FernUniversität in Hagen
Fakultät für Mathematik und Informatik

theoretische informatik

Drawing trees and triangulations with few geometric primitives

Philipp Kindermann
FernUniversität in Hagen

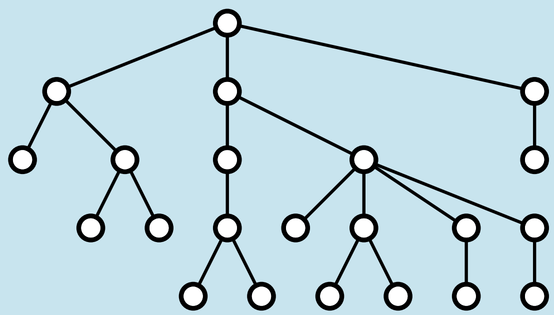
Joint work with
Gregor Hültenschmidt, Wouter Meulemans & André Schulz

Visual Complexity

Number of geometric objects

Visual Complexity

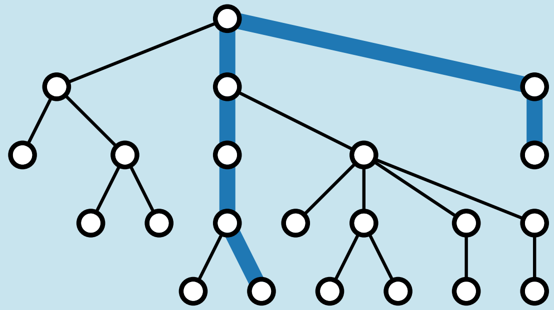
Number of geometric objects



21 edges

Visual Complexity

Number of geometric objects

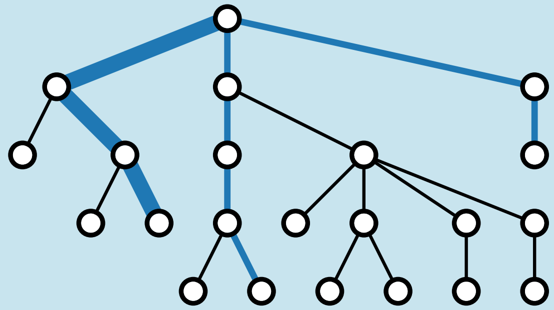


21 edges

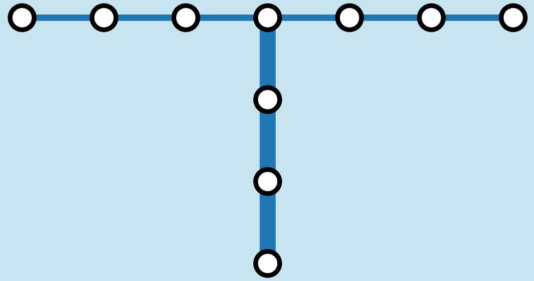
1 segment

Visual Complexity

Number of geometric objects



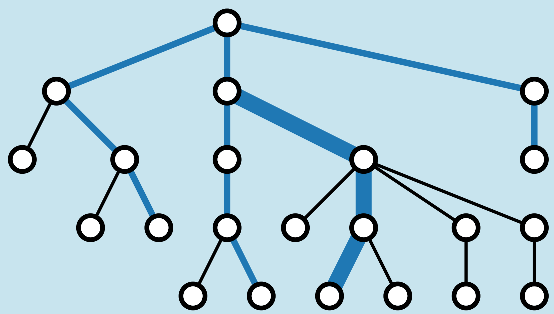
21 edges



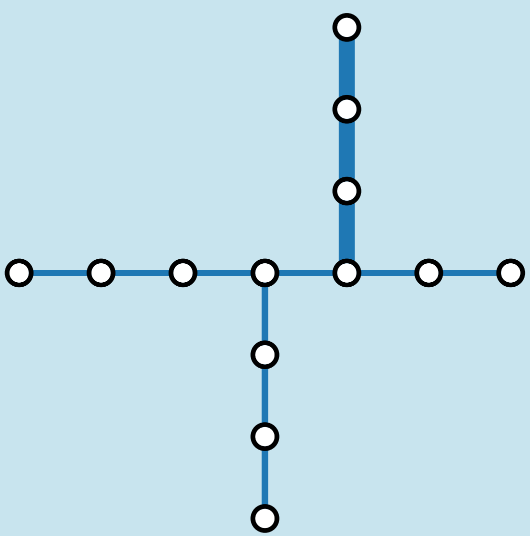
2 segments

Visual Complexity

Number of geometric objects



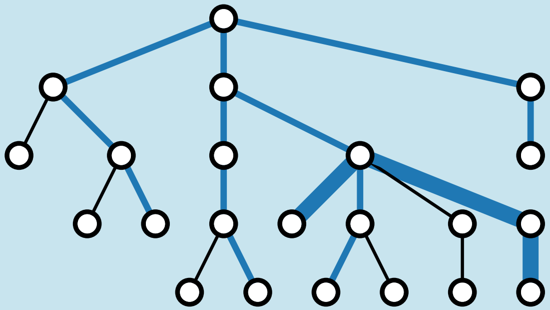
21 edges



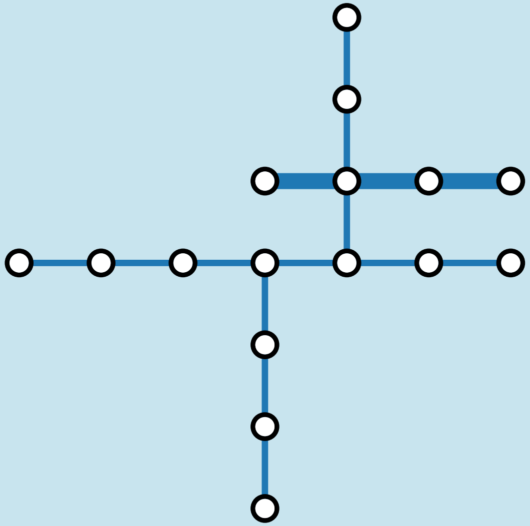
3 segments

Visual Complexity

Number of geometric objects



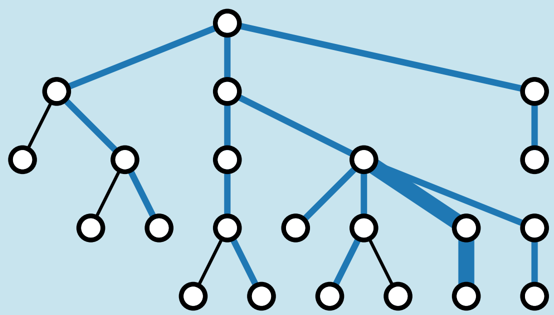
21 edges



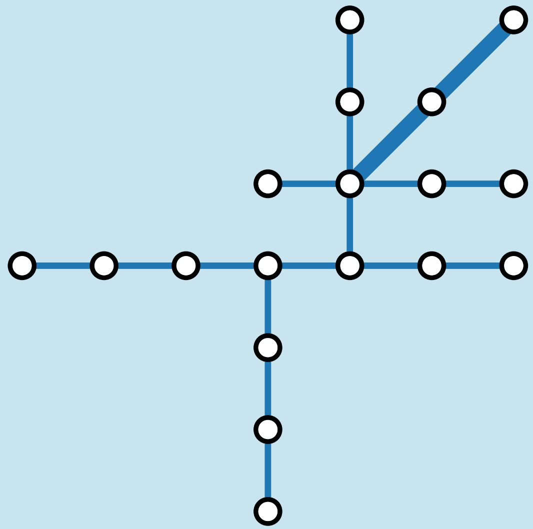
4 segments

Visual Complexity

Number of geometric objects



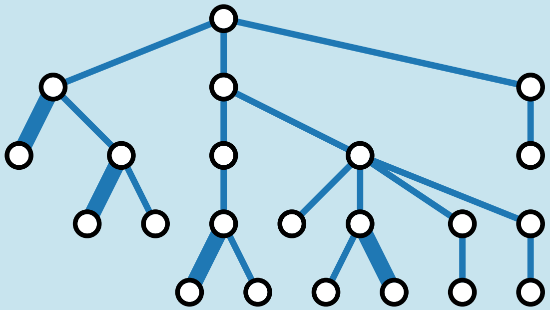
21 edges



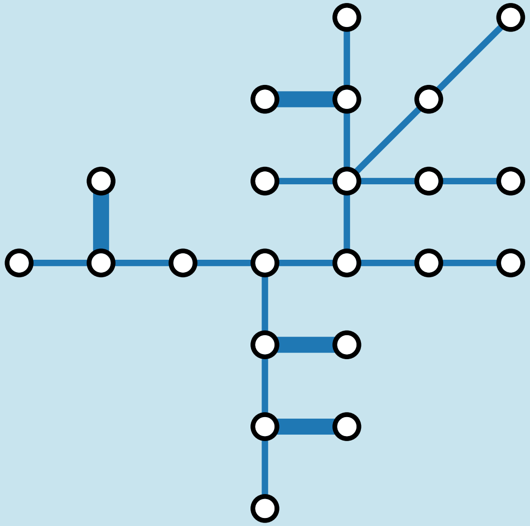
5 segments

Visual Complexity

Number of geometric objects



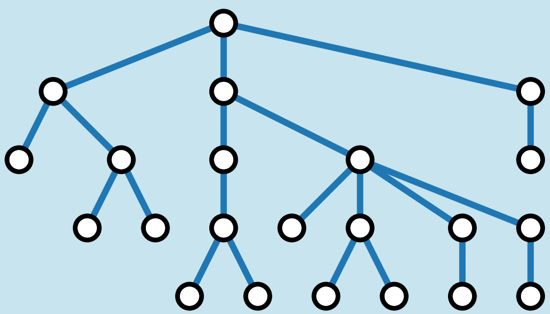
21 edges



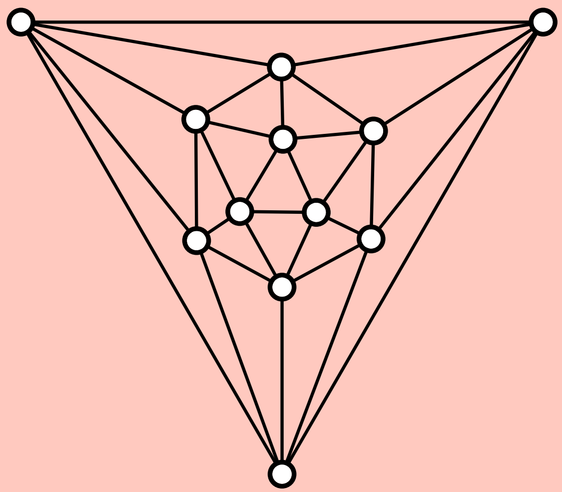
9 segments

Visual Complexity

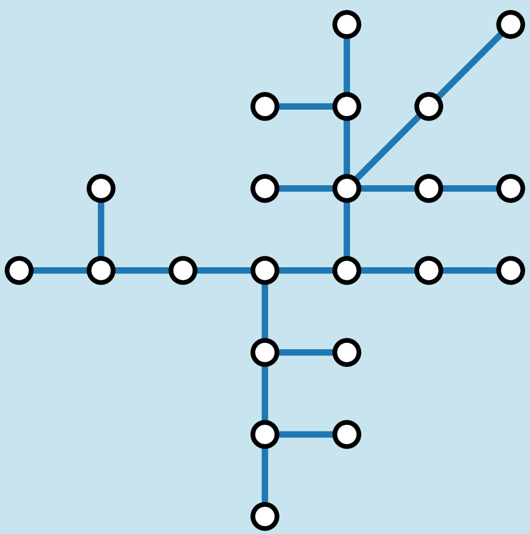
Number of geometric objects



21 edges



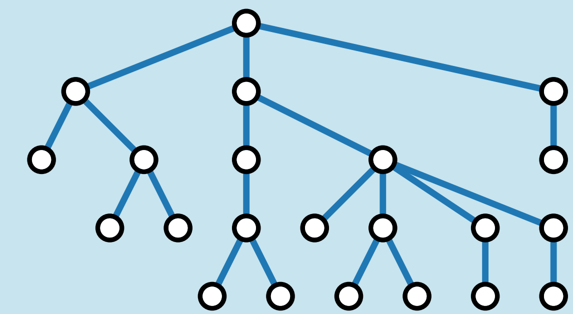
30 edges



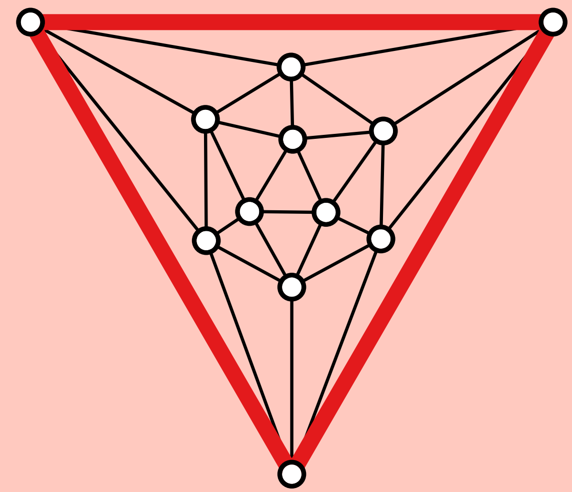
9 segments

Visual Complexity

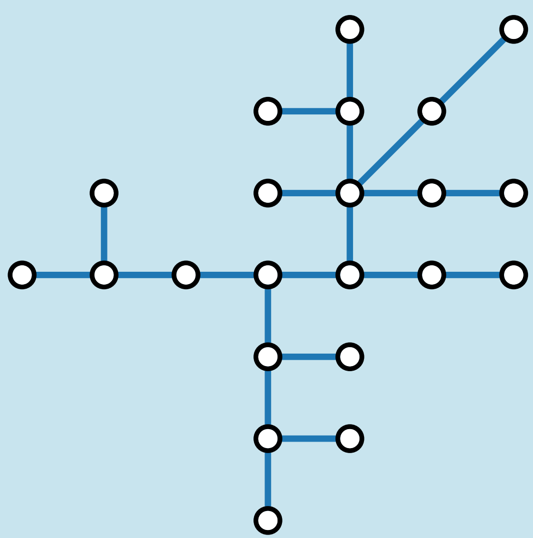
Number of geometric objects



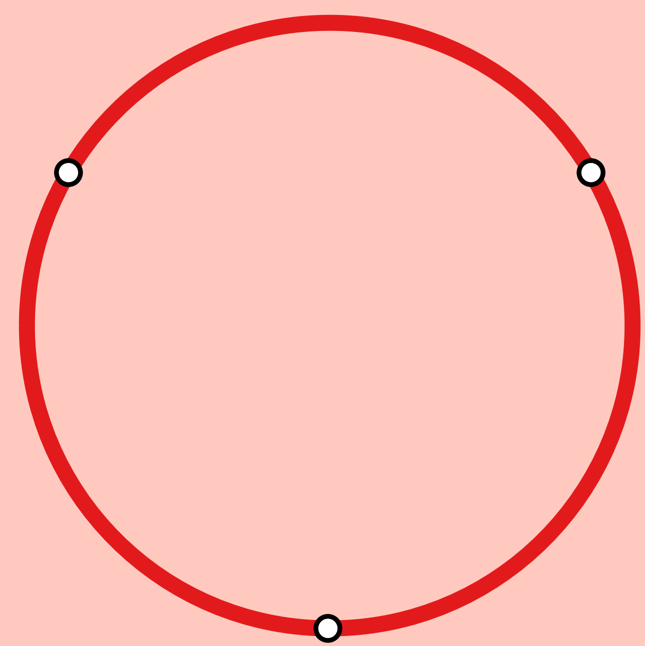
21 edges



30 edges



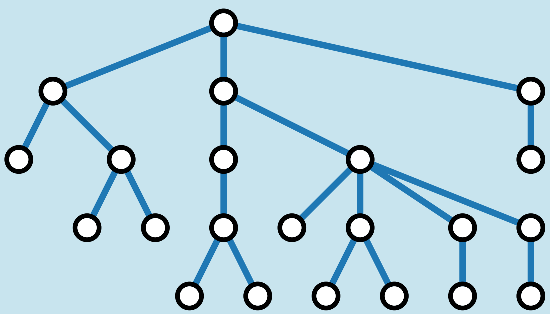
9 segments



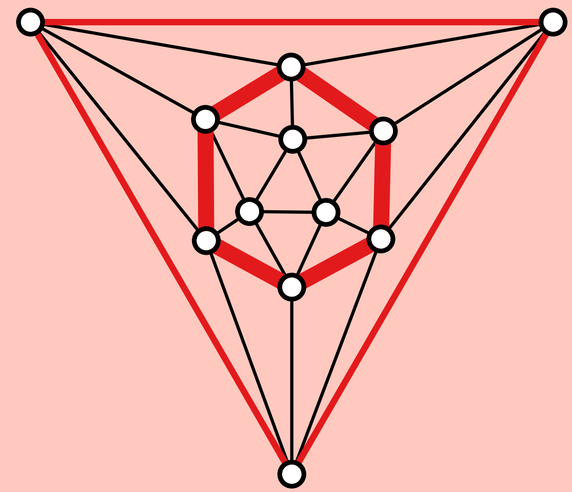
1 arc

Visual Complexity

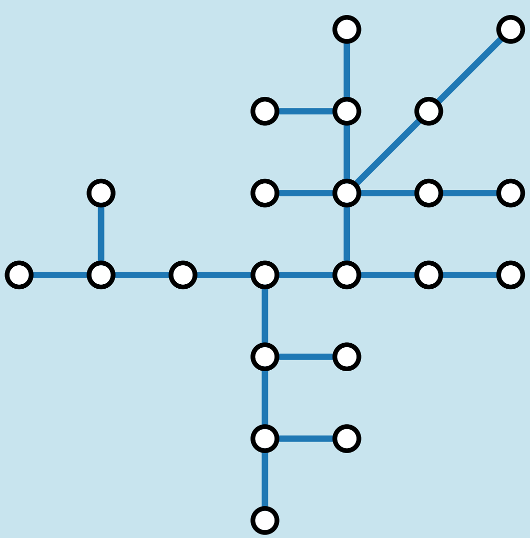
Number of geometric objects



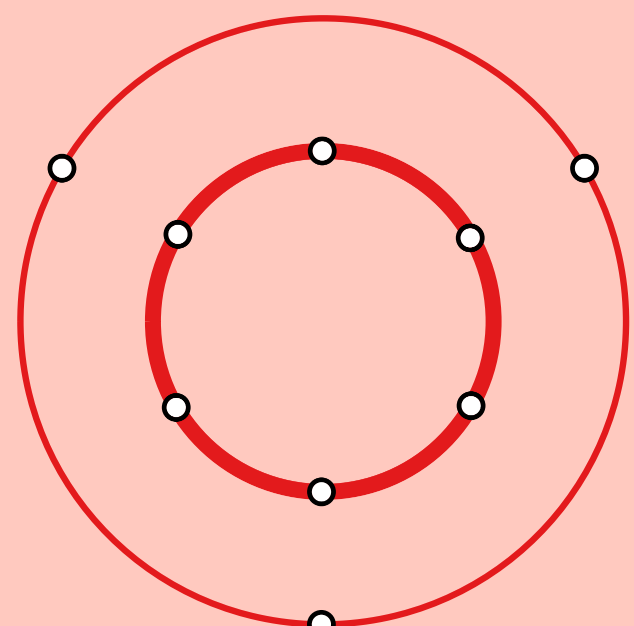
21 edges



30 edges



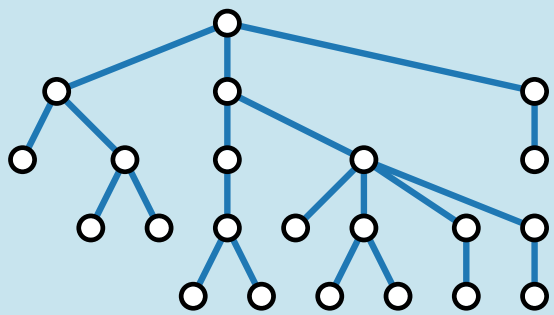
9 segments



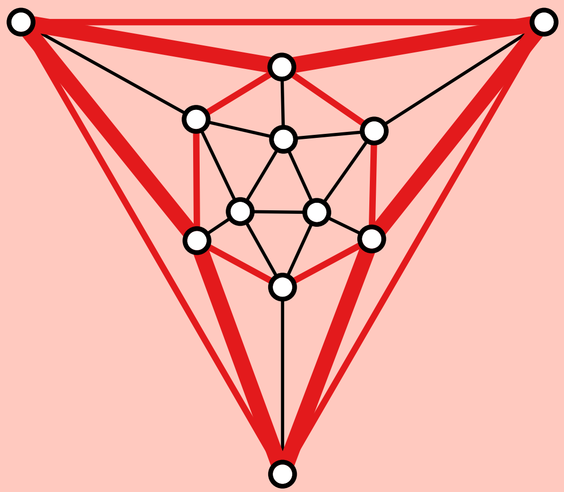
2 arcs

Visual Complexity

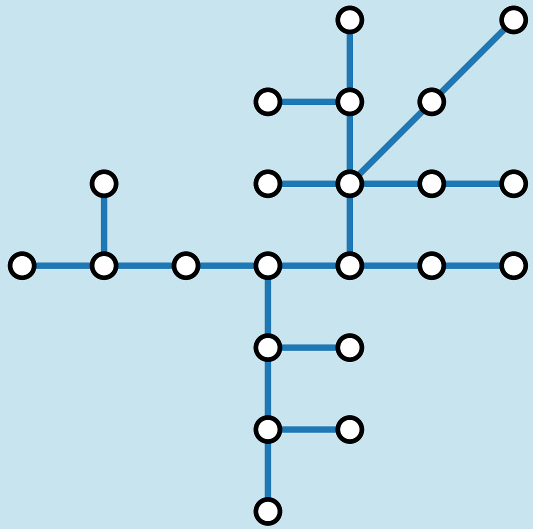
Number of geometric objects



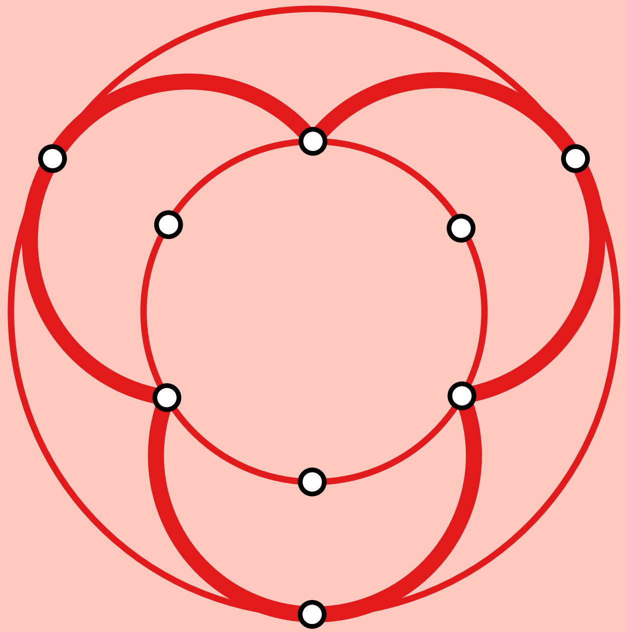
21 edges



30 edges



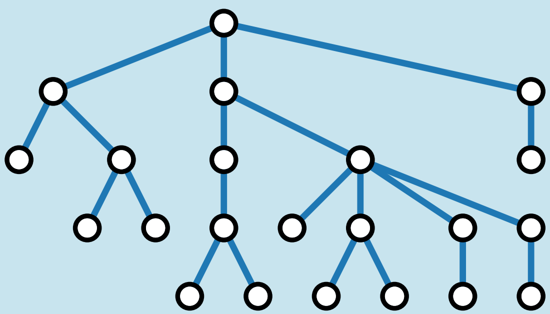
9 segments



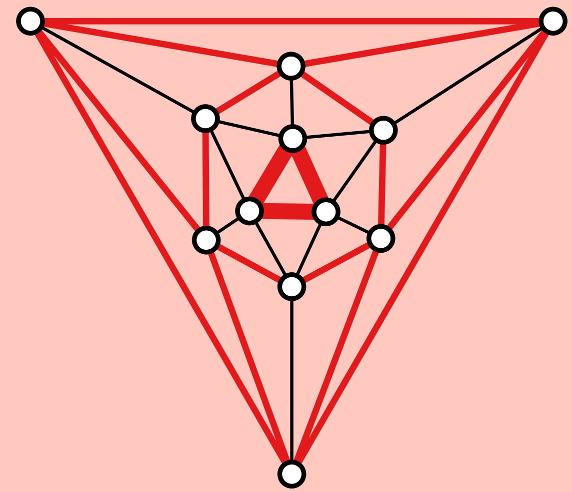
5 arcs

Visual Complexity

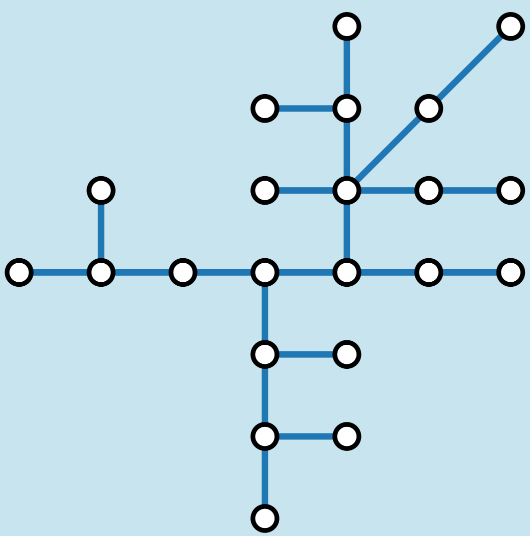
Number of geometric objects



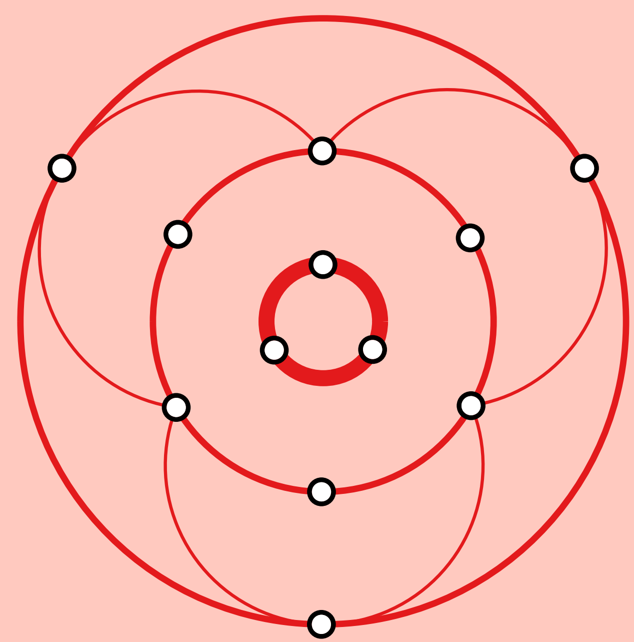
21 edges



30 edges



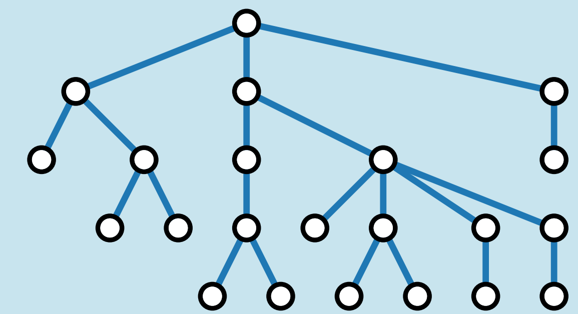
9 segments



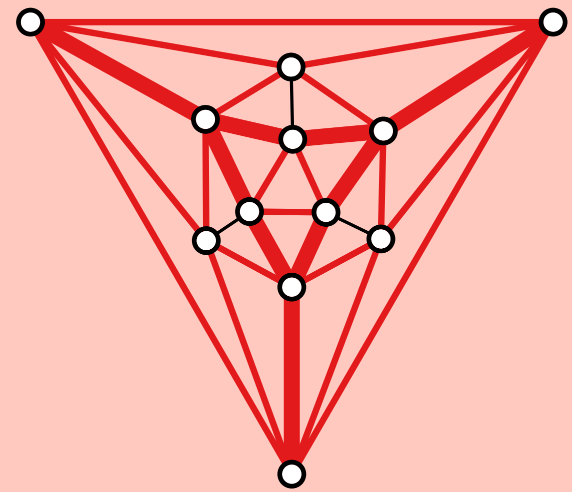
6 arcs

Visual Complexity

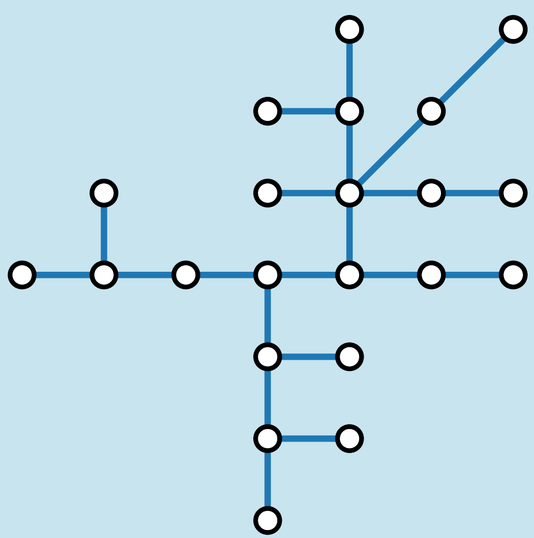
Number of geometric objects



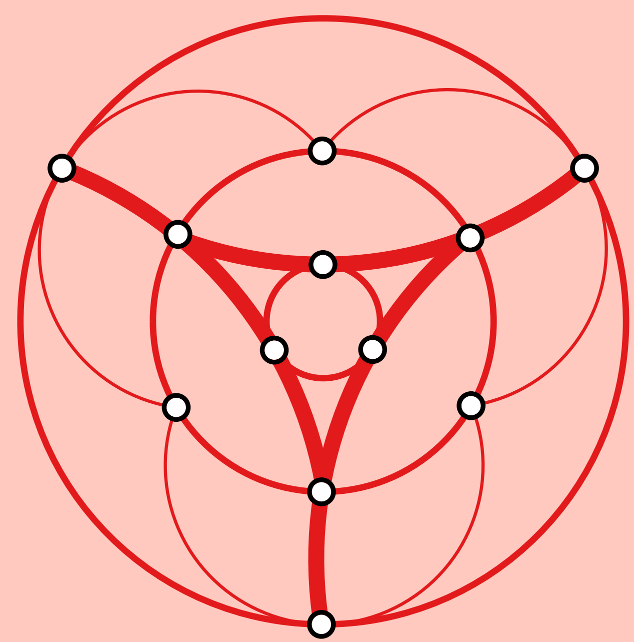
21 edges



30 edges



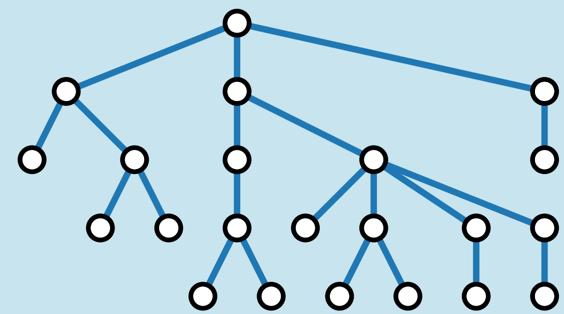
9 segments



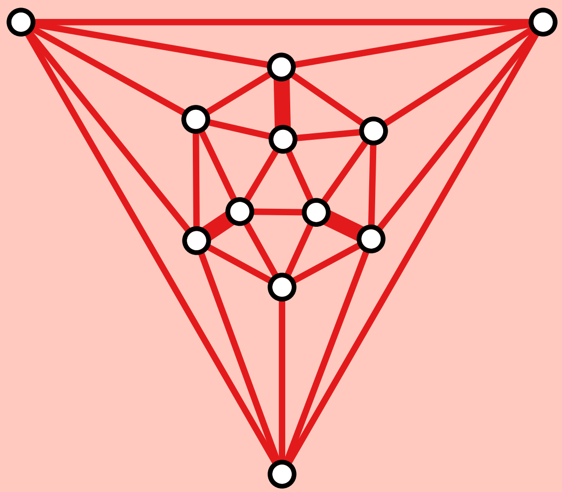
9 arcs

Visual Complexity

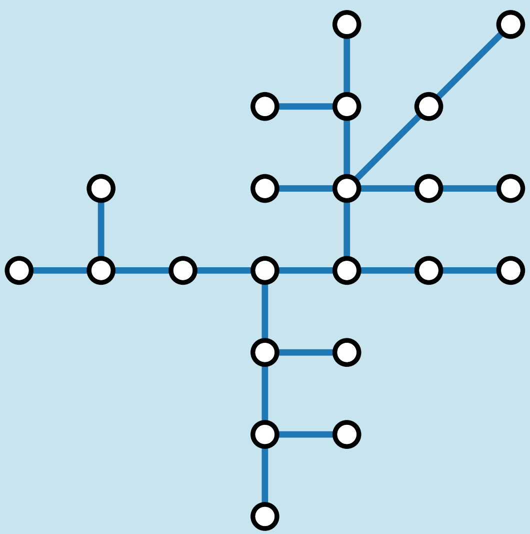
Number of geometric objects



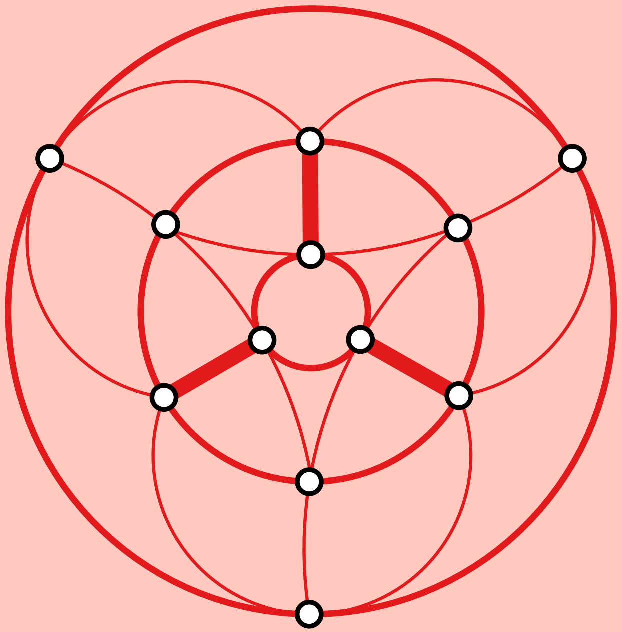
21 edges



30 edges



9 segments



12arcs

Known Results – Segments

Class	Segments	
	Lower	Upper

Known Results – Segments

Class	Segments	
	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]

[1] Dujmović et al. 2007

Known Results – Segments

Class	Segments			
	Lower		Upper	
Tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]
max. outerplanar	n	[1]	n	[1]

[1] Dujmović et al. 2007

Known Results – Segments

Class	Segments	
	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
max. outerplanar	n [1]	n [1]
2-trees	$3n/2$ [1]	$3n/2$ [1]

[1] Dujmović et al. 2007

Known Results – Segments

Class	Segments			
	Lower		Upper	
Tree	$\vartheta/2$	[1]	$\vartheta/2$	[1]
max. outerplanar	n	[1]	n	[1]
2-trees	$3n/2$	[1]	$3n/2$	[1]
3-trees	$2n$	[1]	$2n$	[1]

[1] Dujmović et al. 2007

Known Results – Segments

Class	Segments	
	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
max. outerplanar	n [1]	n [1]
2-trees	$3n/2$ [1]	$3n/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]
2-connected	$2n$ [1]	$16n/3 - e$ [2]

[1] Dujmović et al. 2007 [2] Durocher & Mondal 2014

Known Results – Segments

Class	Segments	
	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
max. outerplanar	n [1]	n [1]
2-trees	$3n/2$ [1]	$3n/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]
2-connected	$2n$ [1]	$16n/3 - e$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]

[1] Dujmović et al. 2007 [2] Durocher & Mondal 2014

Known Results – Segments

Class	Segments	
	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
max. outerplanar	n [1]	n [1]
2-trees	$3n/2$ [1]	$3n/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]
2-connected	$2n$ [1]	$16n/3 - e$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]

[1] Dujmović et al. 2007

[2] Durocher & Mondal 2014

[3] Mondal et al. 2013

[4] Igamberdiev et al. 2015

Known Results – Segments

Class	Segments	
	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
max. outerplanar	n [1]	n [1]
2-trees	$3n/2$ [1]	$3n/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]
2-connected	$2n$ [1]	$16n/3 - e$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]
Triangulation	$2n$ [2]	$7n/3$ [2]

[1] Dujmović et al. 2007

[2] Durocher & Mondal 2014

[3] Mondal et al. 2013

[4] Igamberdiev et al. 2015

Known Results – Segments

Class	Segments	
	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]
max. outerplanar	n [1]	n [1]
2-trees	$3n/2$ [1]	$3n/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]
2-connected	$2n$ [1]	$16n/3 - e$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]
Triangulation	$2n$ [2]	$7n/3$ [2]
Planar	$2n$ [2]	$16n/3 - e$ [2]

[1] Dujmović et al. 2007

[2] Durocher & Mondal 2014

[3] Mondal et al. 2013

[4] Igamberdiev et al. 2015

Known Results – Segments

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]		
max. outerplanar	n [1]	n [1]		
2-trees	$3n/2$ [1]	$3n/2$ [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]	$16n/3 - e$ [2]		
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]		
Triangulation	$2n$ [2]	$7n/3$ [2]		
Planar	$2n$ [2]	$16n/3 - e$ [2]		

[1] Dujmović et al. 2007

[2] Durocher & Mondal 2014

[3] Mondal et al. 2013

[4] Igamberdiev et al. 2015

Known Results – Segments

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]		
max. outerplanar	n [1]	n [1]		
2-trees	$3n/2$ [1]	$3n/2$ [1]		
3-trees	$2n$ [1]	$2n$ [1]		
2-connected	$2n$ [1]	$16n/3 - e$ [2]		
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	$n/2$ [4]	$O(n) \times O(n)$
Triangulation	$2n$ [2]	$7n/3$ [2]		
Planar	$2n$ [2]	$16n/3 - e$ [2]		

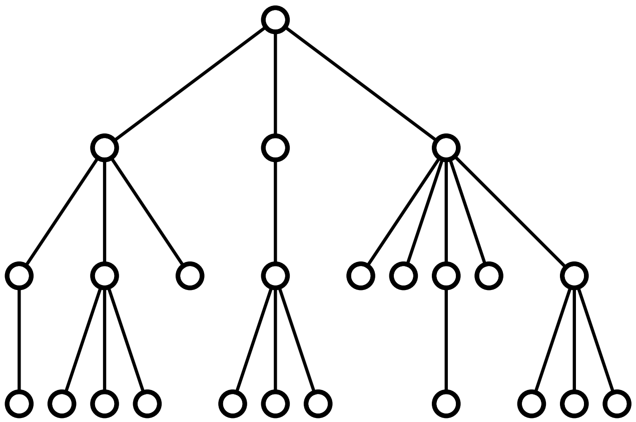
[1] Dujmović et al. 2007
[2] Durocher & Mondal 2014
[3] Mondal et al. 2013
[4] Igamberdiev et al. 2015

Known Results – Segments

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]		?
max. outerplanar	n [1]	n [1]		
2-trees	$3n/2$ [1]	$3n/2$ [1]		
3-trees	$2n$ [1]	$2n$ [1]		?
2-connected	$2n$ [1]	$16n/3 - e$ [2]		
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	$n/2$ [4]	$O(n) \times O(n)$
Triangulation	$2n$ [2]	$7n/3$ [2]		
Planar	$2n$ [2]	$16n/3 - e$ [2]		

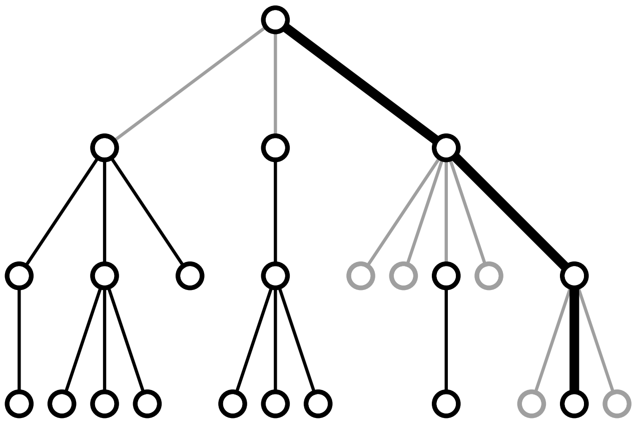
[1] Dujmović et al. 2007
[2] Durocher & Mondal 2014
[3] Mondal et al. 2013
[4] Igamberdiev et al. 2015

Trees on the grid



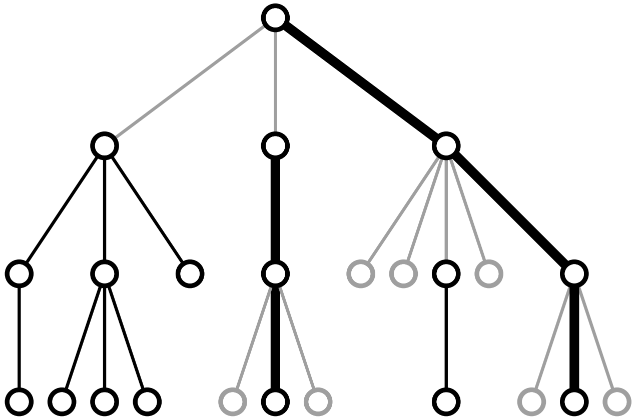
heavy path decomposition

Trees on the grid



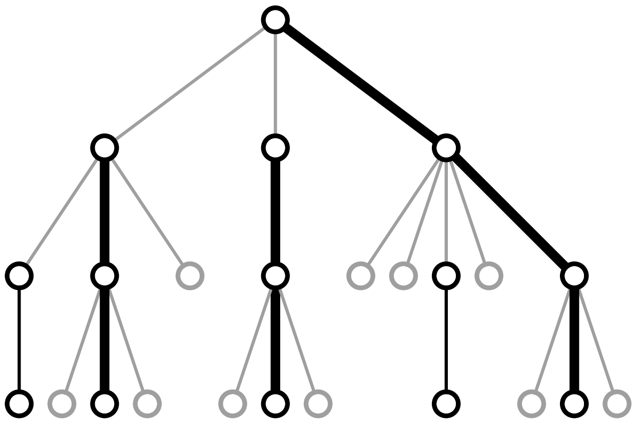
heavy path decomposition

Trees on the grid



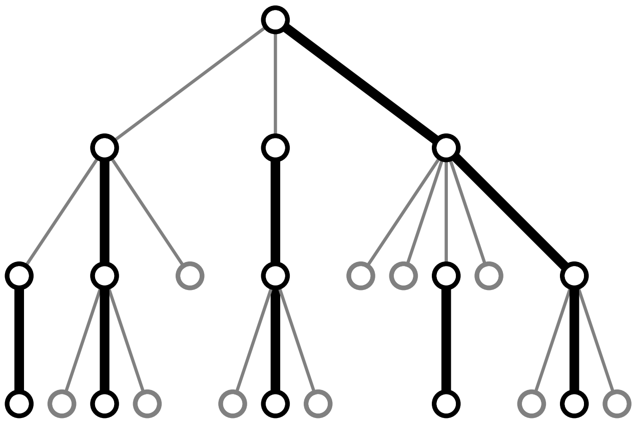
heavy path decomposition

Trees on the grid



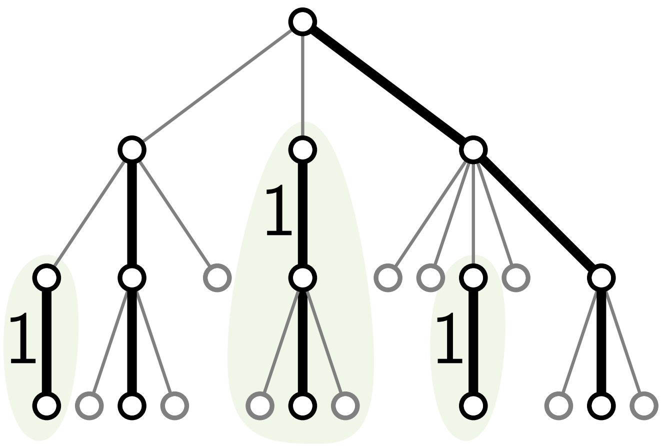
heavy path decomposition

Trees on the grid



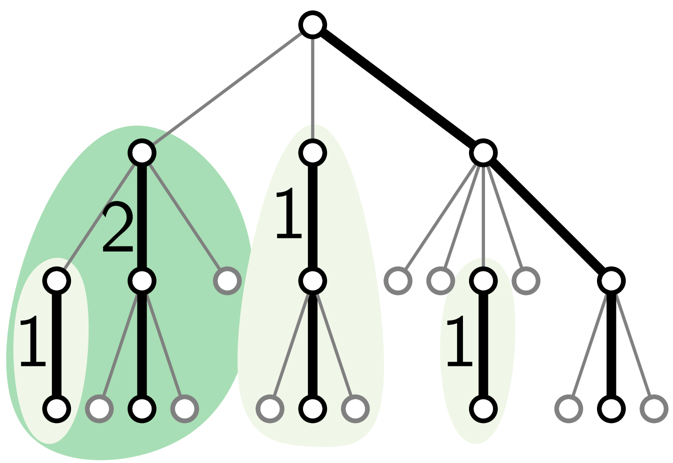
heavy path decomposition

Trees on the grid



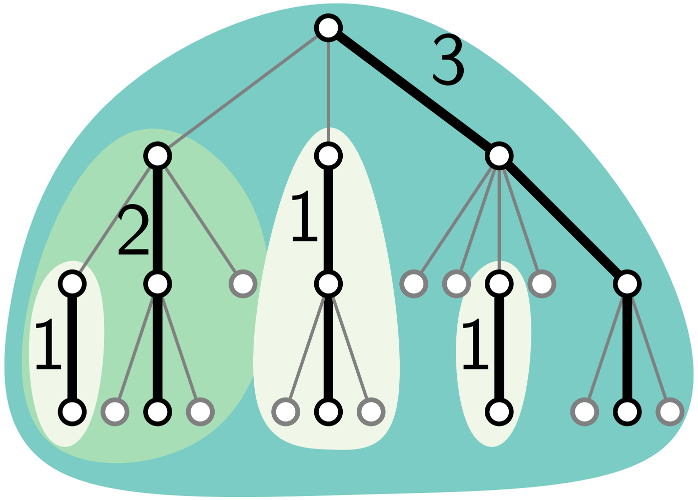
heavy path decomposition

Trees on the grid



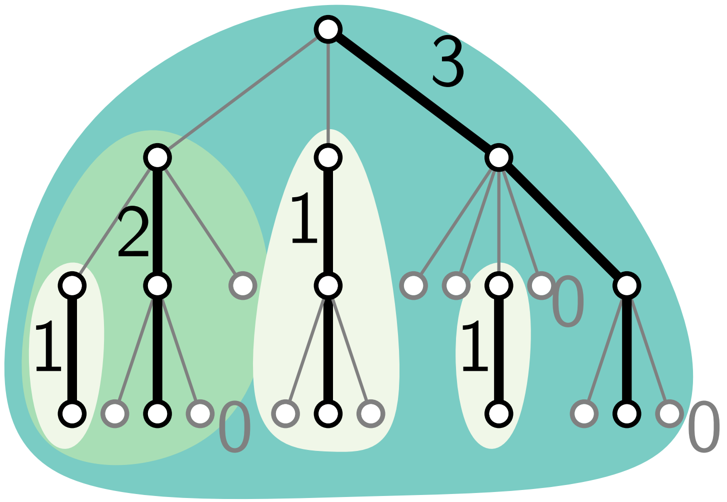
heavy path decomposition

Trees on the grid



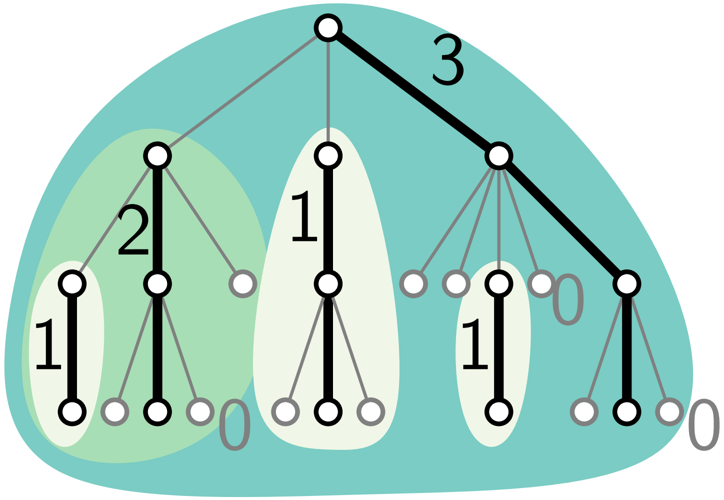
heavy path decomposition

Trees on the grid



heavy path decomposition

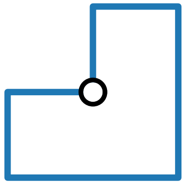
Trees on the grid



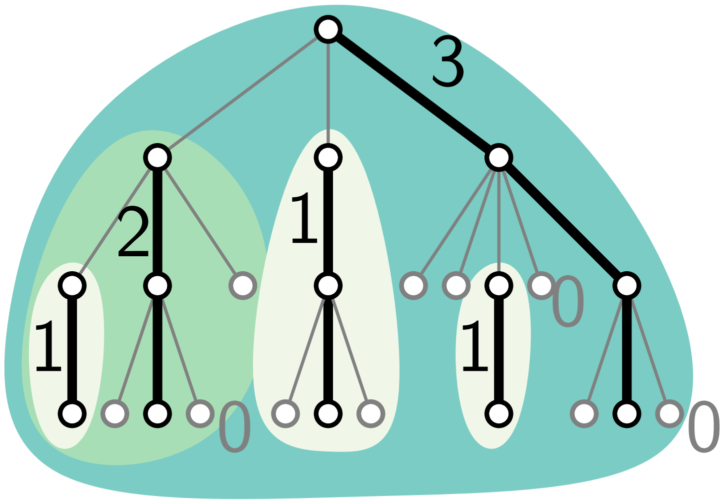
heavy path decomposition

- draw each heavy path tree in a "box"

level 0:



