FernUniversität in Hagen
Fakultät für Mathematik und Informatik

theoretische informatik

Colored Non-Crossing Euclidean Steiner Forest

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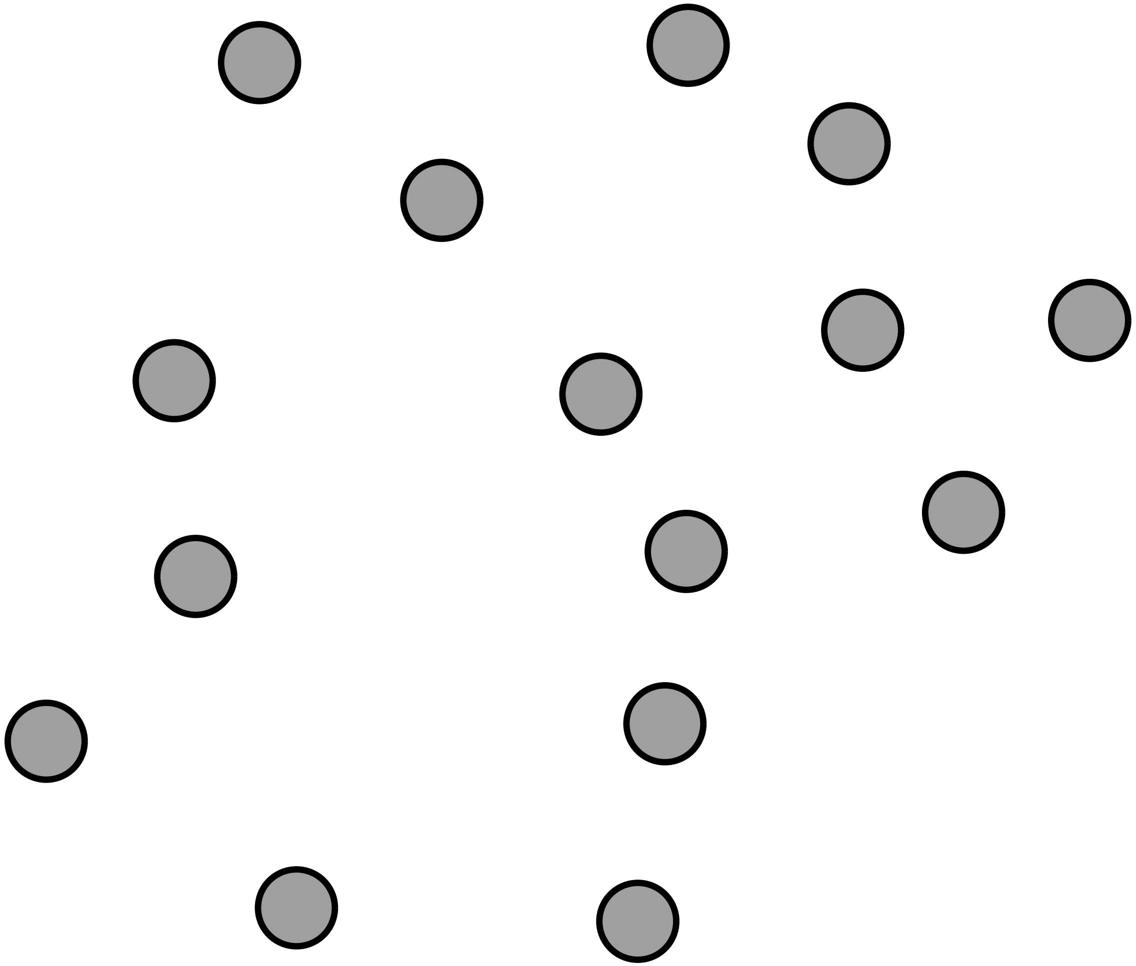
FernUniversität in Hagen

Joint work with
Sergey Bereg, Krzysztof Fleszar, Sergey Pupyrev,
Joachim Spoerhase & Alexander Wolff

Colored Steiner Forest



Colored Steiner Forest

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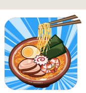 [Icon of Me So Ramen by Moxy Games, LLC]

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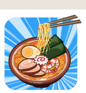 [Icon of Me So Ramen by Moxy Games, LLC]  [Aha-Soft, via seaicons.com]

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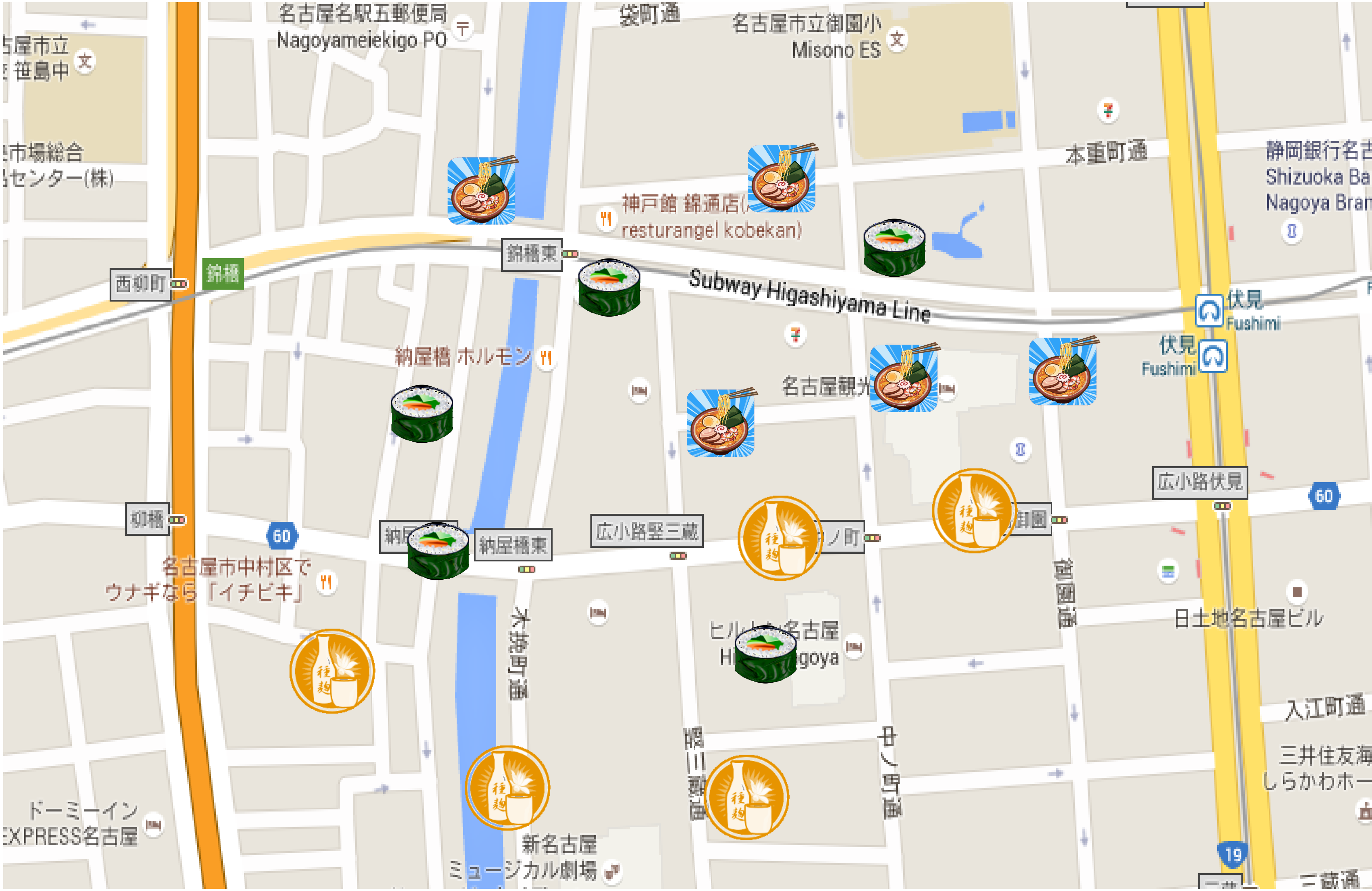

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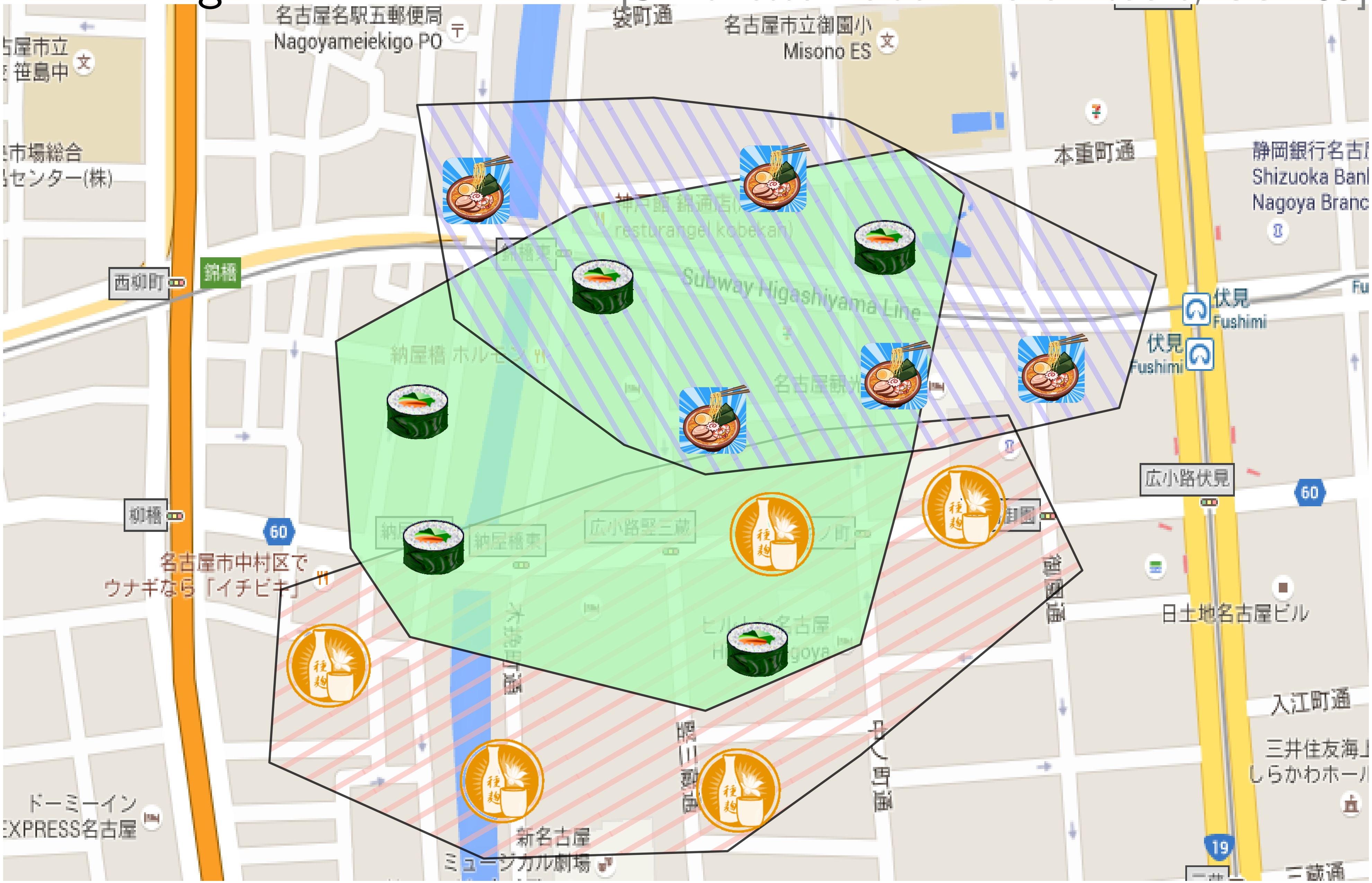
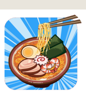
Euler diagrams

[Simonetto Auber Archambault, CGF'09]



 [Icon of Me So Ramen by Moxy Games, LLC]  [Aha-Soft, via seaicons.com]  ©Leeners, LLC

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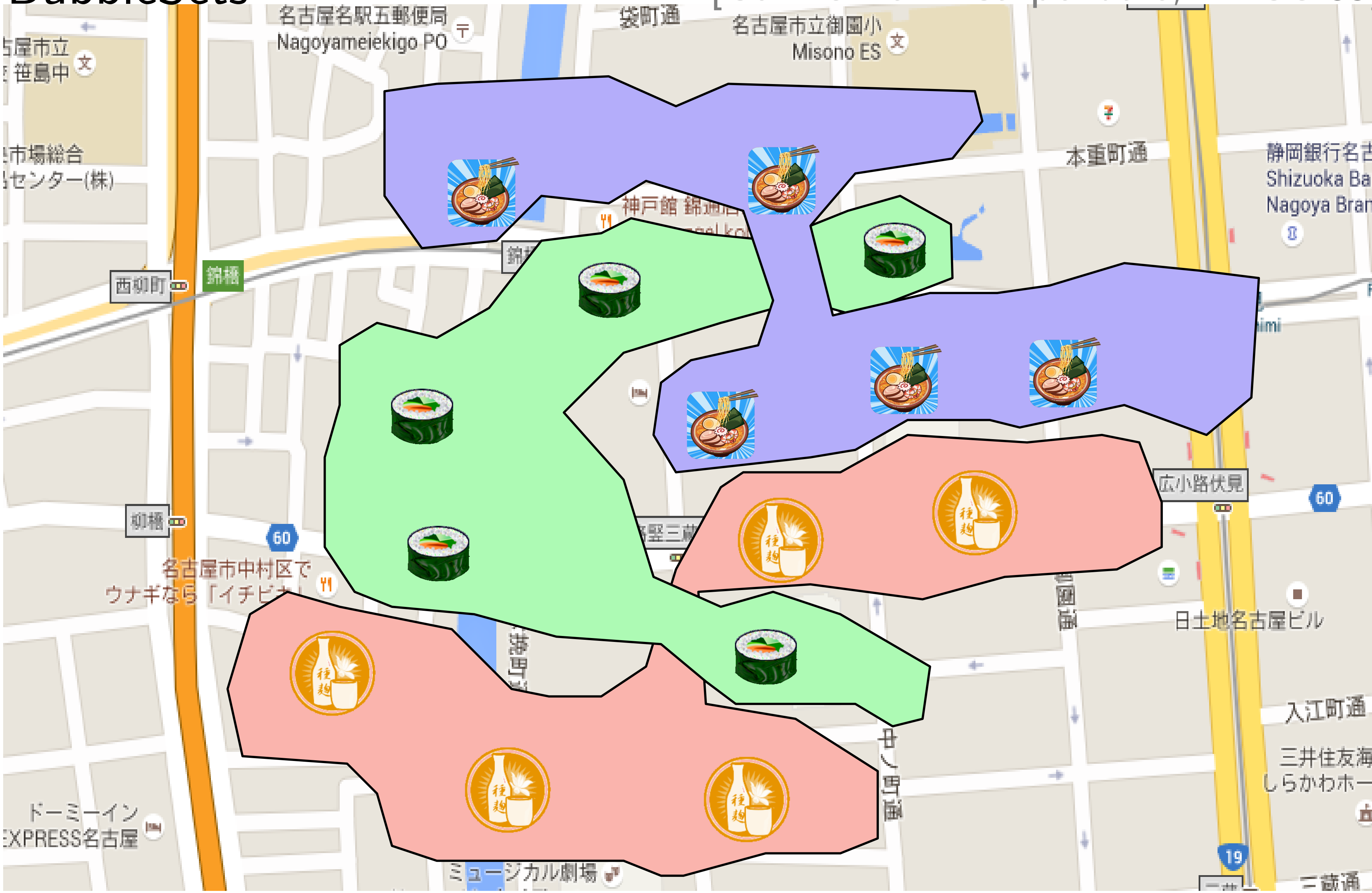
BubbleSets

[Collins Penn Carpendale, TVCG'09]



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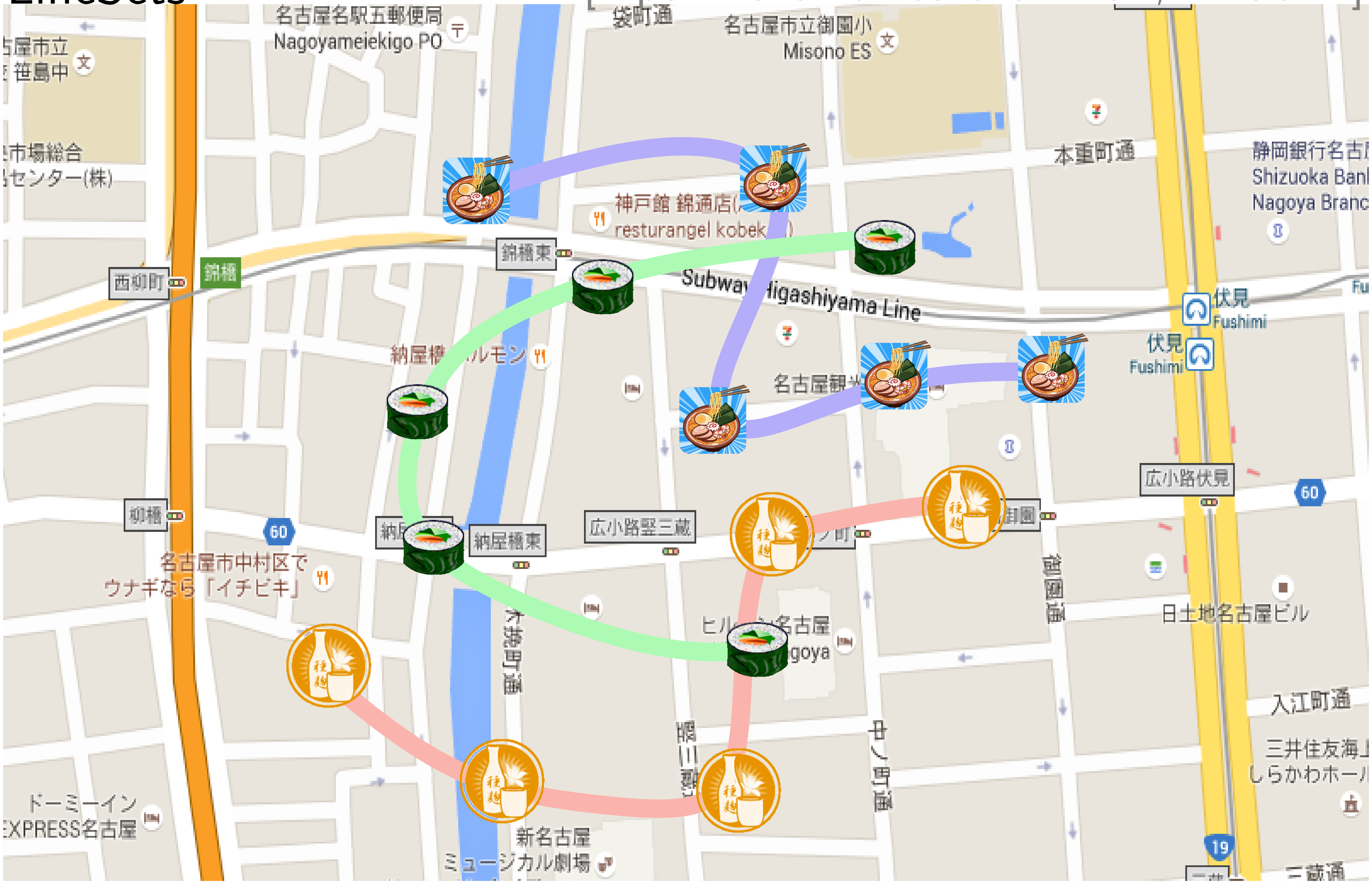
LineSets

[Alper Riche Ramos Czerwinski, TVCG'11]



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KelpFusion

[Meulemans Riche Speckmann Alper Dwyer, TVCG'13]

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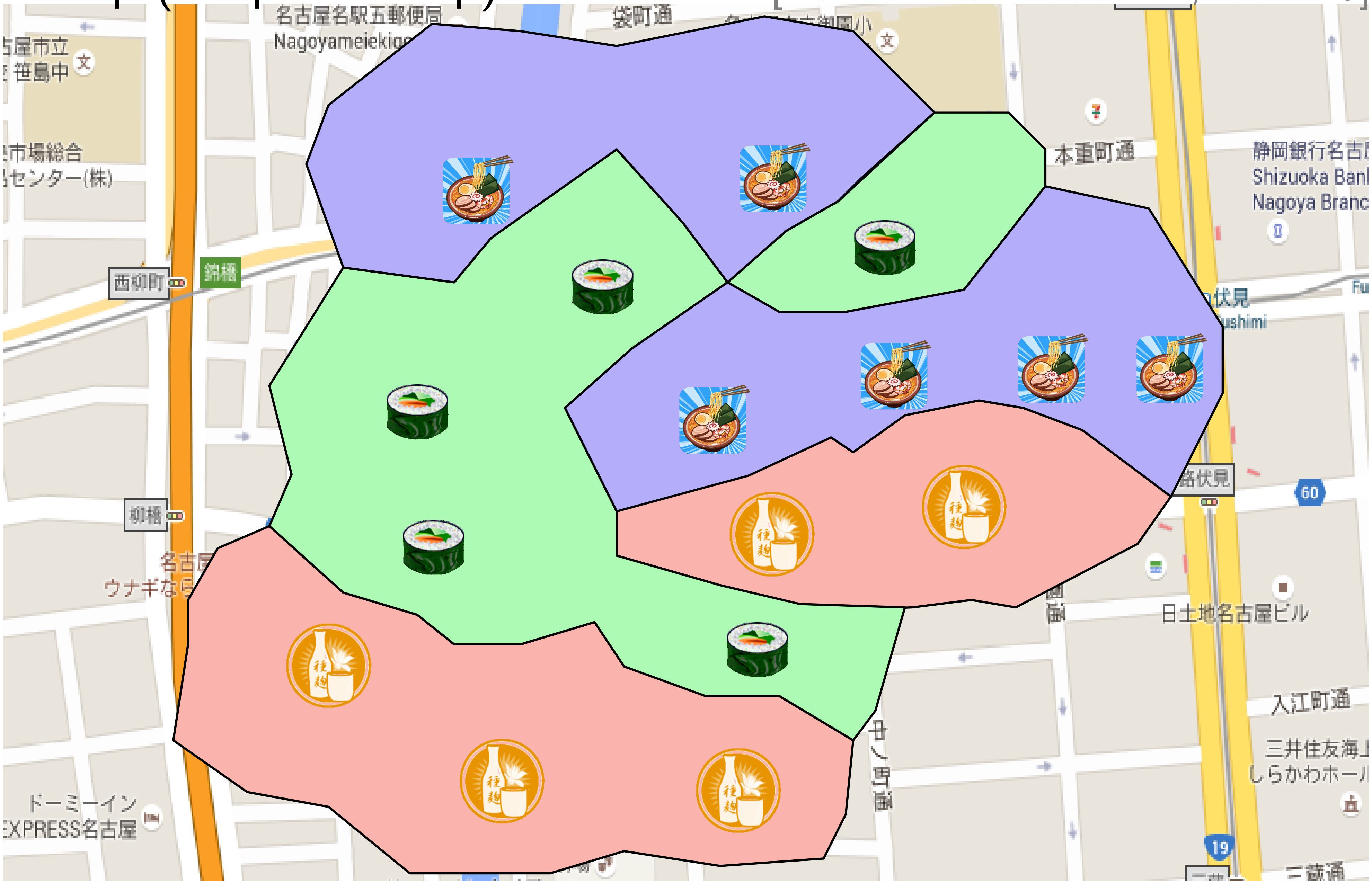
GMap (Graph-to-Map)

[Hu Gansner Kobourov CGA'10]



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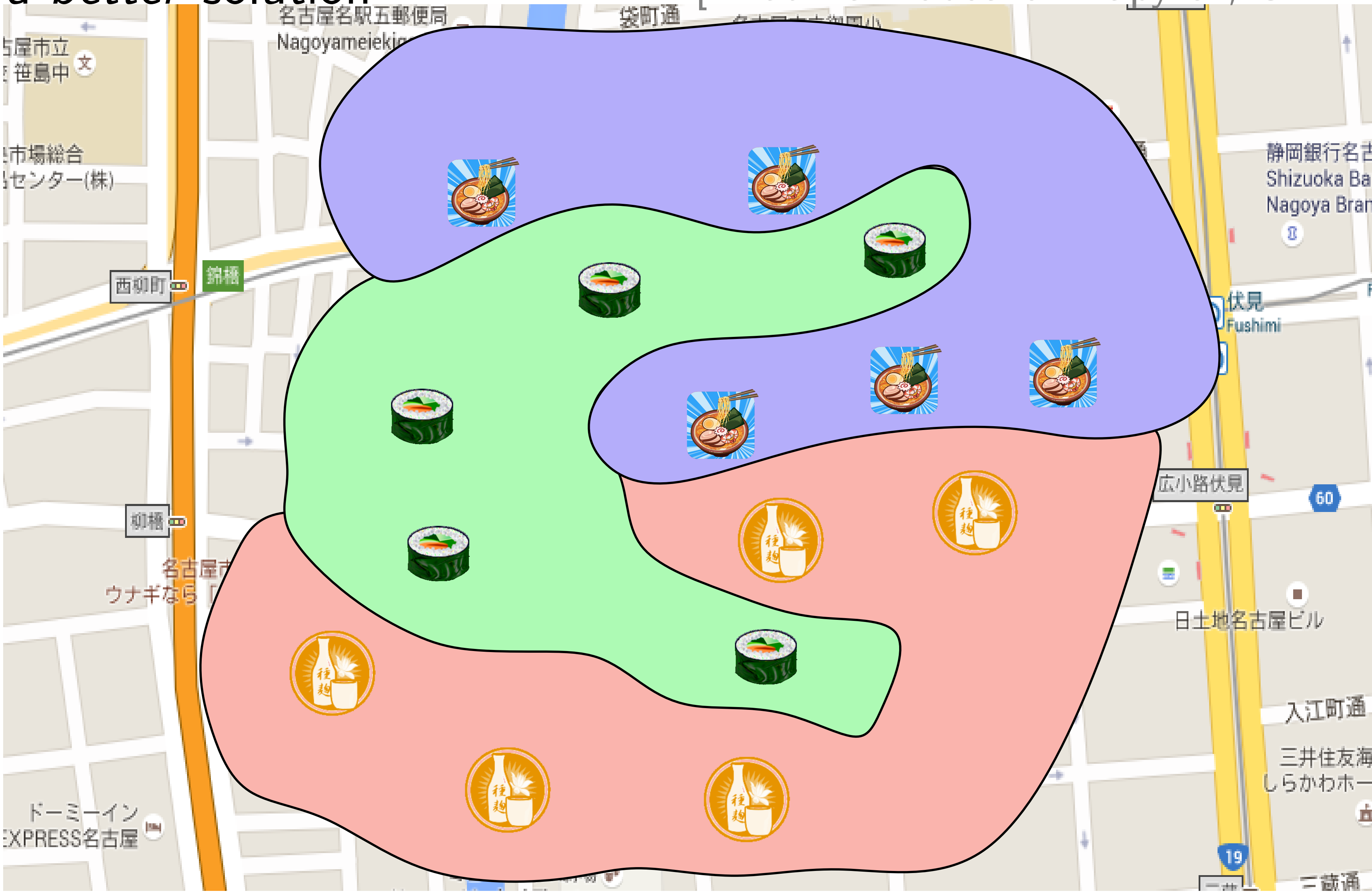
Colored Steiner Forest

a *better* solution

[Efrat Hu Kobourov Pupyrev, GD'14]



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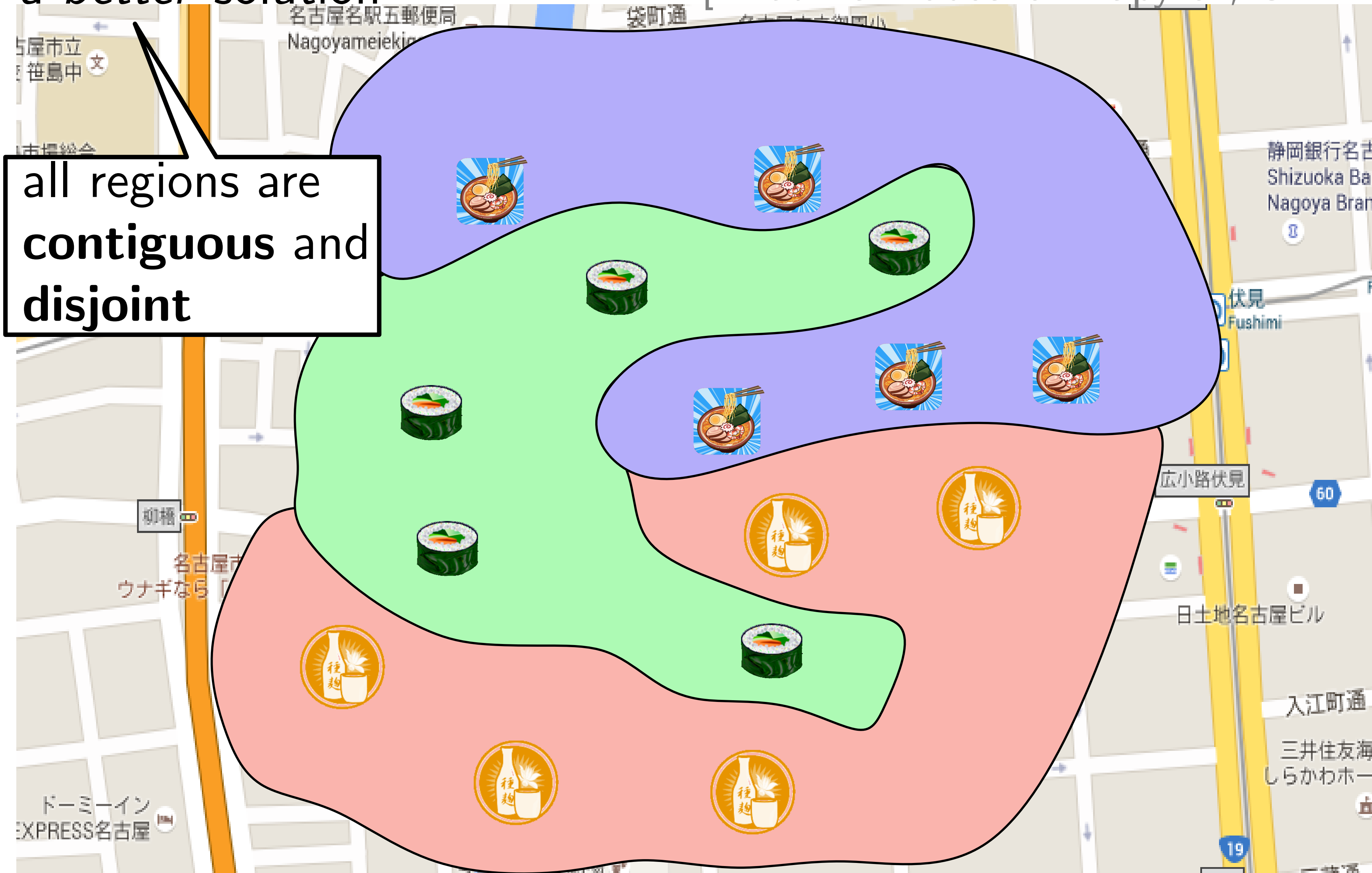
Colored Steiner Forest

a *better* solution

[Efrat Hu Kobourov Pupyrev, GD'14]

all regions are **contiguous** and **disjoint**



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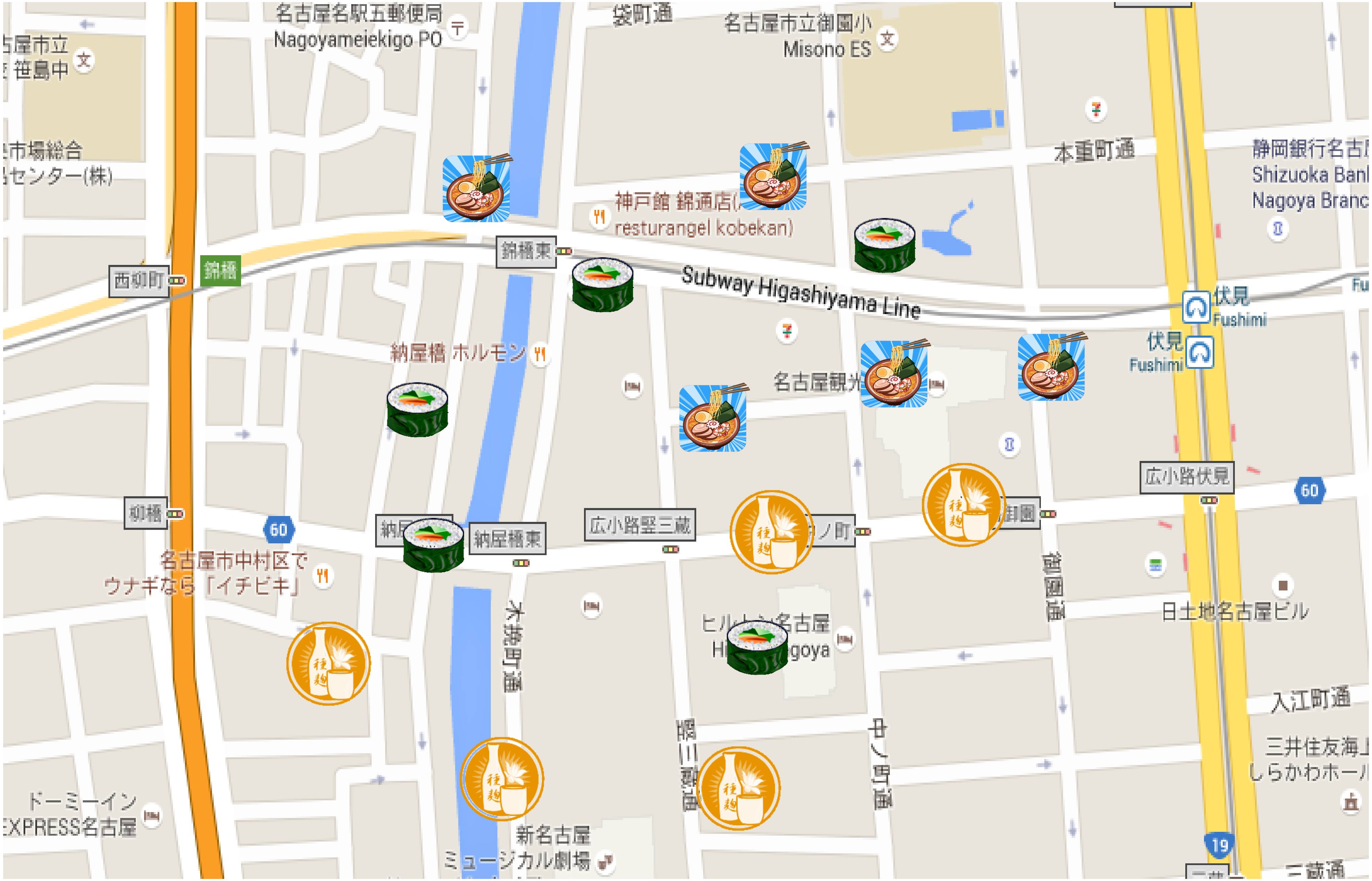

