Drawing trees and triangulations with few geometric primitives

Philipp Kindermann
FernUniversität in Hagen

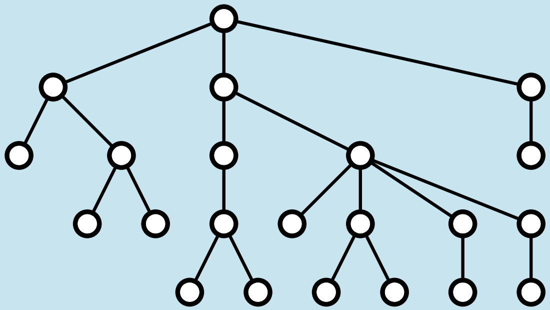
Joint work with
Gregor Hültenschmidt, Wouter Meulemans & André Schulz

Visual Complexity

Number of geometric objects

Visual Complexity

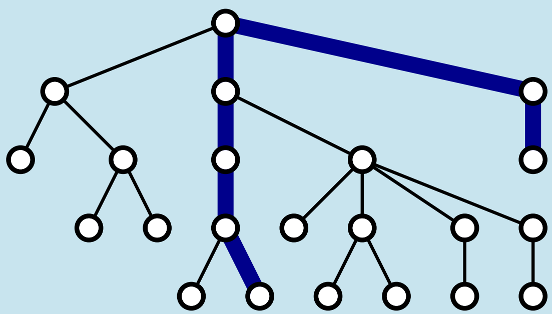
Number of geometric objects



21 edges

Visual Complexity

Number of geometric objects

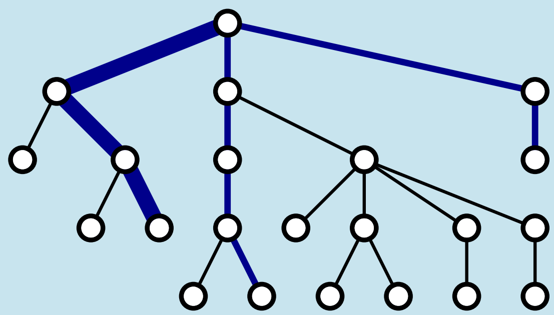


21 edges

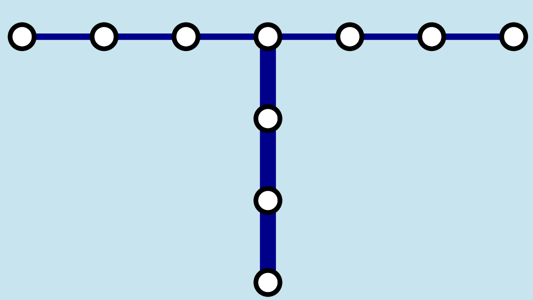
1 segment

Visual Complexity

Number of geometric objects



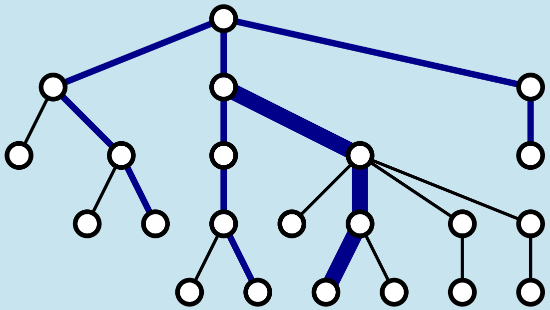
21 edges



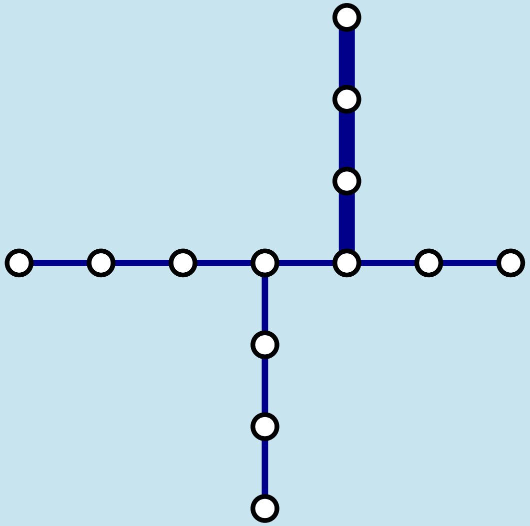
2 segments

Visual Complexity

Number of geometric objects



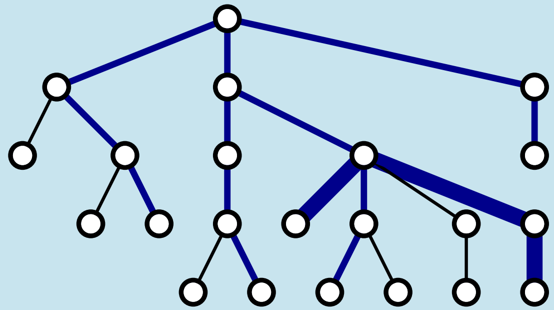
21 edges



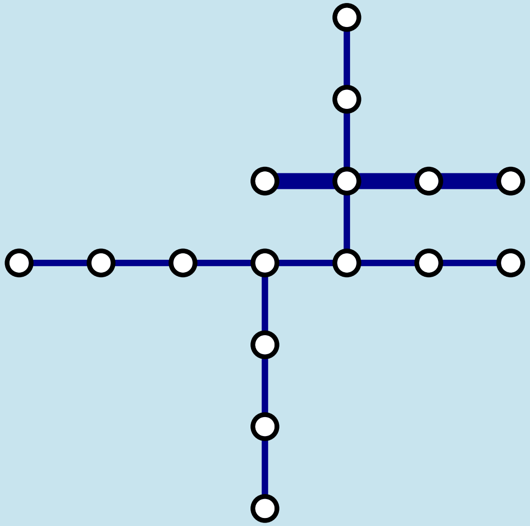
3 segments

Visual Complexity

Number of geometric objects



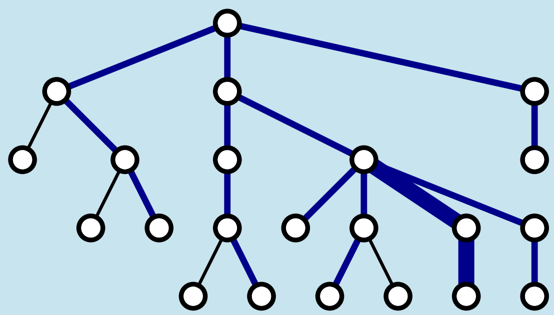
21 edges



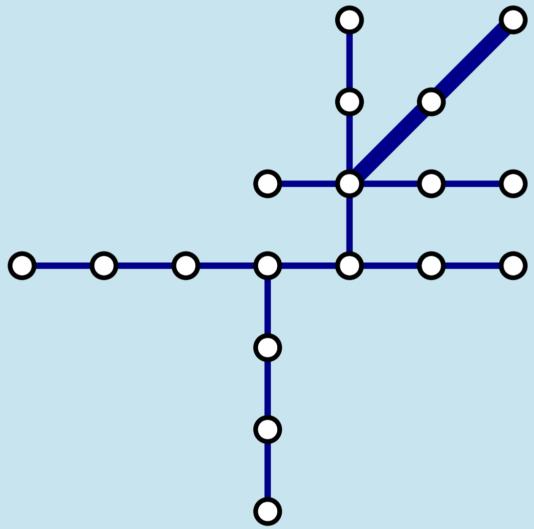
4 segments

Visual Complexity

Number of geometric objects



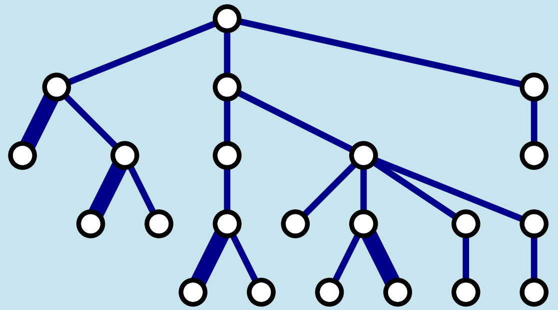
21 edges



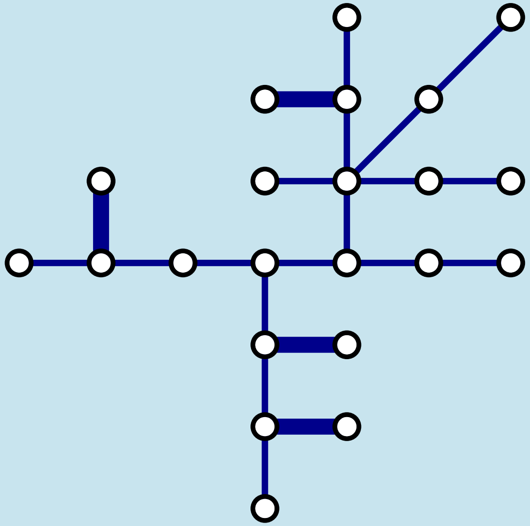
5 segments

Visual Complexity

Number of geometric objects



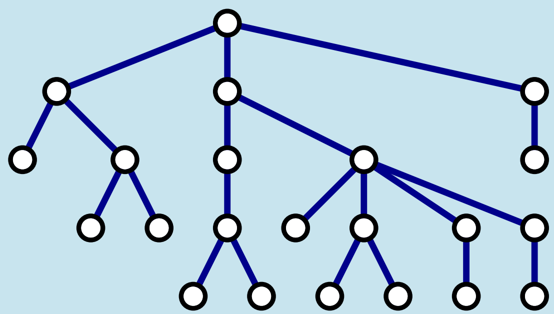
21 edges



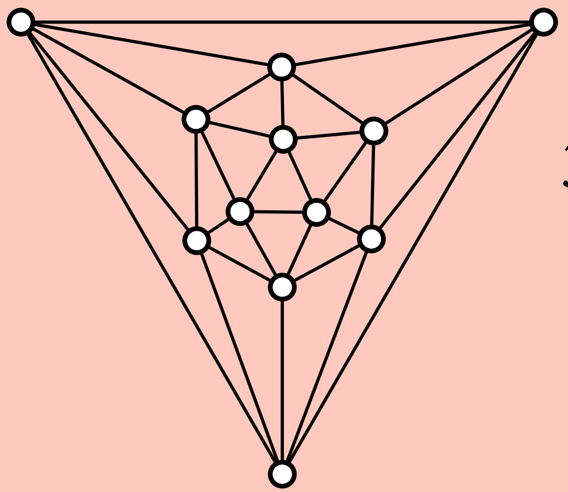
9 segments

Visual Complexity

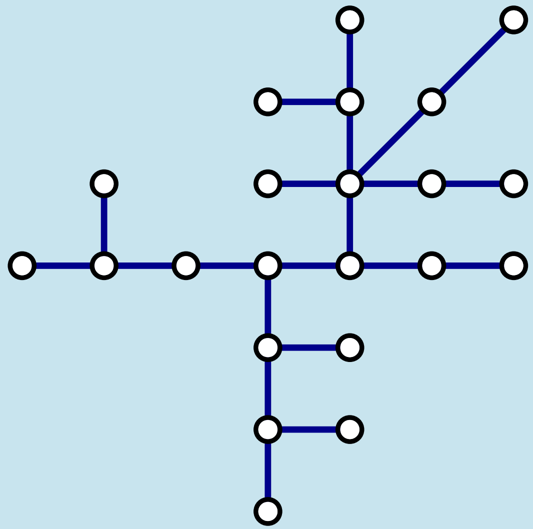
Number of geometric objects



21 edges



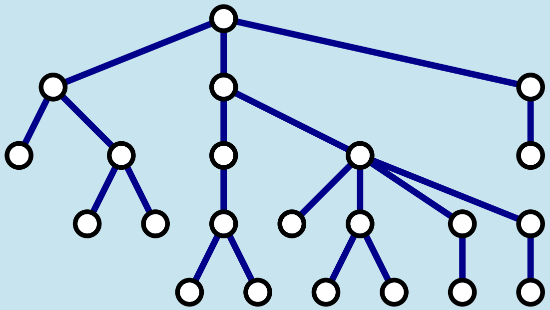
30 edges



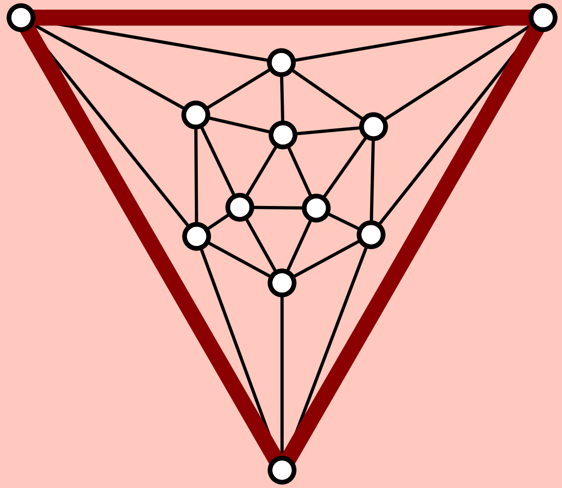
9 segments

Visual Complexity

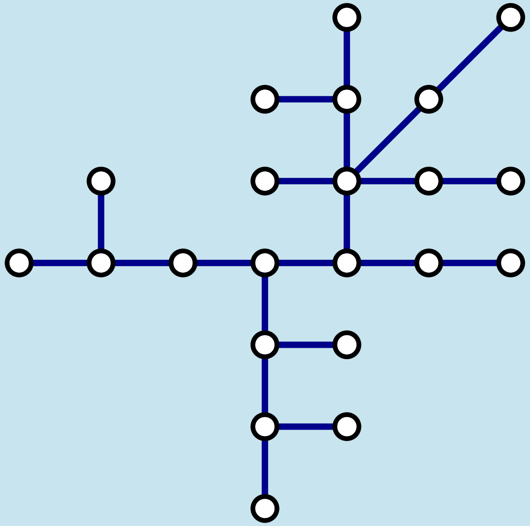
Number of geometric objects



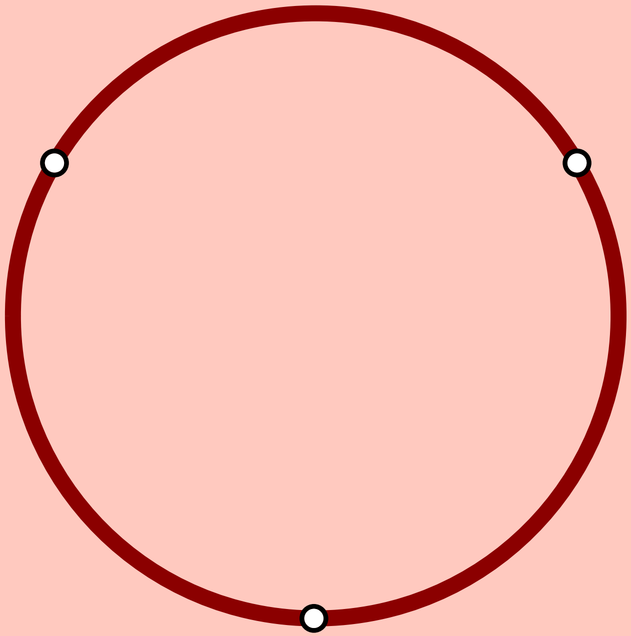
21 edges



30 edges



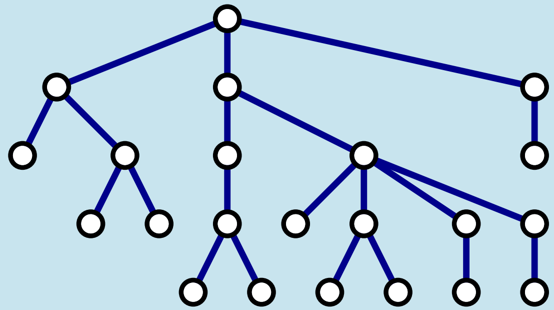
9 segments



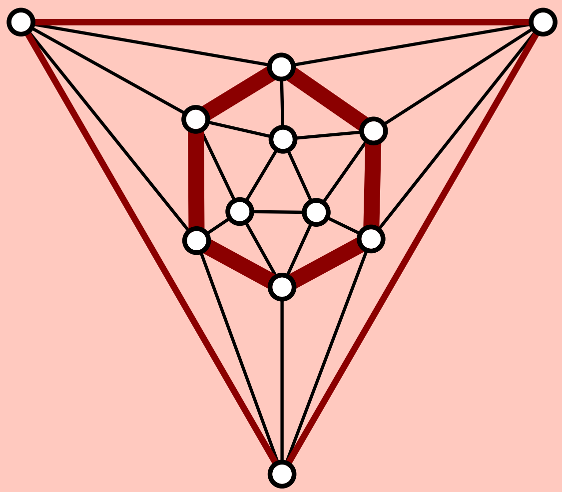
1 arc

Visual Complexity

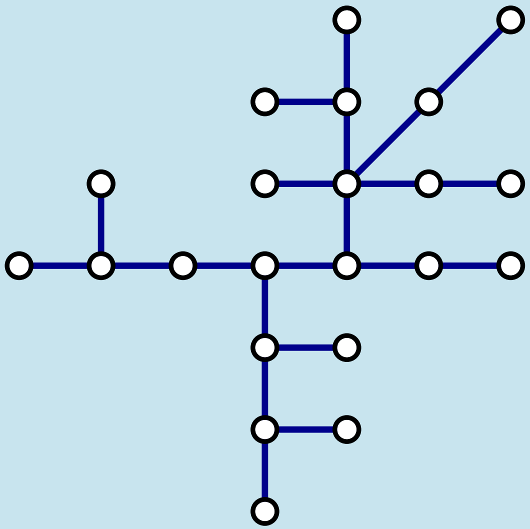
Number of geometric objects



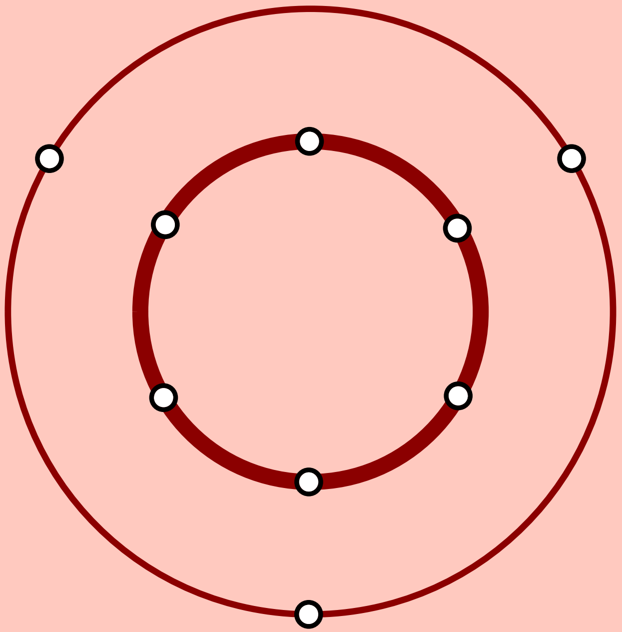
21 edges



30 edges



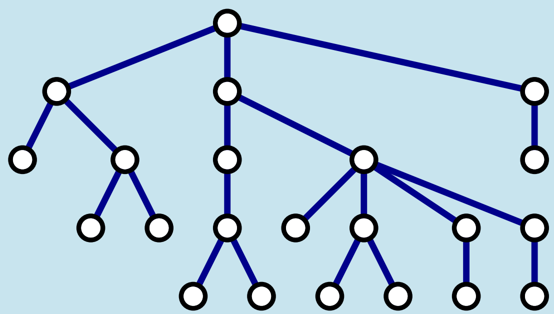
9 segments



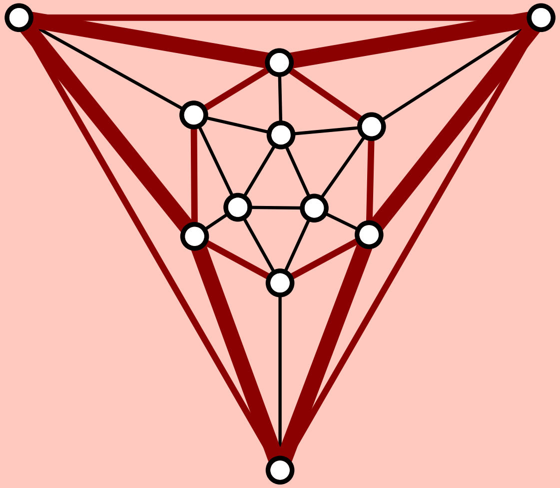
2 arcs

Visual Complexity

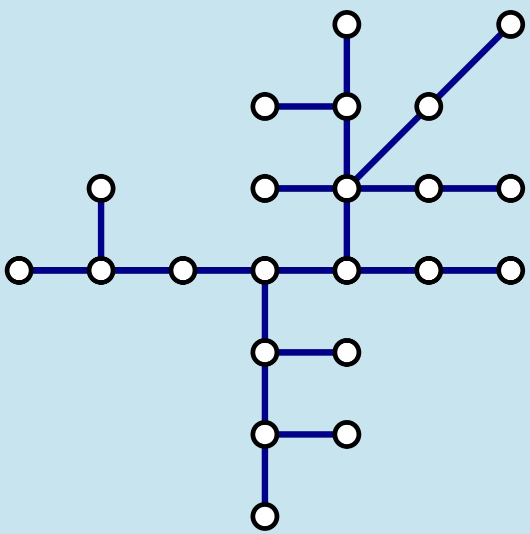
Number of geometric objects



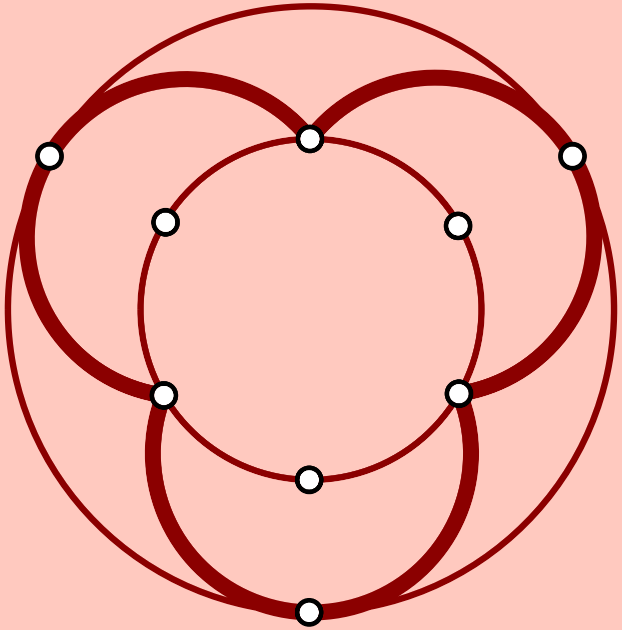
21 edges



30 edges



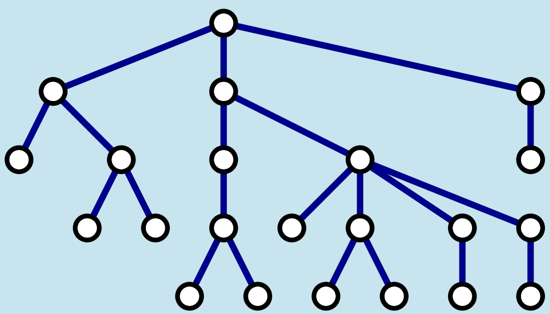
9 segments



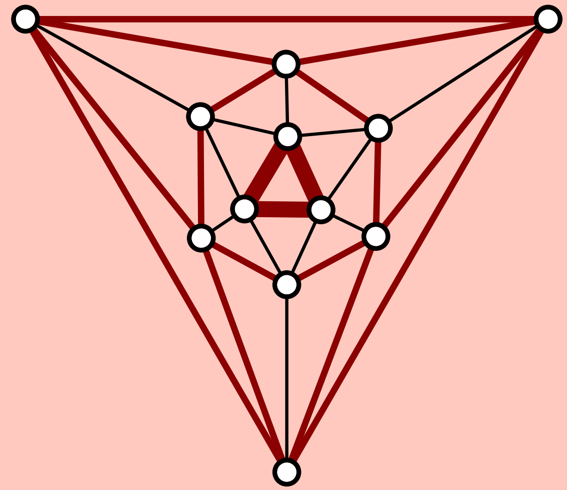
5 arcs

Visual Complexity

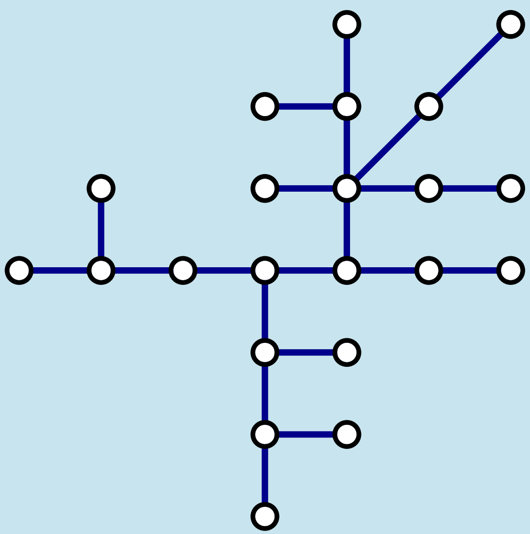
Number of geometric objects



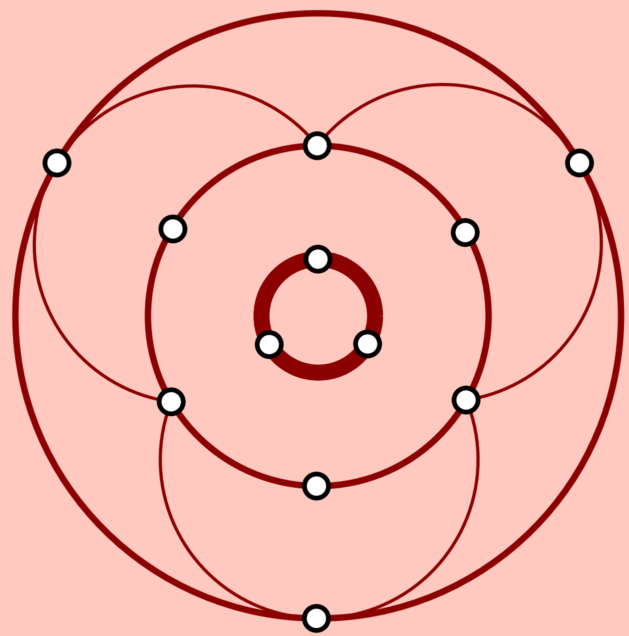
21 edges



30 edges



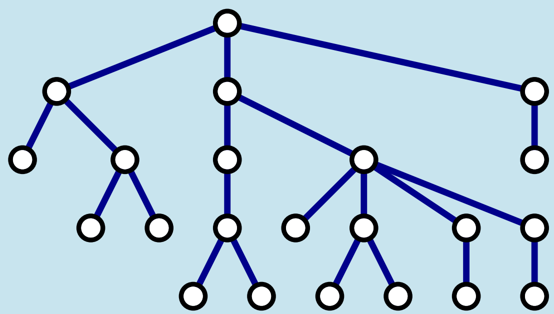
9 segments



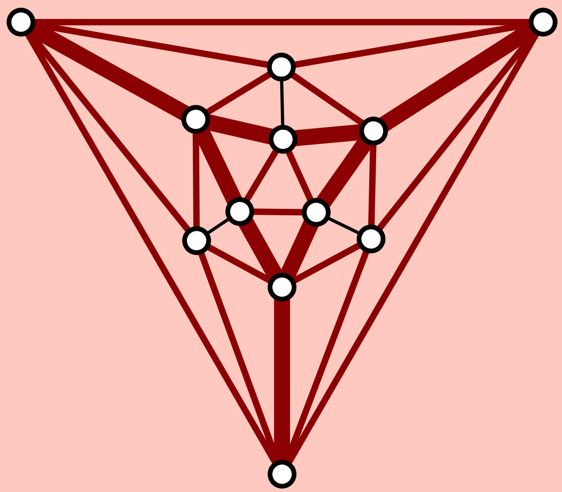
6 arcs

Visual Complexity

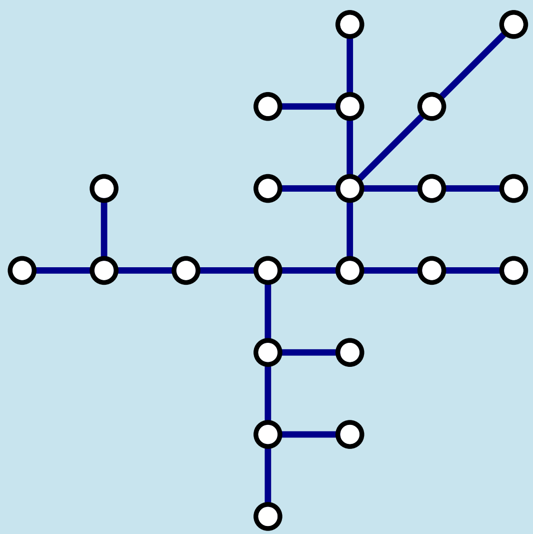
Number of geometric objects



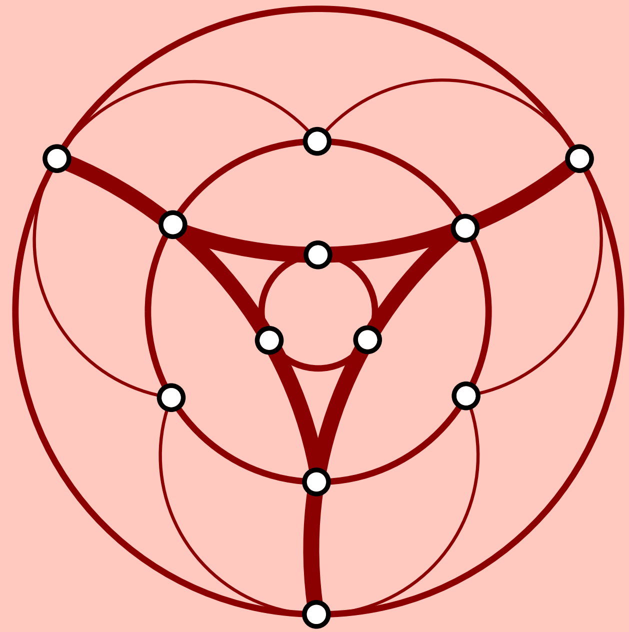
21 edges



30 edges



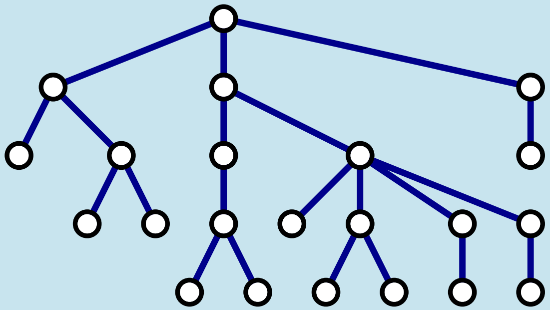
9 segments



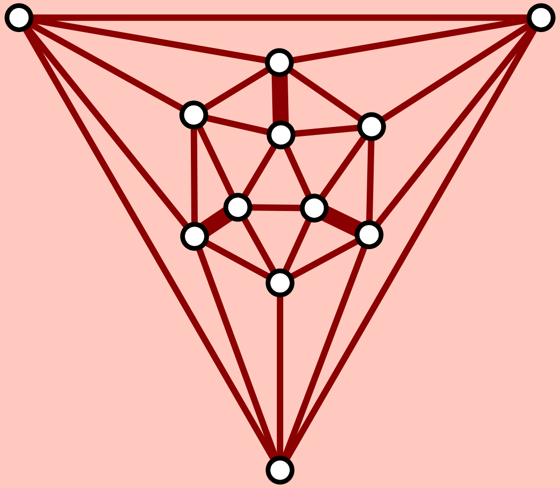
9 arcs

Visual Complexity

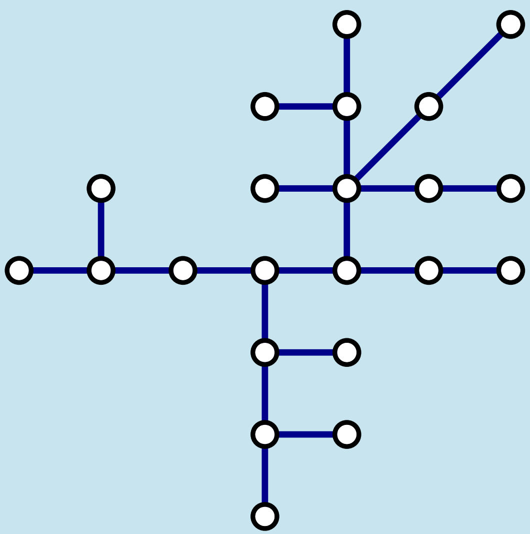
Number of geometric objects



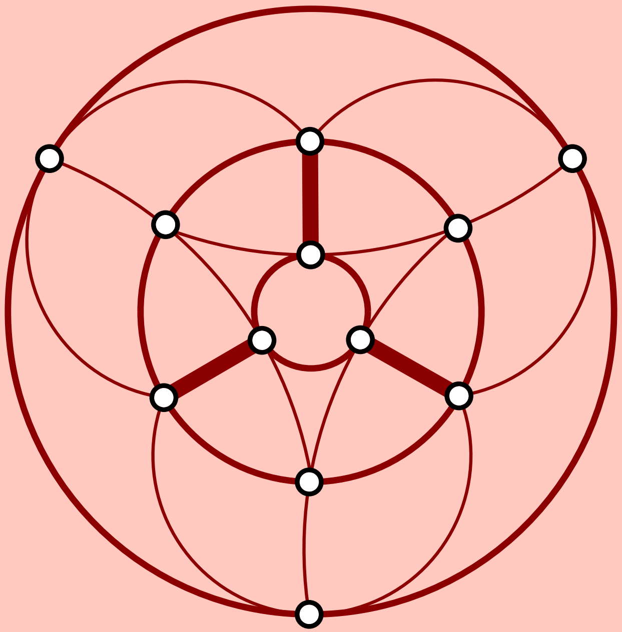
21 edges



30 edges



9 segments



12 arcs

Known Results

Class	Segments		Circular Arcs	
	Lower	Upper	Lower	Upper

Known Results

Class	Segments		Circular Arcs	
	Lower	Upper	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]

