Julius-Maximilians-
UNIVERSITÄT WÜRZBURG
Chair for
INFORMATICS I
Efficient Algorithms and Knowledge-Based Systems
Institute for Informatics

Drawing Planar Graphs with Few Segments on the Grid

Philipp Kindermann
Universität Würzburg

joint work with
Tamara Mchedlidze Thomas Schneck Antonios Symvonis

Karlsruher Institut für Technologie 1825 1956 2009
EBERHARD KARLS UNIVERSITÄT TÜBINGEN
NATIONALE METSOBILE TECHNOLOGIE

Visual Complexity

of geometric entities in a drawing

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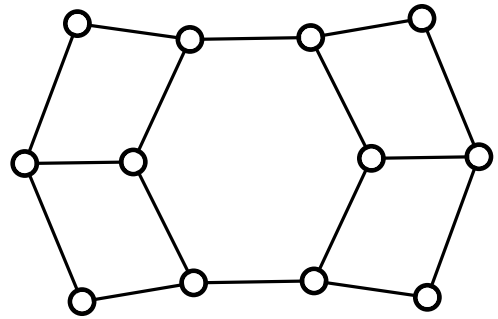

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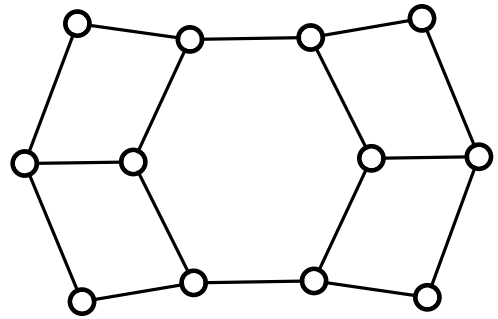

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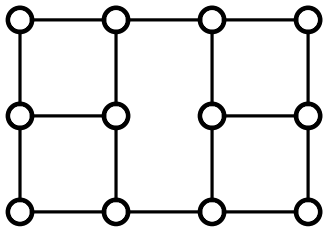

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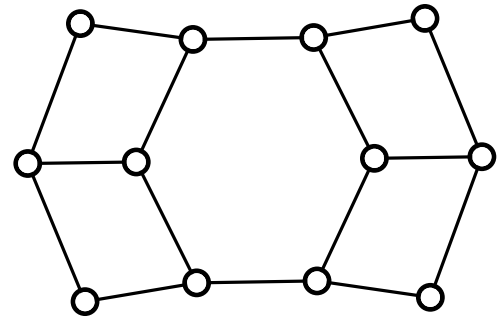

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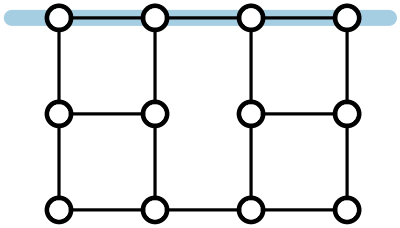

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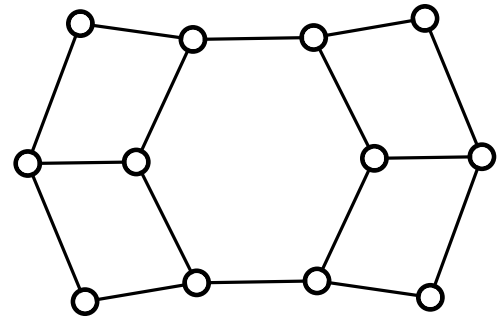

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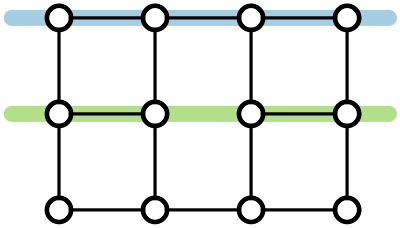

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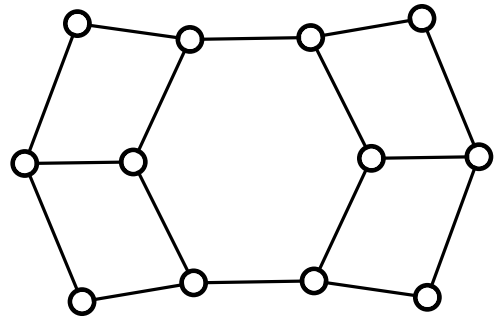

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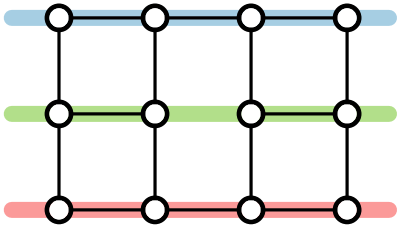

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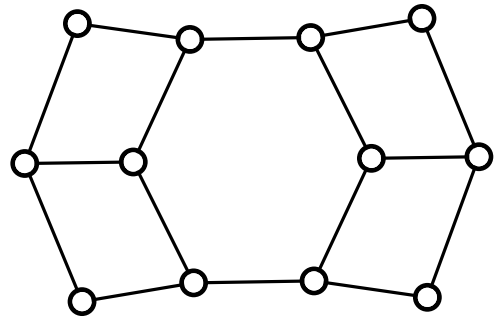

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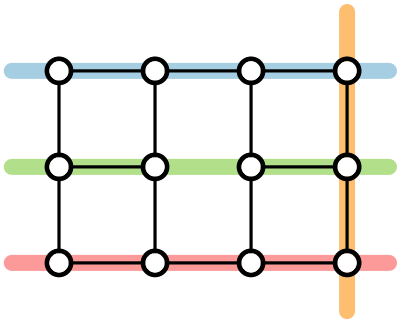

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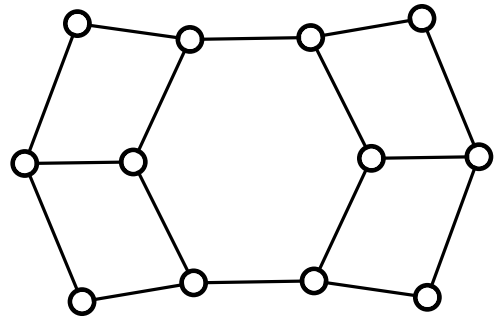

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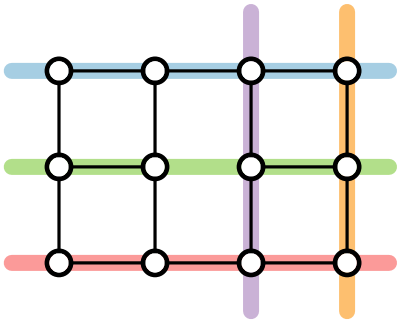

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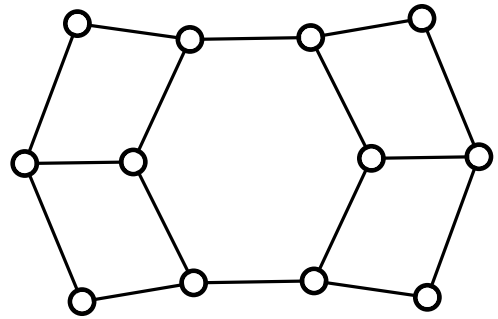

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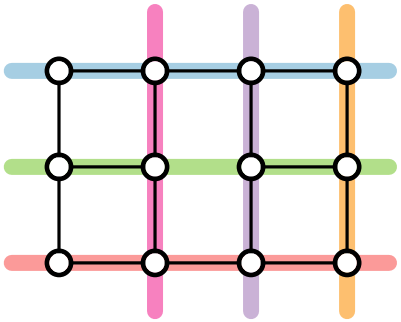

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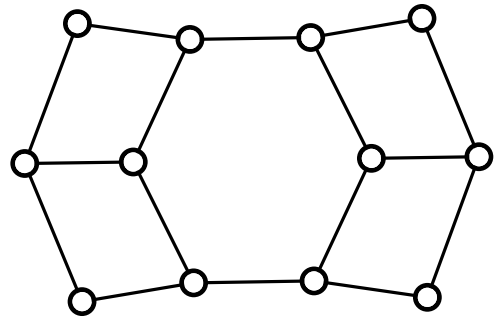

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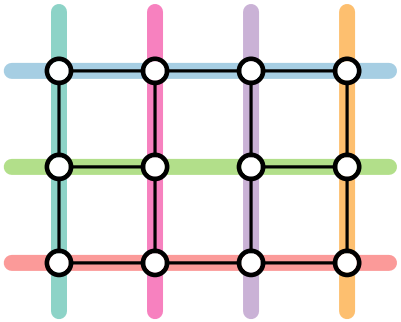

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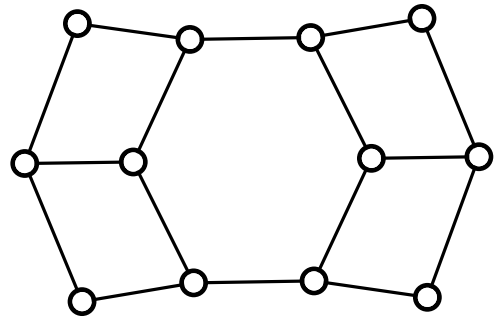

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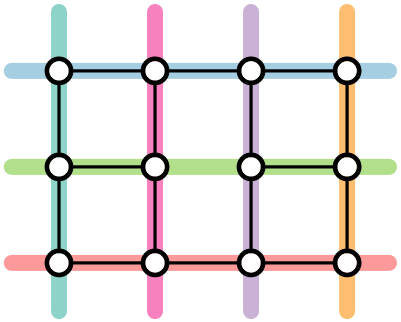

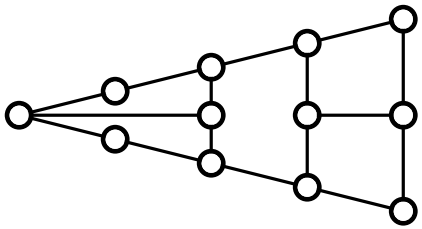

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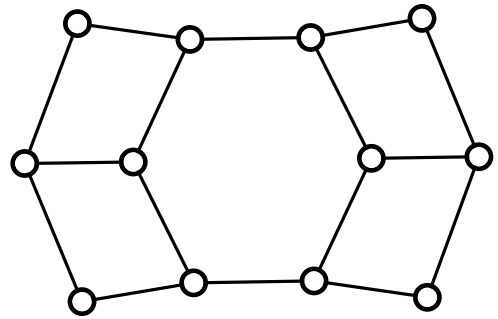

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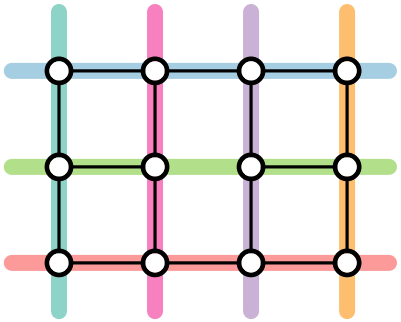

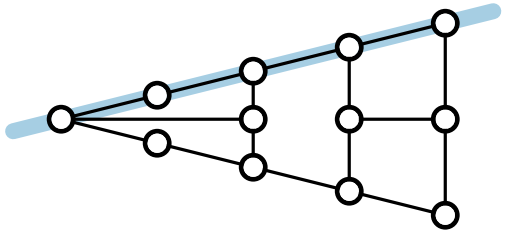

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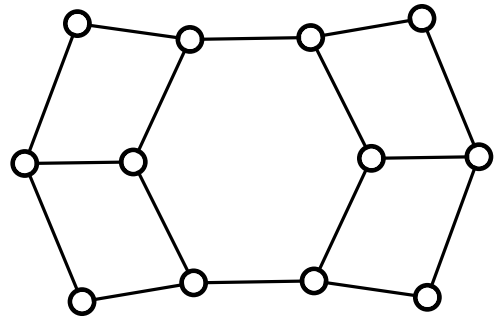

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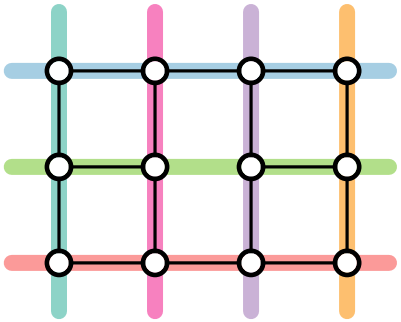

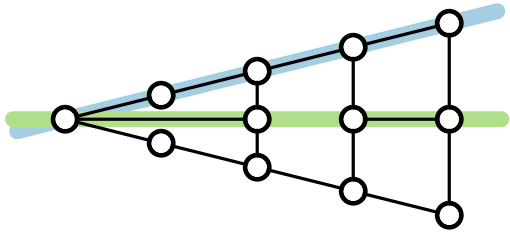

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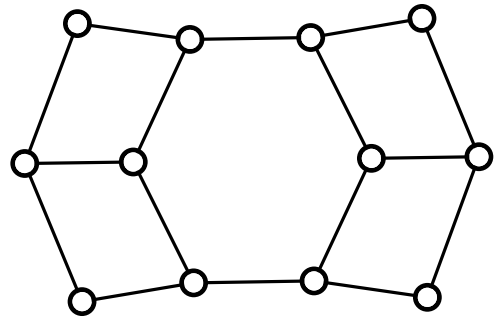

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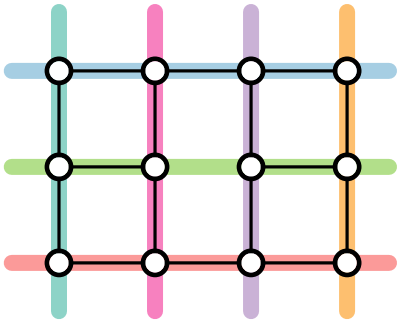

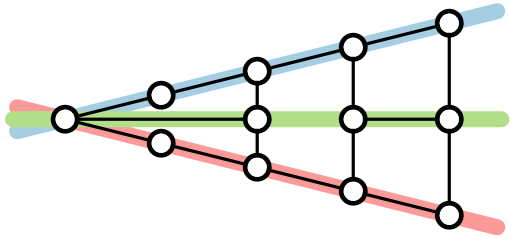

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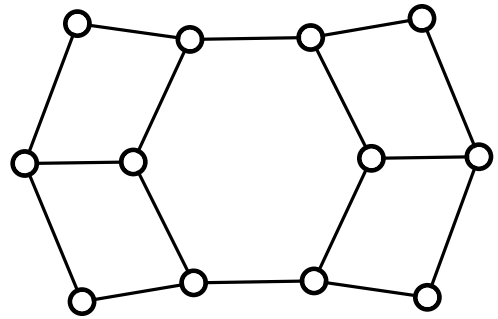

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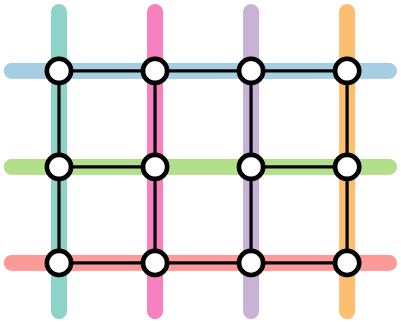

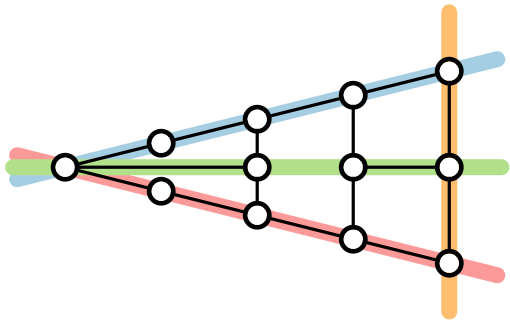

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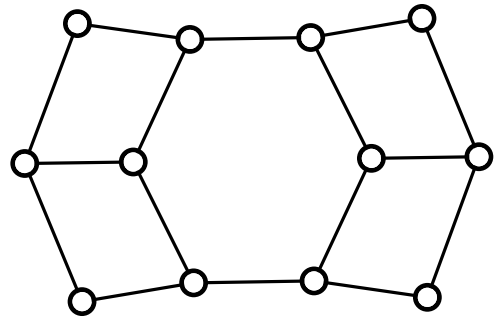

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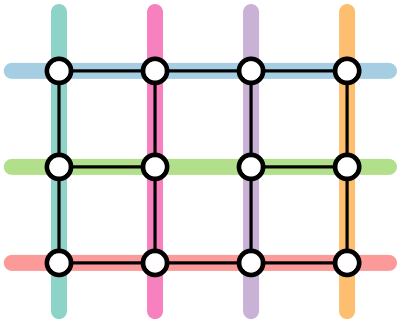

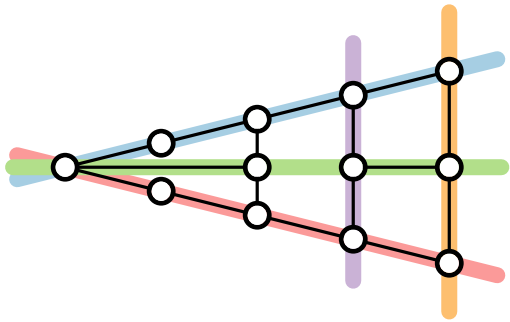

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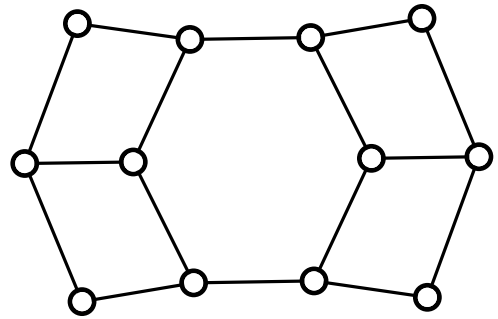

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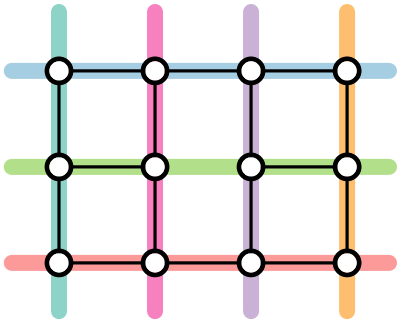

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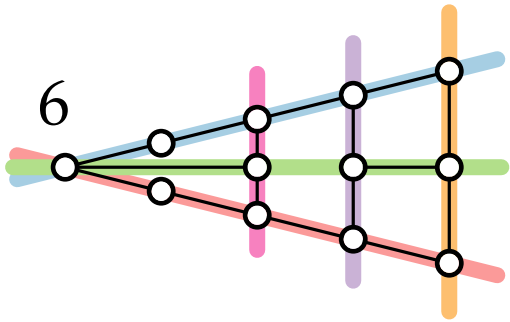

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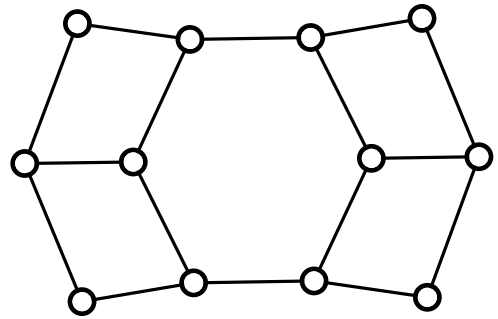

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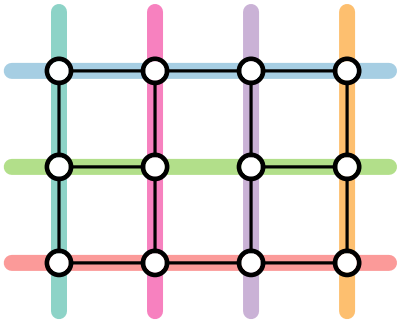

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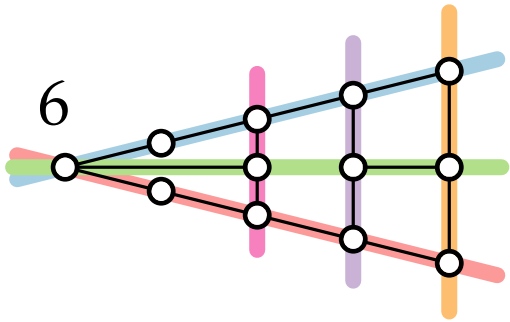

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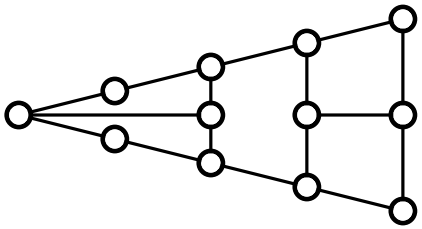

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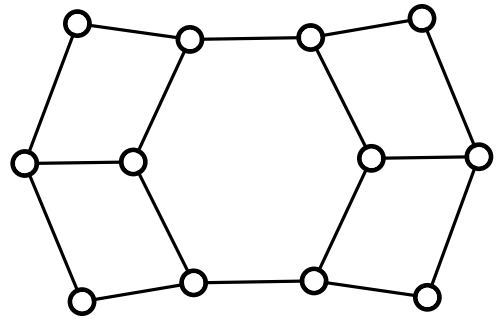

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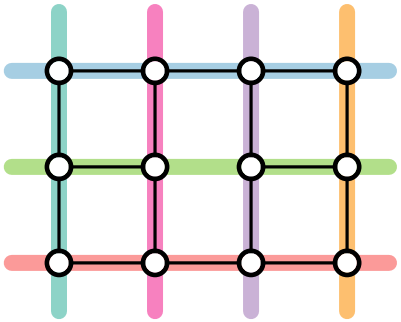

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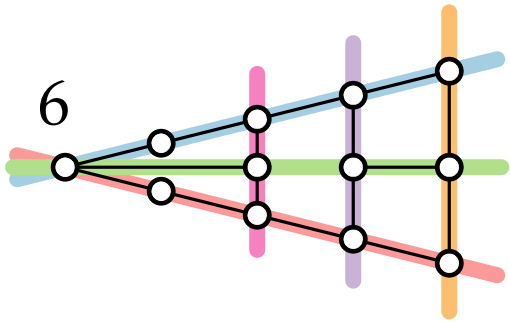

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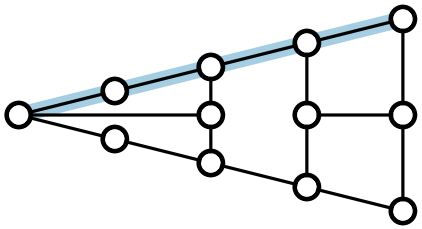

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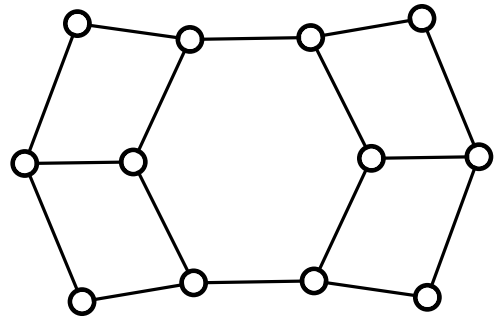

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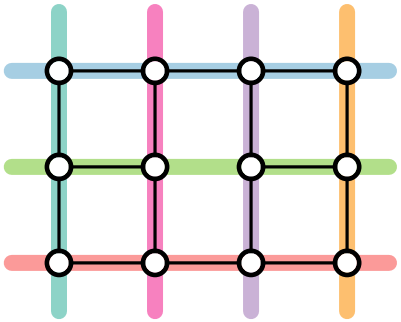

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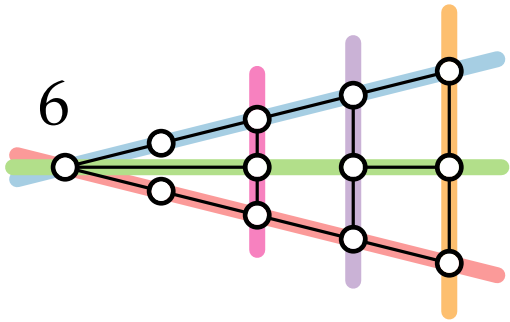

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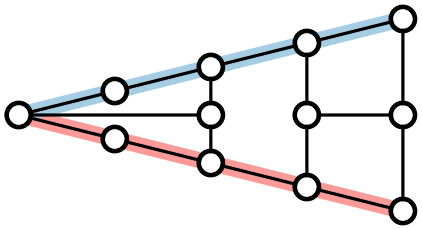

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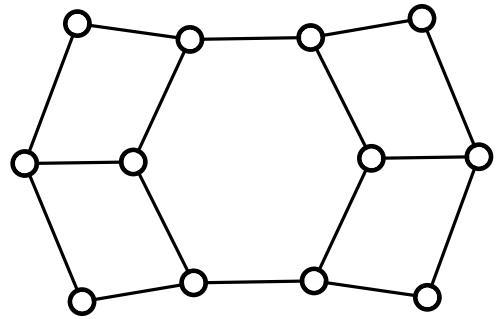

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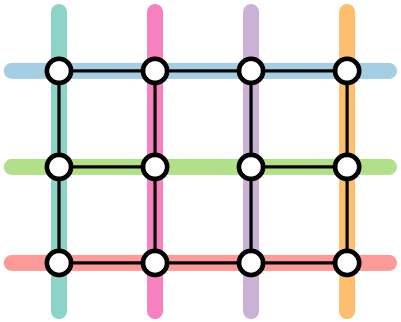

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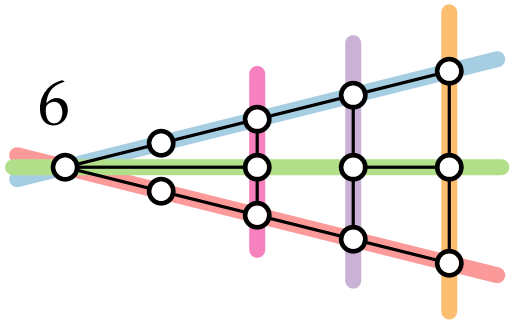

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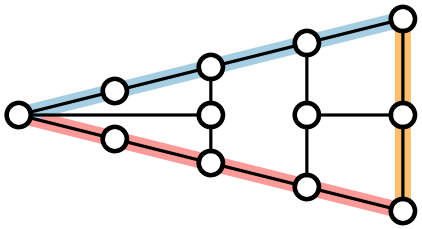

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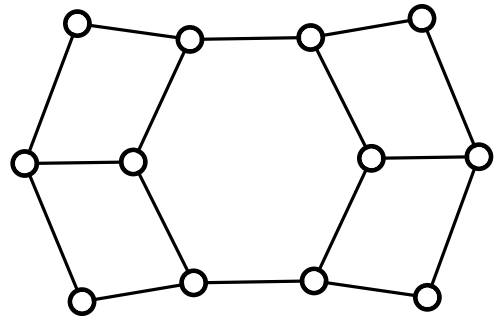

# of geometric entities in a drawing

(strong) line cover number

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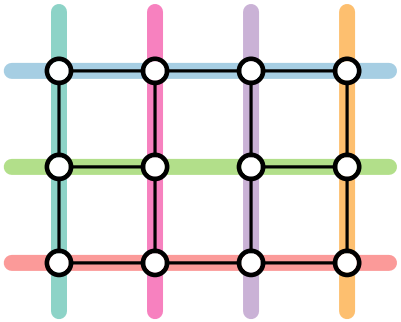

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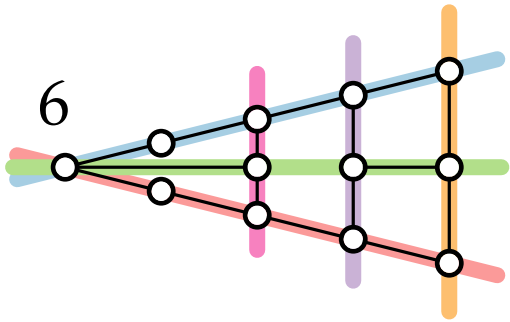

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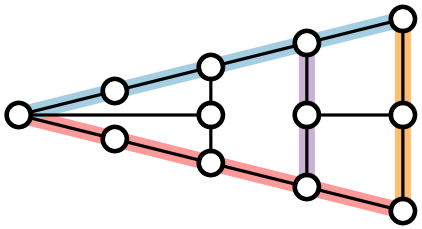

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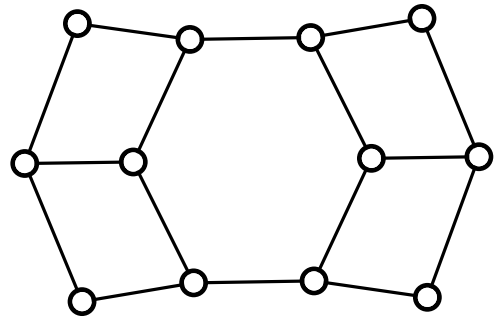

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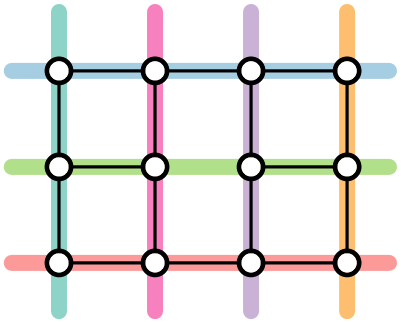

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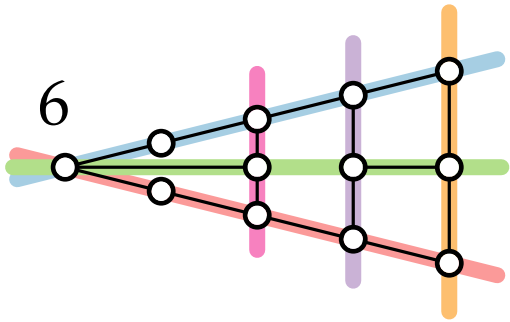

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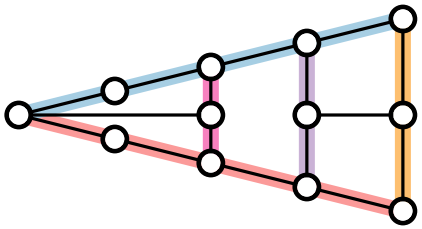

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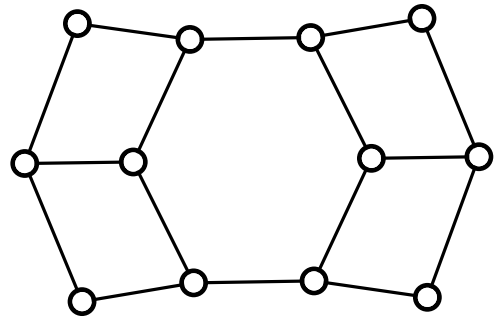

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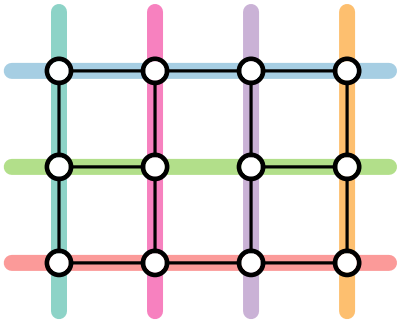

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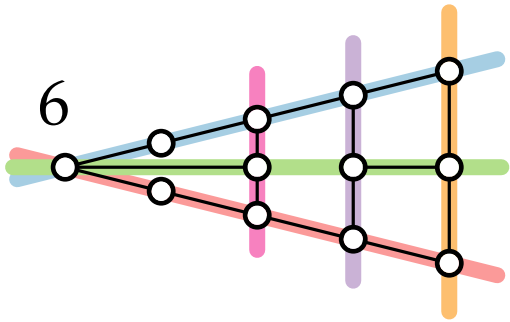

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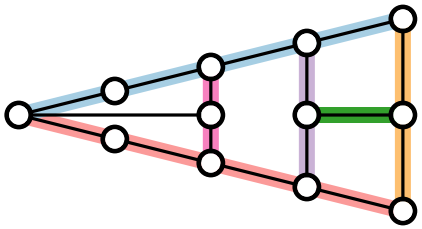

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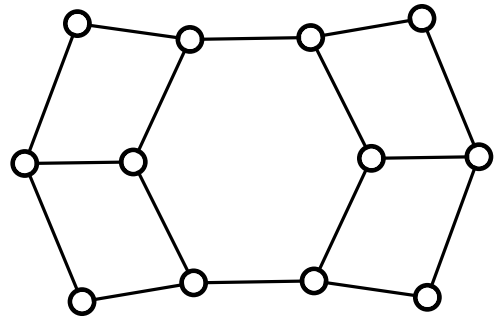

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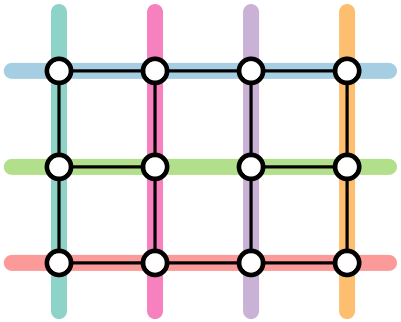

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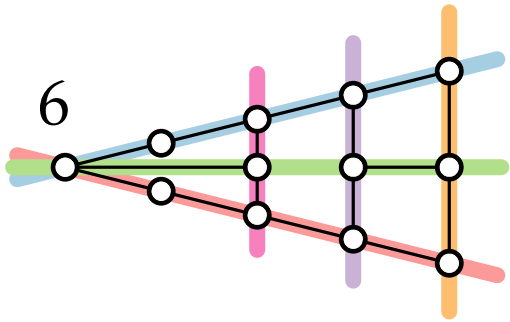

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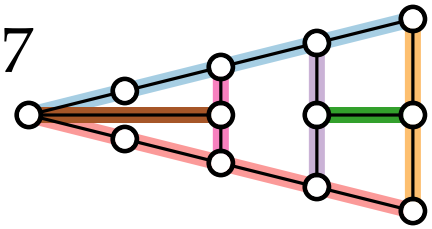

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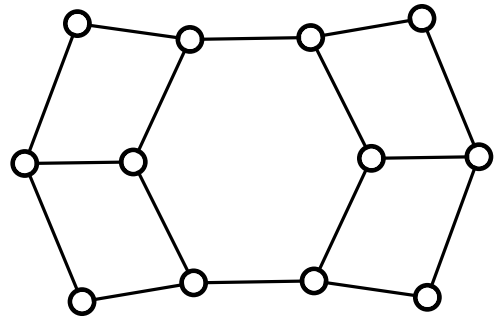

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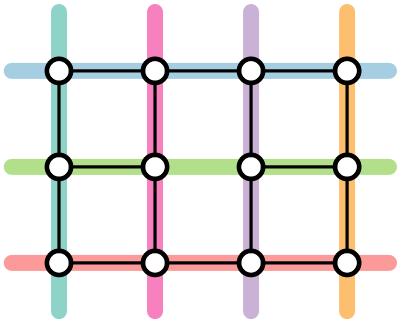

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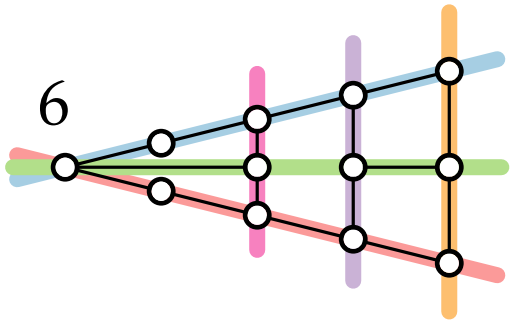

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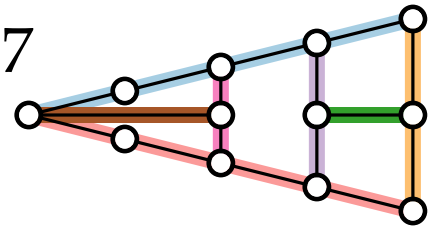

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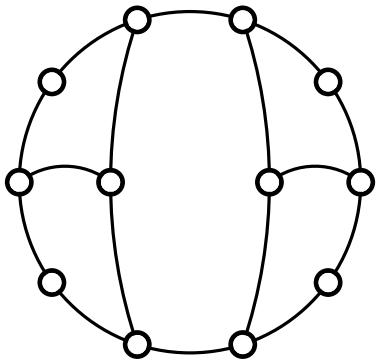

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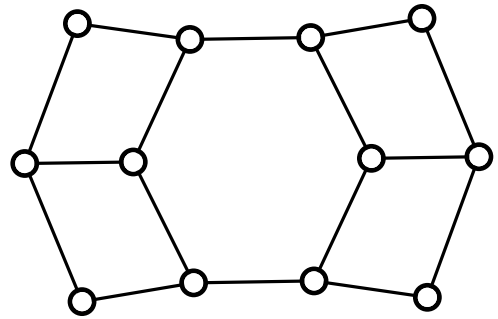

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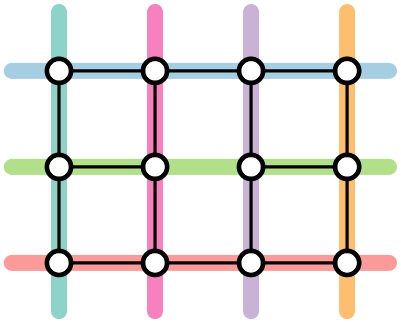

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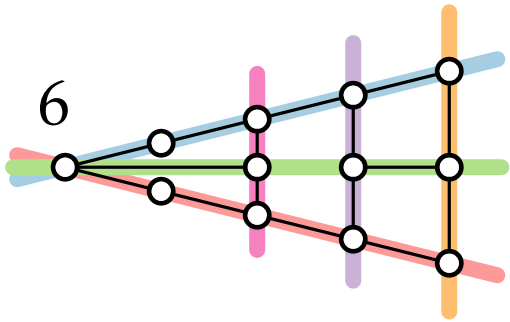

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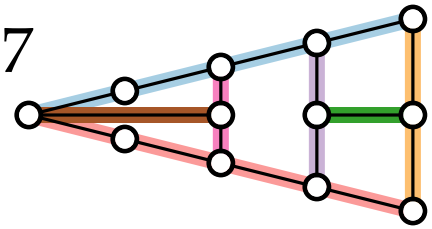

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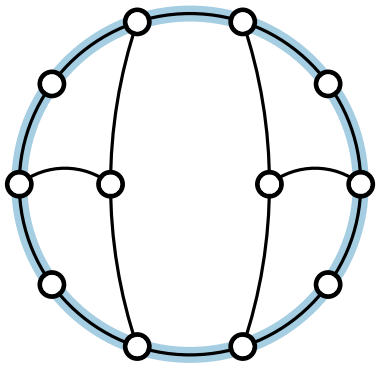

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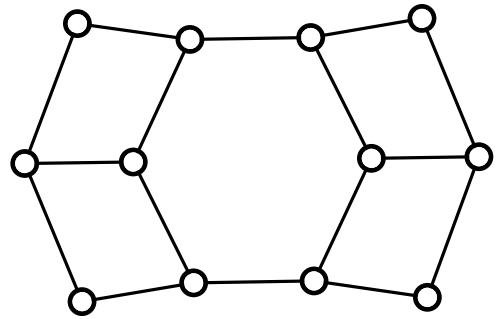

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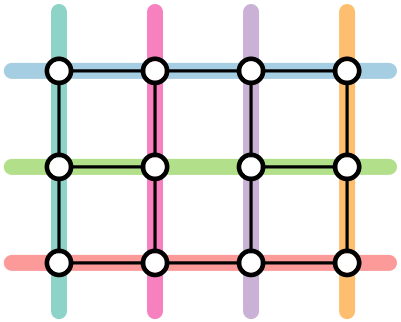

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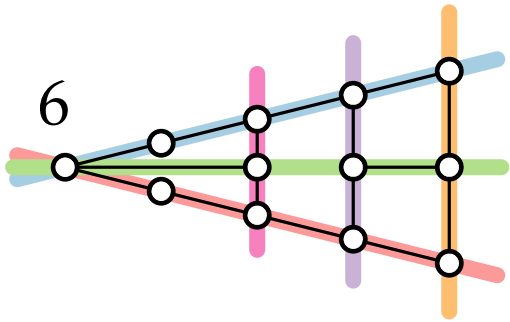

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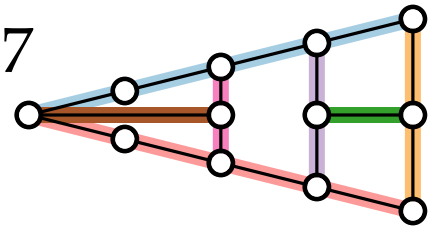

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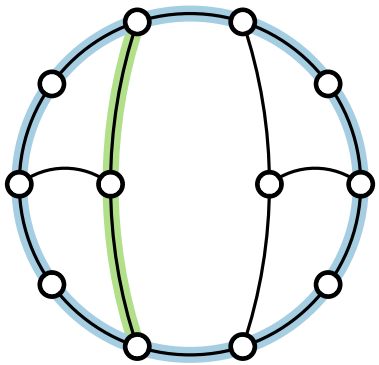
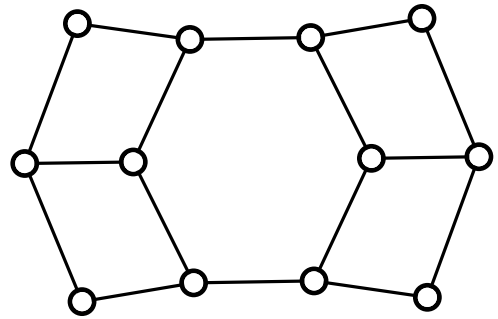


Visual Complexity



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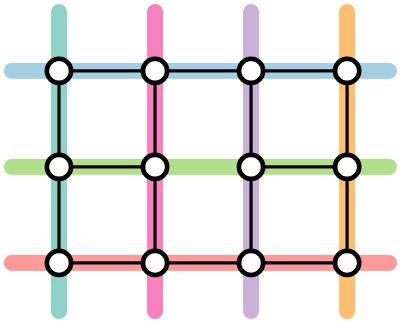

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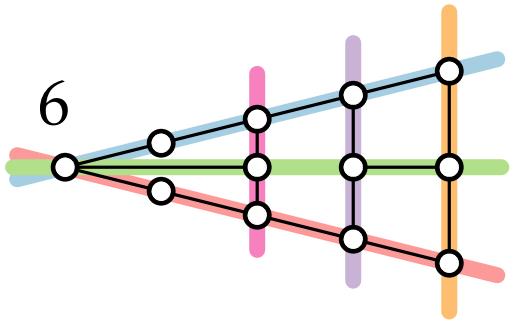

