

Integral Unit

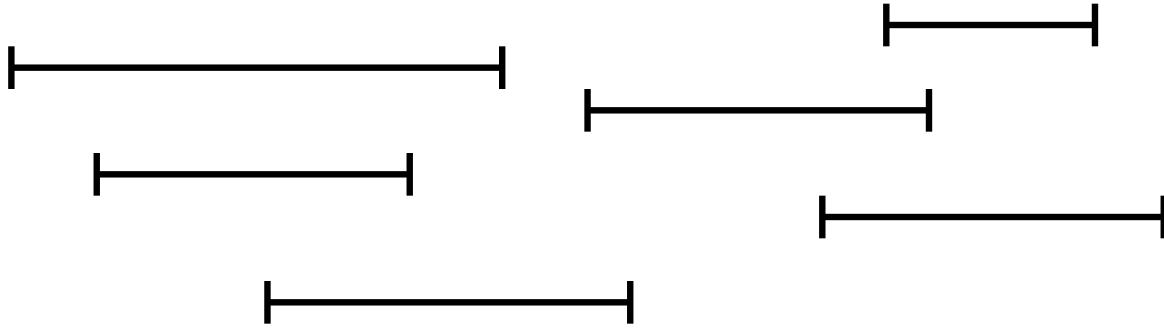
Bar-Visibility Graphs

Therese Biedl Ahmad Biniiaz
Veronika Irvine Philipp Kindermann
Anurag Murty Naredla Alexi Turcotte

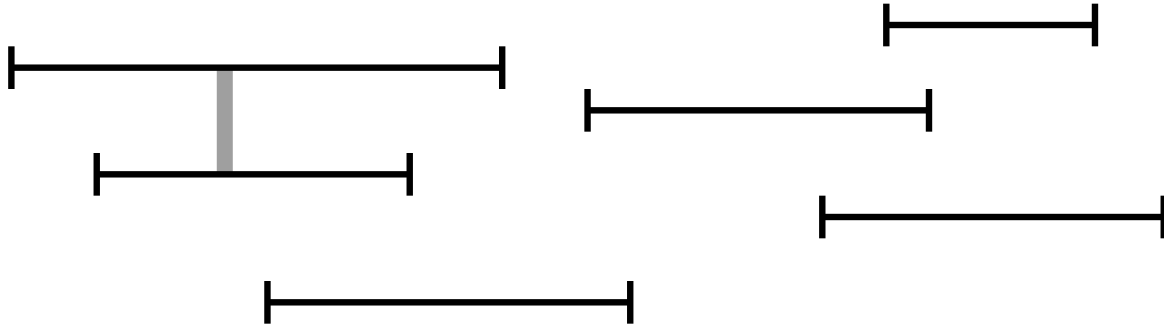


UNIVERSITY OF WATERLOO
FACULTY OF MATHEMATICS
David R. Cheriton School
of Computer Science

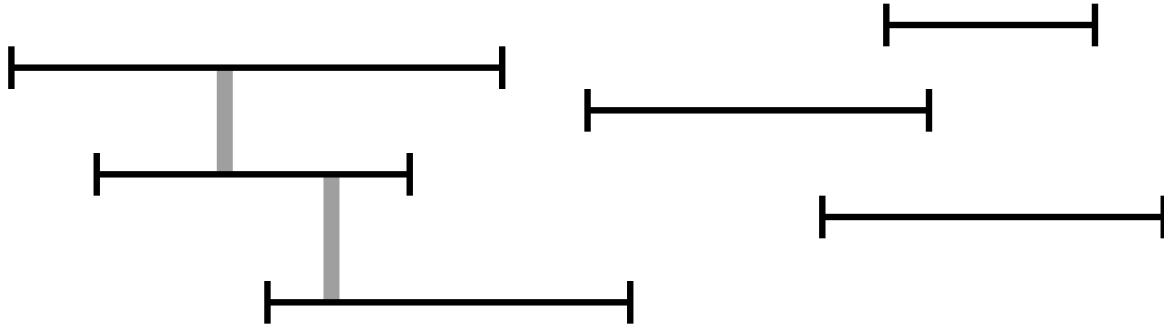
Bar-Visibility



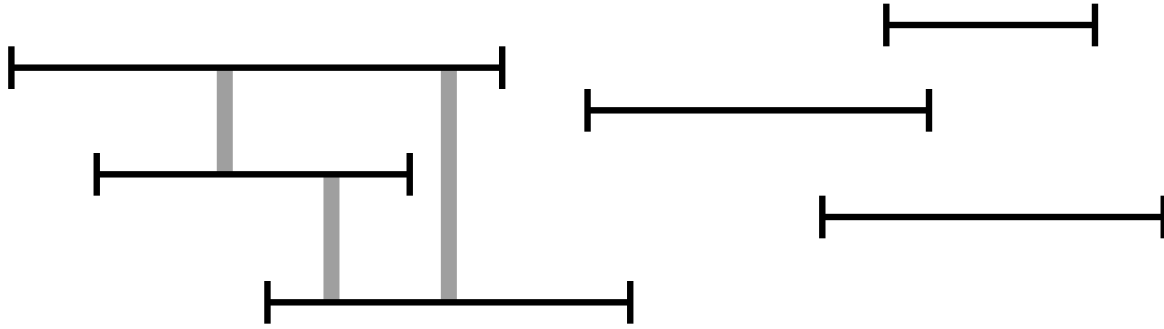
Bar-Visibility



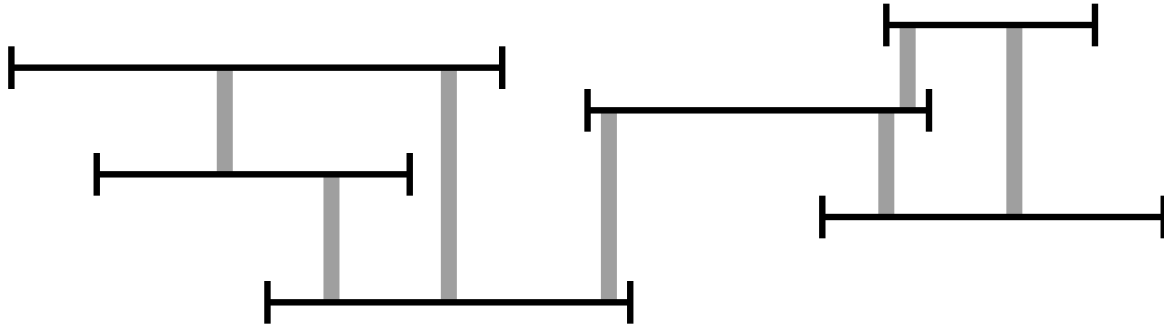
Bar-Visibility



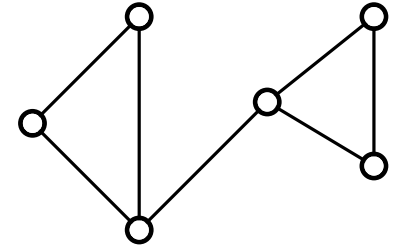
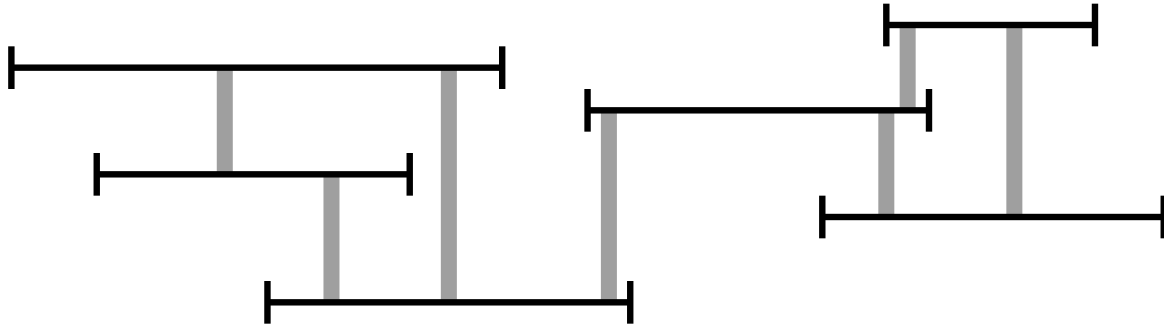
Bar-Visibility



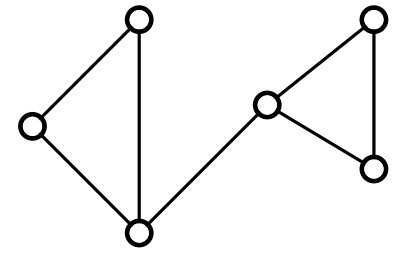
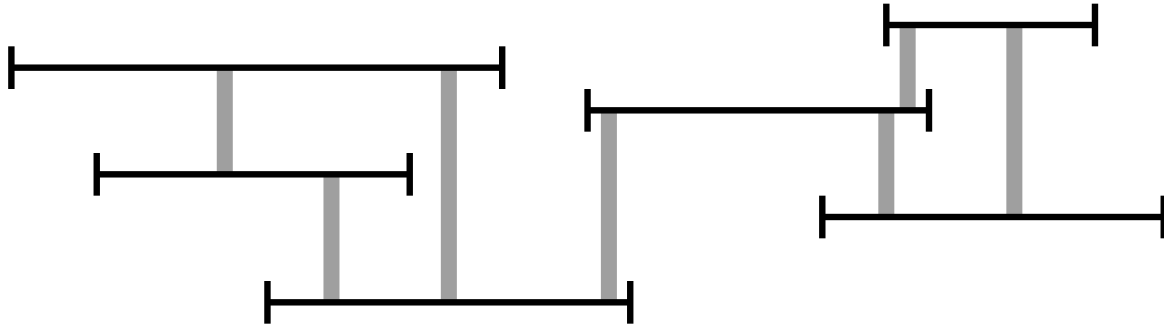
Bar-Visibility



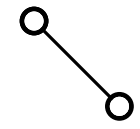
Bar-Visibility



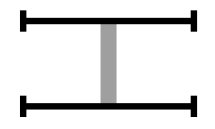
Bar-Visibility



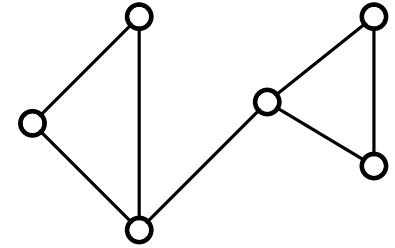
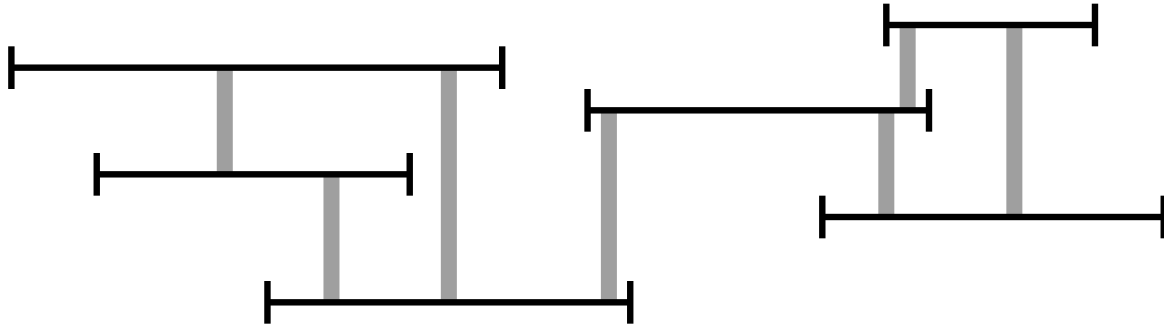
Strong



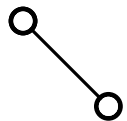
\Leftrightarrow



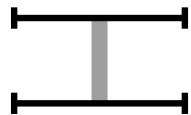
Bar-Visibility



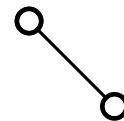
Weak



\Rightarrow



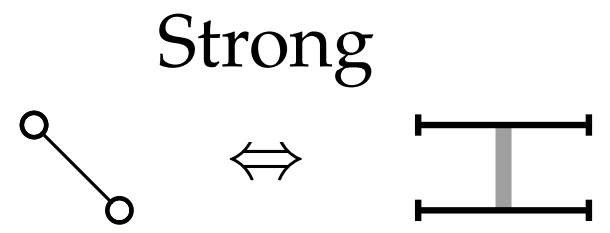
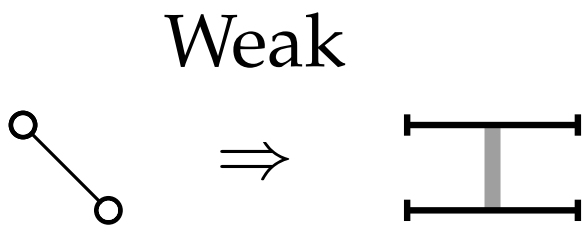
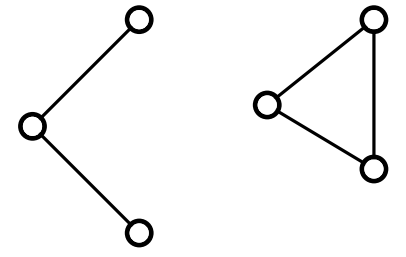
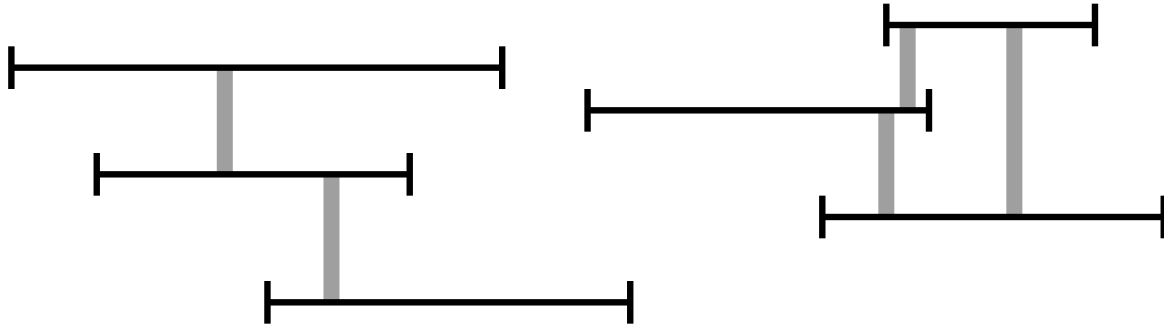
Strong



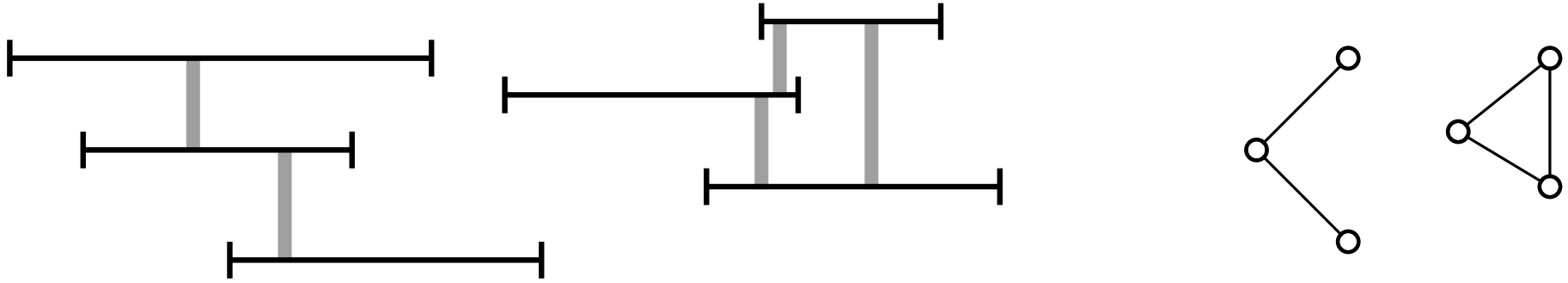
\Leftrightarrow



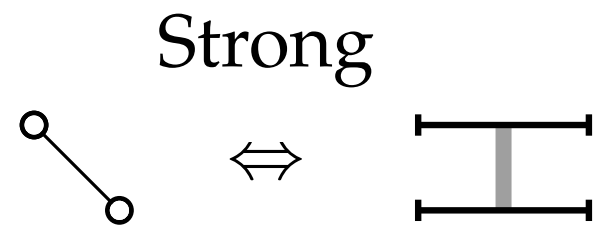
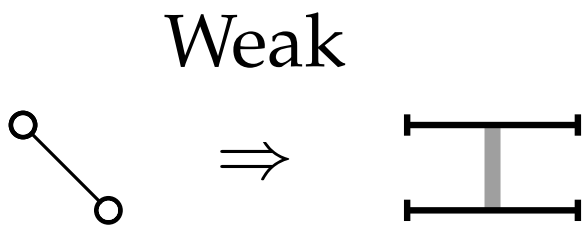
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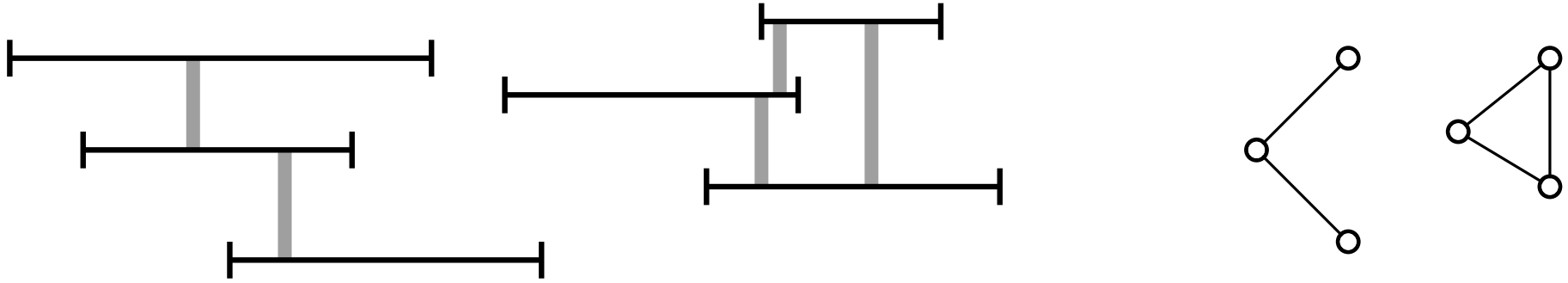
Bar-Visibility



Every planar graph,
 $O(n)$ time

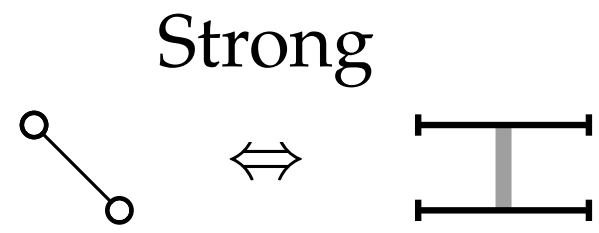
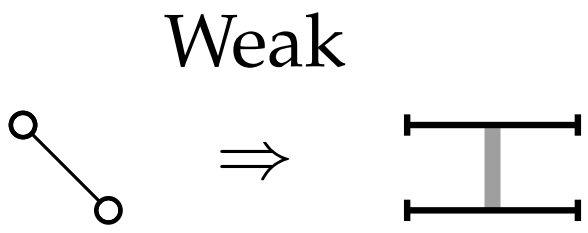


Bar-Visibility

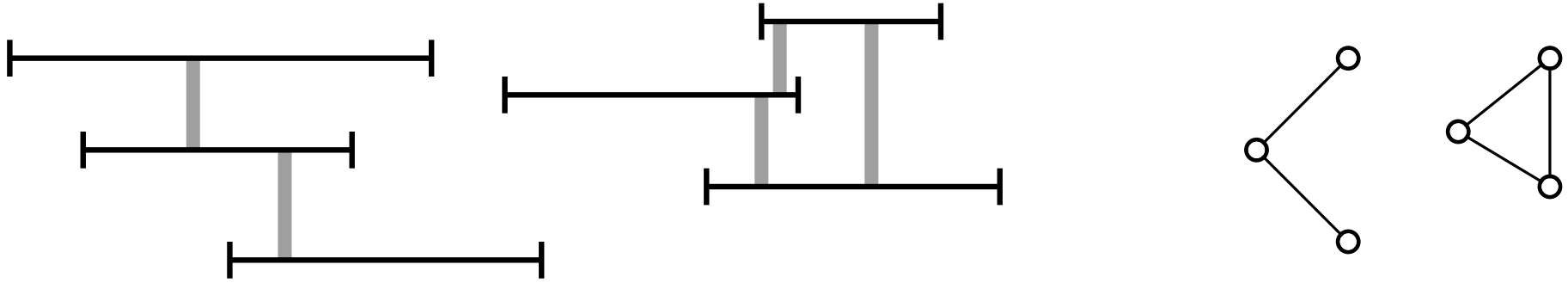


Every planar graph,
 $O(n)$ time

3-connected: NP-hard

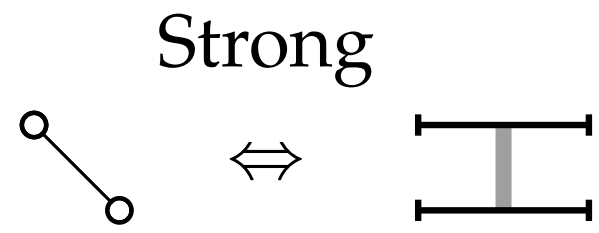
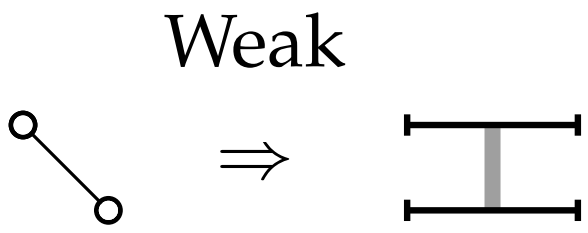


Bar-Visibility

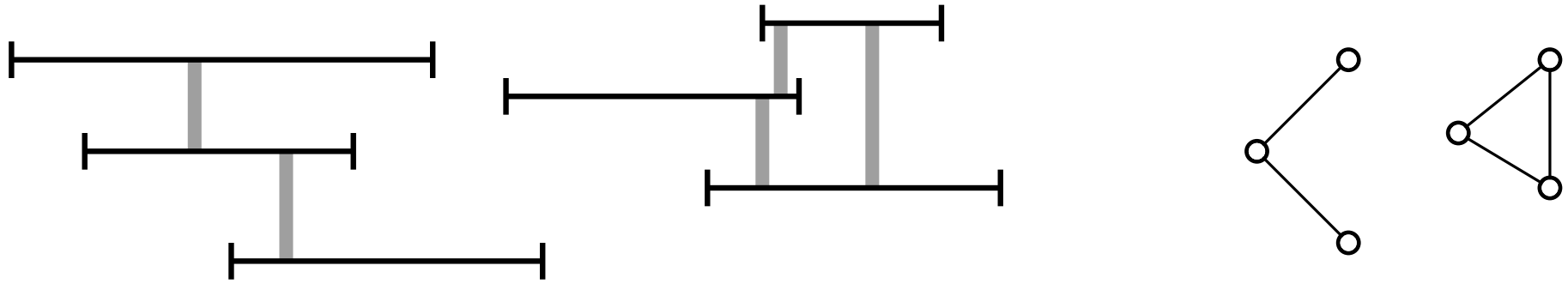


Every planar graph,
 $O(n)$ time

3-connected: NP-hard
Triangulation: $O(n)$ time

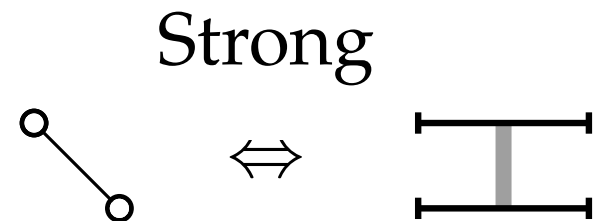
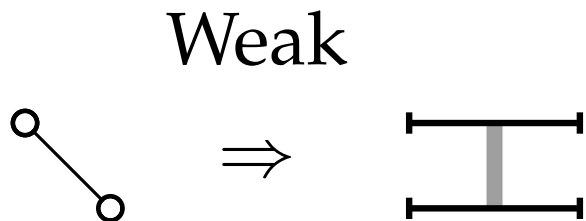


Bar-Visibility

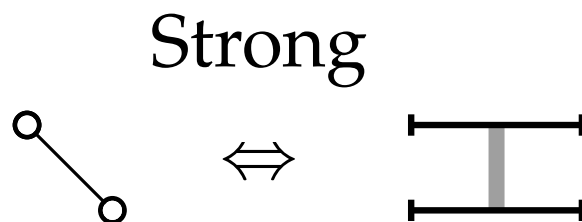
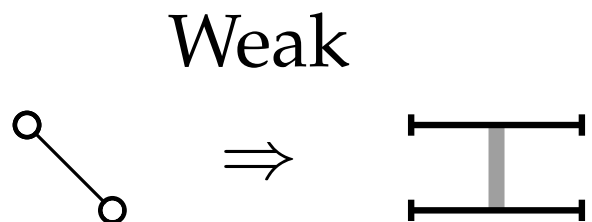
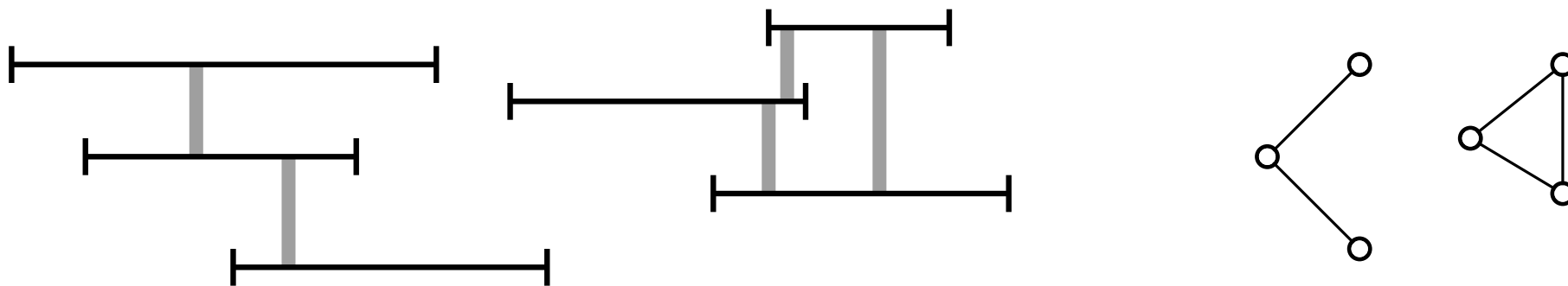


Every planar graph,
 $O(n)$ time

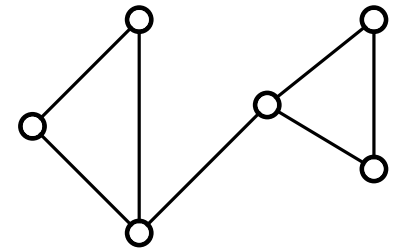
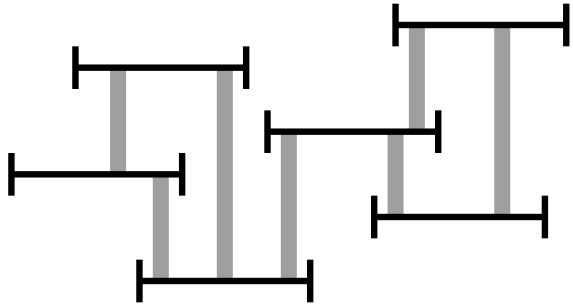
3-connected: NP-hard
Triangulation: $O(n)$ time
4-connected: $O(n)$ time



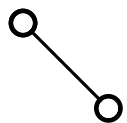
Unit Bar-Visibility



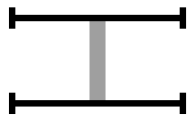
Unit Bar-Visibility



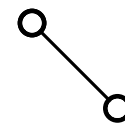
Weak



\Rightarrow



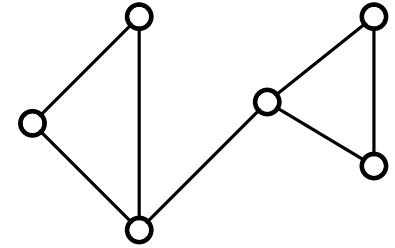
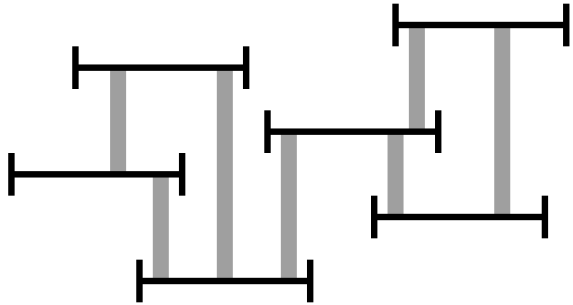
Strong



\Leftrightarrow

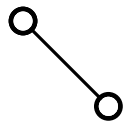


Unit Bar-Visibility



Characterizations for...

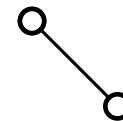
Weak



\Rightarrow



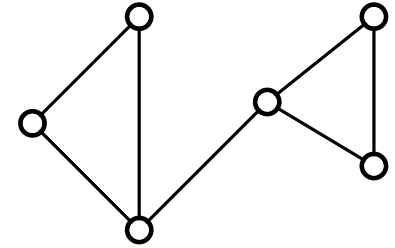
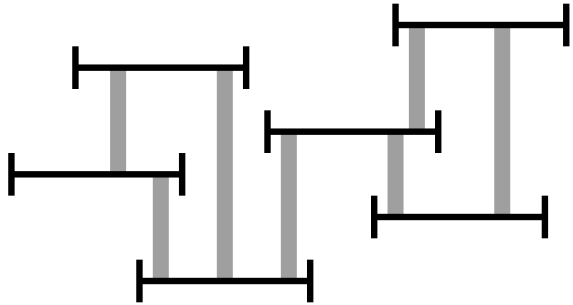
Strong



\Leftrightarrow



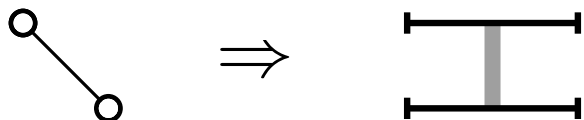
Unit Bar-Visibility



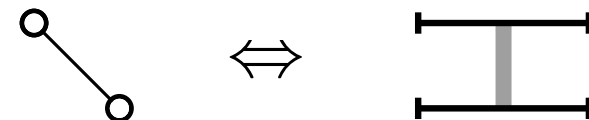
Characterizations for...

- trees

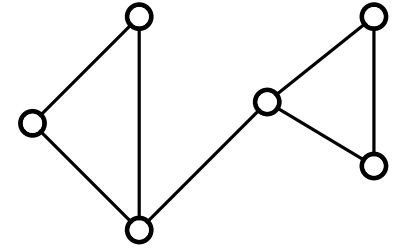
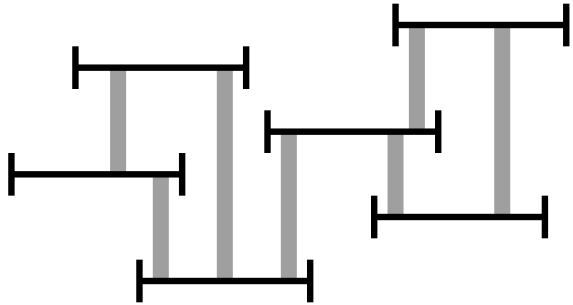
Weak



Strong



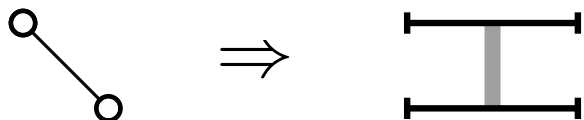
Unit Bar-Visibility



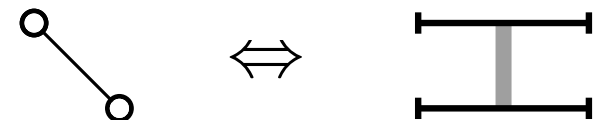
Characterizations for...

- trees
- complete bipartite

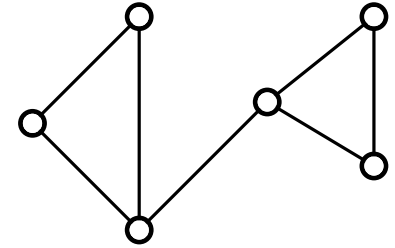
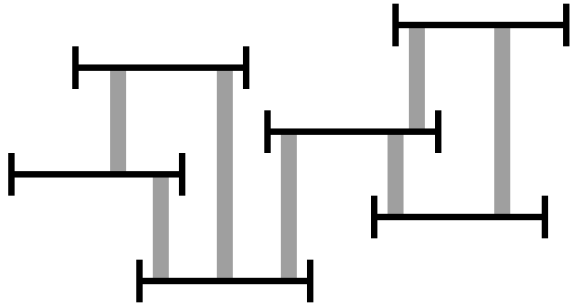
Weak



Strong



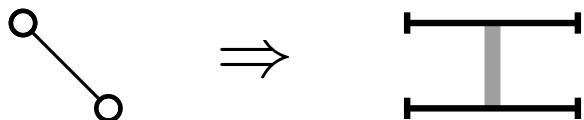
Unit Bar-Visibility



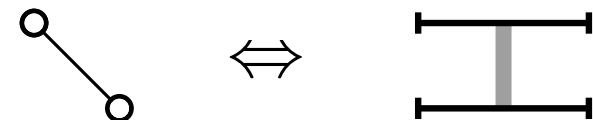
Characterizations for...

- trees
- complete bipartite
- outerplanar

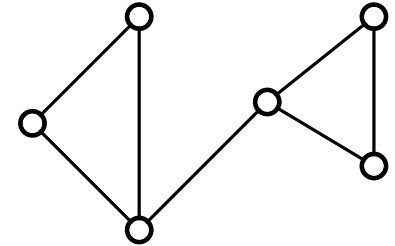
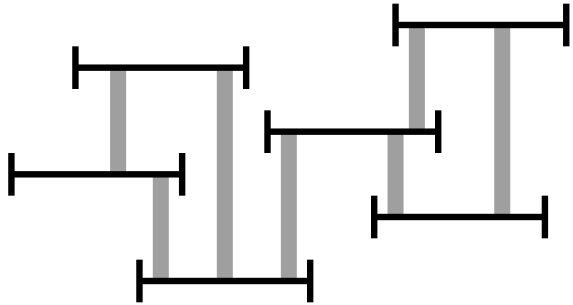
Weak



Strong



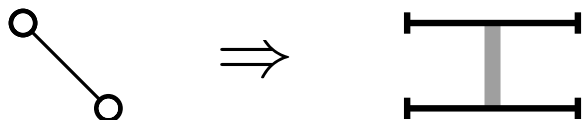
Unit Bar-Visibility



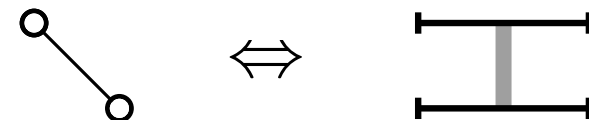
Characterizations for...

- trees
- complete bipartite
- outerplanar
- triangulations

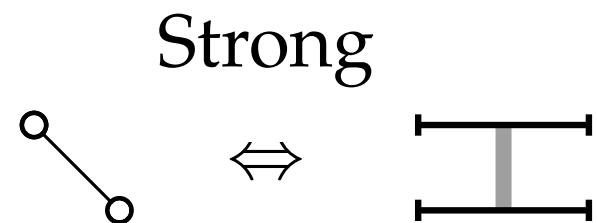
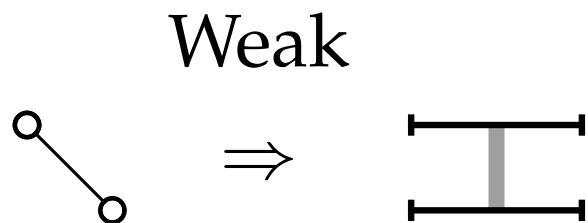
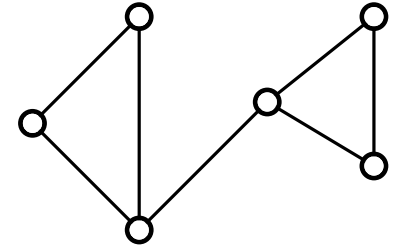
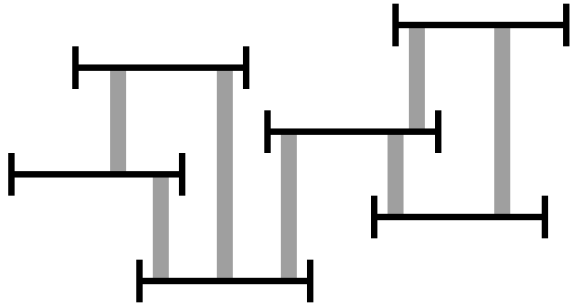
Weak



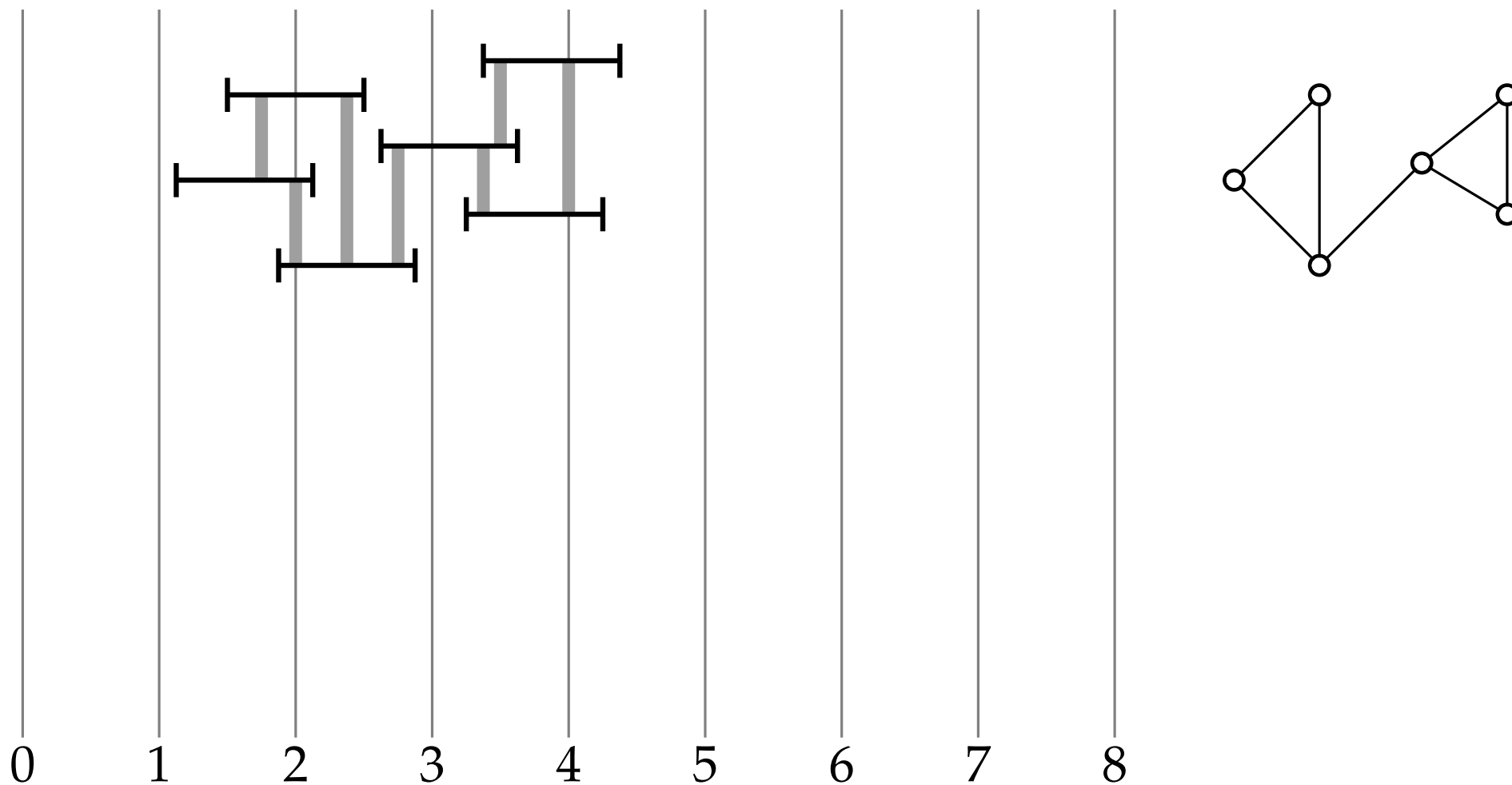
Strong



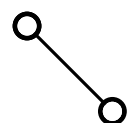
Integral Unit Bar-Visibility



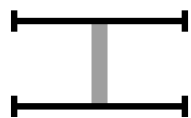
Integral Unit Bar-Visibility



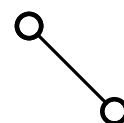
Weak



\Rightarrow



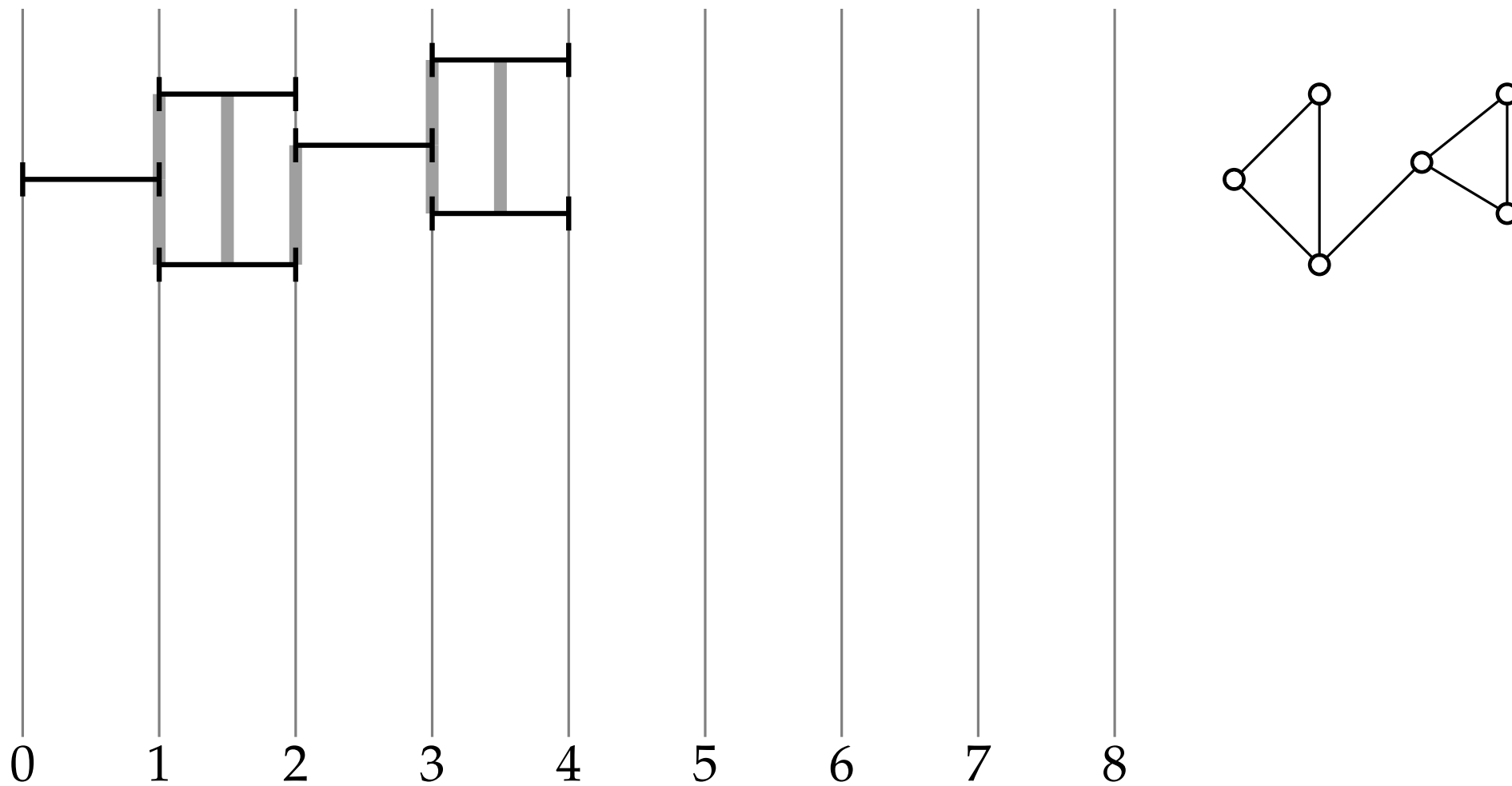
Strong



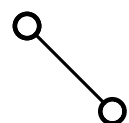
\Leftrightarrow



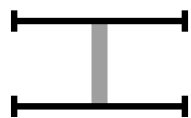
Integral Unit Bar-Visibility



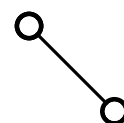
Weak



\Rightarrow



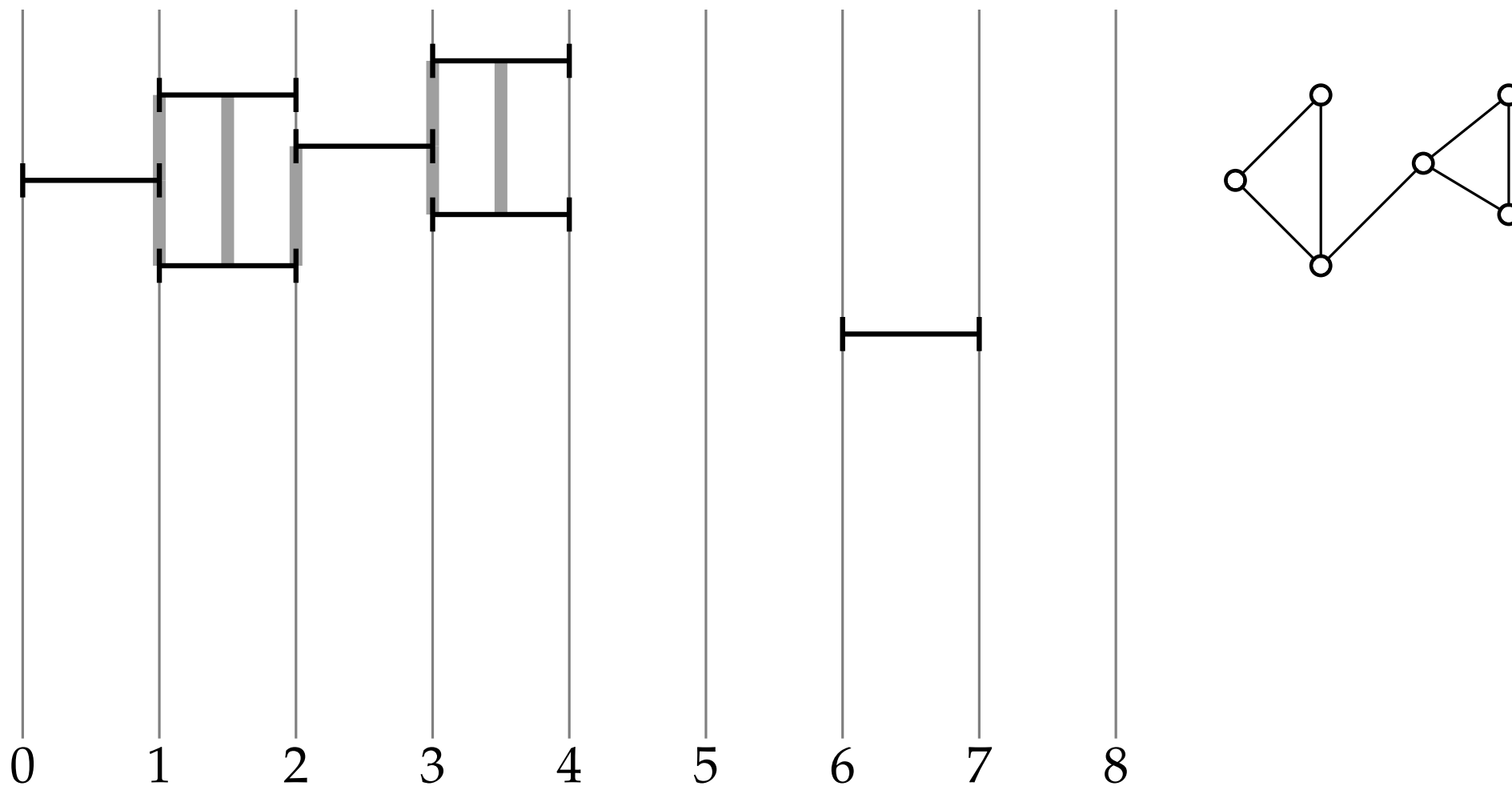
Strong



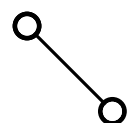
\Leftrightarrow



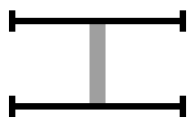
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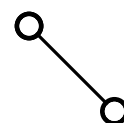
Weak



\Rightarrow



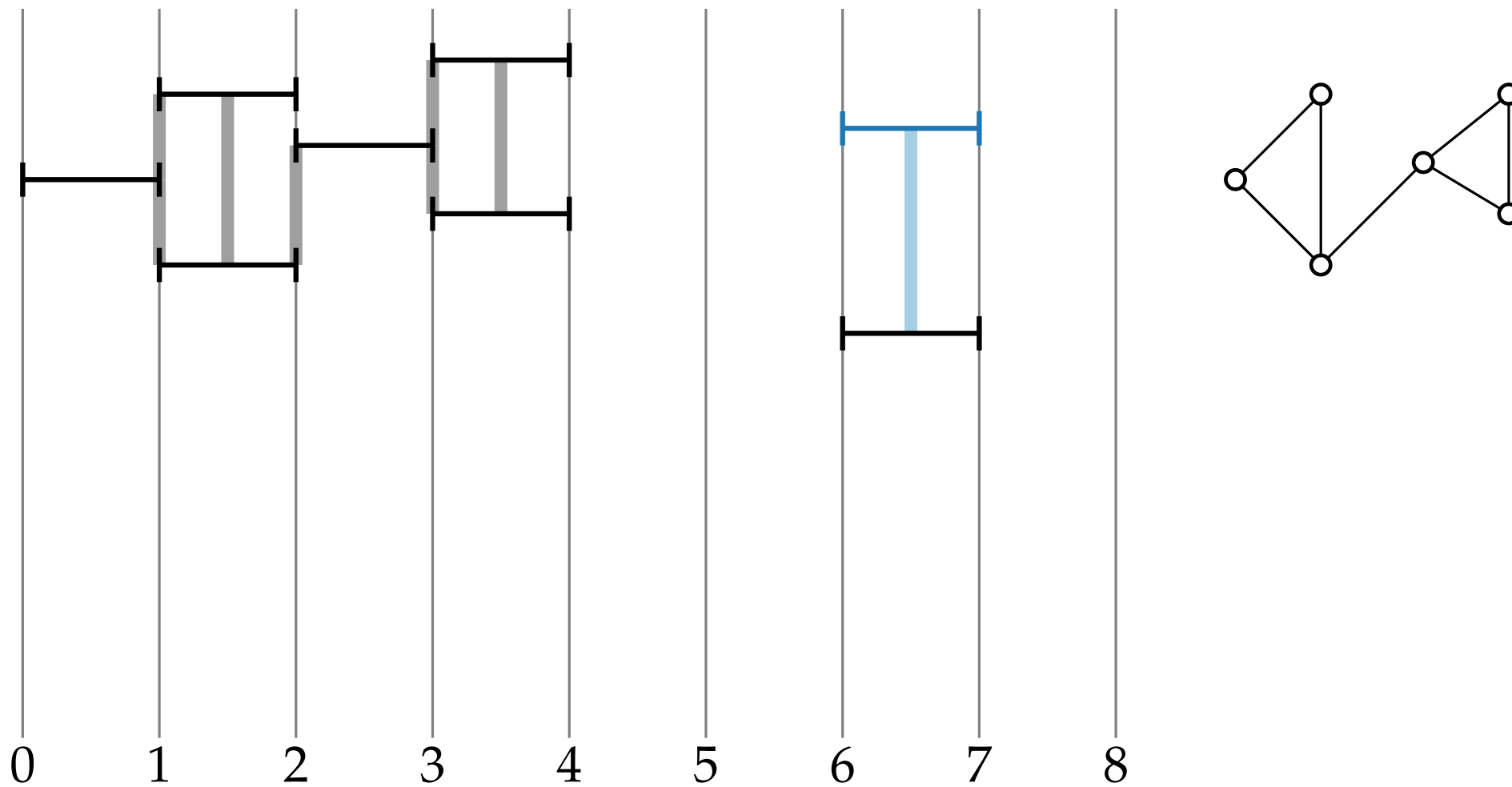
Strong



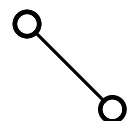
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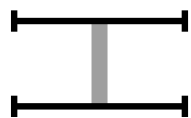
Integral Unit Bar-Visibility



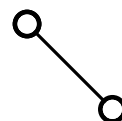
Weak



\Rightarrow



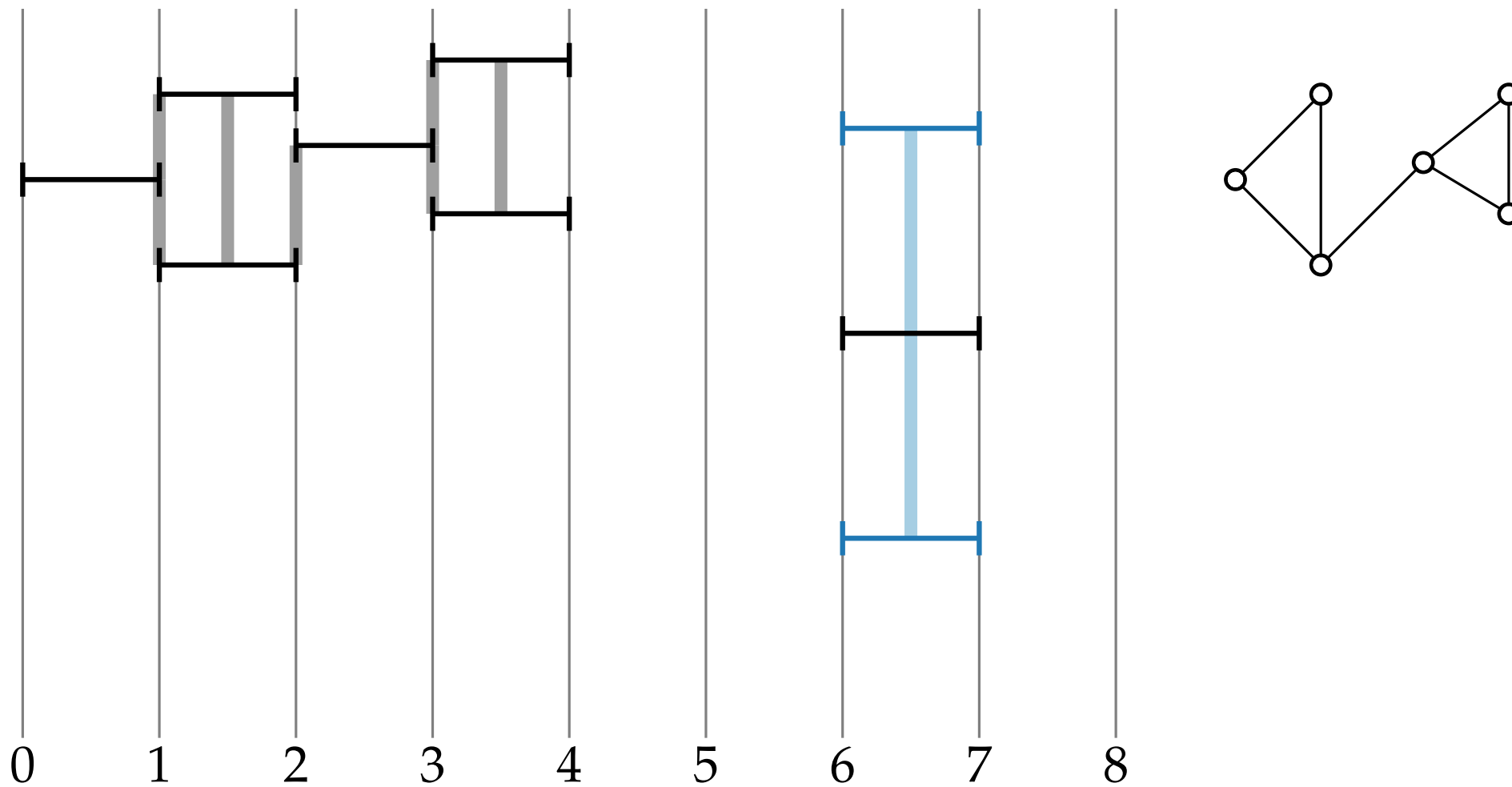
Strong



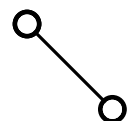
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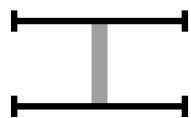
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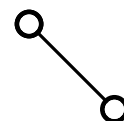
Weak



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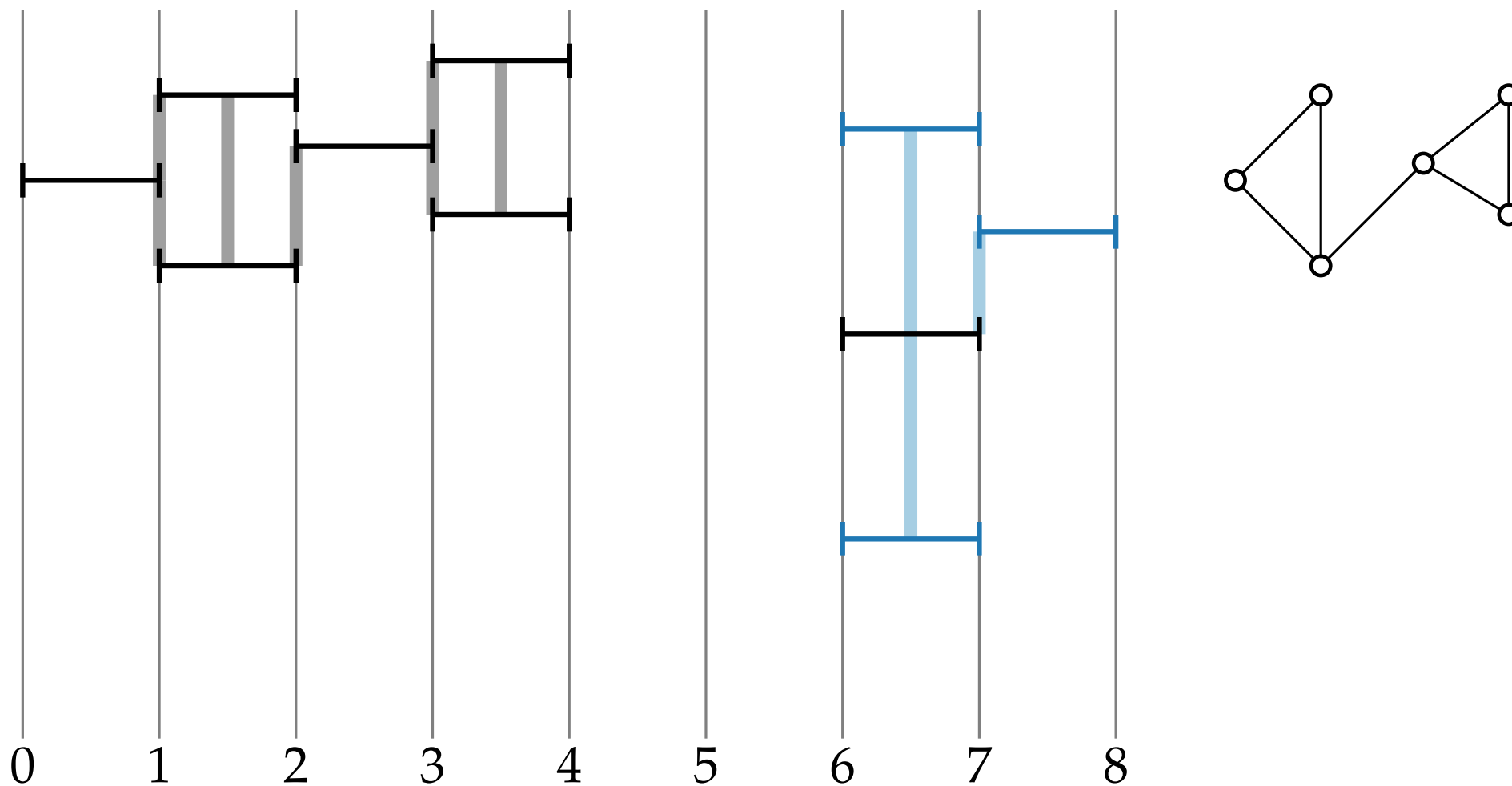
Strong



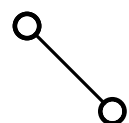
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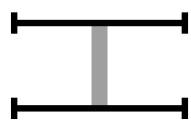
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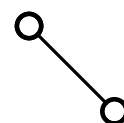
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\Rightarrow



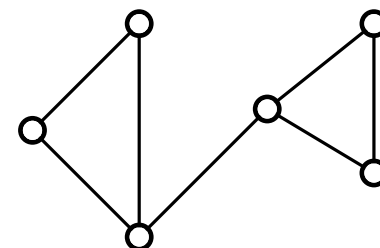
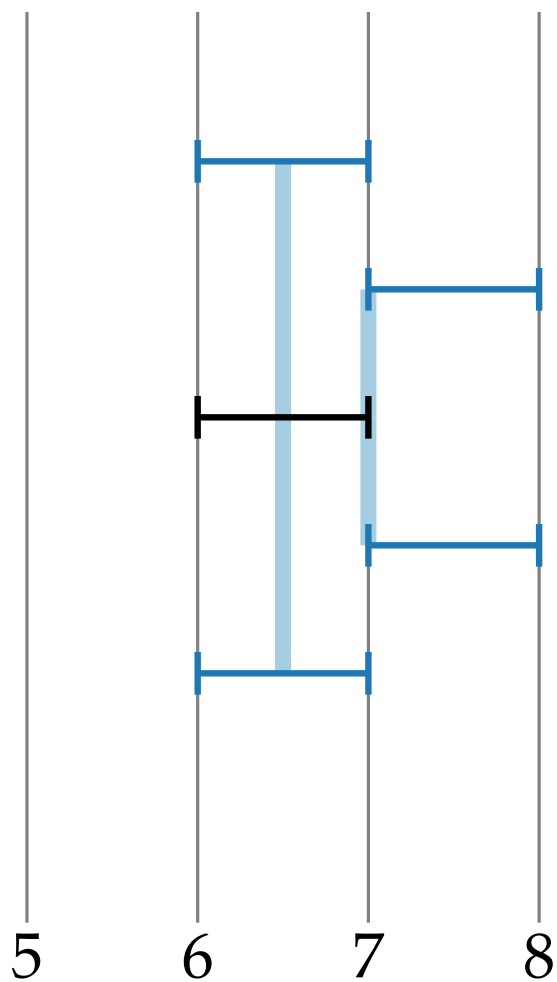
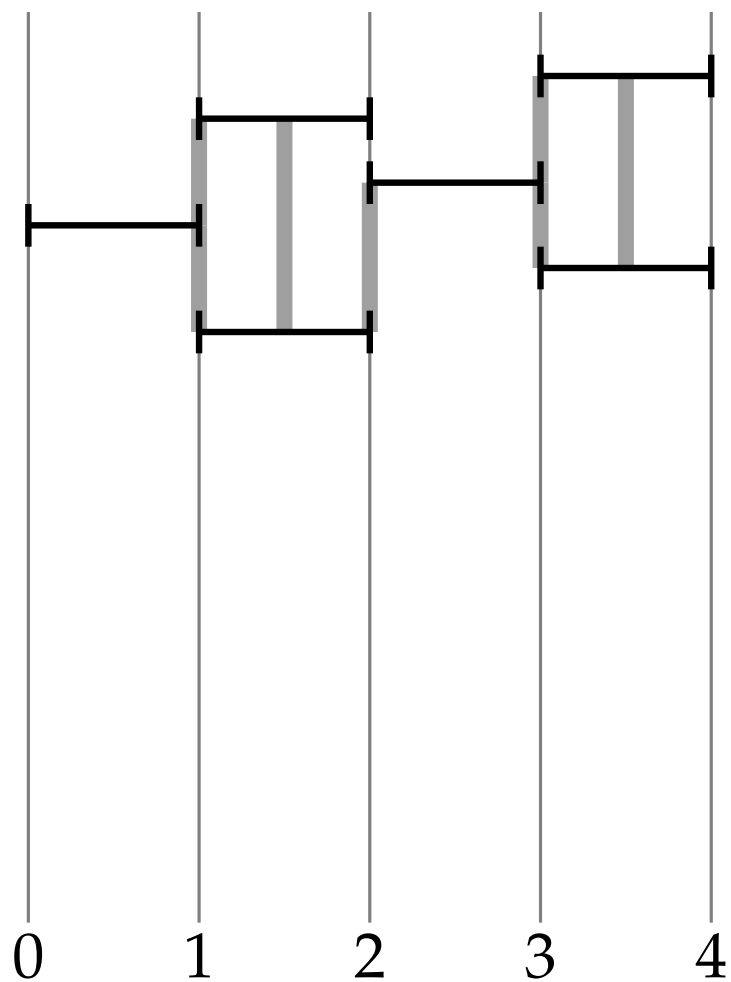
Strong