Trees on the grid



heavy path decomposition

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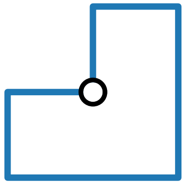

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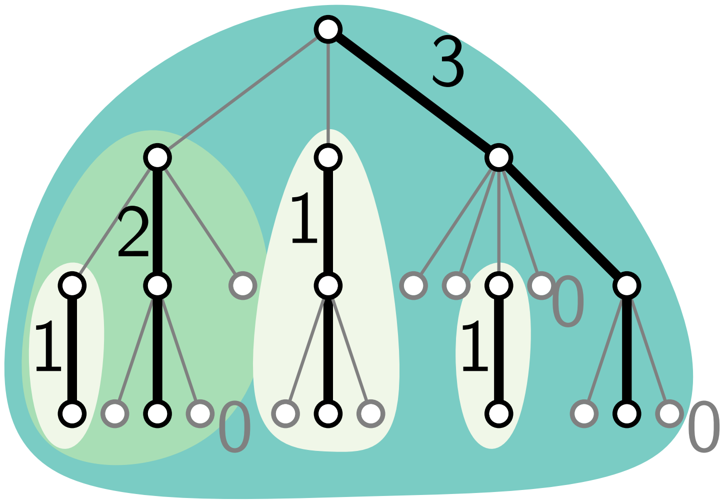

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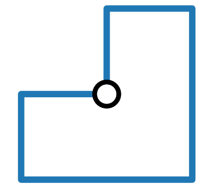
level 0: 

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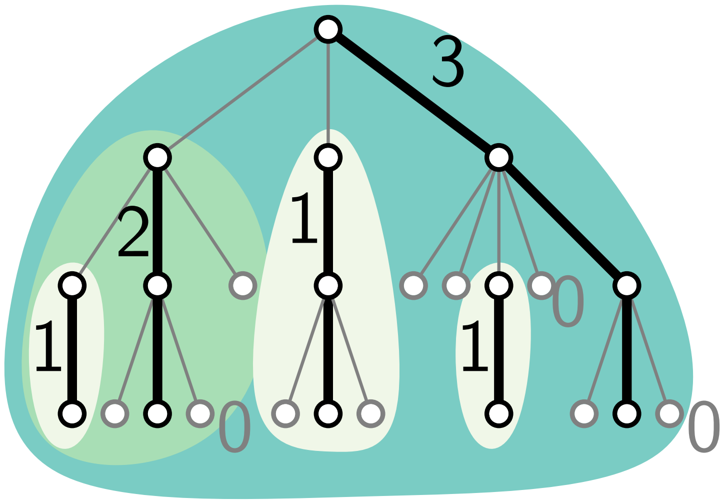

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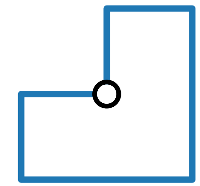
level 0: 

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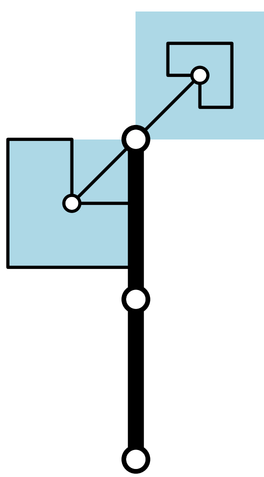

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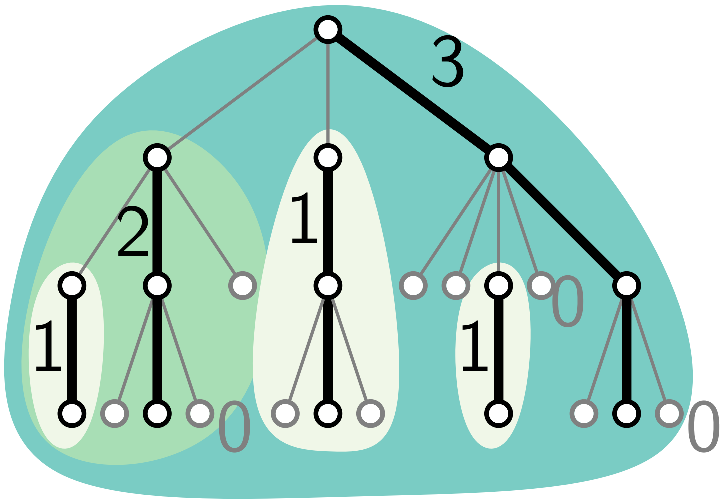

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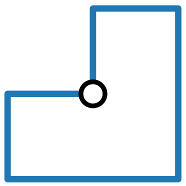

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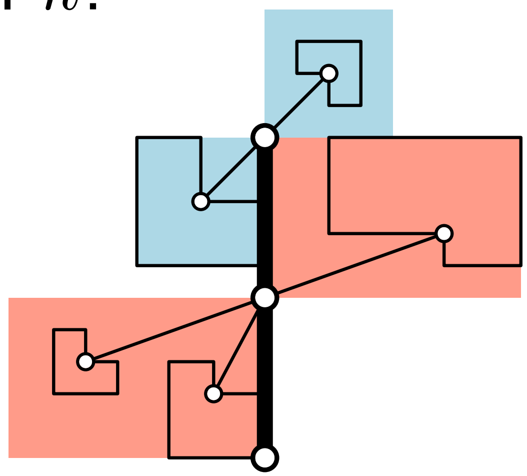

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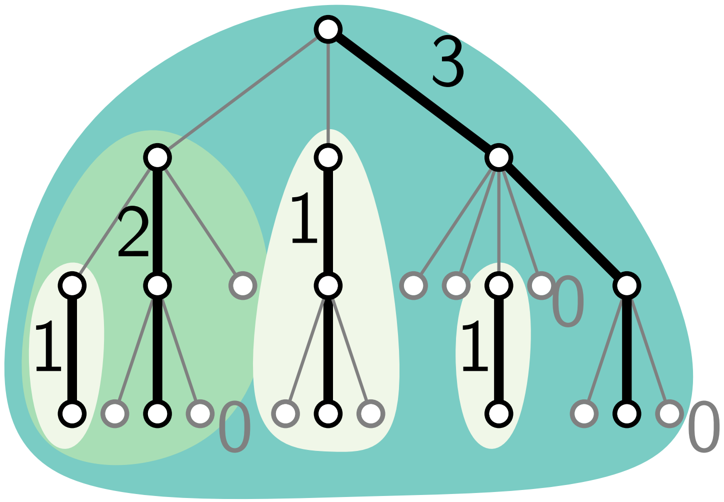

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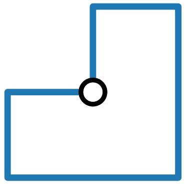

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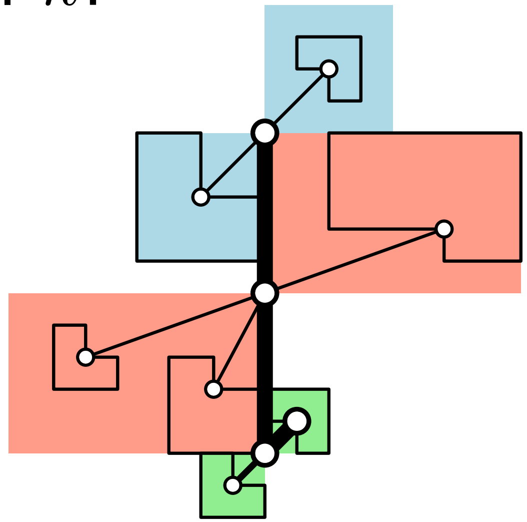

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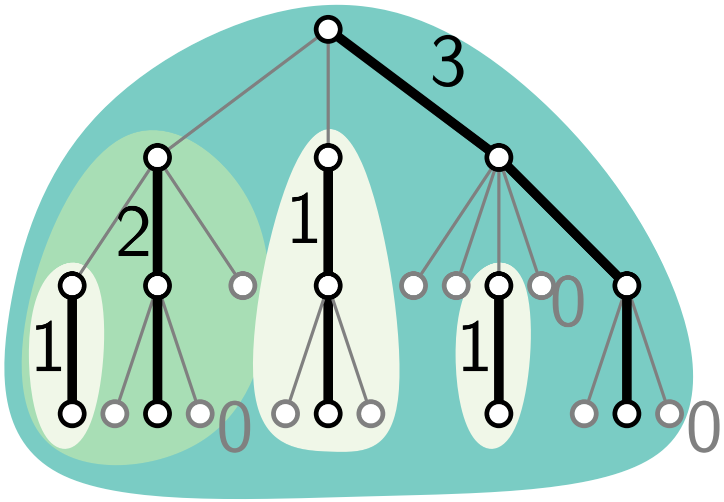

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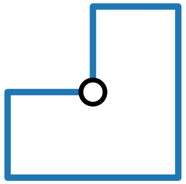
level 0: 

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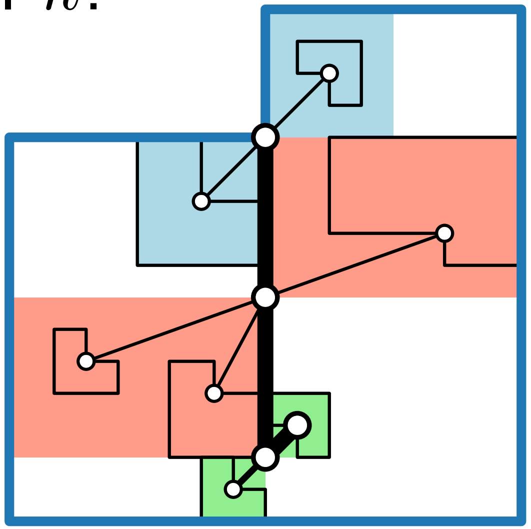

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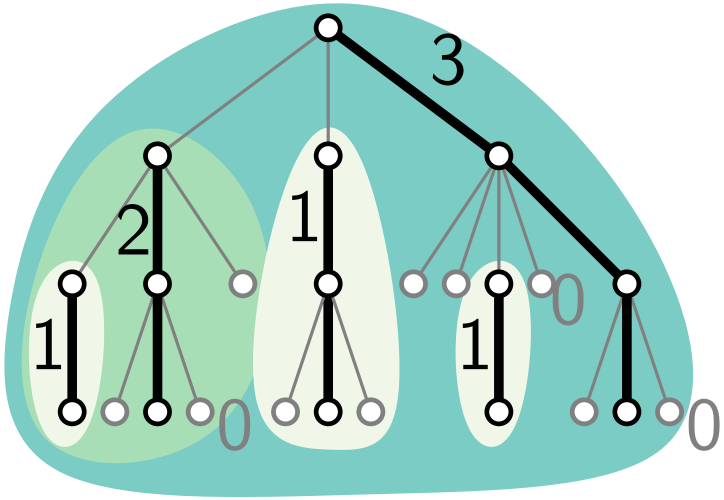

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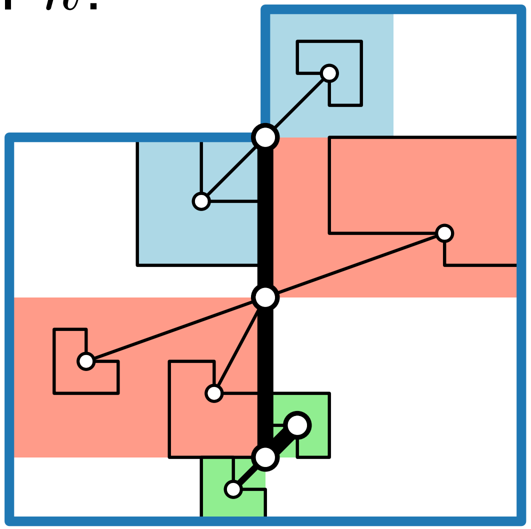

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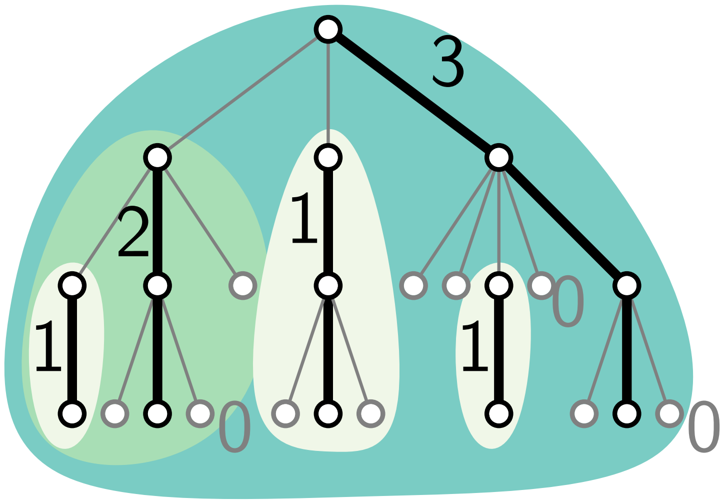

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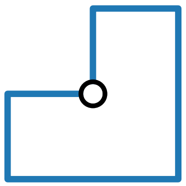
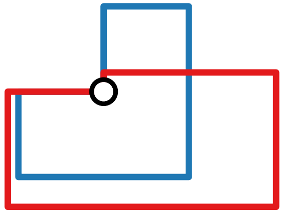
level 0:  

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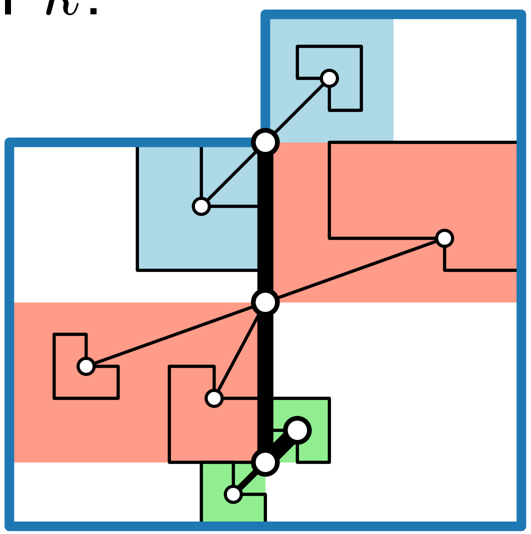

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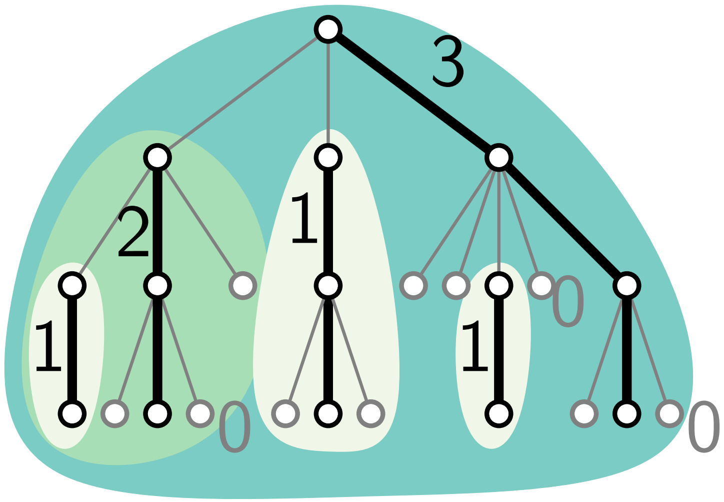

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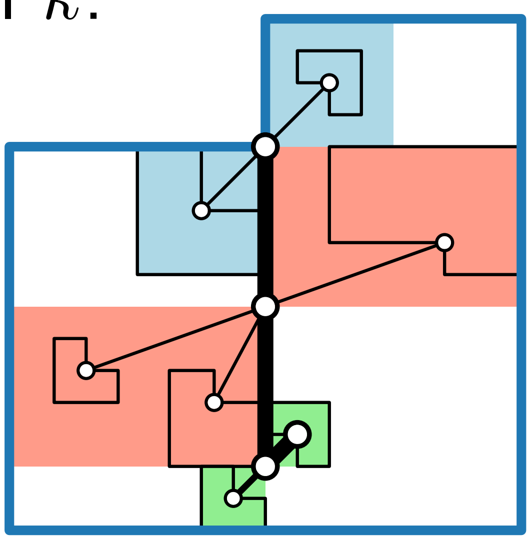

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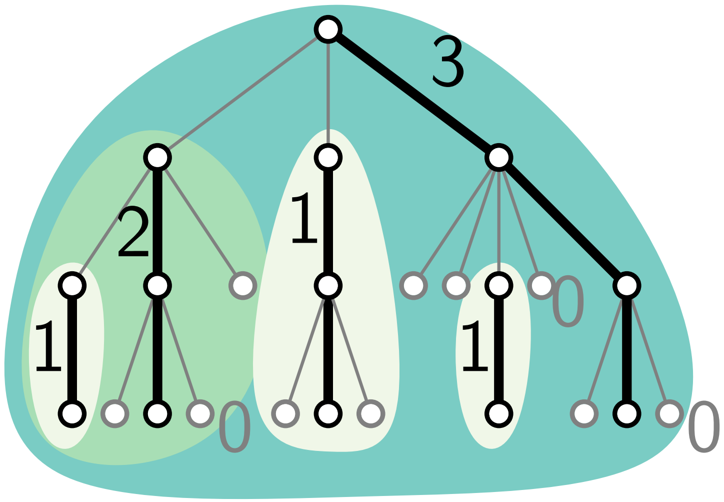

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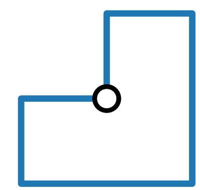

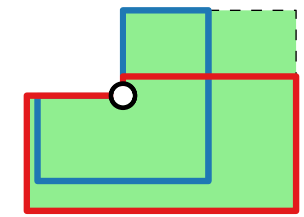

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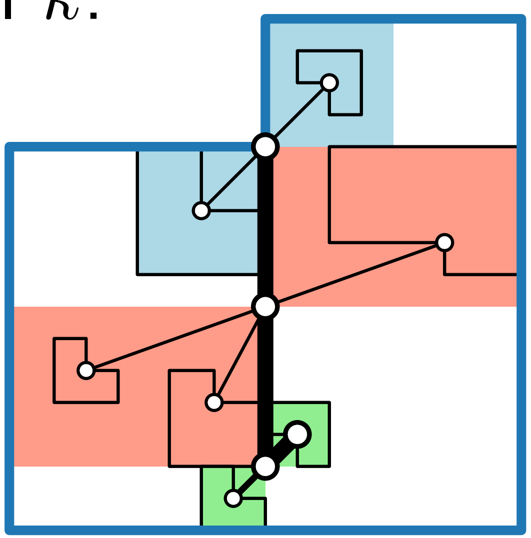

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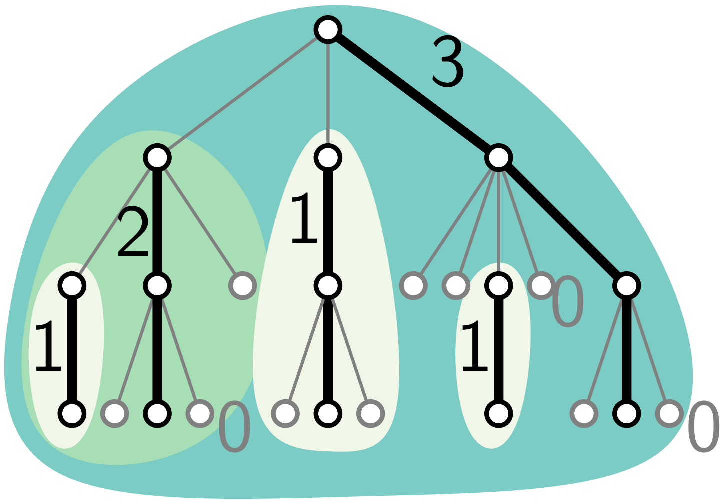

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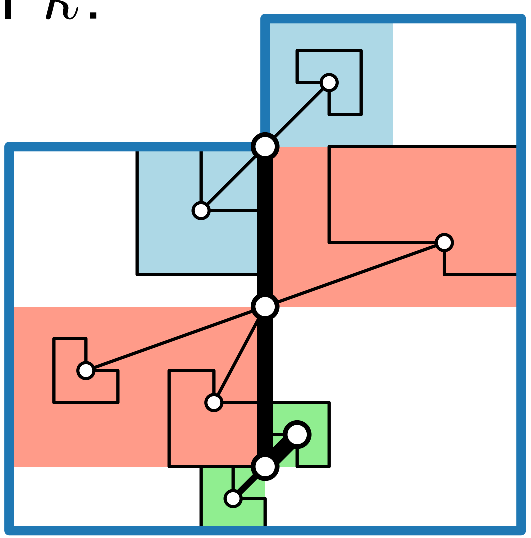

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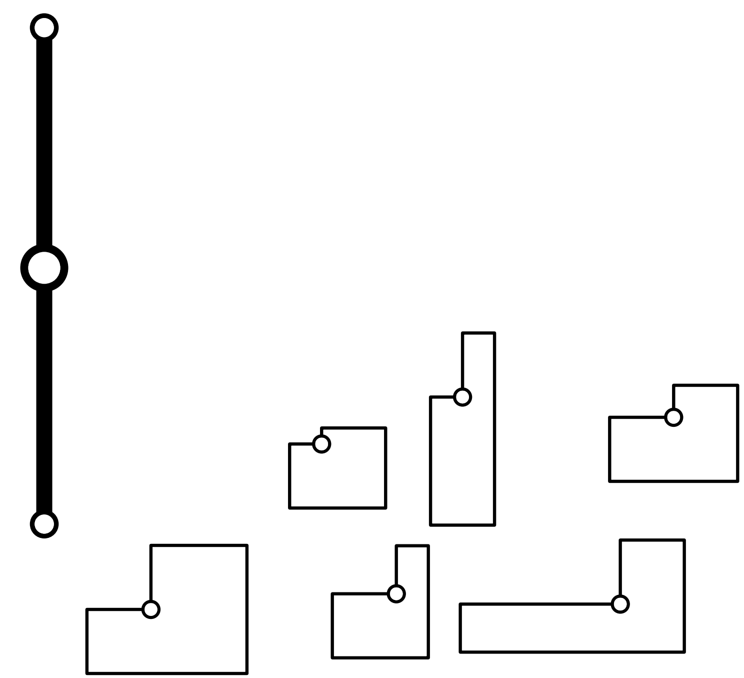

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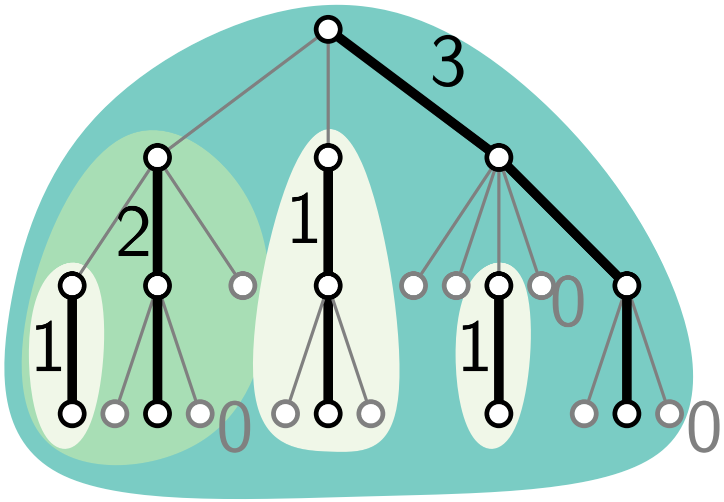

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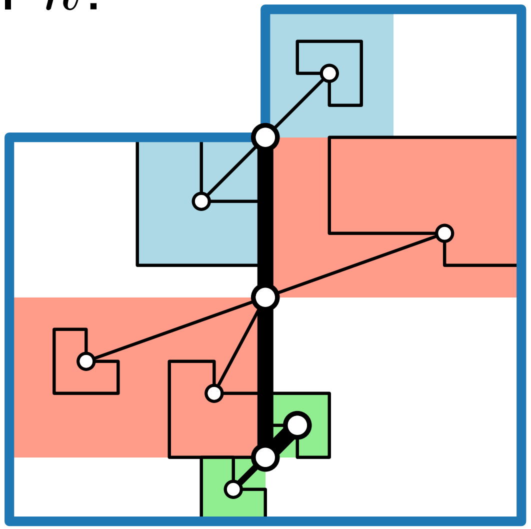

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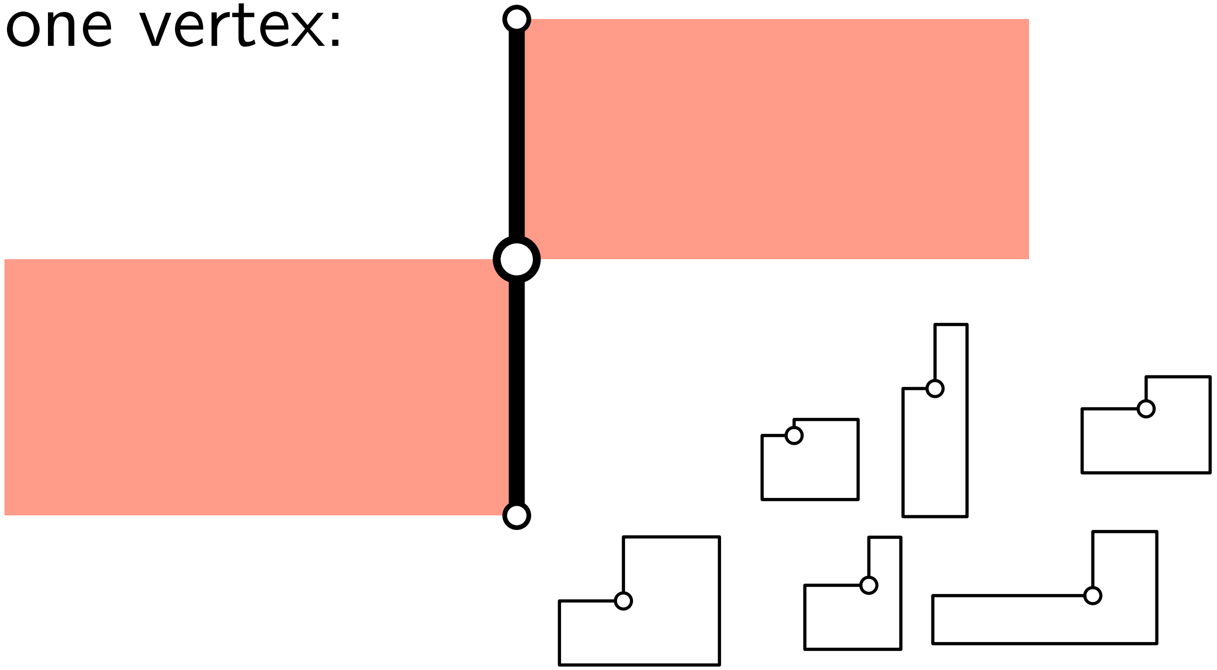

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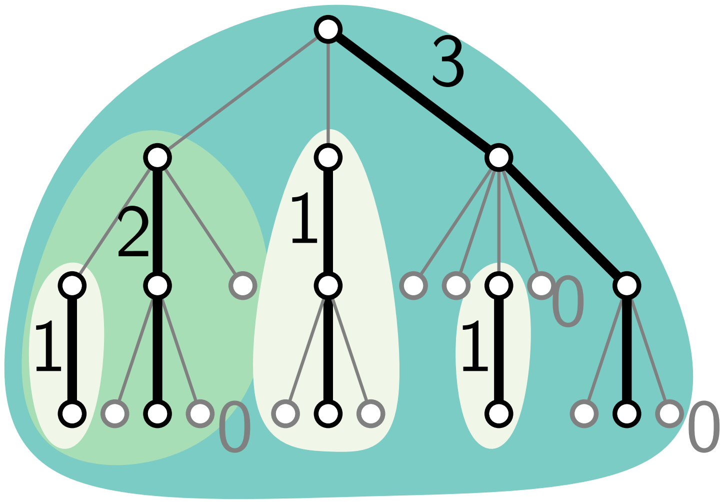

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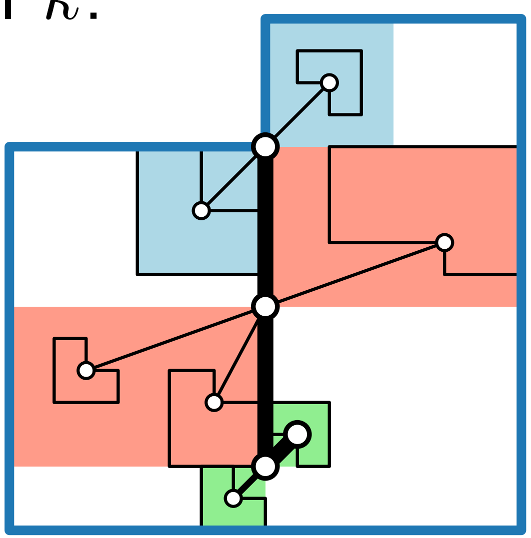

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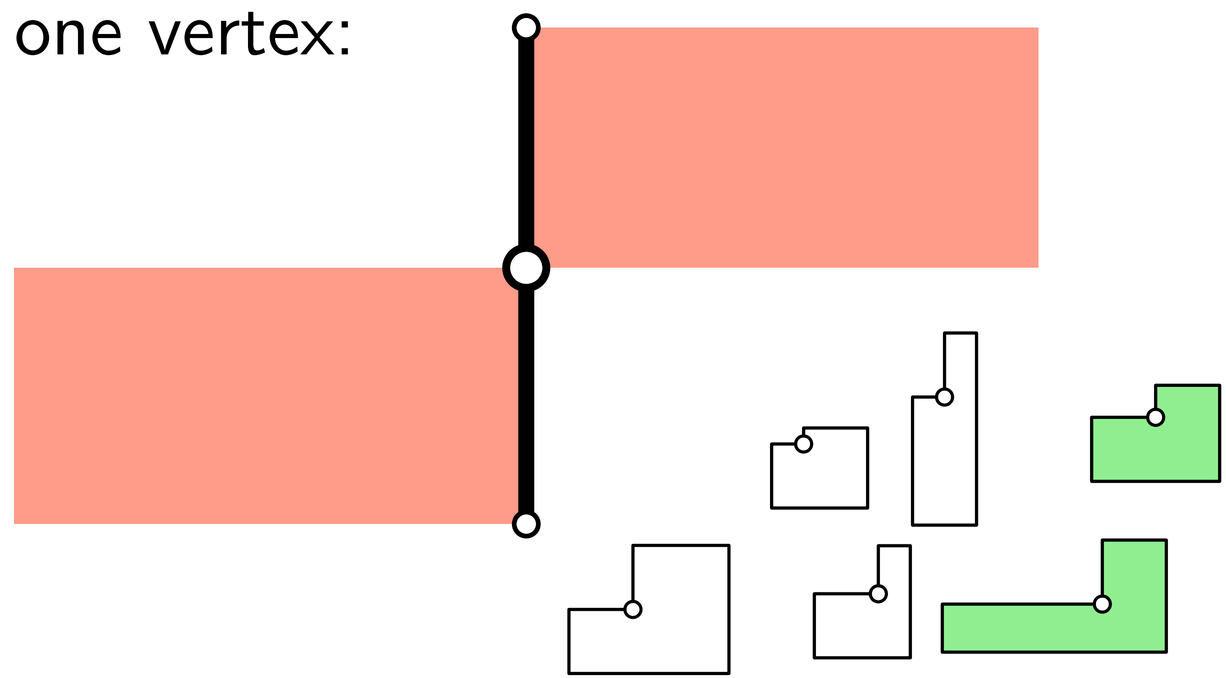

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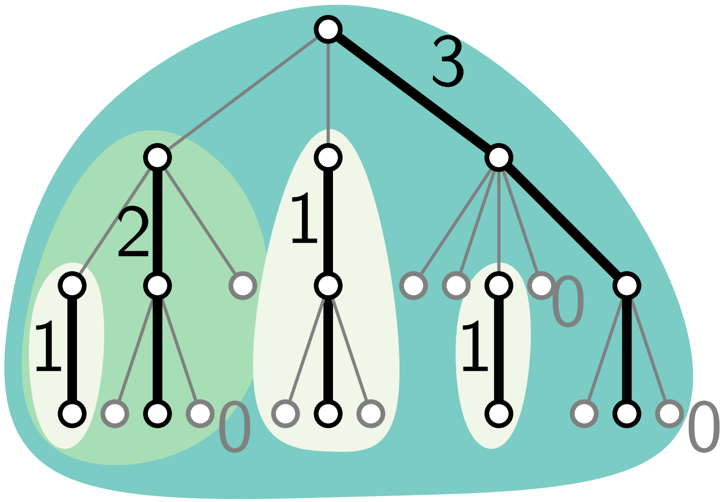

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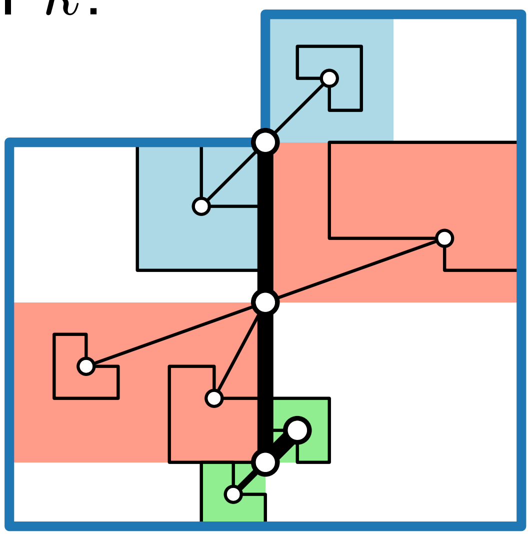

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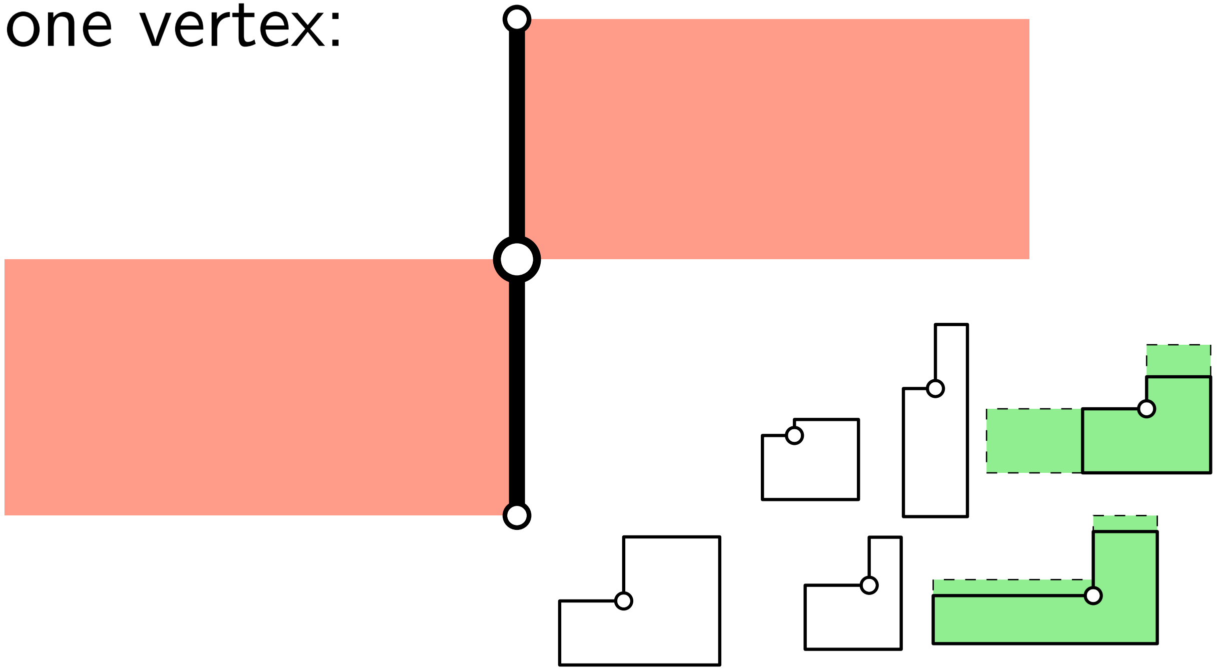

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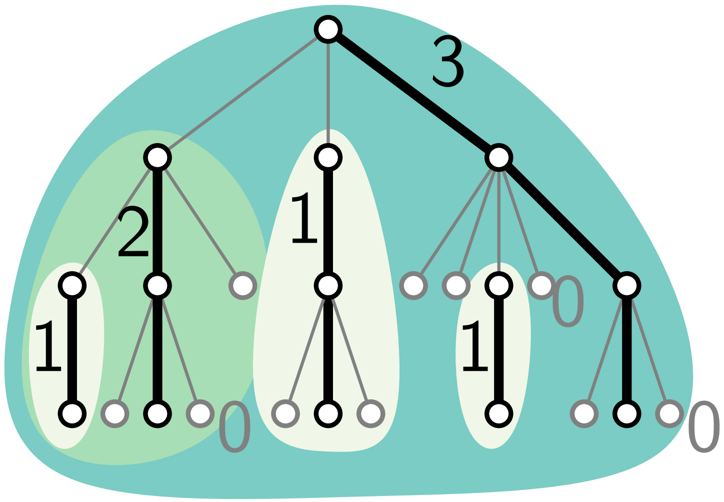

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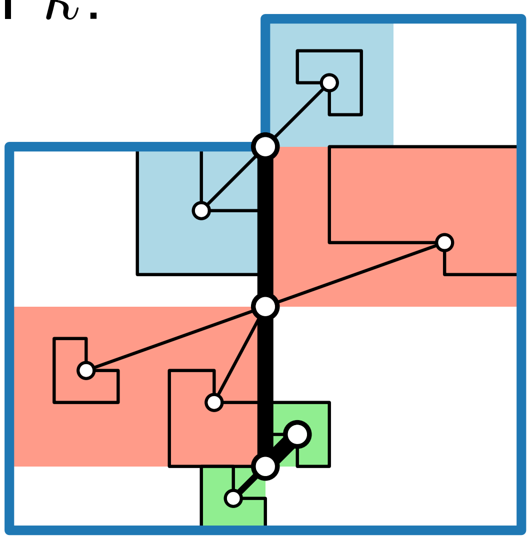

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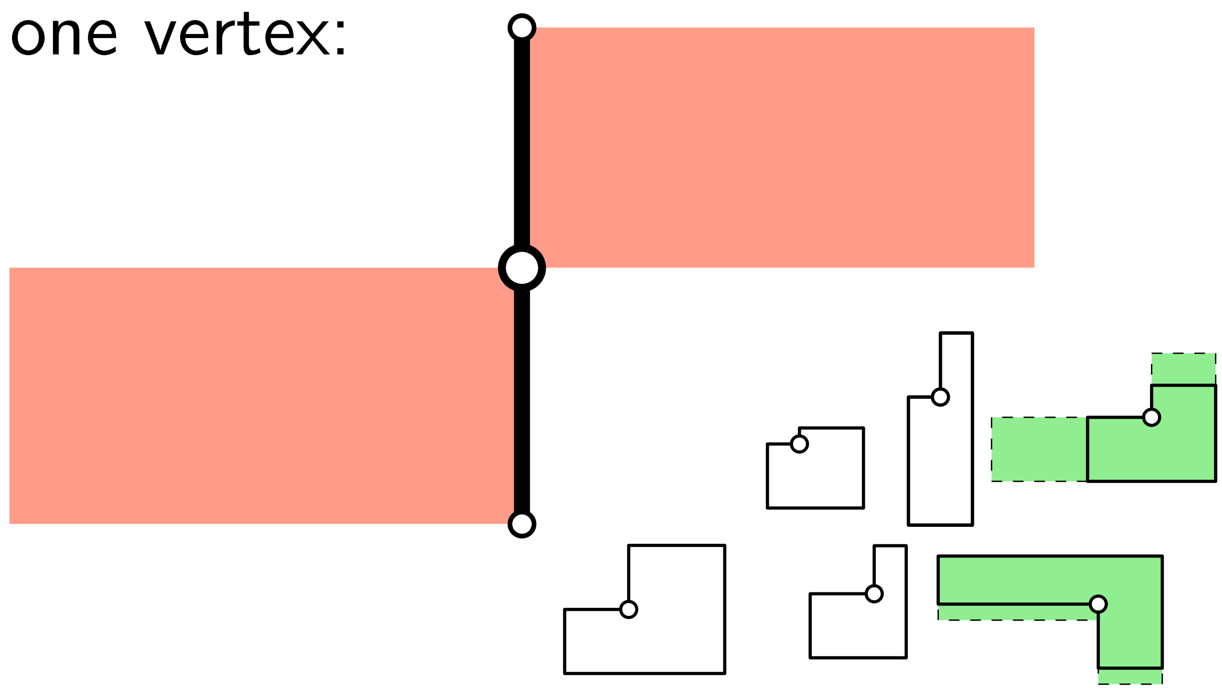

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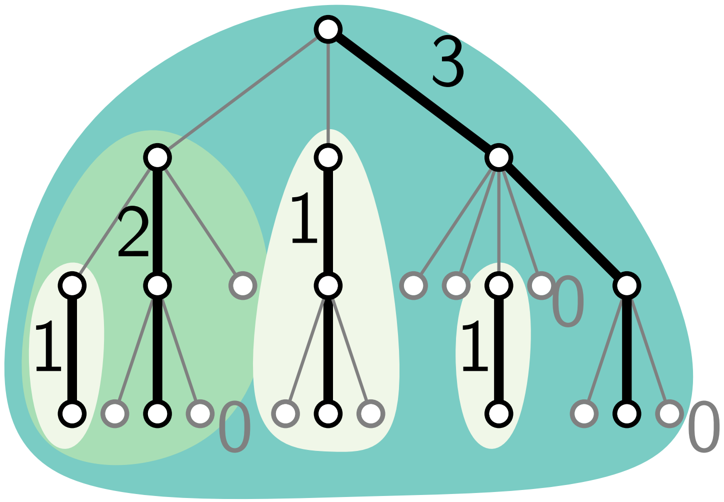

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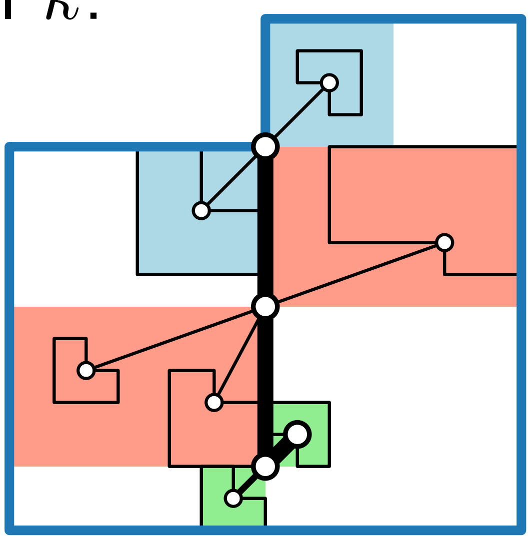

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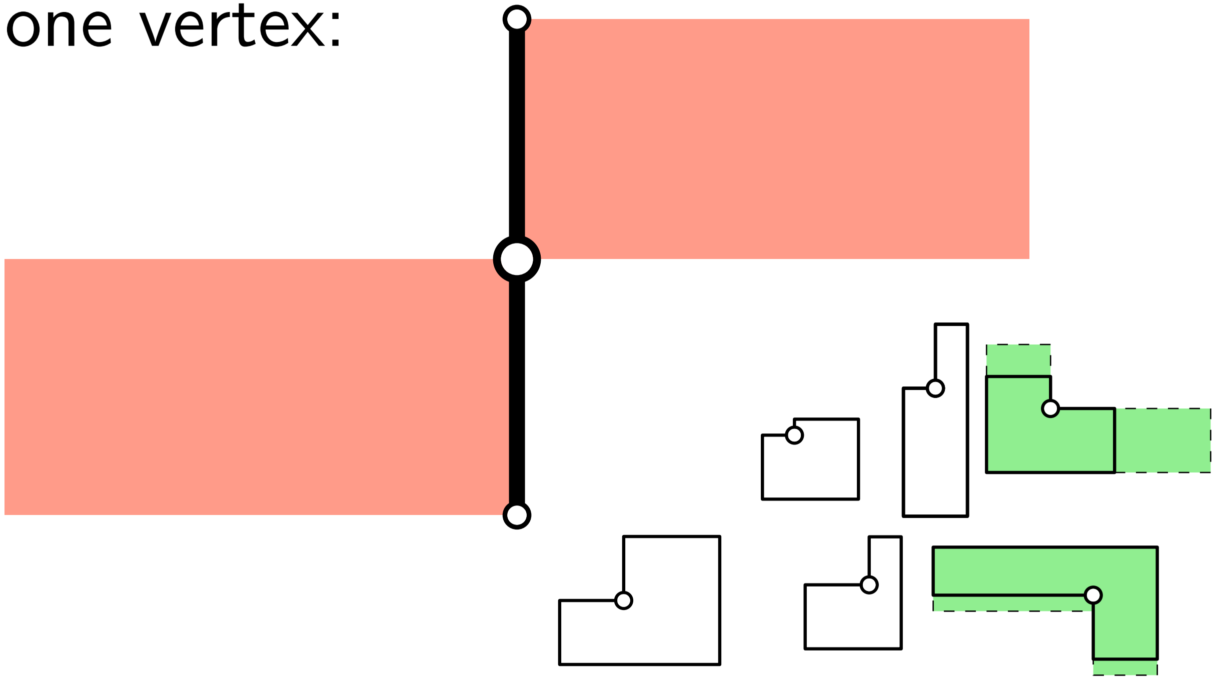