[1] Durocher et al. 2003

Known Results

Class	Segments		Circular Arcs	
	Lower	Upper	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]	$e/6$ [2]	$11e/18$ [2]

[1] Durocher et al. 2003
[2] Schulz 2015

Known Results

Class	Segments		Circular Arcs	
	Lower	Upper	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]	$e/6$ [2]	$11e/18$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]	$e/6$ [2]	$2e/3$ [2]

[1] Durocher et al. 2003

[2] Schulz 2015

Known Results

Class	Segments		Circular Arcs	
	Lower	Upper	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]	$e/6$ [2]	$11e/18$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]	$e/6$ [2]	$2e/3$ [2]
cubic 3-conn.	$n/2$ [4]	$n/2$ [3]	$n/2$ [4]	$n/2$ [3]

[1] Durocher et al. 2003
[2] Schulz 2015
[3] Igamberdiev et al. 2015
[4] Mondal et al. 2013

Known Results

Class	Segments		Circular Arcs	
	Lower	Upper	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]	$e/6$ [2]	$11e/18$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]	$e/6$ [2]	$2e/3$ [2]
cubic 3-conn.	$n/2$ [4]	$n/2$ [3]	$n/2$ [4]	$n/2$ [3]
Triangulation	$2n$ [5]	$7n/3$ [5]		

[1] Durocher et al. 2003
[2] Schulz 2015
[3] Igamberdiev et al. 2015
[4] Mondal et al. 2013
[5] Durocher & Mondal 2014

Known Results

Class	Segments		Circular Arcs	
	Lower	Upper	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]	$e/6$ [2]	$11e/18$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]	$e/6$ [2]	$2e/3$ [2]
cubic 3-conn.	$n/2$ [4]	$n/2$ [3]	$n/2$ [4]	$n/2$ [3]
Triangulation	$2n$ [5]	$7n/3$ [5]		
Planar	$2n$ [5]	$16n/3 - e$ [5]		

[1] Durocher et al. 2003
[2] Schulz 2015
[3] Igamberdiev et al. 2015
[4] Mondal et al. 2013
[5] Durocher & Mondal 2014

Known Results

Class	Segments		Circular Arcs	
	Lower	Upper	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]	$e/6$ [2]	$11e/18$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]	$e/6$ [2]	$2e/3$ [2]
cubic 3-conn.	$n/2$ [4]	$n/2$ [3]	$n/2$ [4]	$n/2$ [3]
Triangulation	$2n$ [5]	$7n/3$ [5]		?
Planar	$2n$ [5]	$16n/3 - e$ [5]		?

[1] Durocher et al. 2003
[2] Schulz 2015
[3] Igamberdiev et al. 2015
[4] Mondal et al. 2013
[5] Durocher & Mondal 2014

Known Results

Grid?

Class	Segments: Lower	Segments: Upper	Circular Arcs: Lower	Circular Arcs: Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]
3-trees	$2n$ [1]	$2n$ [1]	$e/6$ [2]	$11e/18$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]	$e/6$ [2]	$2e/3$ [2]
cubic 3-conn.	$n/2$ [4]	$n/2$ [3]	$n/2$ [4]	$n/2$ [3]
Triangulation	$2n$ [5]	$7n/3$ [5]		?
Planar	$2n$ [5]	$16n/3 - e$ [5]		?

[1] Durocher et al. 2003

[2] Schulz 2015

[3] Igamberdiev et al. 2015

[4] Mondal et al. 2013

[5] Durocher & Mondal 2014

Known Results

Grid?

Class	Segments		Circular Arcs	
	Lower	Upper	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$\vartheta/2$ [1] $3e/4$ [2]
3-trees	$2n$ [1]	$2n$ [1]	$e/6$ [2]	$11e/18$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]	$e/6$ [2]	$2e/3$ [2]
cubic 3-conn.	$n/2$ [4]	$n/2$ [3]	$n/2$ [4]	$n/2$ [3]
Triangulation	$2n$ [5]	$7n/3$ [5]		?
Planar	$2n$ [5]	$16n/3 - e$ [5]		?

[1] Durocher et al. 2003
[2] Schulz 2015
[3] Igamberdiev et al. 2015
[4] Mondal et al. 2013
[5] Durocher & Mondal 2014

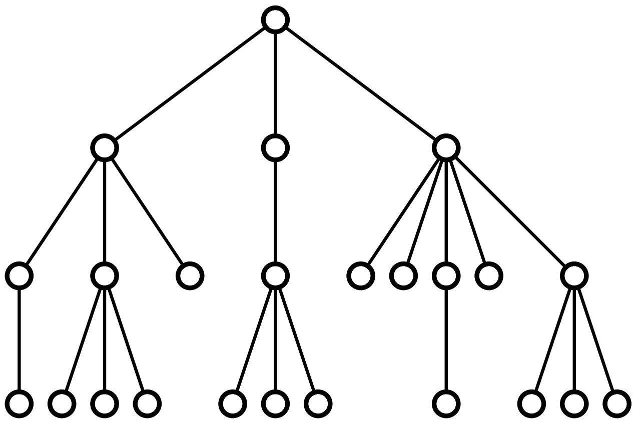
Known Results

Grid?

Class	Segments		Circular Arcs	
	Lower	Upper	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1] ?	$\vartheta/2$ [1]	$\vartheta/2$ [1] $3e/4$ [2]
3-trees	$2n$ [1]	$2n$ [1]	$e/6$ [2]	$11e/18$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]	$e/6$ [2]	$2e/3$ [2]
cubic 3-conn.	$n/2$ [4]	$n/2$ [3]	$n/2$ [4]	$n/2$ [3]
Triangulation	$2n$ [5]	$7n/3$ [5]		?
Planar	$2n$ [5]	$16n/3 - e$ [5]		?

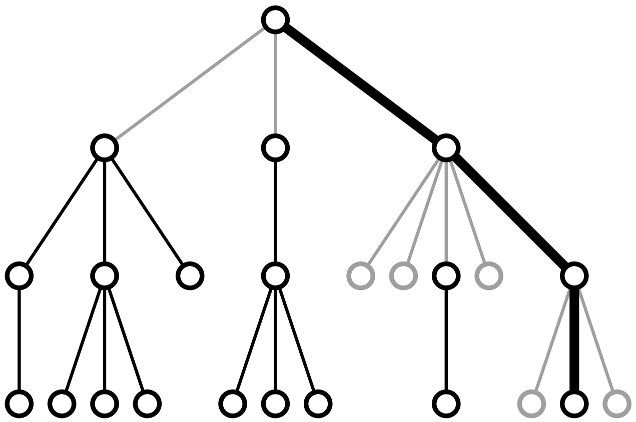
[1] Durocher et al. 2003
[2] Schulz 2015
[3] Igamberdiev et al. 2015
[4] Mondal et al. 2013
[5] Durocher & Mondal 2014

Trees on the grid



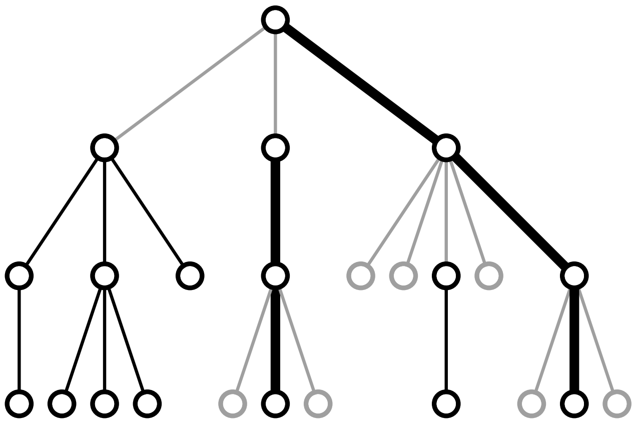
heavy path decomposition

Trees on the grid



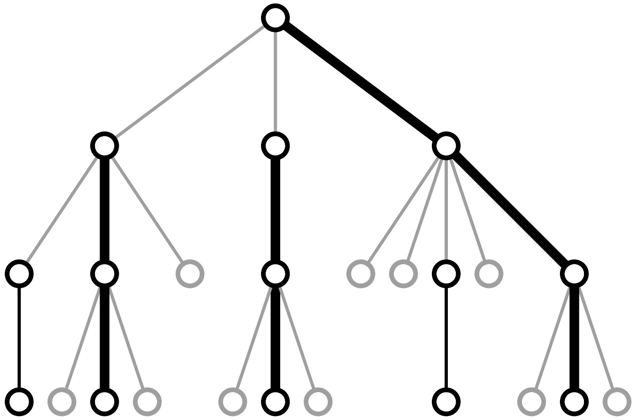
heavy path decomposition

Trees on the grid



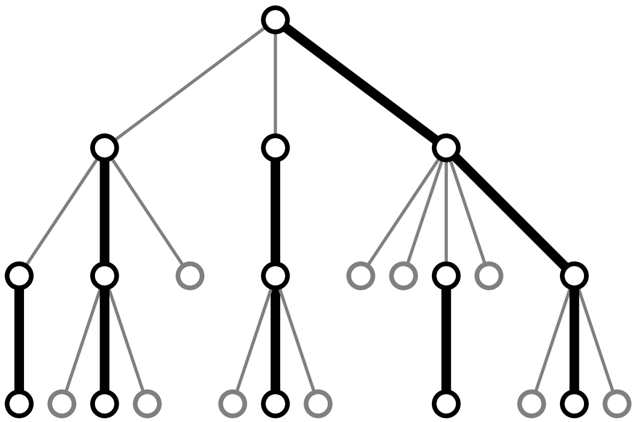
heavy path decomposition

Trees on the grid



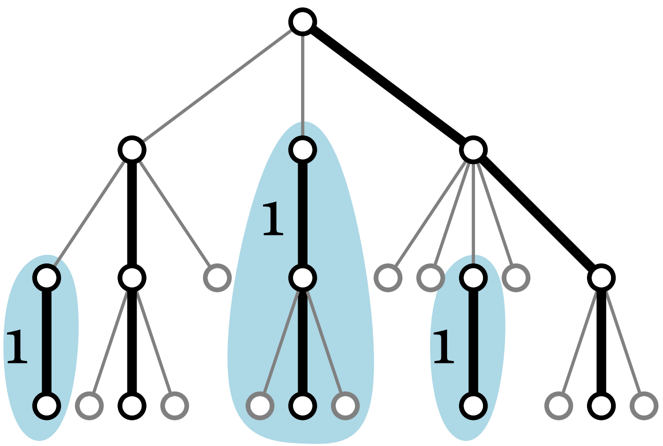
heavy path decomposition

Trees on the grid



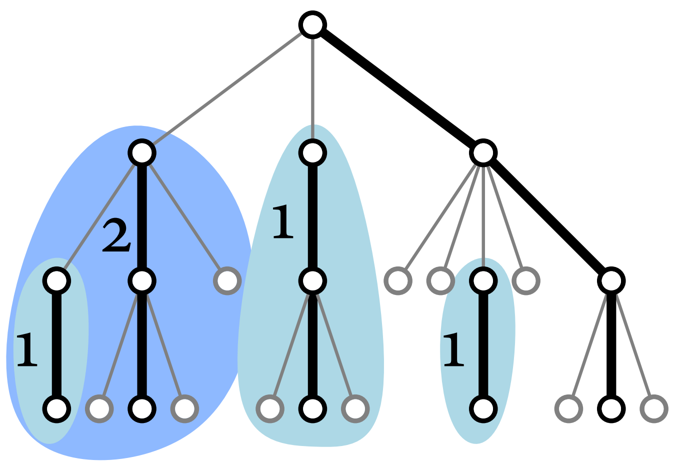
heavy path decomposition

Trees on the grid



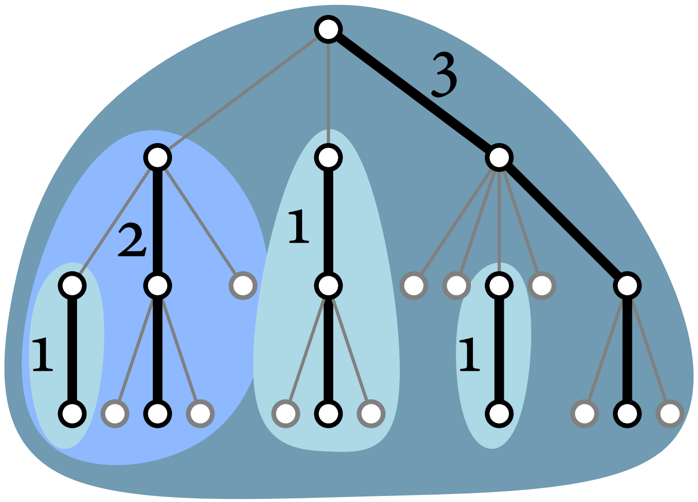
heavy path decomposition

Trees on the grid



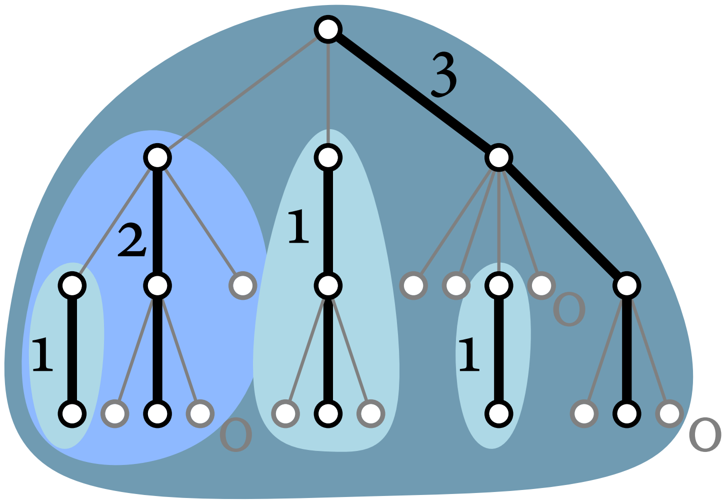
heavy path decomposition

Trees on the grid



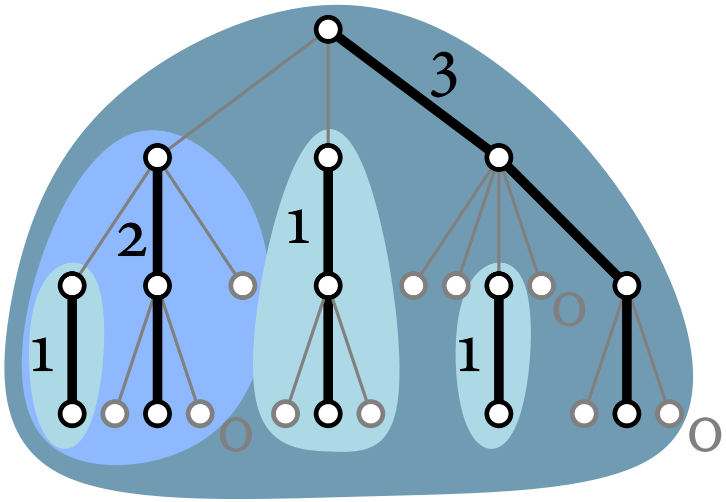
heavy path decomposition

Trees on the grid



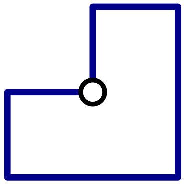
heavy path decomposition

Trees on the grid

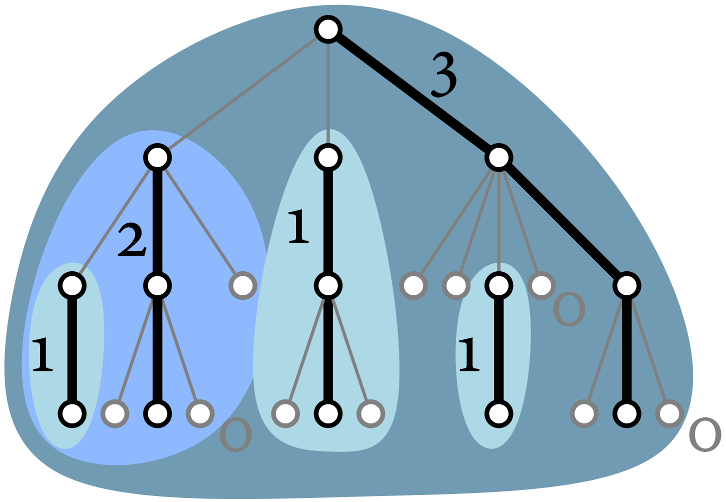


heavy path decomposition

- draw each heavy path tree in a "box"

level 0: 

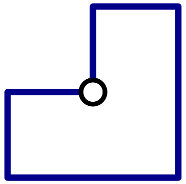
Trees on the grid



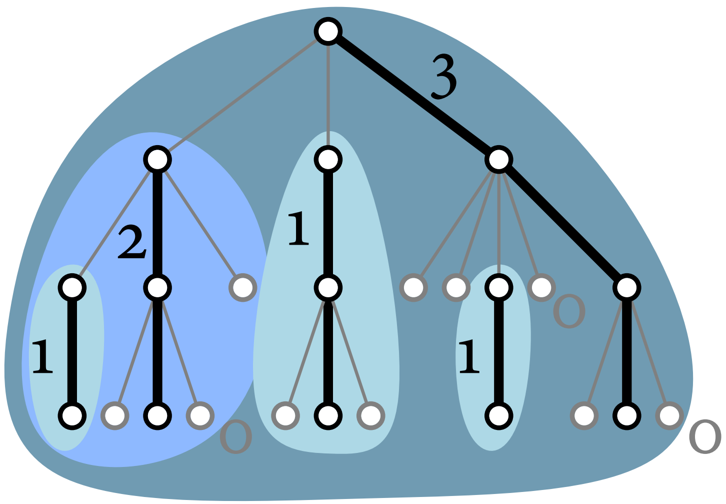
heavy path decomposition

- draw each heavy path tree in a ”box”
- boxes of same level are disjoint

level 0:



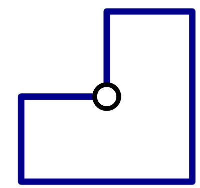
Trees on the grid



heavy path decomposition

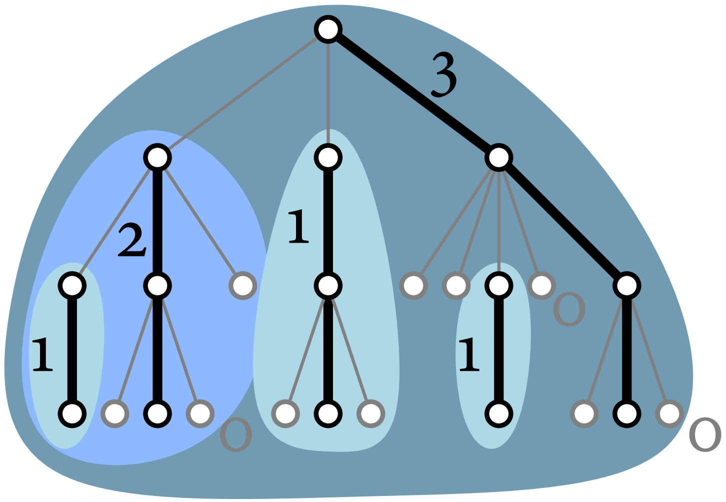
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0:



level k:

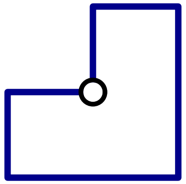
Trees on the grid



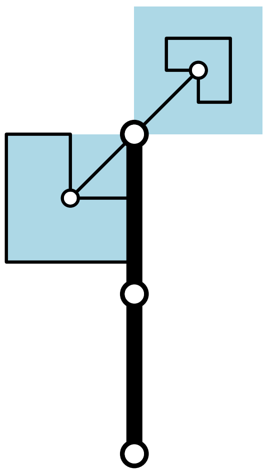
heavy path decomposition

- draw each heavy path tree in a ”box”
- boxes of same level are disjoint

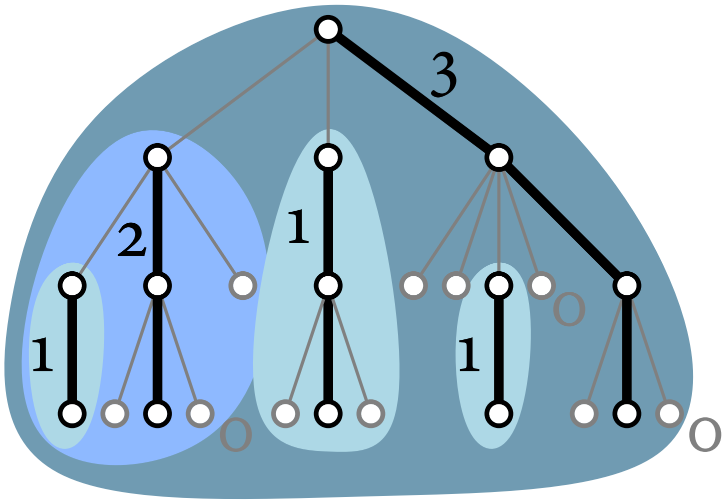
level 0:



level k:

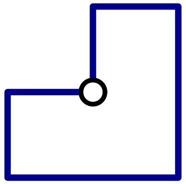


Trees on the grid

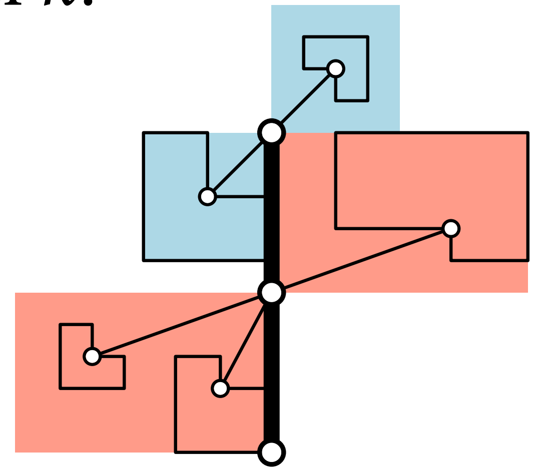


heavy path decomposition

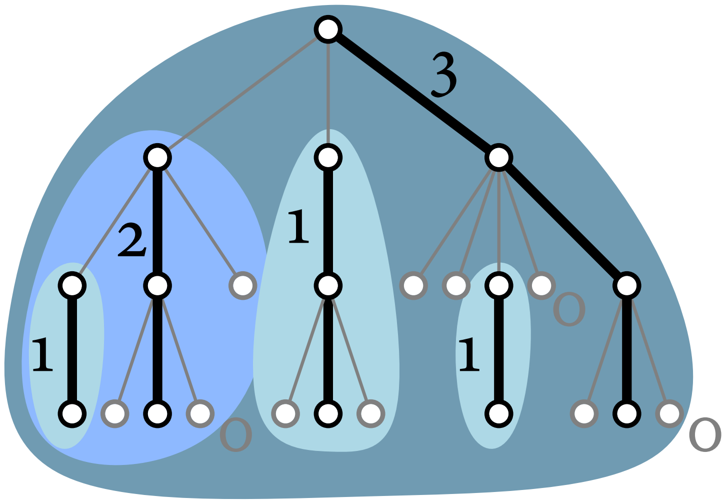
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0: 

level k:



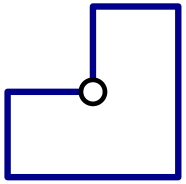
Trees on the grid



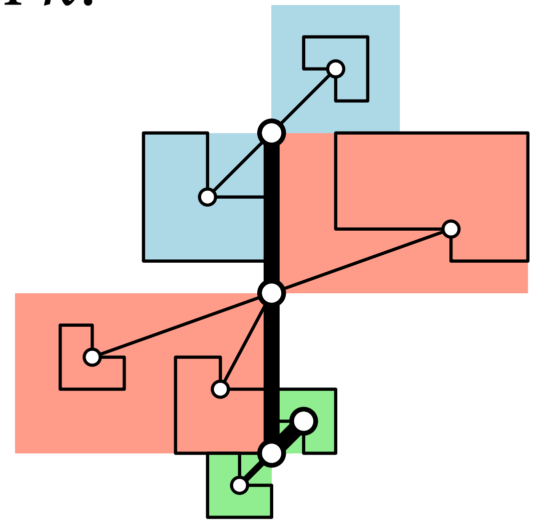
heavy path decomposition

- draw each heavy path tree in a ”box”
- boxes of same level are disjoint

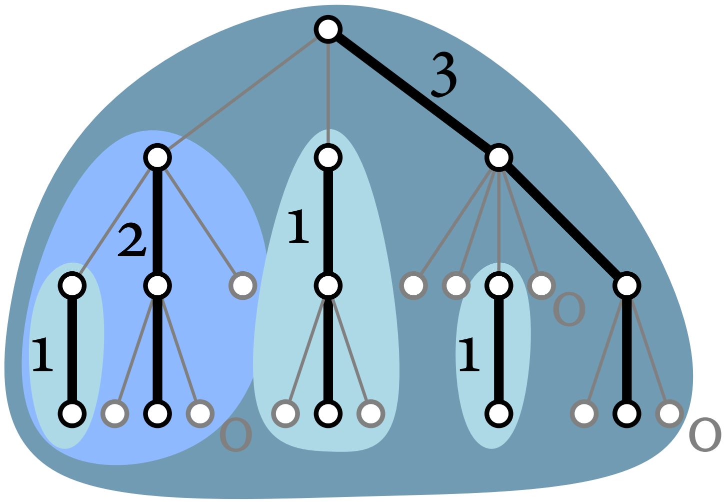
level 0:



level k:

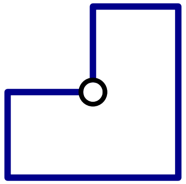


Trees on the grid

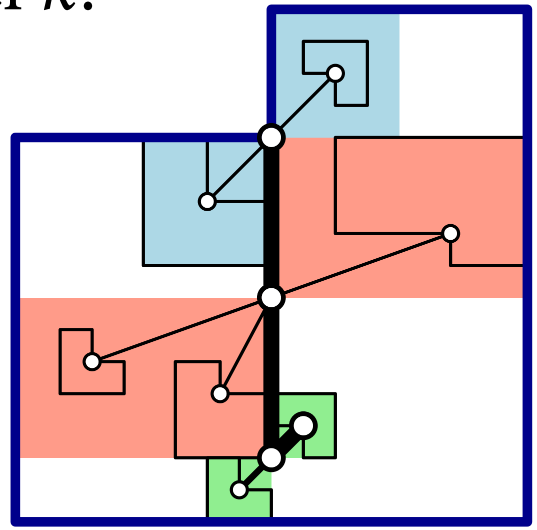


heavy path decomposition

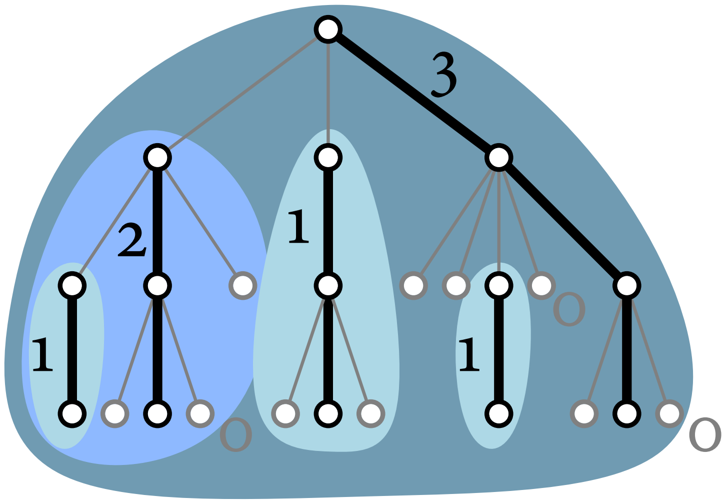
- draw each heavy path tree in a ”box”
- boxes of same level are disjoint

level 0: 

level k:



Trees on the grid

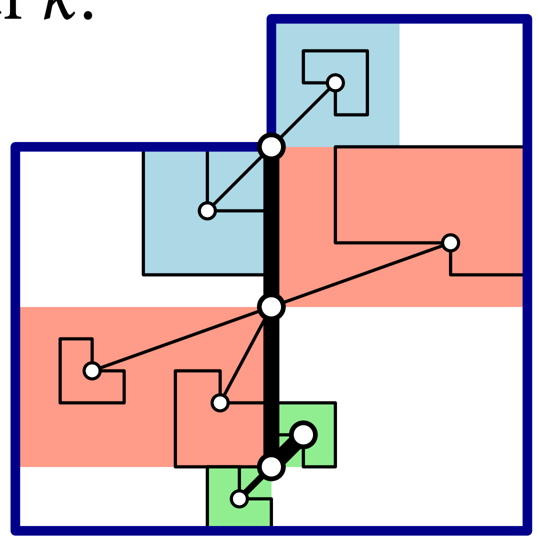


heavy path decomposition

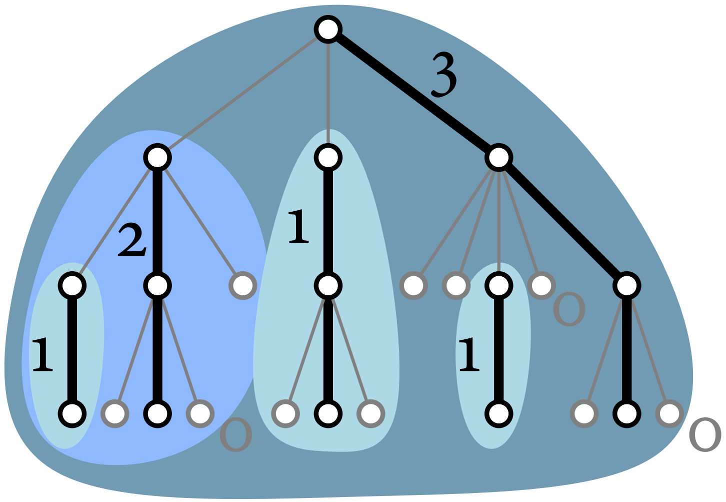
- draw each heavy path tree in a ”box”
- boxes of same level are disjoint

level 0:

level k:



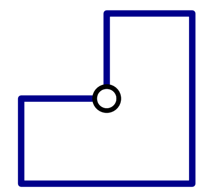
Trees on the grid

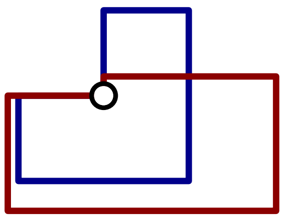


heavy path decomposition

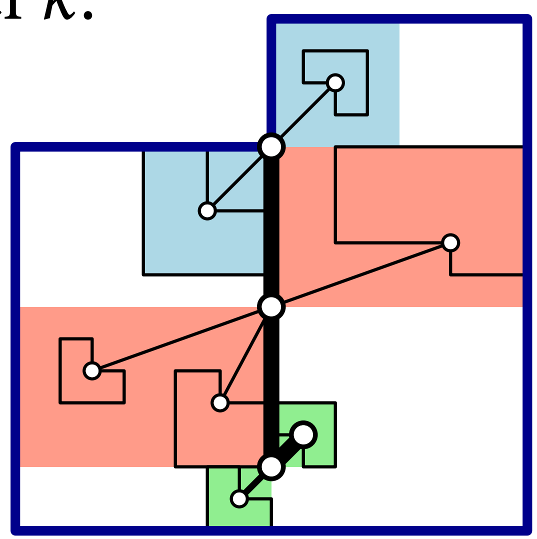
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0:

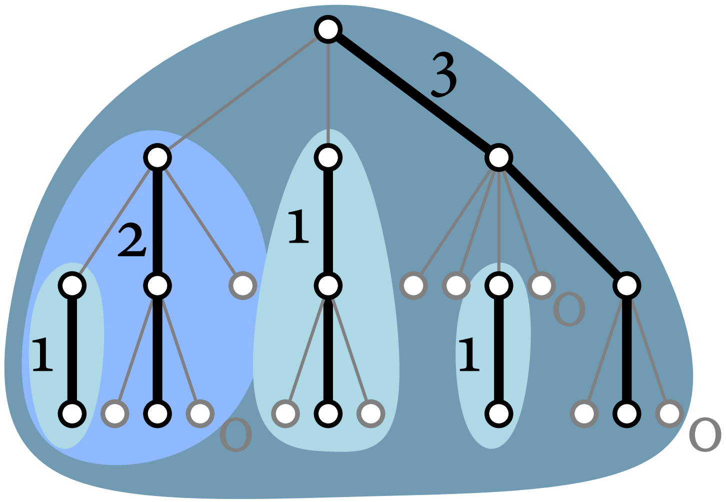




level k:



Trees on the grid

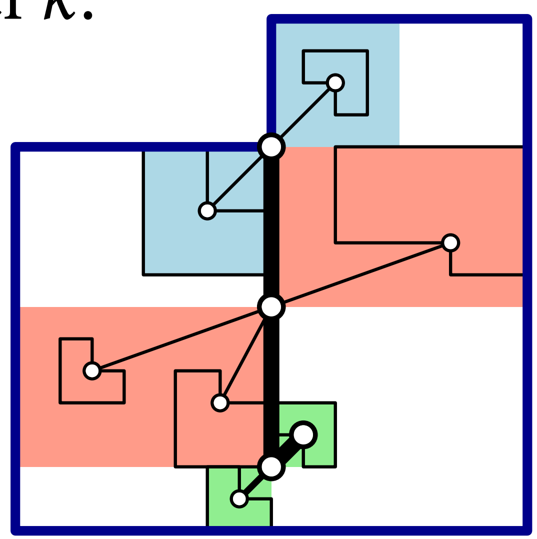


heavy path decomposition

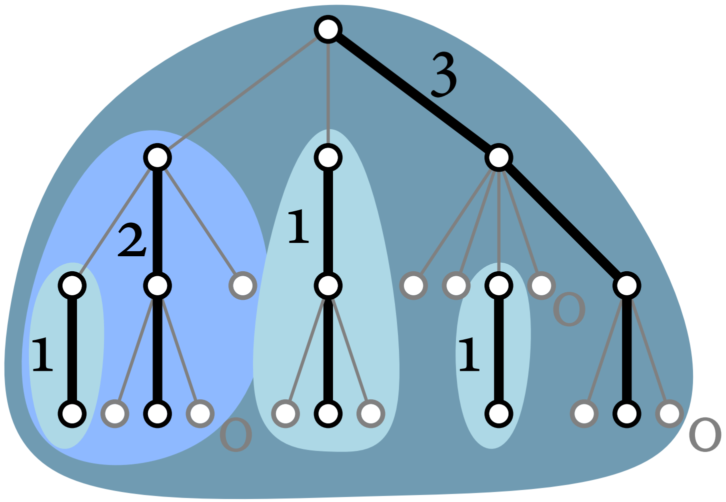
- draw each heavy path tree in a ”box”
- boxes of same level are disjoint

level 0:

level k:



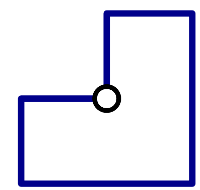
Trees on the grid

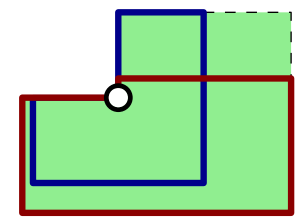


heavy path decomposition

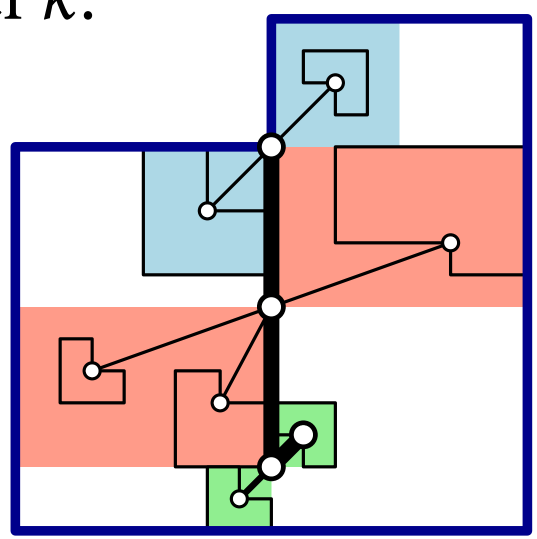
- draw each heavy path tree in a ”box”
- boxes of same level are disjoint

level 0:



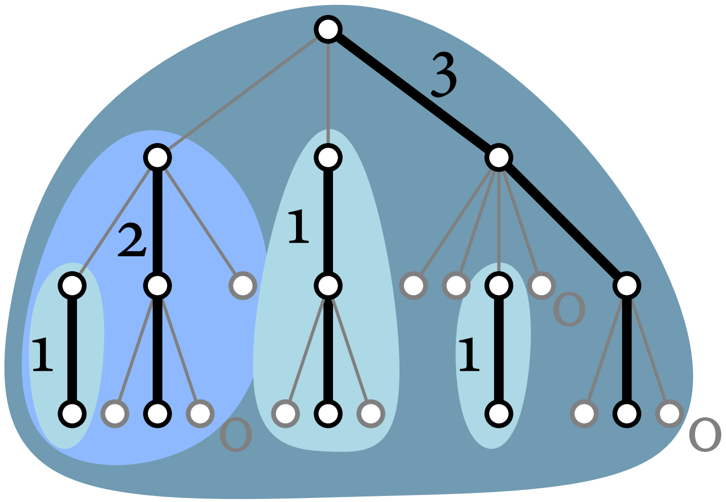


level k:



one vertex:

Trees on the grid

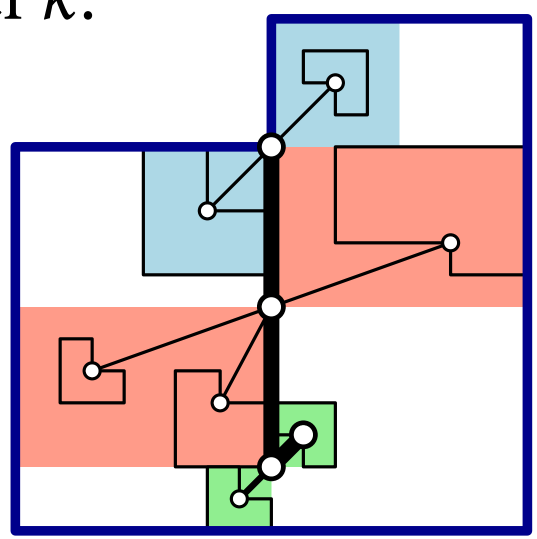


heavy path decomposition

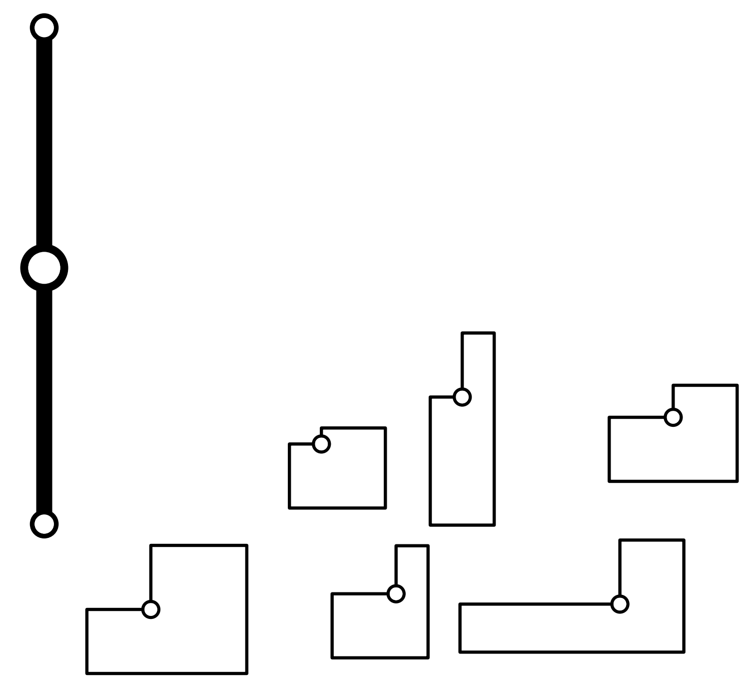
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0:

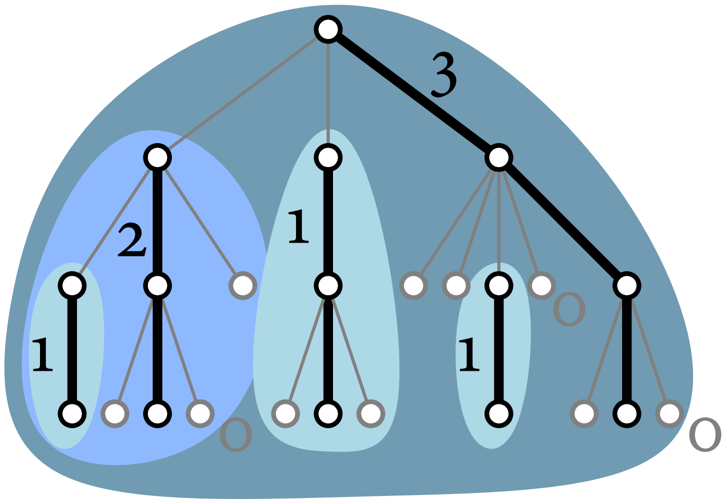
level k:



one vertex:



Trees on the grid

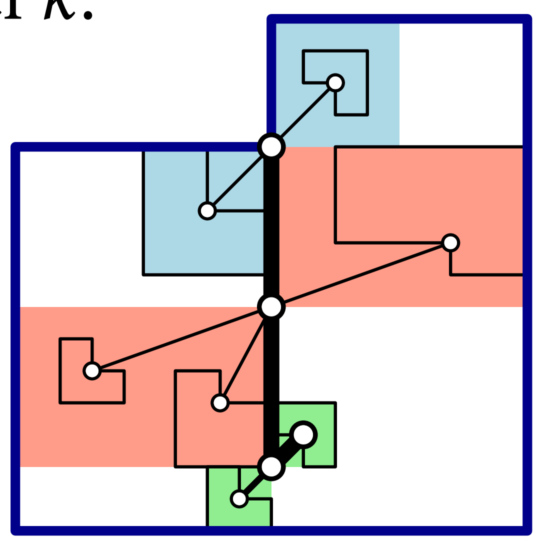


heavy path decomposition

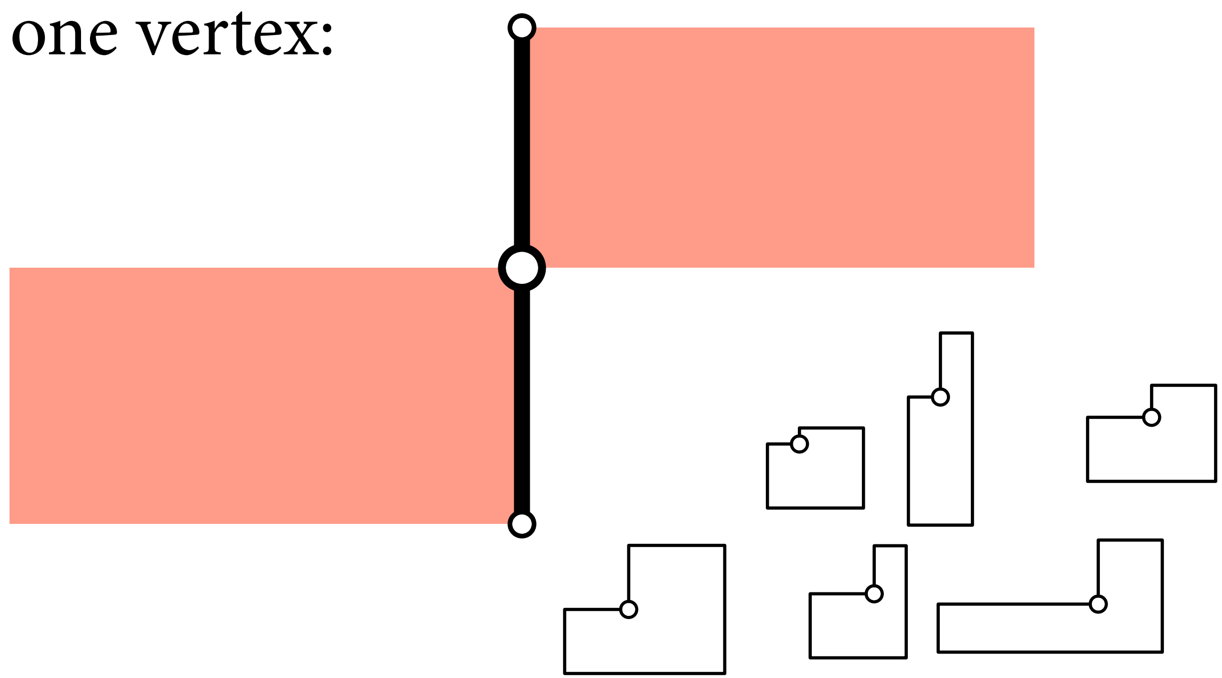
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0:

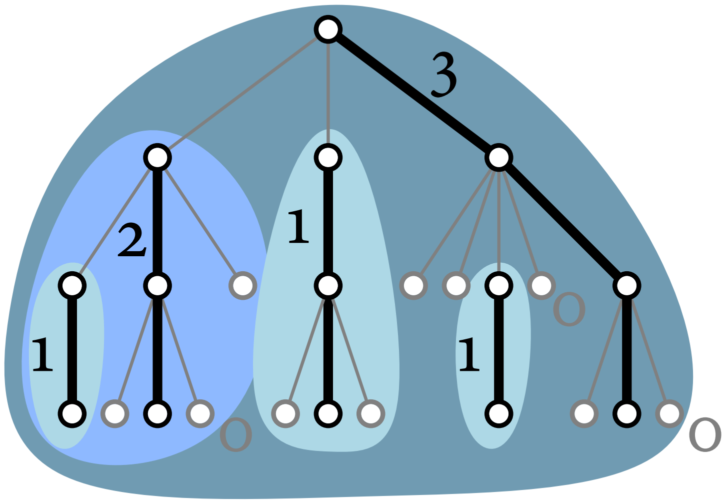
level k:



one vertex:



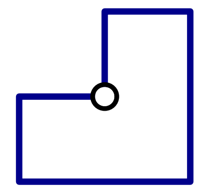
Trees on the grid

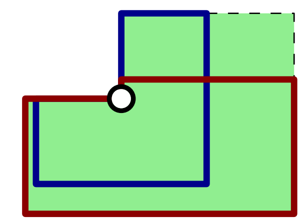


heavy path decomposition

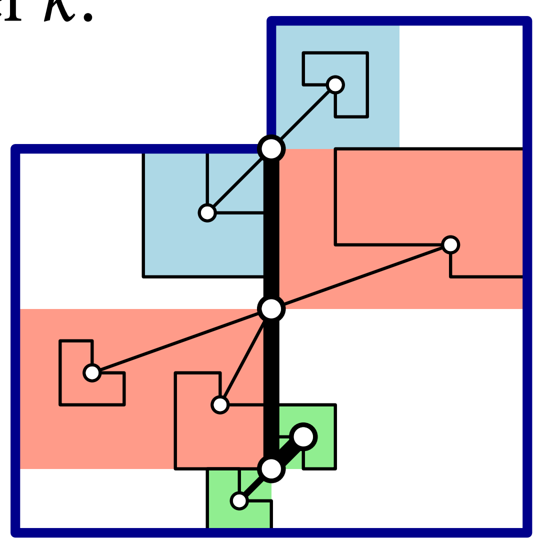
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0:

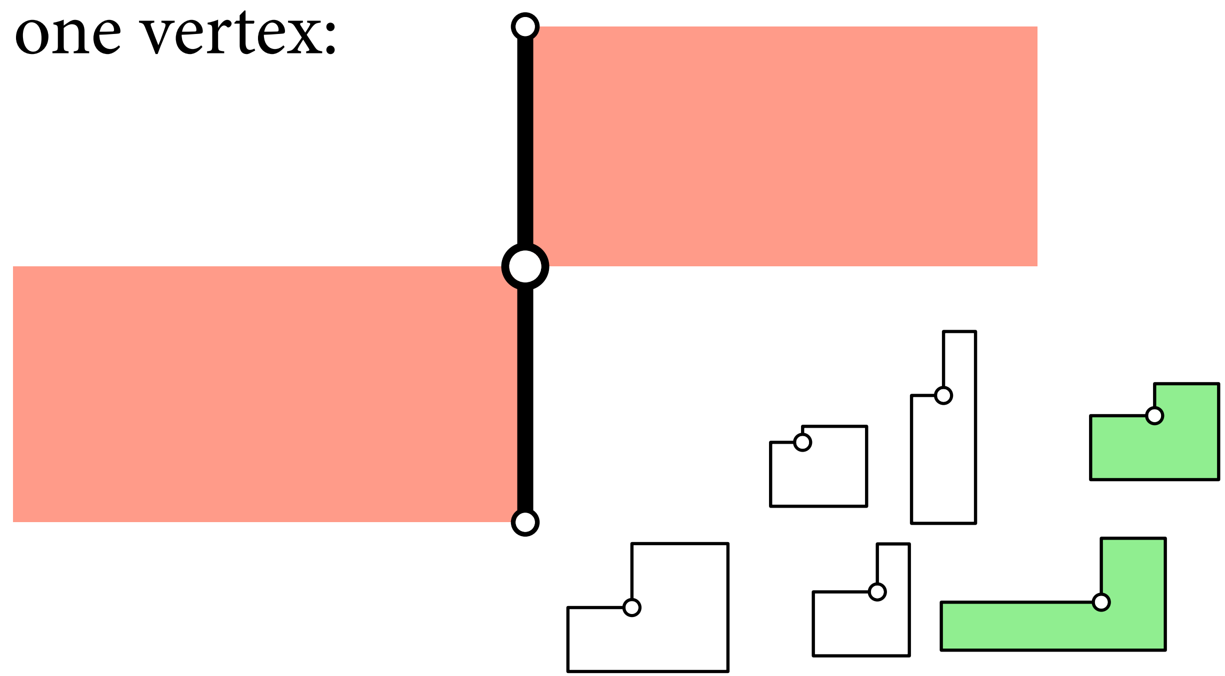




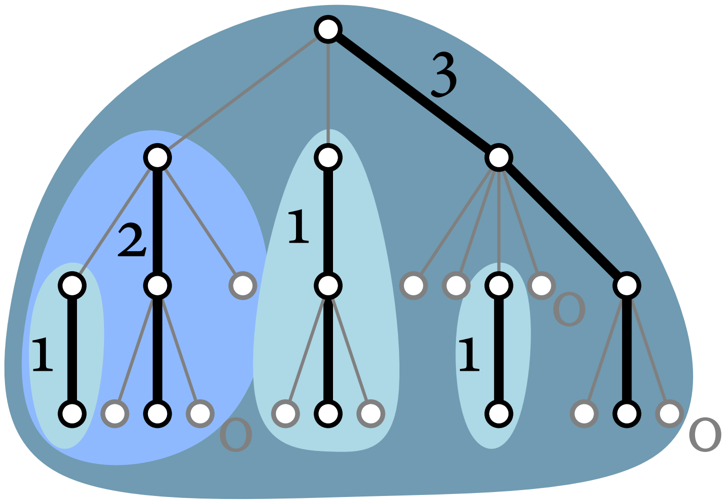
level k:



one vertex:



Trees on the grid

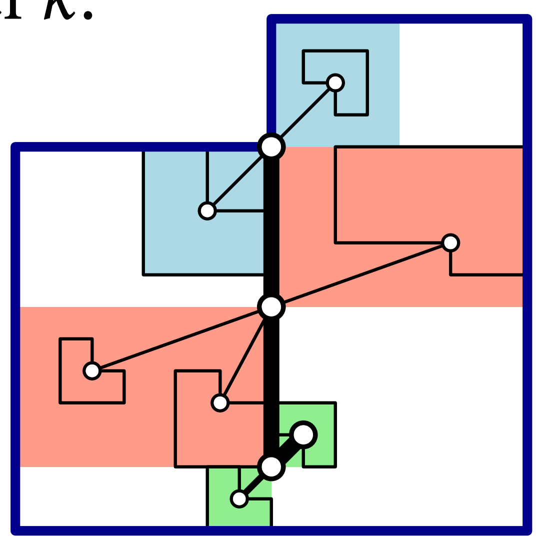


heavy path decomposition

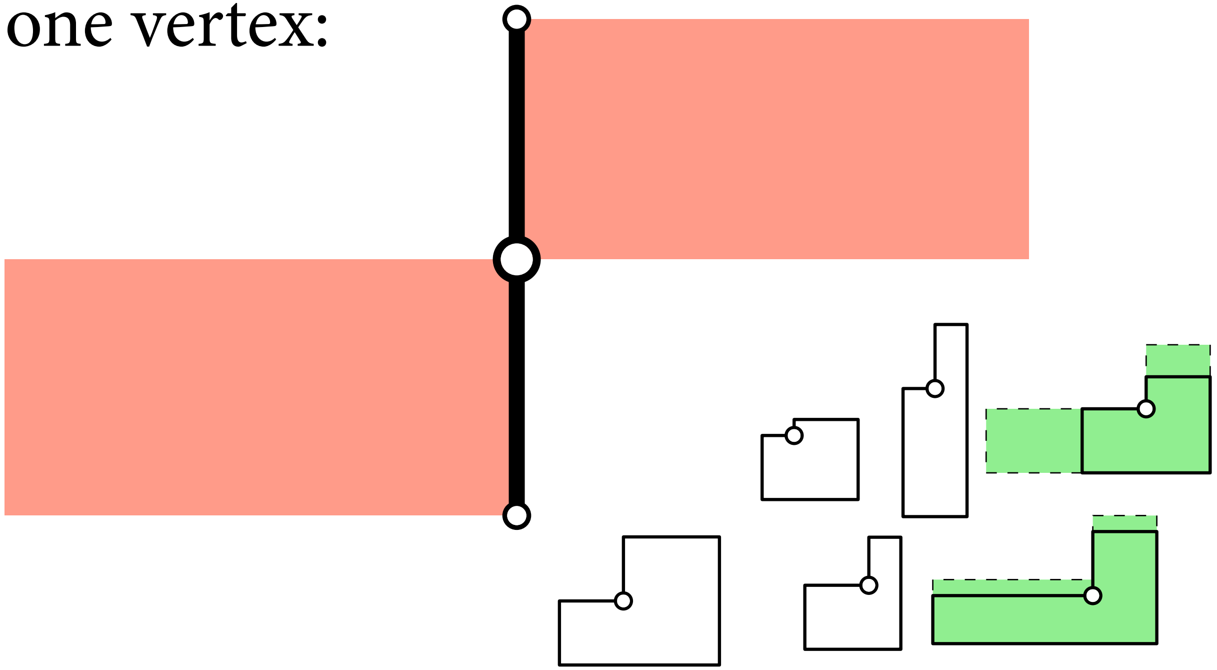
- draw each heavy path tree in a ”box”
- boxes of same level are disjoint

level 0:

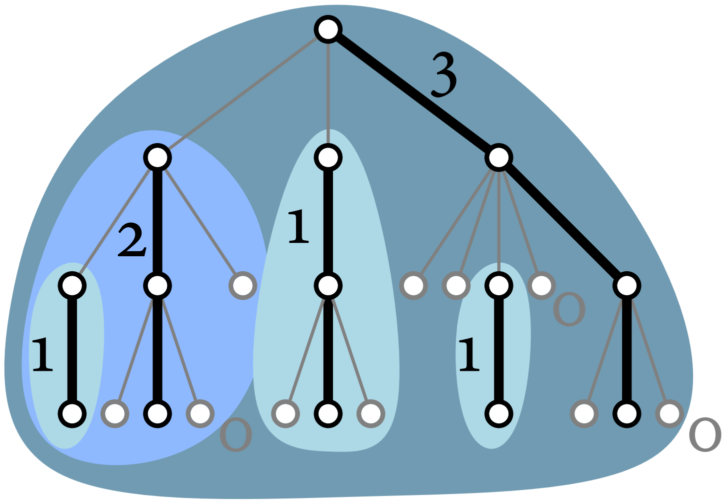
level *k*:



one vertex:



Trees on the grid

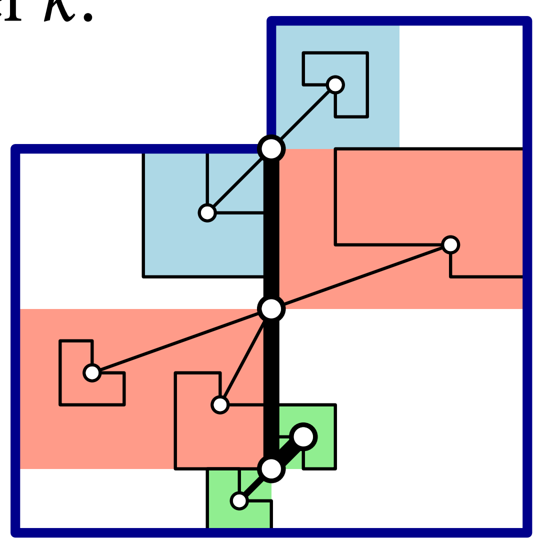


heavy path decomposition

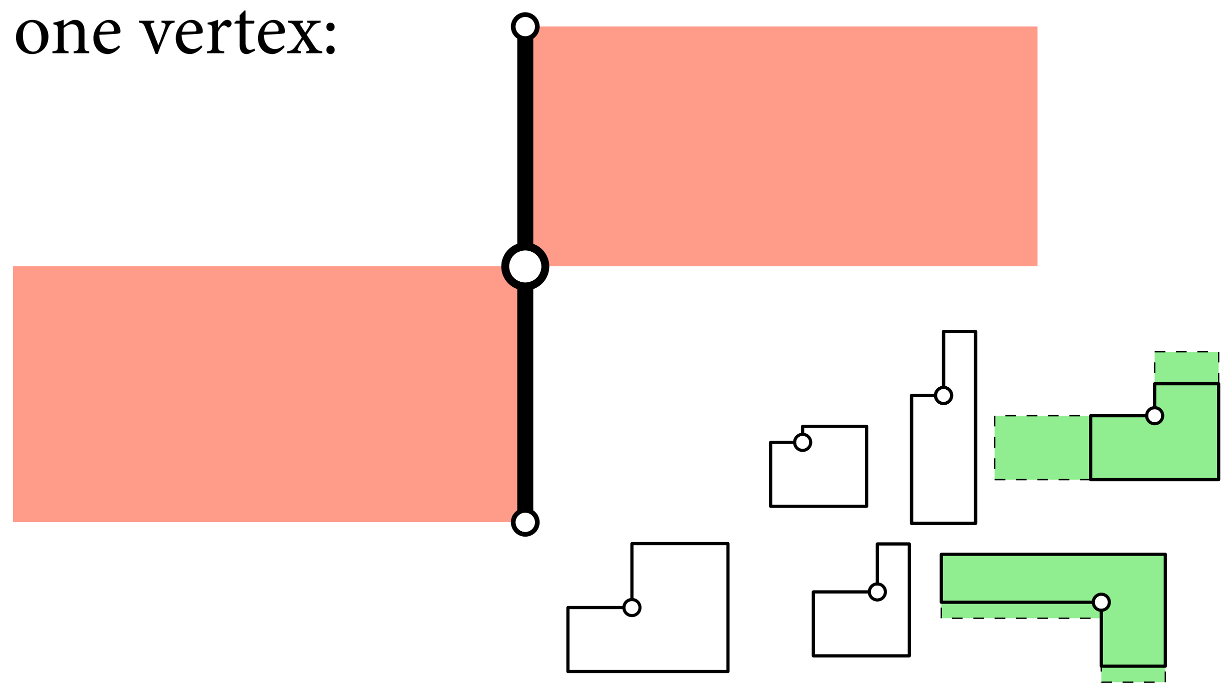
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0:

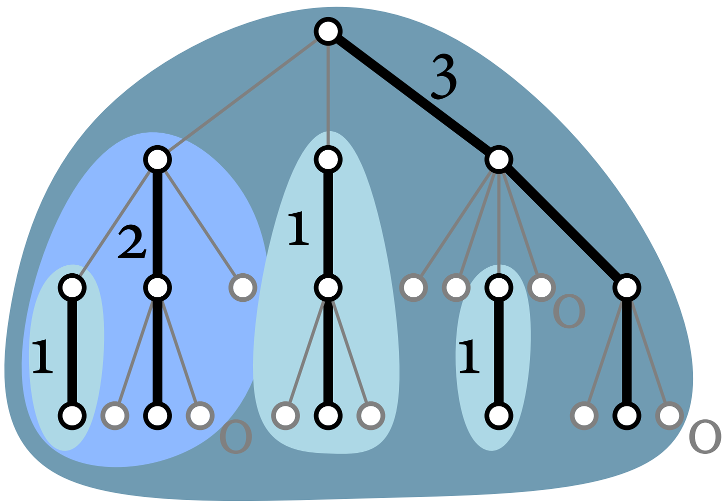
level k:



one vertex:



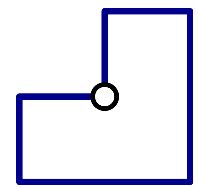
Trees on the grid

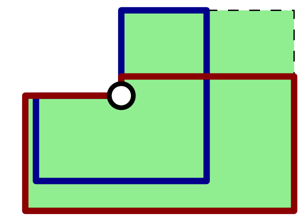


heavy path decomposition

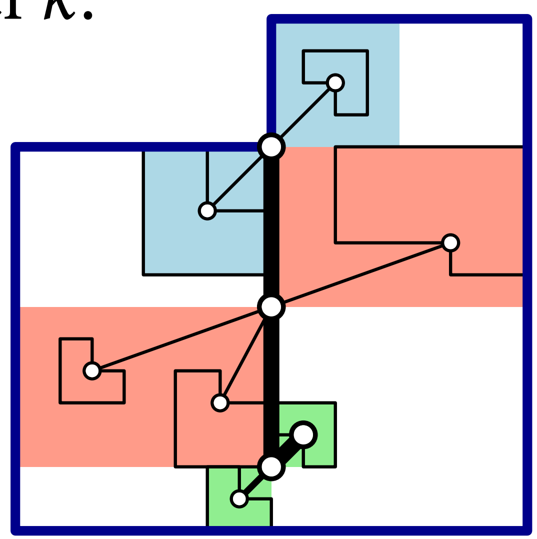
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0:

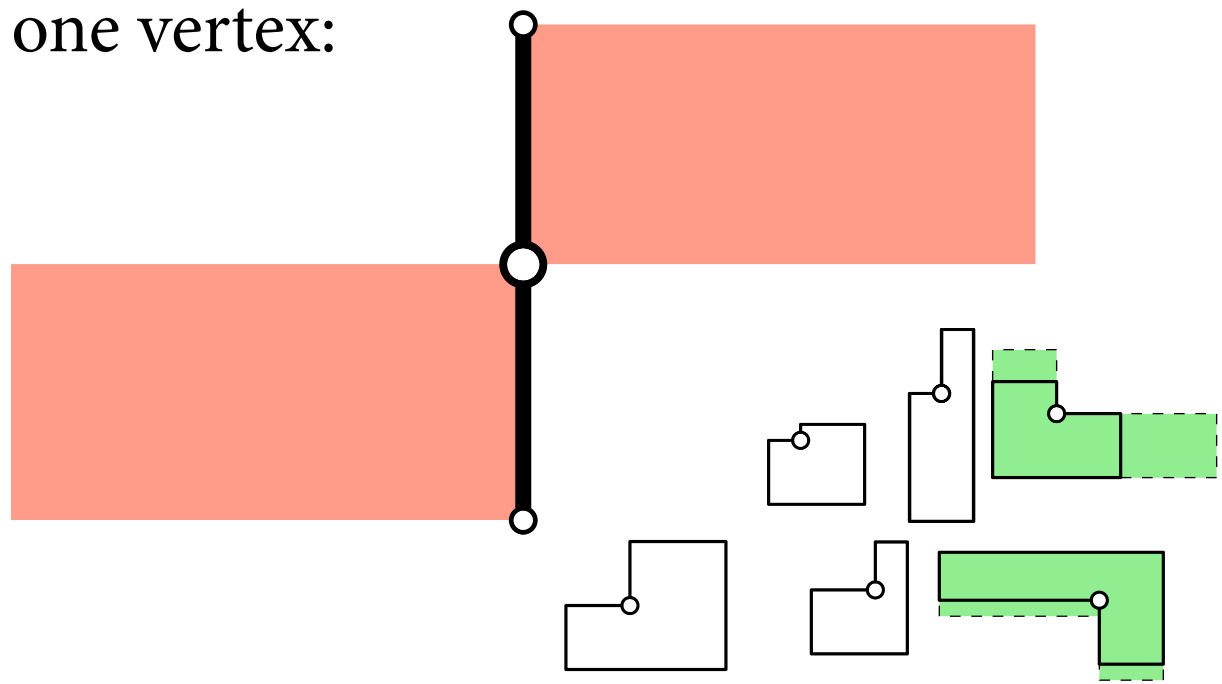




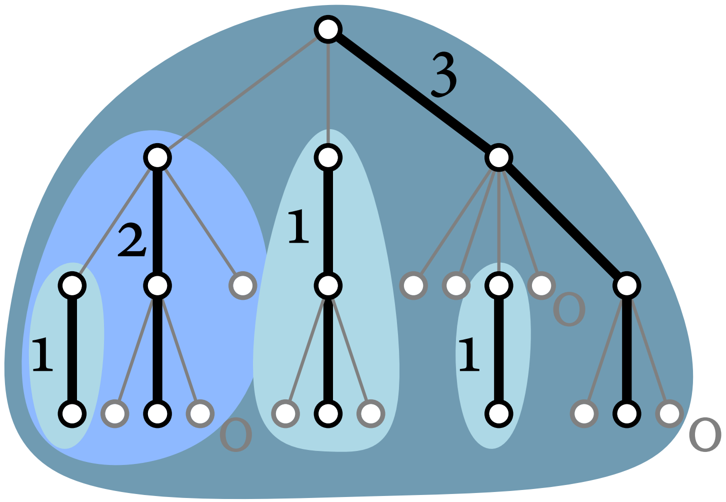
level k:



one vertex:



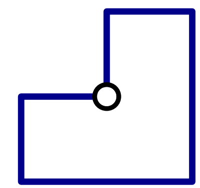
Trees on the grid

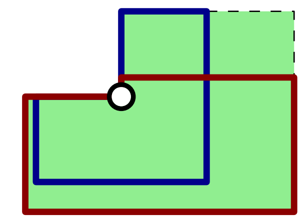


heavy path decomposition

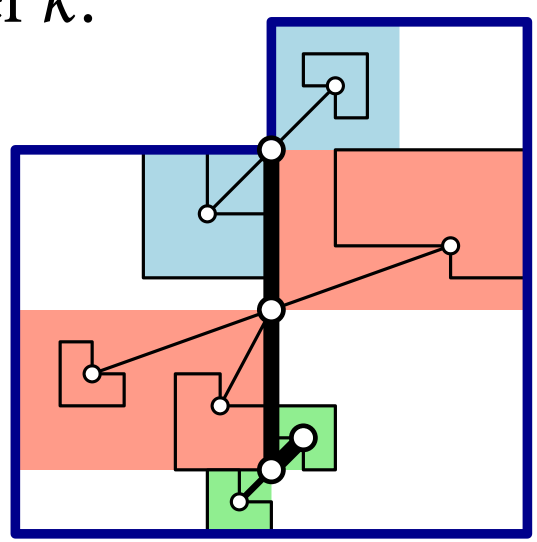
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0:

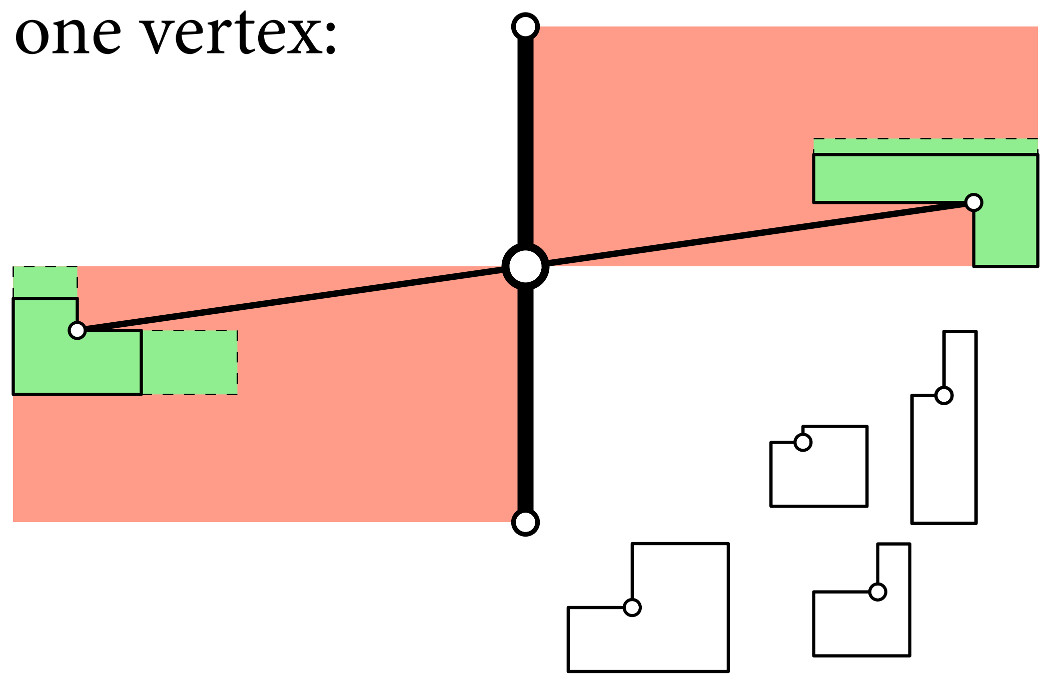




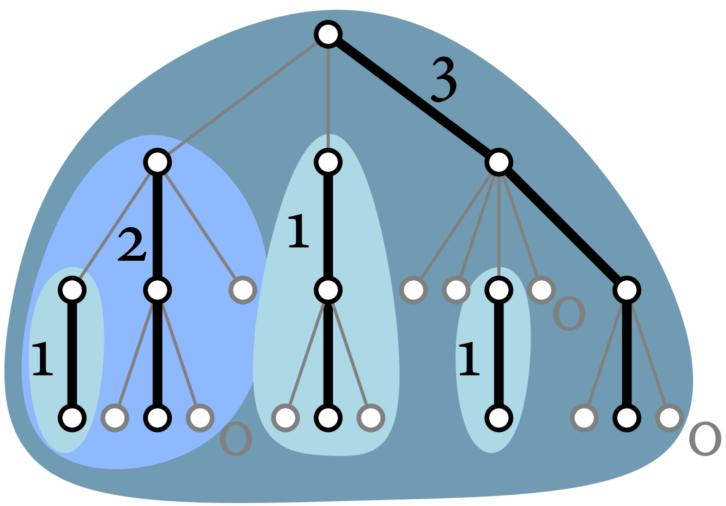
level k:



one vertex:



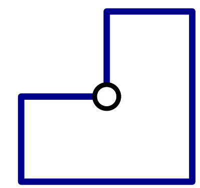
Trees on the grid

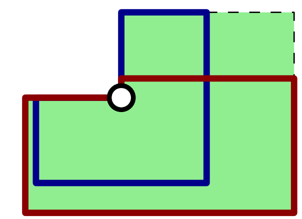


heavy path decomposition

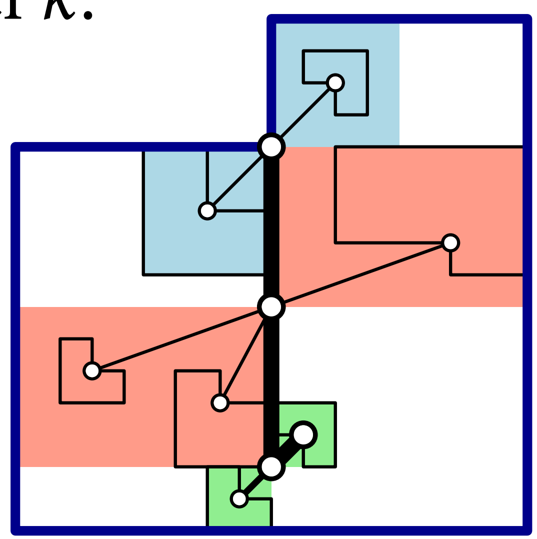
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0:

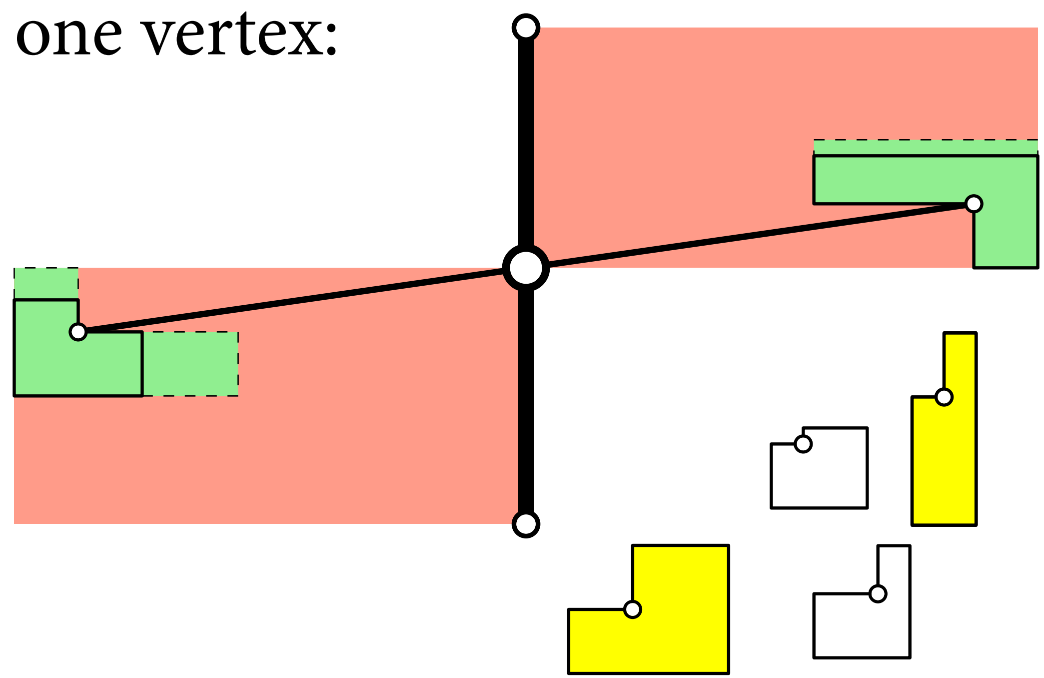




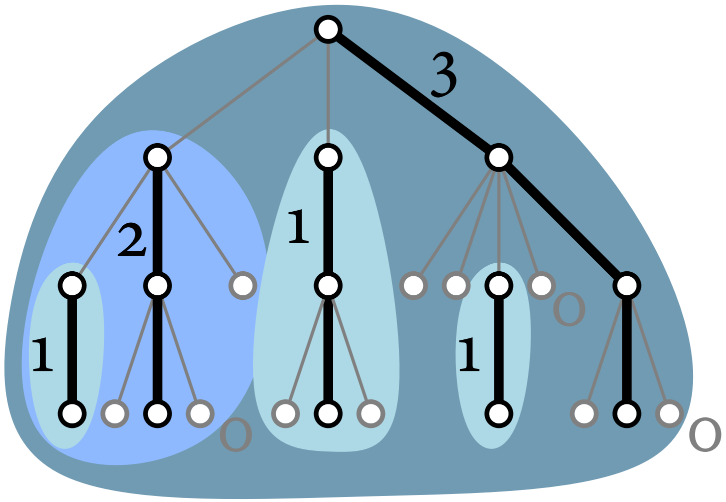
level k:



one vertex:



Trees on the grid

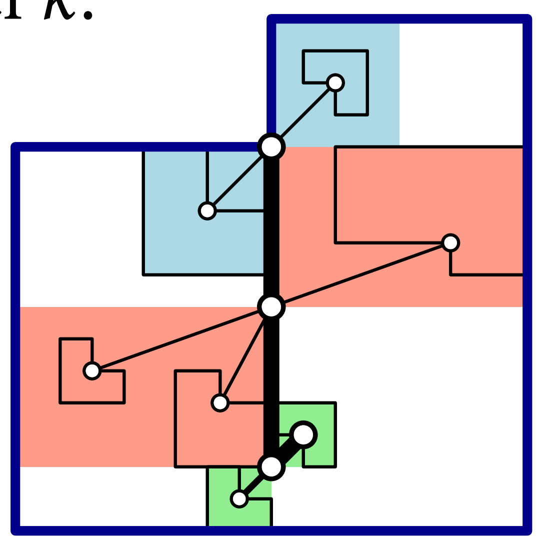


heavy path decomposition

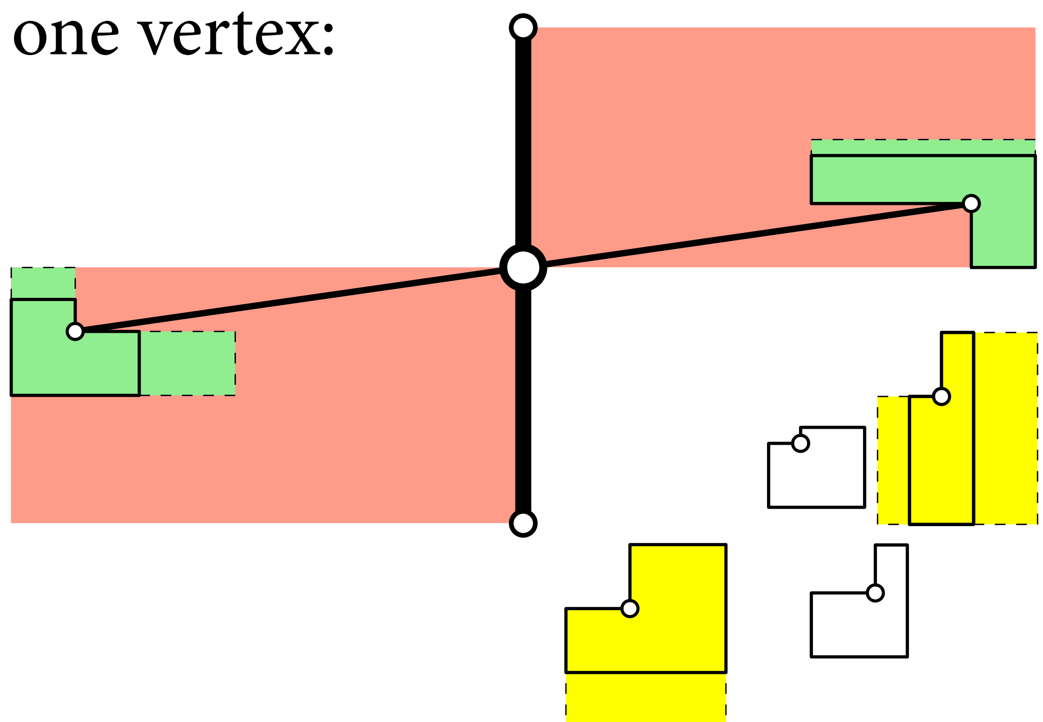
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0:

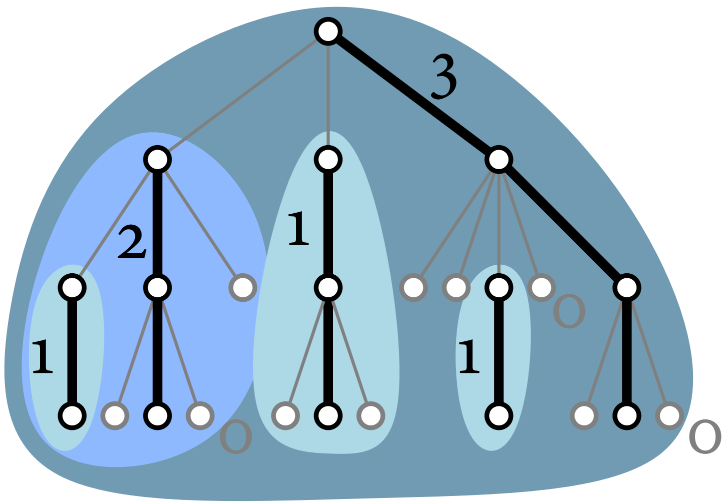
level k:



one vertex:



Trees on the grid

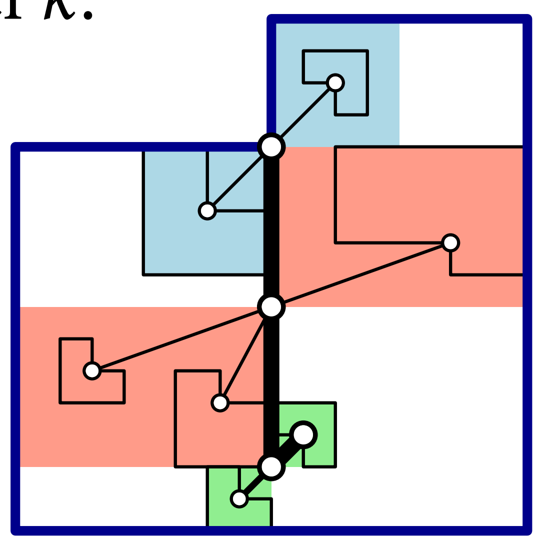


heavy path decomposition

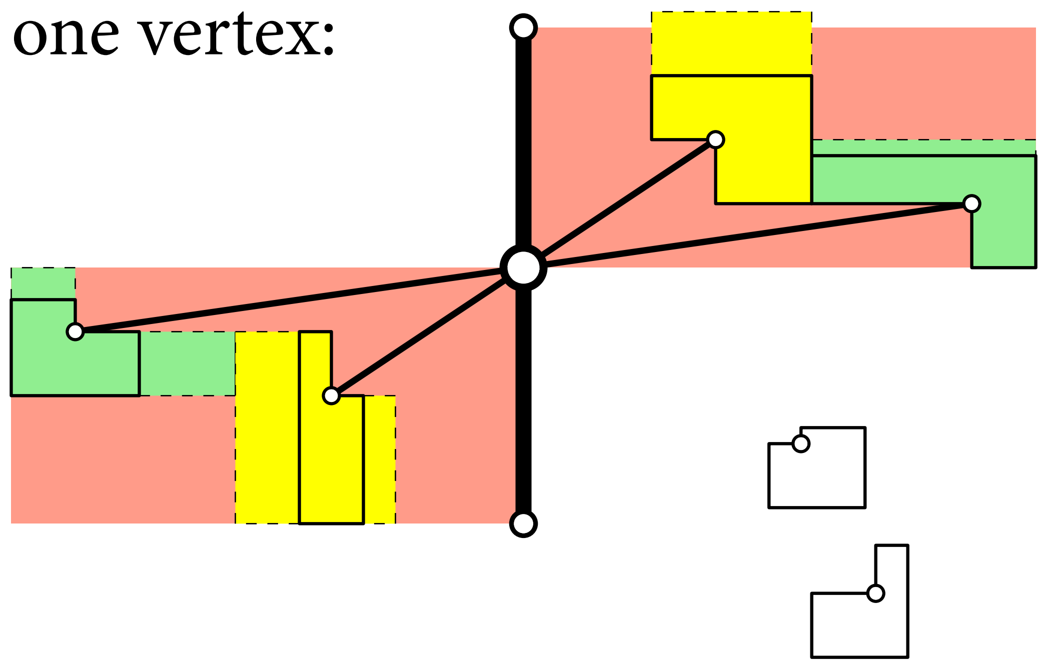
- draw each heavy path tree in a ”box”
- boxes of same level are disjoint

level 0:

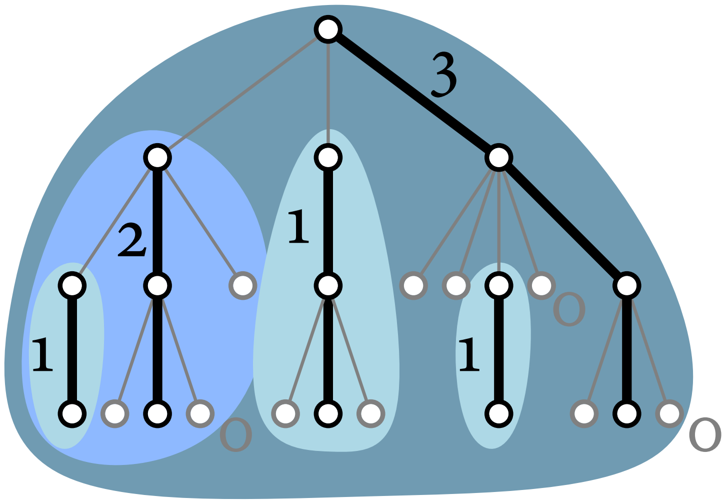
level k:



one vertex:



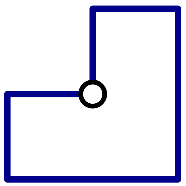
Trees on the grid

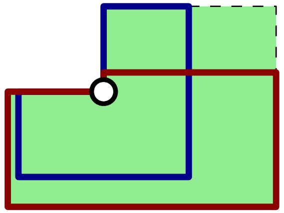


heavy path decomposition

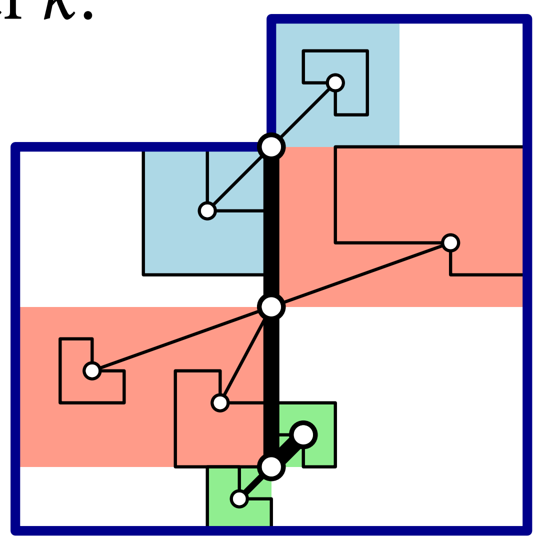
- draw each heavy path tree in a ”box”
- boxes of same level are disjoint

level 0:

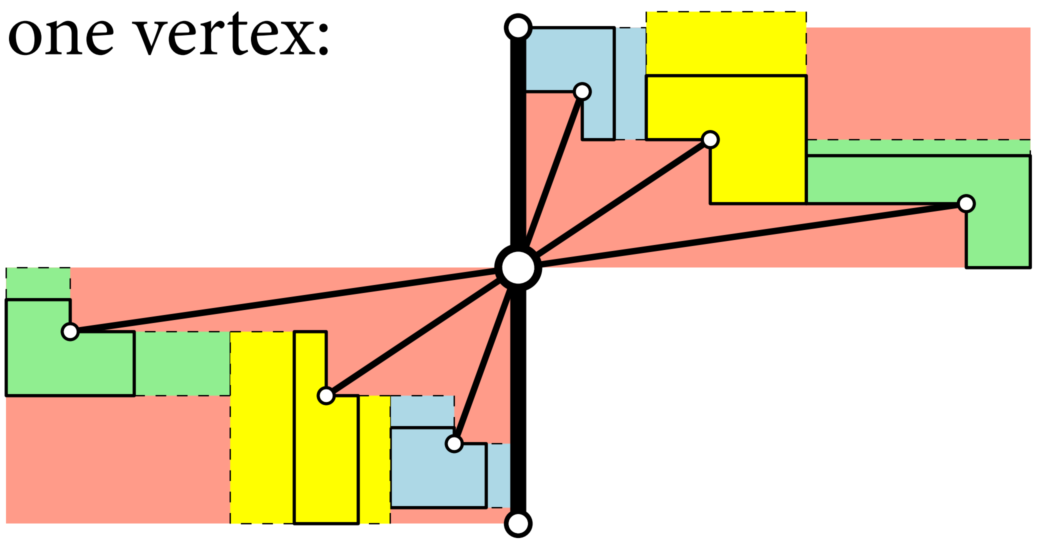




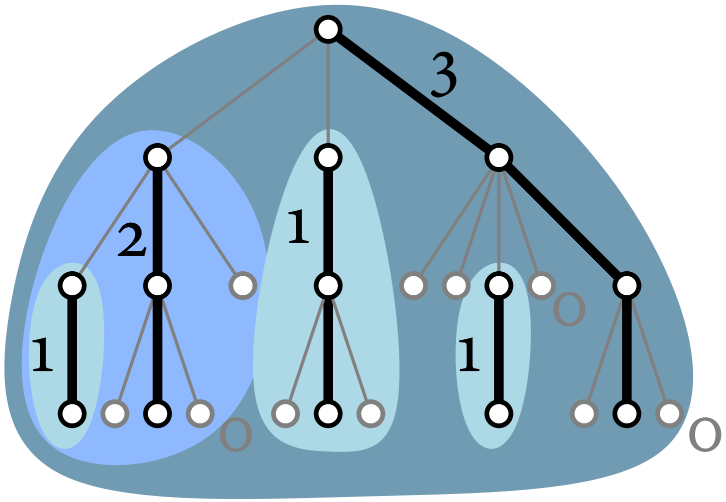
level k:



one vertex:



Trees on the grid

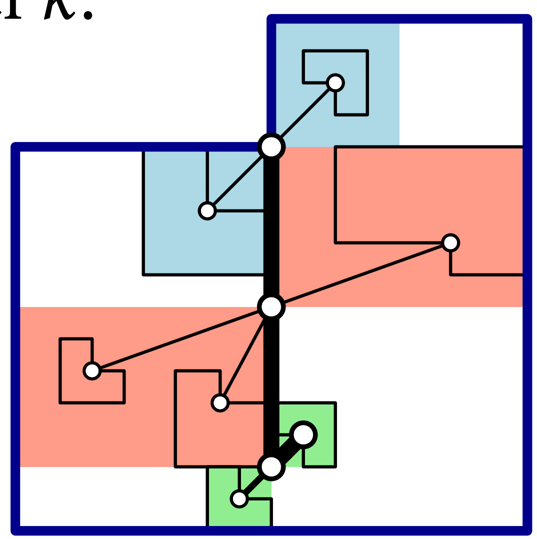


heavy path decomposition

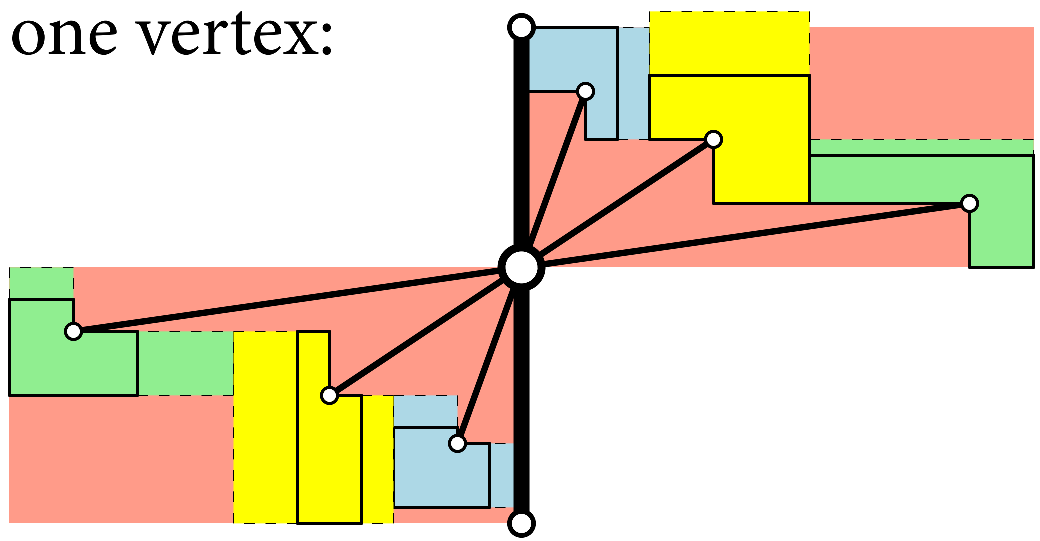
- draw each heavy path tree in a "box"
- boxes of same level are disjoint

level 0:

level k:



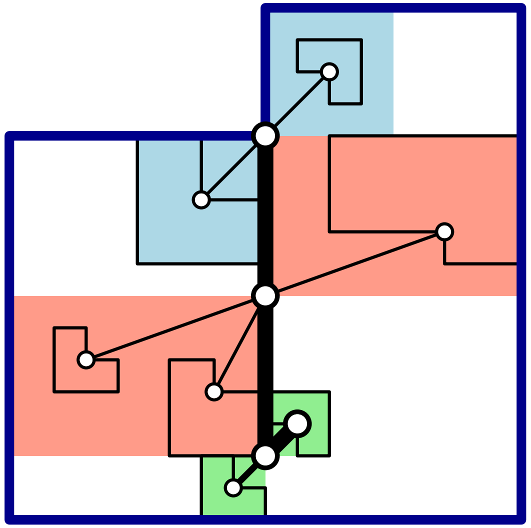
one vertex:



Tree → $3e/4$ arcs, $O(n^2) \times O(n^{1.58})$ grid

Trees on the grid

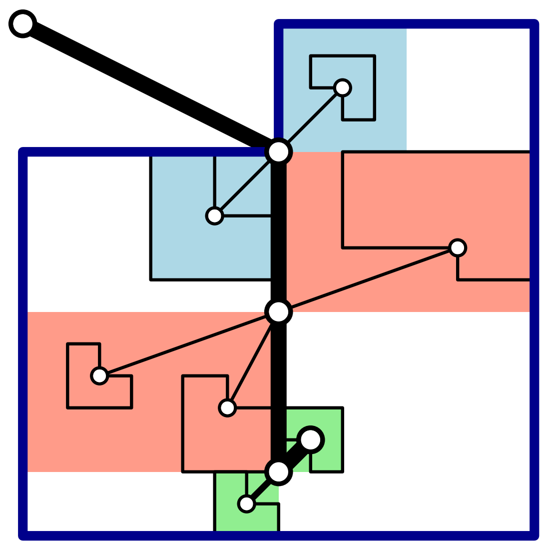
level k:



Tree $\rightarrow 3e/4$ arcs, $O(n^2) \times O(n^{1.58})$ grid

Trees on the grid

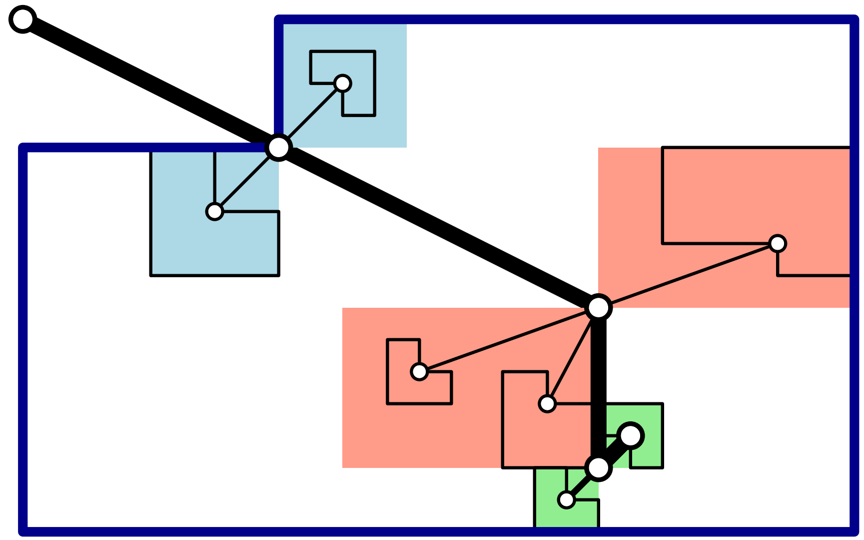
level k:



Tree $\to 3e/4$ arcs, $O(n^2) \times O(n^{1.58})$ grid

Trees on the grid

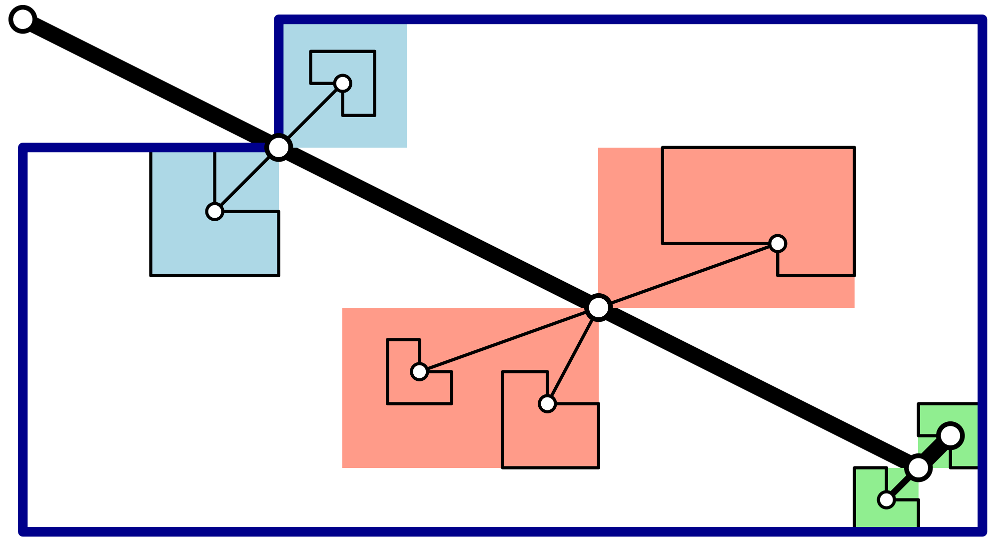
level k:



Tree $\rightarrow 3e/4$ arcs, $O(n^2) \times O(n^{1.58})$ grid

Trees on the grid

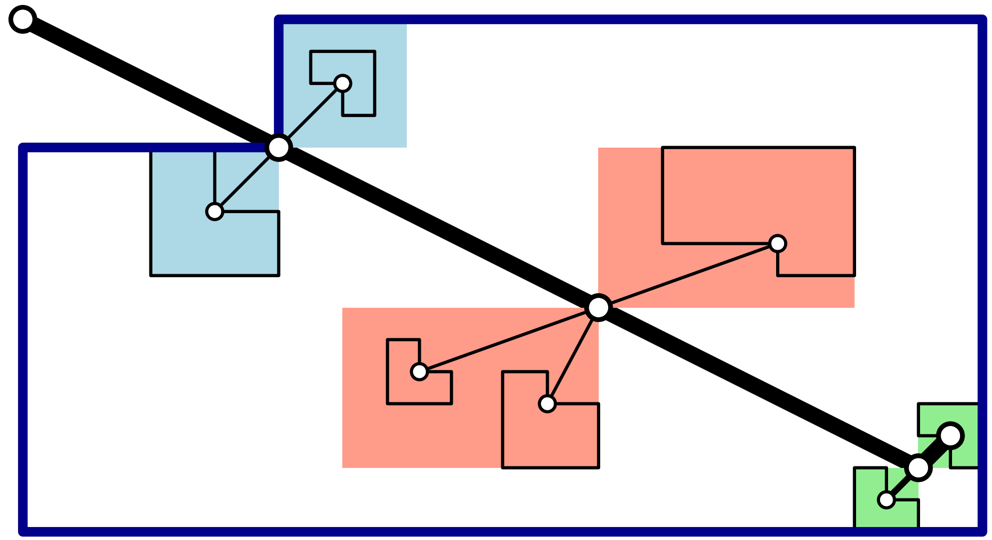
level k:



Tree $\to 3e/4$ arcs, $O(n^2) \times O(n^{1.58})$ grid

Trees on the grid

level k:



Tree $\to \vartheta/2$ arcs, quasipolynomial grid

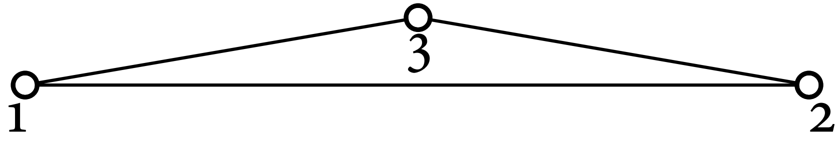
Tree $\to 3e/4$ arcs, $O(n^2) \times O(n^{1.58})$ grid

