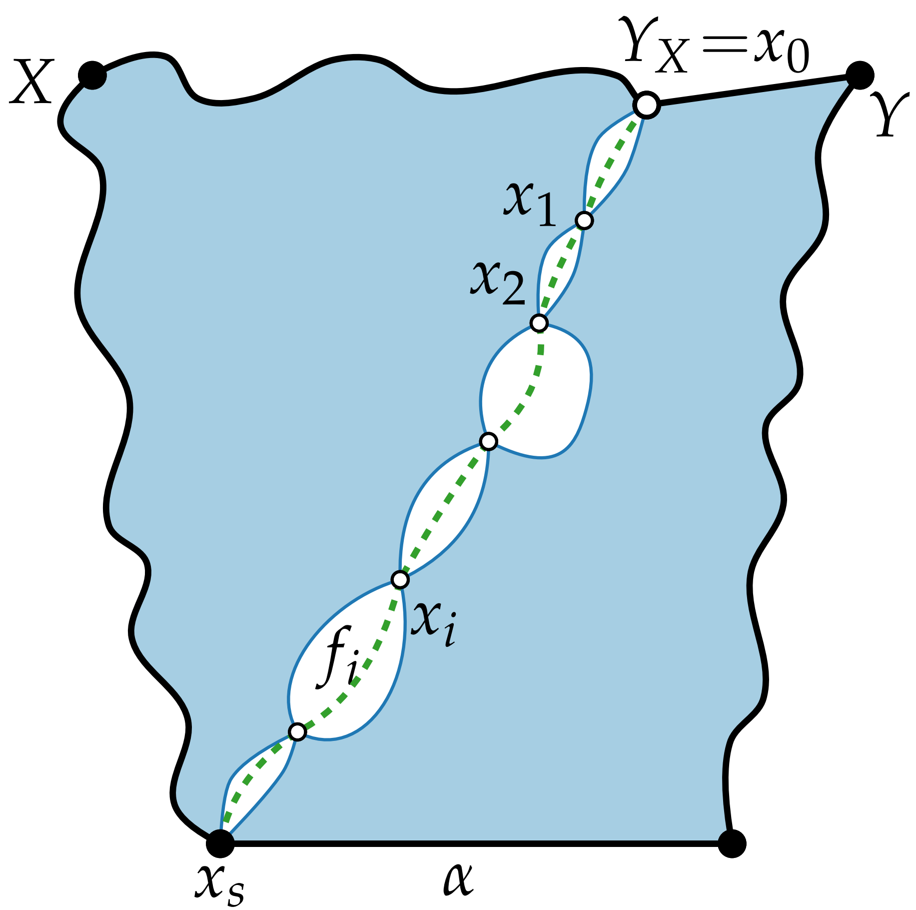
Case 4: No cutting pair

Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$



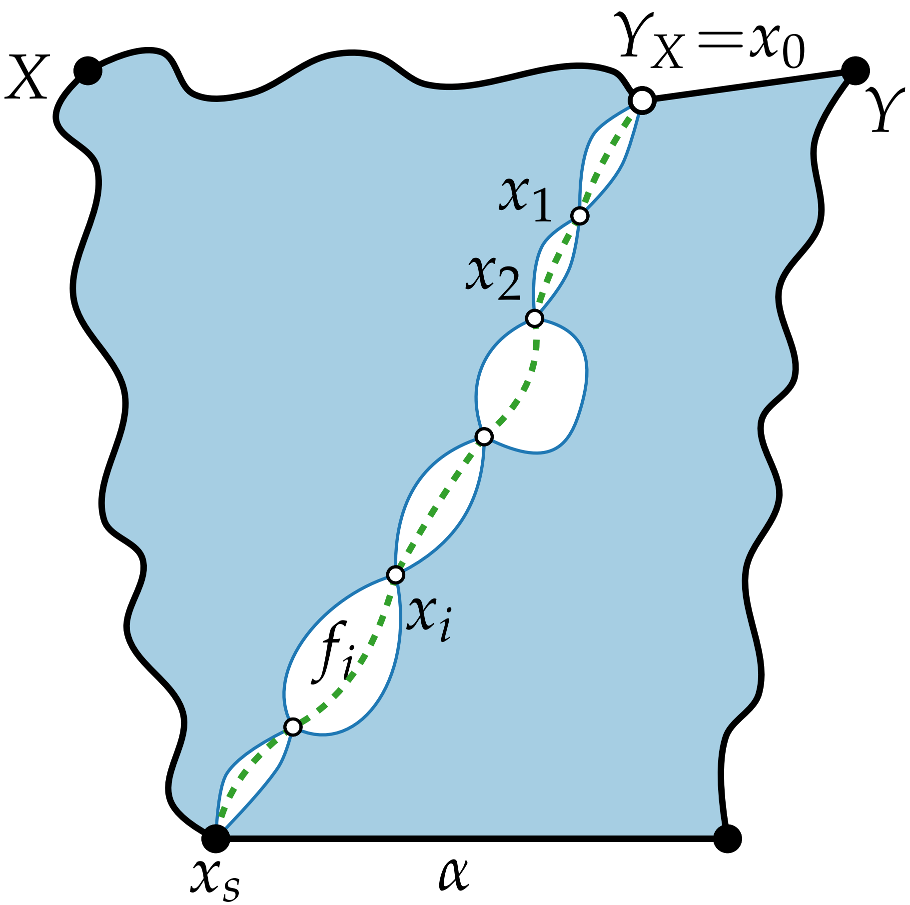
Case 4: No cutting pair

Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$



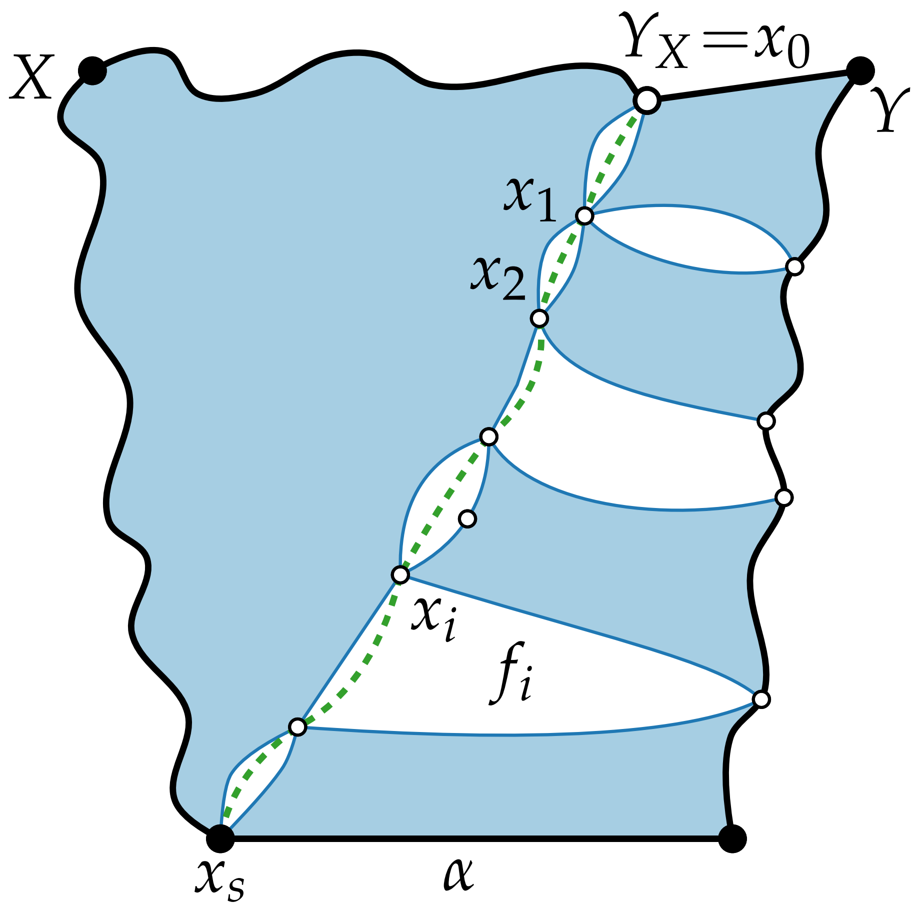
Case 4: No cutting pair

Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$, x_i face-adj. to right side



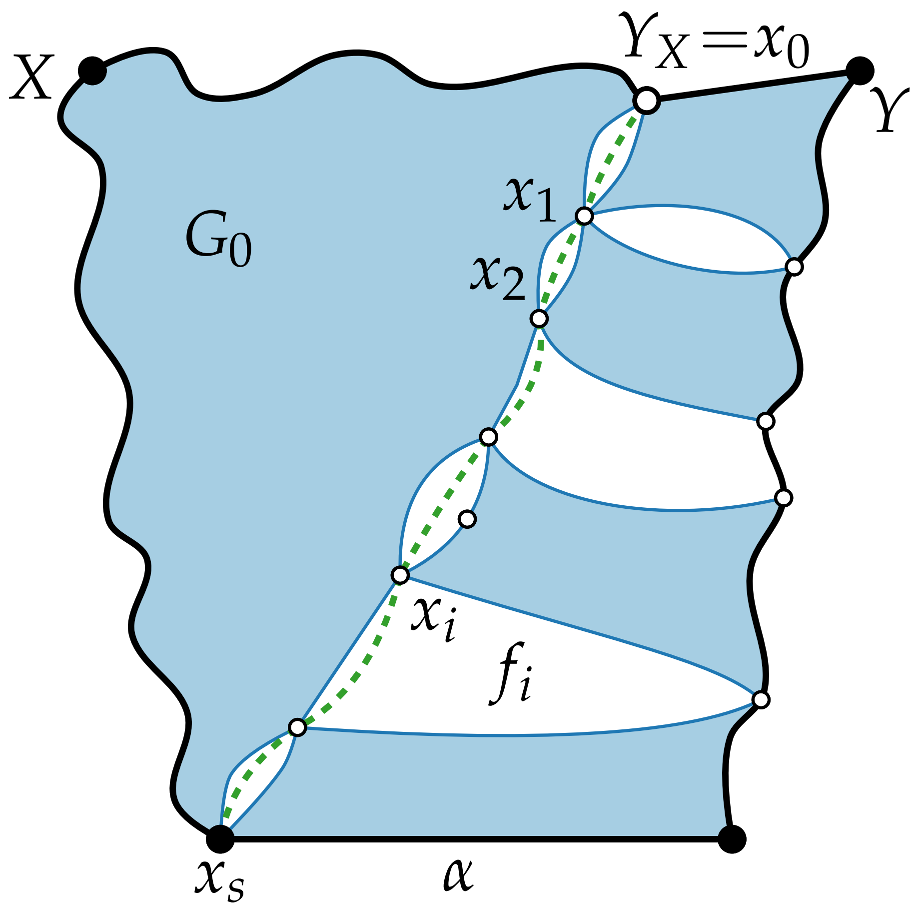
Case 4: No cutting pair

Necklace $\langle Y_X{=}x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side



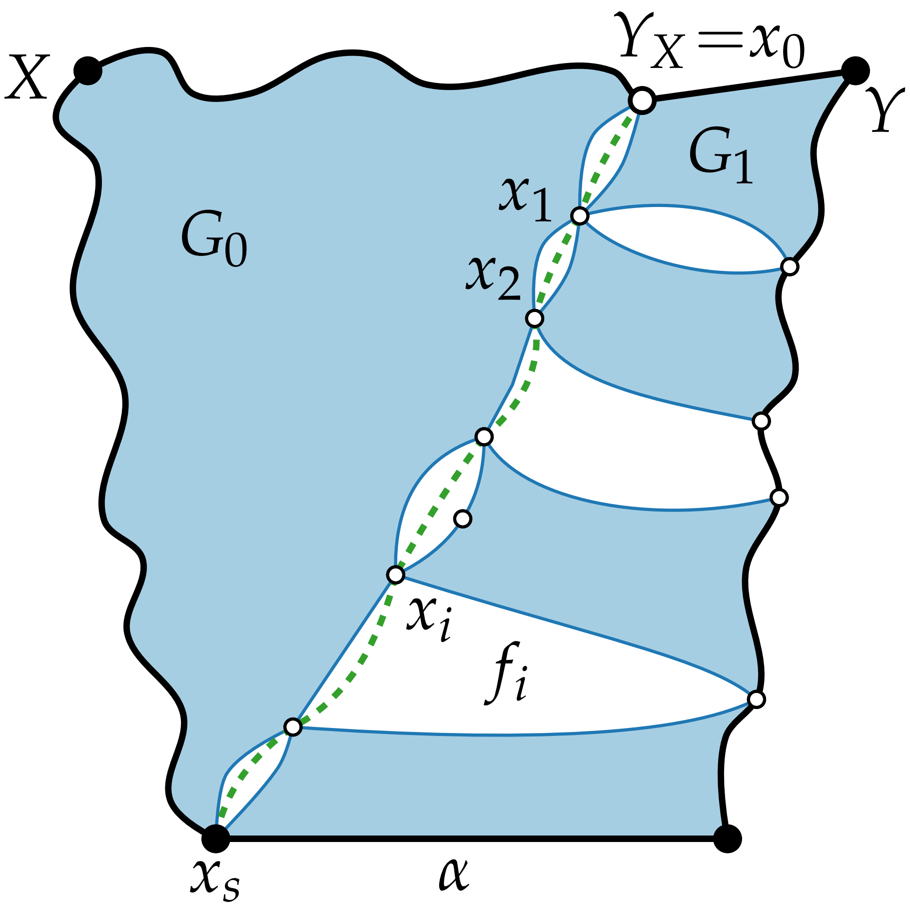
Case 4: No cutting pair

Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side



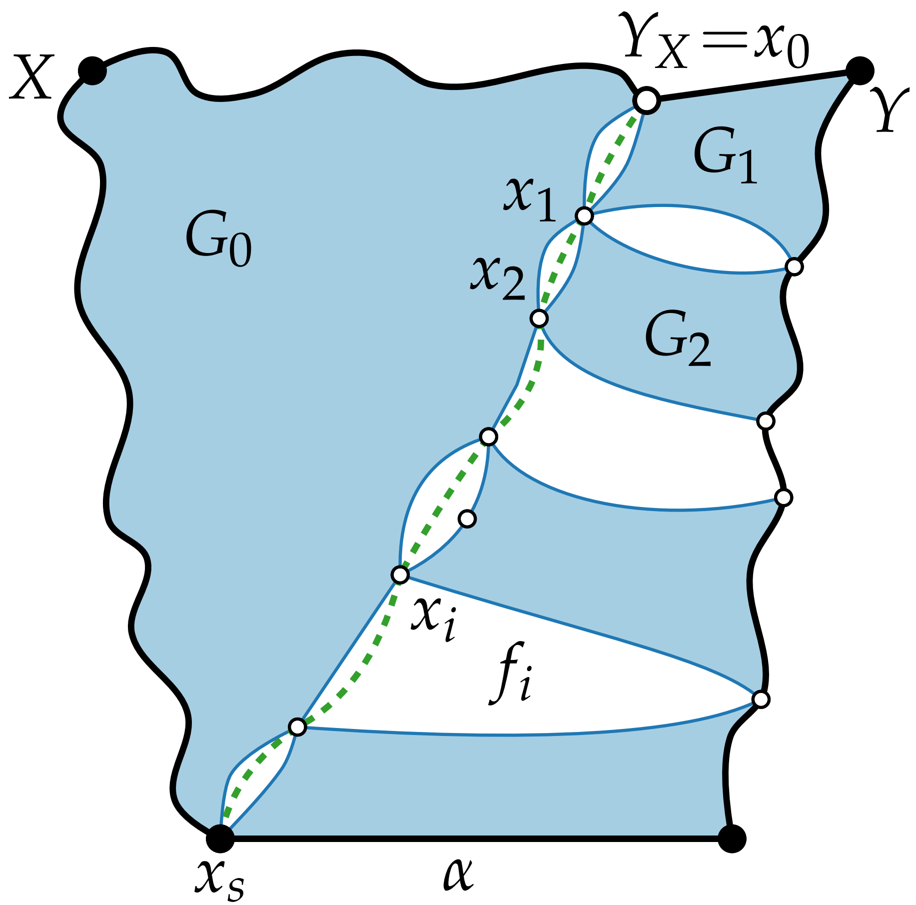
Case 4: No cutting pair

Necklace $\langle Y_X{=}x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side



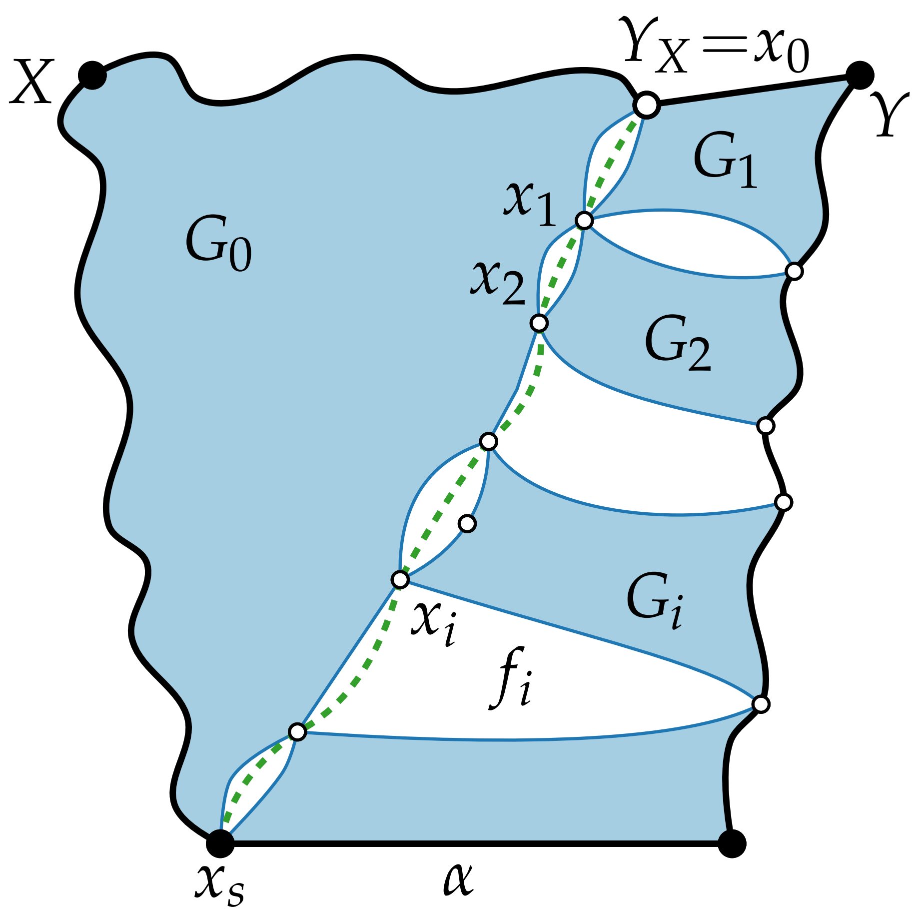
Case 4: No cutting pair

Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side



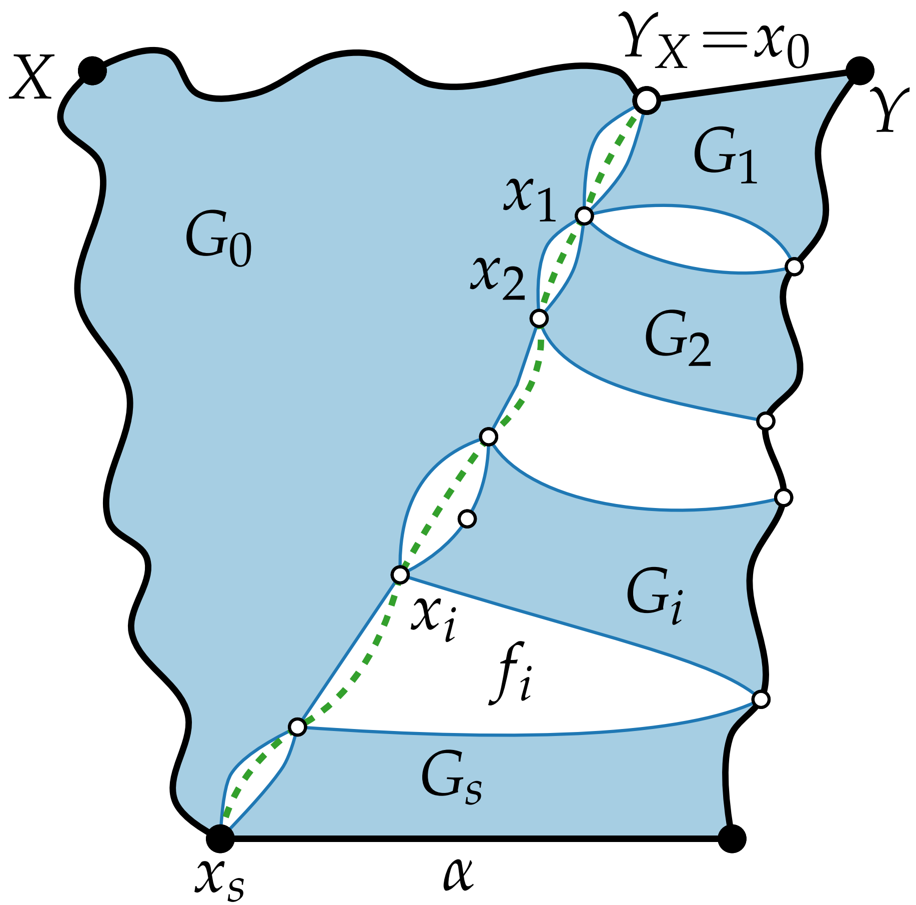
Case 4: No cutting pair

Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side



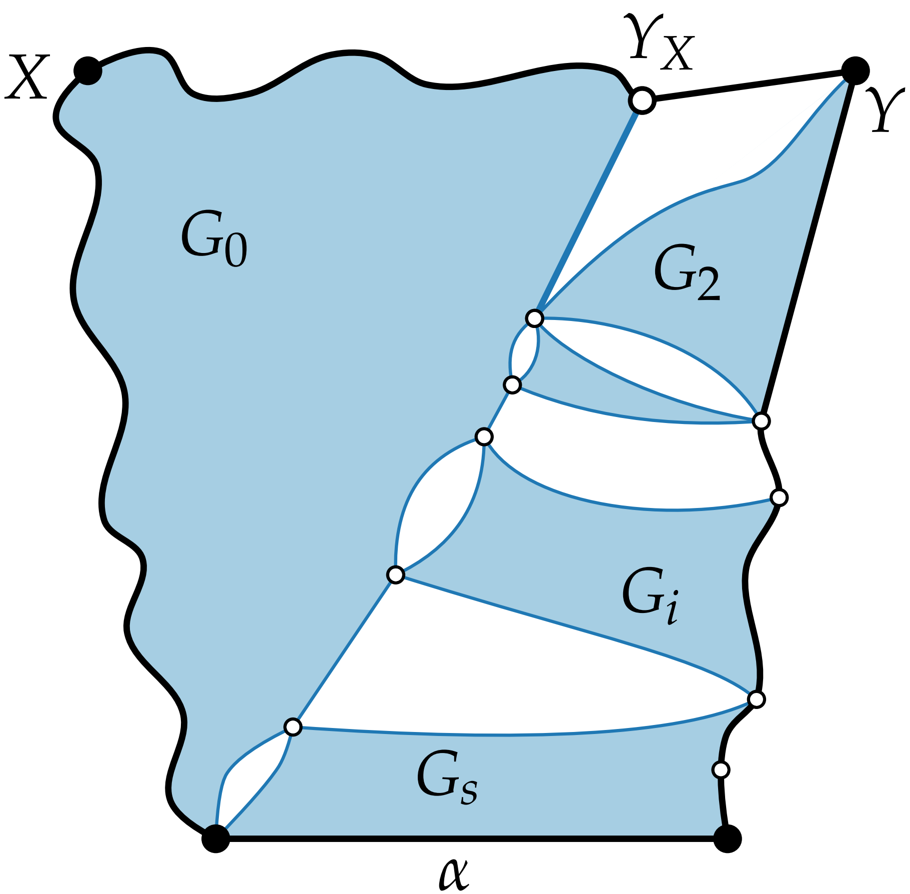
Case 4: No cutting pair

Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$, x_i face-adj. to right side



