Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Recognizing and Drawing IC-planar Graphs

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Universität Würzburg /
FernUniversität in Hagen

Joint work with
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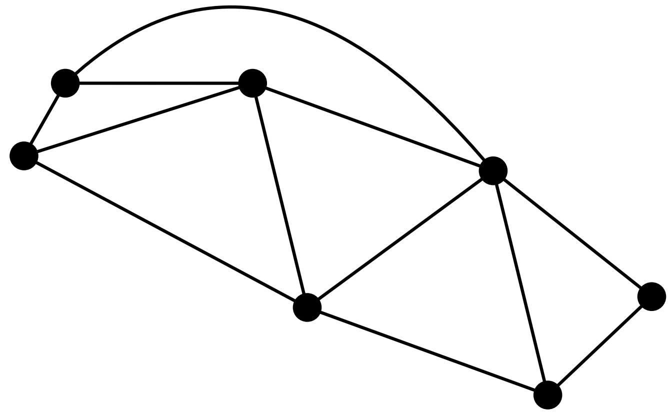
Giuseppe Liotta & Fabrizio Montecchiani

FernUniversität in Hagen
Fakultät für Mathematik und Informatik

theoretische
informatik

1-planar Graphs

Planar graphs: Can be drawn without crossings.



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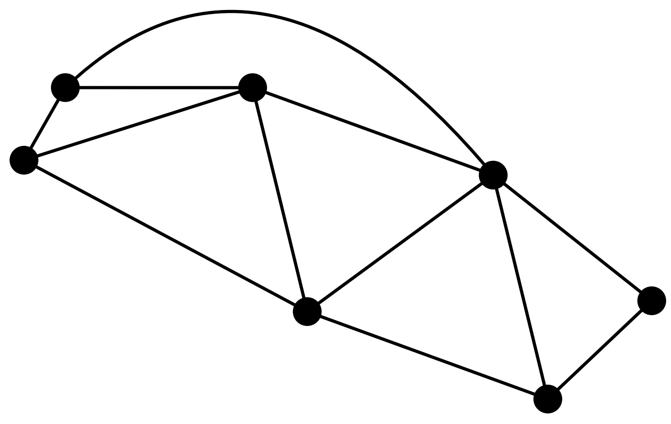

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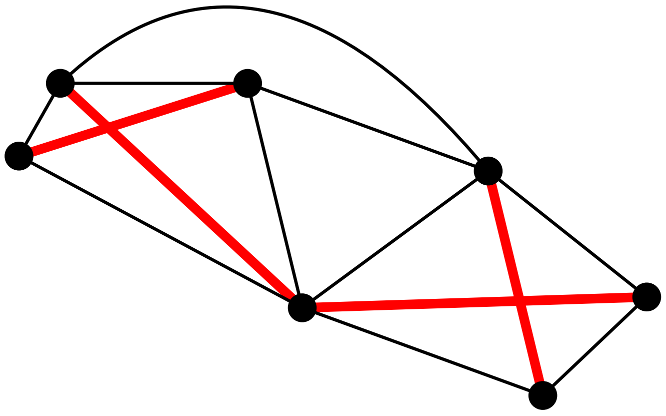

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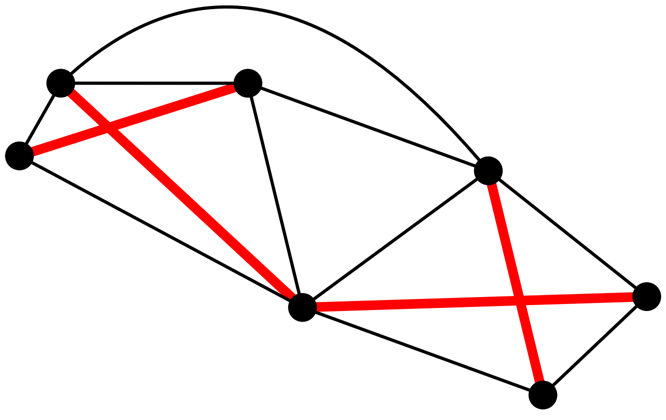
- $\leq 4n - 8$ edges



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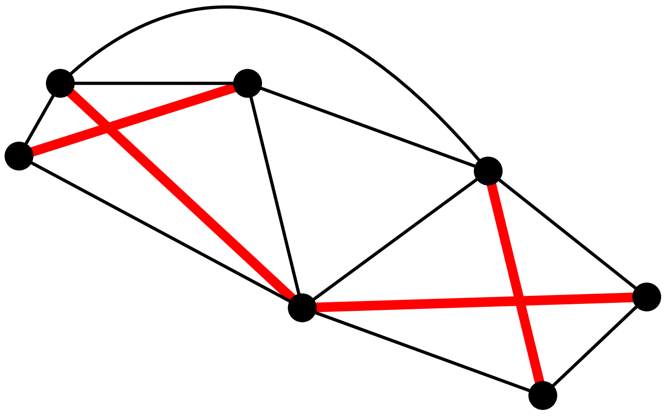
- straight-line: $\leq 4n - 9$ edges



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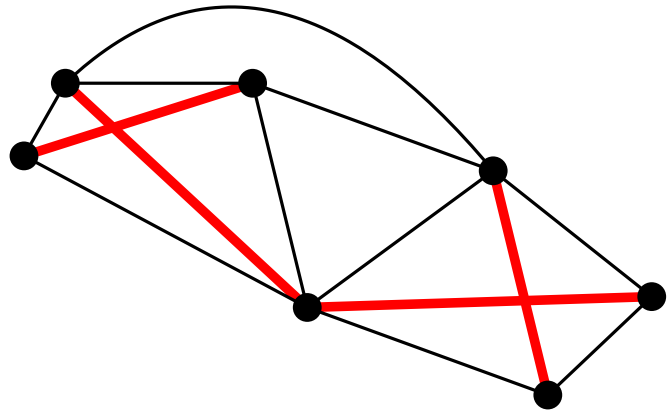

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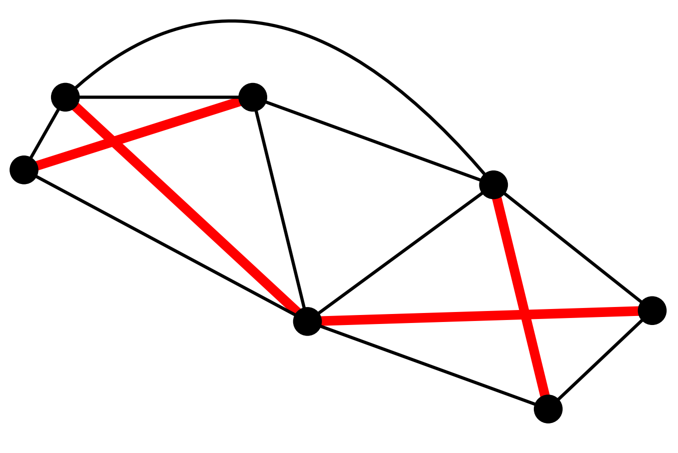

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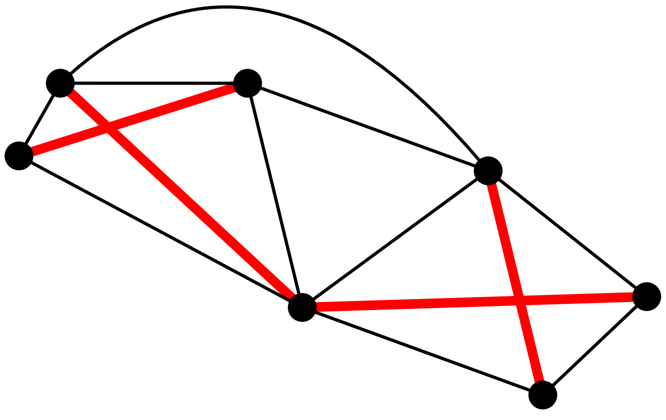

RAC Graphs

RAC graphs: Can be drawn straight-line
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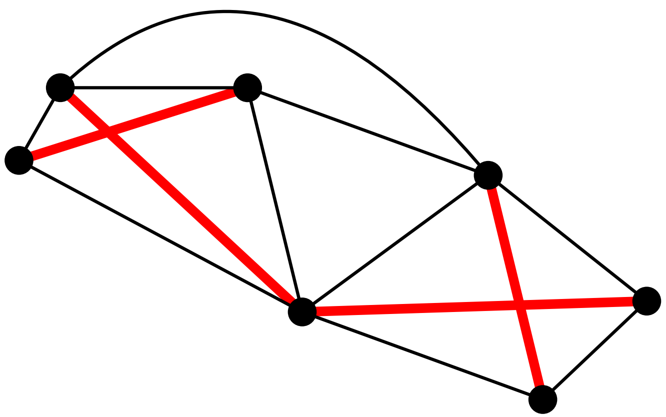

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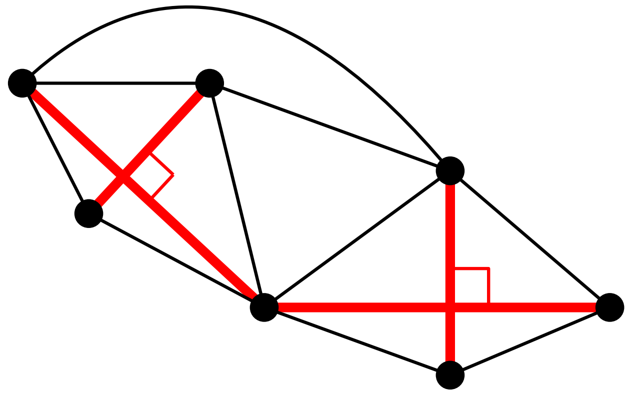

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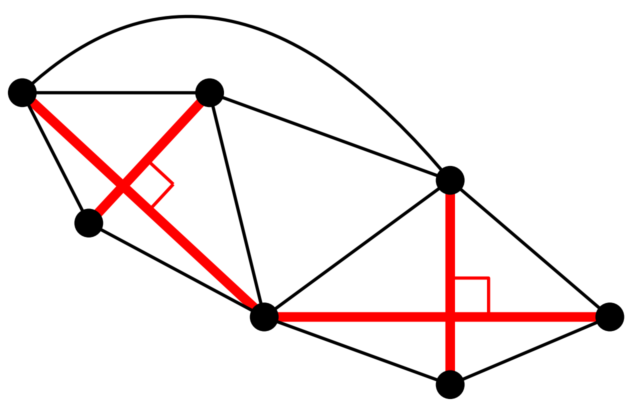

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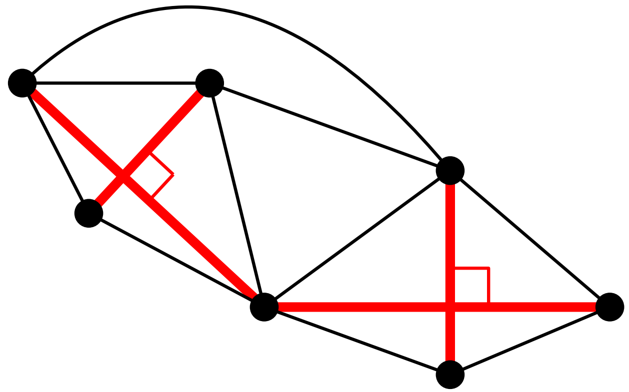

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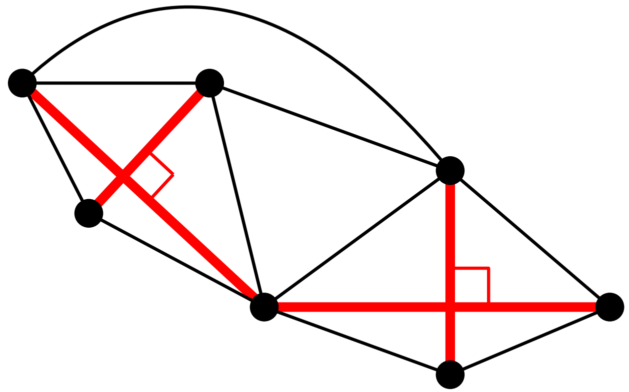

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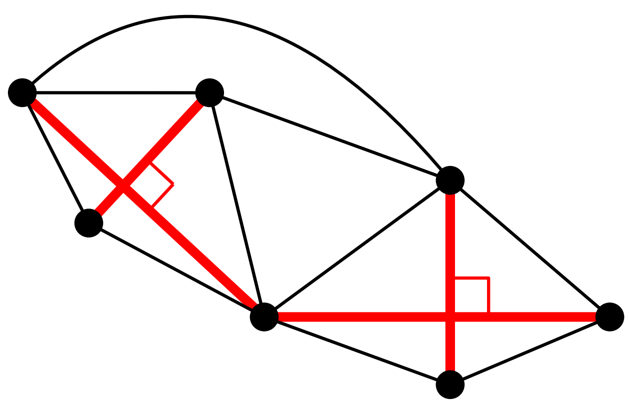
- Recognition: NP-hard [Argyriou et al. JGAA'12]



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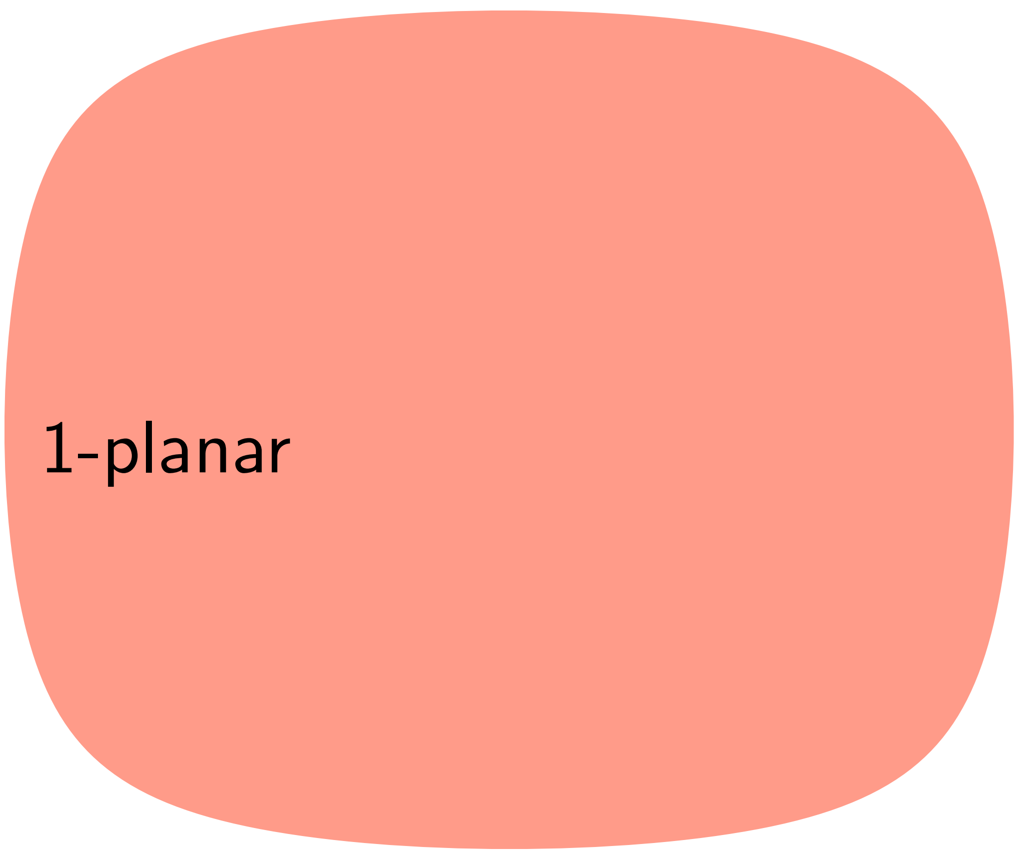

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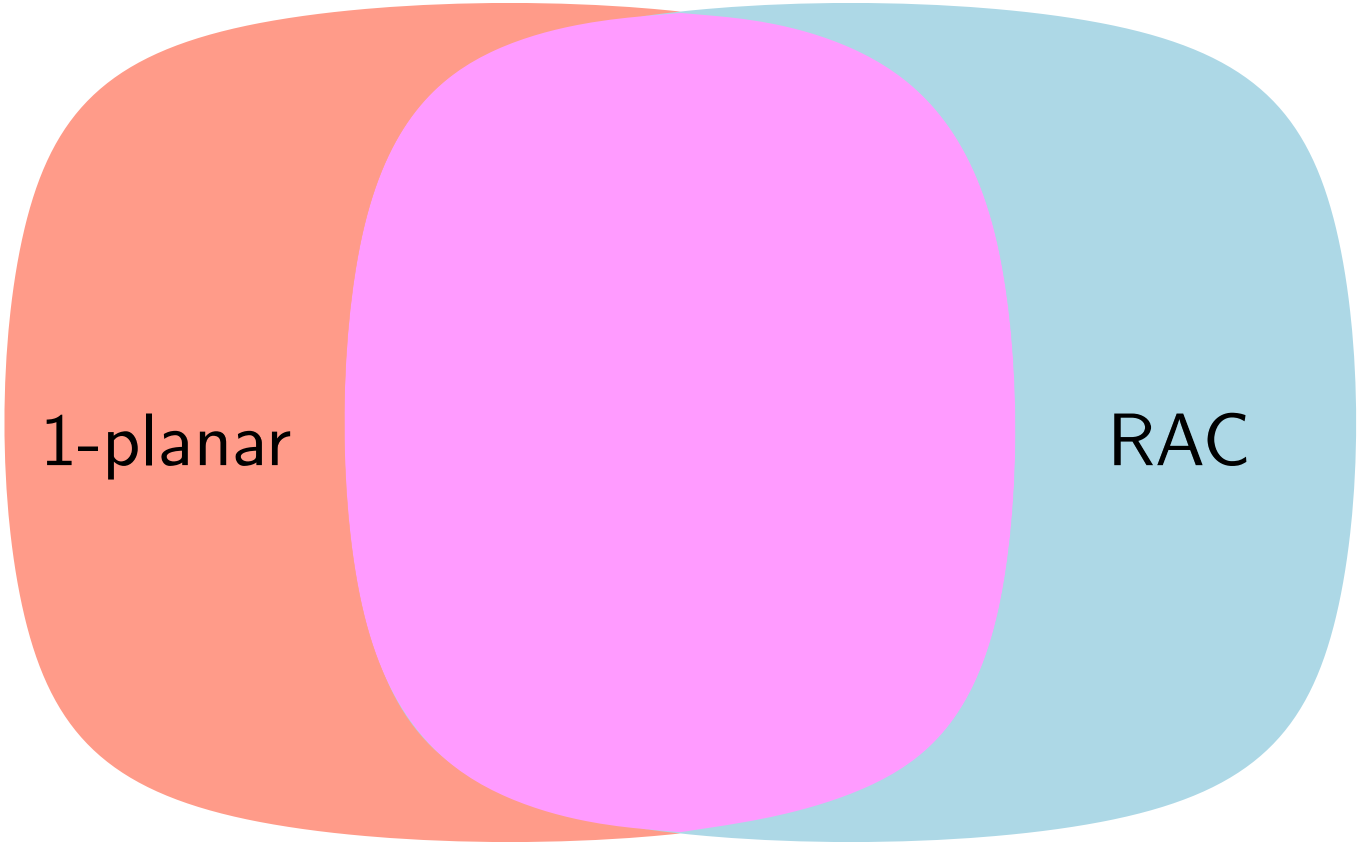
- 1-planar $\neq$ RAC [Eades & Liotta DMA'13]



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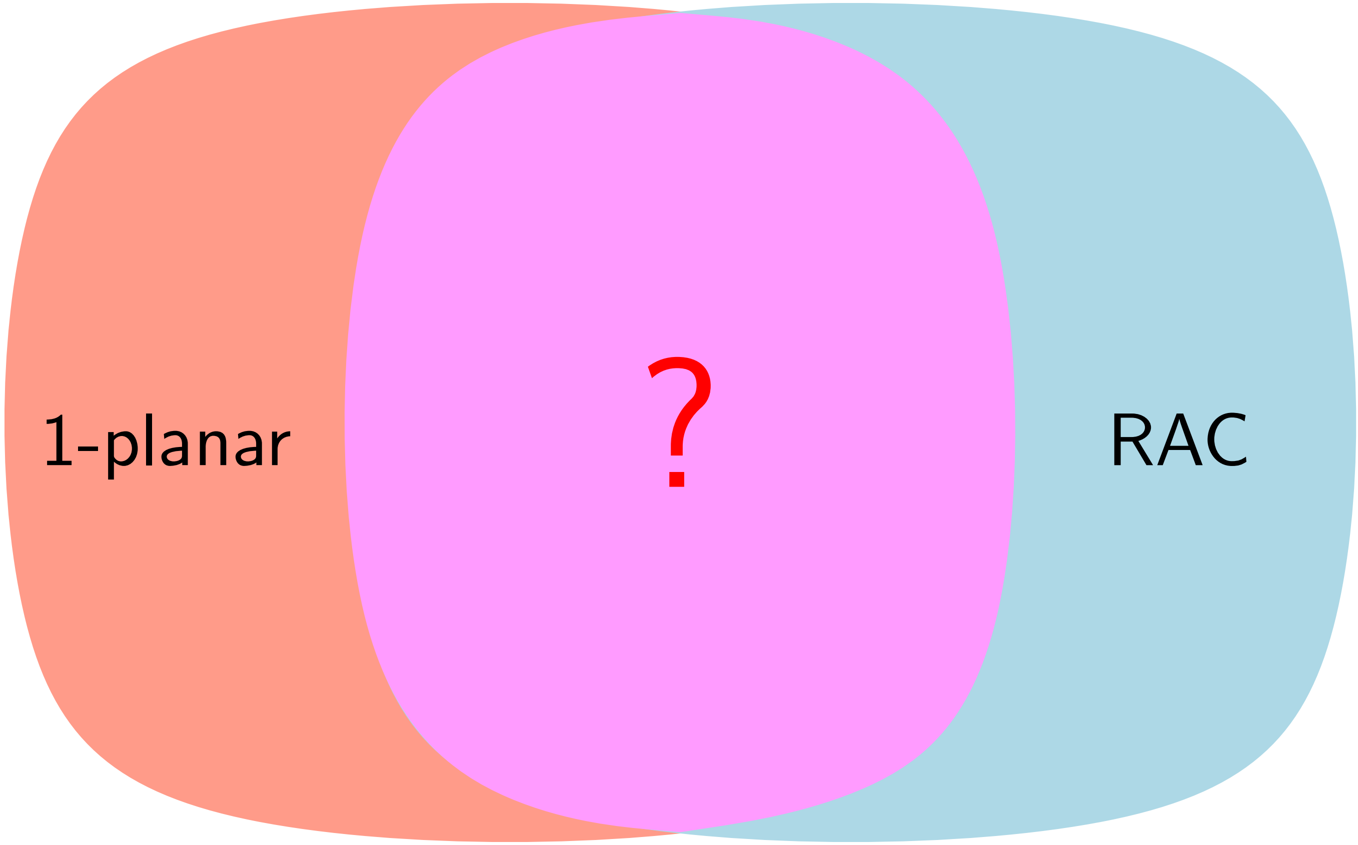