Triangulations with circular arcs

canonical order

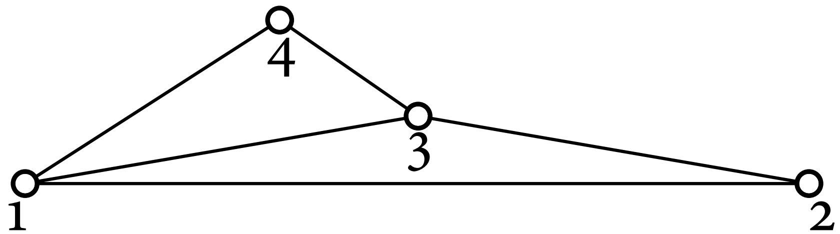
Triangulations with circular arcs

canonical order



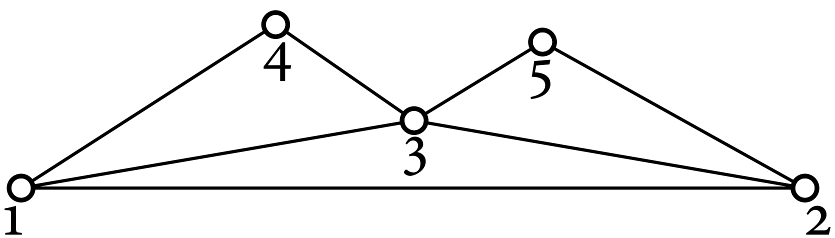
Triangulations with circular arcs

canonical order



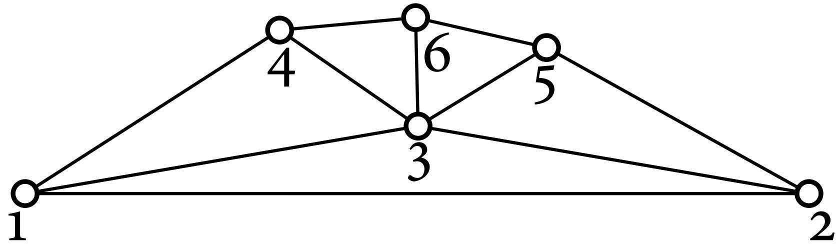
Triangulations with circular arcs

canonical order



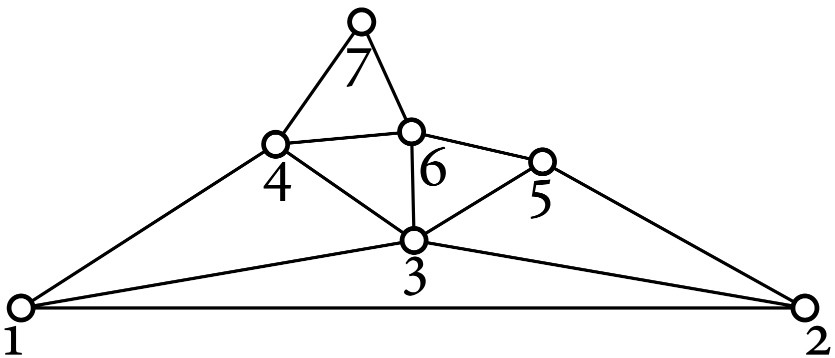
Triangulations with circular arcs

canonical order



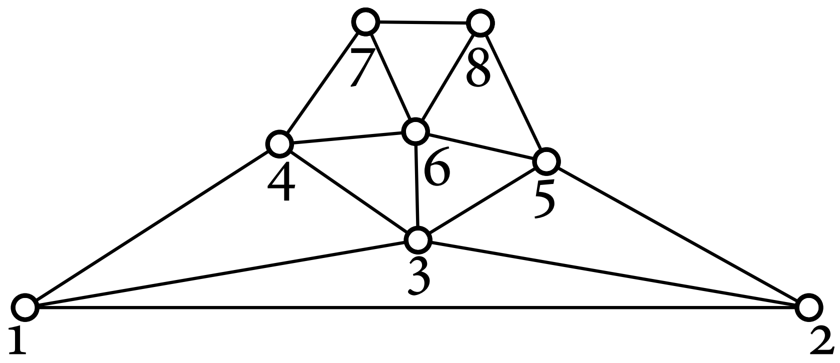
Triangulations with circular arcs

canonical order



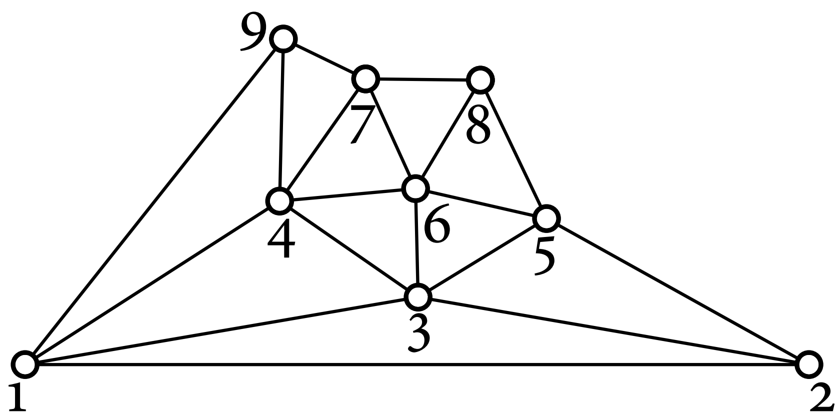
Triangulations with circular arcs

canonical order



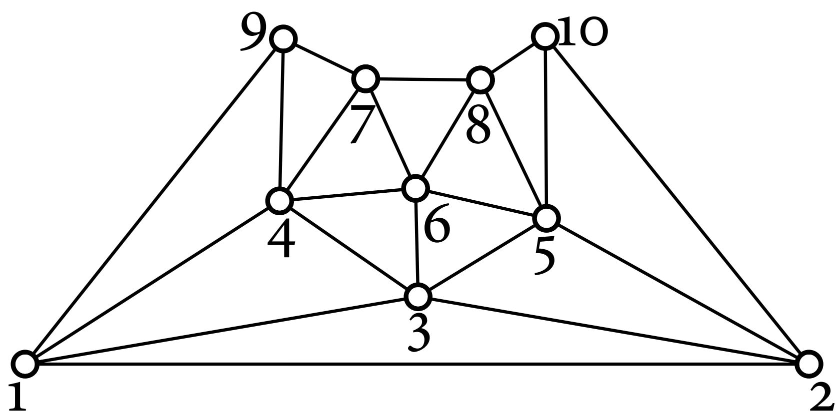
Triangulations with circular arcs

canonical order



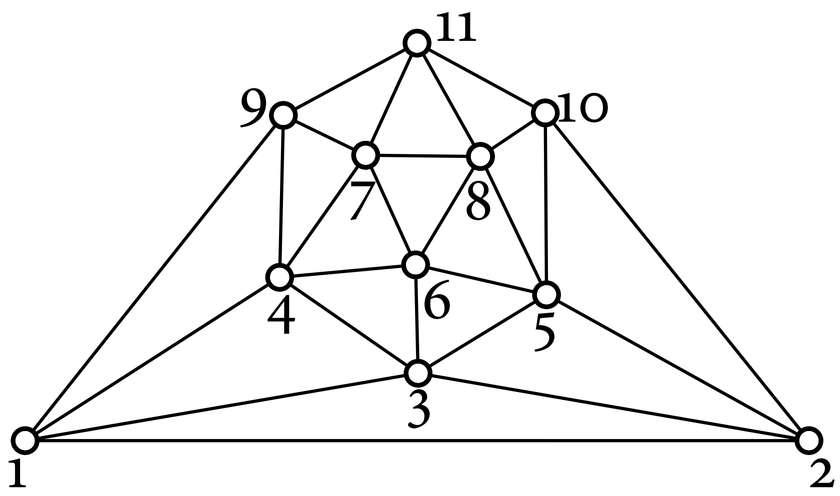
Triangulations with circular arcs

canonical order

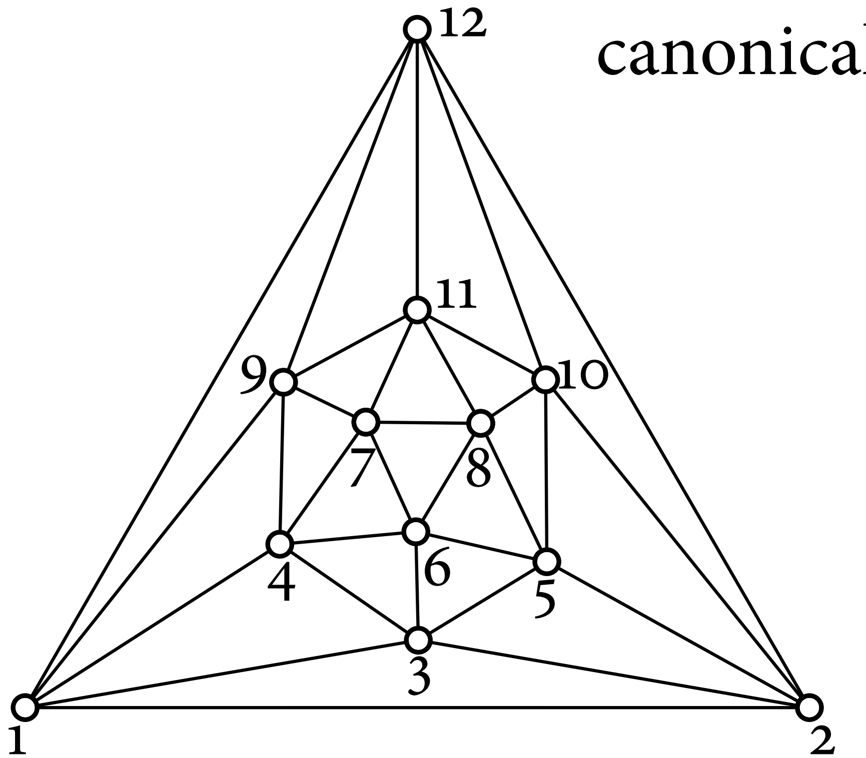


Triangulations with circular arcs

canonical order

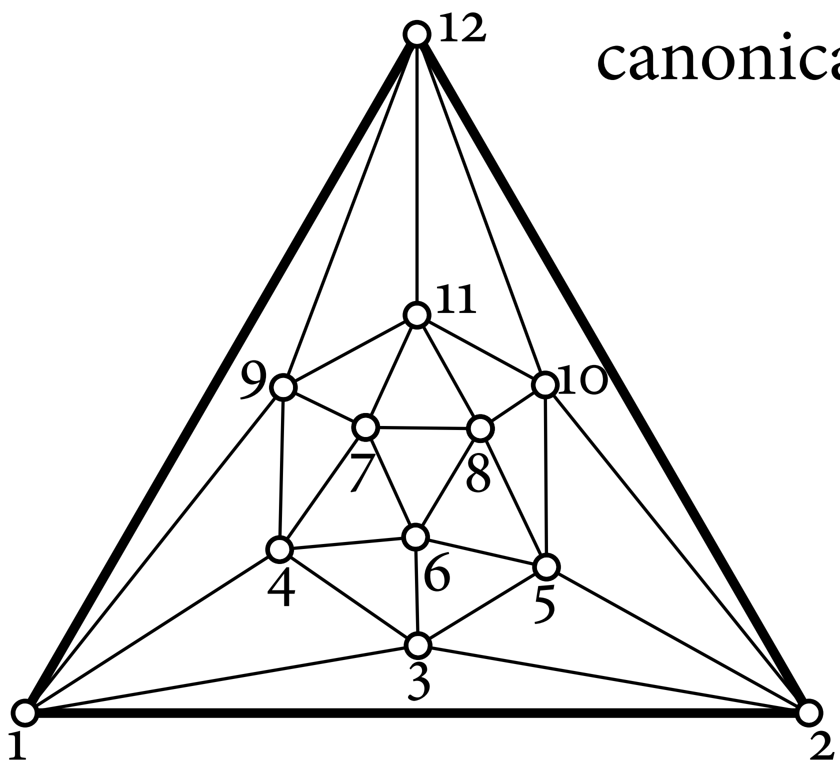


Triangulations with circular arcs

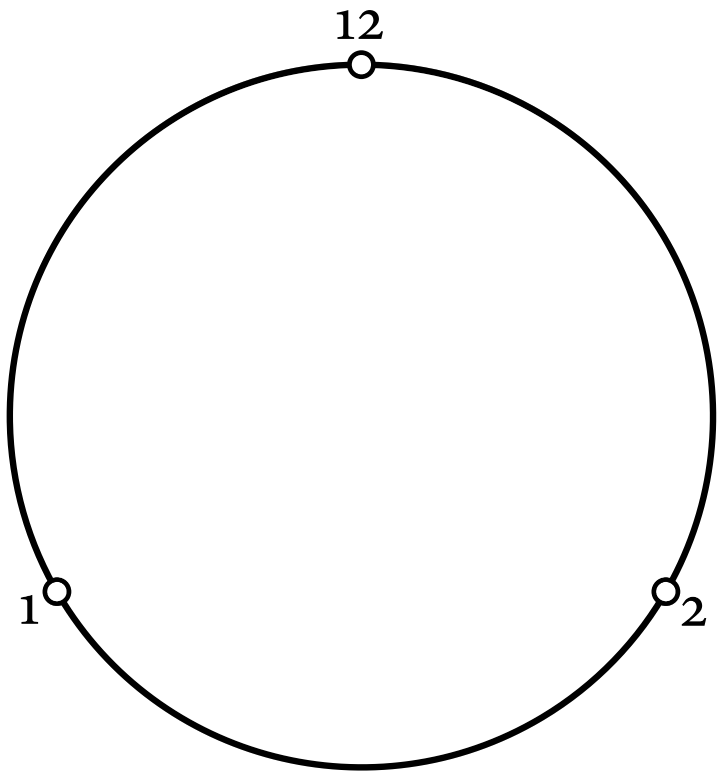


canonical order

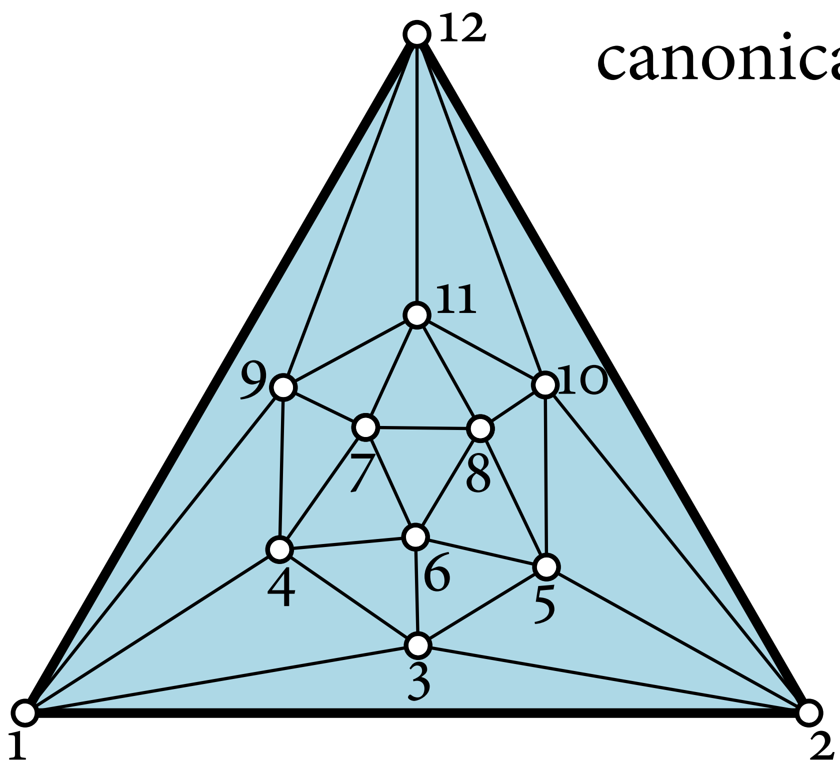
Triangulations with circular arcs



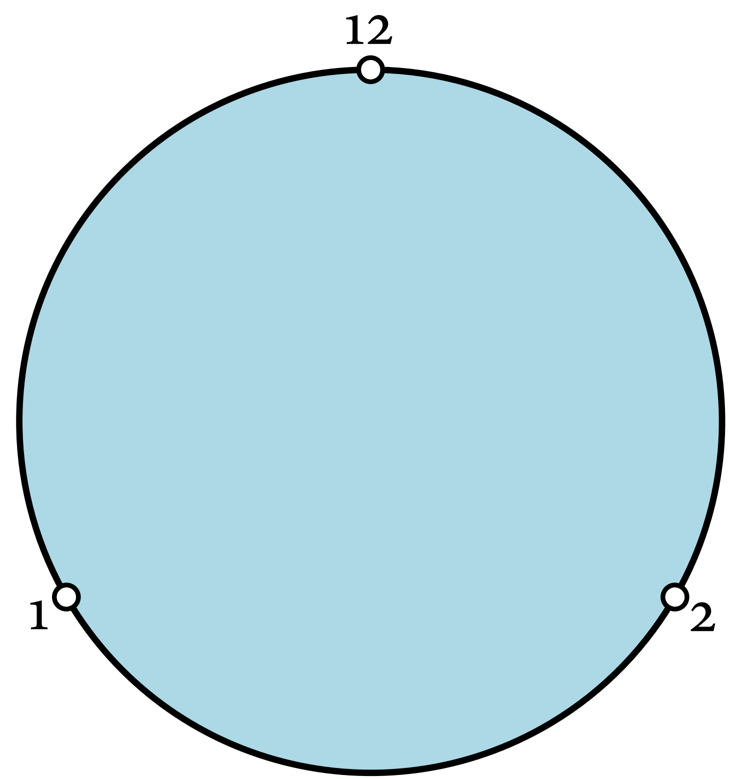
canonical order



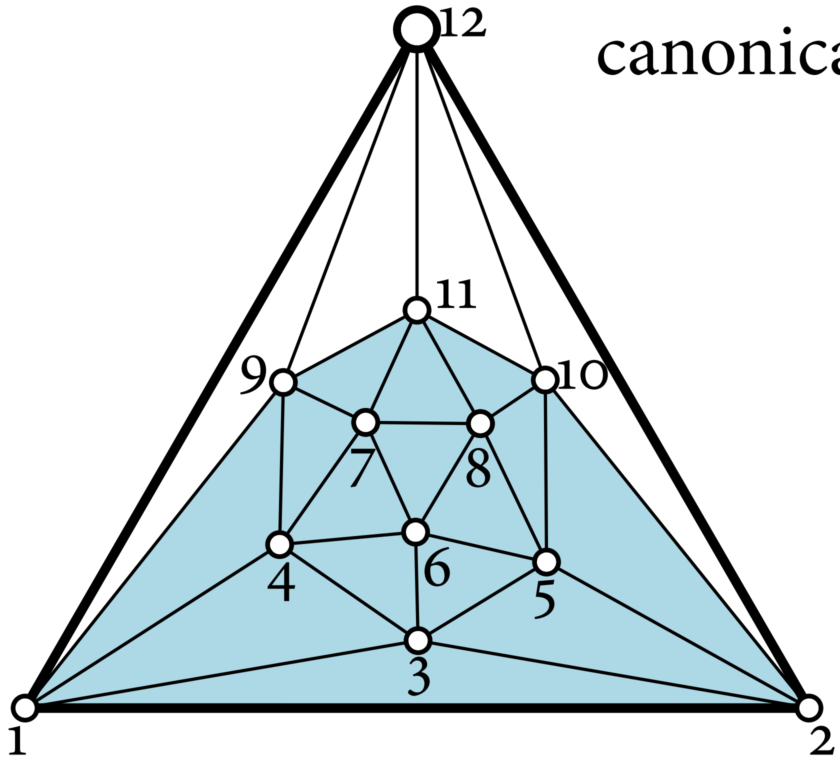
Triangulations with circular arcs



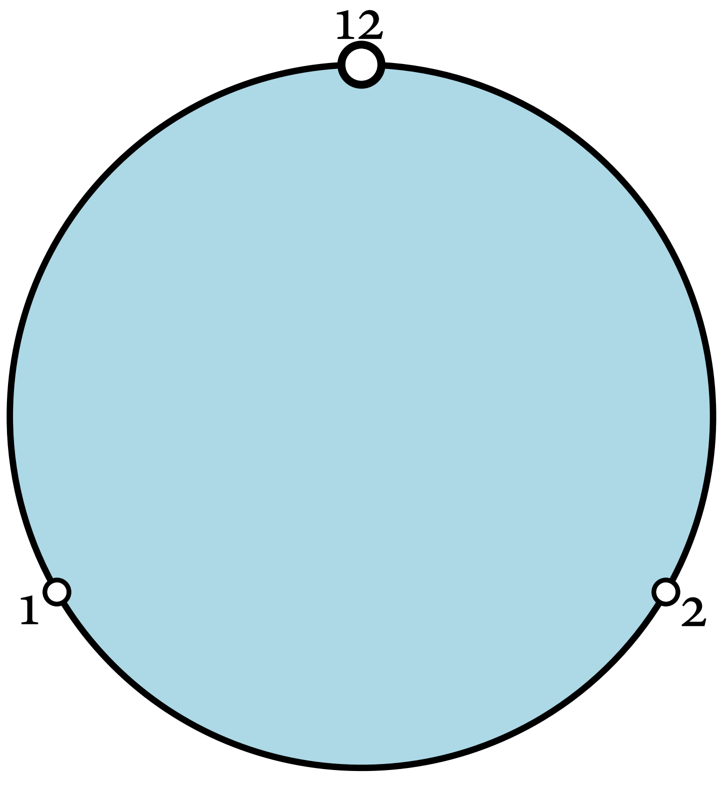
canonical order



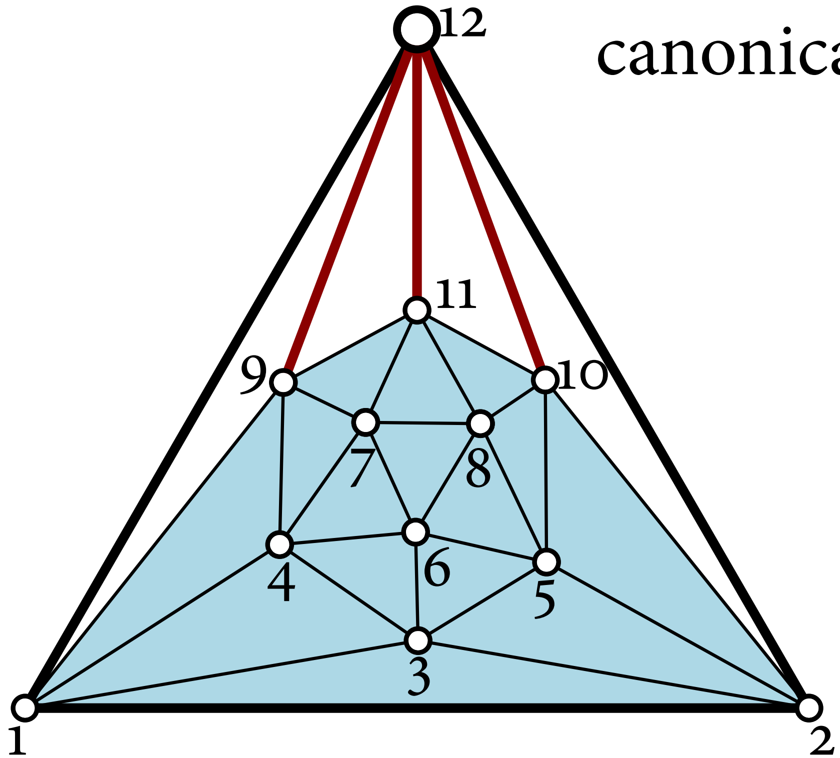
Triangulations with circular arcs



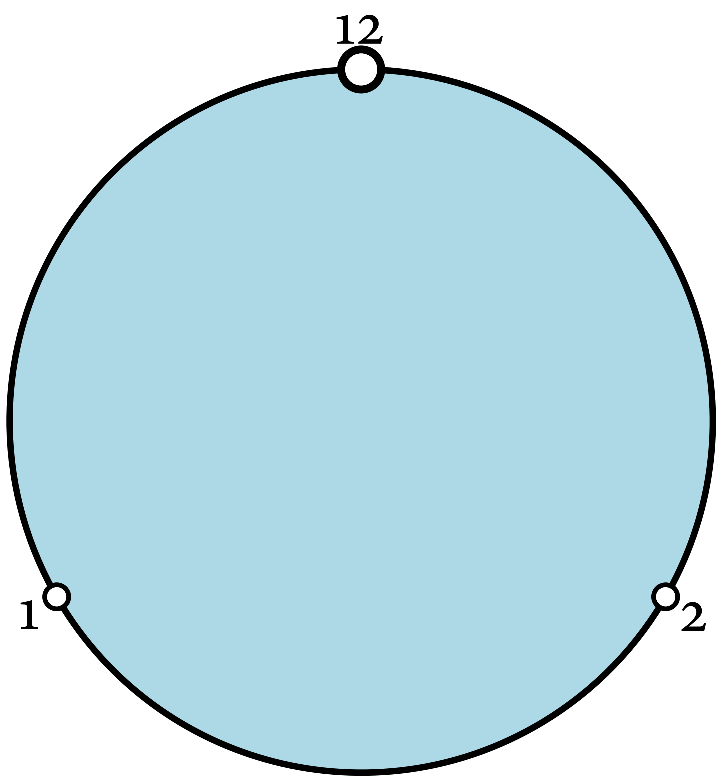
canonical order



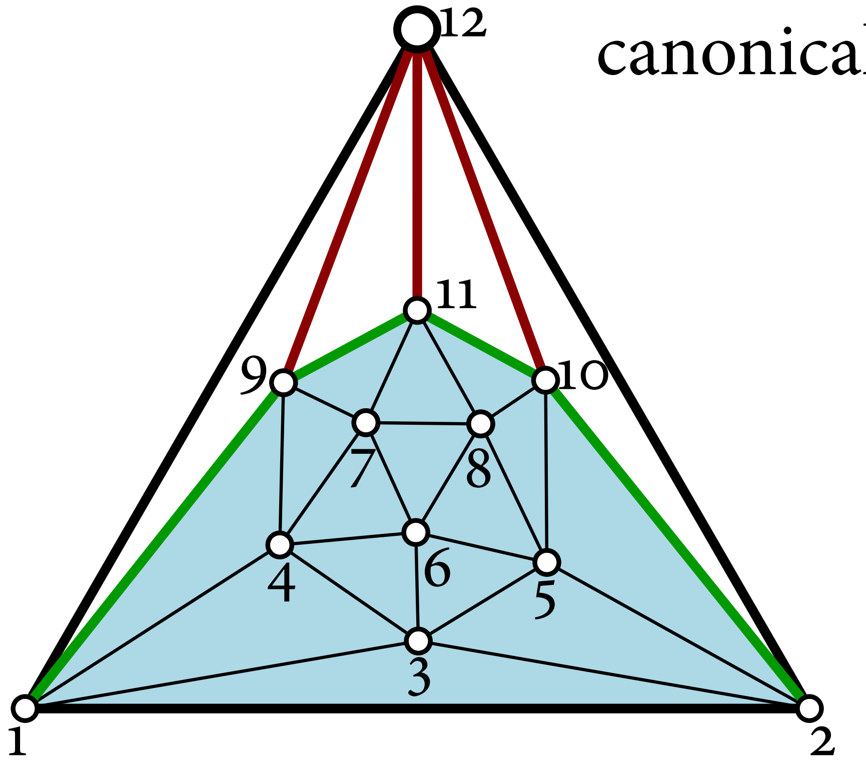
Triangulations with circular arcs



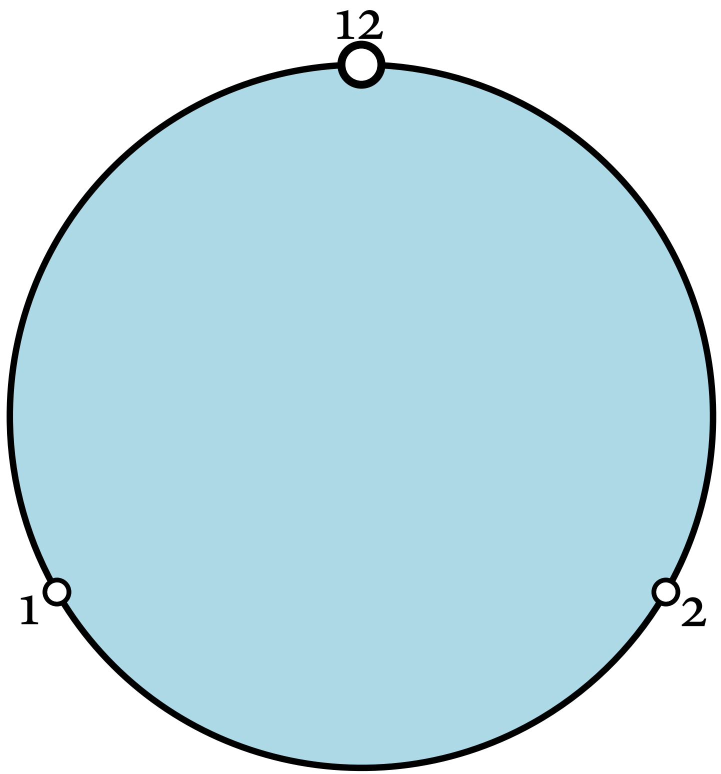
canonical order



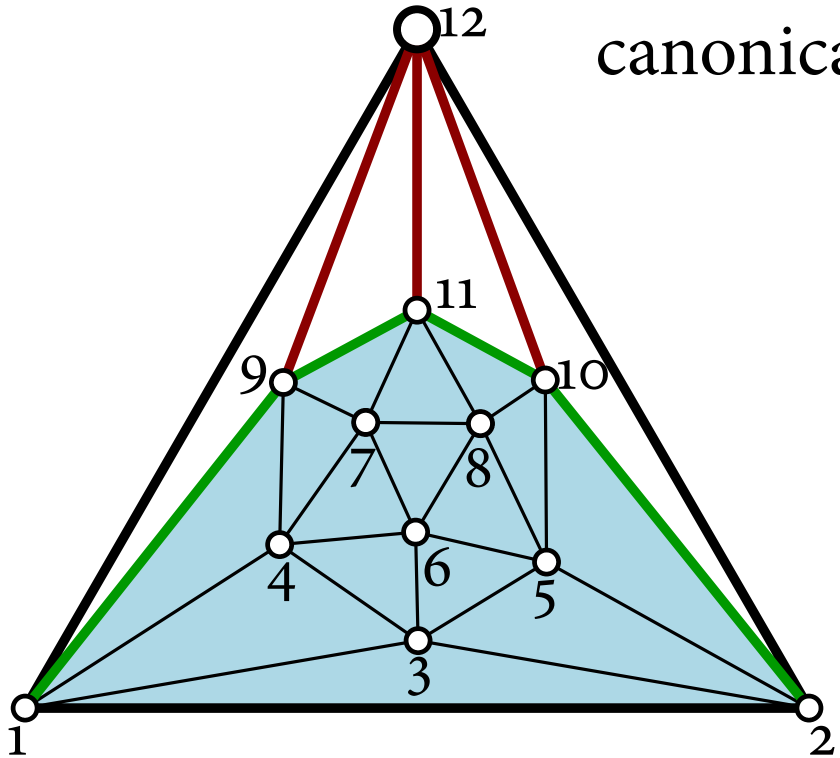
Triangulations with circular arcs



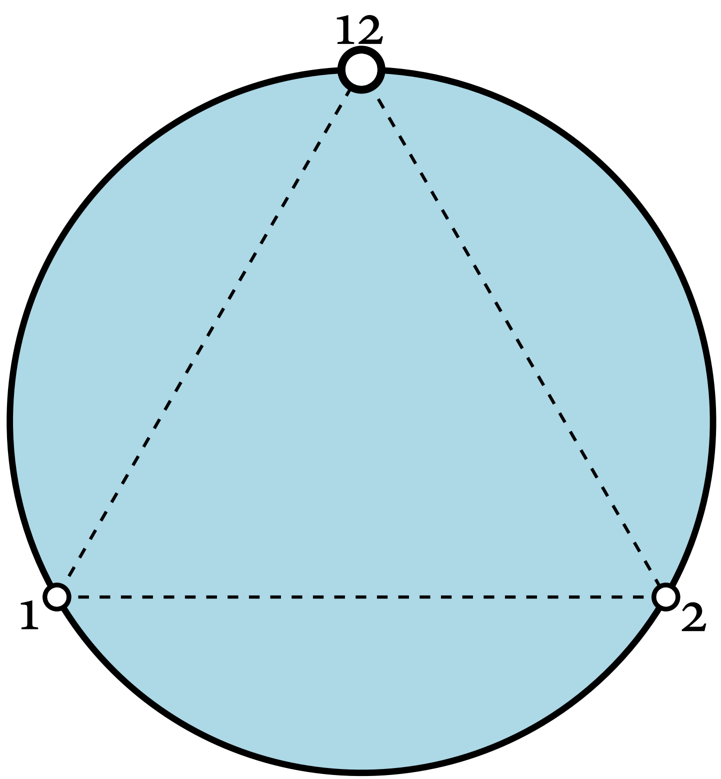
canonical order



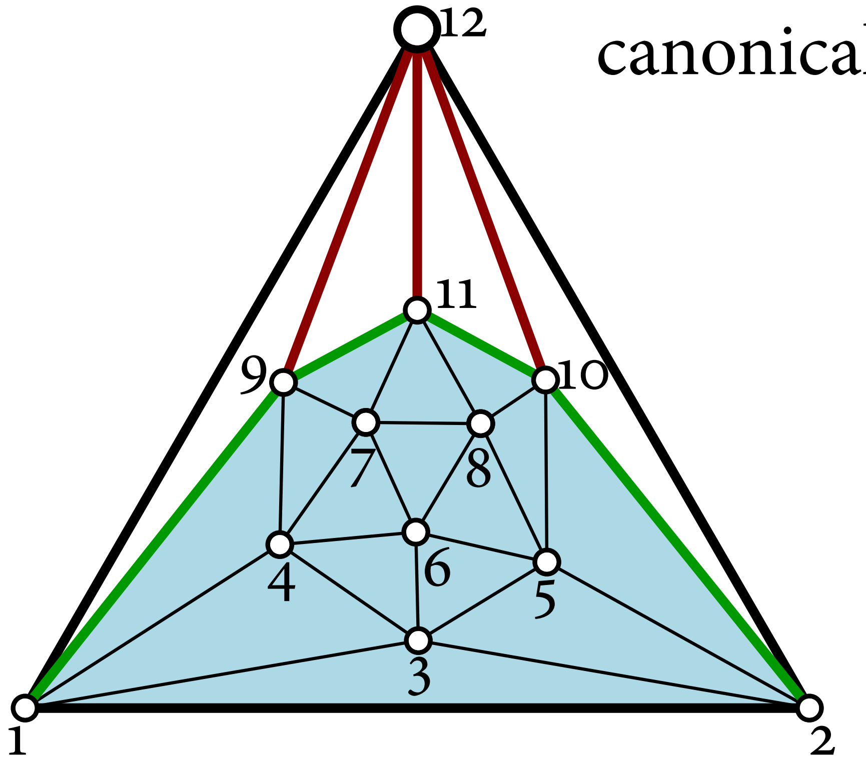
Triangulations with circular arcs



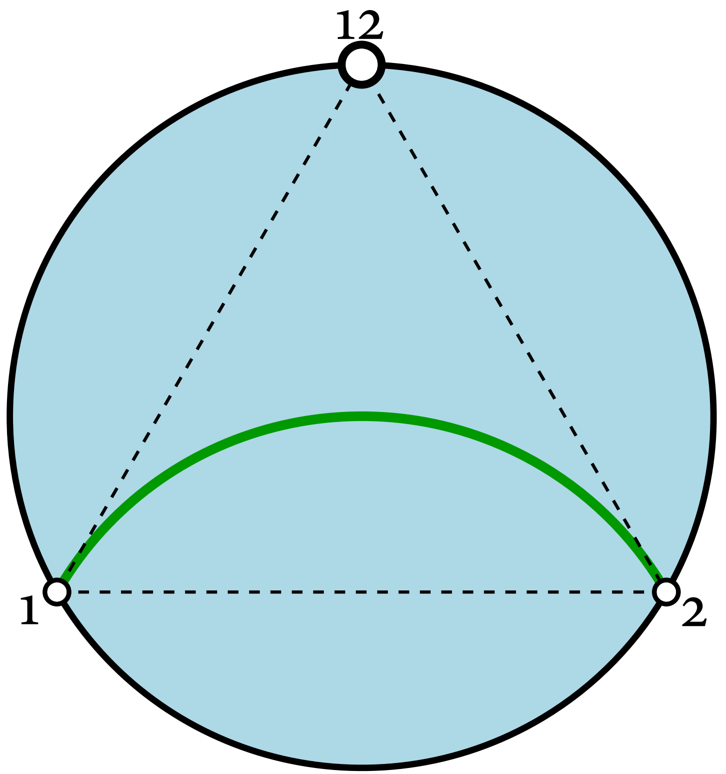
canonical order



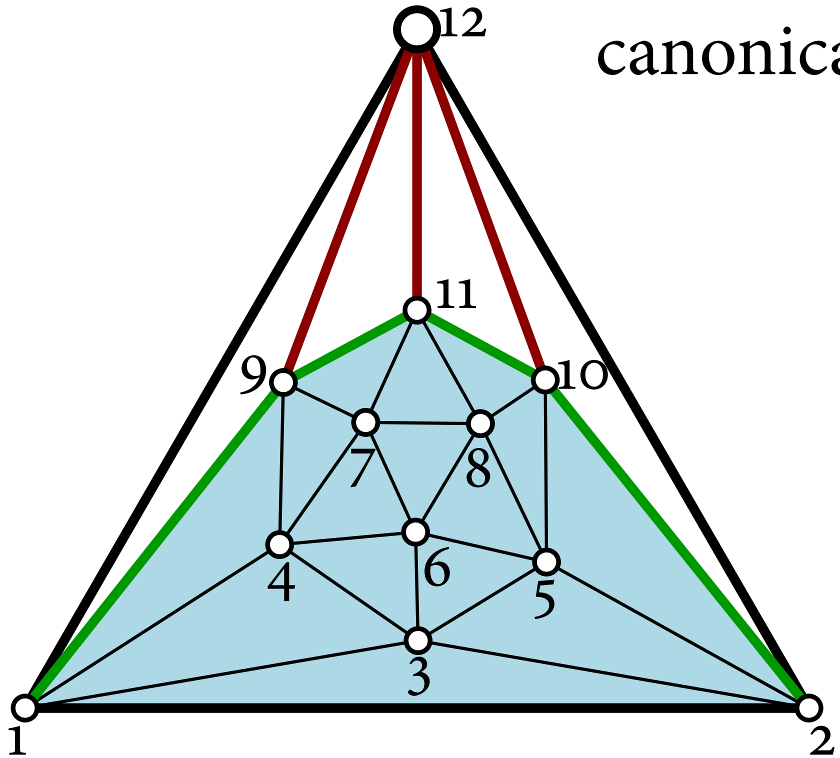
Triangulations with circular arcs



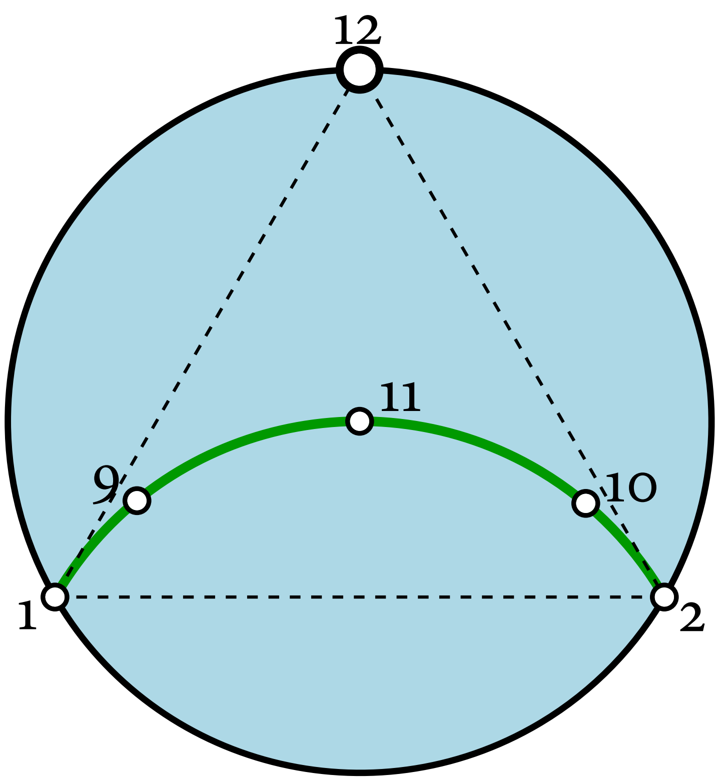
canonical order



Triangulations with circular arcs

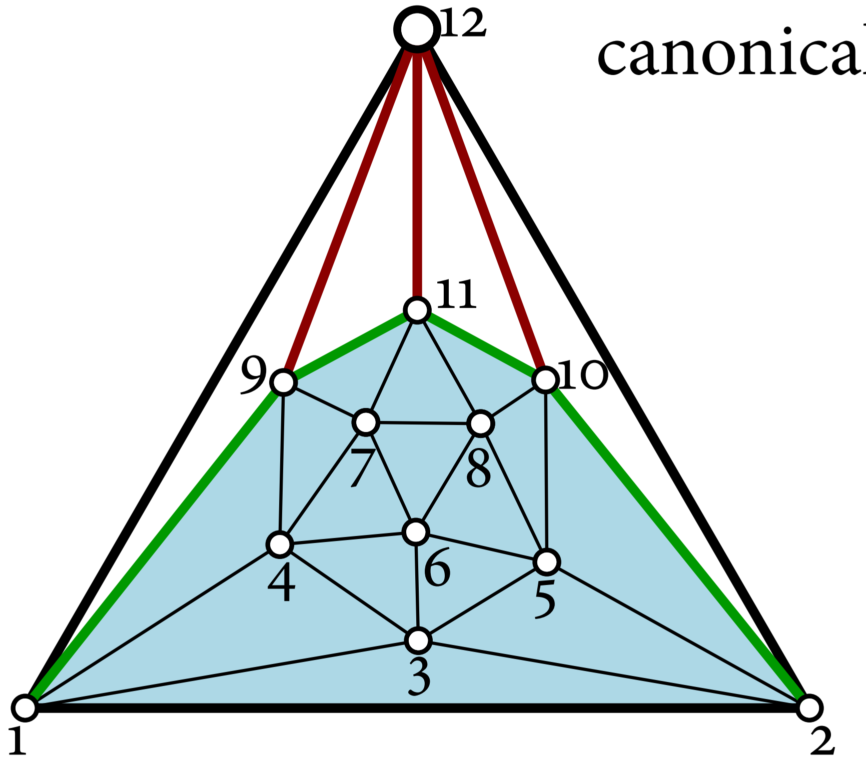


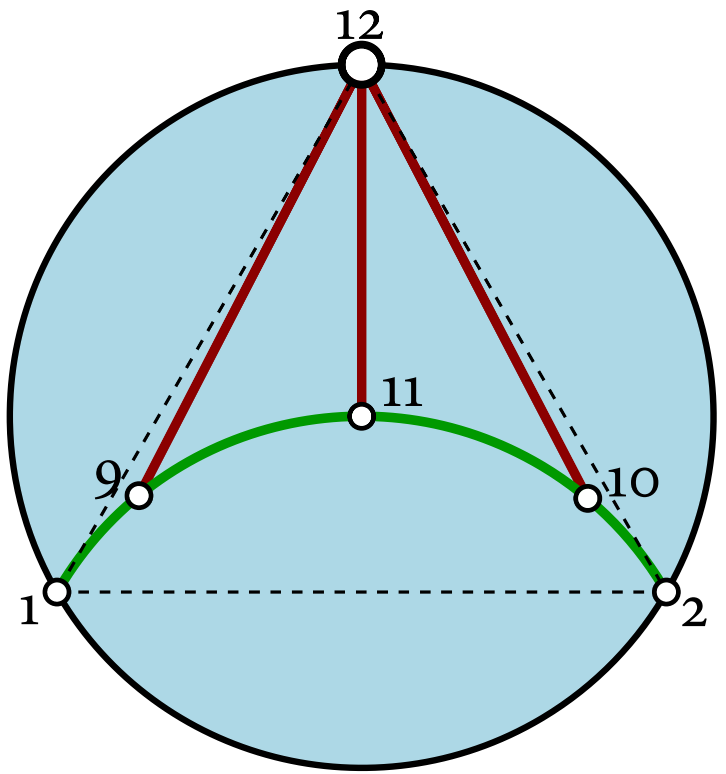
canonical order



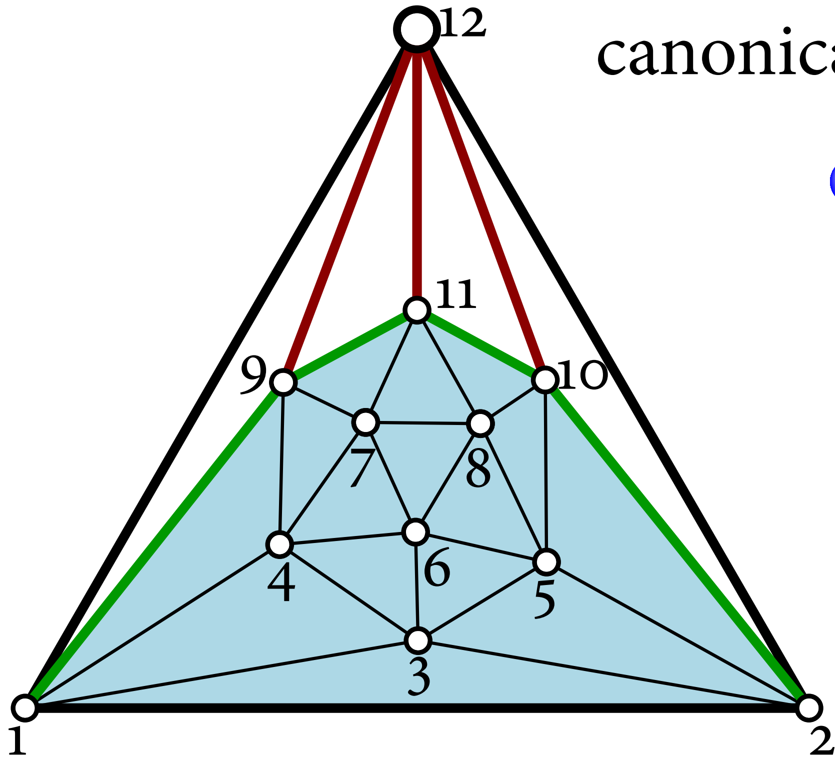
Triangulations with circular arcs

canonical order



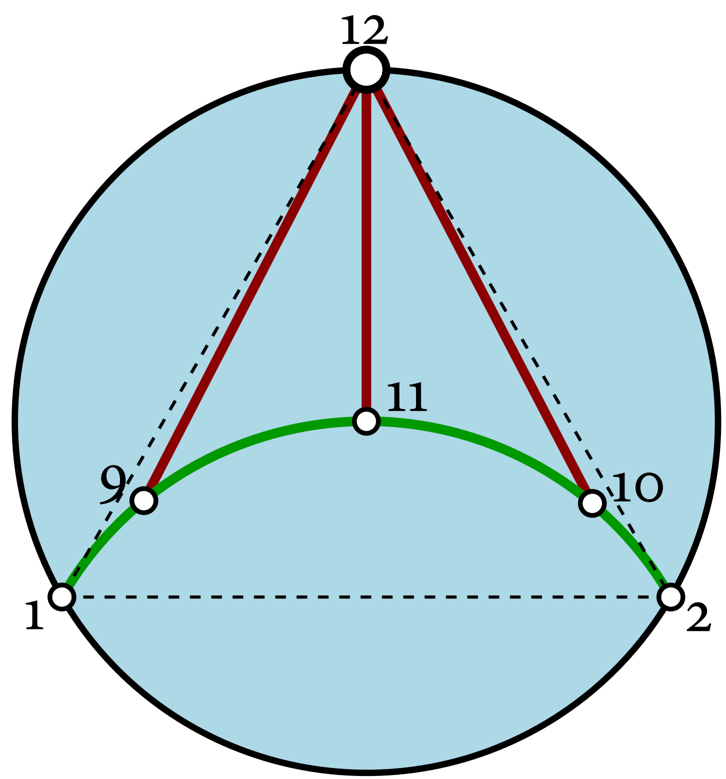


Triangulations with circular arcs

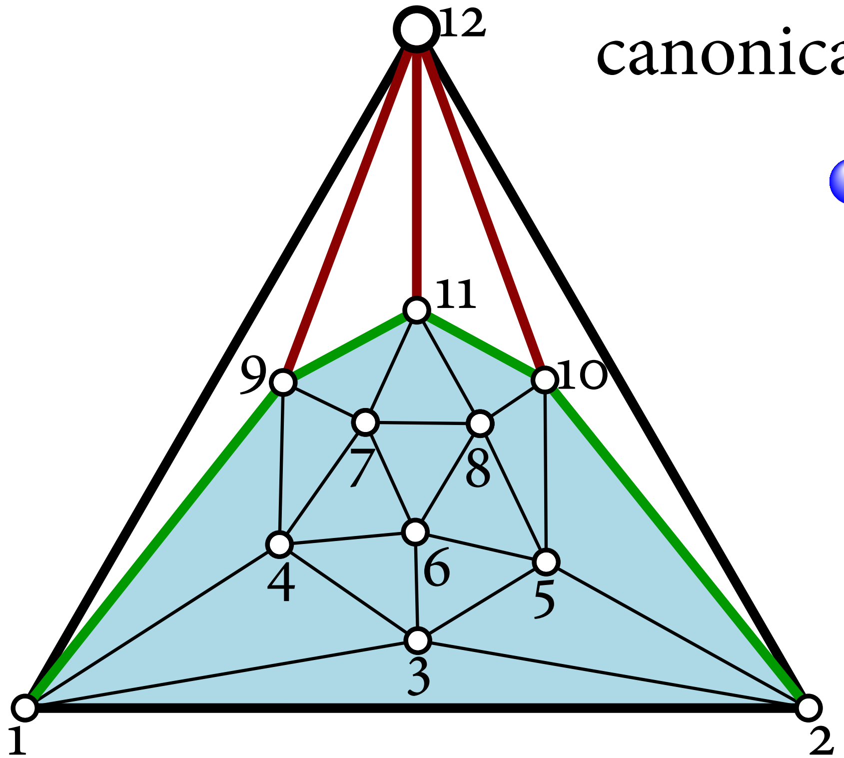


canonical order