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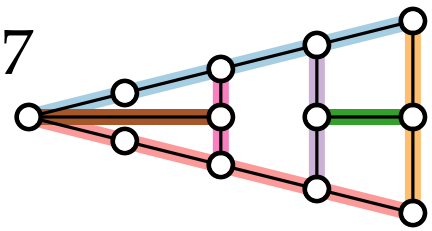

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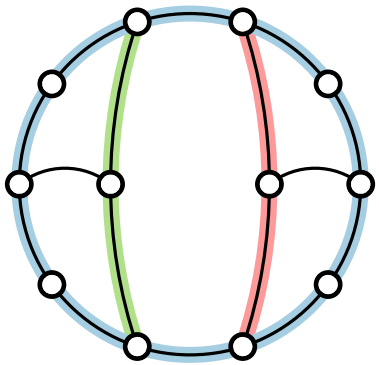

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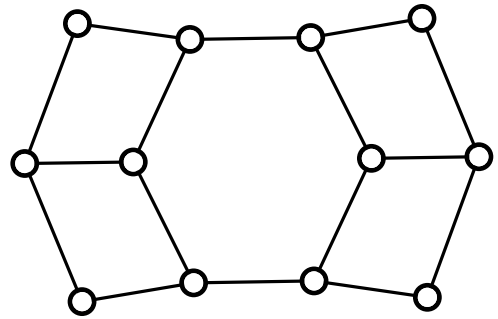

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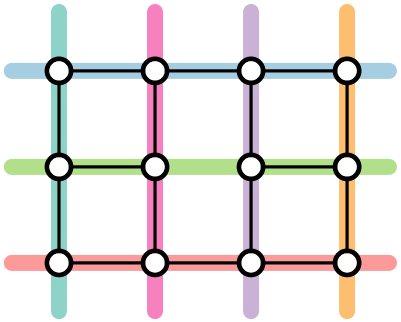

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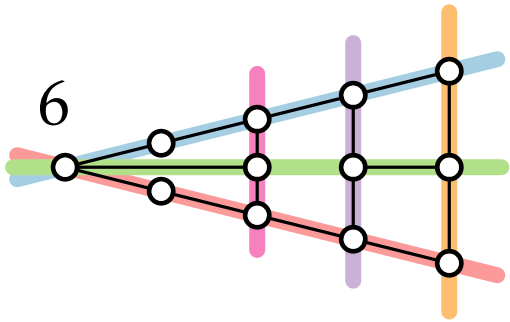

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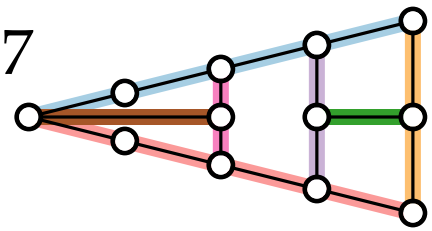

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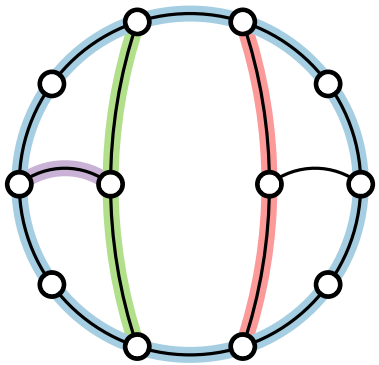

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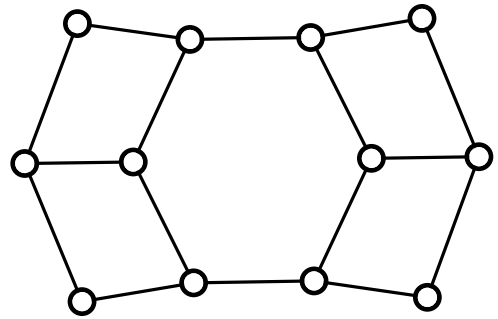

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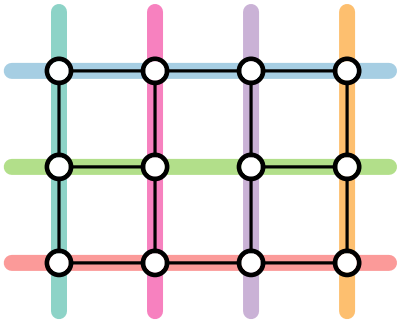

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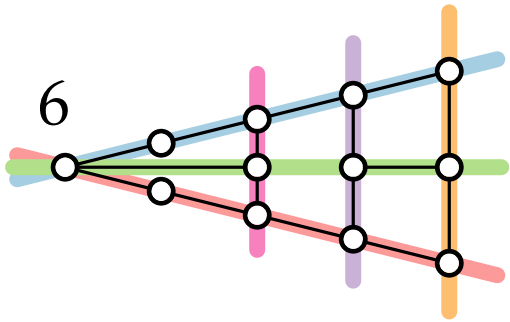

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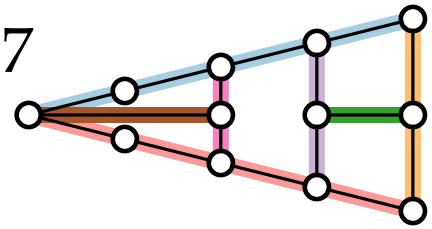

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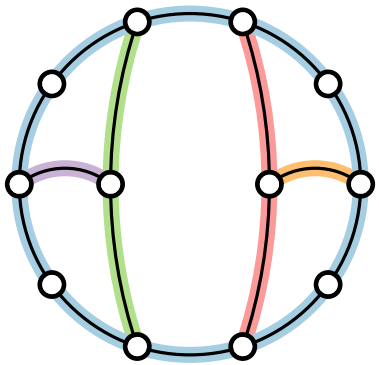
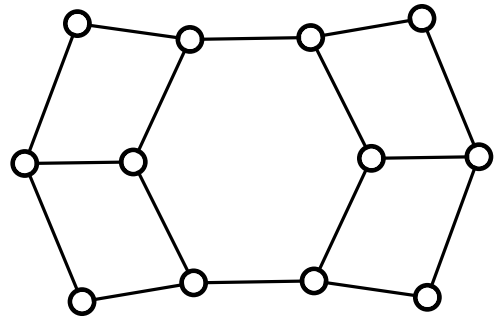


Visual Complexity



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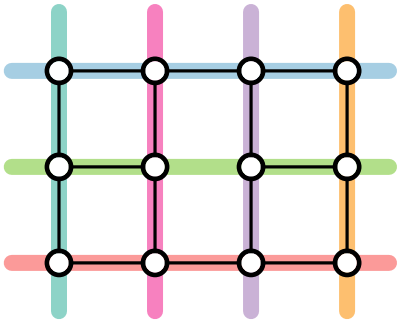

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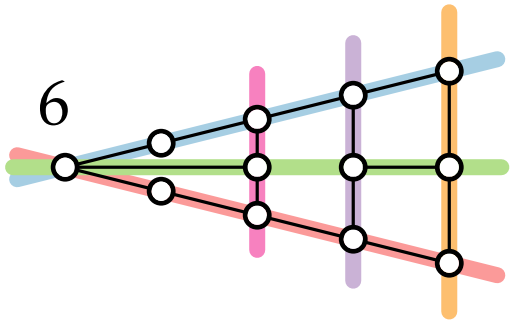

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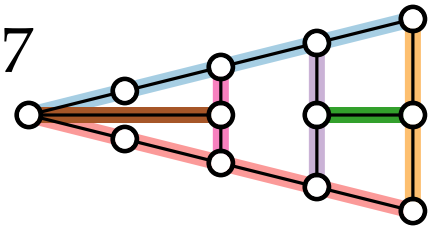

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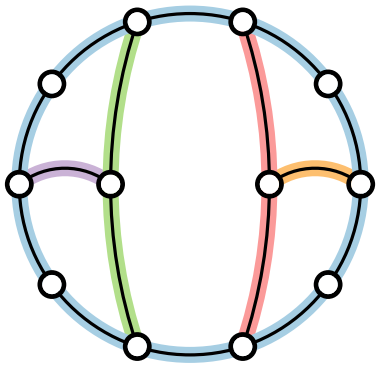

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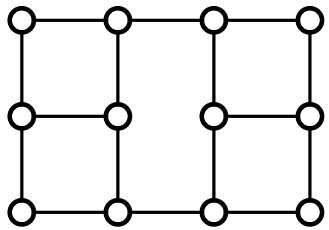
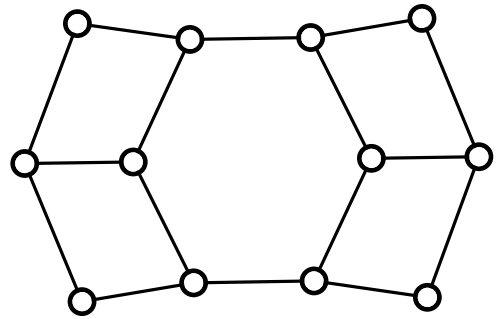


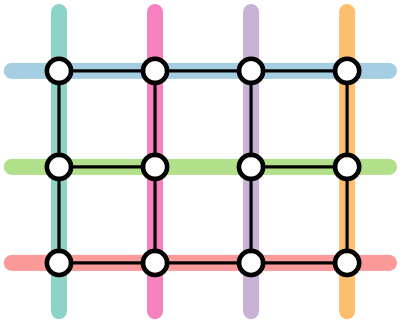
Visual Complexity



# of geometric entities in a drawing

(strong) line cover number

7



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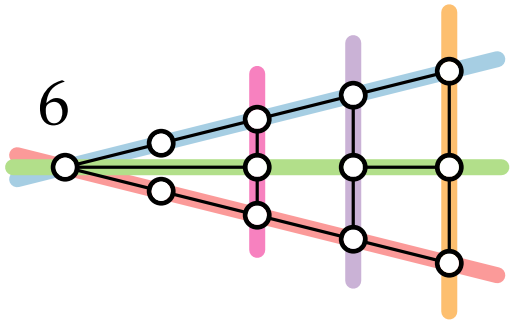
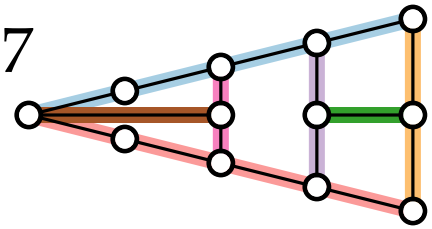


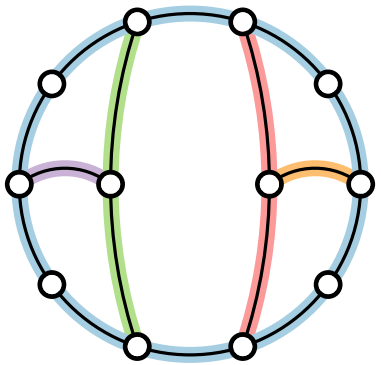
segment number

7

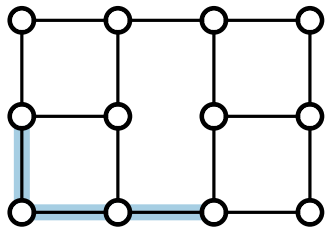


arc number

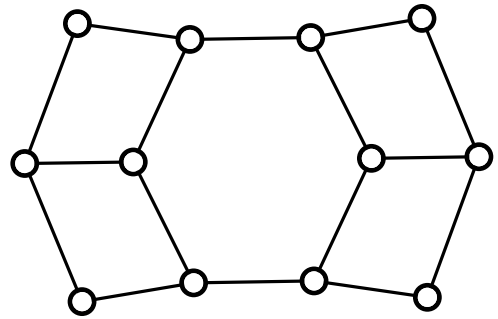
5



path cover number



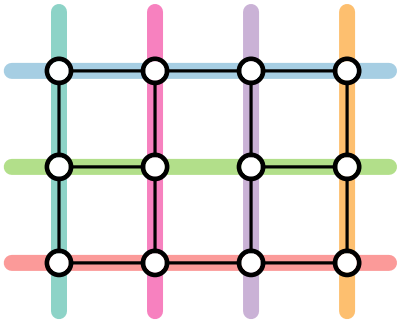
Visual Complexity



# of geometric entities in a drawing

(strong) line cover number

7



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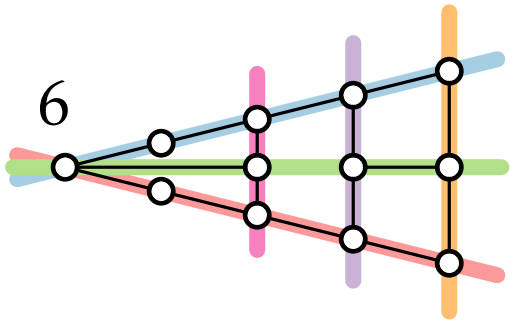

segment number

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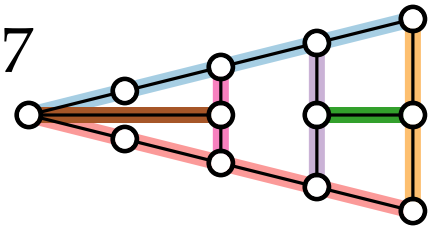

arc number

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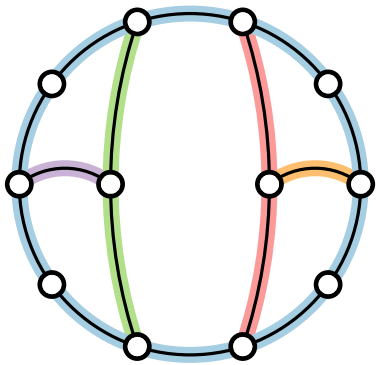
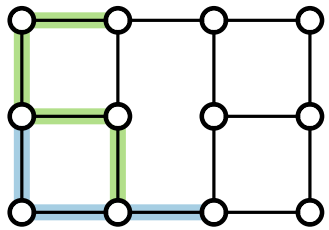
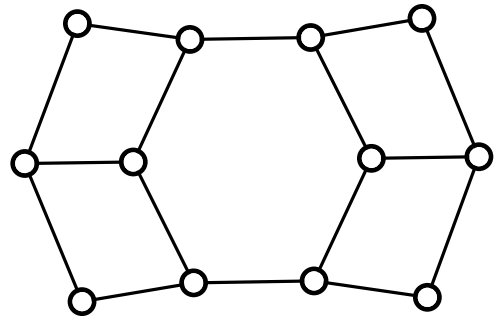


path cover number



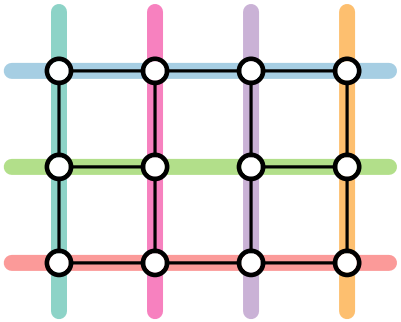
Visual Complexity



# of geometric entities in a drawing

(strong) line cover number

7



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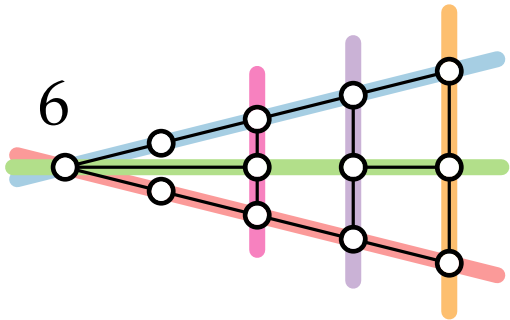

segment number

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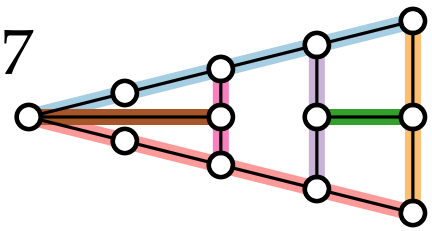

arc number

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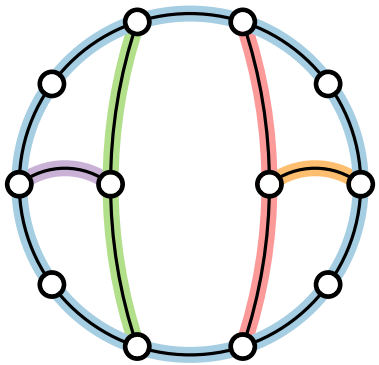
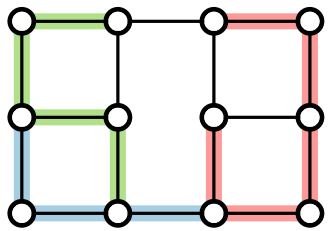
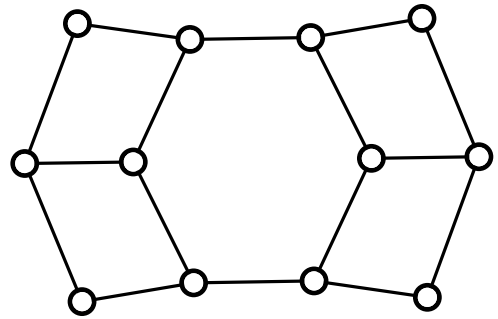


path cover number



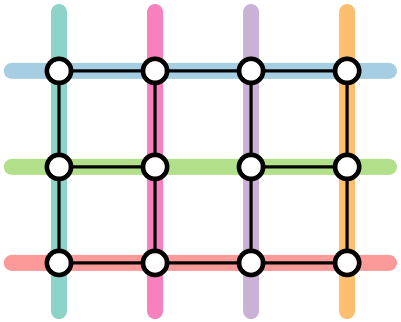
Visual Complexity



# of geometric entities in a drawing

(strong) line cover number

7



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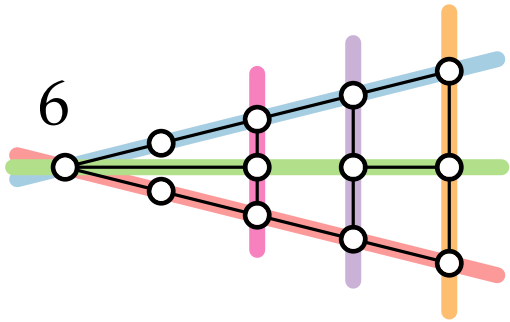

segment number

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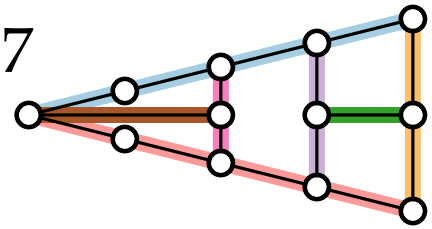

arc number

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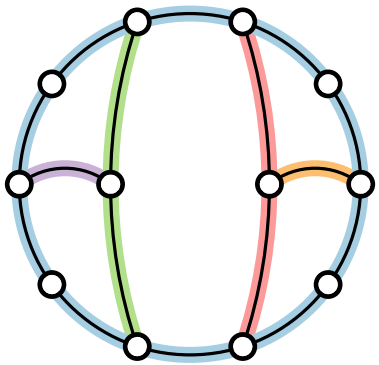
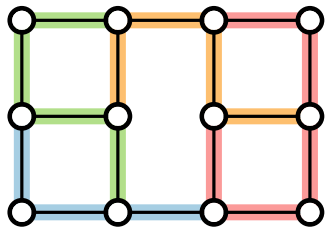
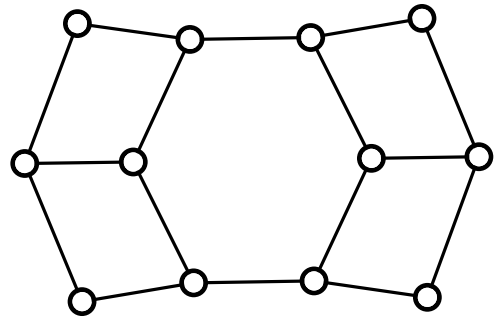


path cover number

4



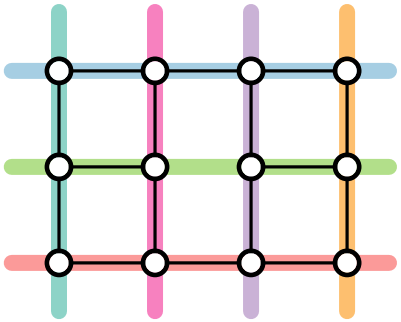
Visual Complexity



# of geometric entities in a drawing

(strong) line cover number

7



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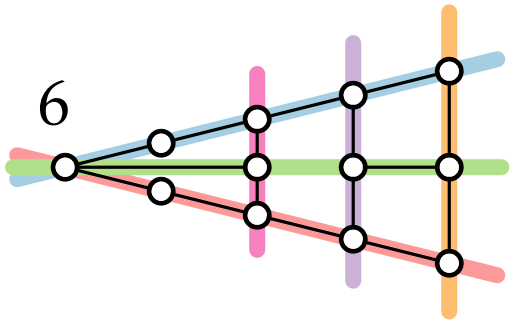
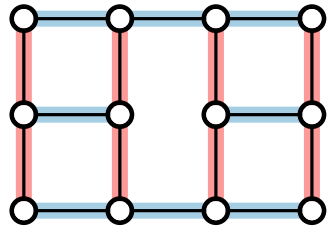


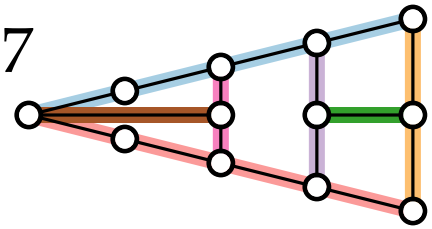
slope number

2



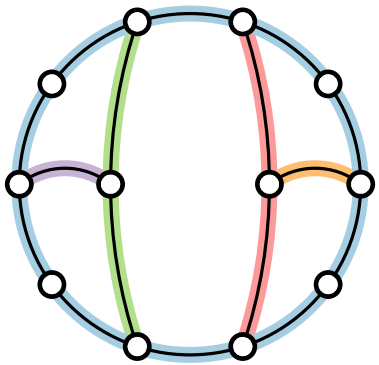
segment number

7



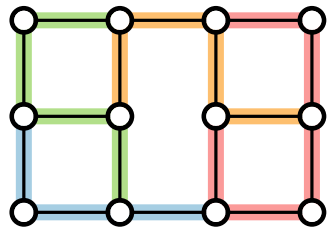
arc number

5



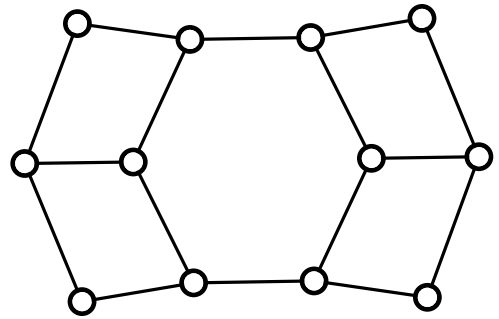
path cover number

4



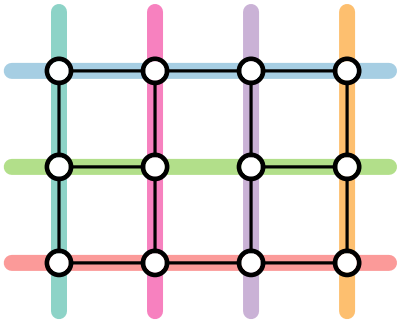
Visual Complexity

of geometric entities in a drawing



(strong) line cover number

7



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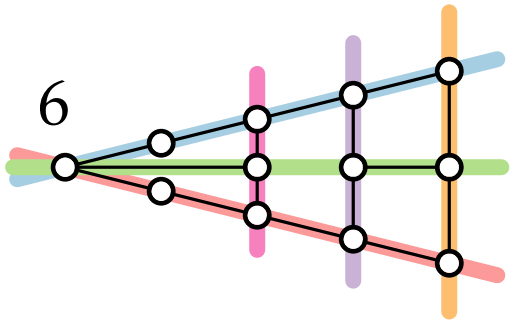
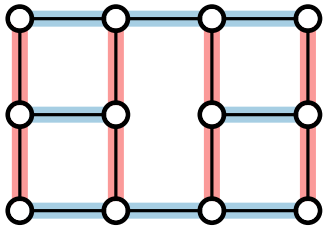


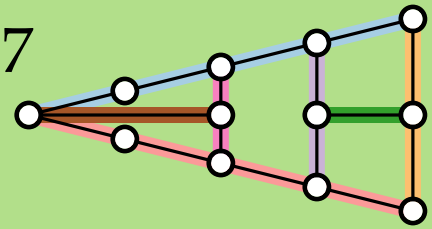
slope number

2



segment number

7



arc number

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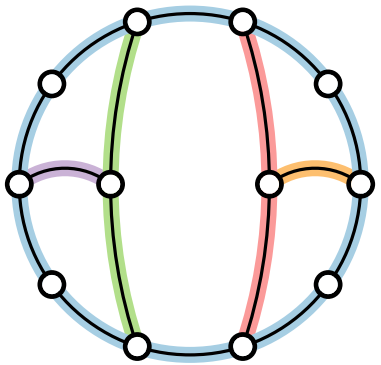
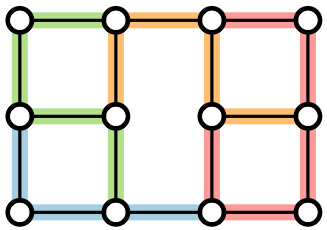


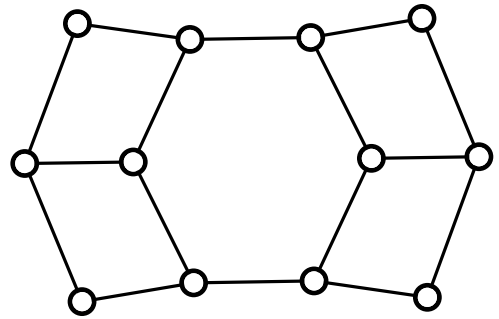
path cover number

4



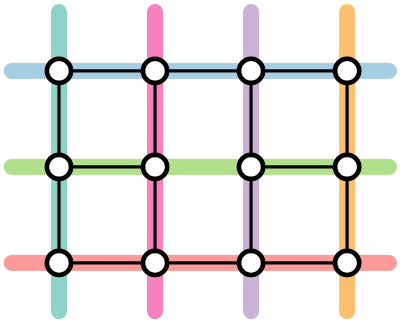
Visual Complexity

of geometric entities in a drawing



(strong) line cover number

7



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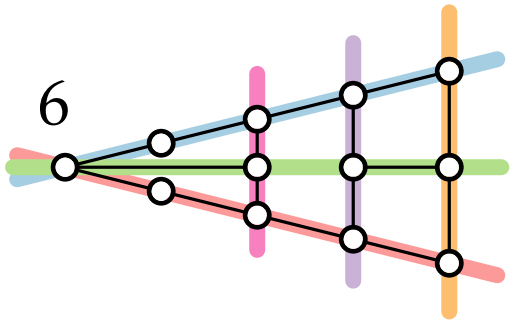
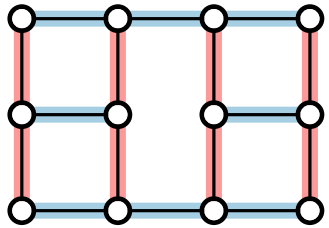


slope number

2



segment number

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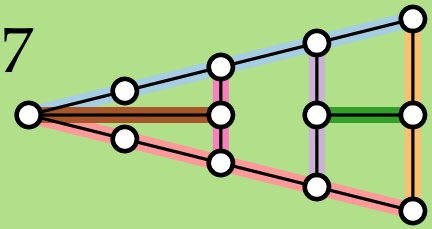

arc number

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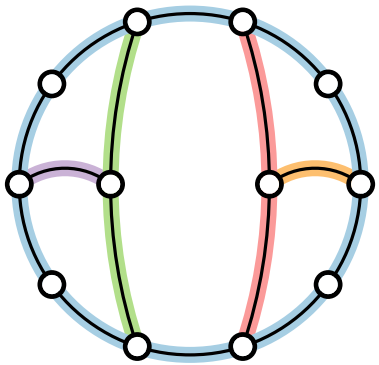
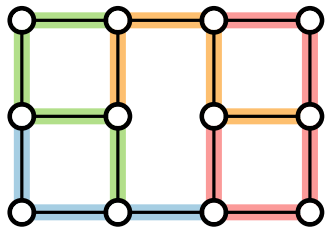


path cover number

4



all other numbers
are lower bounds

(Some) Known Results

Class	Segments	
	Lower	Upper

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
outerplanar	n [1]	

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
outerplanar	n [1]	
max. outerp.	n [1]	n [1]

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
outerplanar	n [1]	
max. outerp.	n [1]	n [1]
3-trees	$2n$ [1]	$2n$ [1]

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
outerplanar	n [1]	
max. outerp.	n [1]	n [1]
3-trees	$2n$ [1]	$2n$ [1]
2-connected	$2n$ [1]	

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
outerplanar	n [1]	
max. outerp.	n [1]	n [1]
3-trees	$2n$ [1]	$2n$ [1]
2-connected	$2n$ [1]	
3-connected	$2n$ [1]	$5n/2$ [1]

[1] Dujmović et al. 2007

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
outerplanar	n [1]	
max. outerp.	n [1]	n [1]
3-trees	$2n$ [1]	$2n$ [1]
2-connected	$2n$ [1]	
3-connected	$2n$ [1]	$5n/2$ [1]
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
outerplanar	n [1]	
max. outerp.	n [1]	n [1]
3-trees	$2n$ [1]	$2n$ [1]
2-connected	$2n$ [1]	
3-connected	$2n$ [1]	$5n/2$ [1]
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]
triangulation	$2n$ [4]	$7n/3$ [4]

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
outerplanar	n [1]	
max. outerp.	n [1]	n [1]
3-trees	$2n$ [1]	$2n$ [1]
2-connected	$2n$ [1]	
3-connected	$2n$ [1]	$5n/2$ [1]
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]
triangulation	$2n$ [4]	$7n/3$ [4]
4-conn. triang.	$2n$ [4]	$9n/4$ [4]

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014

(Some) Known Results

Class	Segments	
	Lower	Upper
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
outerplanar	n [1]	
max. outerp.	n [1]	n [1]
3-trees	$2n$ [1]	$2n$ [1]
2-connected	$2n$ [1]	
3-connected	$2n$ [1]	$5n/2$ [1]
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]
triangulation	$2n$ [4]	$7n/3$ [4]
4-conn. triang.	$2n$ [4]	$9n/4$ [4]
planar	$2n$ [4]	$8n/3$ [4]

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]		
outerplanar	n [1]			
max. outerp.	n [1]	n [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]		
triangulation	$2n$ [4]	$7n/3$ [4]		
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]		
outerplanar	n [1]			
max. outerp.	n [1]	n [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]		
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]		
outerplanar	n [1]			
max. outerp.	n [1]	n [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O(n \log n)}$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013

[4] Durocher & Mondal 2014 [5] Mondal 2016

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6]	$O(n^2) \times O(n^{1.58})$
outerplanar	n [1]			
max. outerp.	n [1]	n [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O(n \log n)}$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013

[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6] $\vartheta/2$ [6]	$O(n^2) \times O(n^{1.58})$ quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O(n \log n)}$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6] $\vartheta/2$ [6]	$O(n^2) \times O(n^{1.58})$ quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O(n \log n)}$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6] $\vartheta/2$ [6]	$O(n^2) \times O(n^{1.58})$ quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [5]	$2^{O(n \log n)}$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6] $\vartheta/2$ [6]	$O(n^2) \times O(n^{1.58})$ quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [6]	$O(n) \times O(n^2)$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6] $\vartheta/2$ [6]	$O(n^2) \times O(n^{1.58})$ quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [6]	$O(n) \times O(n^2)$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

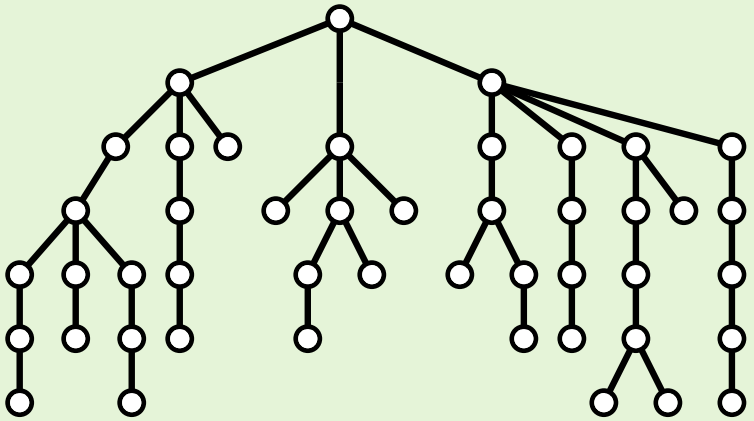
(Some) Known Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6]	$O(n^2) \times O(n^{1.58})$
			$\vartheta/2$ [6]	quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [6]	$O(n) \times O(n^2)$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