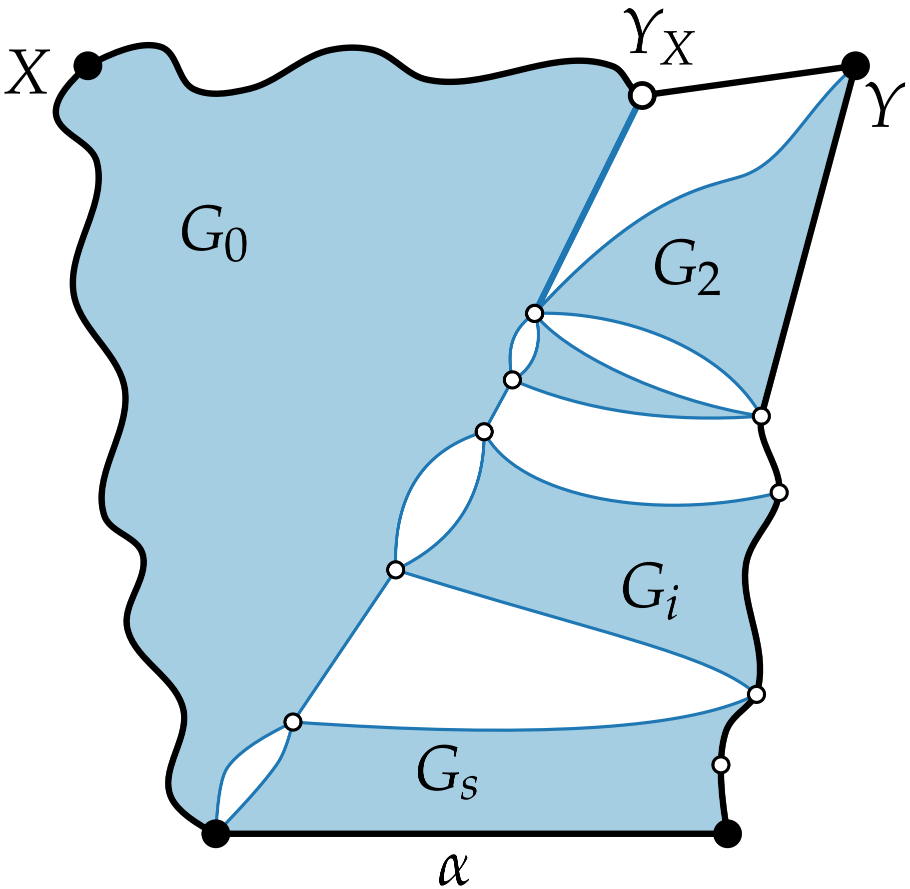
Case 4: No cutting pair

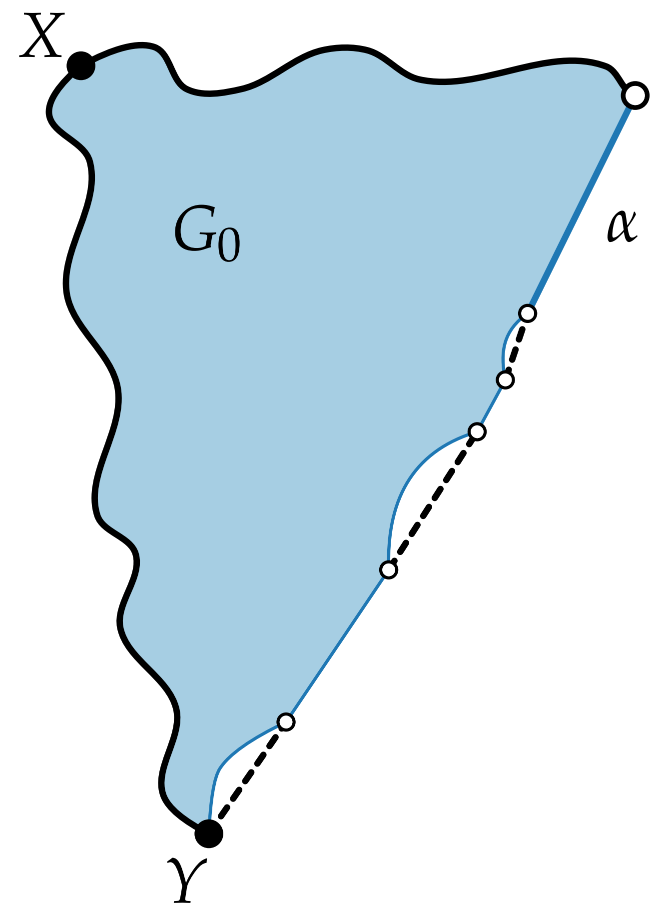
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$



Case 4: No cutting pair

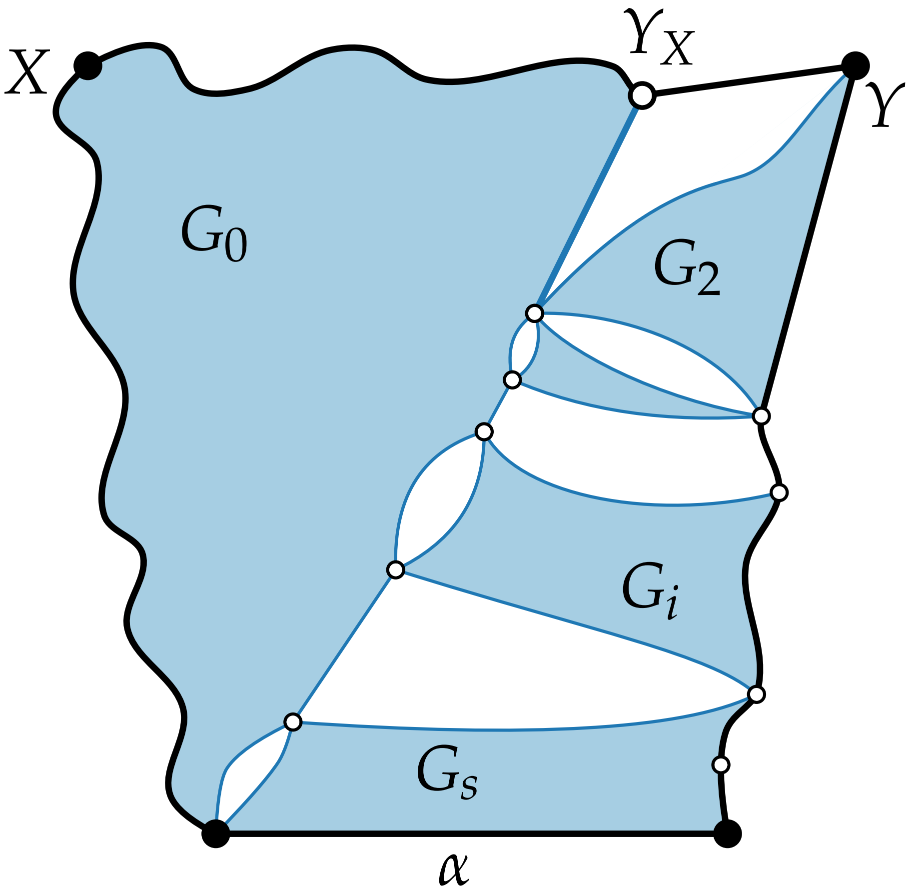
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$

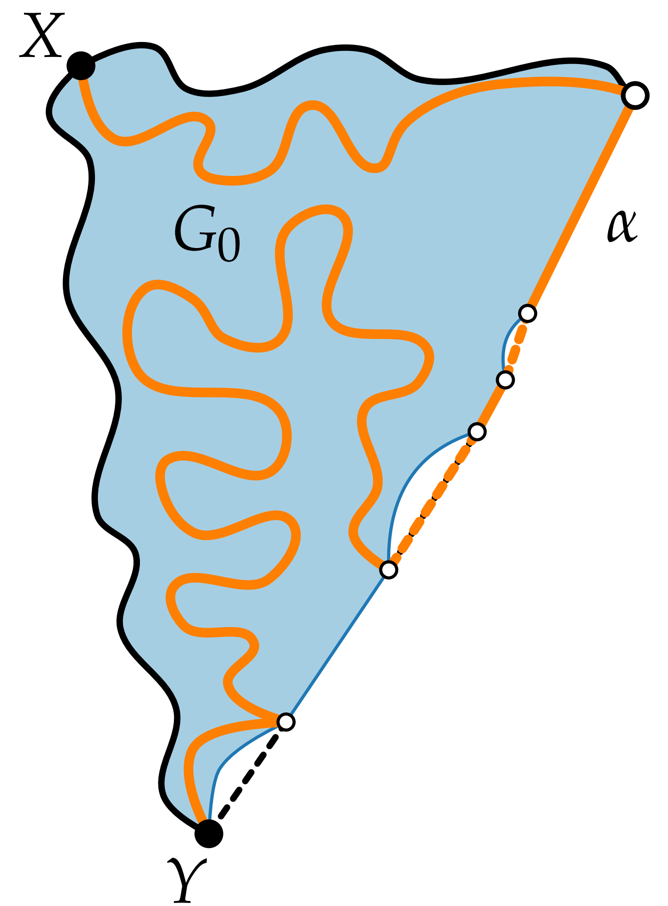




Case 4: No cutting pair

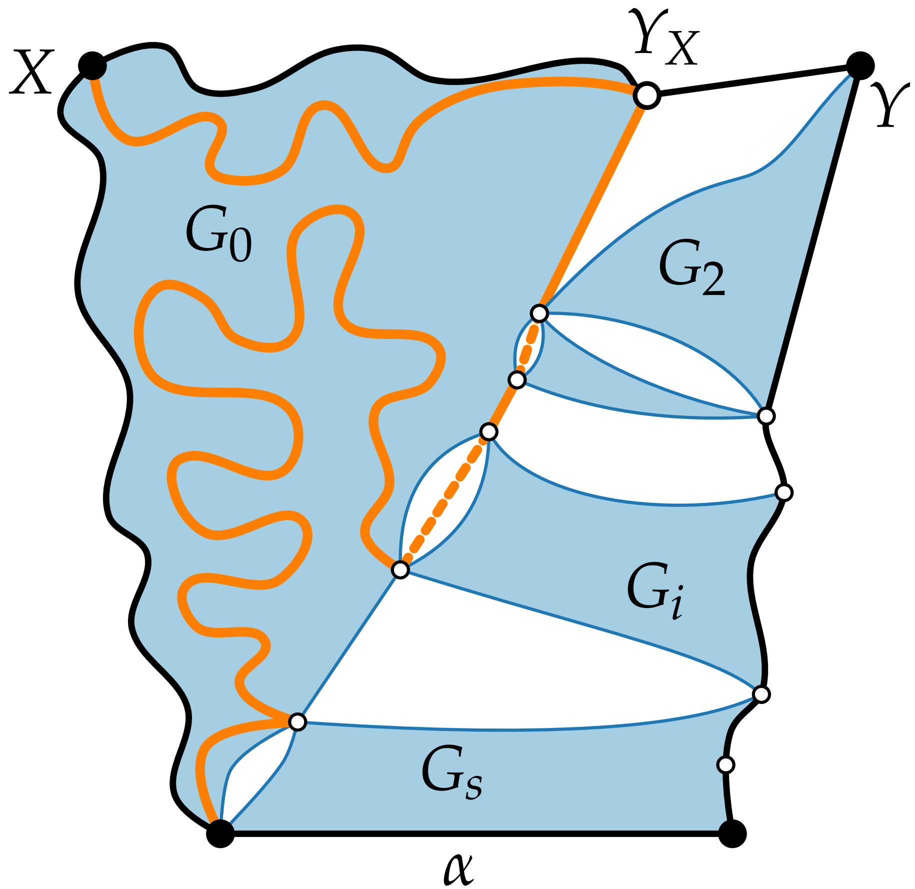
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$

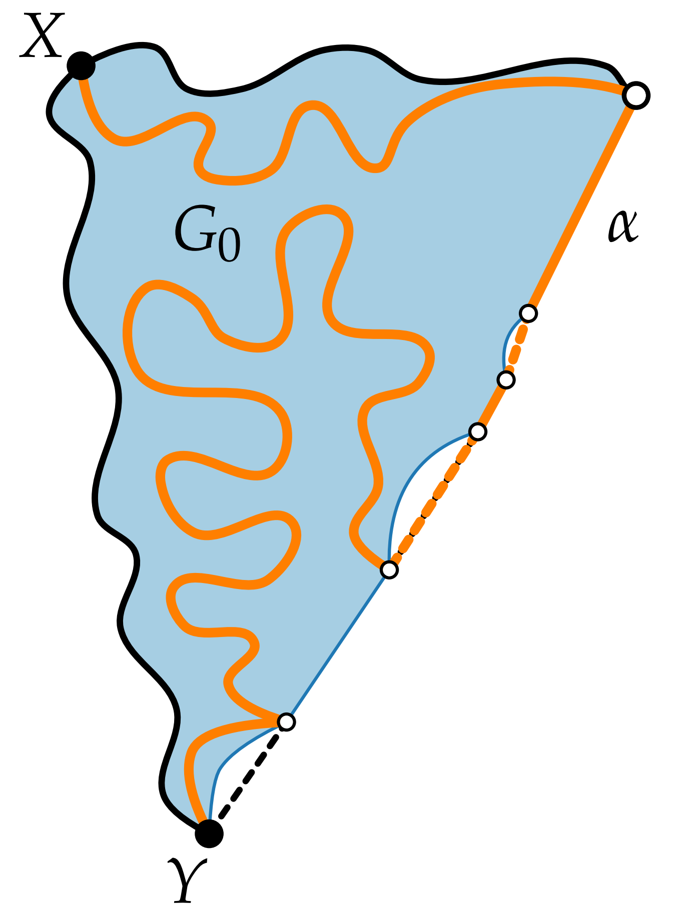




Case 4: No cutting pair

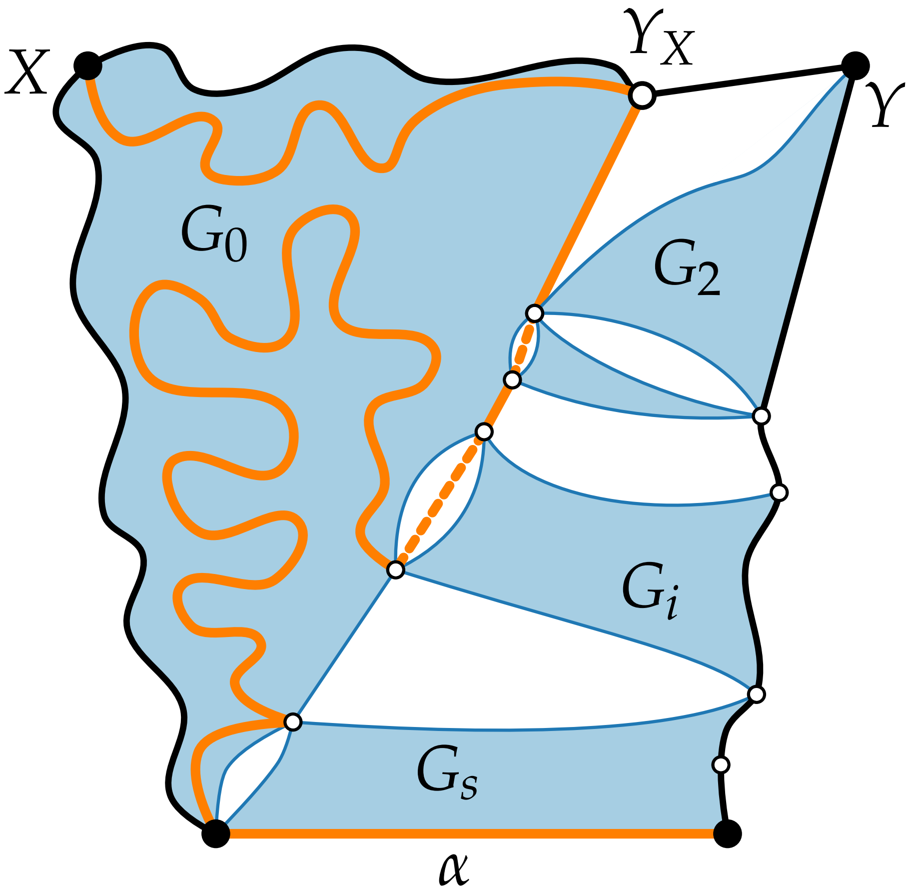
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$

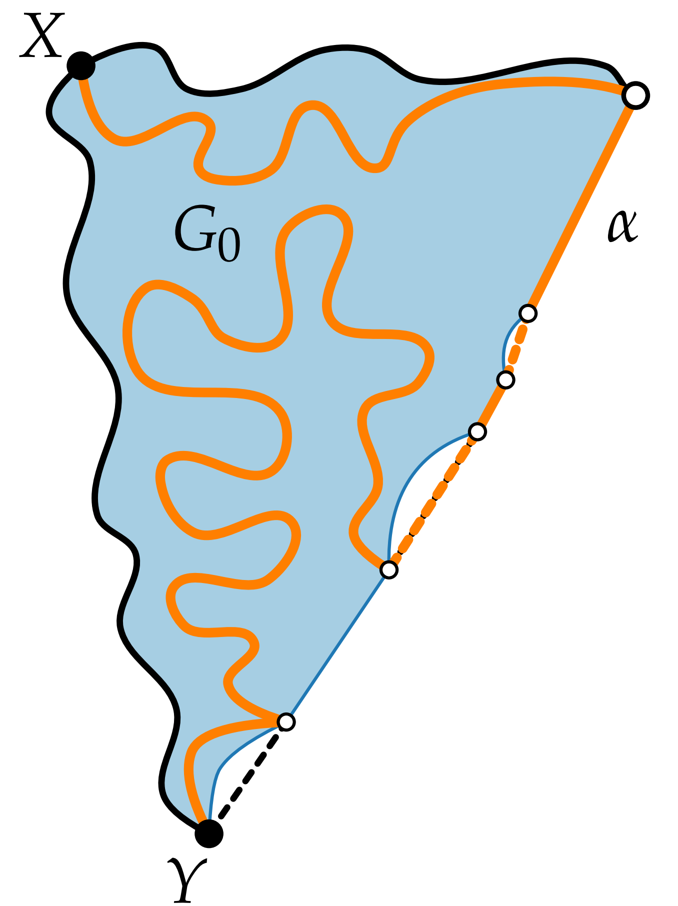




Case 4: No cutting pair

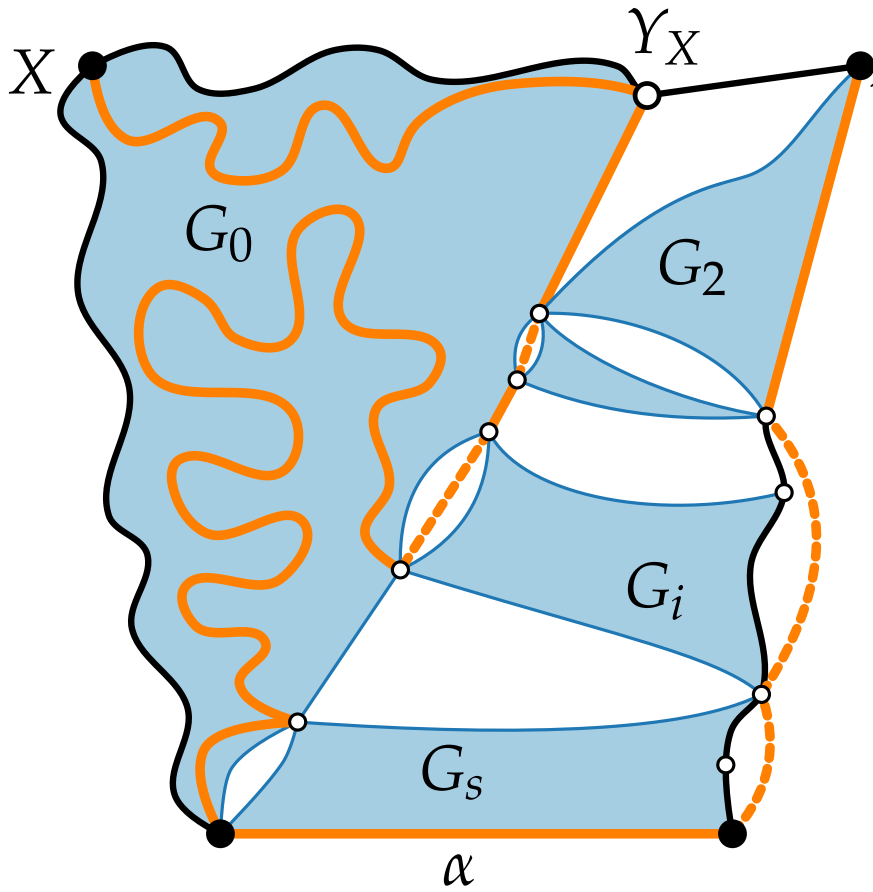
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$

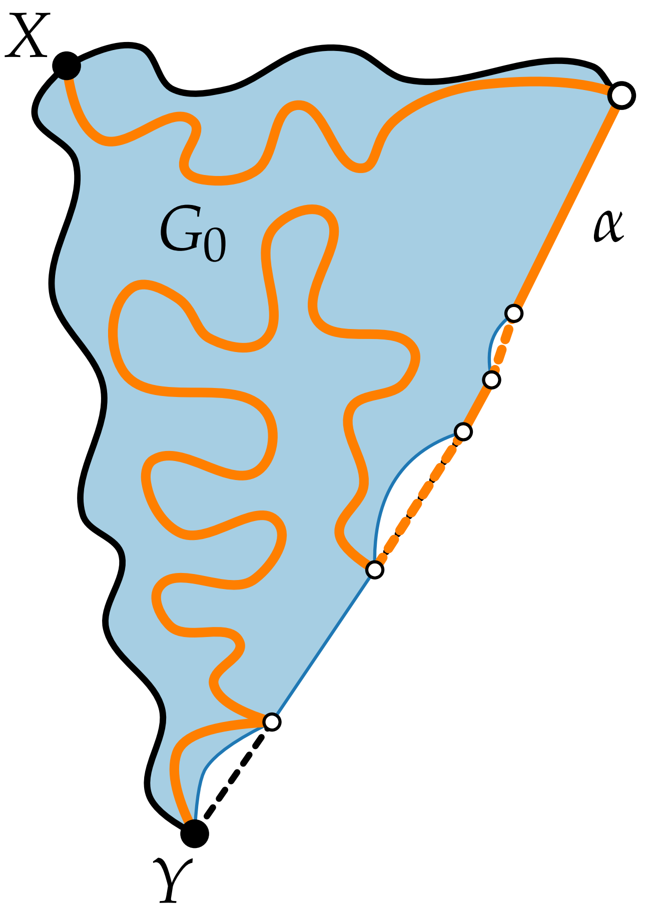




Case 4: No cutting pair

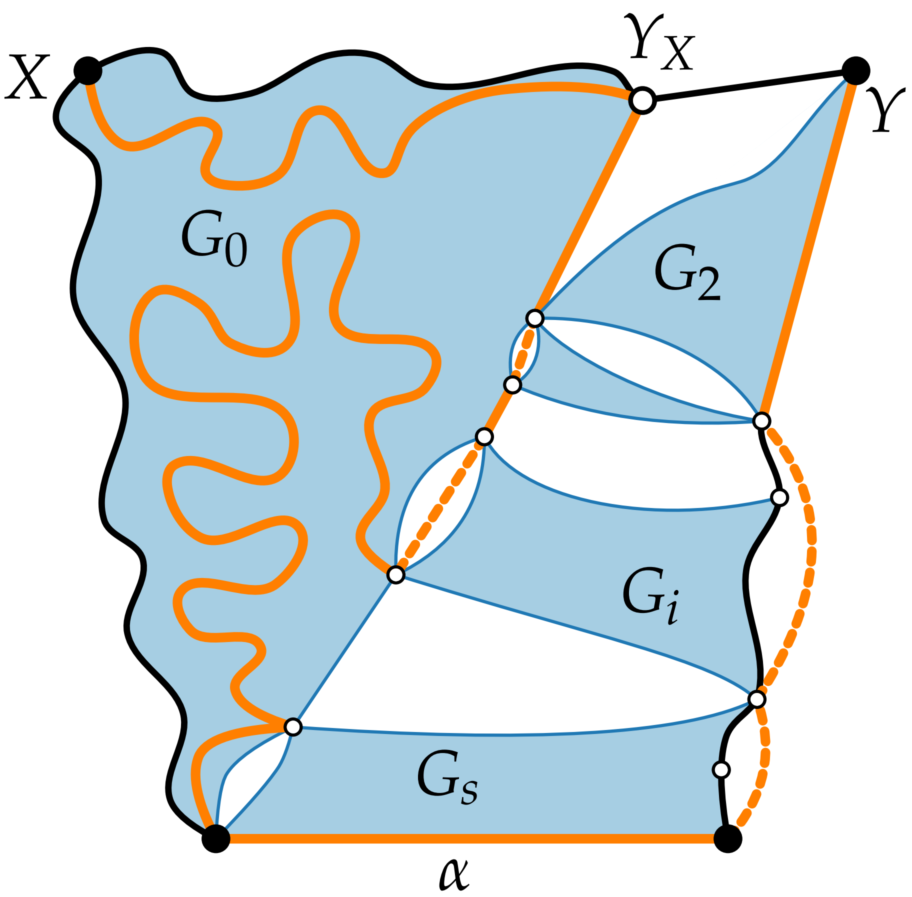
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$, x_i face-adj. to right side, $G_1 = \emptyset$