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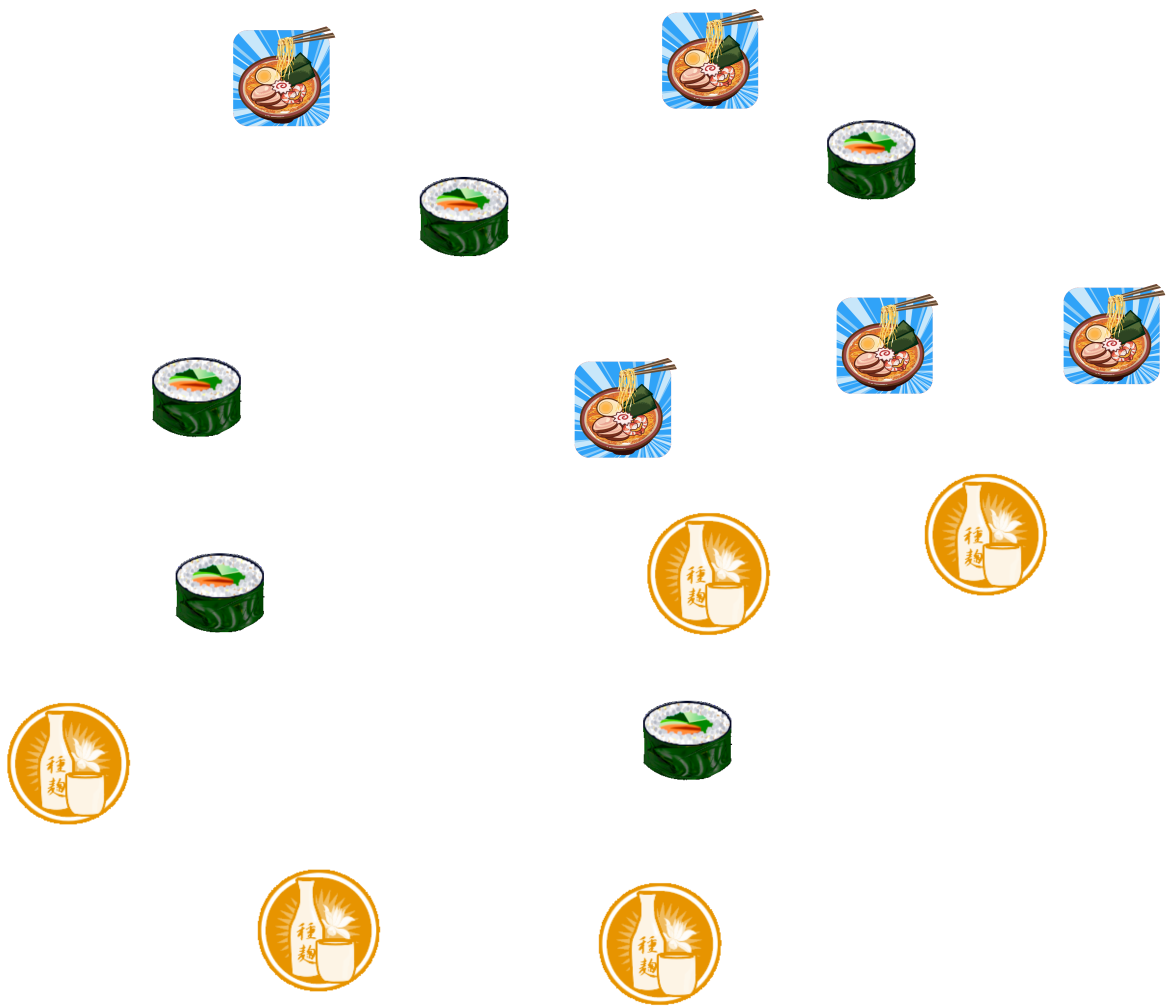
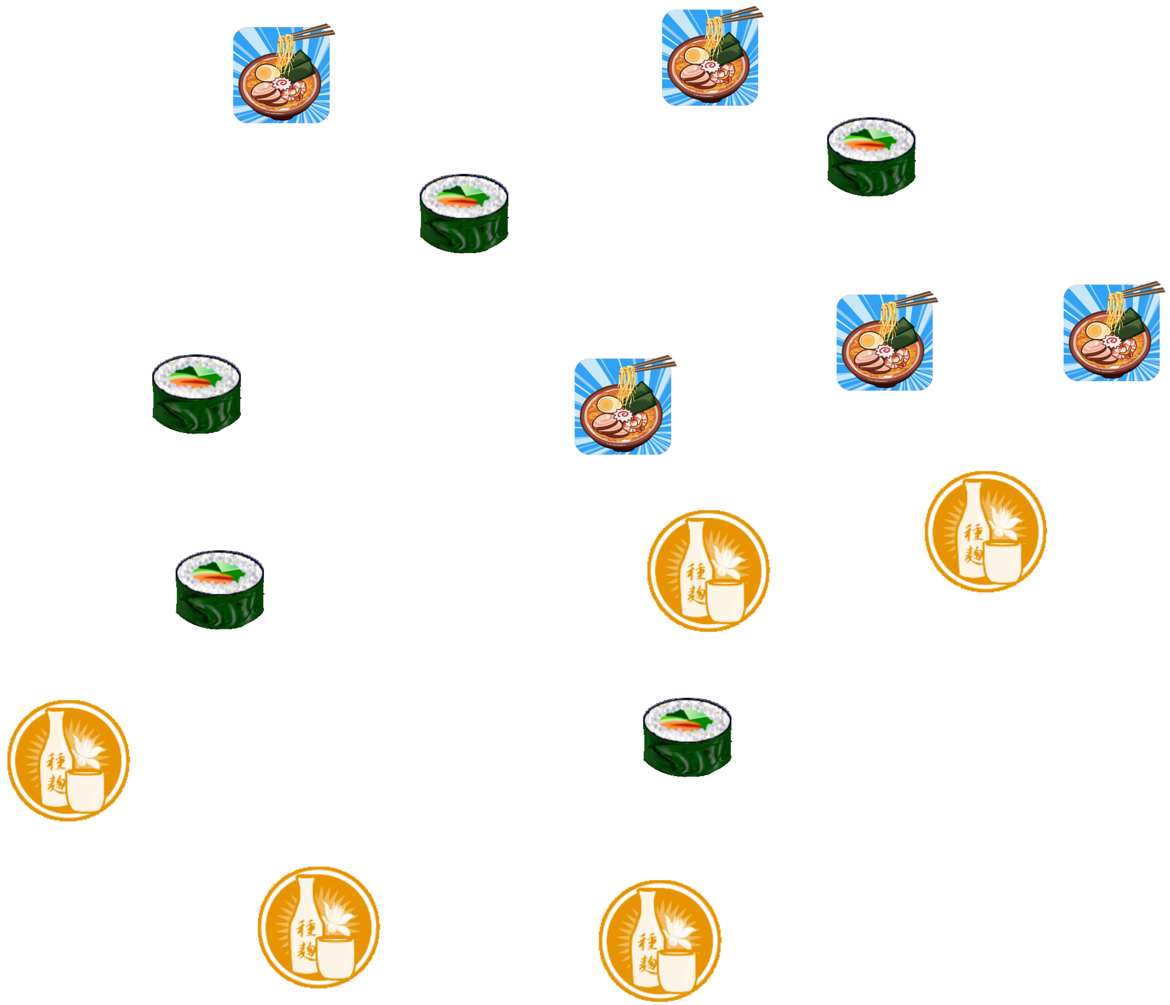


Colored Steiner Forest

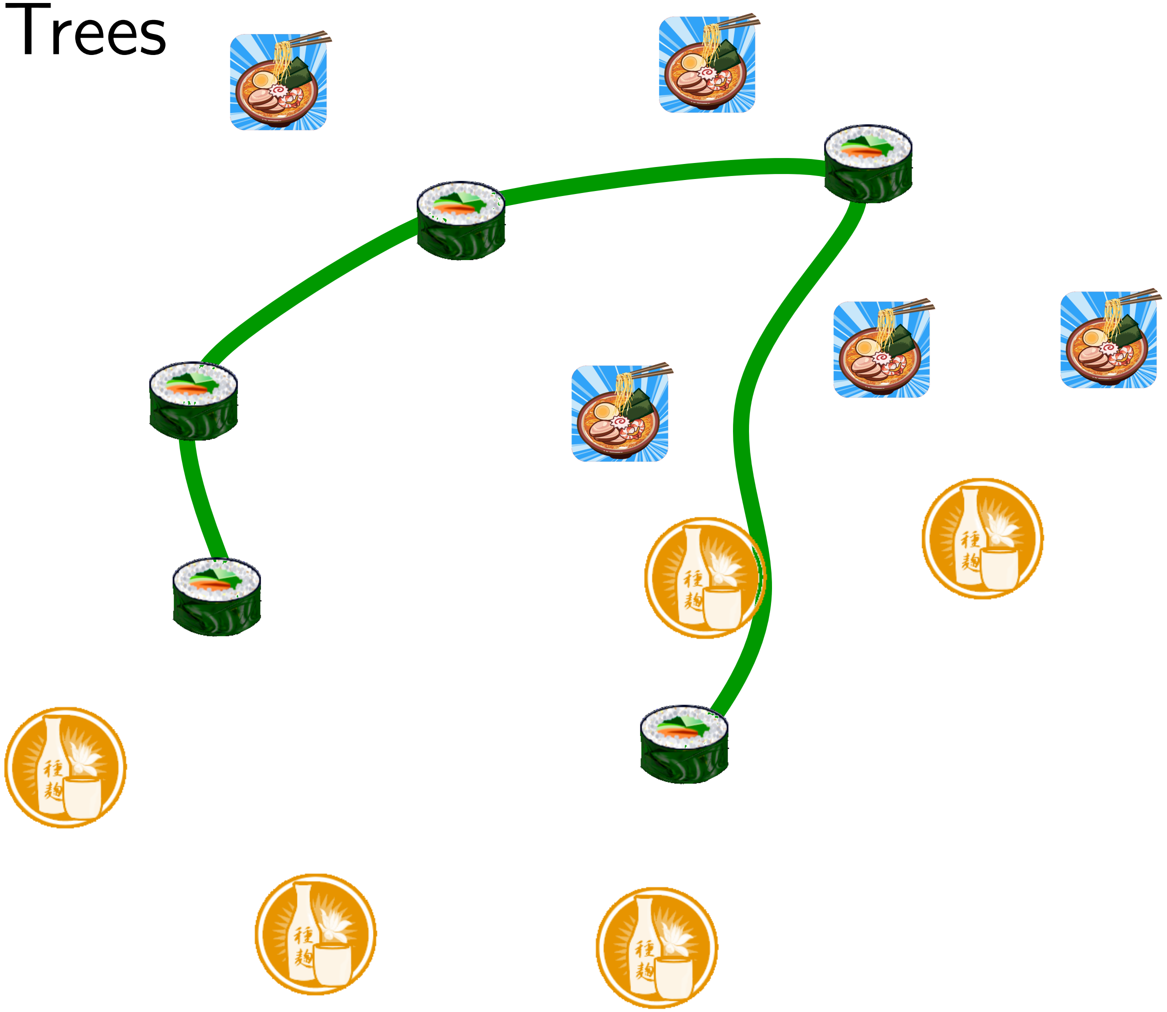
n points, k colors



Colored Steiner Forest

n points, k colors

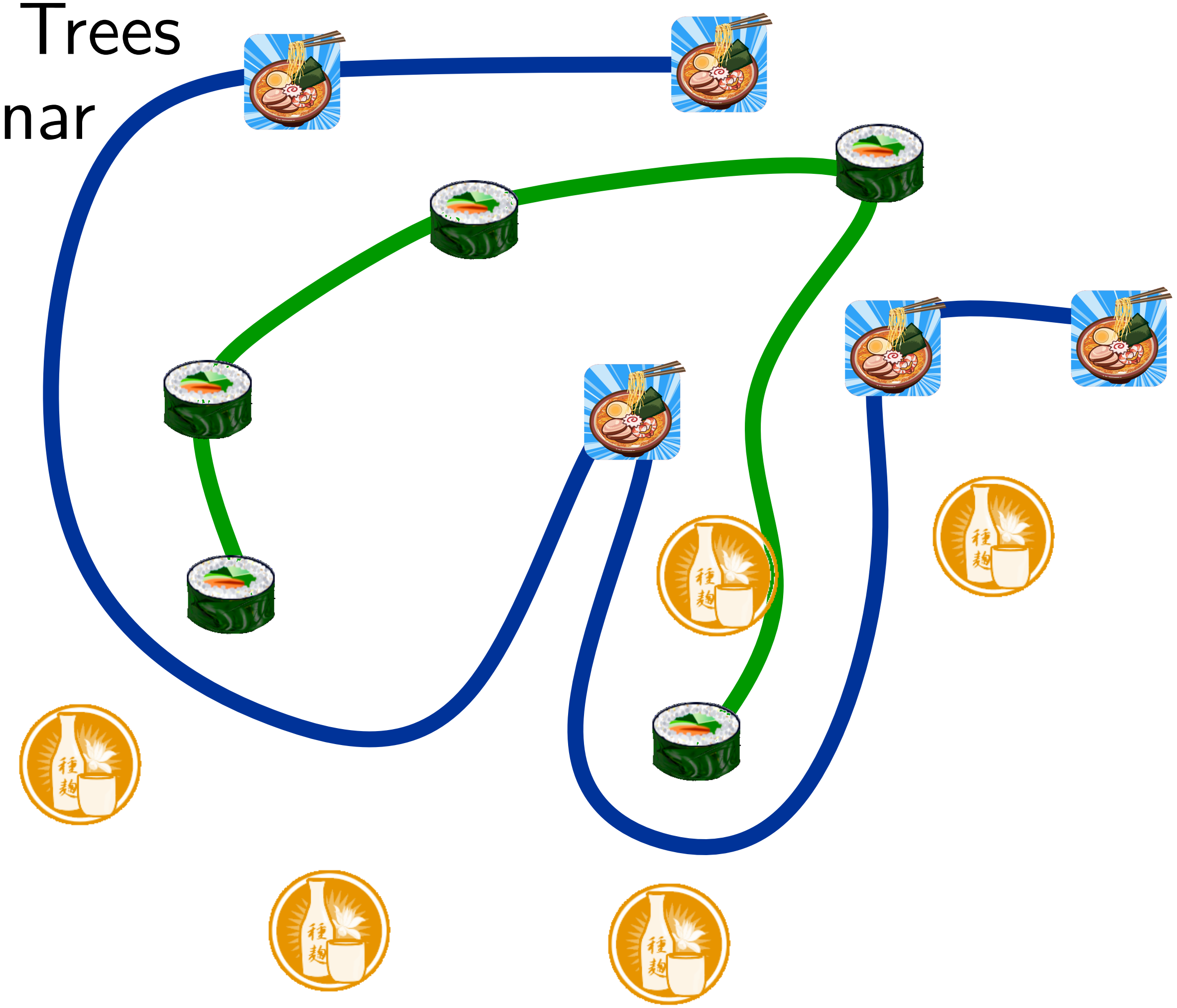
$\rightarrow$ k Steiner Trees



Colored Steiner Forest

n points, *k* colors

→ *k* Steiner Trees
union planar



Colored Steiner Forest

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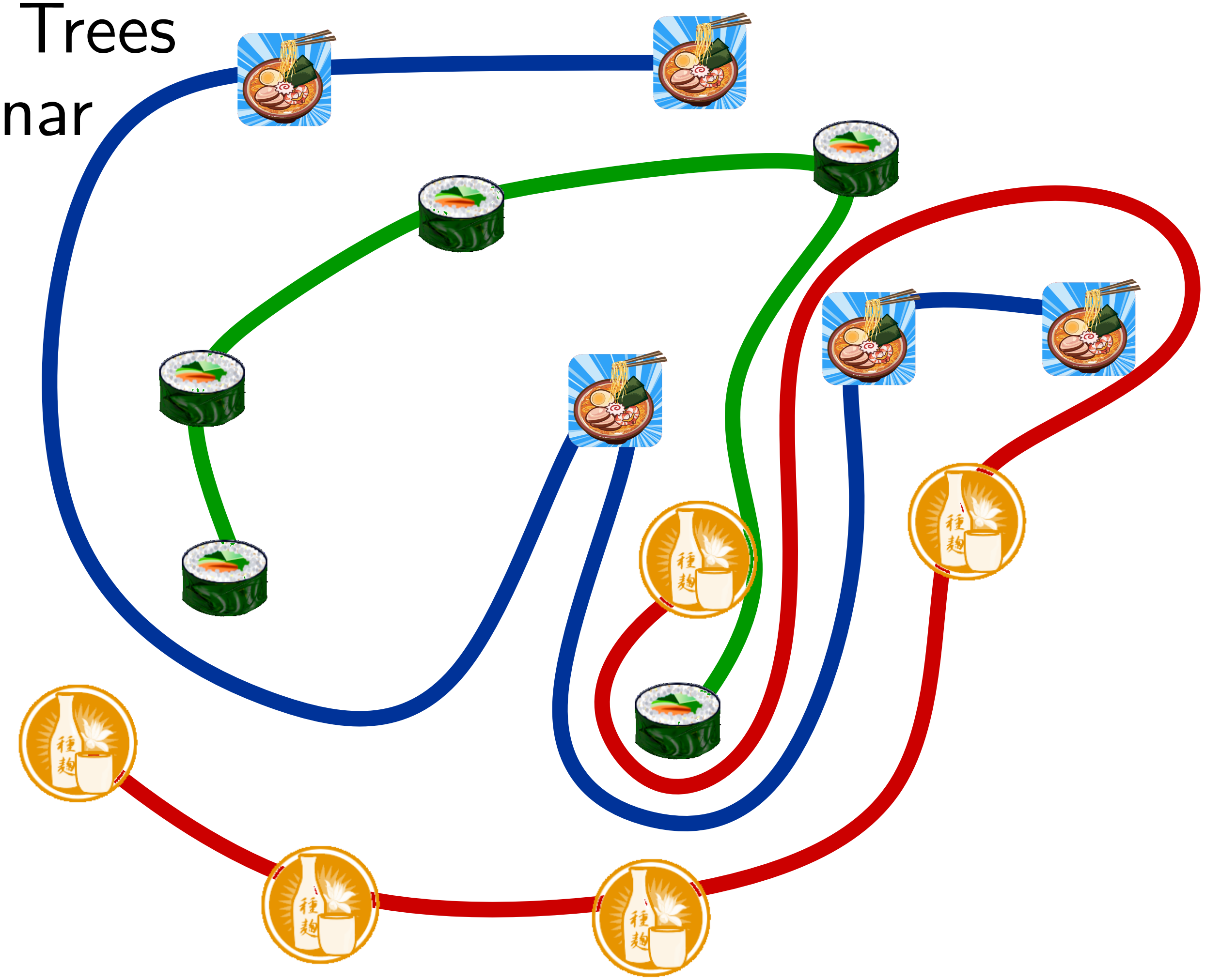

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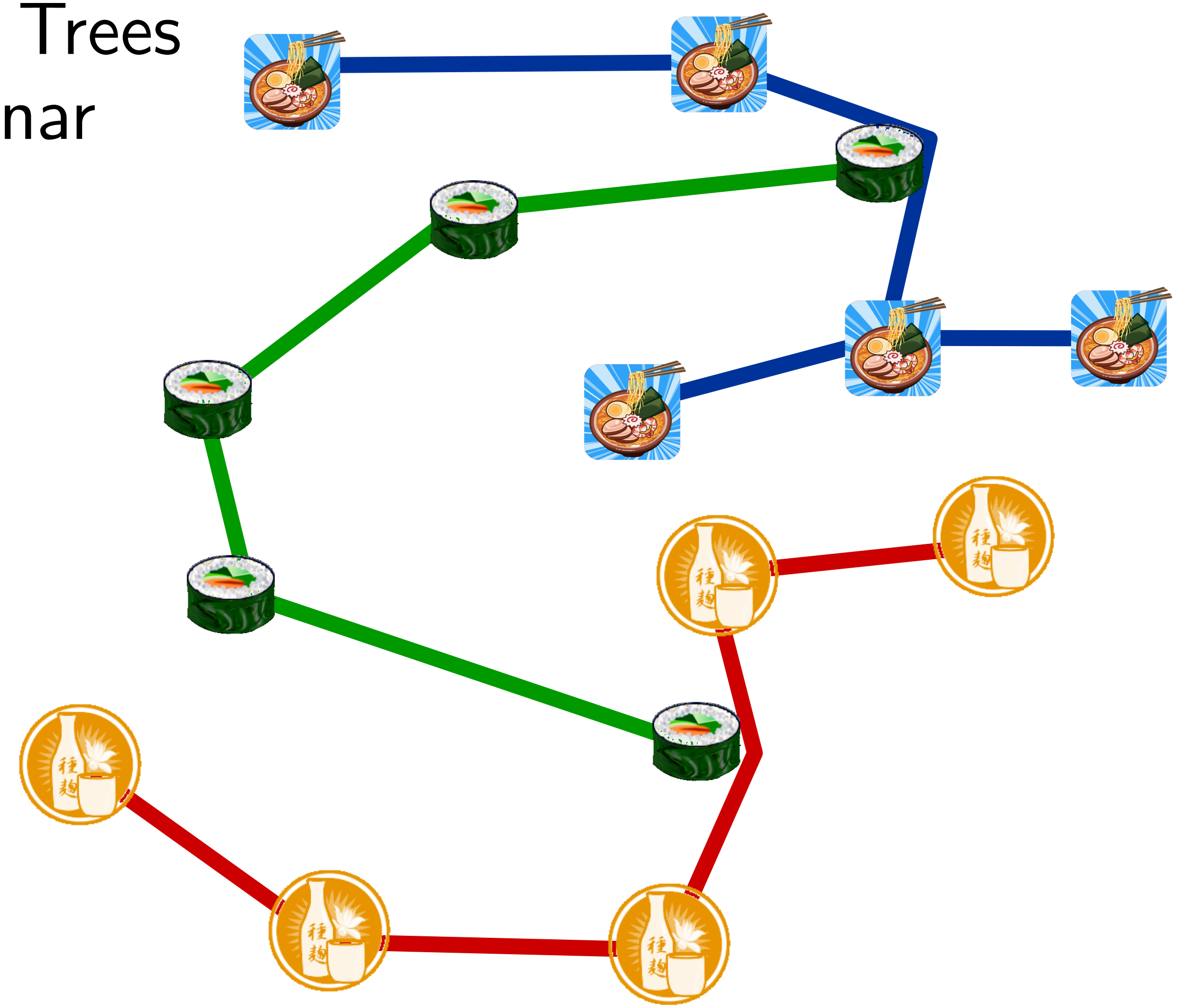
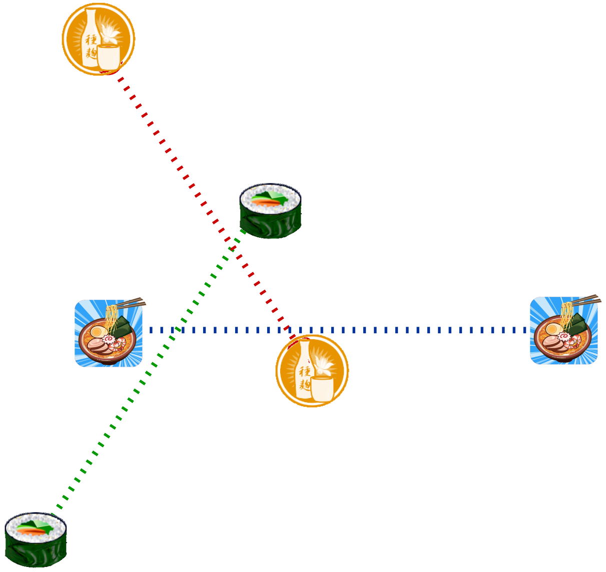


Bad Examples



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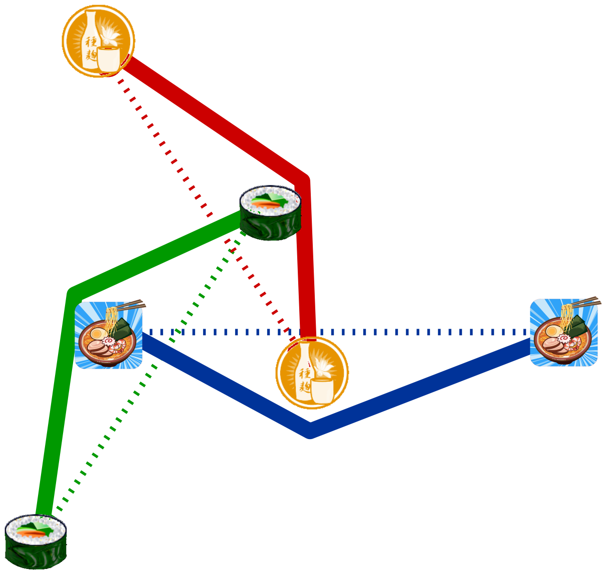

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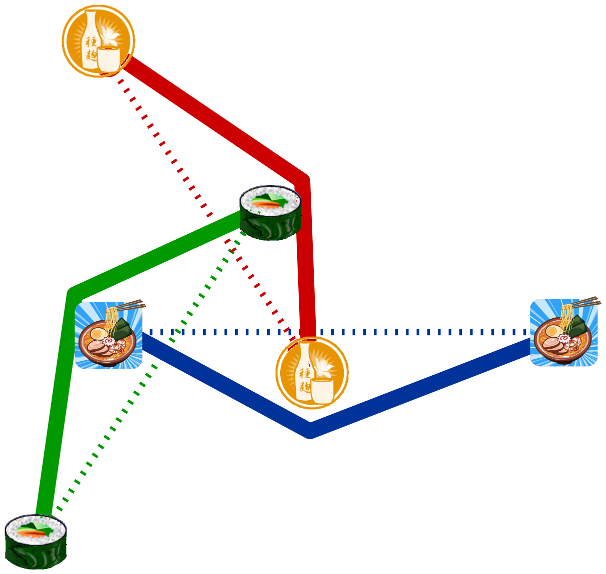

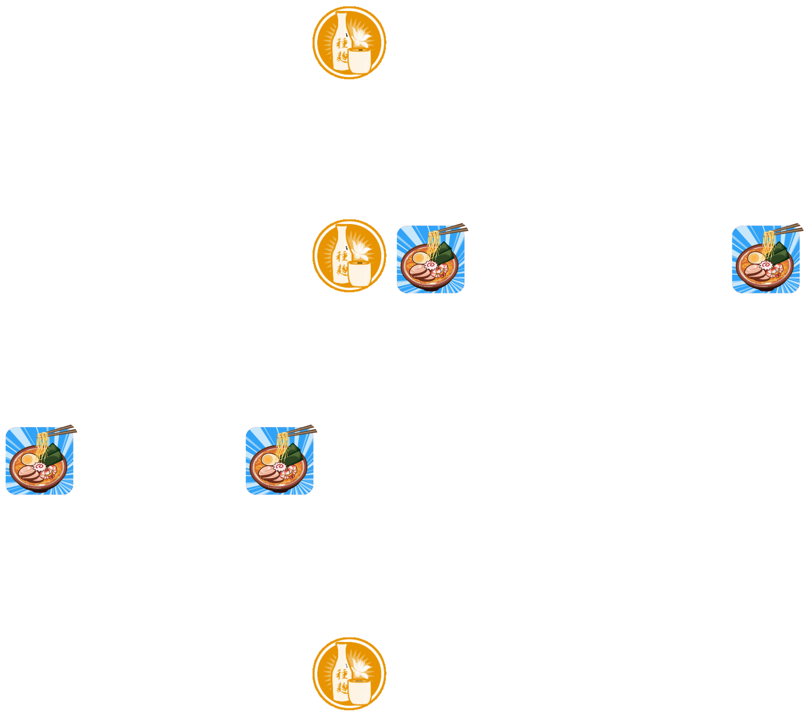

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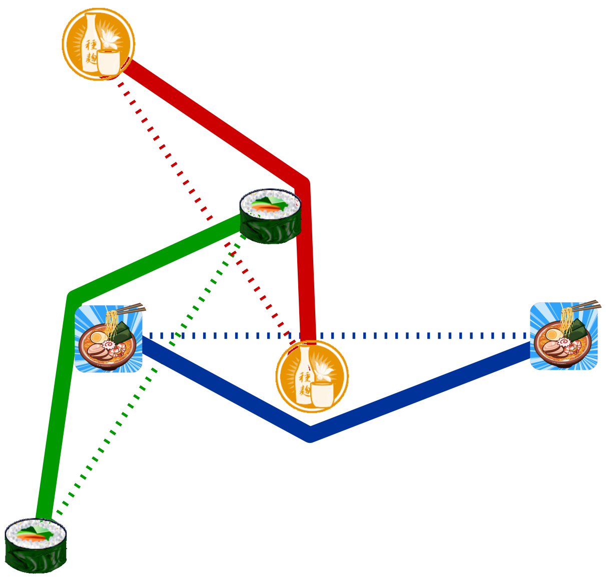

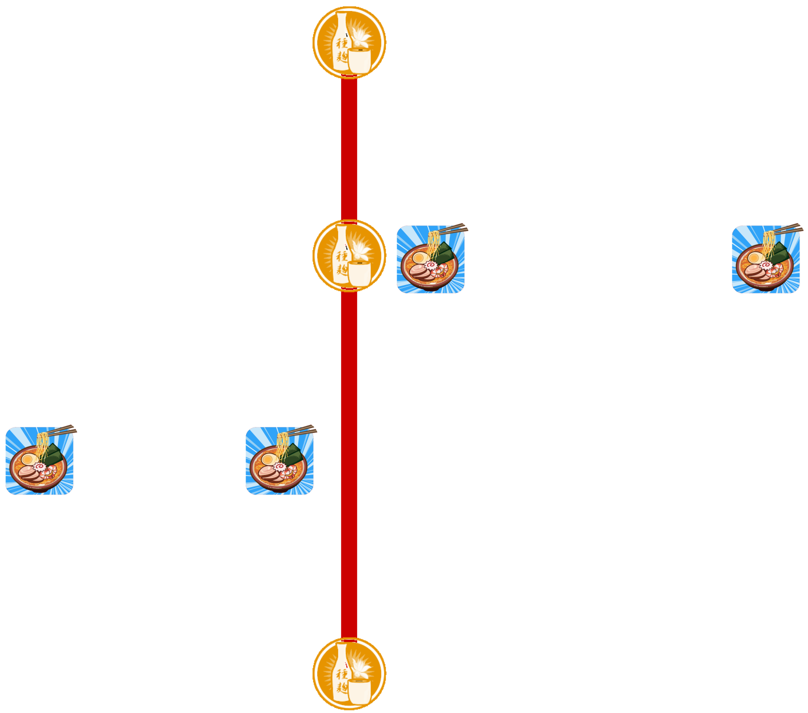

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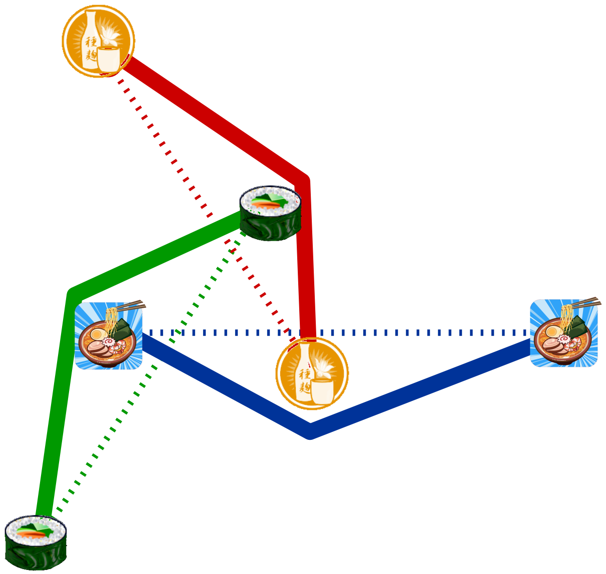

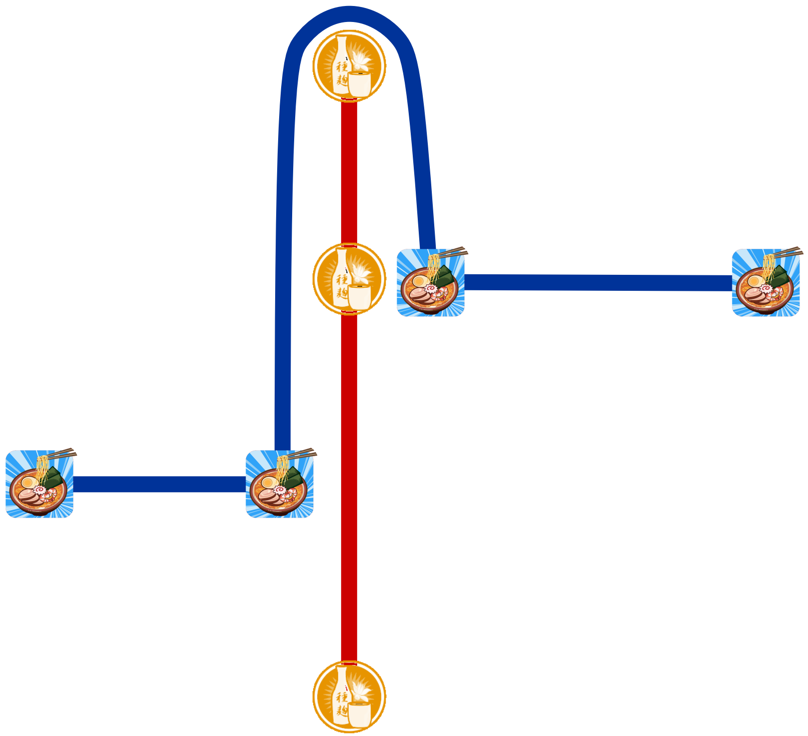

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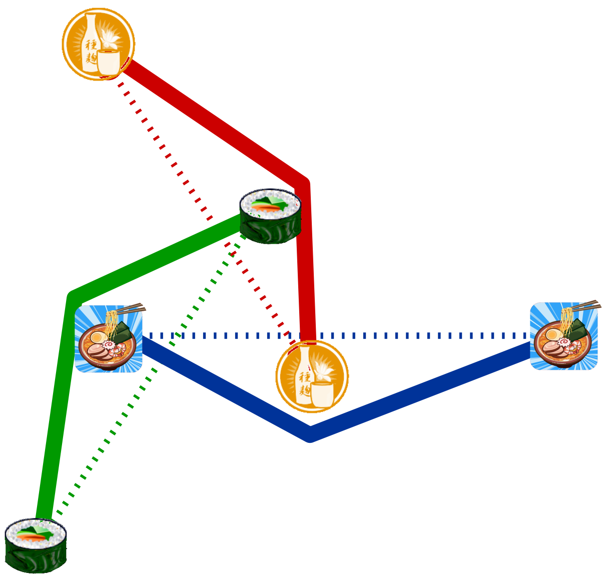

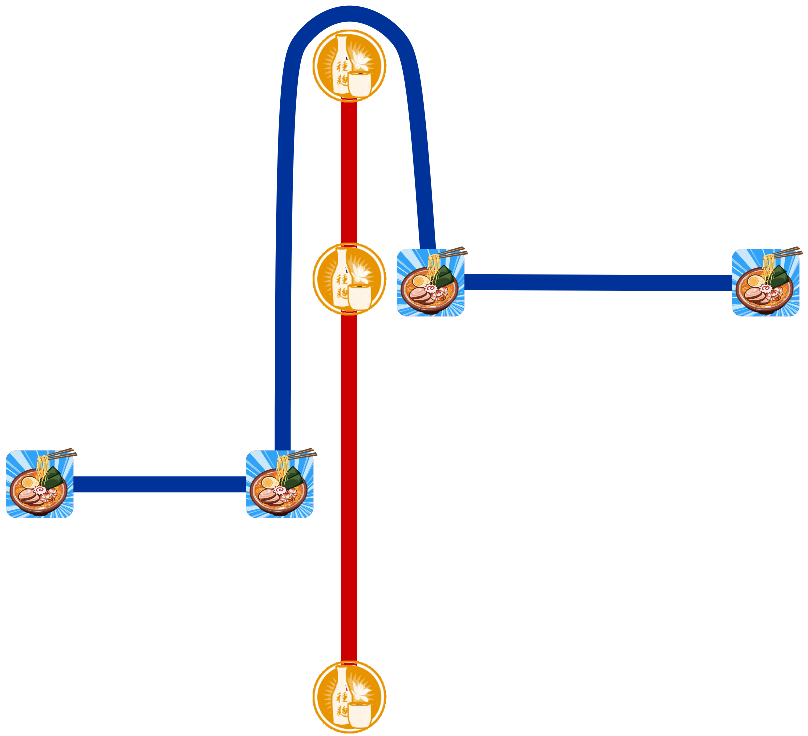

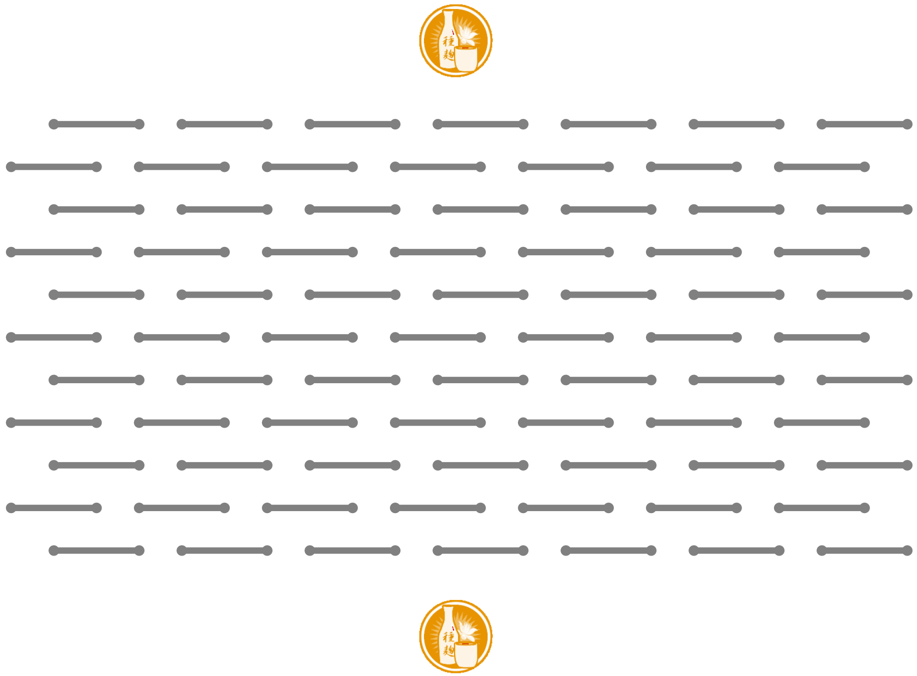

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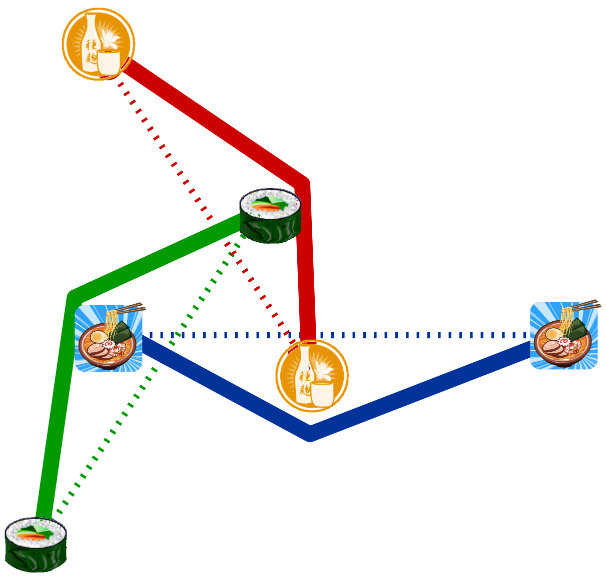

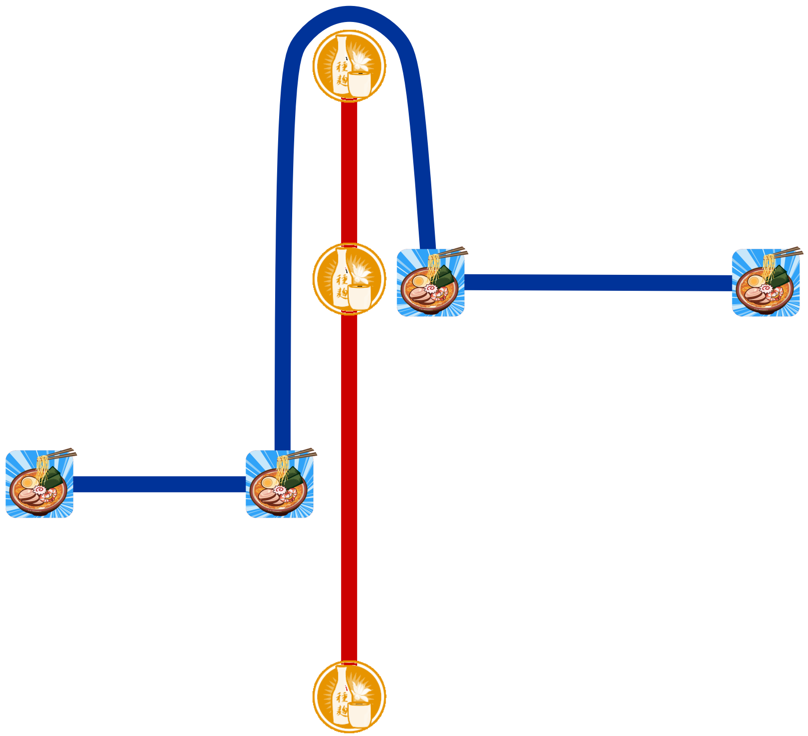

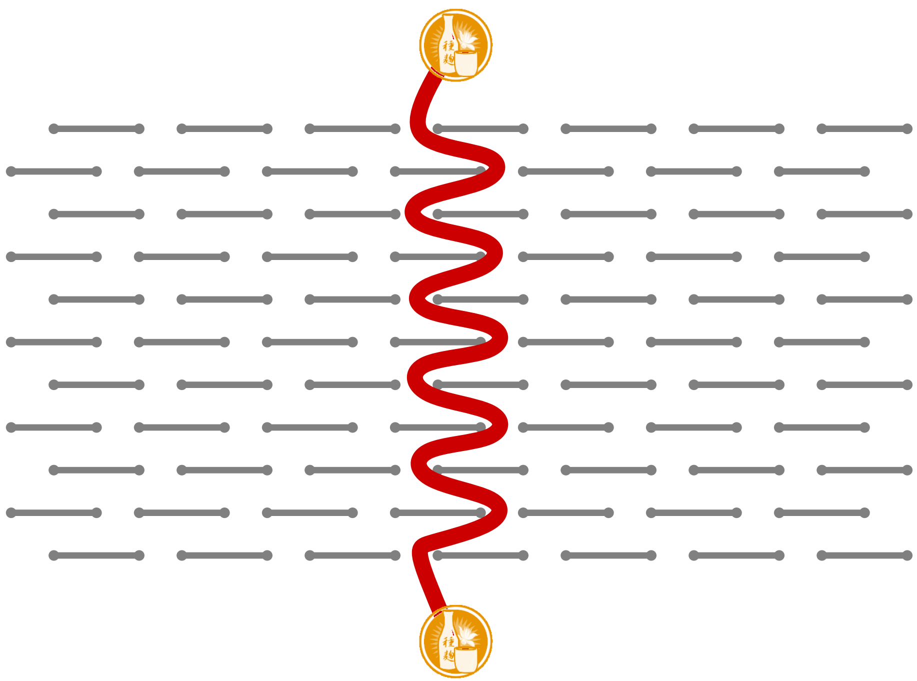

Known Results

1-CESF ($=$ Euclidean Steiner Tree)

