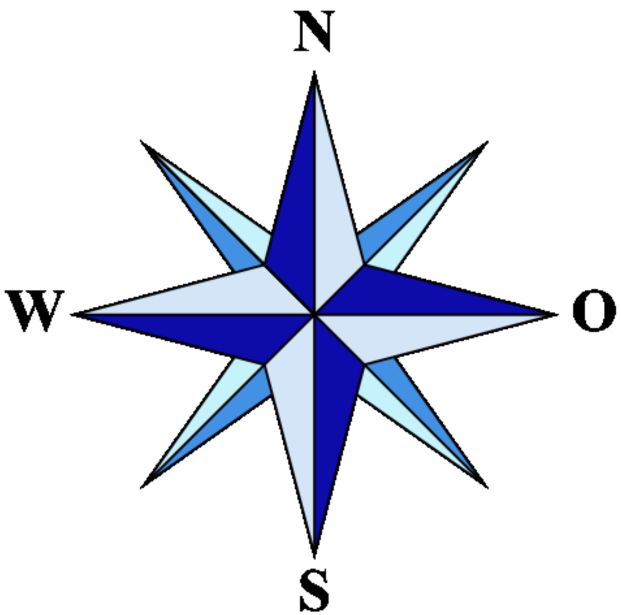
FernUniversität in Hagen
Fakultät für Mathematik und Informatik

theoretische informatik



Windrose Planarity

Embedding Graphs with Direction-Constrained Edges

Philipp Kindermann
LG Theoretische Informatik
FernUniversität in Hagen

Published at SODA'16. Joint work with
Patrizio Angelini, Giordano Da Lozzo, Giuseppe Di Battista,
Valentino Di Donato, Günter Rote & Ignaz Rutter

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An undirected graph is *planar*: no crossings

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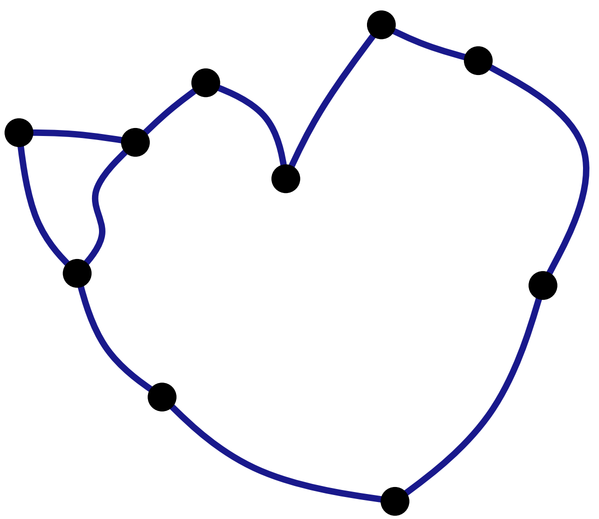

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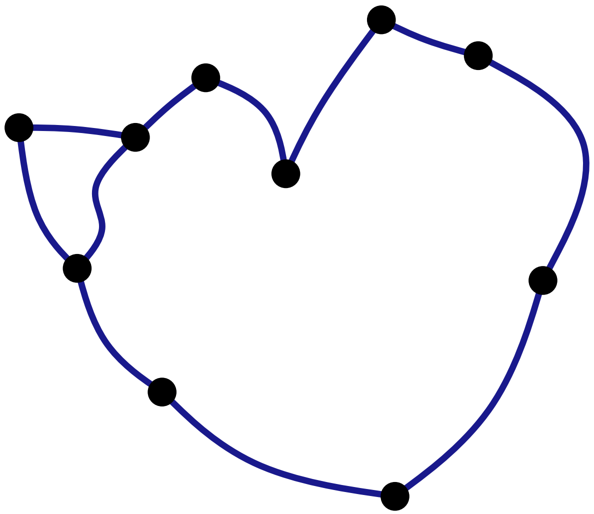

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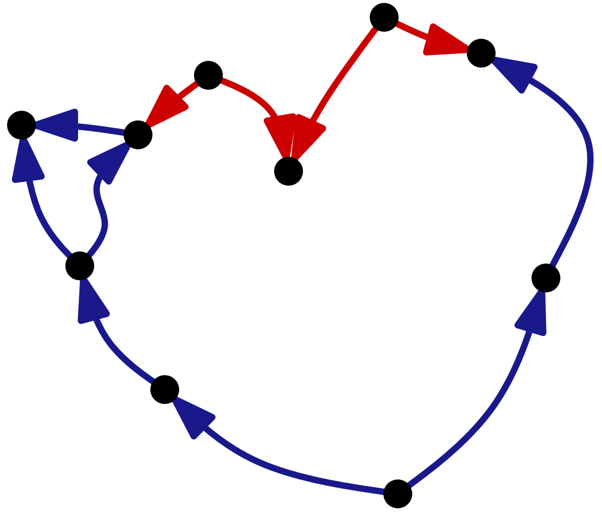

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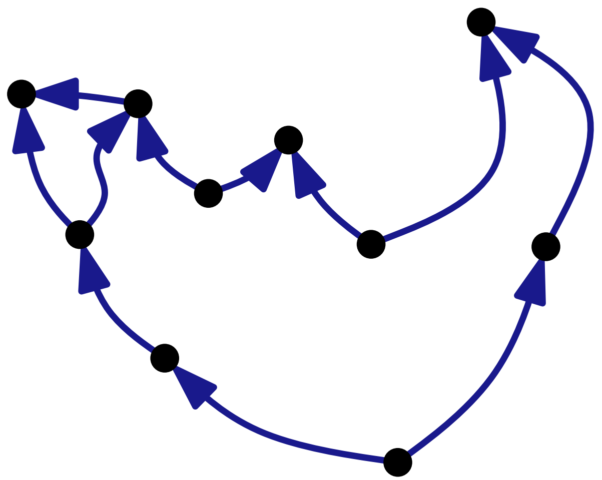

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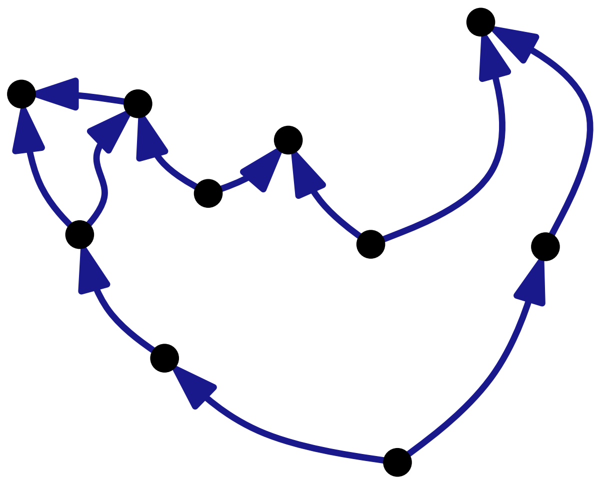

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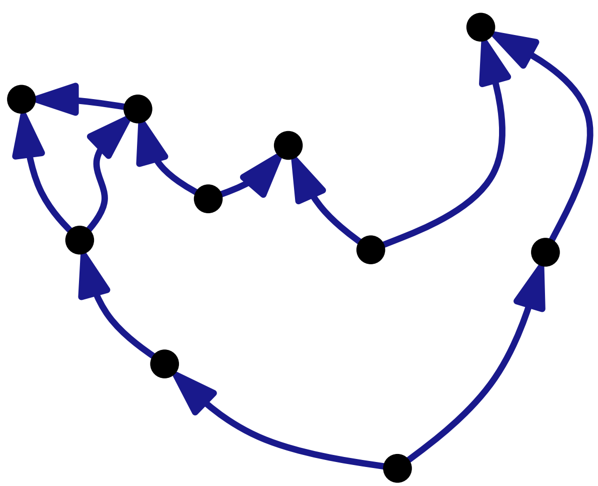

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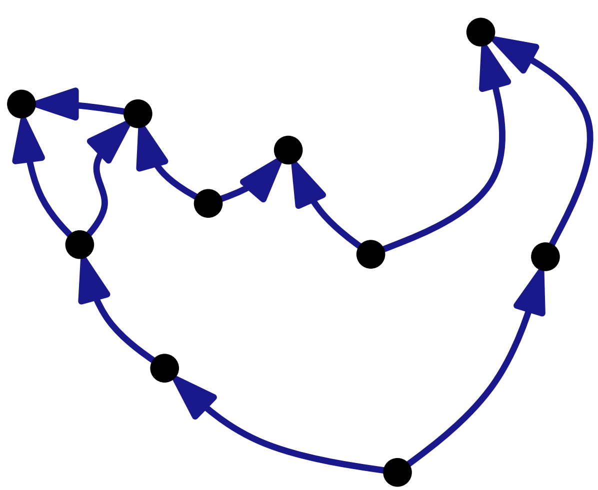

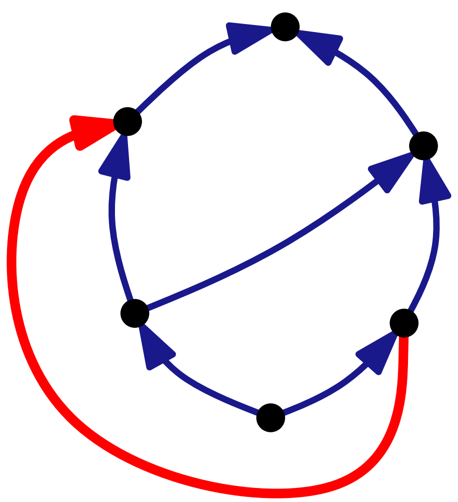

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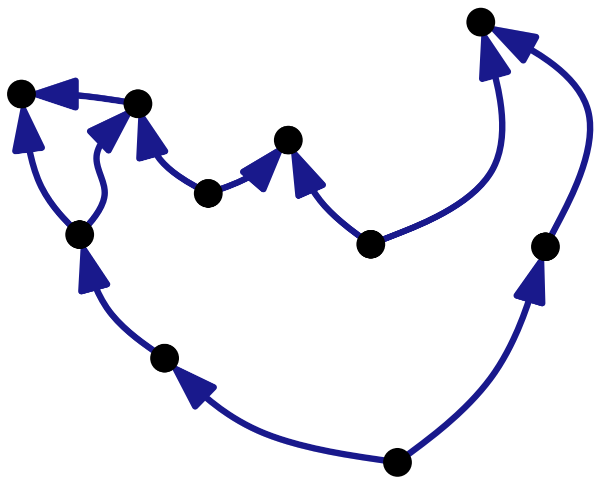

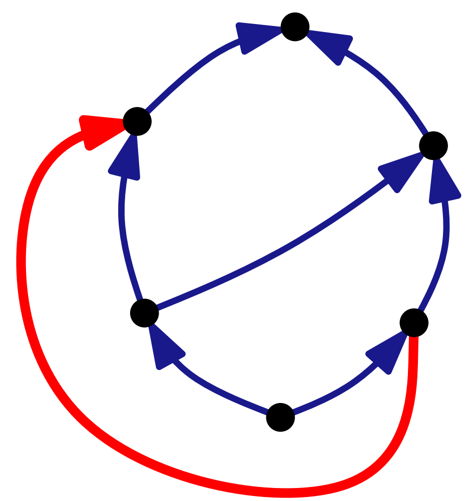

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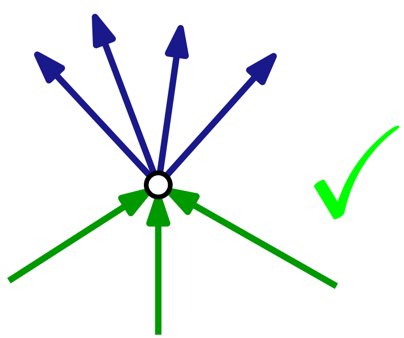

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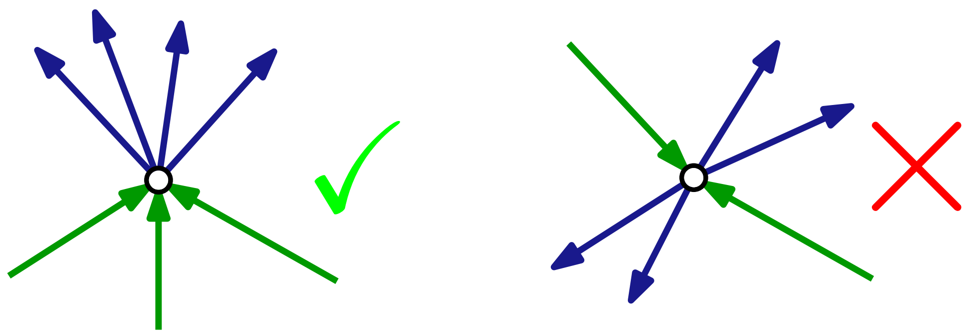

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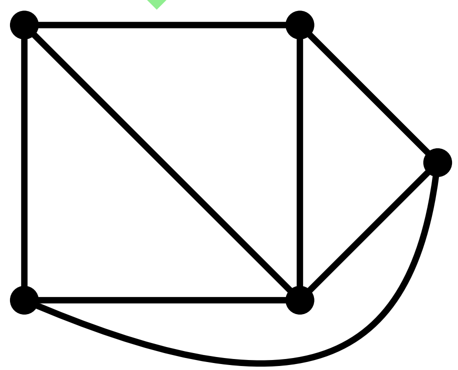

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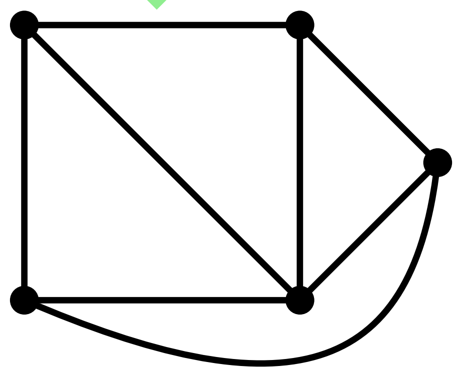

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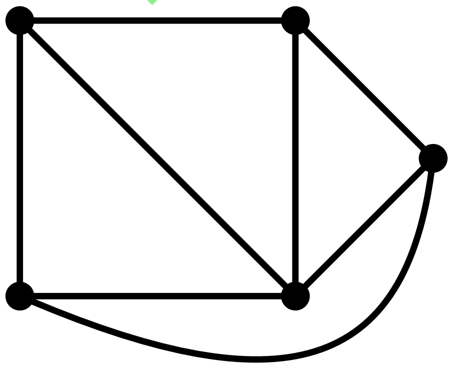

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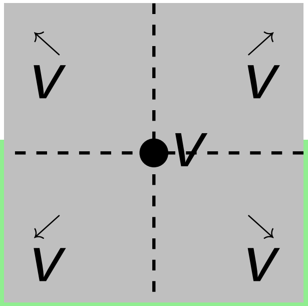

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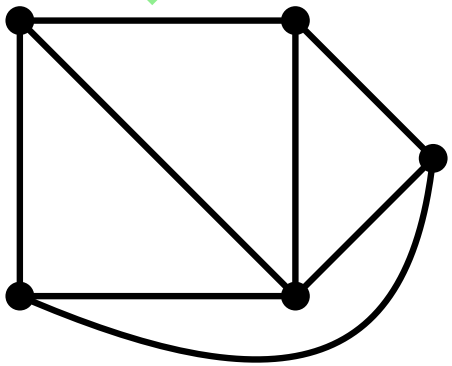

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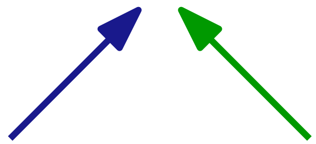
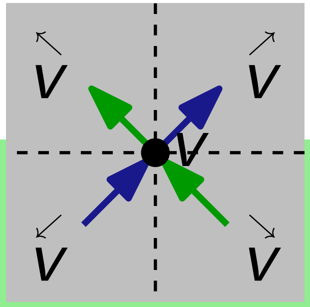
Two directions: 



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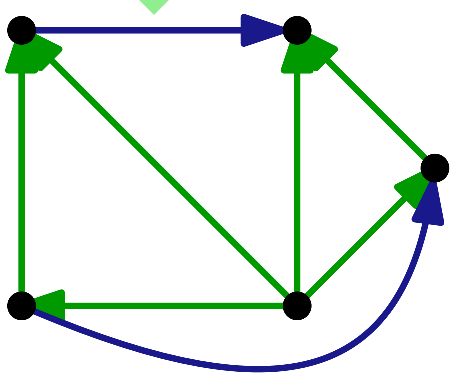

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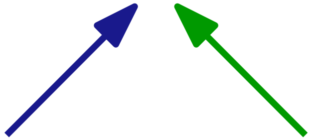
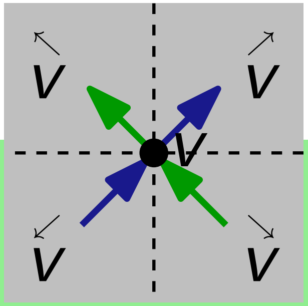
Two directions: 



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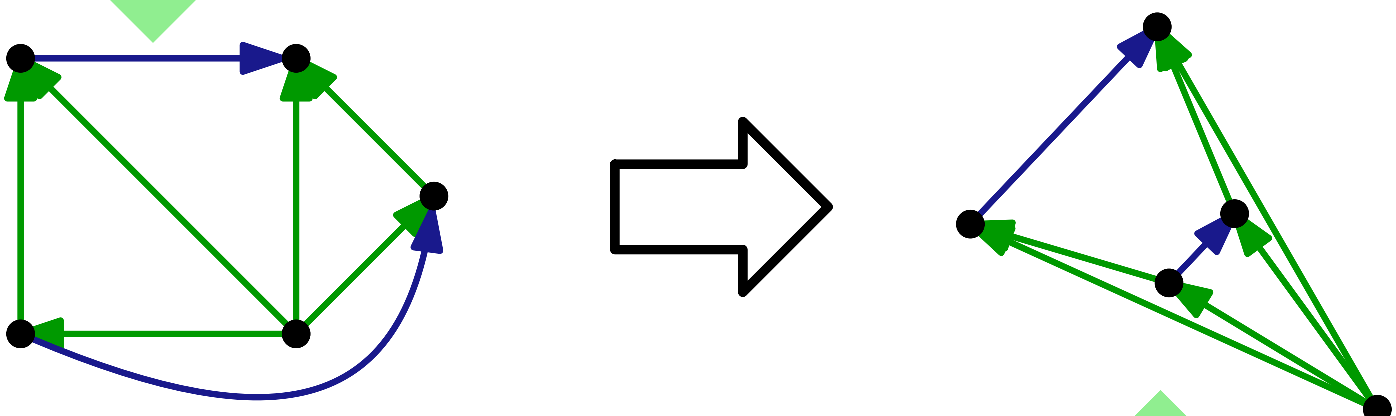

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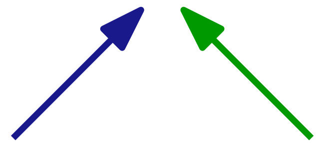
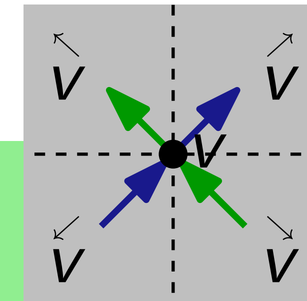
Two directions: 



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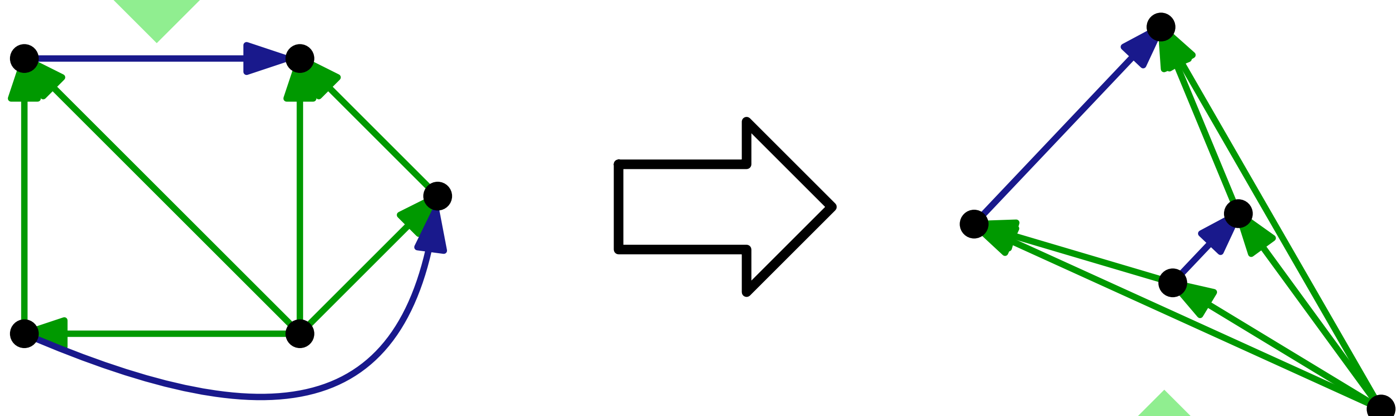

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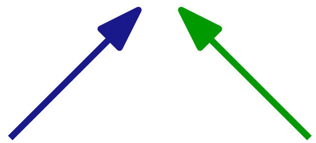
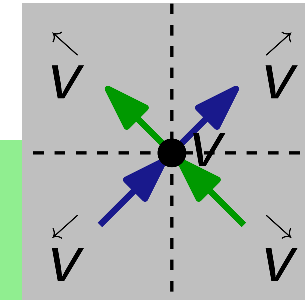
Two directions: 



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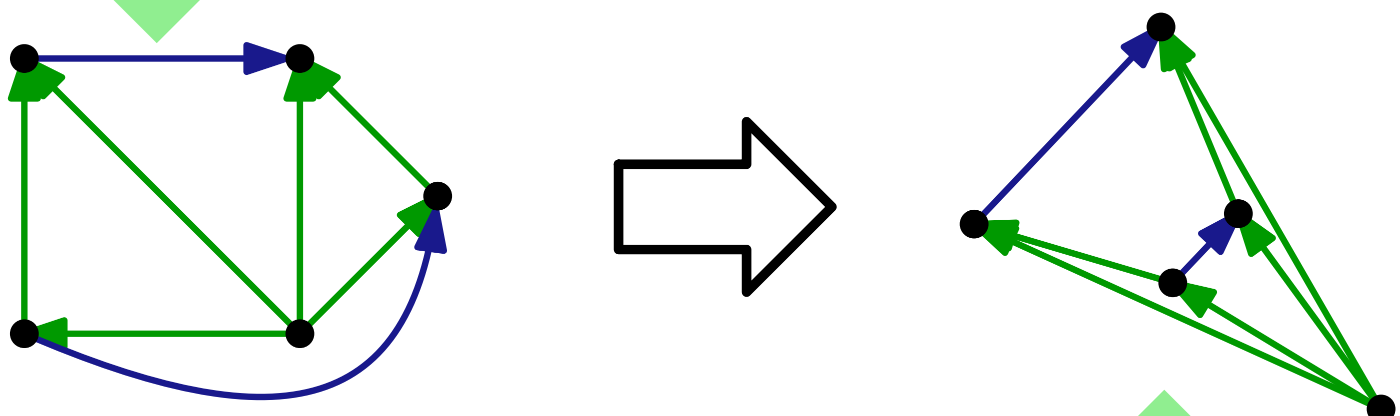

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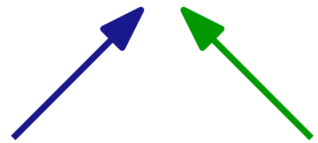
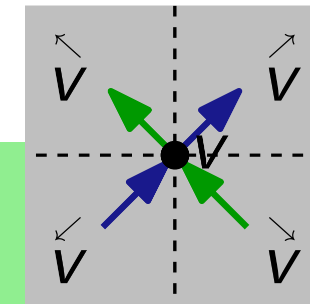
Two directions: 



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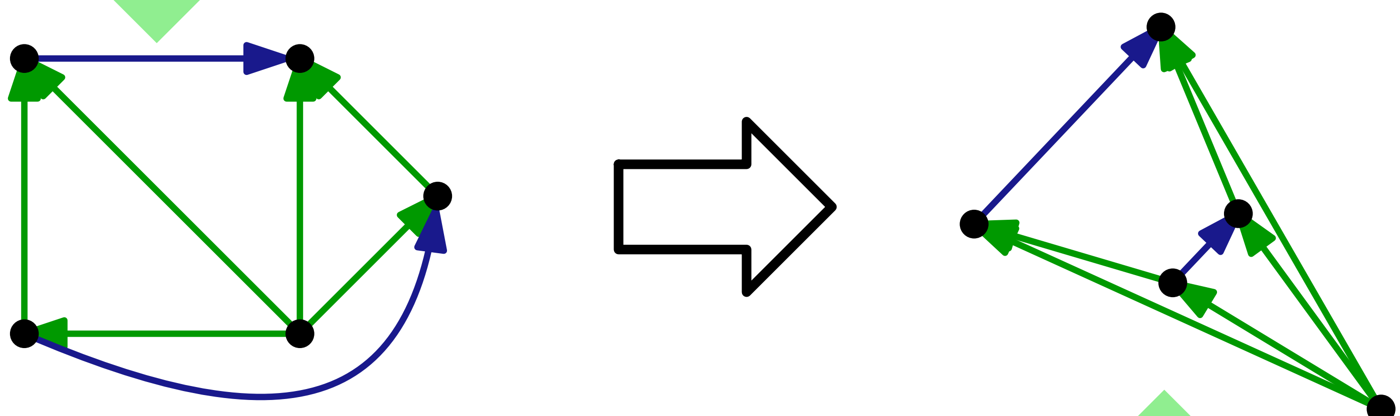

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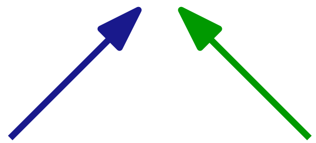
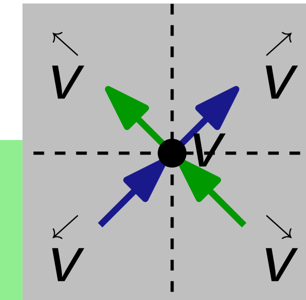
Two directions: 



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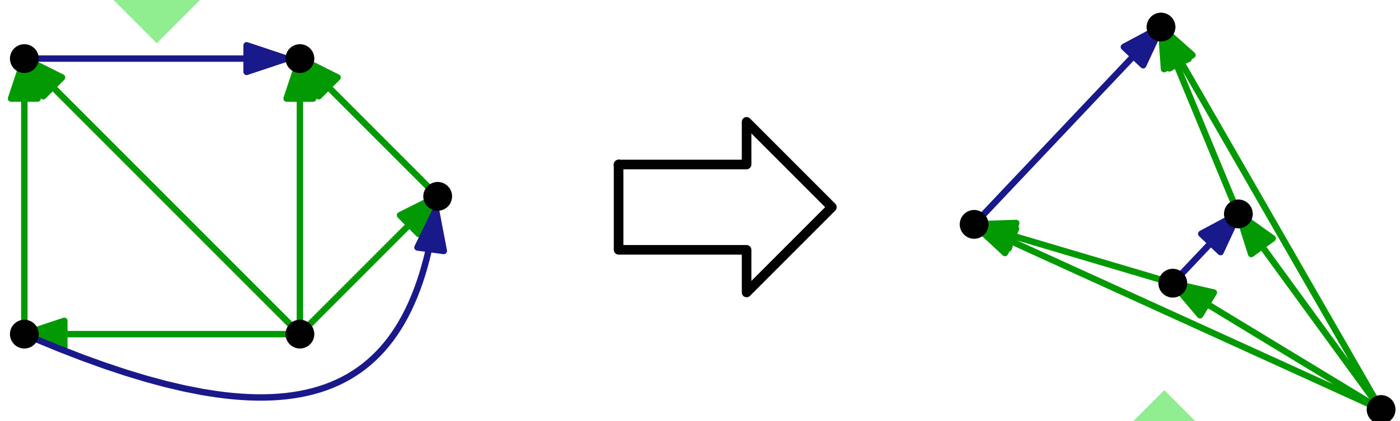

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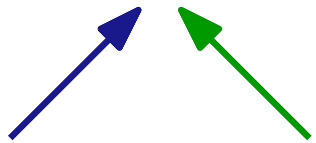
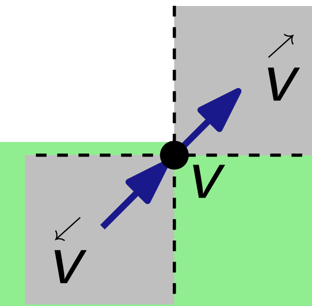
Two directions: 



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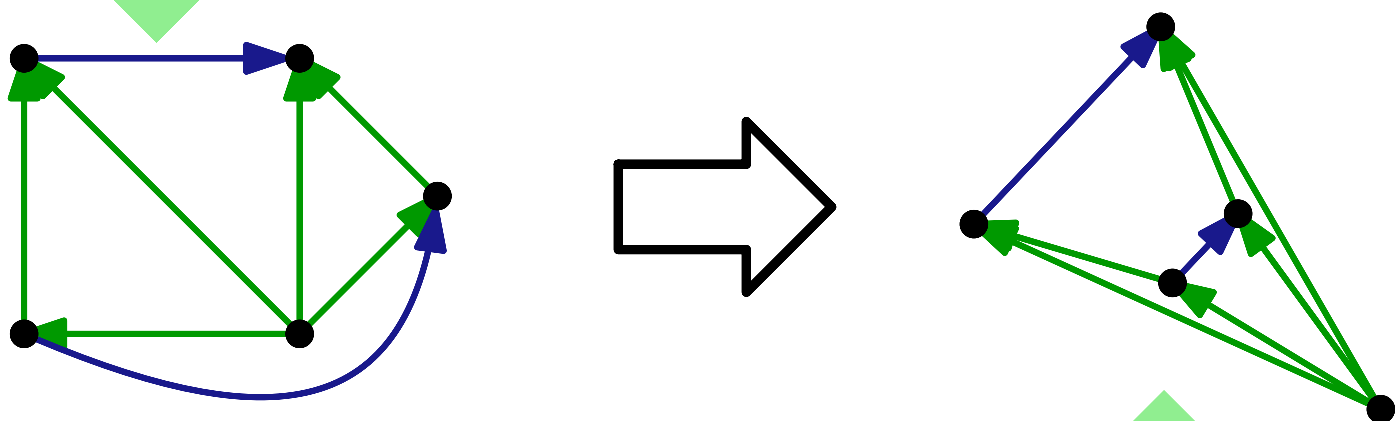

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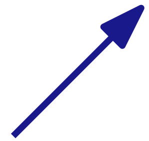
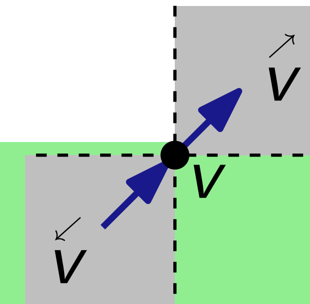
One direction: 



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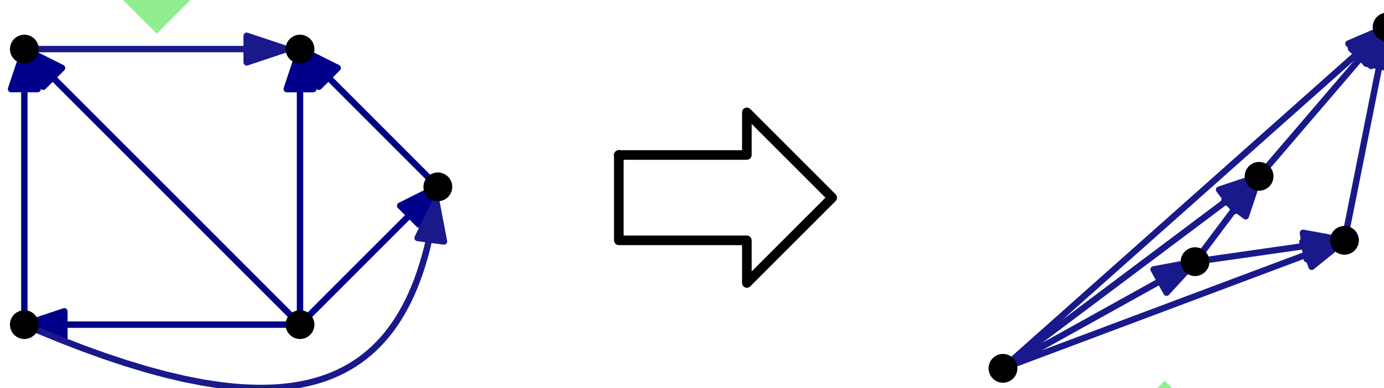

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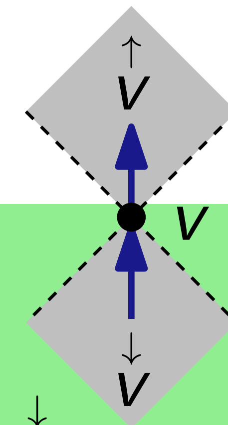

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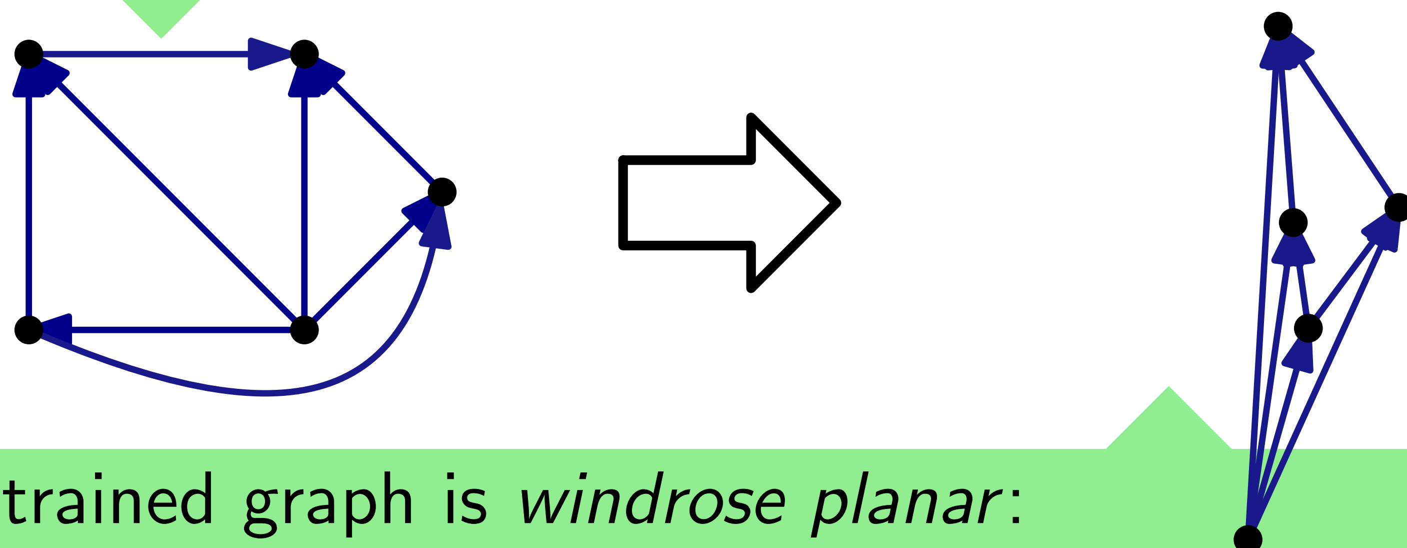