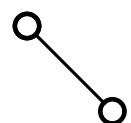
\Leftrightarrow



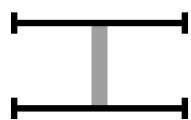
Integral Unit Bar-Visibility



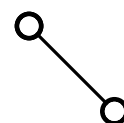
Weak



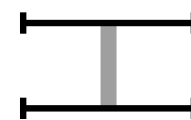
\Rightarrow



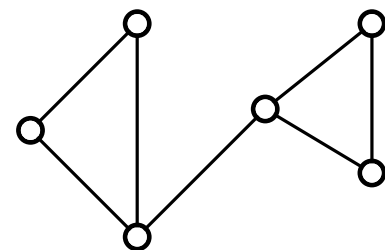
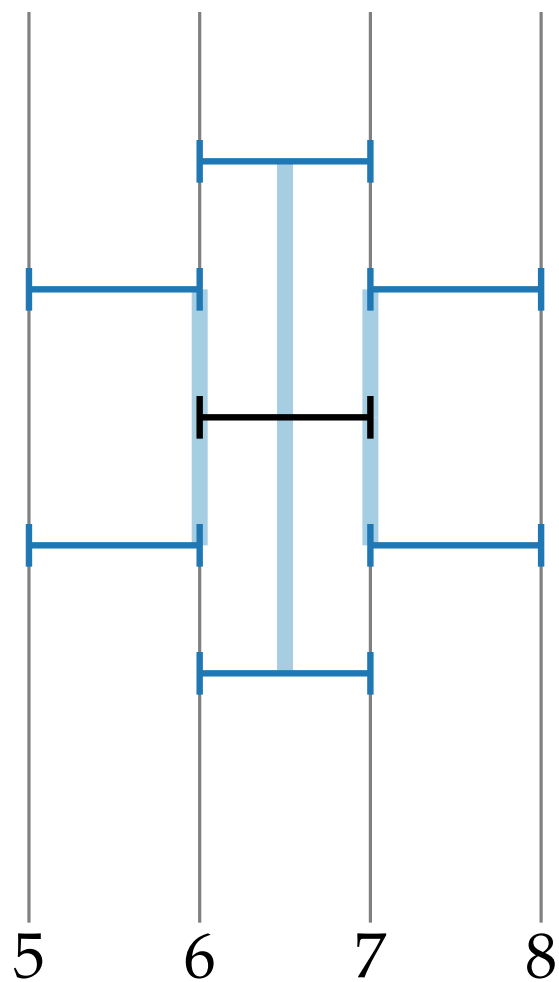
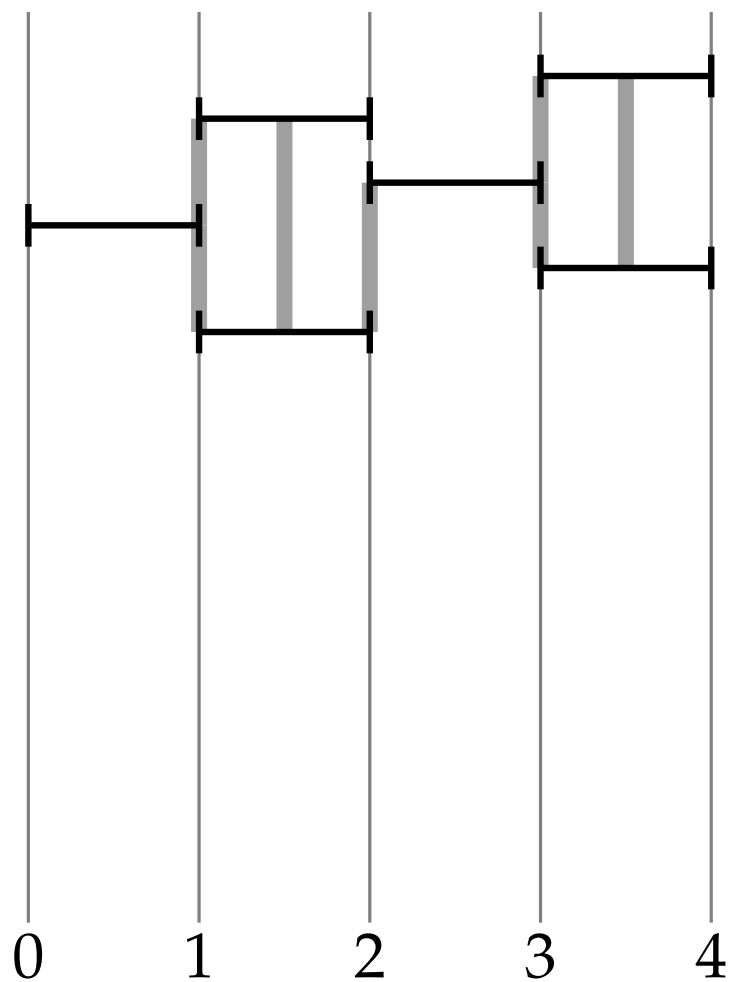
Strong



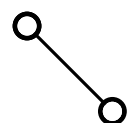
\Leftrightarrow



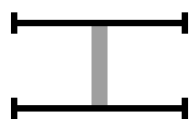
Integral Unit Bar-Visibility



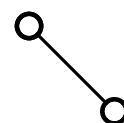
Weak



\Rightarrow



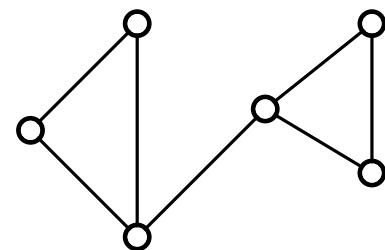
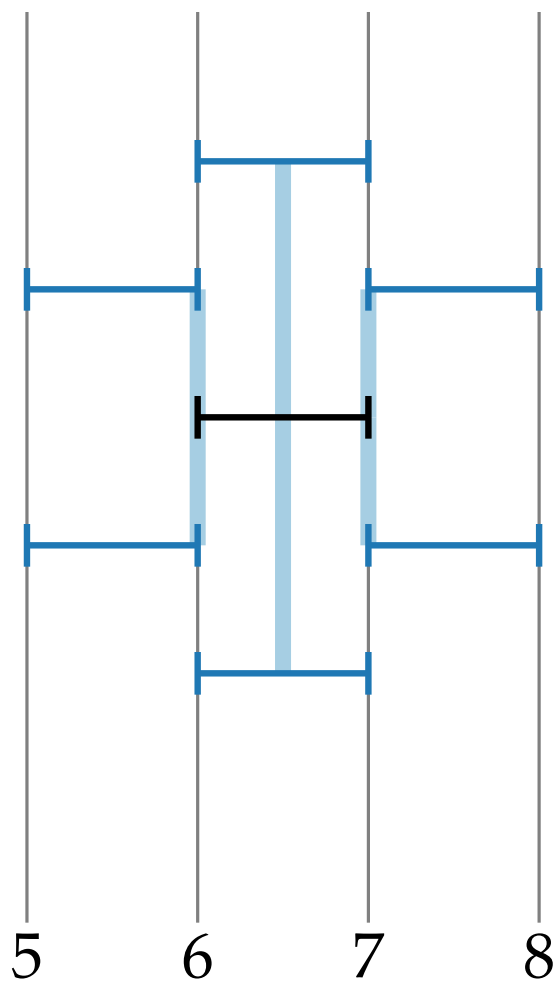
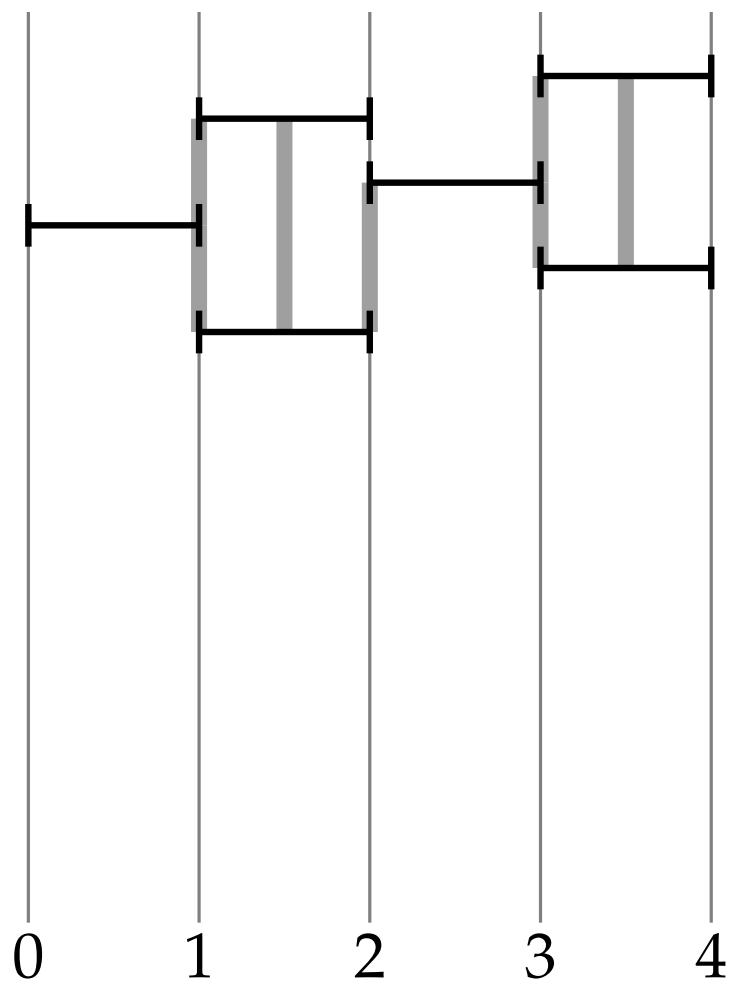
Strong



\Leftrightarrow

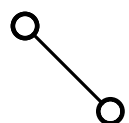


Integral Unit Bar-Visibility

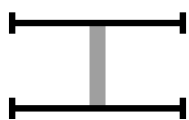


max-deg 6

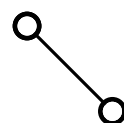
Weak



\Rightarrow



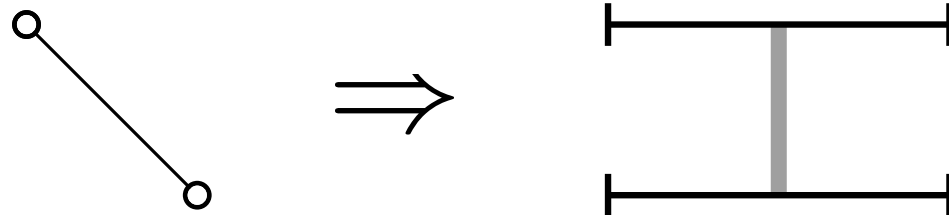
Strong



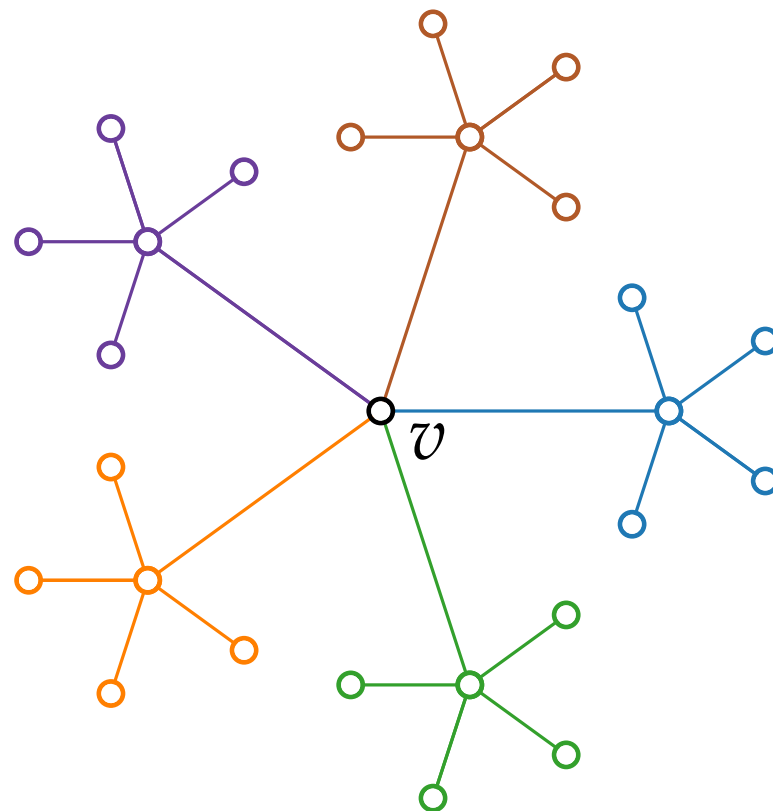
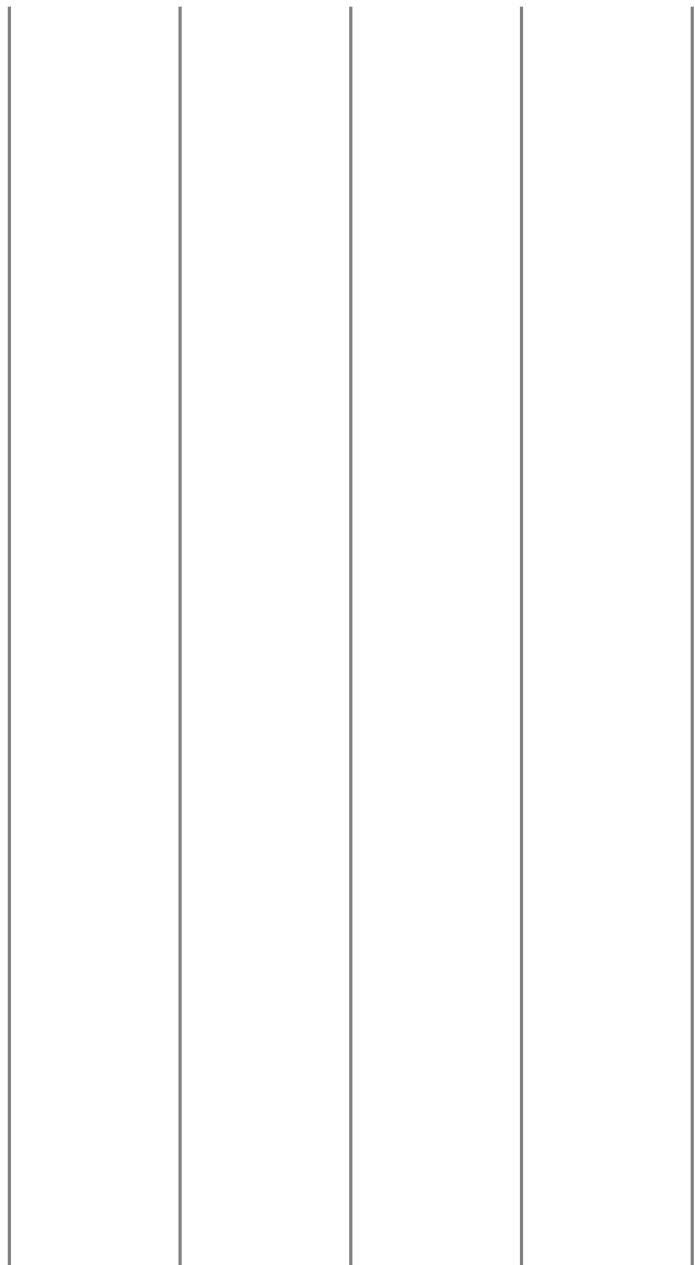
\Leftrightarrow



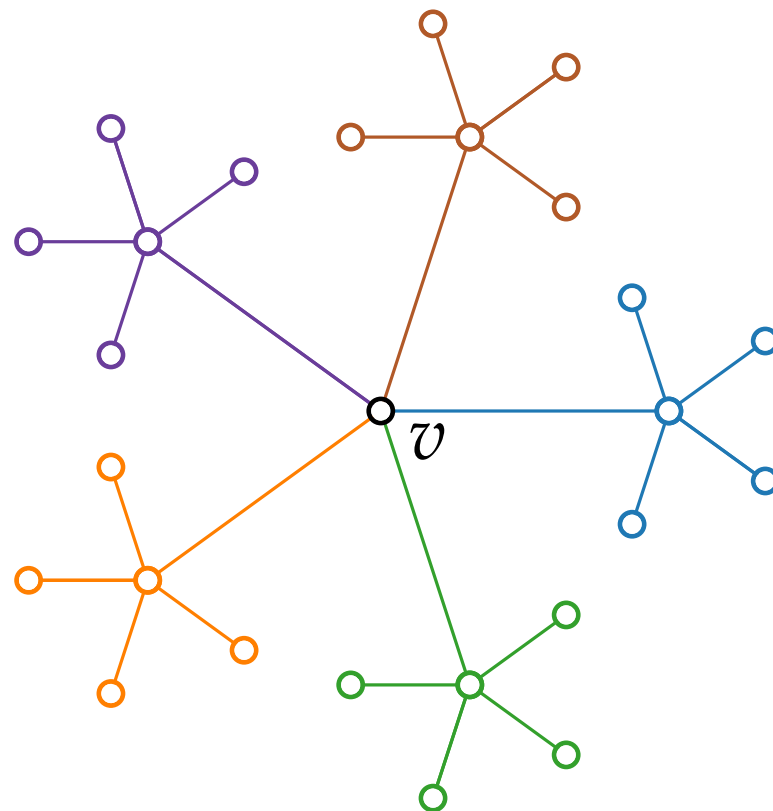
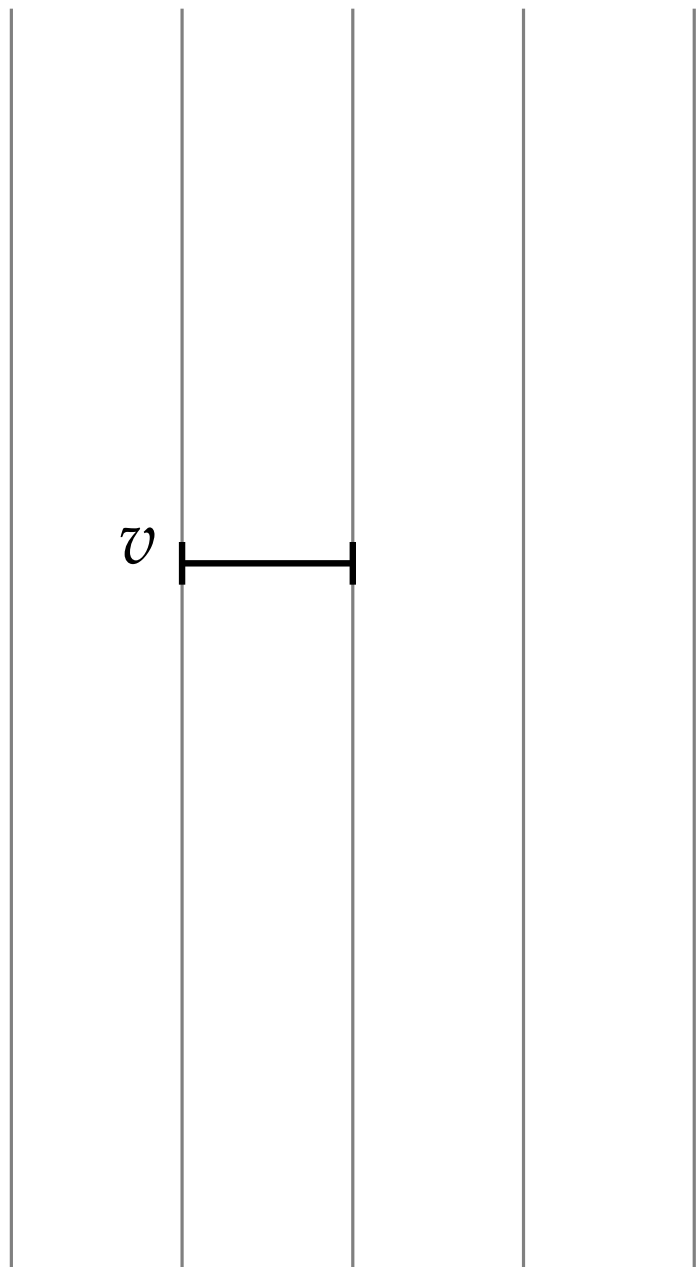
Weak Model



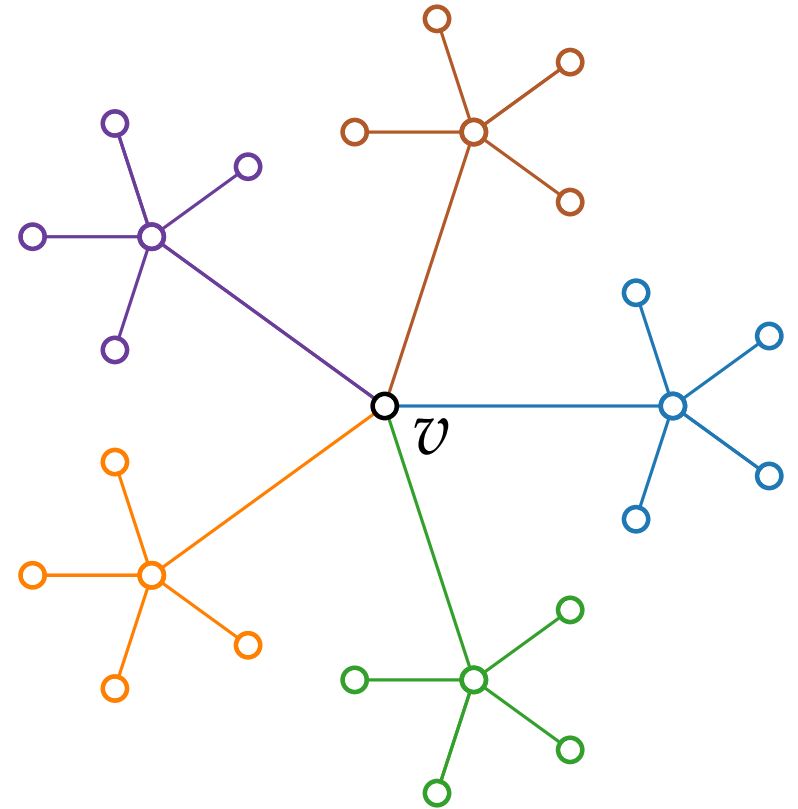
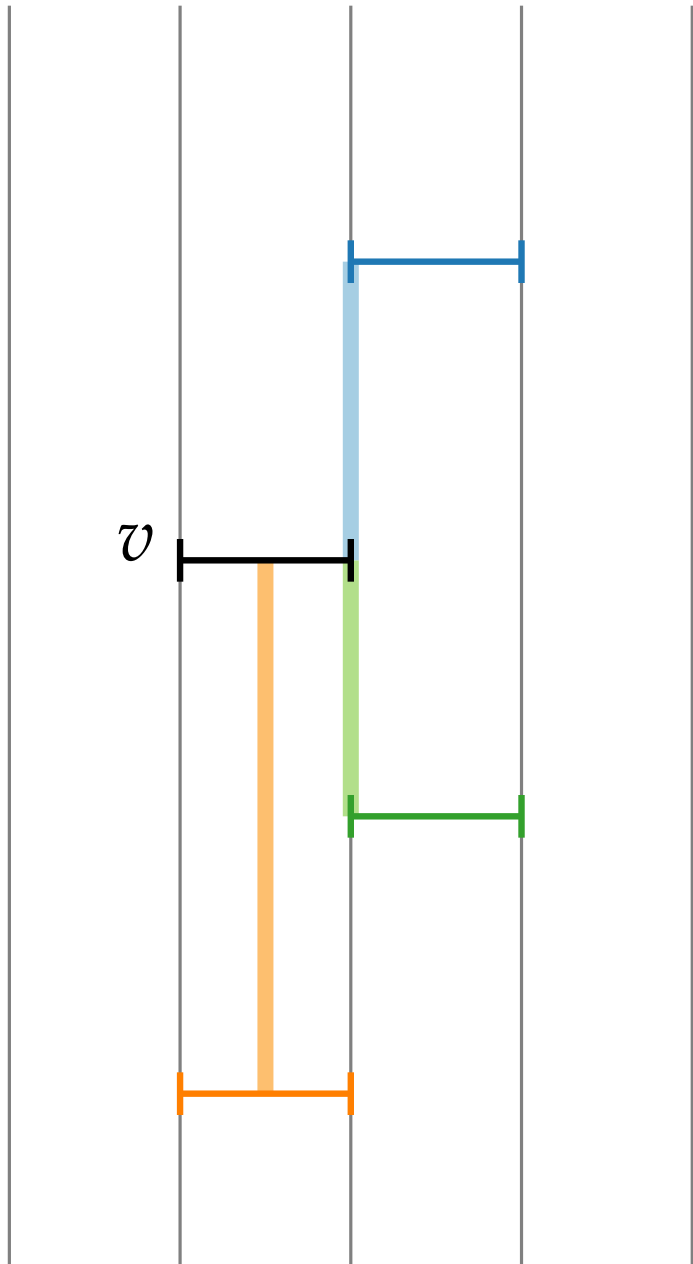
Weak: Trees, maxdeg 5



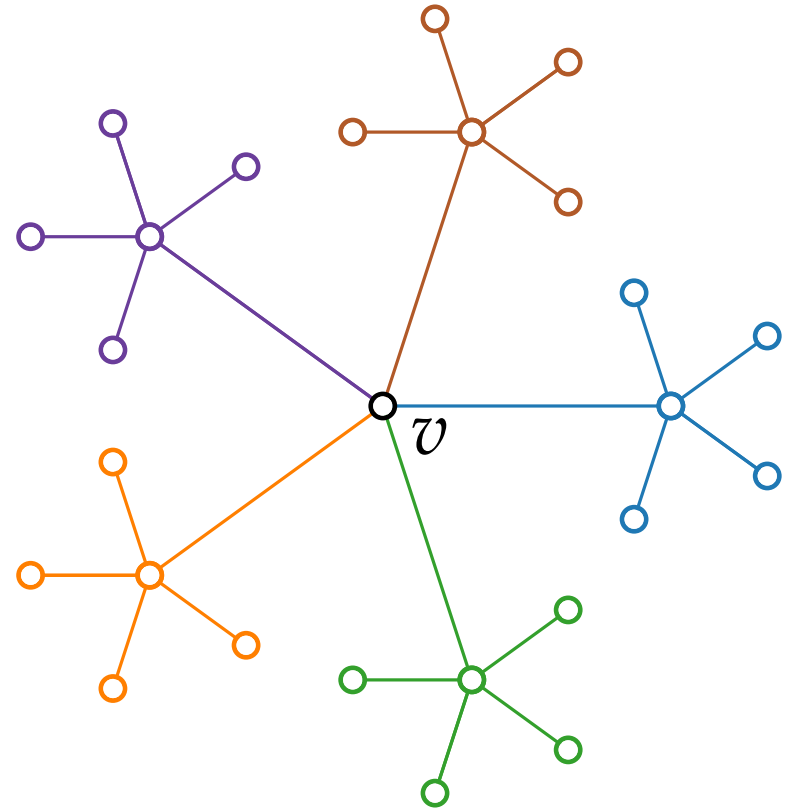
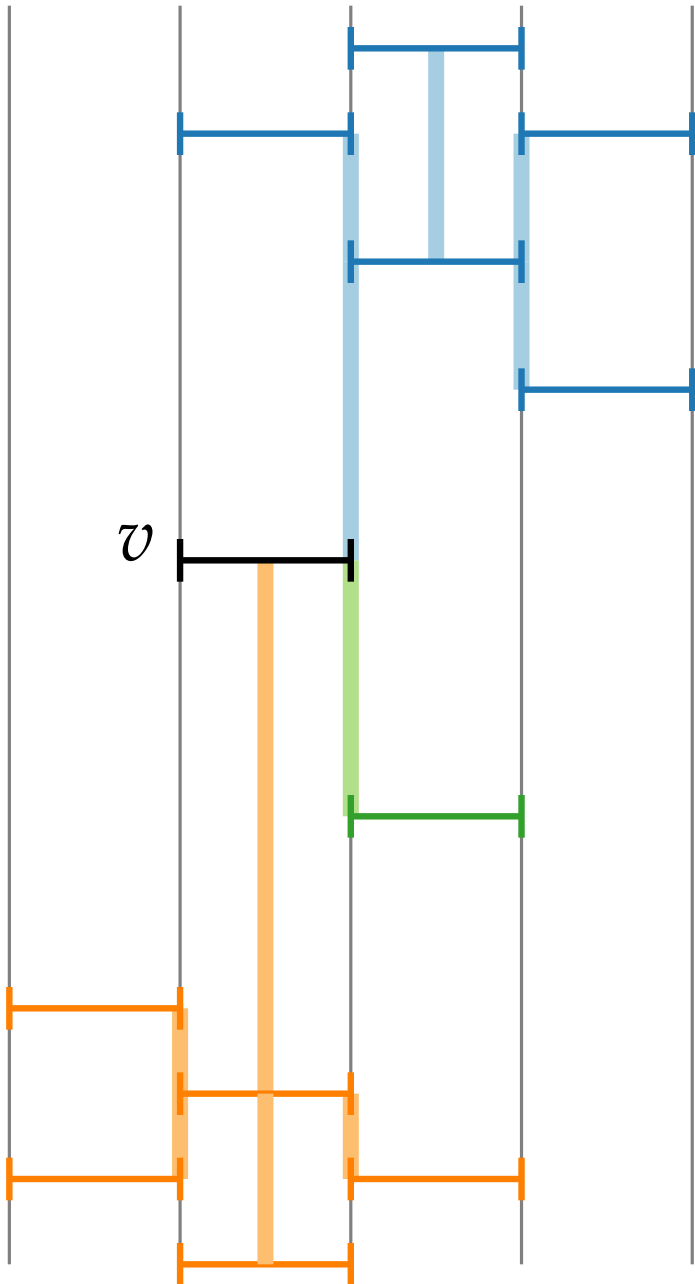
Weak: Trees, maxdeg 5



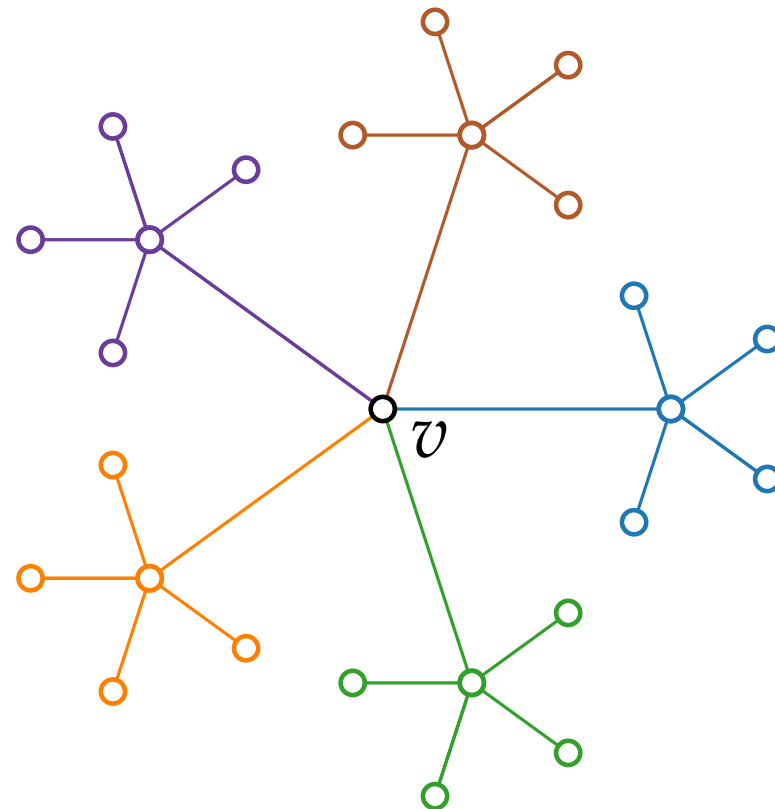
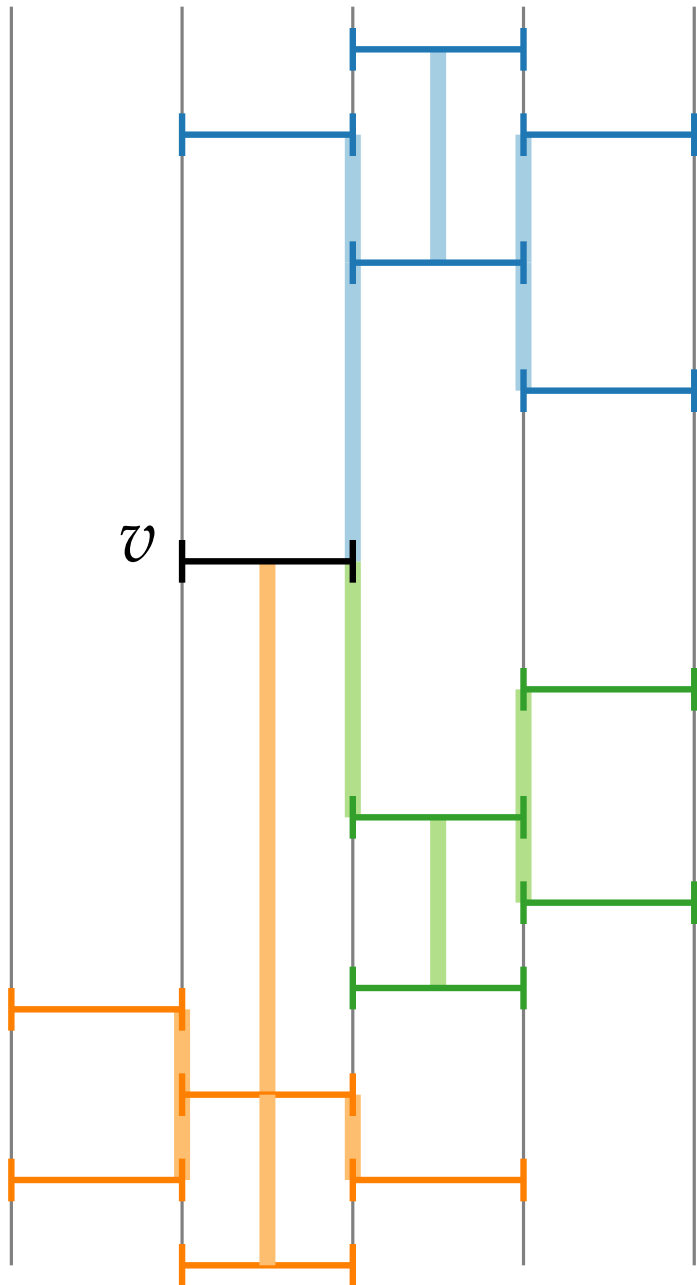
Weak: Trees, maxdeg 5



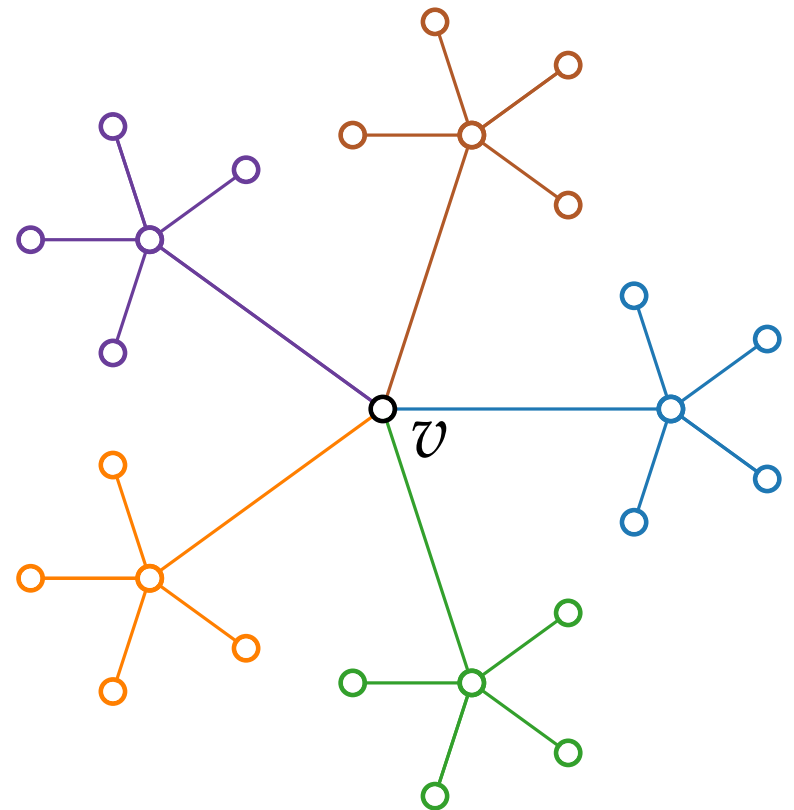
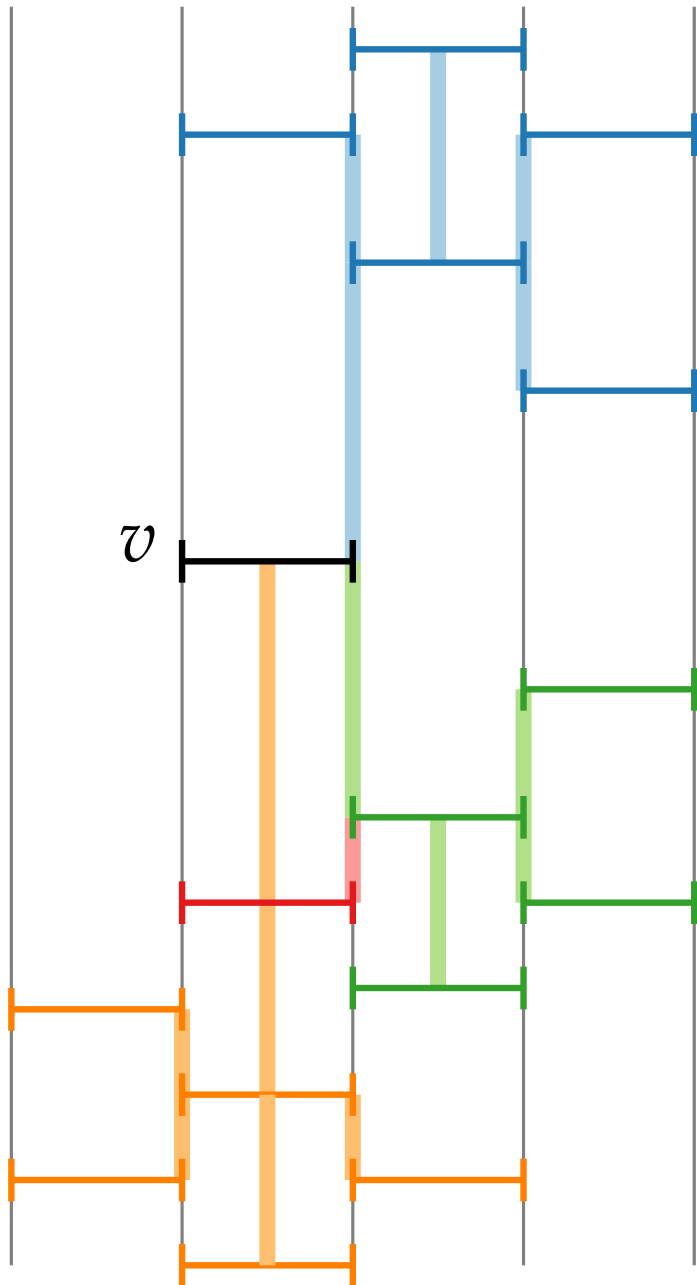
Weak: Trees, maxdeg 5



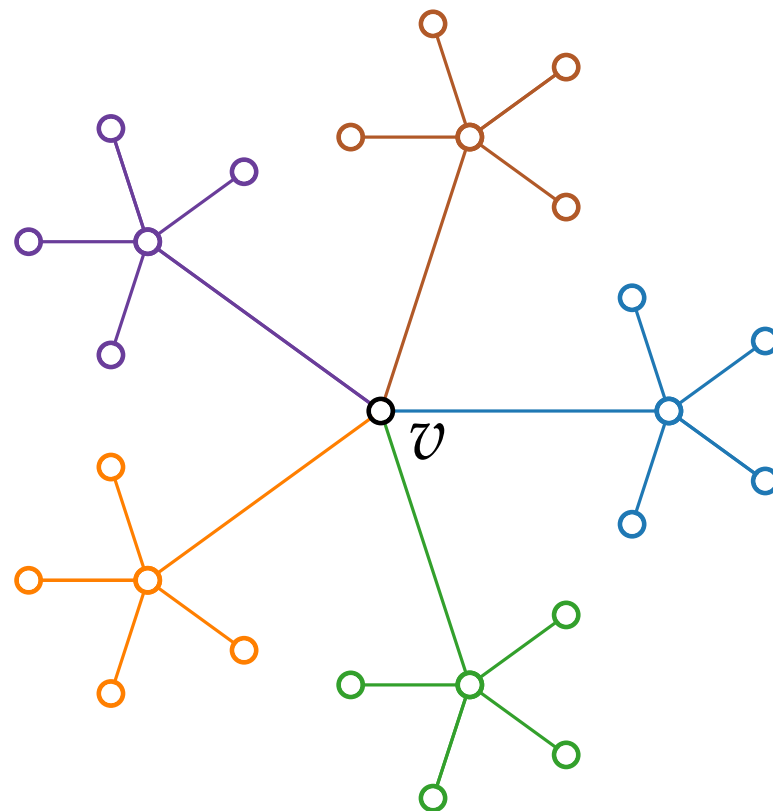
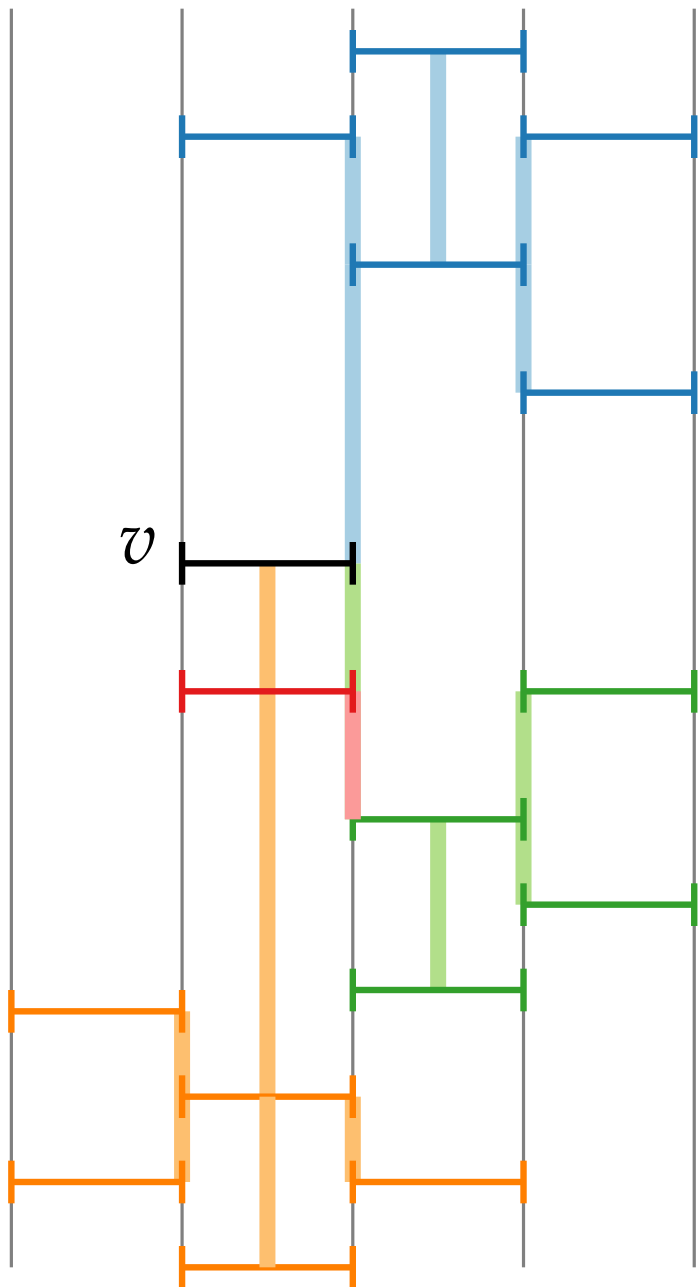
Weak: Trees, maxdeg 5



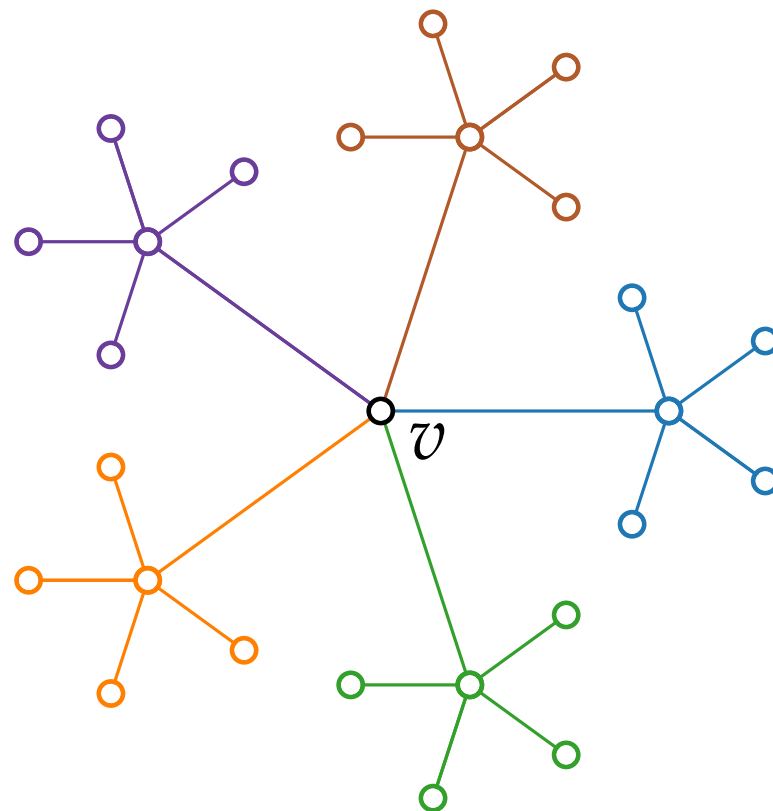
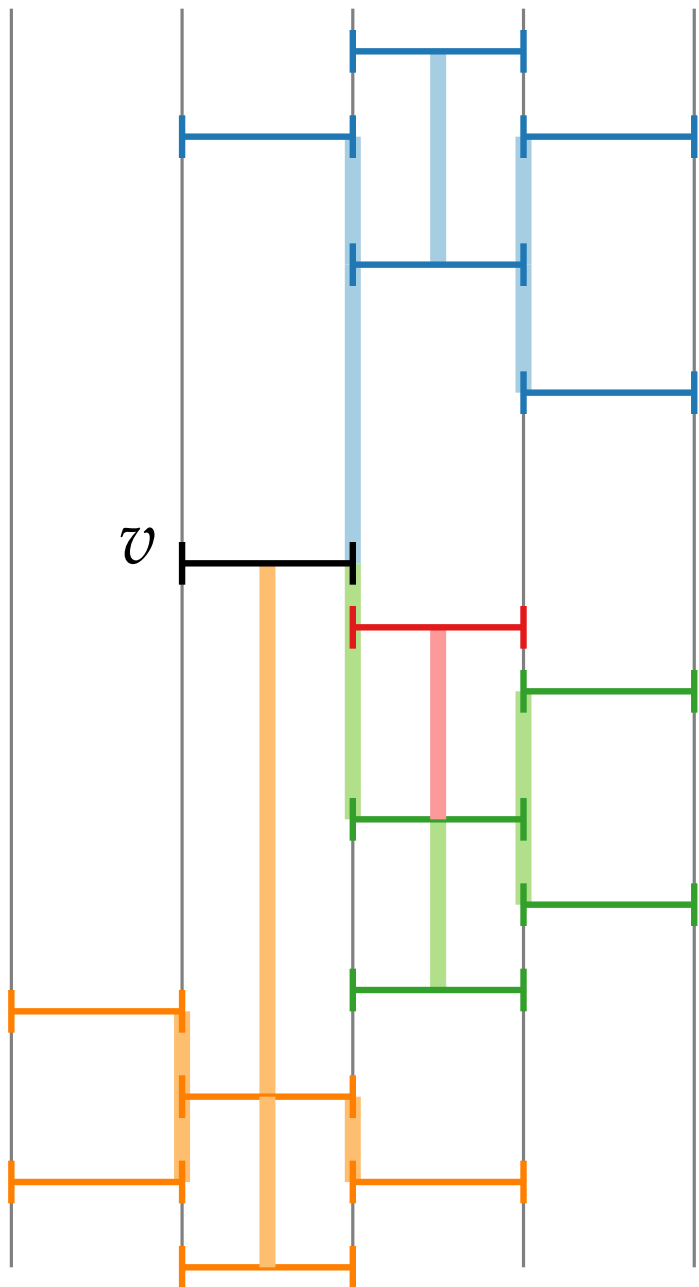
Weak: Trees, maxdeg 5



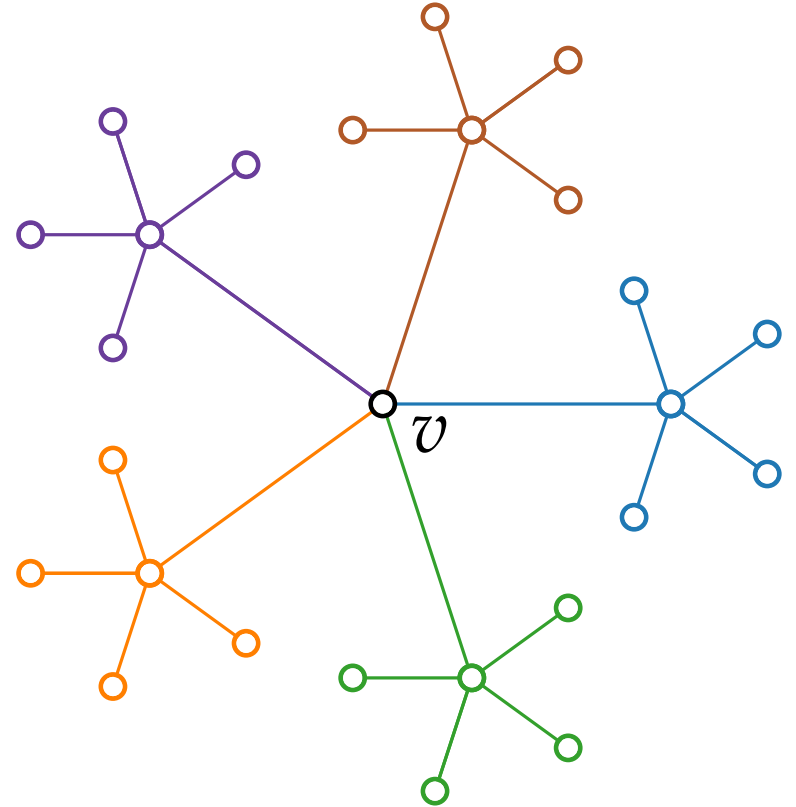
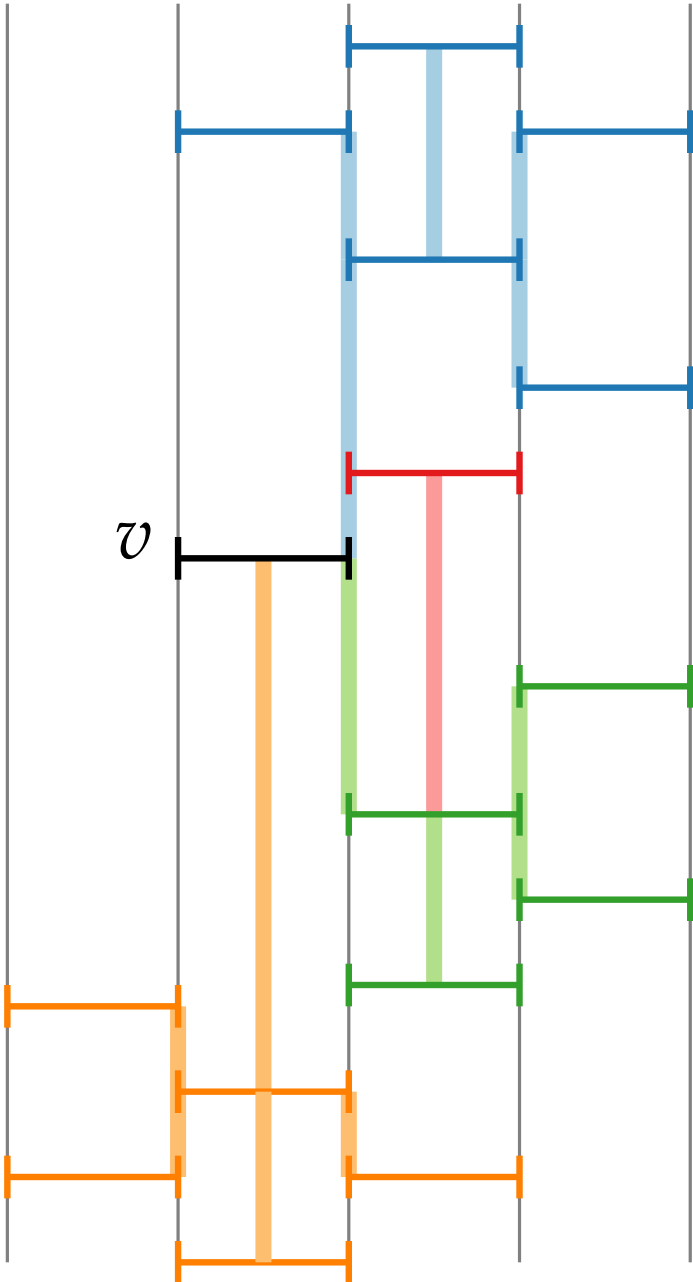
Weak: Trees, maxdeg 5



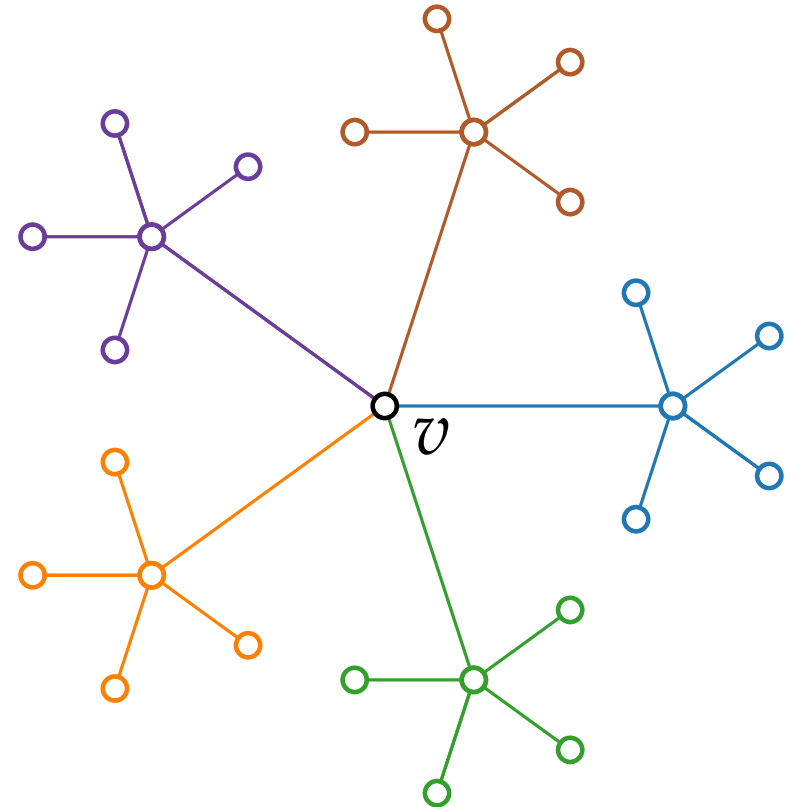
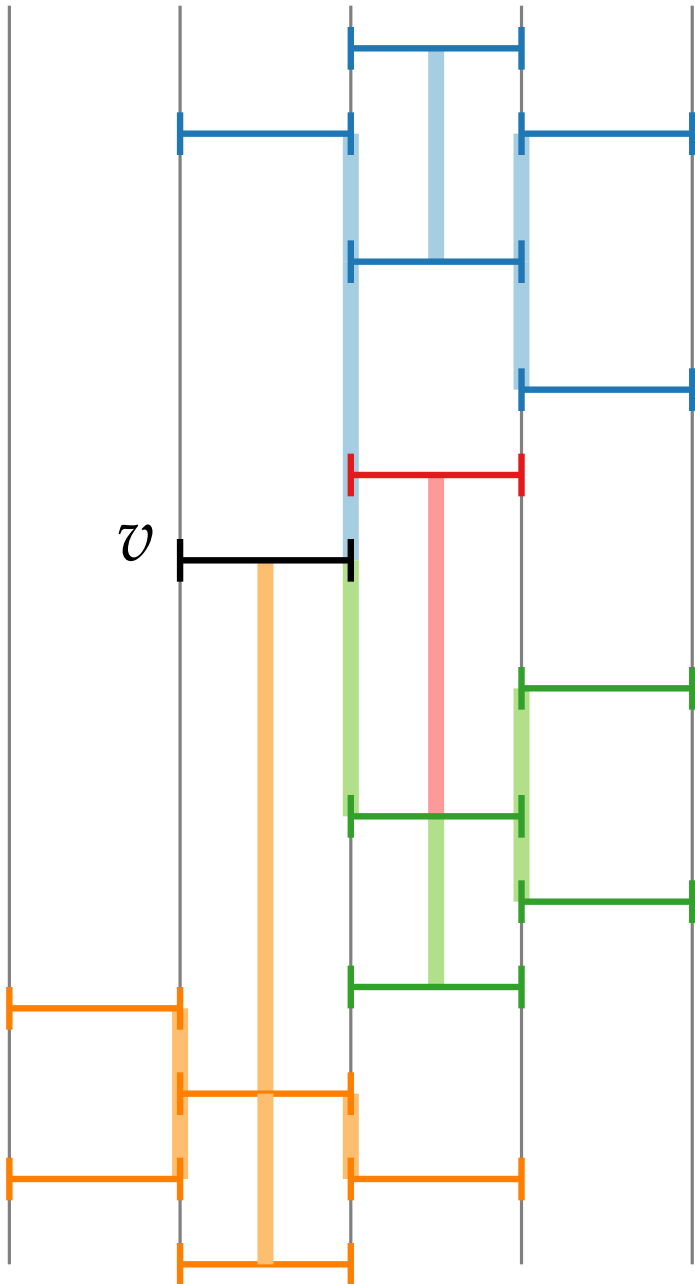
Weak: Trees, maxdeg 5



