Minimum Rectilinear Polygons for Given Angle Sequences

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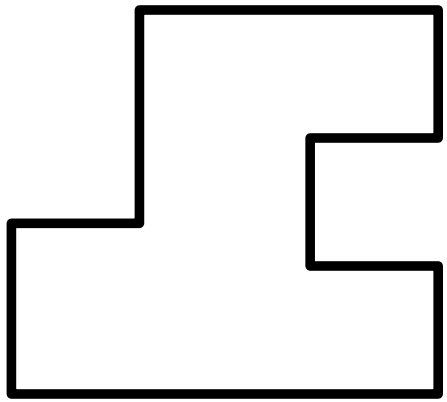
N. Saeedi[1], C.-S. Shin[3], A. Wolff[2]

1. UBC, Canada
2. Würzburg Univ., Germany
3. HUFS, Korea

Problems

Rectilinear polygon P



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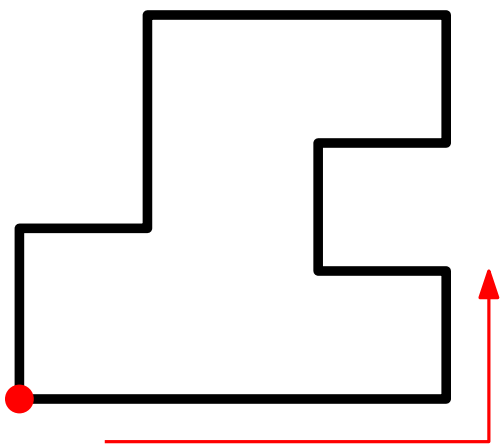

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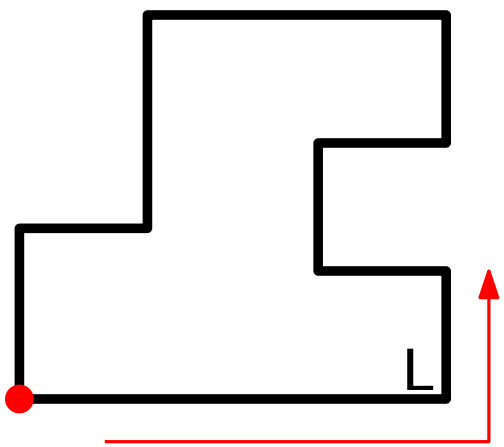

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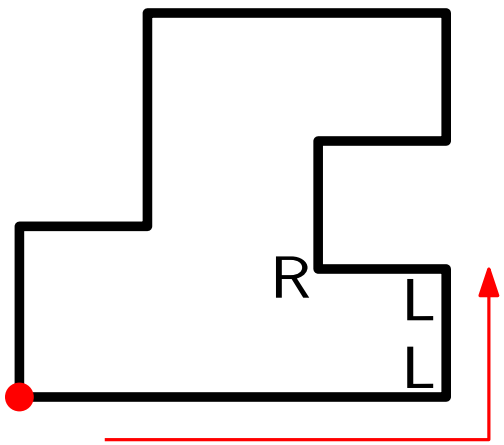

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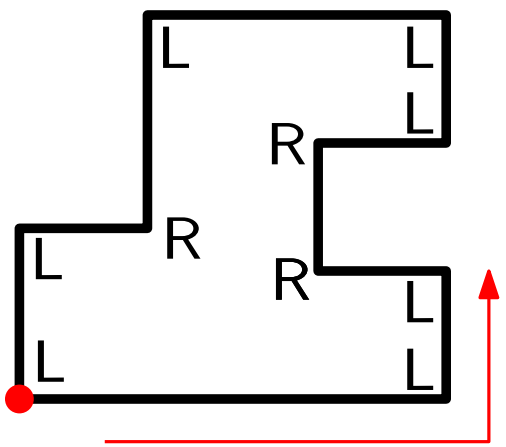
Rectilinear polygon P ⟶ An angle sequence



⟶ LLRRLLLLRLL

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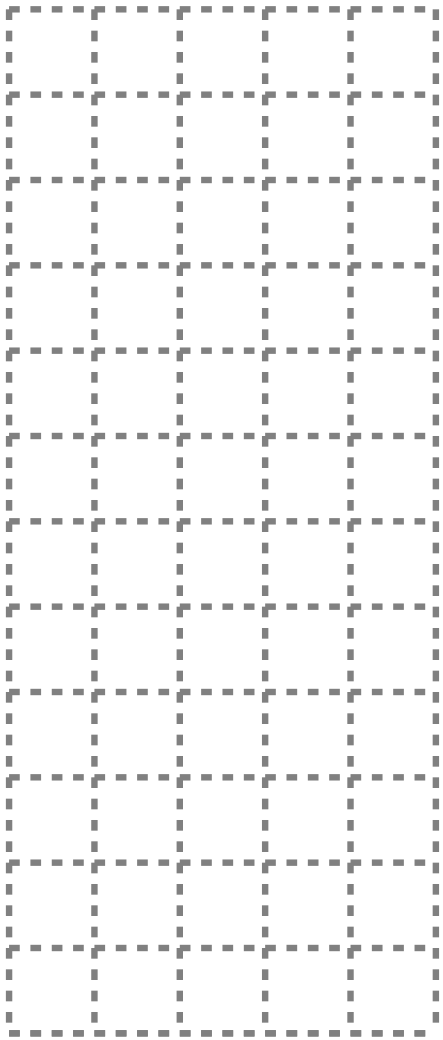

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on integer grid

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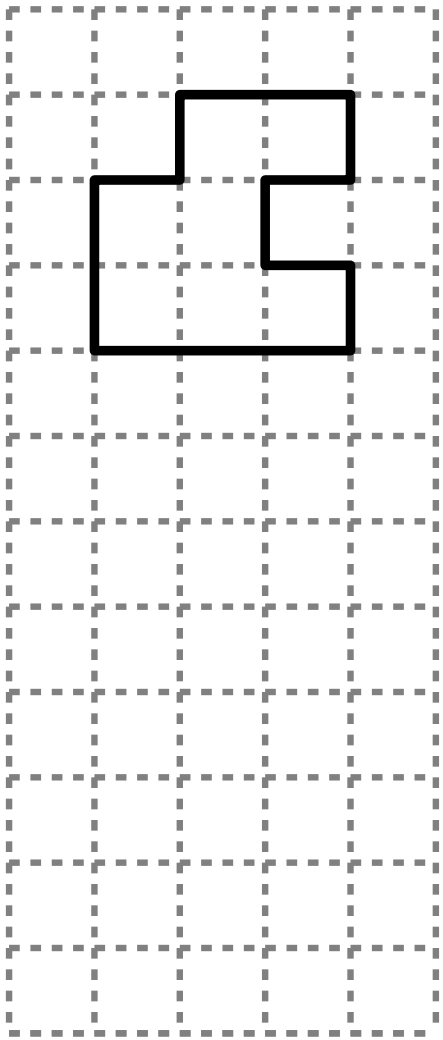

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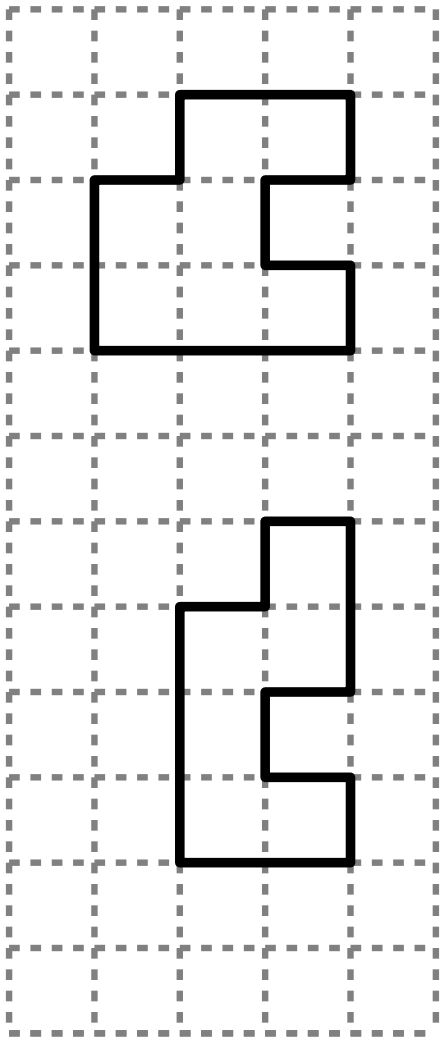

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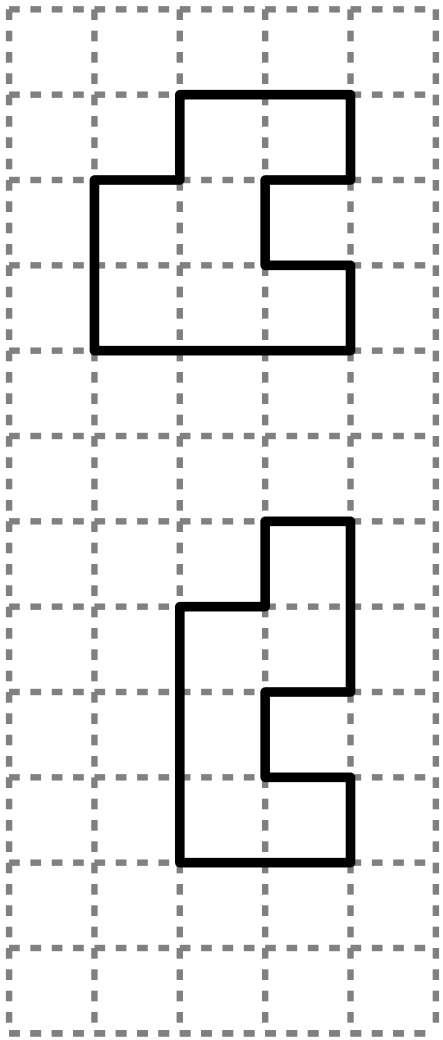

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Realize a given angle sequence as a rectilinear polygpon on the integer grid with ...

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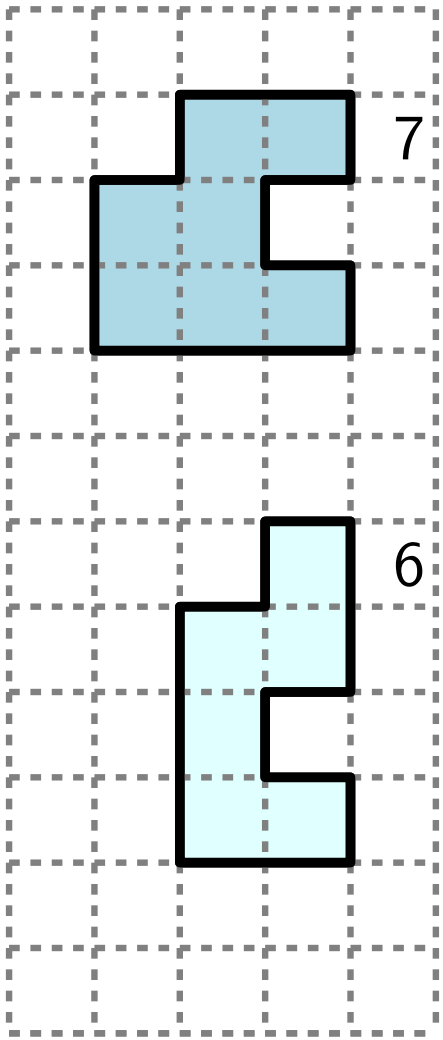

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Realize a given angle sequence as a rectilinear polygpon on the integer grid with minimum area

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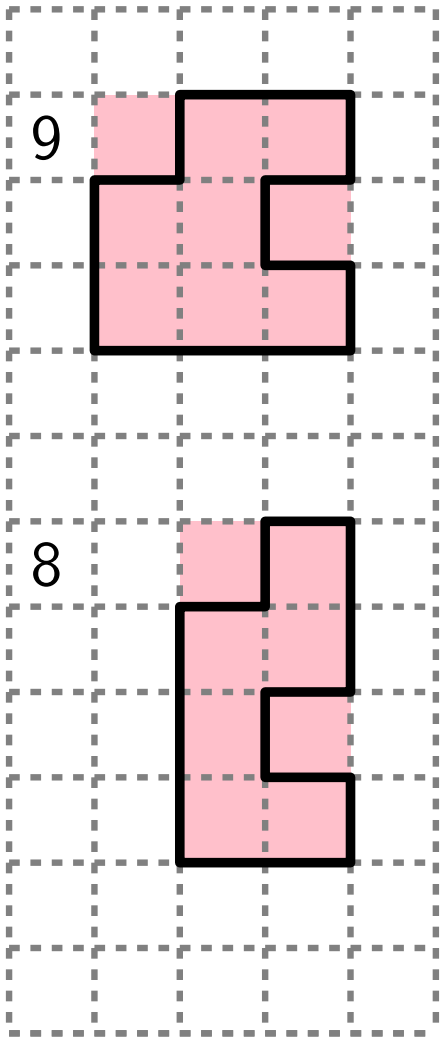

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Realize a given angle sequence as a rectilinear polygpon on the integer grid with minimum bounding box

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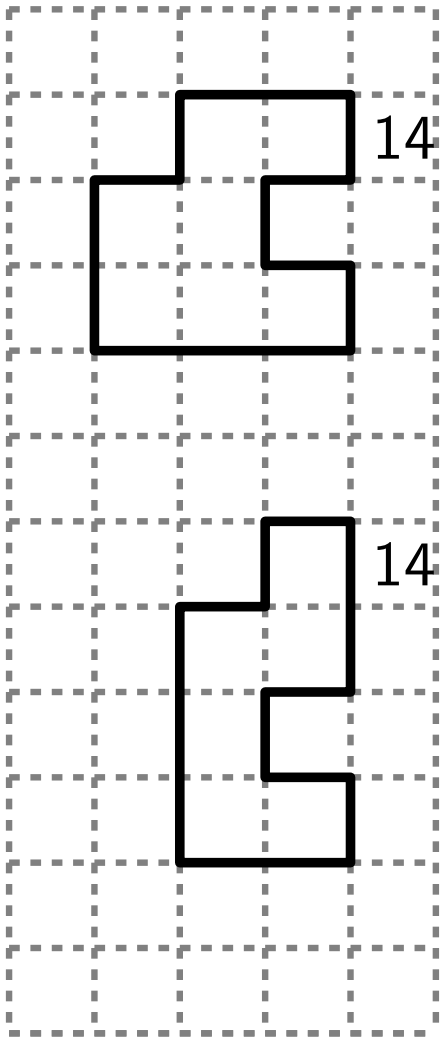

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Realize a given angle sequence as a rectilinear polygpon on the integer grid with minimum perimeter

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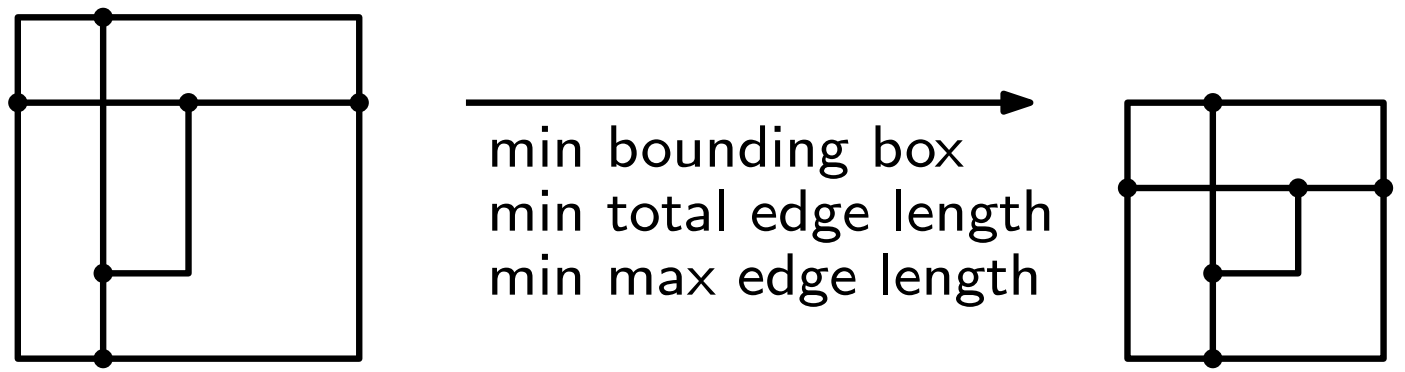

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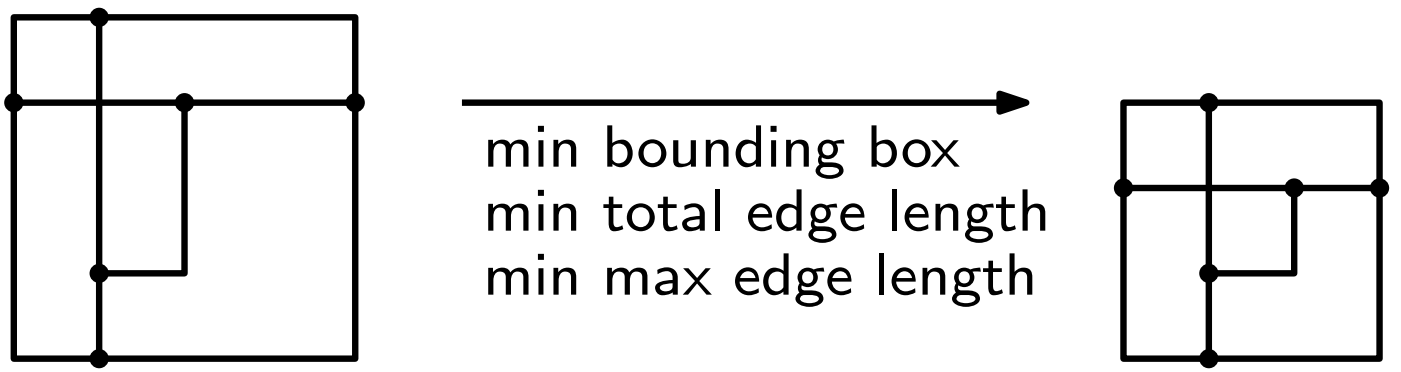

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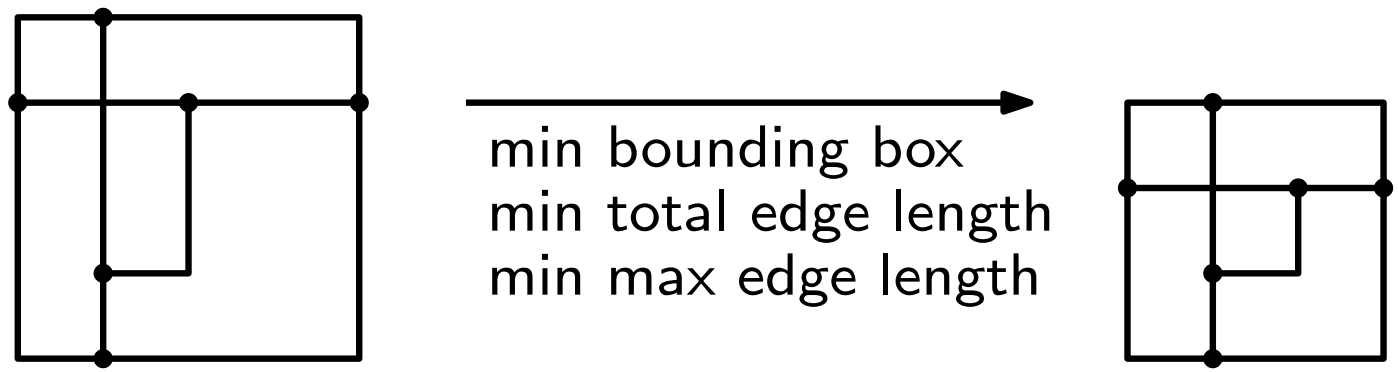

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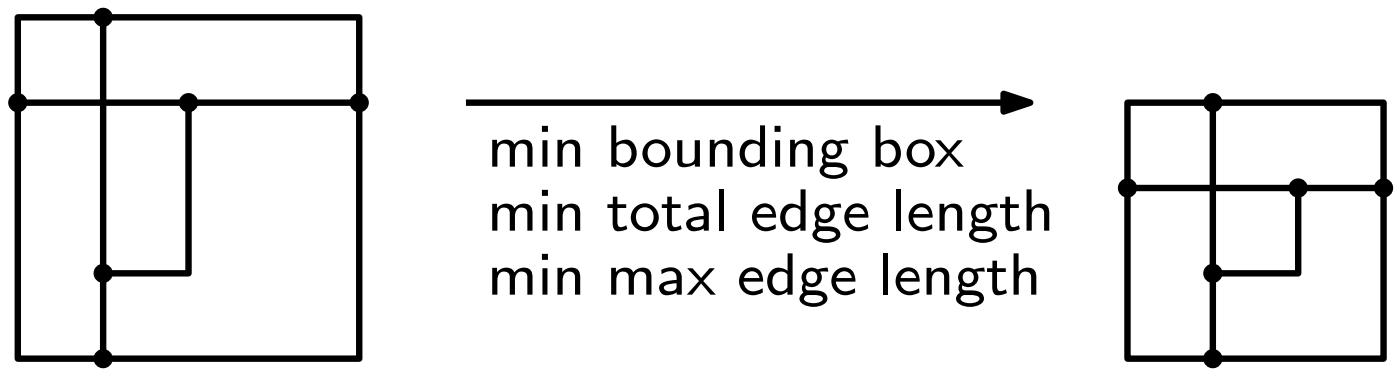

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- No other related results

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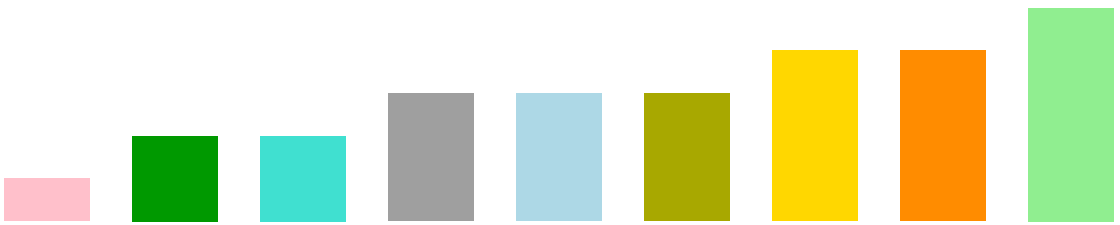
Computing Polygons with Bounding Boxes of Minimum Area

NP-Hardness

We reduce from 3-Partition:

> Given a multiset S of $n = 3m$ integers with $\sum S = mB$, is there a partition of S into m subsets $S_1, \ldots, S_m$ such that $\sum S_i = B$ for each i?



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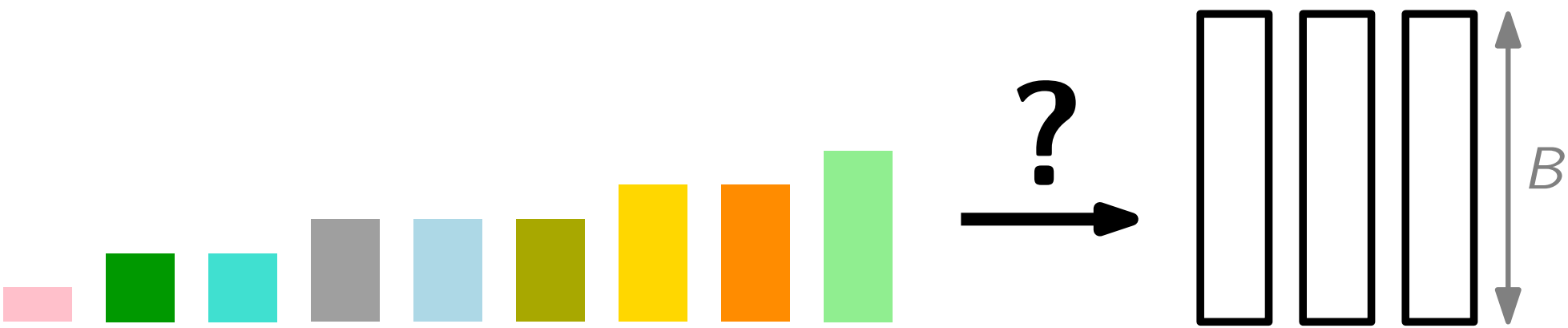

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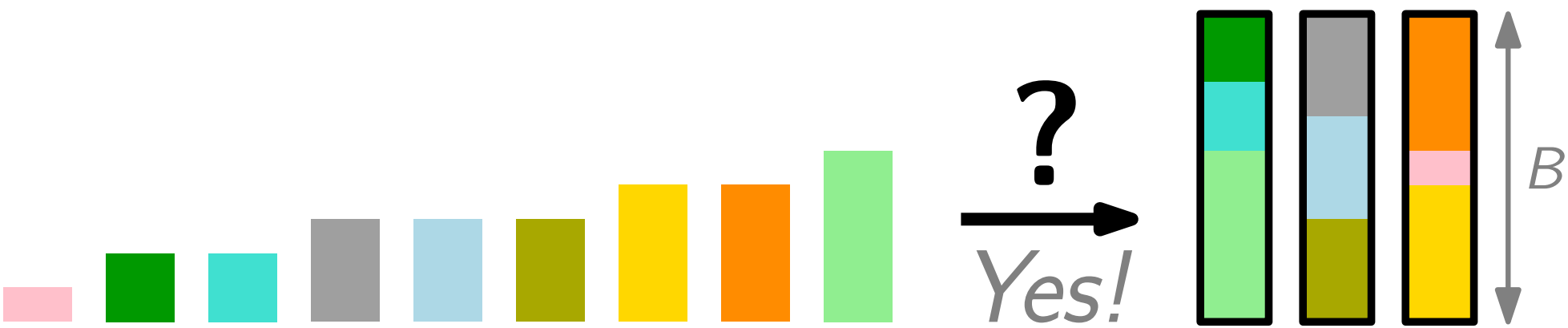

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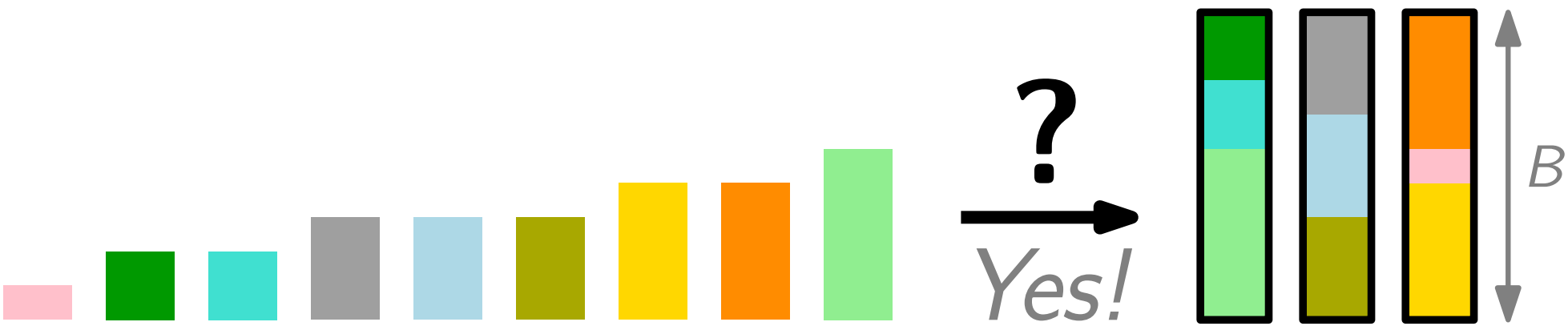

3-Partition is NP-hard even if (i) $B = \text{poly}(m)$
(ii) for each $s \in S$, we have $B/4 < s < B/2$ ($\Rightarrow$ all $|S_i| = 3$).

Reduction (Sketch)

Given an instance $S = (s_1, \ldots, s_{3m})$ of 3-Partition,
compute in poly(m) time $K > 0$ and an instance σ of MinBBLRSequence s.t.

$$\text{area}(\sigma) < K \;\Leftrightarrow\; S \text{ is yes-instance}$$

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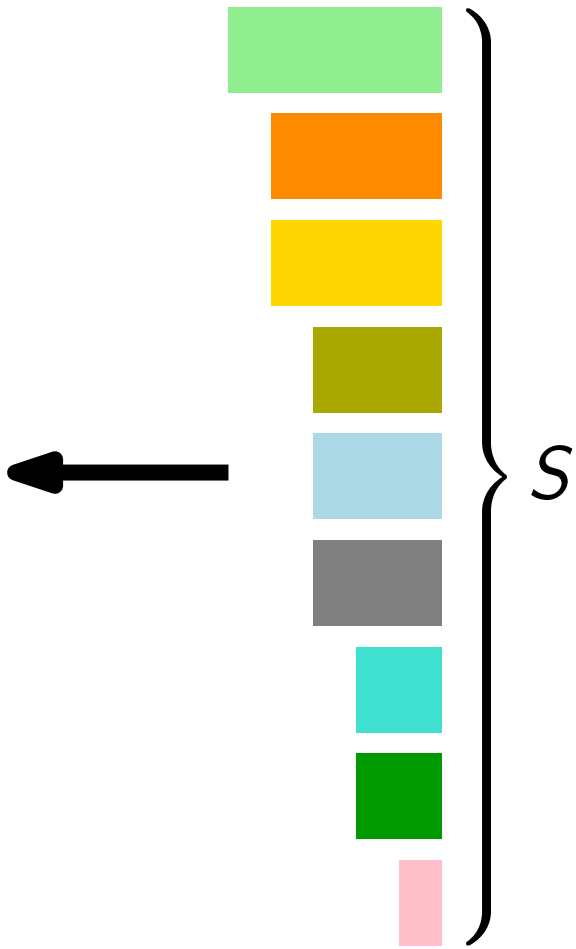

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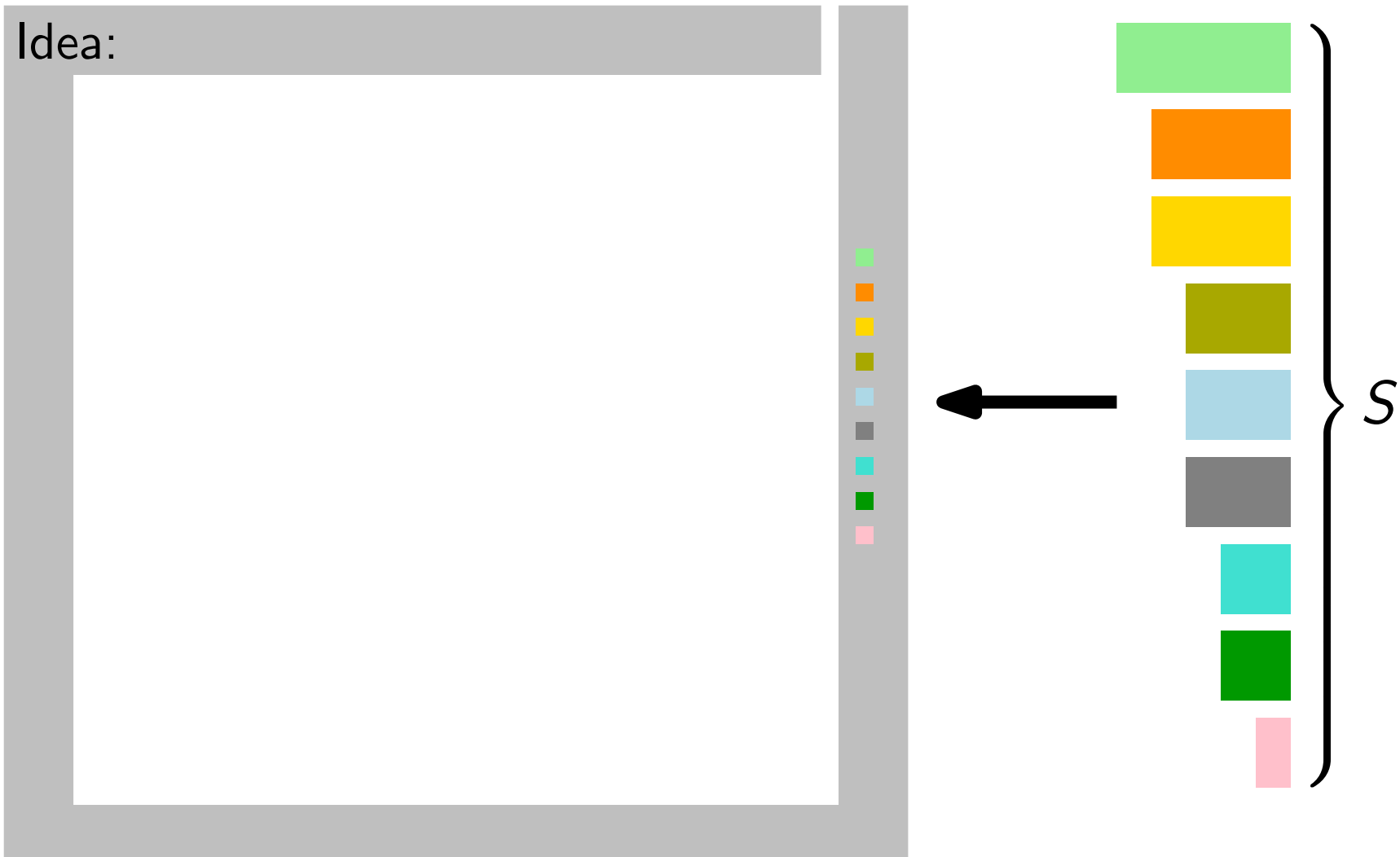

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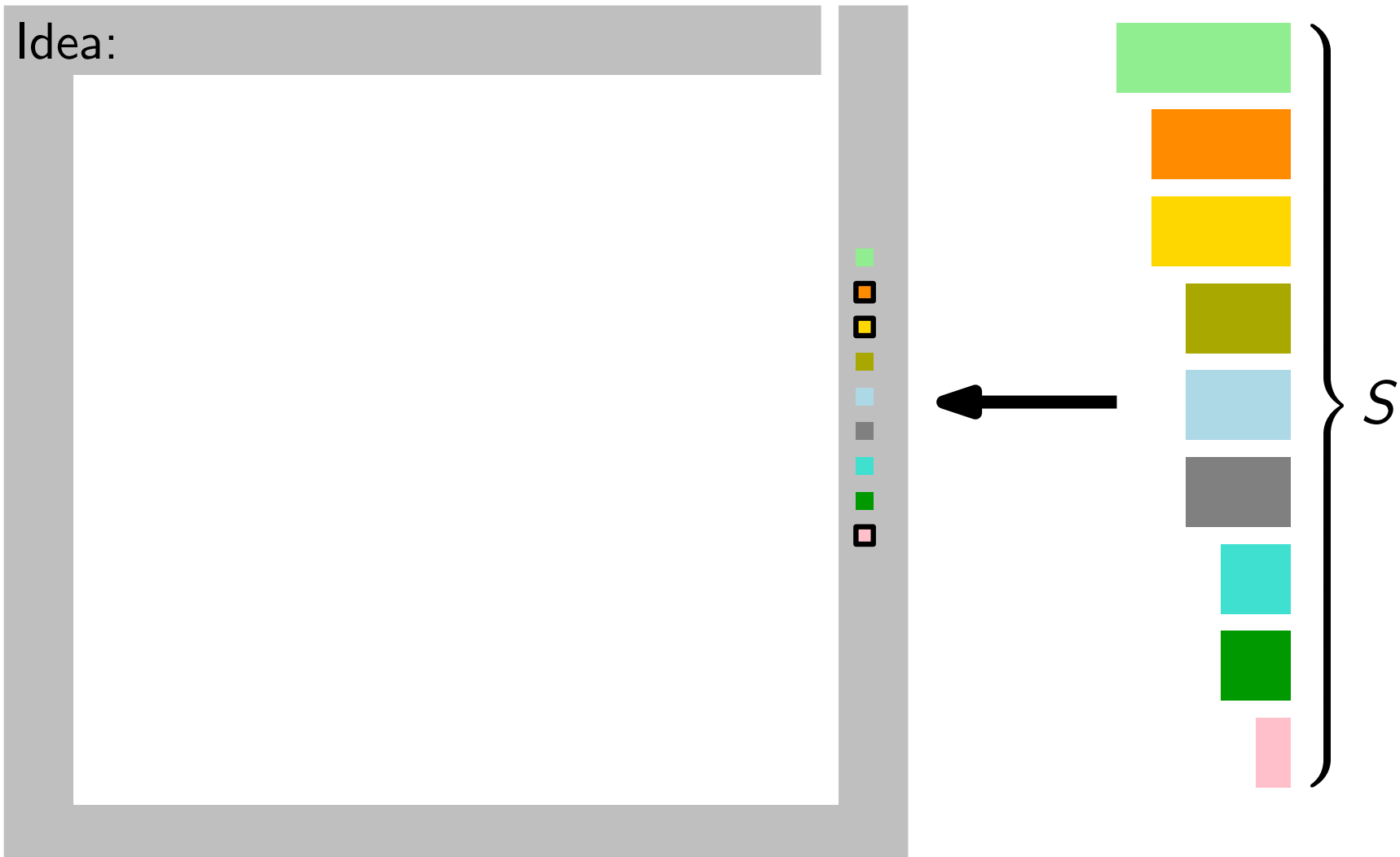