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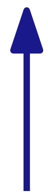

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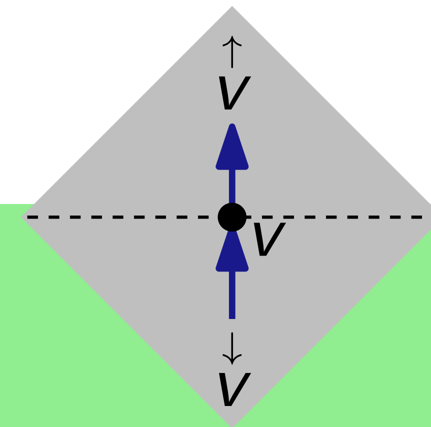

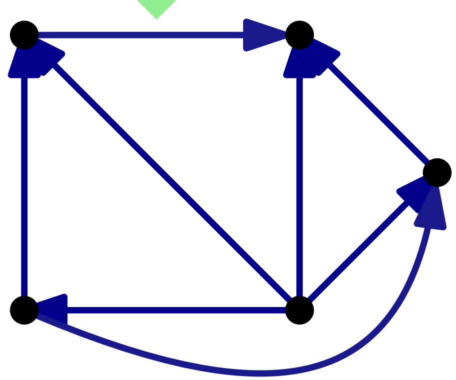

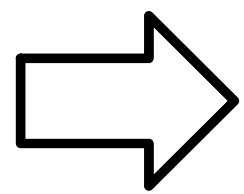

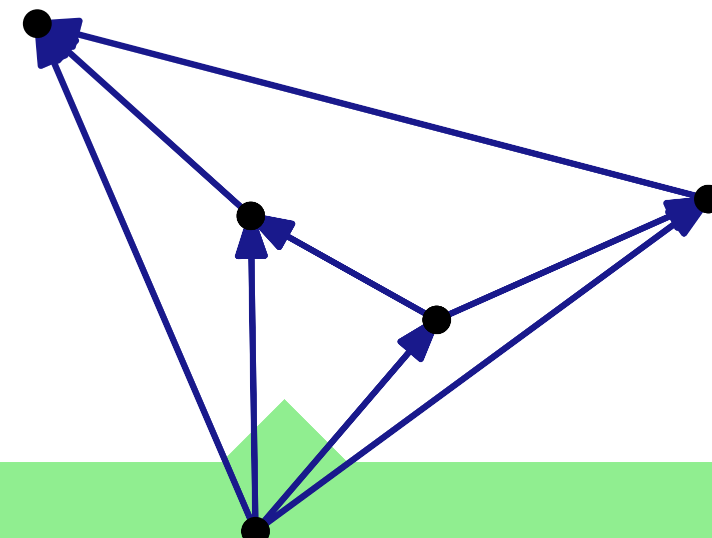

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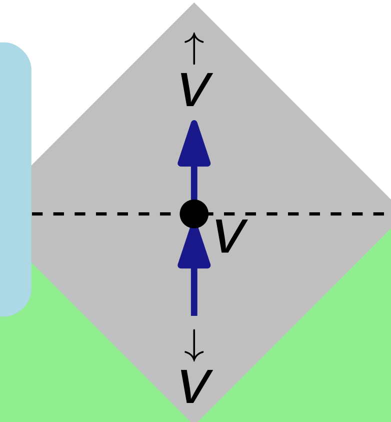
Relationship to Upwards Planarity

One direction: $\uparrow$

Theorem.
Testing Windrose Planarity is NP-complete



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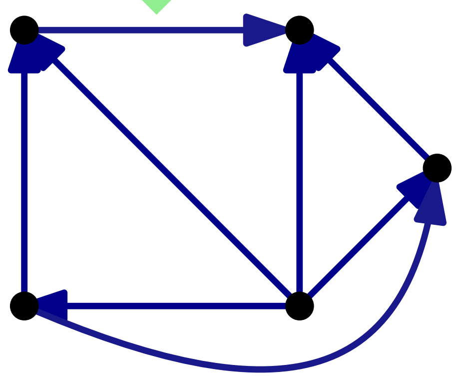

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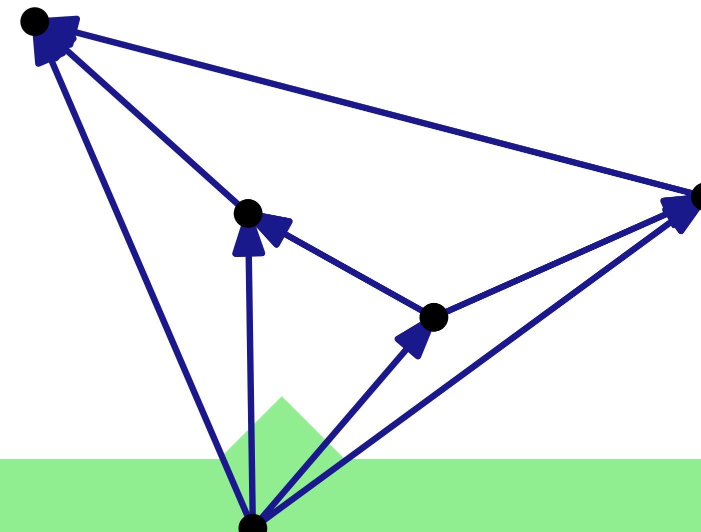

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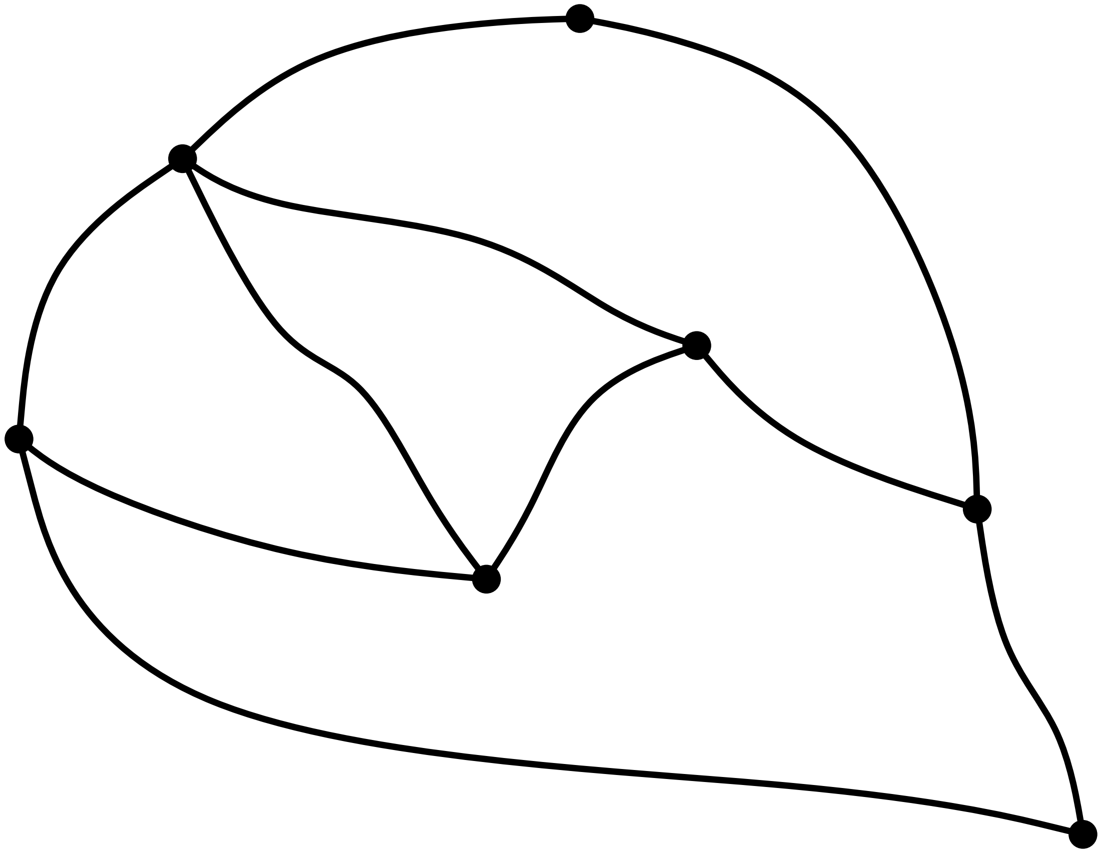

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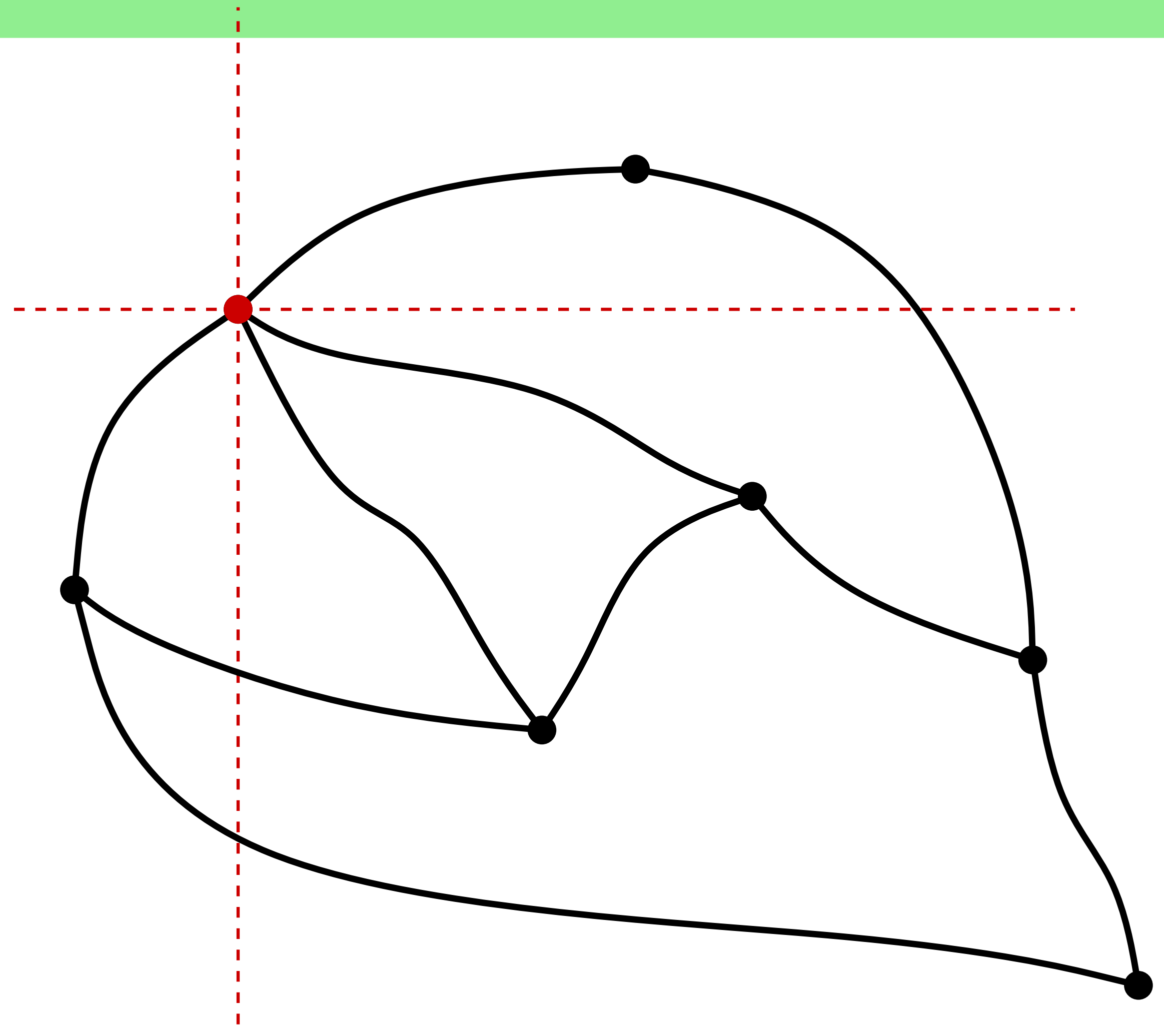

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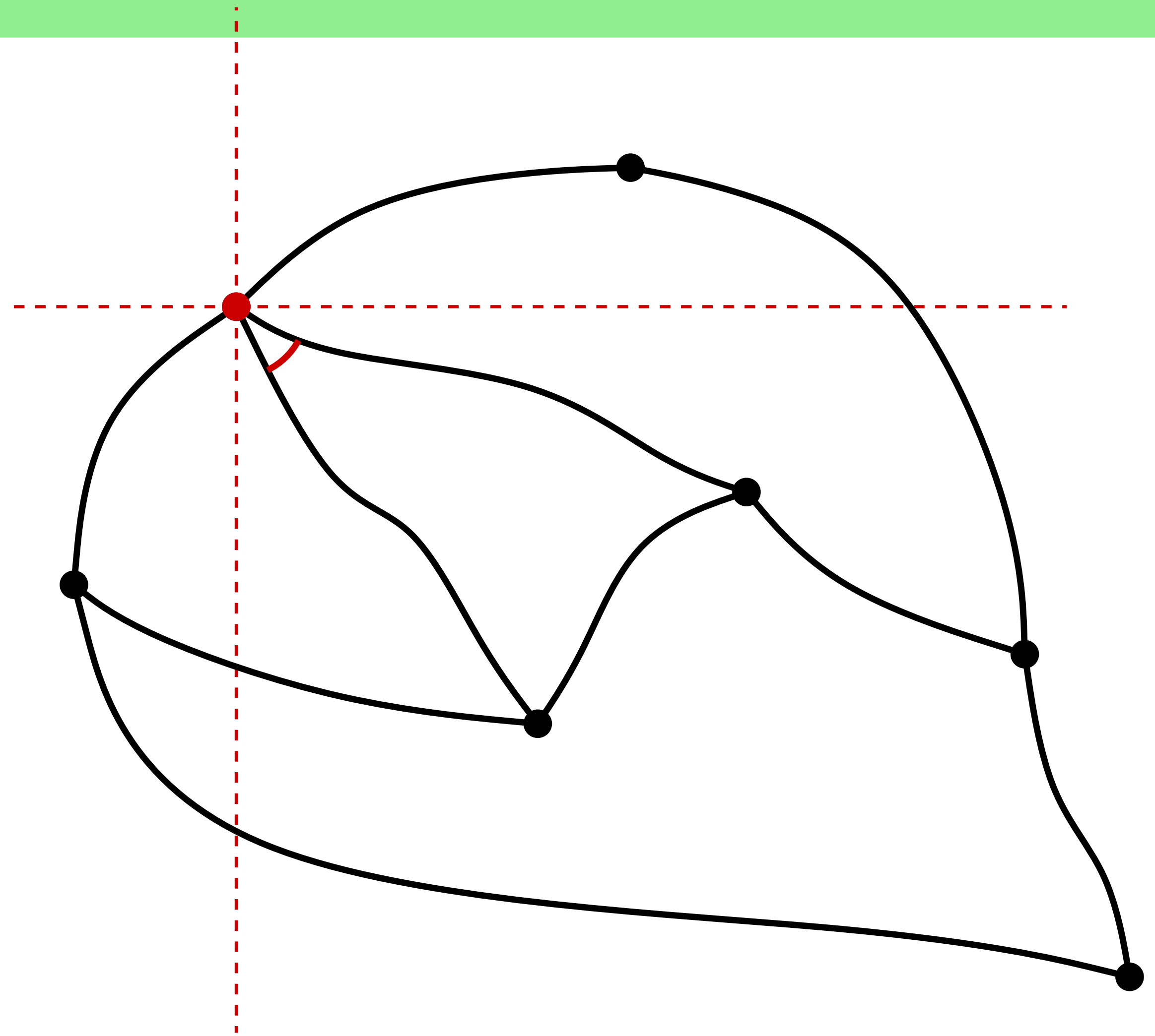

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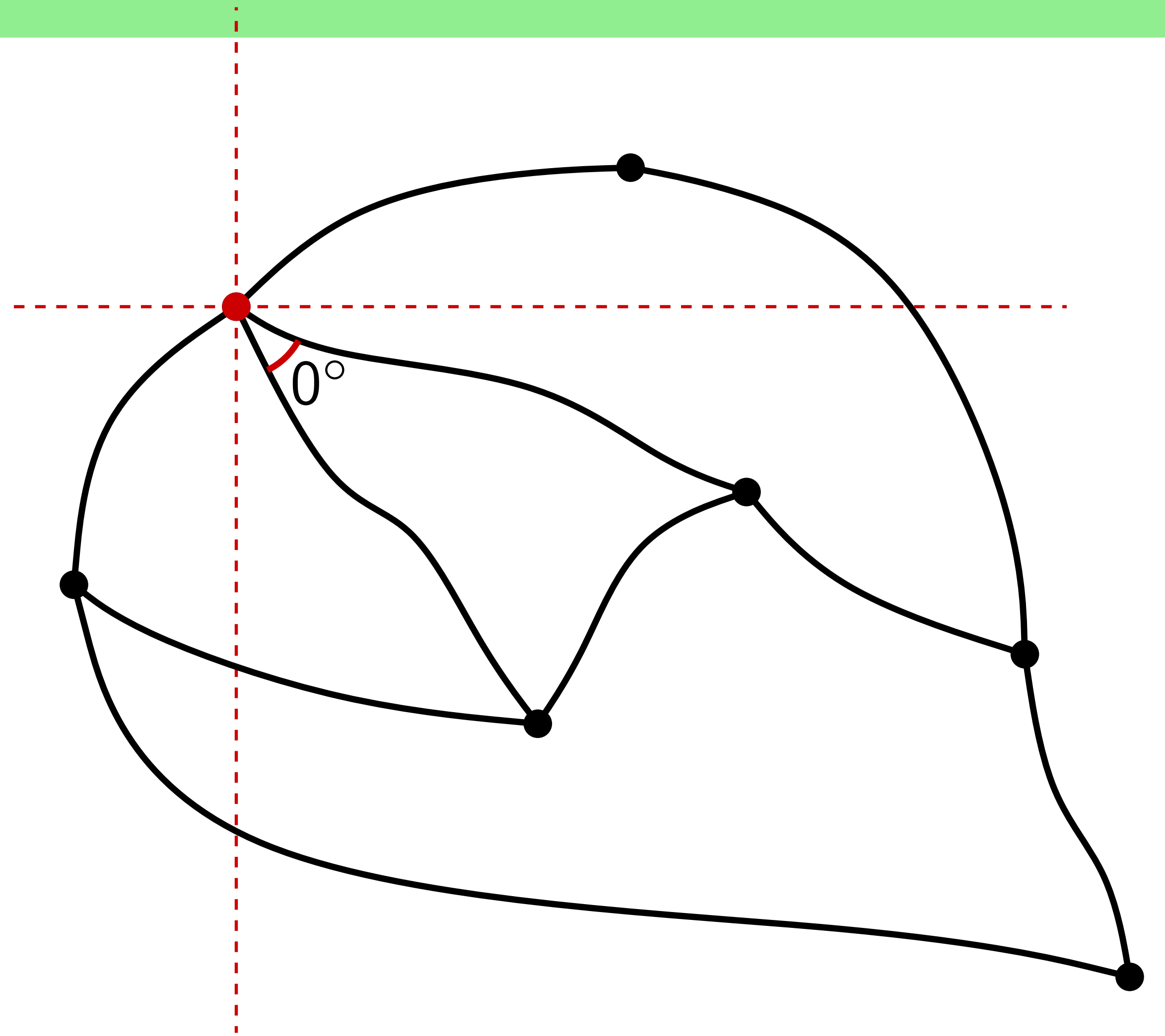

Angular Drawing

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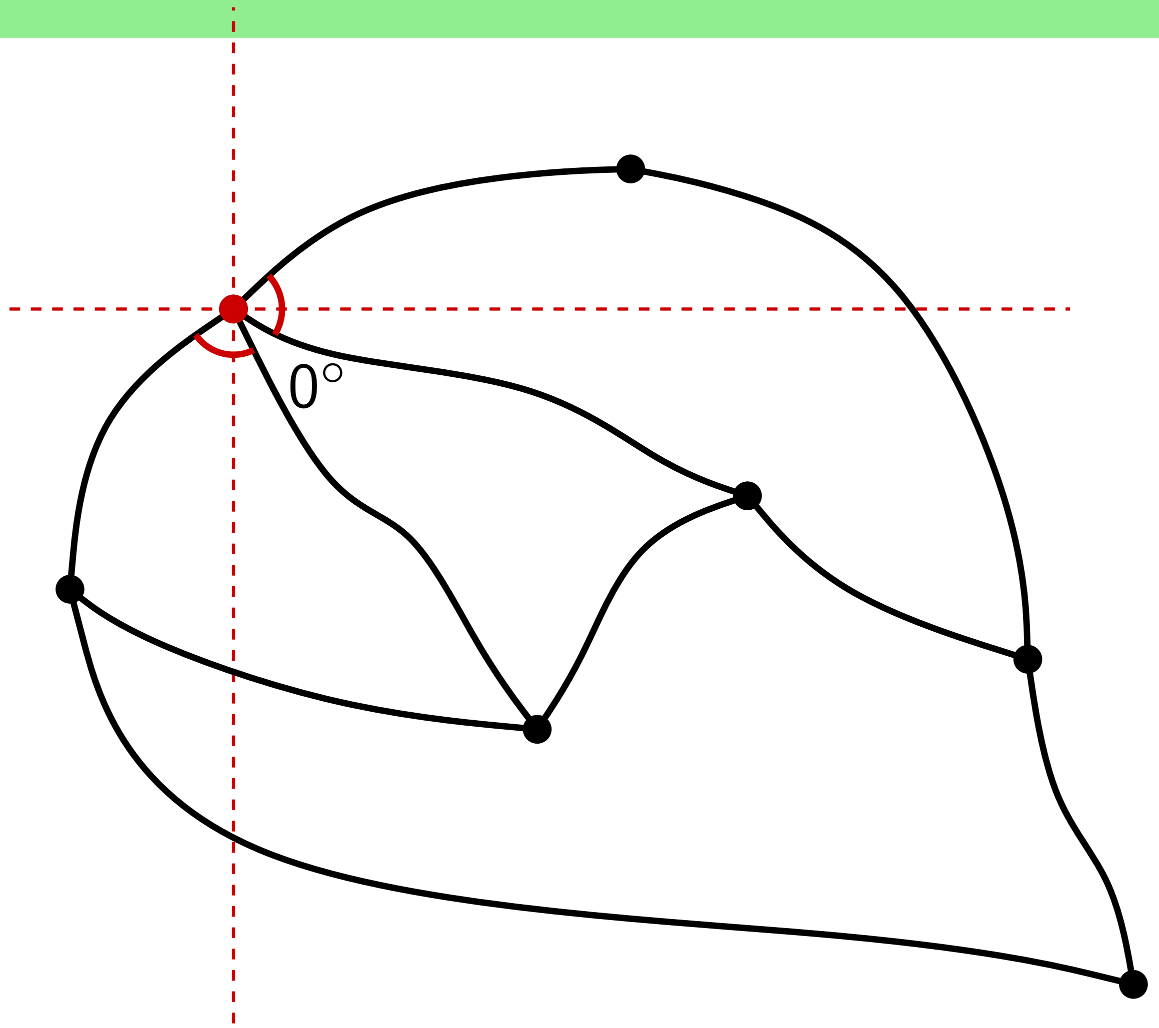

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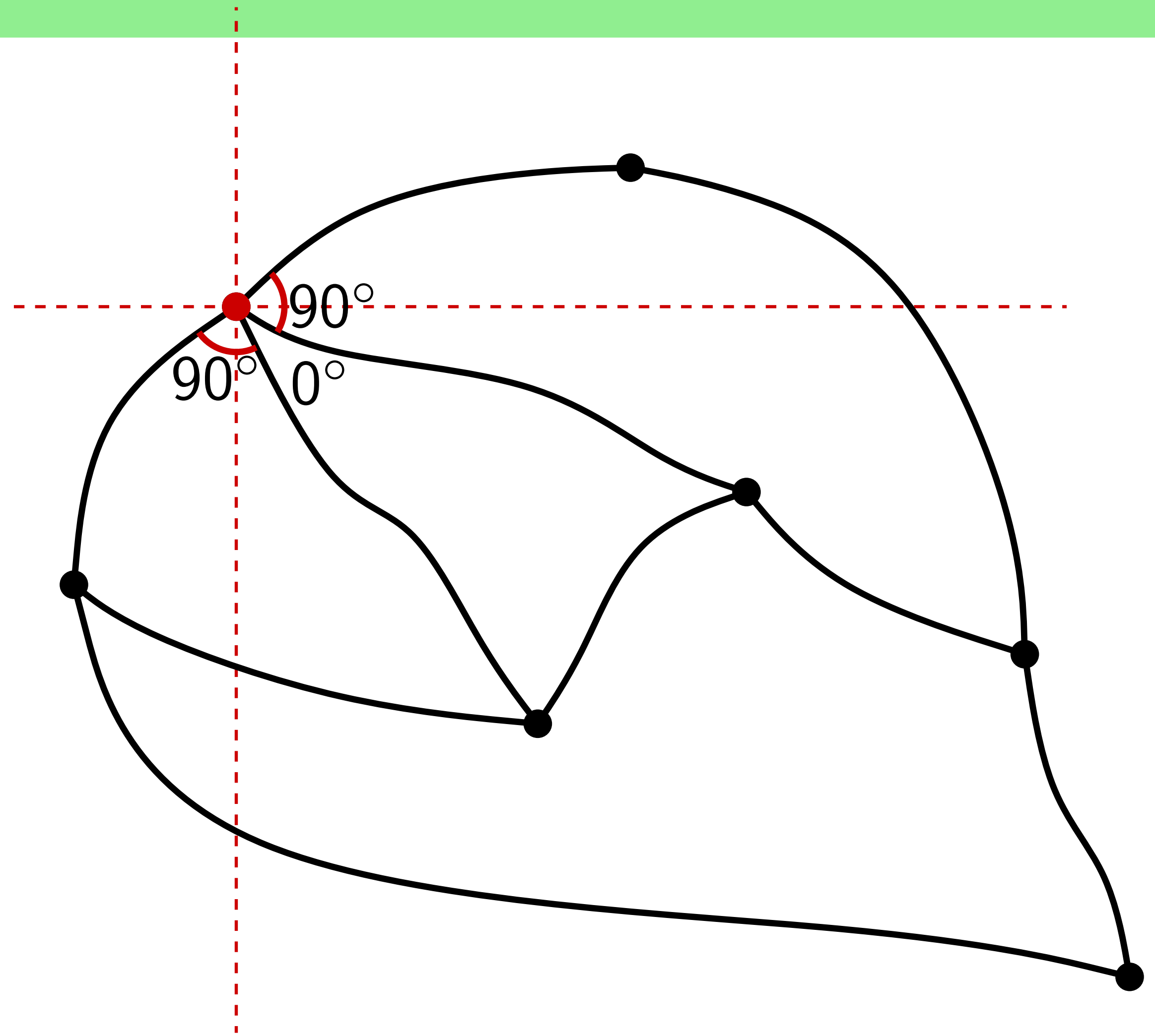

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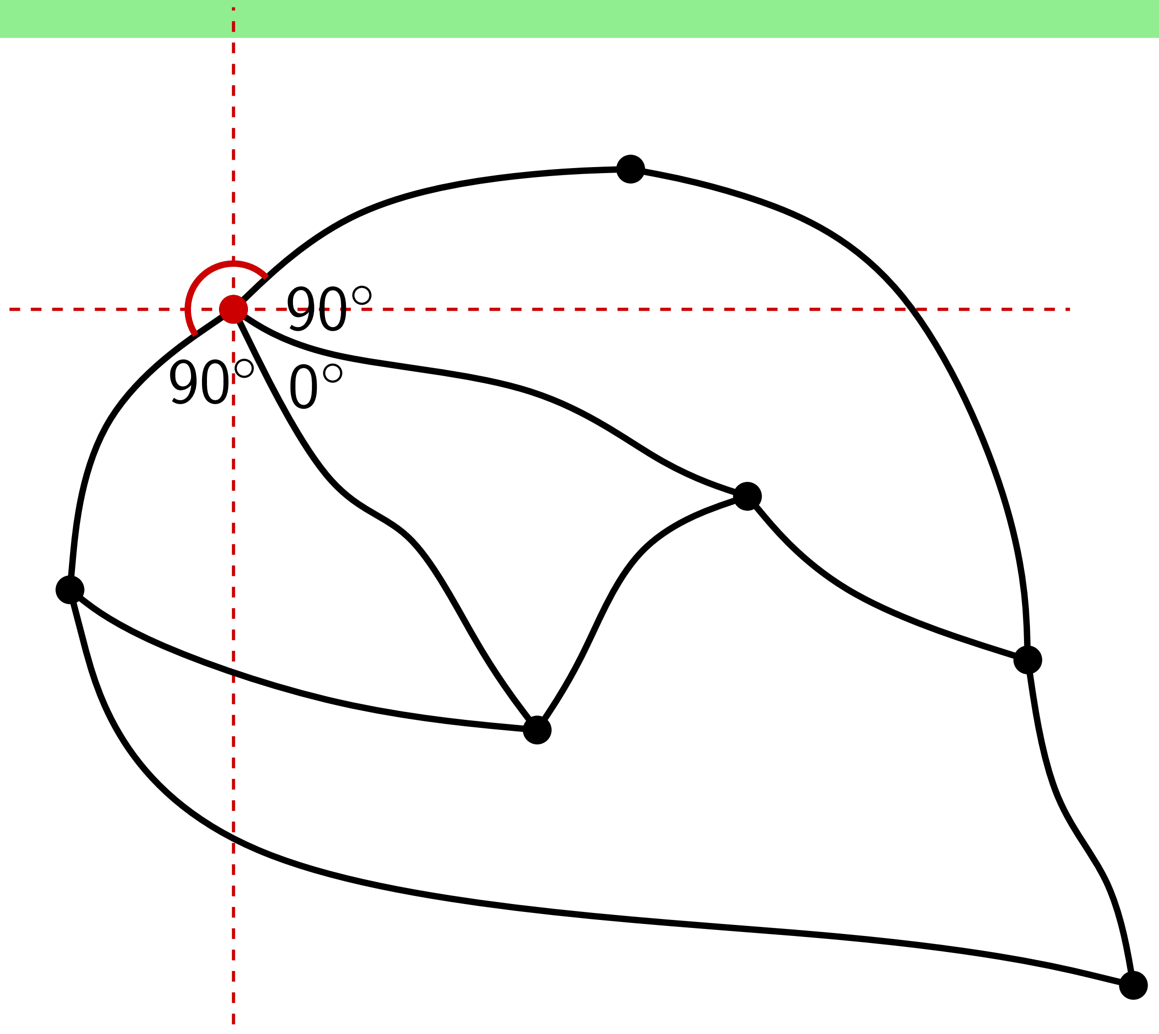

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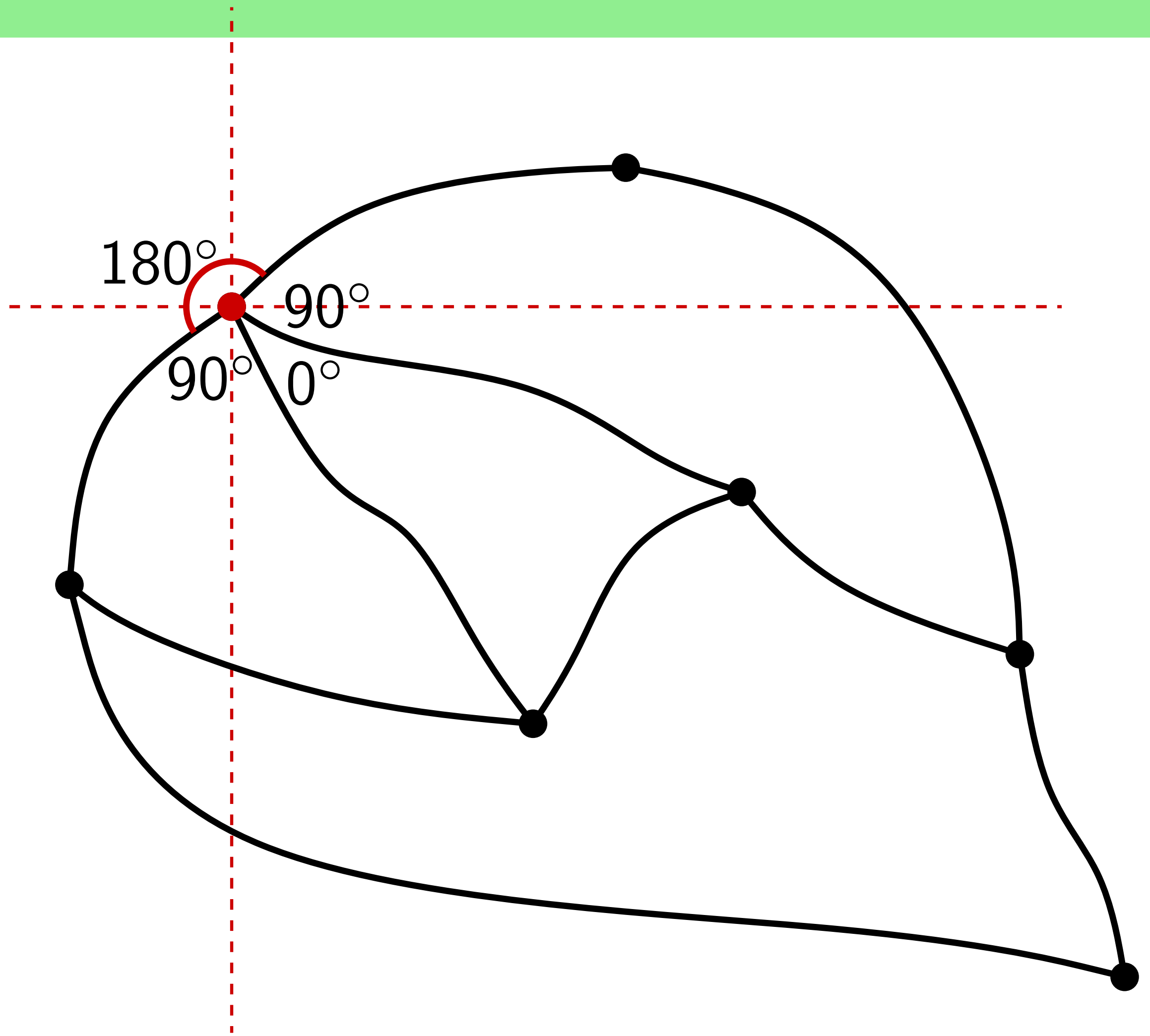