Weak: Trees, maxdeg 5

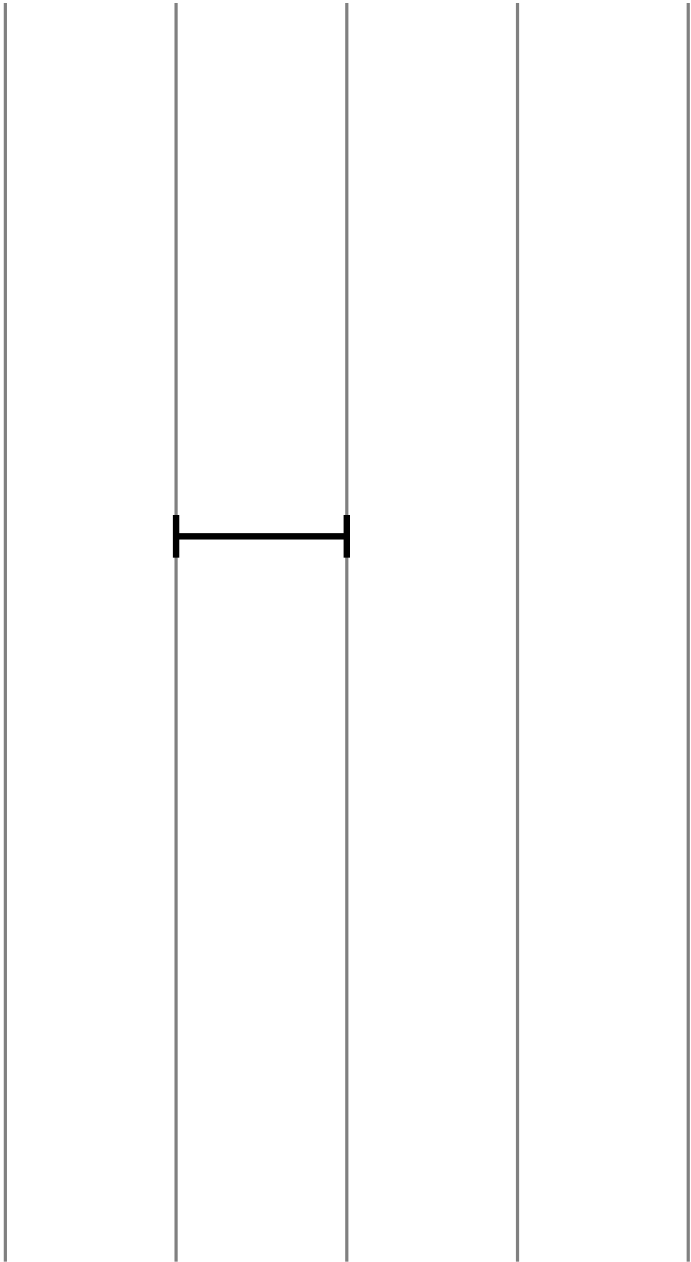


Weak: Trees, maxdeg 5

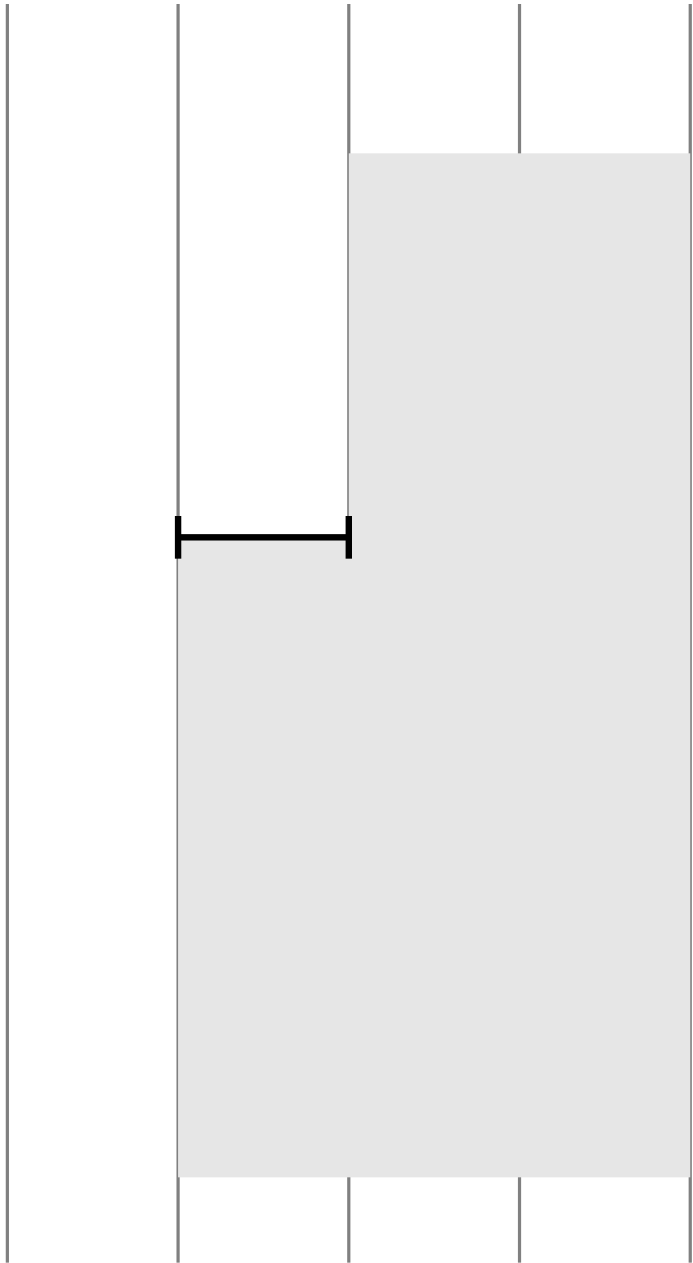


Tree with maxdeg 5 **X**

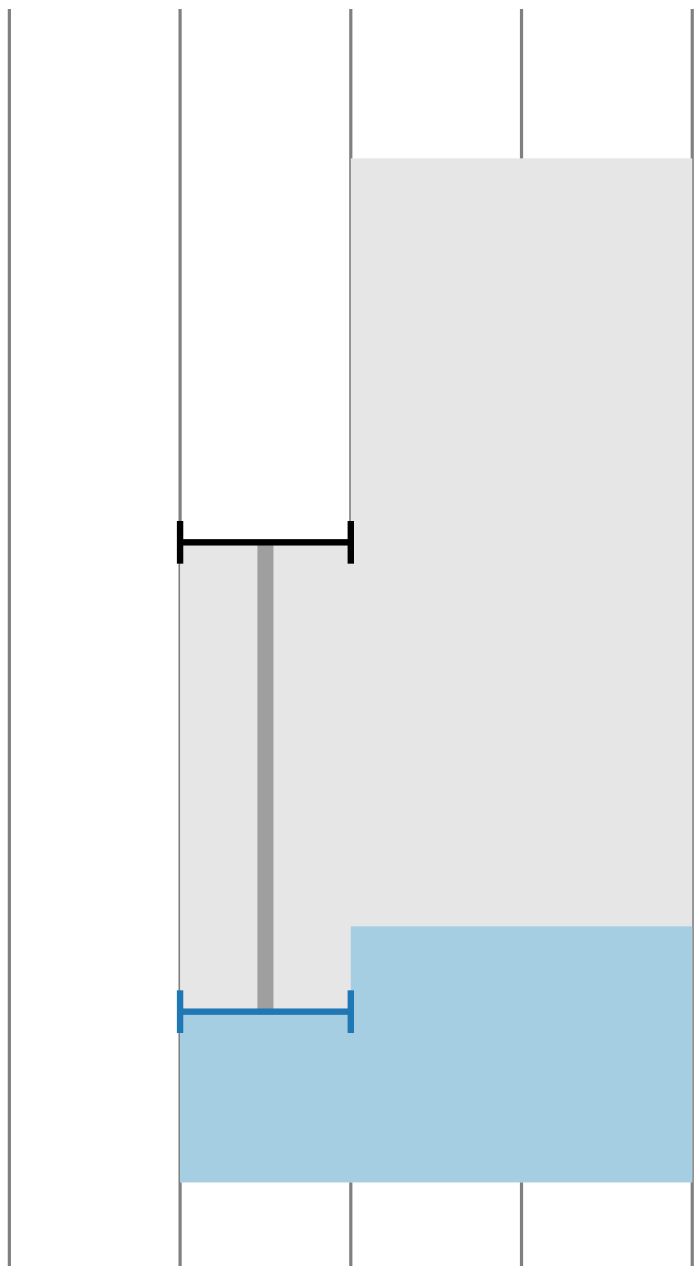
Weak: Trees, maxdeg 4



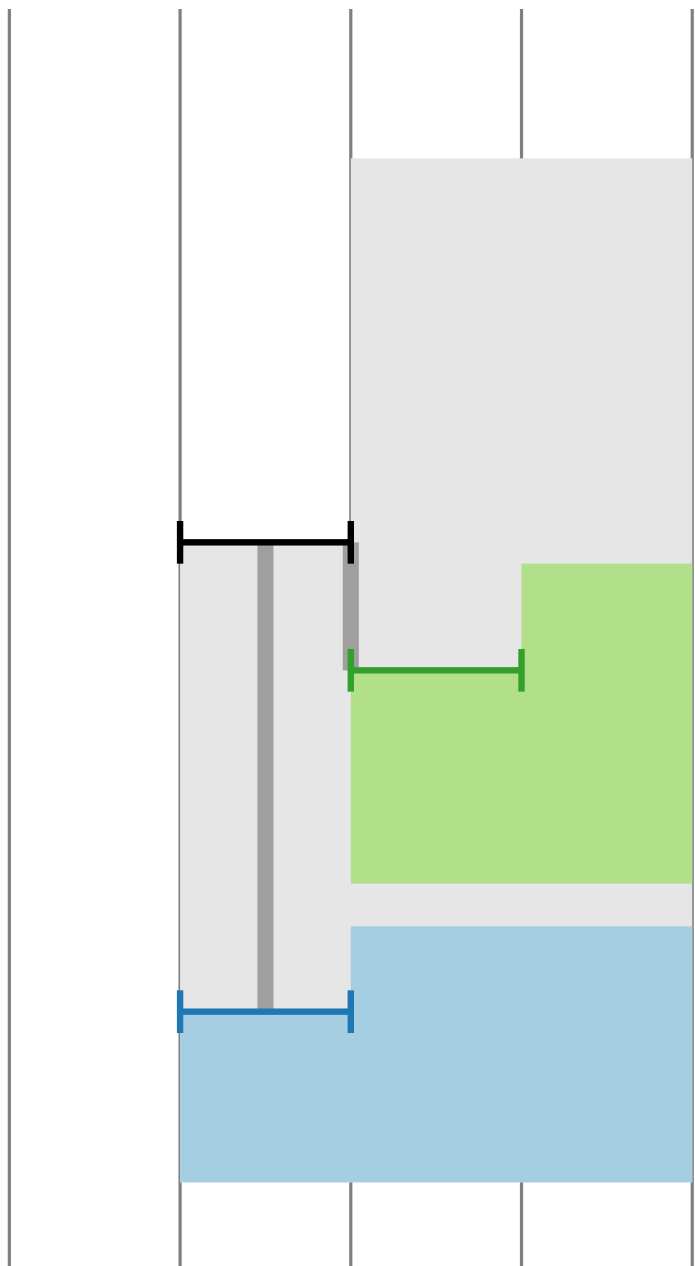
Weak: Trees, maxdeg 4



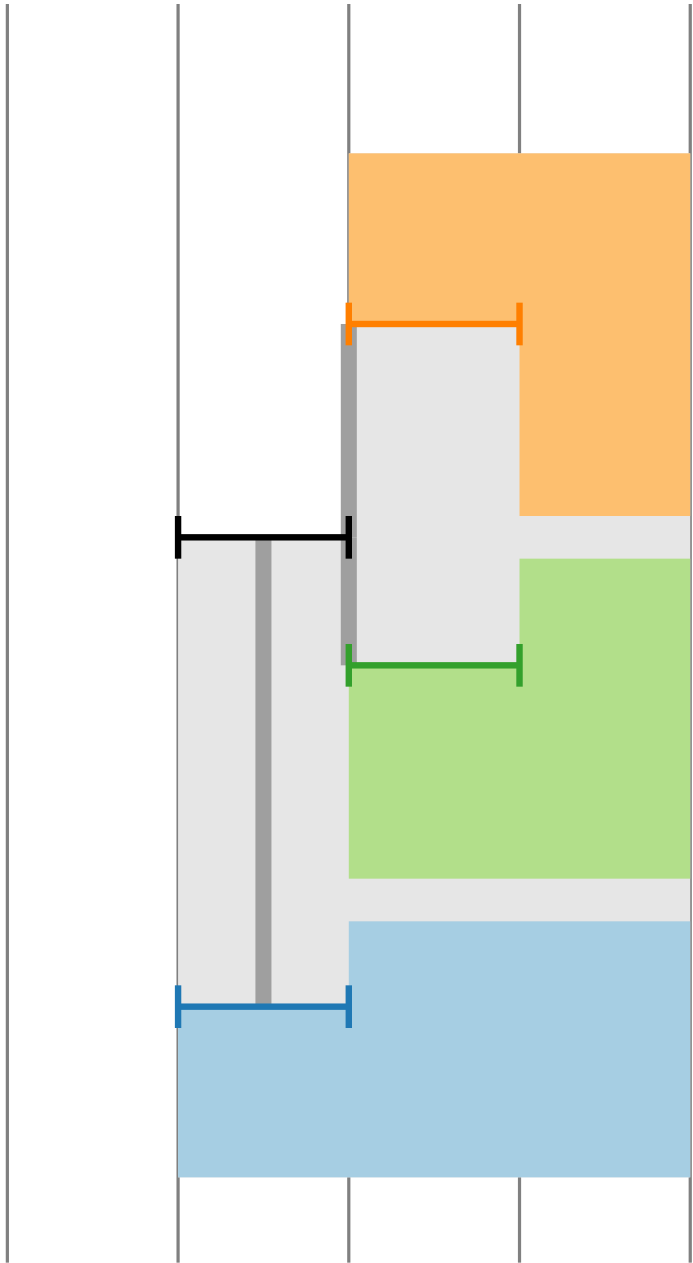
Weak: Trees, maxdeg 4



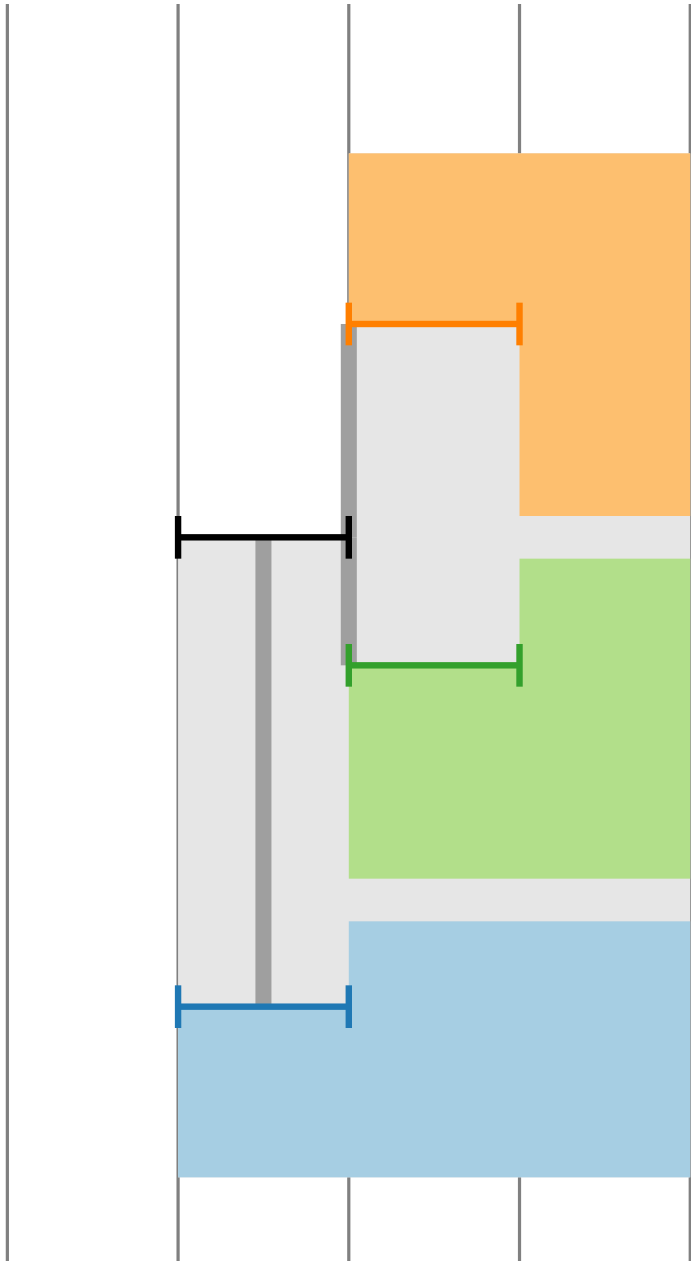
Weak: Trees, maxdeg 4



Weak: Trees, maxdeg 4

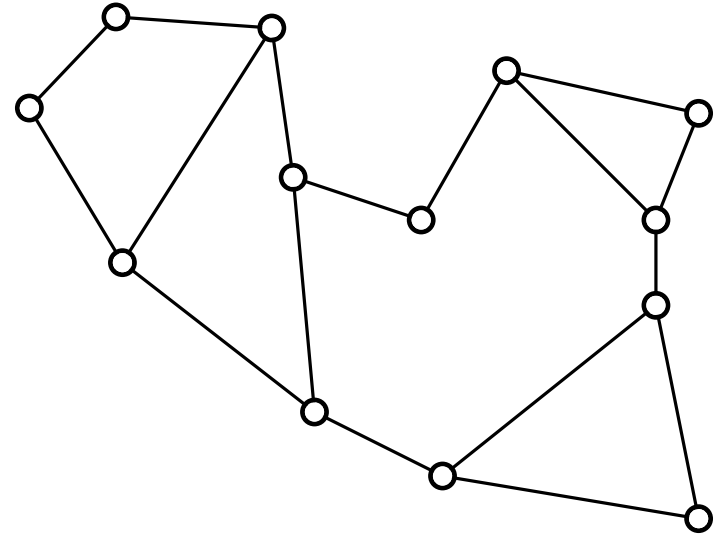
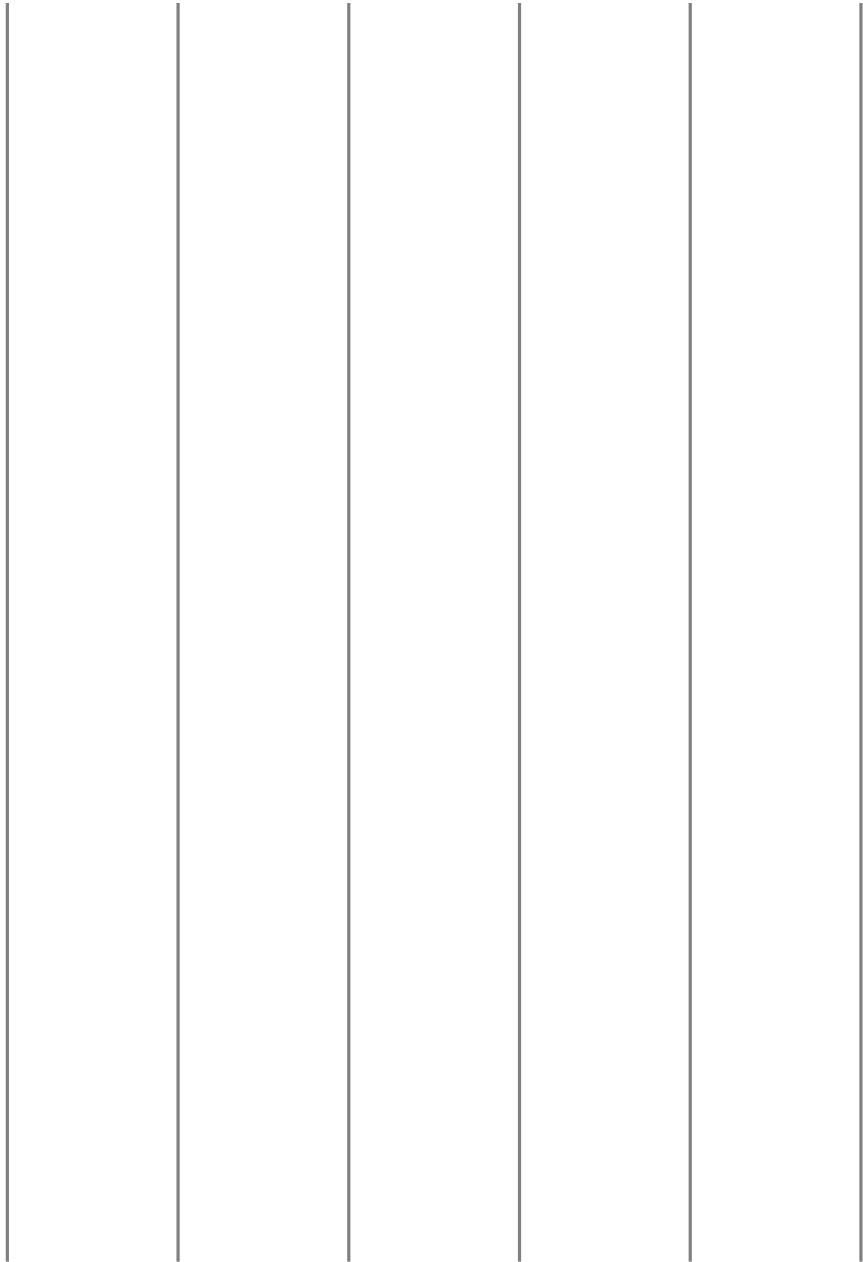


Weak: Trees, maxdeg 4

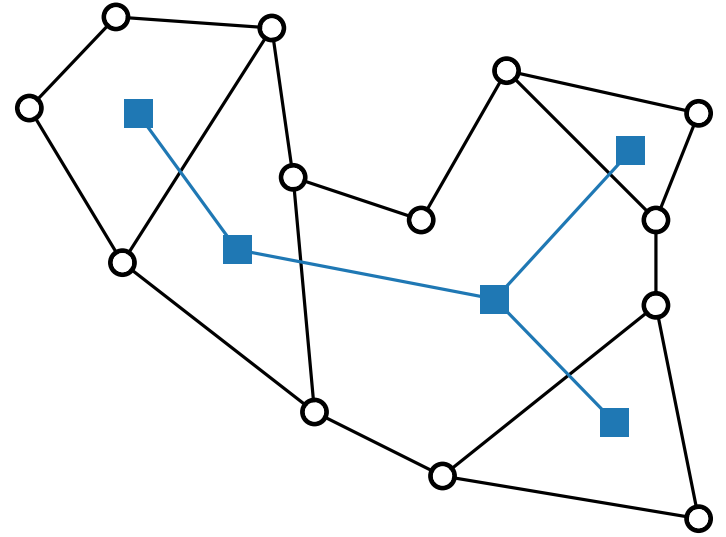
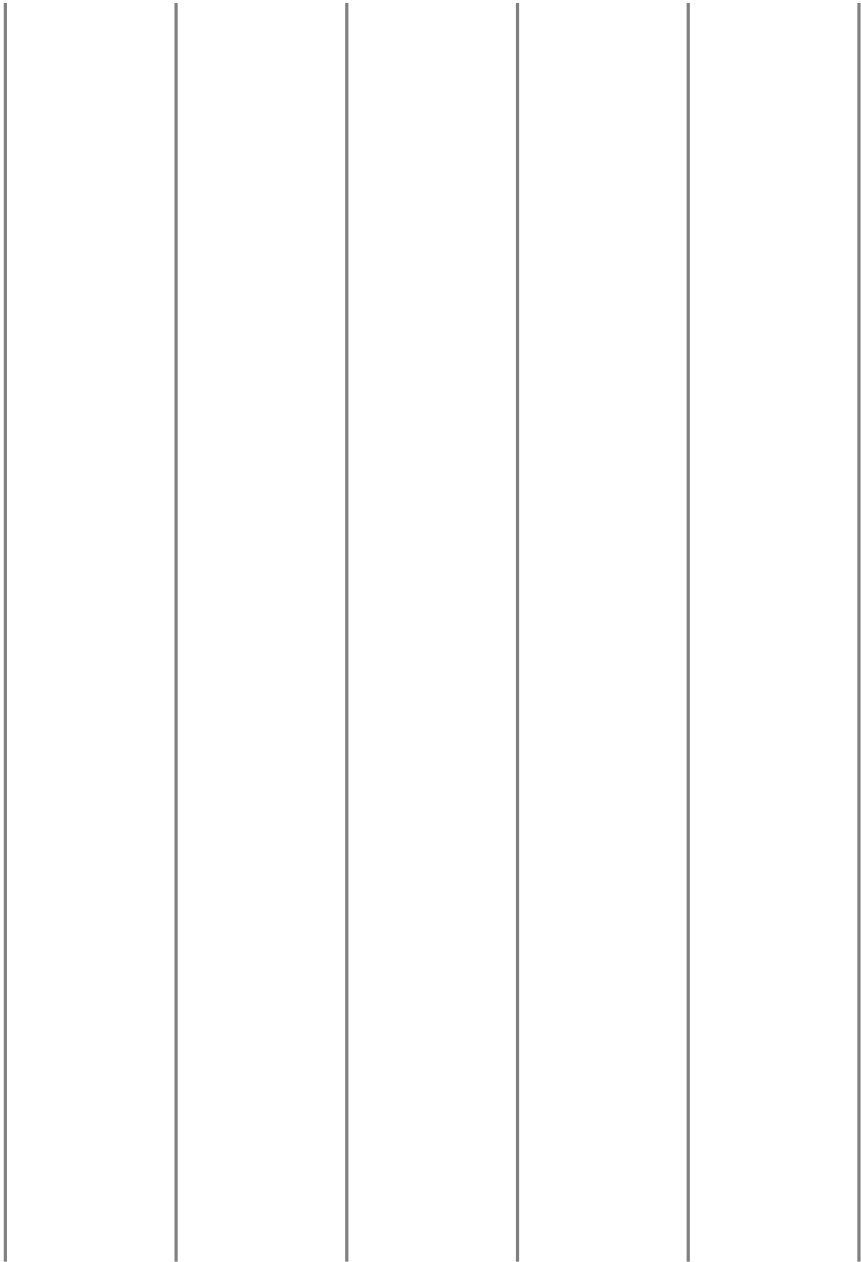


Tree with maxdeg 4 ✓

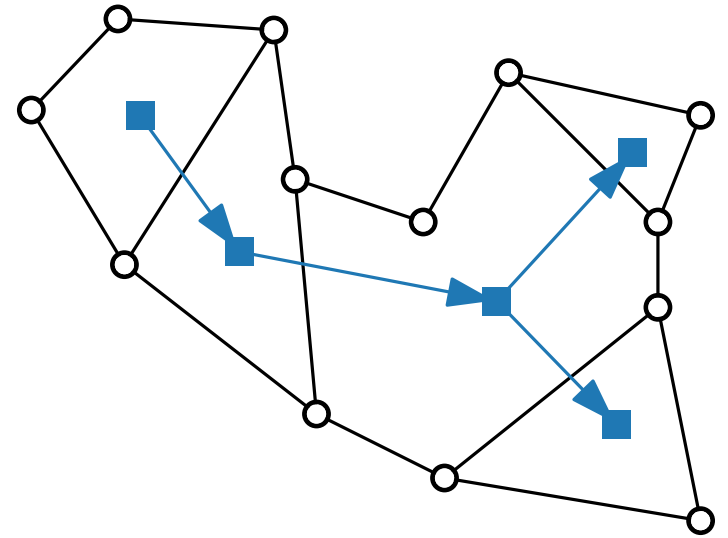
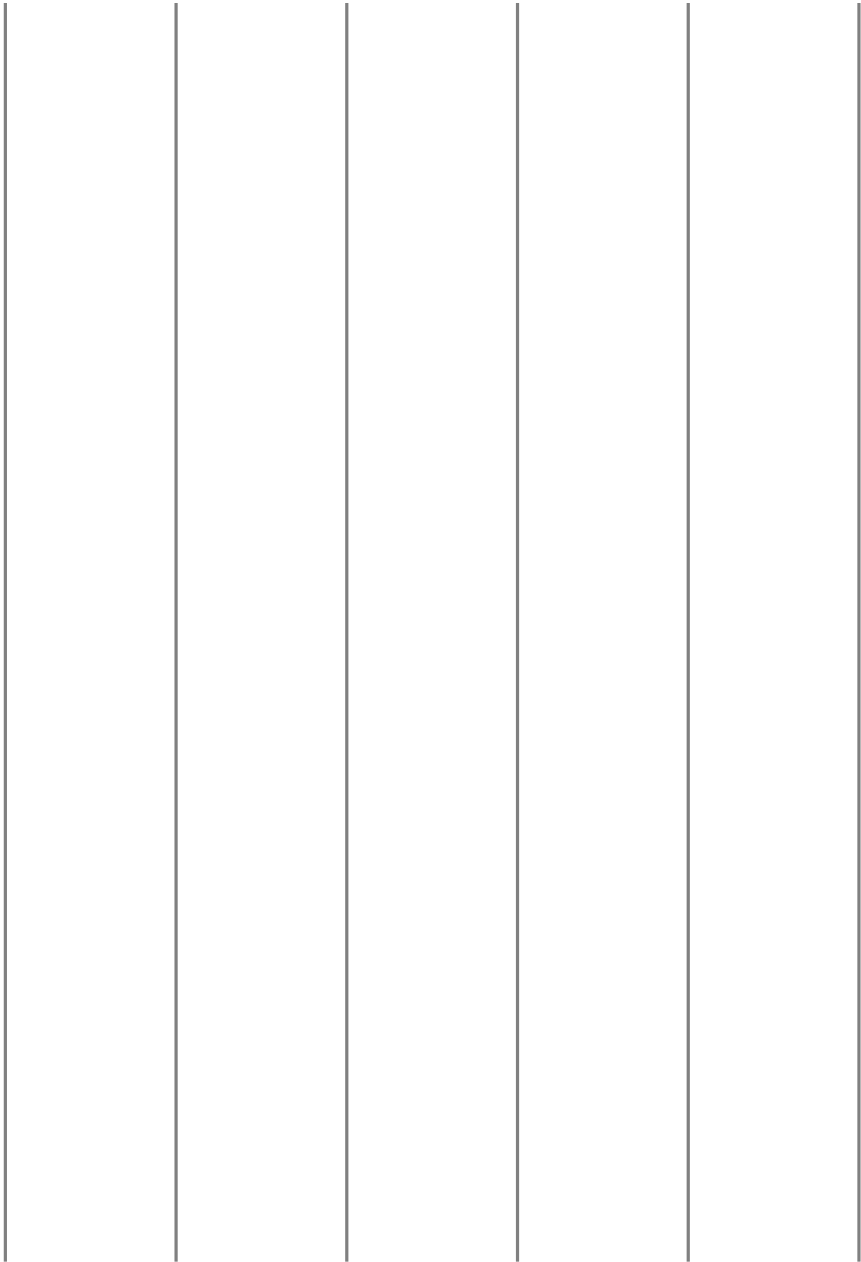
Weak: Biconnected Outerplanar, maxdeg 3



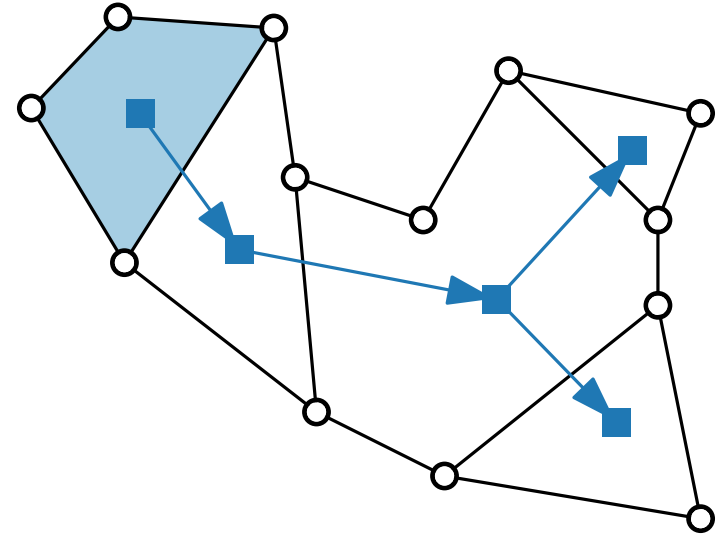
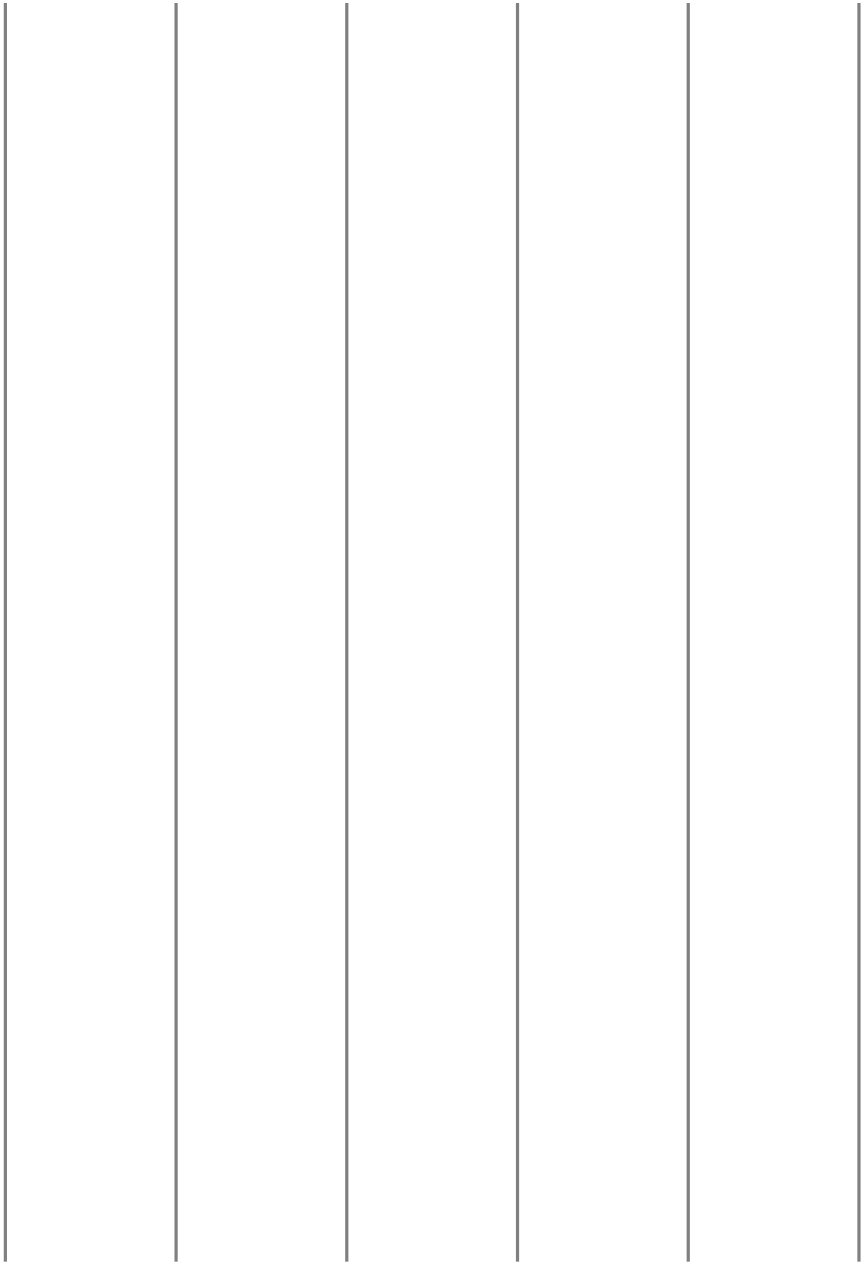
Weak: Biconnected Outerplanar, maxdeg 3



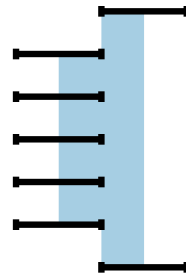
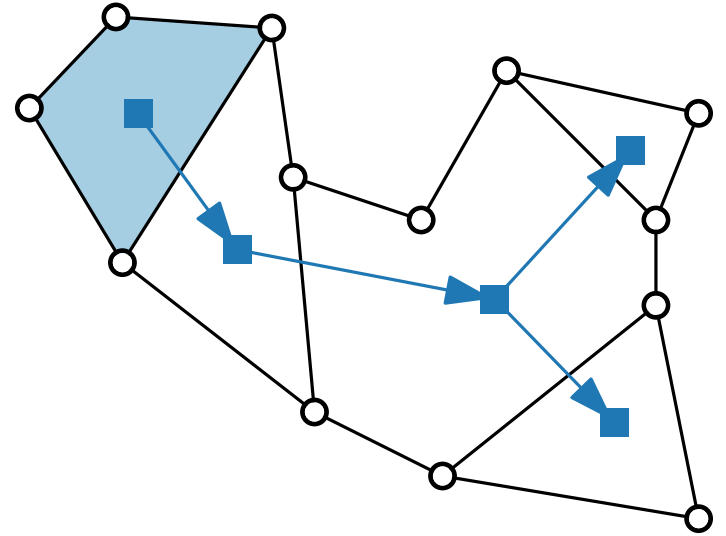
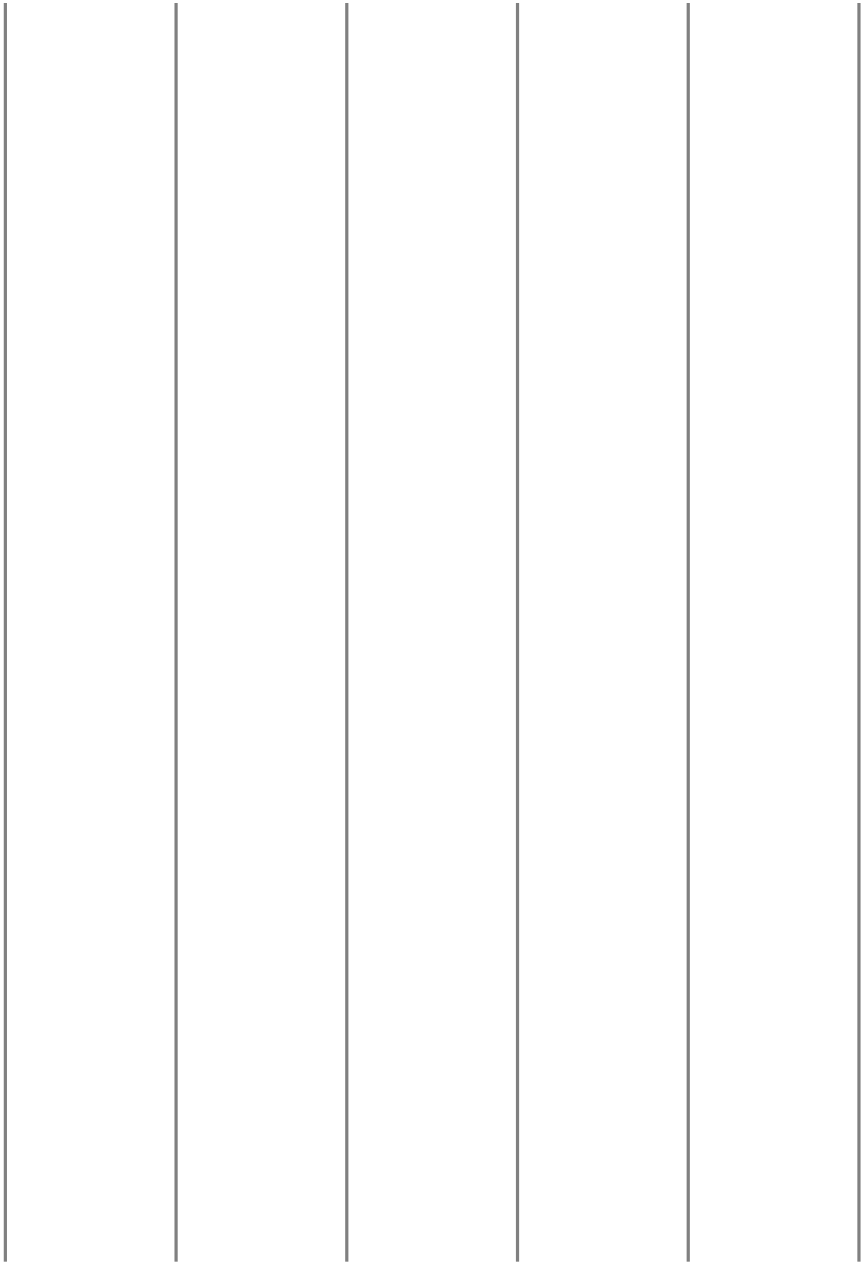
Weak: Biconnected Outerplanar, maxdeg 3



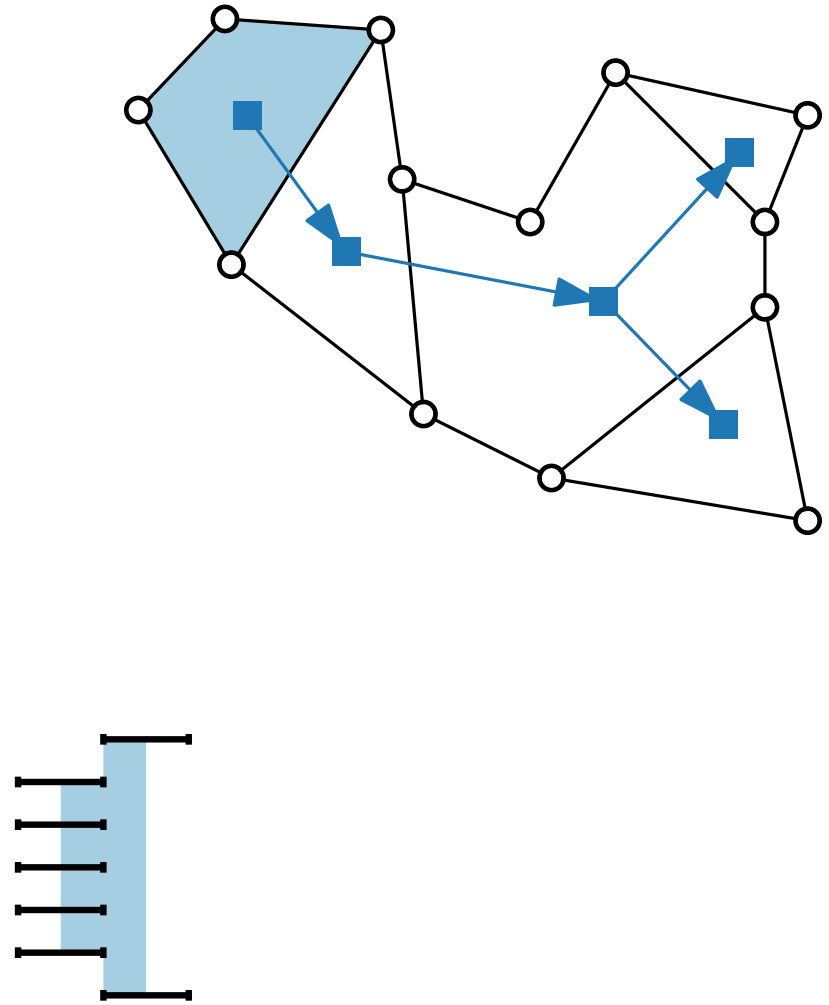
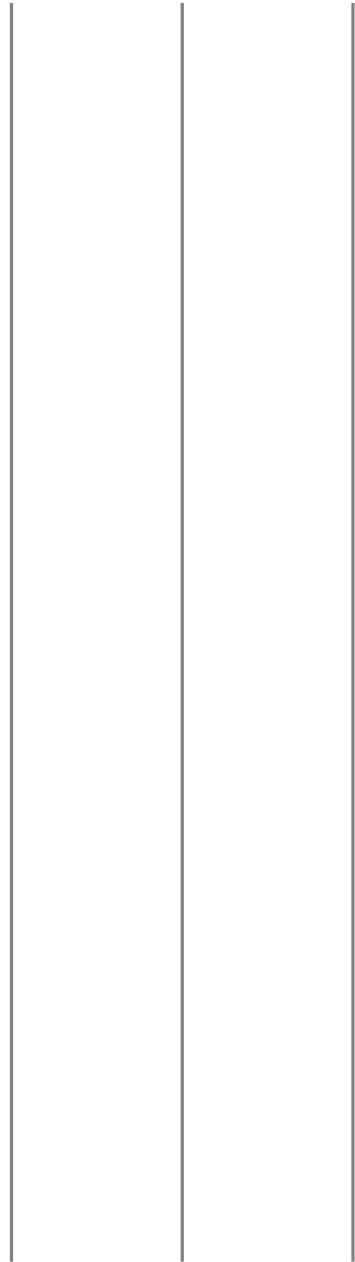
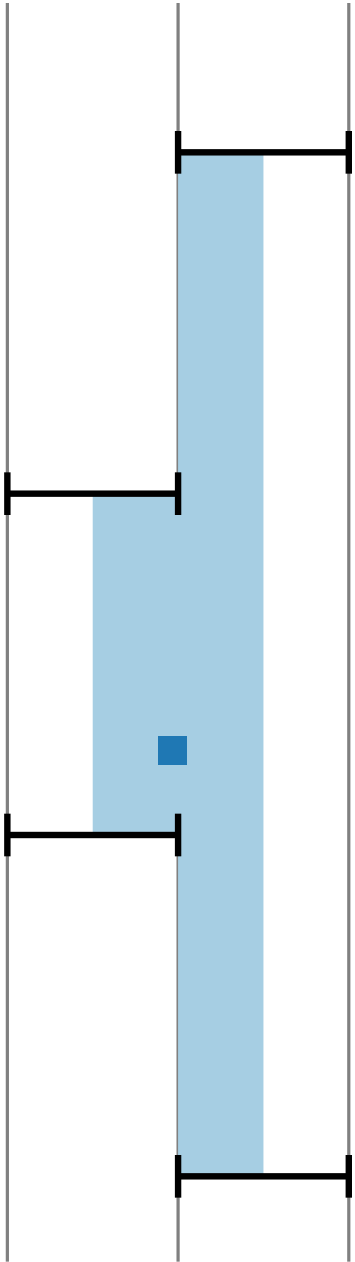
Weak: Biconnected Outerplanar, maxdeg 3



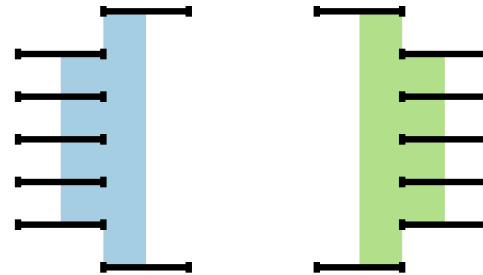
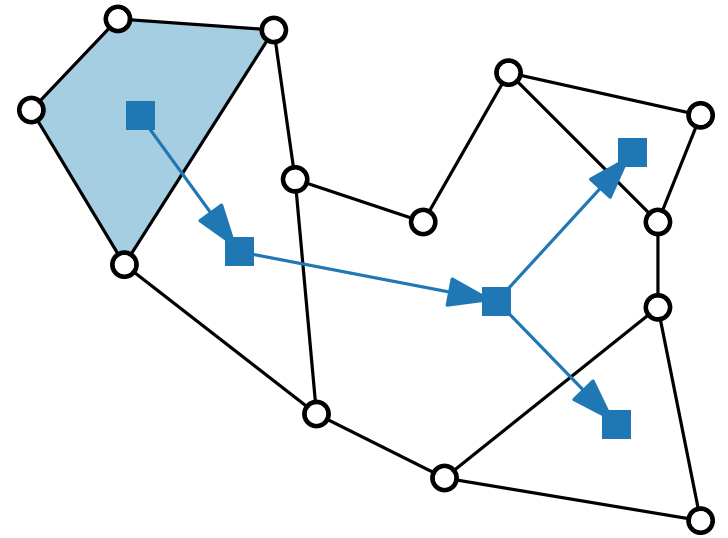
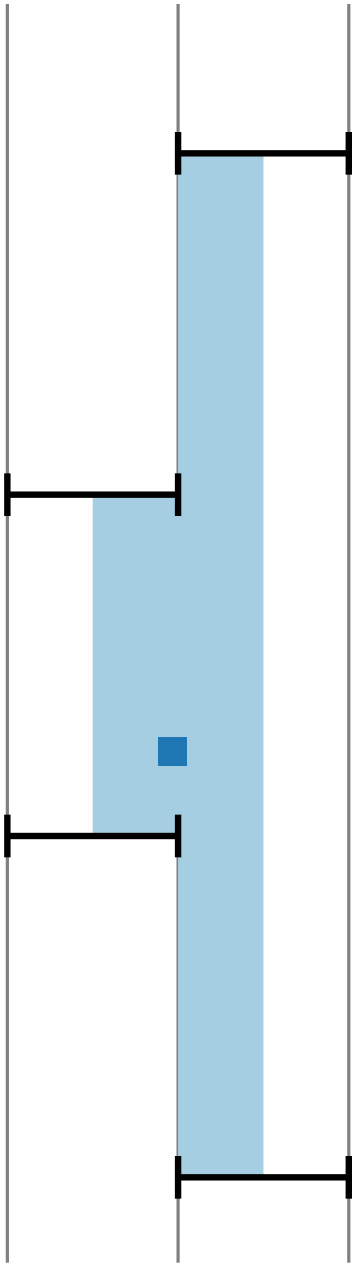
Weak: Biconnected Outerplanar, maxdeg 3



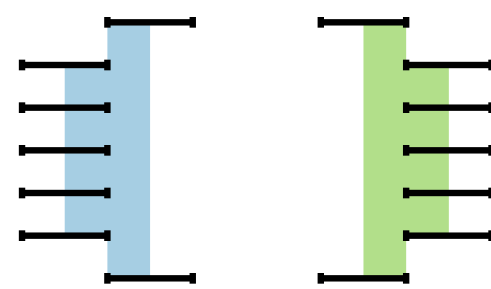
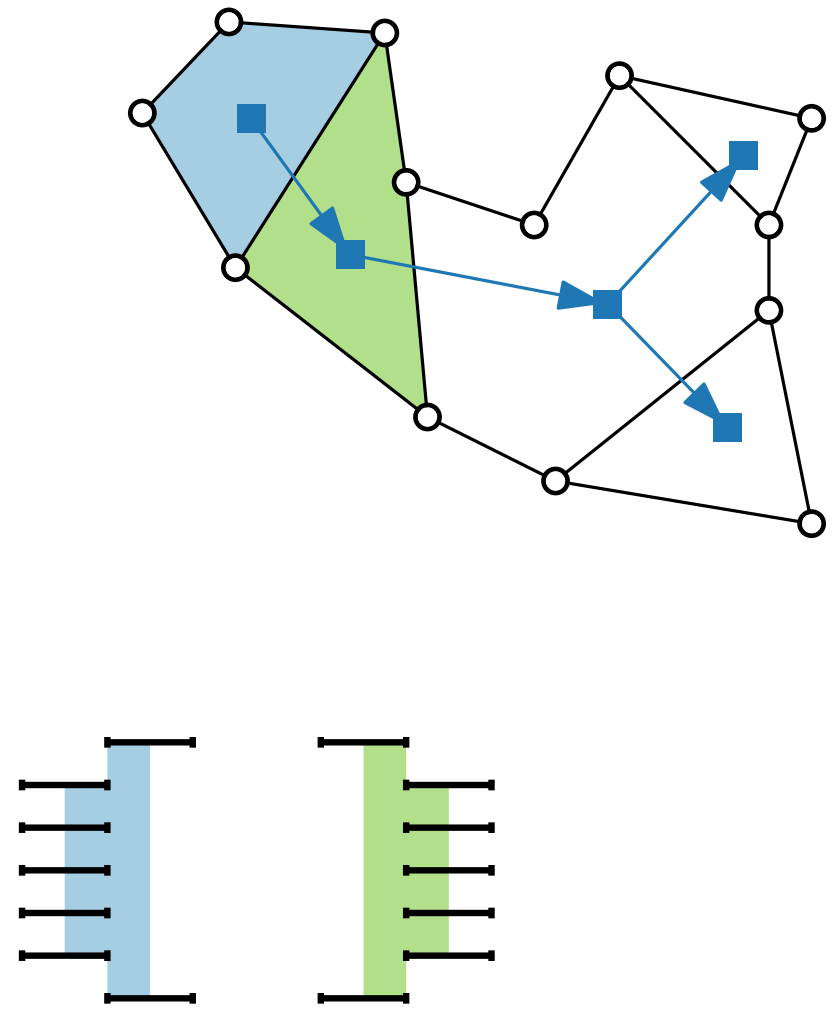
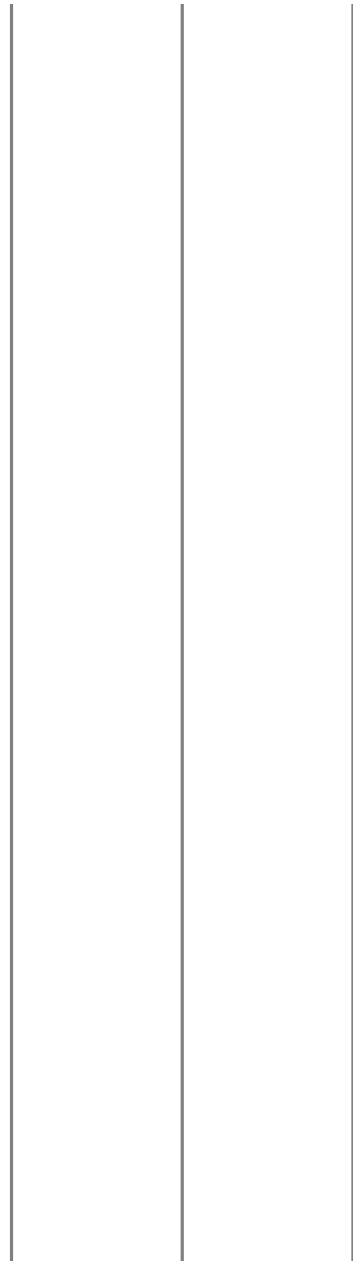
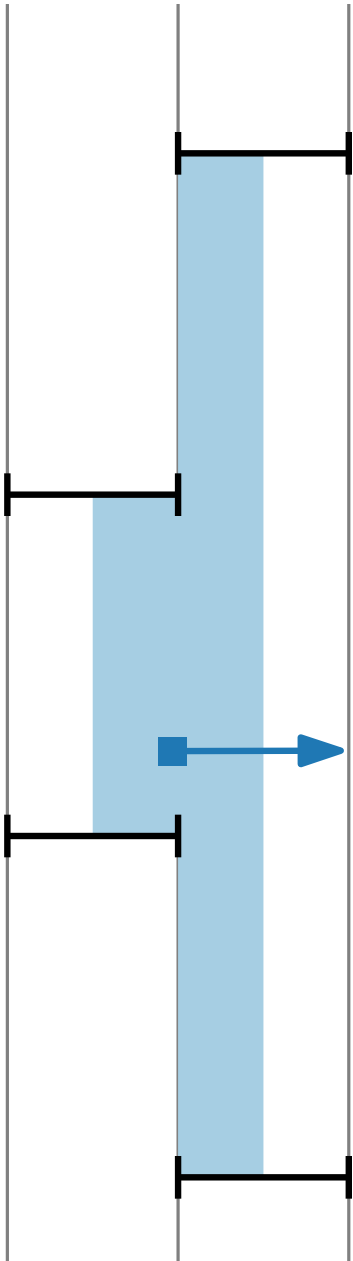
Weak: Biconnected Outerplanar, maxdeg 3



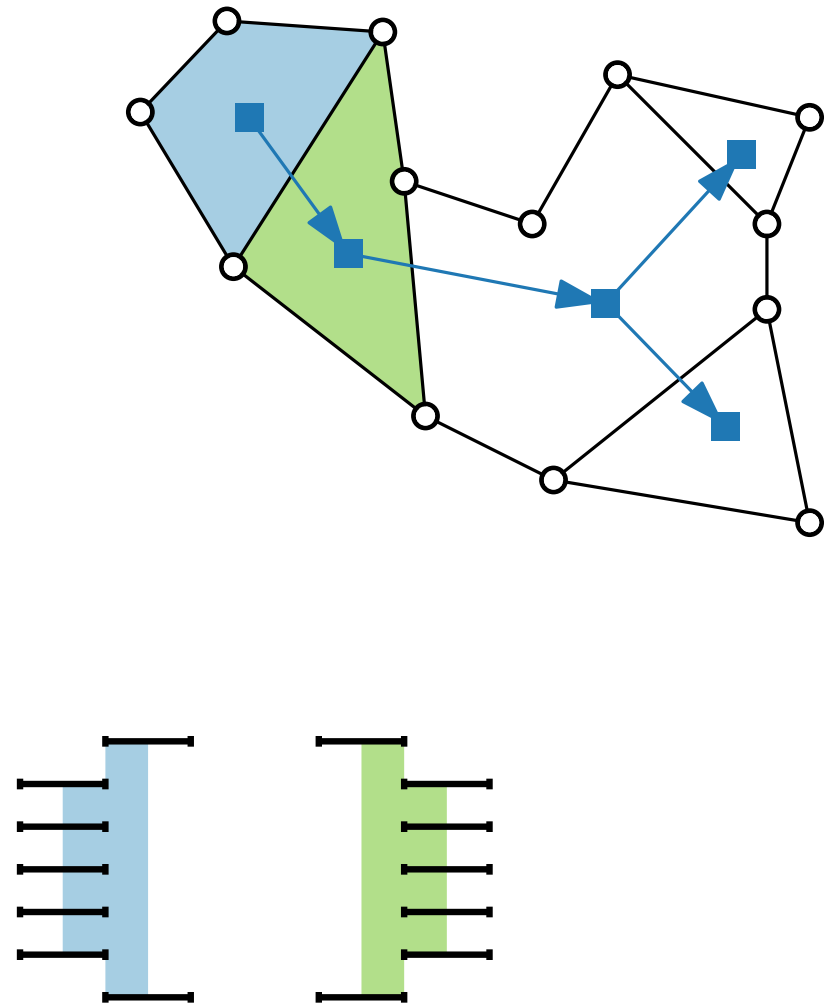
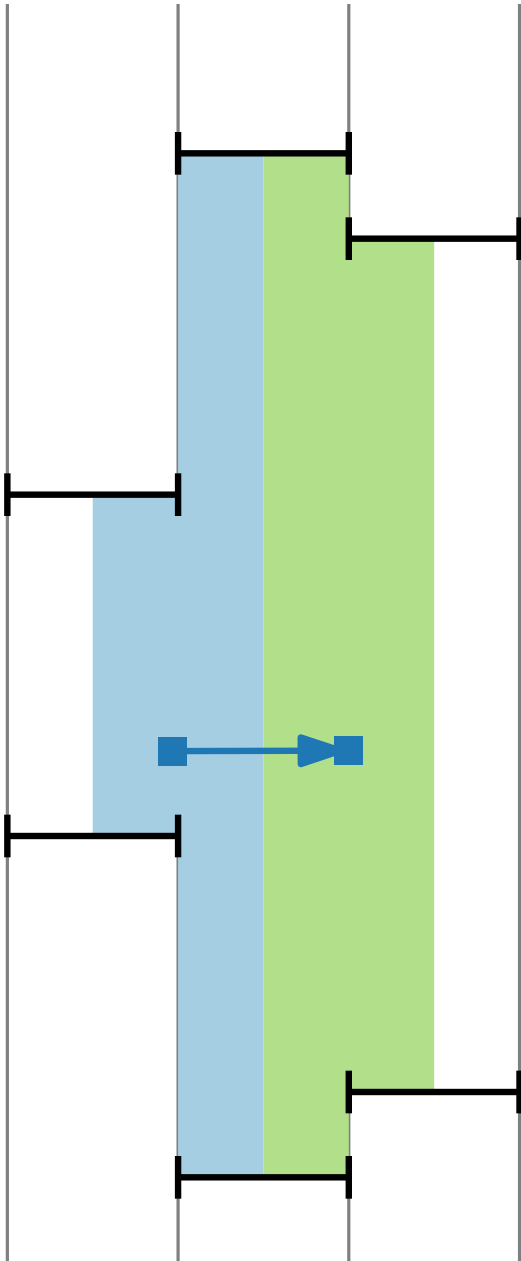
Weak: Biconnected Outerplanar, maxdeg 3



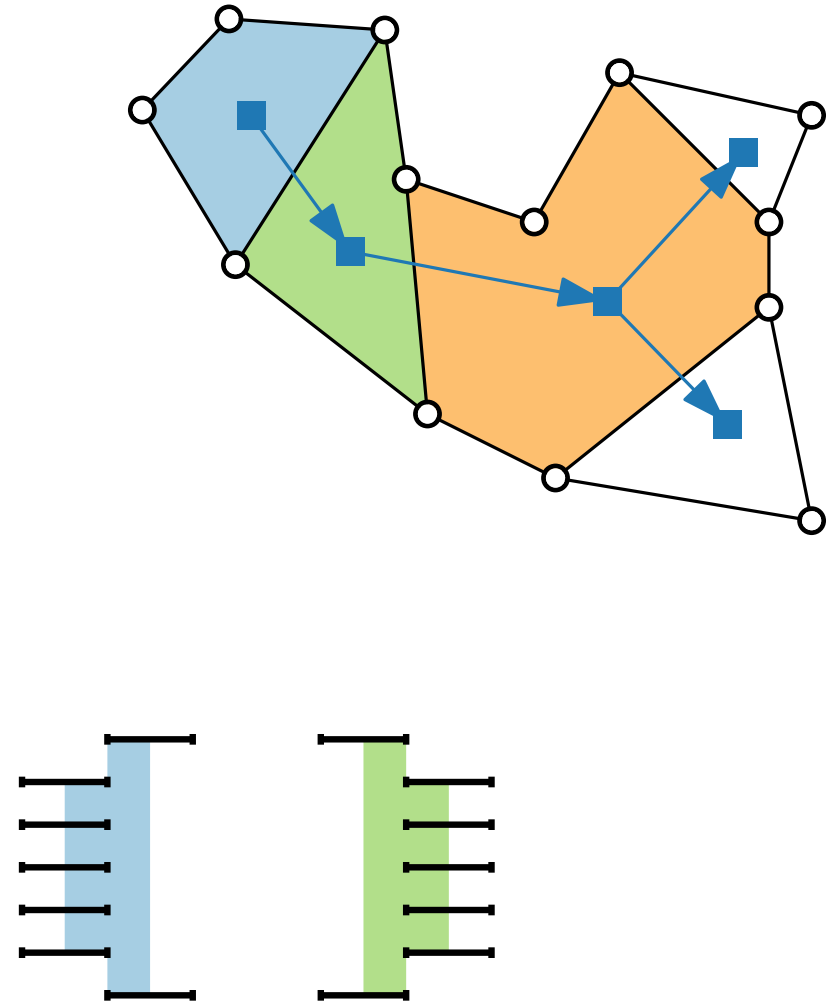
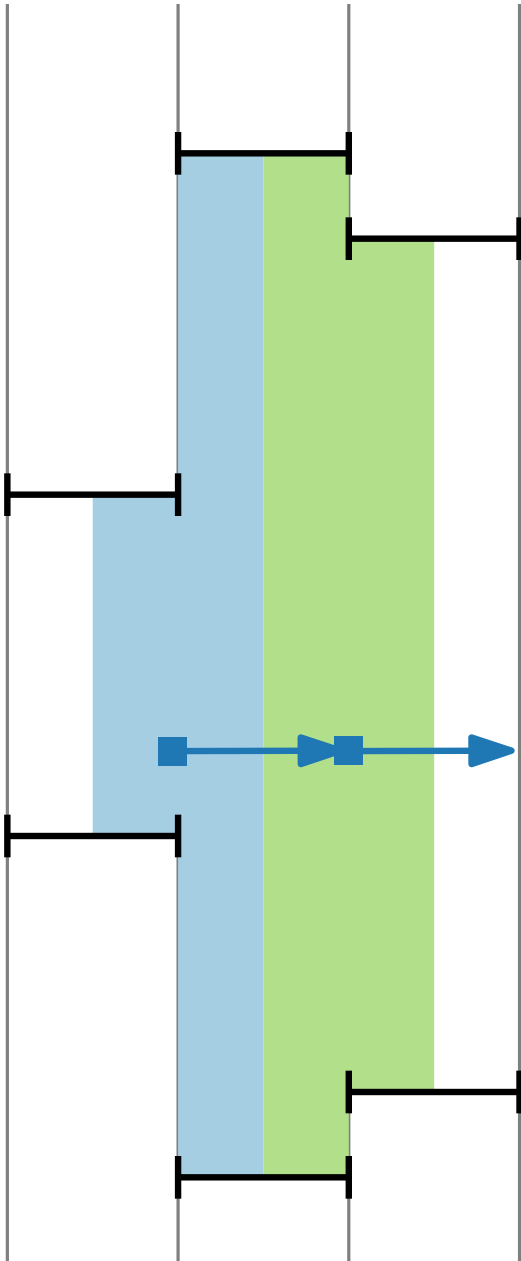
Weak: Biconnected Outerplanar, maxdeg 3



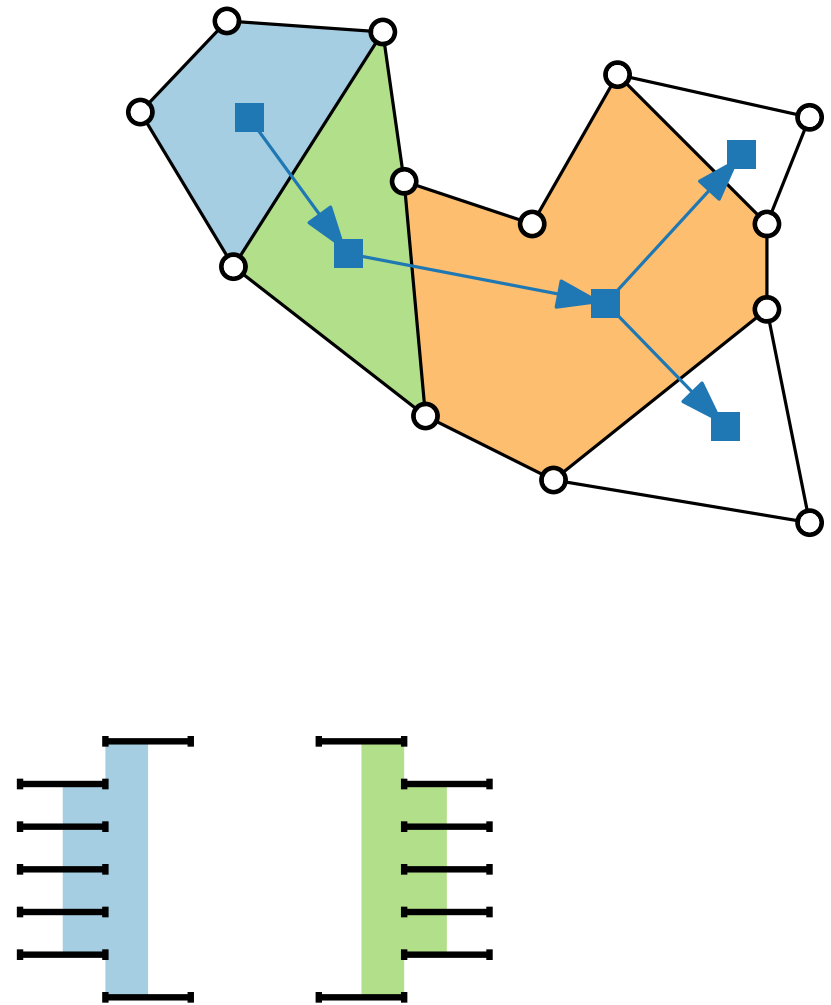
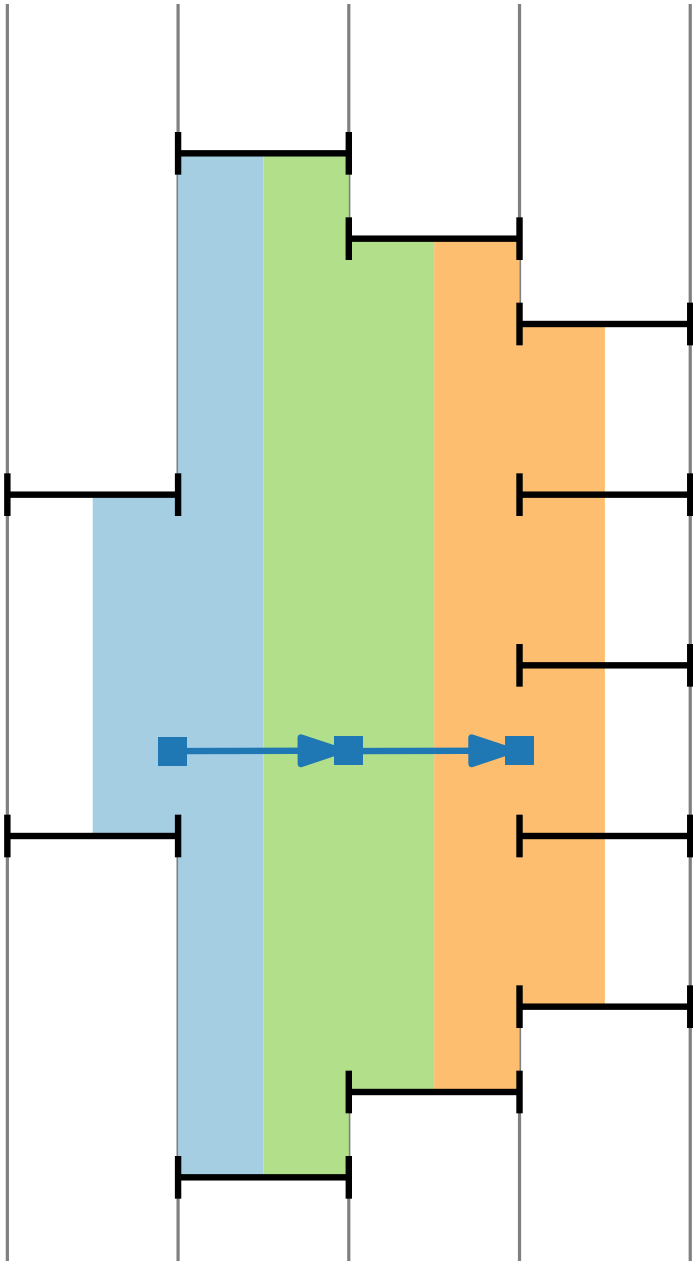
Weak: Biconnected Outerplanar, maxdeg 3



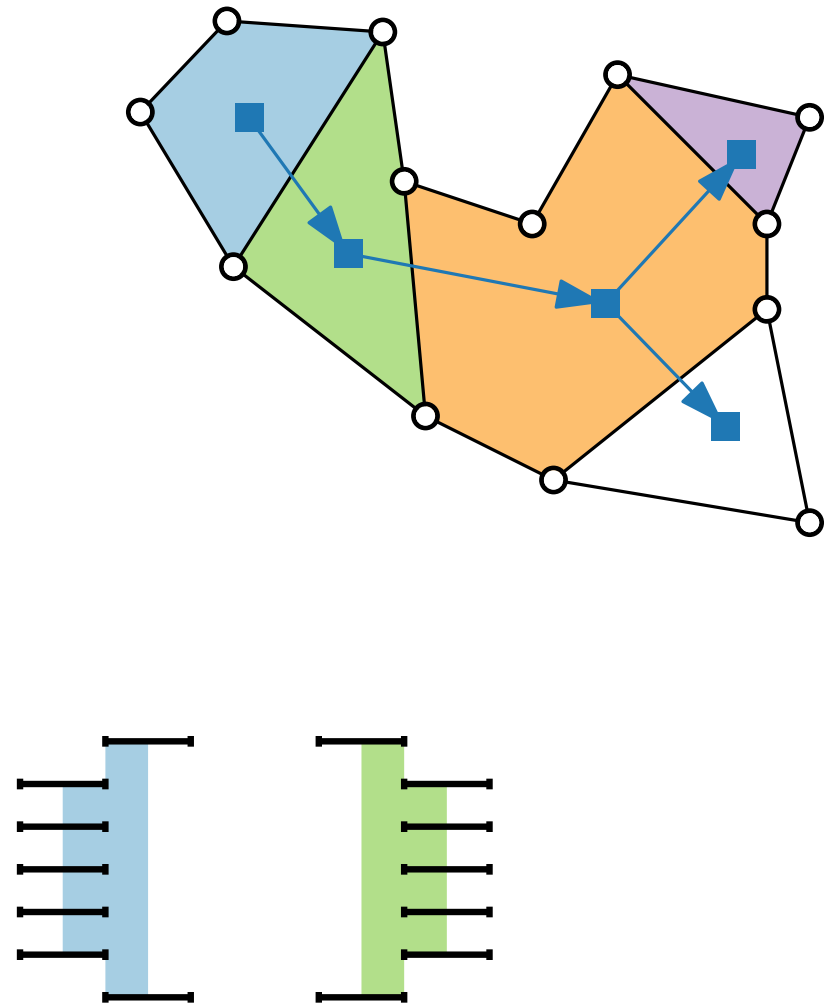
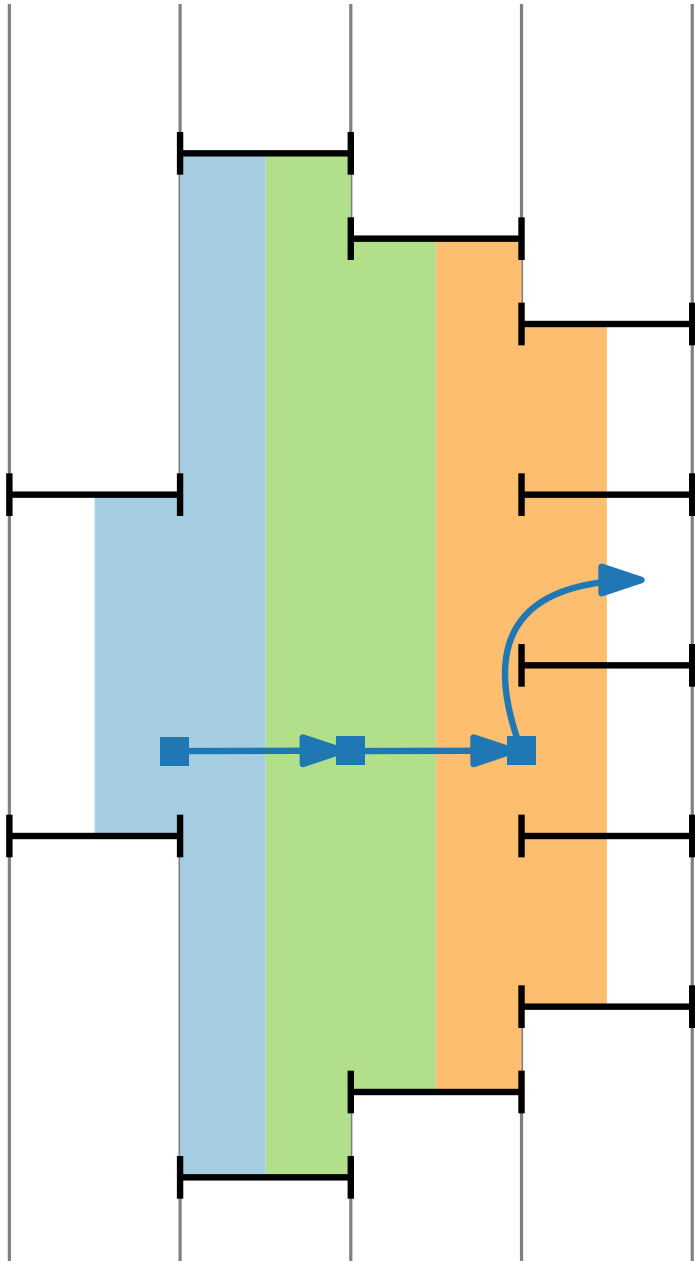
Weak: Biconnected Outerplanar, maxdeg 3



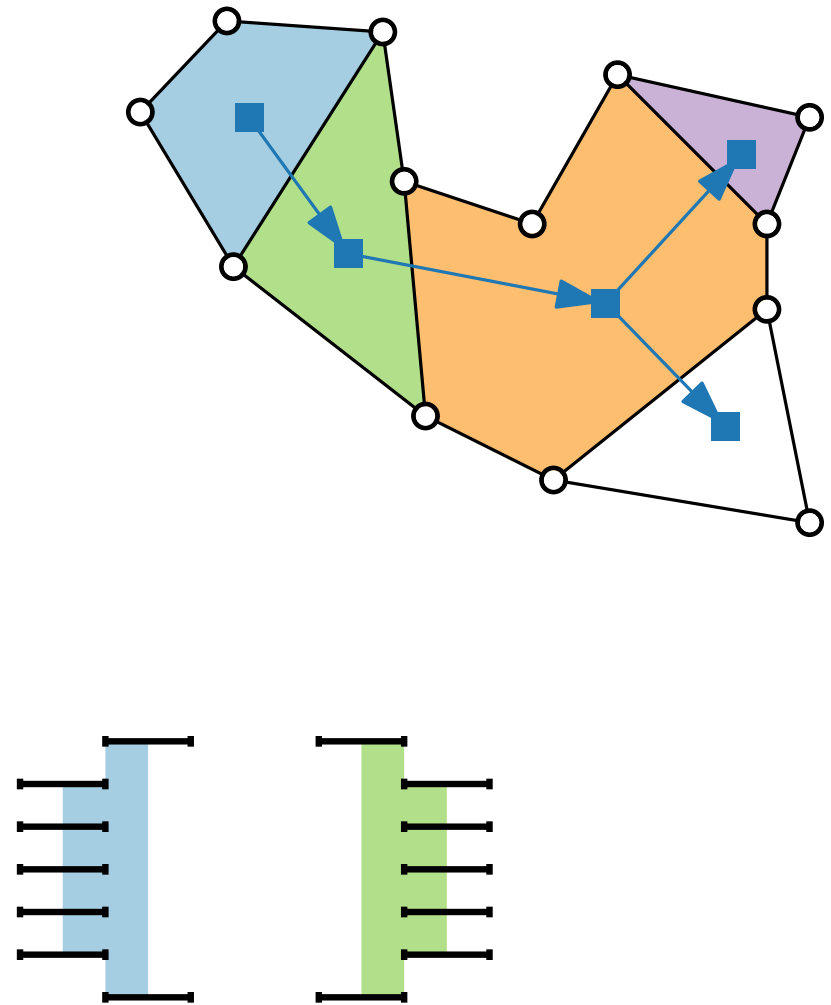
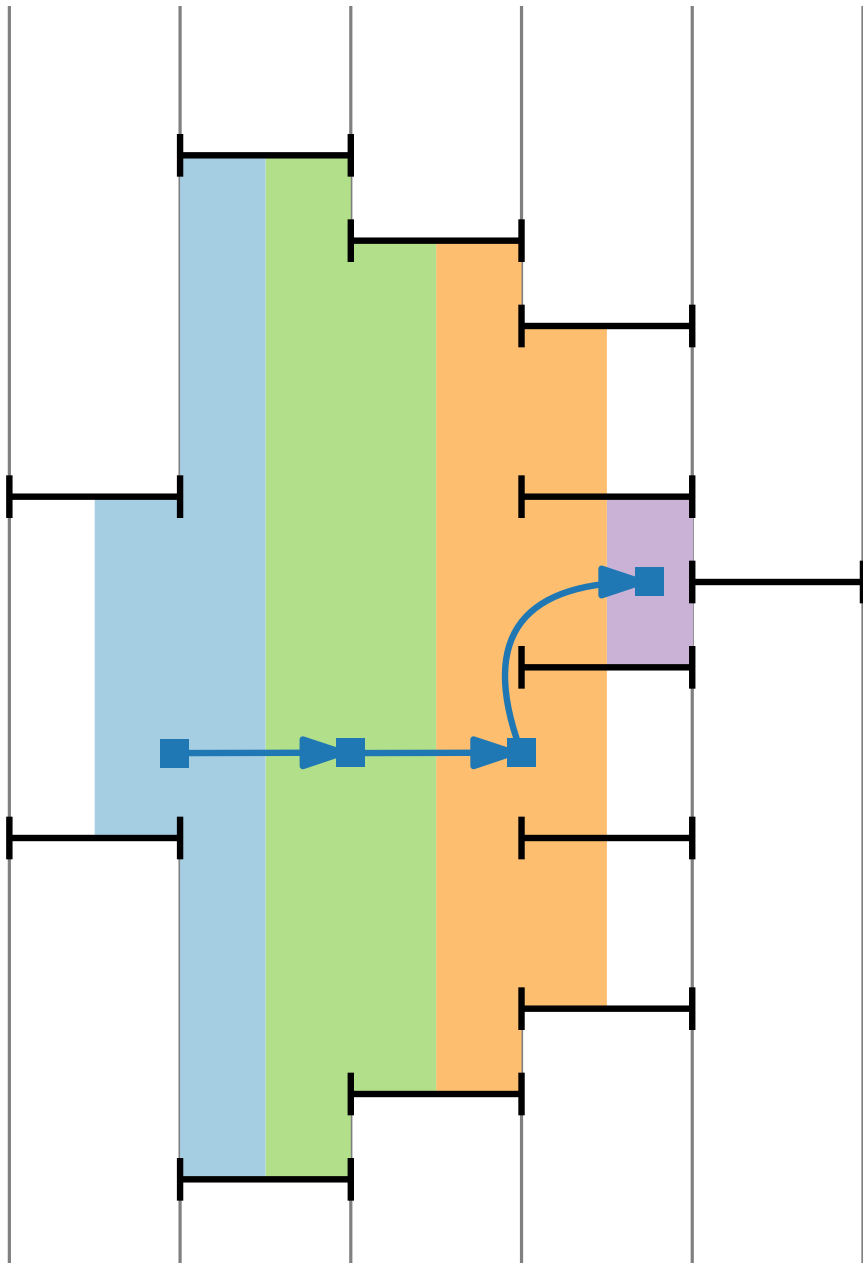
Weak: Biconnected Outerplanar, maxdeg 3