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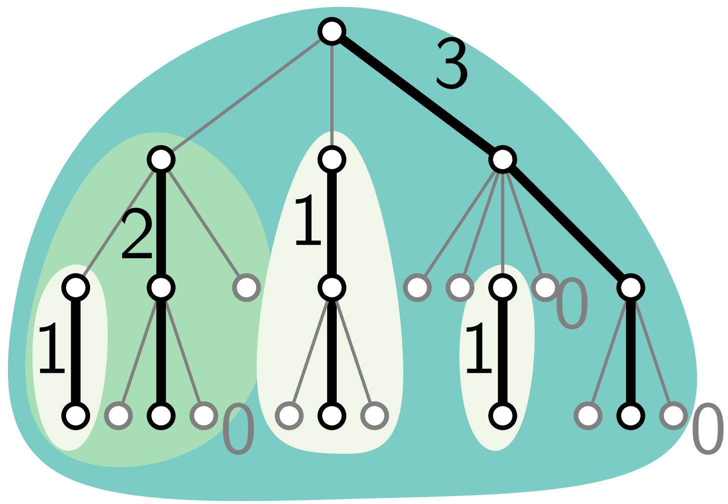

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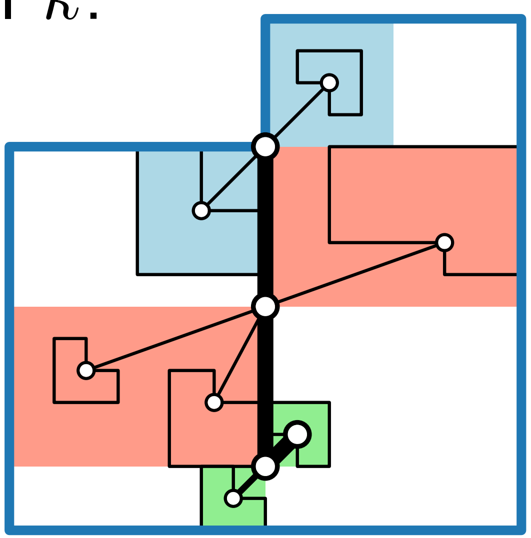

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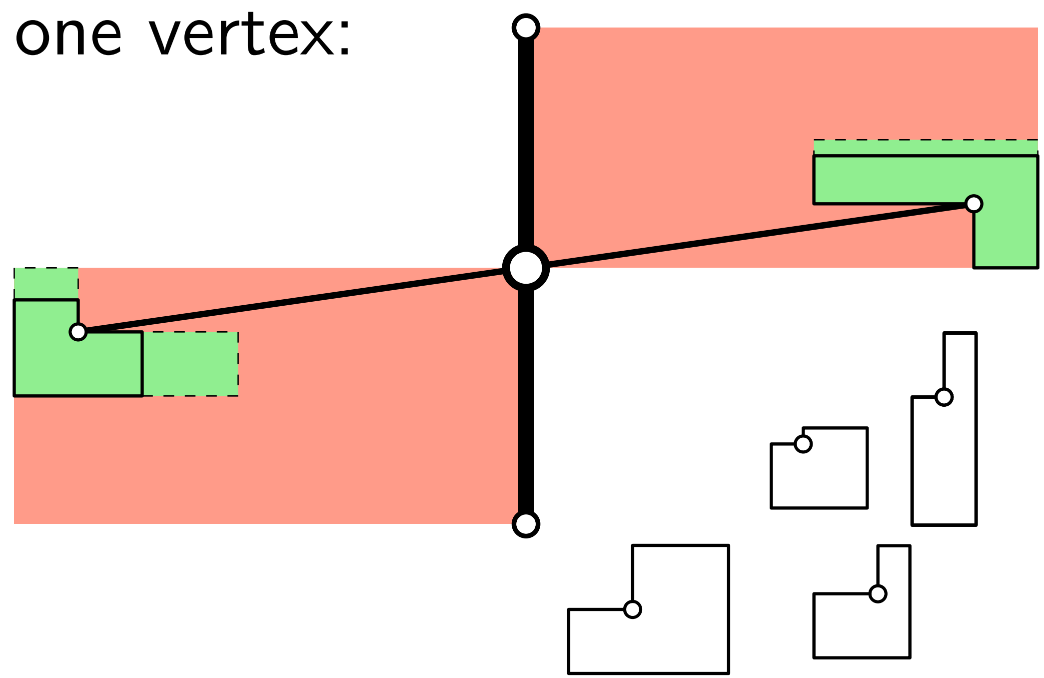

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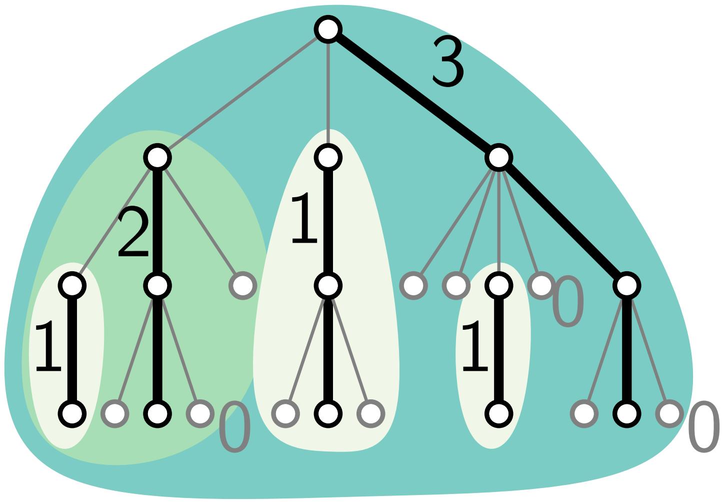

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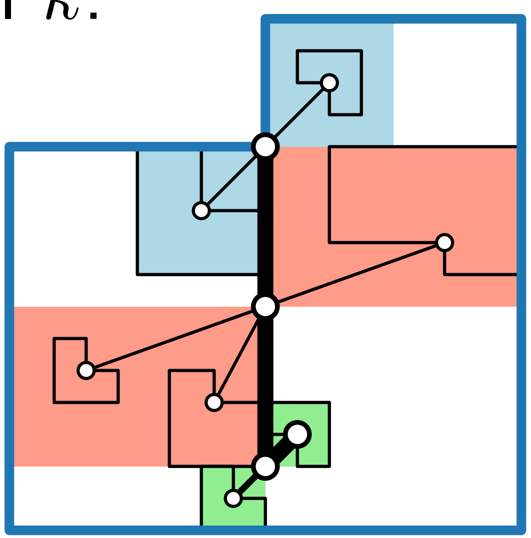

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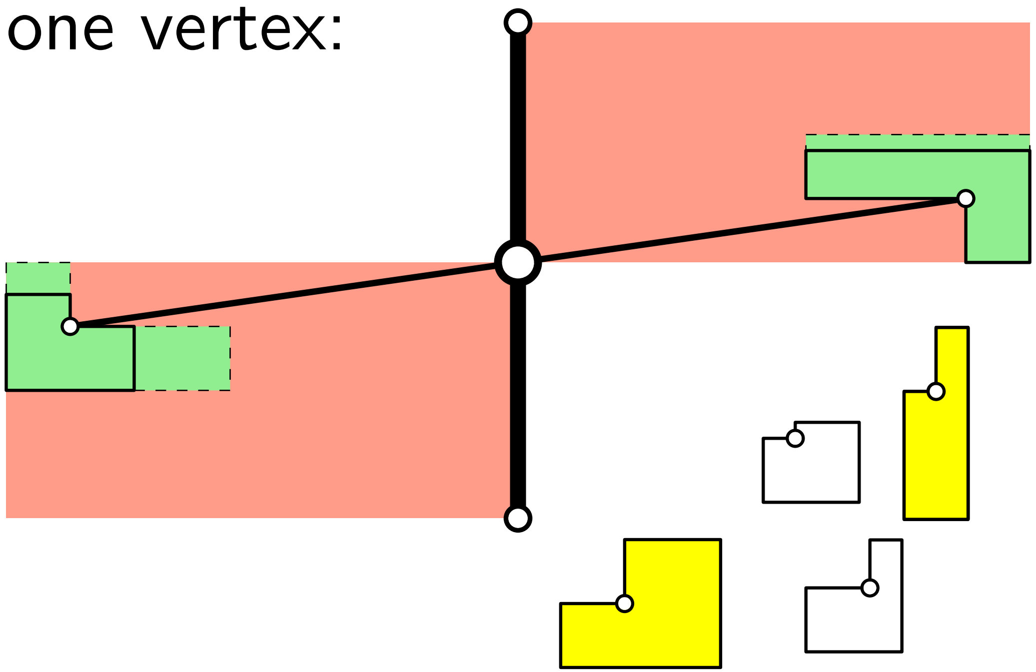

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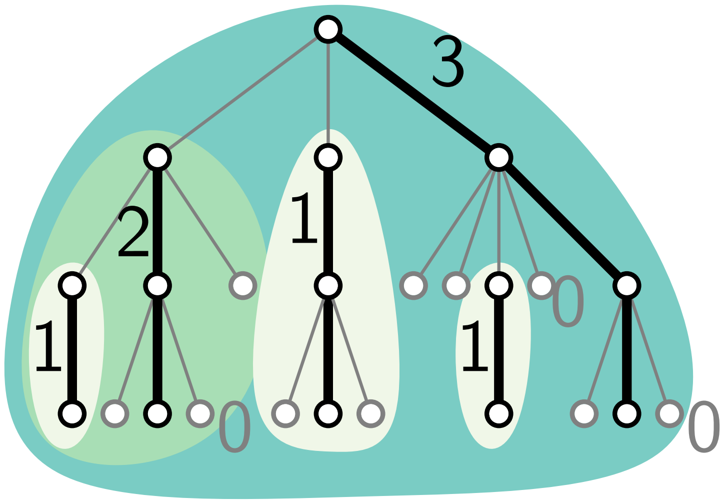

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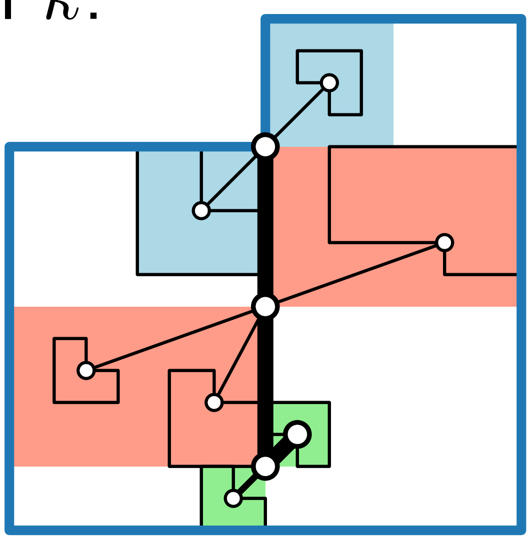

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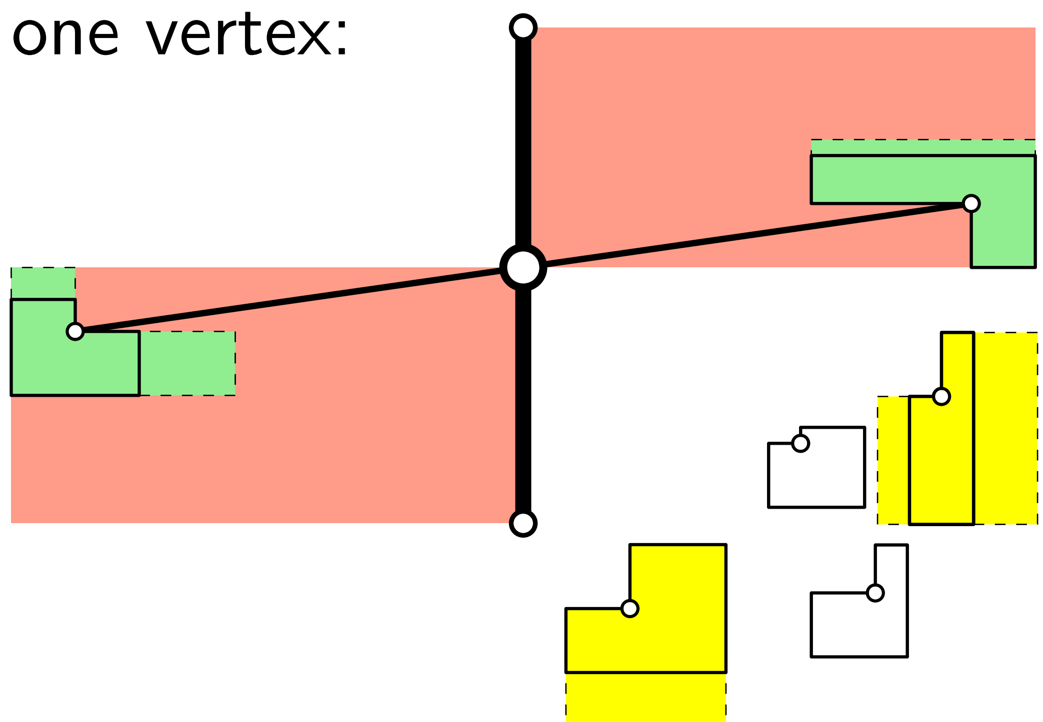

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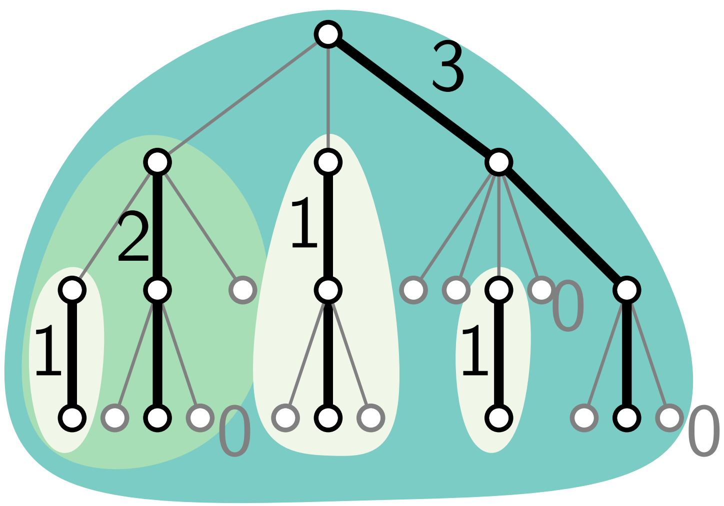

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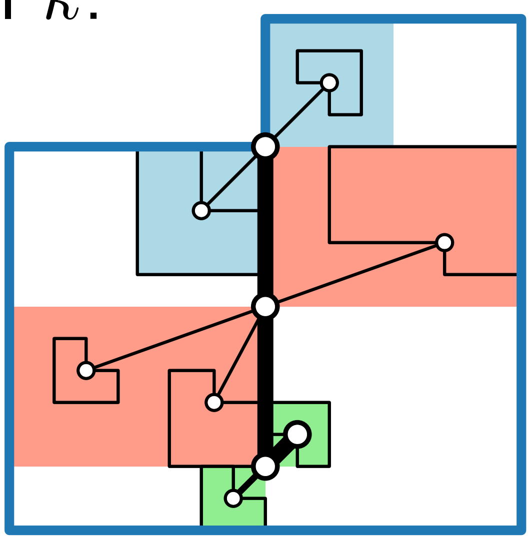

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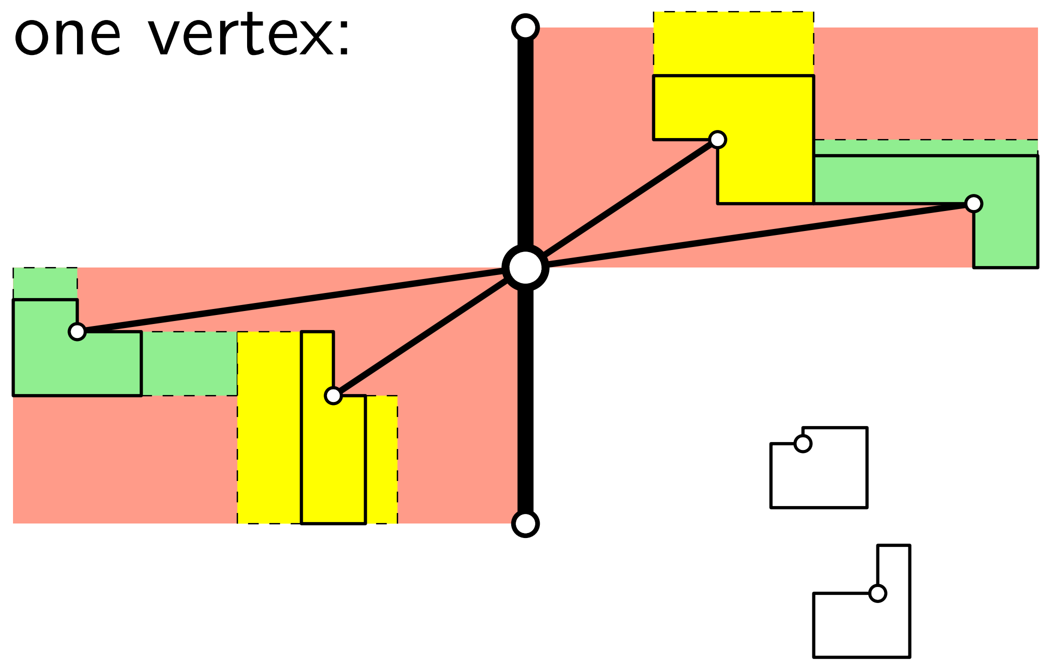

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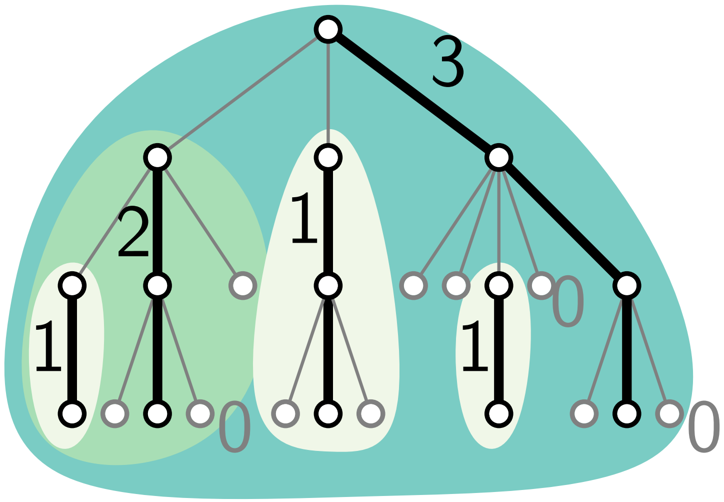

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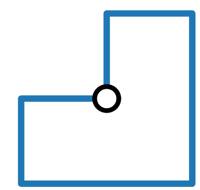

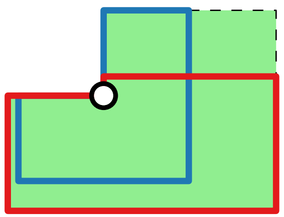

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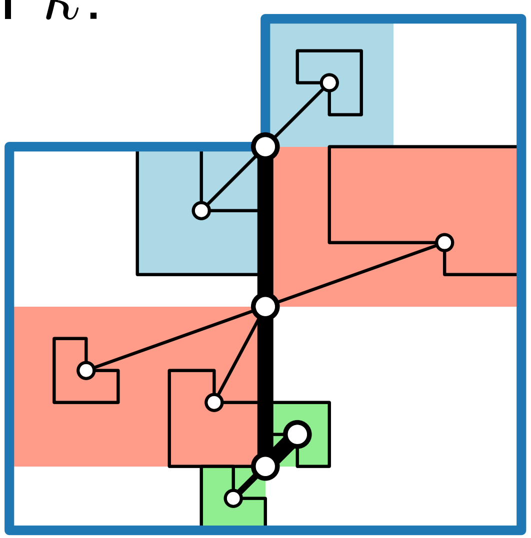

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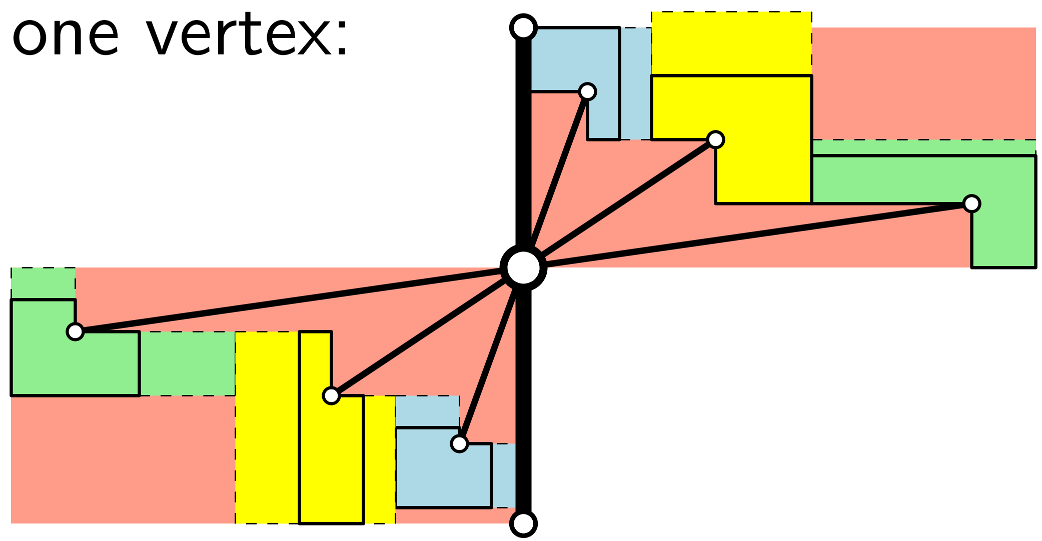

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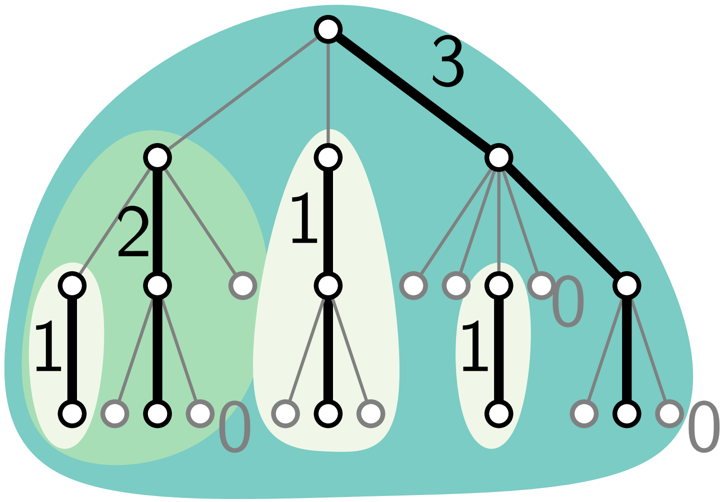

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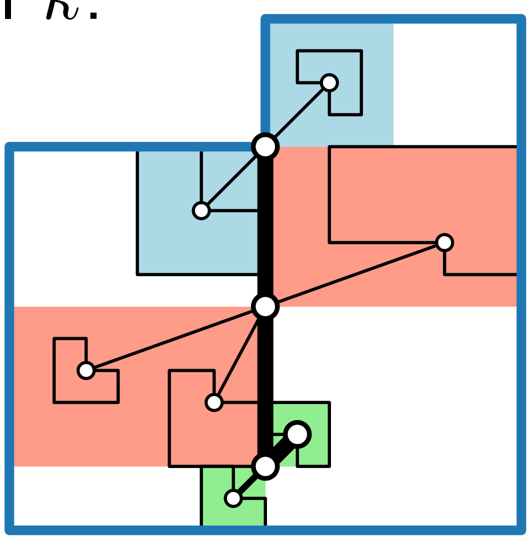

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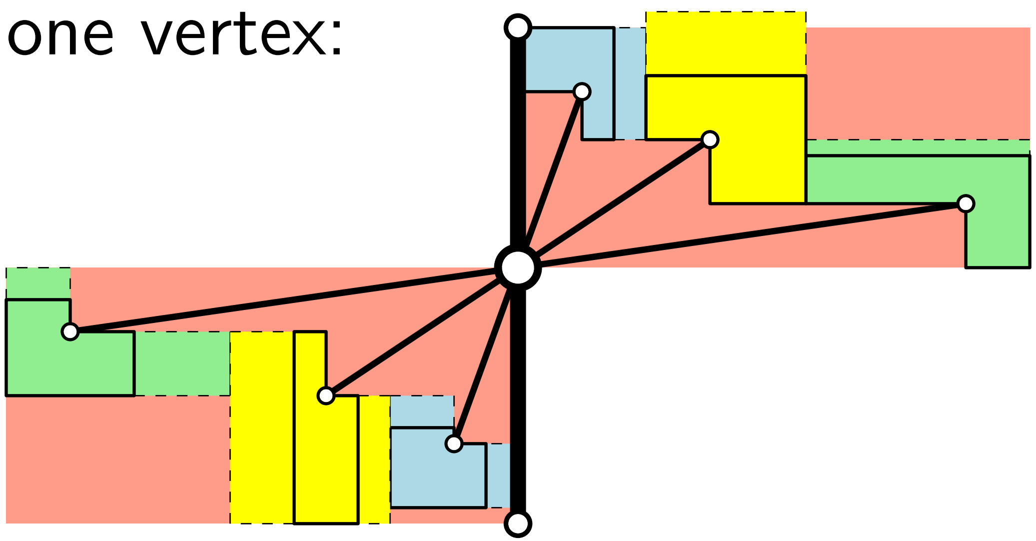
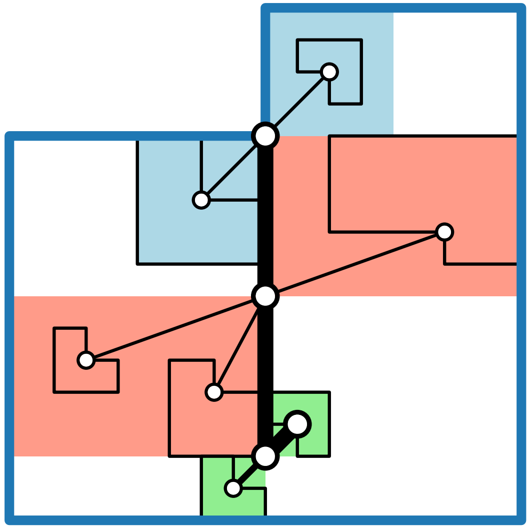


Tree $\rightarrow 3e/4$ segments, $O(n^2) \times O(n^{1.58})$ grid

Trees on the grid

level k:



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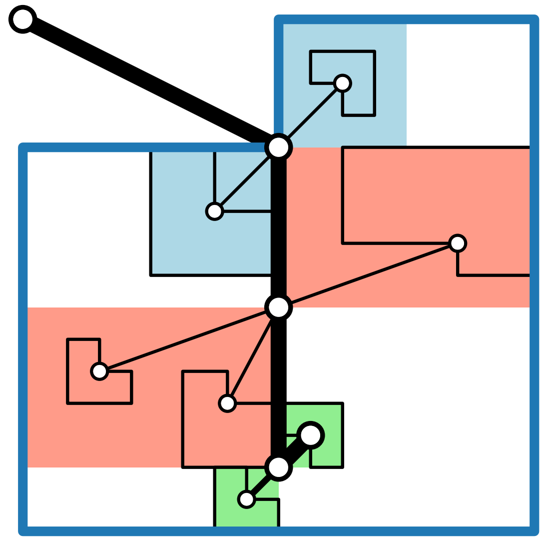
Trees on the grid

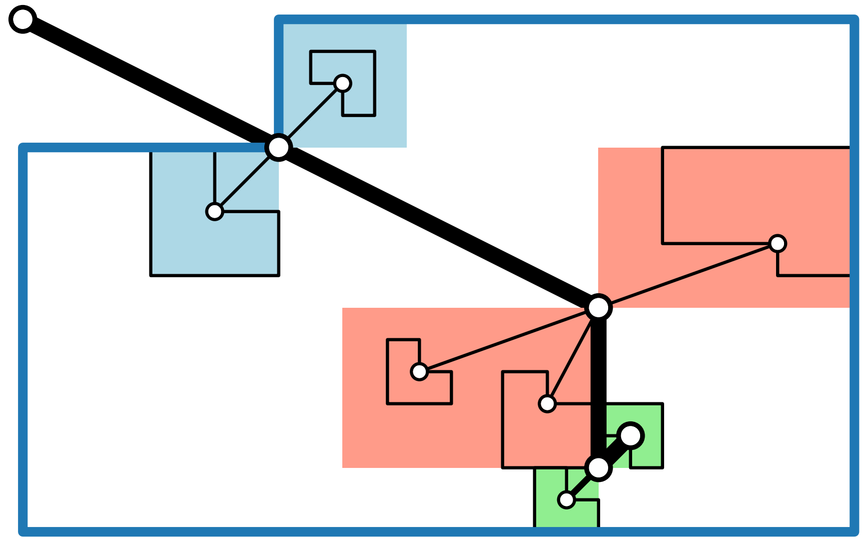
level k:



Tree $\rightarrow 3e/4$ segments, $O(n^2) \times O(n^{1.58})$ grid

Trees on the grid

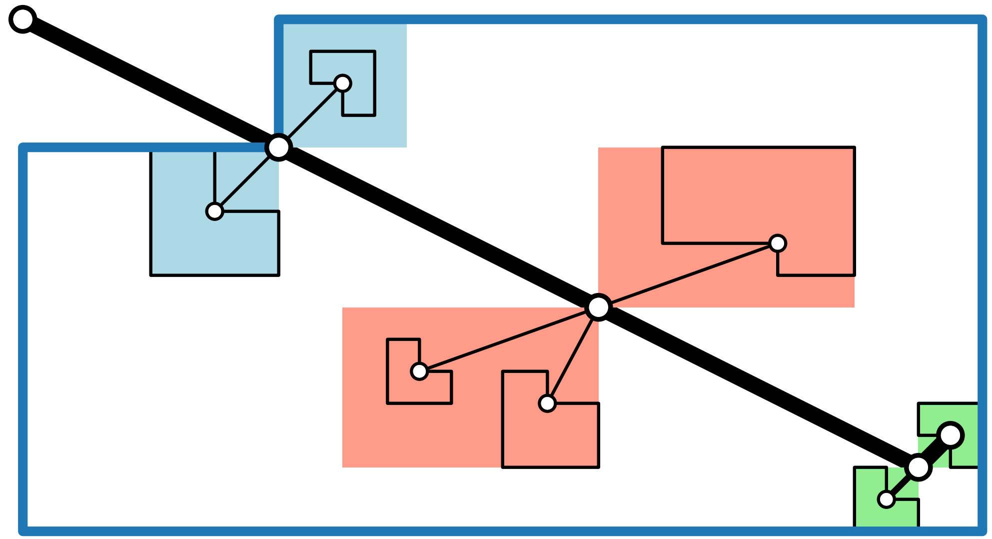
level k:



Tree $\rightarrow 3e/4$ segments, $O(n^2) \times O(n^{1.58})$ grid

Trees on the grid

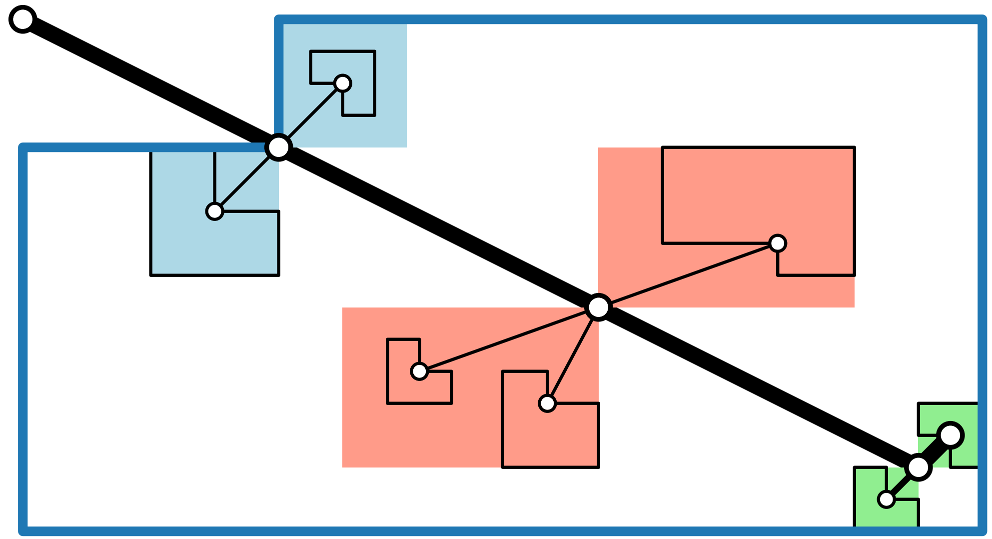
level k:



Tree $\longrightarrow 3e/4$ segments, $O(n^2) \times O(n^{1.58})$ grid

Trees on the grid

level k:

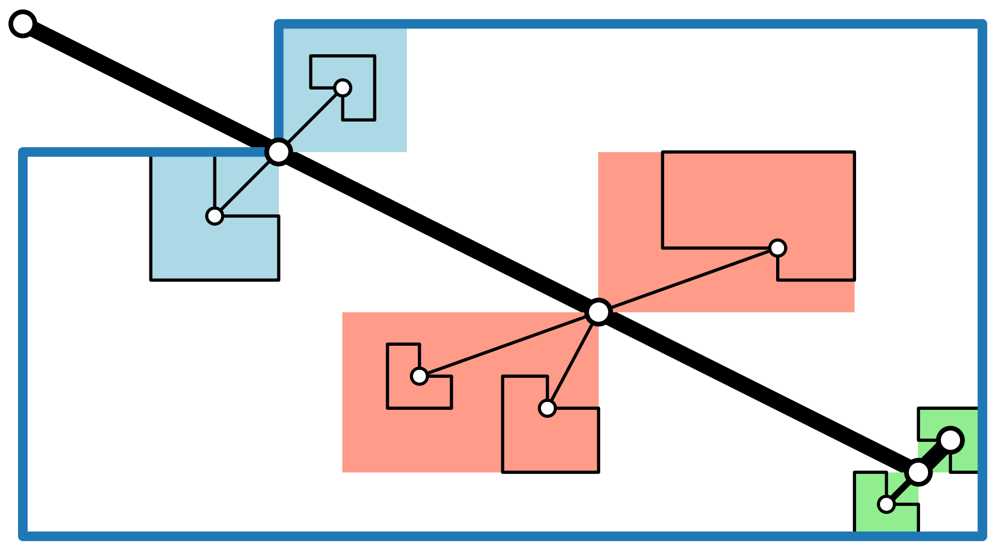


Tree $\rightarrow \vartheta/2$ segments, quasipolynomial grid

Tree $\rightarrow 3e/4$ segments, $O(n^2) \times O(n^{1.58})$ grid

Trees on the grid

level k:



Tree $\rightarrow \vartheta/2$ segments, polynomial grid?

Tree $\rightarrow \vartheta/2$ segments, quasipolynomial grid

Tree $\rightarrow 3e/4$ segments, $O(n^2) \times O(n^{1.58})$ grid

Known Results – Segments

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]		?
max. outerplanar	n [1]	n [1]		
2-trees	$3n/2$ [1]	$3n/2$ [1]		
3-trees	$2n$ [1]	$2n$ [1]		?
2-connected	$2n$ [1]	$16n/3 - e$ [2]		
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	$n/2$ [4]	$O(n) \times O(n)$
Triangulation	$2n$ [2]	$7n/3$ [2]		
Planar	$2n$ [2]	$16n/3 - e$ [2]		

[1] Dujmović et al. 2007
[2] Durocher & Mondal 2014
[3] Mondal et al. 2013
[4] Igamberdiev et al. 2015

Known Results – Segments

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ $\vartheta/2$	$O(n^2) \times O(n^{1.58})$ quasipolynomial
max. outerplanar	n [1]	n [1]		?
2-trees	$3n/2$ [1]	$3n/2$ [1]		
3-trees	$2n$ [1]	$2n$ [1]		?
2-connected	$2n$ [1]	$16n/3 - e$ [2]		
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	$n/2$ [4]	$O(n) \times O(n)$
Triangulation	$2n$ [2]	$7n/3$ [2]		
Planar	$2n$ [2]	$16n/3 - e$ [2]		

[1] Dujmović et al. 2007
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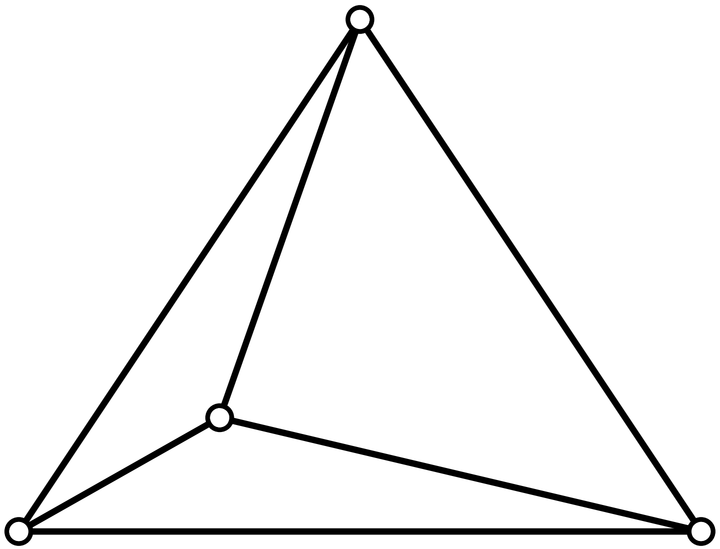
Planar 3-trees on the grid



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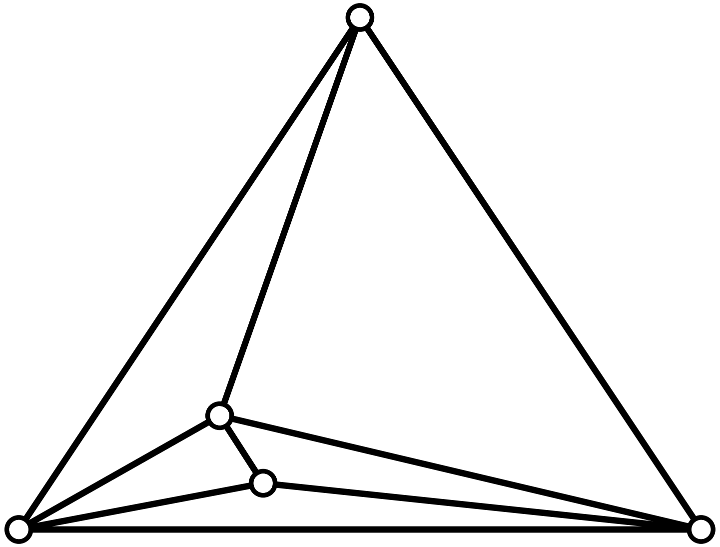
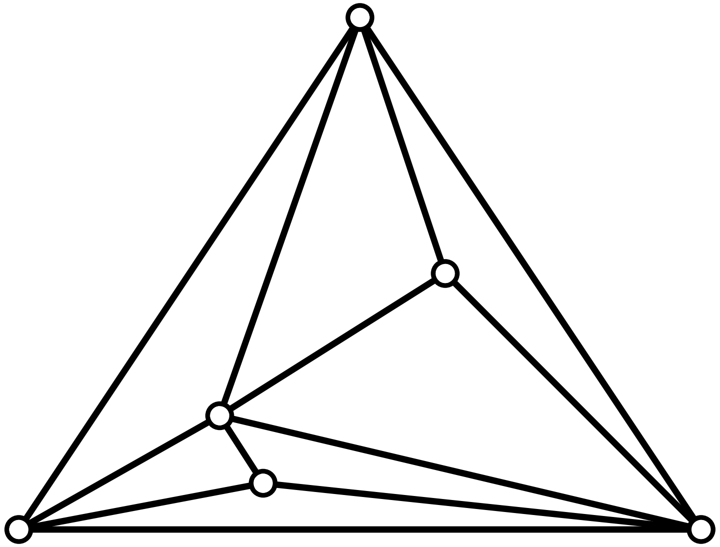


Planar 3-trees on the grid



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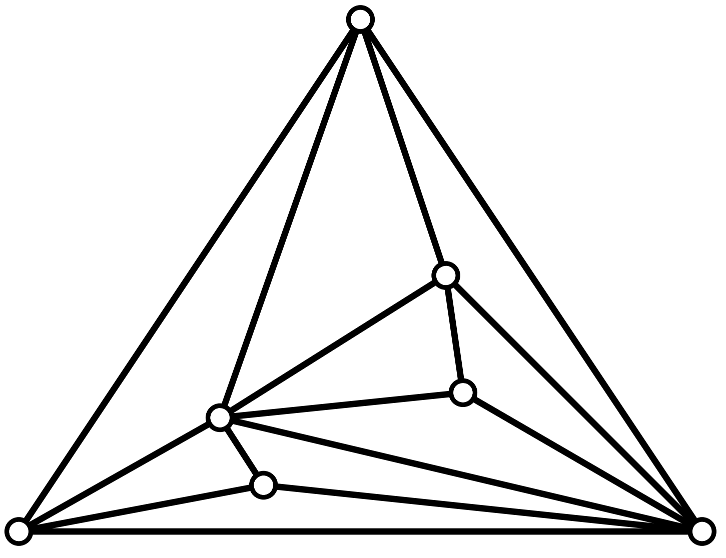

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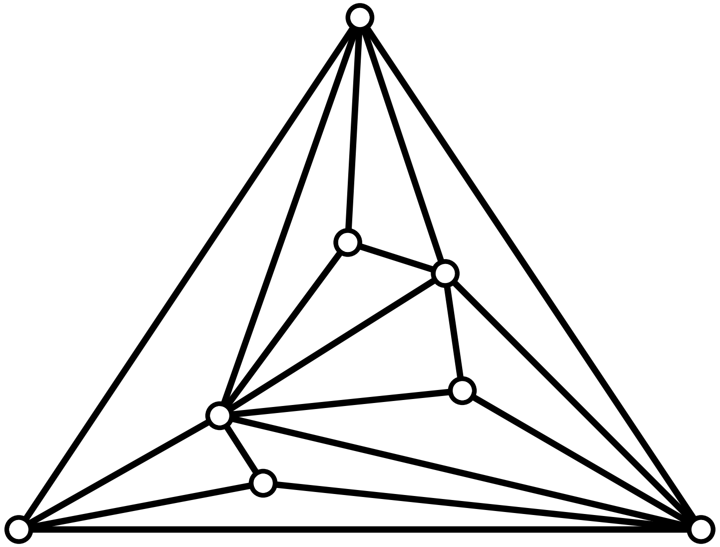
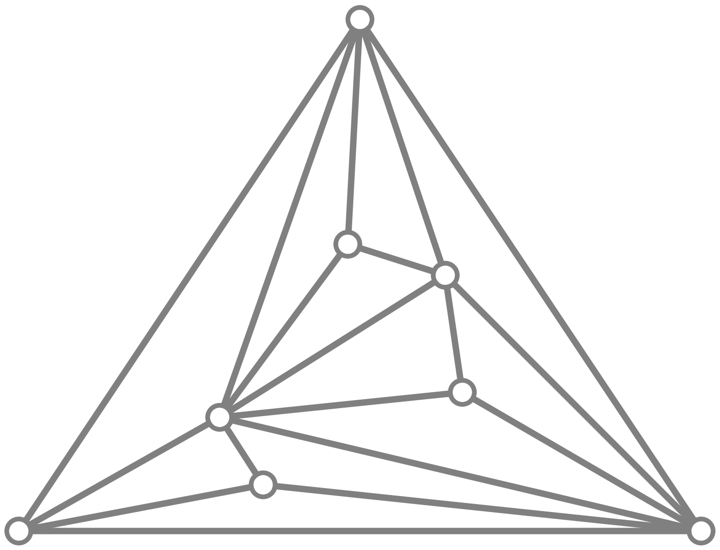
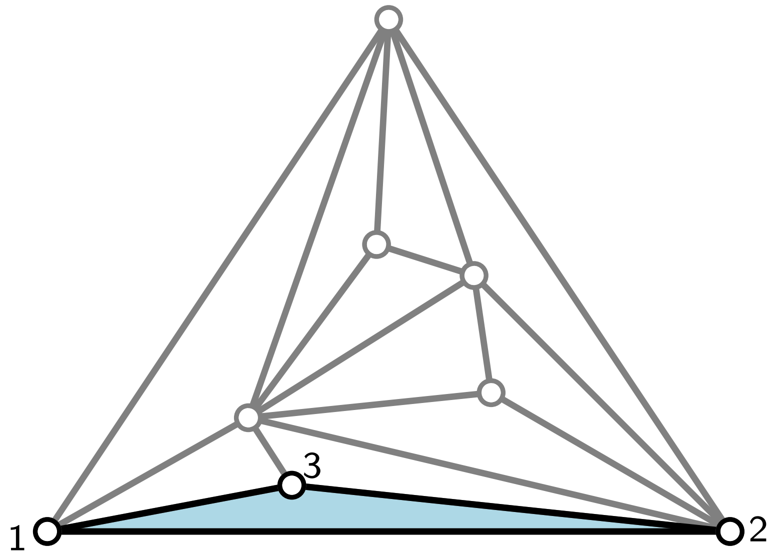


Planar 3-trees on the grid



canonical order

Planar 3-trees on the grid



canonical order

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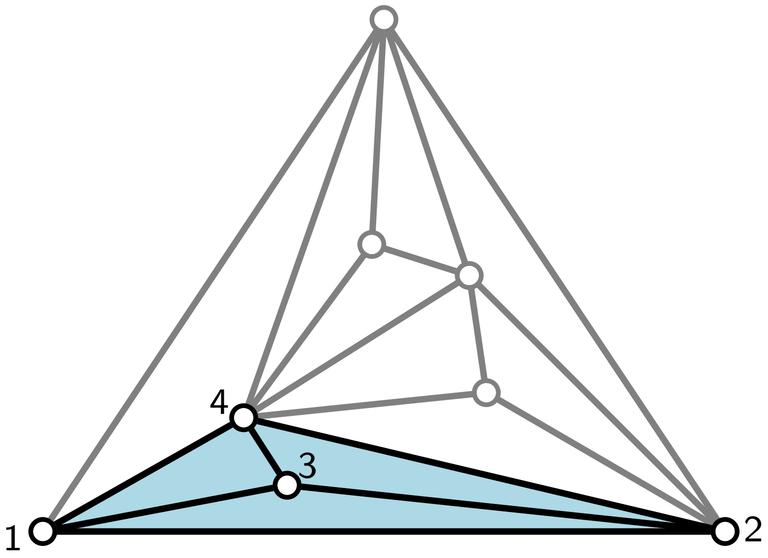
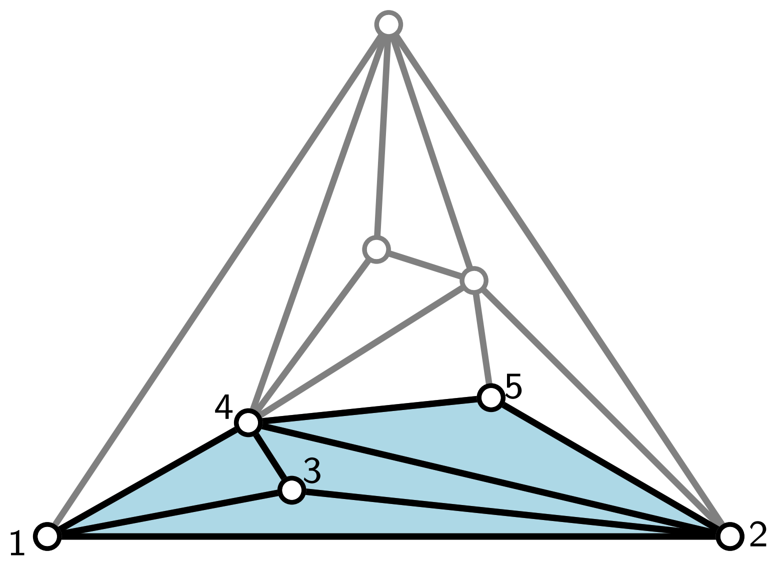