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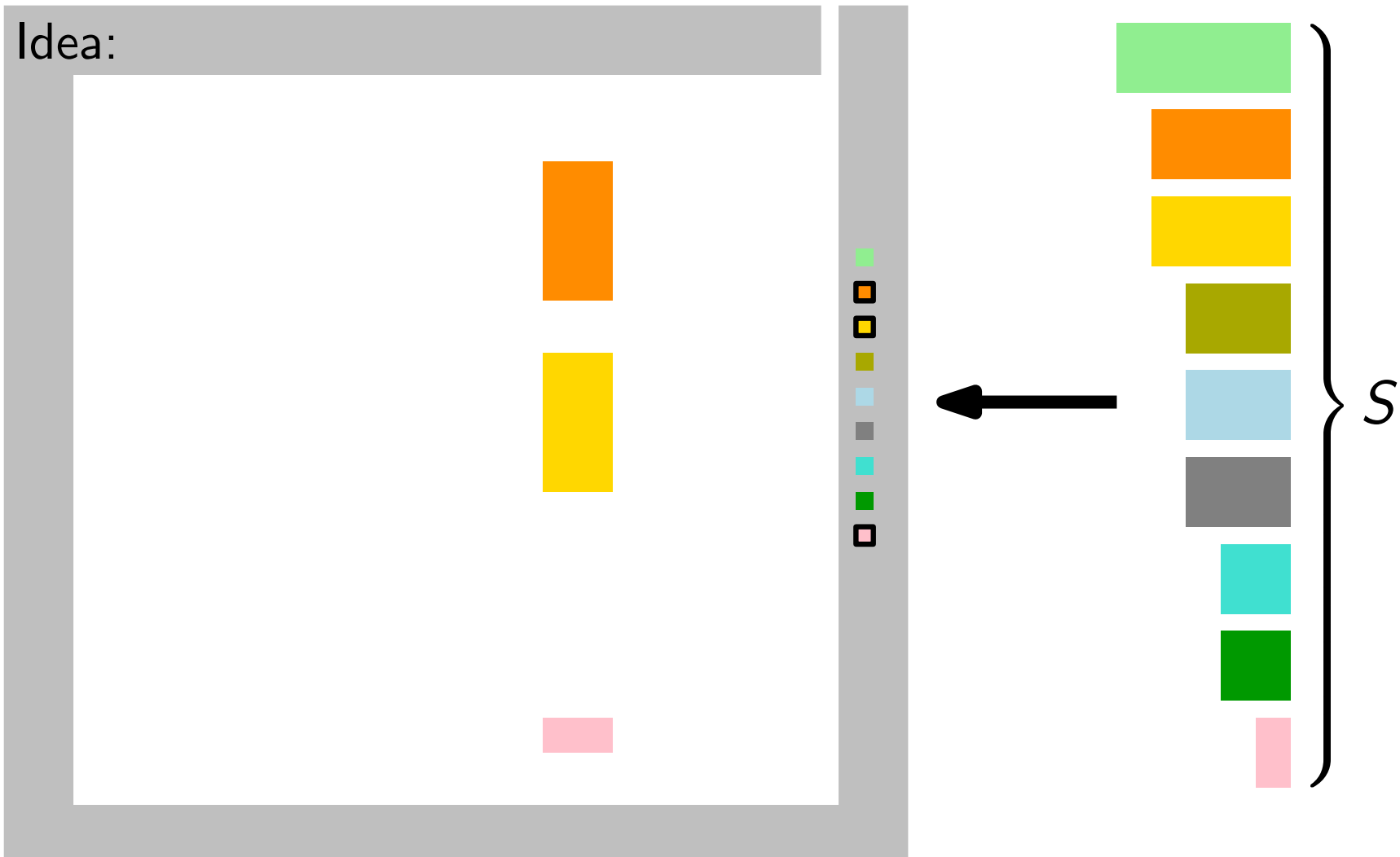

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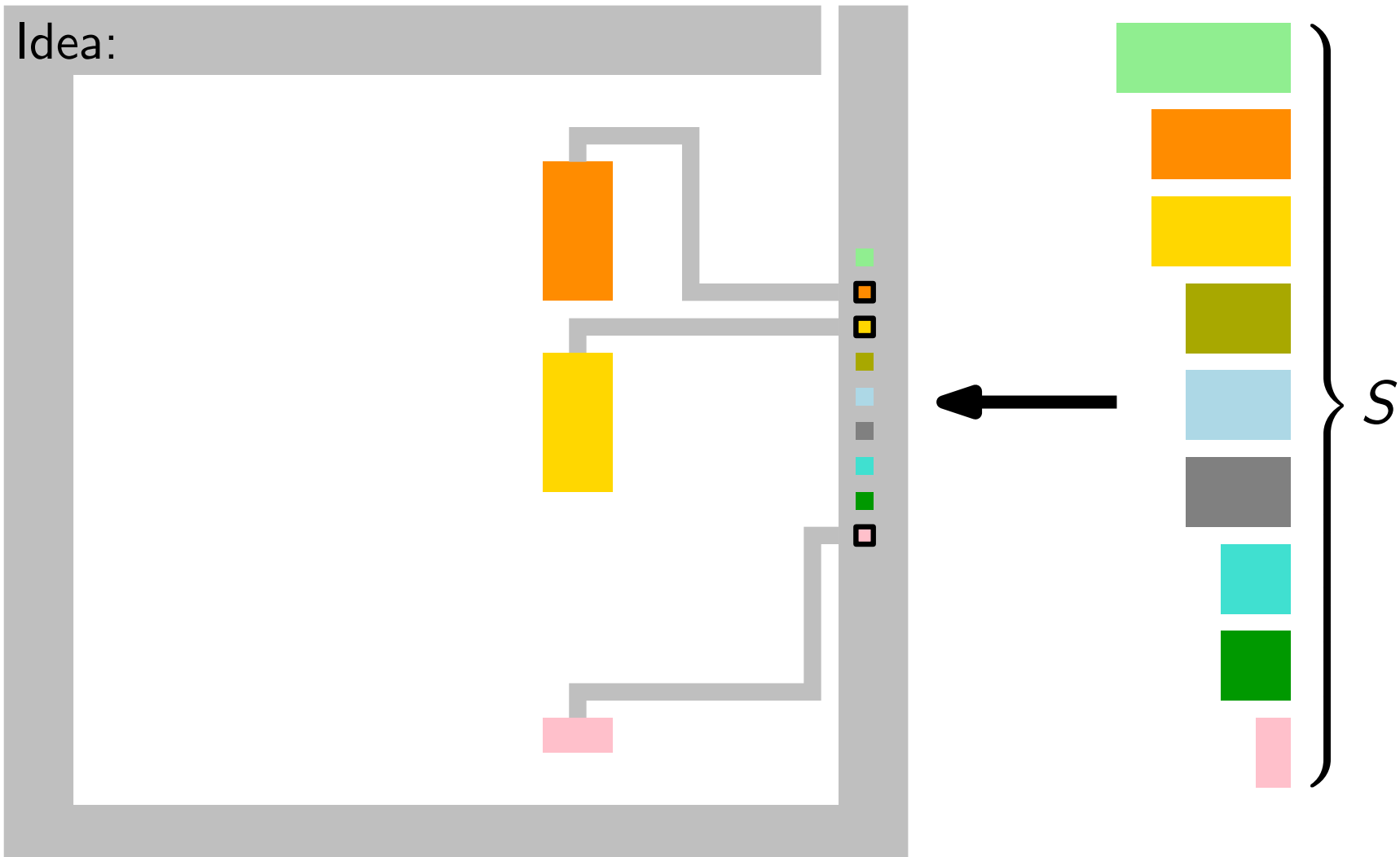

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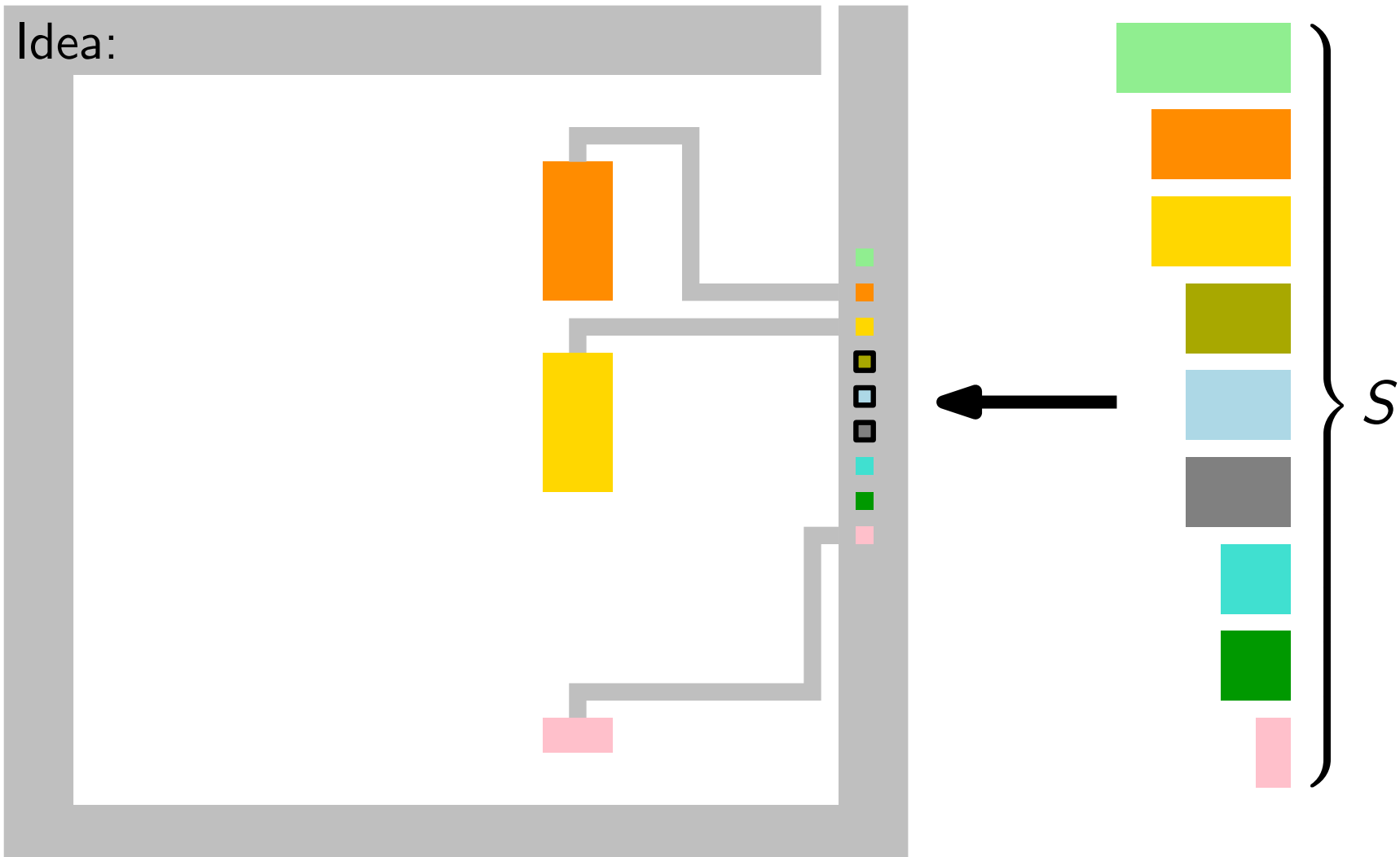

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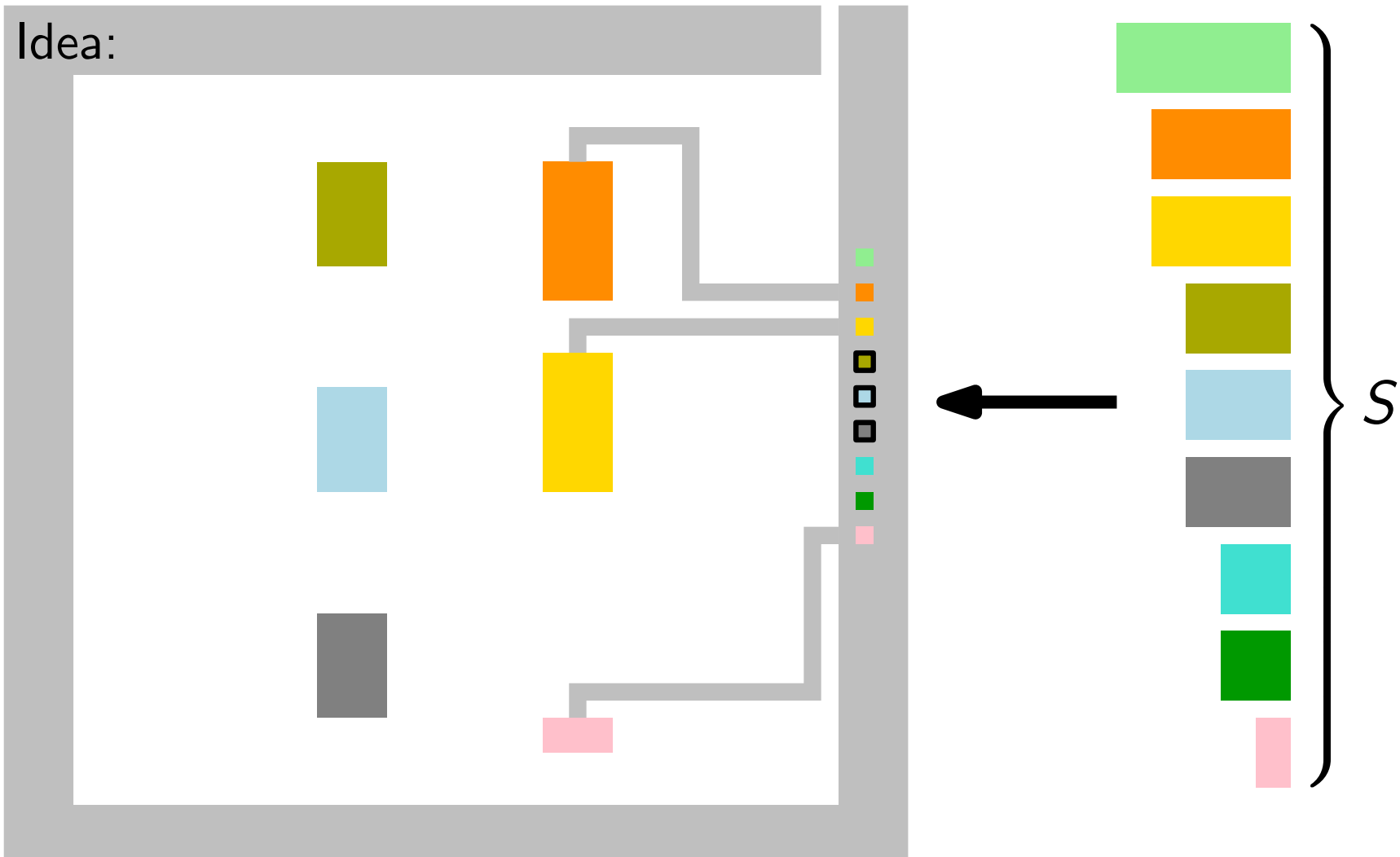

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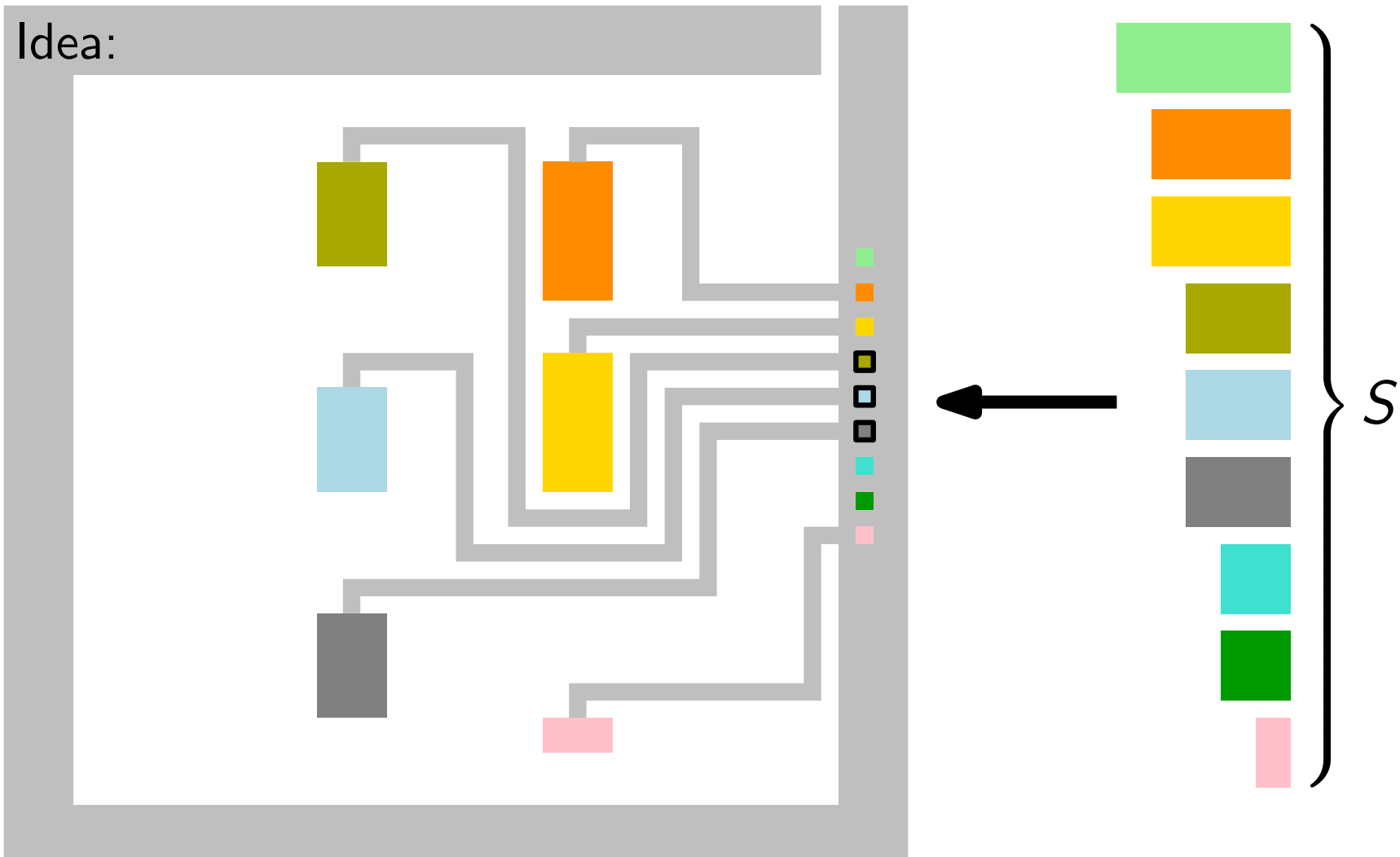

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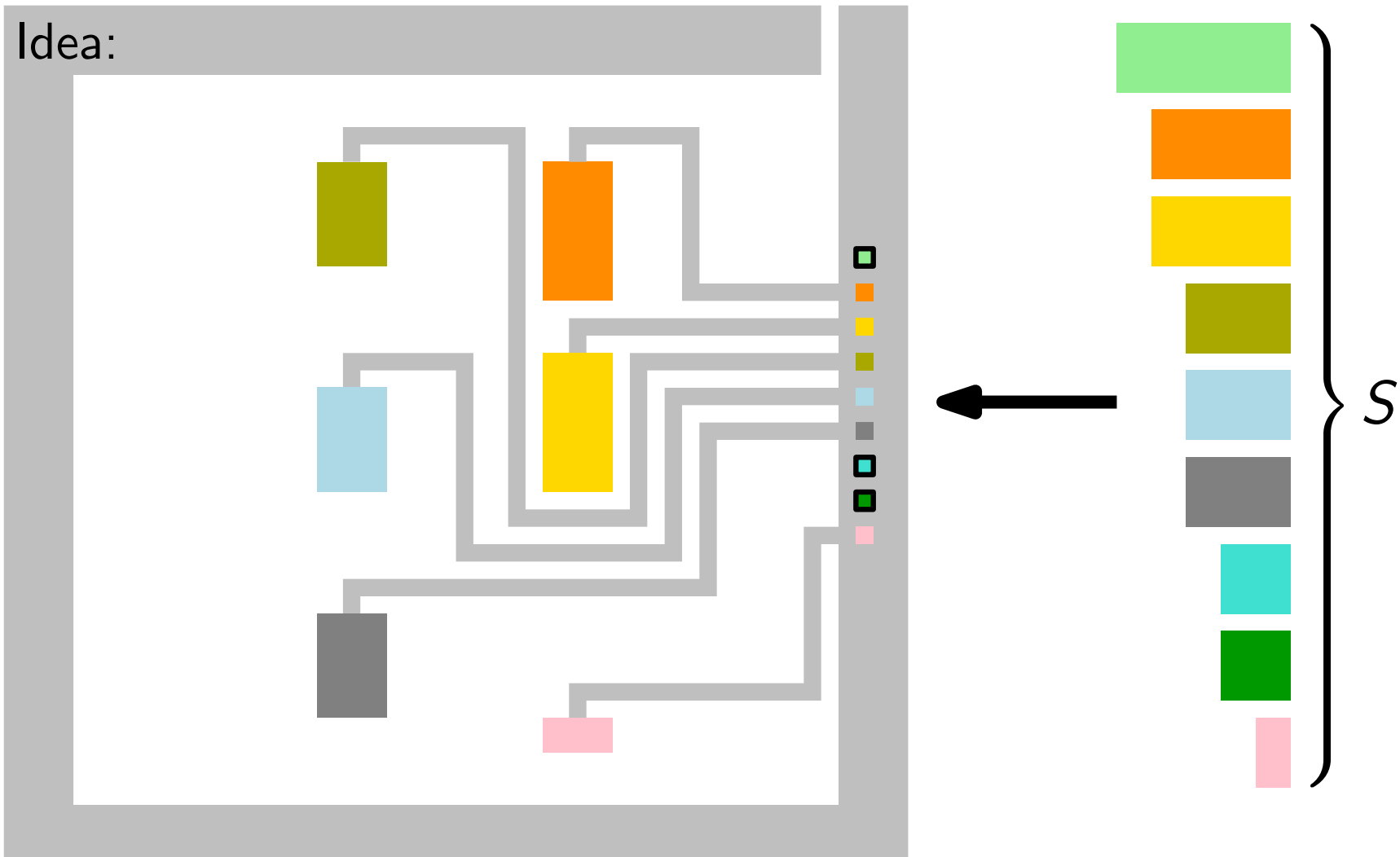

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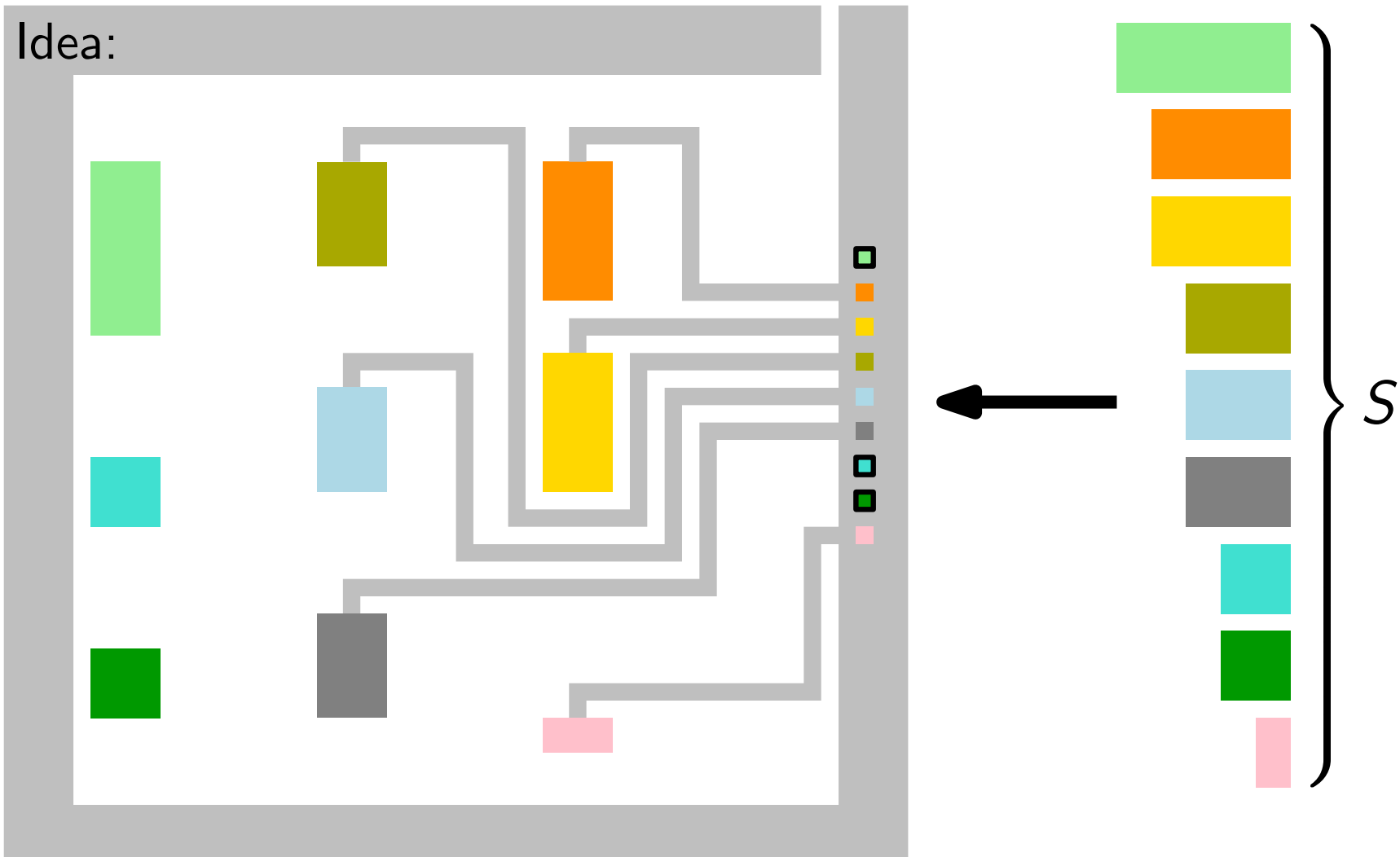

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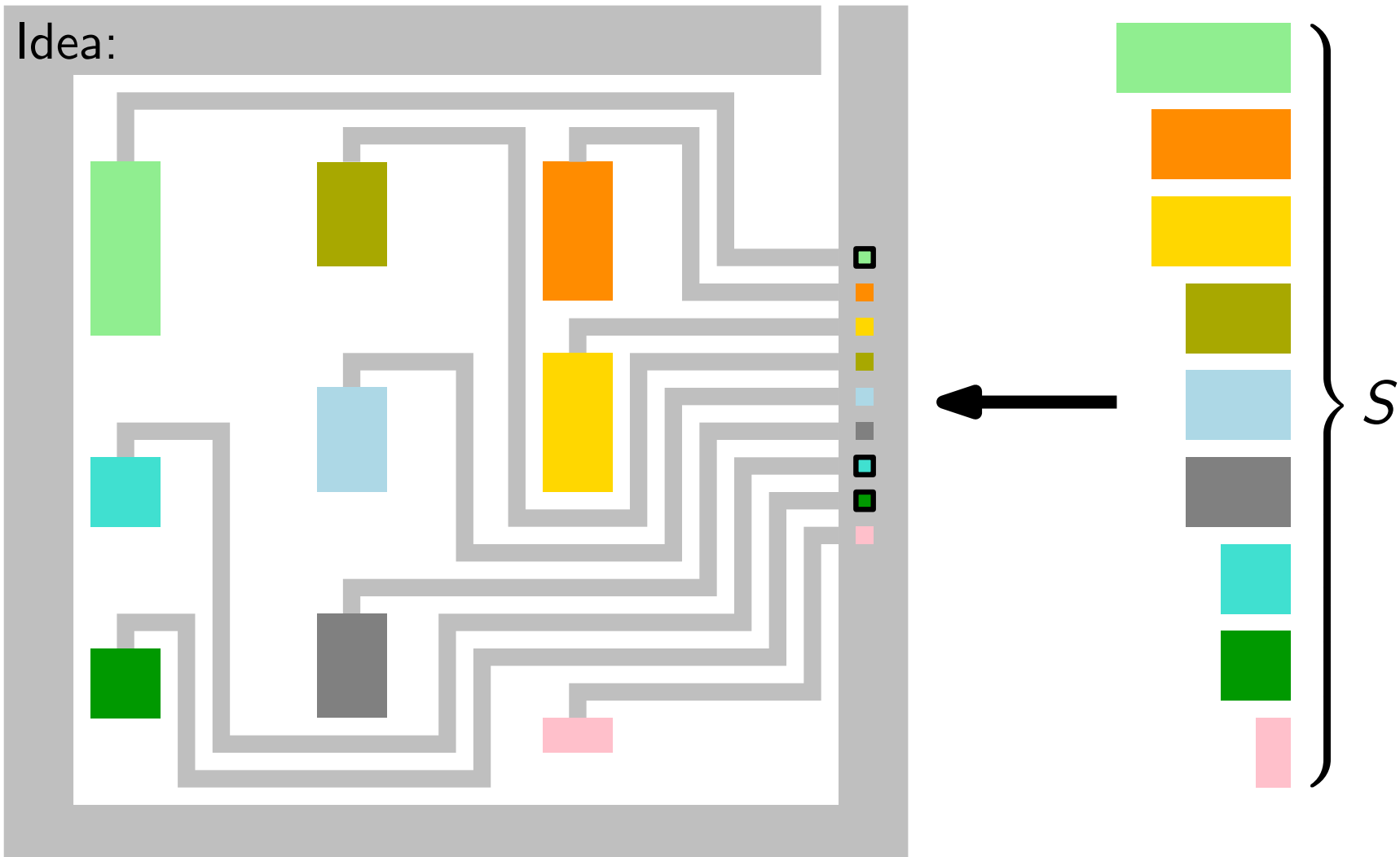

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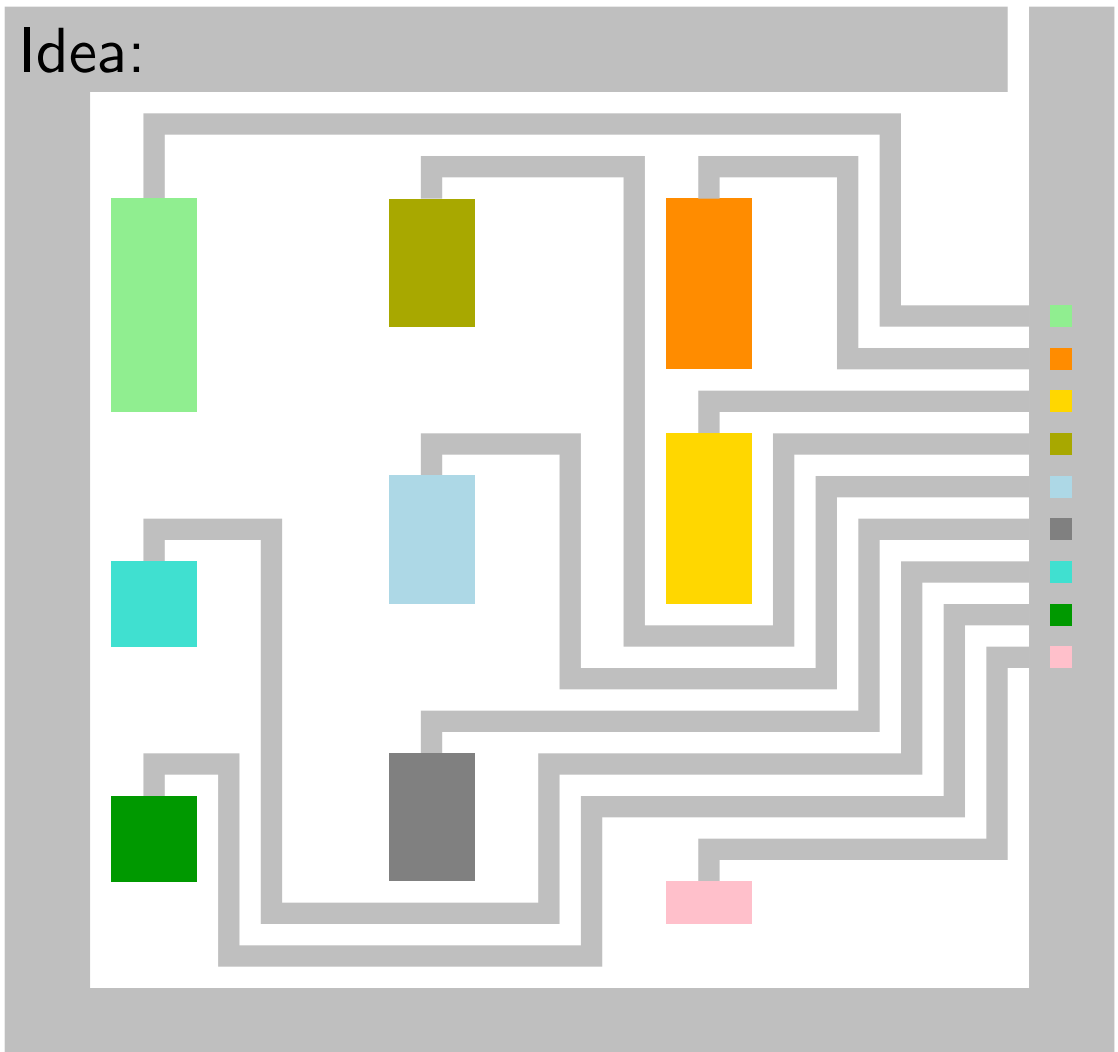

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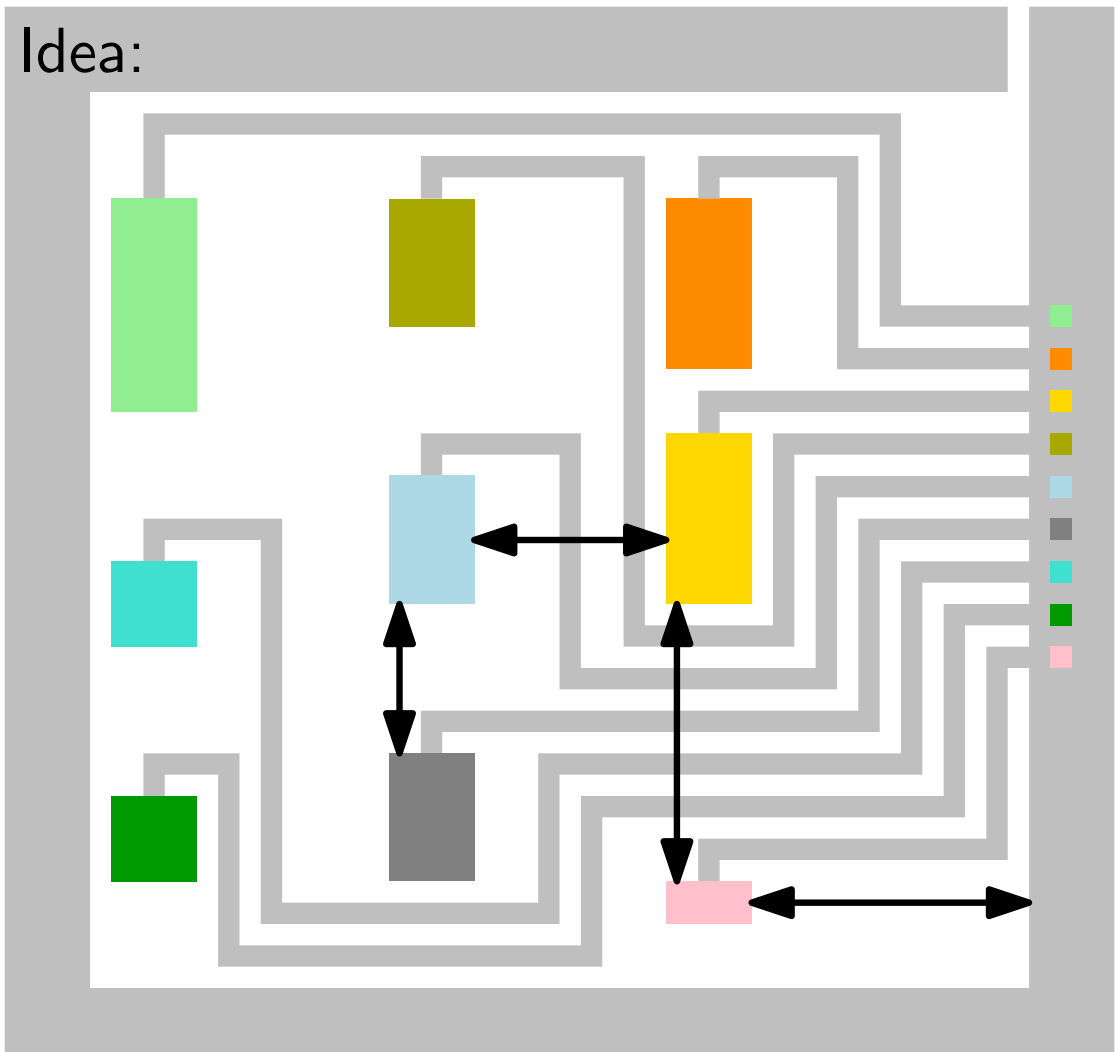

- Distance always $2 \cdot 3m$

Reduction (Sketch)

Given an instance $S = (s_1, \ldots, s_{3m})$ of 3-Partition,
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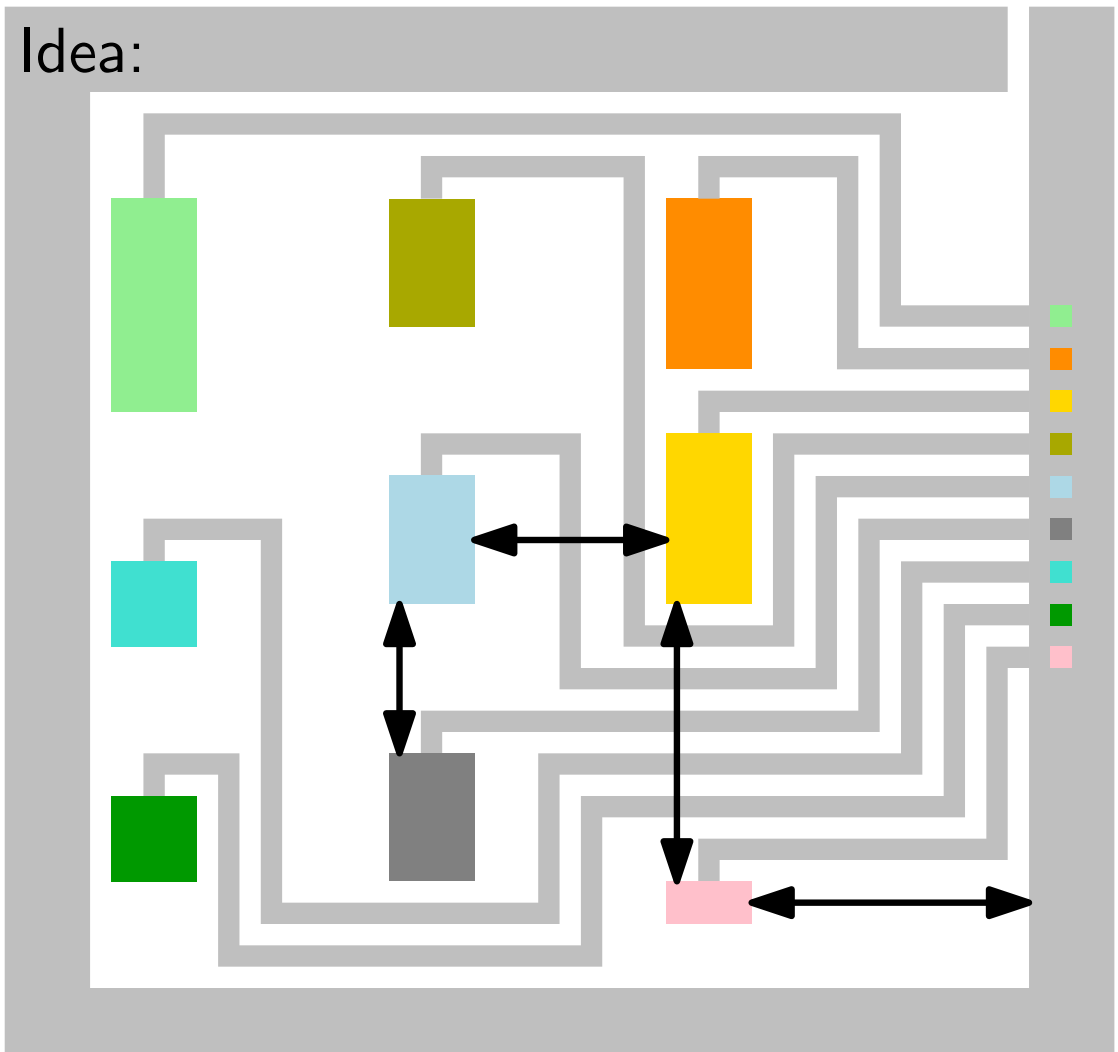