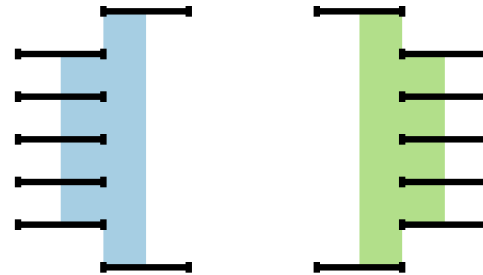
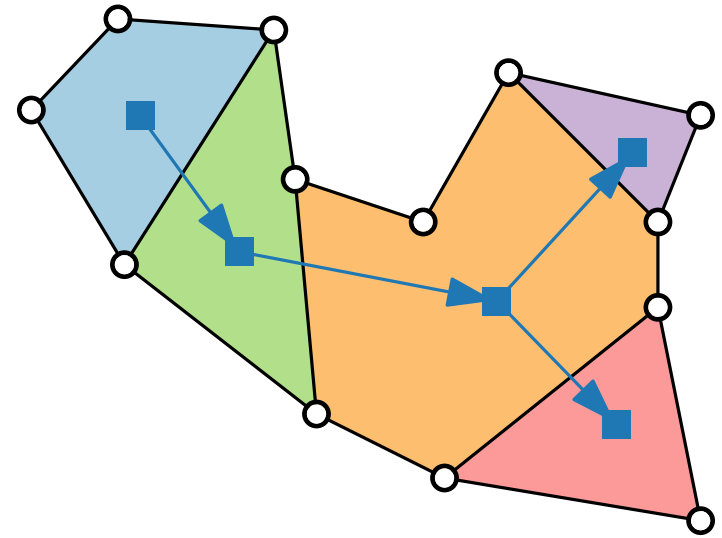
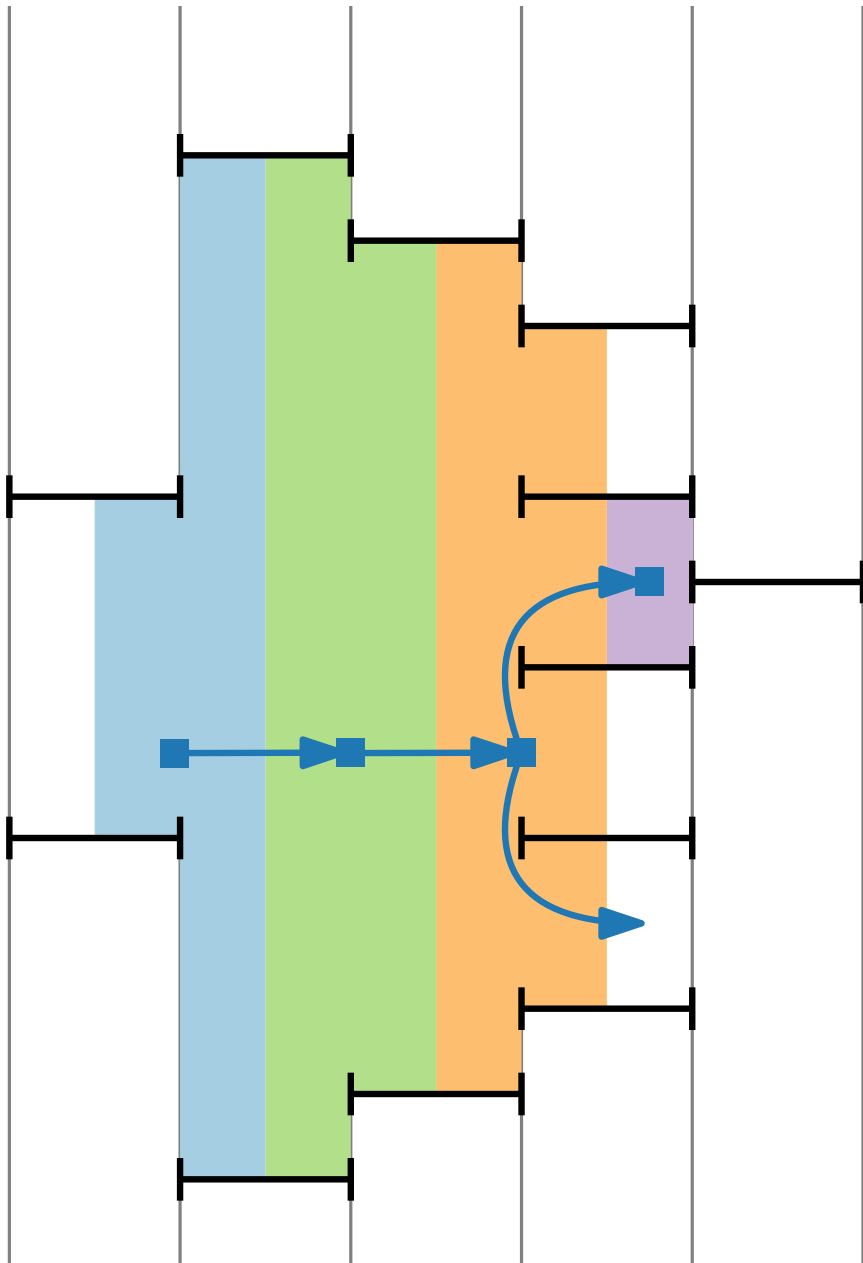
Weak: Biconnected Outerplanar, maxdeg 3



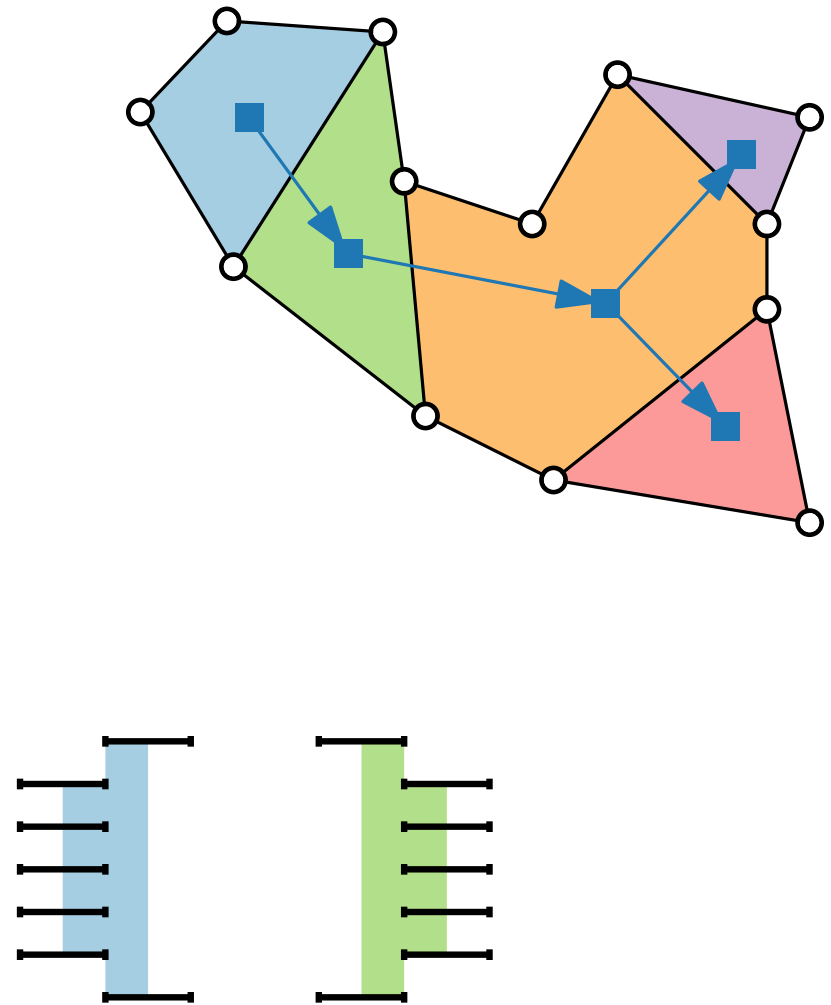
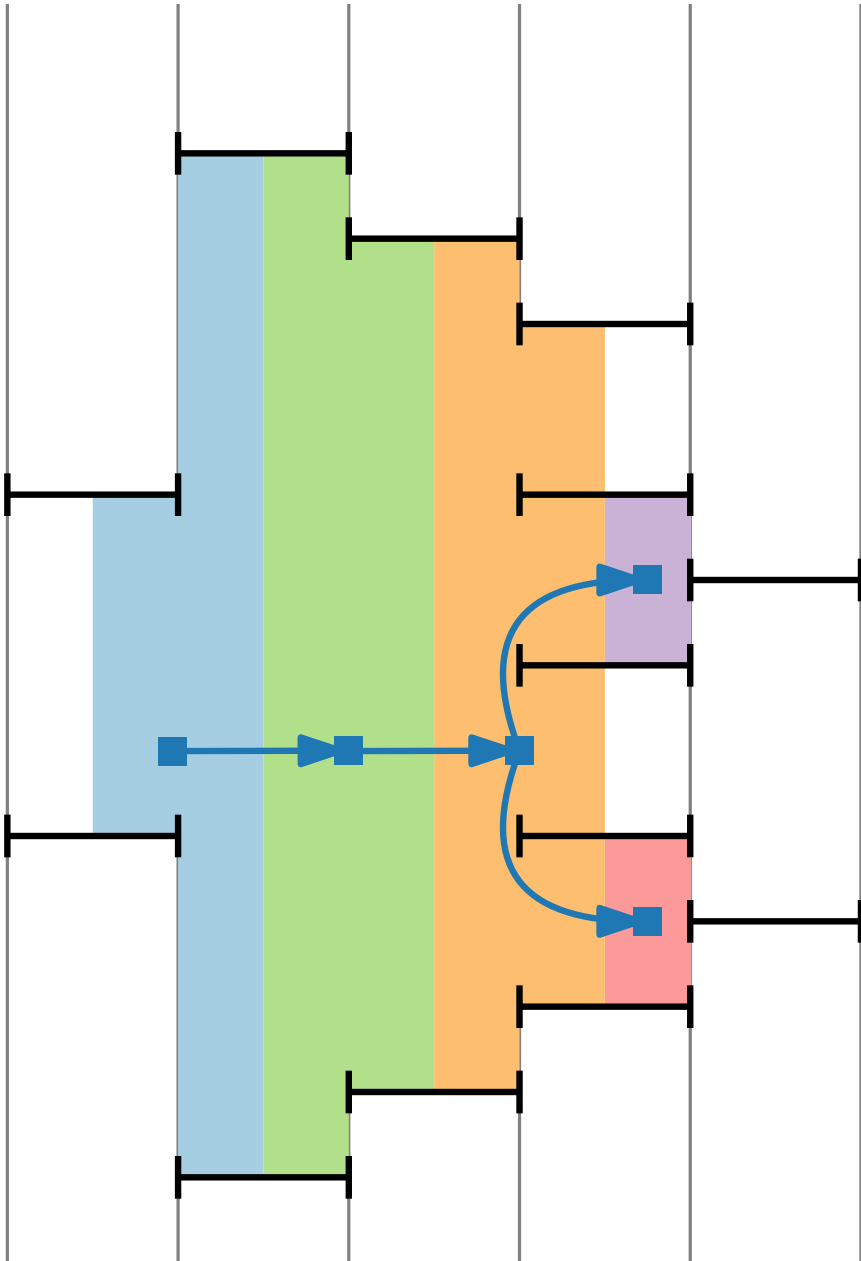
Weak: Biconnected Outerplanar, maxdeg 3



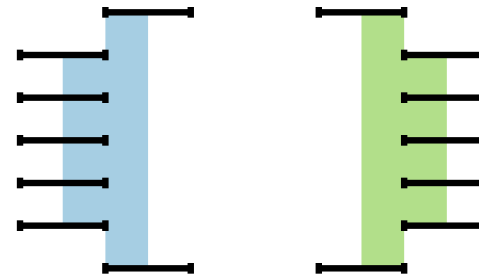
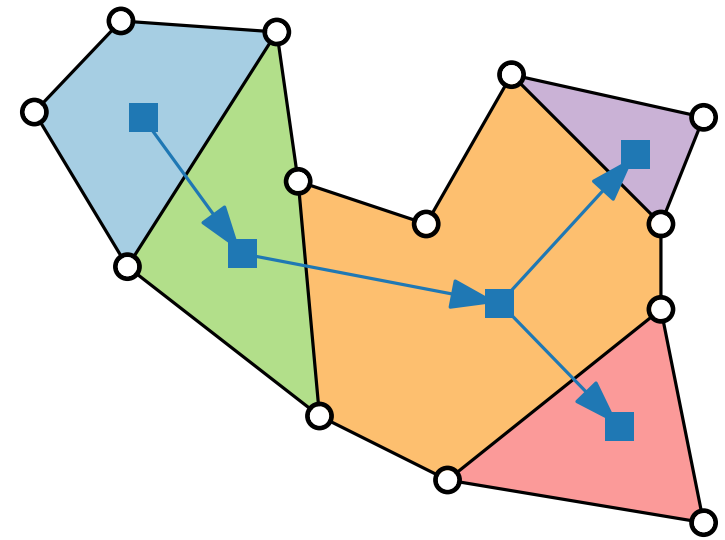
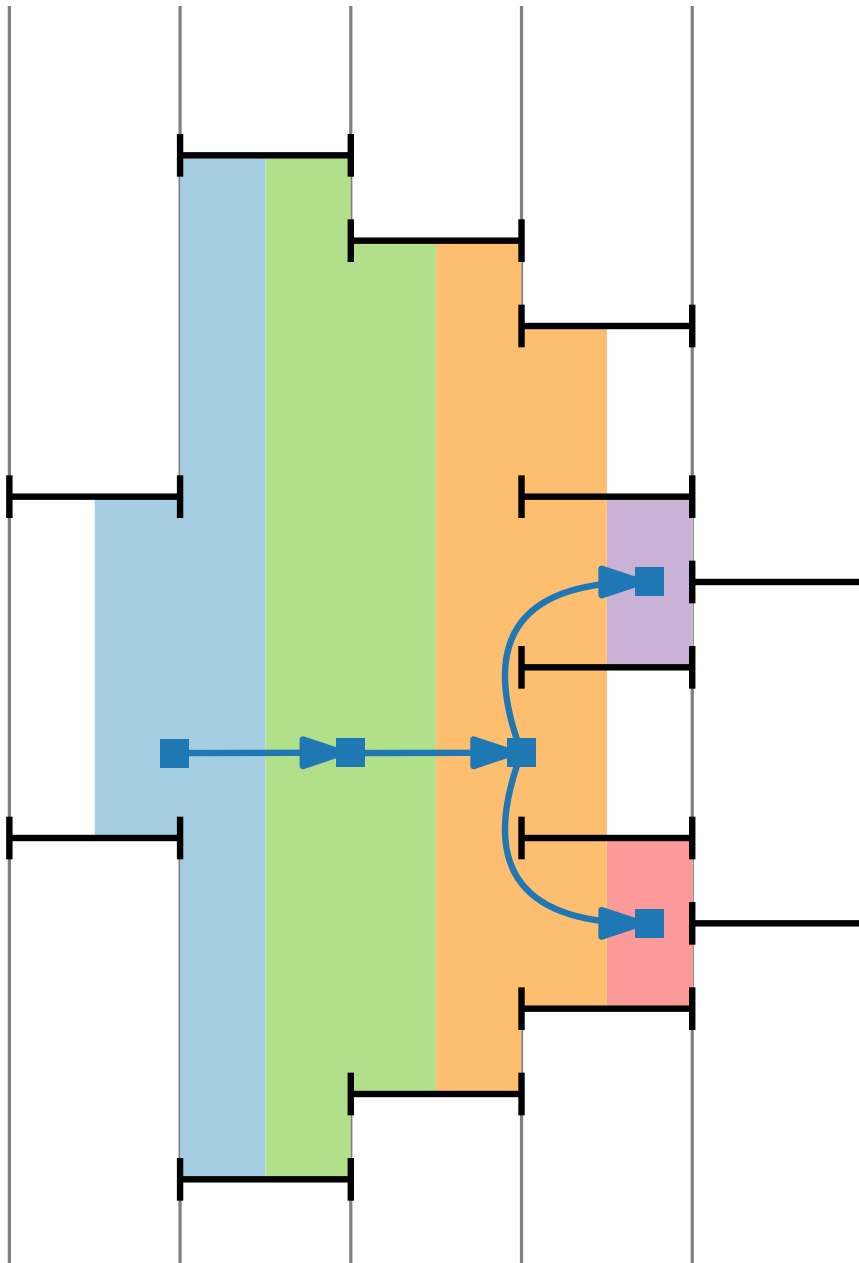
Weak: Biconnected Outerplanar, maxdeg 3



Weak: Biconnected Outerplanar, maxdeg 3

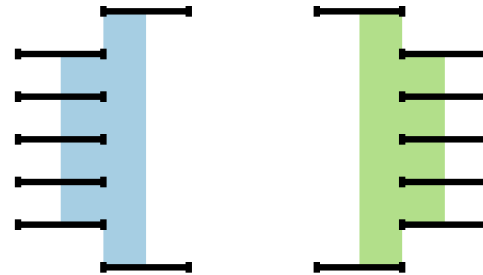
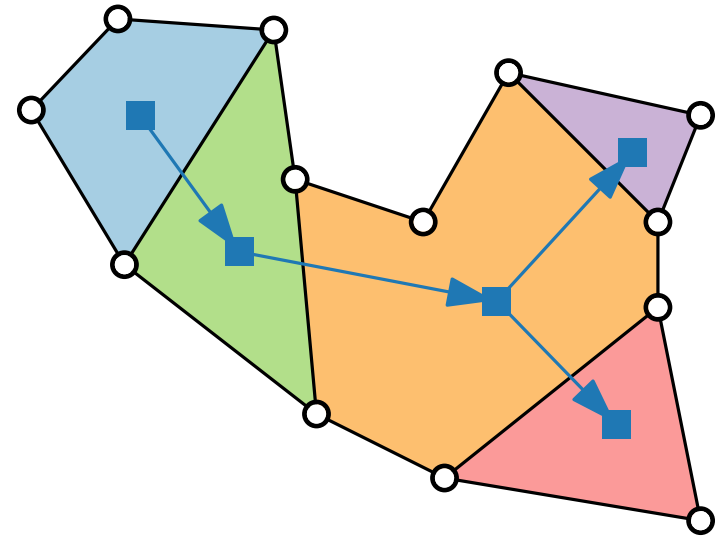
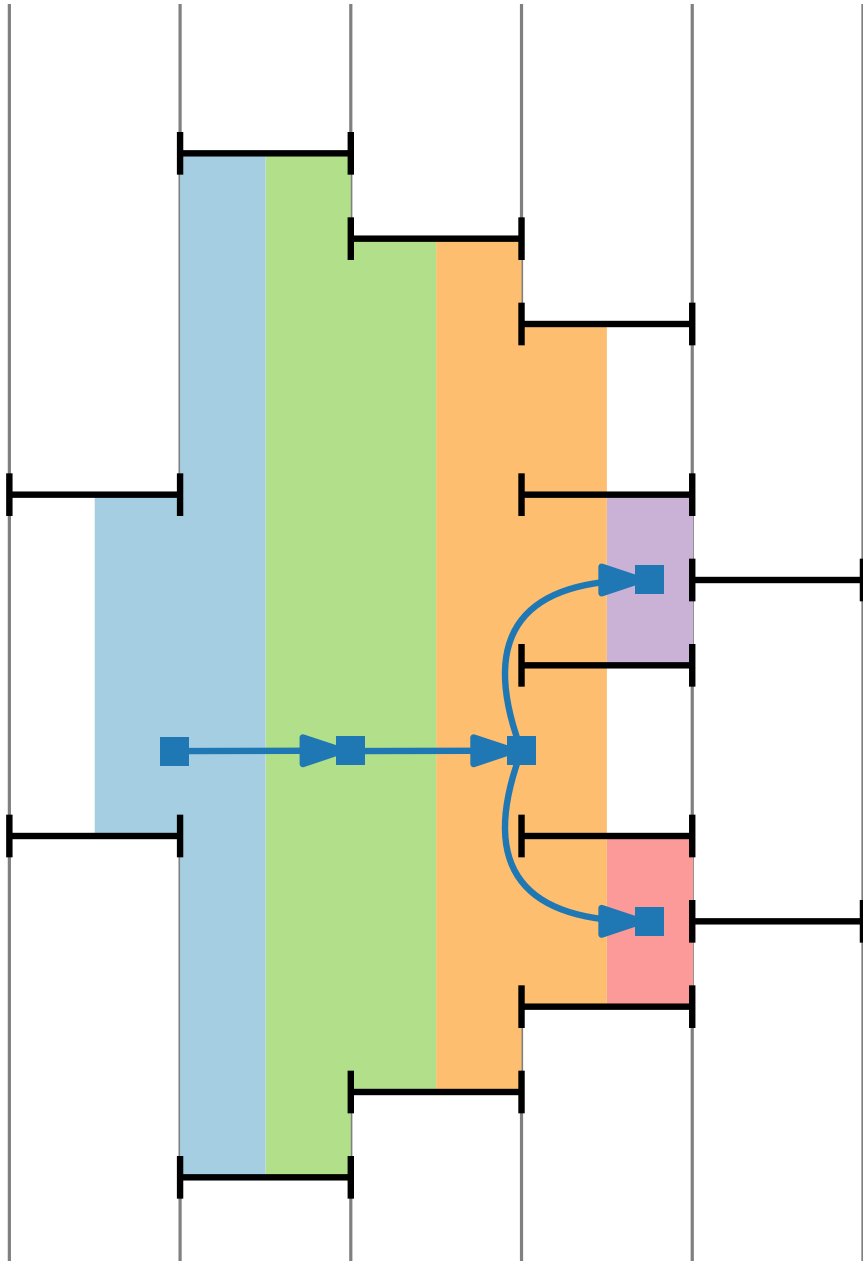


Weak: Biconnected Outerplanar, maxdeg 3

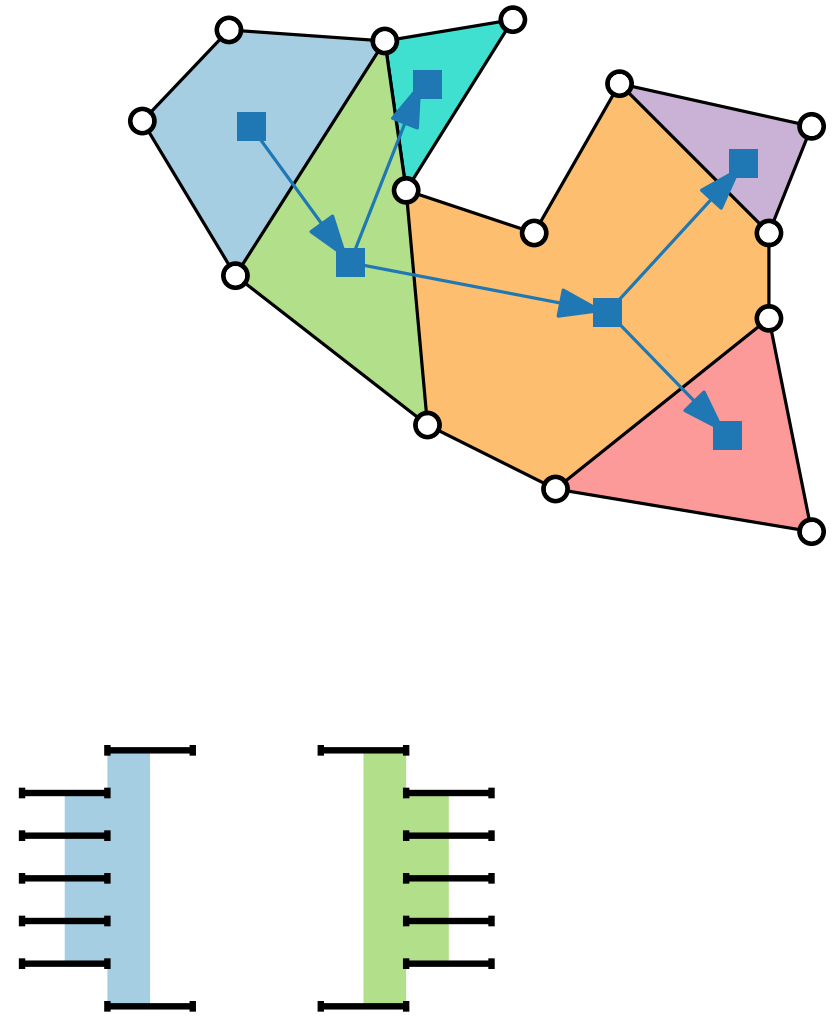
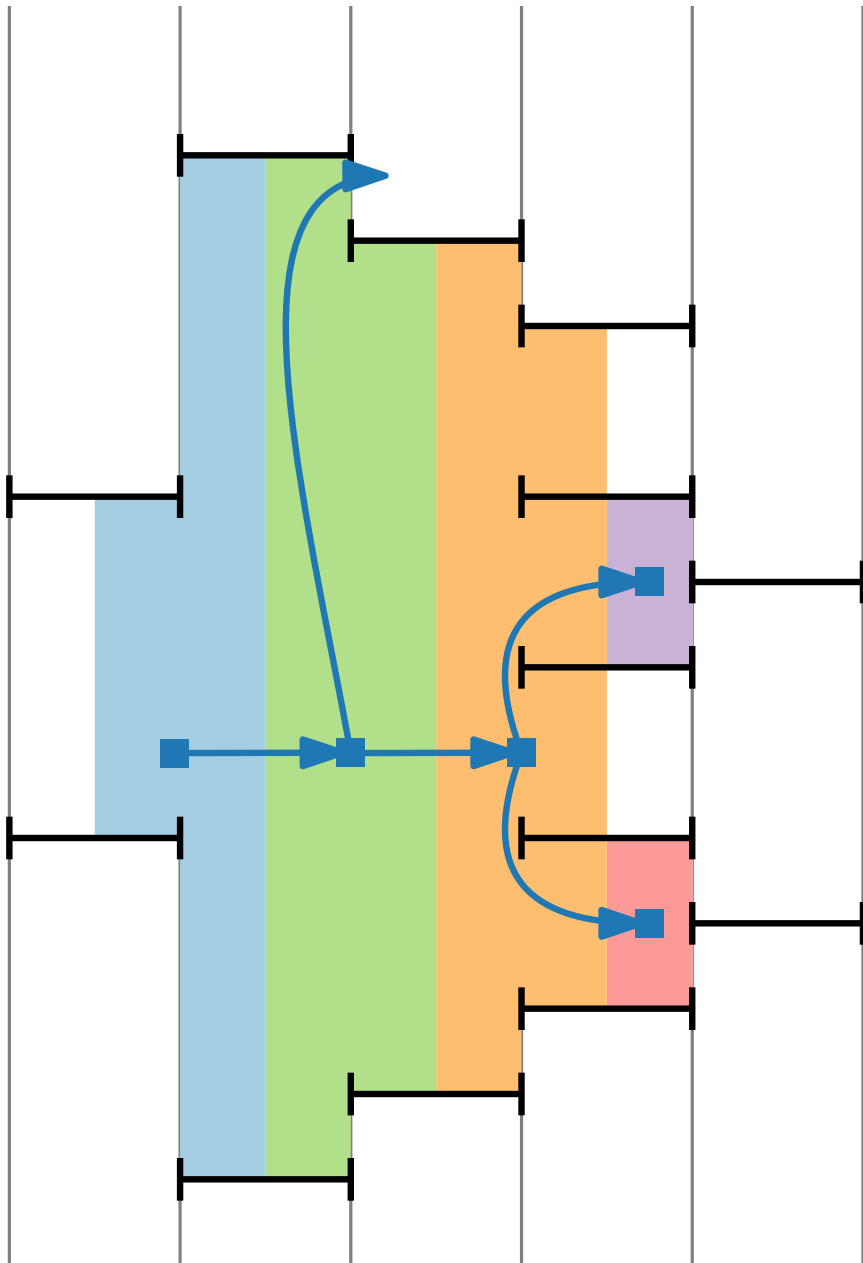


Biconnected Outerplanar
with maxdeg 3 ✓

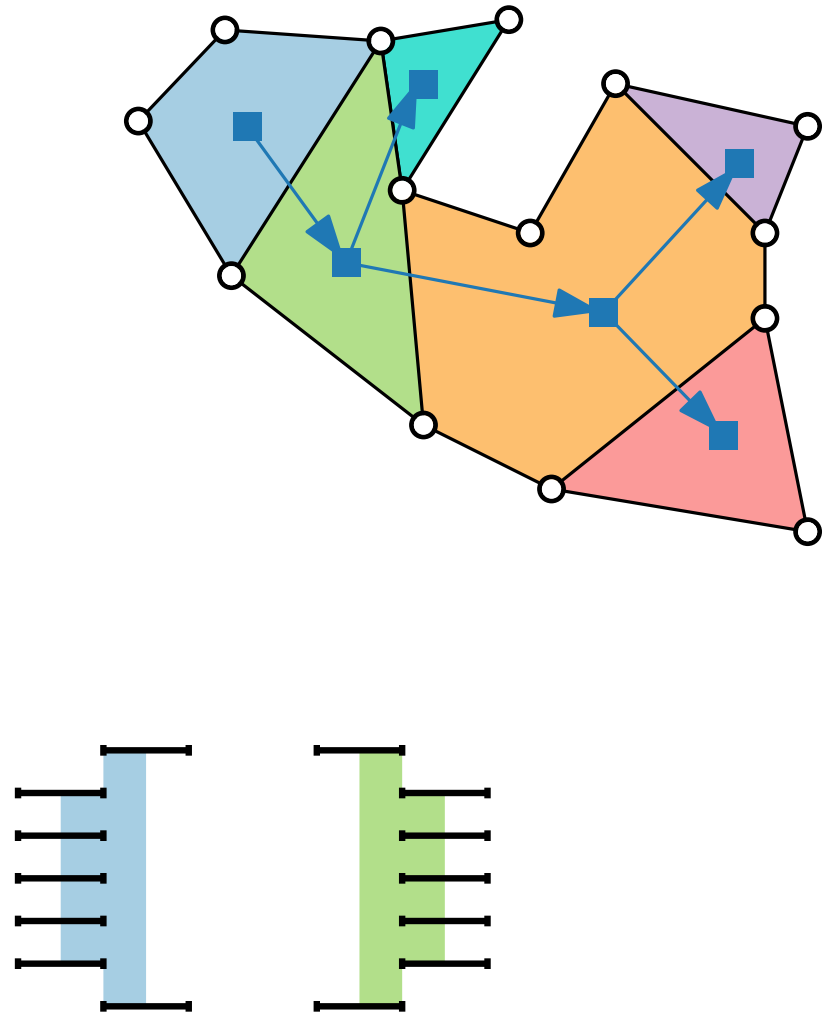
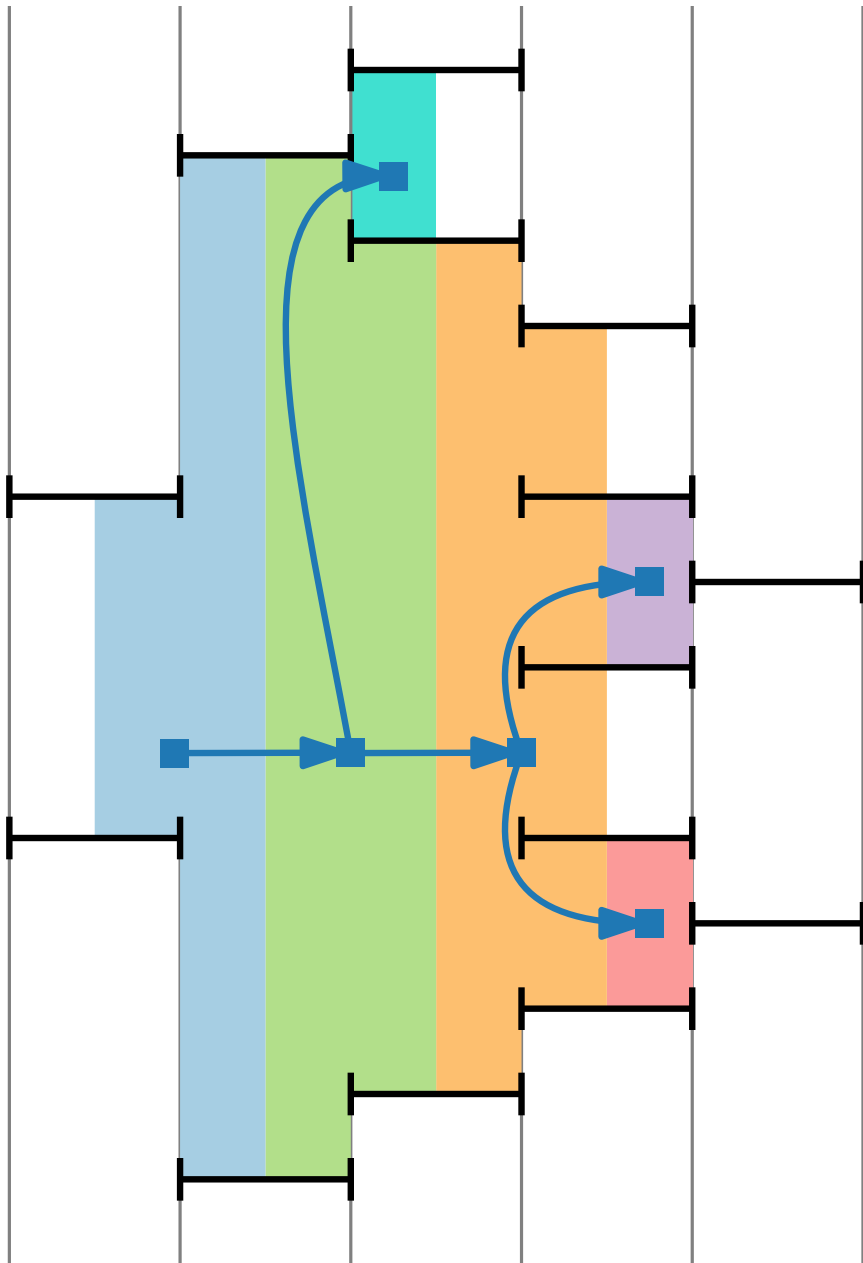
Weak: Biconnected Outerplanar, maxdeg 4



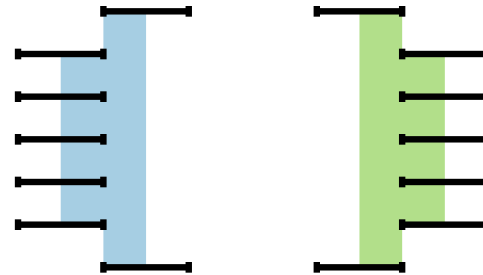
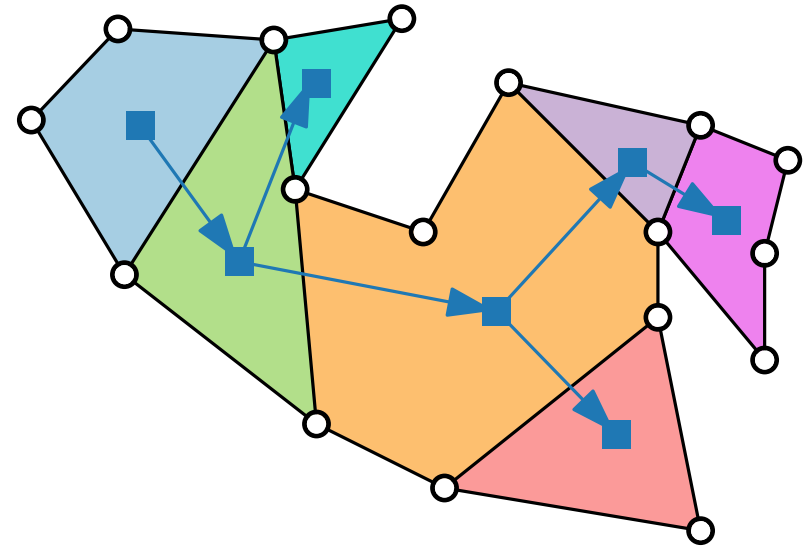
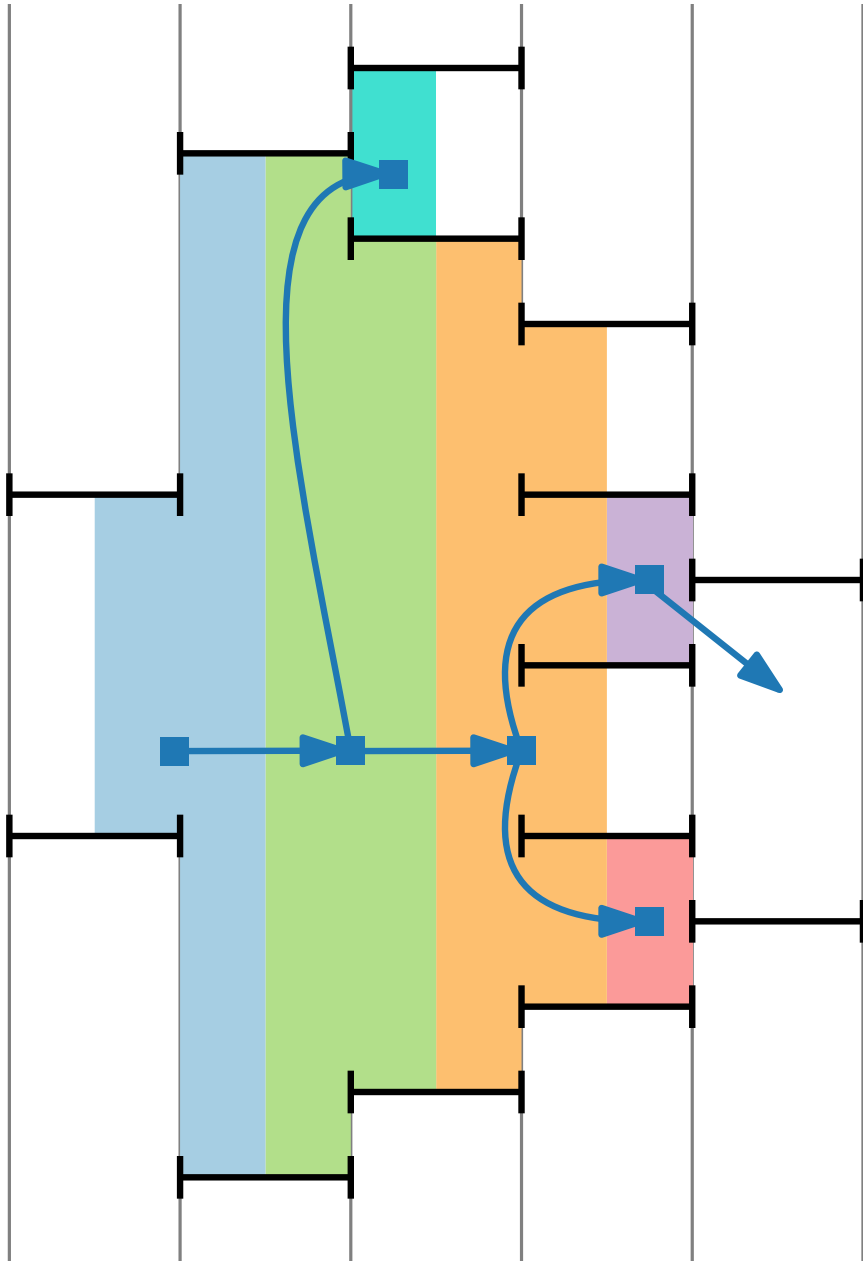
Weak: Biconnected Outerplanar, maxdeg 4



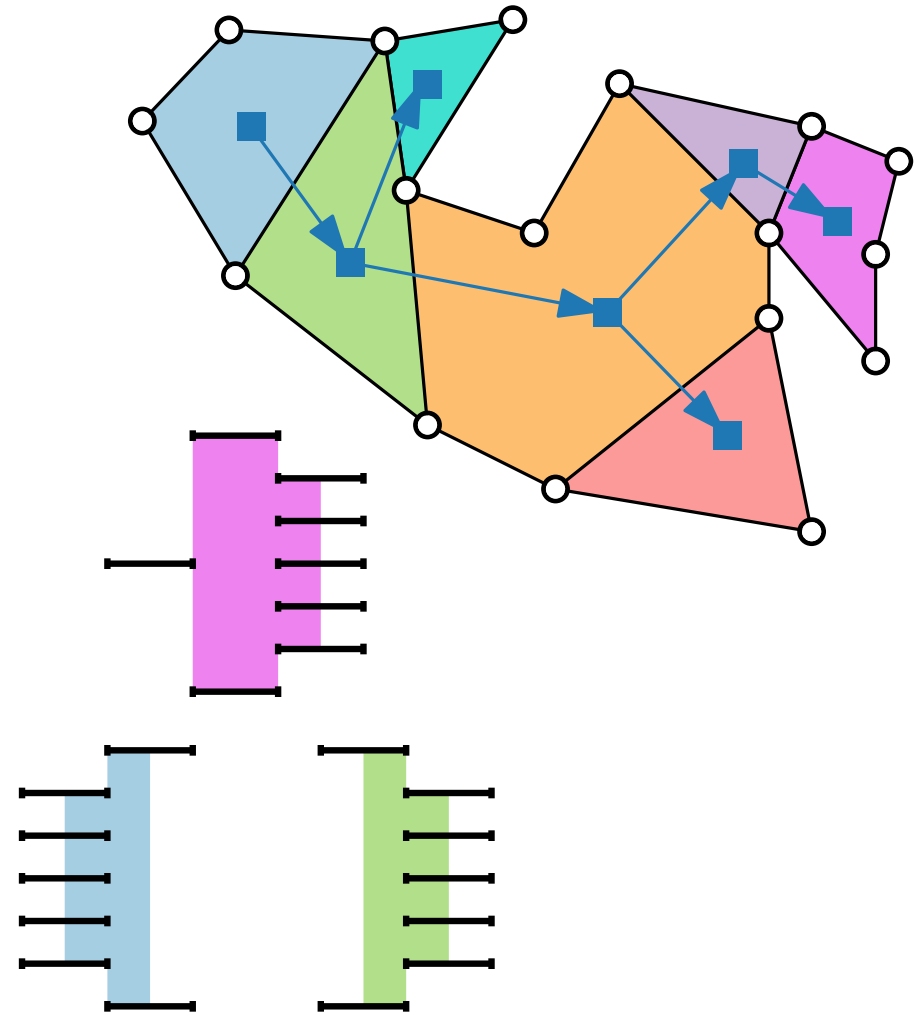
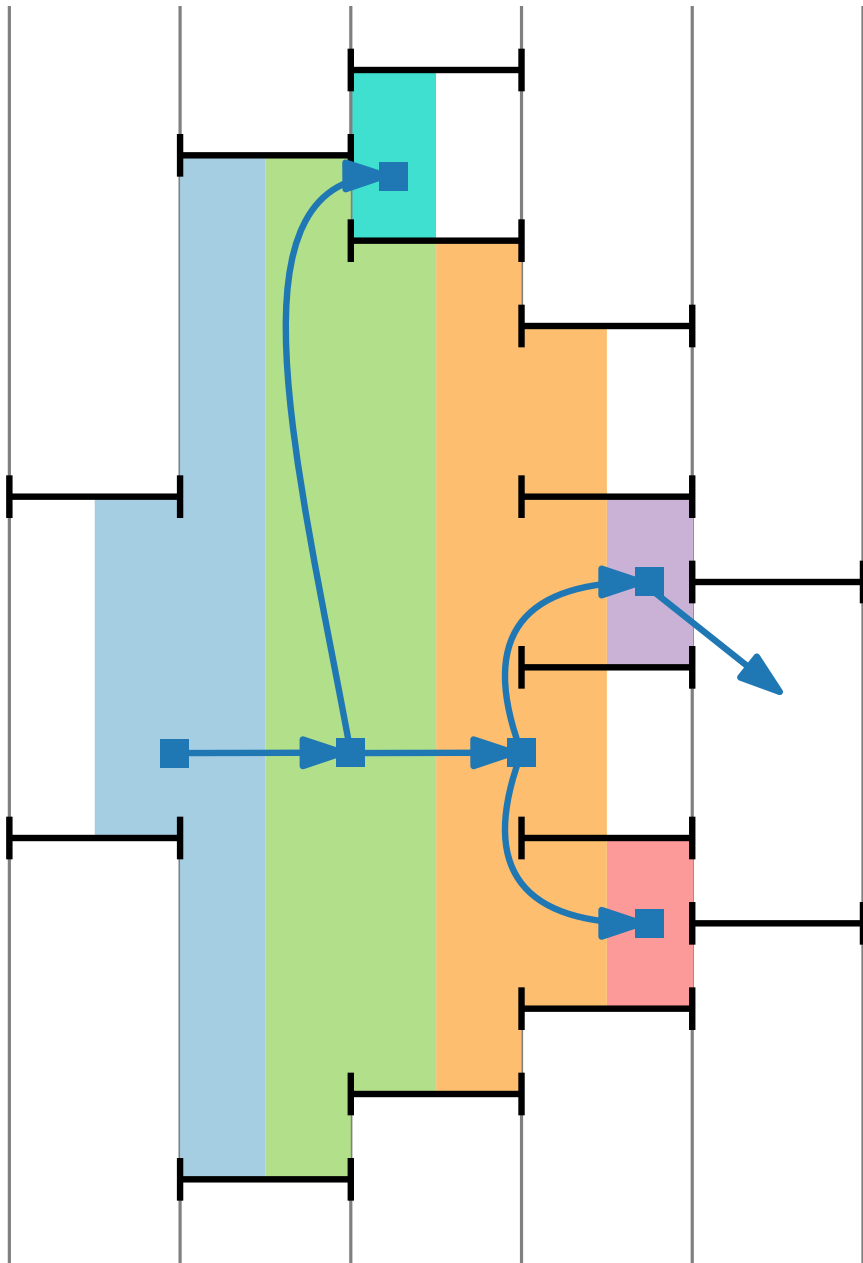
Weak: Biconnected Outerplanar, maxdeg 4



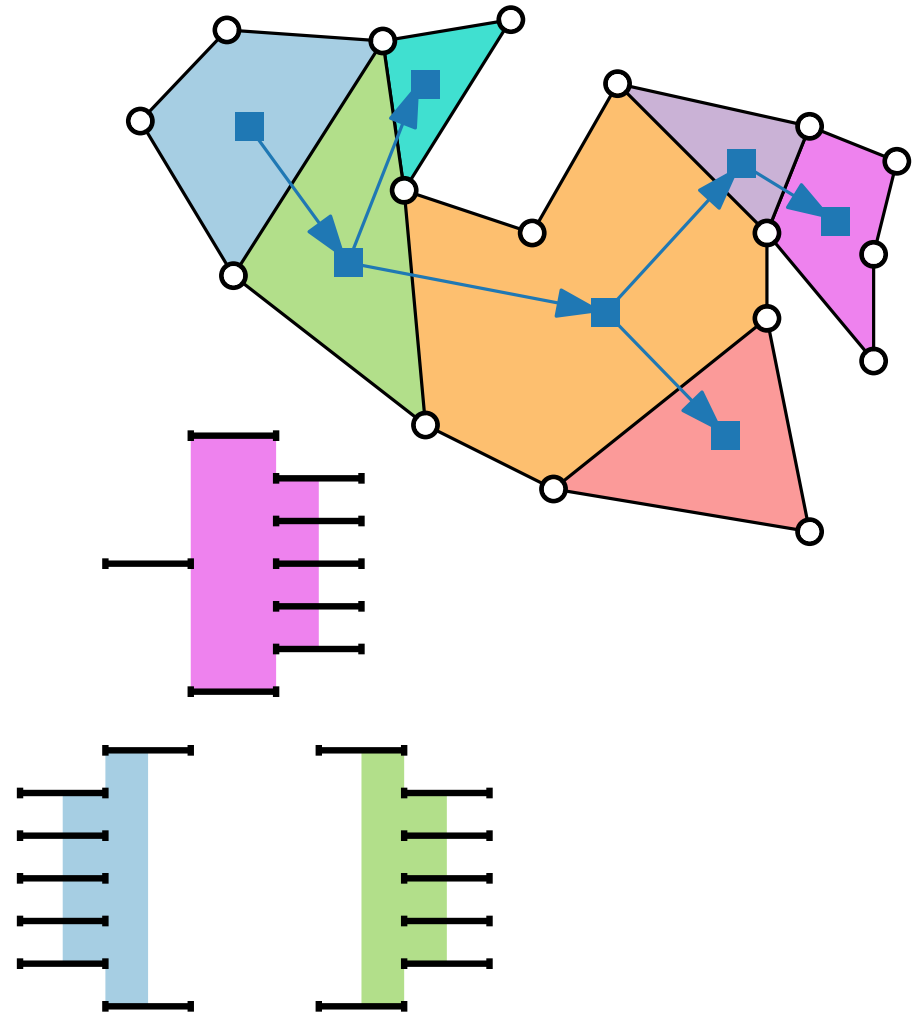
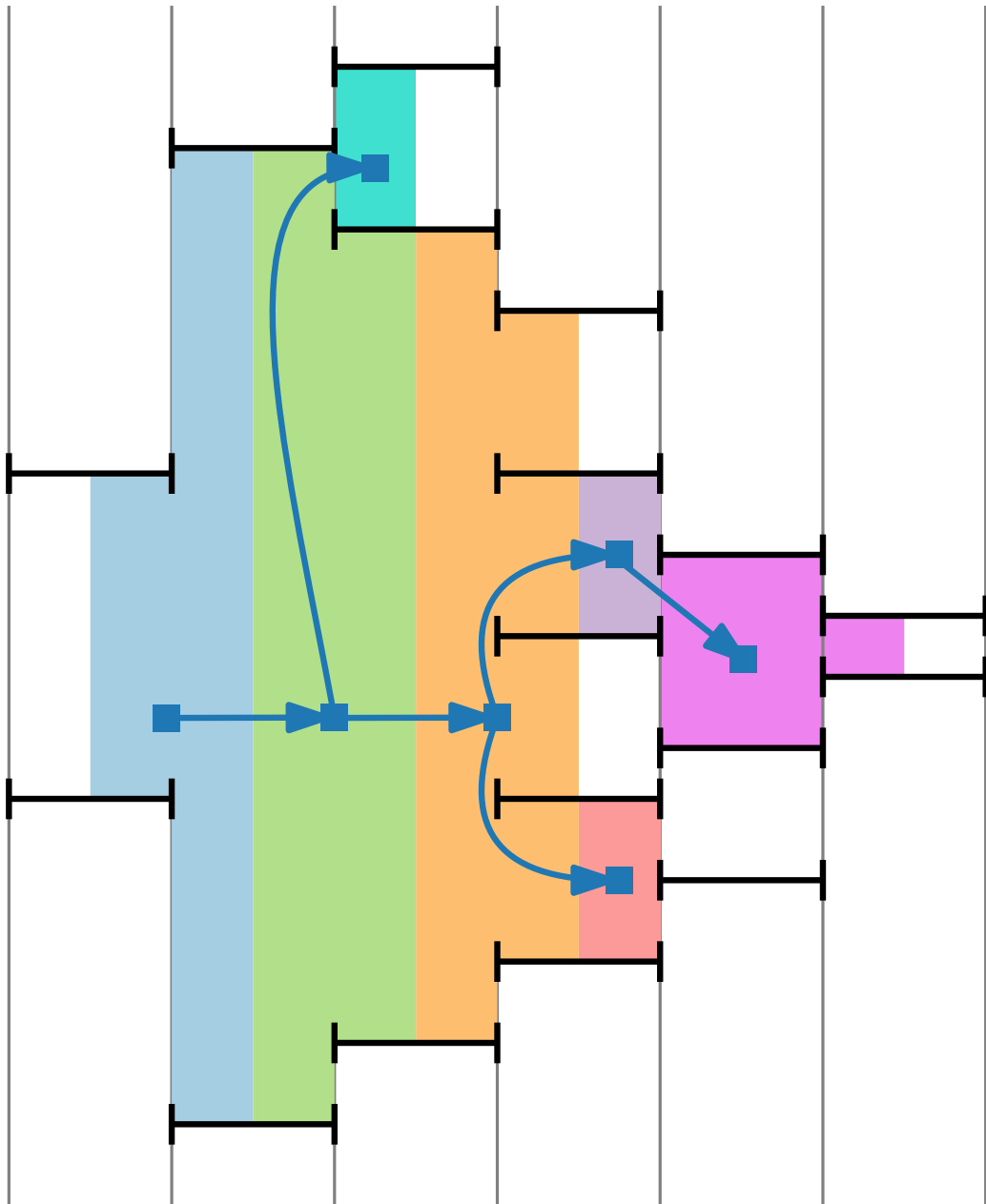
Weak: Biconnected Outerplanar, maxdeg 4



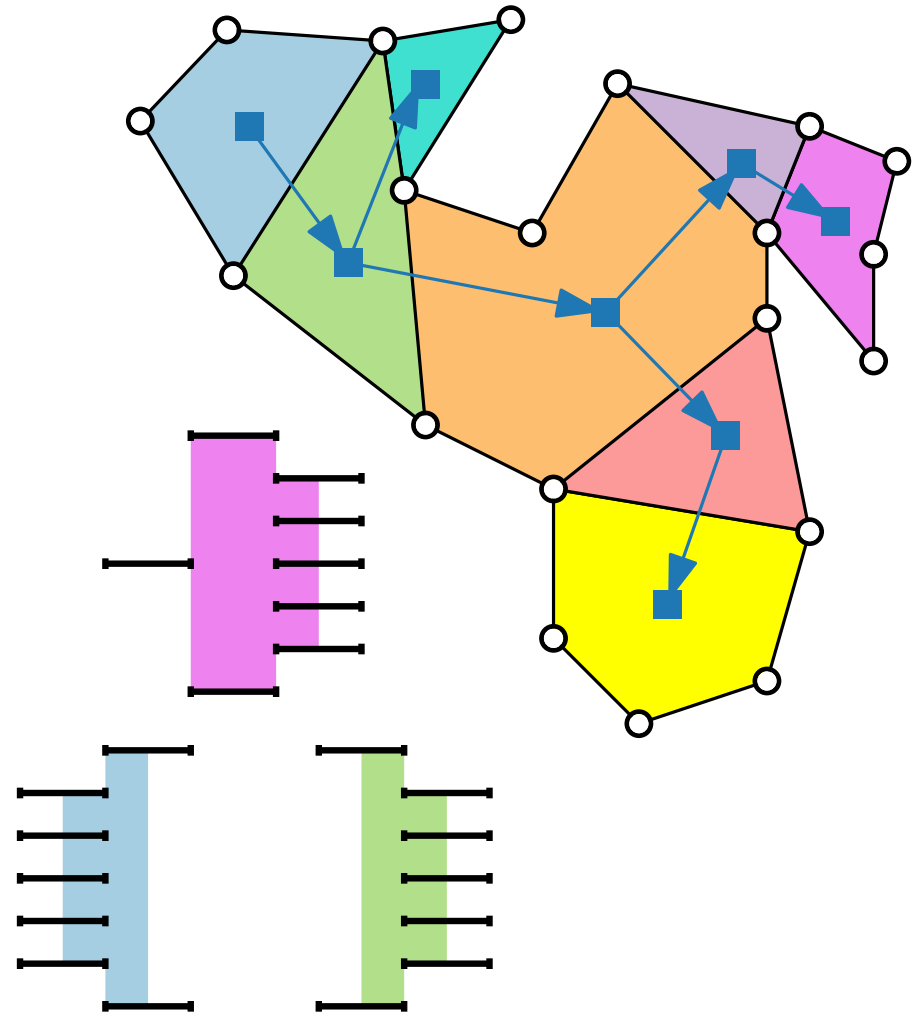
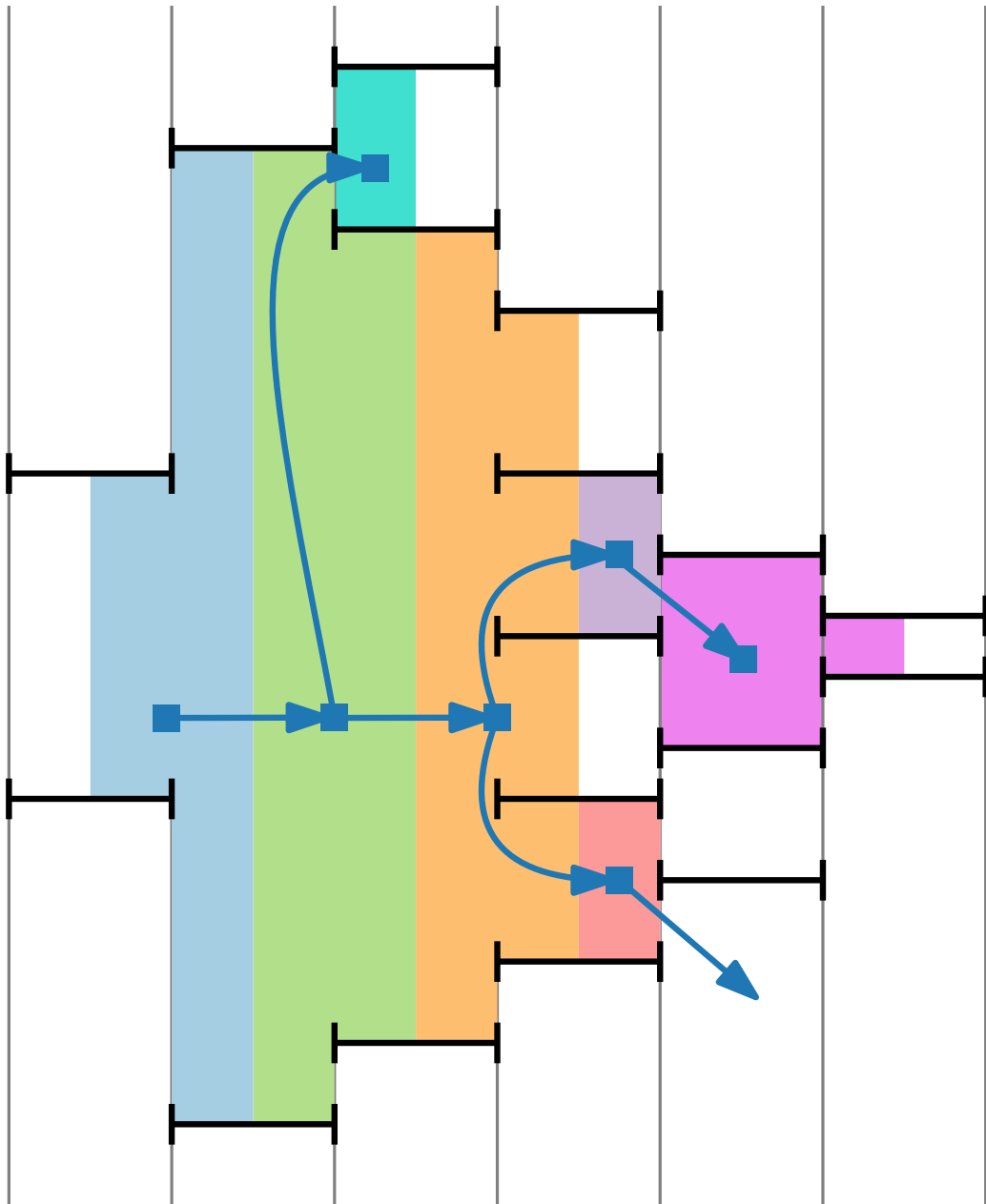
Weak: Biconnected Outerplanar, maxdeg 4



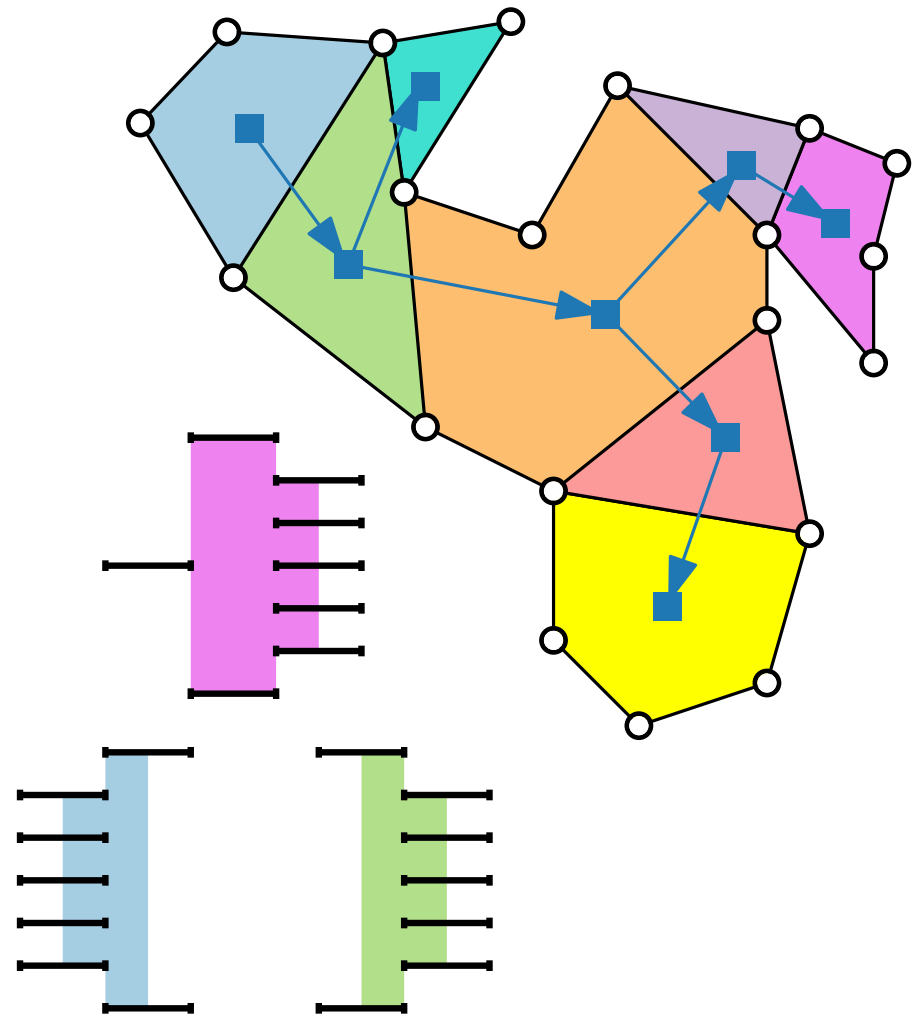
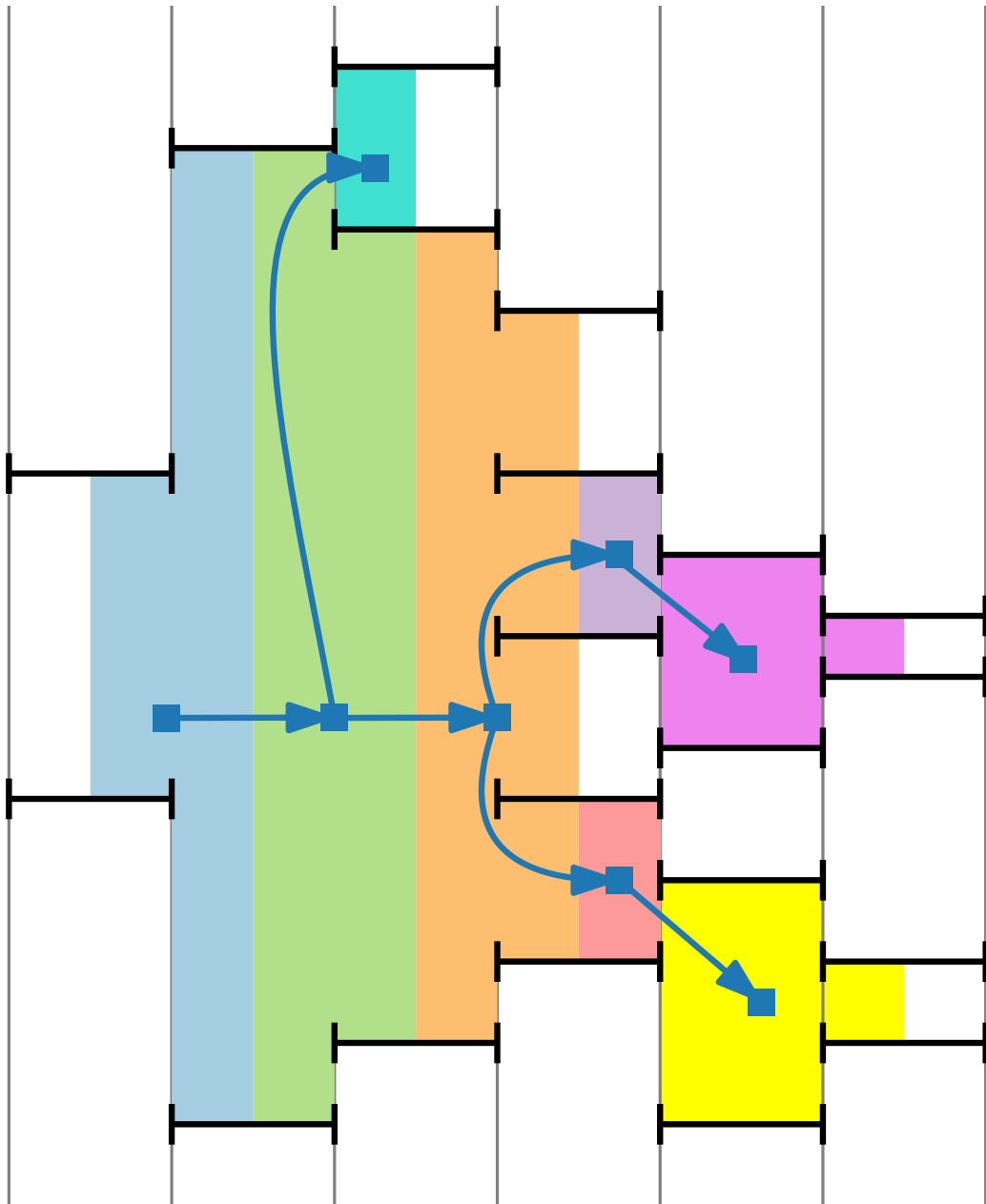
Weak: Biconnected Outerplanar, maxdeg 4