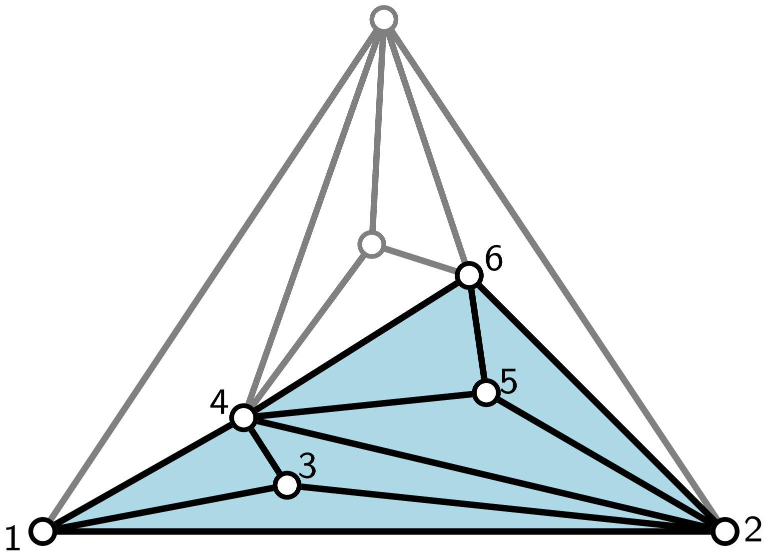
canonical order

Planar 3-trees on the grid



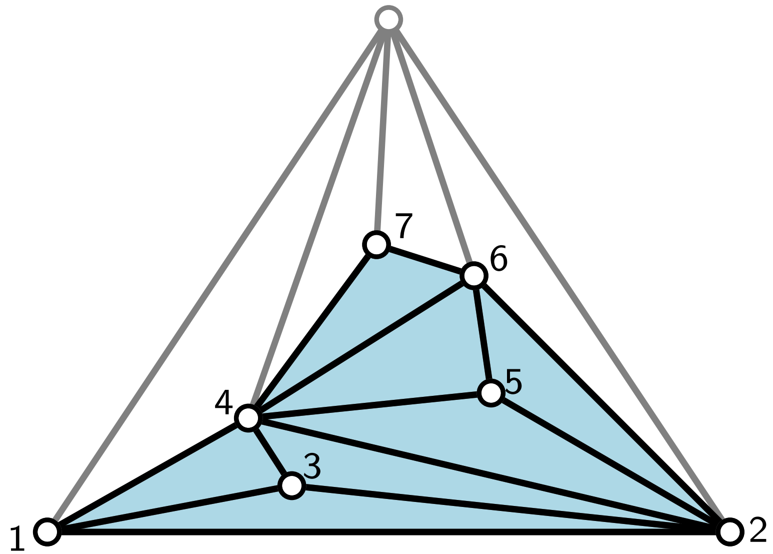
canonical order

Planar 3-trees on the grid



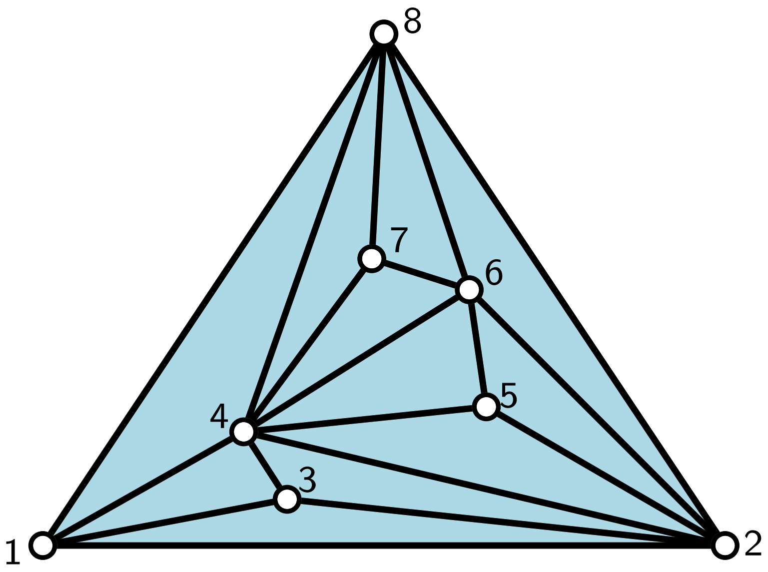
canonical order

Planar 3-trees on the grid



canonical order

Planar 3-trees on the grid



canonical order

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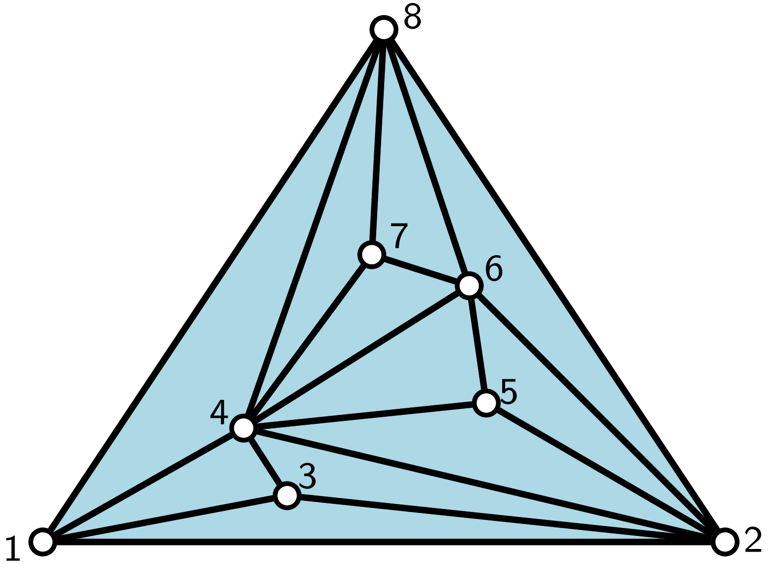
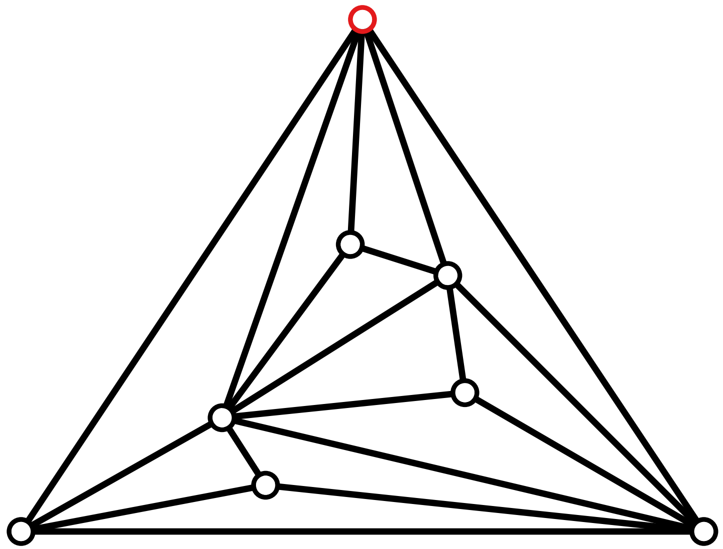
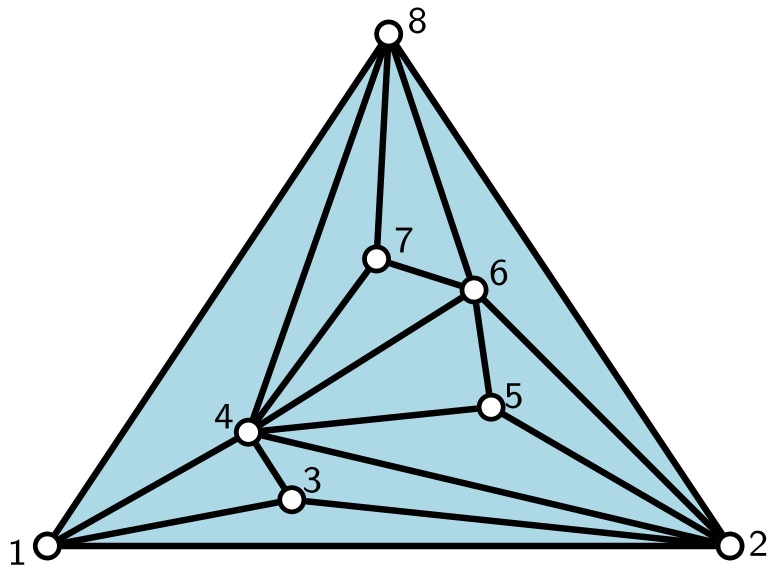


canonical order

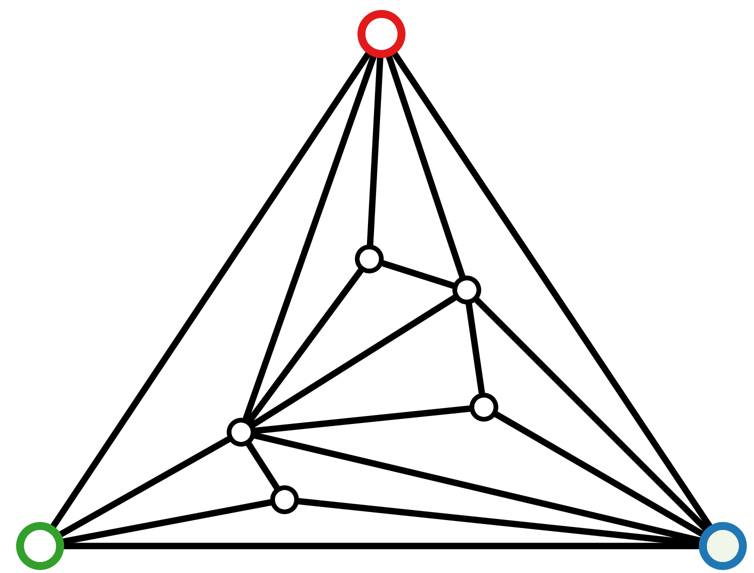


Schnyder realizer

Planar 3-trees on the grid



canonical order



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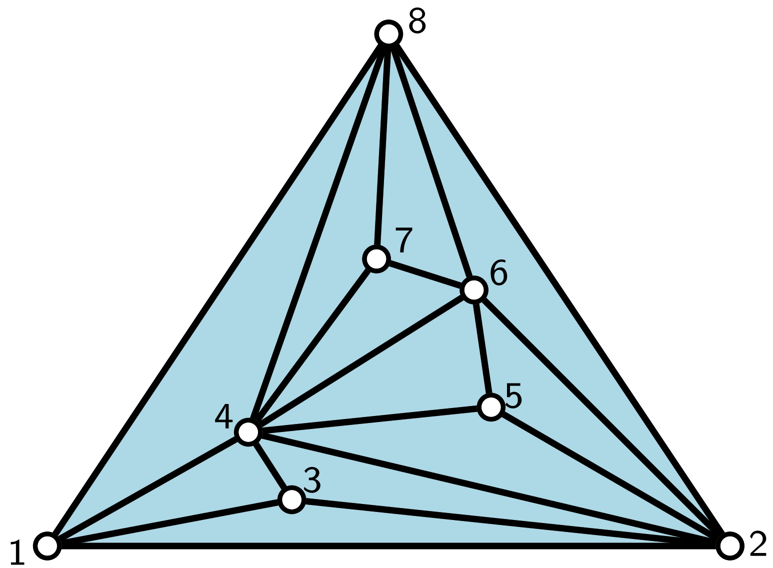
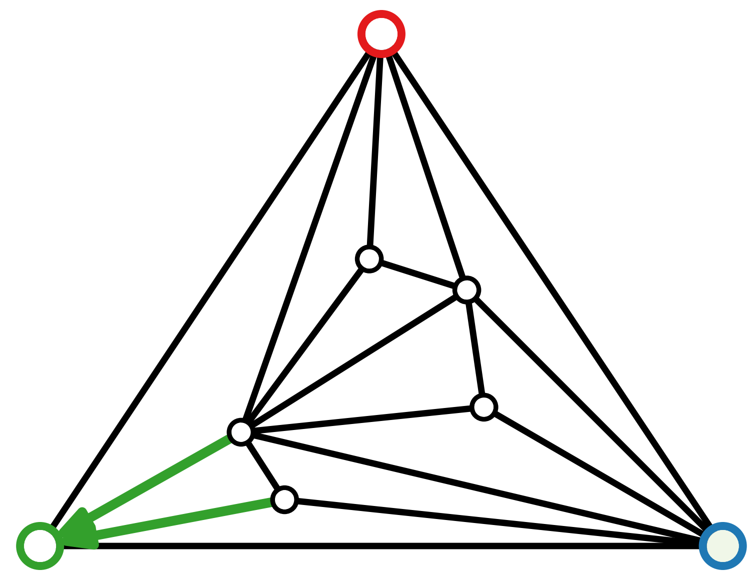


canonical order



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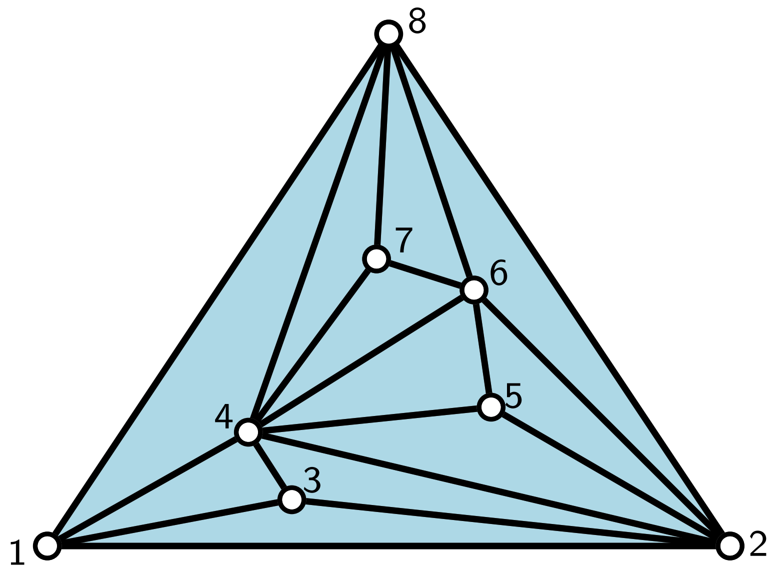
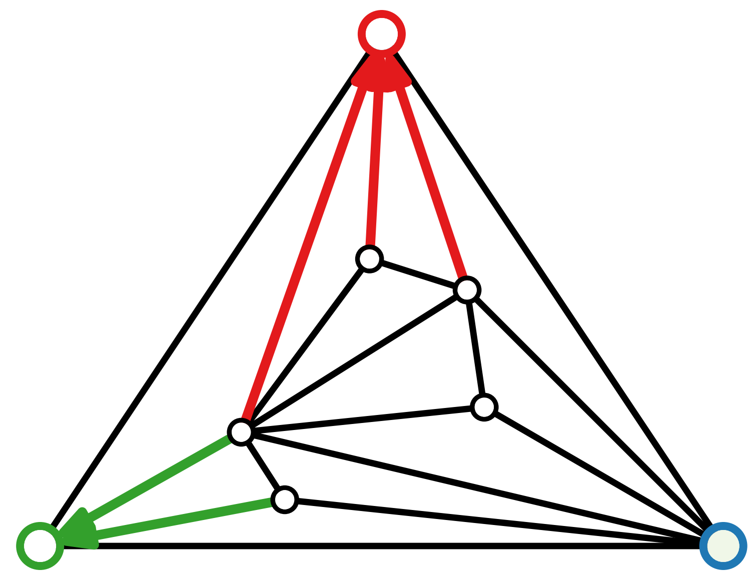


canonical order



Schnyder realizer

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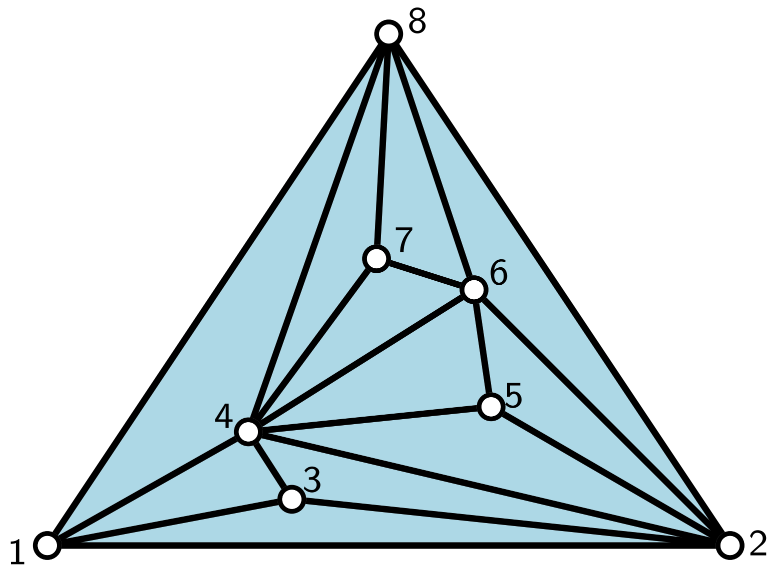
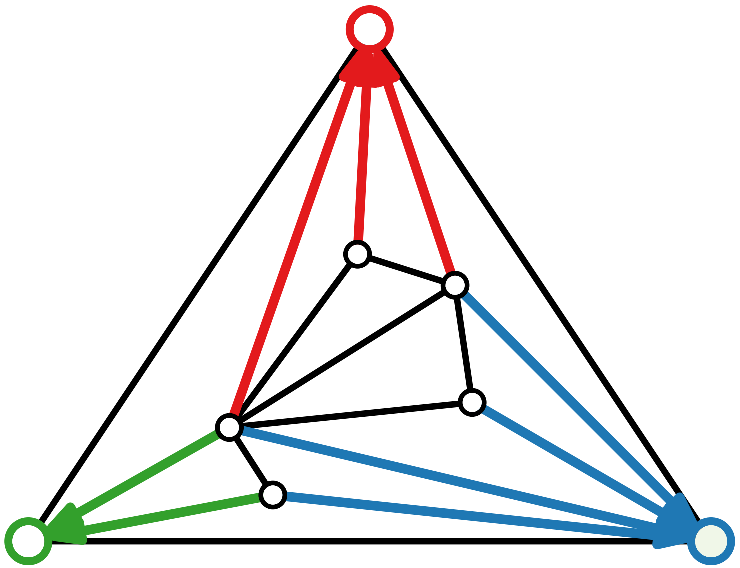


canonical order



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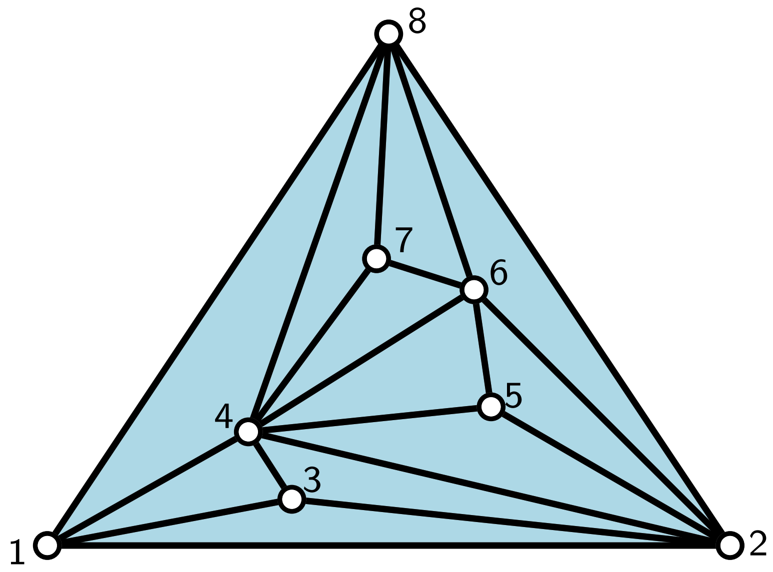

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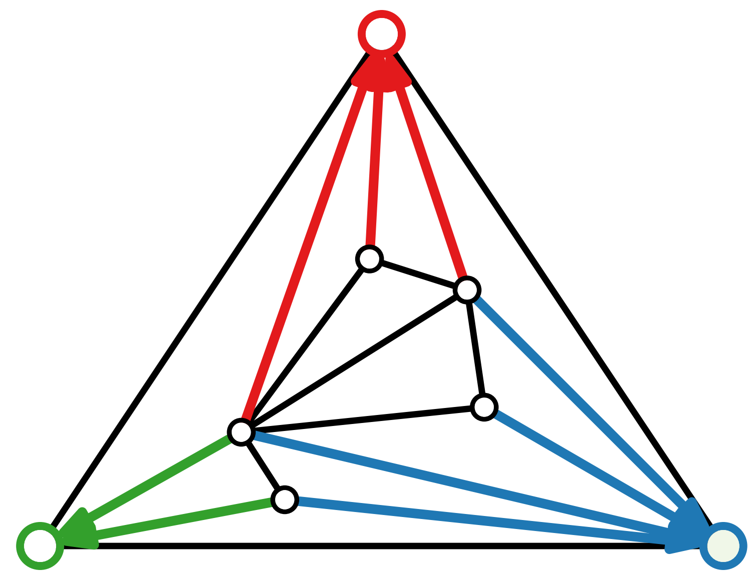

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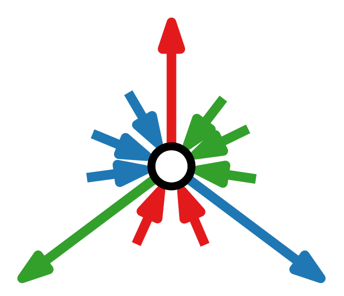

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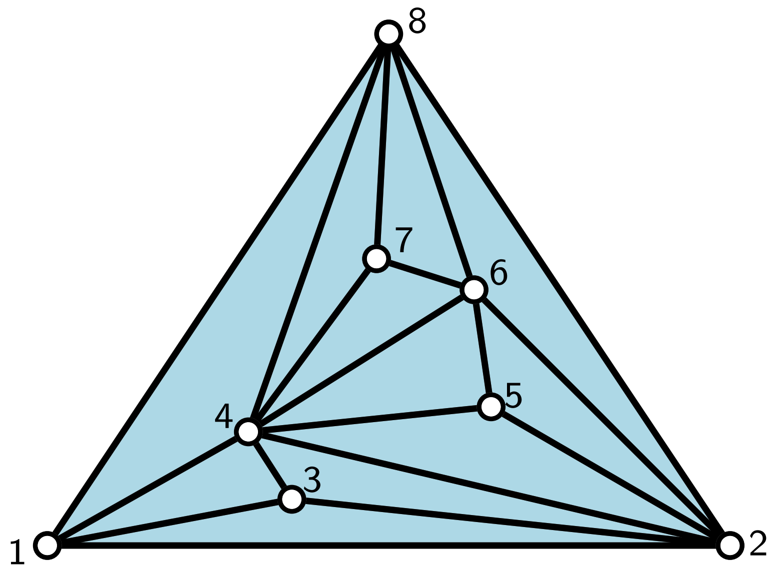
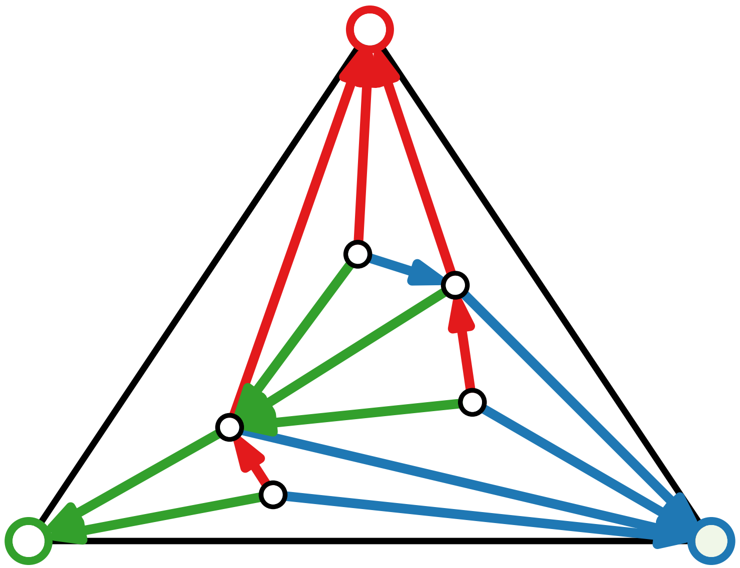


canonical order



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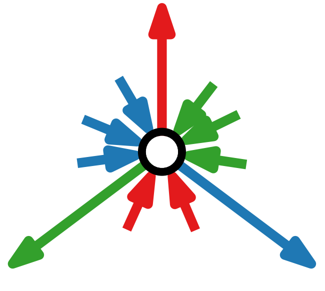

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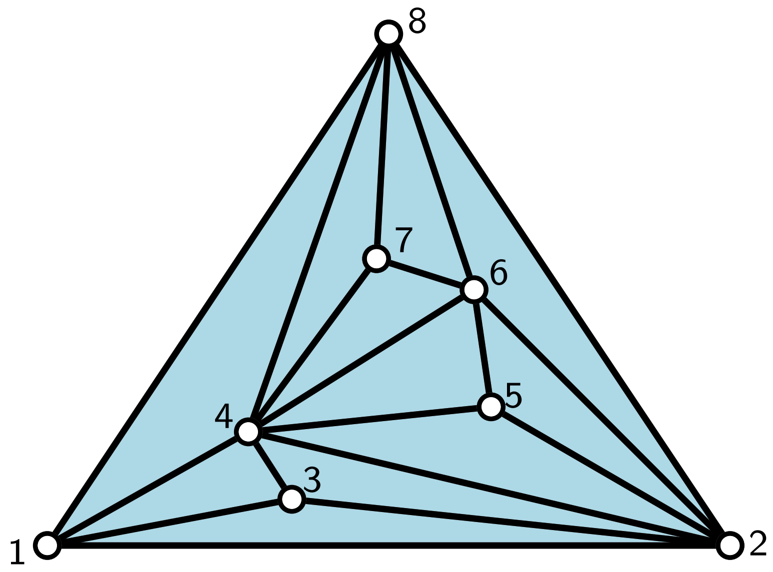

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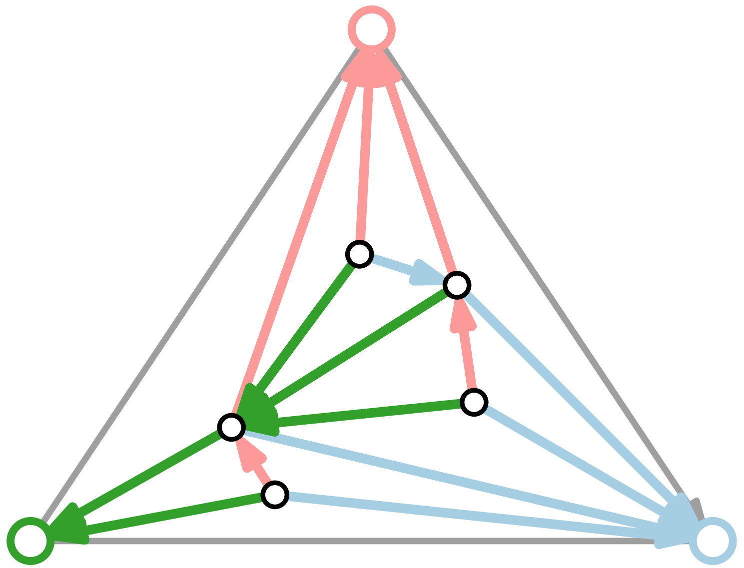
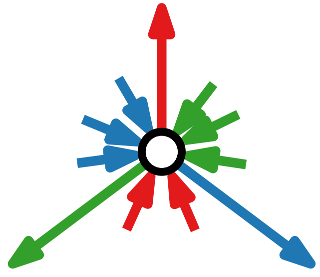


Schnyder realizer



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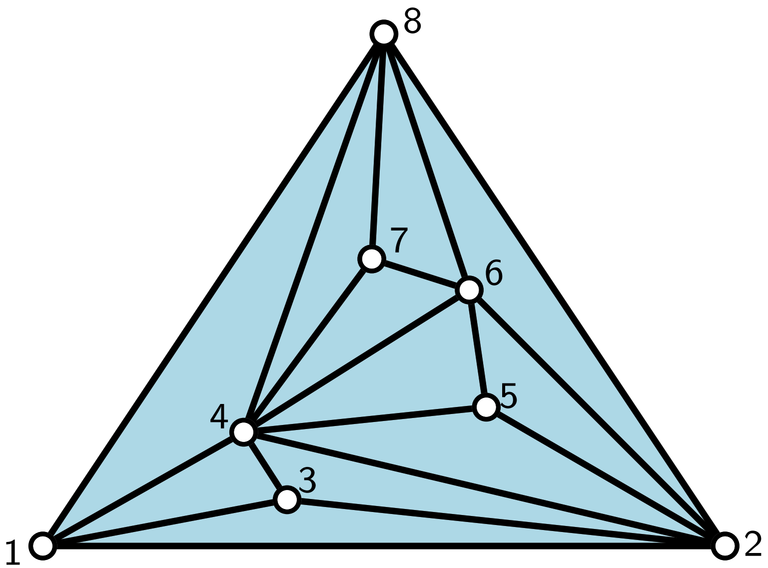

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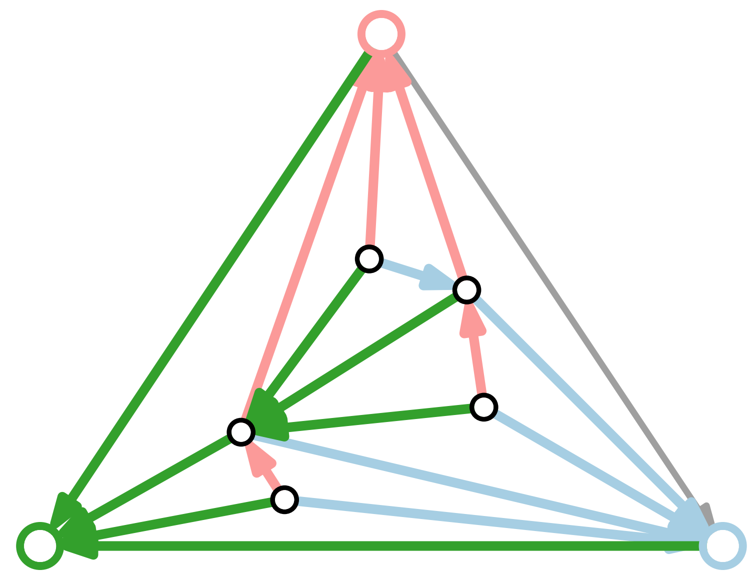

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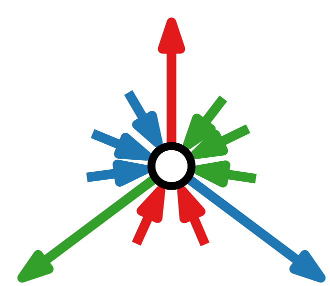
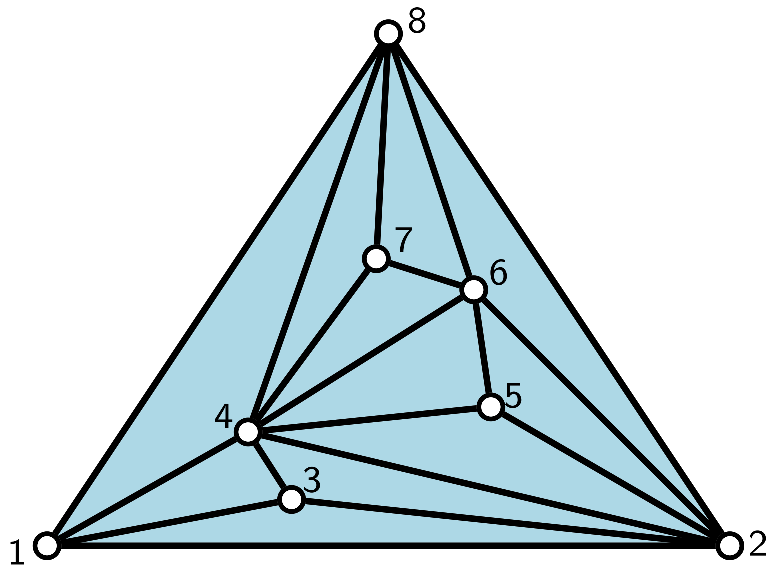


Planar 3-trees on the grid



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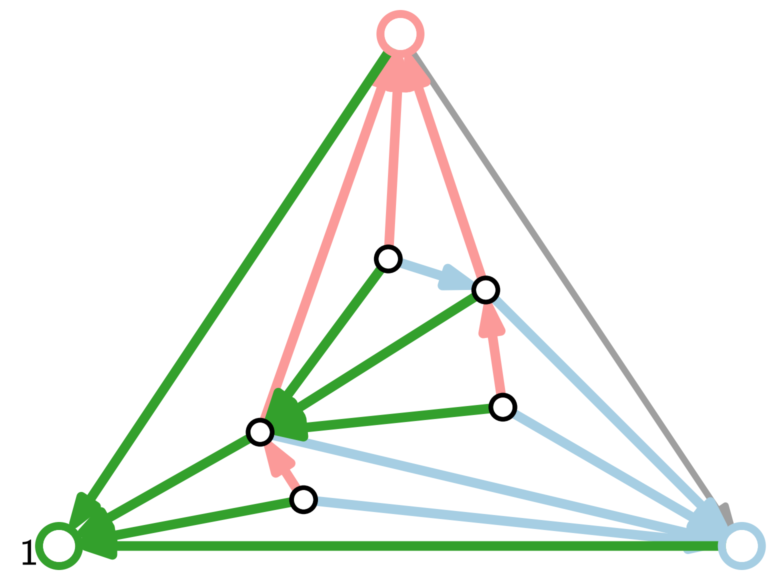
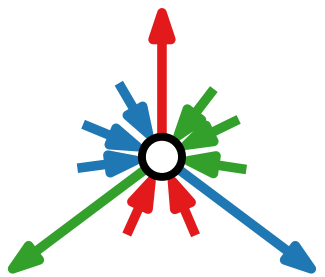


Schnyder realizer



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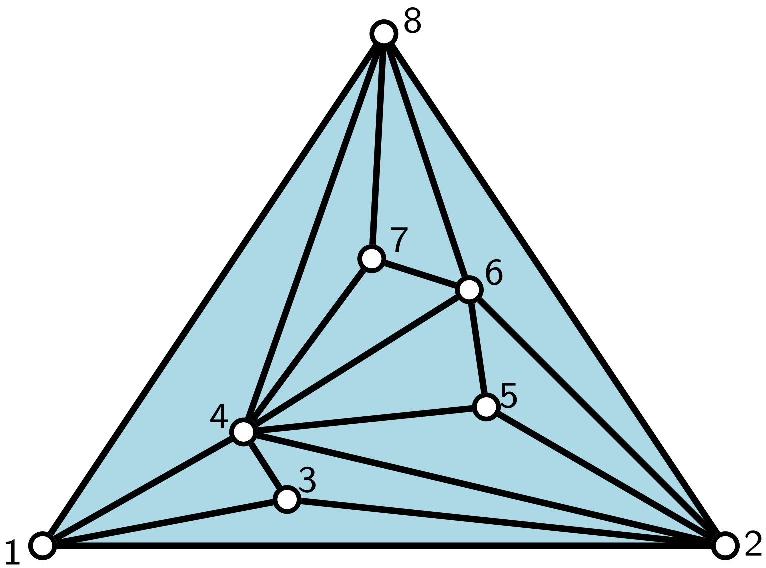

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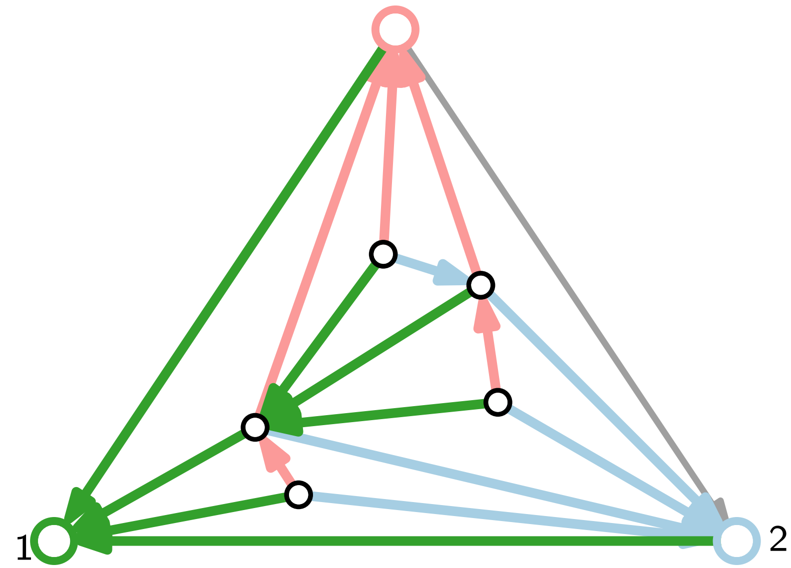
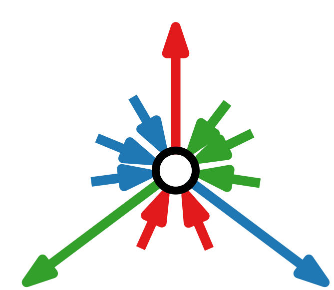


Schnyder realizer



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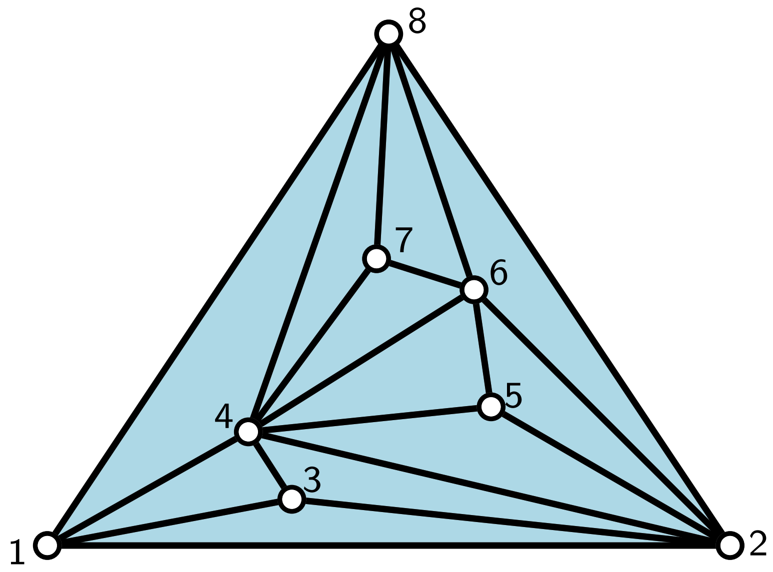

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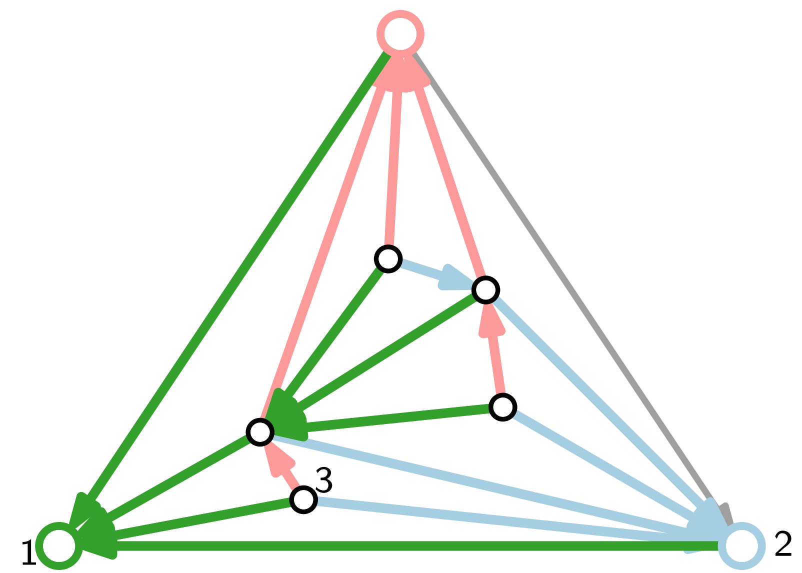
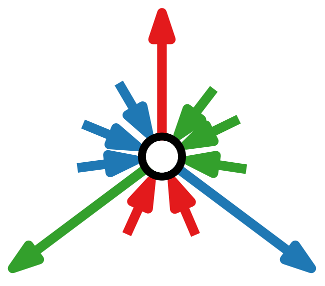
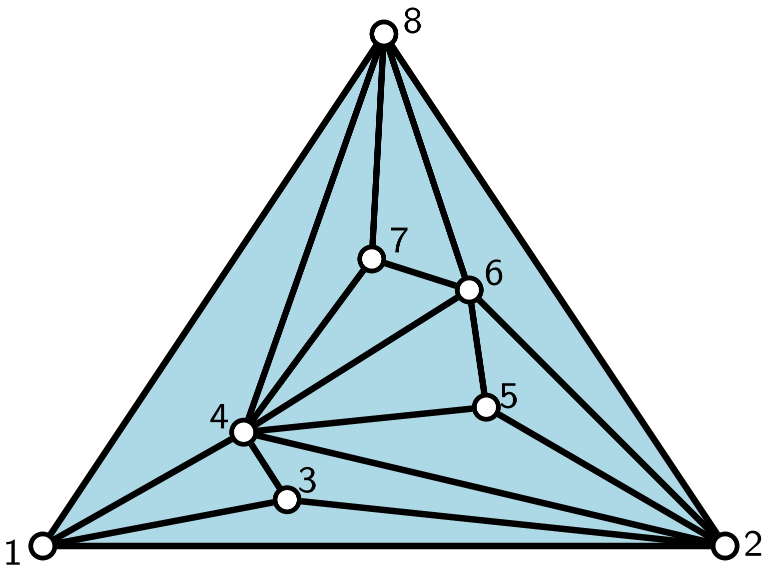


Schnyder realizer



Planar 3-trees on the grid



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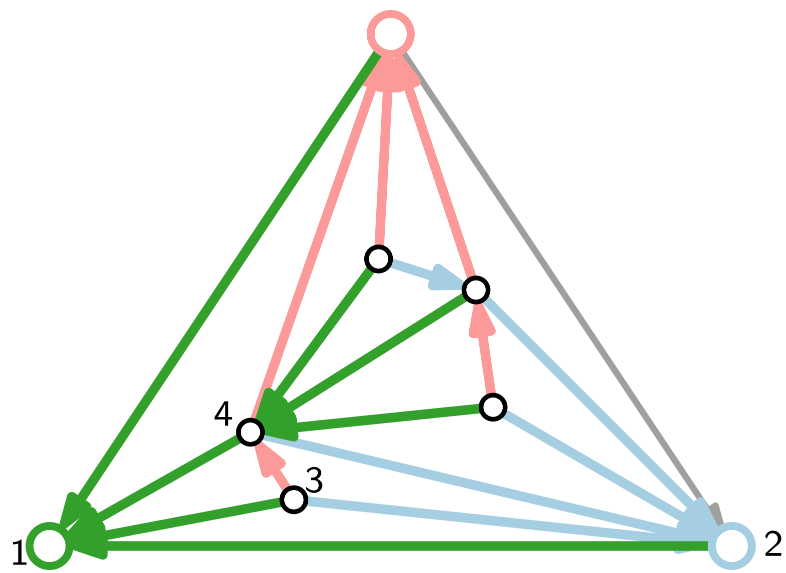
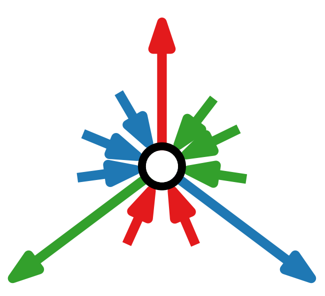


Schnyder realizer



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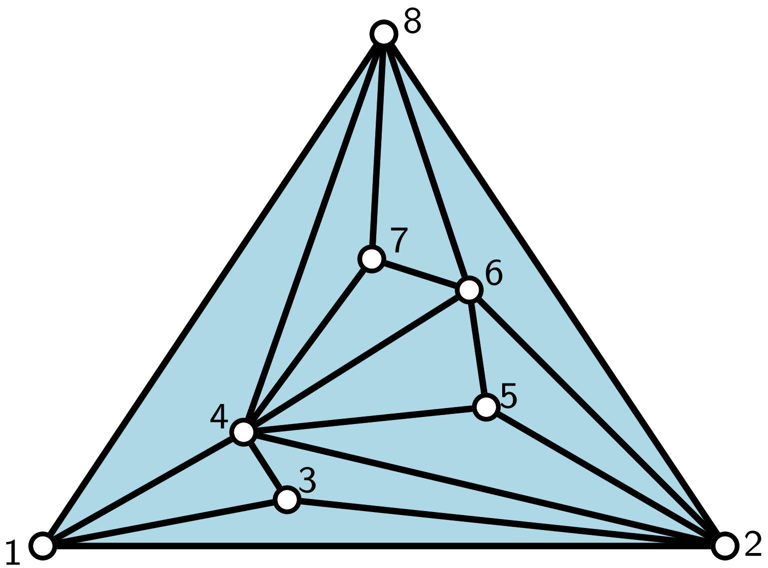
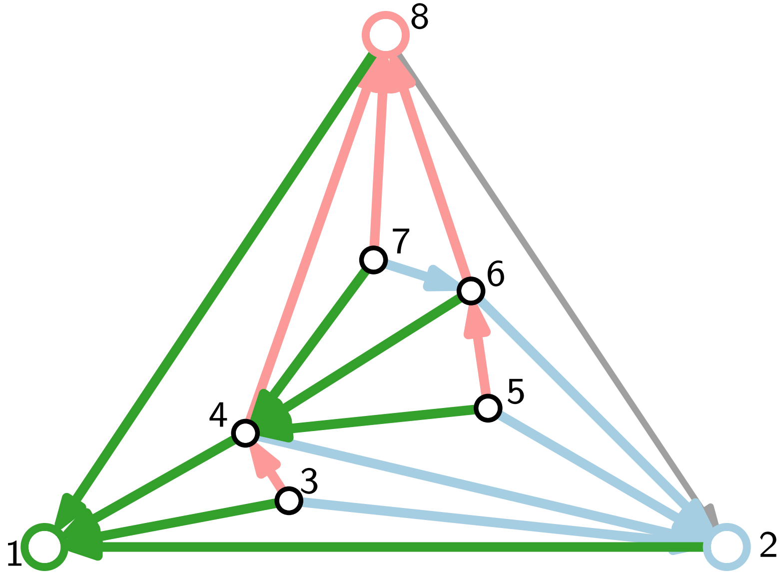
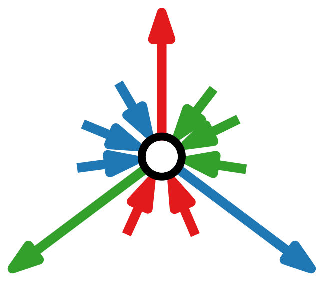


canonical order



Schnyder realizer



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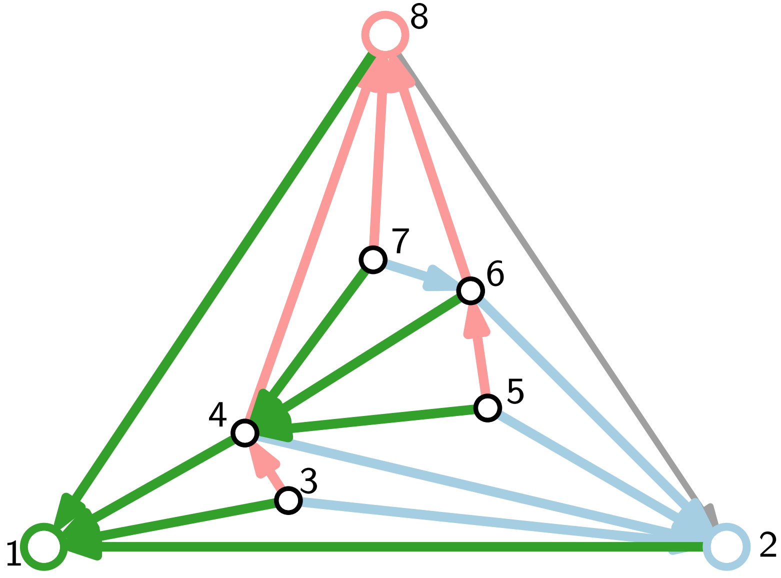

