

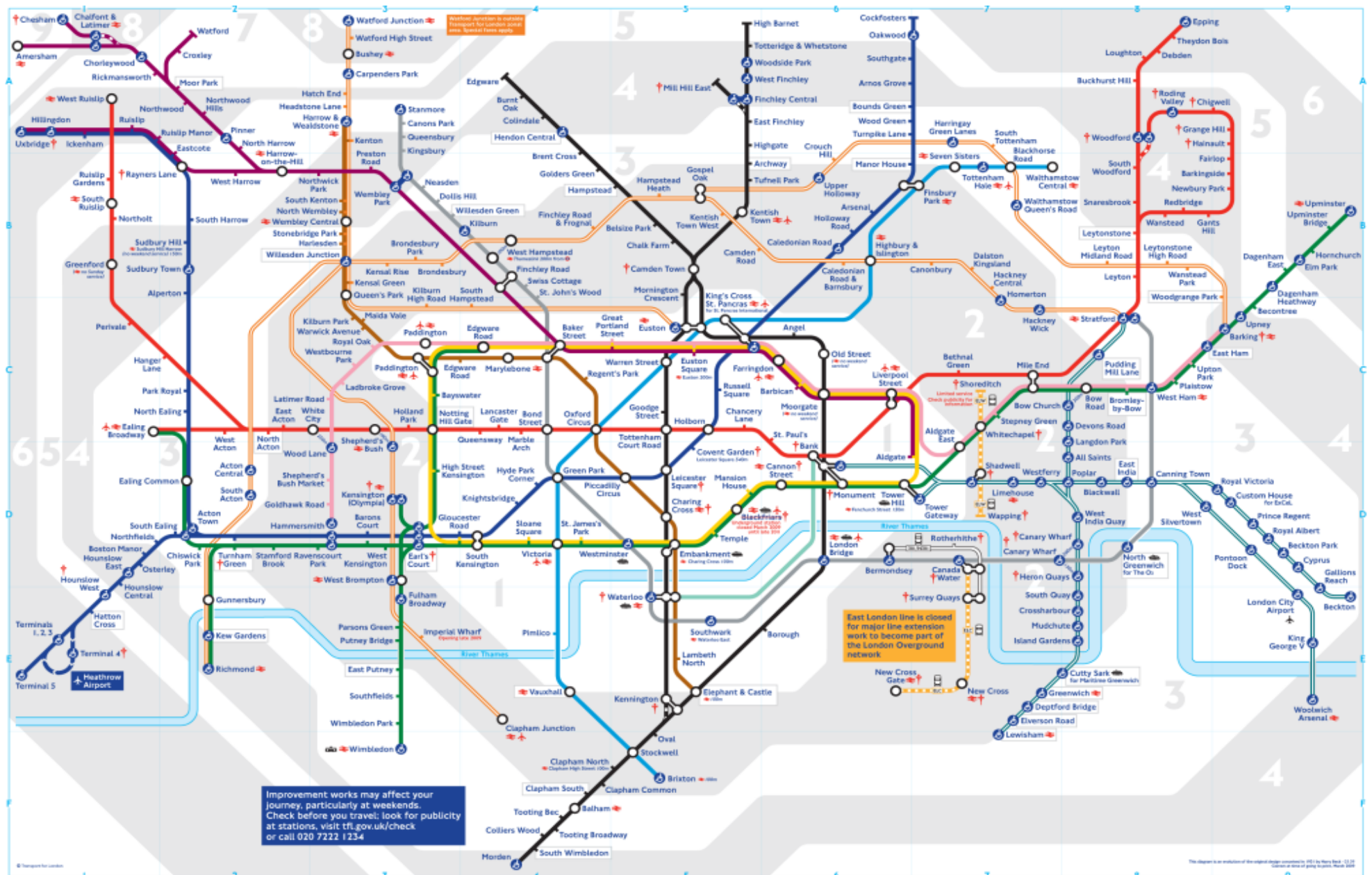
# Winkelschematisierung im Graphenzeichnen



Philipp Kindermann



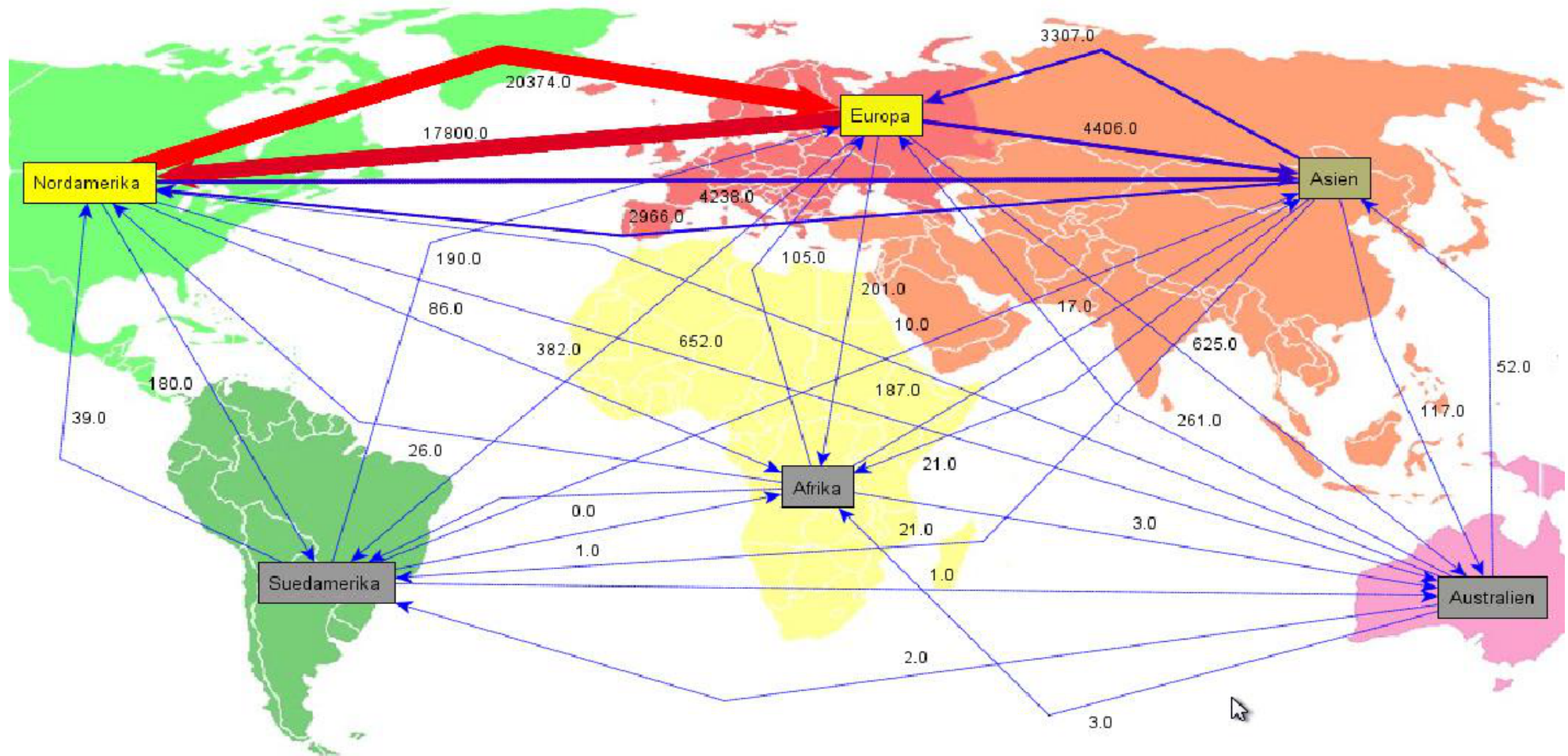
# Graphenzeichnen?



# Graphenzeichnen?

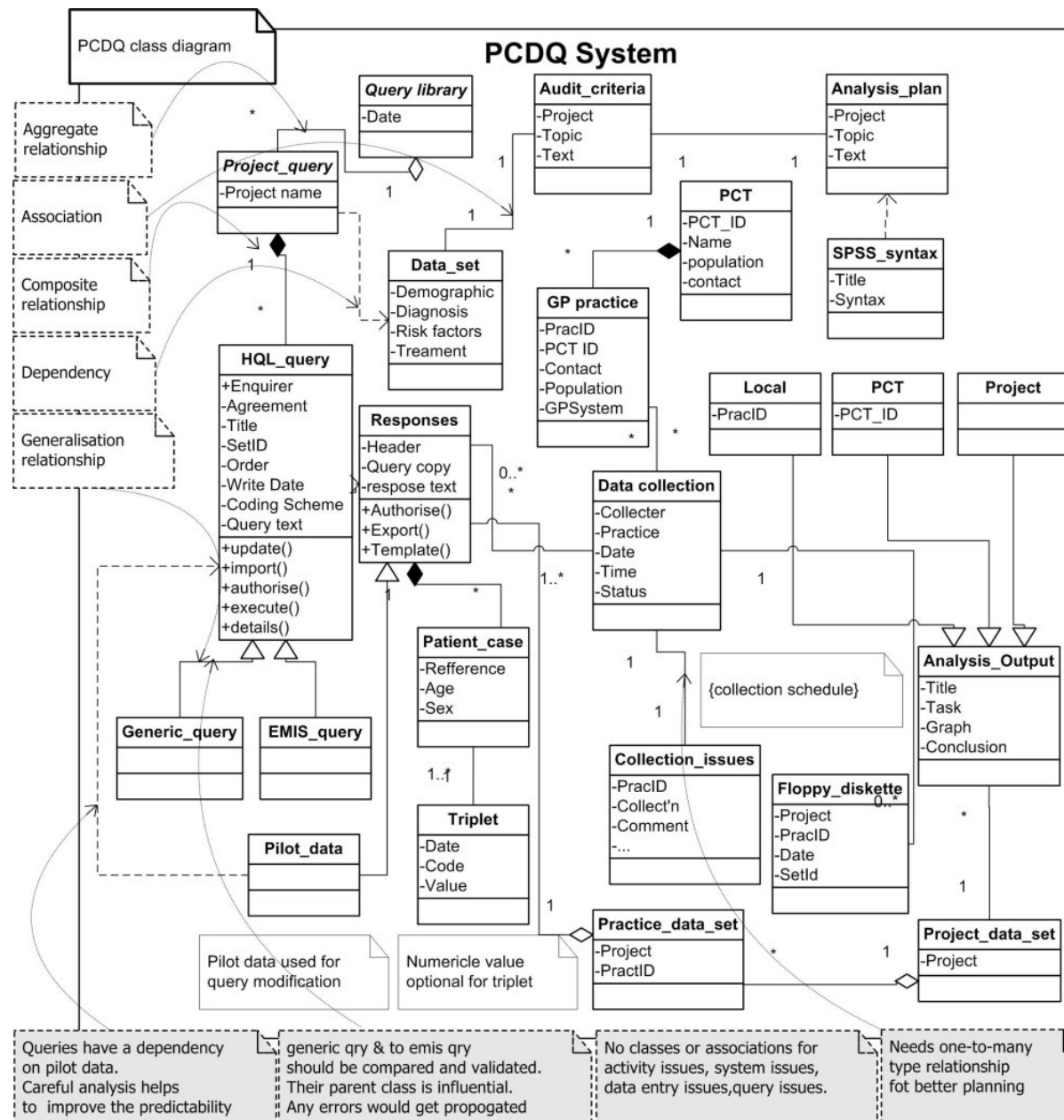
## Kontinentale Aufteilung

2001 - 2005

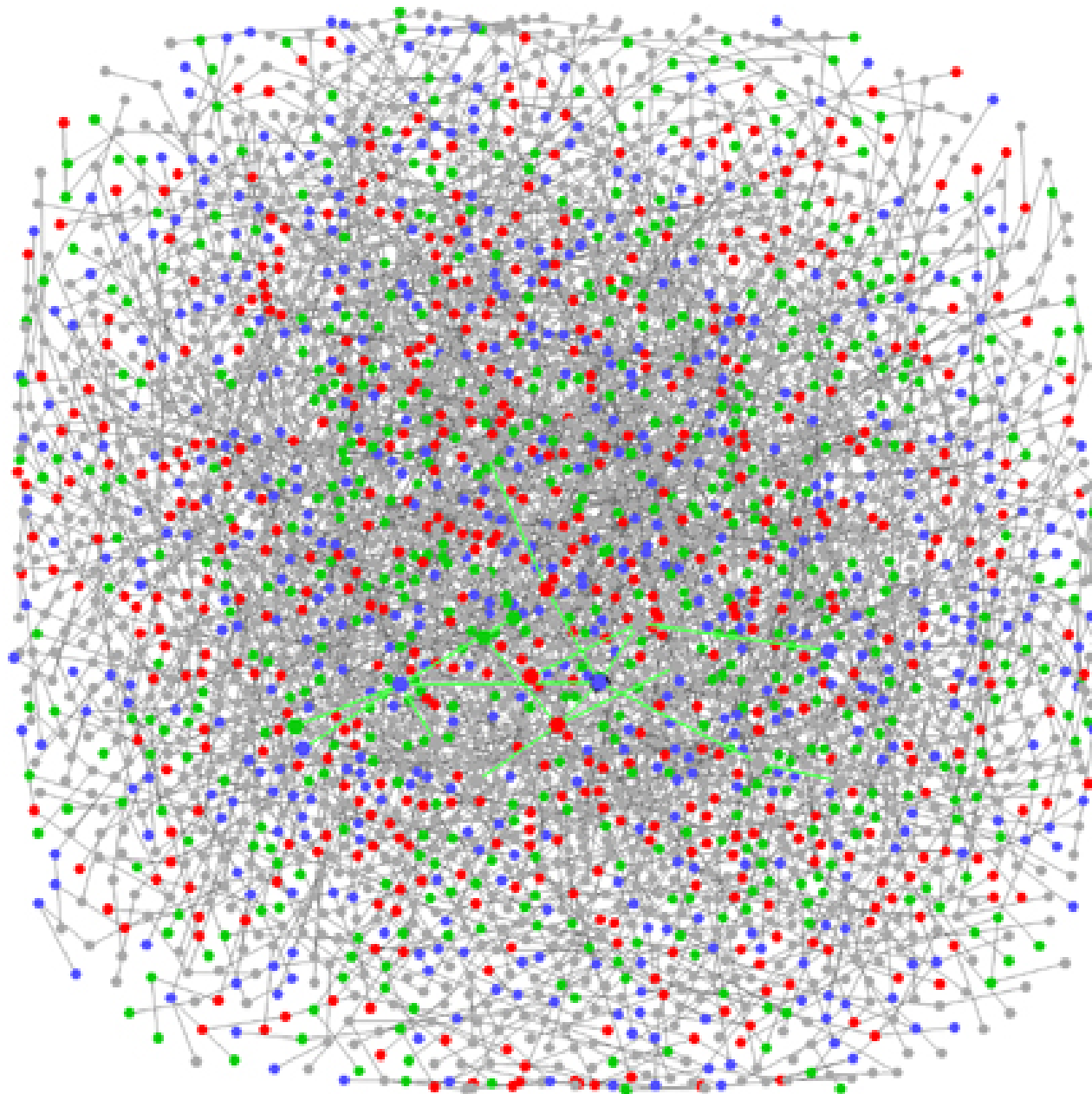




# Graphenzeichnen?



# Graphenzeichnen?



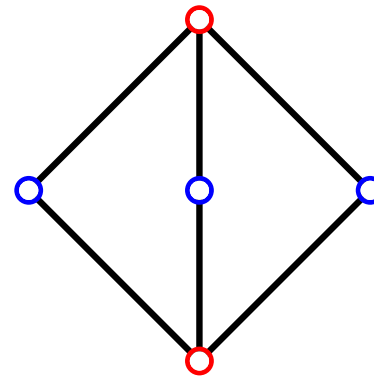
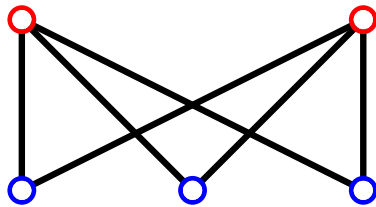
# Winkelschematisierung

Schöne Zeichnungen?

# Winkelschematisierung

Schöne Zeichnungen?

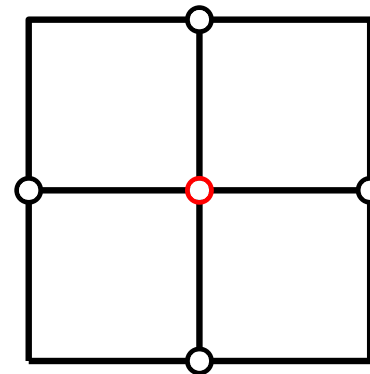
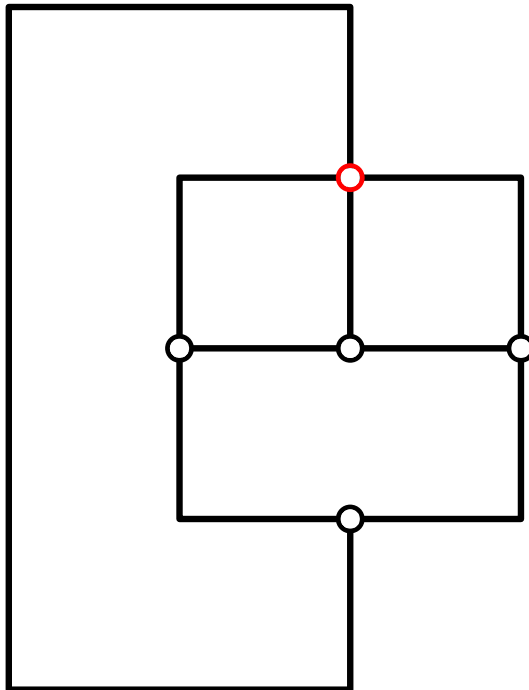
– Kreuzungsminimierung



# Winkelschematisierung

# Schöne Zeichnungen?

- Kreuzungsminimierung
- Knickminimierung

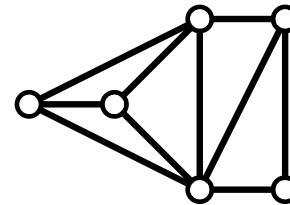
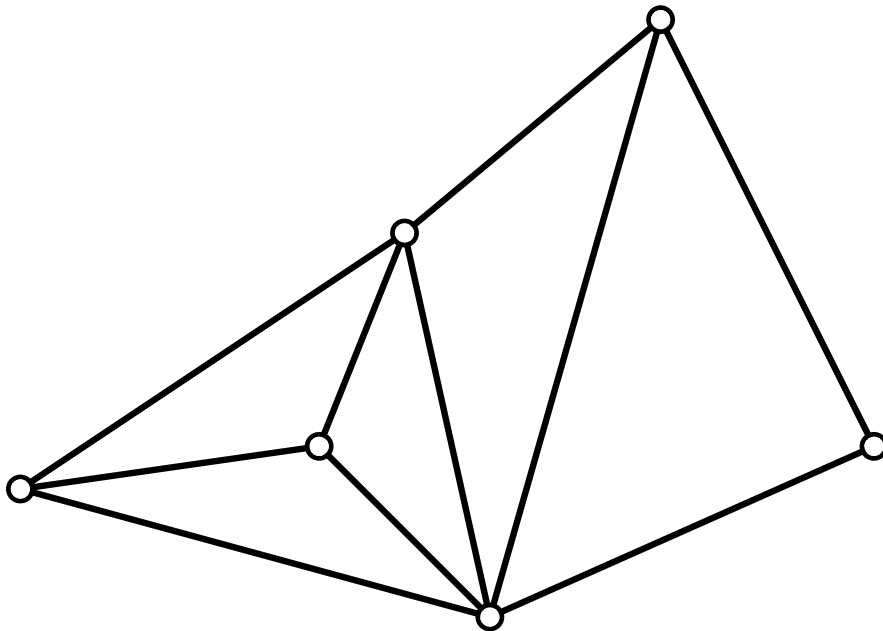




# Winkelschematisierung

Schöne Zeichnungen?

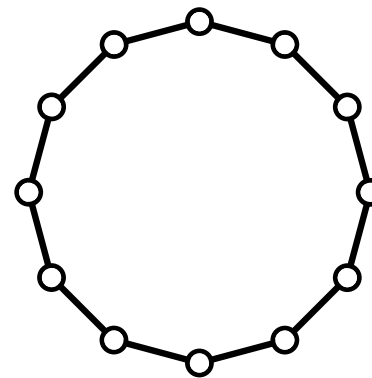
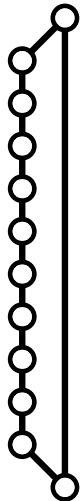
- Kreuzungsminimierung
- Knickminimierung
- minimale Fläche



# Winkelschematisierung

Schöne Zeichnungen?

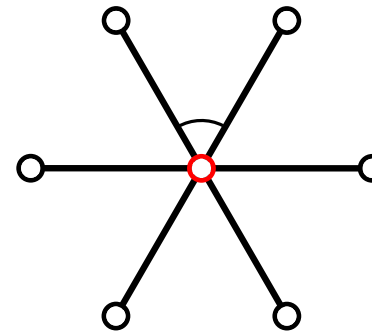
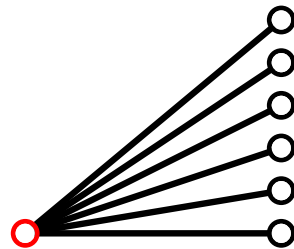
- Kreuzungsminimierung
- Knickminimierung
- minimale Fläche
- gleichmäßige Kantenlängen



# Winkelschematisierung

Schöne Zeichnungen?

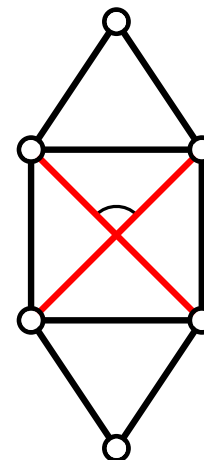
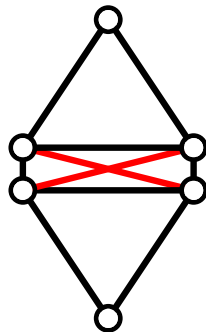
- Kreuzungsminimierung
- Knickminimierung
- minimale Fläche
- gleichmäßige Kantenlängen
- große Winkel
  - an Knoten



# Winkelschematisierung

Schöne Zeichnungen?

- Kreuzungsminimierung
- Knickminimierung
- minimale Fläche
- gleichmäßige Kantenlängen
- große Winkel
  - an Knoten
  - an Kreuzungen



# Gliederung

## I. Platzieren von Boxen

- 1) Mehrseitige Randbeschriftungen
- 2) Approximationsalgorithmen für Rechtecks-Kontakt-Repräsentationen

## II. Visuelle Führung

- 3) Glatt-orthogonale Darstellungen planarer Graphen
- 4) Monotone Zeichnungen von Bäumen

## III. Kreuzungen mit großen Winkeln

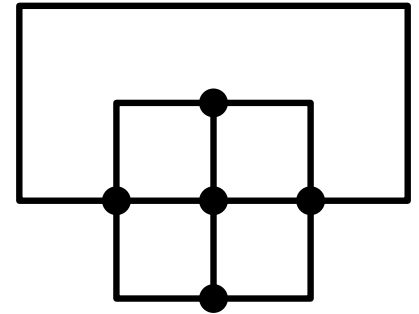
- 5) Gleichzeitiges Zeichnen planarer Graphen mit rechtwinkligen Kreuzungen und wenig Knicken
- 6) Erkennen und Zeichnen von Graphen mit unabhängigen Kreuzungen

# **Glatt-orthogonale Darstellungen planarer Graphen**



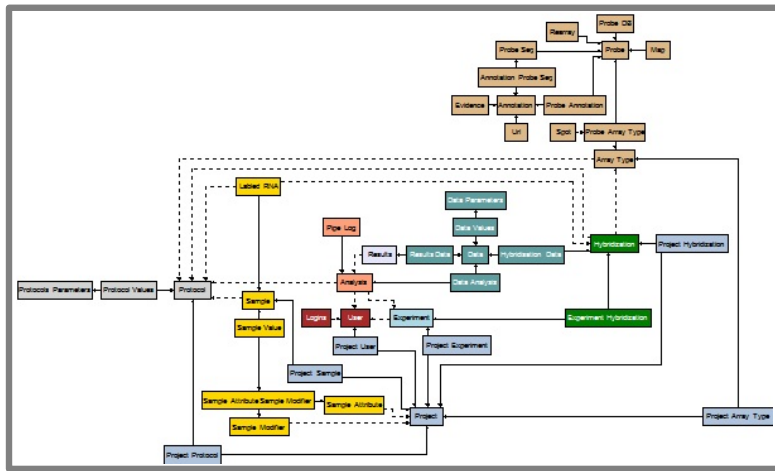
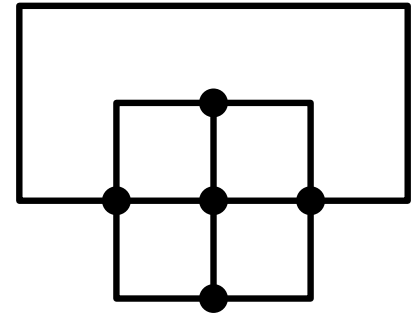
# Orthogonale Darstellungen

- Alle Segmente sind horizontal oder vertikal
- Intensiv untersuchter Zeichenstil
- Viele Anwendungsbeispiele

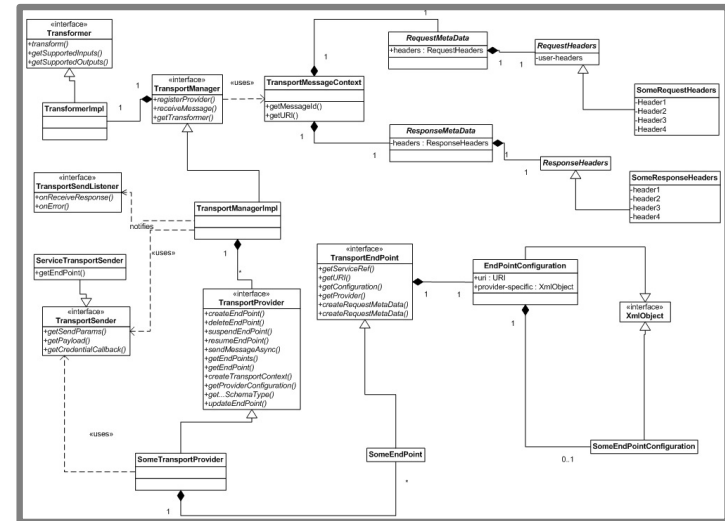


# Orthogonale Darstellungen

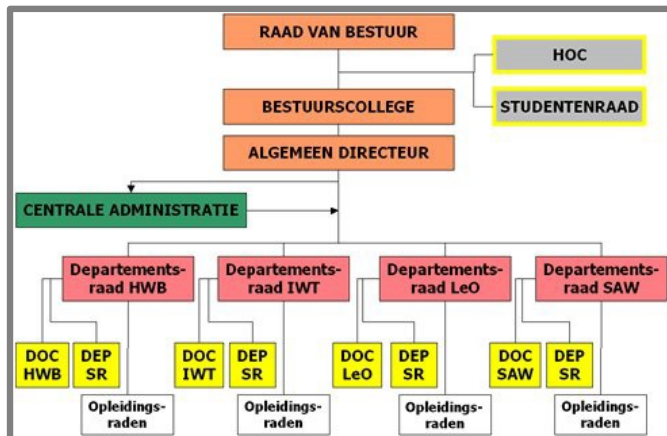
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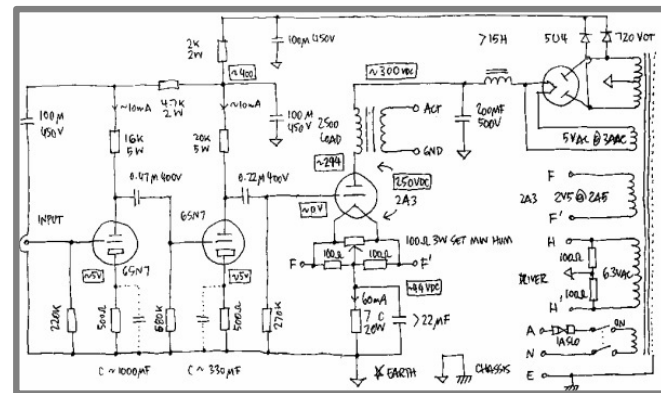
## ER-Diagramm in OGDF



## UML-Diagramm von Oracle



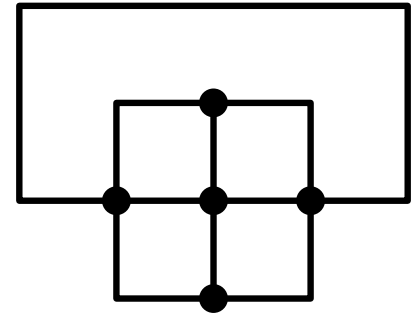
## Organigramm von HS Limburg



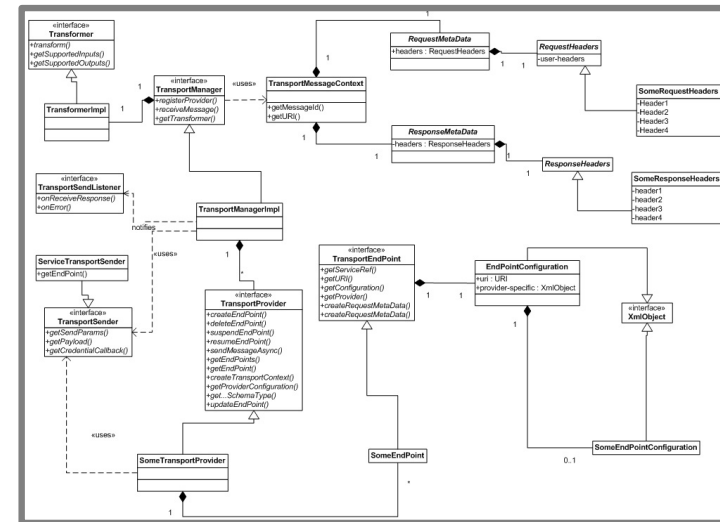
## Schaltkreis-Diagramm von Jeff Atwood

# Orthogonale Darstellungen

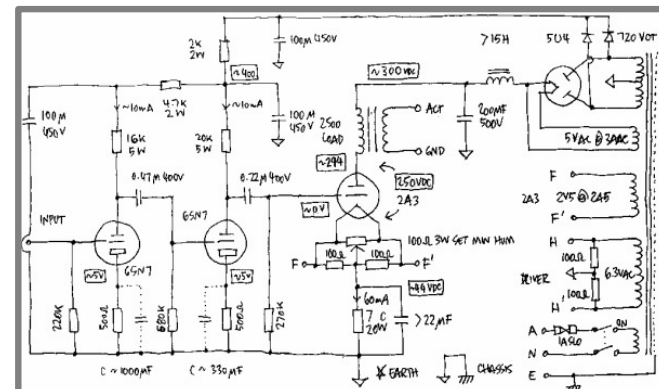
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“Fused Grid” Stadtentwurf



UML-Diagramm von Oracle

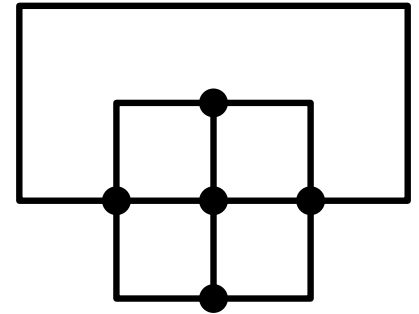


Schaltkreis-Diagramm von Jeff Atwood

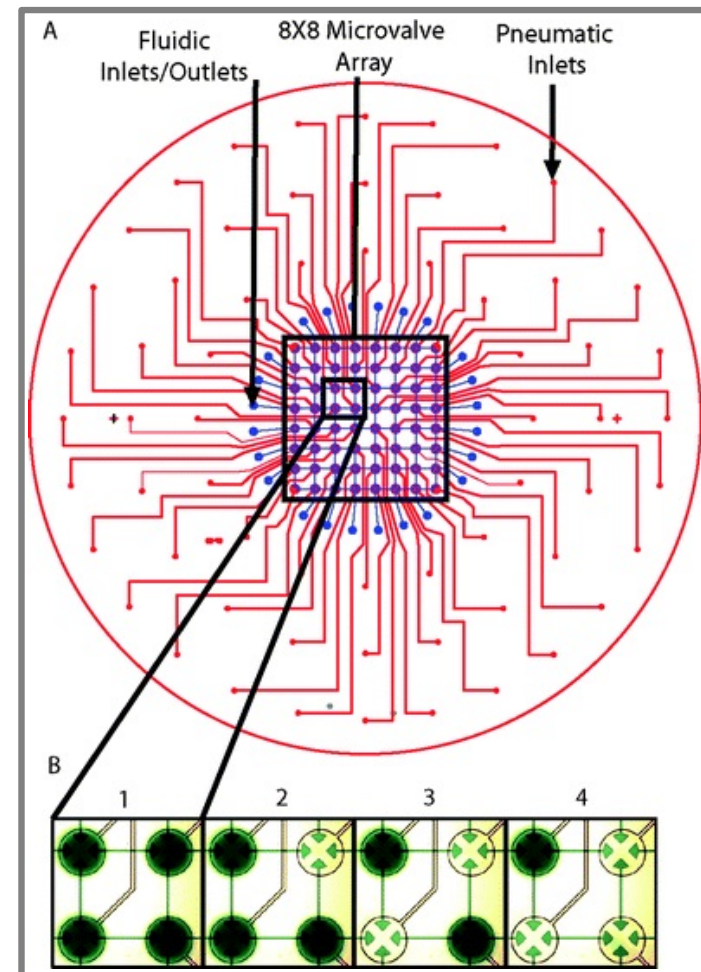


# Orthogonale Darstellungen

- Alle Segmente sind horizontal oder vertikal
- Intensiv untersuchter Zeichenstil
- Viele Anwendungsbeispiele



“Fused Grid” Stadtentwurf



VLSI/PCI Chip Entwurf

# Glatte Zeichnungen



Mercedes W 114, 1967



Nissan Friend-ME, 2013



# Glatte Zeichnungen

## FOLLOW THE MONEY

## The New Global Wealth Machine

Sovereign wealth funds have emerged in recent months as the world's power brokers. They have used their tremendous wealth to make big cross-border investments and prop up some of Wall Street's best-known firms. The increased activity comes as other kinds of acquirers have been sidelined by the credit crisis. These funds are state-sponsored investment vehicles and have combined assets of \$2 trillion. With that much dry powder, sovereign funds dwarf the formerly booming private equity industry — and in some cases, compete directly with it. The Government of Singapore Investment Corporation has been the most active among the world's sovereign funds, making its deputy chairman, Tony Tan, a major center of gravity. Wall Street veterans always follow the money, so many of the big-name advisers in New York and London have found themselves traveling the globe playing international matchmaker to these funds. But sovereign funds have also learned the downside of deal-making: some of their blockbuster transactions have been big money losers so far. The question is where all that money will go next. **ANDREW ROSS SORKIN**

### The Advisers

Selected financial advisers who worked on more than one of the top 20 deals.

#### CITIGROUP

**Michael Klein, Chairman, institutional clients group**  
One of the firm's highest-profile investment bankers, he advised Carlyle in its stake sale to Mubadala, as well as Citigroup in both of its deals with sovereign wealth funds.

#### GOLDMAN SACHS GROUP

**Richard Ong, Former managing director**  
Mr. Ong left Goldman early this year after the Chinese government refused to allow the firm to promote him to run its Beijing office. Mr. Ong's brother, Charles, was the chief investment officer of Temasek Holdings until 2006.

#### LAZARD

**Gary Parr, Deputy chairman**  
In addition to becoming the key adviser on many of the biggest sovereign wealth deals, Mr. Parr helped advise Bear Stearns on its distressed sale to JPMorgan Chase.

#### MORGAN STANLEY

**Kate Richdale, Managing director**  
The head of Morgan Stanley's Asian general industries group, based in Hong Kong. She previously held a senior position in the investment bank's Southeast Asia group.