- red edges extend into bottom arc

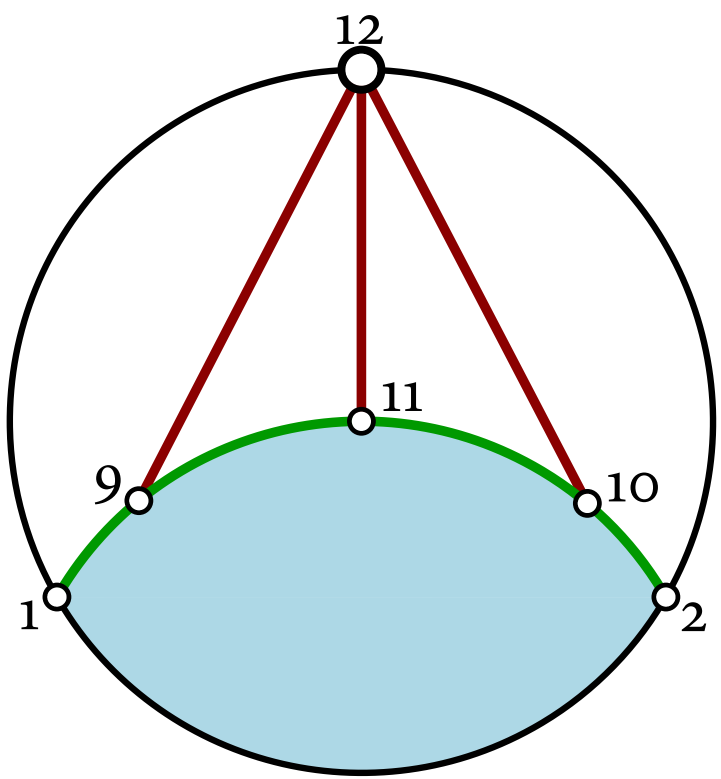


Triangulations with circular arcs

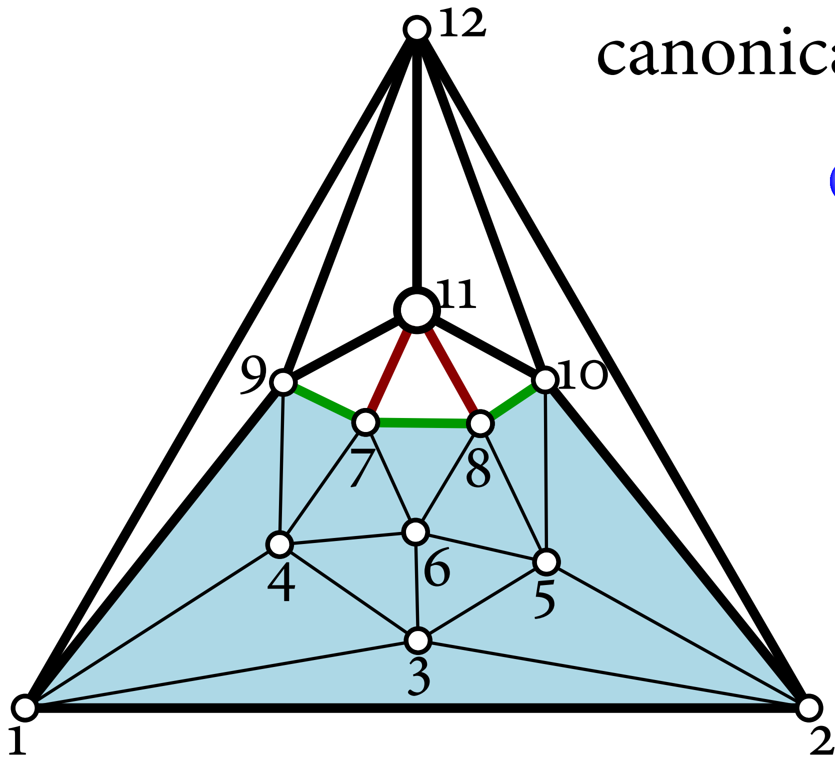


canonical order

- red edges extend into bottom arc

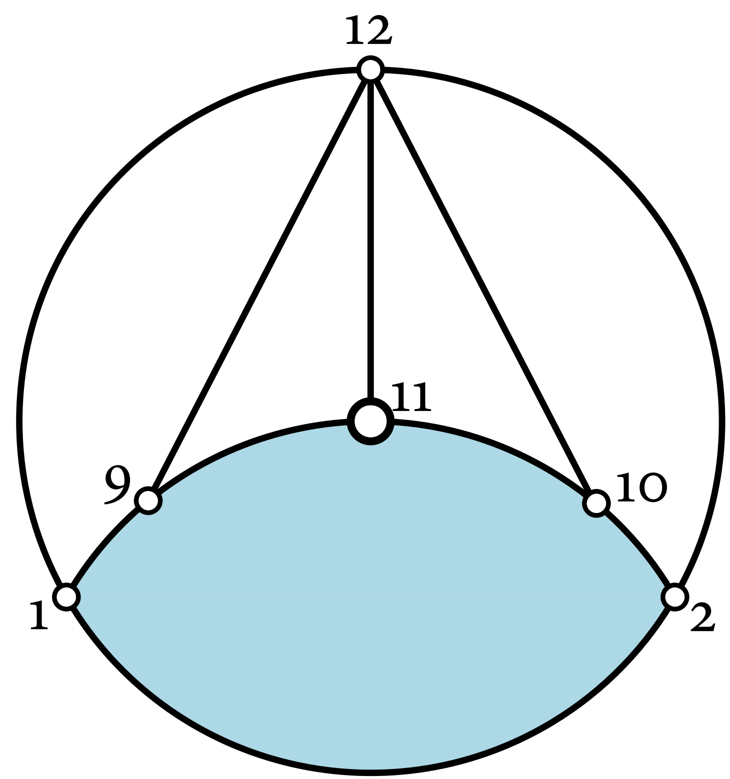


Triangulations with circular arcs

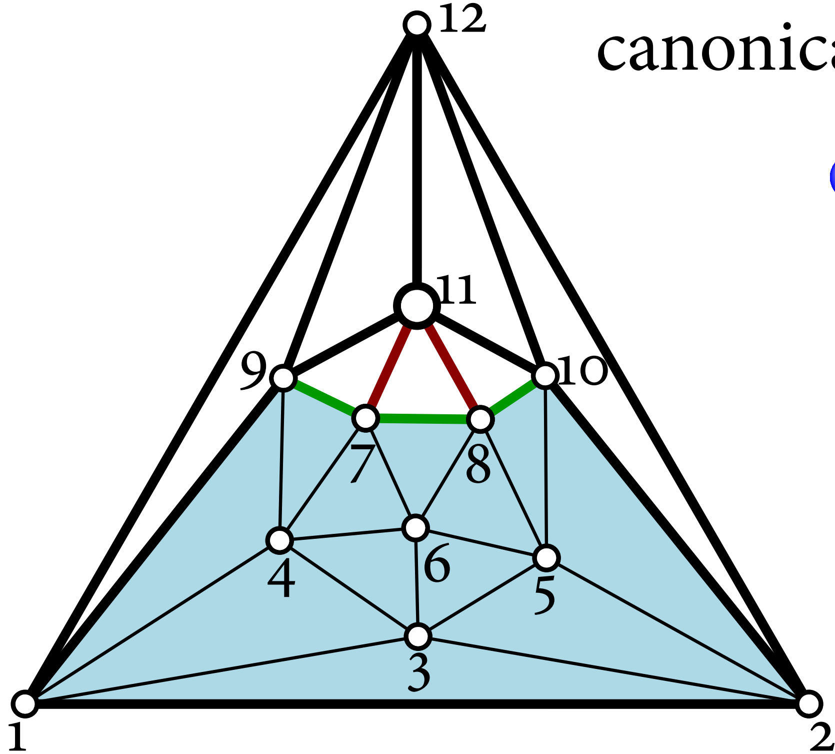


canonical order

- red edges extend into bottom arc

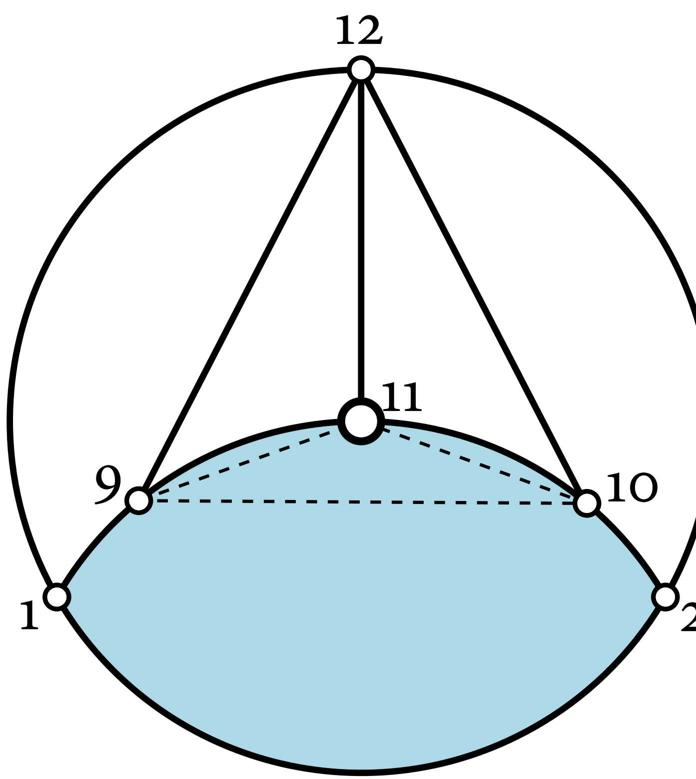


Triangulations with circular arcs

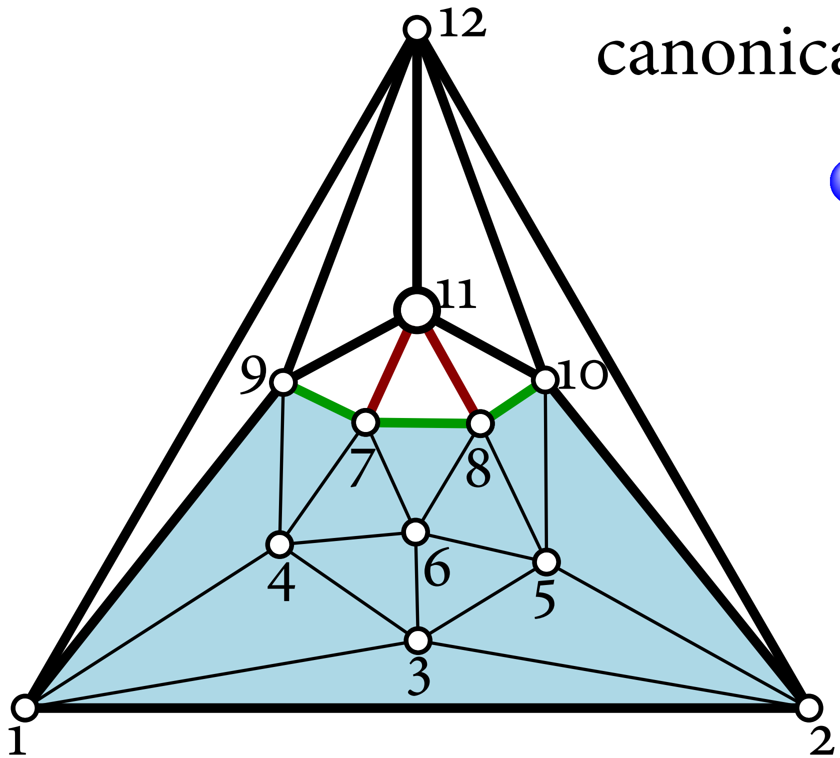


canonical order

- red edges extend into bottom arc

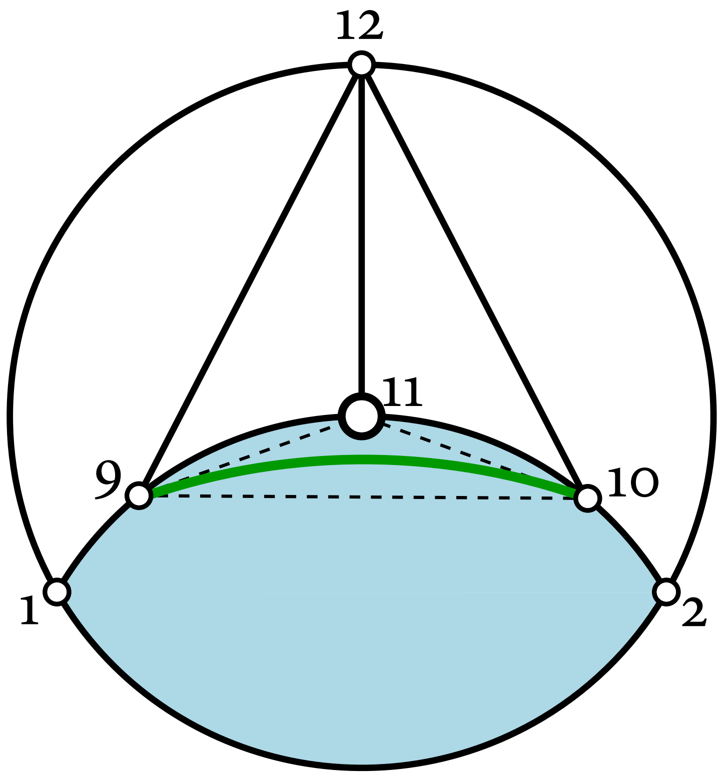


Triangulations with circular arcs

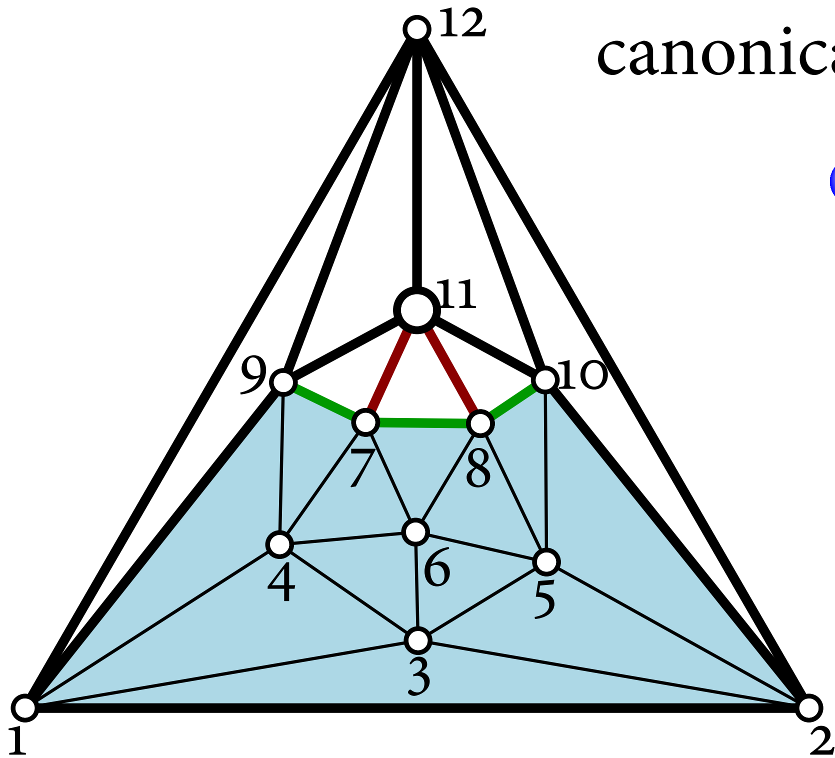


canonical order

- red edges extend into bottom arc

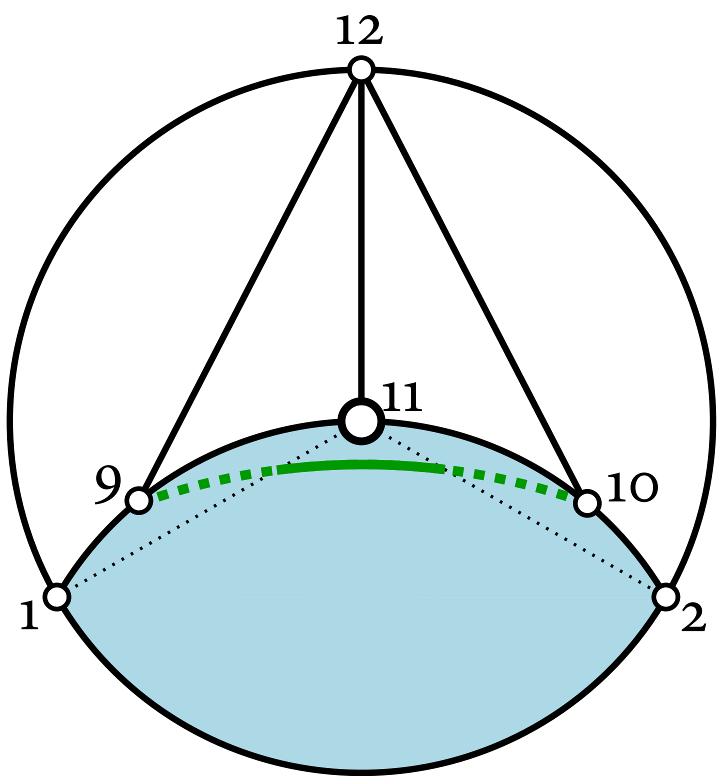


Triangulations with circular arcs

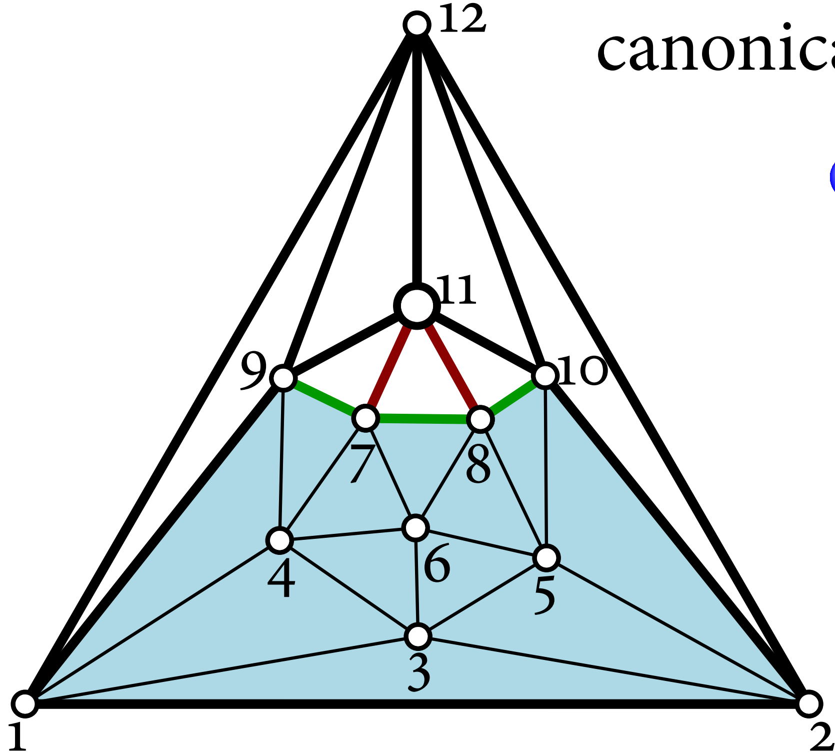


canonical order

- red edges extend into bottom arc

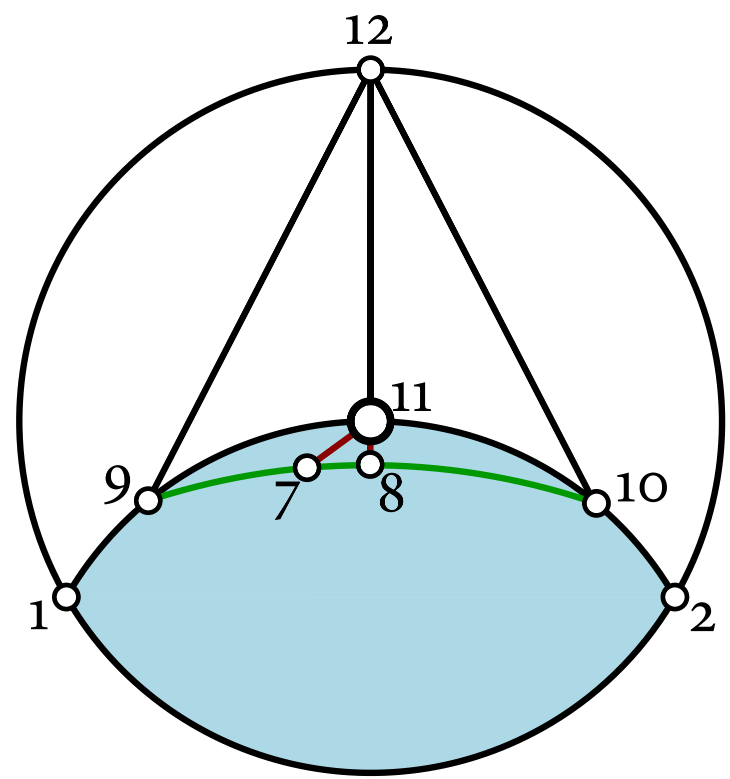


Triangulations with circular arcs

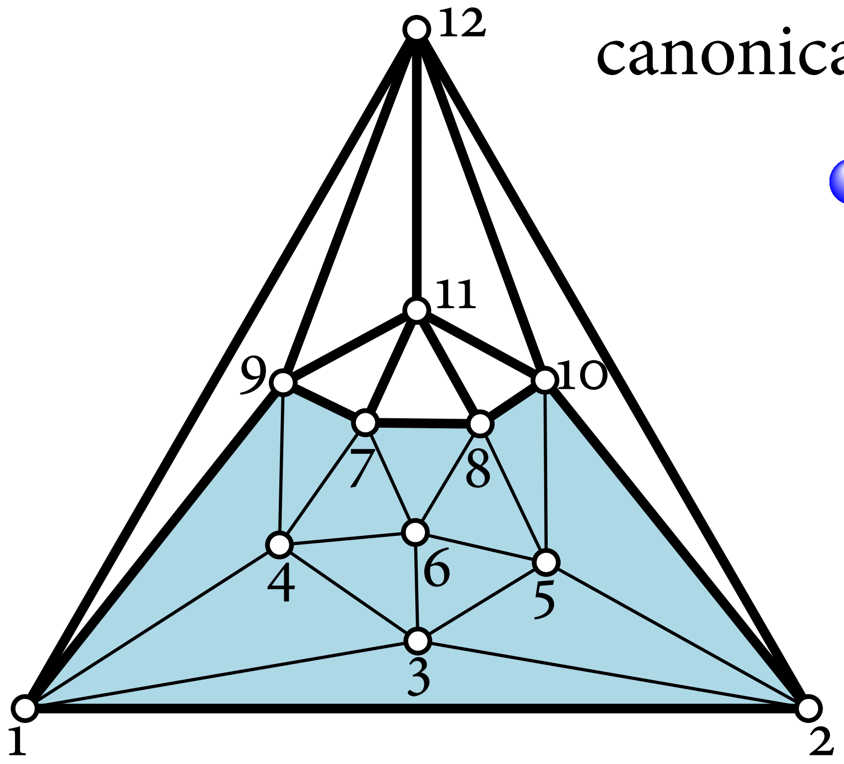


canonical order

- red edges extend into bottom arc

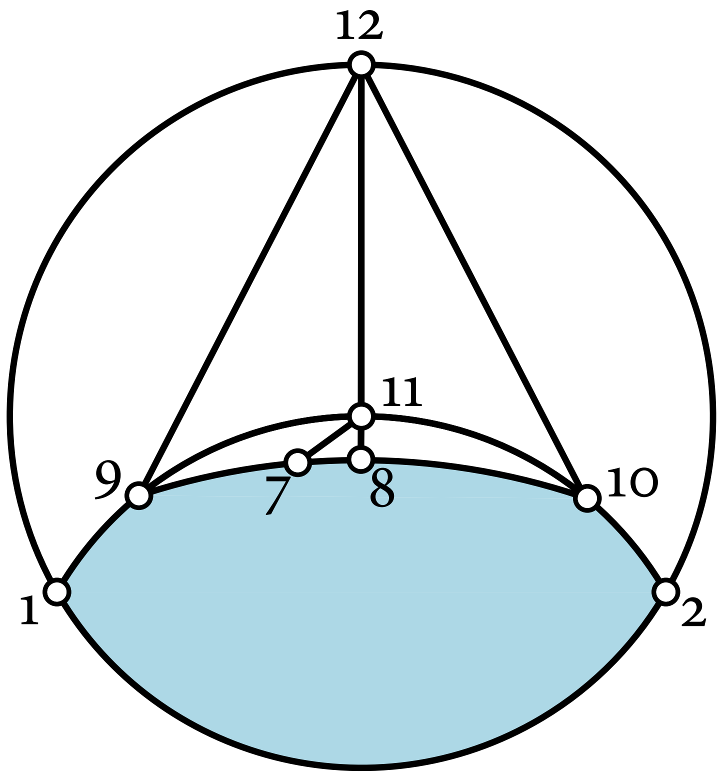


Triangulations with circular arcs

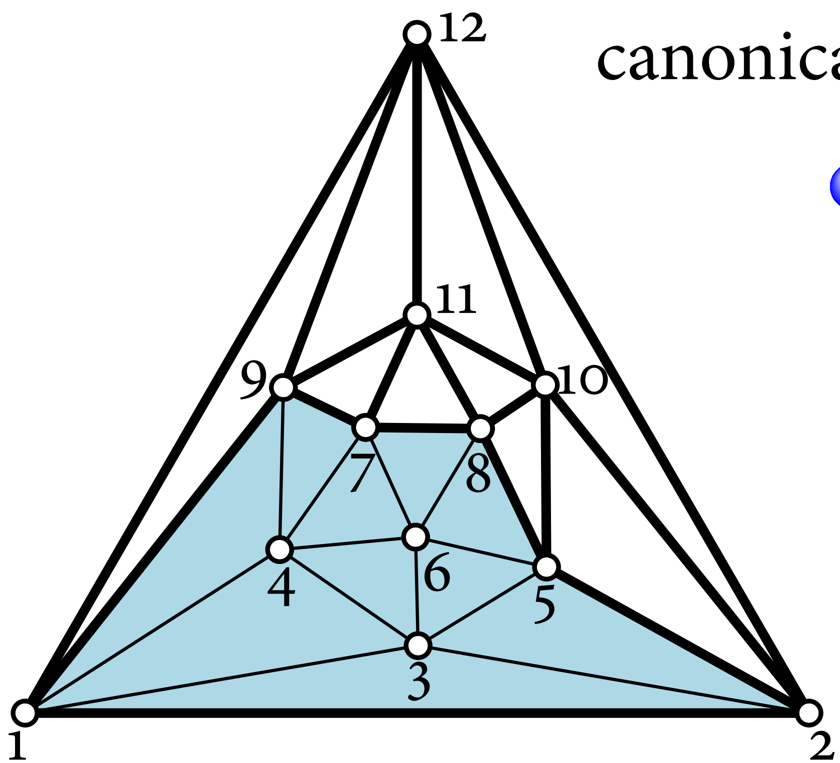


canonical order

- red edges extend into bottom arc

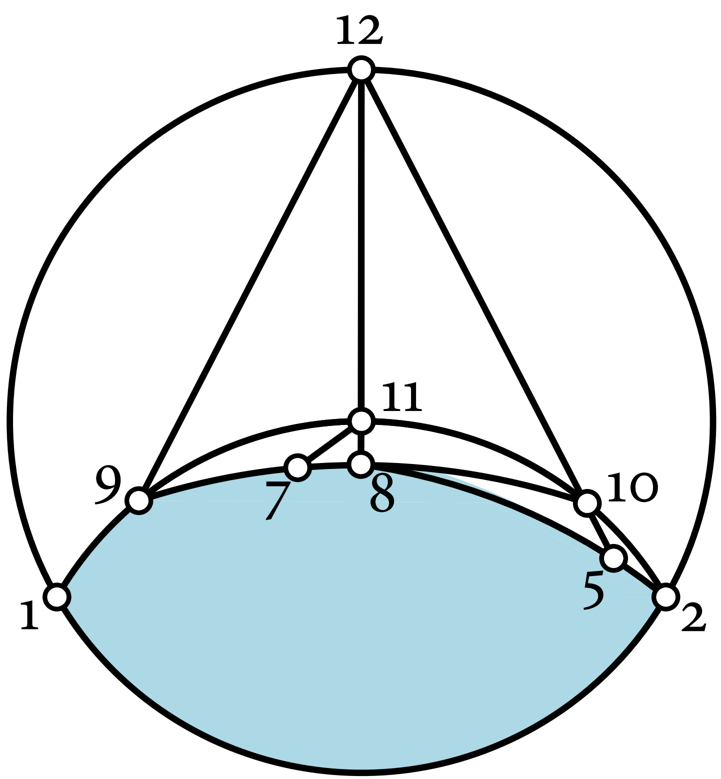


Triangulations with circular arcs

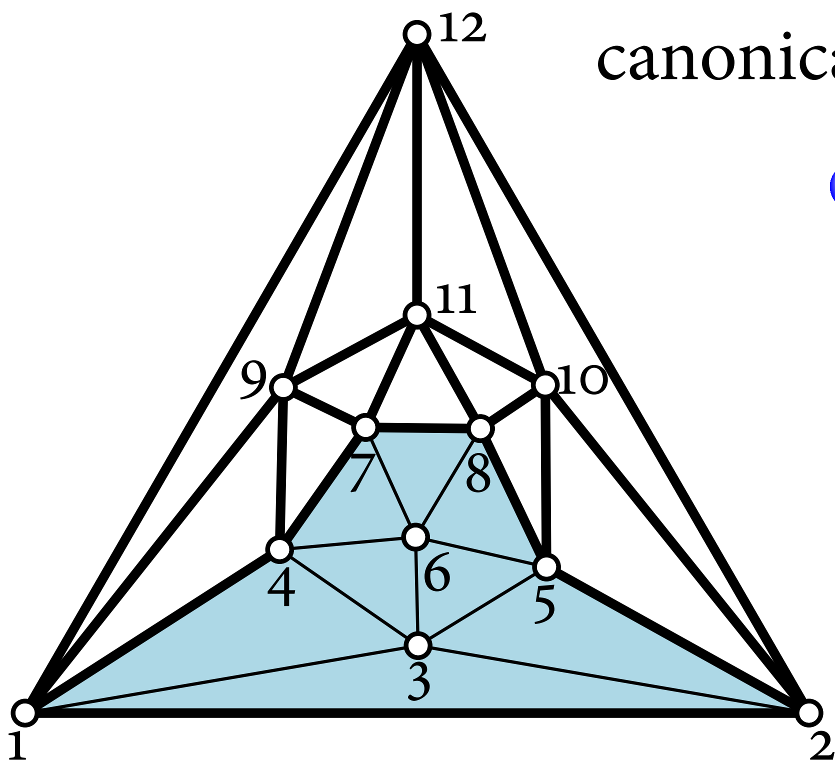


canonical order

- red edges extend into bottom arc

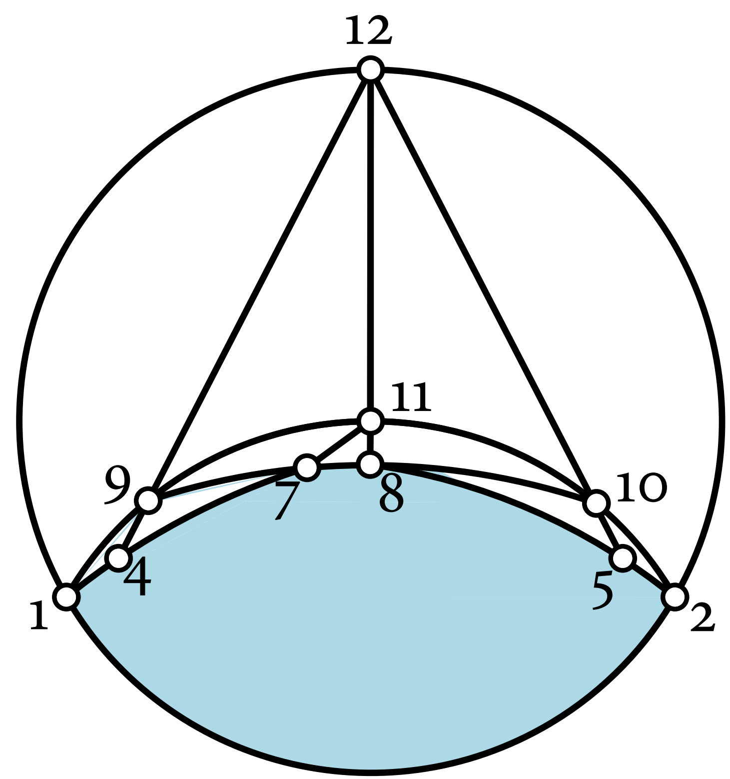


Triangulations with circular arcs

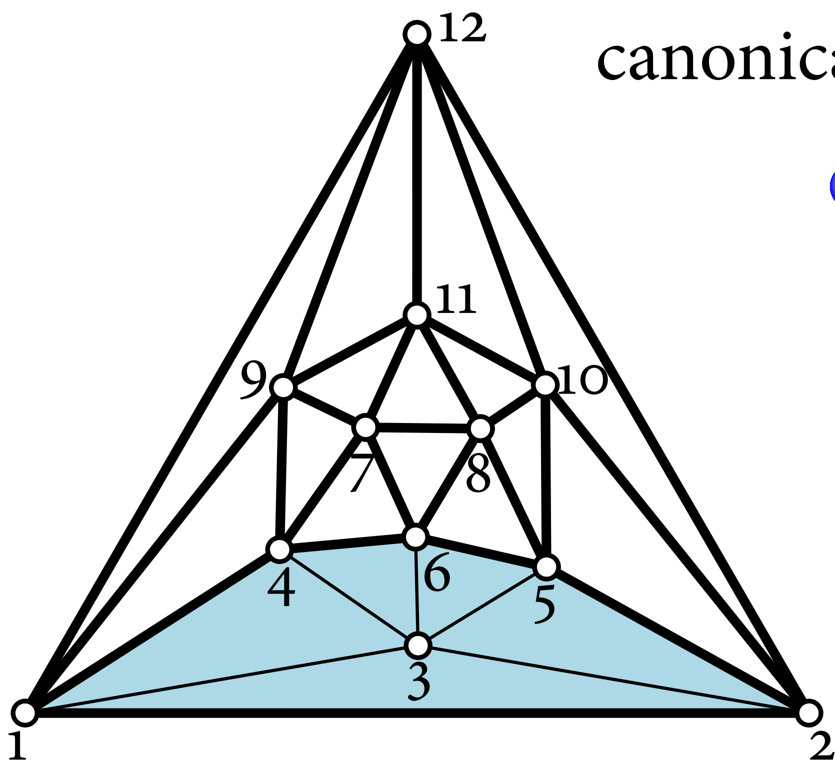


canonical order

- red edges extend into bottom arc

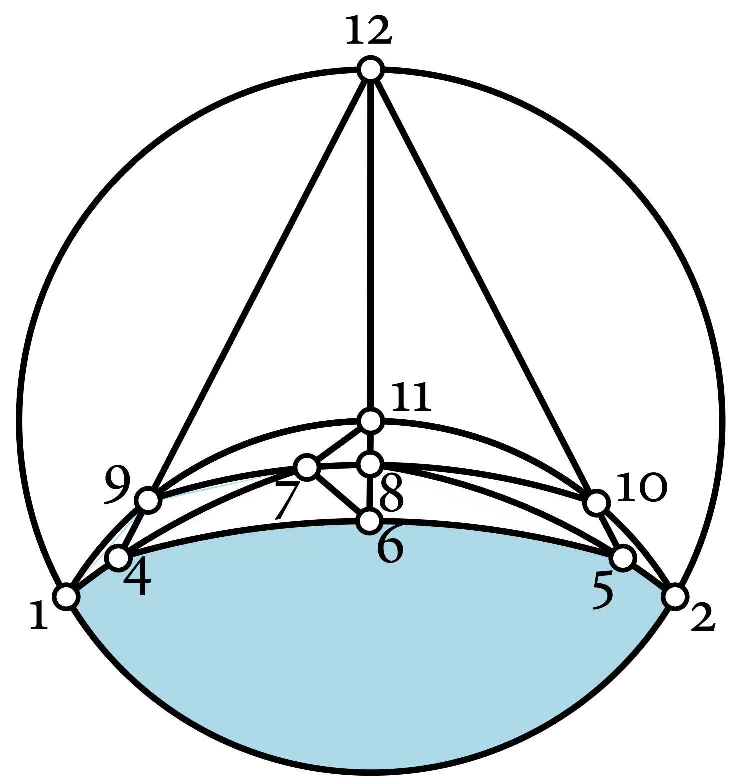


Triangulations with circular arcs

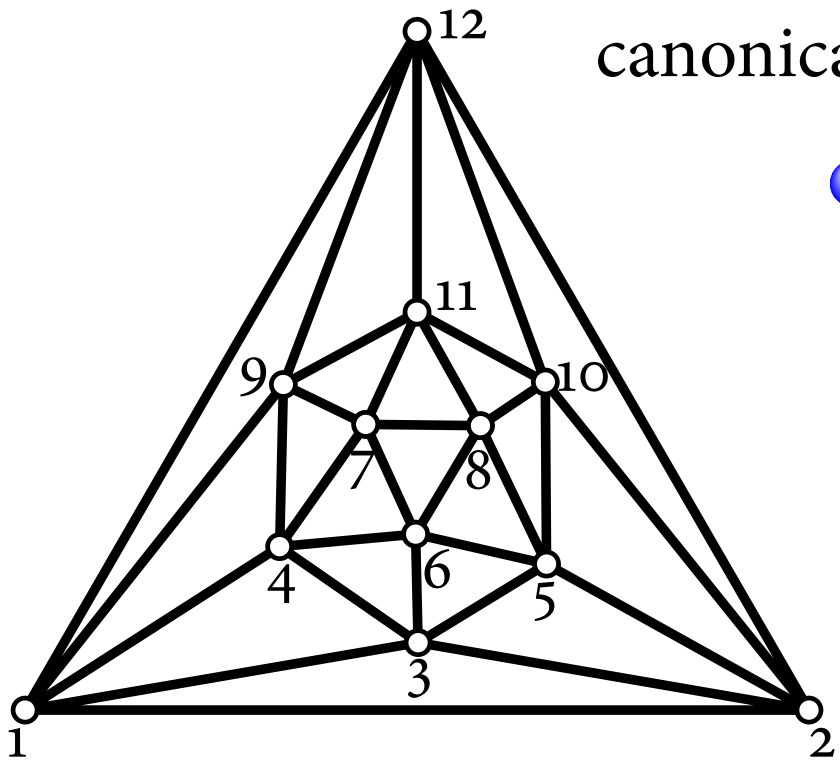


canonical order

- red edges extend into bottom arc

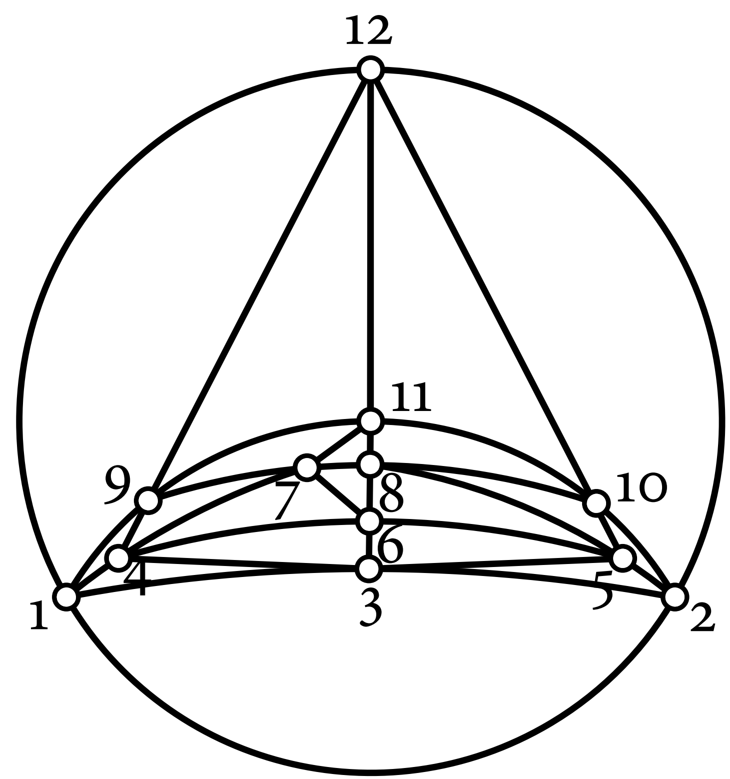


Triangulations with circular arcs

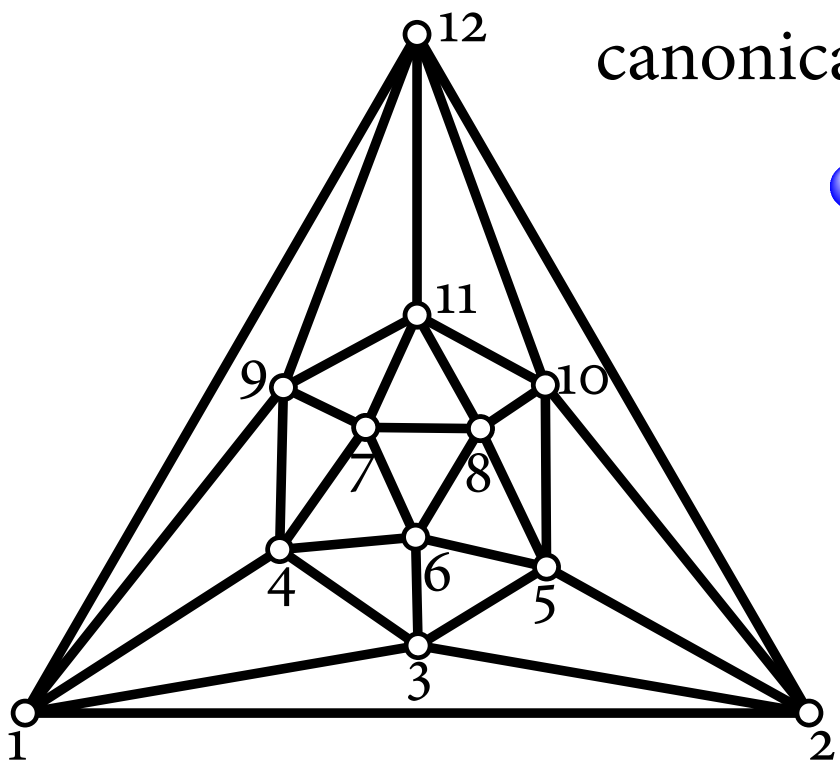


canonical order

- red edges extend into bottom arc

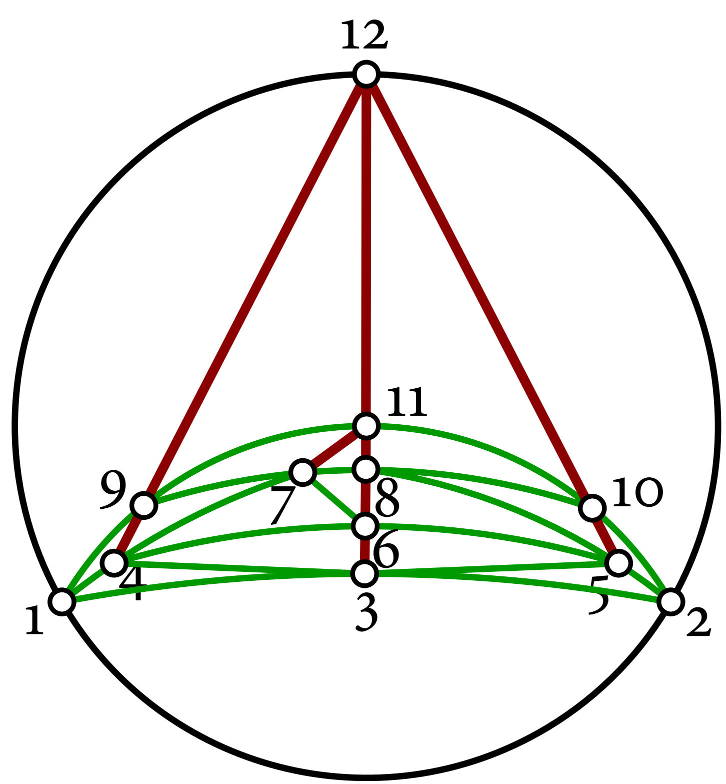


Triangulations with circular arcs



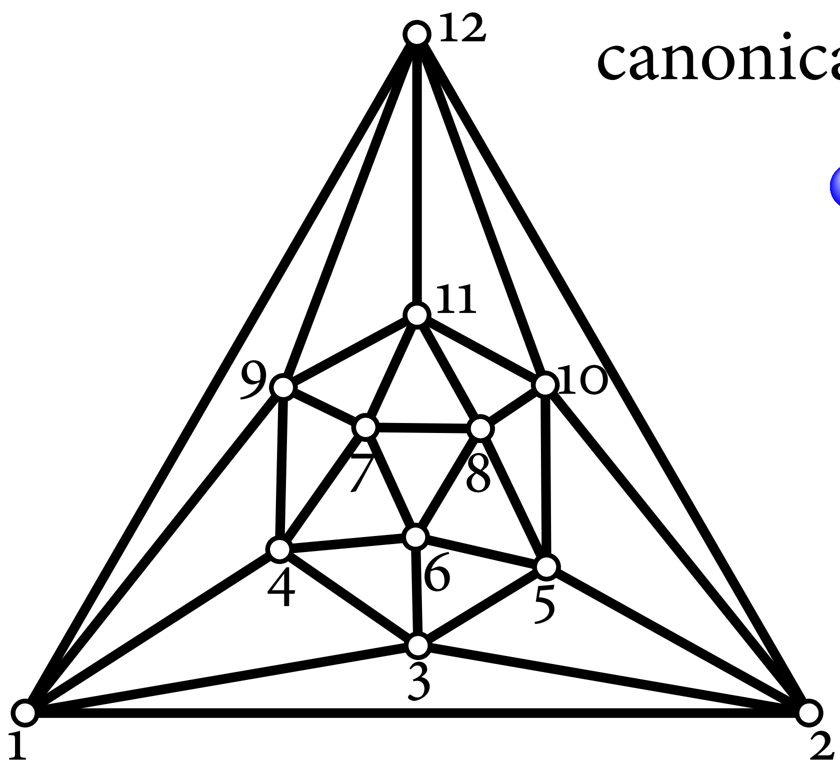
canonical order

- red edges extend into bottom arc



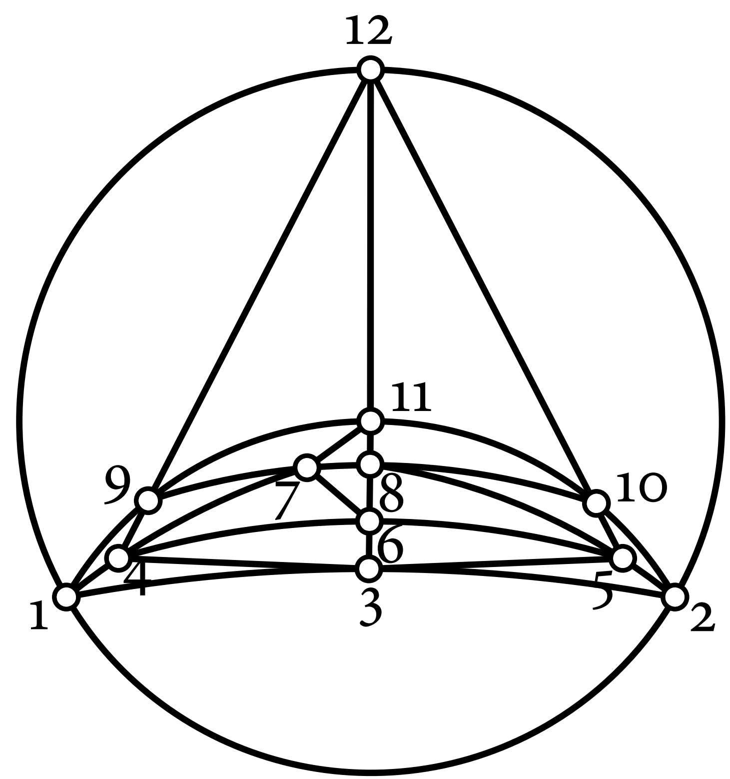
Triangulation → $(5n - 11)/3$ arcs

Planar graphs with circular arcs



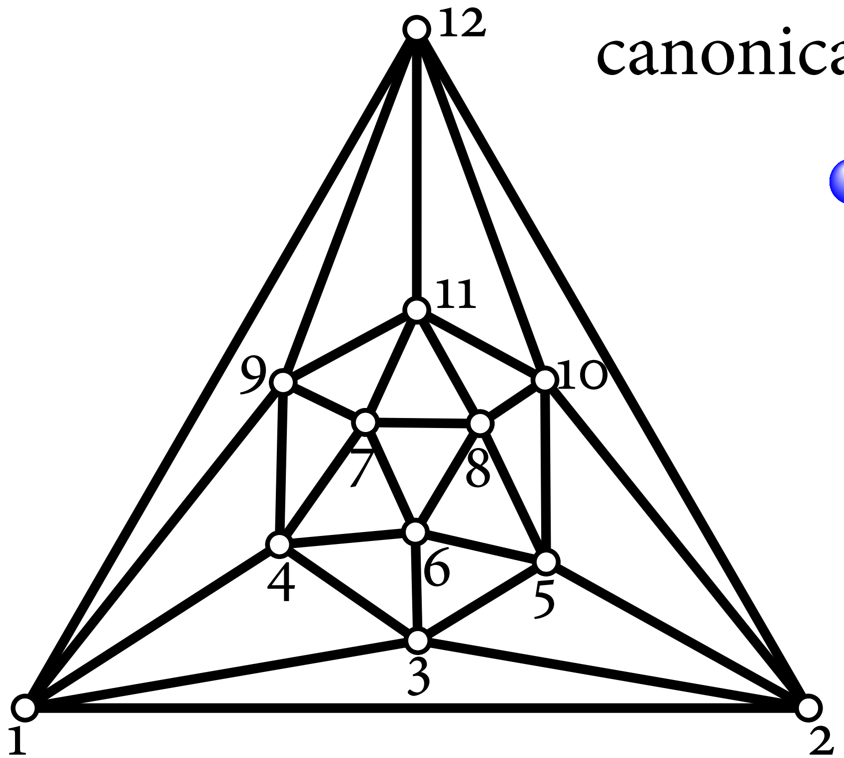
canonical order

- red edges extend into bottom arc



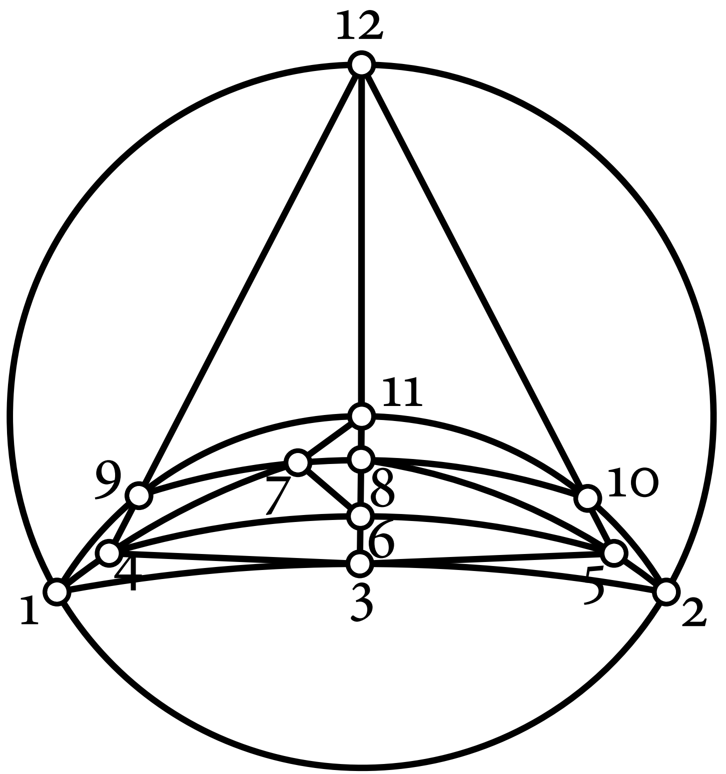
Triangulation → $(5n - 11)/3$ arcs