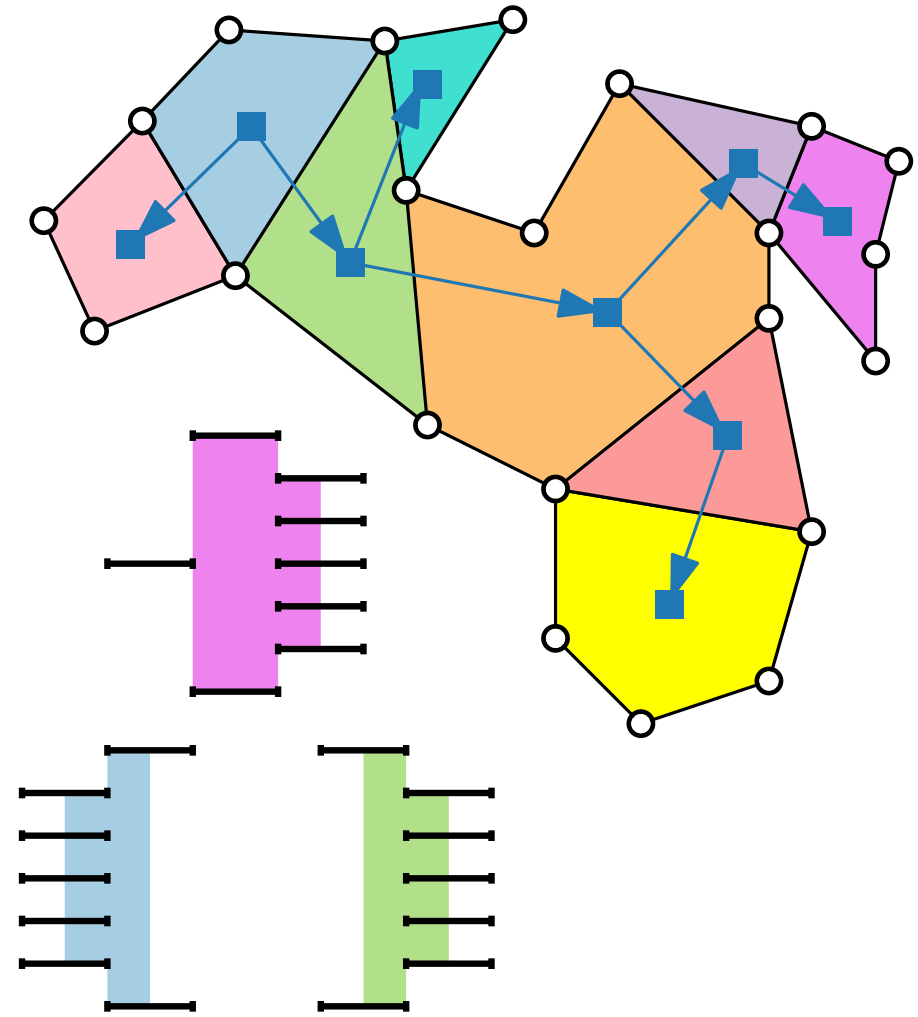
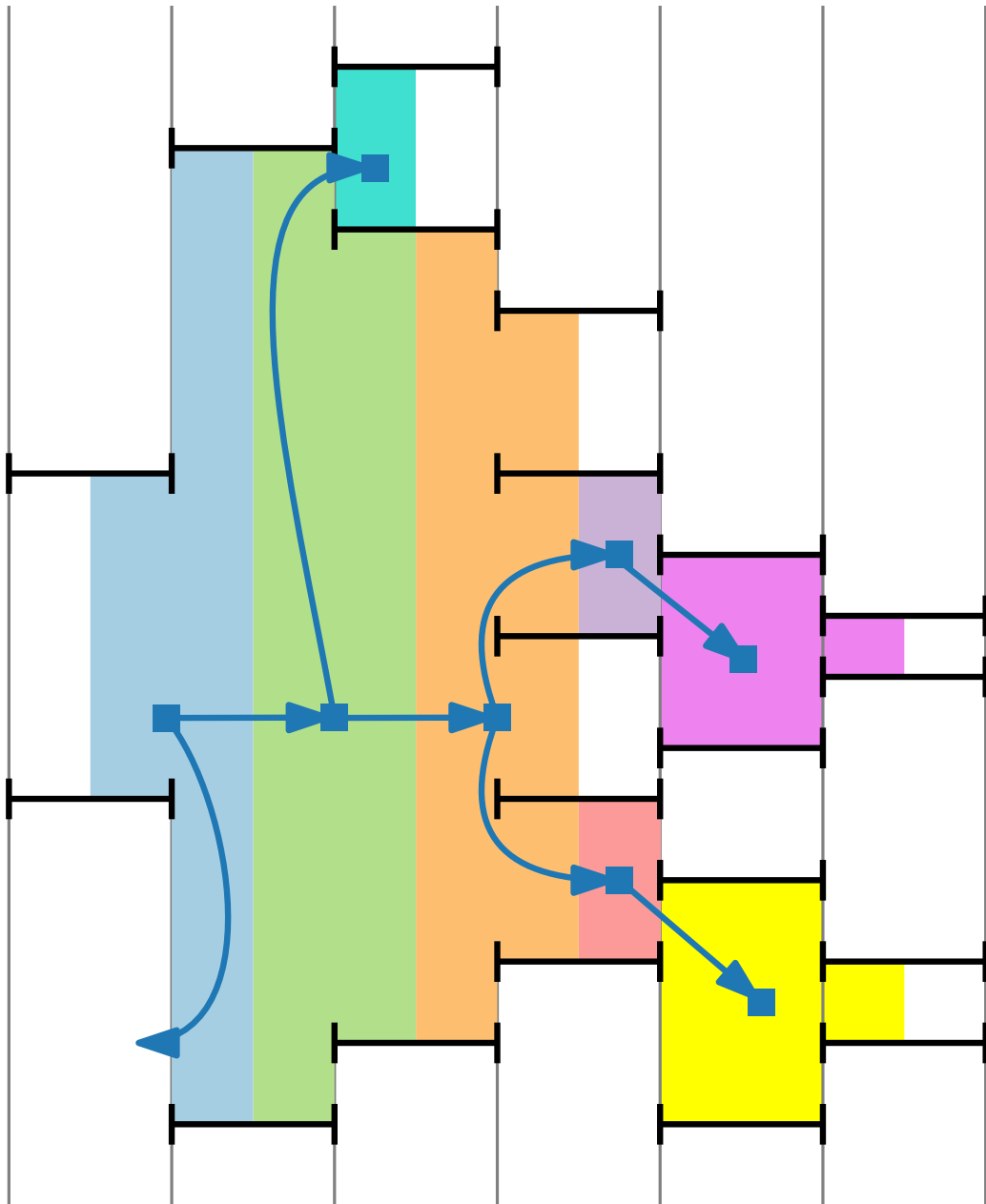
Weak: Biconnected Outerplanar, maxdeg 4



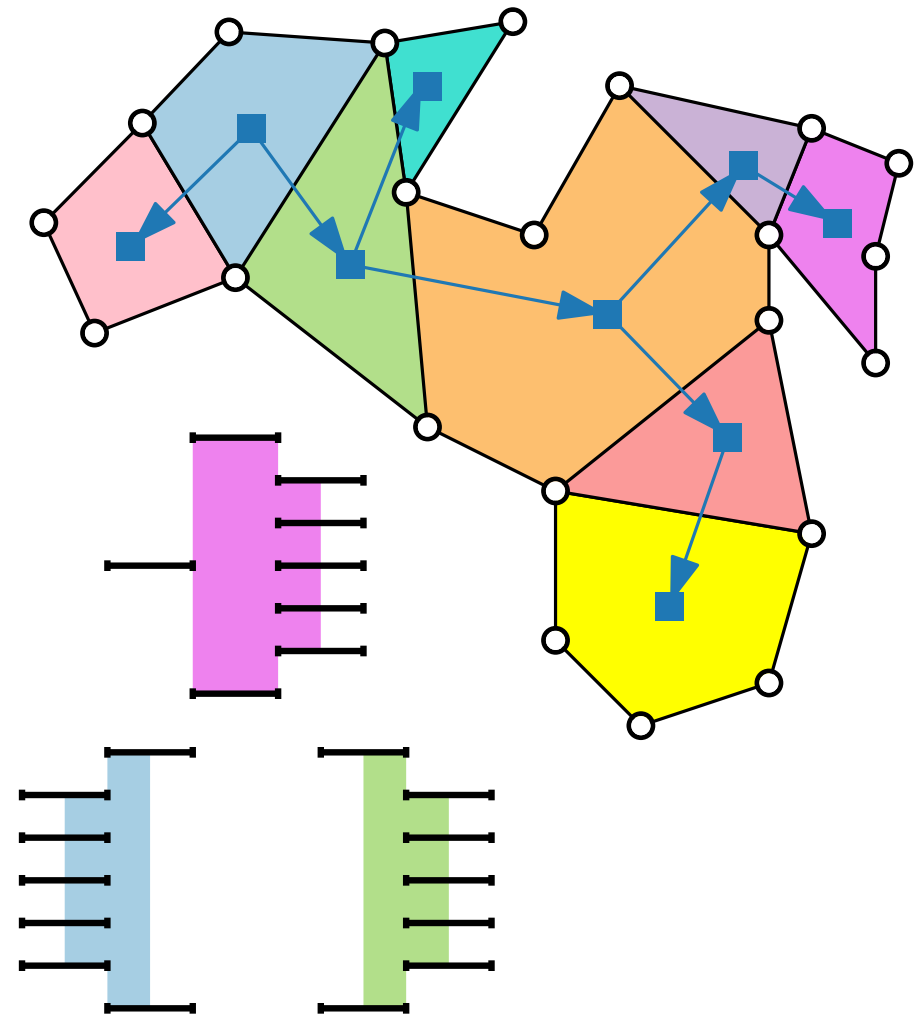
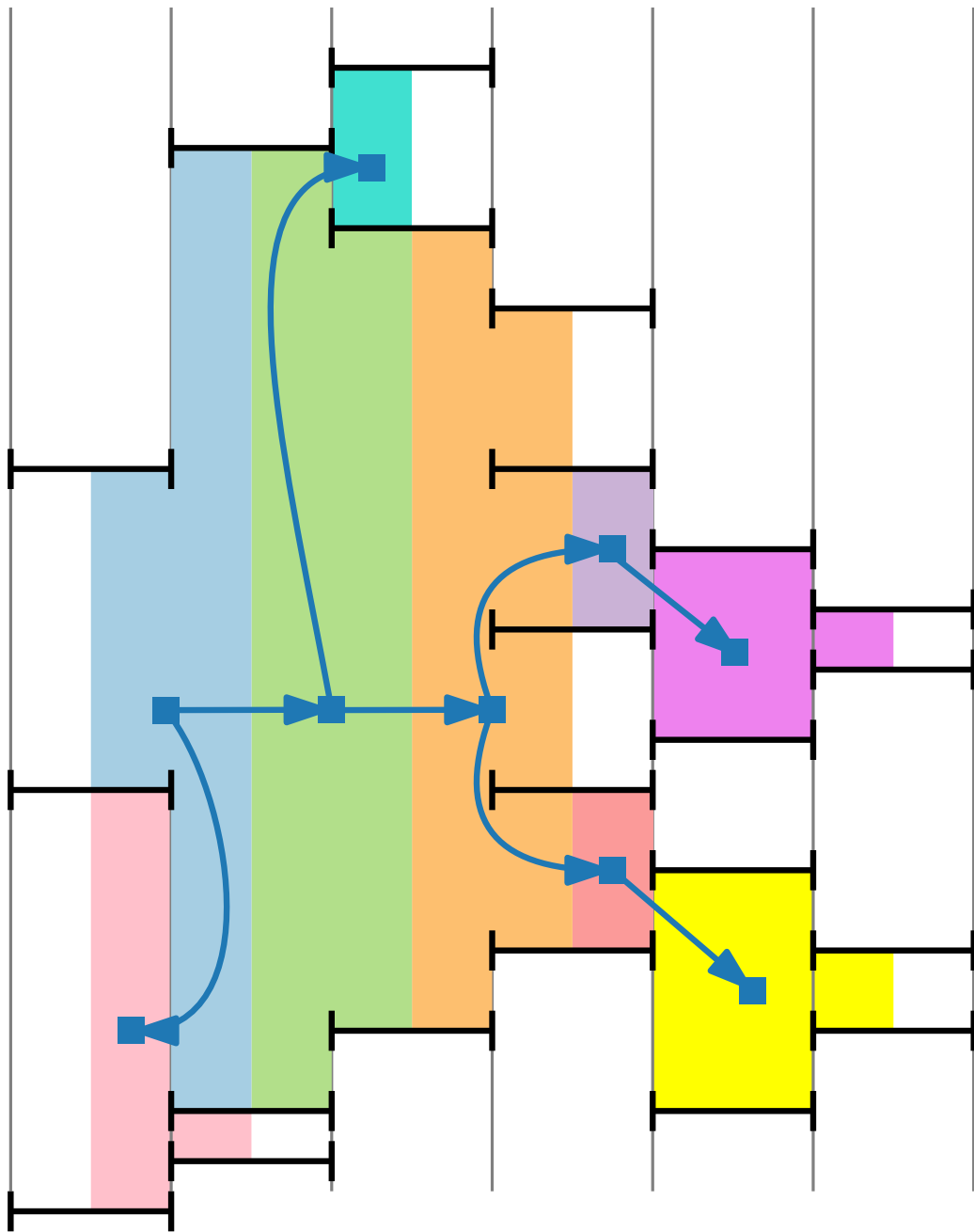
Weak: Biconnected Outerplanar, maxdeg 4



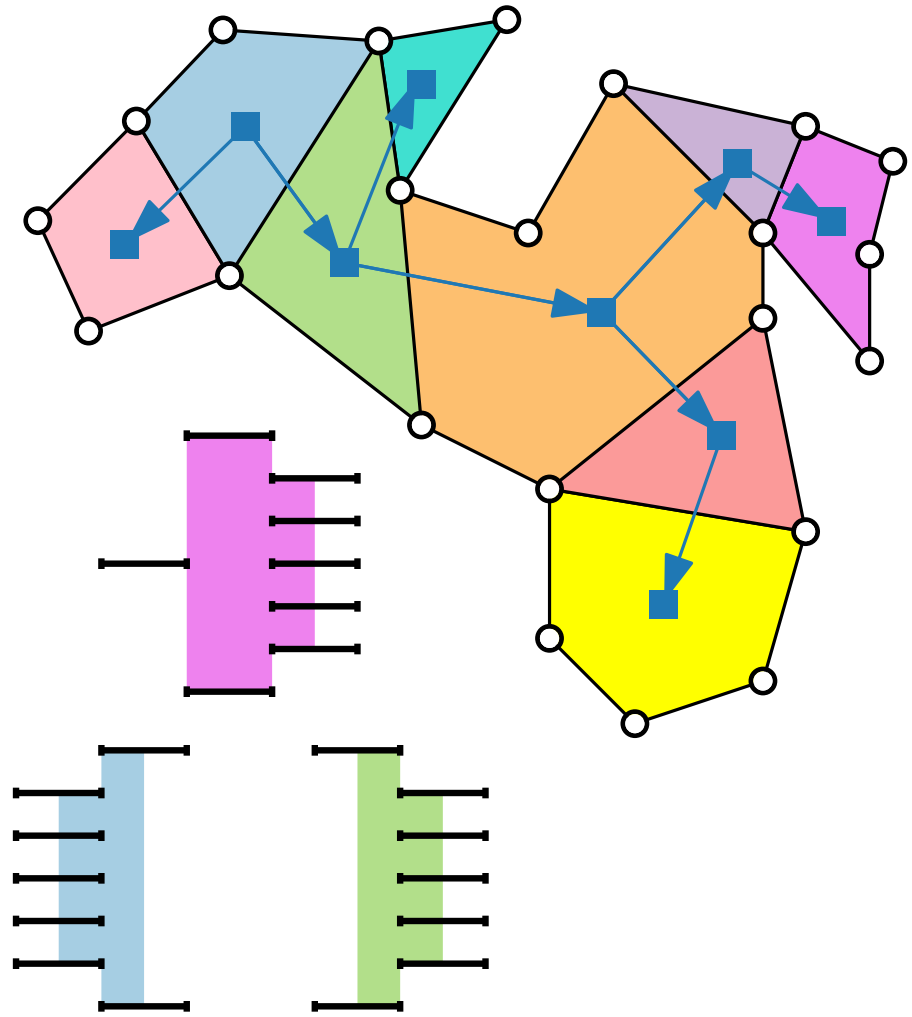
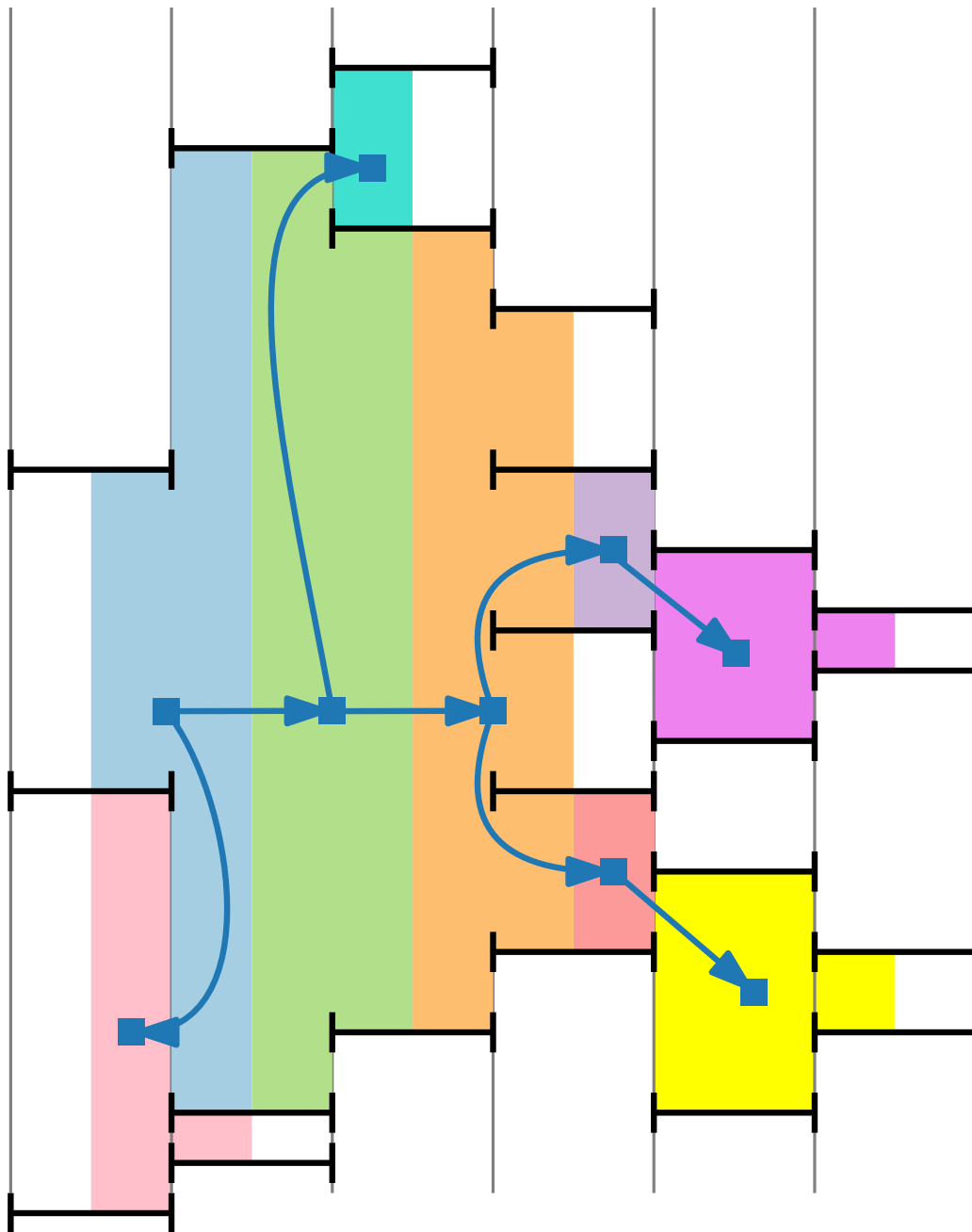
Weak: Biconnected Outerplanar, maxdeg 4



Weak: Biconnected Outerplanar, maxdeg 4

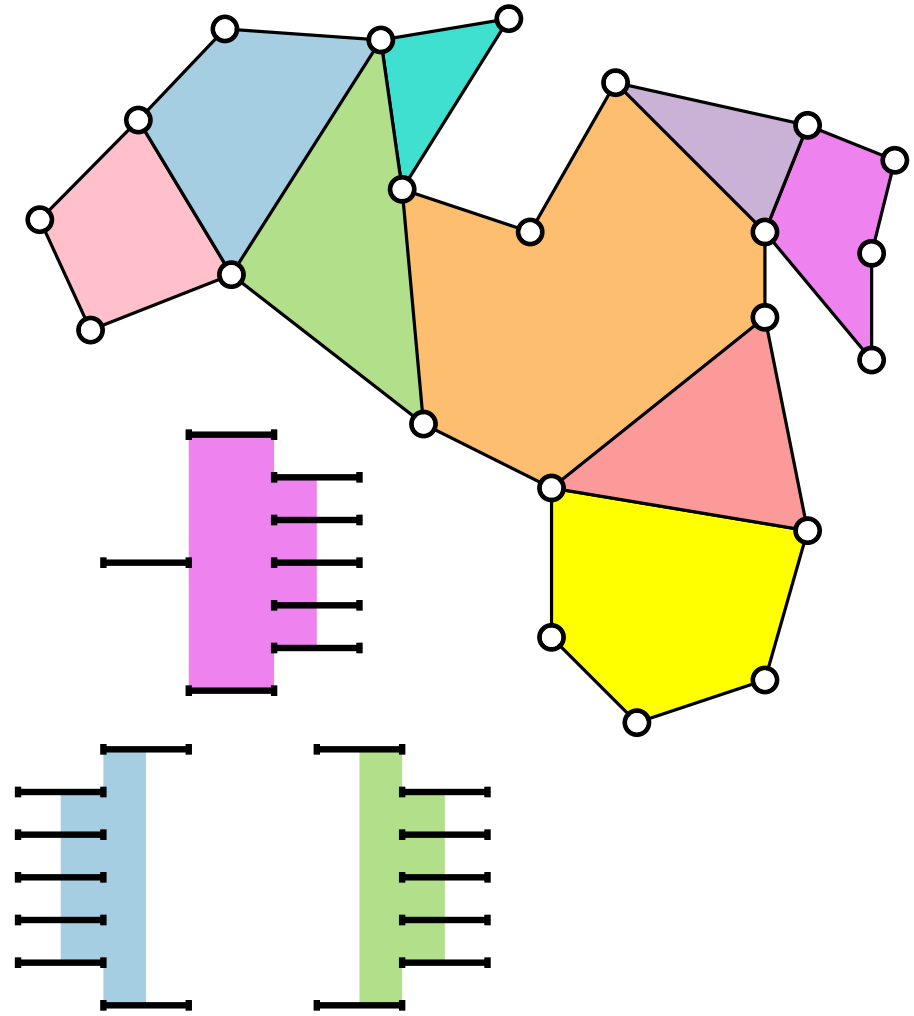
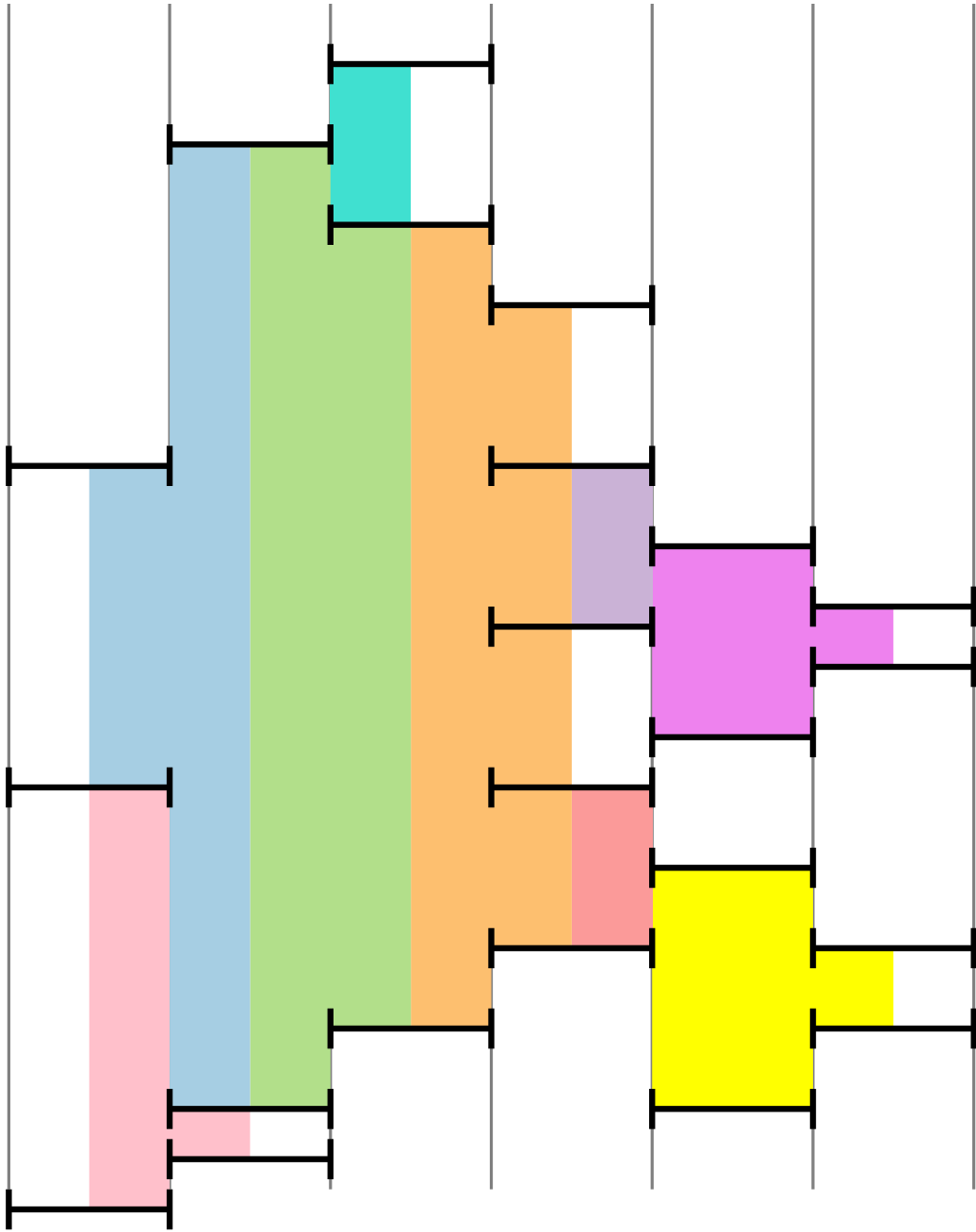


Weak: Biconnected Outerplanar, maxdeg 4

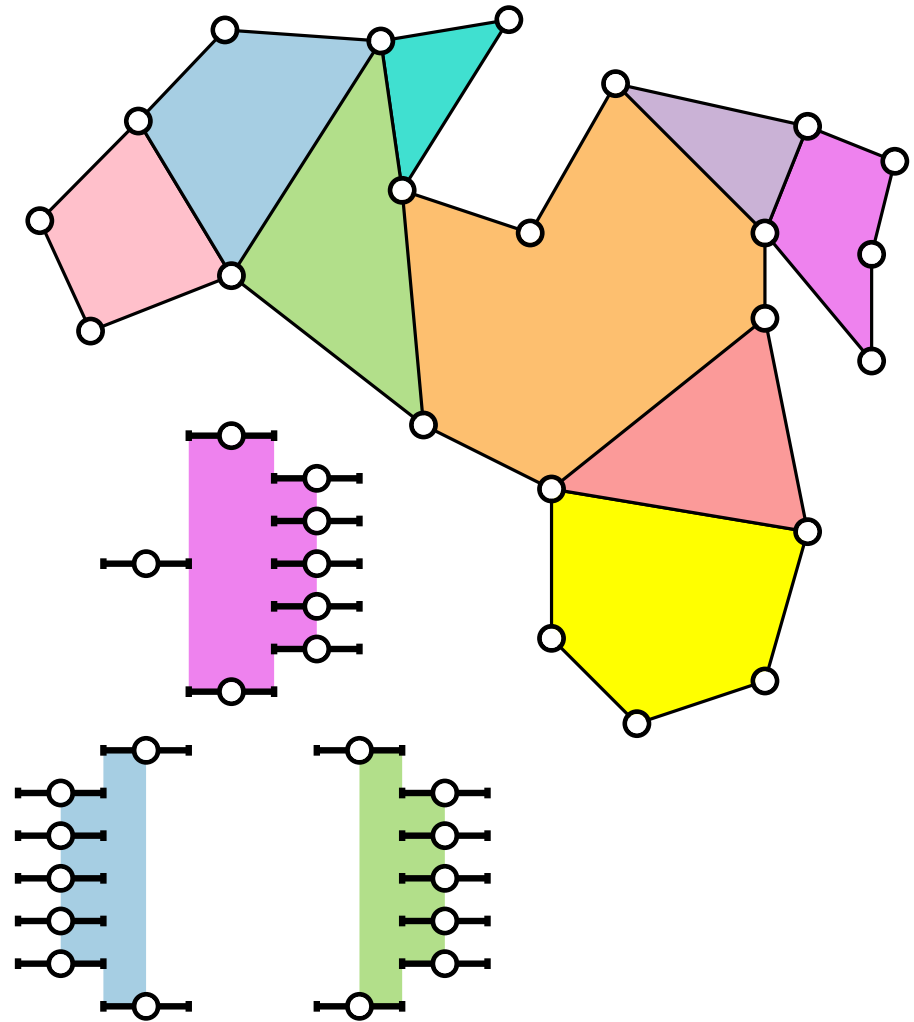
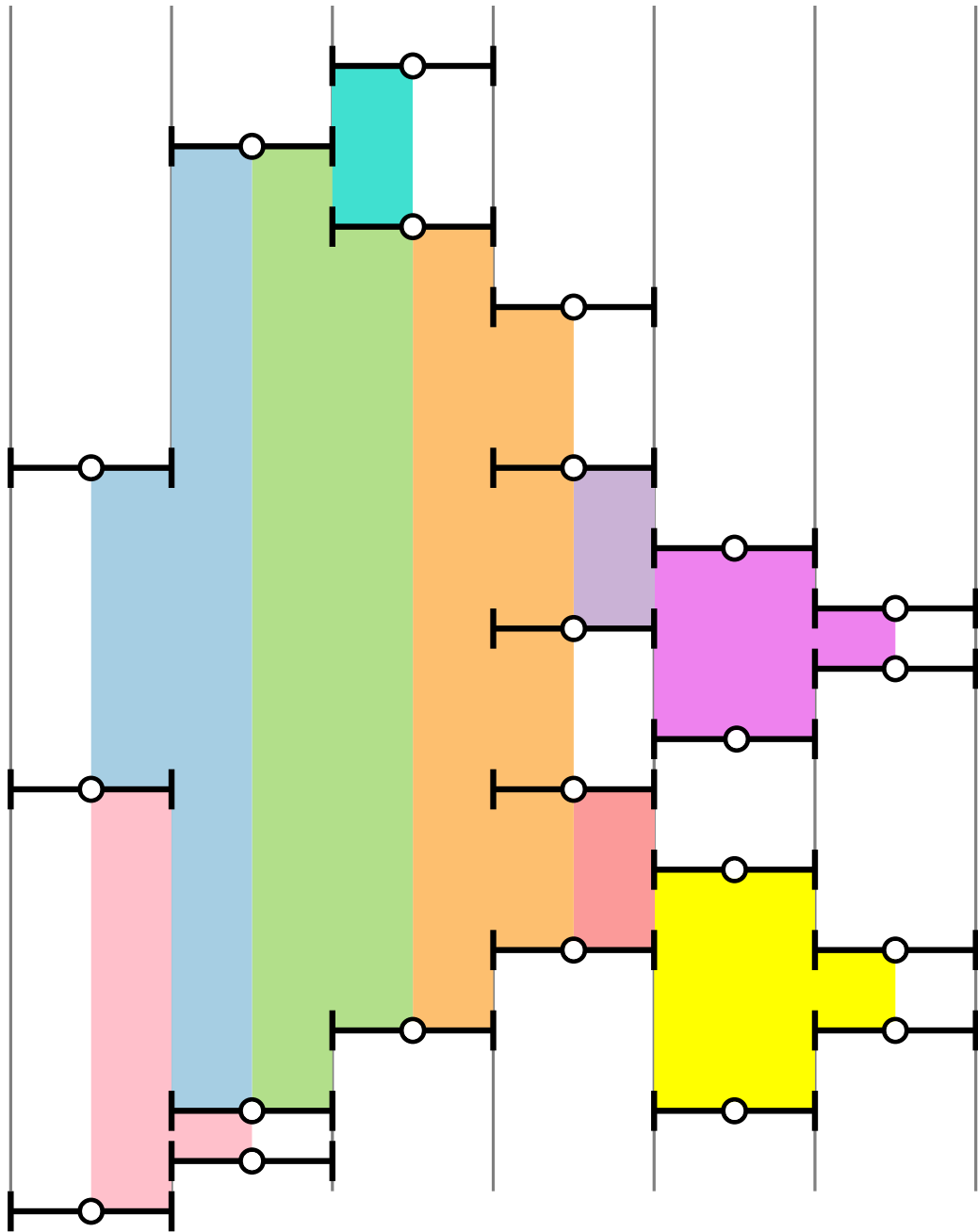


Biconnected Outerplanar
with maxdeg 4 ✓

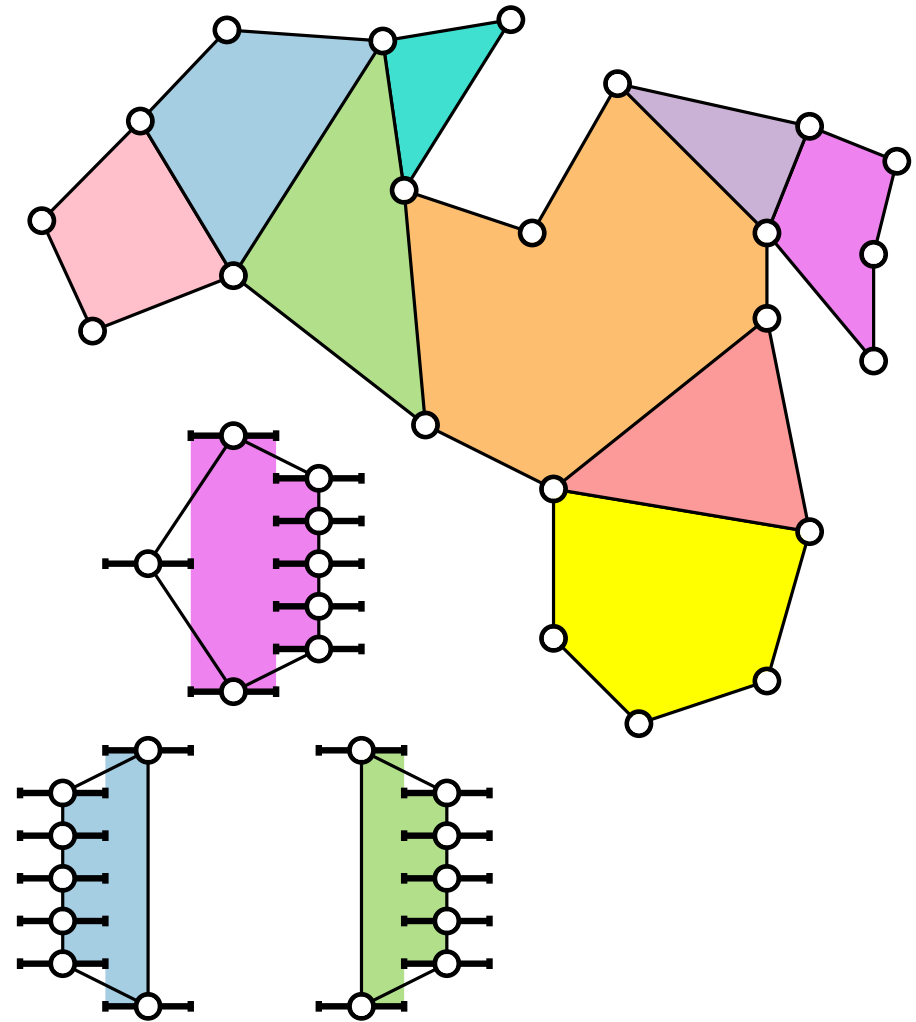
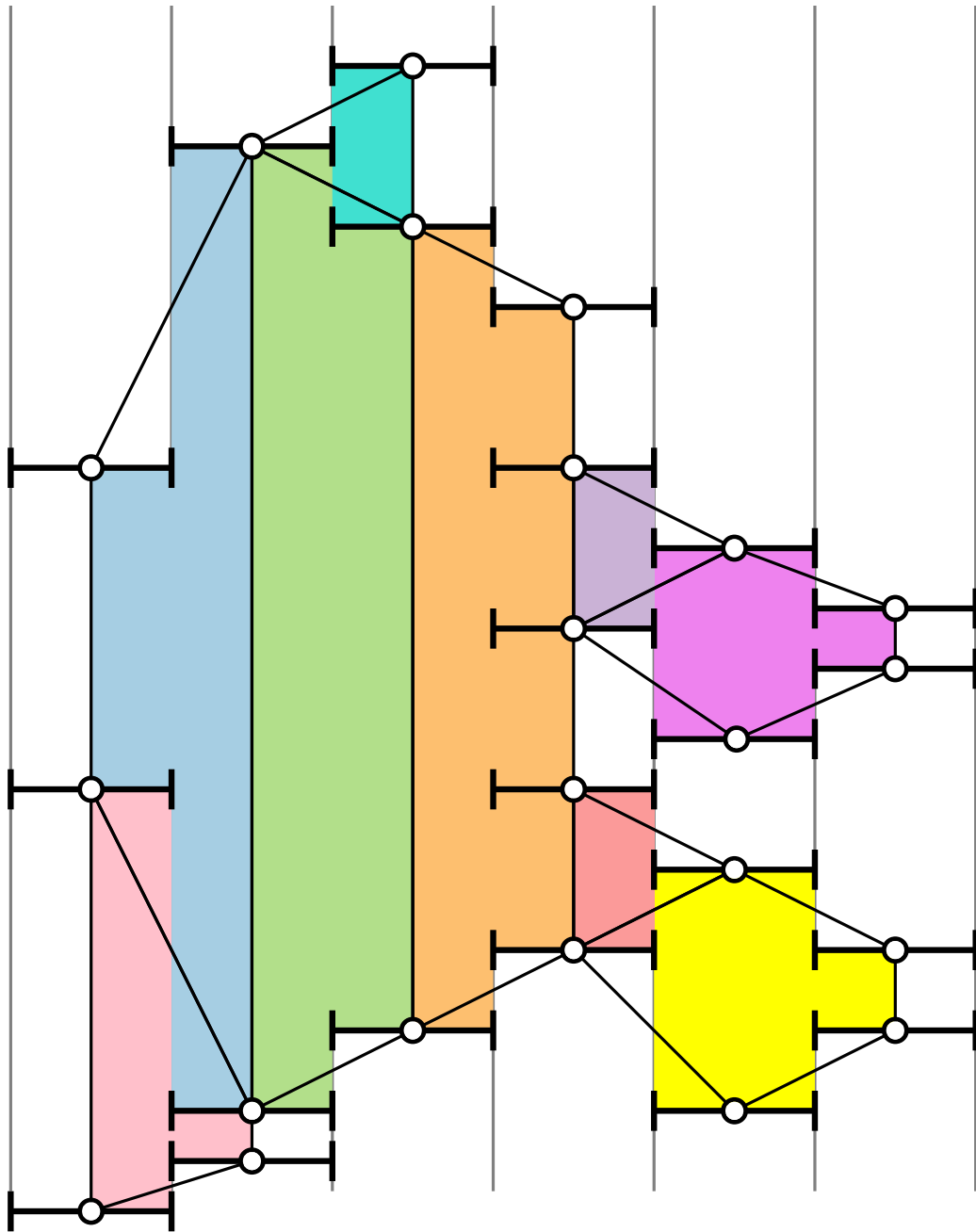
Weak: Biconnected Outerplanar, maxdeg 4



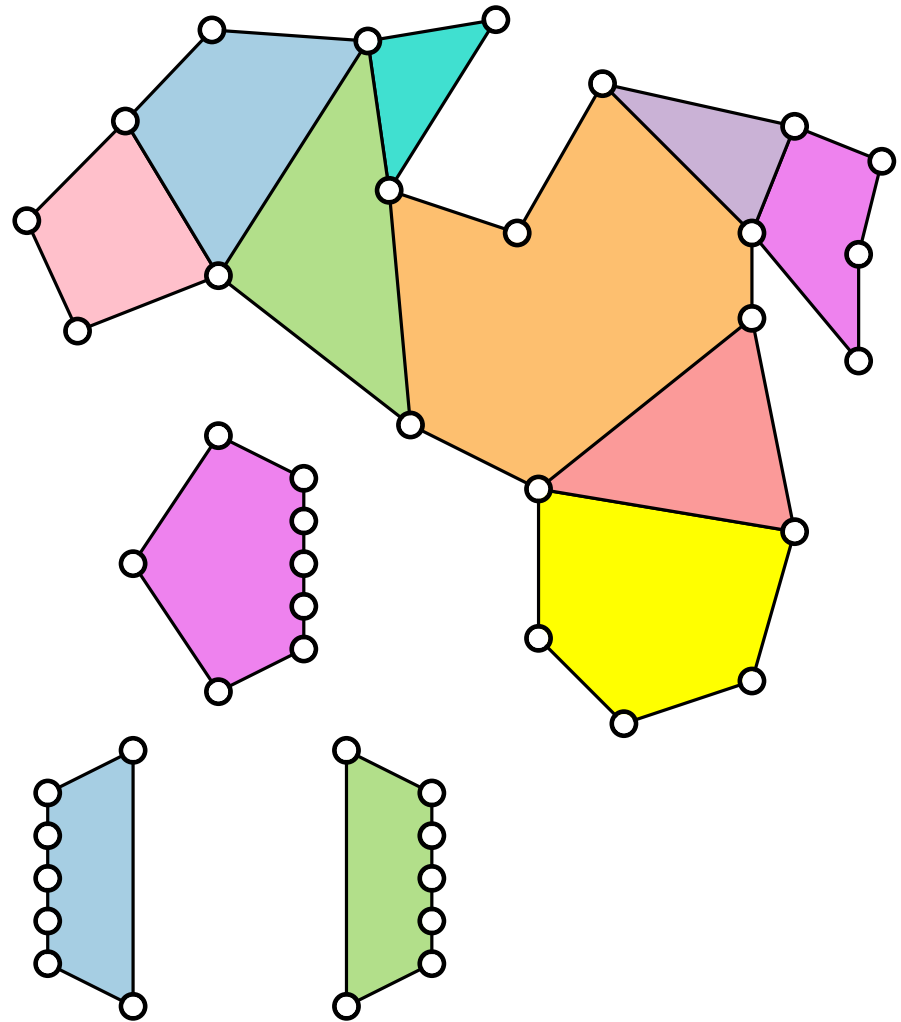
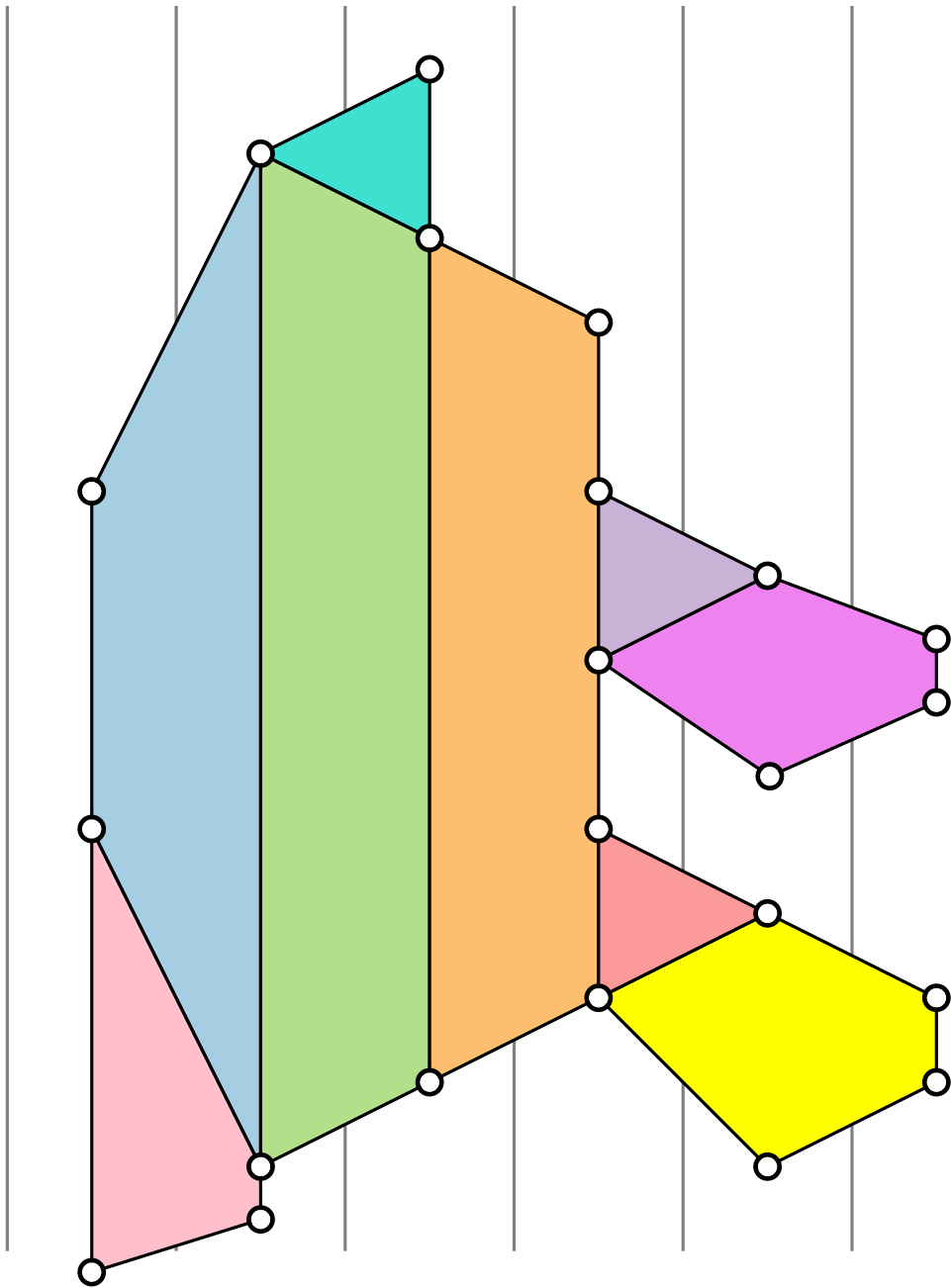
Weak: Biconnected Outerplanar, maxdeg 4



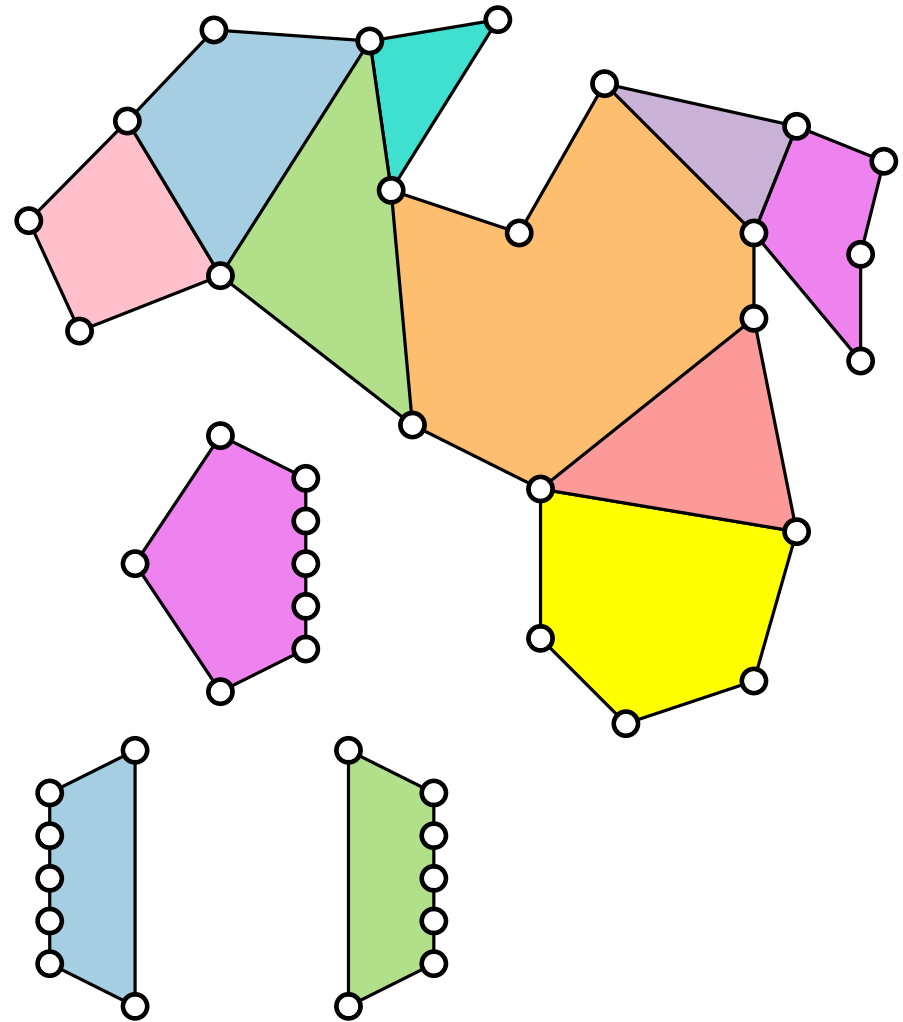
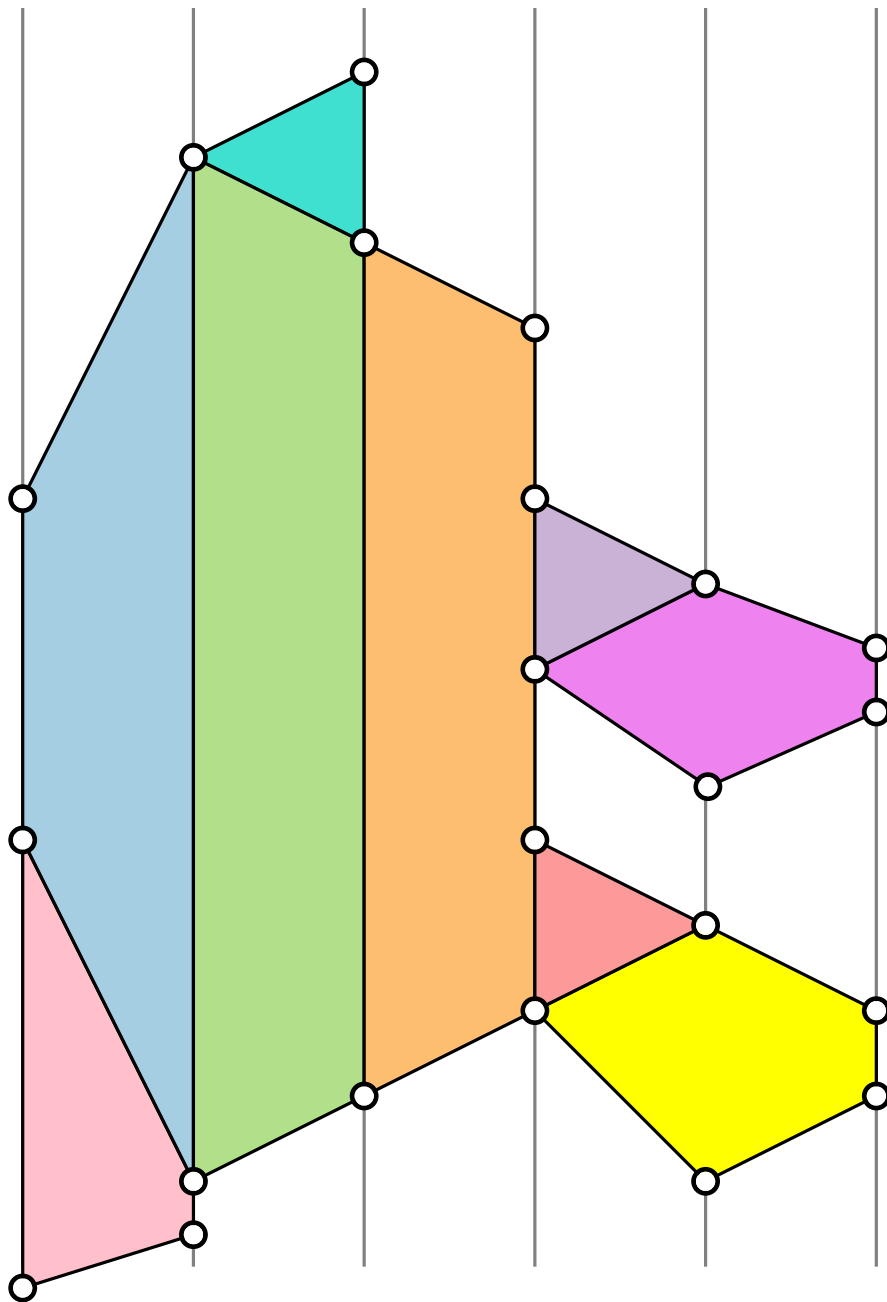
Weak: Biconnected Outerplanar, maxdeg 4



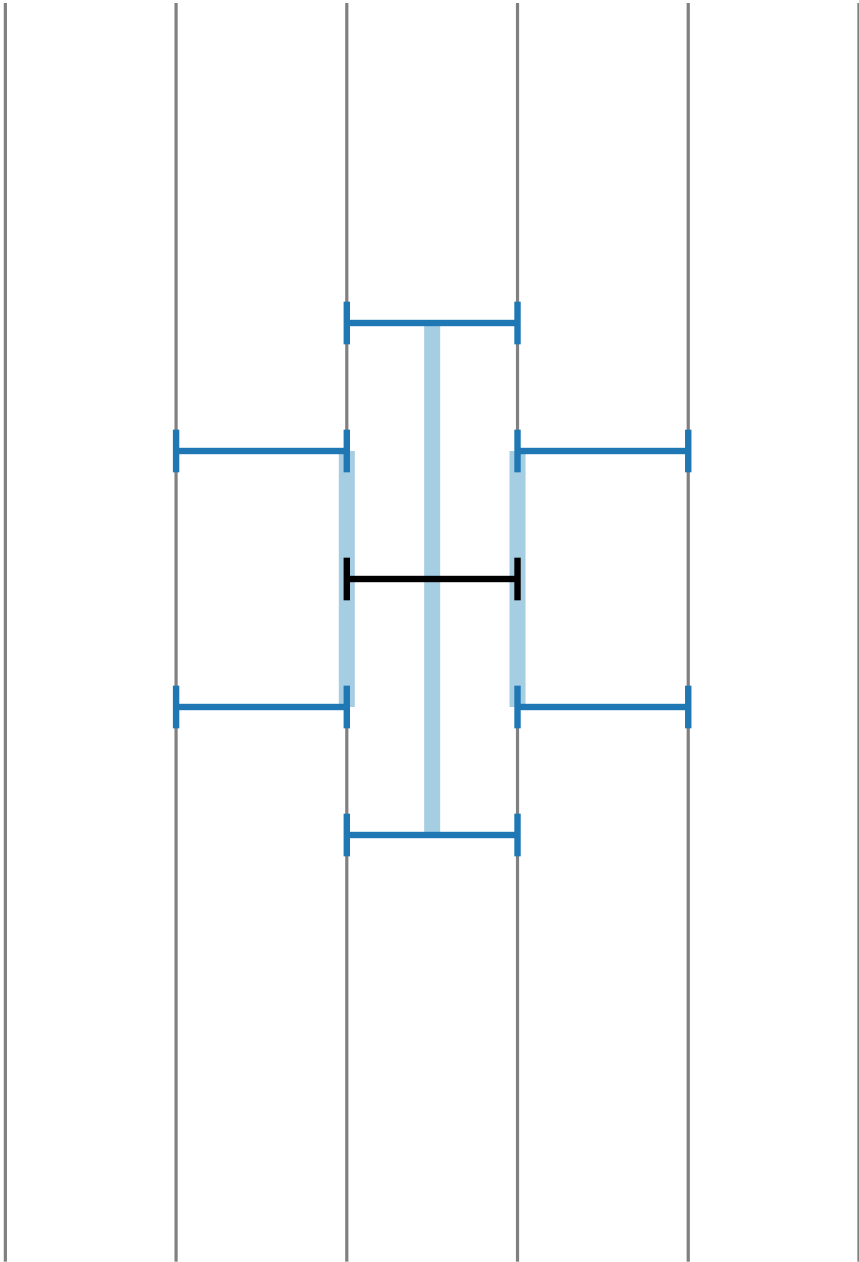
Weak: Biconnected Outerplanar, maxdeg 4



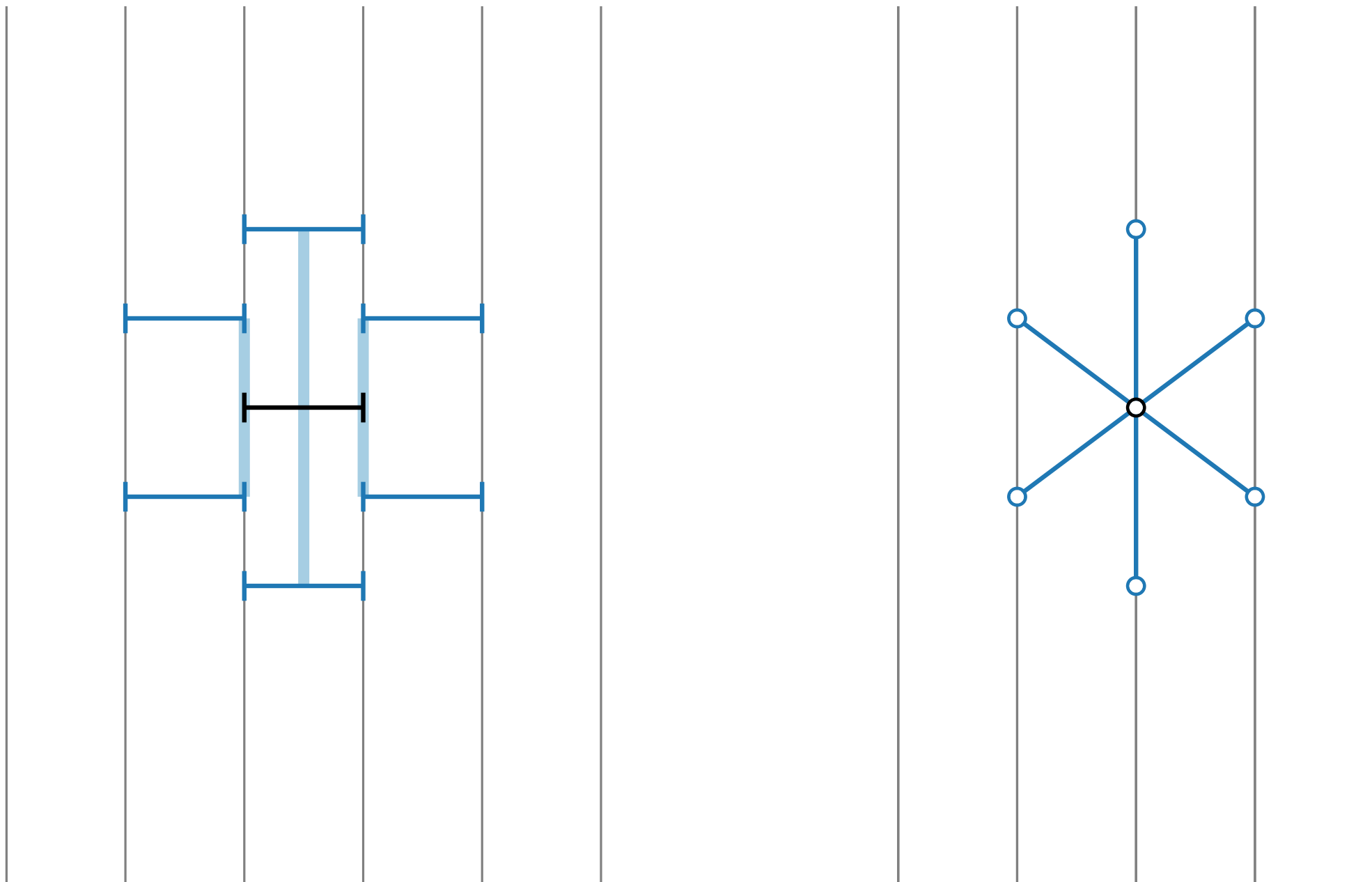
Weak: Biconnected Outerplanar, maxdeg 4



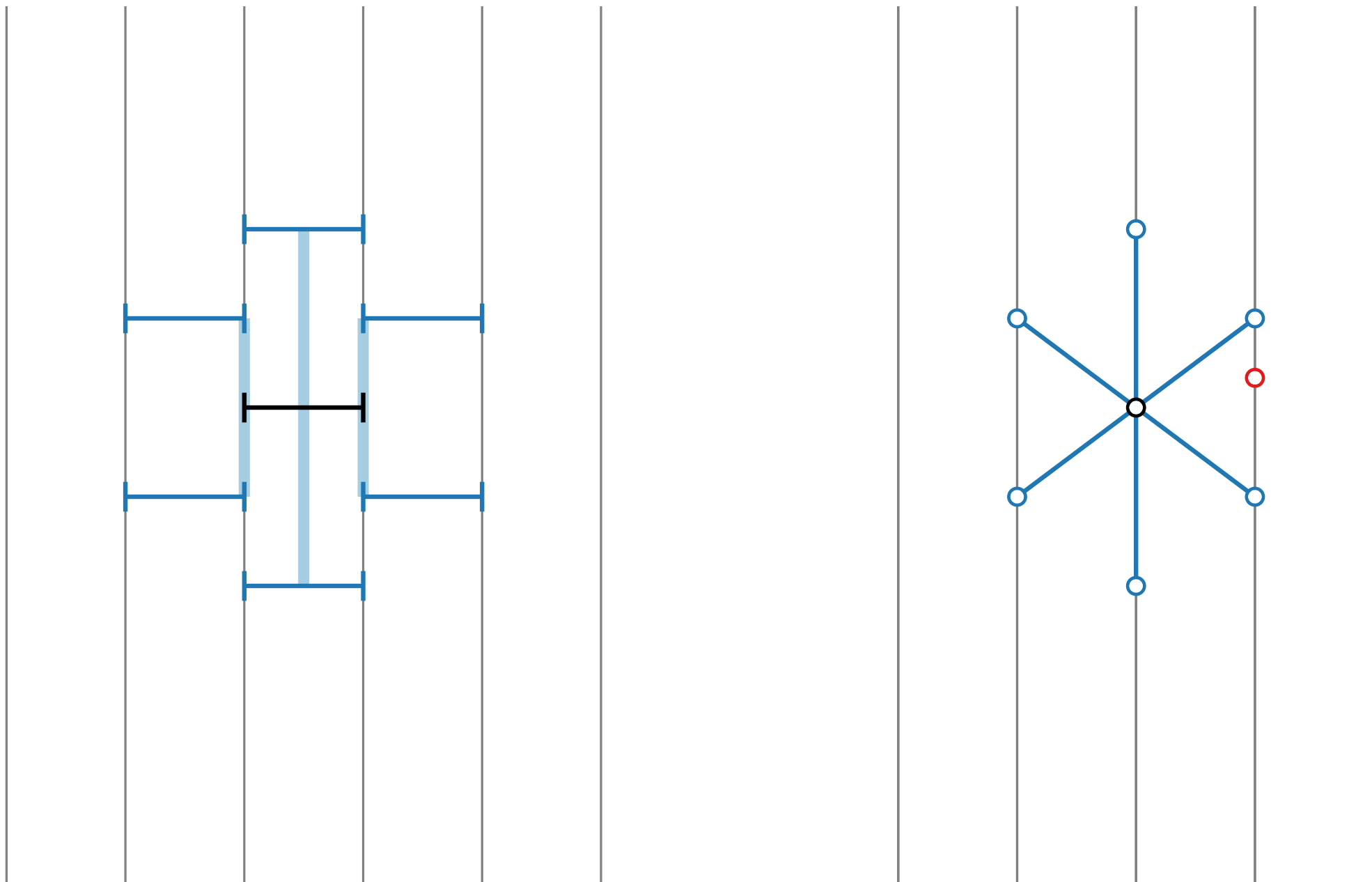
Layered Drawings



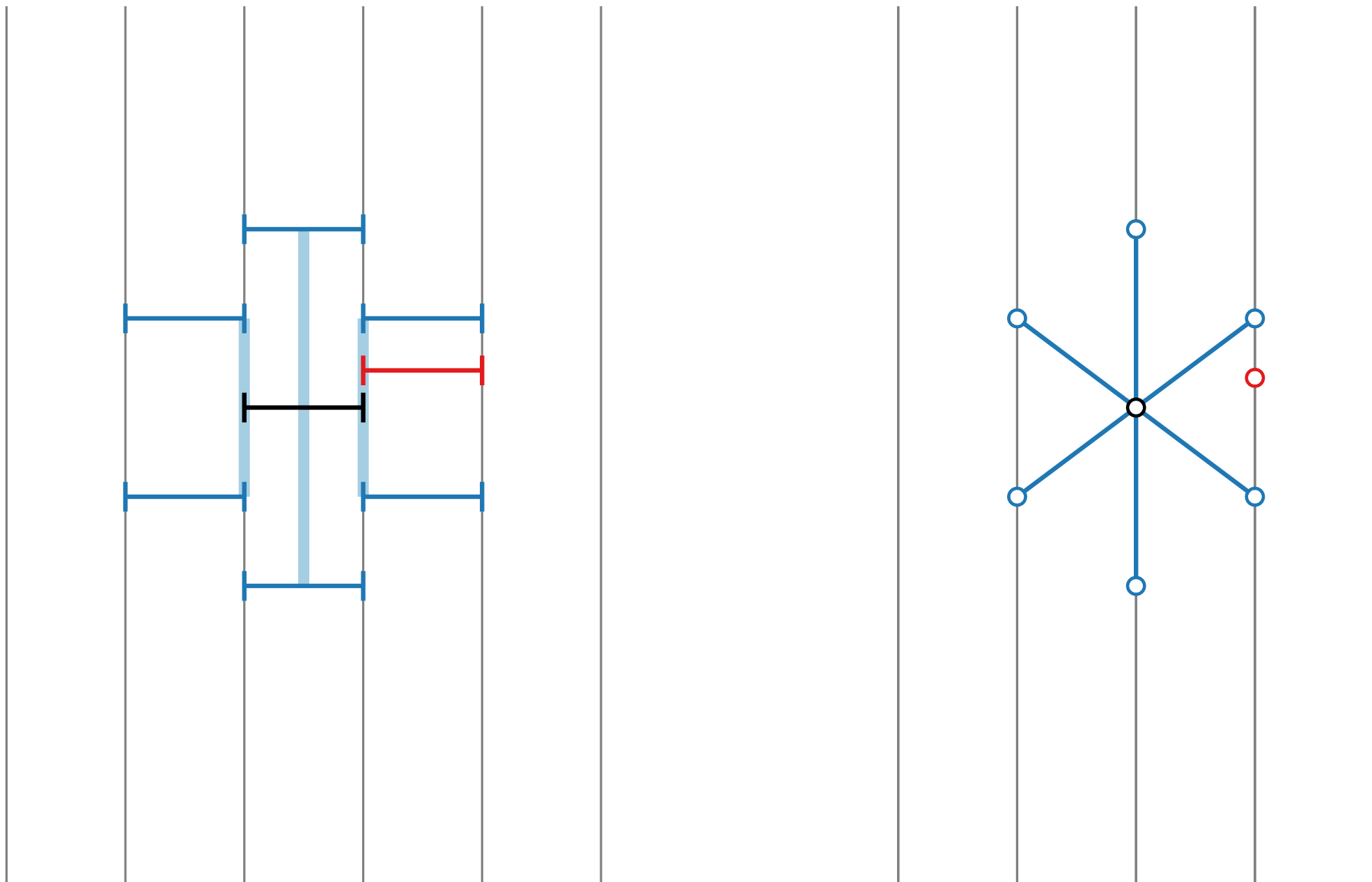
Layered Drawings



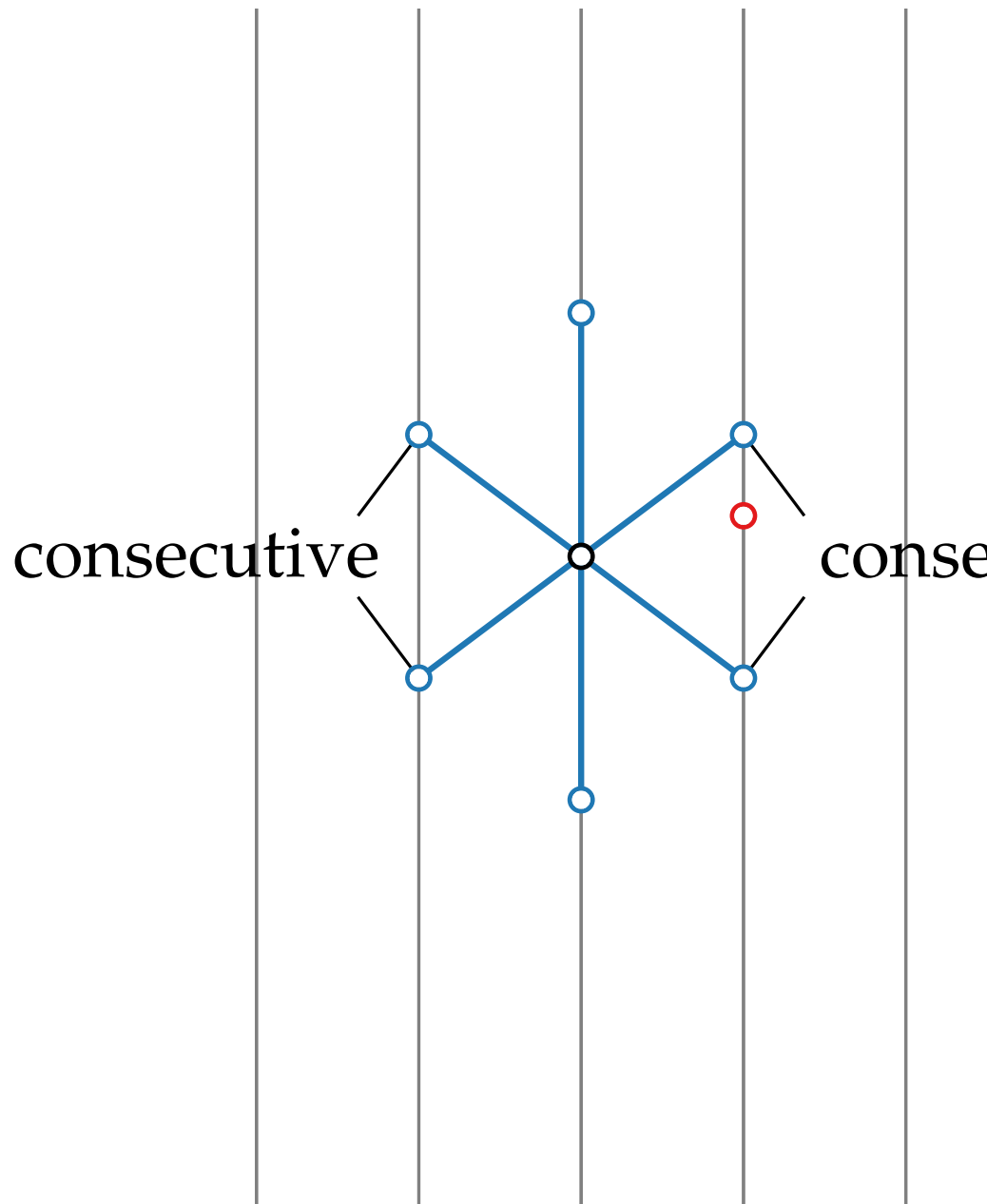
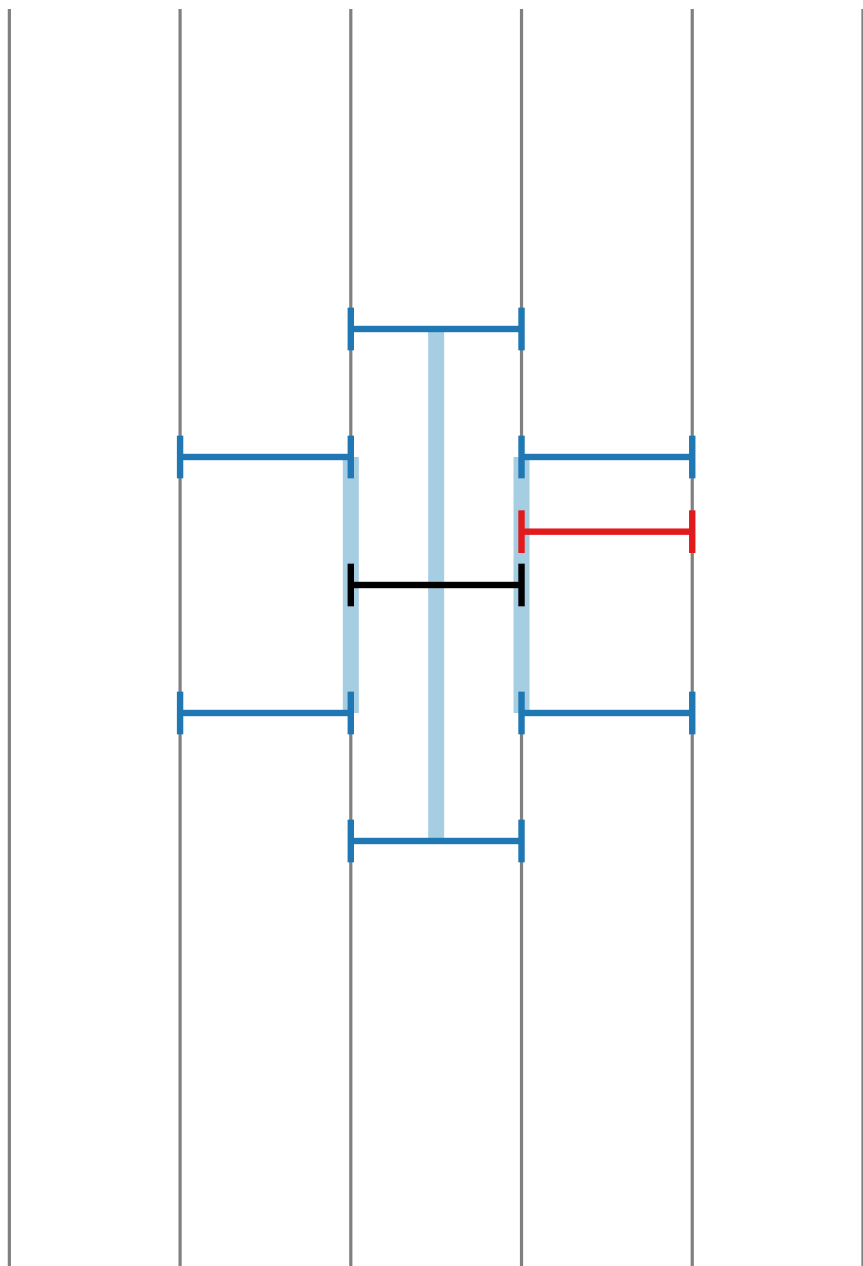
Layered Drawings



