FernUniversität in Hagen
Fakultät für Mathematik und Informatik

theoretische informatik

Simultaneous Orthogonal Drawing

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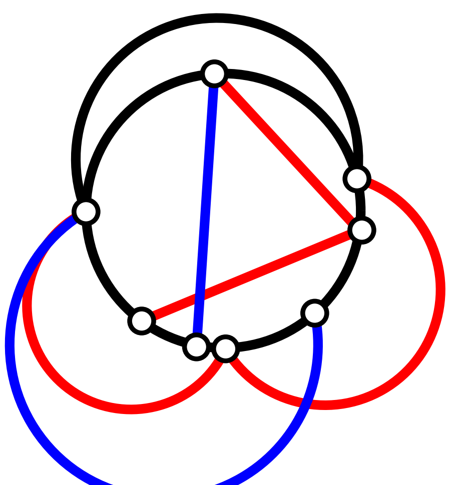
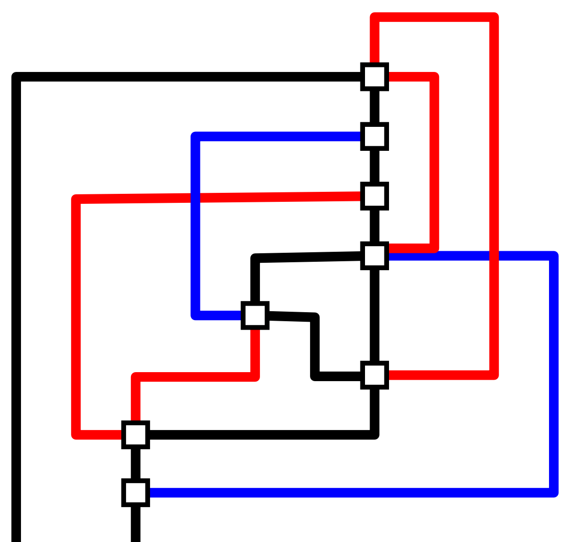
Philipp Kindermann, Jan Kratochvíl, Fabian Lipp, Ignaz Rutter





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Given k graphs $G_i = (V, E_i)$.
Are there drawings of $G_1, \ldots, G_k$ that coincide on G?

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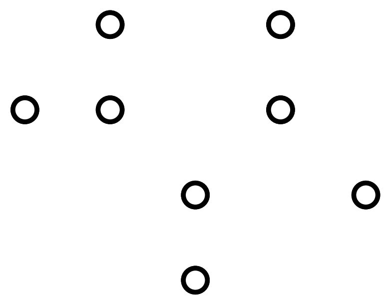
- planar, straight-line



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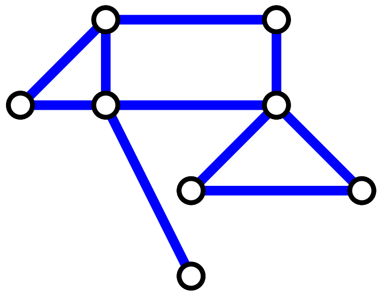

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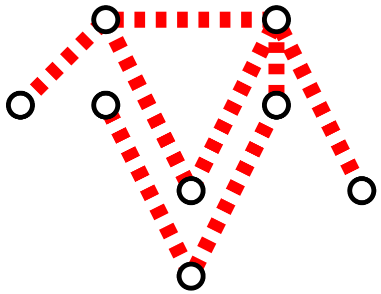

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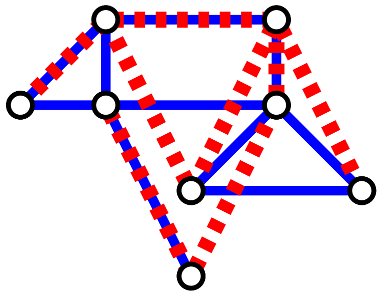

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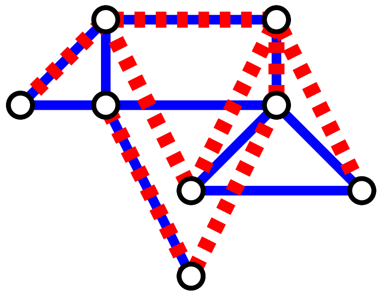

- NP-hard [Estrella-Balderrama et al. '07]

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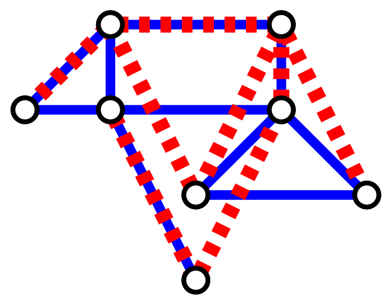

- NP-hard [Estrella-Balderrama et al. '07]
- There exist a tree and a path that don't work [Angelini et al. '12]

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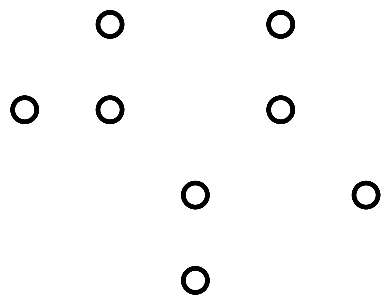

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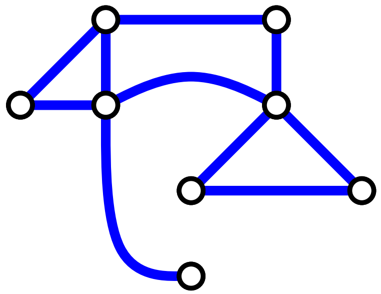

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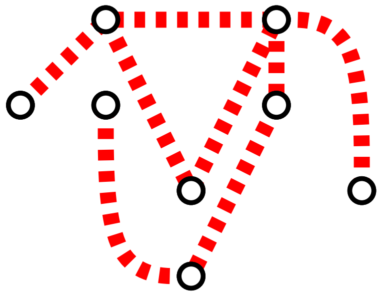

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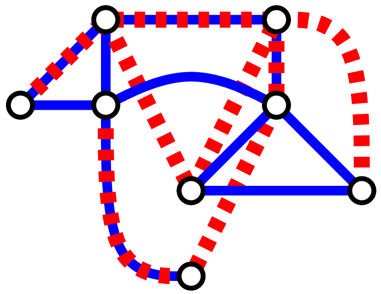

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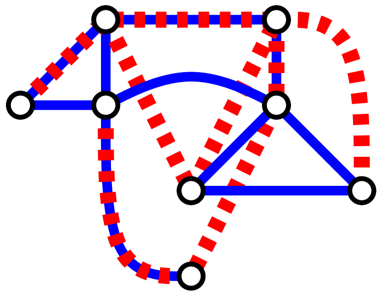

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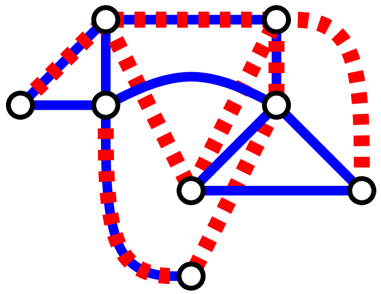

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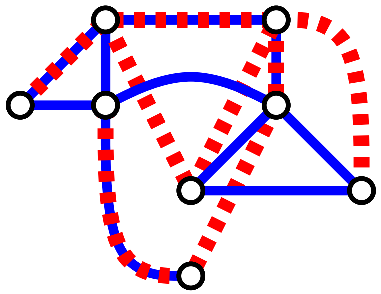

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- G_1, G_2 are biconnected, G is connected [Bläsius & Rutter '16]
- ...

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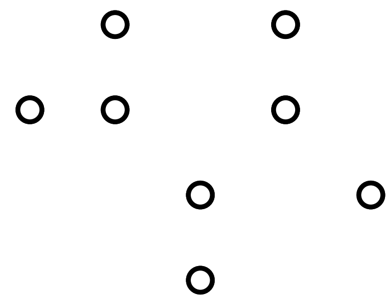

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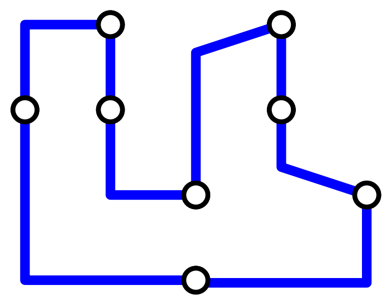

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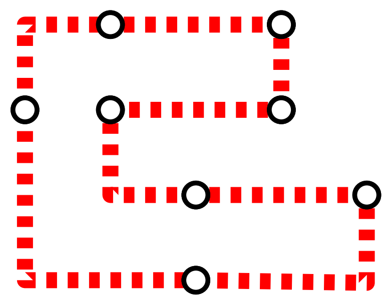

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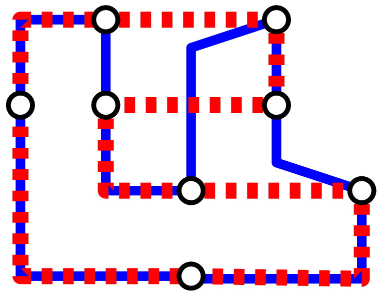

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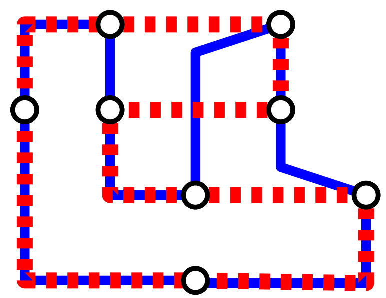

- 6 bends per edge on $O(n) \times O(n)$ grid [Bekos et al. '14]

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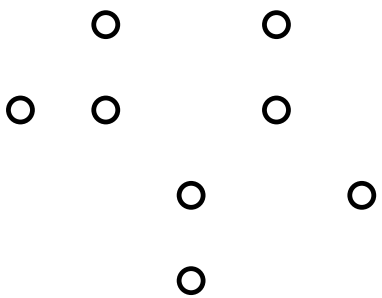

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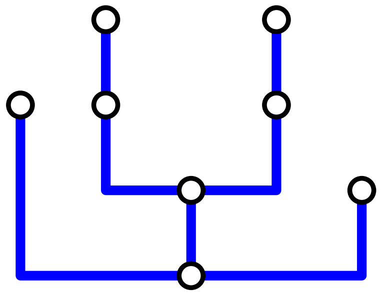

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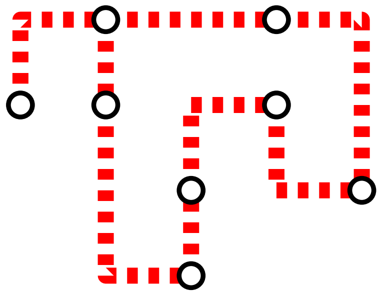

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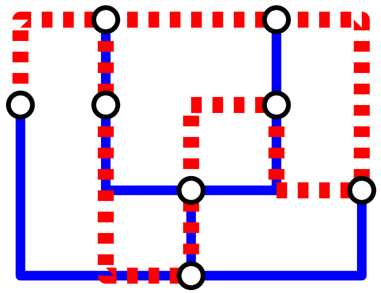

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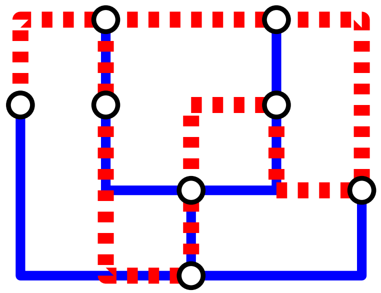

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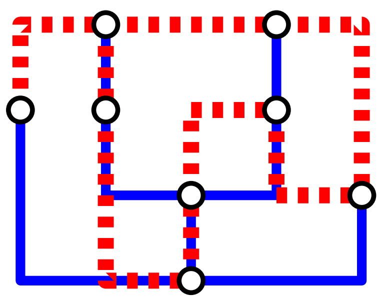

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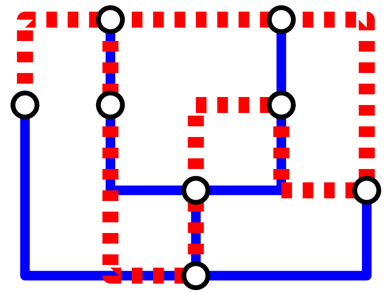

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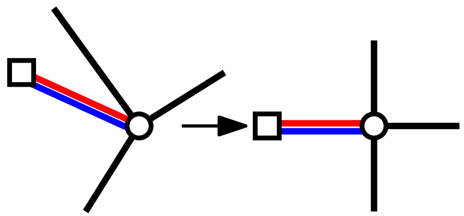

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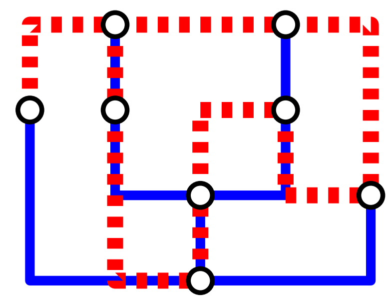

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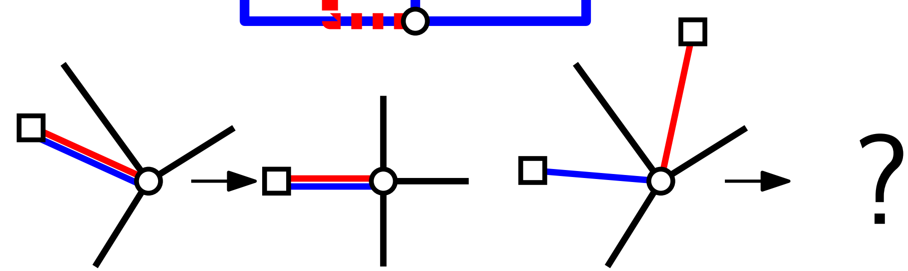

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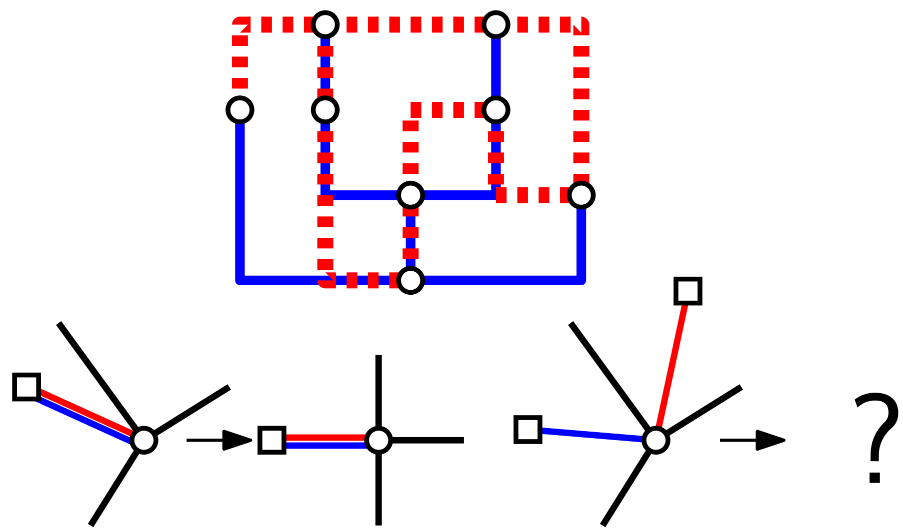

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- combinatorial embeddings of G_1, G_2 coincide on G
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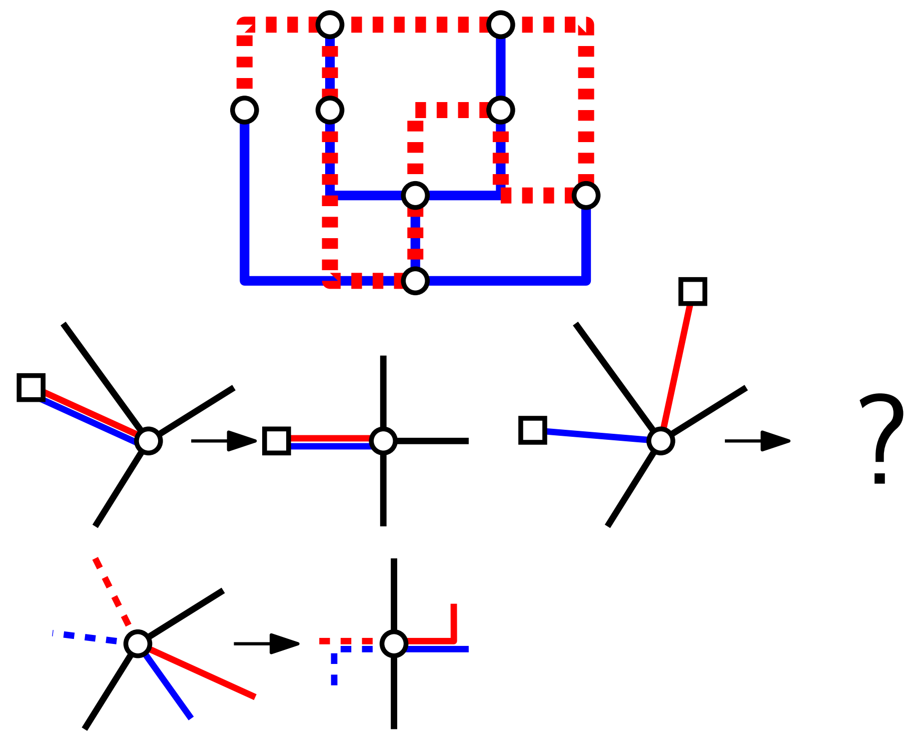

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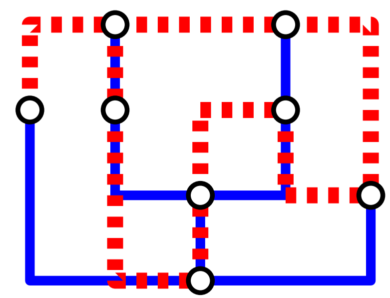

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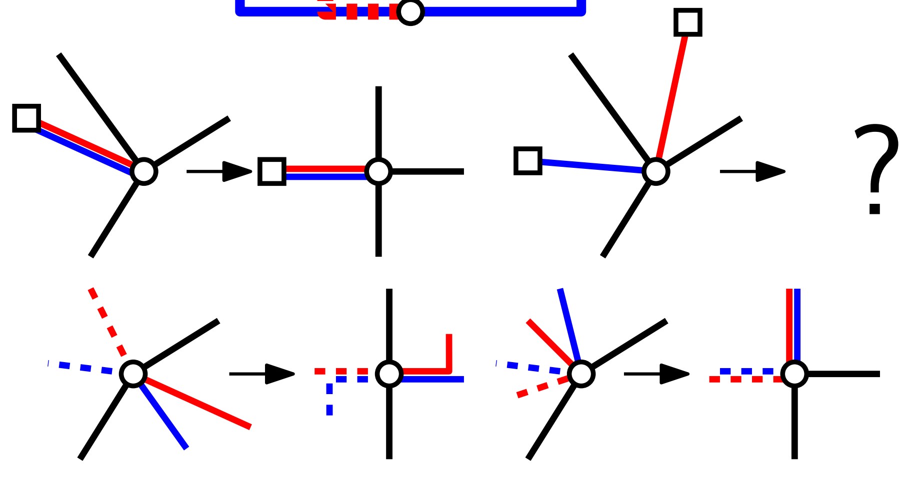

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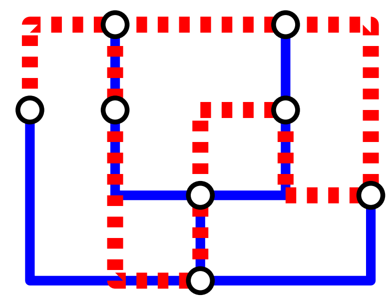

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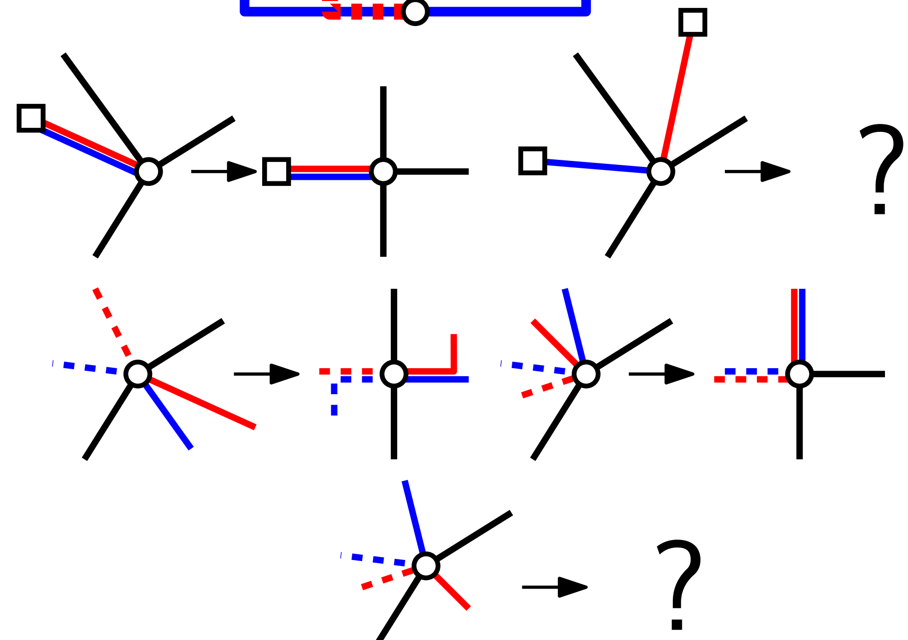

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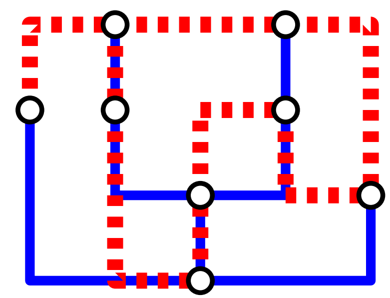

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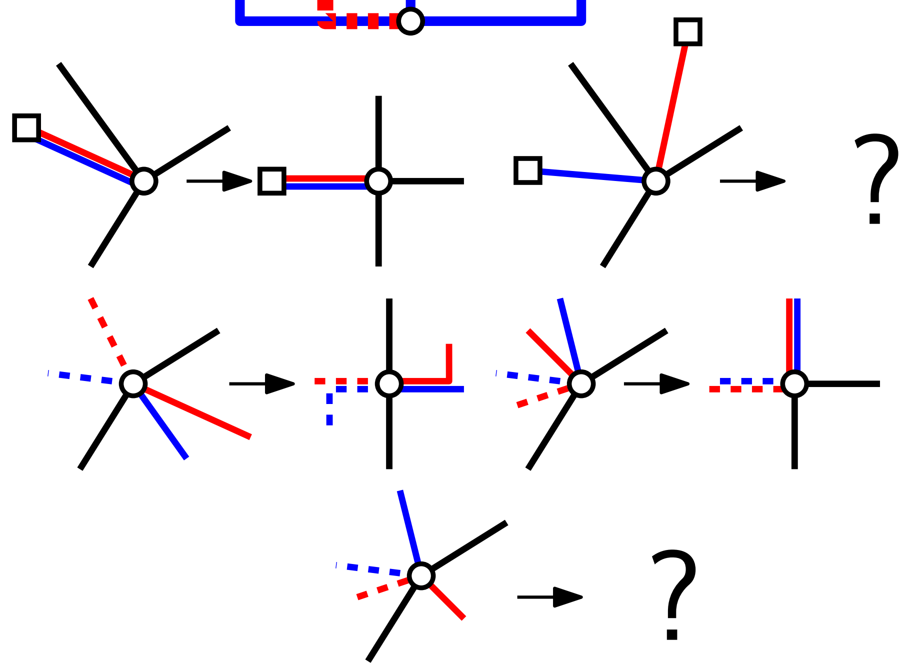

Conditions are sufficient!

Special Case: Common Graph is a Cycle

Isn't this trivial?

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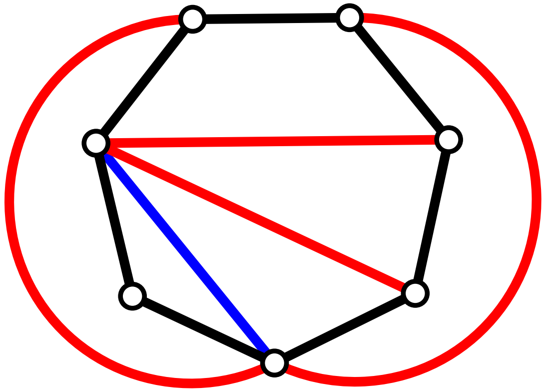
- Instances trivially admit a SEFE...



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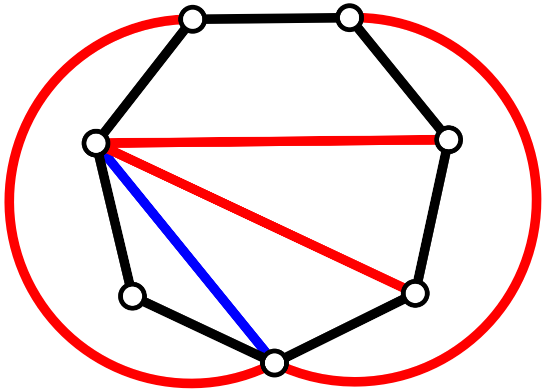

- Instances trivially admit a SEFE. . .
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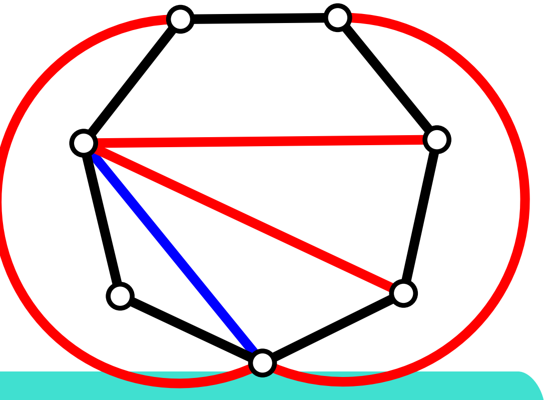

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Theorem.
It is NP-complete to decide whether three graphs G_1, G_2, G_3 whose common graph is a cycle admit an OrthoSEFE.

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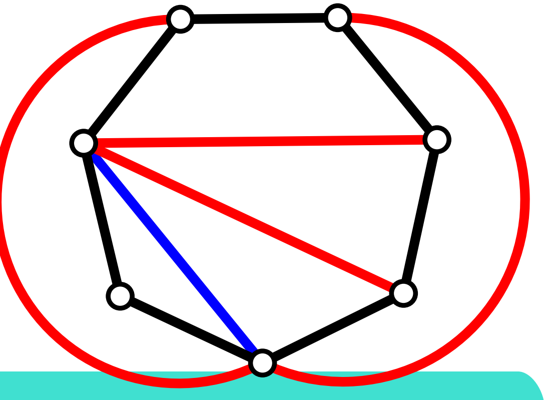

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Proof:
Reduction from Nae-3Sat:

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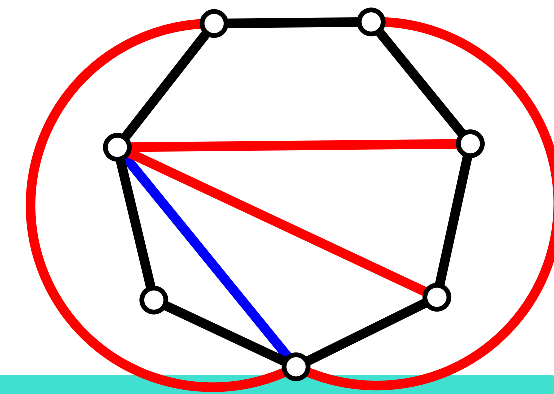

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It is NP-complete to decide whether three graphs G_1, G_2, G_3 whose common graph is a cycle admit an OrthoSEFE.

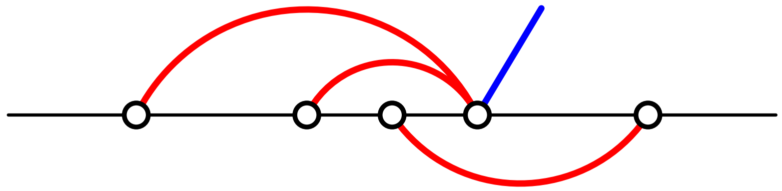
Proof:
Reduction from Nae-3Sat:

Given: X set of variables, C set of clauses each containing 3 literals

Find: Truth assignment such that no clause in C evaluates to (True, True, True) or (False, False, False)

Reduction from NAE-3SAT

Variable gadget for variable x:



$x = \text{TRUE}$

Reduction from NAE-3SAT

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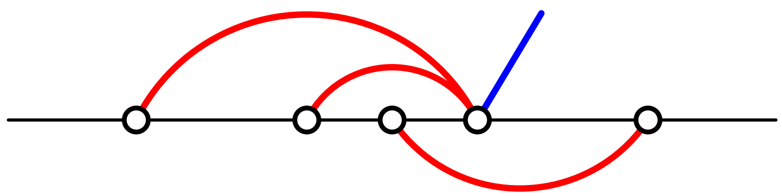

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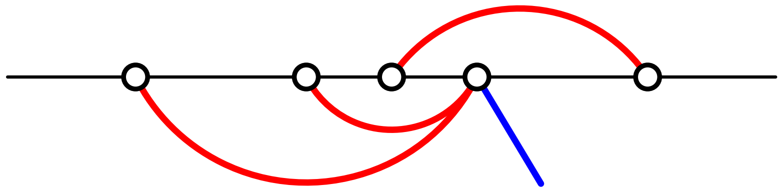

$x = \textsc{False}$

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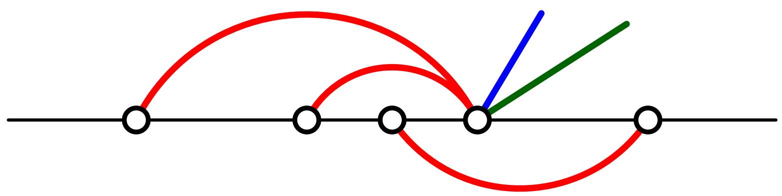

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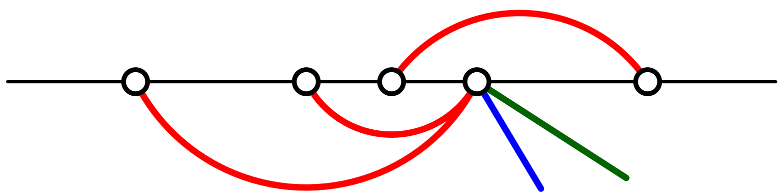
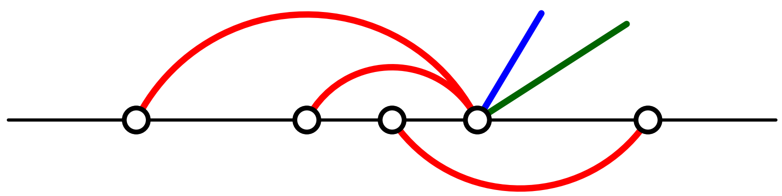


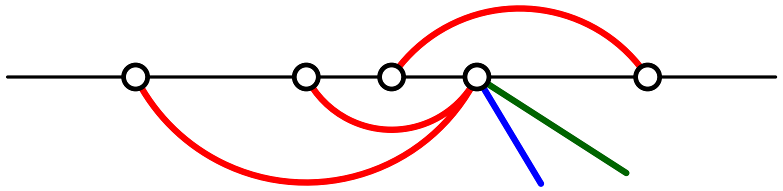
$x = \text{FALSE}$

Reduction from NAE-3SAT

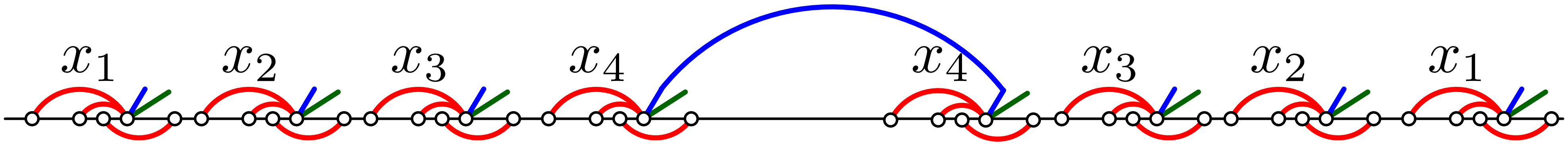
Variable gadget for variable x:



$x = $ TRUE



$x = $ FALSE



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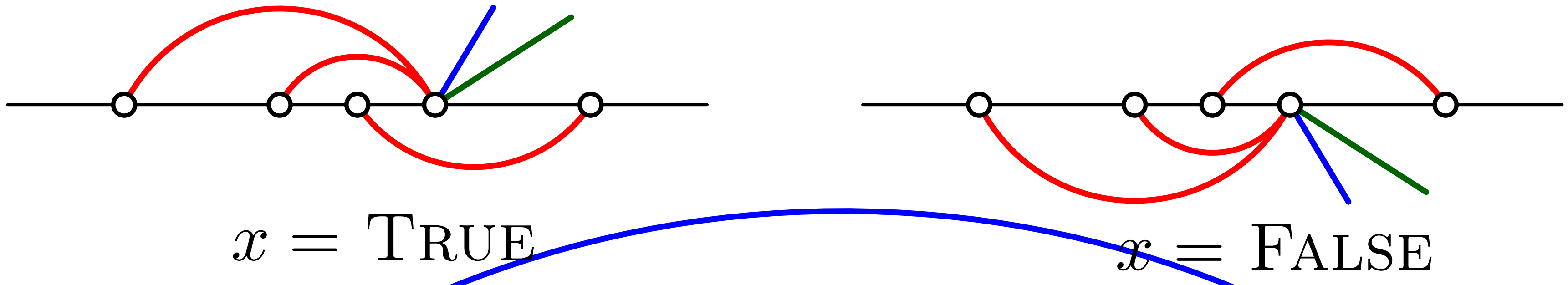