### The Targets

**MORGAN STANLEY**  
John J. Mack, Chairman and C.E.O.

**BLACKSTONE GROUP**  
Stephen A. Schwarzman, Chairman and co-founder

**CITIGROUP**  
Robert E. Rubin, Chairman

**MERRILL LYNCH**  
John A. Thain, Chairman and C.E.O.

**STANDARD CHARTERED BANK**  
Peter Sands, Chief executive

**UBS**  
Marcel Rohner, Chief executive

**Qatar Investment Authority**  
Kenneth Shen, Head of strategic and private equity

**Mubadala Development Co.**  
Khaldoun Khalifa al-Mubarak, C.E.O. and managing director

**Abu Dhabi Investment Authority**  
Sheik Khalifa bin Zayed al-Nahyan, Chairman

**Saudi Arabian Monetary Agency**

**Kuwait Investment Authority**

**China Investment Corp.**  
Lou Jiwei, Chairman

**China Construction Bank**  
Chang Zhenming, (then president)

**BANK OF CHINA**  
Li Lihui, President and vice chairman

**HONGKONG & KOWLOON**  
Eric Ip, Managing director

**SHIN CORP.**  
Bonnie Pang, Director and chairman of the group executive committee

**Govt. of Singapore Investment Corp.**  
Tony Tan, Deputy chairman and executive director

**Temasek Holdings**  
Ho Ching, Executive director and C.E.O.

**JAPAN BANKING CORP.**  
Toshio Masui (then president)

**SHEARMAN & STERLING**

**SULLIVAN & CROMWELL**

**H. Rodgin Cohen, Chairman**

**Richard Good, Partner**

**DAVIS POLK & WARDWELL**

**CLIFFORD CHANCE**

**The Lawyers**

**The 20 Biggest Cross-Border Sovereign Wealth Fund Deals Since 2005**

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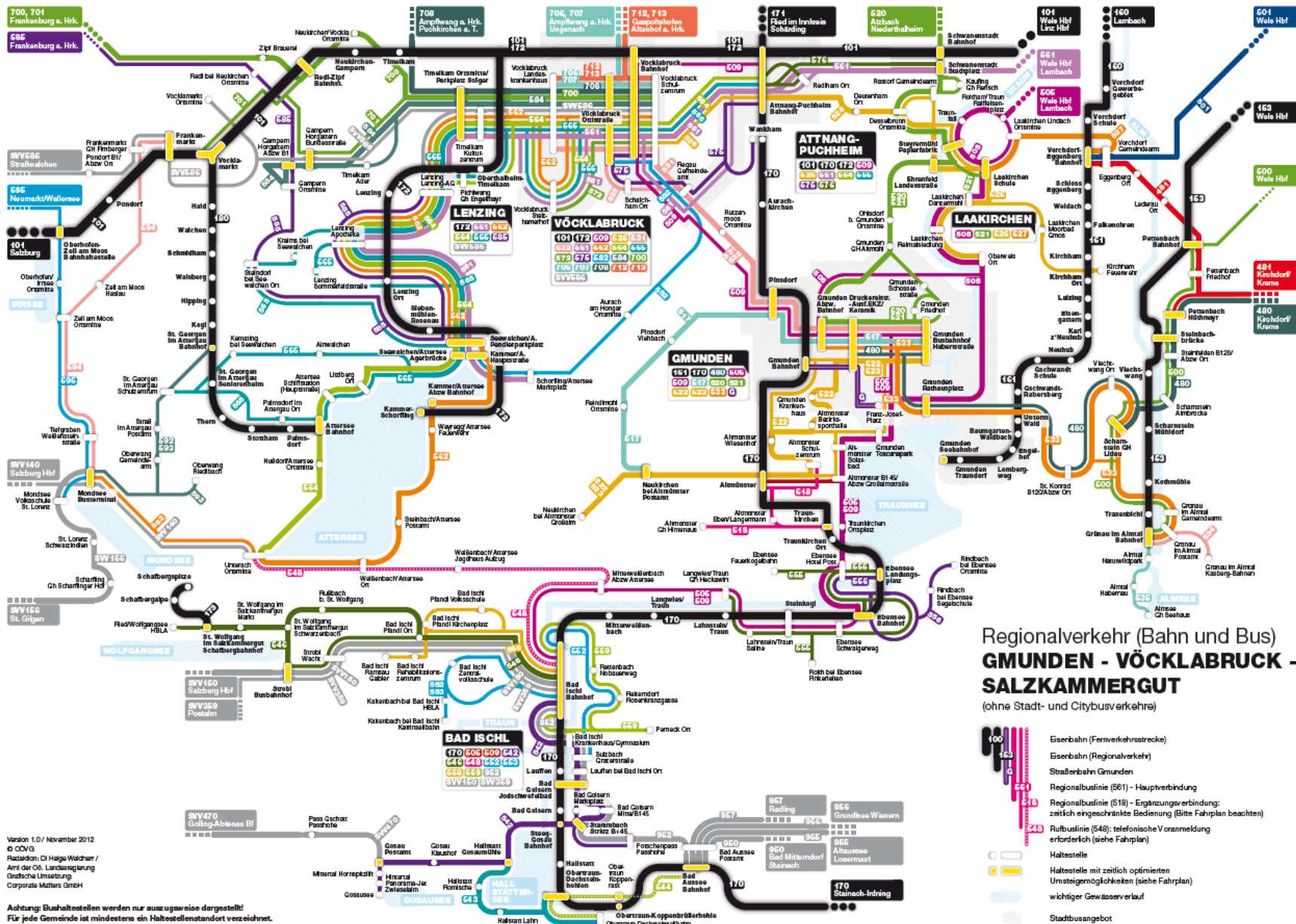


# Glatte Zeichnungen



Städtemodell

# Glatte Zeichnungen

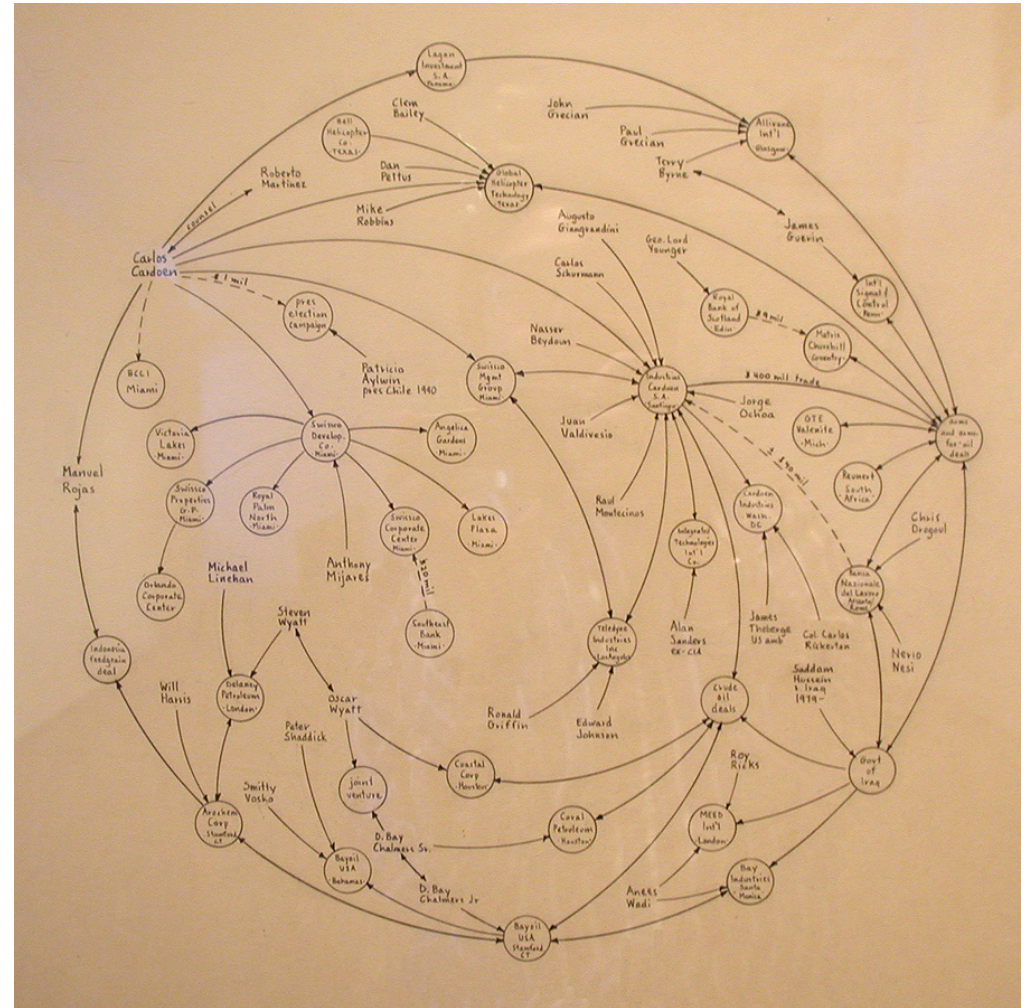


**Achtung:** Bushaltestellen werden nur auszugsweise dargestellt!  
Für jede Gemeinde ist mindestens ein Haltestellenstandort verzeichnet.

# Glatte Zeichnungen

## Lombardi-Zeichnungen

- Kreissegmente
- Perfekte Winkelauflösung



Mark Lombardi  
(1951–2000)



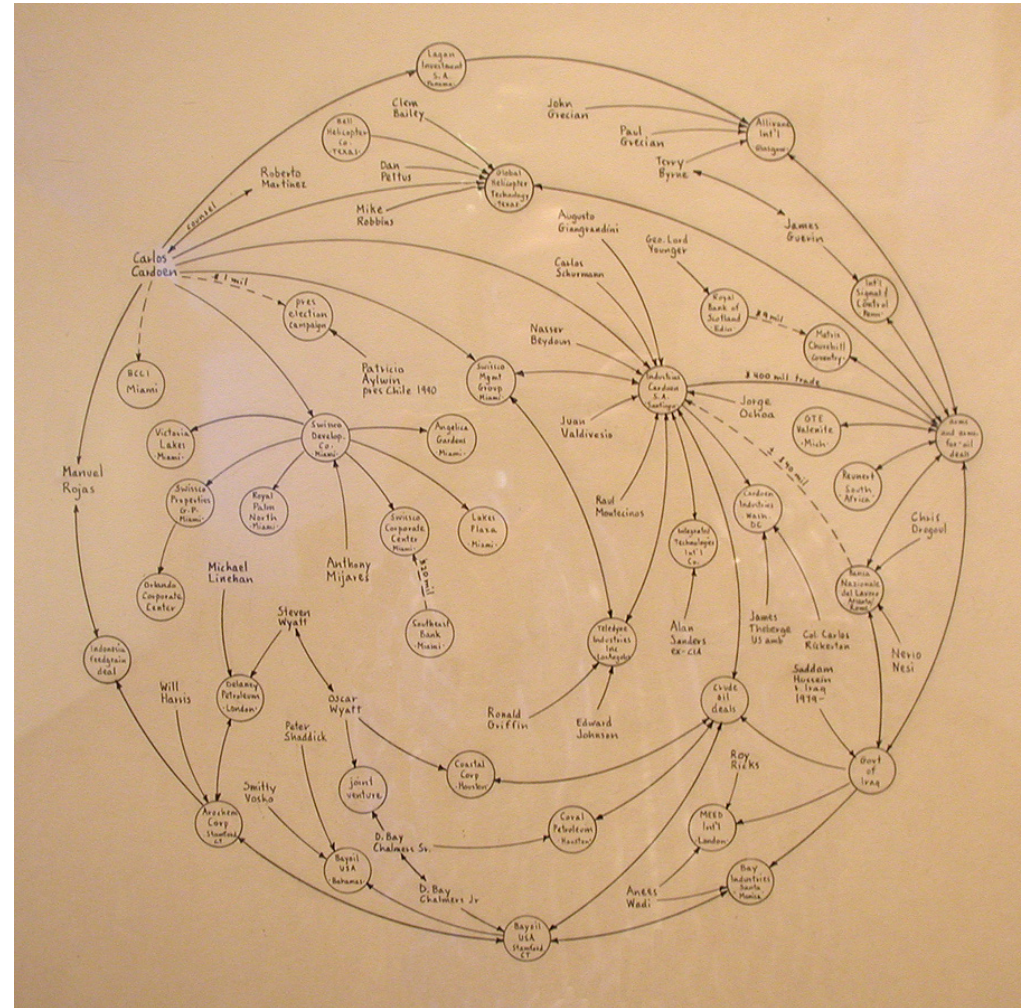
# Glatte Zeichnungen

## Lombardi-Zeichnungen

- Kreissegmente
- Perfekte Winkelauflösung

## $k$ -Lombardi-Zeichnungen

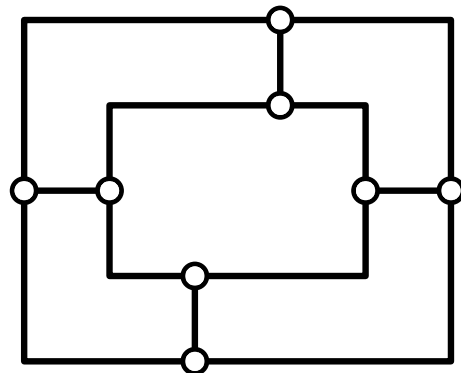
- Kanten bestehen aus  $k$  Kreissegmenten



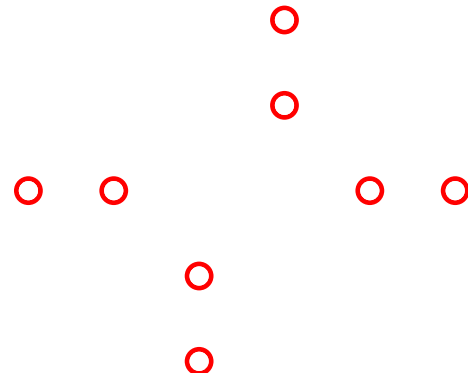
Mark Lombardi  
(1951–2000)

# Glatt-orthogonale Darstellungen

Vereinige beide Welten:



orthogonal

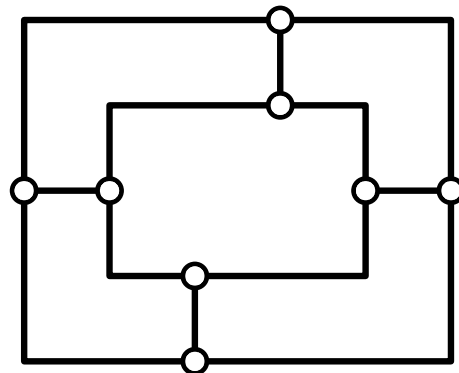


glatt-orthogonal

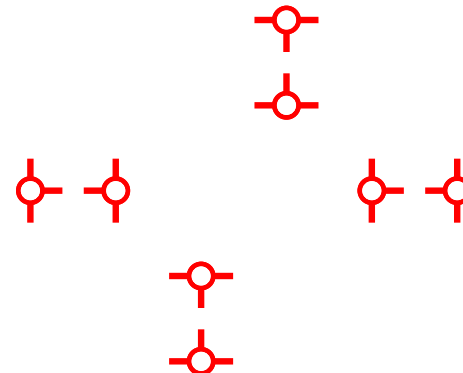
# Glatt-orthogonale Darstellungen

Vereinige beide Welten:

- Kanten treffen horizontal oder vertikal auf Knoten



orthogonal



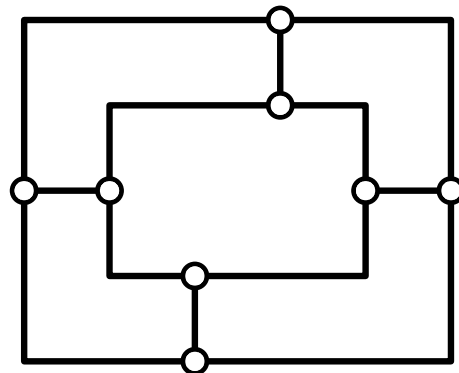
glatt-orthogonal



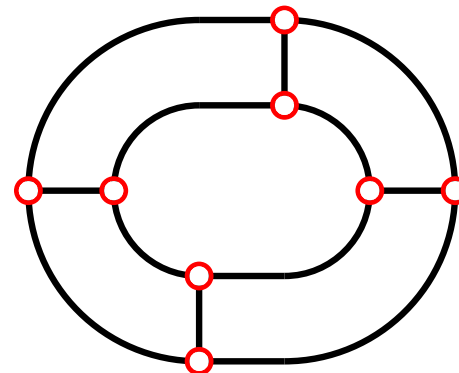
# Glatt-orthogonale Darstellungen

Vereinige beide Welten:

- Kanten treffen horizontal oder vertikal auf Knoten
- Kanten bestehen aus achsen-parallelen Segmenten und Kreissegmenten ohne Knicke



orthogonal

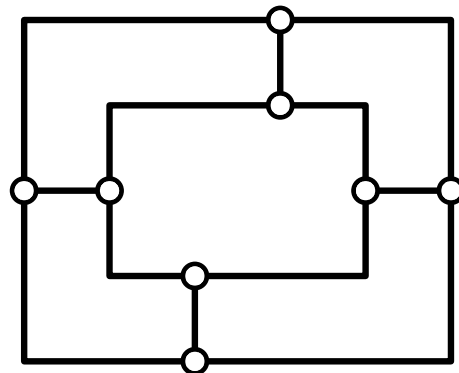


glatt-orthogonal

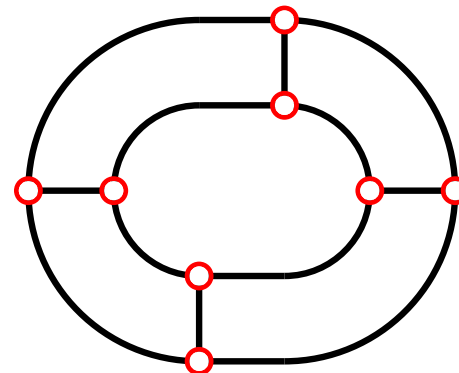
# Glatt-orthogonale Darstellungen

Vereinige beide Welten:

- Kanten treffen horizontal oder vertikal auf Knoten
- Kanten bestehen aus achsen-parallelen Segmenten und Kreissegmenten ohne Knicke
- Es gibt keine Kantenkreuzungen (für planare Graphen)

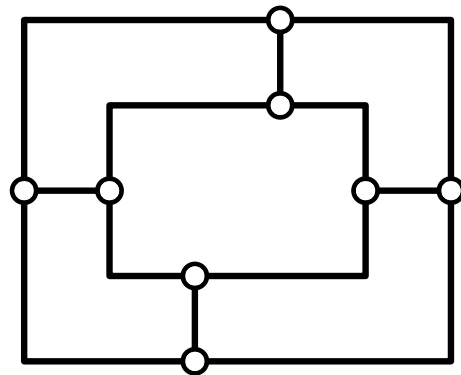


orthogonal

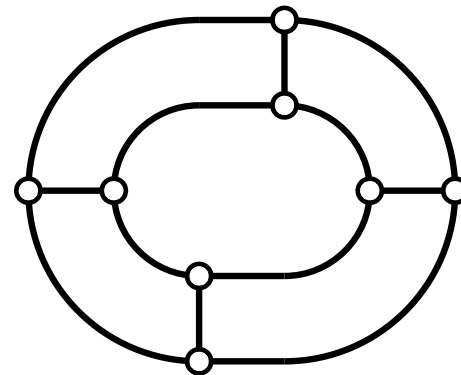


glatt-orthogonal

# Kantenkomplexität



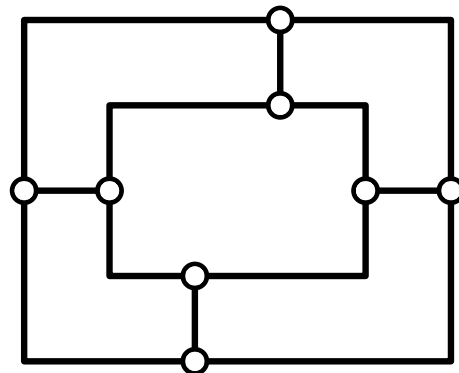
orthogonal



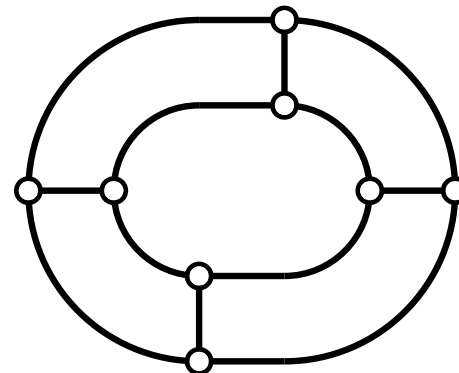
glatt-orthogonal

# Kantenkomplexität

*Komplexität einer Kante:* Anzahl der Segmente



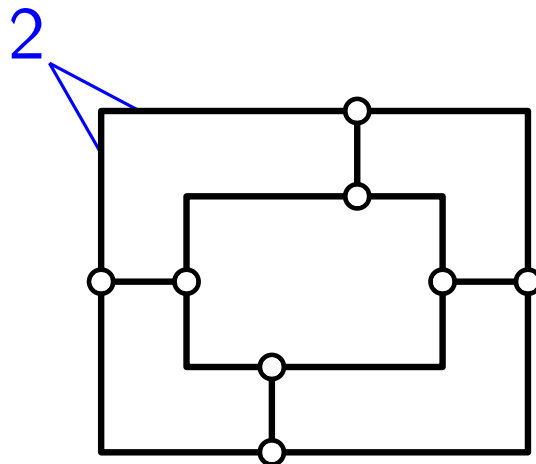
orthogonal



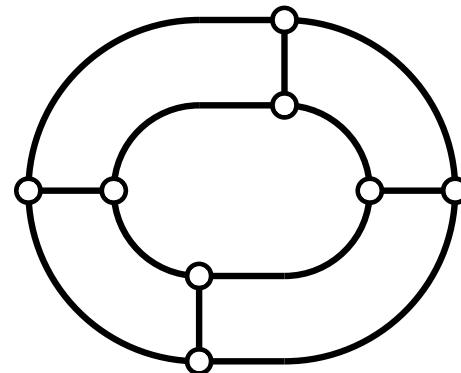
glatt-orthogonal

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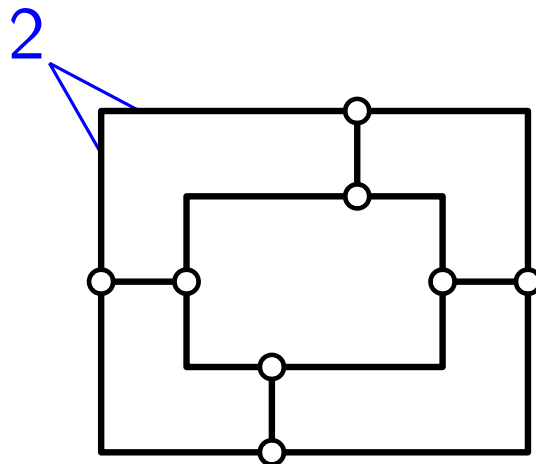
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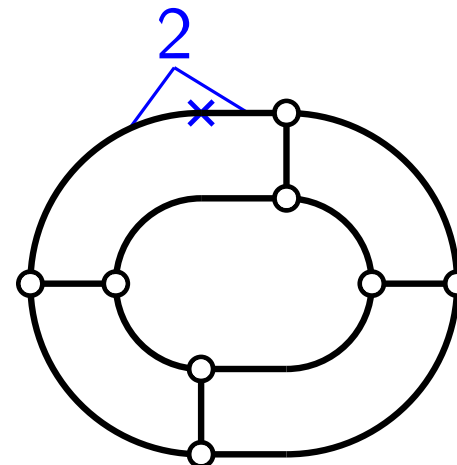
glatt-orthogonal

# Kantenkomplexität

*Komplexität einer Kante: Anzahl der Segmente*



orthogonal



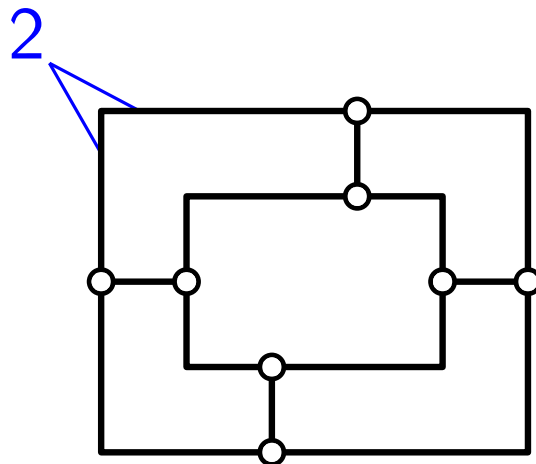
glatt-orthogonal



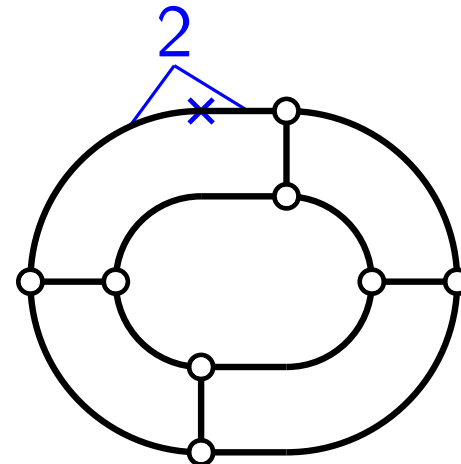
# Kantenkomplexität

*Komplexität einer Kante:* Anzahl der Segmente

*Komplexität einer Zeichnung:* Größte Komplexität aller Kanten



orthogonal

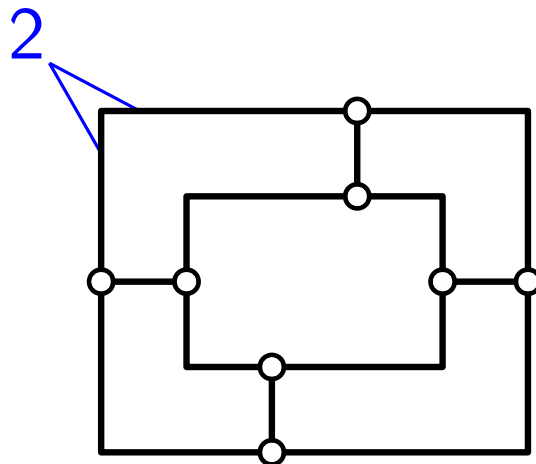
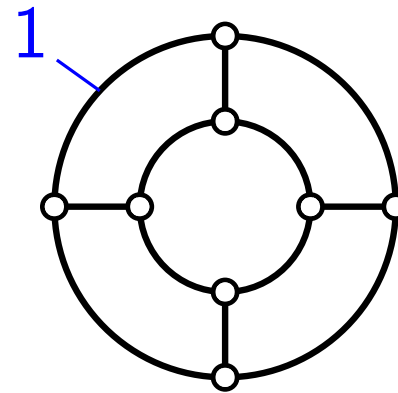
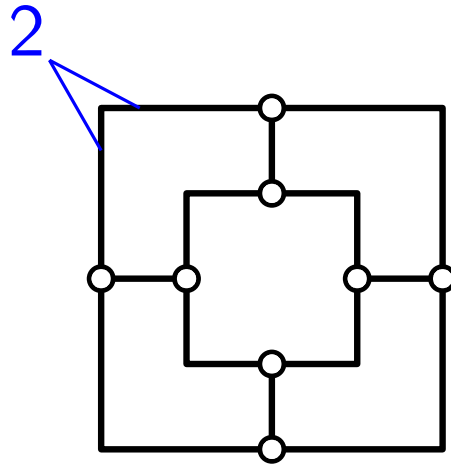


glatt-orthogonal

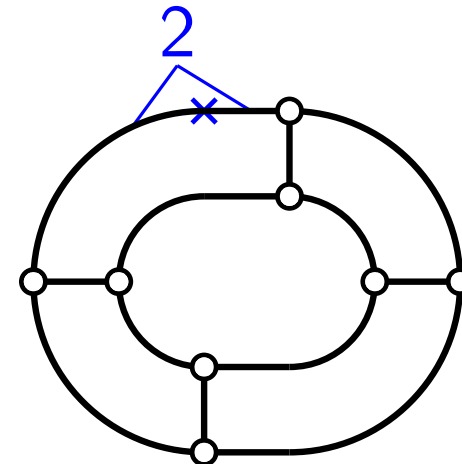
# Kantenkomplexität

*Komplexität einer Kante:* Anzahl der Segmente

*Komplexität einer Zeichnung:* Größte Komplexität aller Kanten



orthogonal



glatt-orthogonal

# Bekannte Resultate

Biedl & Kant [CGTA'98], Liu et al. [DAM'98]  
4-planarer Graph  $\Rightarrow$  orth. Komplexität 3,  $(n \times n)$ -Gitter

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Bekos et al. [GD'12]  
• Zweifach zusammenhängender 4-planarer Graph  $\Rightarrow$   
glatt-orth. Komplexität 3

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- 4-planarer Graph  $\not\Rightarrow$  glatt-orth. Komplexität 1

# Bekannte Resultate

Biedl & Kant [CGTA'98],

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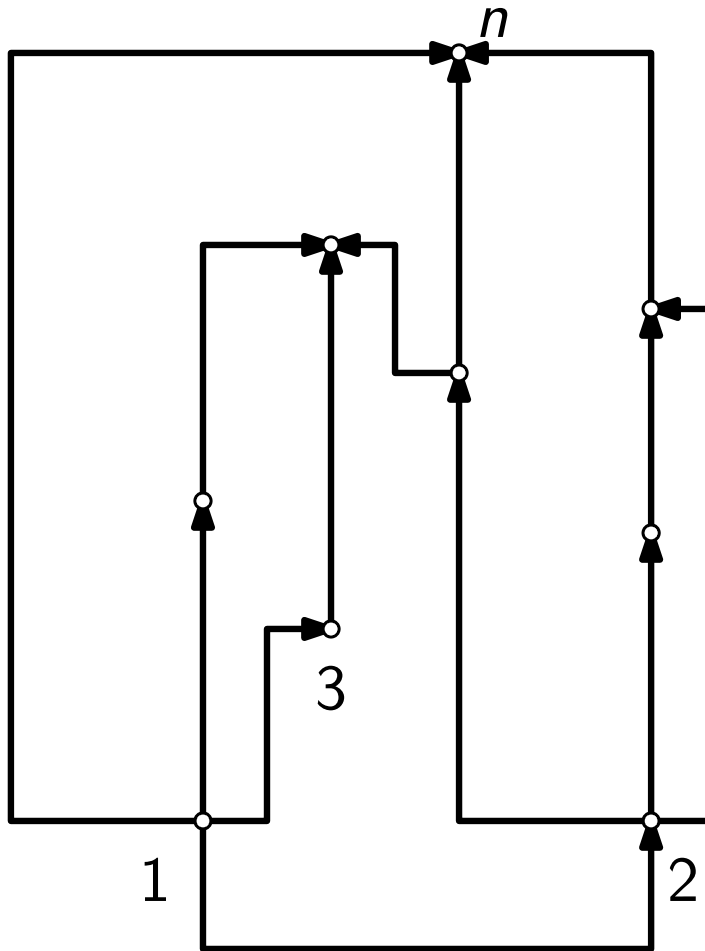
4-planarer Graph  $\Rightarrow$  orth. Komplexität 3,  $(n \times n)$ -Gitter

Bekos et al.

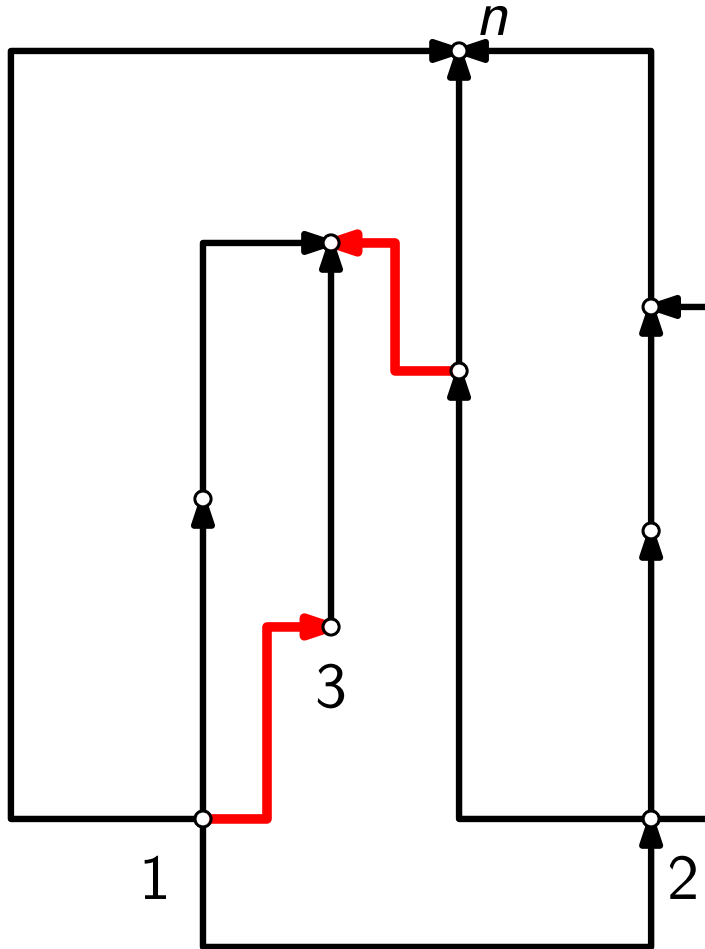
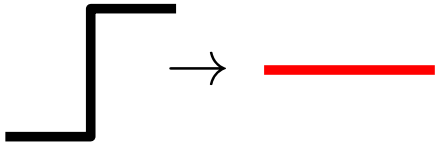
[GD'12]

- Zweifach zusammenhängender 4-planarer Graph  $\Rightarrow$  glatt-orth. Komplexität 3
- 4-planarer Graph  $\not\Rightarrow$  glatt-orth. Komplexität 1

# Komplexität-2 Zeichnungen

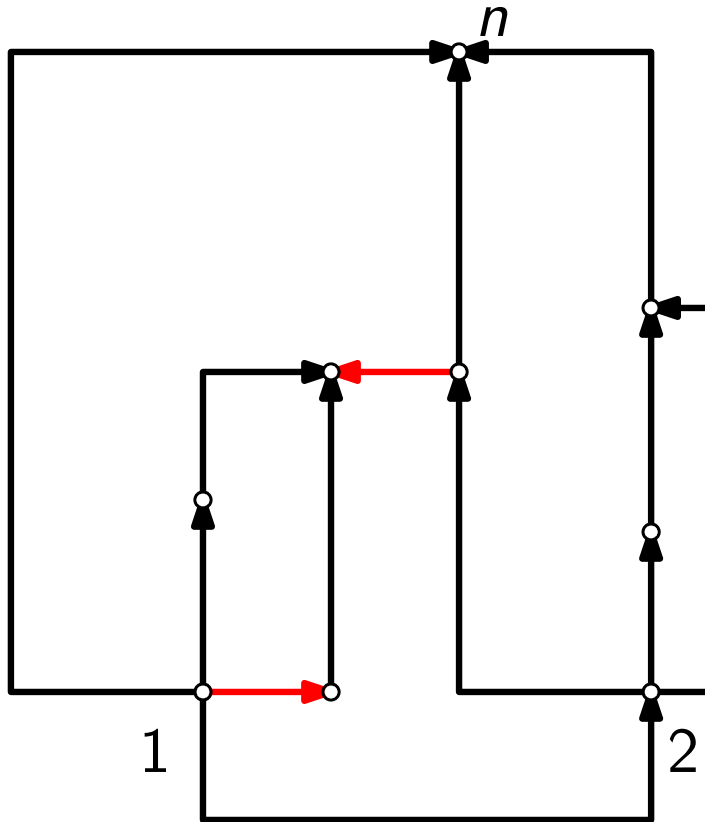
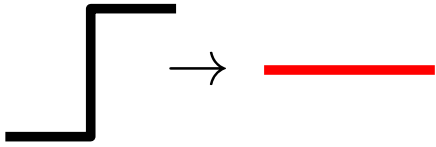


# Komplexität-2 Zeichnungen

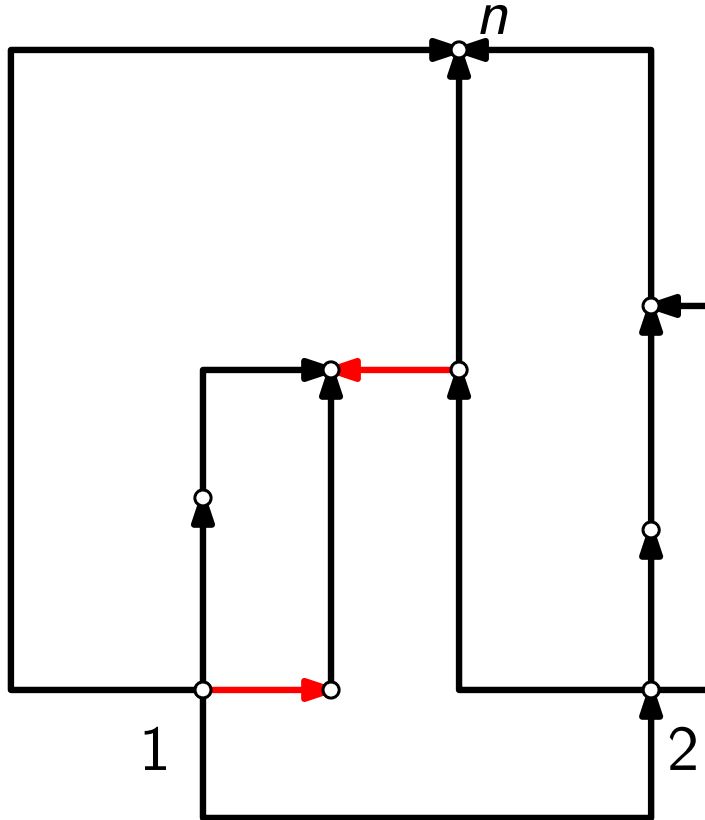
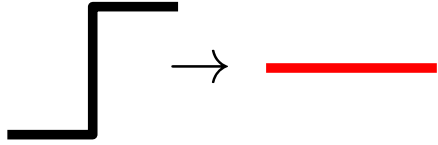
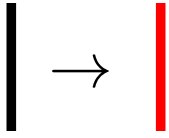




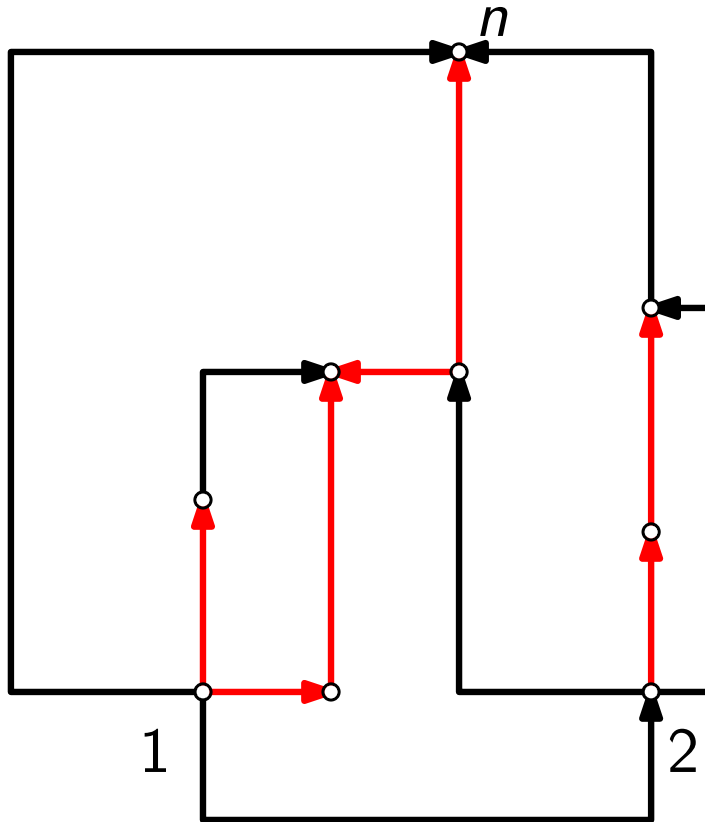
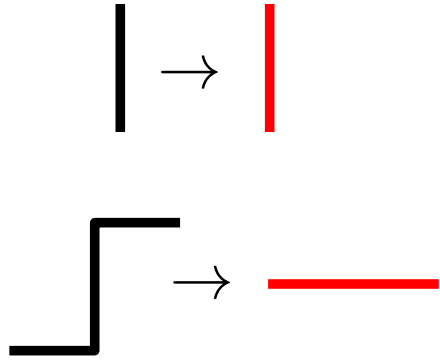
# Komplexität-2 Zeichnungen



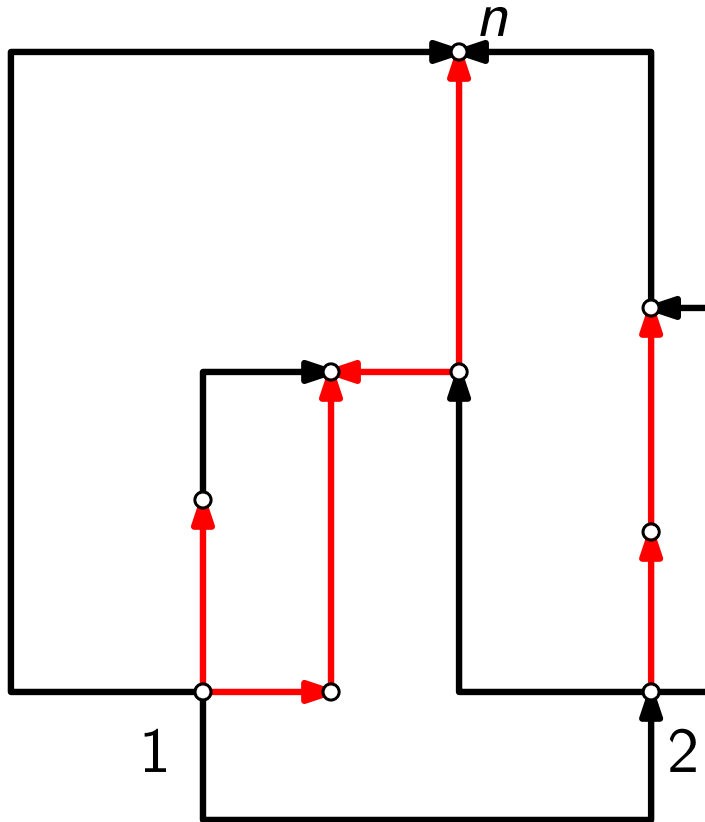
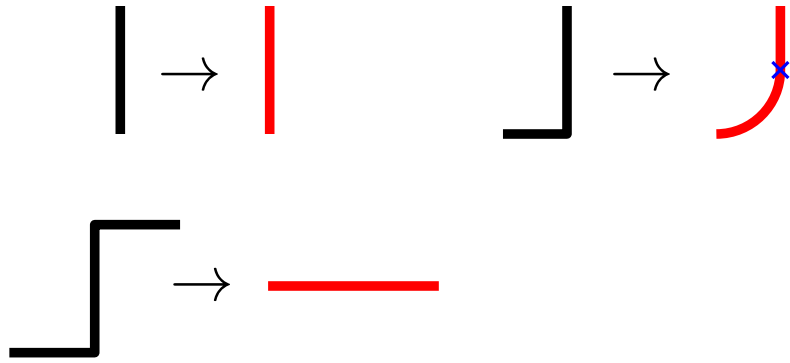
# Komplexität-2 Zeichnungen



