FernUniversität in Hagen
Fakultät für Mathematik und Informatik

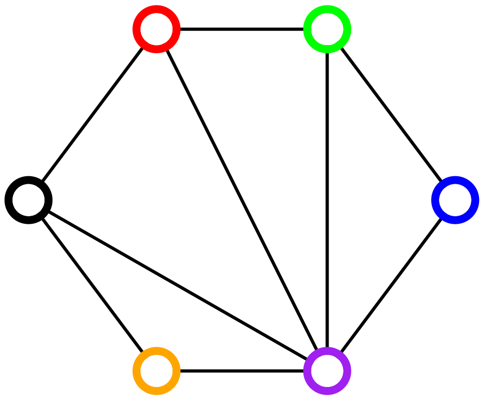
theoretische informatik

The Planar Split Thickness of Graphs

Philipp Kindermann
FernUniversität in Hagen

Joint work with
David Eppstein, Stephen Kobourov, Giuseppe Liotta, Anna Lubiw,
Aude Maignan, Debajyoti Mondal, Hamideh Vosoughpour,
Sue Whitesides, and Stephen Wismath

Split Thickness



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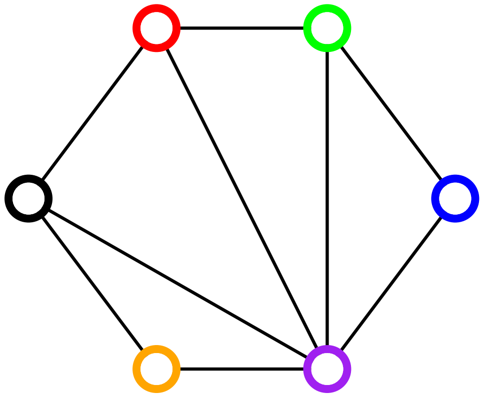

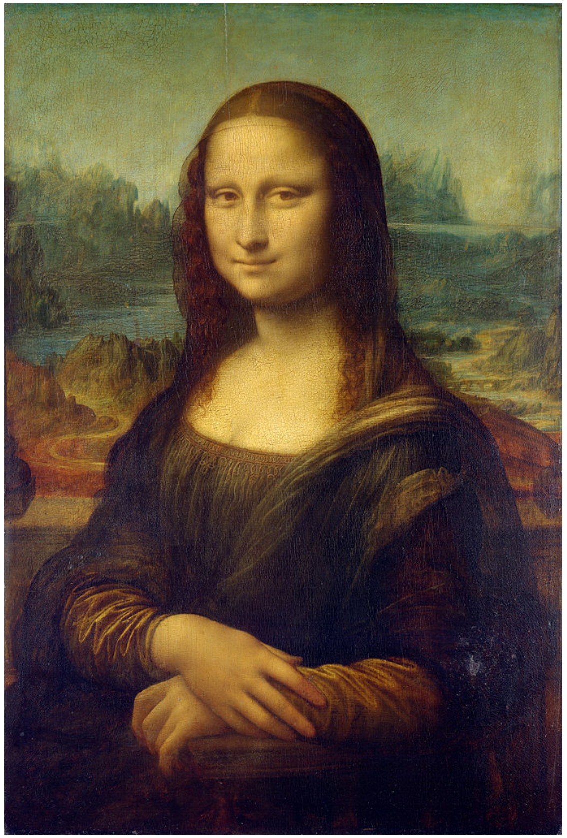

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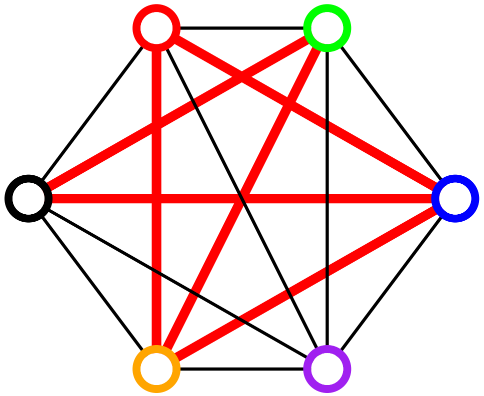

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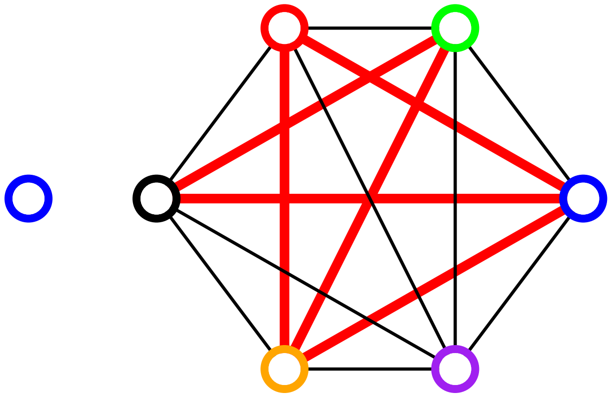

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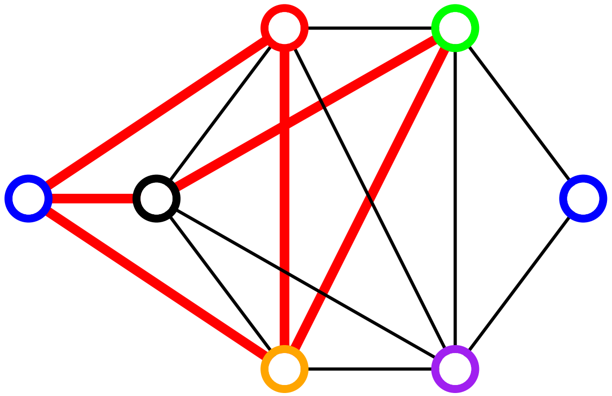

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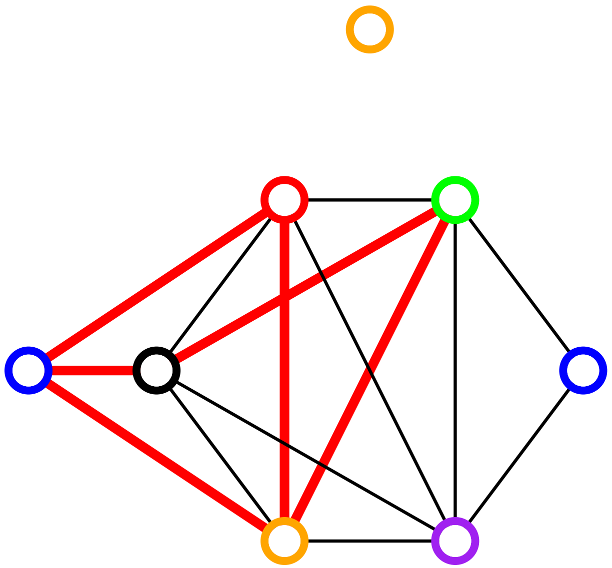

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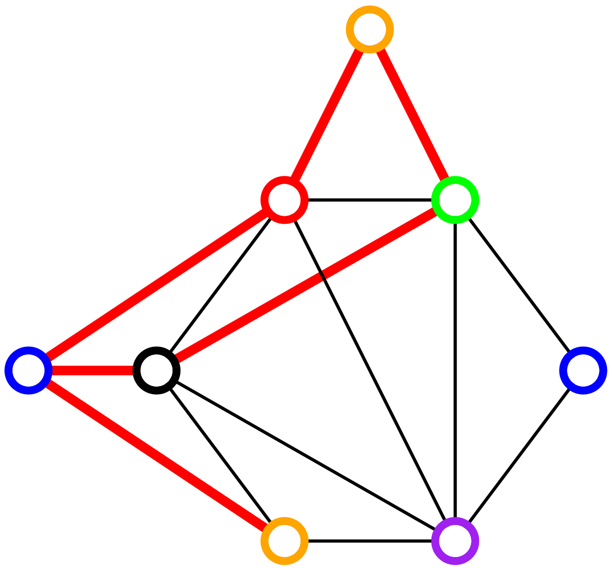
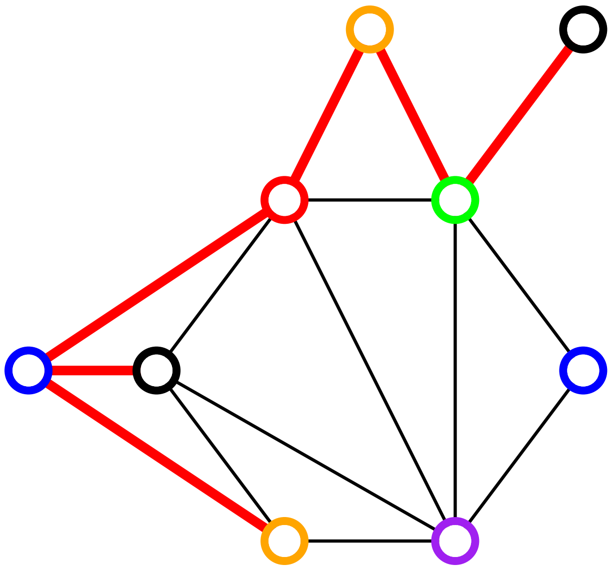
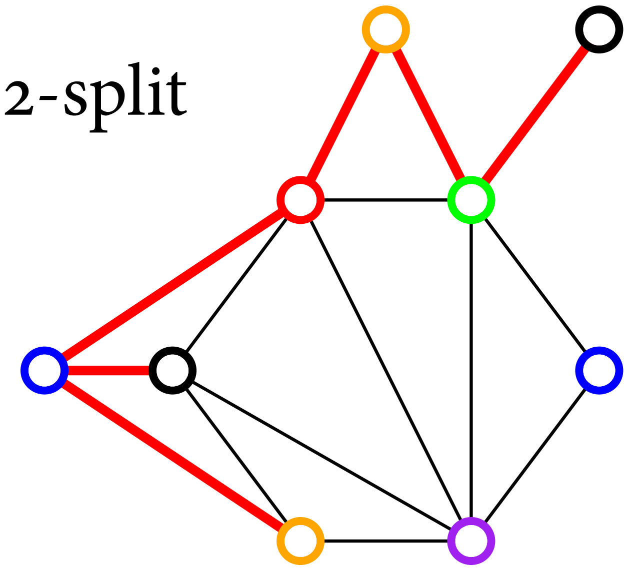


Split Thickness



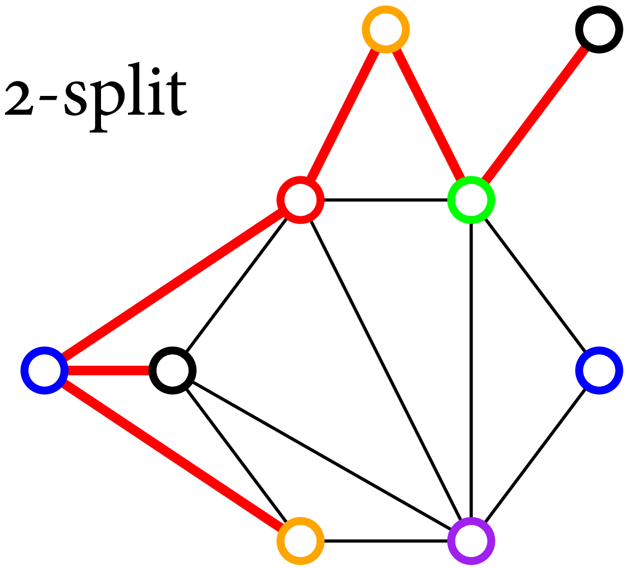
Split Thickness

2-split



Split Thickness

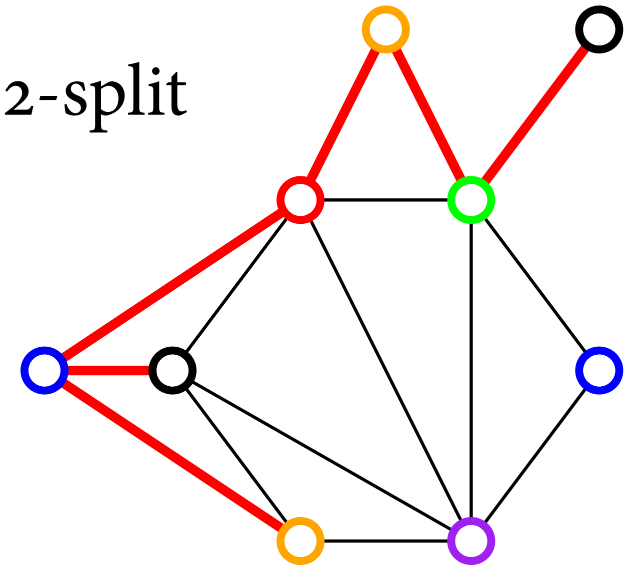
2-split



G has P split thickness k: there is a k-split of G with property P

Split Thickness

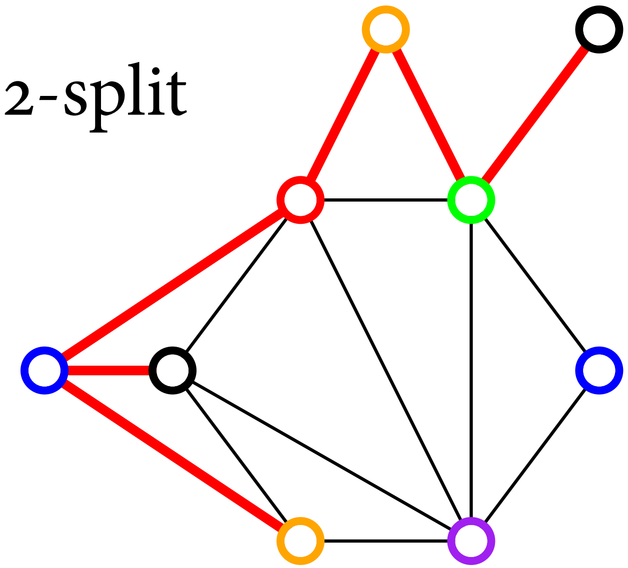
2-split



G has ~~P~~ planar split thickness k: there is a k-split of G ~~with property P~~ which is planar

Split Thickness

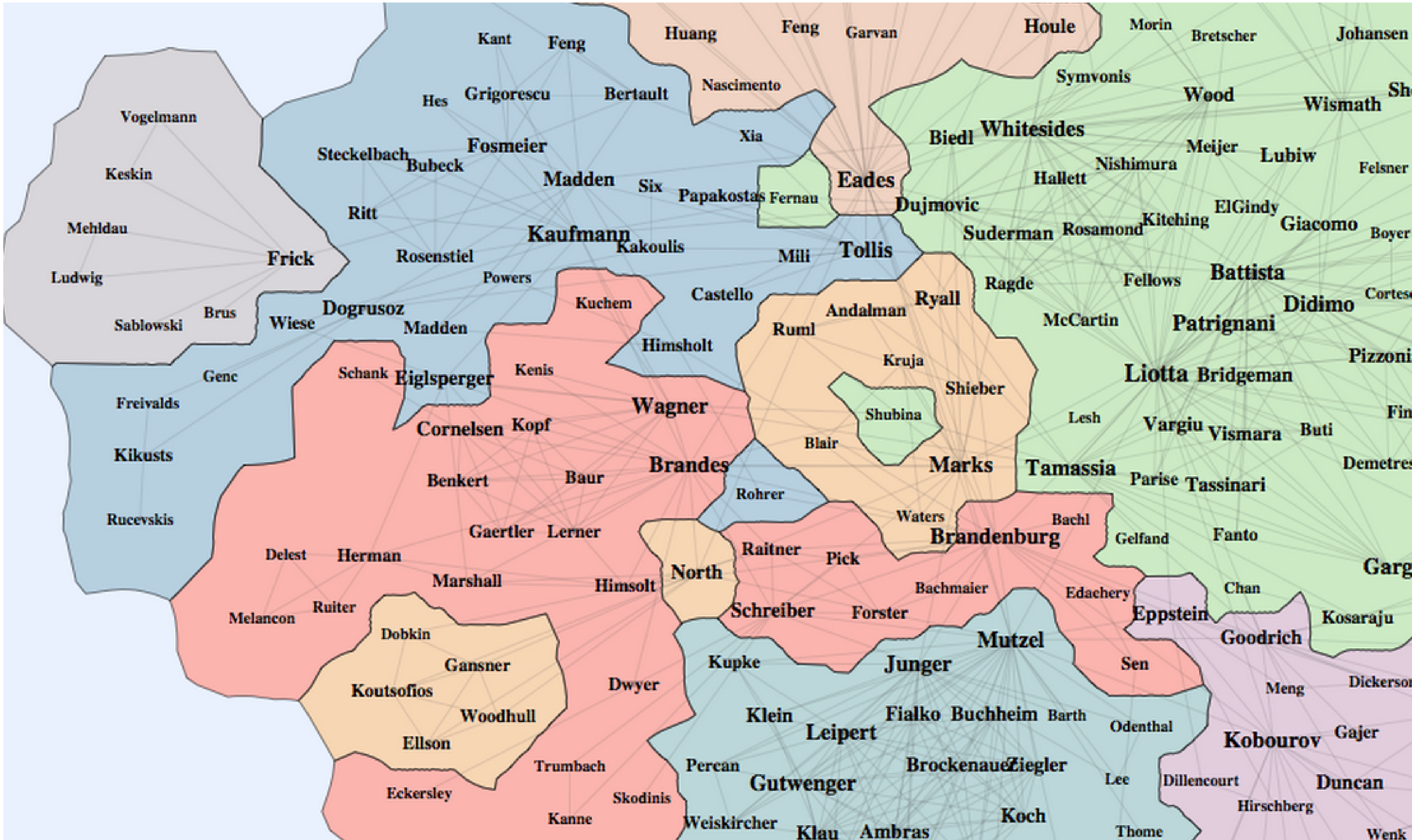
2-split



G has ~~P~~ planar split thickness k: there is a k-split of G ~~with property P~~ which is planar

$\Rightarrow$ G is k-splittable

Maps of clustered social networks



k-split of cluster graph

Heawood's empire problem [1889]

M-pire map: n empires, each at most M components

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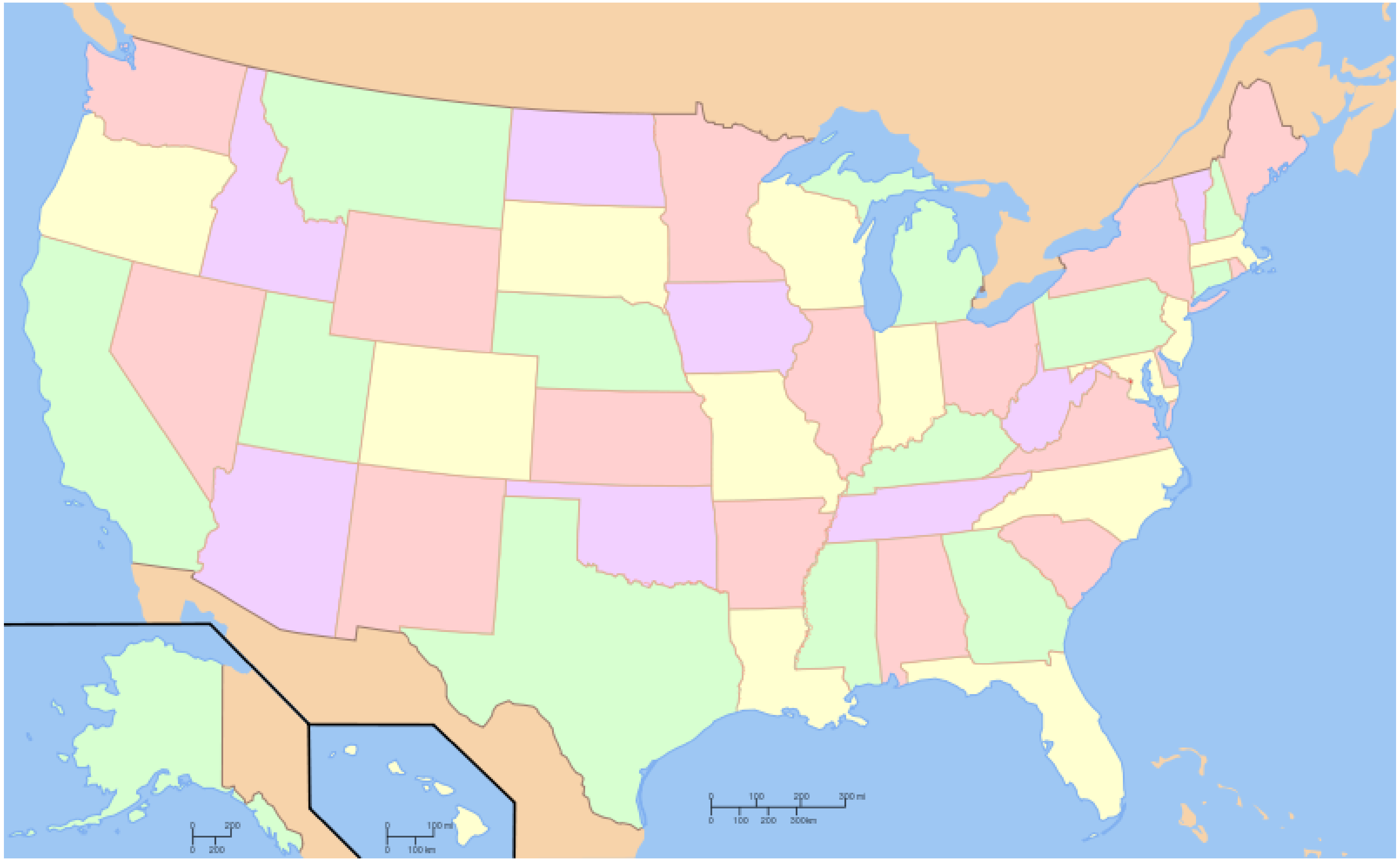
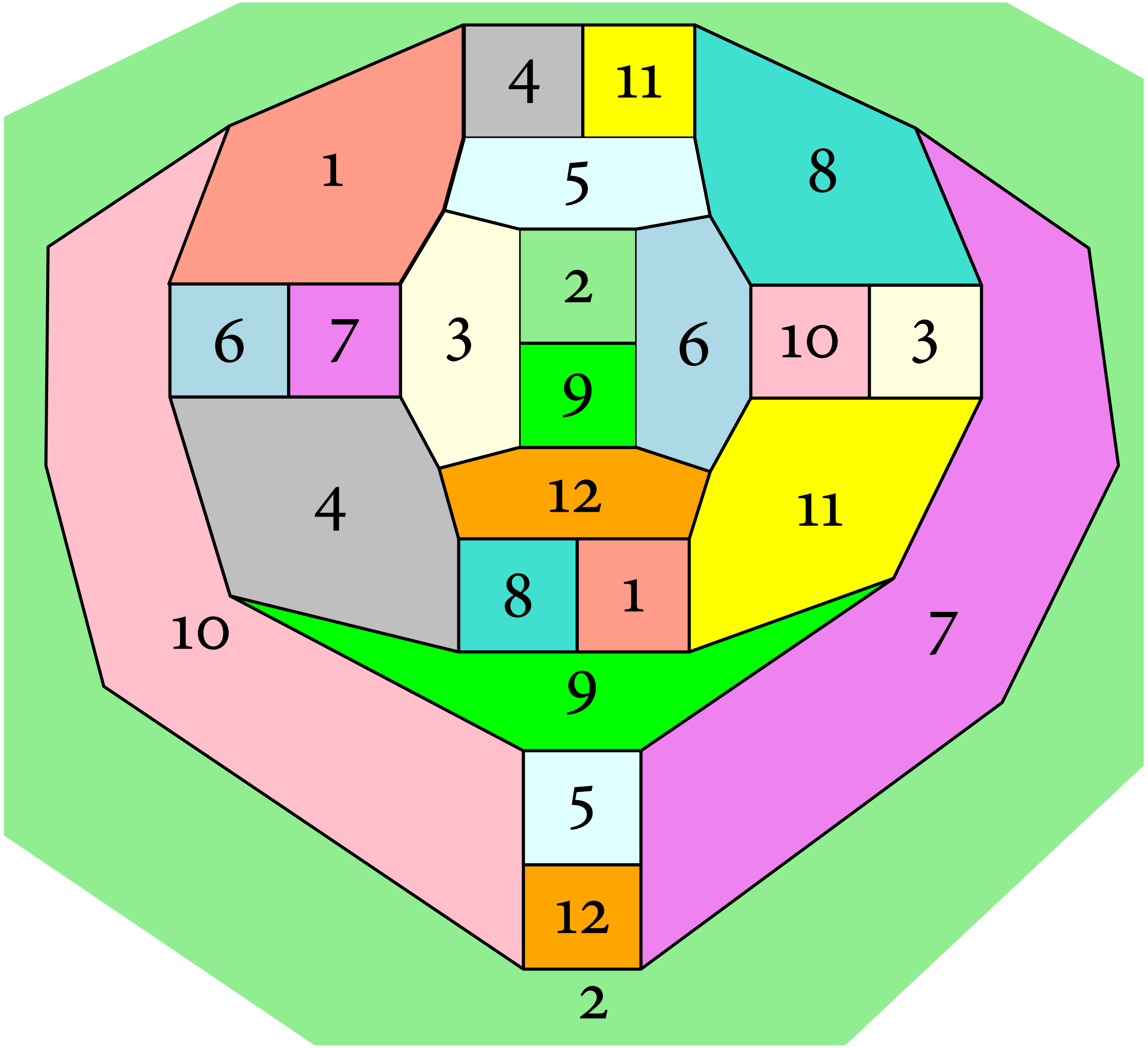
How many colors do you need?
1-pire: 4 colors



Heawood's empire problem [1889]

M-pire map: *n* empires, each at most *M* components
How many colors do you need?
1-pire: 4 colors
2-pire: 12 colors [Kim '??]