Planar graphs with circular arcs



canonical order

- red edges extend into bottom arc



Planar → $(14n - 29)/3 - e$ arcs

Triangulation → $(5n - 11)/3$ arcs

Results

Grid

Class	Segments		Circular Arcs	
	Lower	Upper	Lower	Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1] $3e/4$	$\vartheta/2$ [1]	$\vartheta/2$ [1] $3e/4$ [2]
3-trees	$2n$ [1]	$2n$ [1]	$e/6$ [2]	$11e/8$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]	$e/6$ [2]	$2e/3$ [2]
cubic 3-conn.	$n/2$ [4]	$n/2$ [3]	$n/2$ [4]	$n/2$ [3]
Triangulation	$2n$ [5]	$7n/3$ [5]		$5n/3$
Planar	$2n$ [5]	$16n/3 - e$ [5]		$14n/3 - e$

[1] Durocher et al. 2003
[2] Schulz 2015
[3] Igamberdiev et al. 2015
[4] Mondal et al. 2013
[5] Durocher & Mondal 2014

Results

Grid

Class	Segments Lower	Segments Upper	Circular Arcs Lower	Circular Arcs Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1] $3e/4$	$\vartheta/2$ [1]	$\vartheta/2$ [1] $3e/4$ [2]
3-trees	$2n$ [1]	$2n$ [1]	$e/6$ [2]	$11e/8$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]	$e/6$ [2]	$2e/3$ [2]
cubic 3-conn.	$n/2$ [4]	$n/2$ [3] ?	$n/2$ [4]	$n/2$ [3]
Triangulation	$2n$ [5]	$7n/3$ [5]		$5n/3$
Planar	$2n$ [5]	$16n/3 - e$		$14n/3 - e$

[1] Durocher et al. 2003
[2] Schulz 2015
[3] Igamberdiev et al. 2015
[4] Mondal et al. 2013
[5] Durocher & Mondal 2014

Results

Grid

Class	Segments: Lower	Segments: Upper		Circular Arcs: Lower	Circular Arcs: Upper
Tree	$9/2$ [1]	$9/2$ [1]	$3e/4$	$9/2$ [1]	$9/2$ [1] $3e/4$ [2]
3-trees	$2n$ [1]	$2n$ [1]		$e/6$ [2]	$11e/8$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]		$e/6$ [2]	$2e/3$ [2]
cubic 3-conn.	$n/2$ [4]	$n/2$ [3]	?	$n/2$ [4]	$n/2$ [3]
Triangulation	$2n$ [5]	$7n/3$ [5]		?	$5n/3$
Planar	$2n$ [5]	$16n/3 - e$			$14n/3 - e$

[1] Durocher et al. 2003 [3] Igamberdiev et al. 2015 [5] Durocher & Mondal 2014
[2] Schulz 2015 [4] Mondal et al. 2013

Results

Grid

Class	Segments Lower	Segments Upper		Circular Arcs Lower	Circular Arcs Upper
Tree	$\vartheta/2$ [1]	$\vartheta/2$ [1]	$3e/4$	$\vartheta/2$ [1]	$\vartheta/2$ [1] $3e/4$ [2]
3-trees	$2n$ [1]	$2n$ [1]	?	$e/6$ [2]	$11e/8$ [2]
3-connected	$2n$ [1]	$5n/2$ [1]		$e/6$ [2]	$2e/3$ [2]
cubic 3-conn.	$n/2$ [4]	$n/2$ [3]		$n/2$ [4]	$n/2$ [3]
Triangulation	$2n$ [5]	$7n/3$ [5]		?	$5n/3$
Planar	$2n$ [5]	$16n/3 - e$]			$14n/3 - e$
?					

[1] Durocher et al. 2003
[2] Schulz 2015
[3] Igamberdiev et al. 2015
[4] Mondal et al. 2013
[5] Durocher & Mondal 2014