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Steiner ratio

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k-CESF

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Steiner ratio

$\rho \leq 1.21$ [Chung Graham, ANYAS'85]

2-CESF

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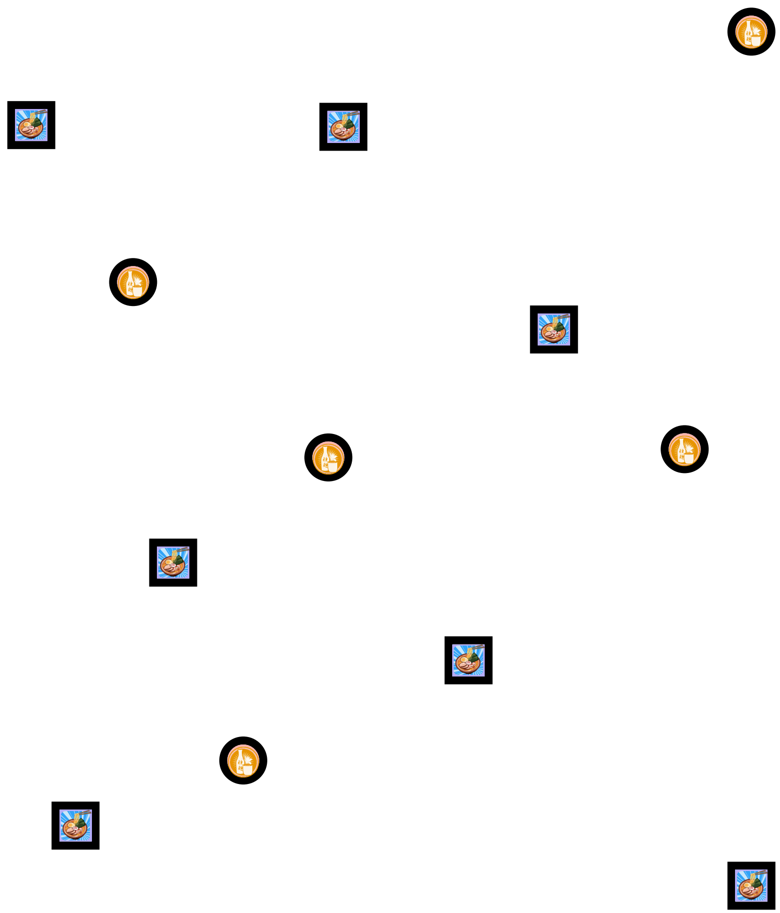
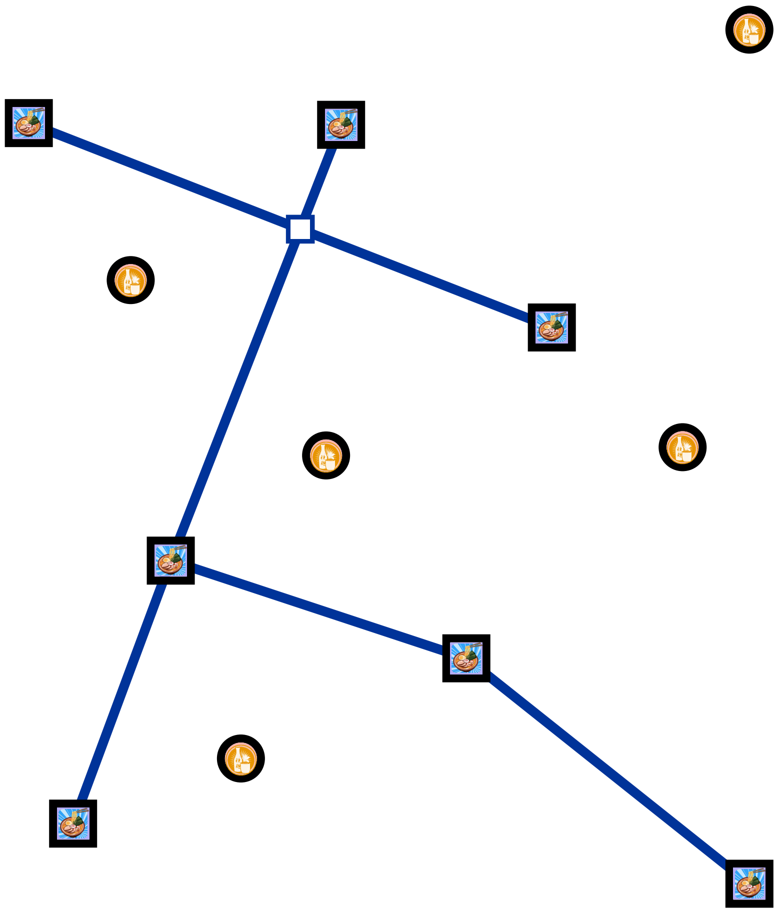
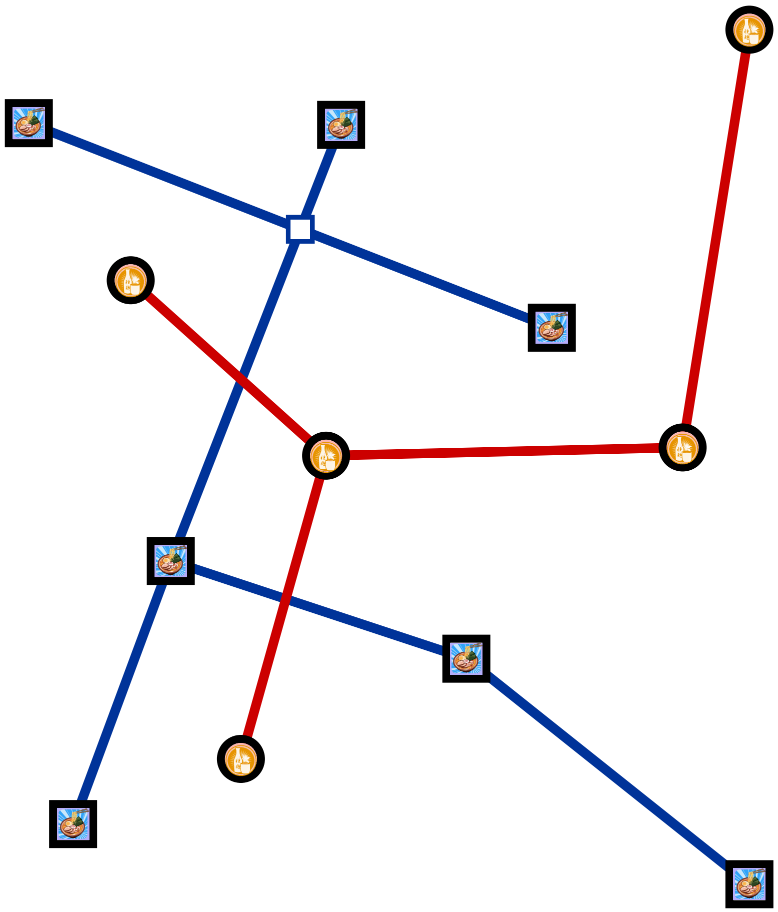


2-CESF



2-CESF



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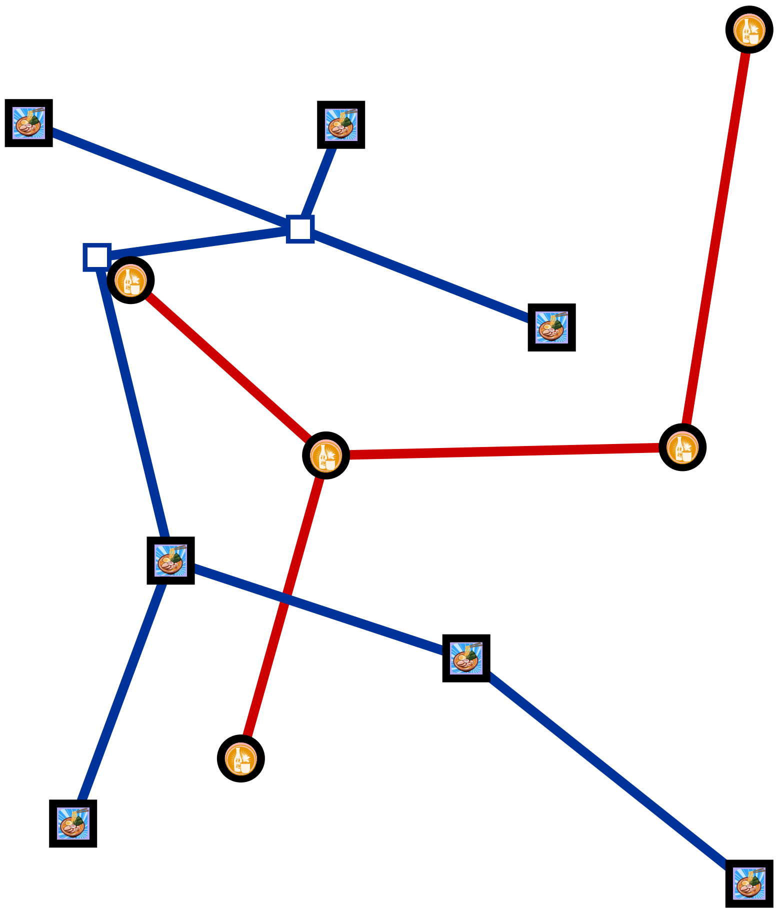

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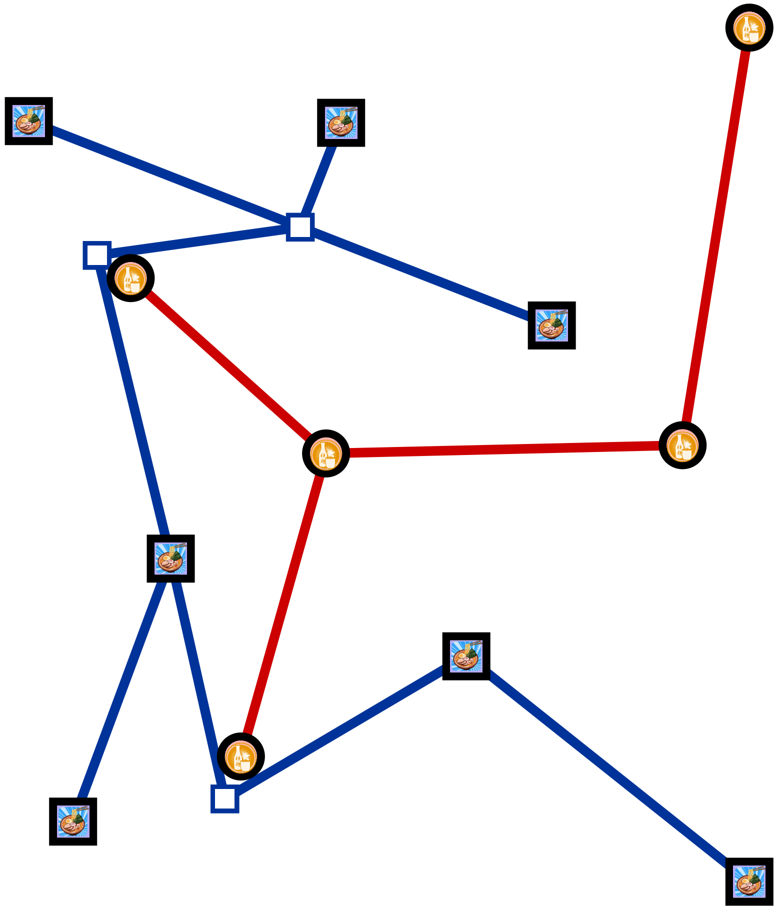
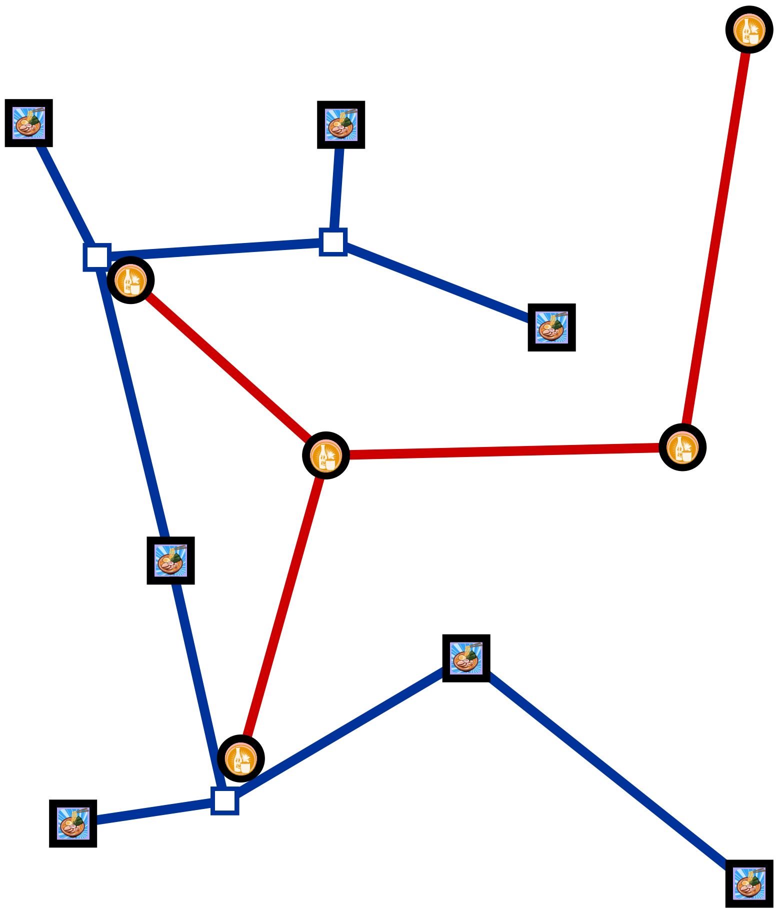
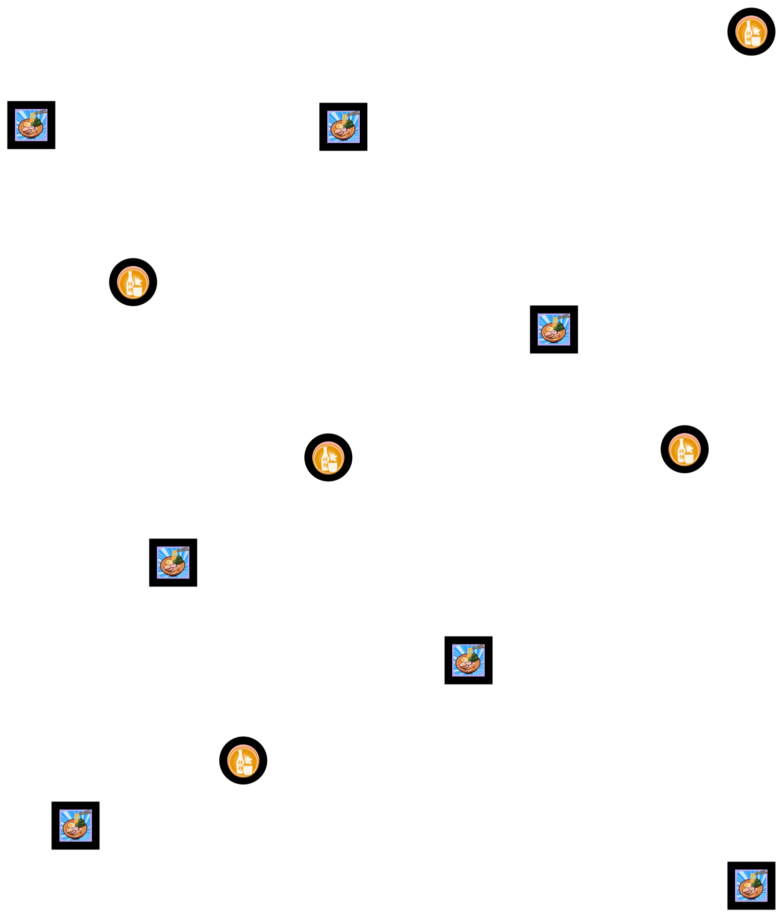


2-CESF



Rounding to the Grid



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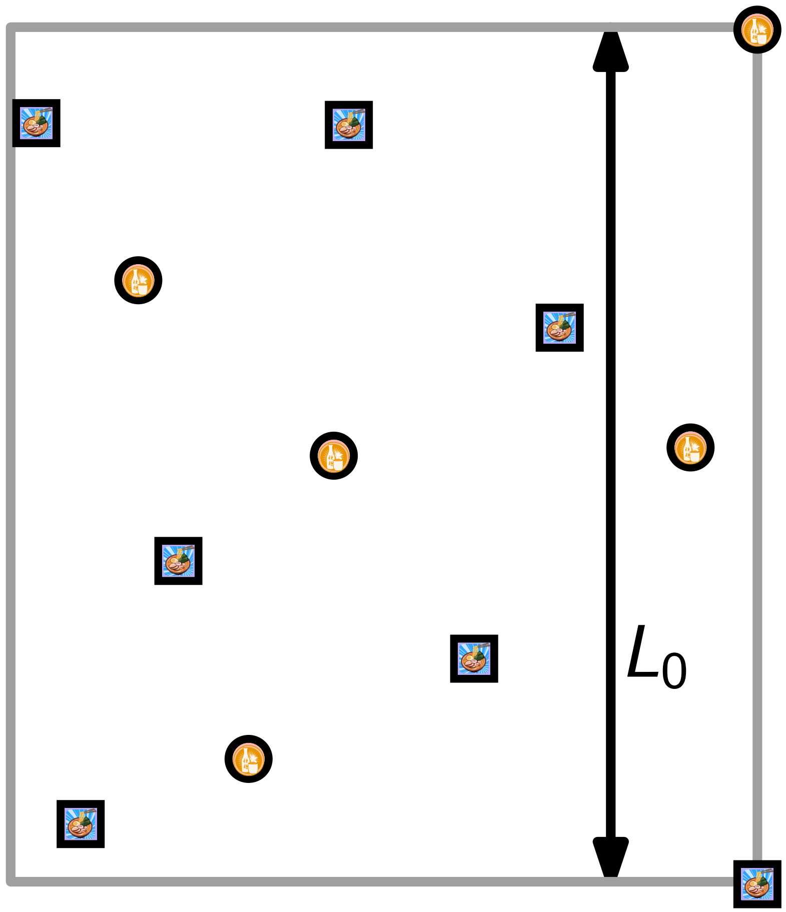
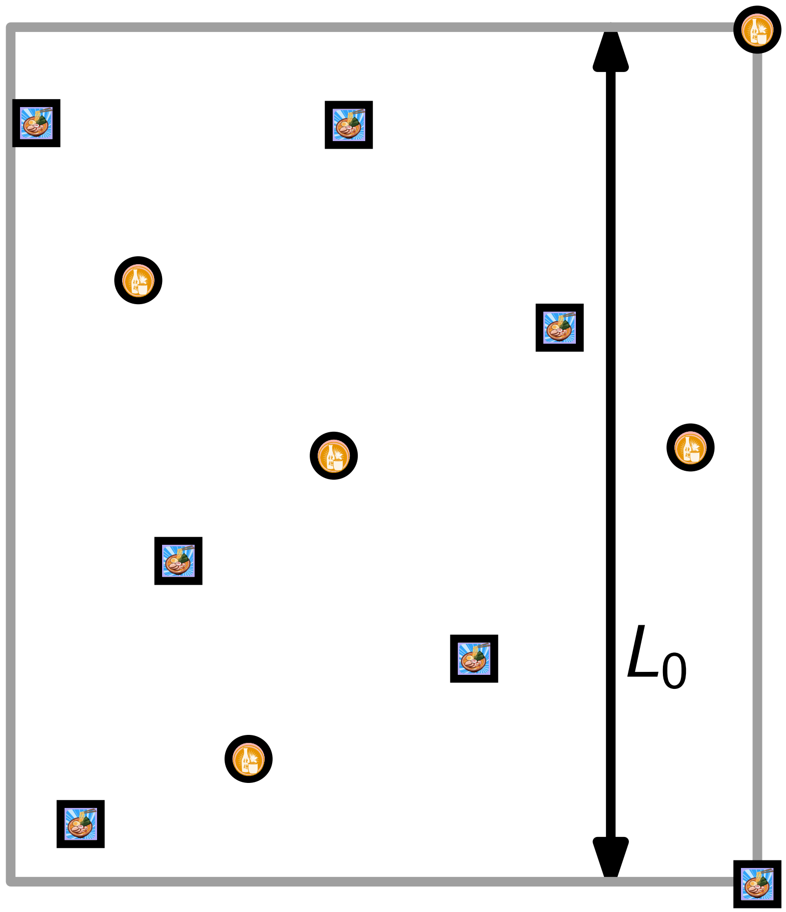


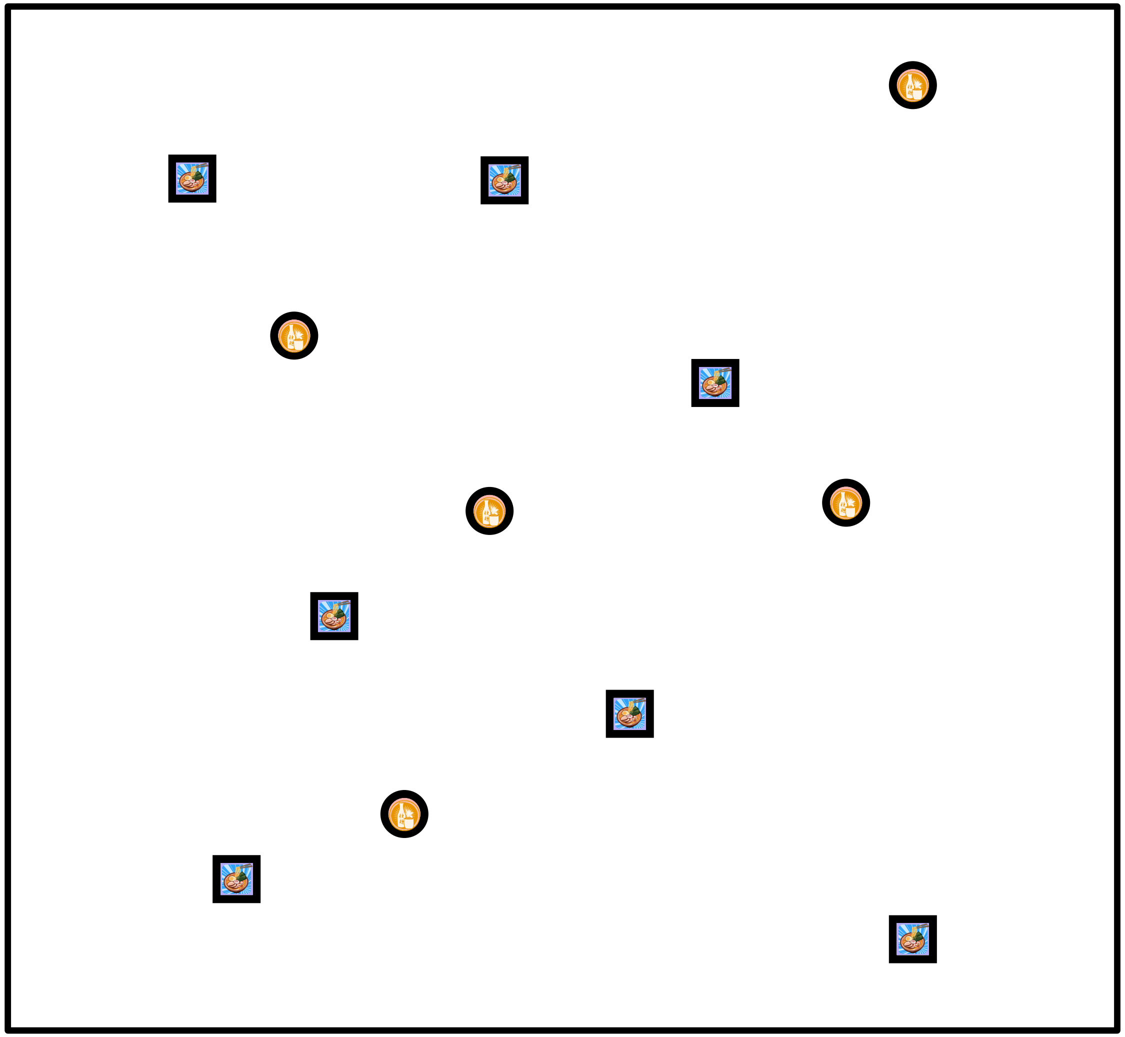
- L_0 diameter of smallest bounding box

Rounding to the Grid



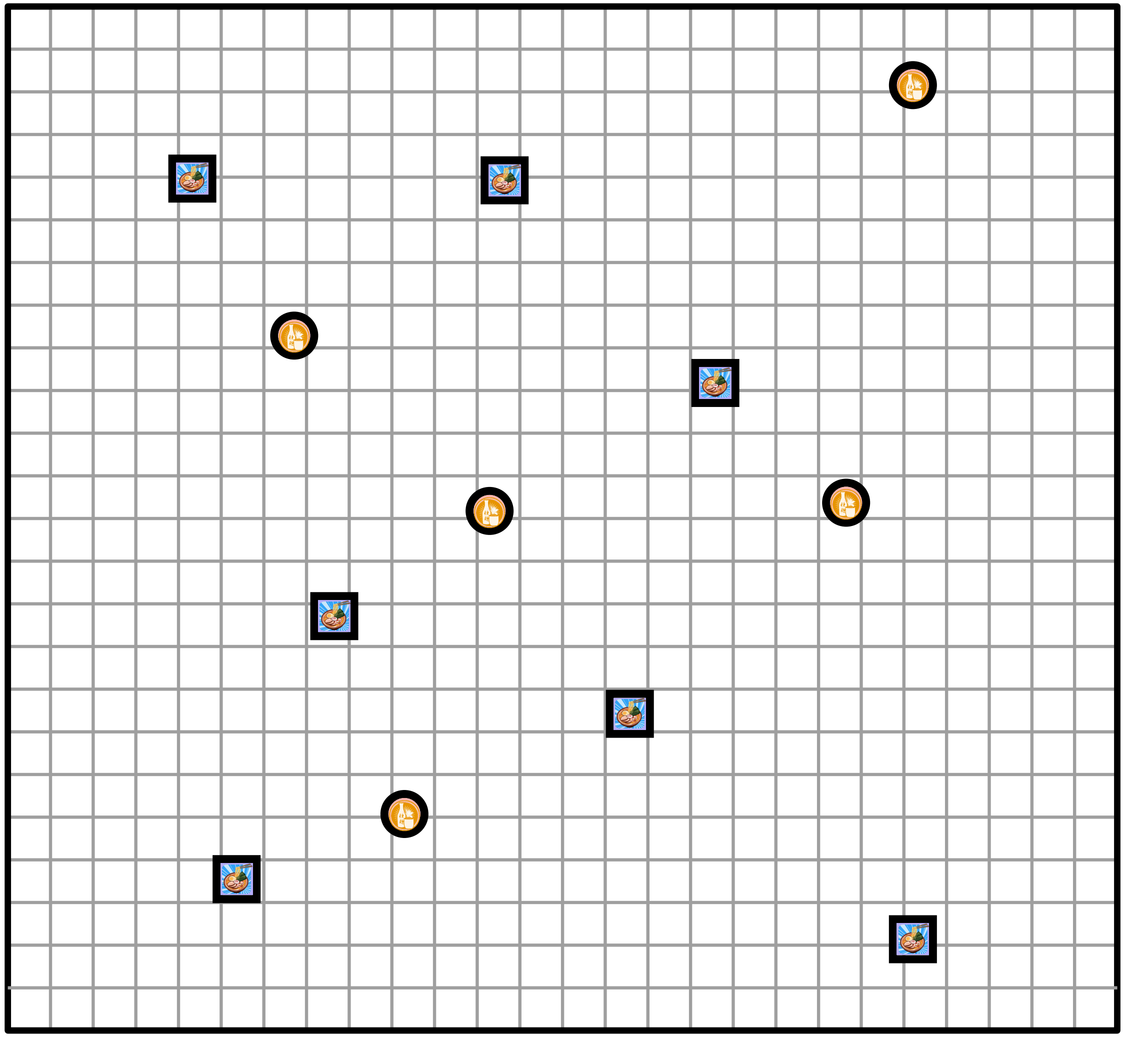
- L_0 diameter of smallest bounding box
- $3\sqrt{2}n/\varepsilon \leq L \leq 6\sqrt{2}n/\varepsilon$

Rounding to the Grid



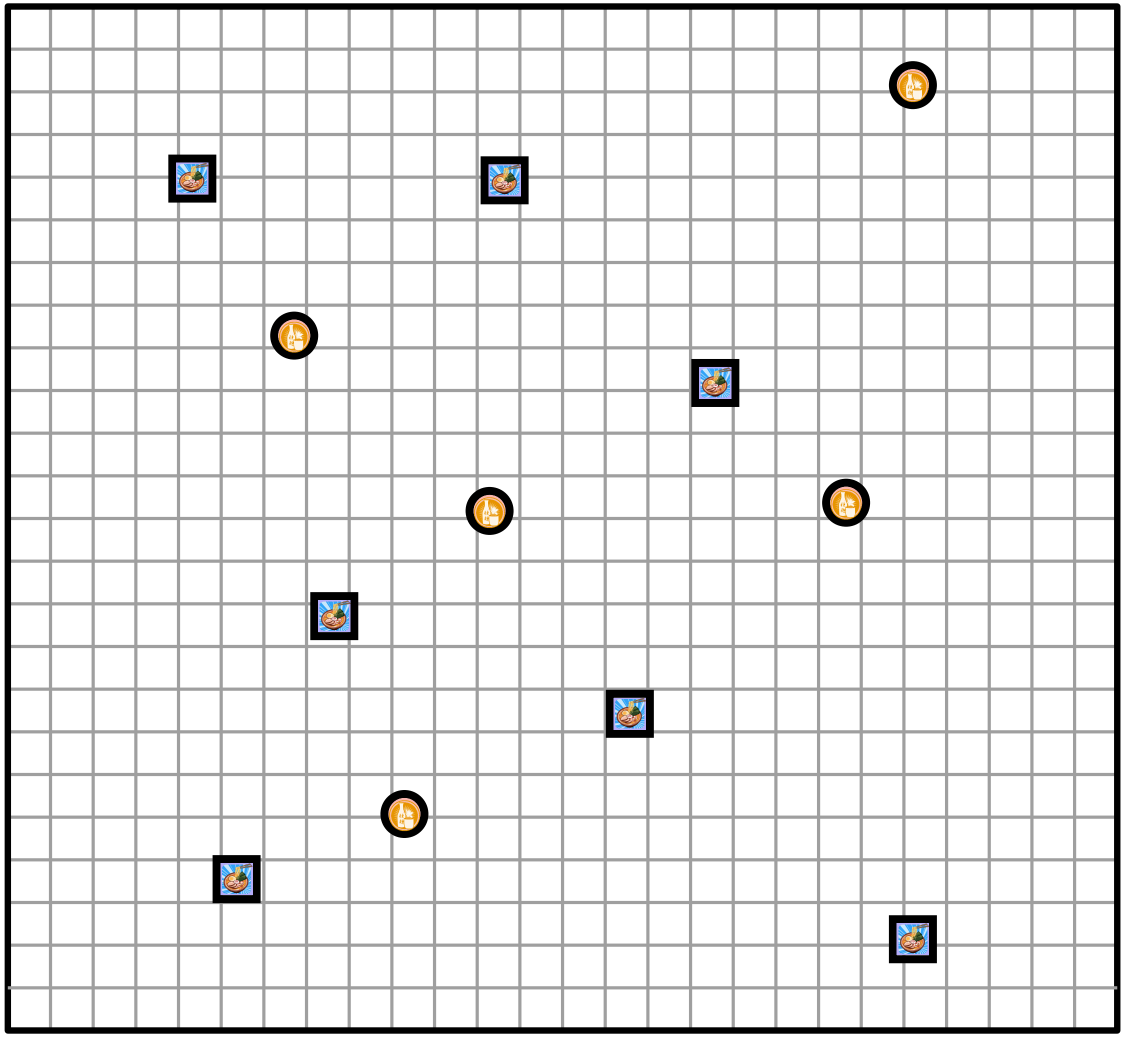
- L_0 diameter of smallest bounding box
- $3\sqrt{2}n/\varepsilon \leq L \leq 6\sqrt{2}n/\varepsilon$
- $(L \times L)$-grid

Rounding to the Grid



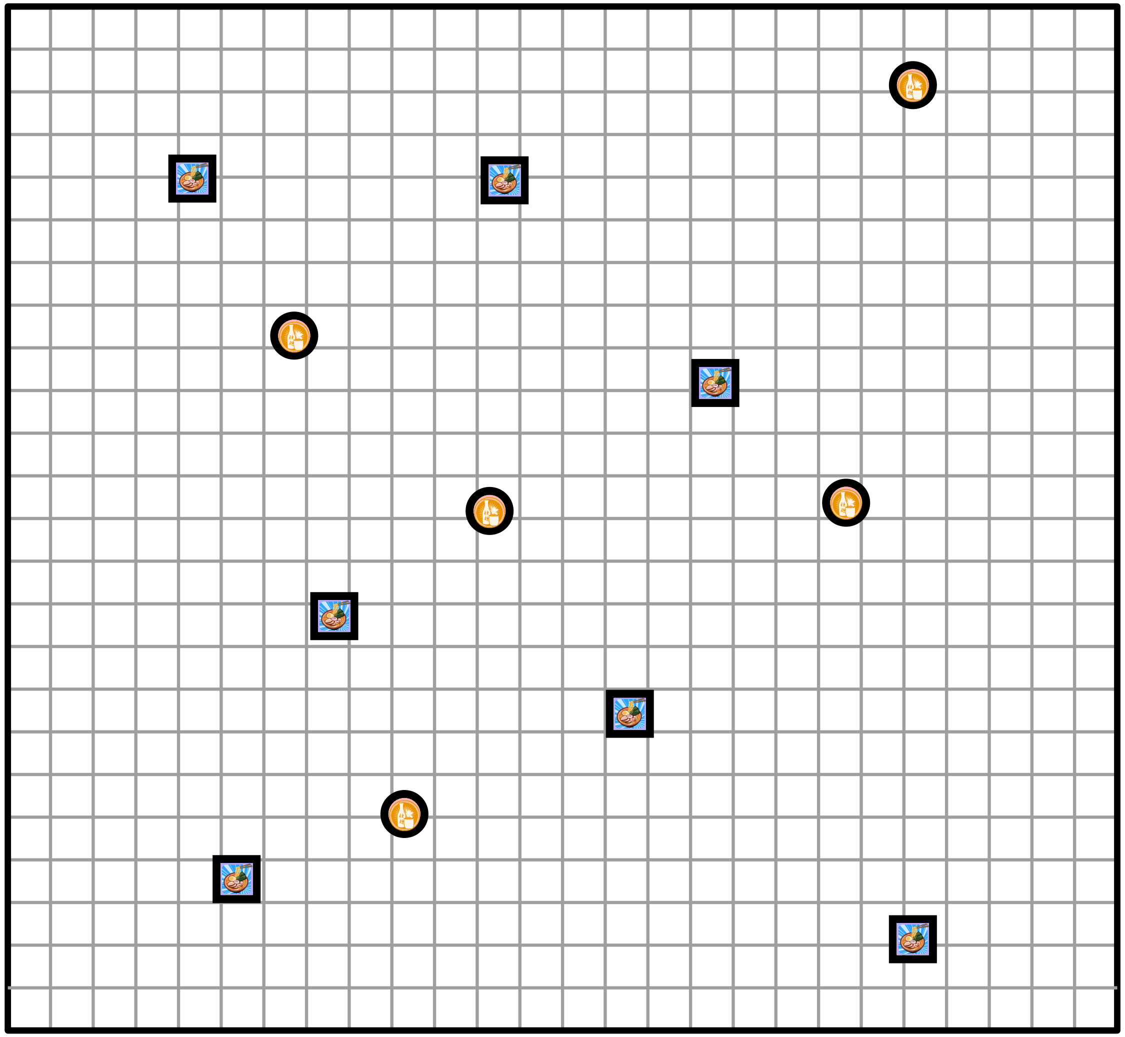
- L_0 diameter of smallest bounding box
- $3\sqrt{2}n/\varepsilon \leq L \leq 6\sqrt{2}n/\varepsilon$
- $(L \times L)$-grid
- granularity L_0/L

Rounding to the Grid



- ~~L_0 diameter of smallest bounding box~~
- $3\sqrt{2}n/\varepsilon \leq L \leq 6\sqrt{2}n/\varepsilon$
- $(L \times L)$-grid
- granularity ~~L_0/L~~ 1