Heawood's empire problem [1889]

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How many colors do you need?
1-pire: 4 colors
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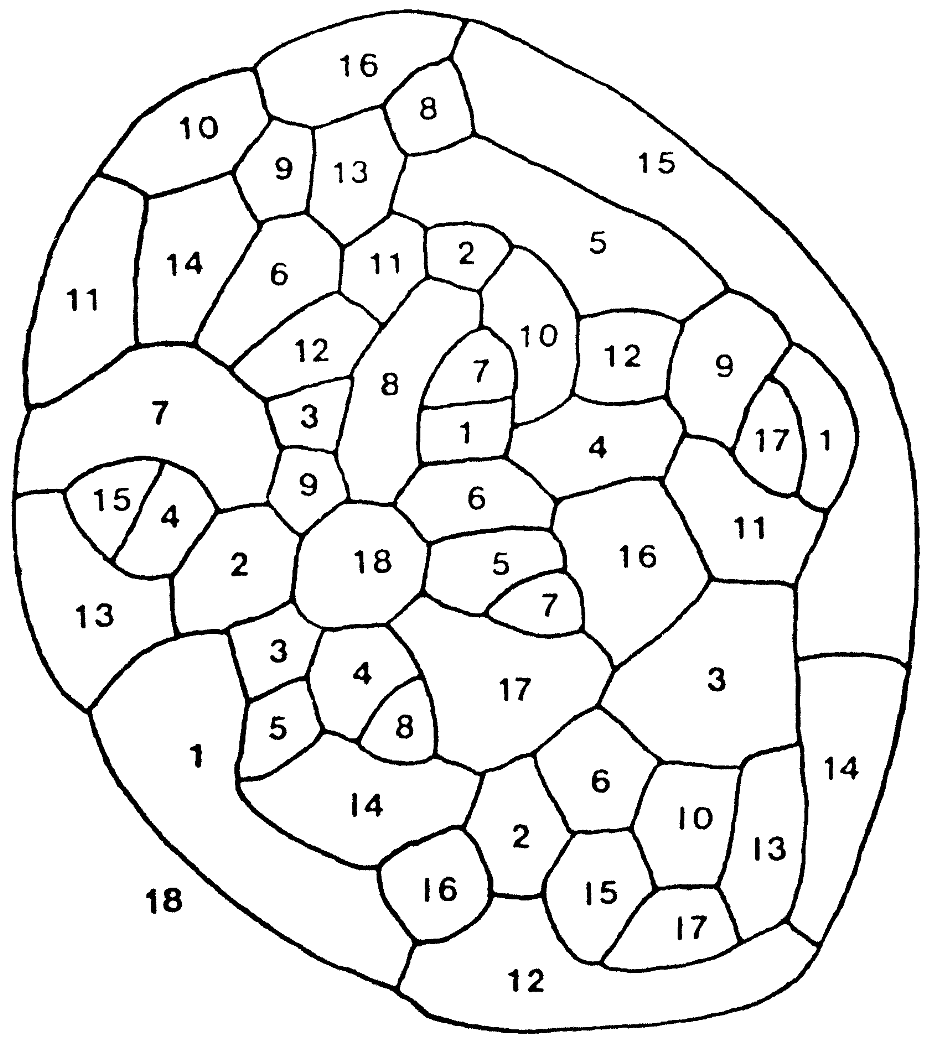
3-pire: 18 colors [Taylor '81]



Heawood's empire problem [1889]

M-pire map: *n* empires, each at most *M* components

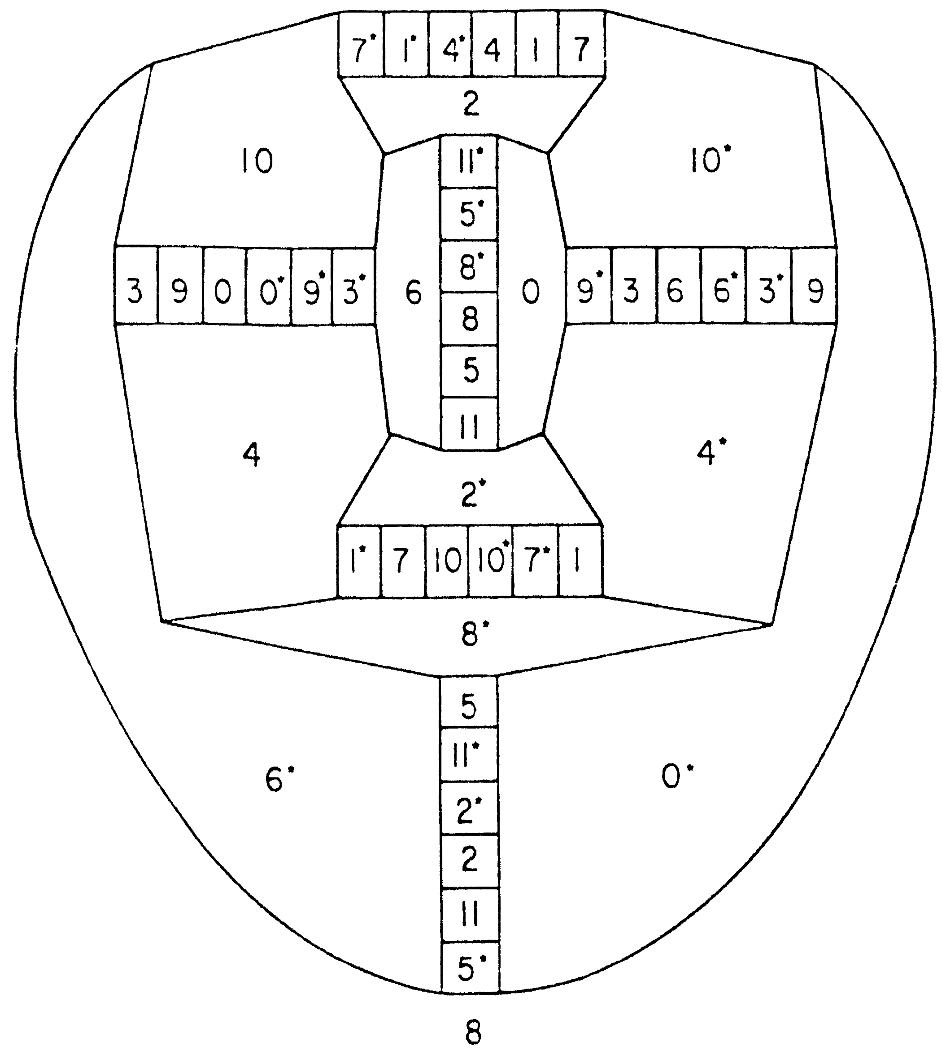
How many colors do you need?

1-pire: 4 colors

2-pire: 12 colors [Kim '??]

3-pire: 18 colors [Taylor '81]

4-pire: 24 colors [Taylor '81]



Heawood's empire problem [1889]

M-pire map: *n* empires, each at most *M* components
How many colors do you need?
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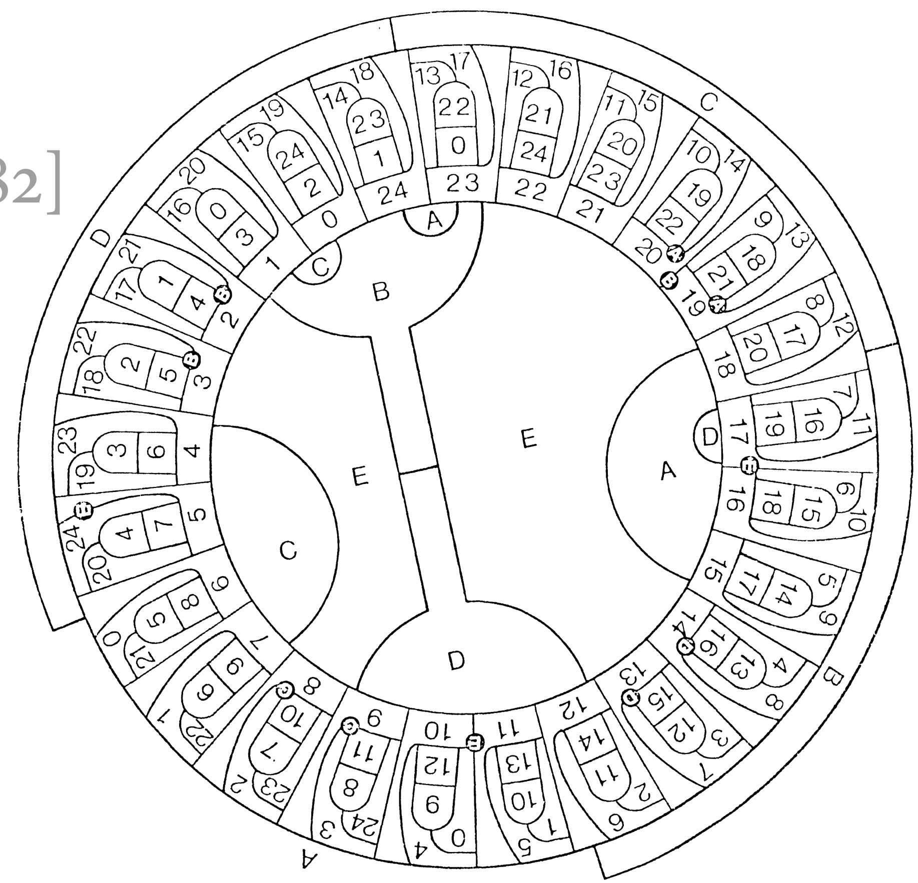
5-pire: 30 colors [Jackson & Ringel '82]



Heawoo

M-pire map

How many c

1-pire: 4 colo

2-pire: 12 col

3-pire: 18 col

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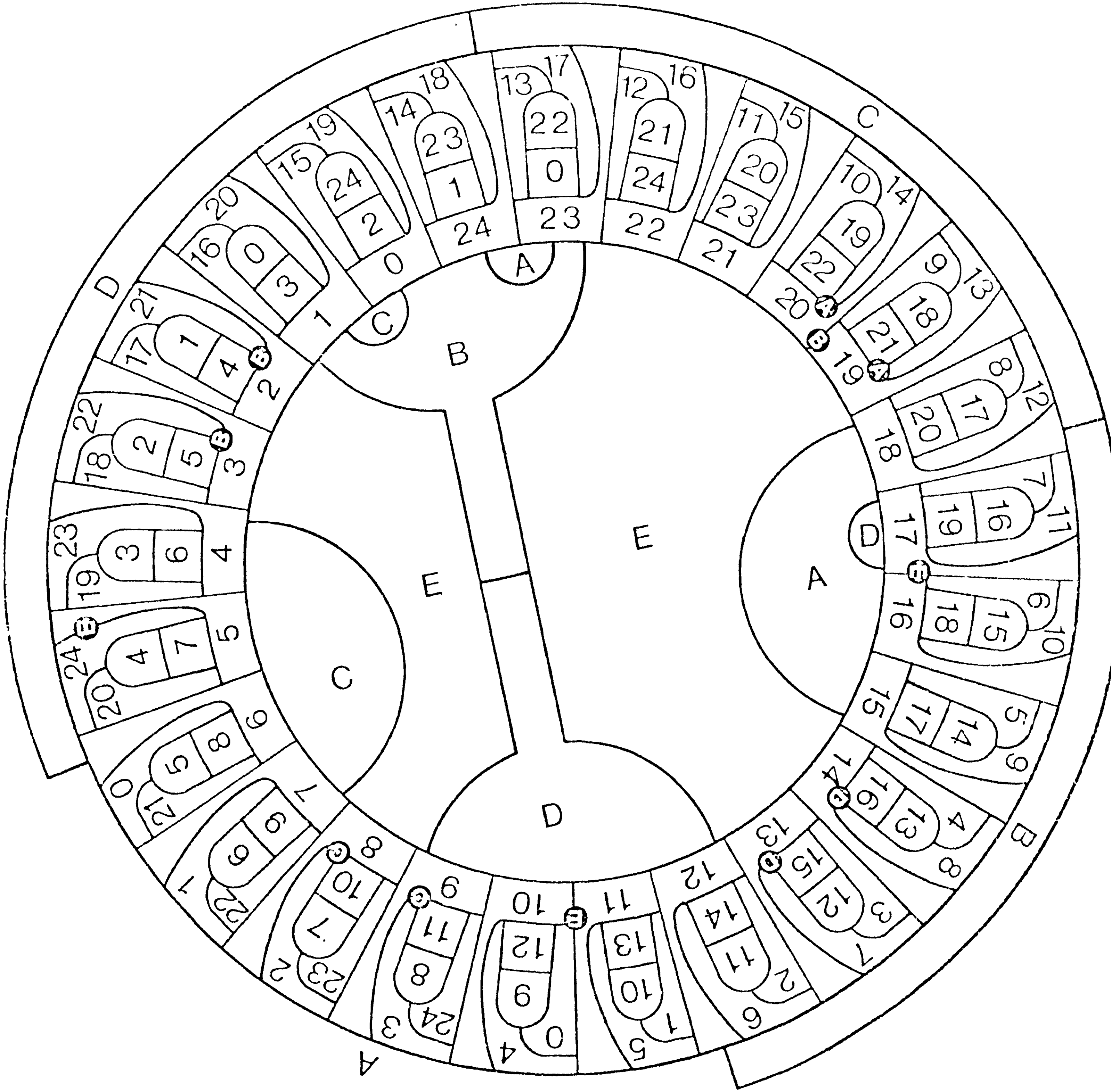
5-pire: 30 col



Heawood's empire problem [1889]

M-pire map: *n* empires, each at most *M* components
How many colors do you need?
1-pire: 4 colors
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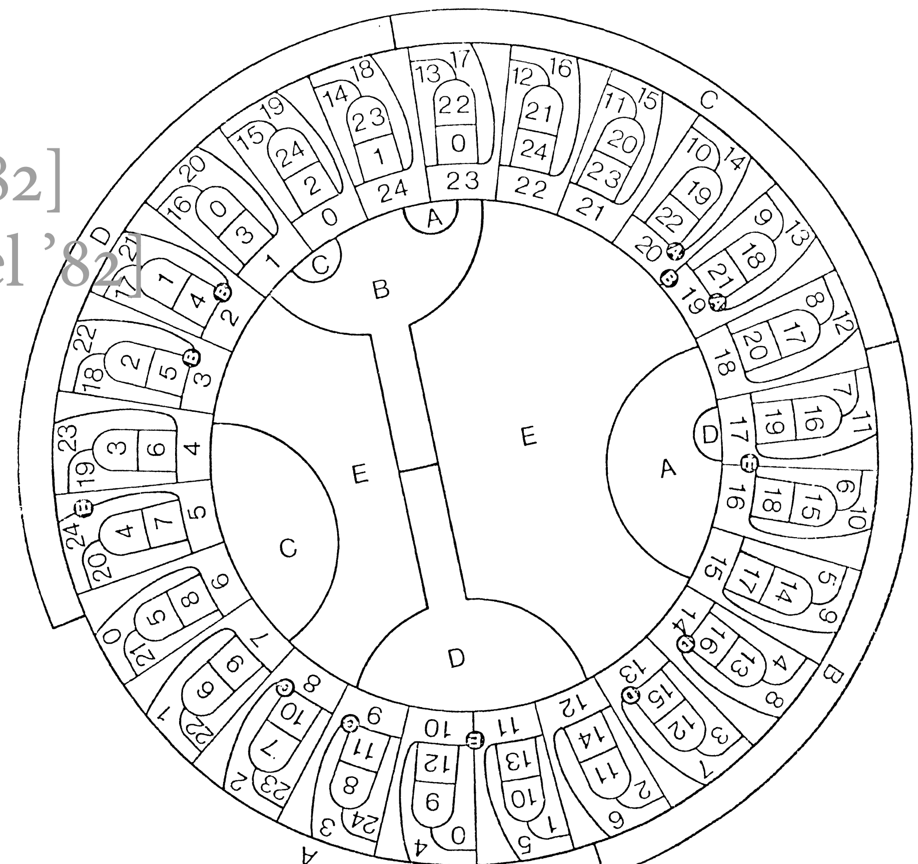
M-pire: 6*M* colors [Jackson & Ringel '82]



Heawood's empire problem [1889]

M-pire map: n empires, each at most M components

How many colors do you need?

1-pire: 4 colors

2-pire: 12 colors [Kim '??]

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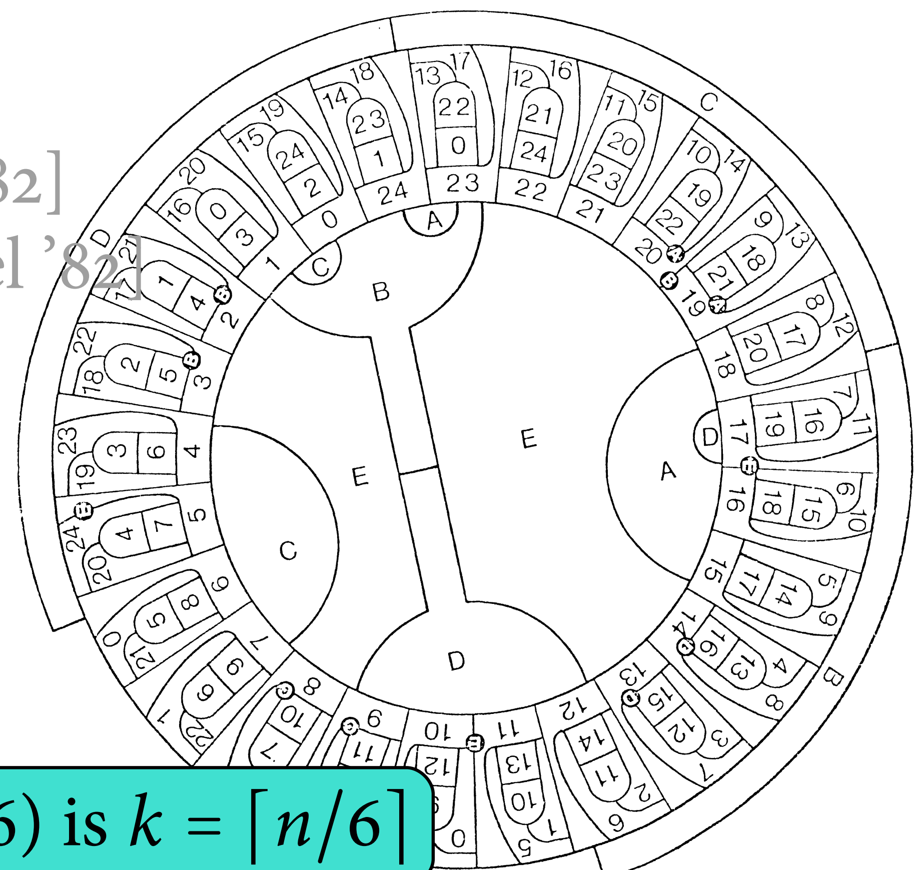
4-pire: 24 colors [Taylor '81]

5-pire: 30 colors [Jackson & Ringel '82]

M-pire: $6M$ colors [Jackson & Ringel '82]



Optimal k-splittability for K_n ($n > 6$) is $k = \lceil n/6 \rceil$

Known Results

Input	#-split	Output	
K_n	$\lceil n/6 \rceil$	planar	[Jackson & Ringel '82]

Known Results

Input	#-split	Output	
K_n	$\lceil n/6 \rceil$	planar	[Jackson & Ringel '82]
outerplanar	2	interval	[Scheinermann & West '83]

Known Results

Input	#-split	Output	
K_n	$\lceil n/6 \rceil$	planar	[Jackson & Ringel ’82]
outerplanar	2	interval	[Scheinermann & West ’83]
planar	3	interval	[Scheinermann & West ’83]

Known Results

Input	#-split	Output	
K_n	$\lceil n/6 \rceil$	planar	[Jackson & Ringel '82]
outerplanar	2	interval	[Scheinermann & West '83]
planar	3	interval	[Scheinermann & West '83]
planar	4	caterpillar forest	[Scheinermann & West '83]

Known Results

Input	#-split	Output	
K_n	$\lceil n/6 \rceil$	planar	[Jackson & Ringel '82]
outerplanar	2	interval	[Scheinermann & West '83]
planar	3	interval	[Scheinermann & West '83]
planar	4	caterpillar forest	[Scheinermann & West '83]
planar	4	star forest	[Knauer & Ueckerdt '16]

Known Results

Input	#-split	Output	
K_n	$\lceil n/6 \rceil$	planar	[Jackson & Ringel '82]
outerplanar	2	interval	[Scheinermann & West '83]
planar	3	interval	[Scheinermann & West '83]
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planar	4	star forest	[Knauer & Ueckerdt '16]
planar bipartite	3	star forest	[Knauer & Ueckerdt '16]

Known Results

Input	#-split	Output	
K_n	$\lceil n/6 \rceil$	planar	[Jackson & Ringel '82]
outerplanar	2	interval	[Scheinermann & West '83]
planar	3	interval	[Scheinermann & West '83]
planar	4	caterpillar forest	[Scheinermann & West '83]
planar	4	star forest	[Knauer & Ueckerdt '16]
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outerplanar	3	star forest	[Knauer & Ueckerdt '16]

Known Results

Input	#-split	Output	
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outerplanar	2	interval	[Scheinermann & West '83]
planar	3	interval	[Scheinermann & West '83]
planar	4	caterpillar forest	[Scheinermann & West '83]
planar	4	star forest	[Knauer & Ueckerdt '16]
planar bipartite	3	star forest	[Knauer & Ueckerdt '16]
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anything	≤ thickness	planar	

Known Results

Input	#-split	Output	
K_n	$\lceil n/6 \rceil$	planar	[Jackson & Ringel '82]
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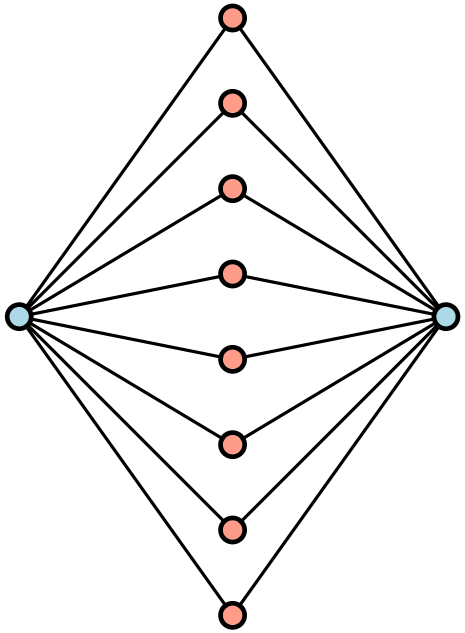
planar	3	interval	[Scheinermann & West '83]
planar	4	caterpillar forest	[Scheinermann & West '83]
planar	4	star forest	[Knauer & Ueckerdt '16]
planar bipartite	3	star forest	[Knauer & Ueckerdt '16]
outerplanar	3	star forest	[Knauer & Ueckerdt '16]
anything	$\leq$ thickness	planar	
anything	$\leq$ arboricity	forest	

2-Splits of Complete Bipartite Graphs

$K_{2,n}$?

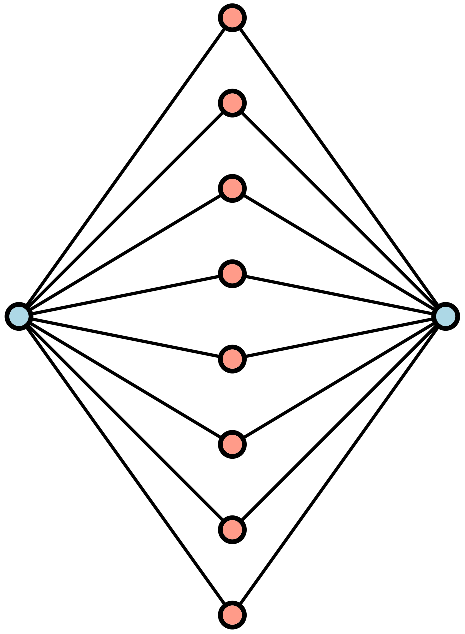
2-Splits of Complete Bipartite Graphs

$K_{2,n}$?



2-Splits of Complete Bipartite Graphs

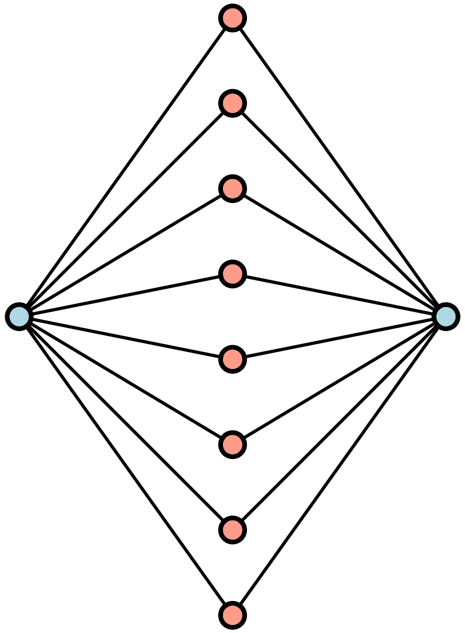
$K_{2,n}$? ✓



2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

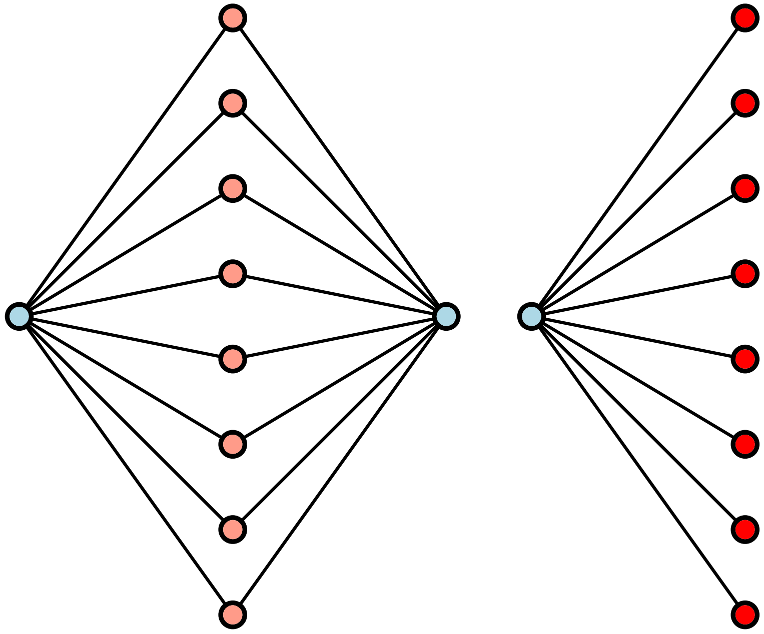
$K_{3,n}$?