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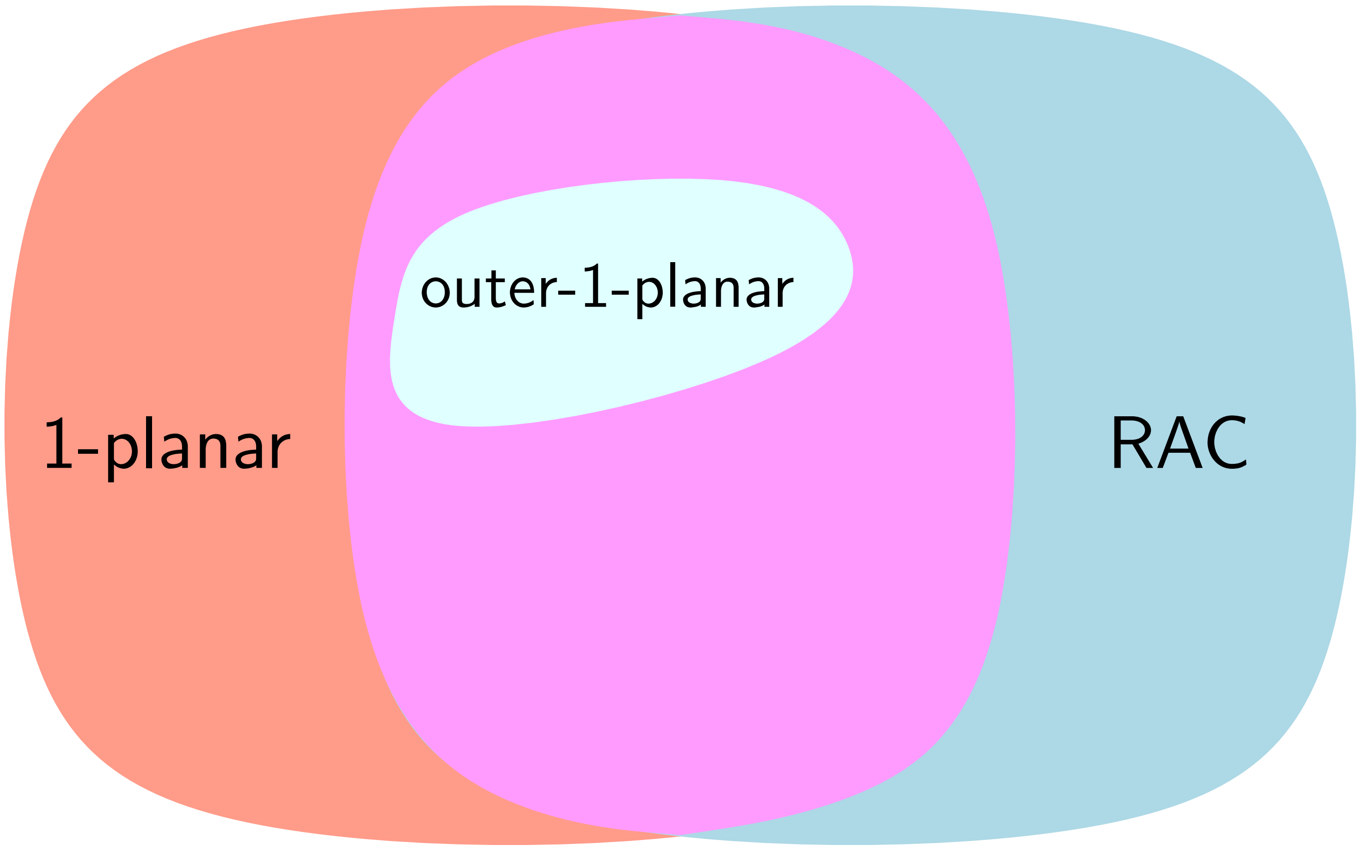

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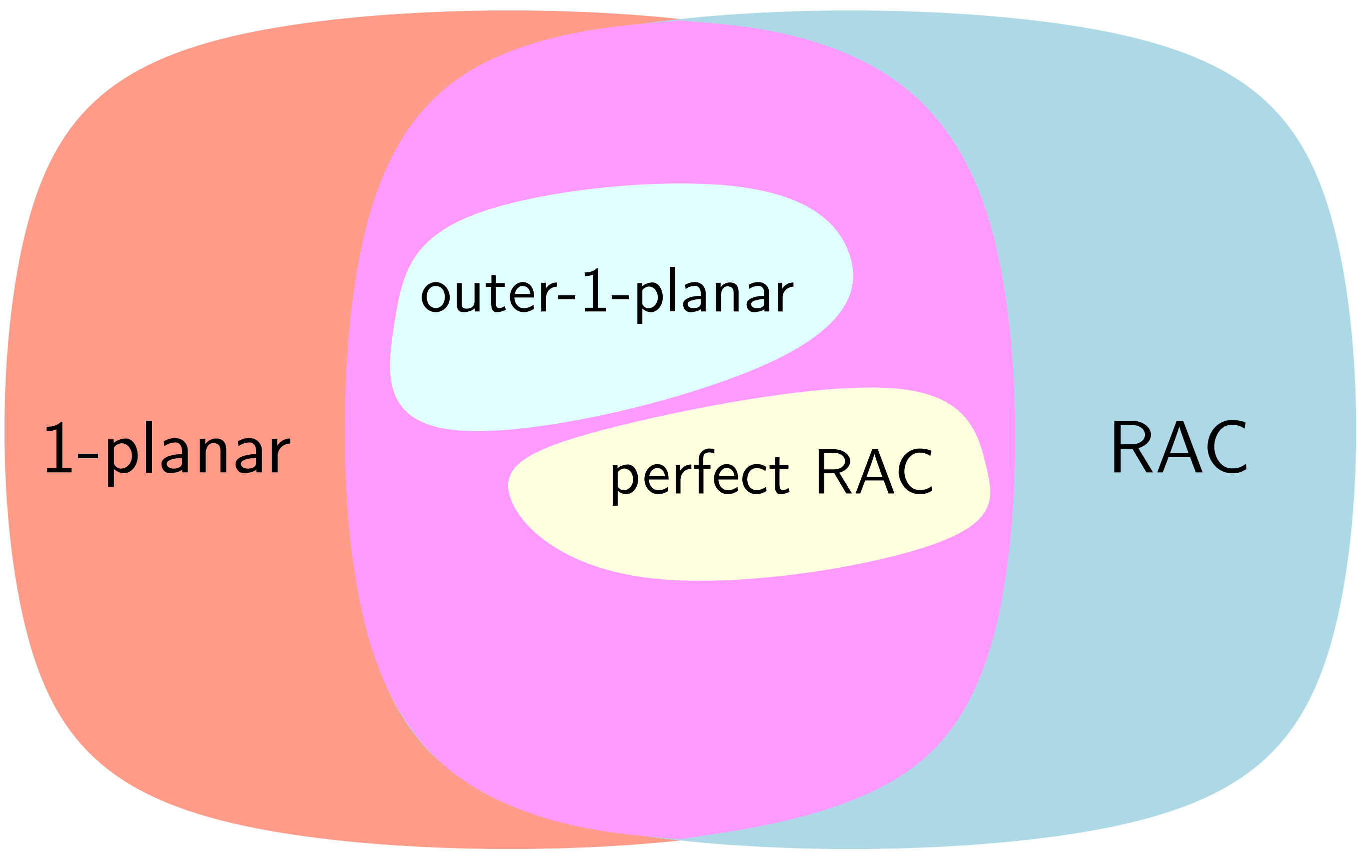
- perfect RAC $\subset$ 1-planar [Eades & Liotta DMA'13]



IC-planar Graphs

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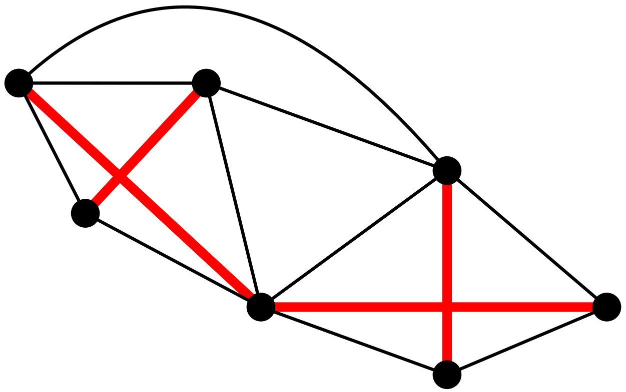
independent crossings



IC-planar Graphs

IC-planar graphs: Each edge is crossed at most once and each vertex is incident to at most one crossing edge.

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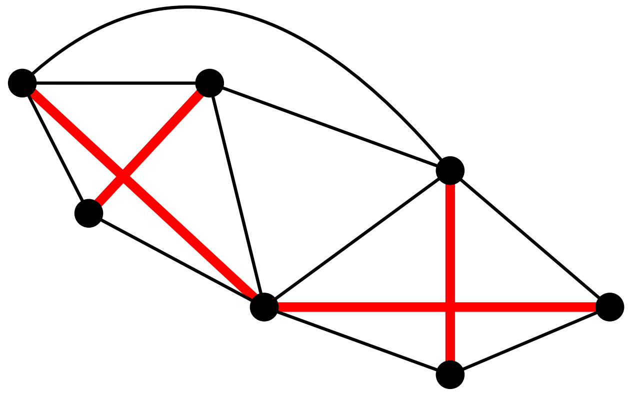

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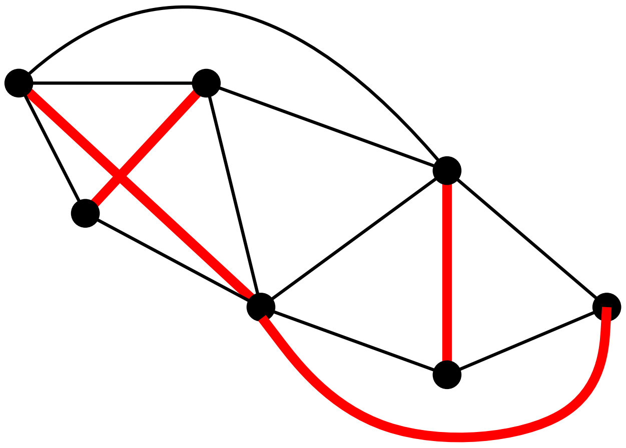

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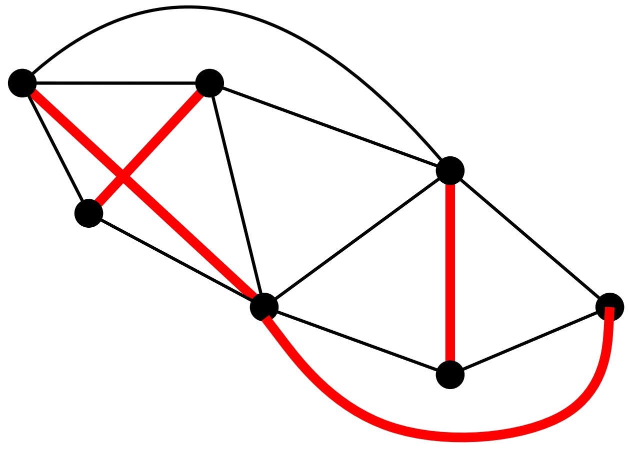
independent crossings

- $\leq 13n/4 - 6$ edges



Recognition

Reduction from 1-planarity testing.

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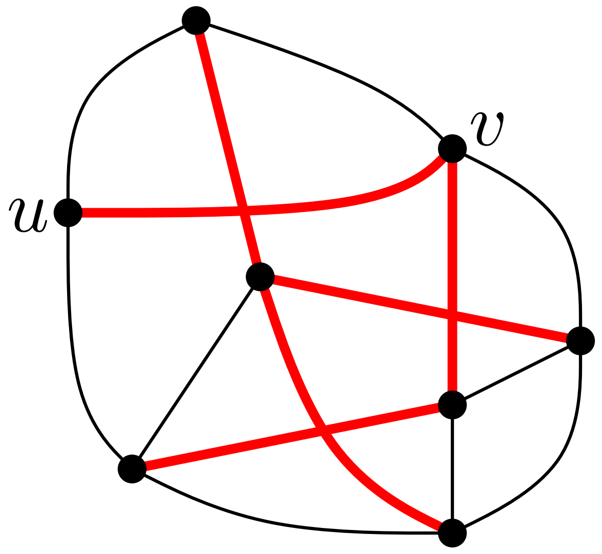

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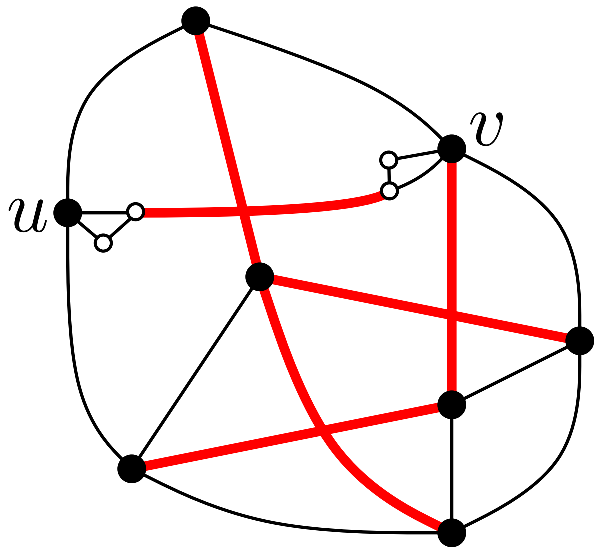

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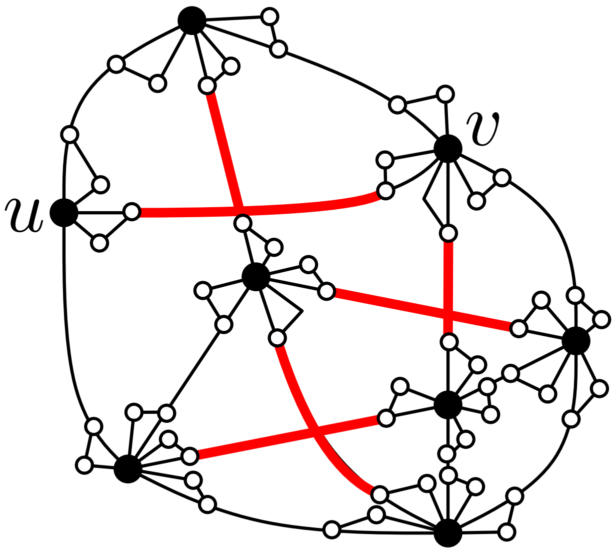

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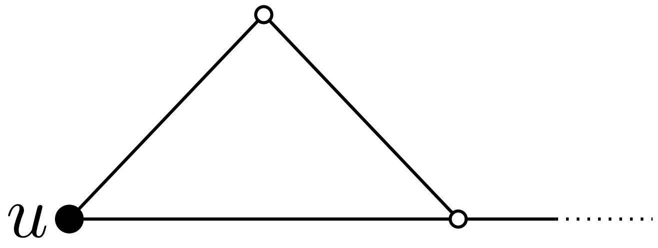

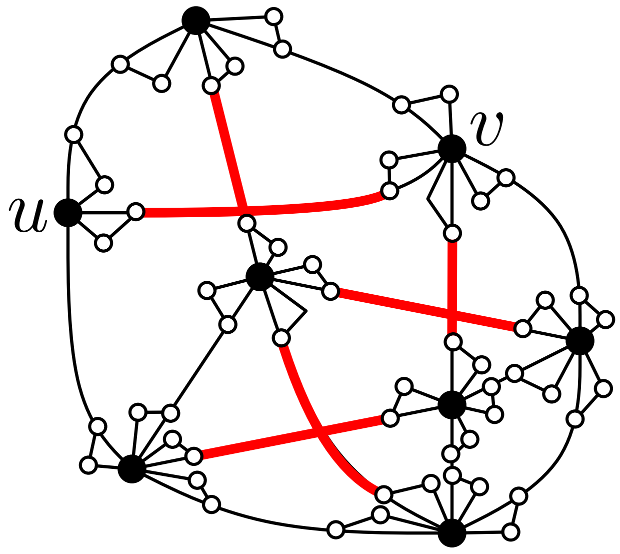

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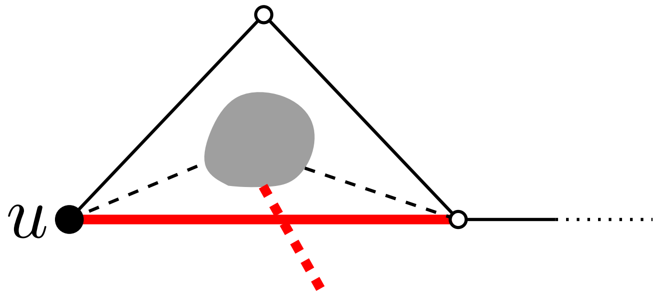

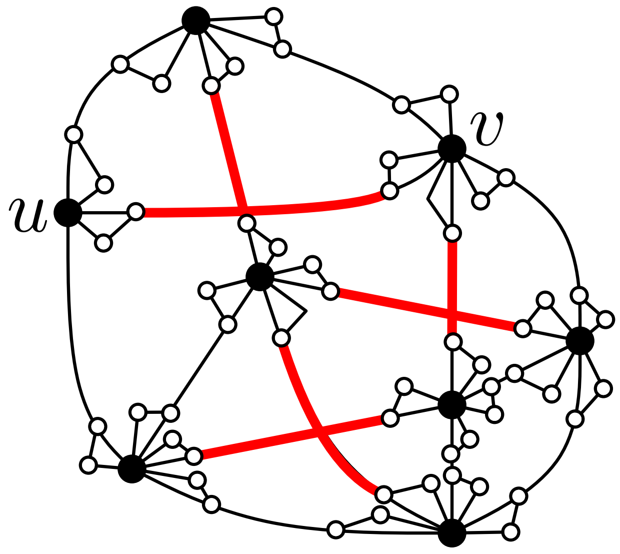

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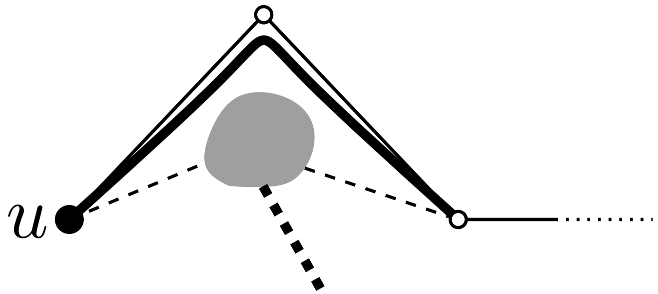

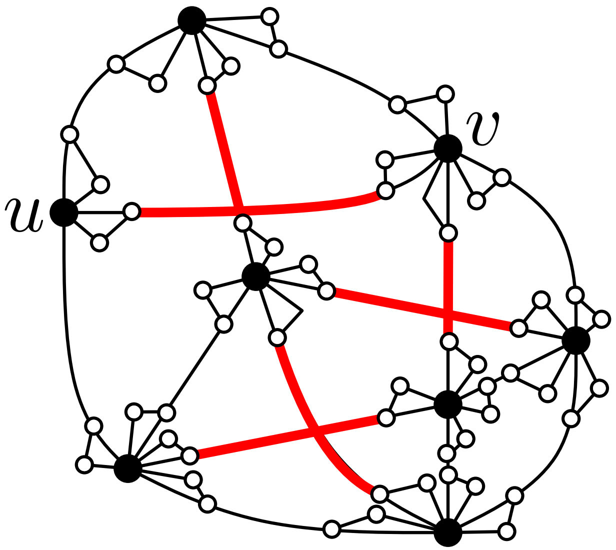

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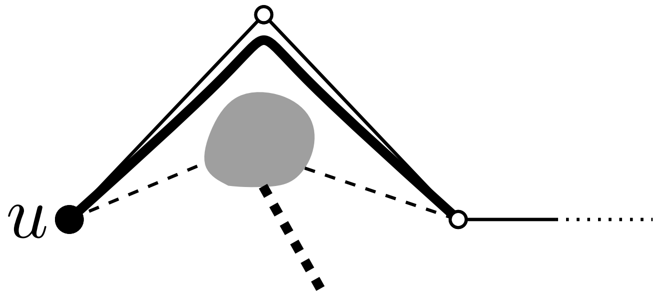

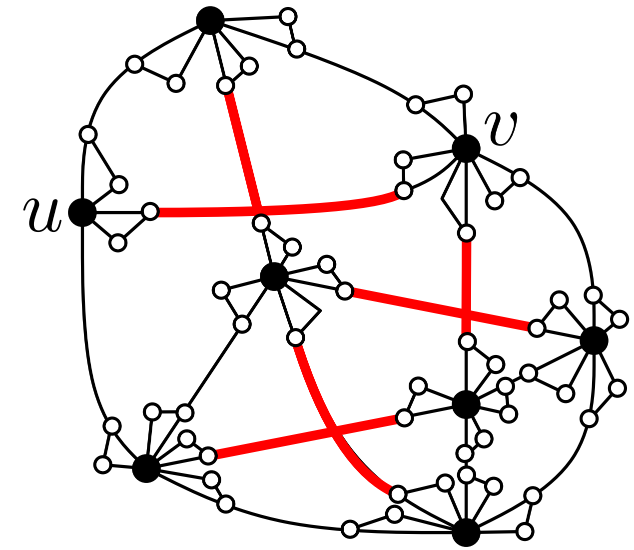

Theorem.
Testing IC-planarity is NP-hard

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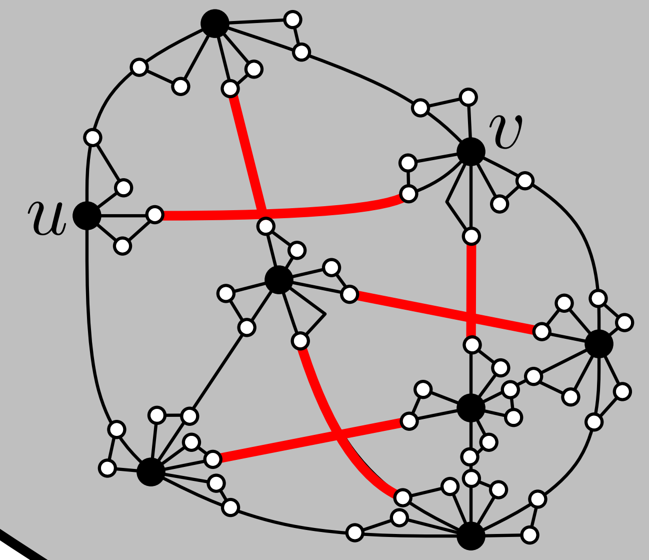