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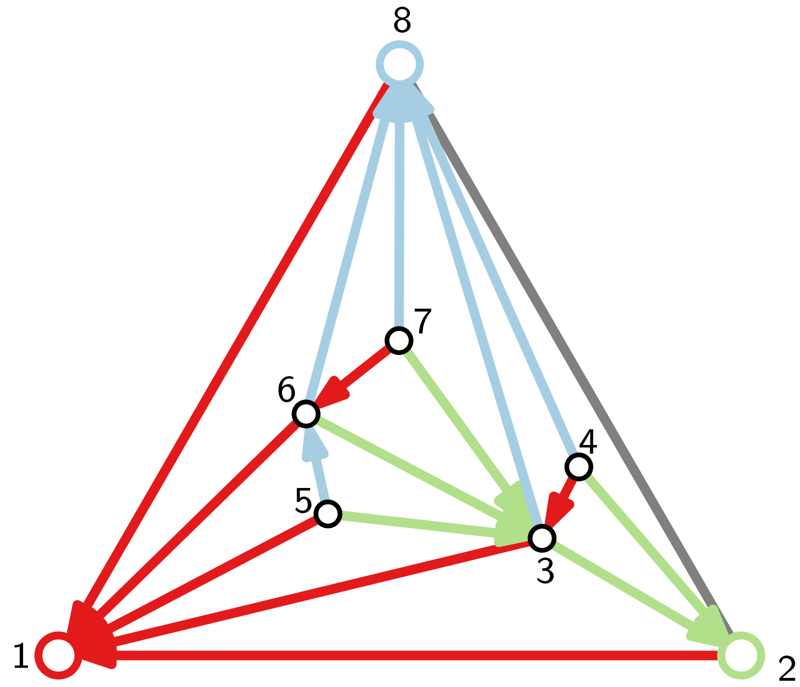

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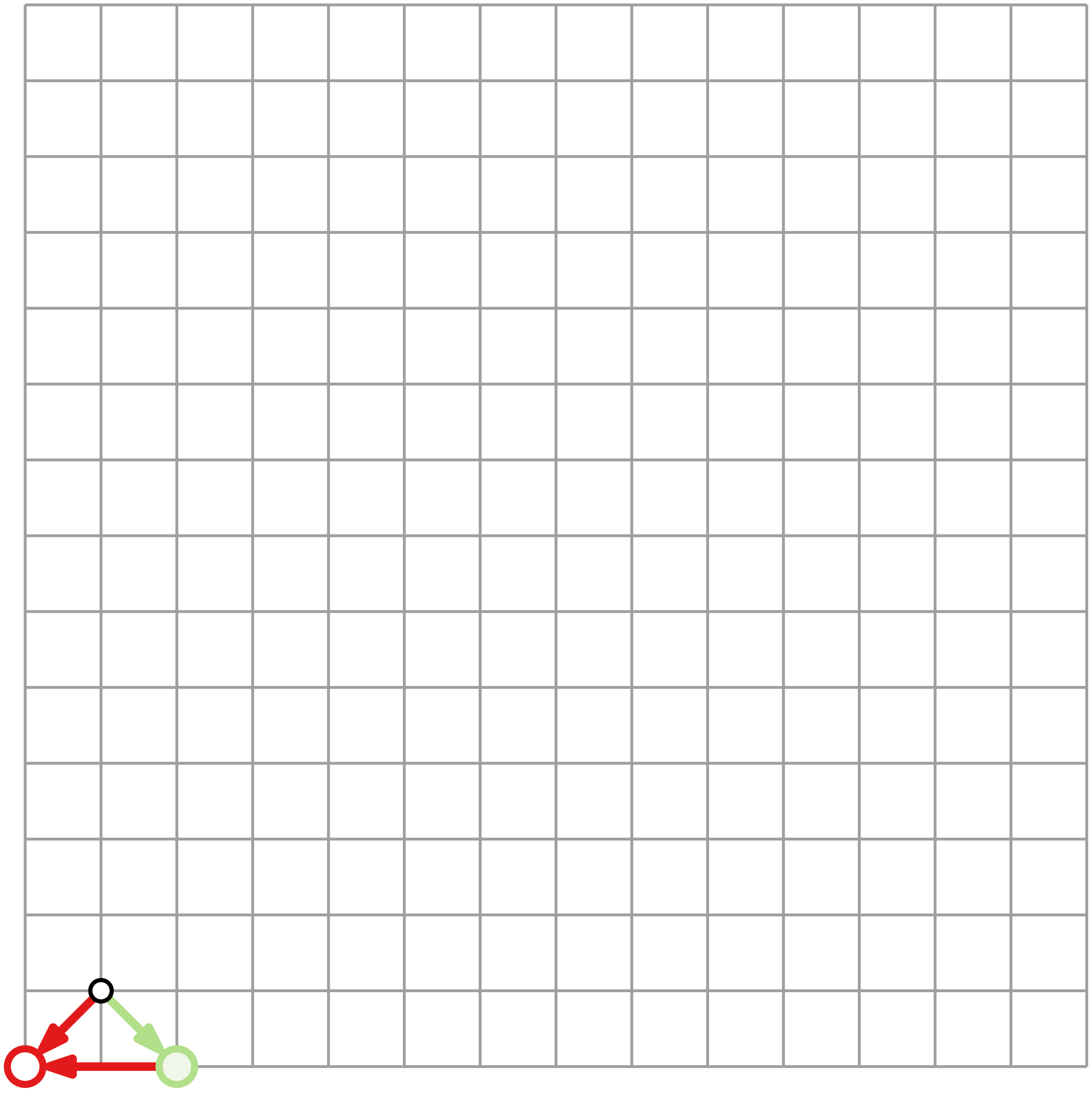

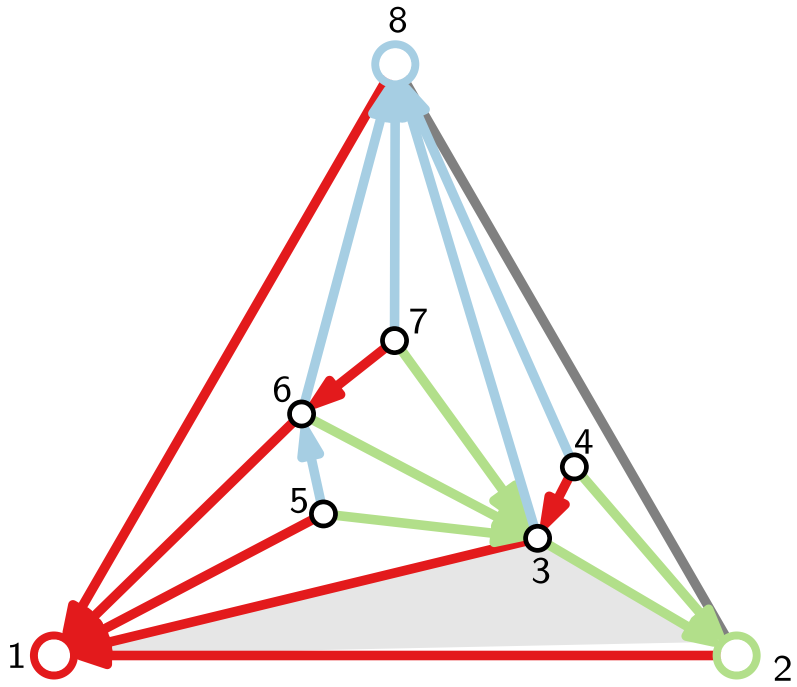

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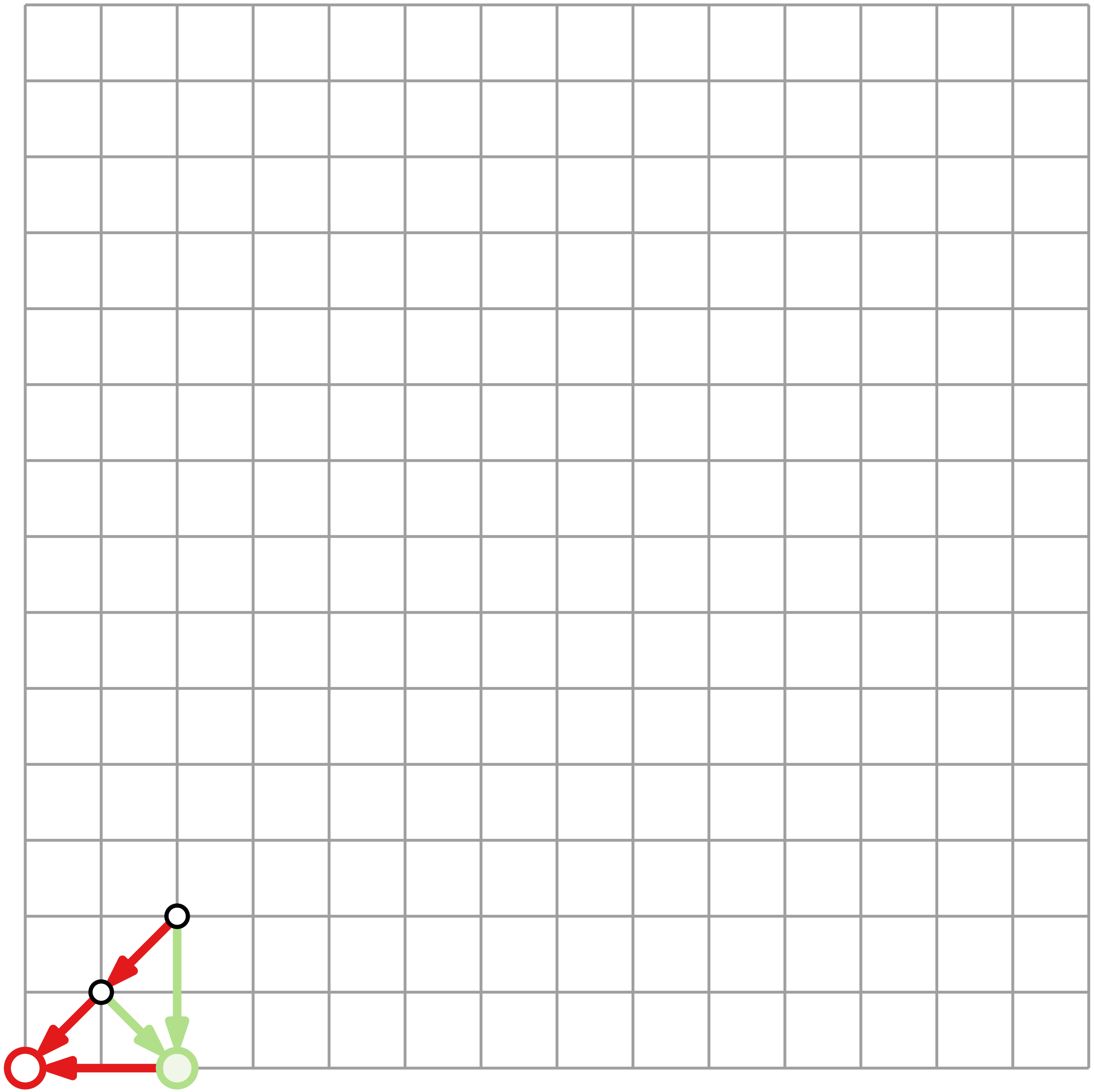

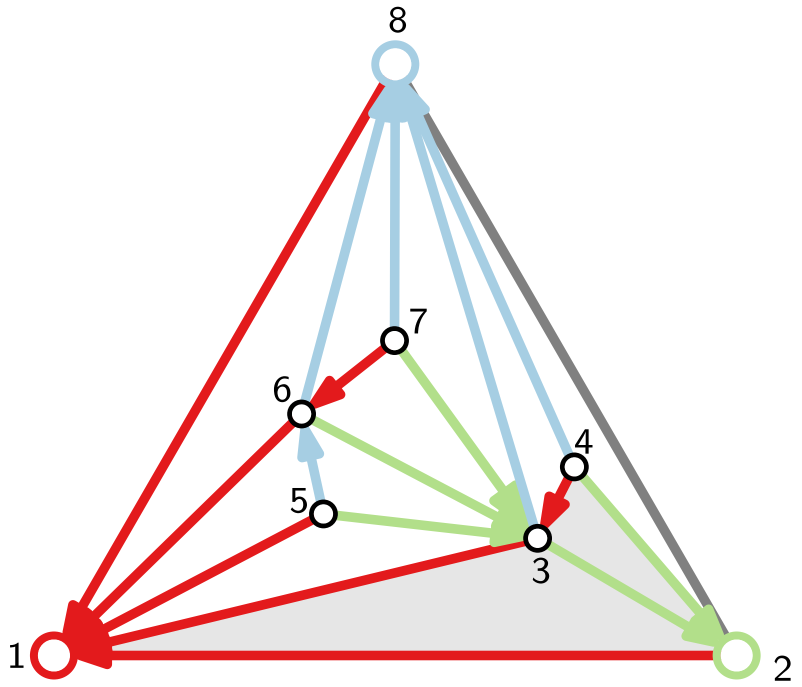

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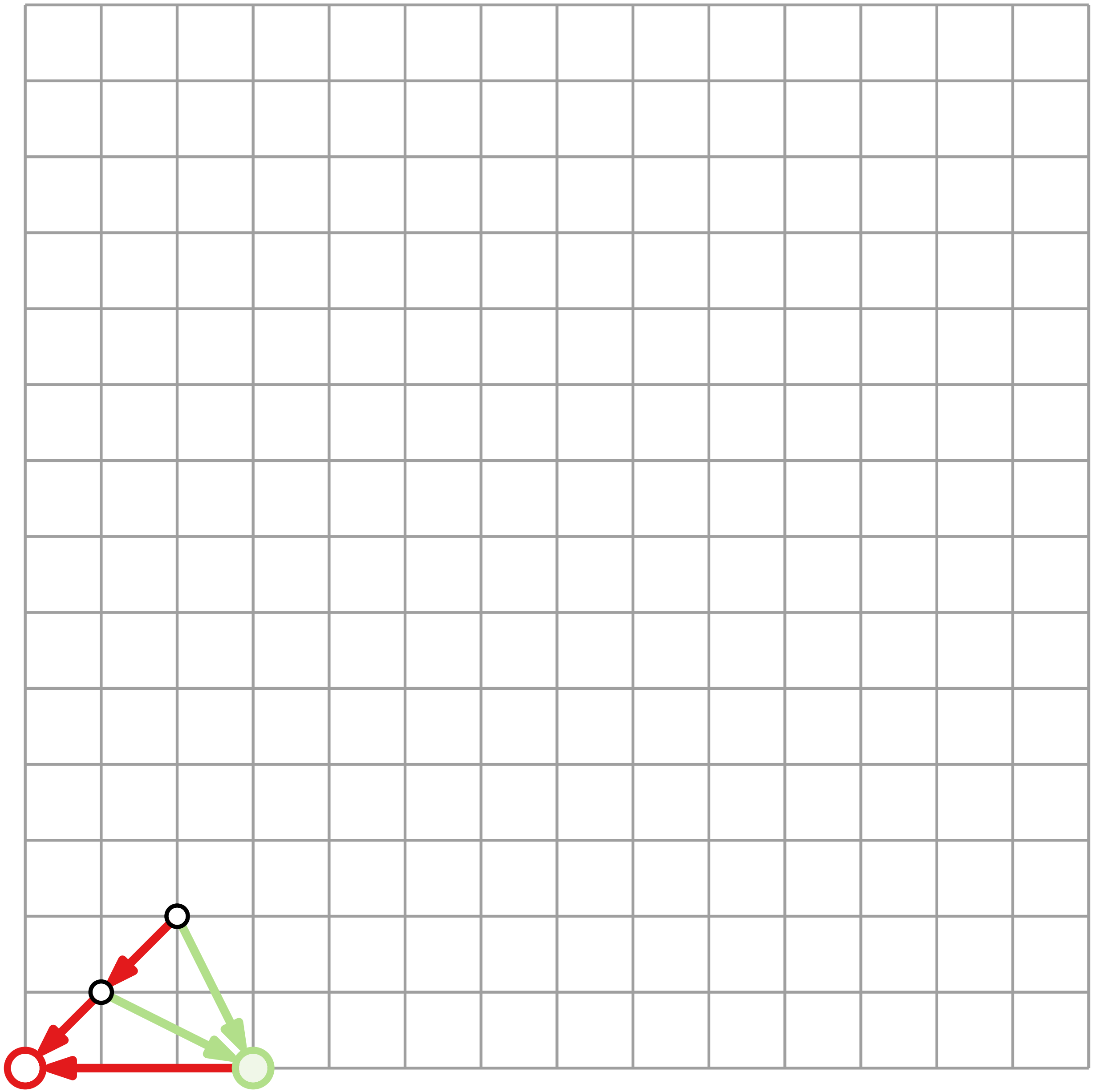

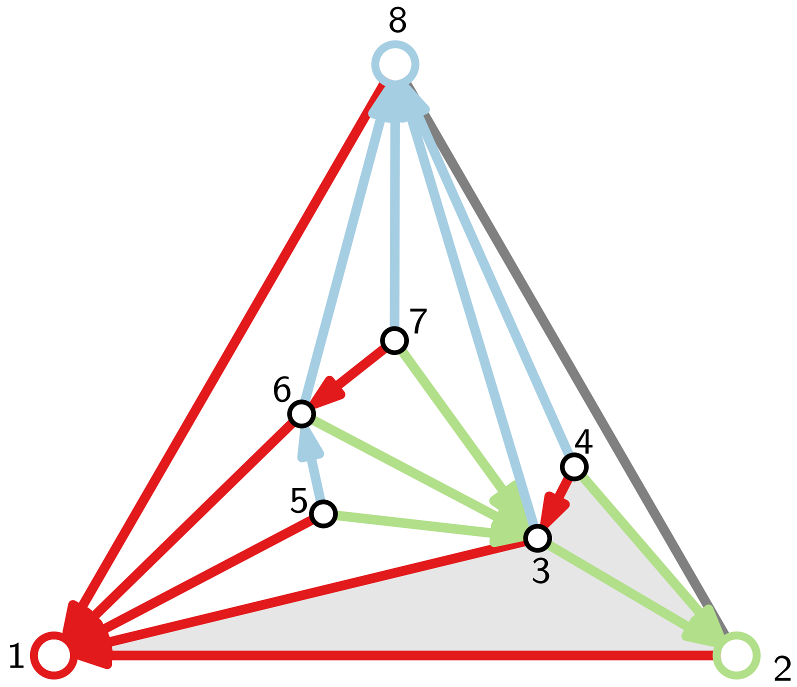

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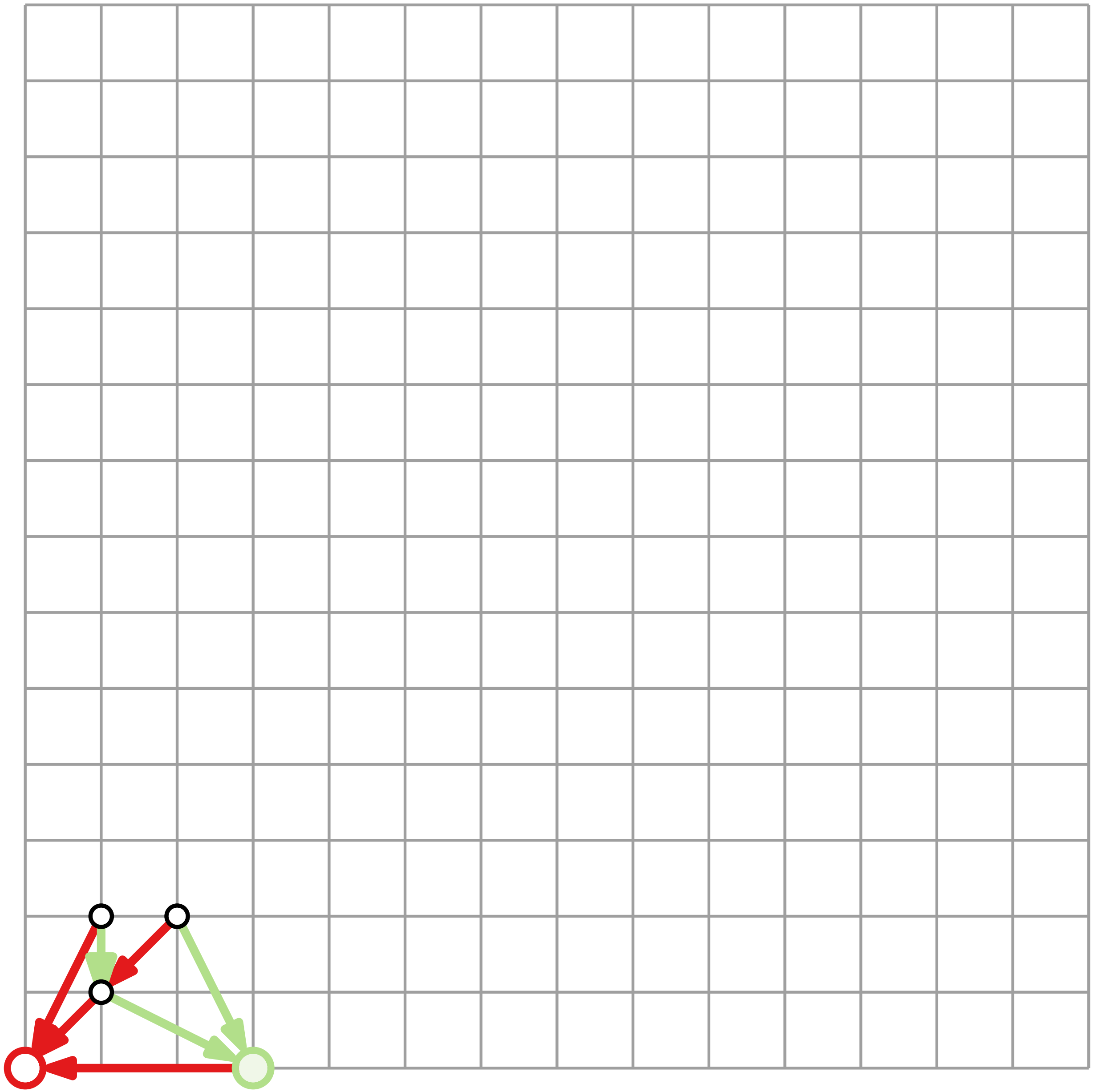
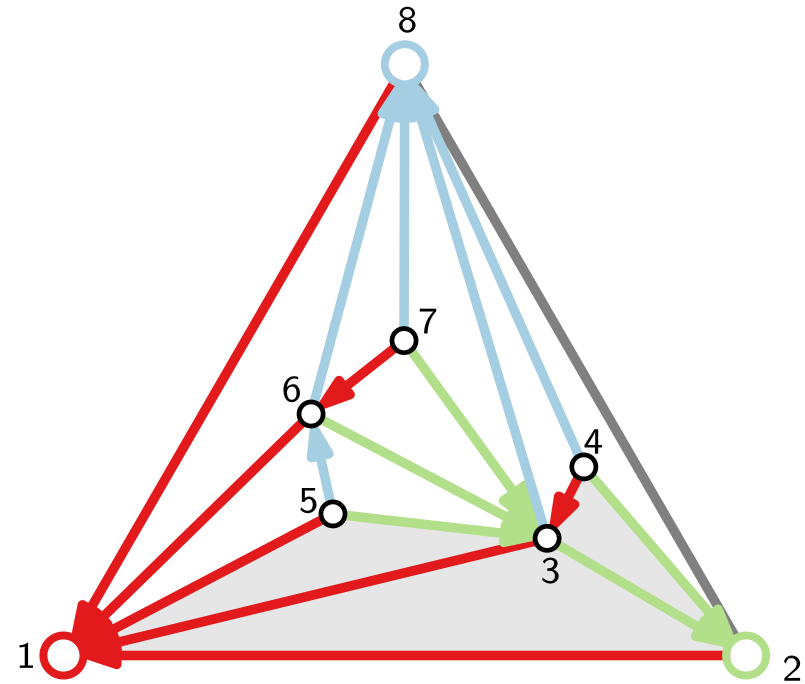
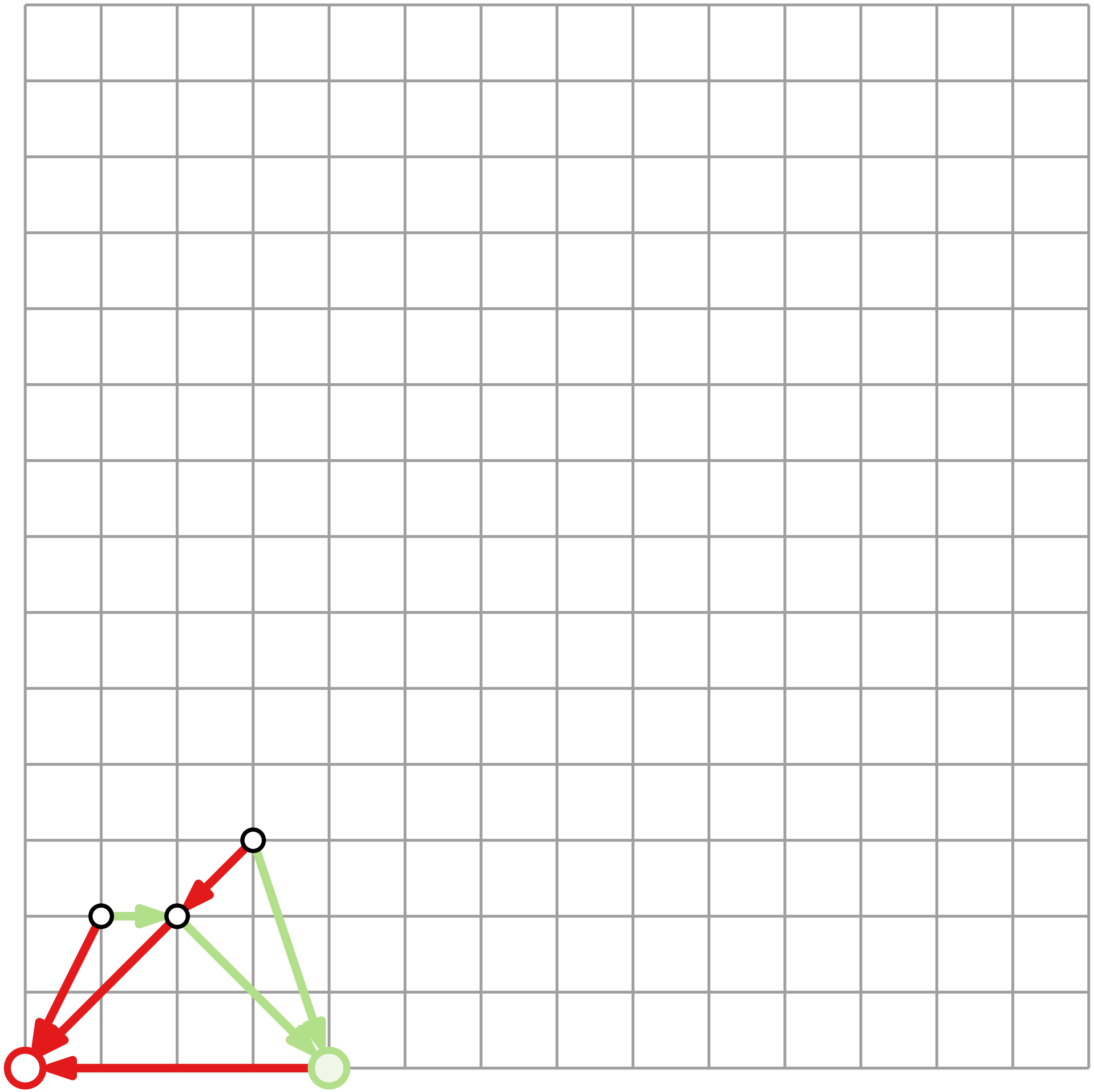
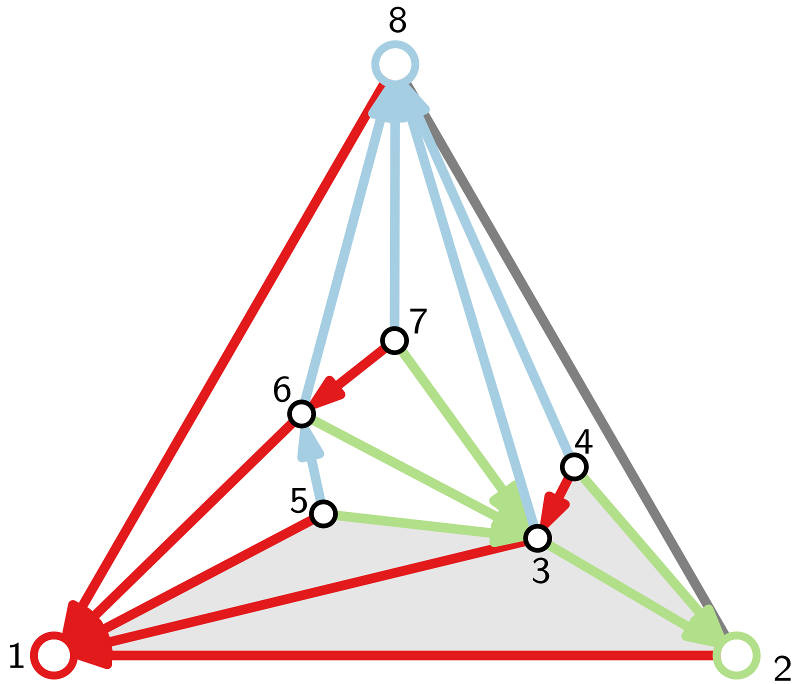