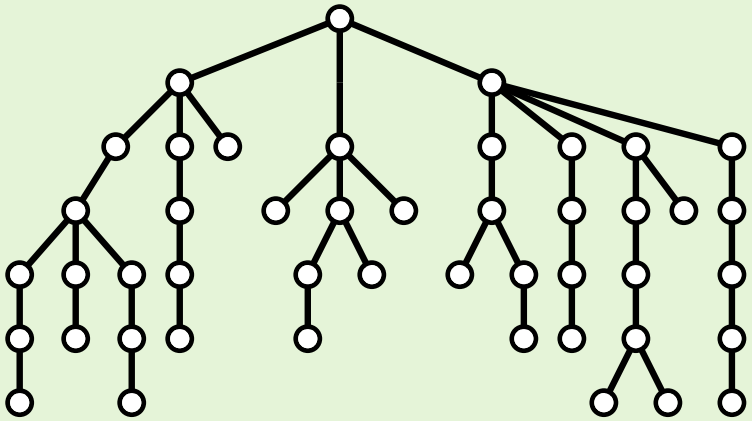
Tree Drawings

Tree T
n vtcs



Tree Drawings

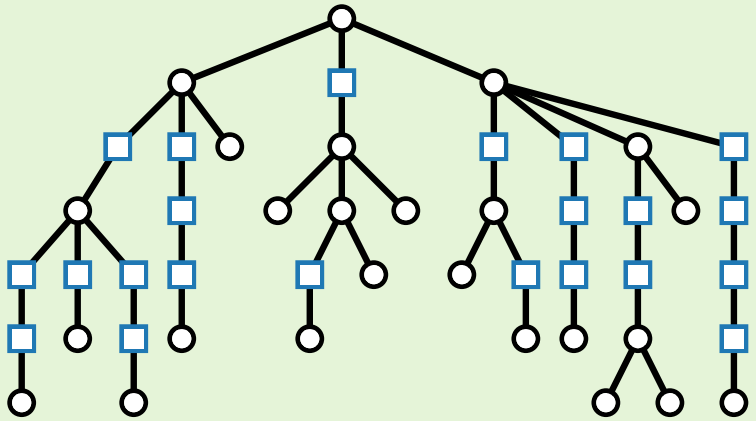
Tree T
n vtcs



β deg-2 vtcs

Tree Drawings

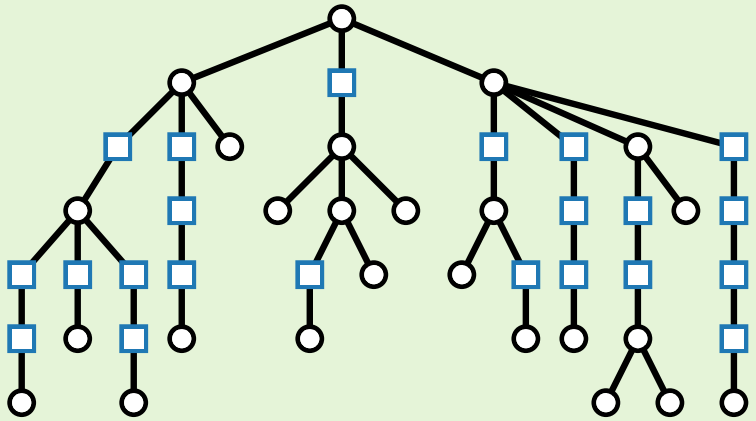
Tree T
n vtcs



β deg-2 vtcs

Tree Drawings

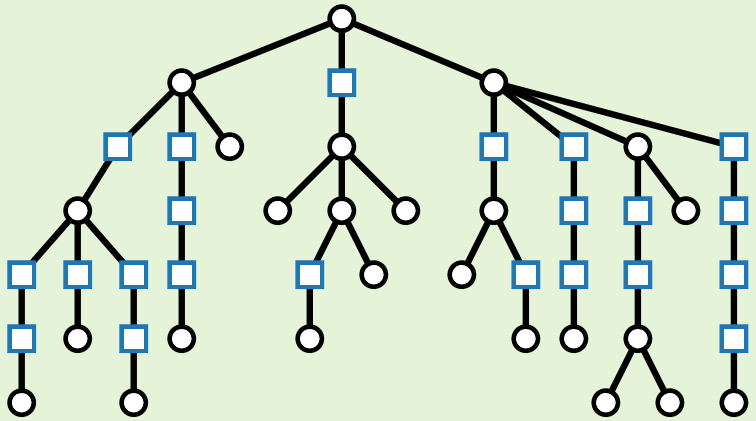
Tree T
n vtcs



Remove β deg-2 vtcs

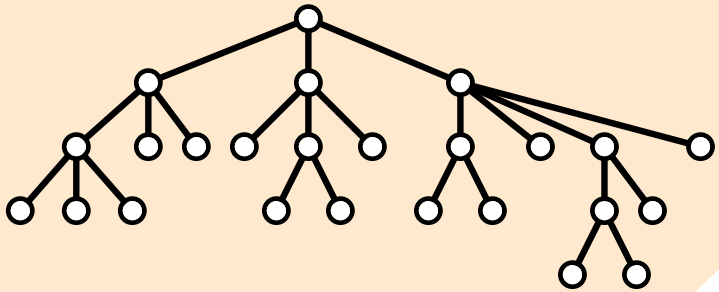
Tree Drawings

Tree T
n vtcs



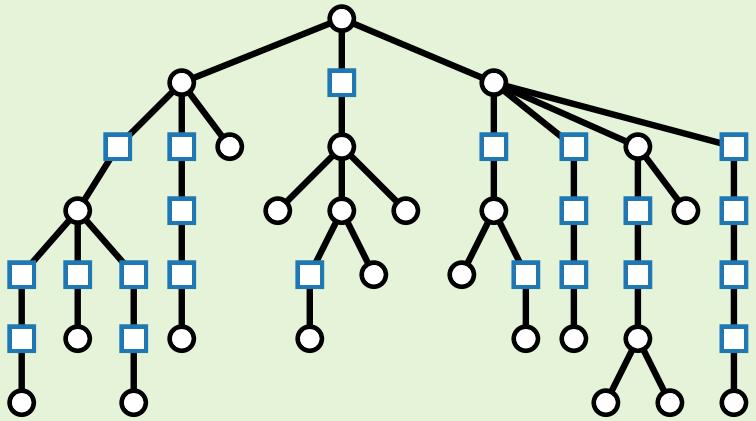
Remove β deg-2 vtcs

$\Rightarrow$ Tree T'
$n-\beta$ vtcs



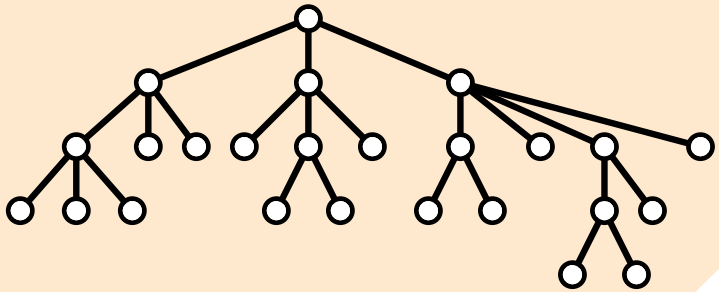
Tree Drawings

Tree T
n vtcs



Remove β deg-2 vtcs

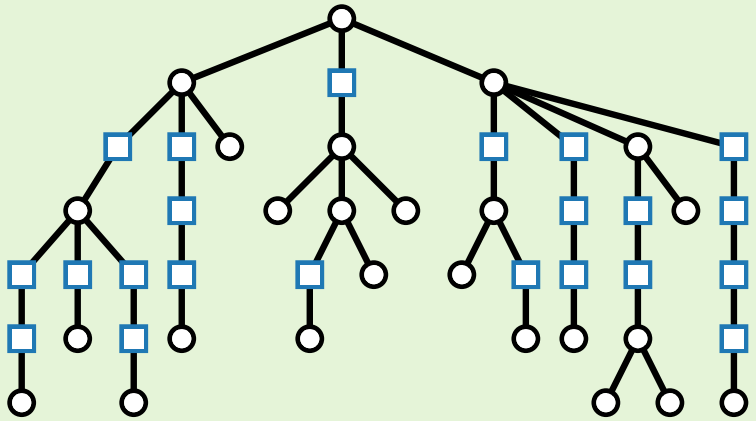
$\Rightarrow$ Tree T'
$n-\beta$ vtcs



α leaves

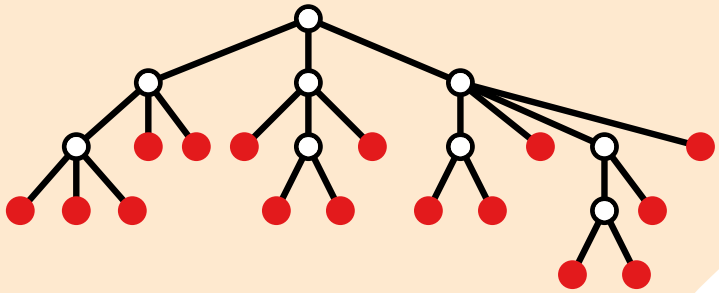
Tree Drawings

Tree T
n vtcs



Remove β deg-2 vtcs

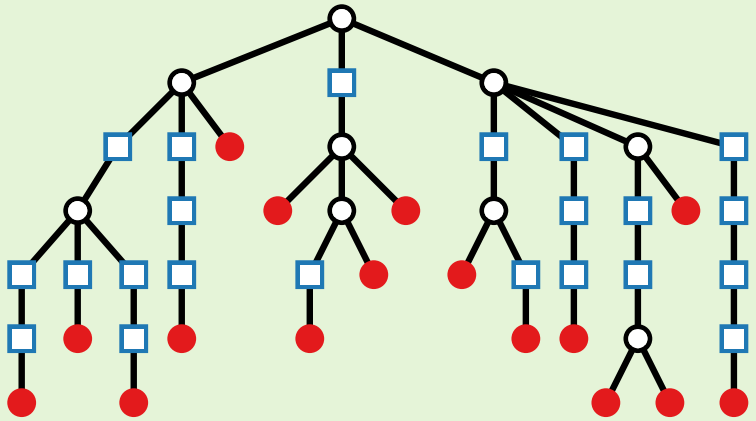
$\Rightarrow$ Tree T'
$n-\beta$ vtcs



α leaves

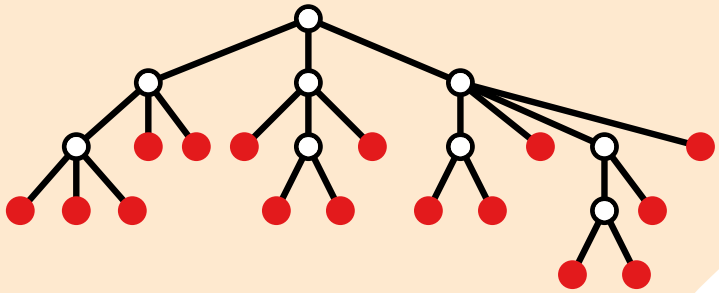
Tree Drawings

Tree T
n vtcs



Remove β deg-2 vtcs

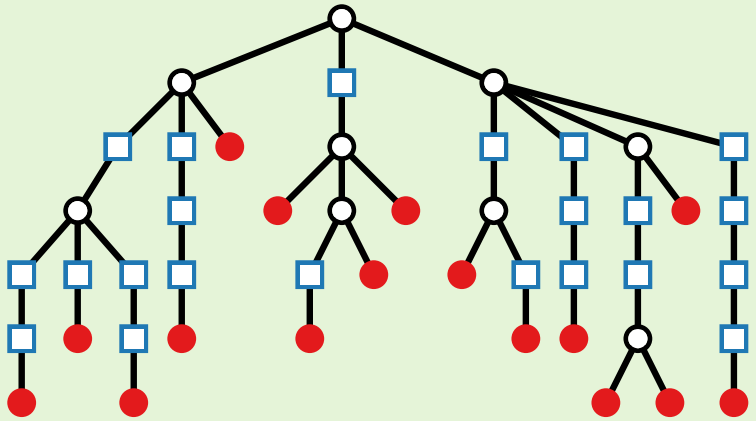
$\Rightarrow$ Tree T'
$n-\beta$ vtcs



α leaves

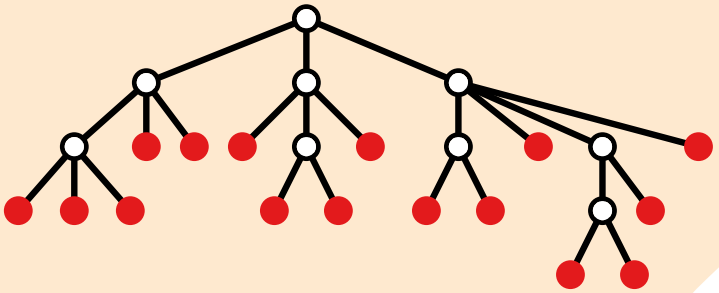
Tree Drawings

Tree T
n vtcs



Remove β deg-2 vtcs

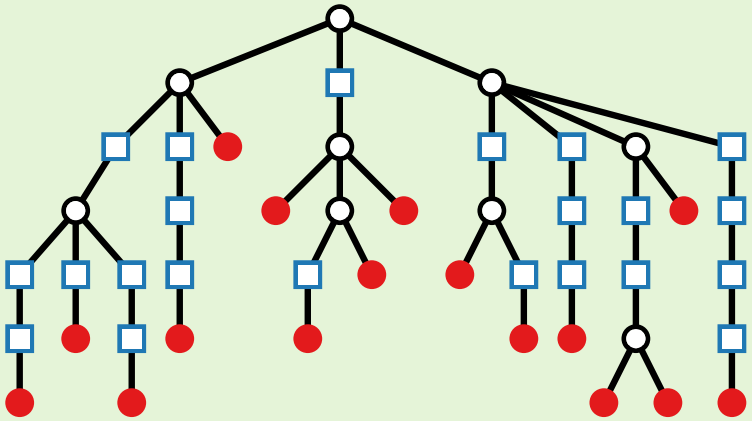
$\Rightarrow$ Tree T'
$n-\beta$ vtcs



Remove α leaves

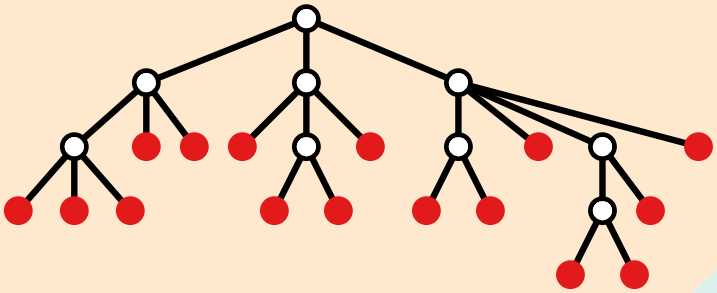
Tree Drawings

Tree T
n vtcs



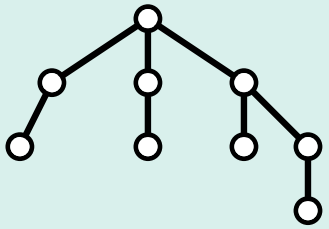
Remove β deg-2 vtcs

$\Rightarrow$ Tree T'
$n-\beta$ vtcs



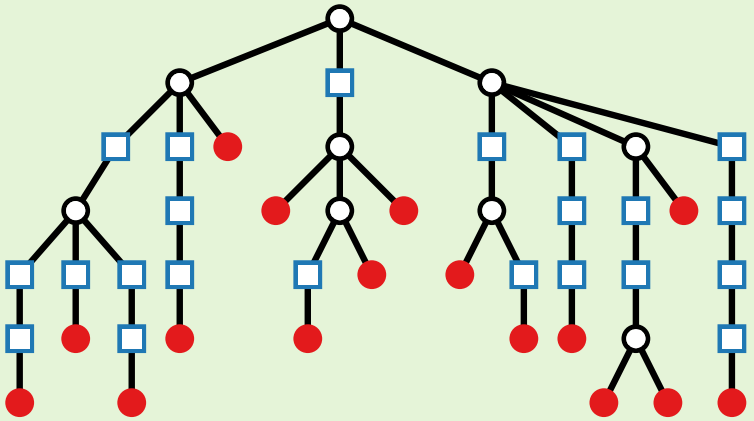
Remove α leaves

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs



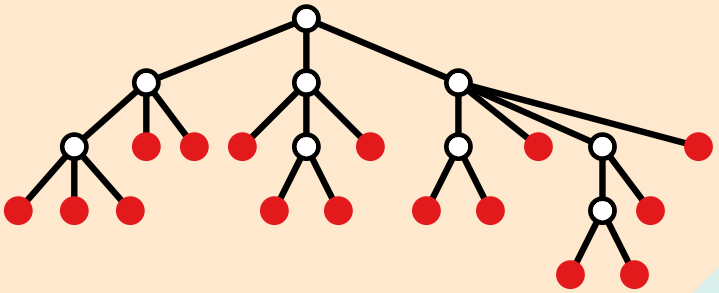
Tree Drawings

Tree T
n vtcs



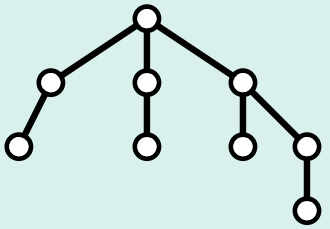
Remove β deg-2 vtcs

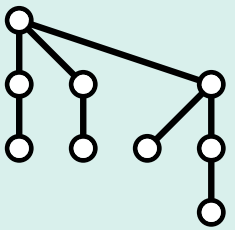
$\Rightarrow$ Tree T'
$n-\beta$ vtcs



Remove α leaves

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

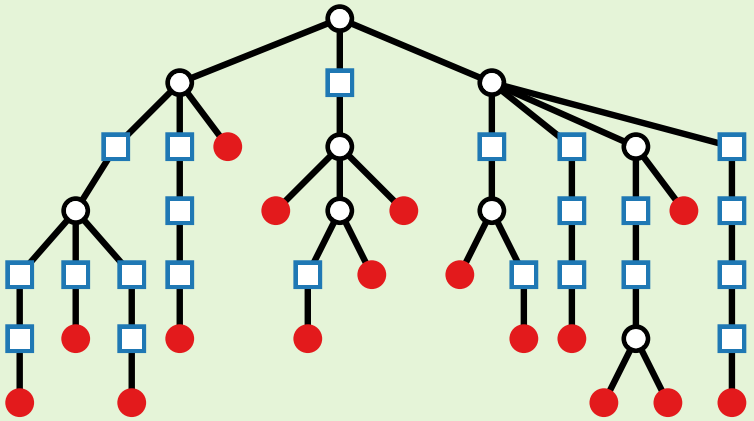




$n-\alpha-\beta$
segment

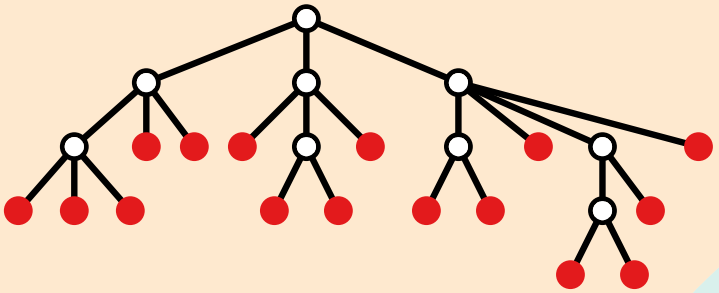
Tree Drawings

Tree T
n vtcs



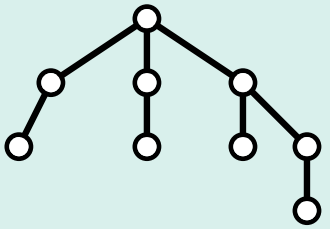
Remove β deg-2 vtcs

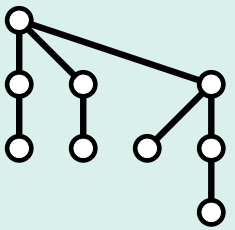
$\Rightarrow$ Tree T'
$n-\beta$ vtcs



Remove α leaves
+ $\alpha/2$ segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