Reduction from planar-3SAT

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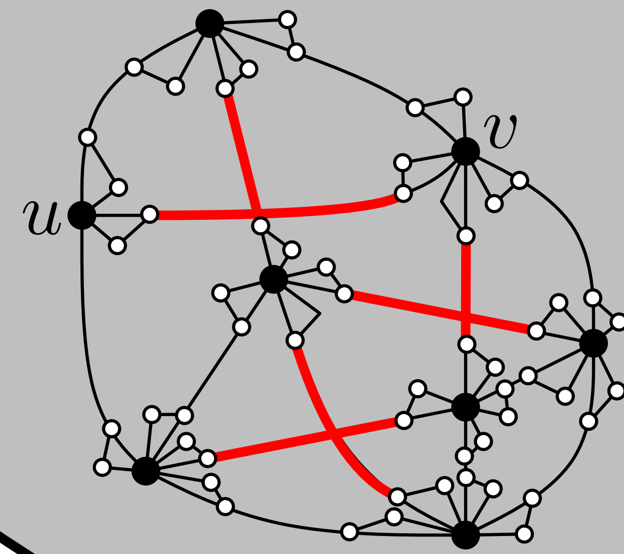

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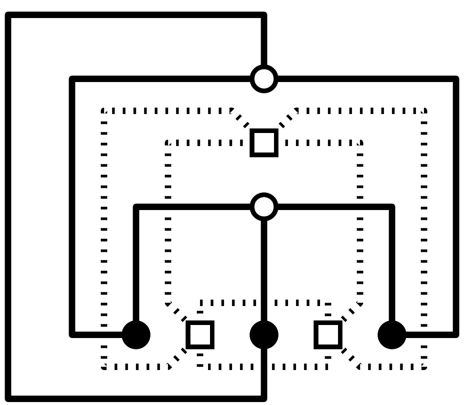

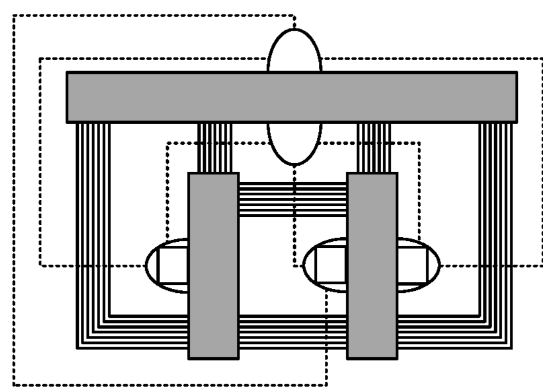

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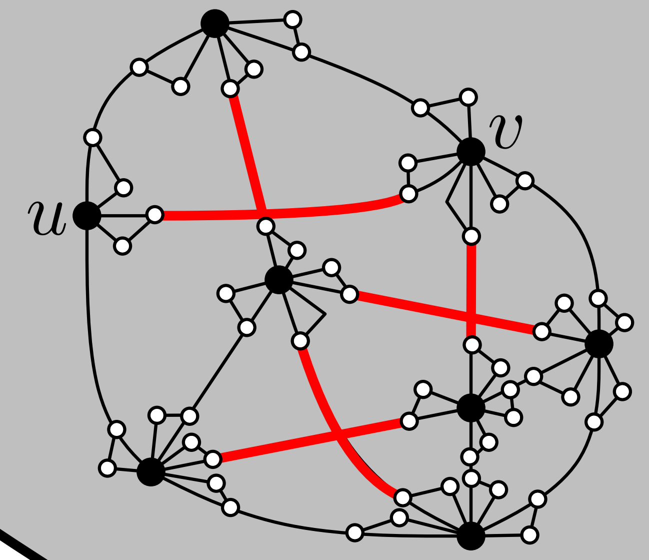

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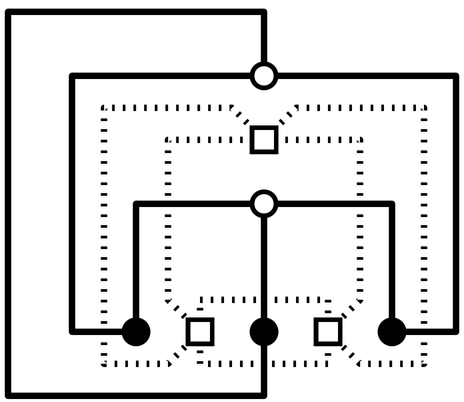

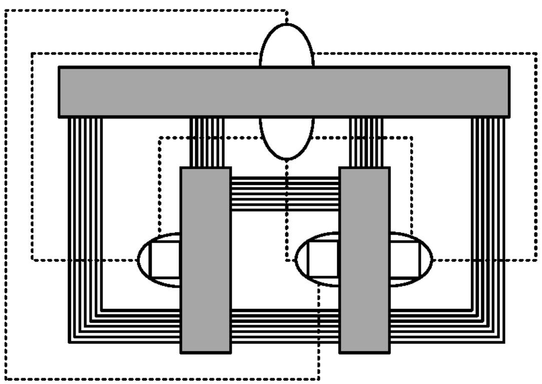

Theorem.
Testing IC-planarity is NP-hard
even if the rotation system is given.

Triangulation + Matching

Given a triconnected plane graph $T = (V, E_T)$
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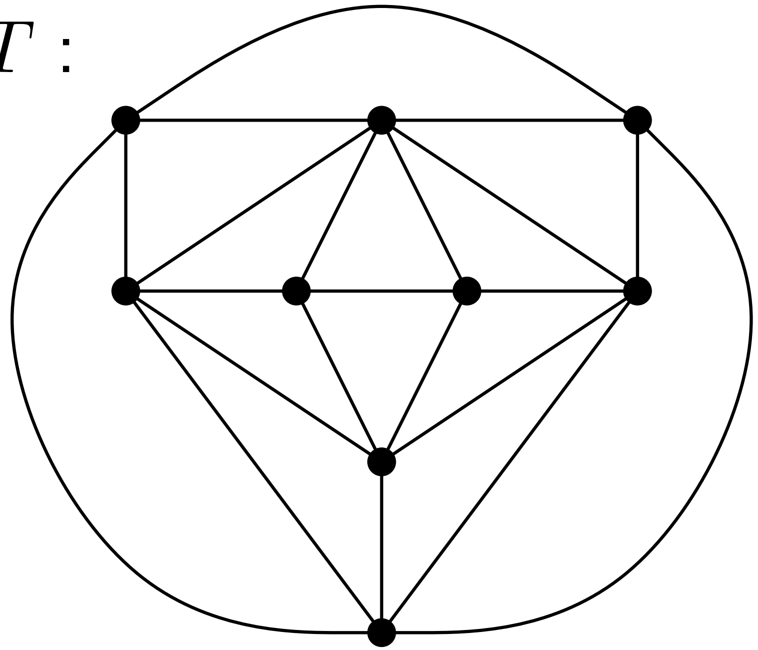

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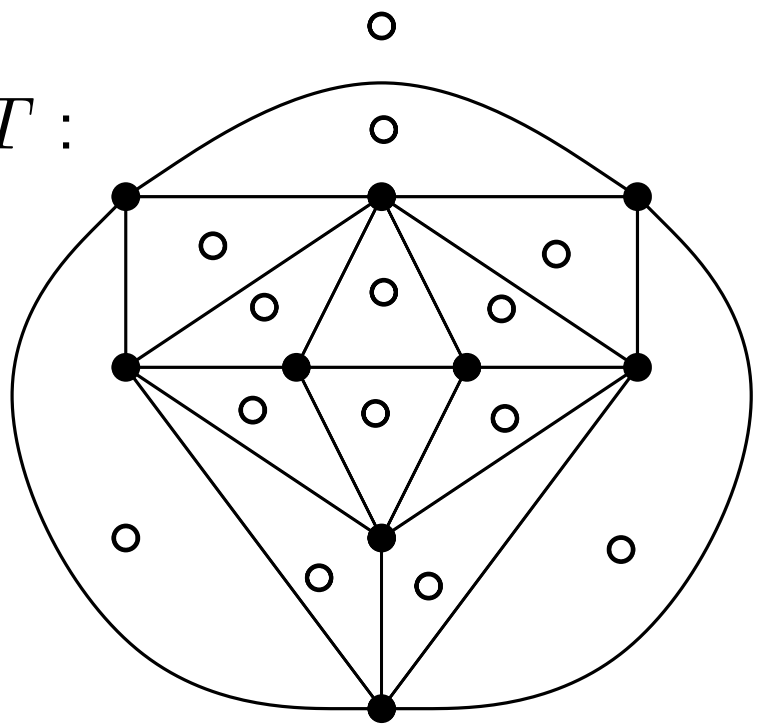

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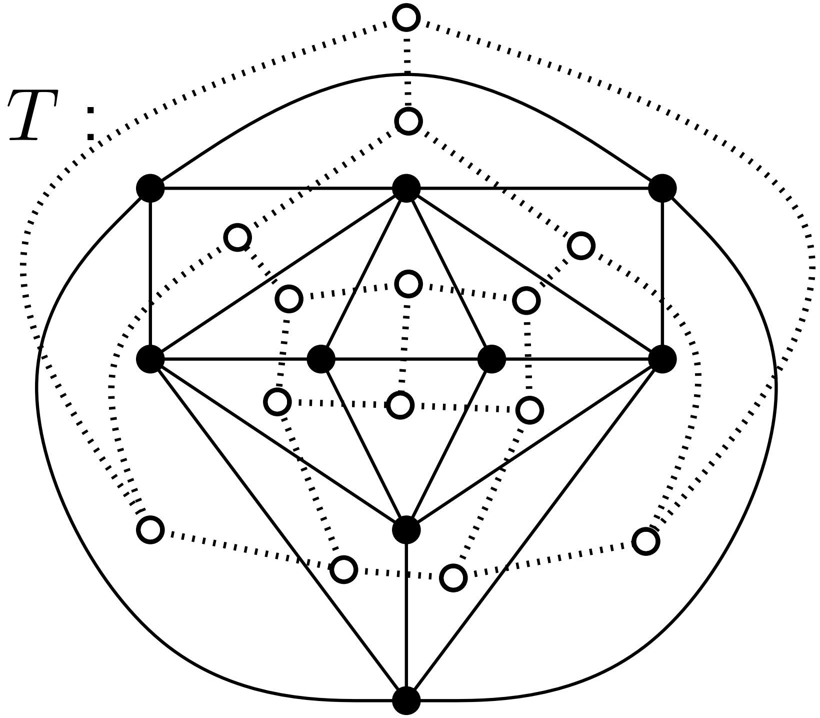

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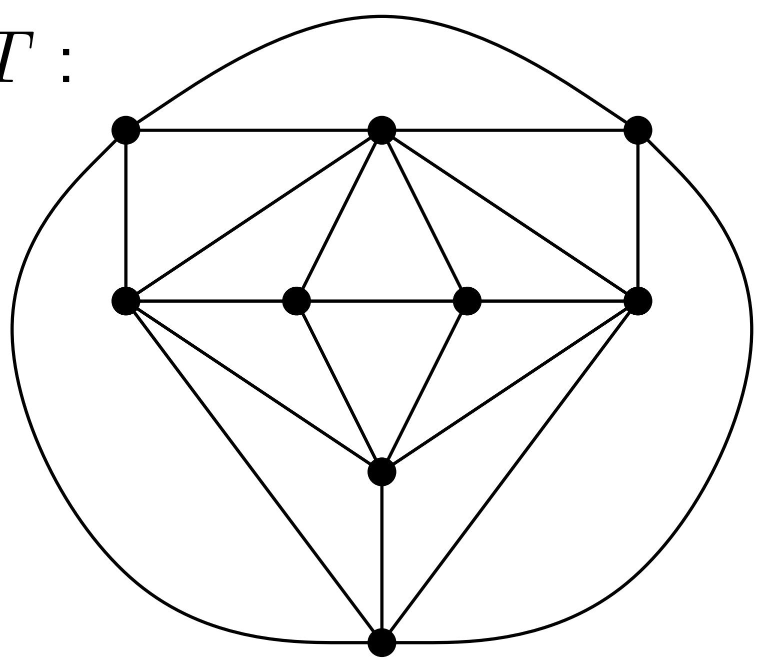

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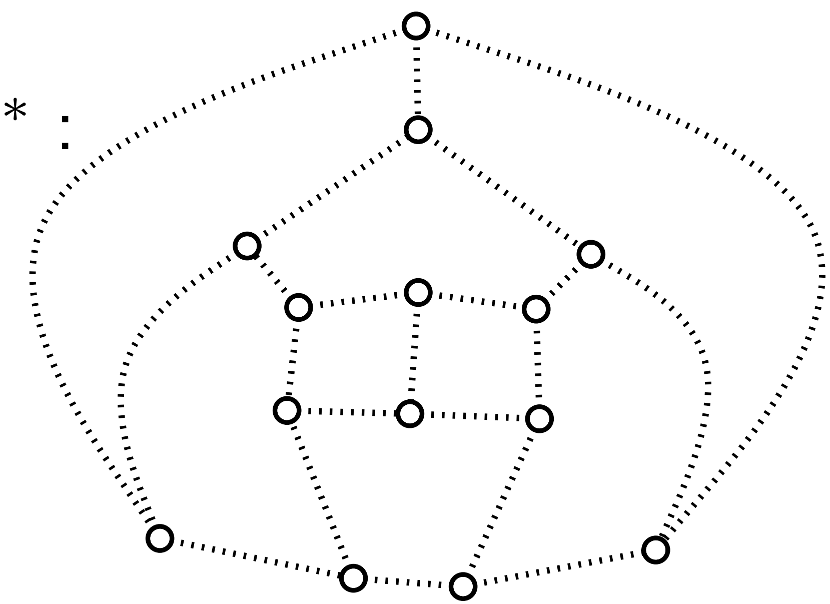

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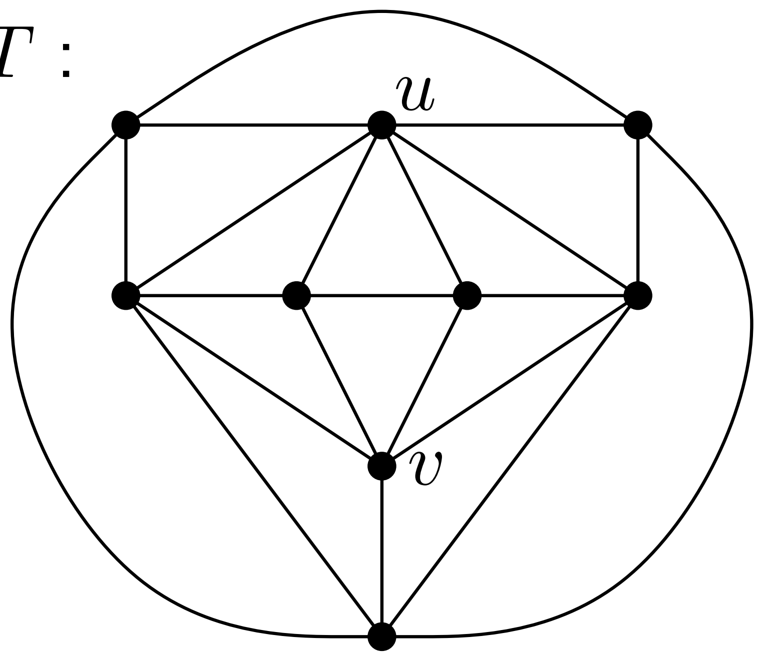

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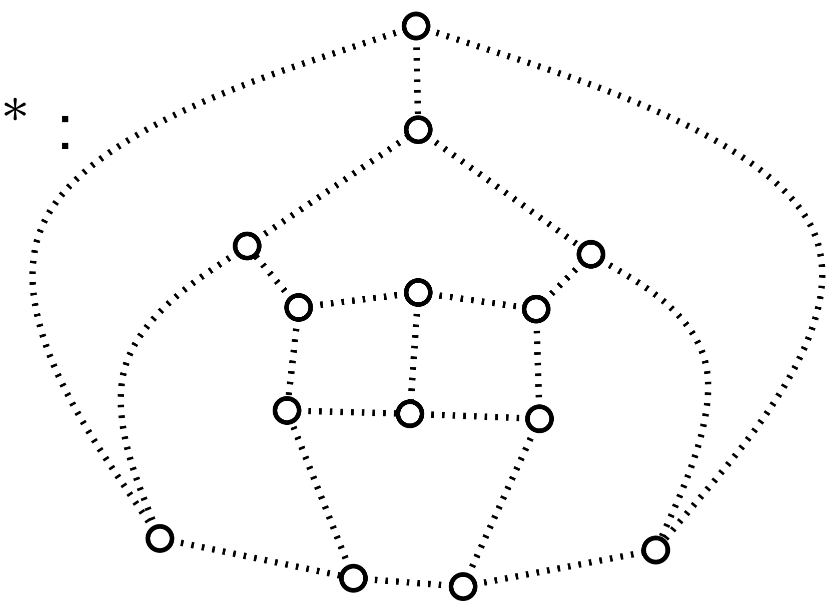

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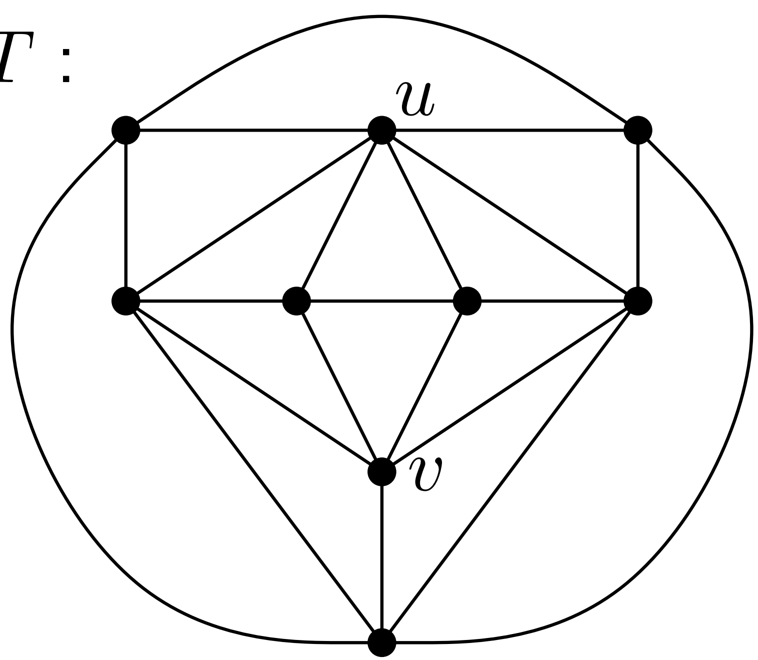

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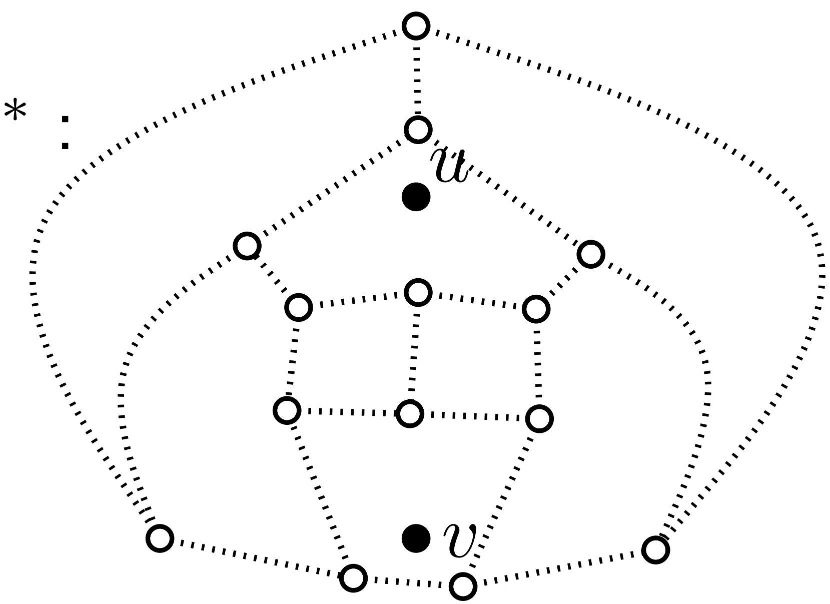

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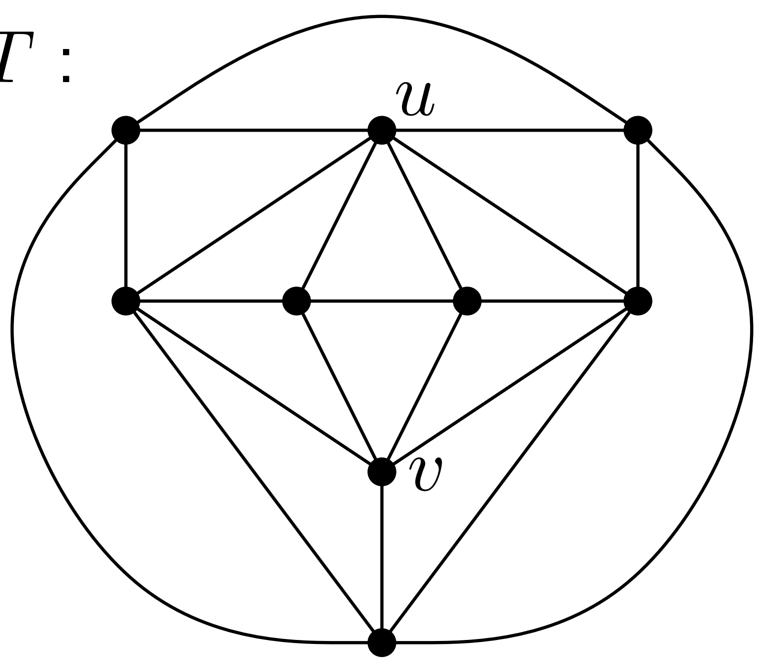

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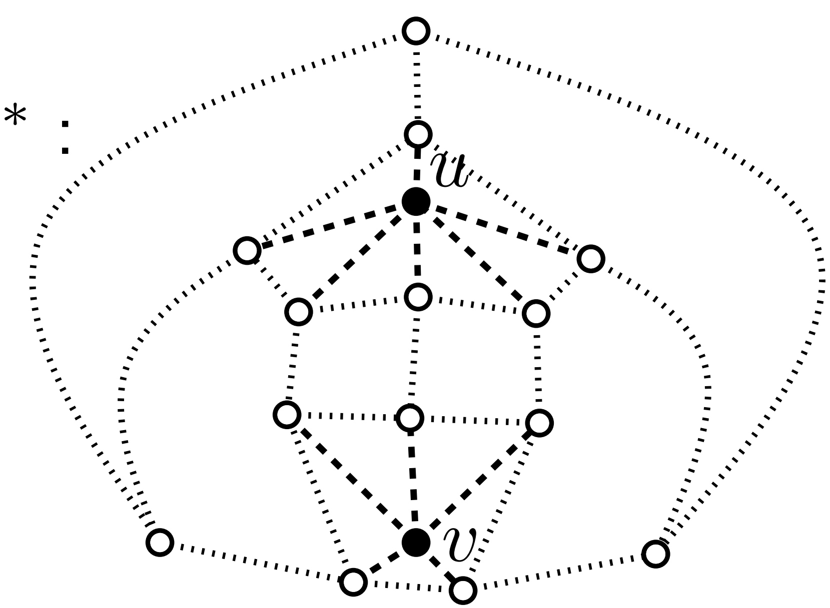