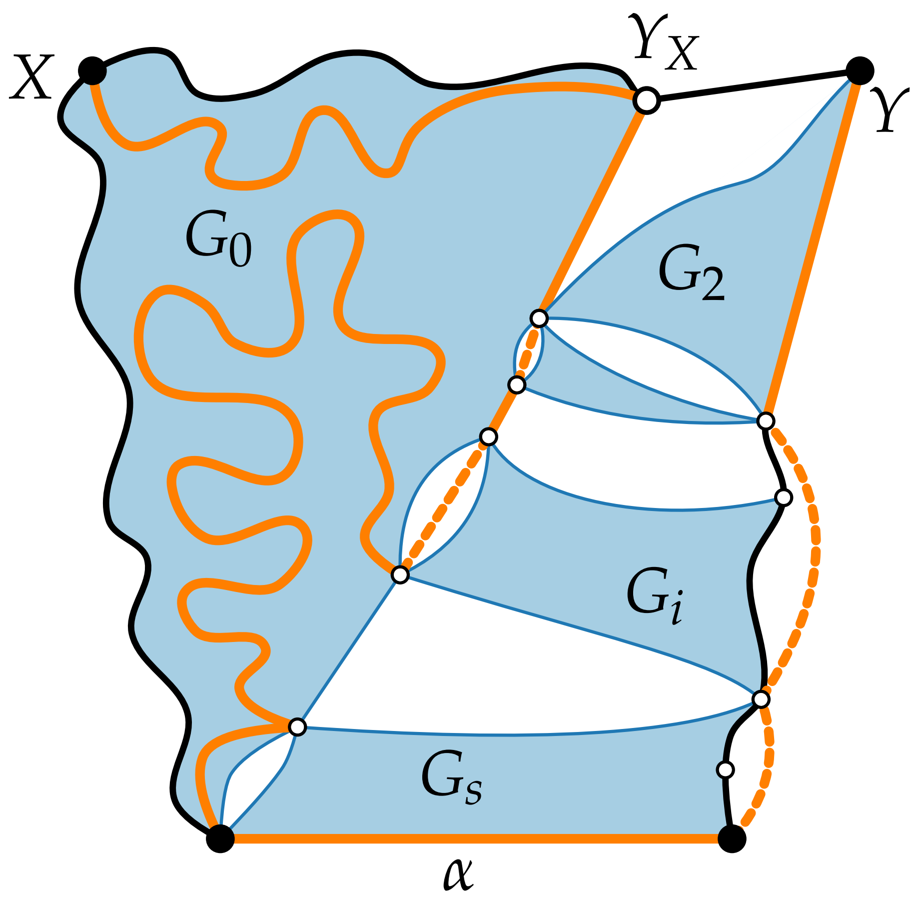
Case 4: No cutting pair

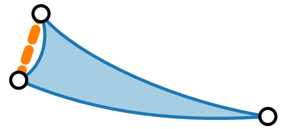
Necklace $\langle Y_X\!=\!x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$



Case 4: No cutting pair

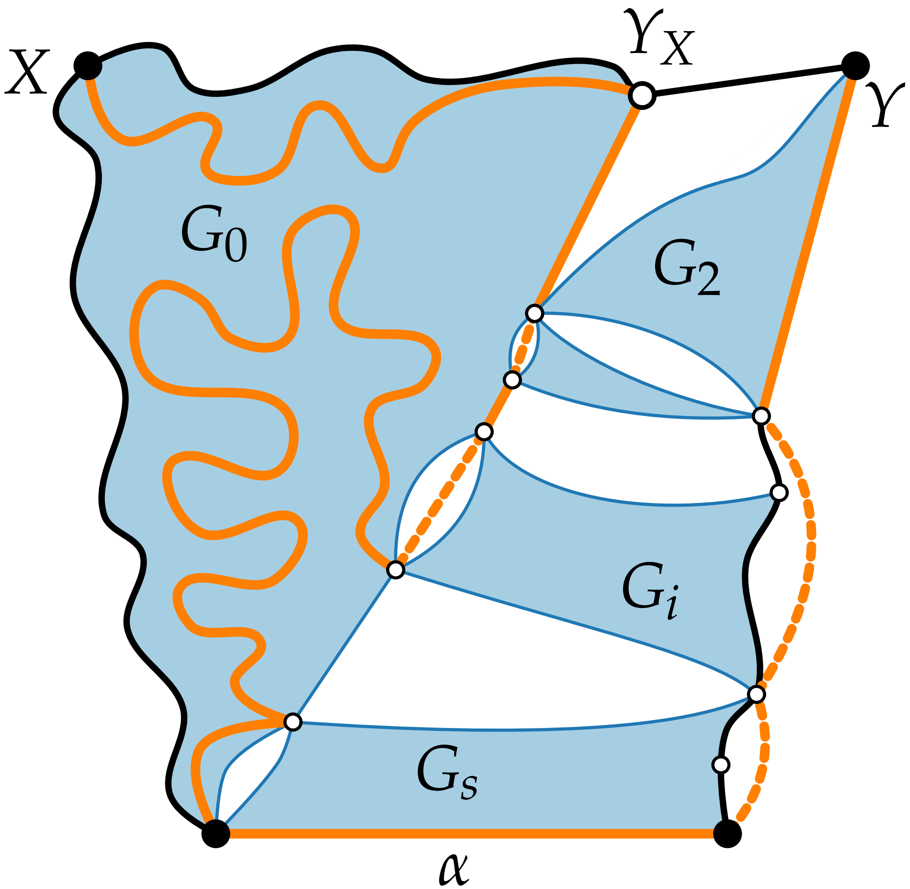
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$

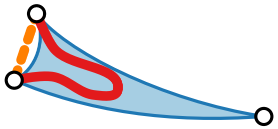




Case 4: No cutting pair

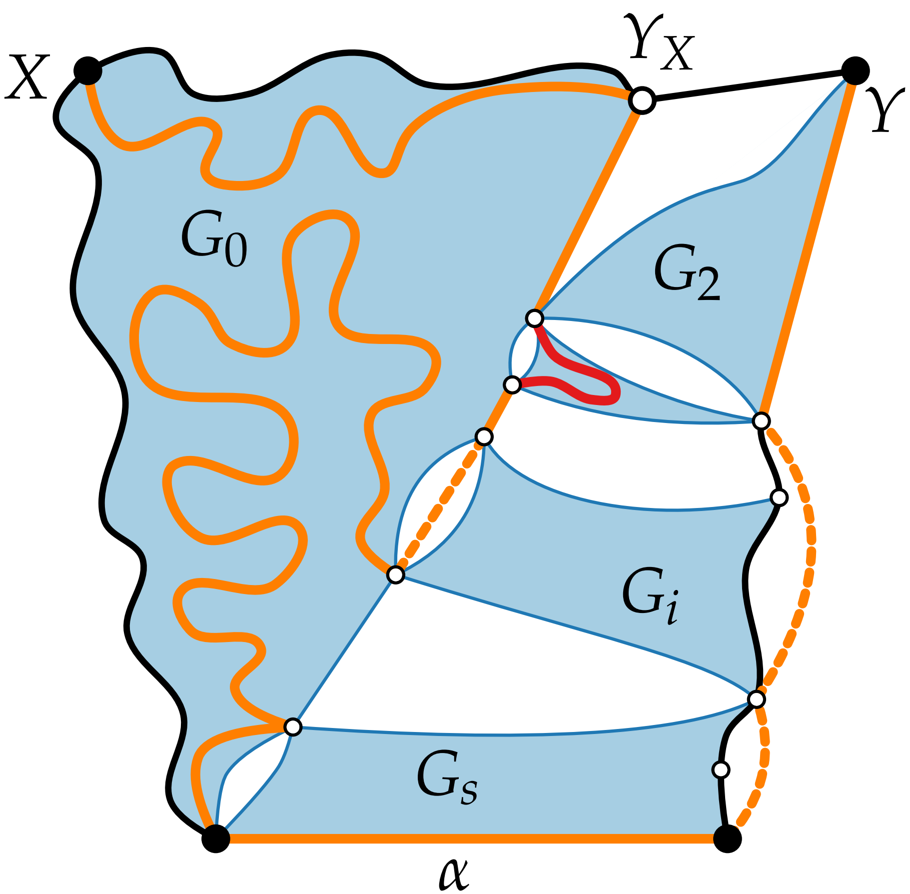
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$

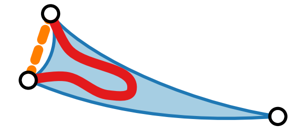




Case 4: No cutting pair

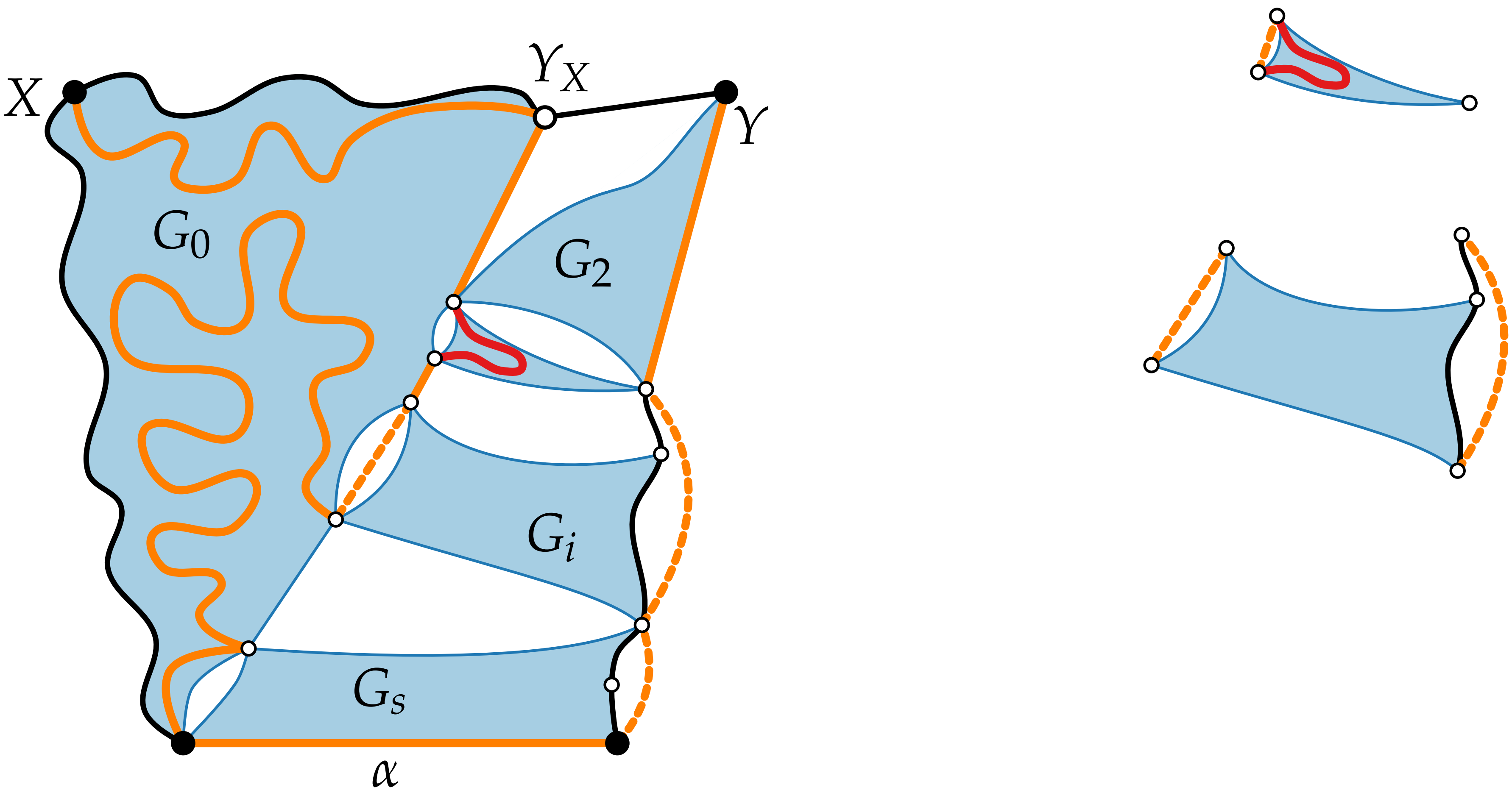
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$





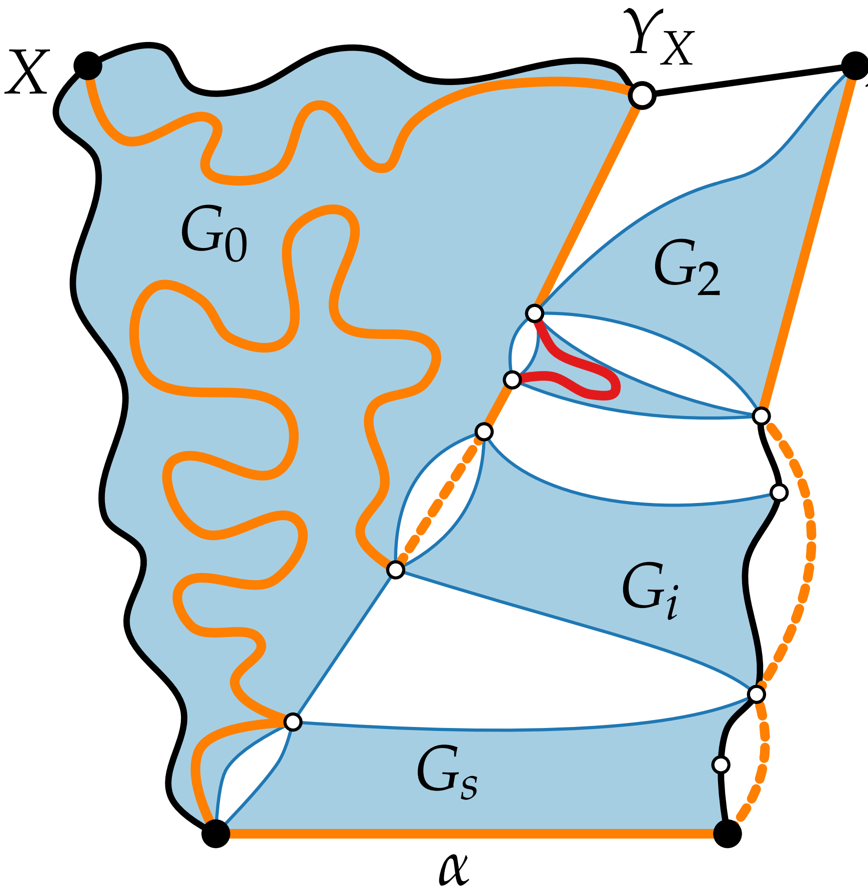
Case 4: No cutting pair

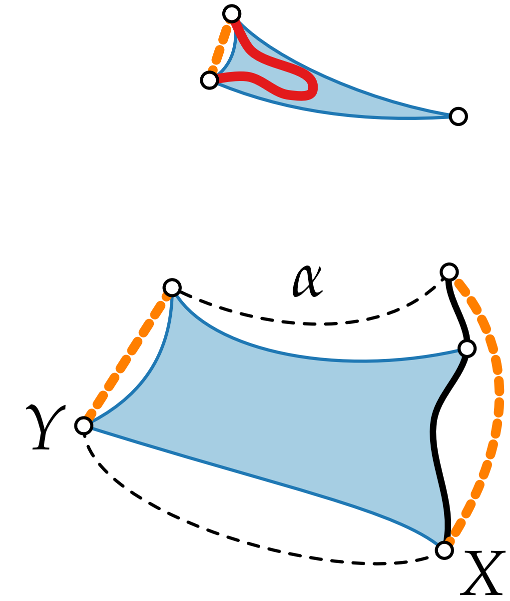
Necklace $\langle Y_X{=}x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s\rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$



Case 4: No cutting pair

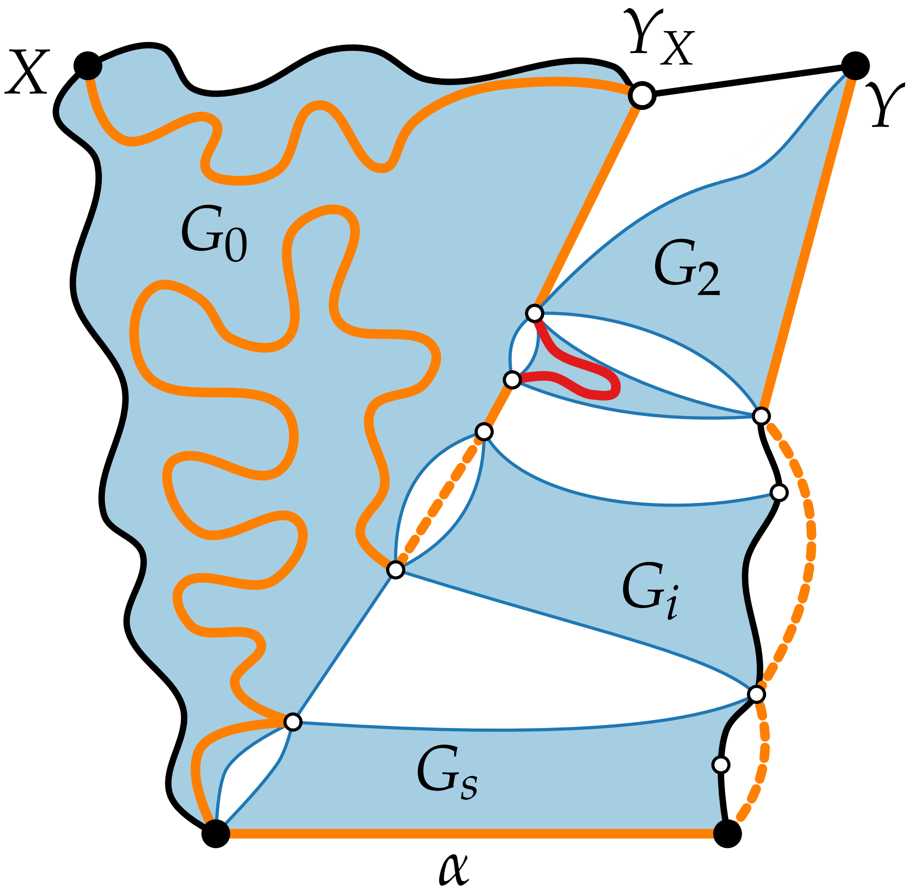
Necklace $\langle Y_X\!=\!x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$

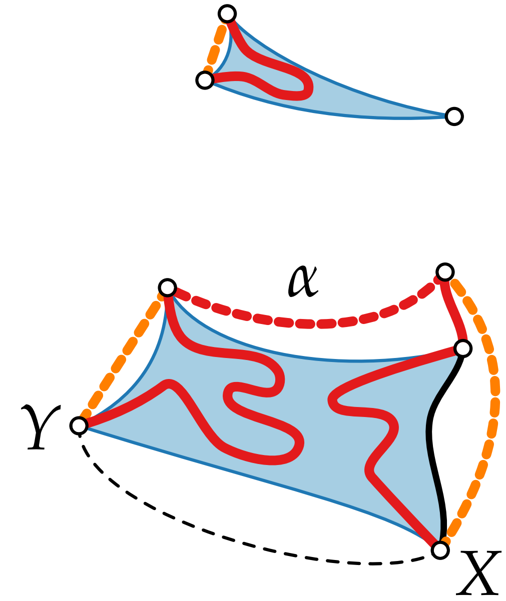




Case 4: No cutting pair

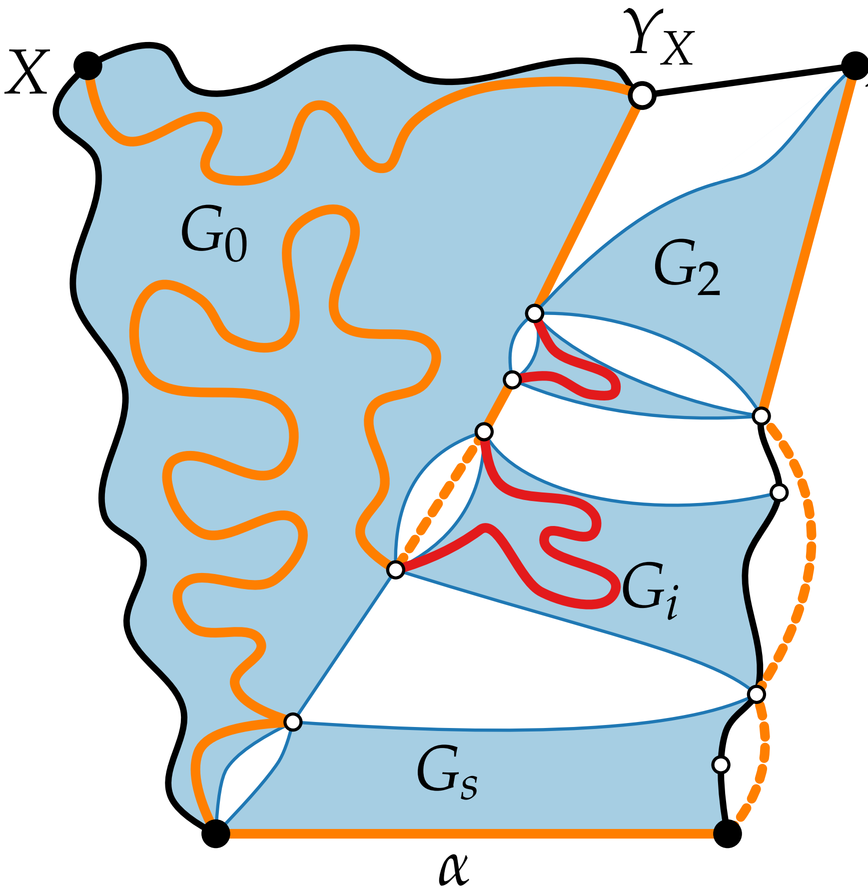
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$