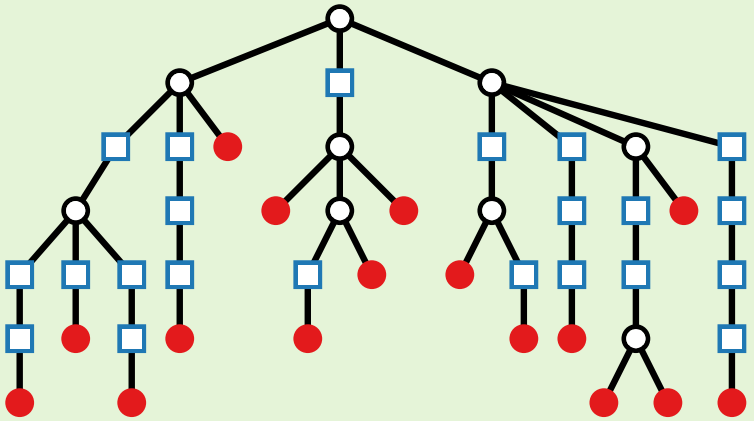




$n-\alpha-\beta$
segment

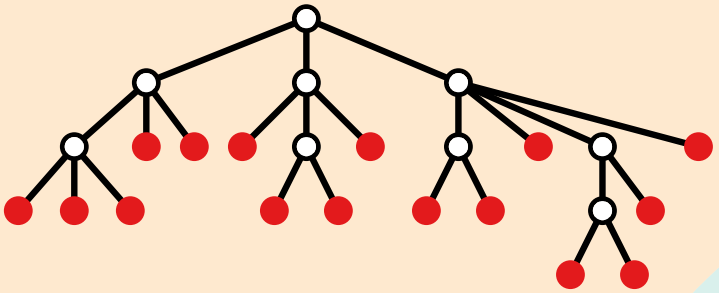
Tree Drawings

Tree T
n vtcs

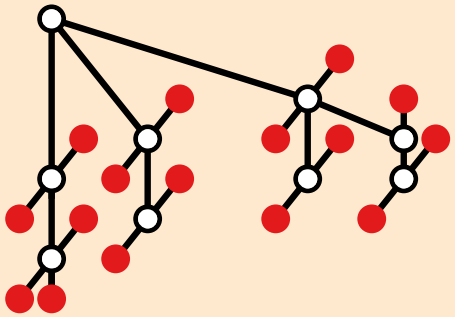


Remove β deg-2 vtcs

$\Rightarrow$ Tree T'
$n-\beta$ vtcs

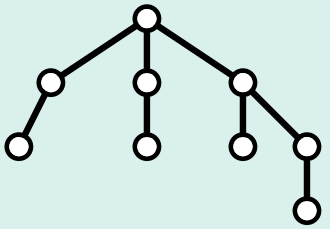


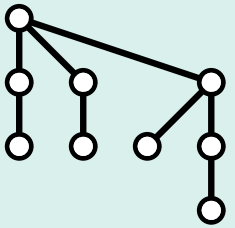
Remove α leaves
+ $\alpha/2$ segments



$n-\alpha/2-\beta$
segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

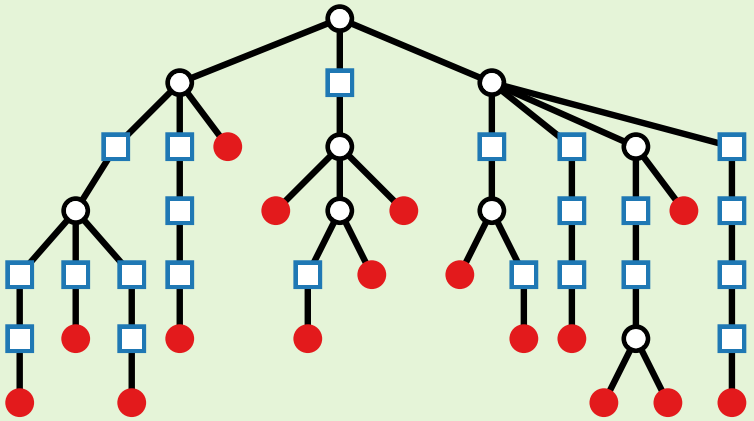




$n-\alpha-\beta$
segment

Tree Drawings

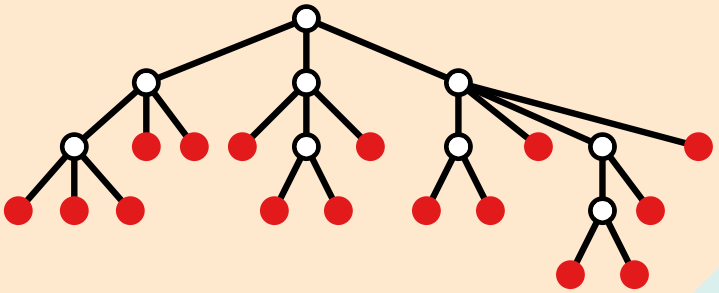
Tree T
n vtcs



Remove β deg-2 vtcs

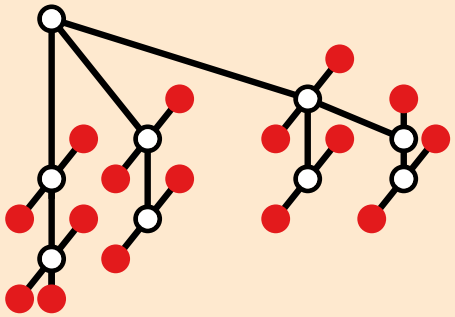
+ 0 segments

$\Rightarrow$ Tree T'
$n-\beta$ vtcs



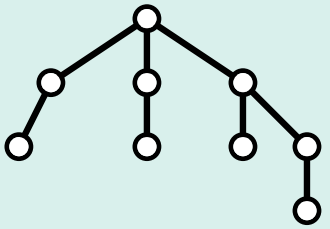
Remove α leaves

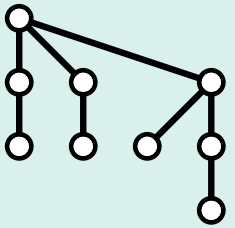
+ $\alpha/2$ segments



$n-\alpha/2-\beta$
segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

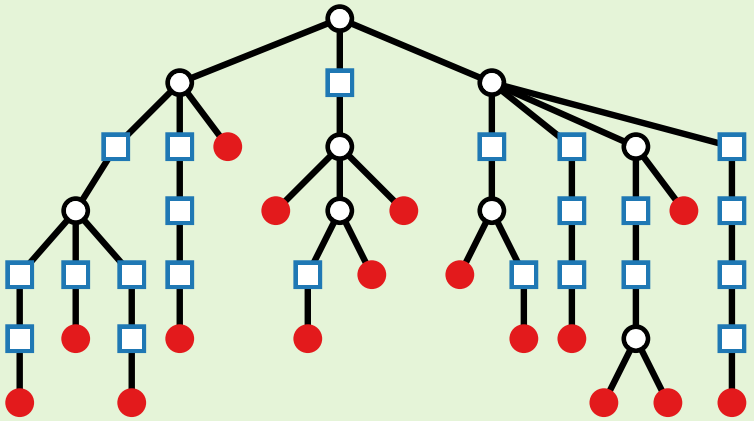




$n-\alpha-\beta$
segment

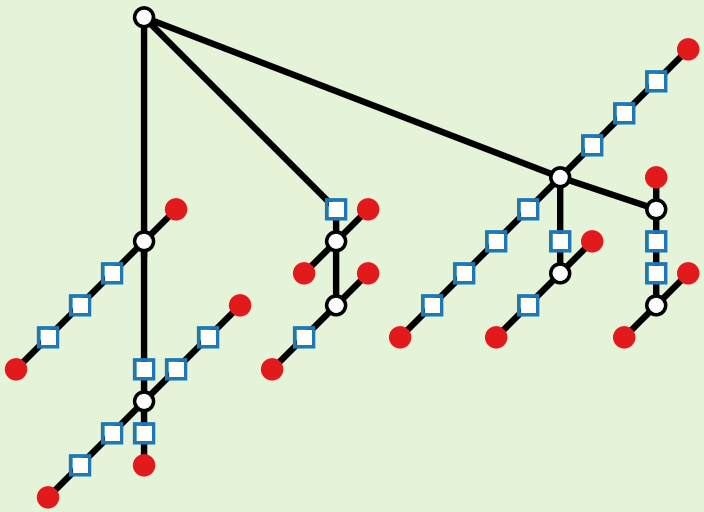
Tree Drawings

Tree T
n vtcs

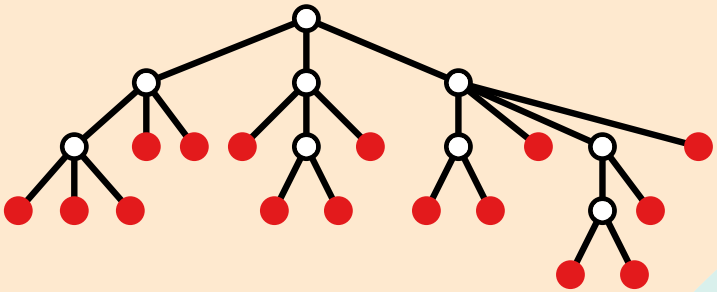


Remove β deg-2 vtcs

+ 0 segments

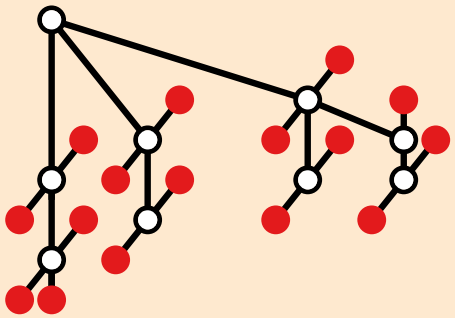


$\Rightarrow$ Tree T'
$n-\beta$ vtcs



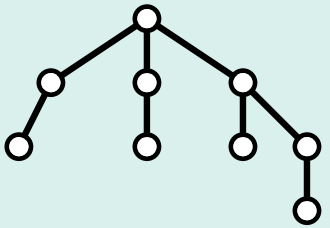
Remove α leaves

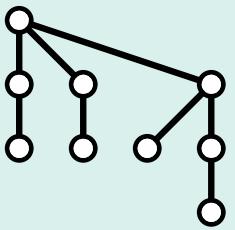
+ $\alpha/2$ segments



$n-\alpha/2-\beta$
segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

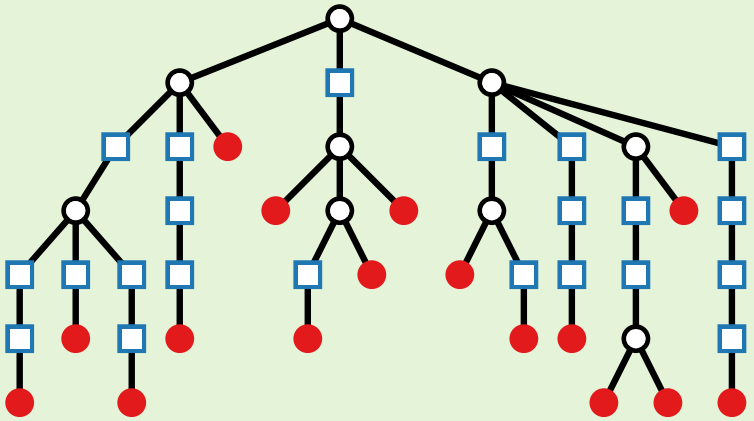




$n-\alpha-\beta$
segment

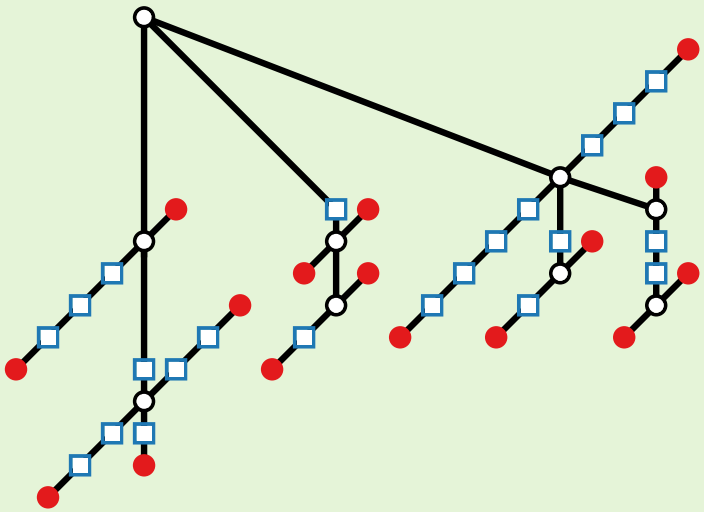
Tree Drawings

Tree T
n vtcs

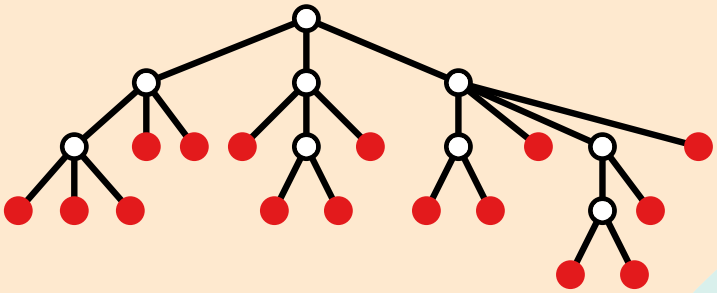


Remove β deg-2 vtcs

+ 0 segments

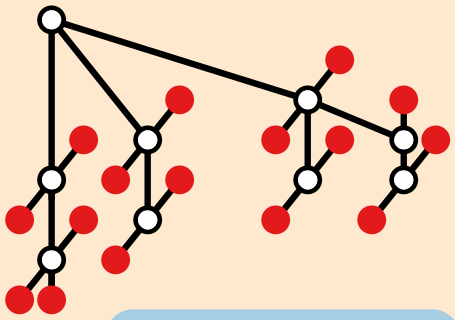


$\Rightarrow$ Tree T'
$n-\beta$ vtcs



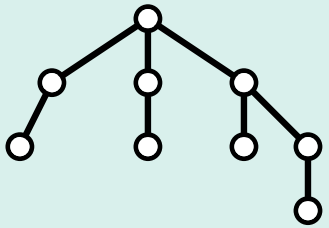
Remove α leaves

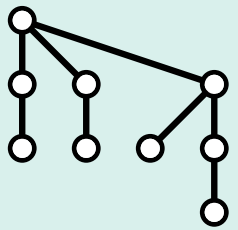
+ $\alpha/2$ segments



$n-\alpha/2-\beta$
segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

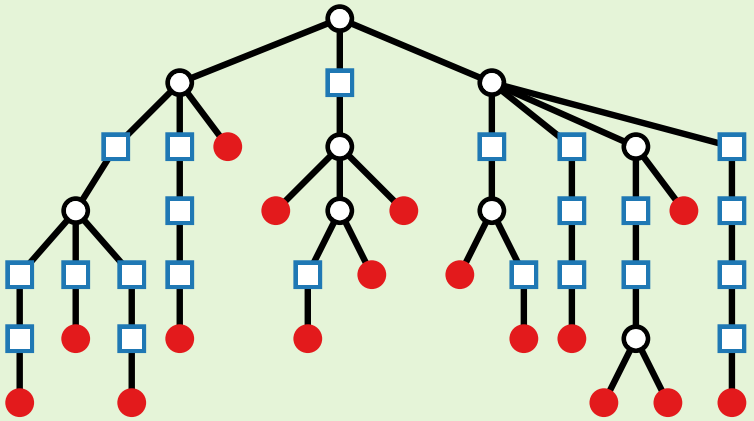




$n-\alpha-\beta$
segment

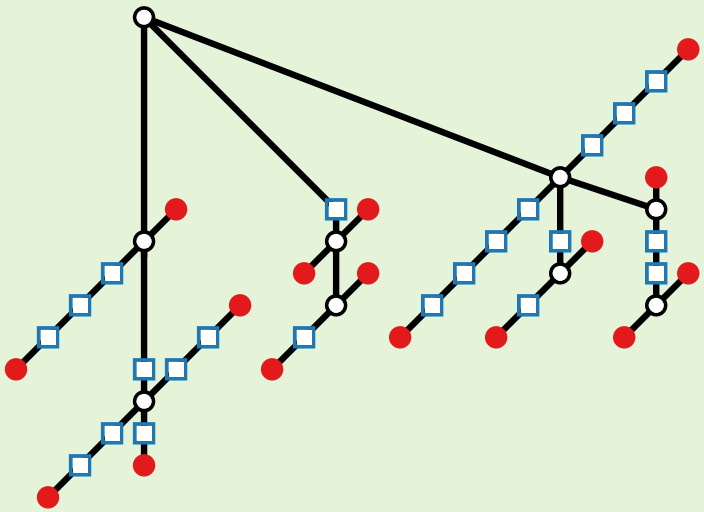
Tree Drawings

Tree T
n vtcs

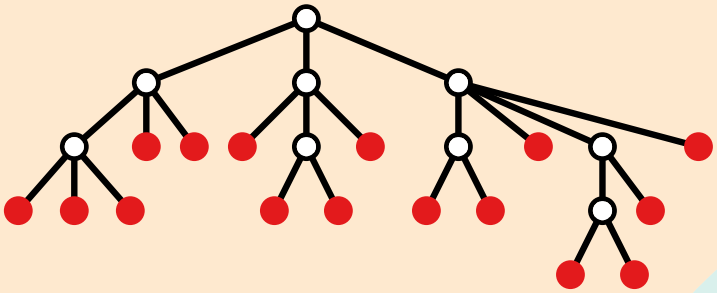


Remove β deg-2 vtcs

+ 0 segments

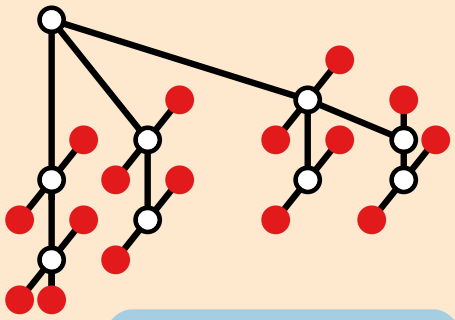


$\Rightarrow$ Tree T'
$n-\beta$ vtcs



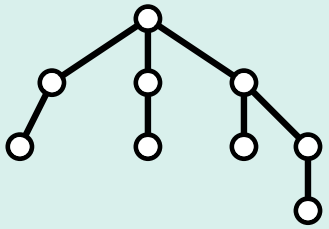
Remove α leaves

+ $\alpha/2$ segments

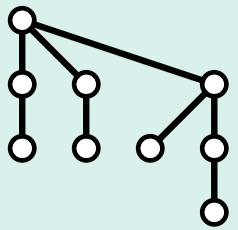


$n-\alpha/2-\beta$
segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs



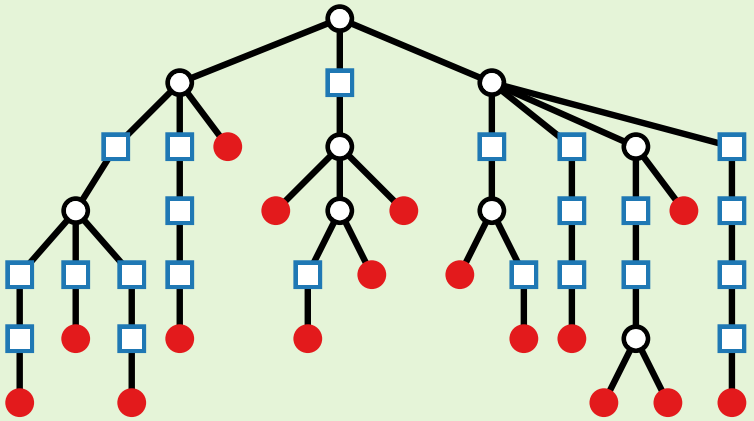
$\alpha > (n-\beta)/2$



$n-\alpha-\beta$
segment

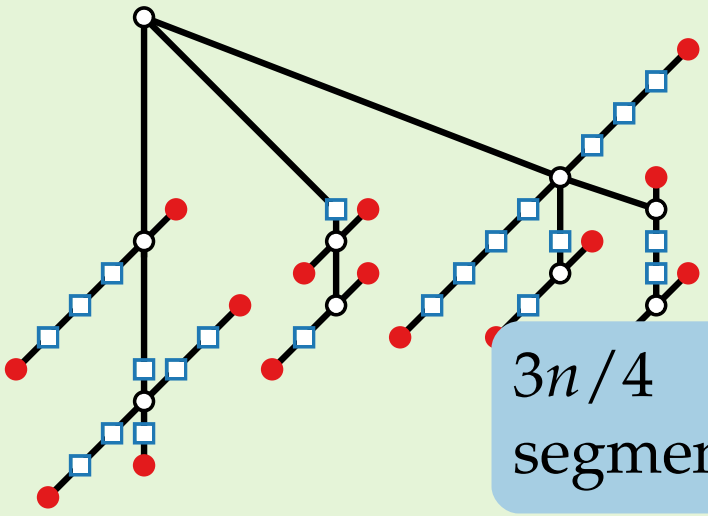
Tree Drawings

Tree T
n vtcs



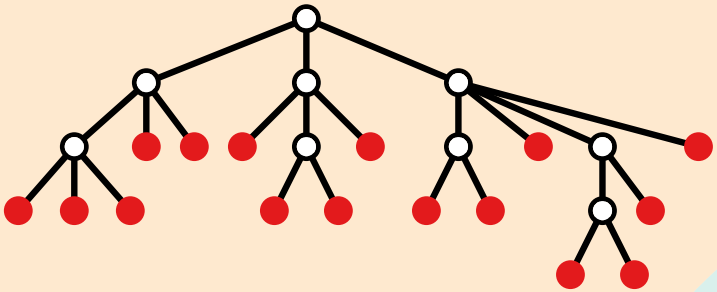
Remove β deg-2 vtcs

+ 0 segments



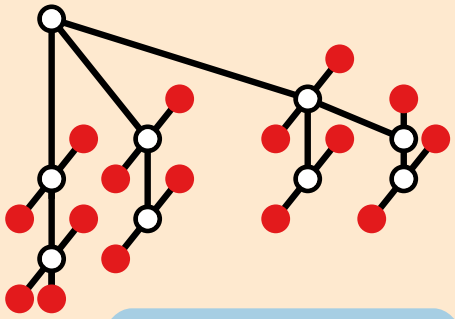
$3n/4$ segments

$\Rightarrow$ Tree T'
$n-\beta$ vtcs



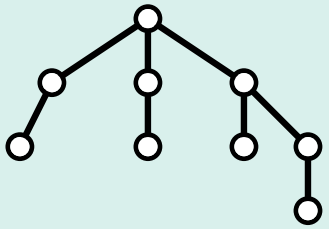
Remove α leaves

+ $\alpha/2$ segments

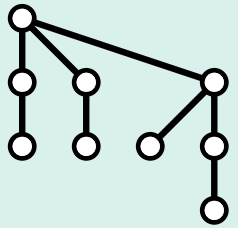


$n-\alpha/2-\beta$ segments

$\Rightarrow$ Tree T''
$n-\alpha-\beta$ vtcs

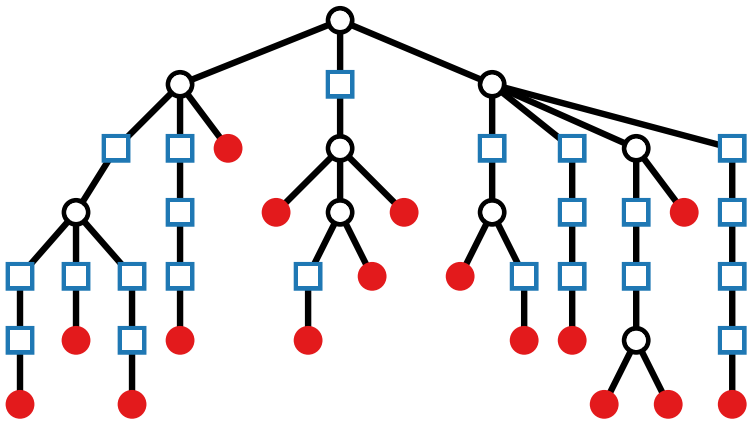


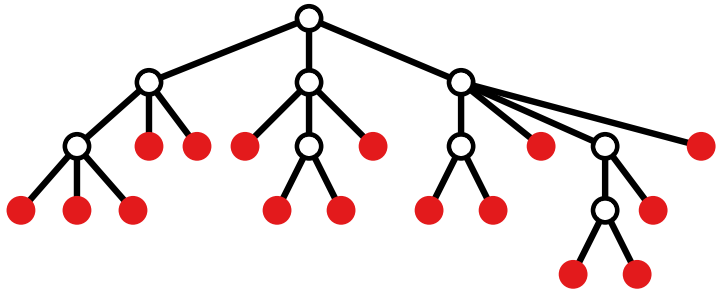
$\alpha > (n-\beta)/2$

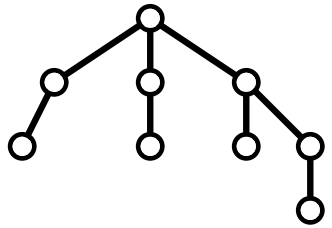


$n-\alpha-\beta$ segment

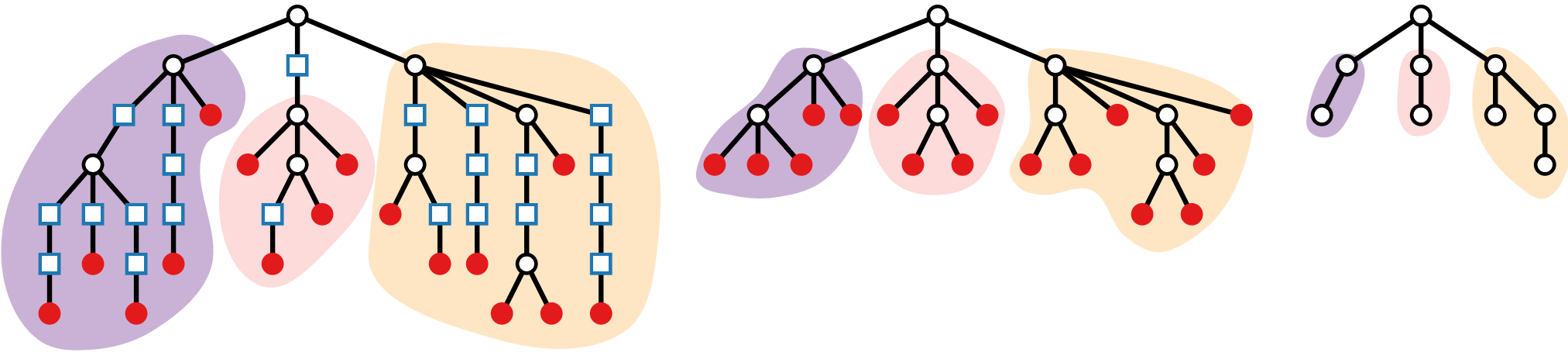
Tree Drawings





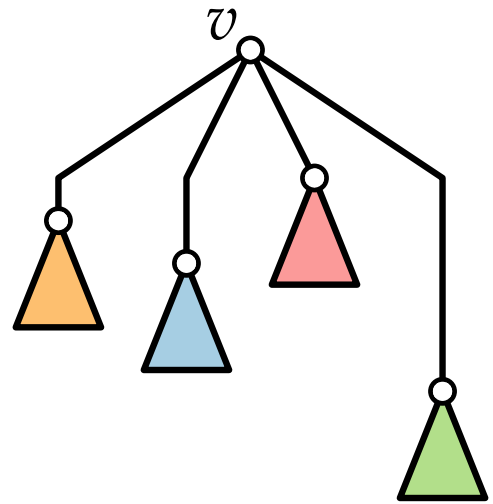


Tree Drawings

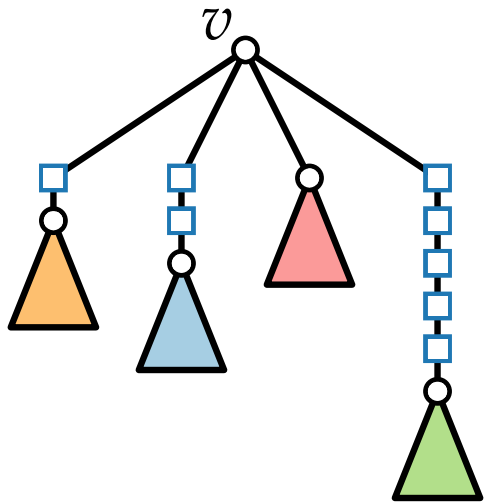


Tree Drawings

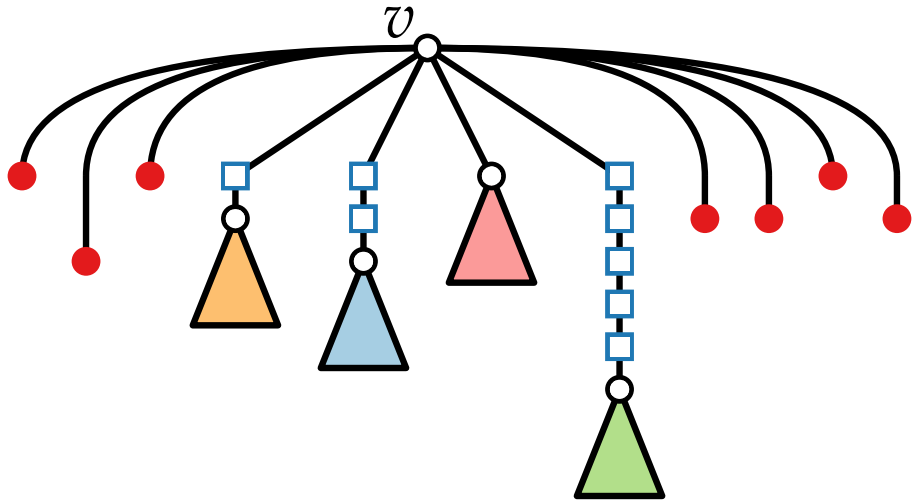
Tree Drawings



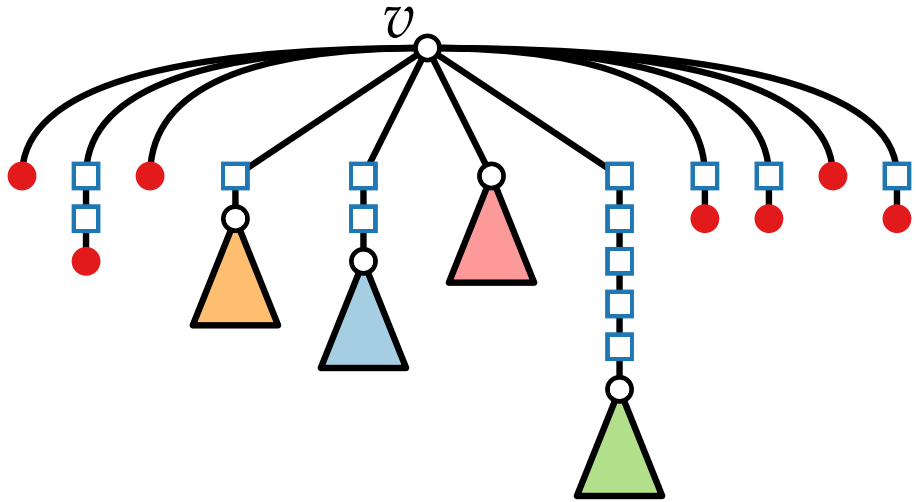
Tree Drawings



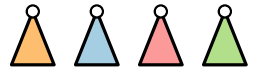
Tree Drawings

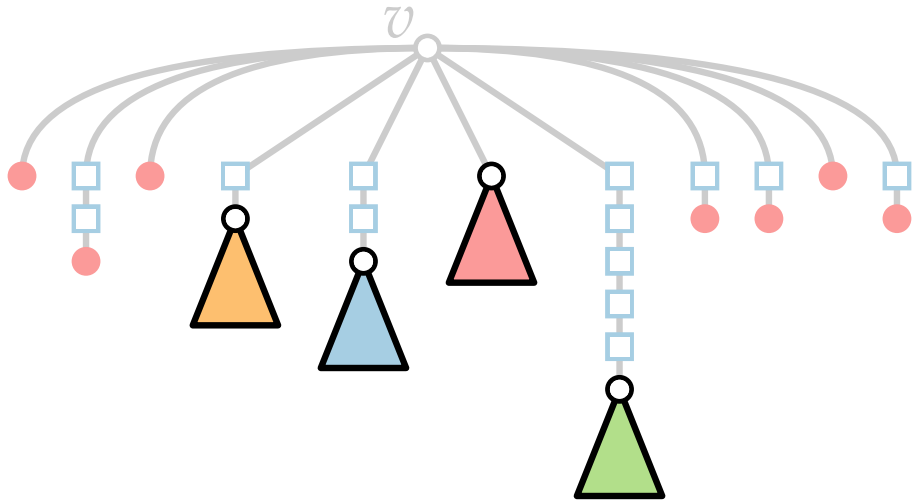


Tree Drawings

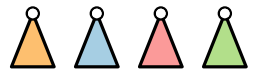


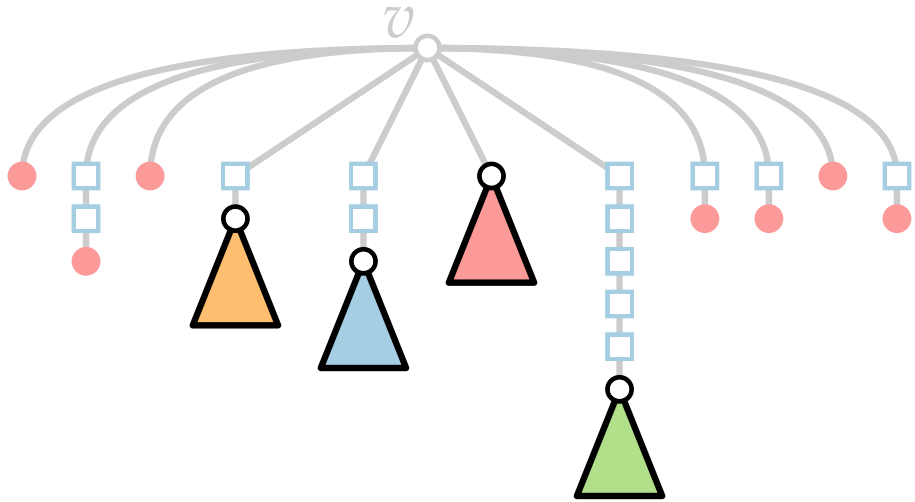
Tree Drawings

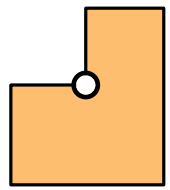
(1) Draw 



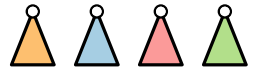
Tree Drawings

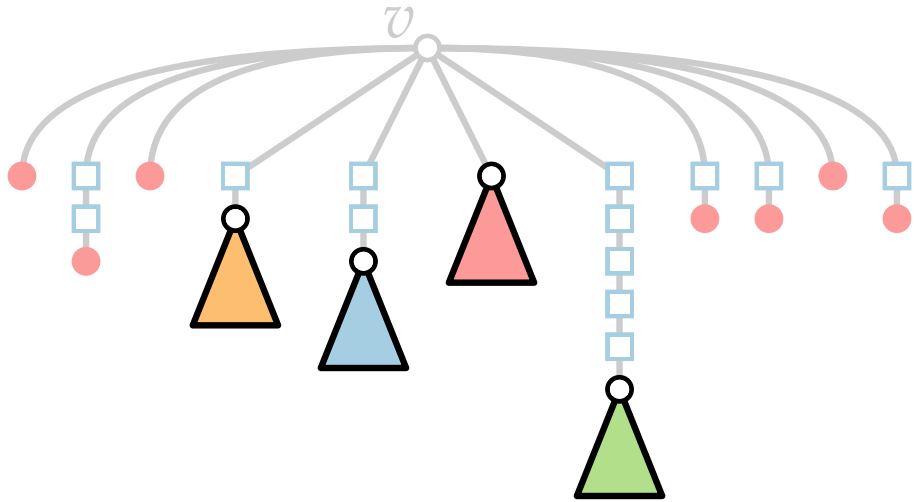
(1) Draw 

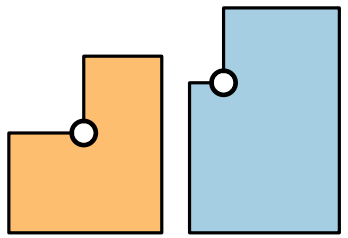




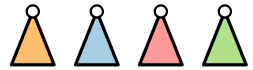
Tree Drawings

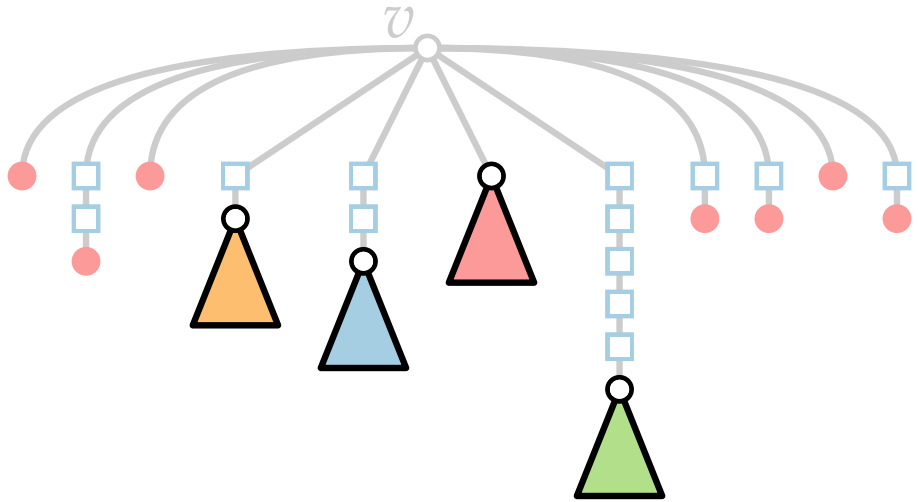
(1) Draw 

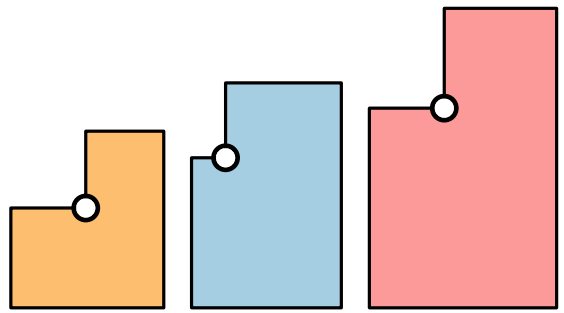




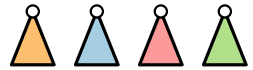
Tree Drawings

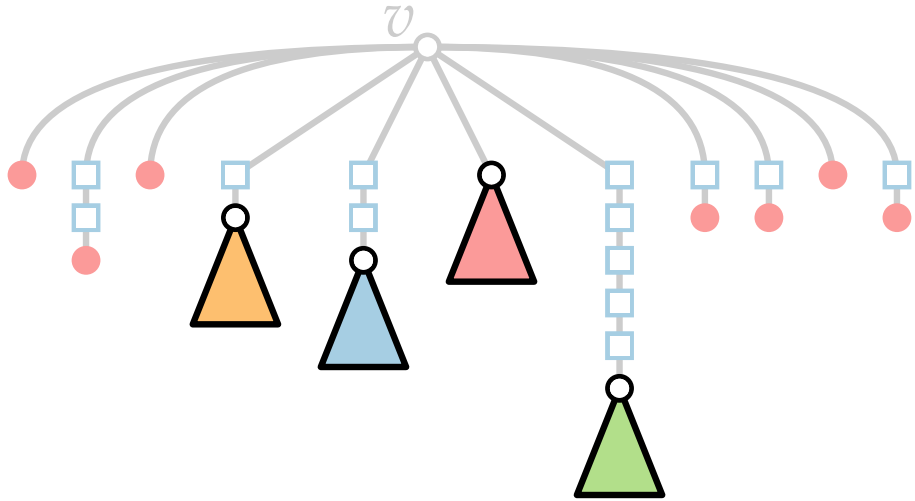
(1) Draw 

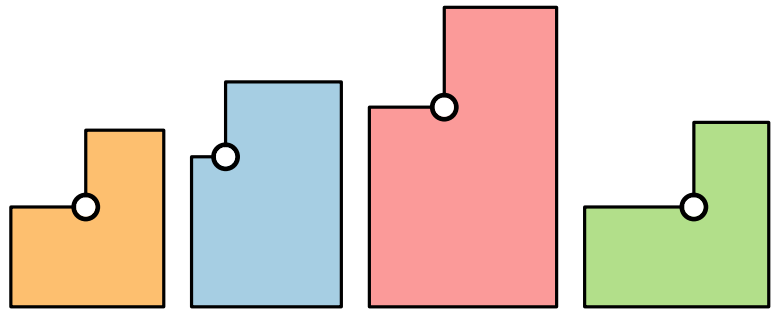




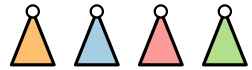
Tree Drawings

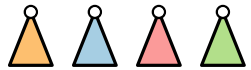
(1) Draw 

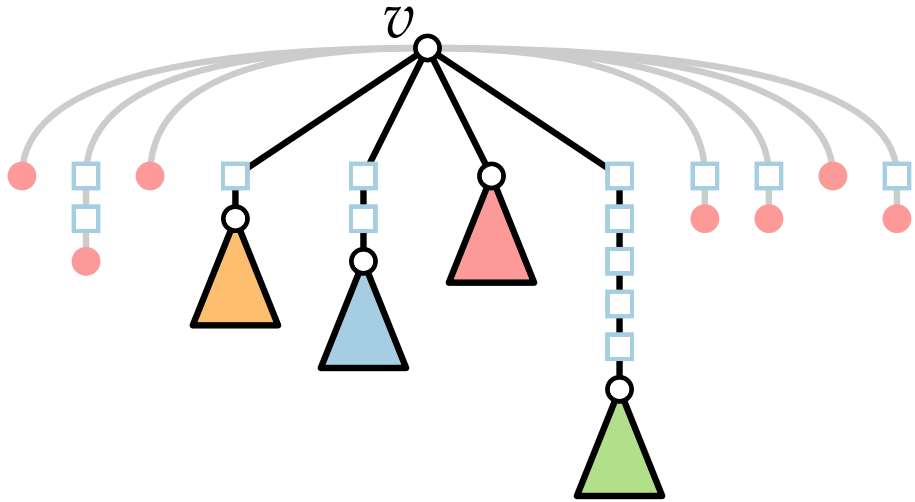


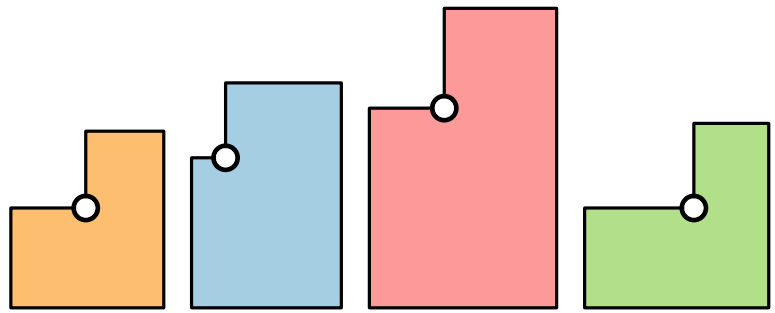


Tree Drawings

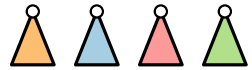
(1) Draw 

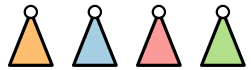
(2) Layout v + 

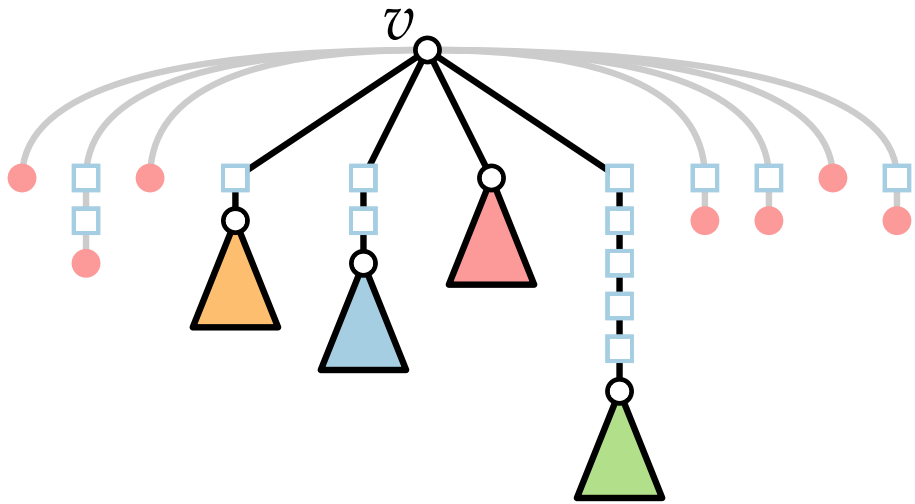


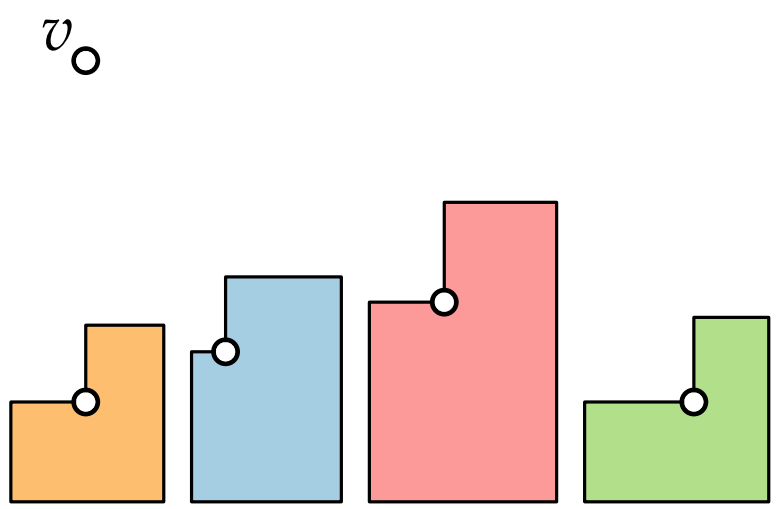


Tree Drawings

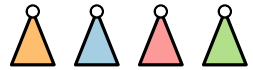
(1) Draw 

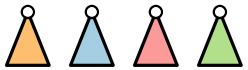
(2) Layout v + 

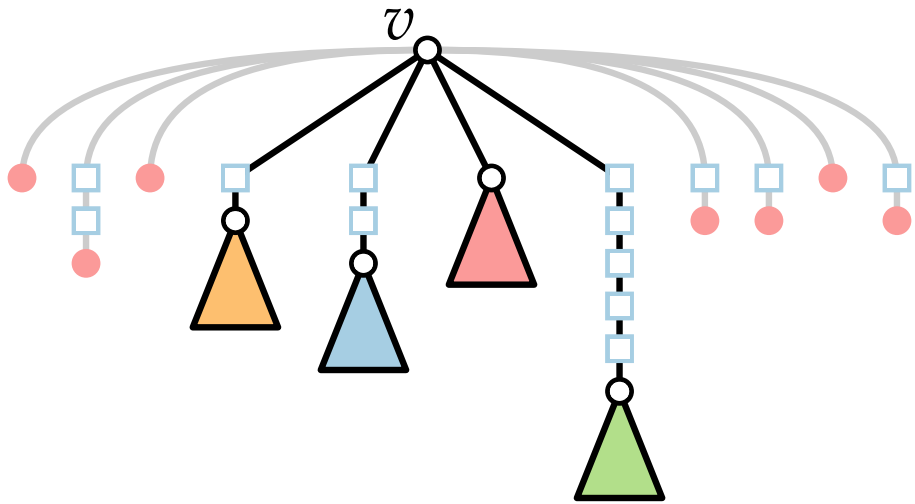


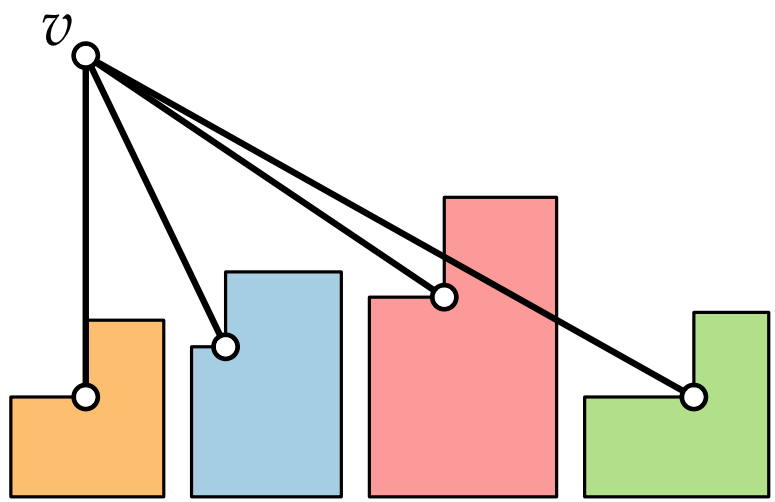


Tree Drawings

(1) Draw 

(2) Layout v + 





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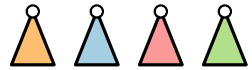
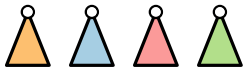
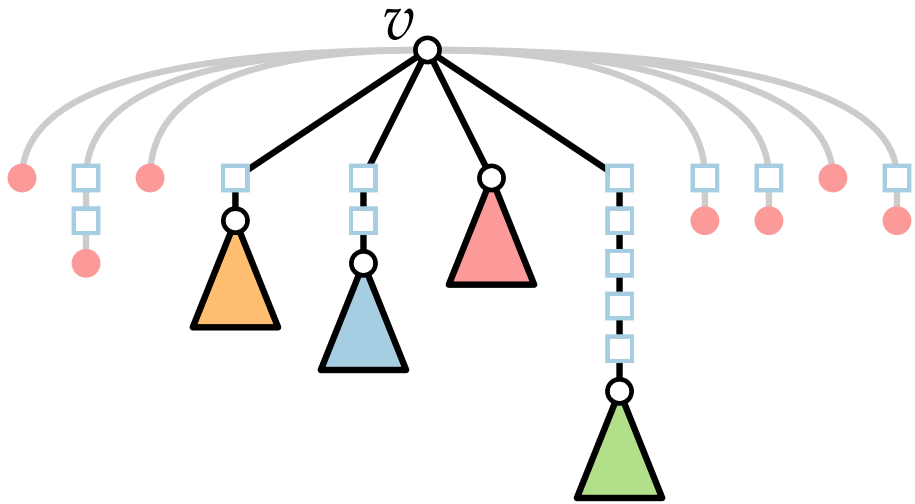
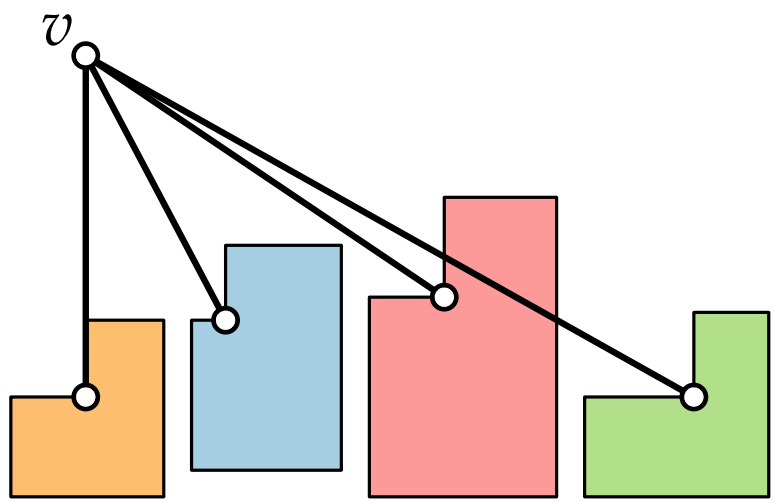
(1) Draw 

(2) Layout v + 





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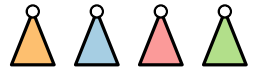
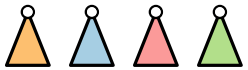
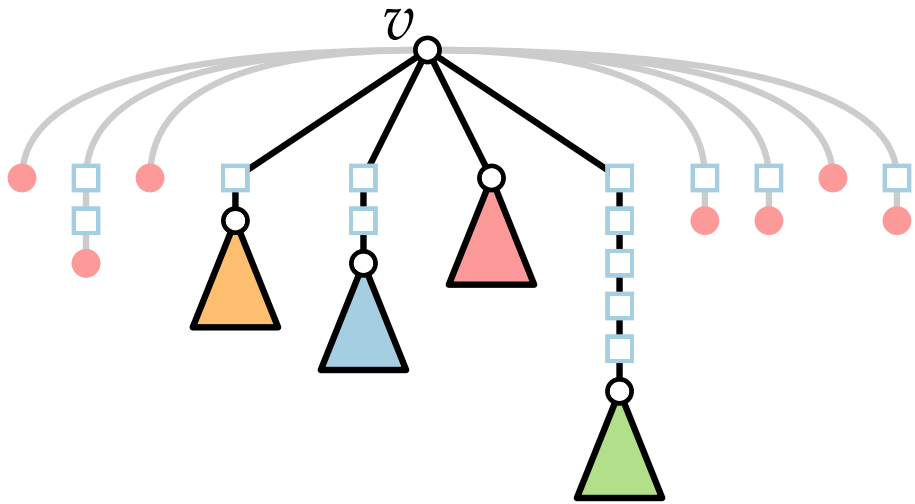
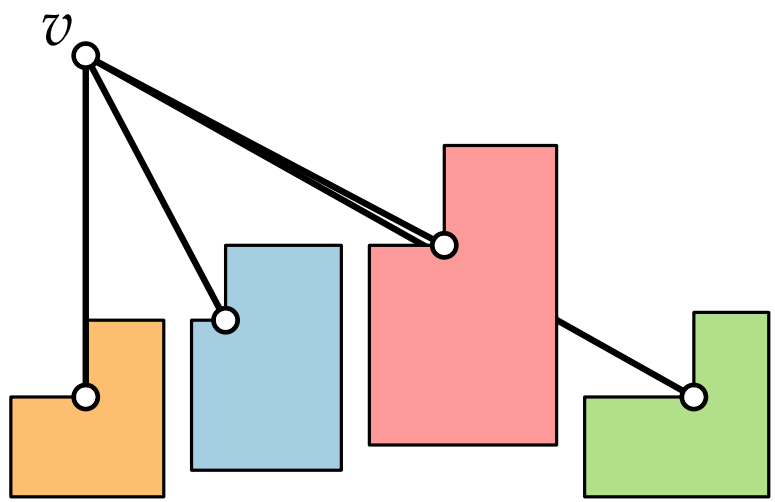
(1) Draw 

(2) Layout v + 





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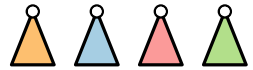
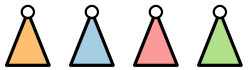
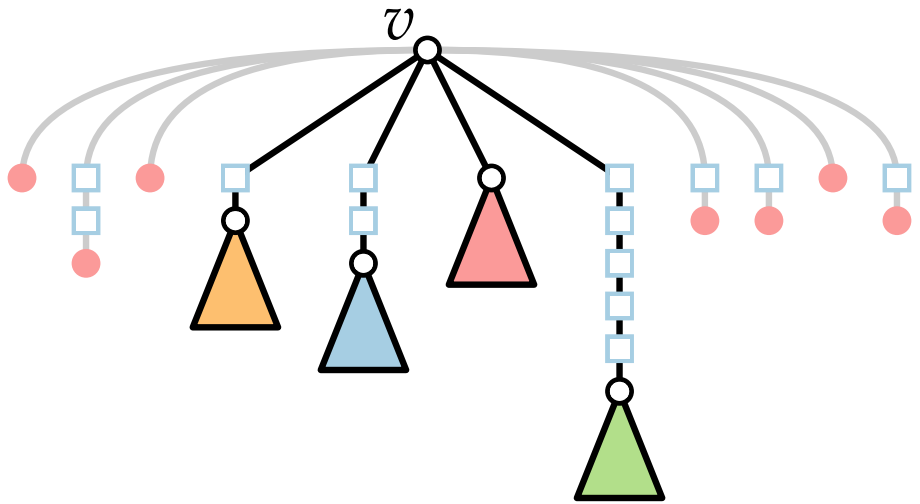
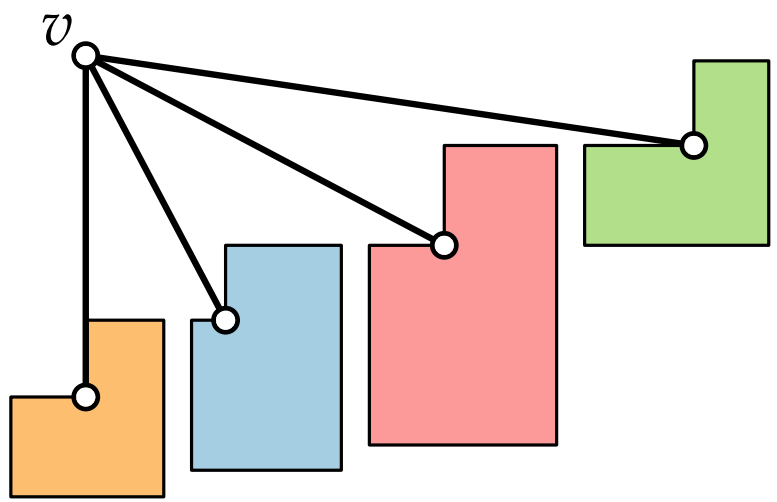
(1) Draw 

(2) Layout v + 





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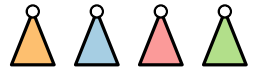
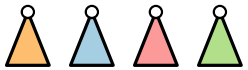
(1) Draw 

(2) Layout v + 

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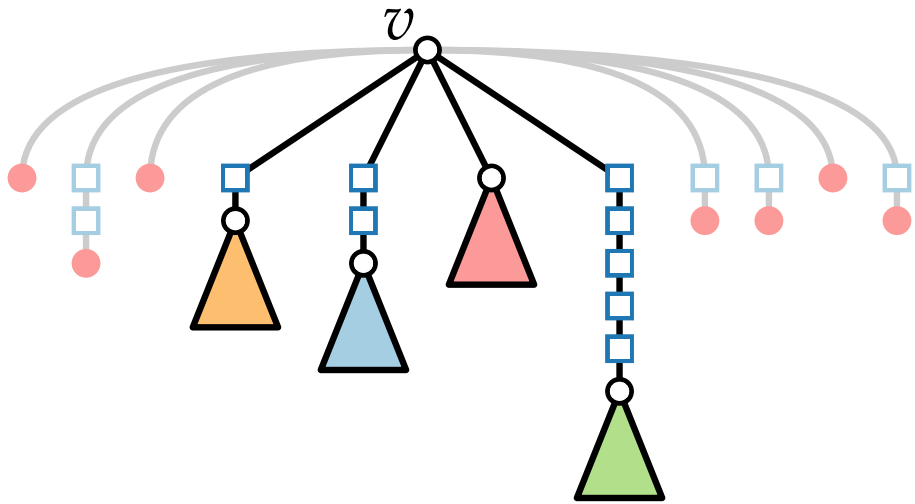

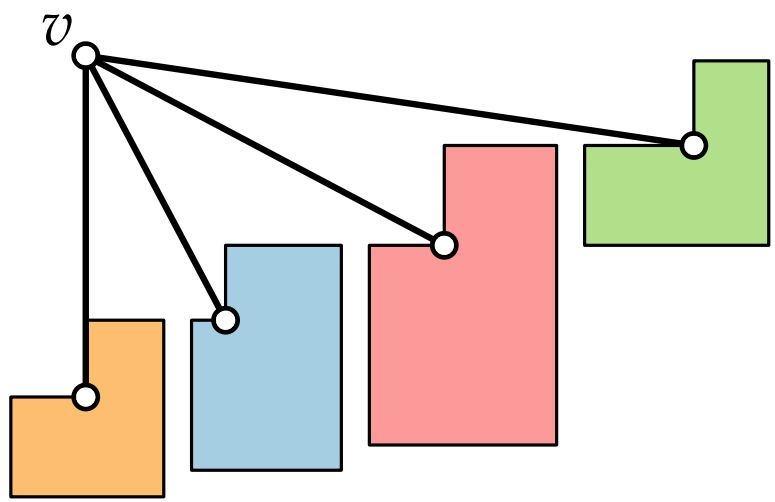

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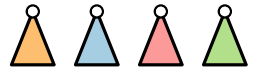
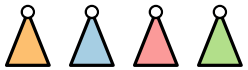
(1) Draw 

(2) Layout v + 

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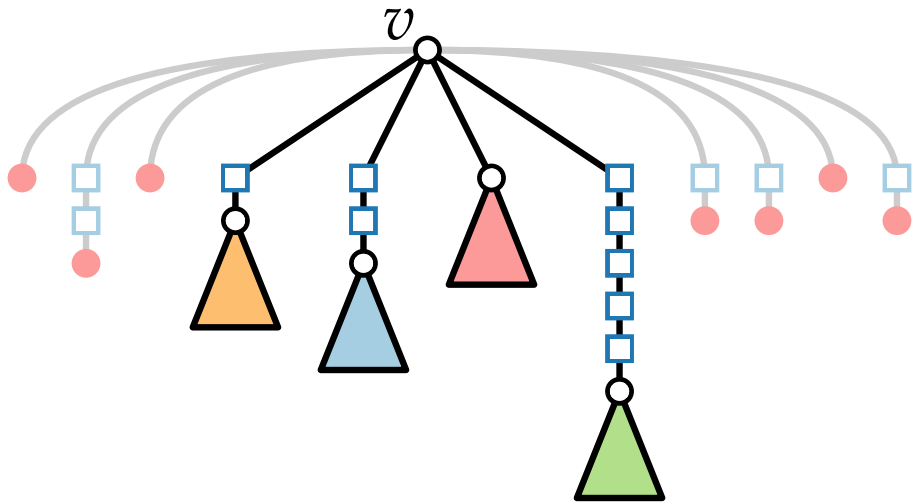

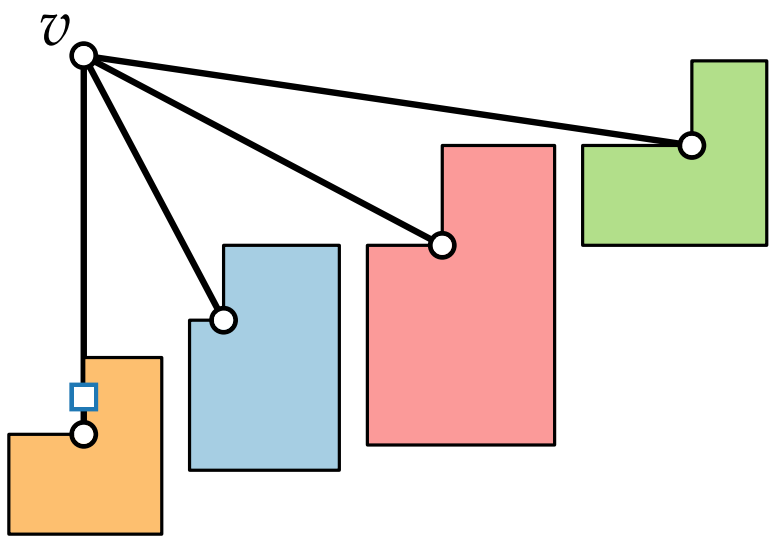

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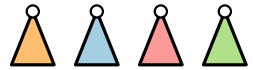
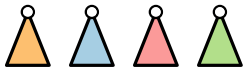
(1) Draw 

(2) Layout v + 

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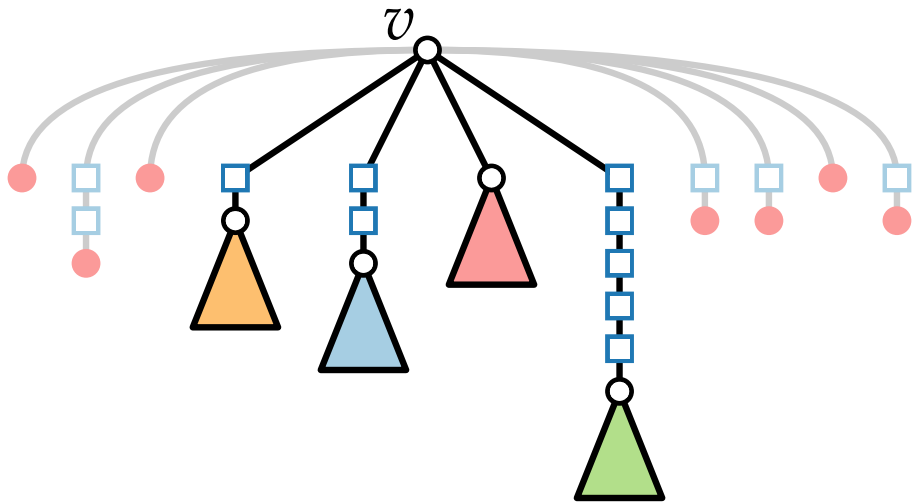
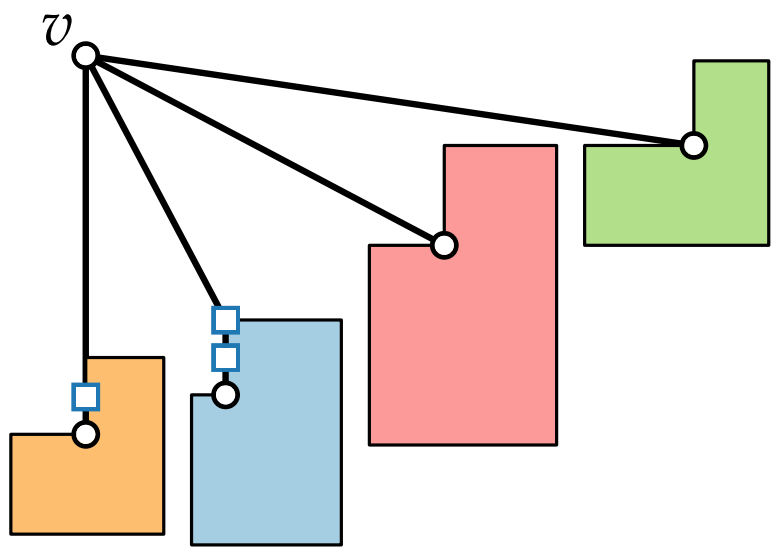
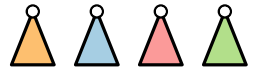
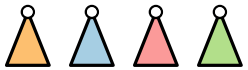




Tree Drawings

(1) Draw 

(2) Layout v + 

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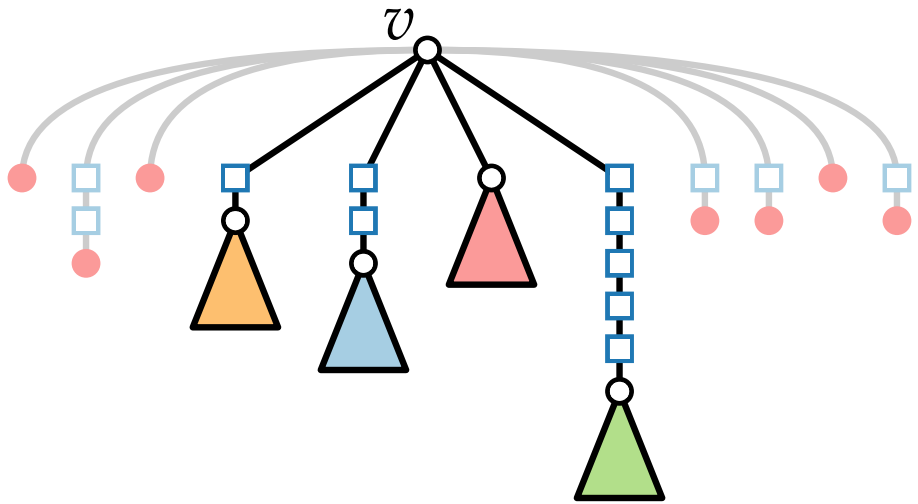
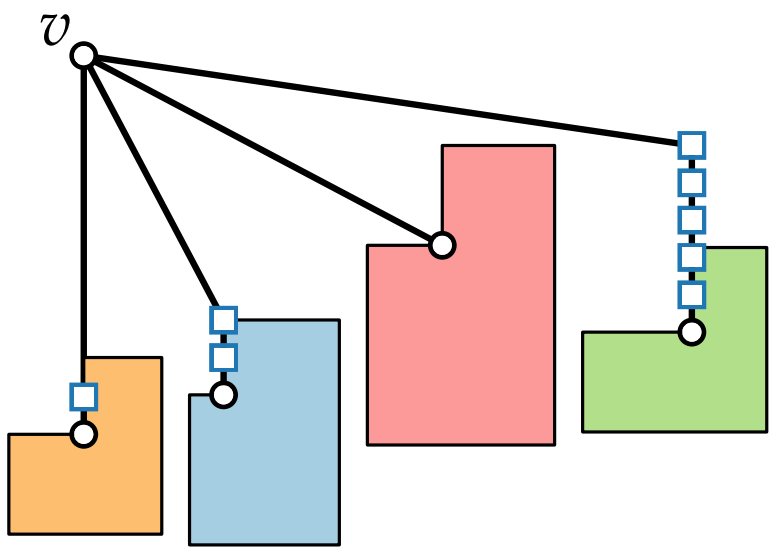
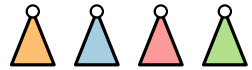
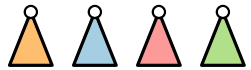




Tree Drawings

(1) Draw 

(2) Layout v + 

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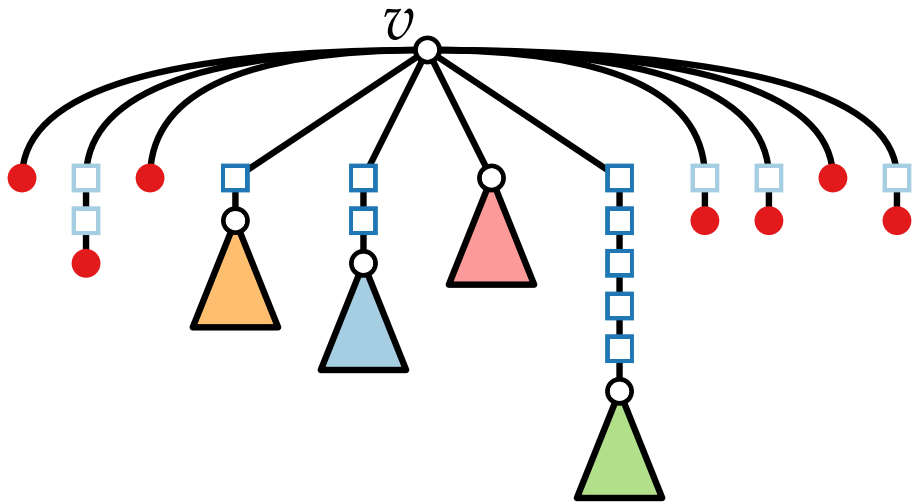
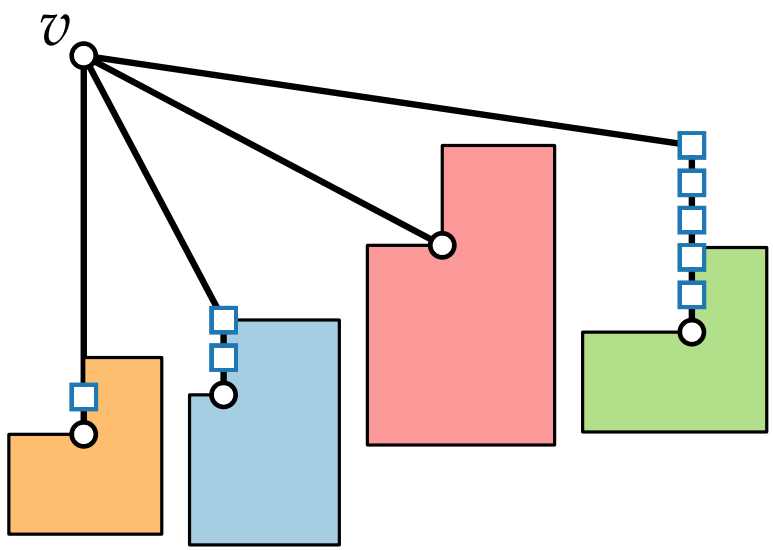
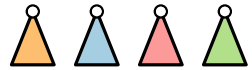
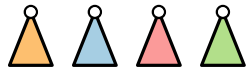
(4) Sort by #