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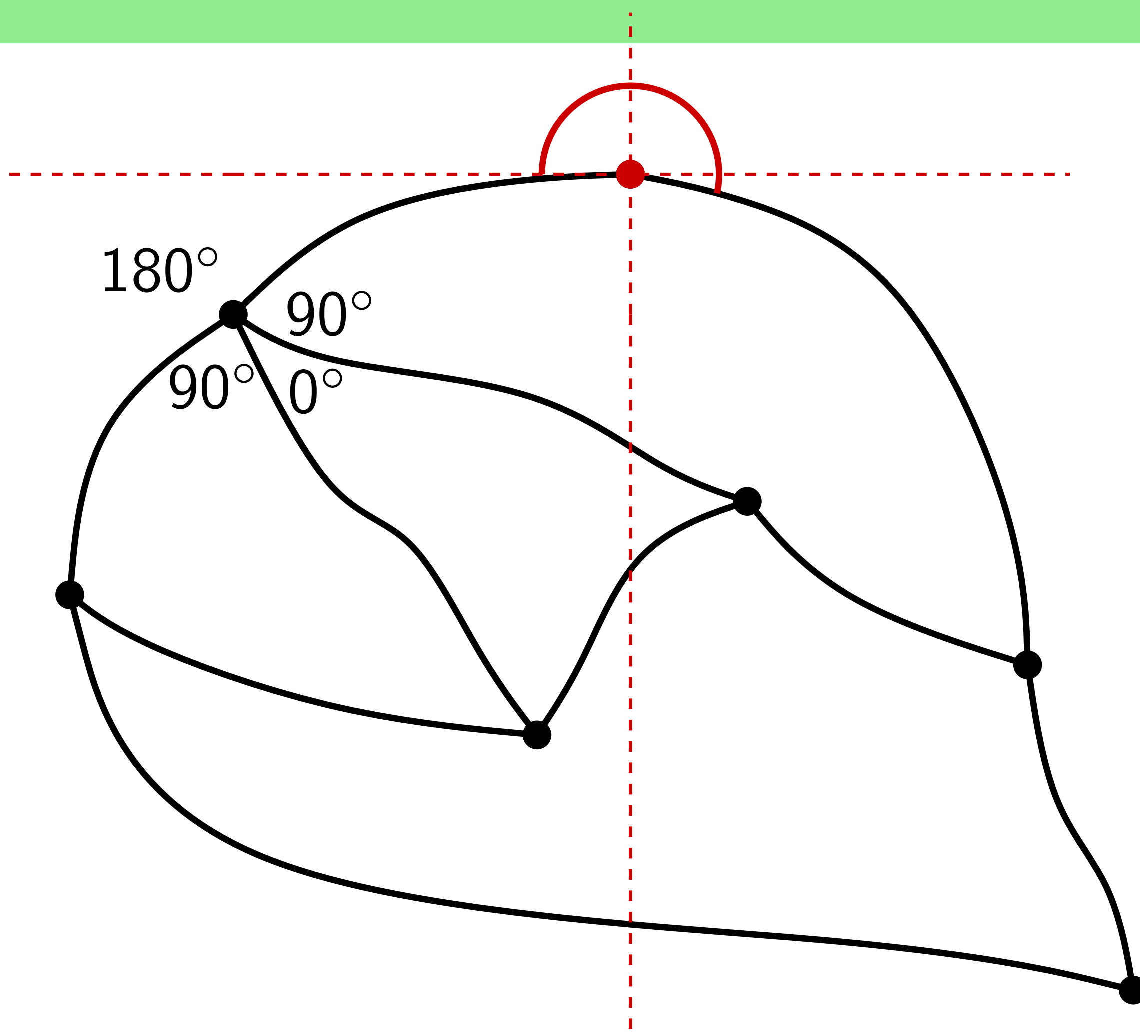

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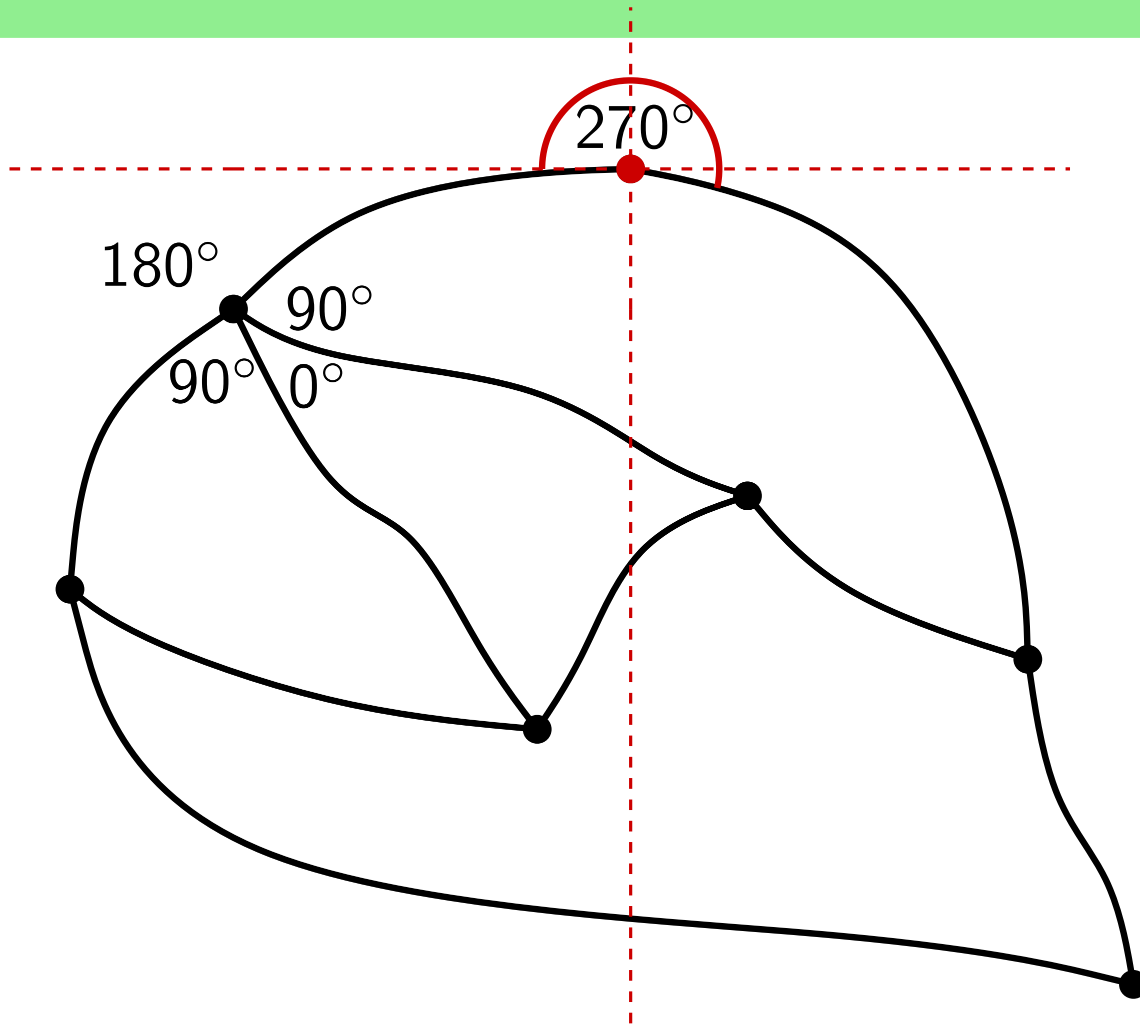

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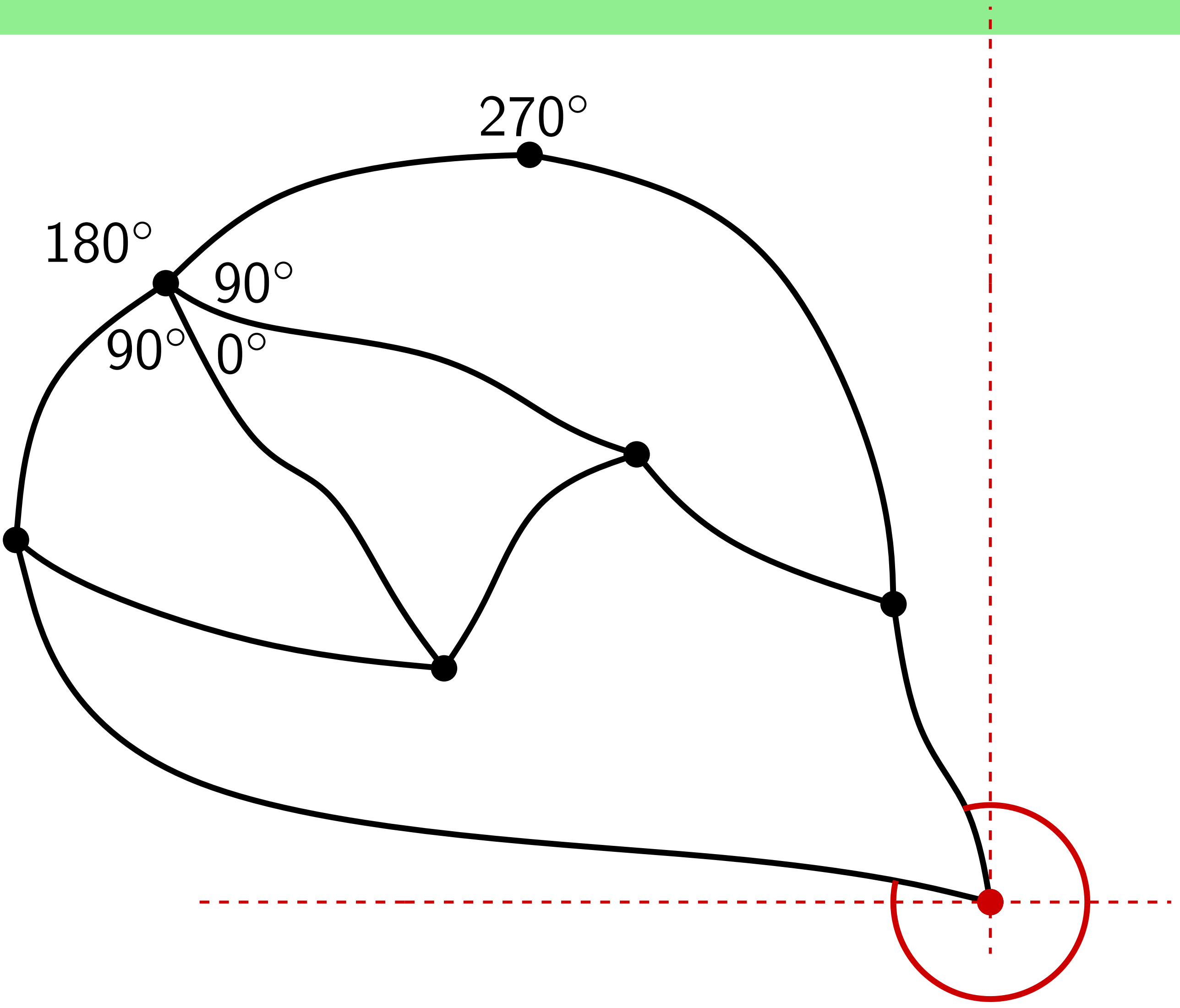

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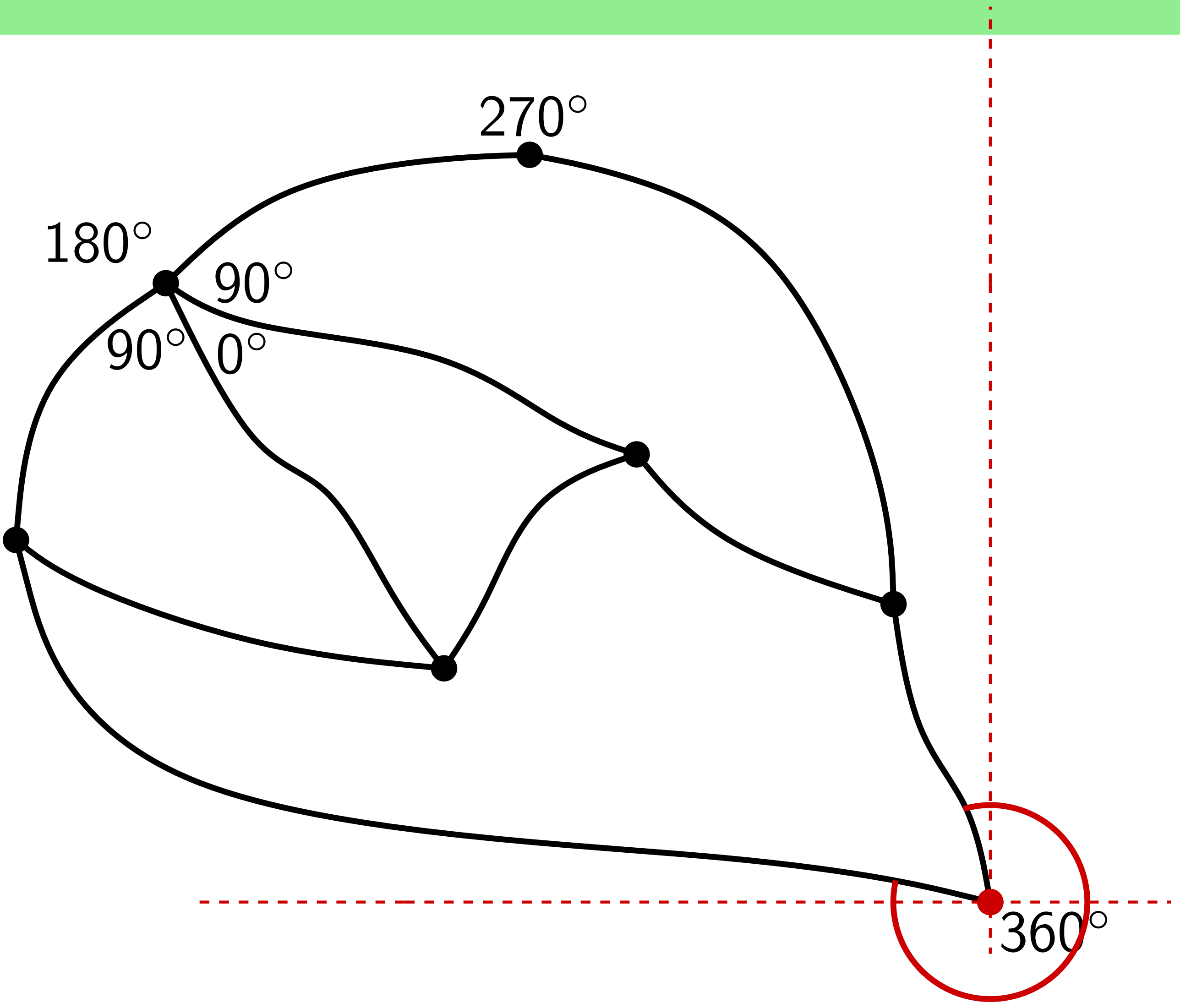

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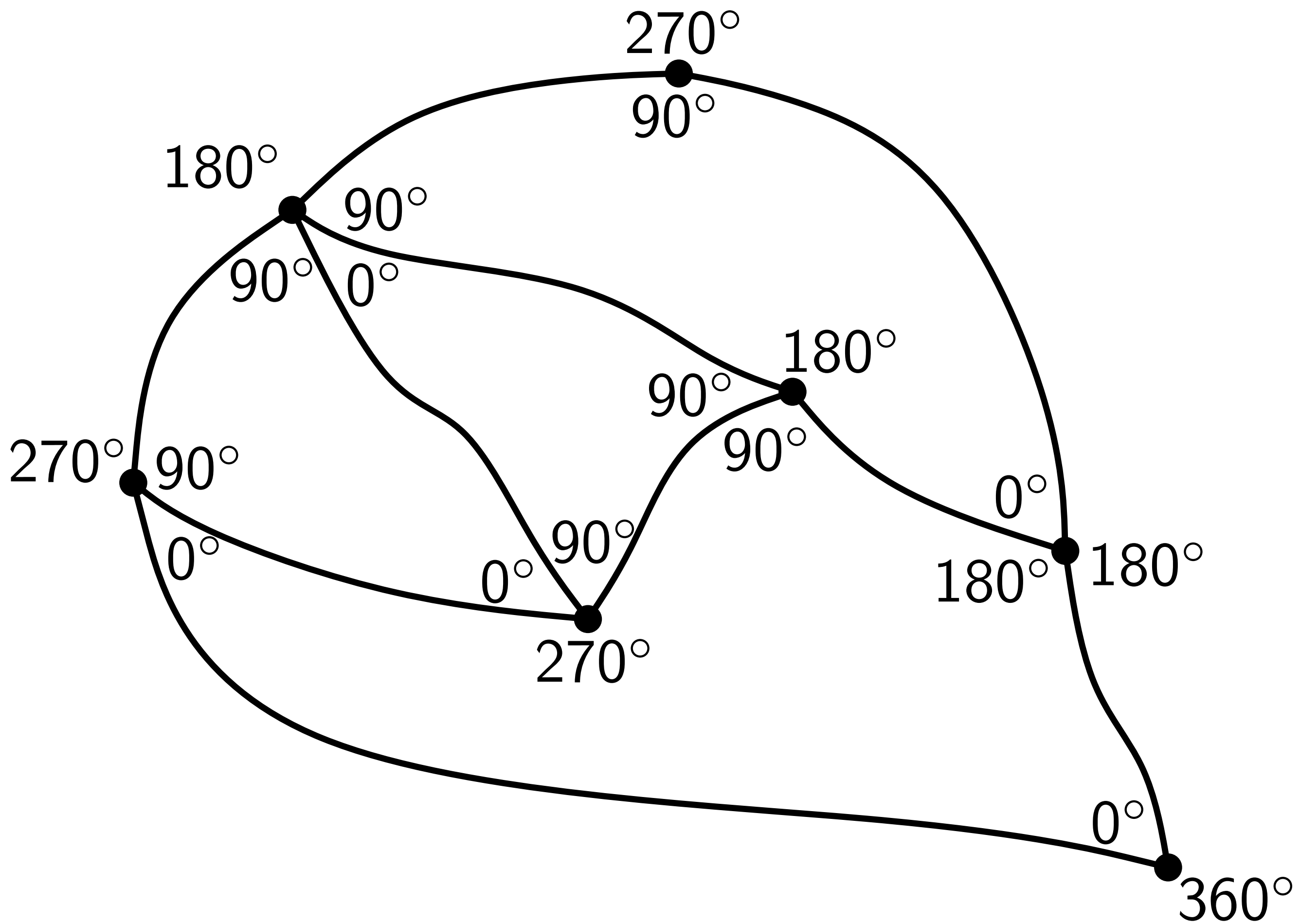
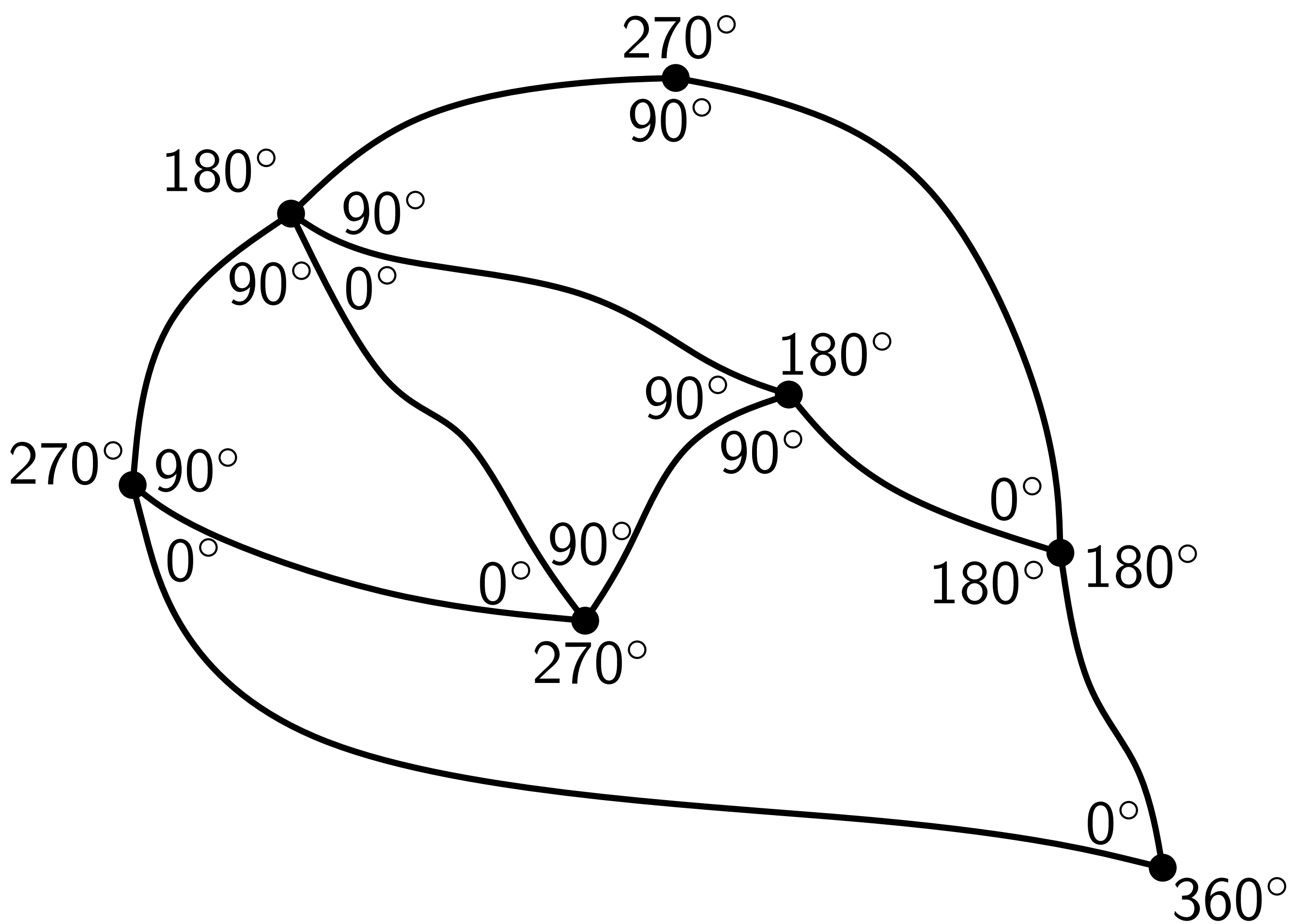


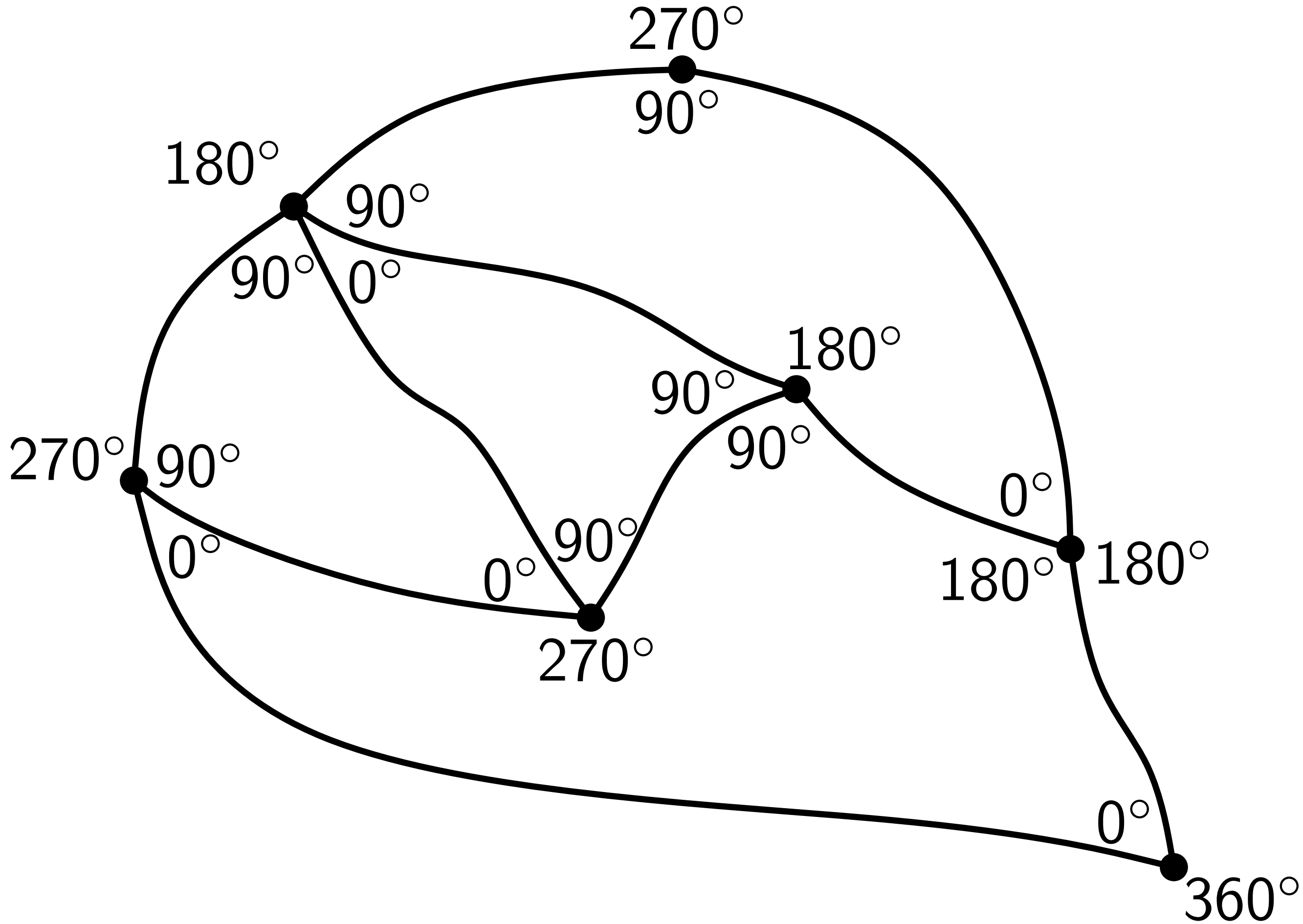
Angular Drawing

Angle categories: 0°, 90°, 180°, 270°, and 360°
Labeled graph (G, A): G plane graph, A labeling of angles



Angular Drawing

Angle categories: $0°$, $90°$, $180°$, $270°$, and $360°$
Labeled graph (G, A): G plane graph, A labeling of angles
Angular drawing: end of segments have slopes $\approx \pm 1$



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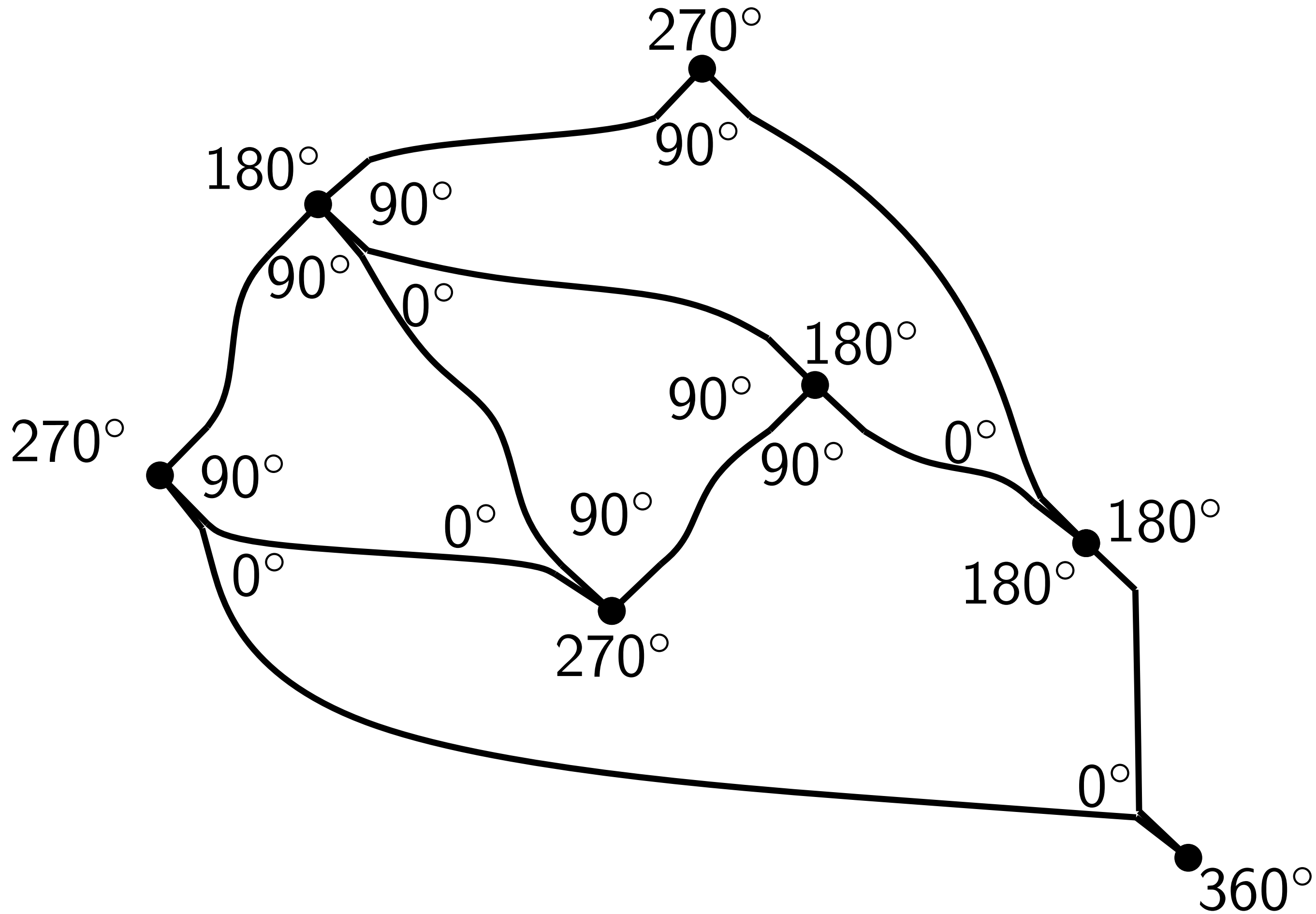

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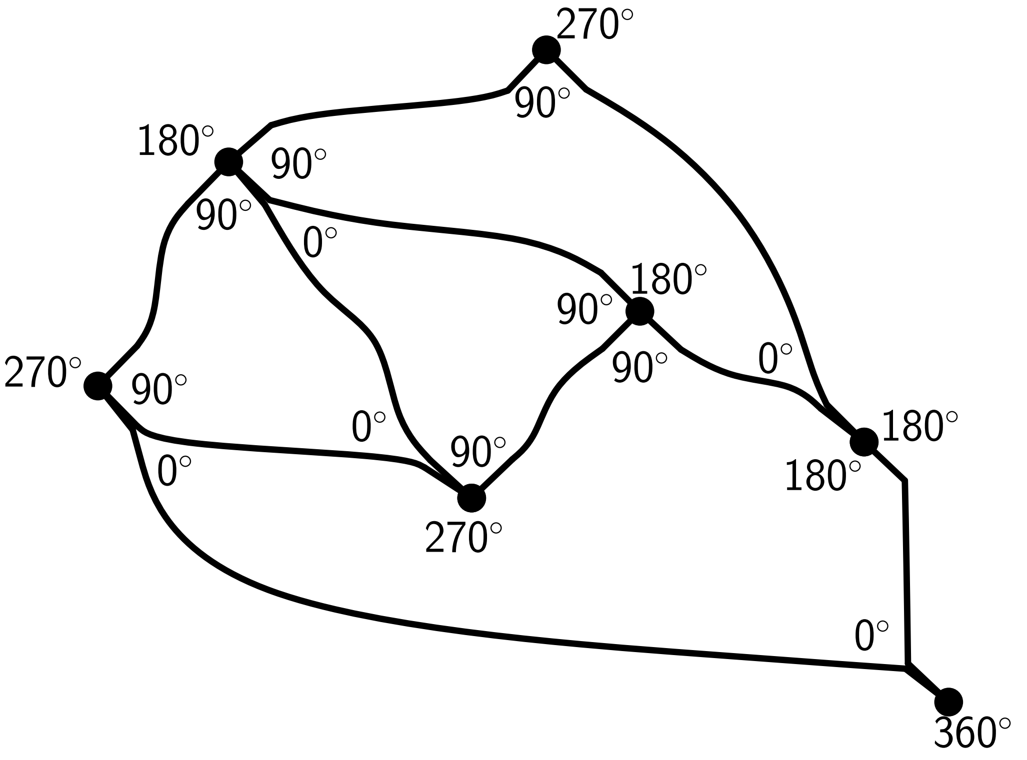