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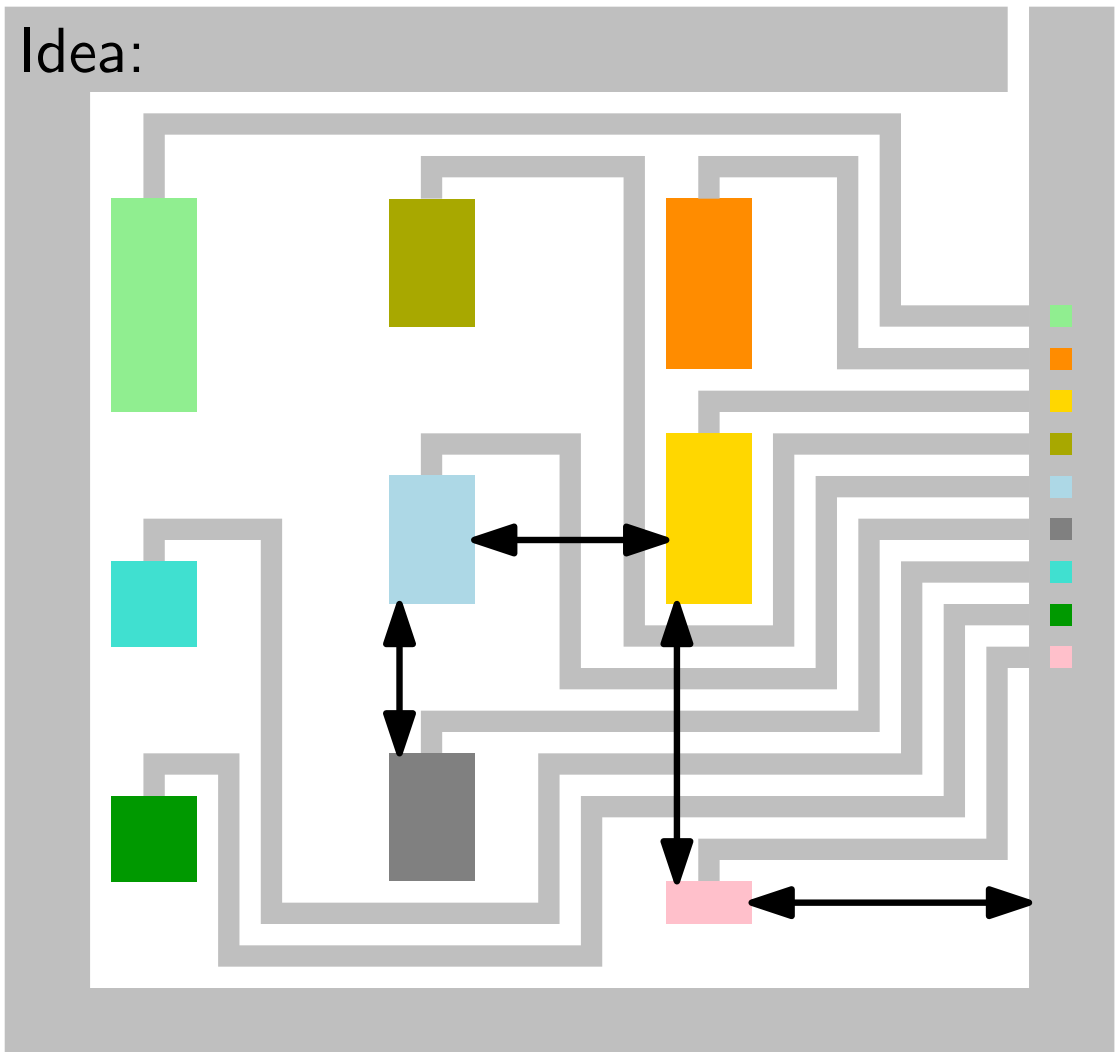

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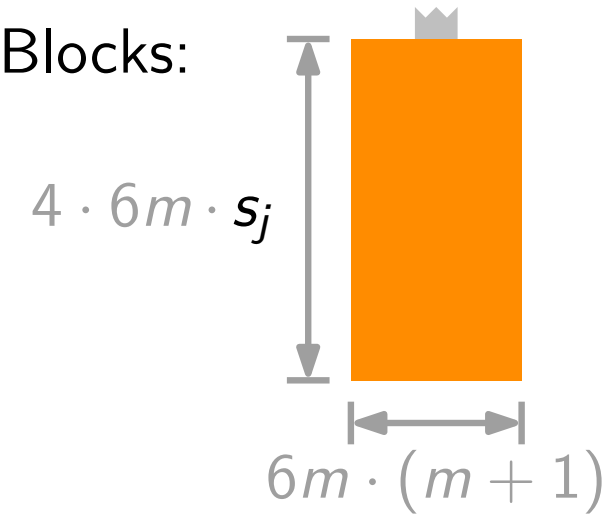

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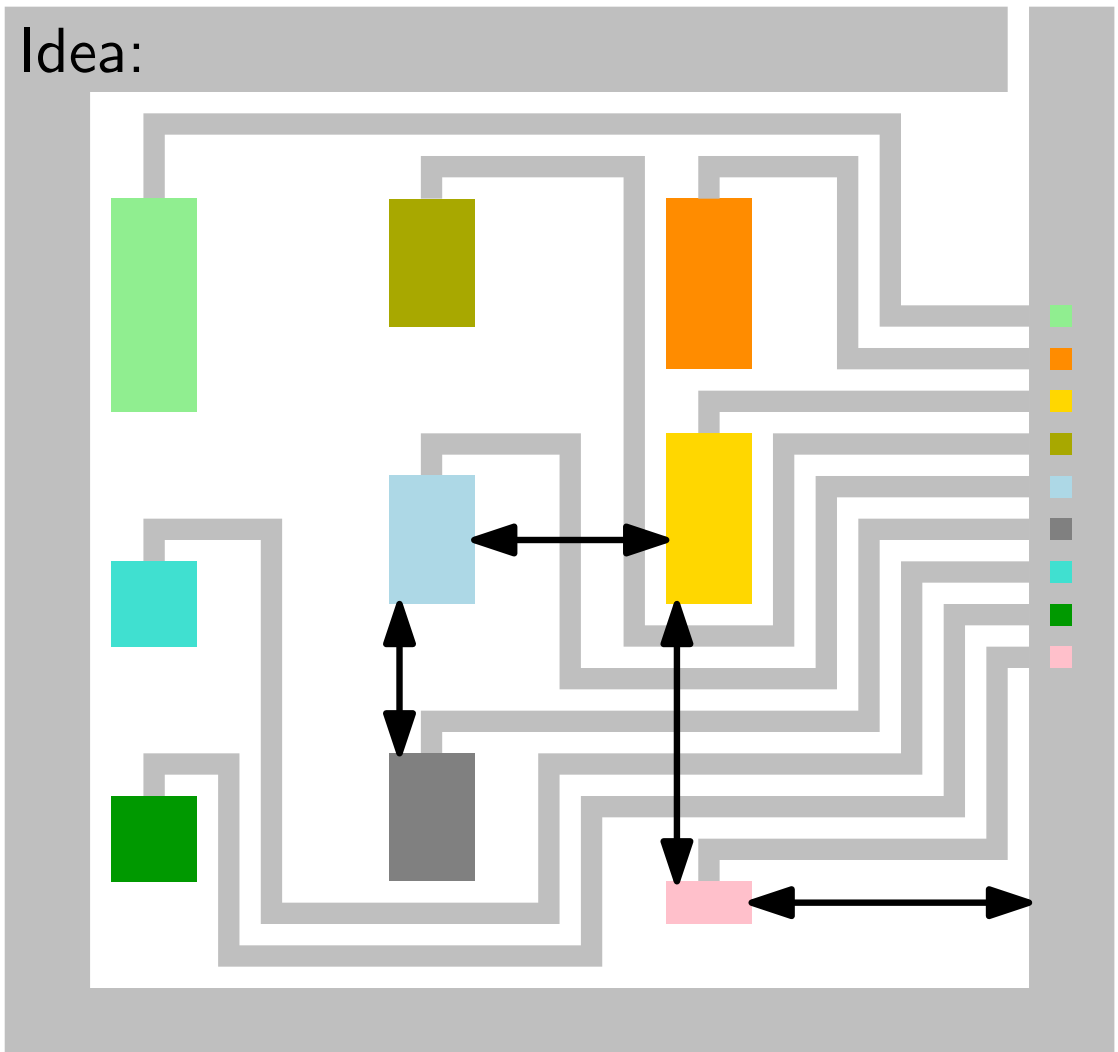

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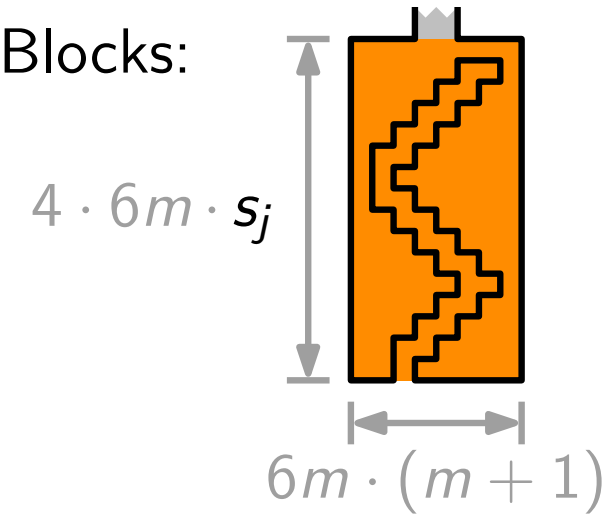

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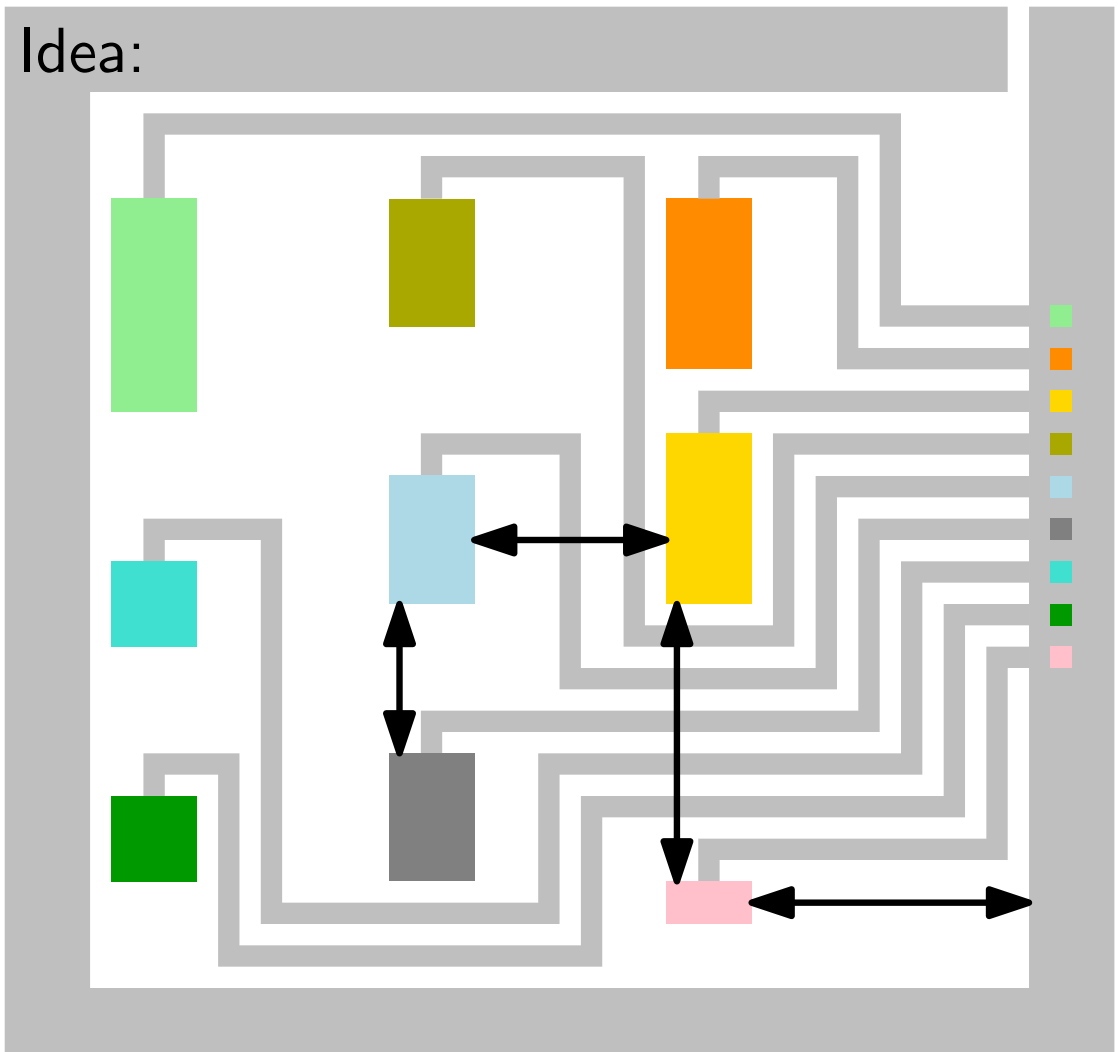

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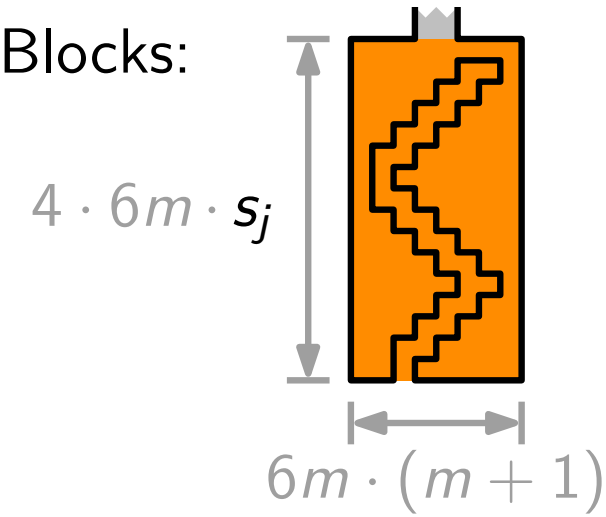

- Walls:

Reduction (Sketch)

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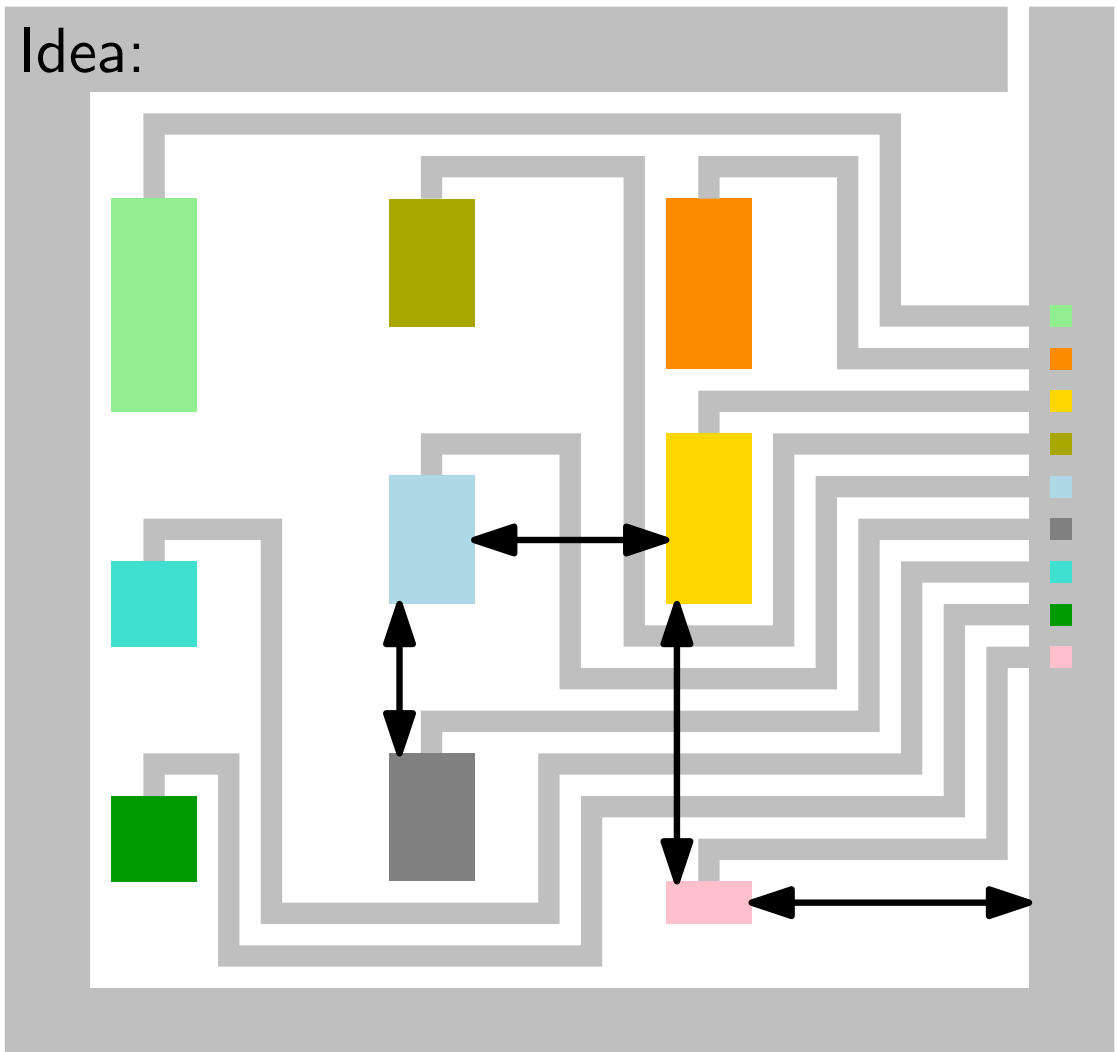

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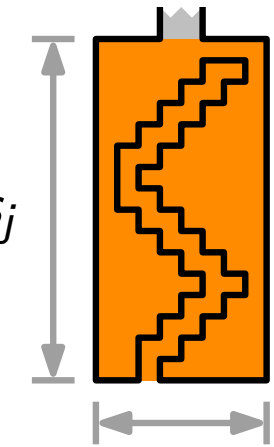

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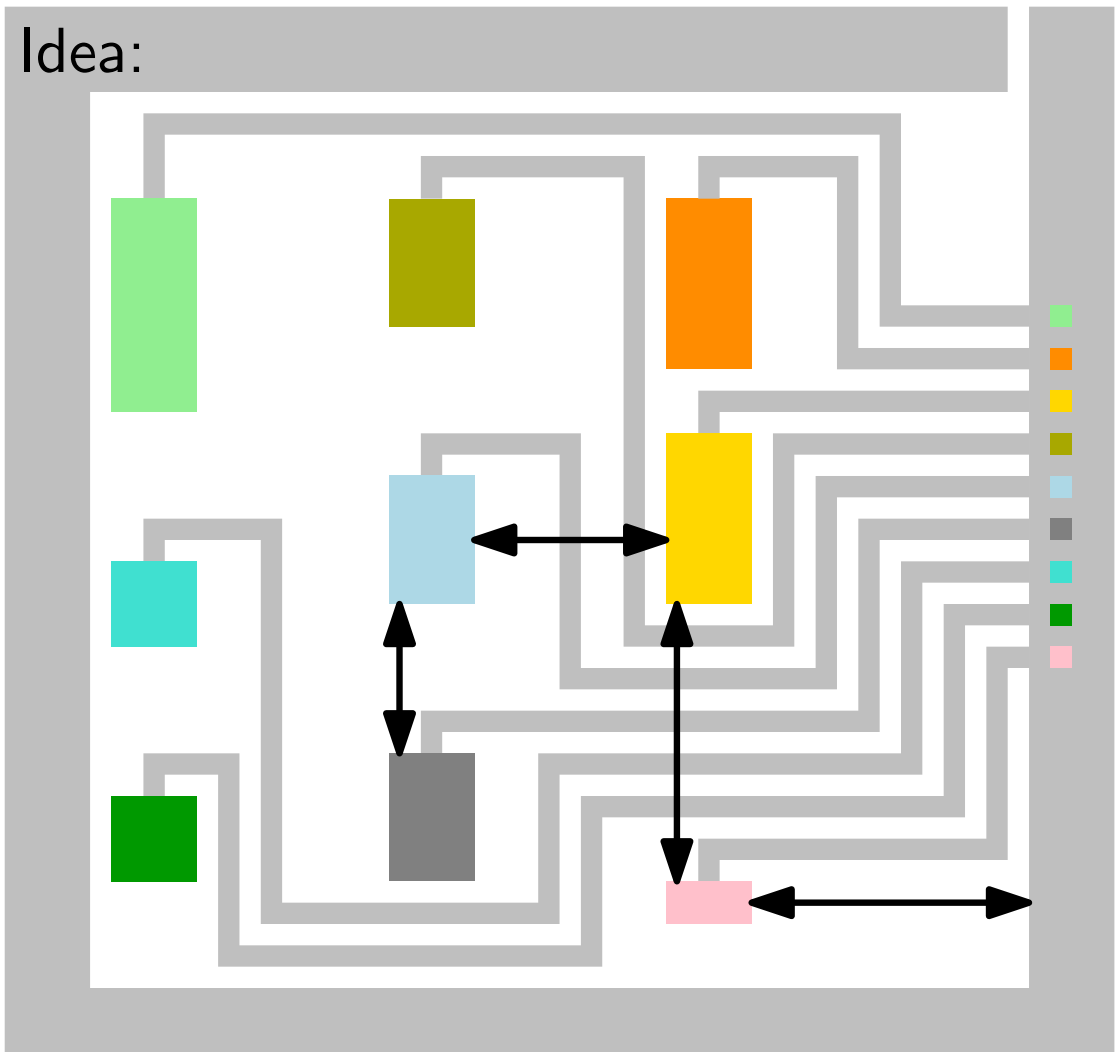

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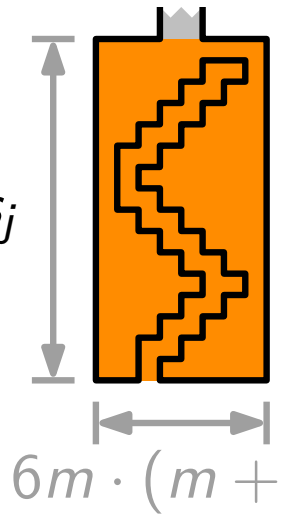
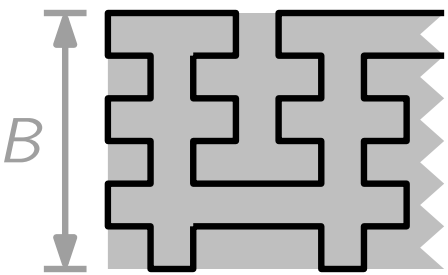


- Walls:



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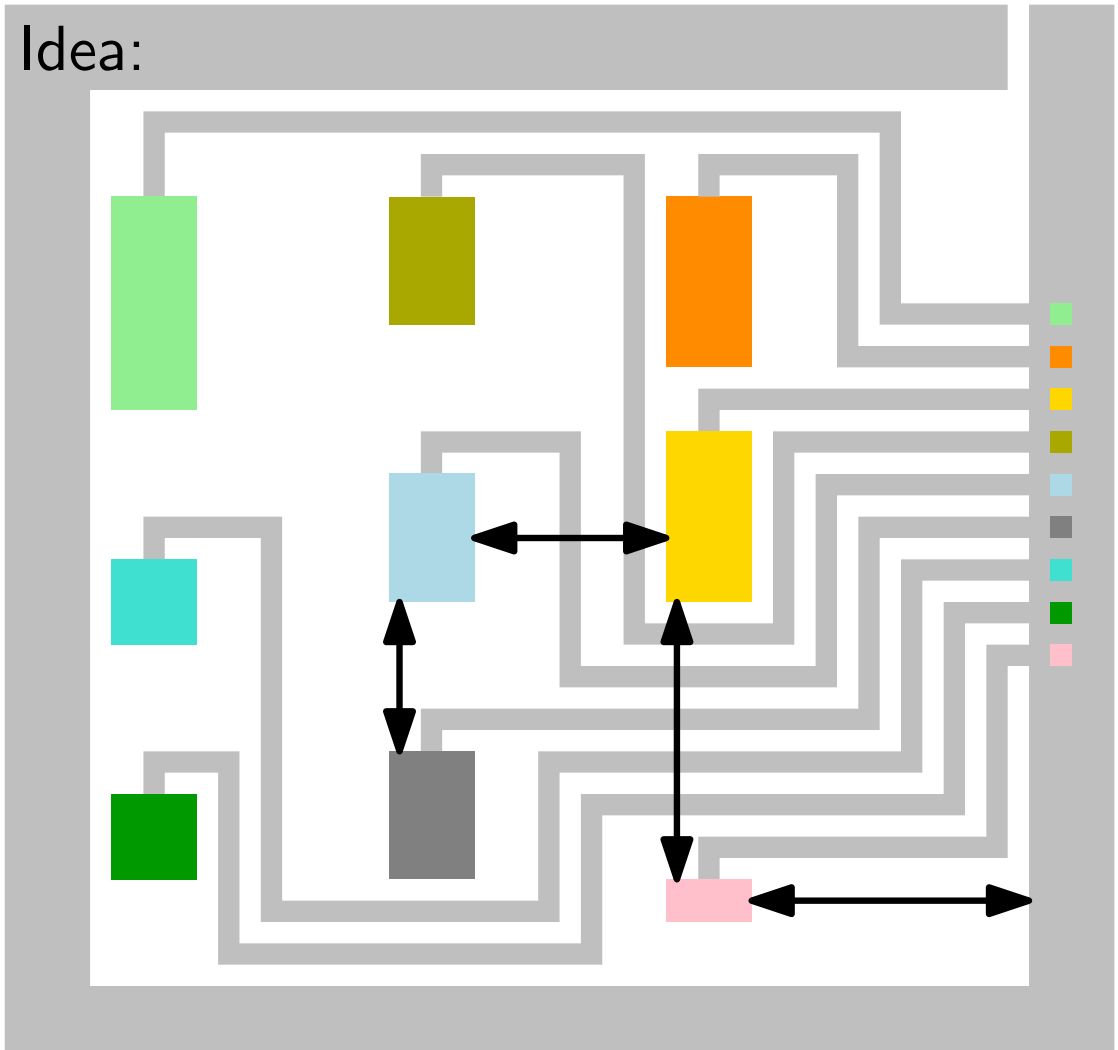

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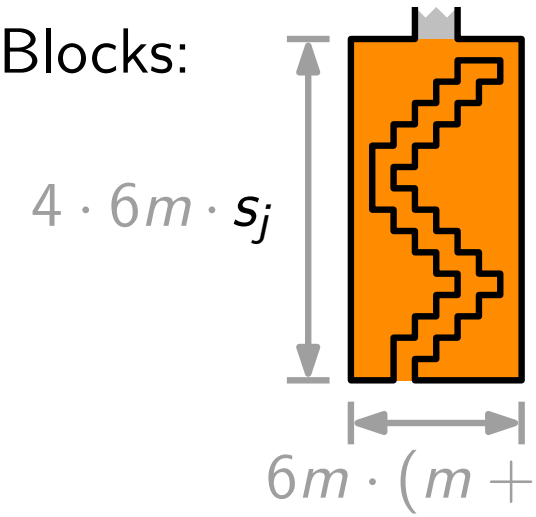

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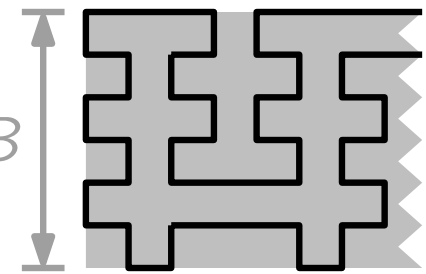
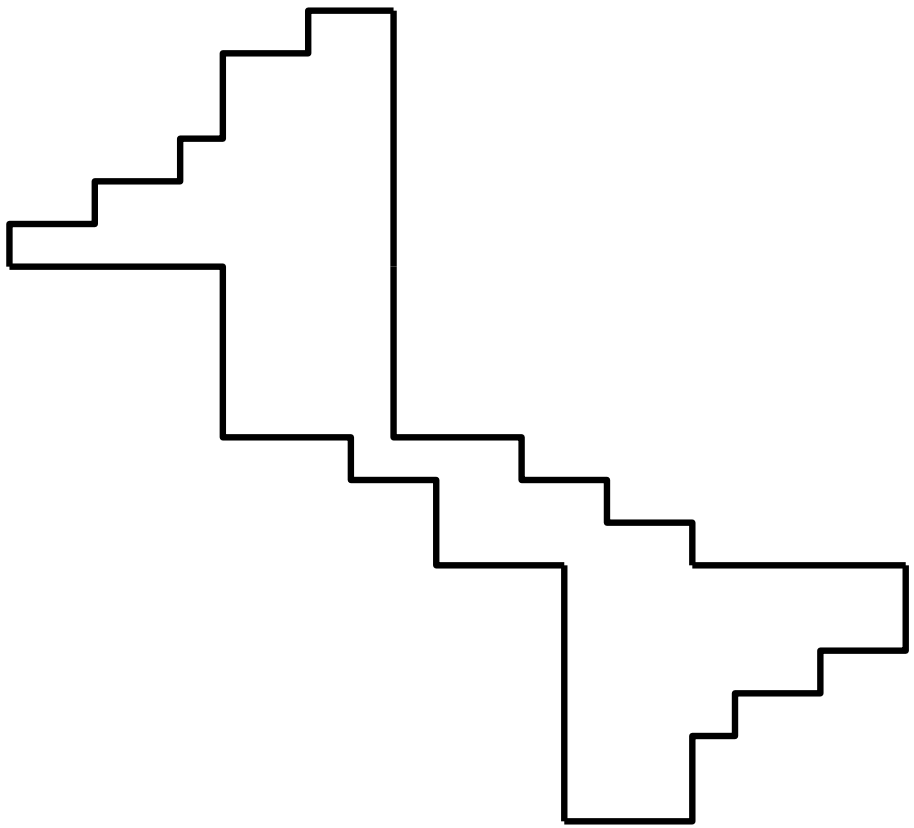


- reduction $\in$ poly(m, B)

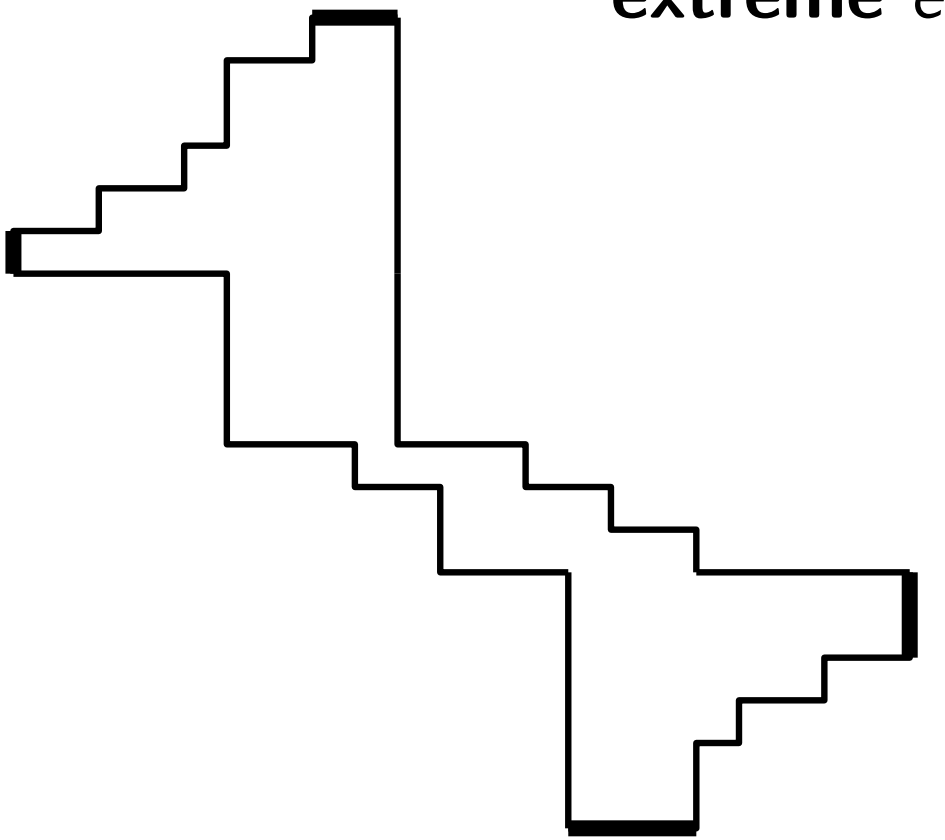
Minimizing the Area of xy-monotone Polygons

Polygon canonical if . . .



Polygon canonical if . . .

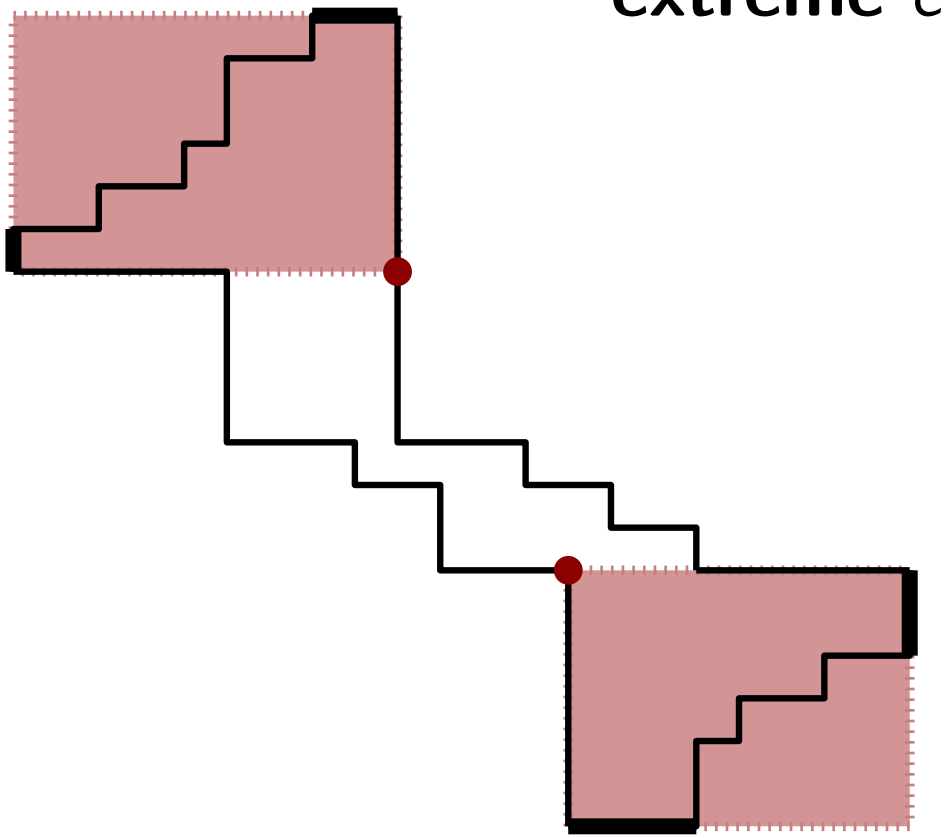
extreme *edges*



Polygon canonical if . . .