Tree Drawings

(1) Draw 

(2) Layout v + 

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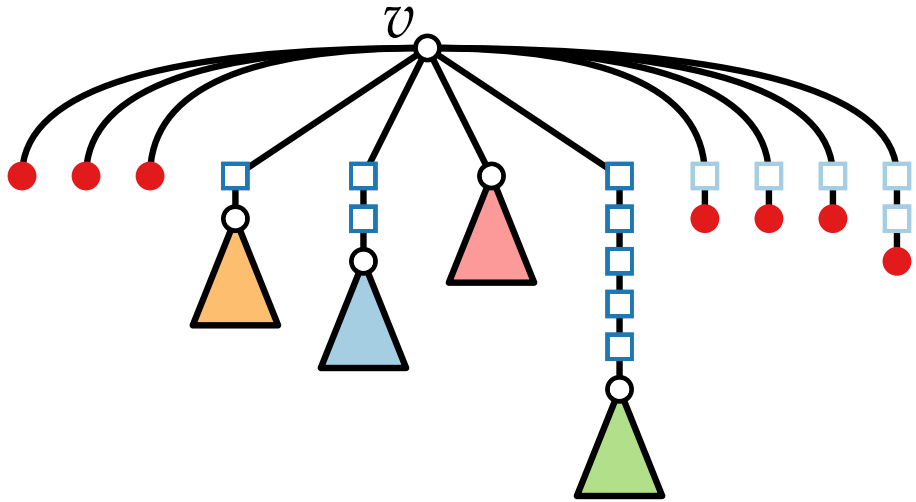
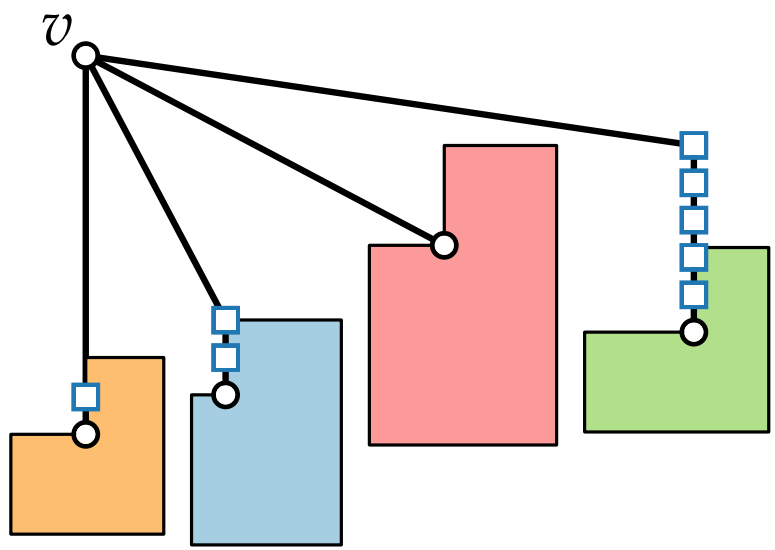
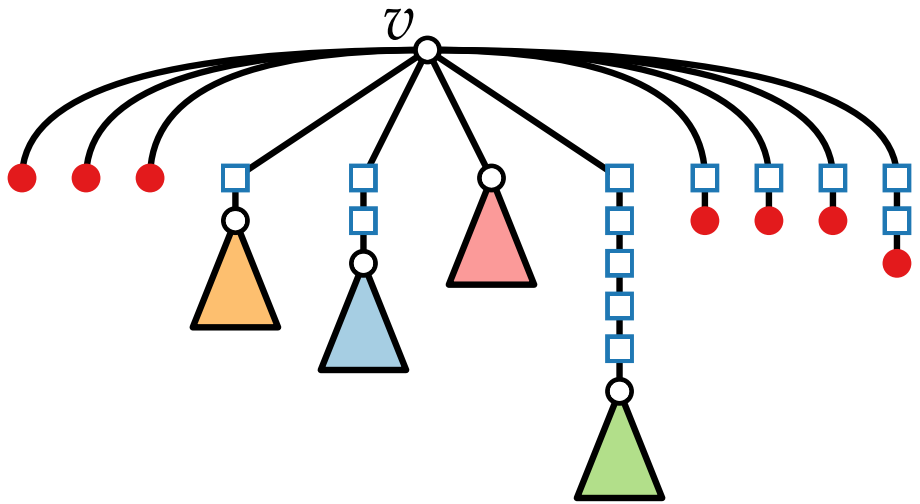
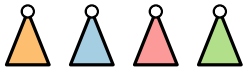
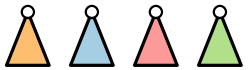




Tree Drawings



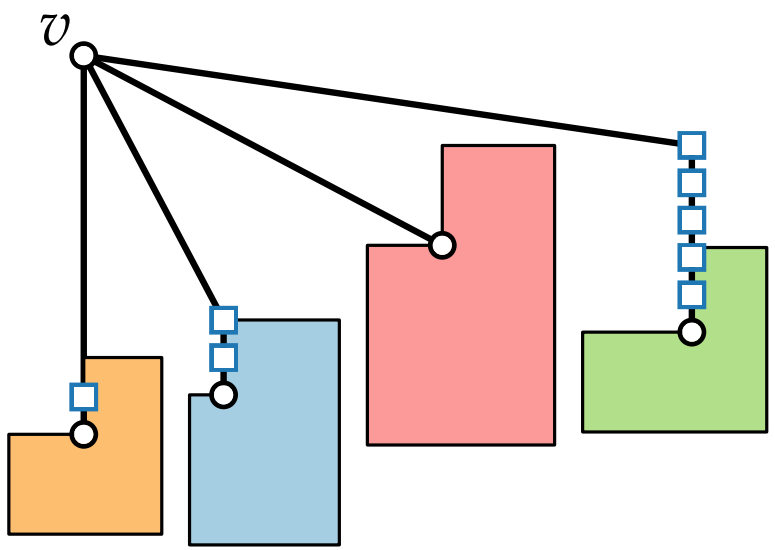
(1) Draw 

(2) Layout v + 

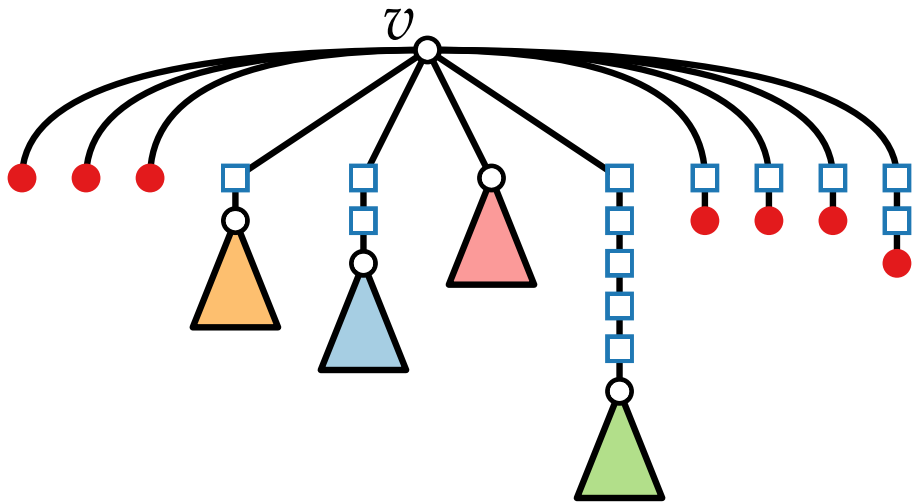
(3) Add □

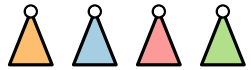
(4) Sort ● by # □

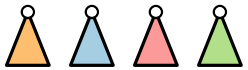
(5) Place ● + □ on common segments in order



Tree Drawings



(1) Draw 

(2) Layout v + 

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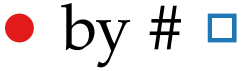
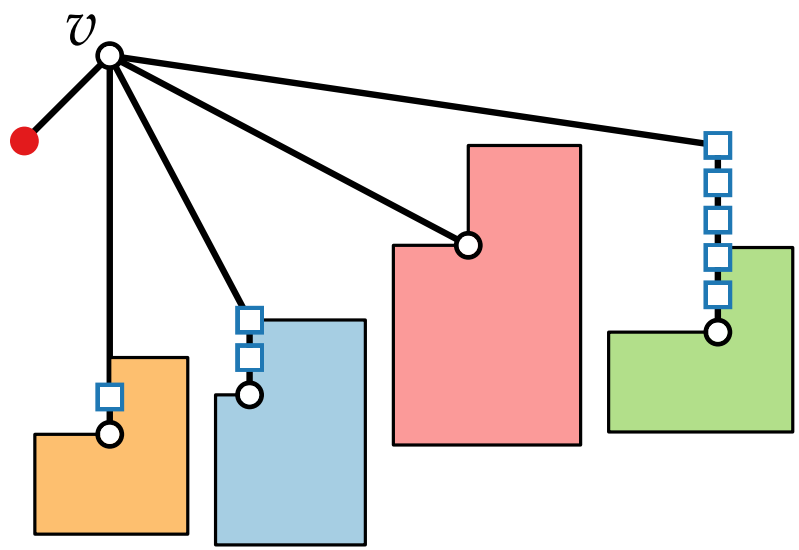
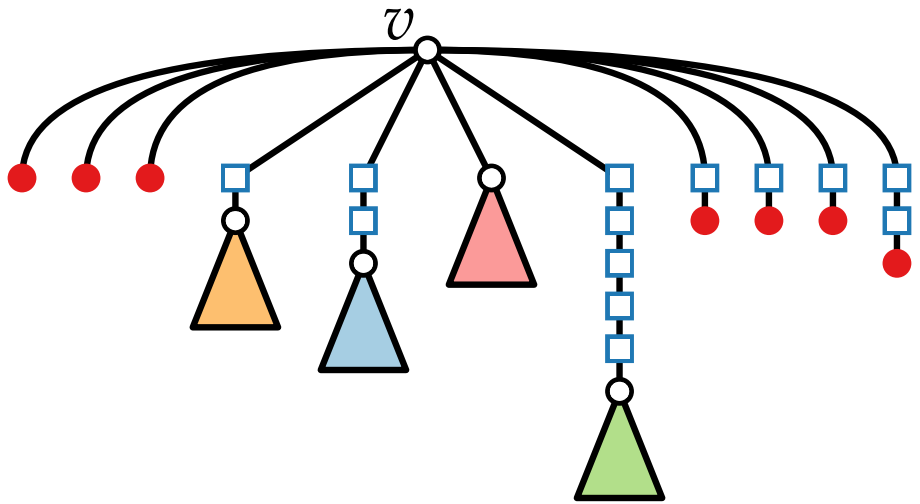
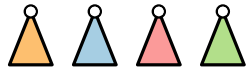
(4) Sort  by #

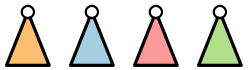
(5) Place + on common segments in order



Tree Drawings



(1) Draw 

(2) Layout v + 

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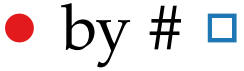
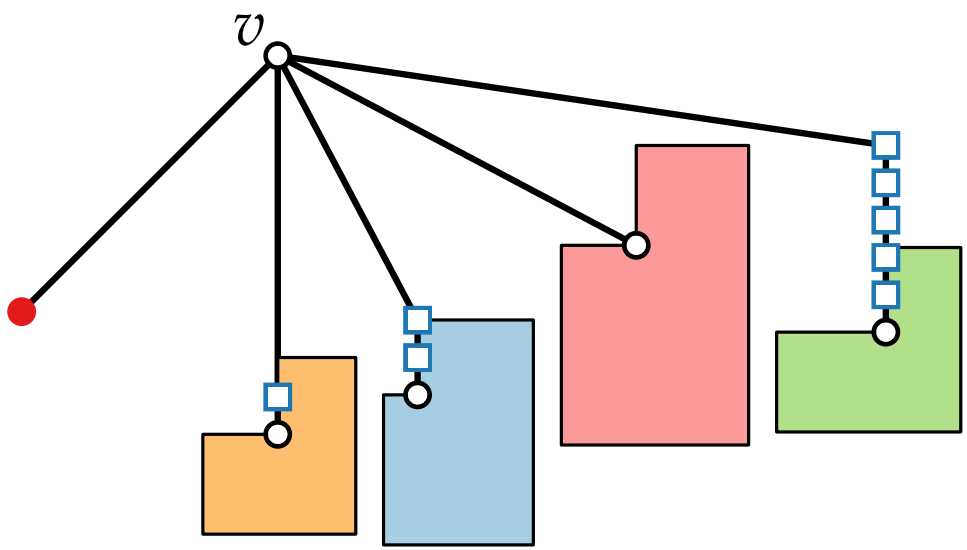
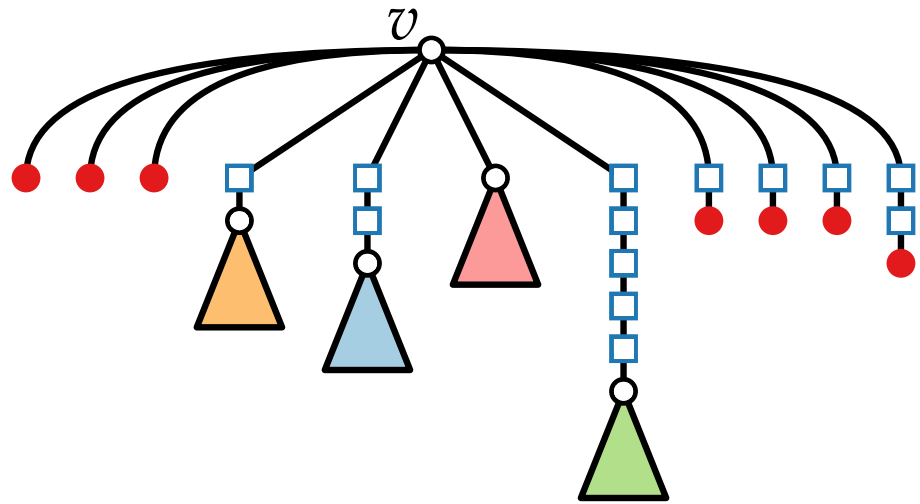
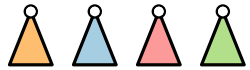
(4) Sort  by #