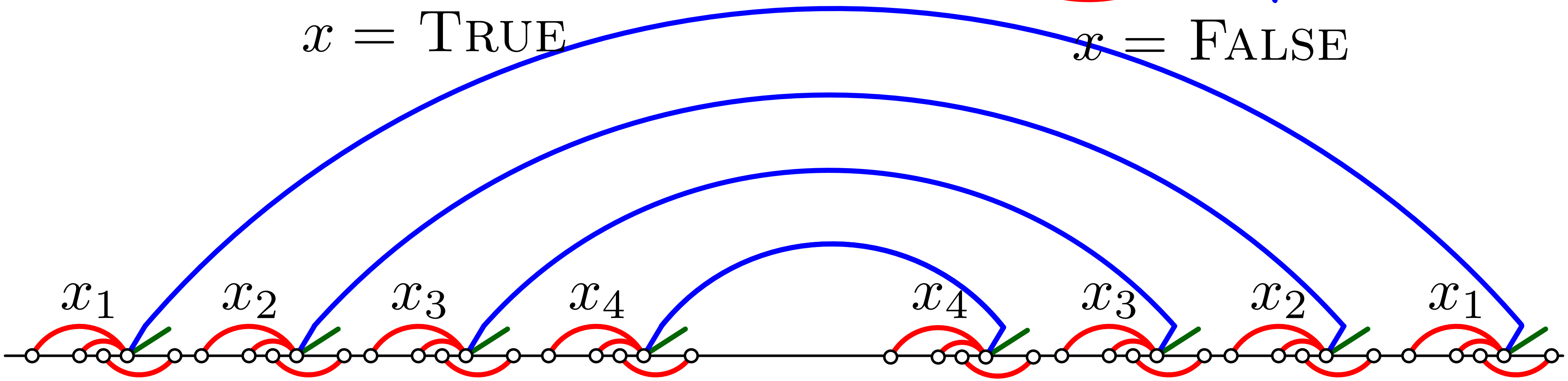

Reduction from $\textsc{Nae-3Sat}$

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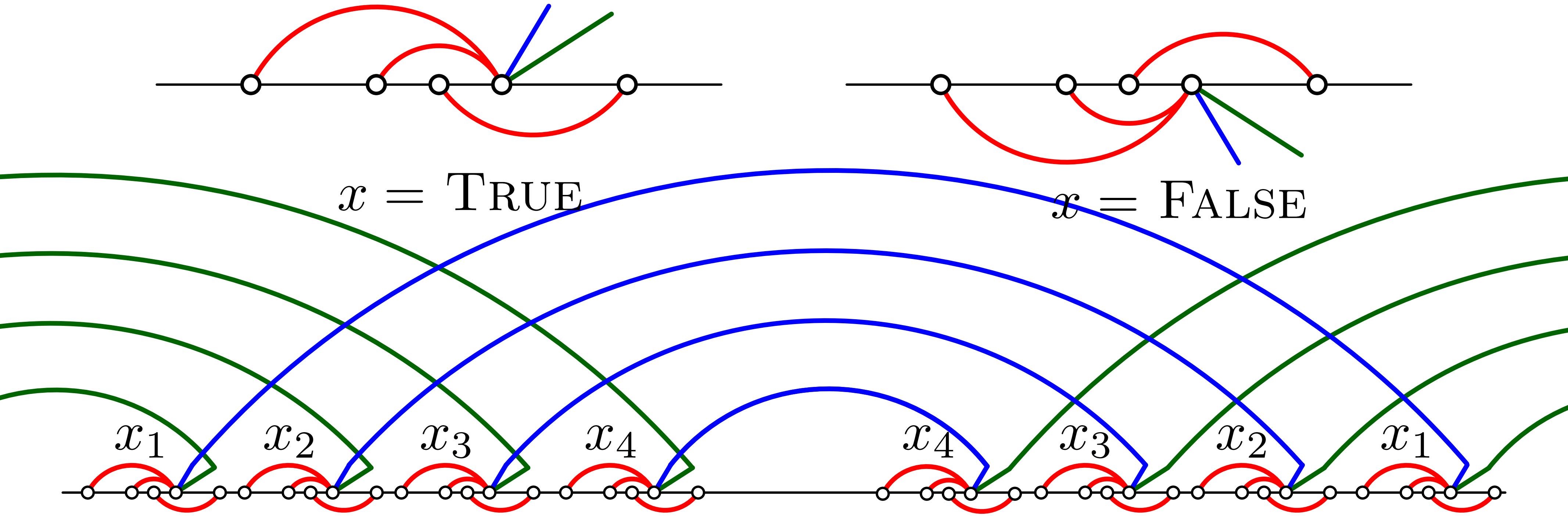

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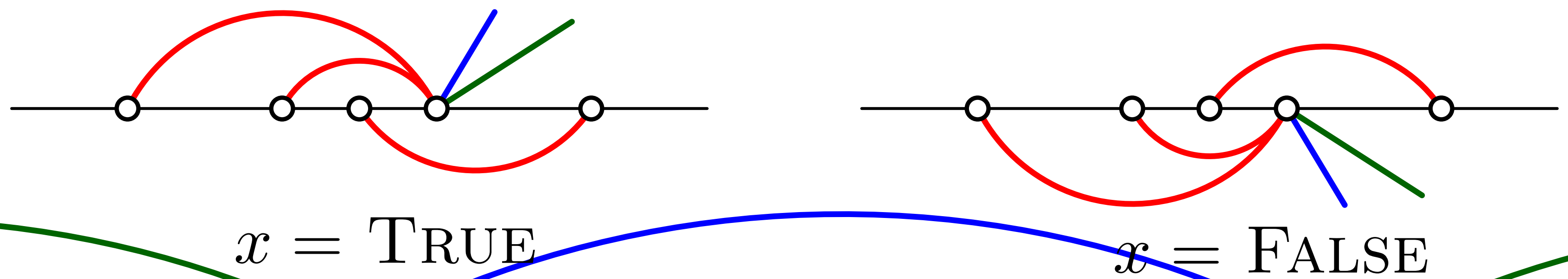

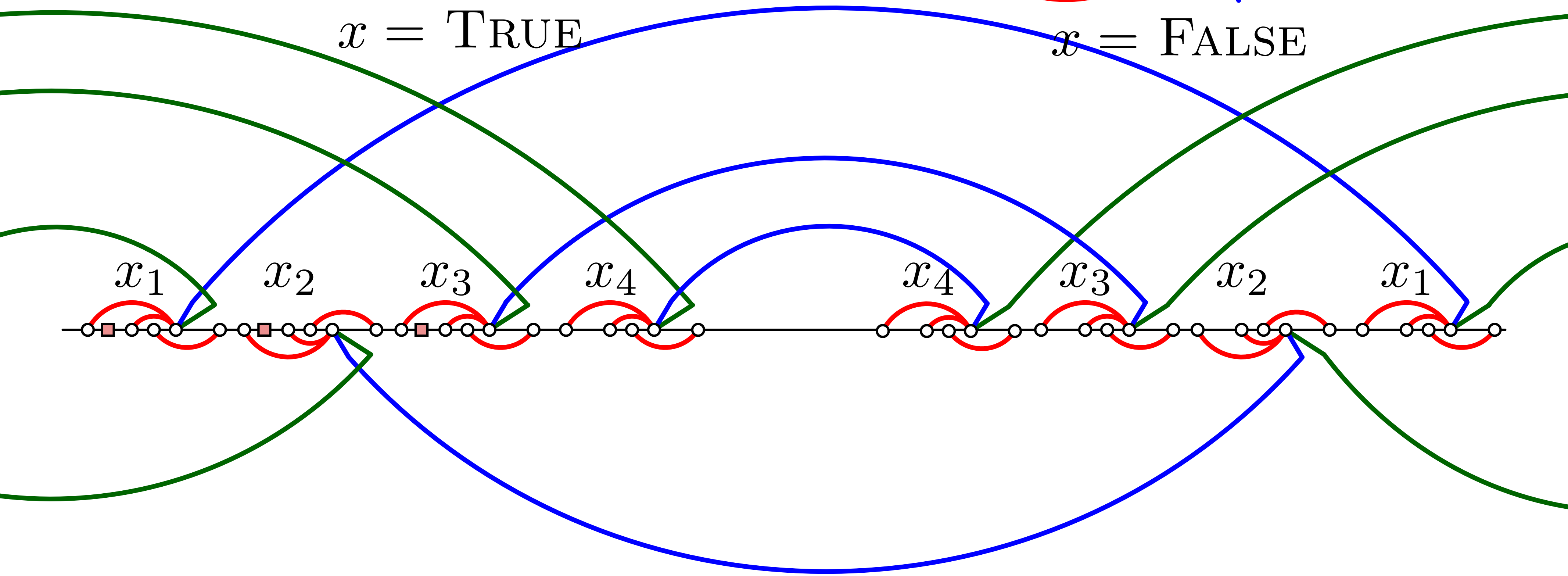

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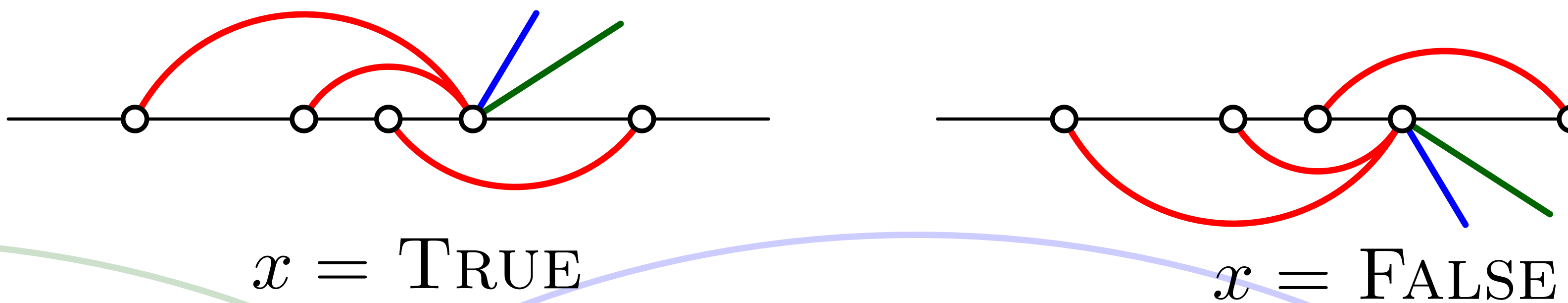

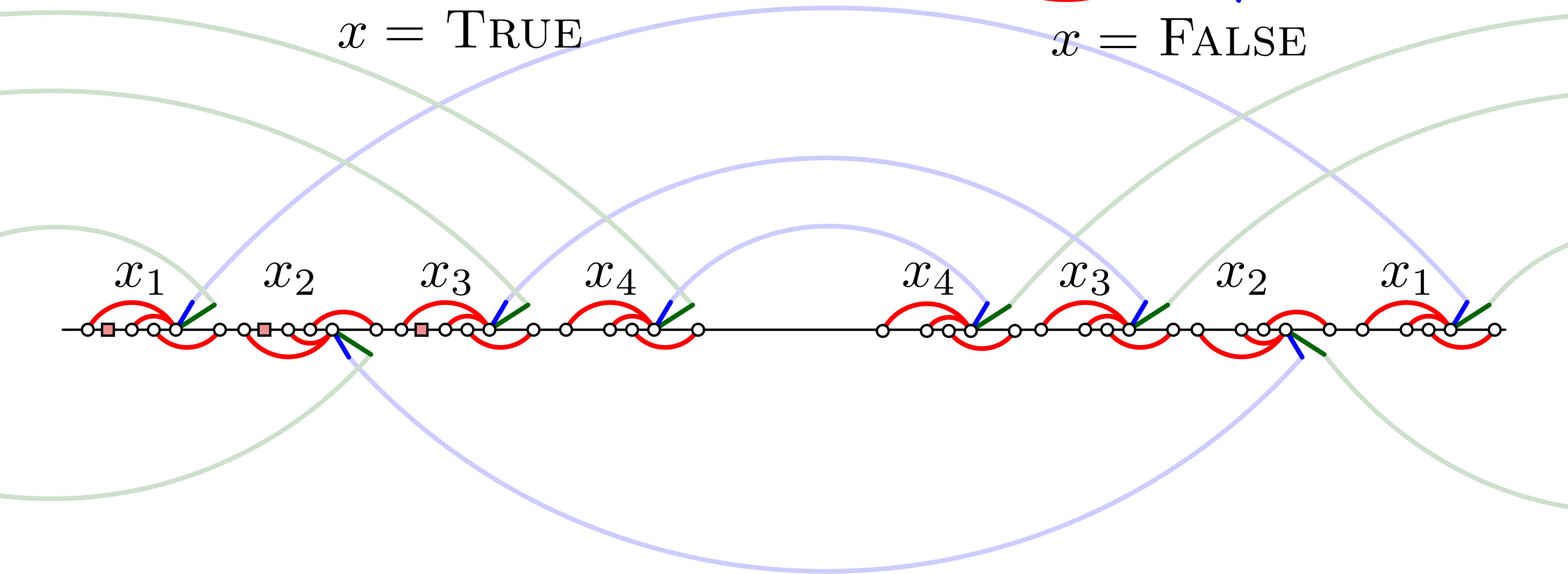

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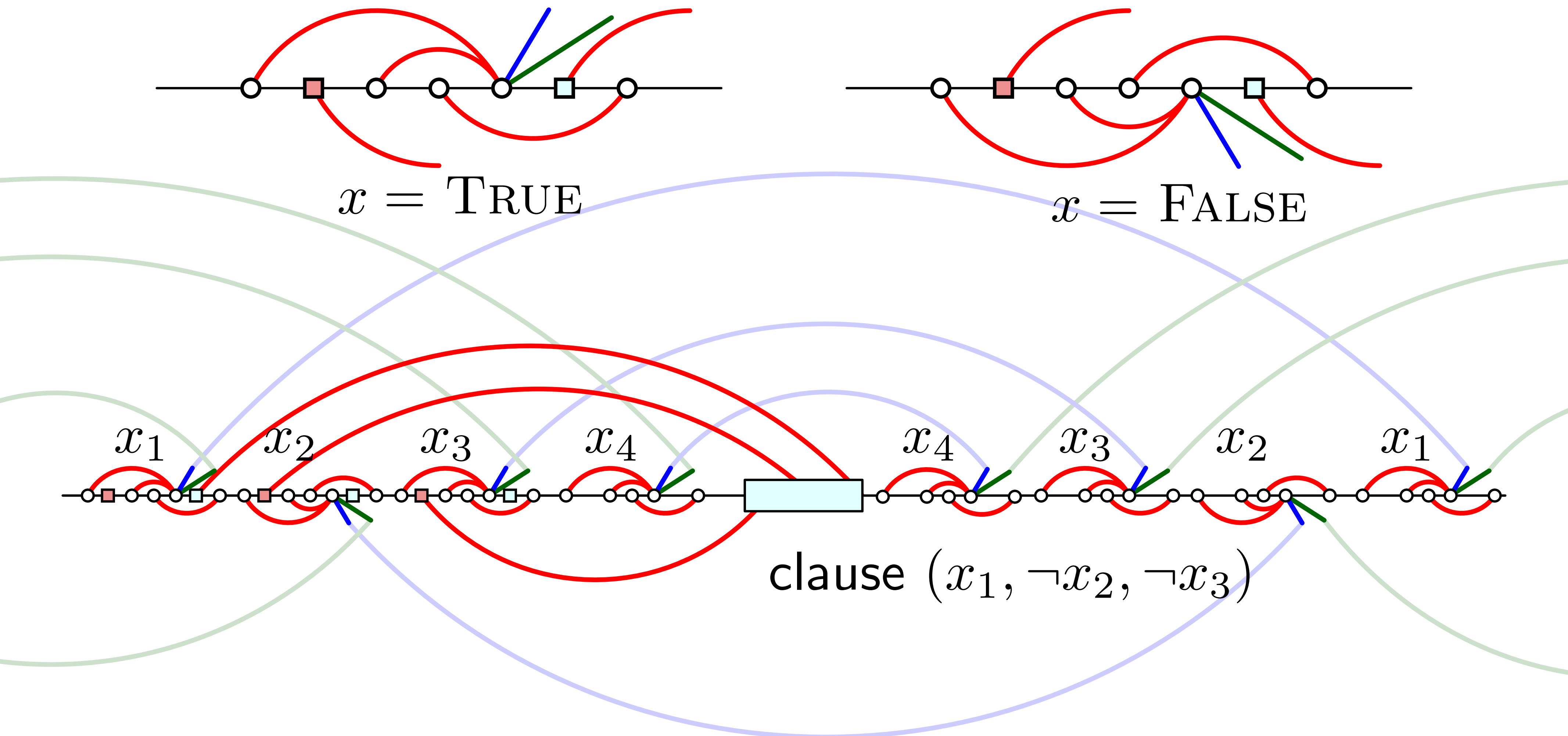

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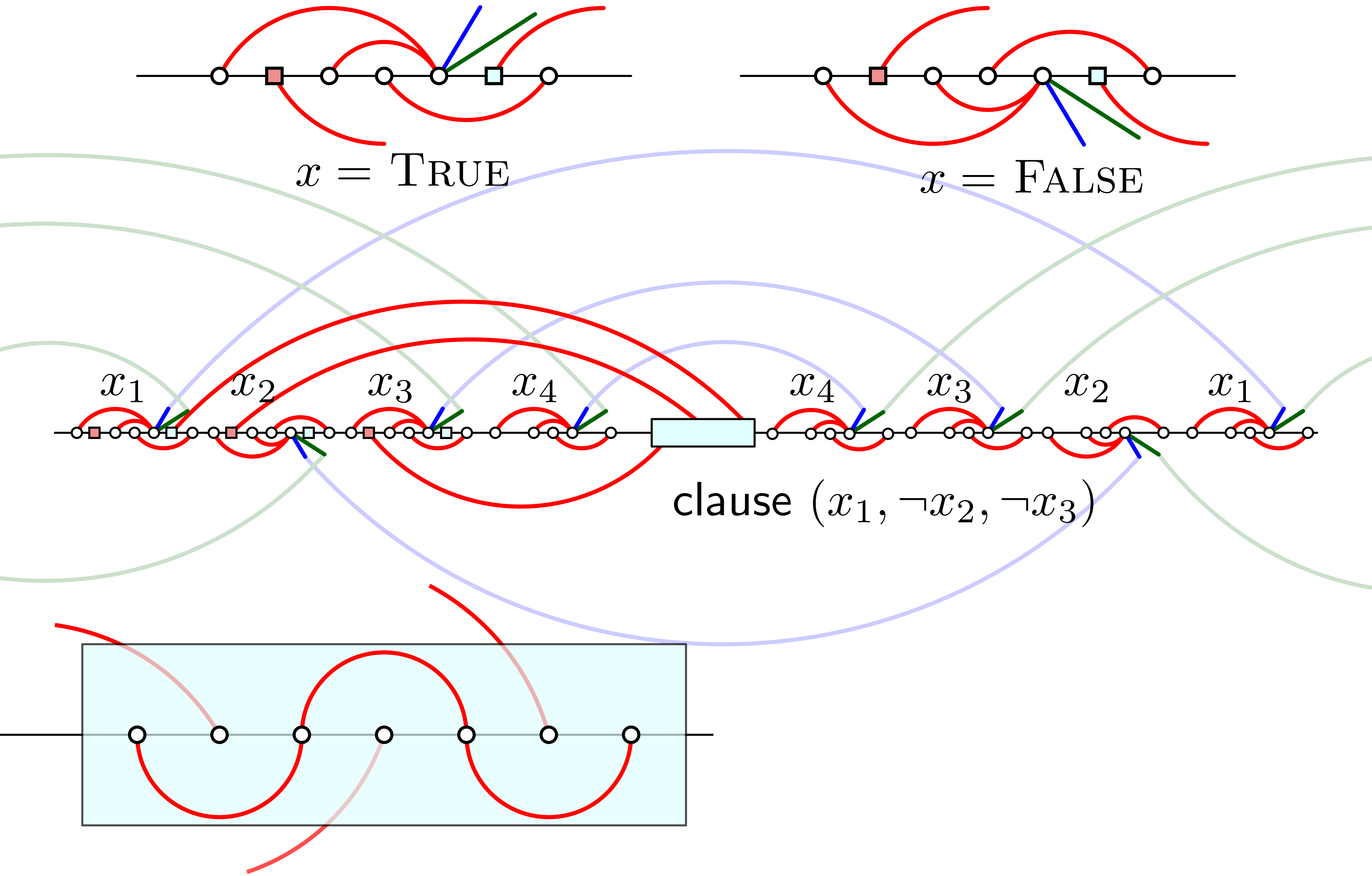

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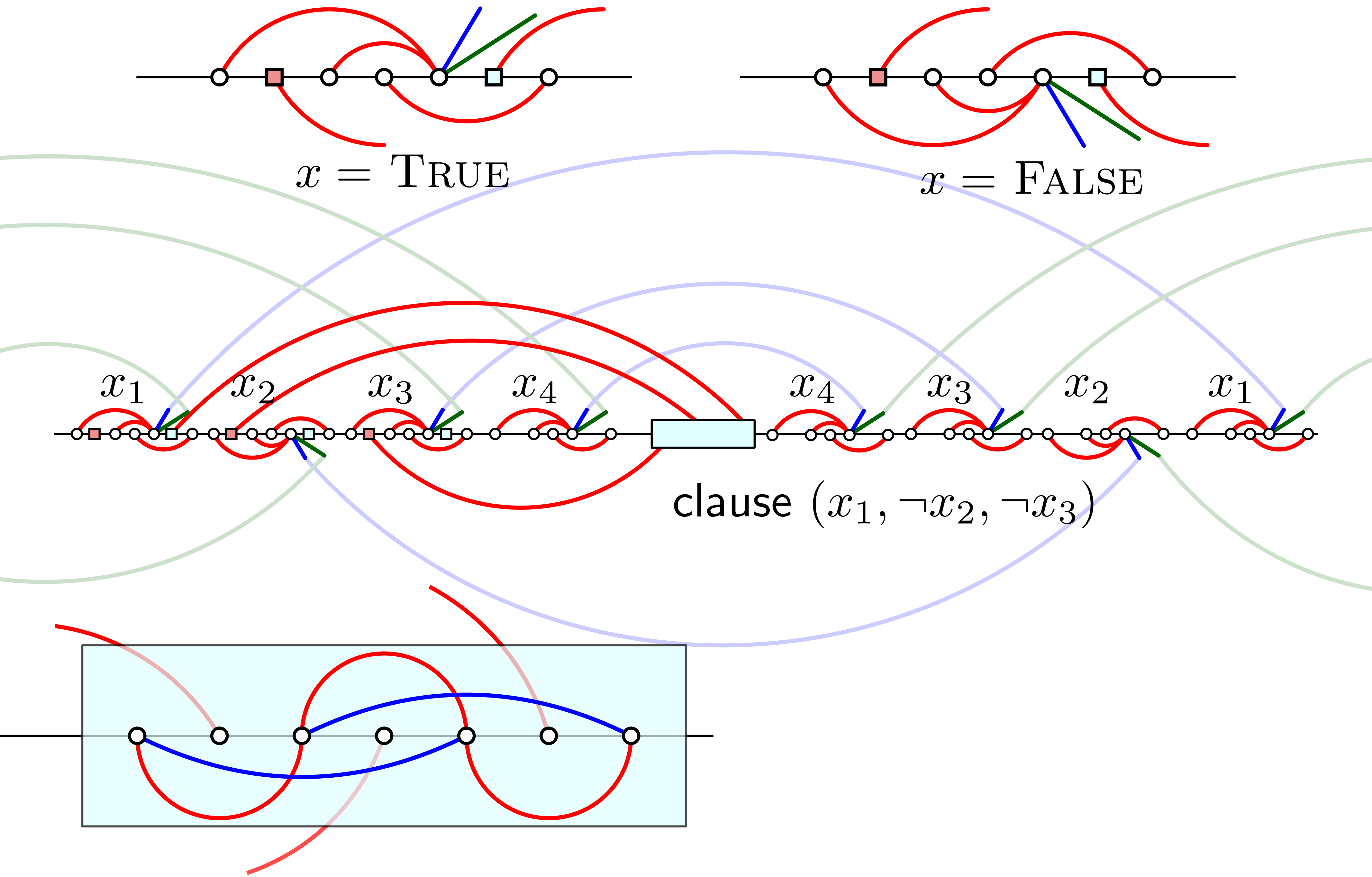

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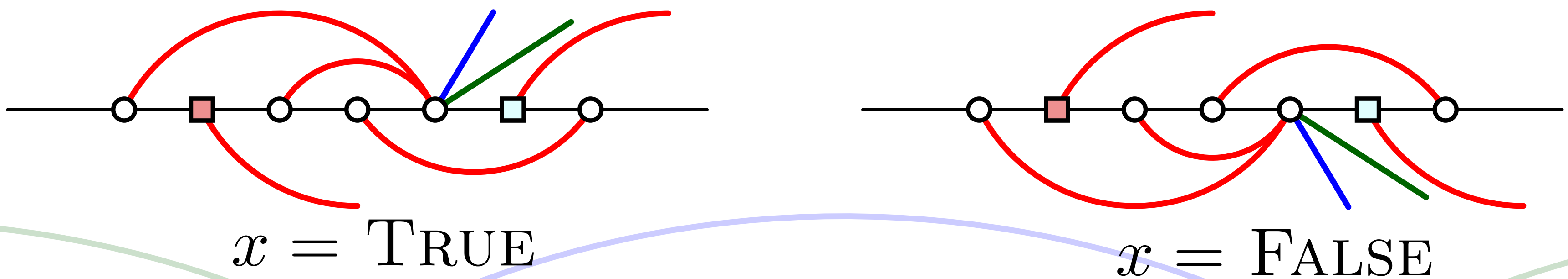

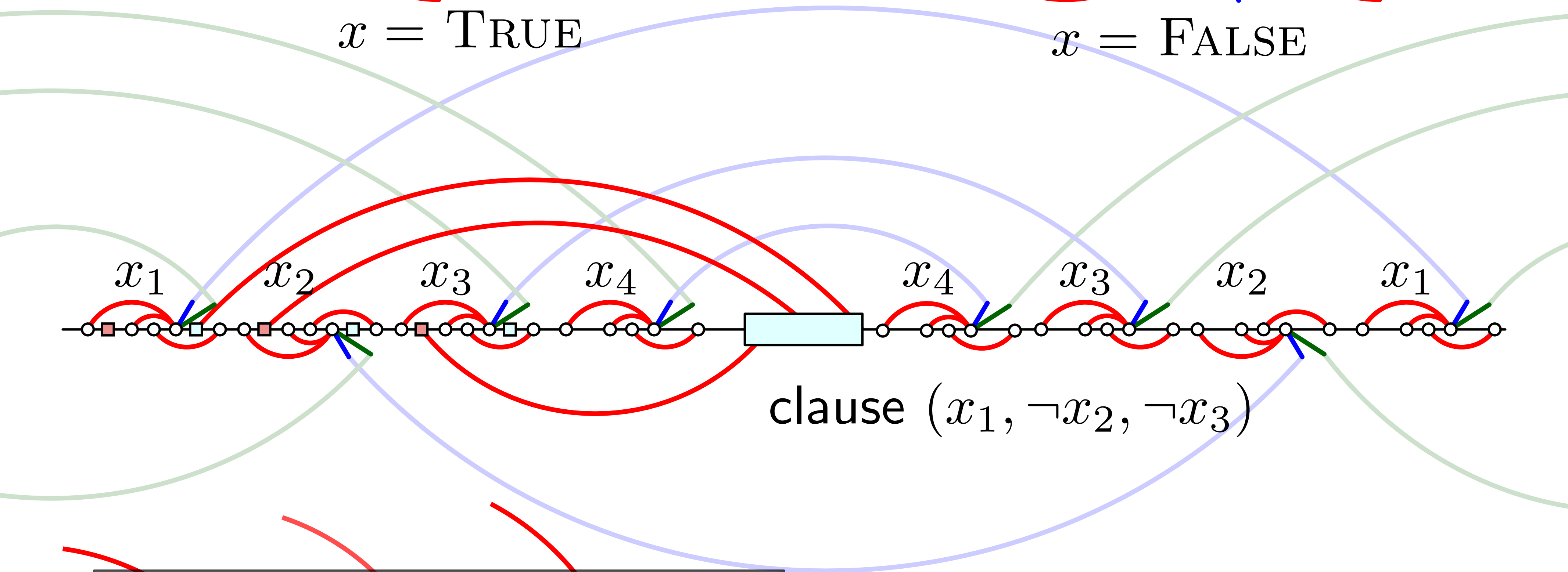

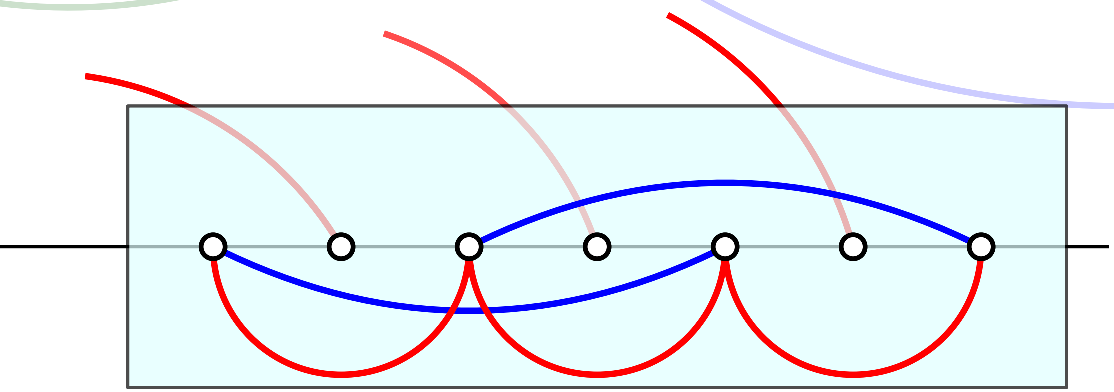

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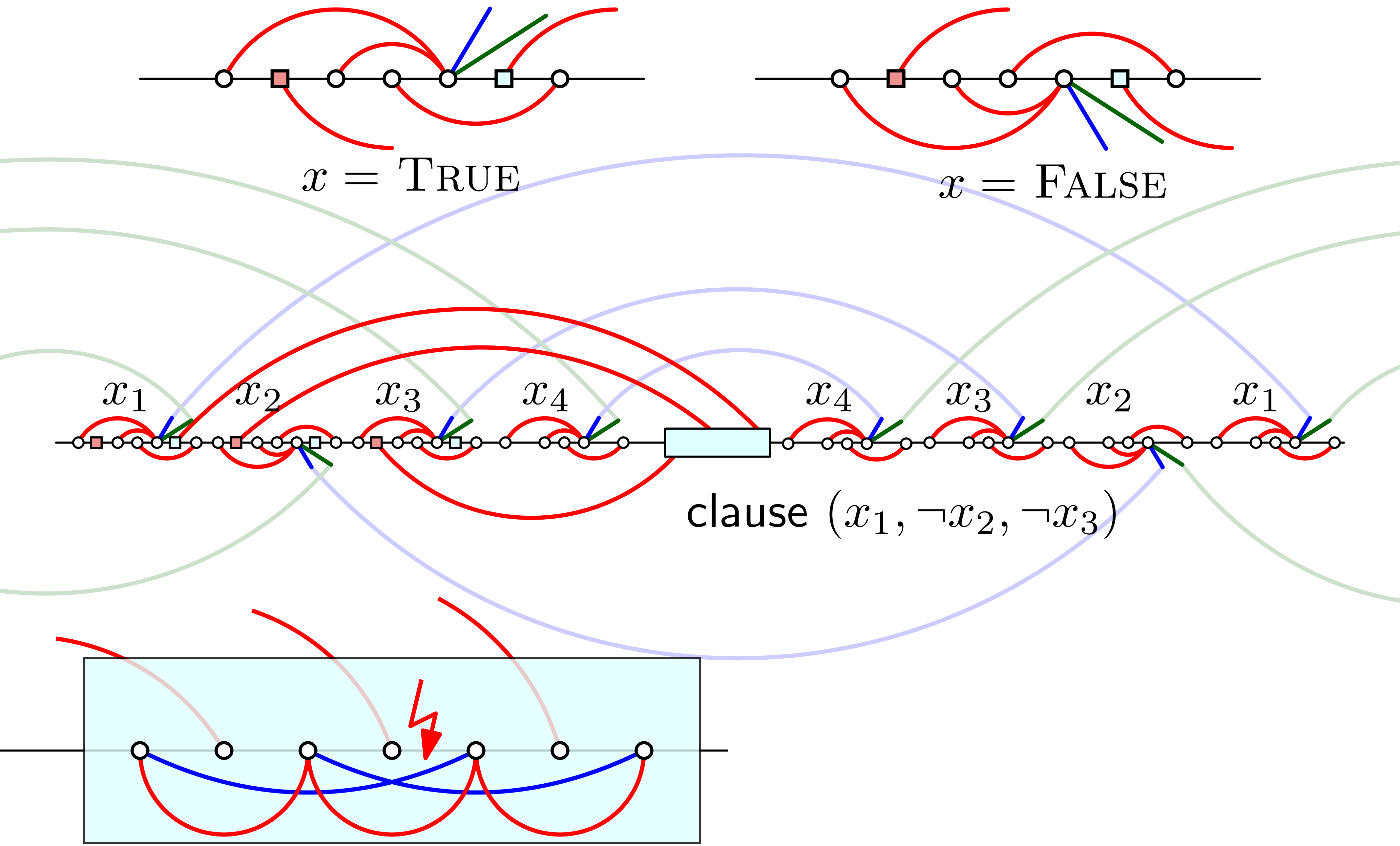

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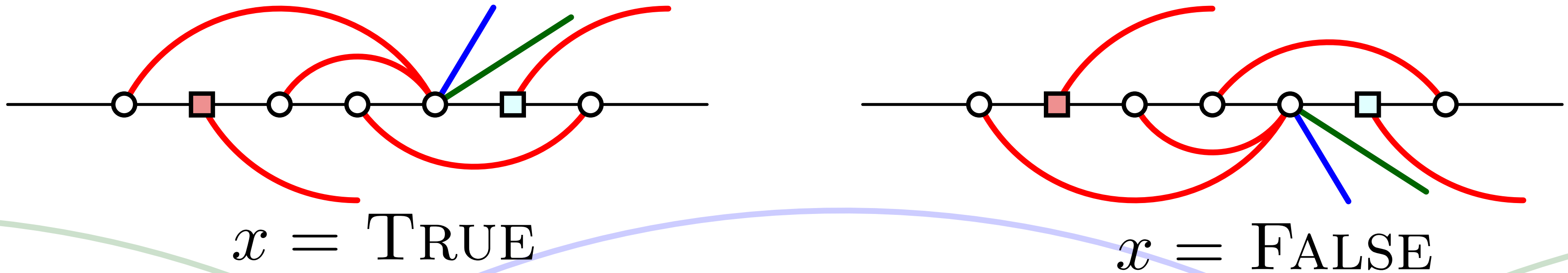

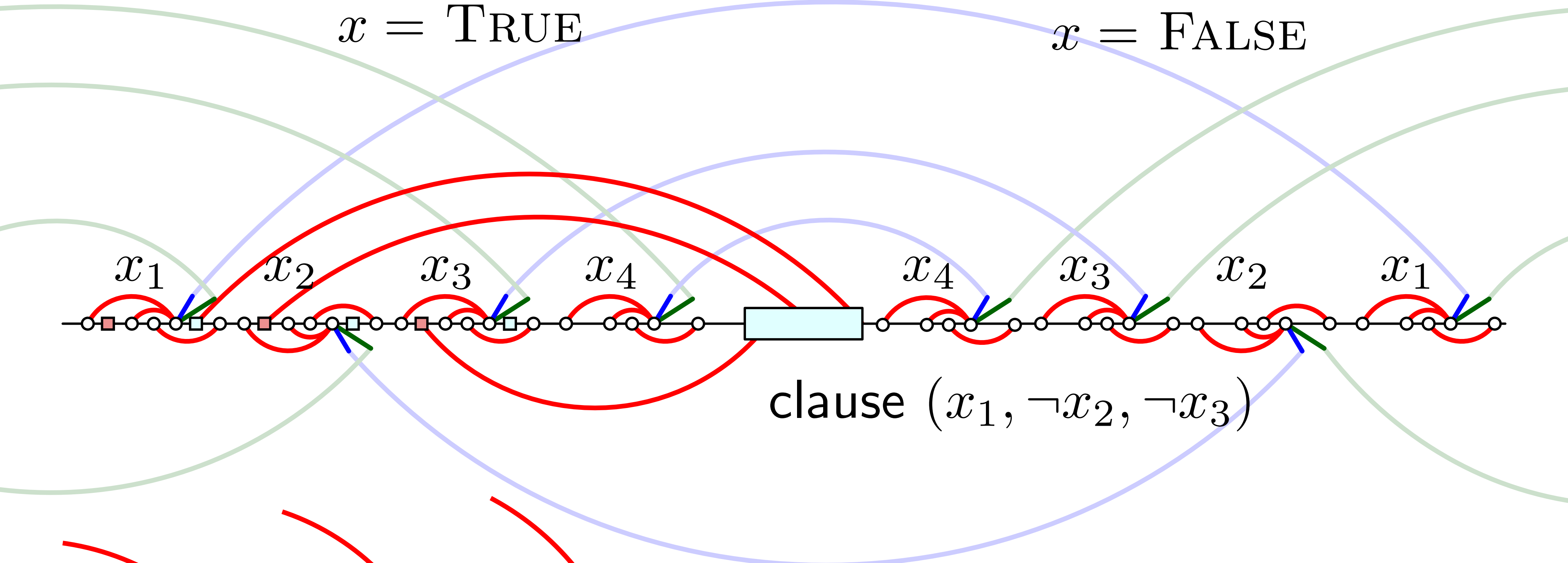

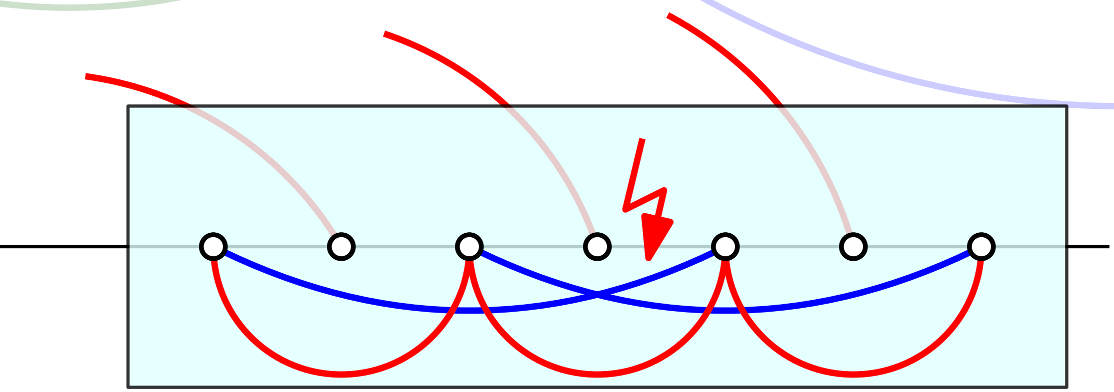

Red edges on different sides

$\Leftrightarrow$

Blue edges on different sides.