2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

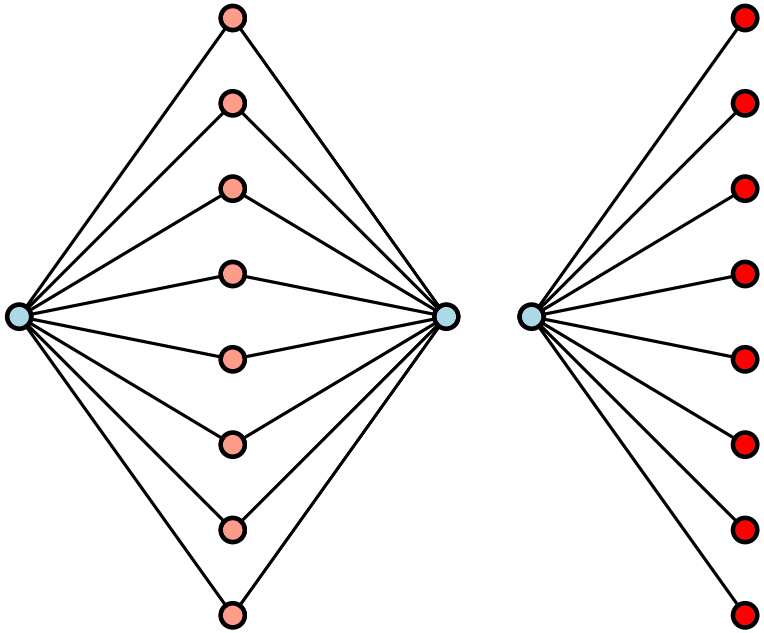
$K_{3,n}$?



2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

$K_{3,n}$? ✓

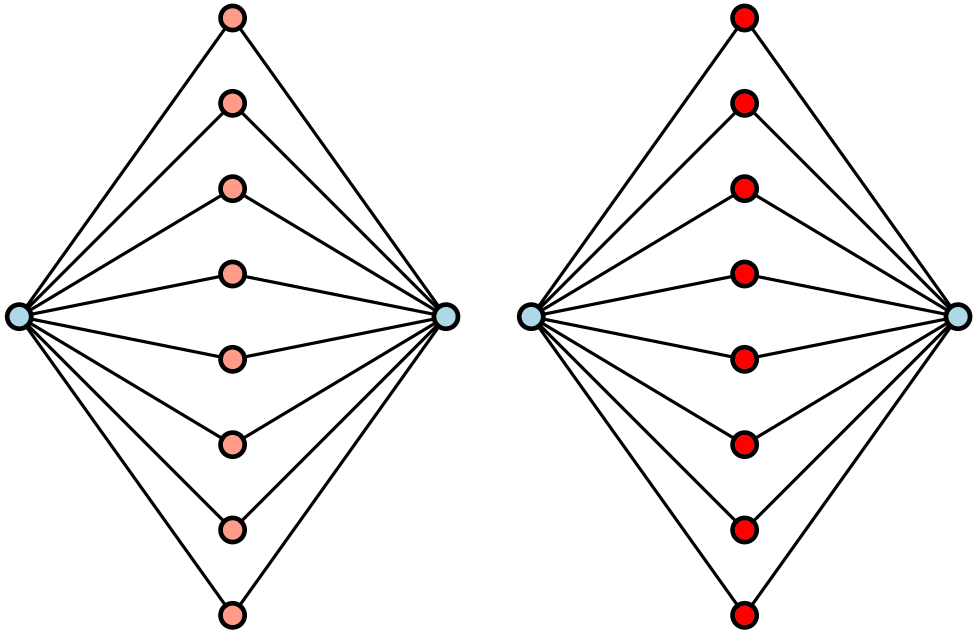


2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

$K_{3,n}$? ✓

$K_{4,n}$? ✓



2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$?

2-Splits of Complete Bipartite Graphs

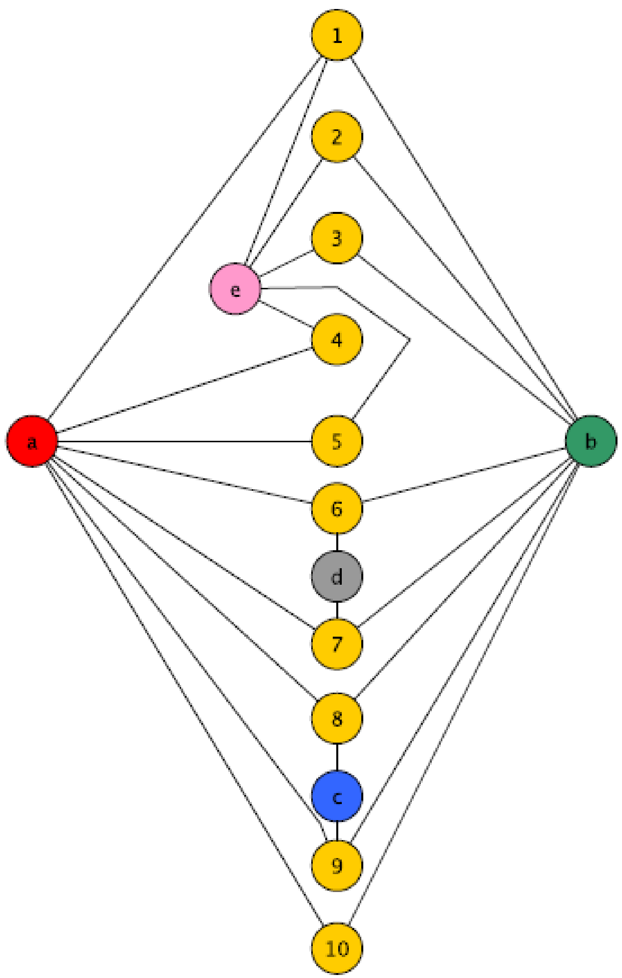
$K_{2,n}$? ✓

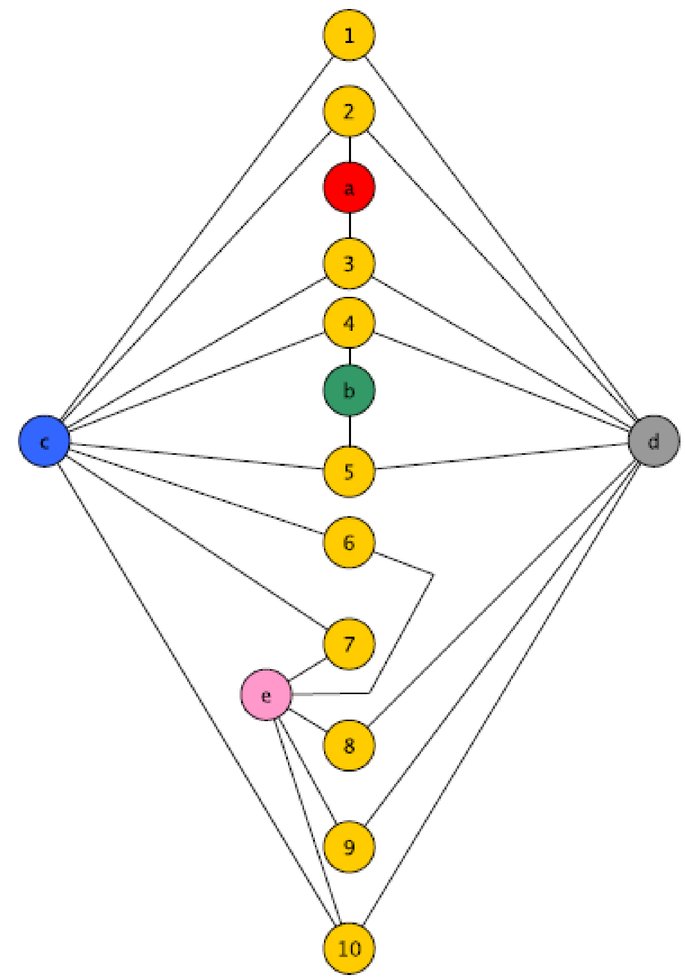
$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$?

$n = 10$





2-Splits of Complete Bipartite Graphs

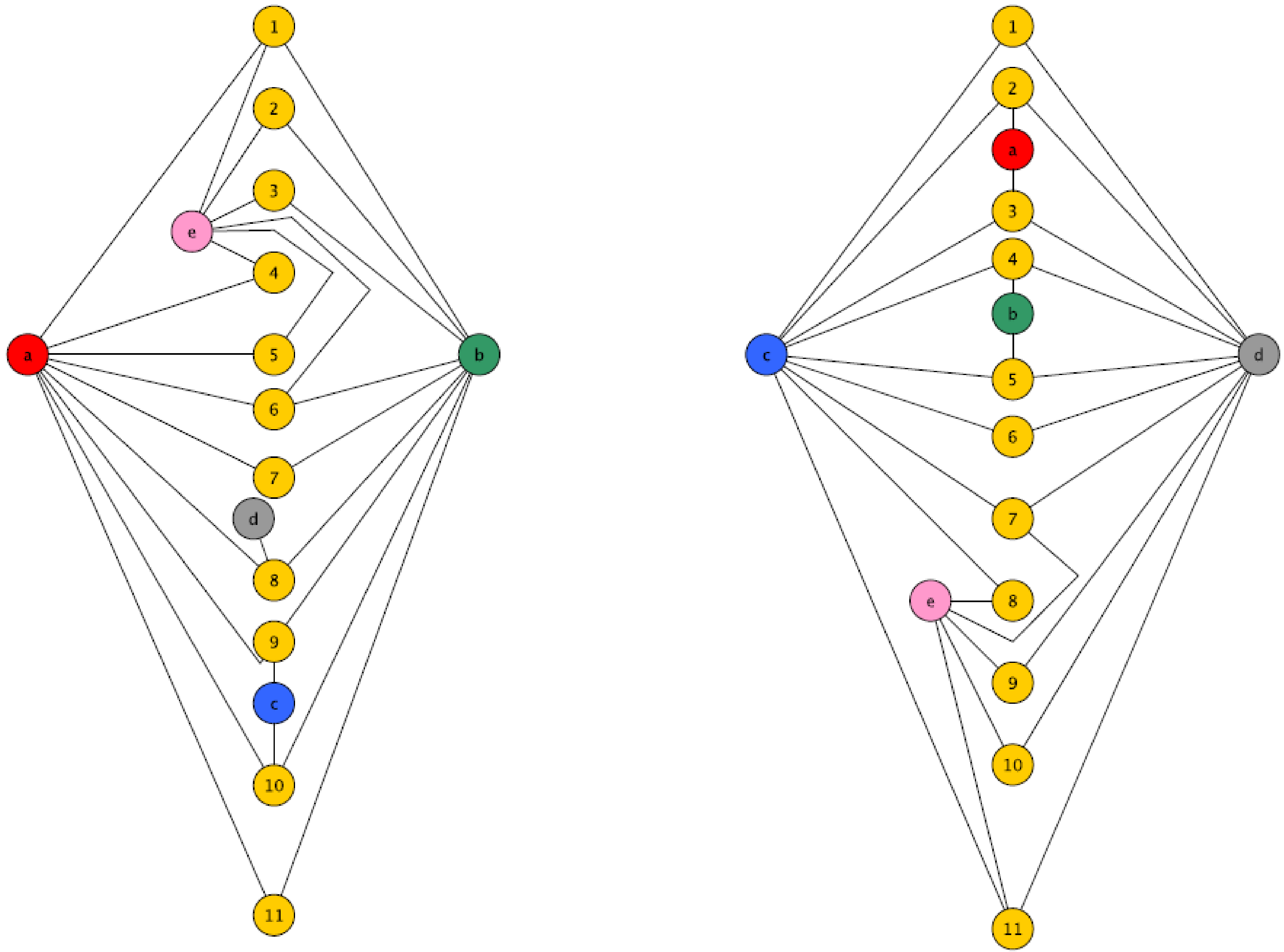
$K_{2,n}$? ✓

$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$?

$n = 11$



2-Splits of Complete Bipartite Graphs

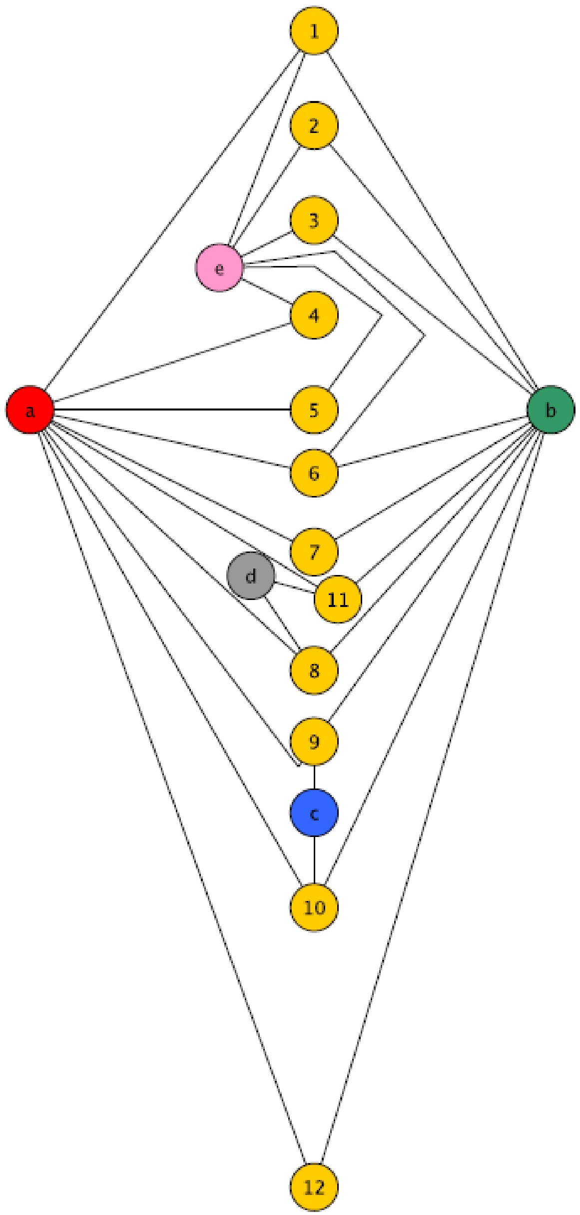
$K_{2,n}$? ✓

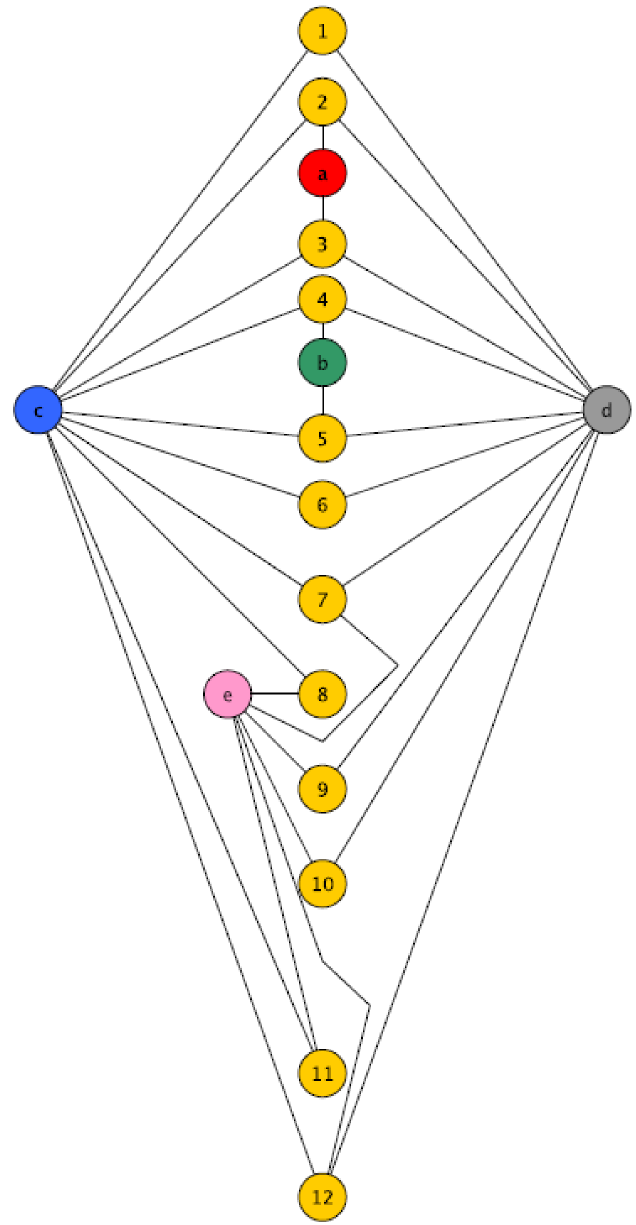
$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$?

$n = 12$





2-Splits of Complete Bipartite Graphs

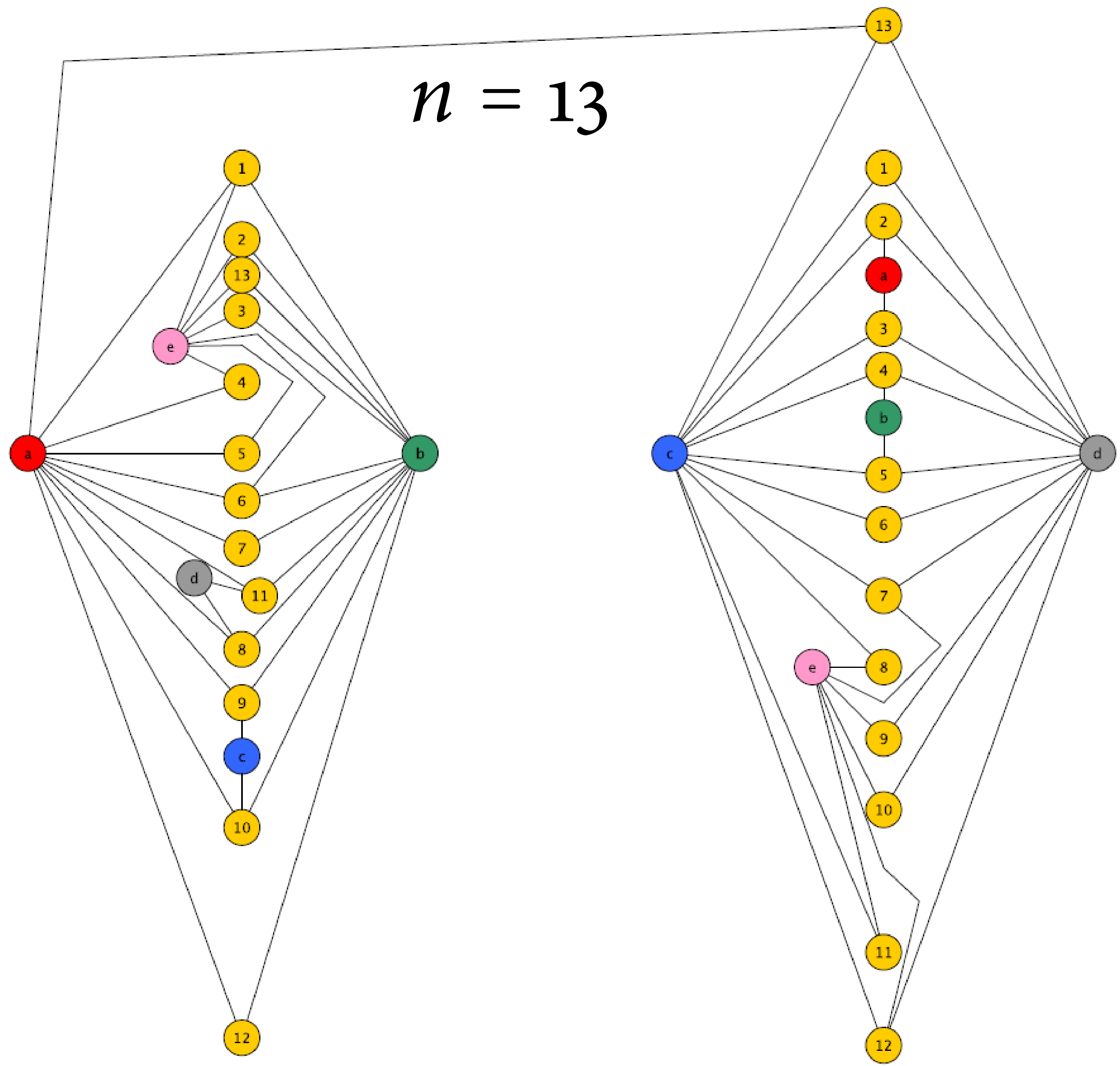
$K_{2,n}$? ✓

$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$?

$n = 13$



2-Splits of Complete Bipartite Graphs

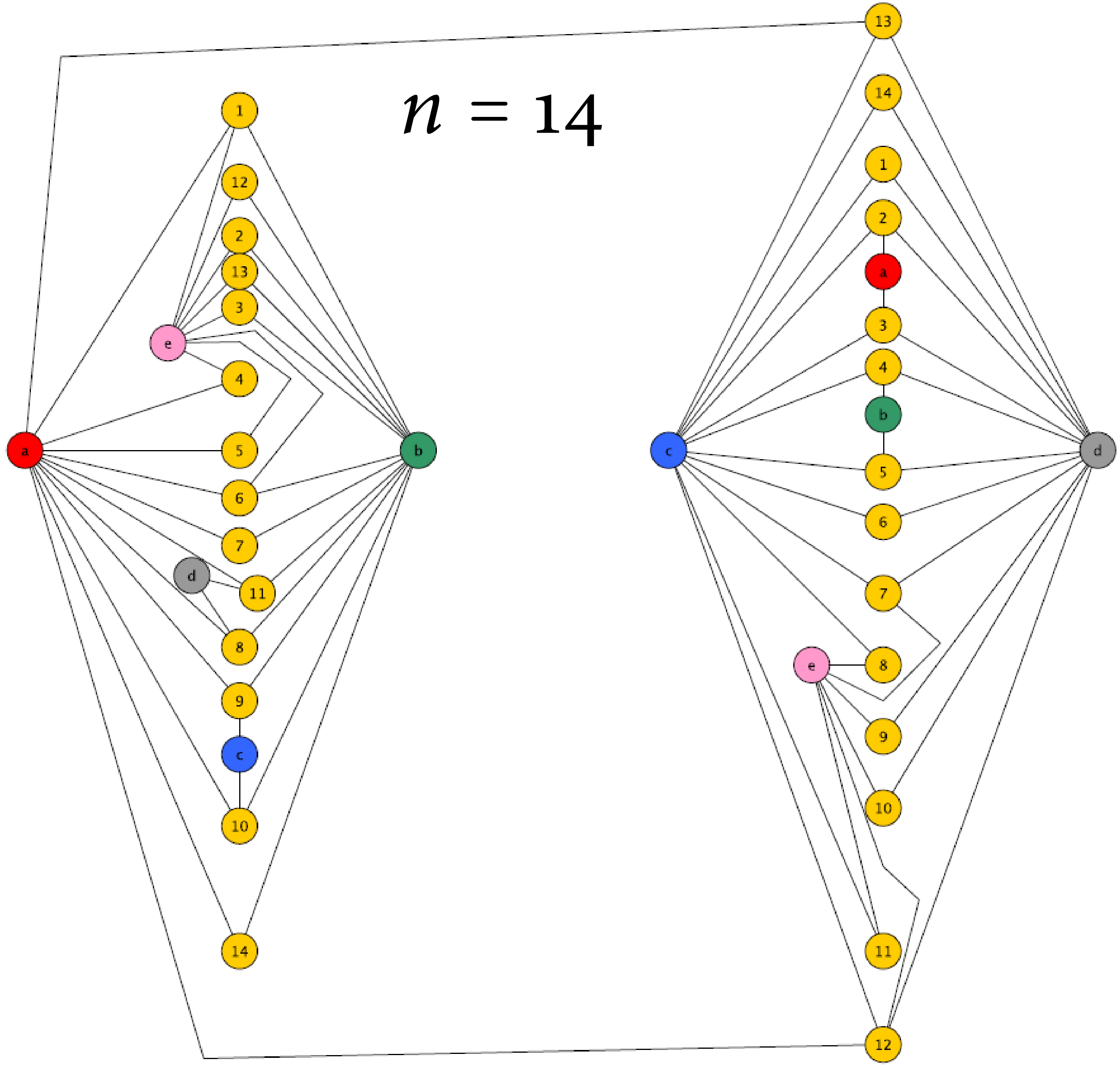
$K_{2,n}$? ✓

$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$?

$n = 14$



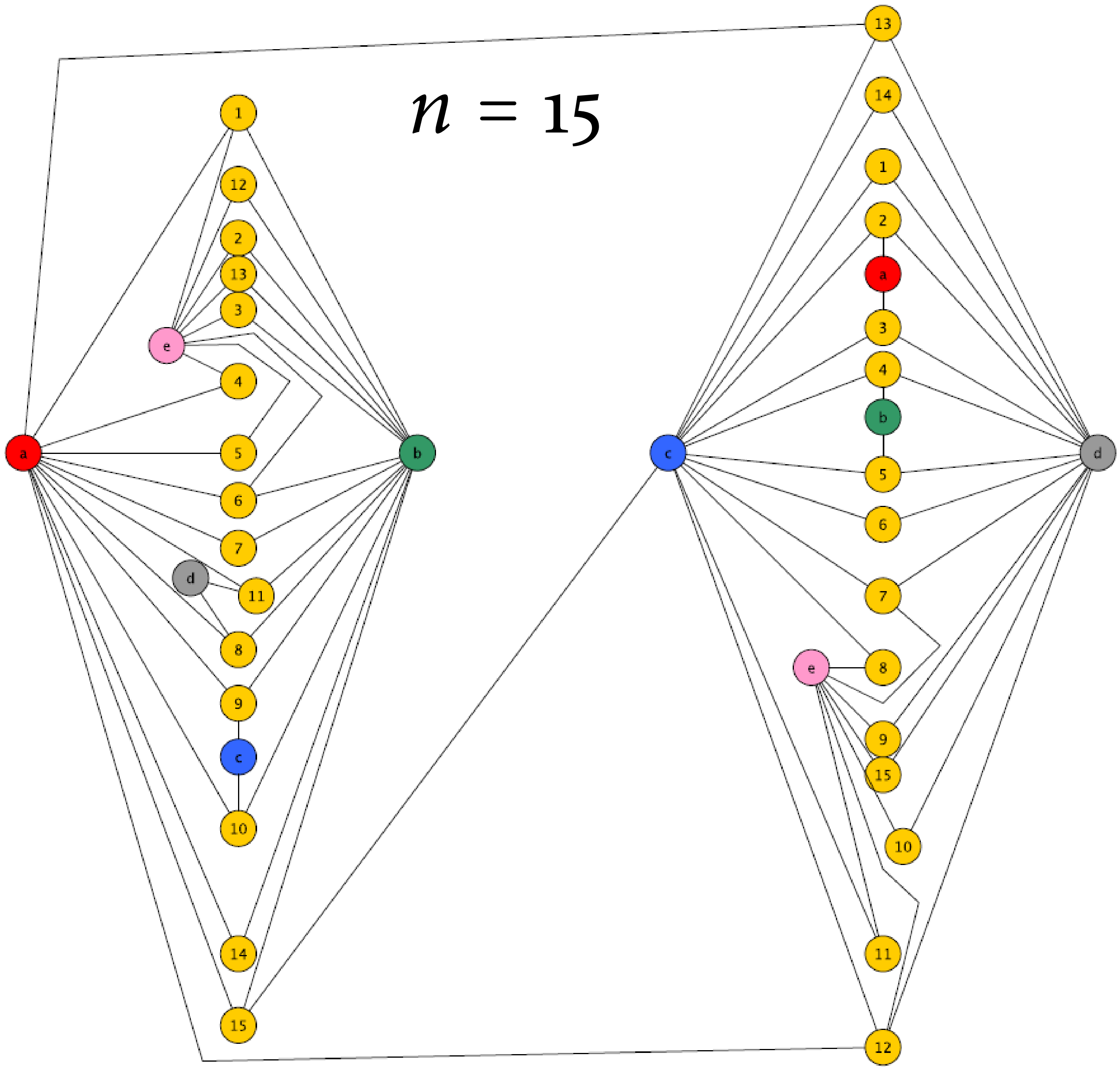
2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$?