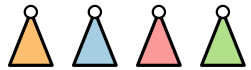
(5) Place + on common segments in order



Tree Drawings



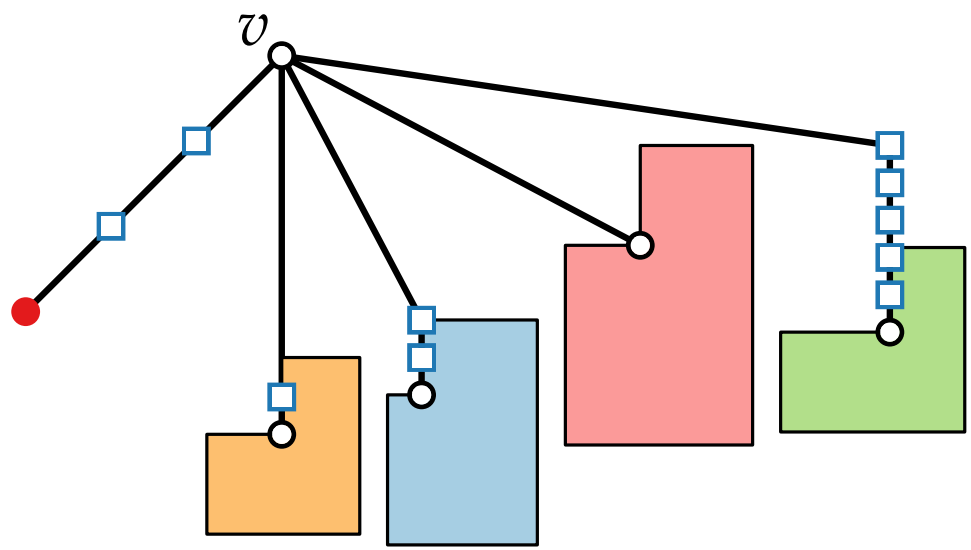
(1) Draw 

(2) Layout v + 

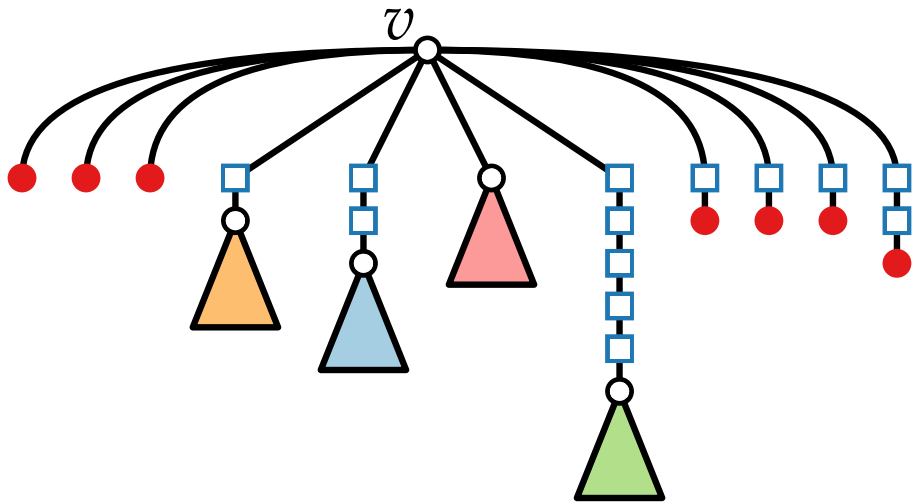
(3) Add

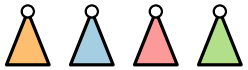
(4) Sort by #

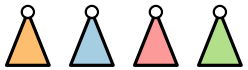
(5) Place + on common segments in order



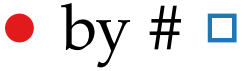
Tree Drawings



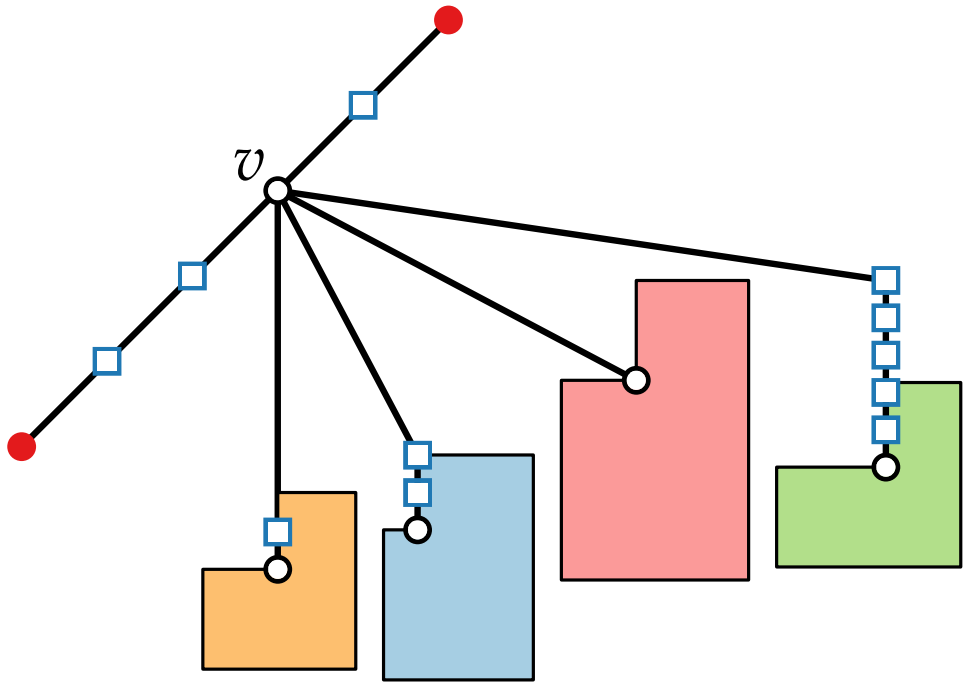
(1) Draw 

(2) Layout v + 

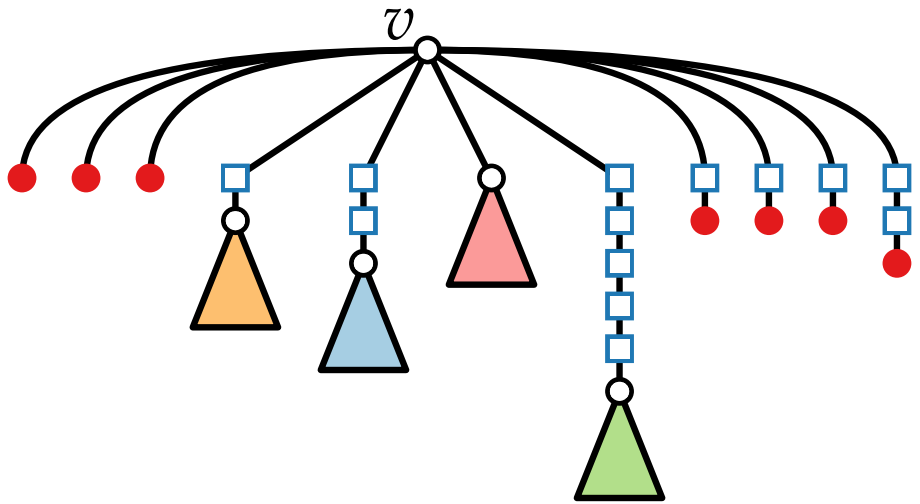
(3) Add

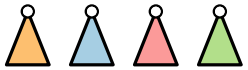
(4) Sort  by #

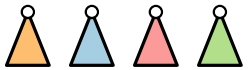
(5) Place + on common segments in order



Tree Drawings



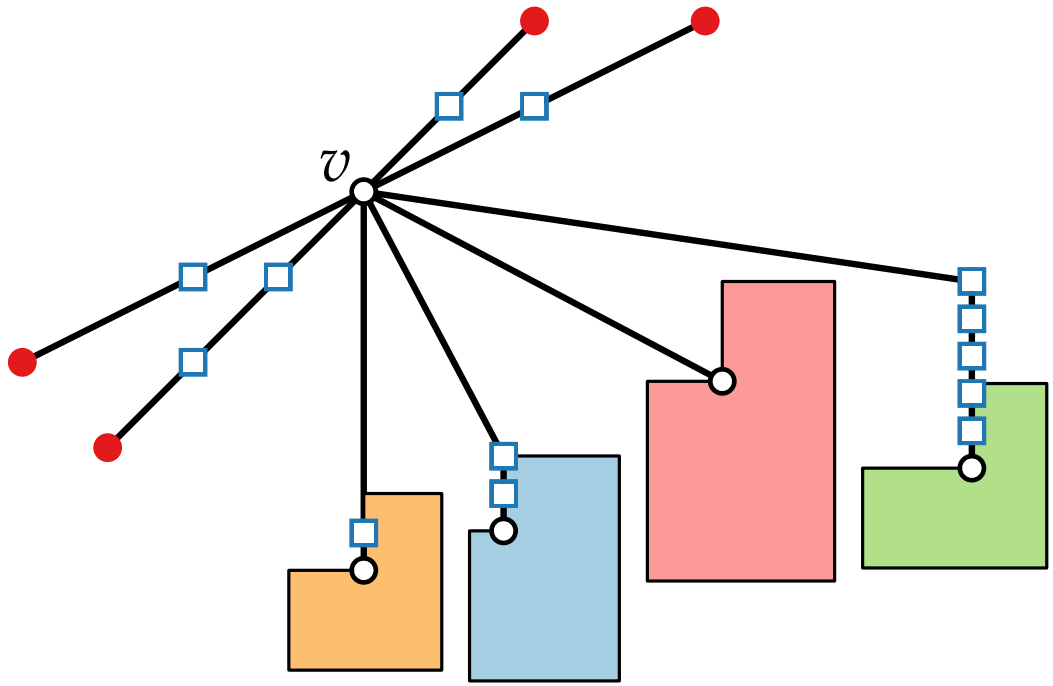
(1) Draw 

(2) Layout v + 

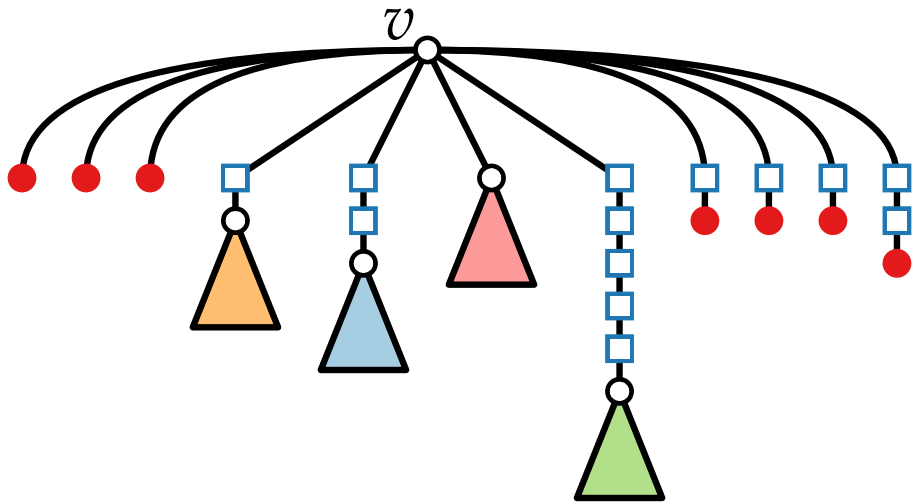
(3) Add

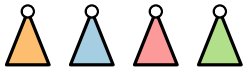
(4) Sort by #

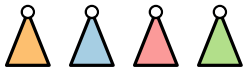
(5) Place + on common segments in order



Tree Drawings



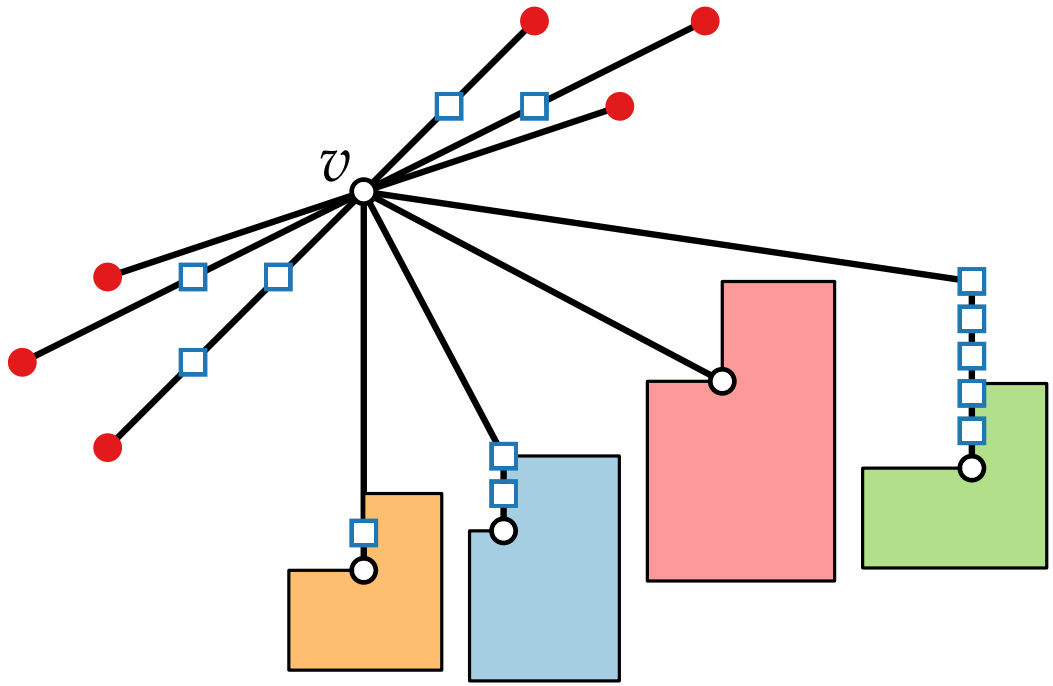
(1) Draw 

(2) Layout v + 

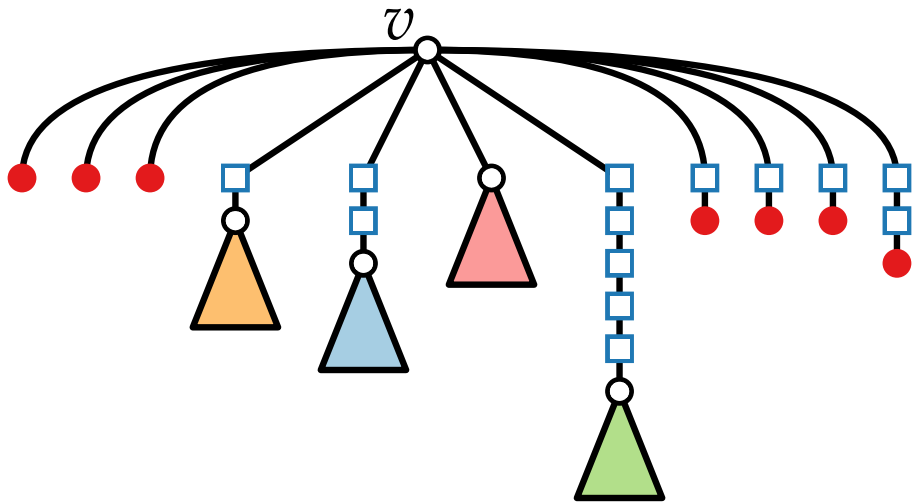
(3) Add

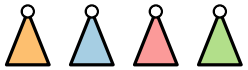
(4) Sort by #

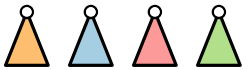
(5) Place + on common segments in order



Tree Drawings



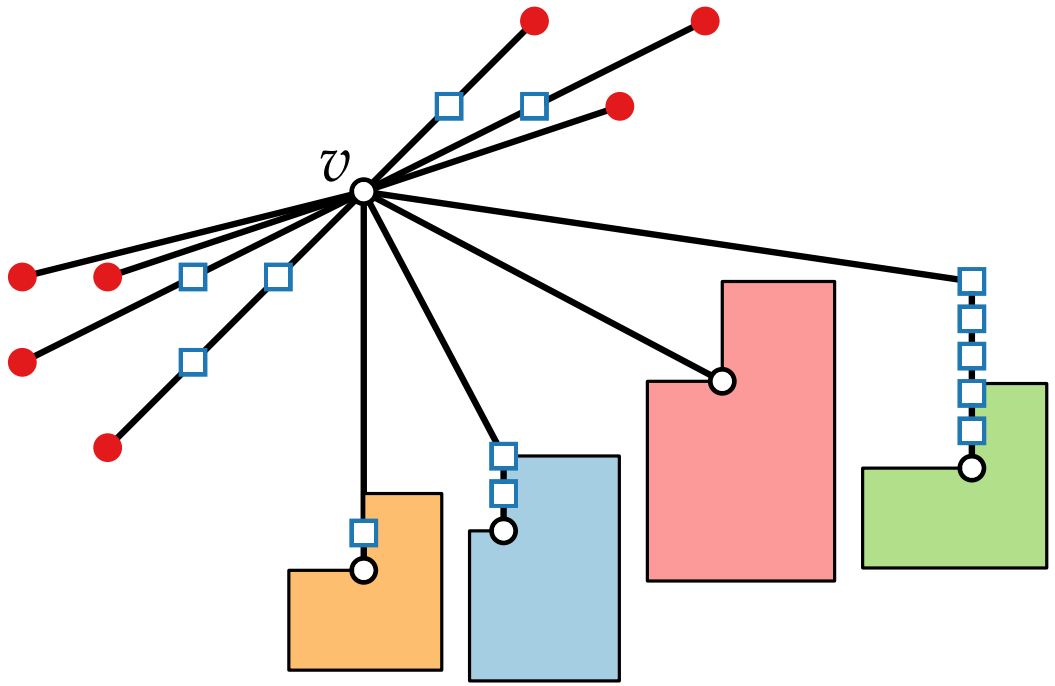
(1) Draw 

(2) Layout v + 

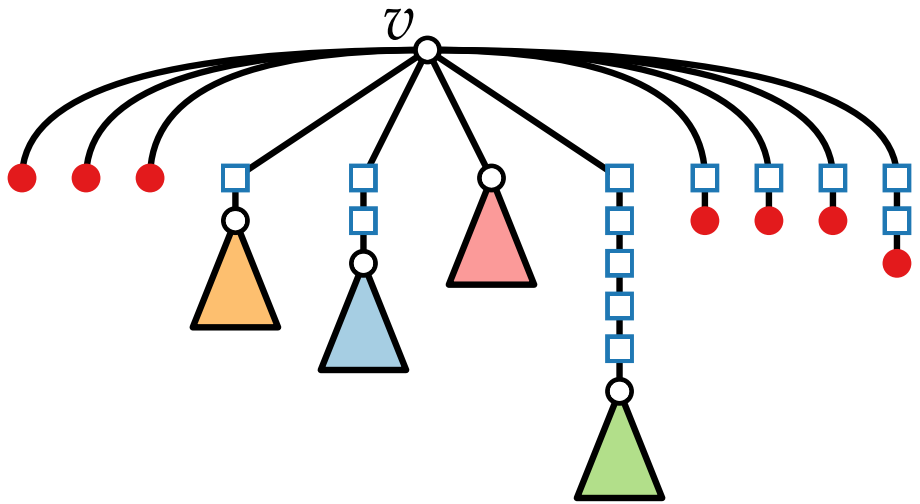
(3) Add

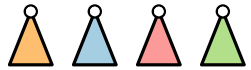
(4) Sort by #

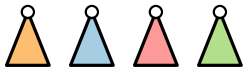
(5) Place + on common segments in order



Tree Drawings



(1) Draw 

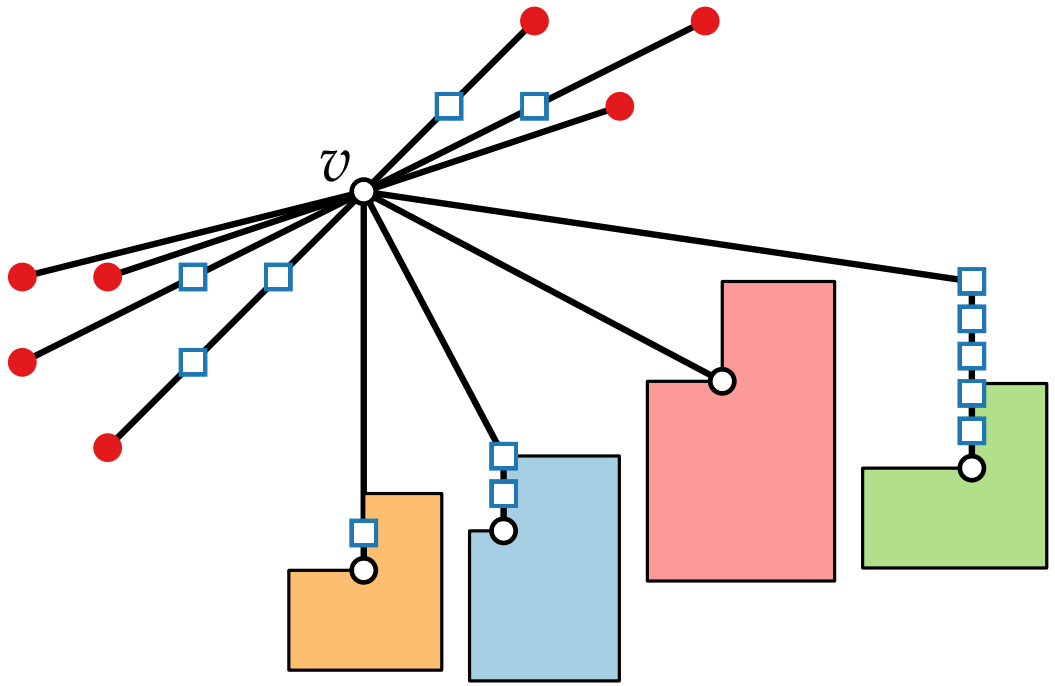
(2) Layout v + 

(3) Add

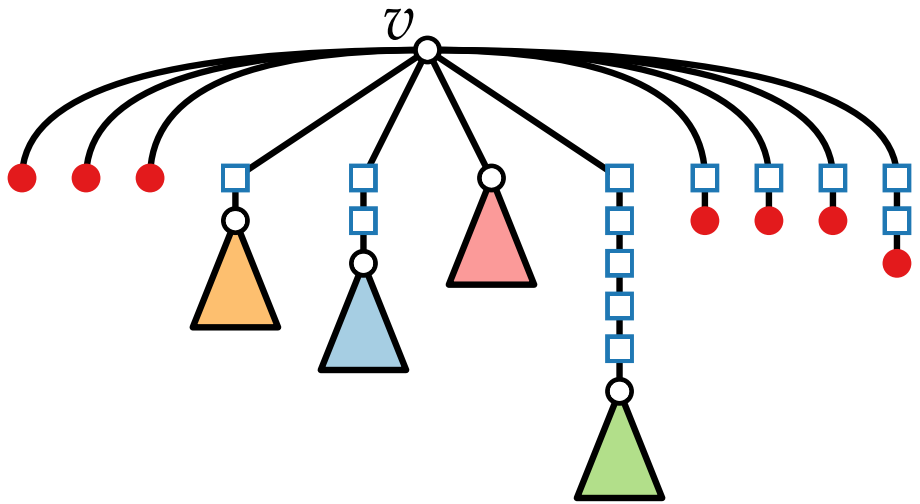
(4) Sort by #

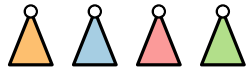
(5) Place + on common segments in order

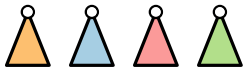
$3n/4$ segments



Tree Drawings



(1) Draw 

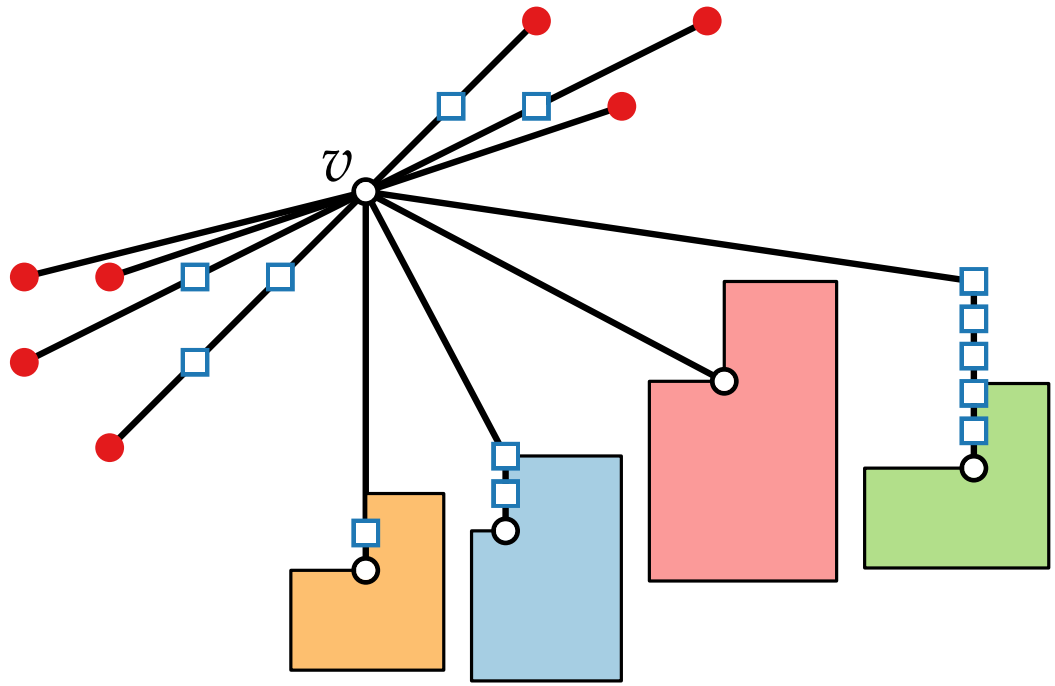
(2) Layout v + 

(3) Add

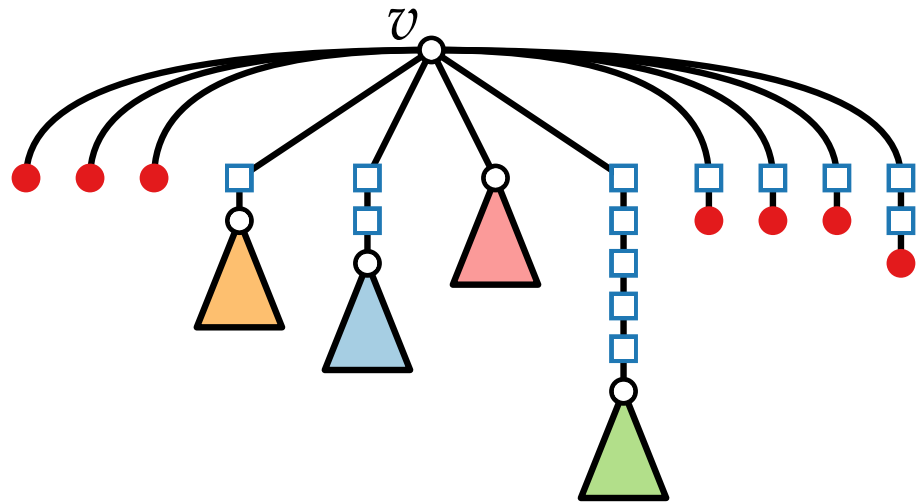
(4) Sort by #

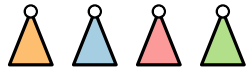
(5) Place + on common segments in order

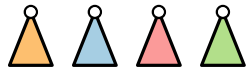
$3n/4$ segments
$n \times n$ grid



Tree Drawings



(1) Draw 

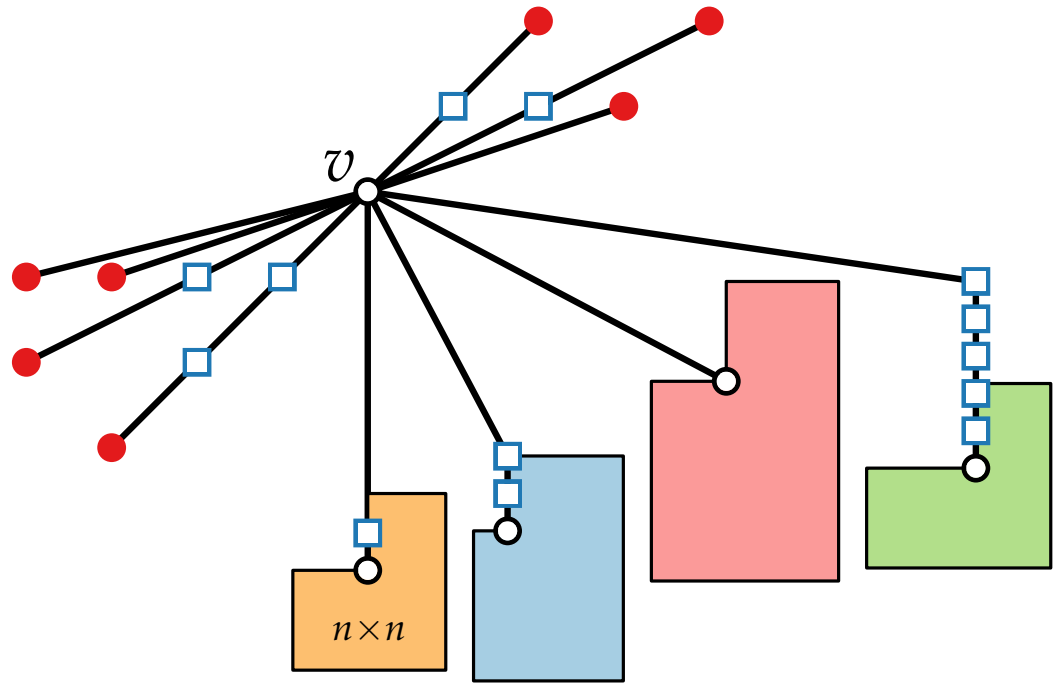
(2) Layout v + 

(3) Add

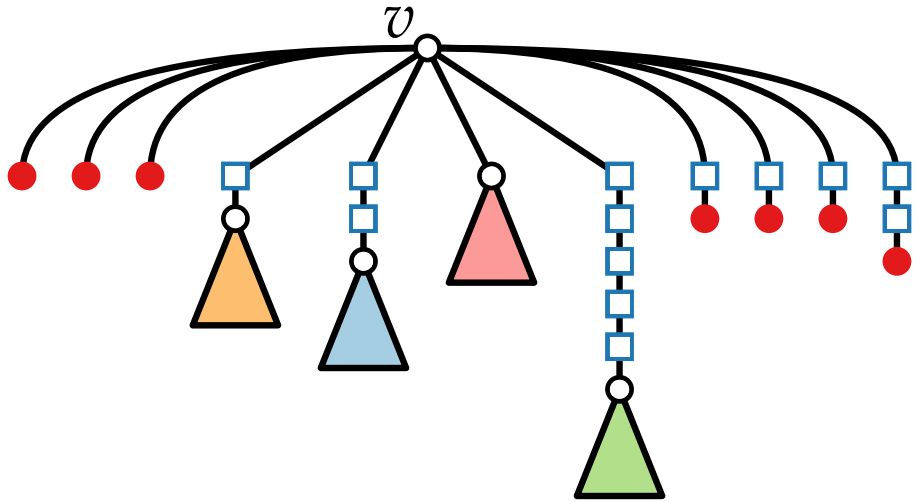
(4) Sort by #

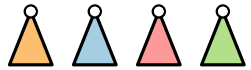
(5) Place + on common segments in order

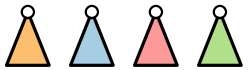
$3n/4$ segments
$n \times n$ grid



Tree Drawings



(1) Draw 

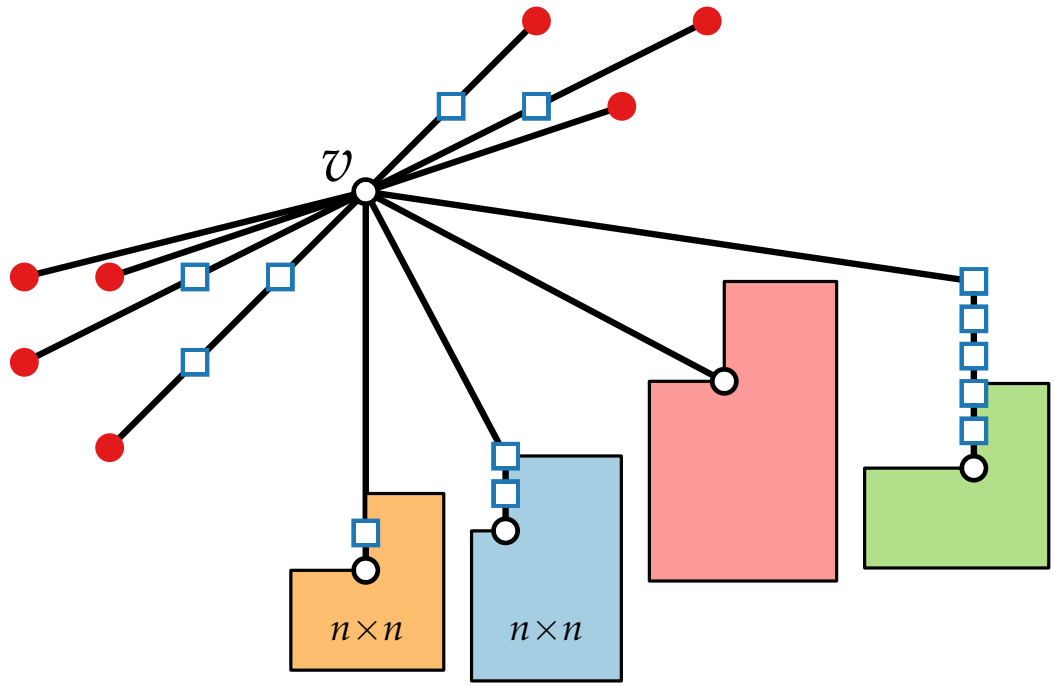
(2) Layout v + 

(3) Add

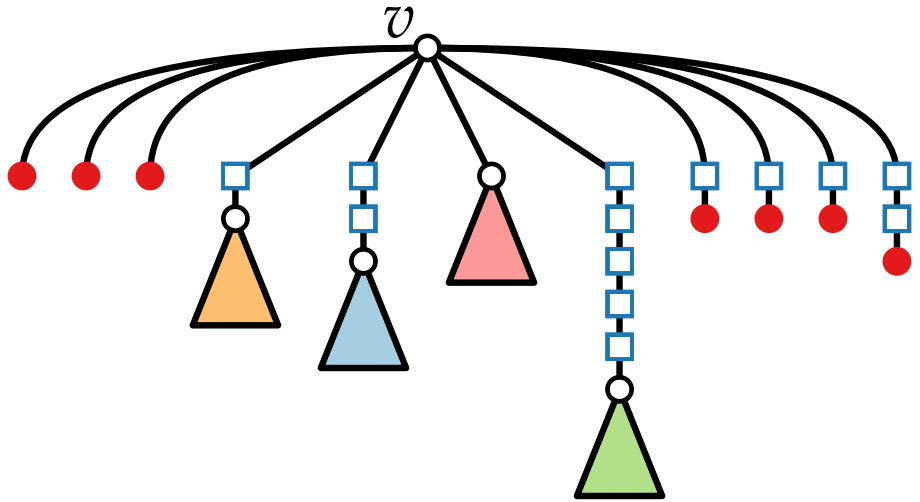
(4) Sort by #

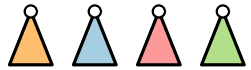
(5) Place + on common segments in order

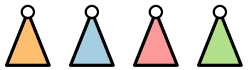
$3n/4$ segments
$n \times n$ grid



Tree Drawings



(1) Draw 

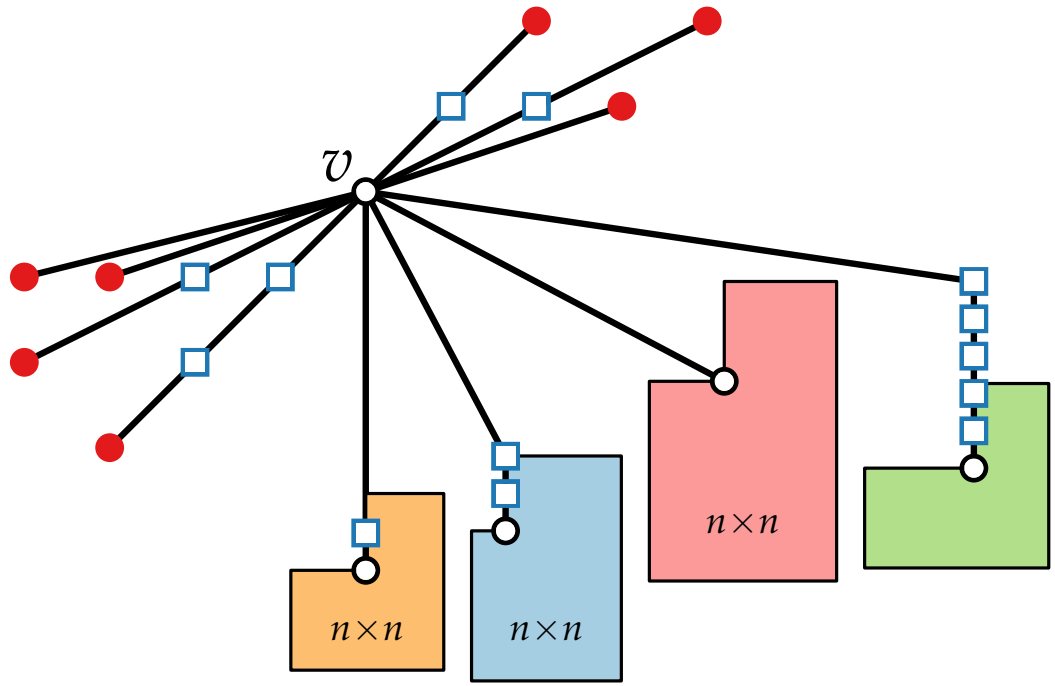
(2) Layout v + 

(3) Add

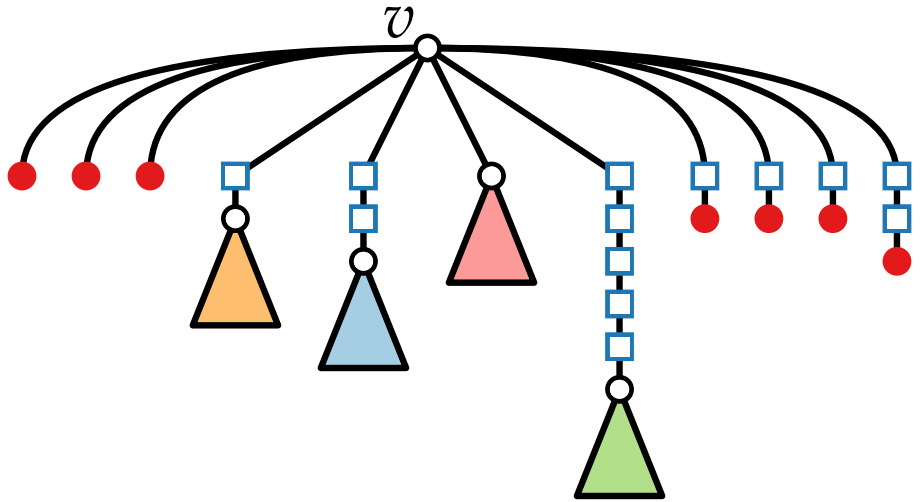
(4) Sort by #

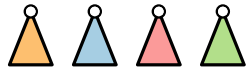
(5) Place + on common segments in order

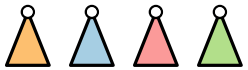
$3n/4$ segments
$n \times n$ grid



Tree Drawings



(1) Draw 

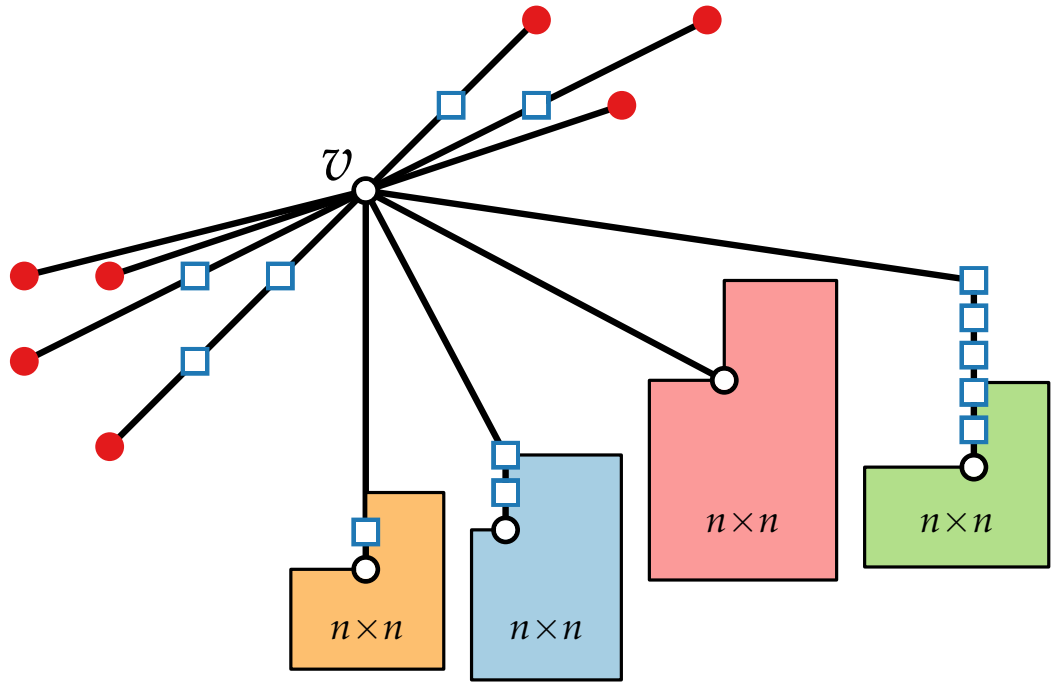
(2) Layout v + 

(3) Add

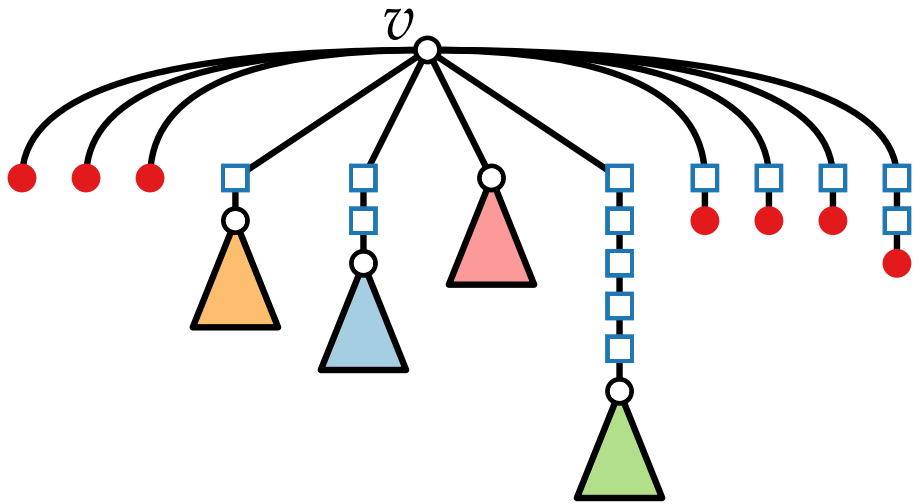
(4) Sort by #

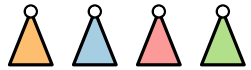
(5) Place + on common segments in order

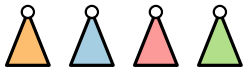
$3n/4$ segments
$n \times n$ grid



Tree Drawings



(1) Draw 

(2) Layout v + 

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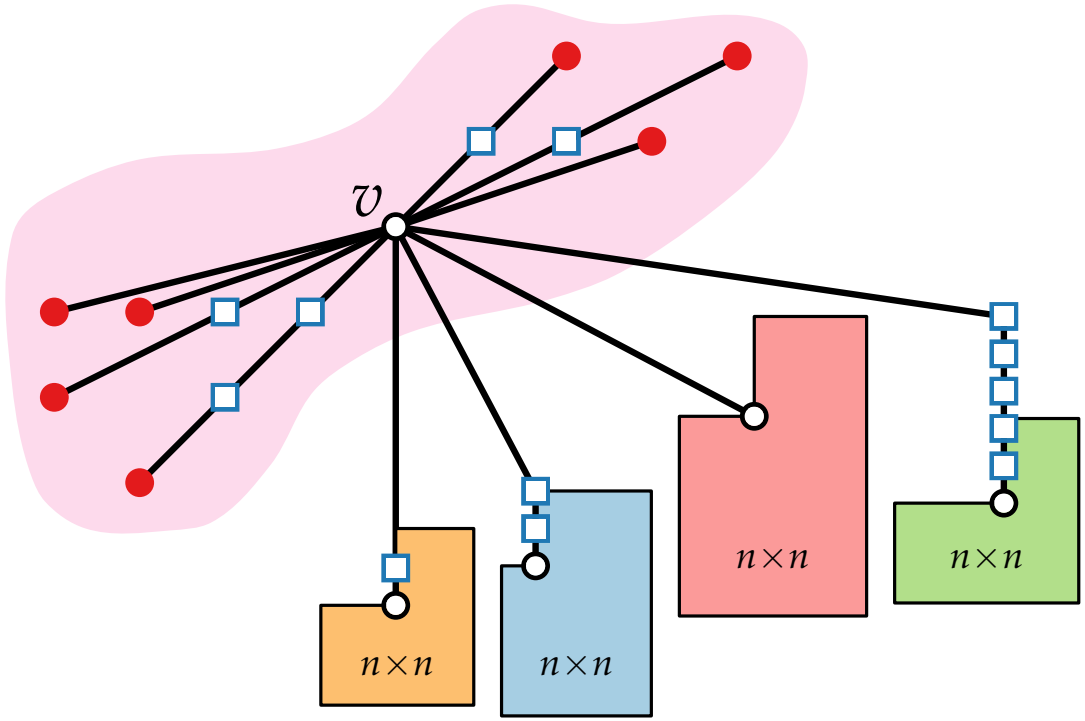
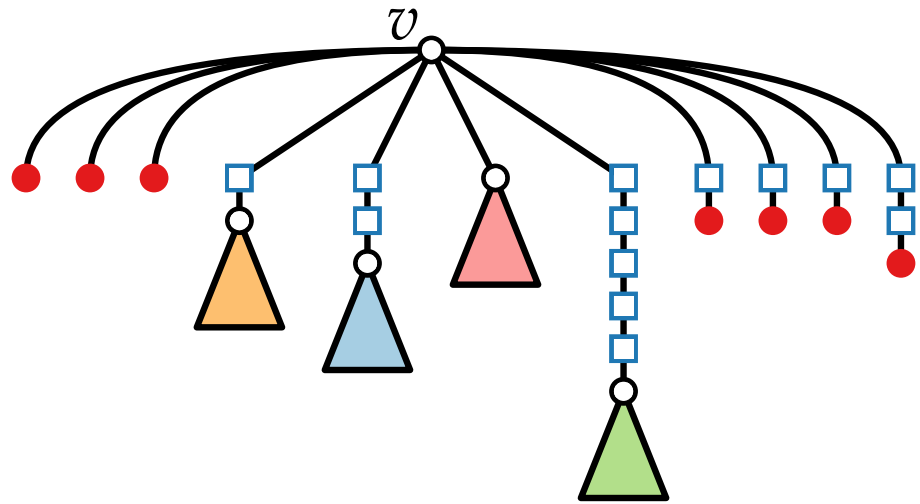
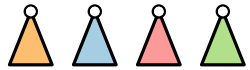
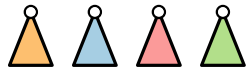


Tree Drawings



(1) Draw 

(2) Layout v + 

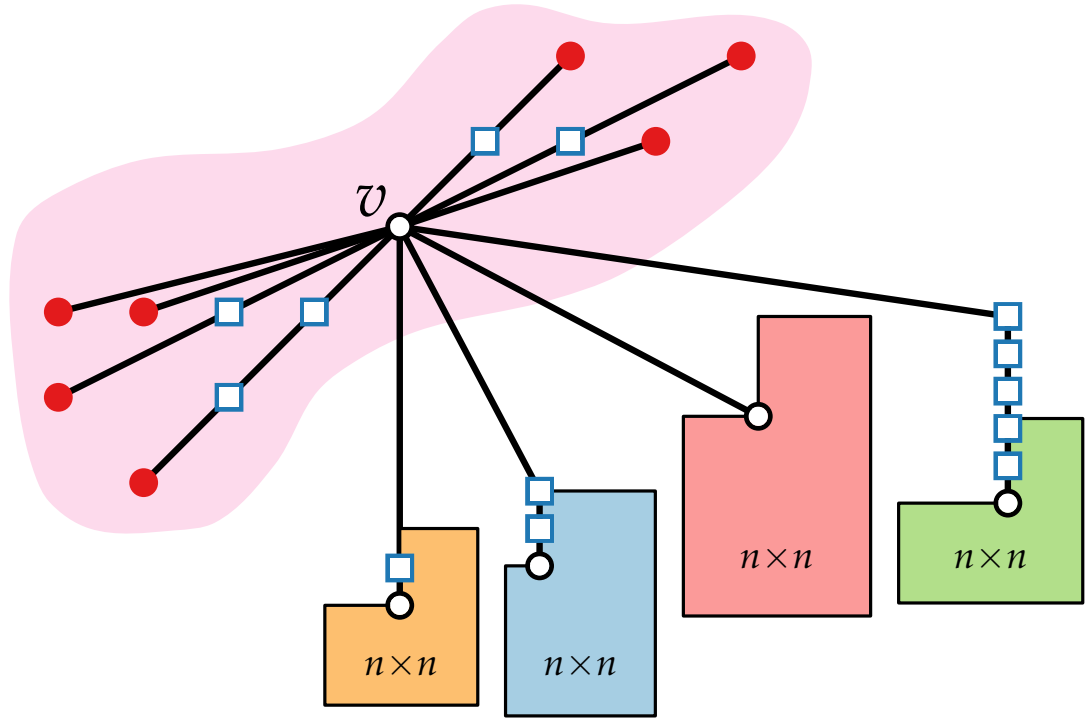
(3) Add

(4) Sort by #

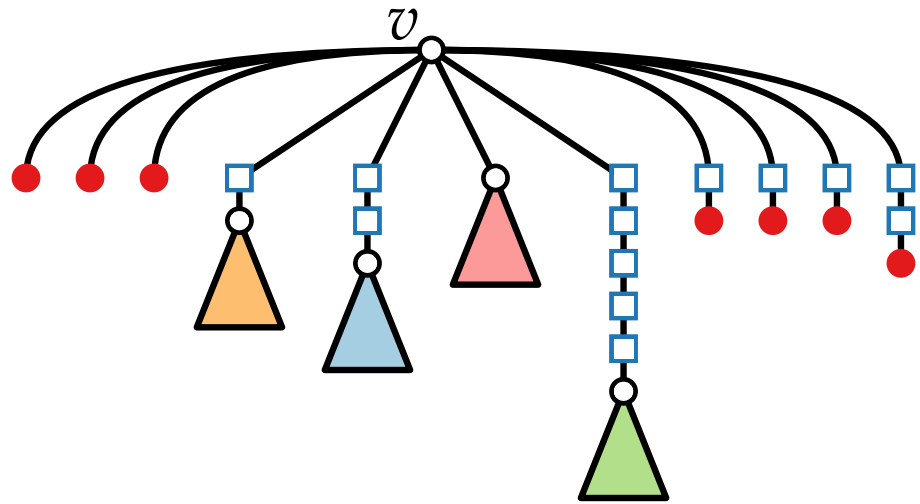
(5) Place + on common segments in order

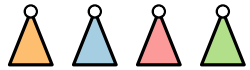
$3n/4$ segments
$n \times n$ grid

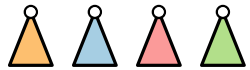
height ✓



Tree Drawings



(1) Draw 

(2) Layout v + 

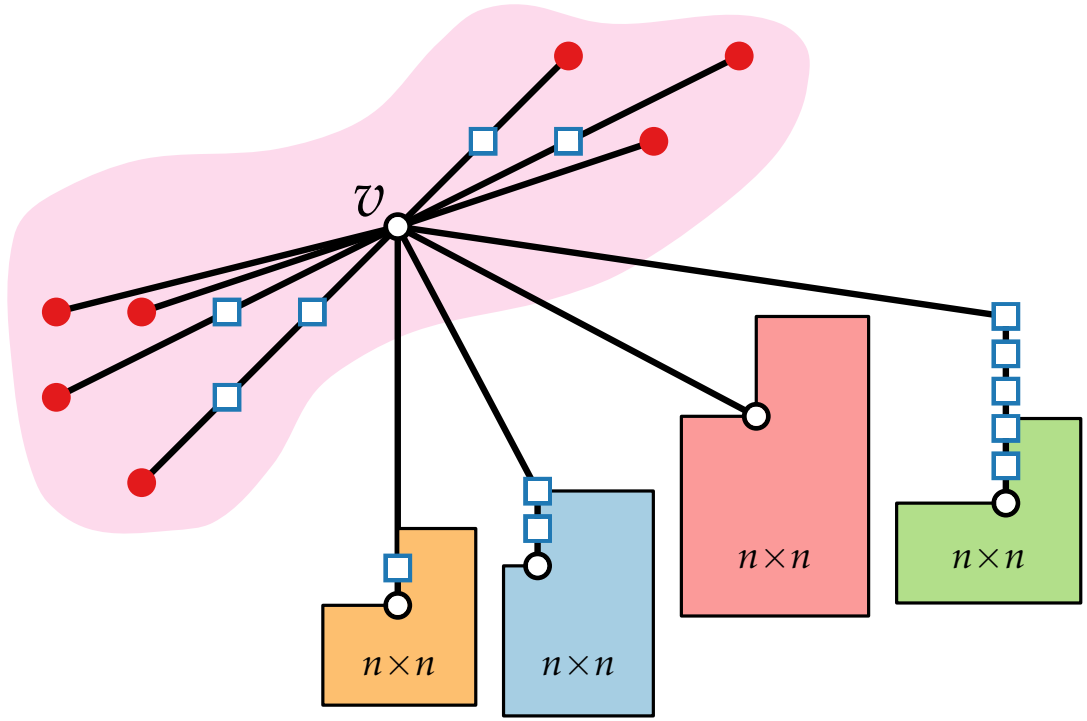
(3) Add

(4) Sort by #

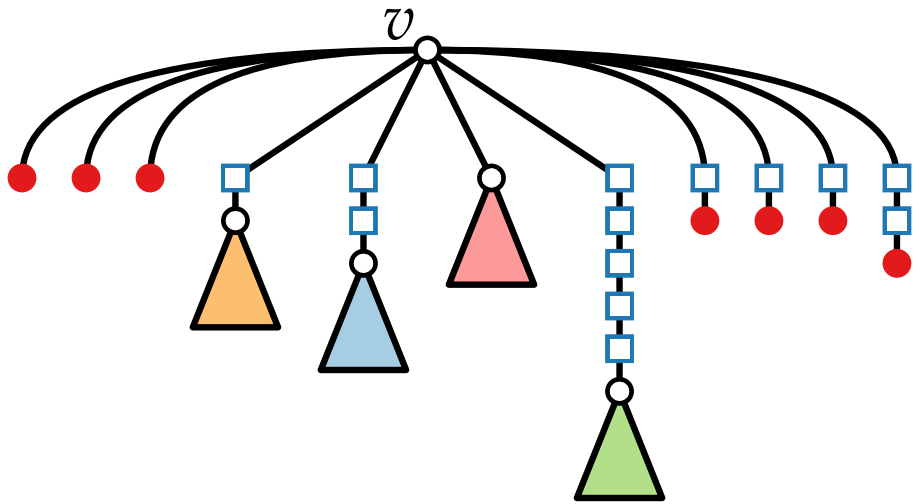
(5) Place + on common segments in order

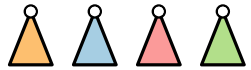
$3n/4$ segments
$n \times n$ grid

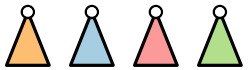
height ✓
width



Tree Drawings



(1) Draw 

(2) Layout v + 

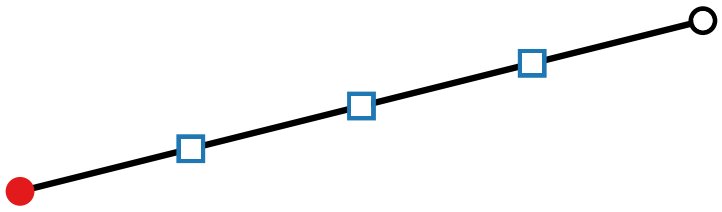
(3) Add

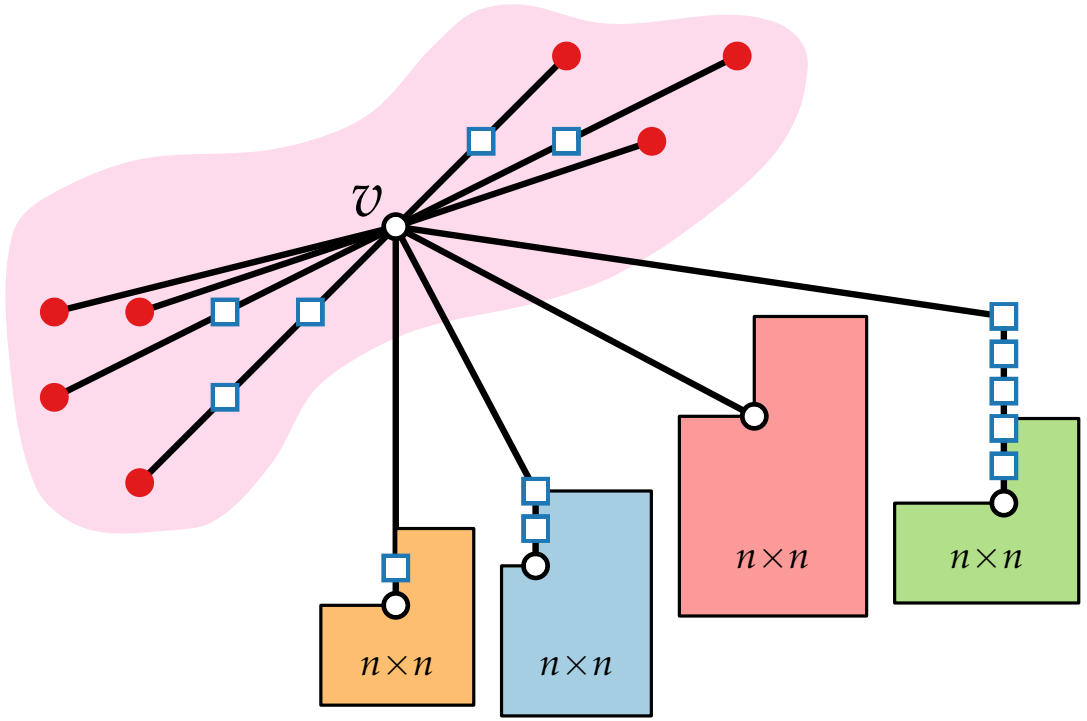
(4) Sort ● by #

(5) Place ● + on common segments in order

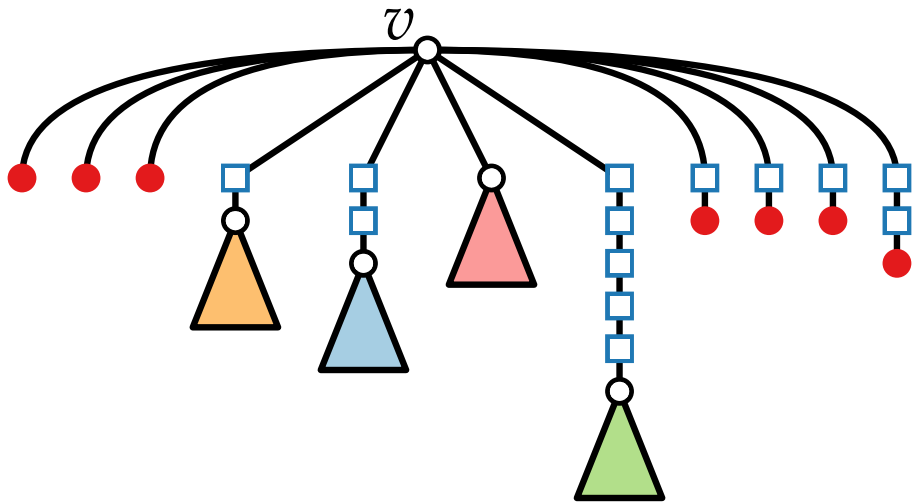
$3n/4$ segments
$n \times n$ grid

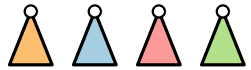
height ✓
width

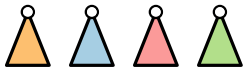




Tree Drawings



(1) Draw 

(2) Layout v + 

(3) Add

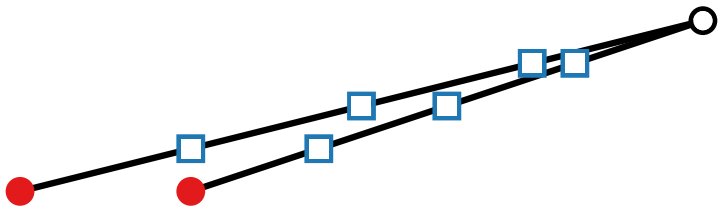
(4) Sort by #

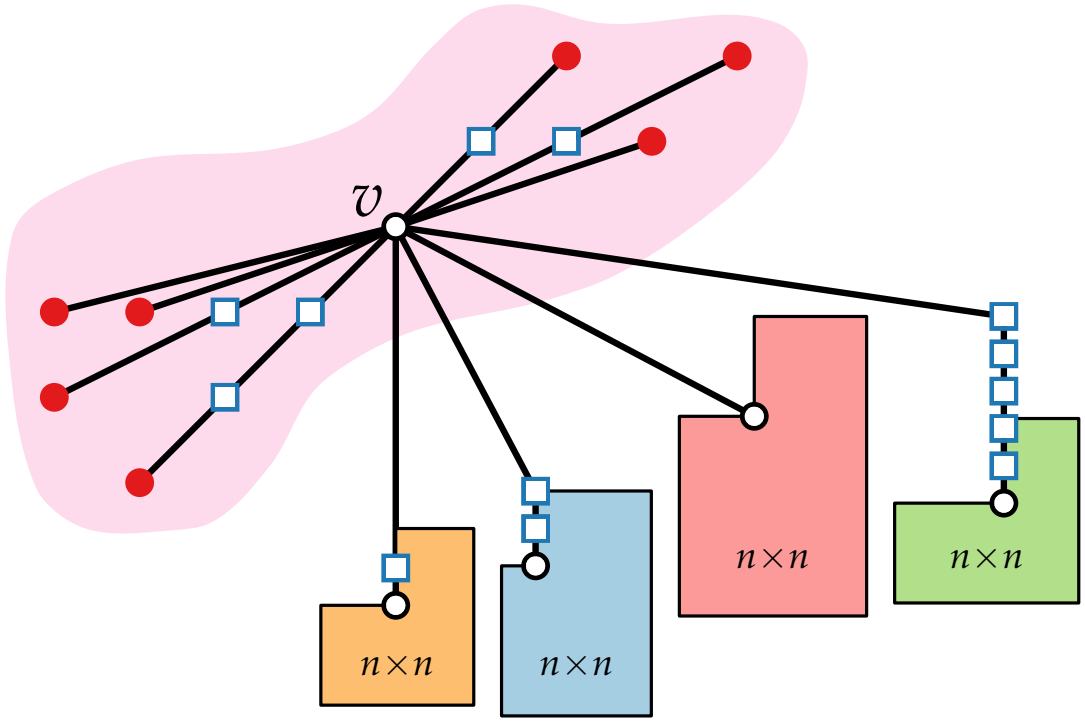
(5) Place + on common segments in order

$3n/4$ segments
$n \times n$ grid

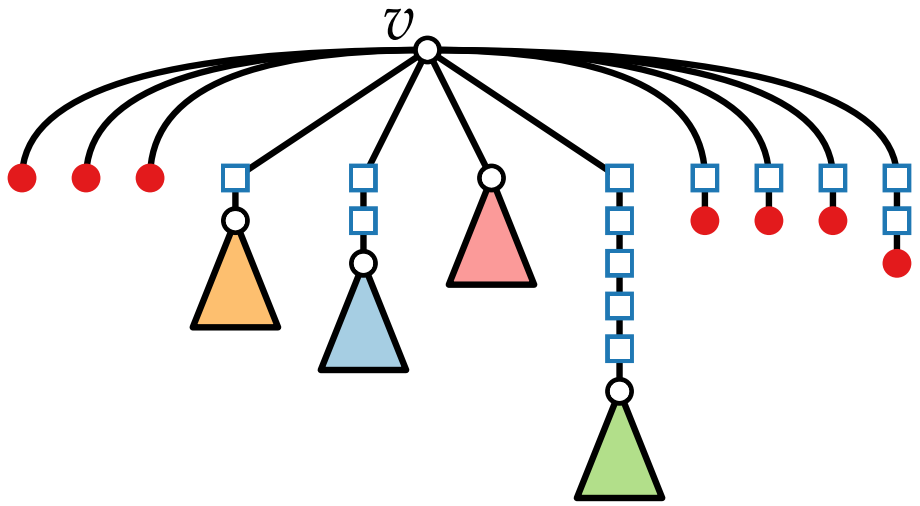
height ✓

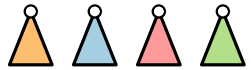
width

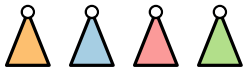




Tree Drawings



(1) Draw 

(2) Layout v + 

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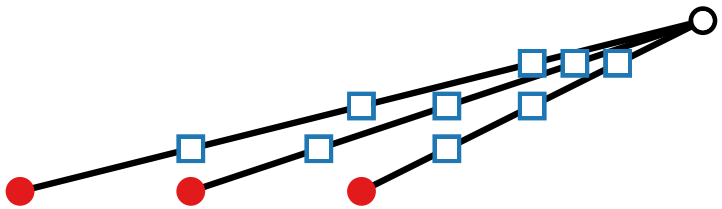
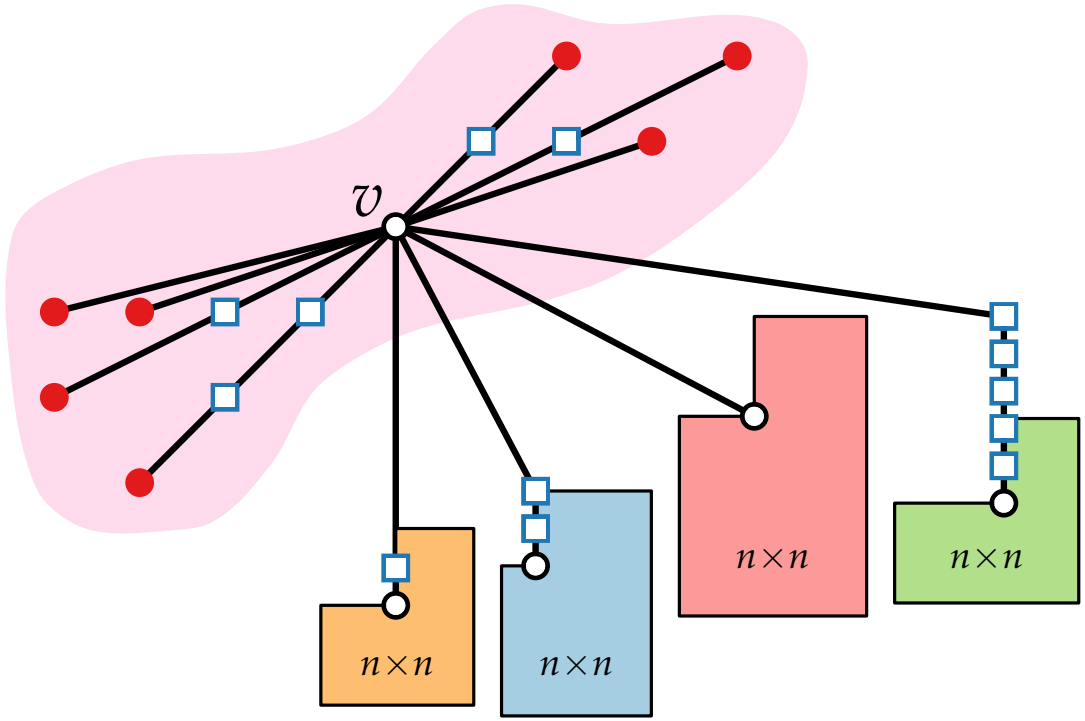
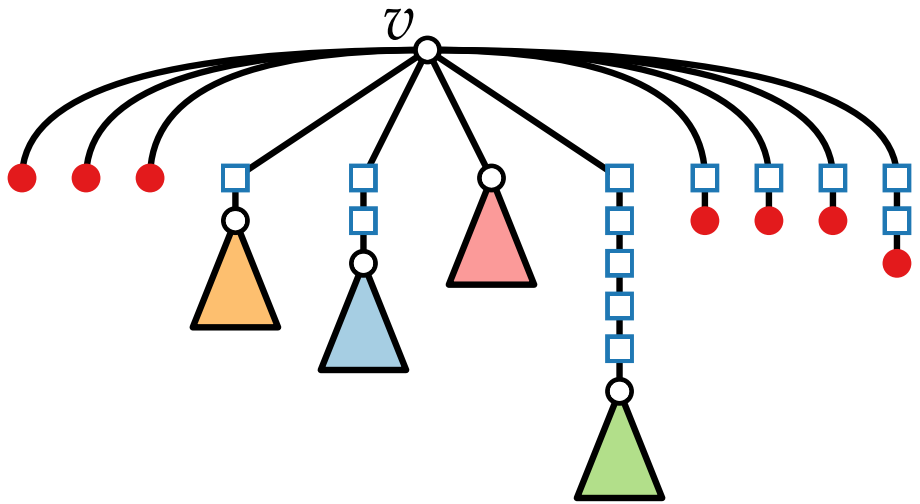
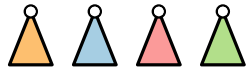
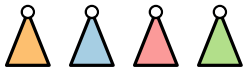