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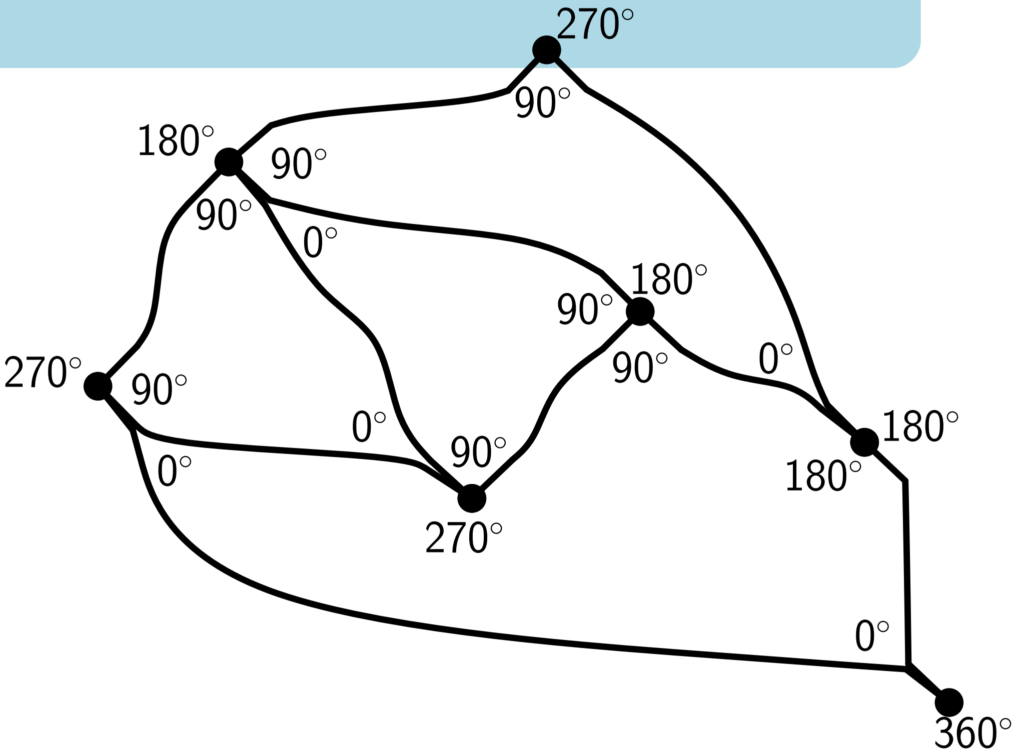
(G, A) admits angular drawing if:



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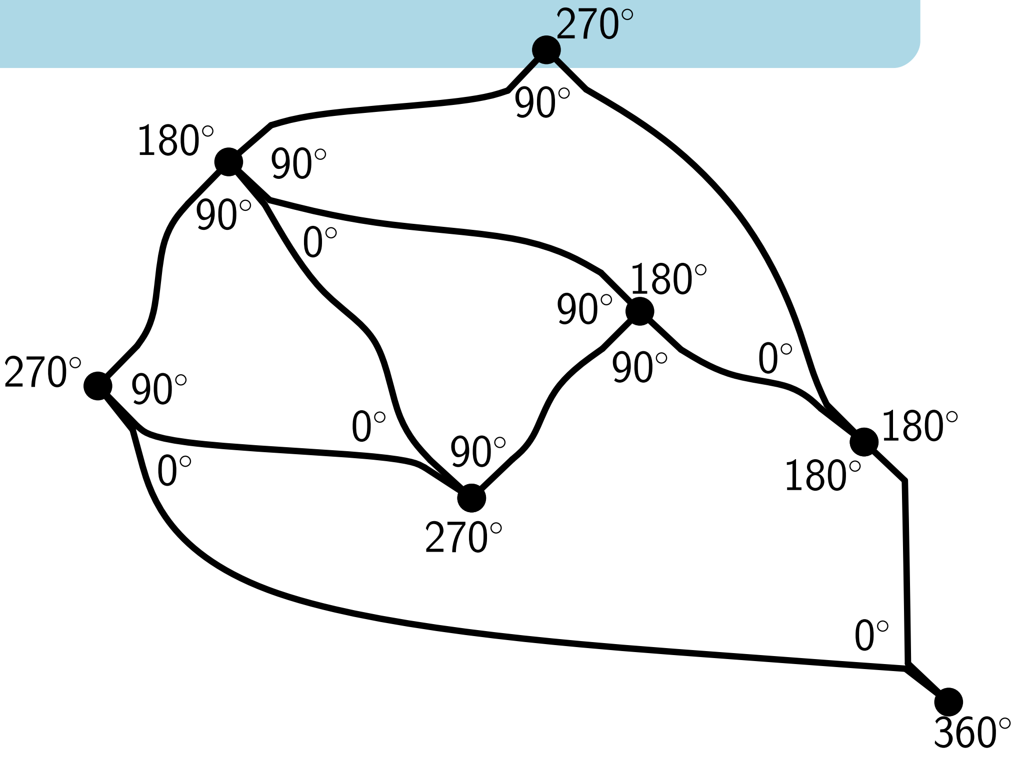

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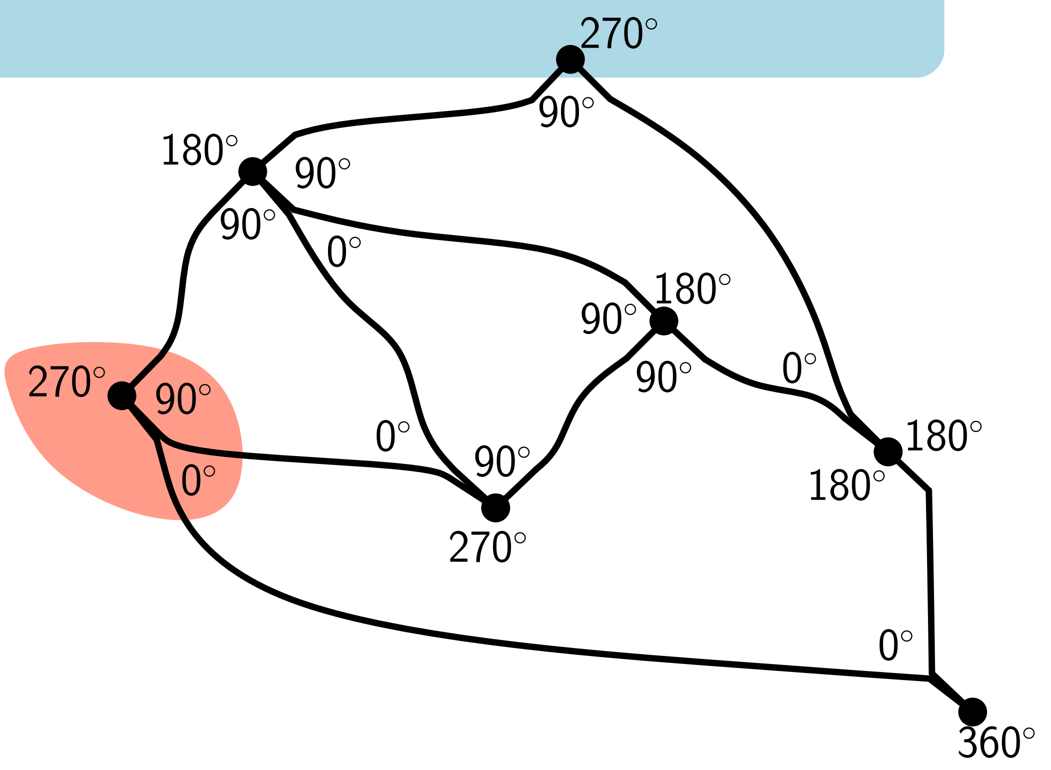

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Labeled graph (G, A): G plane graph, A labeling of angles
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(G, A) admits angular drawing if:

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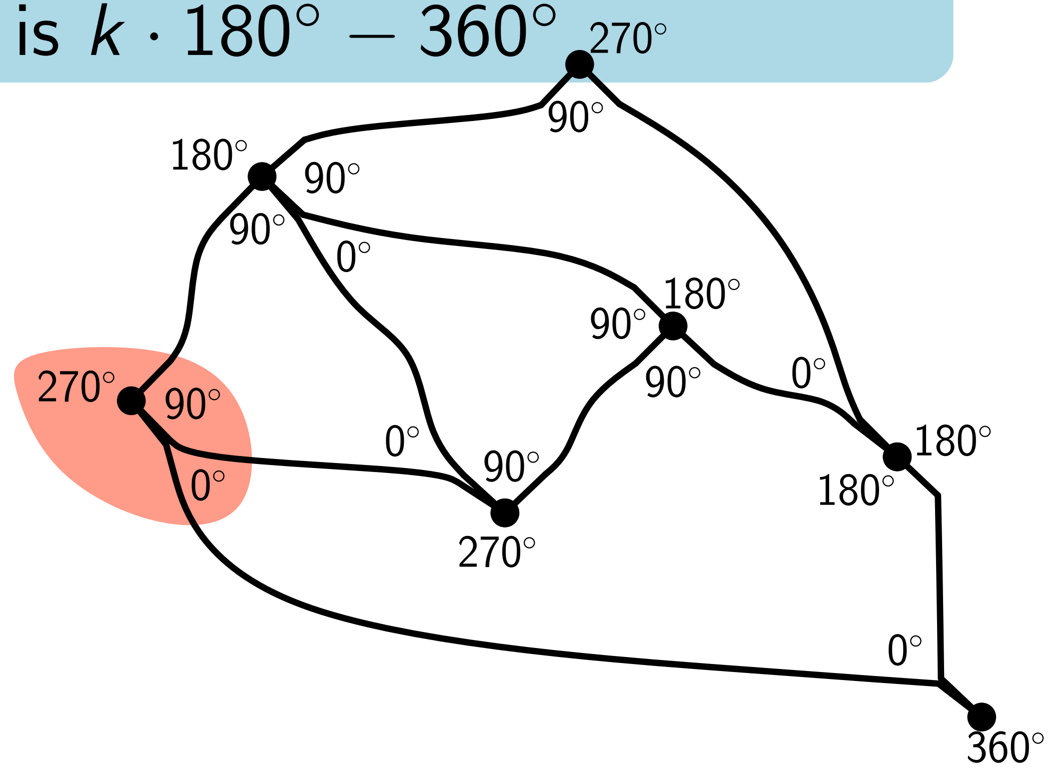

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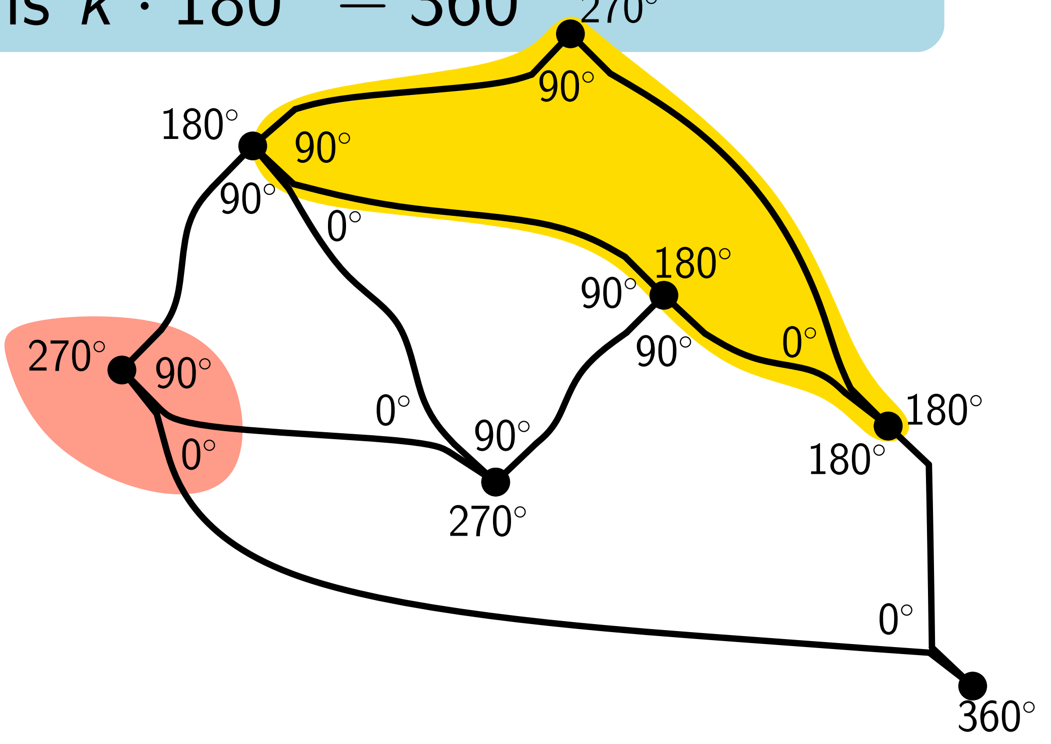

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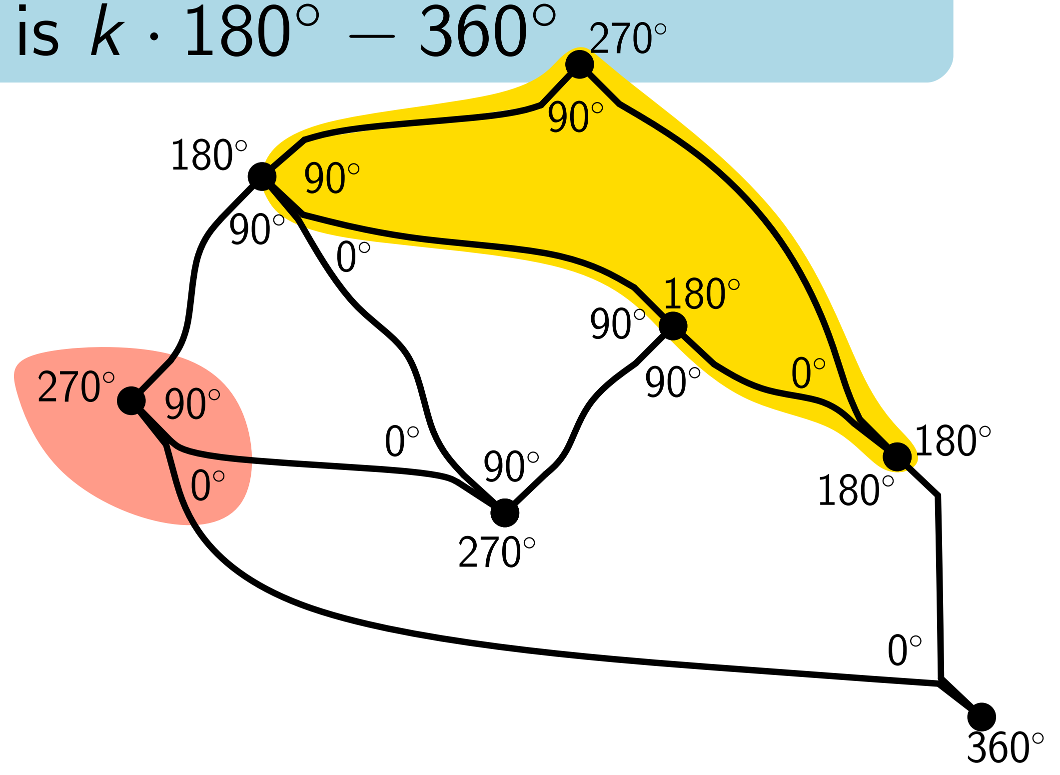

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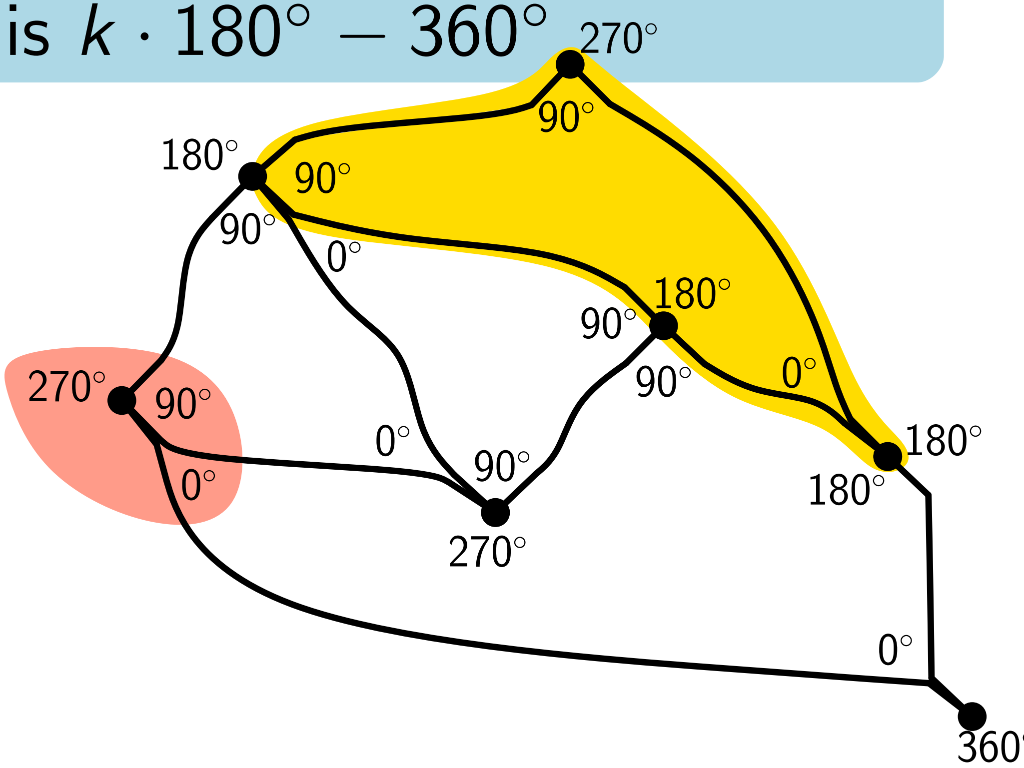
angular labeling A
$\underset{\text{unique}}{\Rightarrow}$ q-constraints Q_A



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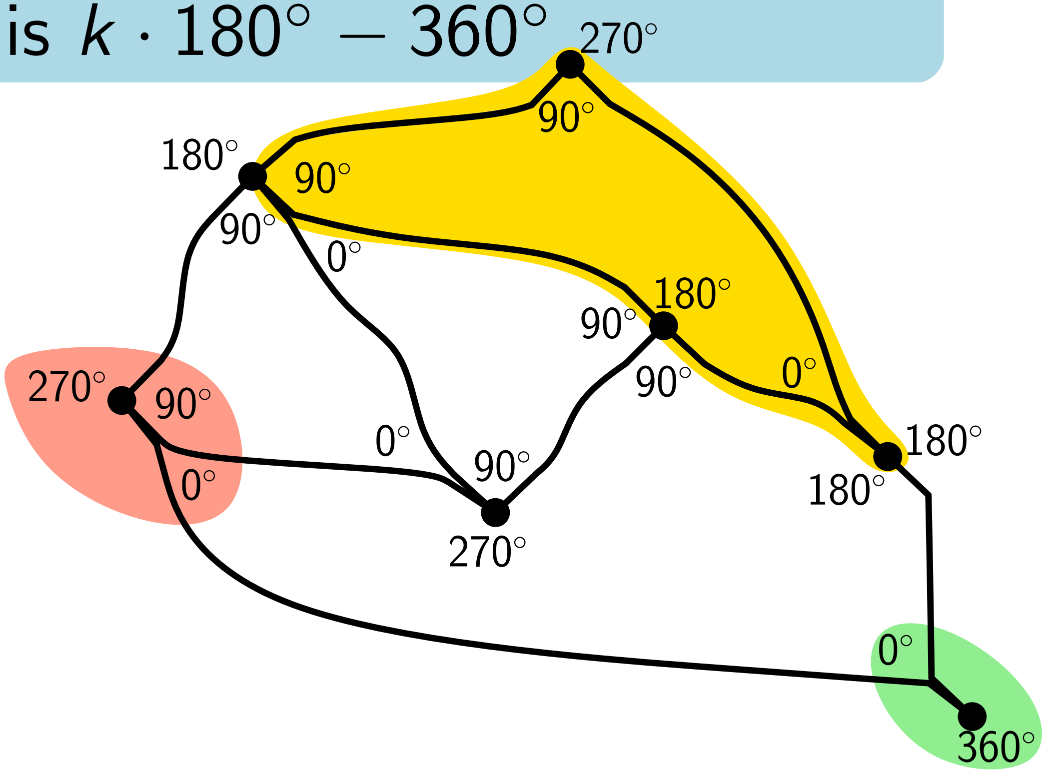

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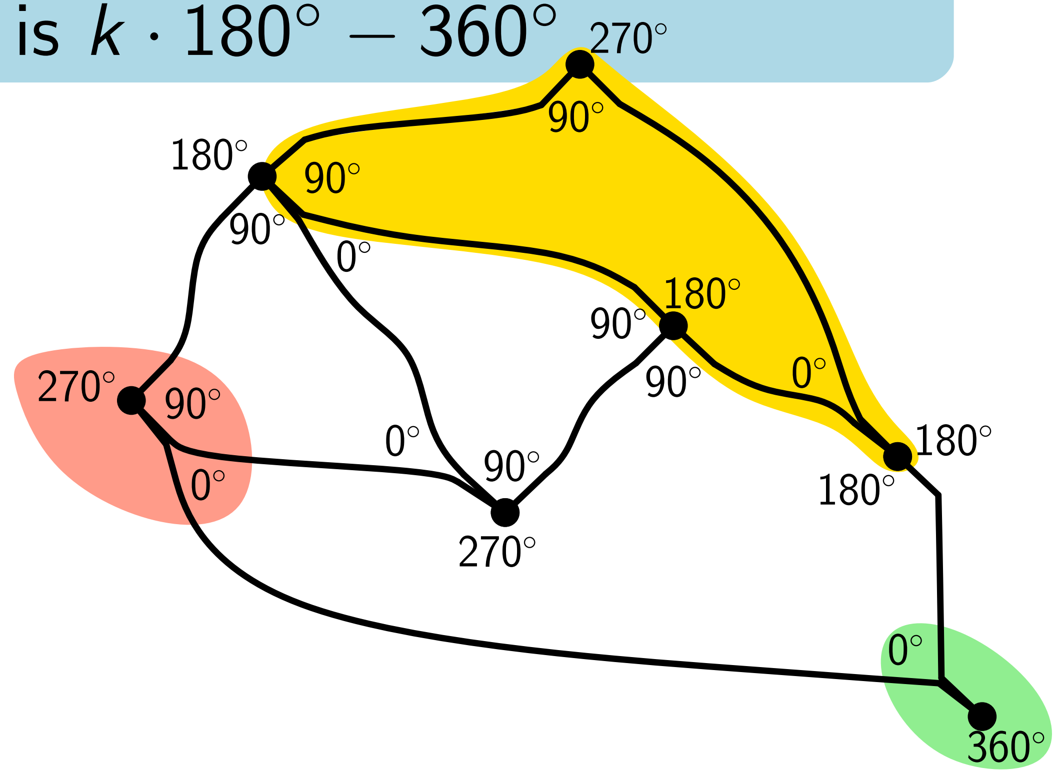
angular labeling A
$\underset{\text{unique}}{\Rightarrow}$ q-constraints Q_A

q-constraints Q
$+$ large-angle assignment L
$\underset{\text{unique}}{\Rightarrow}$ angular labeling $A_{Q,L}$



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angular labeling A
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q-constraints Q
$+$ large-angle assignment L
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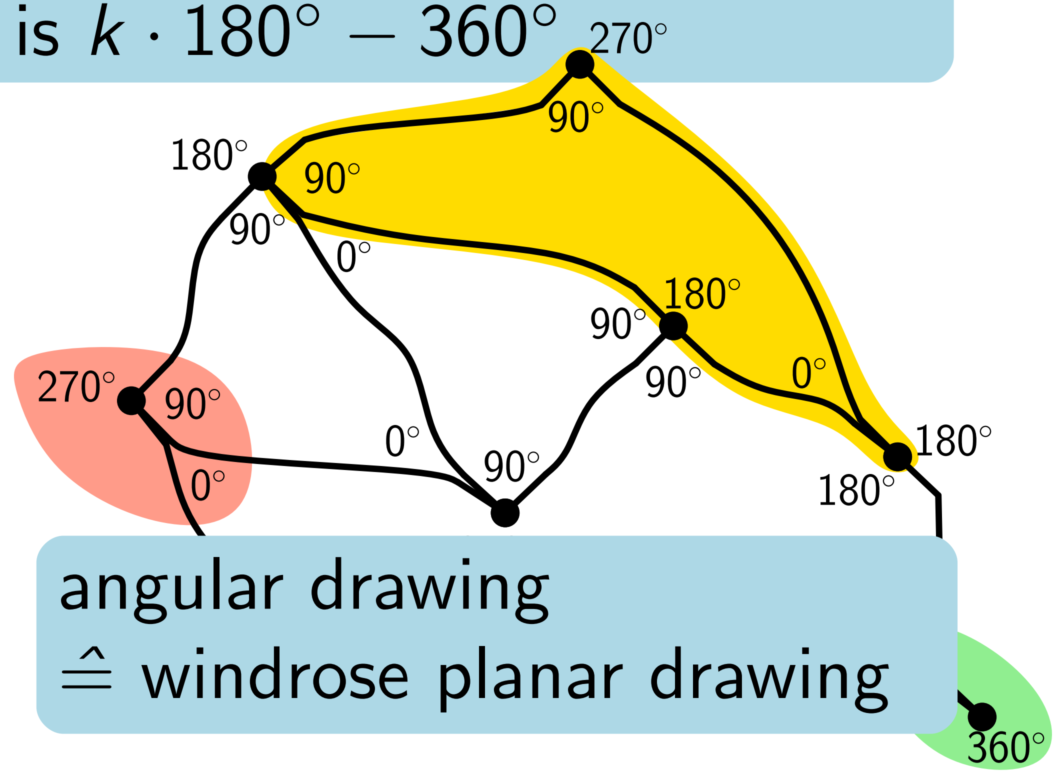
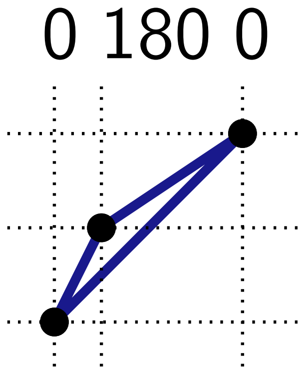


angular drawing
$\hat{=}$ windrose planar drawing

Triangulated Graphs



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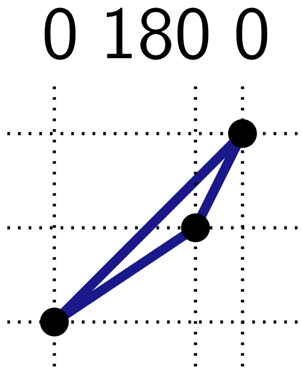
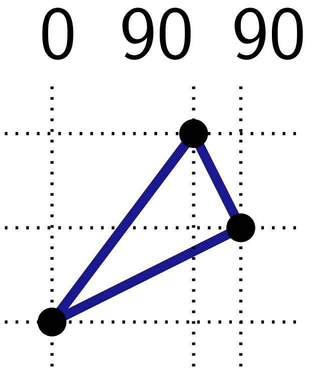
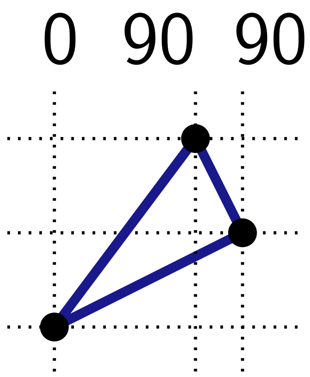


Triangulated Graphs



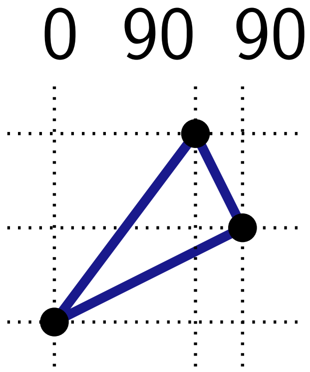
Triangulated Graphs

- No (int.) $> 180^\circ$ angle categories



Triangulated Graphs

- No (int.) $> 180^\circ$ angle categories
- At least one 0° angle category per (int.) face



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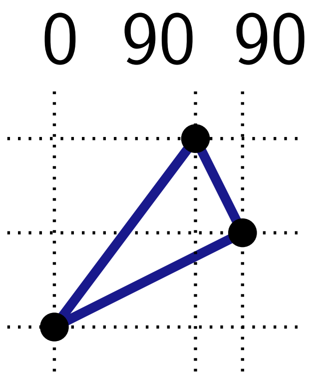

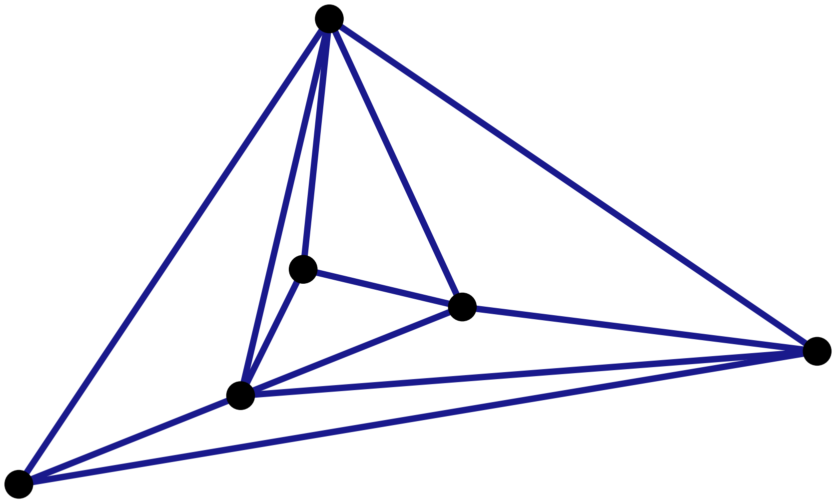

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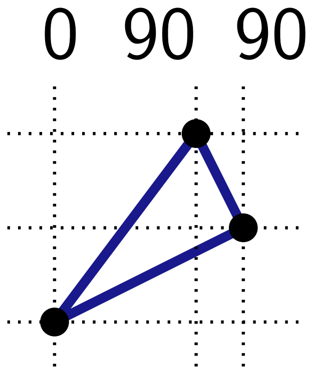

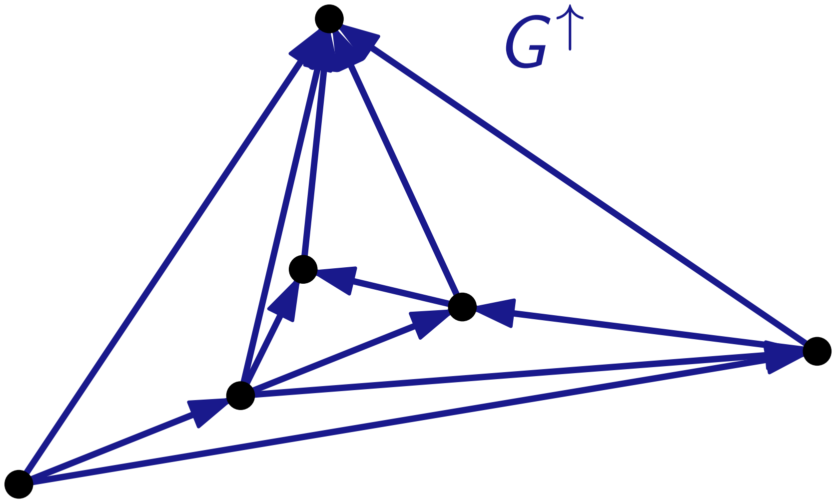

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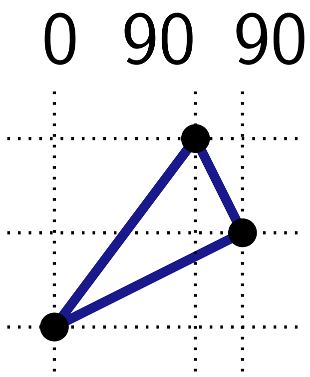
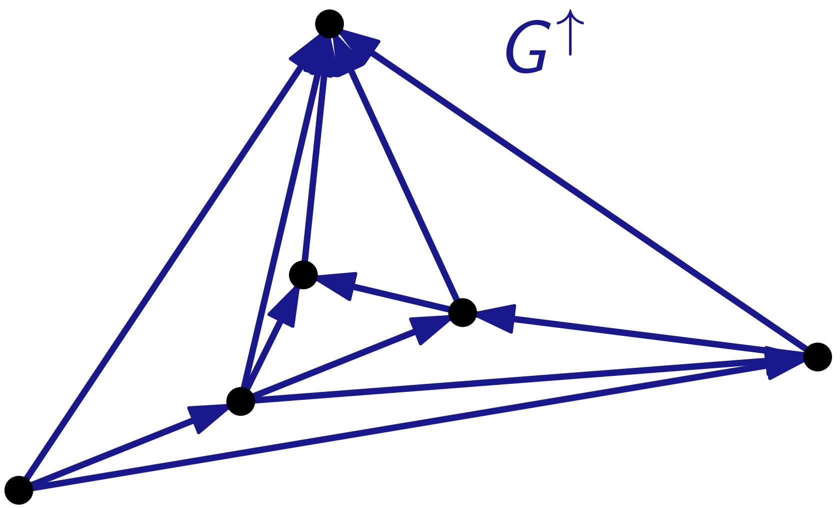
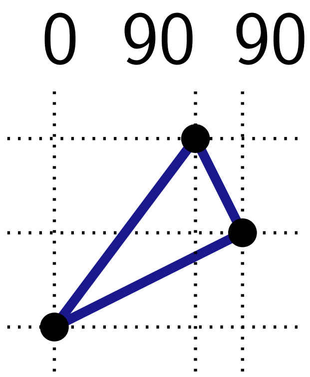


Lemma.
Let (G, A_Q) be a triangulated angular labeled graph. Then, $G^\uparrow$ is acyclic.