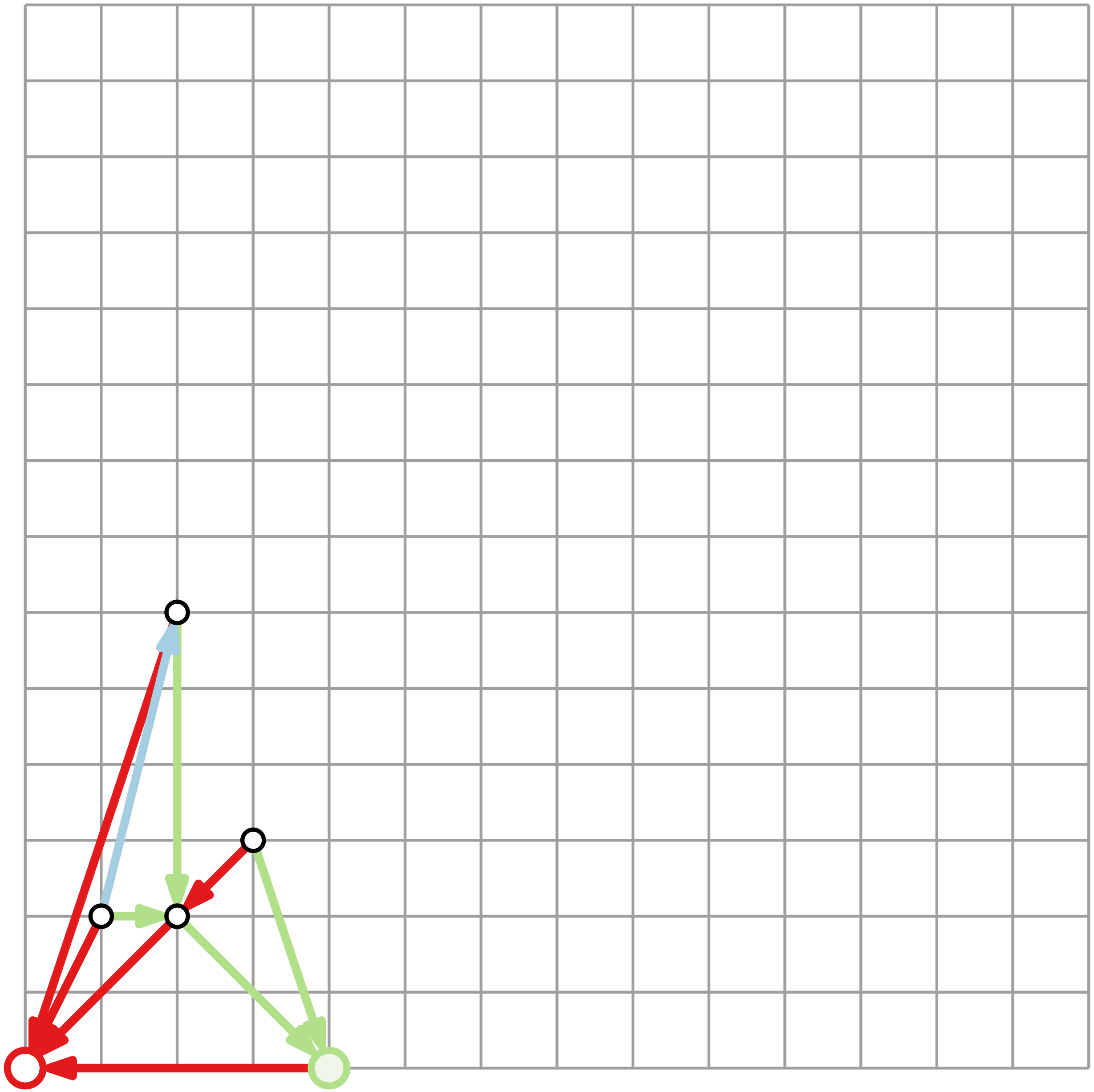


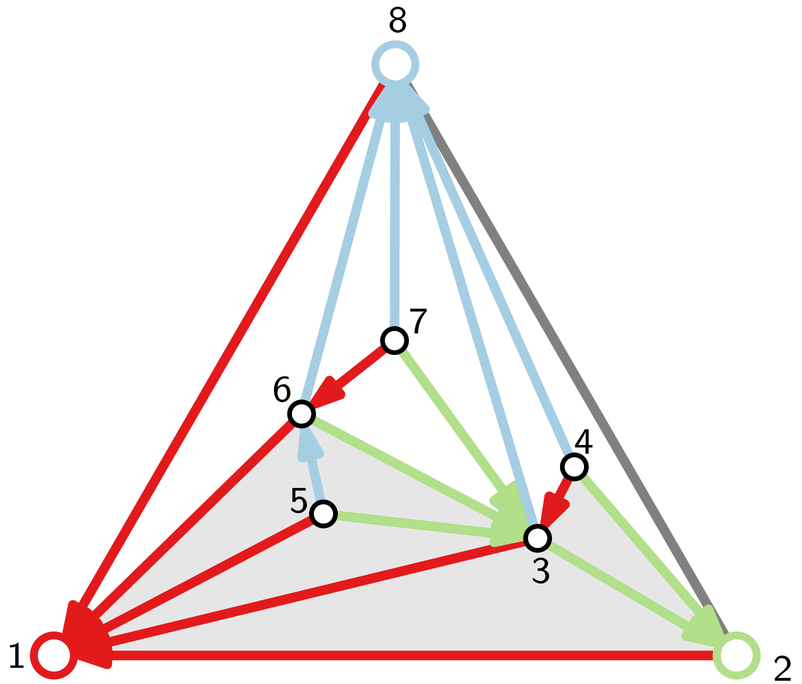
Planar 3-trees on the grid



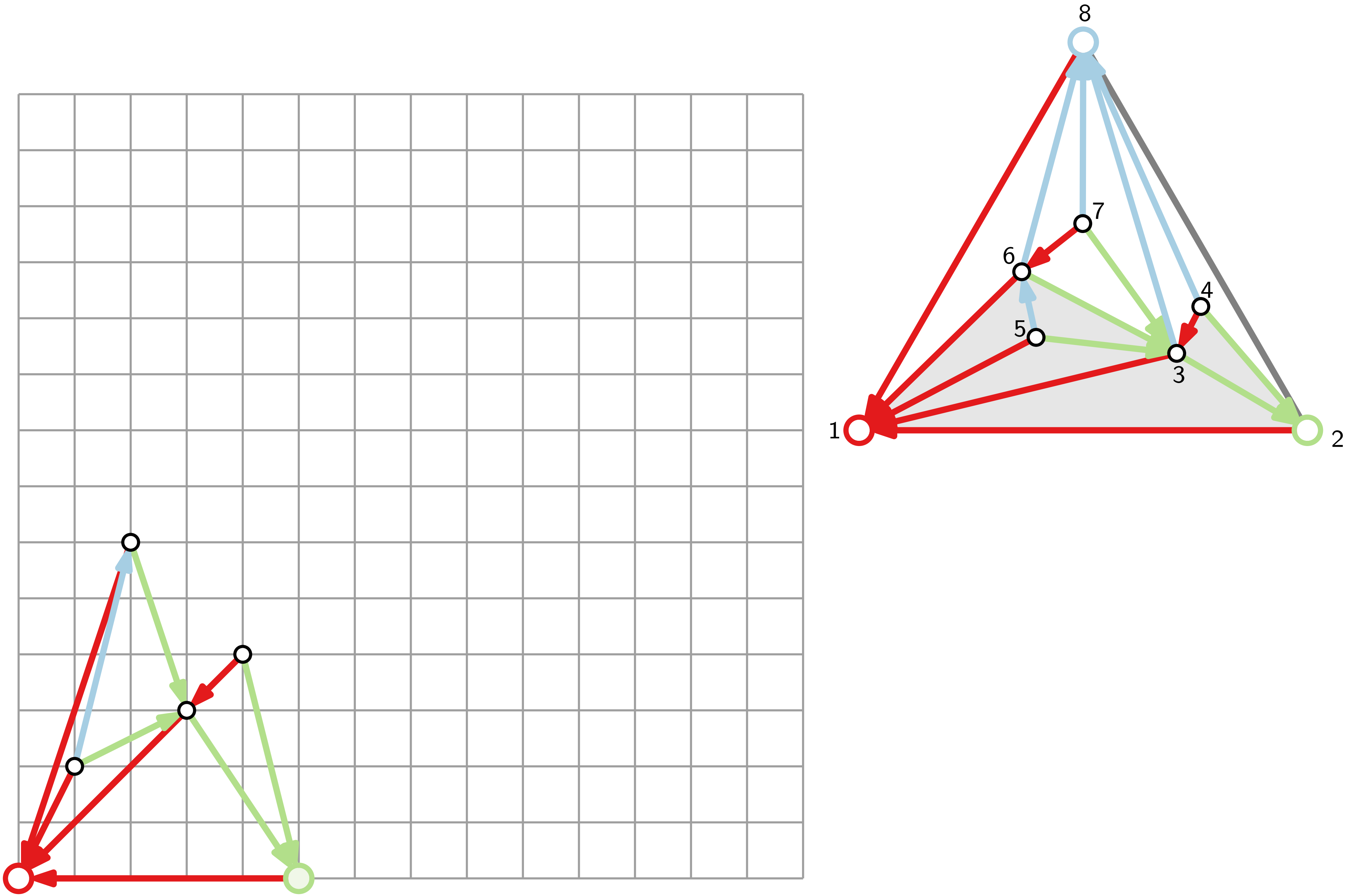


Planar 3-trees on the grid

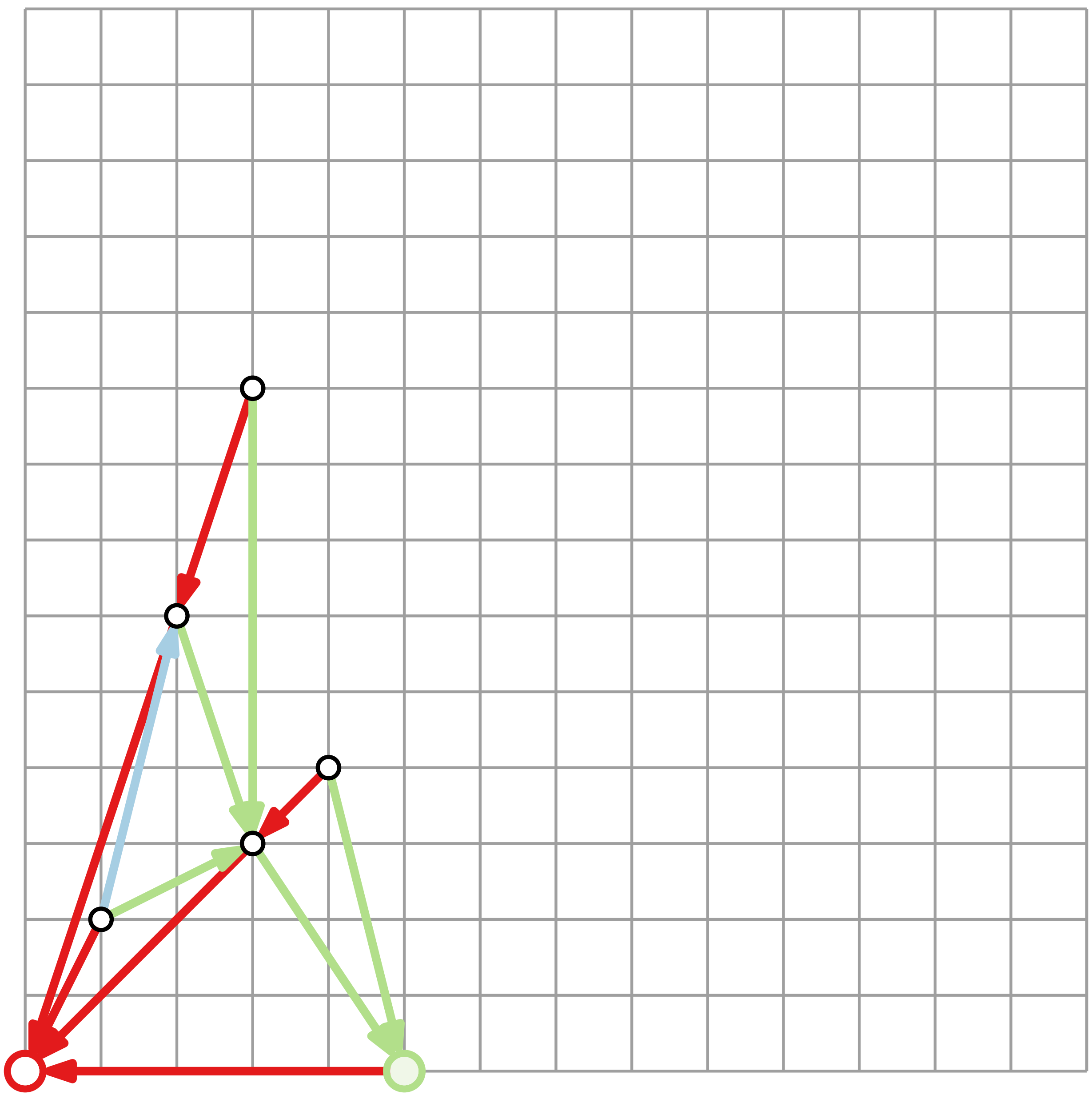


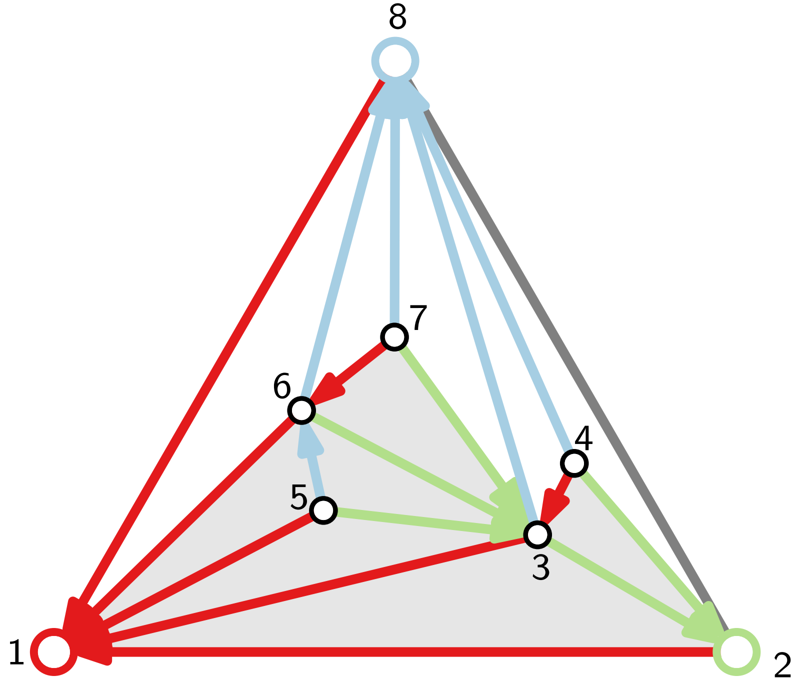


Planar 3-trees on the grid

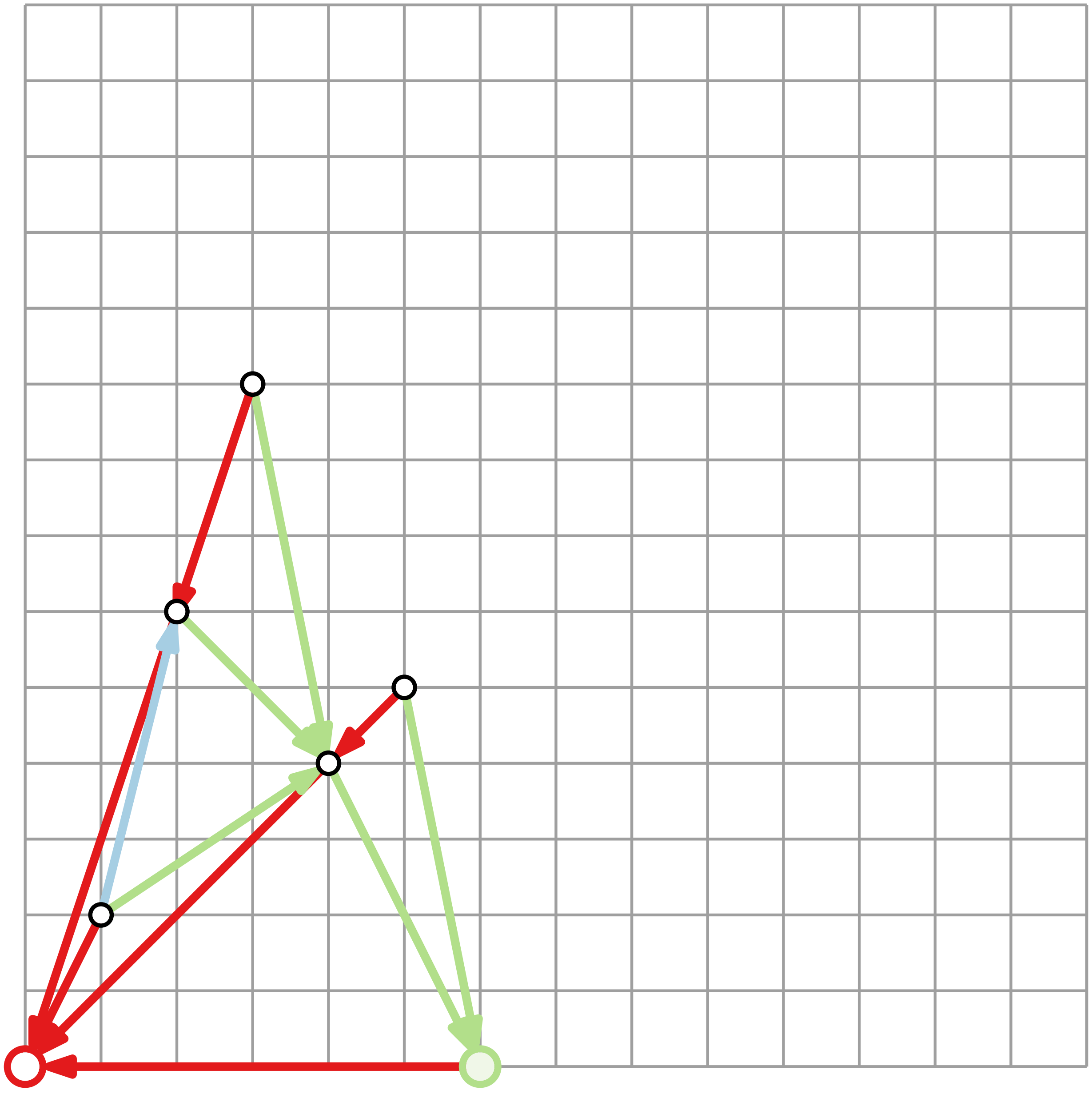


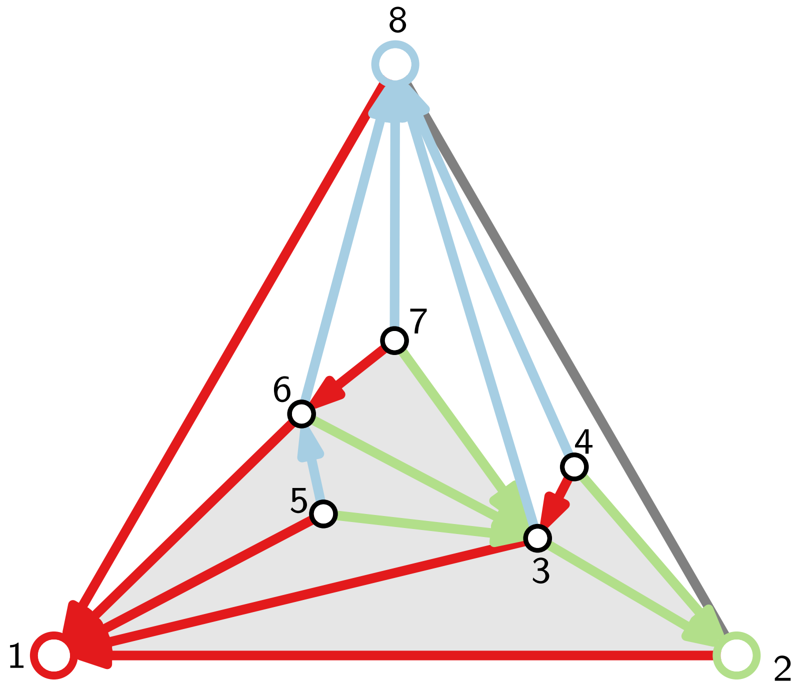
Planar 3-trees on the grid



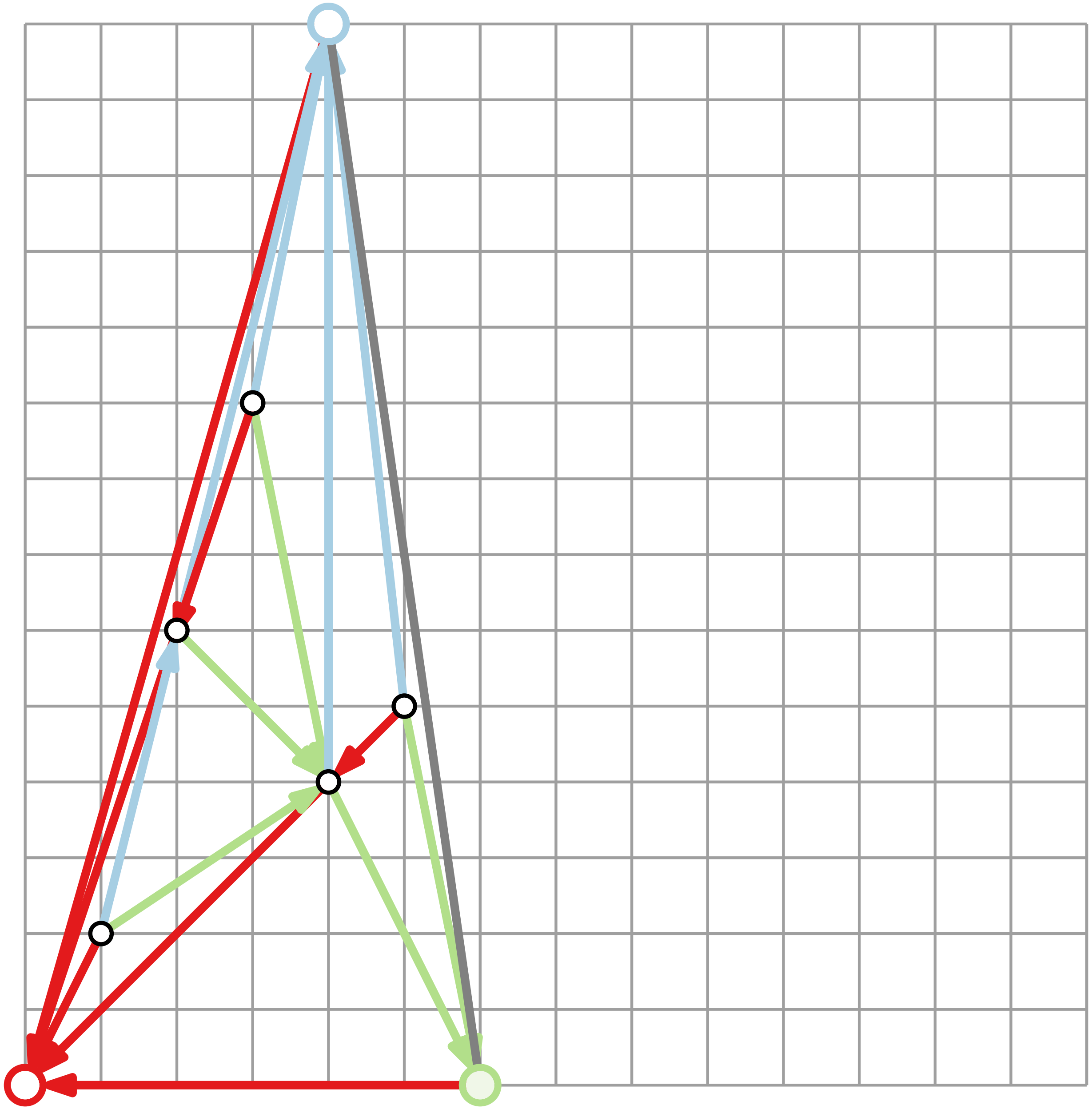


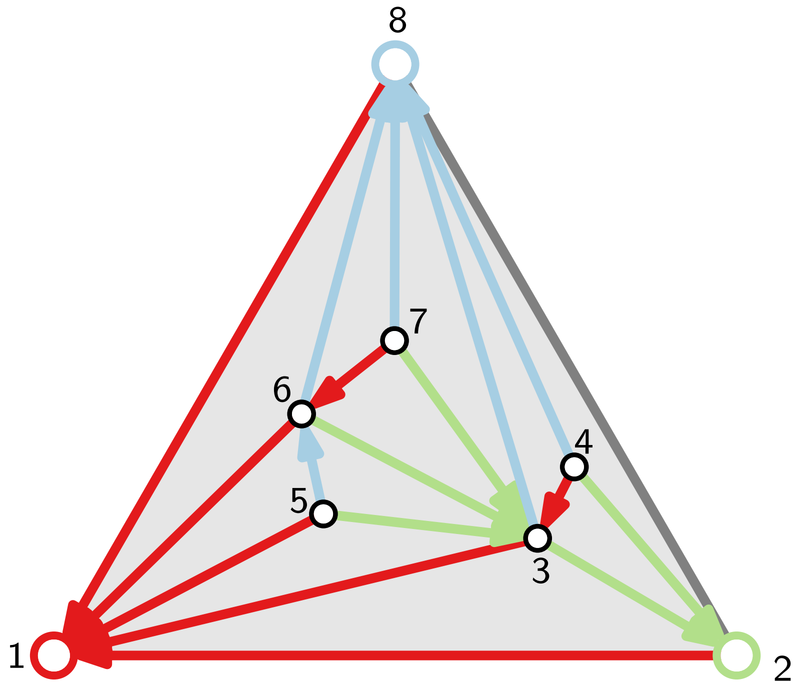
Planar 3-trees on the grid



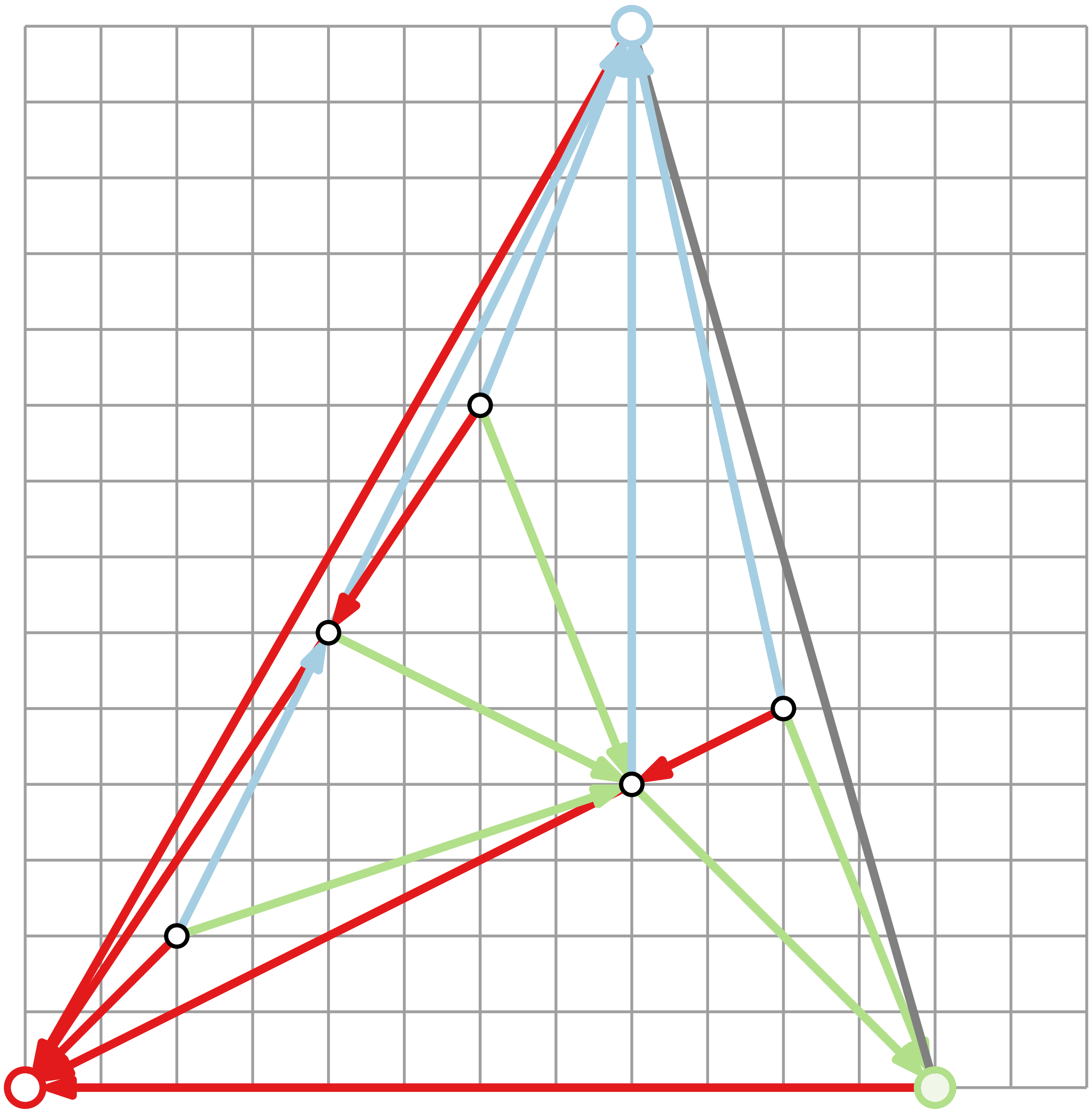


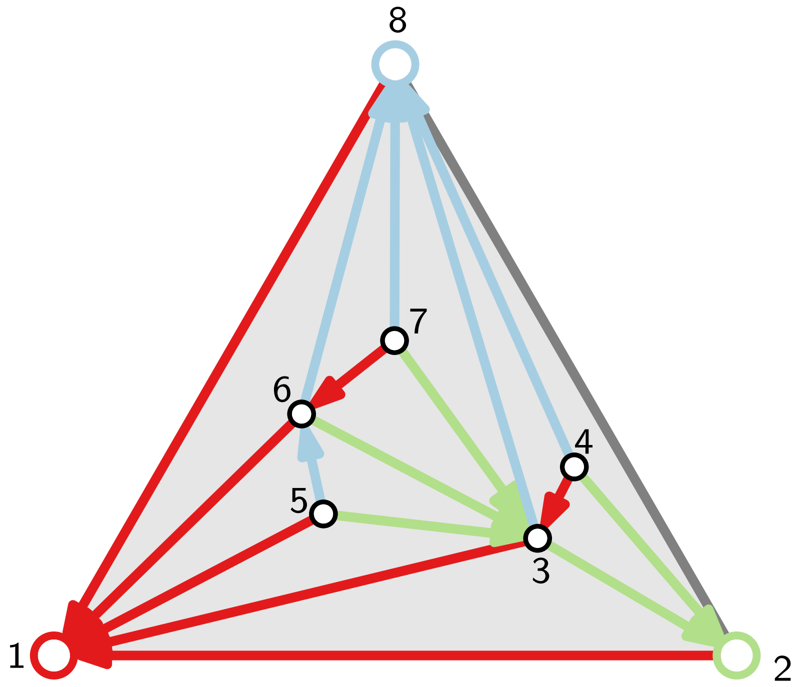
Planar 3-trees on the grid





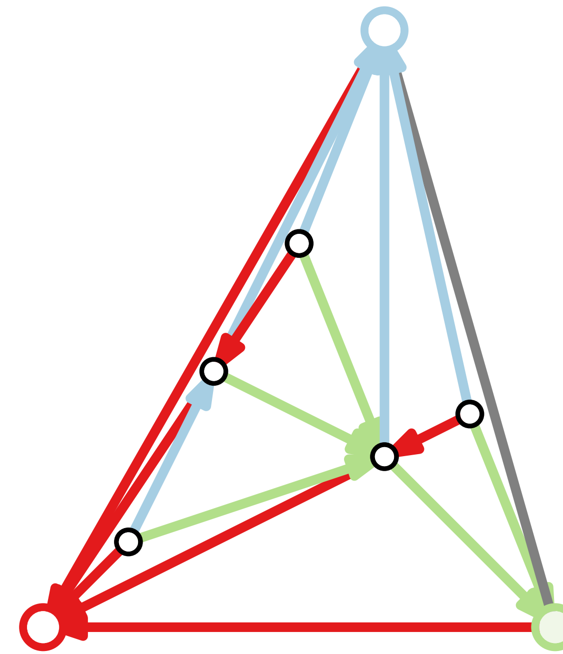
Planar 3-trees on the grid





Planar 3-trees on the grid

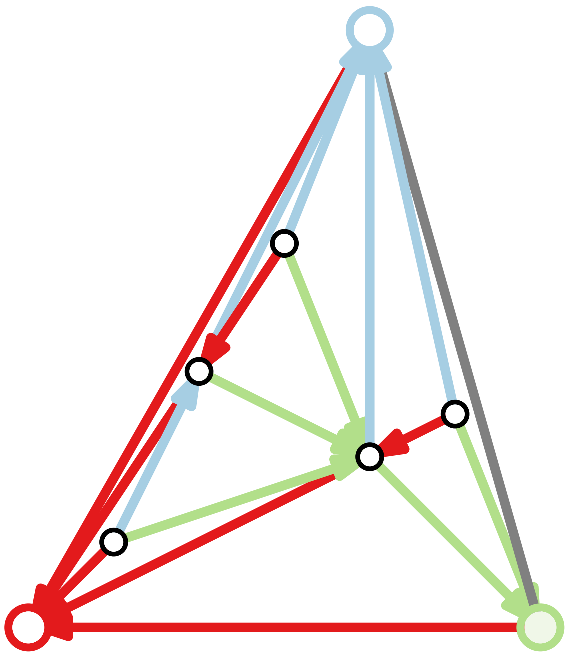
Bonichon et al. [ICALP'02]:
A min. Schnyder realizer has at most $2n-5$ leaves in total



Planar 3-trees on the grid

Bonichon et al. [ICALP'02]:
A min. Schnyder realizer has at most $2n - 5$ leaves in total

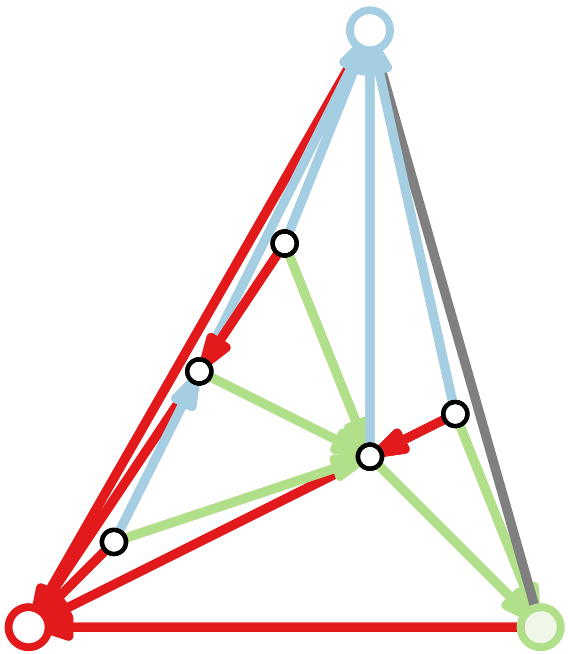
$\Rightarrow$ Red tree has at most $(2n-5)/3$ segments



Planar 3-trees on the grid

Bonichon et al. [ICALP'02]:
A min. Schnyder realizer has at most $2n-5$ leaves in total

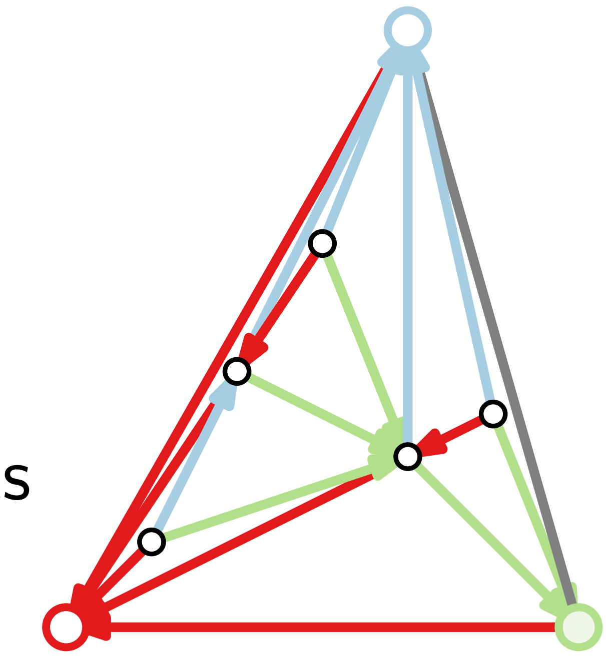
$\Rightarrow$ Red tree has at most $(2n-5)/3$ segments
Blue tree has at most $n-2$ segments



Planar 3-trees on the grid

Bonichon et al. [ICALP'02]:
A min. Schnyder realizer has at most $2n-5$ leaves in total

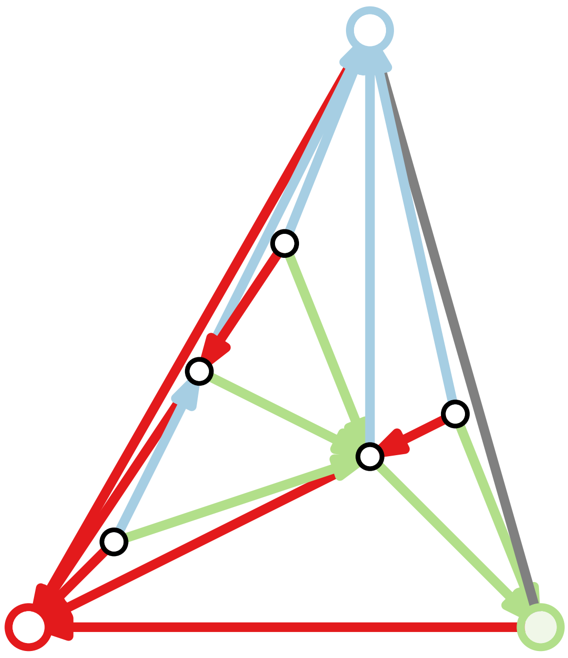
$\Rightarrow$ Red tree has at most $(2n-5)/3$ segments
Blue tree has at most $n-2$ segments
Green tree has at most $n-2$ segments



Planar 3-trees on the grid

Bonichon et al. [ICALP'02]:
A min. Schnyder realizer has at most $2n-5$ leaves in total

$\Rightarrow$ Red tree has at most $(2n-5)/3$ segments
Blue tree has at most $n-2$ segments
Green tree has at most $n-2$ segments
$\Rightarrow$ Drawing has at most $(8n-17)/3$ segments



Planar 3-trees on the grid

Bonichon et al. [ICALP'02]:
A min. Schnyder realizer has at most $2n-5$ leaves in total

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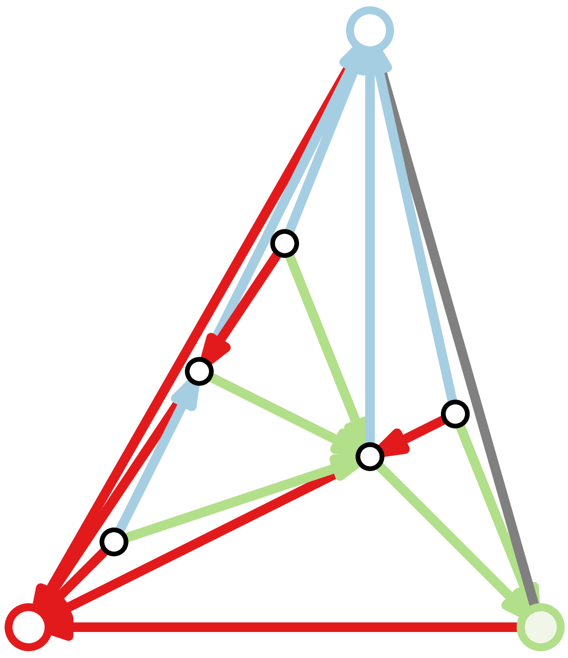
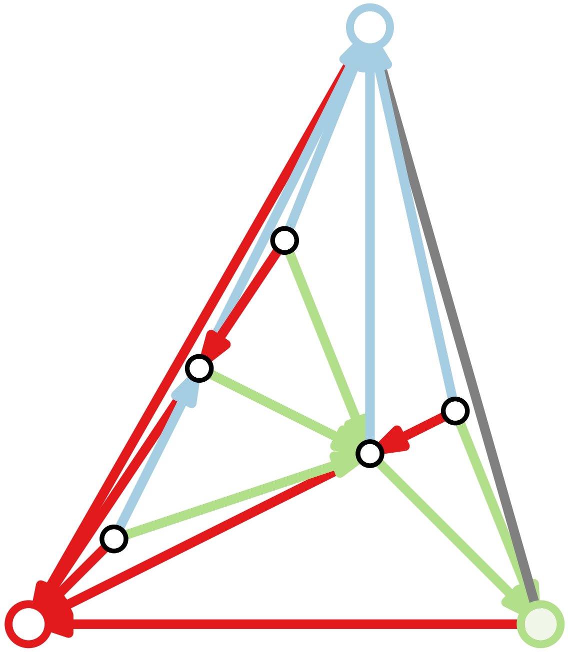


Width: $n-1$

Planar 3-trees on the grid

Bonichon et al. [ICALP'02]:
A min. Schnyder realizer has at most $2n-5$ leaves in total

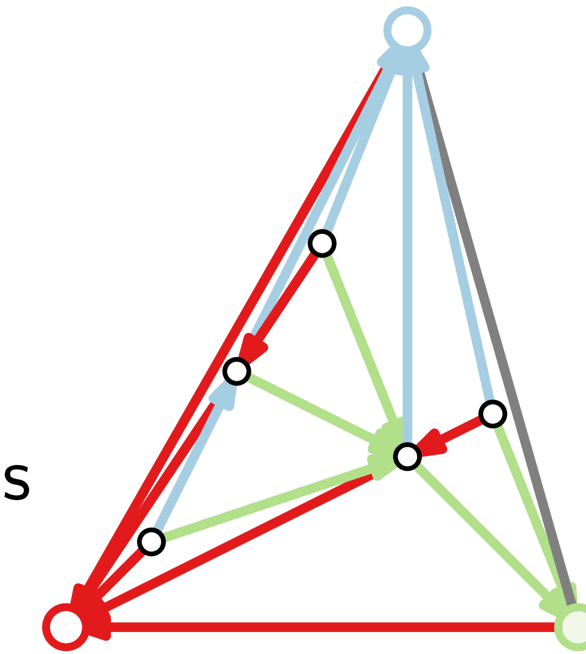


$\Rightarrow$ Red tree has at most $(2n-5)/3$ segments
Blue tree has at most $n-2$ segments
Green tree has at most $n-2$ segments
$\Rightarrow$ Drawing has at most $(8n-17)/3$ segments

Width: $n-1$
Height: $\leq (2n-5)/3 \cdot (n-1)$

Planar 3-trees on the grid

Bonichon et al. [ICALP'02]:
A min. Schnyder realizer has at most $2n-5$ leaves in total

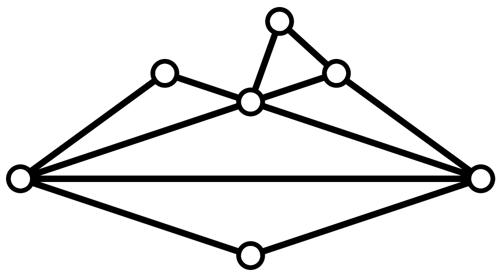


$\Rightarrow$ Red tree has at most $(2n-5)/3$ segments
Blue tree has at most $n-2$ segments
Green tree has at most $n-2$ segments
$\Rightarrow$ Drawing has at most $(8n-17)/3$ segments

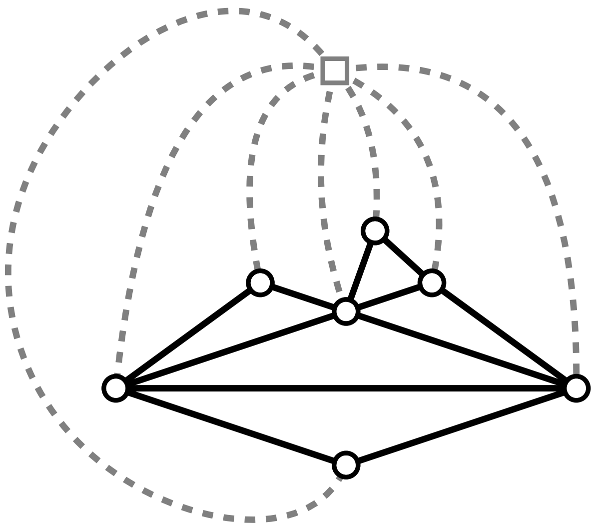
Width: $n-1$
Height: $\leq (2n-5)/3 \cdot (n-1)$

Planar 3-tree $\rightarrow$ $(8n-17)/3$ segments, $O(n) \times O(n^2)$ grid

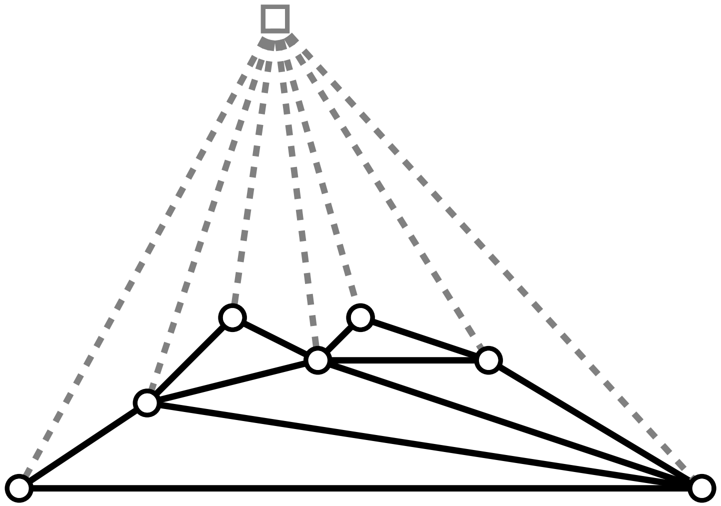
Max. outerplanar graphs on the grid



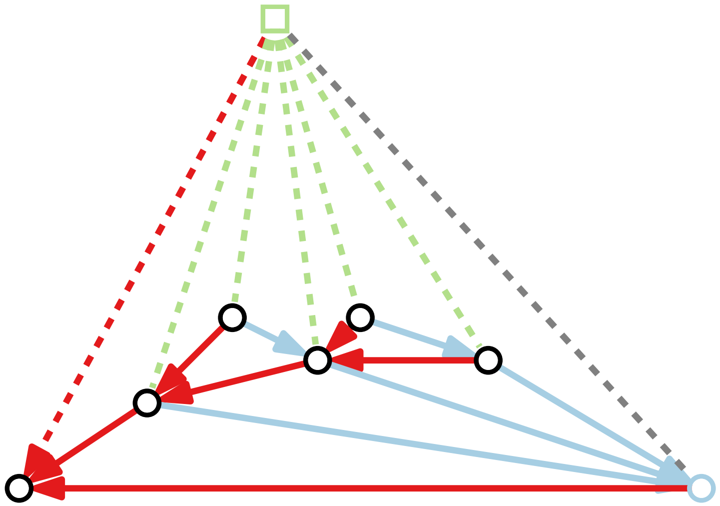
Max. outerplanar graphs on the grid



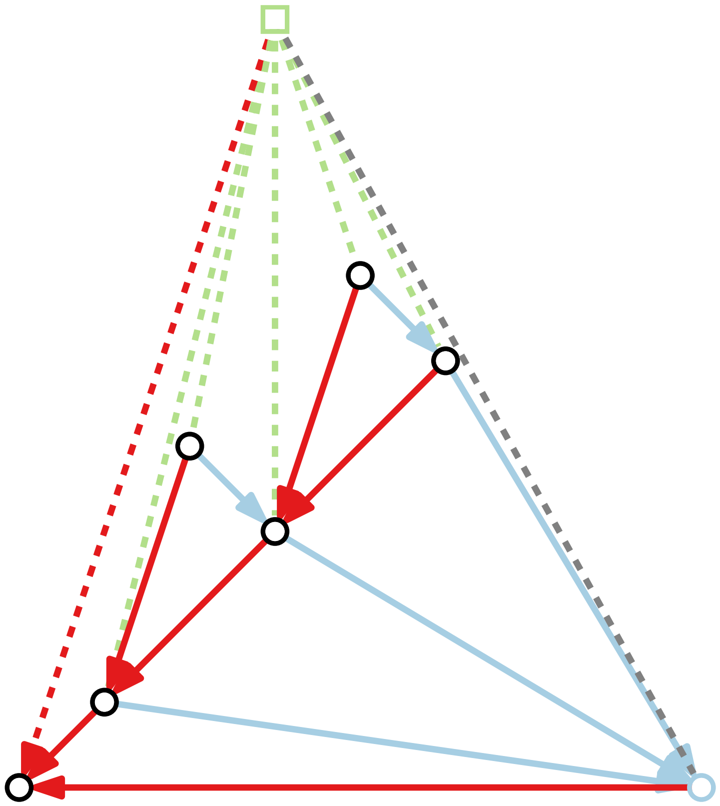
Max. outerplanar graphs on the grid



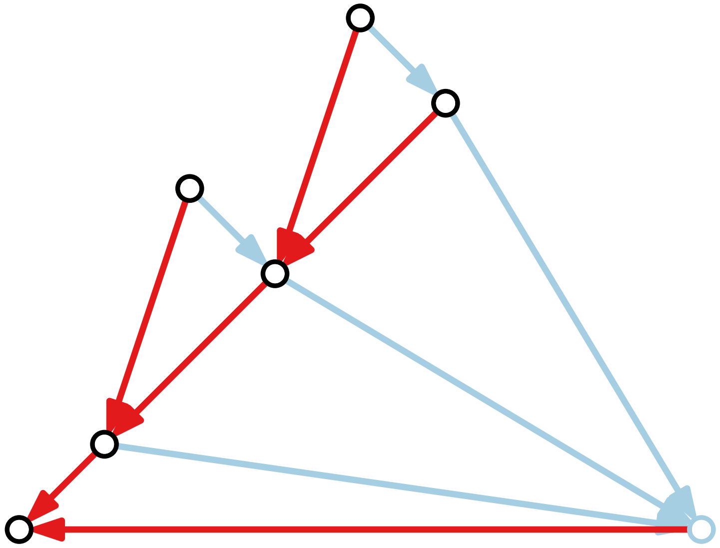
Max. outerplanar graphs on the grid



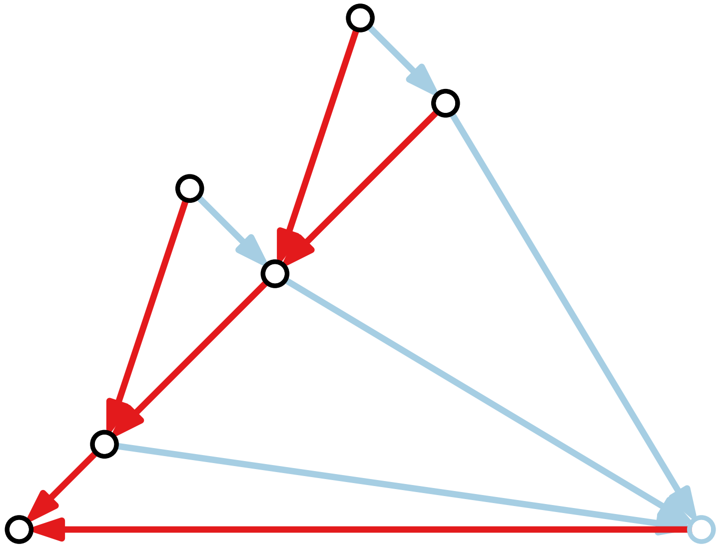
Max. outerplanar graphs on the grid



Max. outerplanar graphs on the grid



Max. outerplanar graphs on the grid



Max. outerplanar $\rightarrow 3n/2$ segments, $O(n) \times O(n^2)$ grid

Known Results – Segments

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ $\vartheta/2$	$O(n^2) \times O(n^{1.58})$ quasipolynomial
max. outerplanar	n [1]	n [1]		?
2-trees	$3n/2$ [1]	$3n/2$ [1]		
3-trees	$2n$ [1]	$2n$ [1]		?
2-connected	$2n$ [1]	$16n/3 - e$ [2]		
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	$n/2$ [4]	$O(n) \times O(n)$
Triangulation	$2n$ [2]	$7n/3$ [2]		
Planar	$2n$ [2]	$16n/3 - e$ [2]		

[1] Dujmović et al. 2007
[2] Durocher & Mondal 2014
[3] Mondal et al. 2013
[4] Igamberdiev et al. 2015

Known Results – Segments

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
			$3n/4$	$O(n^2) \times O(n^{1.58})$
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$	quasipolynomial
max. outerplanar	n [1]	n [1]	$3n/2$	$O(n) \times O(n^2)$
2-trees	$3n/2$ [1]	$3n/2$ [1]		
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$	$O(n) \times O(n^2)$
2-connected	$2n$ [1]	$16n/3 - e$ [2]		
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	$n/2$ [4]	$O(n) \times O(n)$
Triangulation	$2n$ [2]	$7n/3$ [2]		
Planar	$2n$ [2]	$16n/3 - e$ [2]		

[1] Dujmović et al. 2007
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[4] Igamberdiev et al. 2015

Known Results – Segments

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ $\vartheta/2$	$O(n^2) \times O(n^{1.58})$ quasipolynomial
max. outerplanar	n [1]	n [1]	$3n/2$	$O(n) \times O(n^2)$
2-trees	$3n/2$ [1]	$3n/2$ [1]		?
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$	$O(n) \times O(n^2)$
2-connected	$2n$ [1]	$16n/3 - e$ [2]		?
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	$n/2$ [4]	$O(n) \times O(n)$
Triangulation	$2n$ [2]	$7n/3$ [2]		?
Planar	$2n$ [2]	$16n/3 - e$ [2]		

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Known Results – Segments

Class	Segments		Grid Segments	
	Lower	Upper	Segm.	Area
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3n/4$ $\vartheta/2$	$O(n^2) \times O(n^{1.58})$ quasipolynomial
max. outerplanar	n [1]	n [1]	$3n/2$	$O(n) \times O(n^2)$
2-trees	$3n/2$ [1]	$3n/2$ [1]		?
3-trees	$2n$ [1]	$2n$ [1]	$8n/3$	$O(n) \times O(n^2)$
2-connected	$2n$ [1]	$16n/3 - e$ [2]		?
3-connected	$2n$ [1]	$5n/2$ [1]		
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	$n/2$ [4]	$O(n) \times O(n)$
Triangulation	$2n$ [2]	$7n/3$ [2]		?
Planar	$2n$ [2]	$16n/3 - e$ [2]		
Series-parallel, ...				

[1] Dujmović et al. 2007

[2] Durocher & Mondal 2014

[3] Mondal et al. 2013

[4] Igamberdiev et al. 2015

Known Results – Circular Arcs

Class	Segments		Circular Arcs
	Lower	Upper	
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	
max. outerplanar	n [1]	n [1]	
2-trees	$3n/2$ [1]	$3n/2$ [1]	
3-trees	$2n$ [1]	$2n$ [1]	
2-connected	$2n$ [1]	$16n/3 - e$ [2]	
3-connected	$2n$ [1]	$5n/2$ [1]	
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	
Triangulation	$2n$ [2]	$7n/3$ [2]	
Planar	$2n$ [2]	$16n/3 - e$ [2]	

[1] Dujmović et al. 2007
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Known Results – Circular Arcs

Class	Segments		Circular Arcs
	Lower	Upper	
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	
max. outerplanar	n [1]	n [1]	
2-trees	$3n/2$ [1]	$3n/2$ [1]	
3-trees	$2n$ [1]	$2n$ [1]	
2-connected	$2n$ [1]	$16n/3 - e$ [2]	
3-connected	$2n$ [1]	$5n/2$ [1]	
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	
Triangulation	$2n$ [2]	$7n/3$ [2]	
Planar	$2n$ [2]	$16n/3 - e$ [2]	

[1] Dujmović et al. 2007
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[4] Igamberdiev et al. 2015

Known Results – Circular Arcs

Class	Segments		Circular Arcs
	Lower	Upper	
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	←
max. outerplanar	n [1]	n [1]	←
2-trees	$3n/2$ [1]	$3n/2$ [1]	←
3-trees	$2n$ [1]	$2n$ [1]	$11e/18$ [5]
2-connected	$2n$ [1]	$16n/3 - e$ [2]	←
3-connected	$2n$ [1]	$5n/2$ [1]	
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	←
Triangulation	$2n$ [2]	$7n/3$ [2]	←
Planar	$2n$ [2]	$16n/3 - e$ [2]	←

[1] Dujmović et al. 2007
[2] Durocher & Mondal 2014
[5] Schulz 2015
[3] Mondal et al. 2013
[4] Igamberdiev et al. 2015

Known Results – Circular Arcs

Class	Segments		Circular Arcs
	Lower	Upper	
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	
max. outerplanar	n [1]	n [1]	
2-trees	$3n/2$ [1]	$3n/2$ [1]	
3-trees	$2n$ [1]	$2n$ [1]	$11e/18$ [5]
2-connected	$2n$ [1]	$16n/3 - e$ [2]	
3-connected	$2n$ [1]	$5n/2$ [1]	$2e/3$ [5]
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	
Triangulation	$2n$ [2]	$7n/3$ [2]	
Planar	$2n$ [2]	$16n/3 - e$ [2]	

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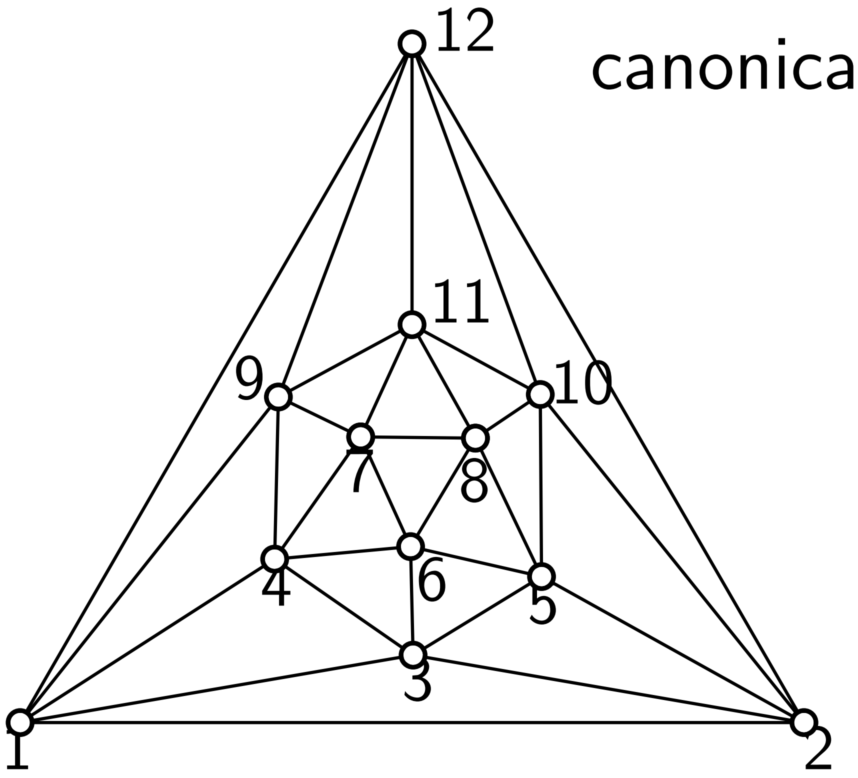
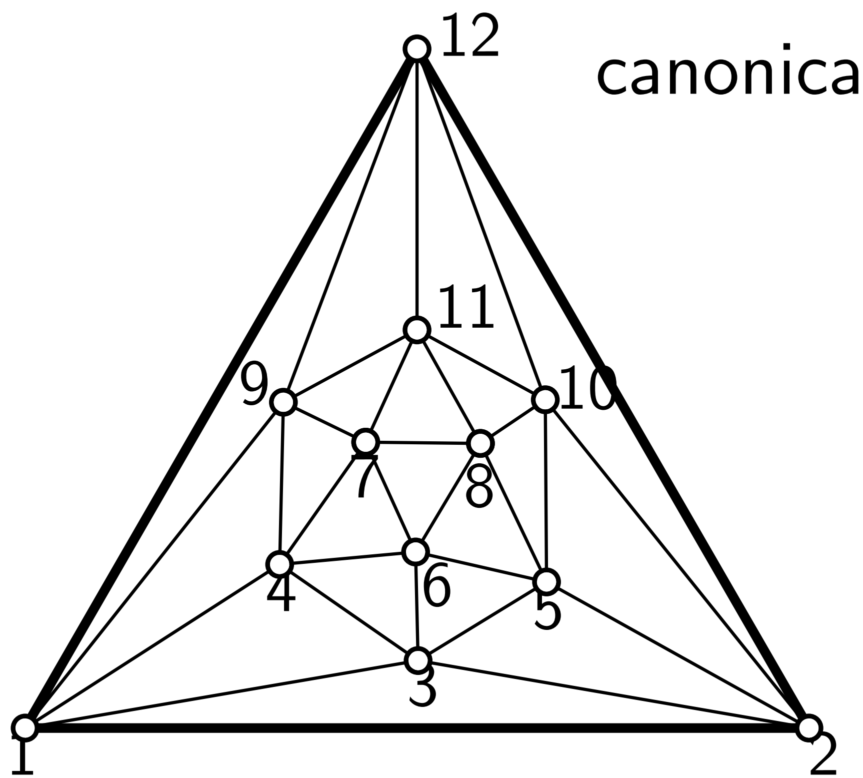
Triangulations with circular arcs