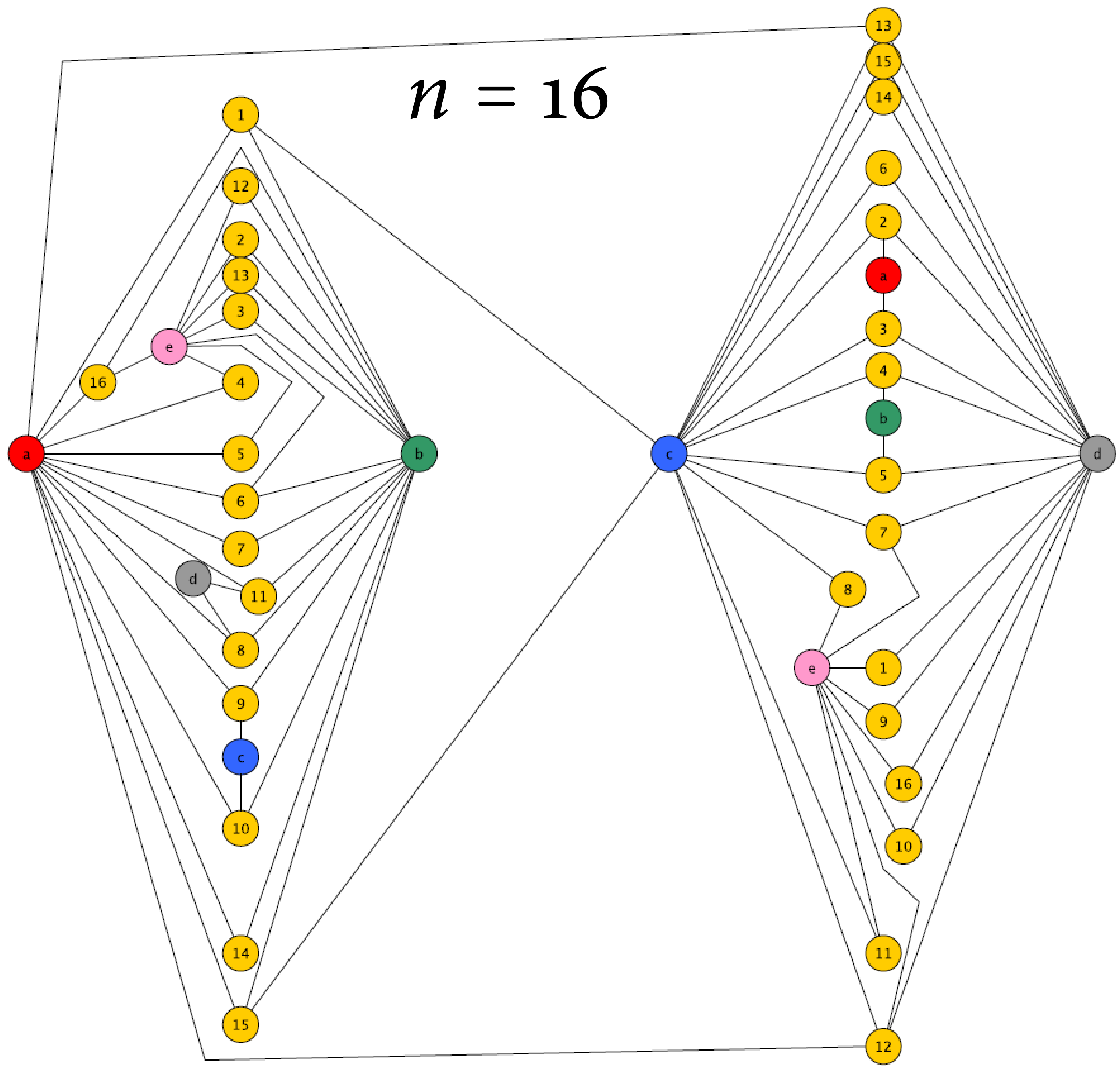
2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$?



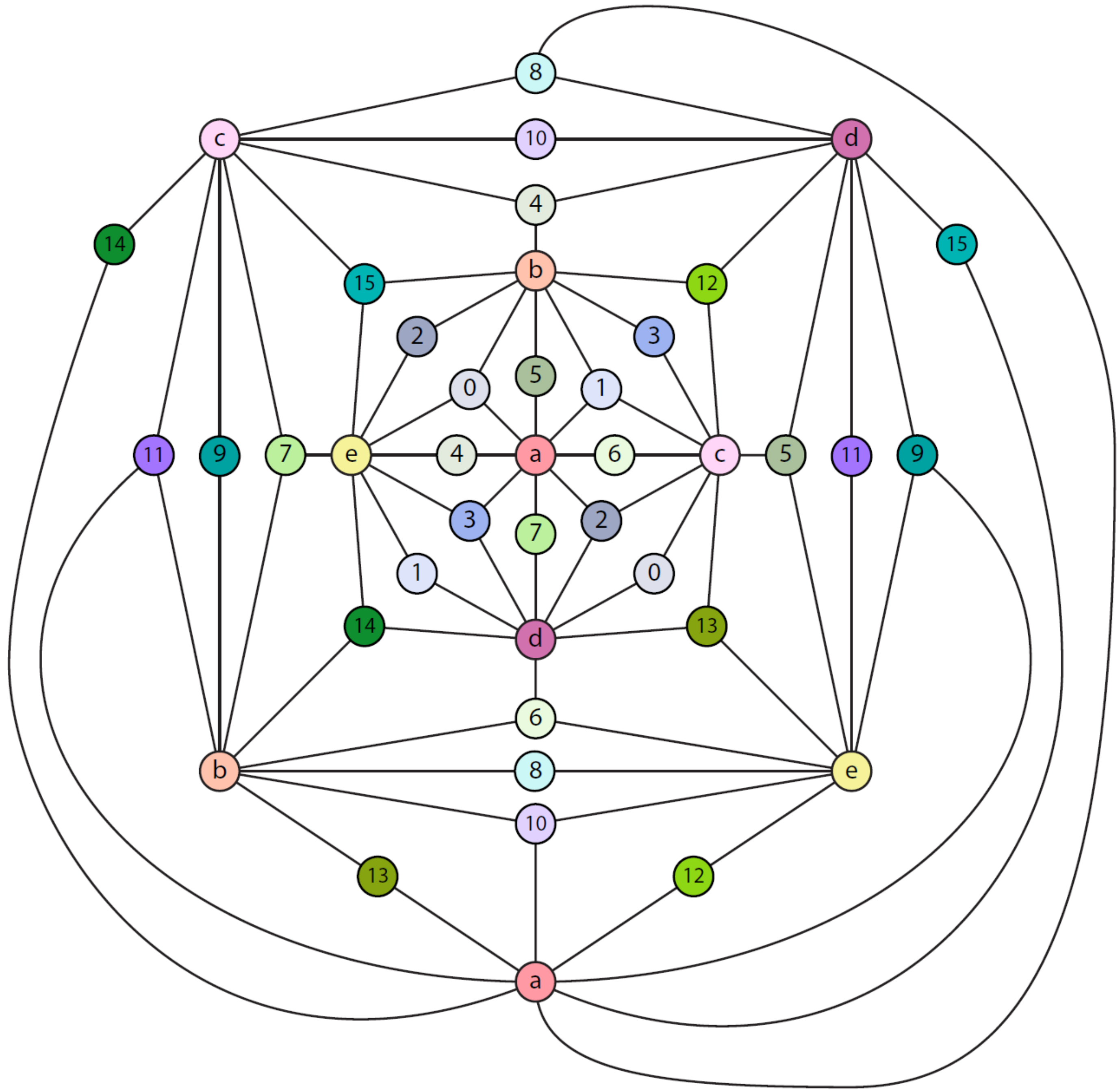
2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$? $n \leq 16$



2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$? $n \leq 16$

$K_{6,n}$?

2-Splits of Complete Bipartite Graphs

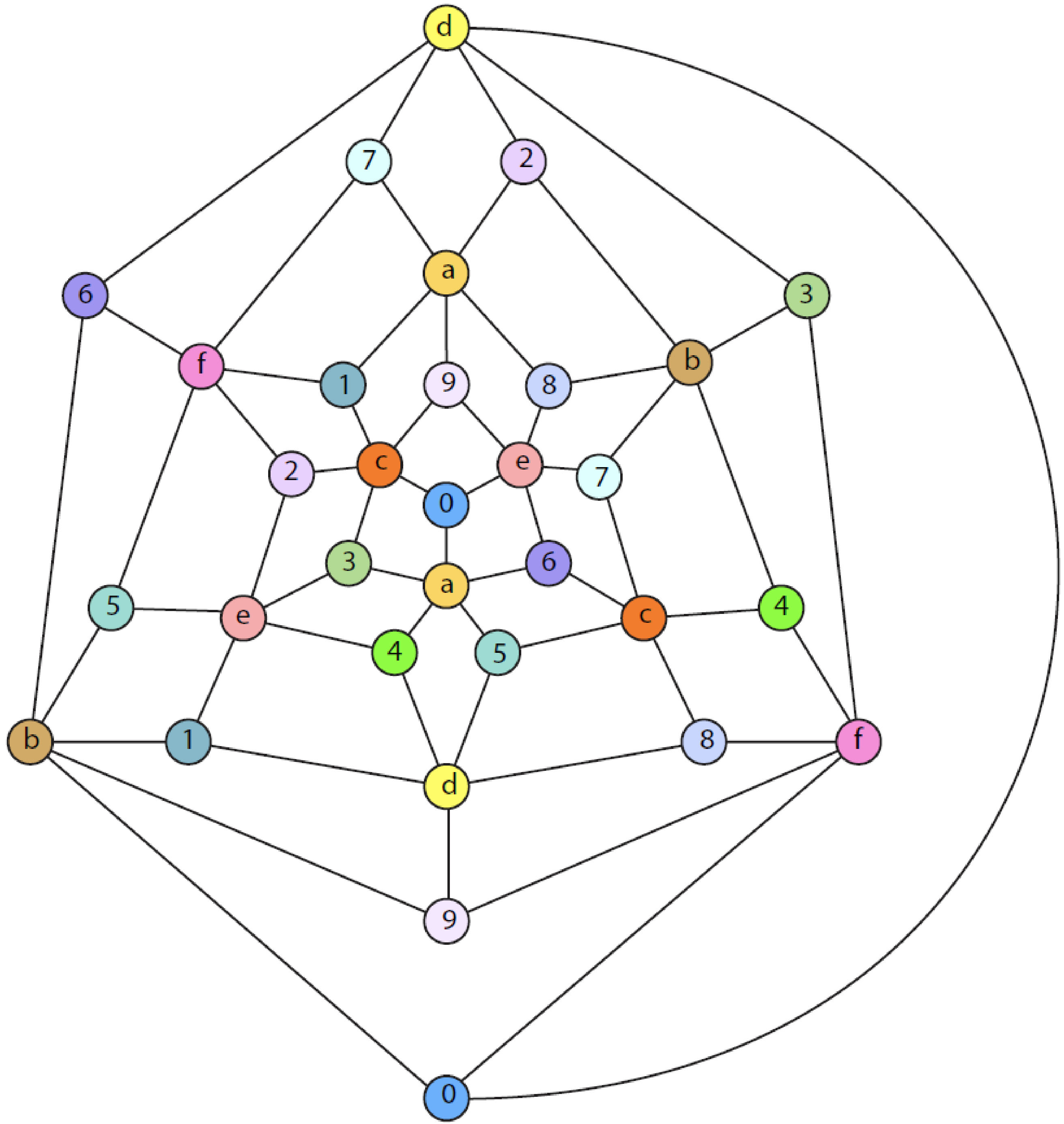
$K_{2,n}$? ✓

$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$? $n \leq 16$

$K_{6,n}$? $n \leq 10$



2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$? $n \leq 16$

$K_{6,n}$? $n \leq 10$

$K_{7,n}$?

2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

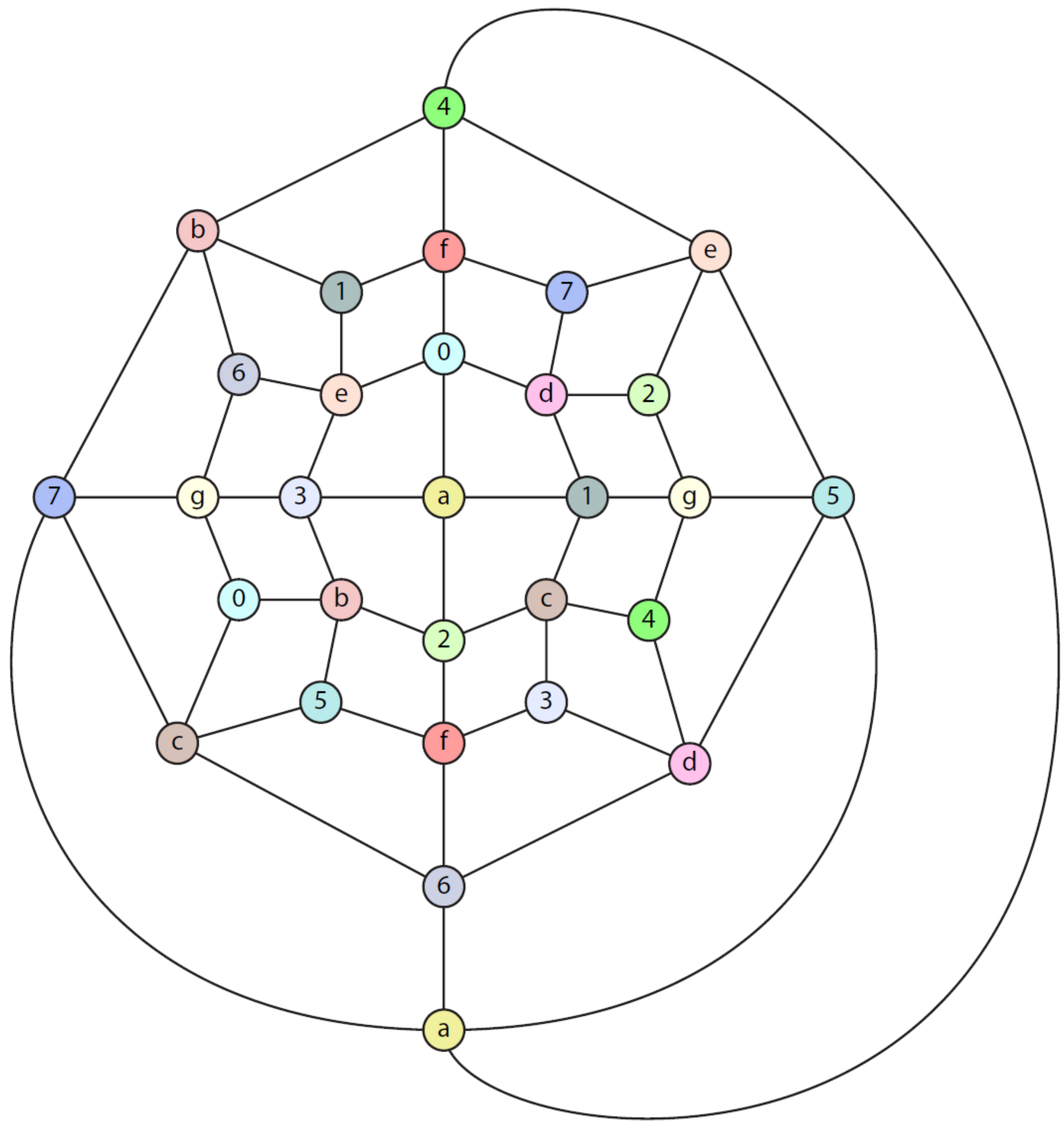
$K_{3,n}$? ✓

$K_{4,n}$? ✓

$K_{5,n}$? $n \leq 16$

$K_{6,n}$? $n \leq 10$

$K_{7,n}$? $n \leq 8$



2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

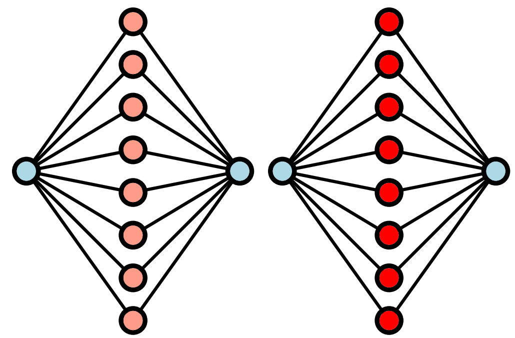
$K_{3,n}$? ✓

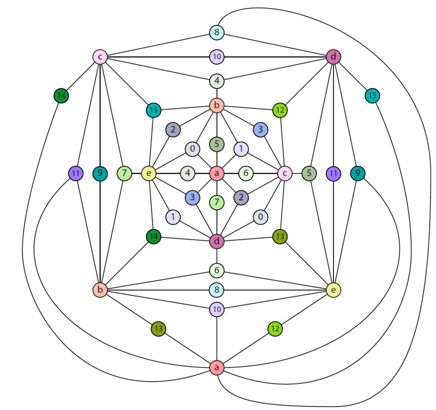
$K_{4,n}$? ✓

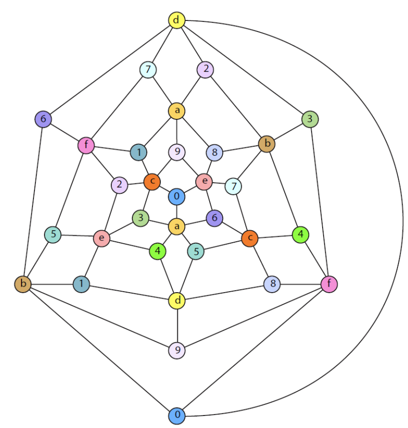
$K_{5,n}$? $n \leq 16$

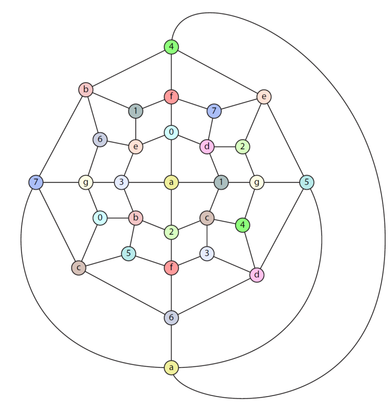
$K_{6,n}$? $n \leq 10$

$K_{7,n}$? $n \leq 8$









$K_{a,b}$ is 2-splittable if and only if $ab \leq 4(a+b) - 4$

2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

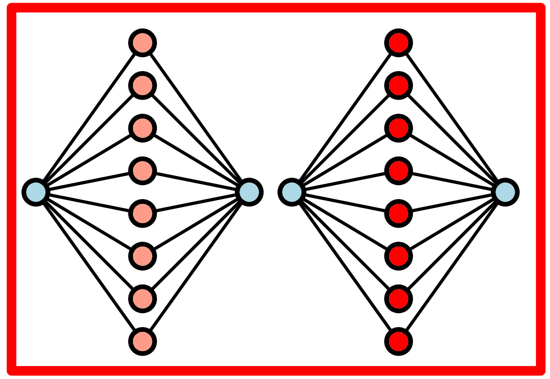
$K_{3,n}$? ✓

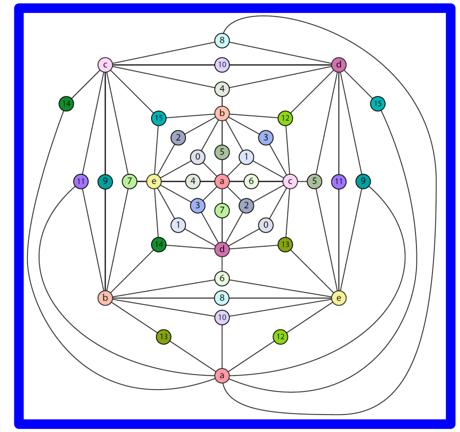
$K_{4,n}$? ✓

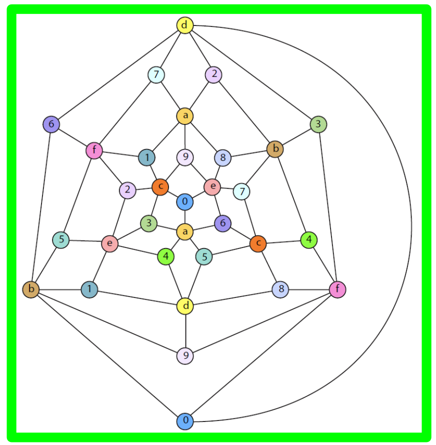
$K_{5,n}$? $n \leq 16$

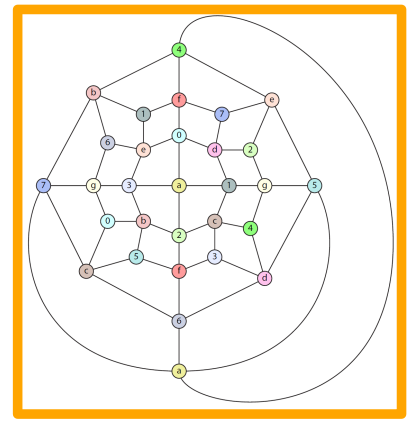
$K_{6,n}$? $n \leq 10$

$K_{7,n}$? $n \leq 8$









$K_{a,b}$ is 2-splittable if and only if $ab \leq 4(a+b) - 4$

Proof: $ab \leq 4(a+b) - 4 \Rightarrow G \subseteq K_{4,b}$, $K_{5,16}$, $K_{6,10}$, or $K_{7,8}$

2-Splits of Complete Bipartite Graphs

$K_{2,n}$? ✓

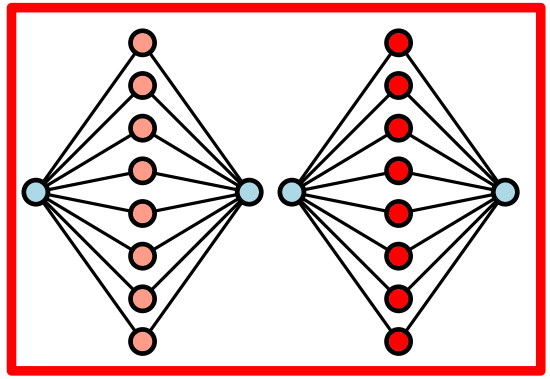
$K_{3,n}$? ✓

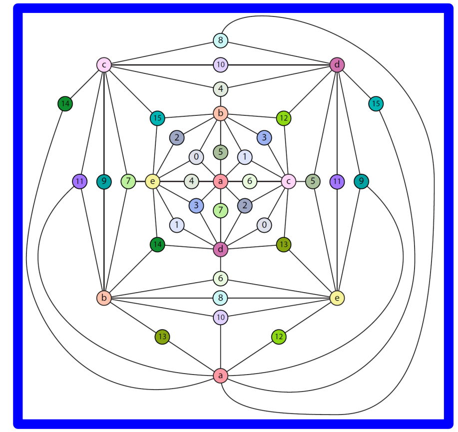
$K_{4,n}$? ✓

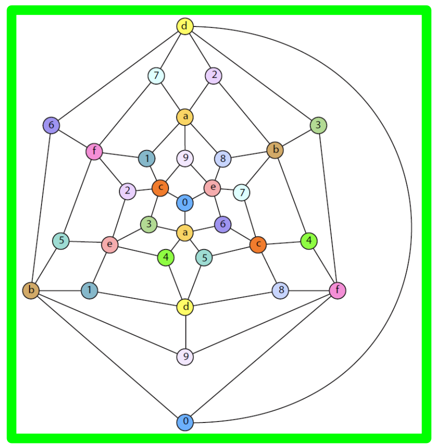
$K_{5,n}$? $n \leq 16$

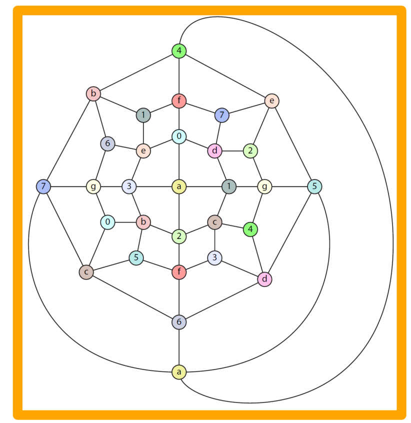
$K_{6,n}$? $n \leq 10$

$K_{7,n}$? $n \leq 8$









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Proof: $ab \leq 4(a+b) - 4 \Rightarrow G \subseteq K_{4,b}$, $K_{5,16}$, $K_{6,10}$, or $K_{7,8}$

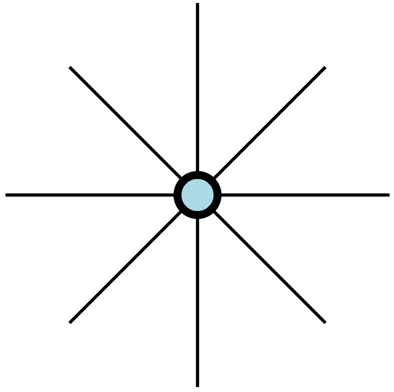
$ab > 4(a+b) - 4 \Rightarrow$ too many edges (Euler)