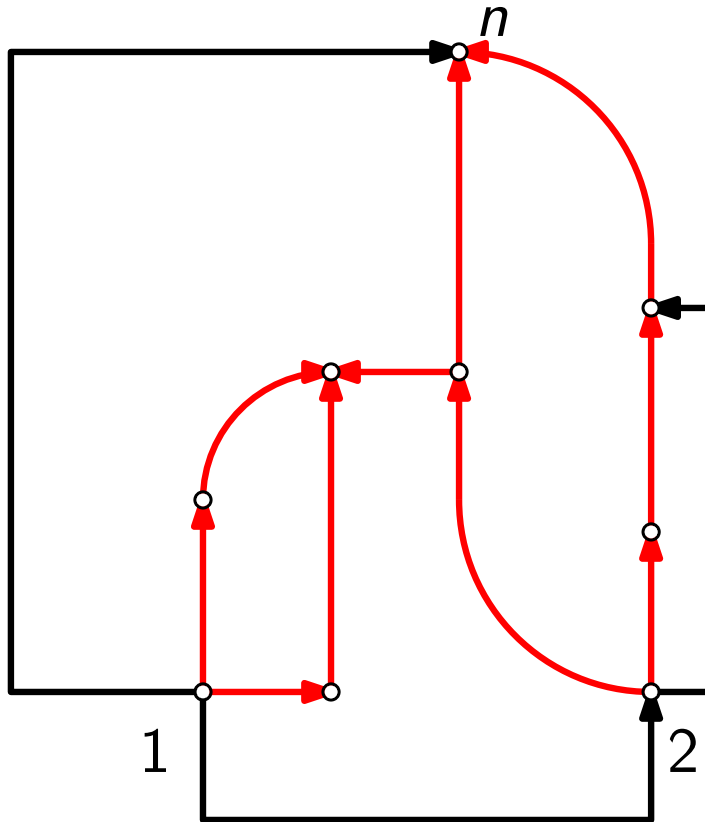
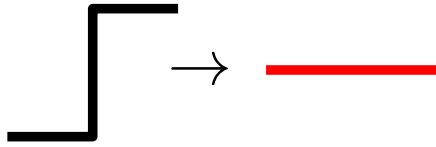
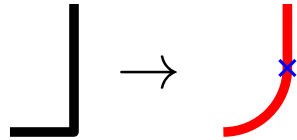
# Komplexität-2 Zeichnungen



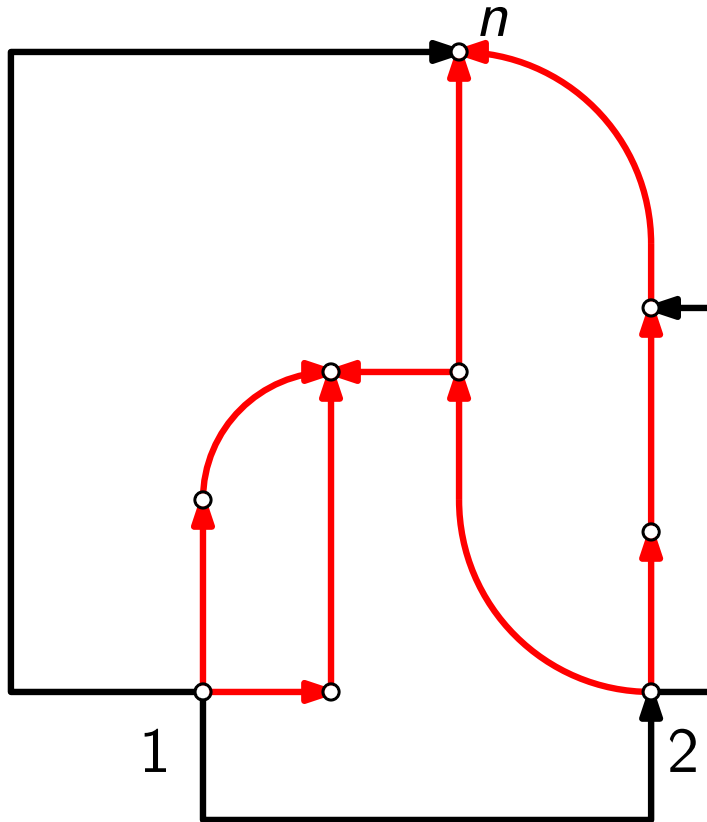
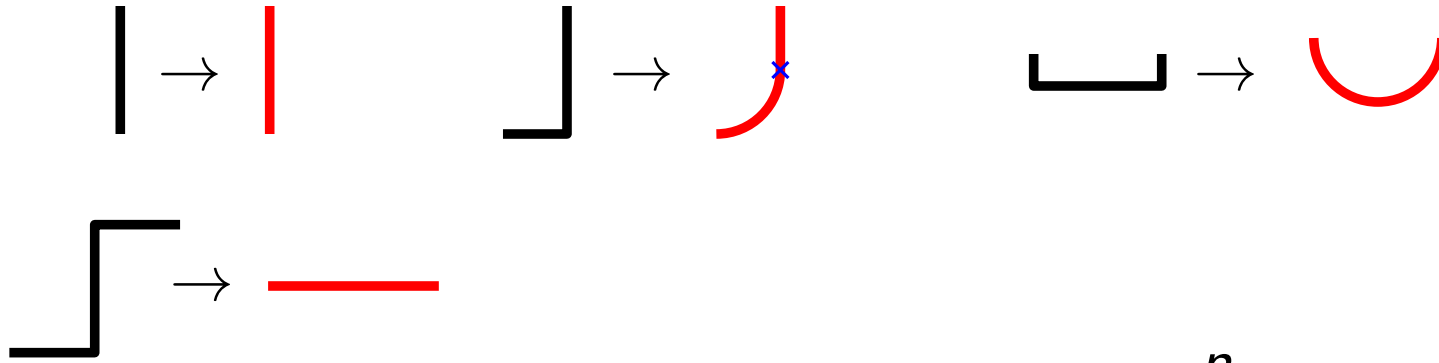
# Komplexität-2 Zeichnungen



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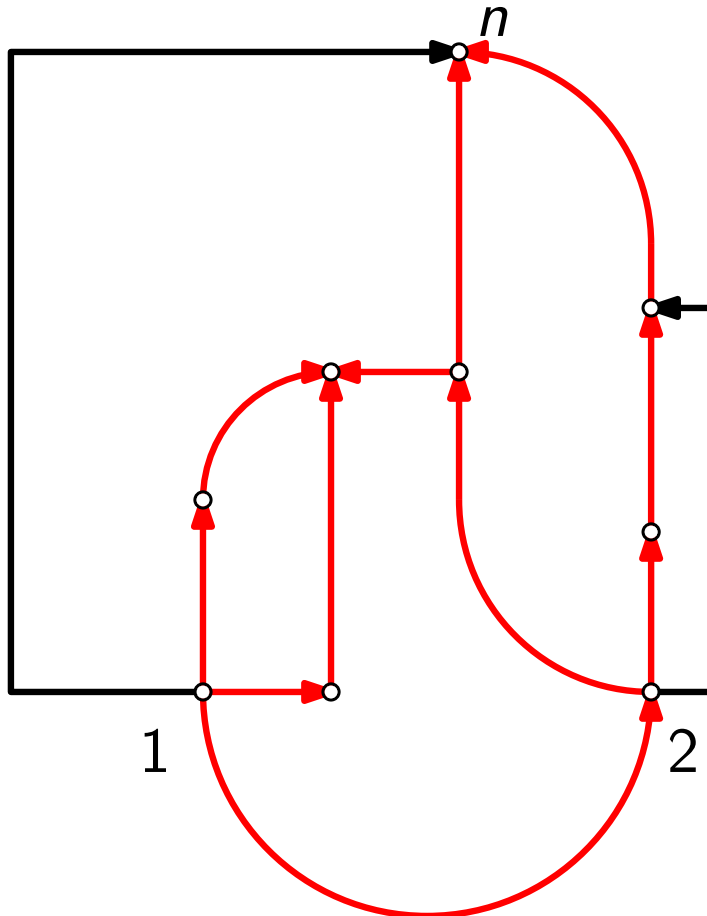
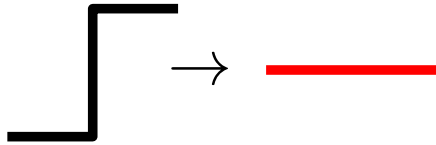
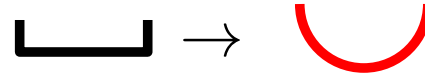
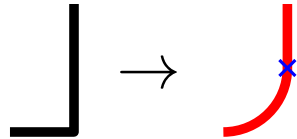
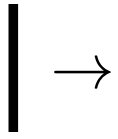


# Komplexität-2 Zeichnungen

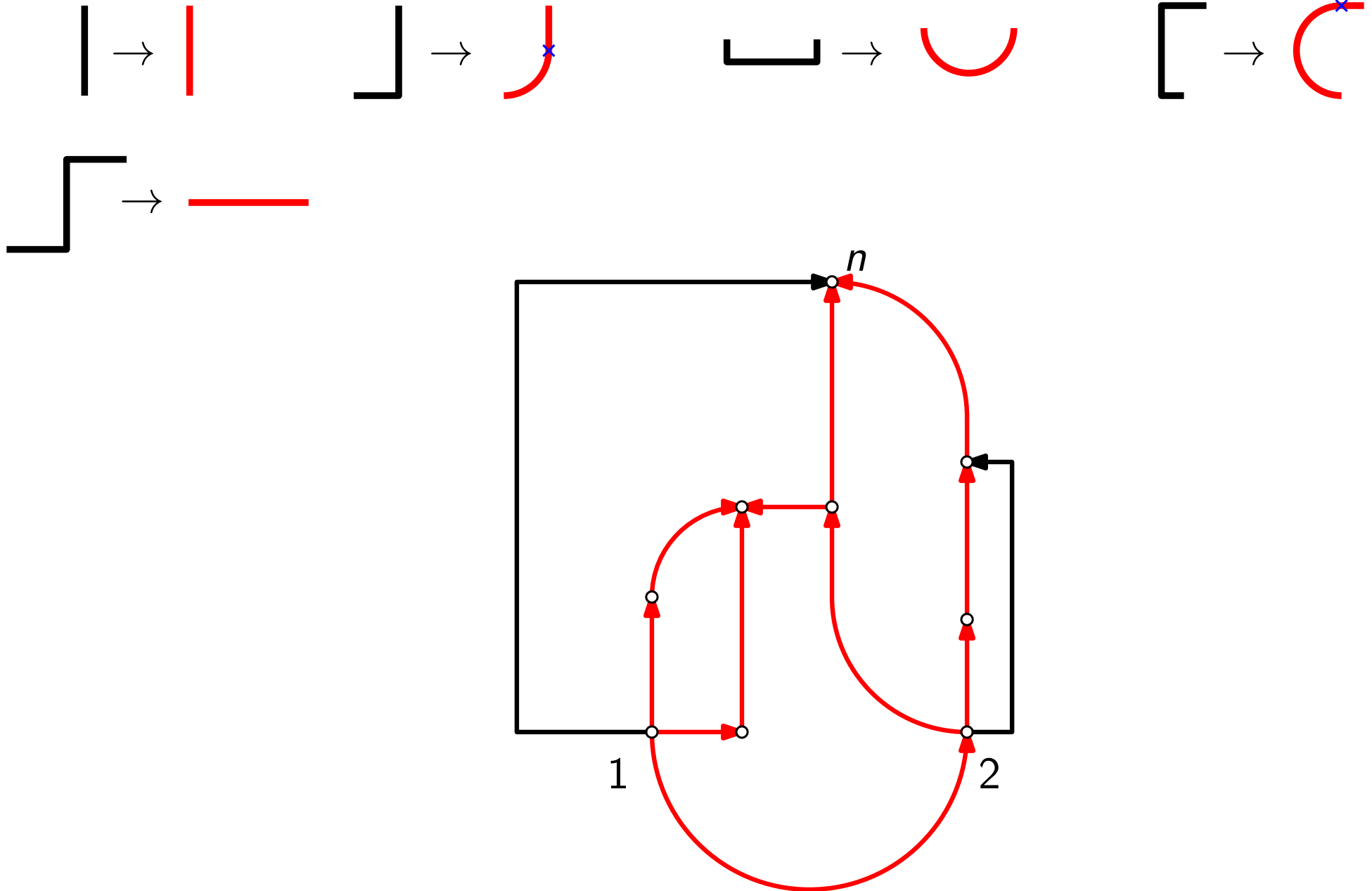




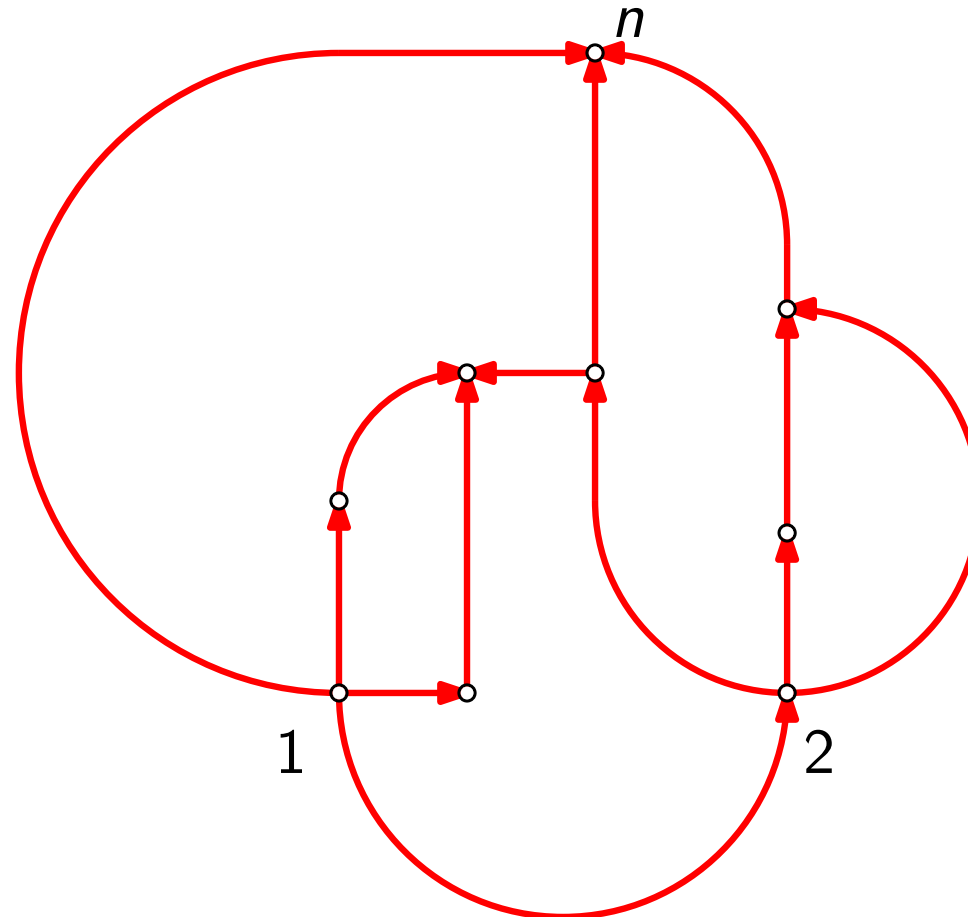
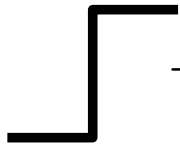
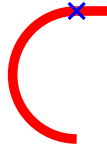
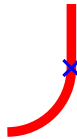
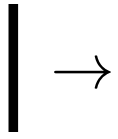
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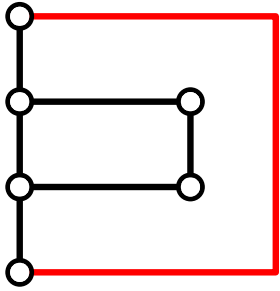
# Komplexität-2 Zeichnungen



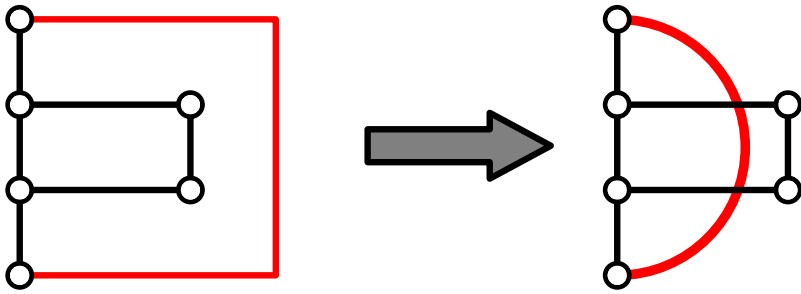
# Komplexität-2 Zeichnungen



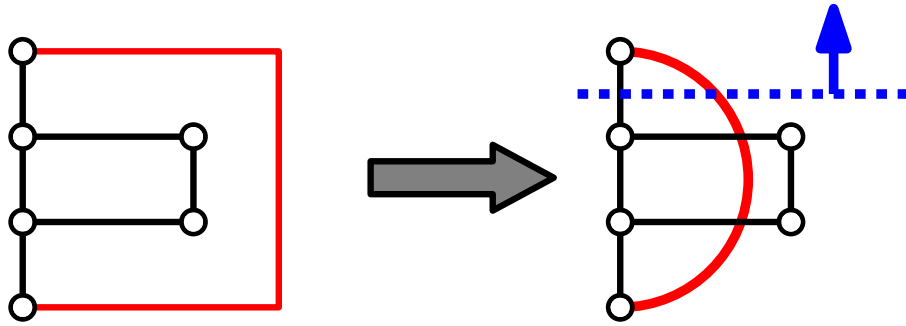
# Kreuzungen



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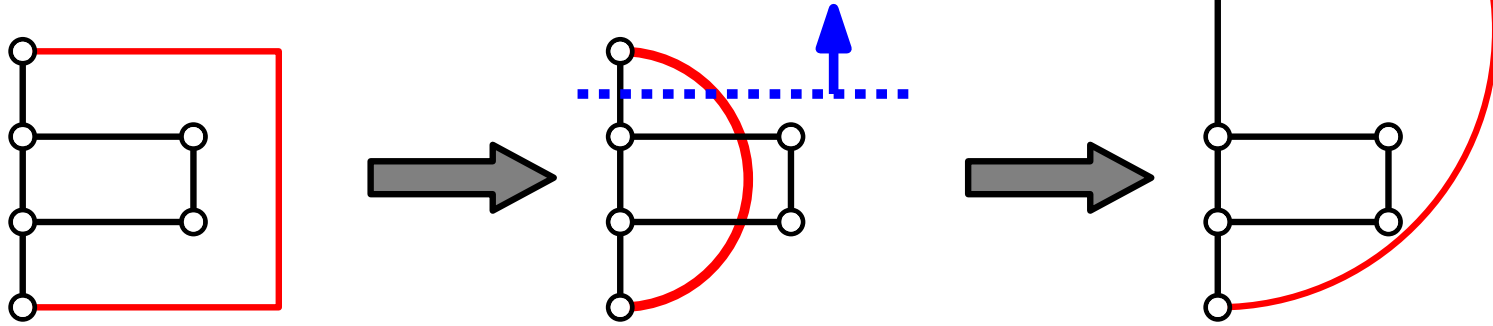


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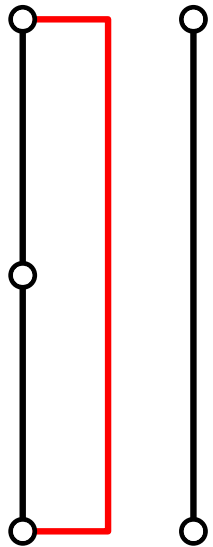
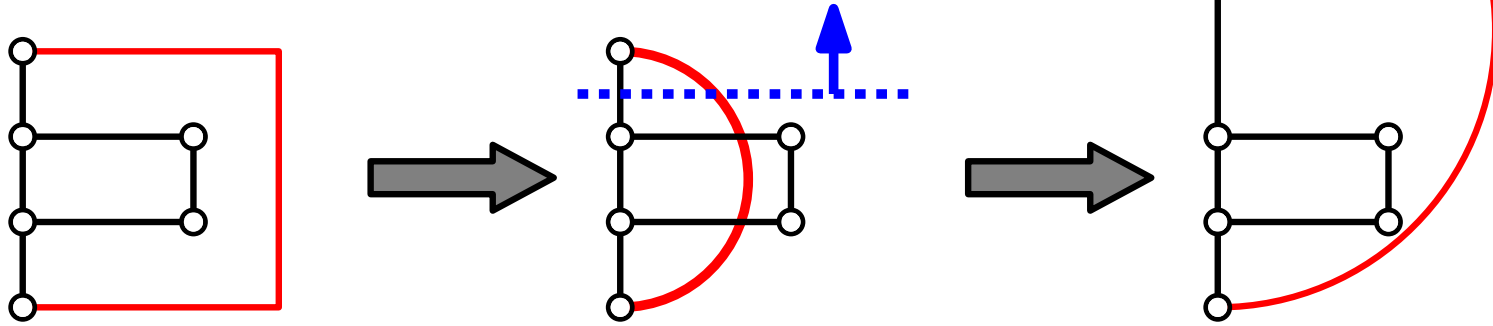




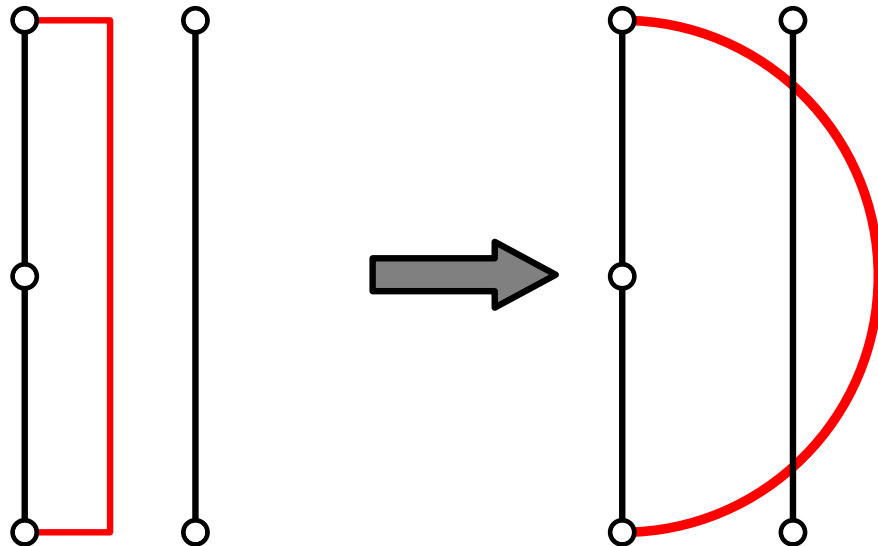
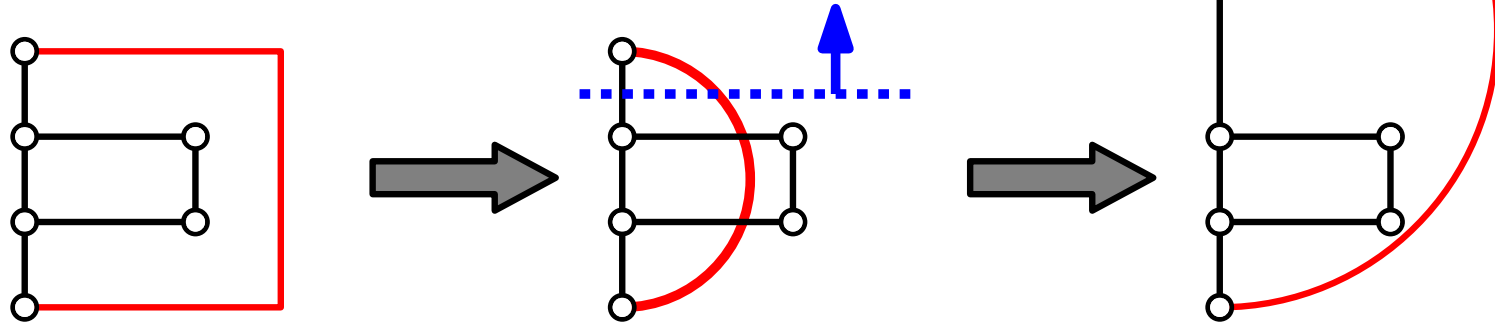
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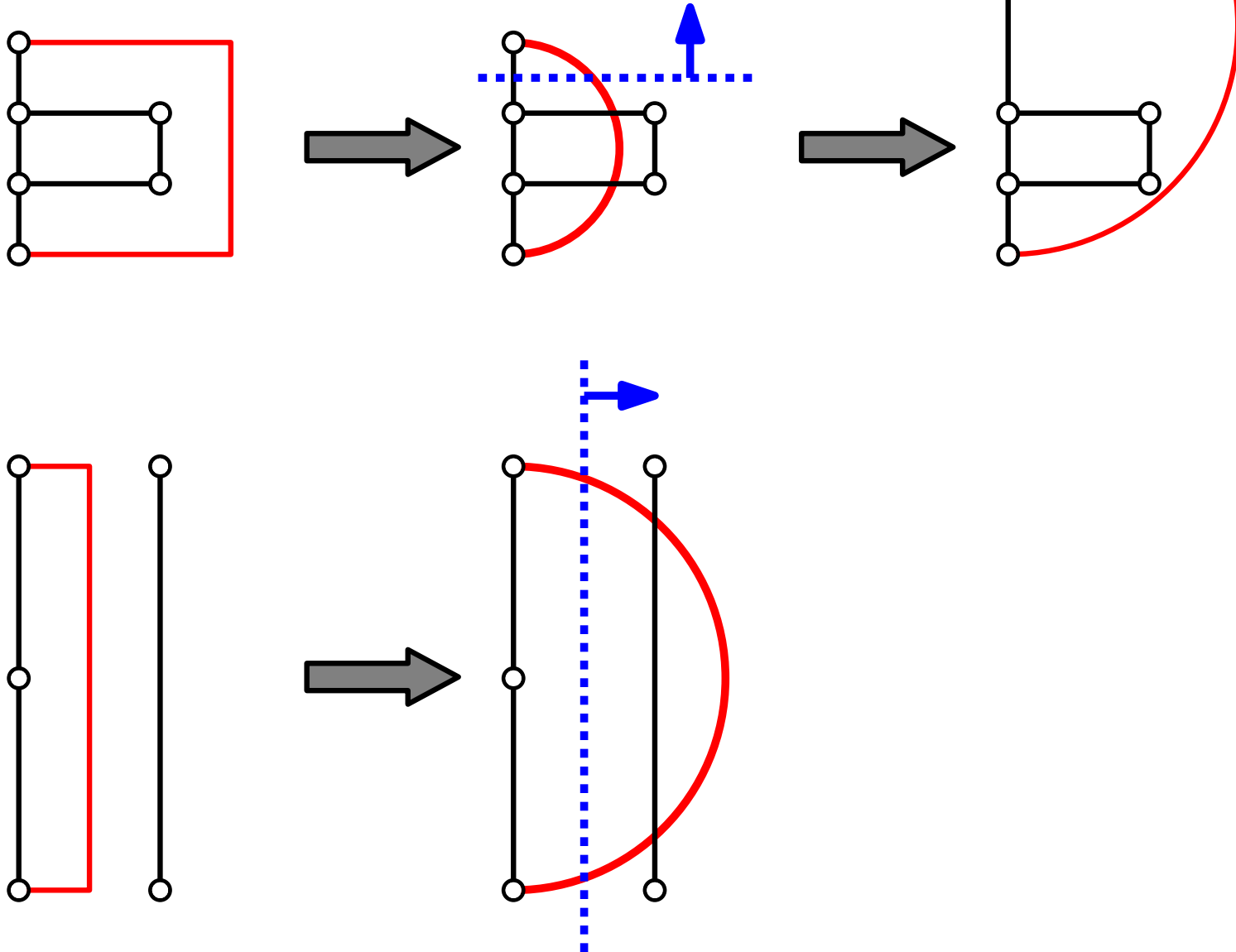
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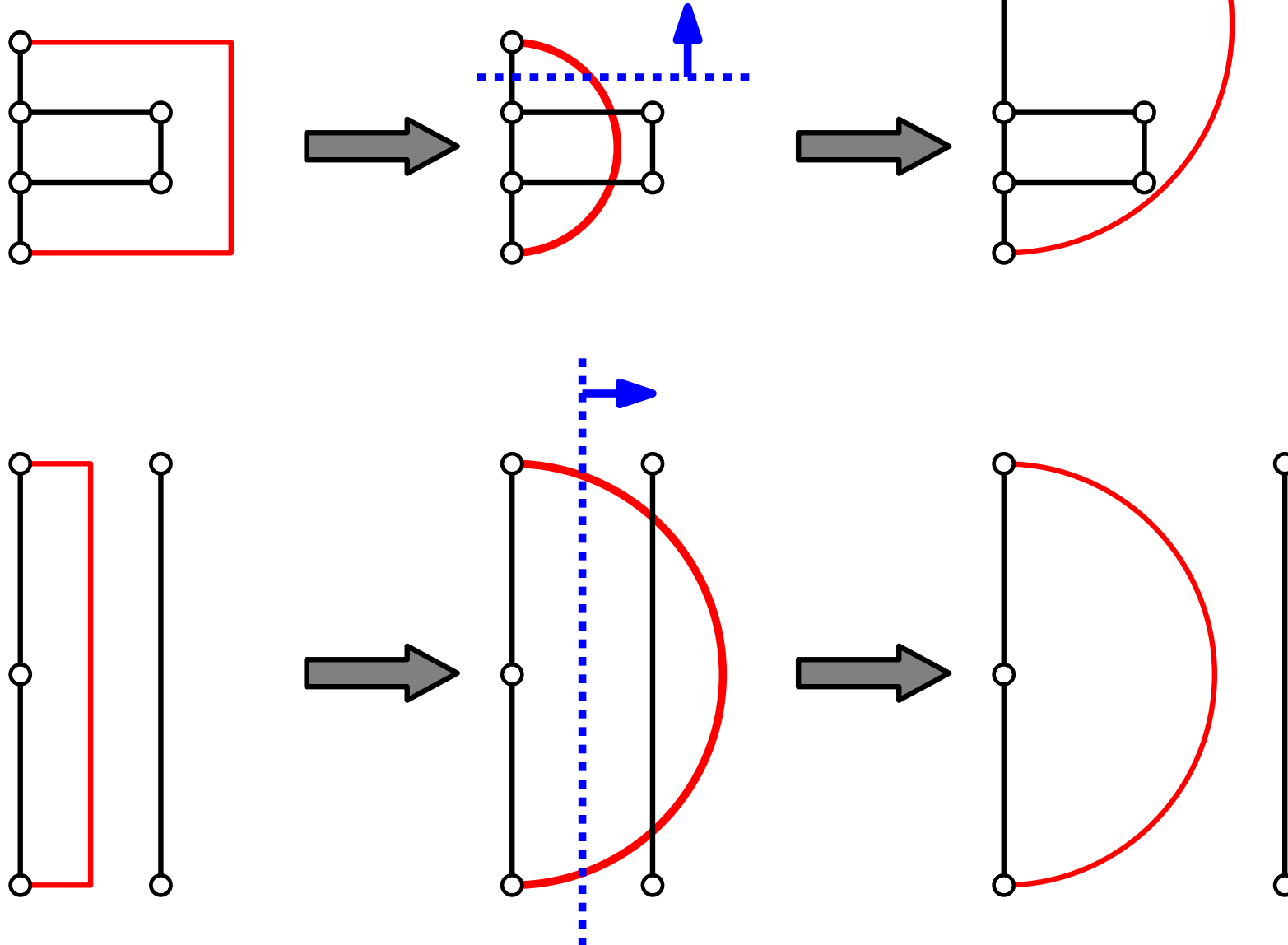
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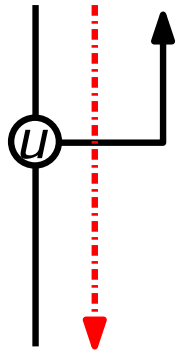
# Schnitte

Def.



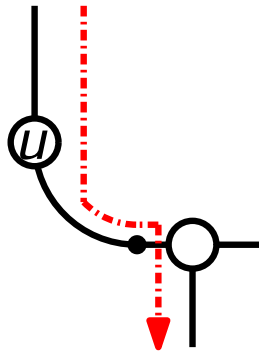
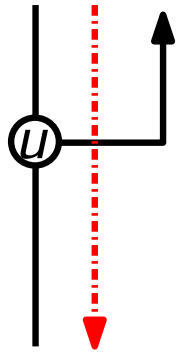
# Schnitte

Def. ●  $y$ -monotone Kurve



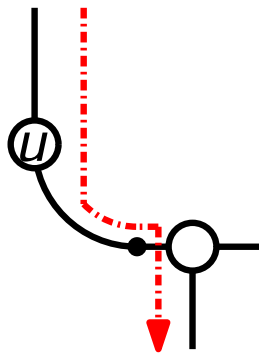
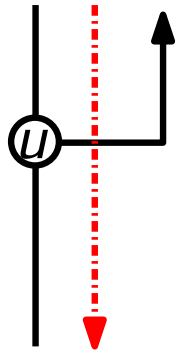
# Schnitte

- Def. ●  $y$ -monotone Kurve
- horizontale, vertikale und Kreissegmente



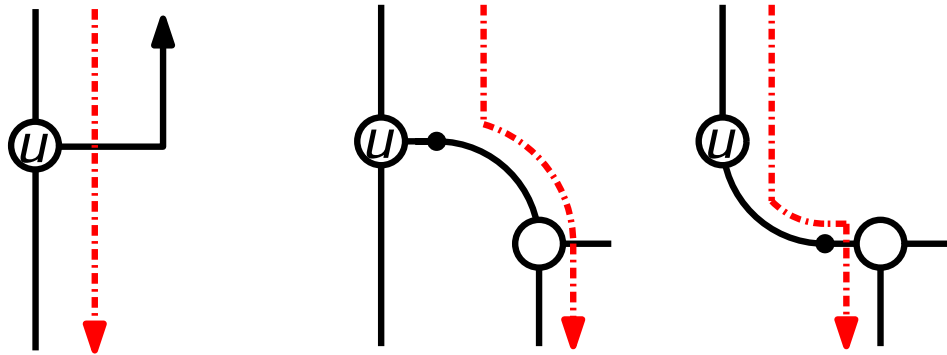
# Schnitte

- Def.
- $y$ -monotone Kurve
  - horizontale, vertikale und Kreissegmente
  - Teilt Zeichnung in linken und rechten Teil



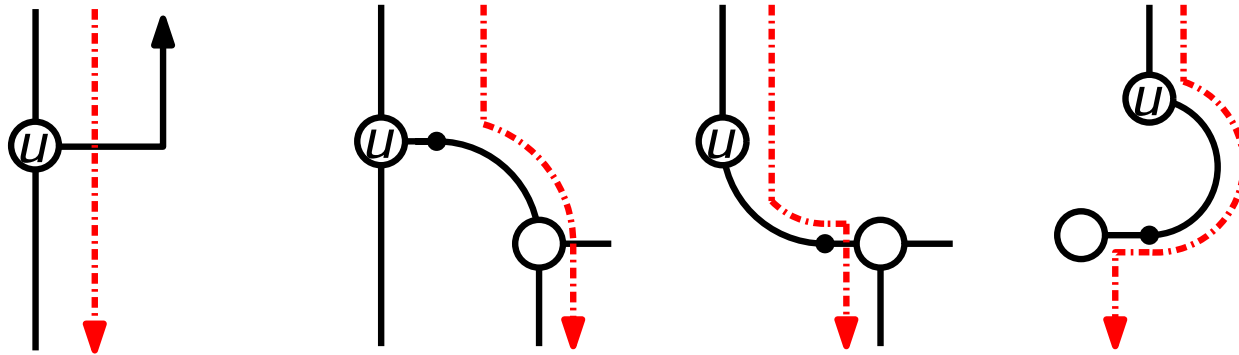
# Schnitte

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- $y$ -monotone Kurve
  - horizontale, vertikale und Kreissegmente
  - Teilt Zeichnung in linken und rechten Teil
  - Schneidet nur horizontale Segmente



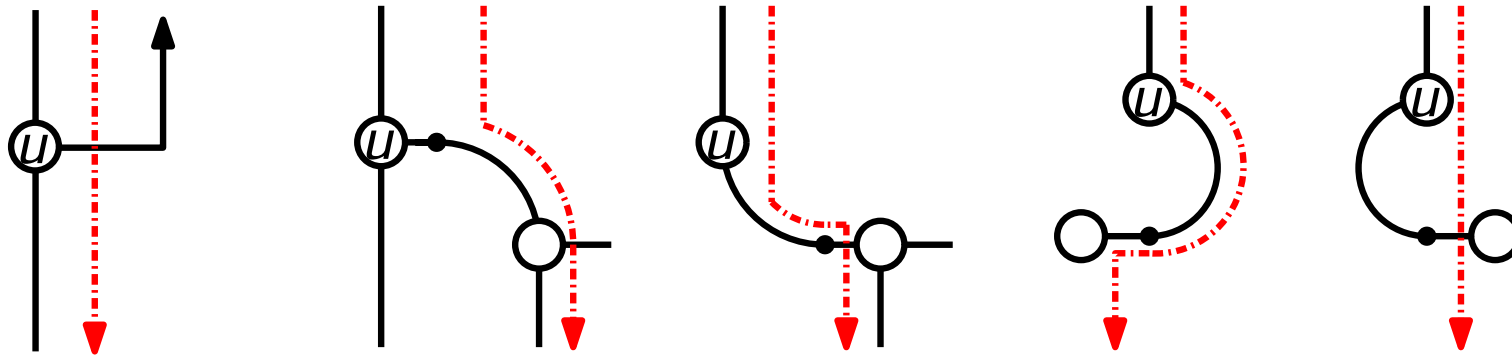
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- Def.
- y-monotone Kurve
  - horizontale, vertikale und Kreissegmente
  - Teilt Zeichnung in linken und rechten Teil
  - Schneidet nur horizontale Segmente



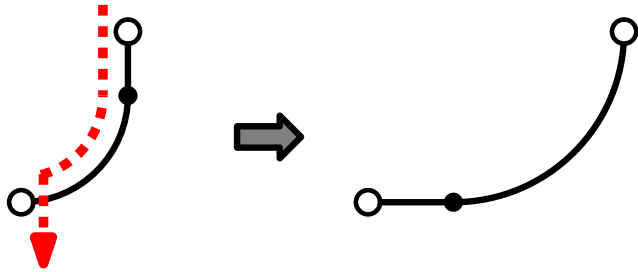
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- Def.
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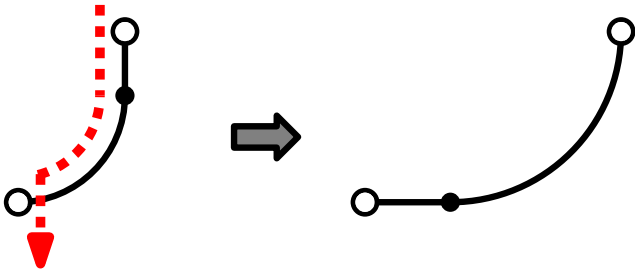
# Maintain invariants