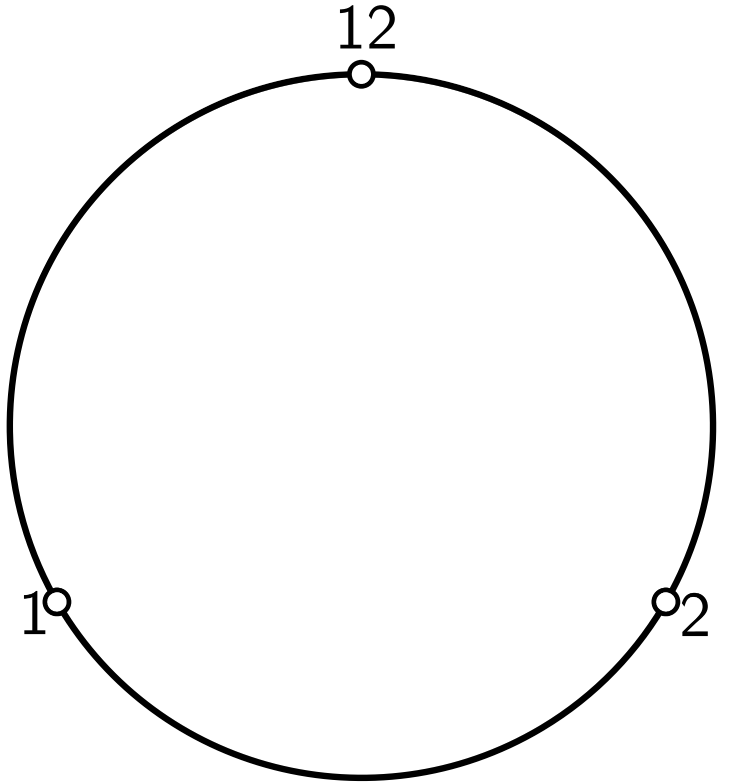


canonical order

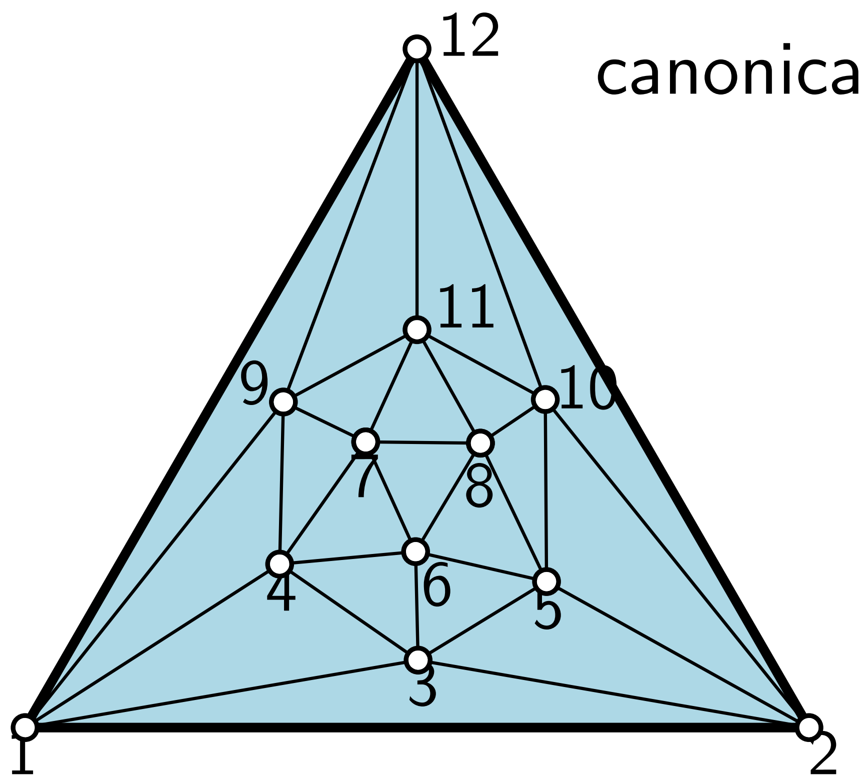
Triangulations with circular arcs



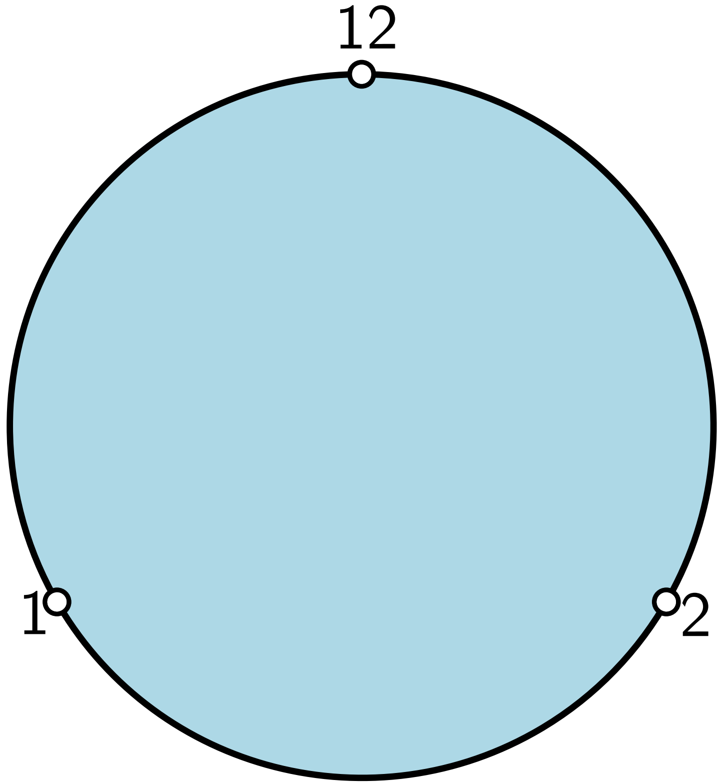
canonical order



Triangulations with circular arcs

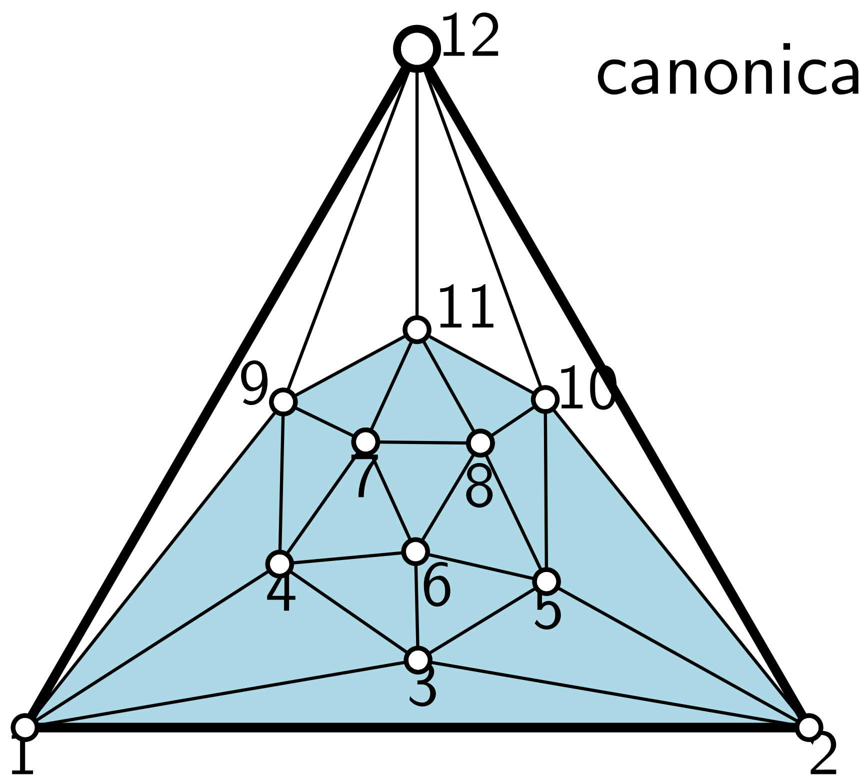


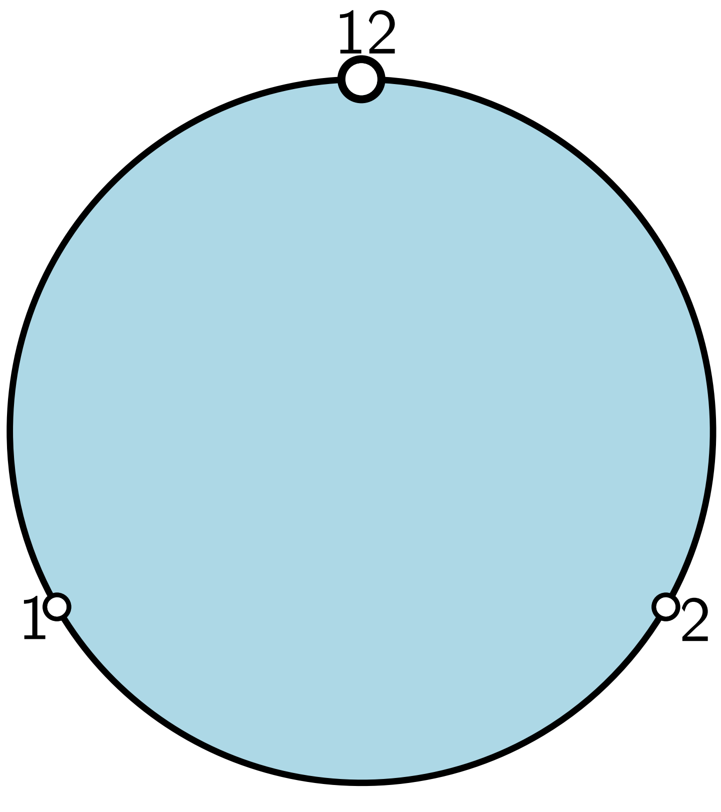
canonical order



Triangulations with circular arcs

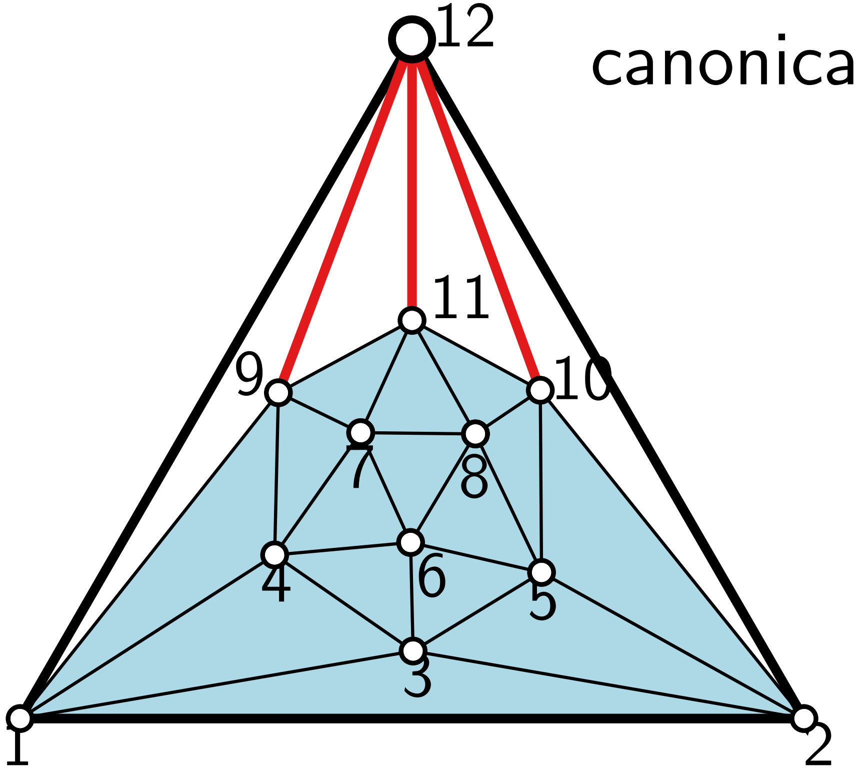
canonical order

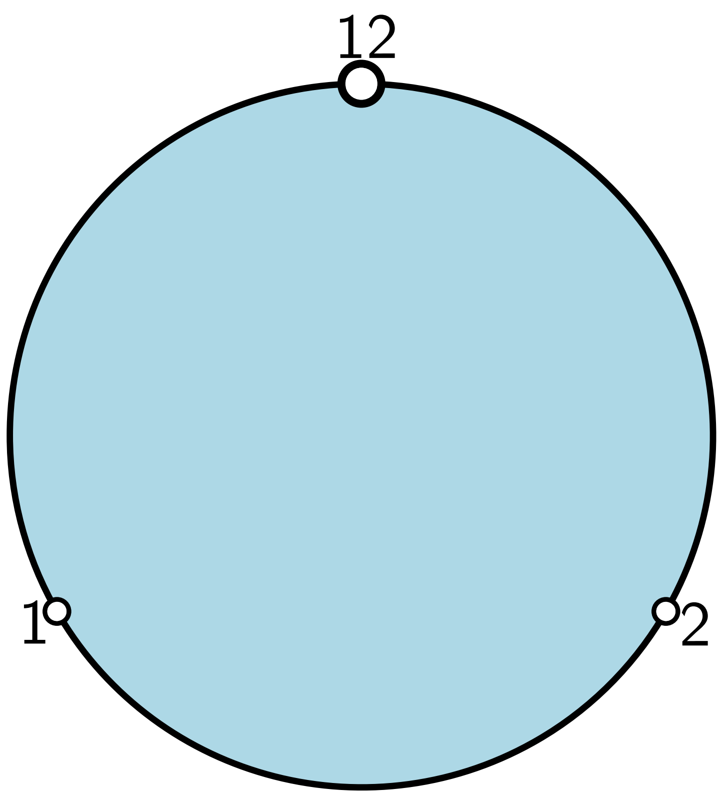




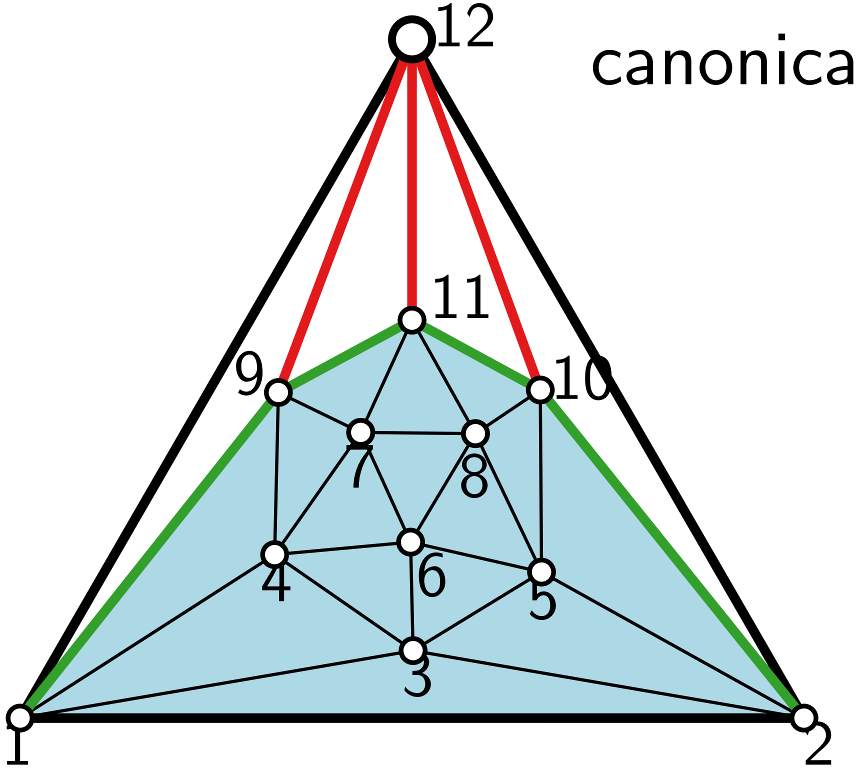
Triangulations with circular arcs

canonical order





Triangulations with circular arcs



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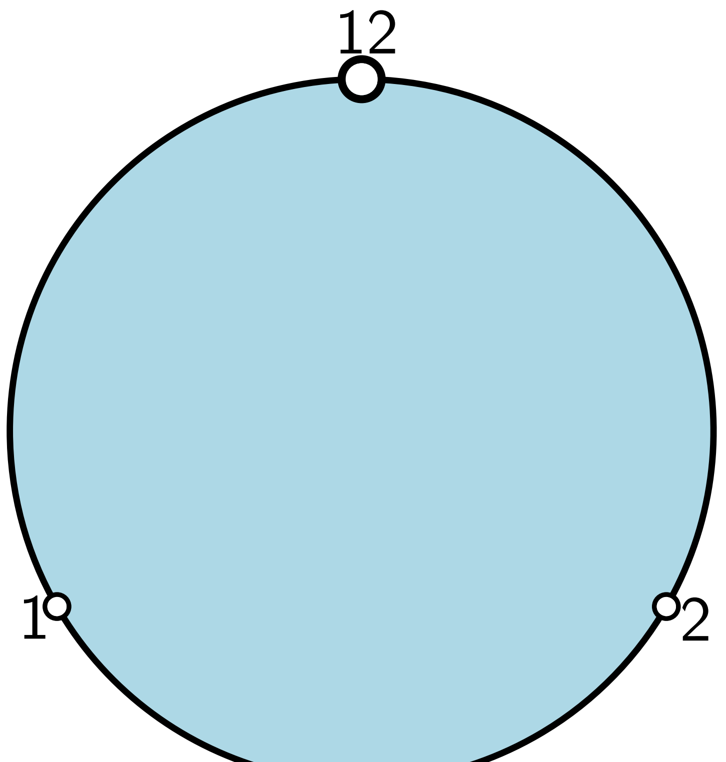
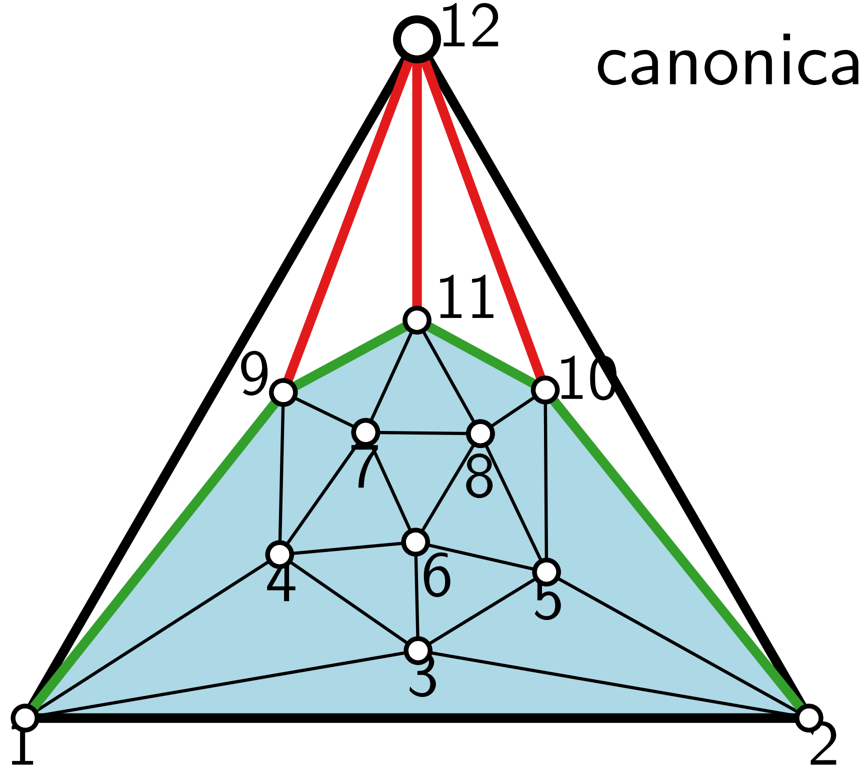
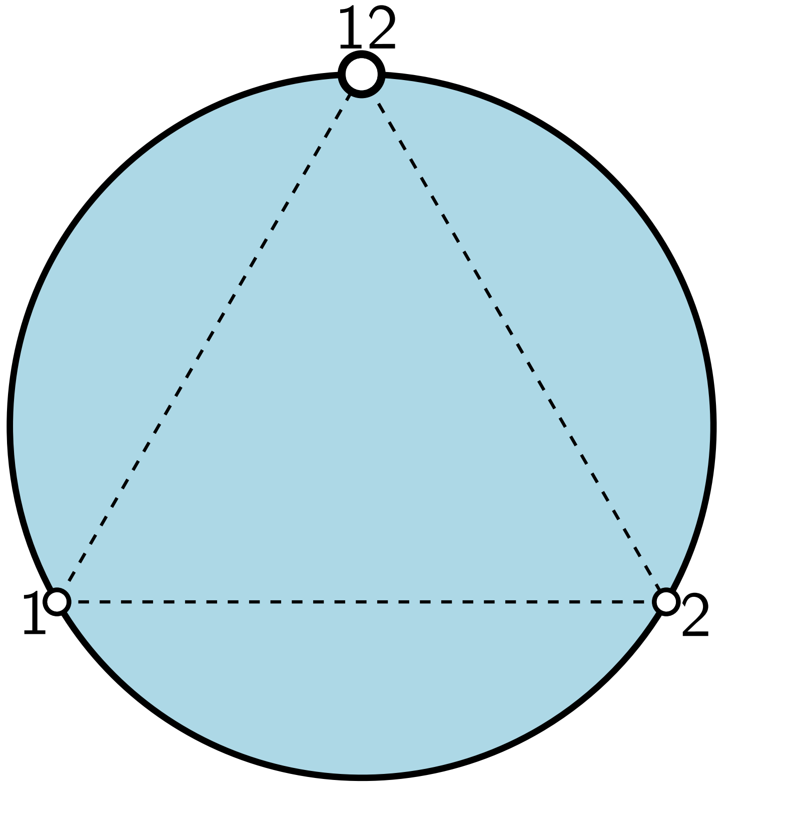