Max-Degree-Δ Graphs

Every max-degree-Δ graph is $\lceil \Delta/2 \rceil$-splittable

Max-Degree-Δ Graphs

Every max-degree-Δ graph is $\lceil \Delta/2 \rceil$-splittable



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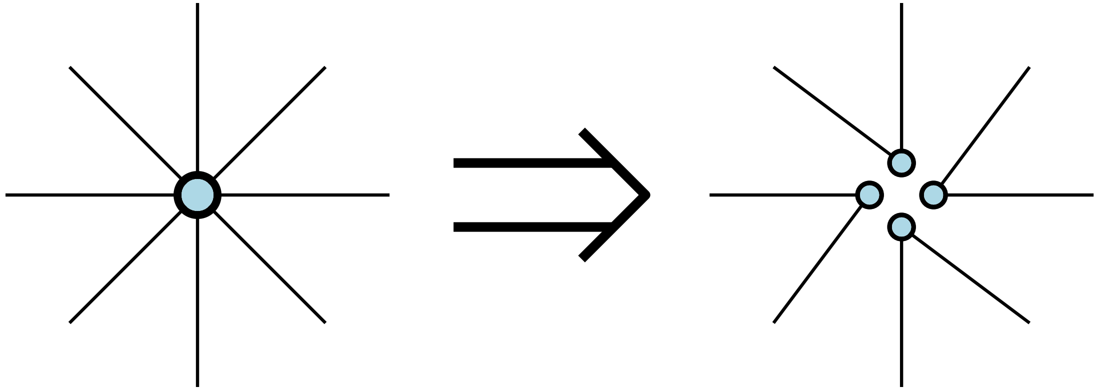

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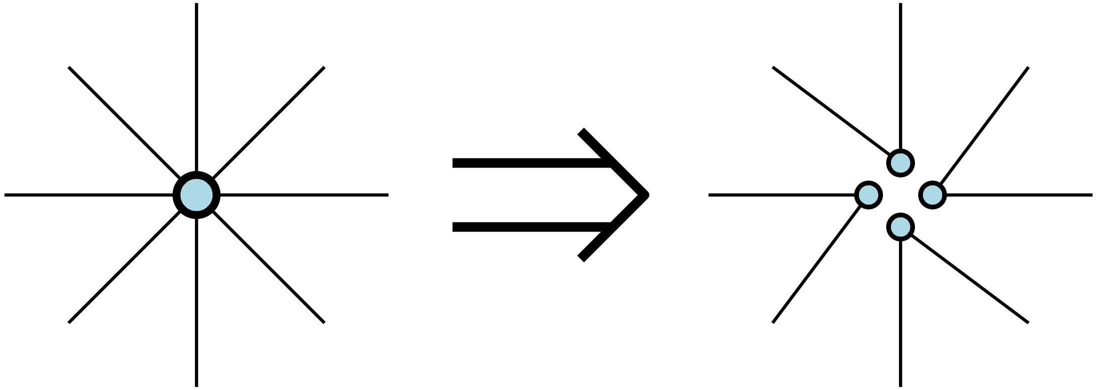

$\Rightarrow$ max-degree 2

Max-Degree-Δ Graphs

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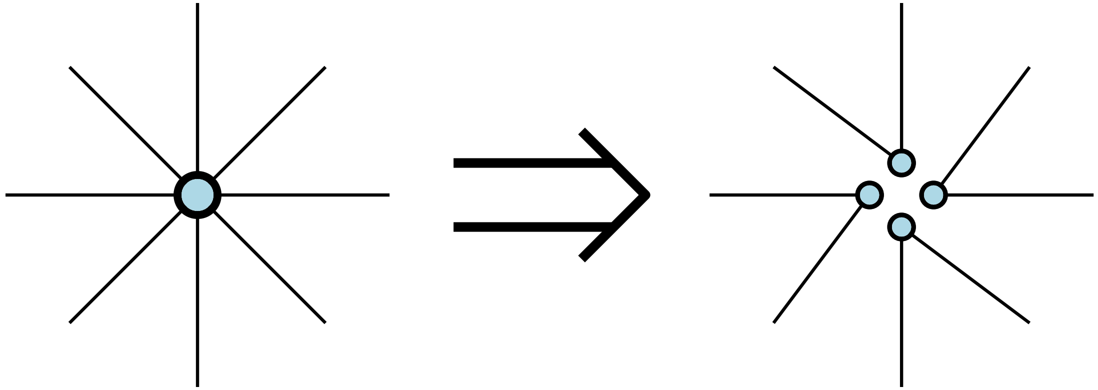

⇒ max-degree 2

⇒ planar

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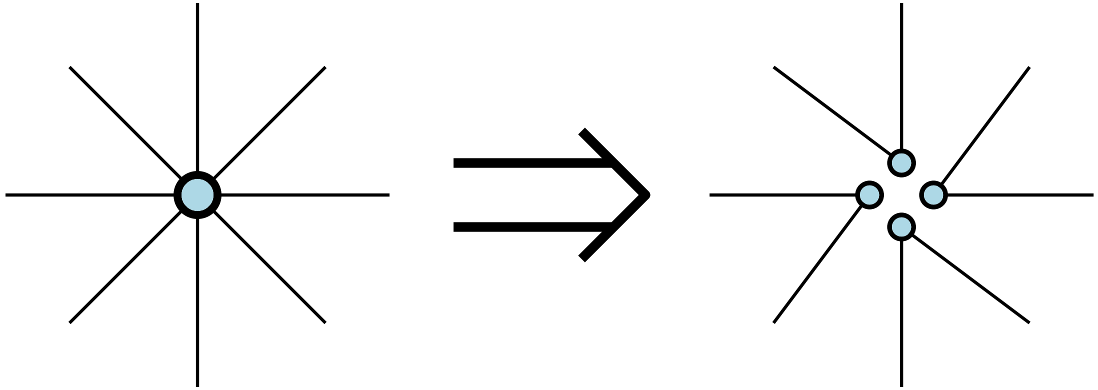

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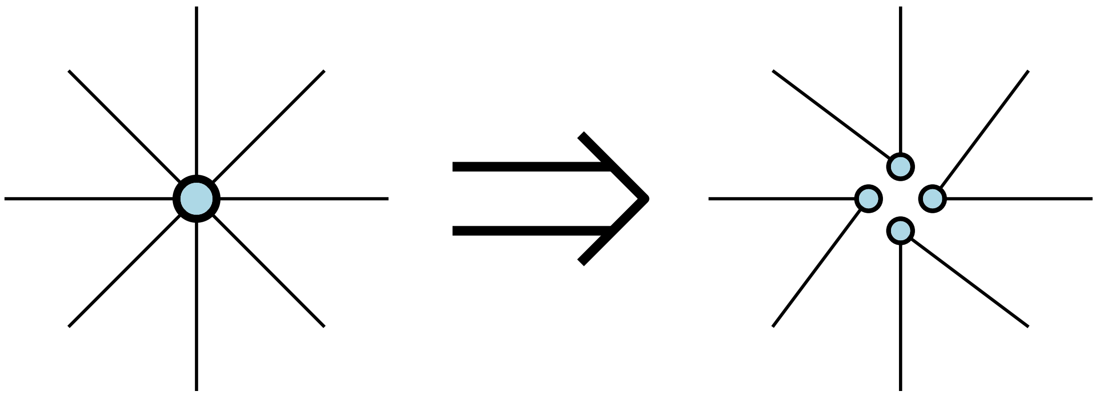

$\Rightarrow$ max-degree 2

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Not every max-degree-Δ graph is $\lfloor \Delta/2 \rfloor$-splittable

Max-Degree-Δ Graphs

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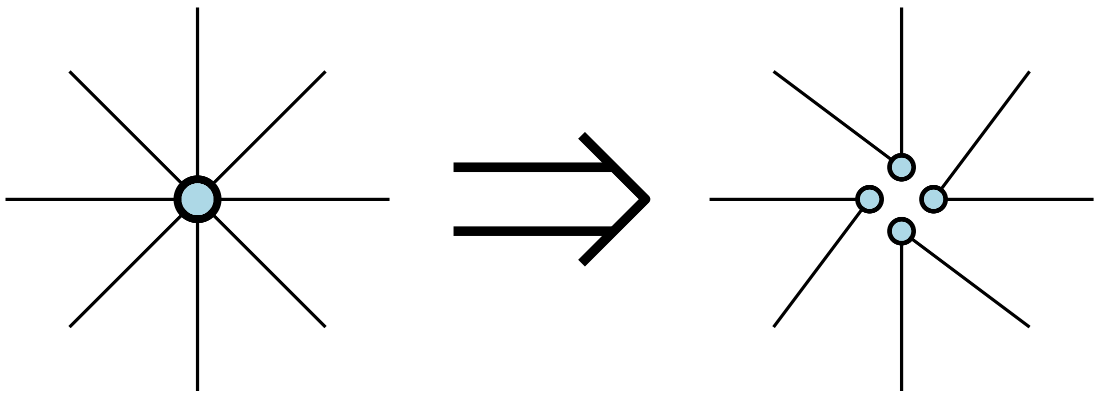

⇒ max-degree 2

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Not every max-degree-Δ graph is $\lfloor \Delta/2 \rfloor$-splittable

1. $\Delta \geq 5 \Rightarrow \exists$ Δ-regular graphs of size n with girth $\Omega(\log n)$

Max-Degree-Δ Graphs

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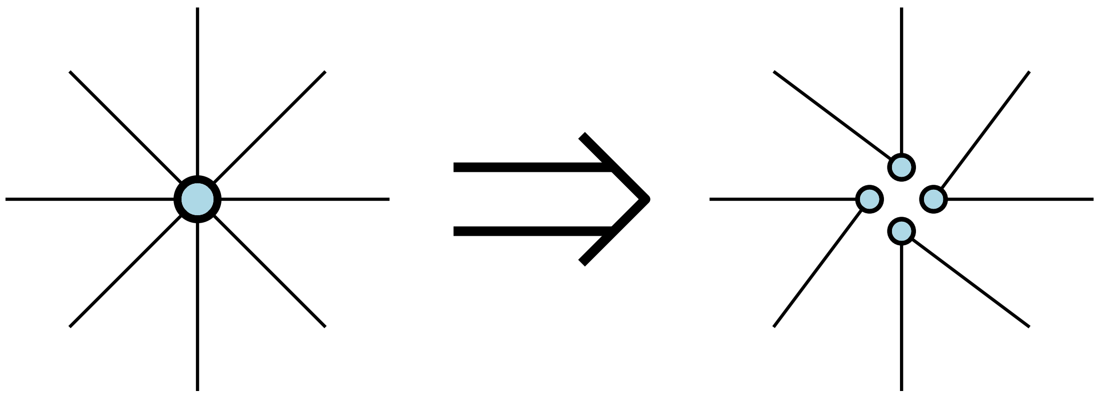

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length of smallest cycle

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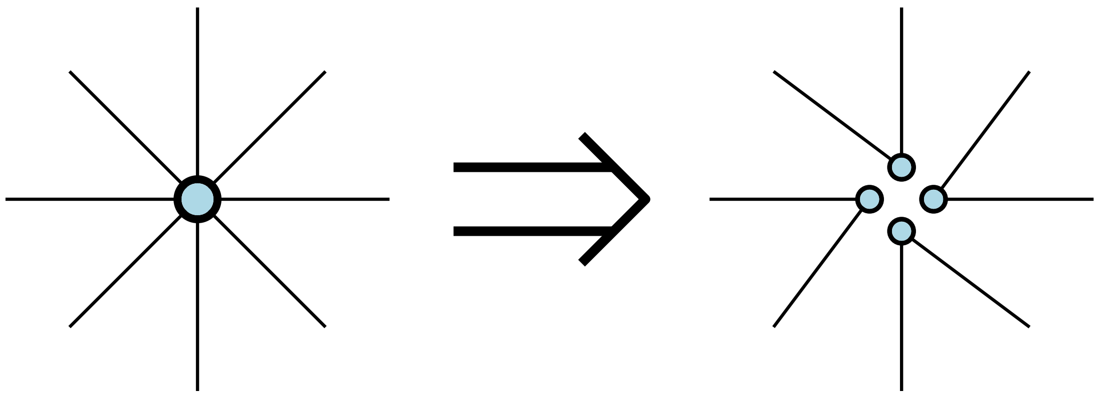

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Not every max-degree-Δ graph is $\lfloor \Delta/2 \rfloor$-splittable

1. $\Delta \geq 5 \Rightarrow \exists$ Δ-regular graphs of size n with girth $\Omega(\log n)$
2. Splitting a graph cannot decrease its girth.

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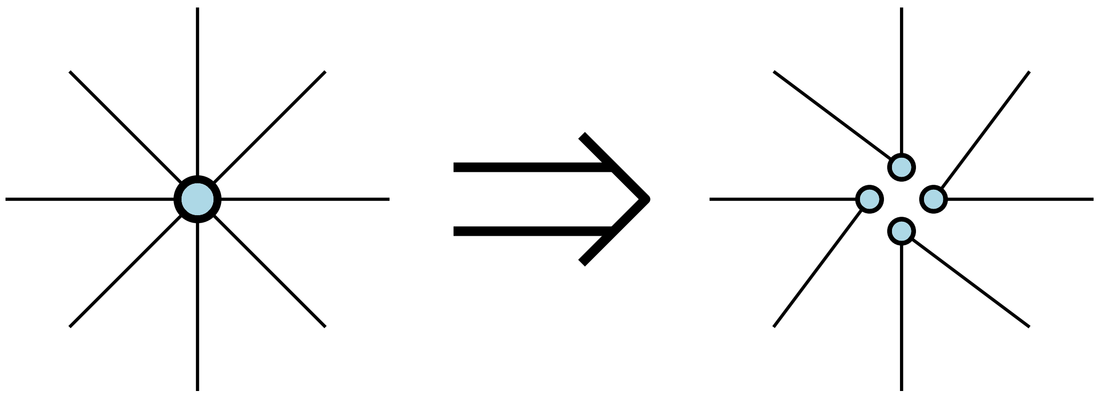

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   length of smallest cycle

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3. High-girth planar graphs have $\leq (1 + o(1))n$ edges

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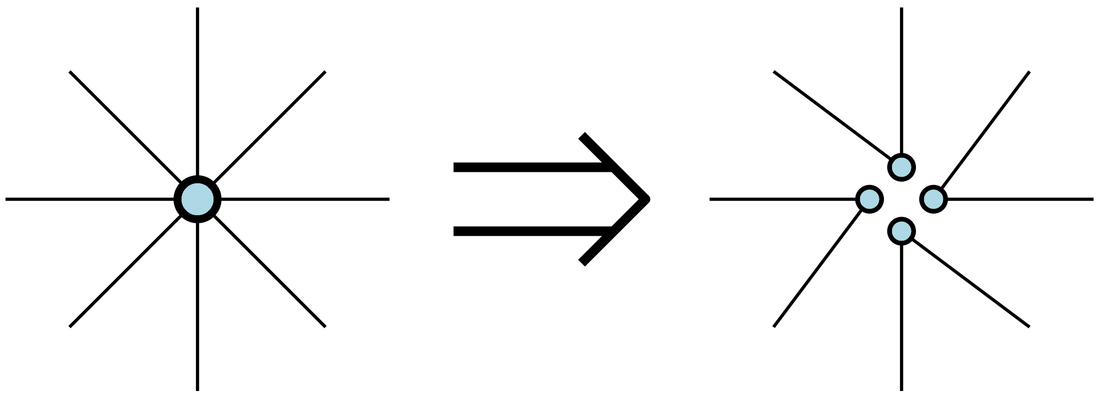

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1. $\Delta \geq 5 \Rightarrow \exists$ Δ-regular graphs of size n with girth $\Omega(\log n)$ (girth: length of smallest cycle)
2. Splitting a graph cannot decrease its girth.
3. High-girth planar graphs have $\leq (1 + o(1))n$ edges
4. Any $\lfloor \Delta/2 \rfloor$-split would have $(1 + \frac{1}{\Delta - 1})n$ edges

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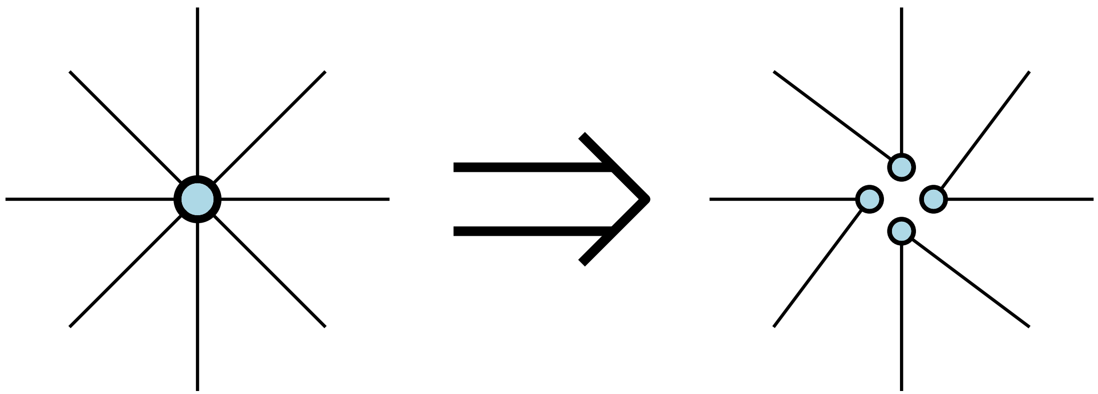

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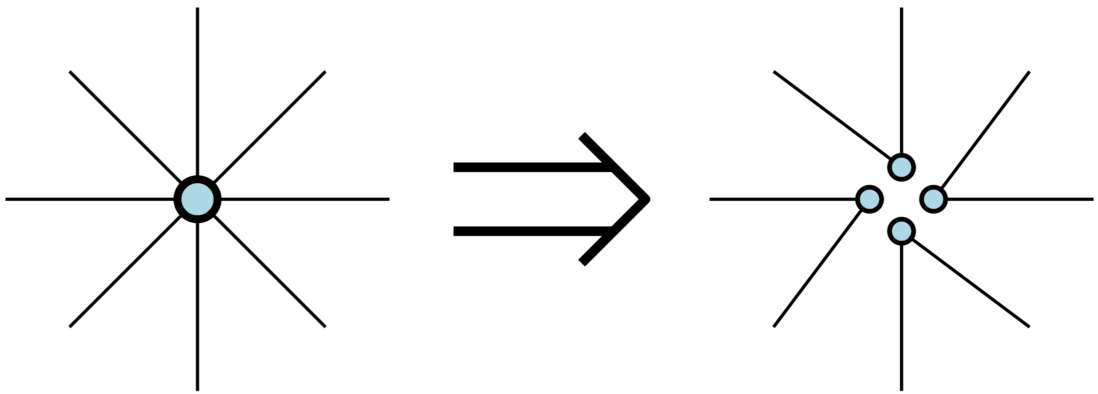

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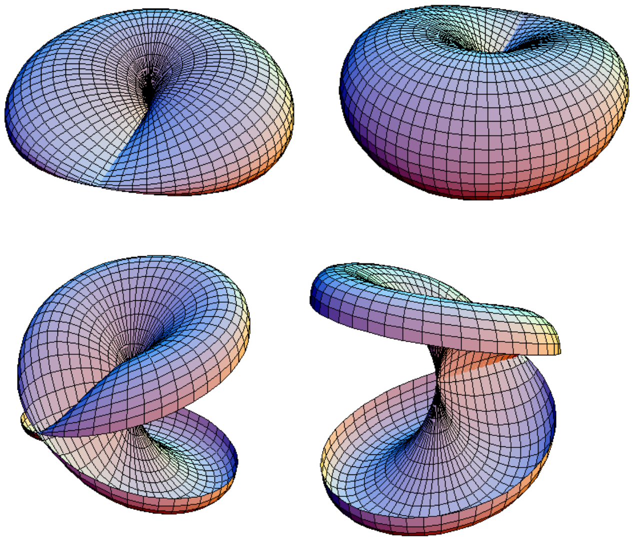
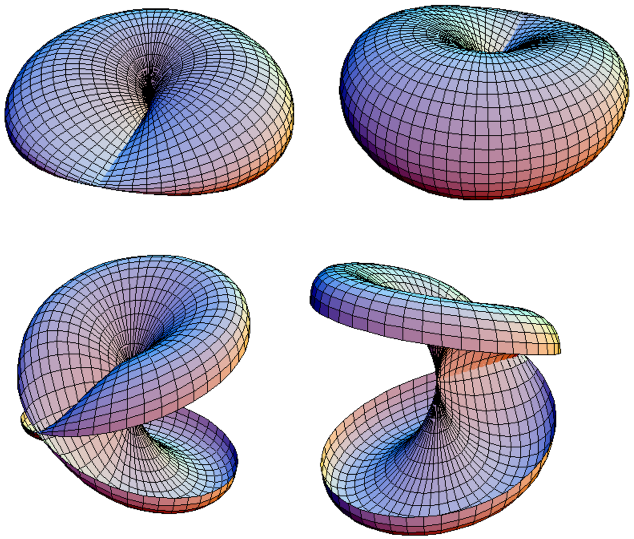
Lower bound holds for every minor-free graph class

Genus-1-Planar Graphs

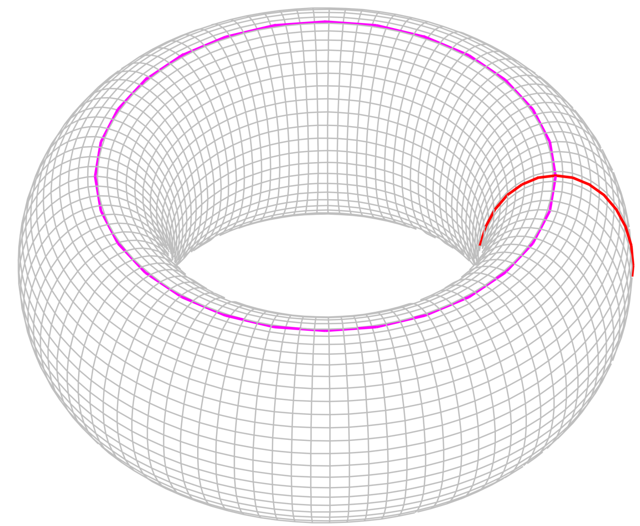


projective plane

Genus-1-Planar Graphs

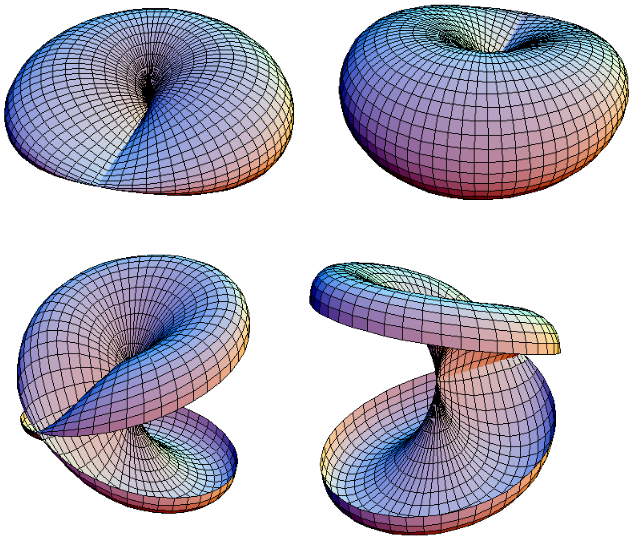


projective plane



torus

Genus-1-Planar Graphs



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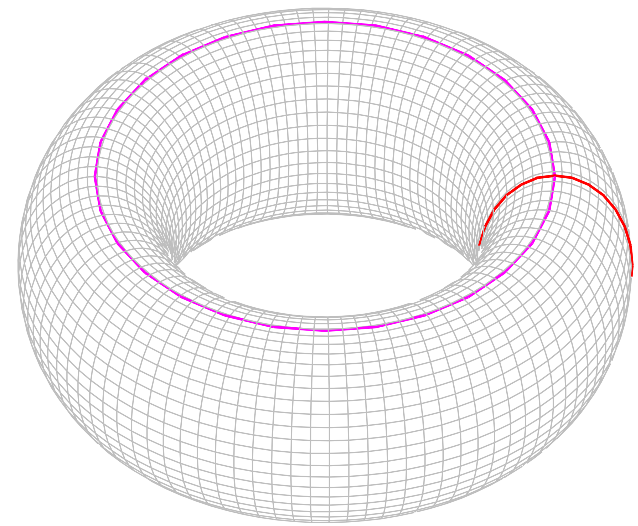

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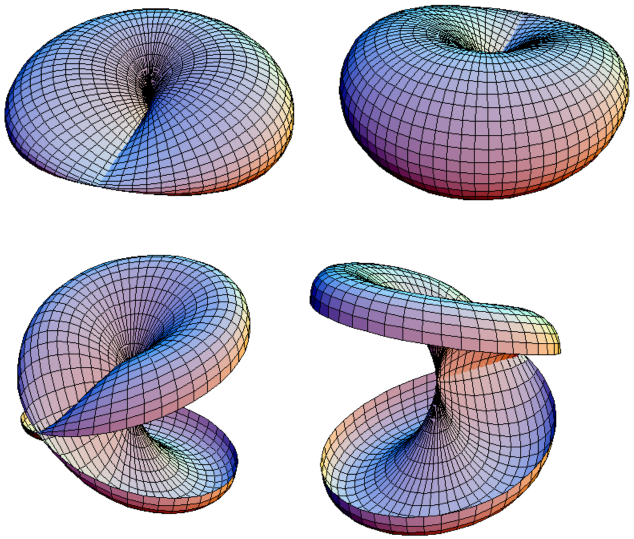