Rounding to the Grid



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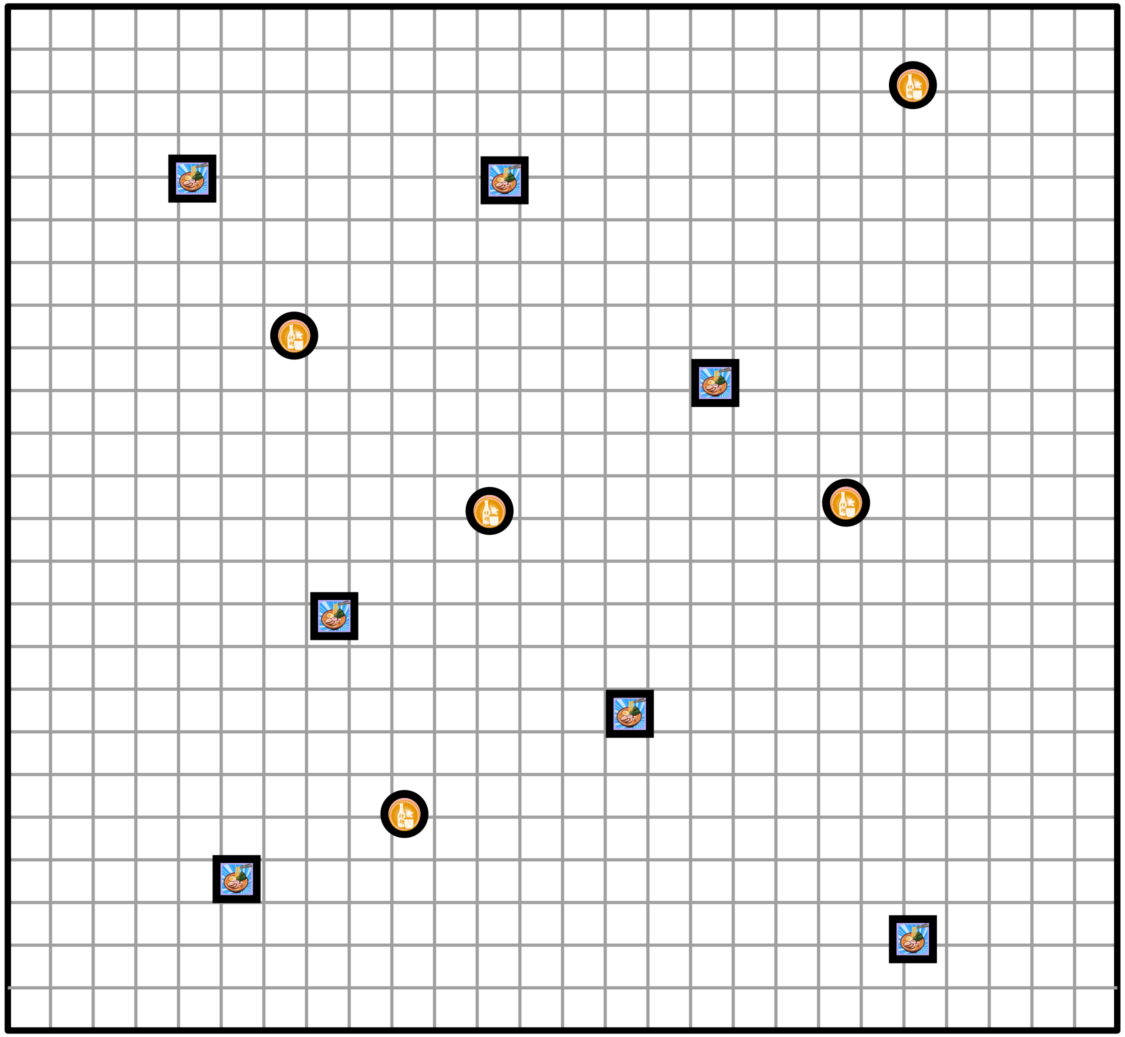

- $3\sqrt{2}n/\varepsilon \leq L \leq 6\sqrt{2}n/\varepsilon$
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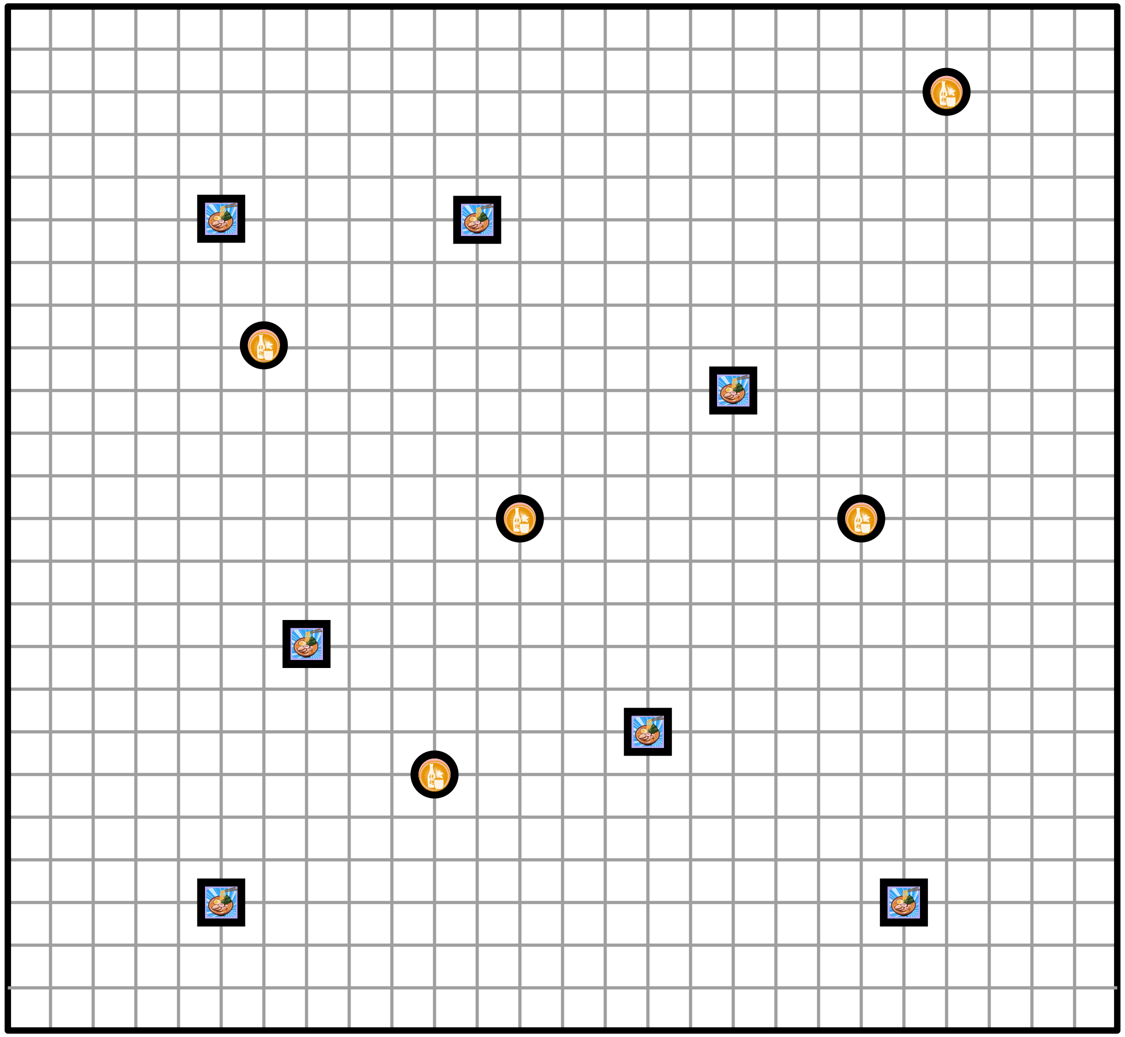
 $\rightarrow$ (even,even)

 $\rightarrow$ (odd,odd)

Rounding to the Grid



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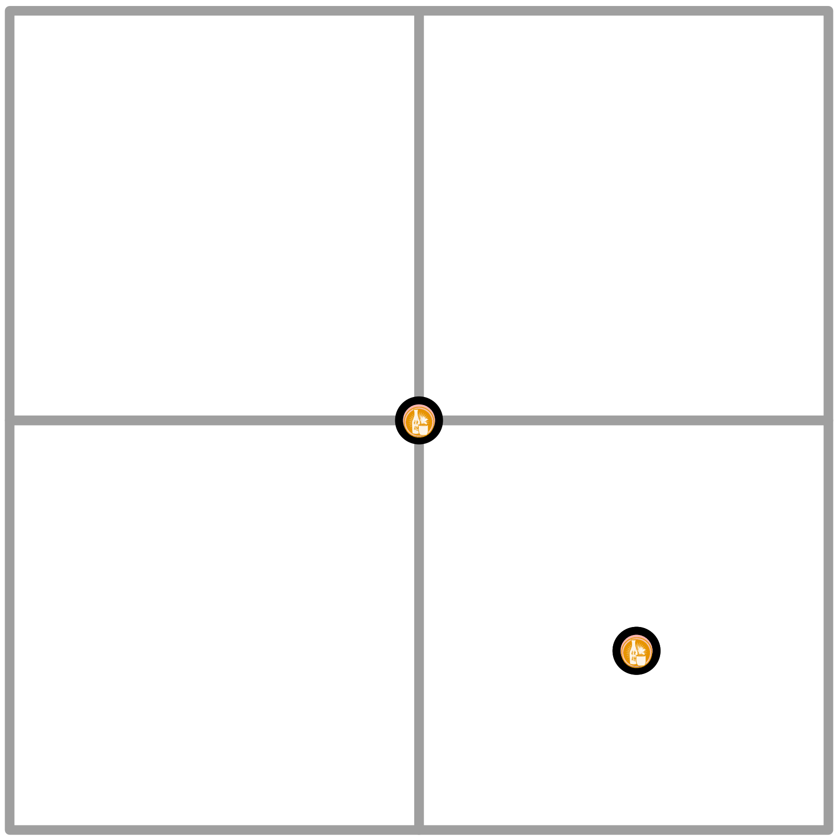
Going Back



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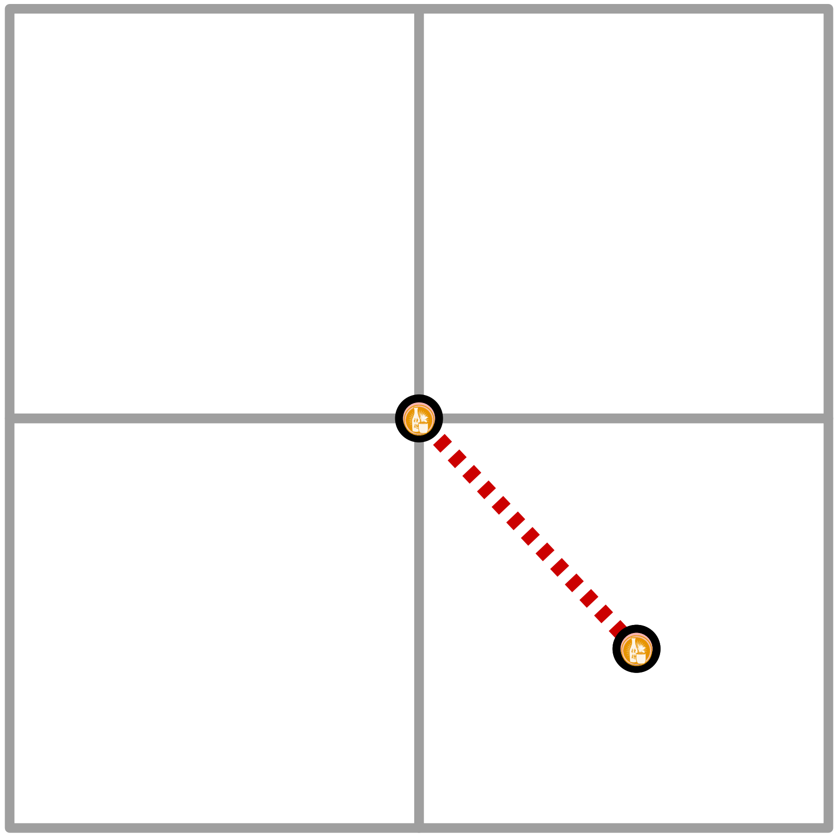
Going Back



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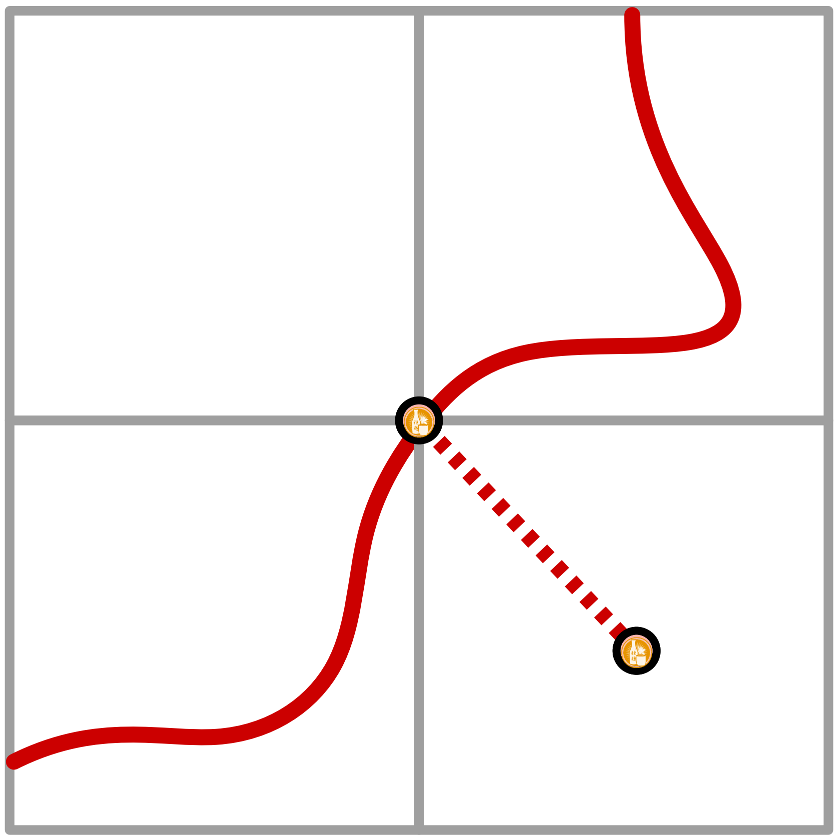

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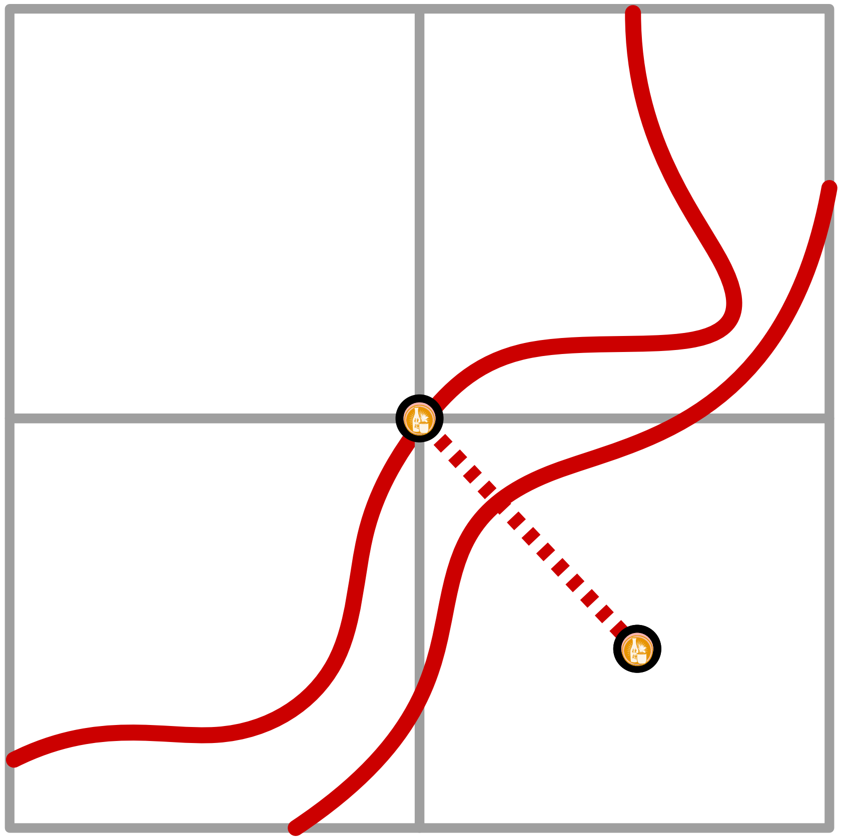

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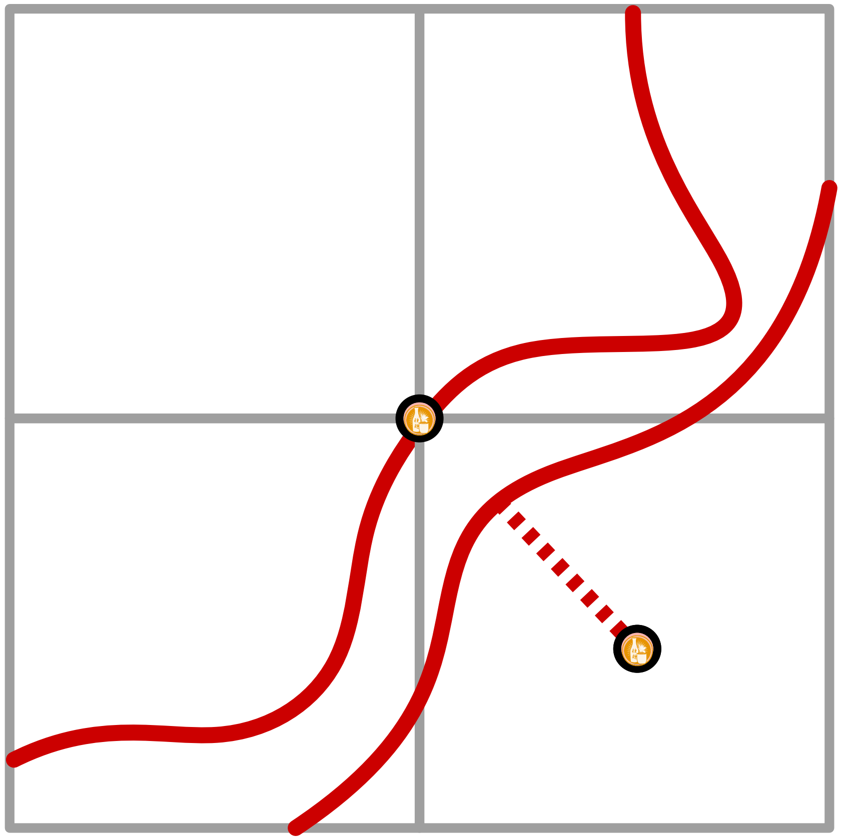

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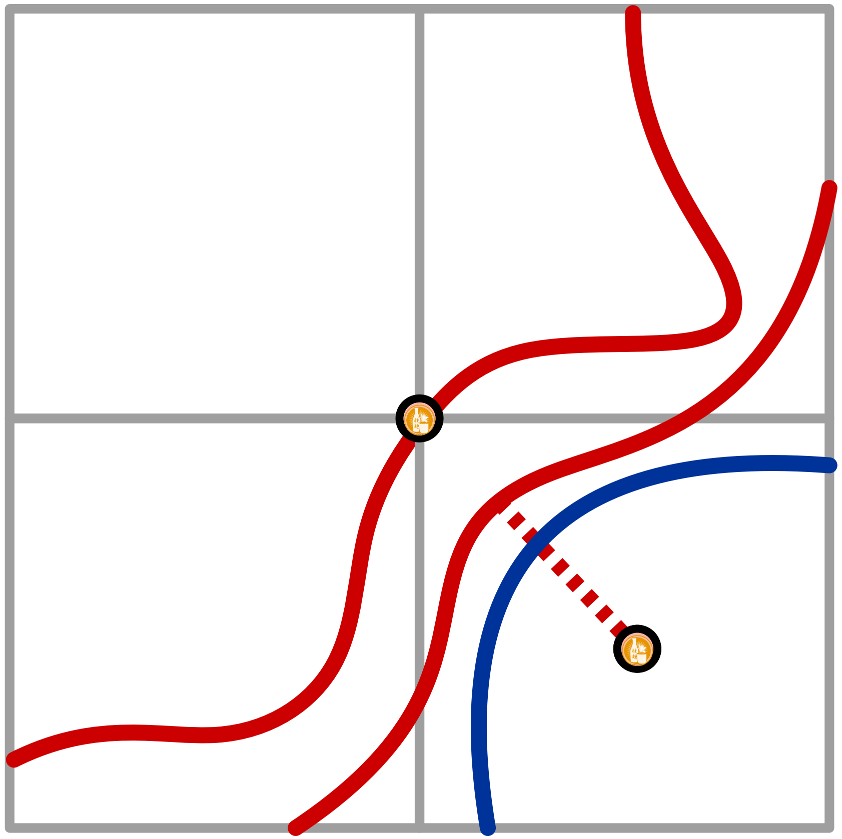

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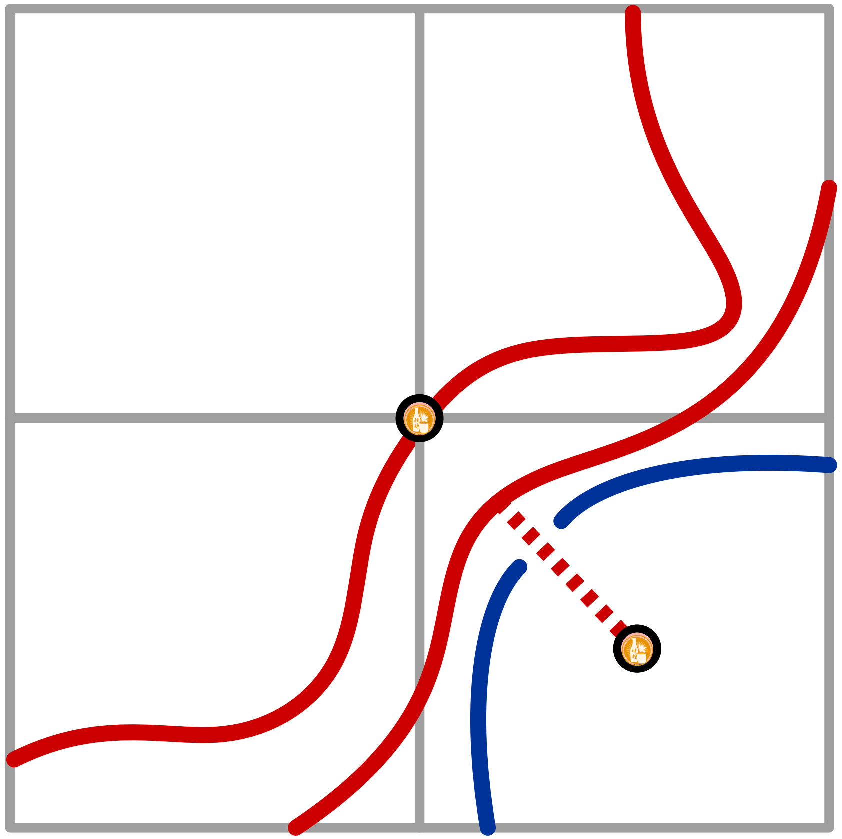
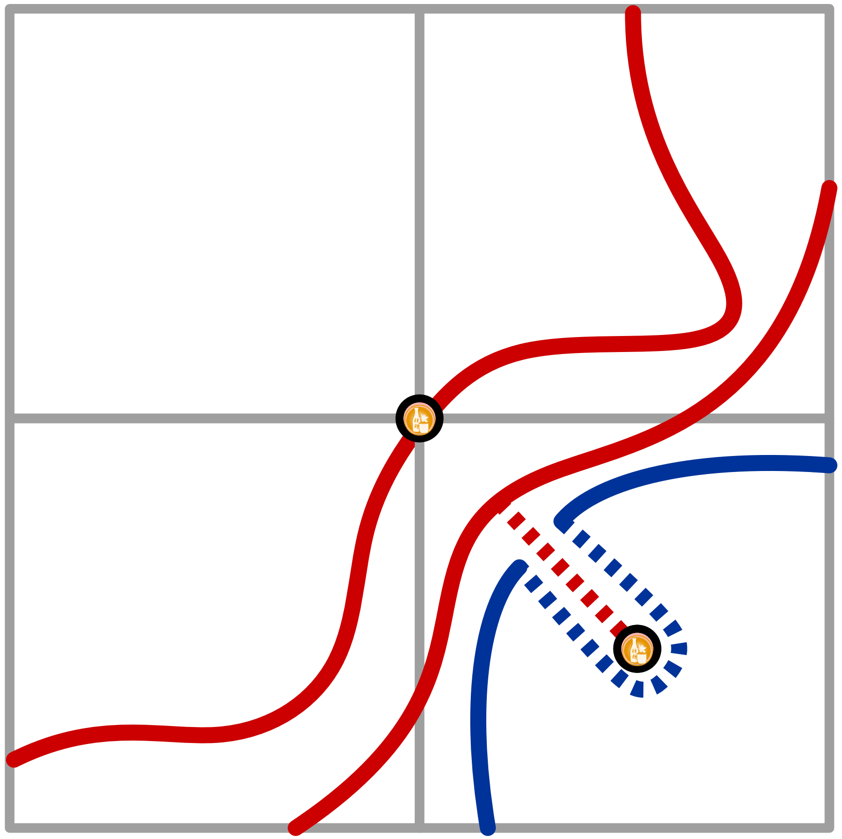
Going Back



- $3\sqrt{2}n/\varepsilon \leq L \leq 6\sqrt{2}n/\varepsilon$
- $(L \times L)$-grid
- granularity 1

Going Back



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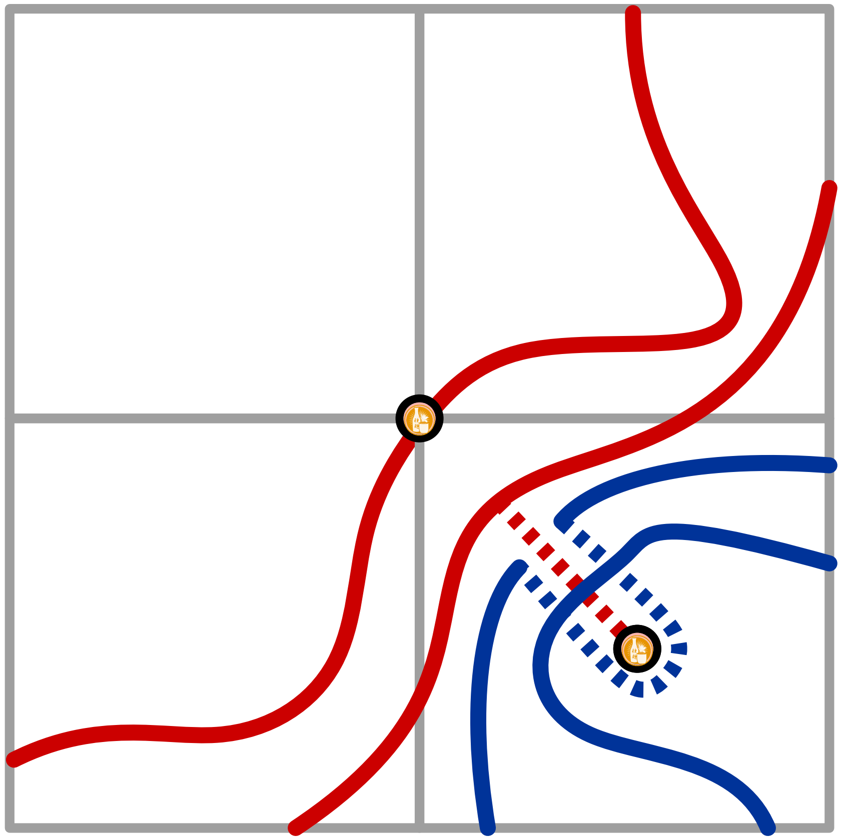

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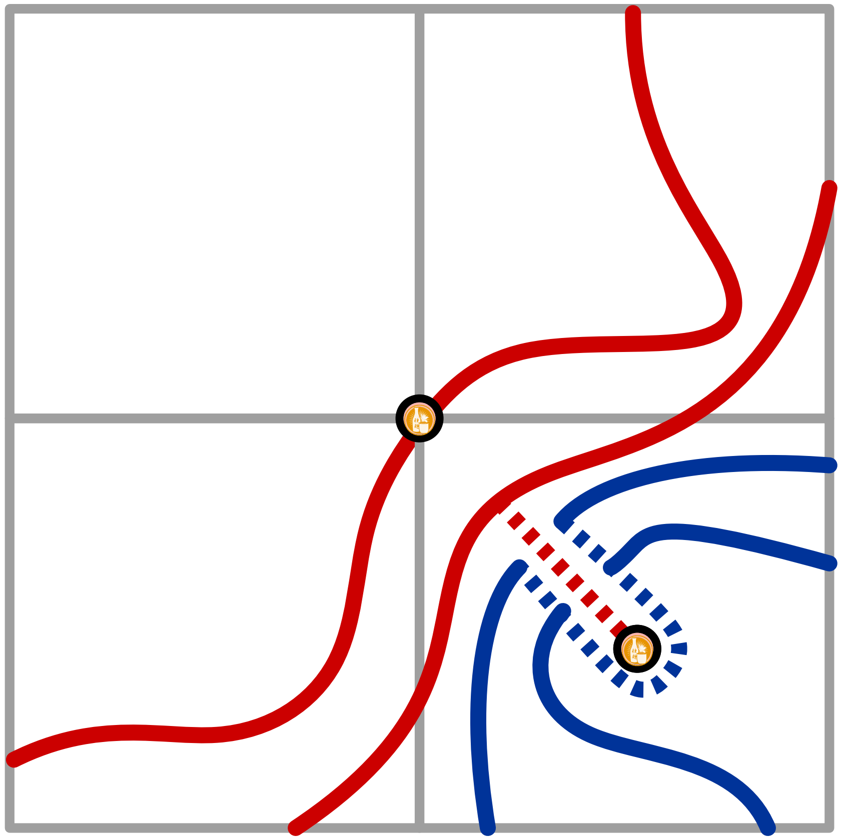

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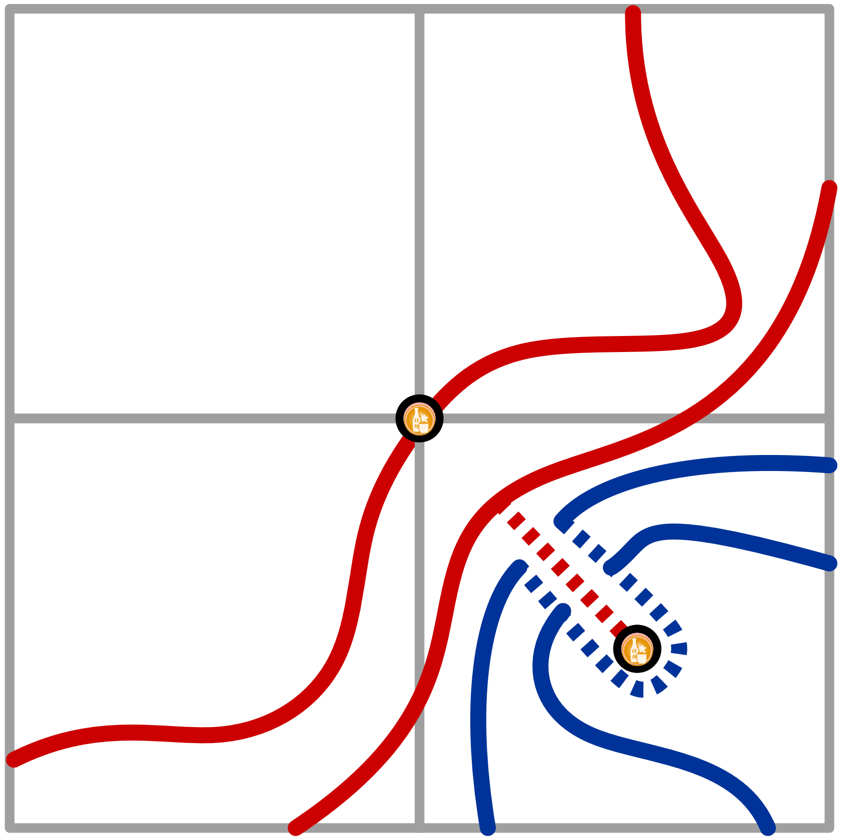

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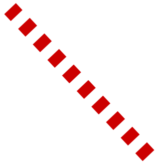
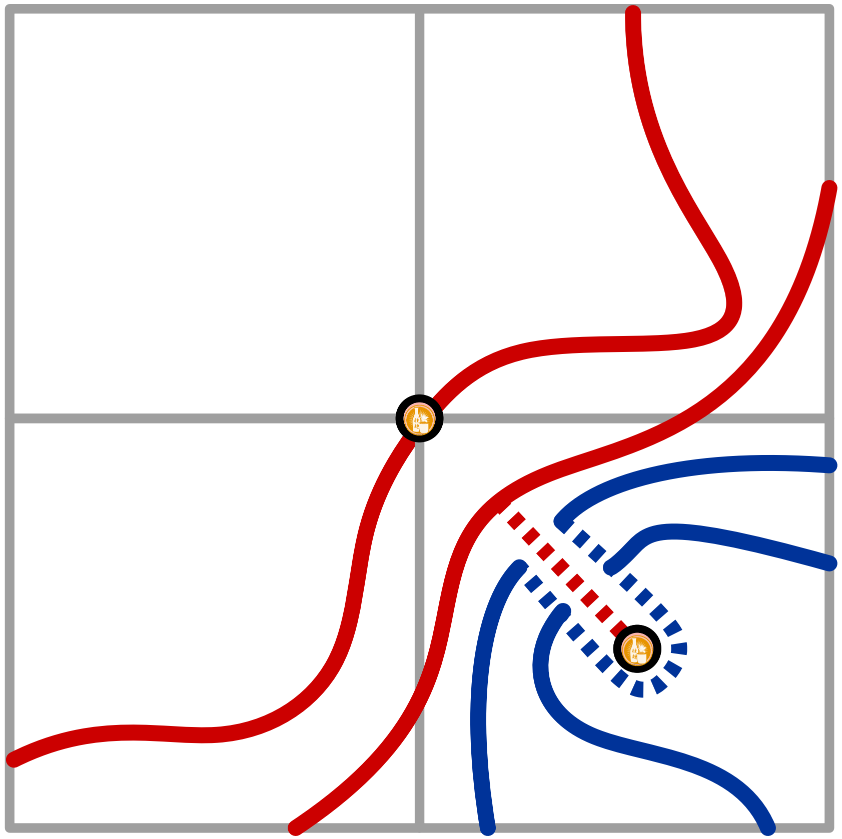


$\leq \sqrt{2}$

Going Back



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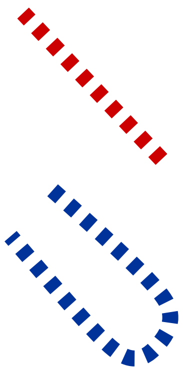
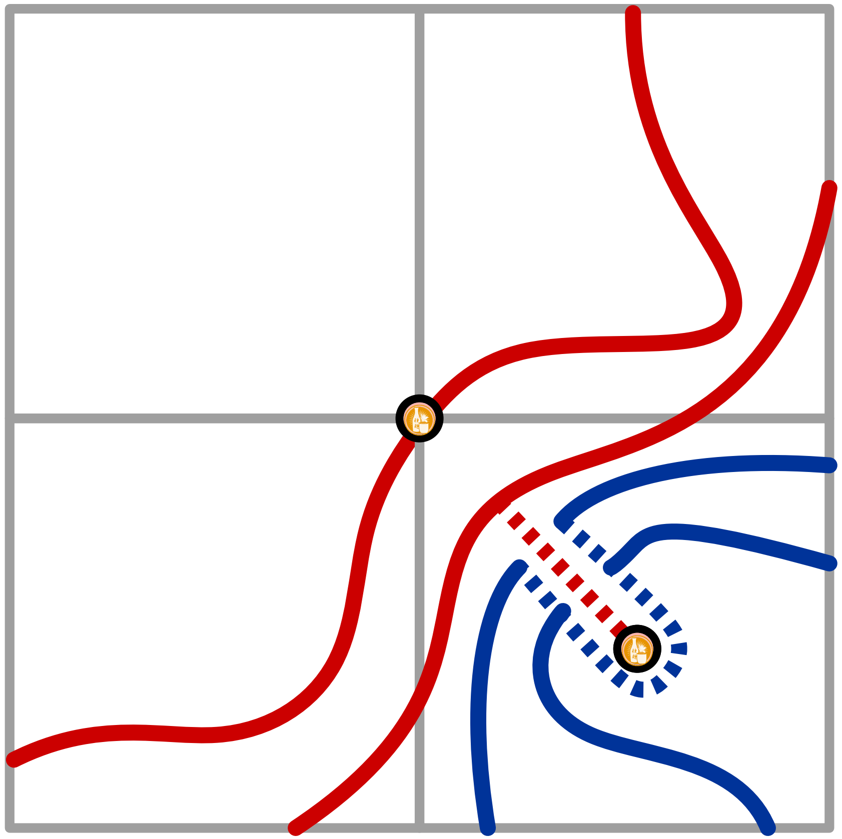