Tree Drawings



(1) Draw 

(2) Layout v + 

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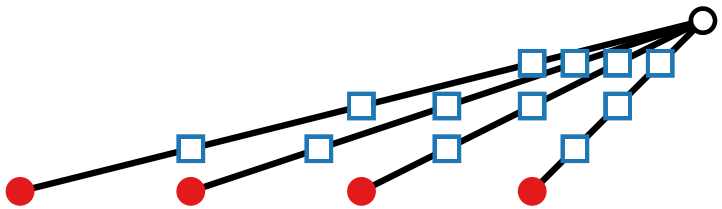
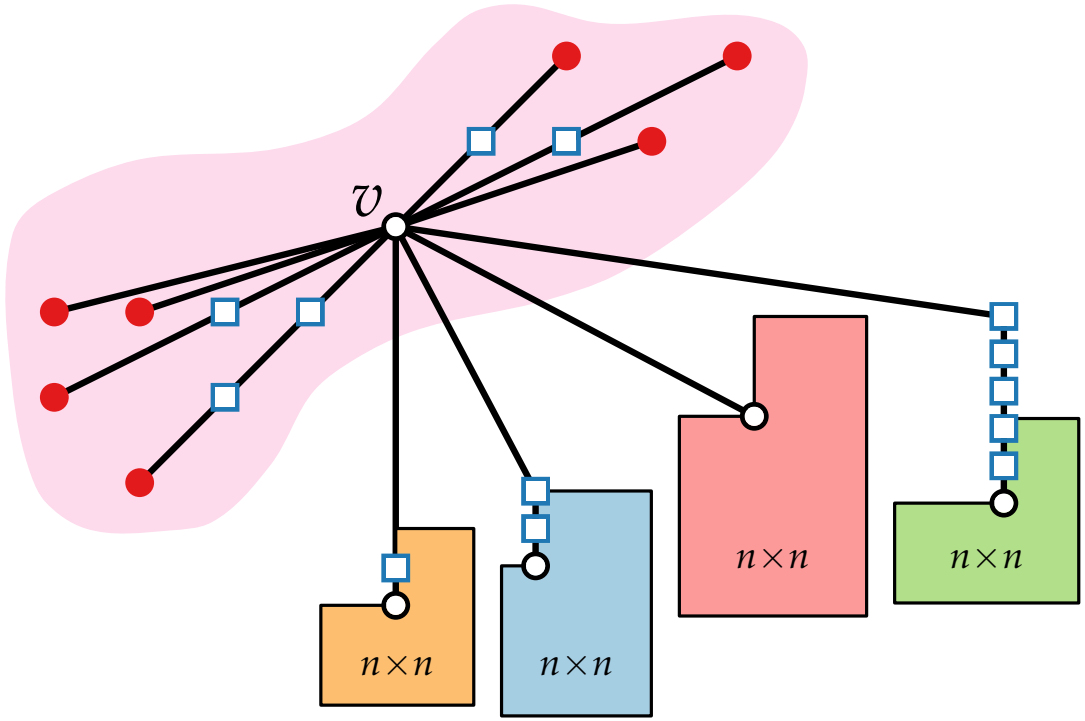
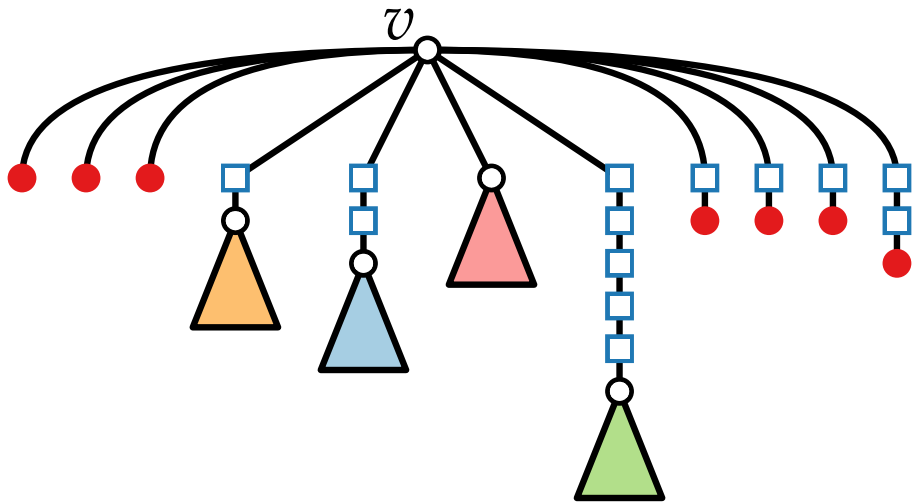
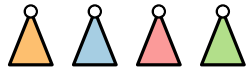
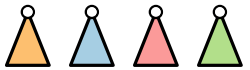




Tree Drawings



(1) Draw 

(2) Layout v + 

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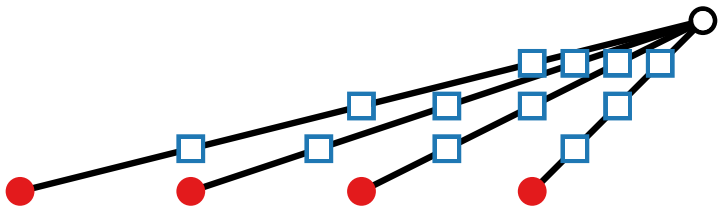
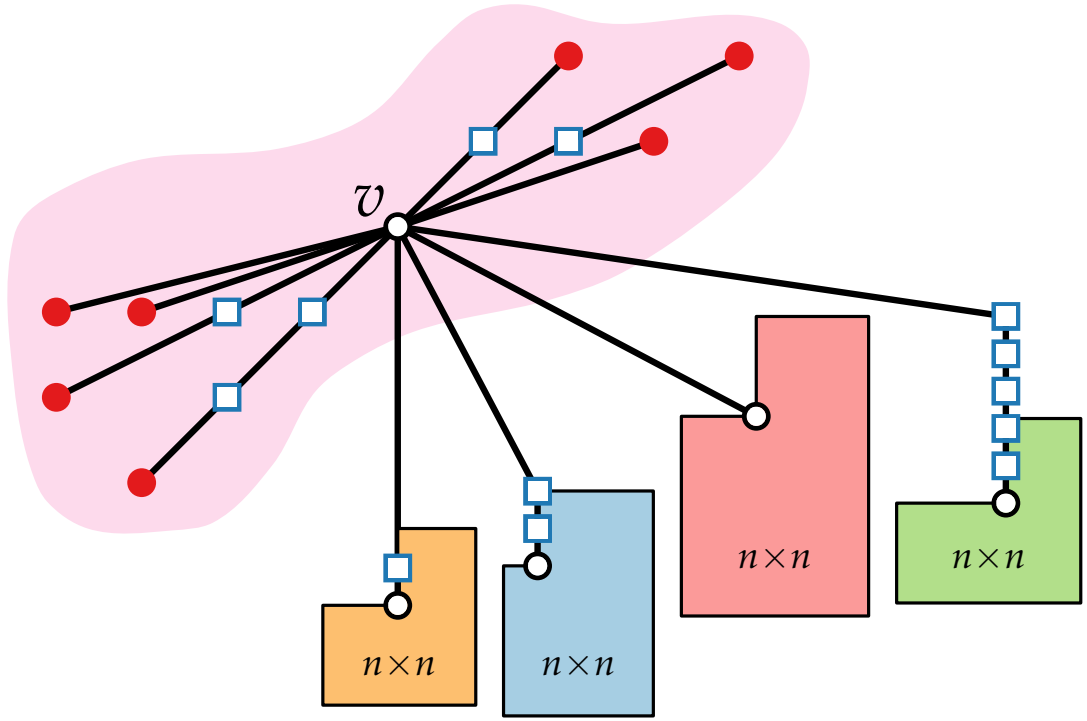
height ✓

width ✓





Improved Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ [6]	$O(n^2) \times O(n^{1.58})$
			$\vartheta/2$ [6]	quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [6]	$O(n) \times O(n^2)$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

Improved Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$	$n \times n$
			$\vartheta/2$ [6]	quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [6]	$O(n) \times O(n^2)$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
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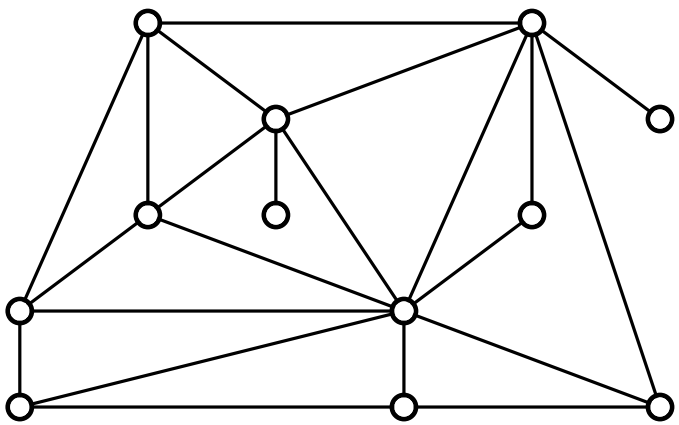
Orderly Spanning Trees

[Chiang, Lin, Lu '05]



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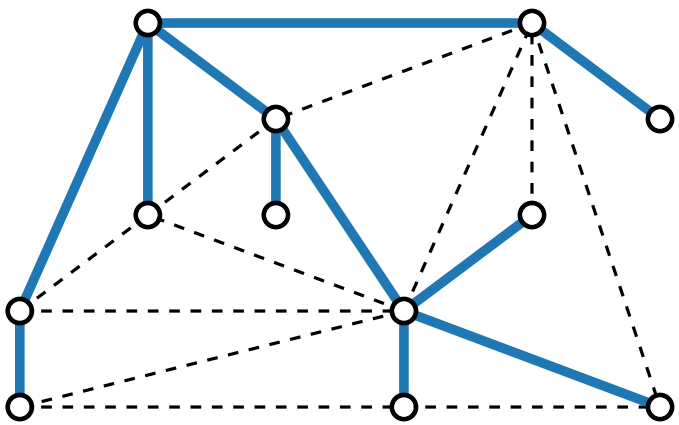

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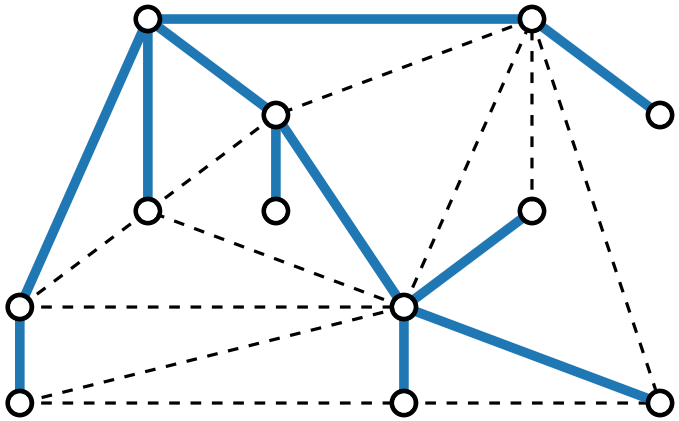

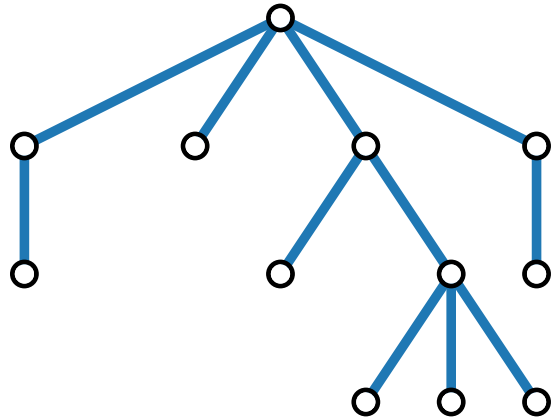

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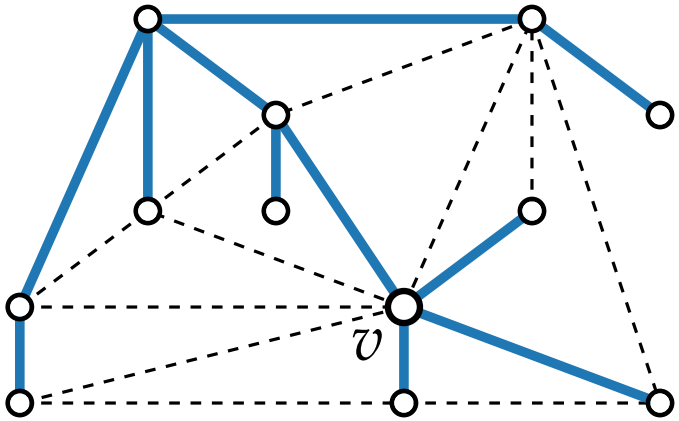
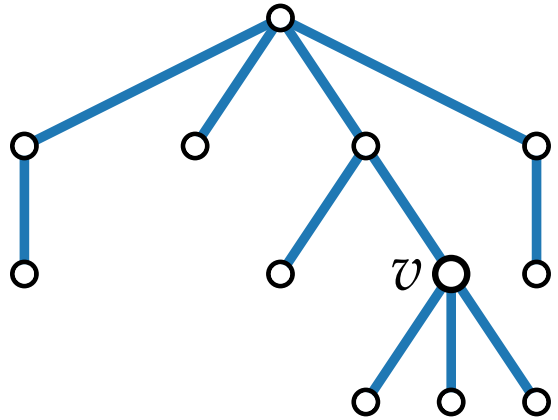
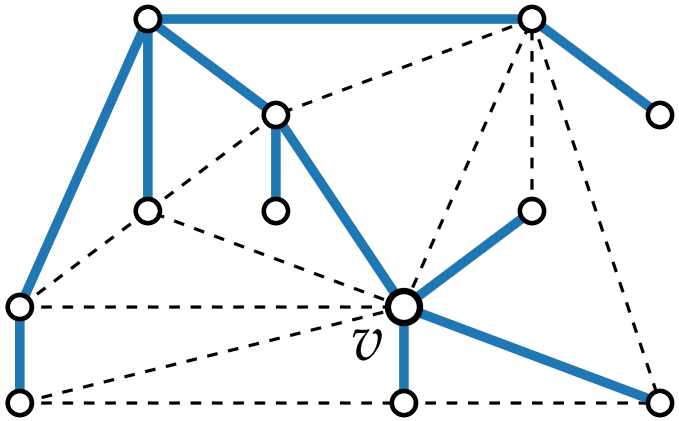
[Chiang, Lin, Lu '05]

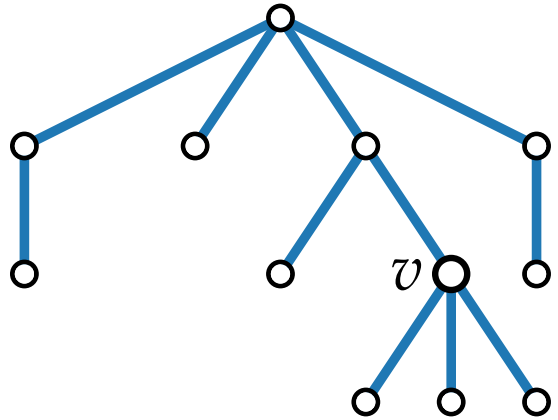




Orderly Spanning Trees

[Chiang, Lin, Lu '05]

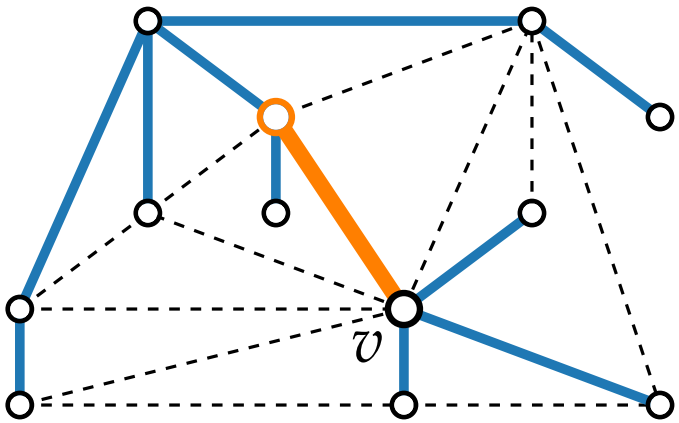


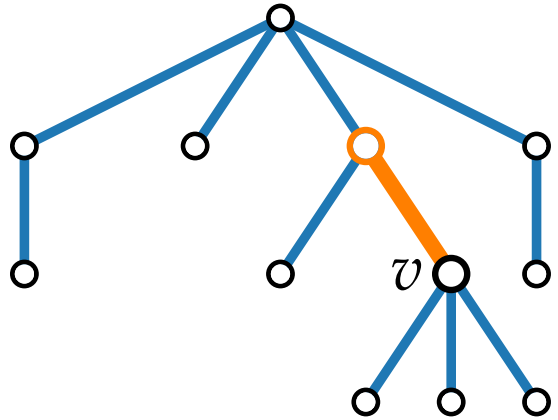


neighbors of v in circ. order:

Orderly Spanning Trees

[Chiang, Lin, Lu '05]





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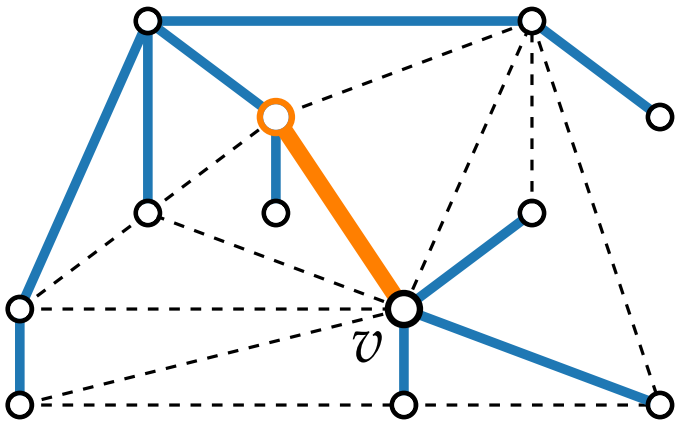
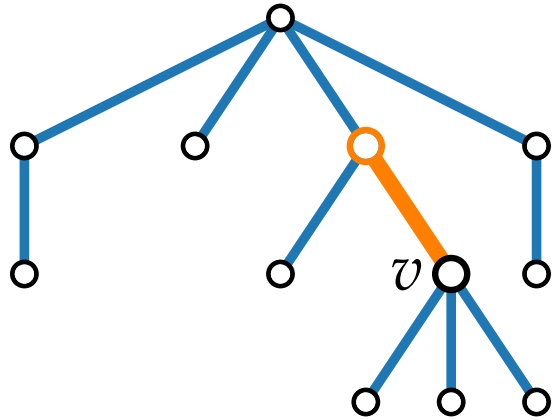
Orderly Spanning Trees

[Chiang, Lin, Lu '05]





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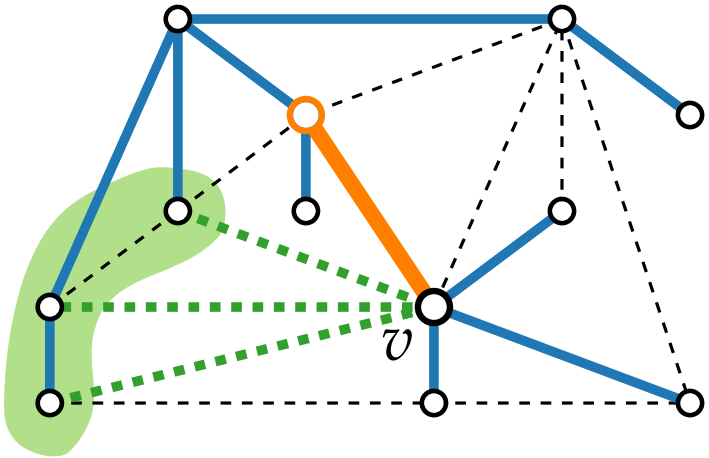
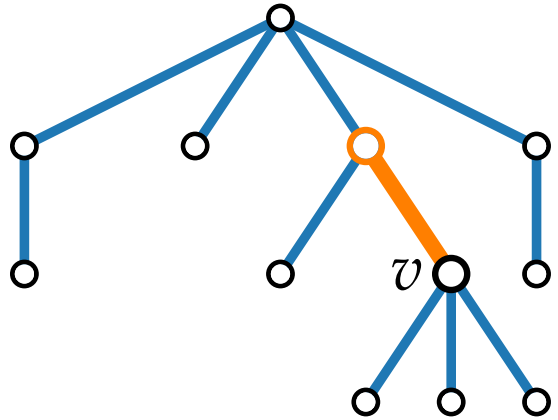
Orderly Spanning Trees

[Chiang, Lin, Lu '05]





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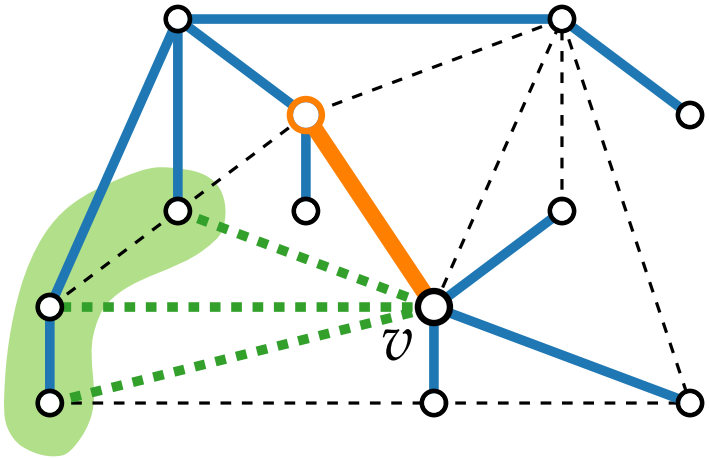

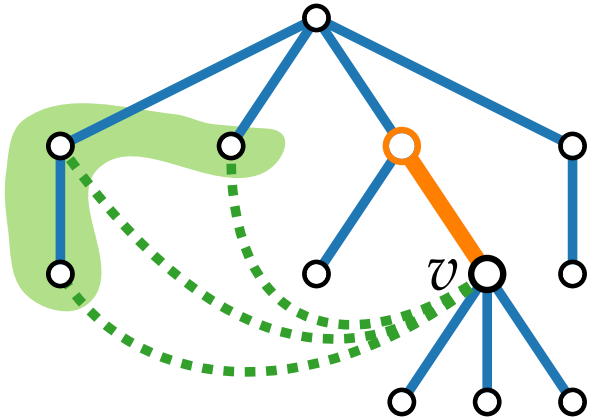

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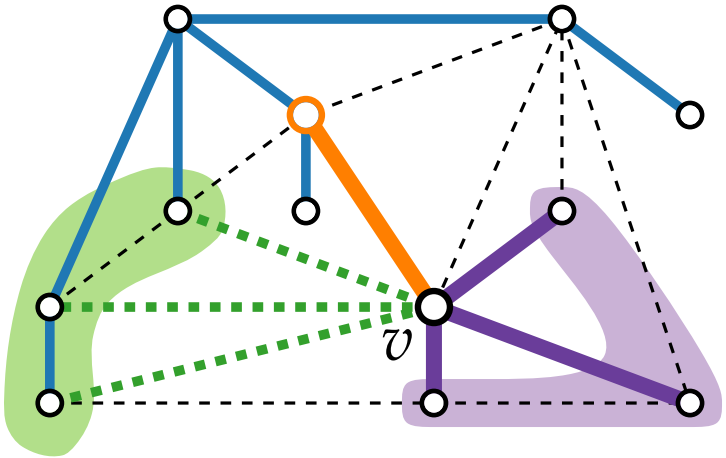

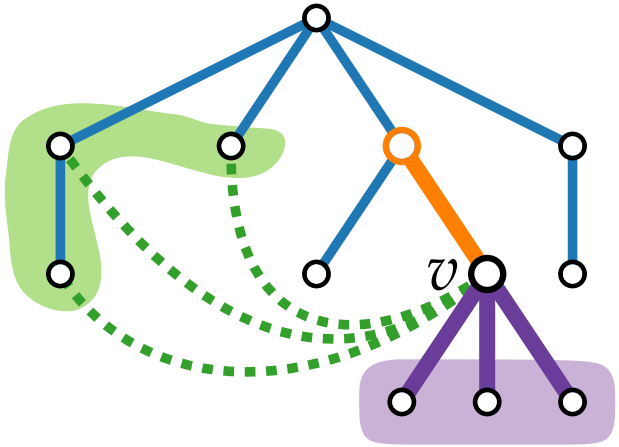

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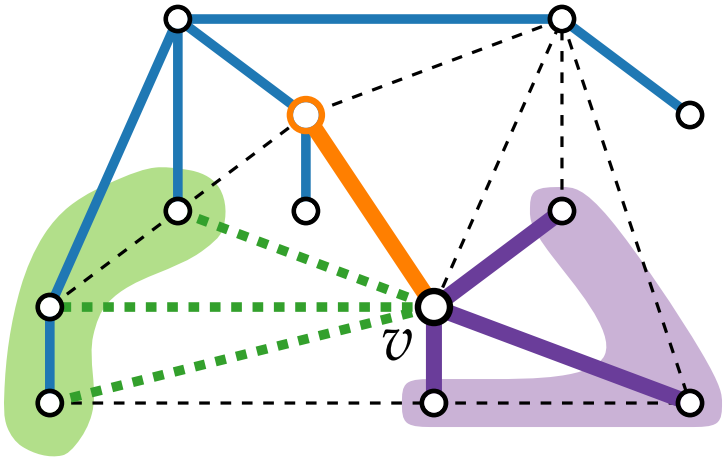
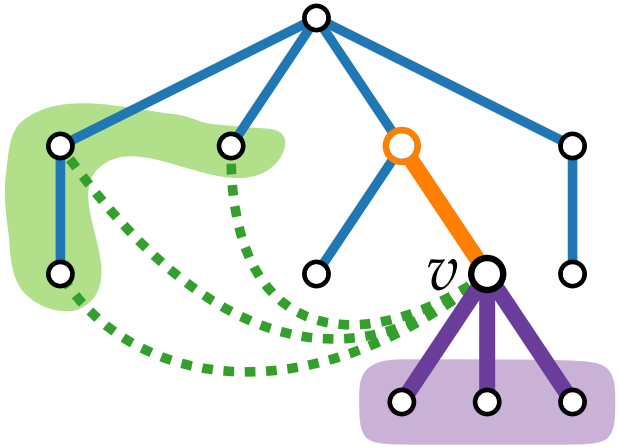
(3) children

Orderly Spanning Trees

[Chiang, Lin, Lu '05]





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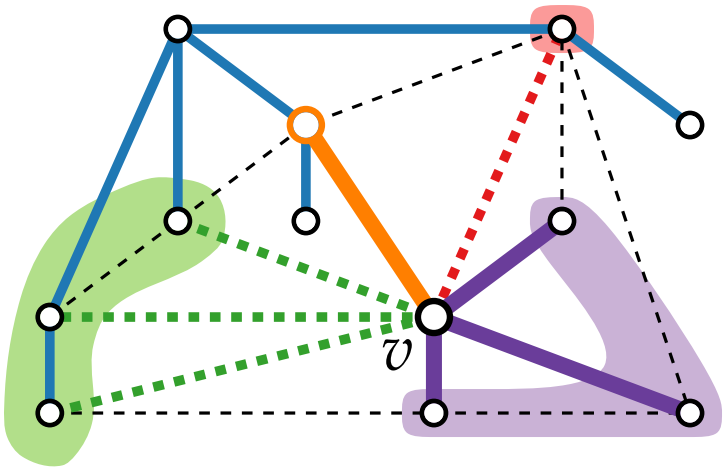
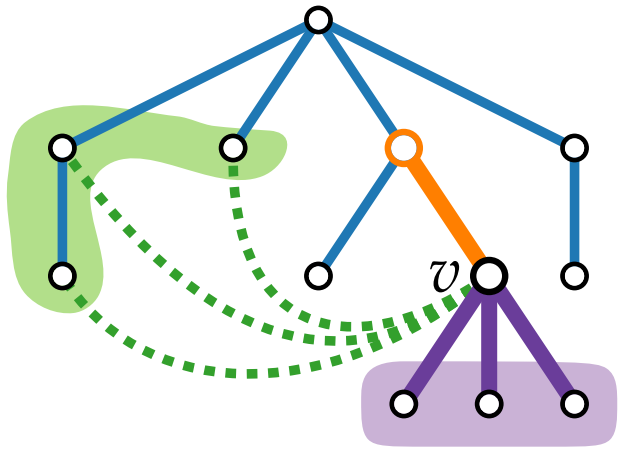
Orderly Spanning Trees

[Chiang, Lin, Lu '05]





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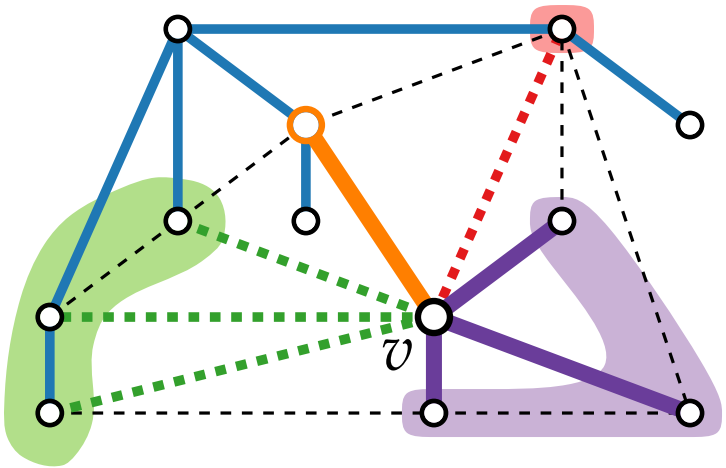

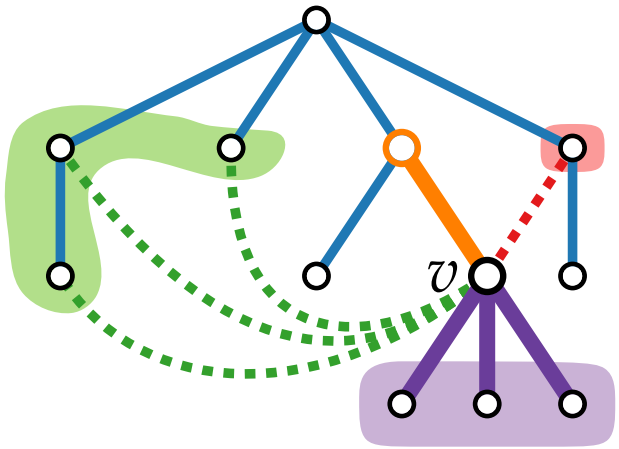

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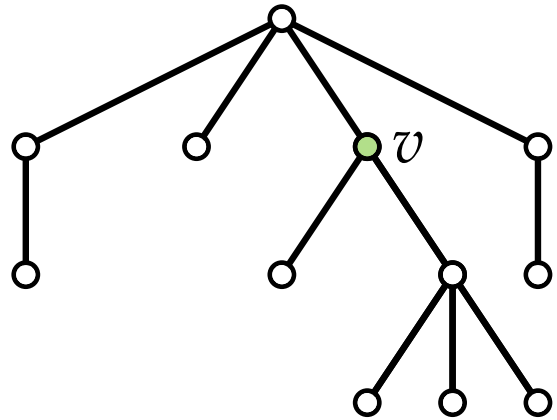
Slope-Disjoint Drawing of a Tree

[Angelini et al. '12]

Assign angle interval to each vtx



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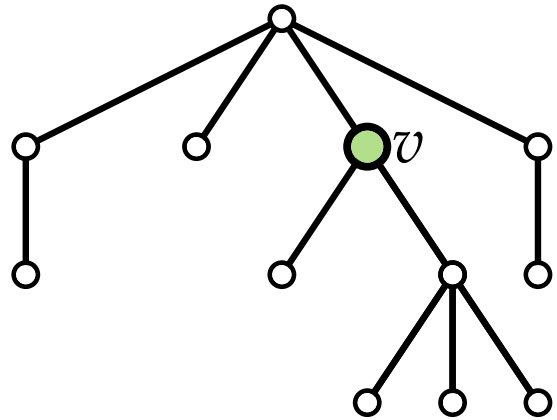

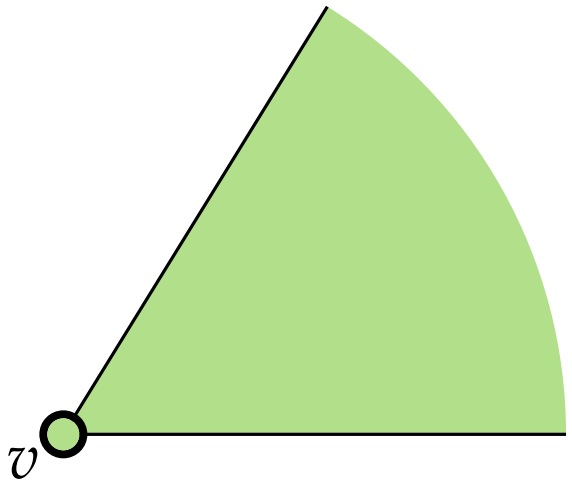

Slope-Disjoint Drawing of a Tree

[Angelini et al. '12]

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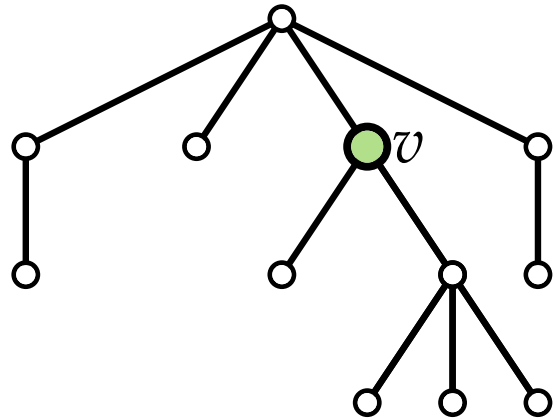
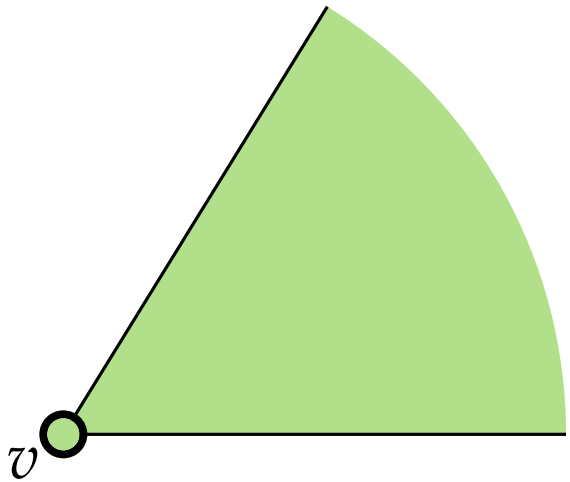
All segments in $T[v]$ in interval





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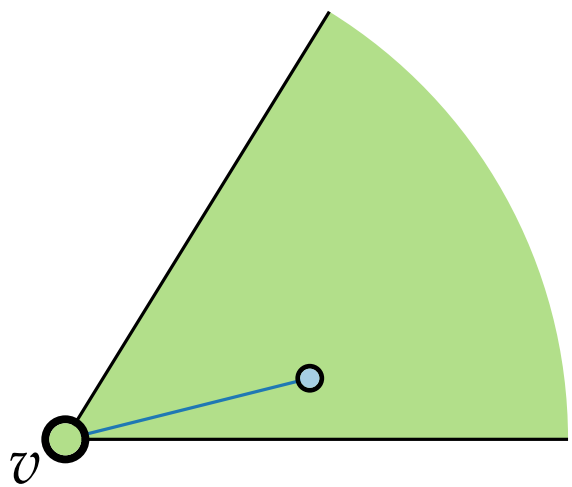

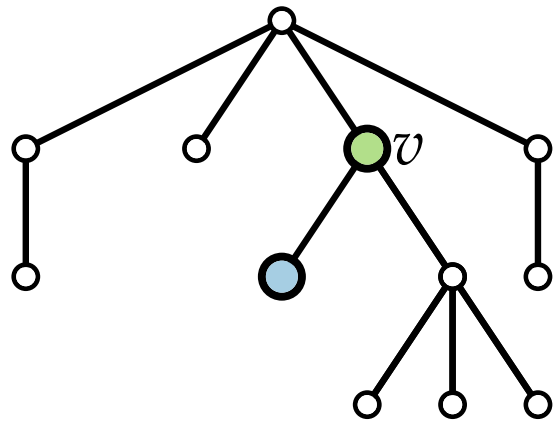

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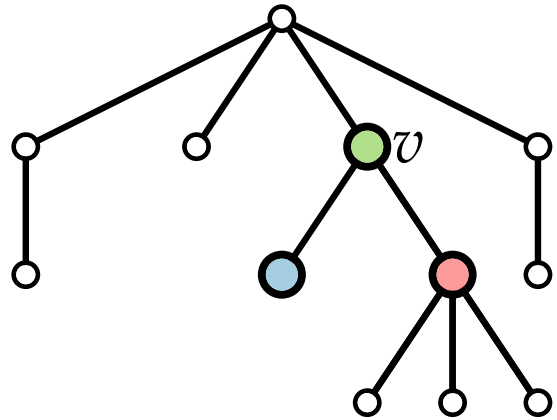

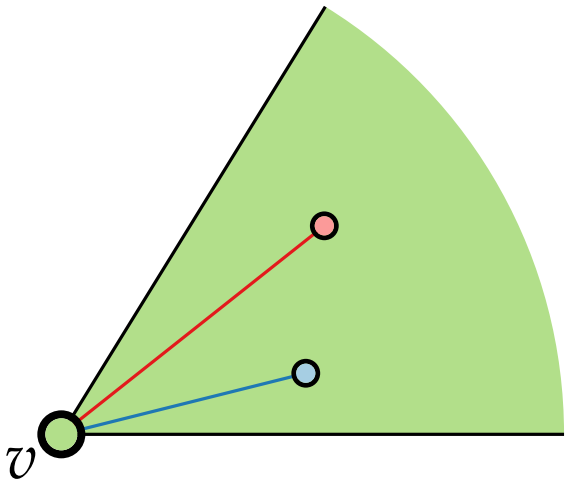

Slope-Disjoint Drawing of a Tree

[Angelini et al. '12]

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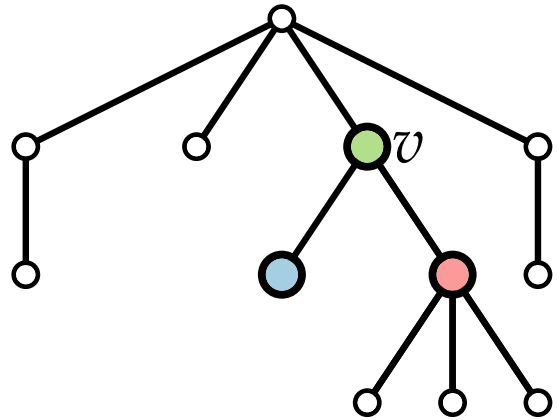
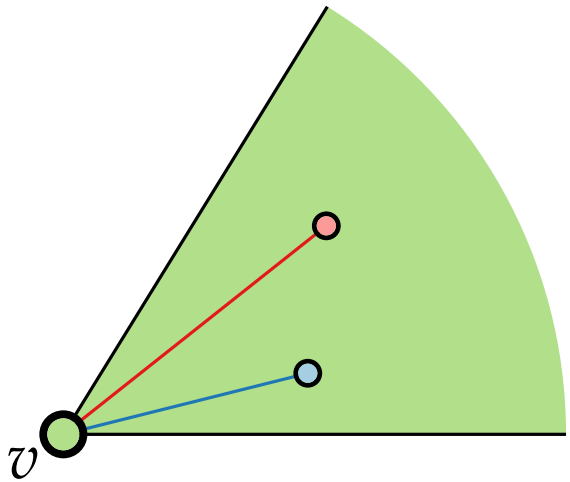
Intervals of children: disjoint subintervals that contain parent edge





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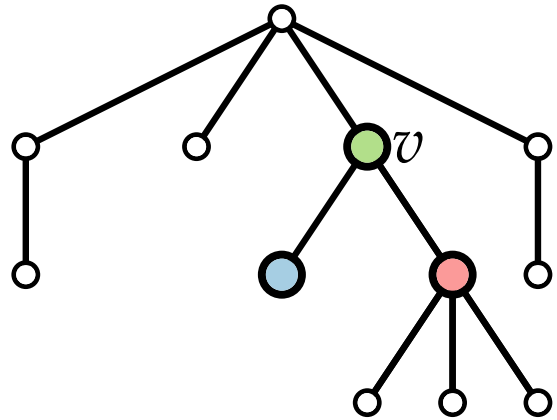

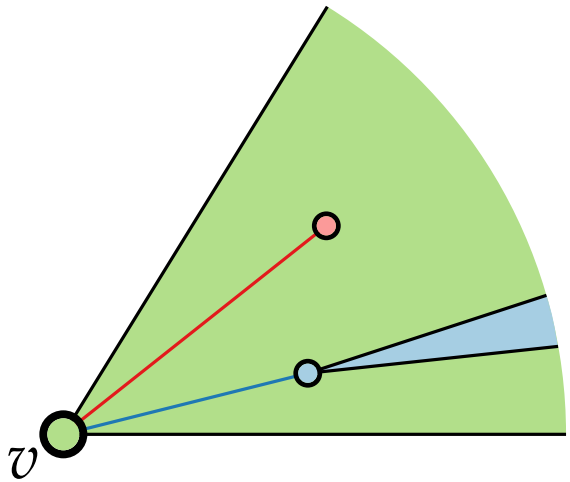

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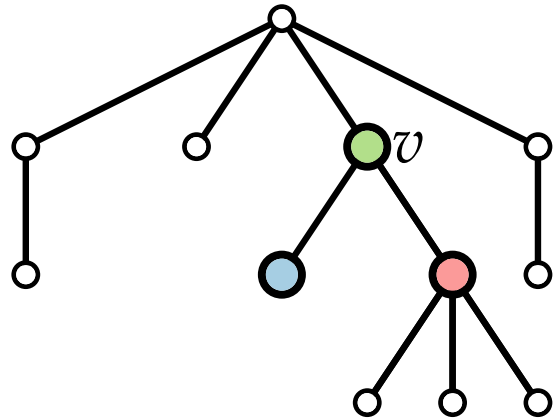

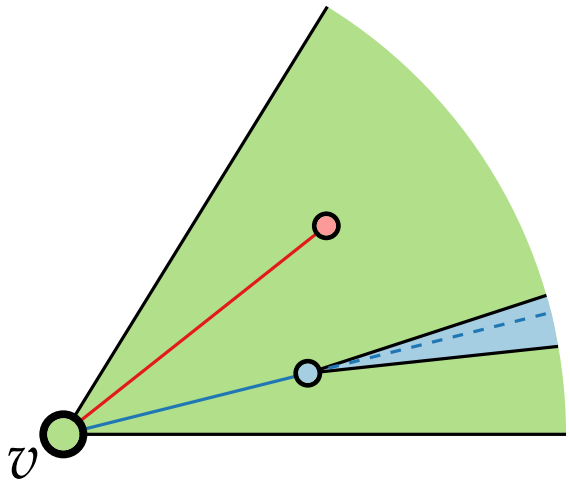

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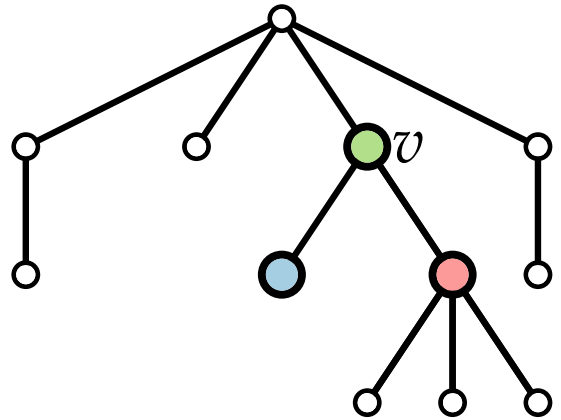

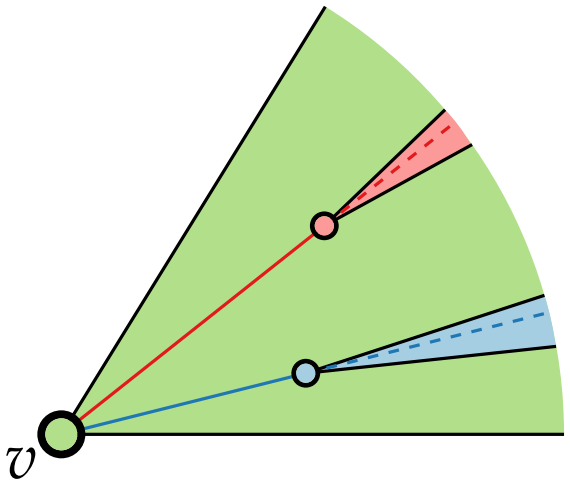

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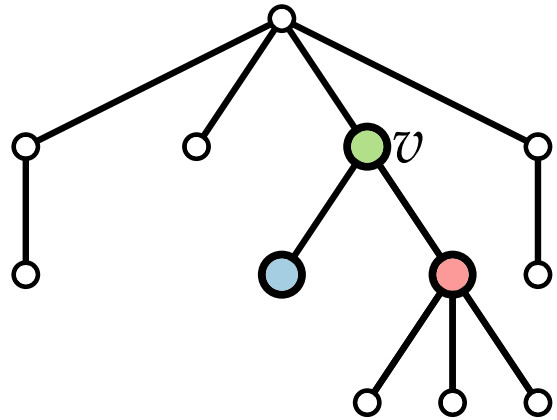

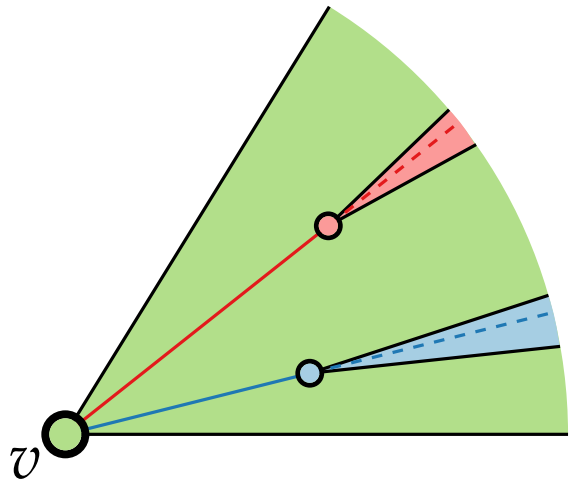

[Hossain & Rahman '15]

Slope-disjoint drawing of orderly spanning tree on $O(n) \times O(n^2)$ grid
$\Rightarrow$ planar (monotone) drawing on $O(n) \times O(n^2)$ grid

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[Angelini et al. '12]

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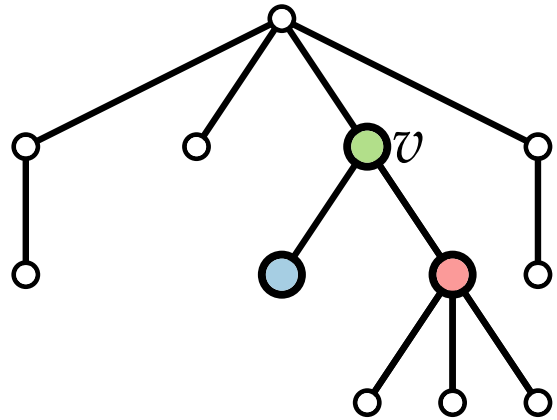

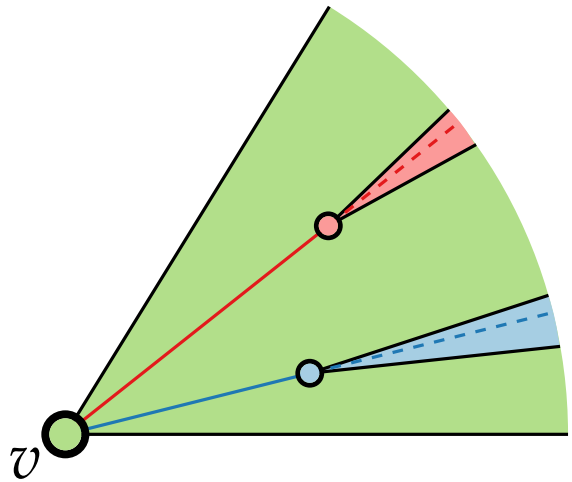

[Hossain & Rahman '15]

Slope-disjoint drawing of orderly spanning tree on $O(n) \times O(n^2)$ grid
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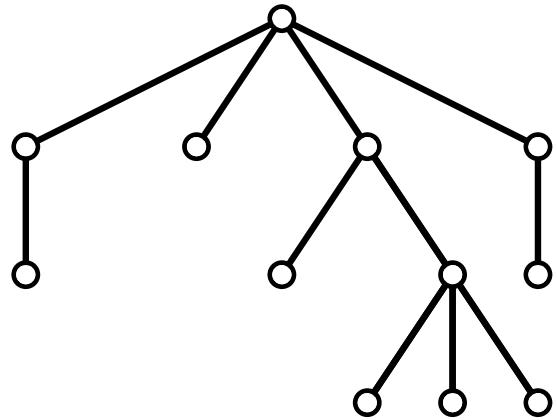
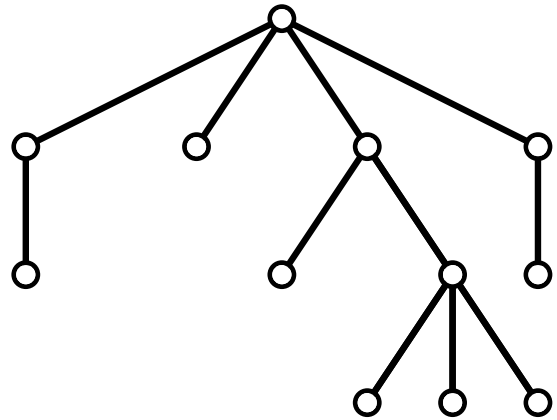
doesn't change the slopes!