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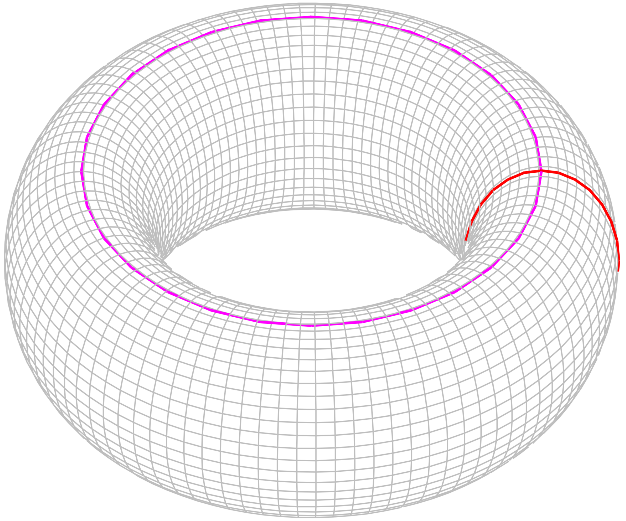
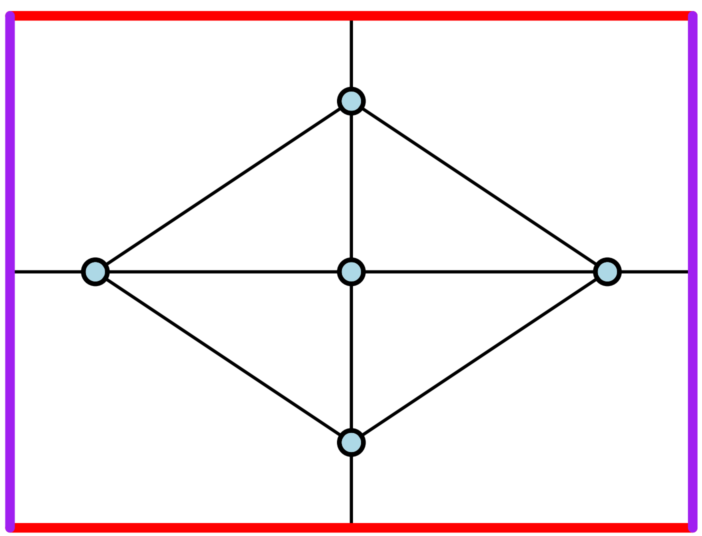


torus



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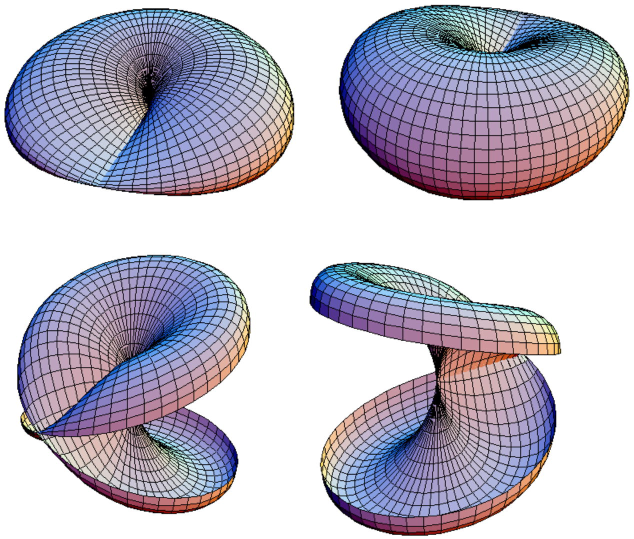

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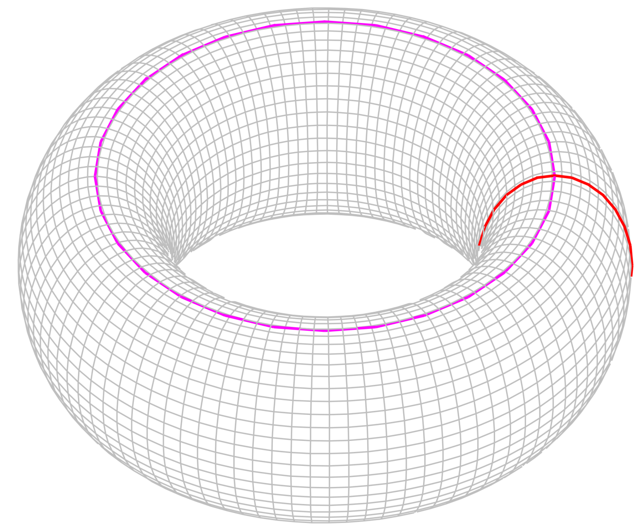
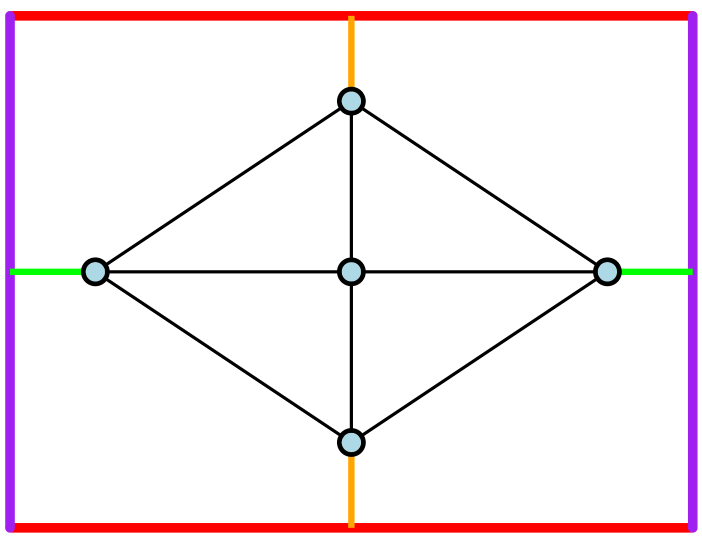


torus



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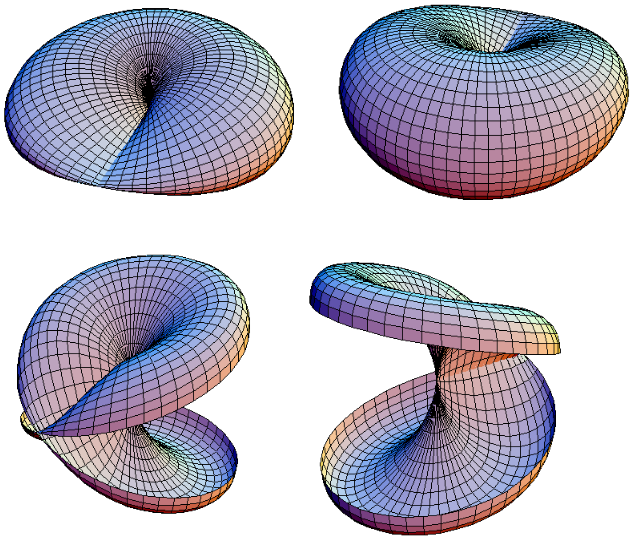

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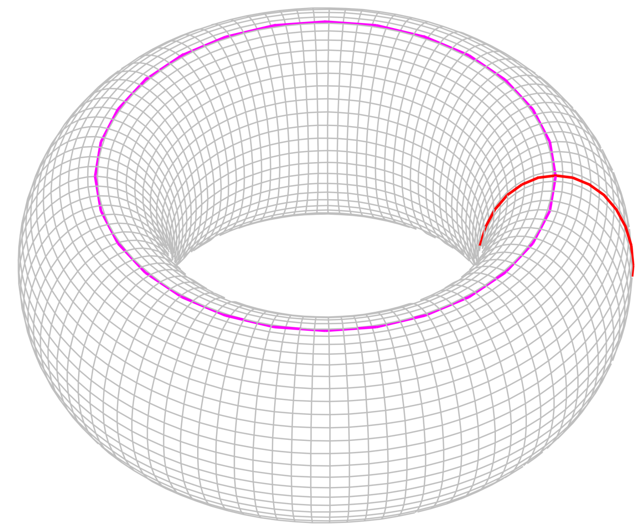

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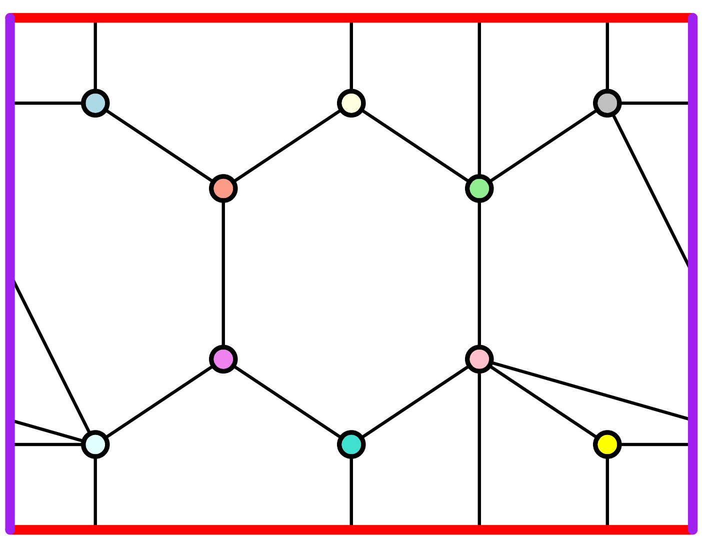

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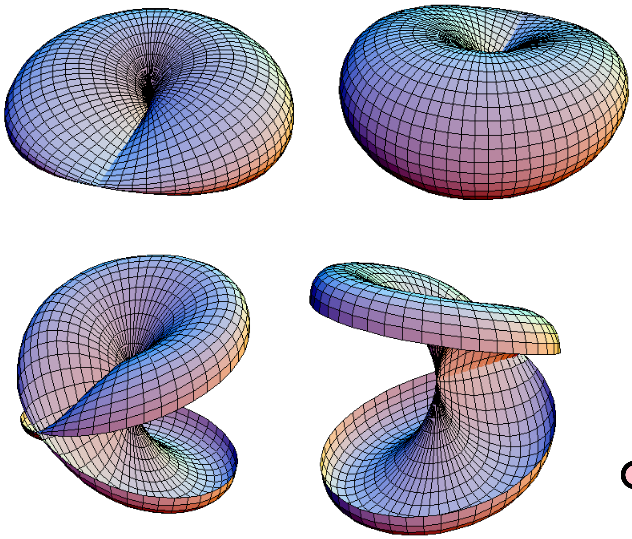
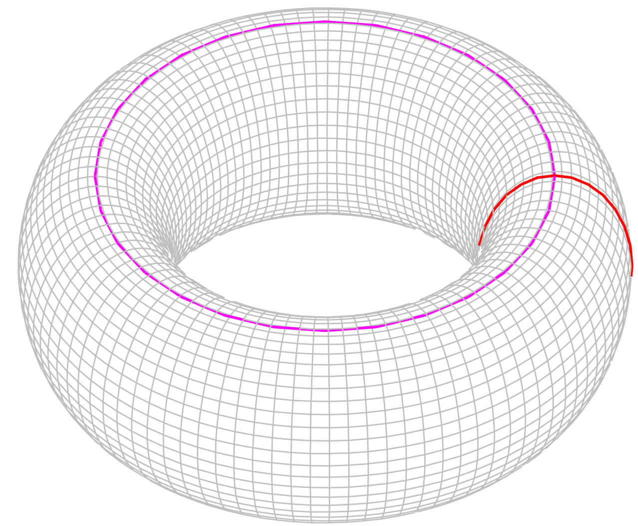
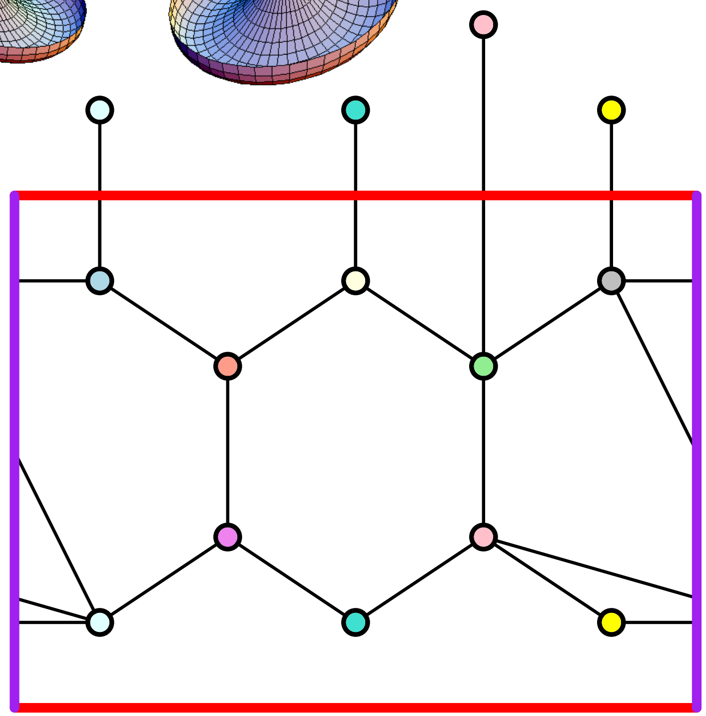


projective plane



torus



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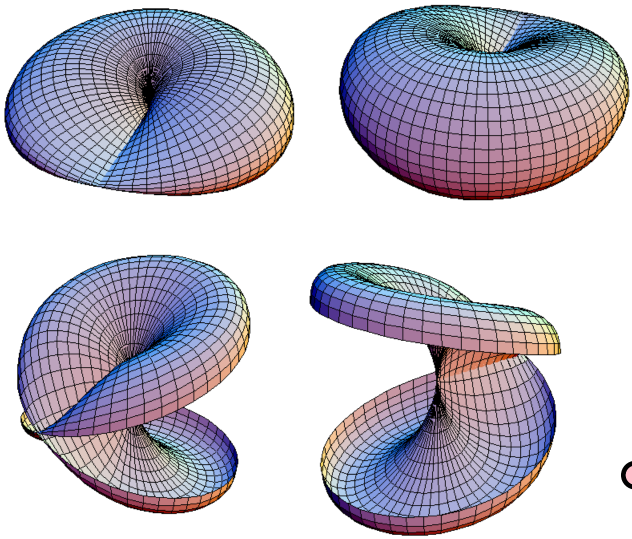
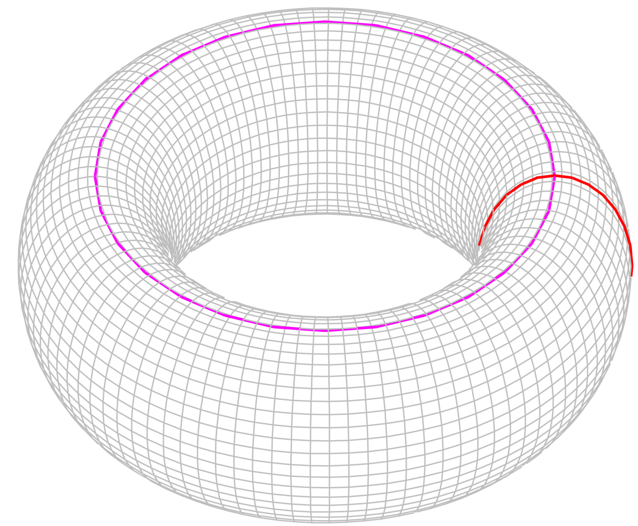


projective plane



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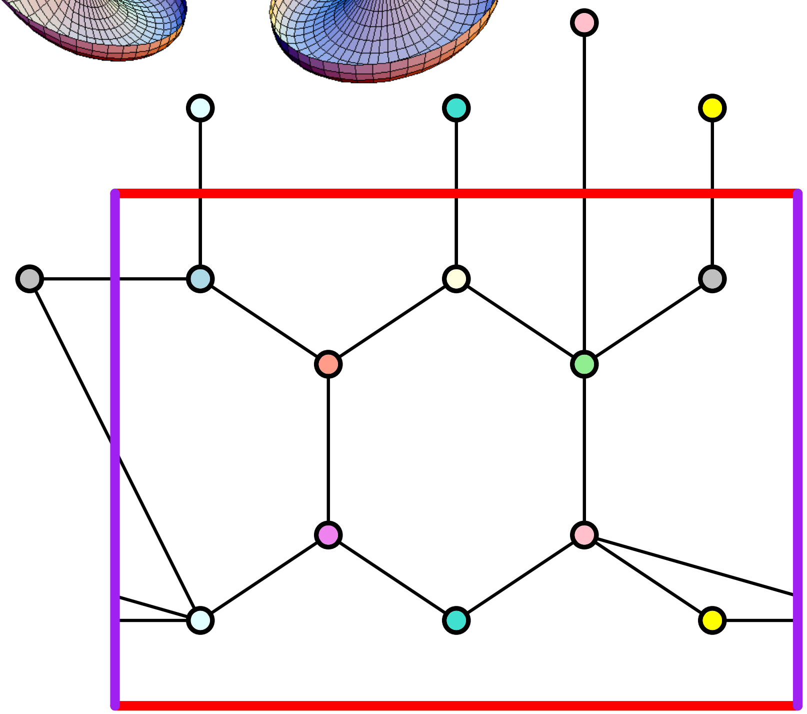

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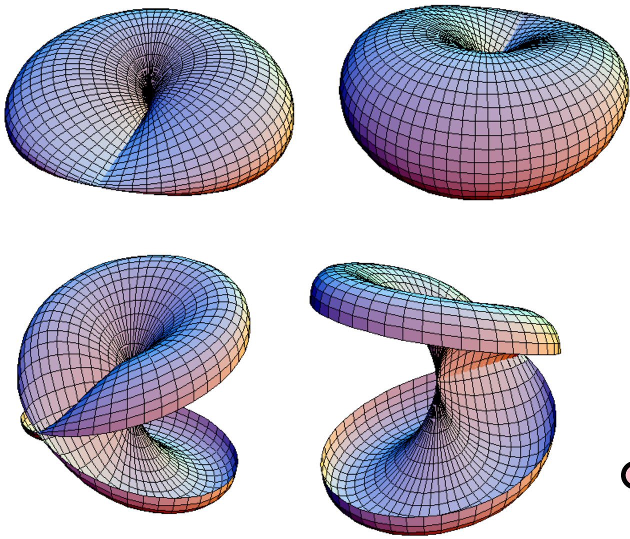

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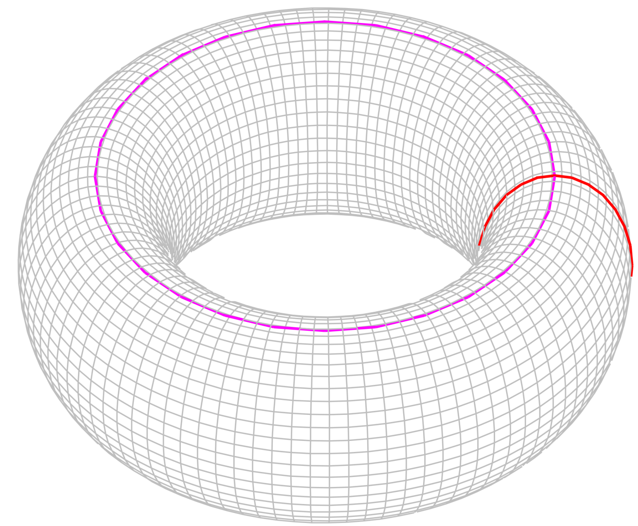

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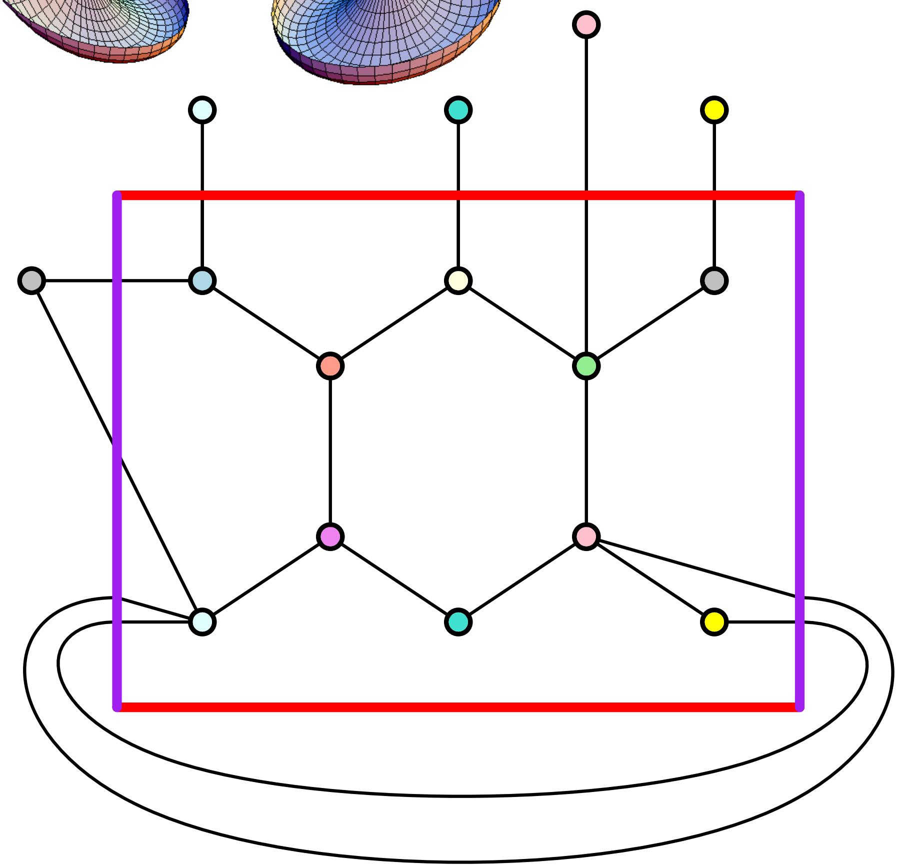

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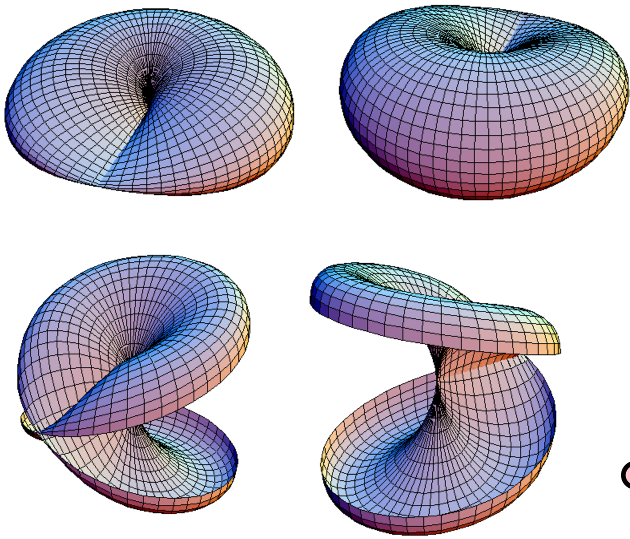

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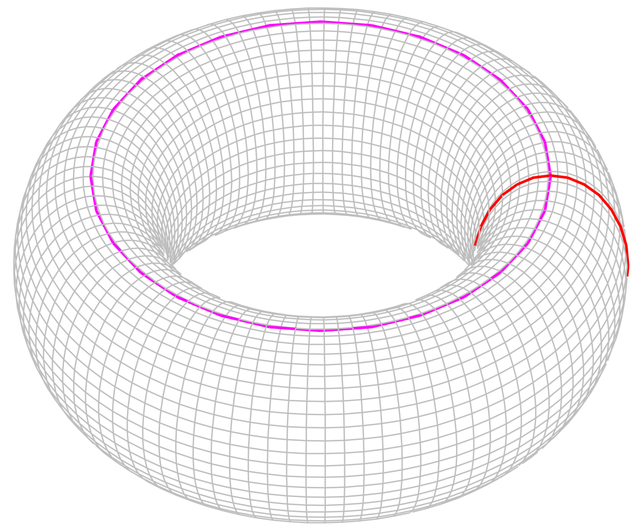

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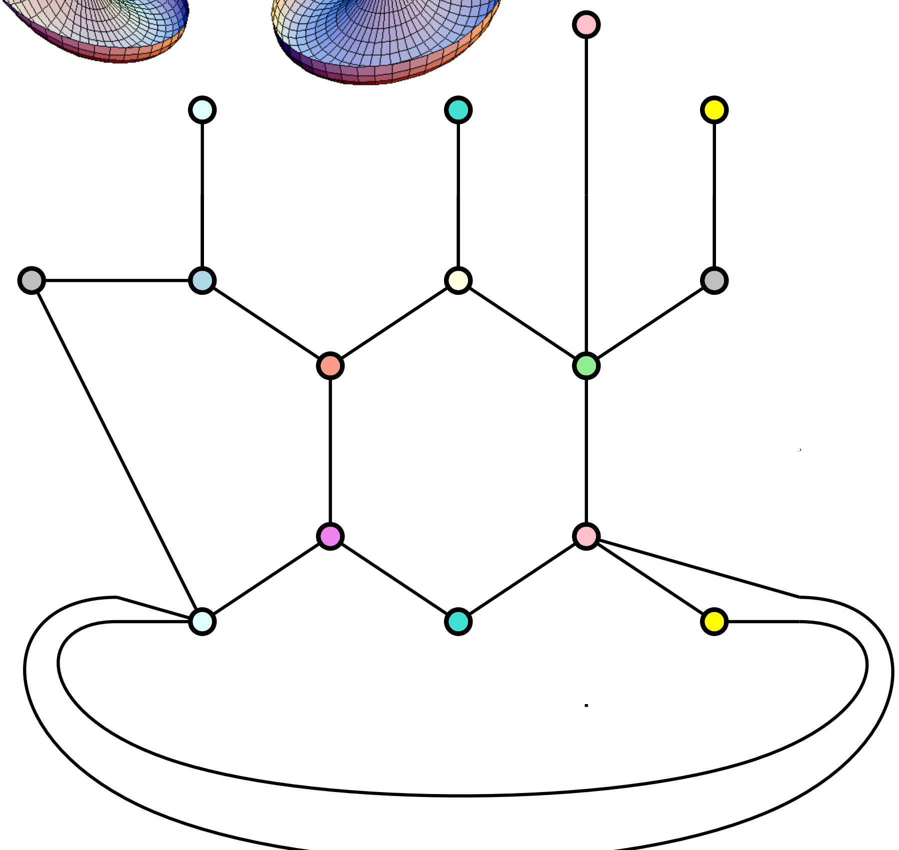

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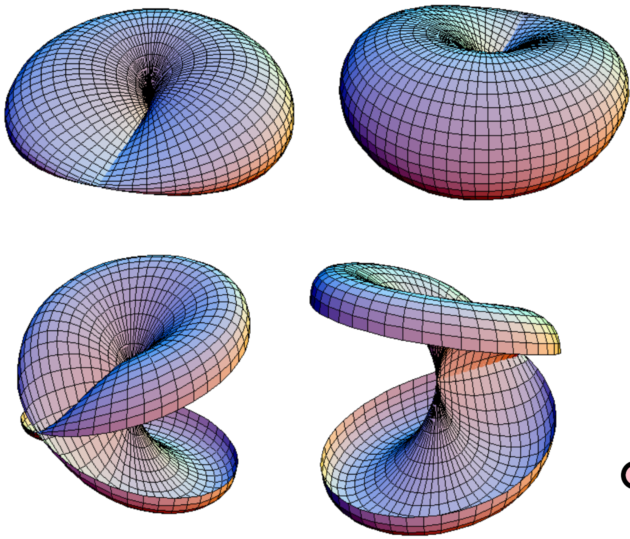