Reduction from NAE-3SAT

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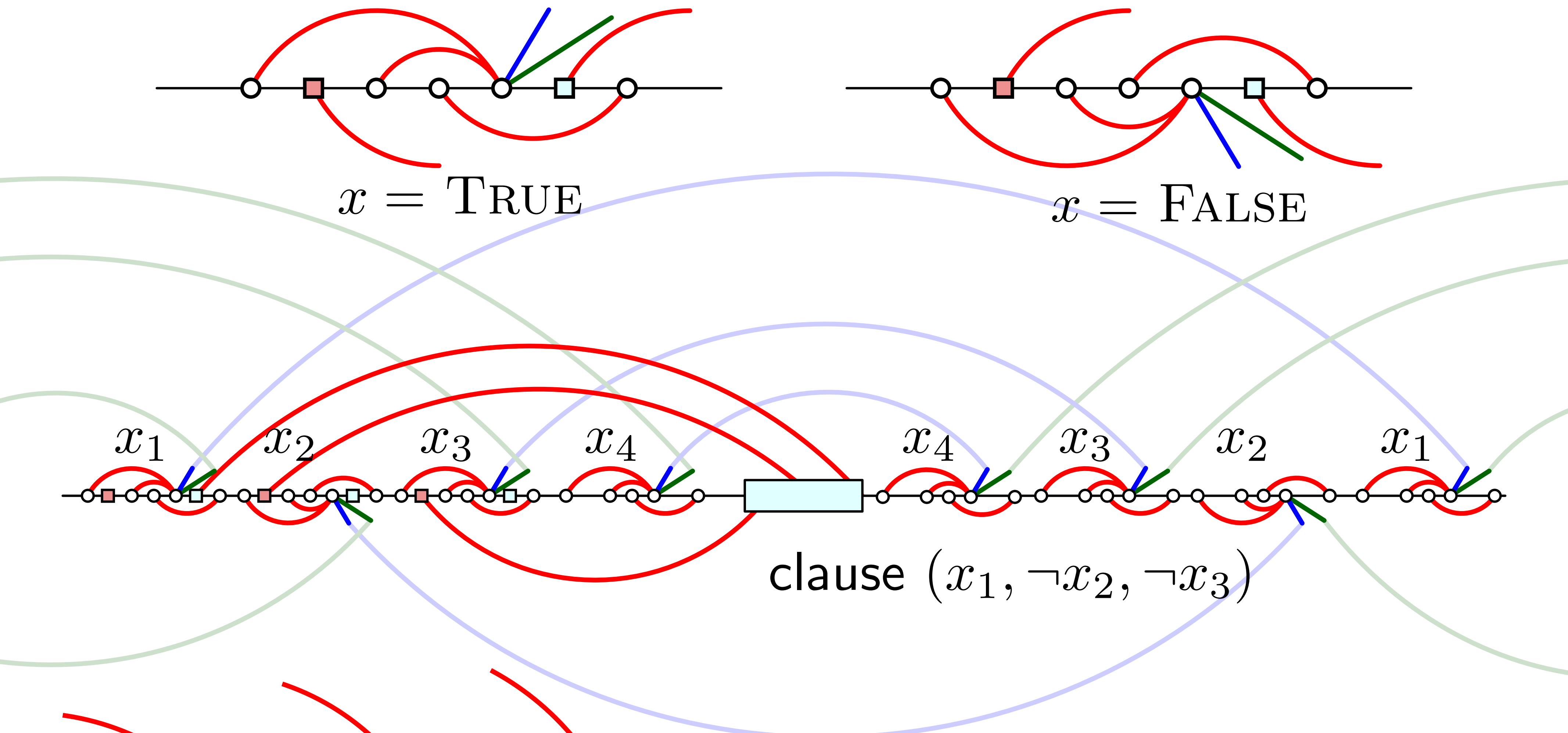

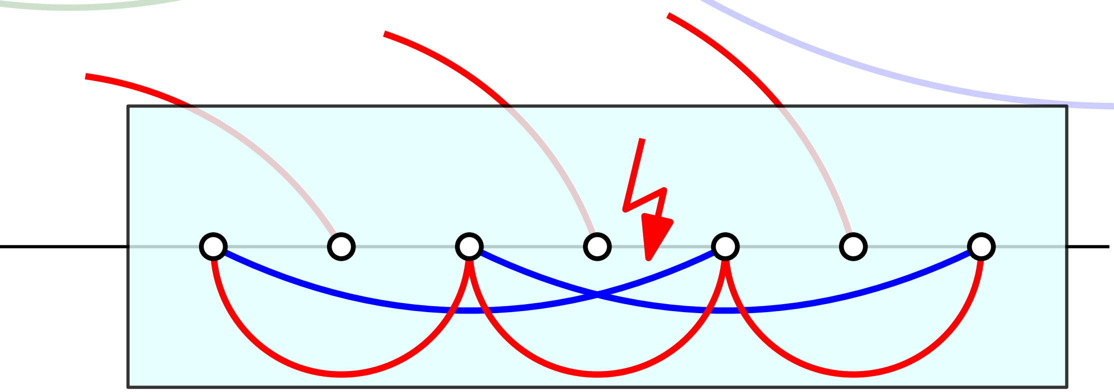

Red edges on different sides
⇔
Blue edges on different sides.

- Blue edges cross if and only if all literals equal. □

Reduction from NAE-3SAT

Reduction also works for two colors:

Reduction from NAE-3SAT

Reduction also works for two colors:

- subdivide some edges and use different colors

Reduction from Nae-3Sat

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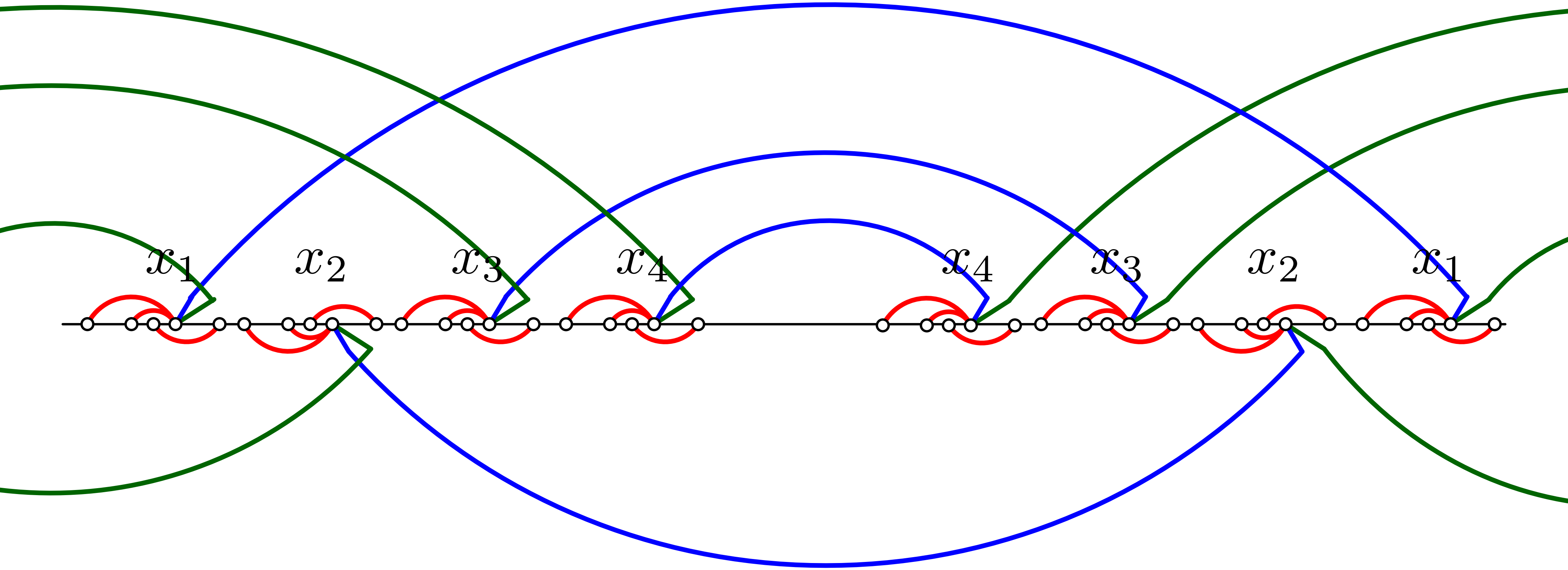

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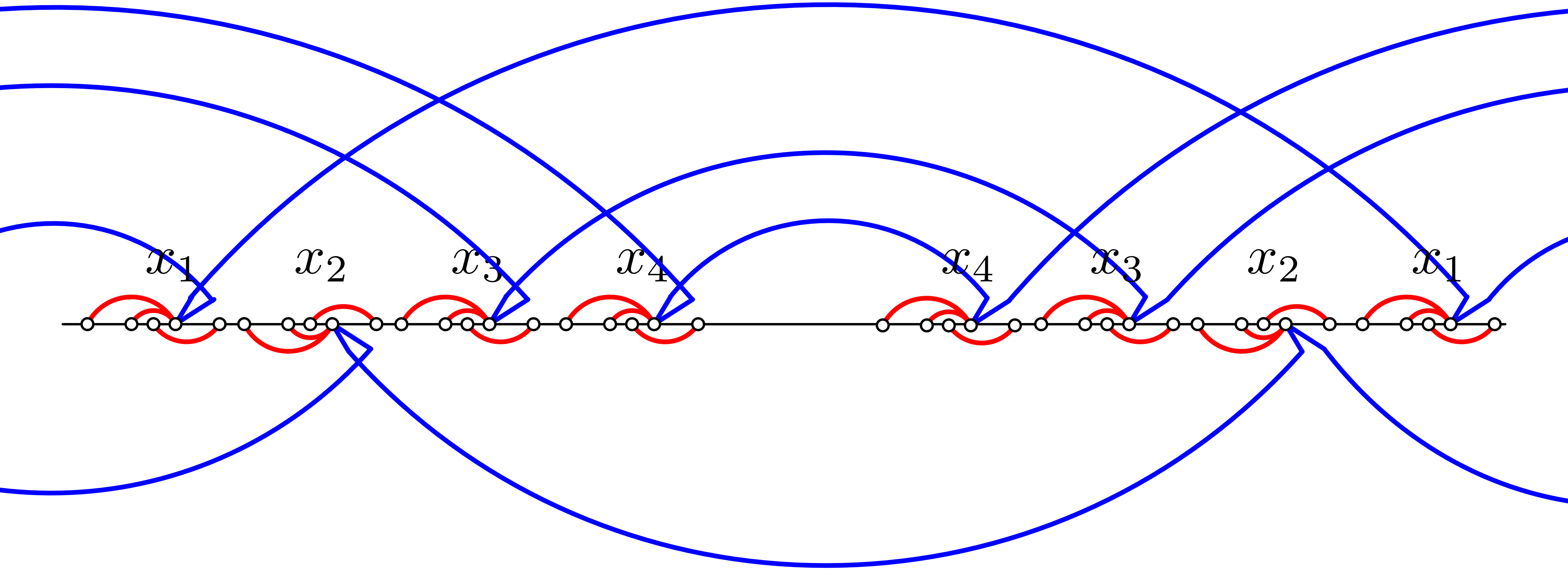

Reduction from Nae-3Sat

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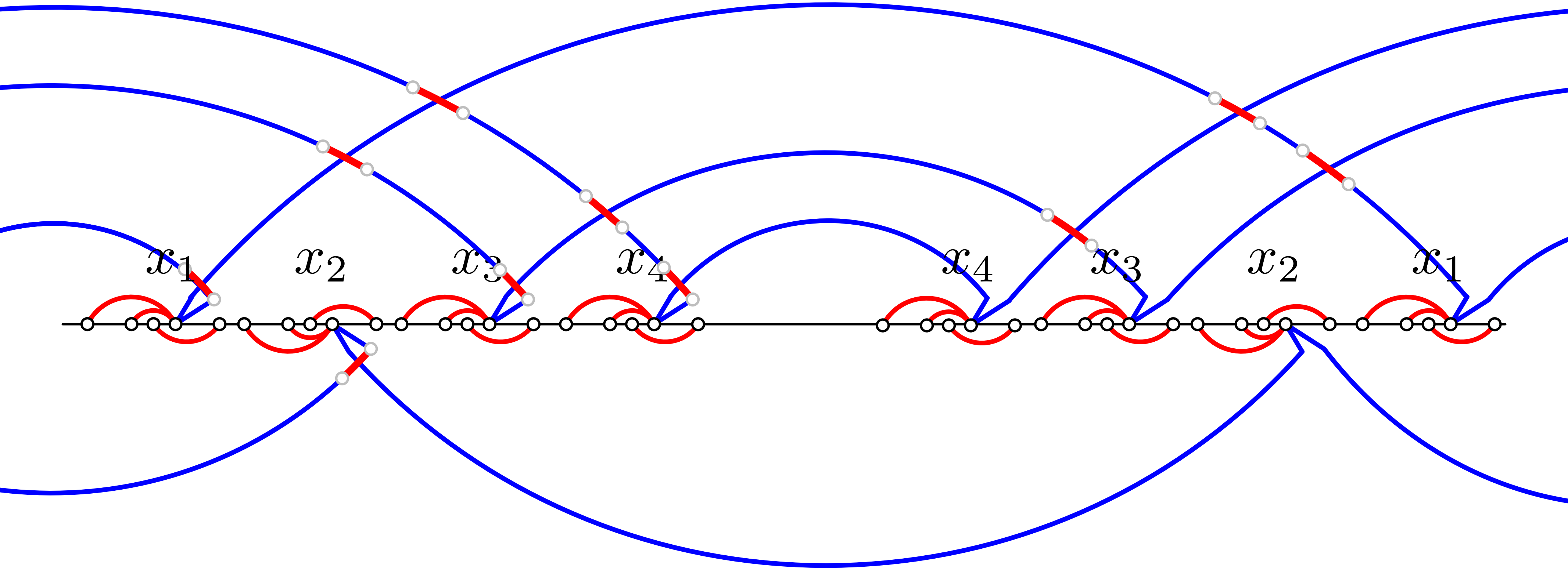
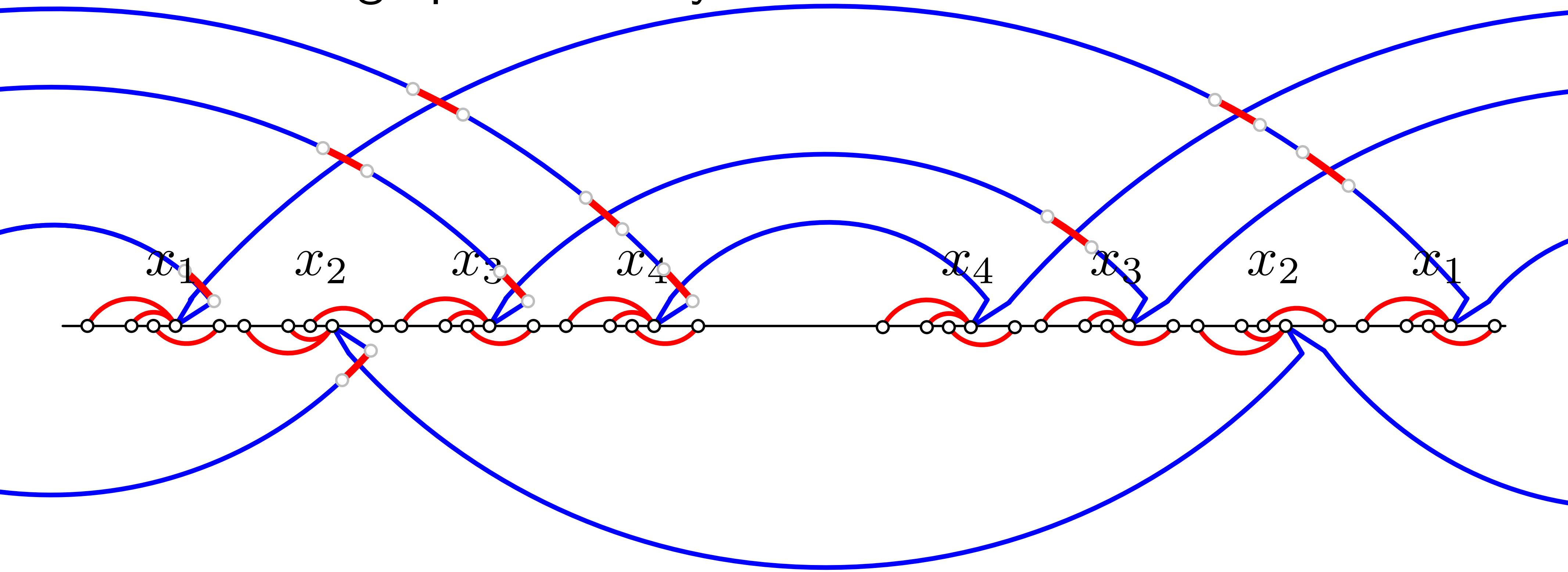


Reduction from Nae-3Sat

Reduction also works for two colors:

- subdivide some edges and use different colors
- common graph now is cycle + isolated vertices



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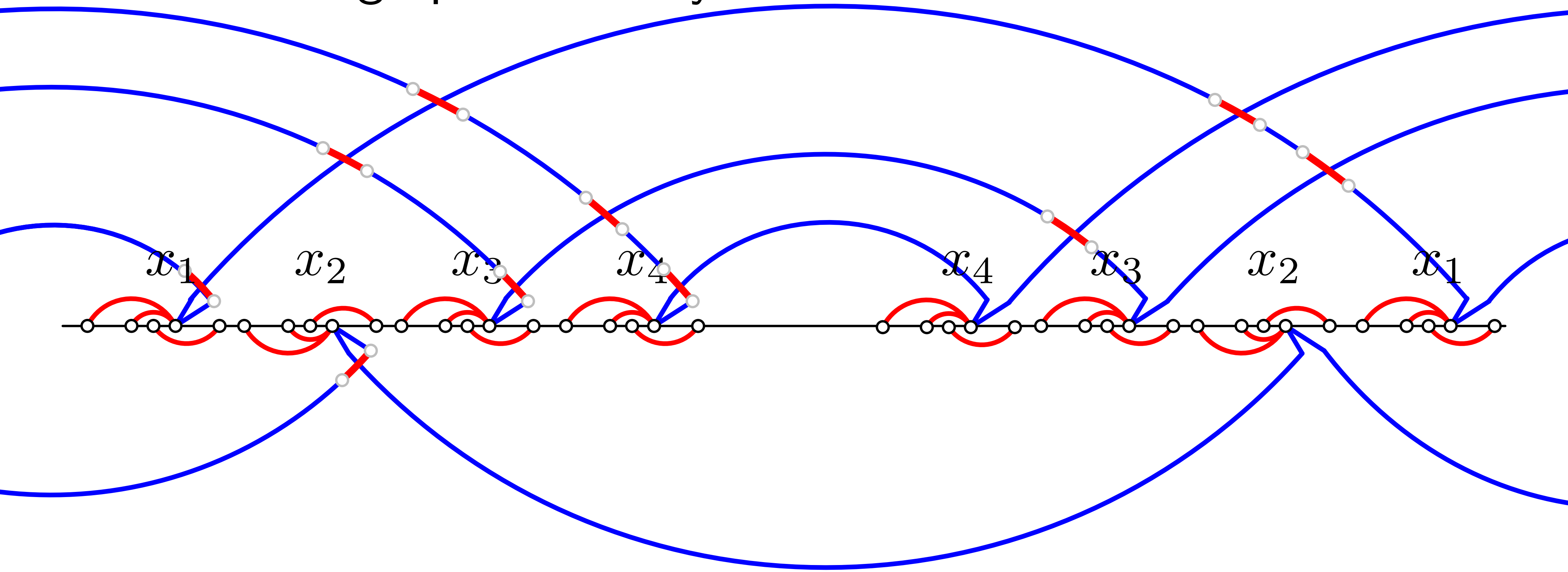
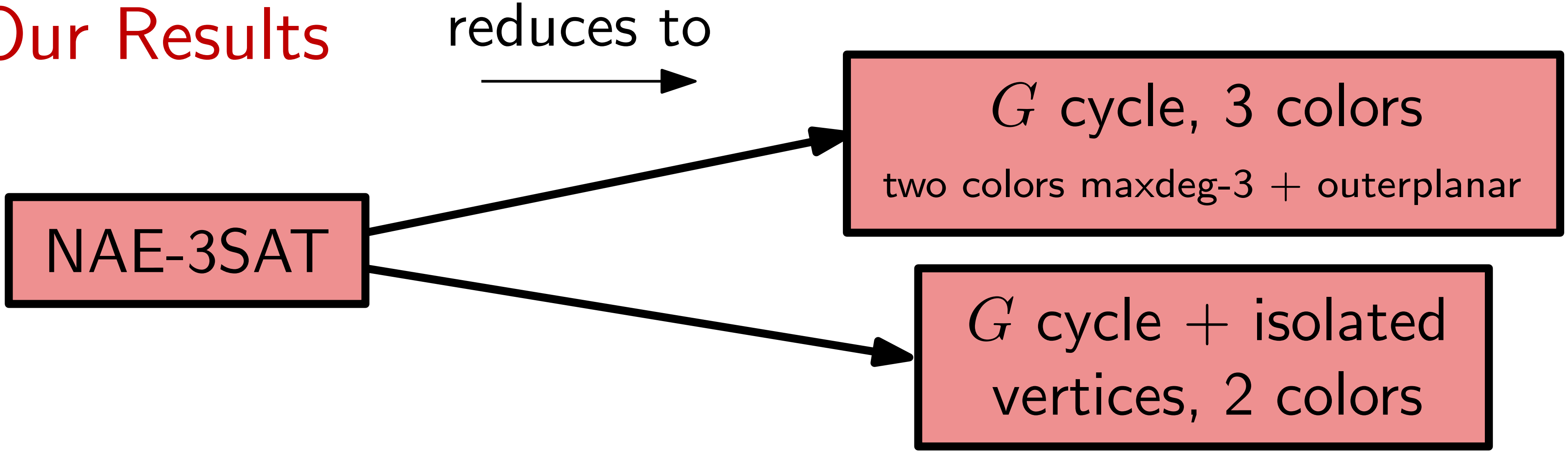


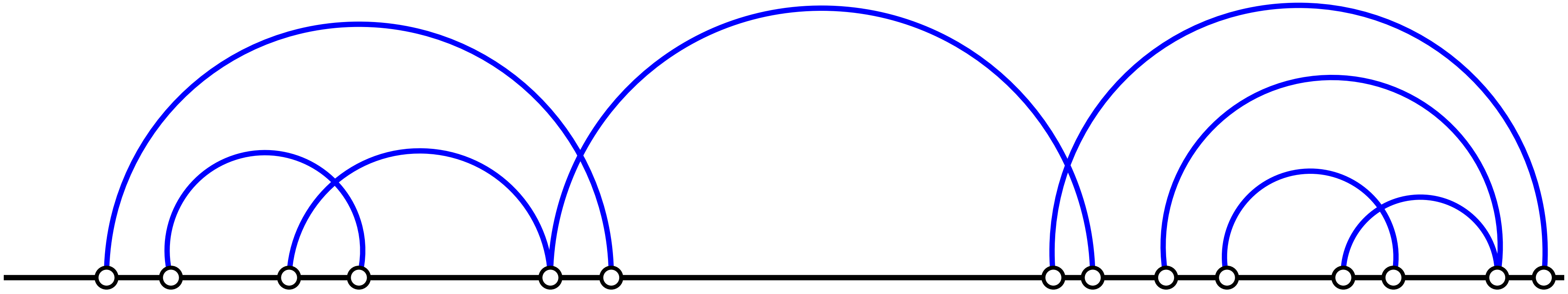
Theorem.
It is NP-complete to decide whether two graphs G_1, G_2 whose common graph consists of a cycle plus isolated vertices admit an ORTHOSEFE.

Our Results


reduces to
NAE-3SAT
G cycle, 3 colors
two colors maxdeg-3 + outerplanar
G cycle + isolated vertices, 2 colors

G cycle; G_2 outerplanar, deg ≤ 3

- Consider G on a line and G_1 above.
  - Nested intersection components
  - Bipartition of intersecting edges



G cycle; G_2 outerplanar, deg ≤ 3

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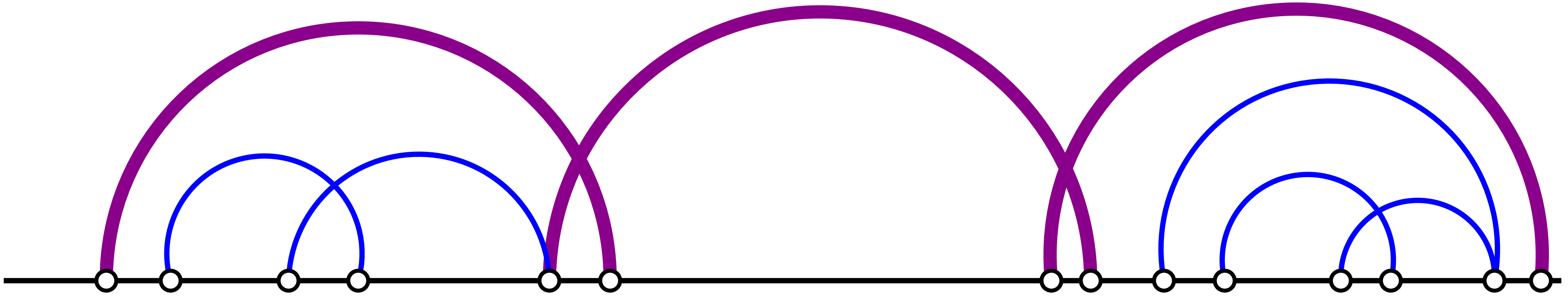

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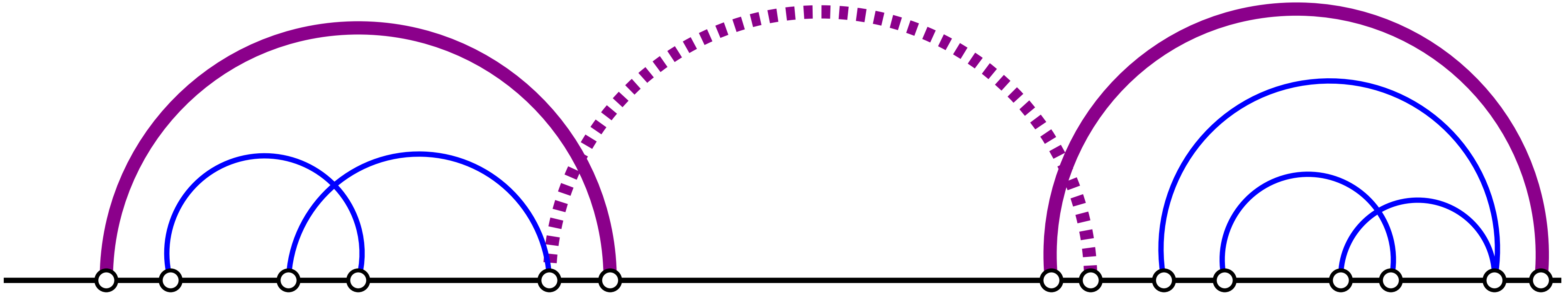

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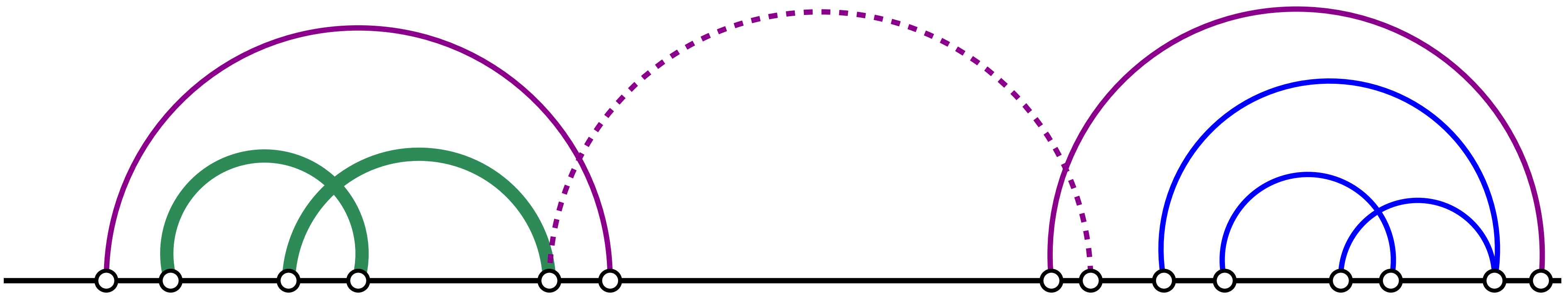

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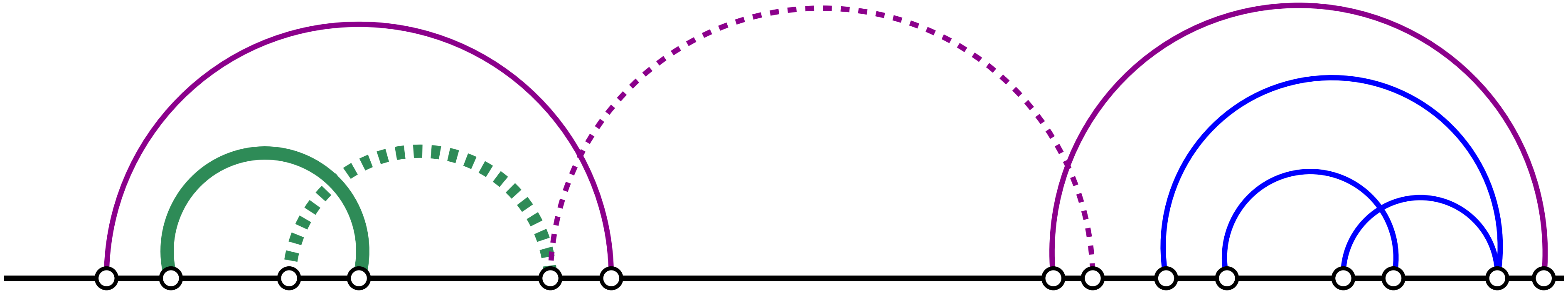