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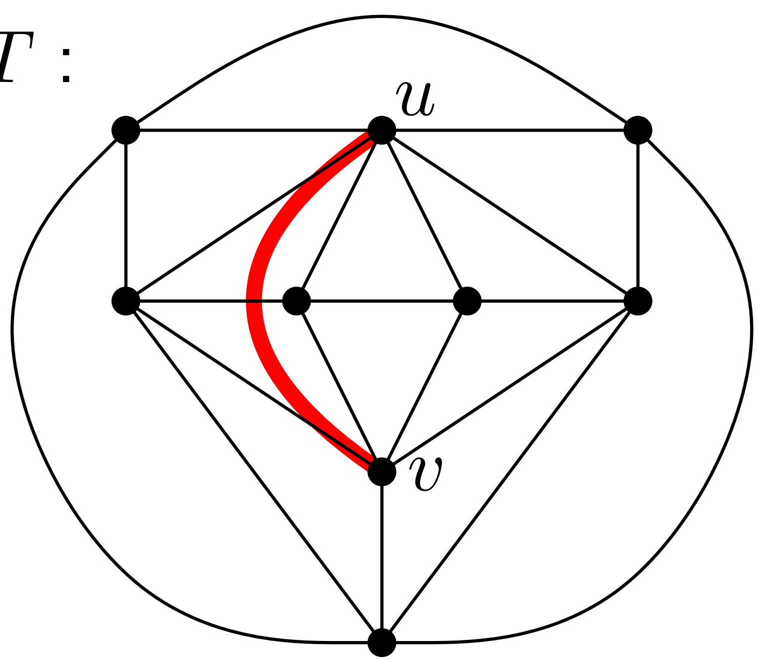
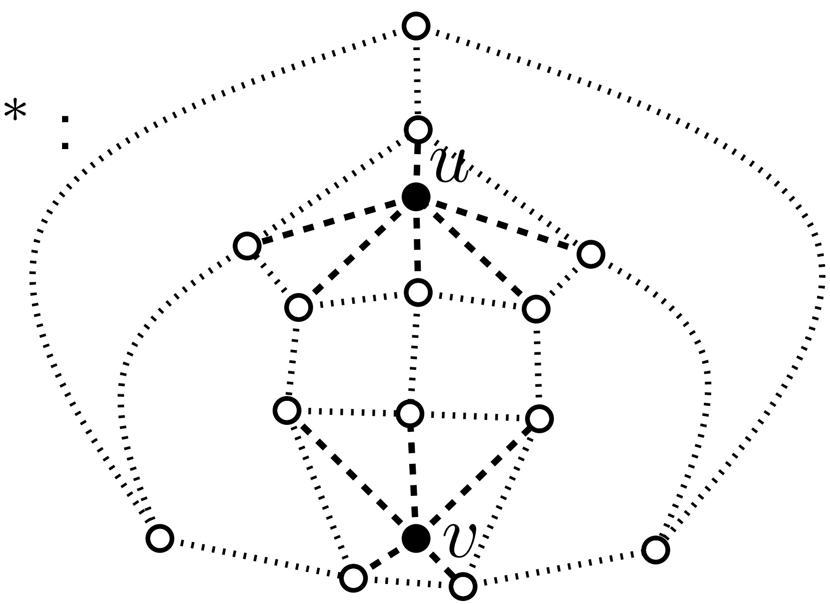


T^* :



$(u, v) \in E_M$

Triangulation + Matching

Given a triconnected plane graph $T = (V, E_T)$
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Task: Find a valid routing for each matching edge!
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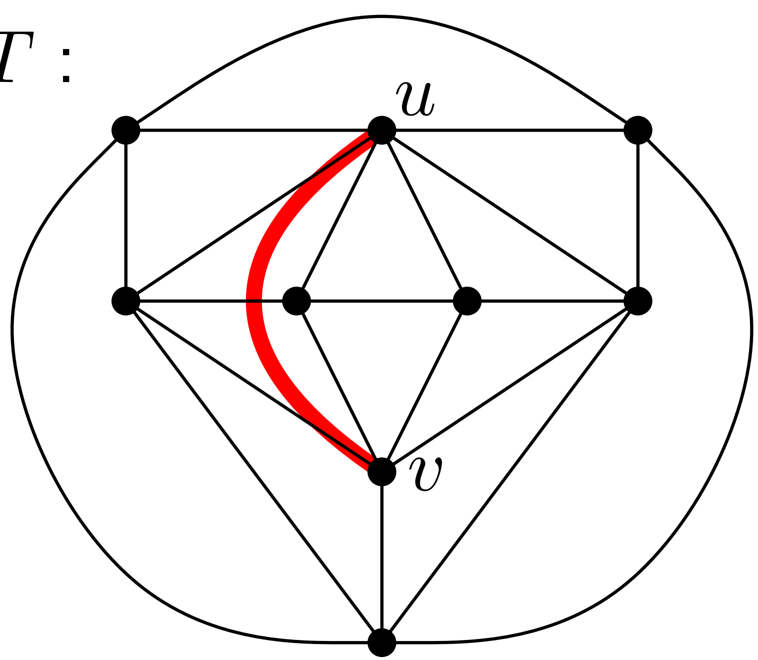

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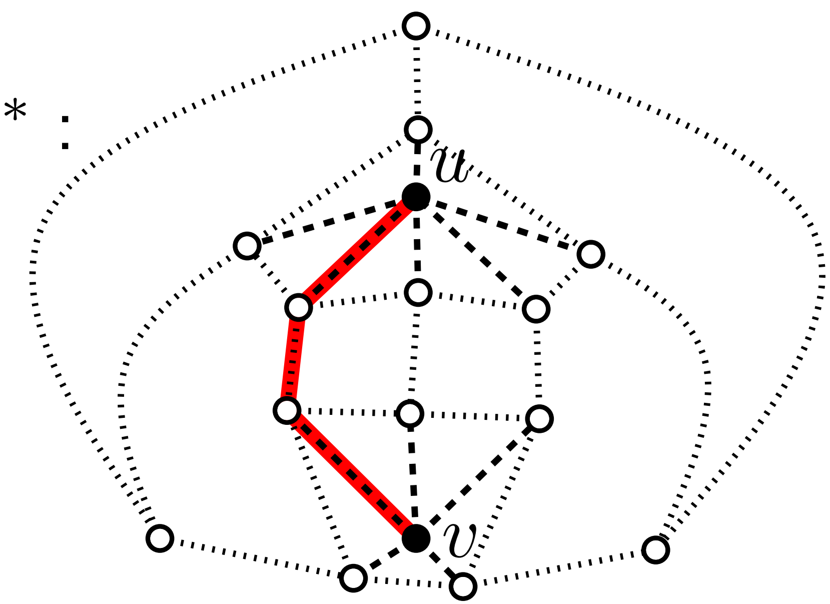

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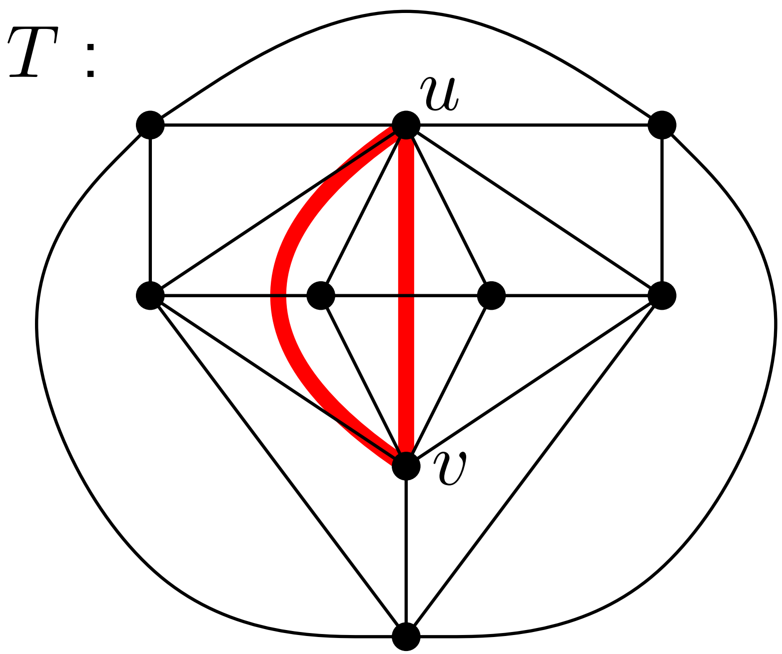

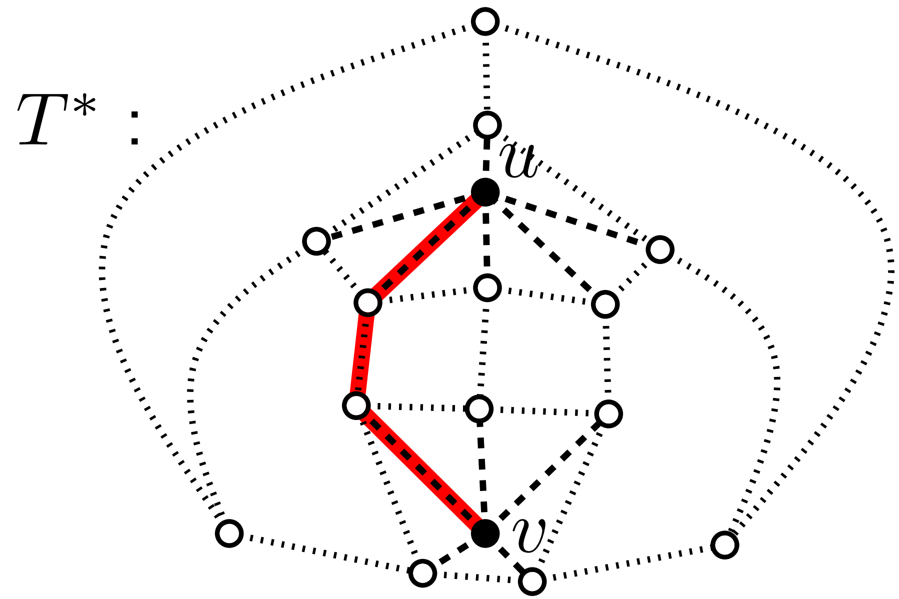

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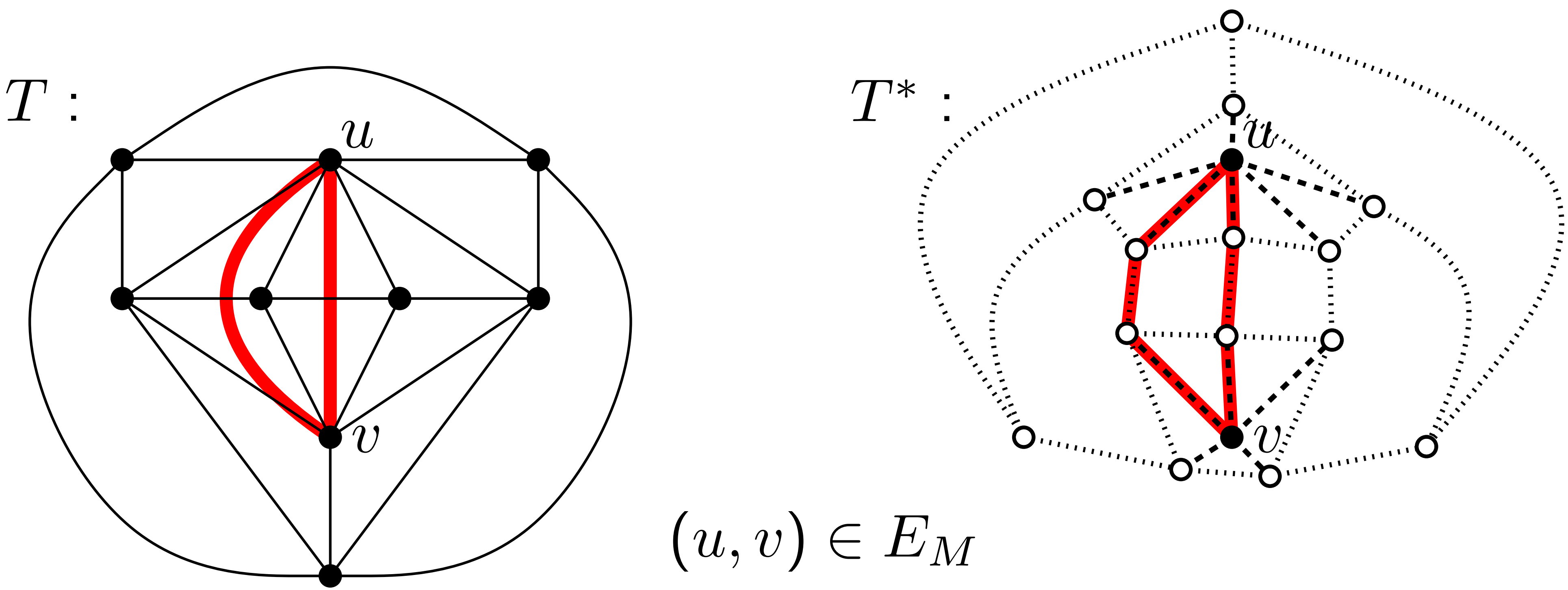

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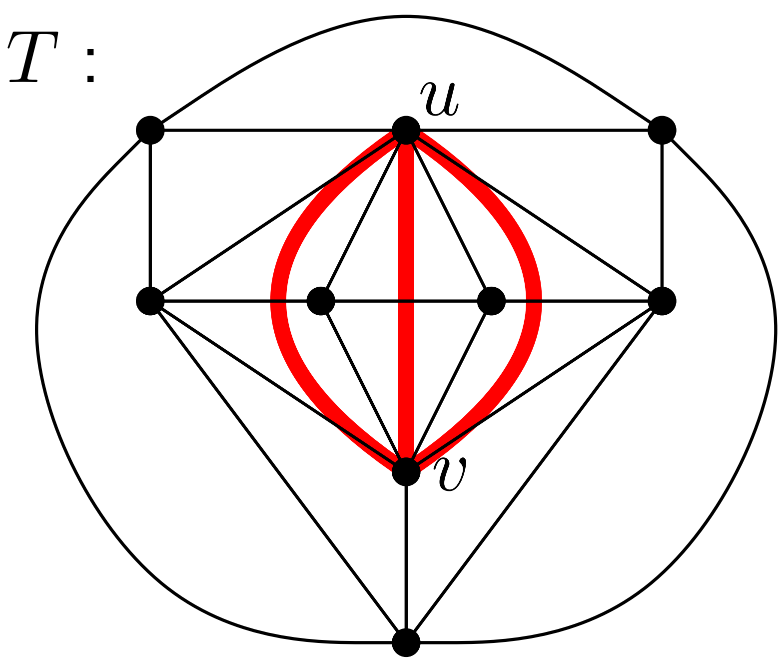

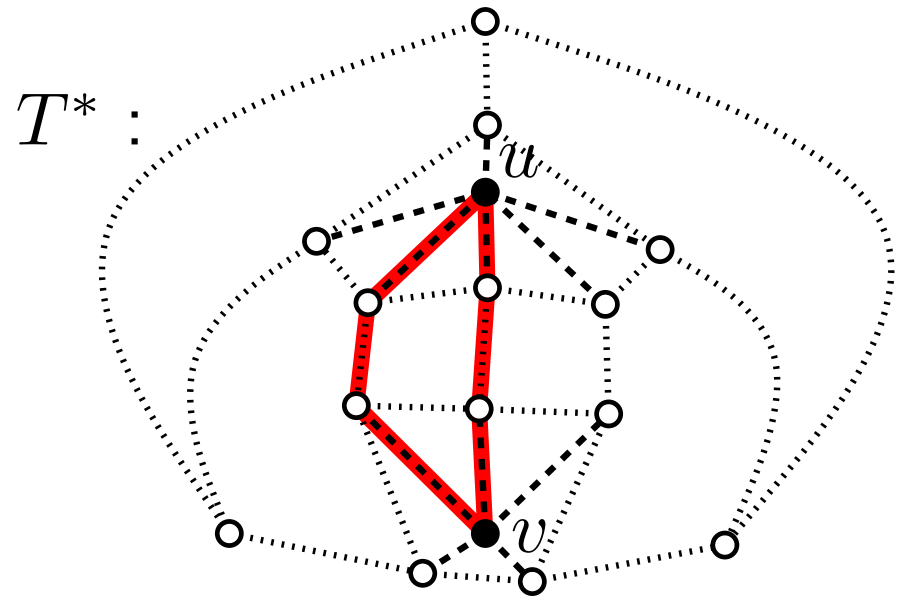

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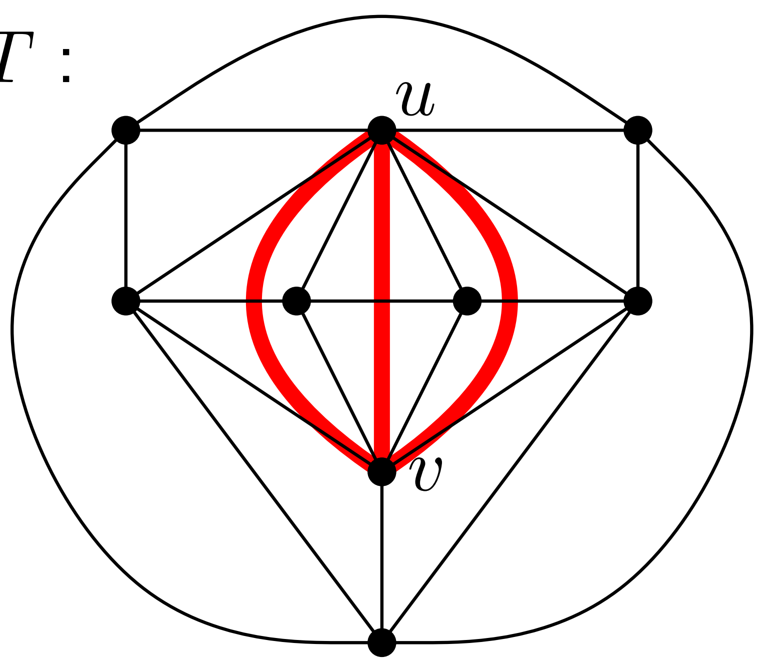

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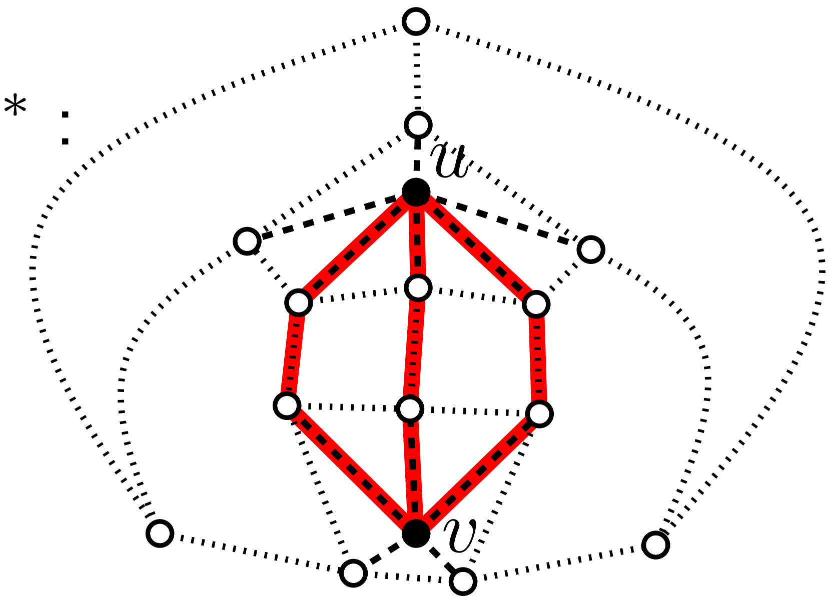

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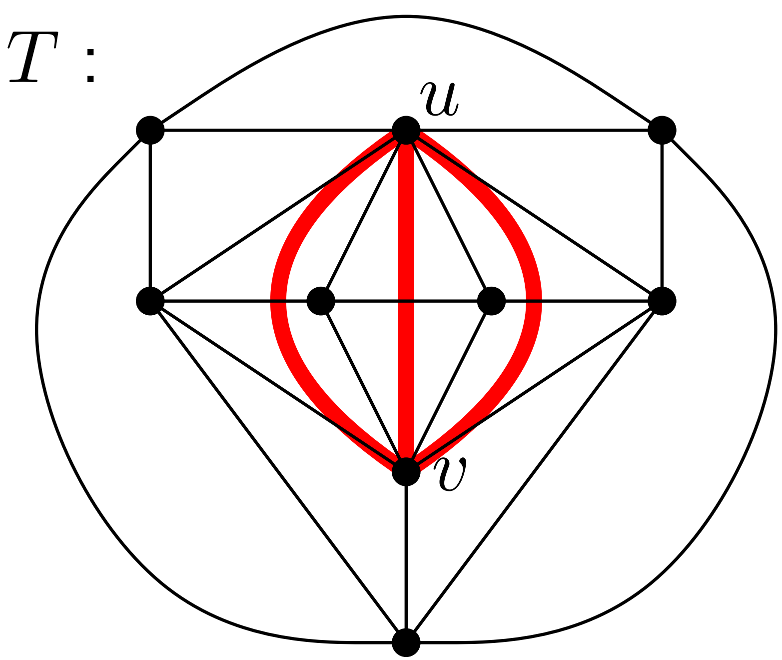
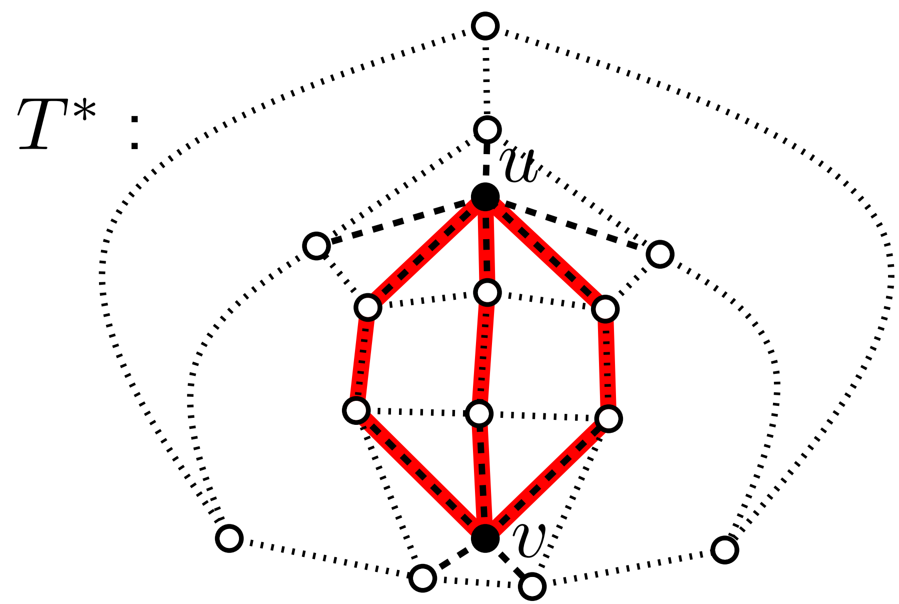
Task: Find a valid routing for each matching edge!
Compute extended dual T^* of T.
Routing in $T \triangleq$ path of length 3 in T^*