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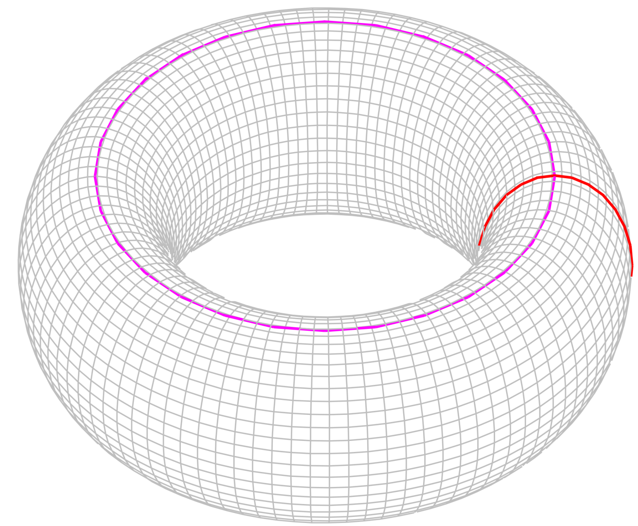

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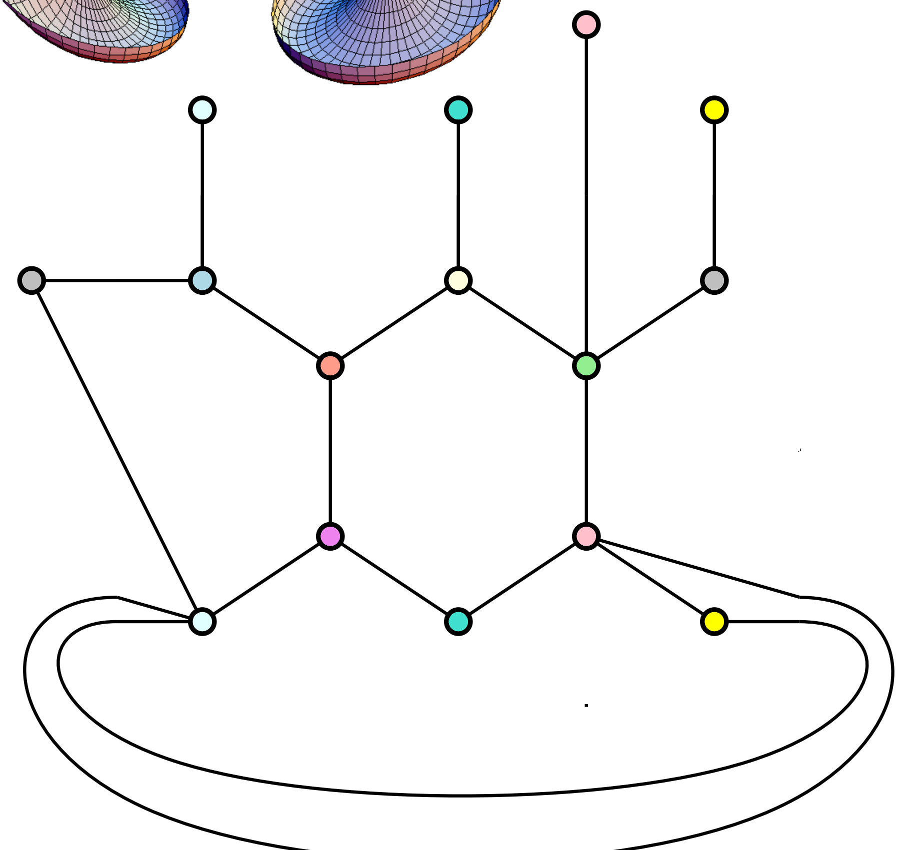
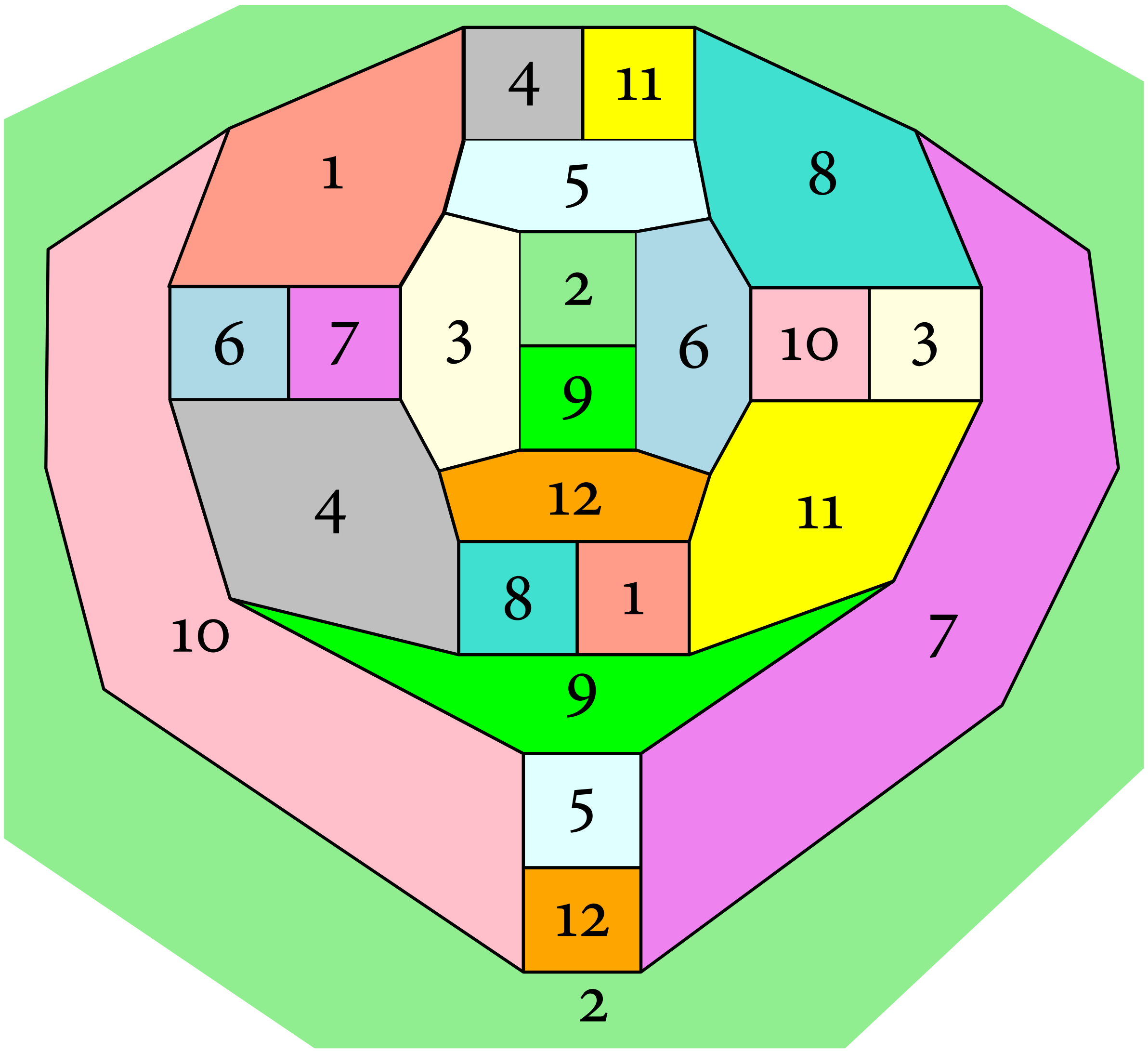
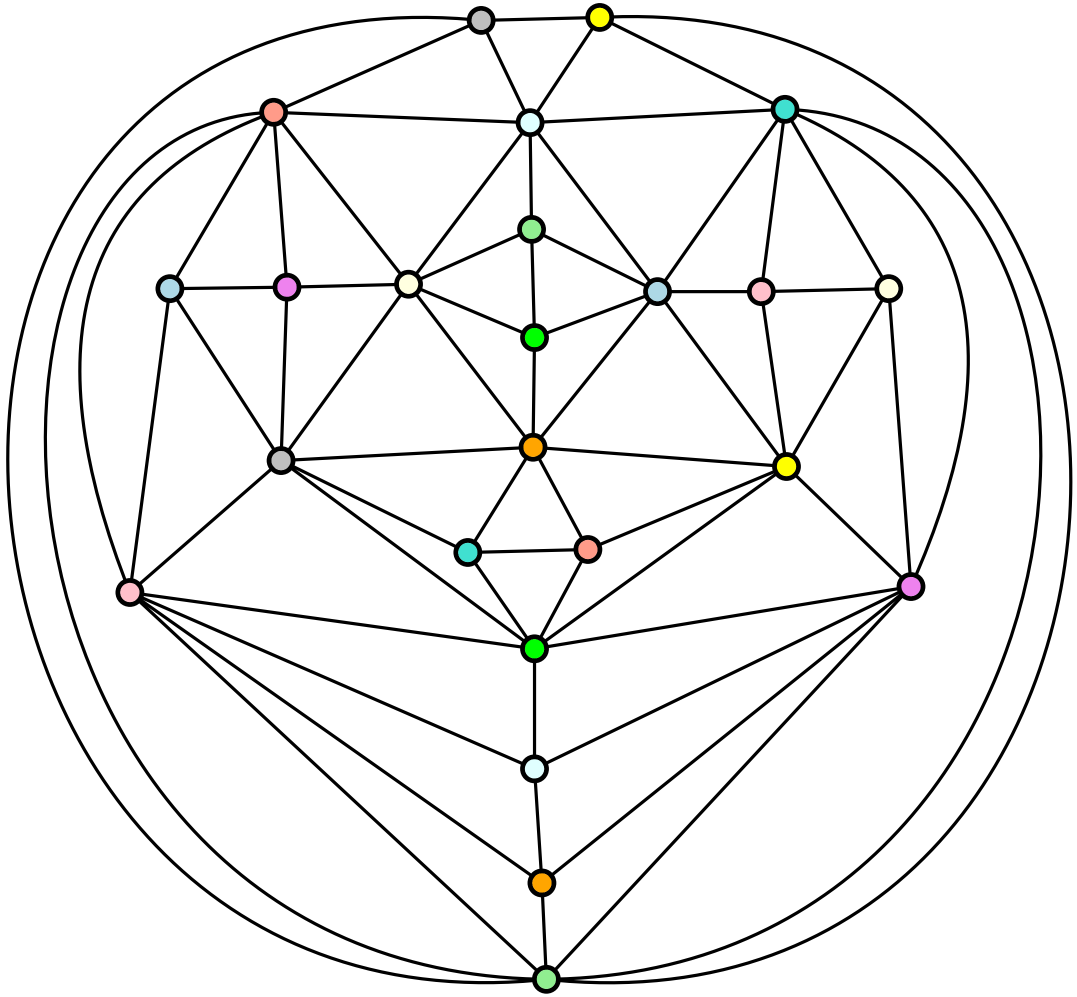


Projective-planar and toroidal graphs are 2-splittable

NP-hardness of 2-Splittability



NP-hardness of 2-Splittability



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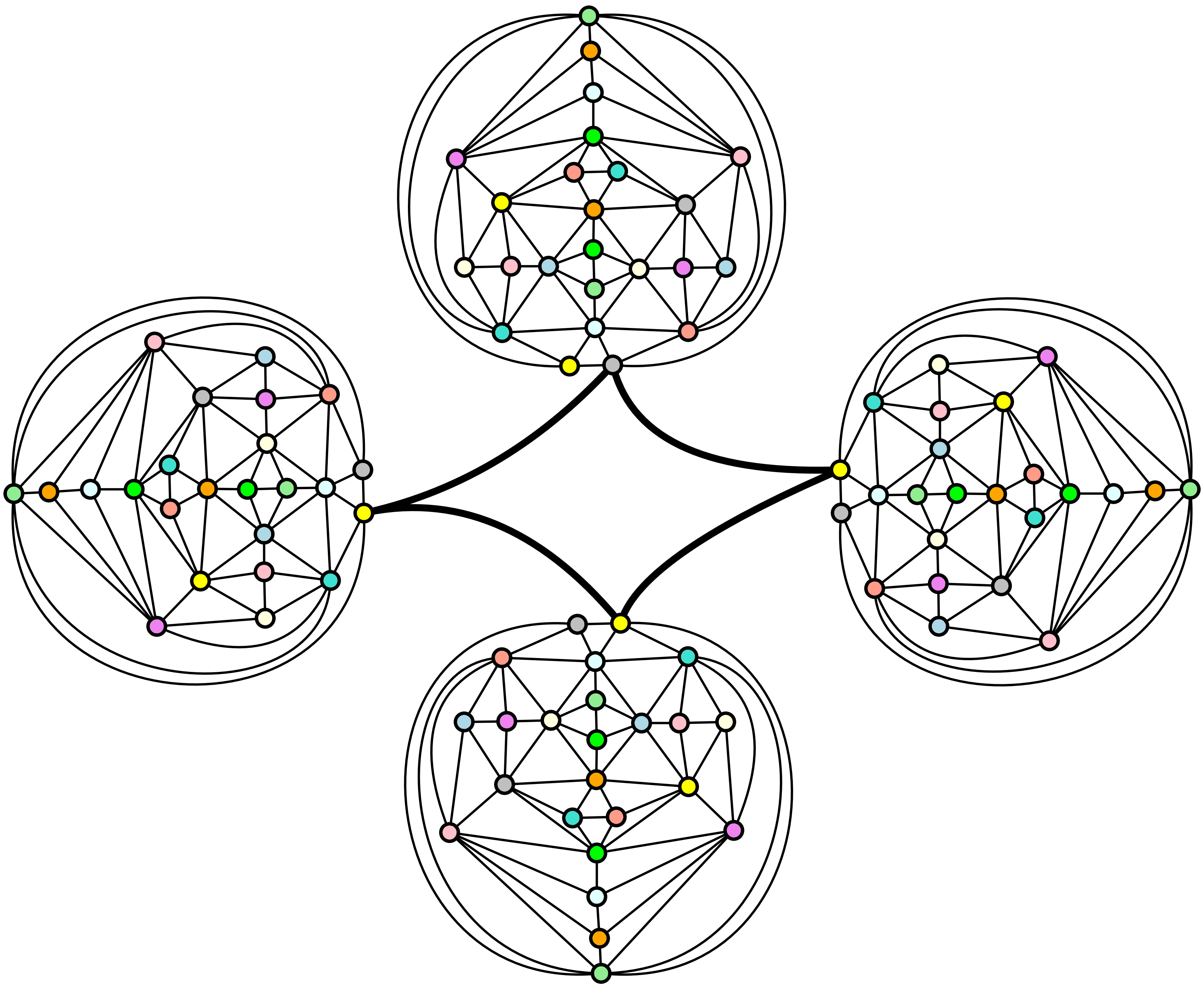

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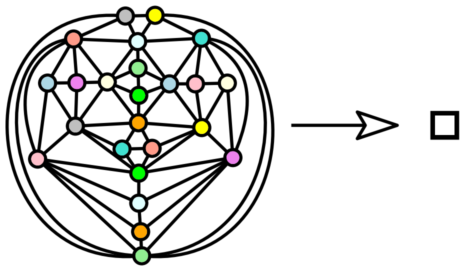
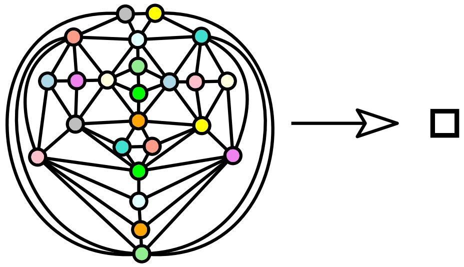


NP-hardness of 2-Splittability

Reduction from planar 3-SAT with a cycle through clause vertices

[Kratochvíl, Lubiw & Nešetřil 1991]



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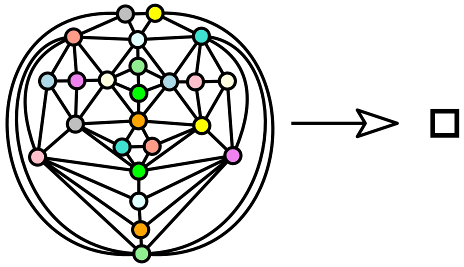
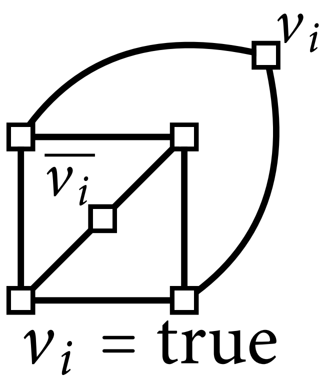


Variable:



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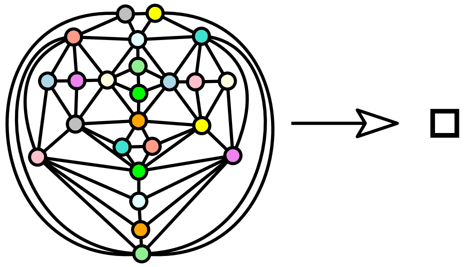

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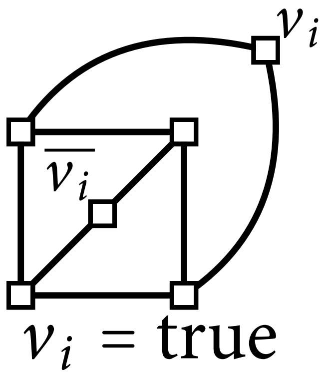

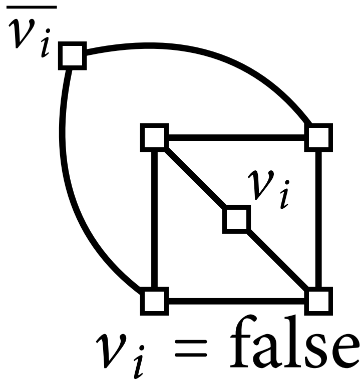

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Reduction from planar 3-SAT with a cycle through clause vertices

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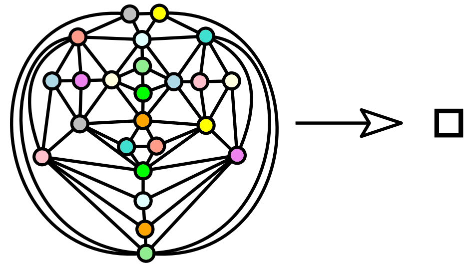

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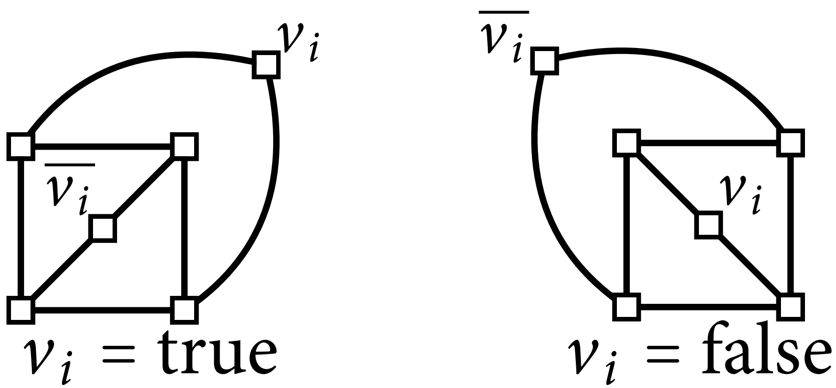

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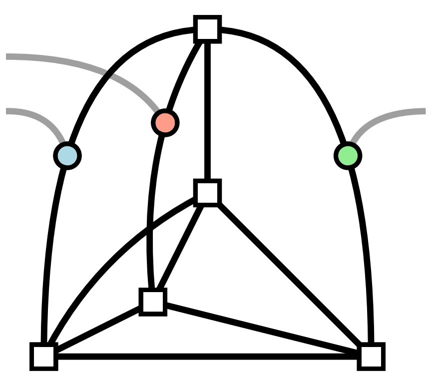

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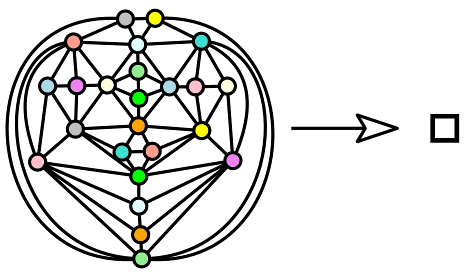

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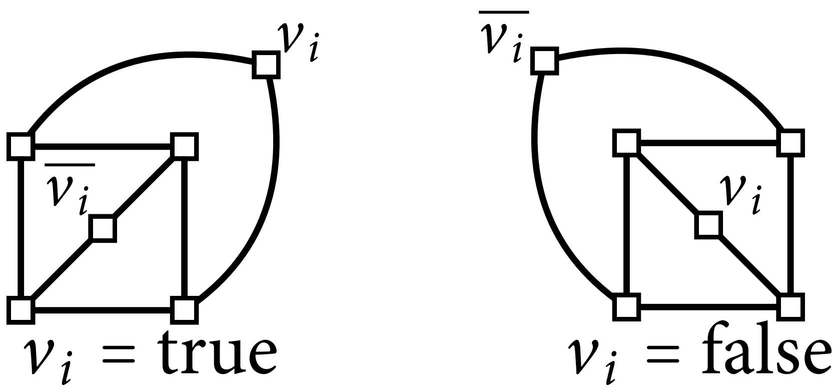

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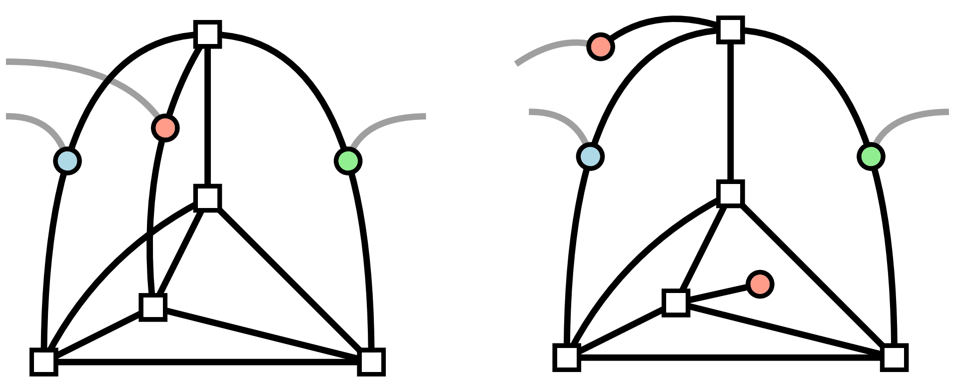

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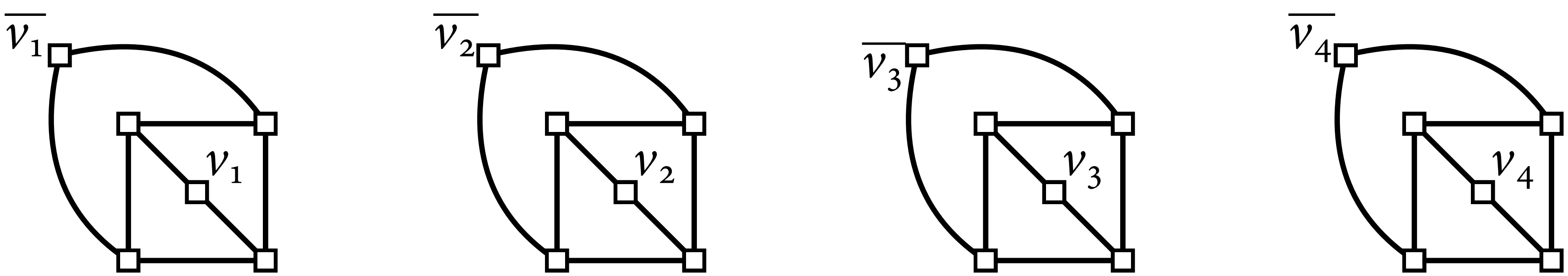

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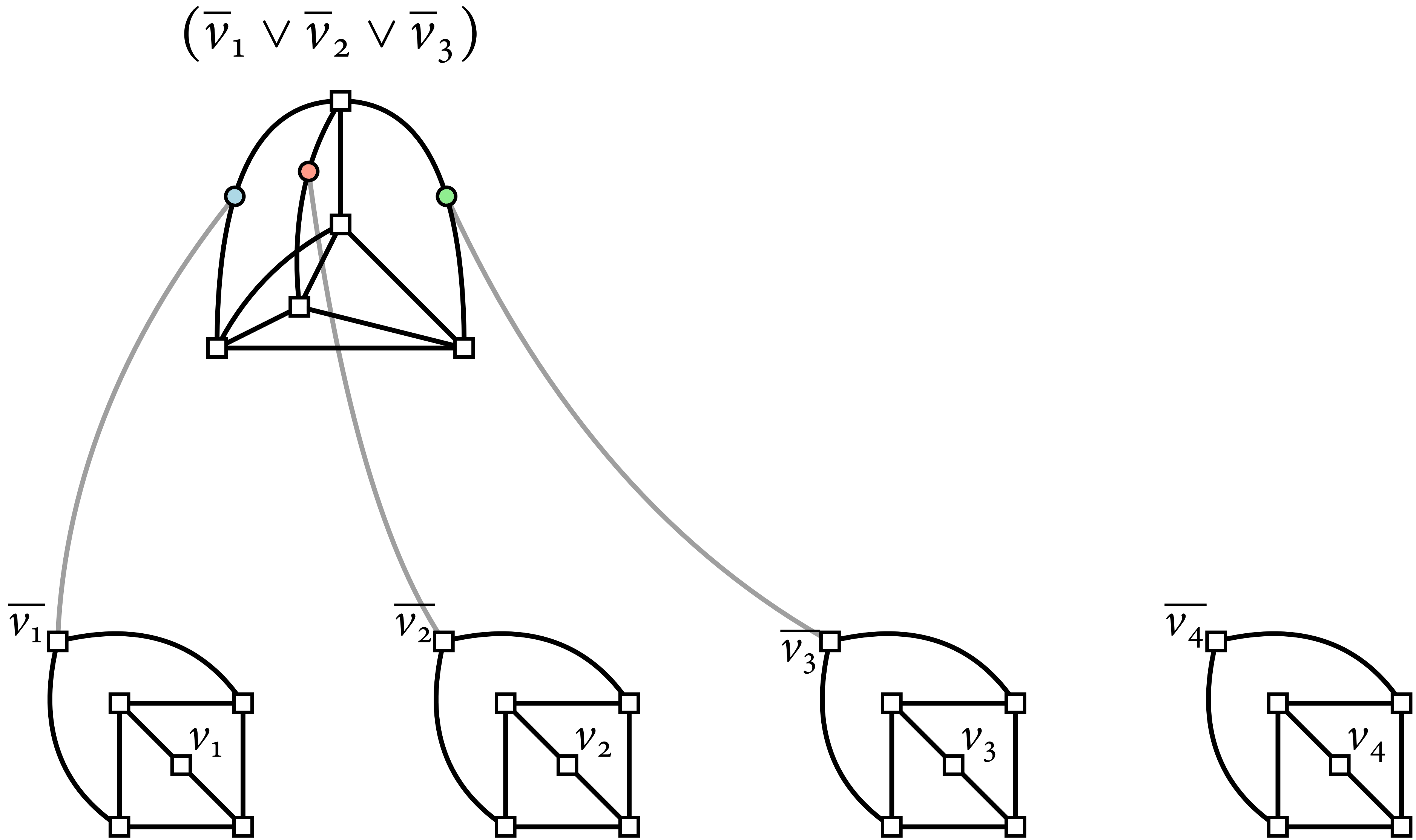