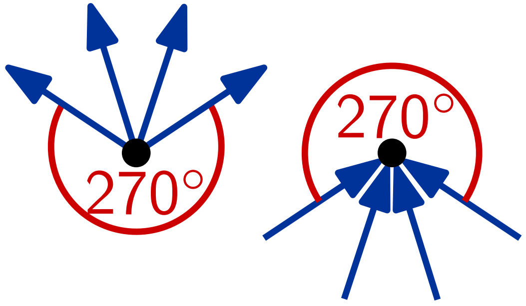


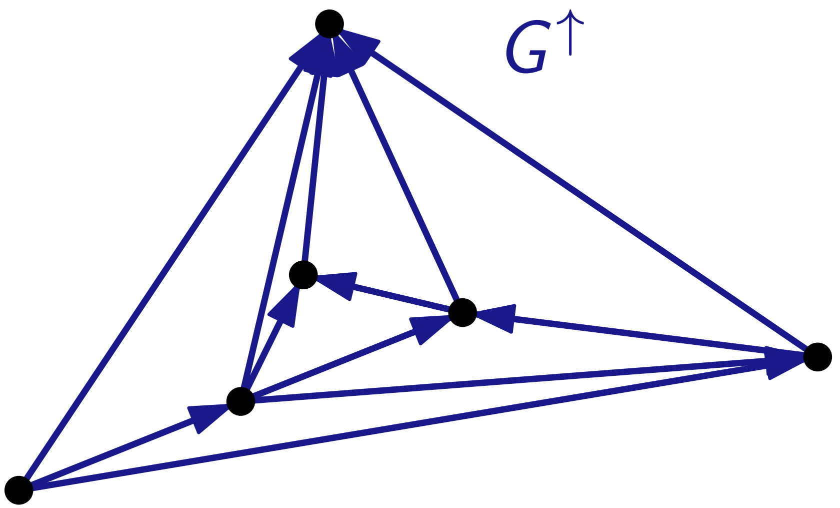
Triangulated Graphs

- No (int.) $> 180°$ angle categories
- At least one $0°$ angle category per (int.) face

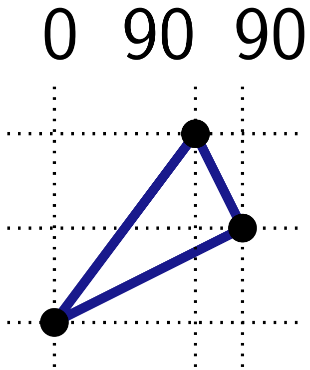


Lemma.
Let (G, A_Q) be a triangulated angular labeled graph. Then, $G^{\uparrow}$ is acyclic and has no internal sources or sinks.





Triangulated Graphs



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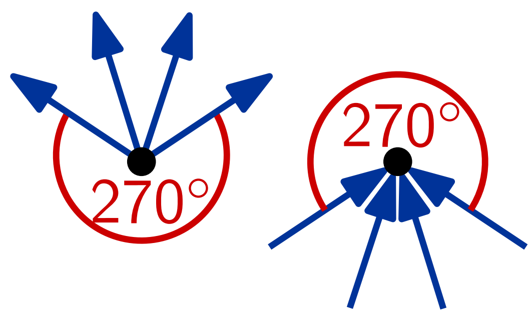

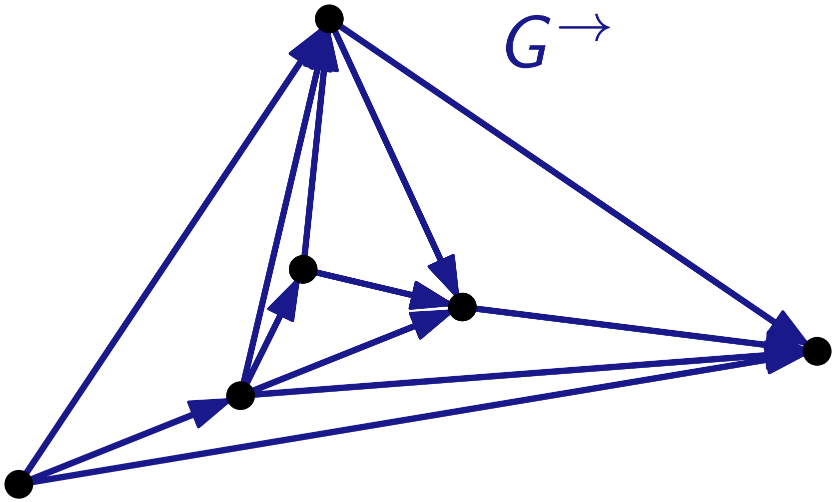

Triangulated Graphs

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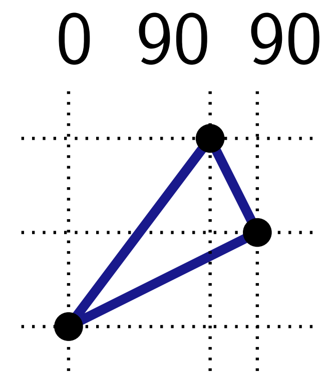
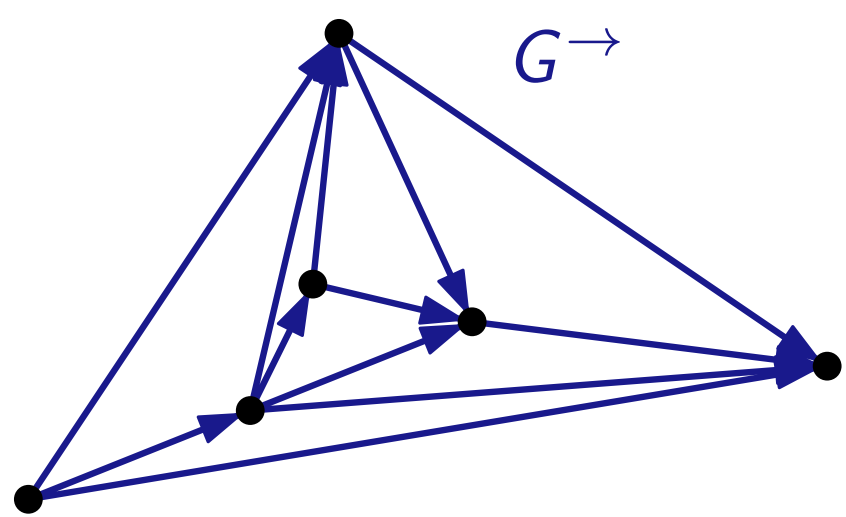


Lemma.
Let (G, A_Q) be a triangulated angular labeled graph. Then, $G^{\uparrow}$ and $G^{\rightarrow}$ are acyclic and have no internal sources or sinks.



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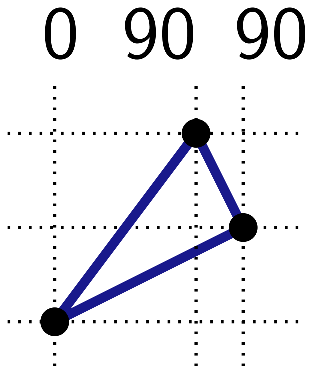

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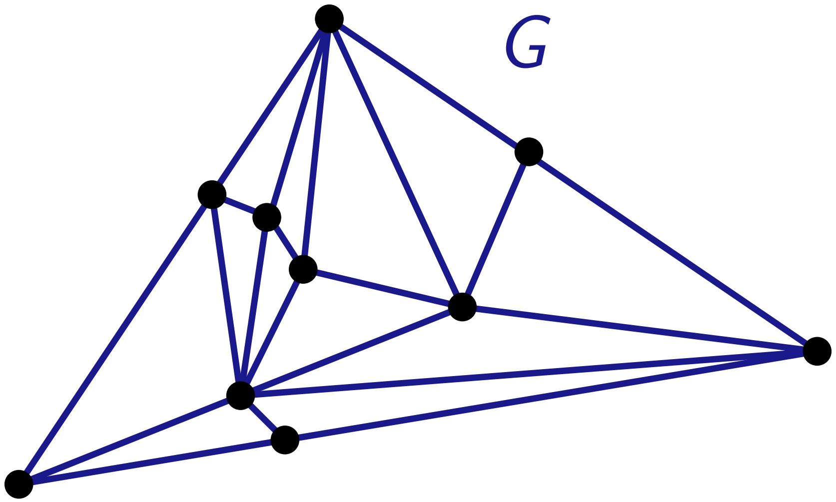

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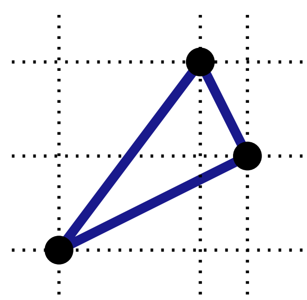
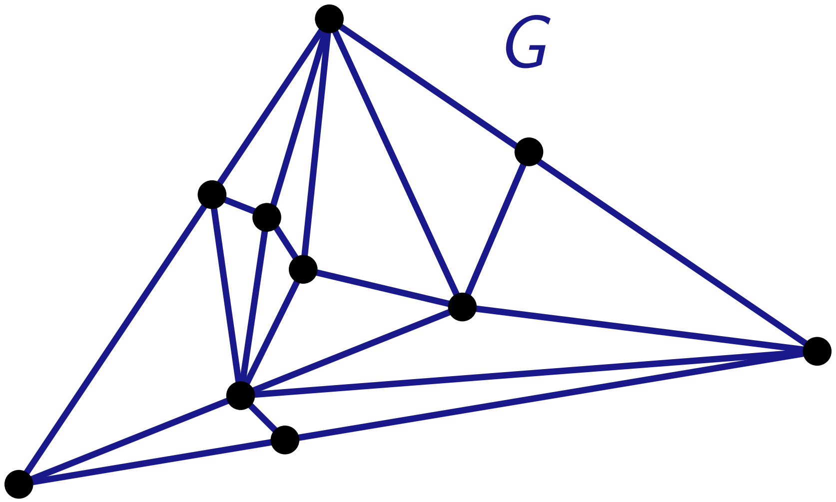


Lemma.
Let (G, A_Q) be a internally triangulated angular labeled graph. Then, $G^{\uparrow}$ and $G^{\rightarrow}$ are acyclic and have no internal sources or sinks.



Triangulated Graphs

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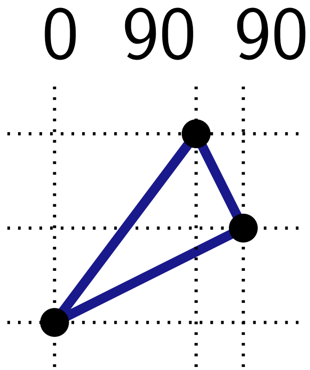
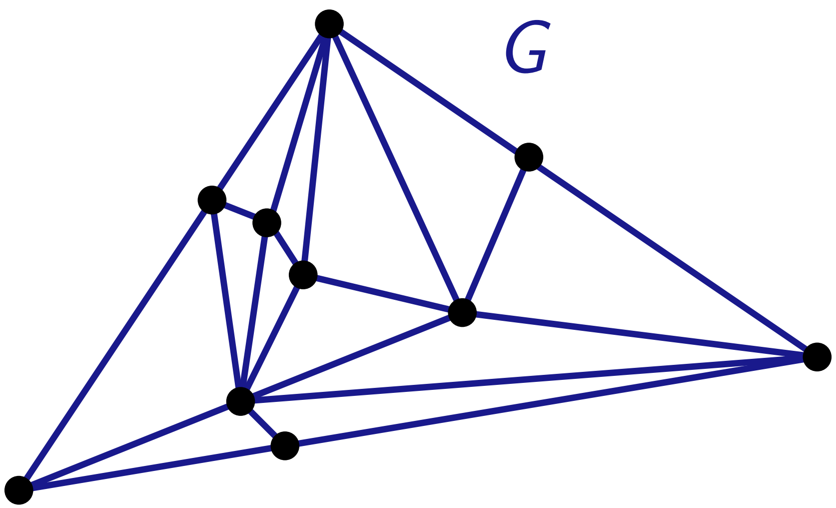


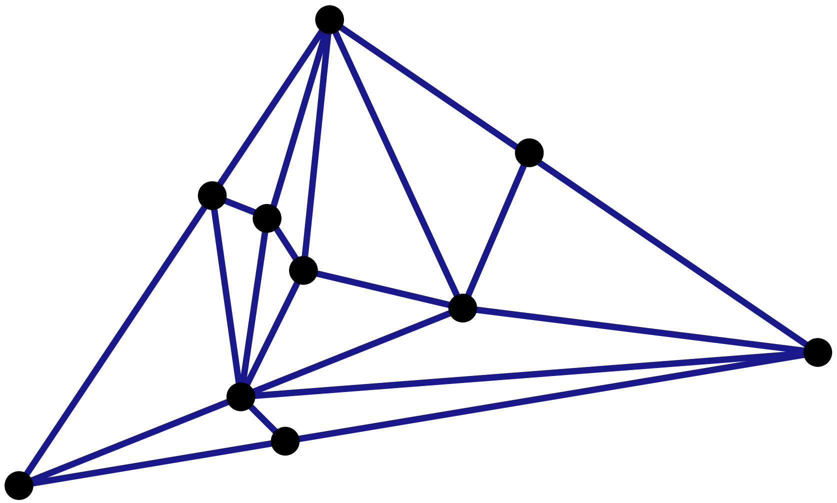
Lemma.
Let (G, A_Q) be a internally triangulated angular labeled graph. Then, $G^{\uparrow}$ and $G^{\rightarrow}$ are acyclic and have no internal sources or sinks.

What if there are no (int.) 180° angle categories?



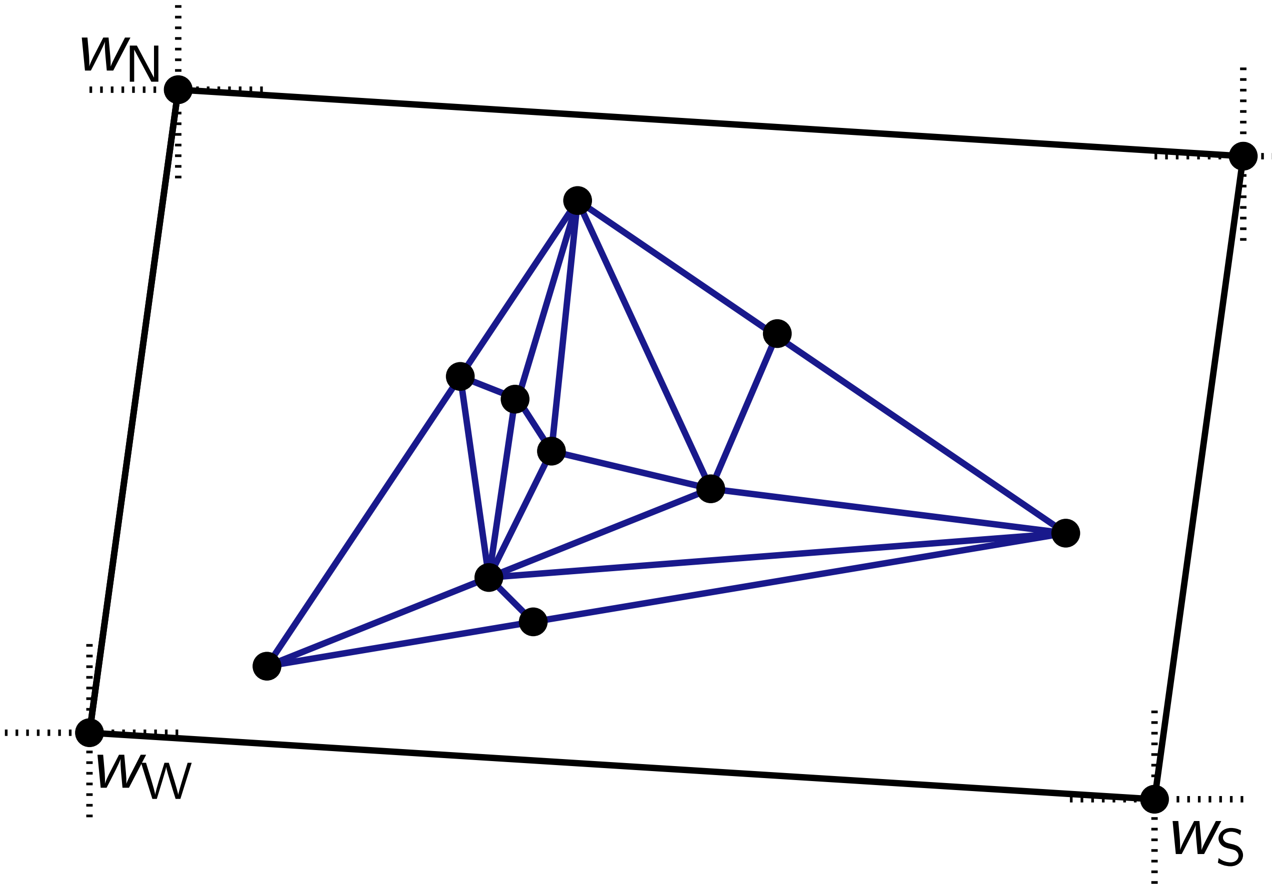
Quasi-triangulated Graphs

What if there are no (int.) 180° angle categories?



Quasi-triangulated Graphs

What if there are no (int.) 180° angle categories?



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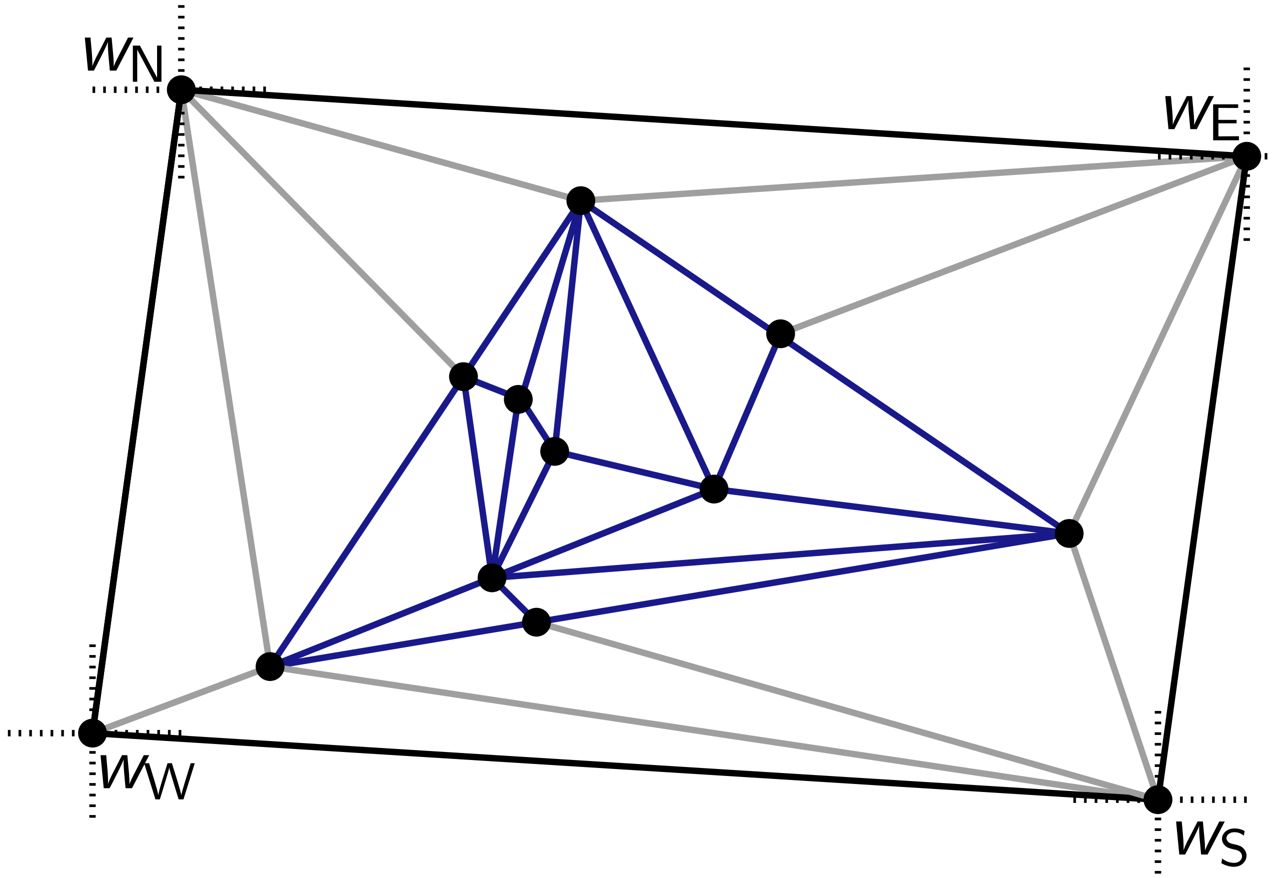

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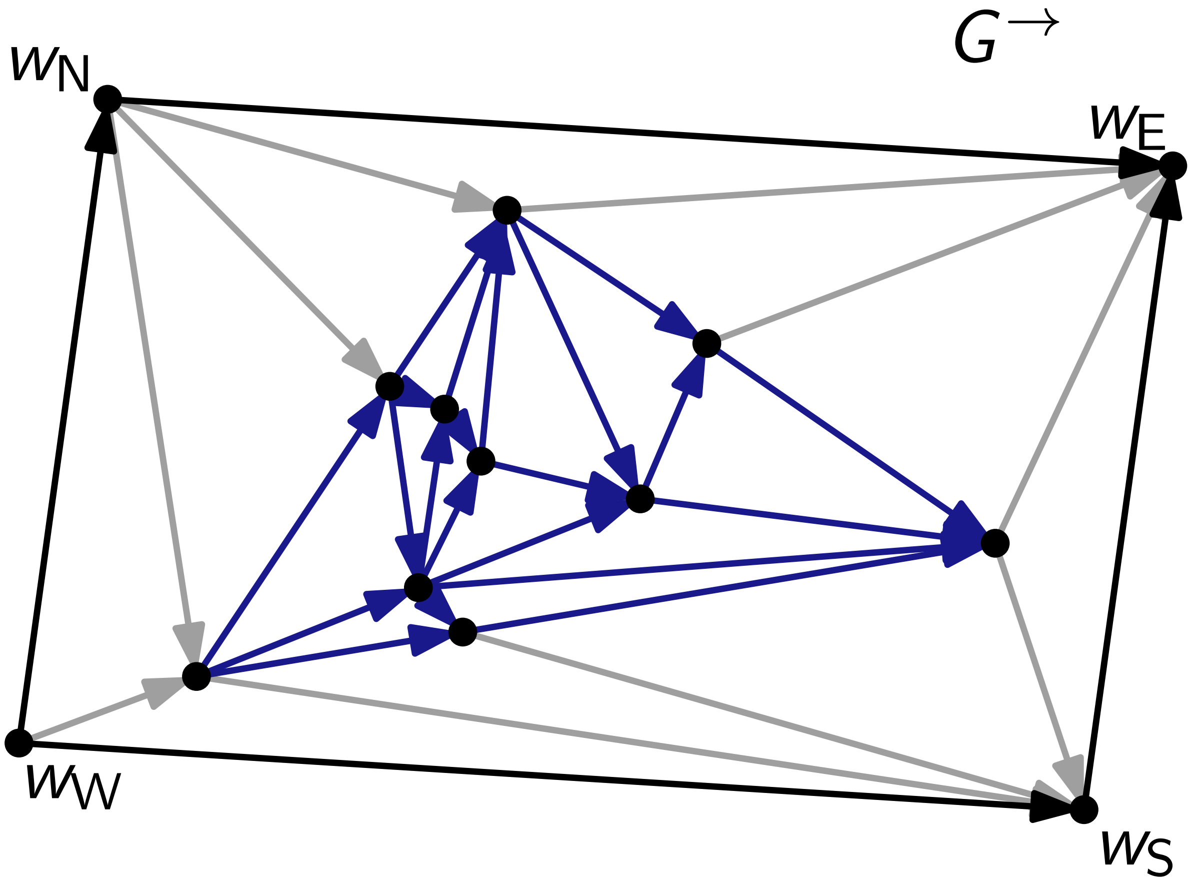
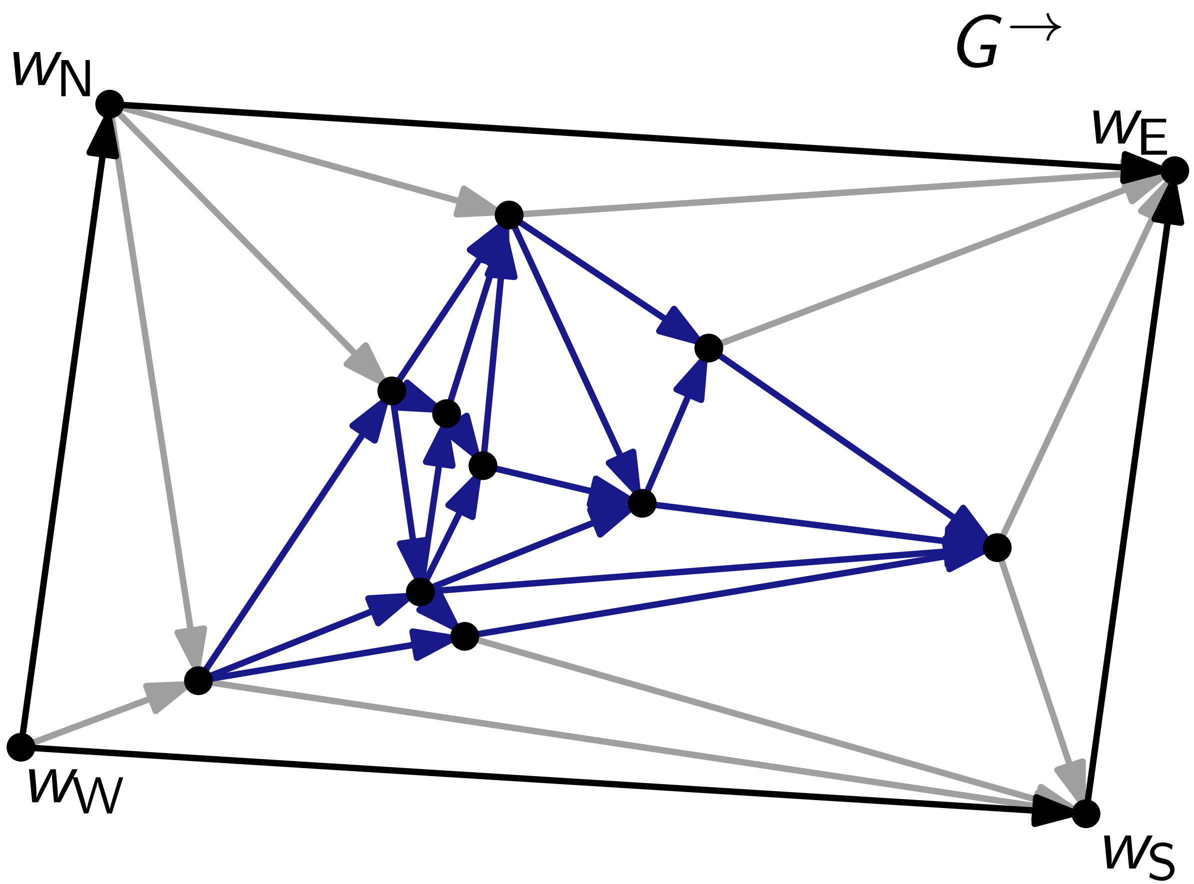


Quasi-triangulated Graphs

What if there are no (int.) 180° angle categories?

- topological order on $G^{\rightarrow}$: x-coordinates



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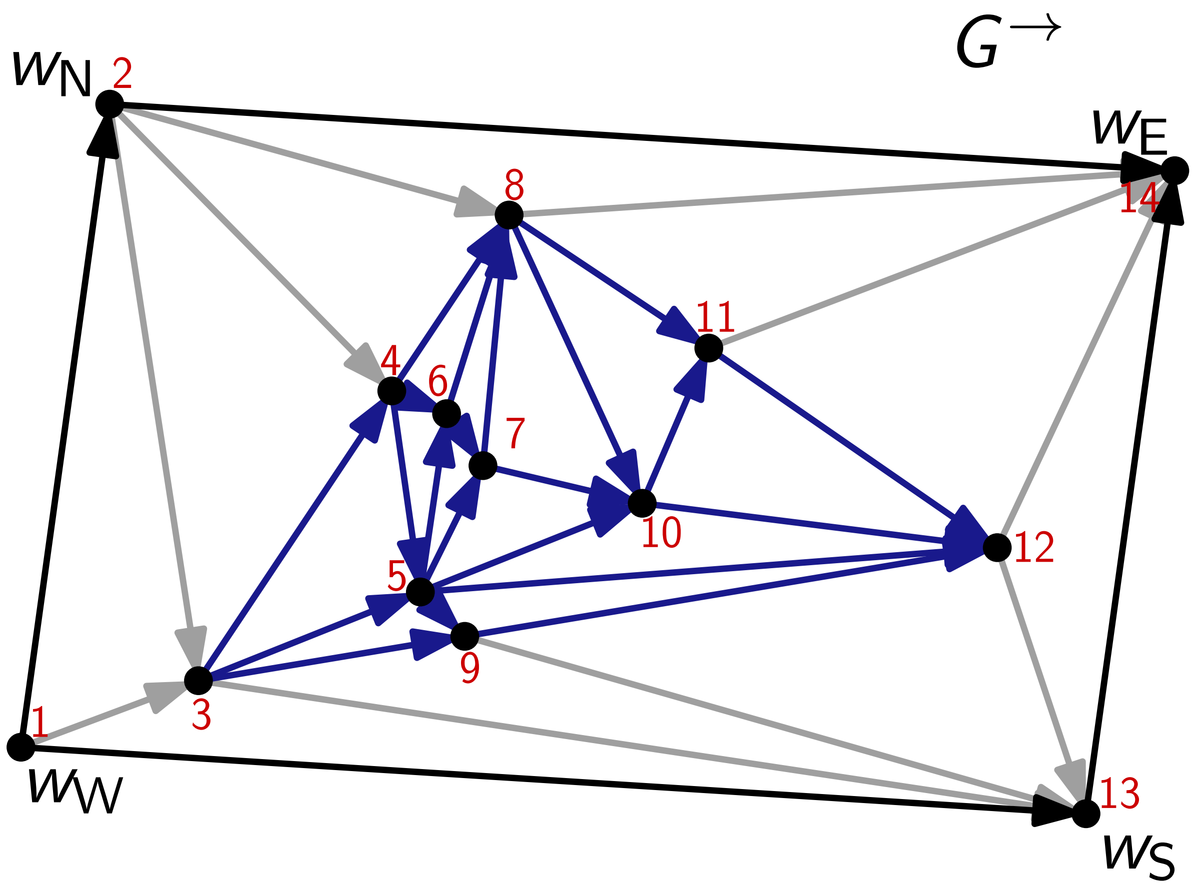

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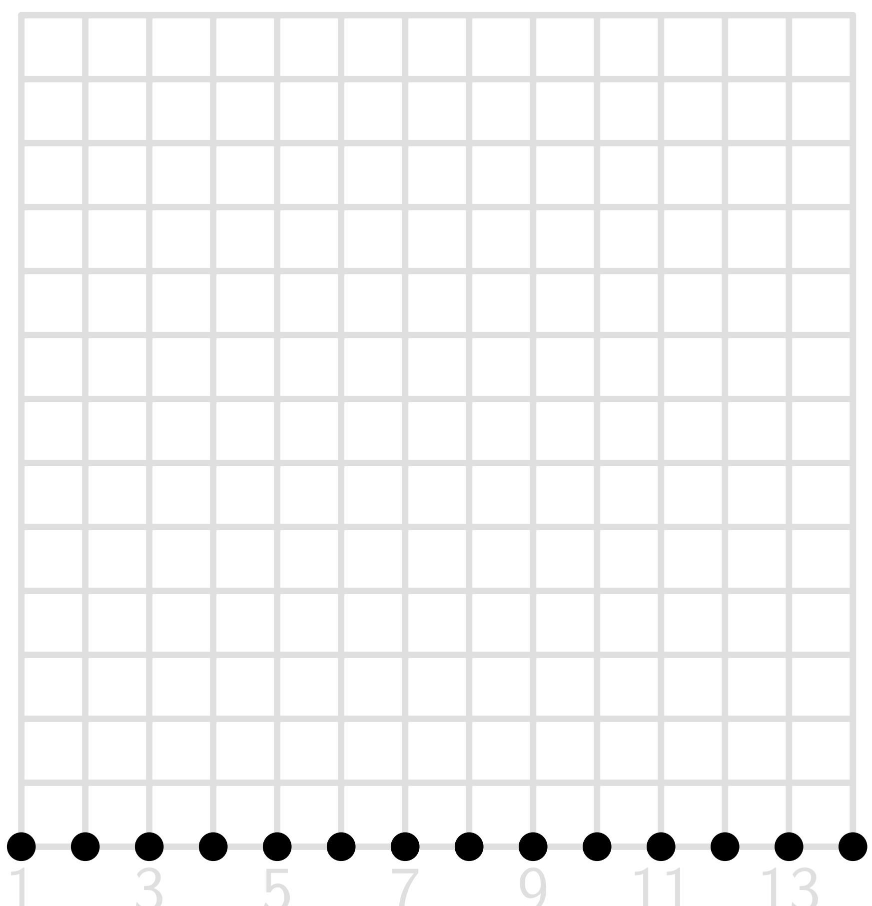