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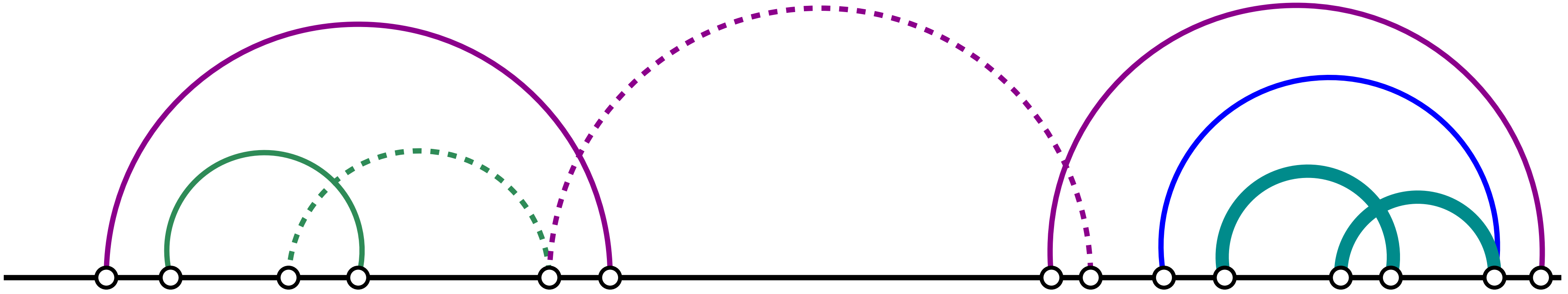

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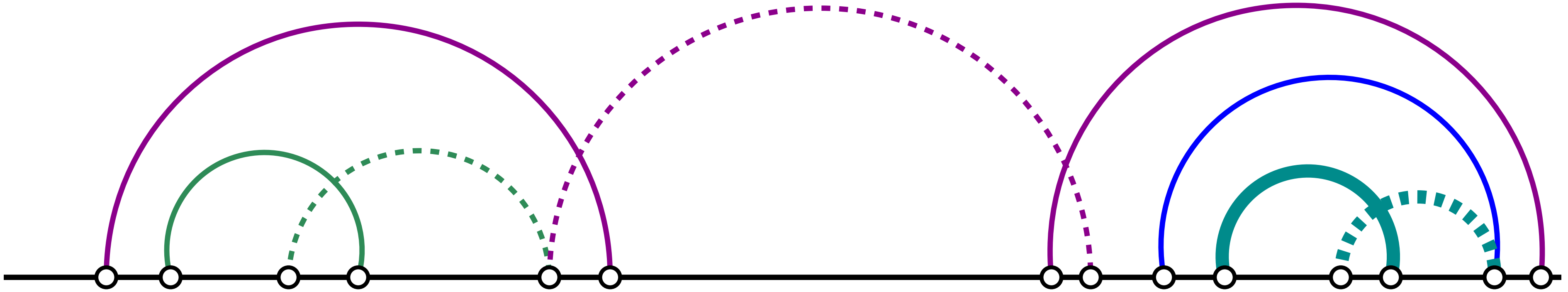

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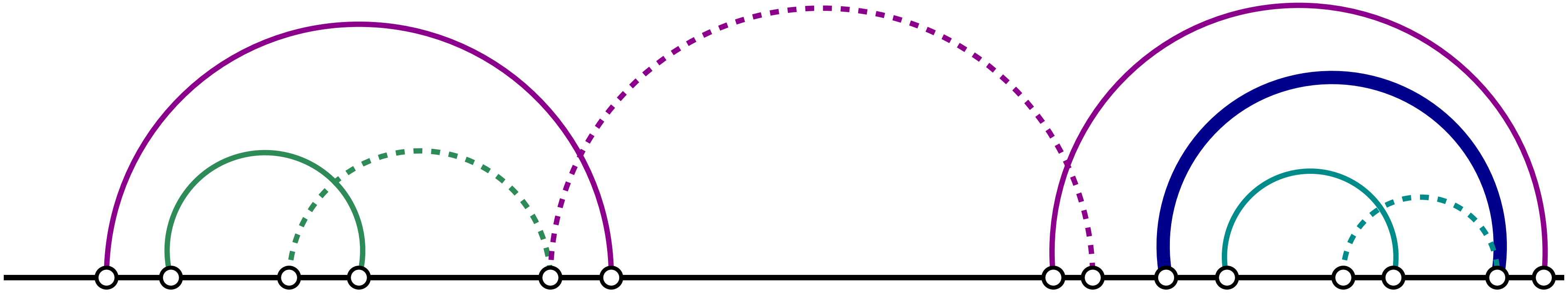

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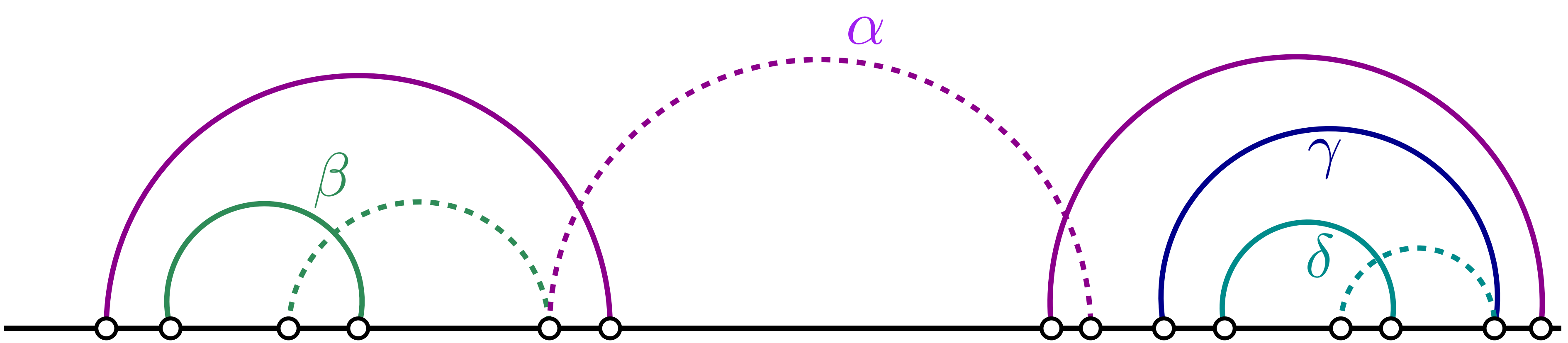

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G cycle; G_2 outerplanar, deg ≤ 3

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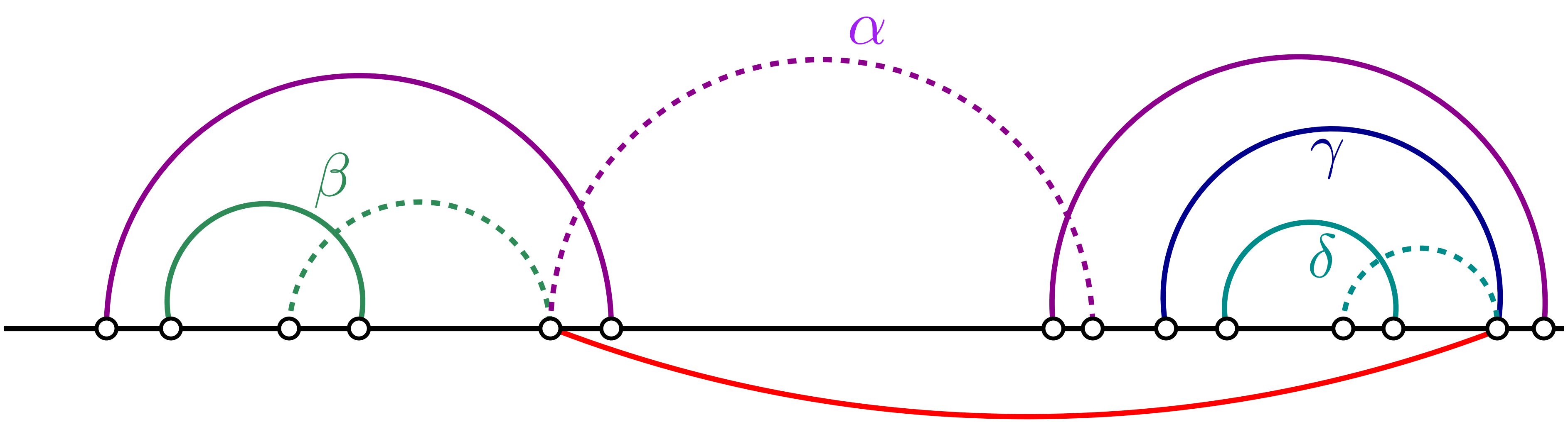

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- Red can be inserted iff not one end vertex up, one down
  $\neg((\overline{\beta} \wedge \overline{\alpha} \wedge \overline{\gamma} \wedge \delta) \vee (\beta \wedge \alpha \wedge \gamma \wedge \overline{\delta}))$

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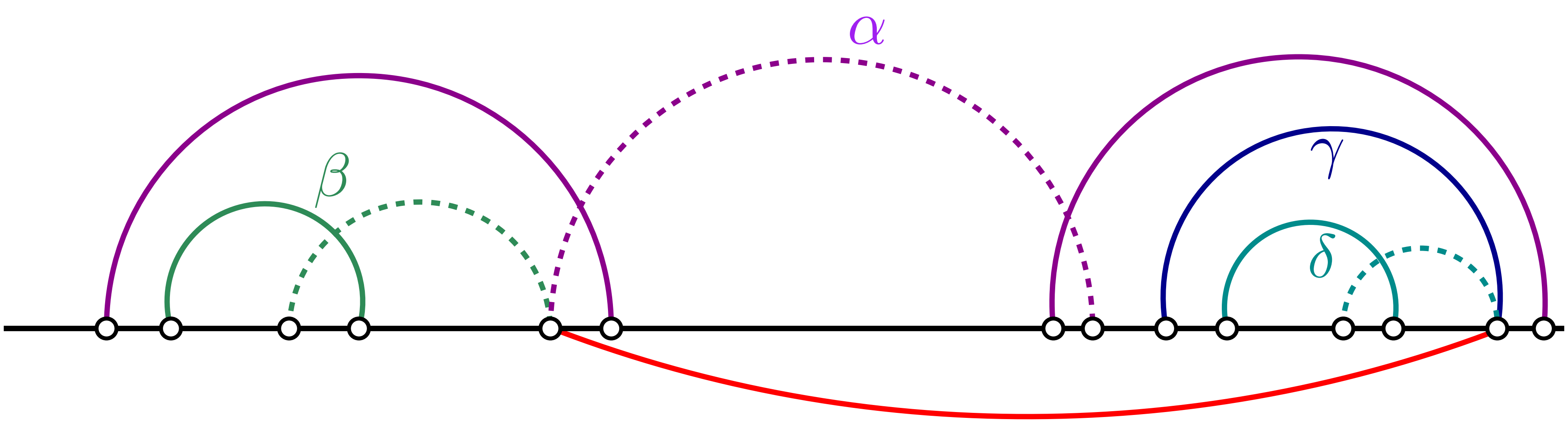

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- not-all-equal SAT

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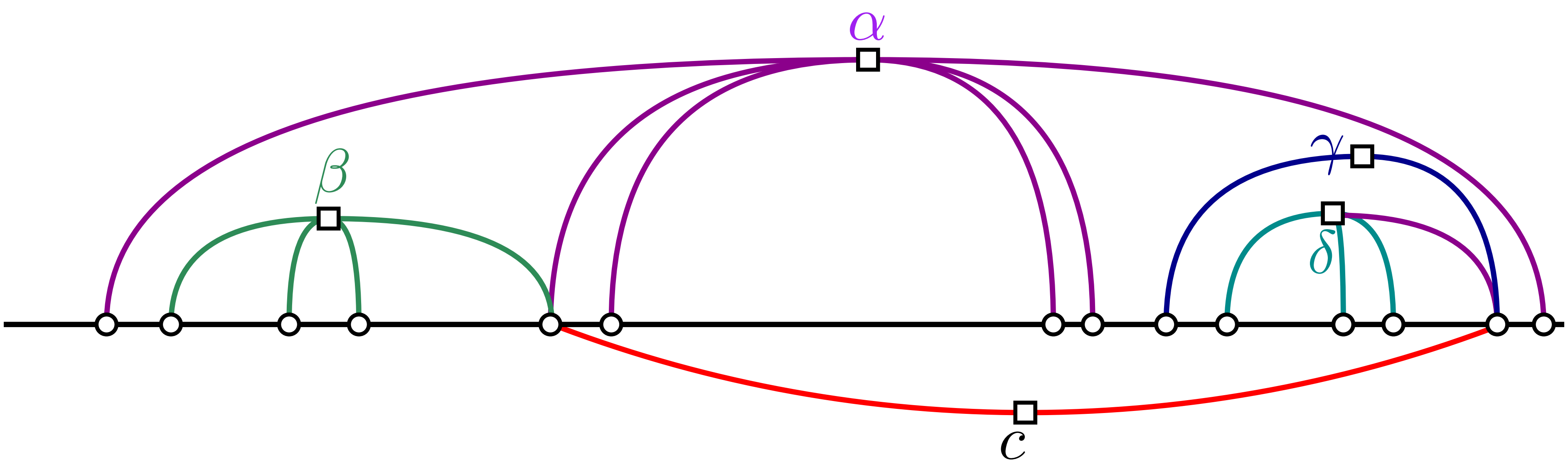

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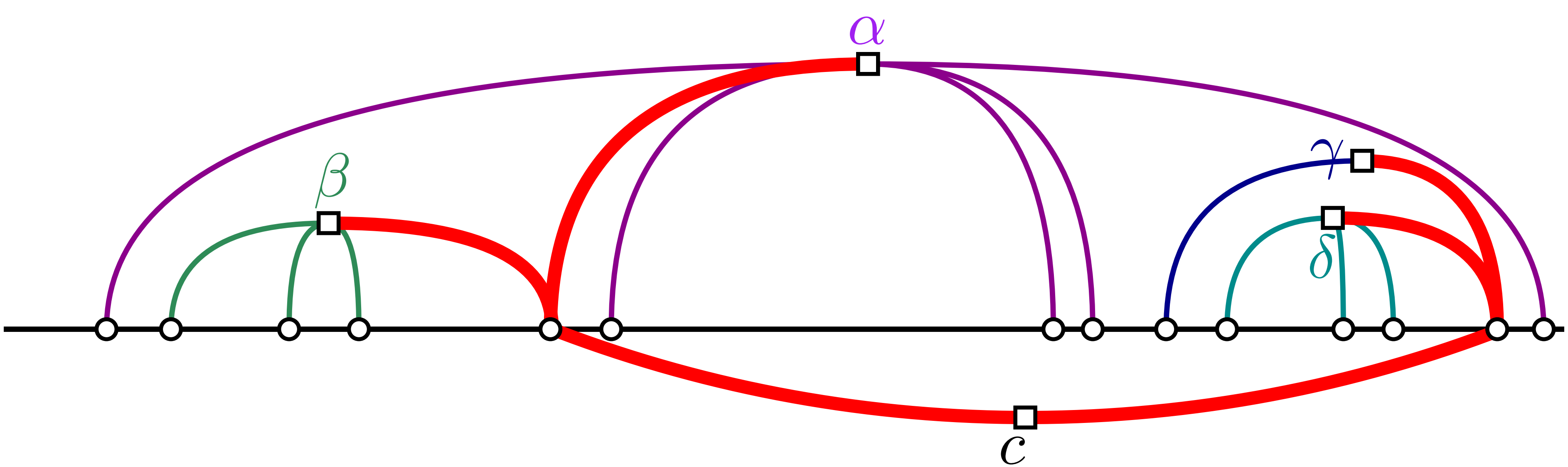

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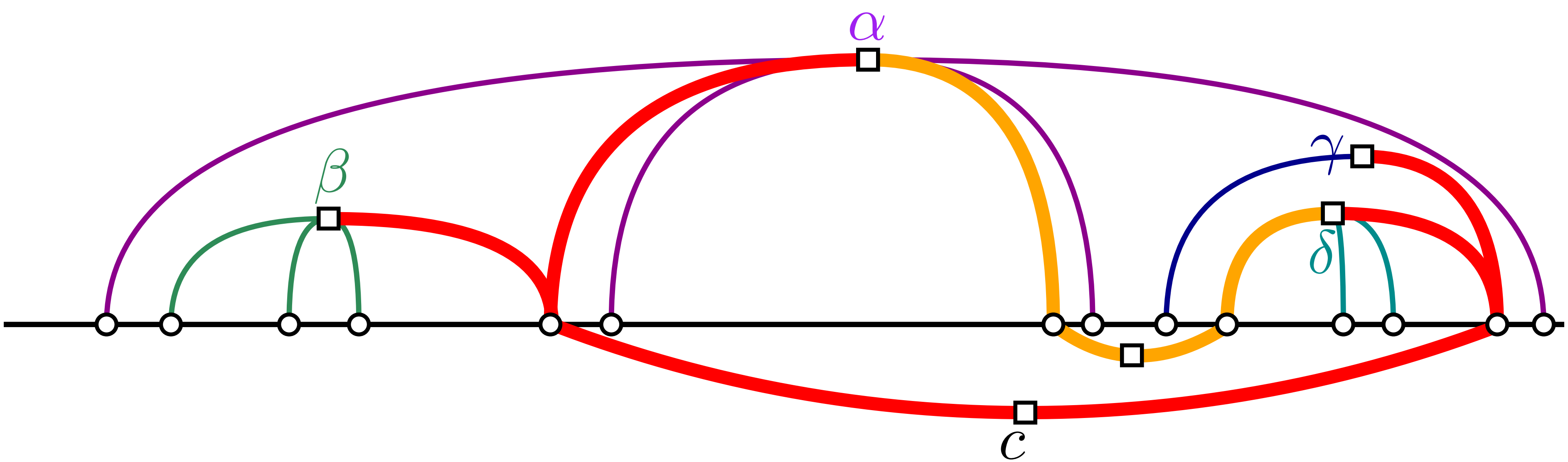

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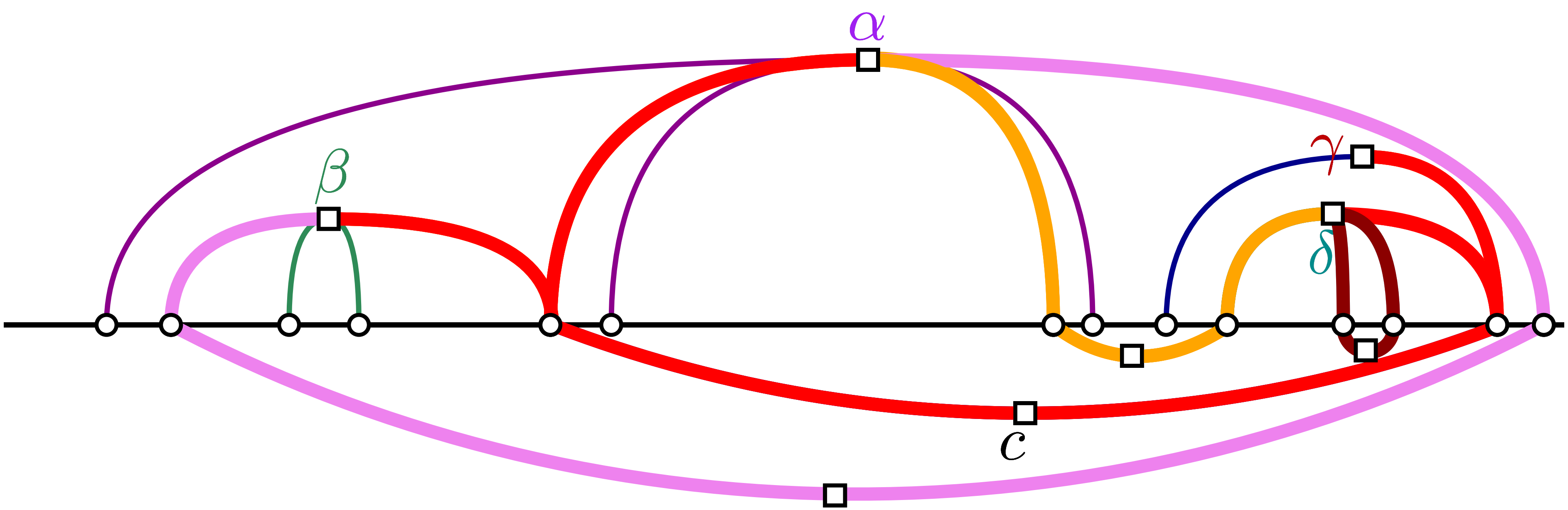

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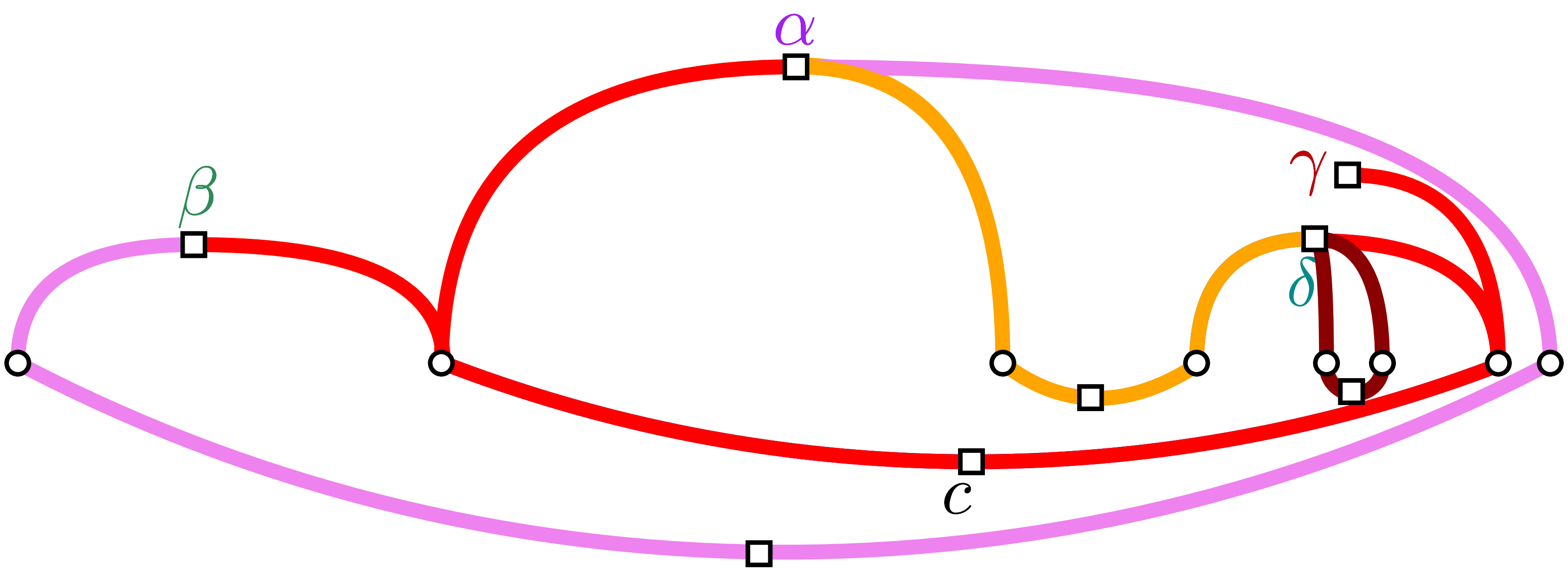
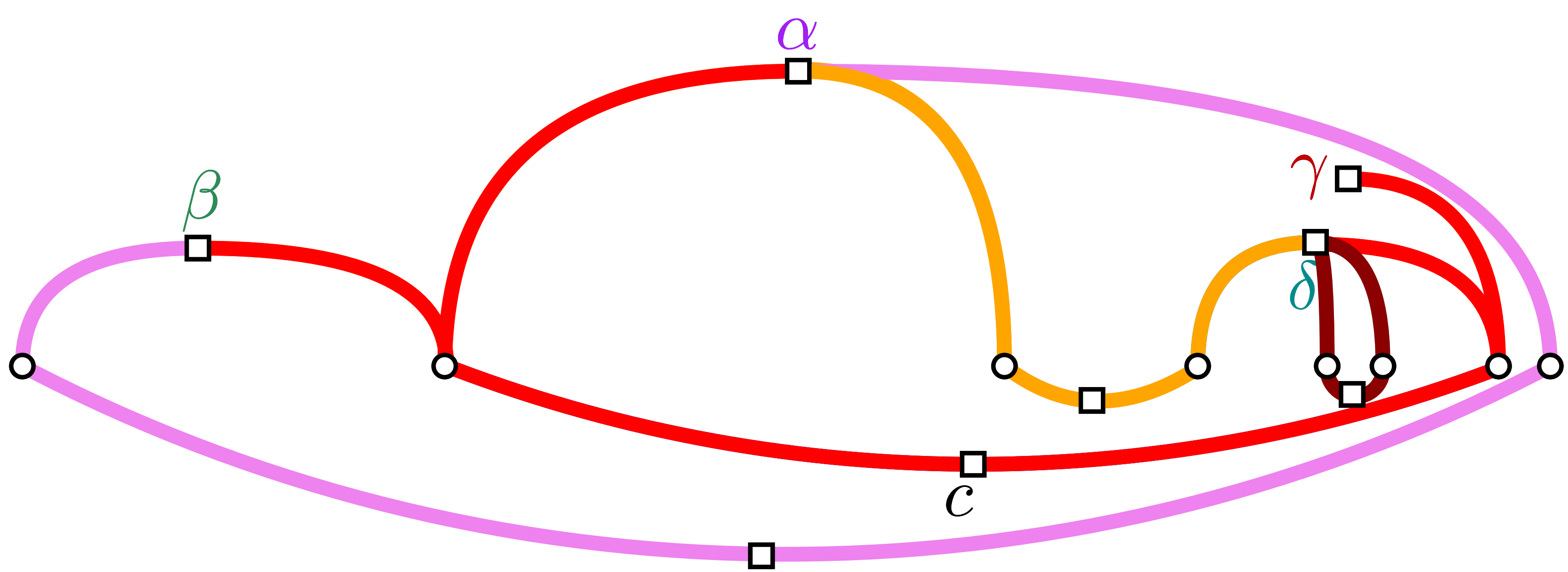


- Boolean variable per class: dashed up $=$ false
- Red can be inserted iff not one end vertex up, one down
$$\neg((\overline{\beta} \wedge \overline{\alpha} \wedge \overline{\gamma} \wedge \delta) \vee (\beta \wedge \alpha \wedge \gamma \wedge \overline{\delta}))$$
- planar not-all-equal SAT

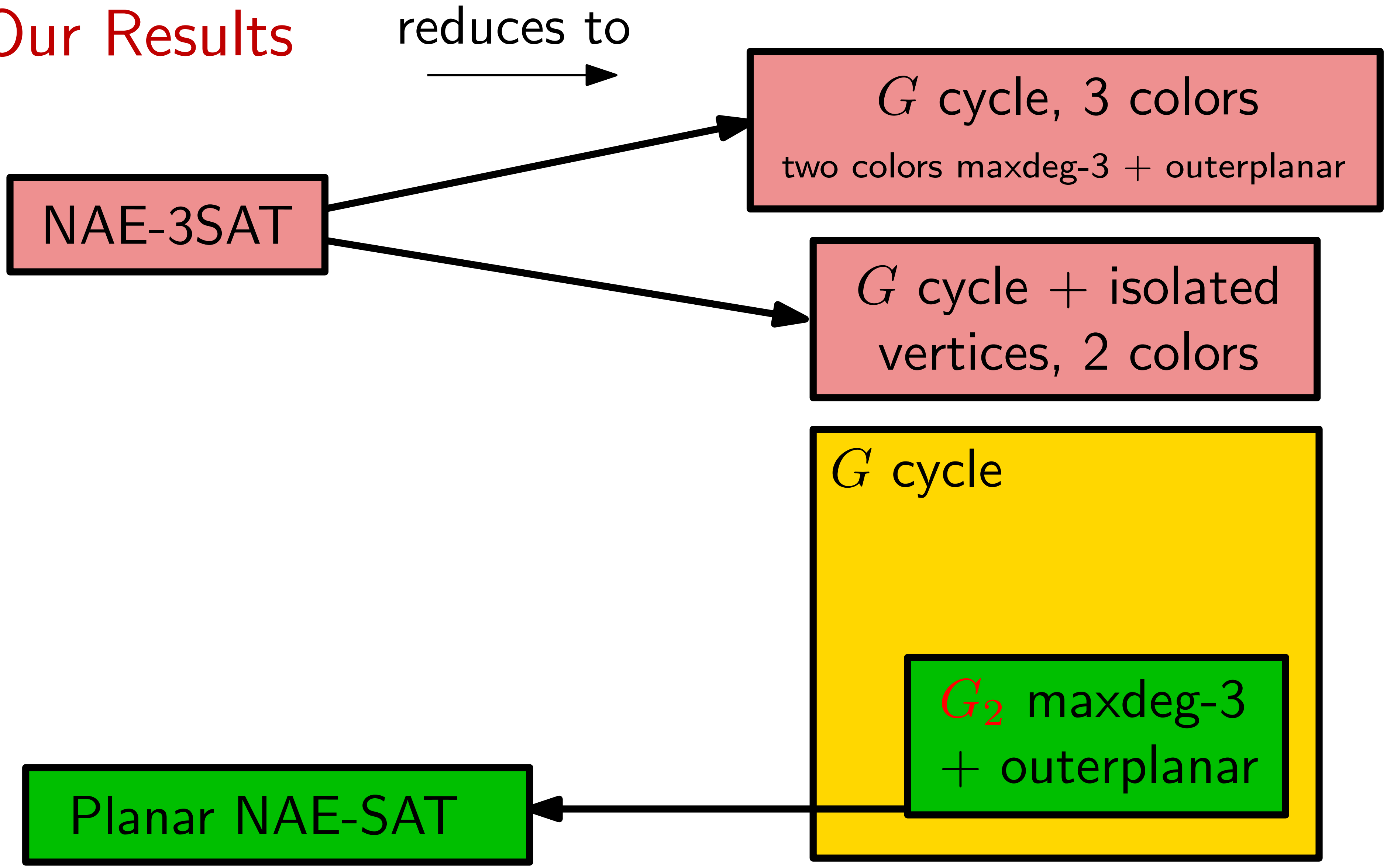
G cycle; G_2 outerplanar, deg ≤ 3

- Consider G on a line and G_1 above.
  - Nested intersection components
  - Bipartition of intersecting edges



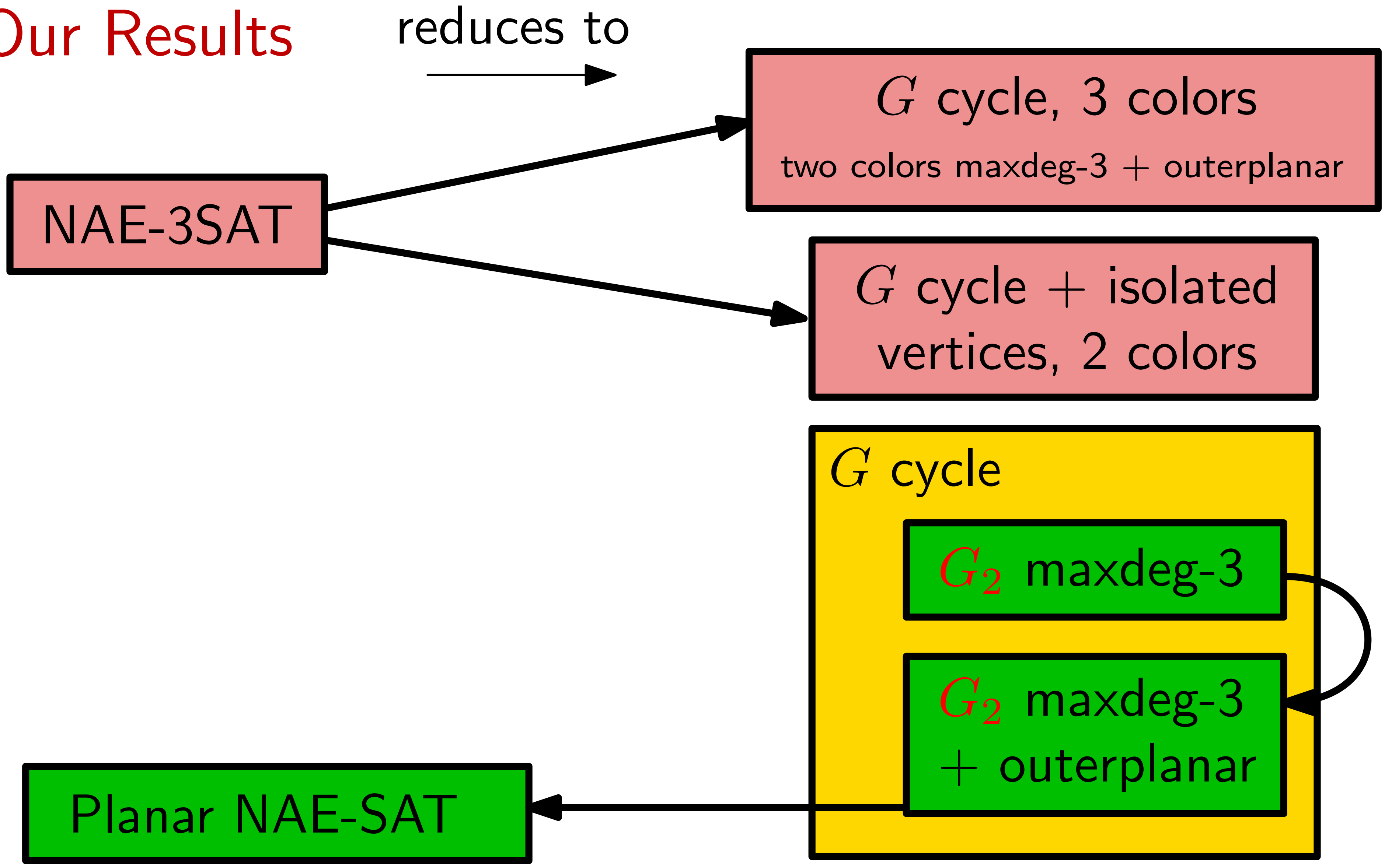
- Boolean variable per class: dashed up = false
- Red can be inserted iff not one end vertex up, one down
  $$\neg((\overline{\beta} \wedge \overline{\alpha} \wedge \overline{\gamma} \wedge \delta) \vee (\beta \wedge \alpha \wedge \gamma \wedge \overline{\delta}))$$
- planar not-all-equal SAT, which is in $\mathcal{P}$! [Moret '88]

Our Results



reduces to
NAE-3SAT
G cycle, 3 colors
two colors maxdeg-3 + outerplanar
G cycle + isolated vertices, 2 colors
G cycle
G_2 maxdeg-3 + outerplanar
Planar NAE-SAT

Our Results



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G cycle, 3 colors
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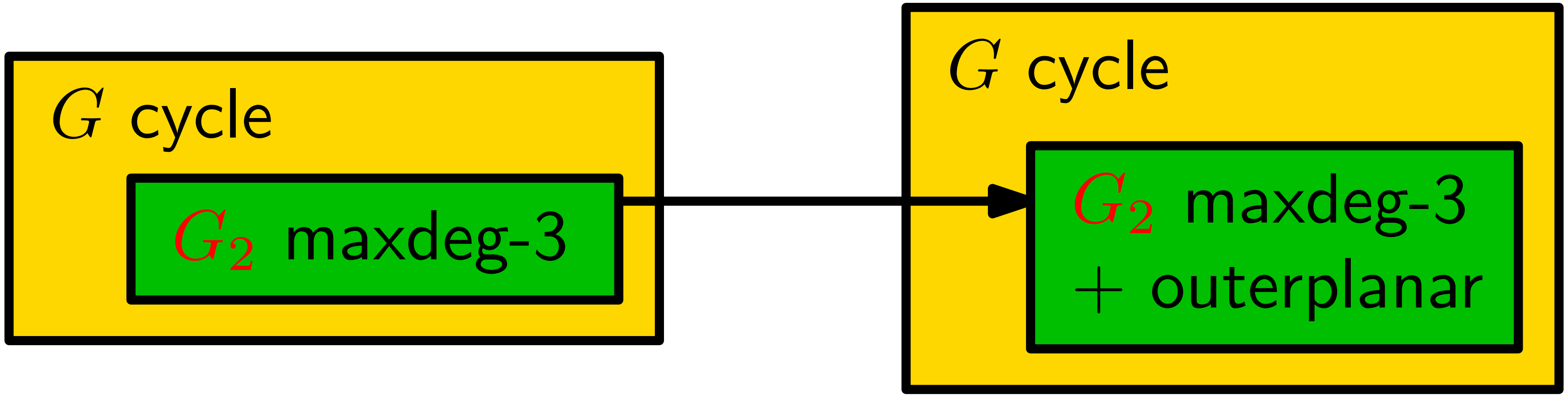
G cycle + isolated vertices, 2 colors
G cycle
G₂ maxdeg-3
G₂ maxdeg-3 + outerplanar
Planar NAE-SAT

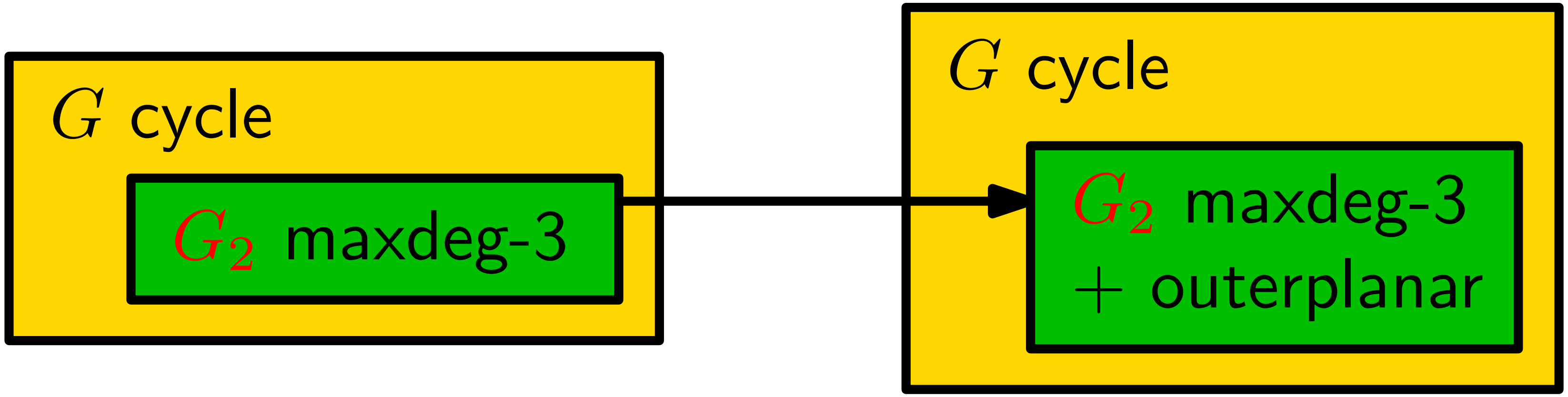
Making a Maxdeg-3 Graph Outerplanar

Theorem.