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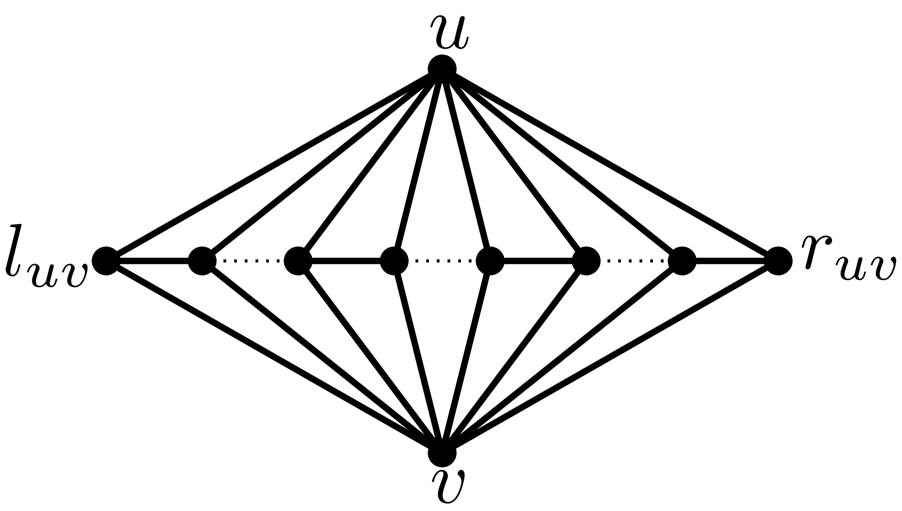

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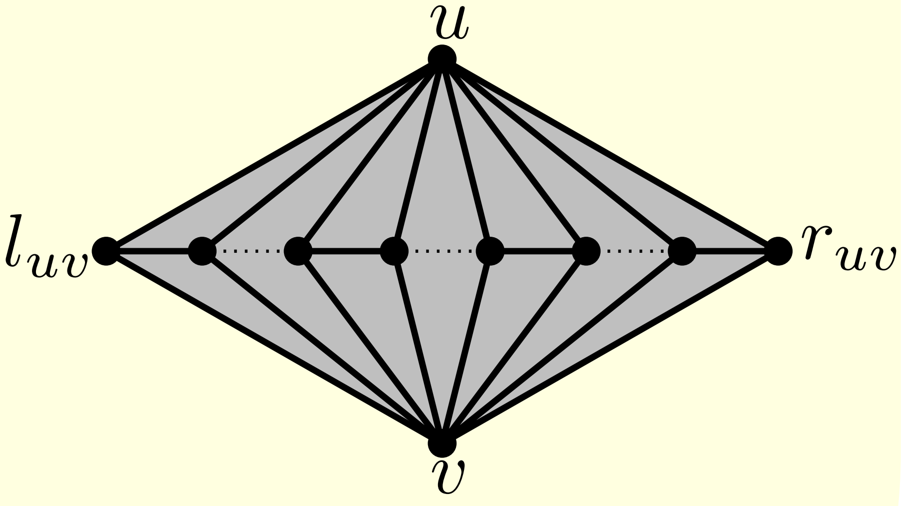
Interior $\mathcal{I}(u, v)$



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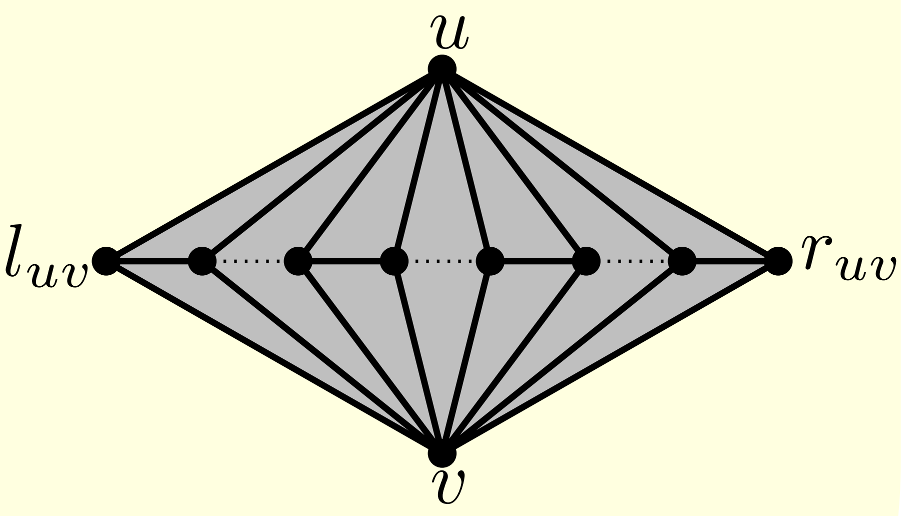

The boundaries of two interiors may not intersect.

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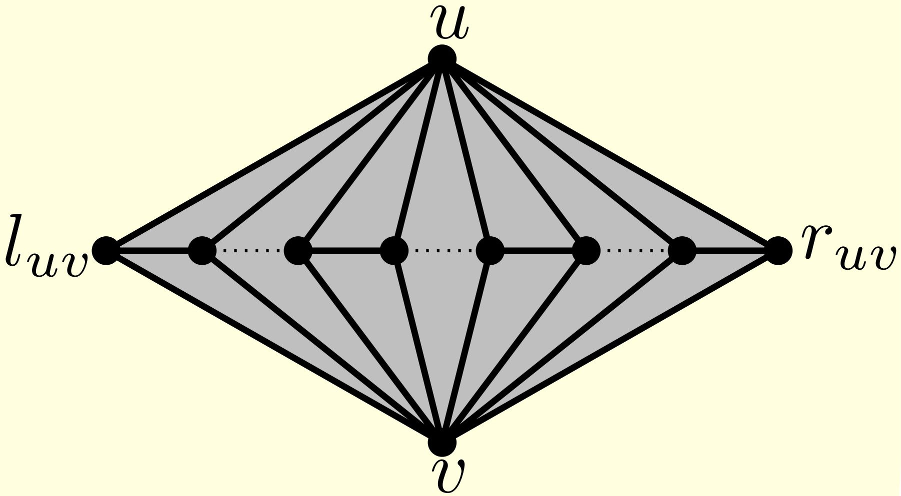

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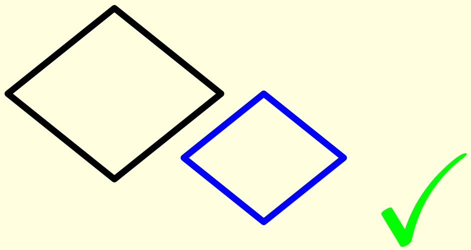

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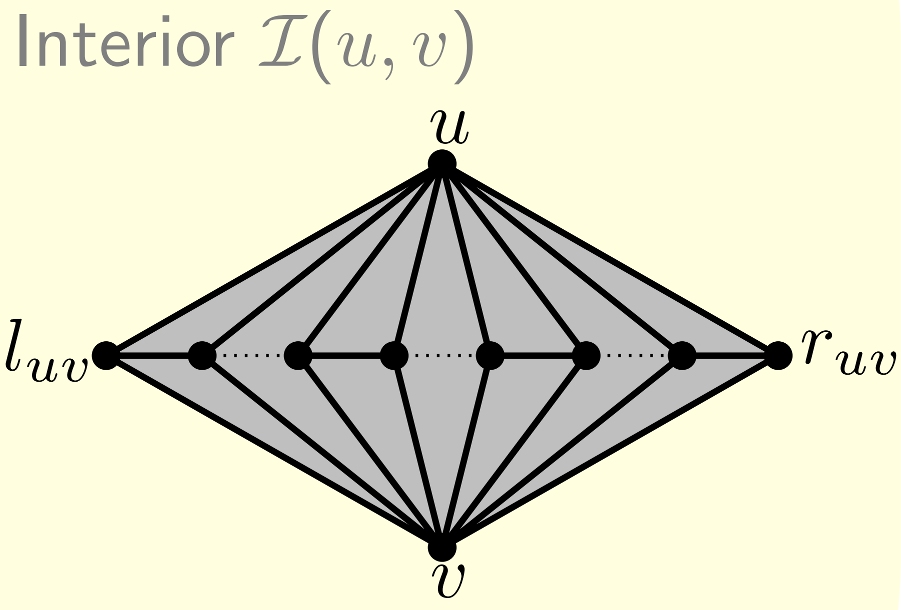

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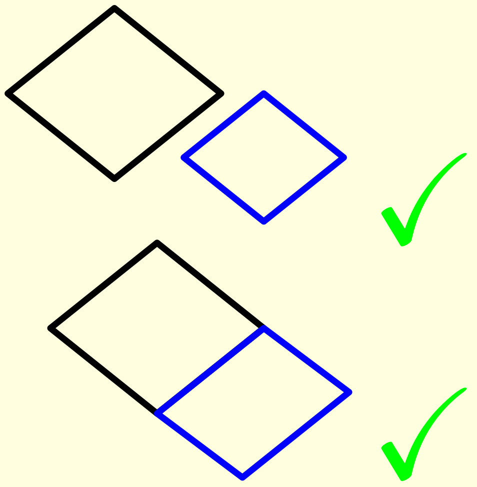

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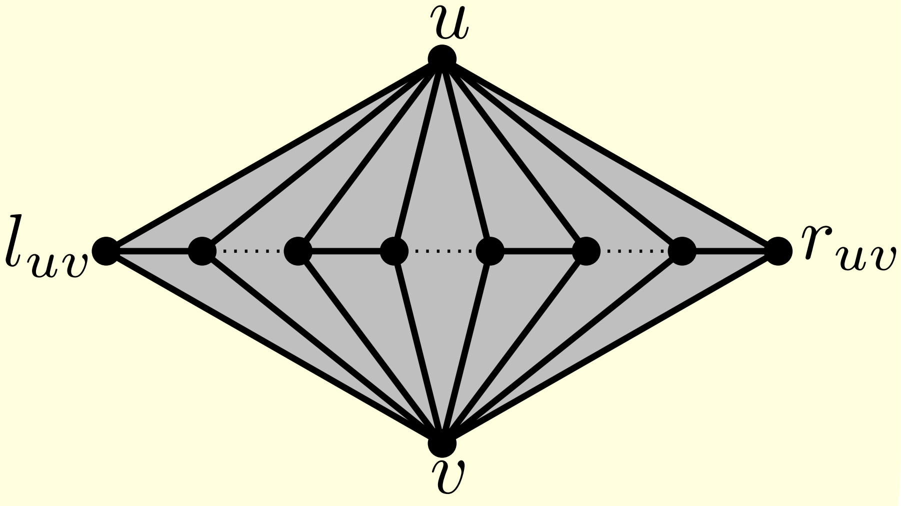

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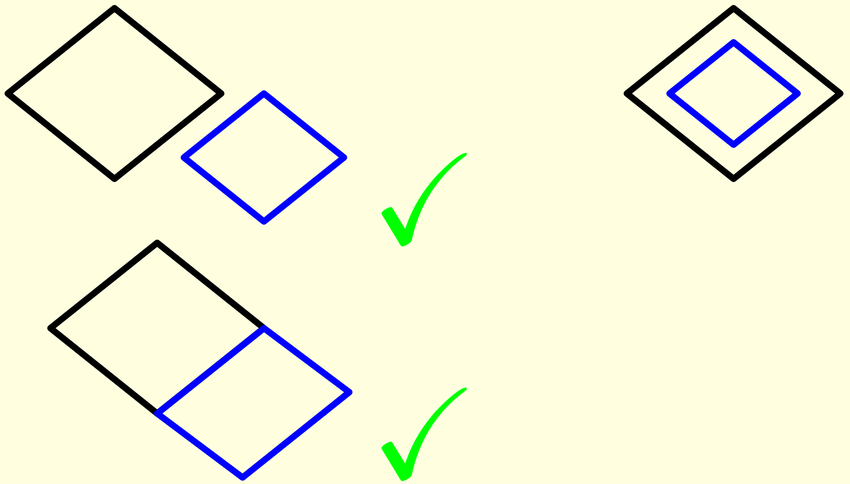

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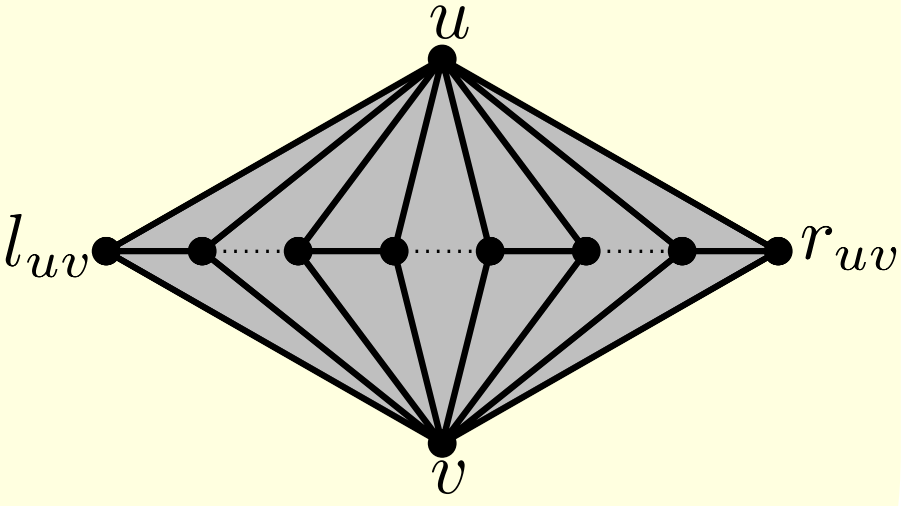

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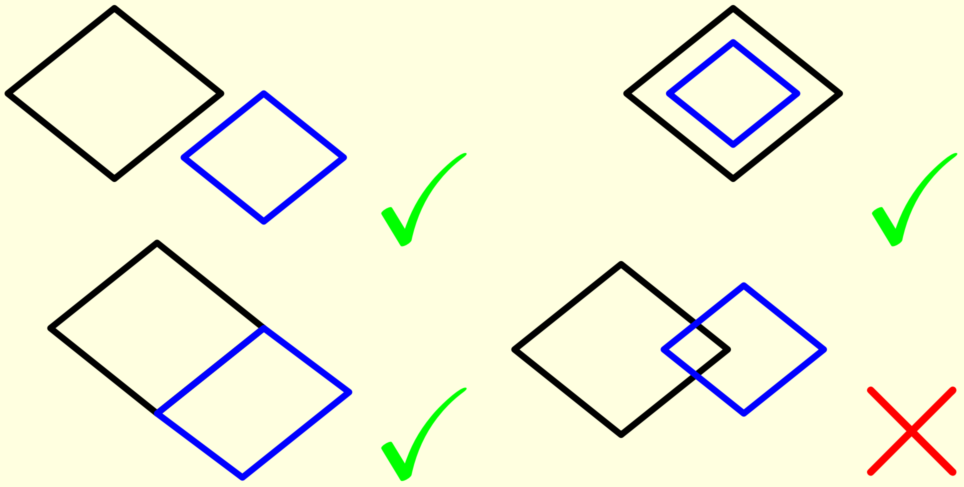

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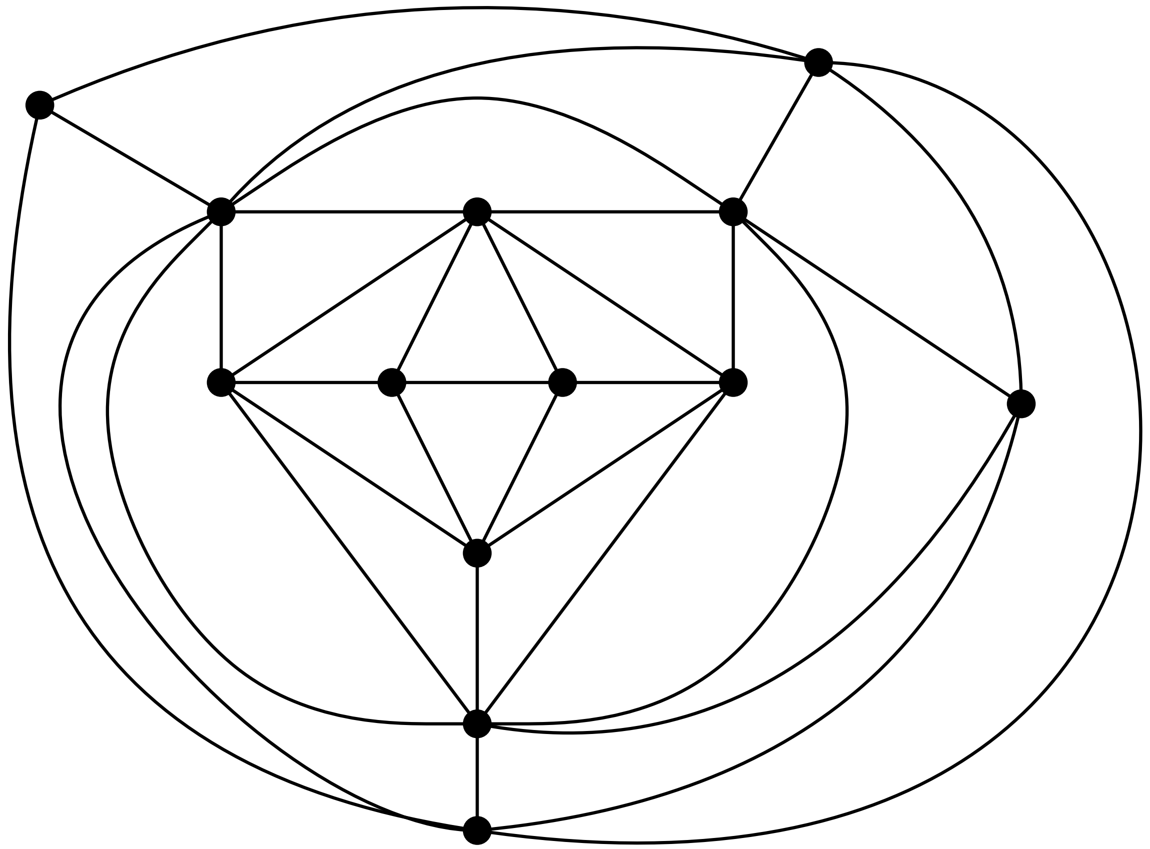

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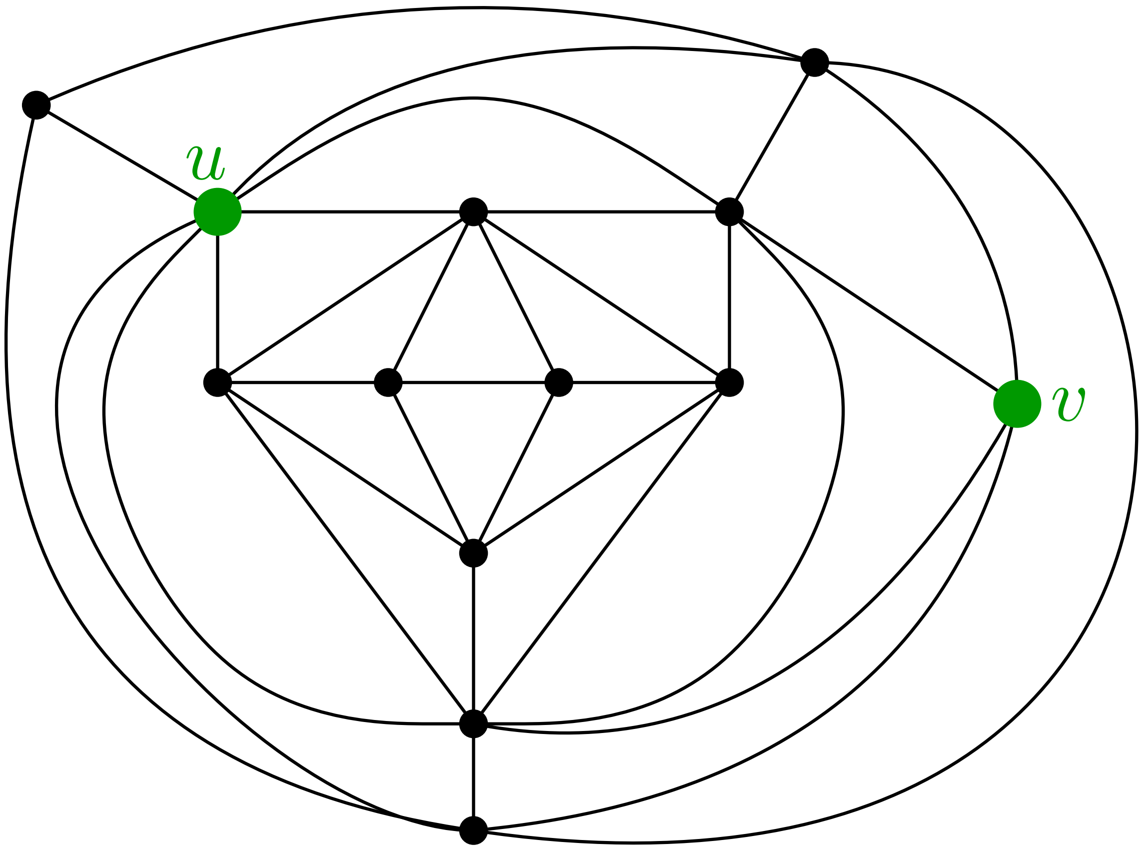

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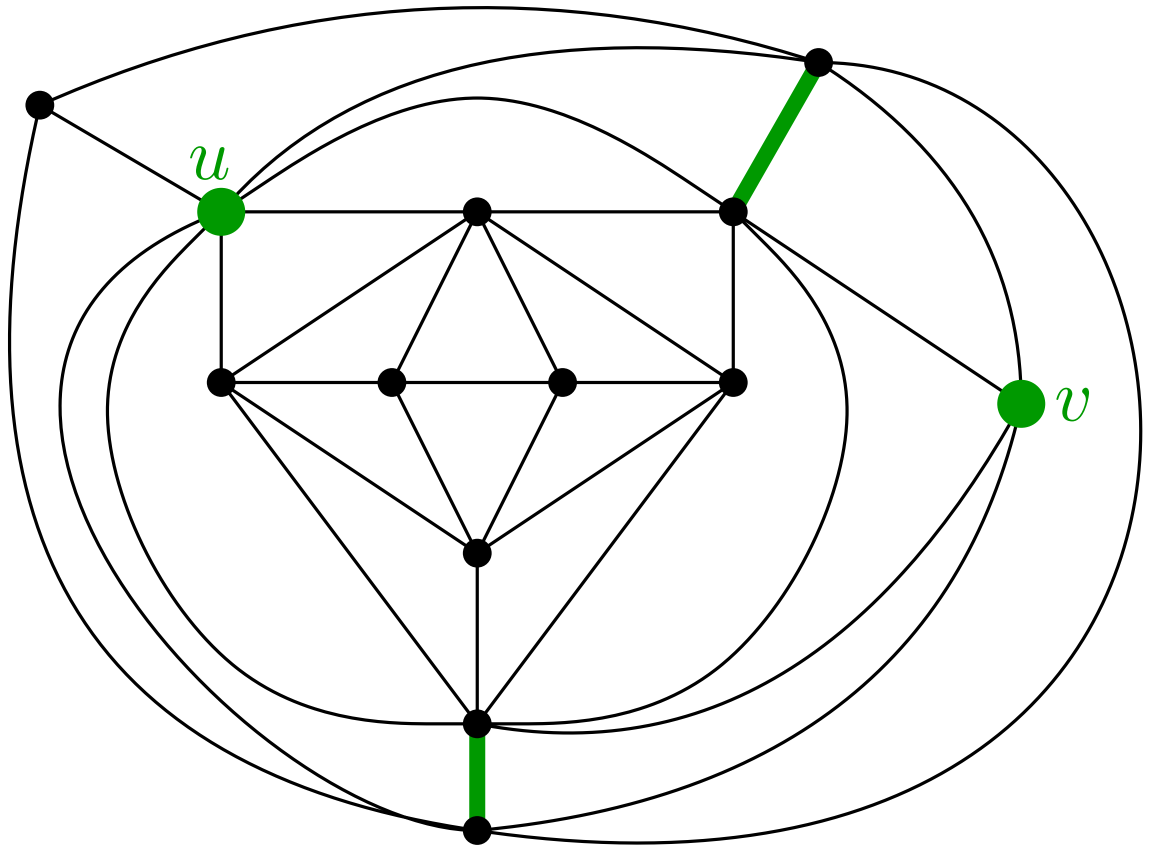