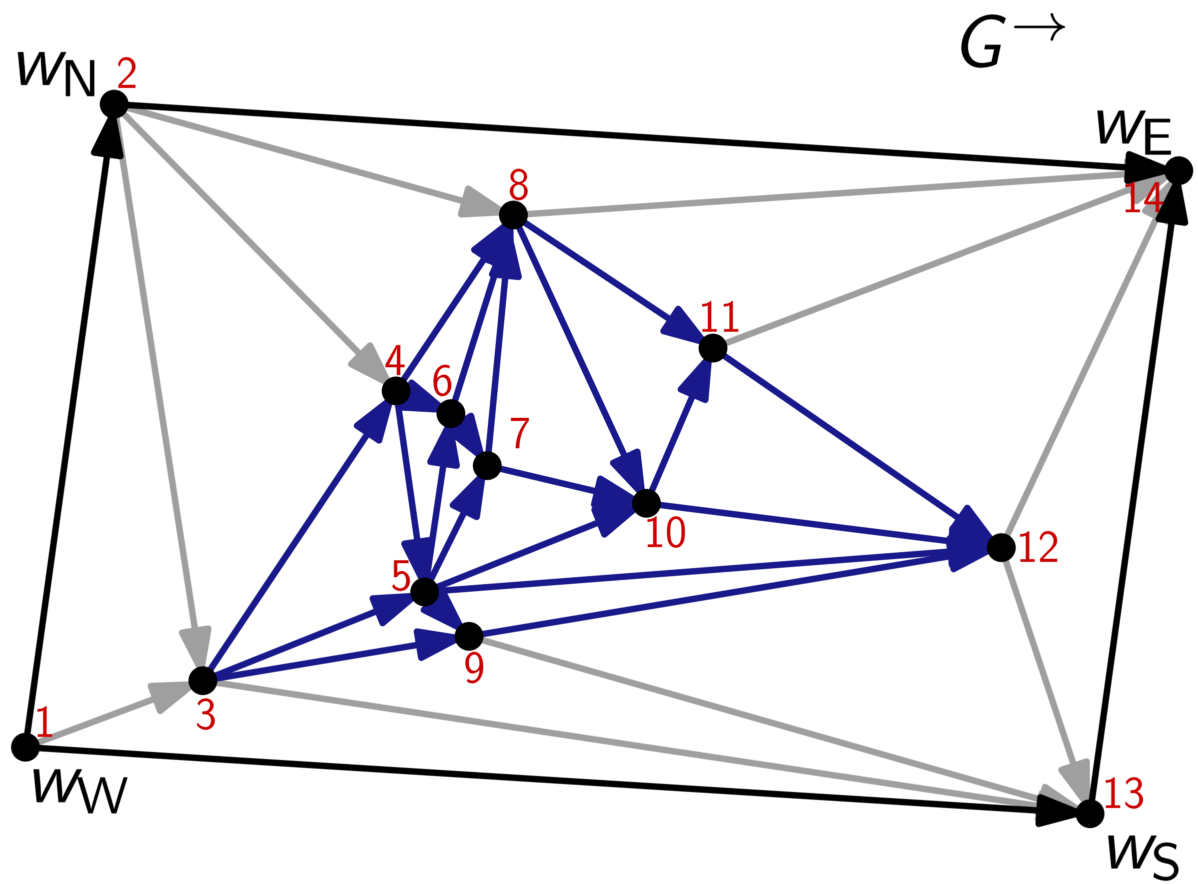

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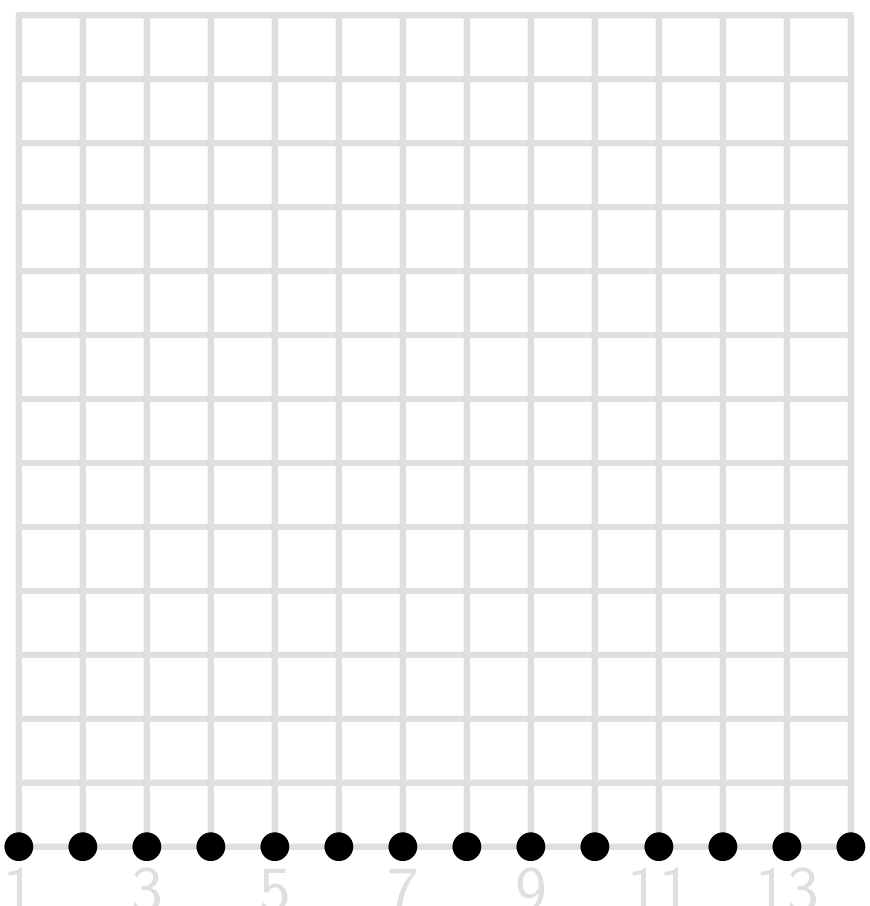

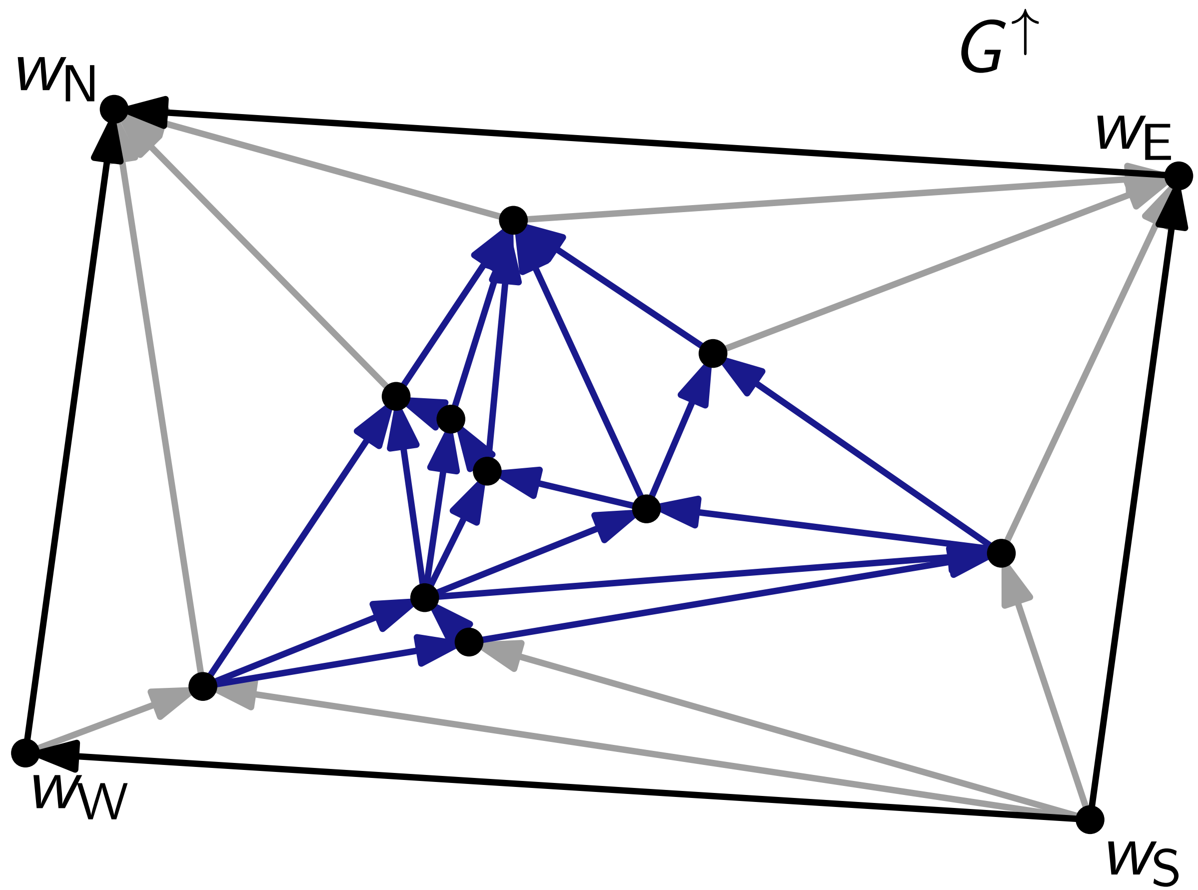

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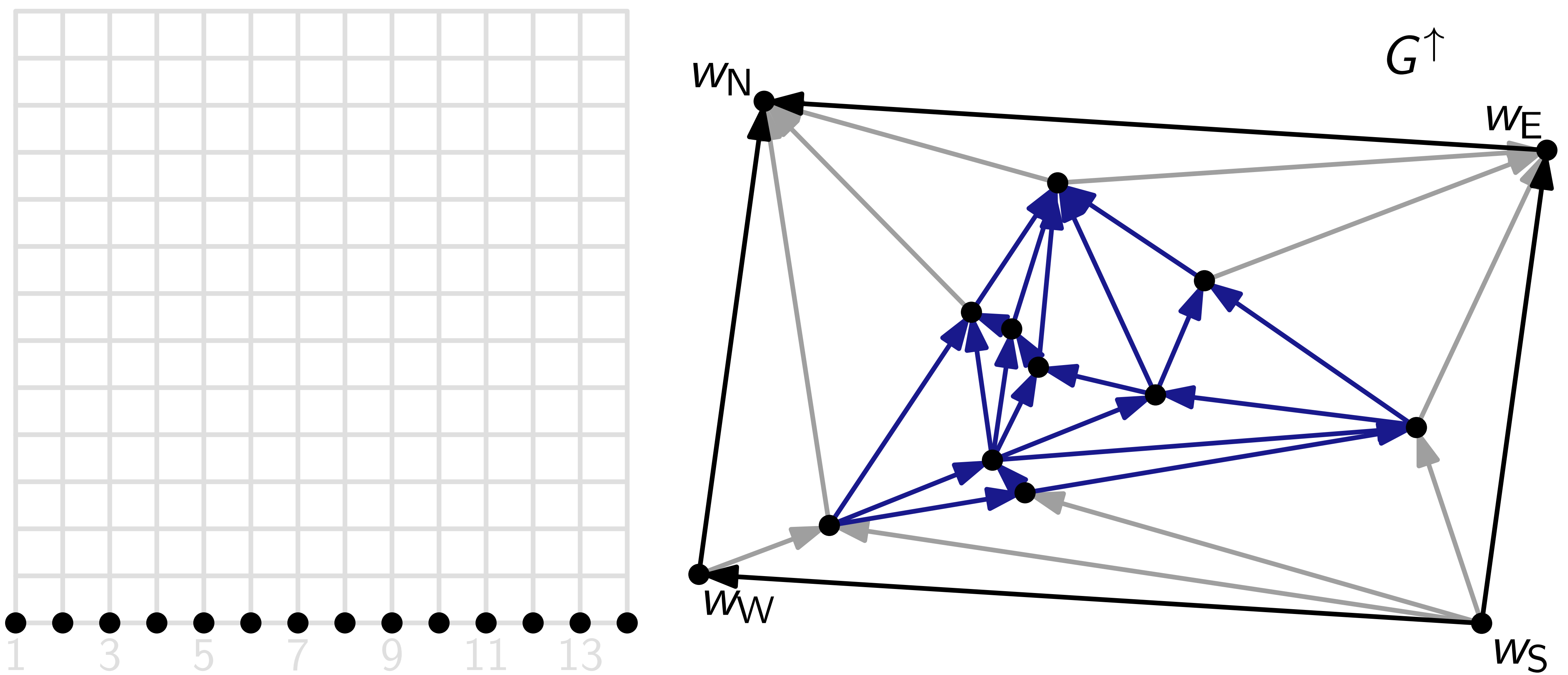

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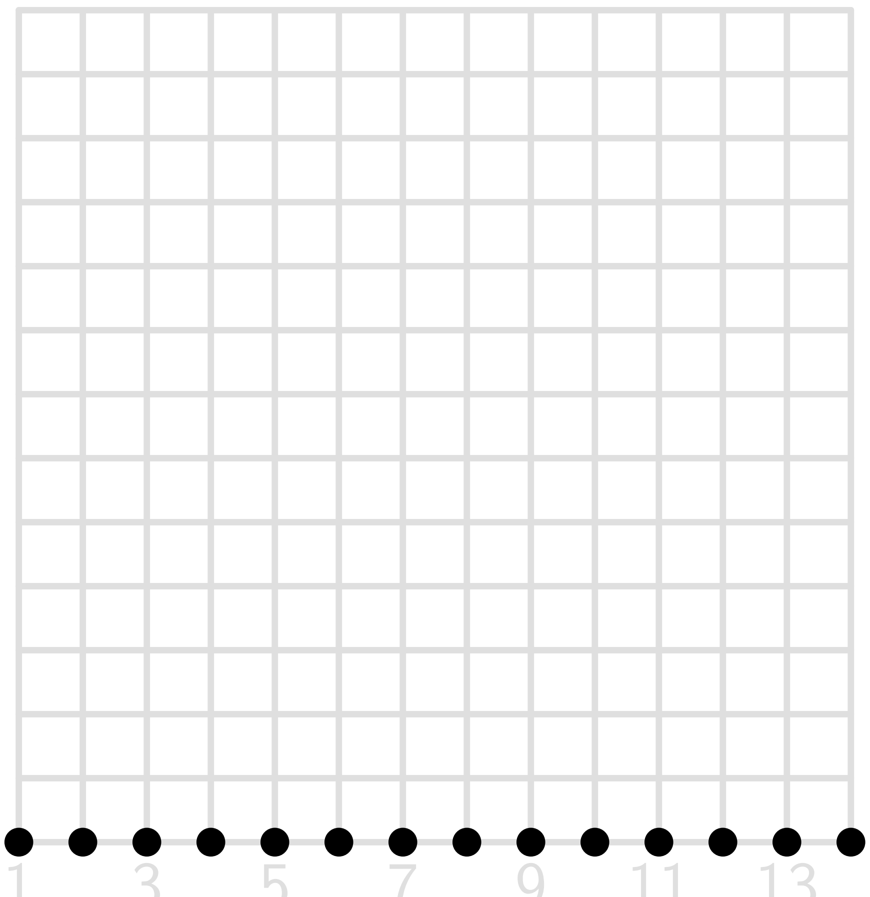

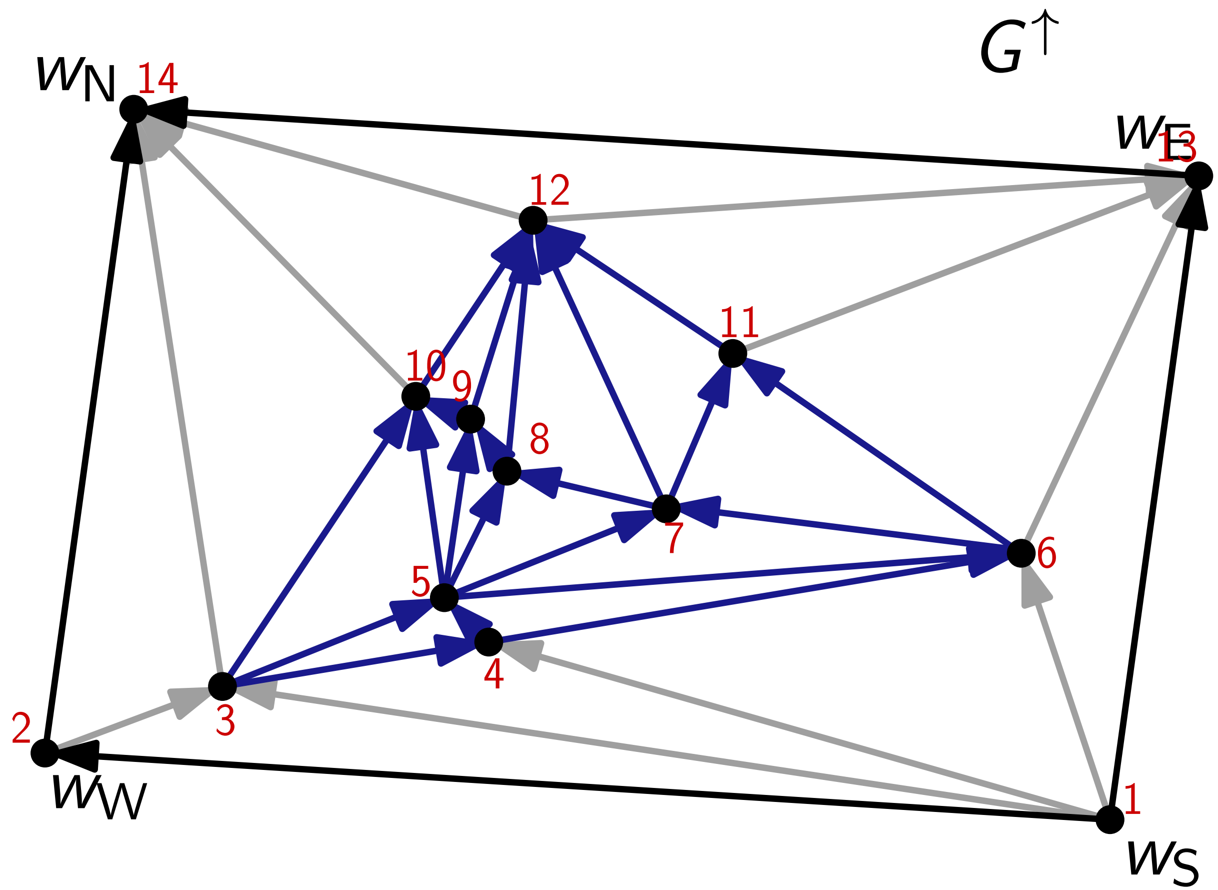

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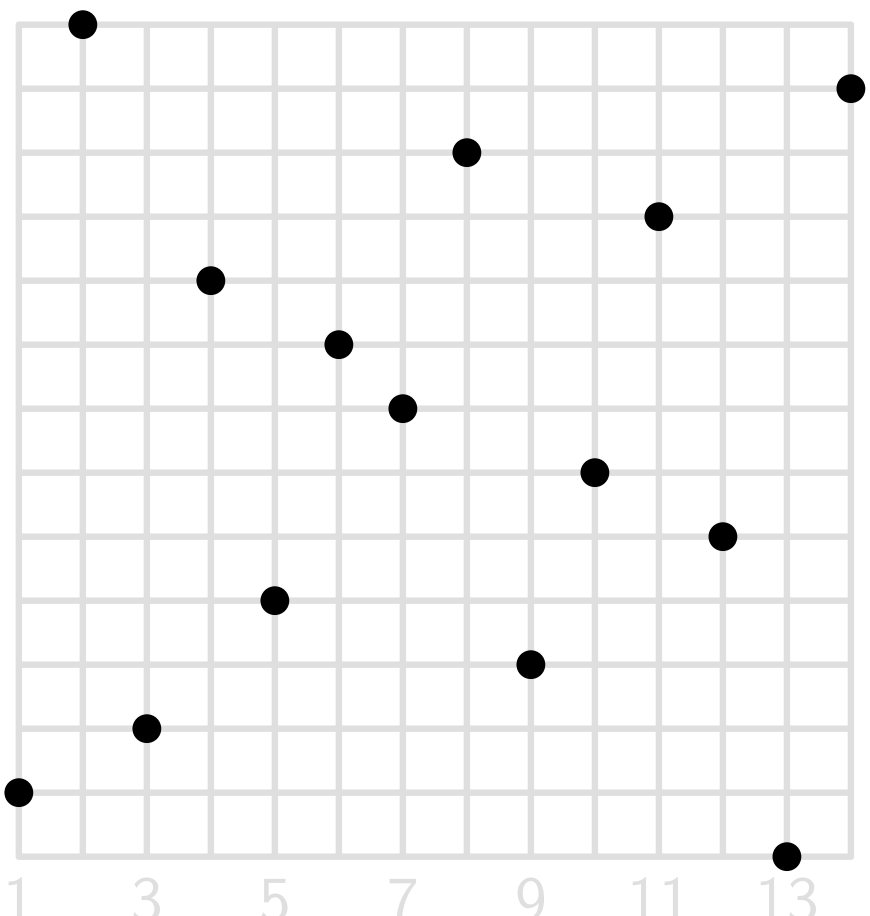

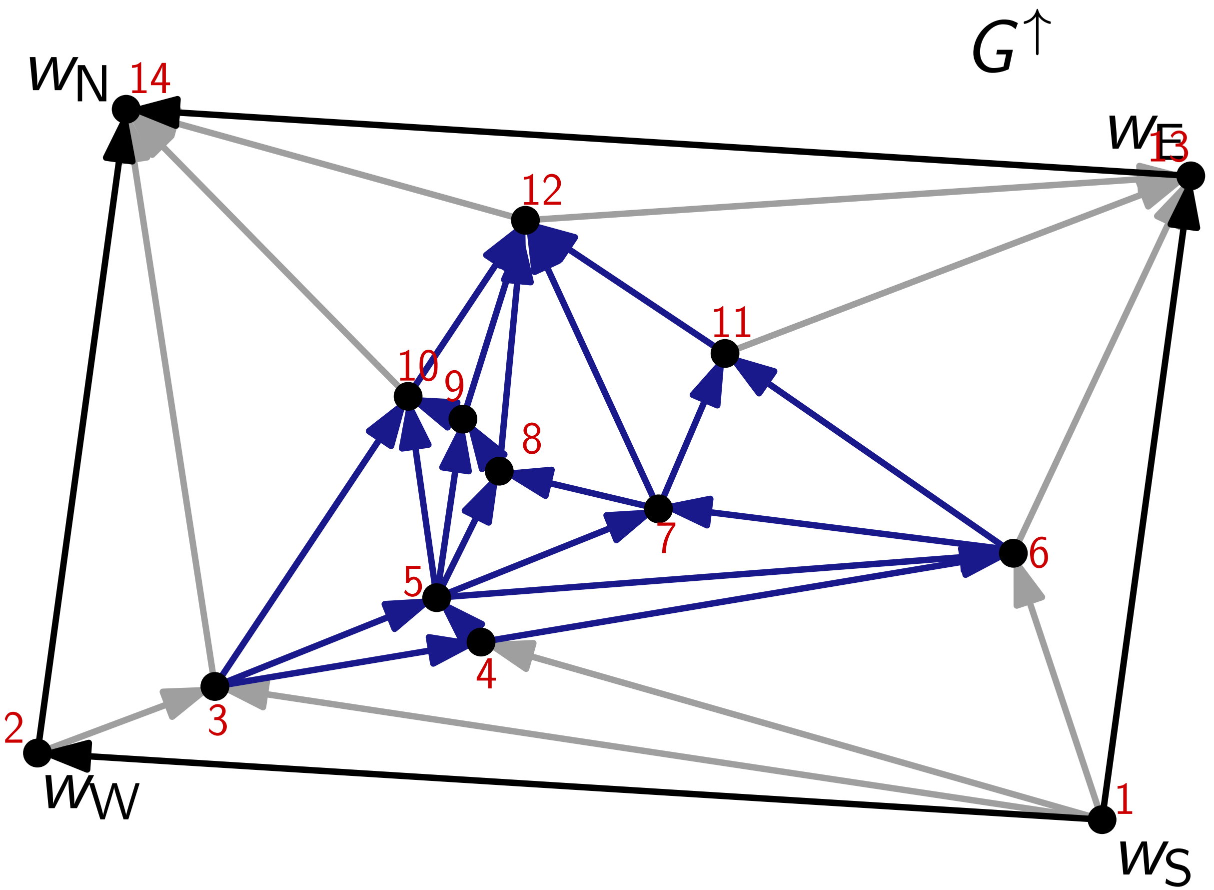

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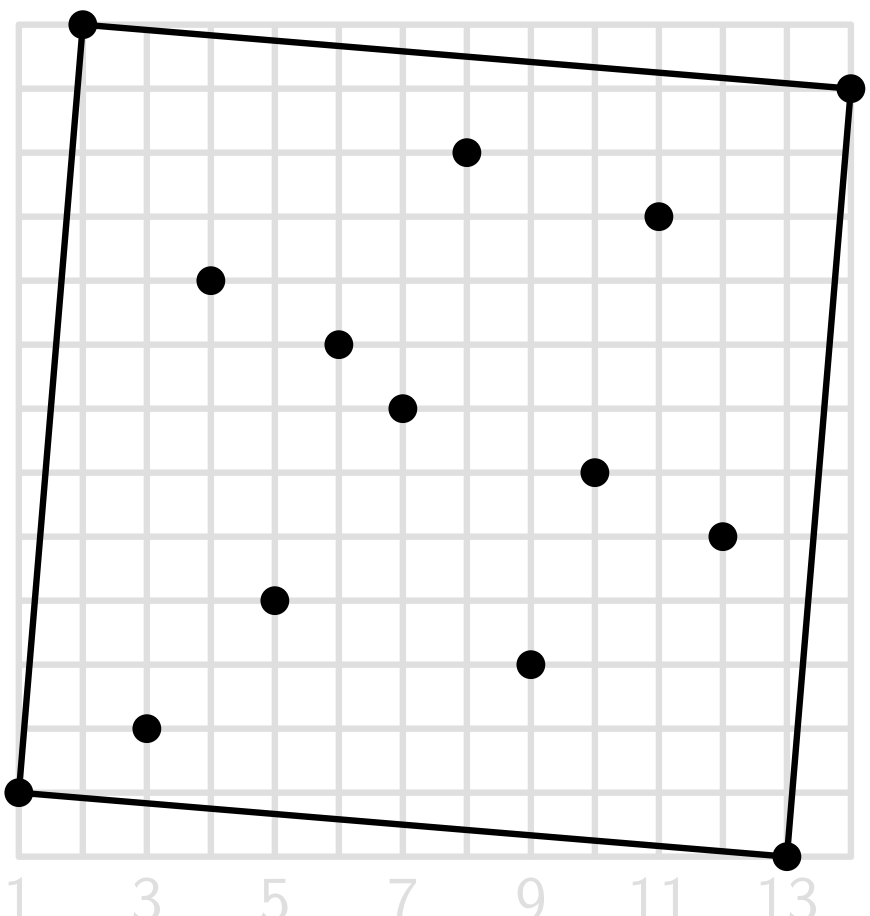

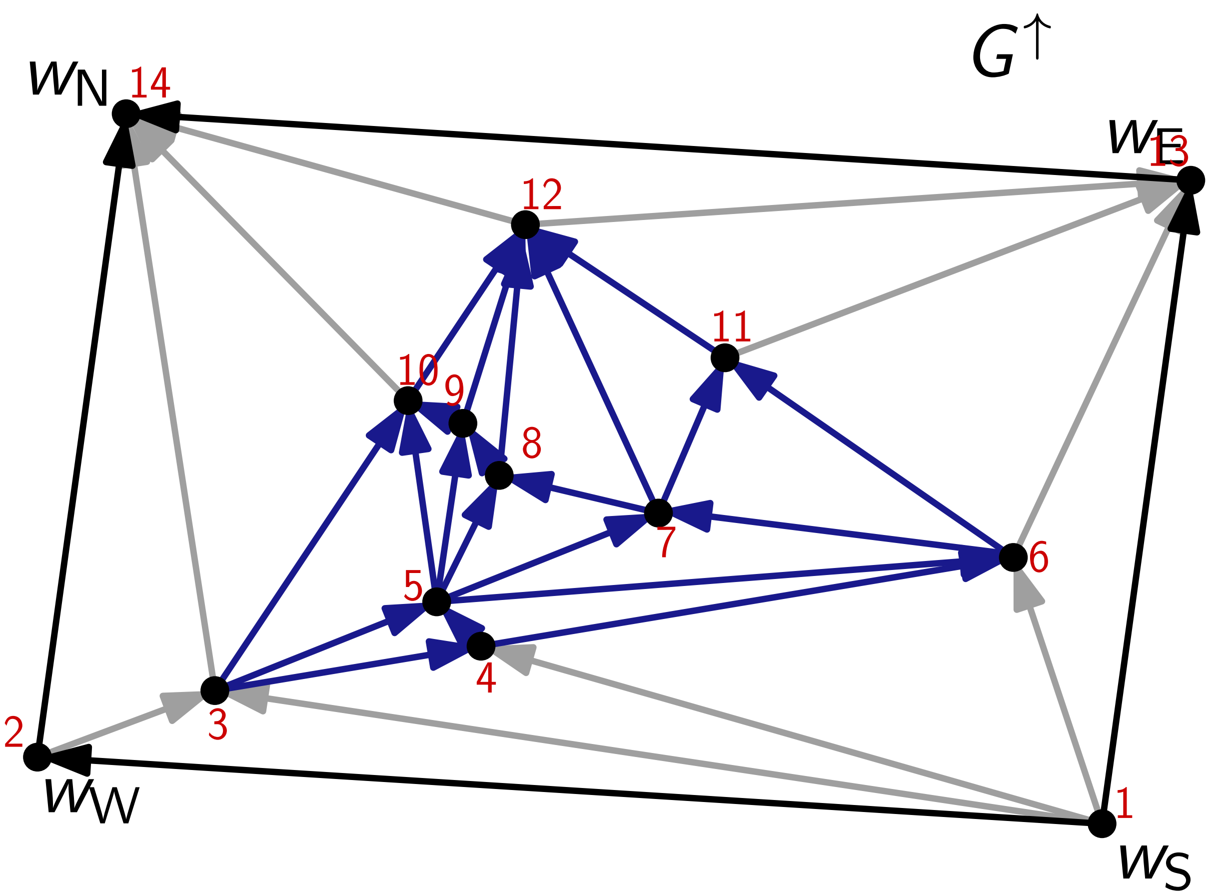

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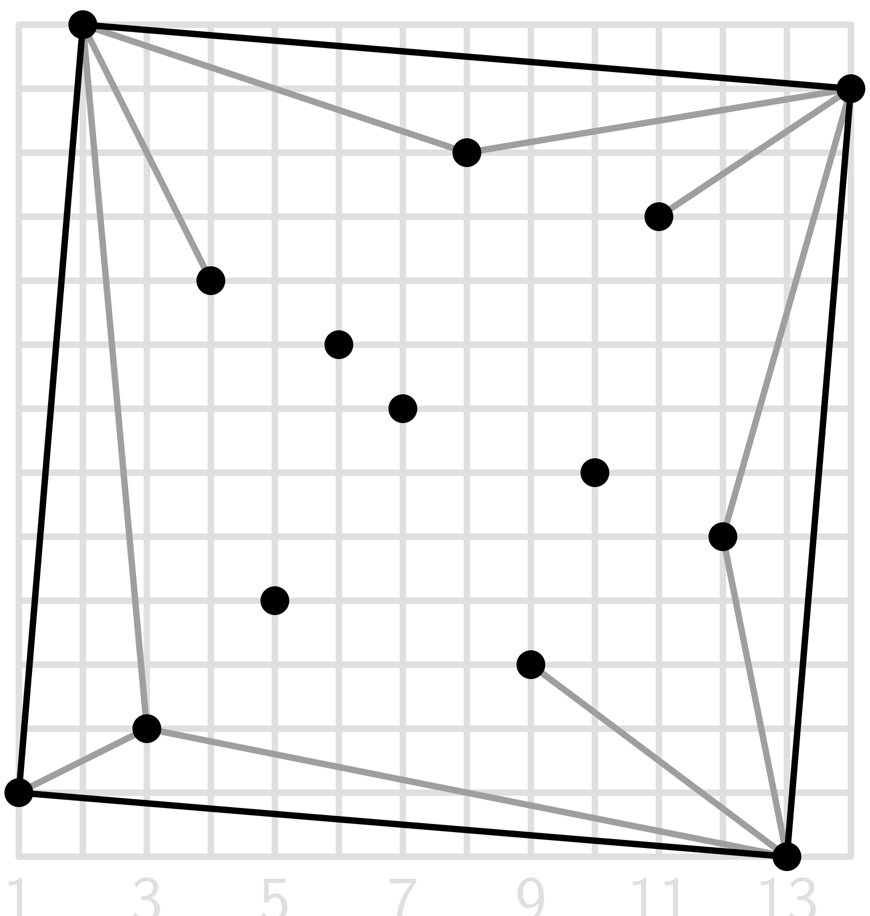

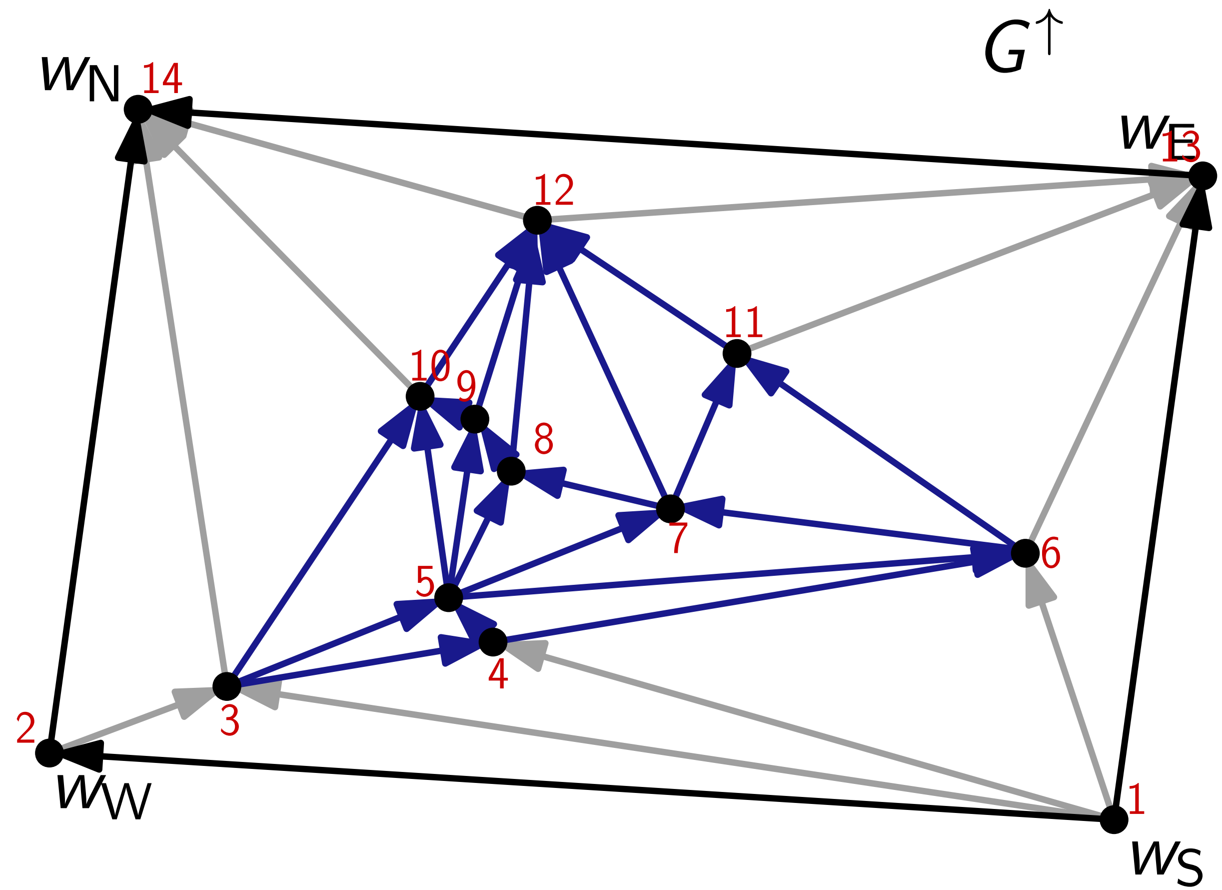

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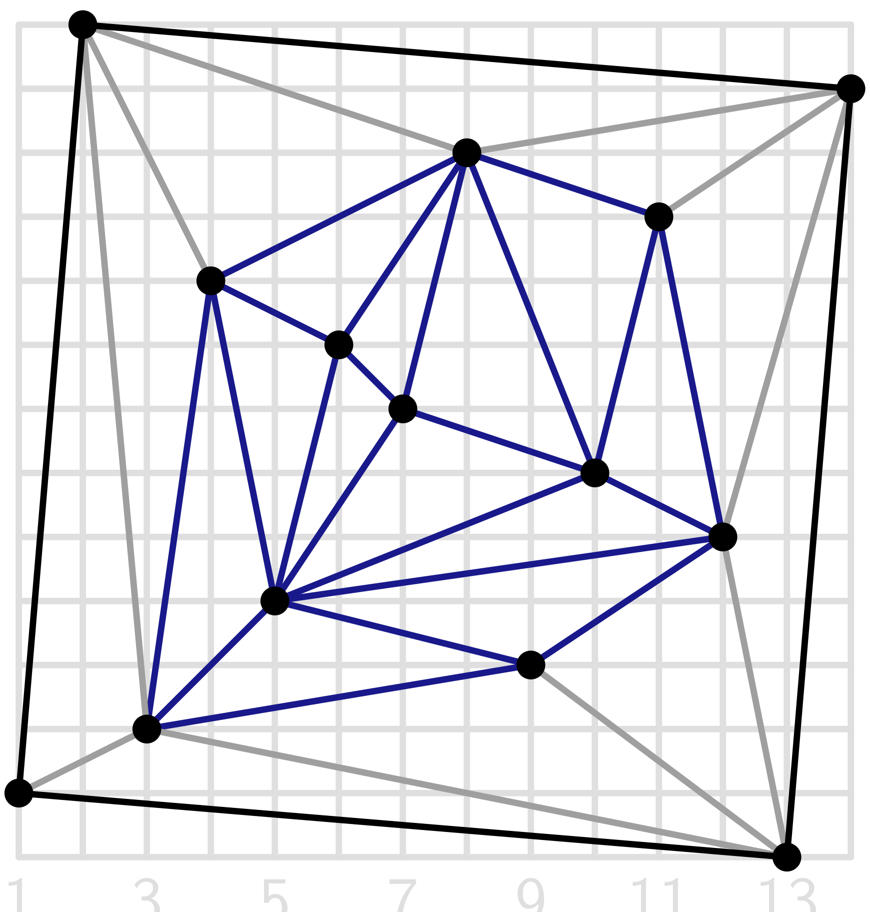