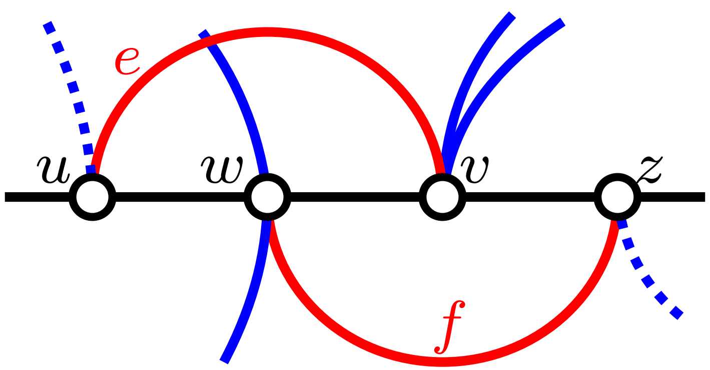


Making a Maxdeg-3 Graph Outerplanar

Theorem.



Proof:



Making a Maxdeg-3 Graph Outerplanar

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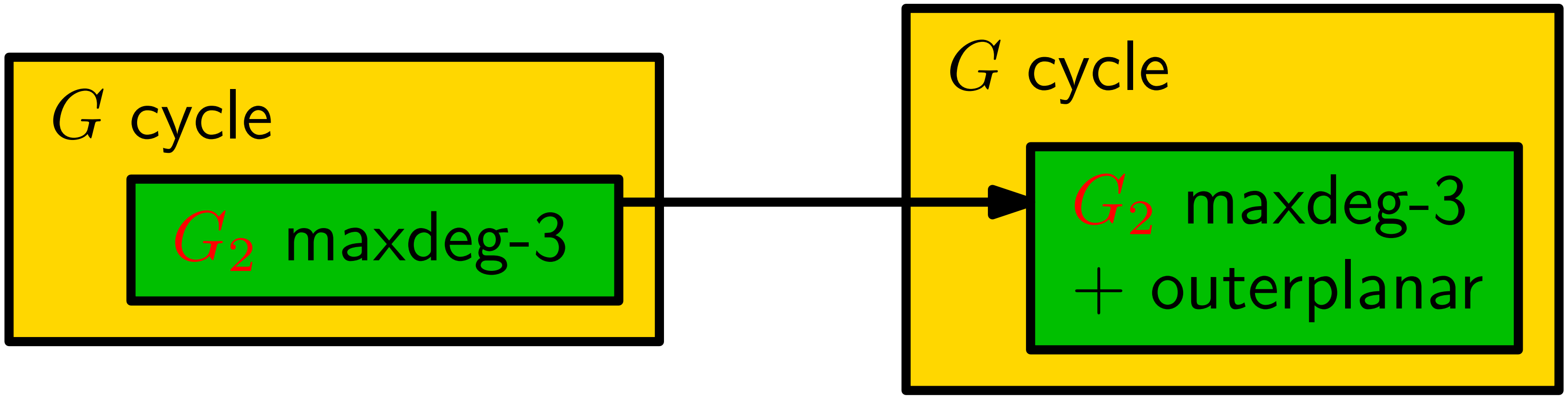
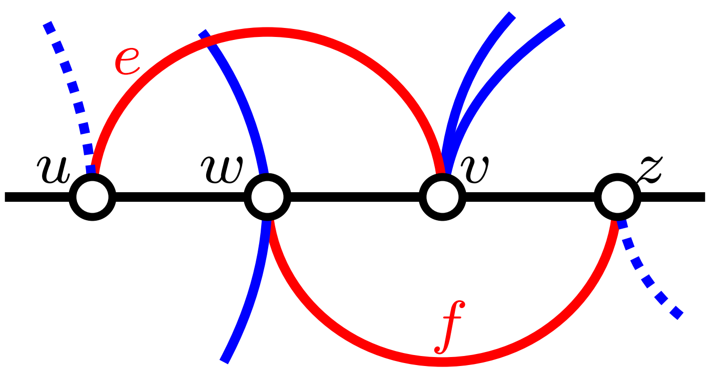


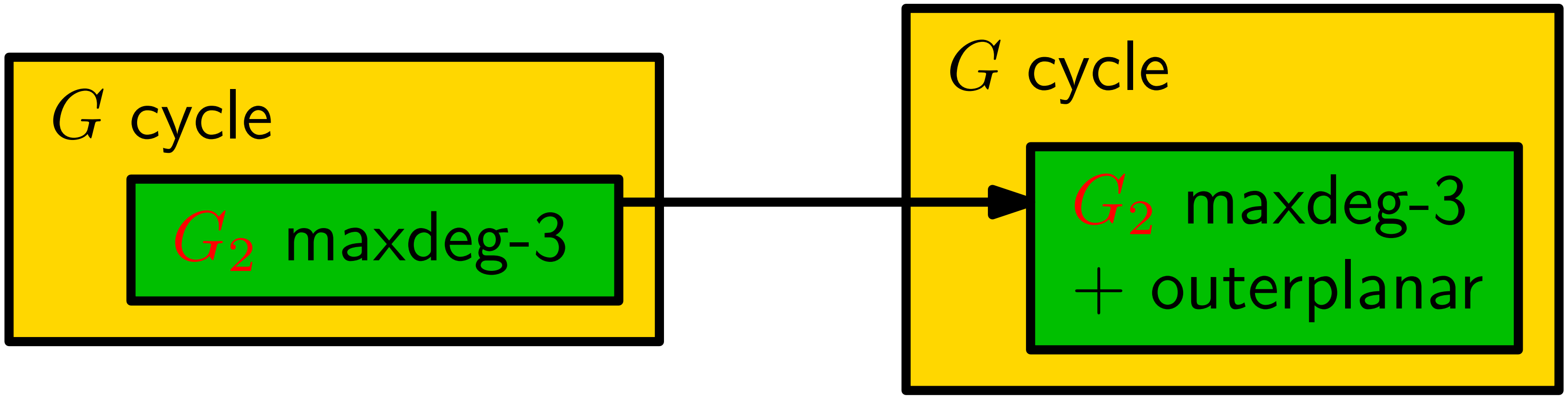
Proof:



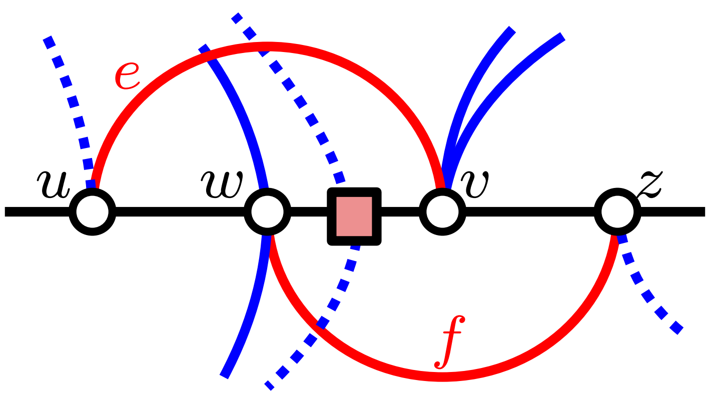
pick u, z as close as possible

Making a Maxdeg-3 Graph Outerplanar

Theorem.



Proof:



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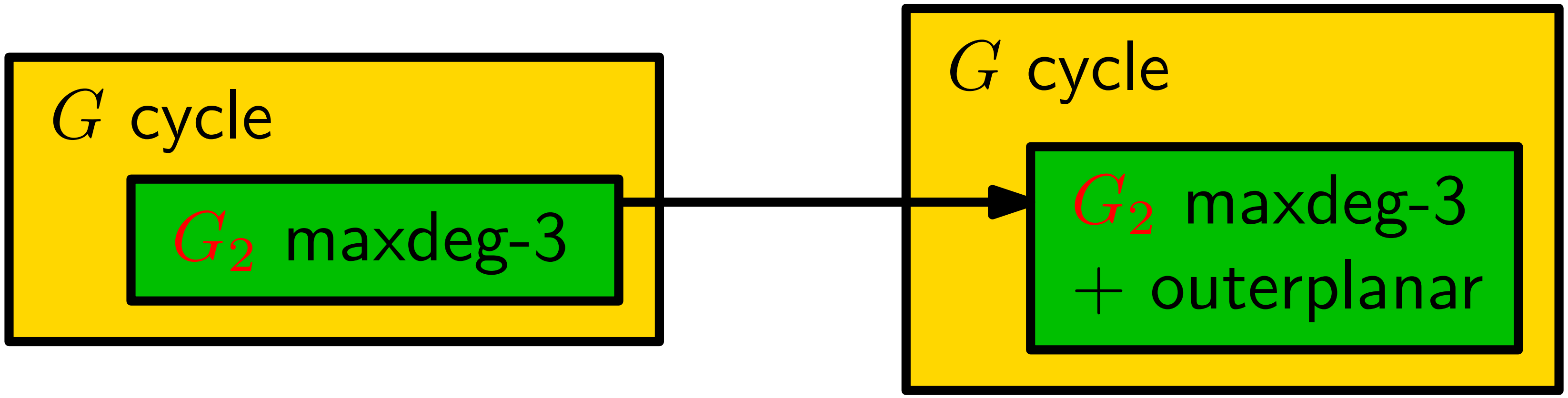
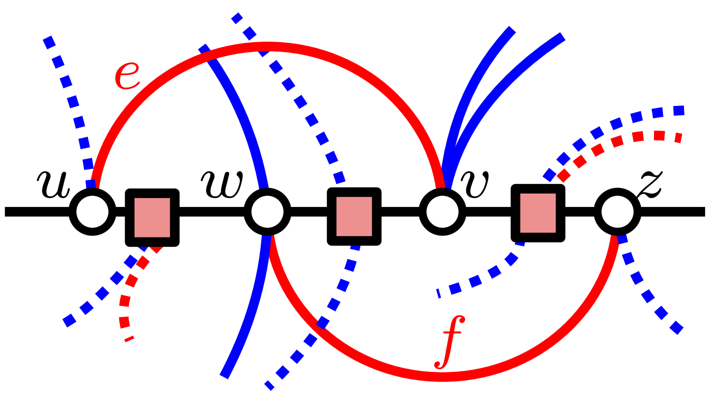


Proof:



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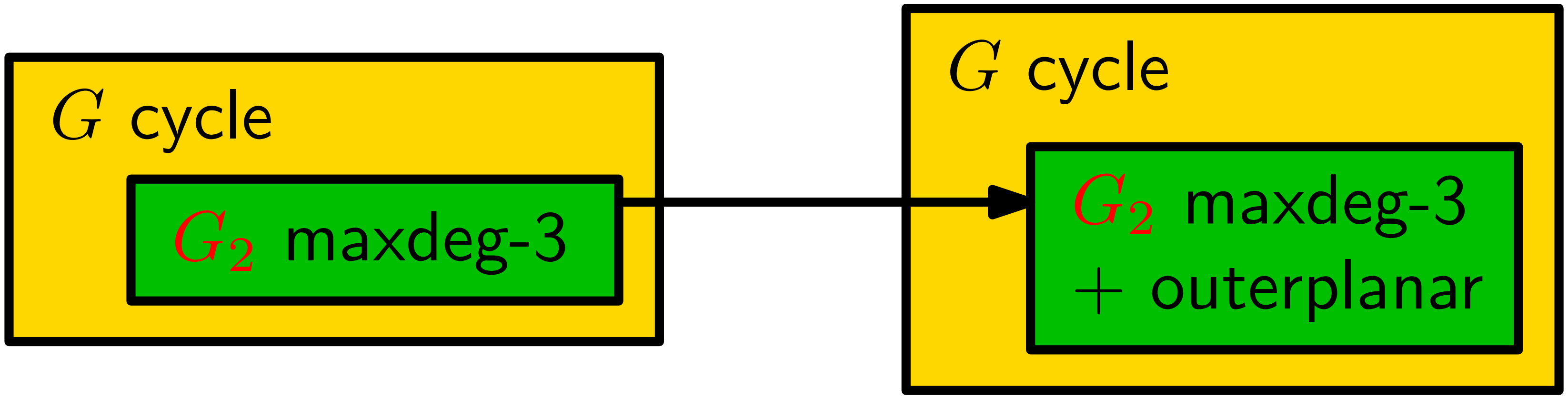

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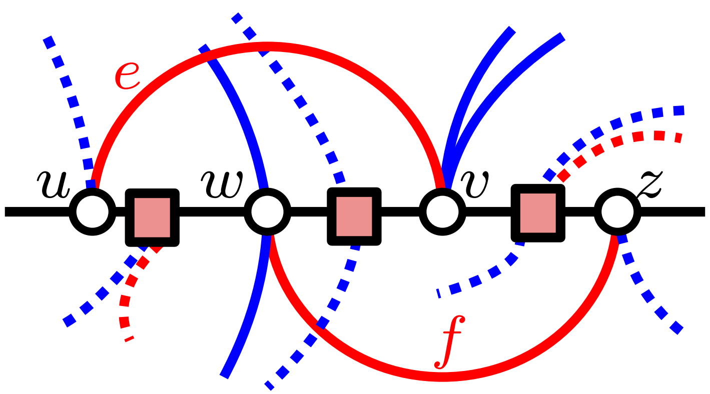
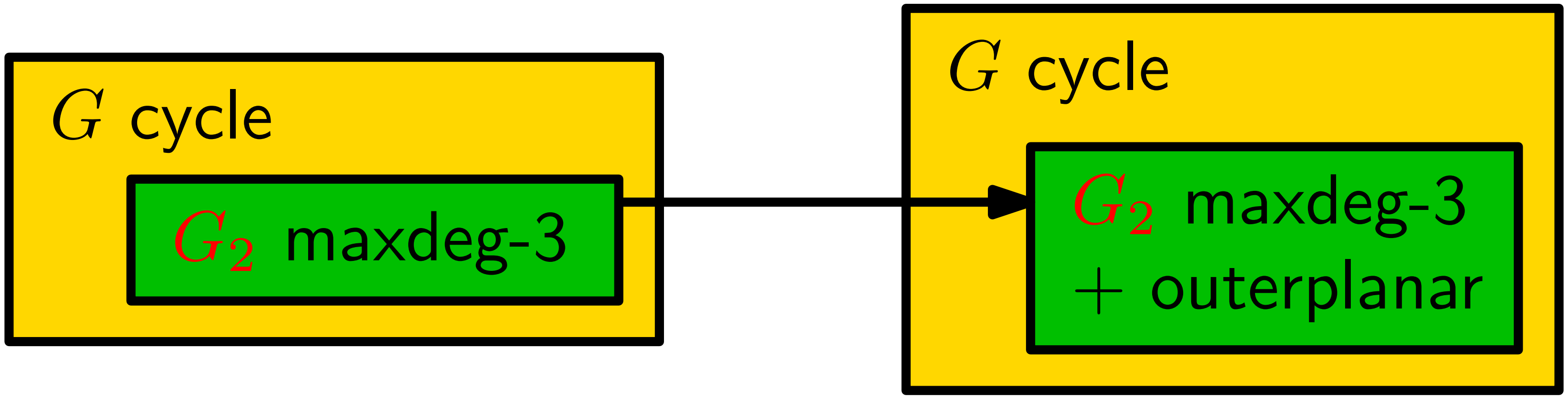


pick u, z as close as possible

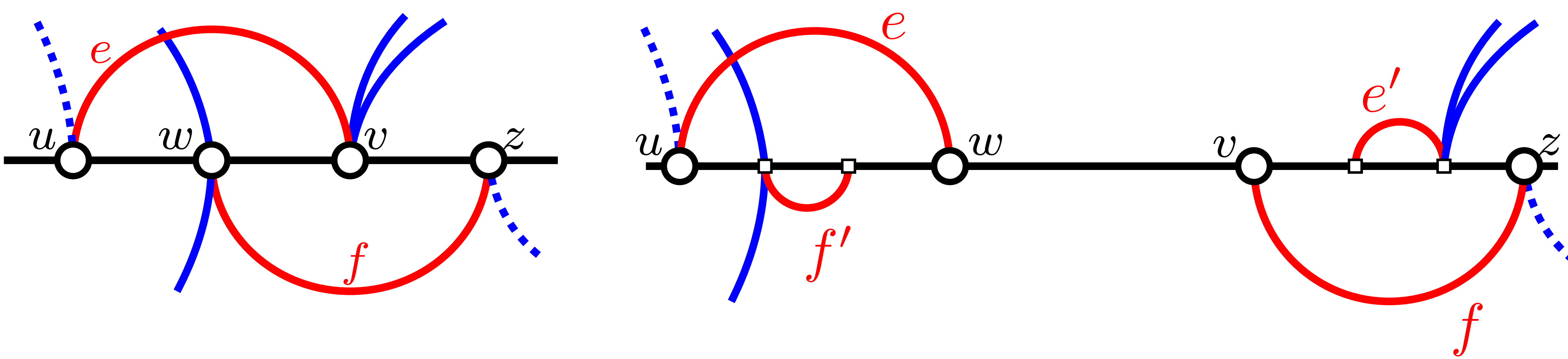
- ■ outerplanar,
- no edge between ■ and u, z

Making a Maxdeg-3 Graph Outerplanar

Theorem.

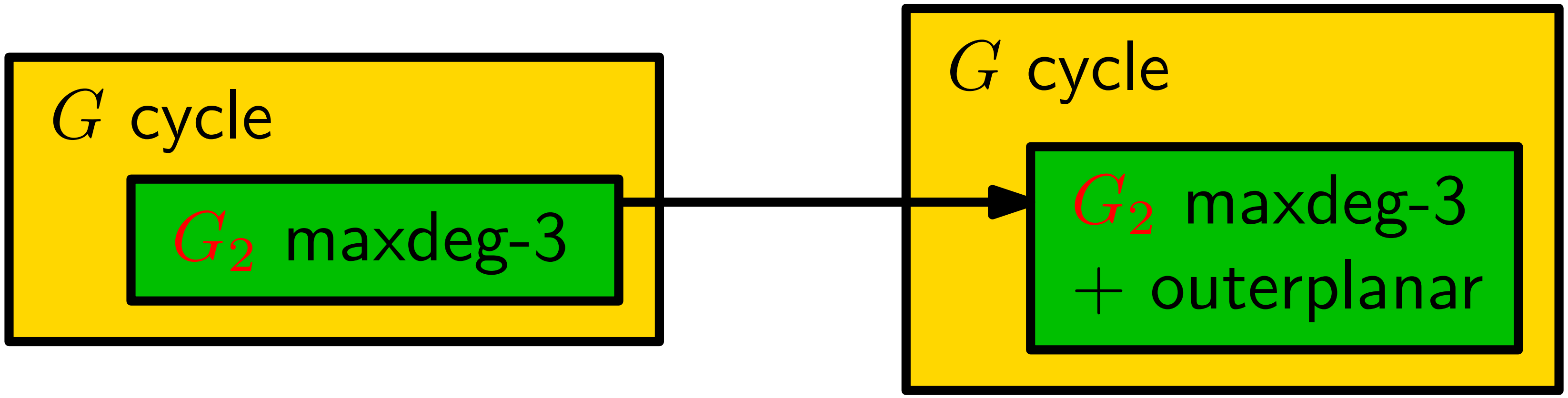


Proof:

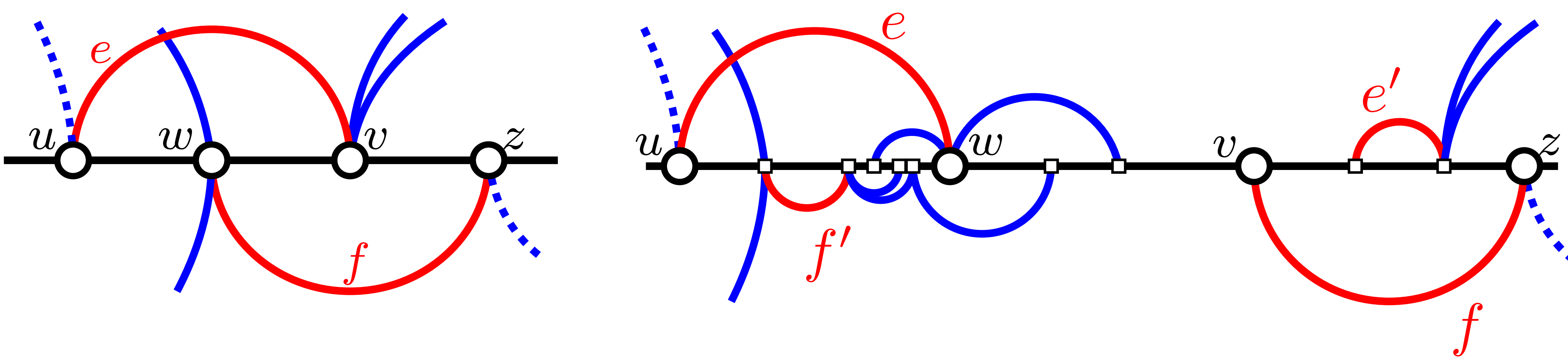


Making a Maxdeg-3 Graph Outerplanar

Theorem.

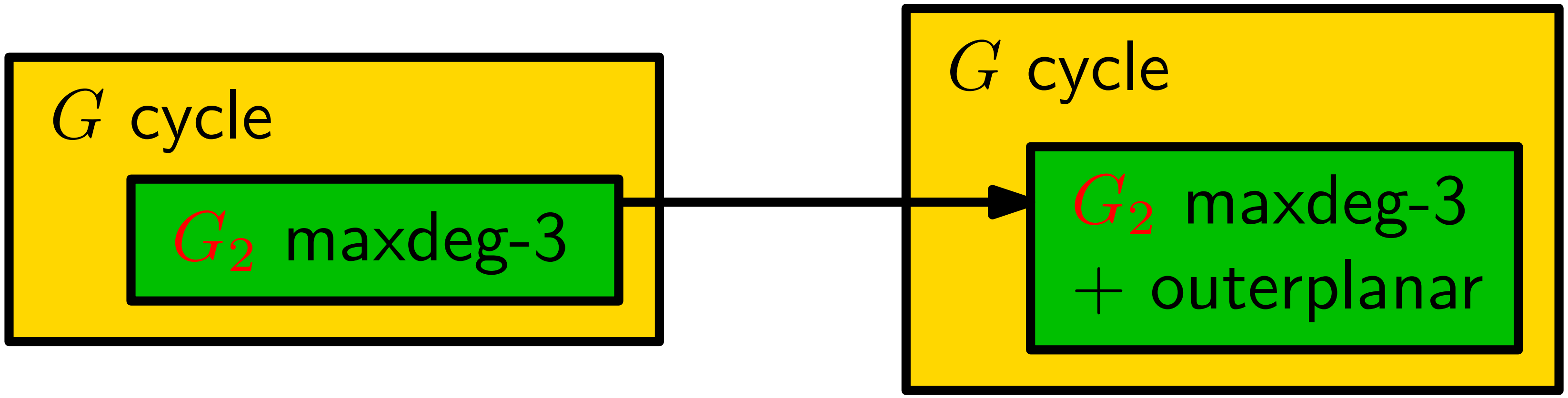


Proof:



Making a Maxdeg-3 Graph Outerplanar

Theorem.



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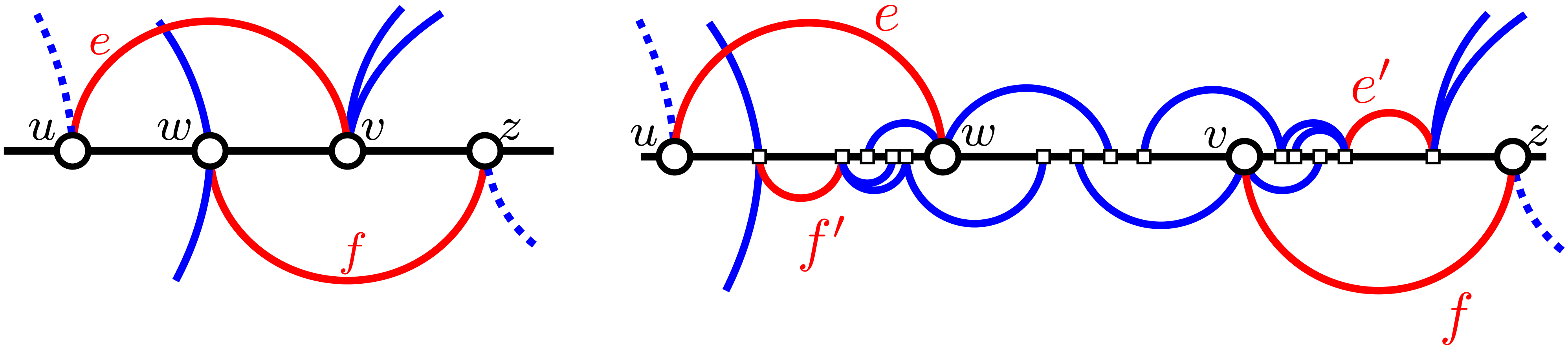
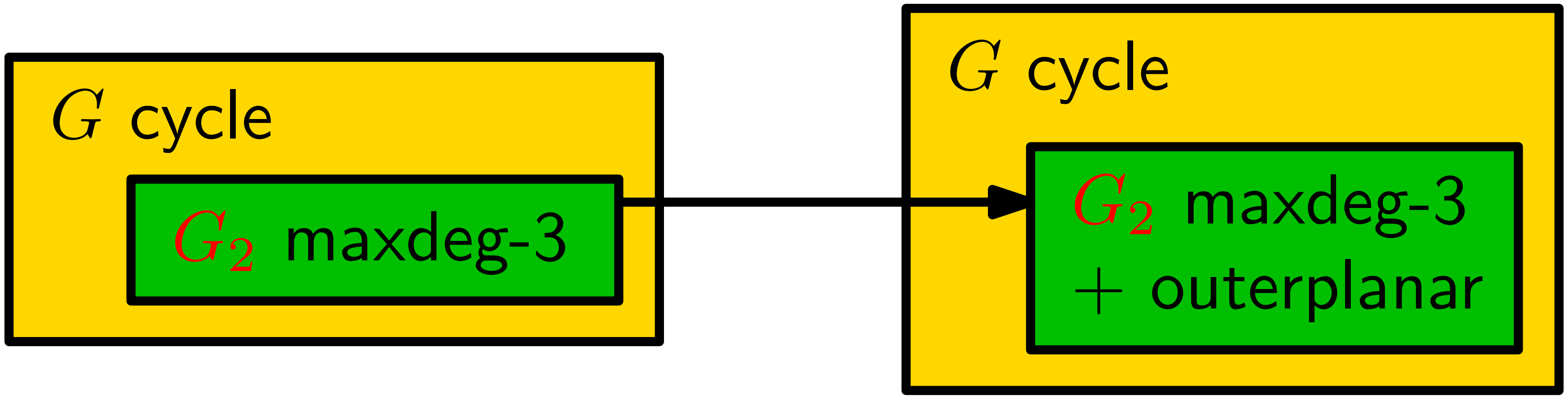
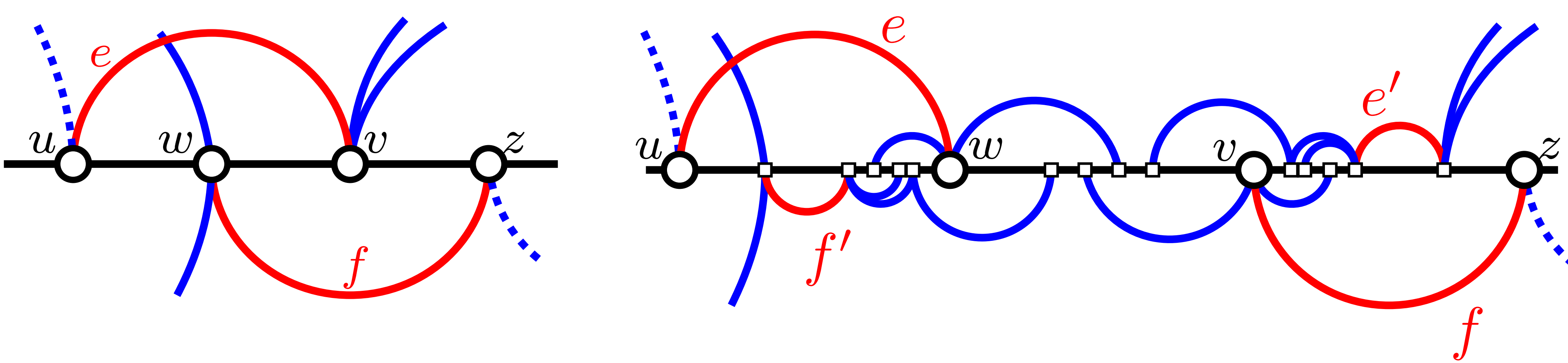


Making a Maxdeg-3 Graph Outerplanar

Theorem.