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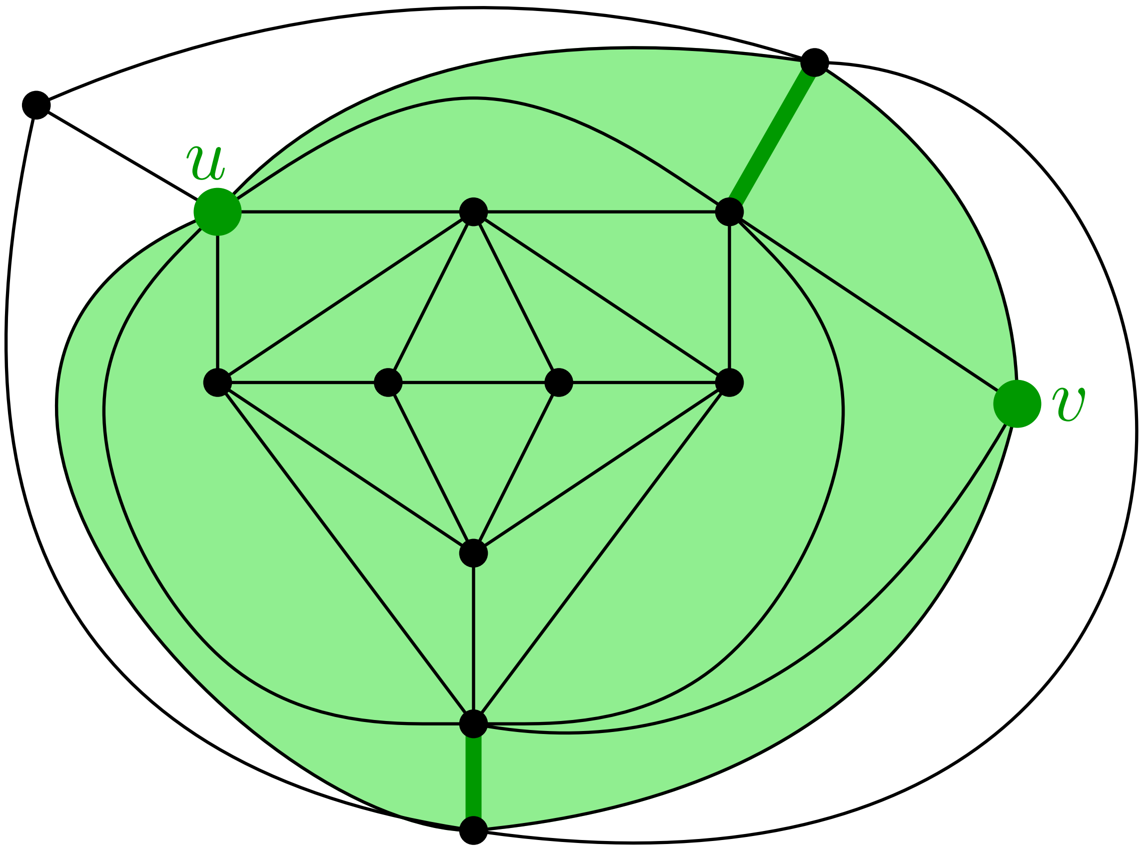

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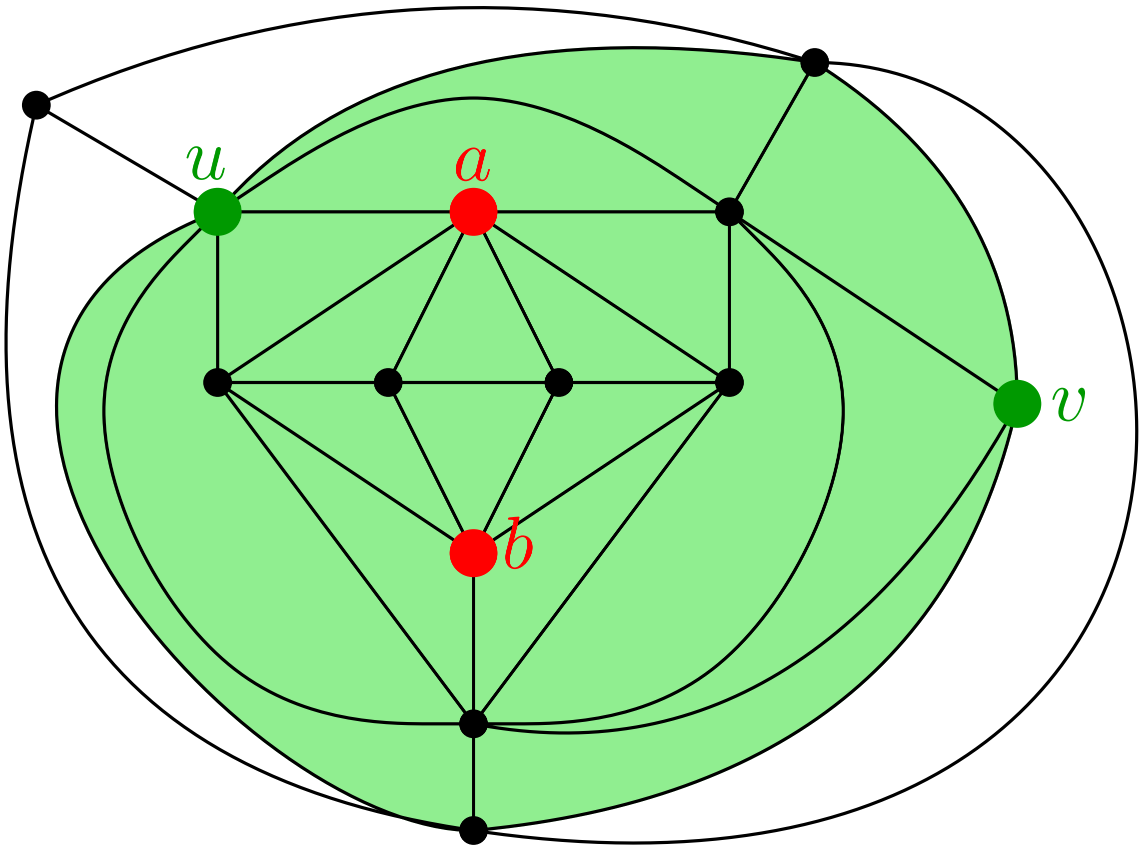

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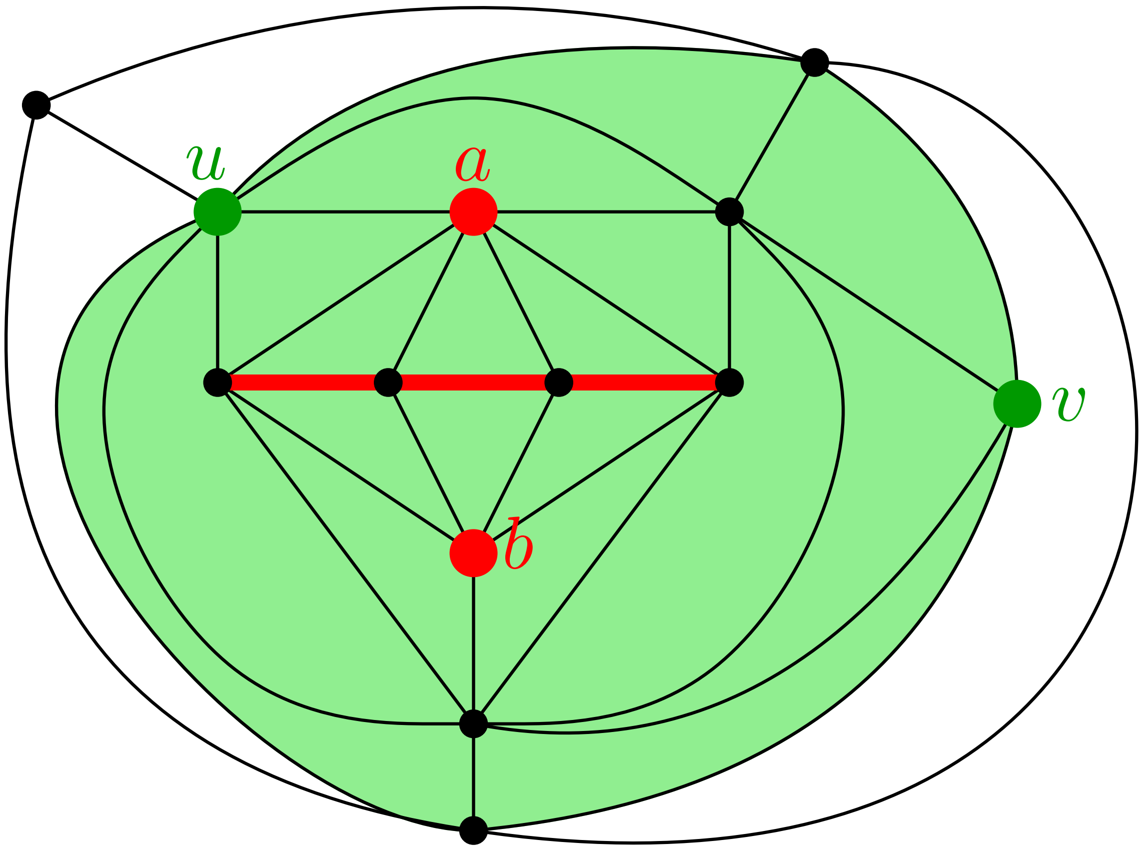

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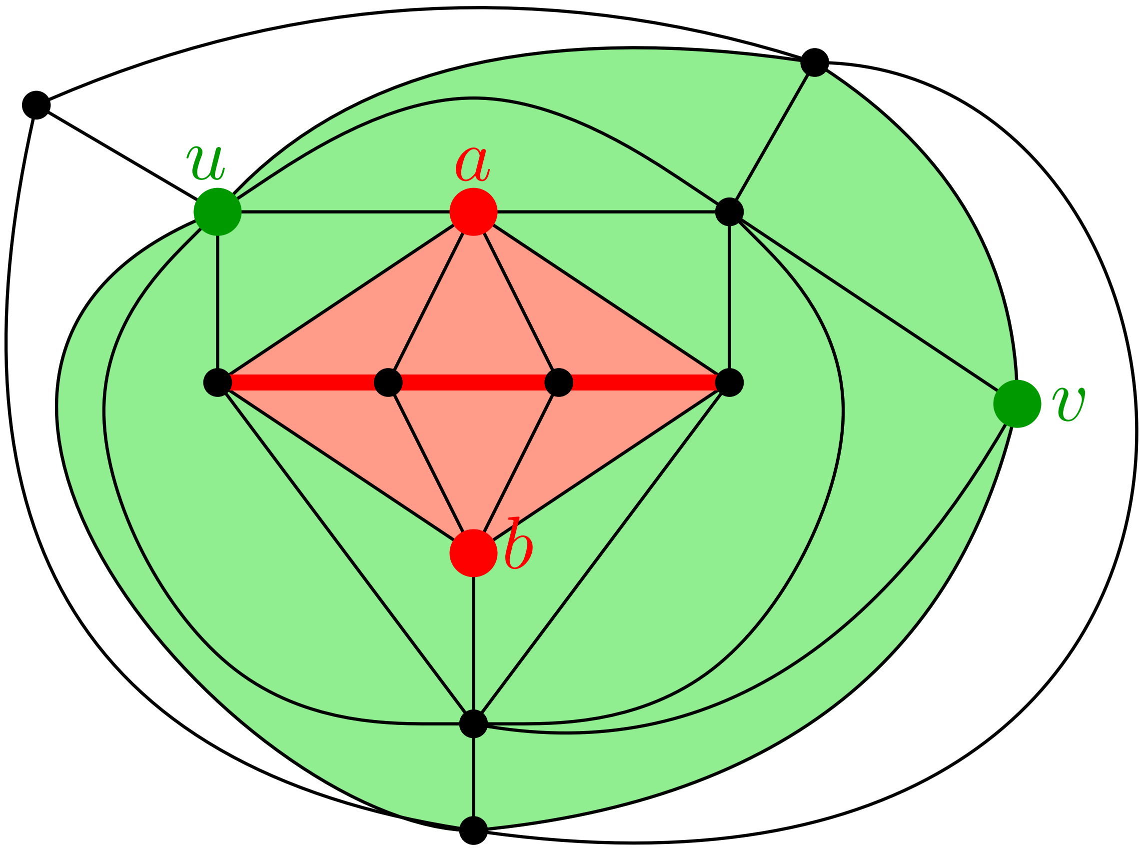

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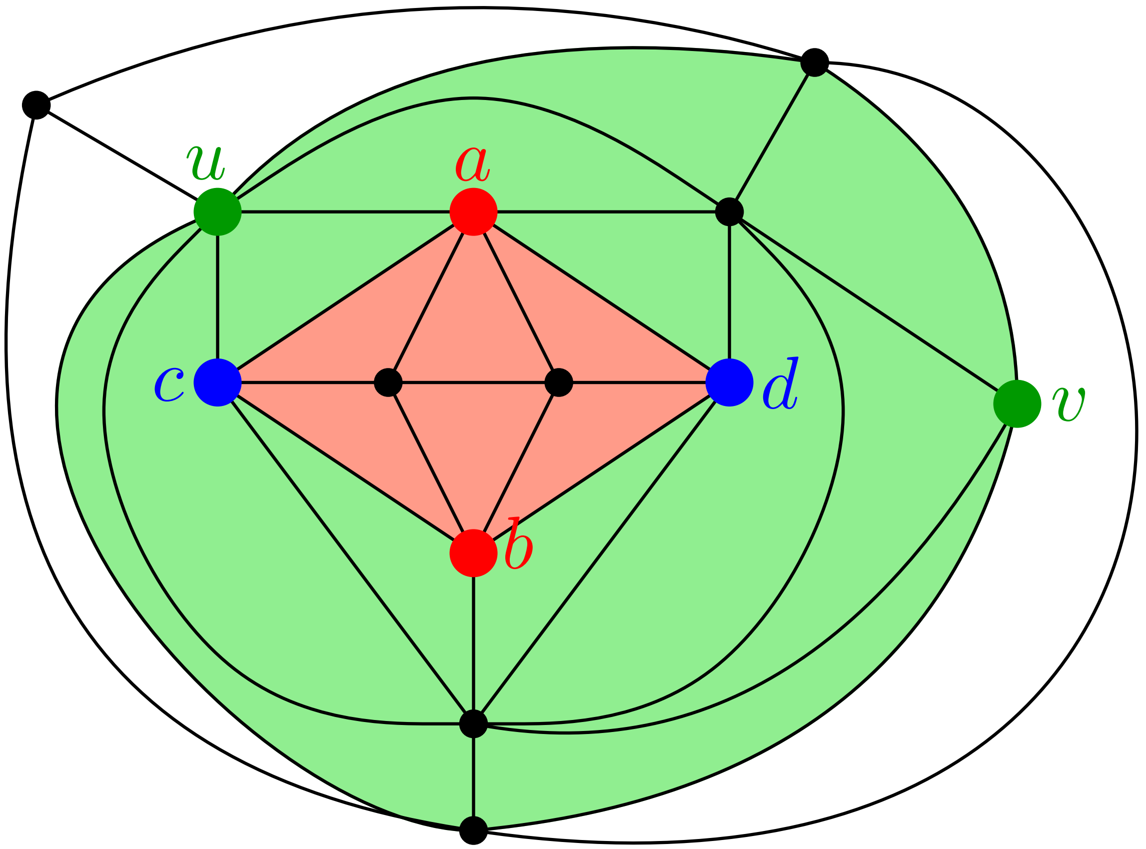

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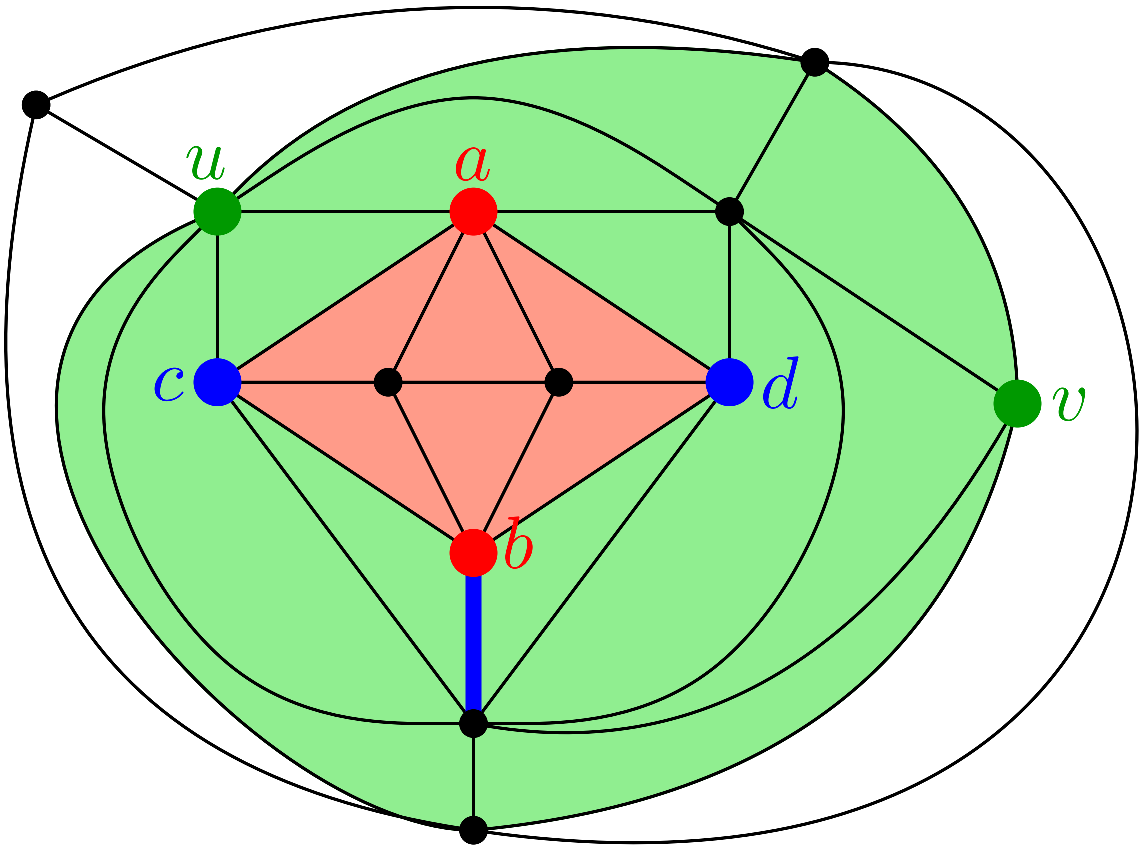

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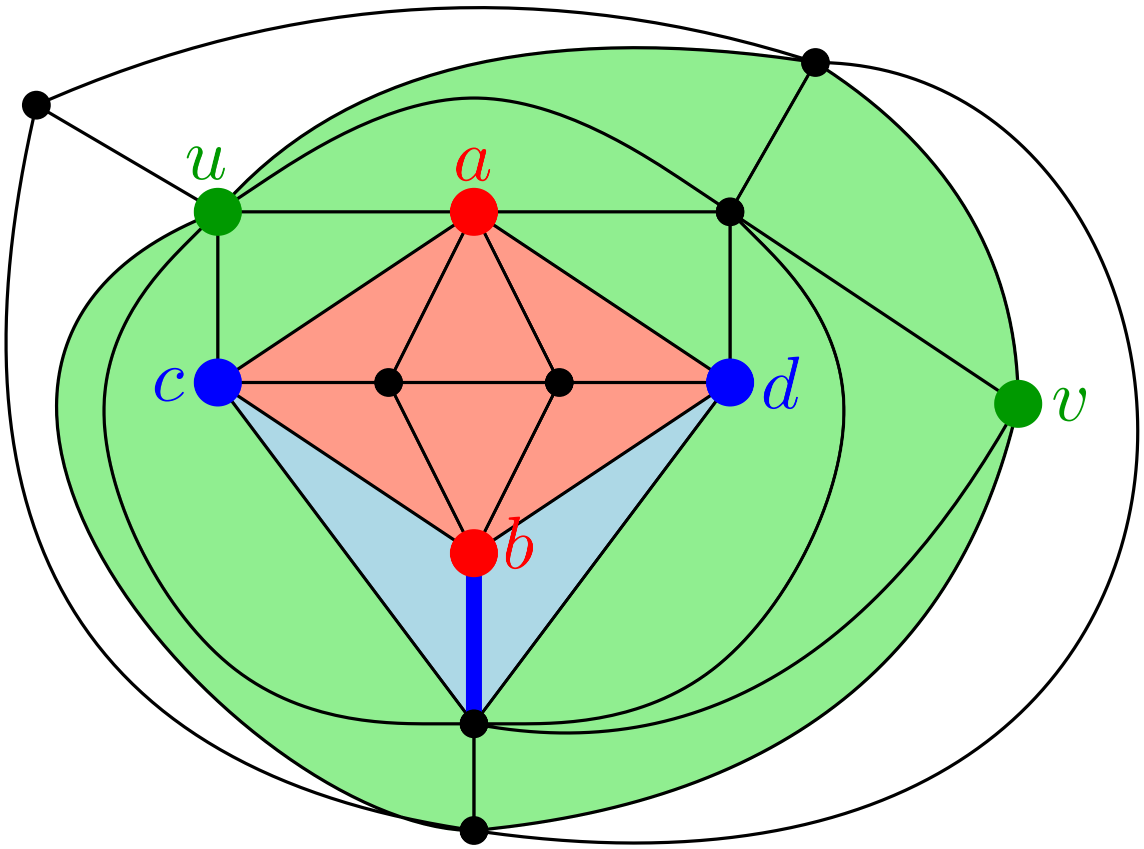