$\leq \sqrt{2}$

$\leq 2\sqrt{2}$

Going Back



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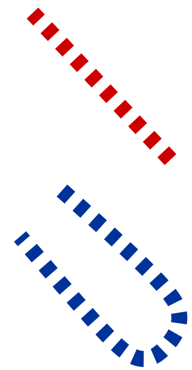
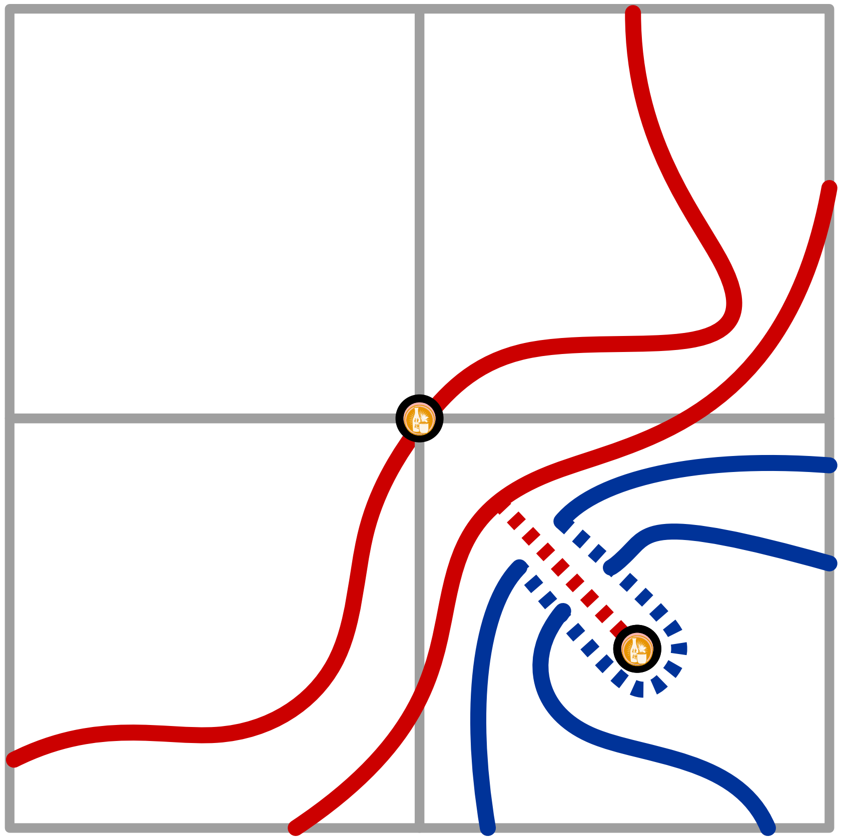


$\leq \sqrt{2}$

$\leq 2\sqrt{2}$

in total $\leq 3\sqrt{2}n$

Going Back



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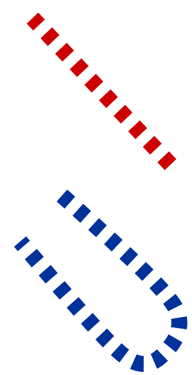
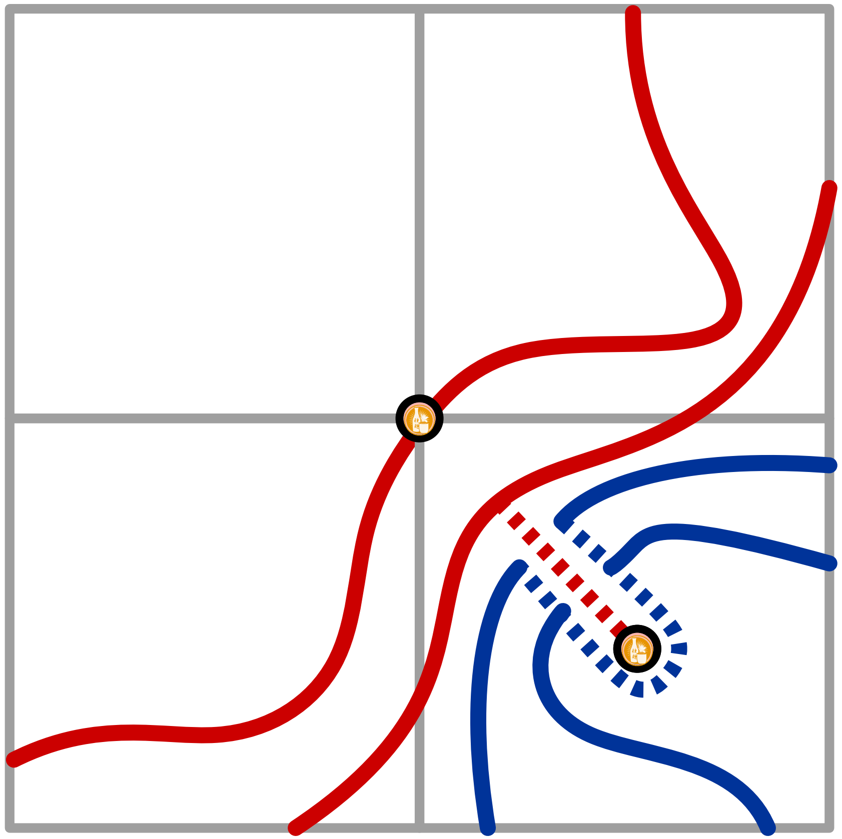


$\leq \sqrt{2}$

$\leq 2\sqrt{2}$

in total $\leq 3\sqrt{2}n \leq \varepsilon L$

Going Back



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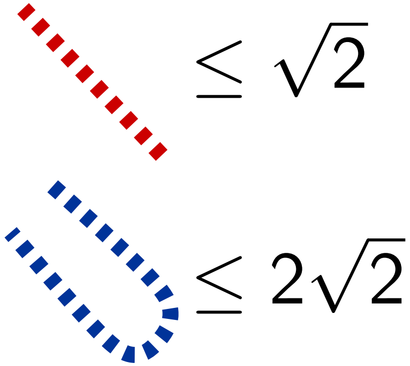

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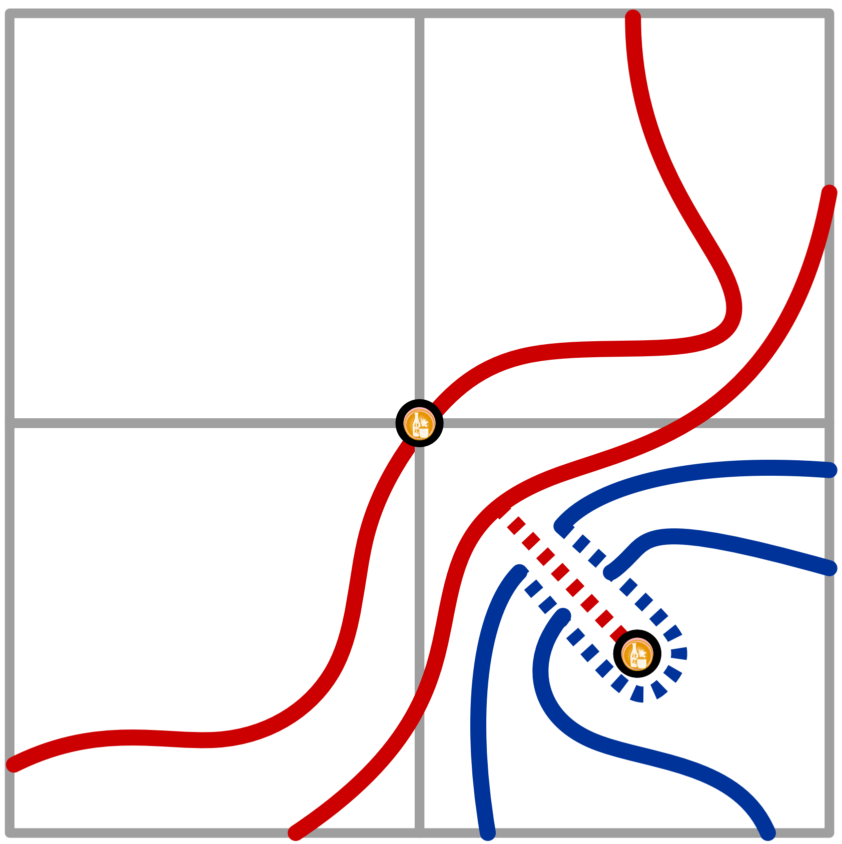
Going Back



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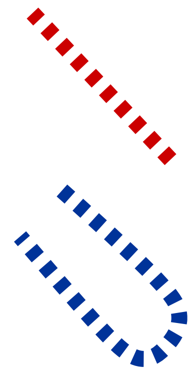


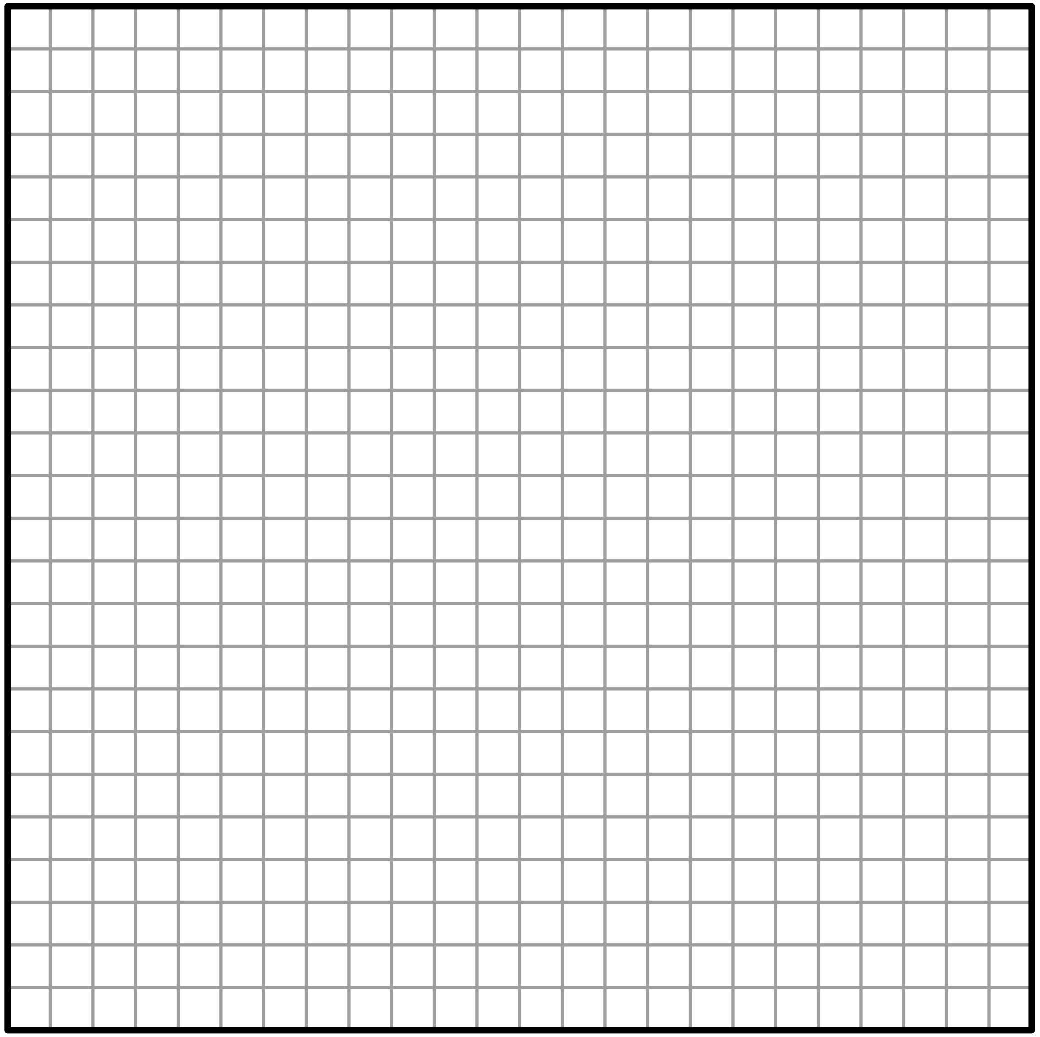
$\leq \sqrt{2}$

$\leq 2\sqrt{2}$

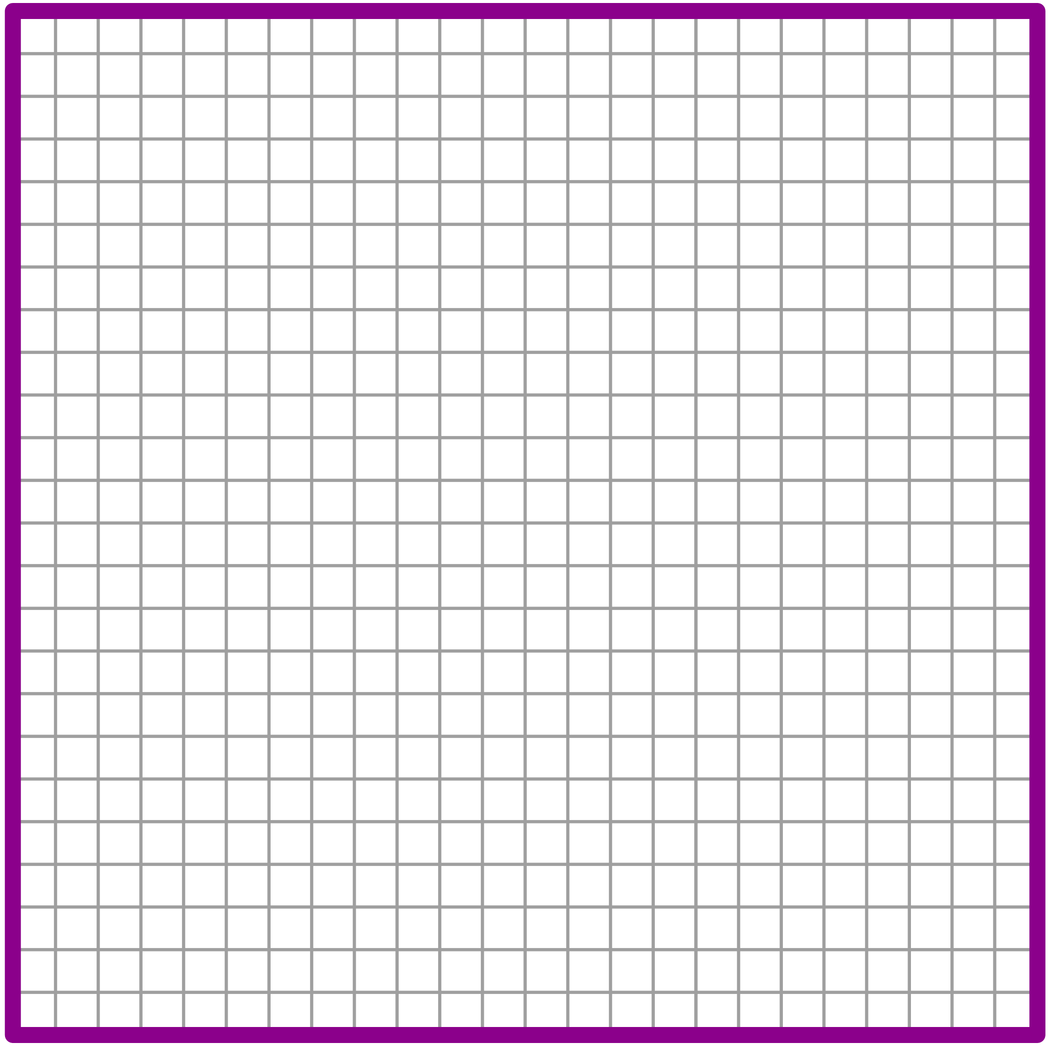
in total $\leq 3\sqrt{2}n \leq \varepsilon L \leq \varepsilon \mathrm{OPT}$

2-CESF instance I $\rightarrow$ rounded instance I^* $\rightarrow$ solution $\mathcal{L}_I$
$|\mathcal{L}_I| \leq (1+\varepsilon)\mathrm{OPT}_{I^*} \leq (1+\varepsilon)^2\mathrm{OPT}_I$

Quadtree Placement

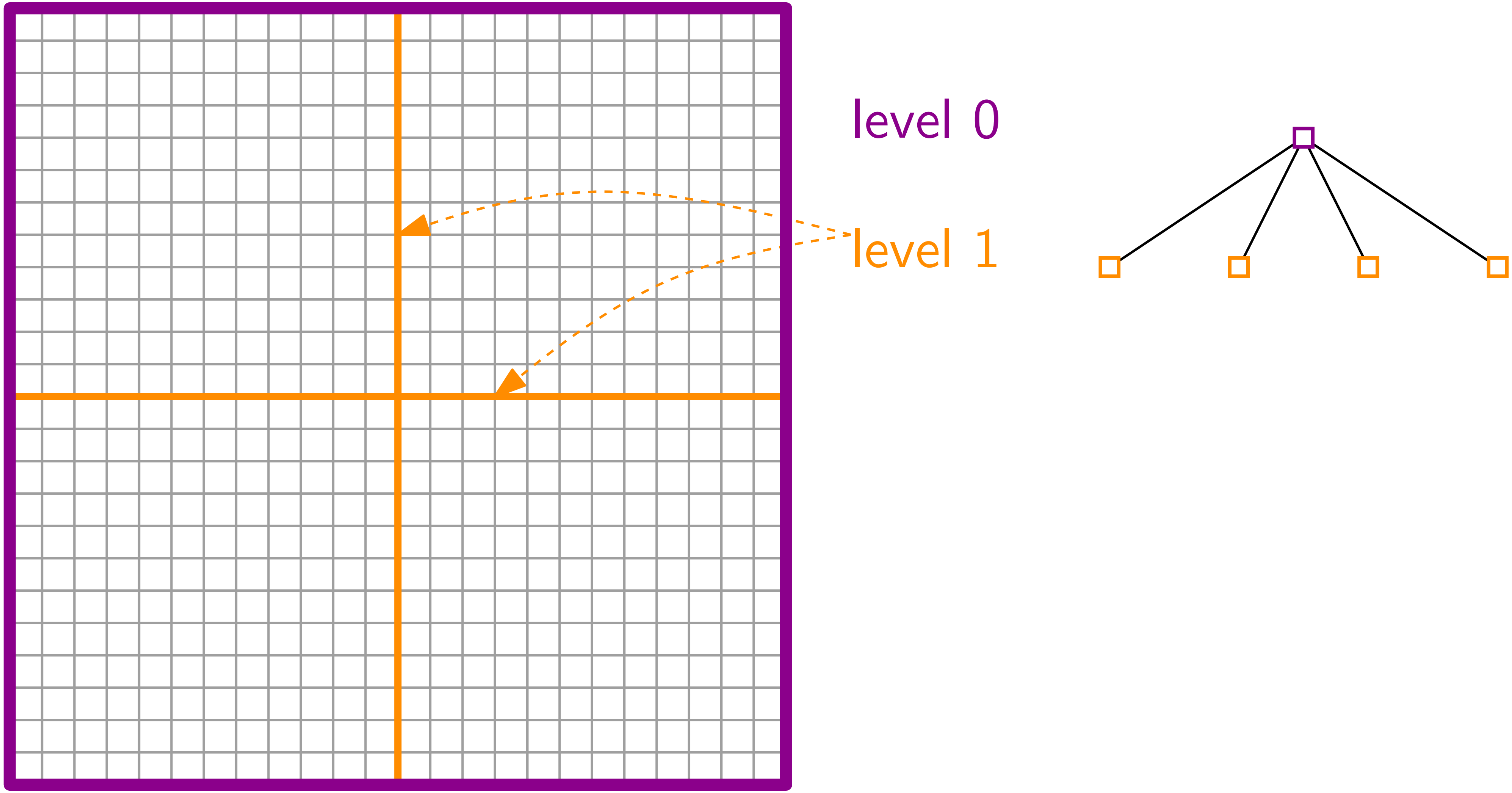


Quadtree Placement



level 0

Quadtree Placement



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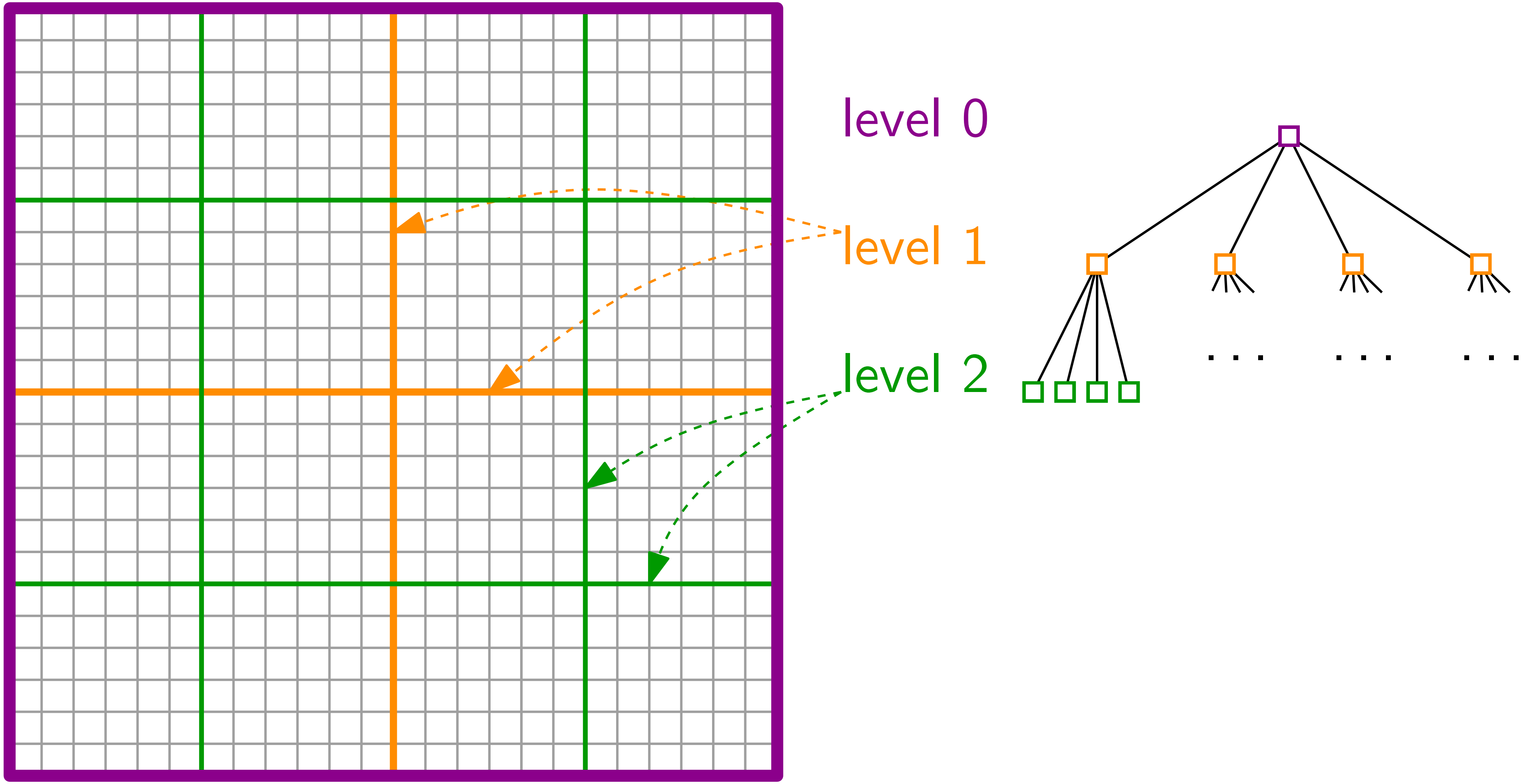
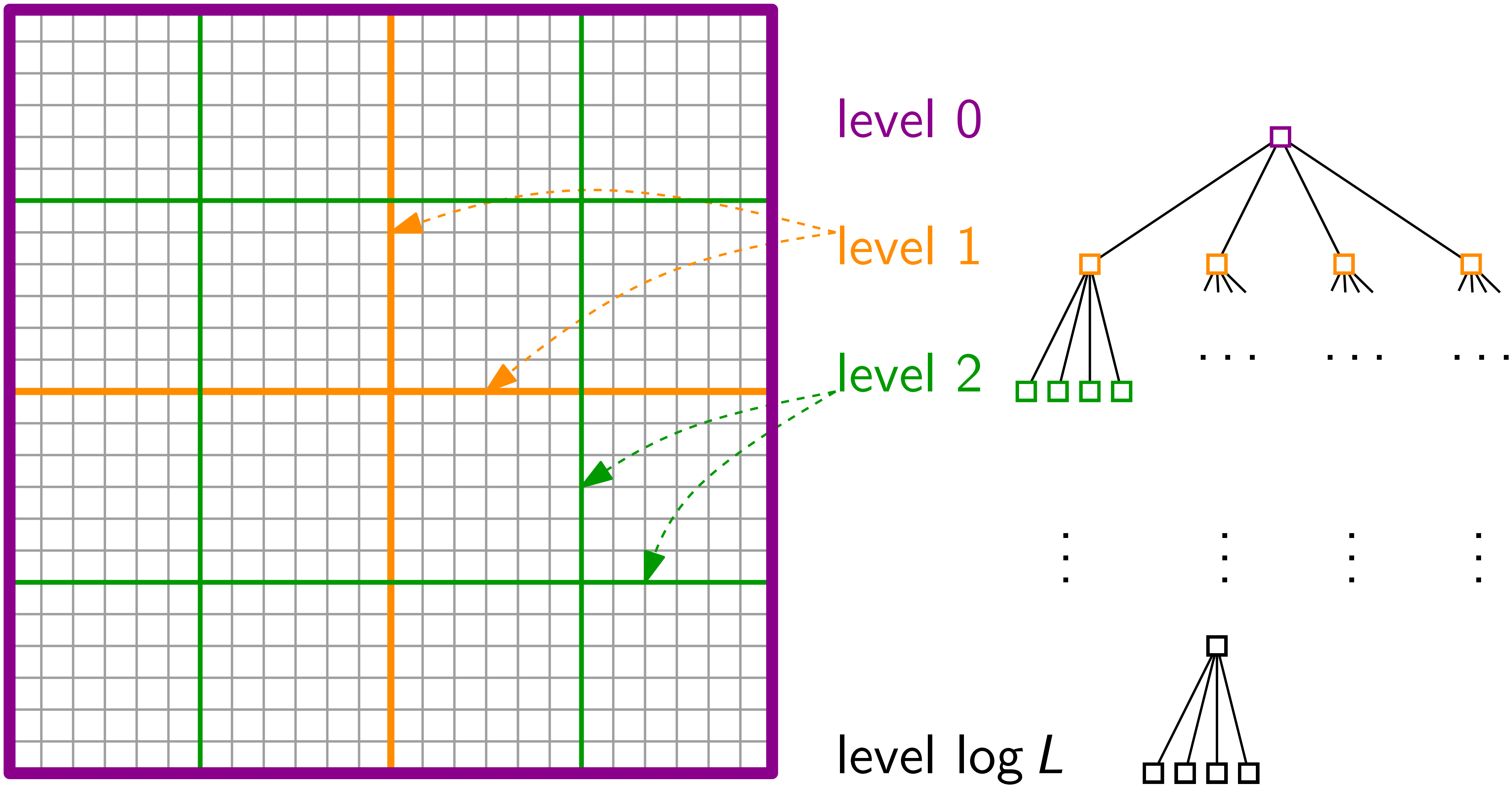
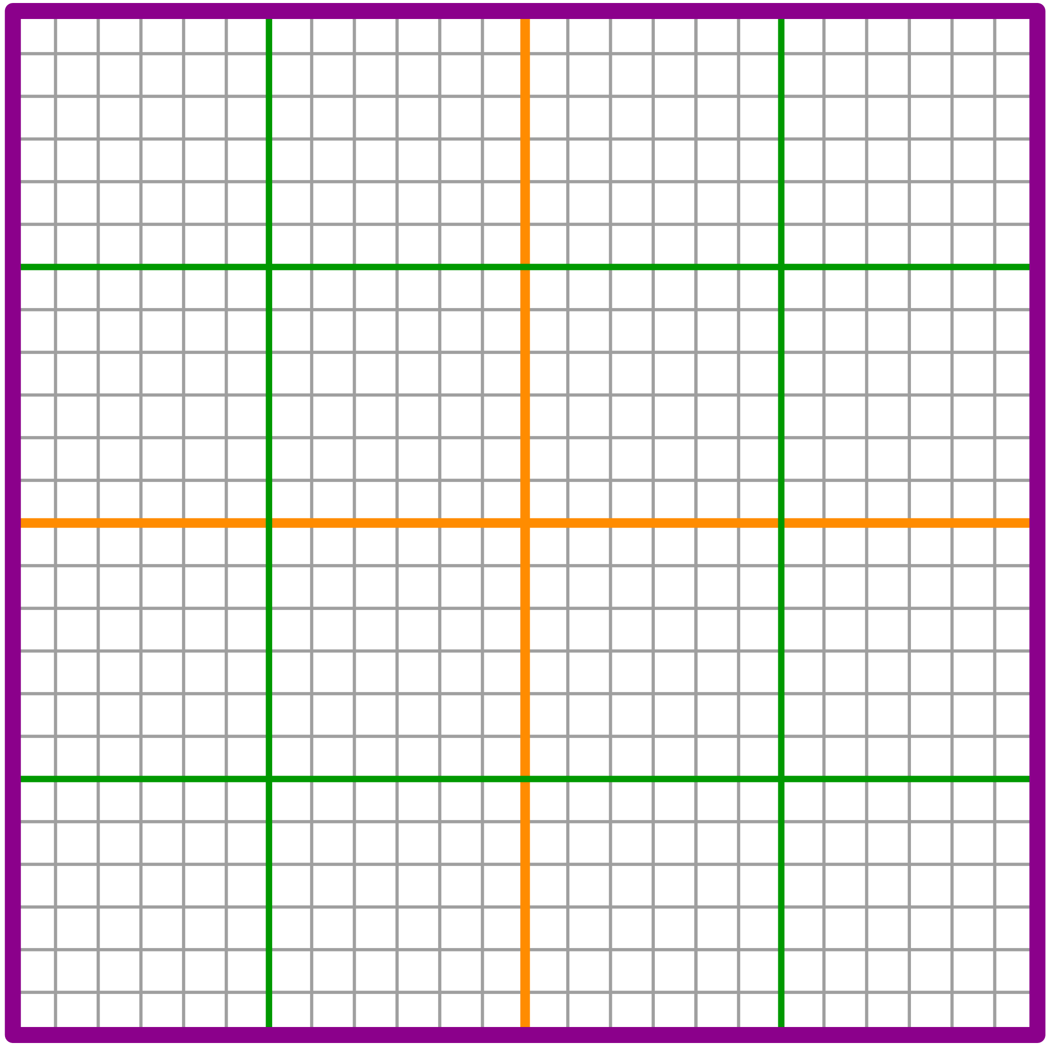


Quadtree Placement

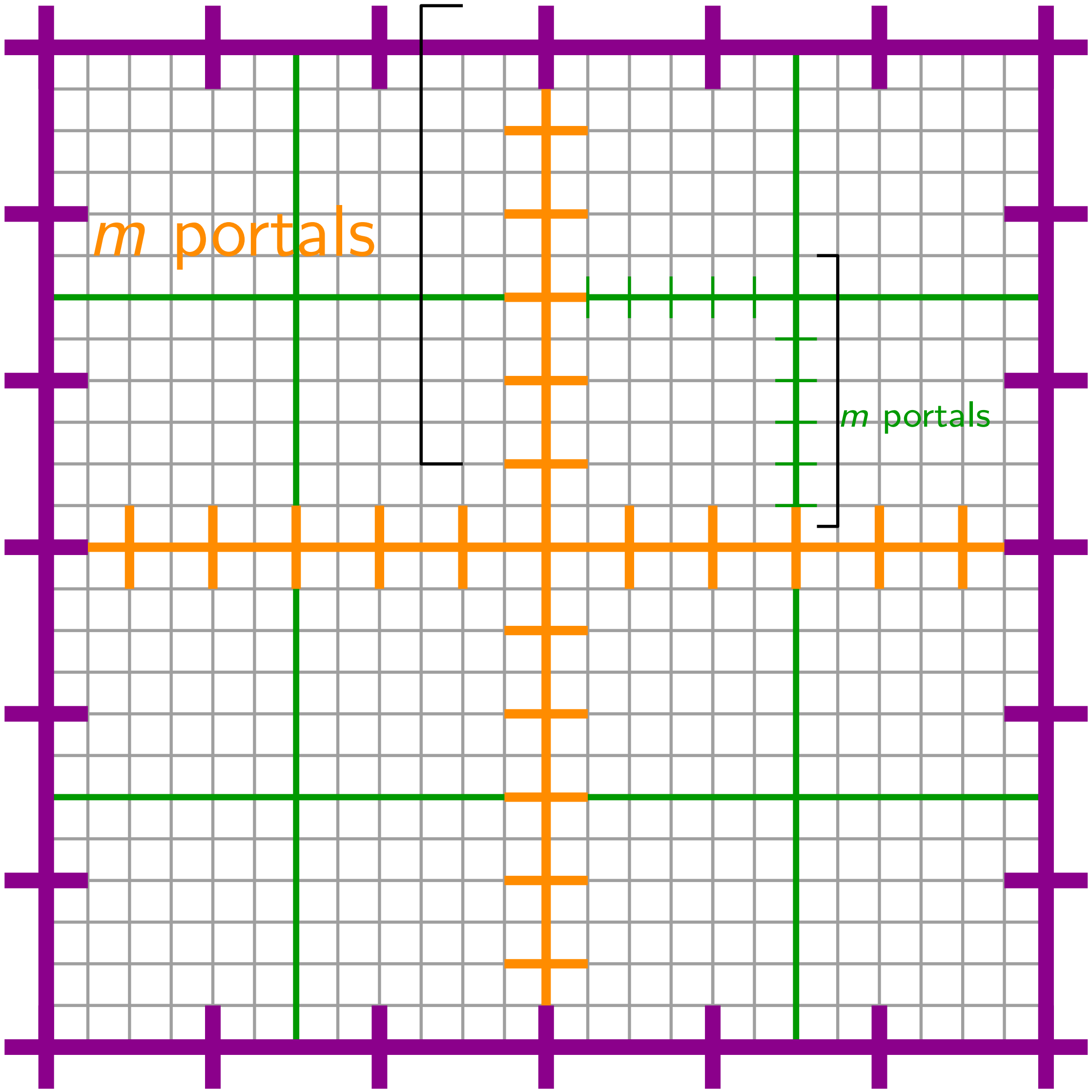


Quadtree Placement



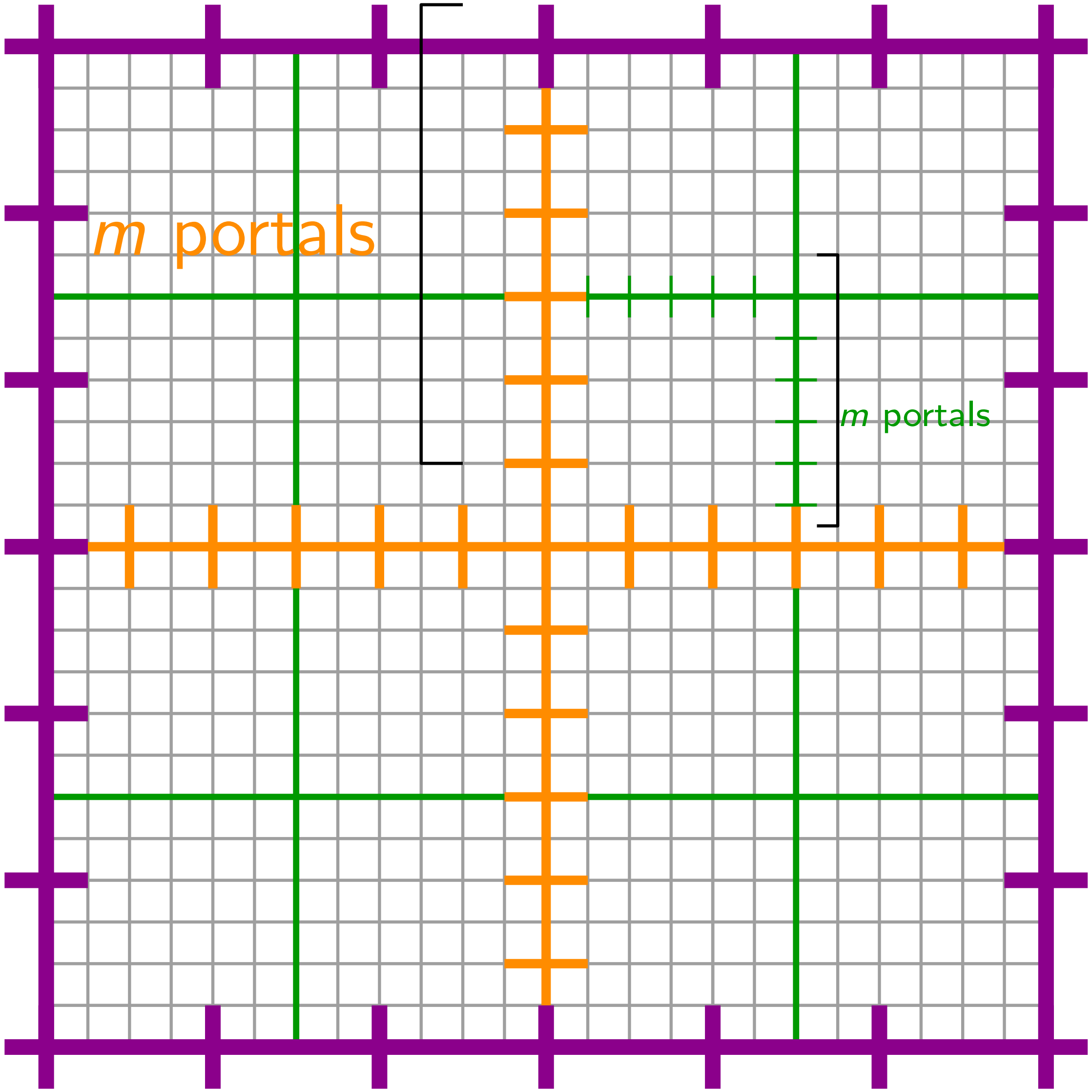
- $m = 4 \log(L)/\varepsilon$

Quadtree Placement



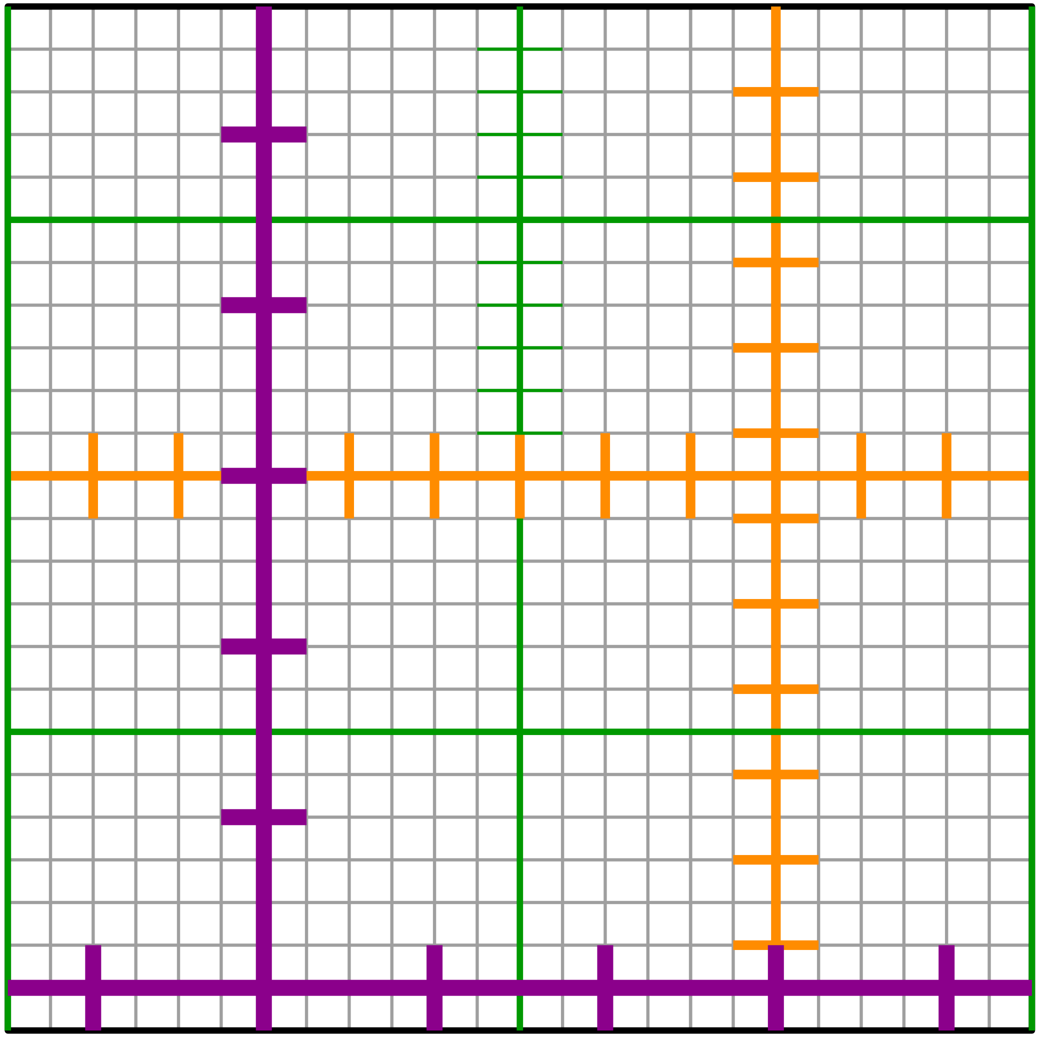
- $m = 4\log(L)/\varepsilon$
- portals on level-i-line with distance $L/(2^i m)$