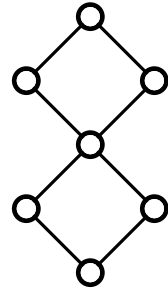
Layered Drawings



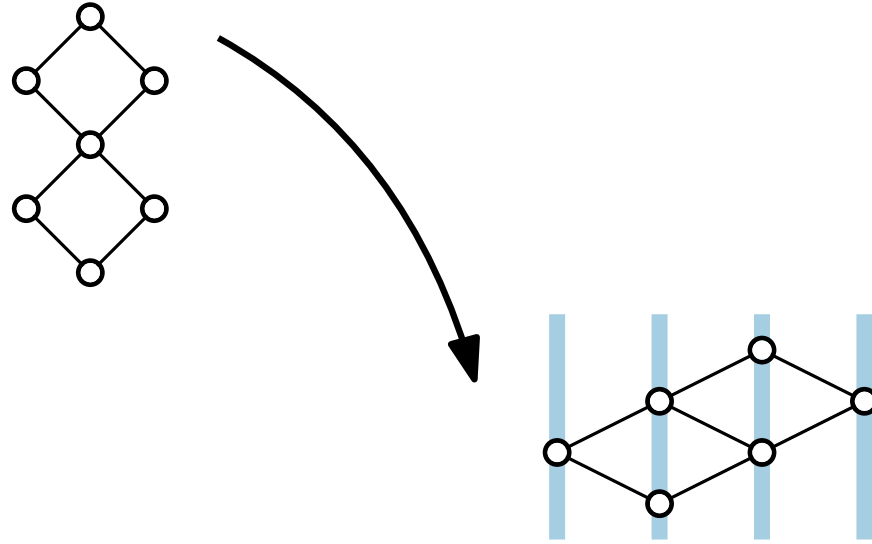
Layered Drawings



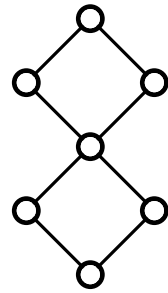
Weak: NP-hardness



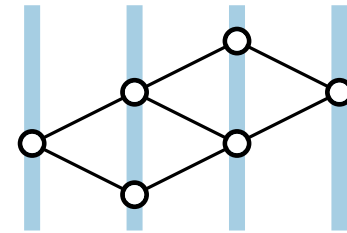
Weak: NP-hardness



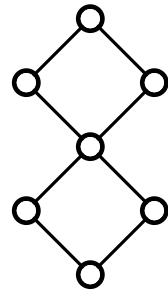
Weak: NP-hardness



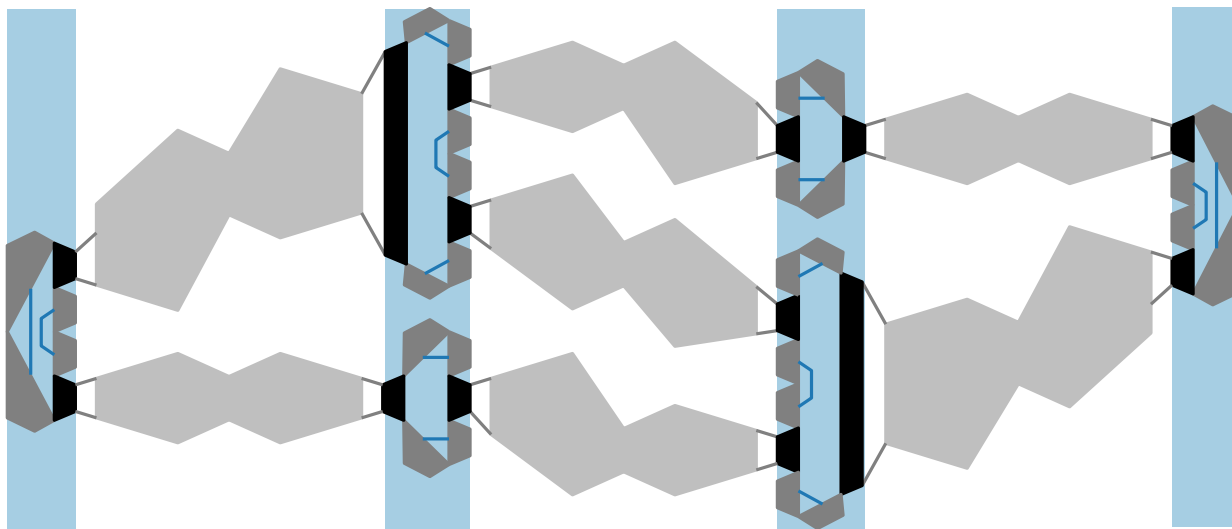
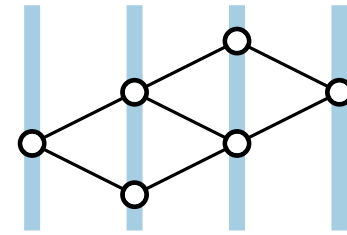
NP-hard
[Heath & Rosenberg]



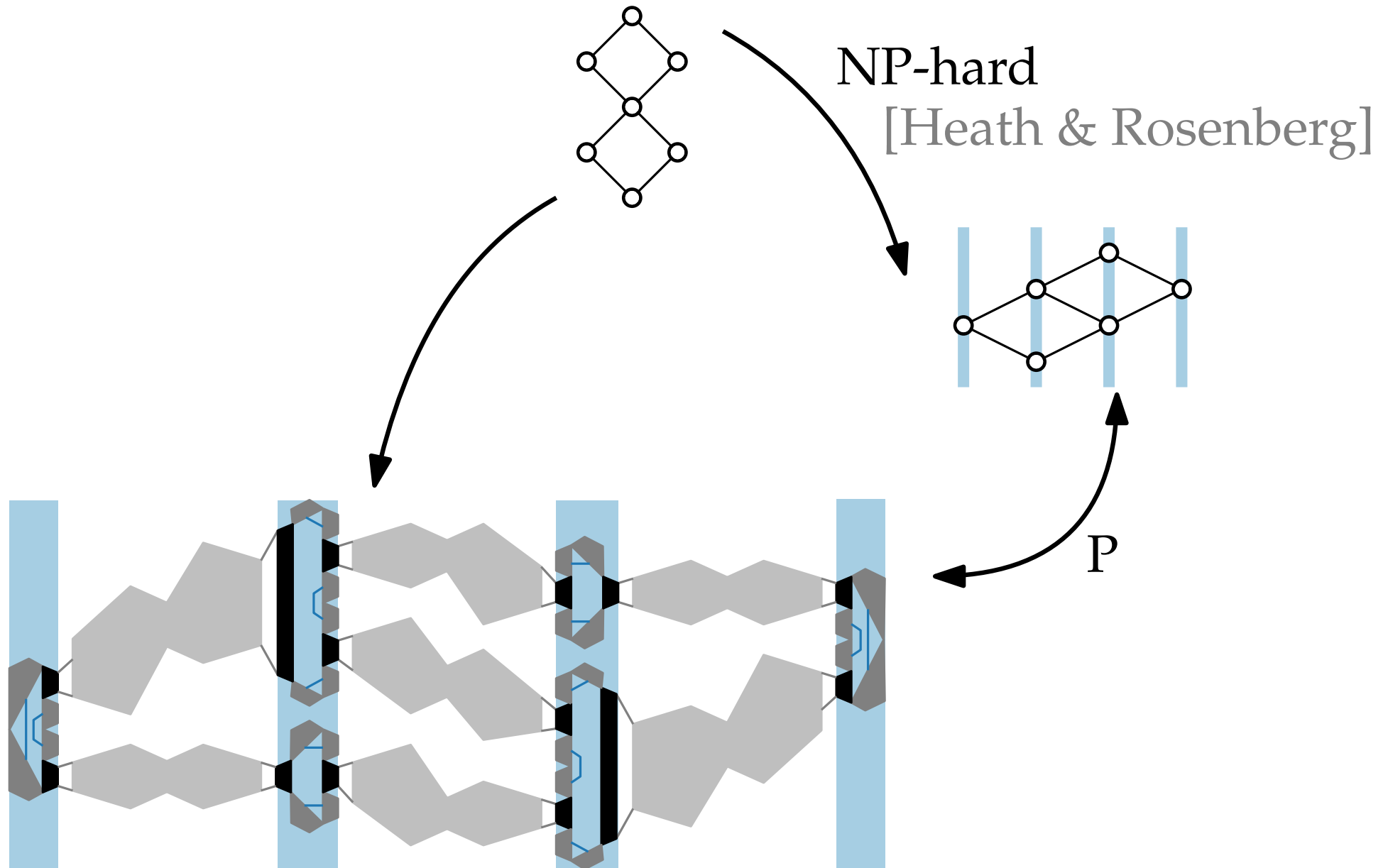
Weak: NP-hardness



NP-hard
[Heath & Rosenberg]

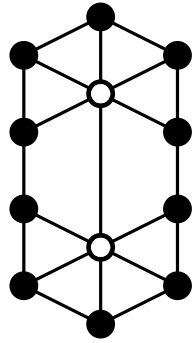


Weak: NP-hardness



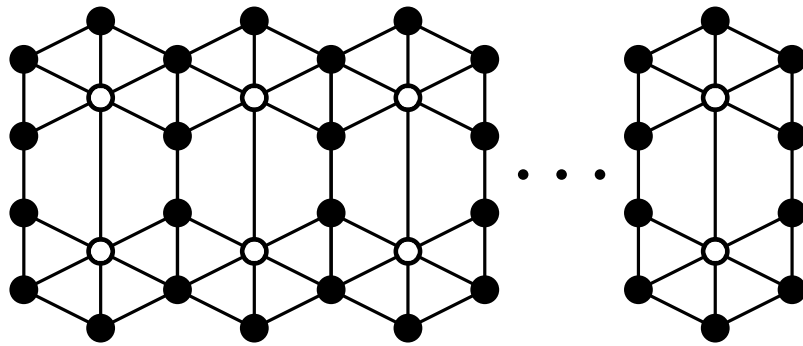
Weak: NP-hardness

Edges:



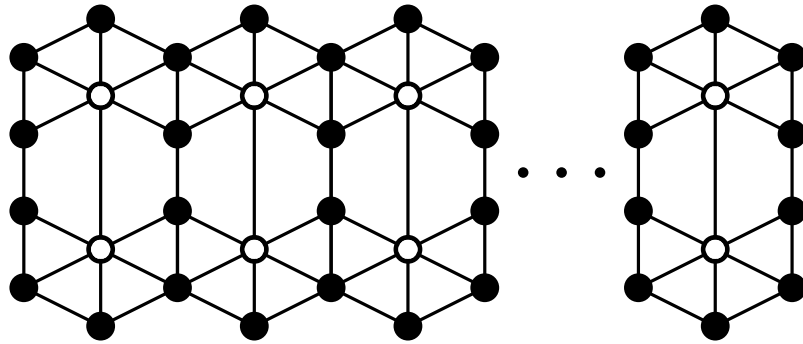
Weak: NP-hardness

Edges:

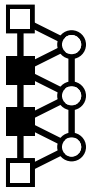


Weak: NP-hardness

Edges:

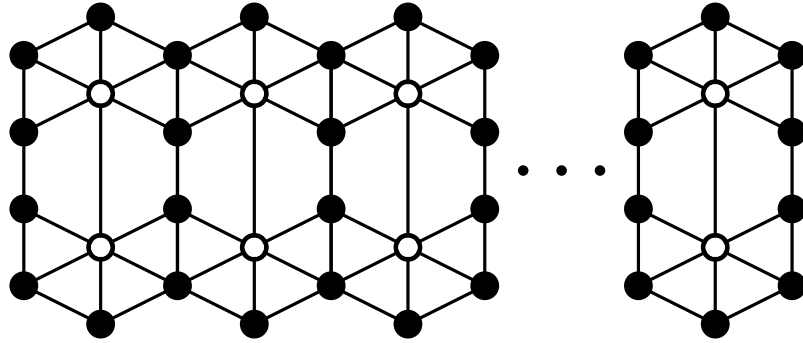


Node block:

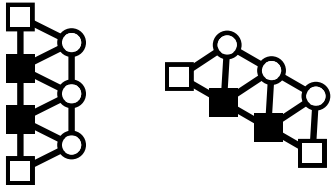


Weak: NP-hardness

Edges:

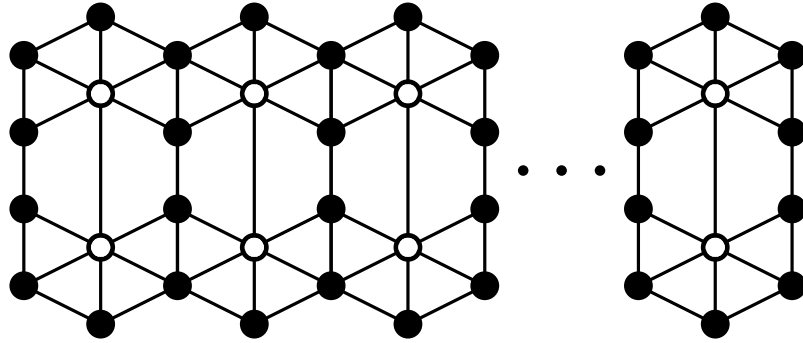


Node block:

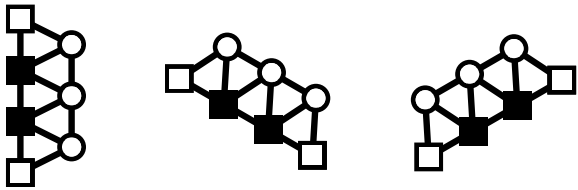


Weak: NP-hardness

Edges:

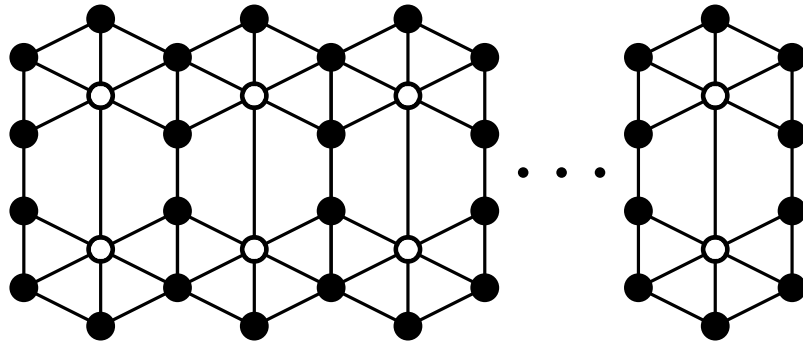


Node block:

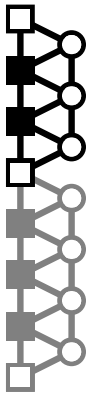


Weak: NP-hardness

Edges:

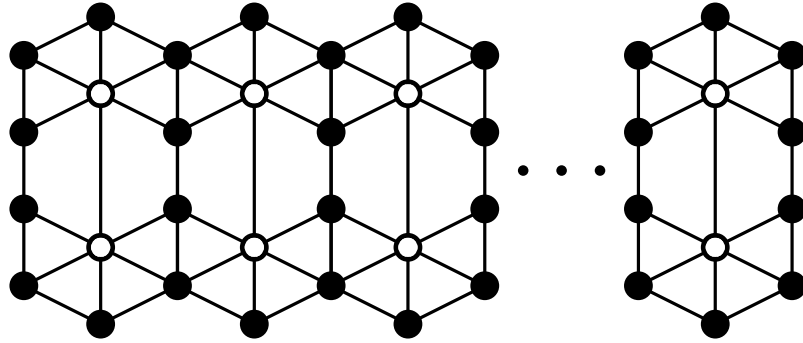


Node block:



Weak: NP-hardness

Edges:

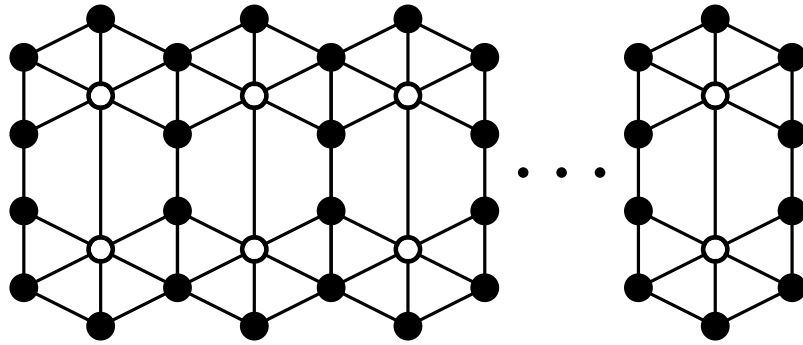


Node block:

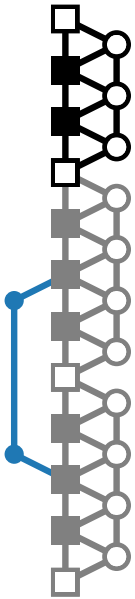


Weak: NP-hardness

Edges:

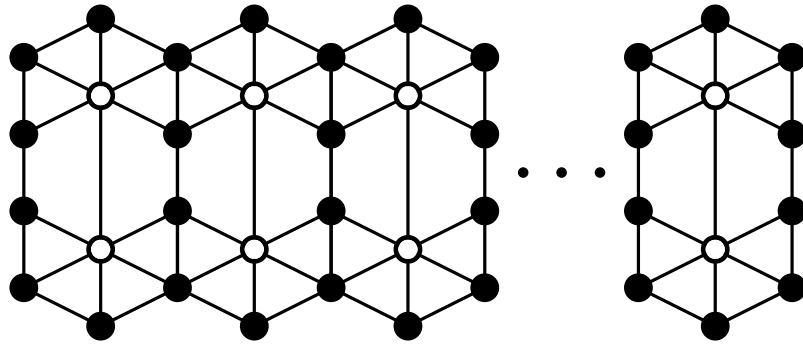


Node block:

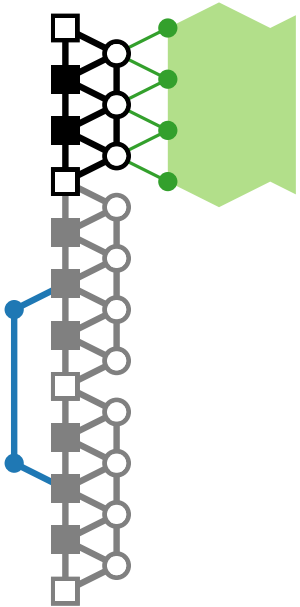


Weak: NP-hardness

Edges:

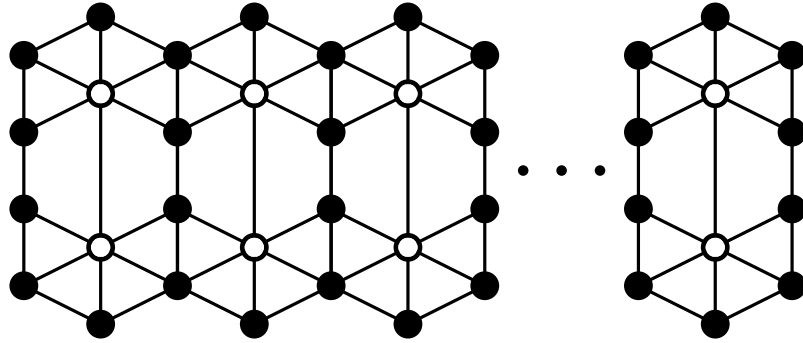


Node block:

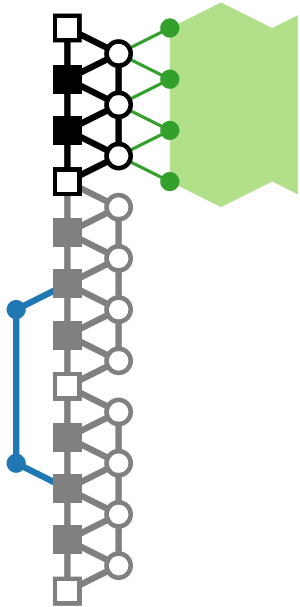


Weak: NP-hardness

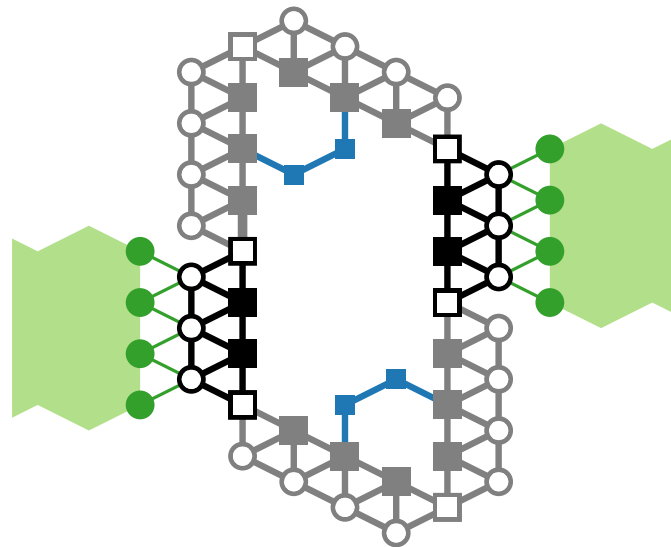
Edges:



Node block:

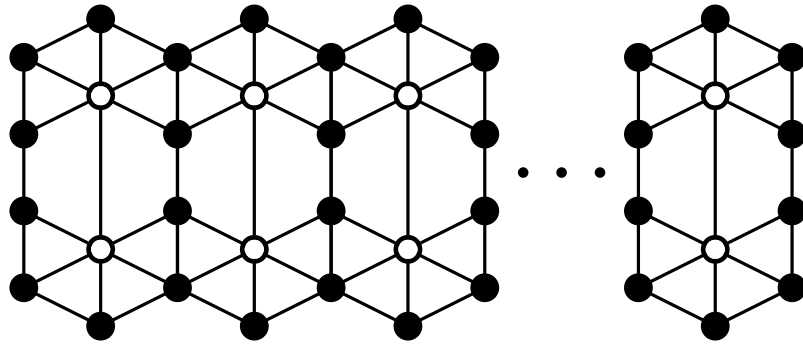


Nodes:

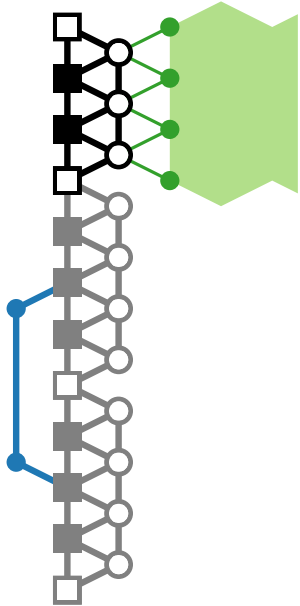


Weak: NP-hardness

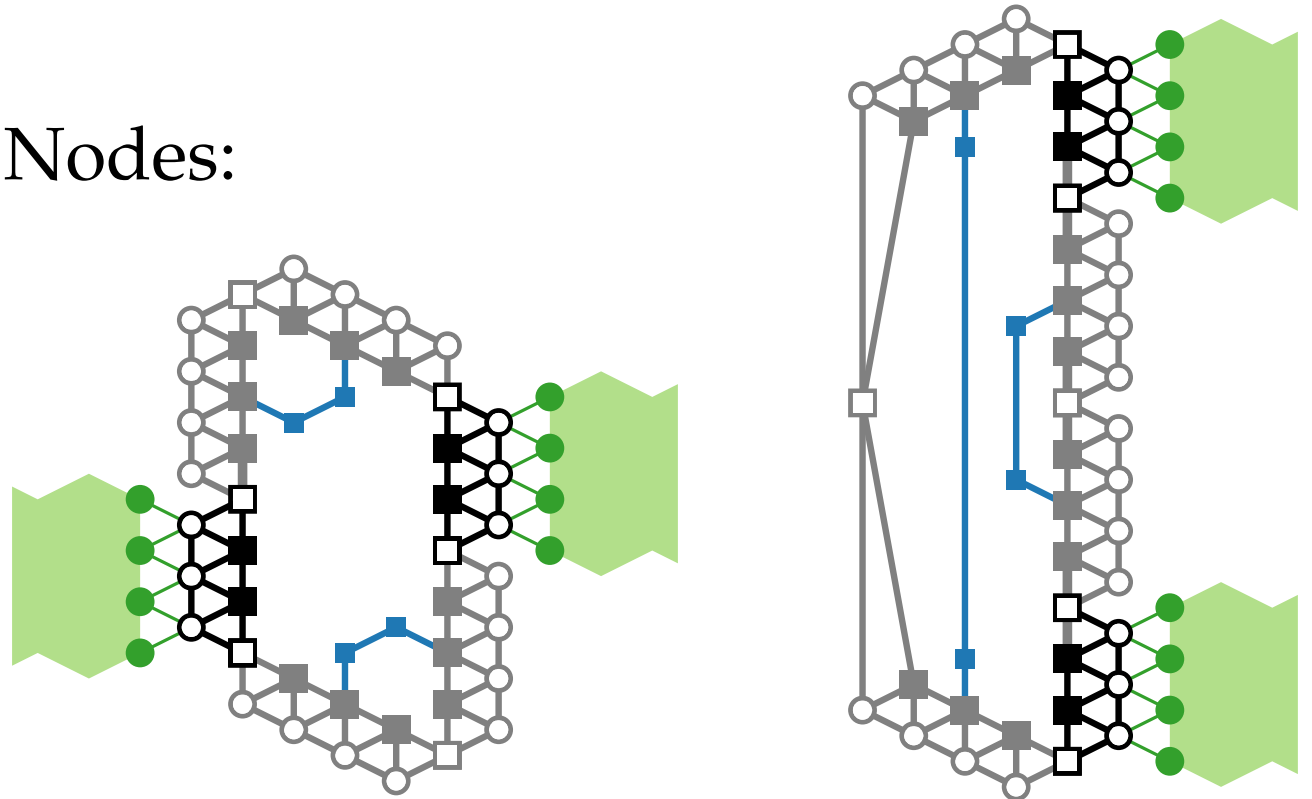
Edges:



Node block:

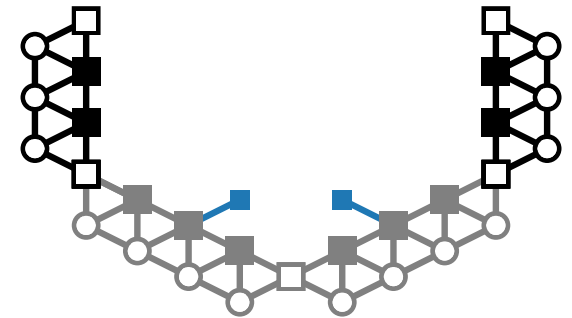
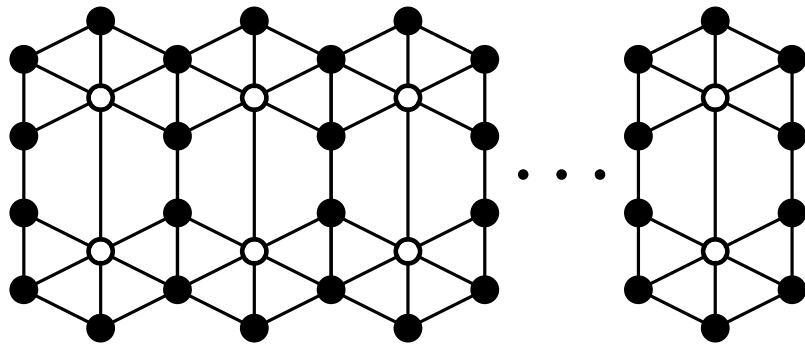


Nodes:

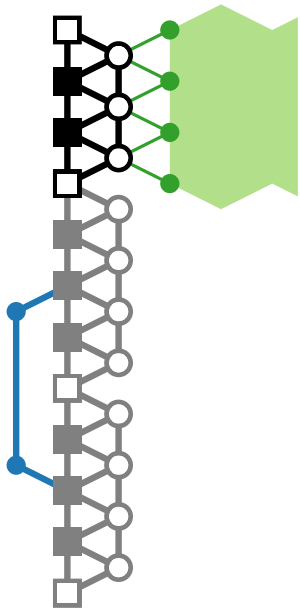


Weak: NP-hardness

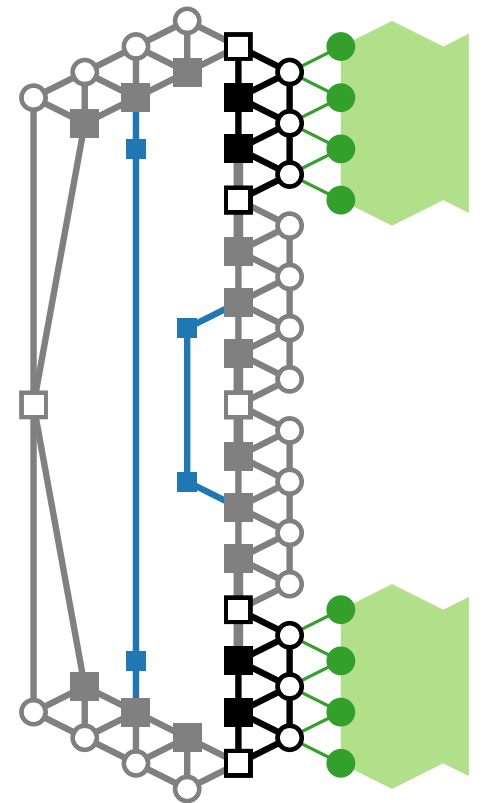
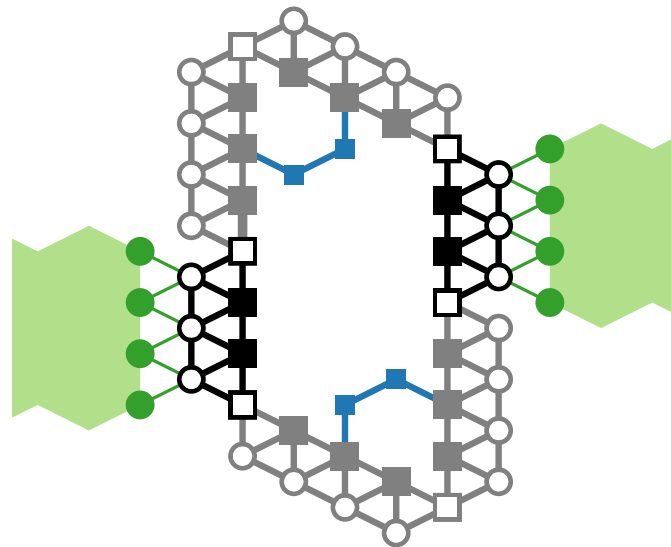
Edges:



Node block:

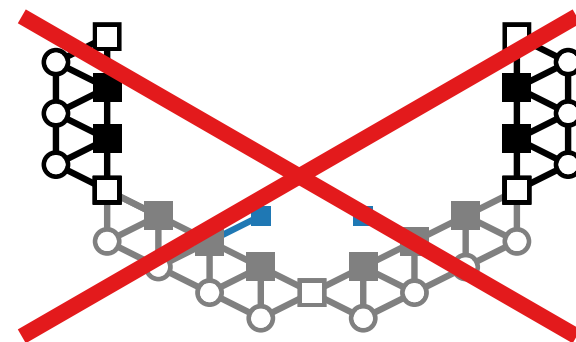
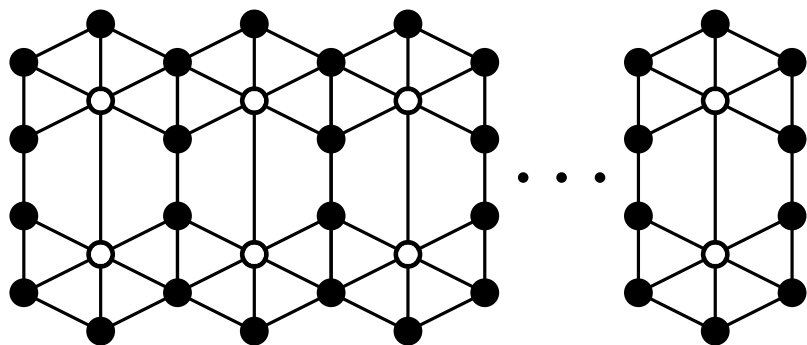


Nodes:

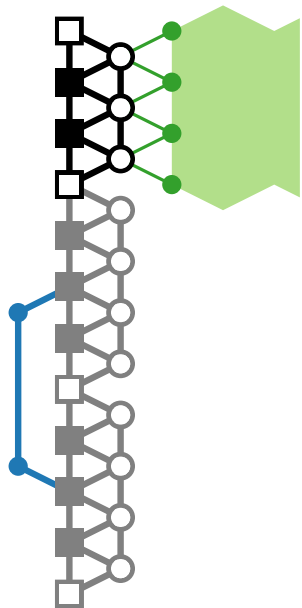


Weak: NP-hardness

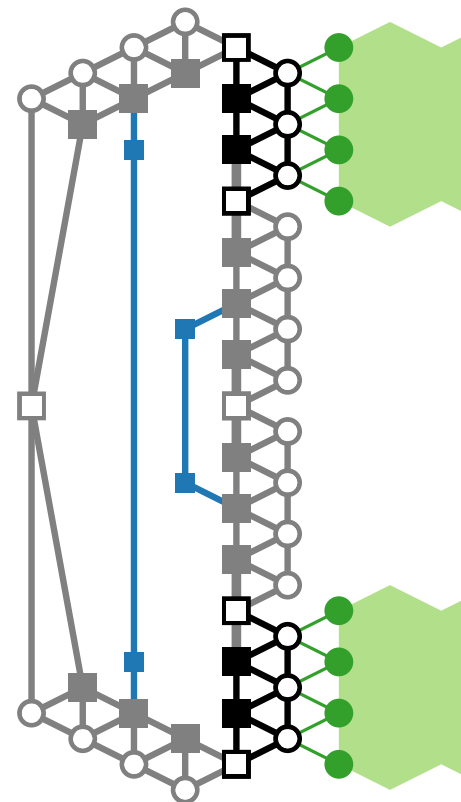
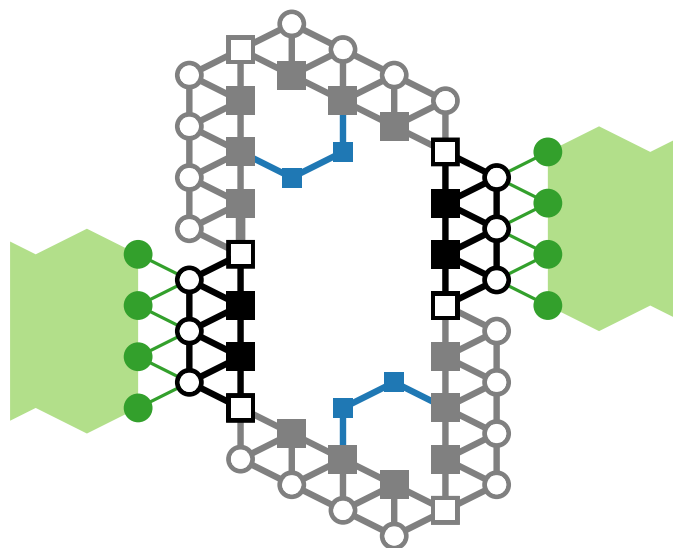
Edges:



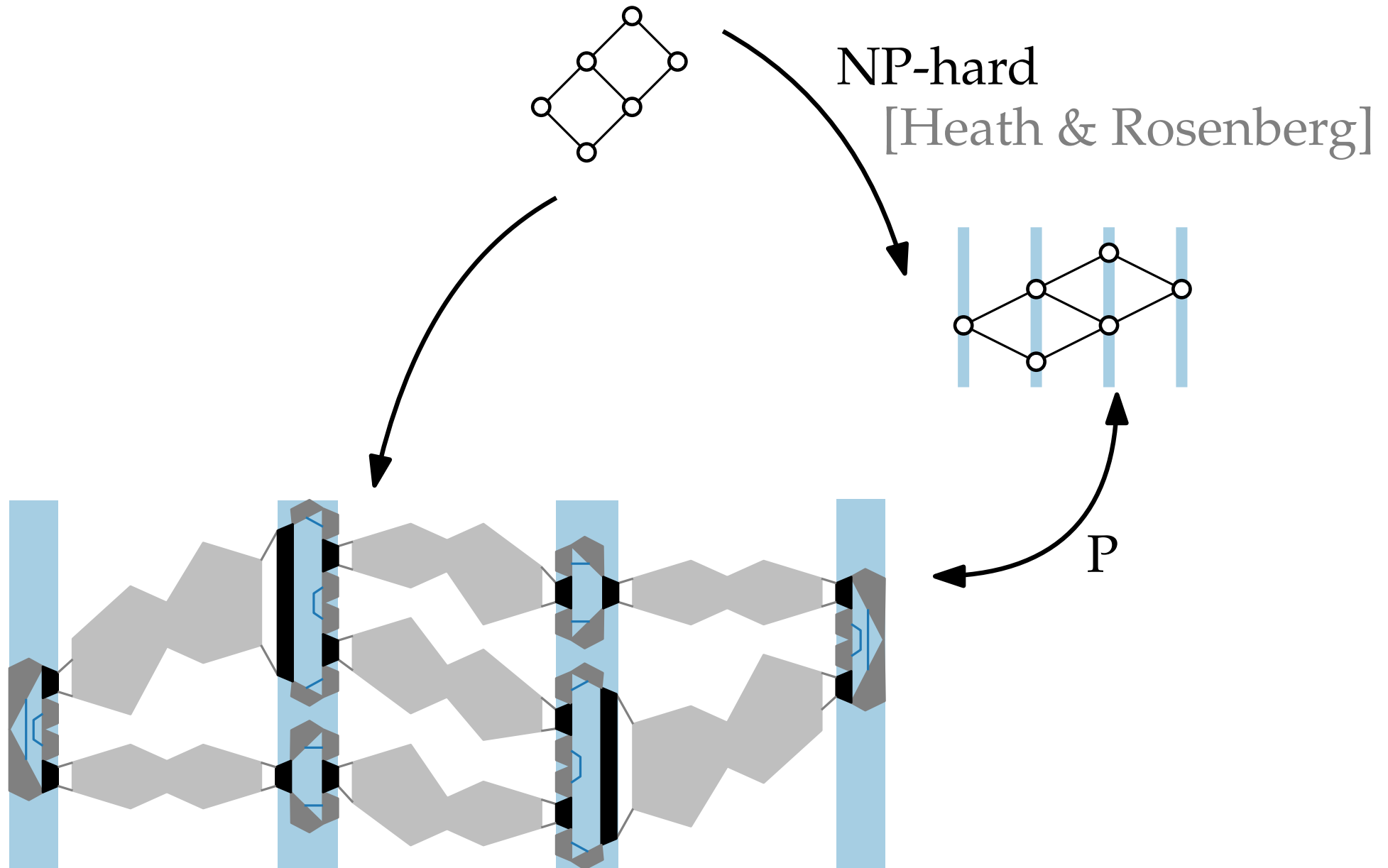
Node block:



Nodes:

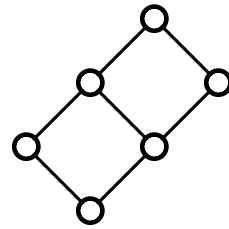


Weak: NP-hardness

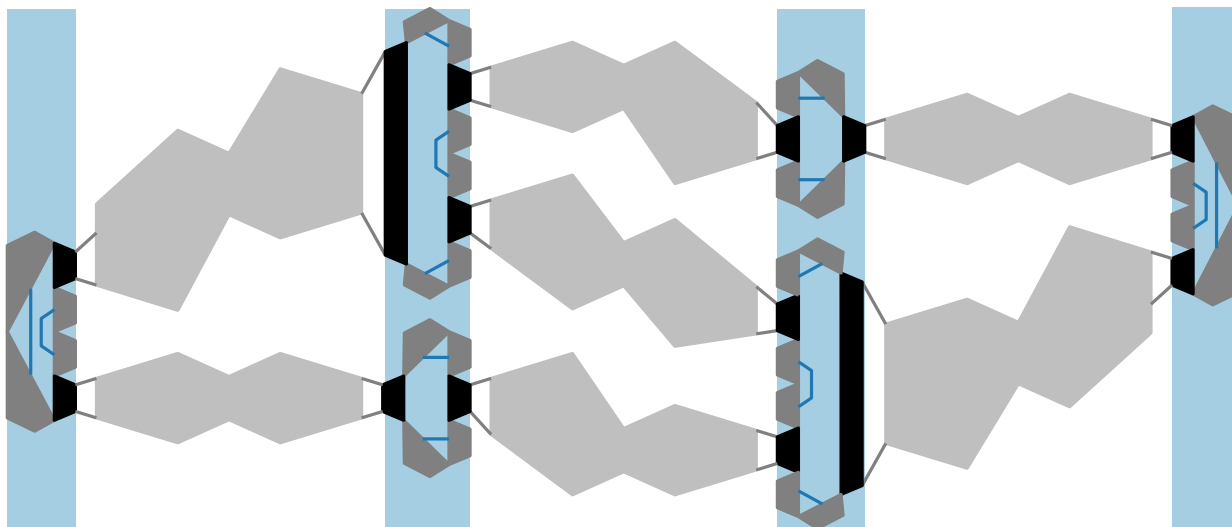
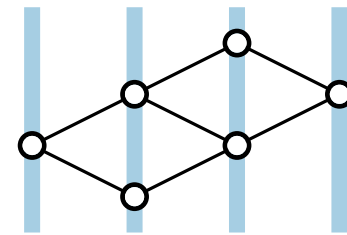


Weak: NP-hardness

Recognition:
NP-hard

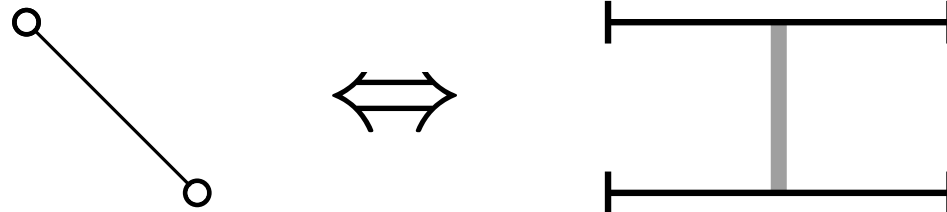


NP-hard
[Heath & Rosenberg]



P

Strong Model



Strong: Trees

[Dean & Veytsel]

Unit Bar-Visibility Representation

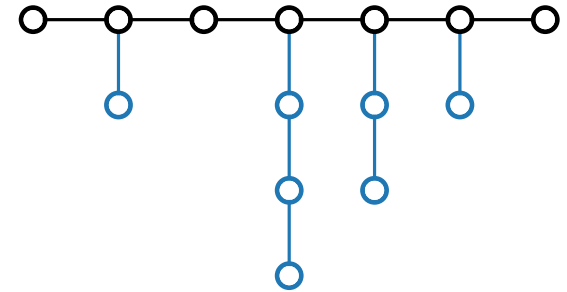
\Leftrightarrow maxdeg 3, subdivision of caterpillar

Strong: Trees

[Dean & Veytsel]

Unit Bar-Visibility Representation

\Leftrightarrow maxdeg 3, subdivision of caterpillar

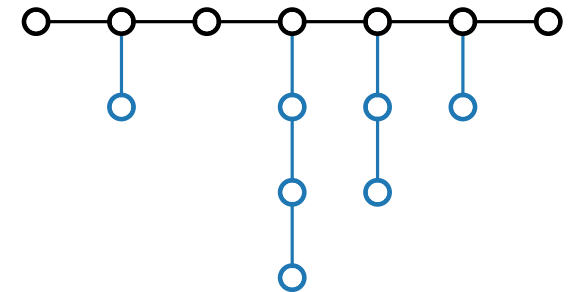
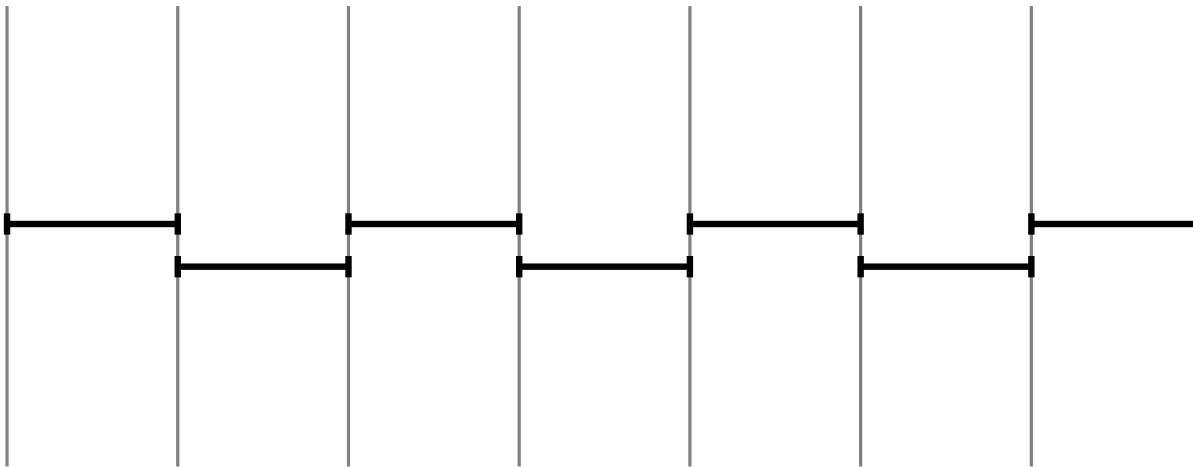


Strong: Trees

[Dean & Veytsel]

Unit Bar-Visibility Representation

\Leftrightarrow maxdeg 3, subdivision of caterpillar

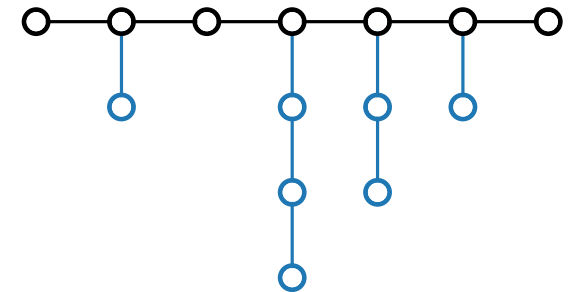
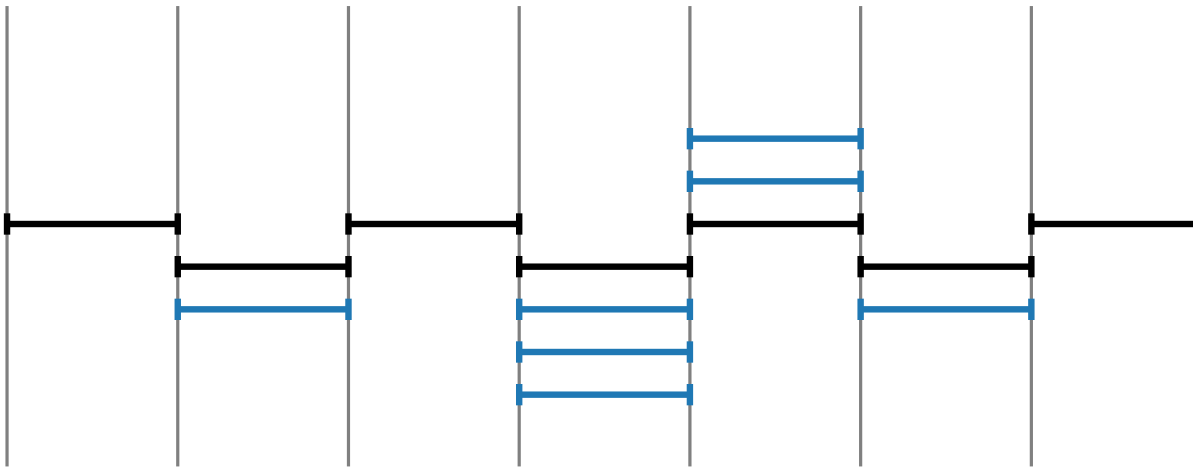


Strong: Trees

[Dean & Veytsel]

Unit Bar-Visibility Representation

\Leftrightarrow maxdeg 3, subdivision of caterpillar

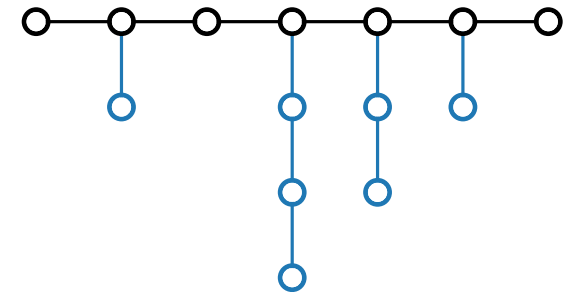
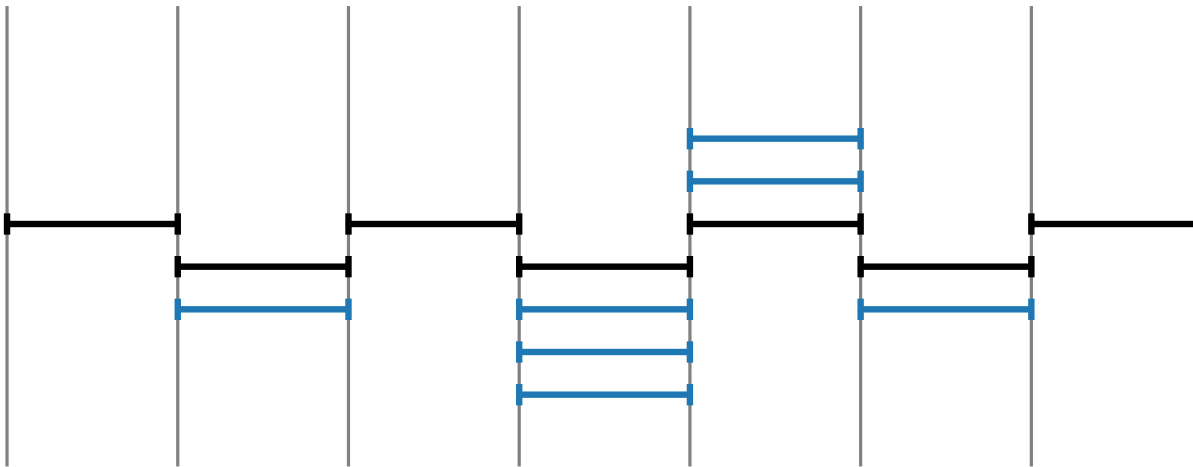


Strong: Trees

[Dean & Veytsel]