L-Form

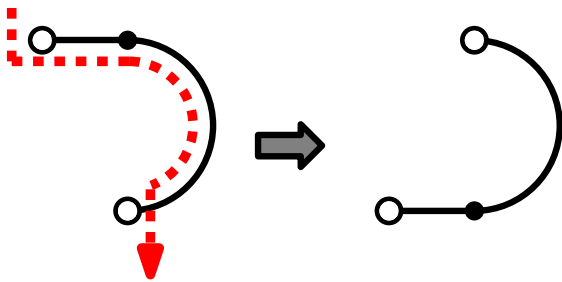


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L-Form



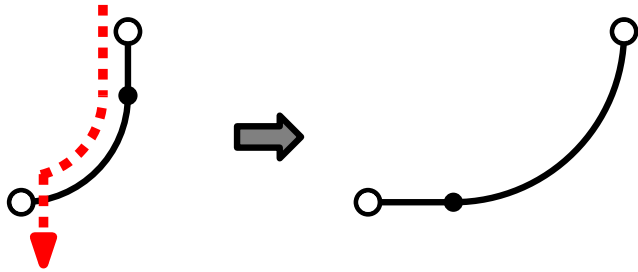
C-Form



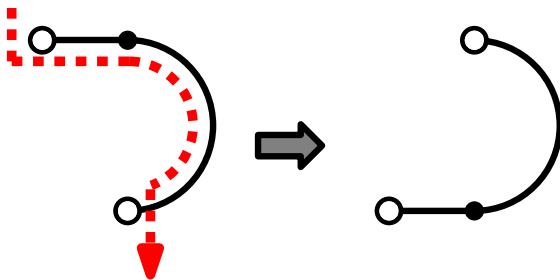


# Maintain invariants

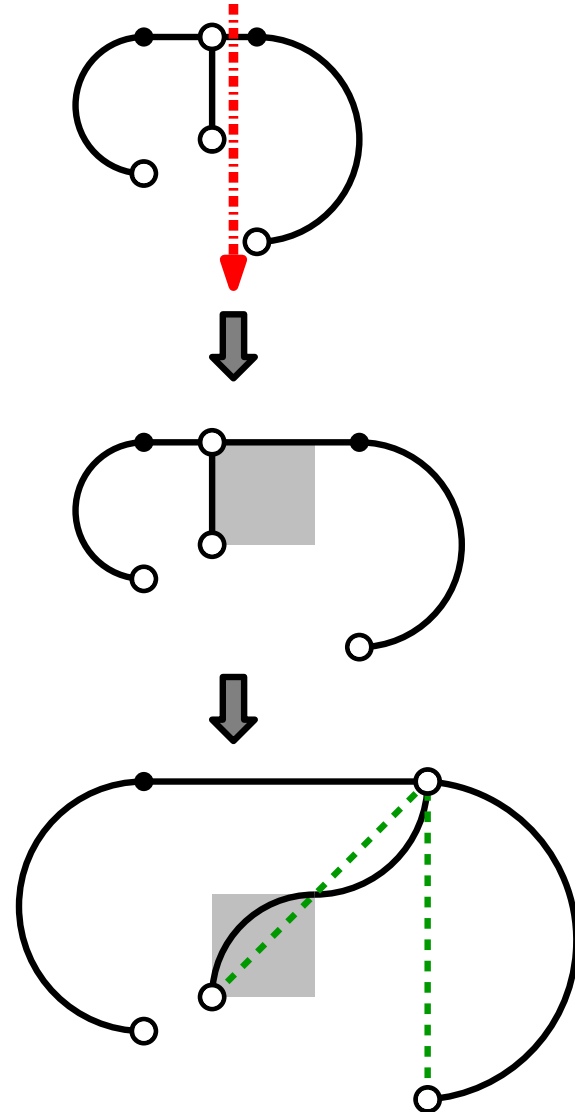
L-Form



C-Form

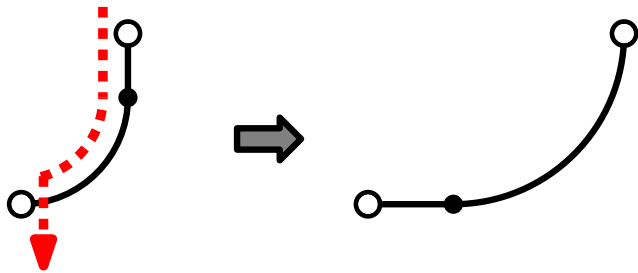


Doppelte C-Form

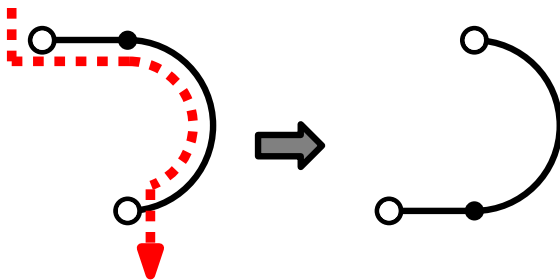


# Maintain invariants

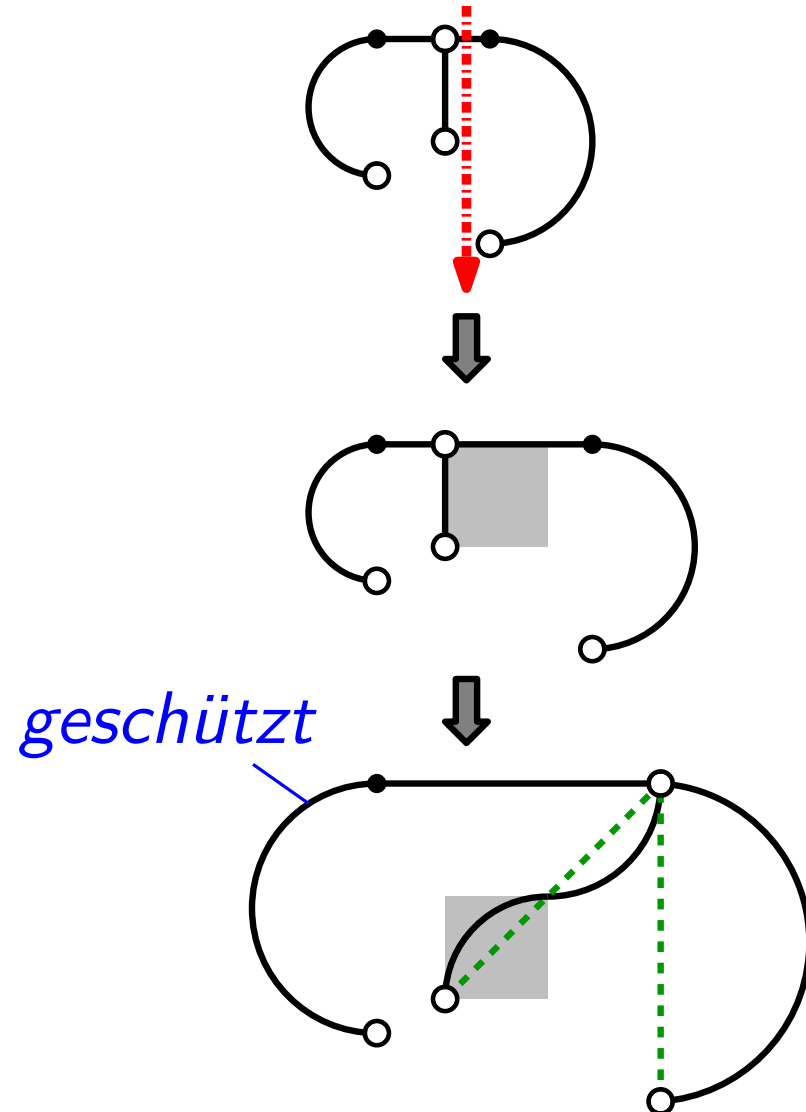
L-Form



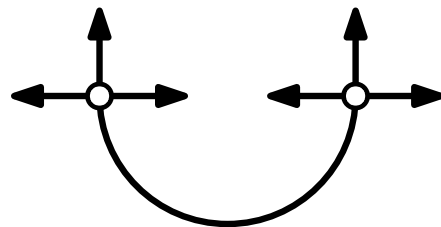
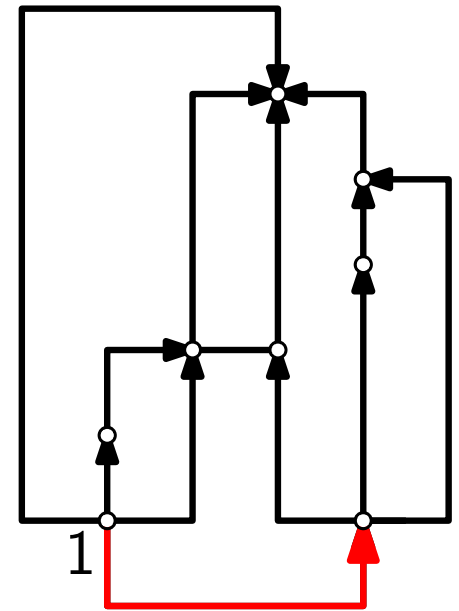
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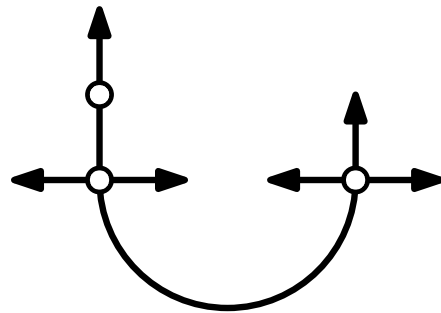
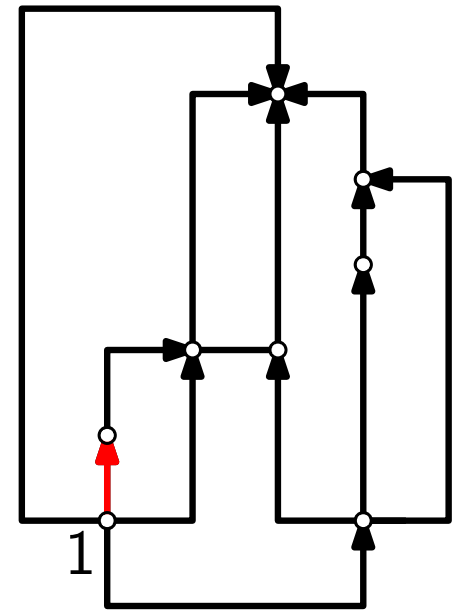
Doppelte C-Form



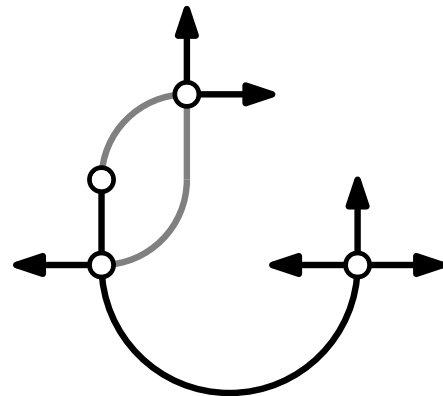
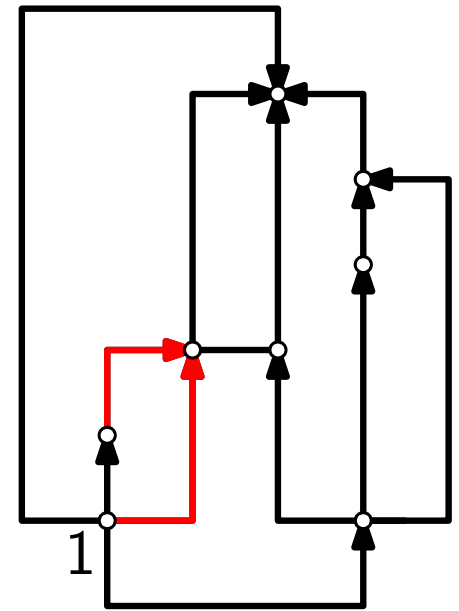
# Beispiel



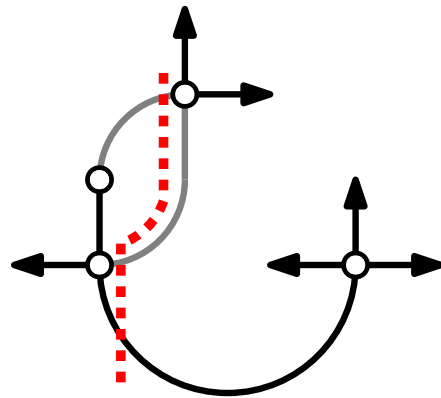
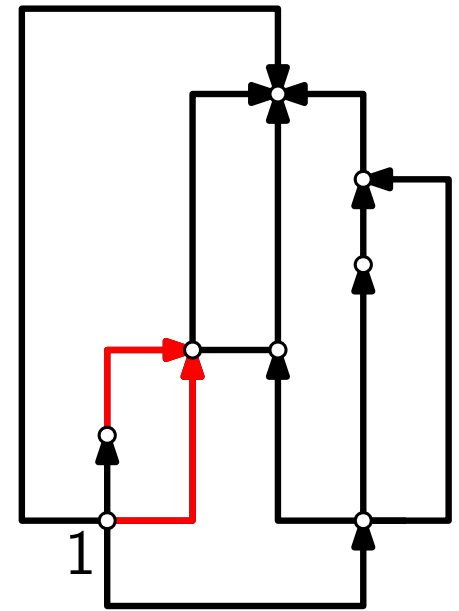
# Beispiel



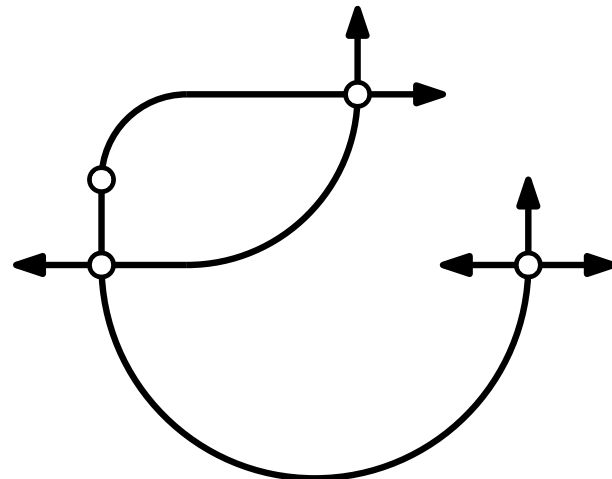
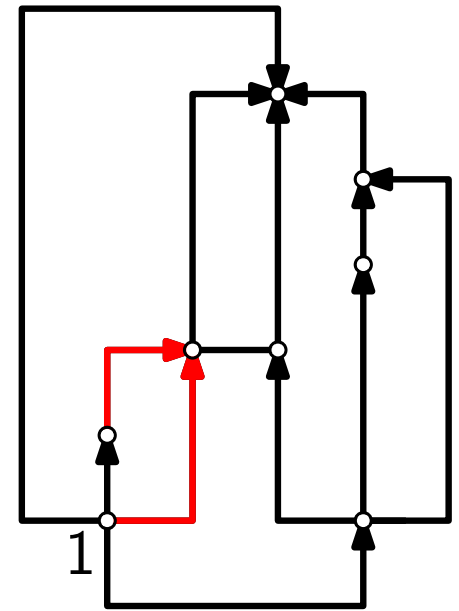
# Beispiel



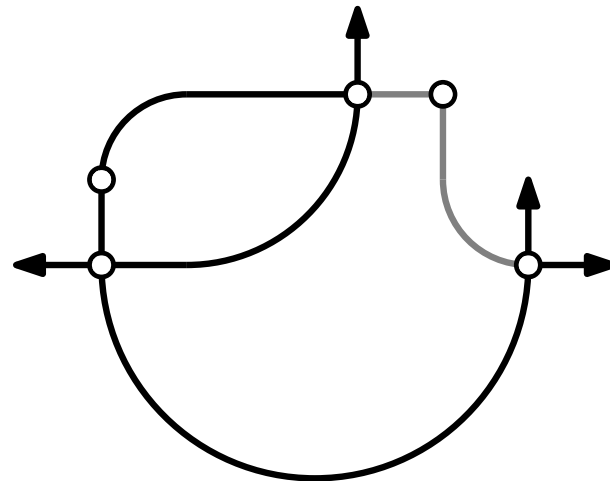
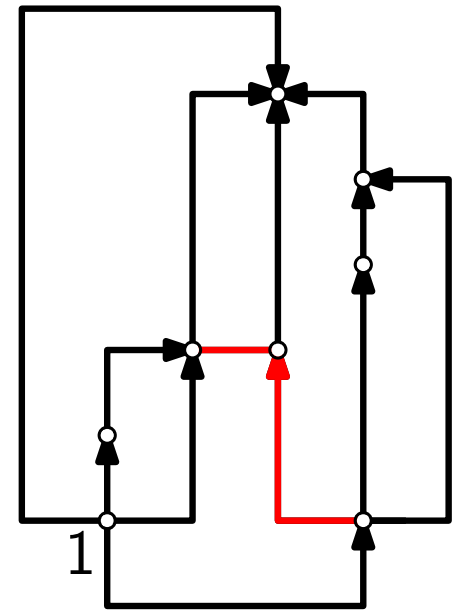
# Beispiel



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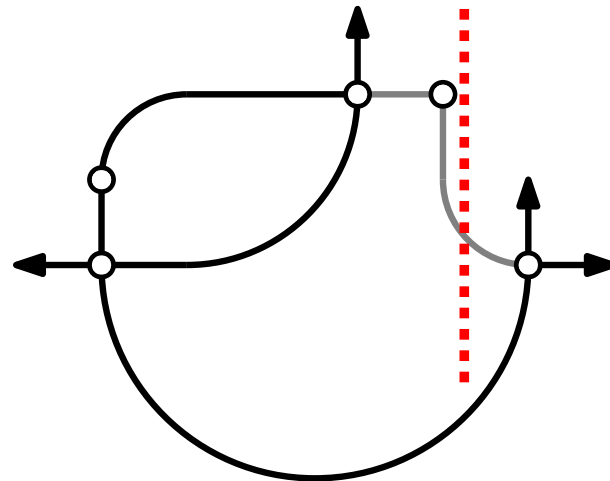
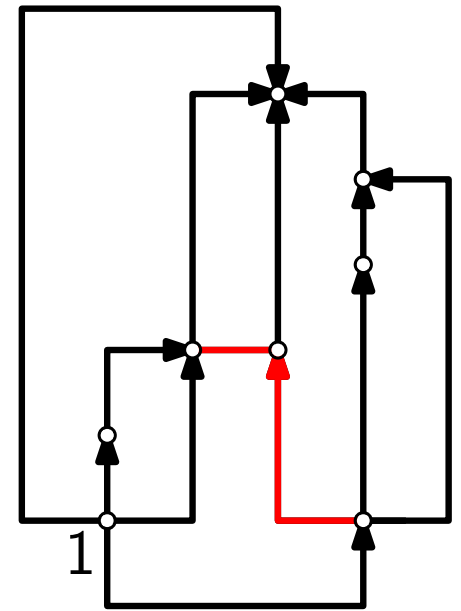


# Beispiel

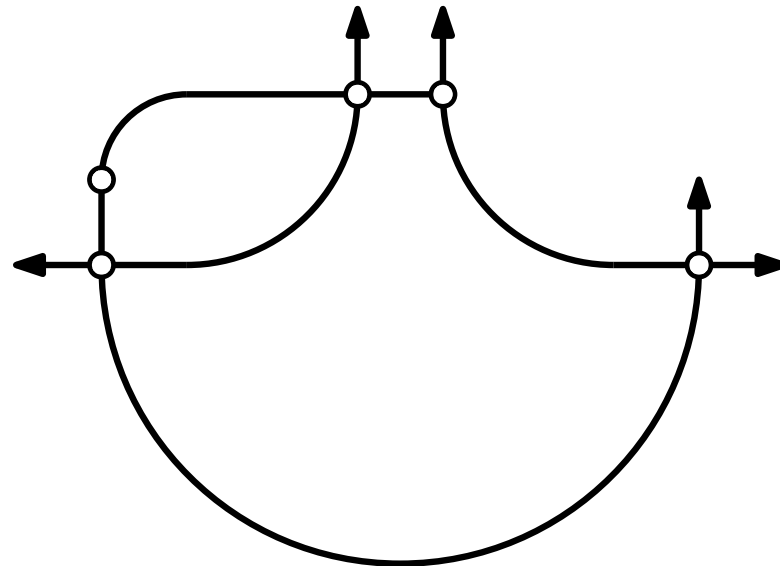
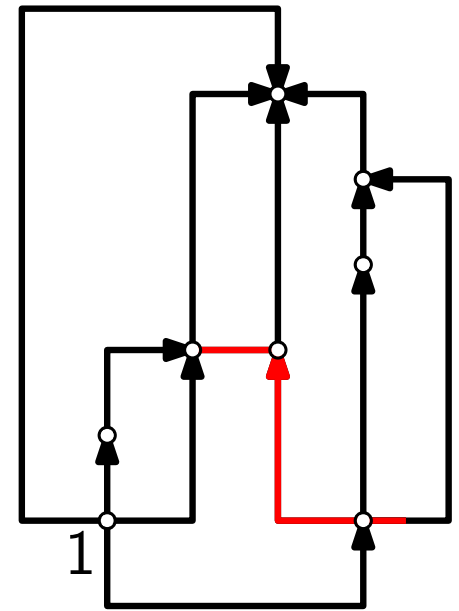




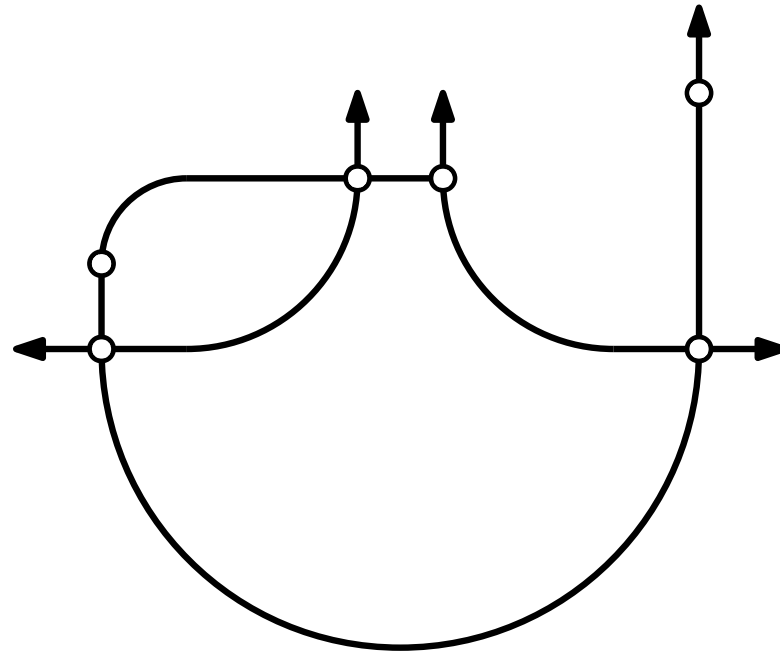
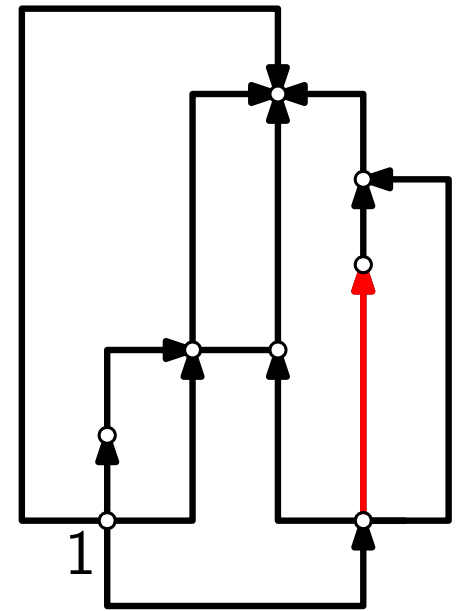
# Beispiel



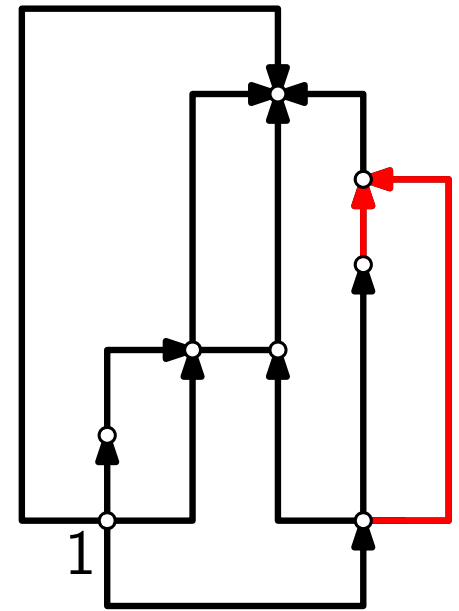
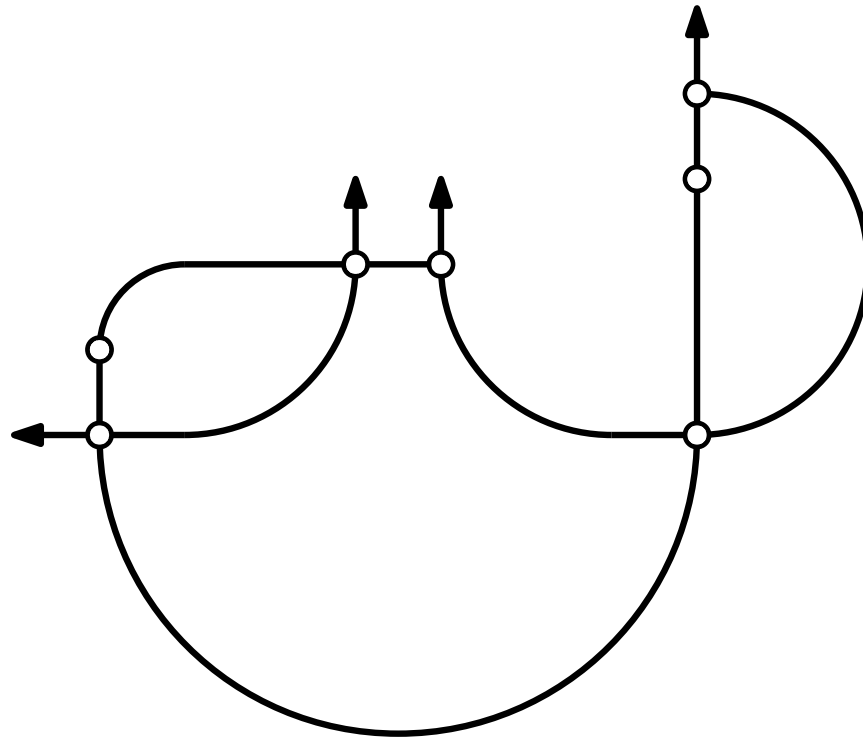
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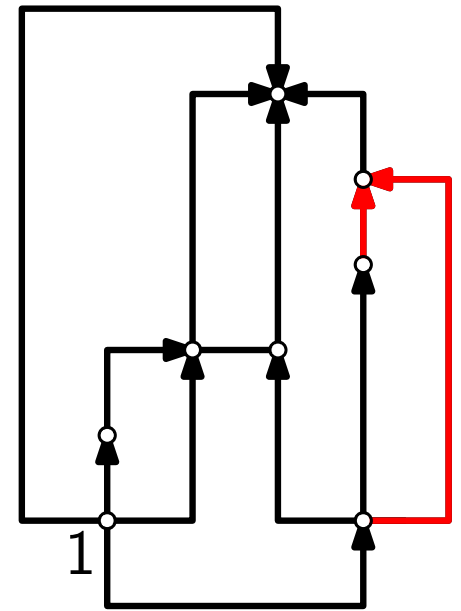
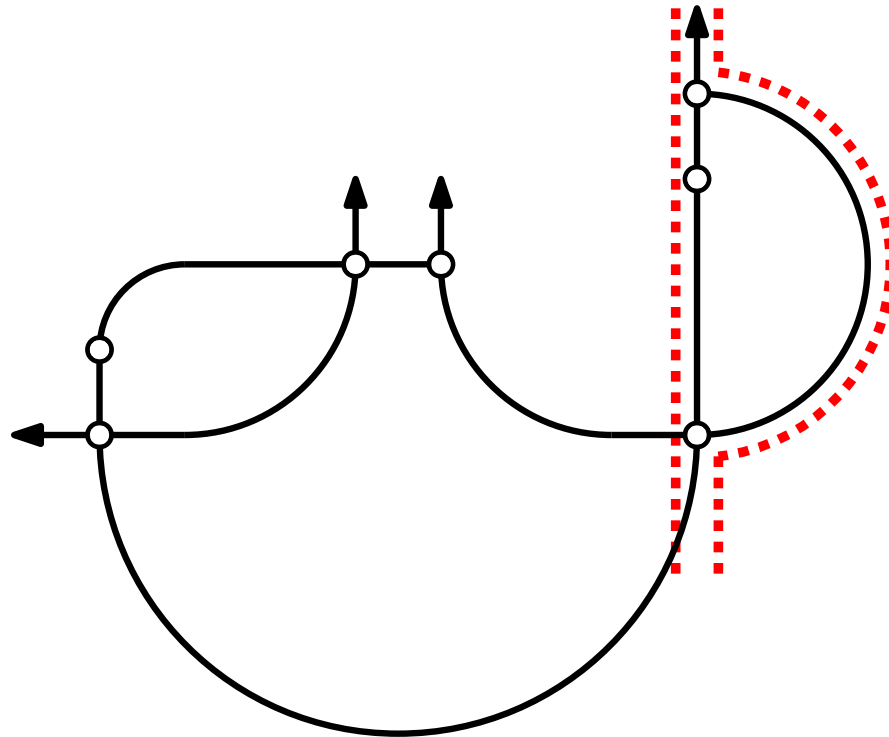
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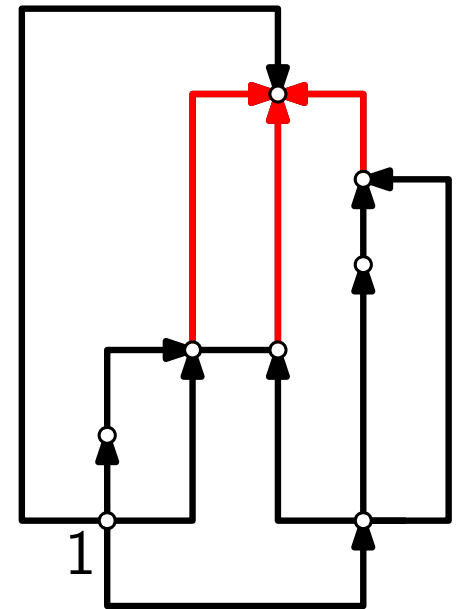
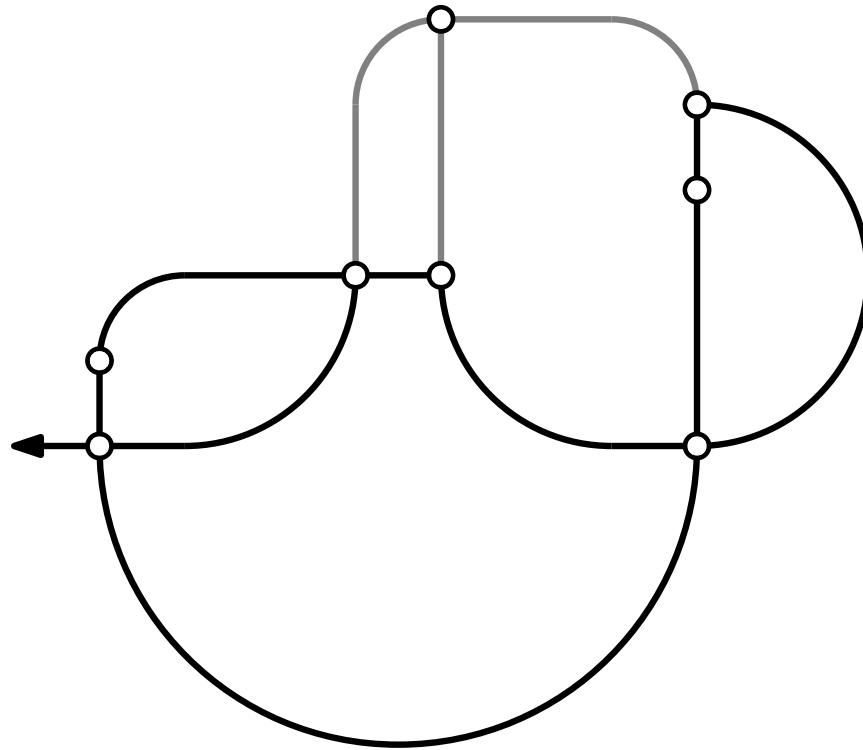
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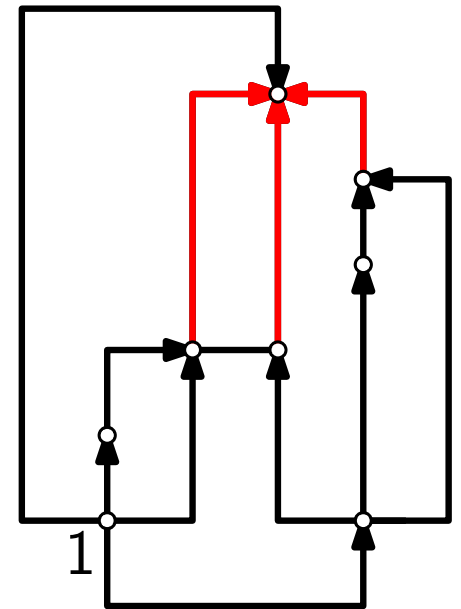
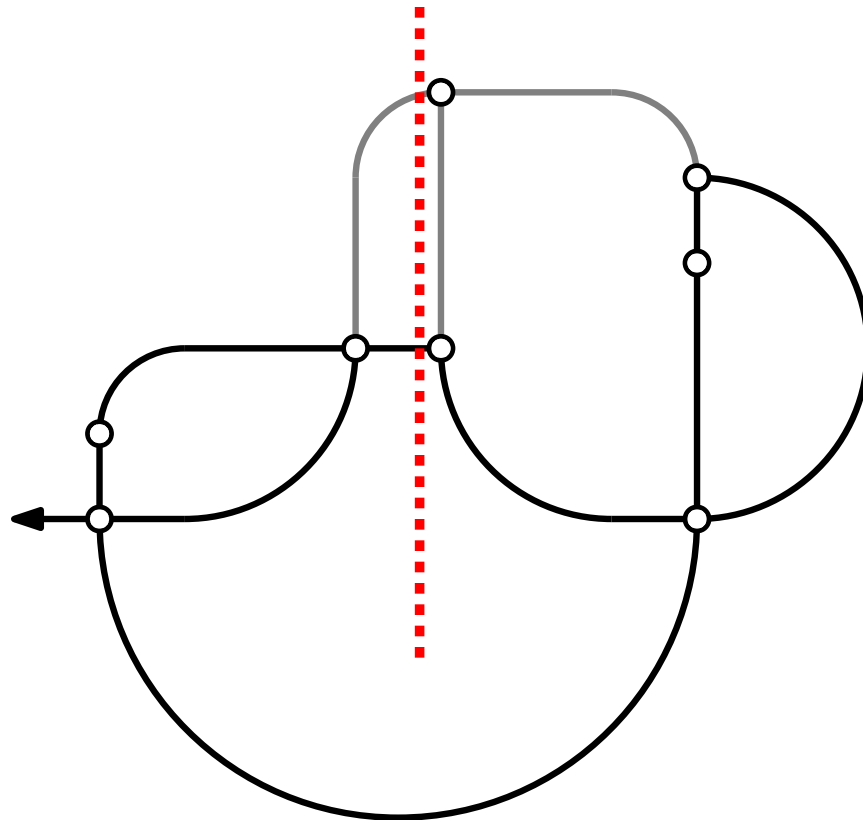
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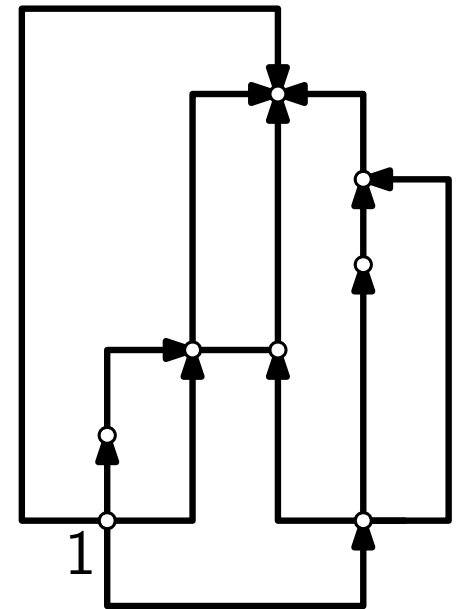
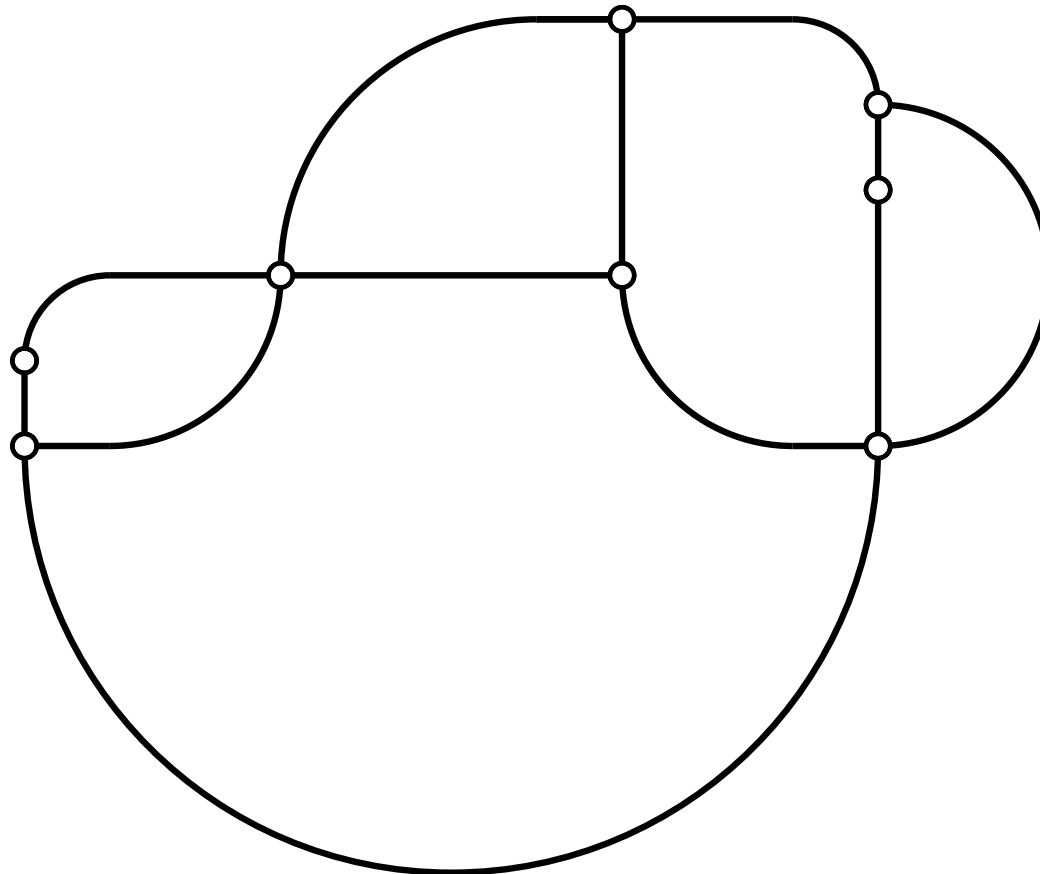
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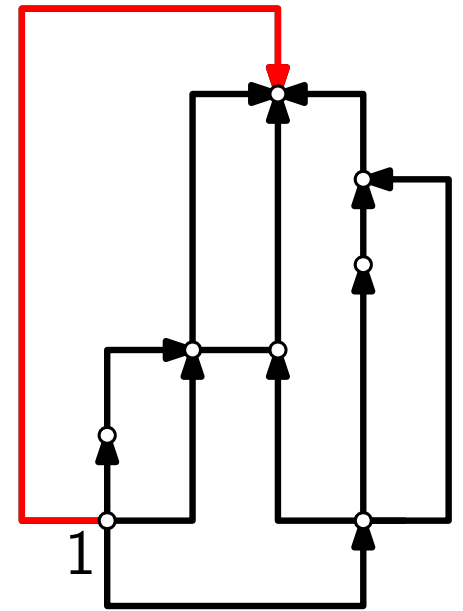
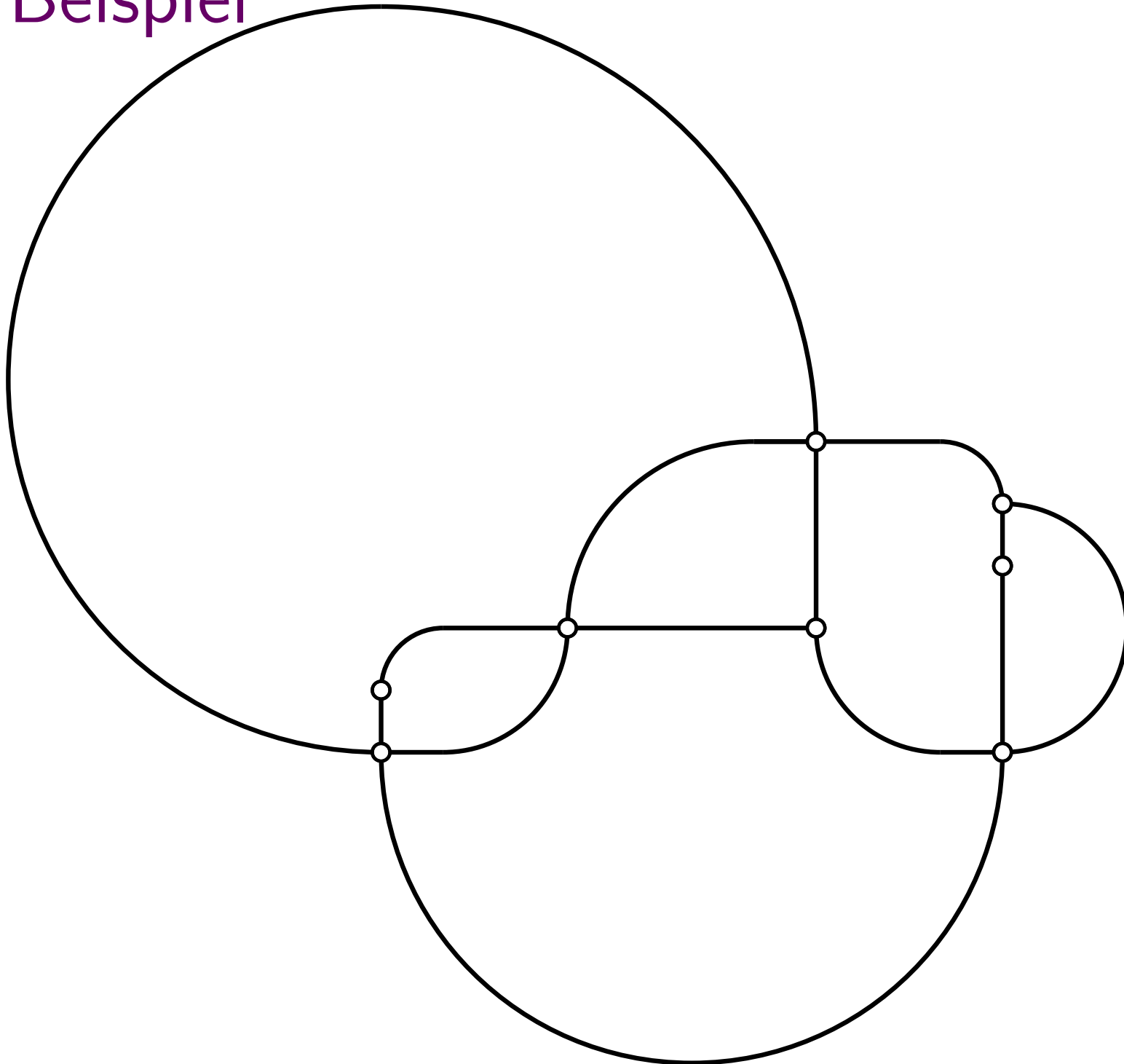