Quadtree Placement



- $m = 4\log(L)/\varepsilon$
- portals on level-i-line with distance $L/(2^i m)$
- level-i-square has at most $4m$ portals on its margin

Quadtree Placement



- $m = 4\log(L)/\varepsilon$
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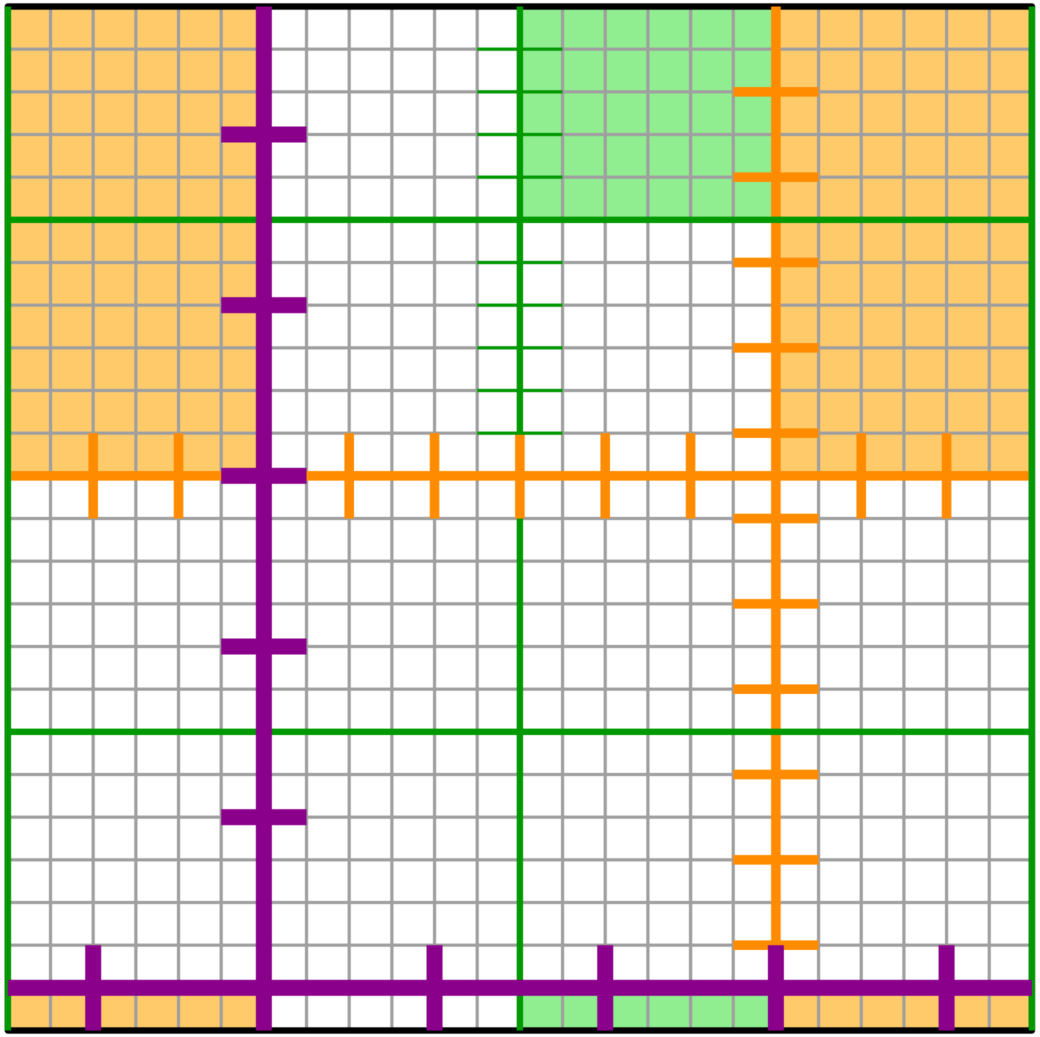
- place origin uniformly at random

Quadtree Placement



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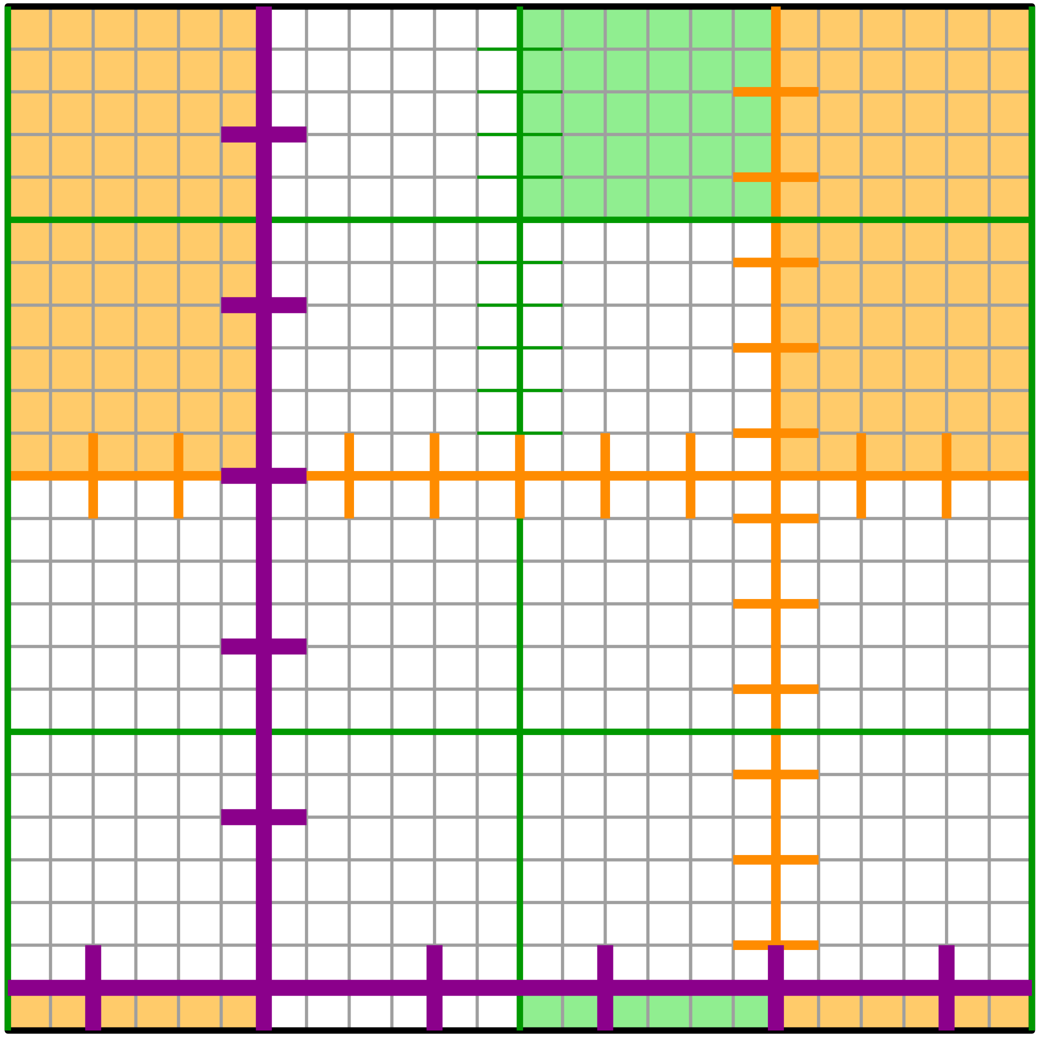

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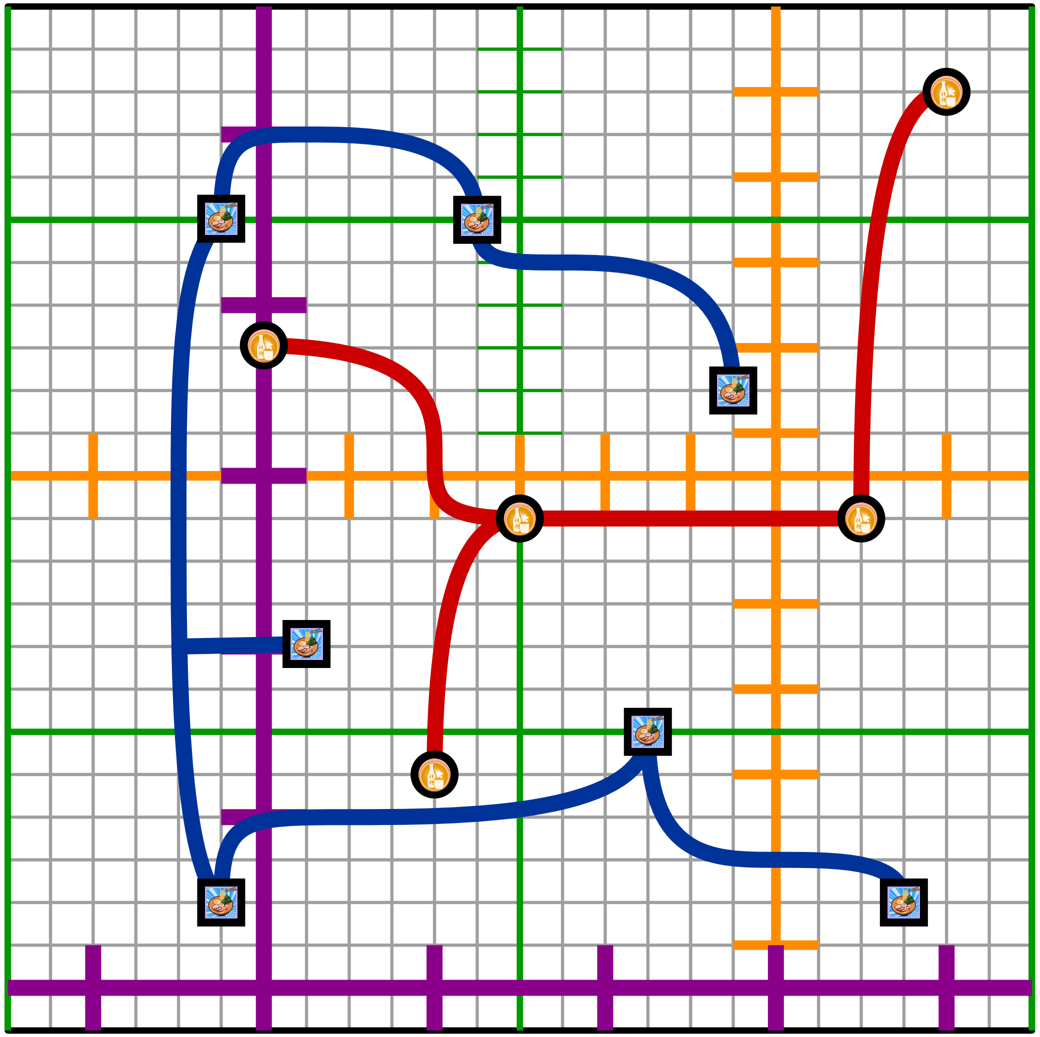
portal-respecting solution: crosses grid lines only at portals

Quadtree Placement



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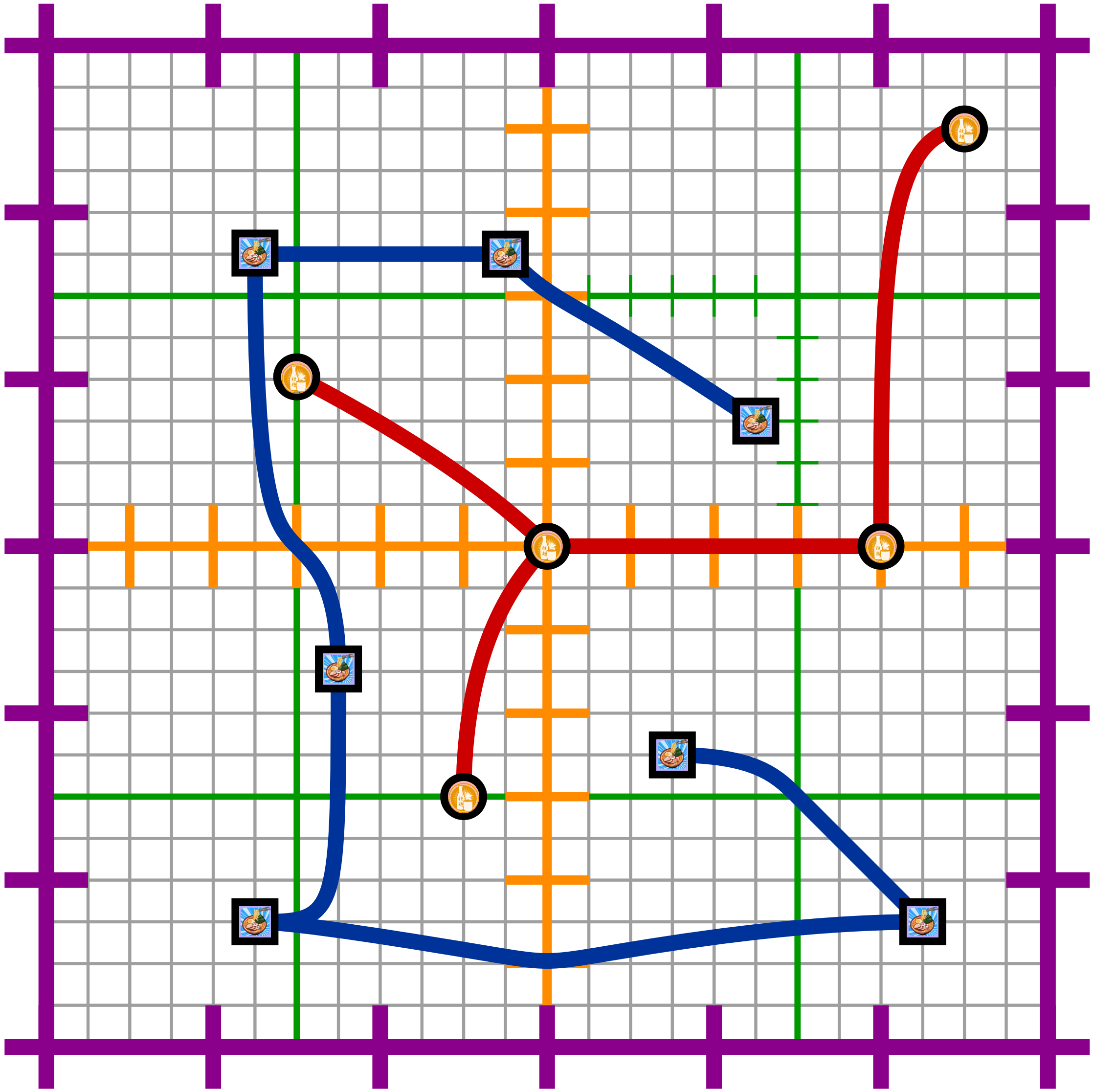

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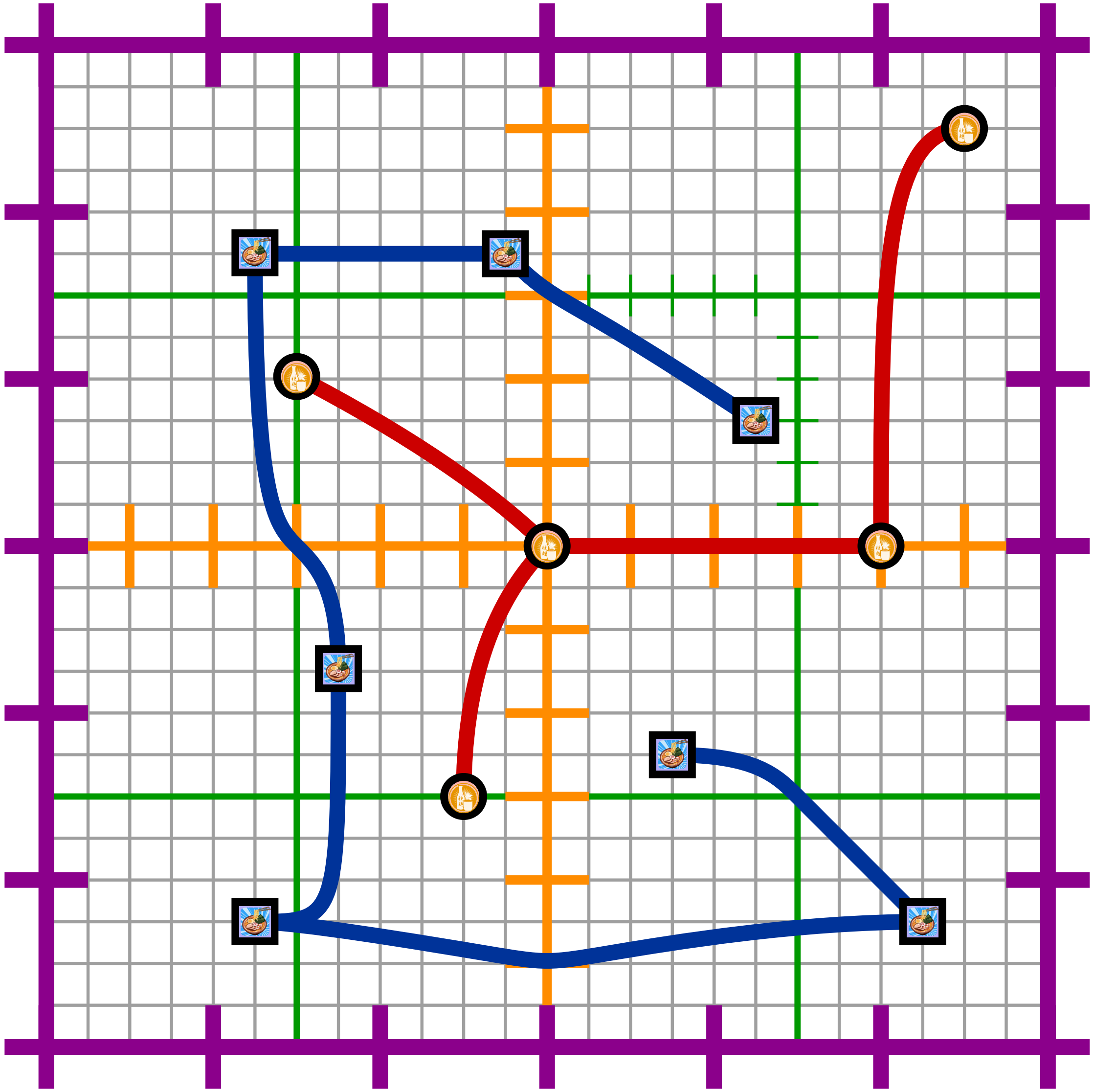

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line ℓ crosses drawing $t(\ell)$ times; expected length increase: $\leq \varepsilon \frac{t(\ell)}{4}$

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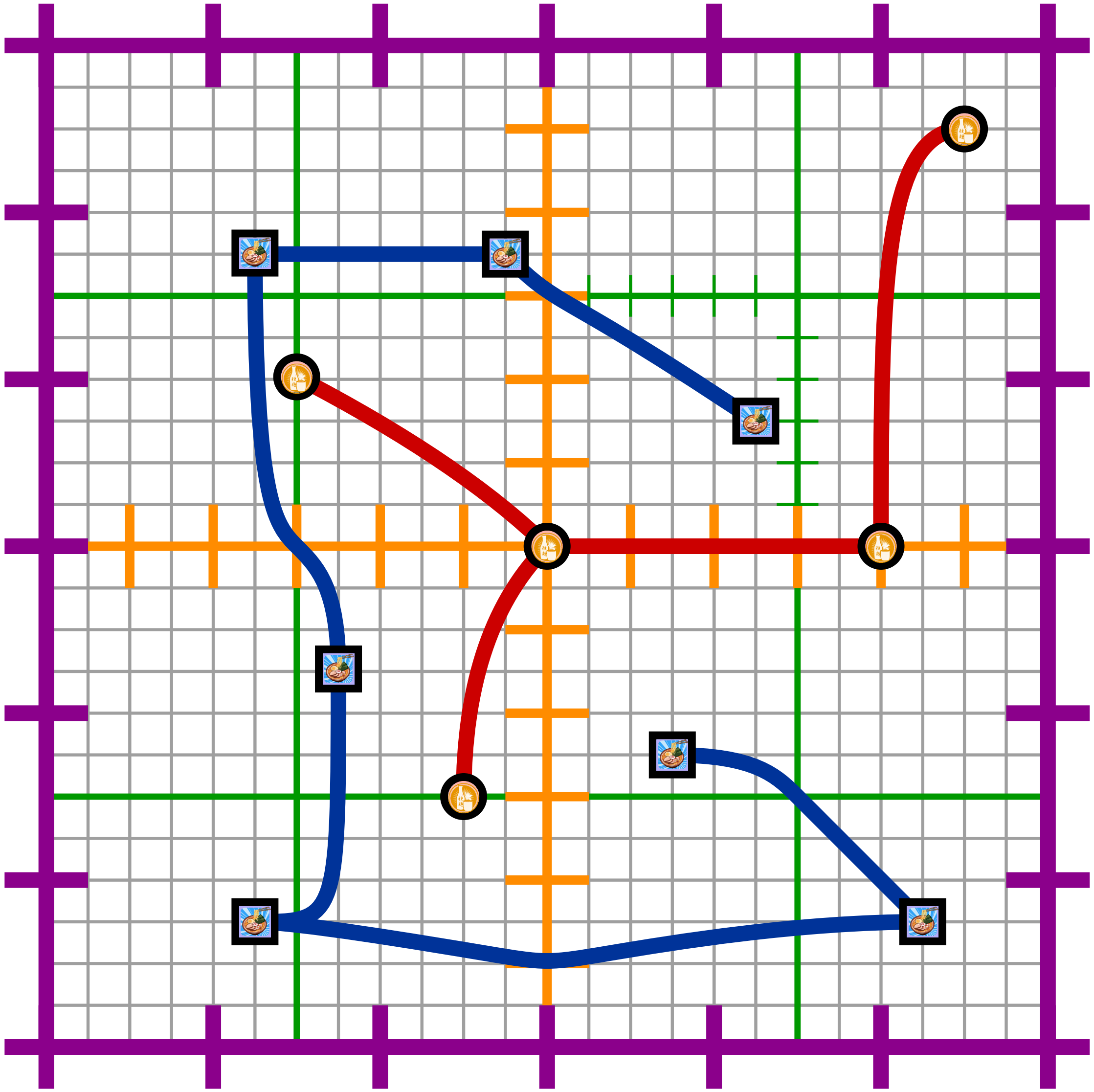

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2-CESF instance $I \rightarrow$ portal-respecting solution $\mathcal{L}$
$|\mathcal{L}| \leq (1+\varepsilon)^3 \mathrm{OPT}_I$

3-Light Solution

3-light solution: each portal is crossed at most 3 times

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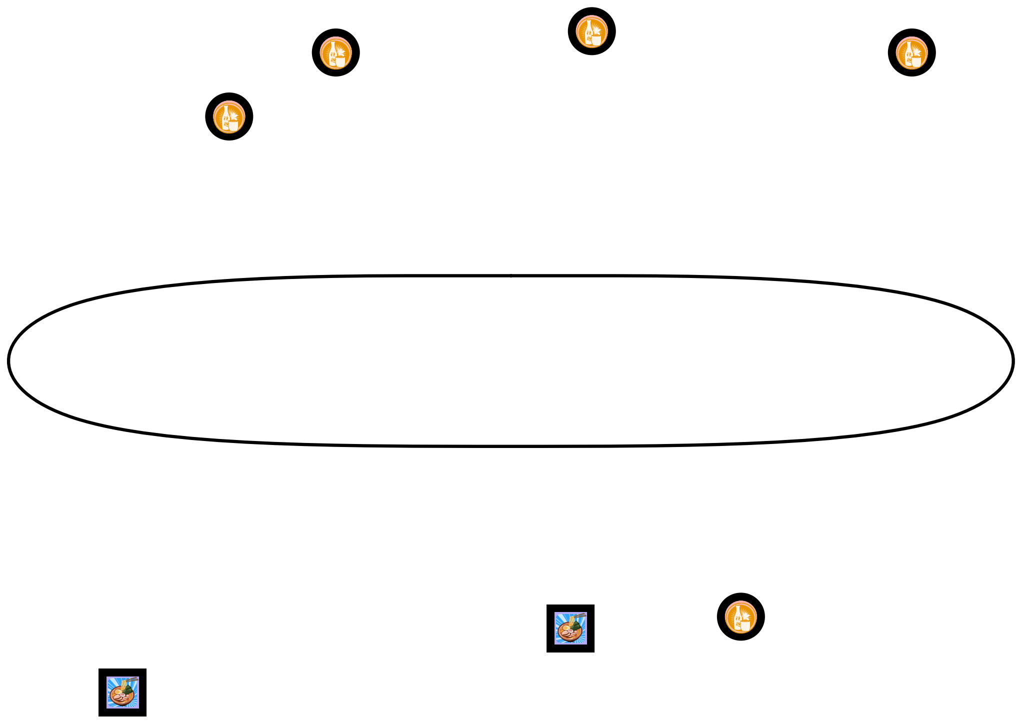

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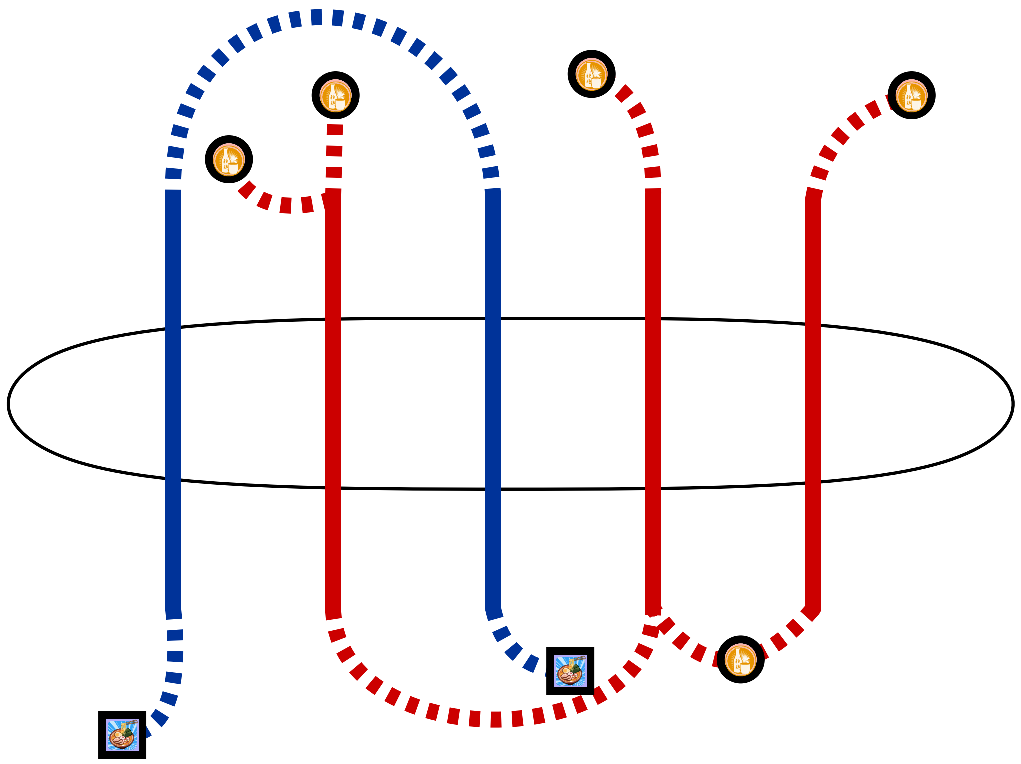

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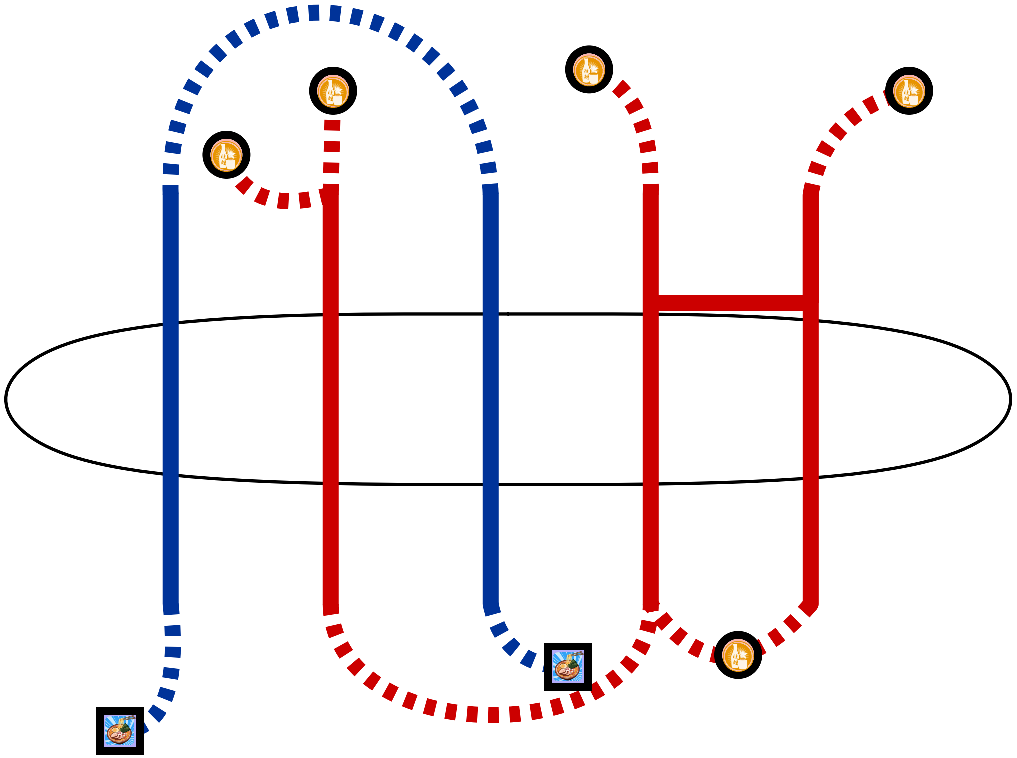

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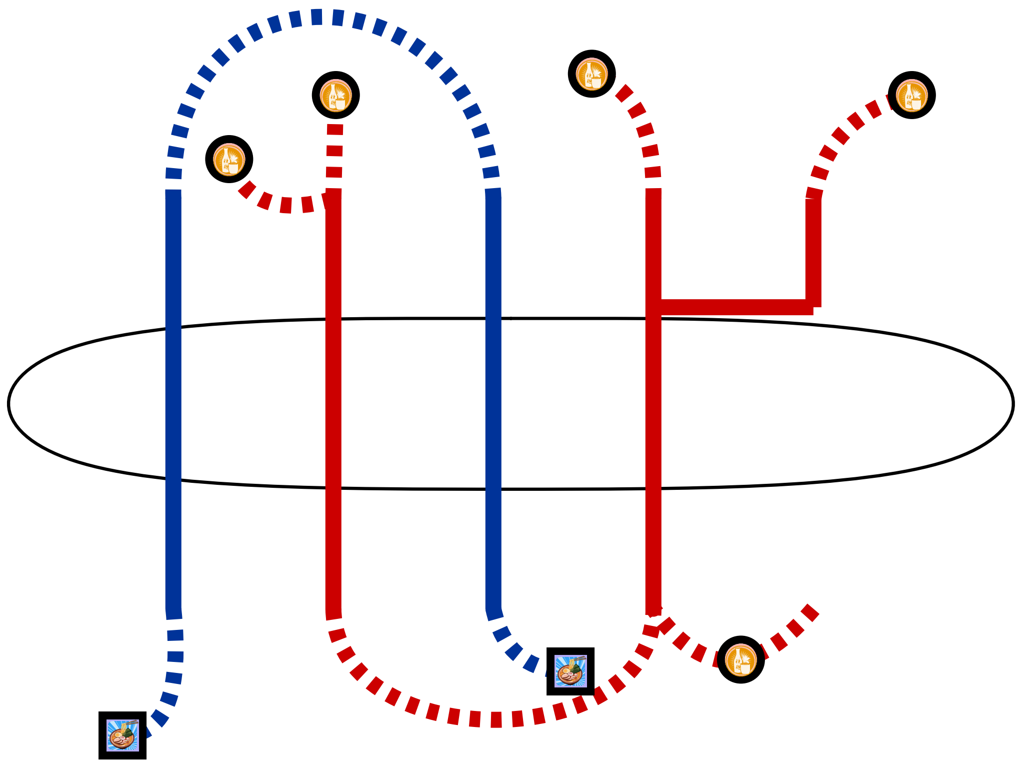

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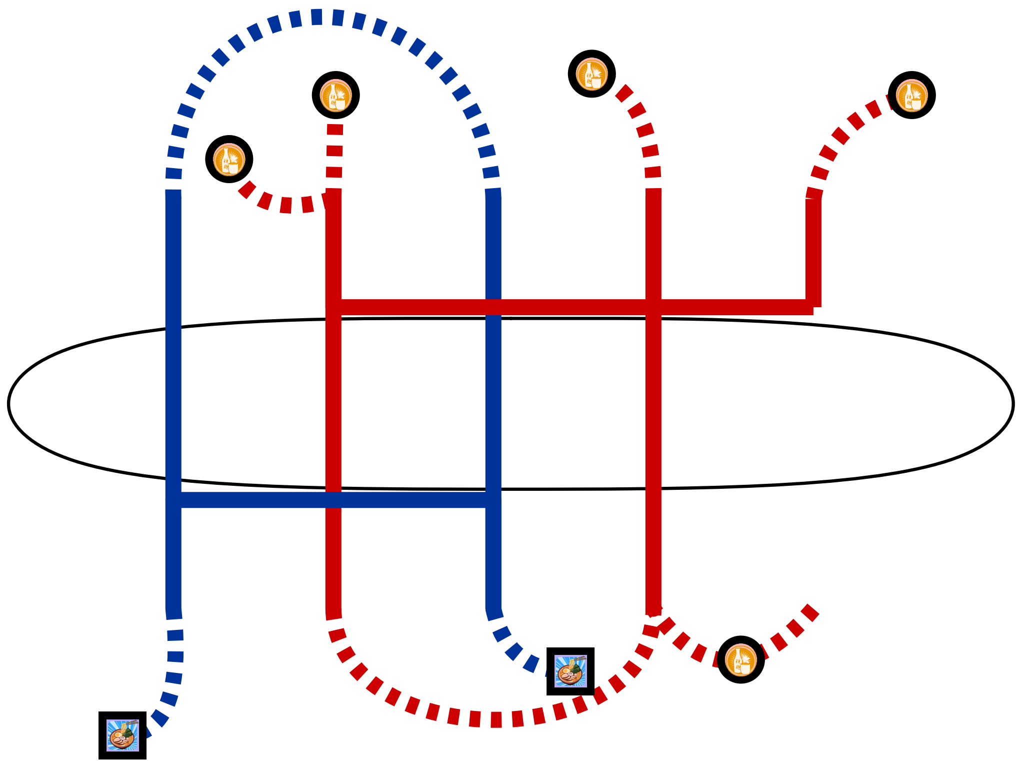

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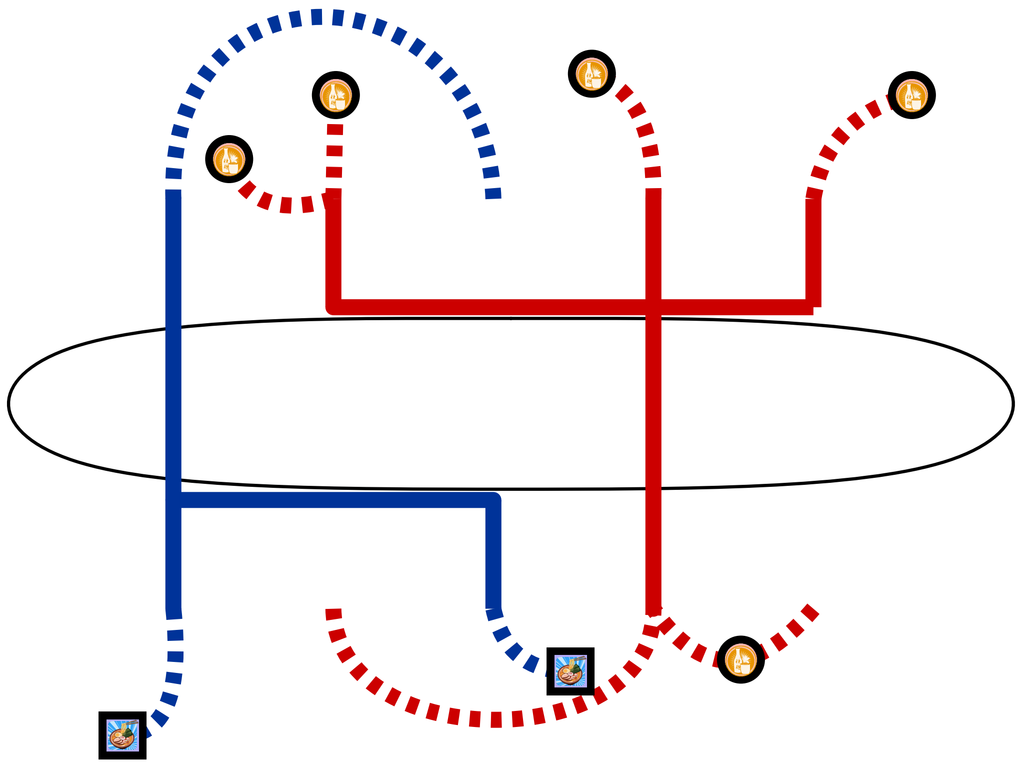