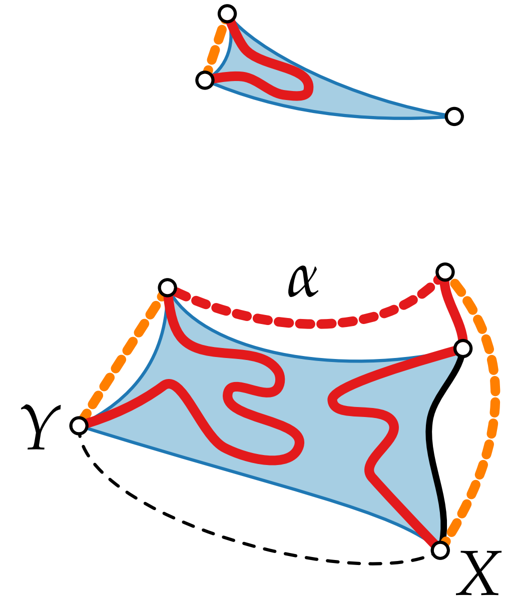




Case 4: No cutting pair

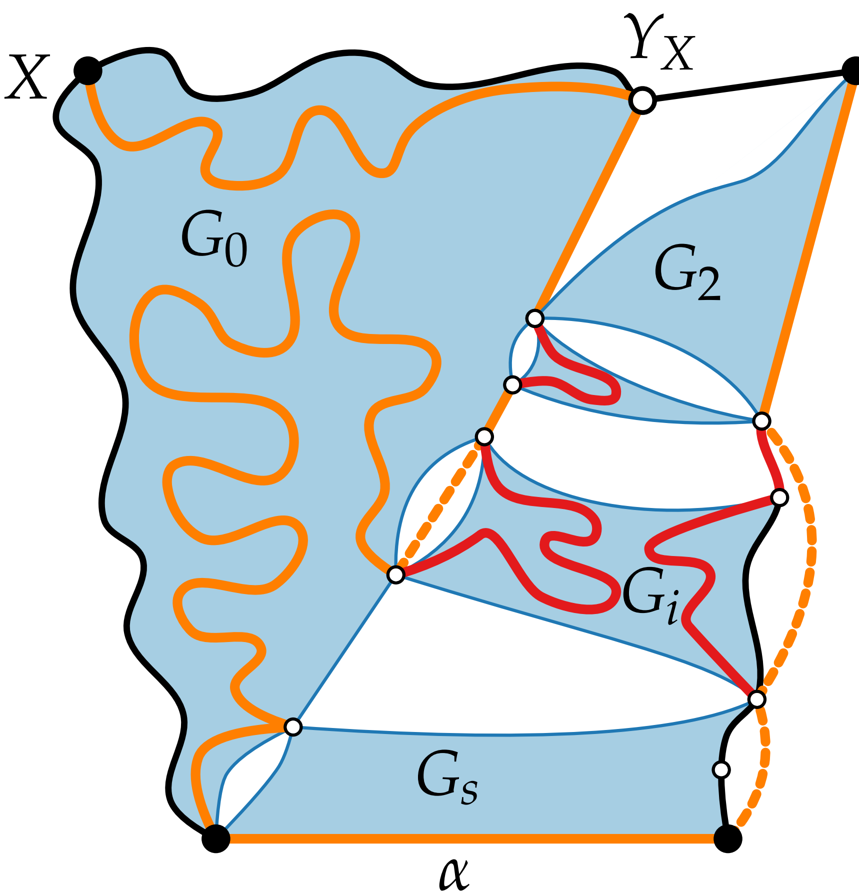
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$

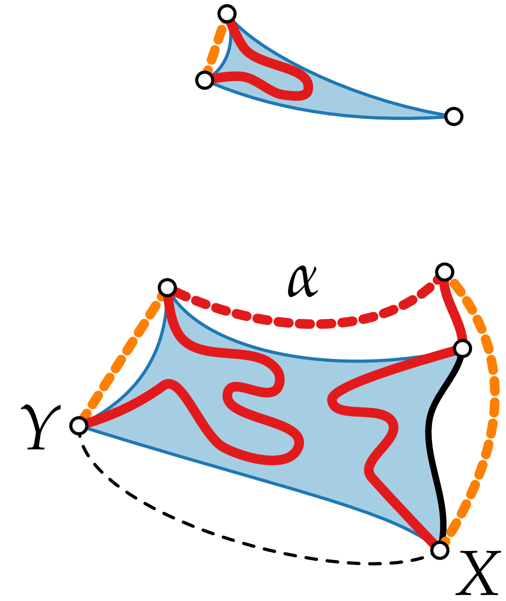




Case 4: No cutting pair

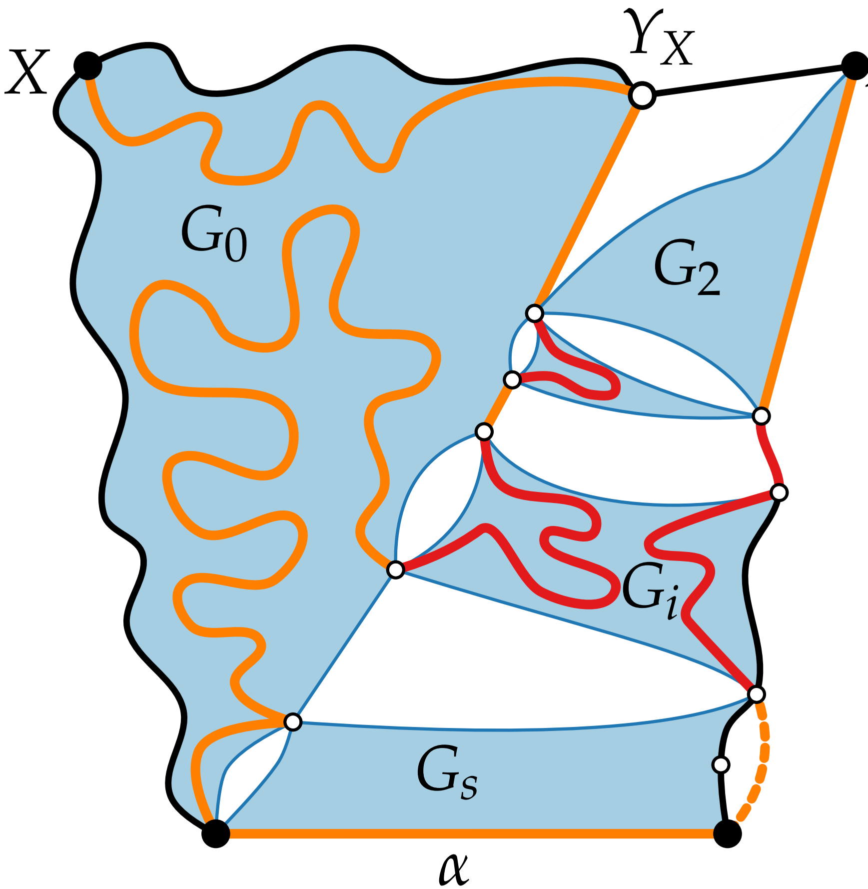
Necklace $\langle Y_X{=}x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$

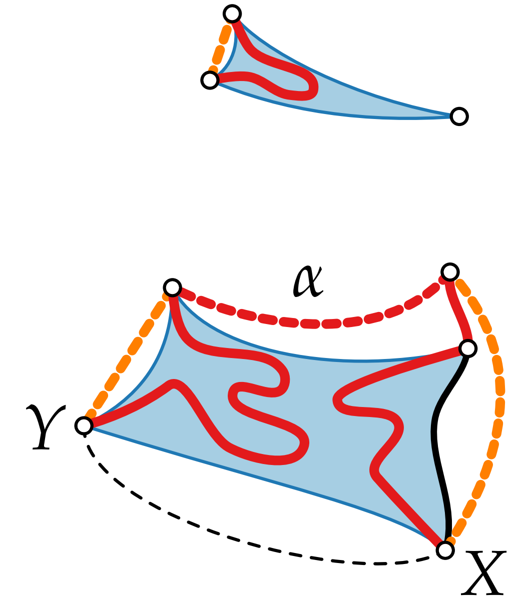




Case 4: No cutting pair

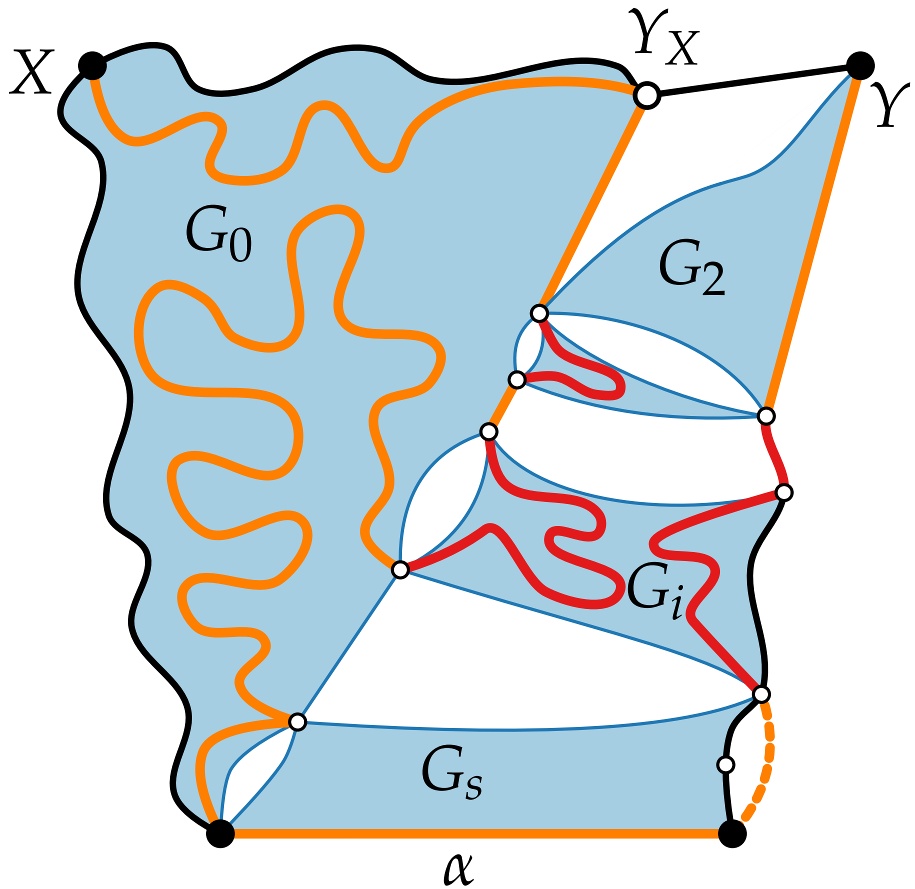
Necklace $\langle Y_X\!=\!x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$

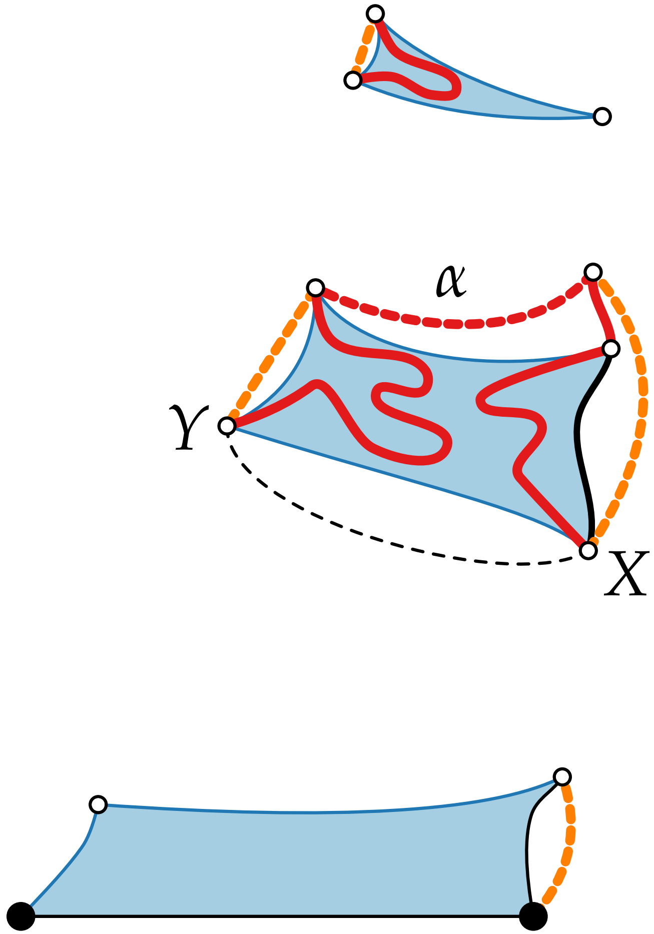




Case 4: No cutting pair

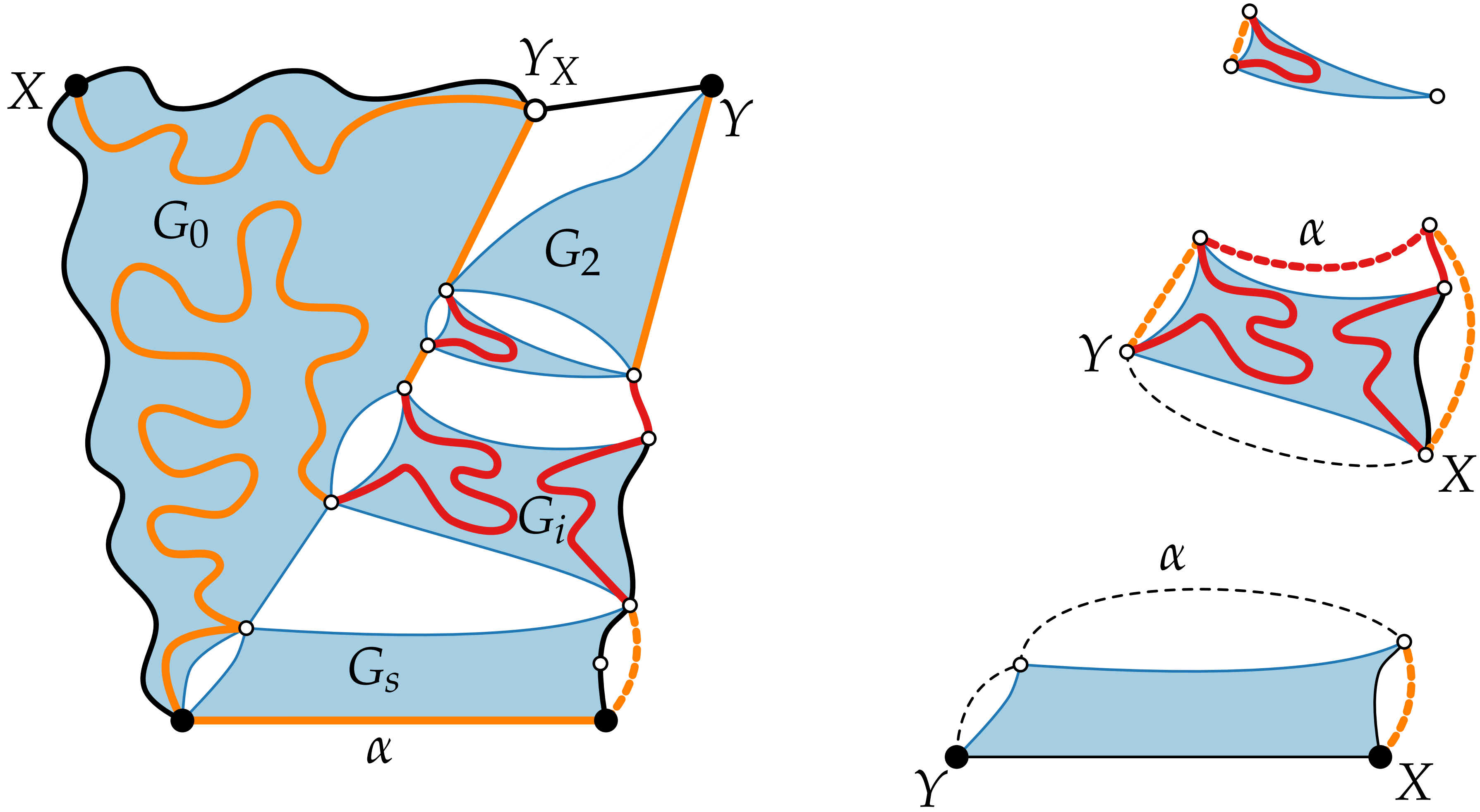
Necklace $\langle Y_X = x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s \rangle$, x_i face-adj. to right side, $G_1 = \emptyset$





Case 4: No cutting pair

Necklace $\langle Y_X{=}x_0, f_1, x_1, \ldots, x_{s-1}, f_s, x_s\rangle$,x_i face-adj. to right side ,$G_1 = \emptyset$



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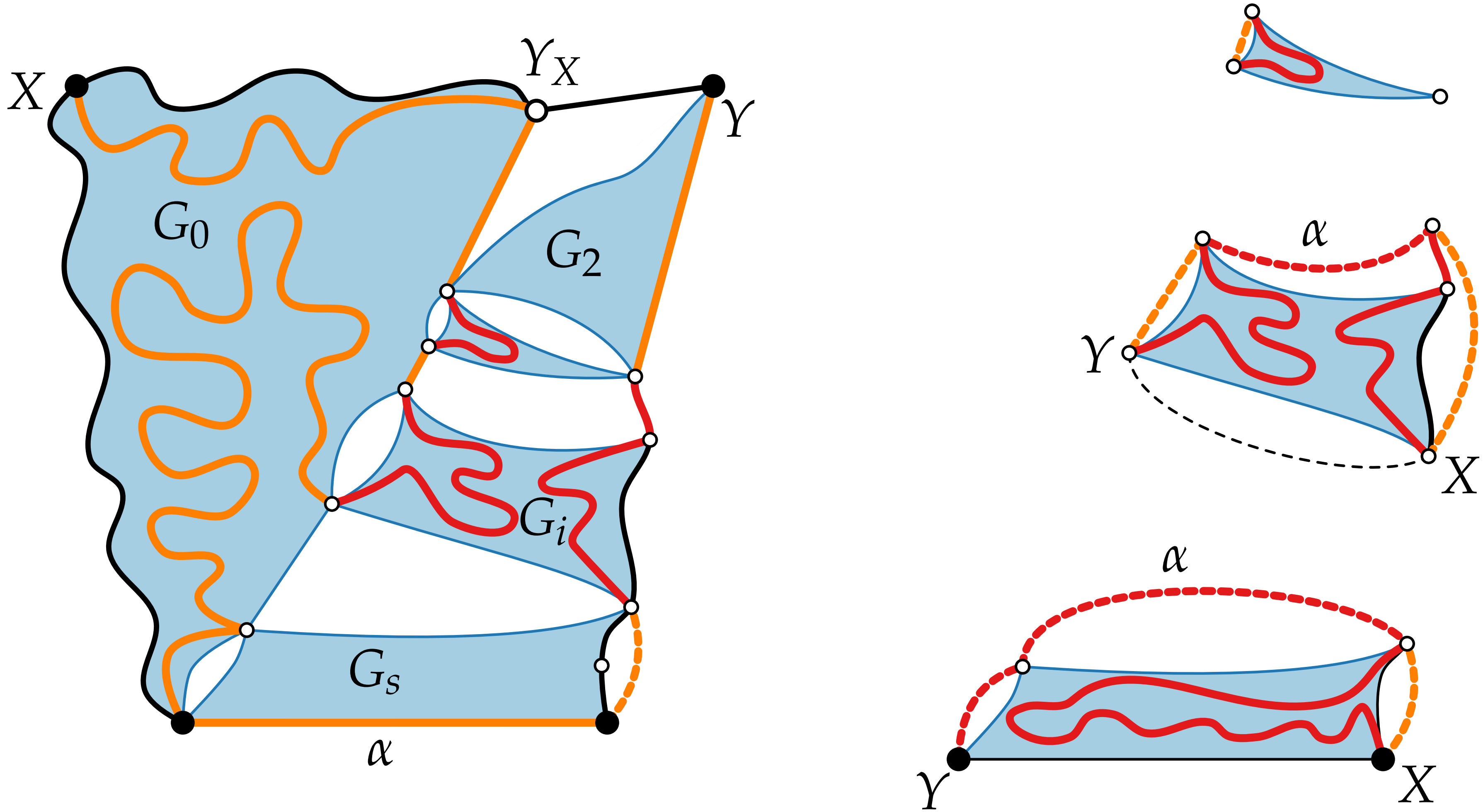

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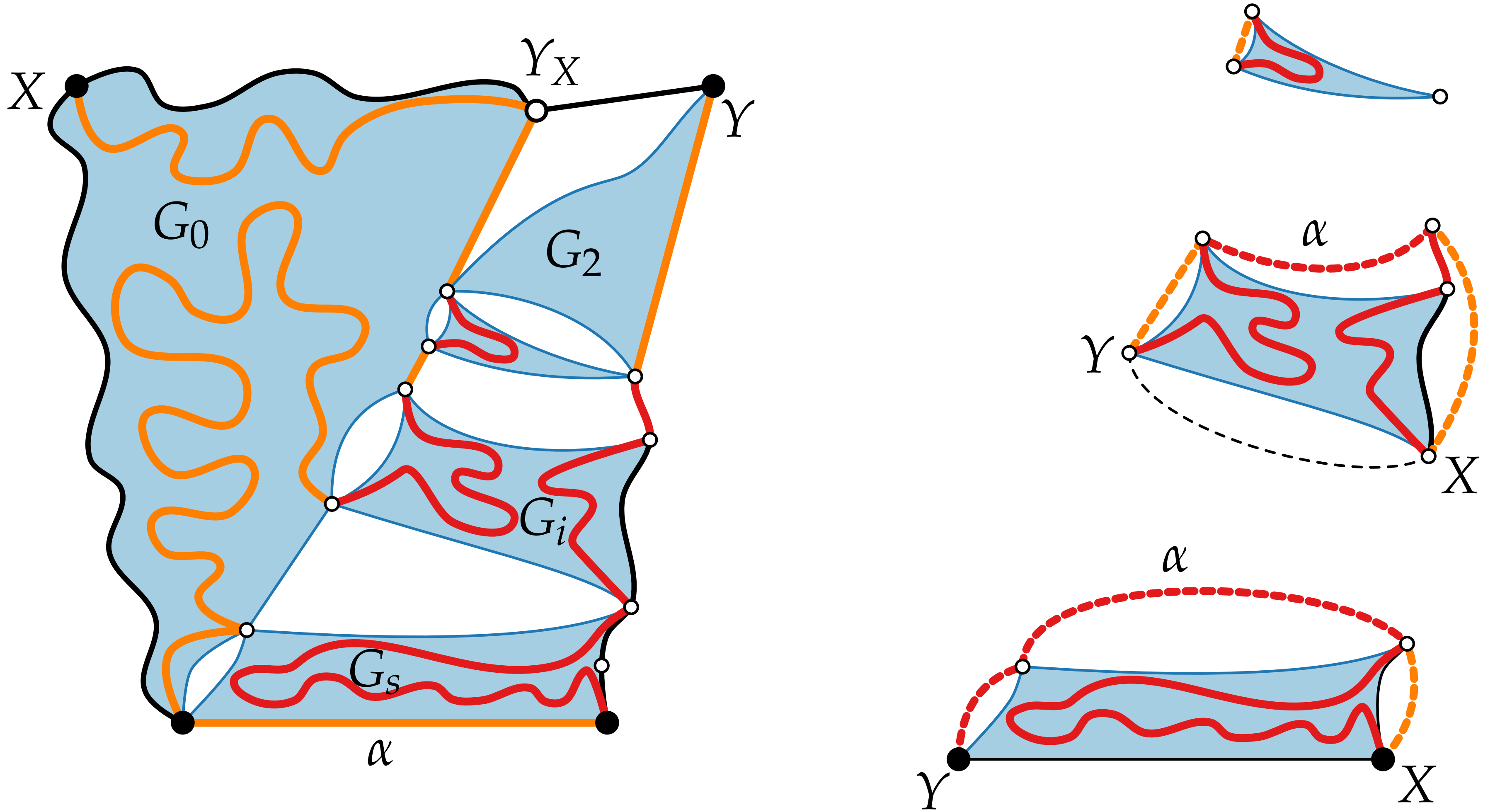