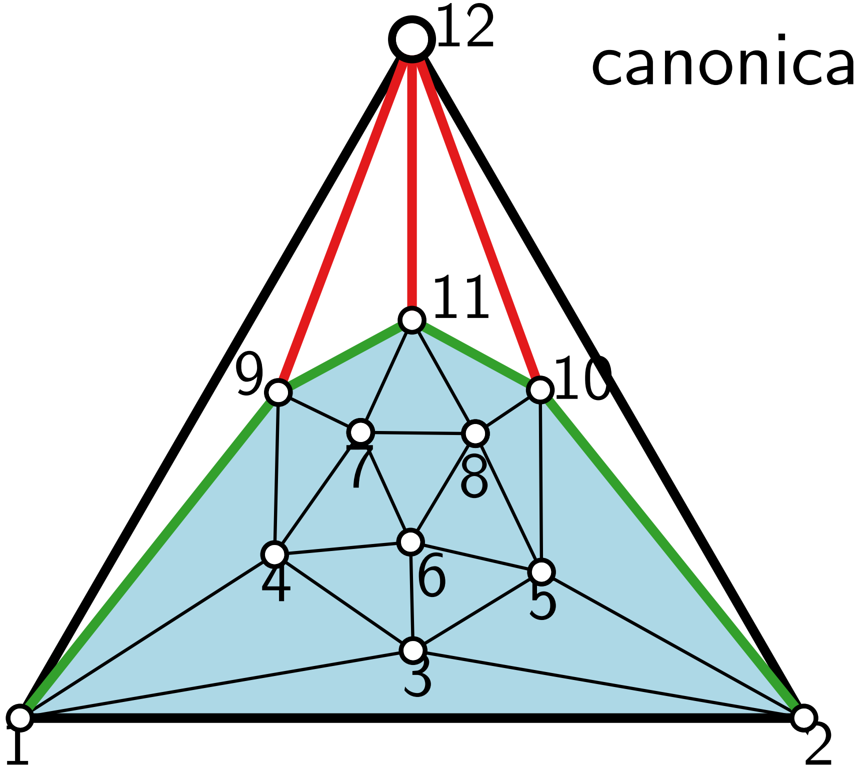
Triangulations with circular arcs

canonical order

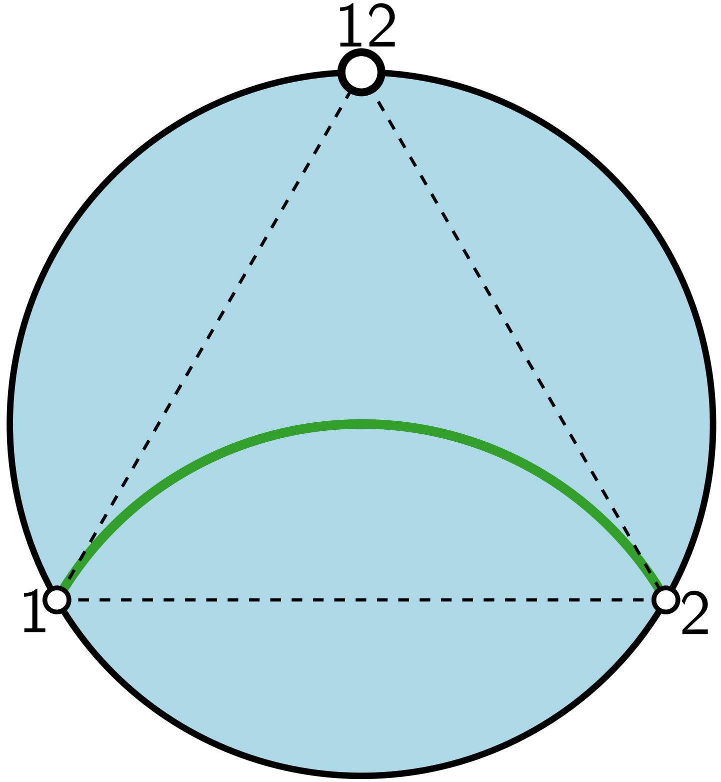




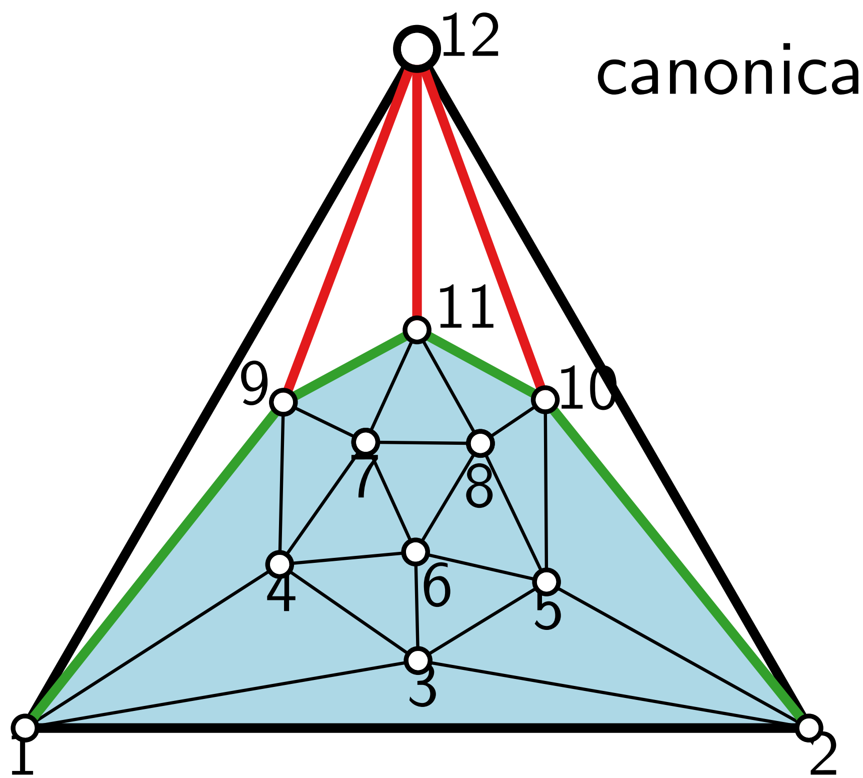
Triangulations with circular arcs



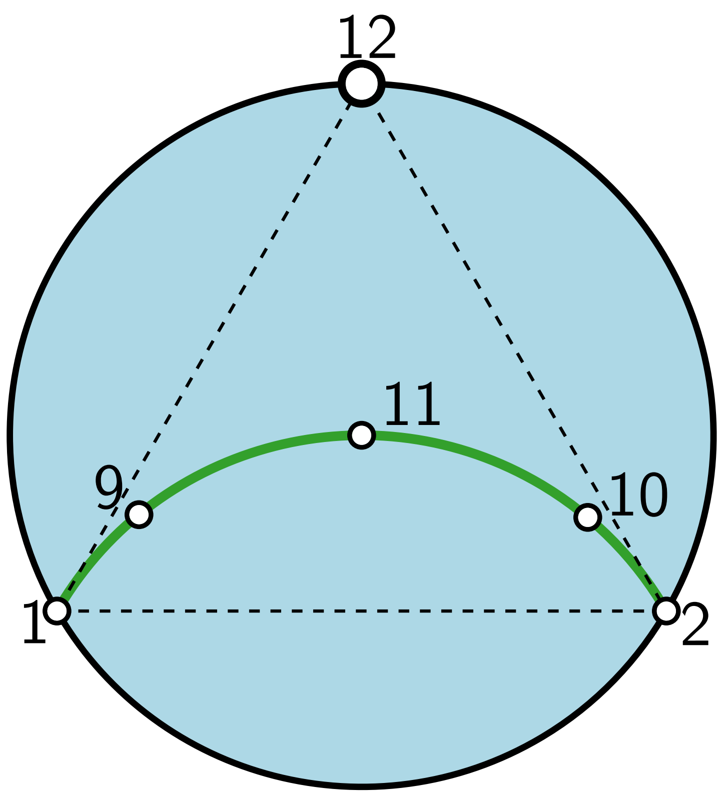
canonical order



Triangulations with circular arcs

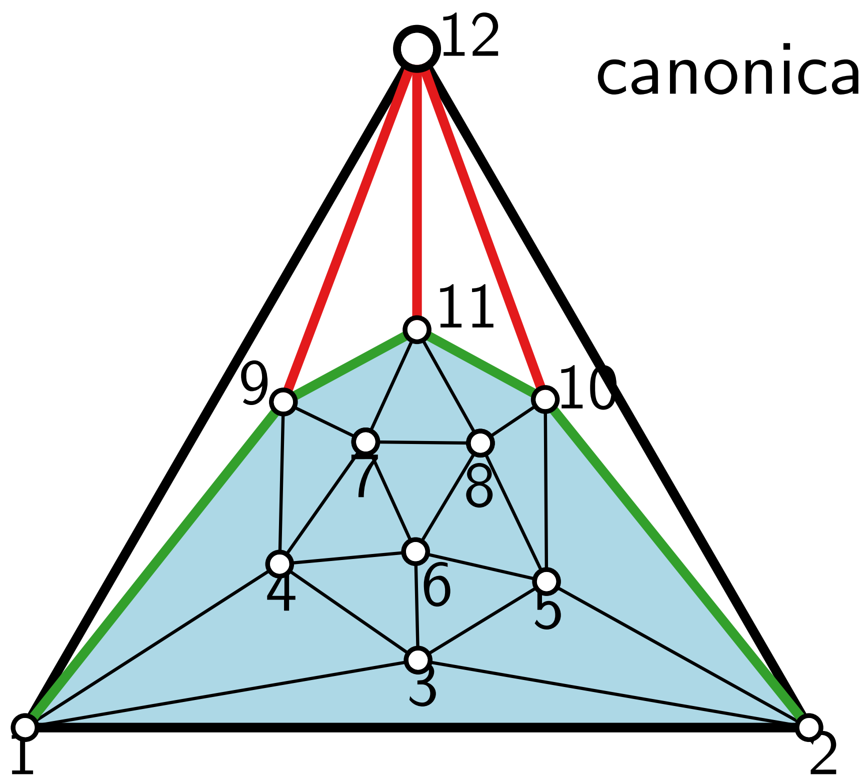


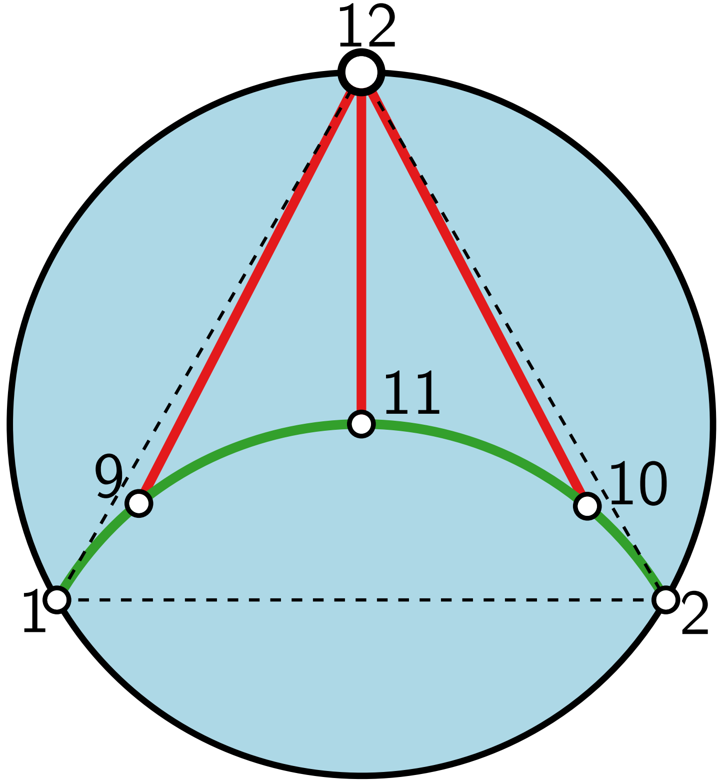
canonical order



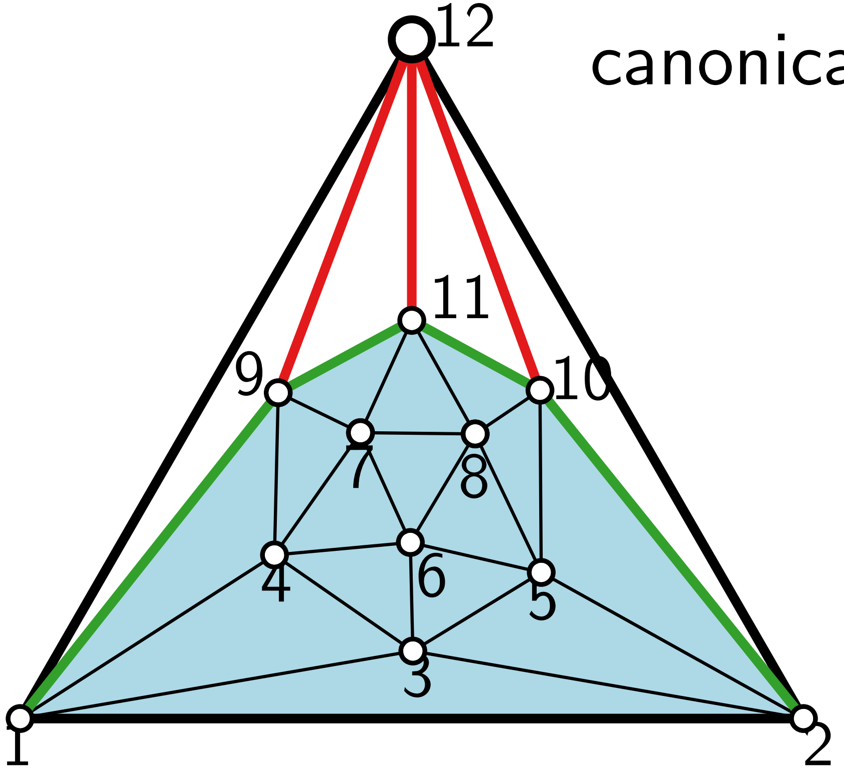
Triangulations with circular arcs

canonical order



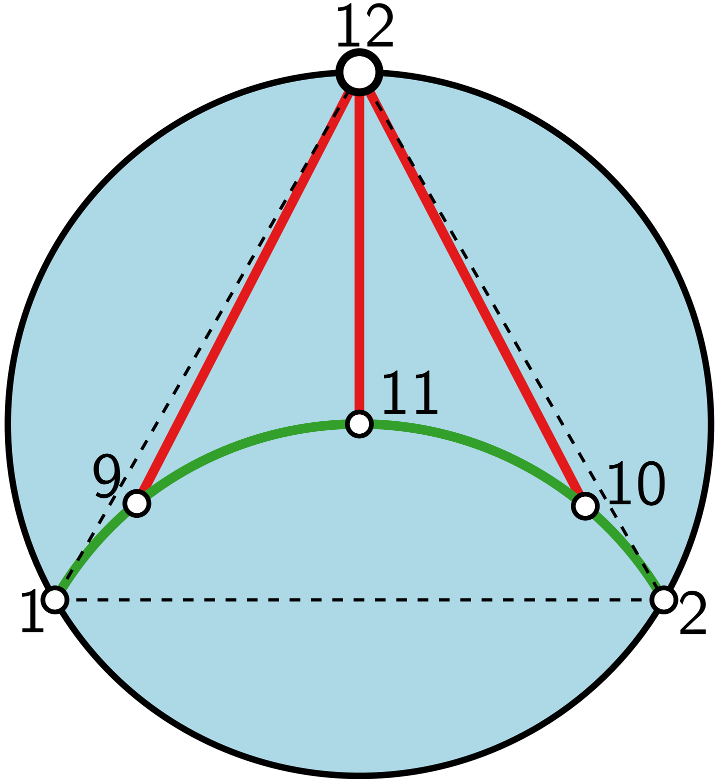


Triangulations with circular arcs

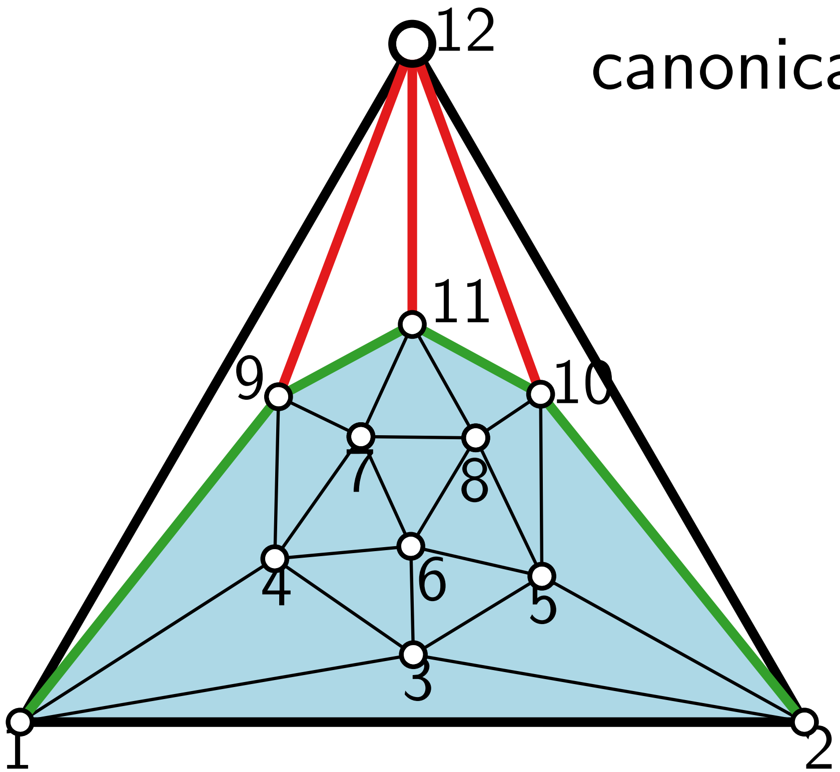


canonical order

- red edges extend into bottom arc

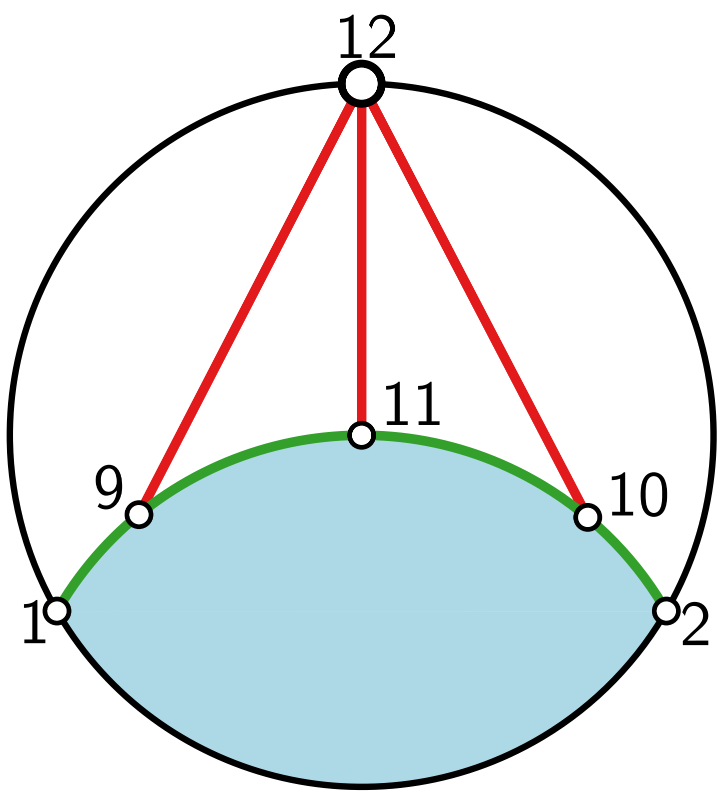


Triangulations with circular arcs

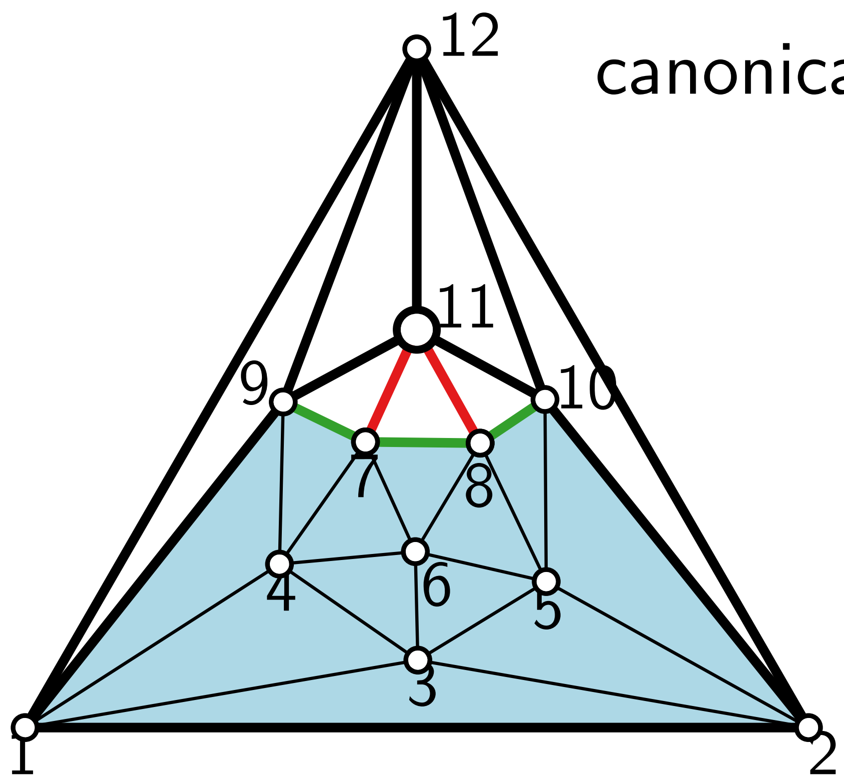


canonical order

- red edges extend into bottom arc

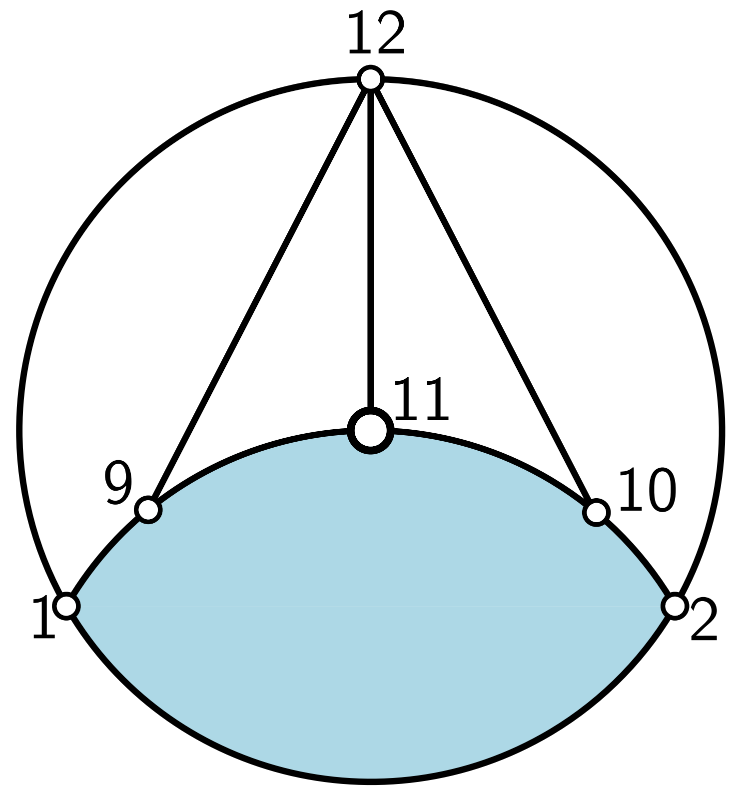


Triangulations with circular arcs

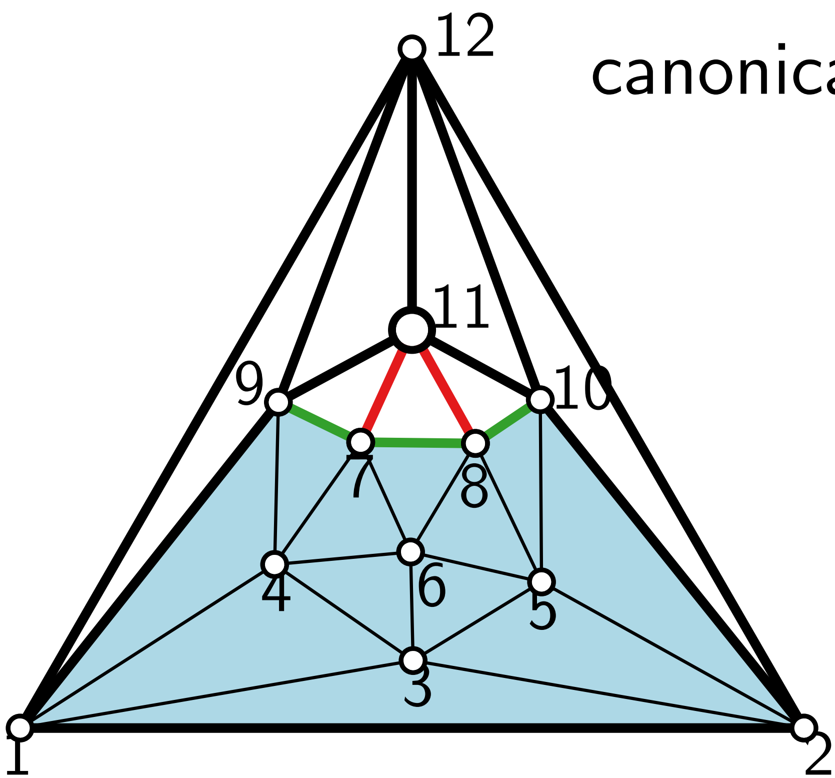


canonical order

- red edges extend into bottom arc

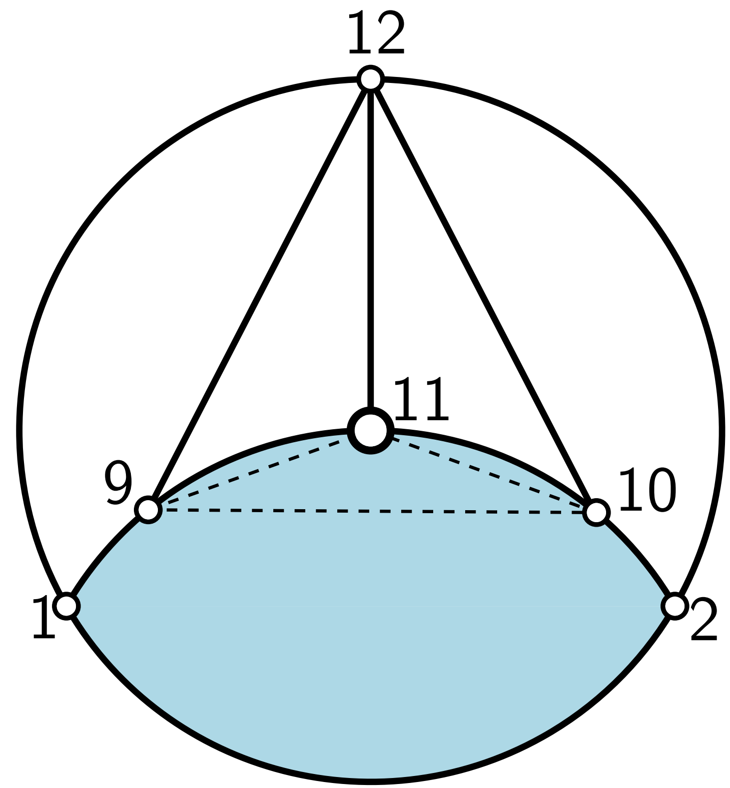


Triangulations with circular arcs

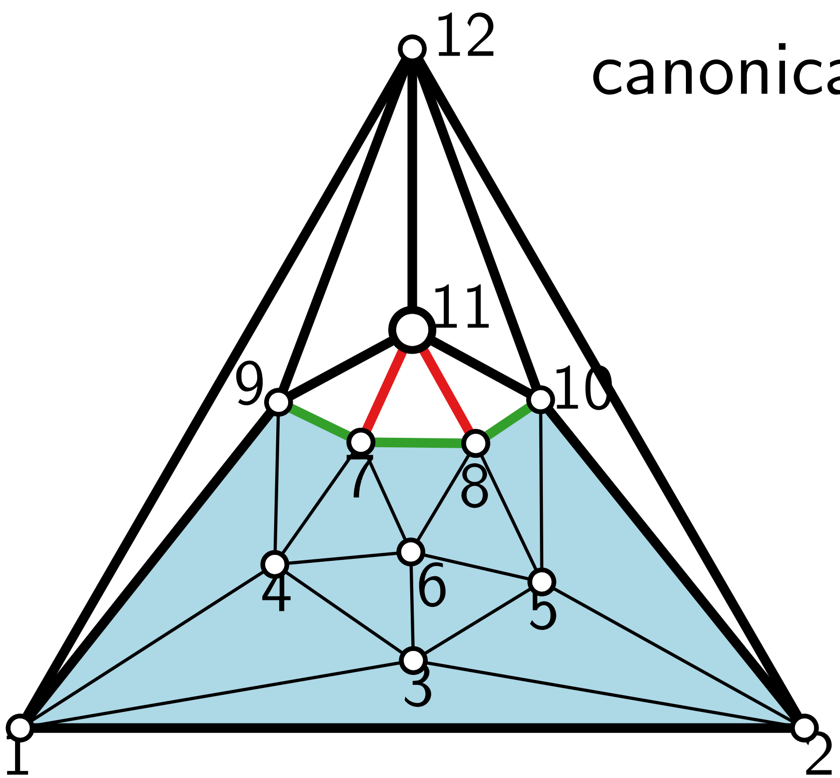


canonical order

- red edges extend into bottom arc

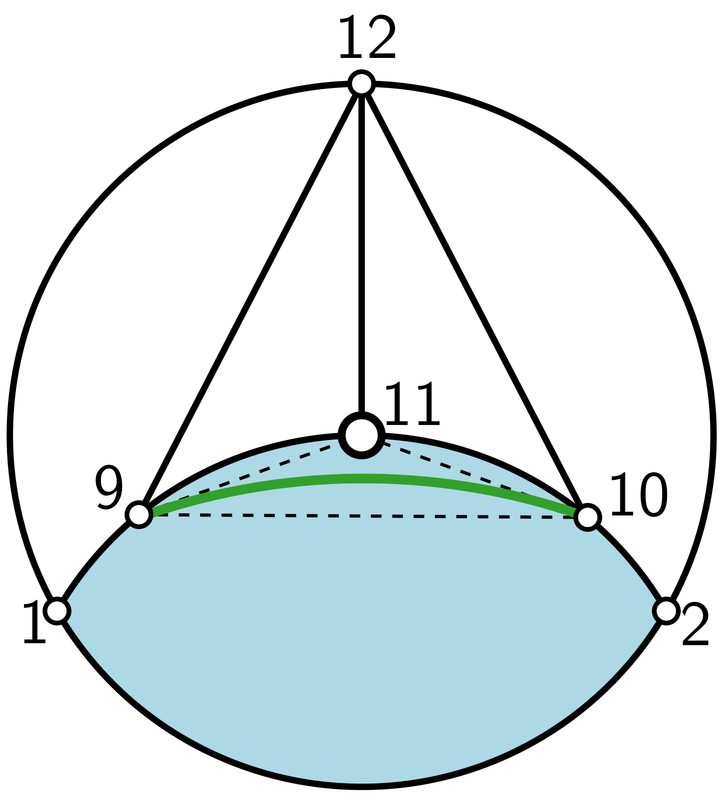


Triangulations with circular arcs

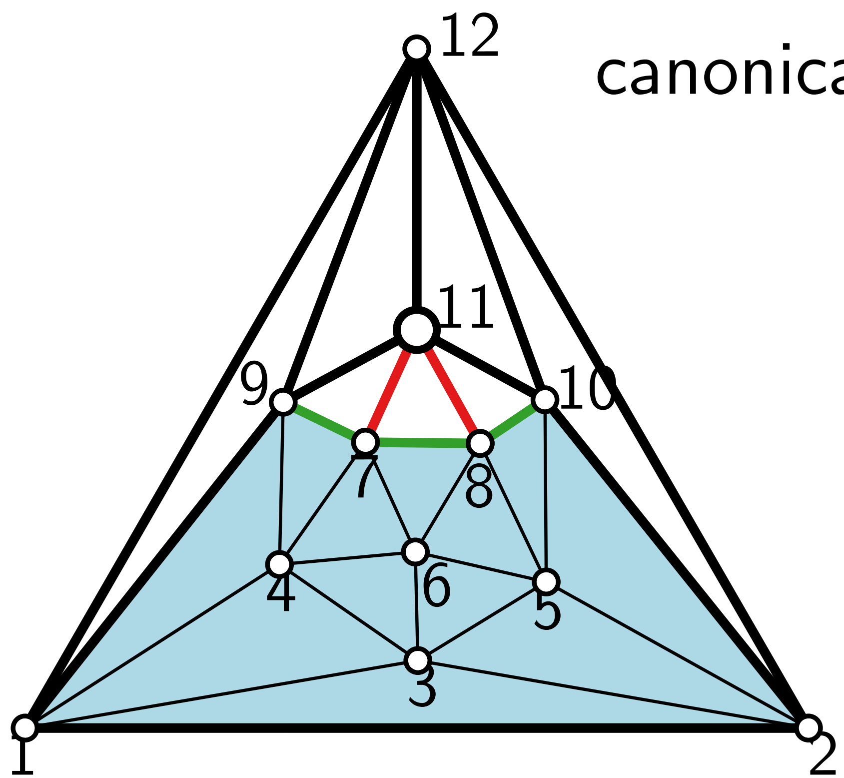


canonical order

- red edges extend into bottom arc

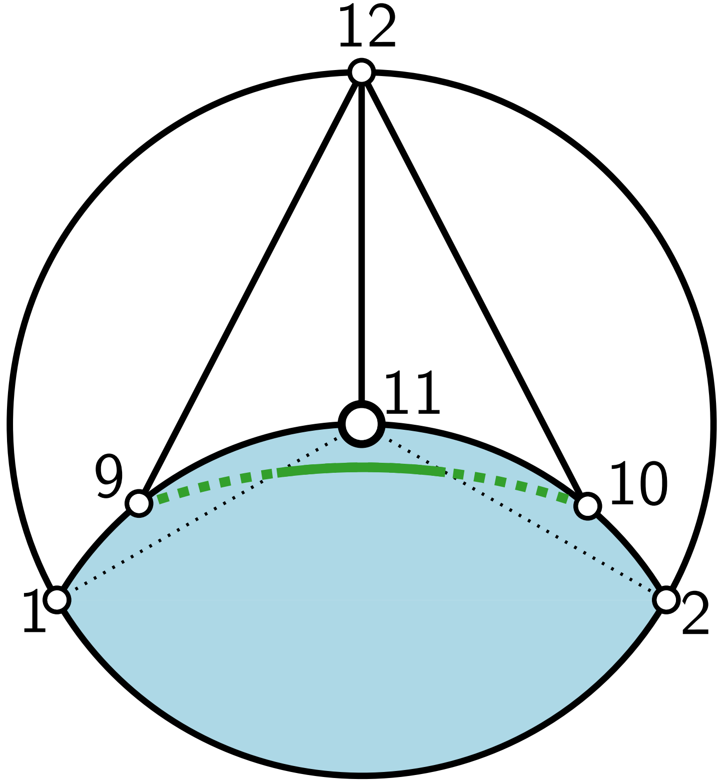


Triangulations with circular arcs

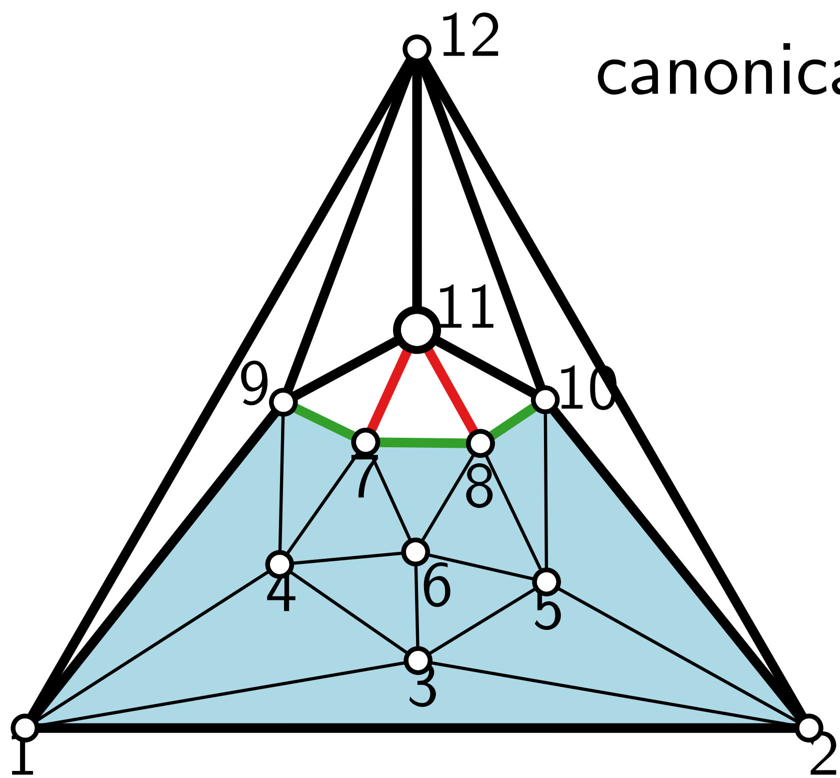


canonical order

- red edges extend into bottom arc

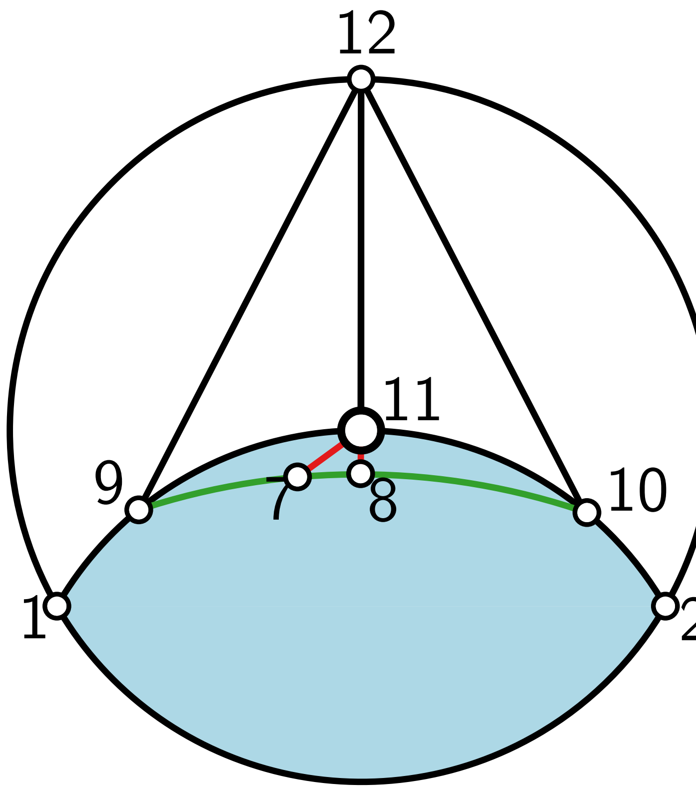


Triangulations with circular arcs

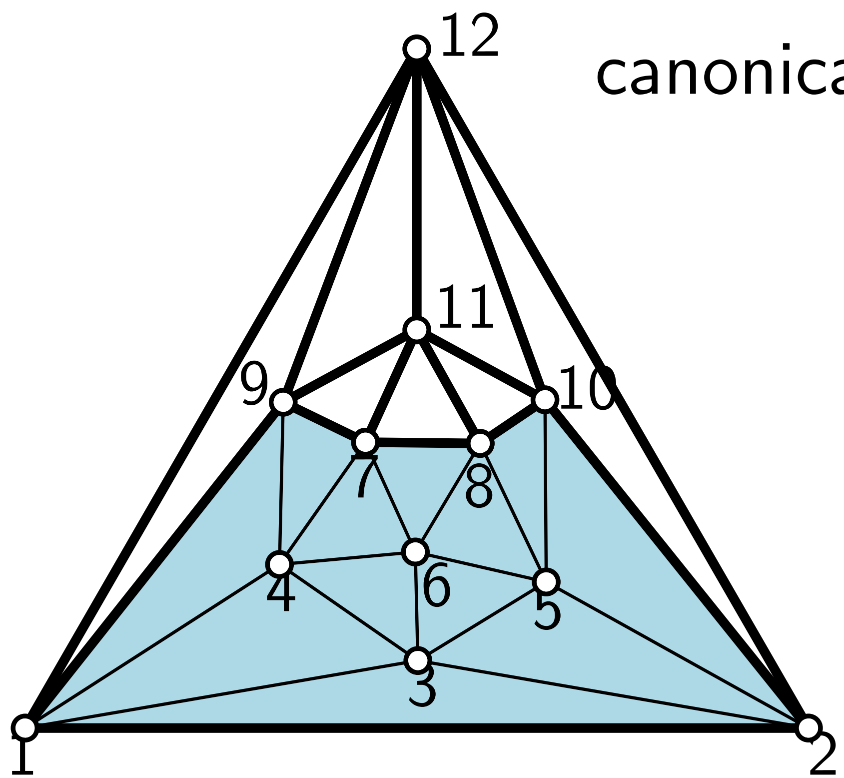


canonical order

- red edges extend into bottom arc

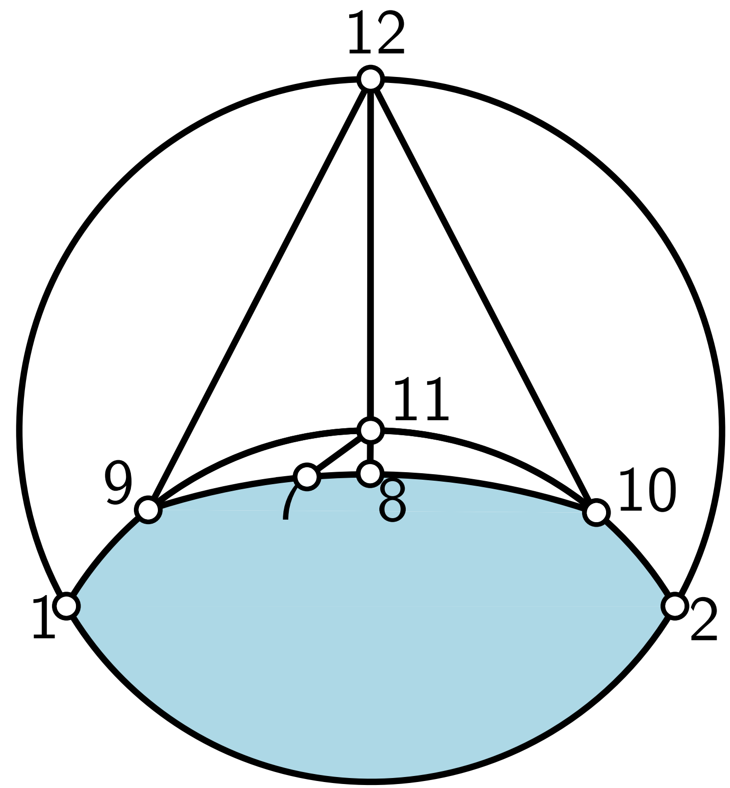


Triangulations with circular arcs

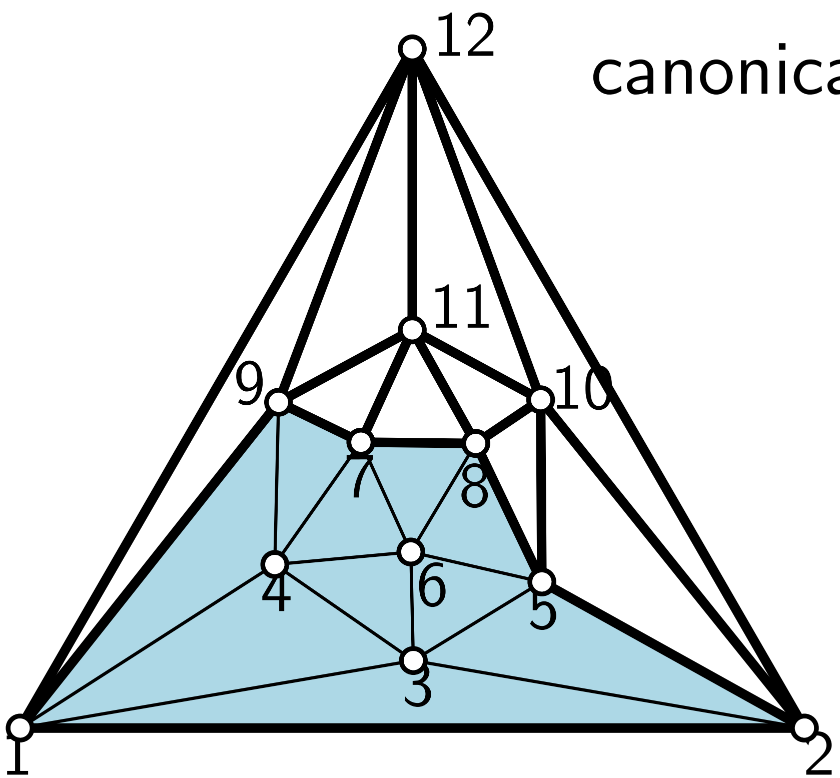


canonical order

- red edges extend into bottom arc

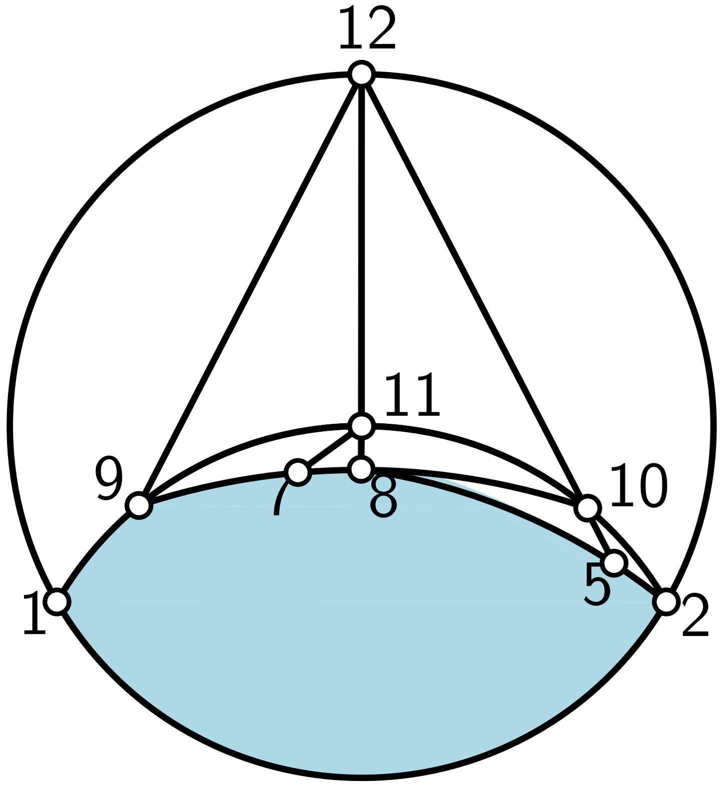


Triangulations with circular arcs

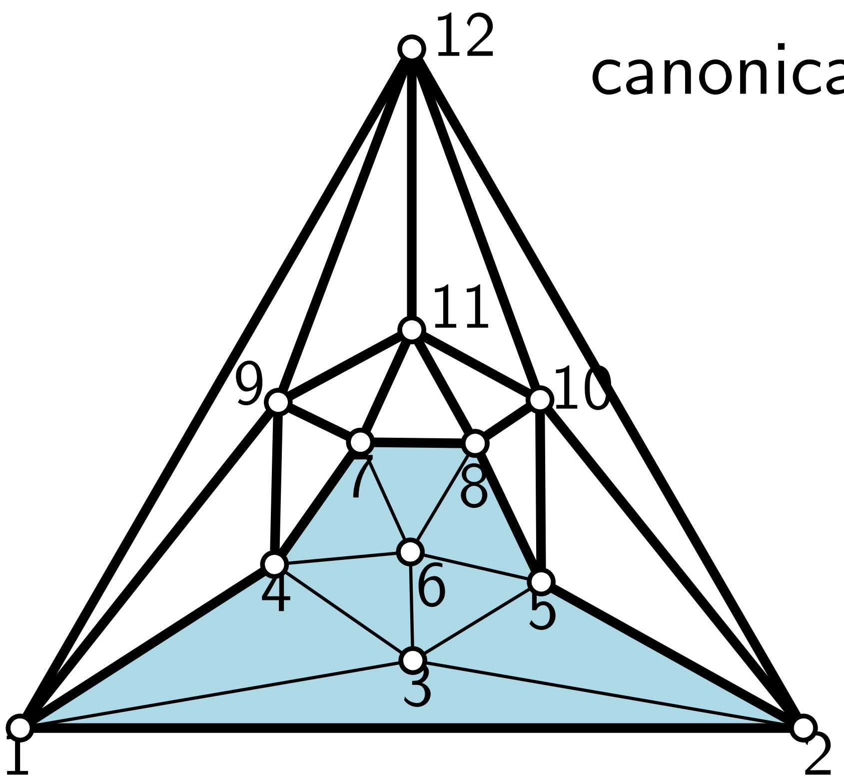


canonical order

- red edges extend into bottom arc

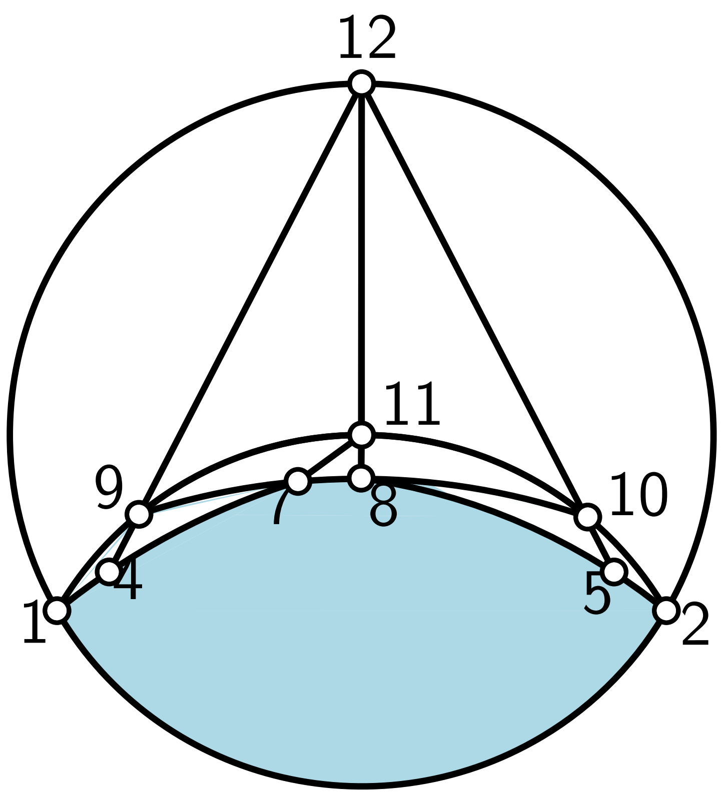


Triangulations with circular arcs

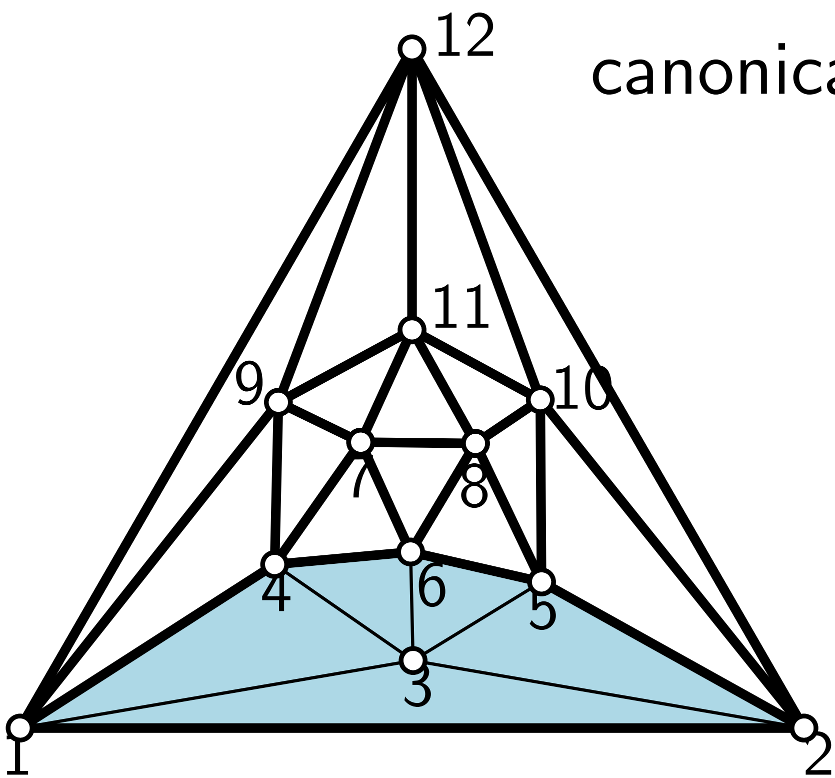


canonical order

- red edges extend into bottom arc

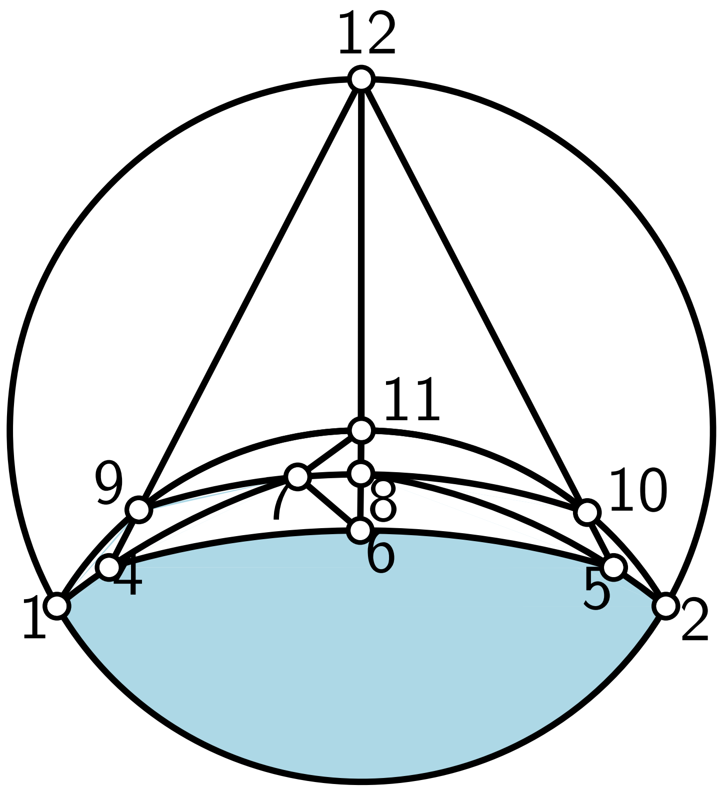


Triangulations with circular arcs

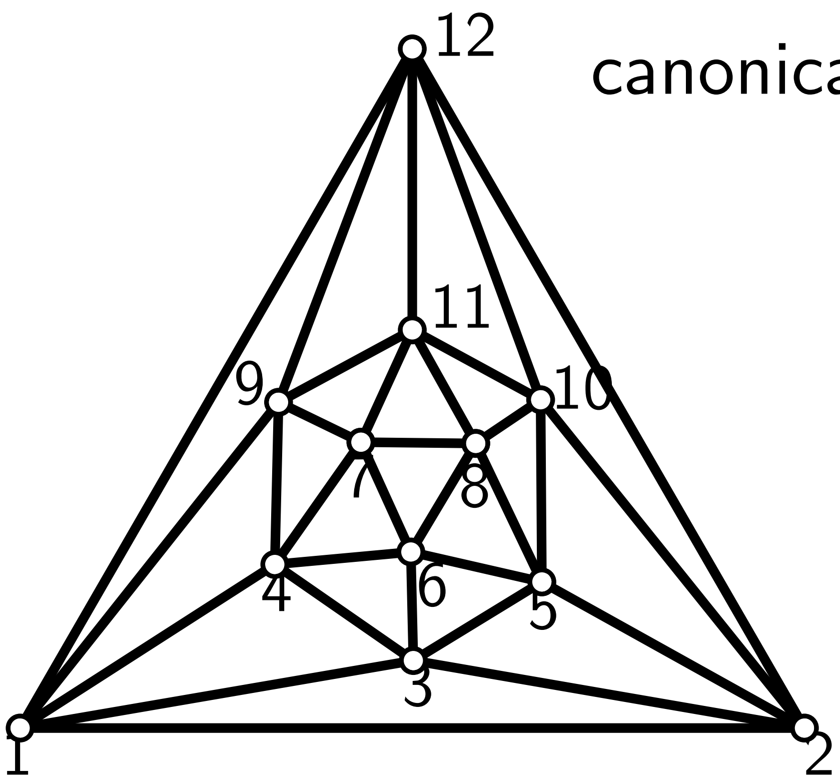


canonical order

- red edges extend into bottom arc

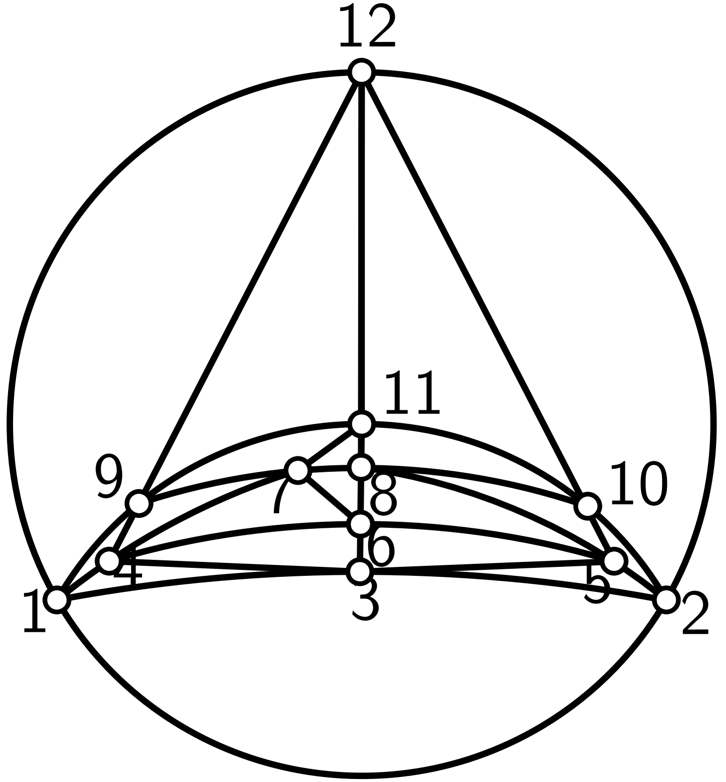


Triangulations with circular arcs

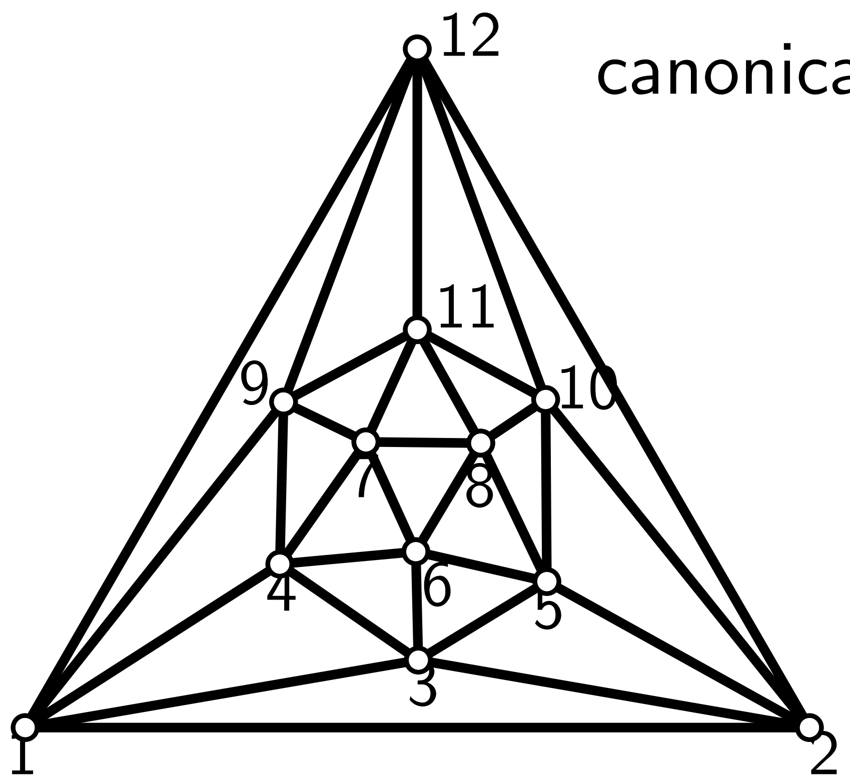


canonical order

- red edges extend into bottom arc

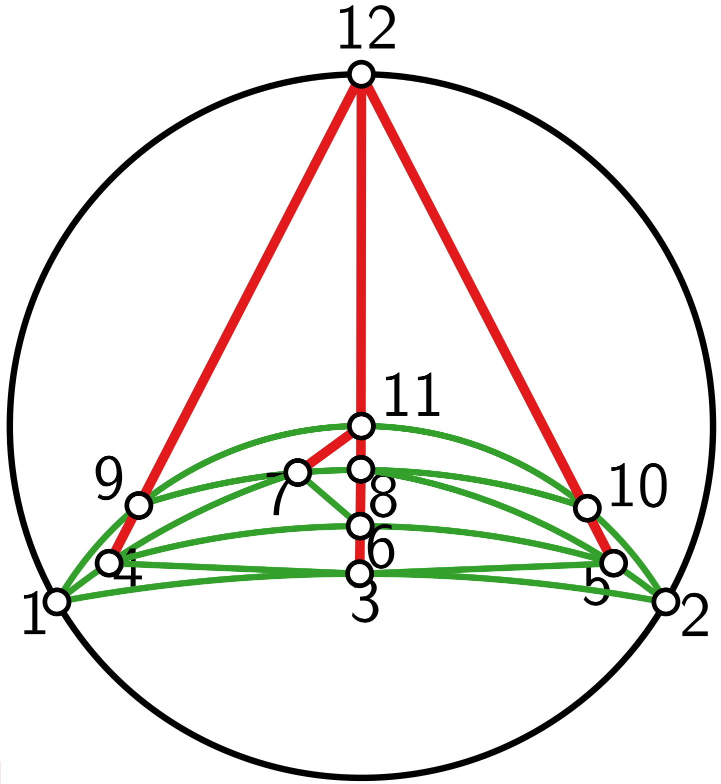


Triangulations with circular arcs



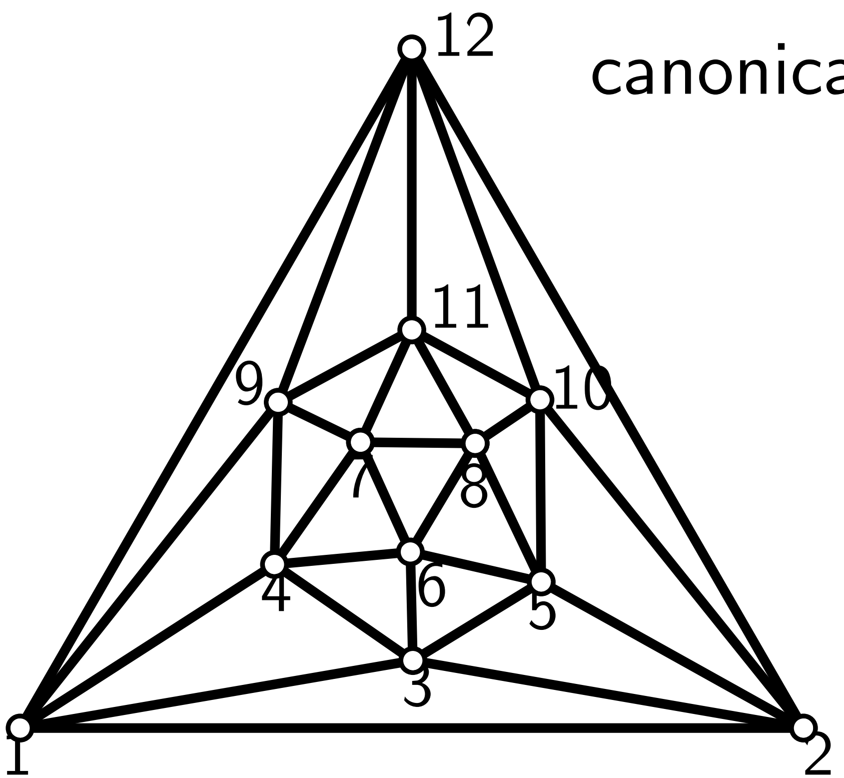
canonical order

- red edges extend into bottom arc



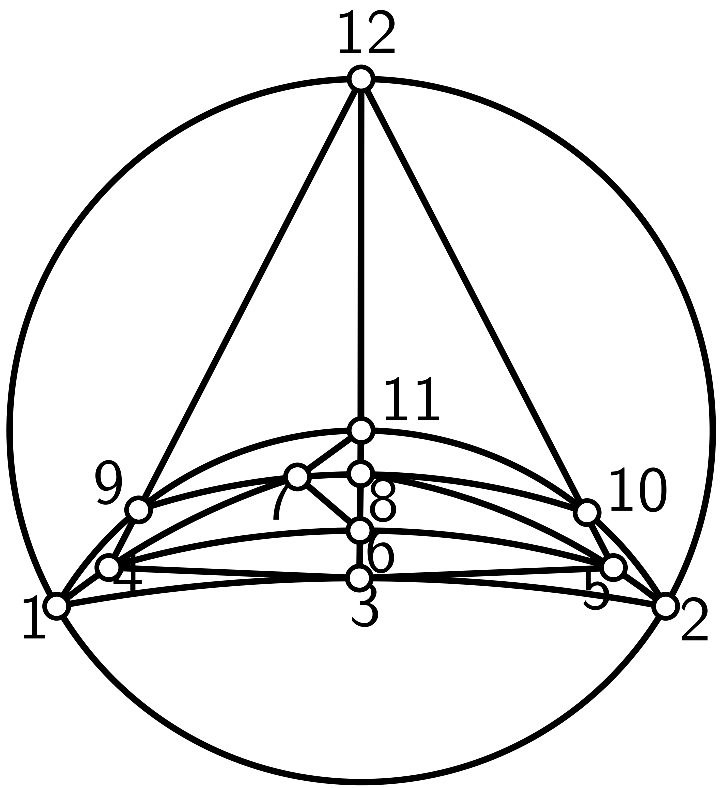
Triangulation $\rightarrow (5n-11)/3$ arcs

Planar graphs with circular arcs



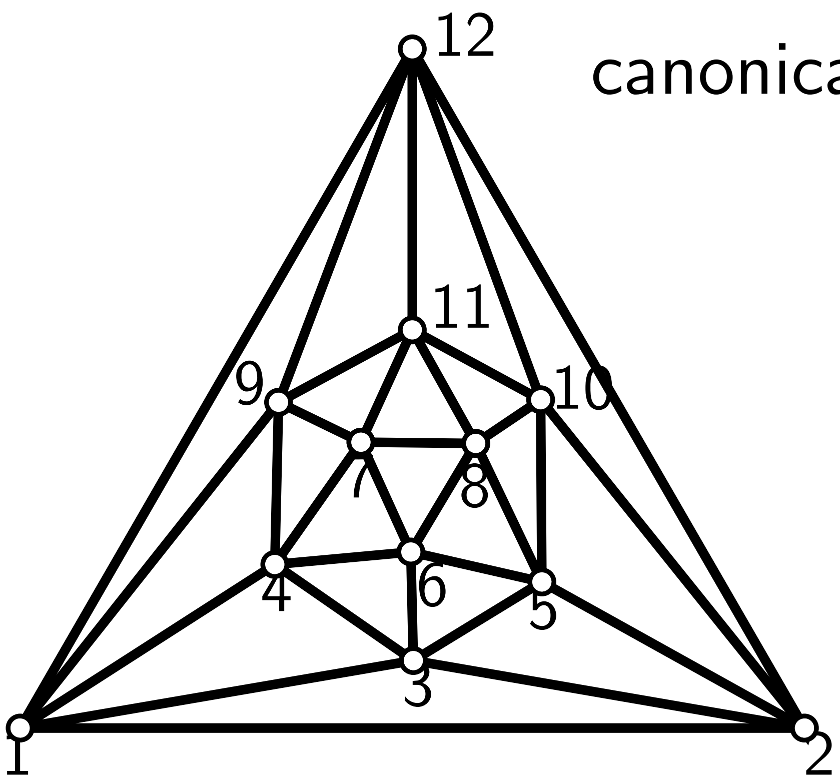
canonical order

- red edges extend into bottom arc



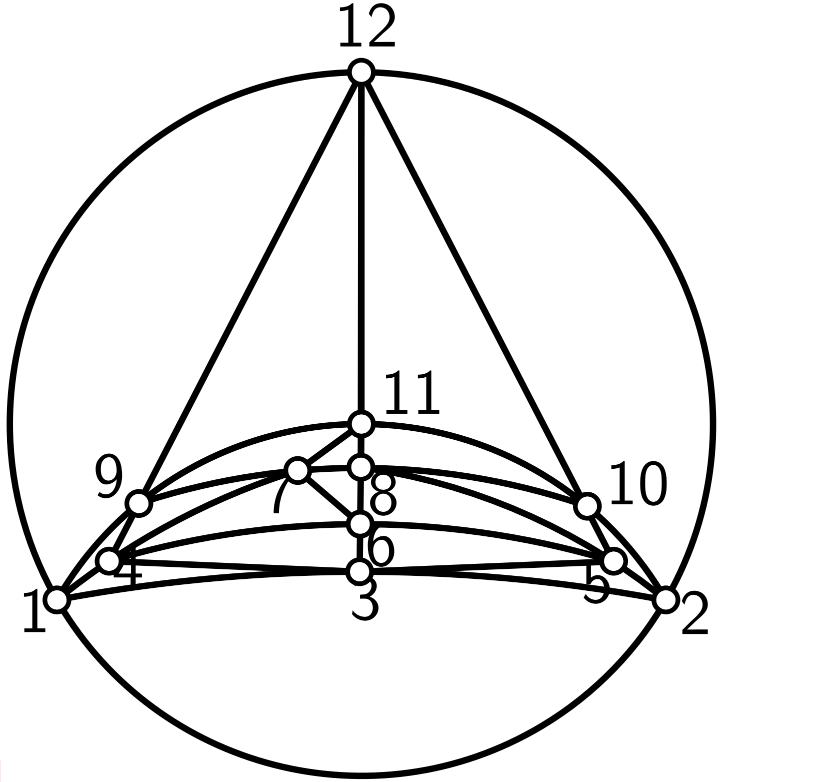
Triangulation $\rightarrow (5n-11)/3$ arcs

Planar graphs with circular arcs



canonical order

- red edges extend into bottom arc



Planar $\rightarrow (14n - 29)/3 - e$ arcs

Triangulation $\rightarrow (5n - 11)/3$ arcs

Class	Segments		Circular Arcs
	Lower	Upper	
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	
max. outerplanar	n [1]	n [1]	
2-trees	$3n/2$ [1]	$3n/2$ [1]	
3-trees	$2n$ [1]	$2n$ [1]	$11e/18$ [5]
2-connected	$2n$ [1]	$16n/3-e$ [2]	
3-connected	$2n$ [1]	$5n/2$ [1]	$2e/3$ [5]
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	
Triangulation	$2n$ [2]	$7n/3$ [2]	
Planar	$2n$ [2]	$16n/3-e$ [2]	

[1] Dujmović et al. 2007

[2] Durocher & Mondal 2014

[5] Schulz 2015

[3] Mondal et al. 2013

[4] Igamberdiev et al. 2015

Class	Segments: Lower	Segments: Upper	Circular Arcs
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	
max. outerplanar	n [1]	n [1]	
2-trees	$3n/2$ [1]	$3n/2$ [1]	
3-trees	$2n$ [1]	$2n$ [1]	$11e/18$ [5]
2-connected	$2n$ [1]	$16n/3 - e$ [2]	
3-connected	$2n$ [1]	$5n/2$ [1]	$2e/3$ [5]
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	
Triangulation	$2n$ [2]	$7n/3$ [2]	$(5n-11)/3$
Planar	$2n$ [2]	$16n/3 - e$ [2]	

[1] Dujmović et al. 2007
[2] Durocher & Mondal 2014
[3] Mondal et al. 2013
[4] Igamberdiev et al. 2015
[5] Schulz 2015

Class	Segments		Circular Arcs
	Lower	Upper	
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	
max. outerplanar	n [1]	n [1]	
2-trees	$3n/2$ [1]	$3n/2$ [1]	
3-trees	$2n$ [1]	$2n$ [1]	$11e/18$ [5]
2-connected	$2n$ [1]	$16n/3 - e$ [2]	
3-connected	$2n$ [1]	$5n/2$ [1]	$2e/3$ [5]
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	
Triangulation	$2n$ [2]	$7n/3$ [2]	$(5n-11)/3$
Planar	$2n$ [2]	$16n/3 - e$ [2]	$(14n-29)/3 - e$

[1] Dujmović et al. 2007

[2] Durocher & Mondal 2014

[5] Schulz 2015

[3] Mondal et al. 2013

[4] Igamberdiev et al. 2015

Class	Segments		Circular Arcs
	Lower	Upper	
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	
max. outerplanar	n [1]	n [1]	
2-trees	$3n/2$ [1]	$3n/2$ [1]	
3-trees	$2n$ [1]	$2n$ [1]	$11e/18$ [5]
2-connected	$2n$ [1]	$16n/3 - e$ [2]	
3-connected	$2n$ [1]	$5n/2$ [1]	$2e/3$ [5]
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	
Triangulation	$2n$ [2]	$7n/3$ [2]	$(5n - 11)/3$
Planar	$2n$ [2]	$16n/3 - e$ [2]	$(14n - 29)/3 - e$
4-connected	$2n$ [1]	$9n/4$ [2]	$9n/2 - e$
4-conn. triang.	$2n$ [2]	$9n/4$ [2]	$3n/2$

[1] Dujmović et al. 2007
[2] Durocher & Mondal 2014
[3] Mondal et al. 2013
[4] Igamberdiev et al. 2015
[5] Schulz 2015

Class	Segments Lower	Segments Upper	Circular Arcs
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	
max. outerplanar	n [1]	n [1]	?
2-trees	$3n/2$ [1]	$3n/2$ [1]	
3-trees	$2n$ [1]	$2n$ [1]	$11e/18$ [5]
2-connected	$2n$ [1]	$16n/3 - e$ [2]	?
3-connected	$2n$ [1]	$5n/2$ [1]	$2e/3$ [5]
cubic 3-conn.	$n/2$ [3]	$n/2$ [4]	?
Triangulation	$2n$ [2]	$7n/3$ [2]	$(5n-11)/3$
Planar	$2n$ [2]	$16n/3 - e$ [2]	$(14n-29)/3 - e$
4-connected	$2n$ [1]	$9n/4$ [2]	$9n/2 - e$
4-conn. triang.	$2n$ [2]	$9n/4$ [2]	$3n/2$

[1] Dujmović et al. 2007
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