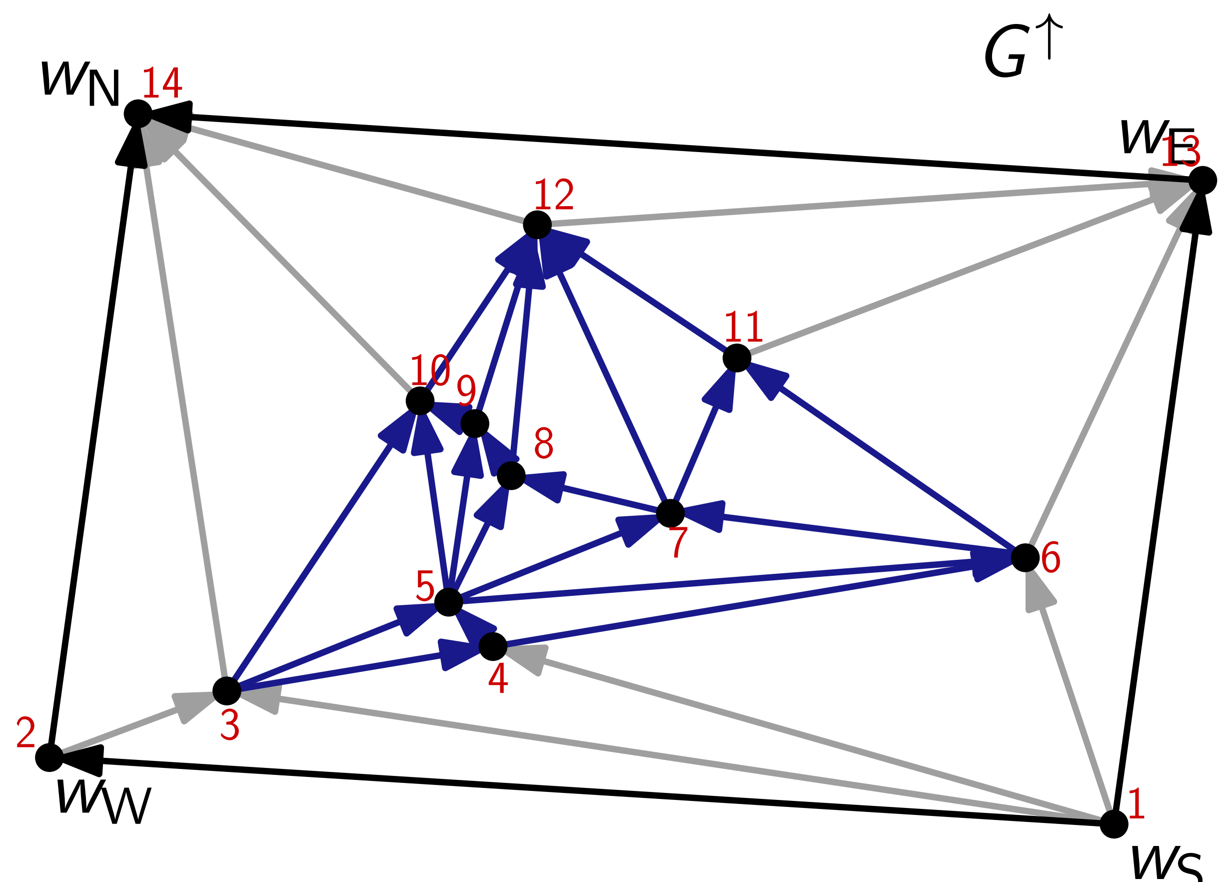

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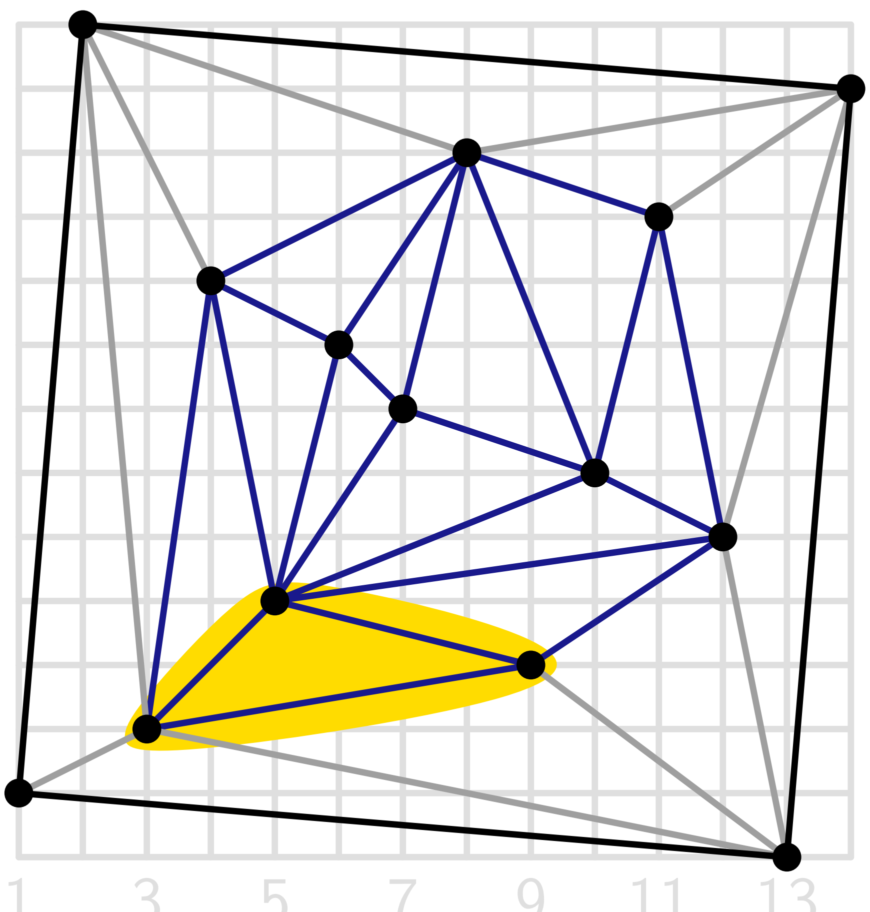

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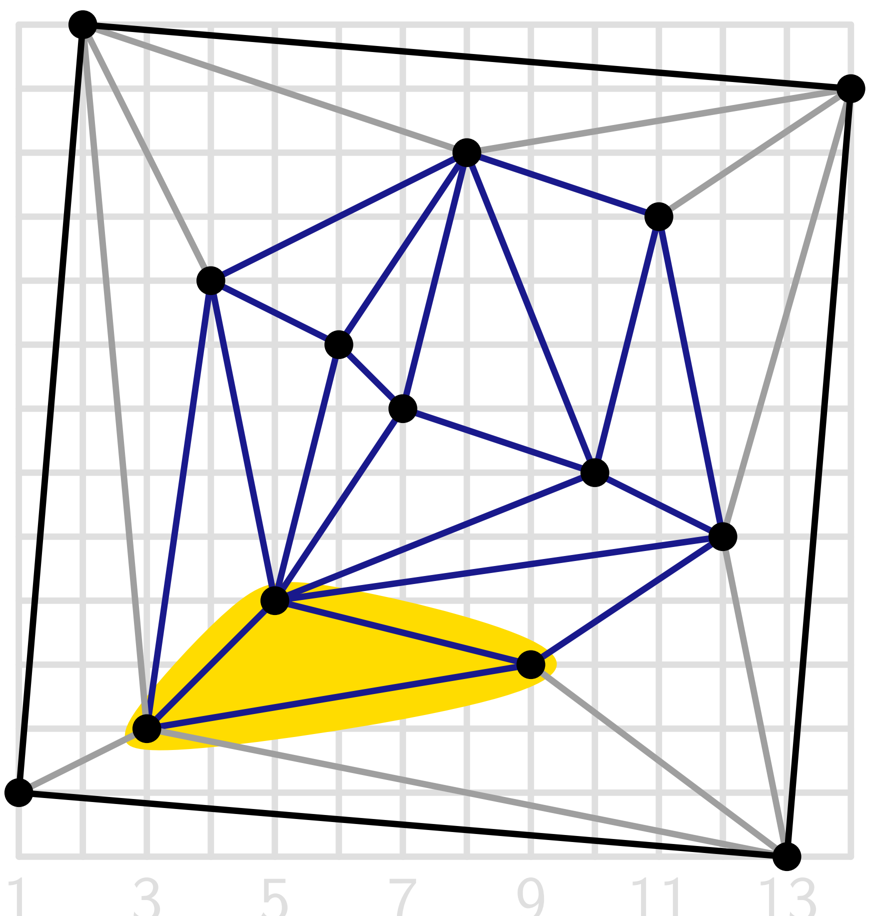

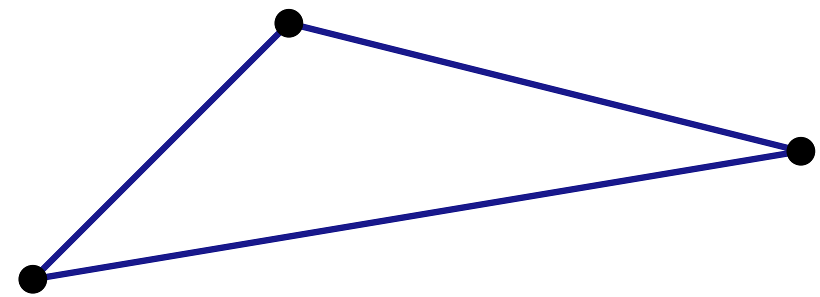

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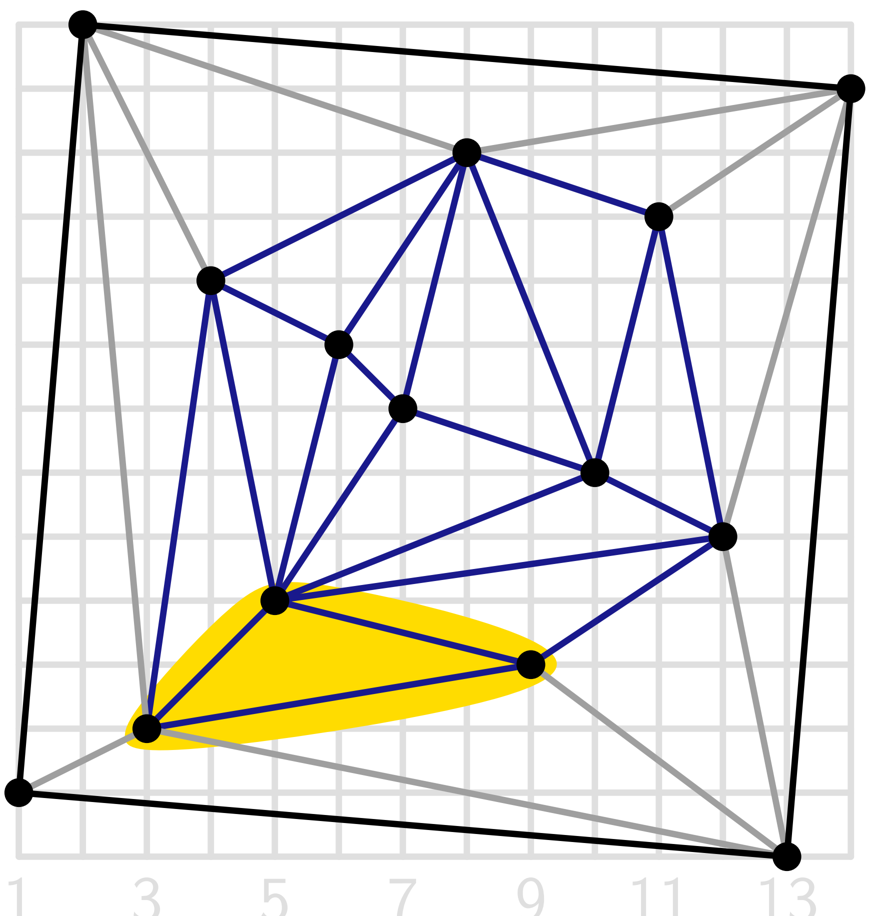

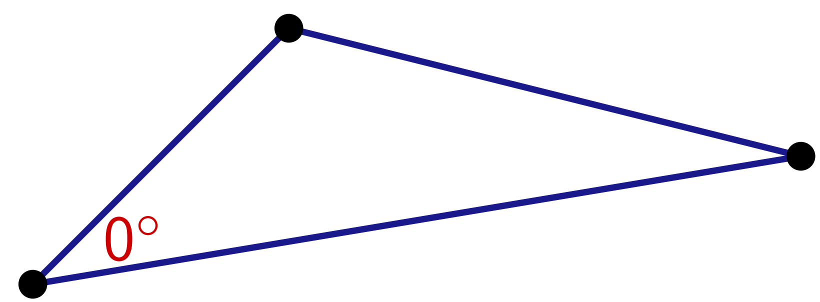

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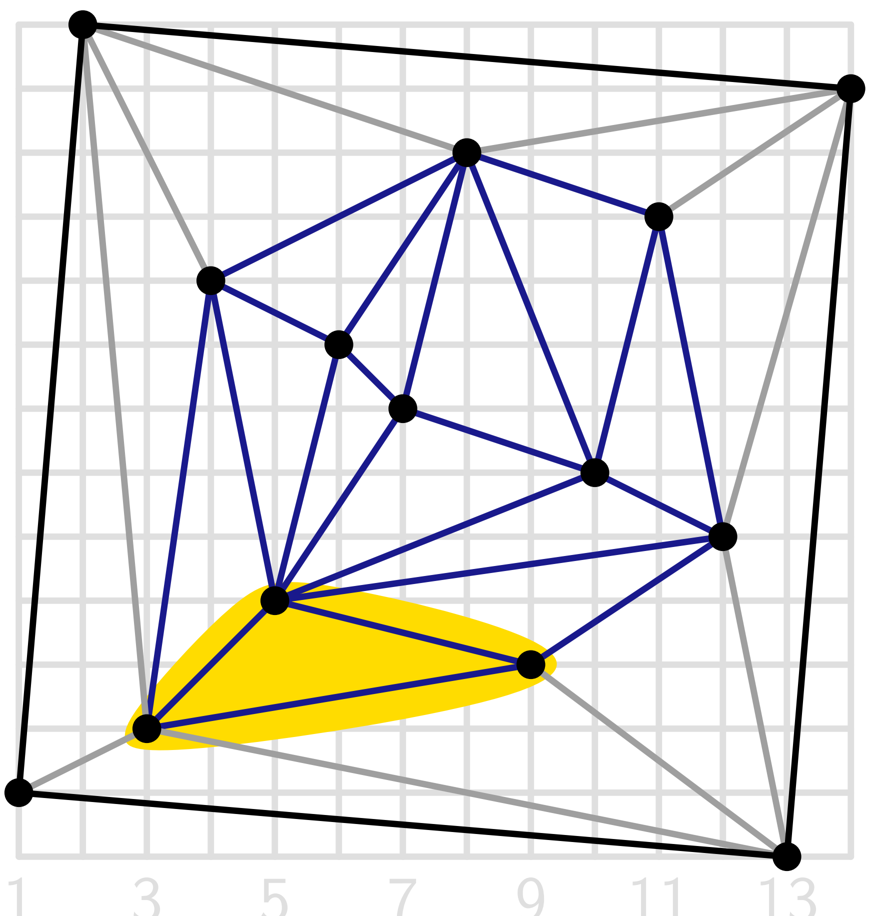

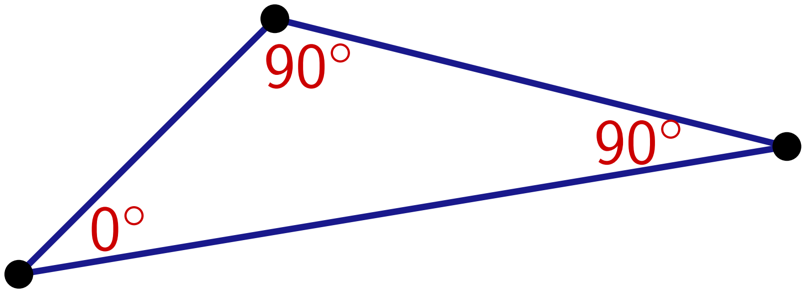

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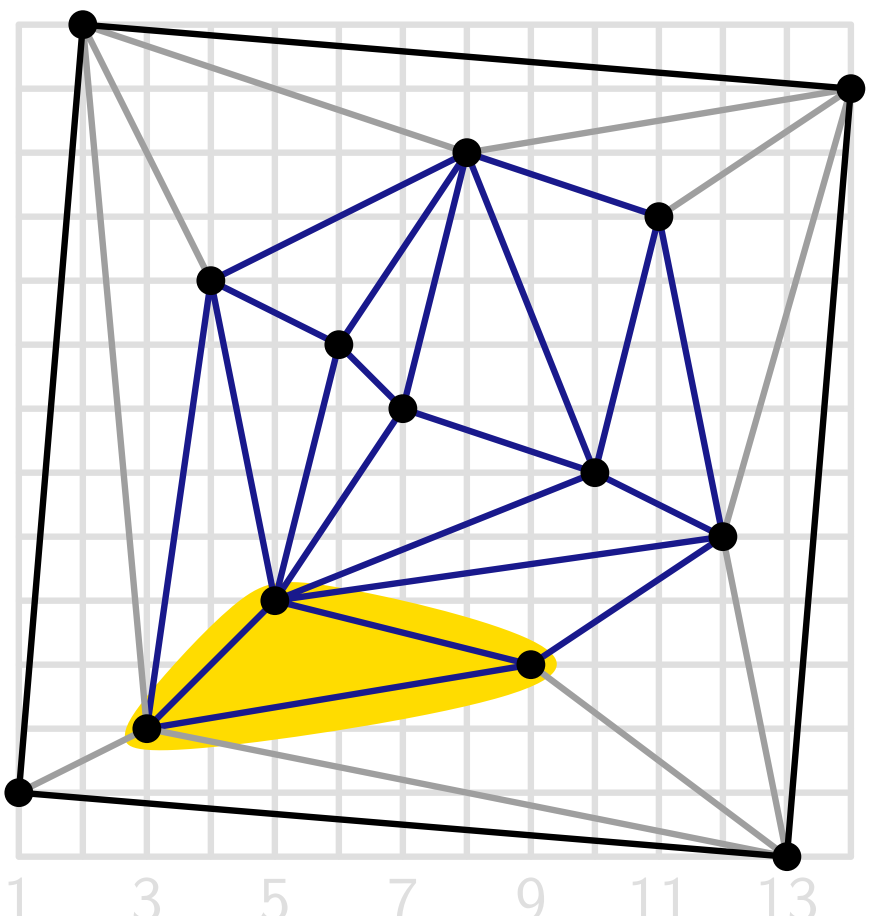

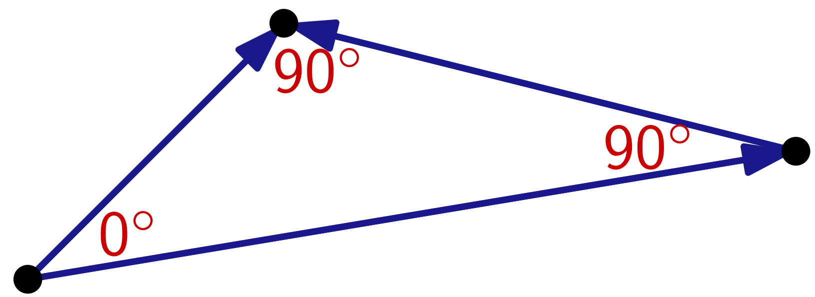

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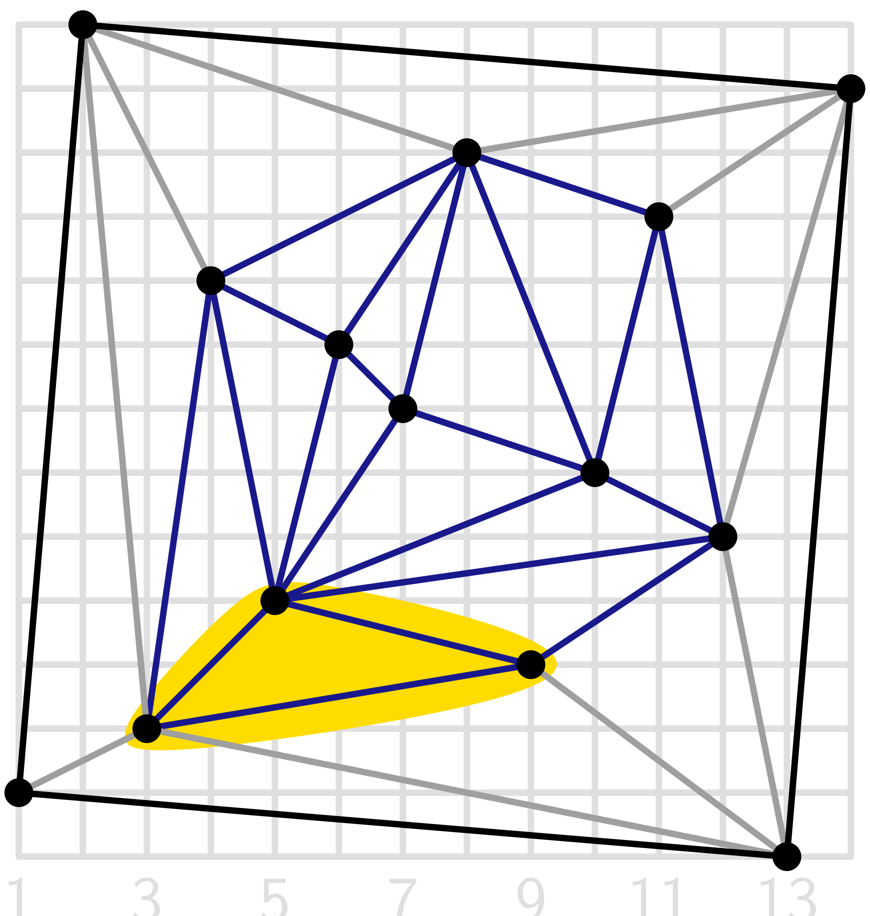

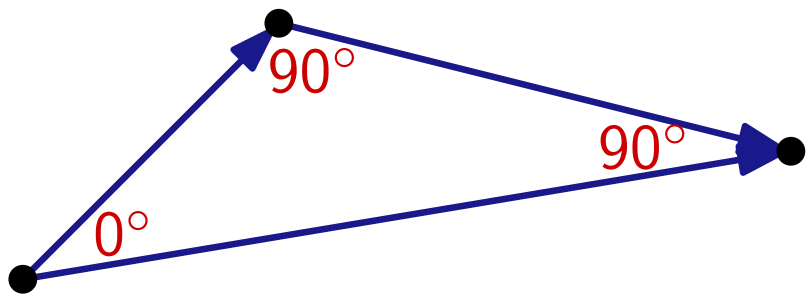

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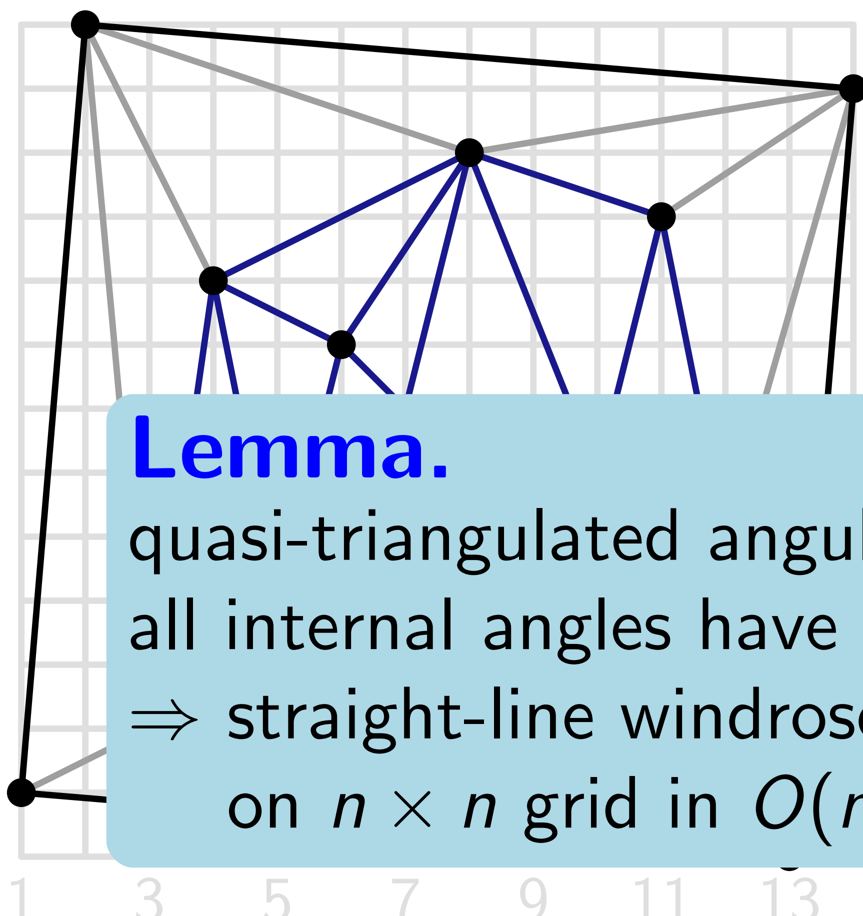

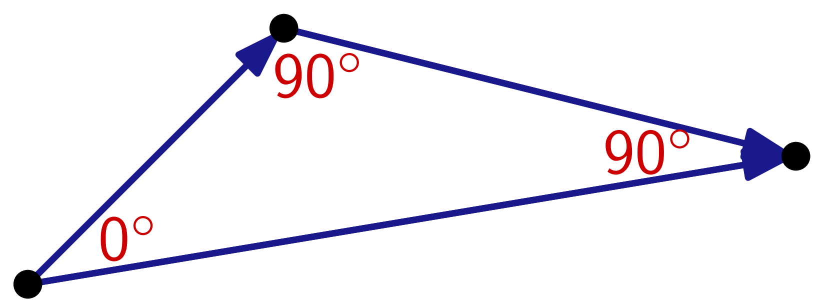

Lemma.
quasi-triangulated angular labeled graph (G, A_Q),
all internal angles have category 0° or 90°
$\Rightarrow$ straight-line windrose planar drawing
on $n \times n$ grid in $O(n)$ time

Triangulated graphs

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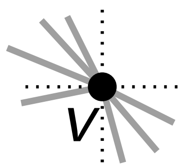

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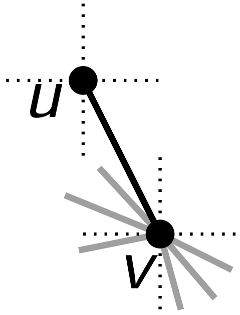

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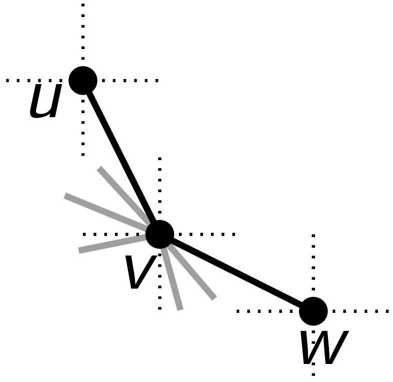

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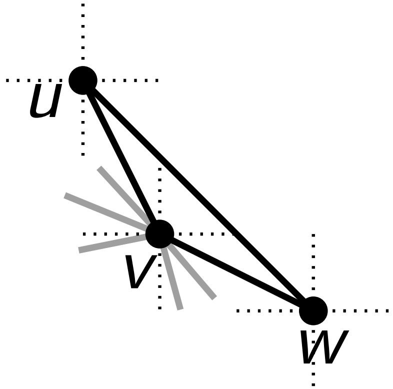

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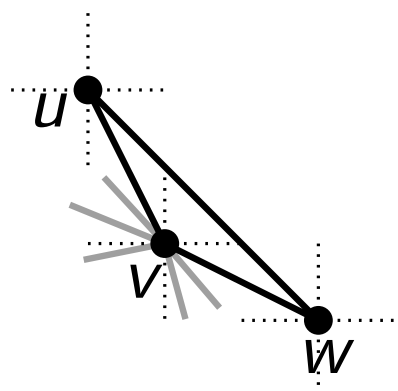

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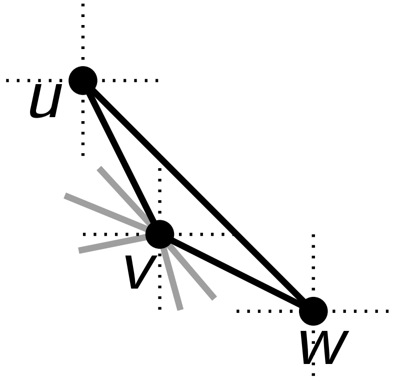

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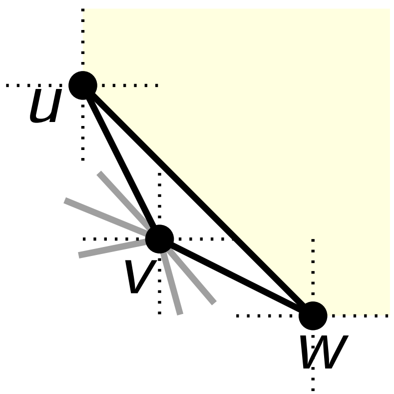

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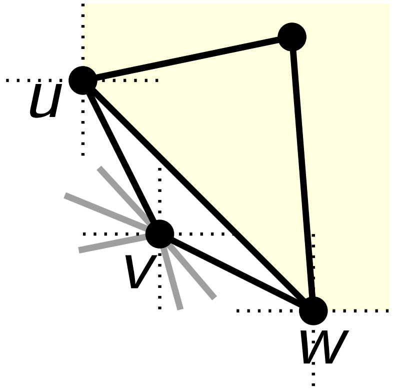

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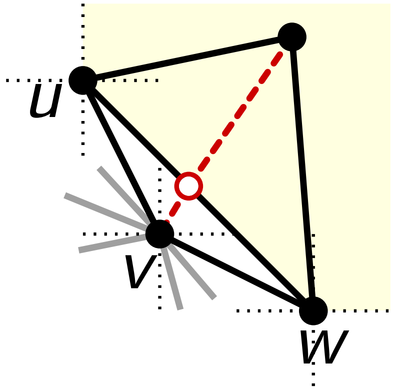

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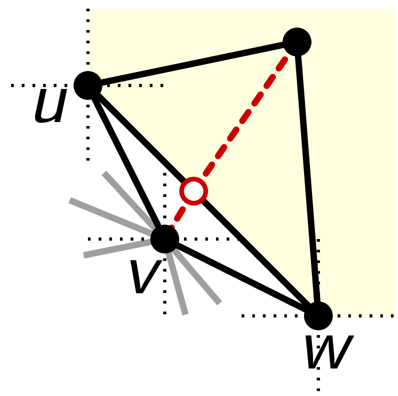

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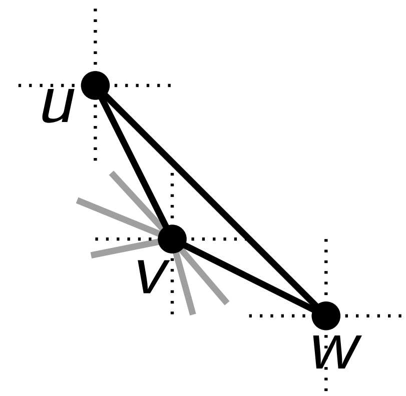

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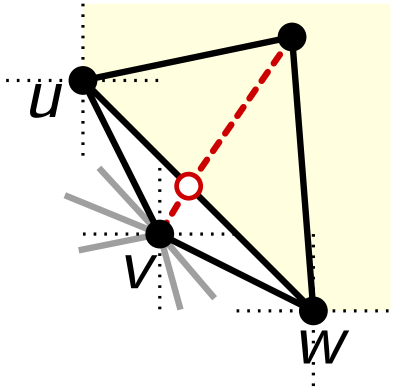

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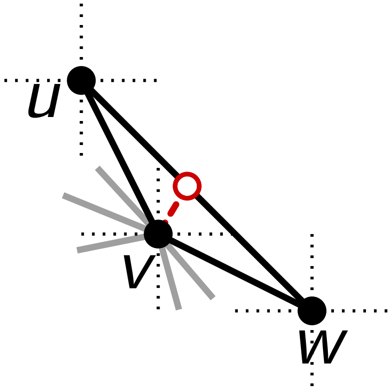