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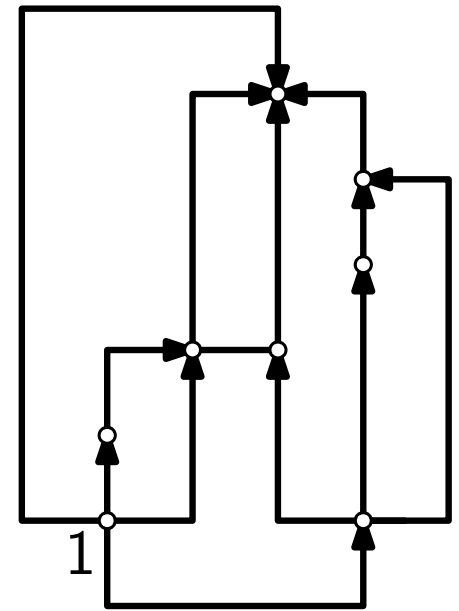
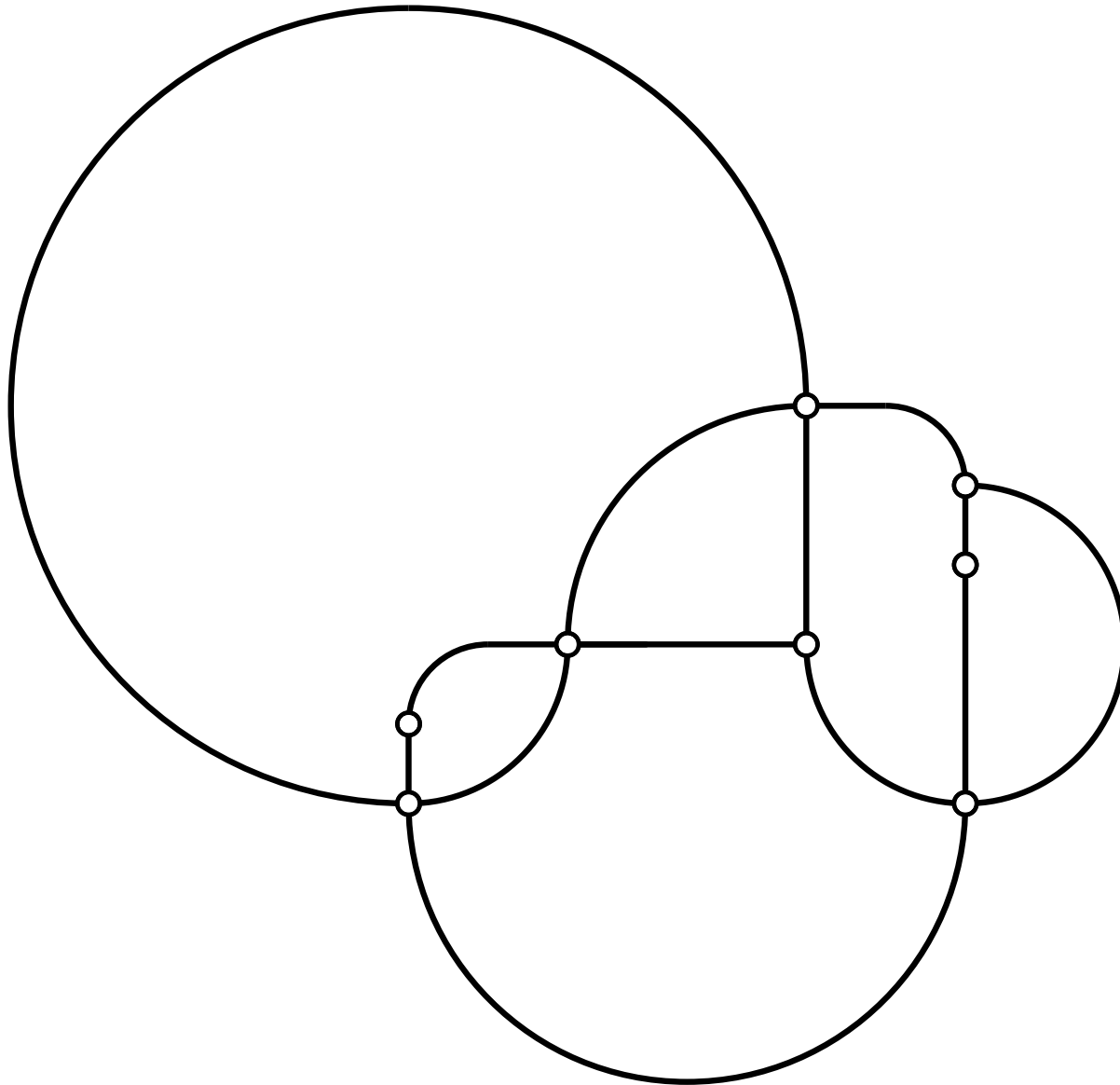


Beispiel





# Beispiel



# Glatt-orthogonale Zeichnungen – Ergebnisse

Zweifach zusammenhängender 4-planarer Graph  $\Rightarrow$   
glatt-orth. Komplexität 3

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~~Zweifach zusammenhängender~~ 4-planarer Graph  $\Rightarrow$   
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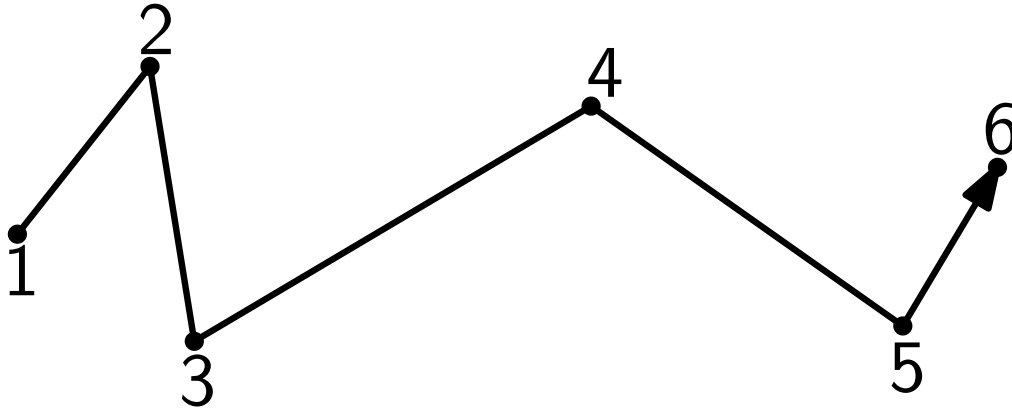
# Monotone Zeichnungen von Bäumen

# Monotone Zeichnungen

*Monotoner* Pfad:  $\exists$  Richtung  $d$ , so dass  
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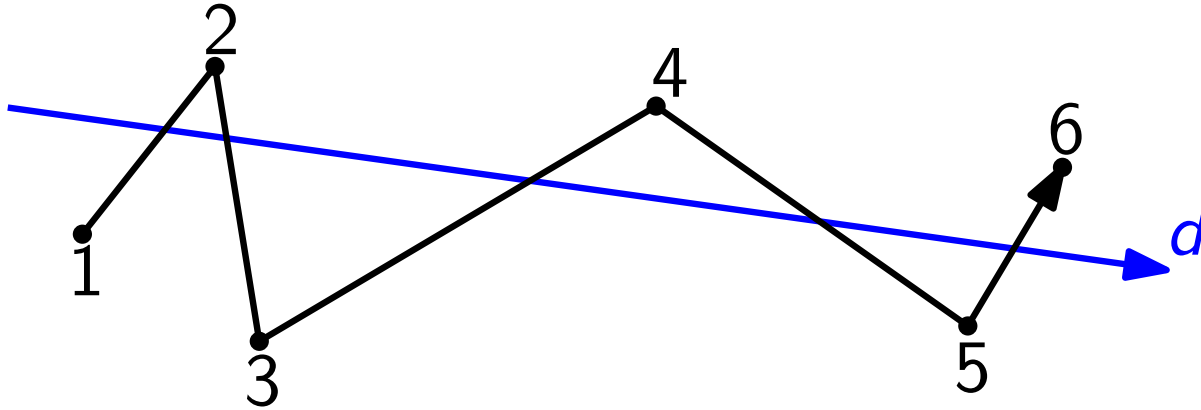
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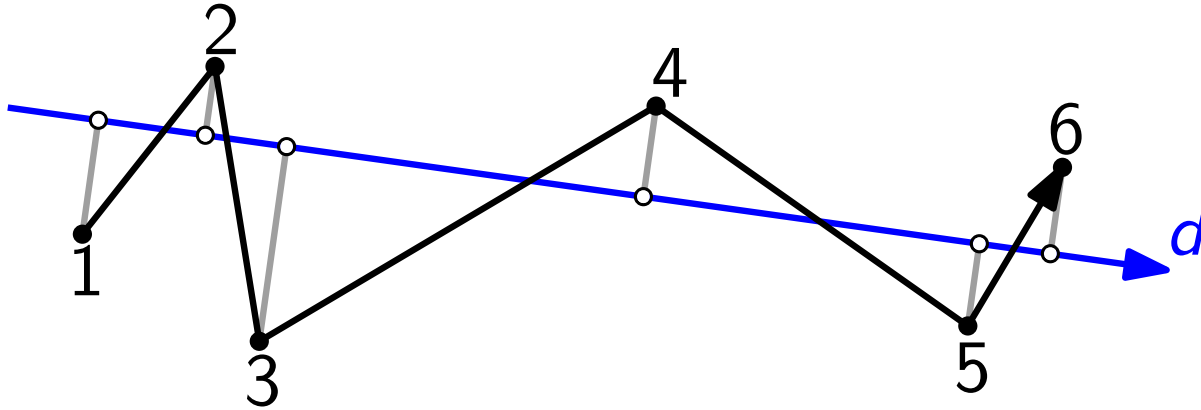
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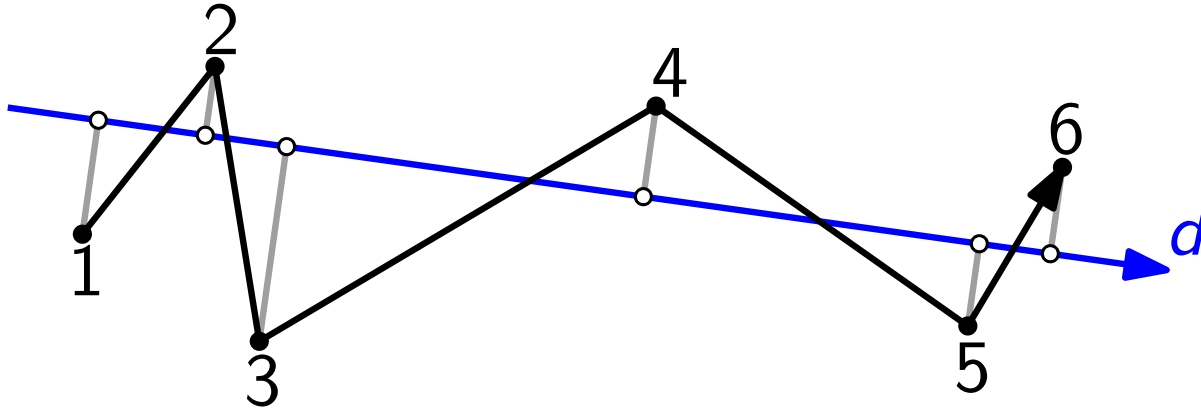
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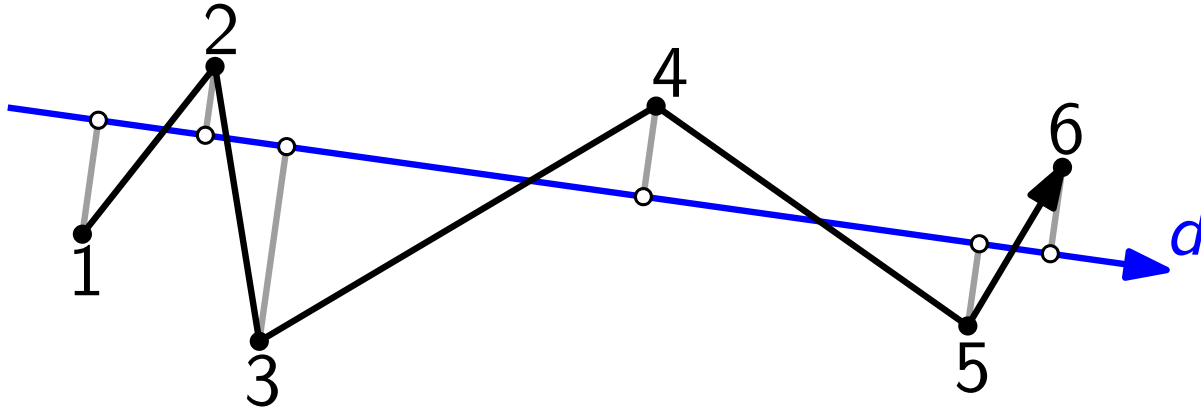


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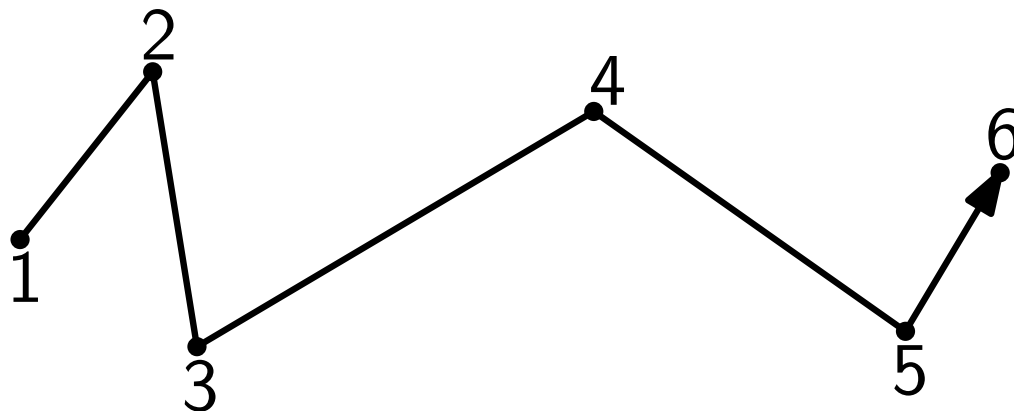
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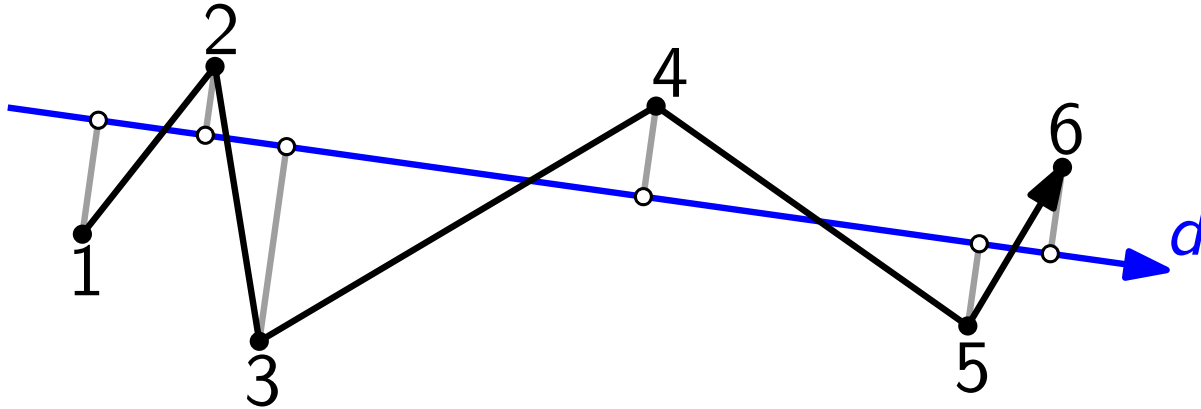
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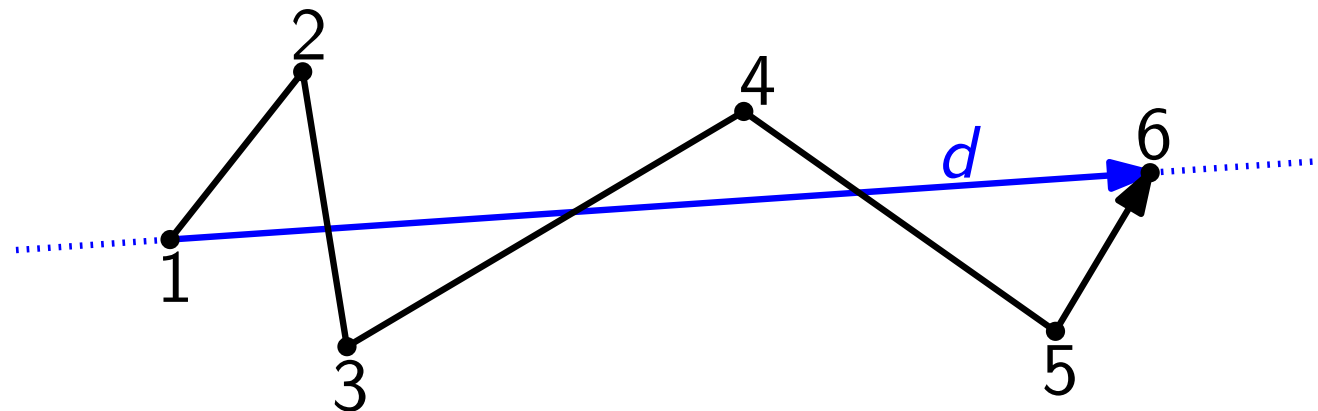
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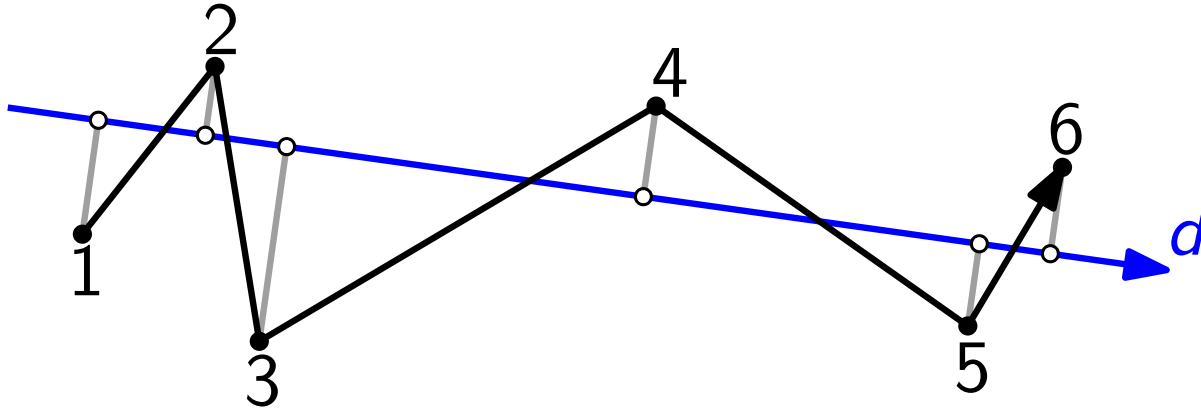
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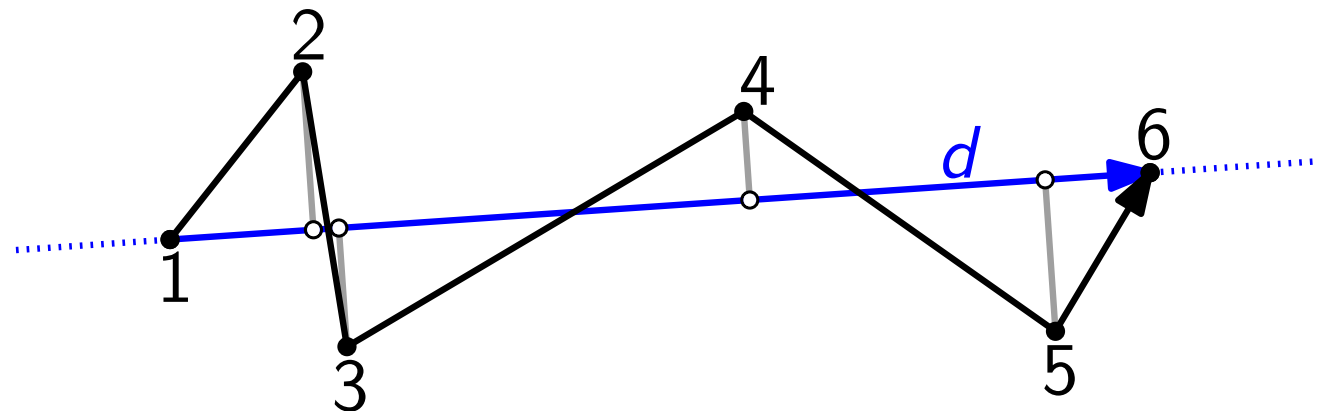
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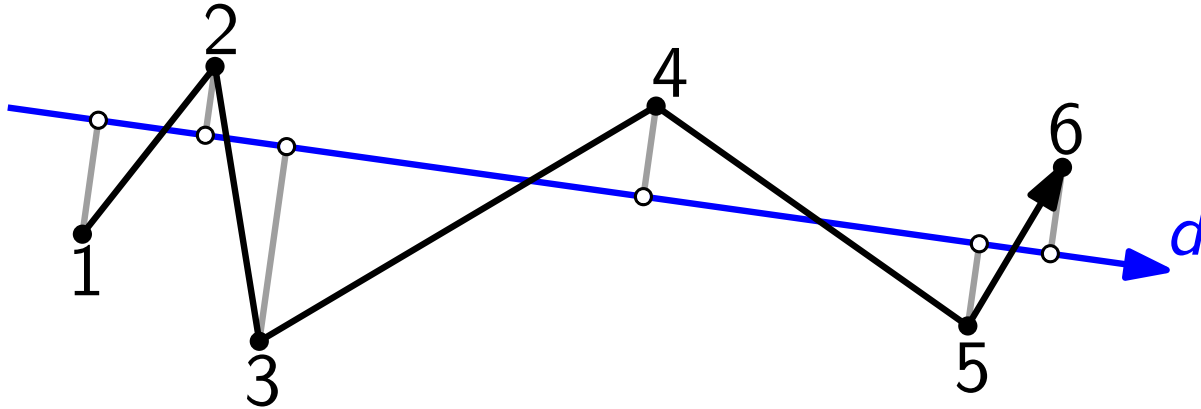
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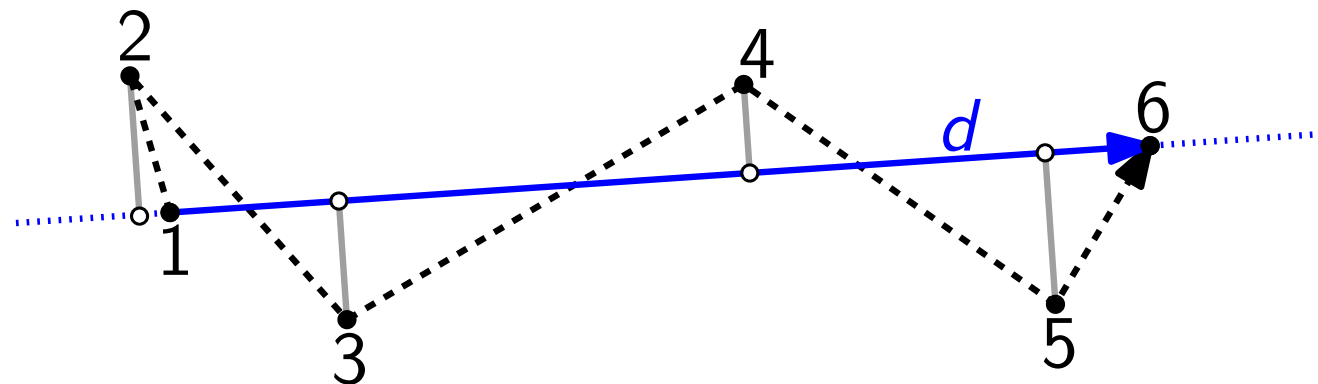
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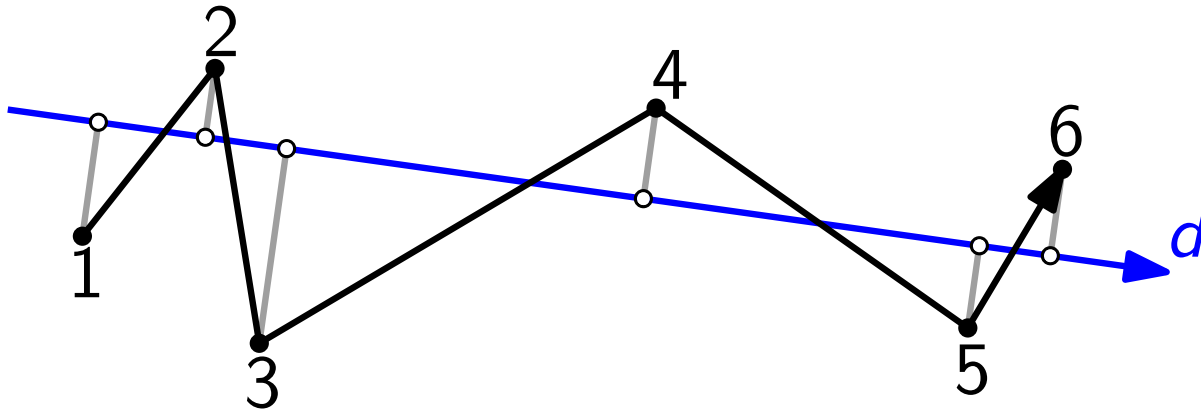
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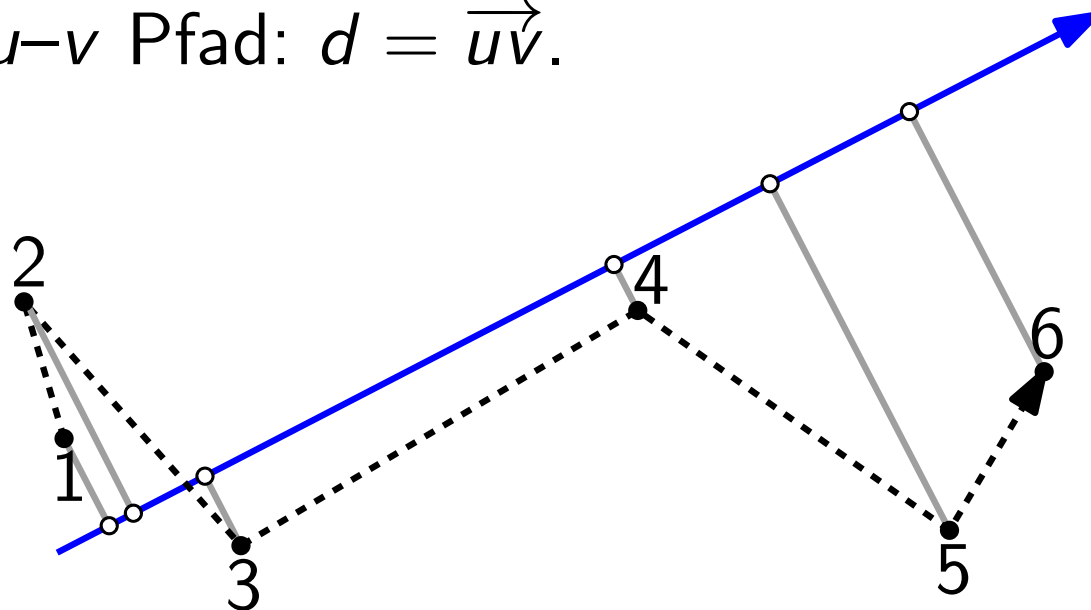
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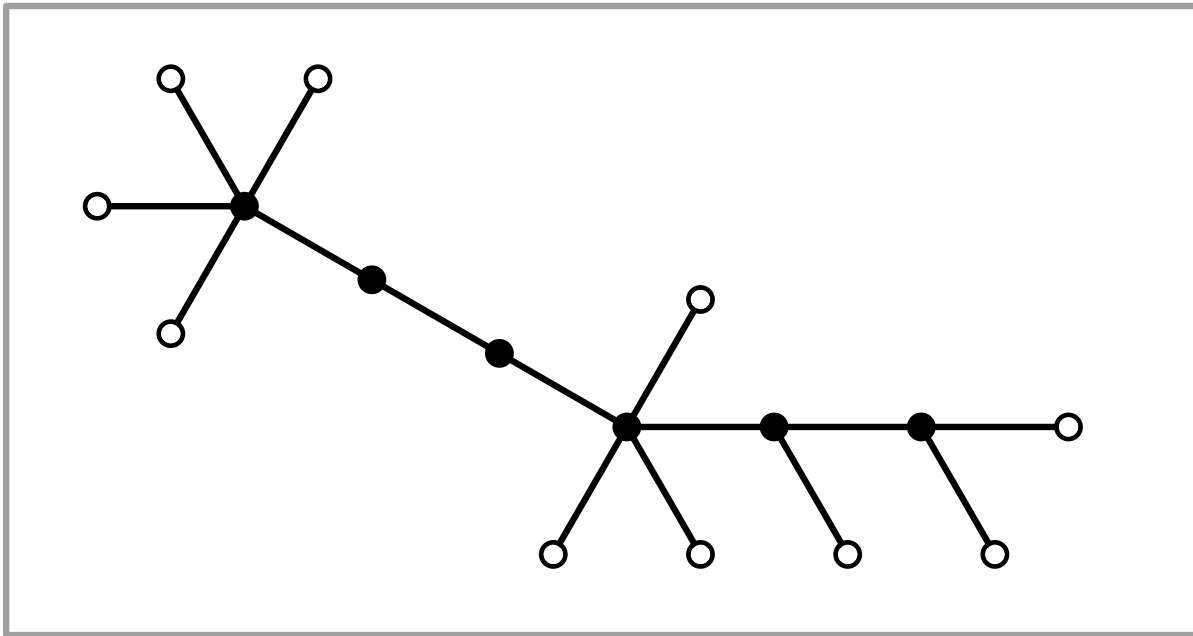


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*Konvexe* Zeichnung: Jede Facette ist konvex.