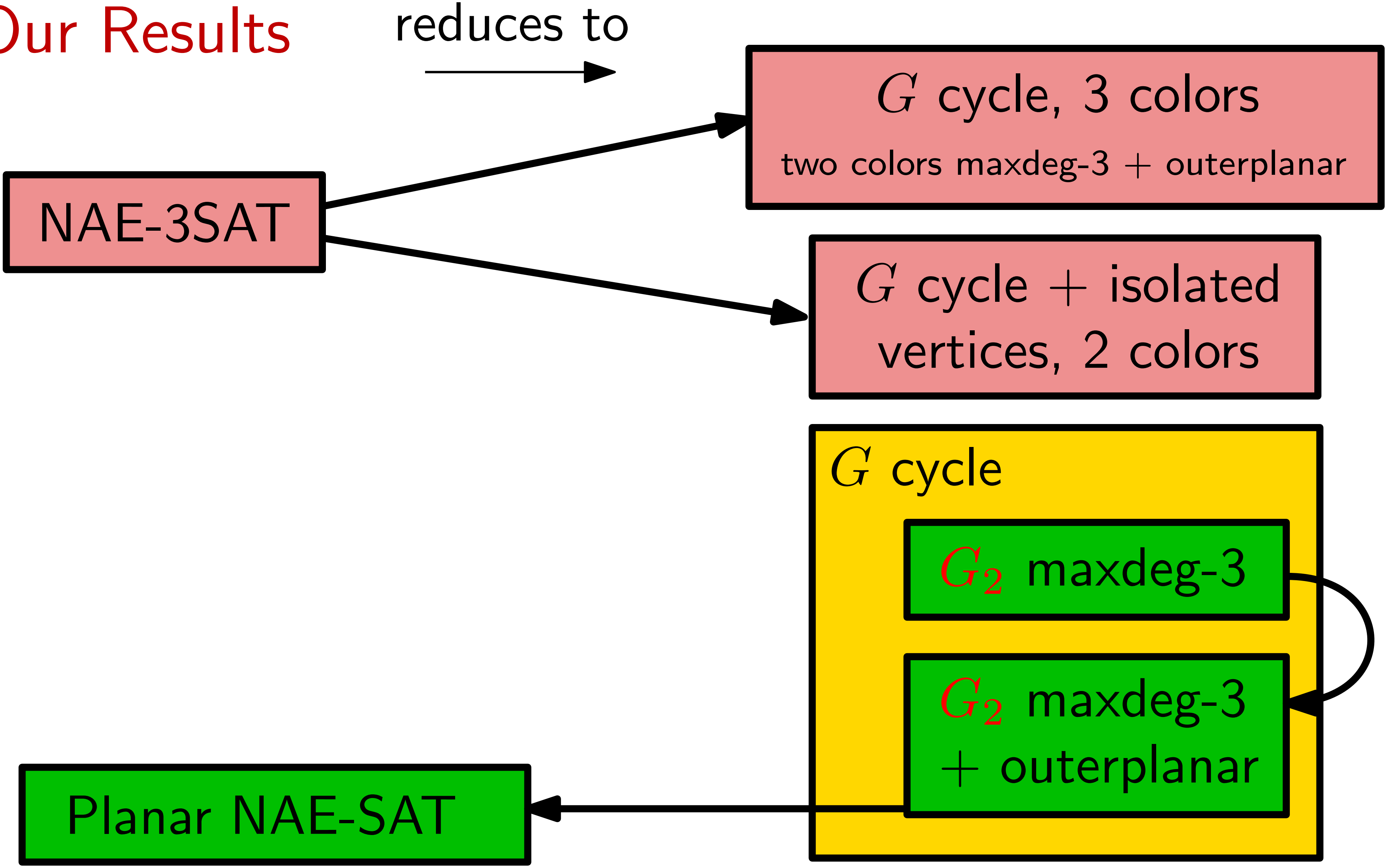


Proof:



□

Our Results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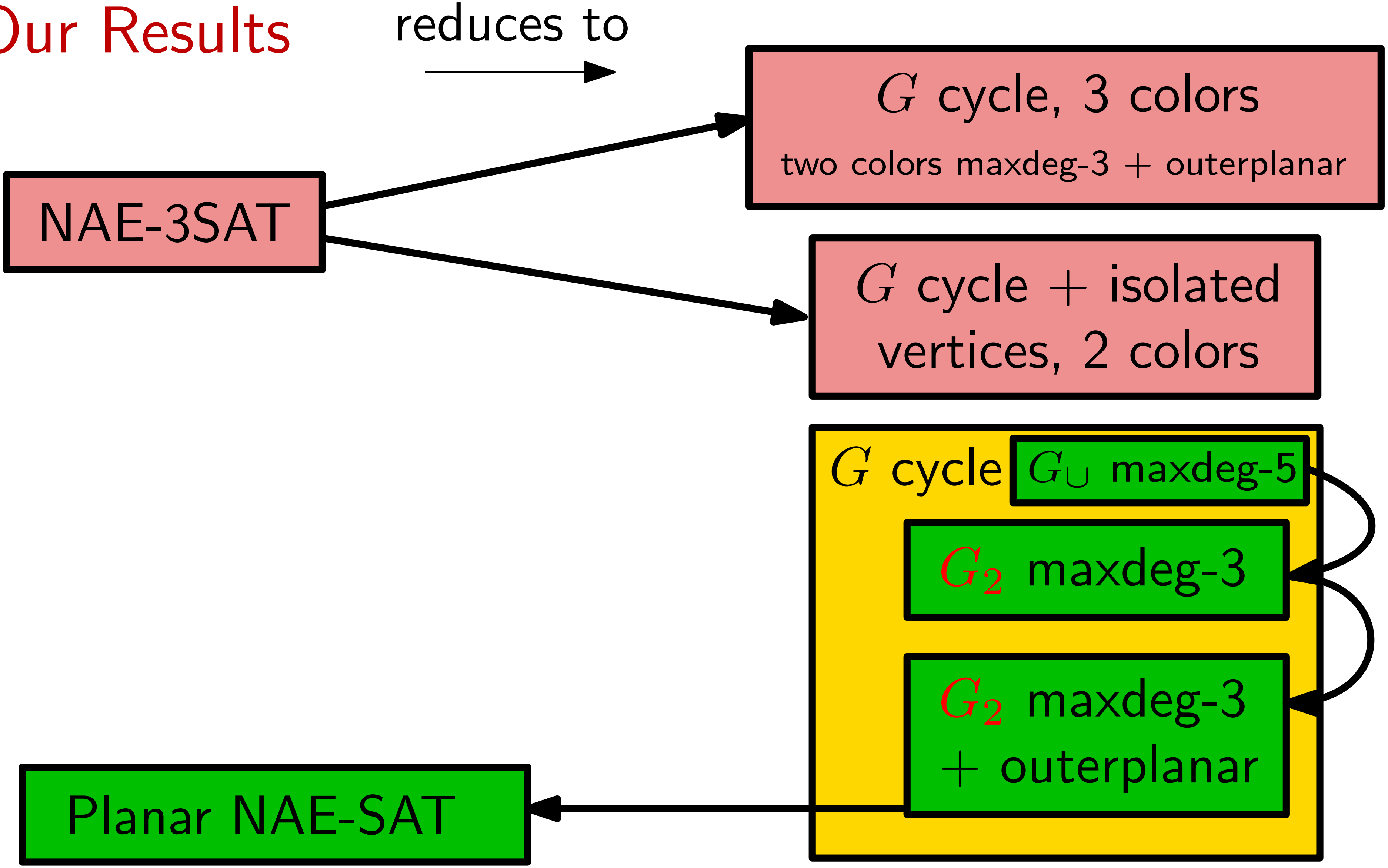
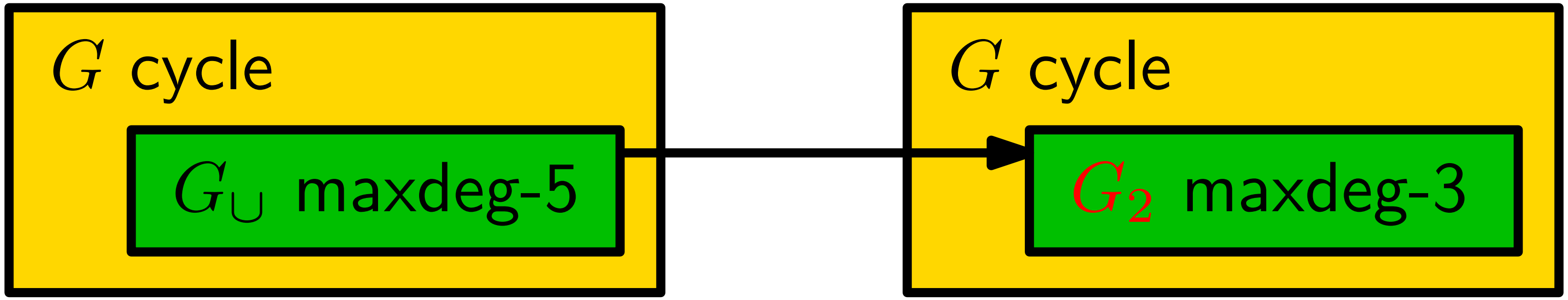
two colors maxdeg-3 + outerplanar
G cycle + isolated vertices, 2 colors
G cycle
G_2 maxdeg-3
G_2 maxdeg-3 + outerplanar
Planar NAE-SAT

Our Results


reduces to
NAE-3SAT
G cycle, 3 colors
two colors maxdeg-3 + outerplanar
G cycle + isolated vertices, 2 colors
G cycle
G∪ maxdeg-5
G2 maxdeg-3
G2 maxdeg-3 + outerplanar
Planar NAE-SAT

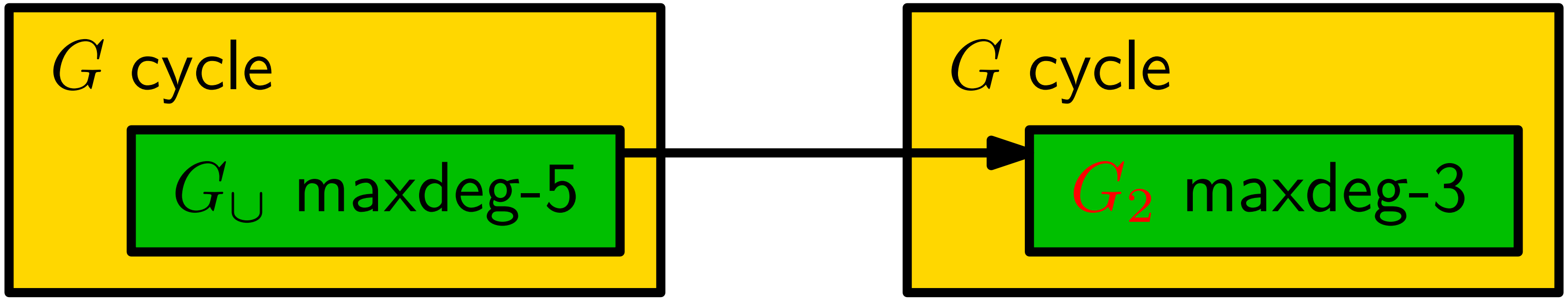
From $G_\cap$ maxdeg-5 to G_2 maxdeg-3

Theorem.

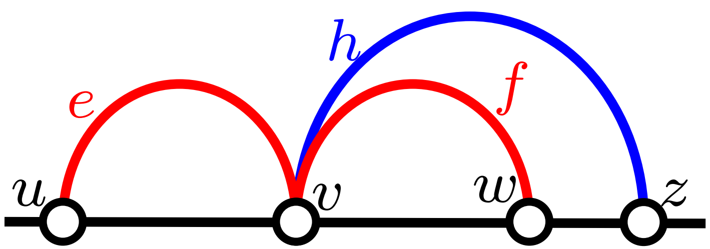


From $G_\cap$ maxdeg-5 to G_2 maxdeg-3

Theorem.

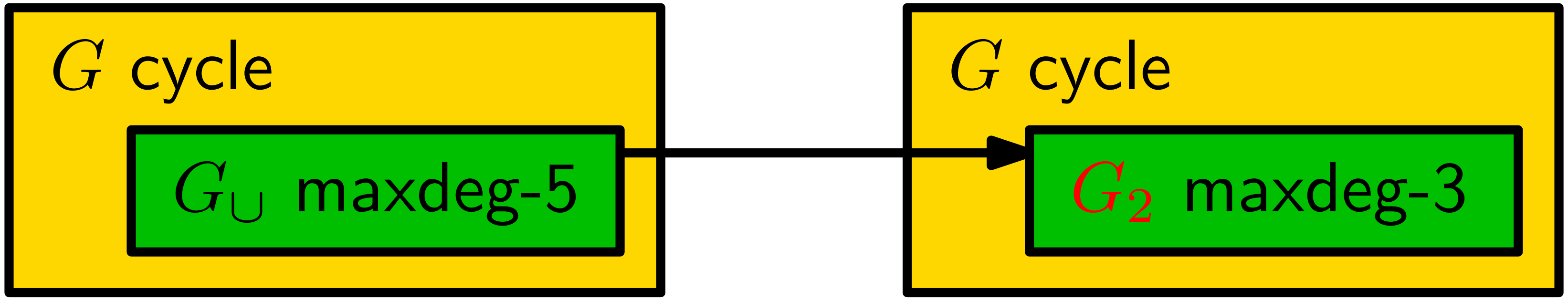


Proof:

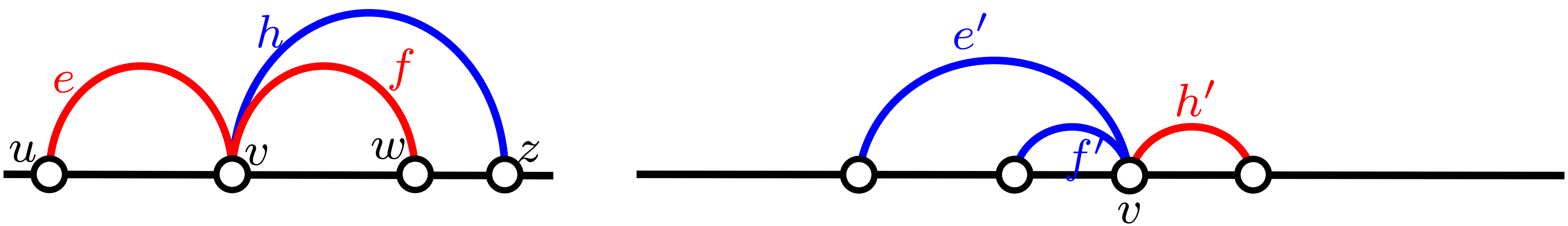


From $G_\cap$ maxdeg-5 to G_2 maxdeg-3

Theorem.



Proof:



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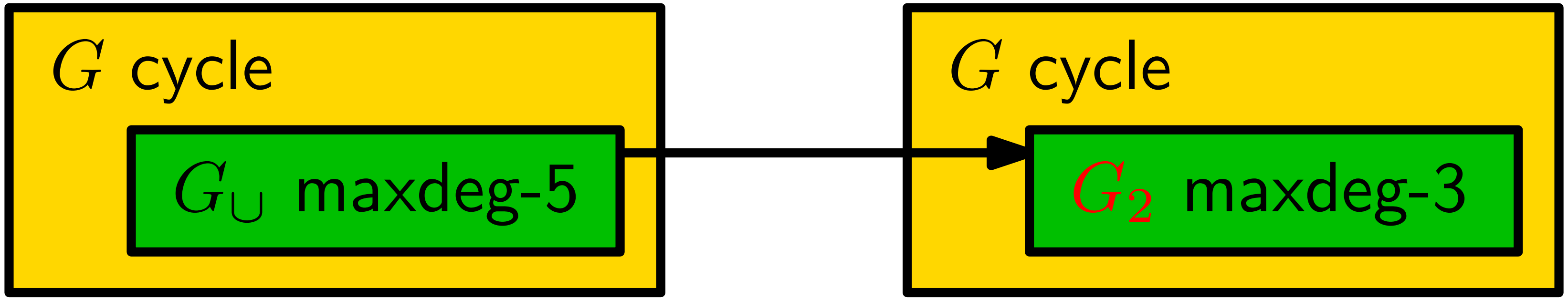

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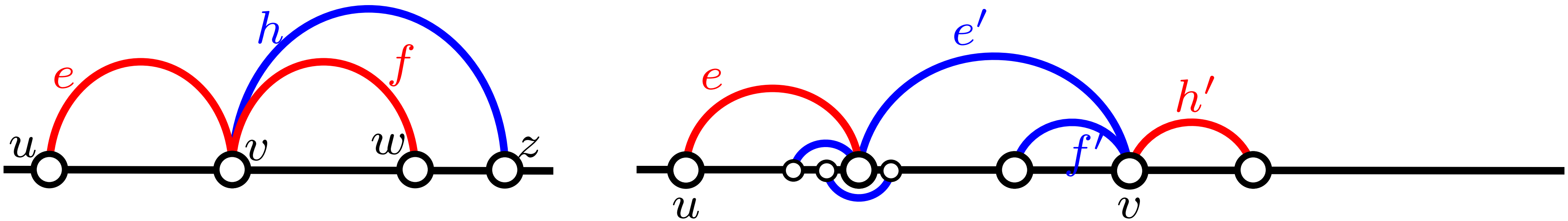

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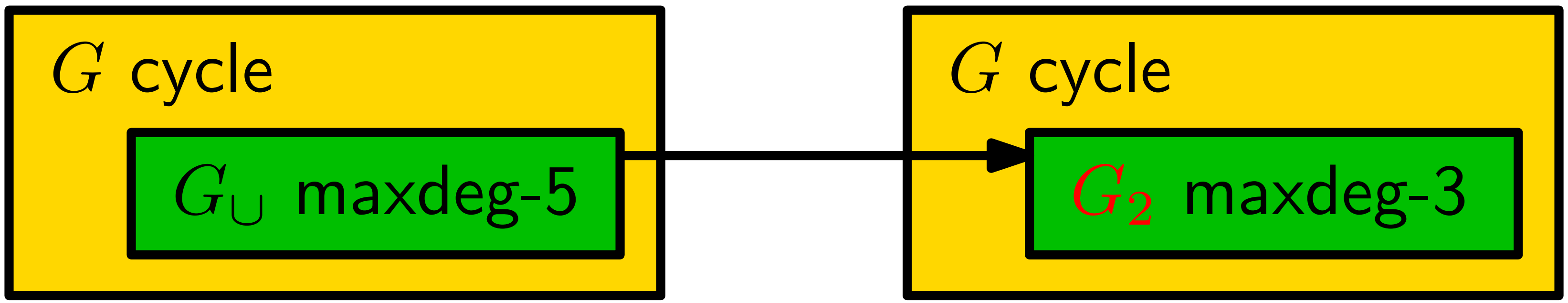
Theorem.



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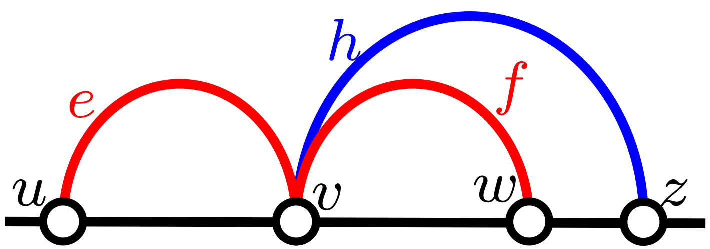
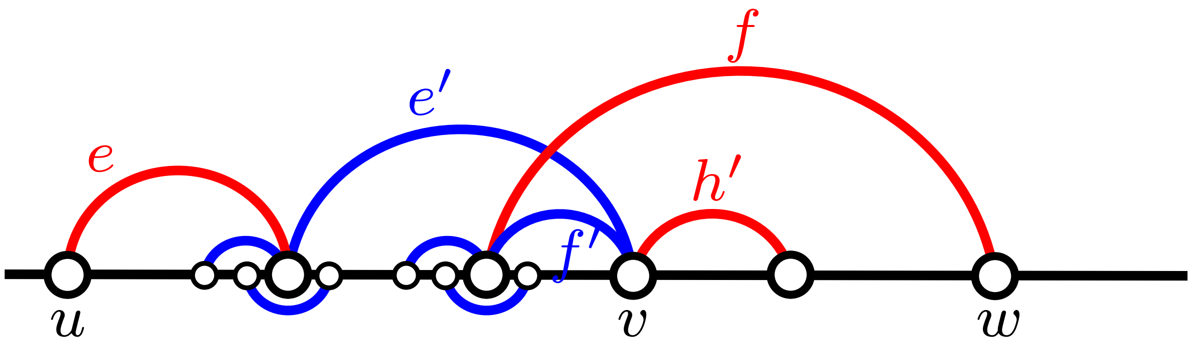
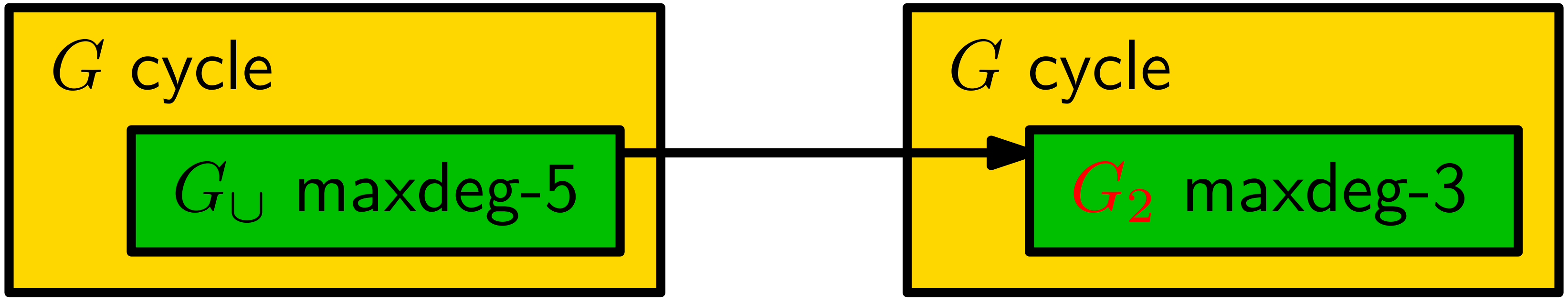




From $G_\cap$ maxdeg-5 to G_2 maxdeg-3

Theorem.



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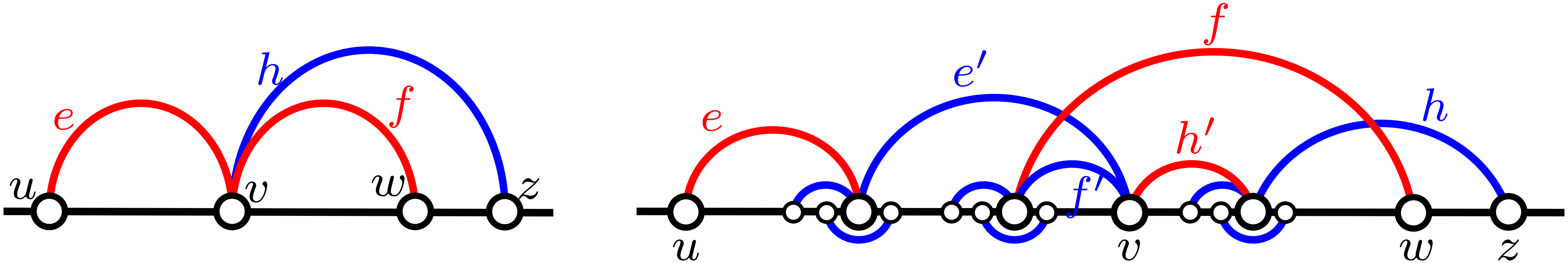
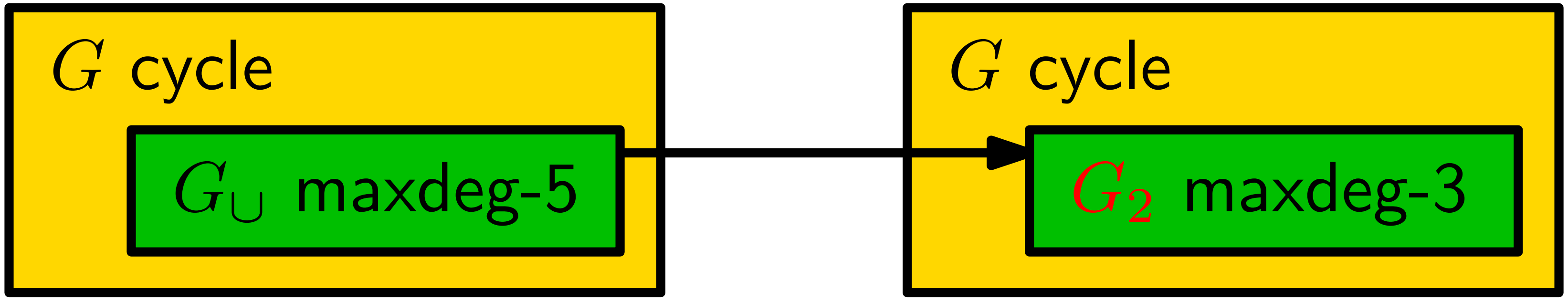
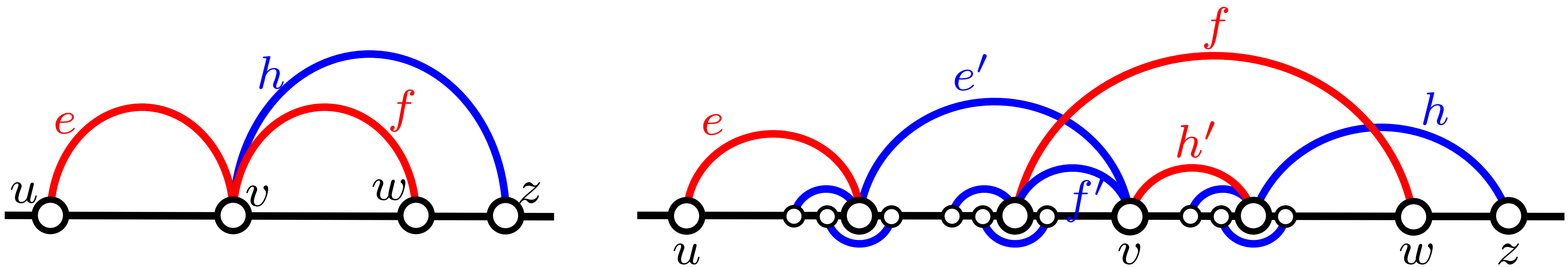


From $G_\cap$ maxdeg-5 to G_2 maxdeg-3

Theorem.

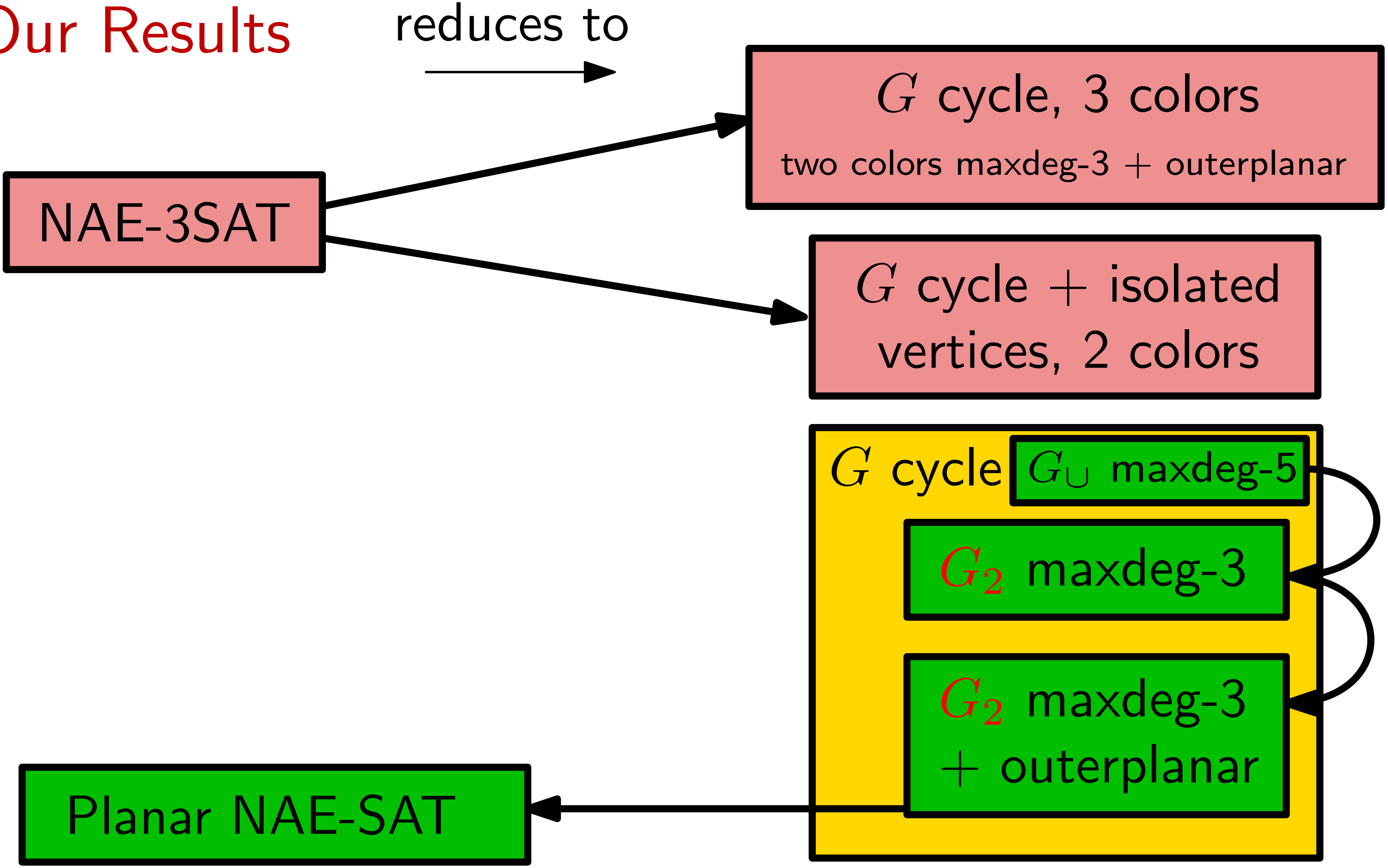


Proof:



□

Our Results


reduces to
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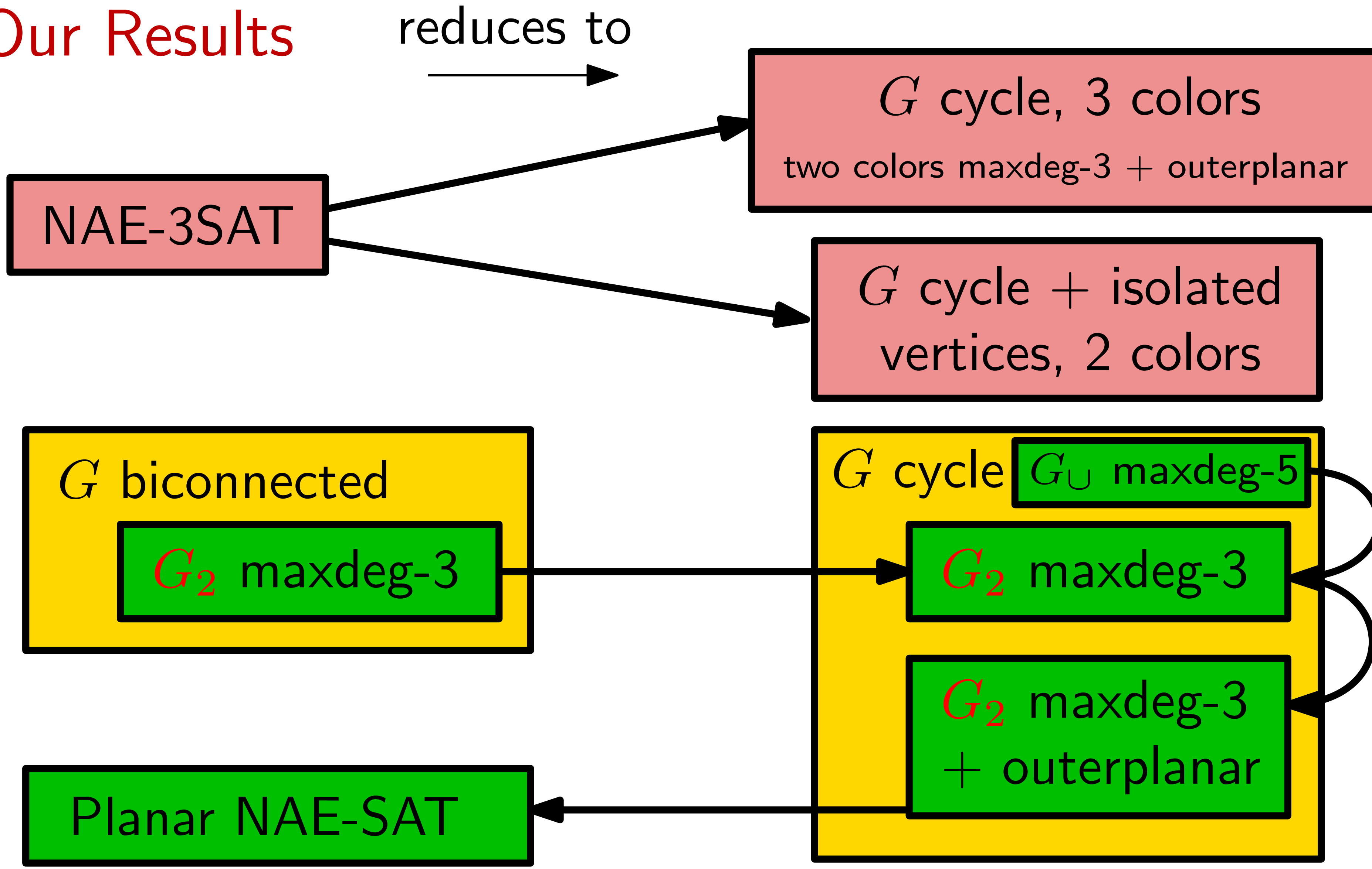
G cycle + isolated vertices, 2 colors
G cycle
$G_\cup$ maxdeg-5
G_2 maxdeg-3
G_2 maxdeg-3 + outerplanar
Planar NAE-SAT

Our Results



reduces to
NAE-3SAT
G cycle, 3 colors
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G cycle + isolated vertices, 2 colors
G biconnected
G_2 maxdeg-3
G cycle
$G_\cup$ maxdeg-5
G_2 maxdeg-3
G_2 maxdeg-3 + outerplanar
Planar NAE-SAT

From biconnected to cycle

Theorem.