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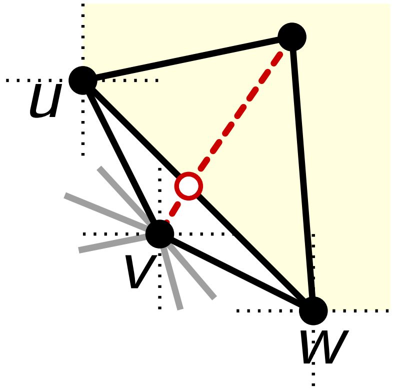

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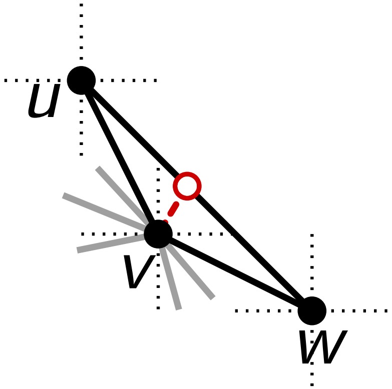

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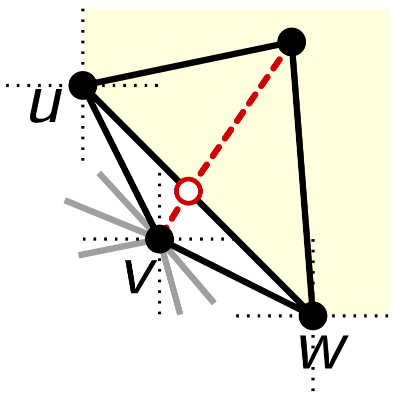

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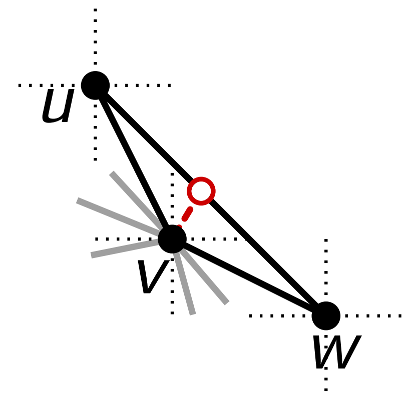

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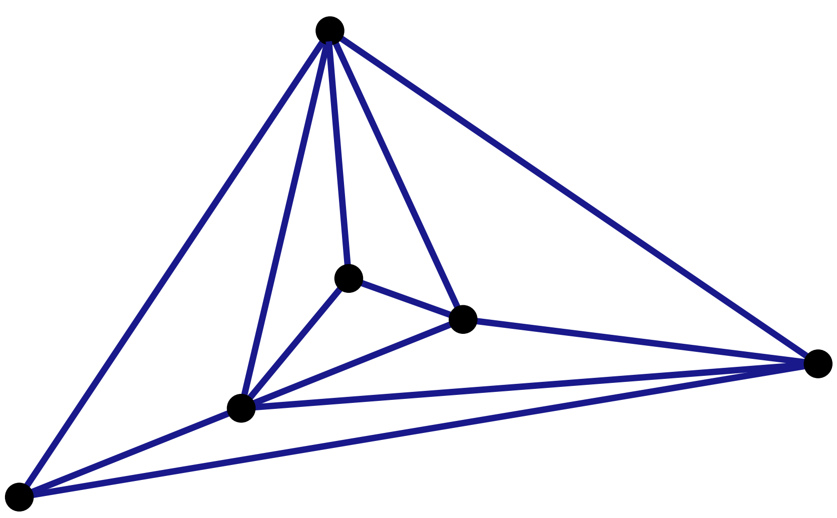

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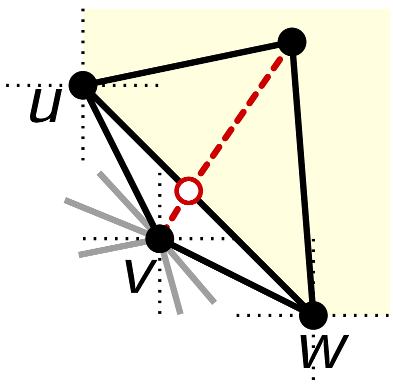

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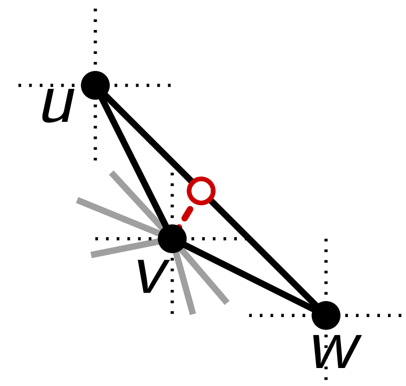

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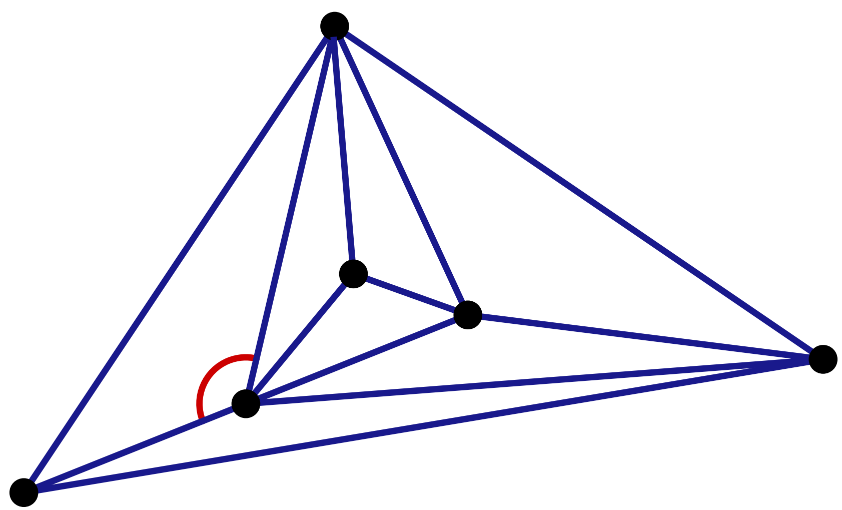

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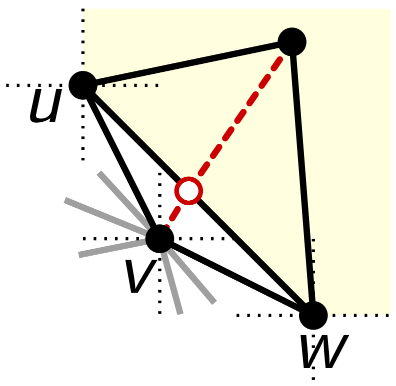

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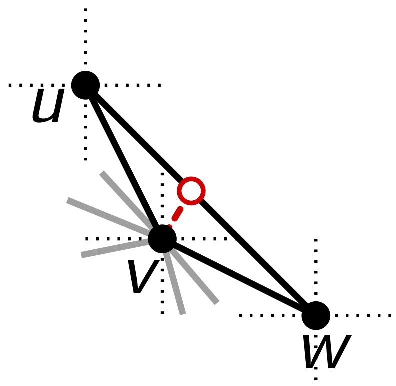

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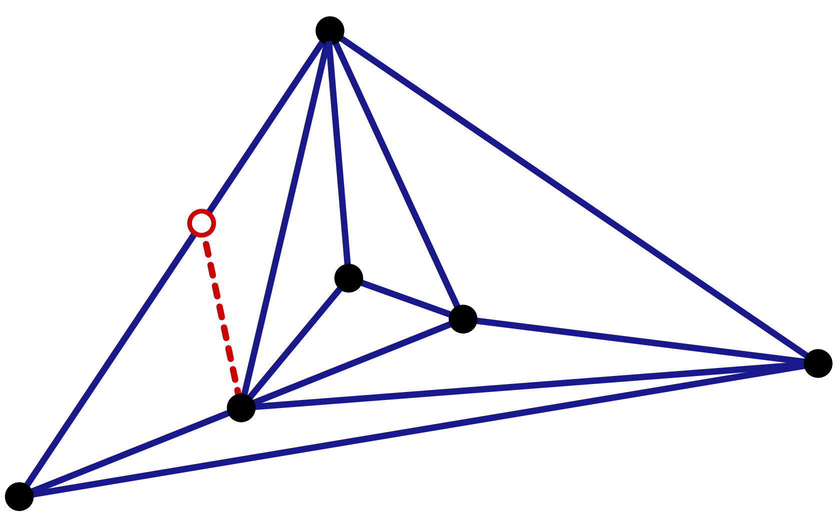

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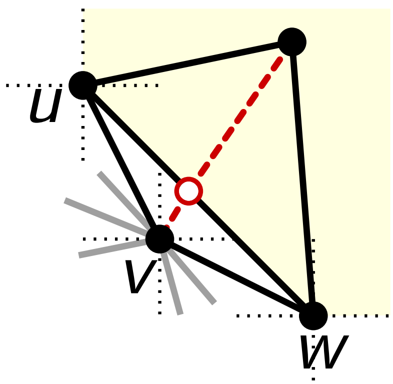

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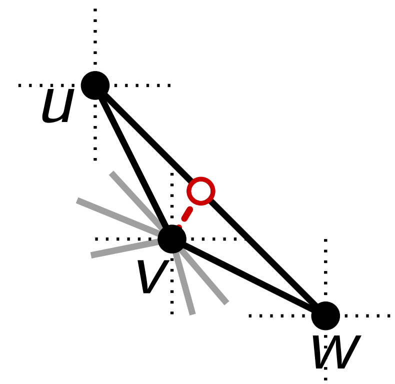

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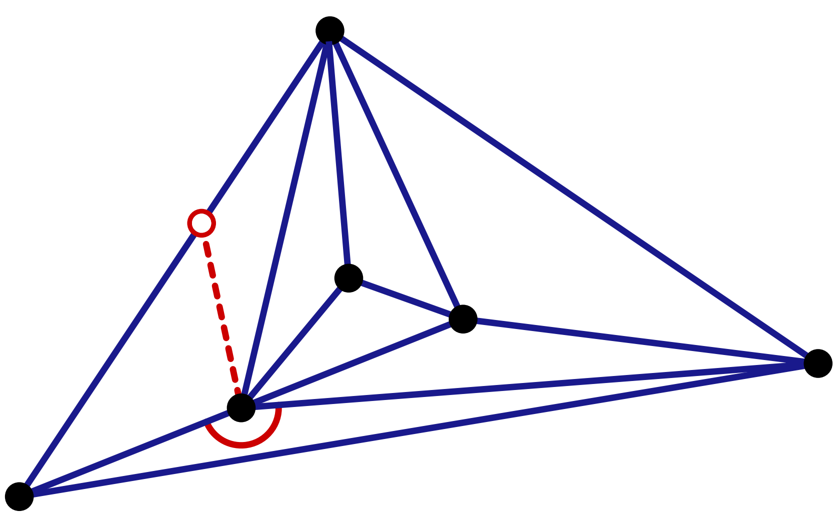

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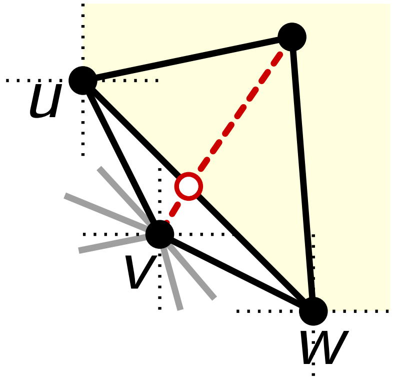

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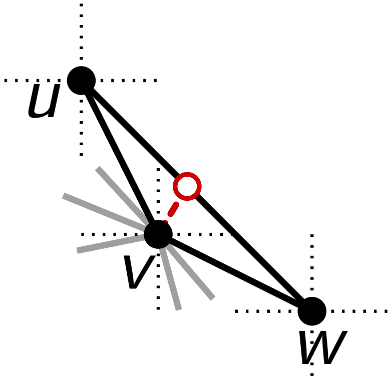

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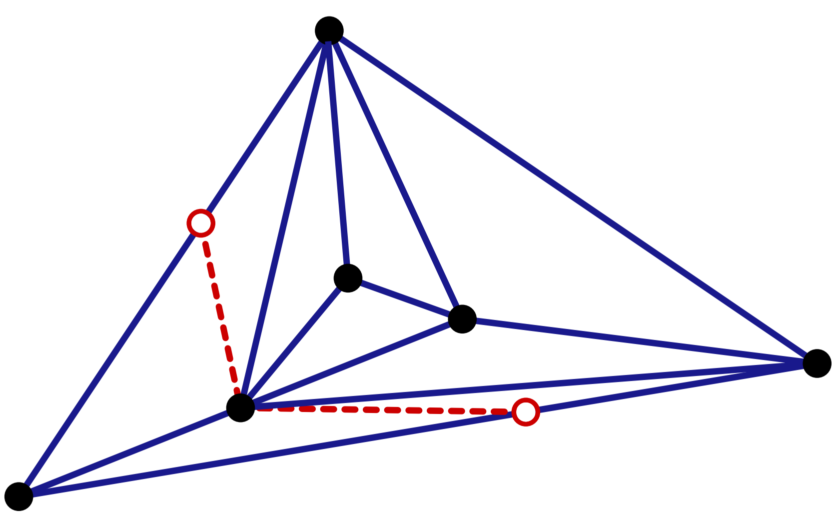

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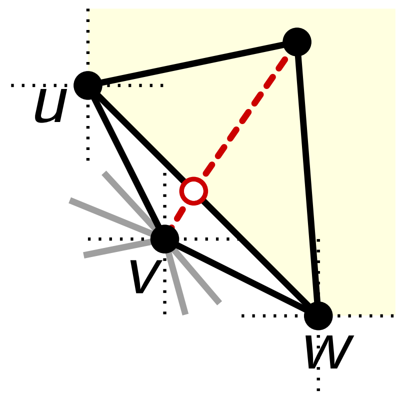

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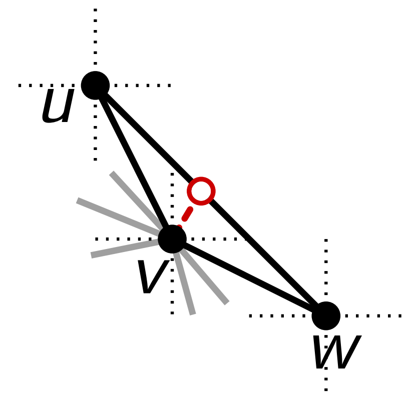

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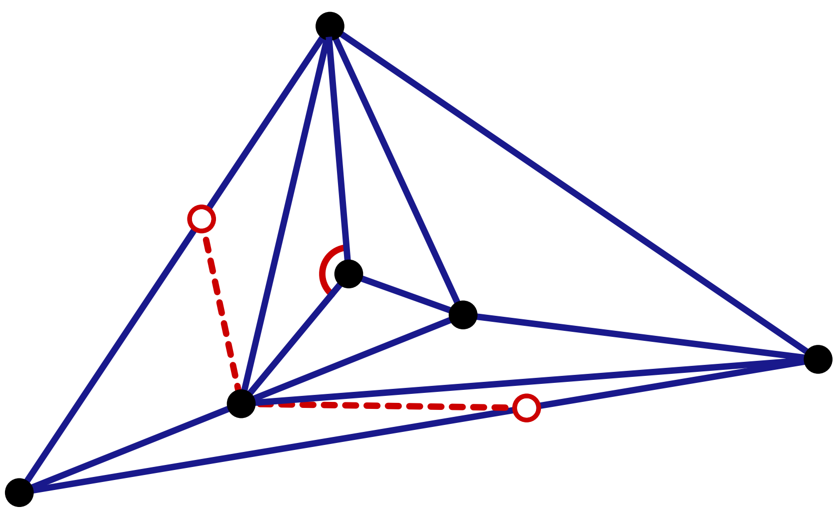

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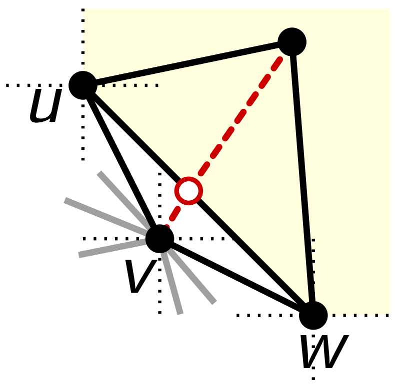

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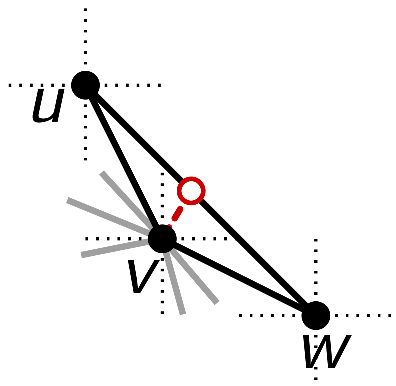

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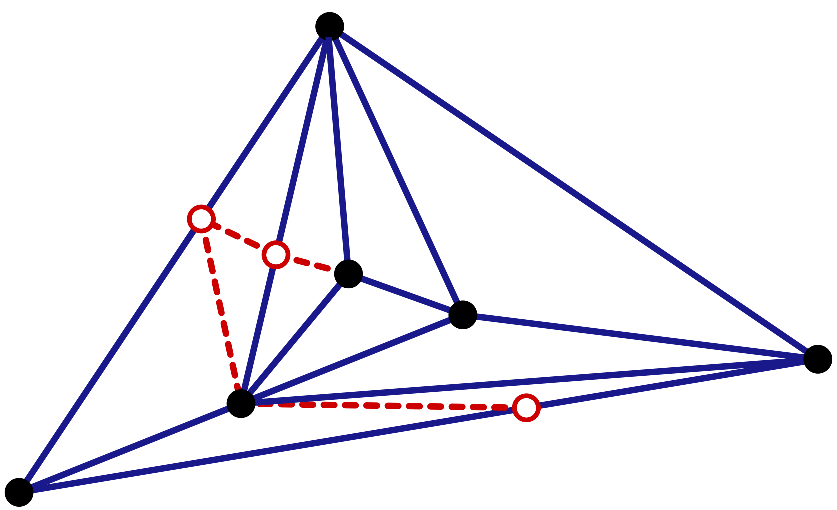

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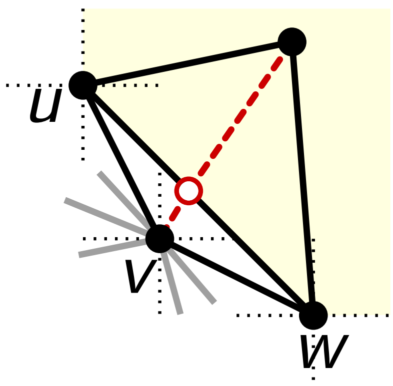

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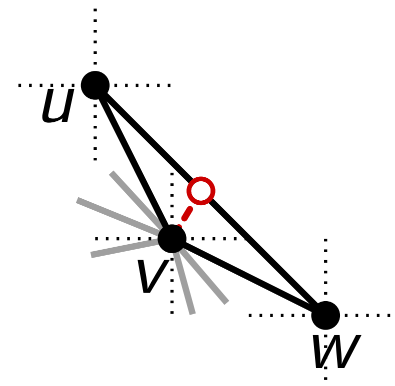

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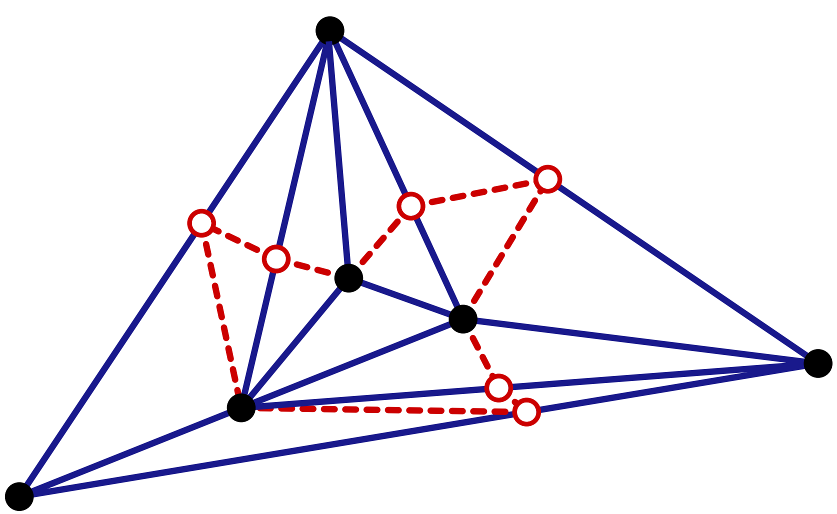

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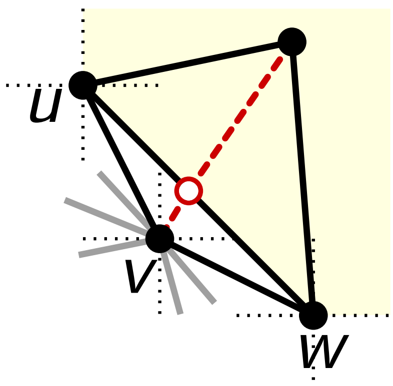

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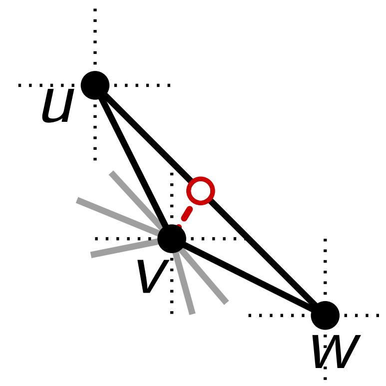

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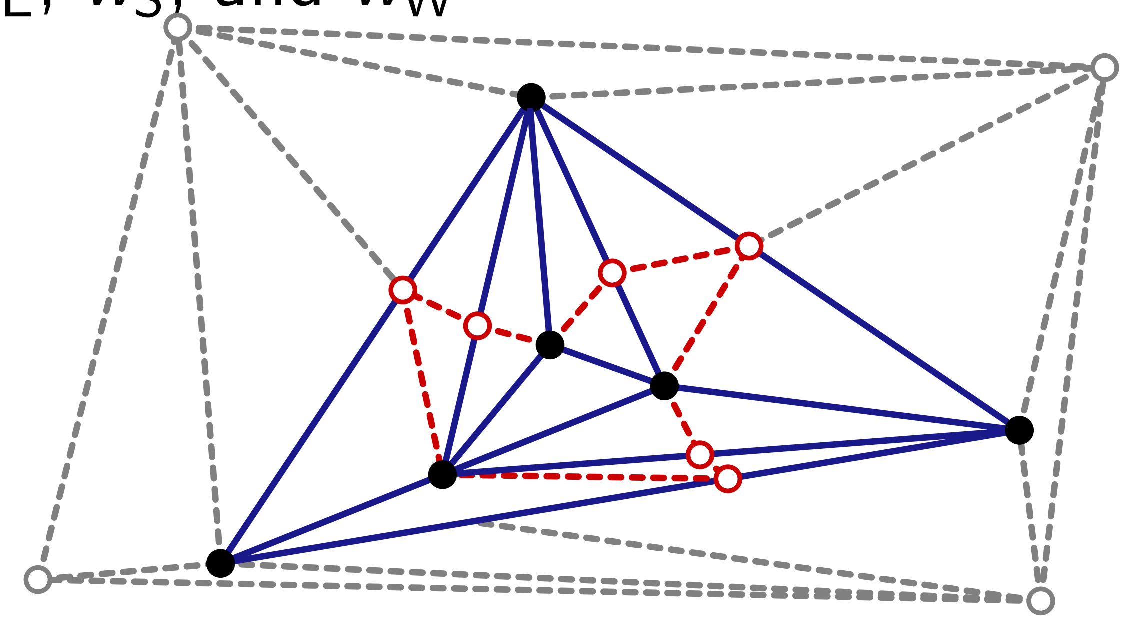

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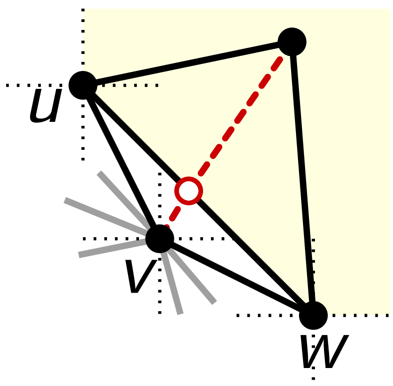

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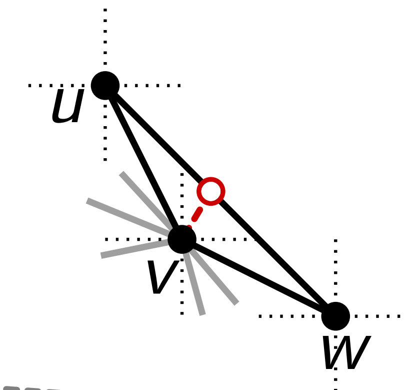

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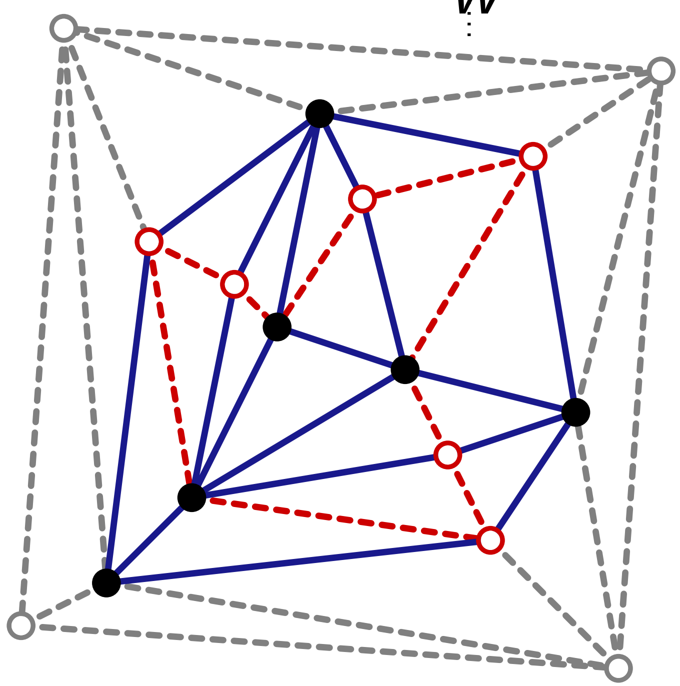

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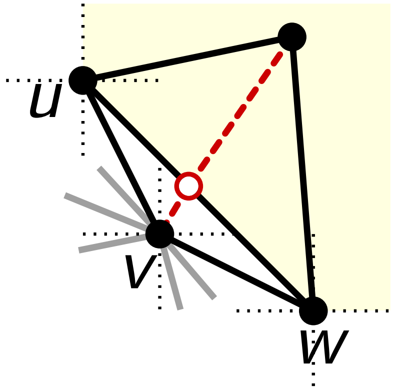

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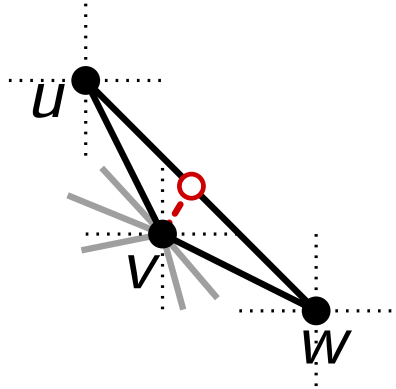

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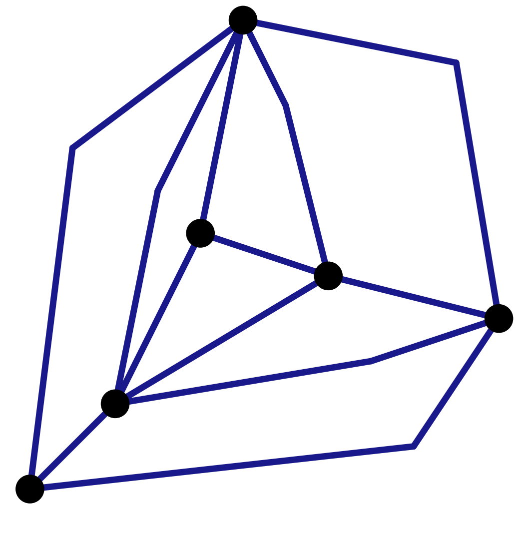

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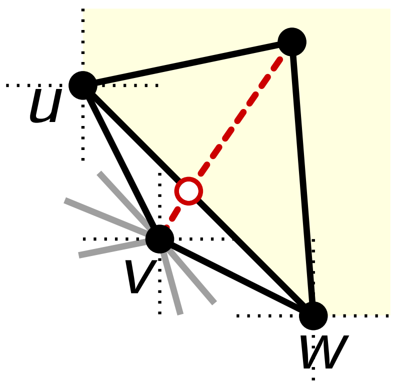

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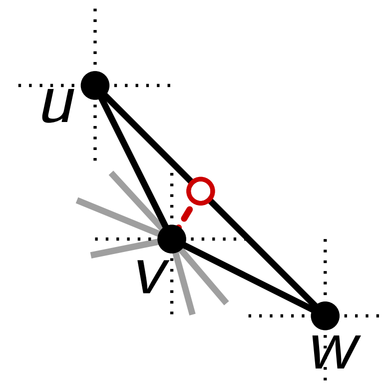

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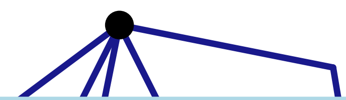

Theorem.
A triangulated q-constrained graph (G, Q) is windrose planar
$\Leftrightarrow$ A_Q is angular
$\rightarrow$ draw with 1 bend per edge
on an $O(n) \times O(n)$ grid in $O(n)$ time

Plane Graphs

Lemma.
plane q-constrained graph (G, Q)
$\Rightarrow$ find a large-angle assignment L such that
$A_{Q,L}$ is angular (if it exists) in $O(n \log^3 n)$ time

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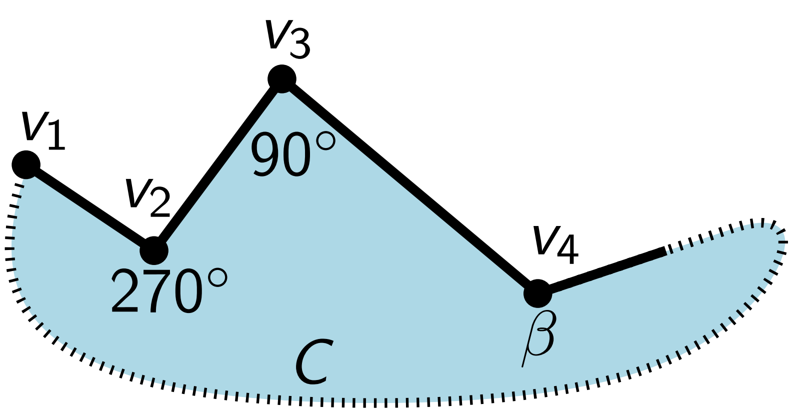

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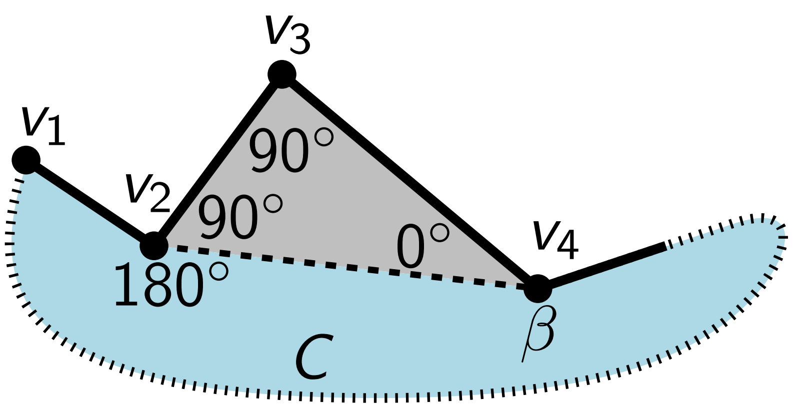

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Further Results

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Windrose planar q-constrained graph (G, Q) whose blocks are either edges or planar 3-trees
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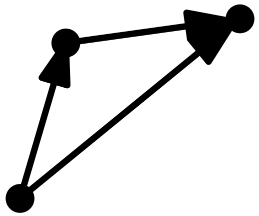

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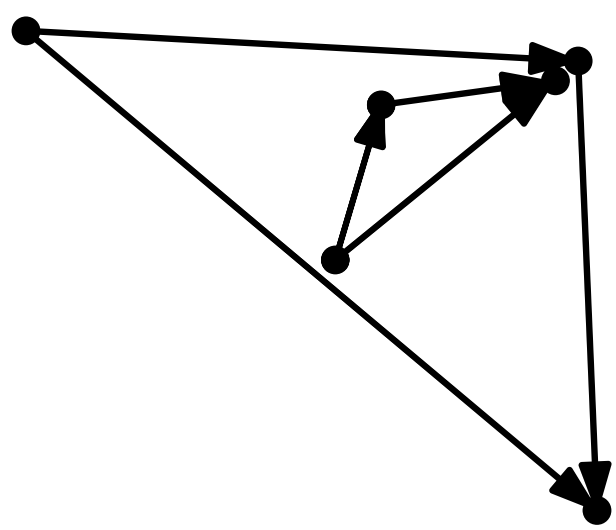

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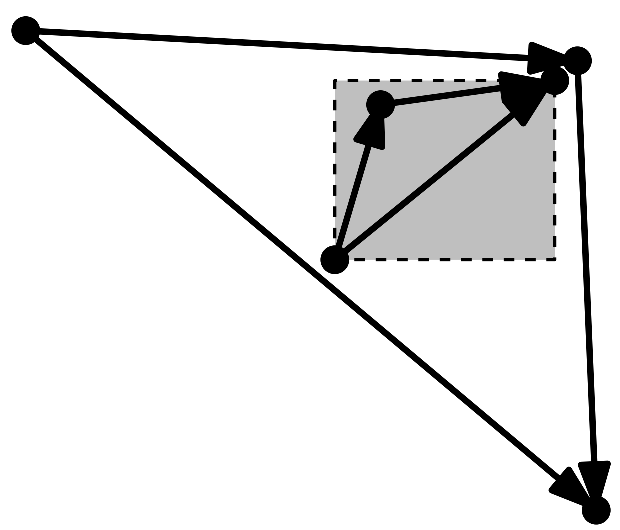

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