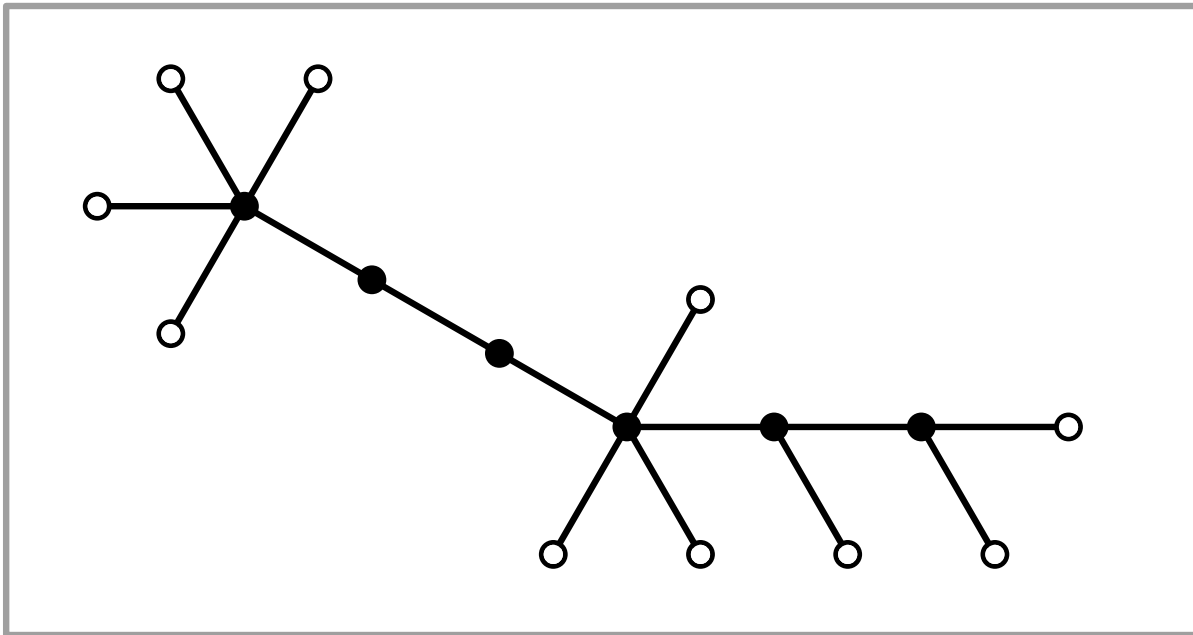
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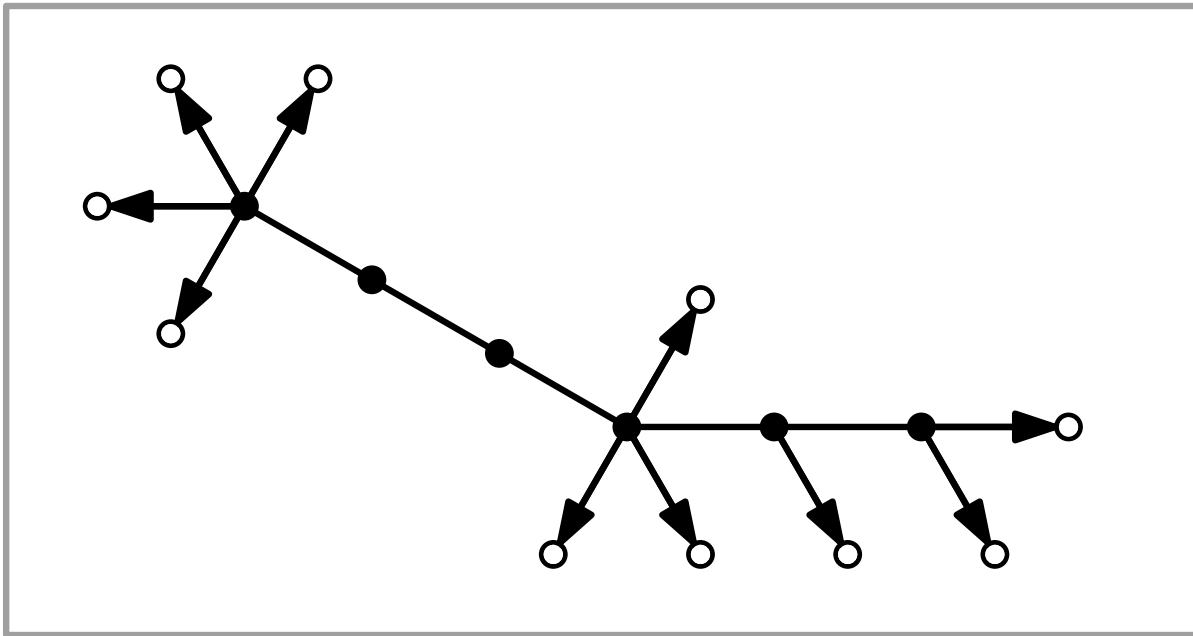
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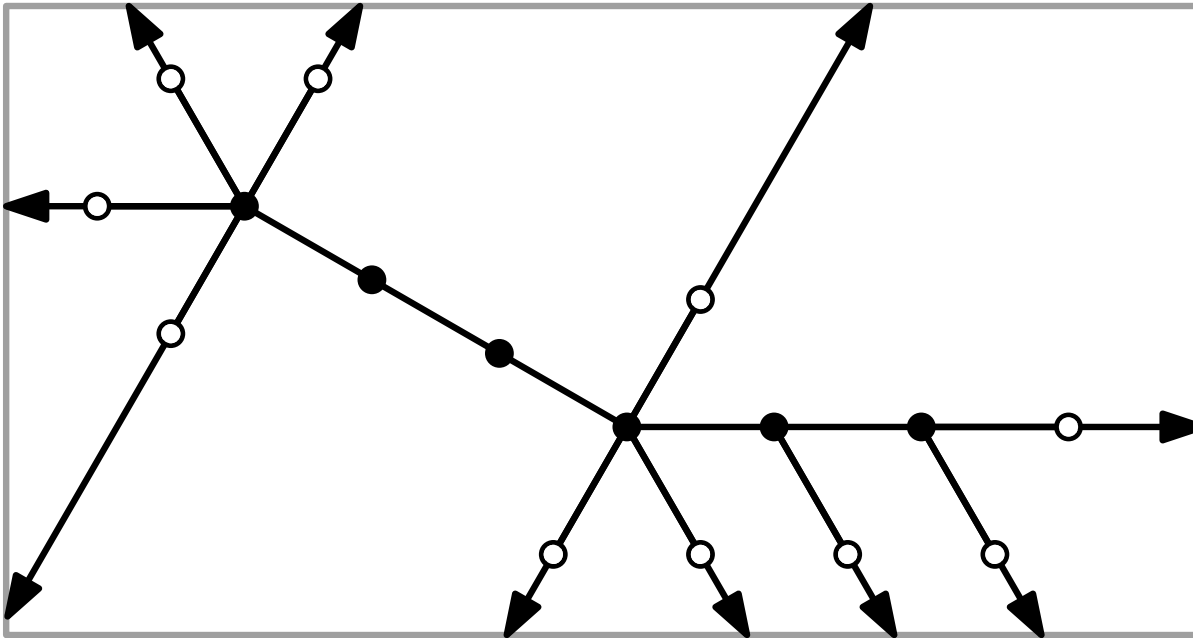
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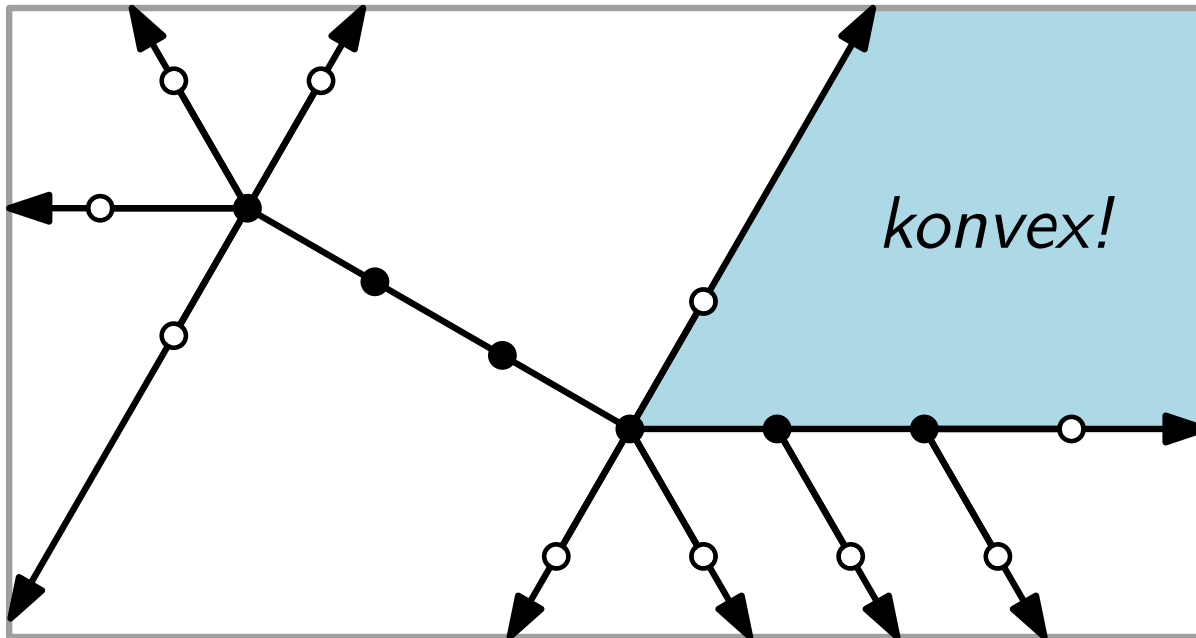
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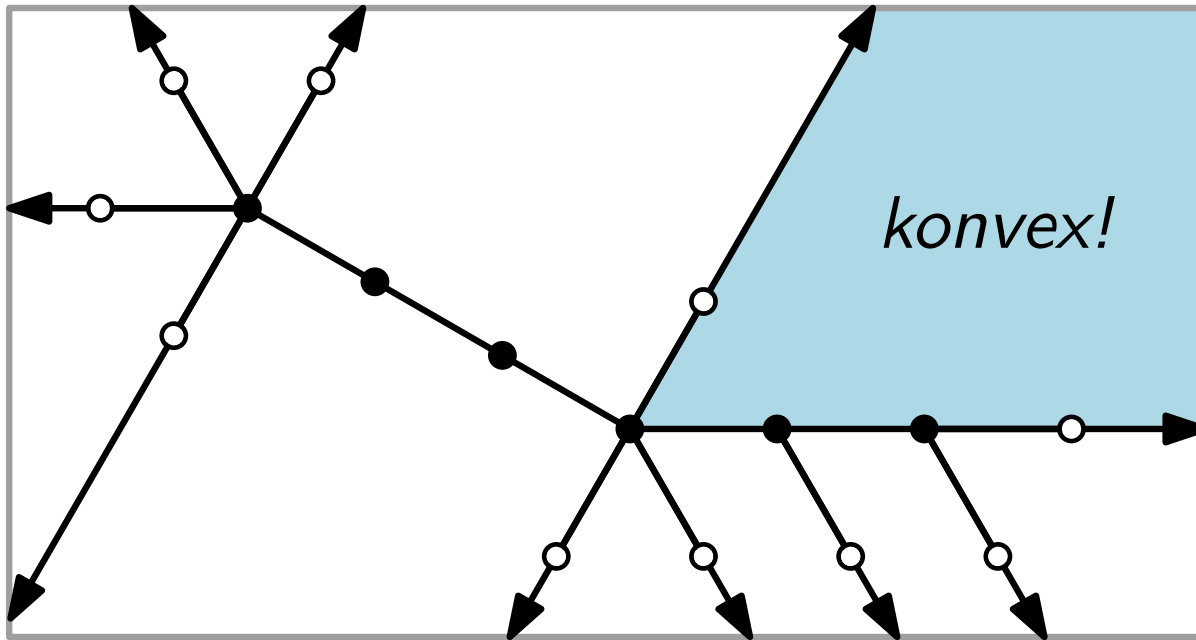
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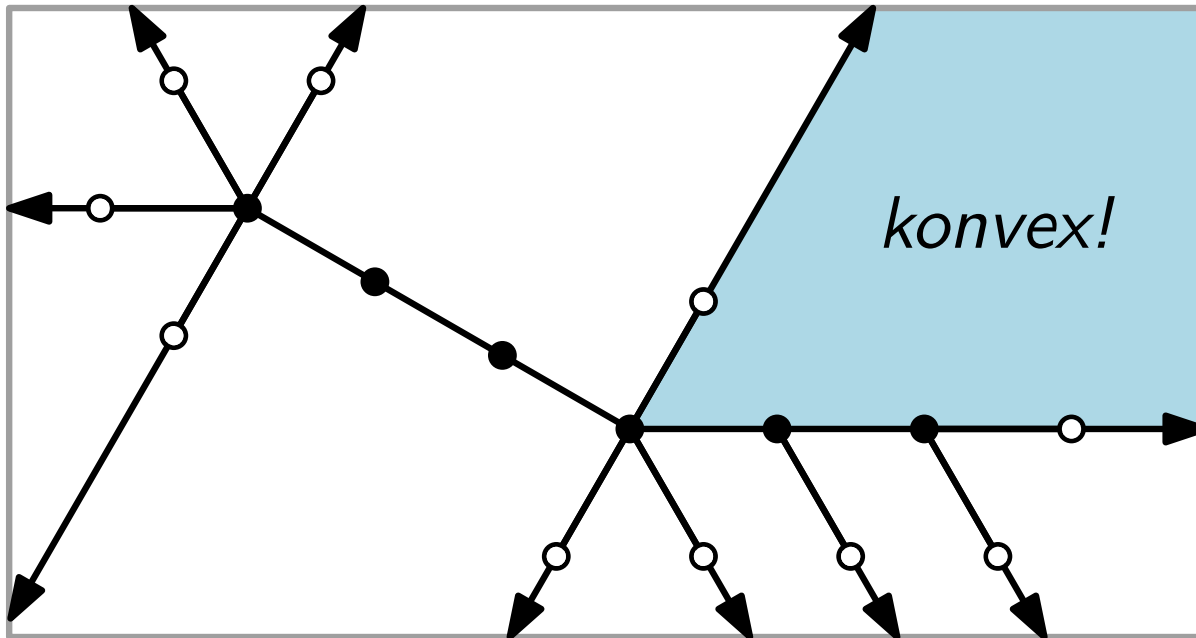
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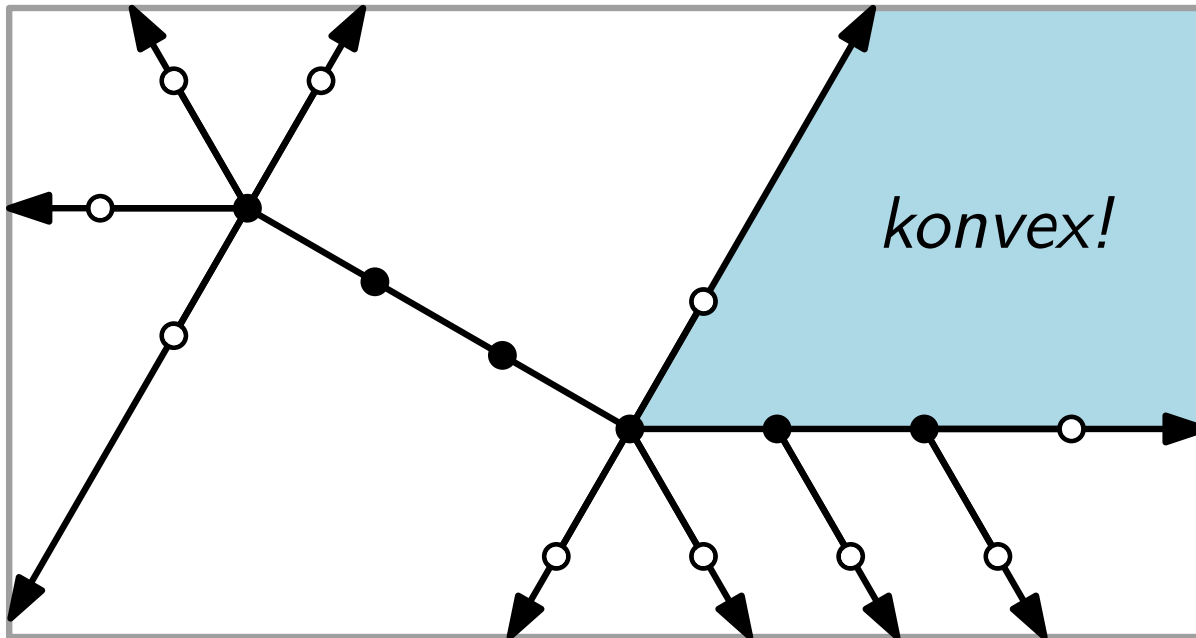


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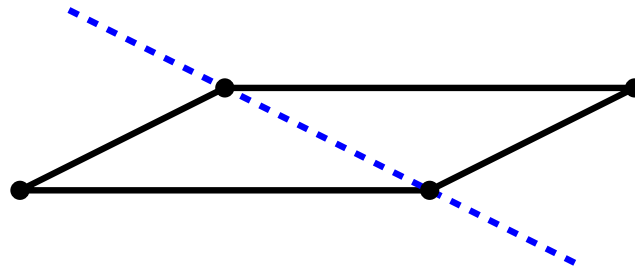


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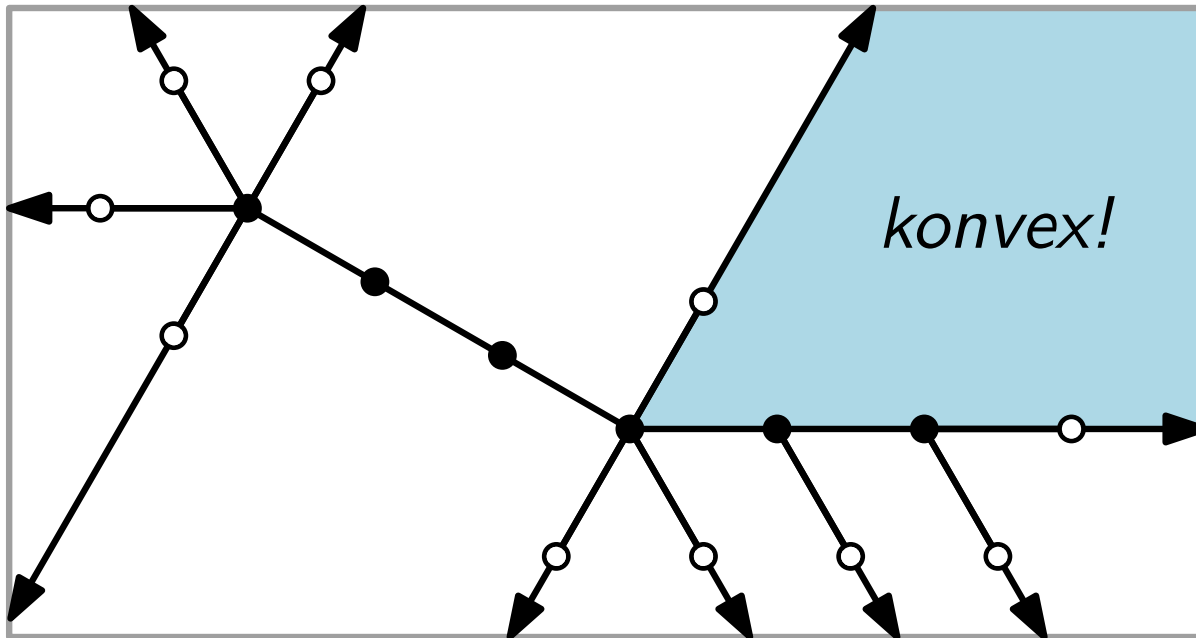


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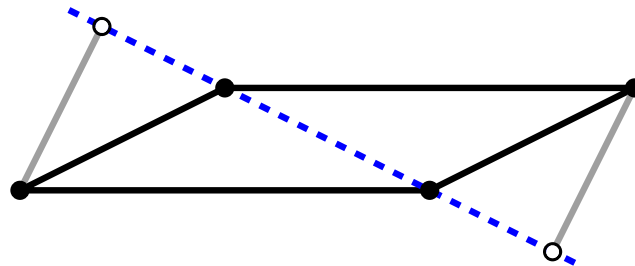


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Angelini et al.

[JGAA'12]

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Hossain & Rahman

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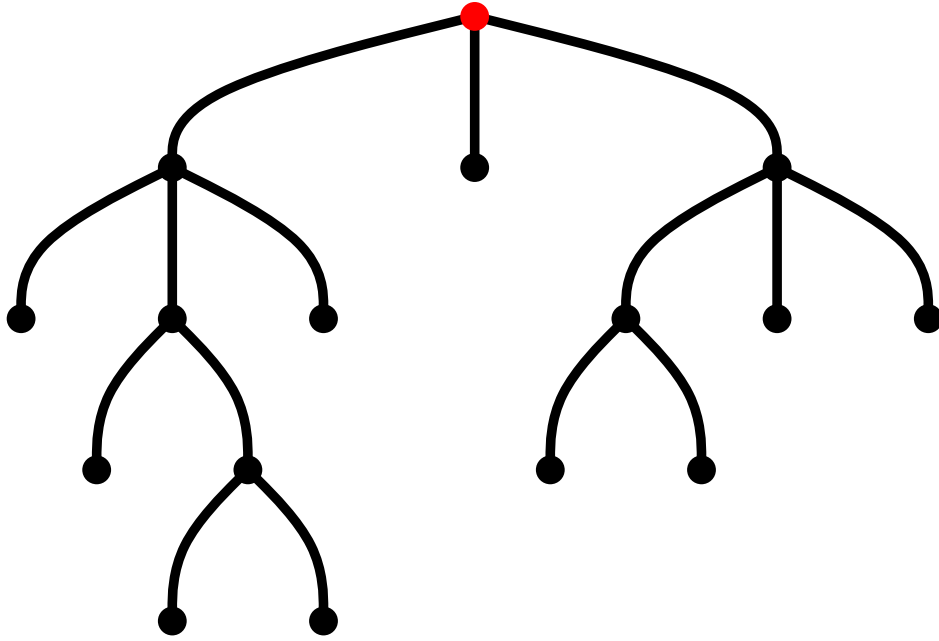
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Carlson & Eppstein

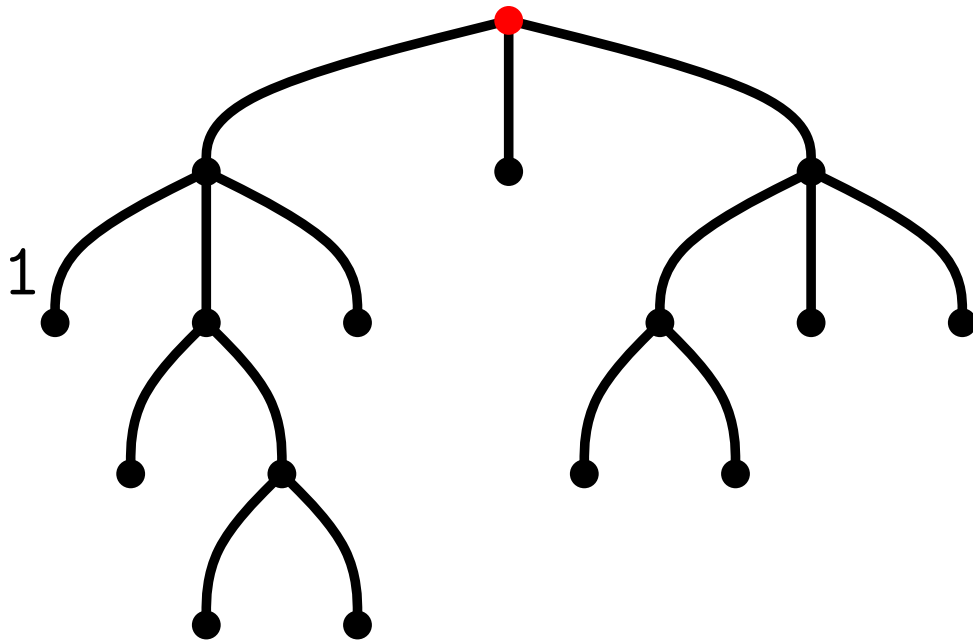
[GD'06]

*Baum*  $\Rightarrow$  konvexe Zeichnung, *optimaler Winkelauflösung*

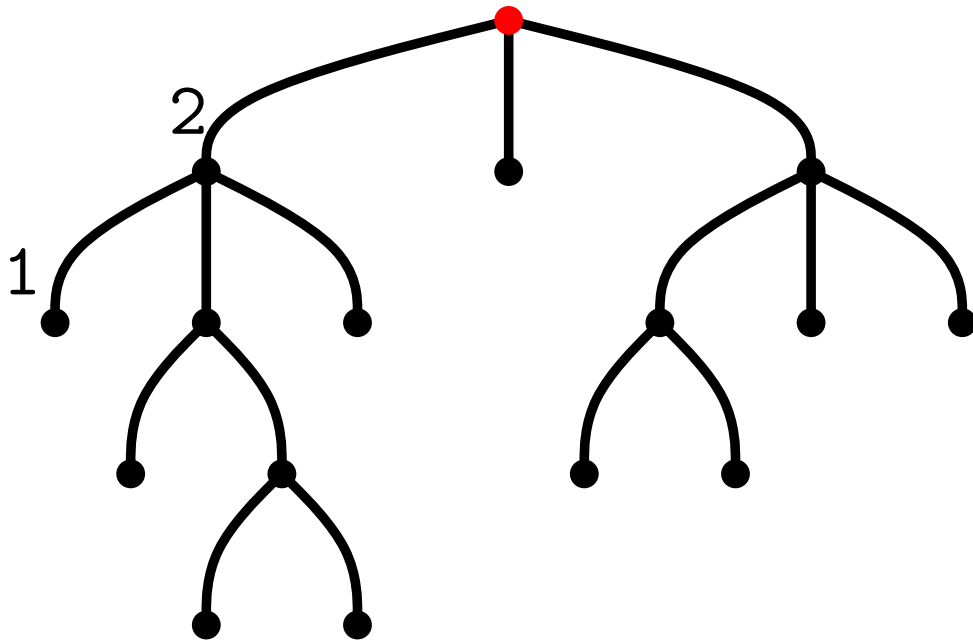
## Schritt I: Beschriftete Kanten



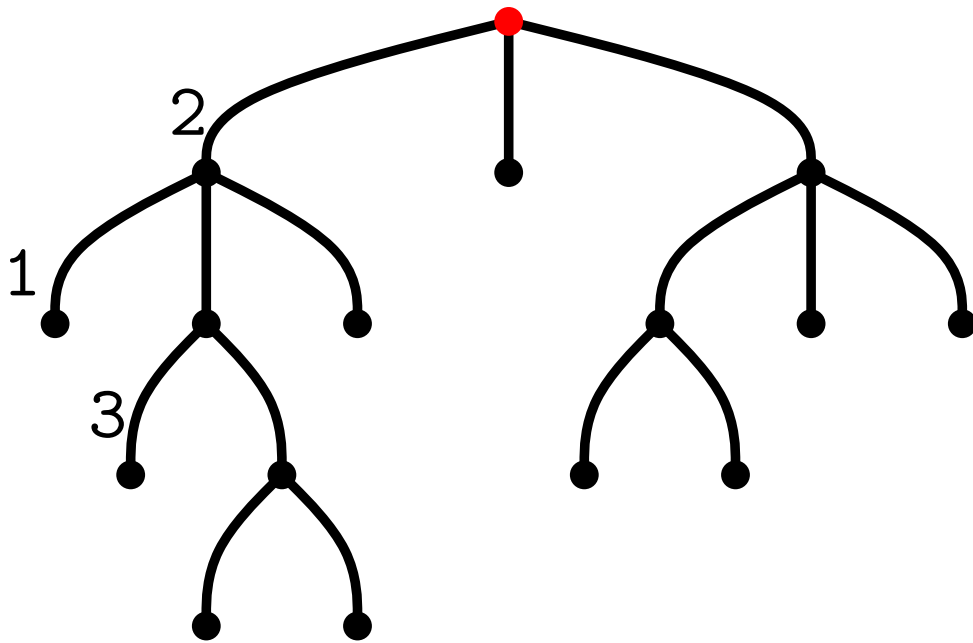
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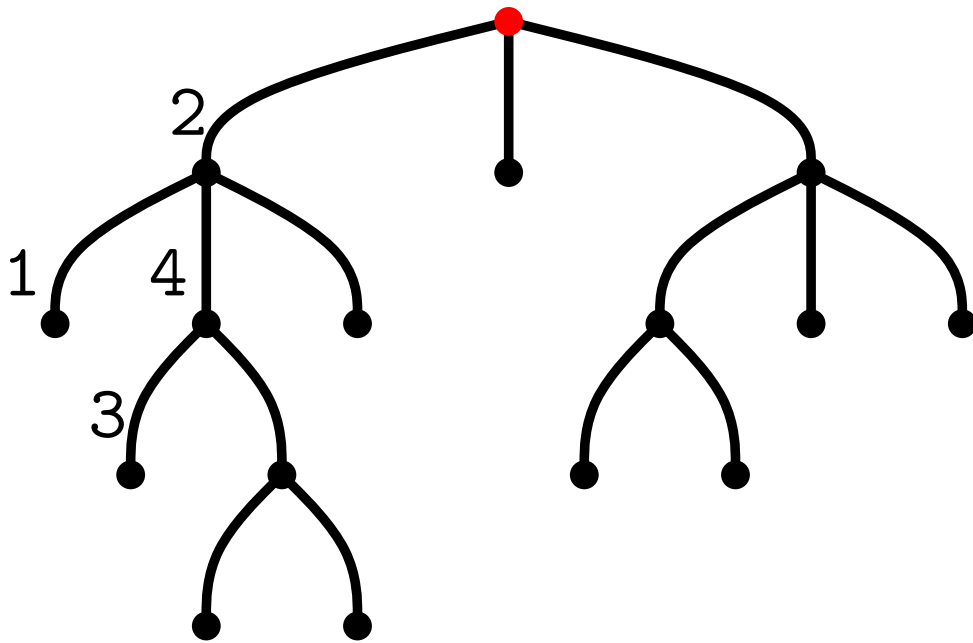


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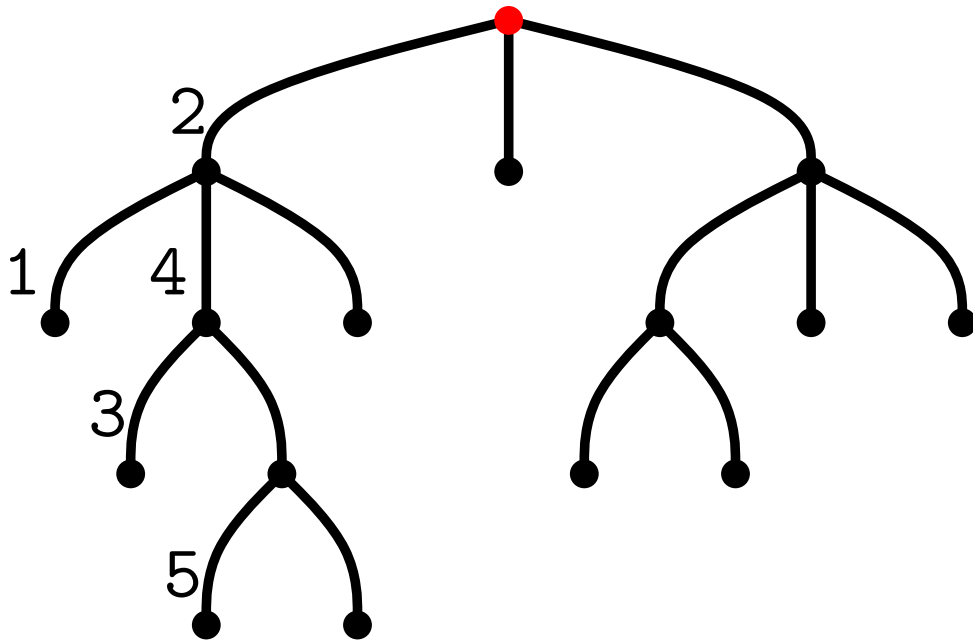




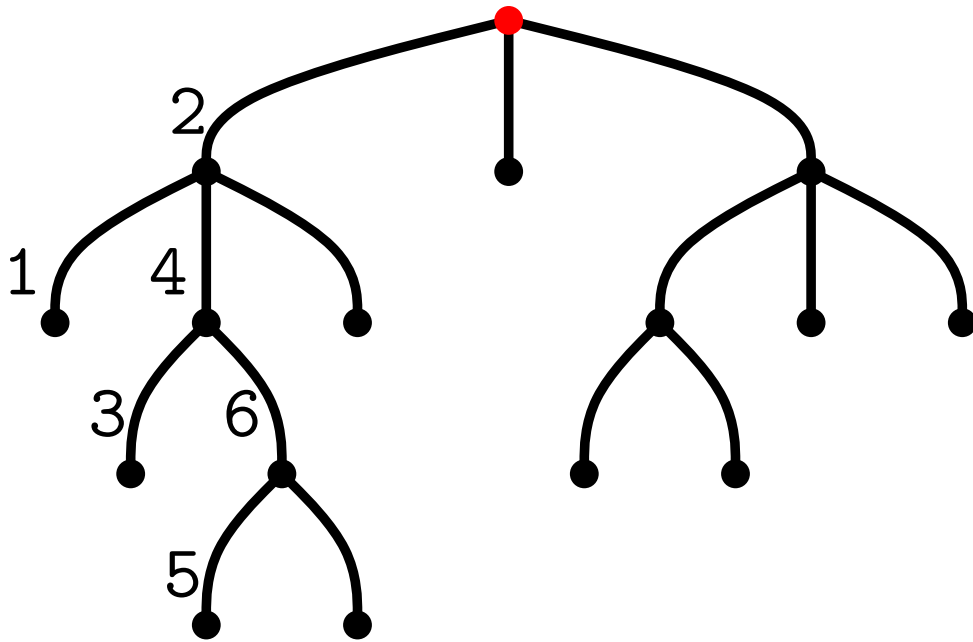
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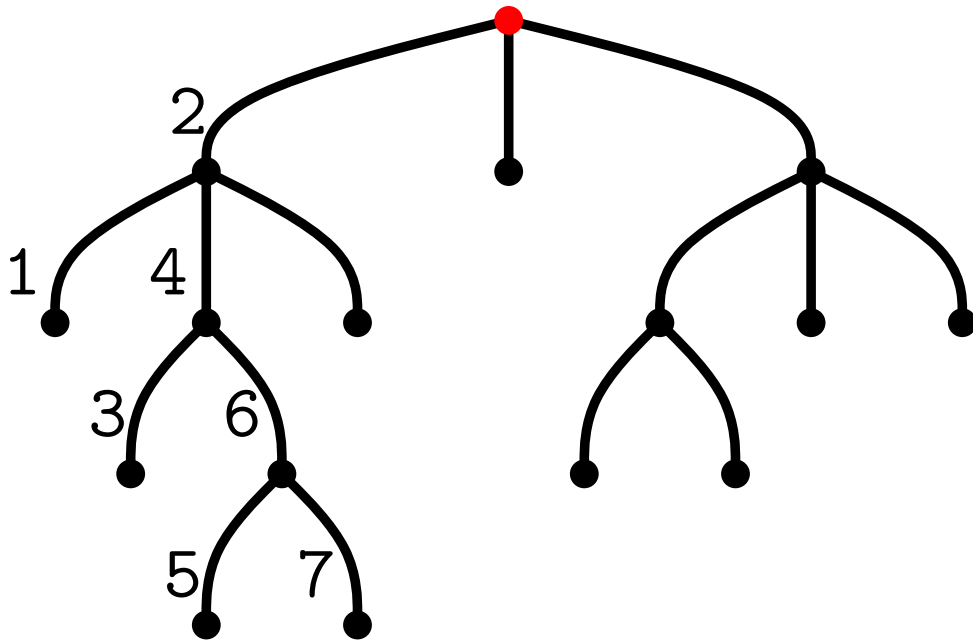
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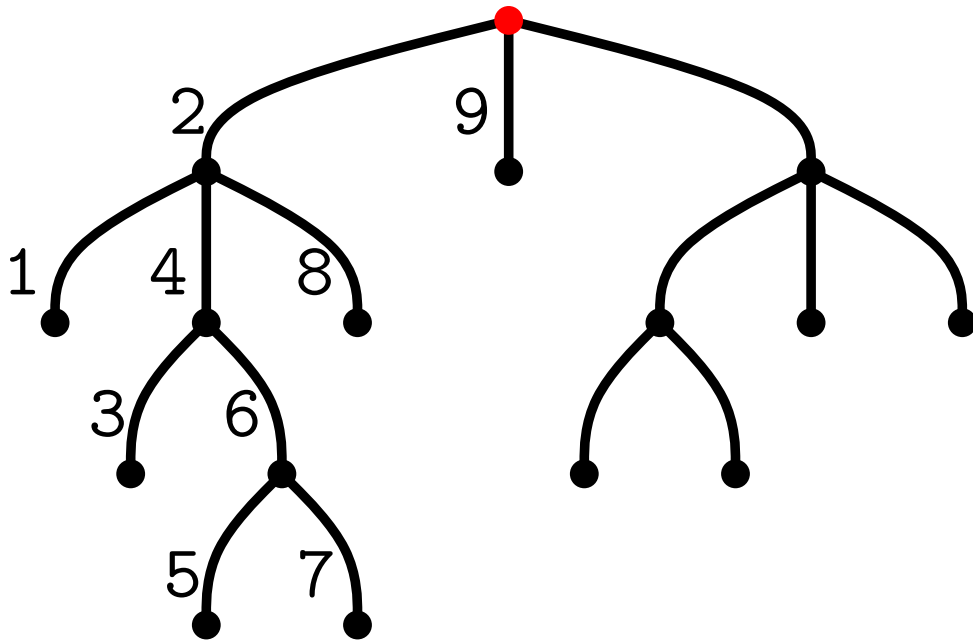
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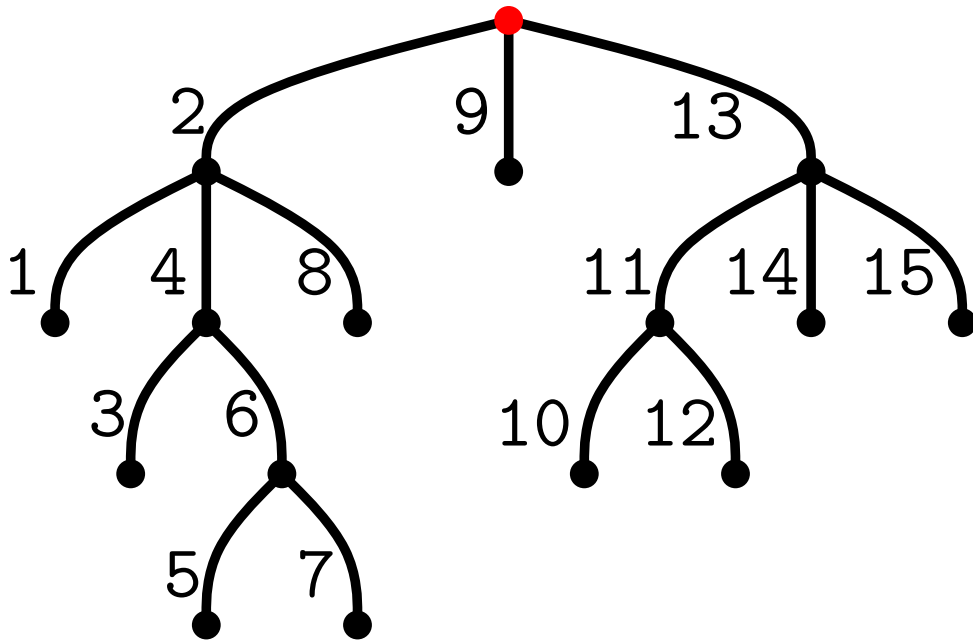
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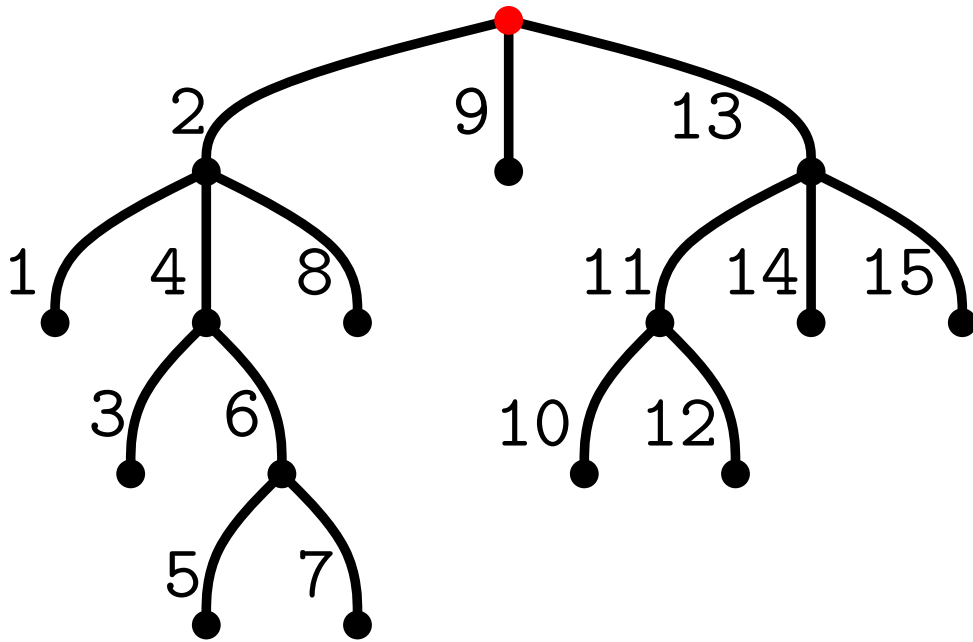
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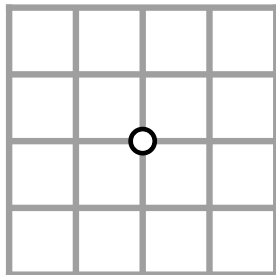
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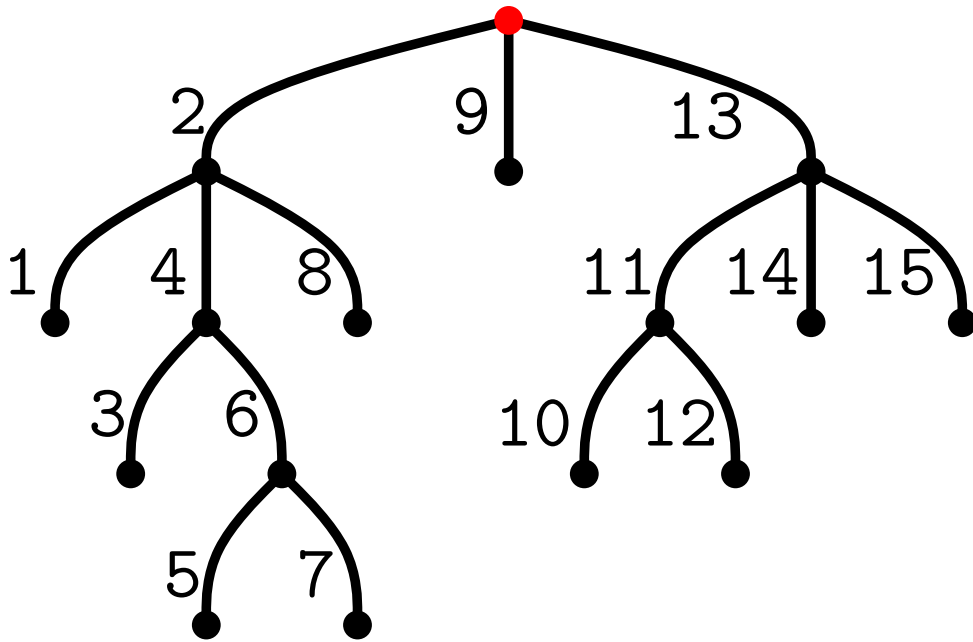
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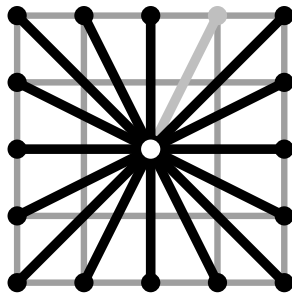
## Schritt II: Wähle Vektoren