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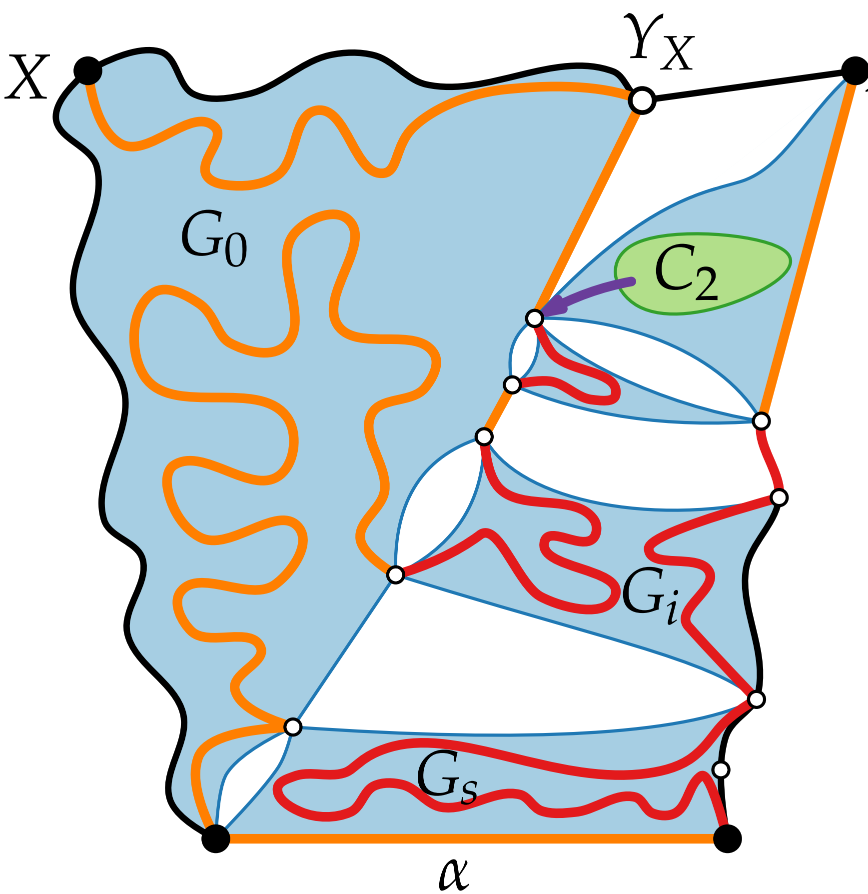
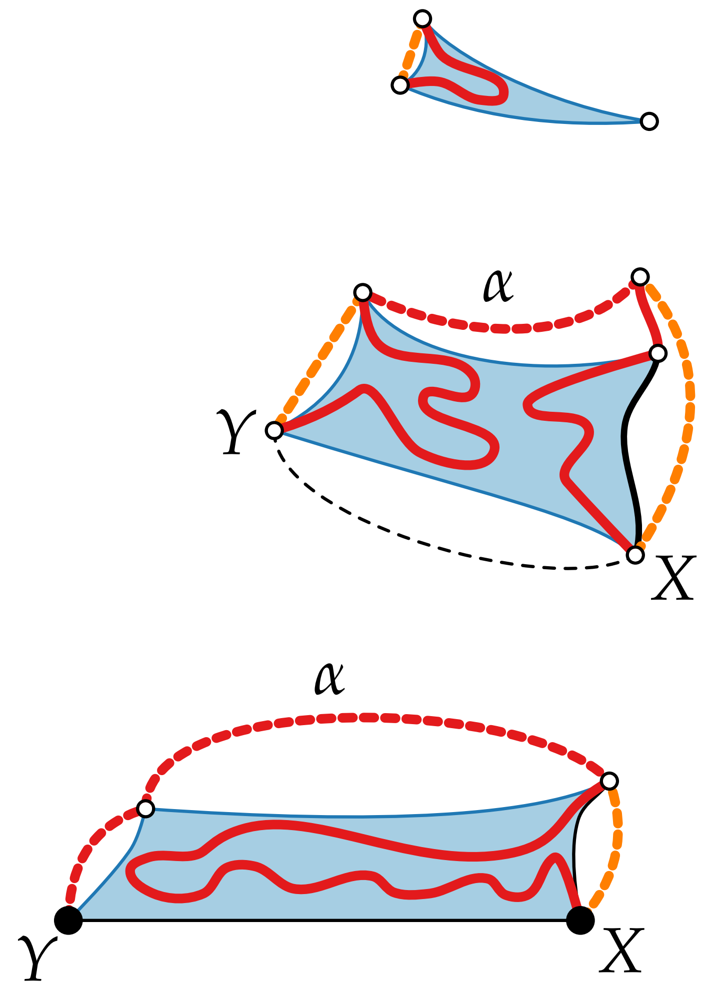
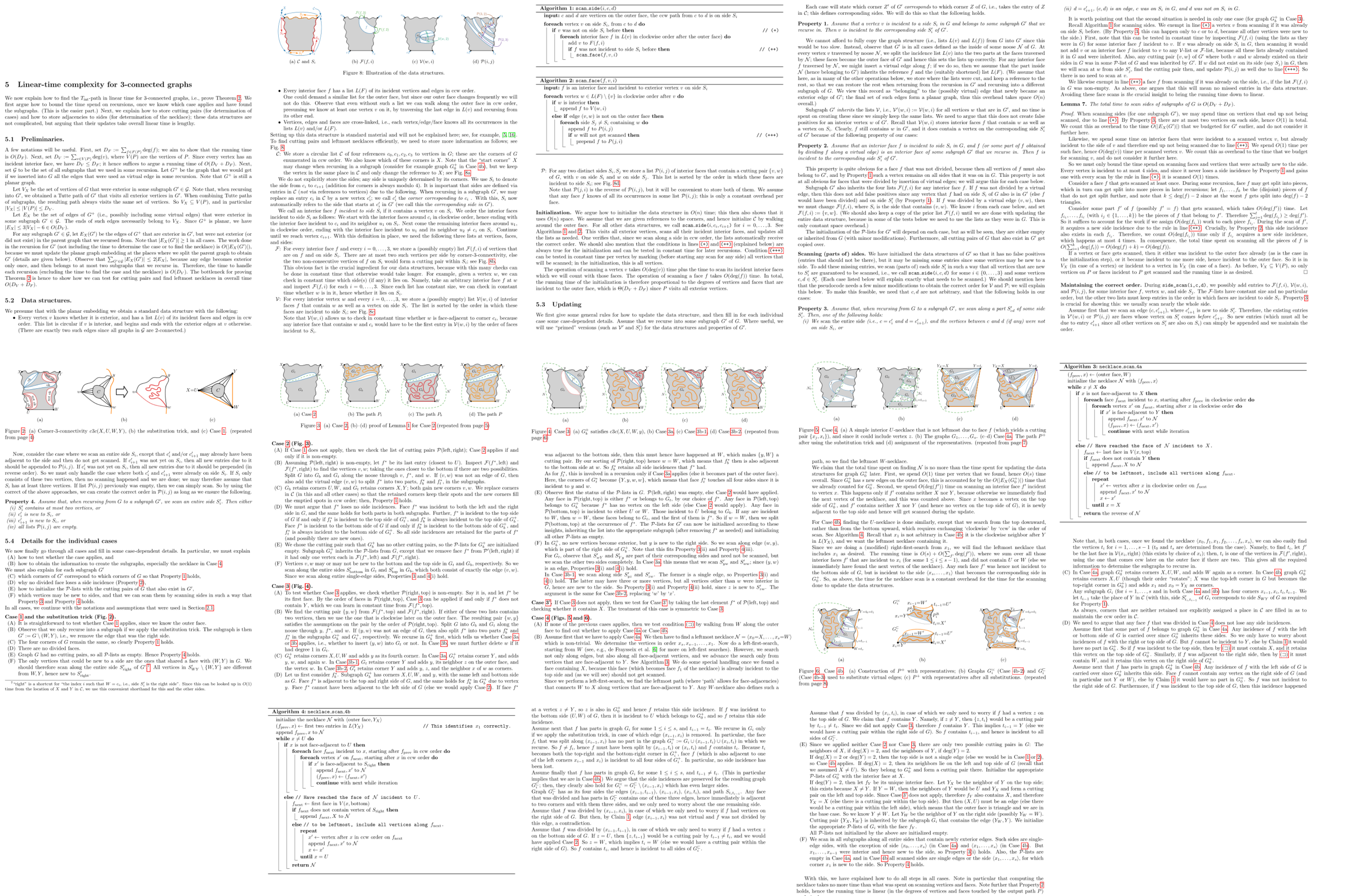




Running Time

Running Time



Running Time

Store:

Running Time

Store:

- corners

Running Time

Store:

- corners
- faces: all vtcs on each side

Running Time

Store:

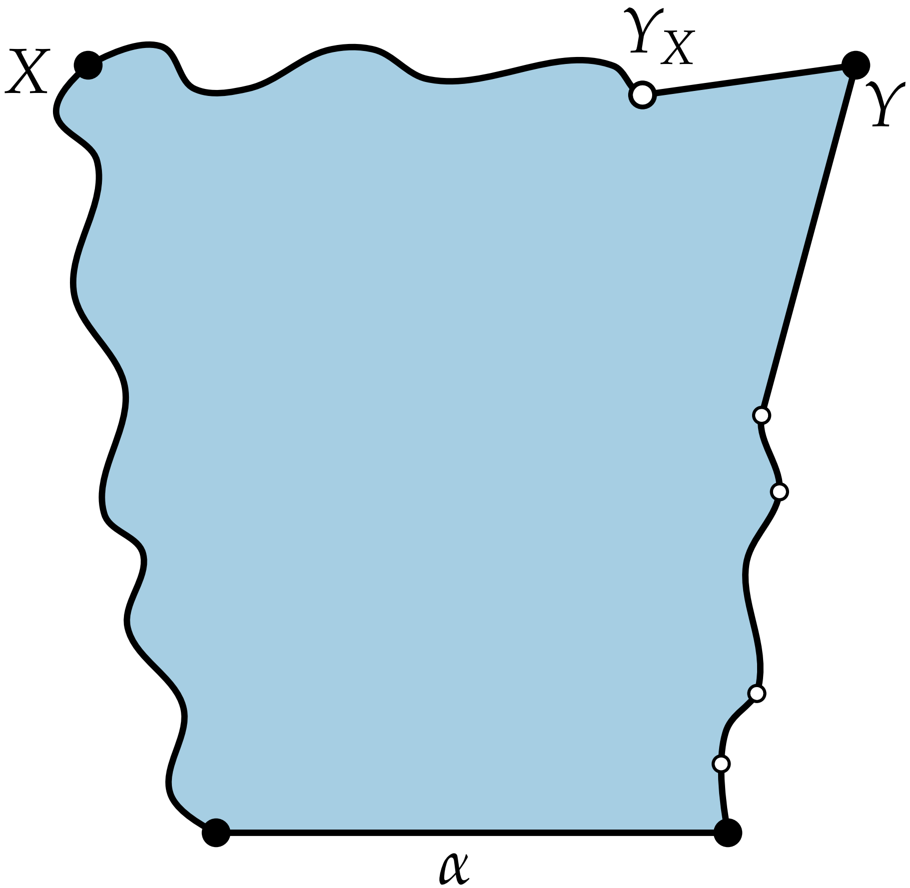
- corners
- faces: all vtcs on each side
- vtcs: all face-incidences to each side

Running Time

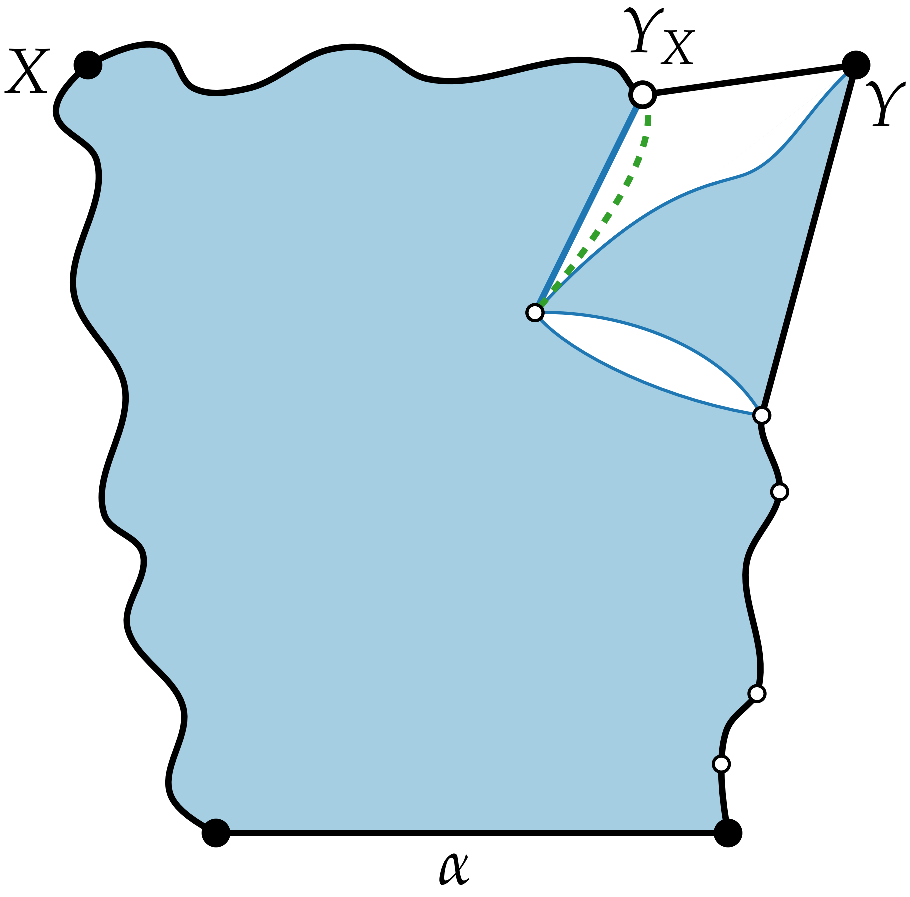
Store:

- corners
- faces: all vtcs on each side
- vtcs: all face-incidences to each side
- sides: all cutting pairs