Obtaining a slope-disjoint drawing



Obtaining a slope-disjoint drawing

ccw pre-order traversal



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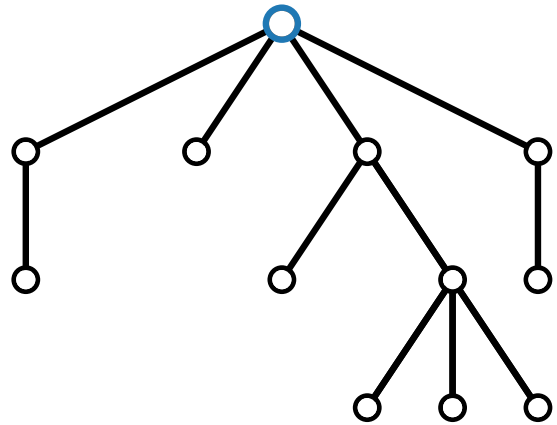

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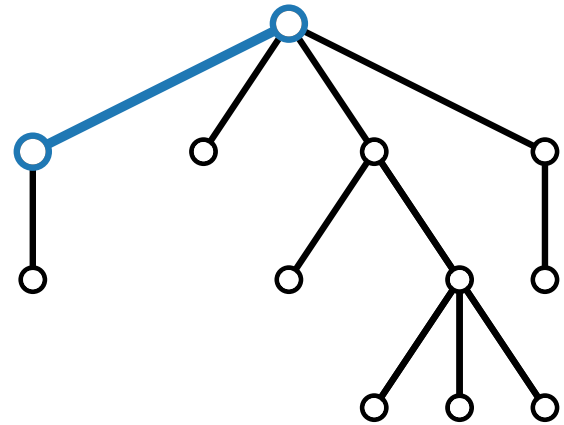

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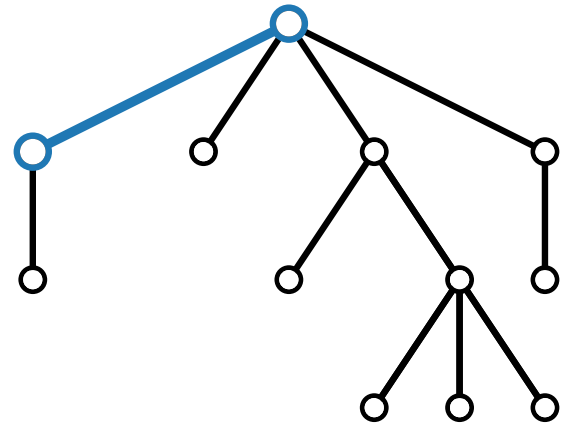

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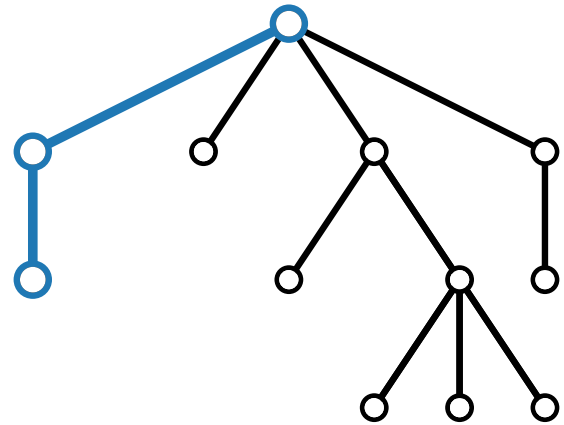
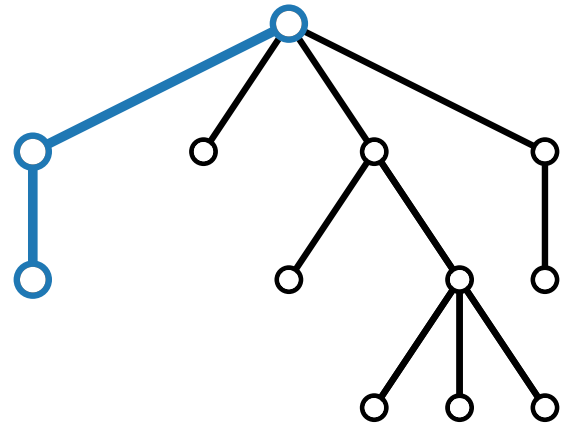


Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible



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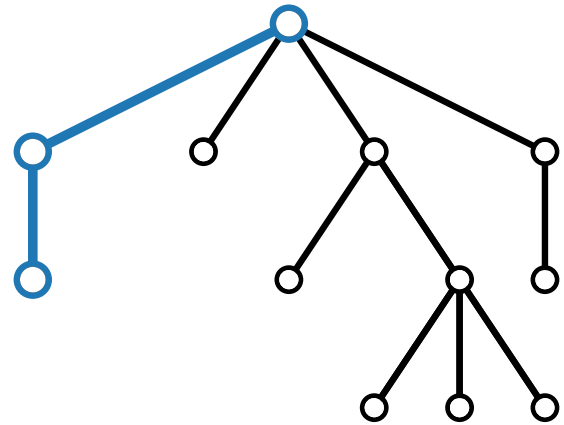

Obtaining a slope-disjoint drawing

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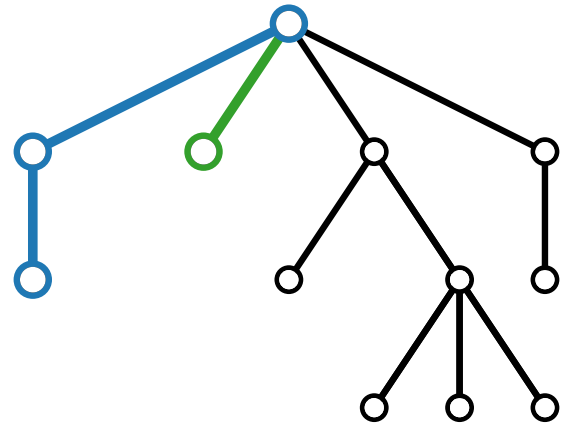

Obtaining a slope-disjoint drawing

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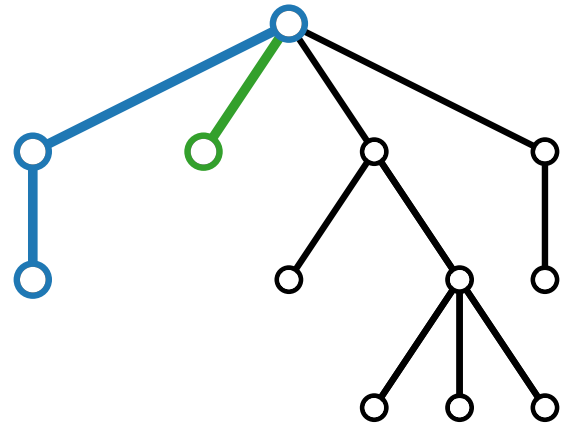

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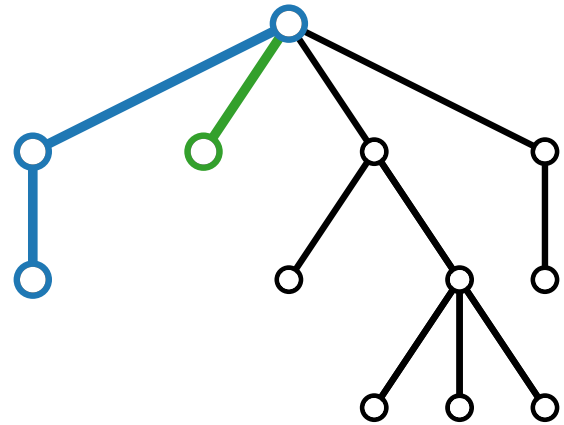

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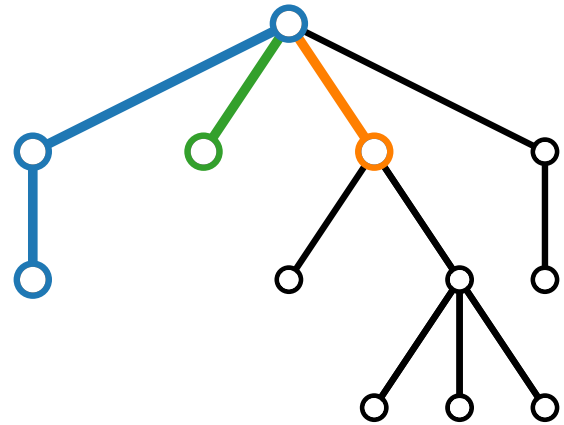

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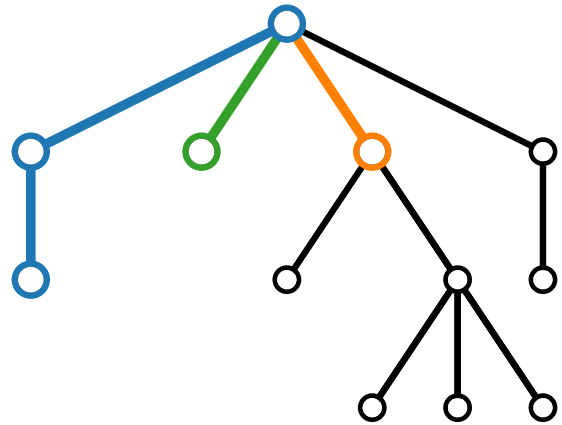

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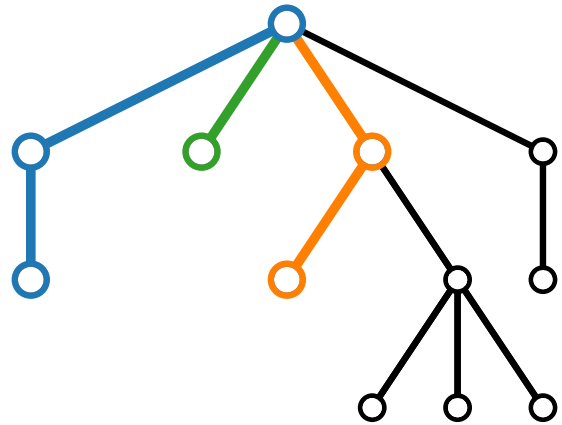

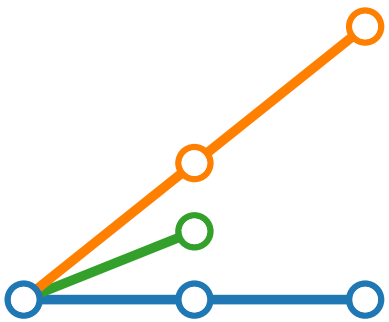

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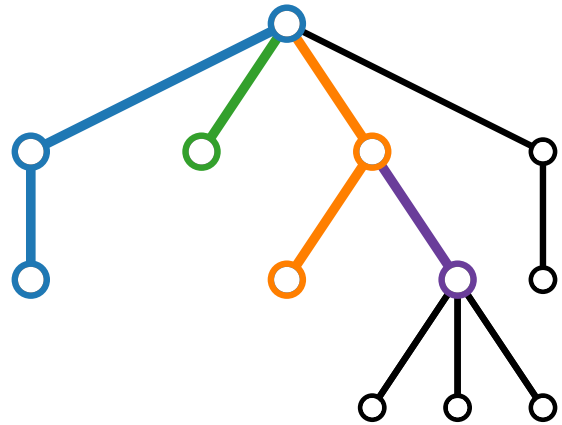

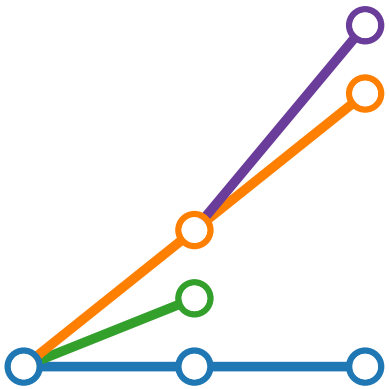

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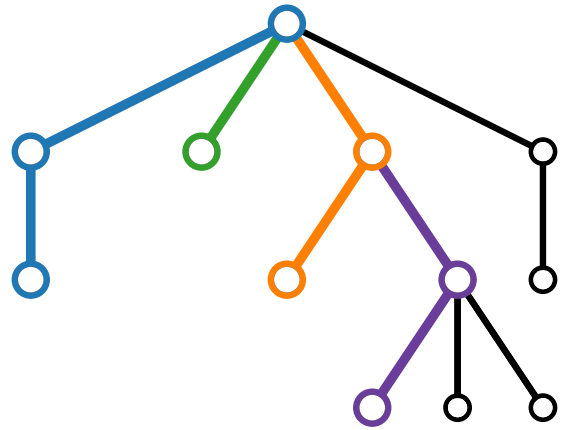

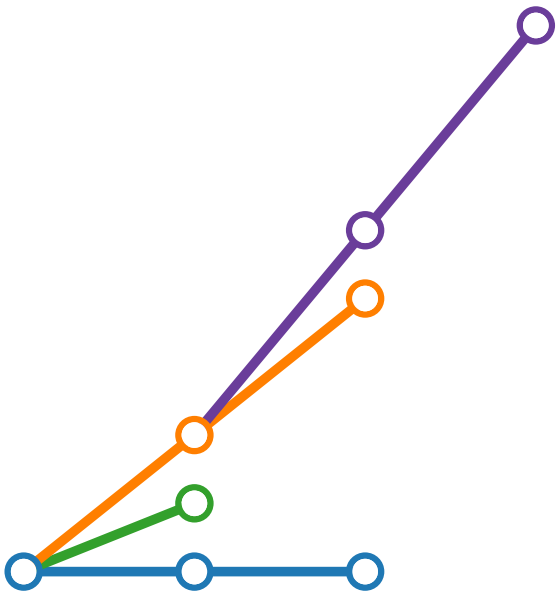

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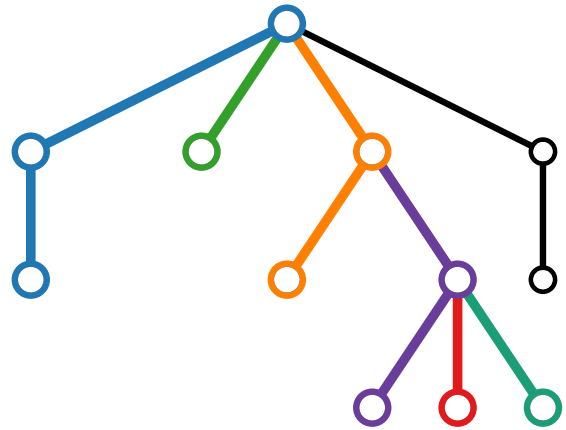

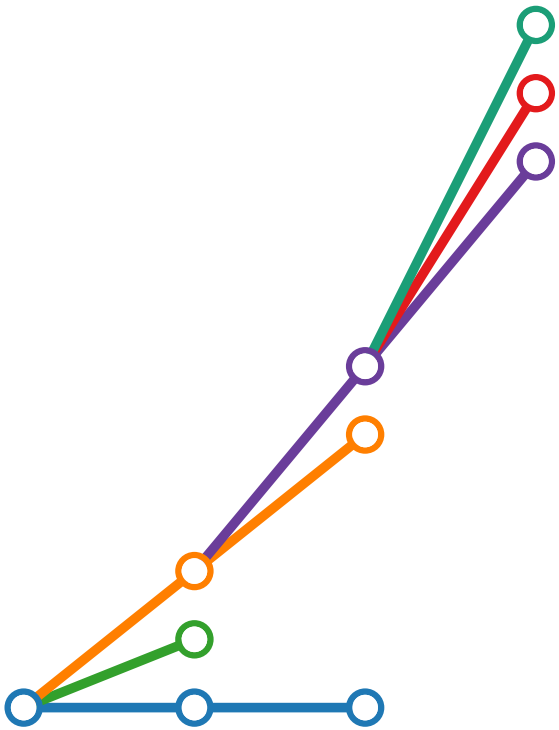

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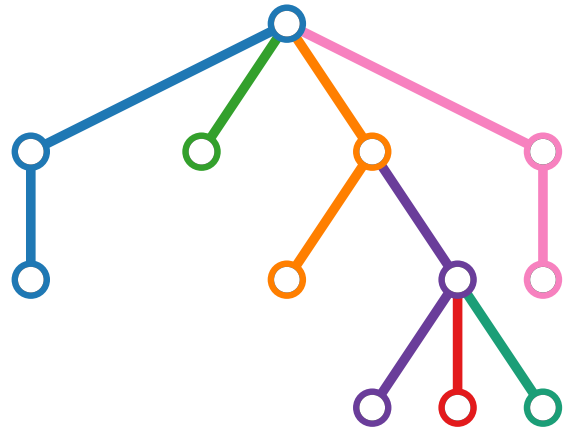
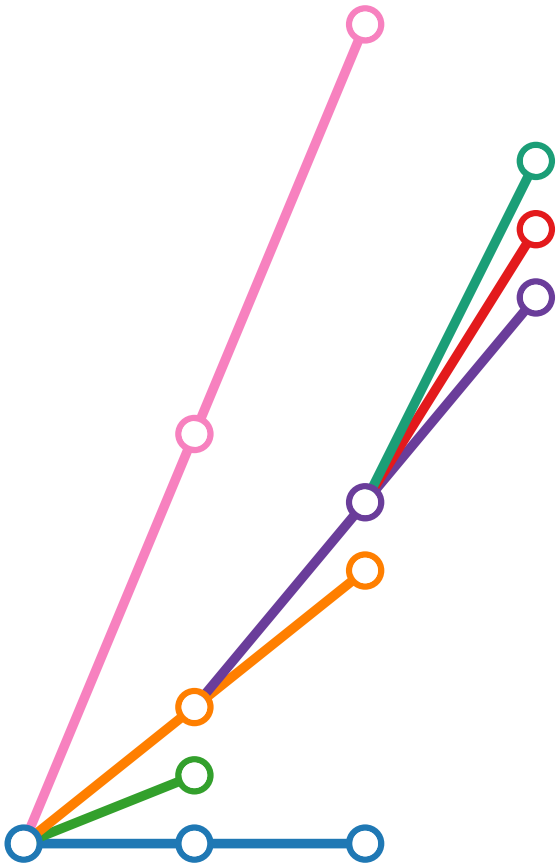
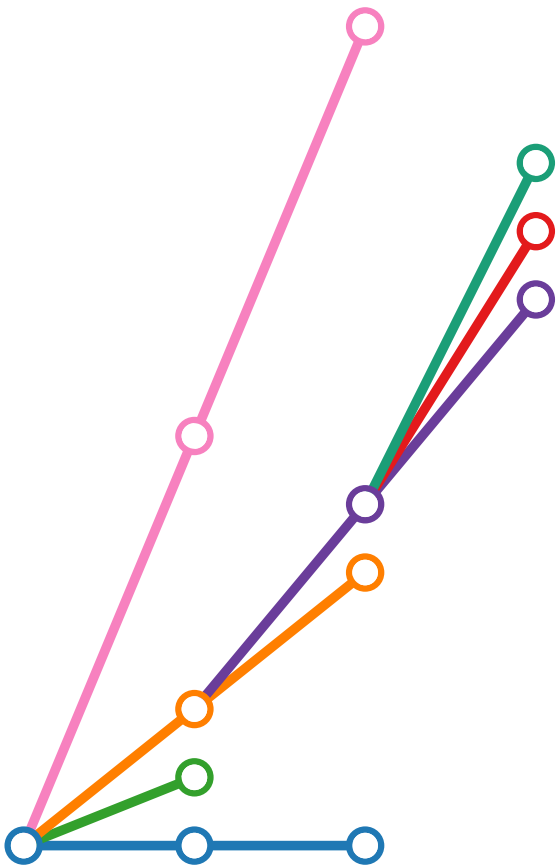


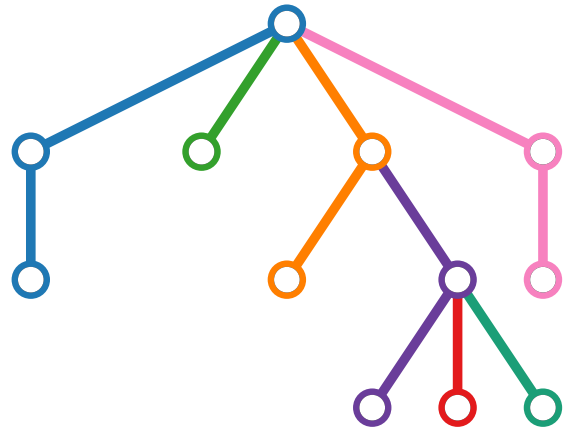


Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
otherwise use highest slope +1

highest slope: n

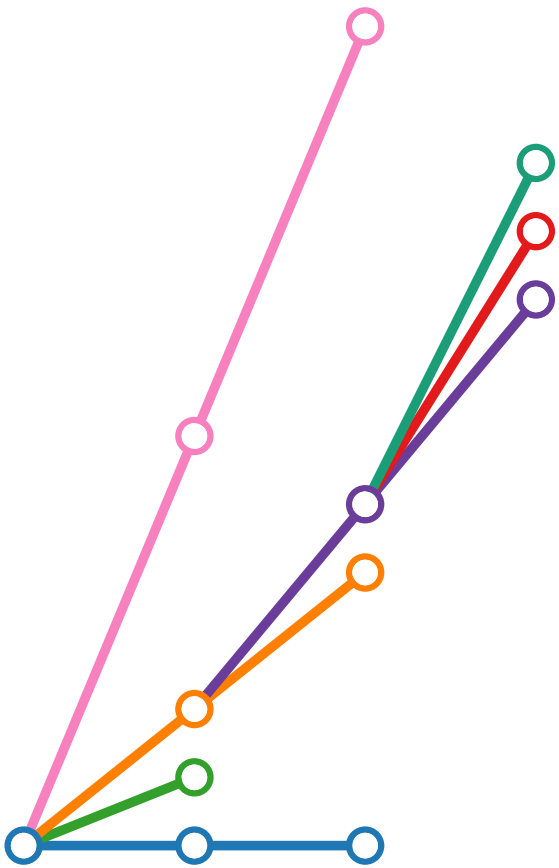


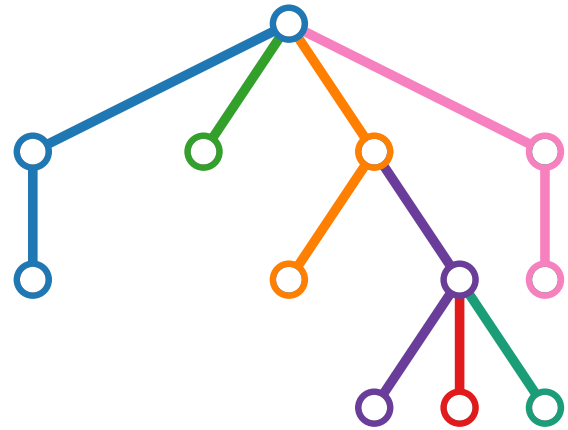


Obtaining a slope-disjoint drawing

ccw pre-order traversal
reuse slope whenever possible
otherwise use highest slope +1

highest slope: n
max. width: n





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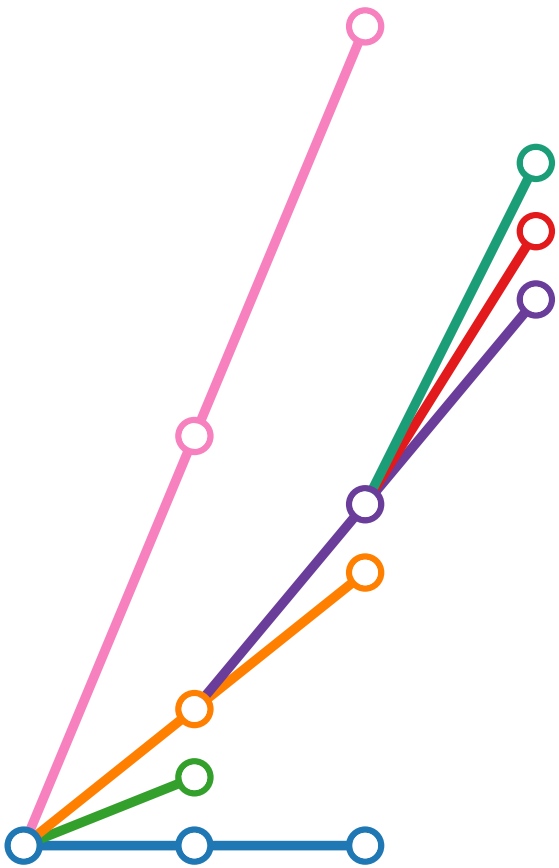
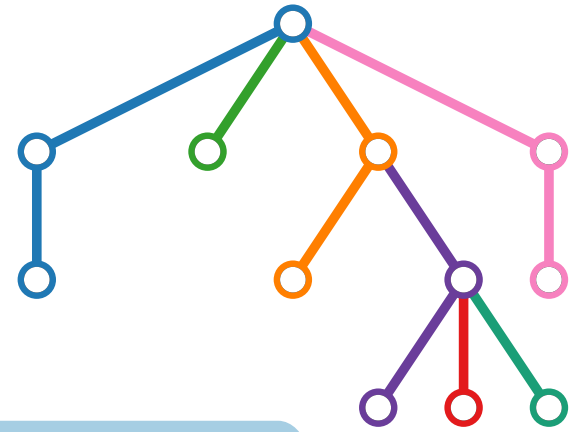


highest slope: n
max. width: n

$\Rightarrow n \times n^2$ grid



Obtaining a slope-disjoint drawing

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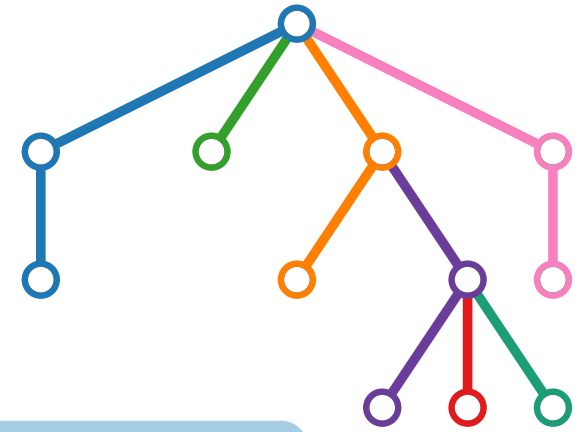

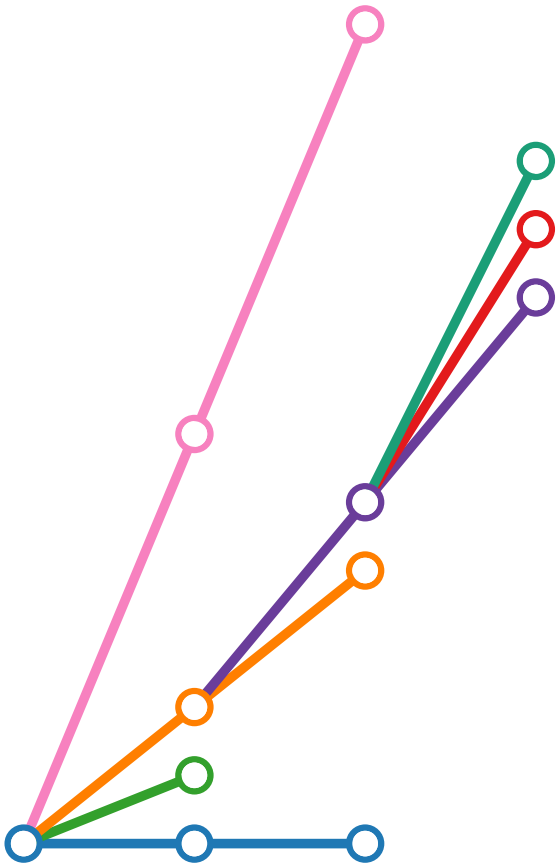

highest slope: n
max. width: n

$\Rightarrow n \times n^2$ grid , 1 segment per leaf

Obtaining a slope-disjoint drawing

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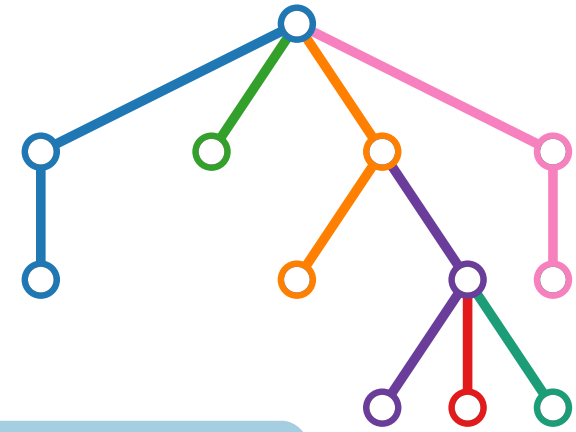

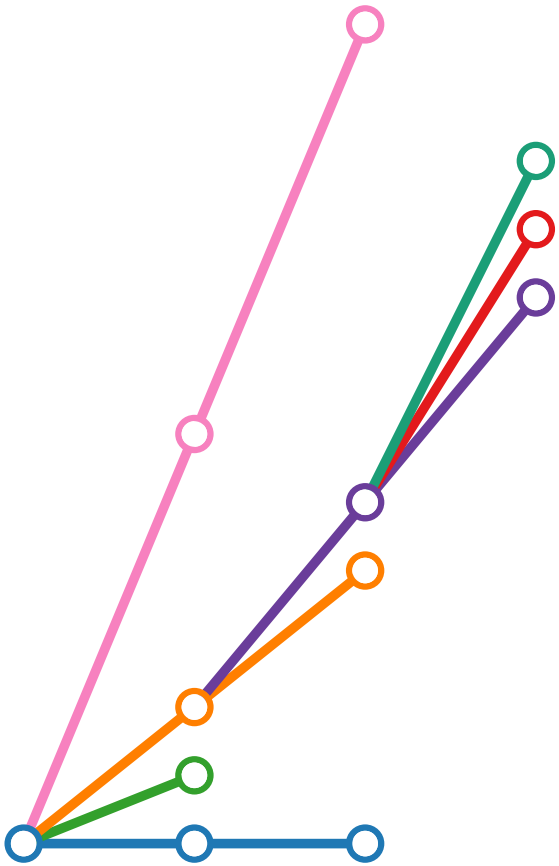

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[Miura, Azuma, Nishizeki '05]
Every Schnyder tree is an orderly spanning tree

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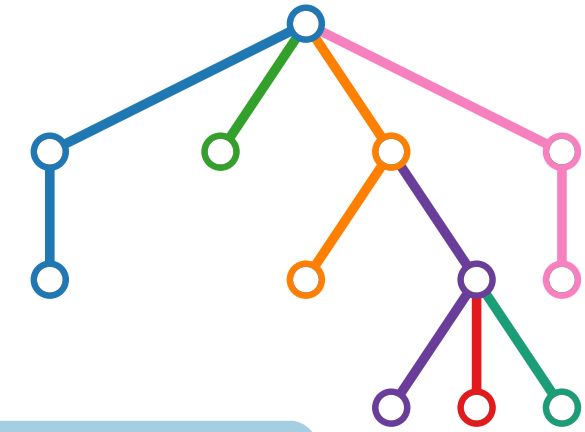

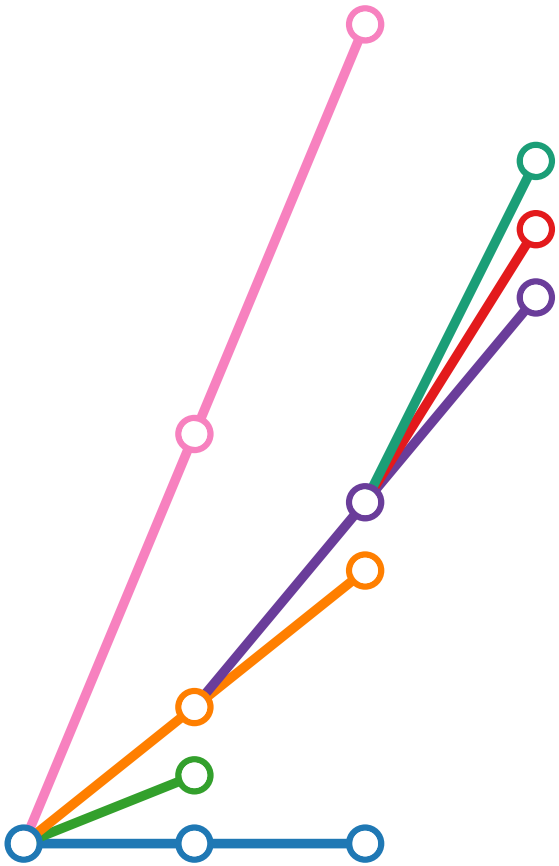

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T_1, T_2, T_3 Schnyder realizer of 3-conn. planar graph
$\Rightarrow \leq 2n + 1$ leaves in total in T_1, T_2, T_3

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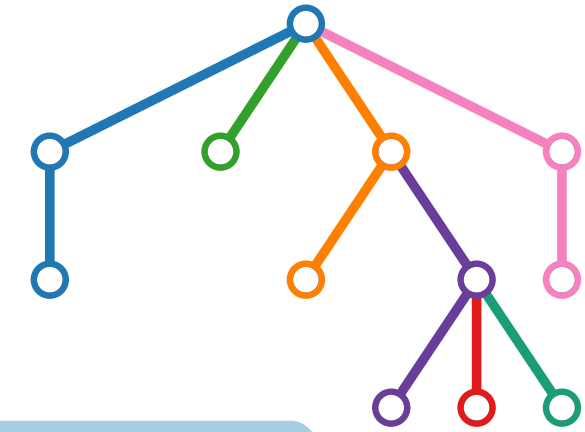

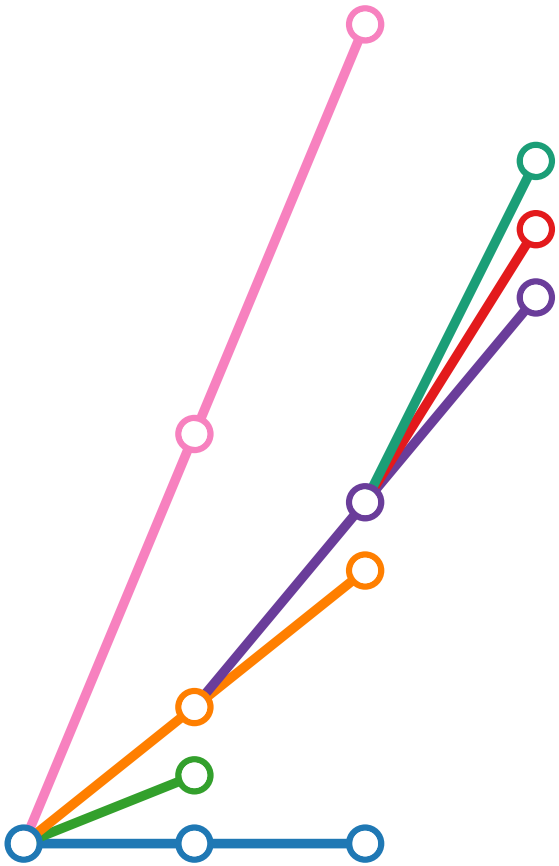

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3-conn. planar graph
$\Rightarrow (8n-14)/3$ segments, $O(n) \times O(n^2)$ grid

New Results

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$	$n \times n$
			$\vartheta/2$ [6]	quasipolynomial
outerplanar	n [1]			
max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [6]	$O(n) \times O(n^2)$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]		
planar	$2n$ [4]	$8n/3$ [4]		

[1] Dujmović et al. 2007 [2] Igamberdiev et al. 2015 [3] Mondal et al. 2013
[4] Durocher & Mondal 2014 [5] Mondal 2016 [6] Hültenschmidt et al. 2017

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max. outerp.	n [1]	n [1]	$3n/2$ [6]	$O(n) \times O(n^2)$
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$ [6]	$O(n) \times O(n^2)$
2-connected	$2n$ [1]			
3-connected	$2n$ [1]	$5n/2$ [1]	$8n/3$	$O(n) \times O(n^2)$
cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
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cubic 3-conn.	$n/2$ [3]	$n/2$ [2]	$n/2$ [2]	$O(n) \times O(n)$
triangulation	$2n$ [4]	$7n/3$ [4]	$8n/3$ [6]	$O(n) \times O(n^2)$
4-conn. triang.	$2n$ [4]	$9n/4$ [4]	$5n/2$	$O(n) \times O(n^2)$
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