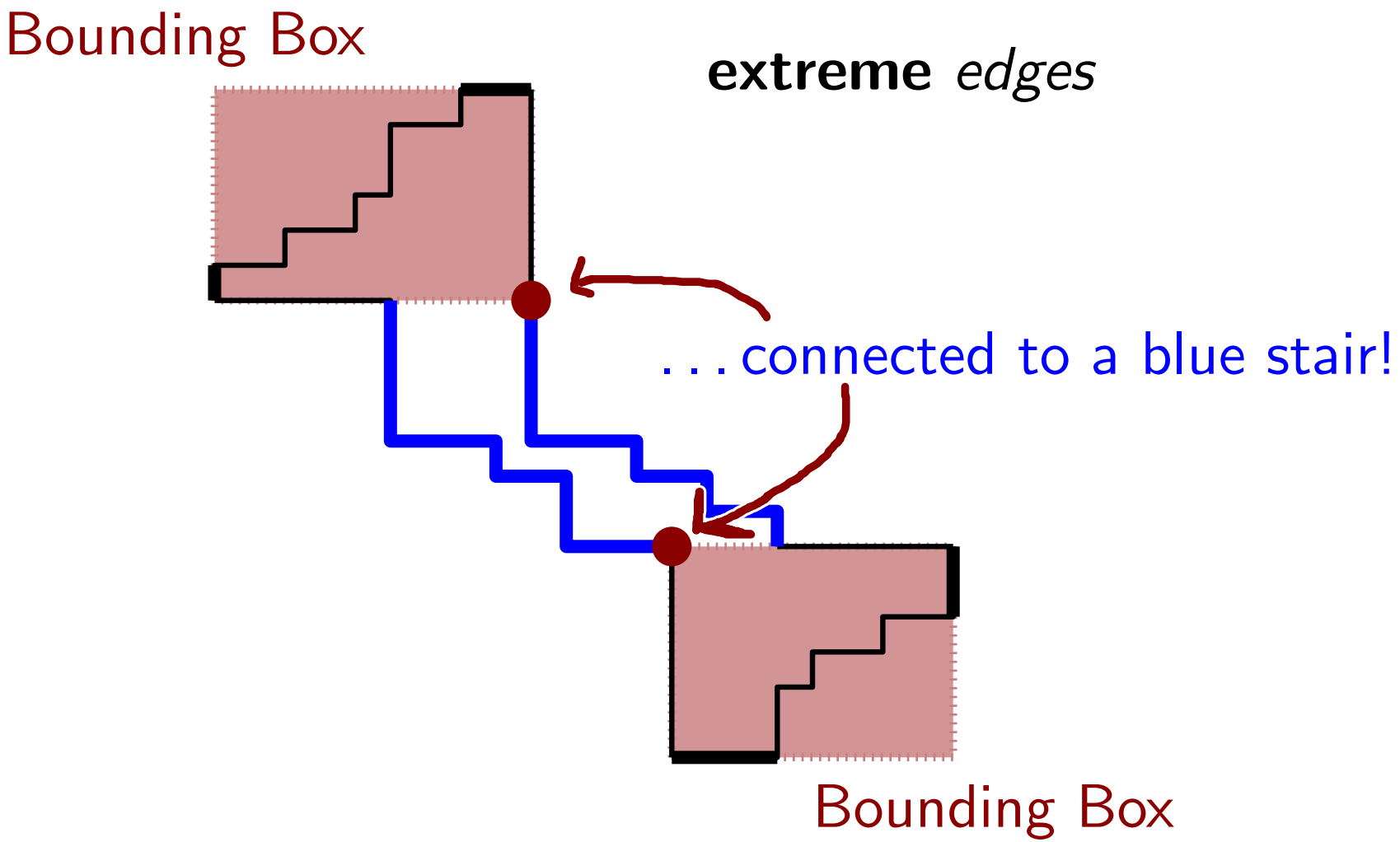
Bounding Box

extreme *edges*

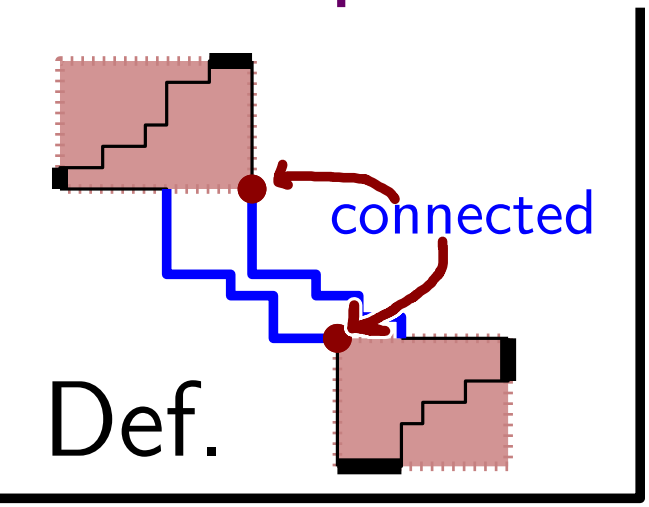


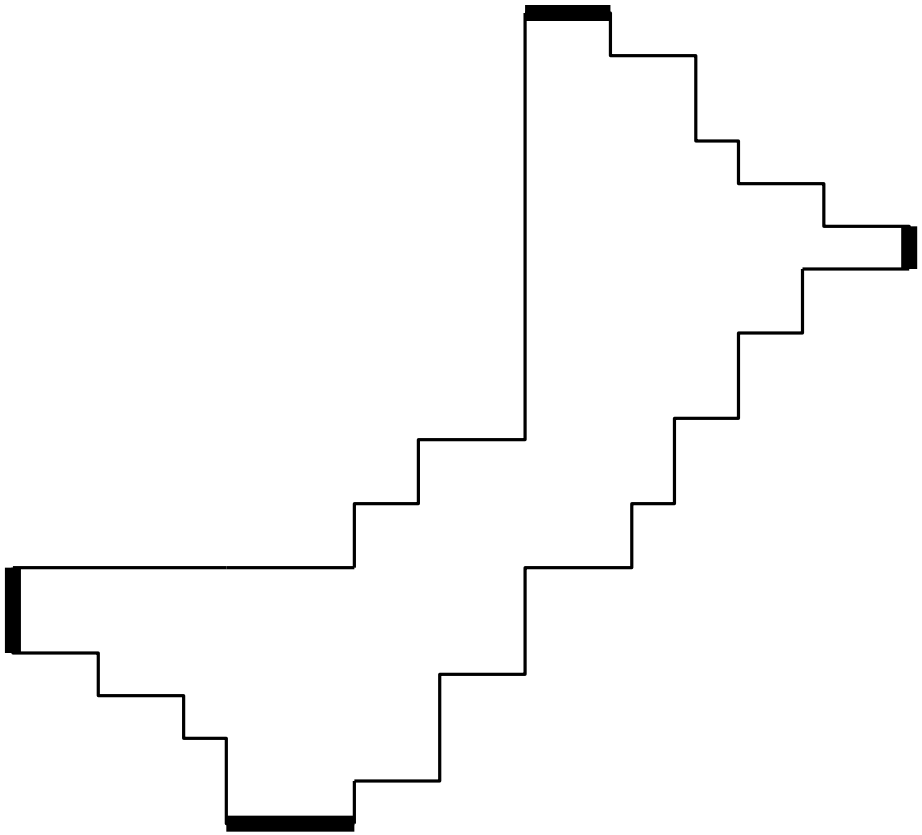
Bounding Box

Polygon canonical if . . .



Example: Canonical Polygon





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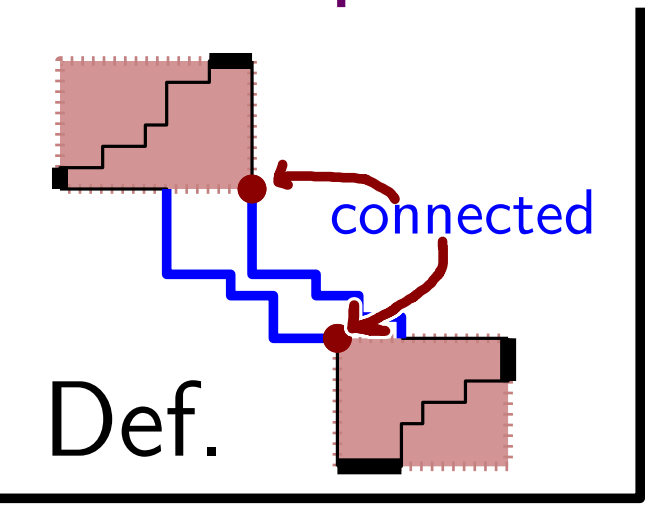

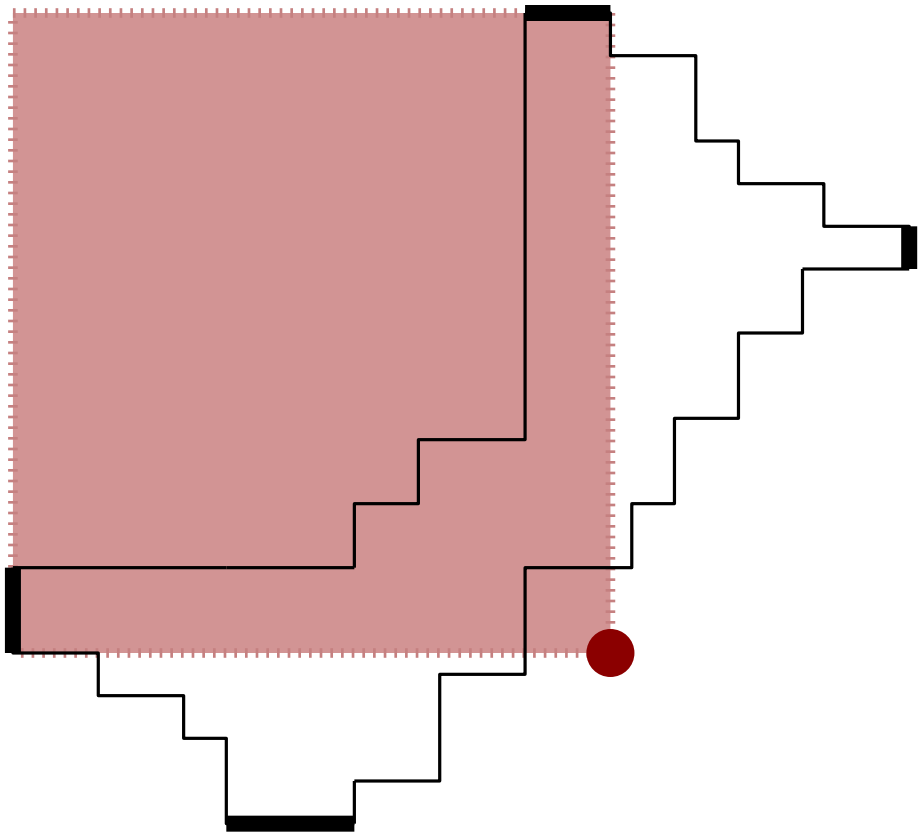

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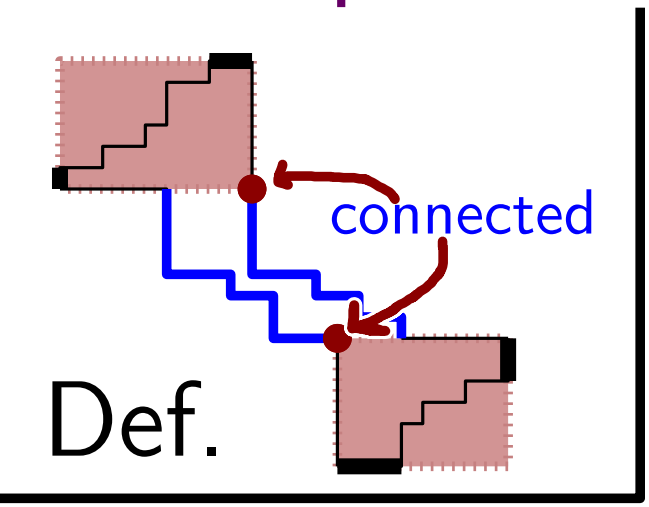

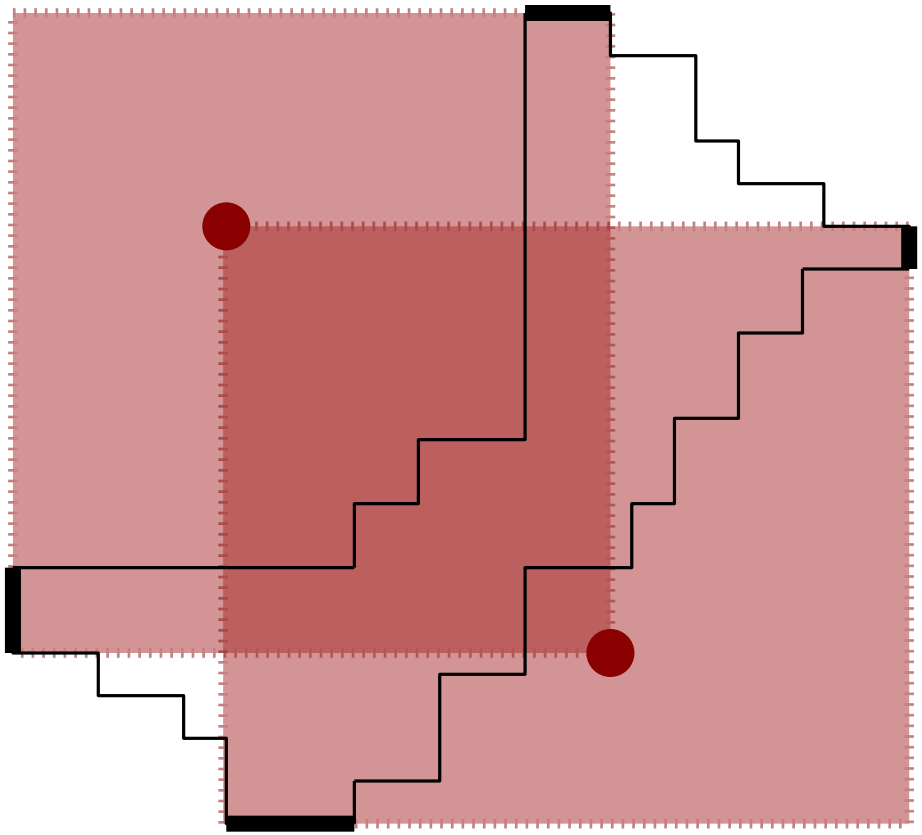

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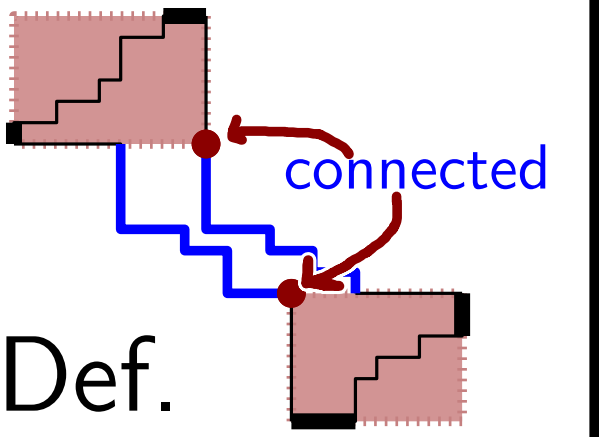
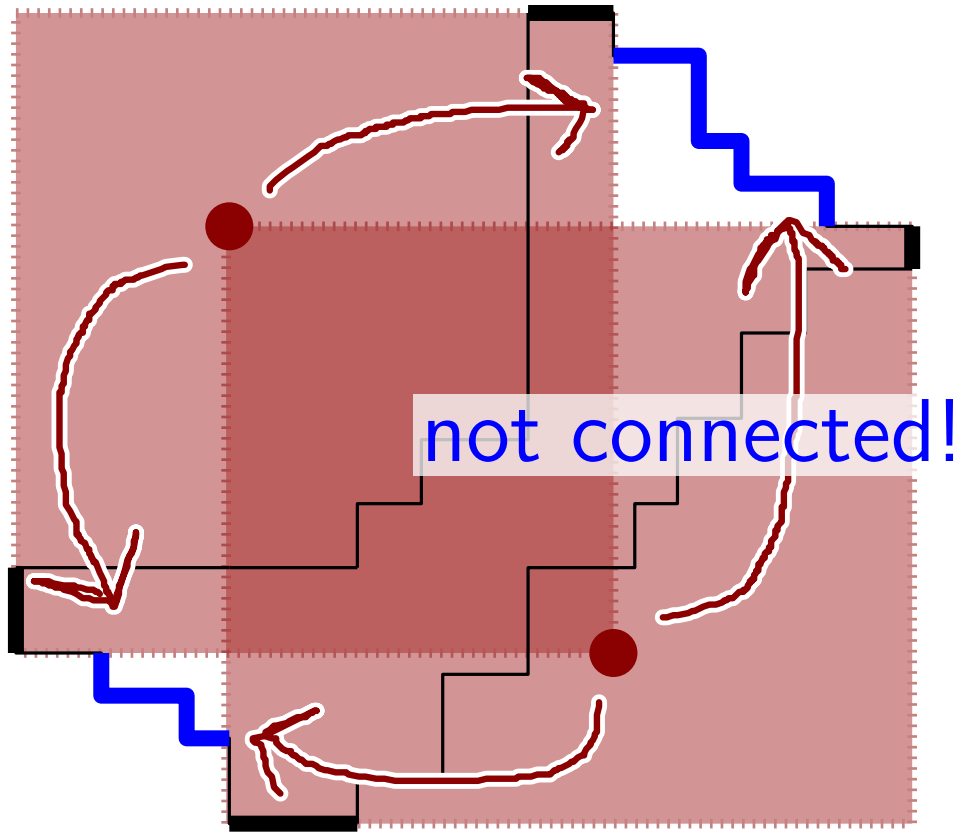


Def.



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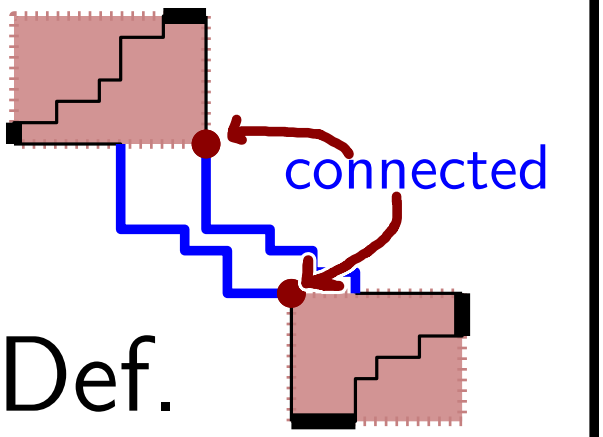
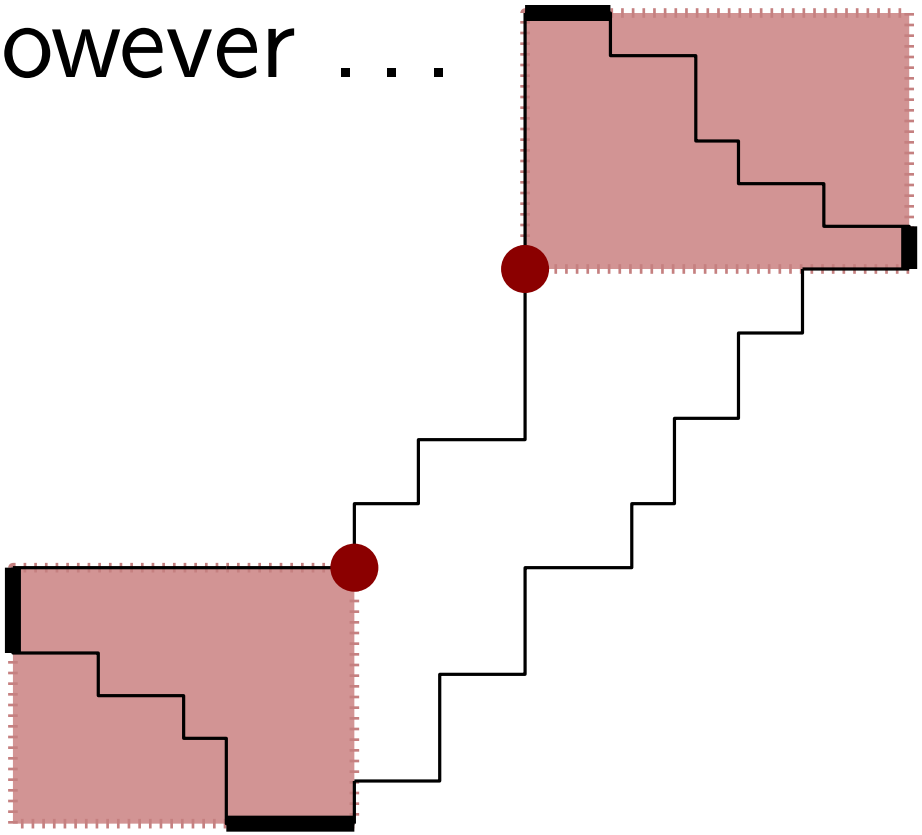


Def.

However . . .



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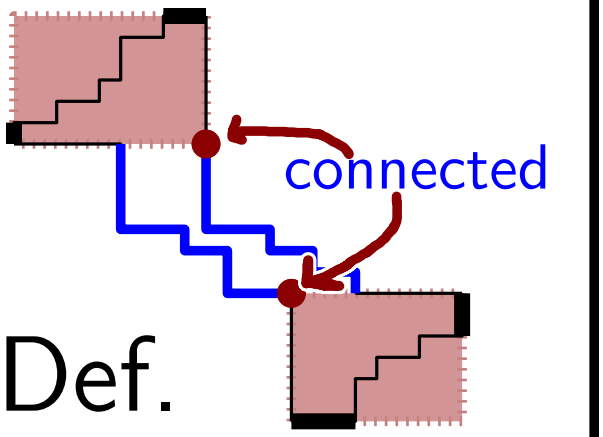

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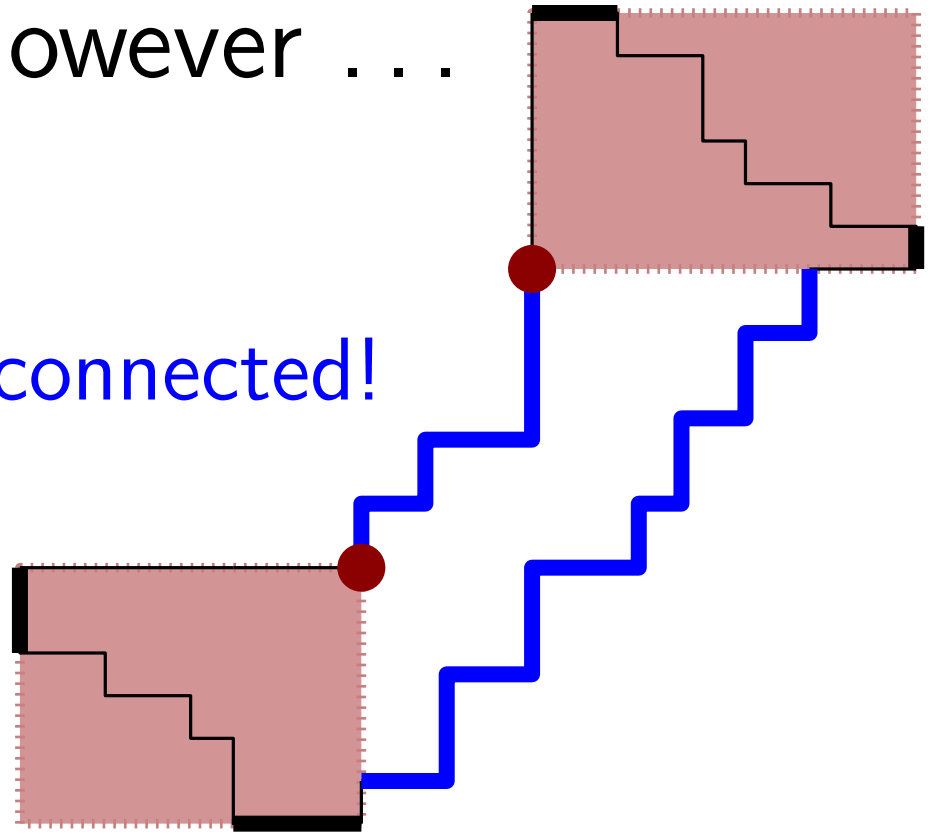
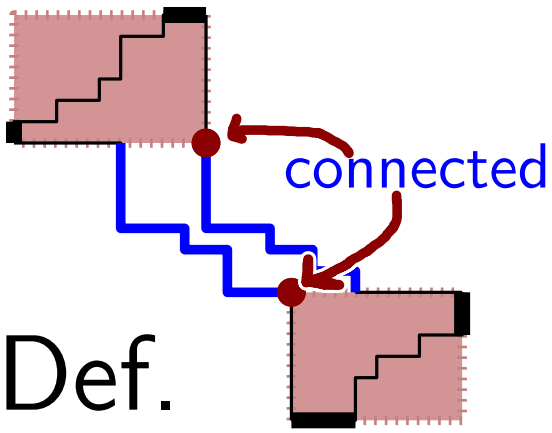


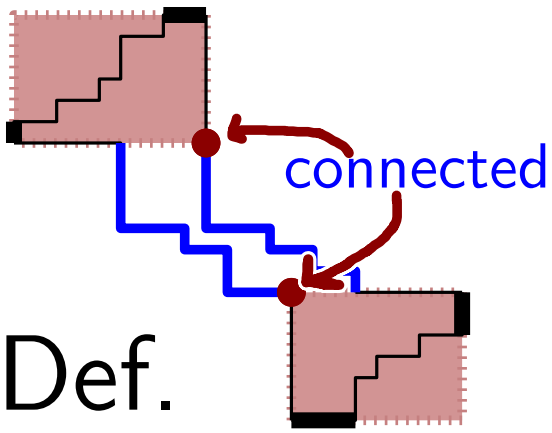
. . . is canonical.



Def.

Lemma:

There is an optimum canonical polygon!



Def.

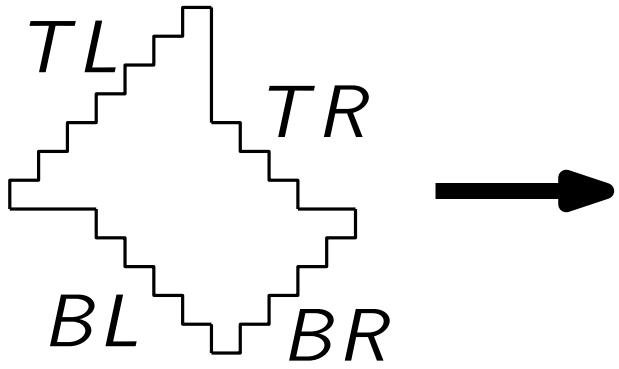
Lemma:

There is an optimum canonical polygon!

How to find it?

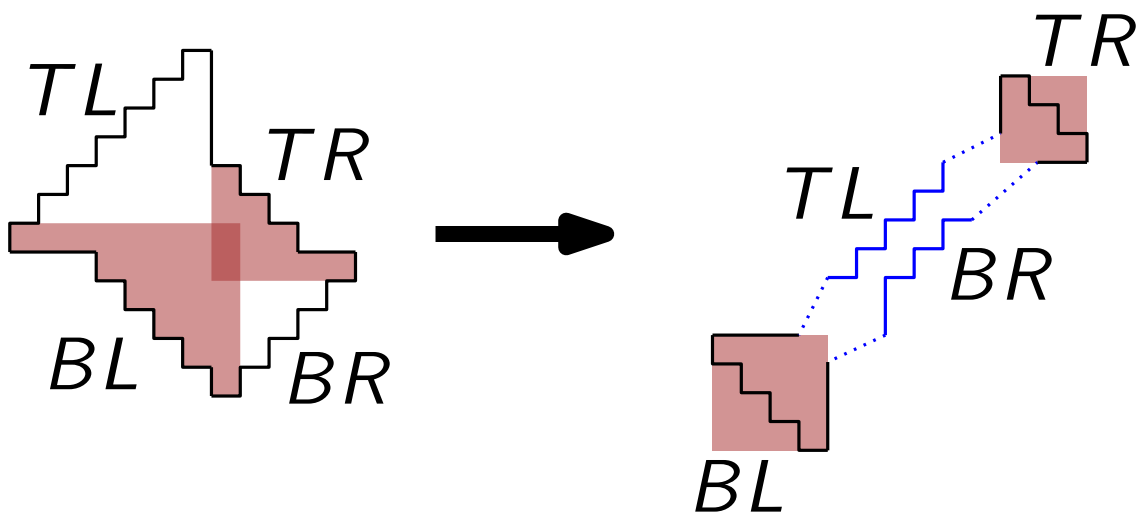
Computing Optimum Canonical Polygon

Given angle sequence . . .



Computing Optimum Canonical Polygon

Given angle sequence . . . . . $O(1)$ candidates for OPT.



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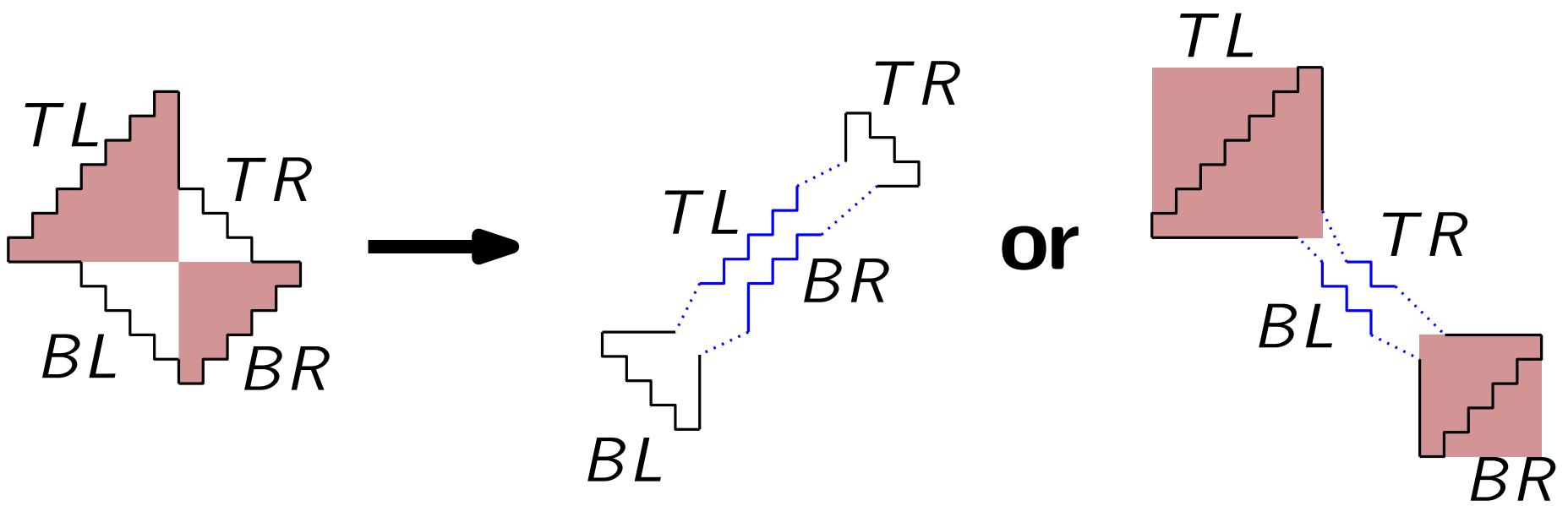

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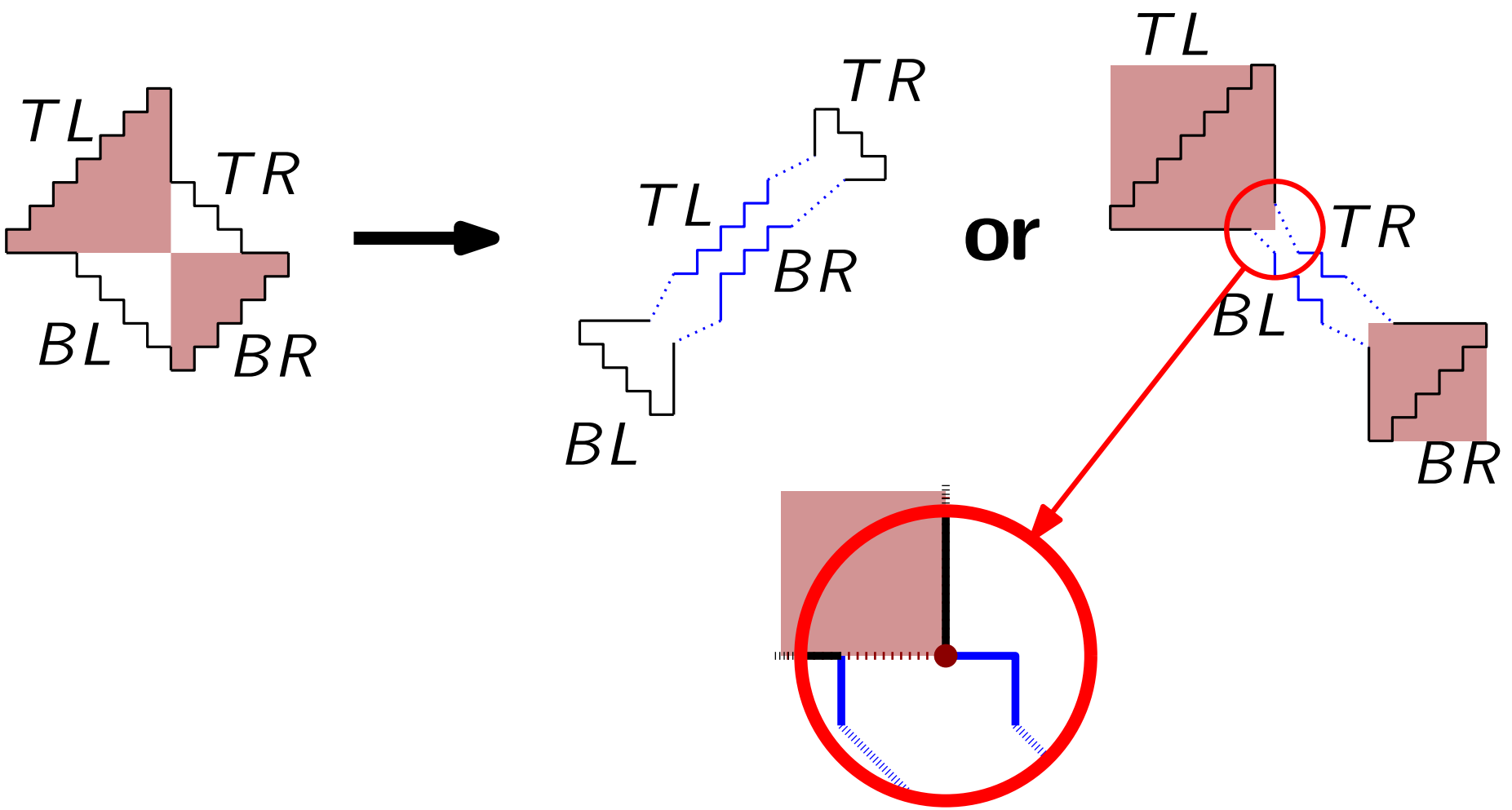

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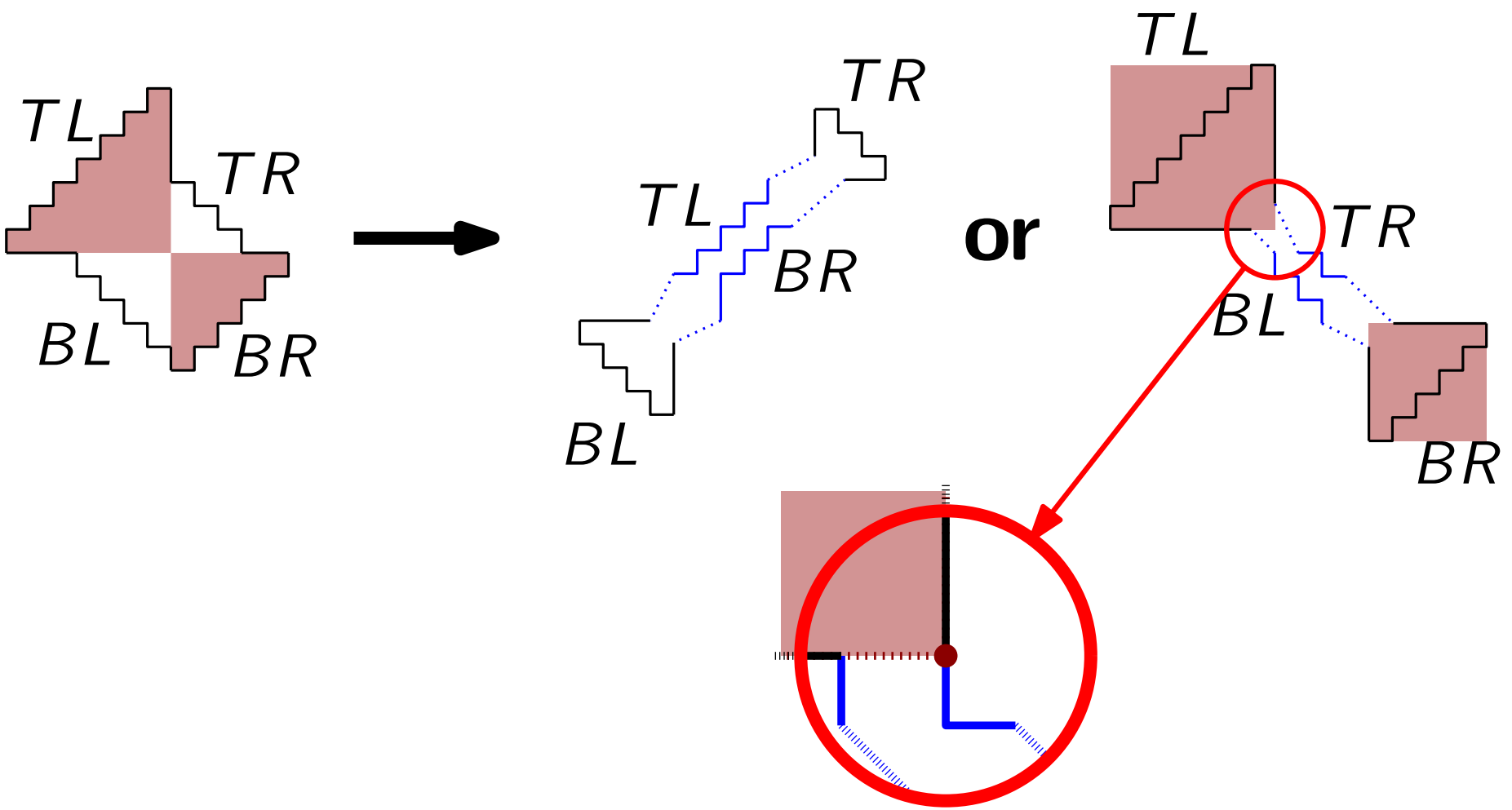