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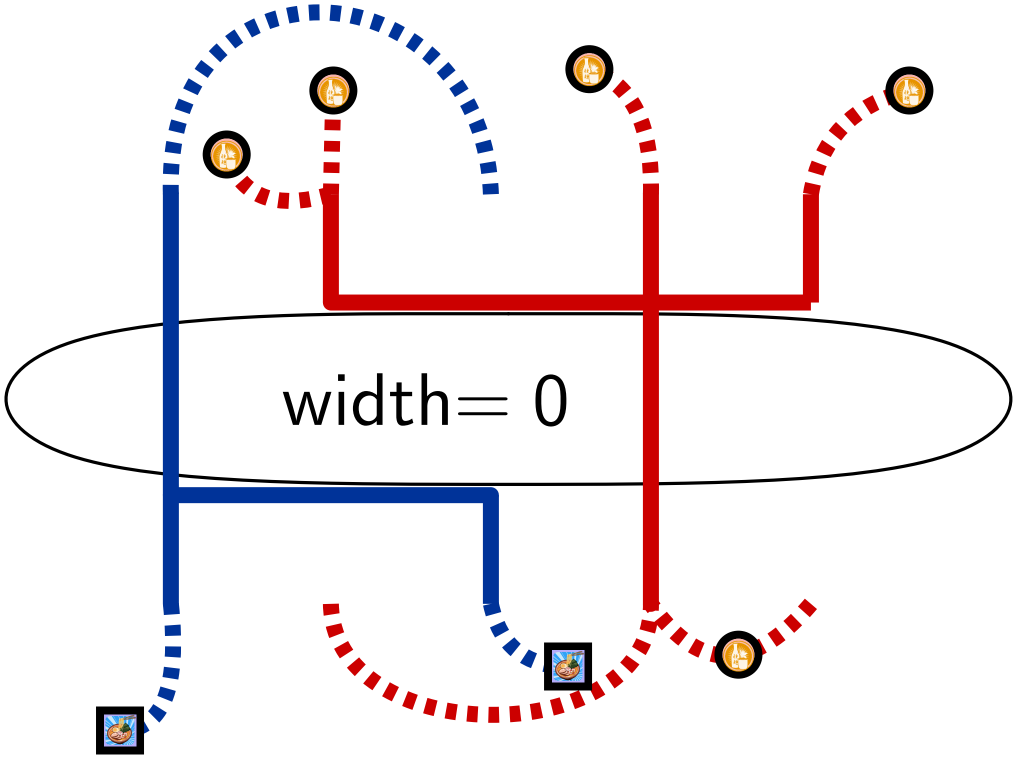

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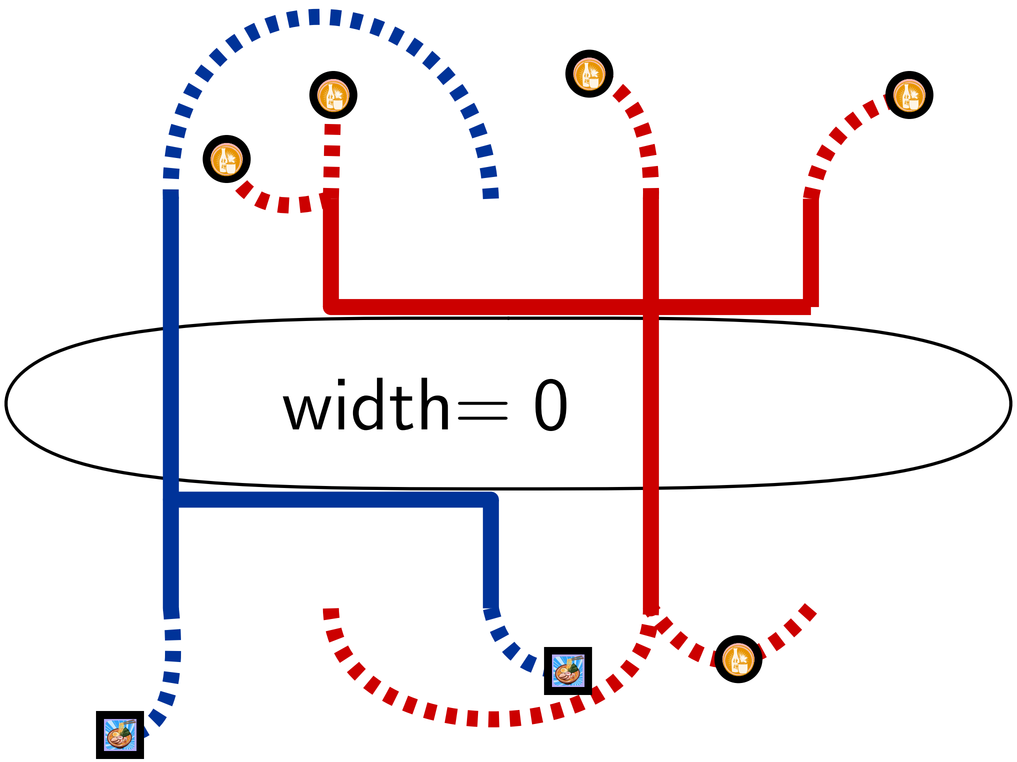

2-CESF instance $I \rightarrow$ portal-respecting 3-light solution $\mathcal{L}^*$
$|\mathcal{L}^*| \leq (1+\varepsilon)^3 \mathsf{OPT}_I$

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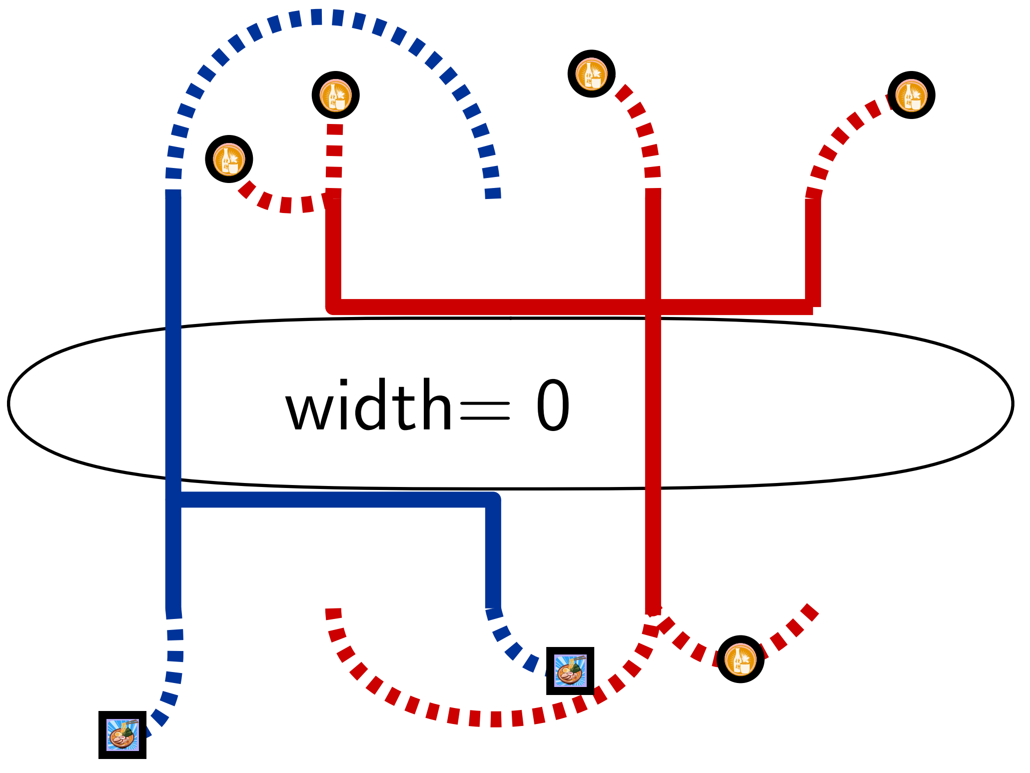

2-CESF instance $I \rightarrow$ portal-respecting 3-light solution $\mathcal{L}^*$
$|\mathcal{L}^*| \leq (1+\varepsilon)^3 \mathrm{OPT}_I \leq (1+\varepsilon')\mathrm{OPT}_I$

Putting Things Together

Use a dynamic program!

Putting Things Together

Use a dynamic program! A subproblem consists of:

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- a square of the quadtree

Putting Things Together

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- up to three red and blue points on each portal

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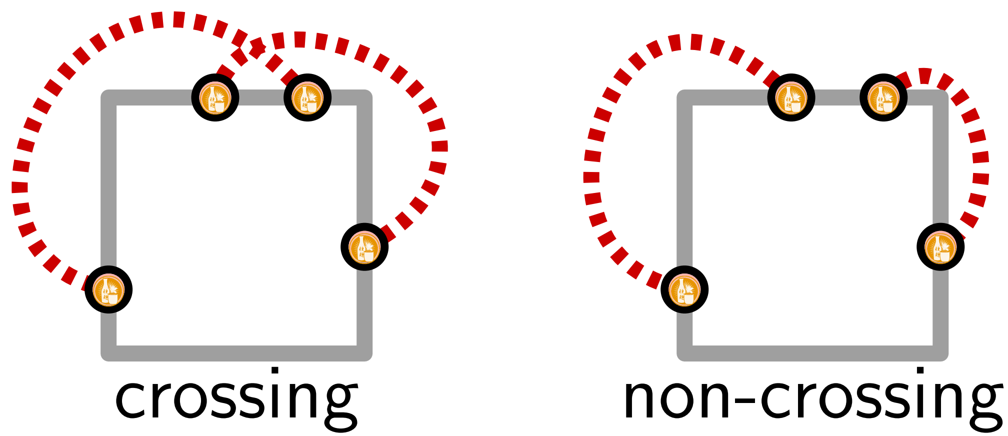

Putting Things Together

Use a dynamic program! A subproblem consists of:

- a square of the quadtree $O(n^2)$
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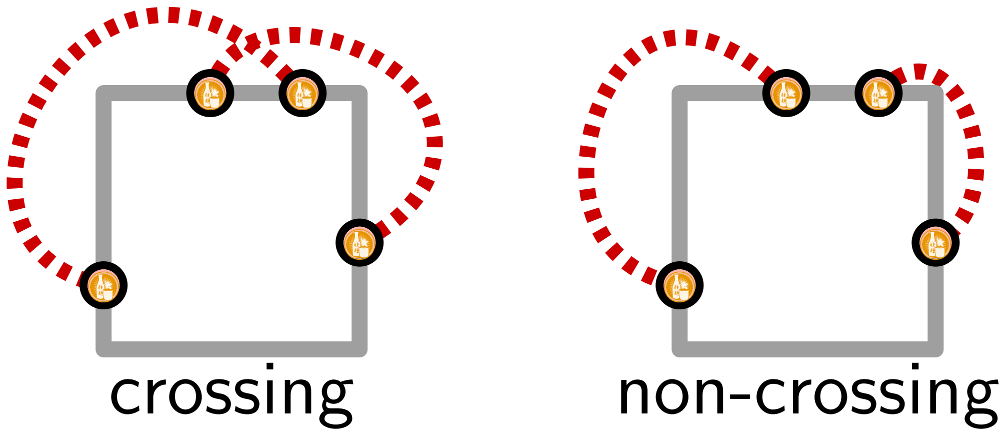
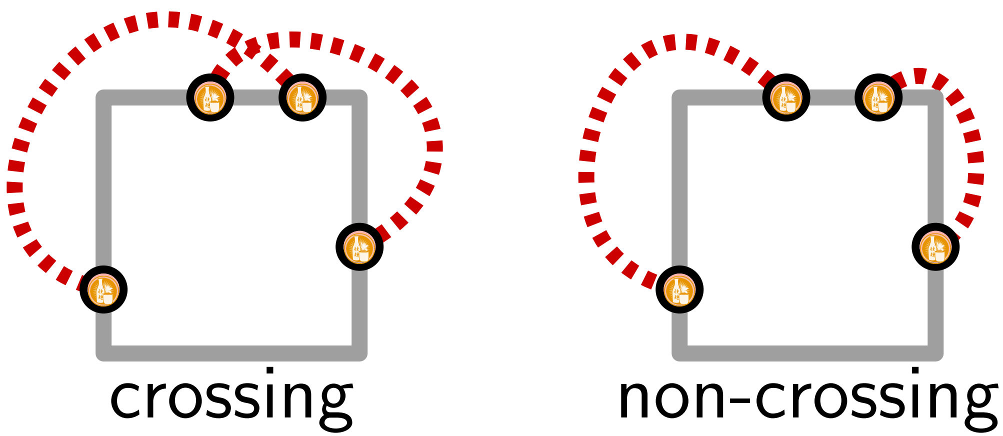


Putting Things Together

Use a dynamic program! A subproblem consists of:

- a square of the quadtree $O(n^2)$
- up to three red and blue points on each portal $2^{O(\log n/\varepsilon)} = n^{O(1/\varepsilon)}$
- non-crossing partition of the points into sets of same color



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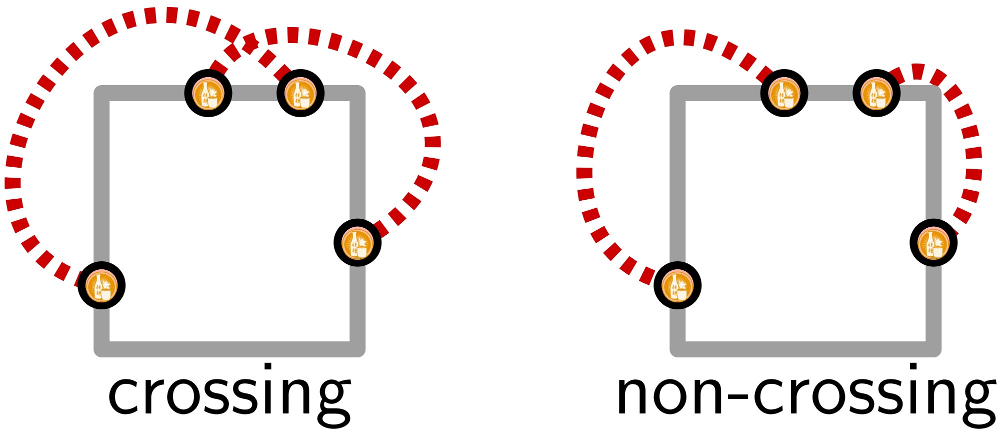
$C_{O(\log n/\varepsilon)} = n^{O(1/\varepsilon)}$



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Base case: unit square

Putting Things Together

Use a dynamic program! A subproblem consists of:

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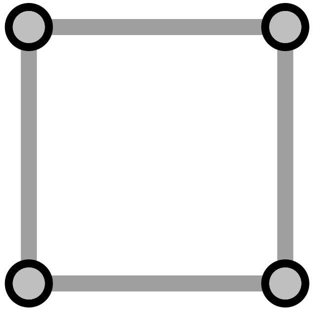
Base case: unit square

- portals (and points) only in corners



Putting Things Together

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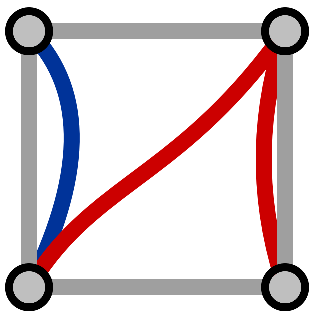
Base case: unit square

- portals (and points) only in corners
- solve with PTAS for EST



Putting Things Together

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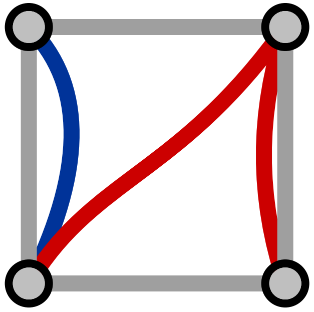

Composite squares:

Putting Things Together

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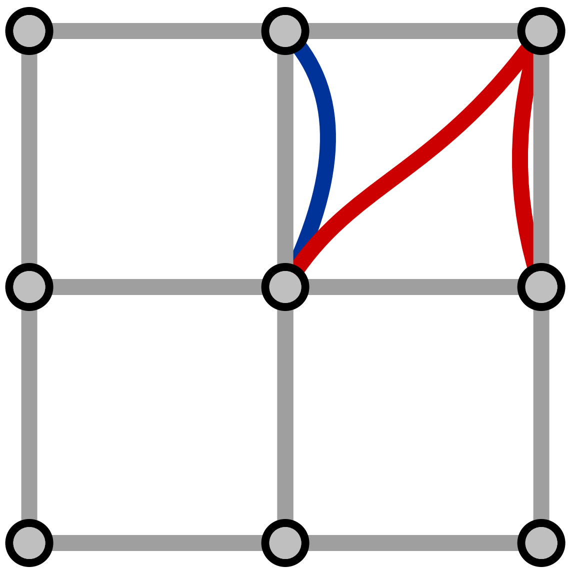
Composite squares:

- divide into squares (acc. to quadtree)



Putting Things Together

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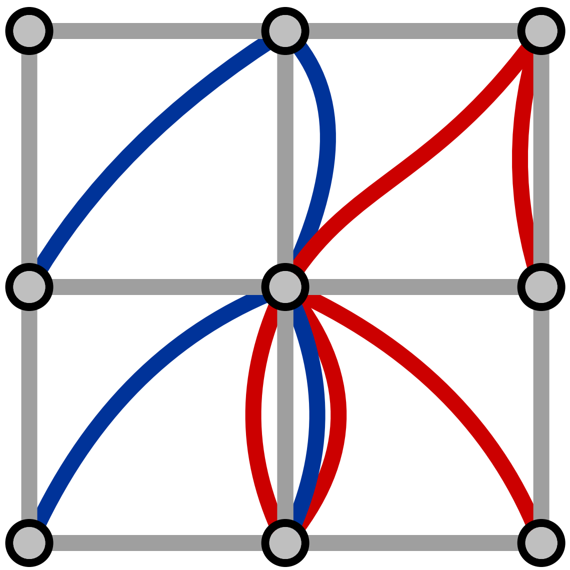
Base case: unit square

- portals (and points) only in corners
- solve with PTAS for EST

Composite squares:

- divide into squares (acc. to quadtree)
- solve each combination of $n^{O(1/\varepsilon)}$ compatible subproblems



Putting Things Together

Use a dynamic program! A subproblem consists of:

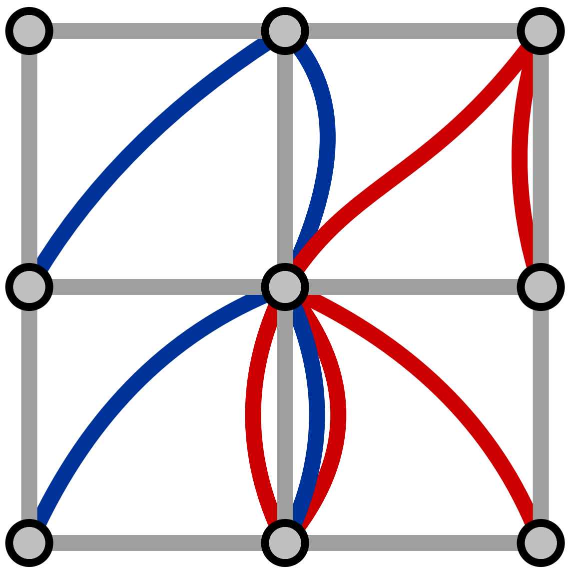
- a square of the quadtree $O(n^2)$
- up to three red and blue points on each portal $2^{O(\log n/\varepsilon)} = n^{O(1/\varepsilon)}$
- non-crossing partition of the points into sets of same color $C_{O(\log n/\varepsilon)} = n^{O(1/\varepsilon)}$

Base case: unit square

- portals (and points) only in corners
- solve with PTAS for EST

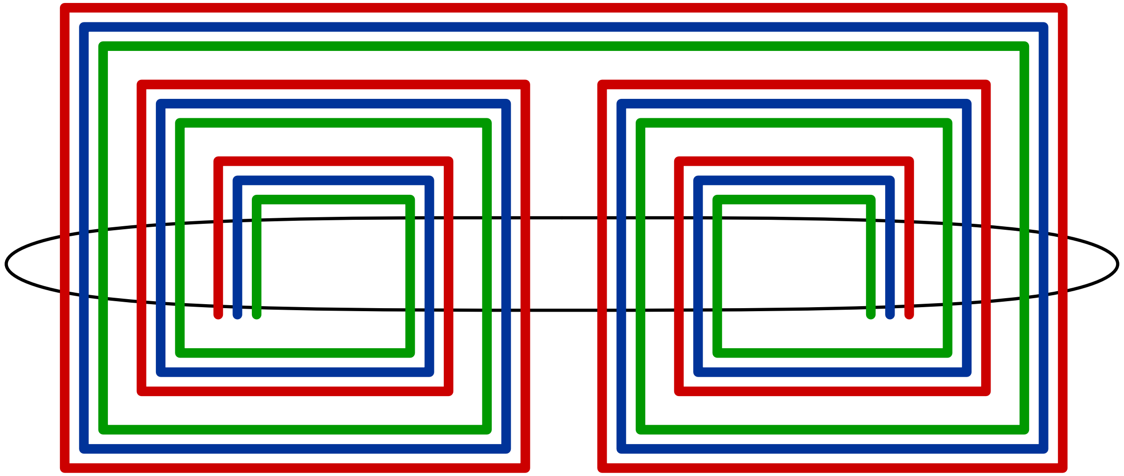
Composite squares:

- divide into squares (acc. to quadtree)
- solve each combination of $n^{O(1/\varepsilon)}$ compatible subproblems

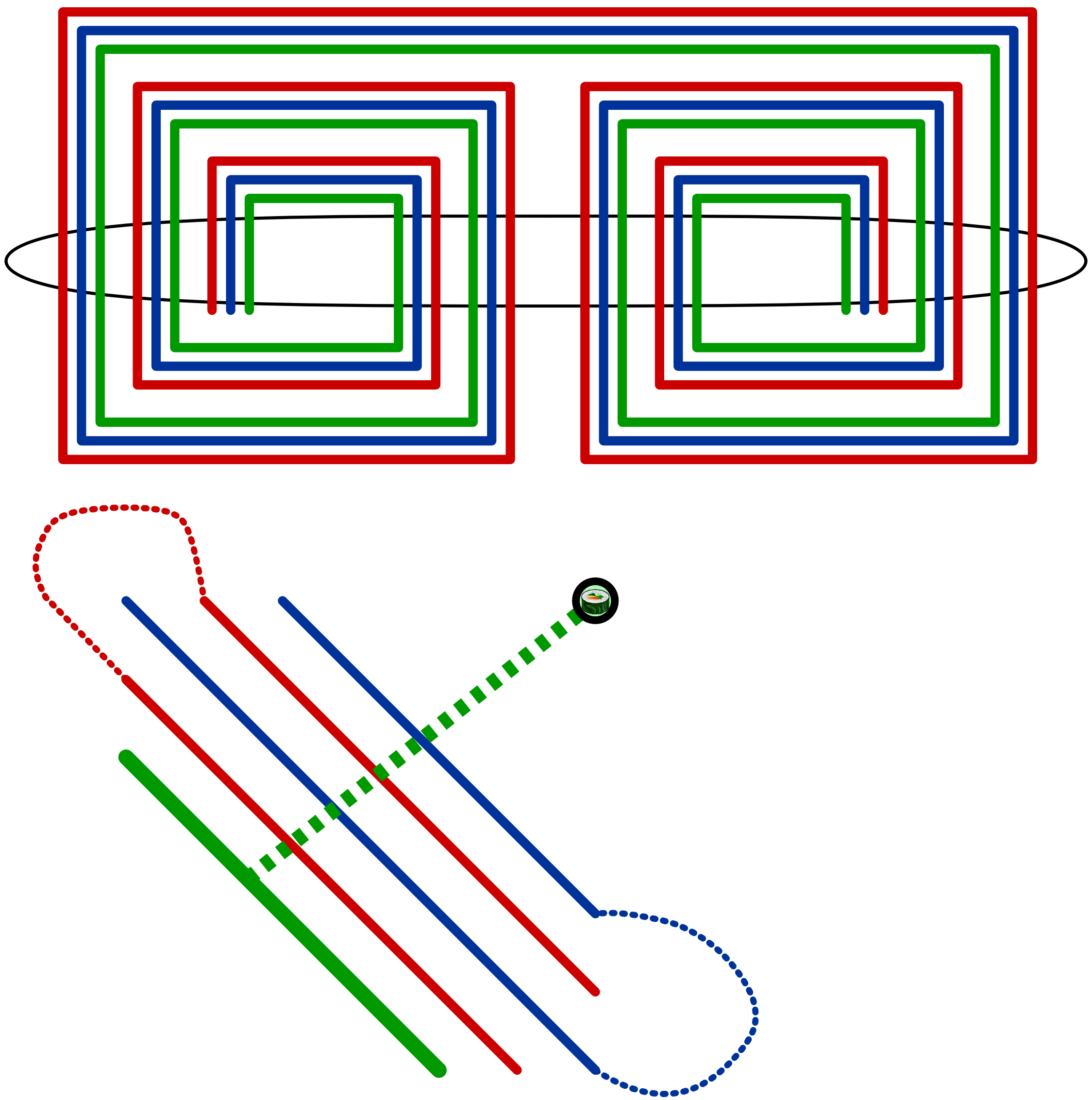


2-CESF admits a PTAS.

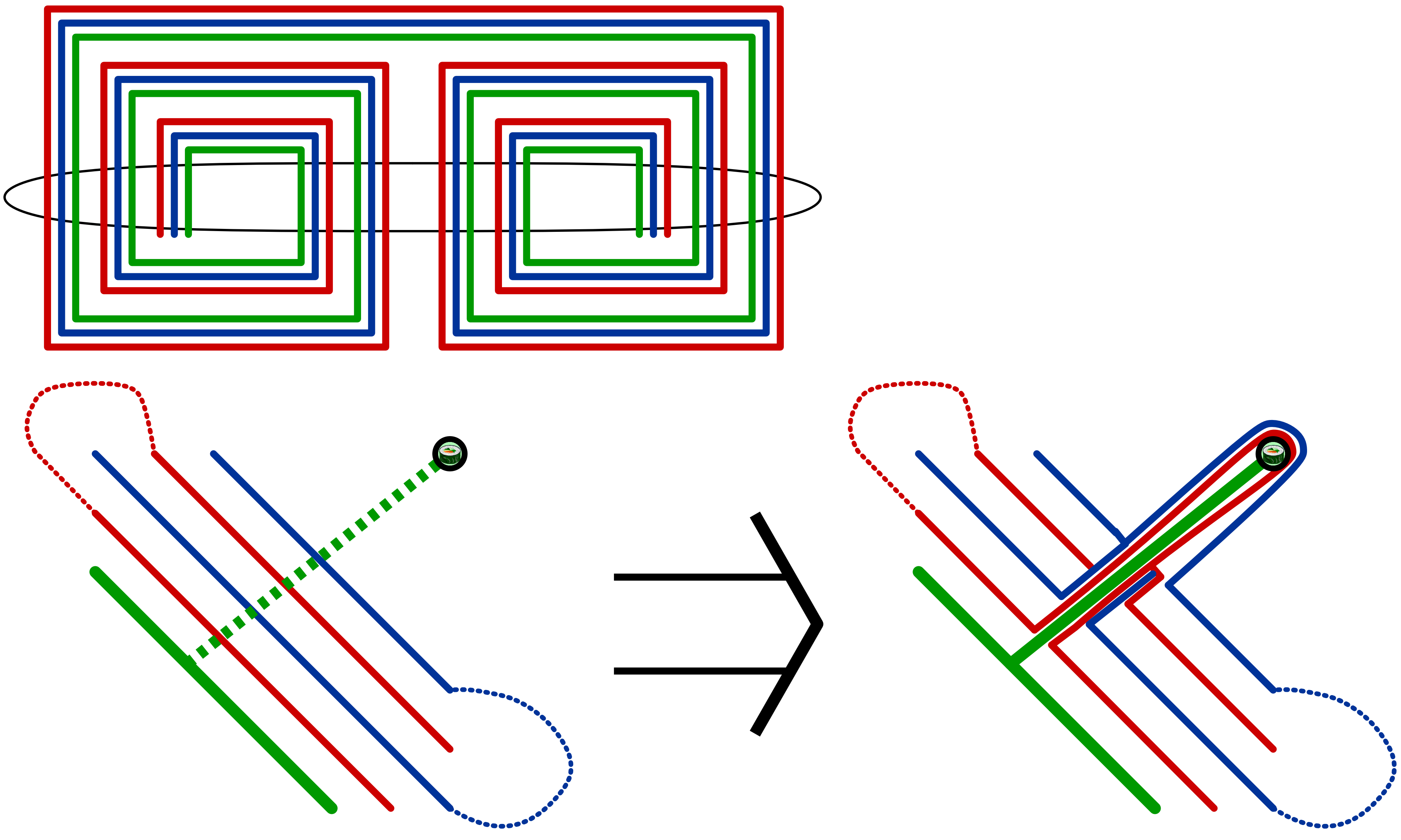
3-CESF



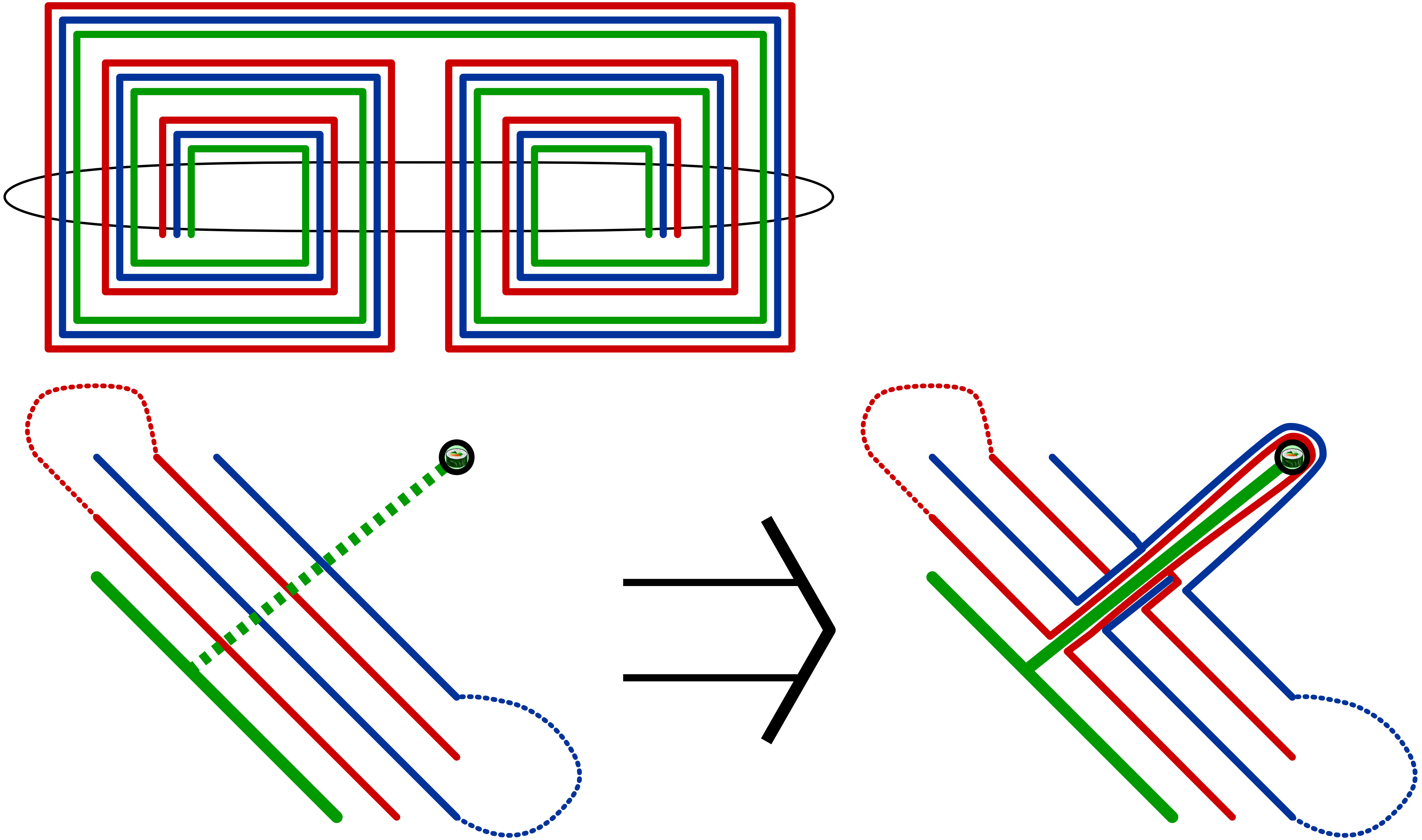
3-CESF



3-CESF

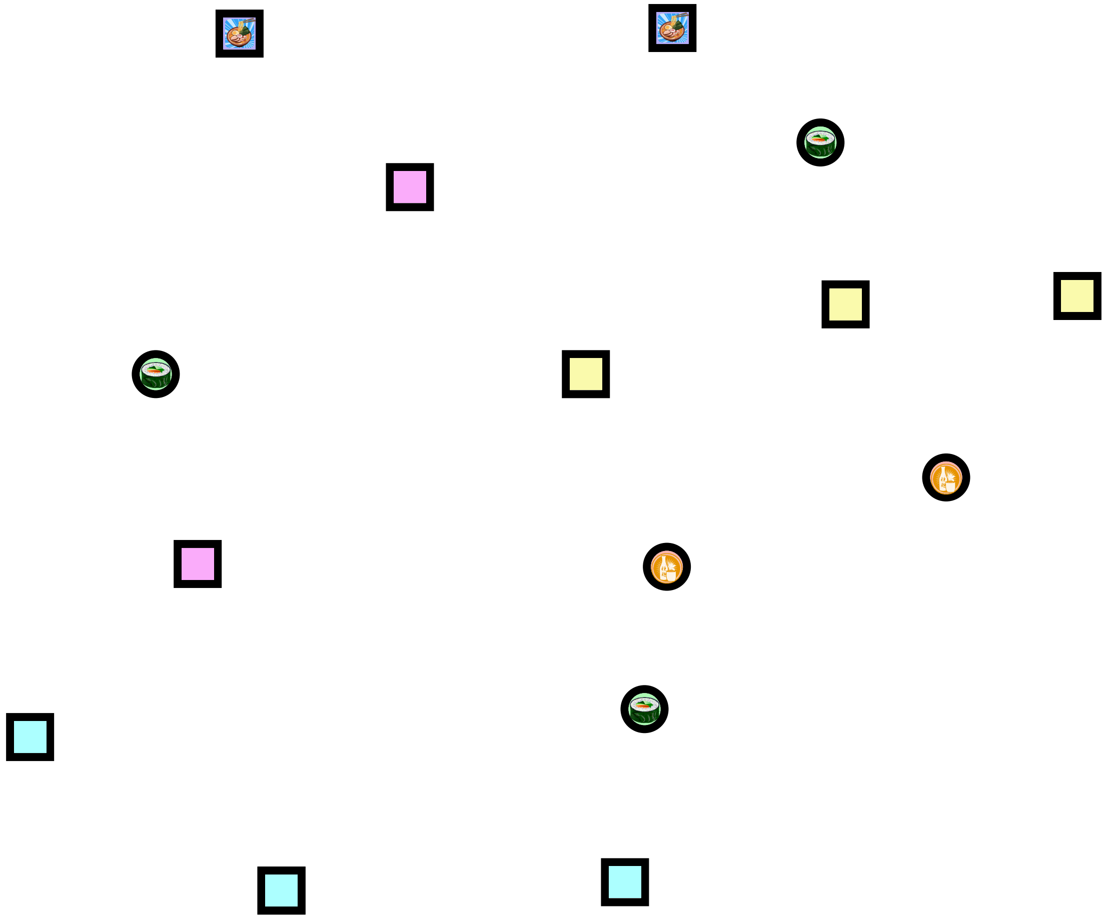


3-CESF



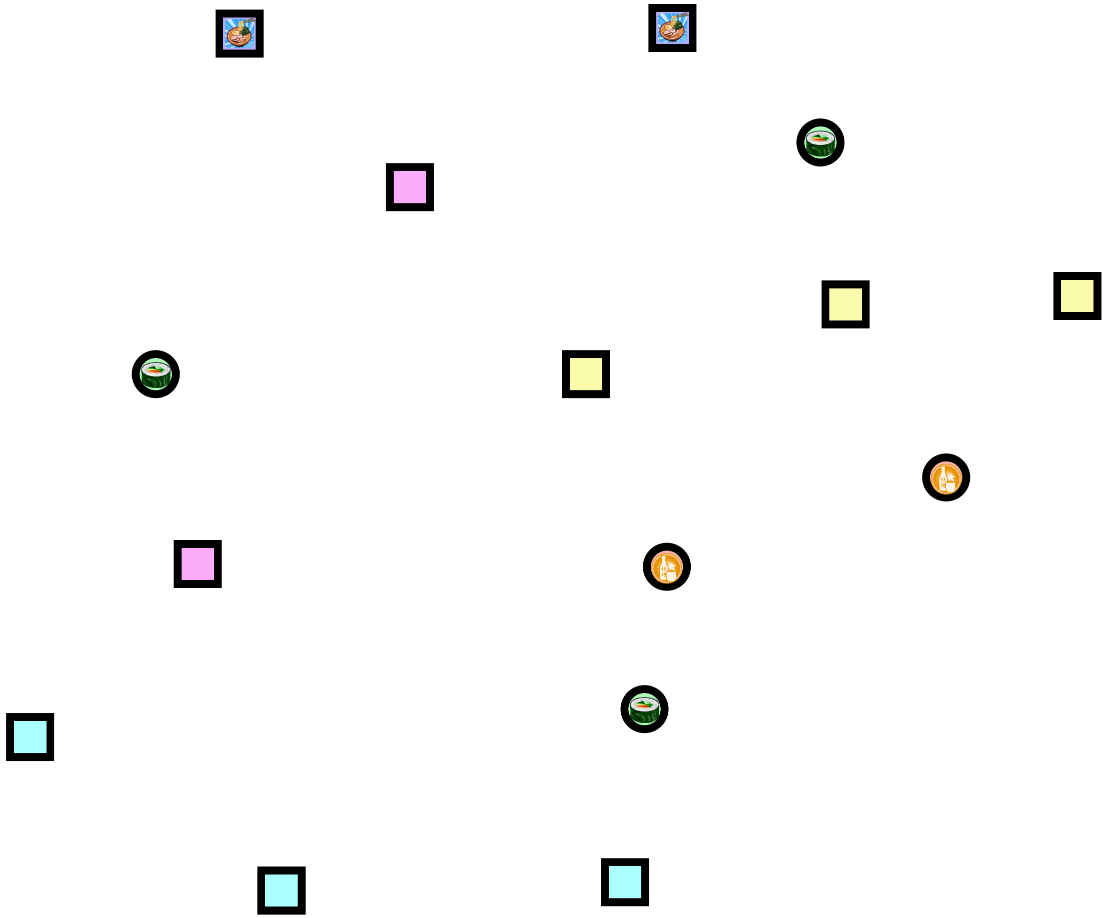
3-CESF admits a $(5/3+\varepsilon)$-approximation algorithm.