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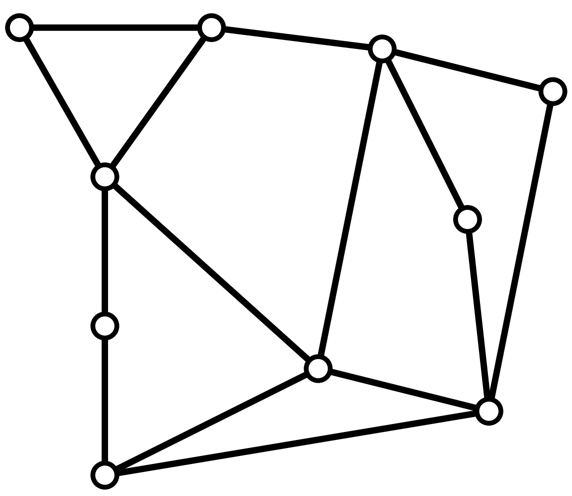
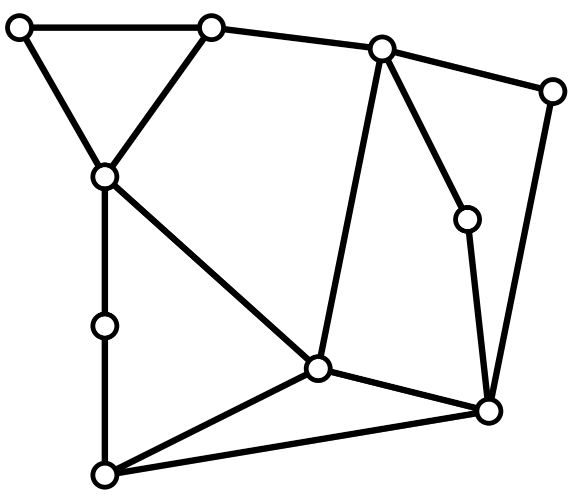


G biconnected

From biconnected to cycle

Theorem.



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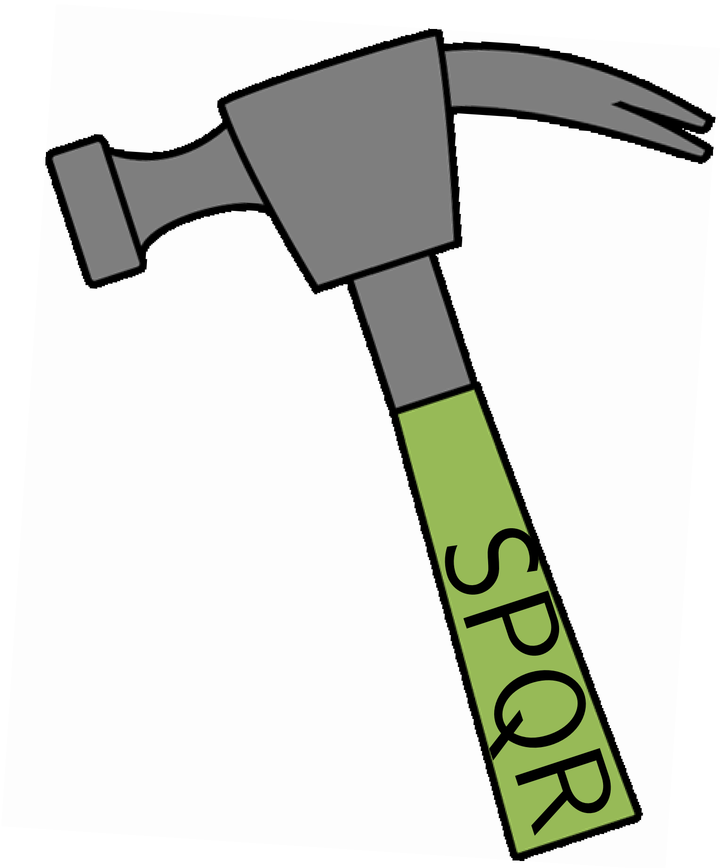

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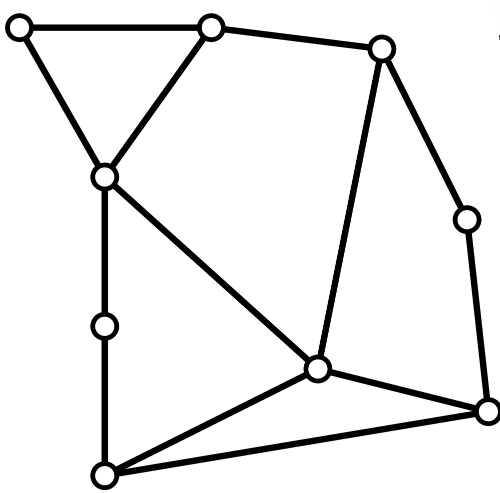

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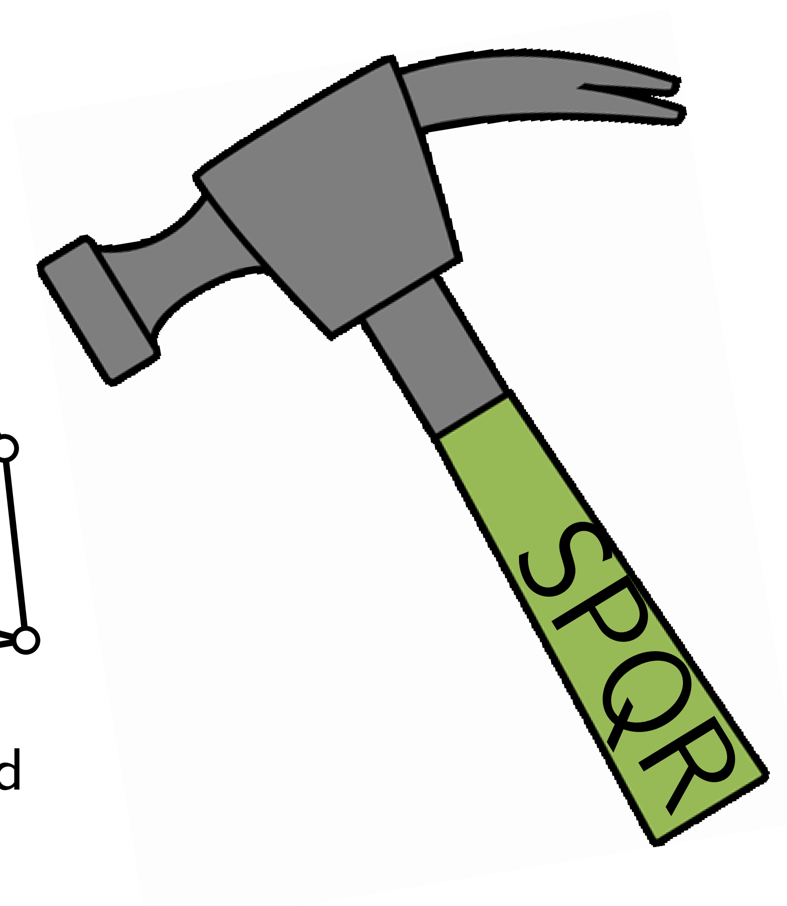

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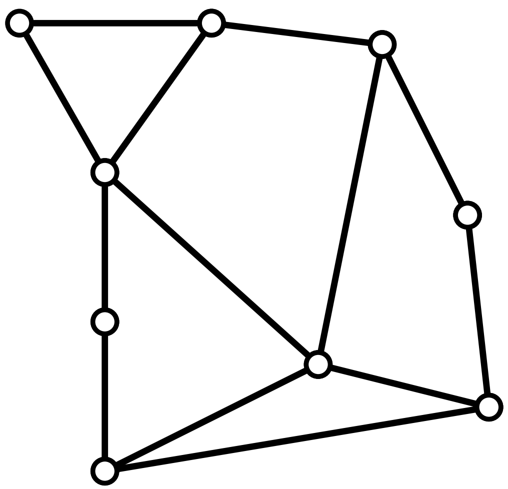

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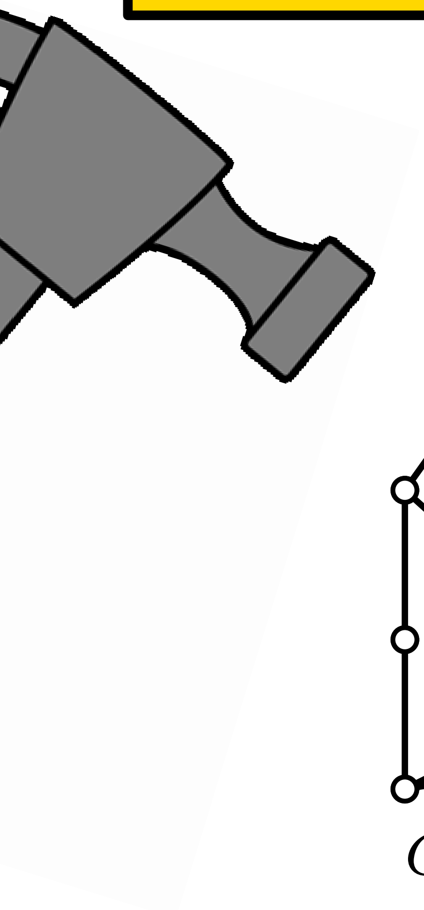

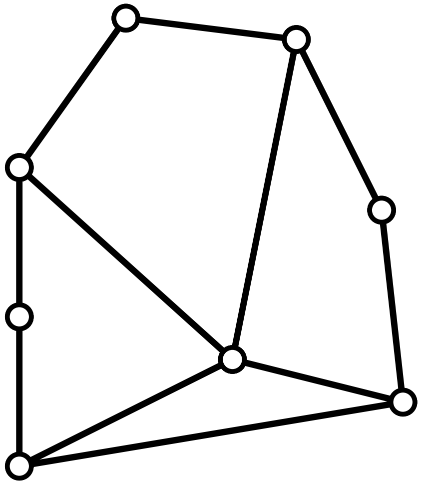

G biconnected

From biconnected to cycle

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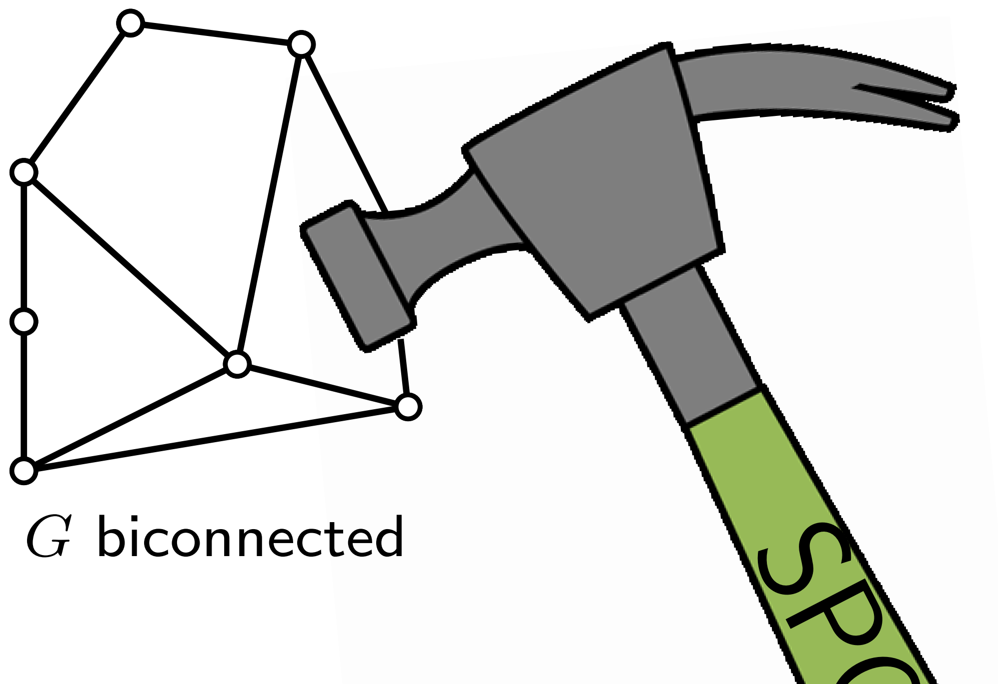
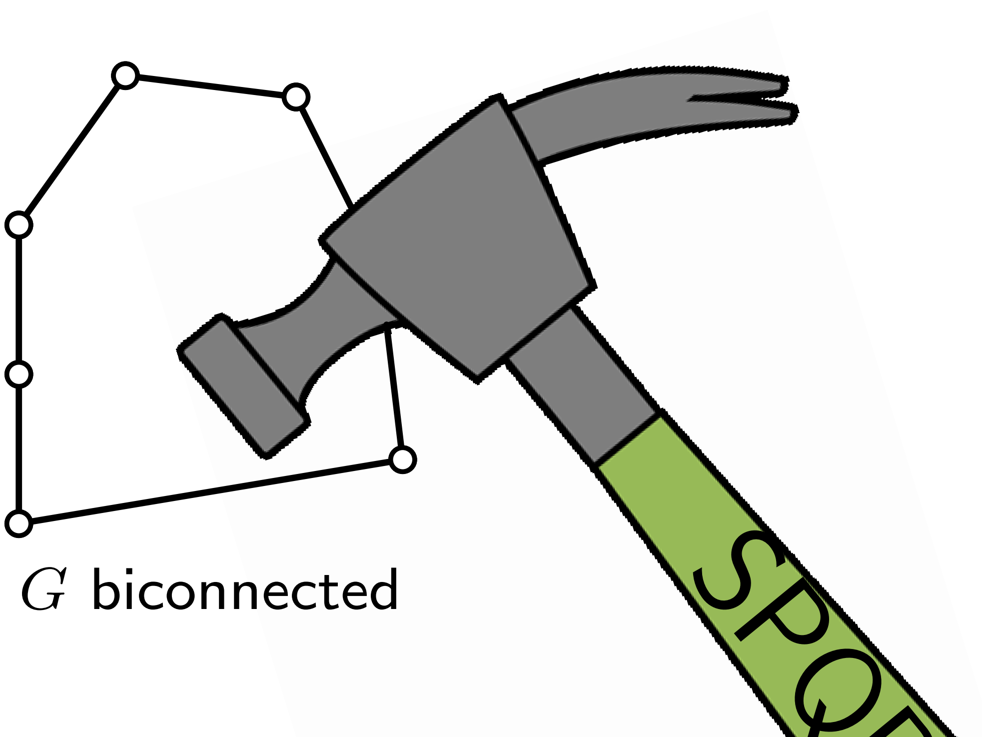


From biconnected to cycle

Theorem.



From biconnected to cycle

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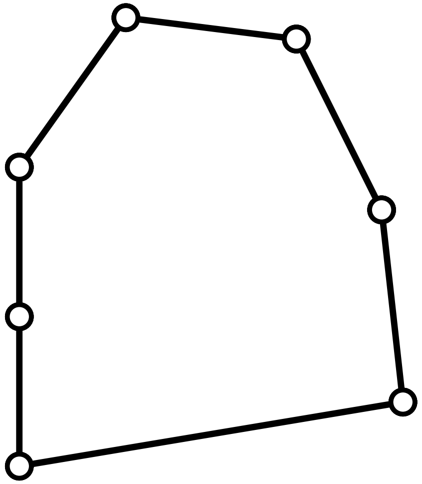
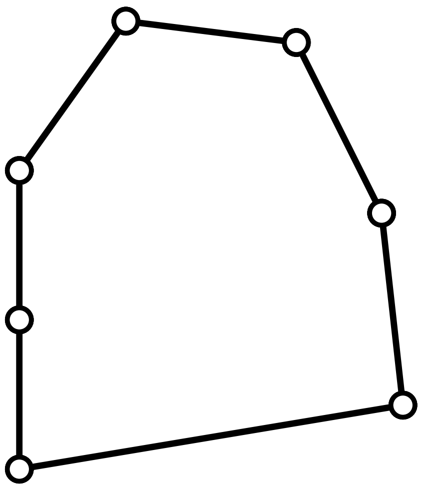


G cycle

From biconnected to cycle

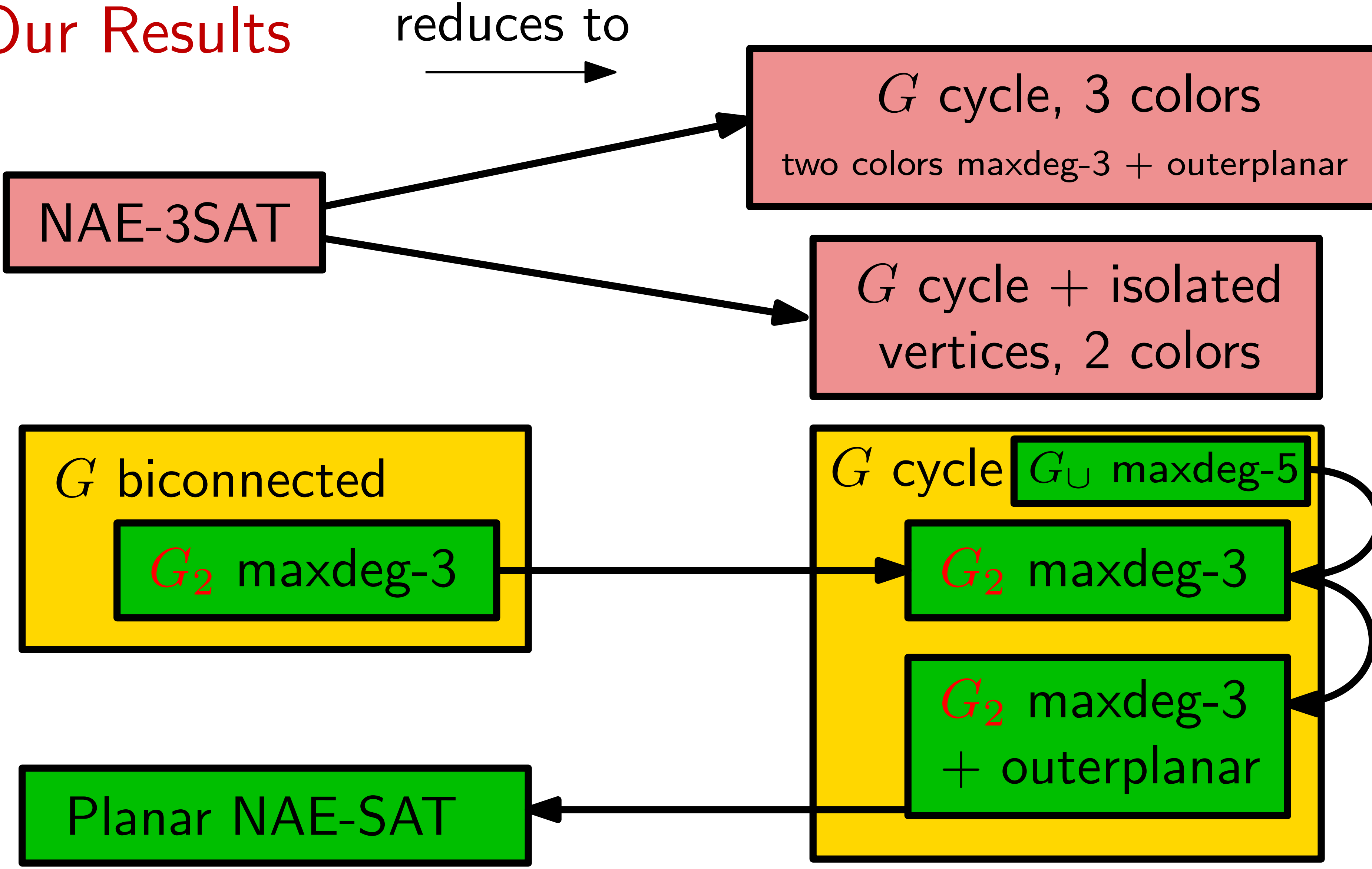
Theorem.



G cycle

□

Our Results



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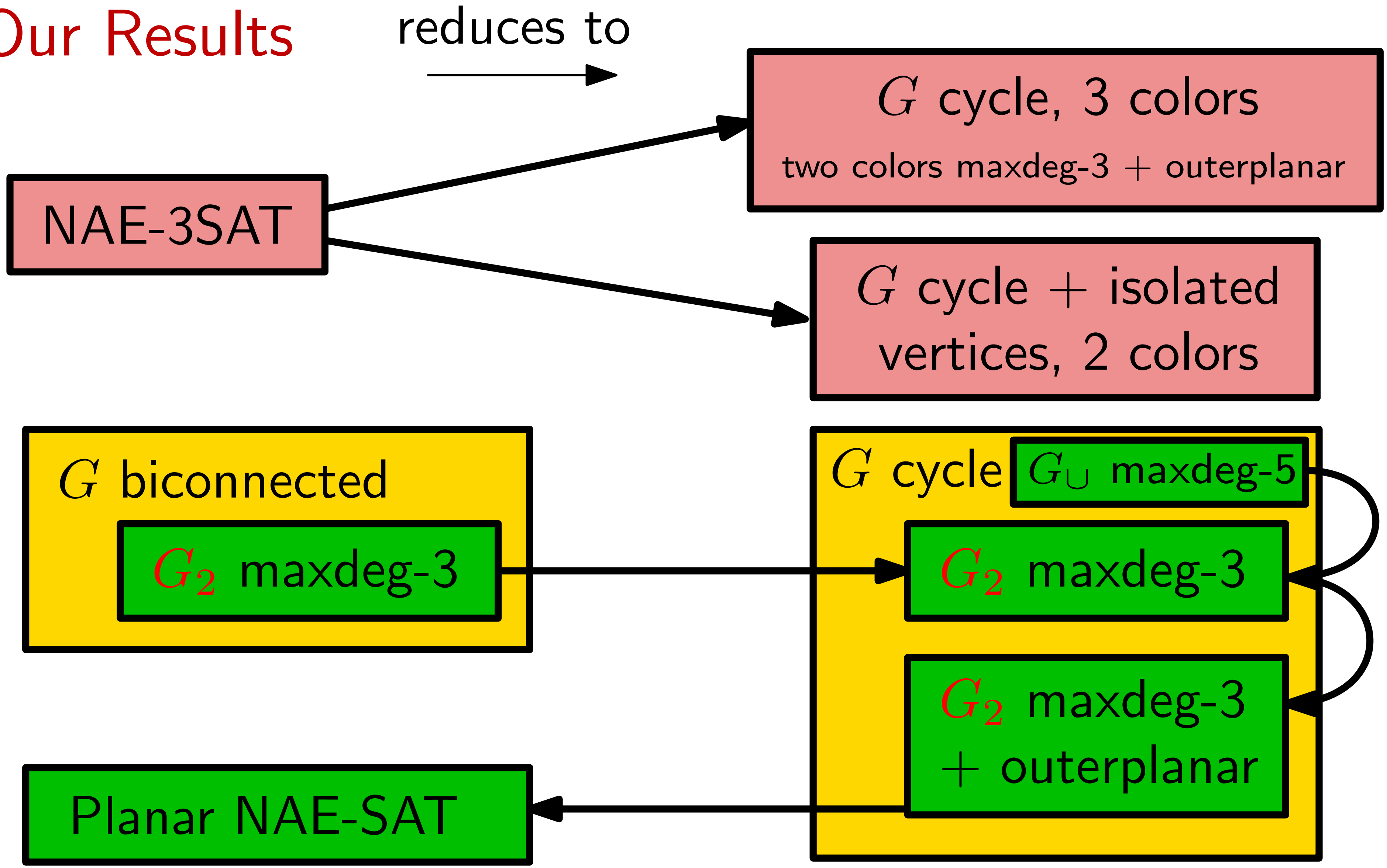
Planar NAE-SAT

Our Results



G biconnected $\Rightarrow$ can draw simultaneous orthogonal embedding with ≤ 3 bends per edge

Drawing Algorithm

- Based on Biedl & Kant [ESA '94, Comput. Geom. '98]

Drawing Algorithm

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- Place vertices bottom-to-top by s-t-ordering on G

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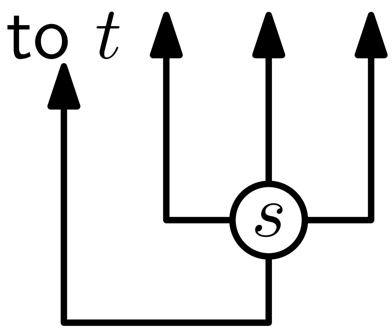

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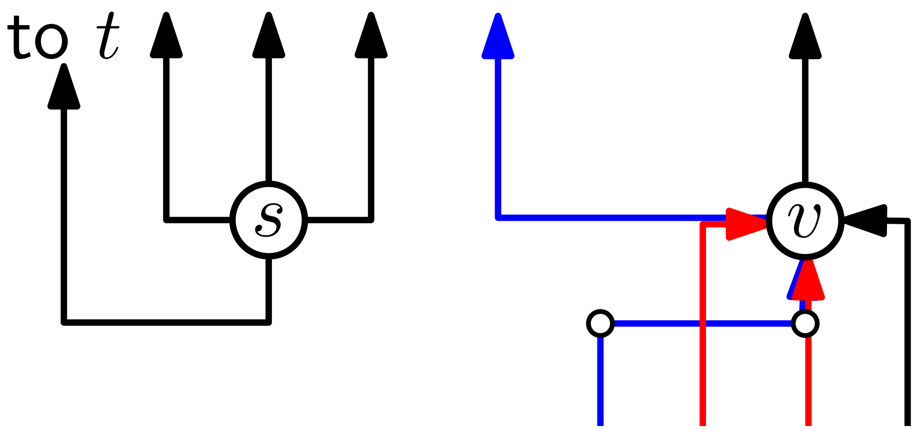

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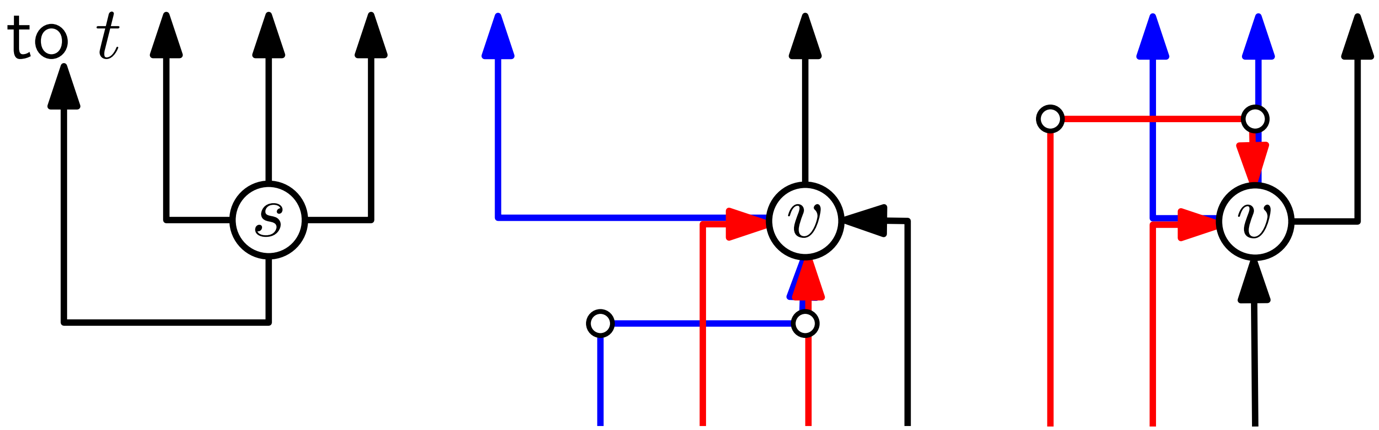

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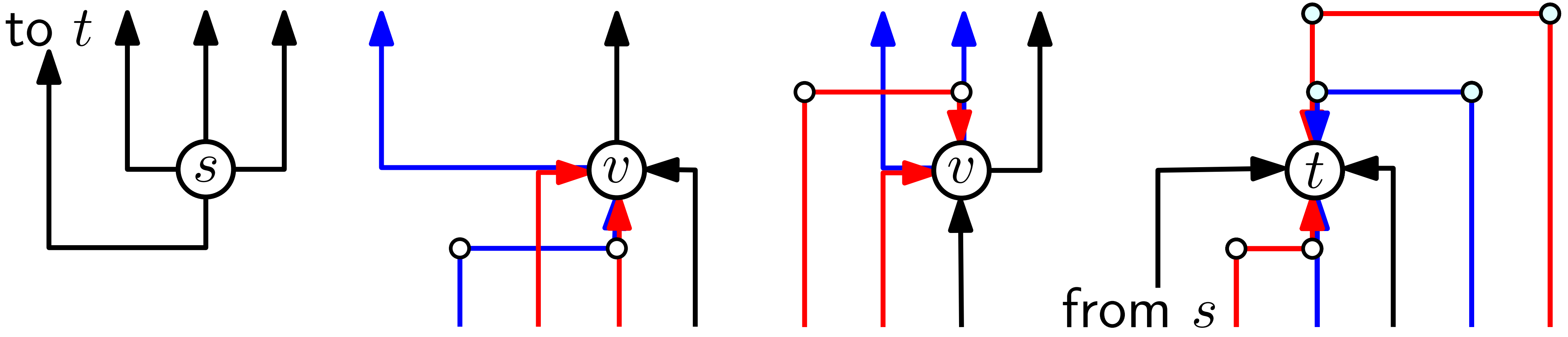

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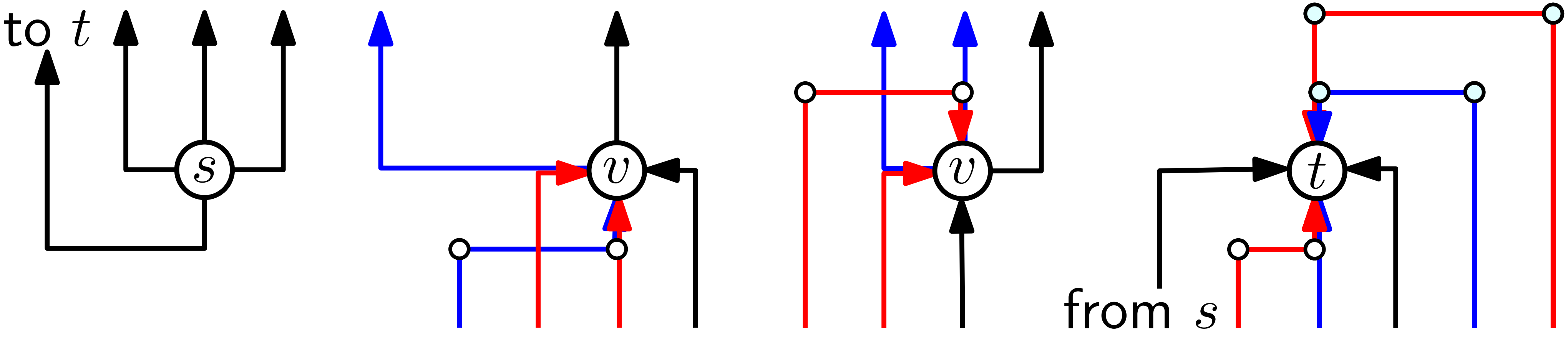

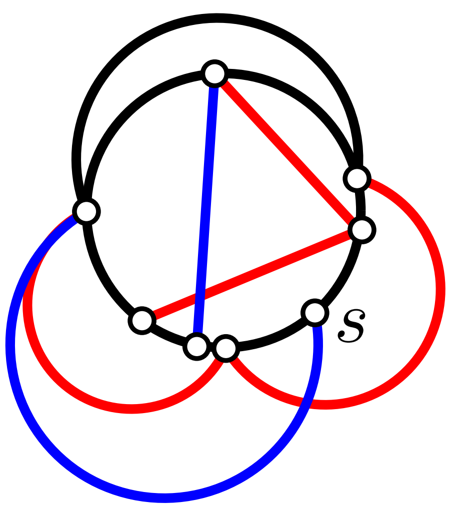

Drawing Algorithm

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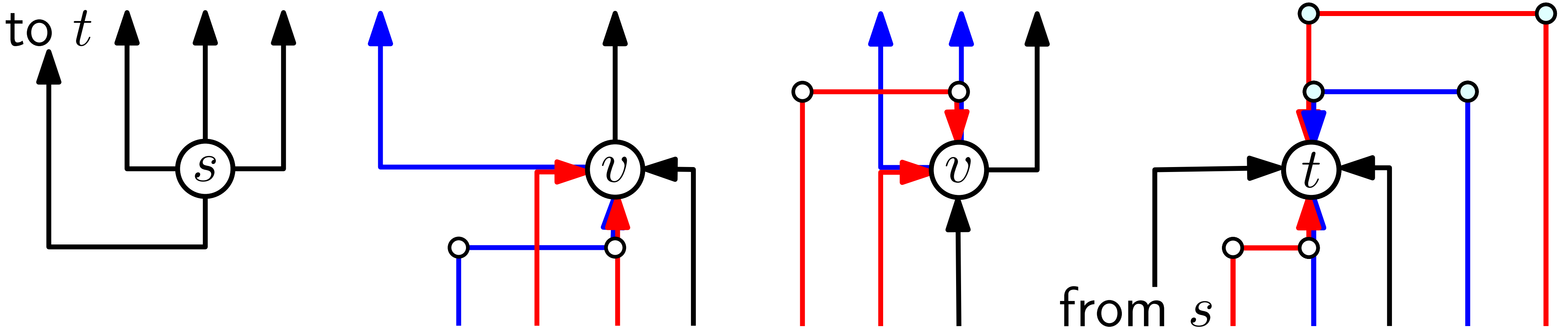

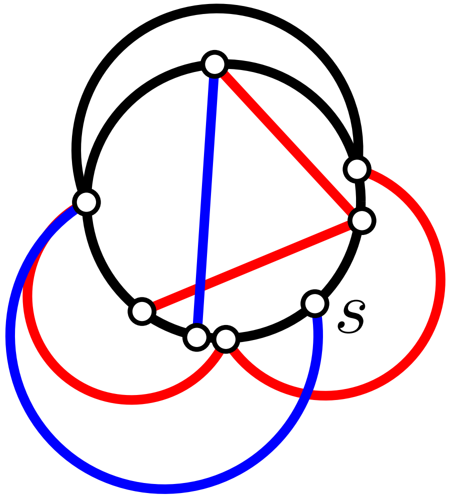

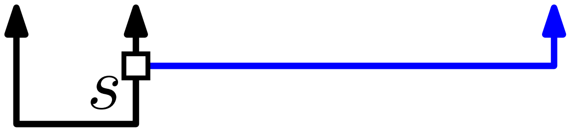

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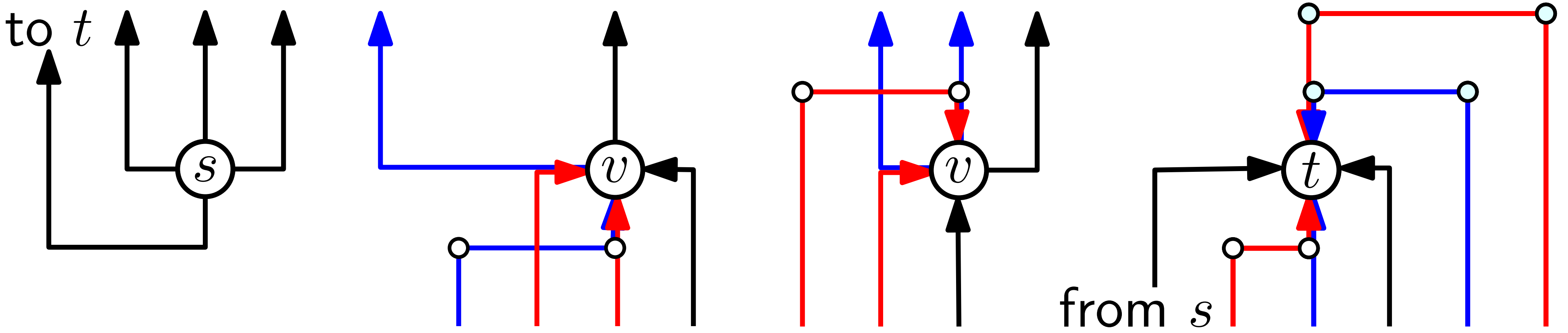