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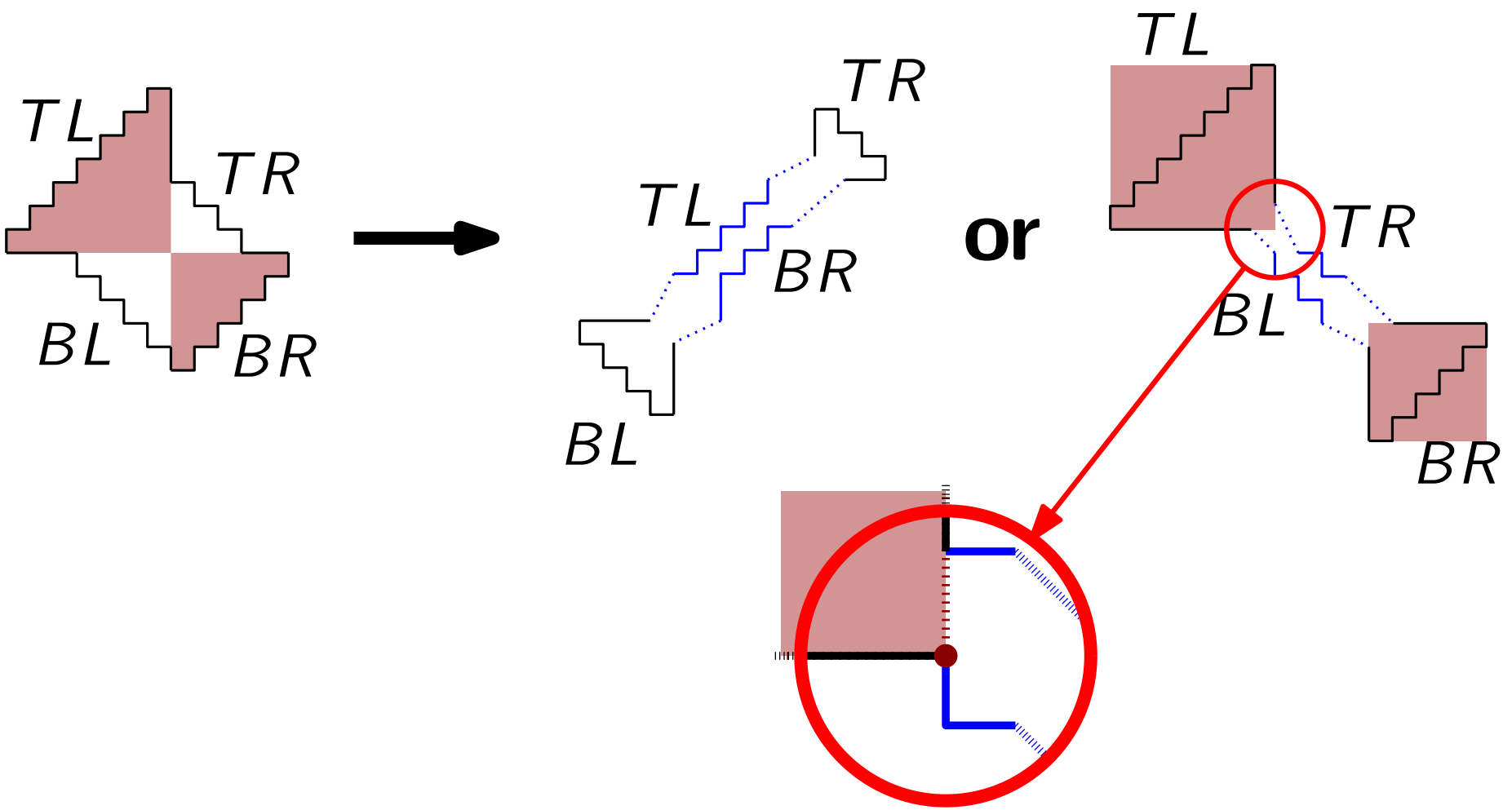

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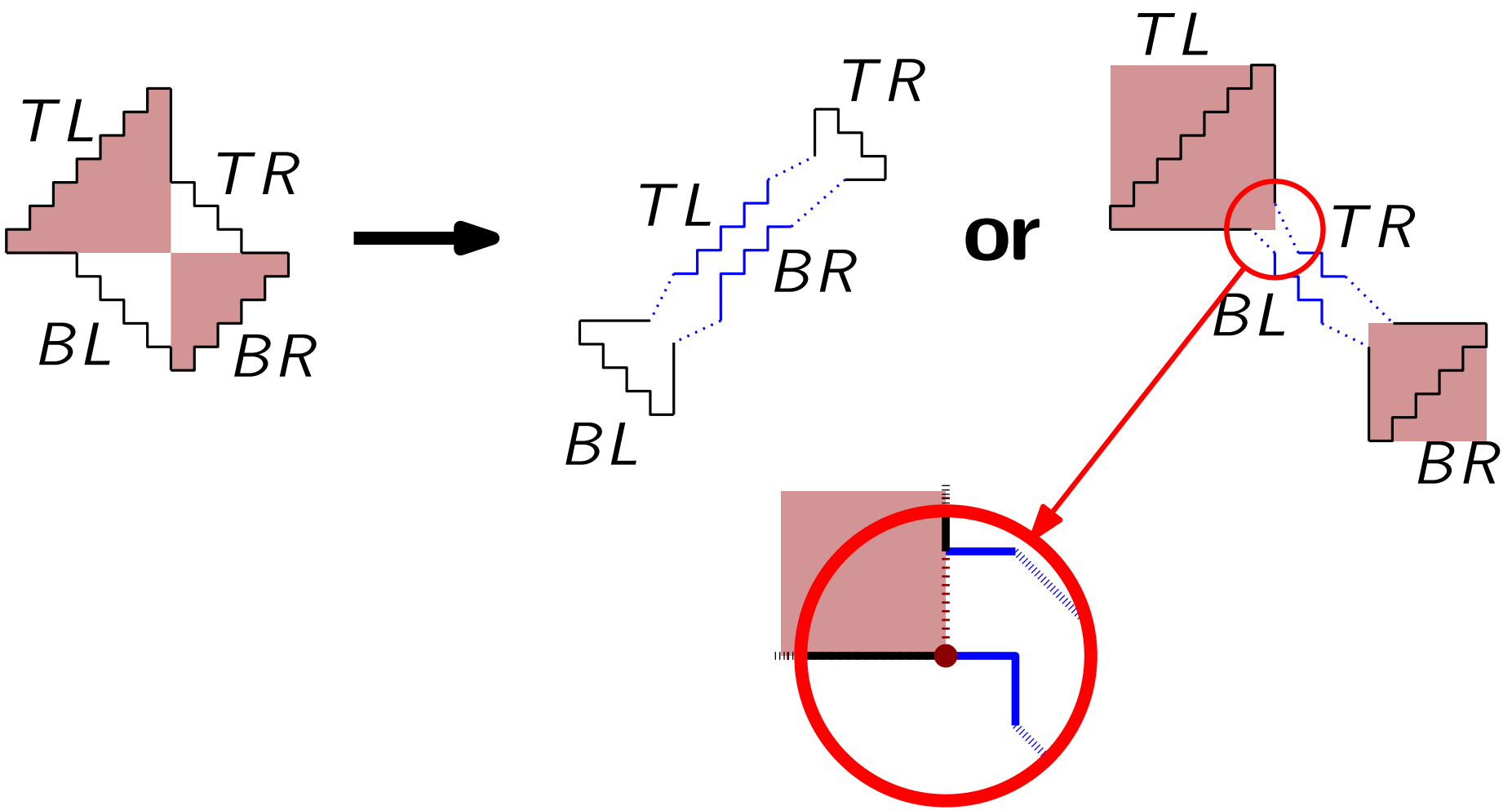
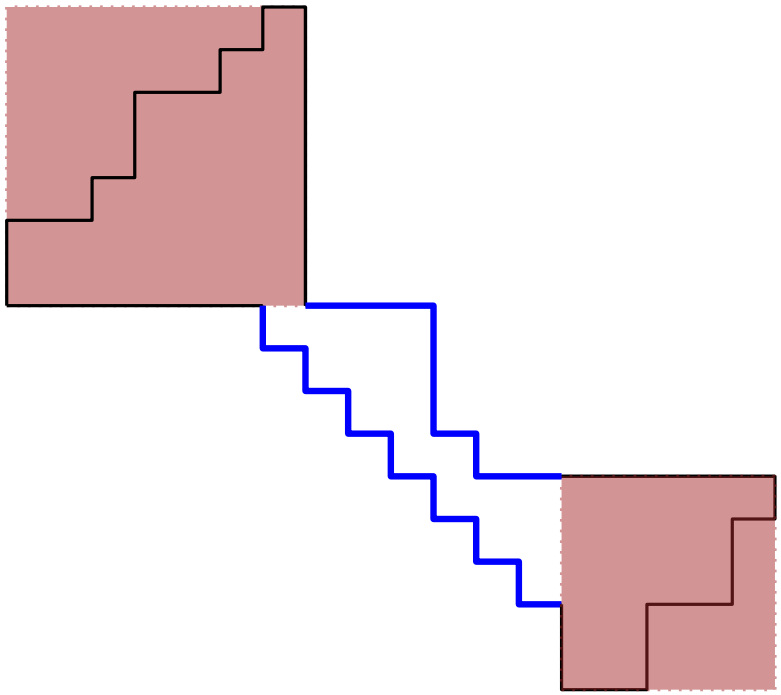


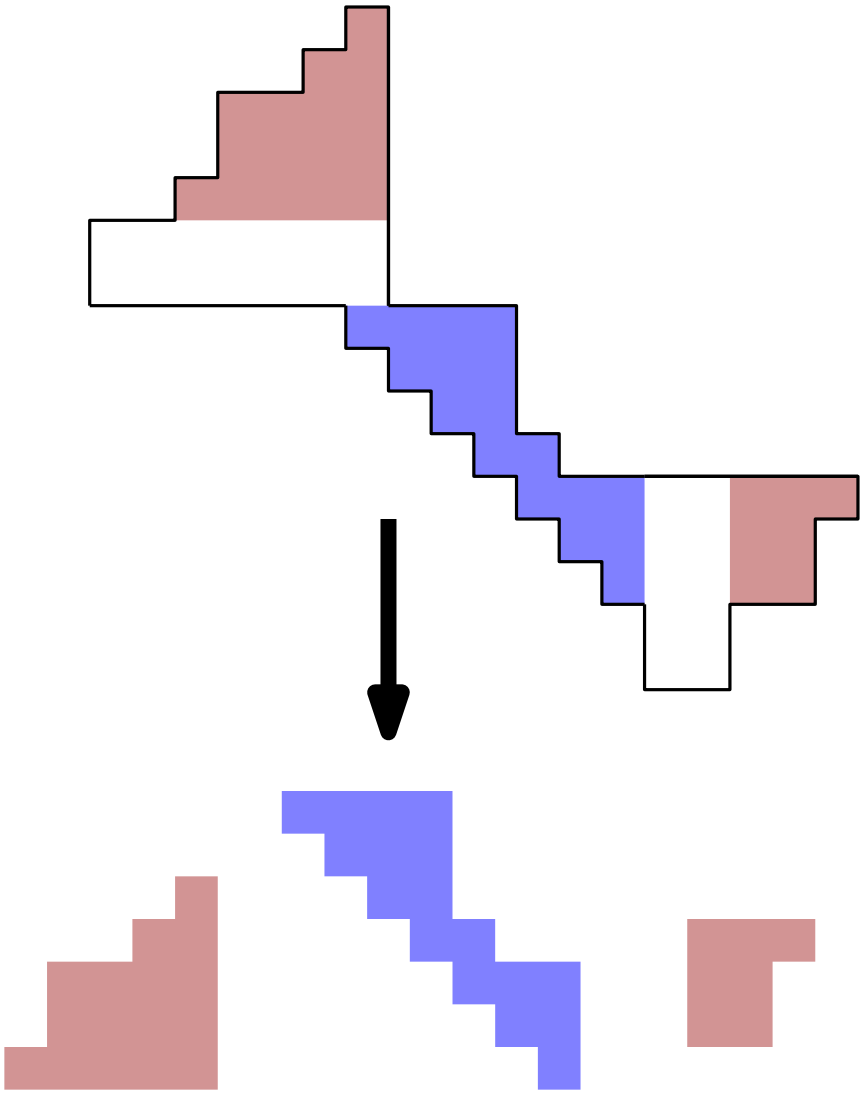
Computing Area of a Candidate

Given a candidate . . .



Computing Area of a Candidate

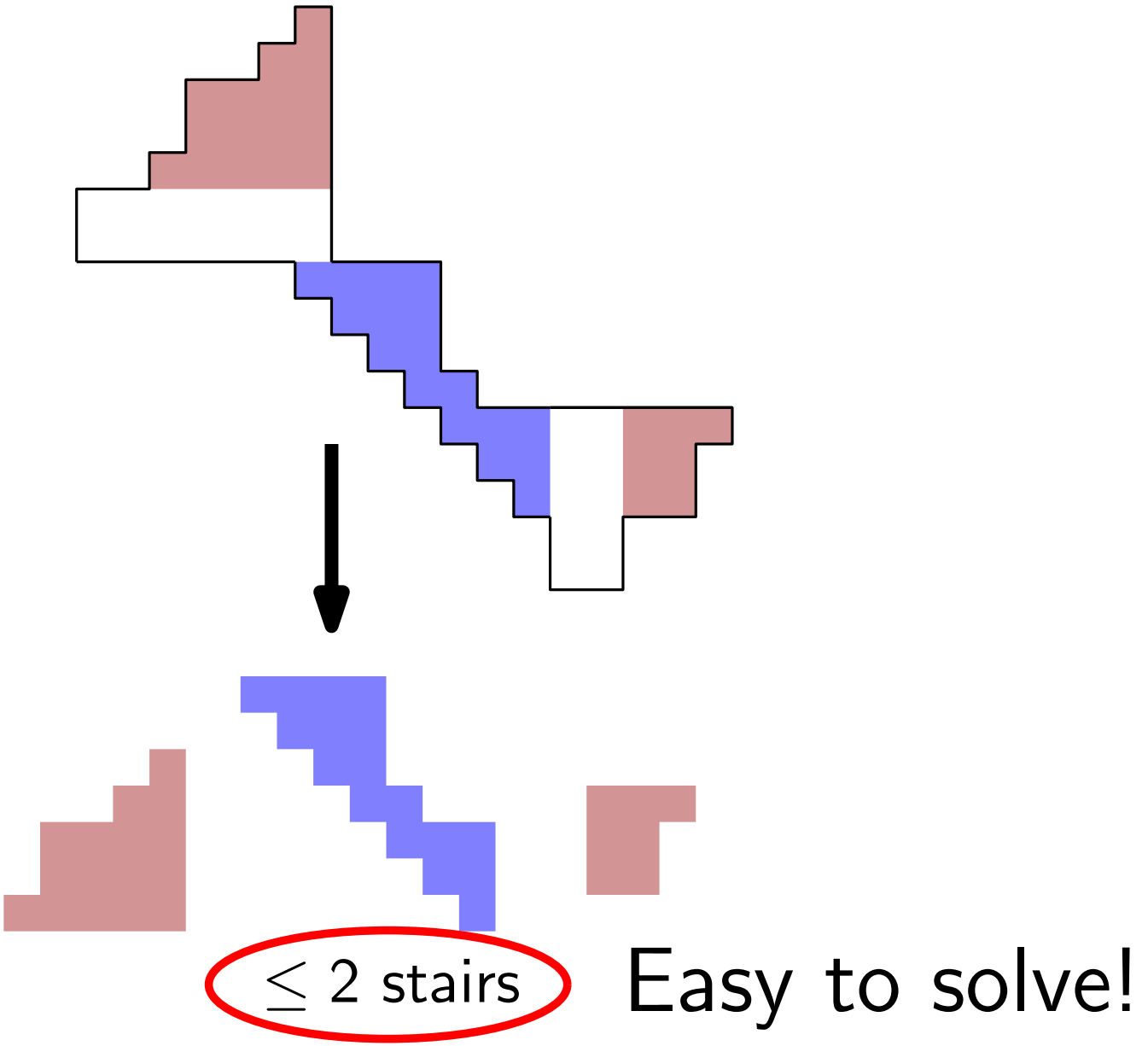
Given a candidate . . .



≤ 2 stairs

Computing Area of a Candidate

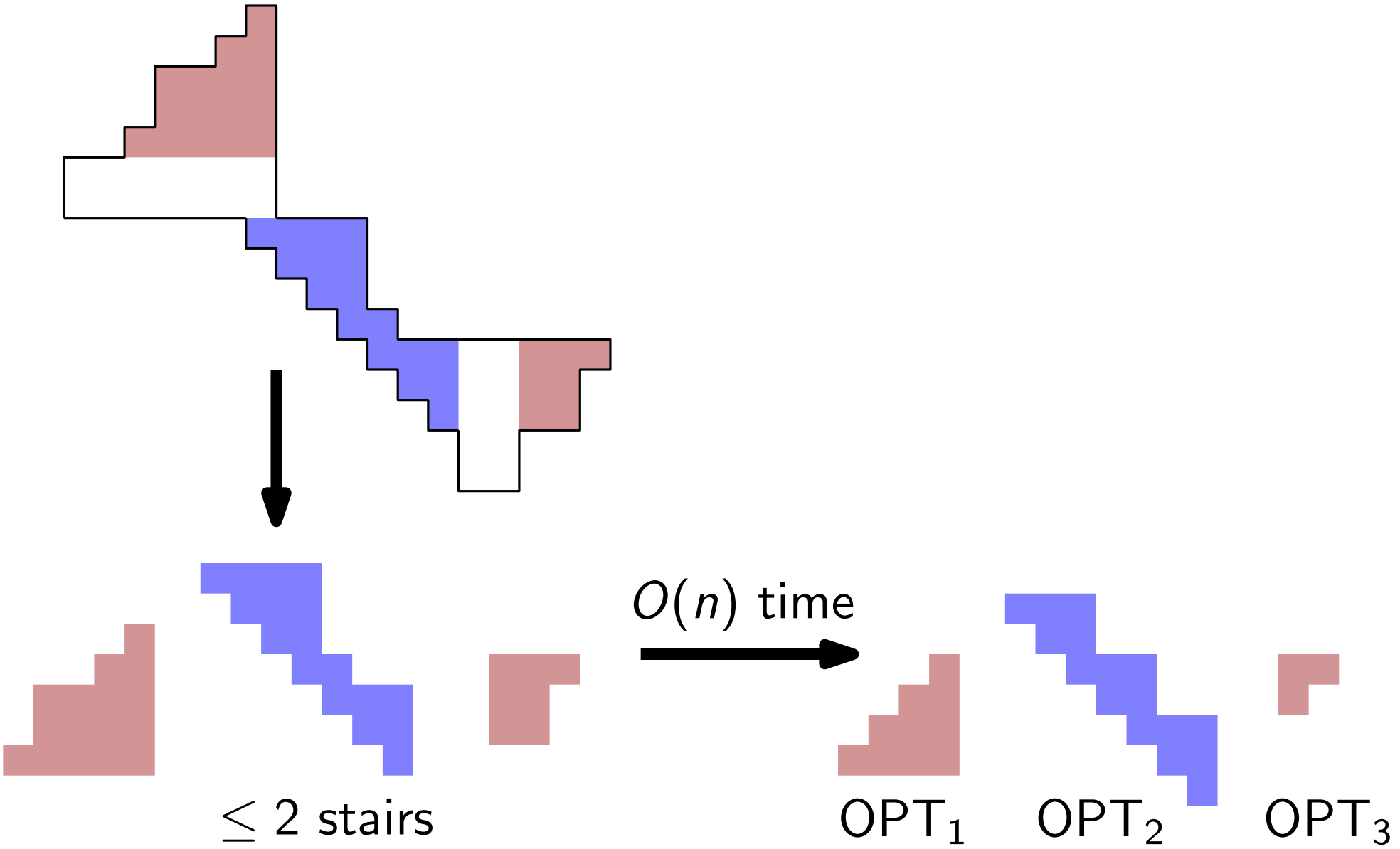
Given a candidate . . .



Easy to solve!

Computing Area of a Candidate

Given a candidate . . .



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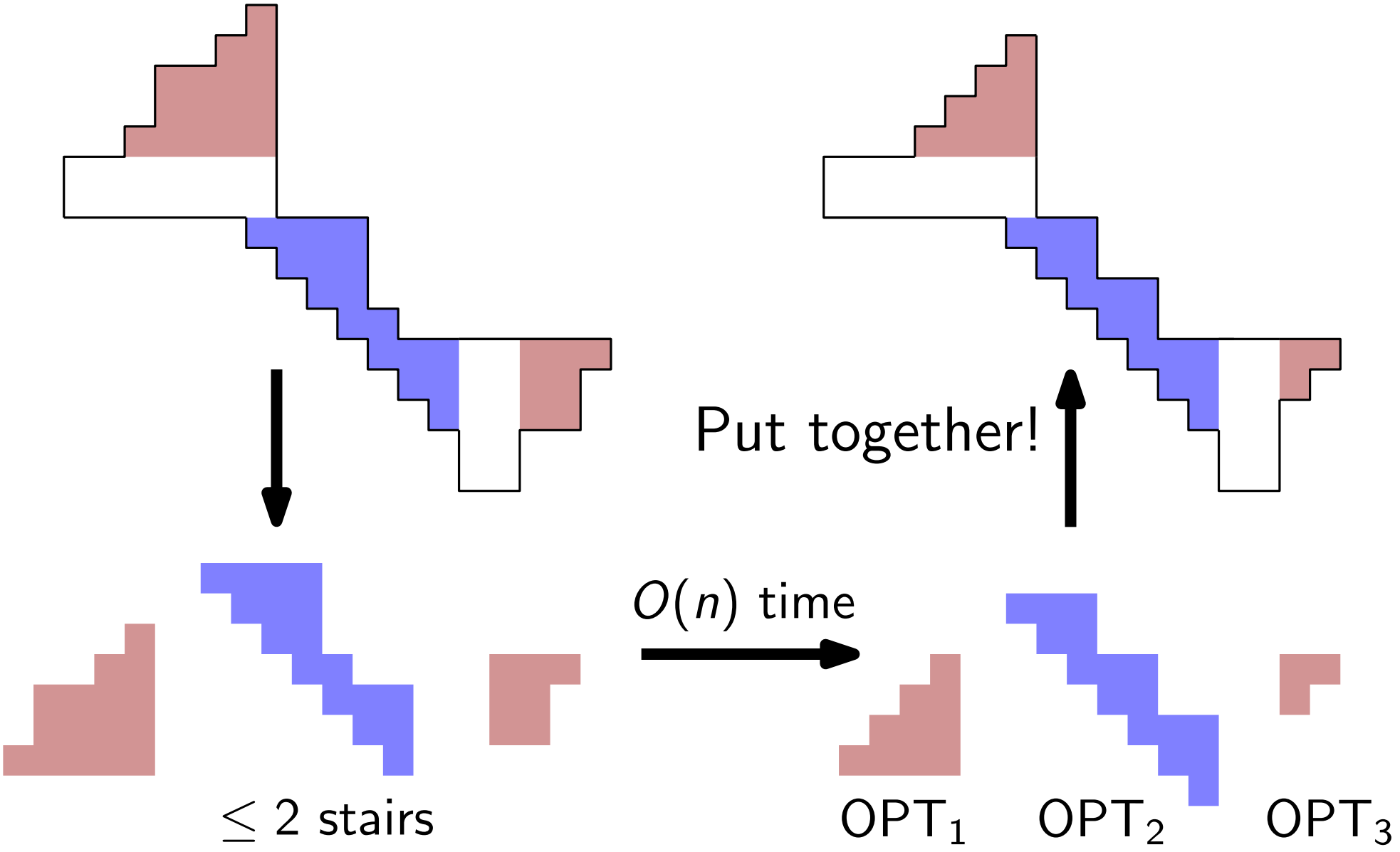

Computing Area of a Candidate

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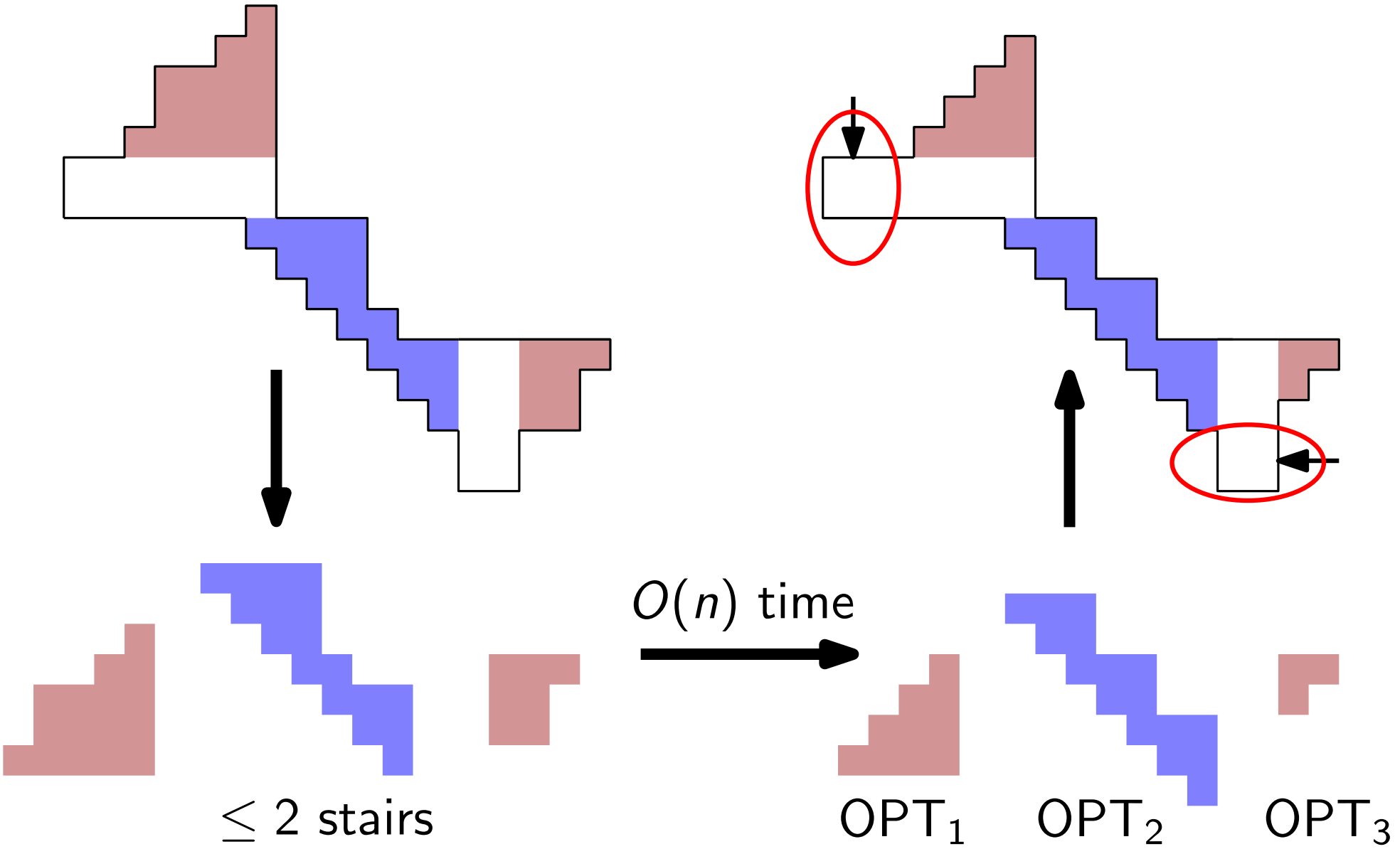
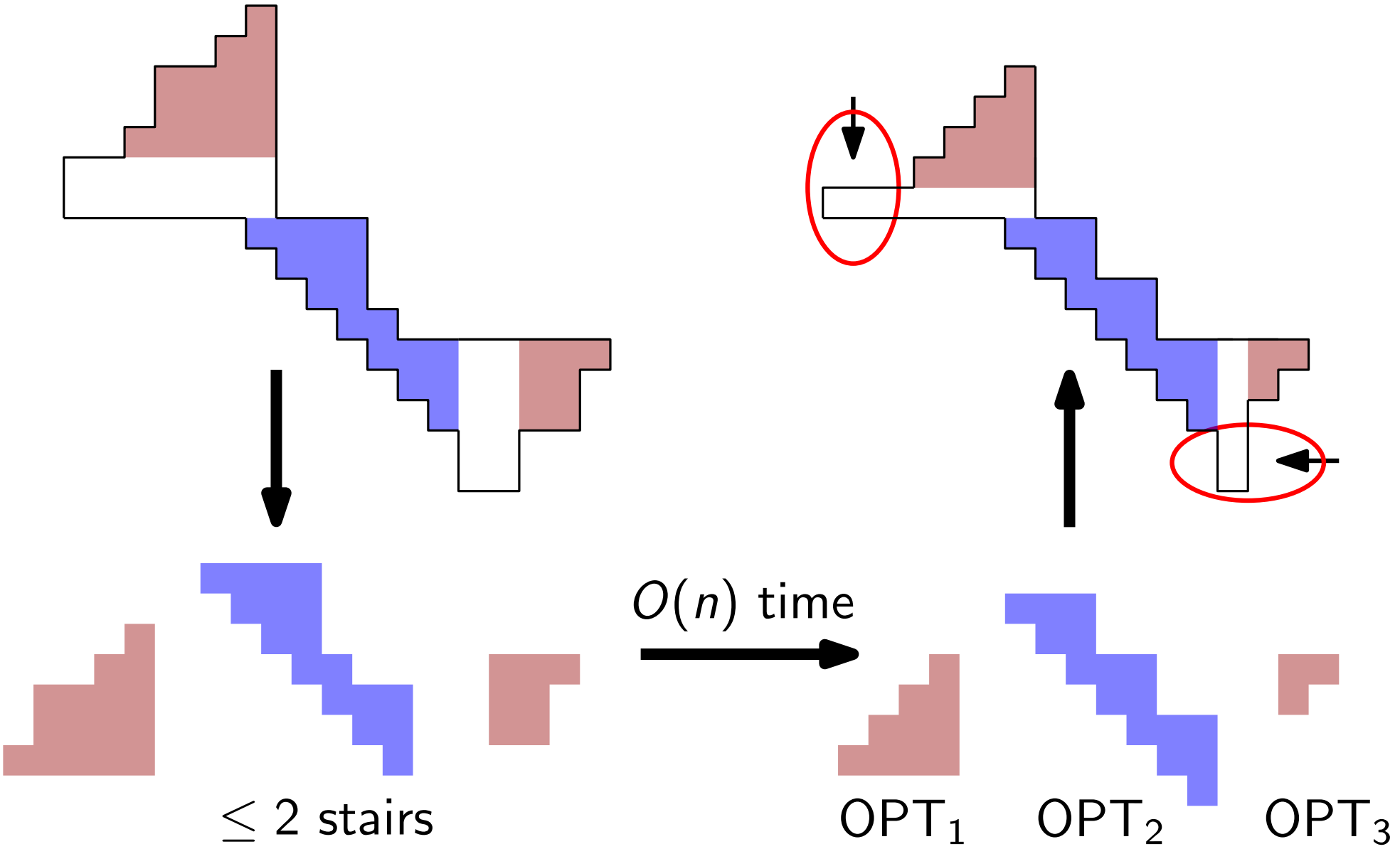


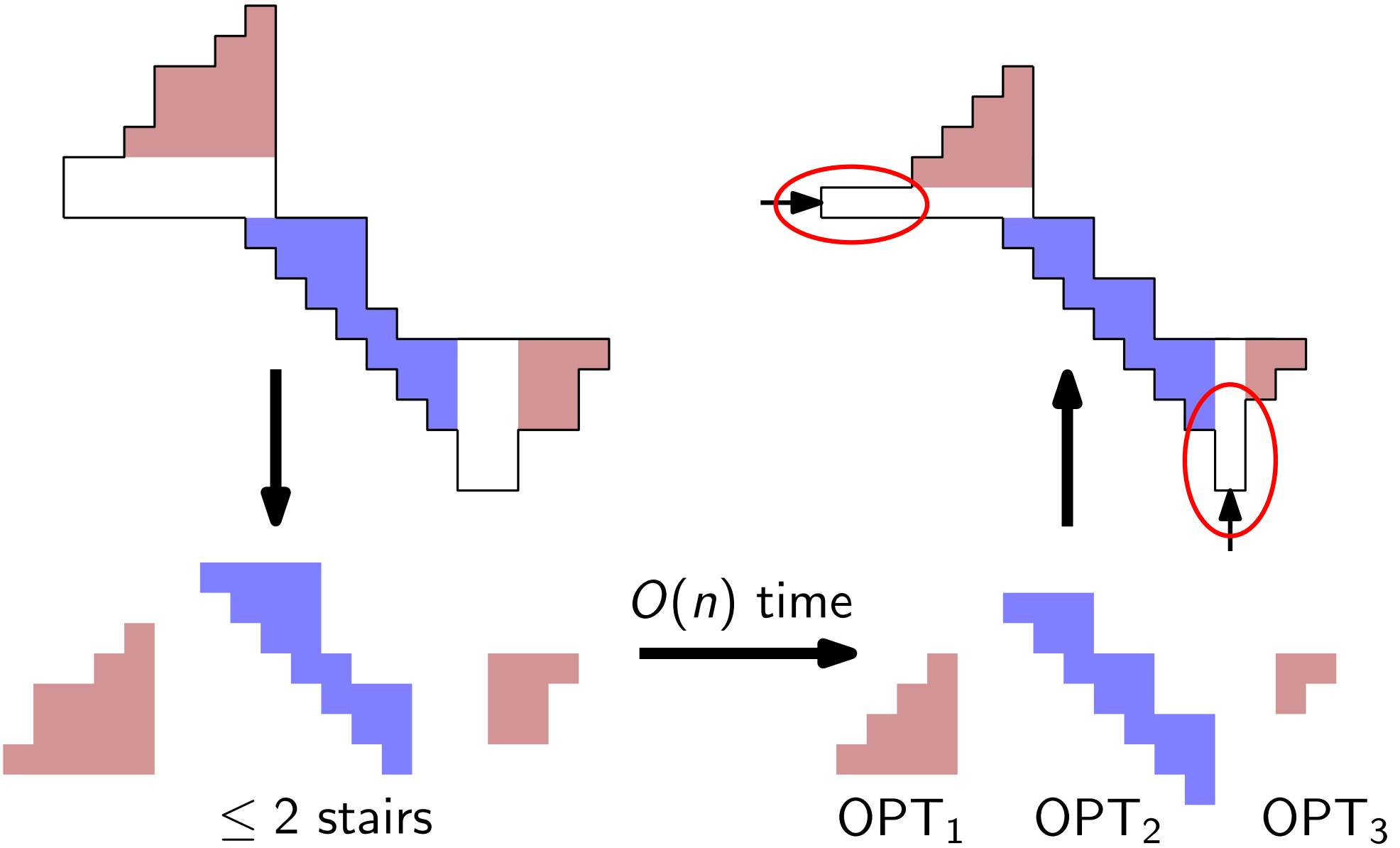
Computing Area of a Candidate

Given a candidate . . .



Computing Area of a Candidate

Given a candidate . . .