k-CESF



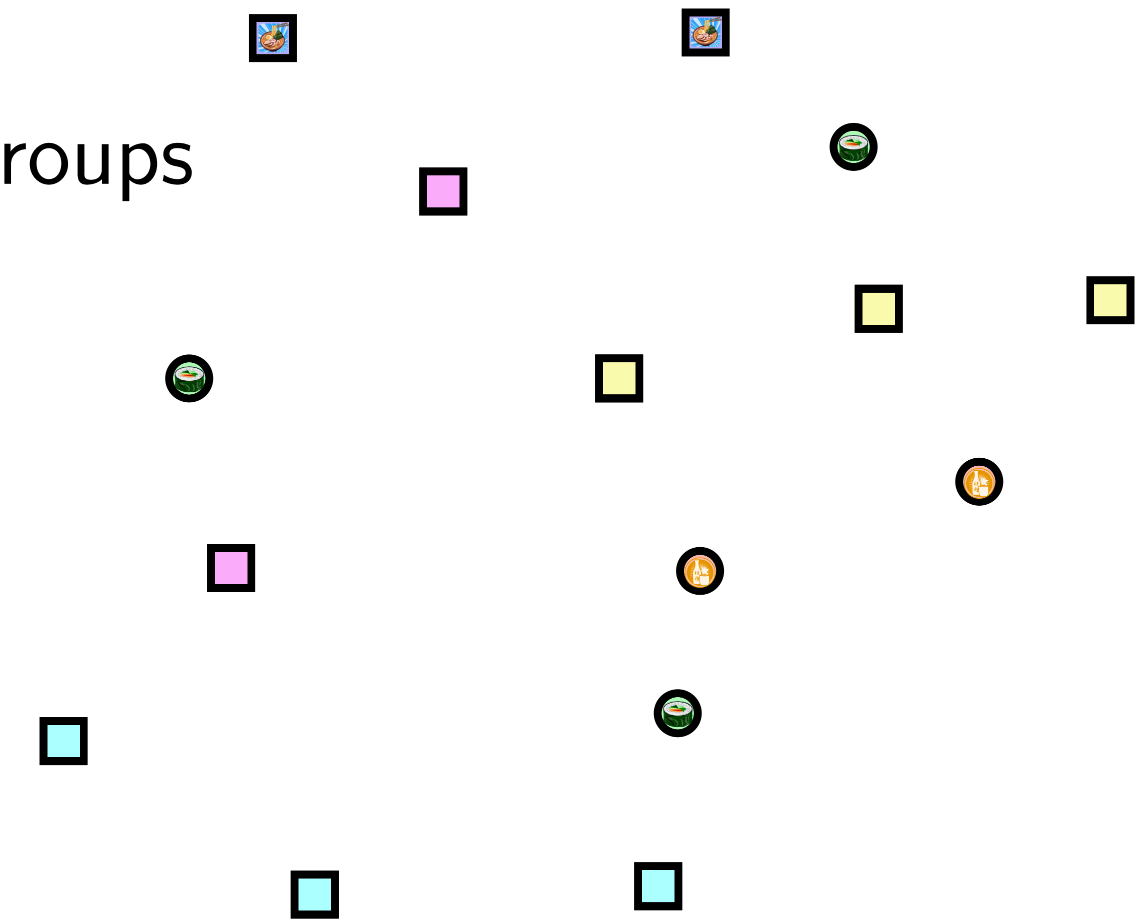
k-CESF

- split into 2 groups



k-CESF

- split into 2 groups
- use PTAS for the groups



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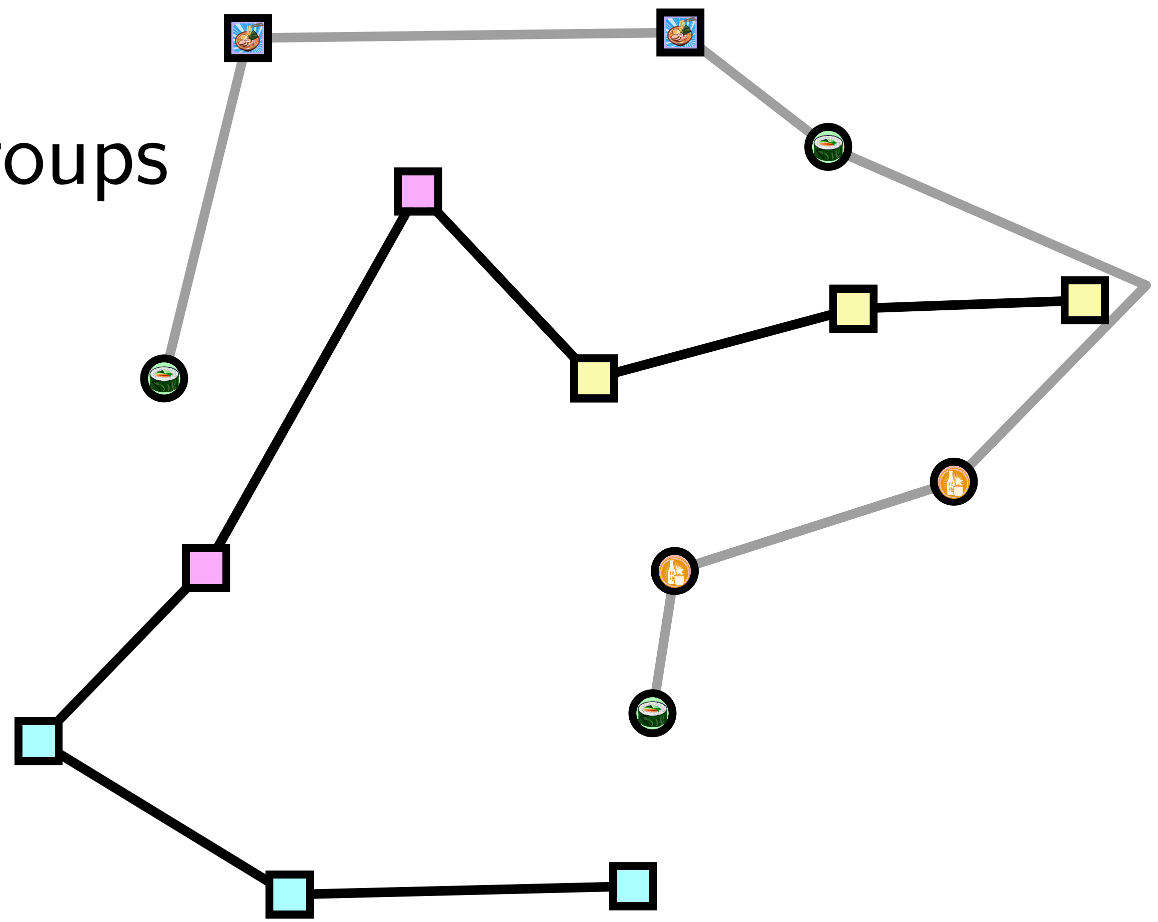
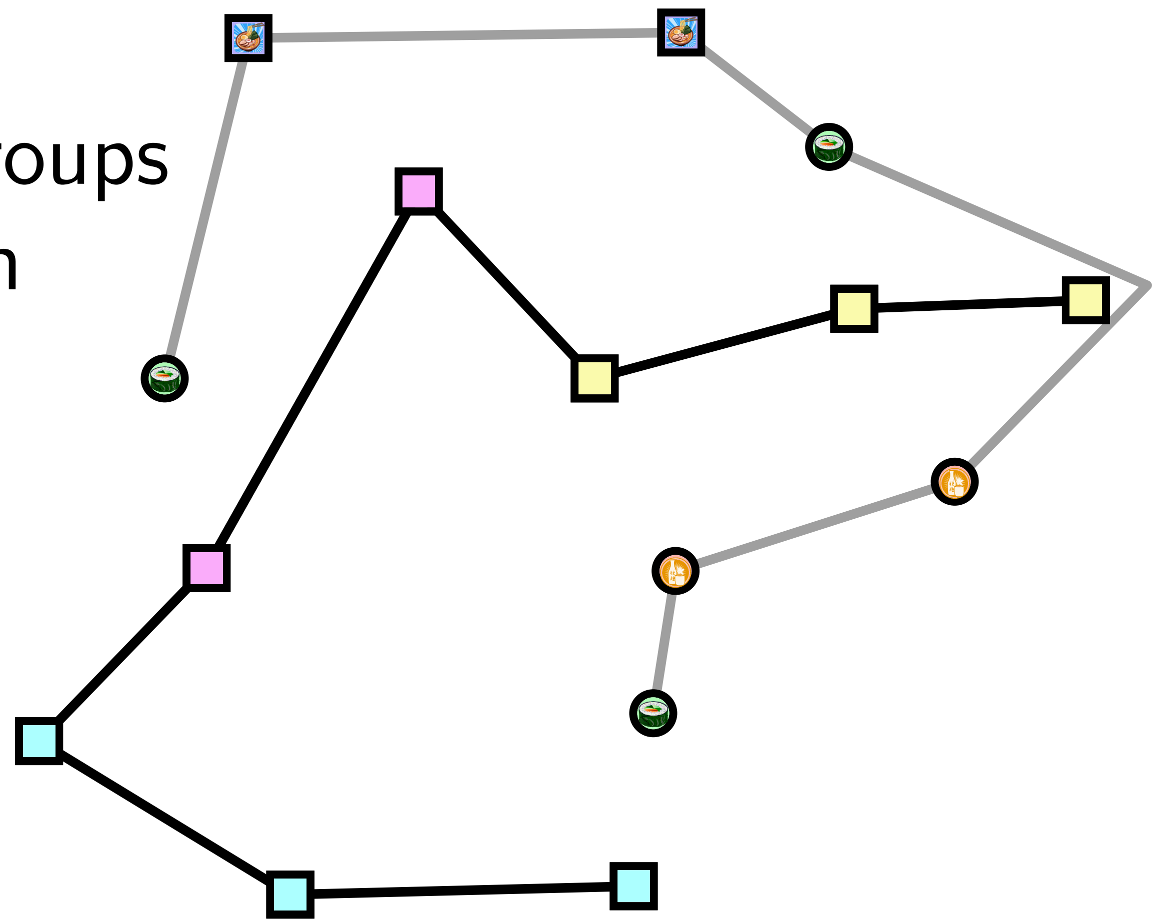


k-CESF

- split into 2 groups
- use PTAS for the groups
- Construct trees from this "super-tree"



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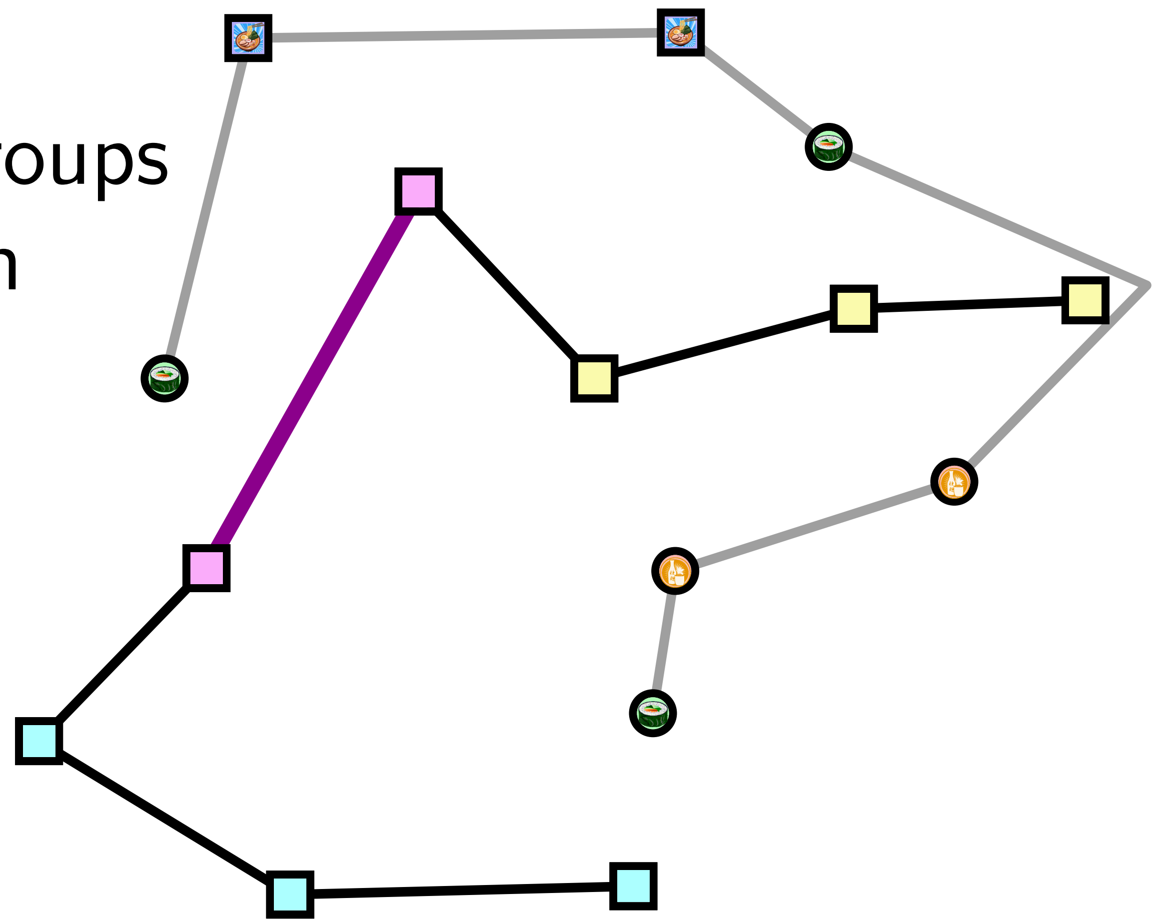

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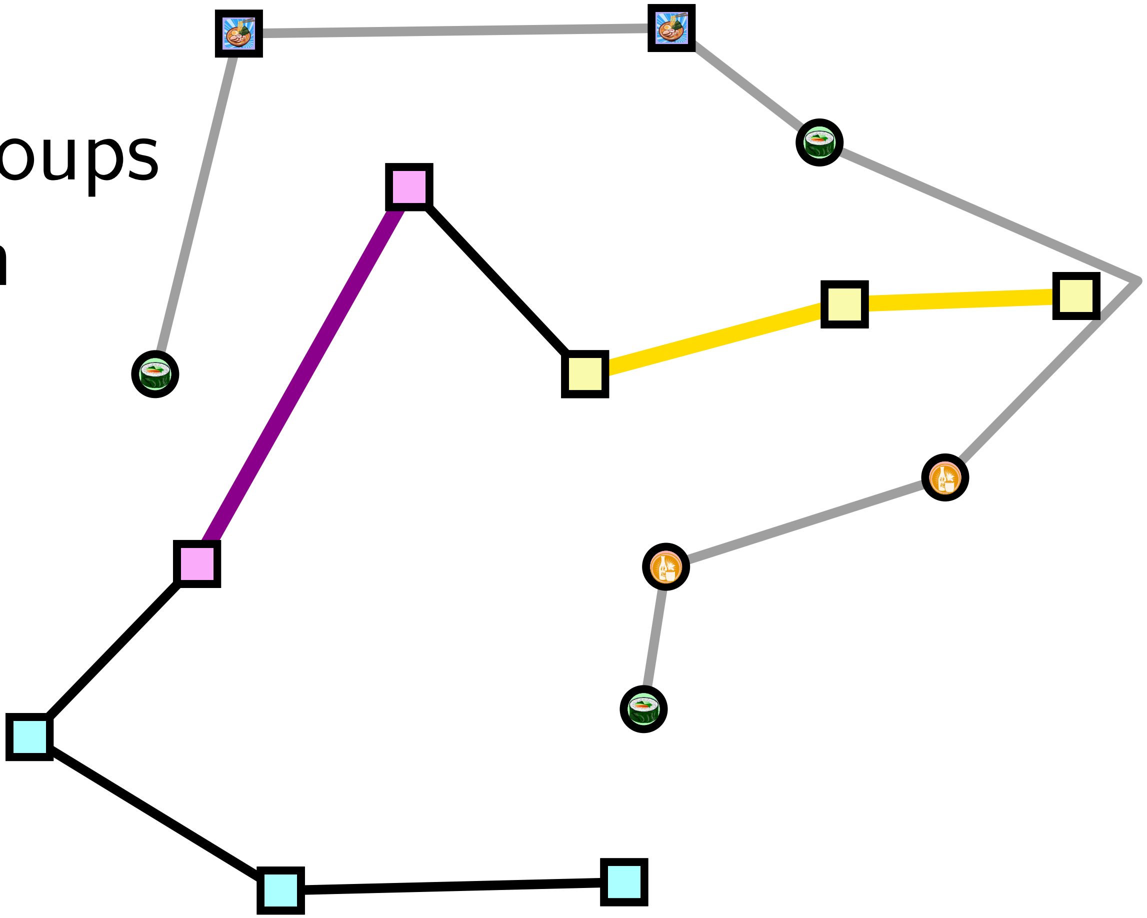

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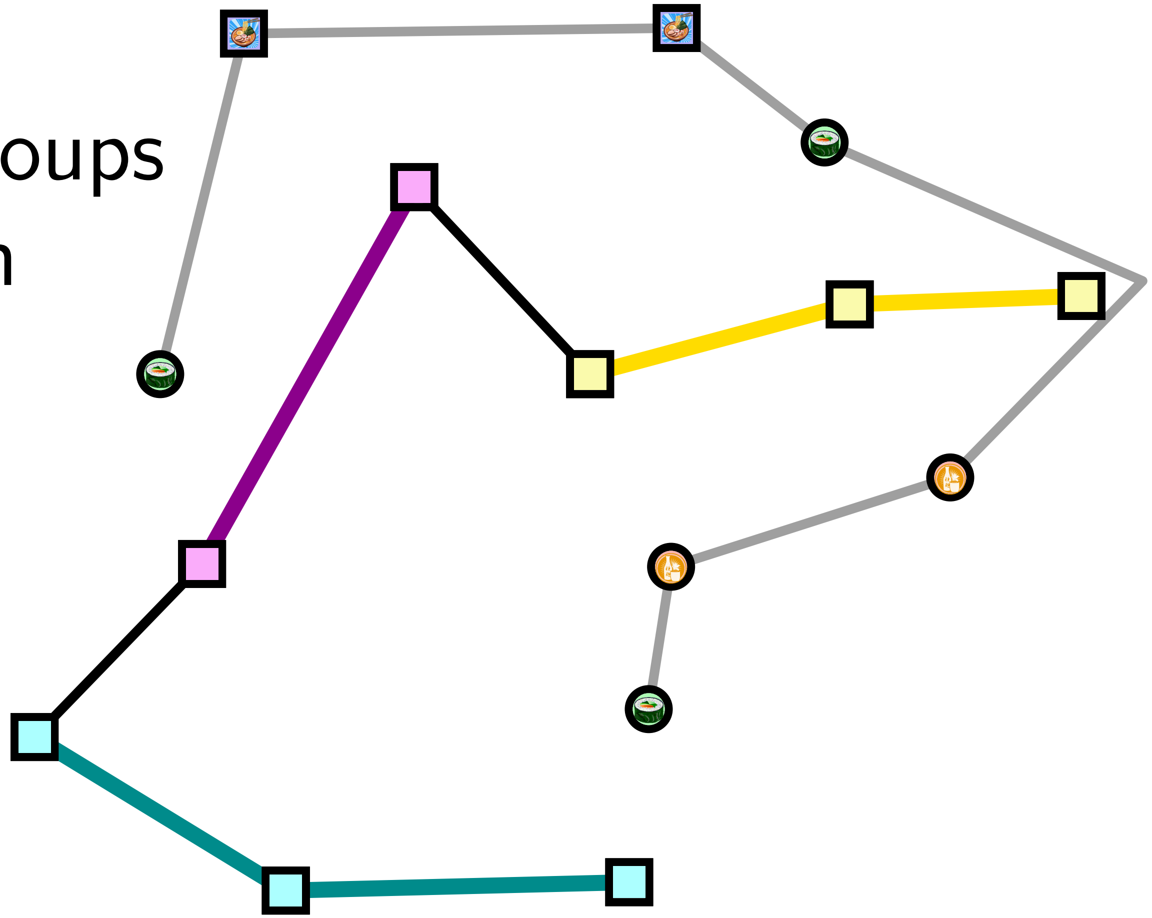

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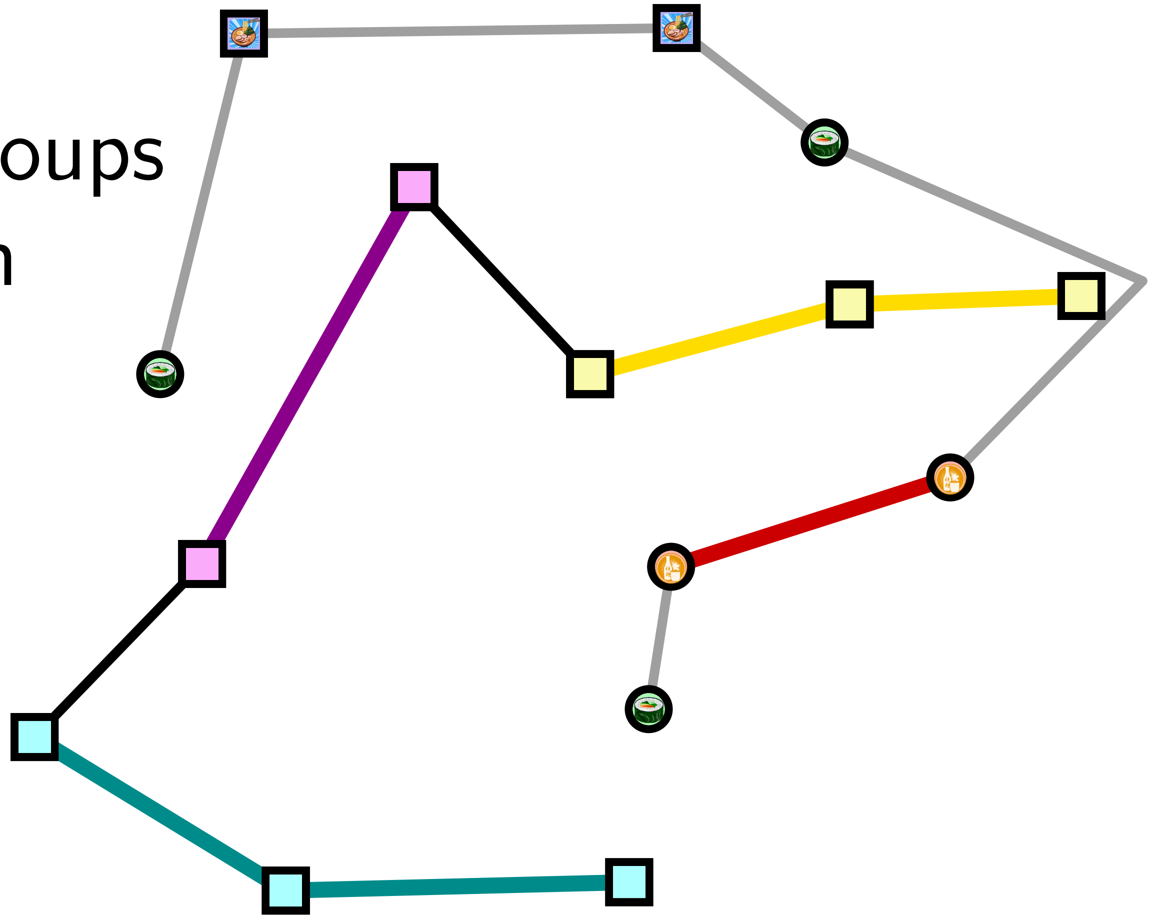

k-CESF

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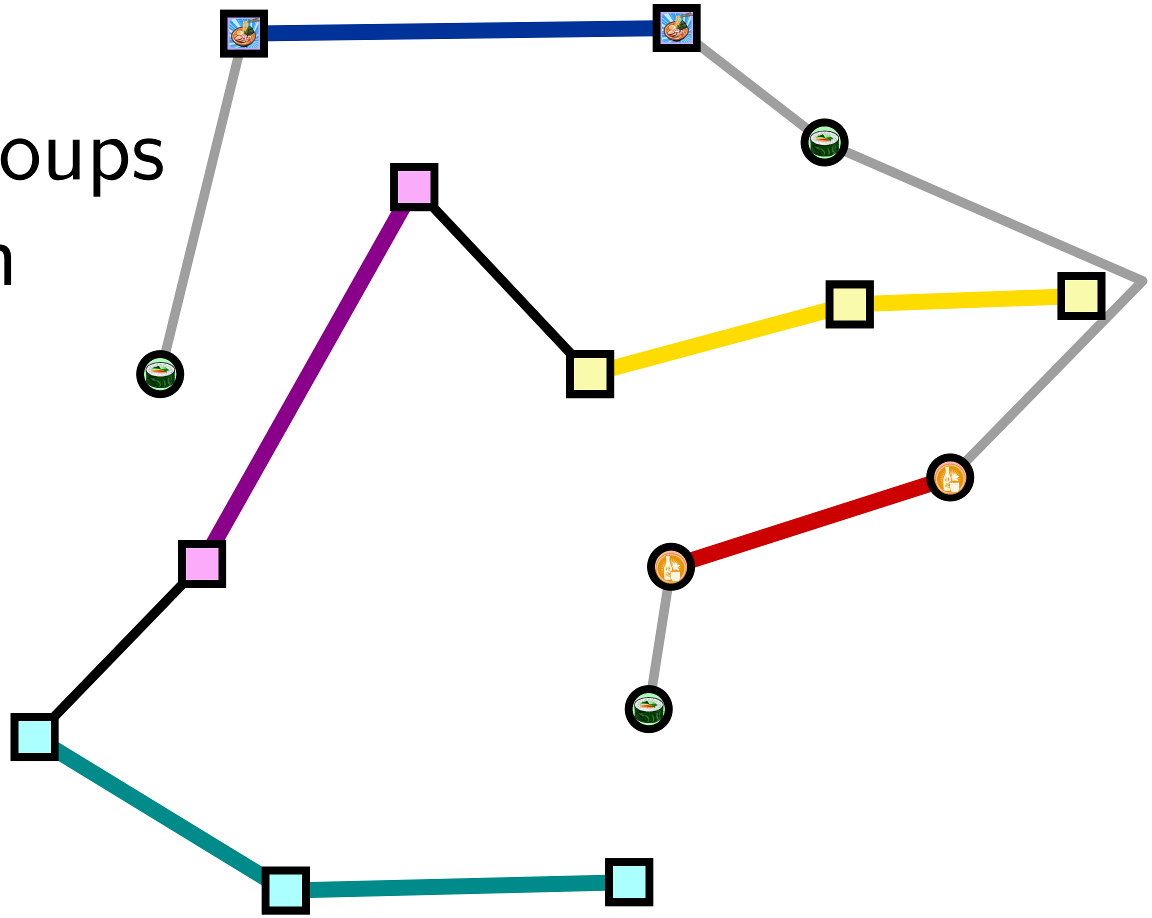

k-CESF

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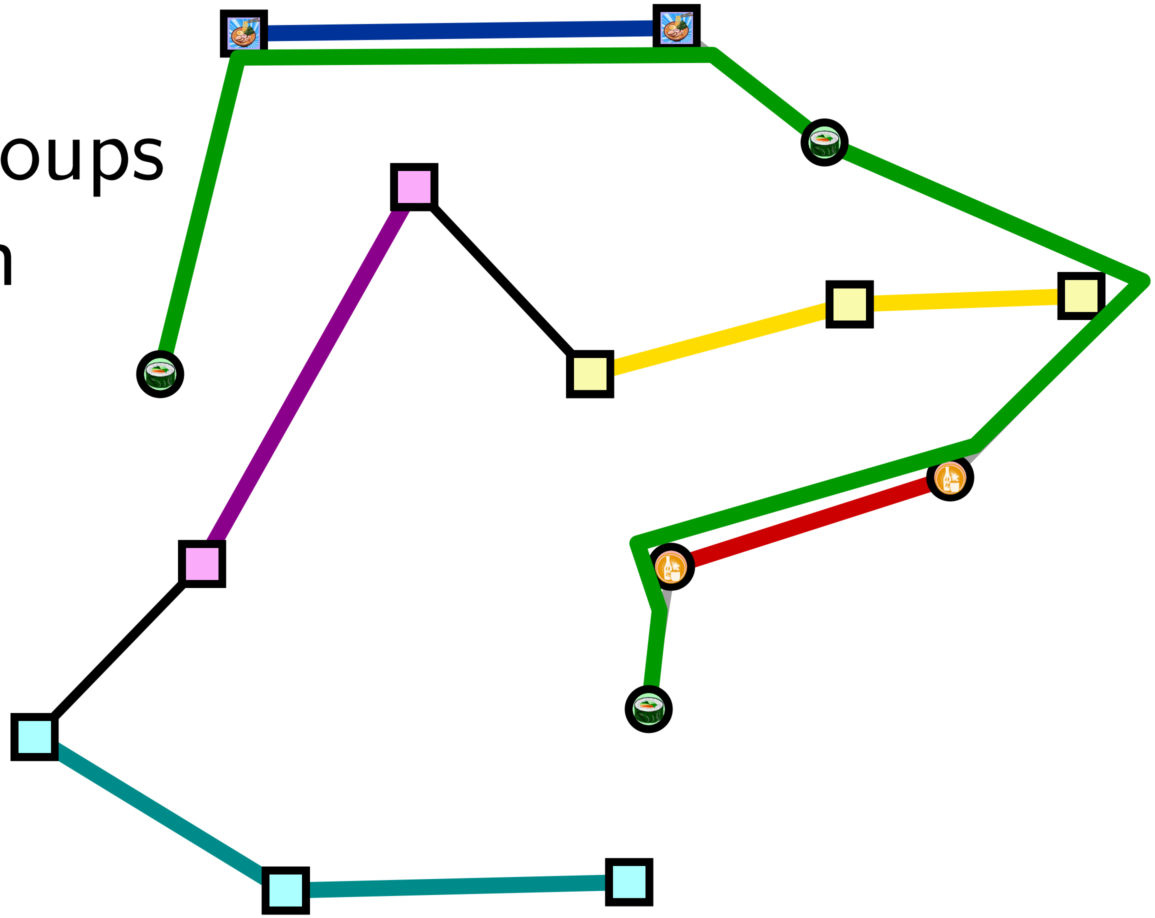

k-CESF

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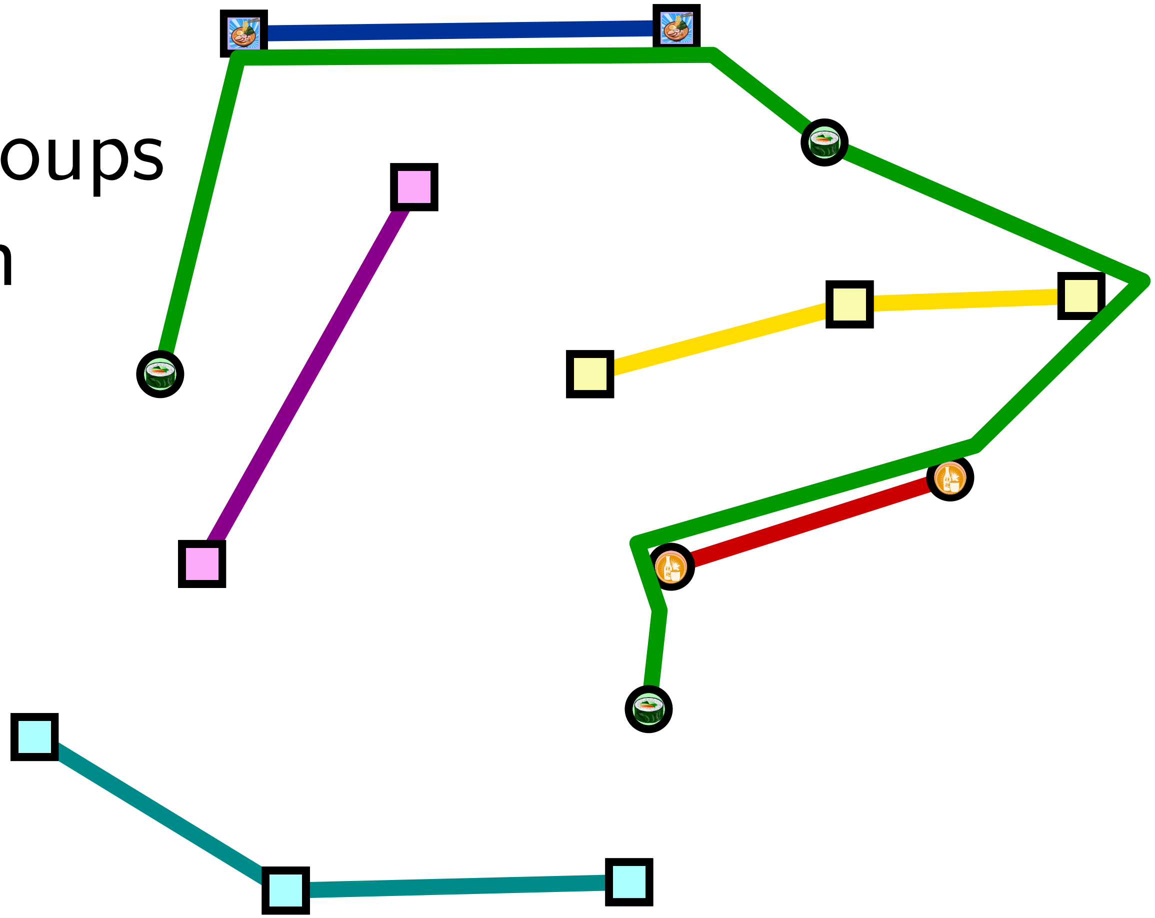

k-CESF

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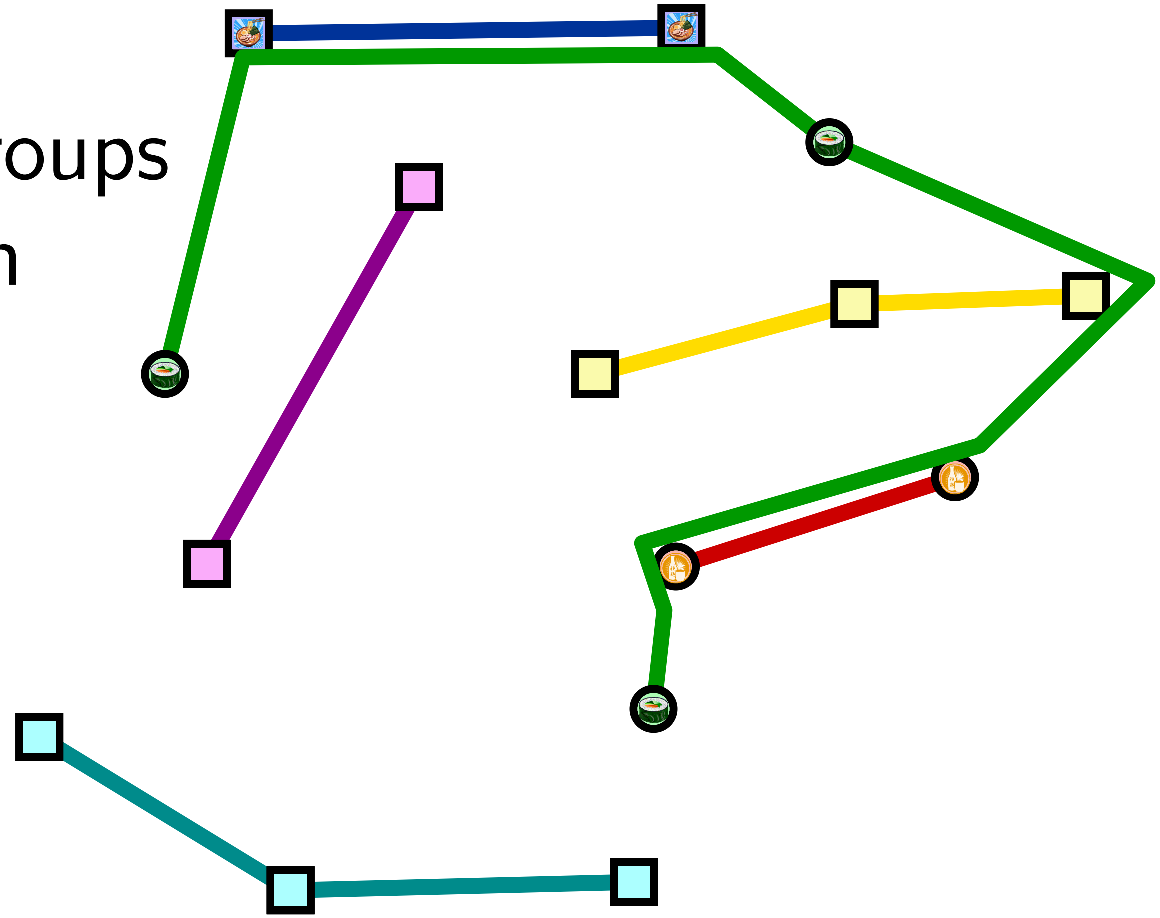

k-CESF admits an

- $(k + \varepsilon)$-approximation algorithm is k is odd
- $(k - 1 + \varepsilon)$-approximation algorithm is k is even