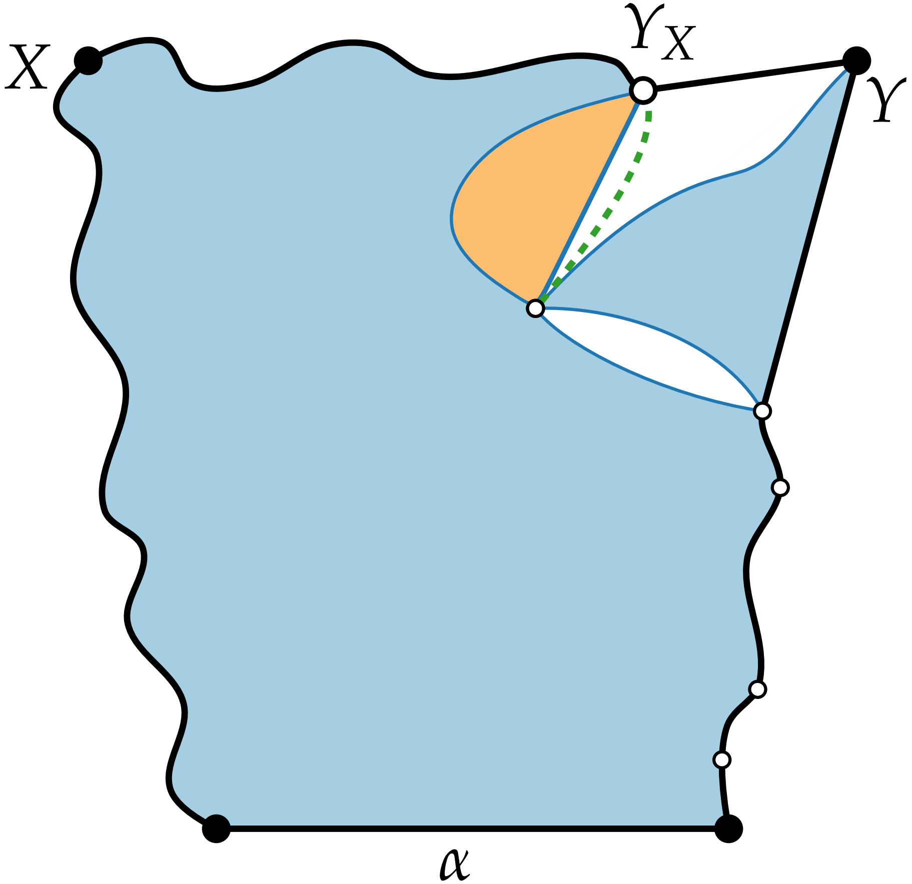
Necklace scan



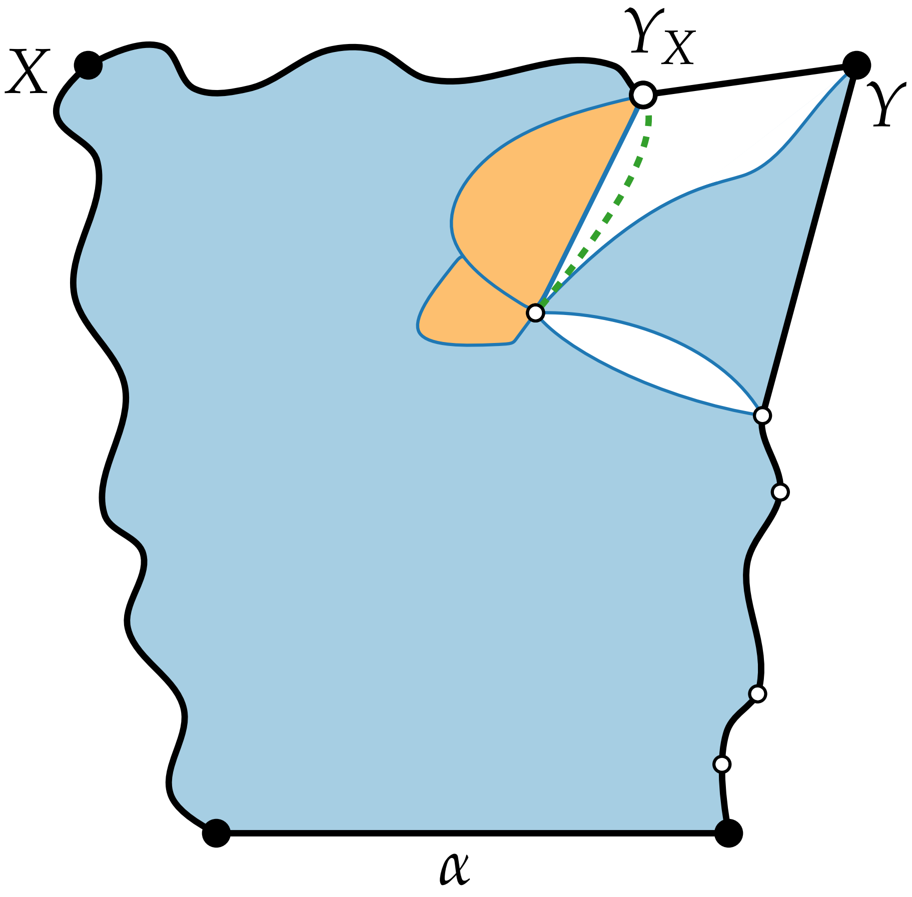
Necklace scan



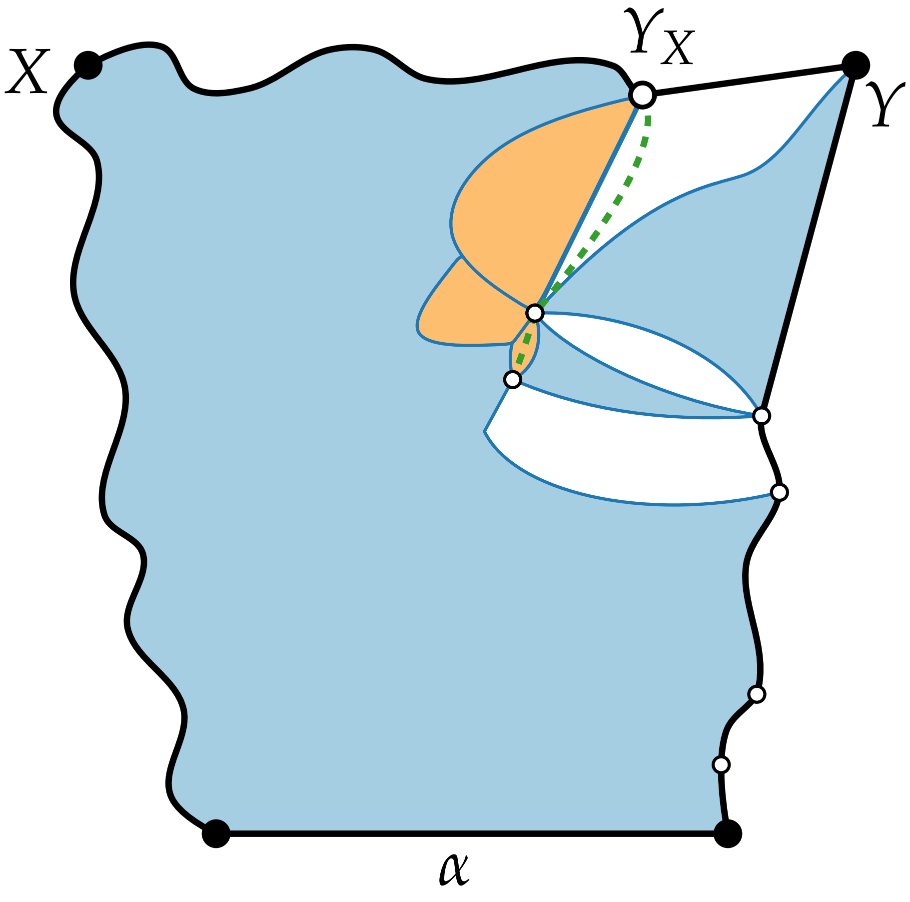
Necklace scan



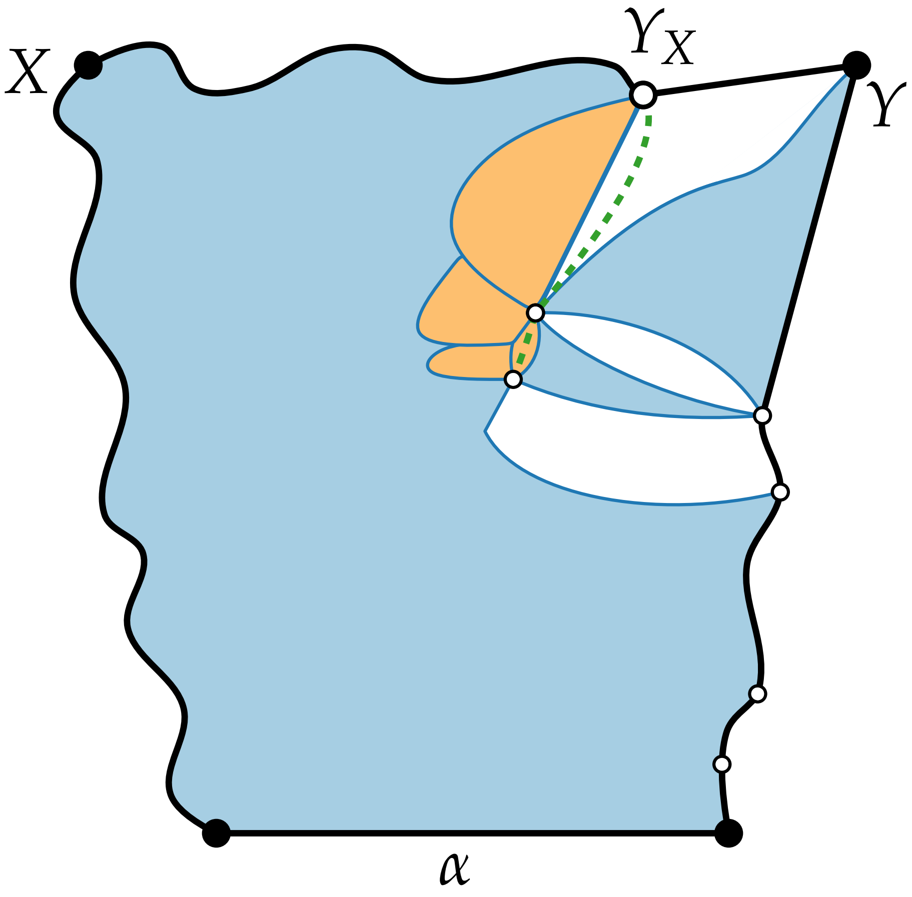
Necklace scan



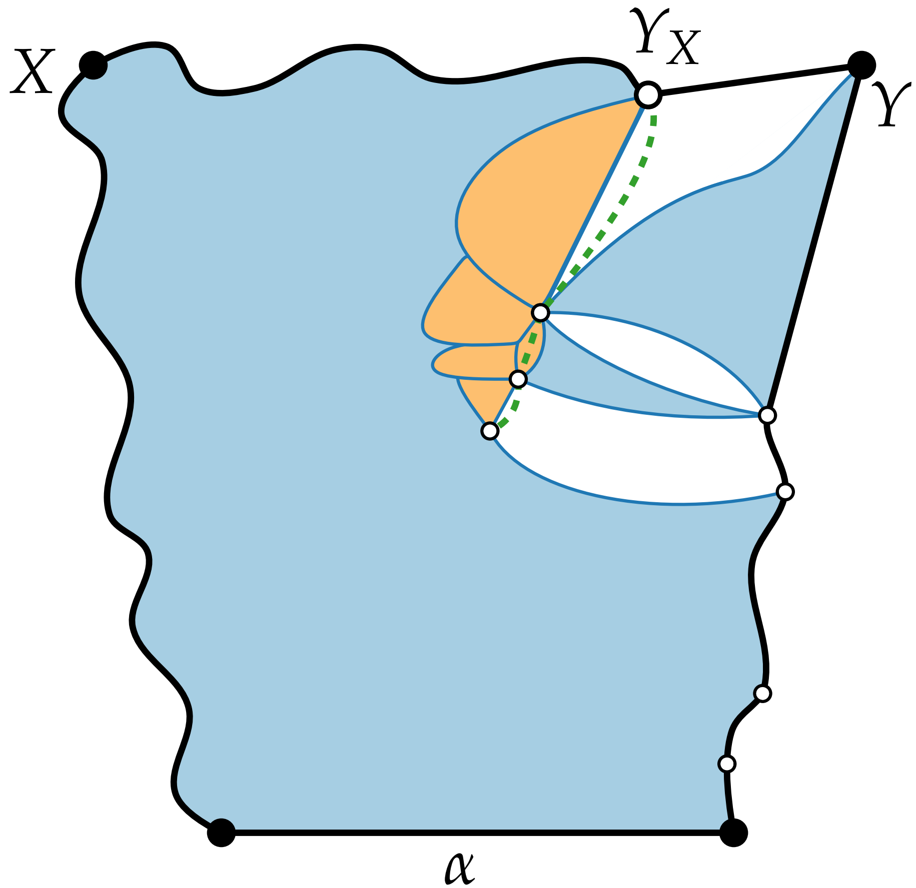
Necklace scan



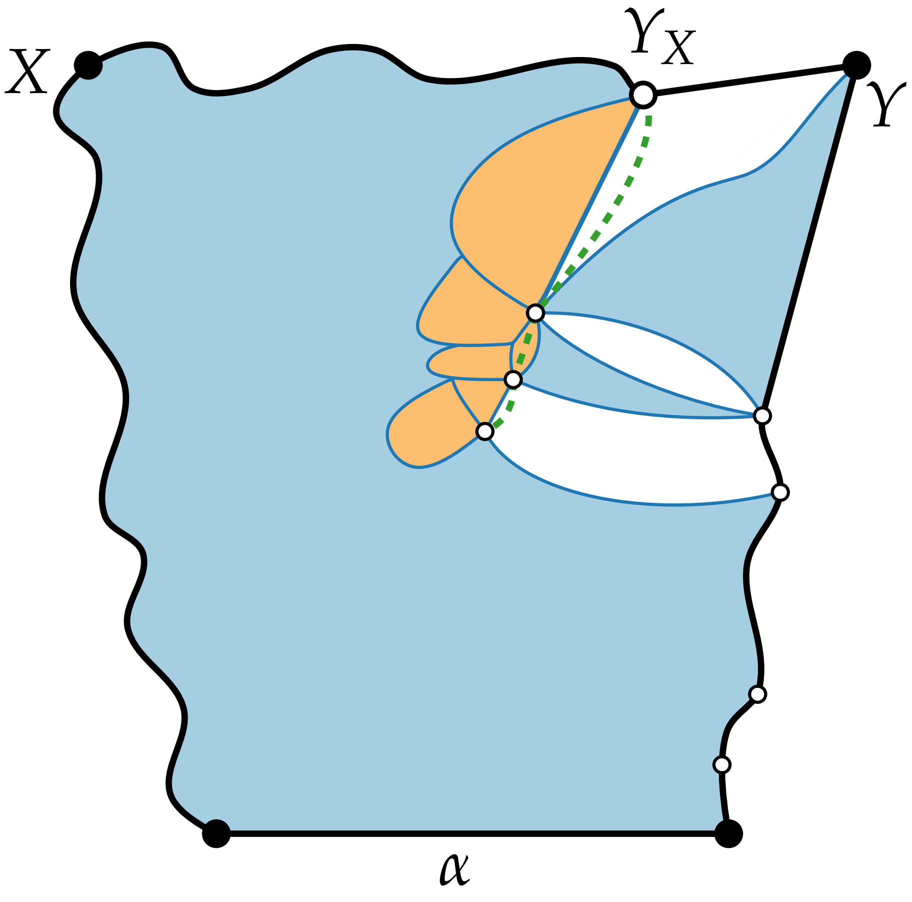
Necklace scan



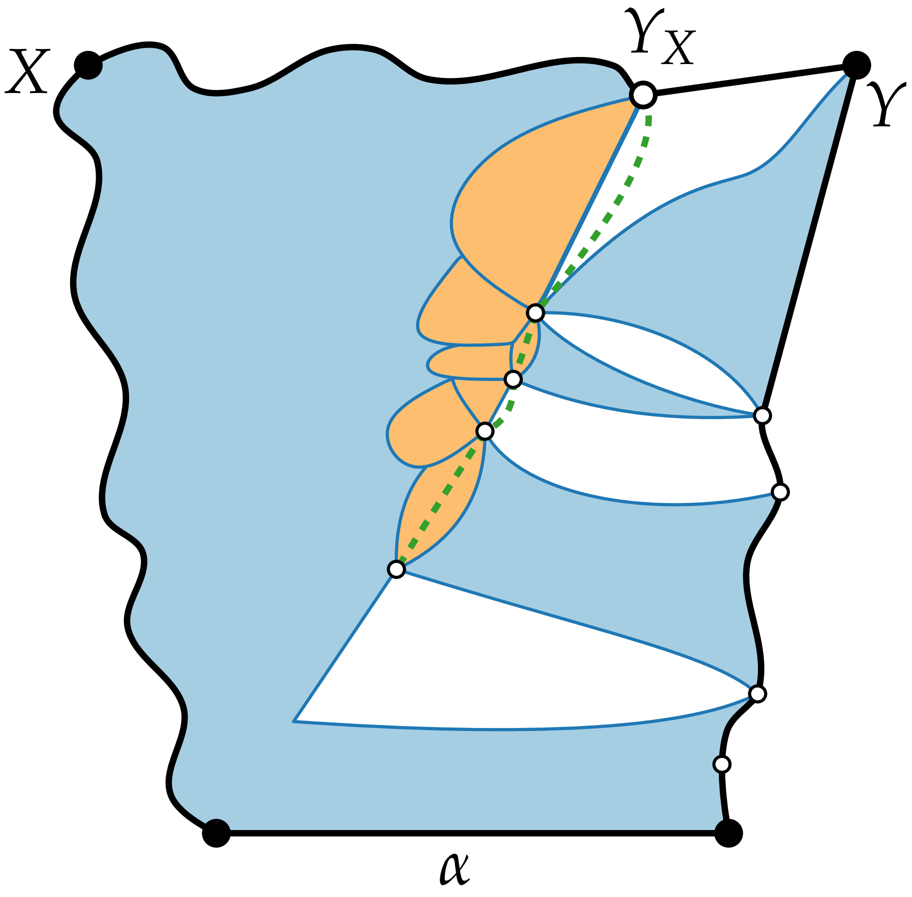
Necklace scan



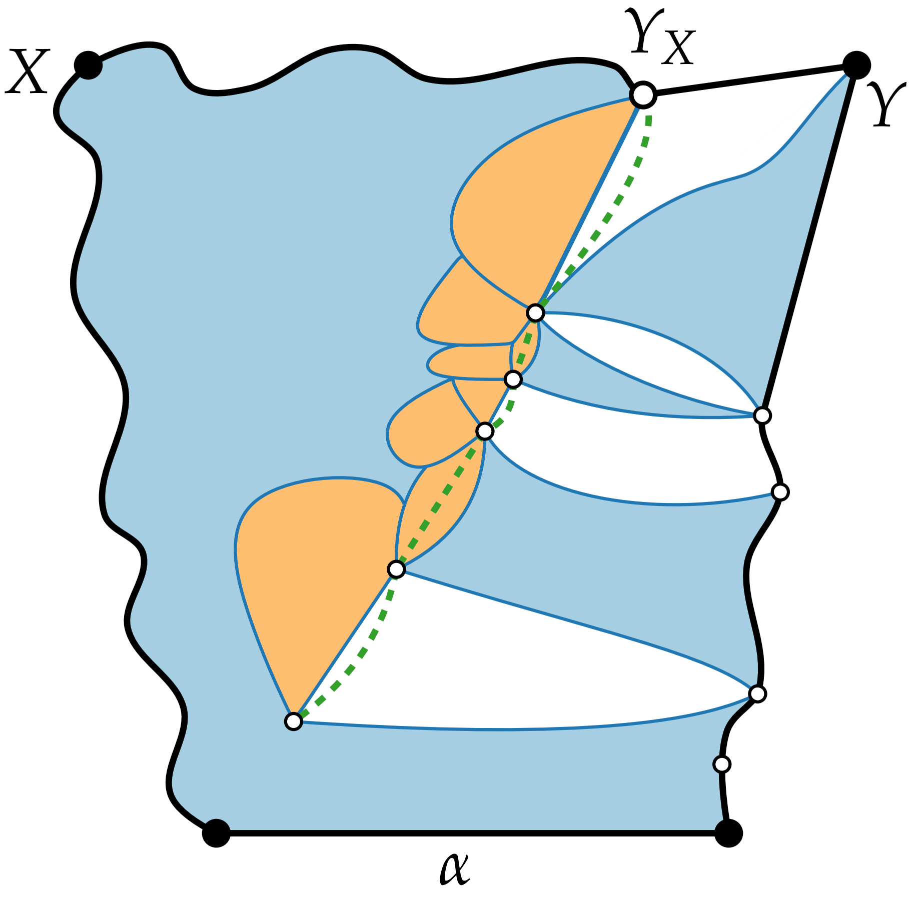
Necklace scan



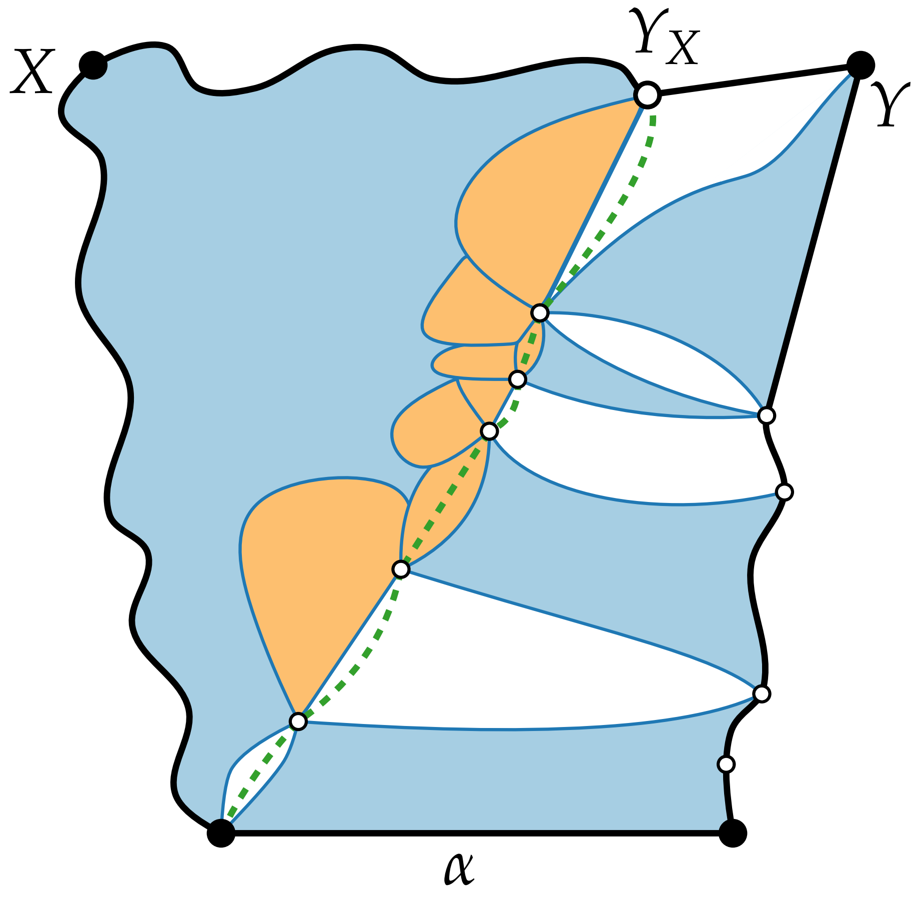
Necklace scan



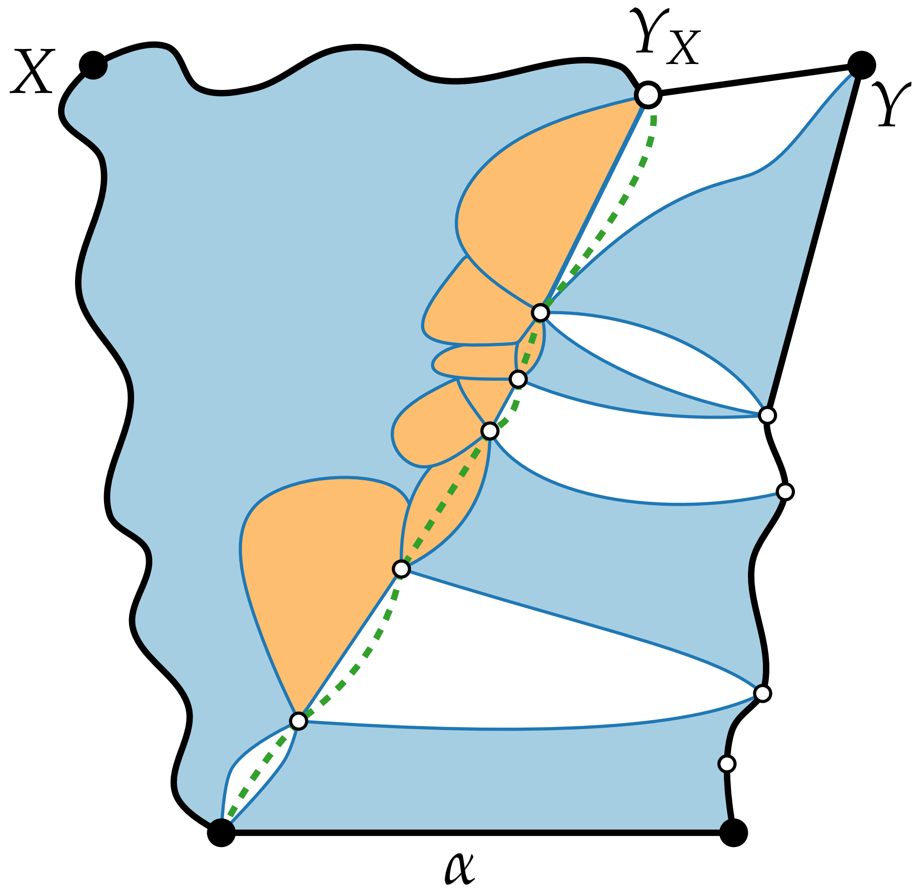
Necklace scan



Necklace scan

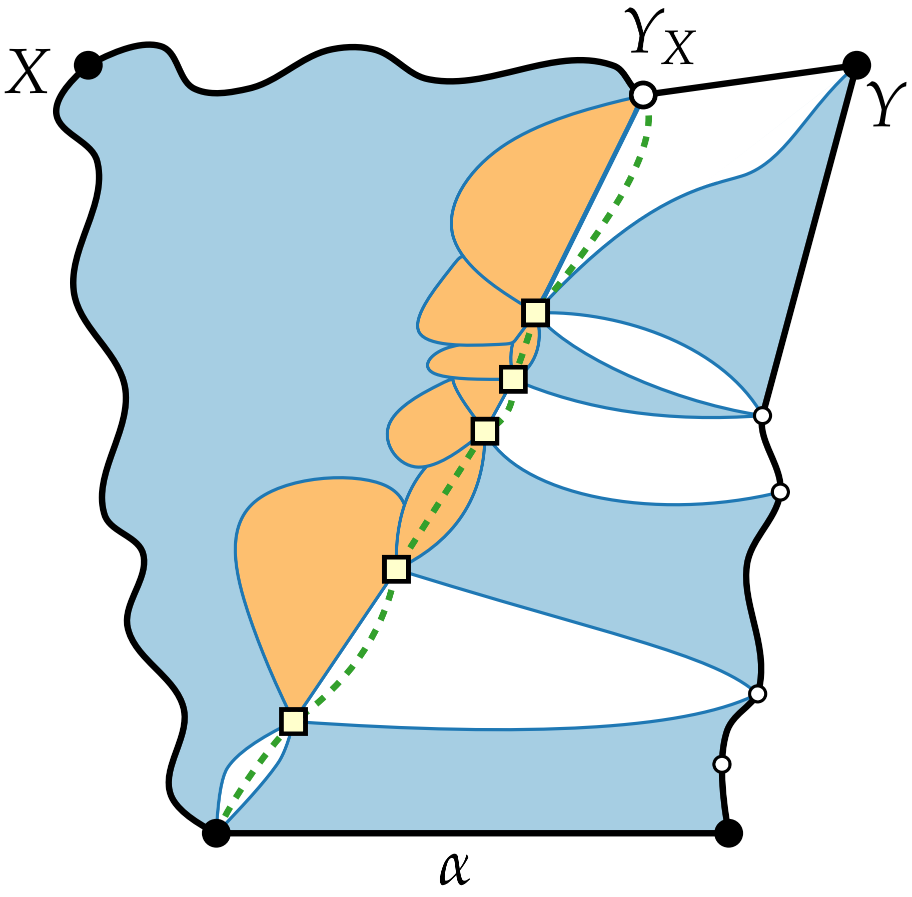


Necklace scan



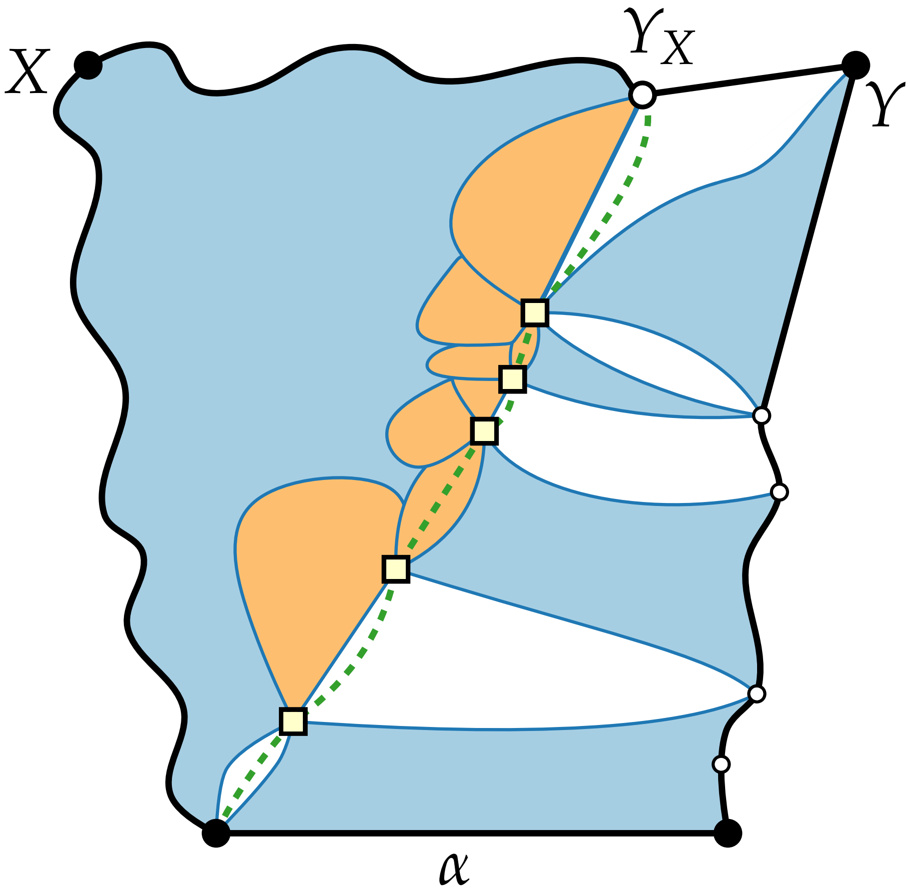
face gets scanned:

Necklace scan



face gets scanned:

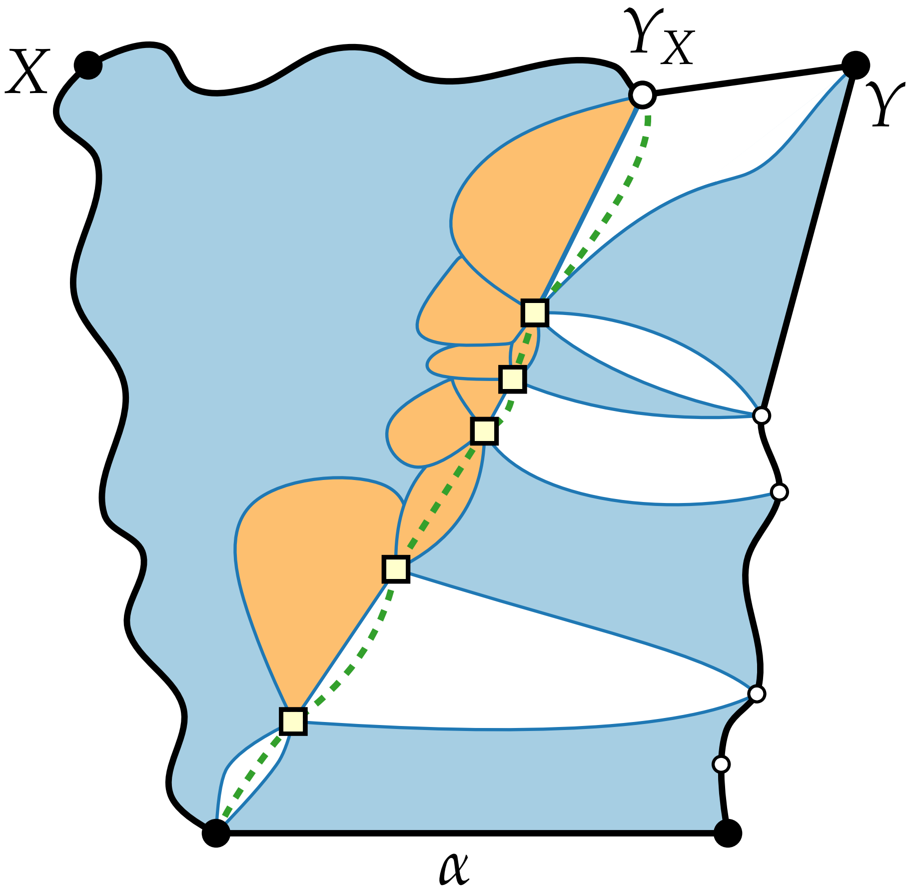
Necklace scan

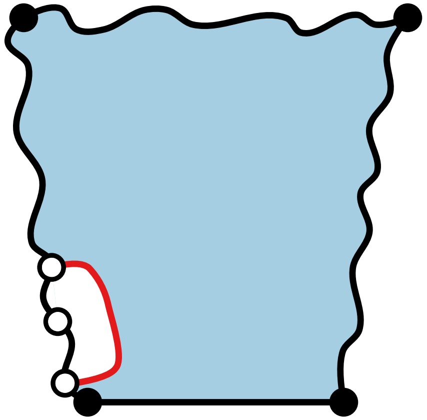


face gets scanned:

⇒ one vtx becomes outer

Necklace scan

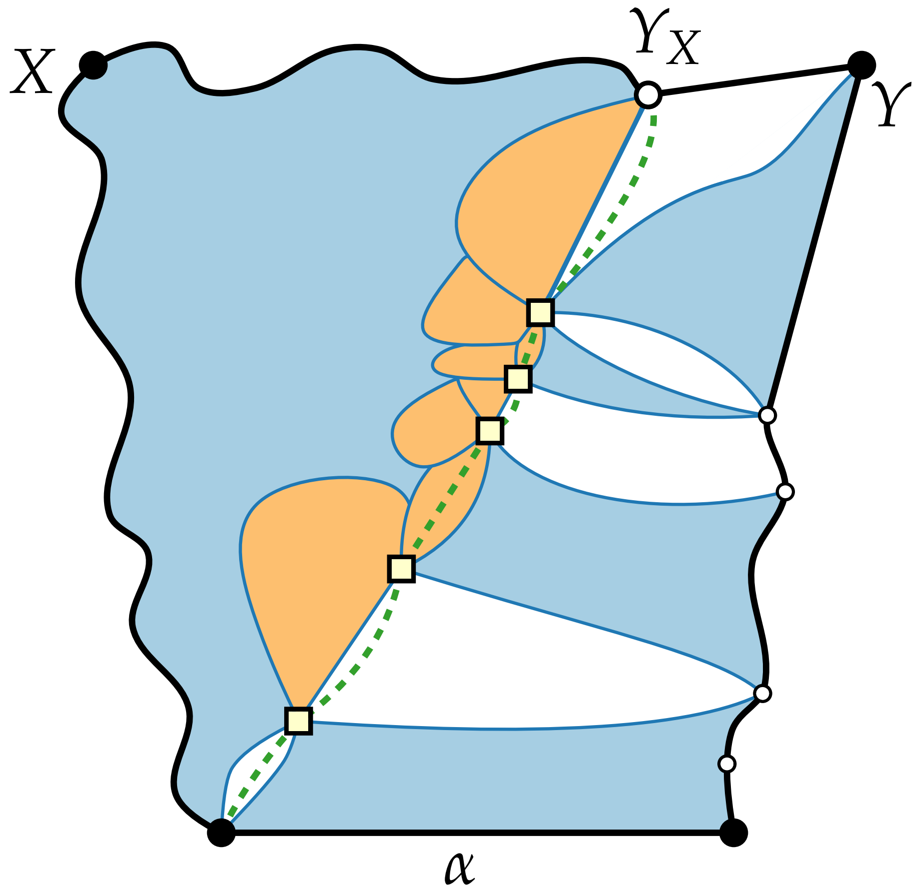


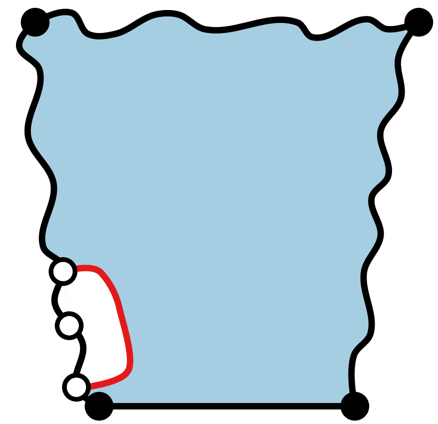


face gets scanned:

$\Rightarrow$ one vtx becomes outer

Necklace scan



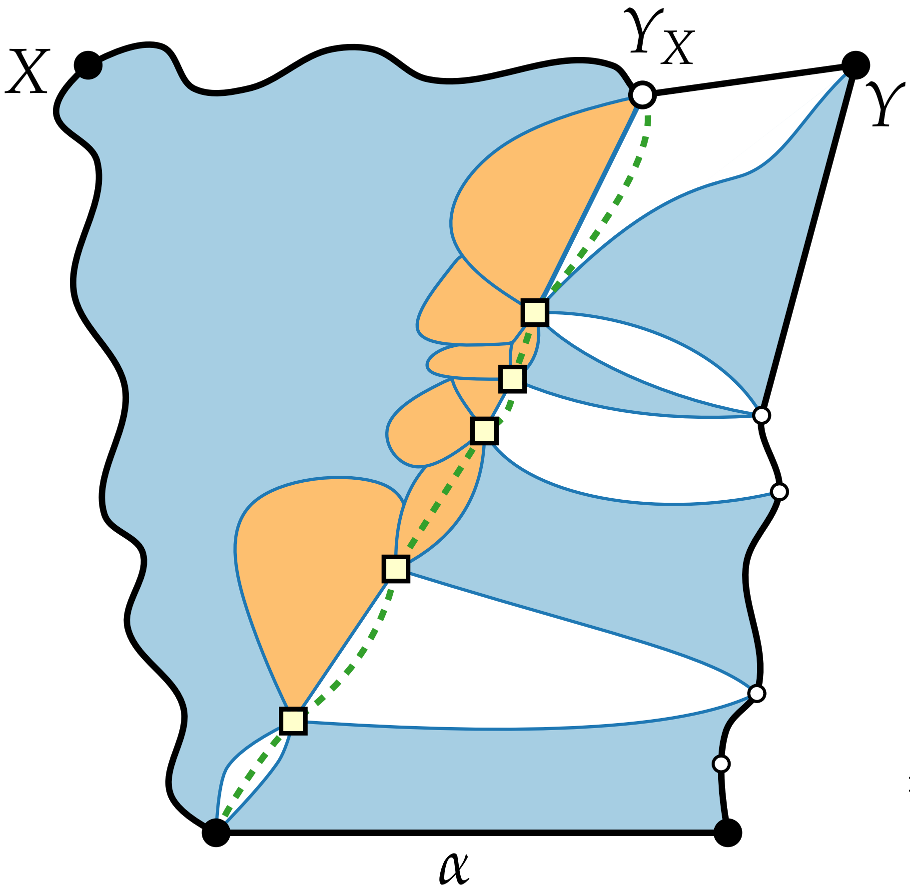


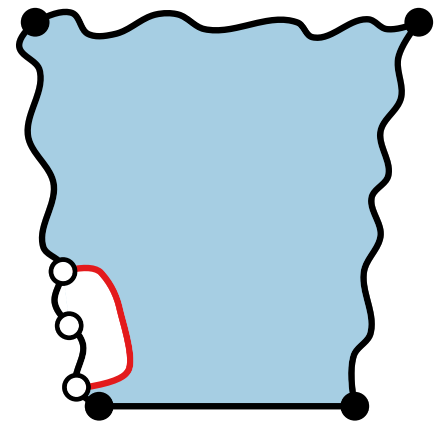
face gets scanned:

$\Rightarrow$ one vtx becomes outer

$\Rightarrow$ $O(1)$ times

Necklace scan





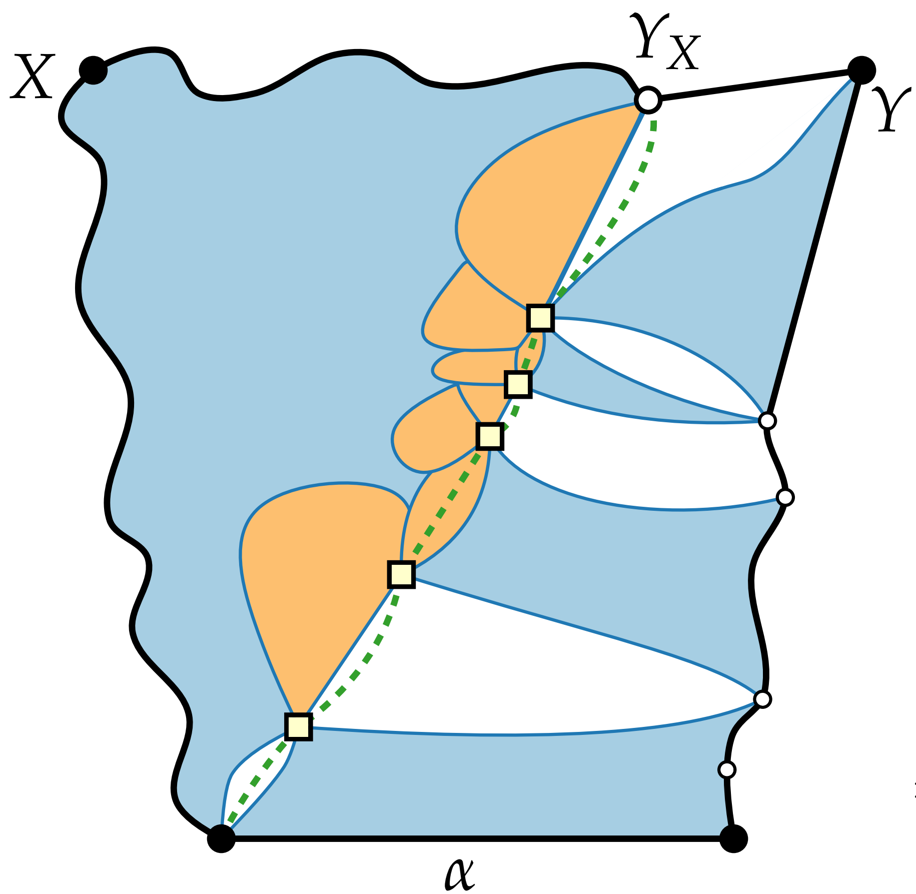
face gets scanned:

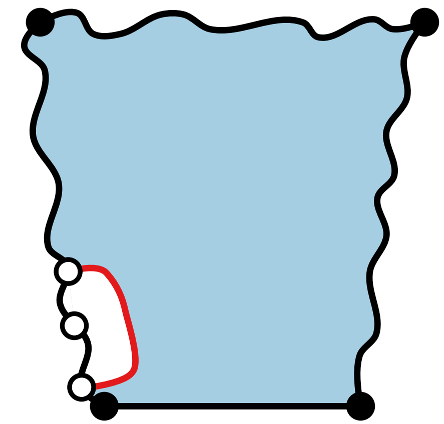
$\Rightarrow$ one vtx becomes outer

$\Rightarrow O(1)$ times

$\Rightarrow O(\sum_f \deg(f)) = O(n)$ time

Necklace scan





face gets scanned:

$\Rightarrow$ one vtx becomes outer

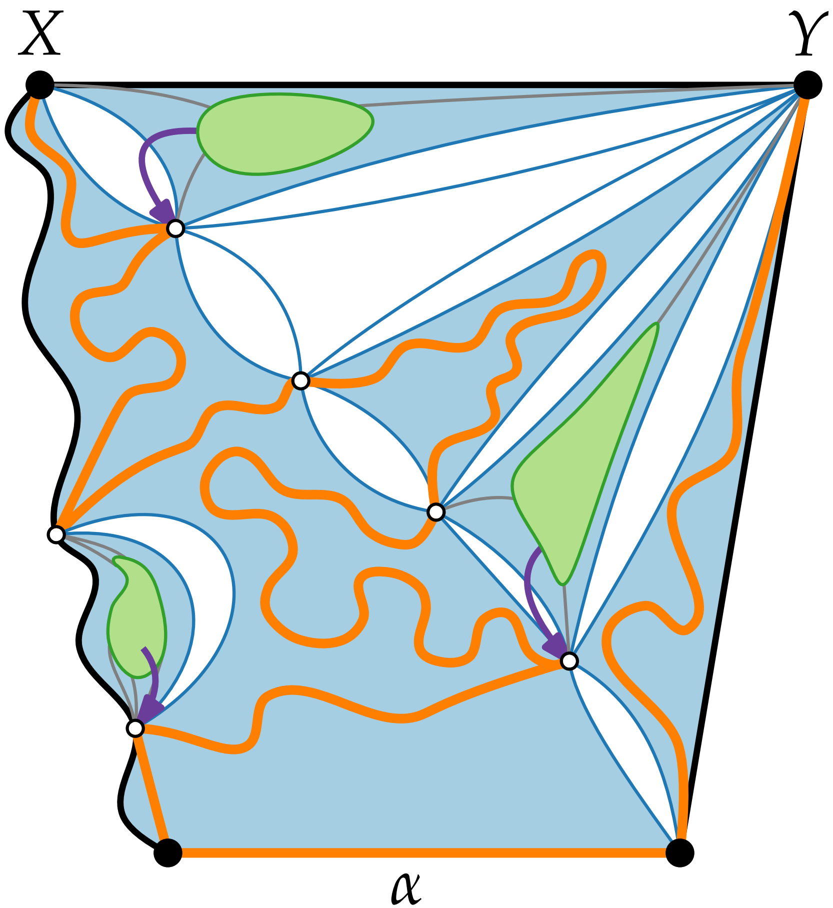
$\Rightarrow$ $O(1)$ times

$\Rightarrow$ $O(\sum_f \deg(f)) = O(n)$ time

Theorem.
G int. 3-conn., X, Y, α on outer face $\Rightarrow$ T_{int}-path in $O(n)$ time.

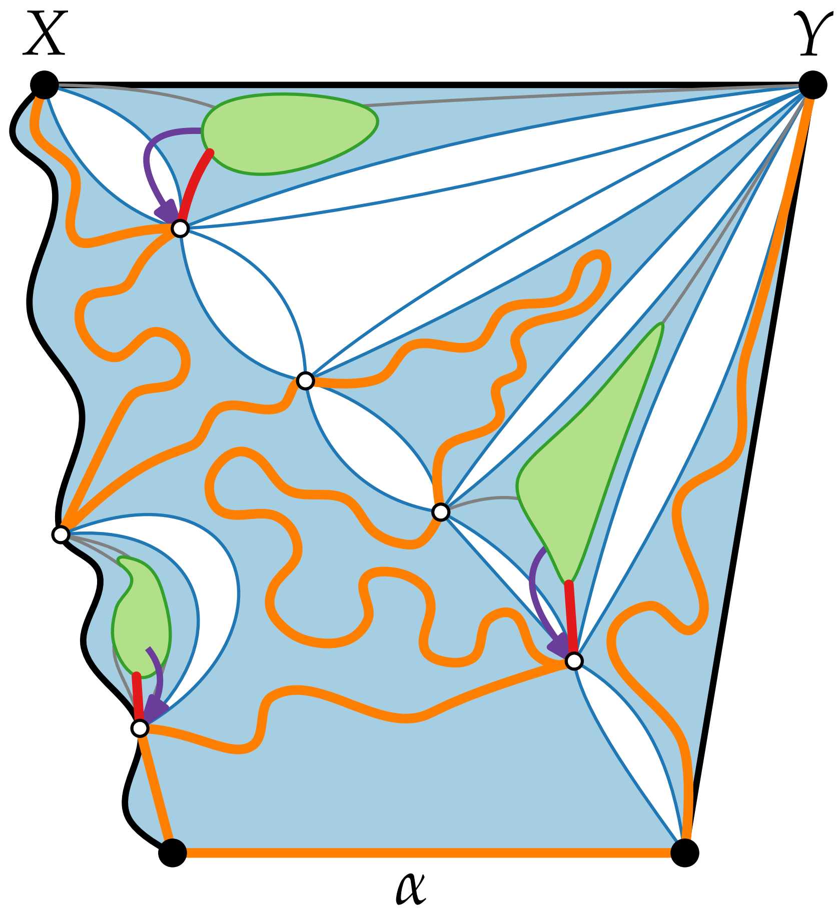
Applications

Applications



Theorem.
G int. 3-conn. $\Rightarrow$ binary spanning tree in $O(n)$ time.

Applications



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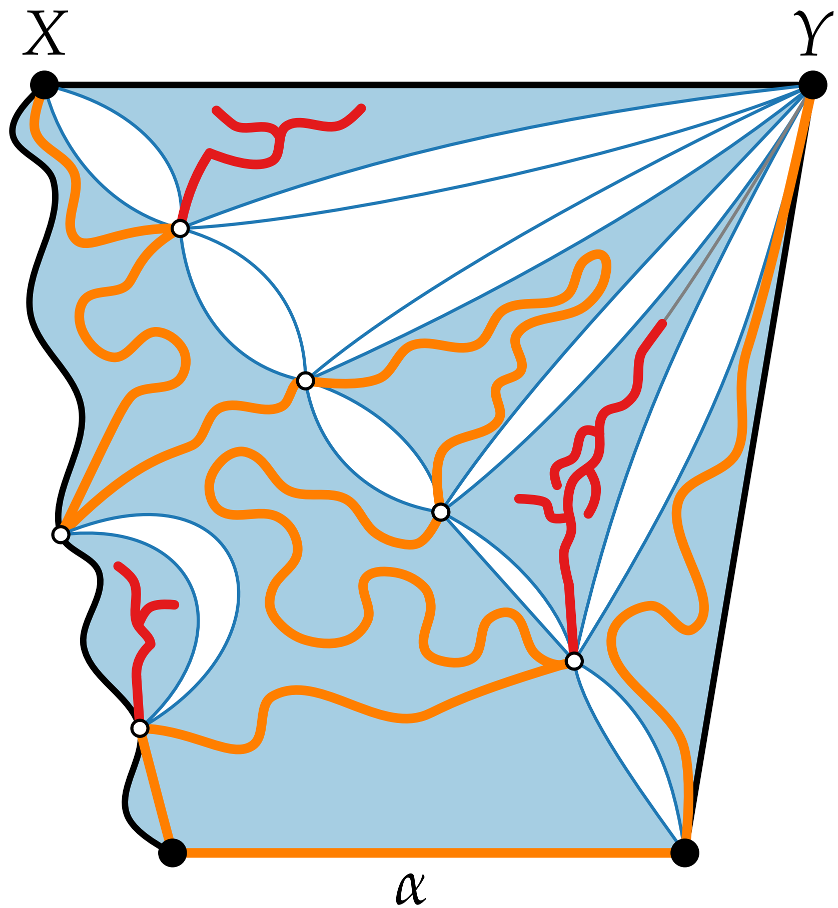

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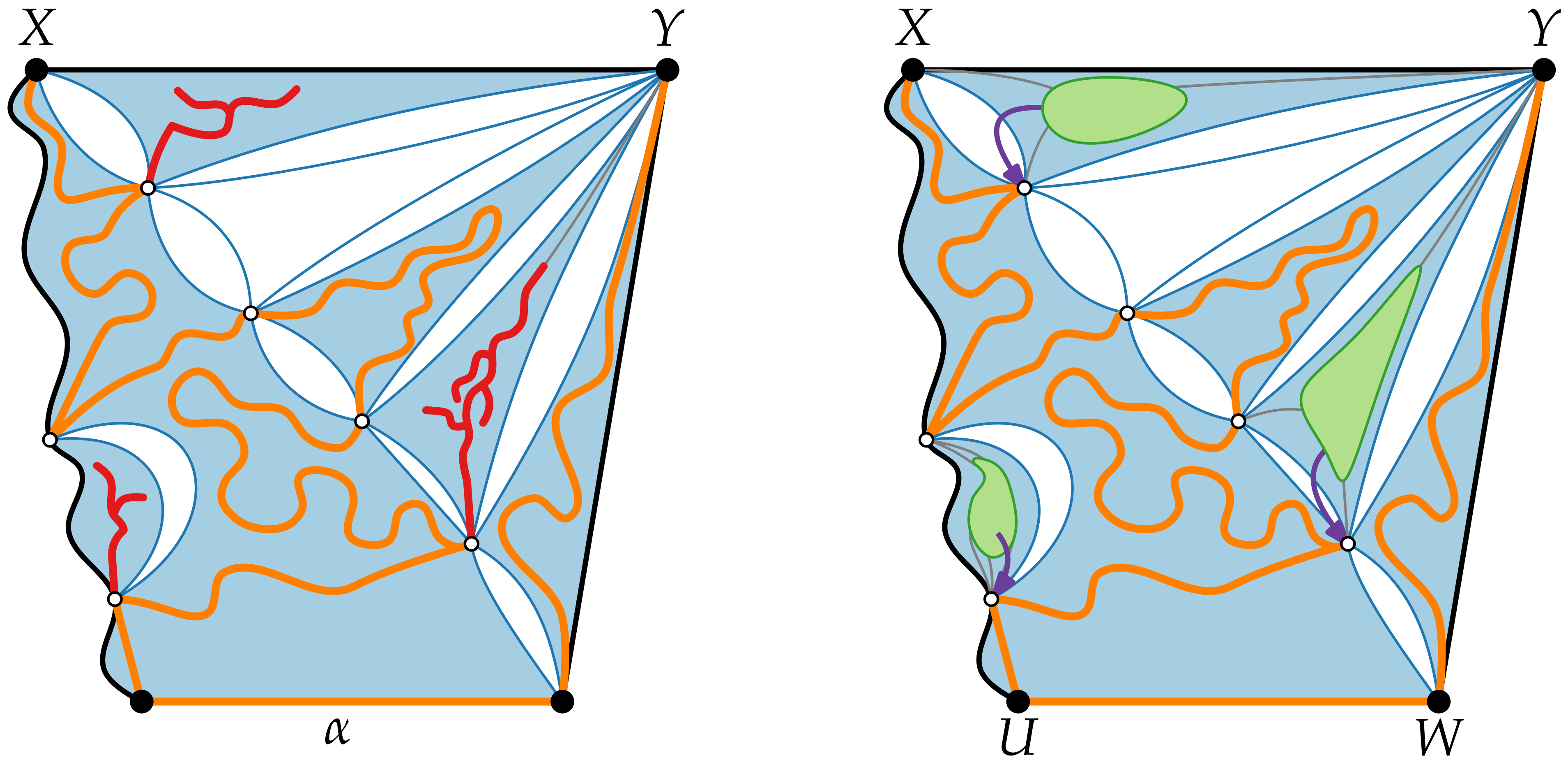
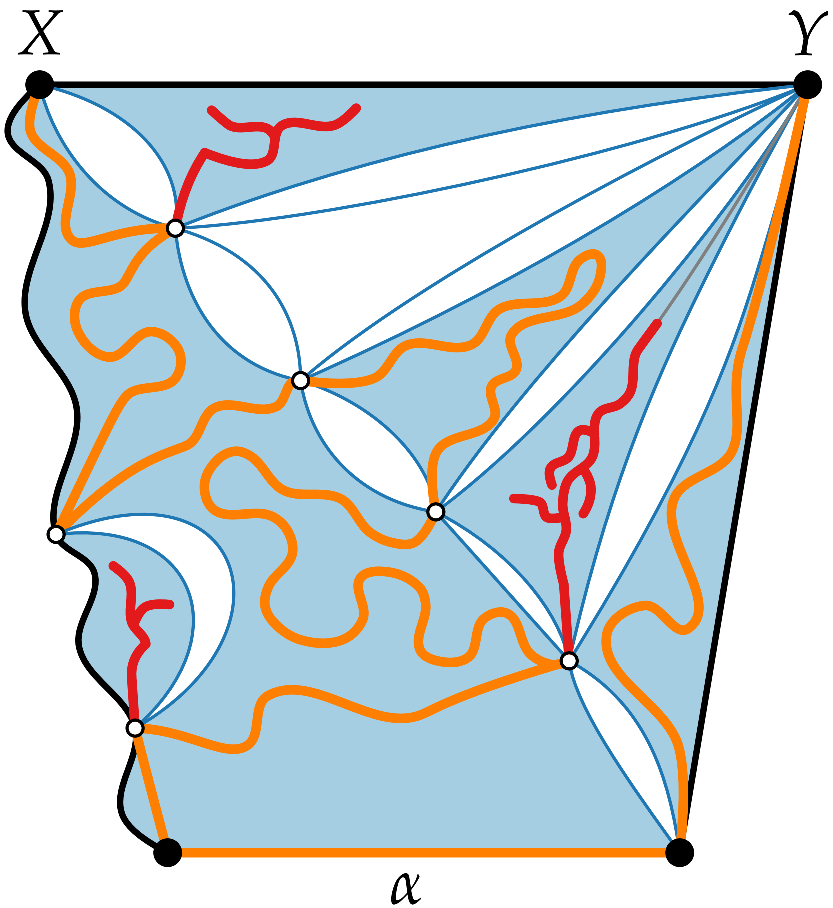
Applications