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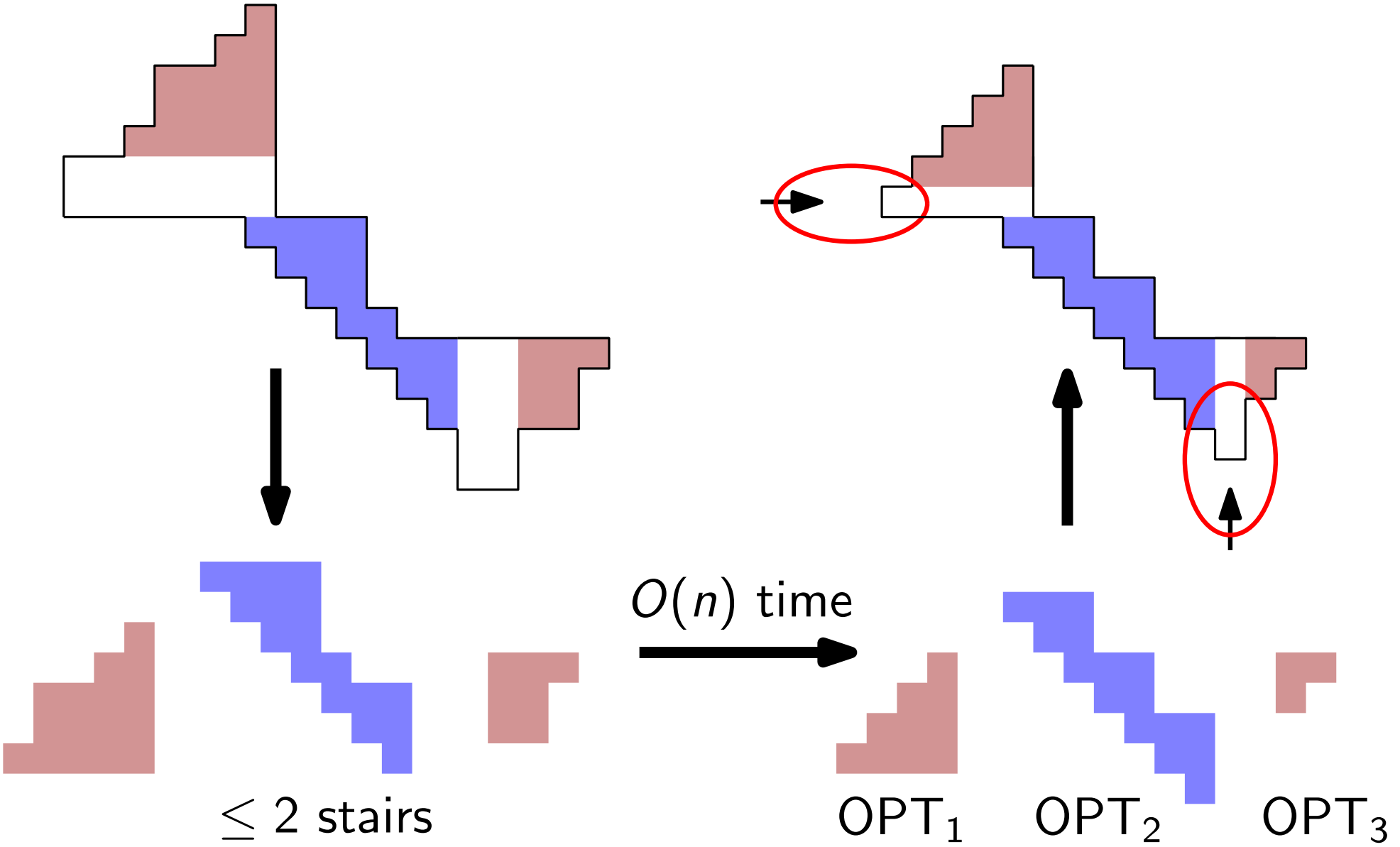


Algorithm

Algorithm:

For every candidate $\times O(1)$

compute $\mathrm{OPT}_{\mathrm{candidate}}$ $O(n)$

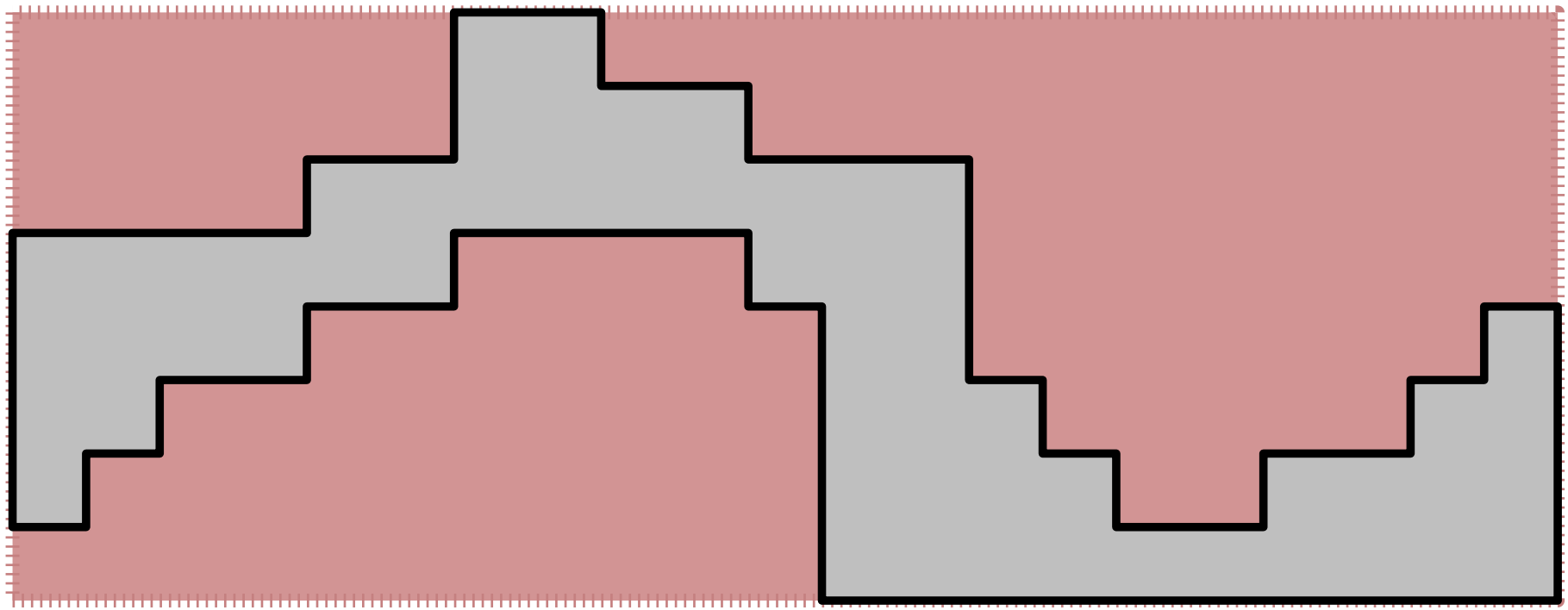
$$\mathrm{OPT} = \min \mathrm{OPT}_{\mathrm{candidate}}$$

$O(n)$

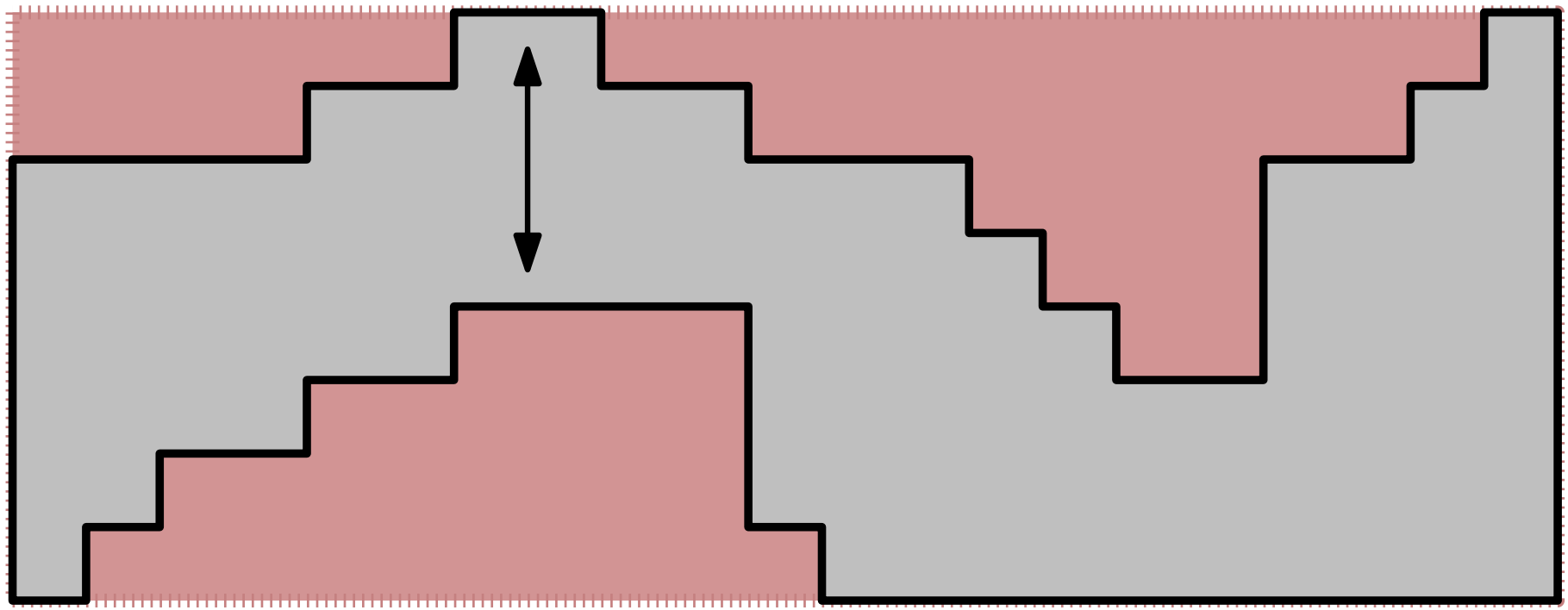
Minimizing Bounding Box of x-monotone Polygons

Canonical x-monotone Polygons

Bounding Box (BB)



Canonical x-monotone Polygons



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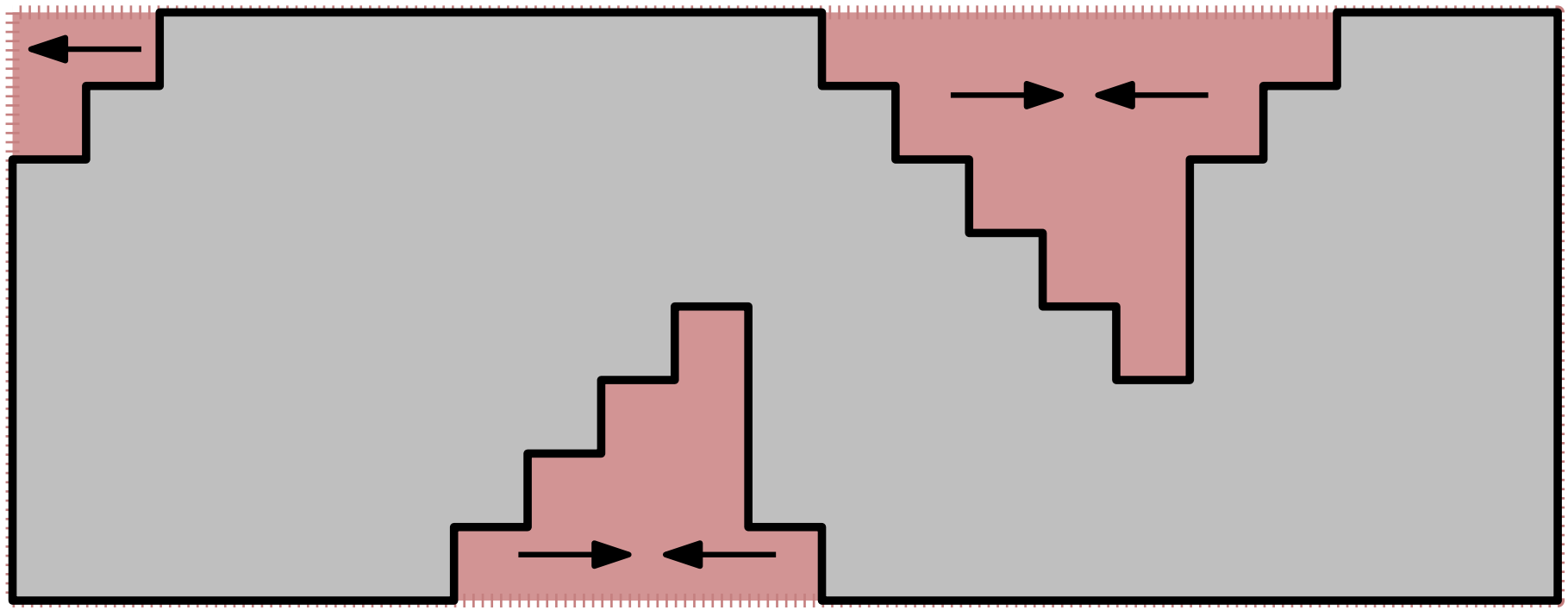

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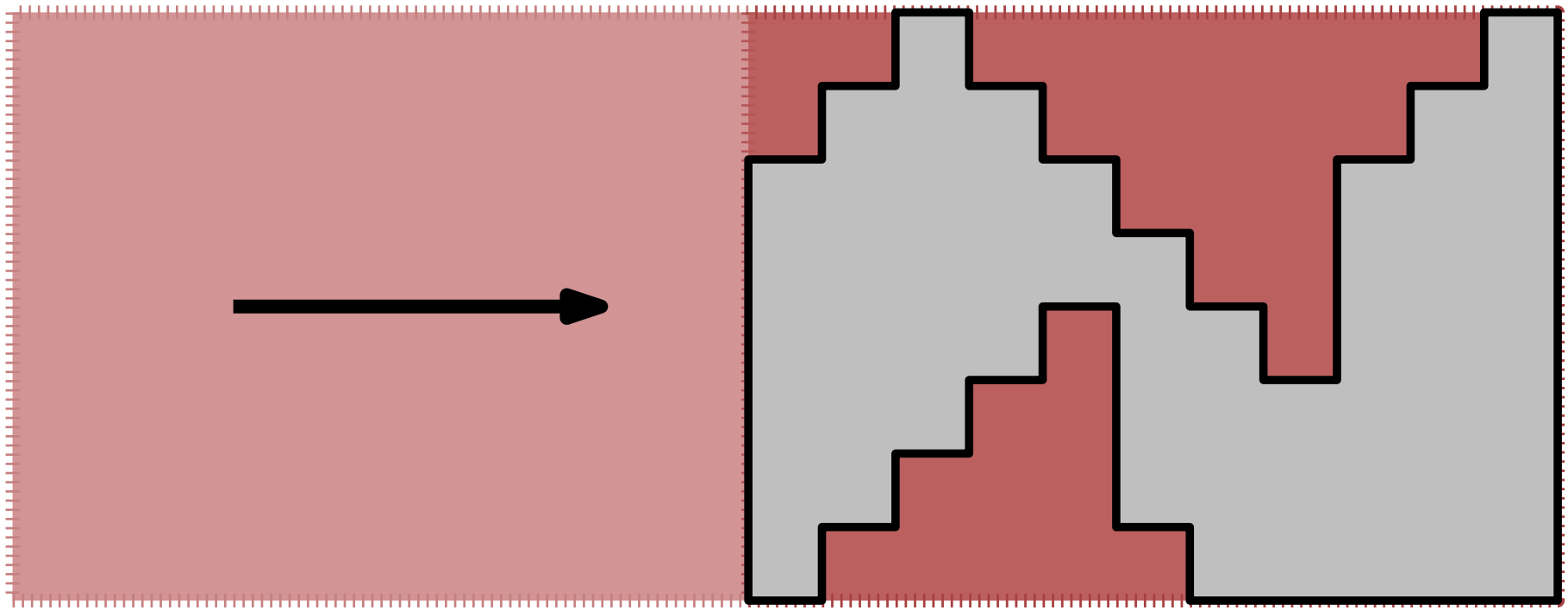
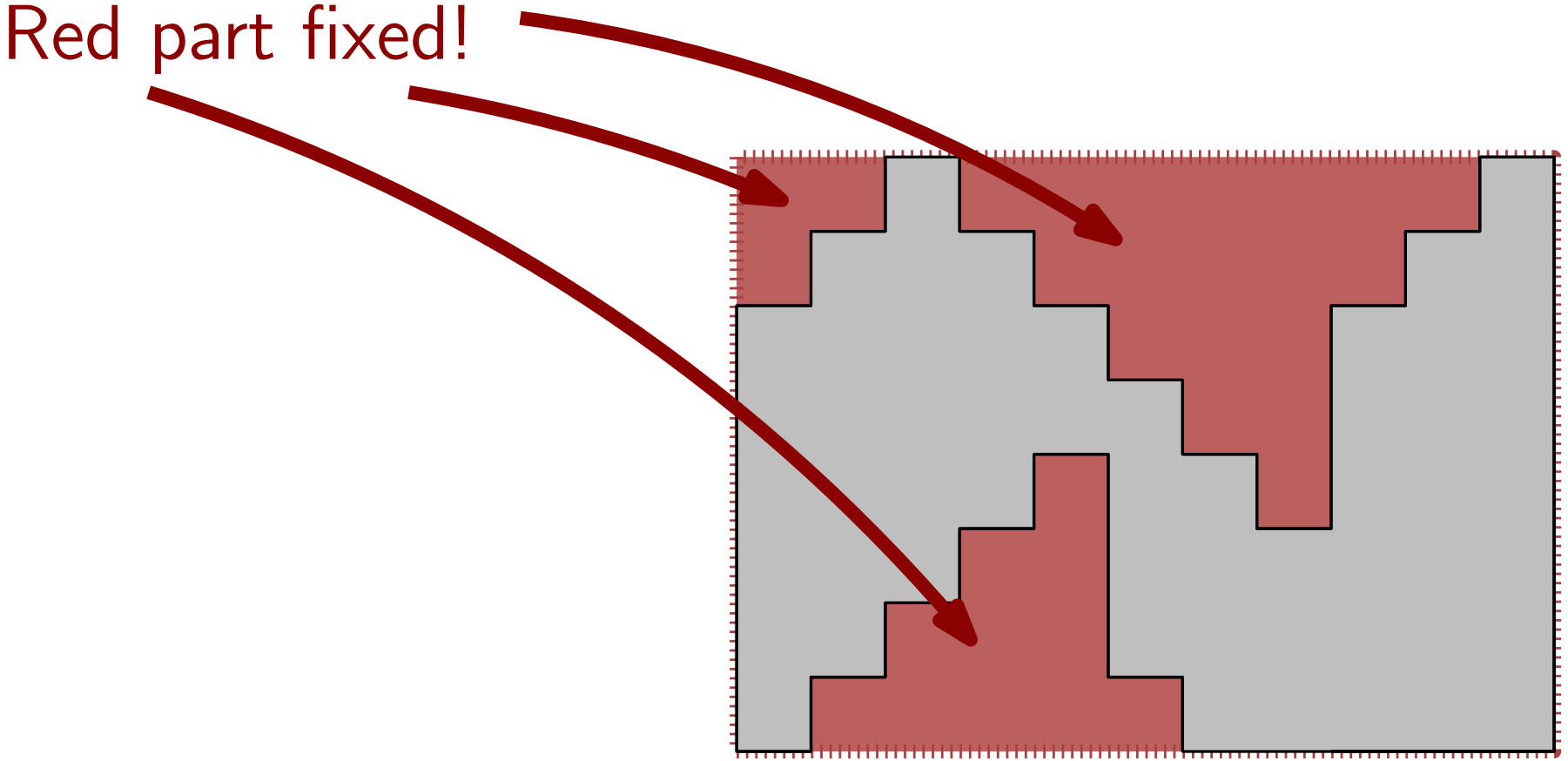


Canonical

Canonical x-monotone Polygons

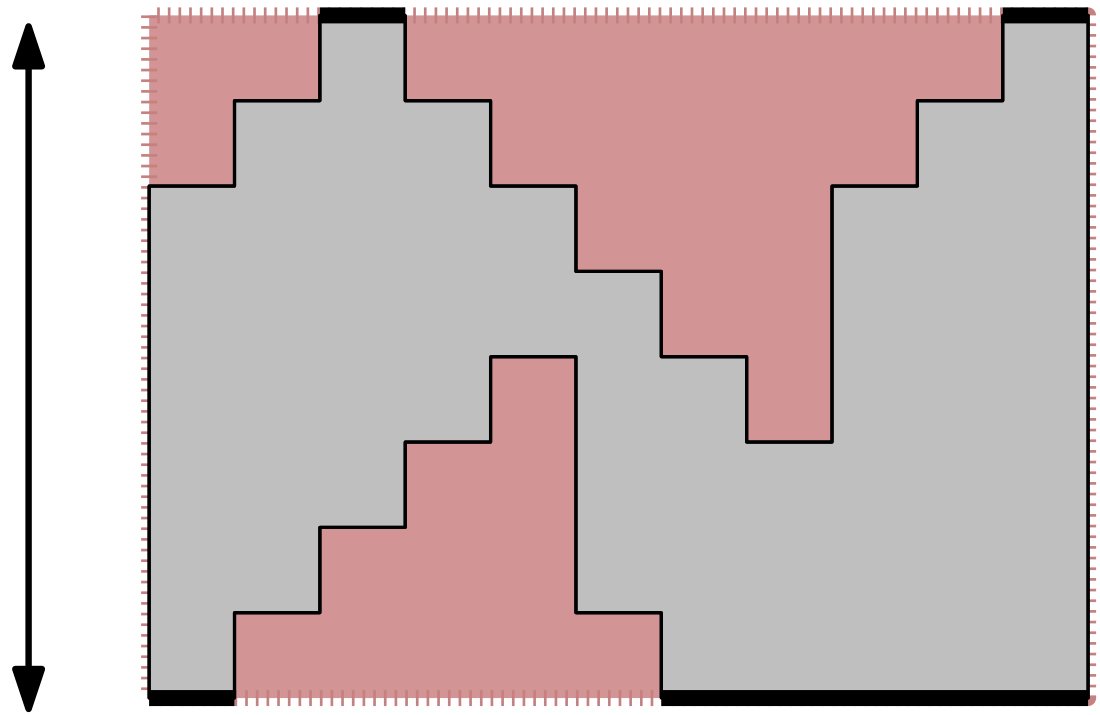


Canonical x-monotone Polygons

Red part fixed!

Only unknowns:

a) Height of BB



b) Width of extreme edges

Canonical x-monotone Polygons

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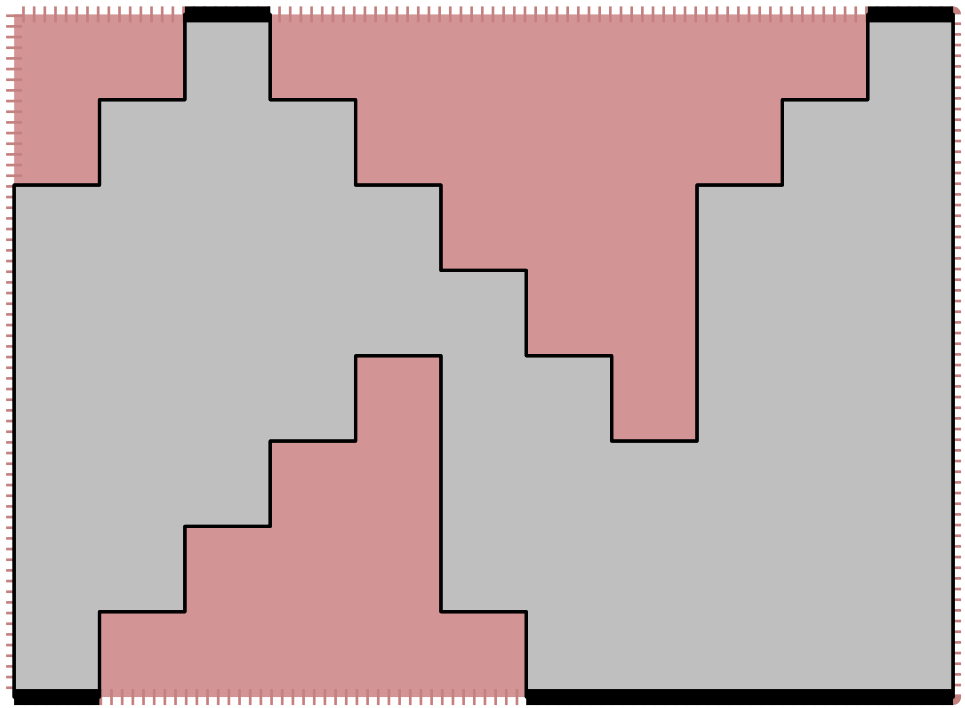


b) Width of extreme edges

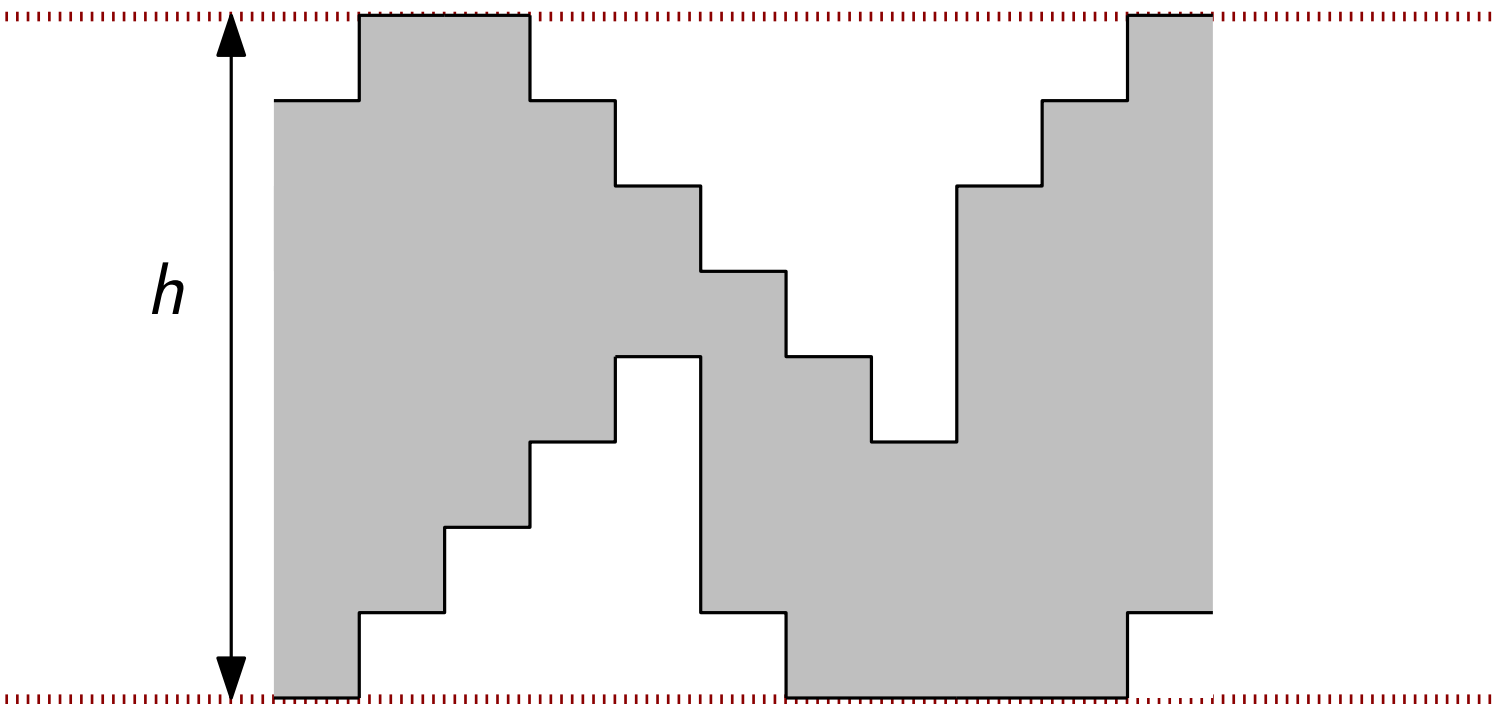
Algorithm:

Guess height of BB

Use DP to find widths

DP Algorithm

$T[\ ,\ ]$



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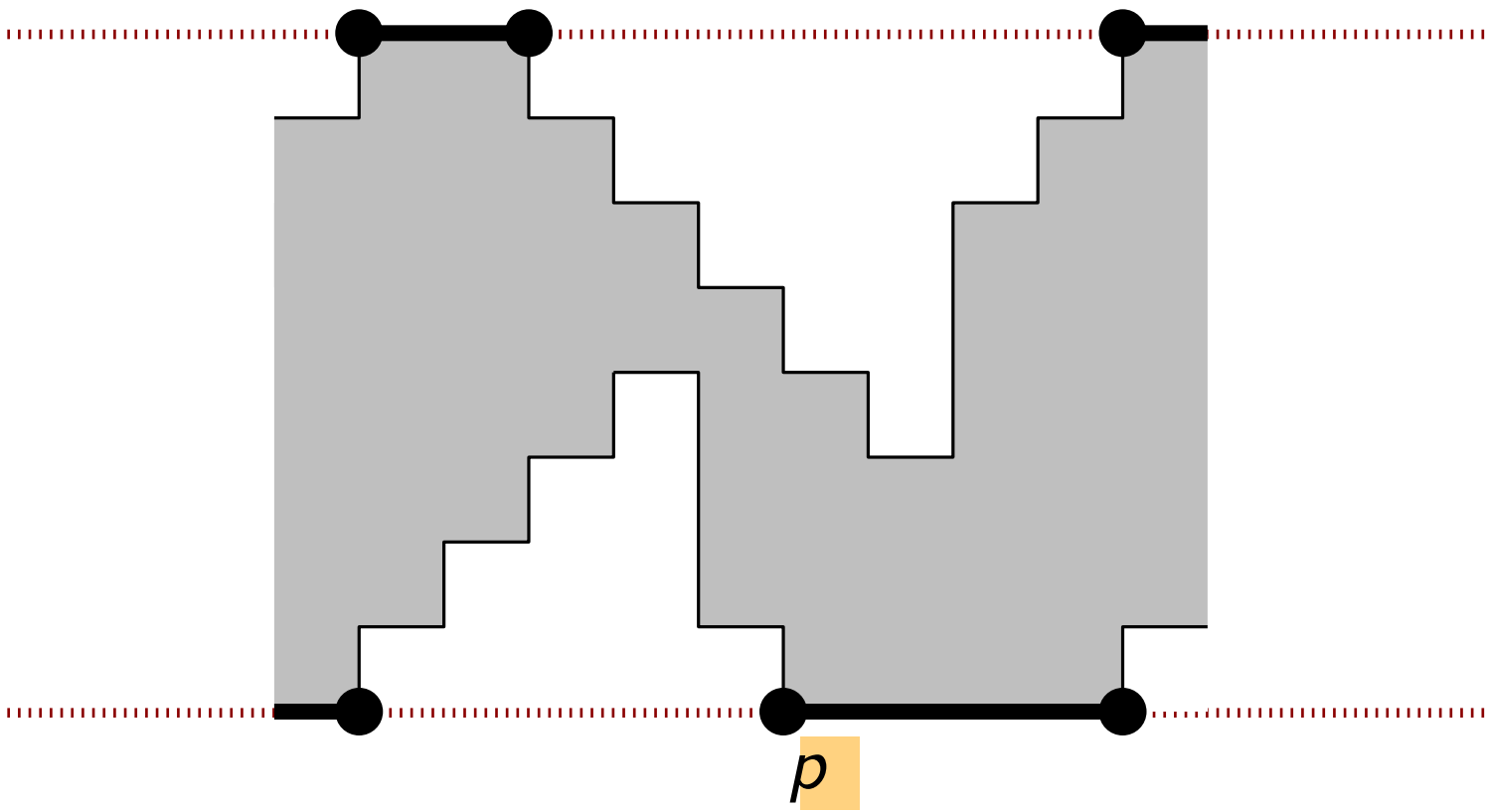
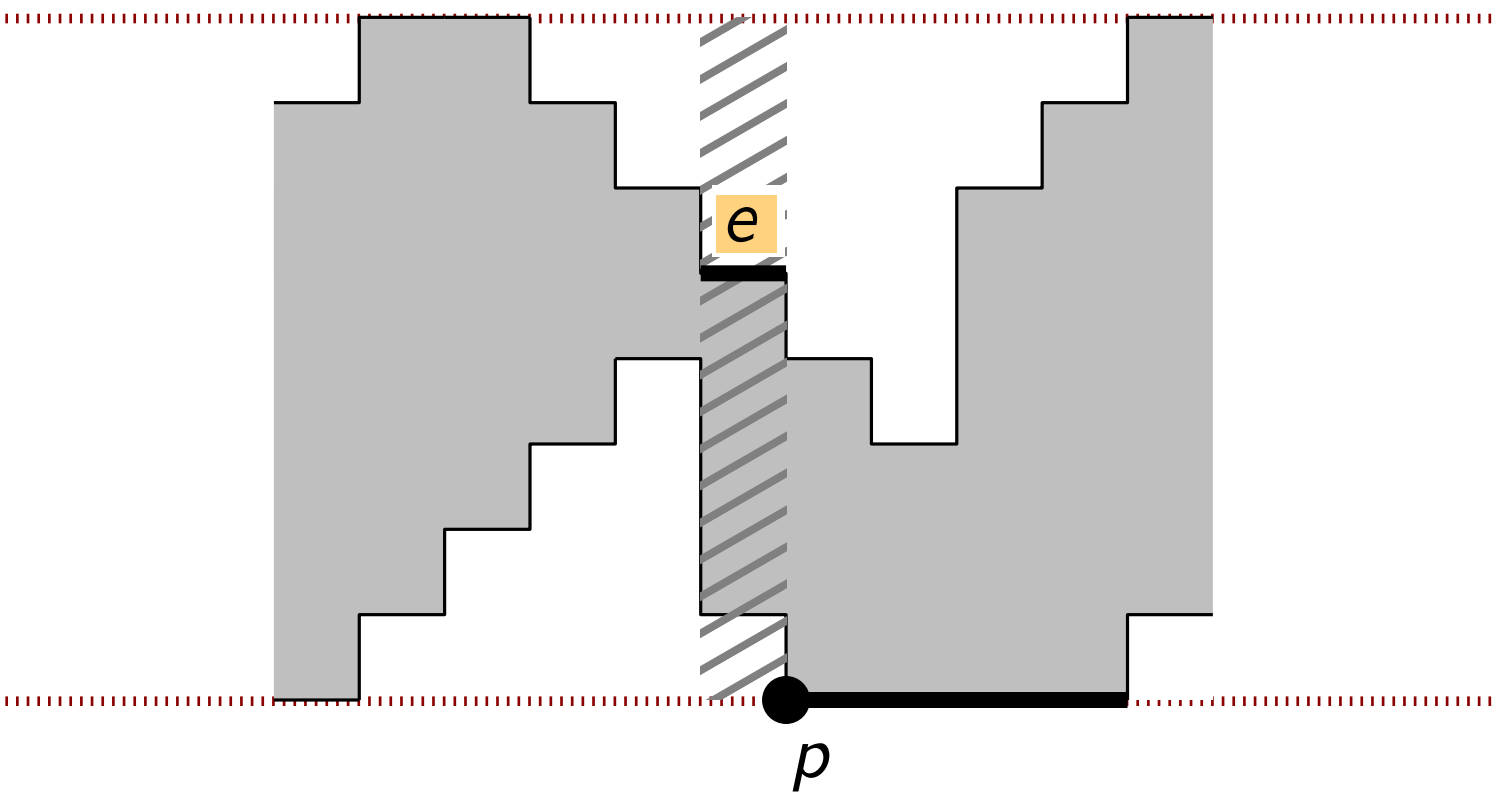
$T[p, \ ]$



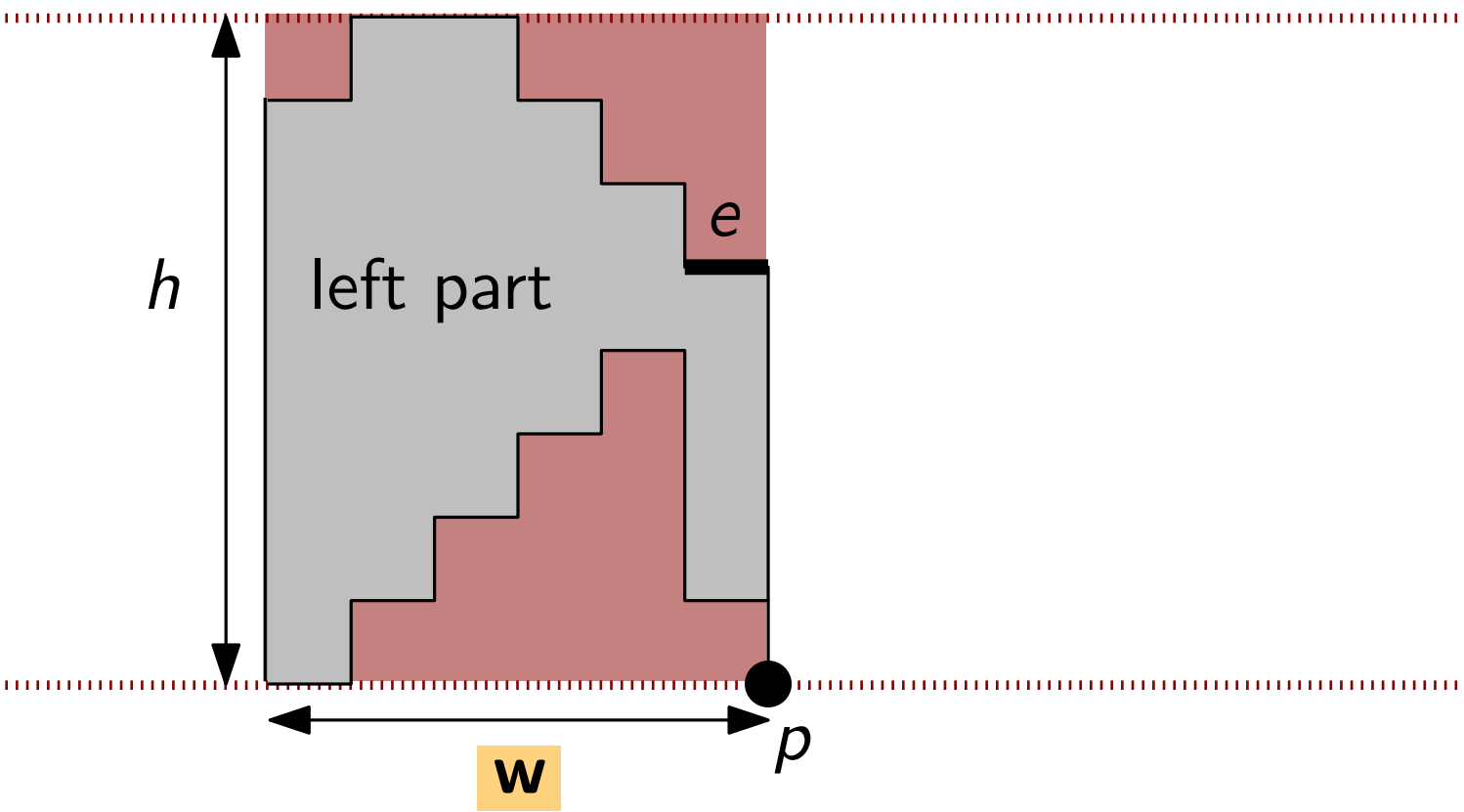
DP Algorithm

$T[p, e]$



DP Algorithm

$$T[p, e] = \text{minimum } \mathbf{w} \text{ s.t. left part} \in \text{BB}(\mathbf{w} \times h)$$



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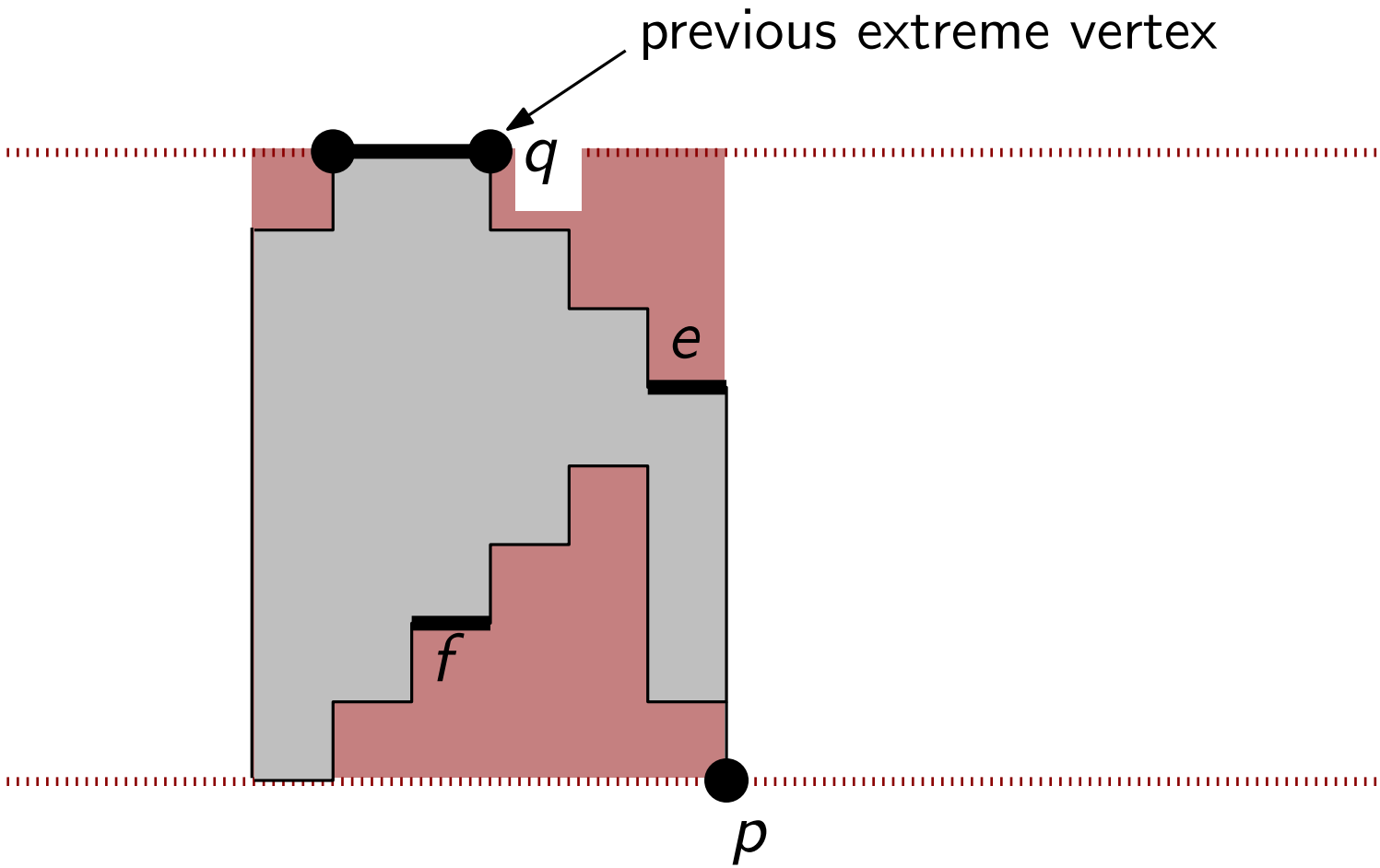