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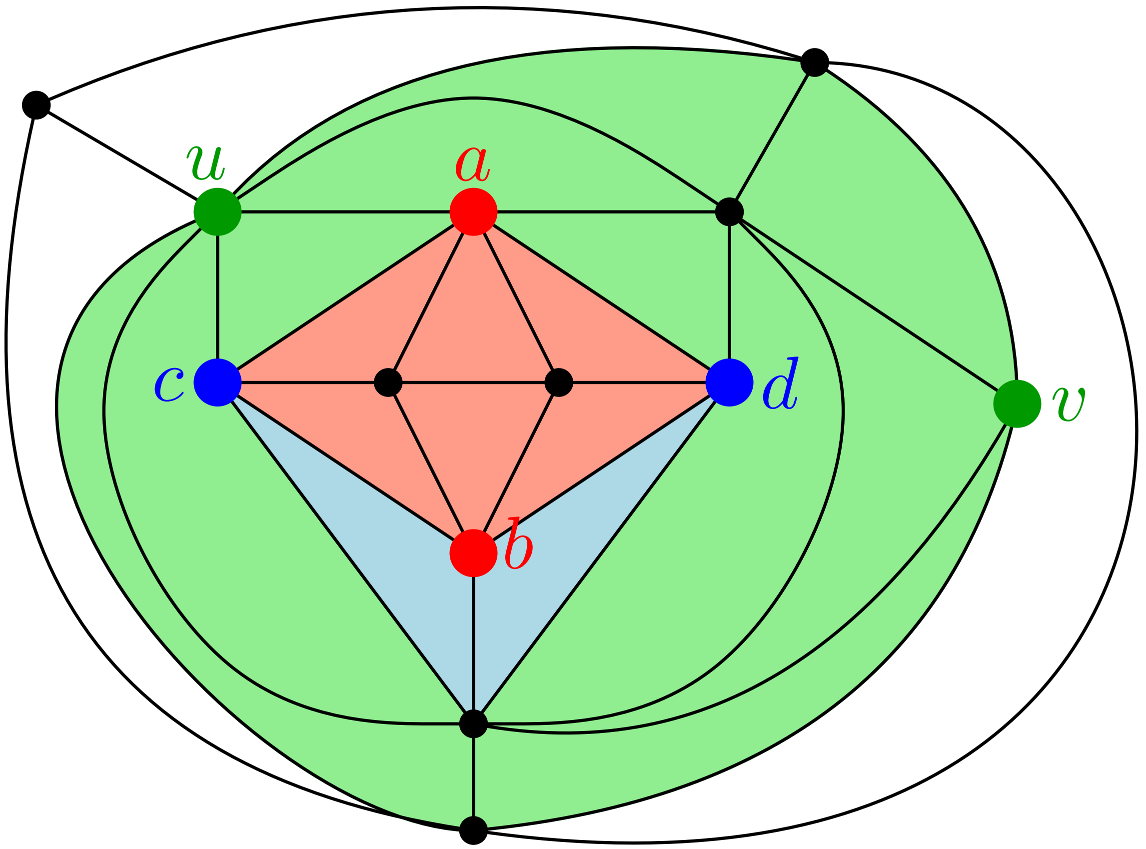
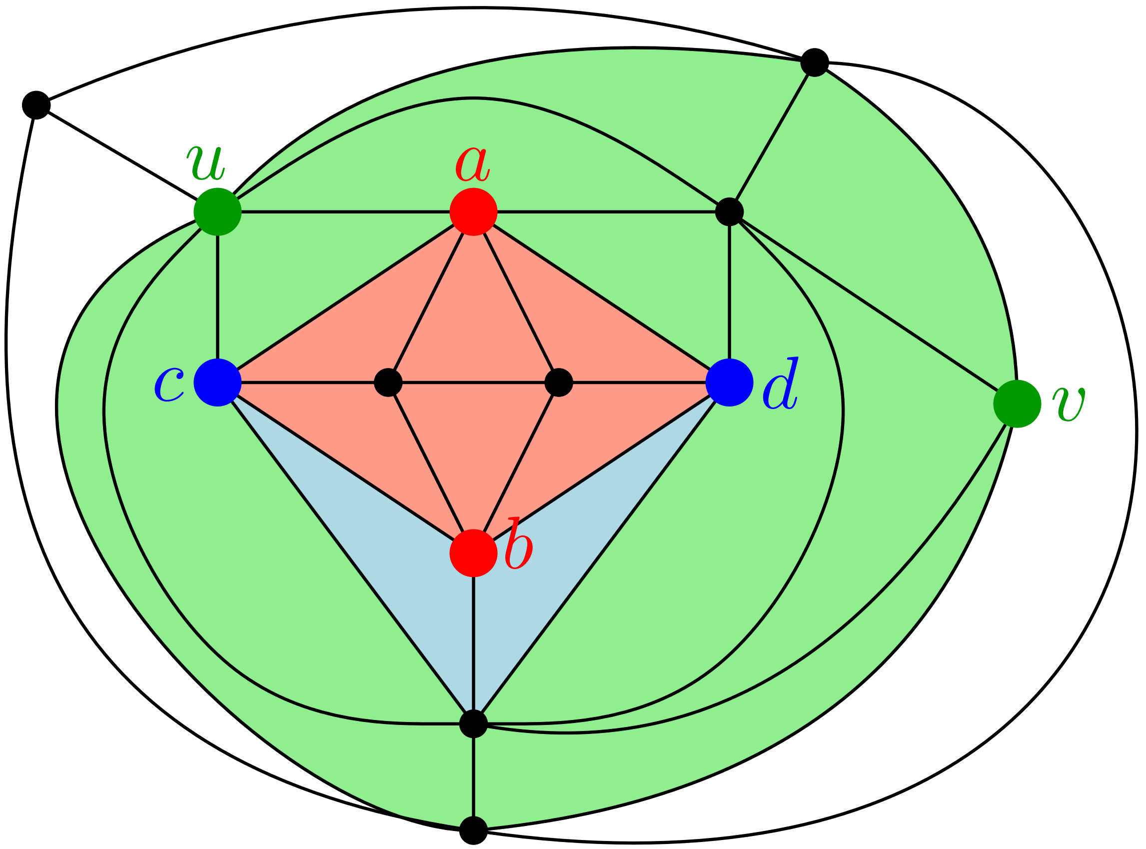


H:

Hierarchical structure: Tree $H = (V_H, E_H)$

Triangulation + Matching



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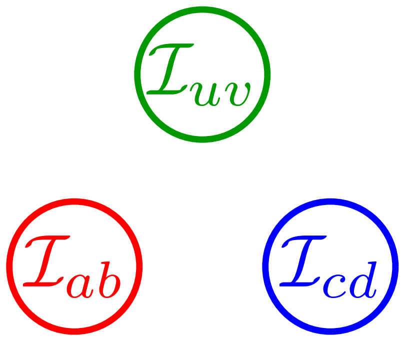

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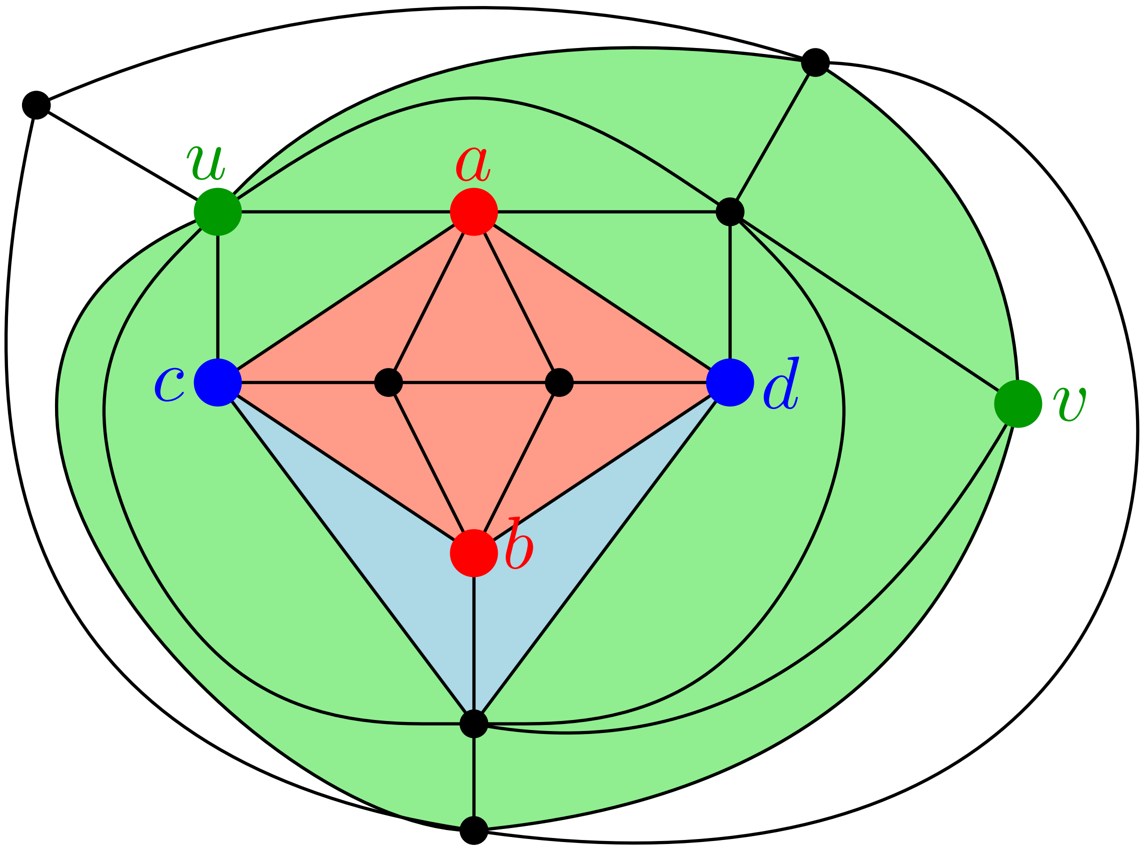
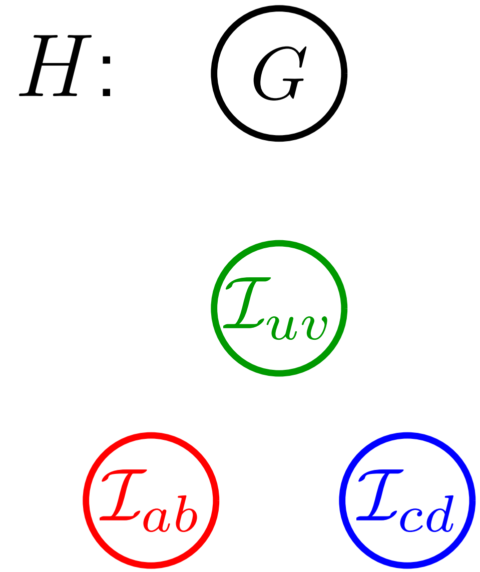
Triangulation + Matching





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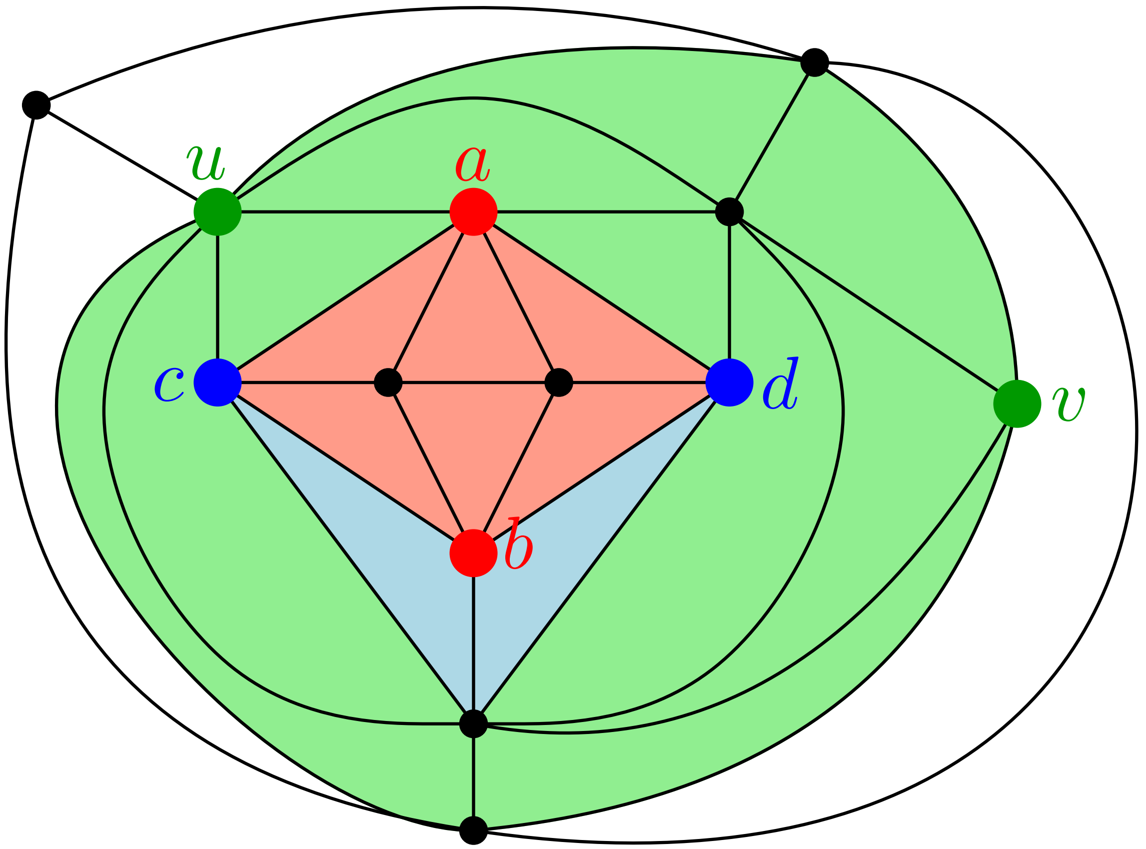
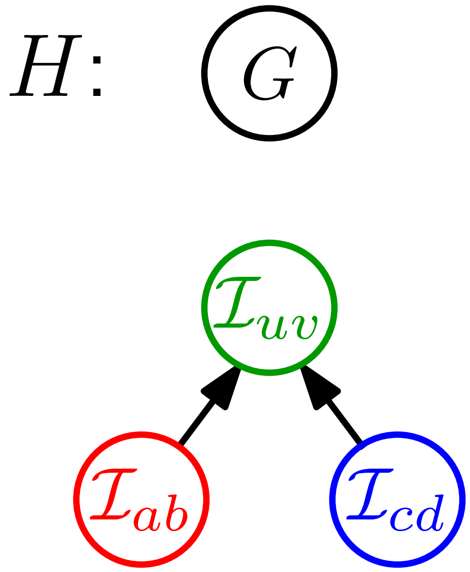
Triangulation + Matching





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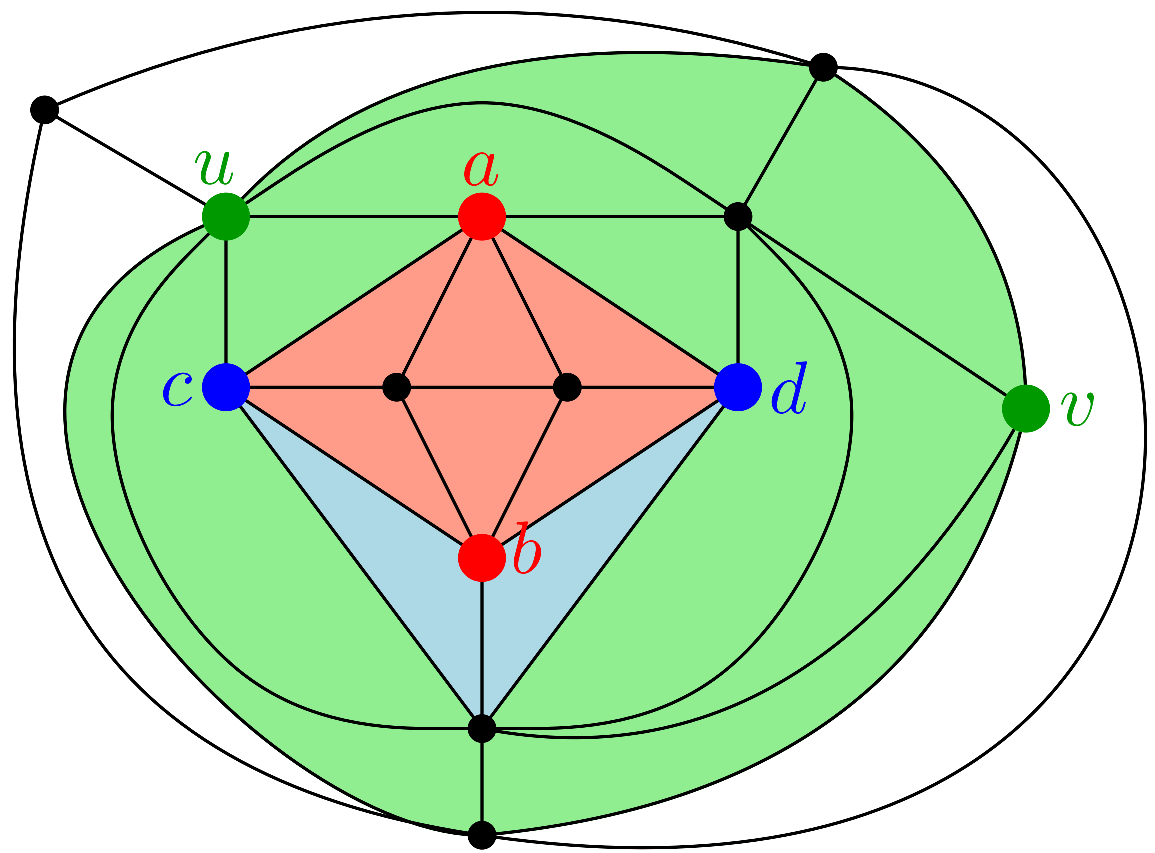
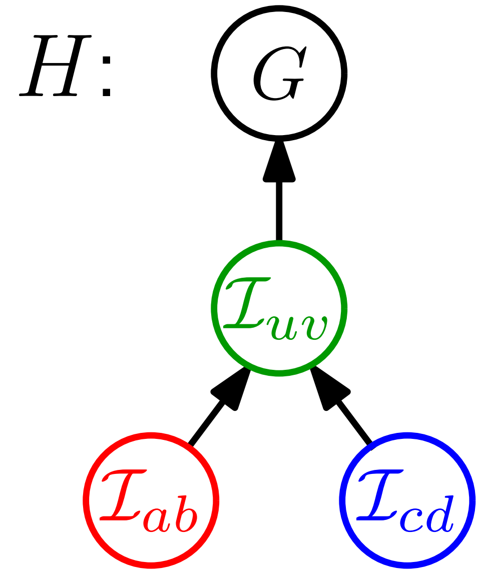
$(\mathcal{I}_{uv}, \mathcal{I}_{ab}) \in E_H \Leftrightarrow \mathcal{I}_{uv} \subset \mathcal{I}_{ab}$

Triangulation + Matching





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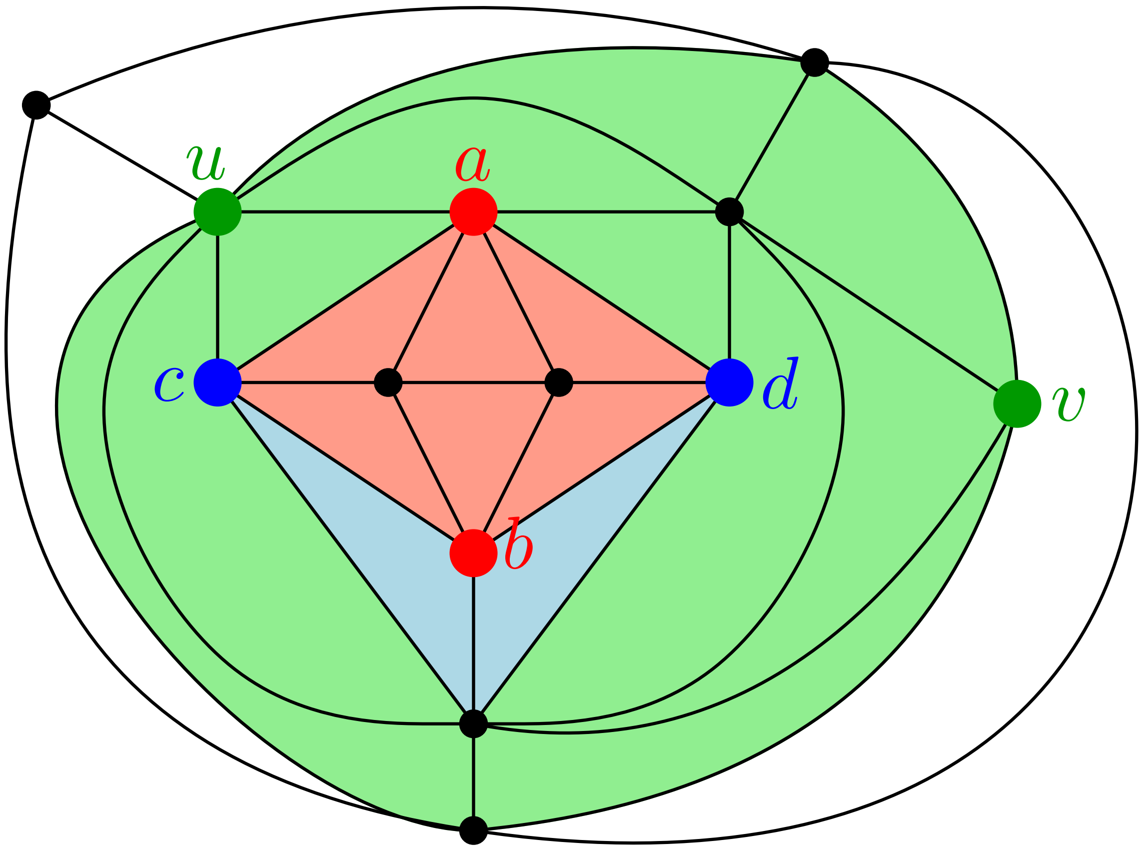
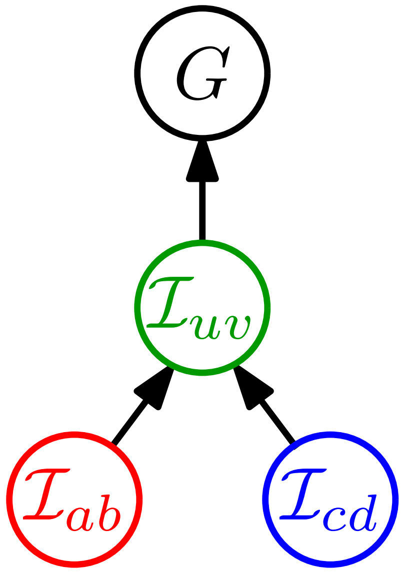
$\text{outdeg}(\mathcal{I}_{uv}) = 0 \Rightarrow (\mathcal{I}_{uv}, G) \in E_H$