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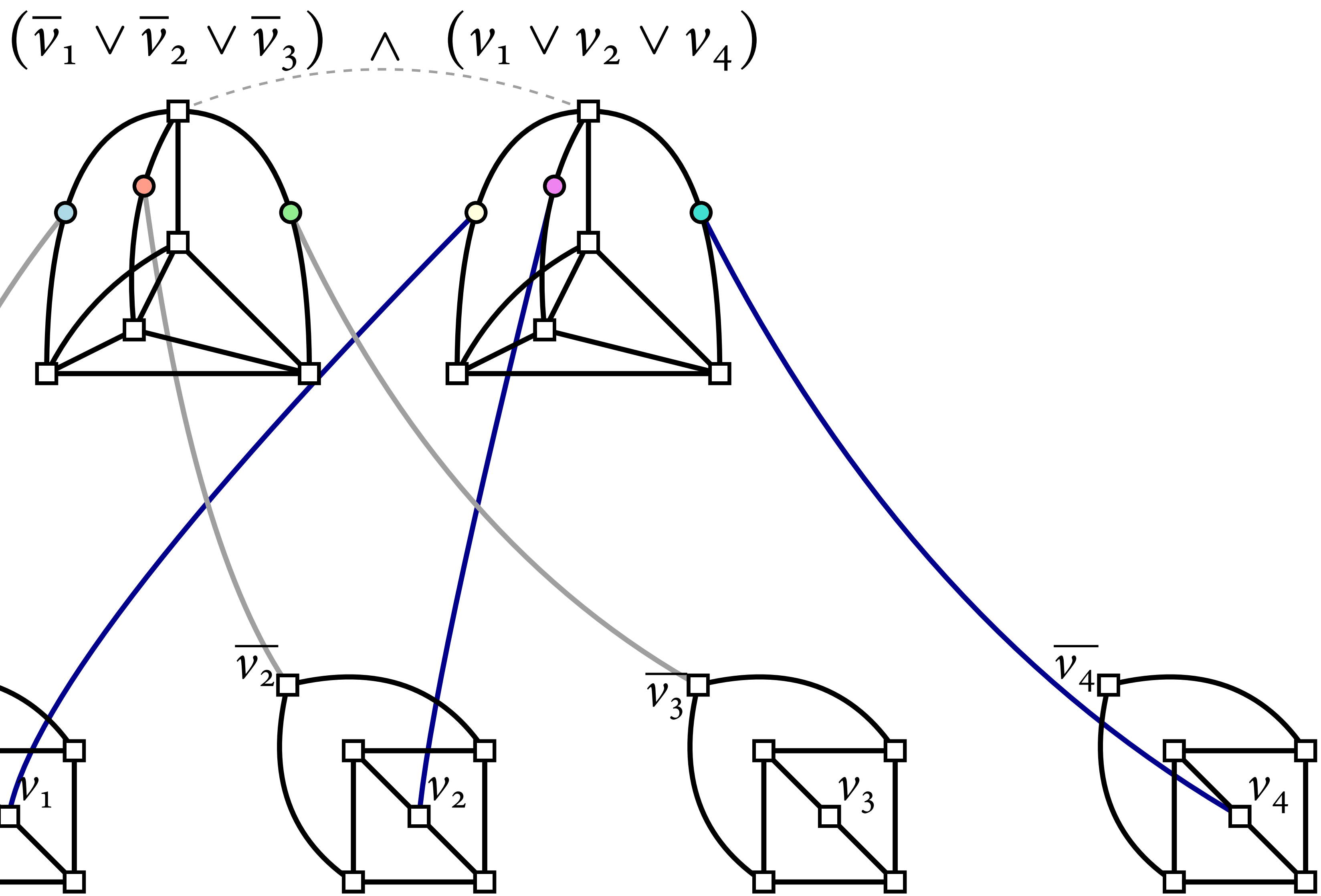

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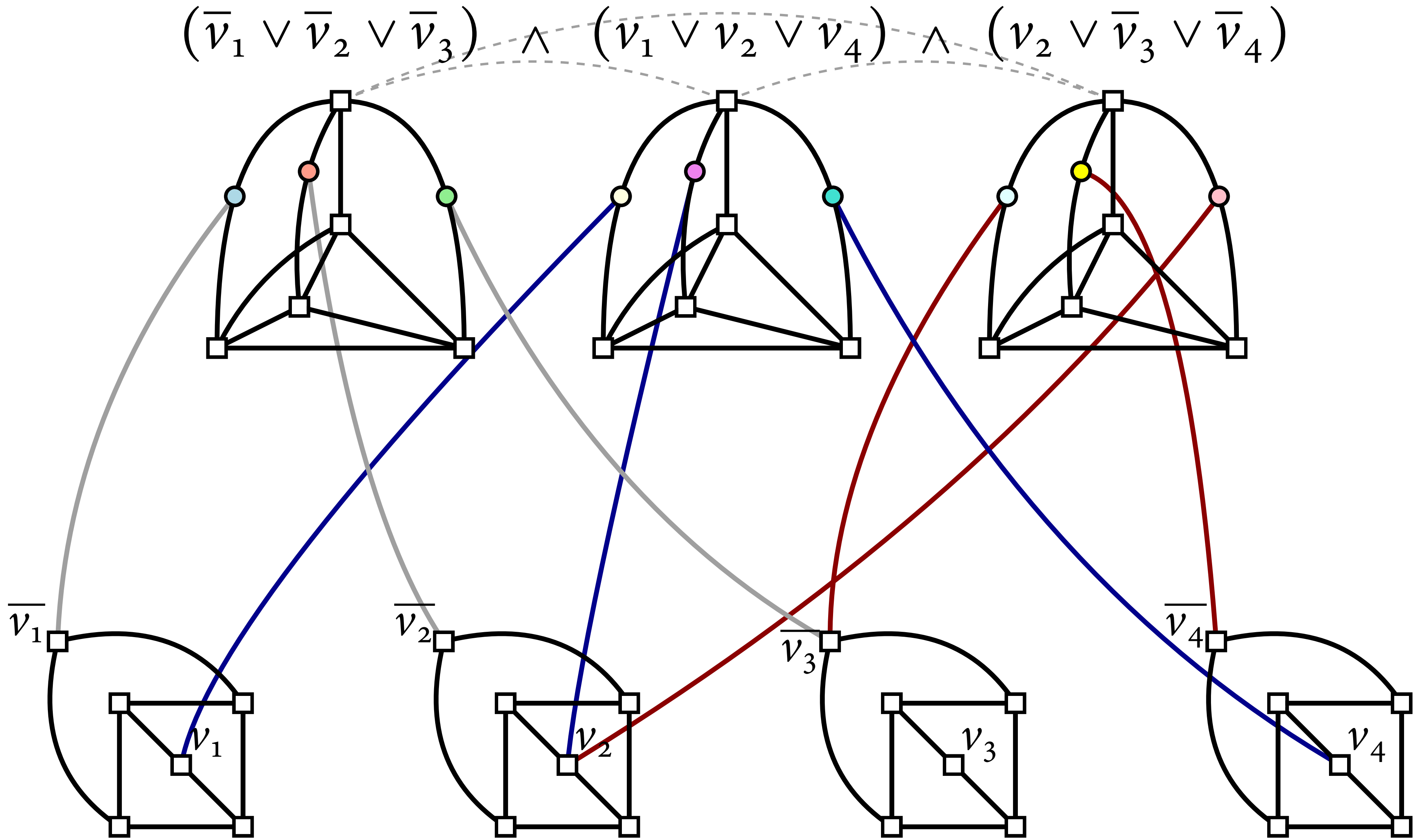

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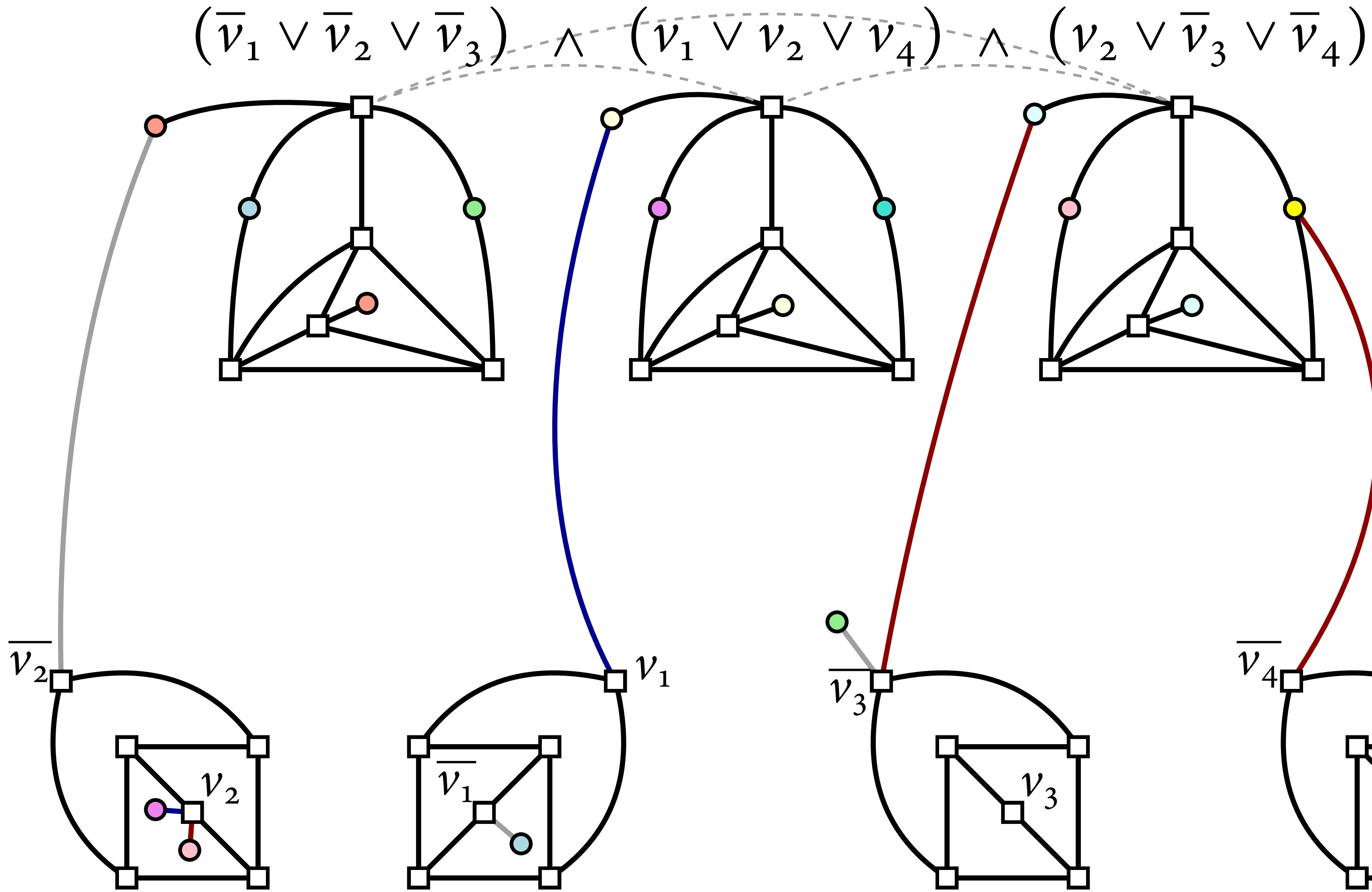

$v_1 \wedge \overline{v}_2 \wedge \overline{v}_3 \wedge \overline{v}_4$

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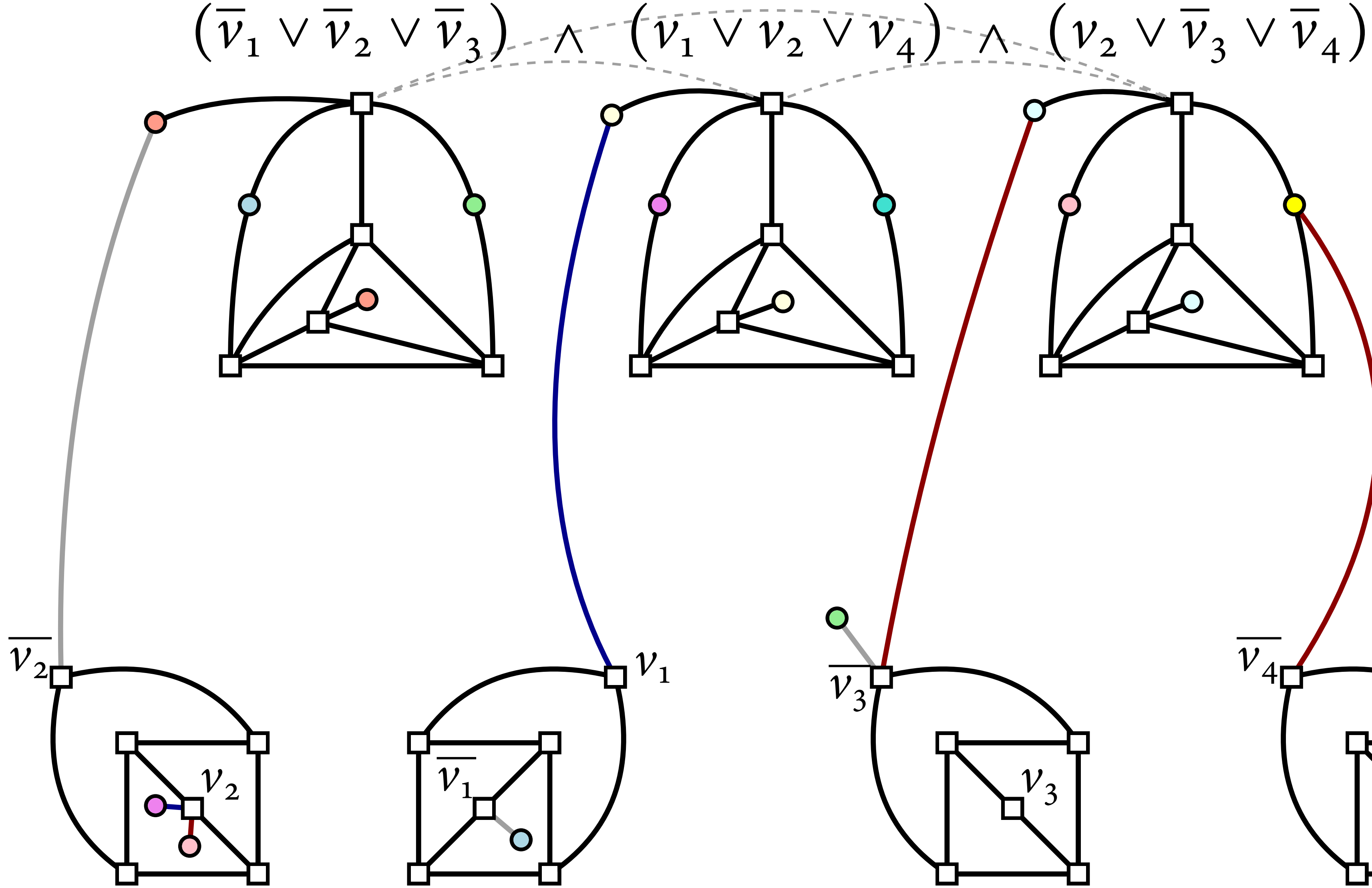
$(\overline{v}_1 \vee \overline{v}_2 \vee \overline{v}_3) \wedge (v_1 \vee v_2 \vee v_4) \wedge (v_2 \vee \overline{v}_3 \vee \overline{v}_4)$



2-splittability is NP-complete

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Pseudoarboricity $pa(G)$: minimum # pseudotrees whose union is the given graph

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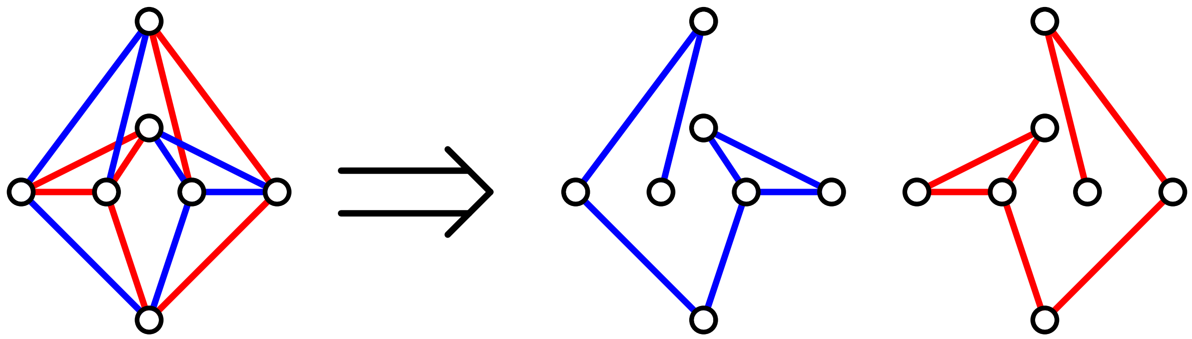

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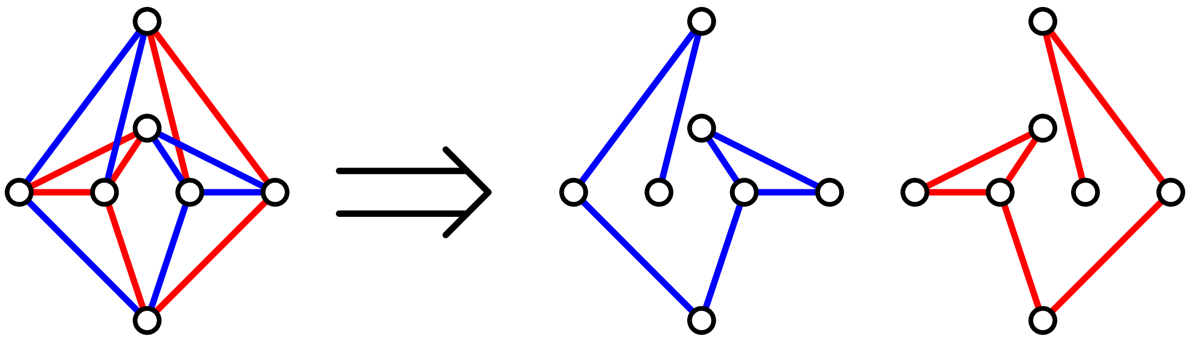

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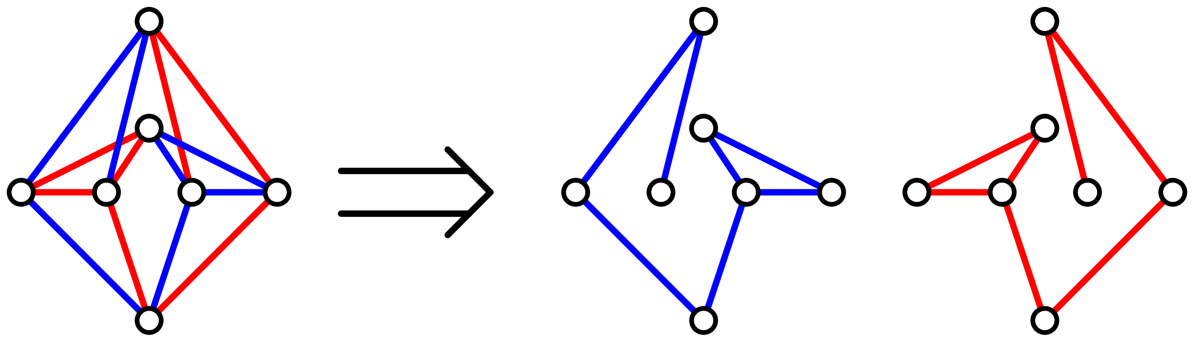

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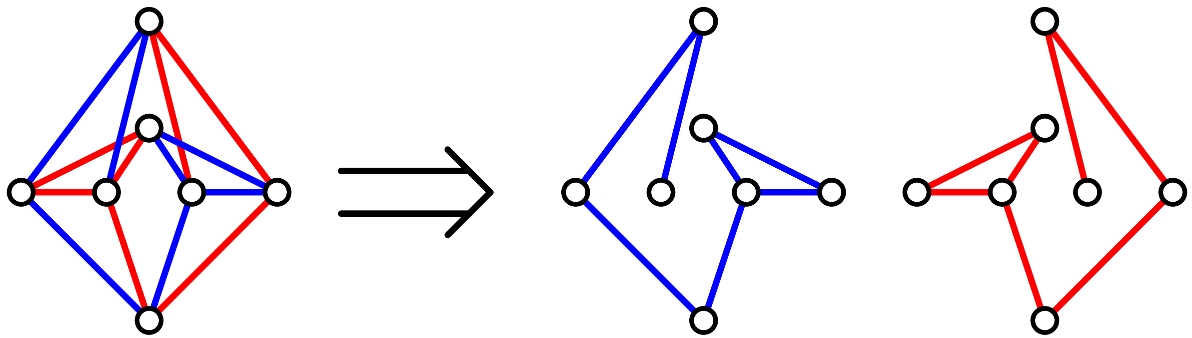

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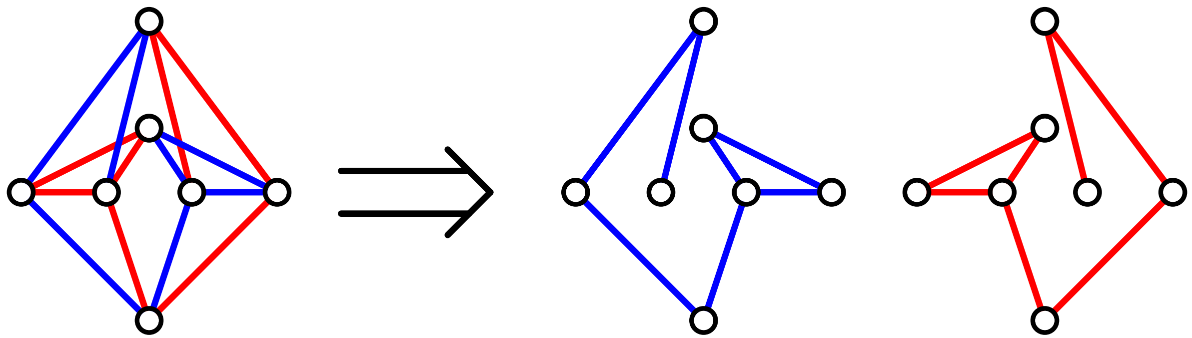

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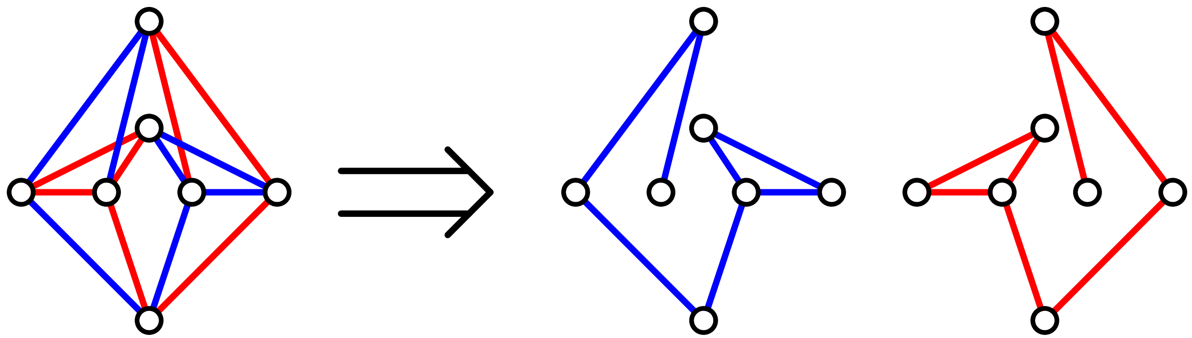

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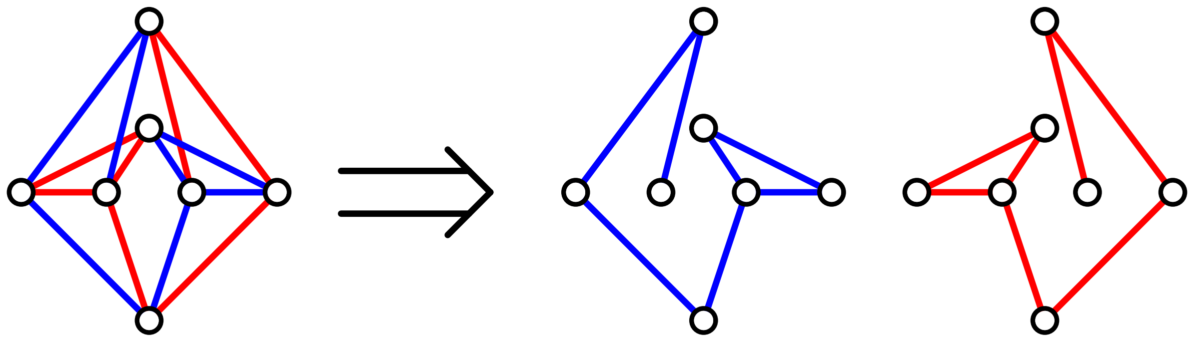

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Pseudoarboricity approximates splittability with factor 3

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$$\forall S \subseteq V : \neg(S \equiv \varnothing) \rightarrow \exists v \in S : \neg(w_1, w_2 \in S : \neg(w_1 \equiv w_2) \rightarrow \neg((v, w_1), (v, w_2) \in E))$$
$$\forall (v, w) \in E : \neg((v, w) \in T) \rightarrow \exists S \subset T : \exists y \in V : \exists (y, *) \in S \wedge (\neg(y \equiv r) \rightarrow \forall z \in V :$$
$$\neg(z \equiv r, y) \rightarrow \neg(\exists (a, z) \in T) \vee (\exists (a, z), (b, z) \in T)) \wedge \exists (v, *) \in S \wedge \exists (w, *) \in S)$$

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Can test k-splittability of graphs of treewidth $\leq w$ in time $O(f(k,w) \cdot n)$

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New concept of k-splittability
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Open Problems: anything you want!