Unit Bar-Visibility Representation

\Leftrightarrow maxdeg 3, subdivision of caterpillar

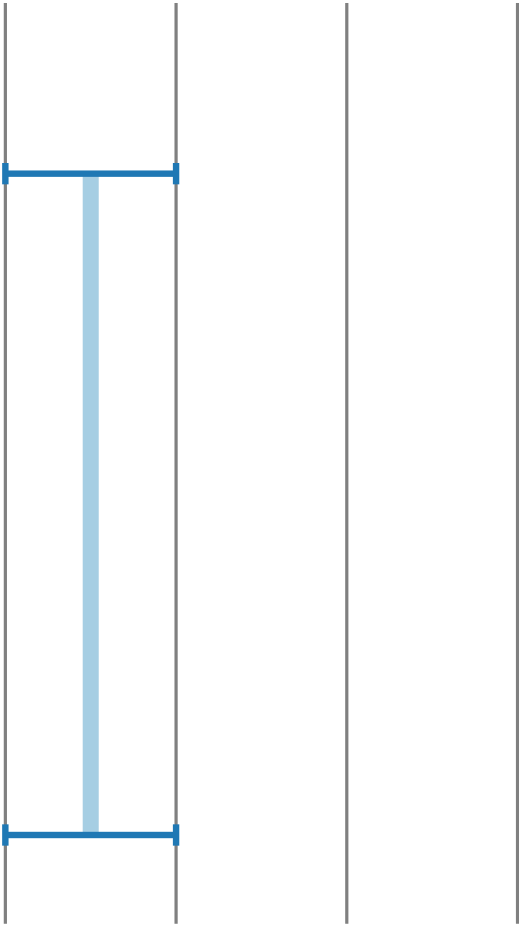


Integral

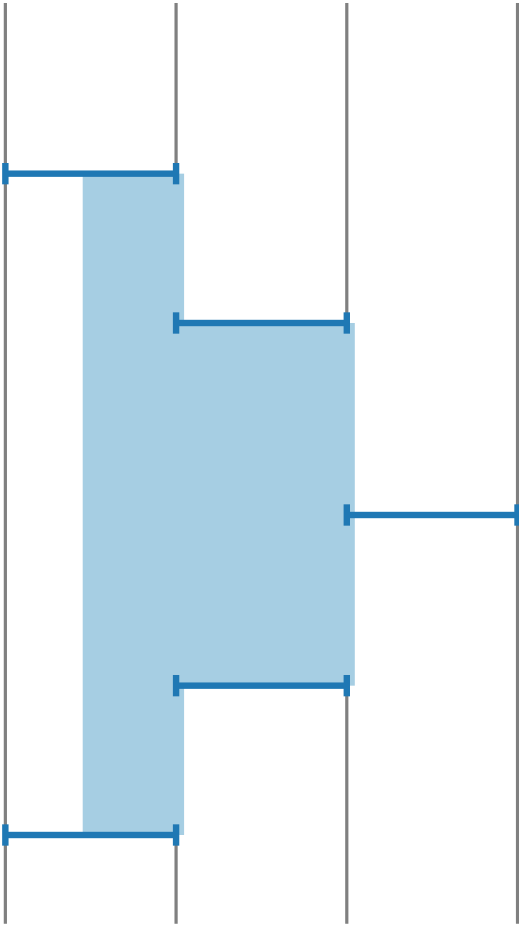
Unit Bar-Visibility Representation

\Leftrightarrow maxdeg 3, subdivision of caterpillar

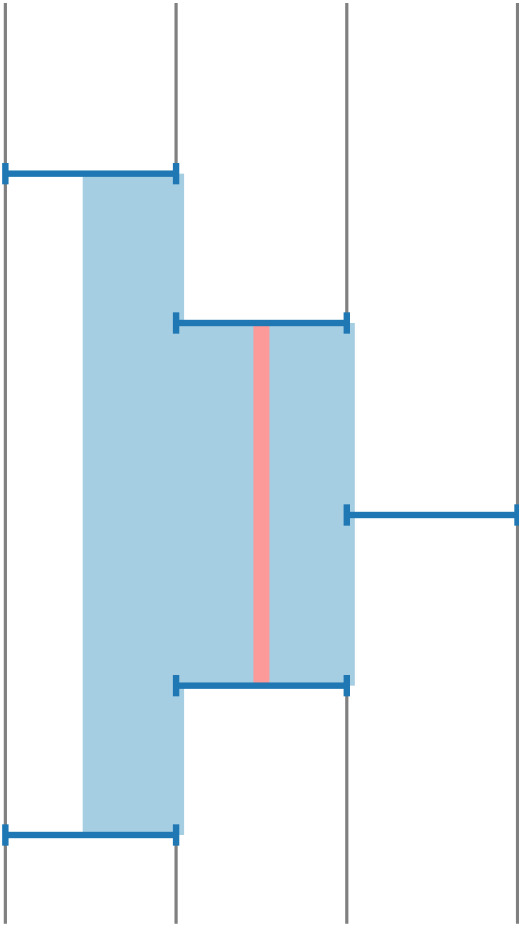
Strong: Biconnected Graphs



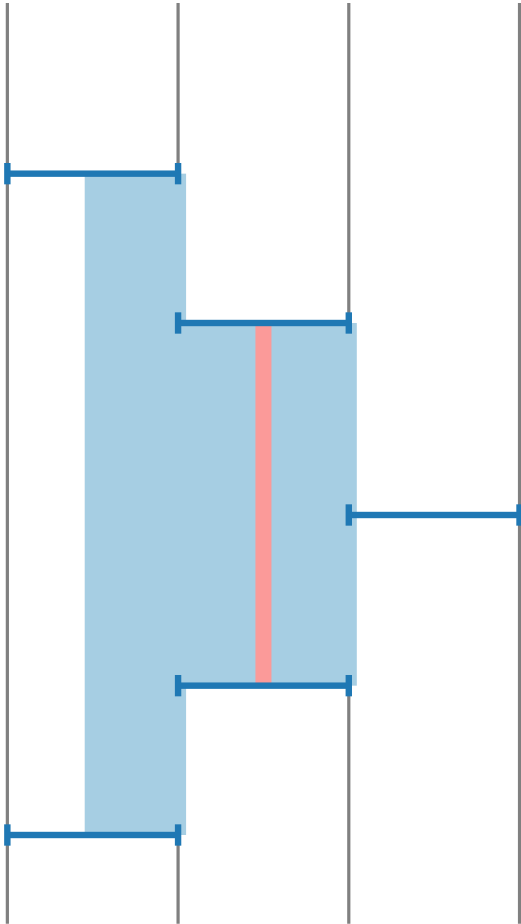
Strong: Biconnected Graphs



Strong: Biconnected Graphs

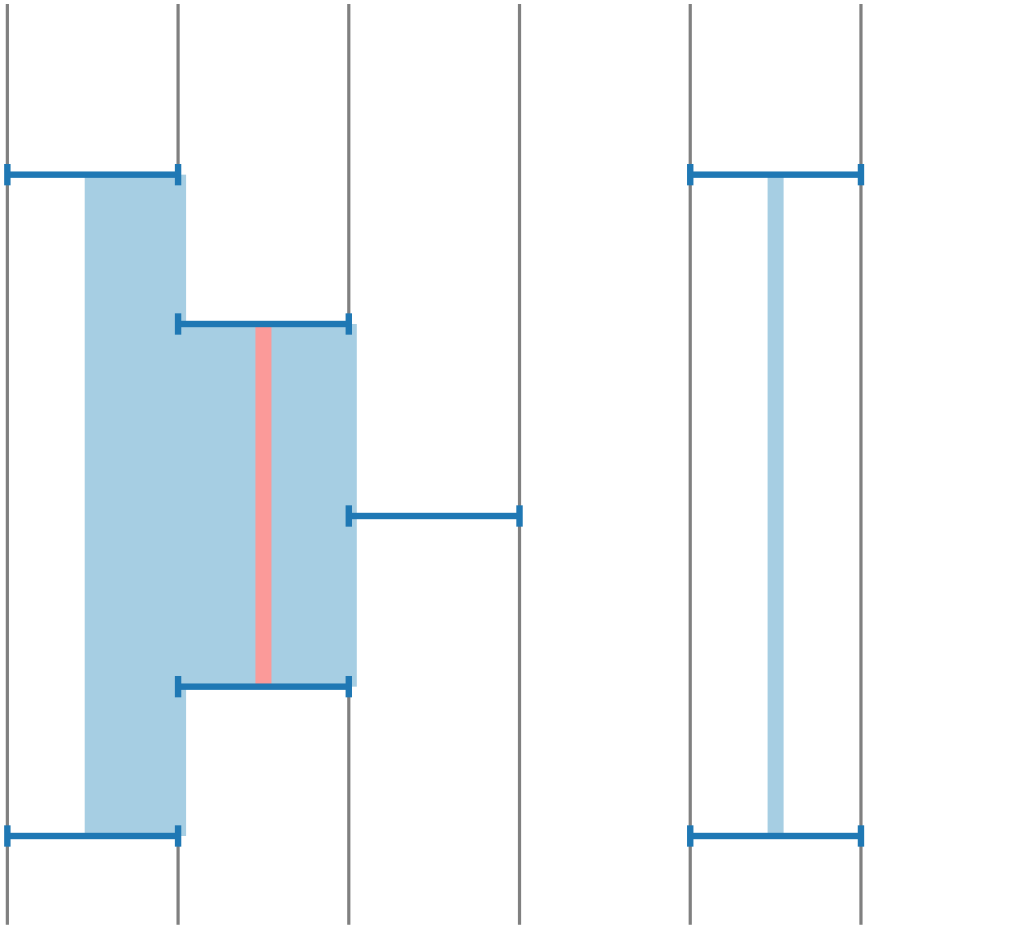


Strong: Biconnected Graphs



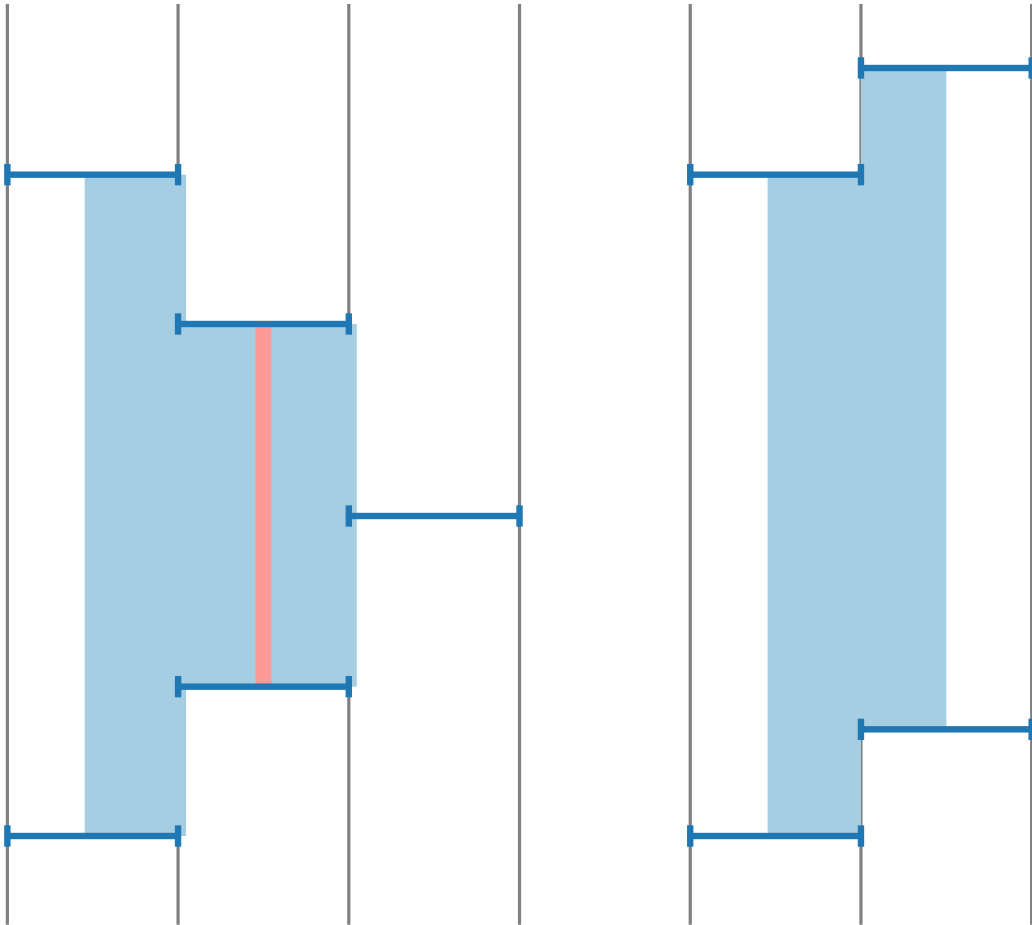
Every face spans two slabs

Strong: Biconnected Graphs



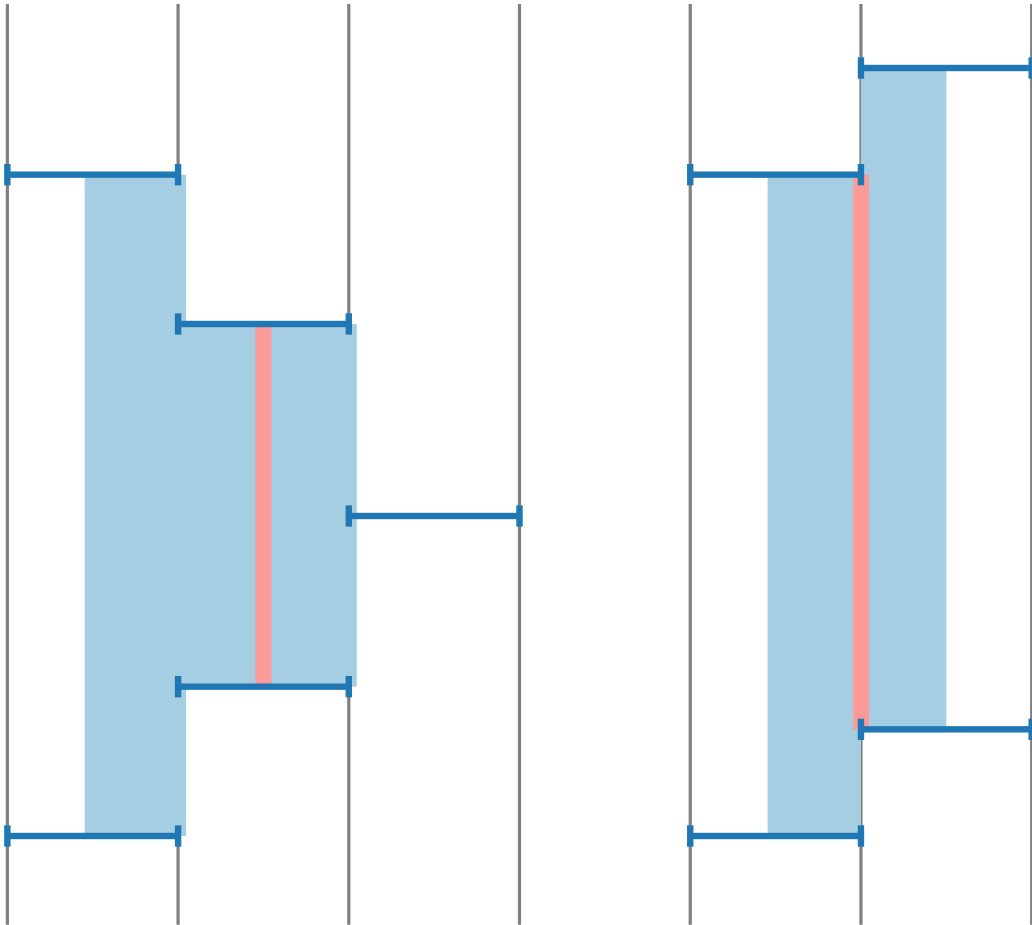
Every face spans two slabs

Strong: Biconnected Graphs



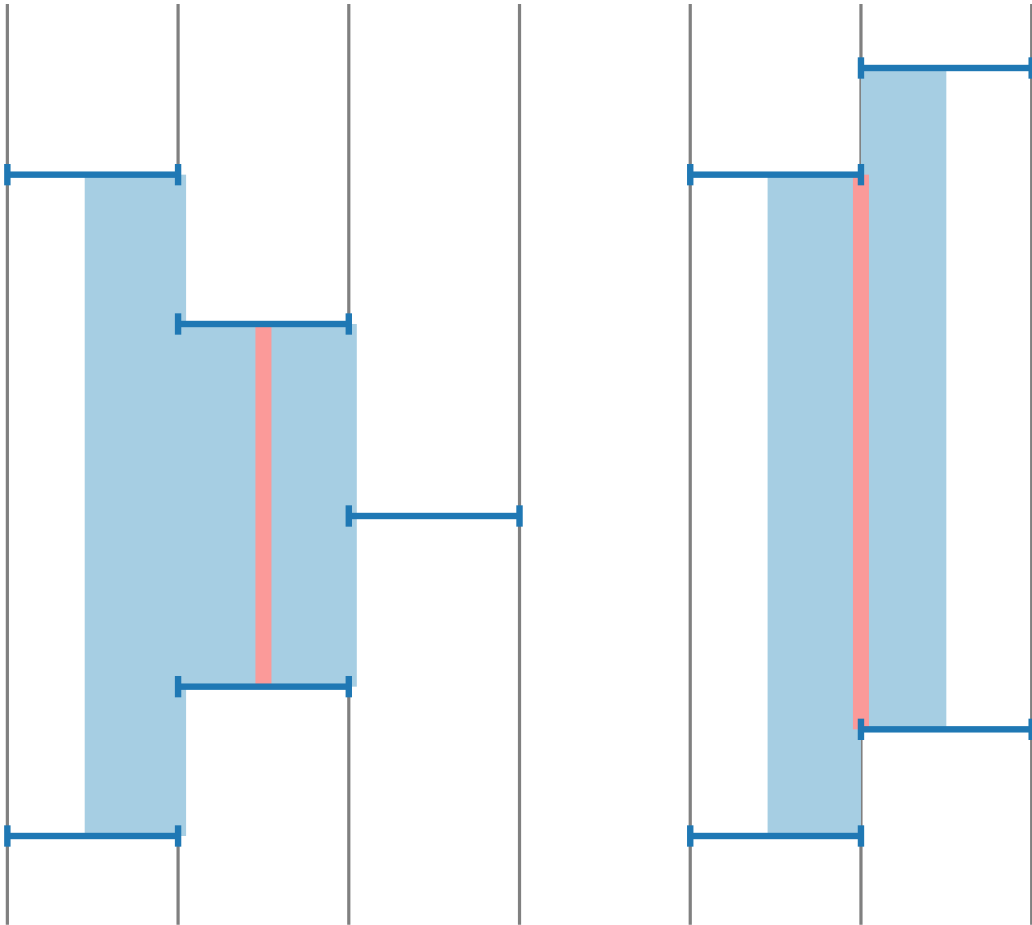
Every face spans two slabs

Strong: Biconnected Graphs



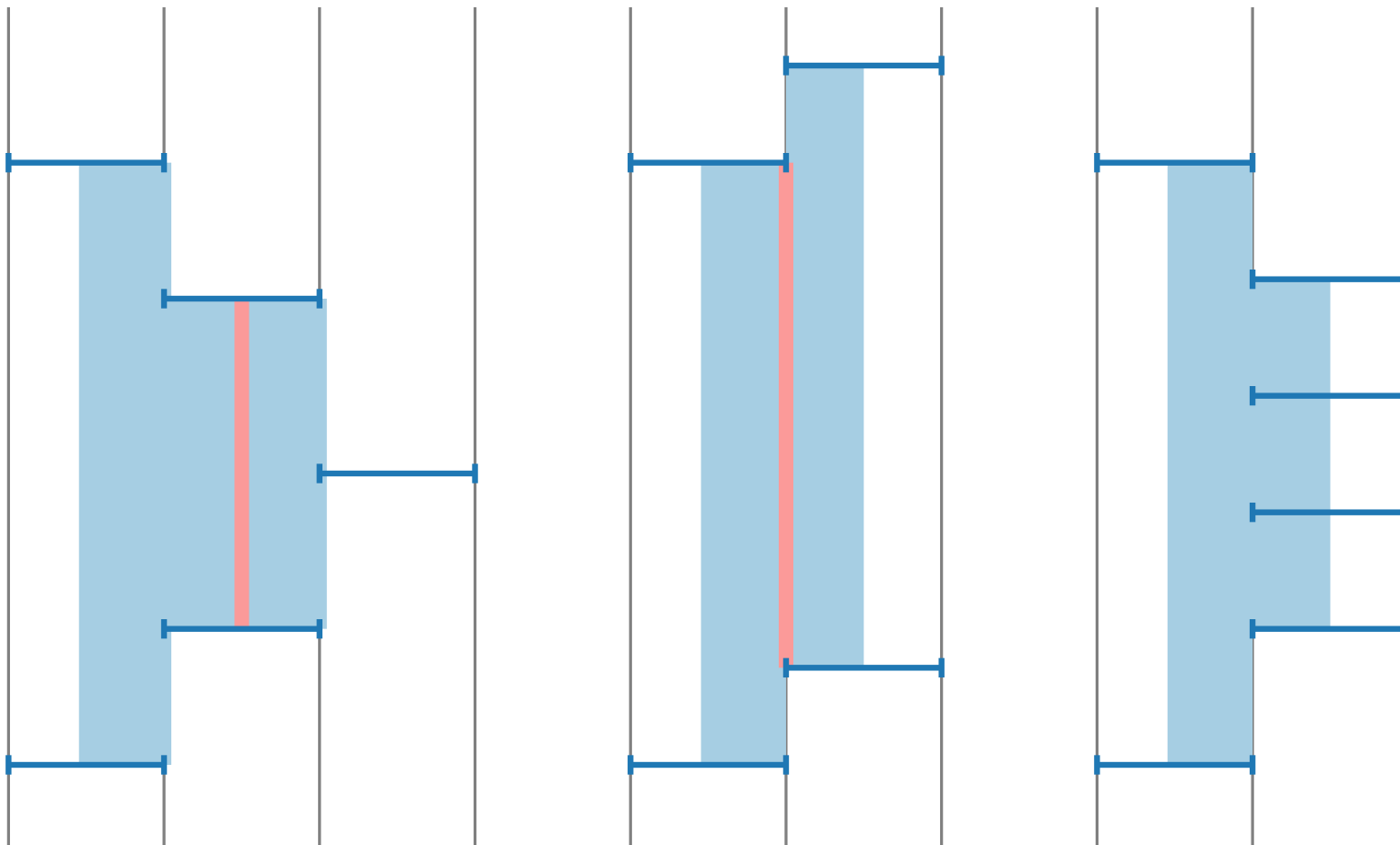
Every face spans two slabs

Strong: Biconnected Graphs



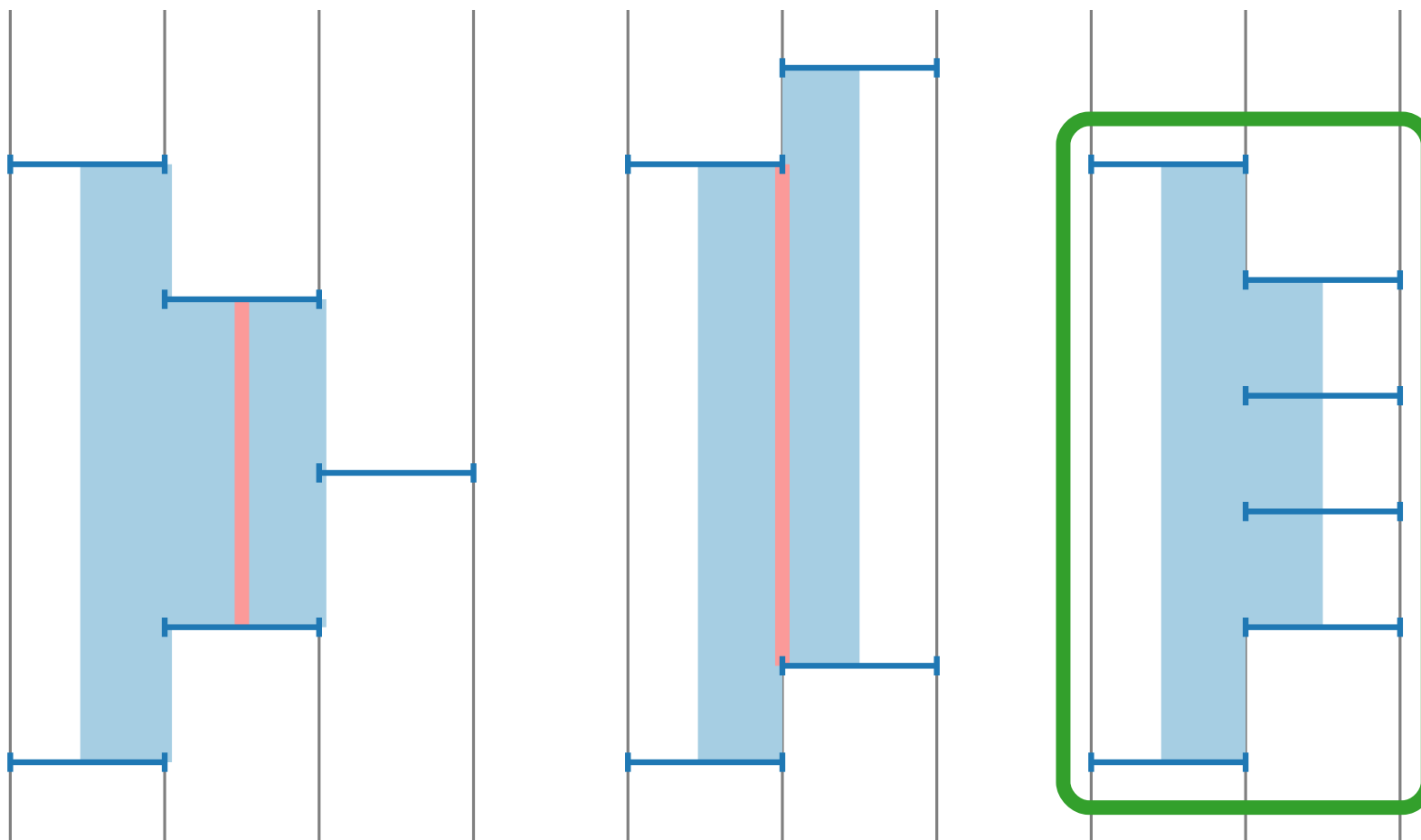
Every face spans two slabs

Strong: Biconnected Graphs



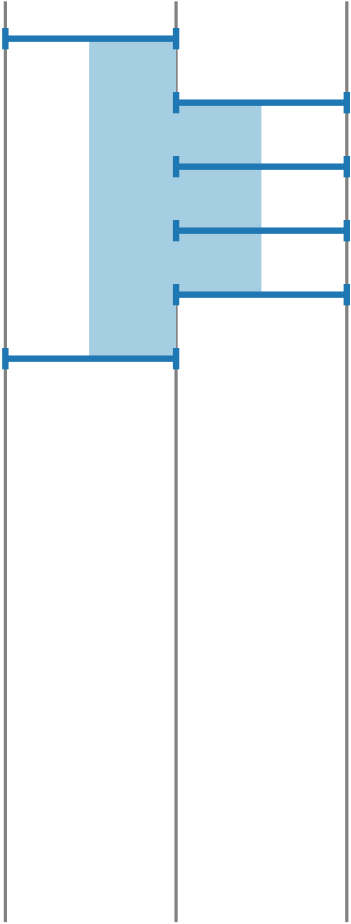
Every face spans two slabs

Strong: Biconnected Graphs

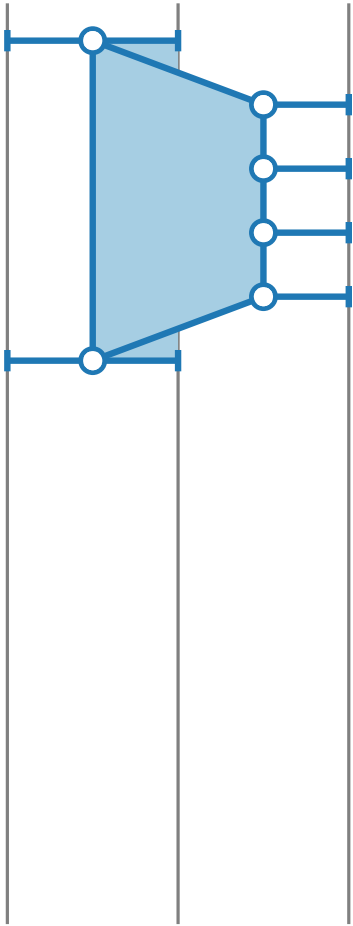


Every face spans two slabs and is a trapezoid

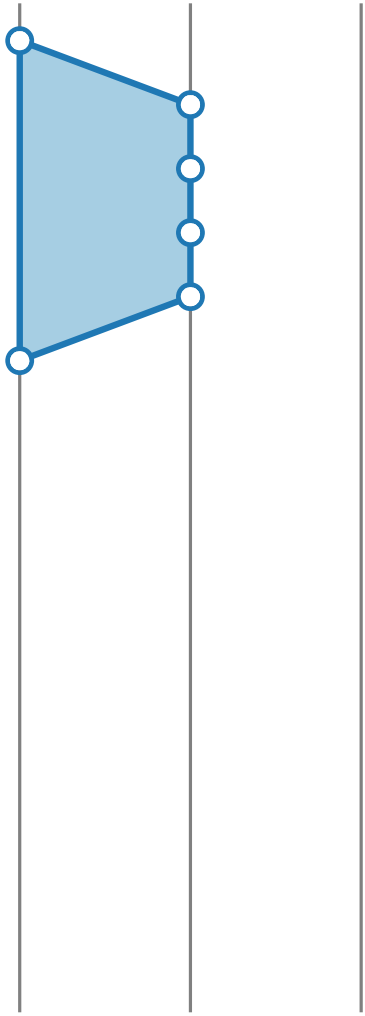
Strong: Biconnected Graphs



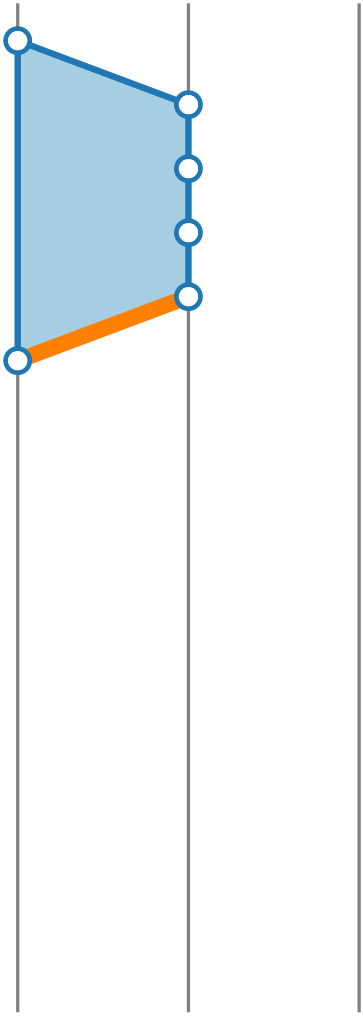
Strong: Biconnected Graphs



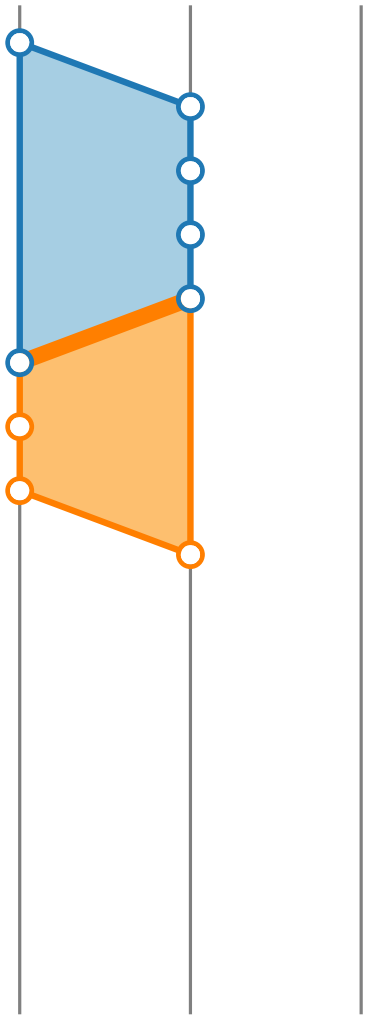
Strong: Biconnected Graphs



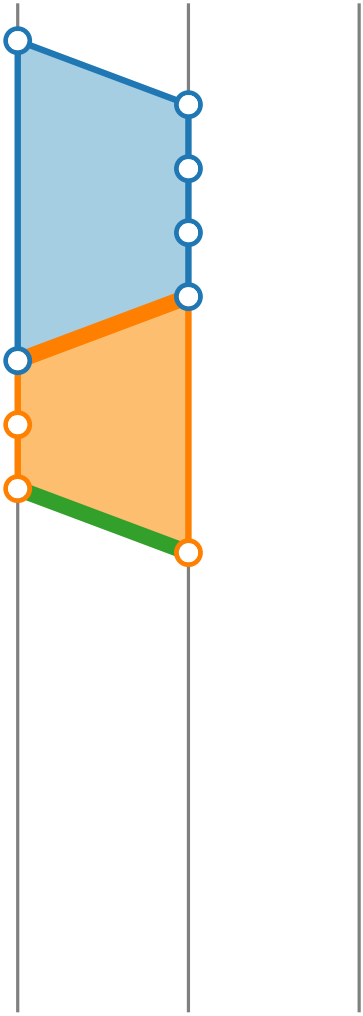
Strong: Biconnected Graphs



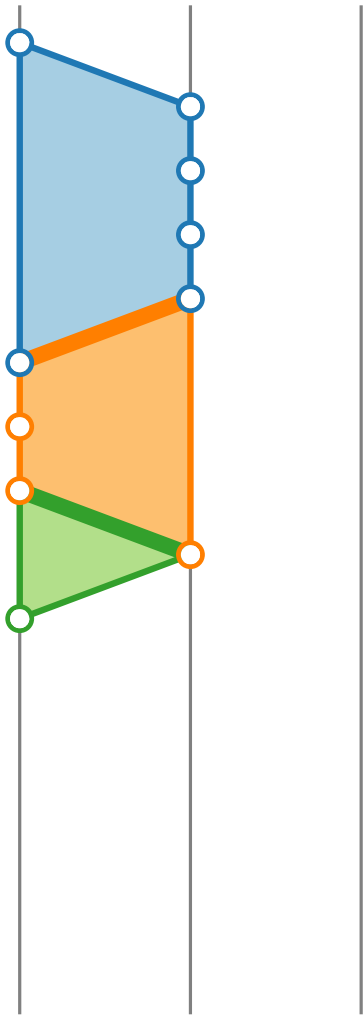
Strong: Biconnected Graphs



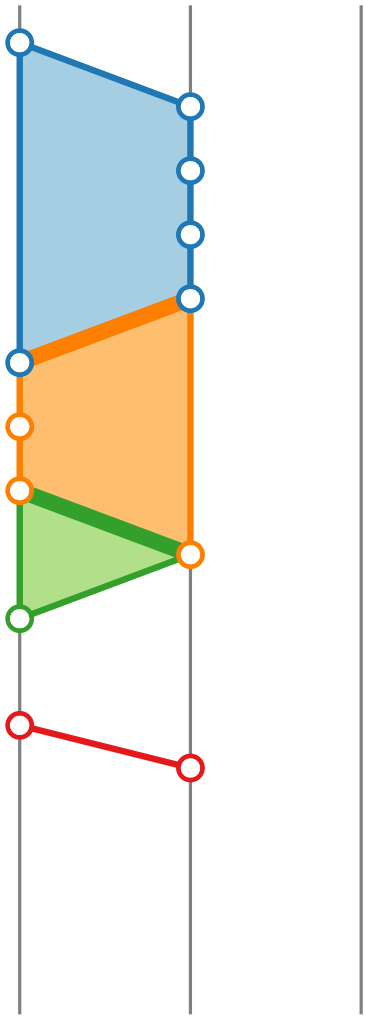
Strong: Biconnected Graphs



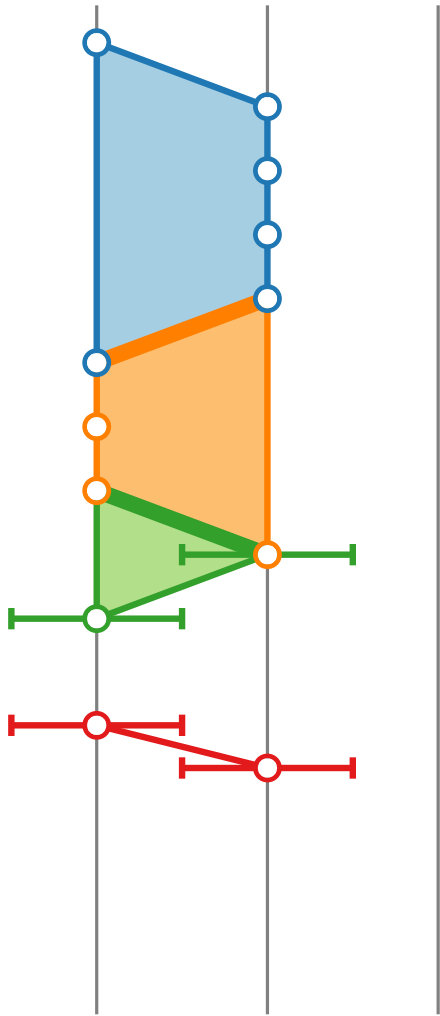
Strong: Biconnected Graphs



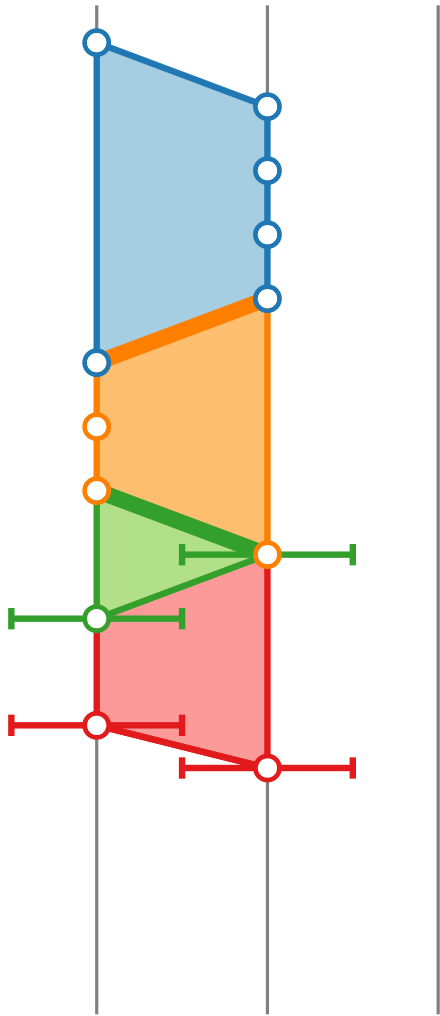
Strong: Biconnected Graphs



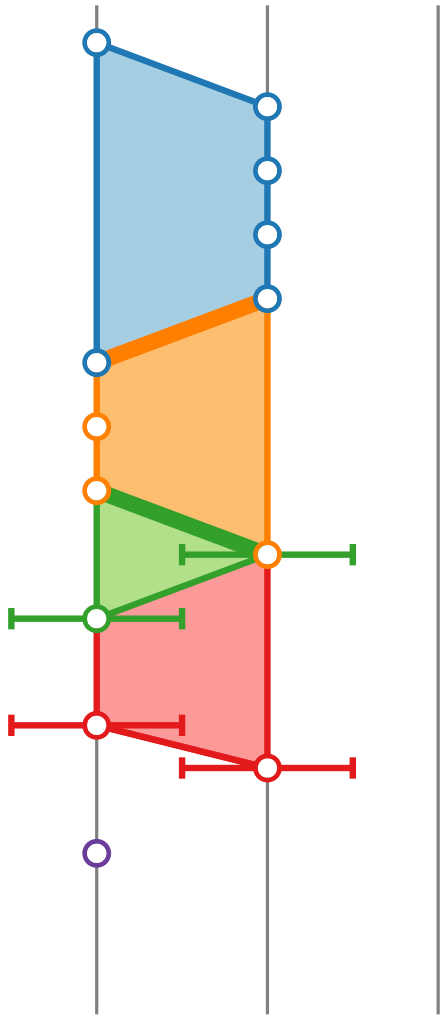
Strong: Biconnected Graphs



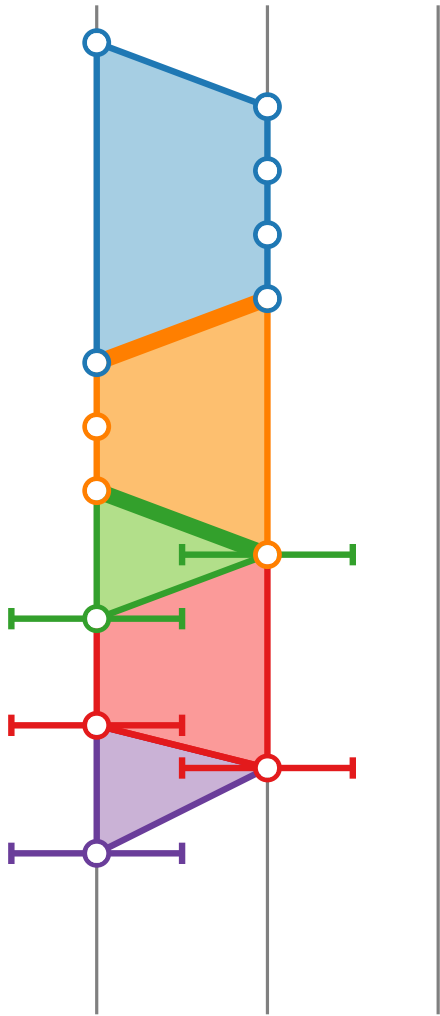
Strong: Biconnected Graphs



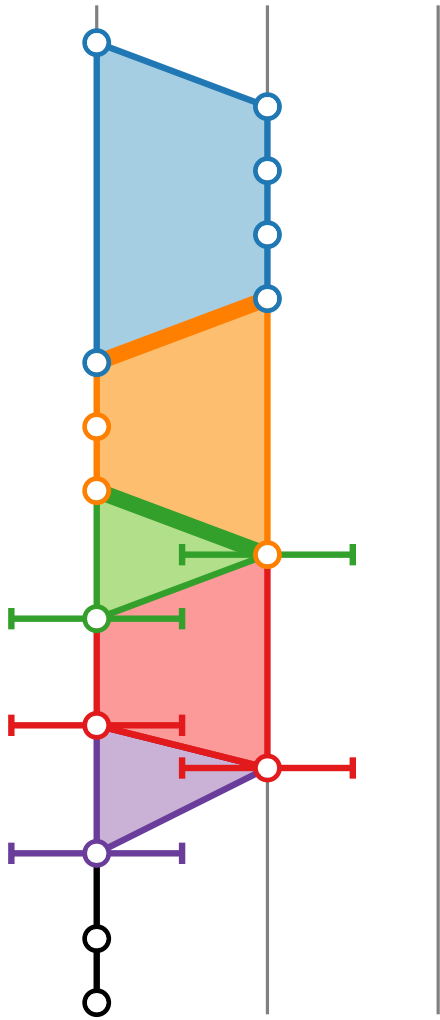
Strong: Biconnected Graphs



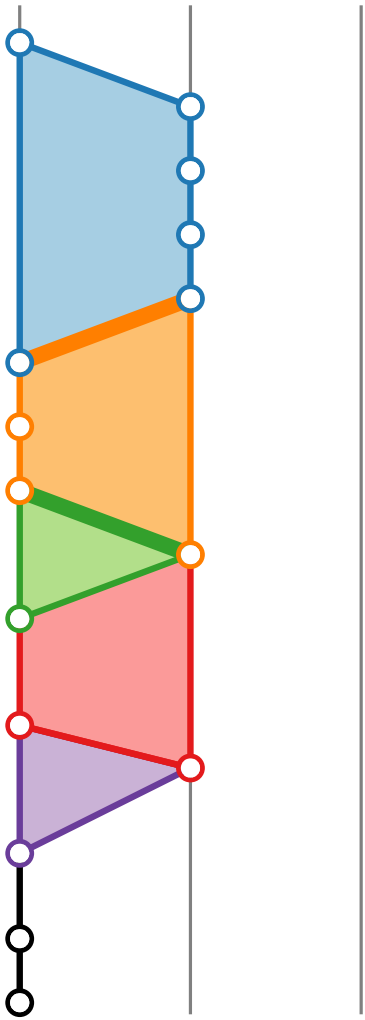
Strong: Biconnected Graphs



Strong: Biconnected Graphs

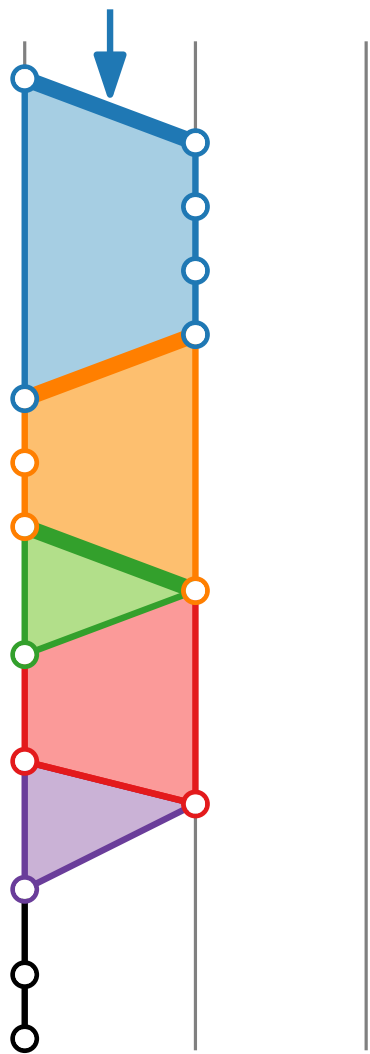


Strong: Biconnected Graphs



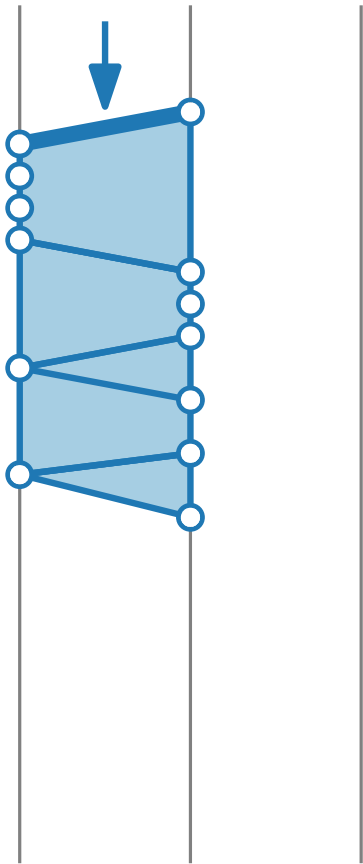
Fixing one diagonal fixes the whole slab

Strong: Biconnected Graphs



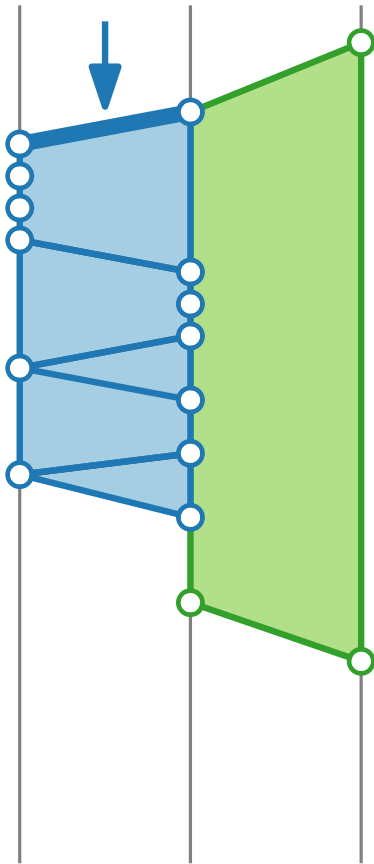
Fixing one diagonal fixes the whole slab
Idea: guess the topmost diagonal

Strong: Biconnected Graphs



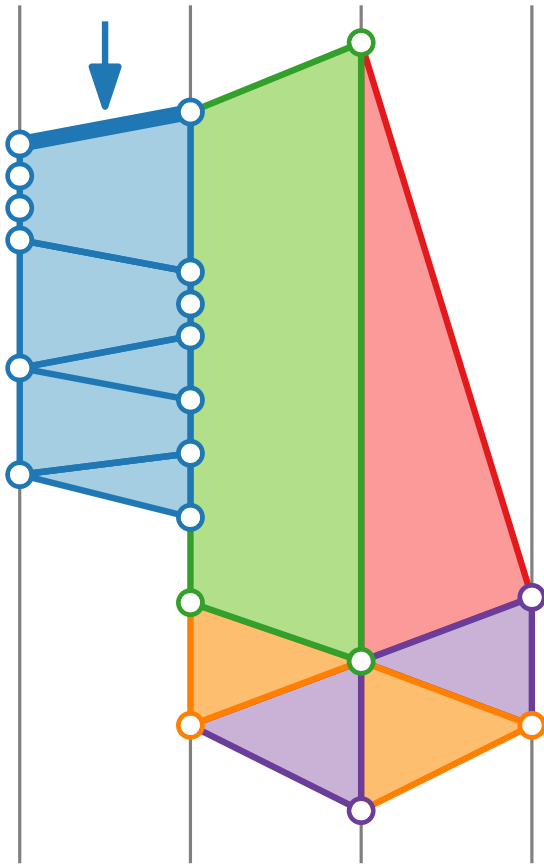
Fixing one diagonal fixes the whole slab
Idea: guess the topmost diagonal

Strong: Biconnected Graphs



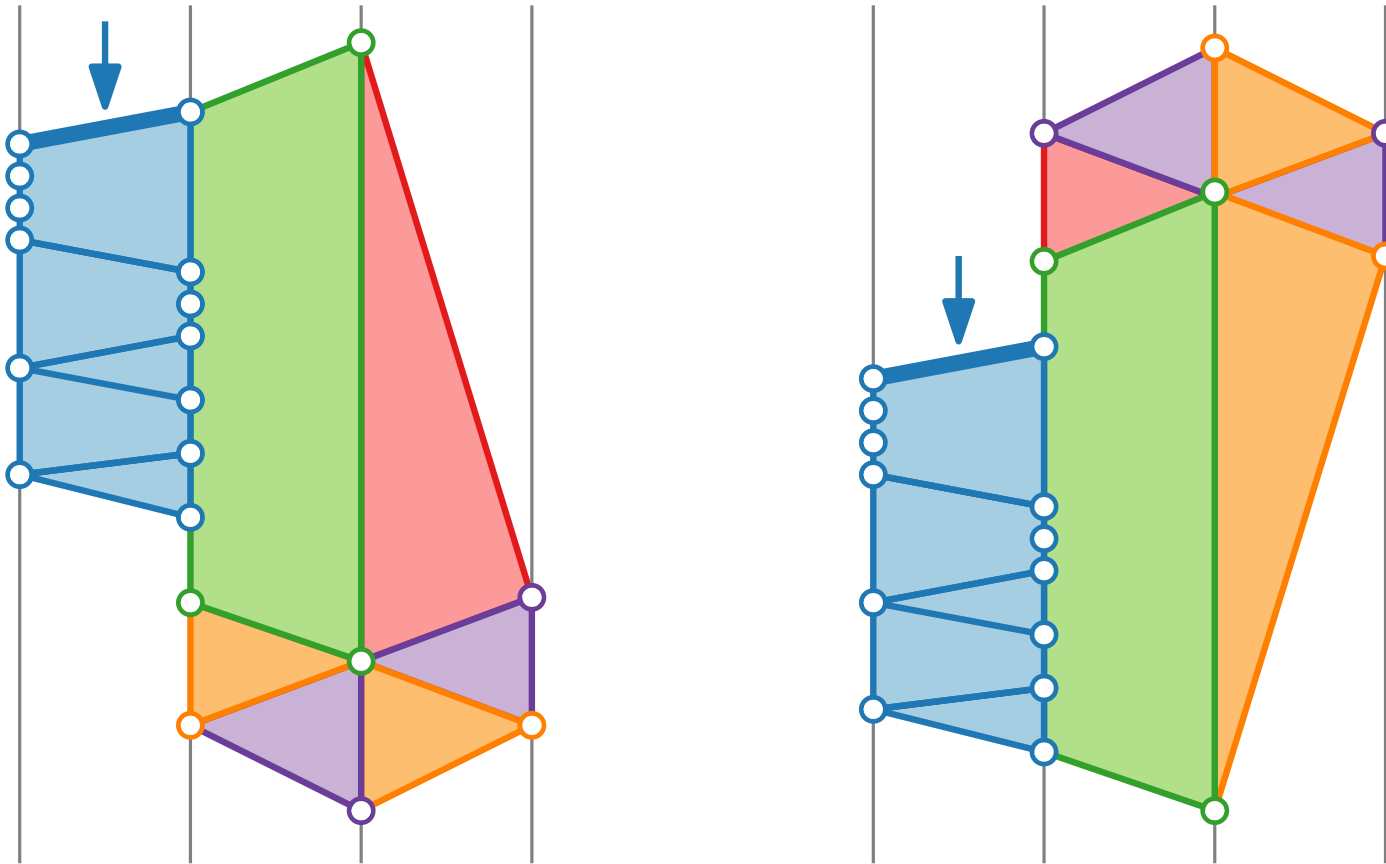
Fixing one diagonal fixes the whole slab
Idea: guess the topmost diagonal

Strong: Biconnected Graphs



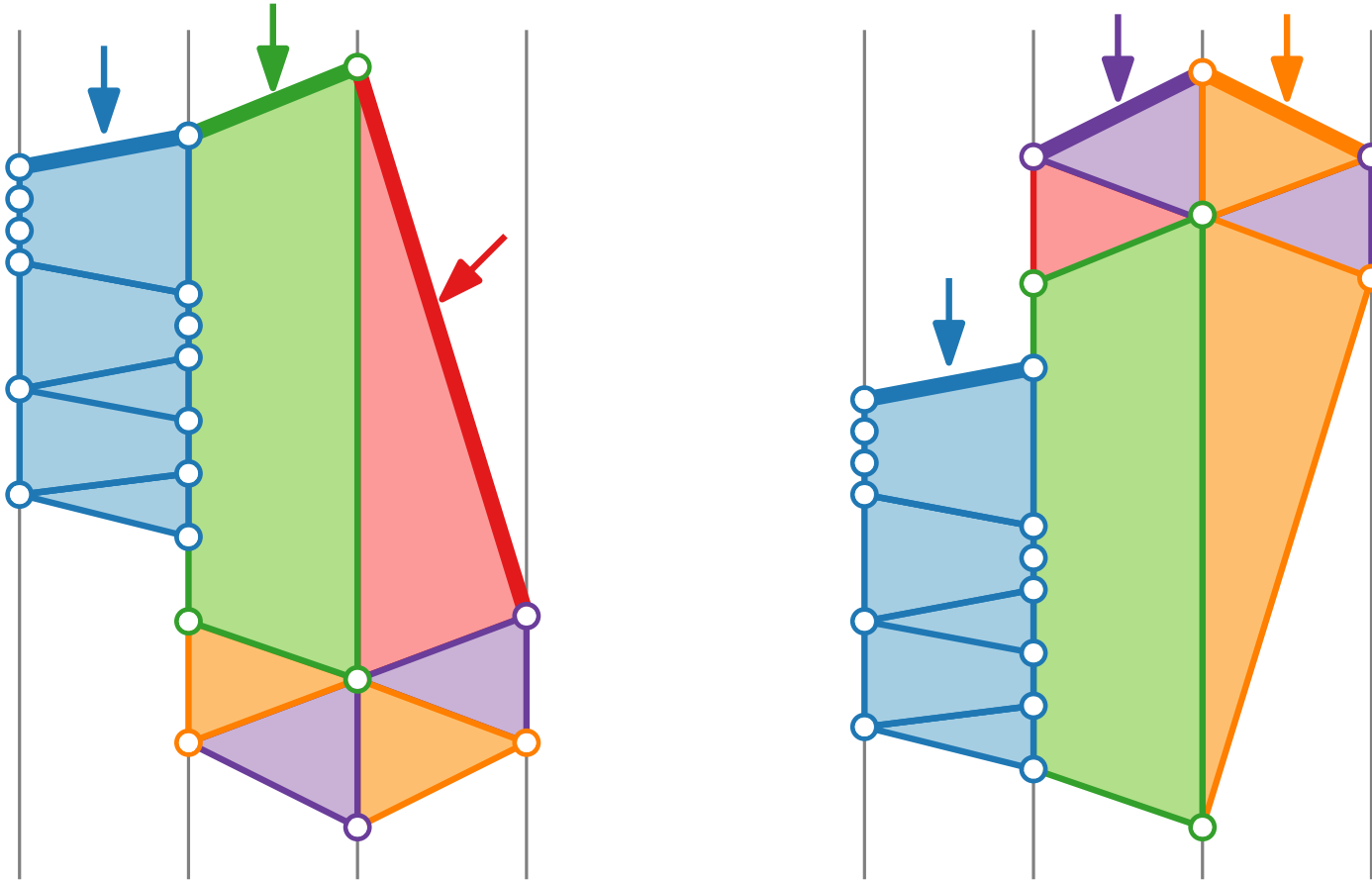
Fixing one diagonal fixes the whole slab
Idea: guess the topmost diagonal

Strong: Biconnected Graphs



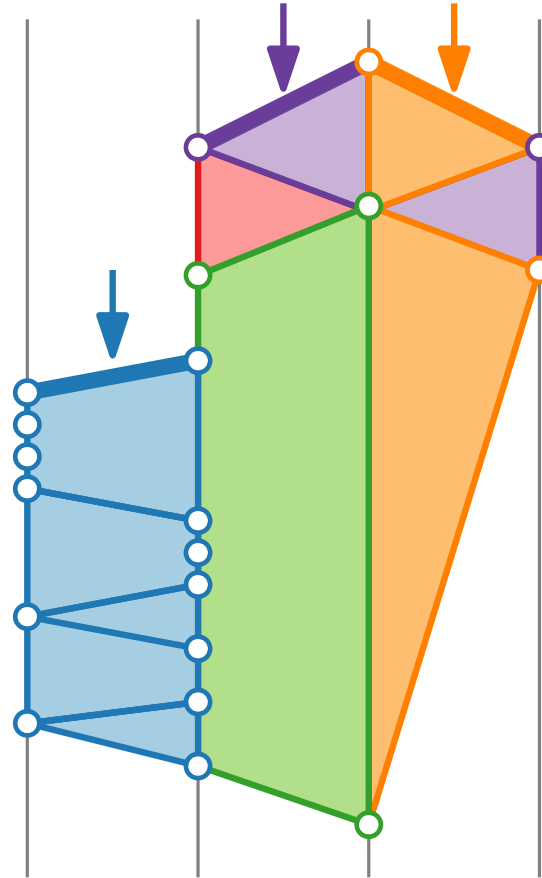
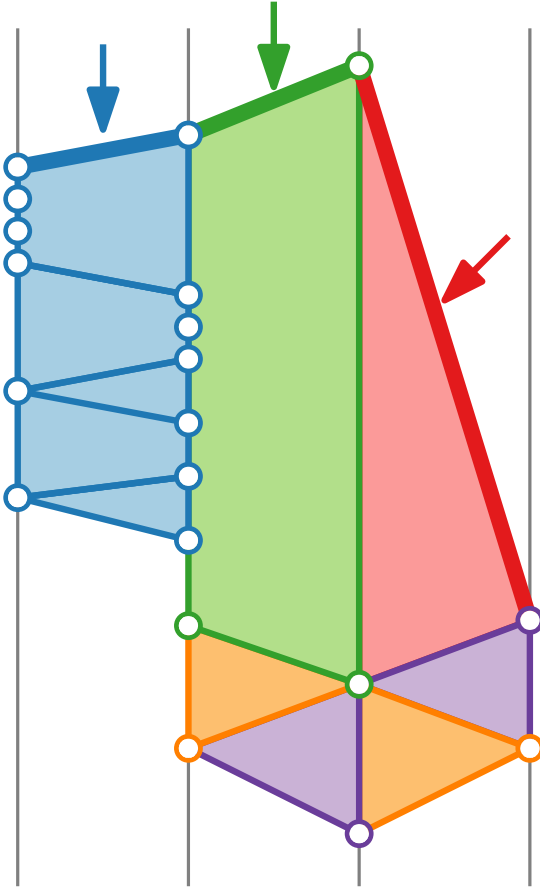
Fixing one diagonal fixes the whole slab
Idea: guess the topmost diagonal

Strong: Biconnected Graphs

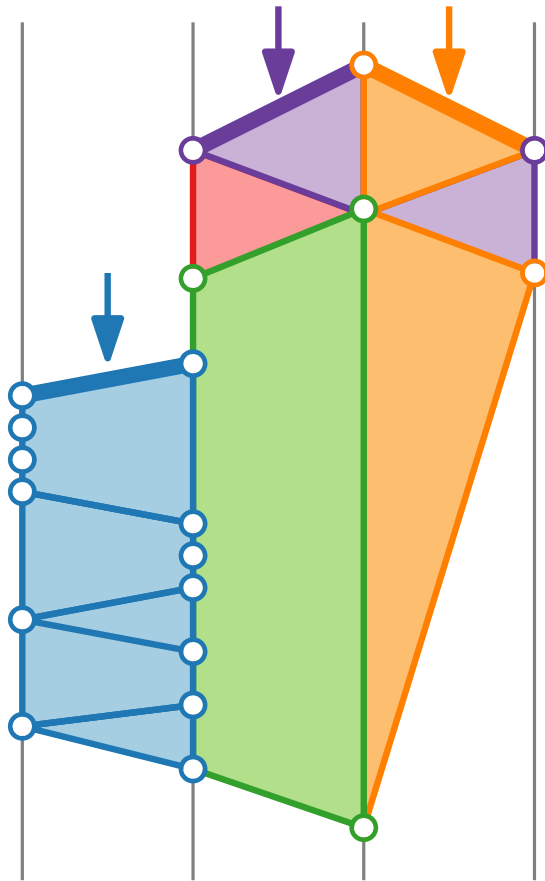
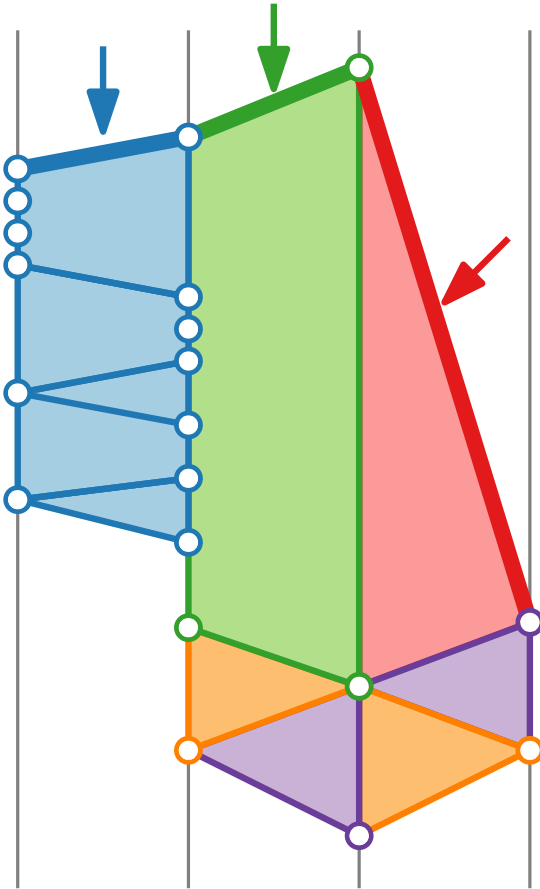


Fixing one diagonal fixes the whole slab
Idea: guess the topmost diagonal

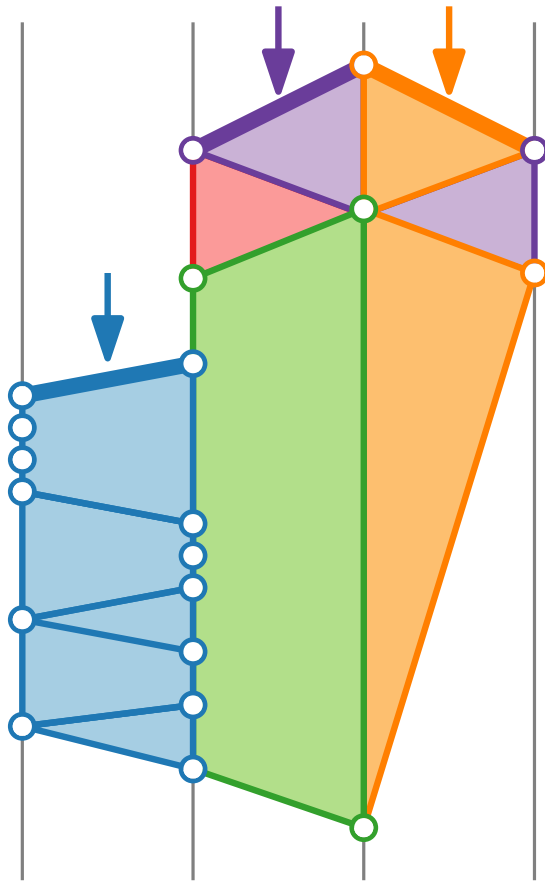
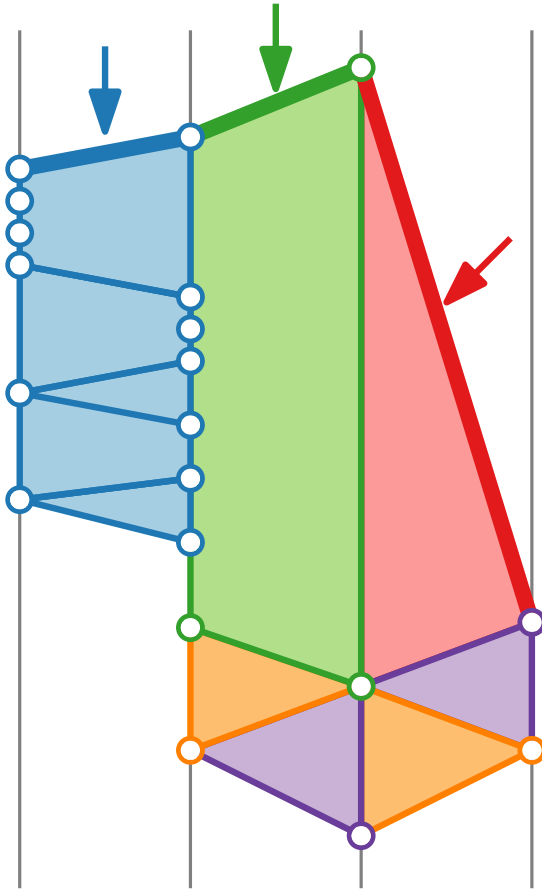
Strong: Biconnected Graphs



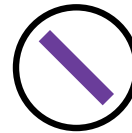
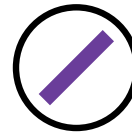
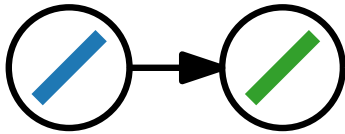
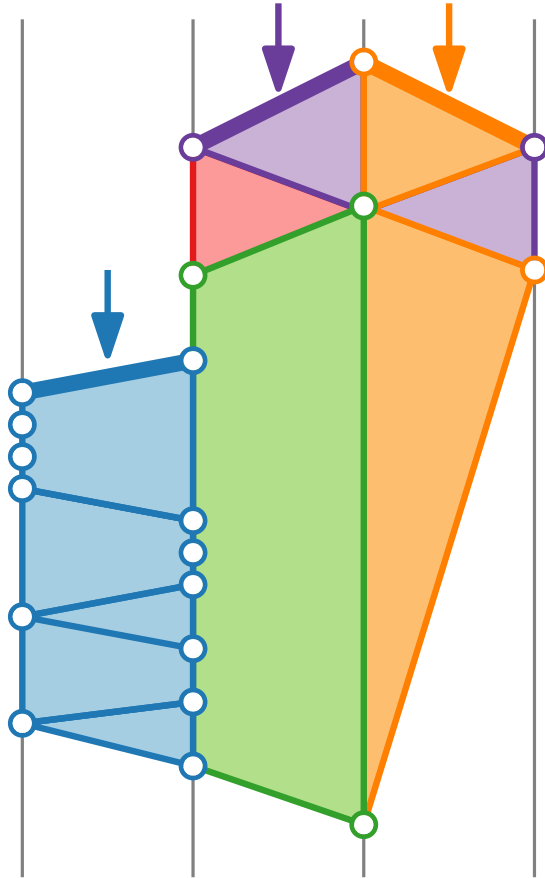
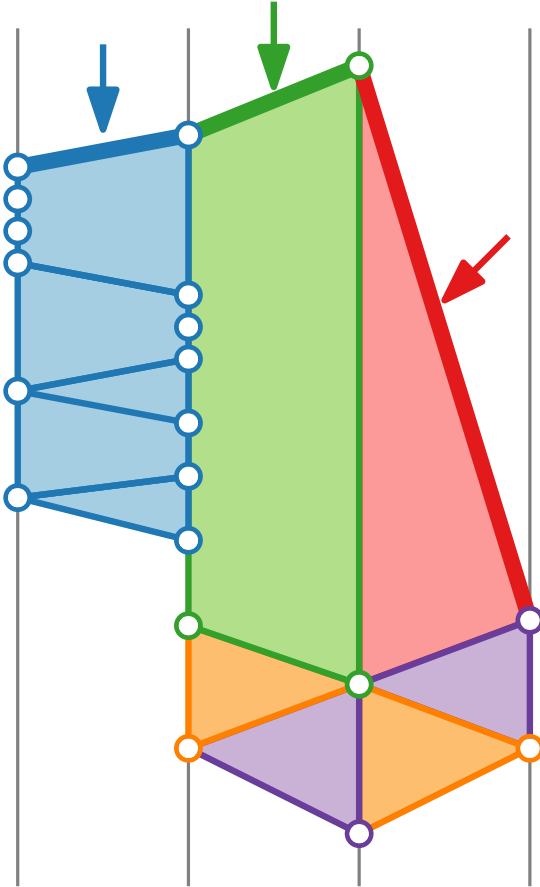
Strong: Biconnected Graphs



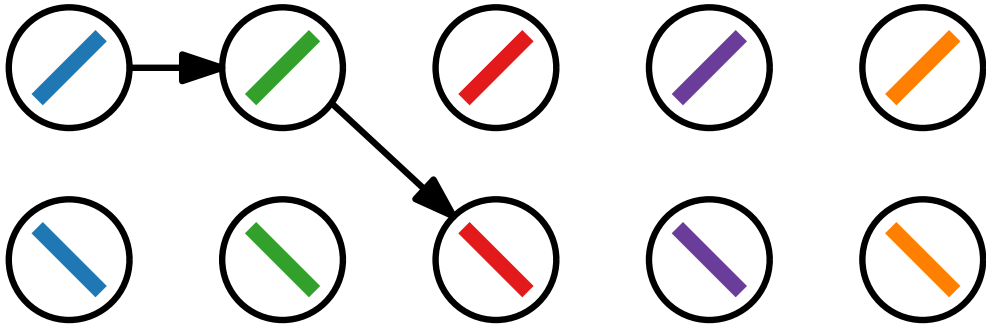
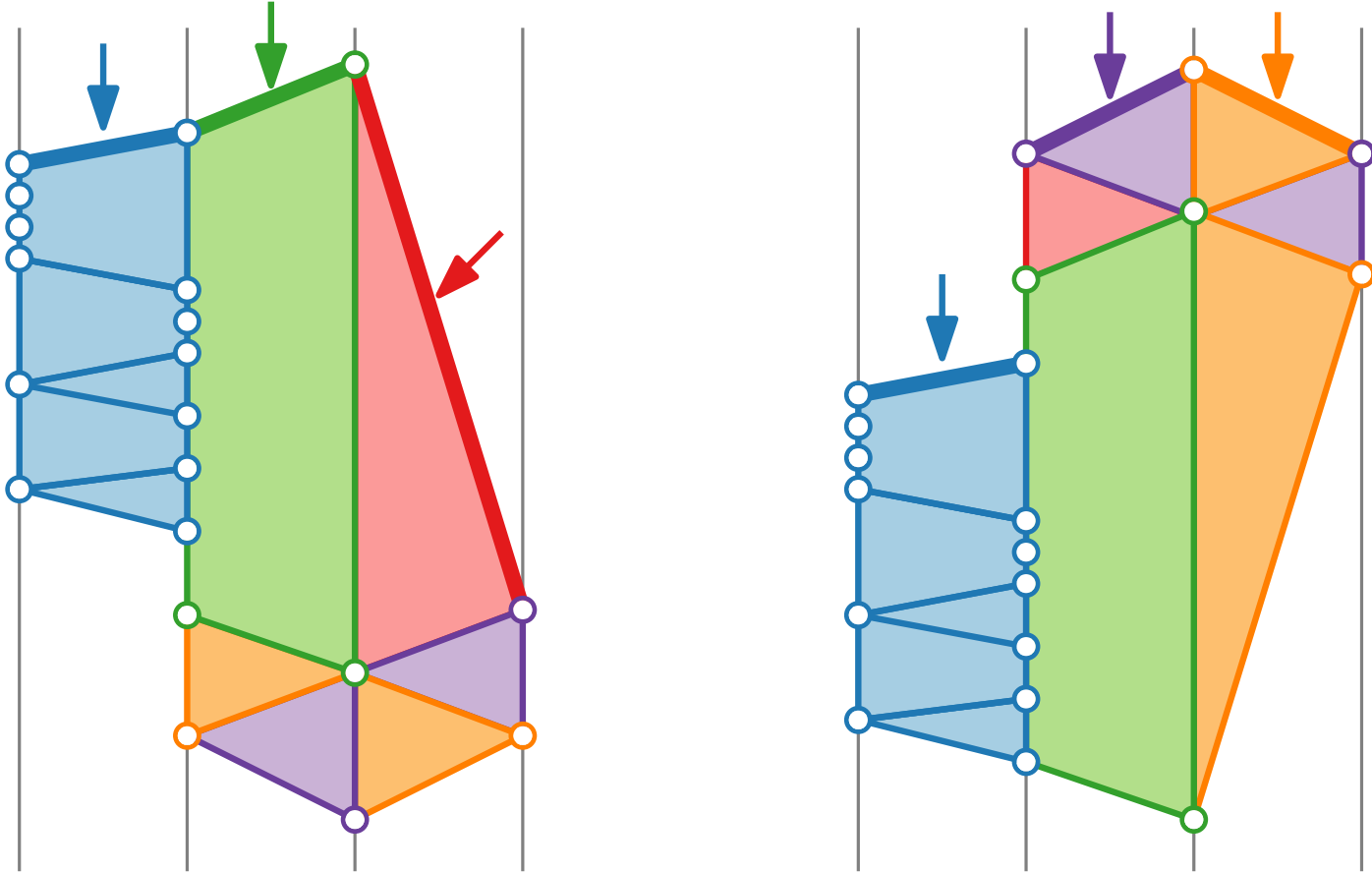
Strong: Biconnected Graphs