DP Algorithm

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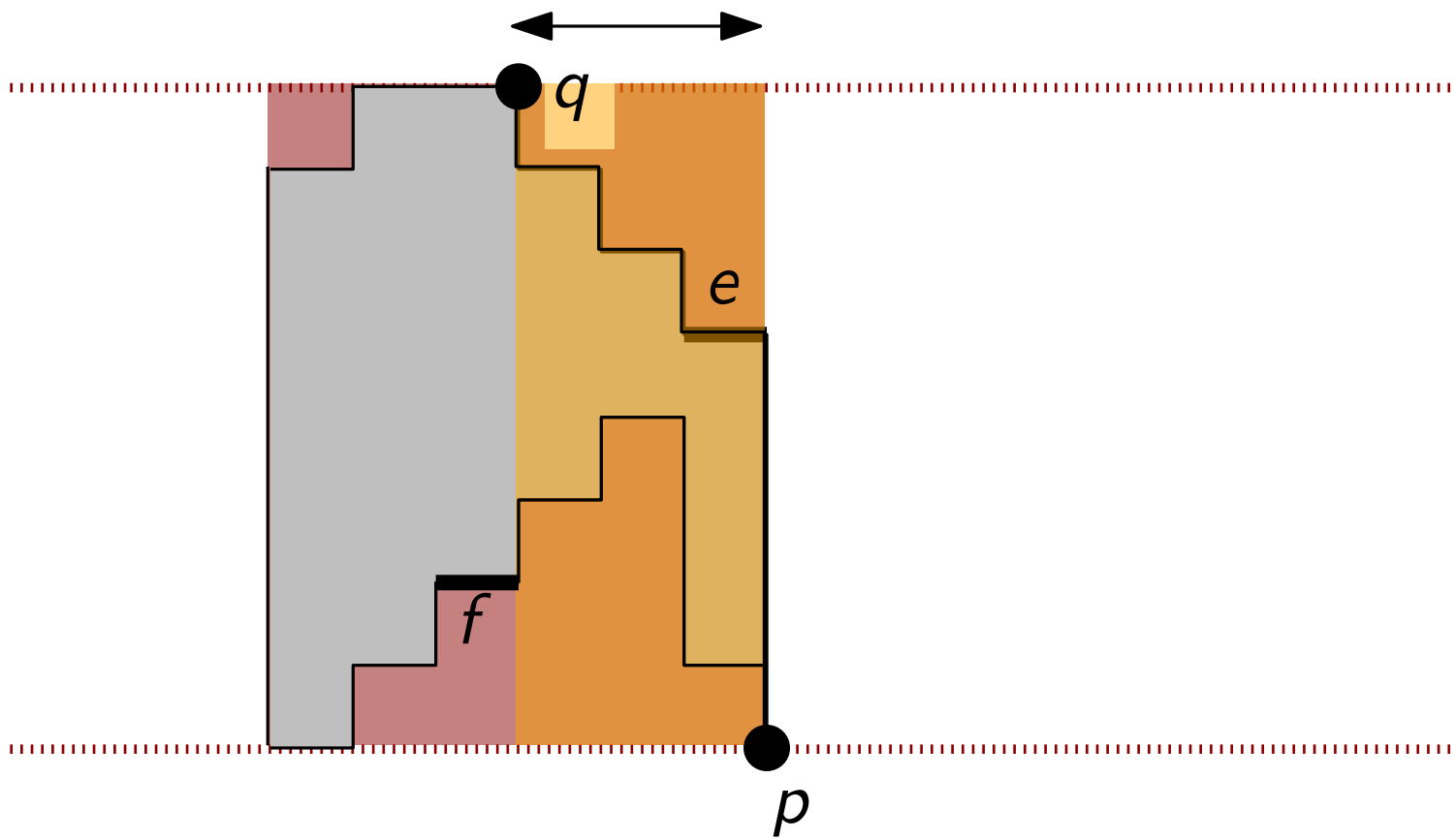
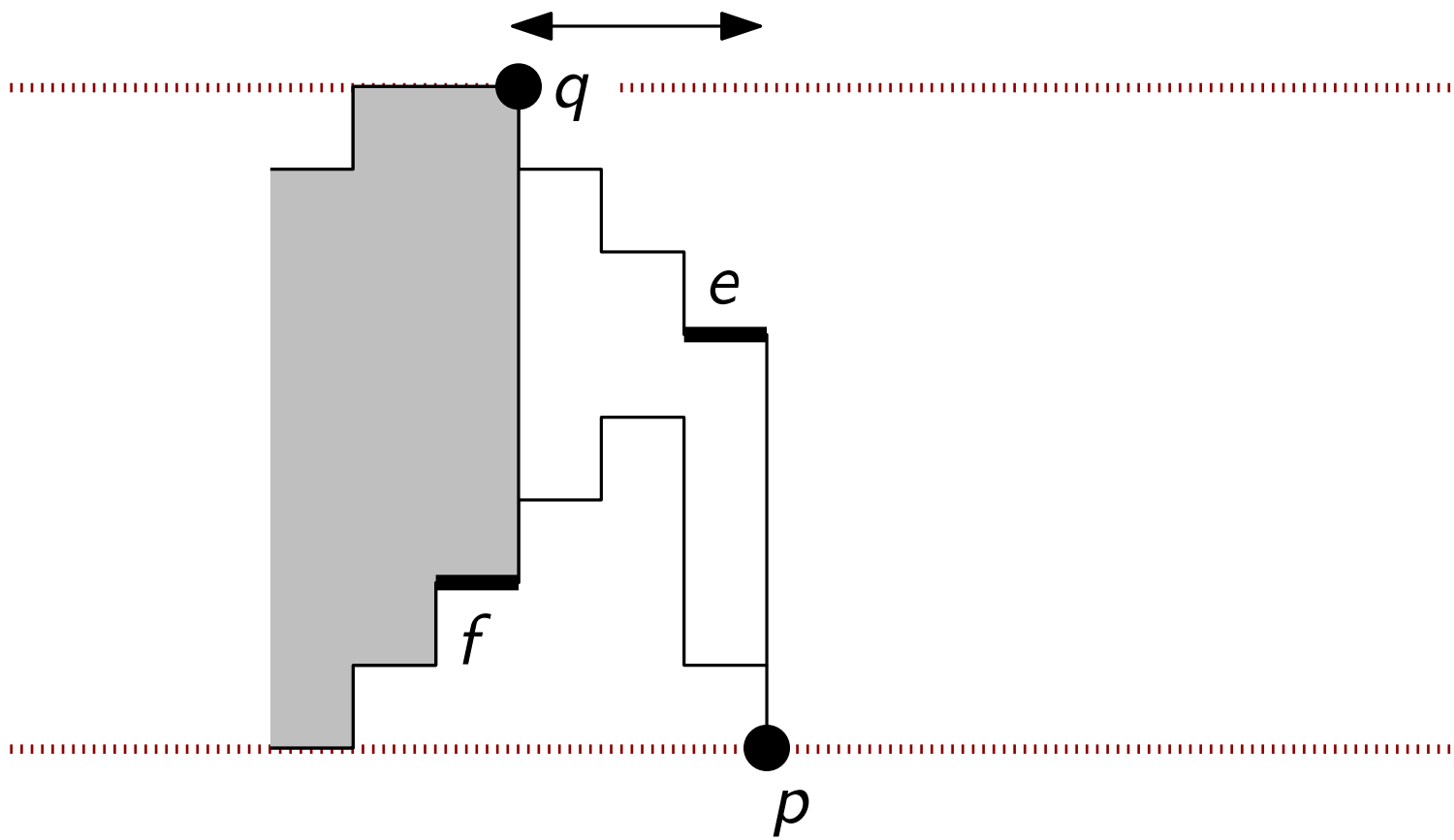
w' is fixed



DP Algorithm

$$T[p, e] = \text{minimum } \mathbf{w} \text{ s.t. left part} \in \text{BB}(\mathbf{w} \times h)$$
$$= \quad T[q, f] \quad + \quad w'$$



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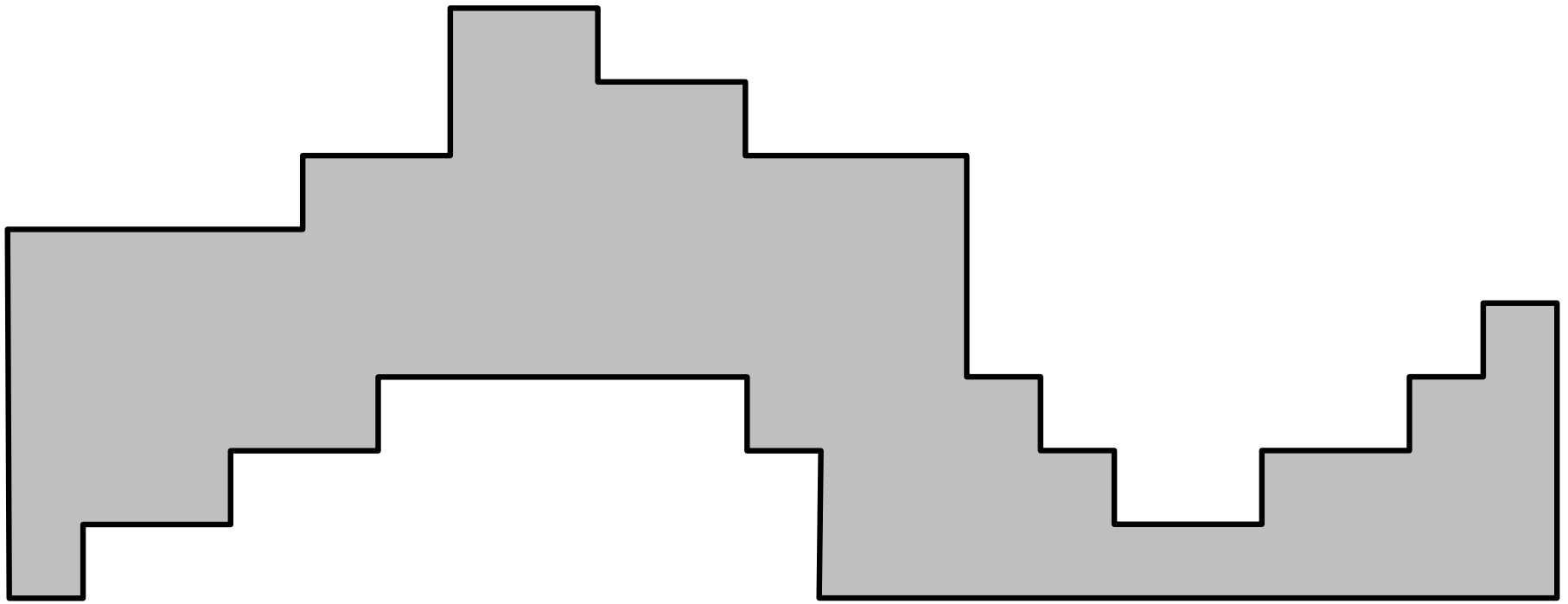
Algorithm:

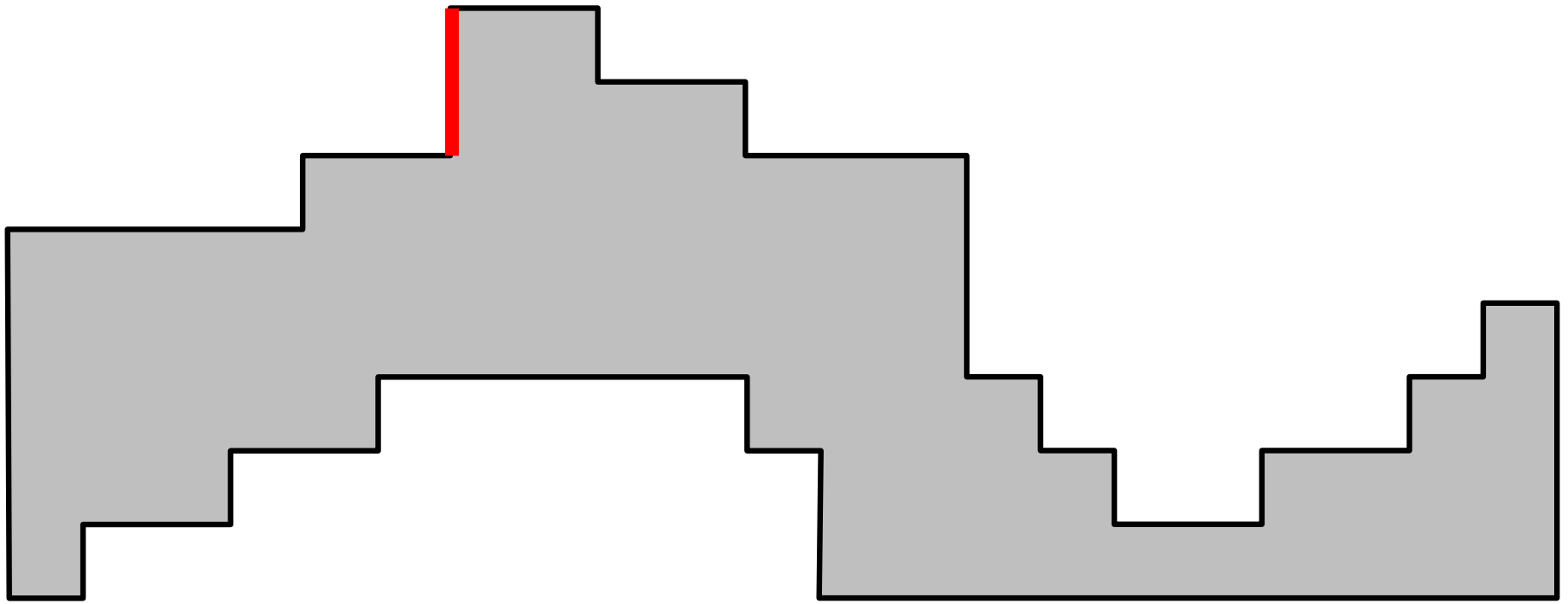
Guess height of BB	$O(n)$
Use DP to find widths	$O(n^2)$
	$O(n^3)$

Minimizing the Perimeter of x-monotone Polygons

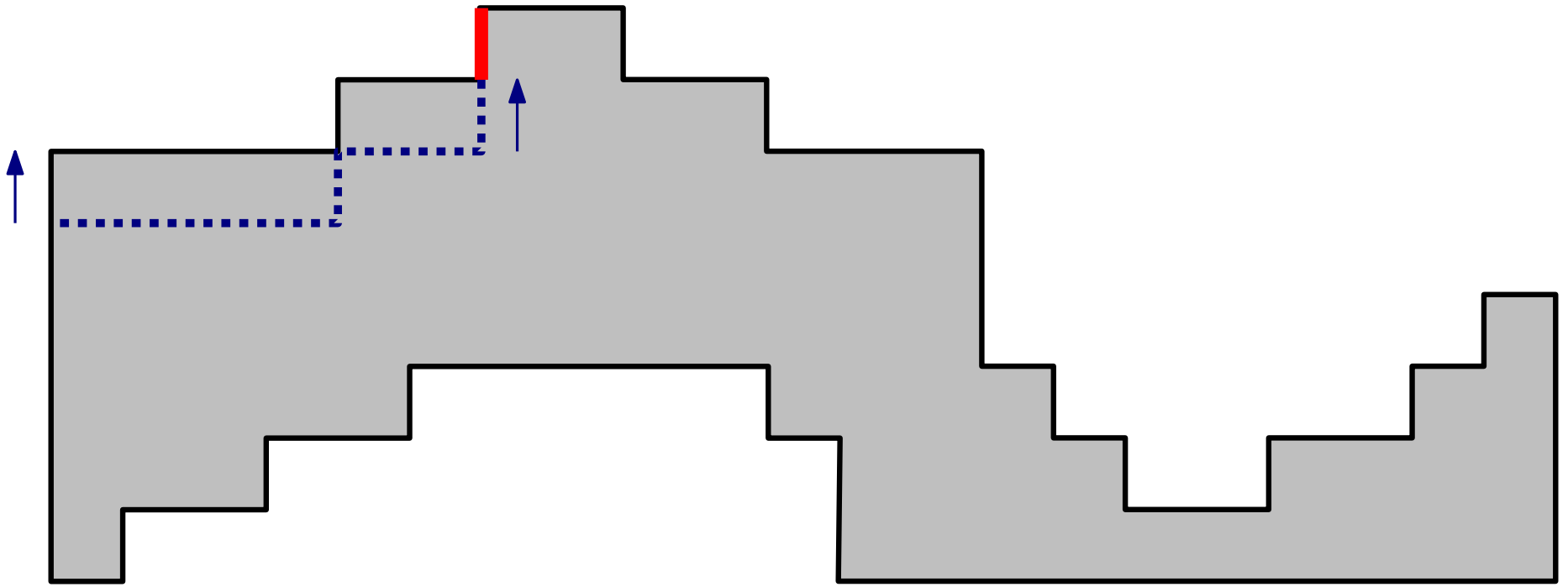
Canonical x-monotone Polygons



Canonical x-monotone Polygons



Canonical x-monotone Polygons



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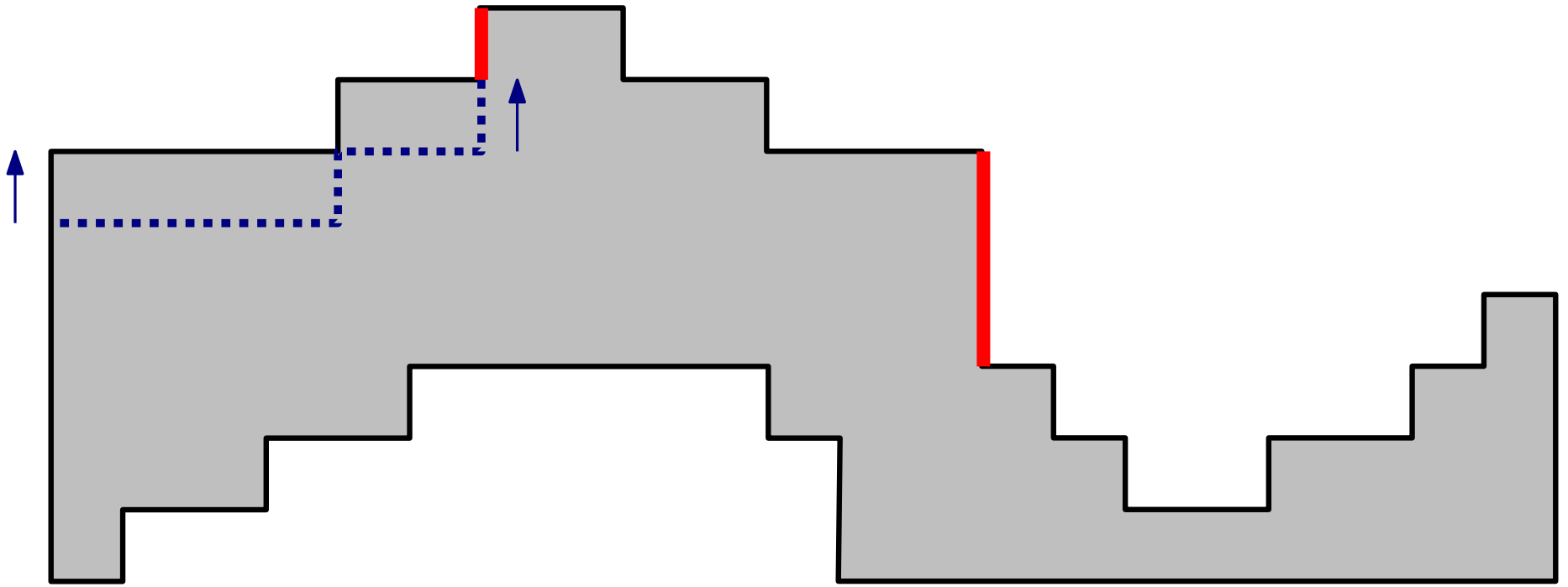
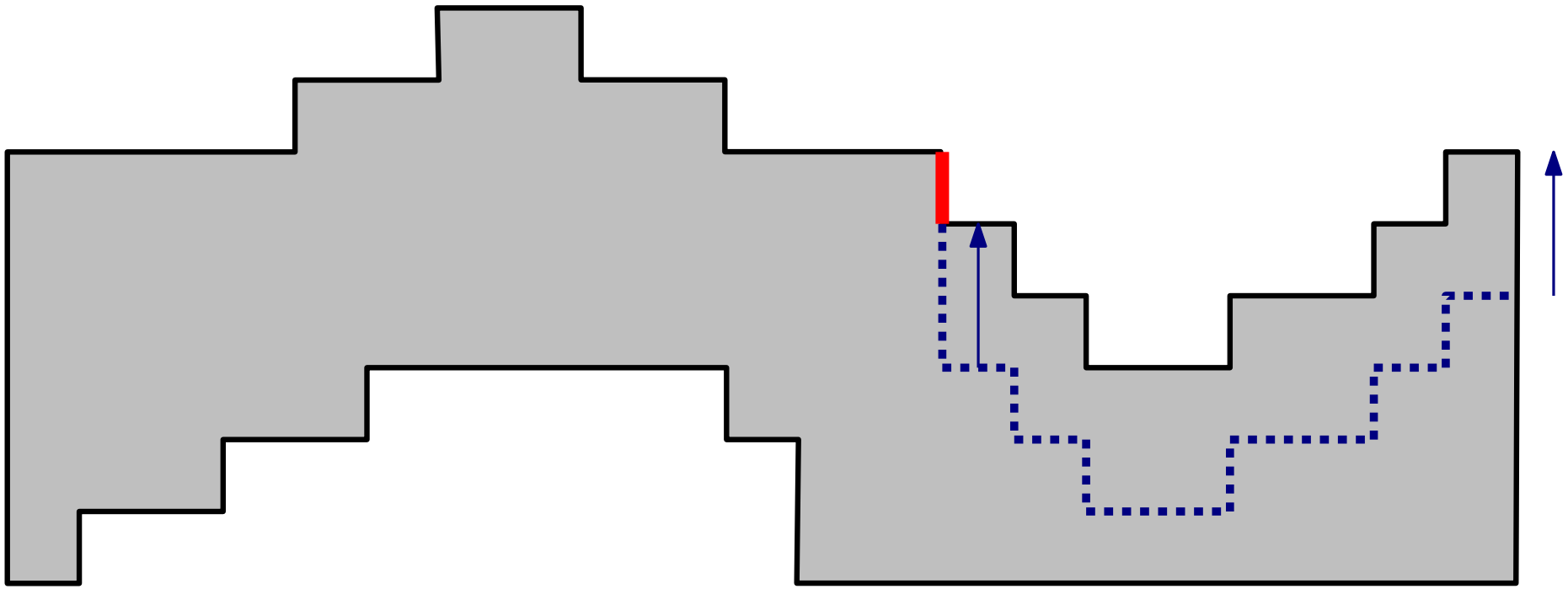


Canonical x-monotone Polygons



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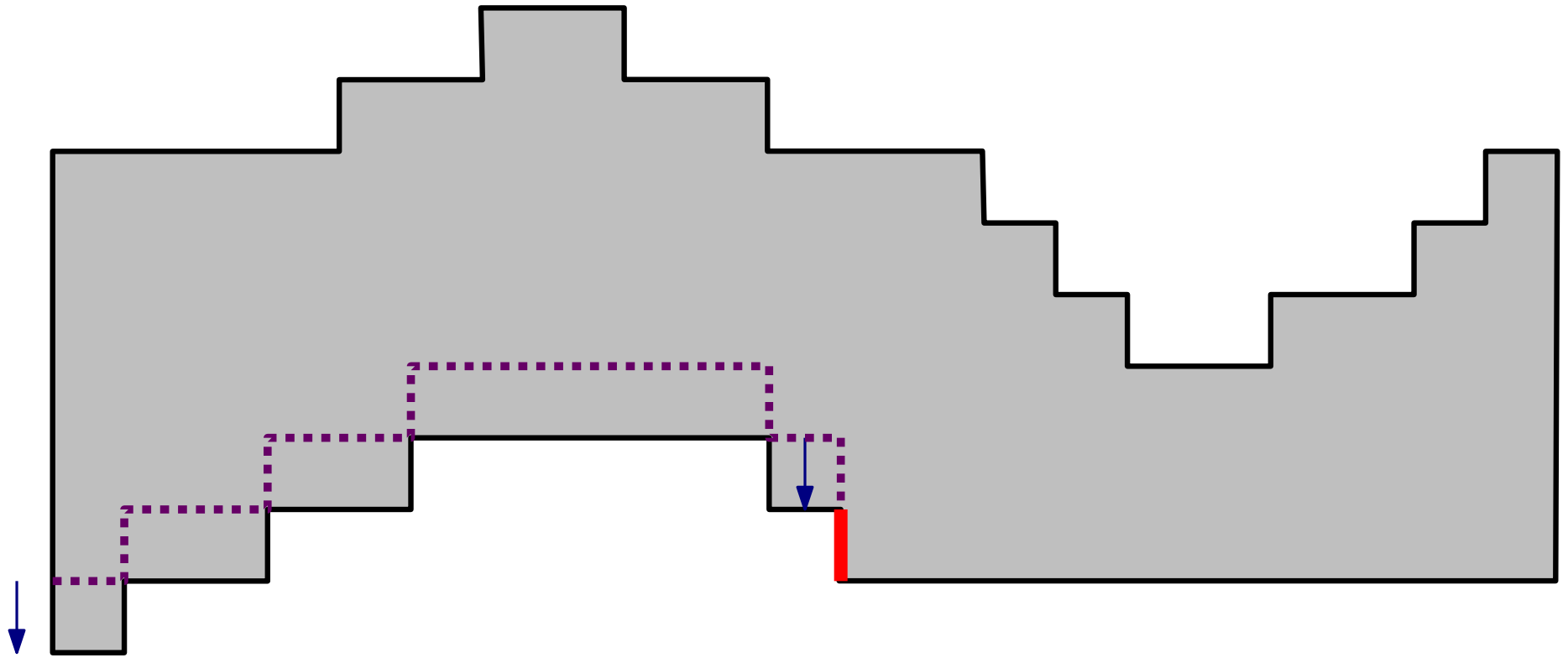
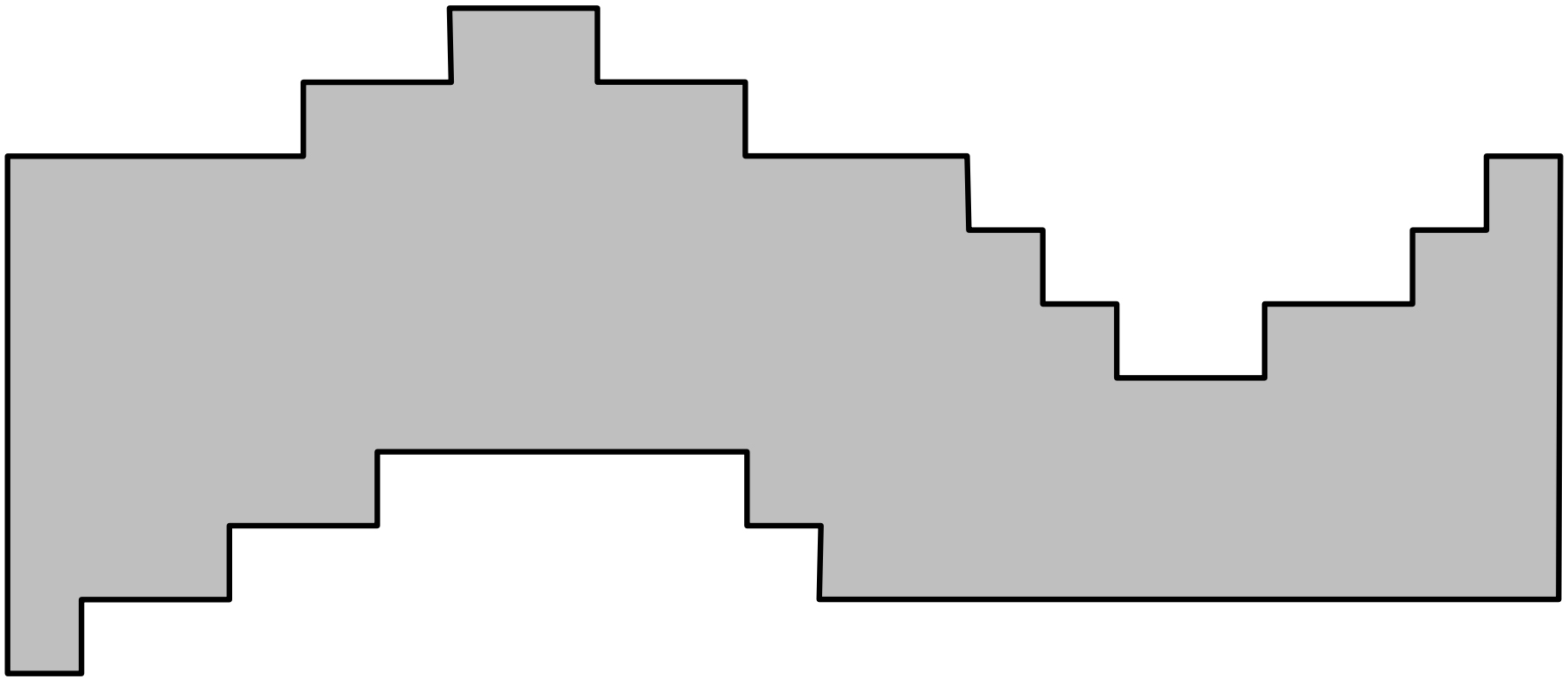


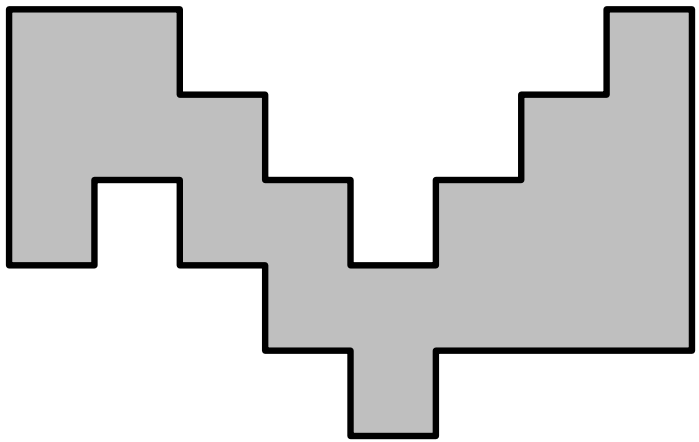
Canonical x-monotone Polygons

All vertical edges (except two extremes) are of unit-length



Canonical x-monotone Polygons

Assume that upper chain is longer (with more reflex vertices).



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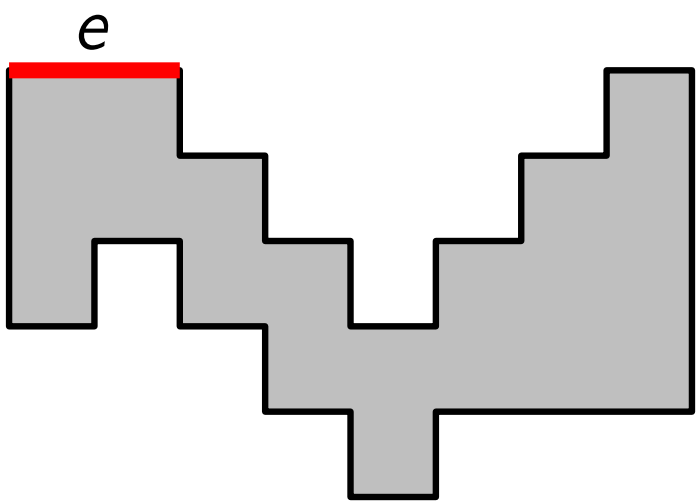

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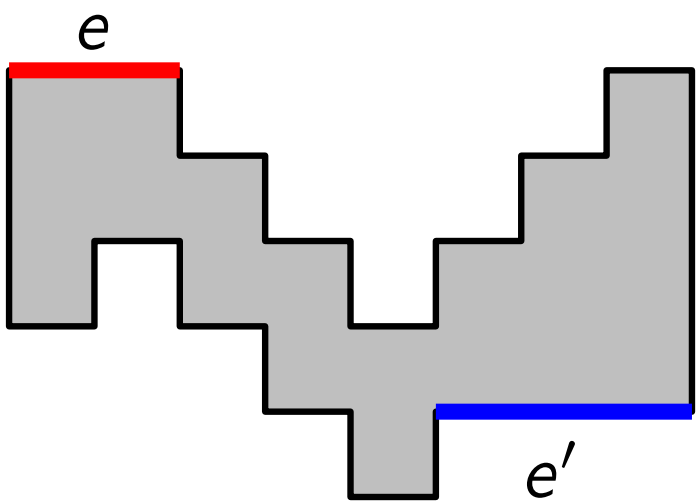

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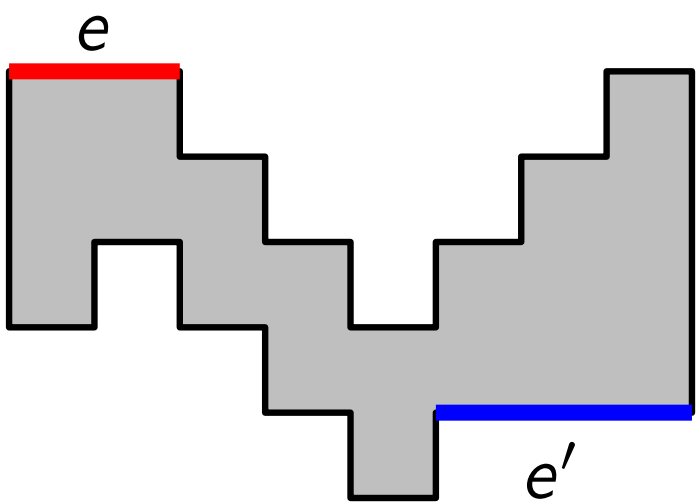

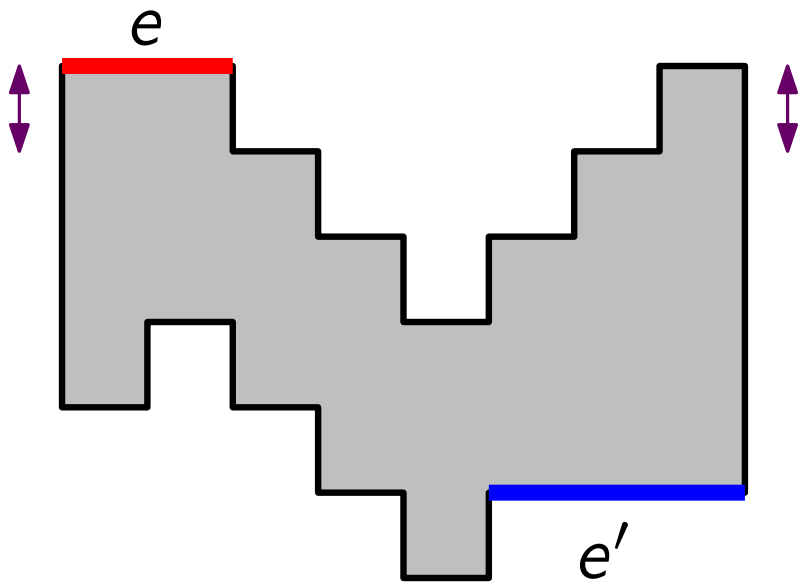

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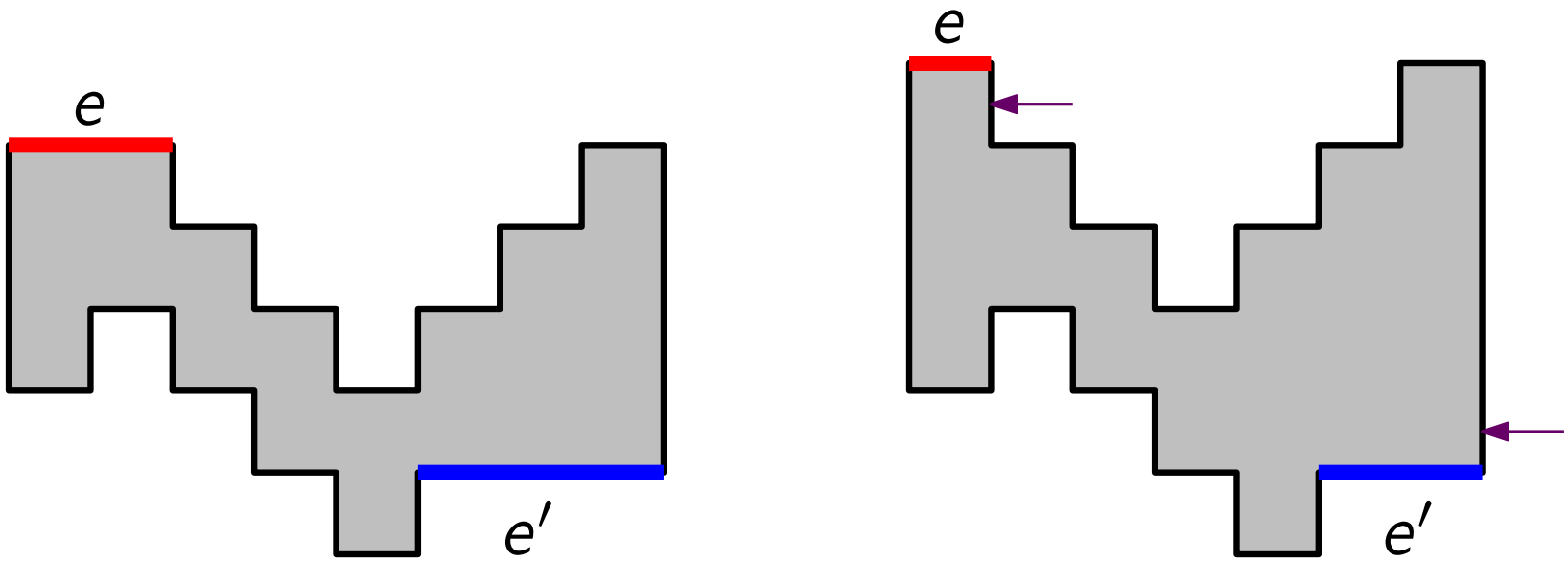