## Schritt I: Beschrifte Kanten

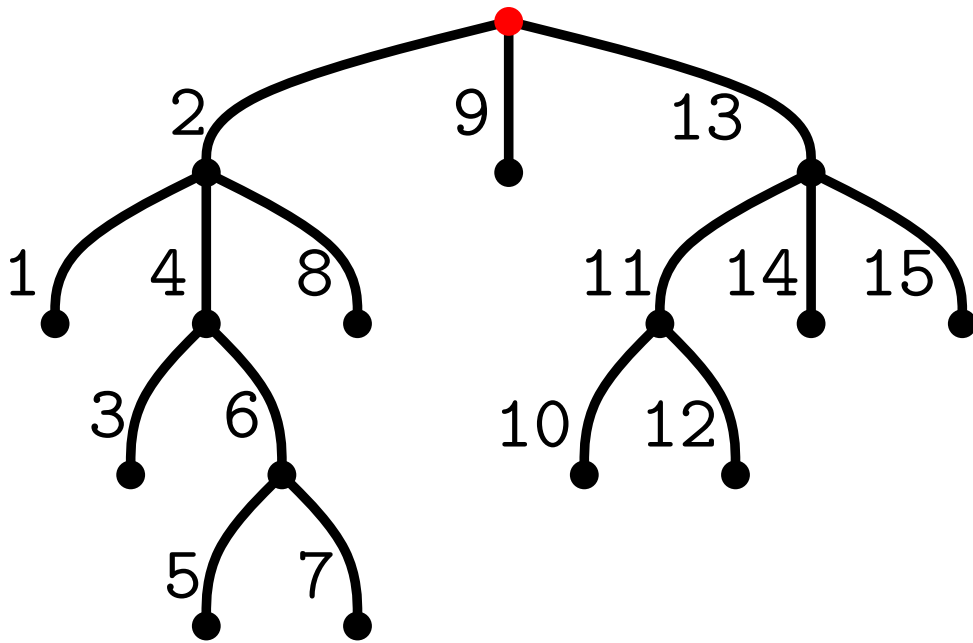


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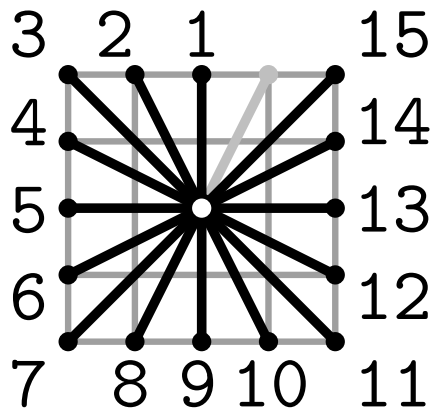




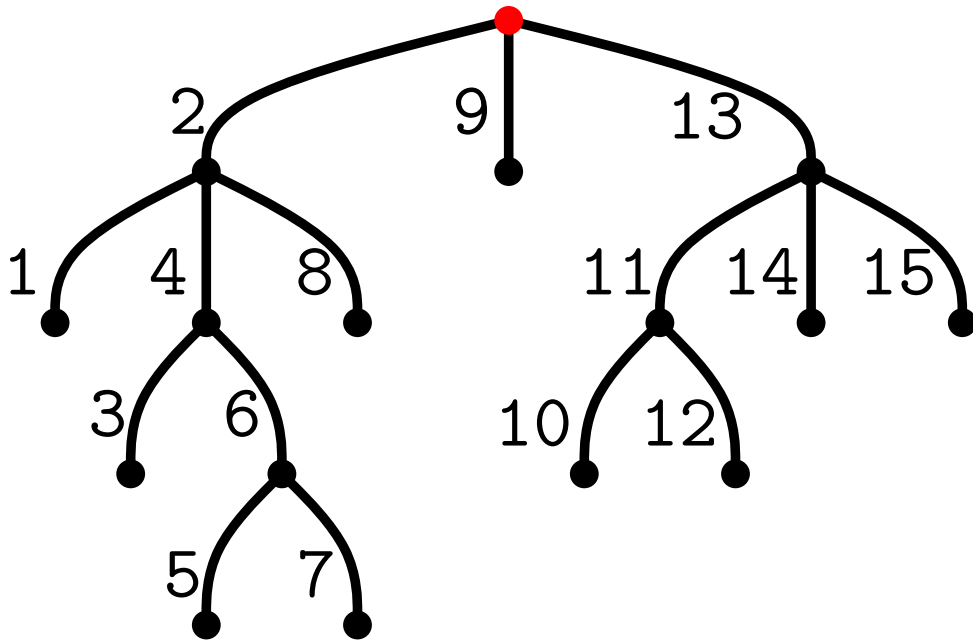
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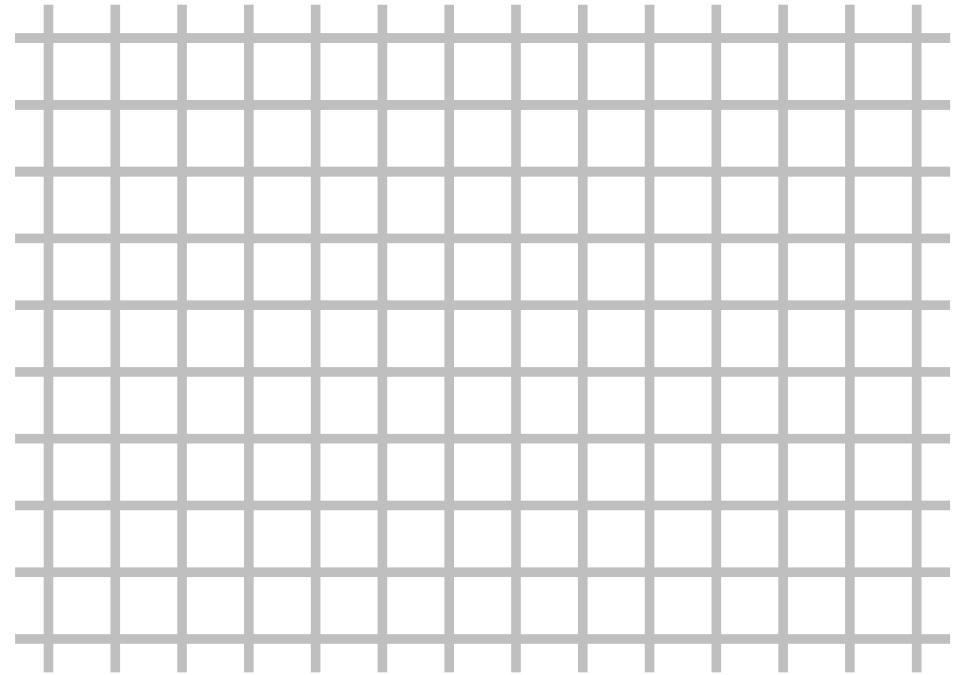
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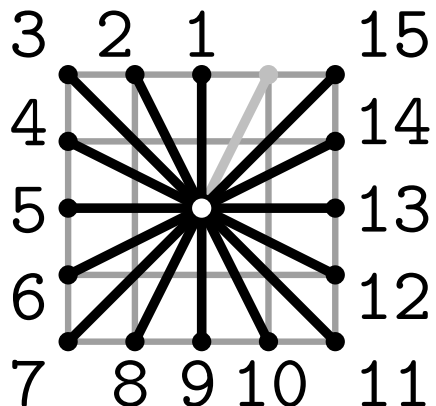
## Schritt I: Beschriftete Kanten



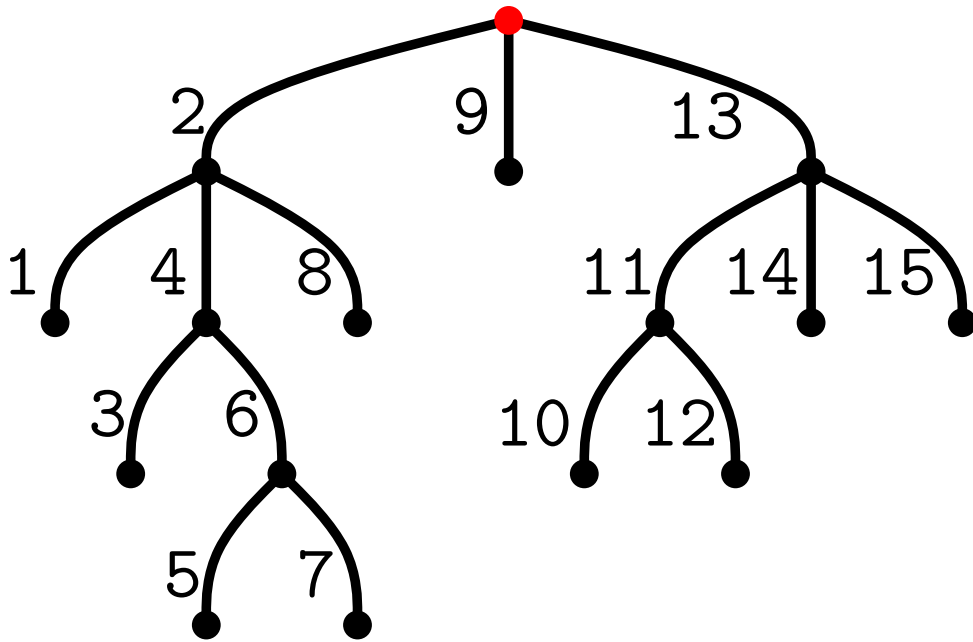
## Schritt III: Zeichne Baum



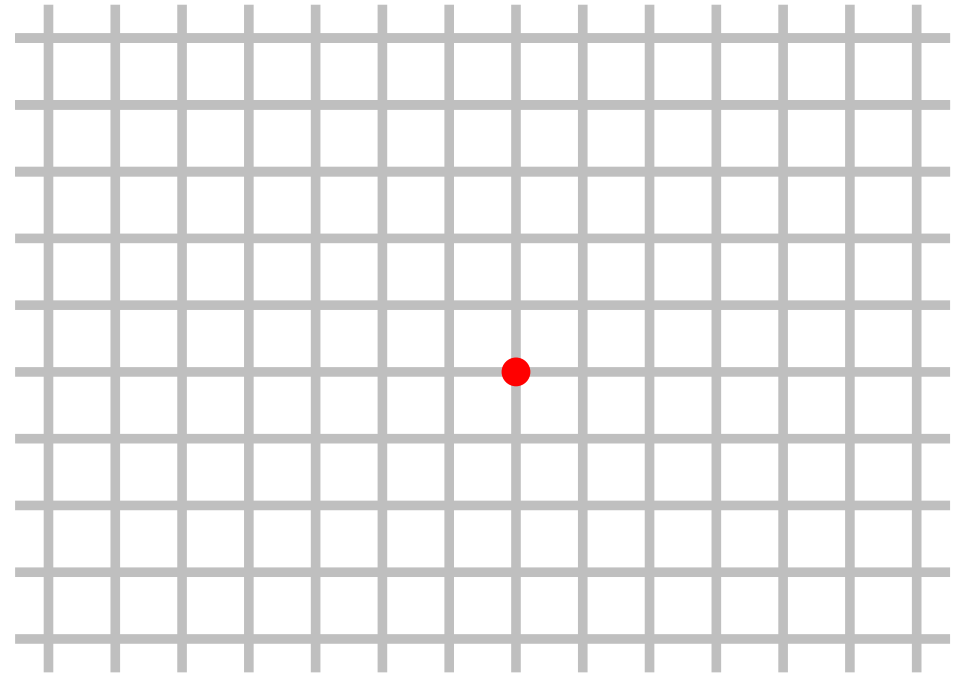
## Schritt II: Wähle Vektoren



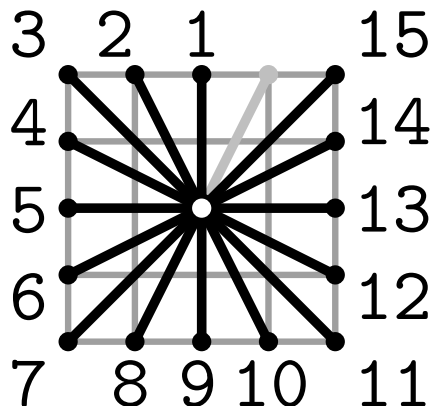
## Schritt I: Beschriftete Kanten



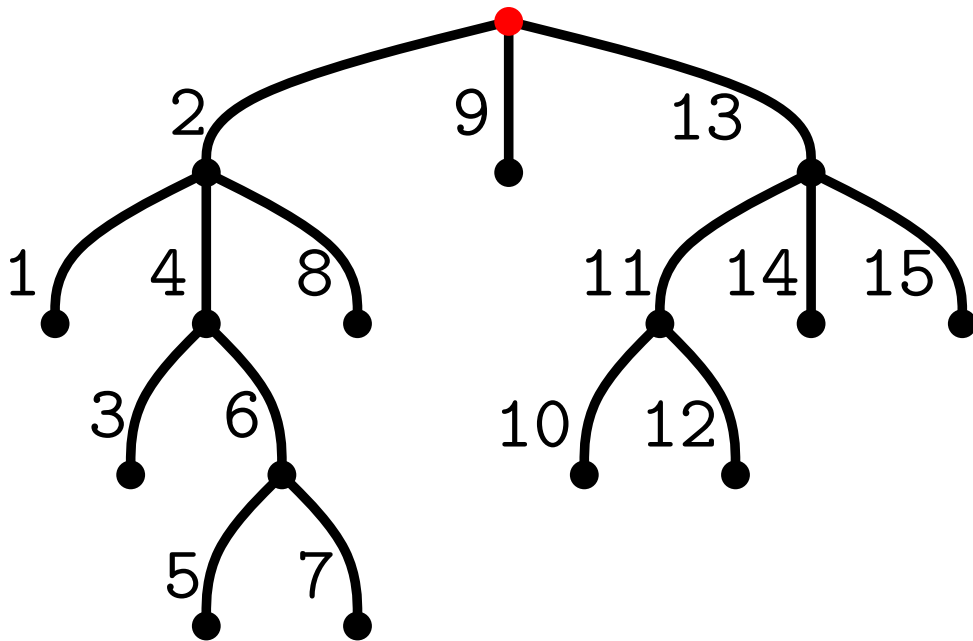
## Schritt III: Zeichne Baum



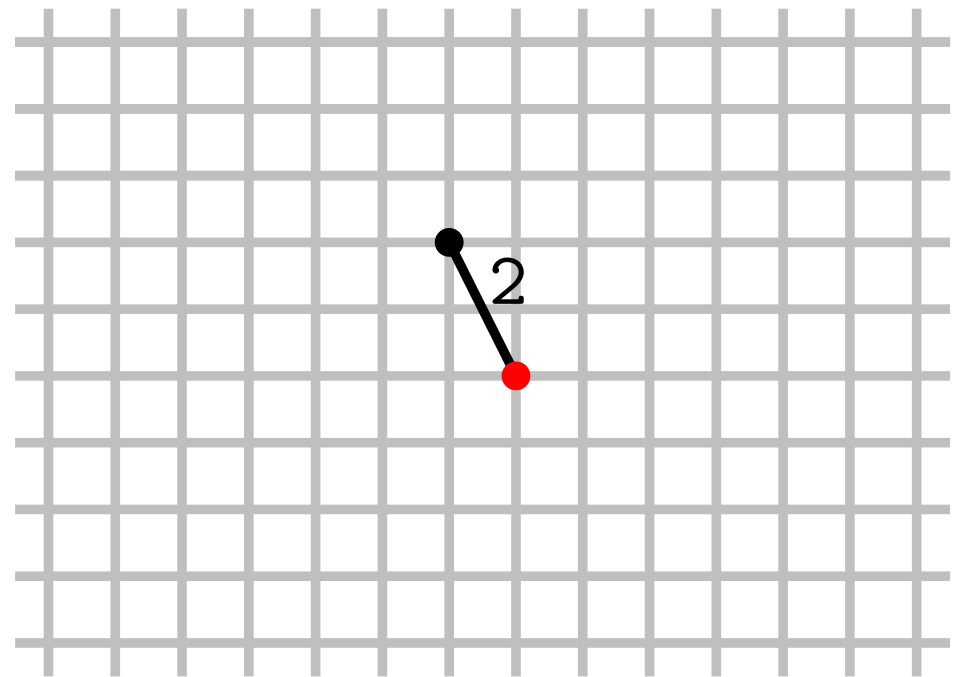
## Schritt II: Wähle Vektoren



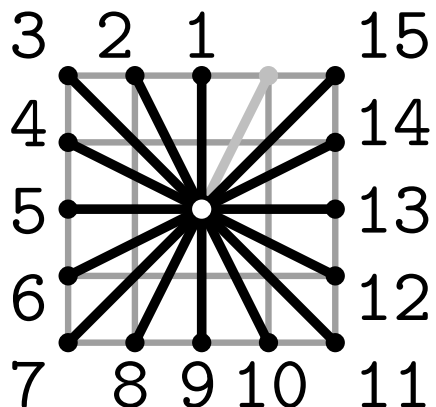
## Schritt I: Beschriftete Kanten



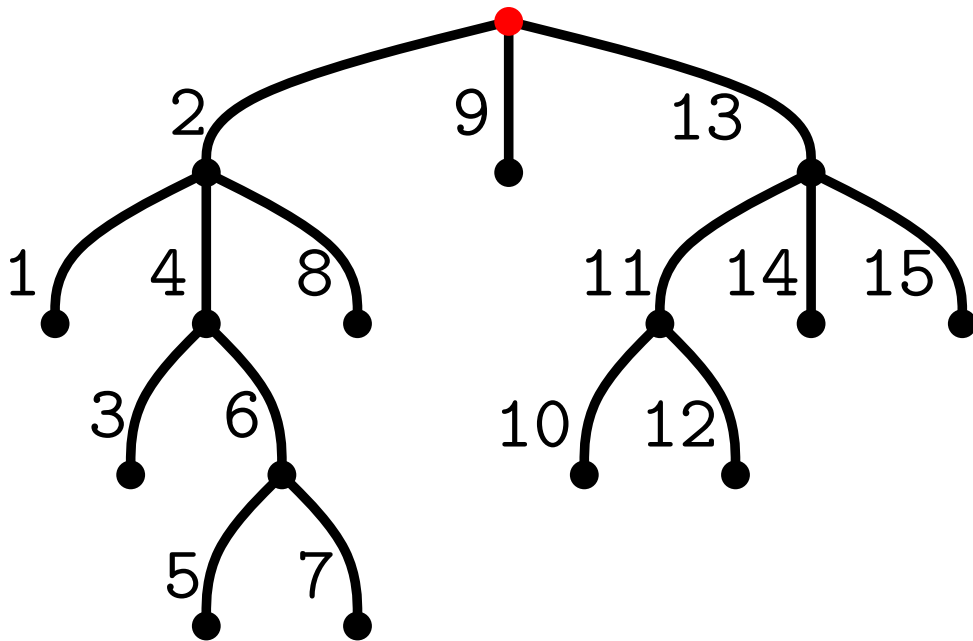
## Schritt III: Zeichne Baum



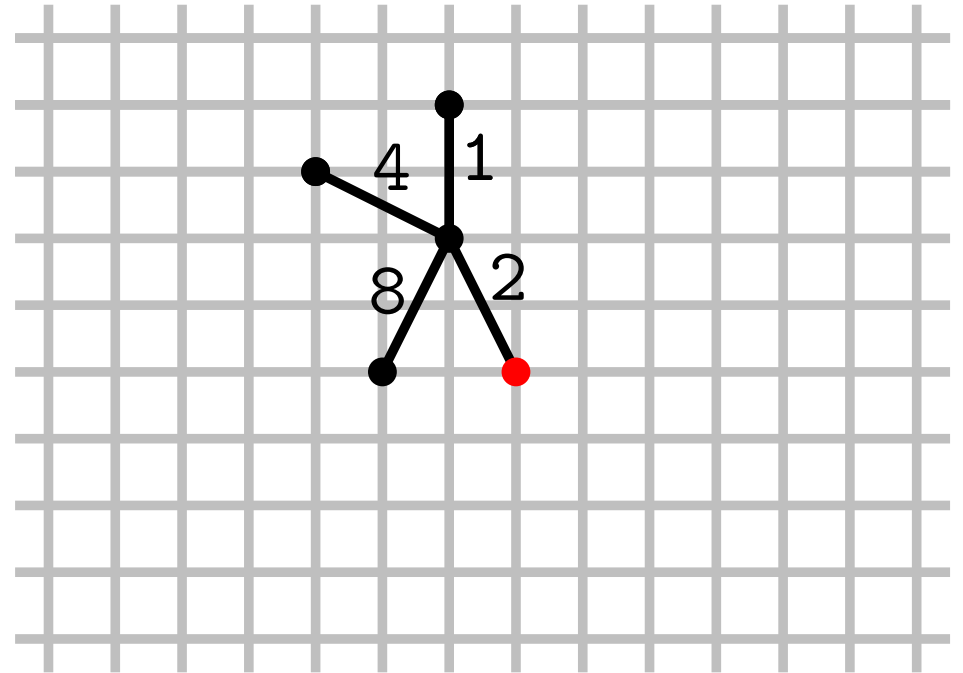
## Schritt II: Wähle Vektoren



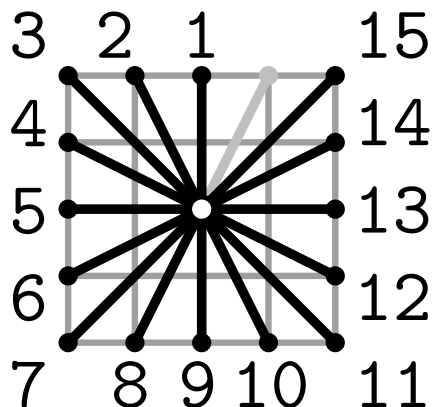
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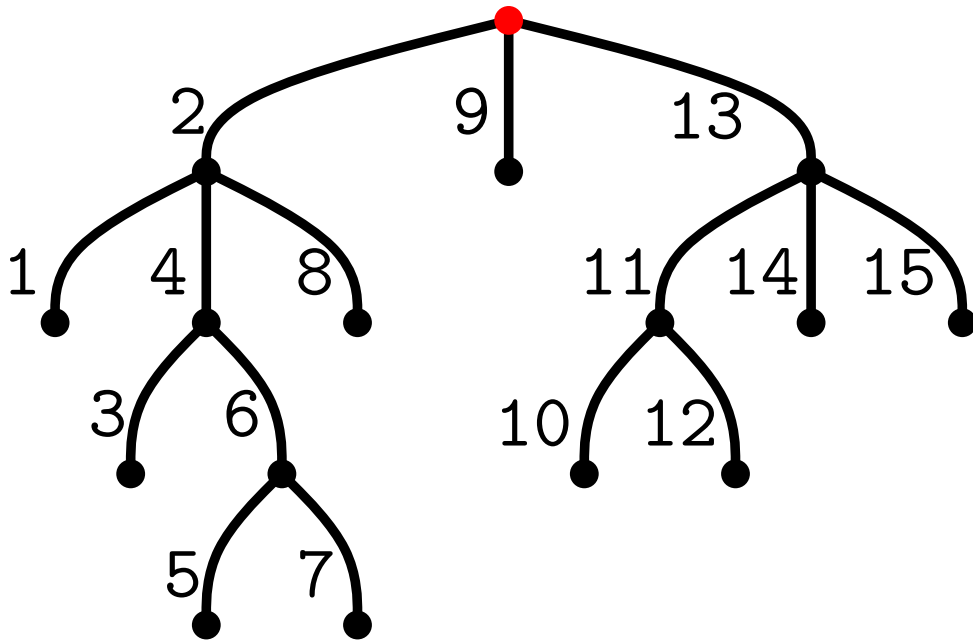
## Schritt III: Zeichne Baum



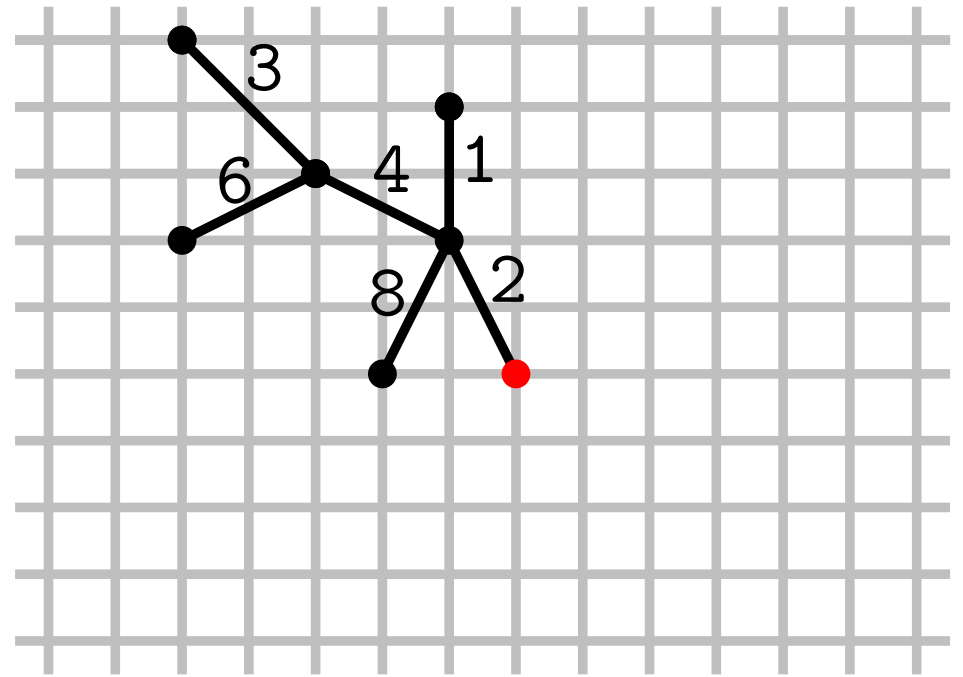
## Schritt II: Wähle Vektoren



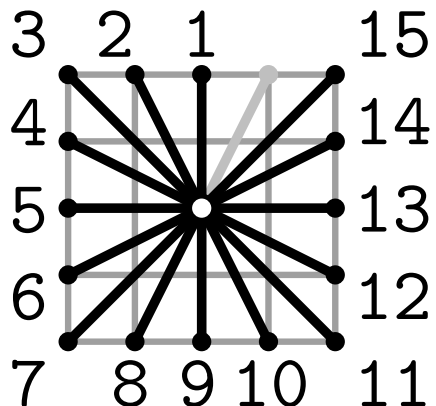
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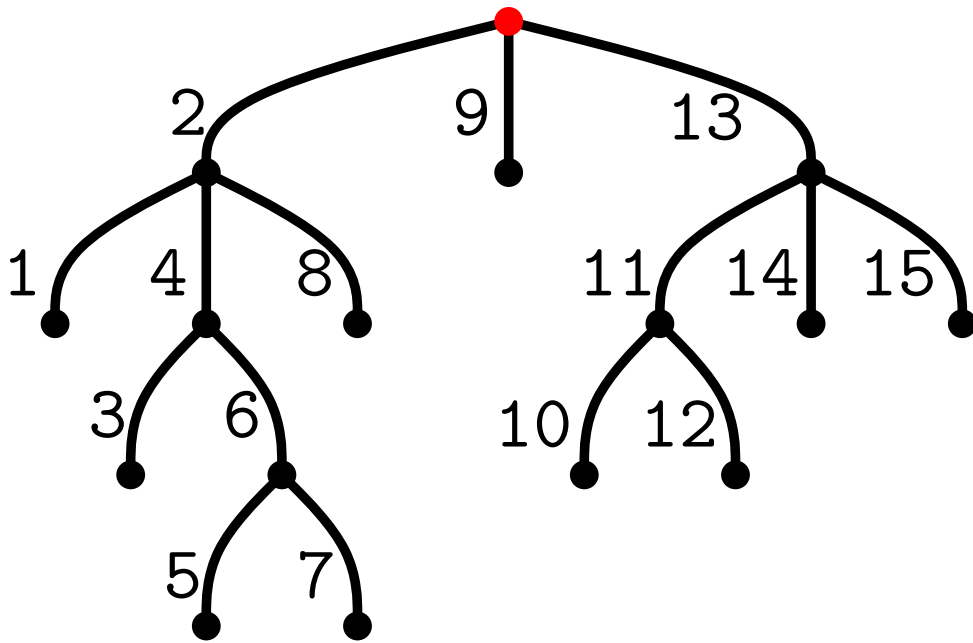
## Schritt III: Zeichne Baum



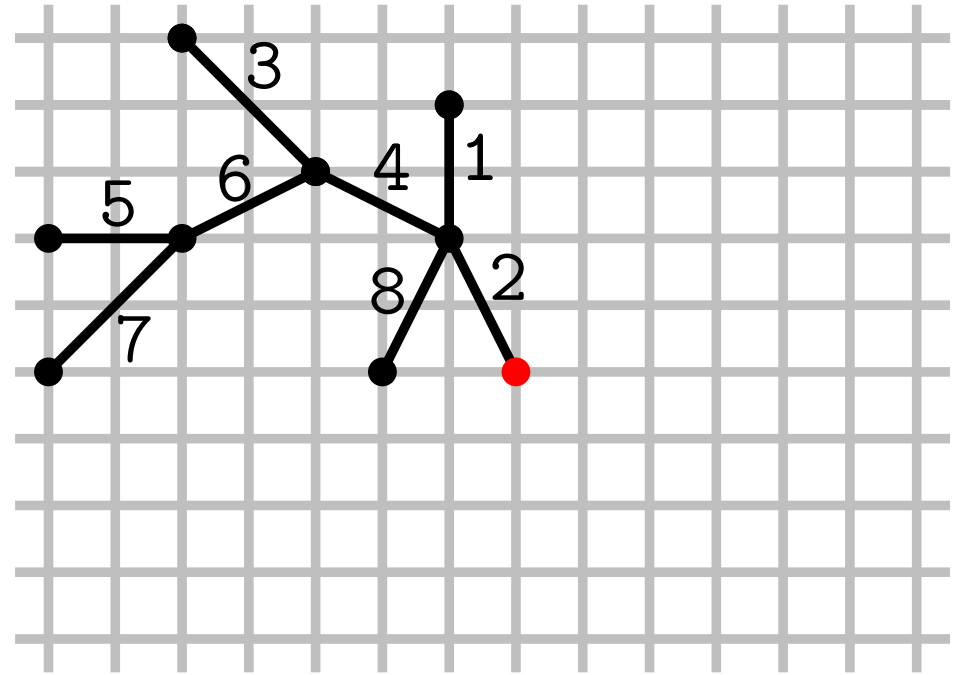
## Schritt II: Wähle Vektoren



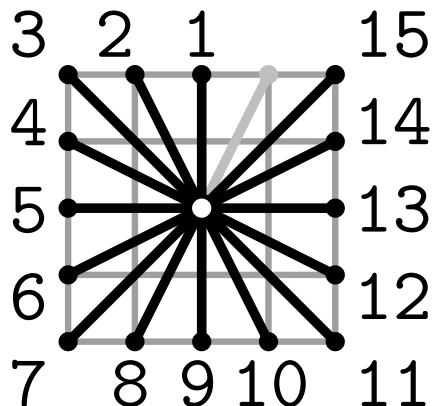
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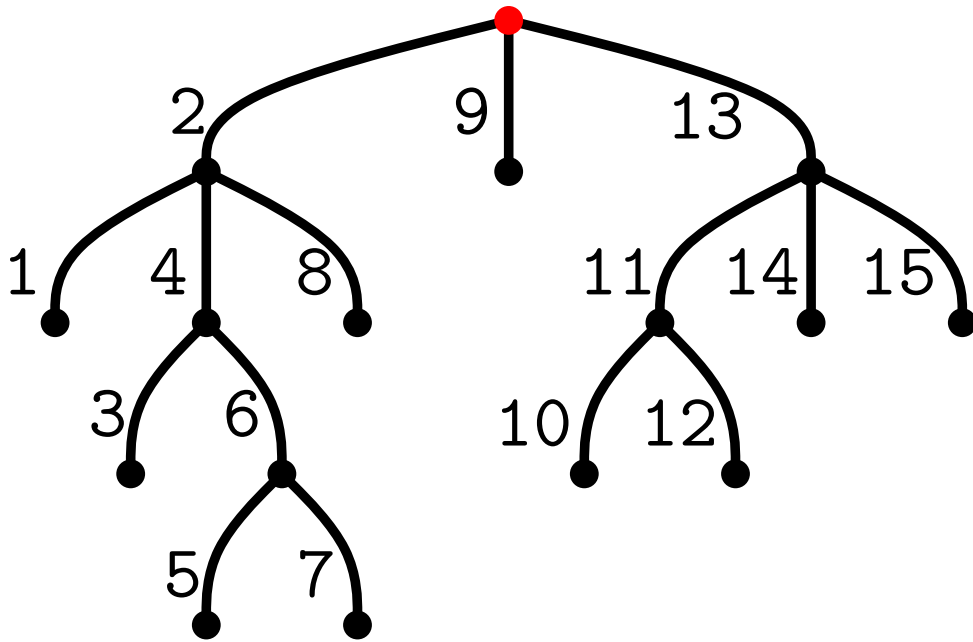
## Schritt III: Zeichne Baum



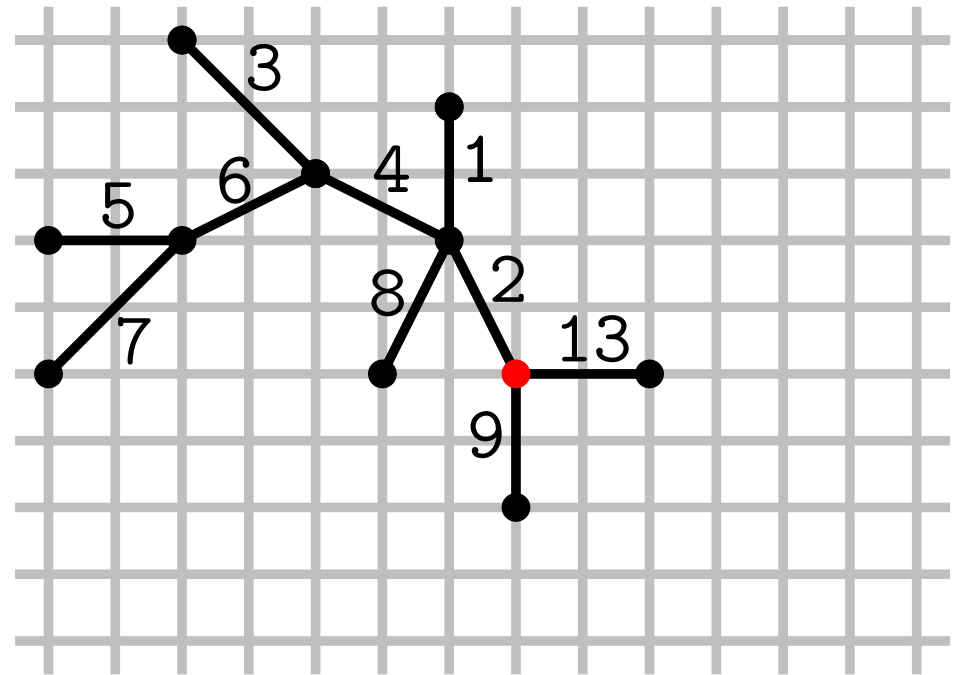
## Schritt II: Wähle Vektoren



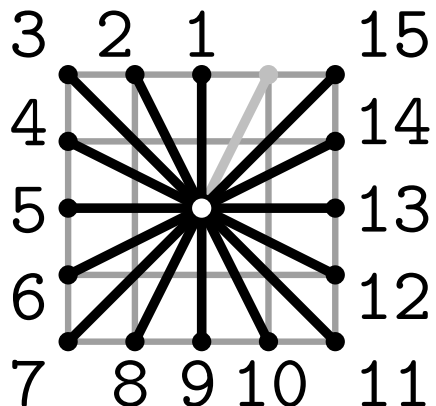
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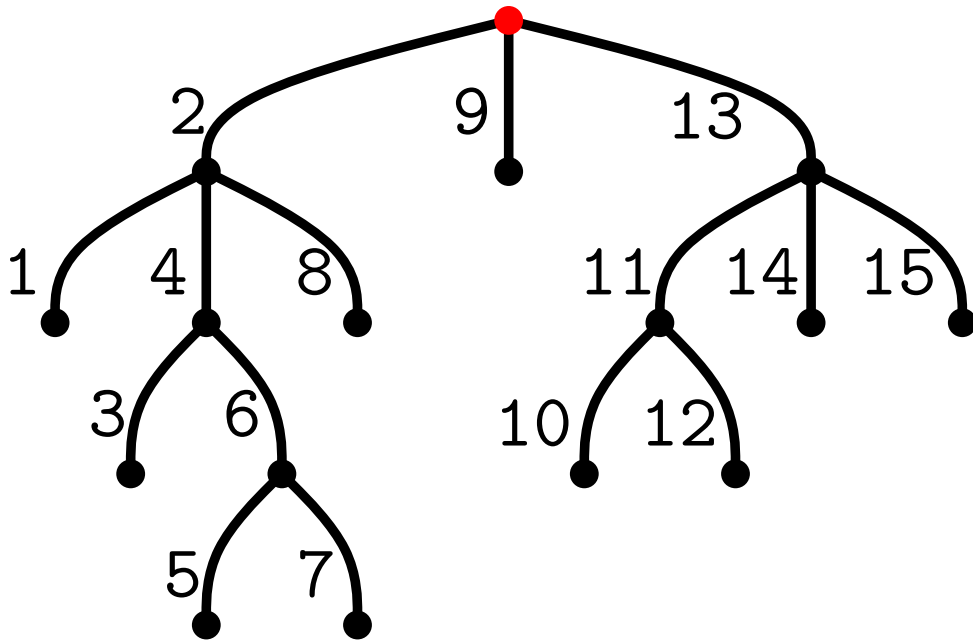


## Schritt II: Wähle Vektoren

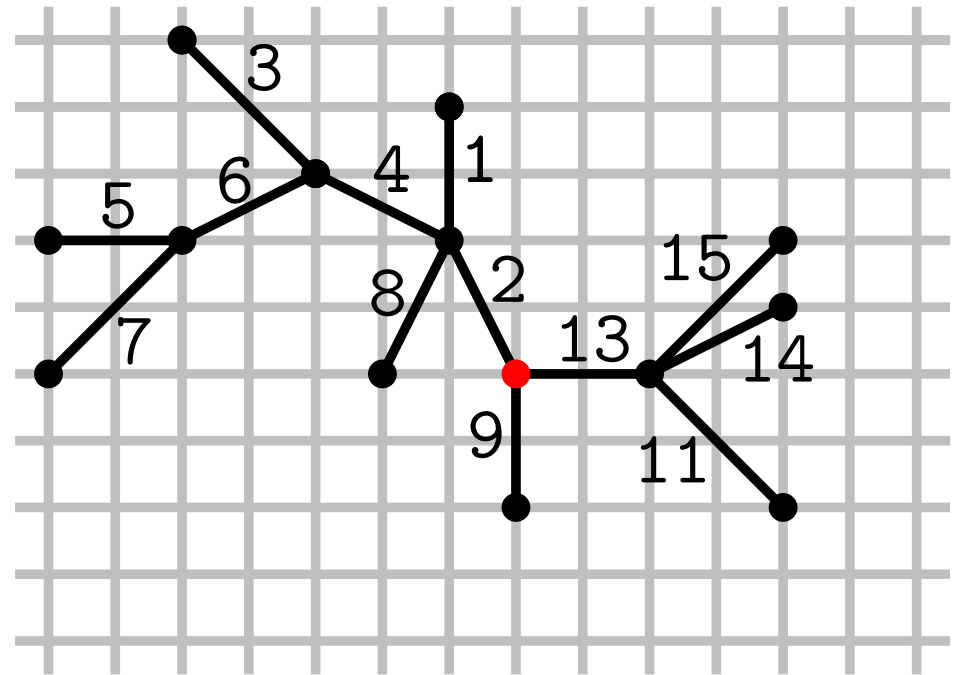




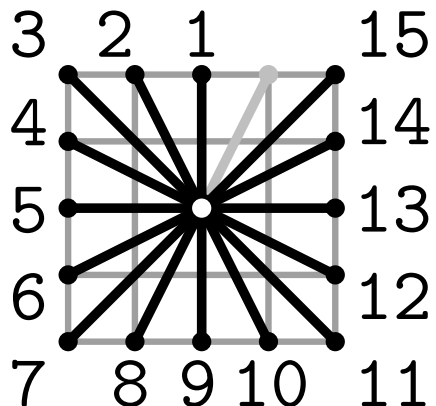
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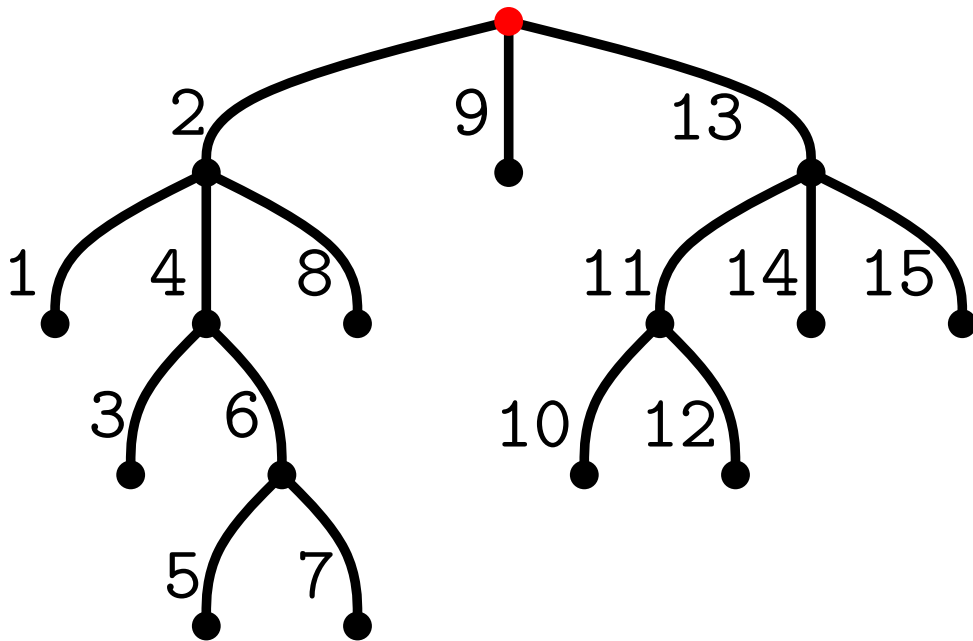
## Schritt III: Zeichne Baum