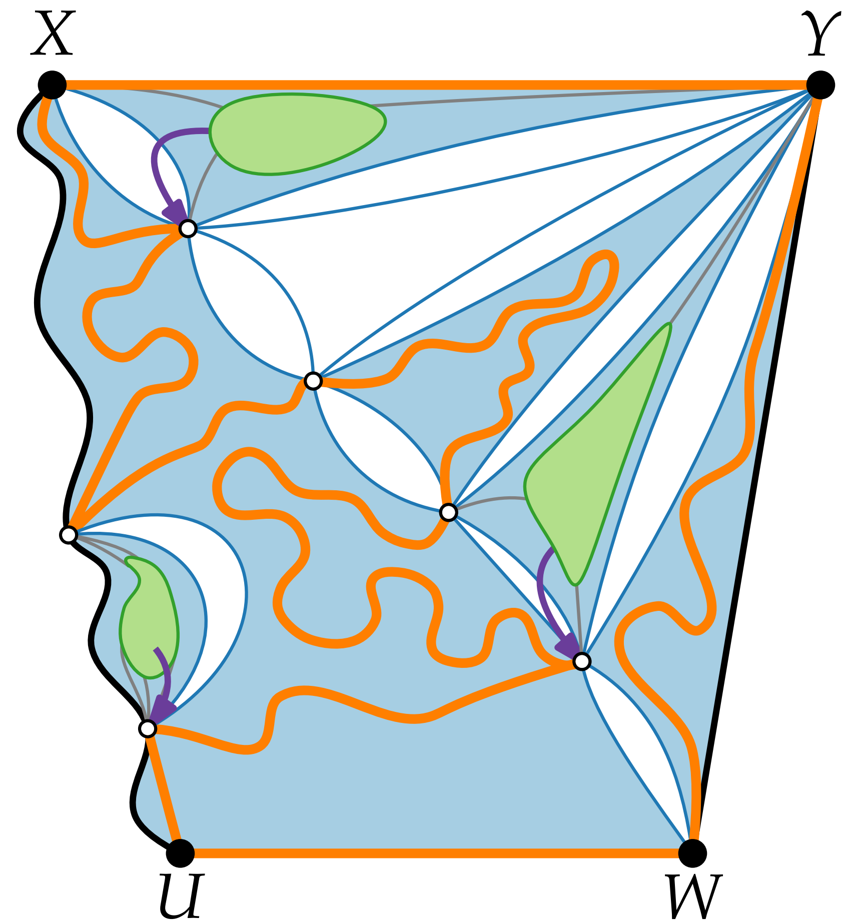


Theorem.
G int. 3-conn. ⇒ binary spanning tree in $O(n)$ time.

Theorem.
G int. 3-conn. ⇒ 2-circuit in $O(n)$ time.

Applications





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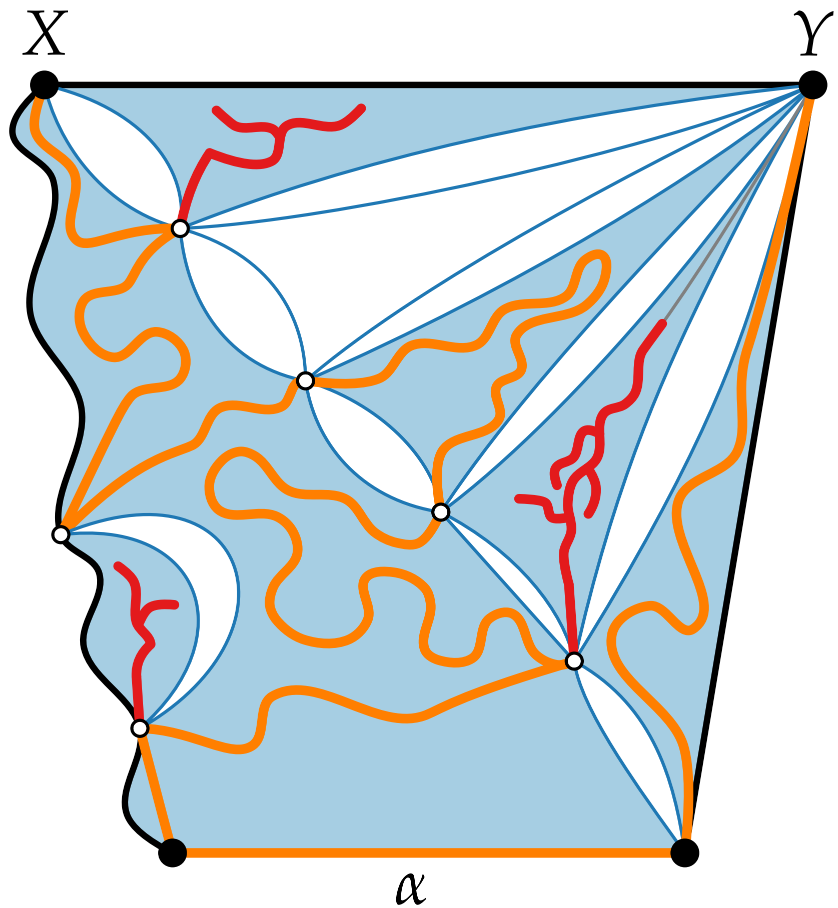
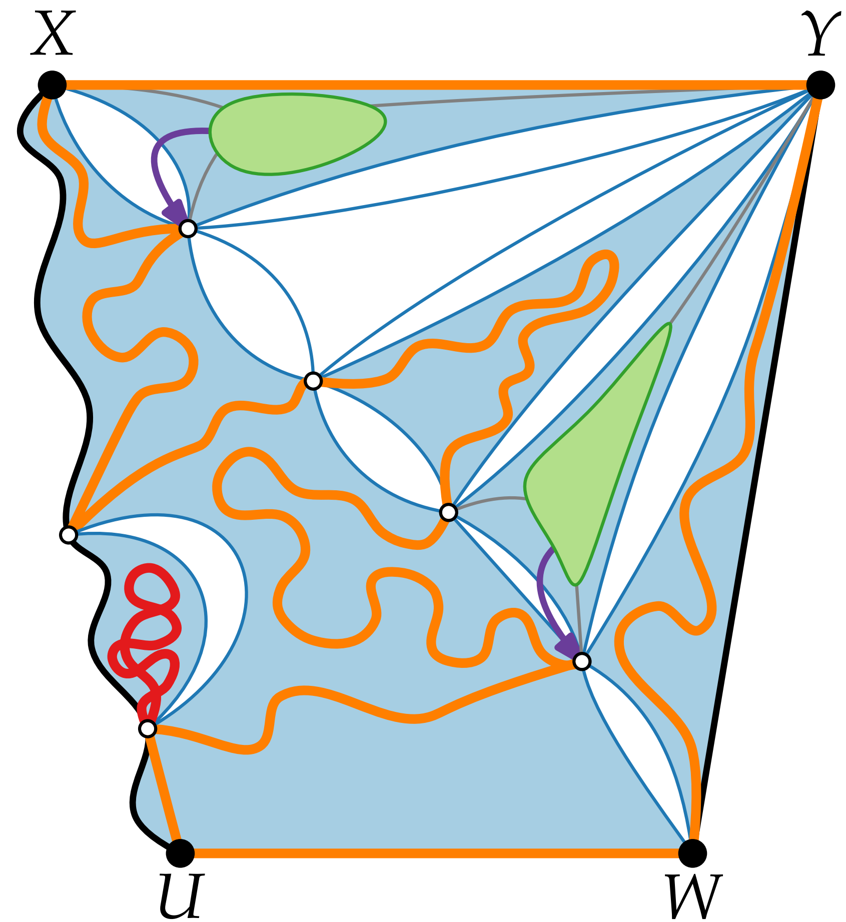
Applications





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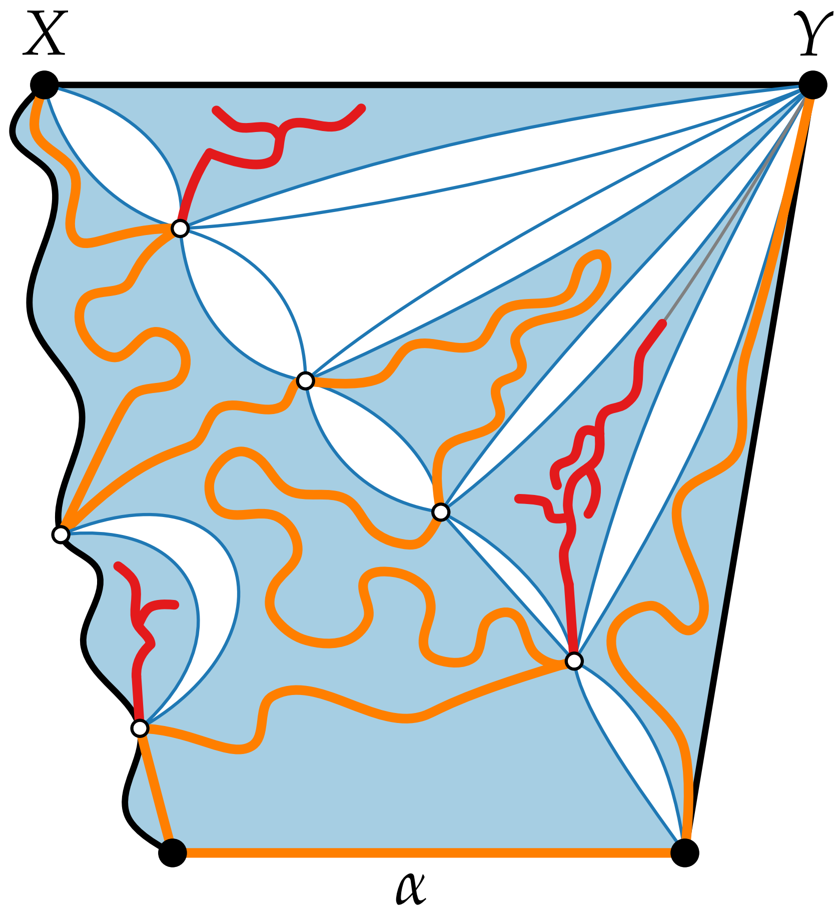

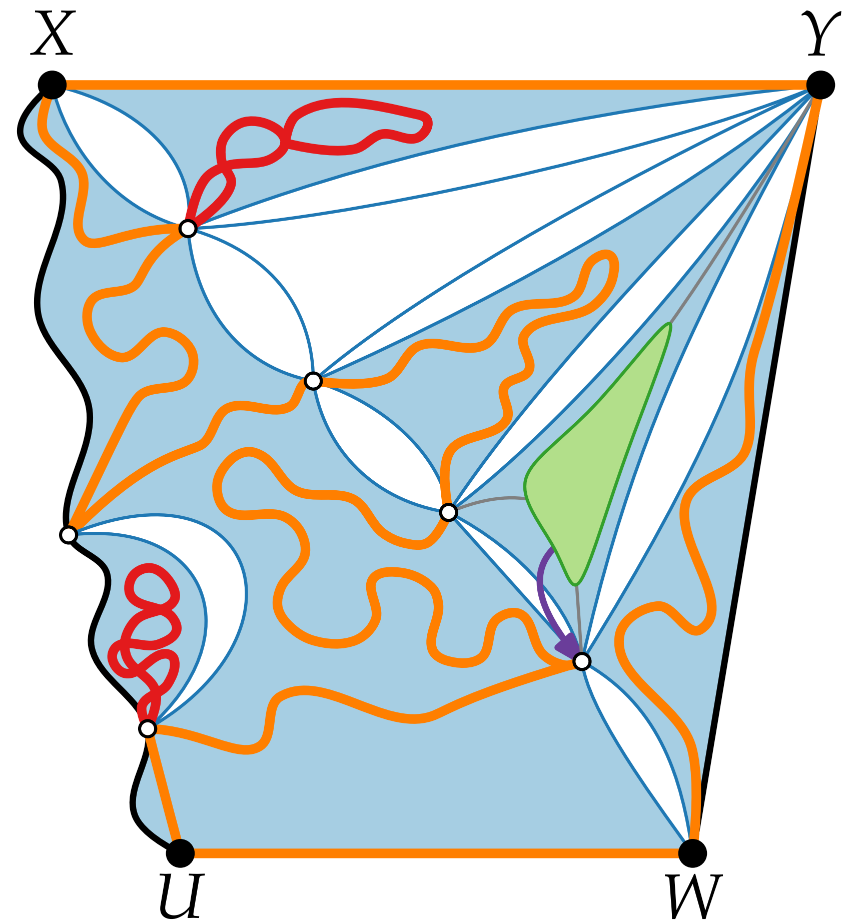

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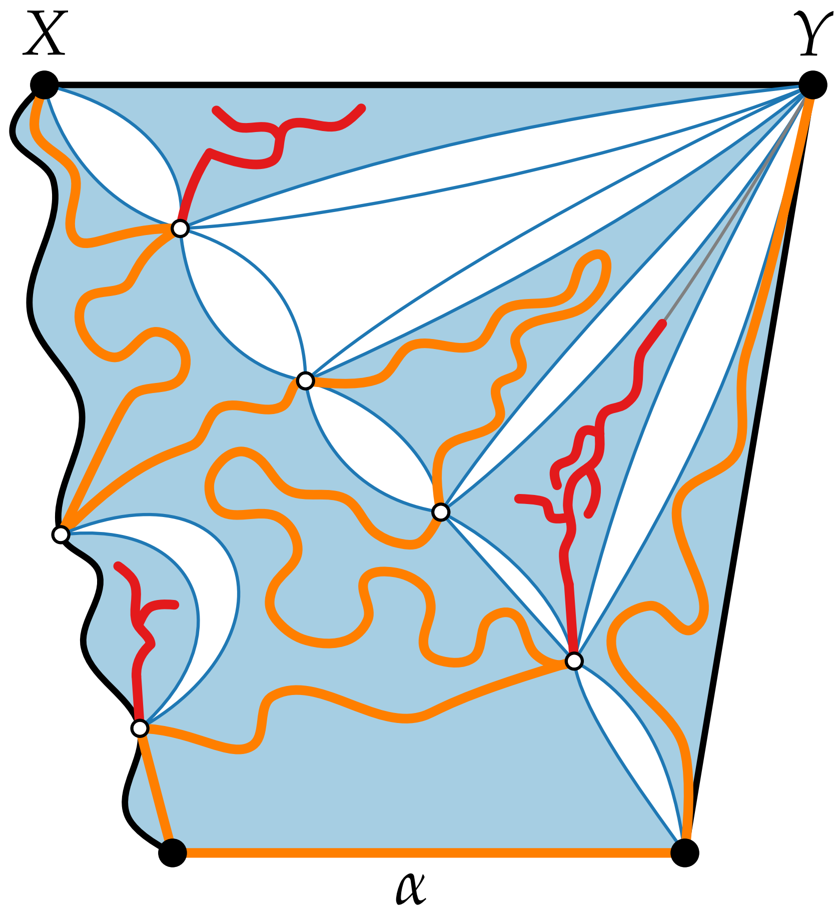

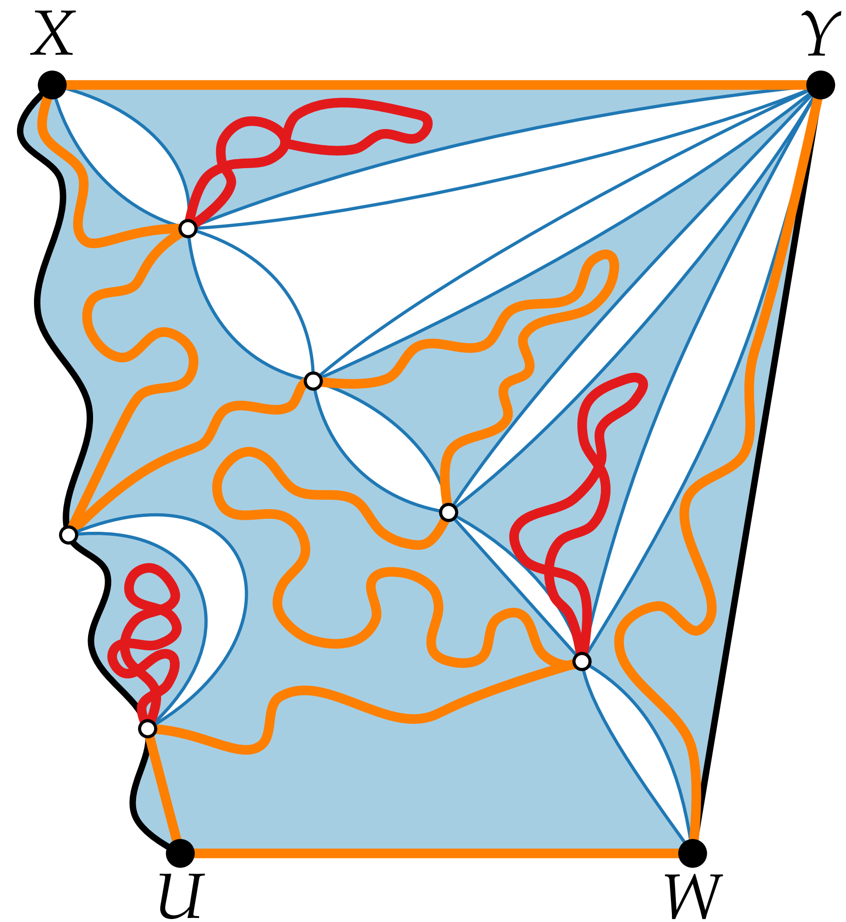

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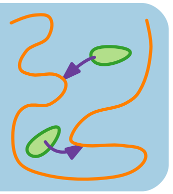
G int. 3-conn. ⇒ binary spanning tree in $O(n)$ time.

Theorem.
G int. 3-conn. ⇒ 2-circuit in $O(n)$ time.

Conclusion

Conclusion

Theorem.
G int. 3-conn., X, Y, α on outer face $\Rightarrow$ T_{int}-path in $O(n)$ time.



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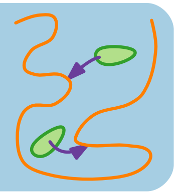
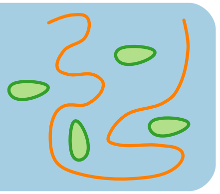


Theorem.
G 2-conn., X, Y, α on outer face $\Rightarrow$ Tutte path in $O(n)$ time.



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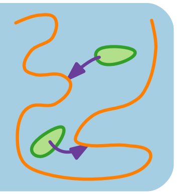

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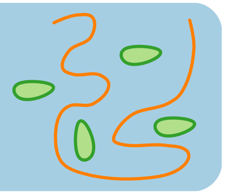

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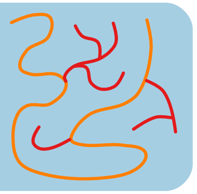

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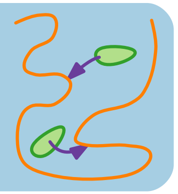

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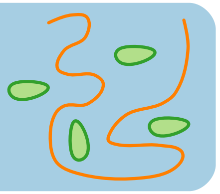

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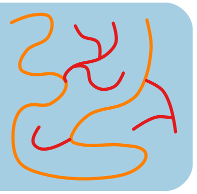
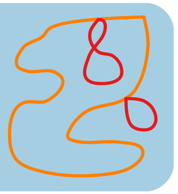


Theorem.
G int. 3-conn. $\Rightarrow$ 2-circuit in $O(n)$ time.



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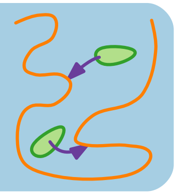

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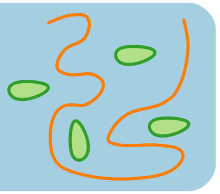

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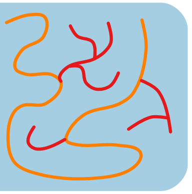

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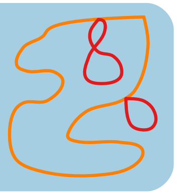
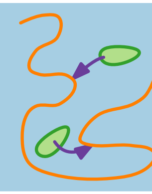


X, Y, α on different faces?

Conclusion

Theorem.
G int. 3-conn., X, Y, α on outer face $\Rightarrow$ T_{int}-path in $O(n)$ time.



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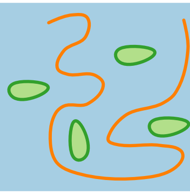

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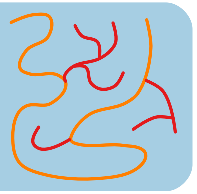

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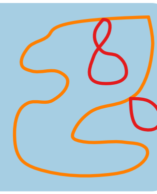

X, Y, α on different faces?

Non-planar graphs?

Conclusion

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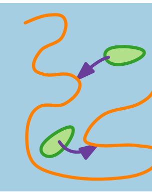
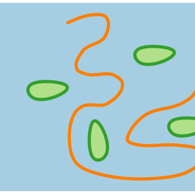
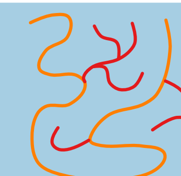


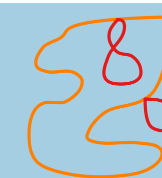
Theorem.
G 2-conn., X, Y, α on outer face $\Rightarrow$ Tutte path in $O(n)$ time.



Theorem.
G int. 3-conn. $\Rightarrow$ binary spanning tree in $O(n)$ time.



Theorem.
G int. 3-conn. $\Rightarrow$ 2-circuit in $O(n)$ time.



X, Y, α on different faces?

Non-planar graphs?