Canonical x-monotone Polygons

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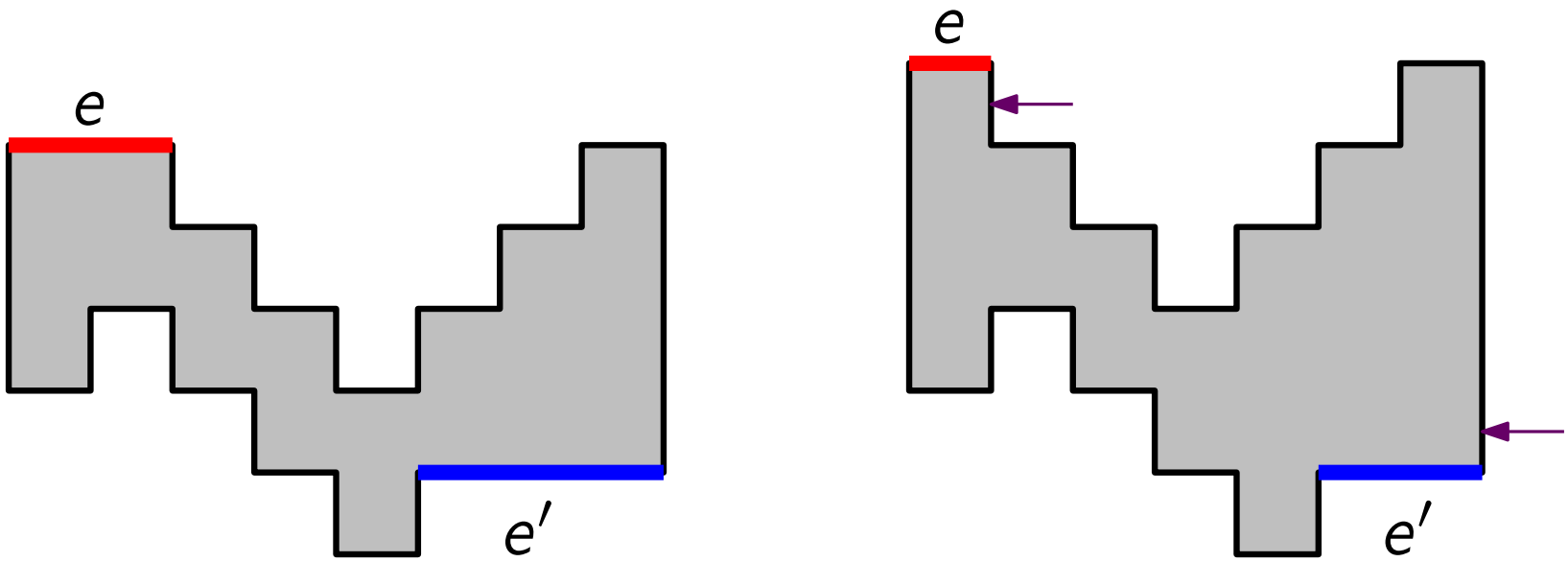
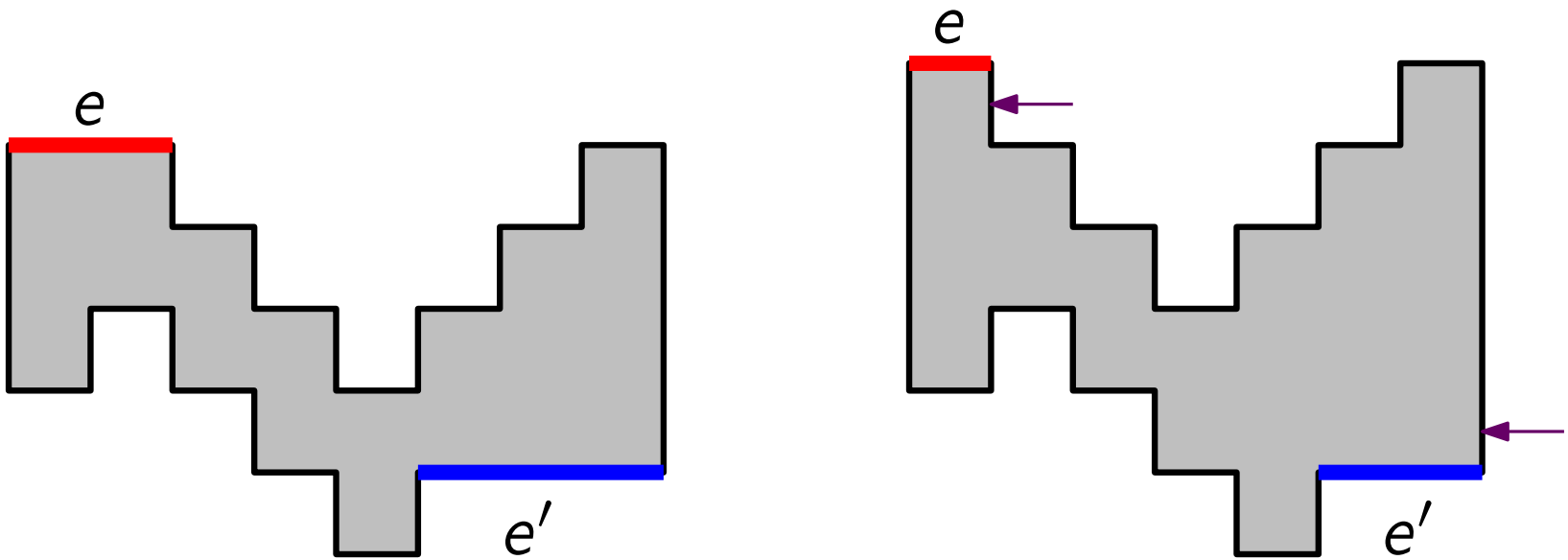


All horizontal edges in the longer chain are of unit-length

Canonical x-monotone Polygons

Assume that upper chain is longer (with more reflex vertices).

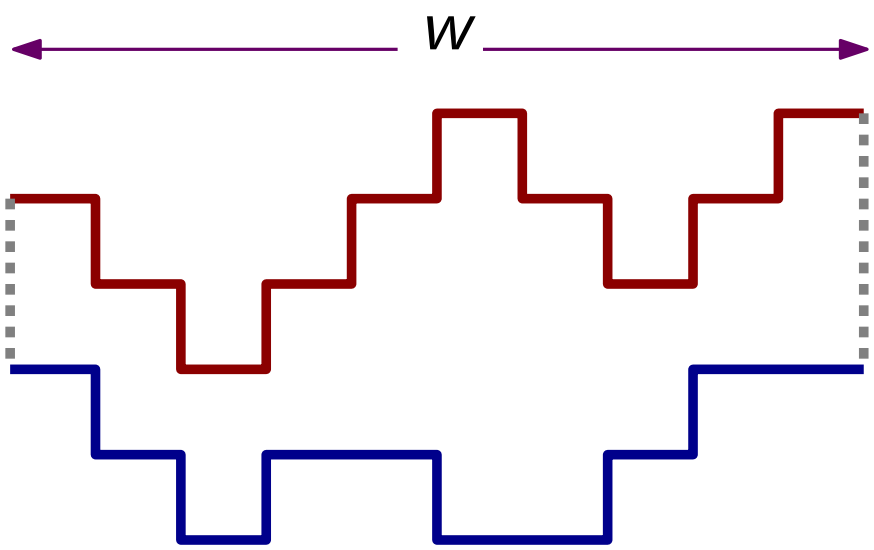


Canonical x-monotone polygon if

1. all vertical edges (except two extremes) are of unit-length, and
2. all horizontal edges in the longer chain are of unit-length

DP Algorithm

The upper (longer) chain is fixed, i.e., width w is fixed



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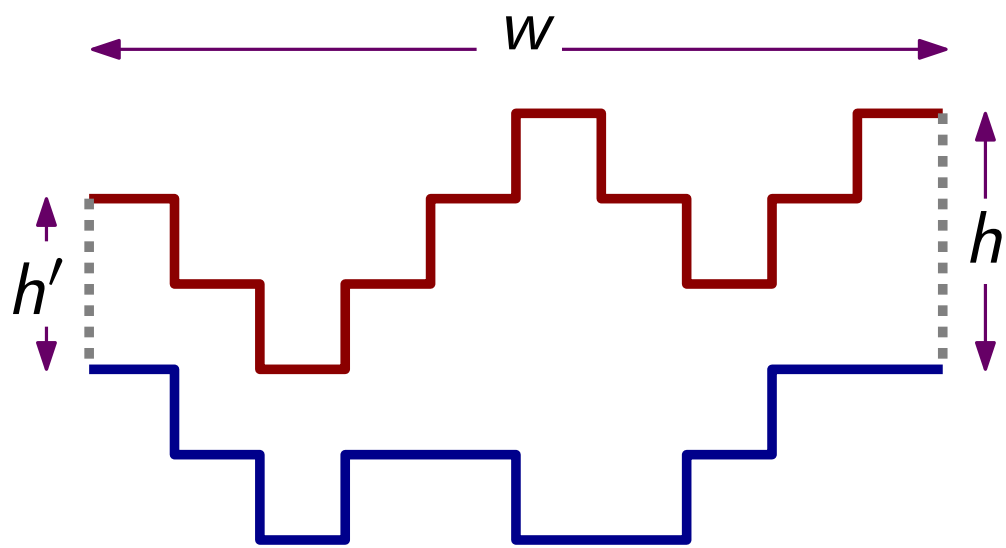

perimeter $= 2w +$ (# of vertical edges) $+ h + h'$

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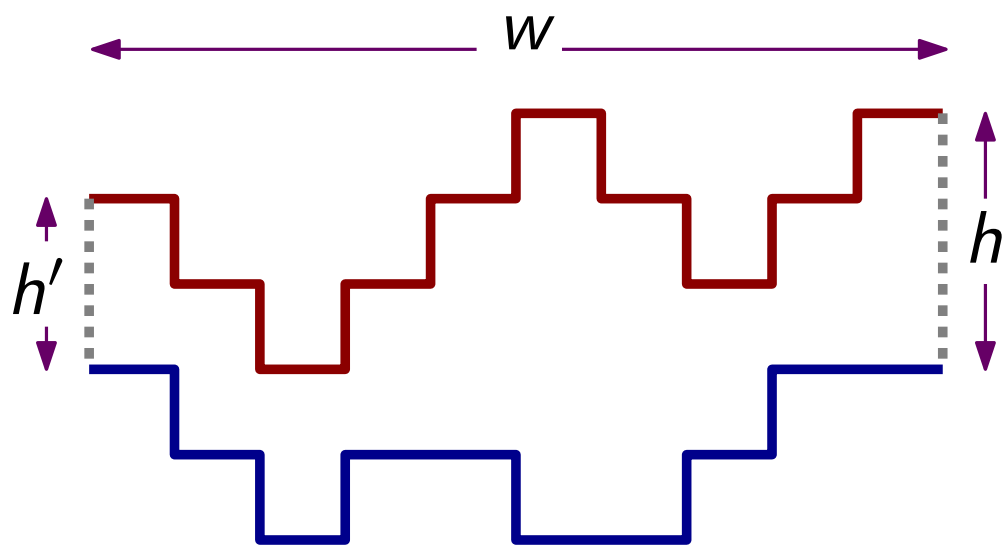

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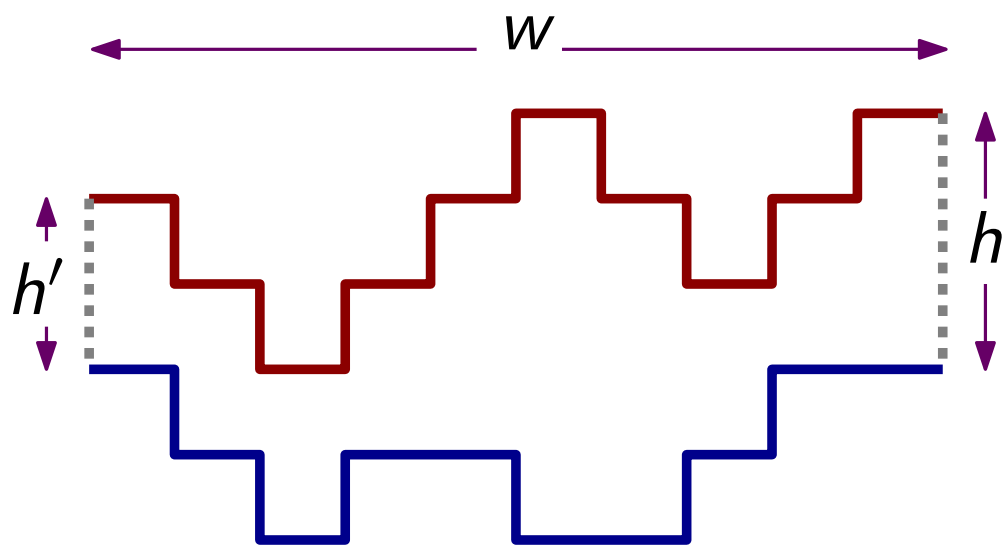

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DP Algorithm

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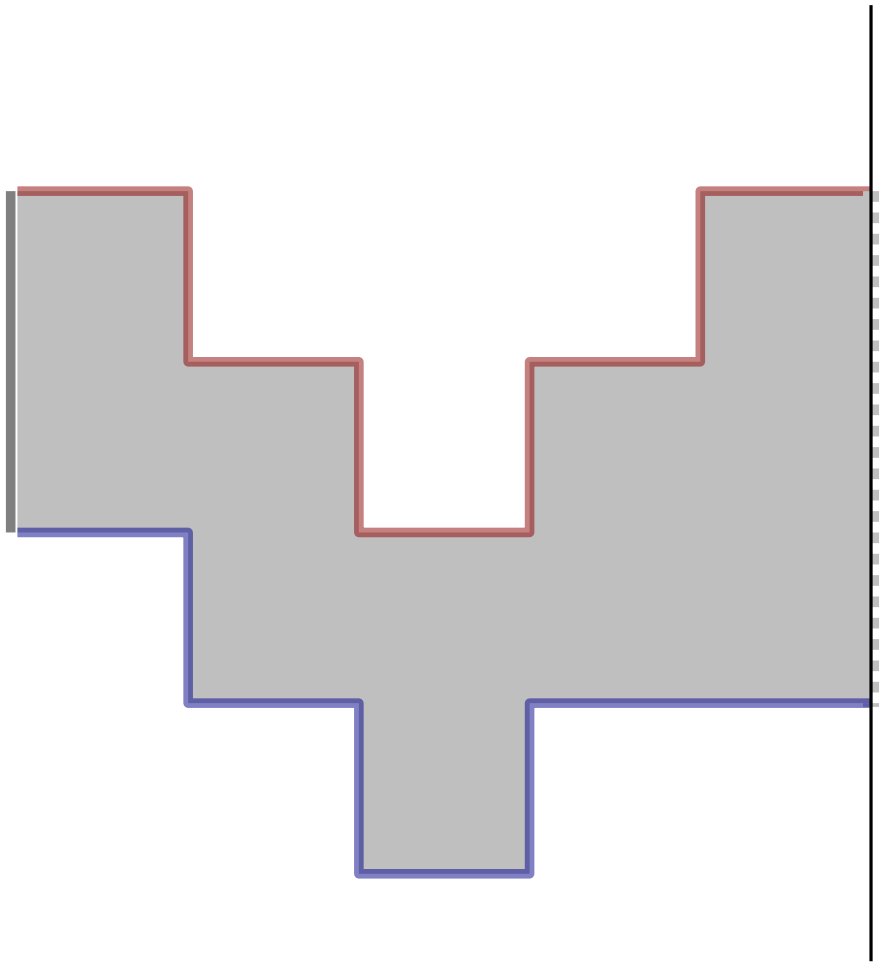
DP Algorithm

min perimeter $=$ min h



DP Algorithm

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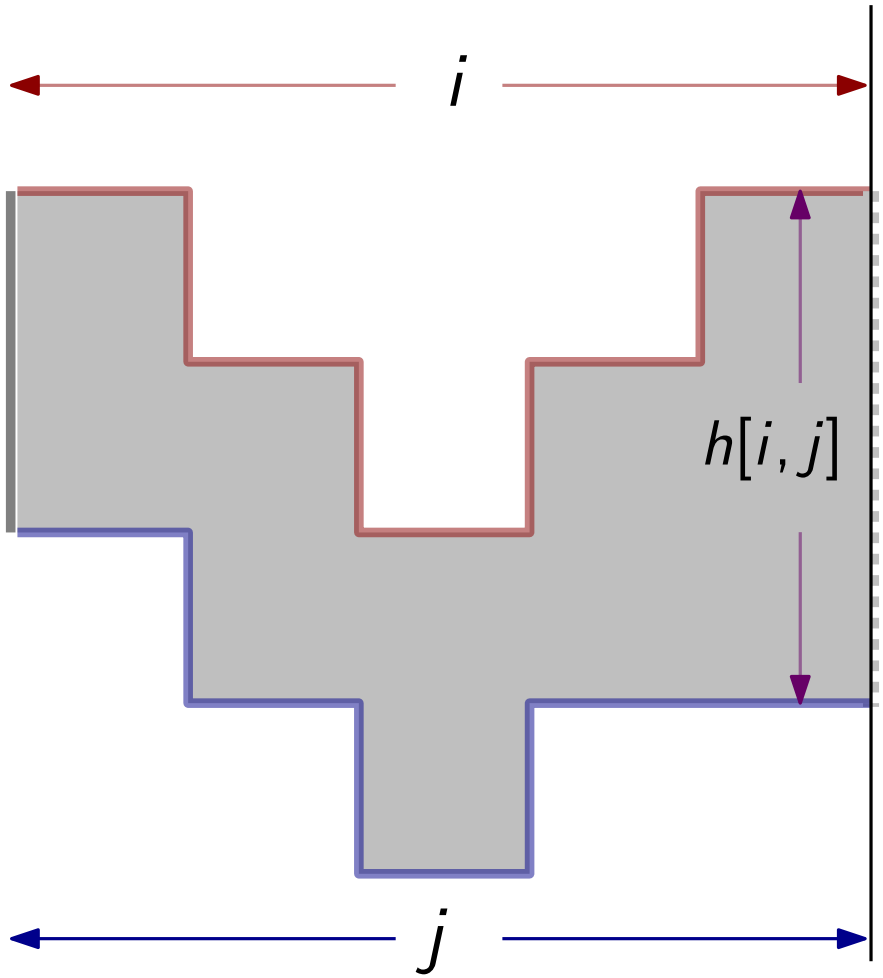

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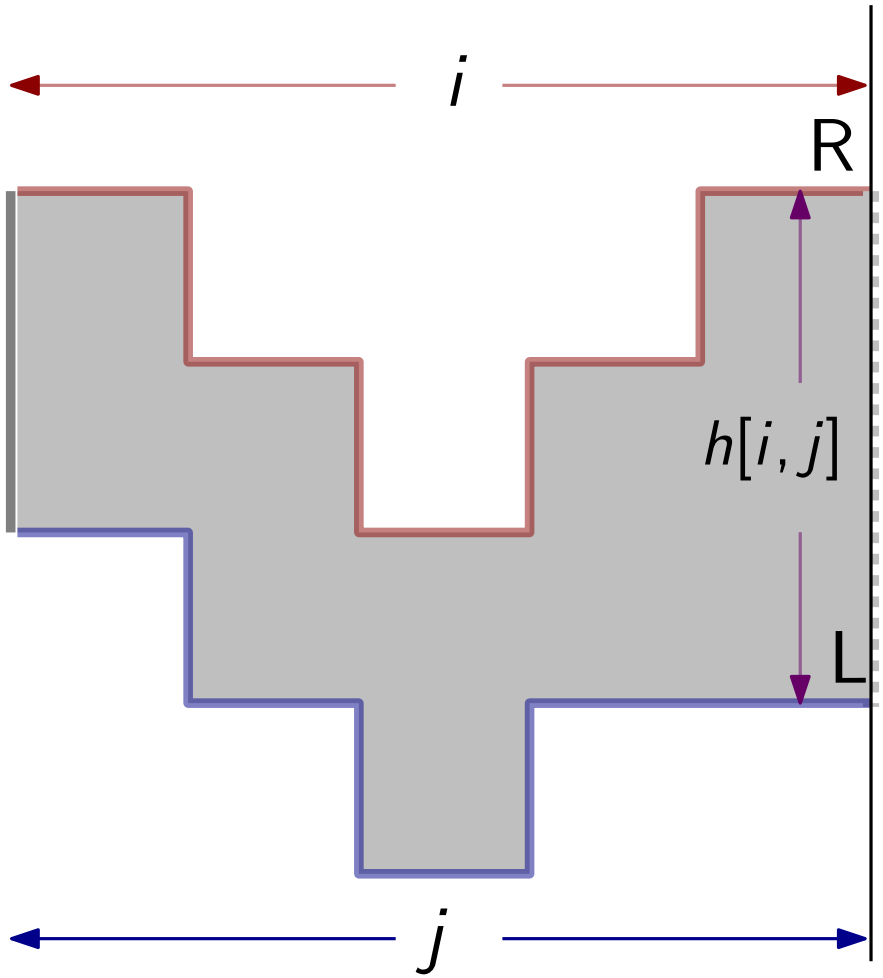

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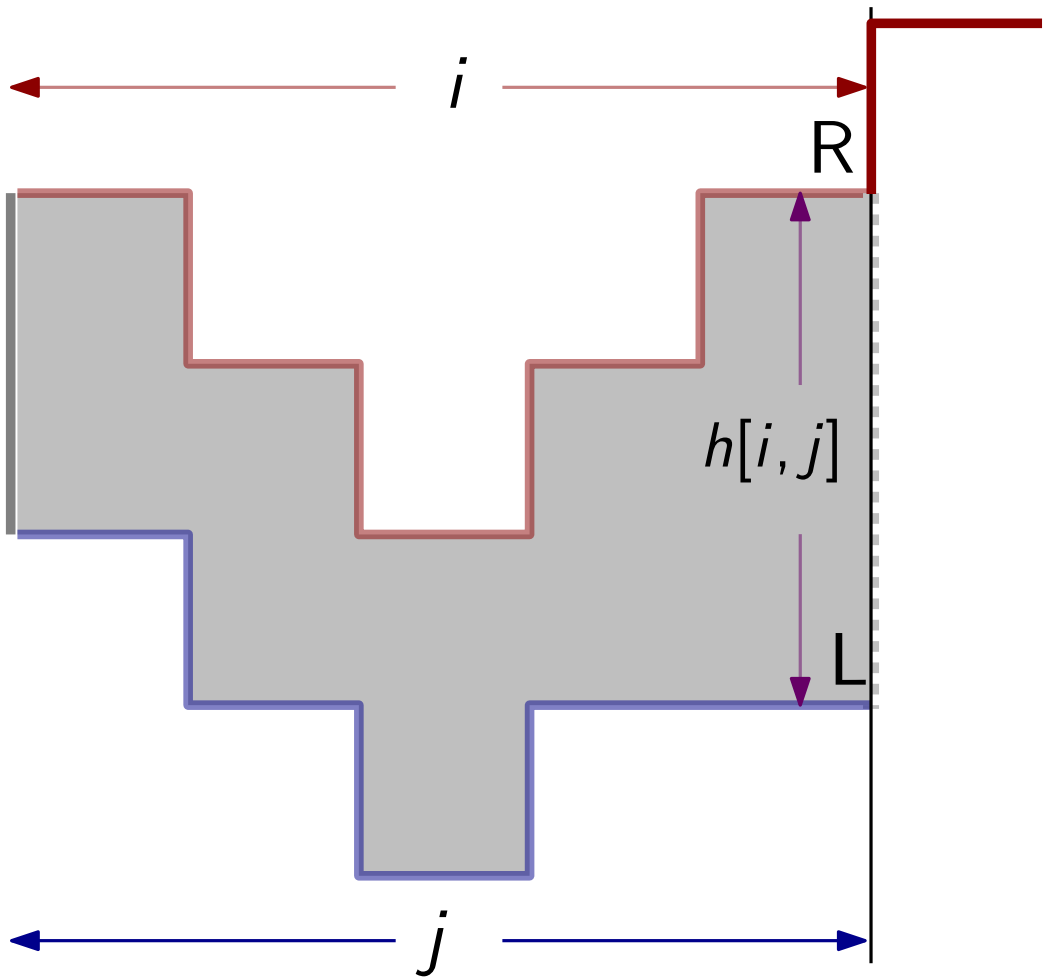

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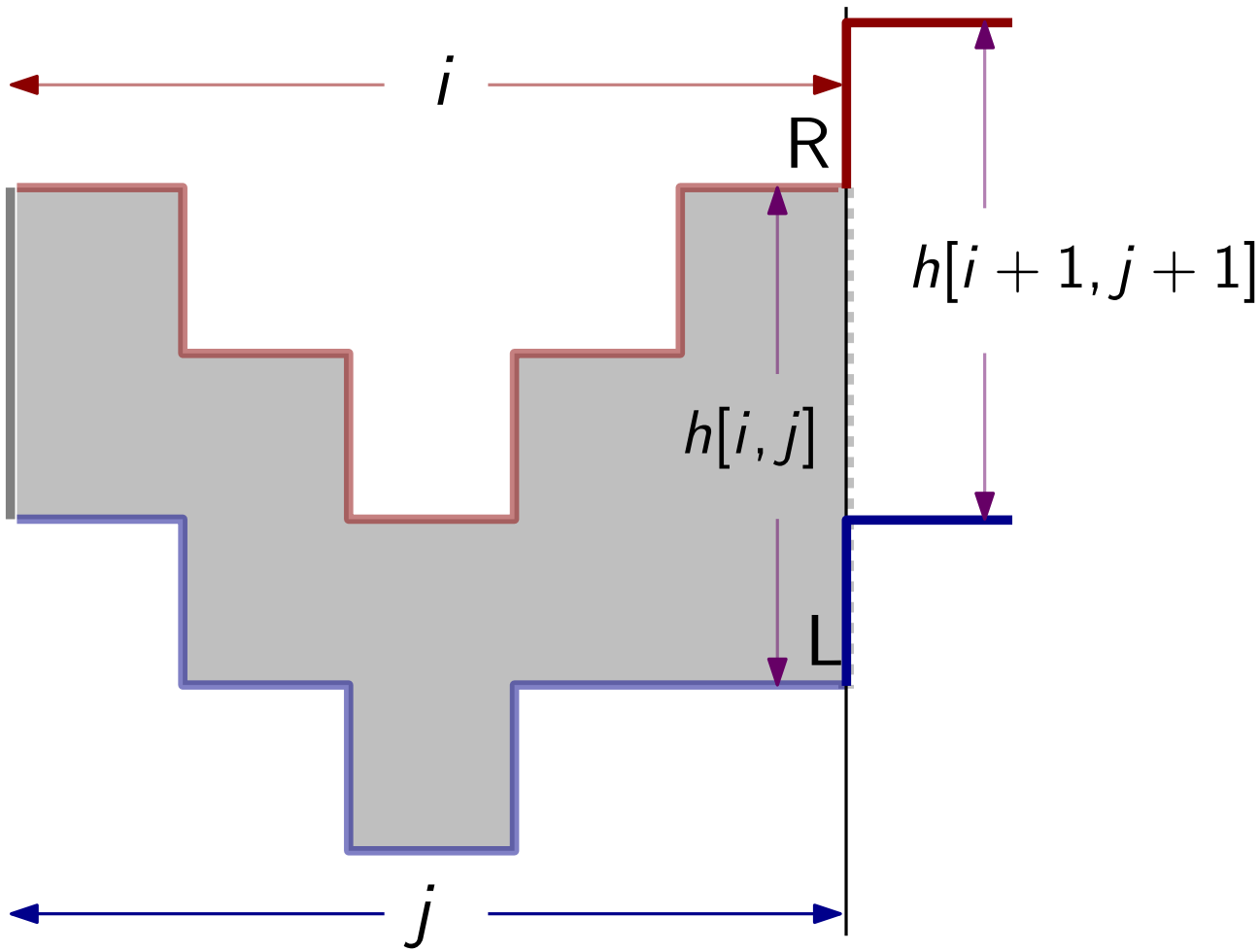

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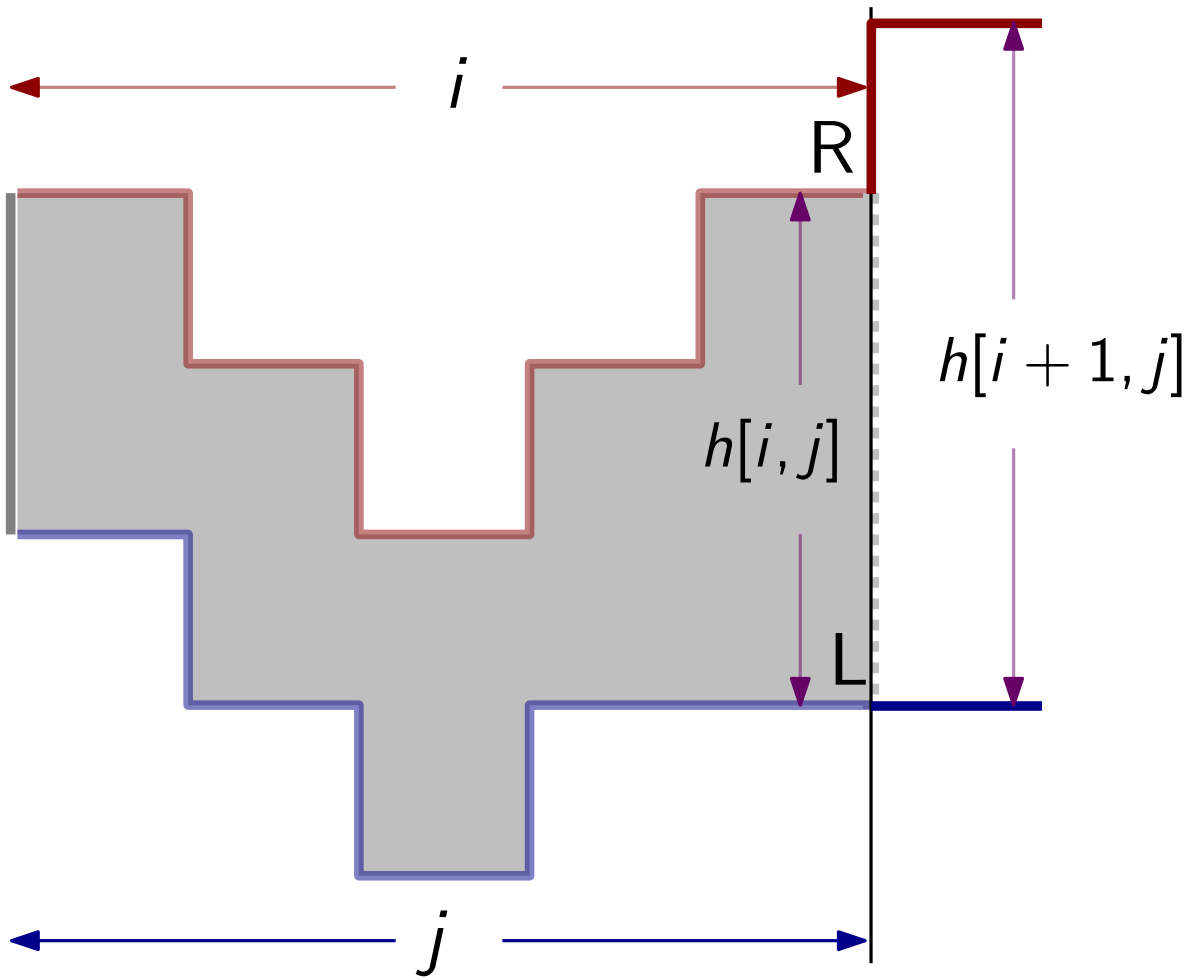

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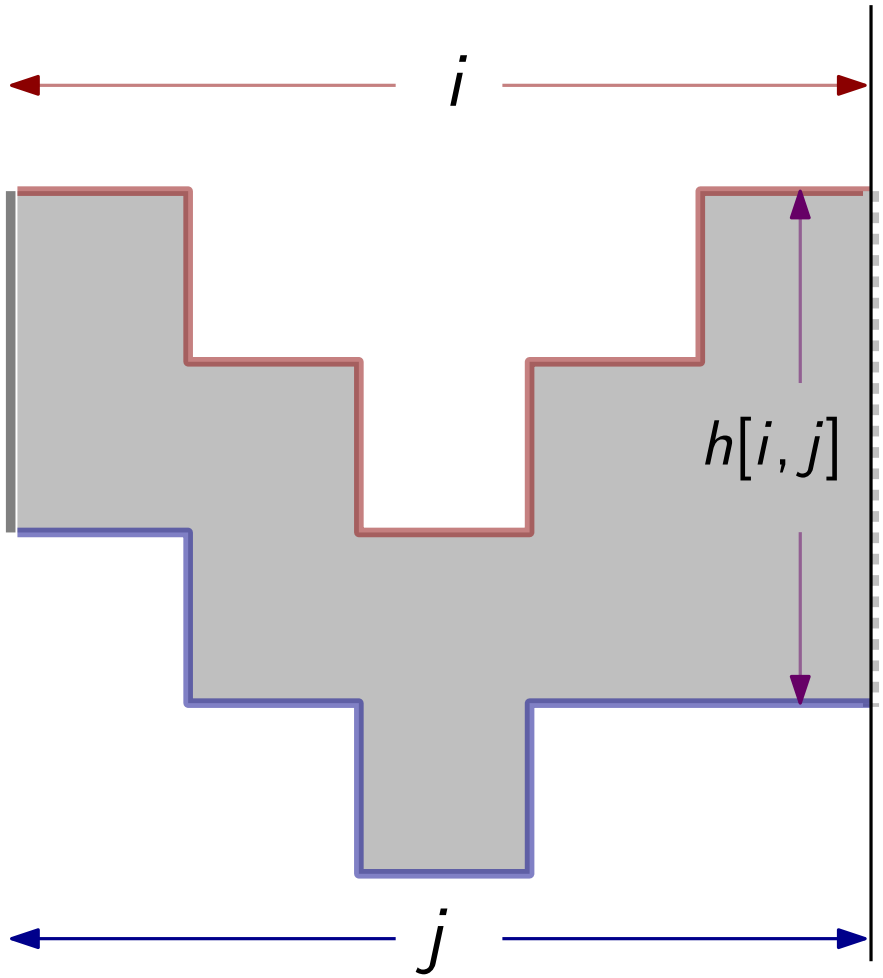
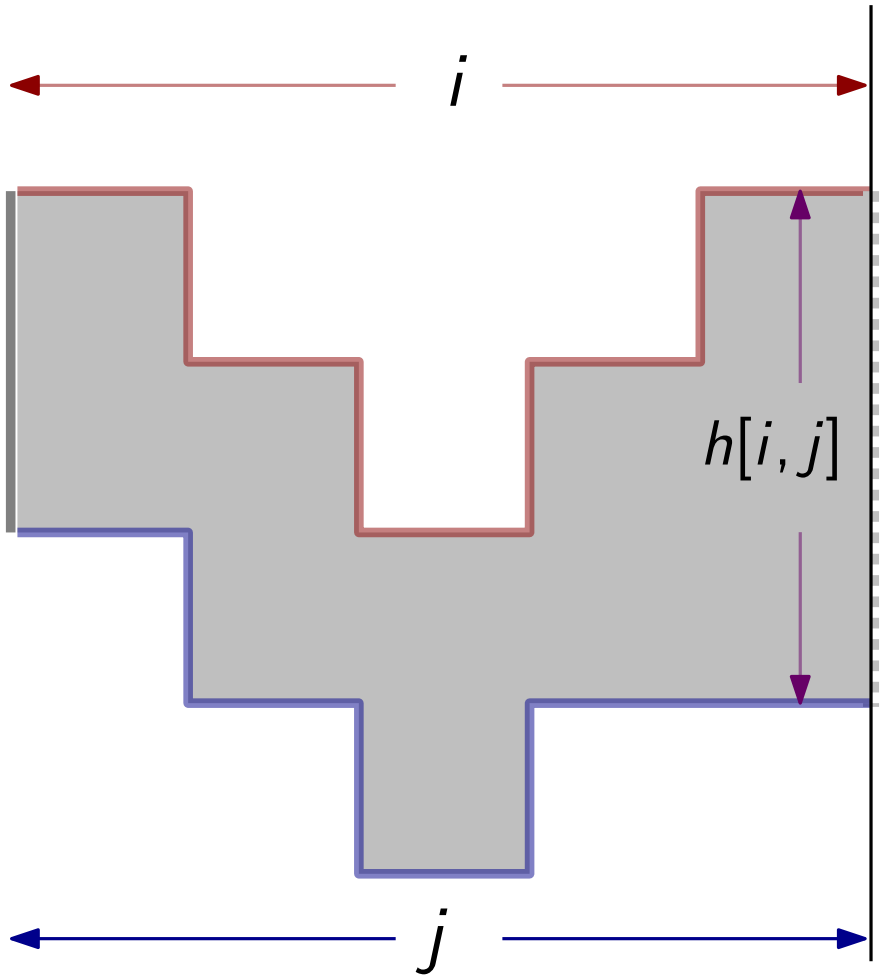


For (i,j) pairs:

Update $h[i,j]$

DP Algorithm

min perimeter = min h



For (i,j) pairs:	$O(n^2)$
Update $h[i,j]$	$O(1)$
	$O(n^2)$

Conclusions

1. Showed that realizing angles sequences optimally is NP-hard.
2. Designed algorithms for monotone sequences.

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Thank you