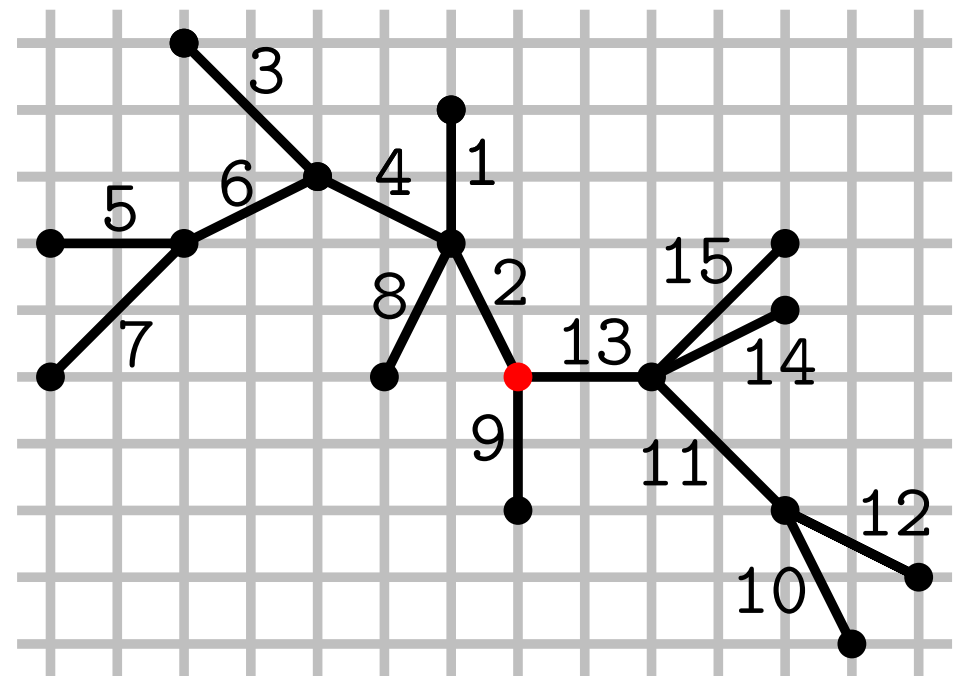
## Schritt II: Wähle Vektoren



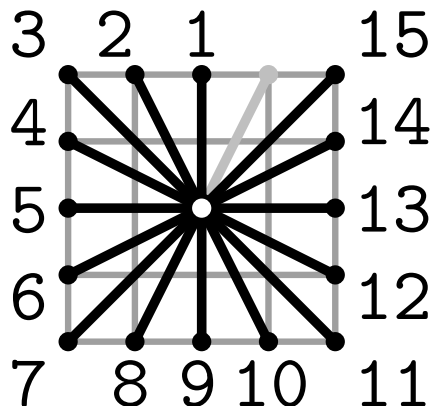
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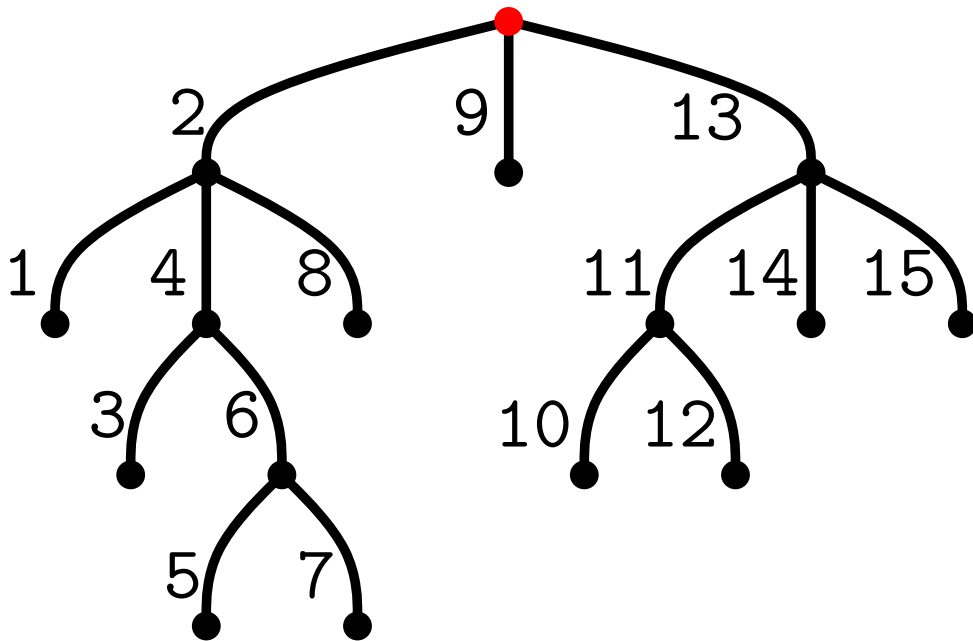
## Schritt III: Zeichne Baum



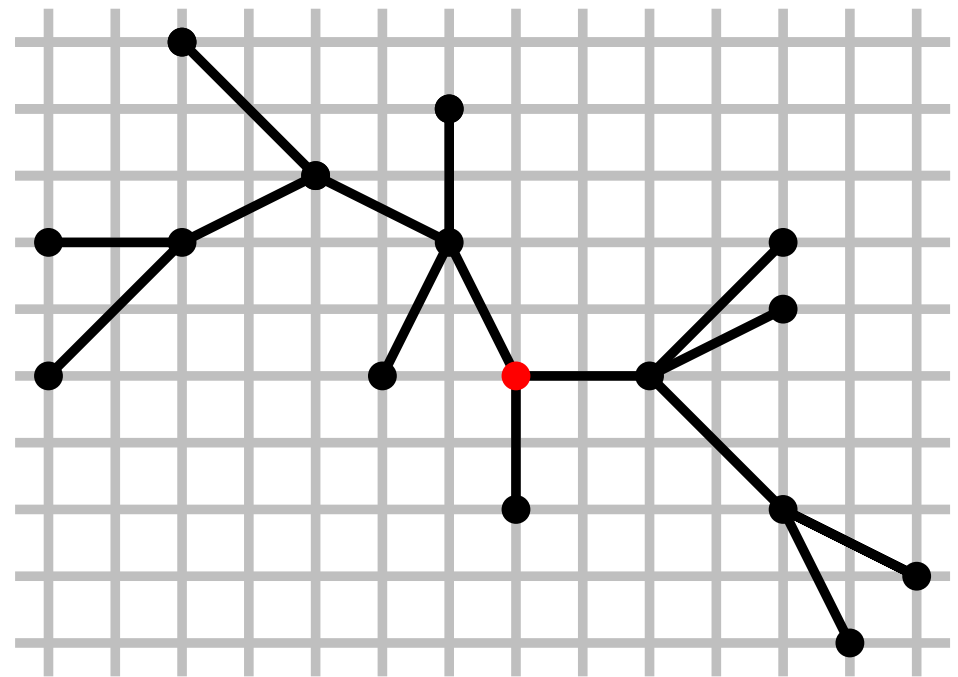
## Schritt II: Wähle Vektoren



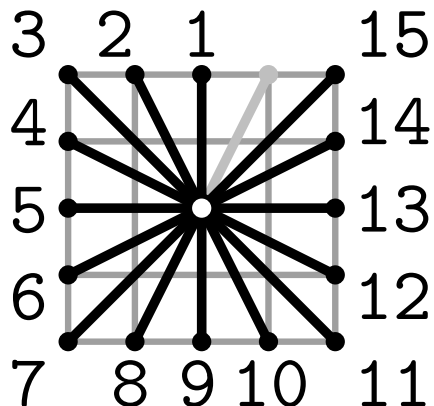
## Schritt I: Beschriftete Kanten



## Schritt III: Zeichne Baum



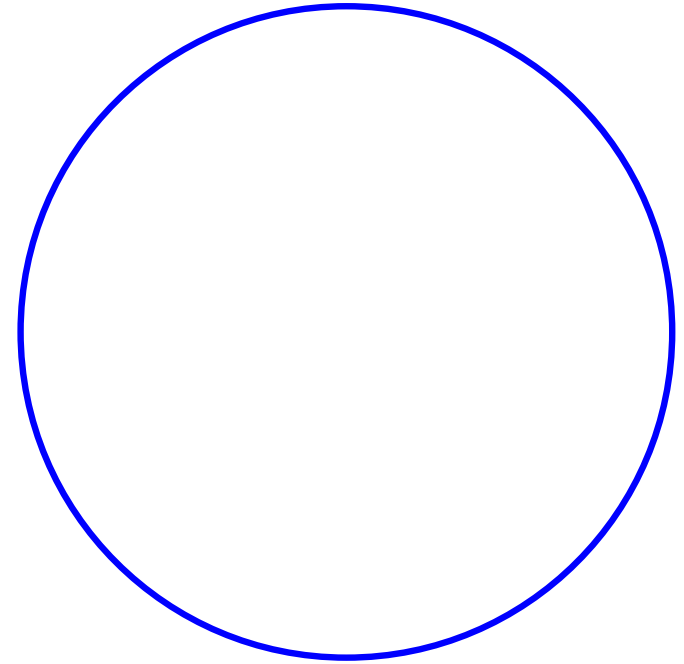
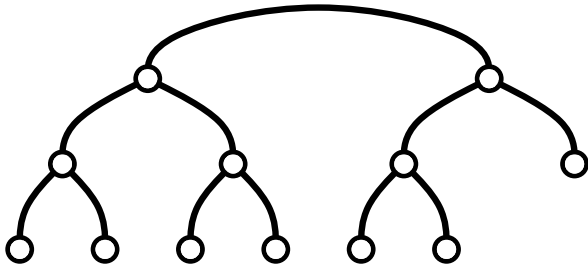
## Schritt II: Wähle Vektoren



Baum  $\Rightarrow$   
monotone konvexe Zeichnung,  
 $O(n^{1.5}) \times O(n^{1.5})$  Gitter

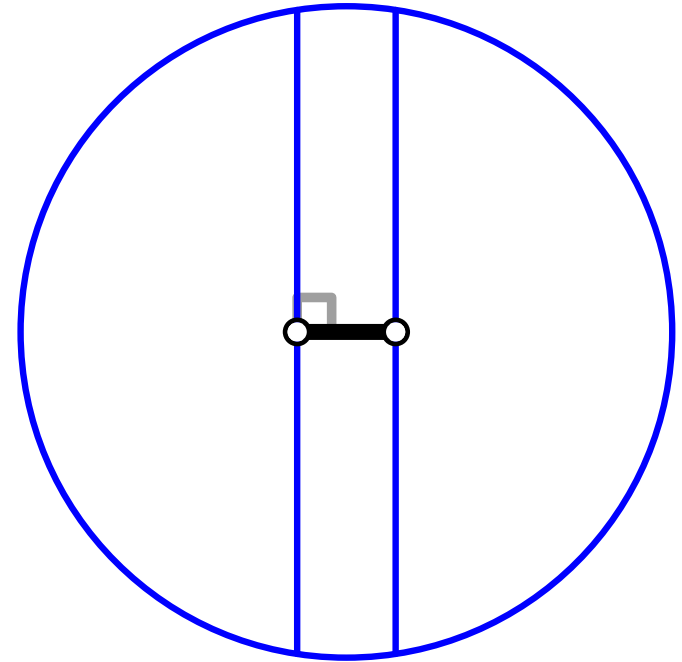
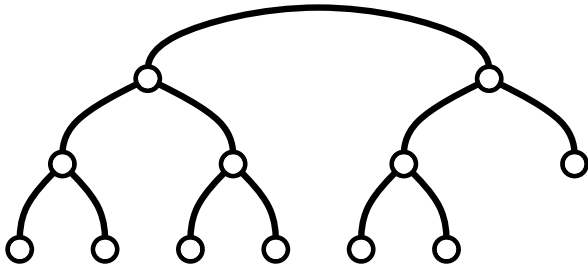
# Streng Monotone Zeichnungen

*Ordentlicher Binärbaum:*  
Kein Grad-2 Knoten



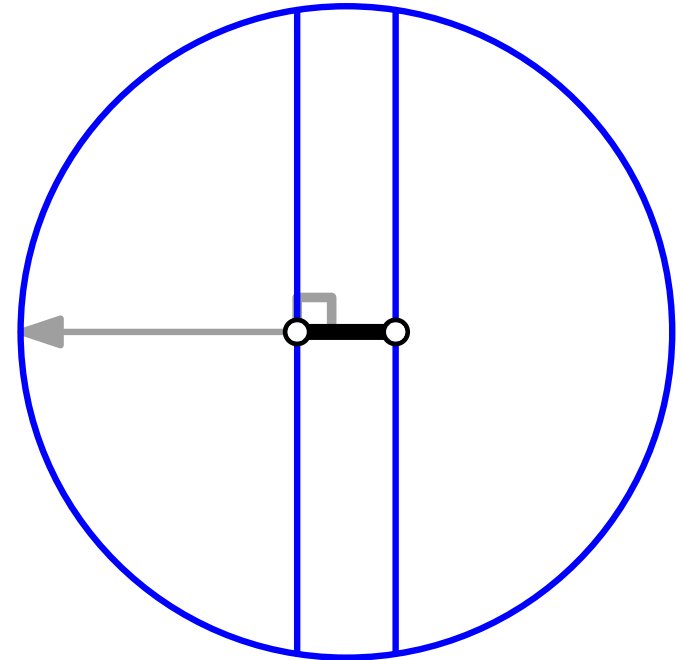
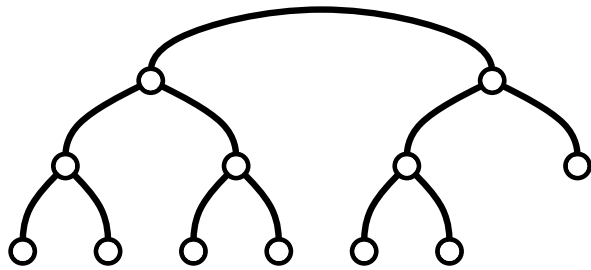
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Kein Grad-2 Knoten



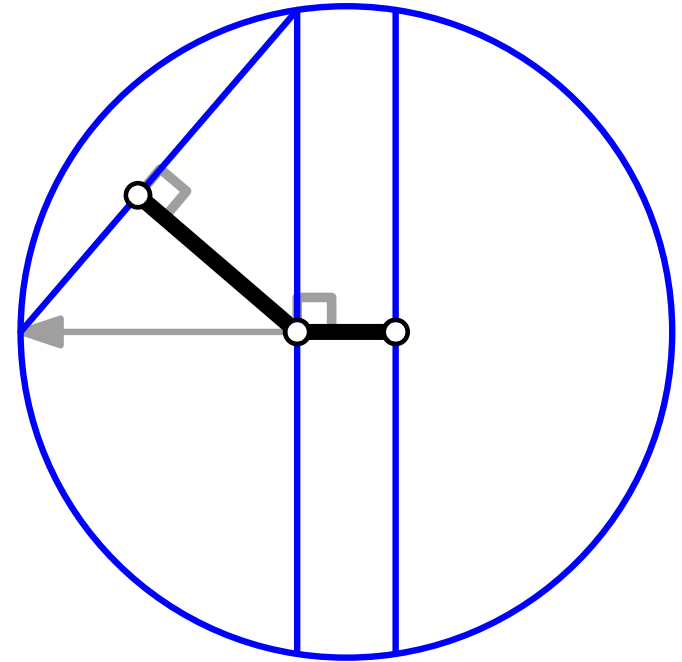
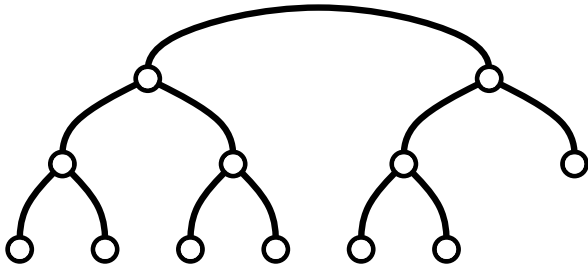
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*Ordentlicher Binärbaum:*  
Kein Grad-2 Knoten



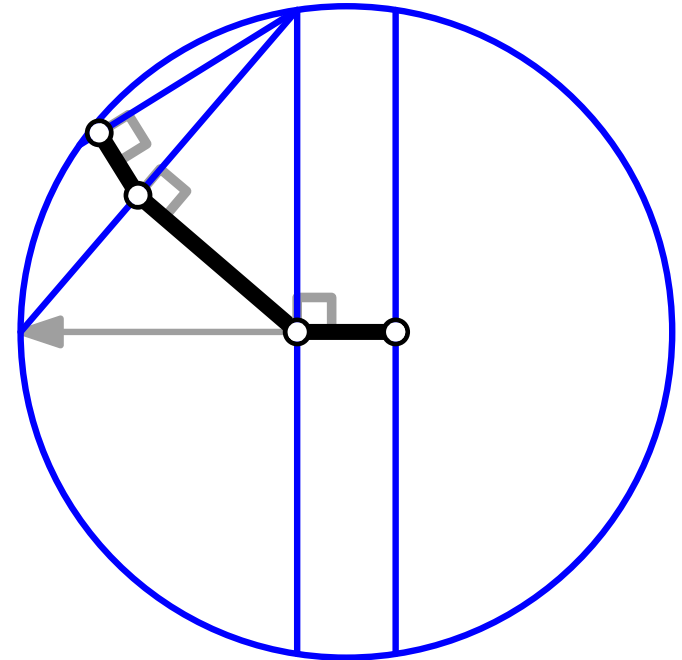
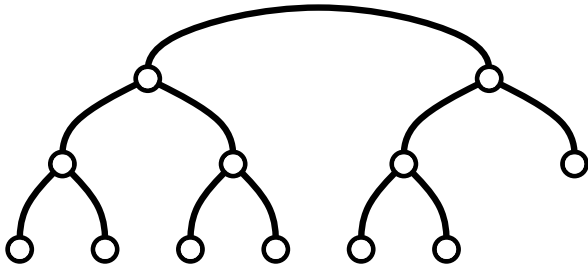
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*Ordentlicher Binärbaum:*  
Kein Grad-2 Knoten



# Streng Monotone Zeichnungen

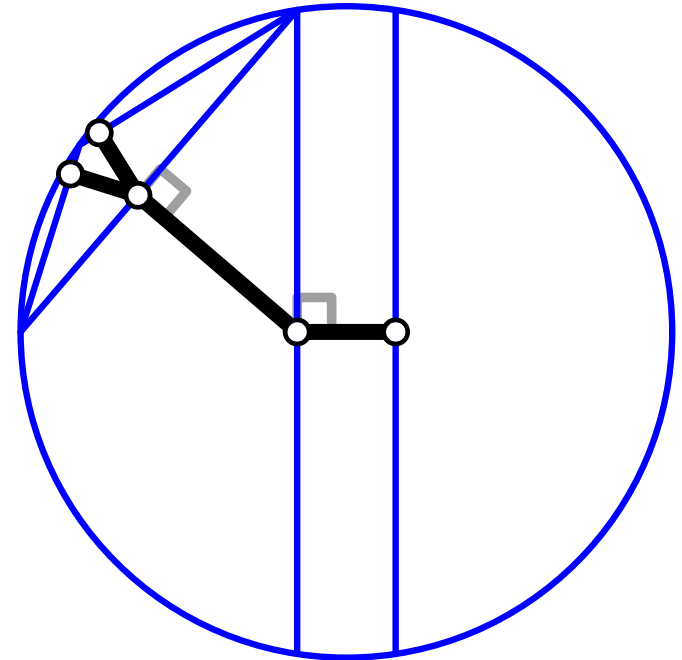
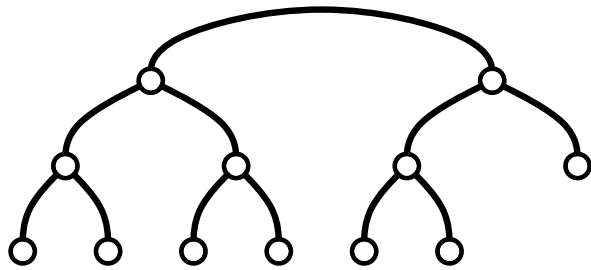
*Ordentlicher Binärbaum:*  
Kein Grad-2 Knoten





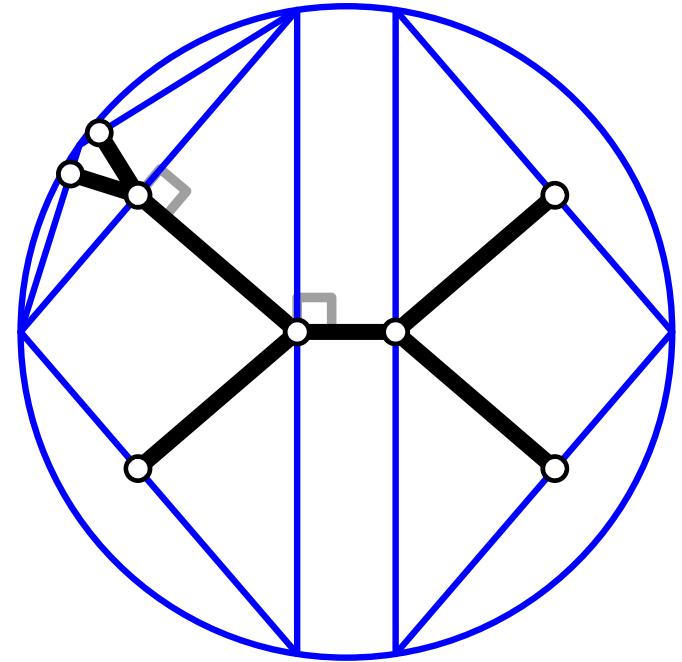
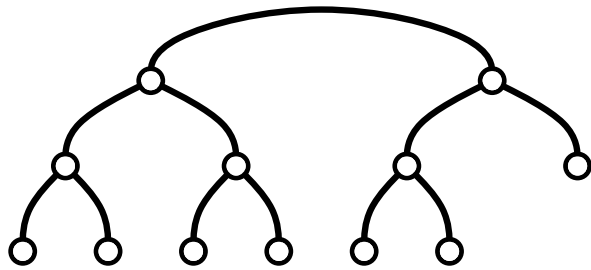
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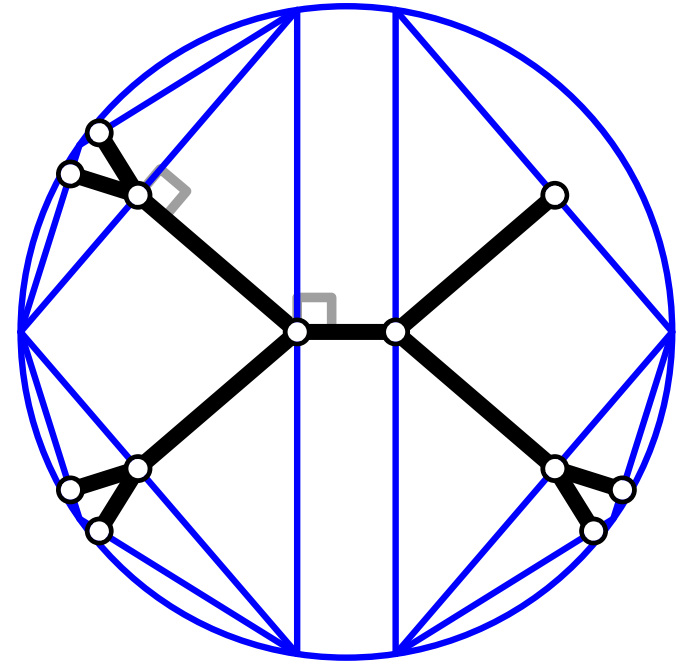
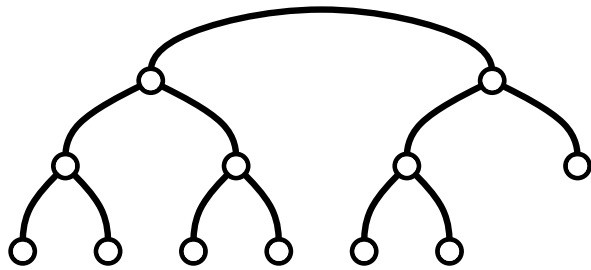
# Streng Monotone Zeichnungen

*Ordentlicher Binärbaum:*  
Kein Grad-2 Knoten



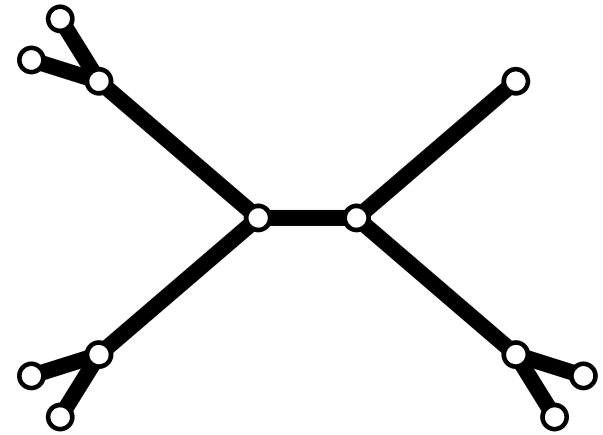
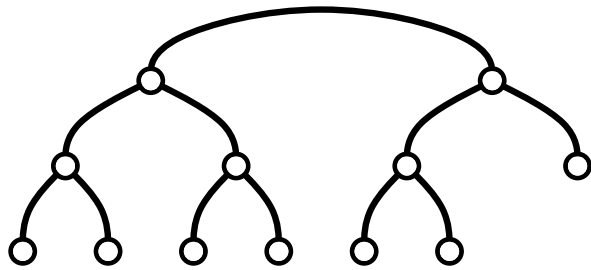
# Streng Monotone Zeichnungen

*Ordentlicher Binärbaum:*  
Kein Grad-2 Knoten



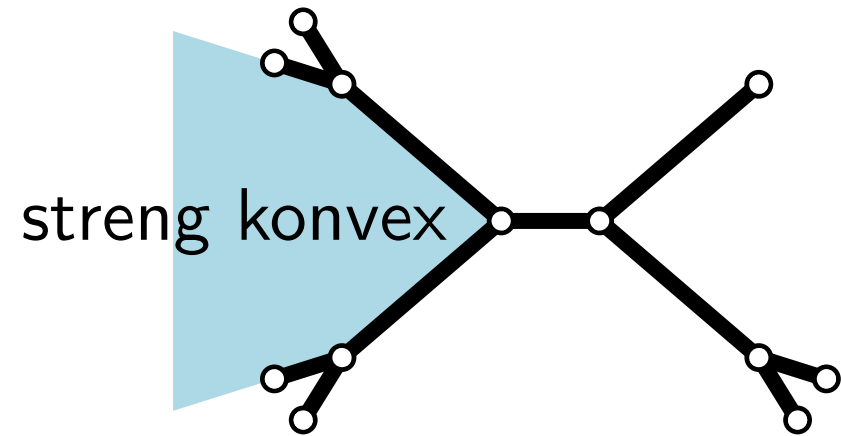
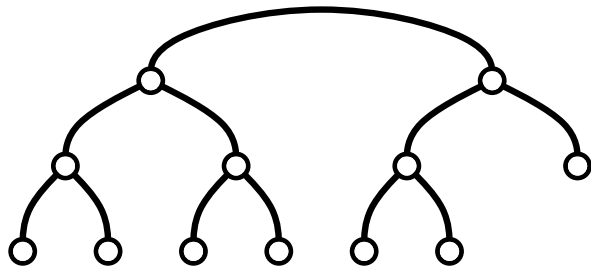
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*Ordentlicher Binärbaum:*  
Kein Grad-2 Knoten



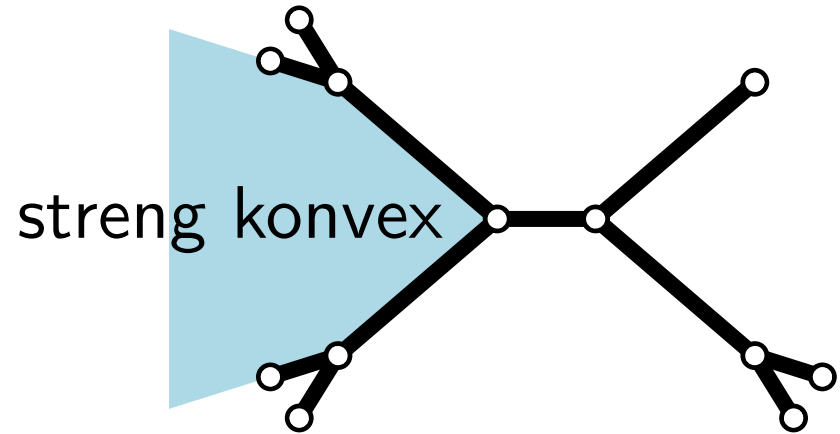
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*Ordentlicher Binärbaum:*  
Kein Grad-2 Knoten



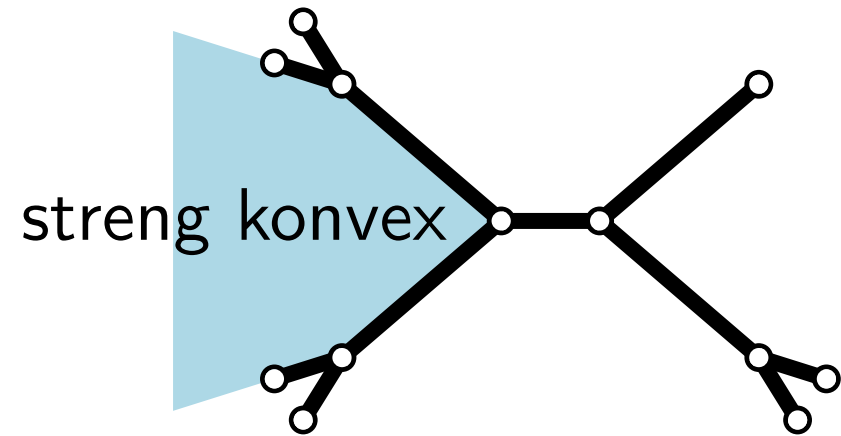
# Streng Monotone Zeichnungen

*Ordentlicher Binärbaum*  $\Rightarrow$   
streng monotone,  
streng konvexe Zeichnung,  
exponentielle Fläche



# Streng Monotone Zeichnungen

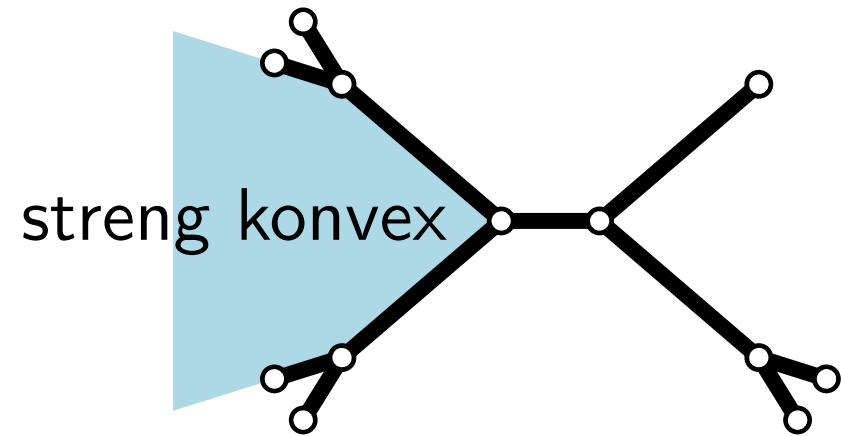
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Baum  $\Rightarrow$  streng monotone Zeichnung, exponentielle Fläche

# Streng Monotone Zeichnungen

*Ordentlicher Binärbaum*  $\Rightarrow$   
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Baum  $\Rightarrow$  streng monotone Zeichnung, exponentielle Fläche

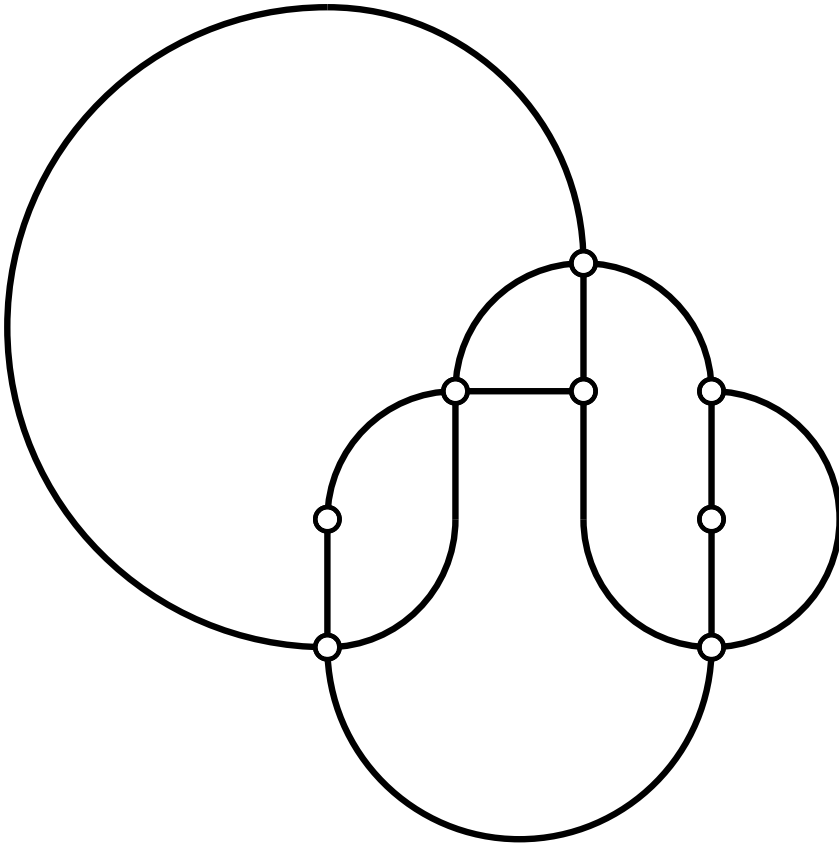
Nöllenburg et al.  
Exponentielle Fläche wird benötigt.

[arXiv'14]

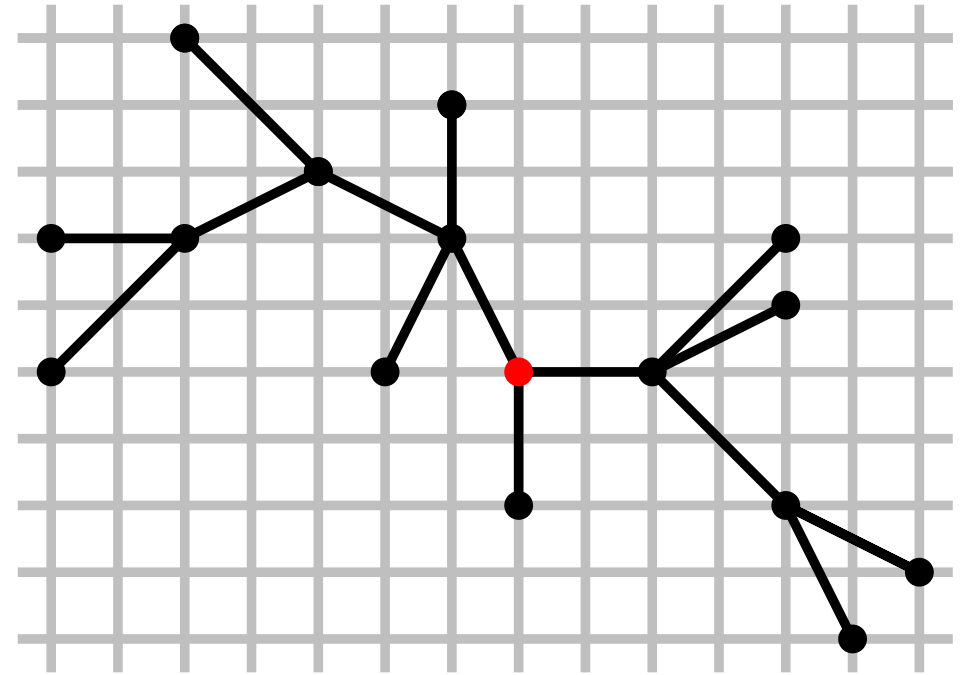
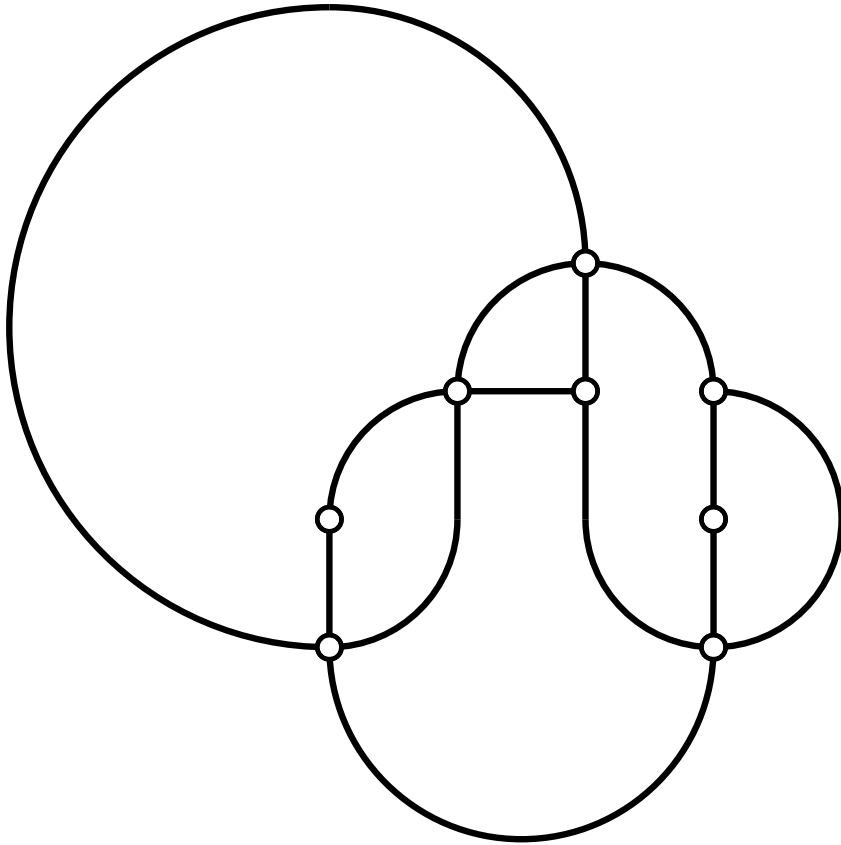


# Zusammenfassung

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