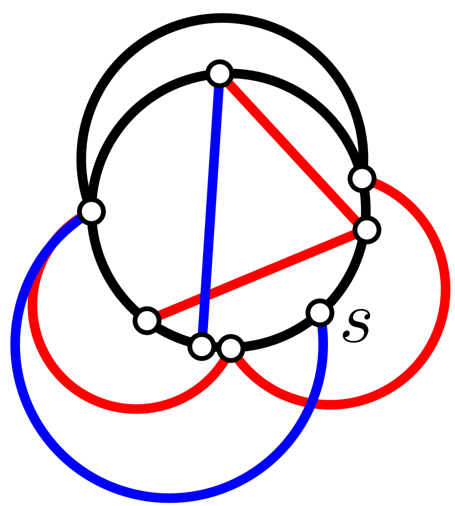

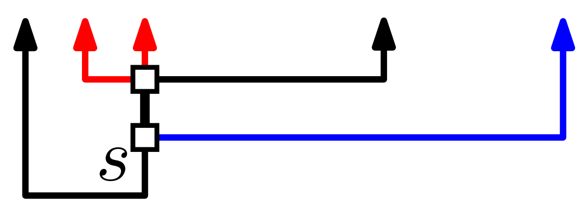

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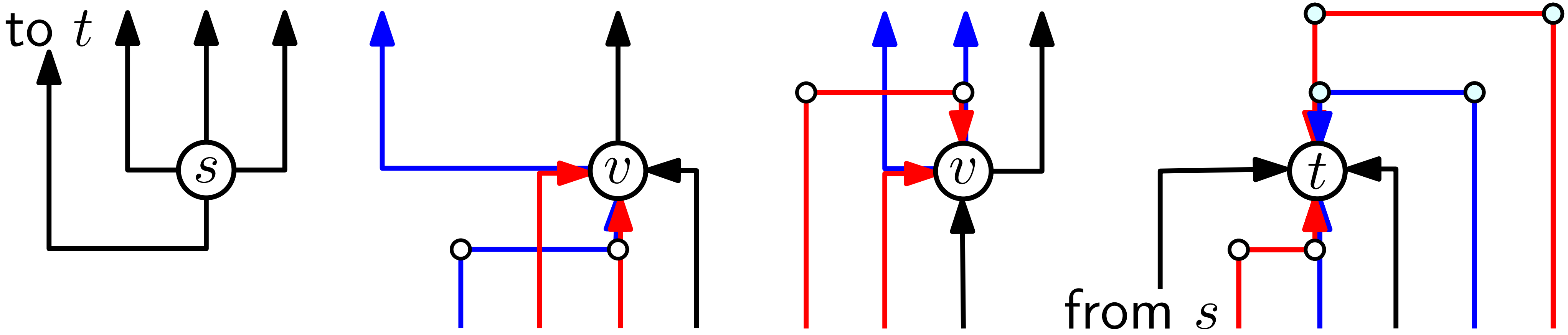

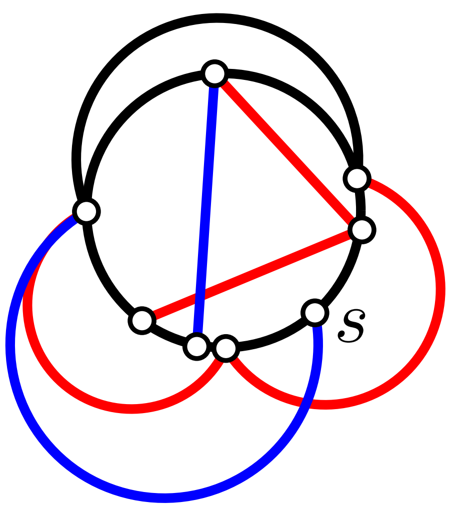

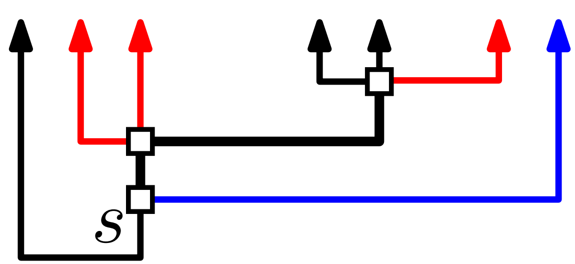

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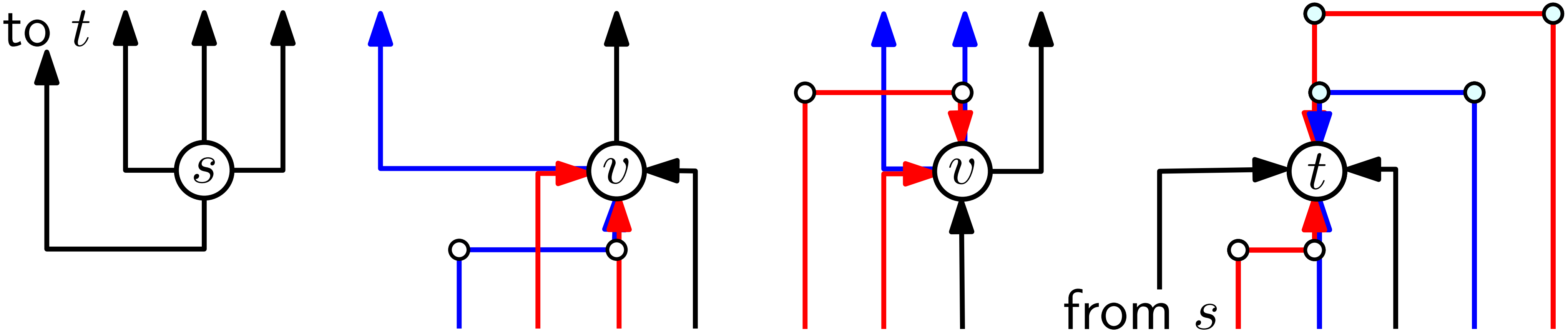

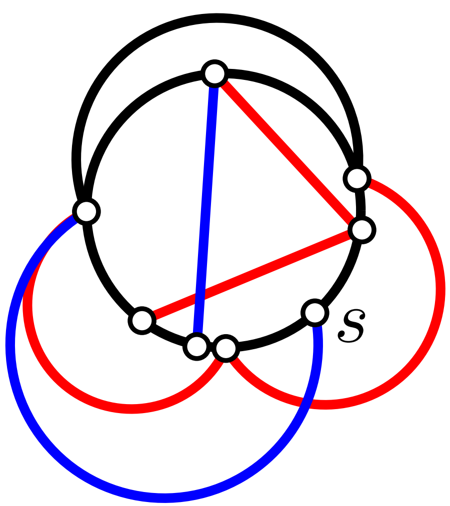

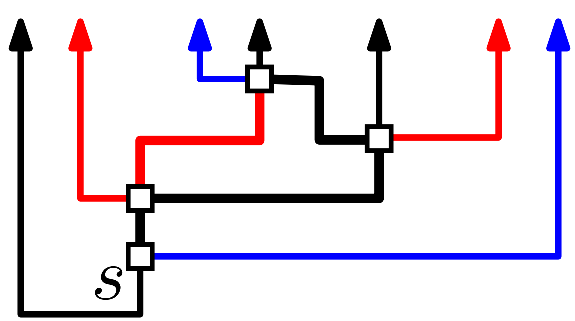

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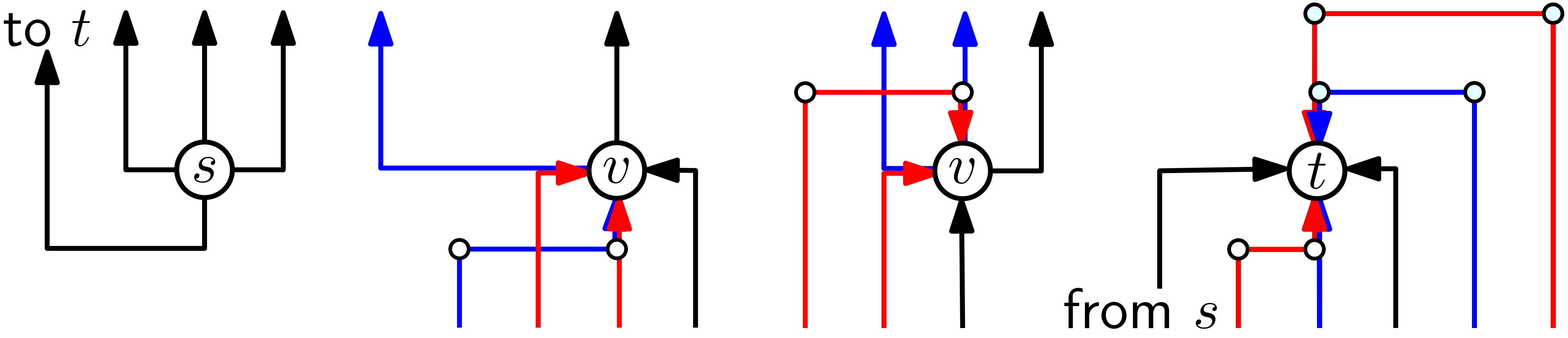

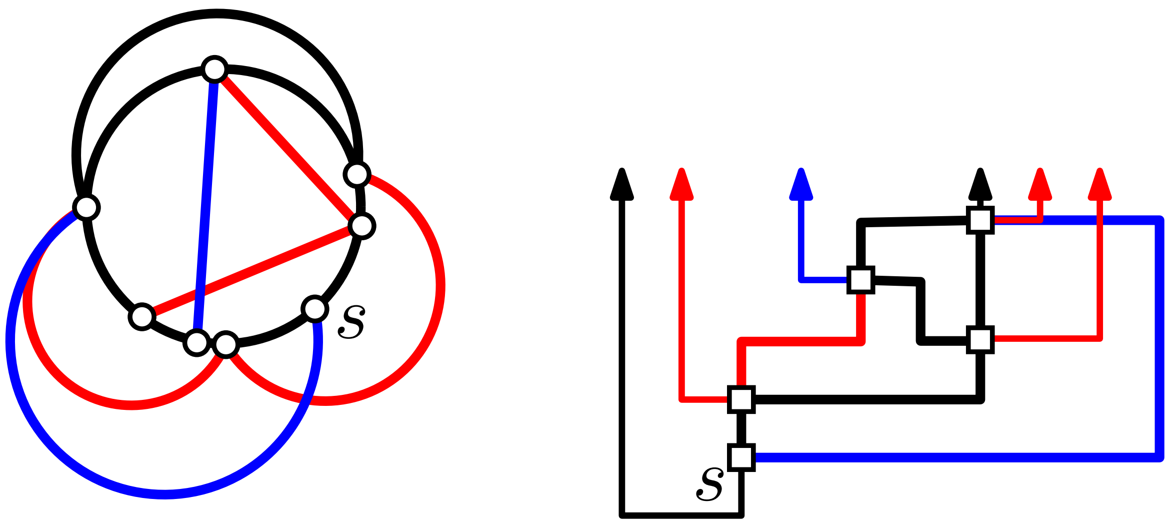

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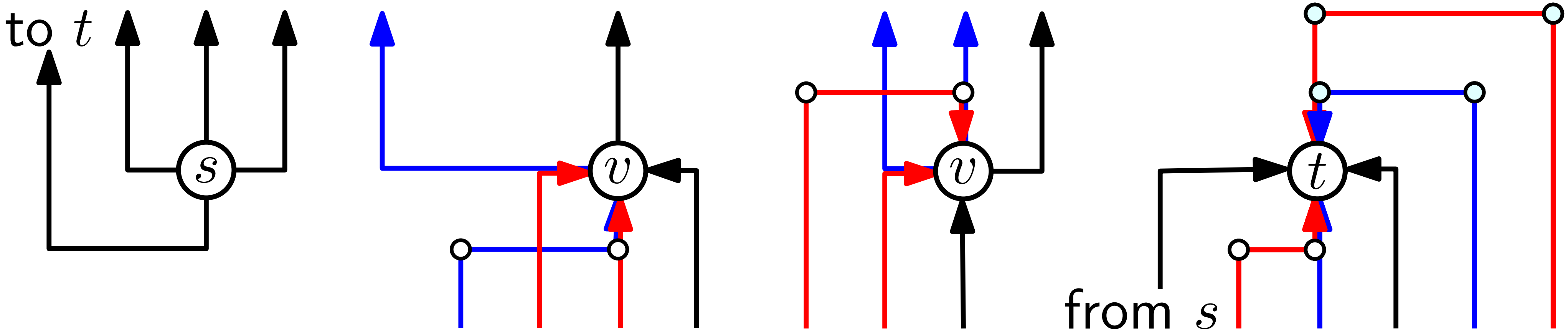

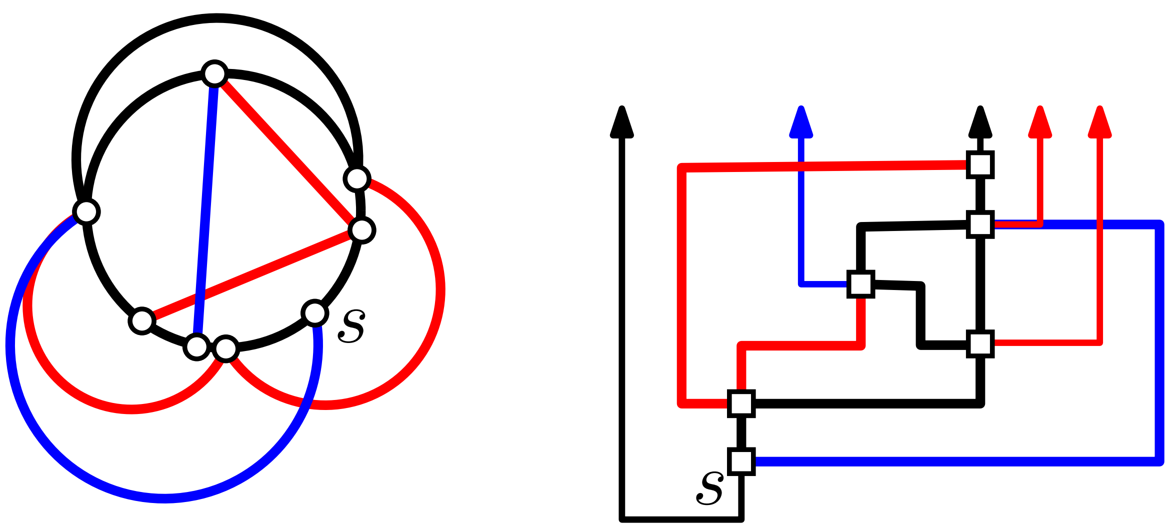

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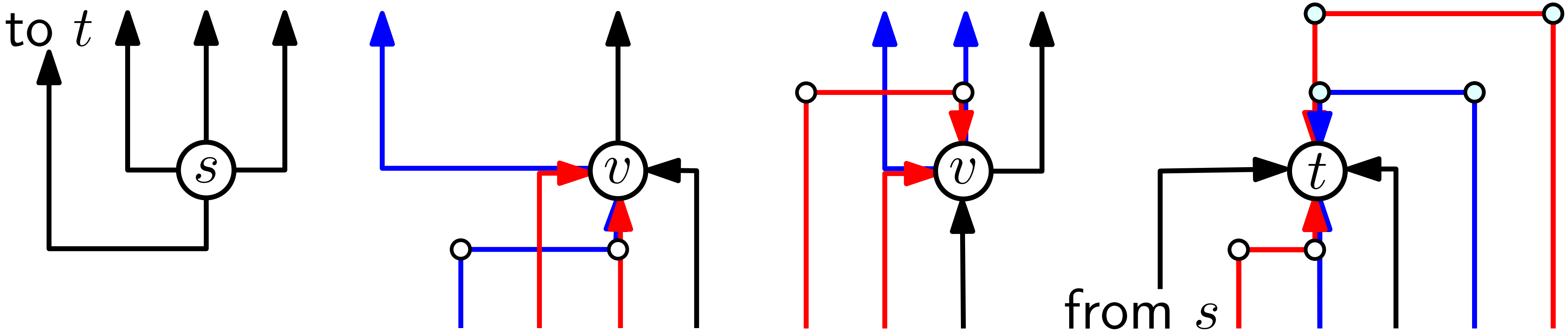

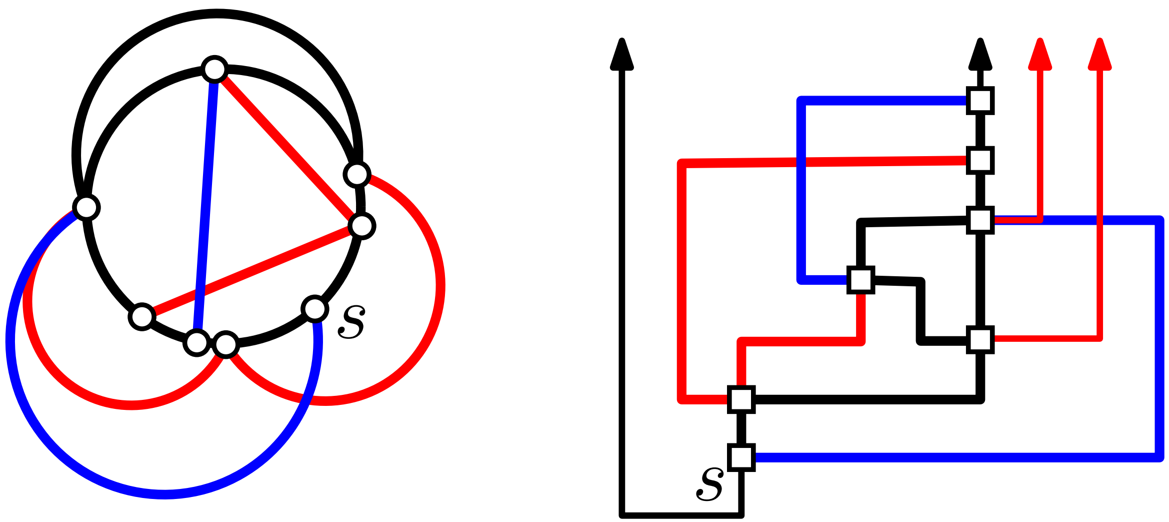

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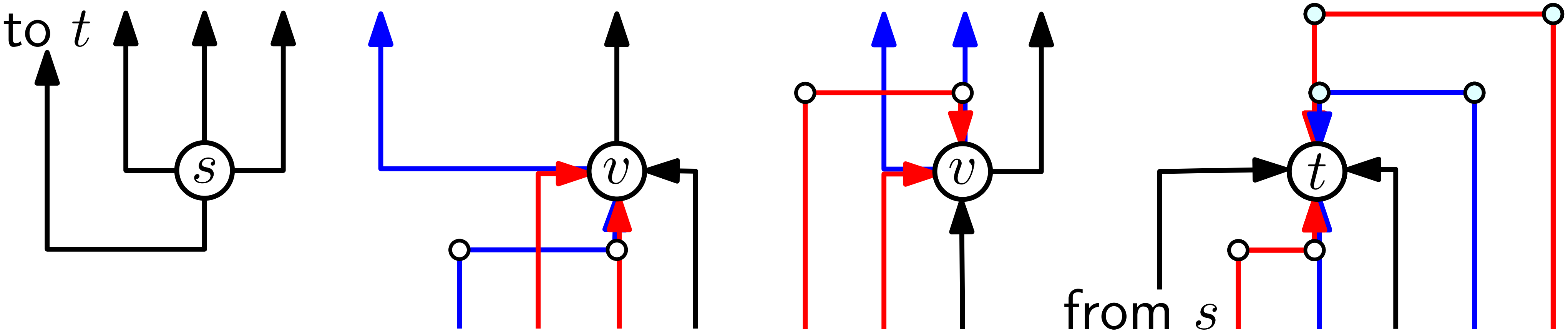
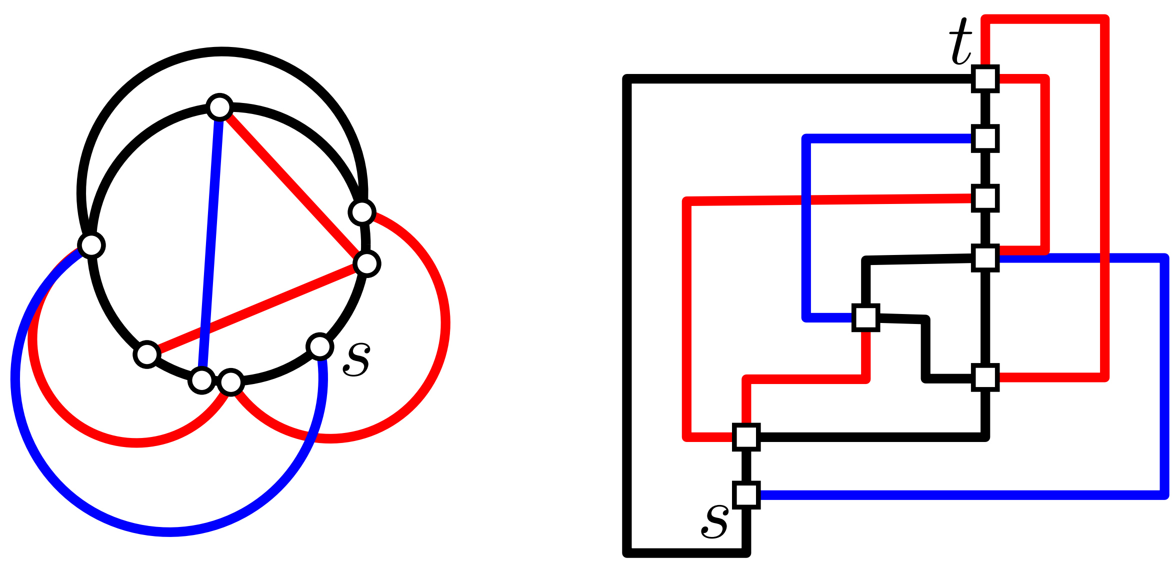
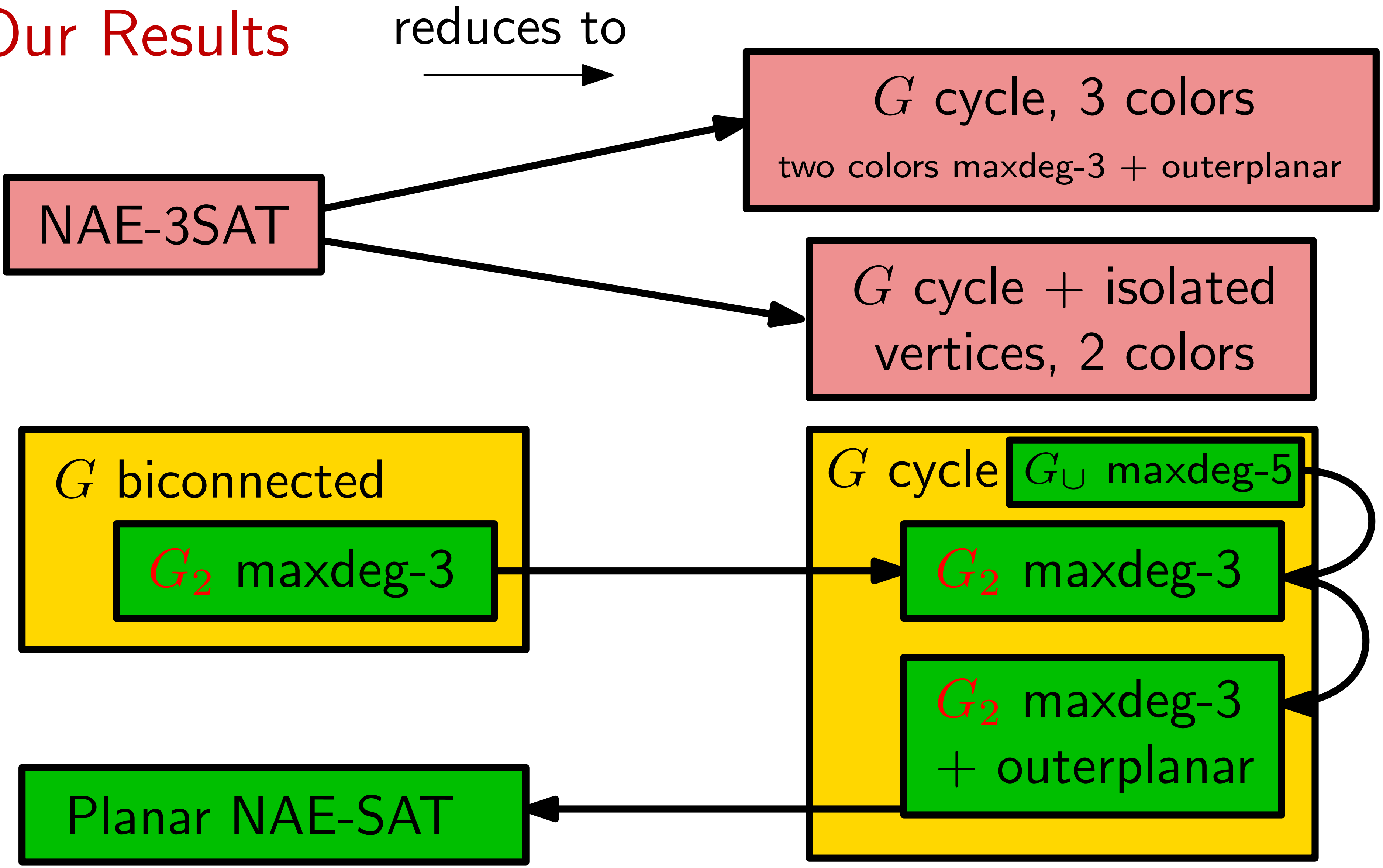




Our Results



G biconnected $\Rightarrow$ can draw simultaneous orthogonal embedding with ≤ 3 bends per edge

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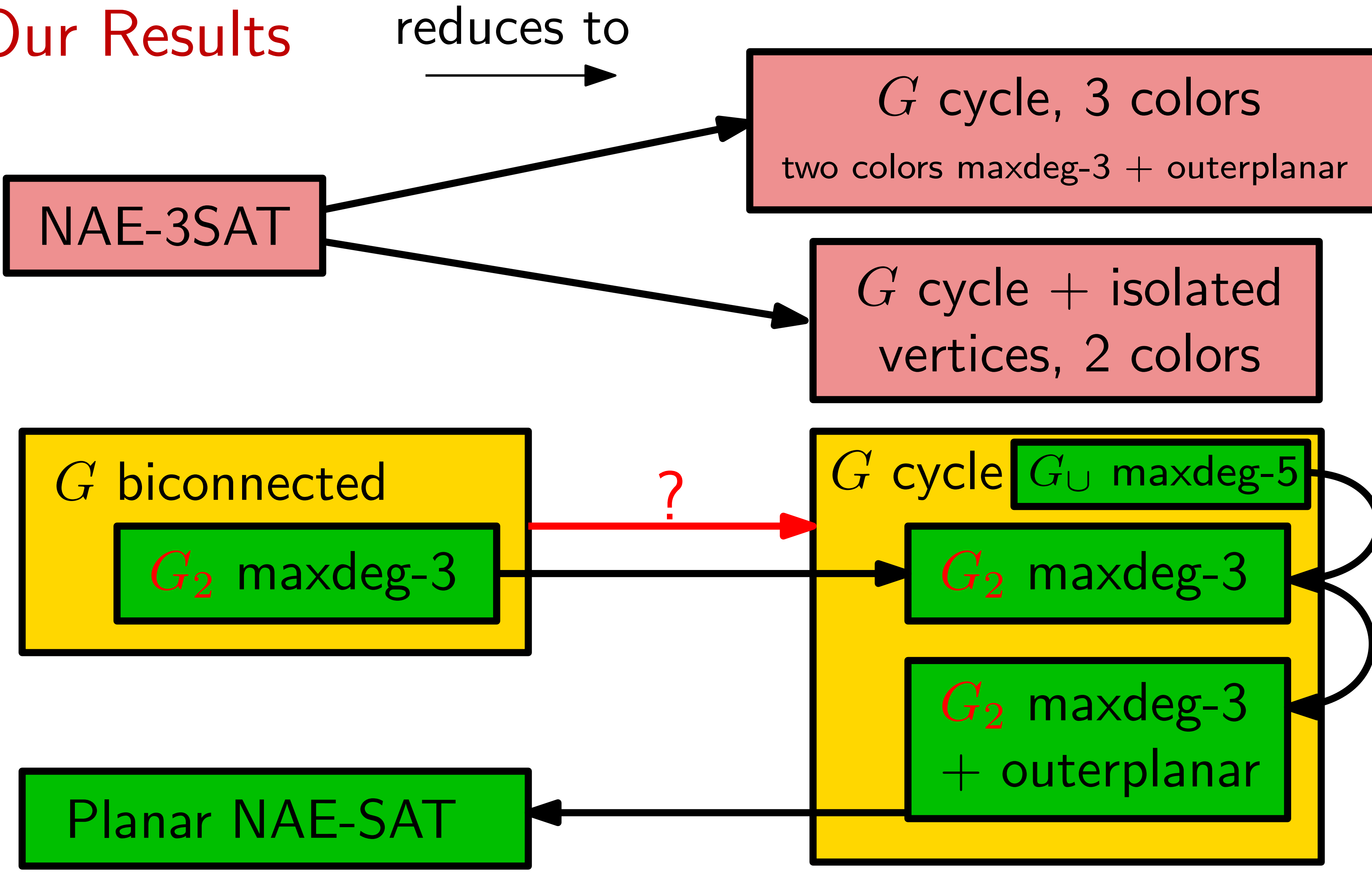

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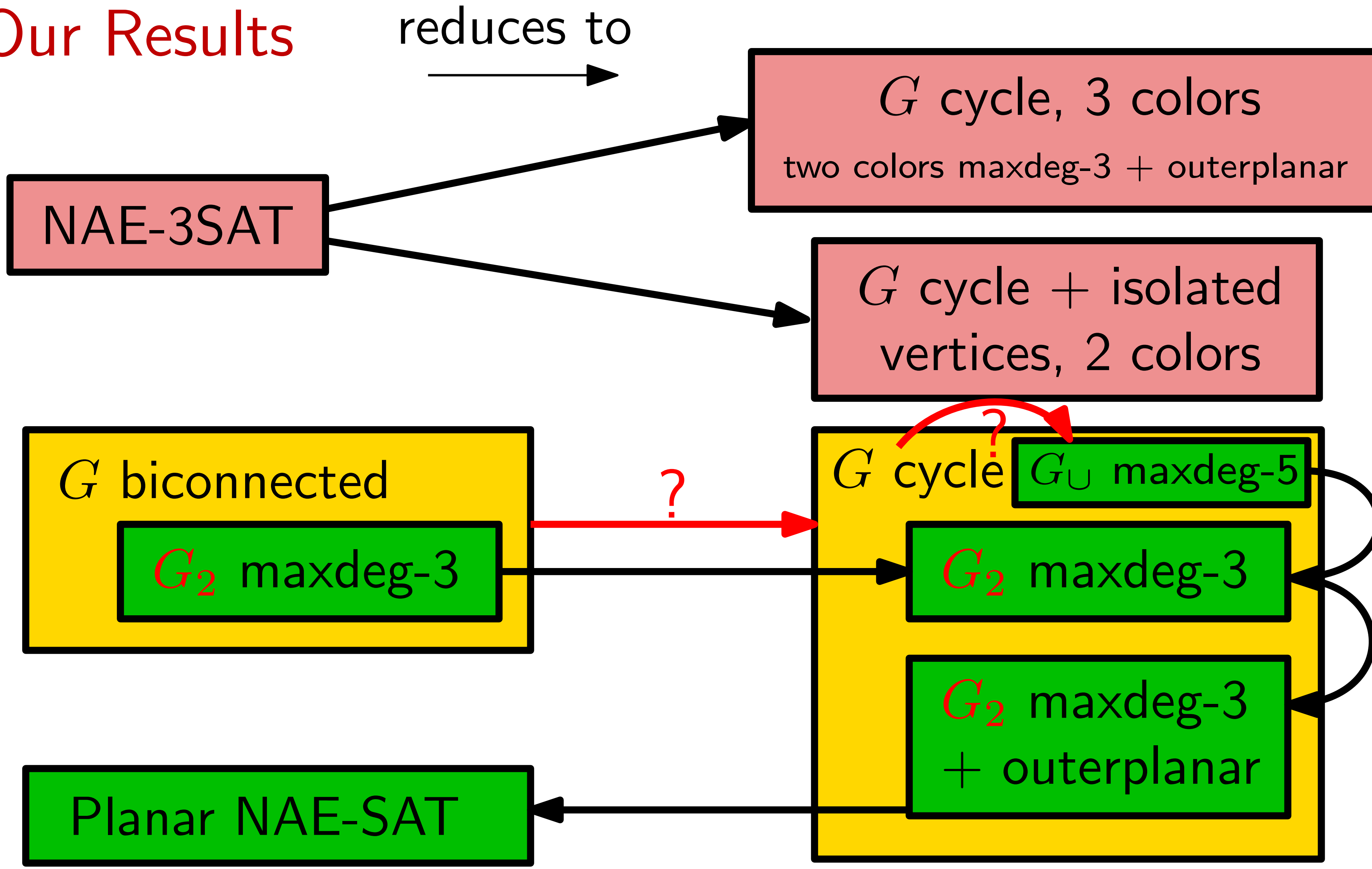

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