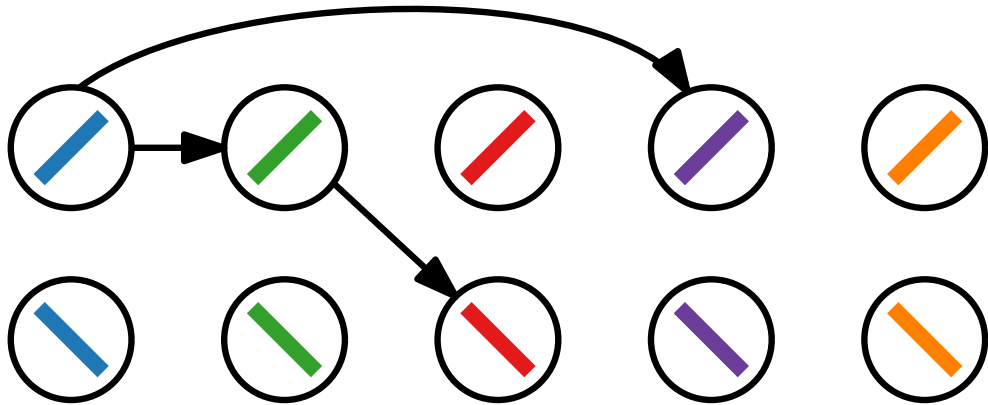
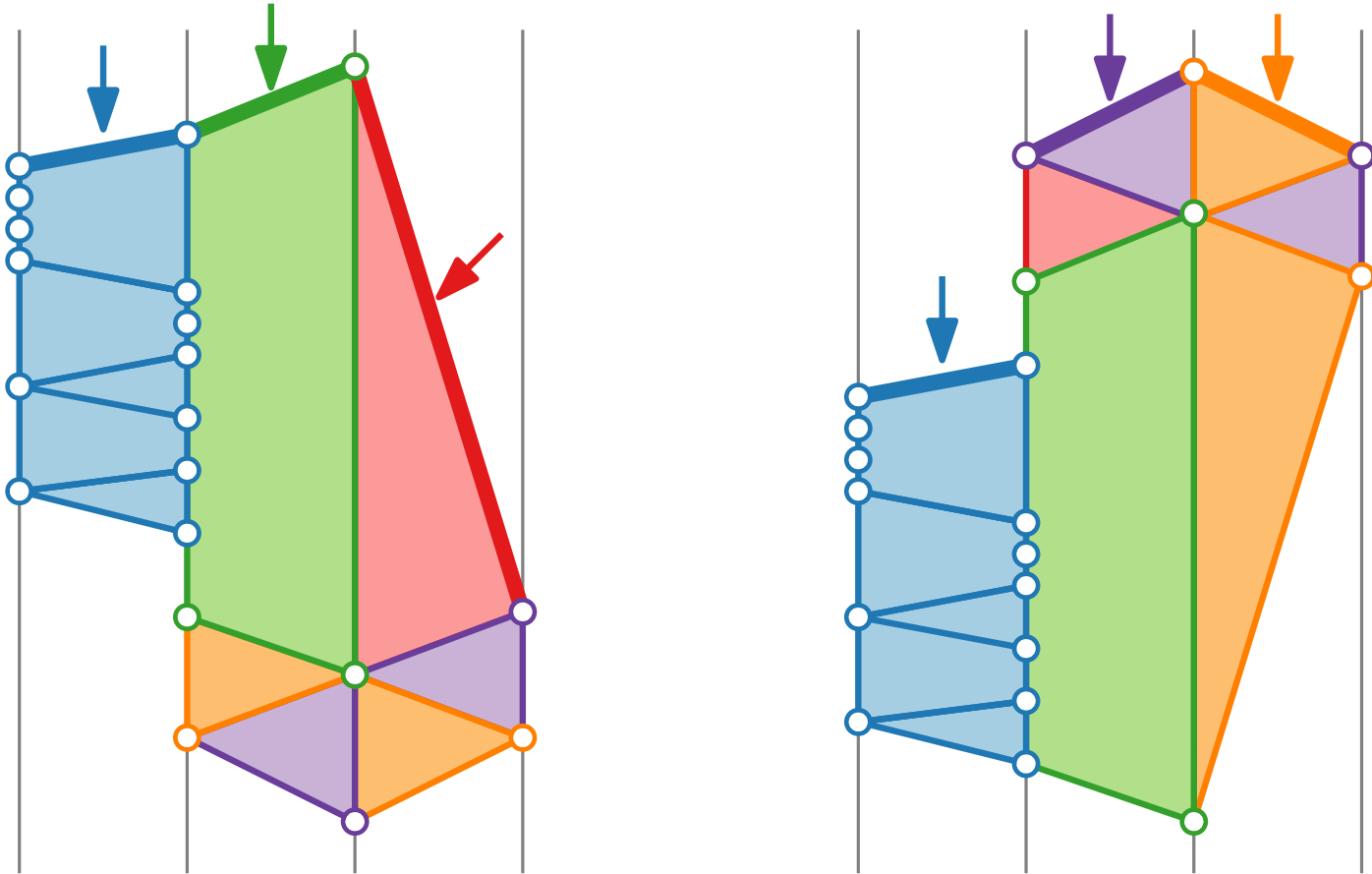
Strong: Biconnected Graphs



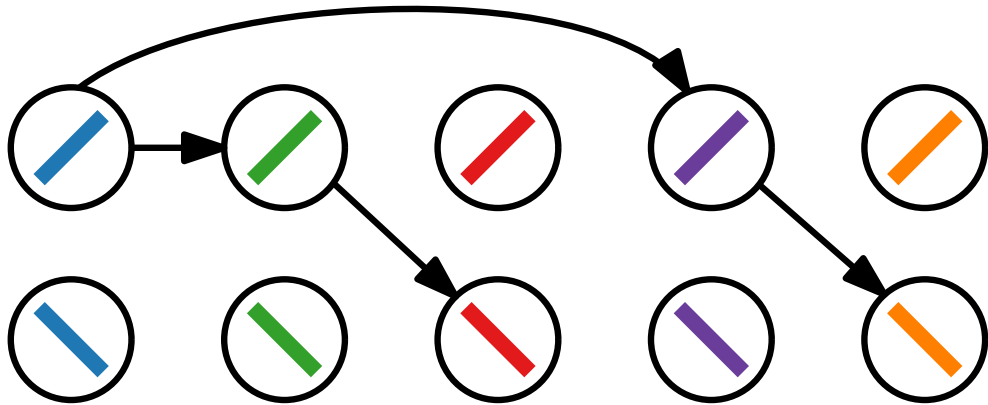
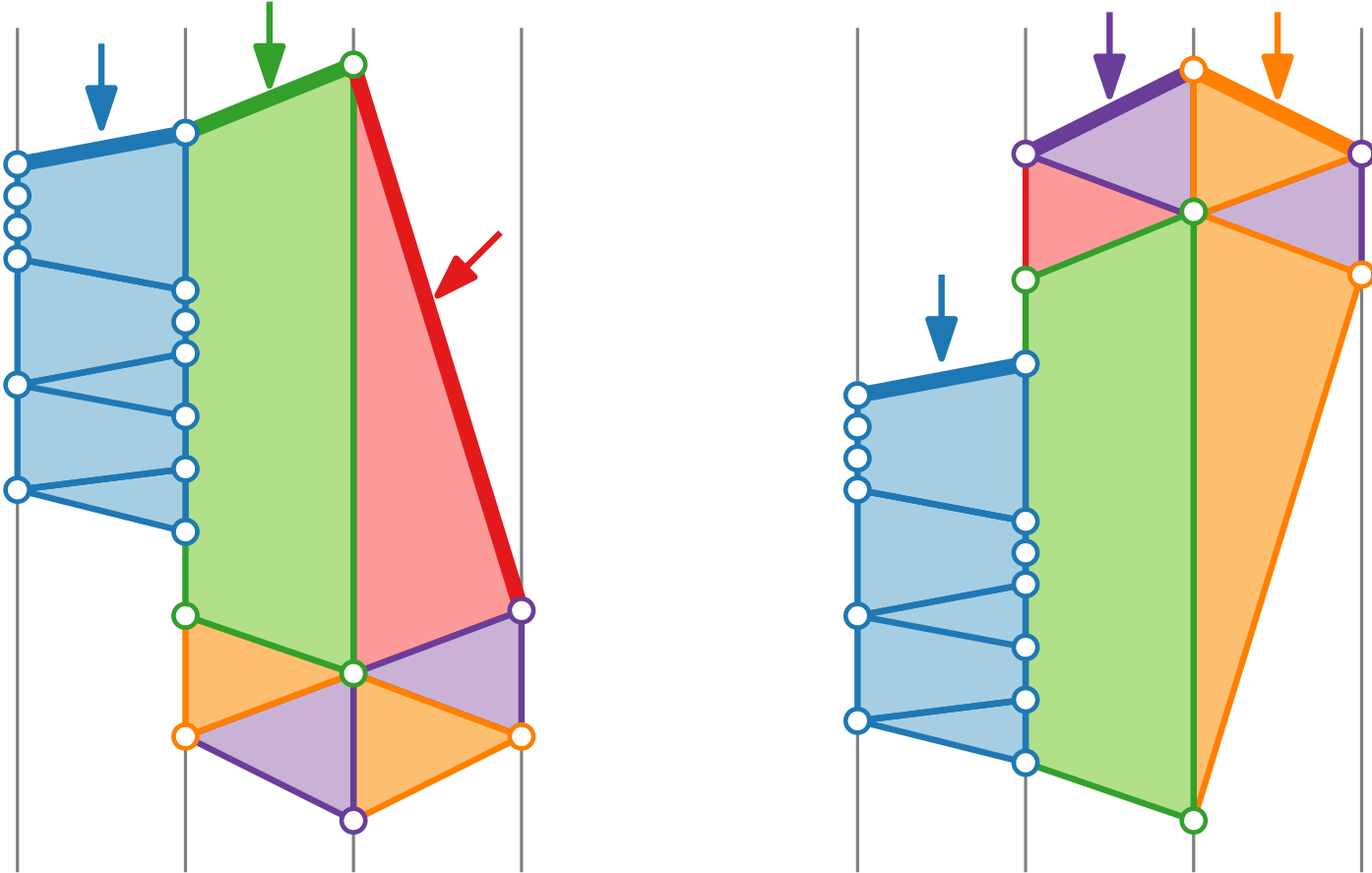
Strong: Biconnected Graphs



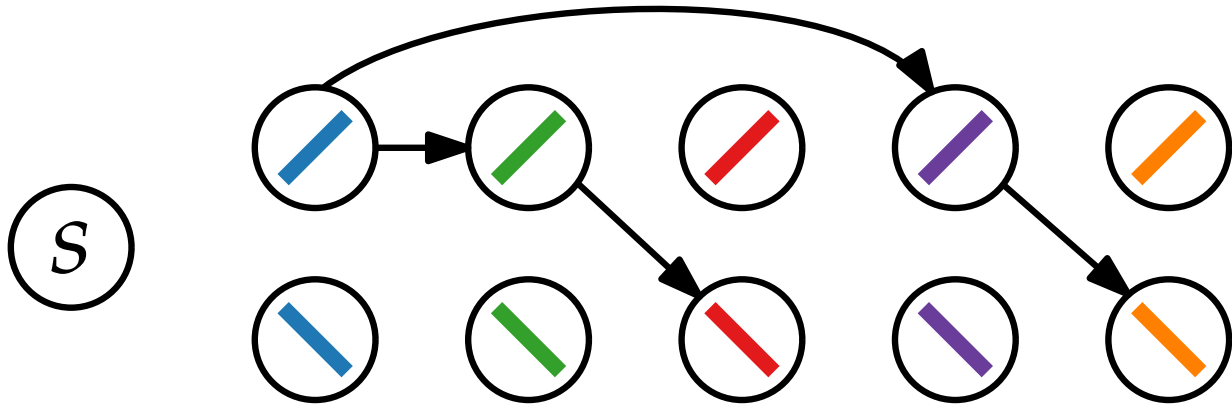
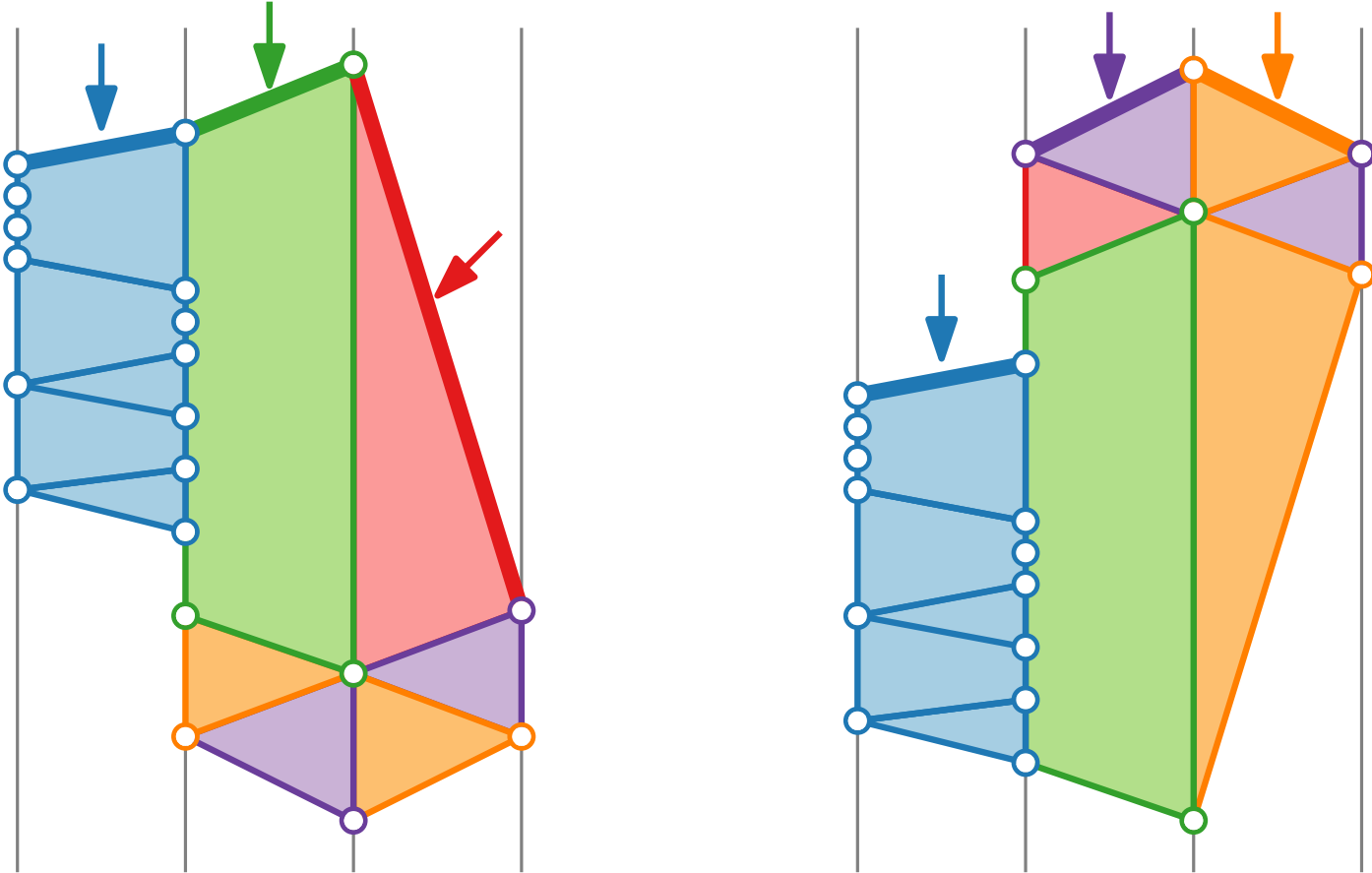
Strong: Biconnected Graphs



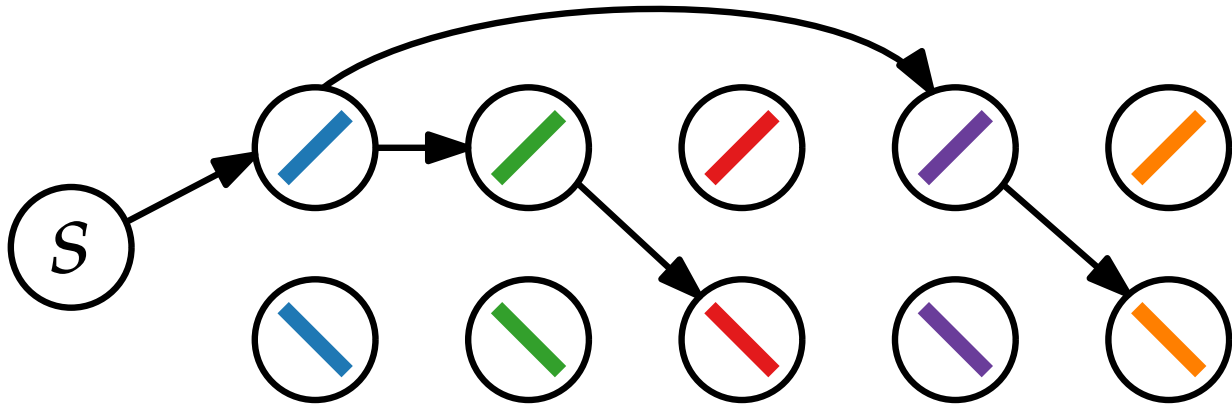
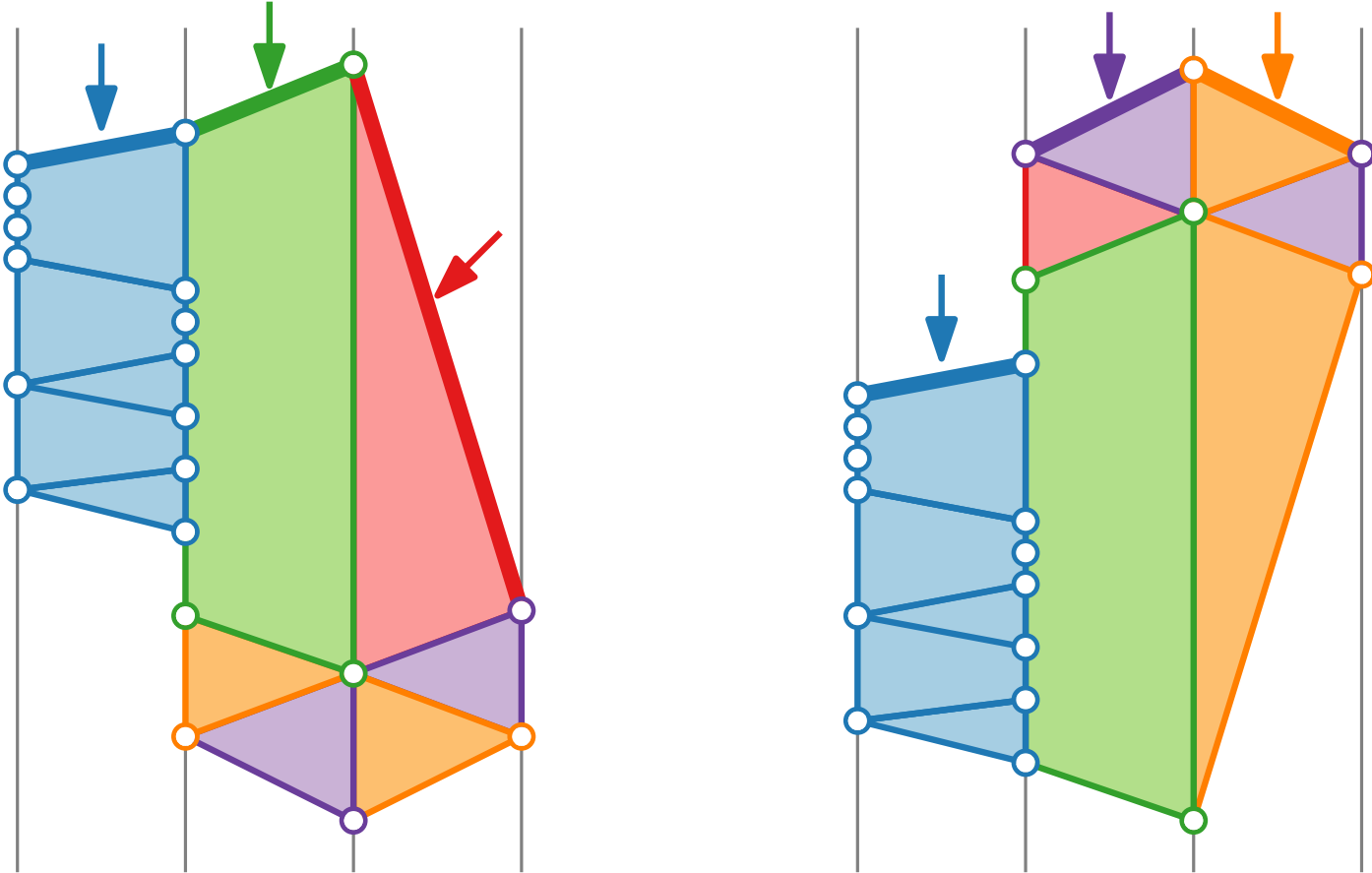
Strong: Biconnected Graphs



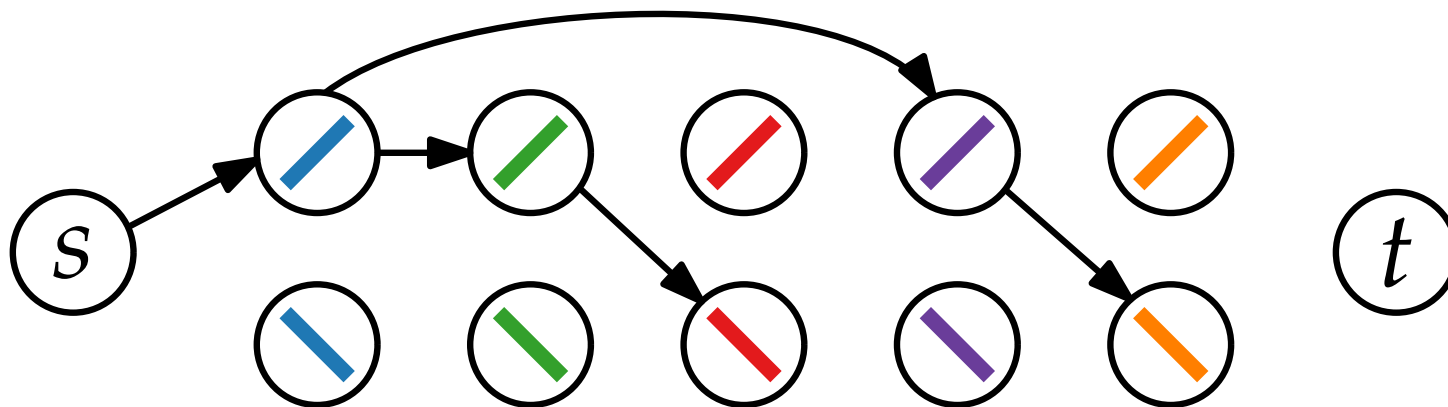
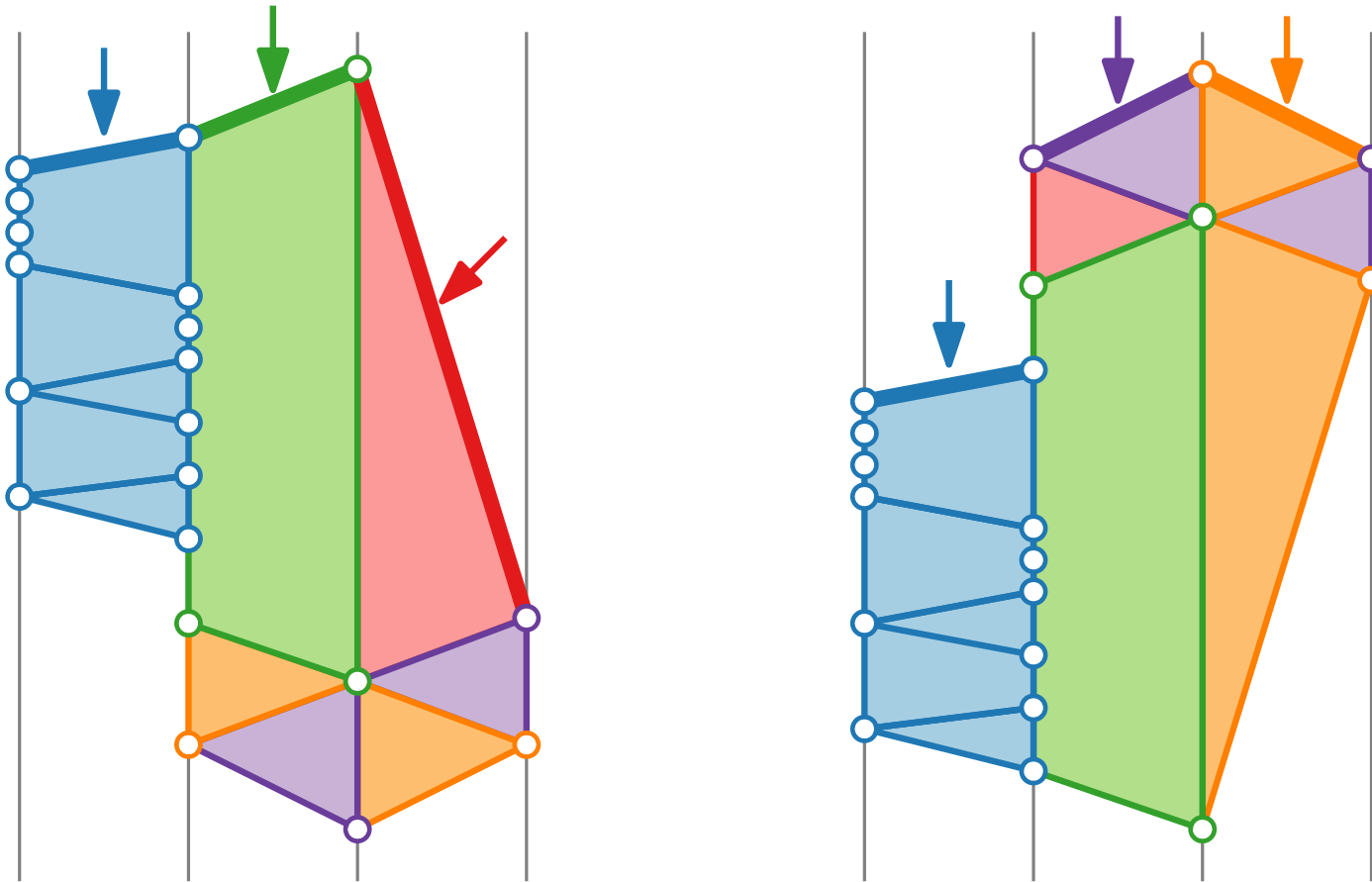
Strong: Biconnected Graphs



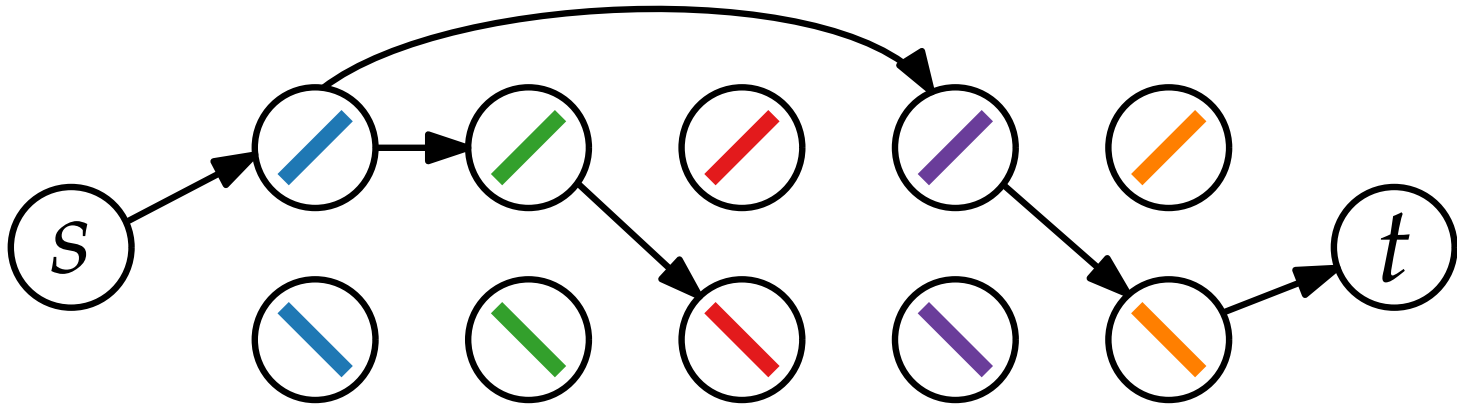
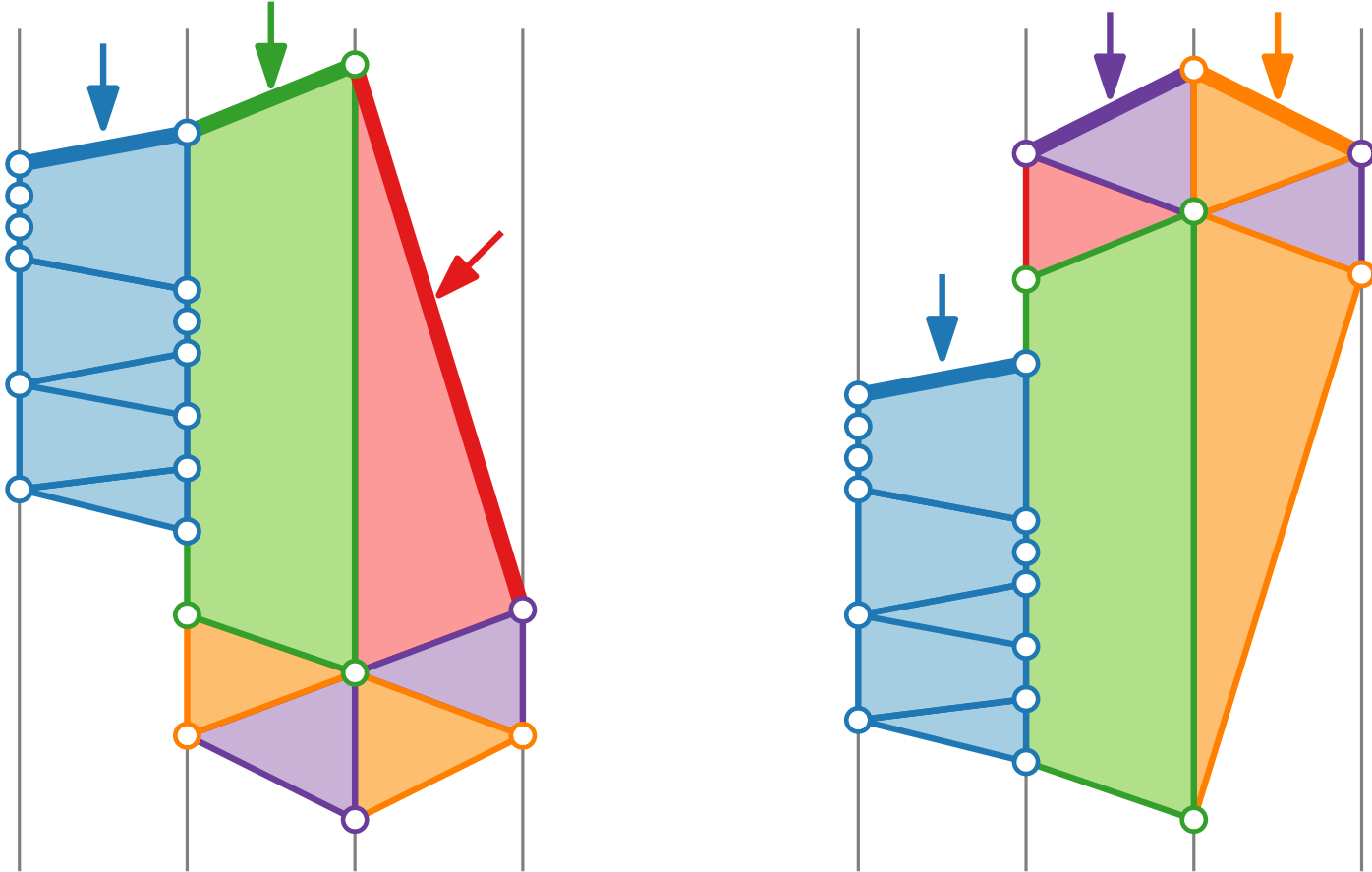
Strong: Biconnected Graphs



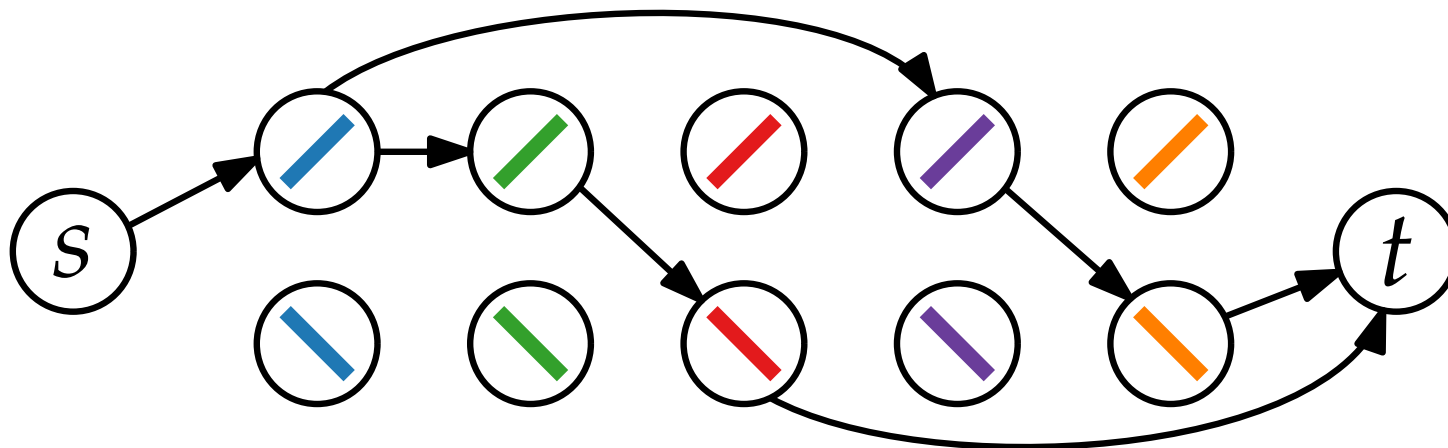
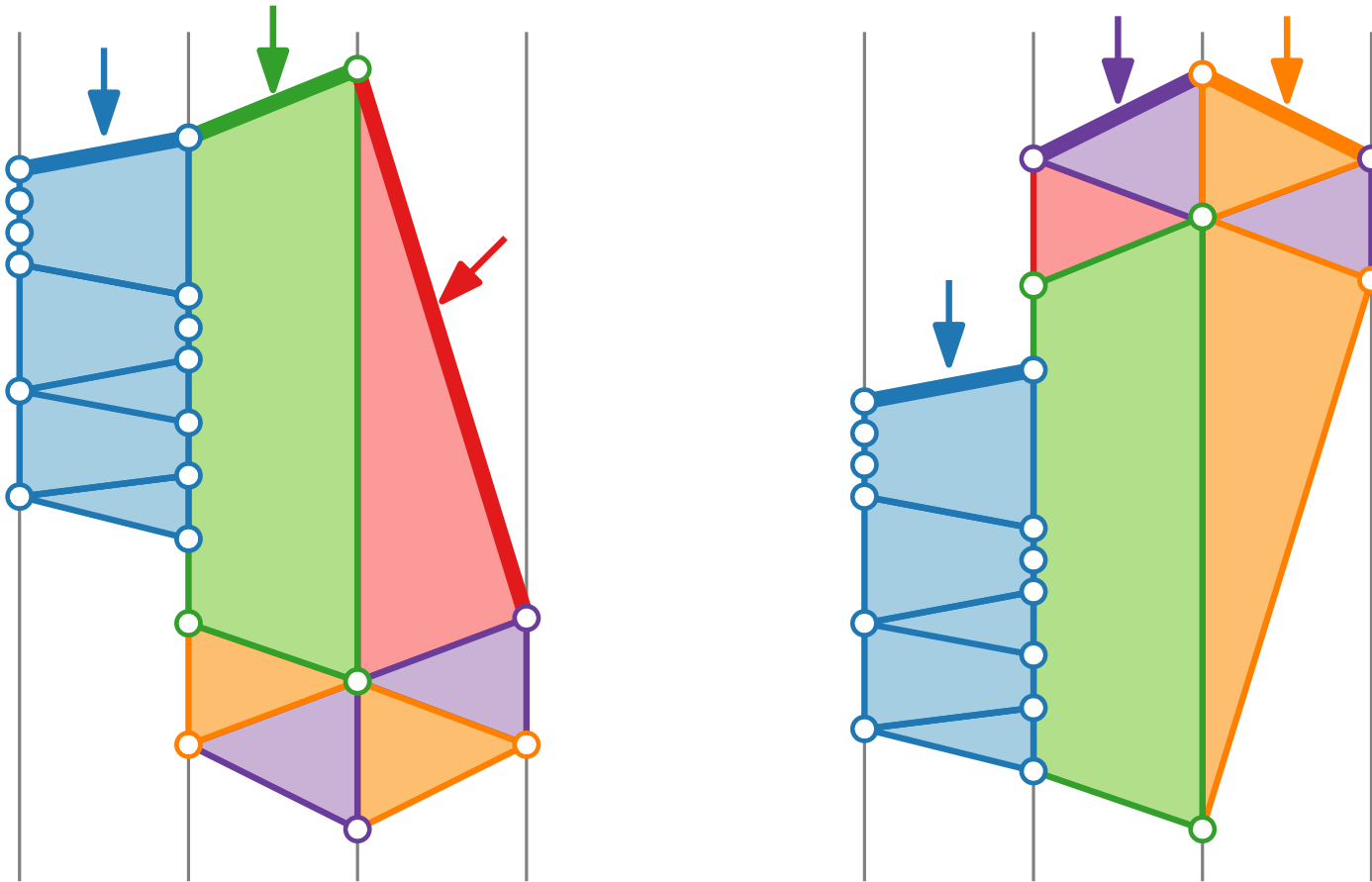
Strong: Biconnected Graphs



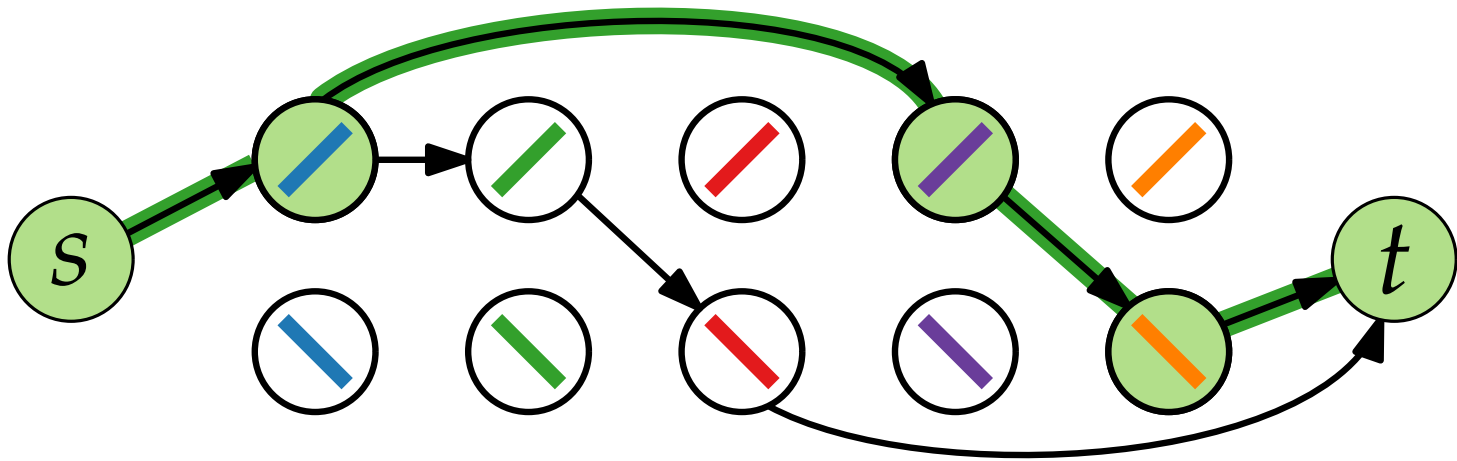
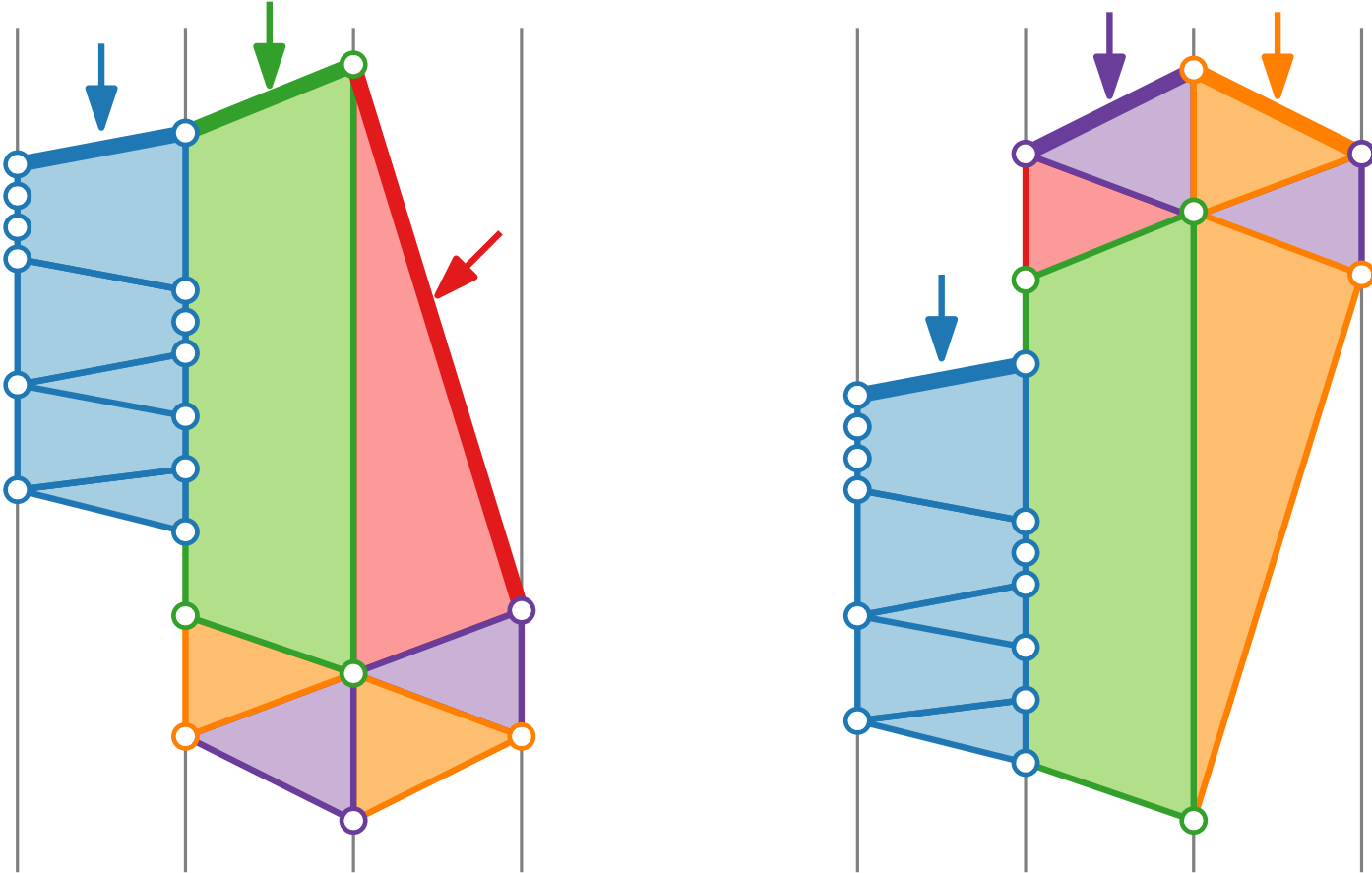
Strong: Biconnected Graphs



Strong: Biconnected Graphs

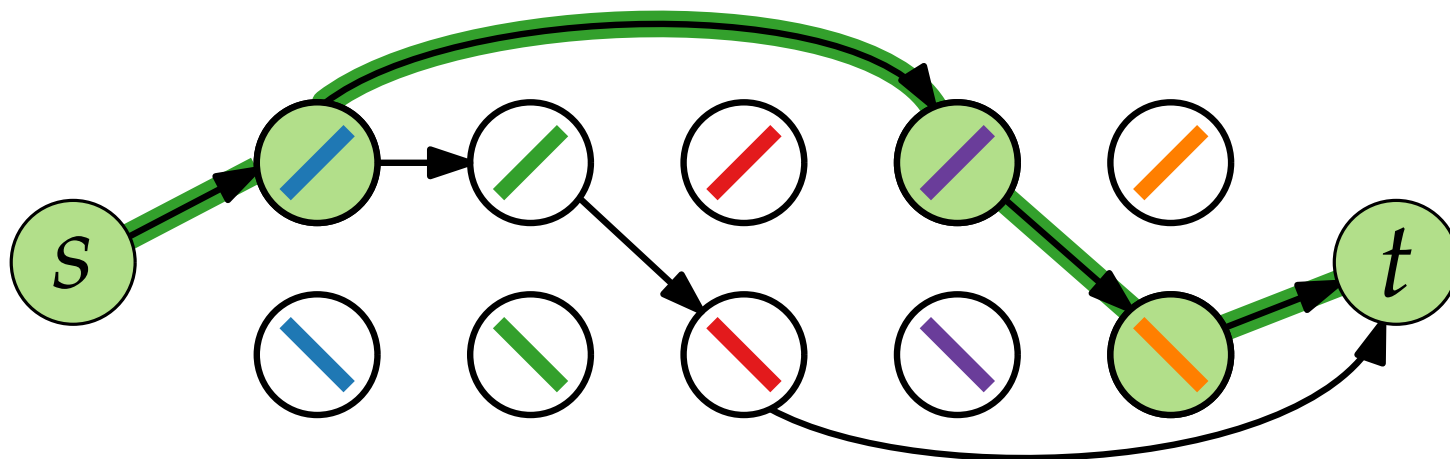
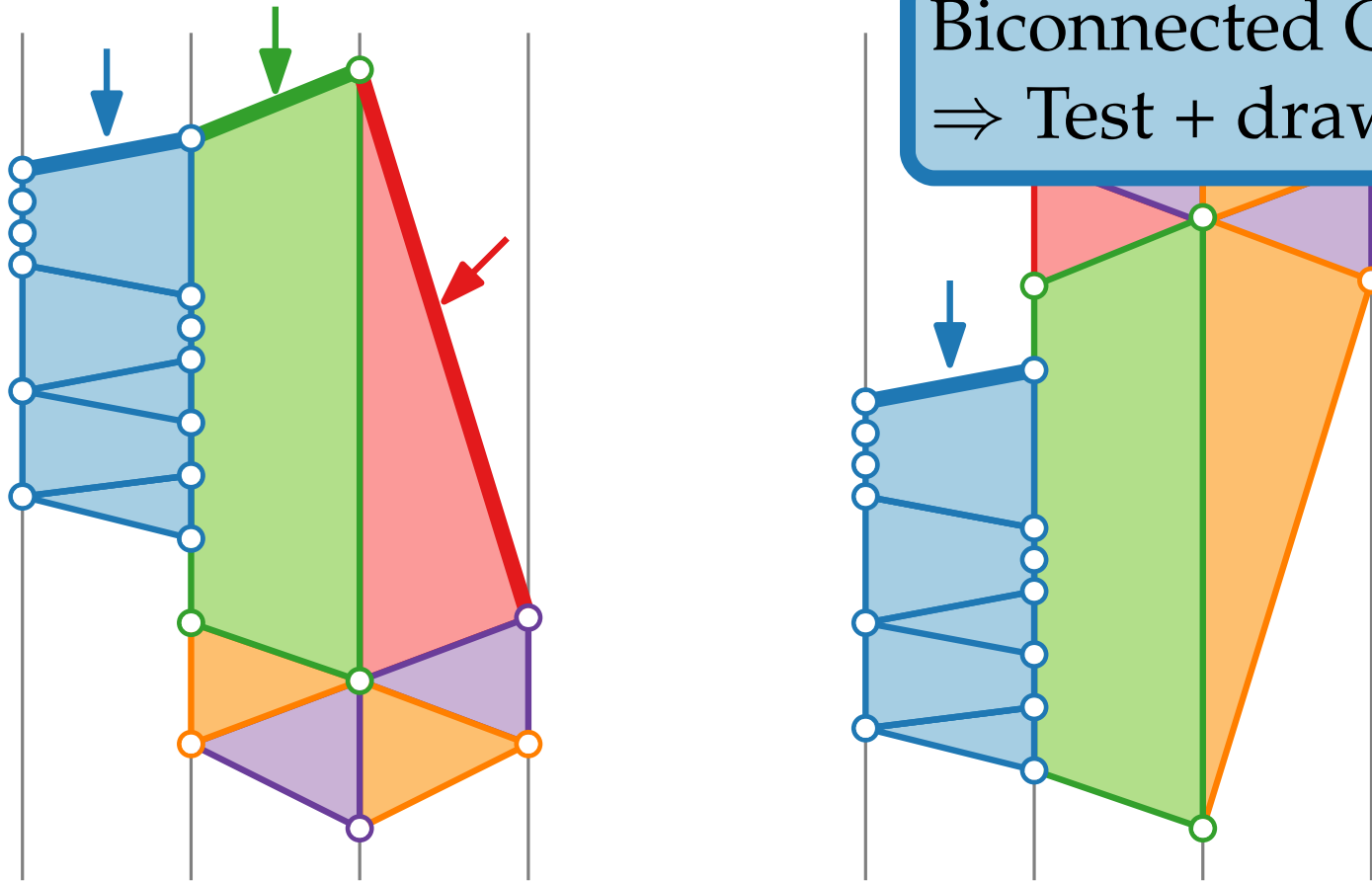


Strong: Biconnected Graphs



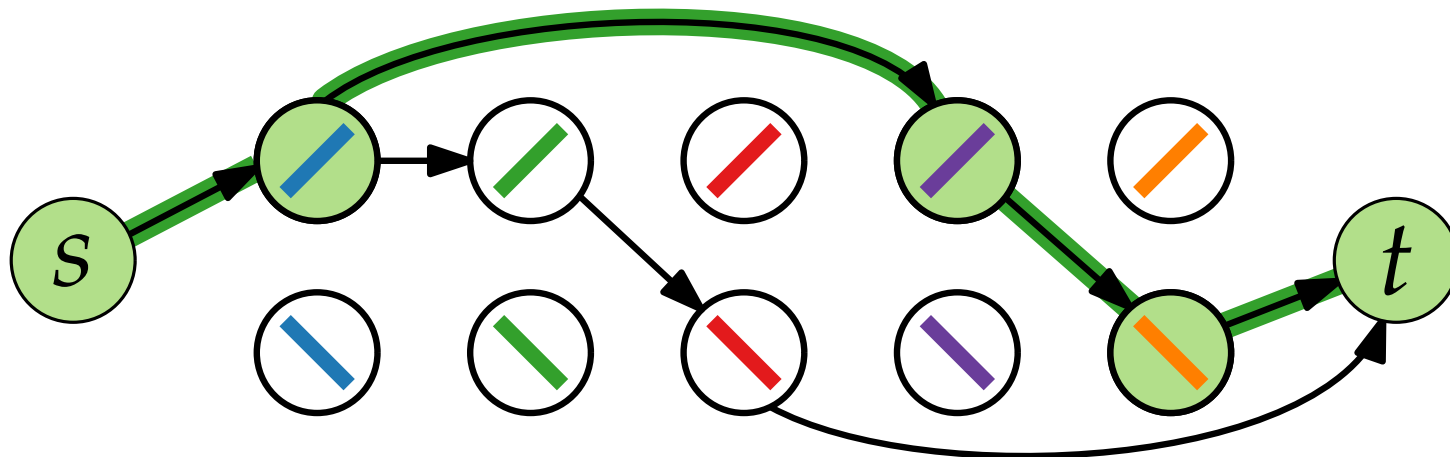
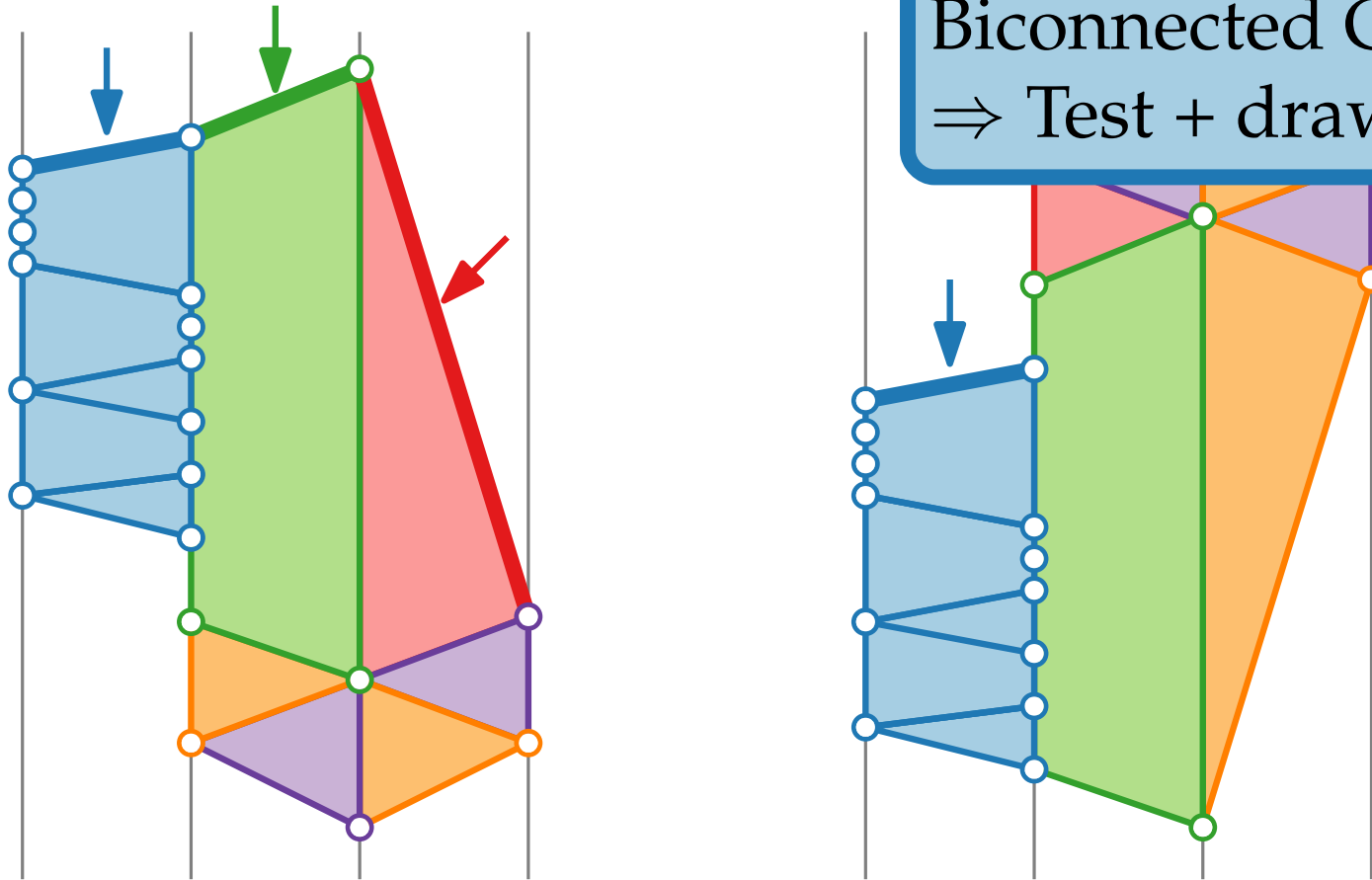
Strong: Biconnected Graphs

Biconnected Graph
⇒ Test + draw in $O(n^3)$ time

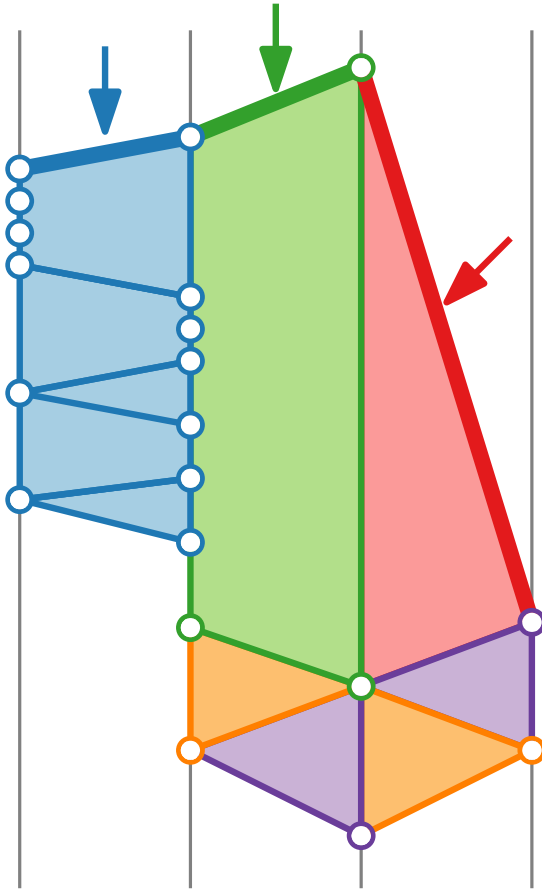


Strong: Biconnected Graphs

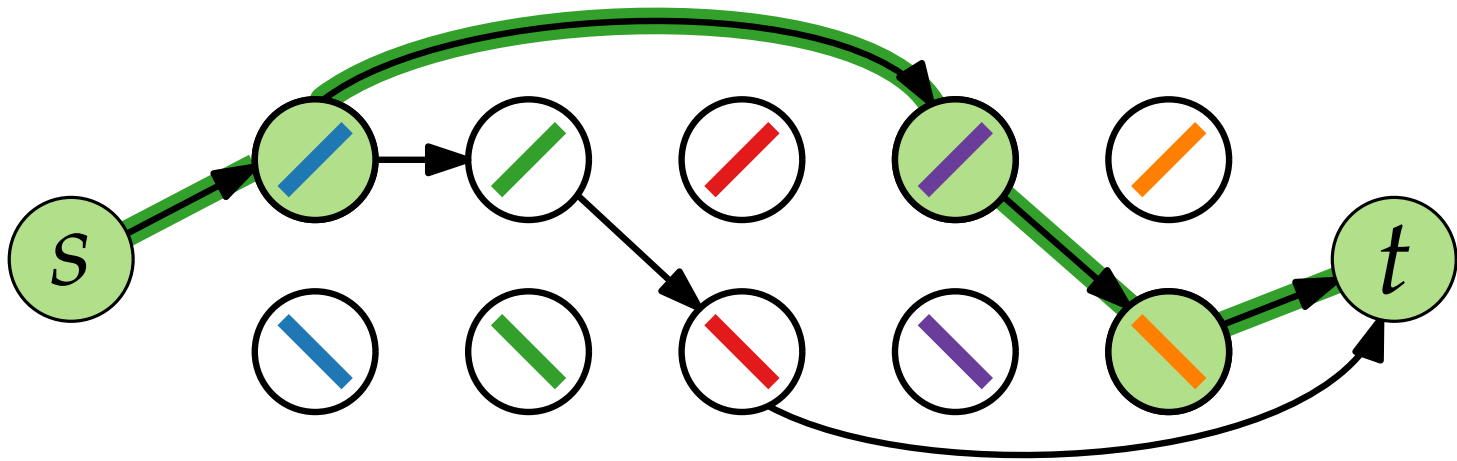
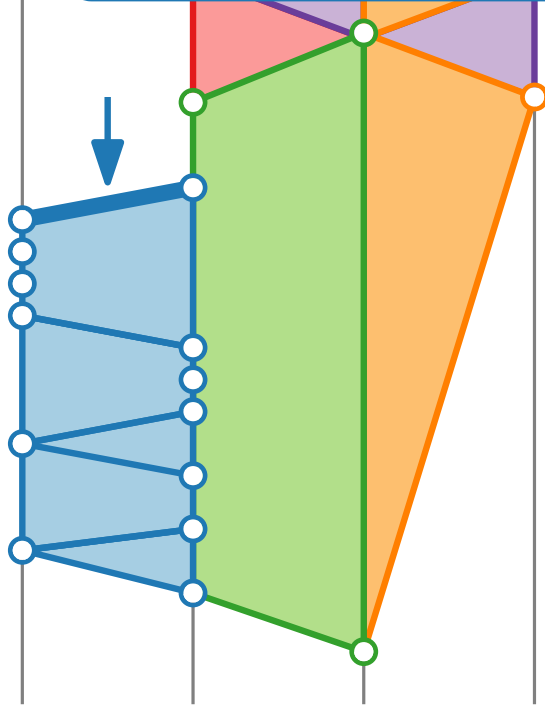
Biconnected Graph
⇒ Test + draw in $O(n^{\cancel{3}2})$ time



Strong: ~~Biconnected~~ Graphs

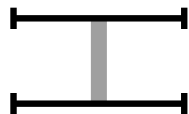
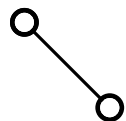


~~Biconnected~~ Graph
⇒ Test + draw in $O(n^{\cancel{3}2})$ time

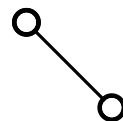


Summary

Weak

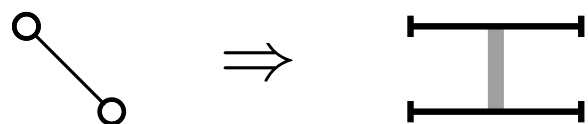


Strong



Summary

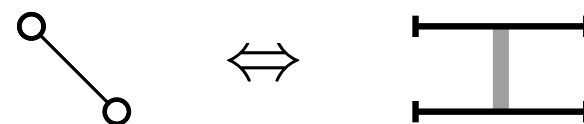
Weak



~~X~~ Trees, maxdeg 5

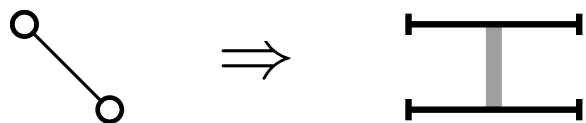


Strong



Summary

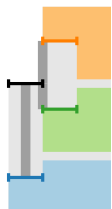
Weak



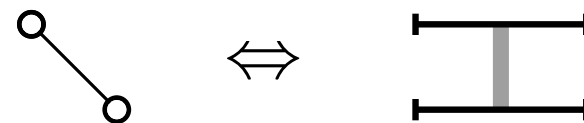
X Trees, maxdeg 5



✓ Trees, maxdeg 4

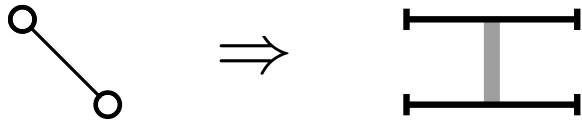


Strong

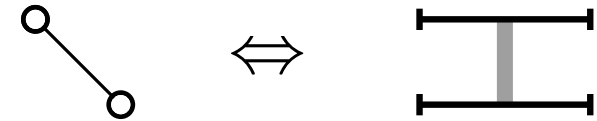


Summary

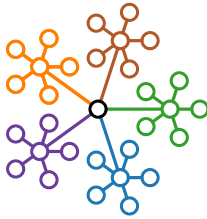
Weak



Strong



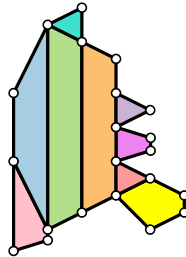
X Trees, maxdeg 5



✓ Trees, maxdeg 4

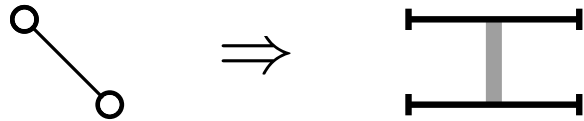


✓ Biconn. outerplanar, maxdeg 4

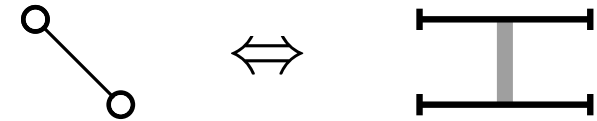


Summary

Weak



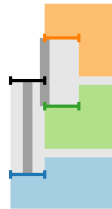
Strong



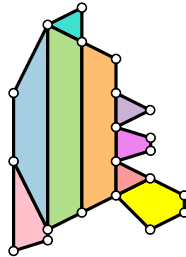
X Trees, maxdeg 5



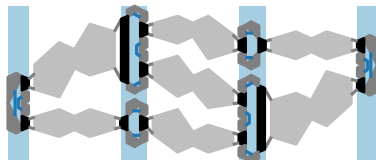
✓ Trees, maxdeg 4



✓ Biconn. outerplanar, maxdeg 4

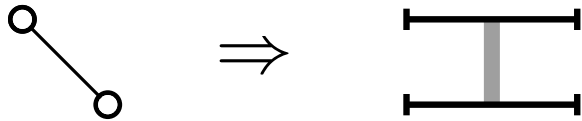


General: NP-hard

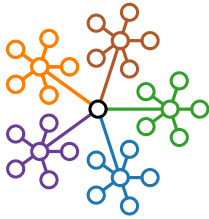


Summary

Weak



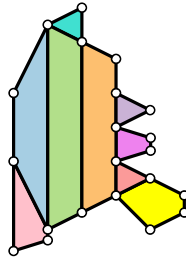
X Trees, maxdeg 5



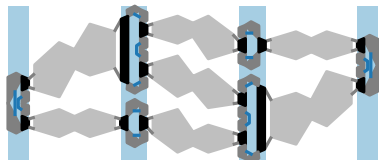
✓ Trees, maxdeg 4



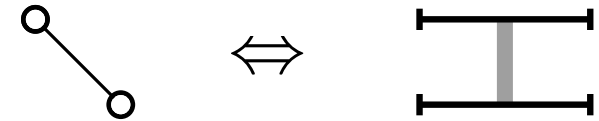
✓ Biconn. outerplanar, maxdeg 4



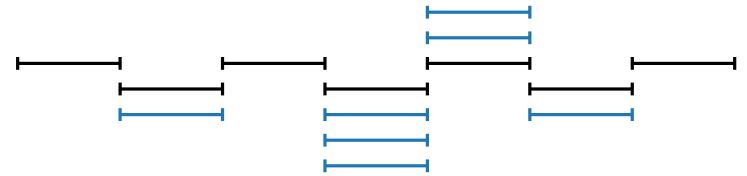
General: NP-hard



Strong

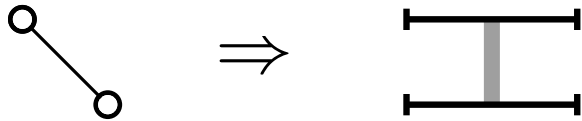


Trees: maxdeg 3,
subdiv. of caterpillar



Summary

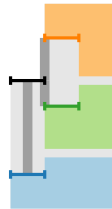
Weak



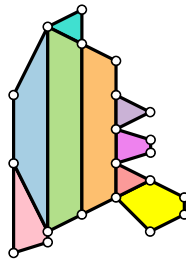
X Trees, maxdeg 5



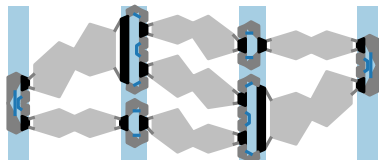
✓ Trees, maxdeg 4



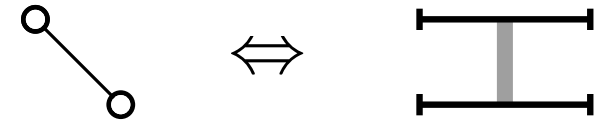
✓ Biconn. outerplanar, maxdeg 4



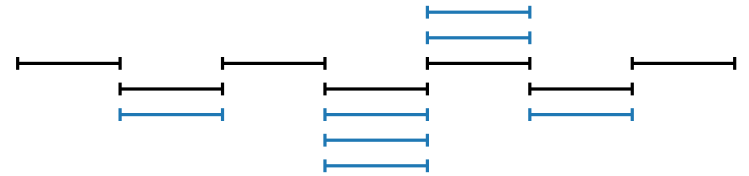
General: NP-hard



Strong



Trees: maxdeg 3, subdiv. of caterpillar



General:
 $O(n^2)$ time recognition