Triangulation + Matching





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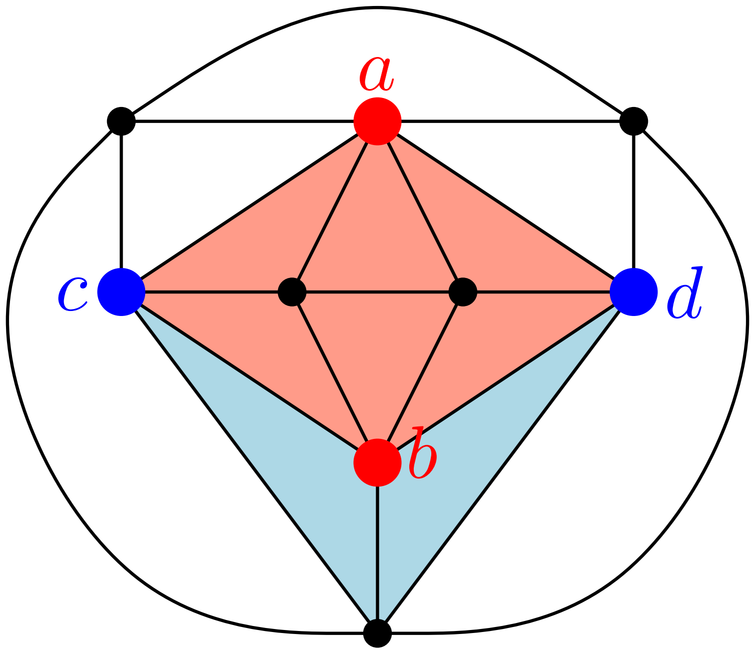

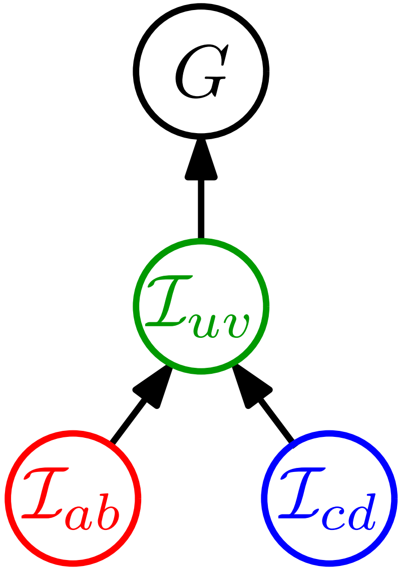

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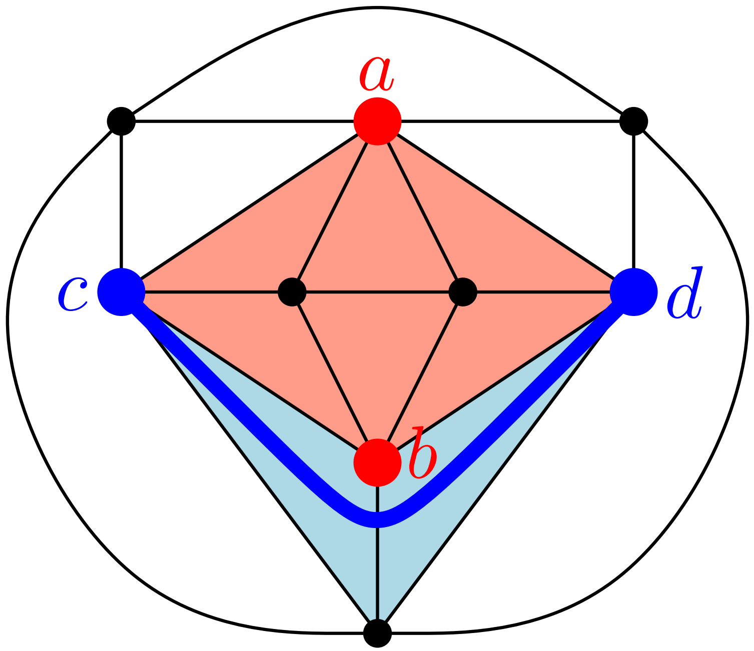

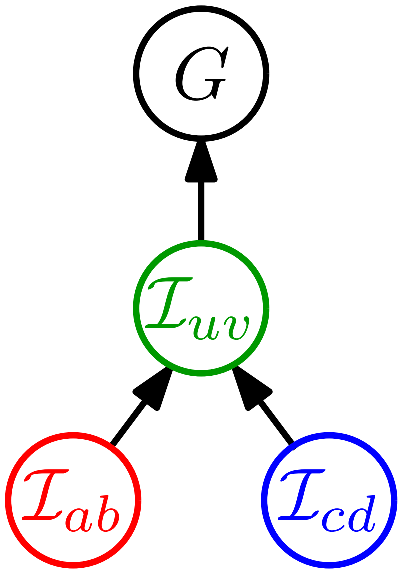

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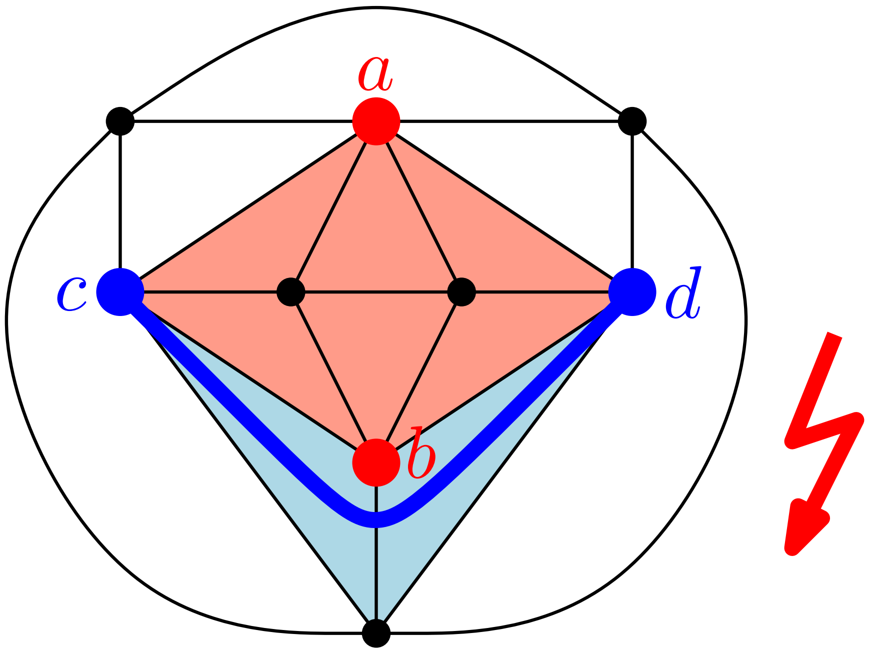

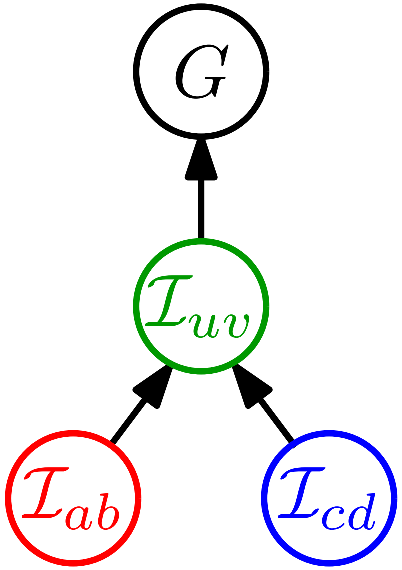

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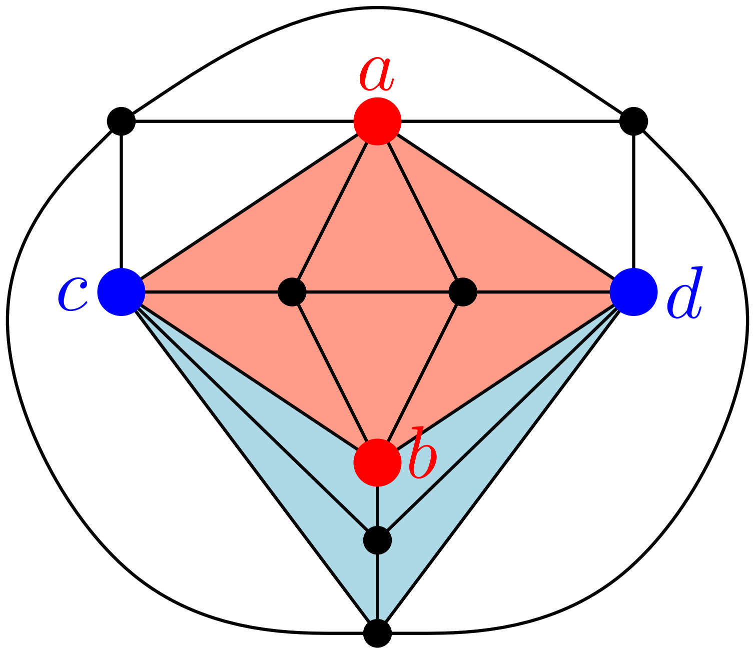

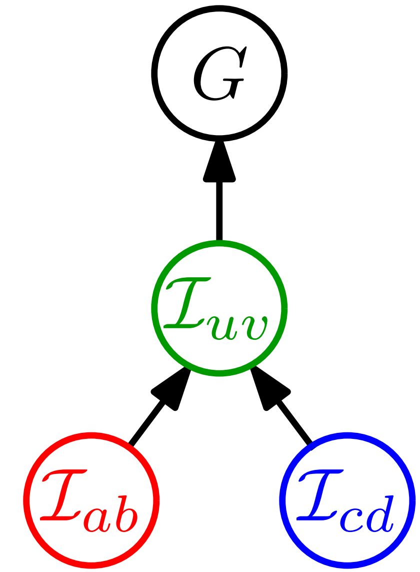

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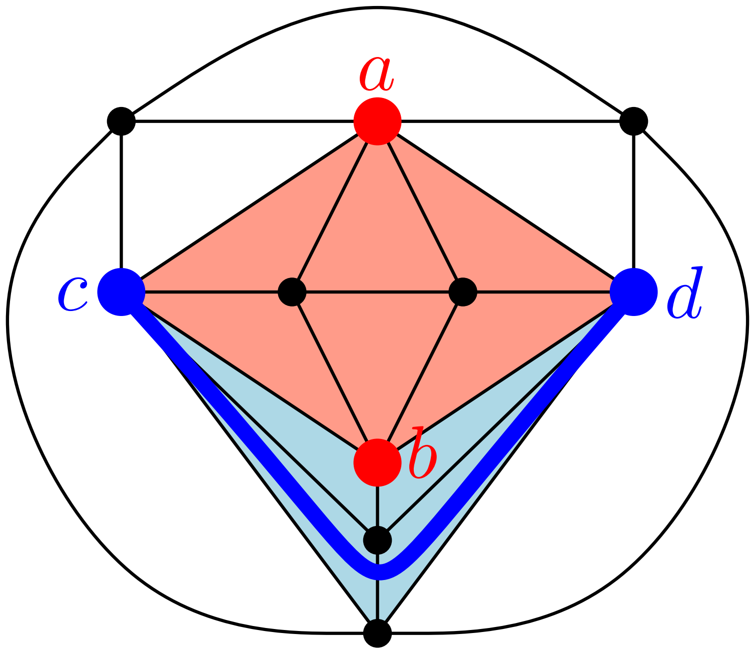

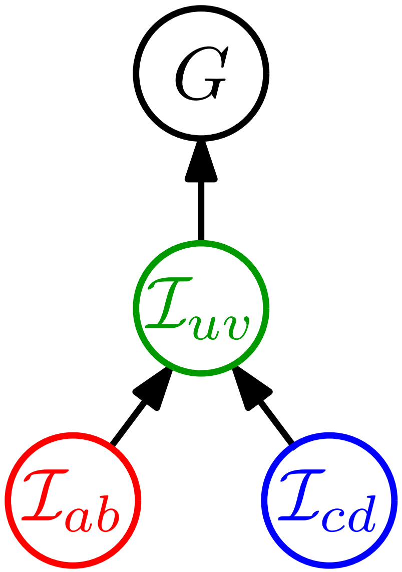

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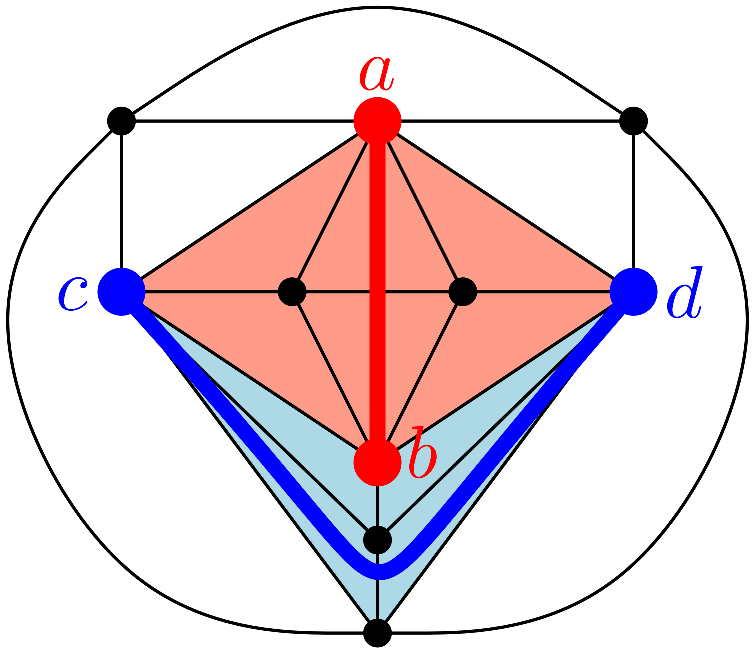

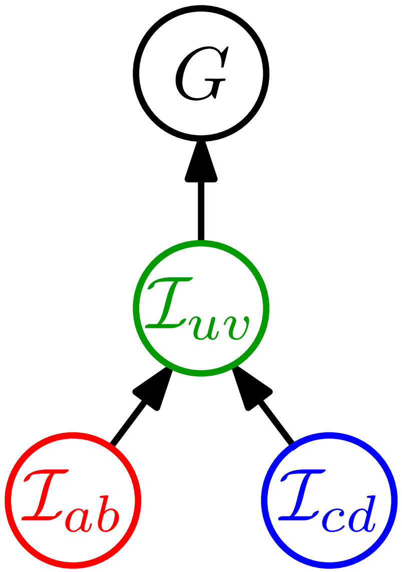

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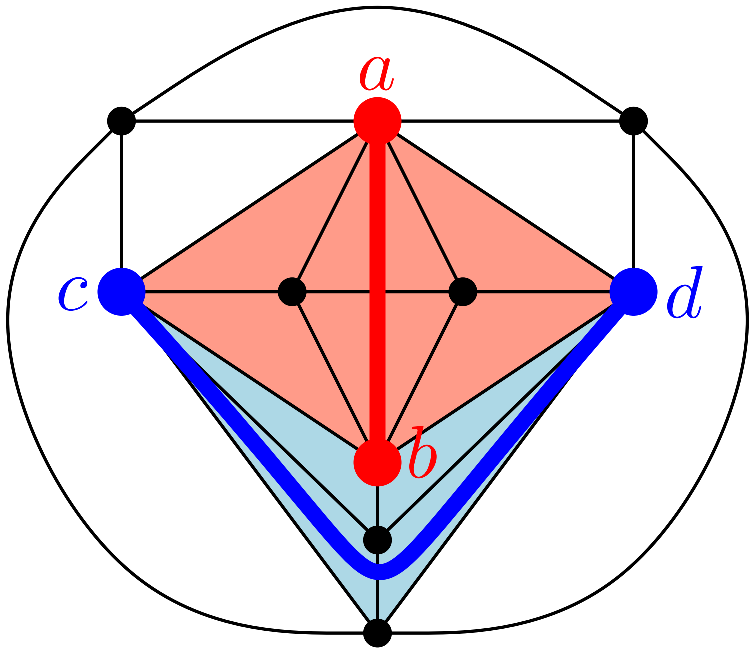
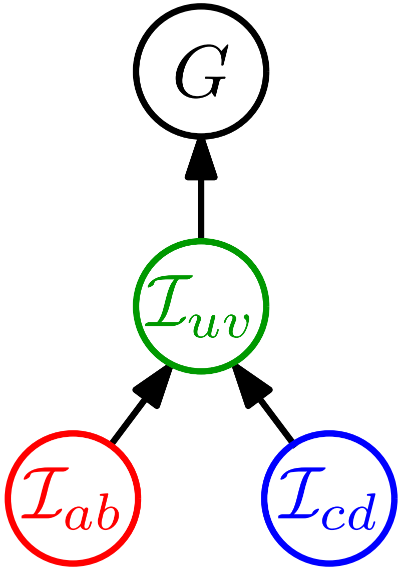
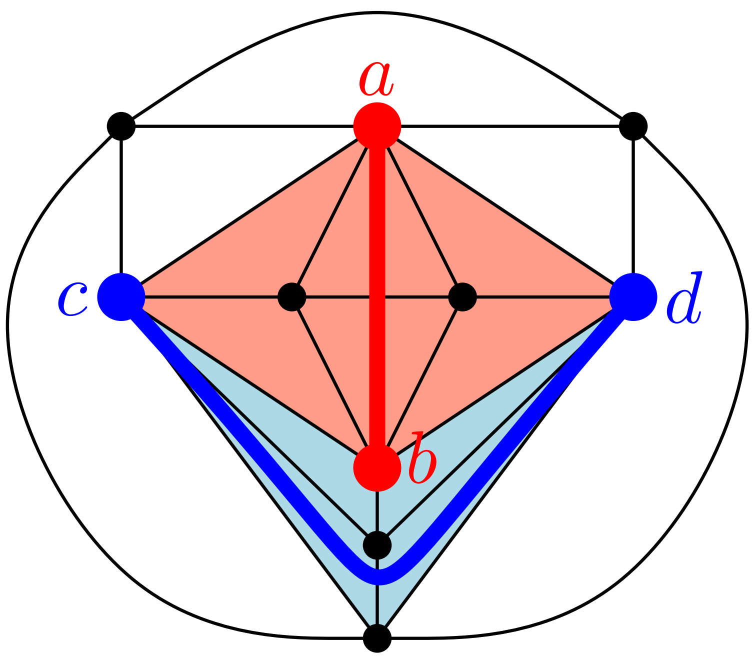
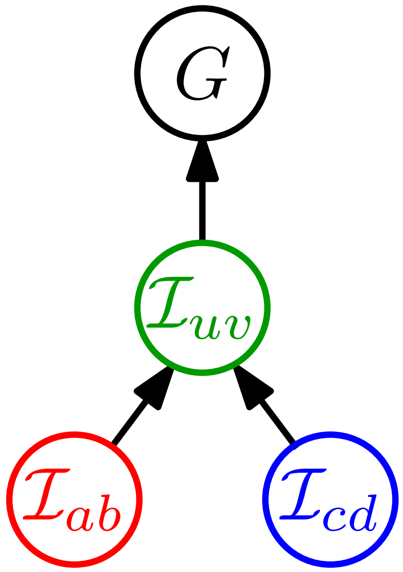




- Always pick “middle” routing

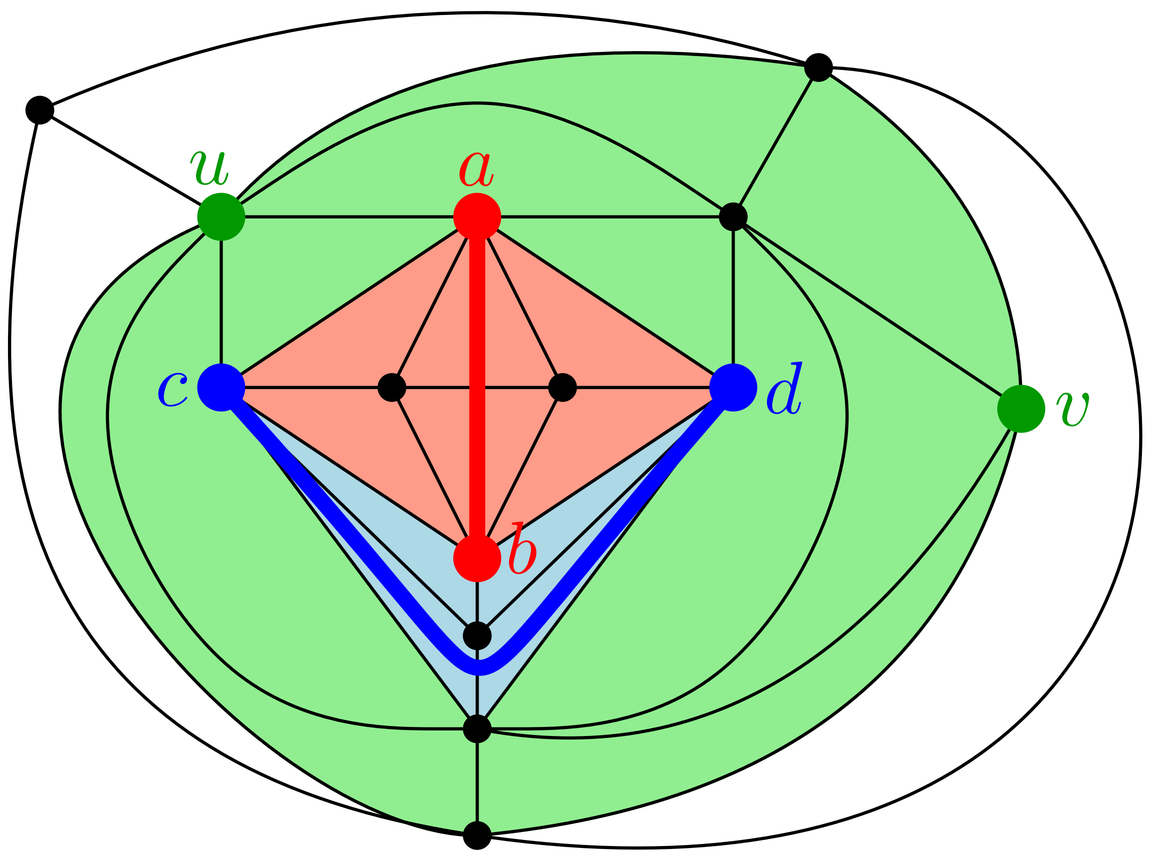
Triangulation + Matching

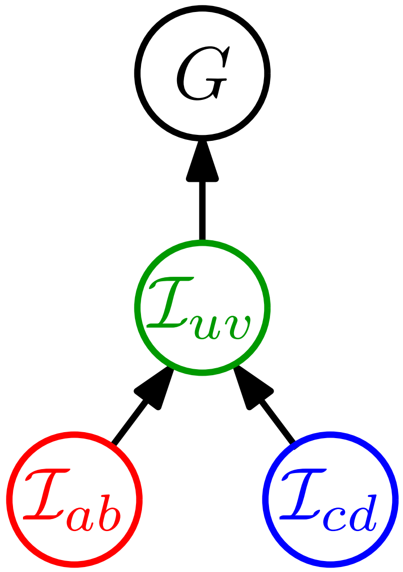




- Always pick “middle” routing
- Solve rest with 2SAT

Triangulation + Matching





- Always pick “middle” routing
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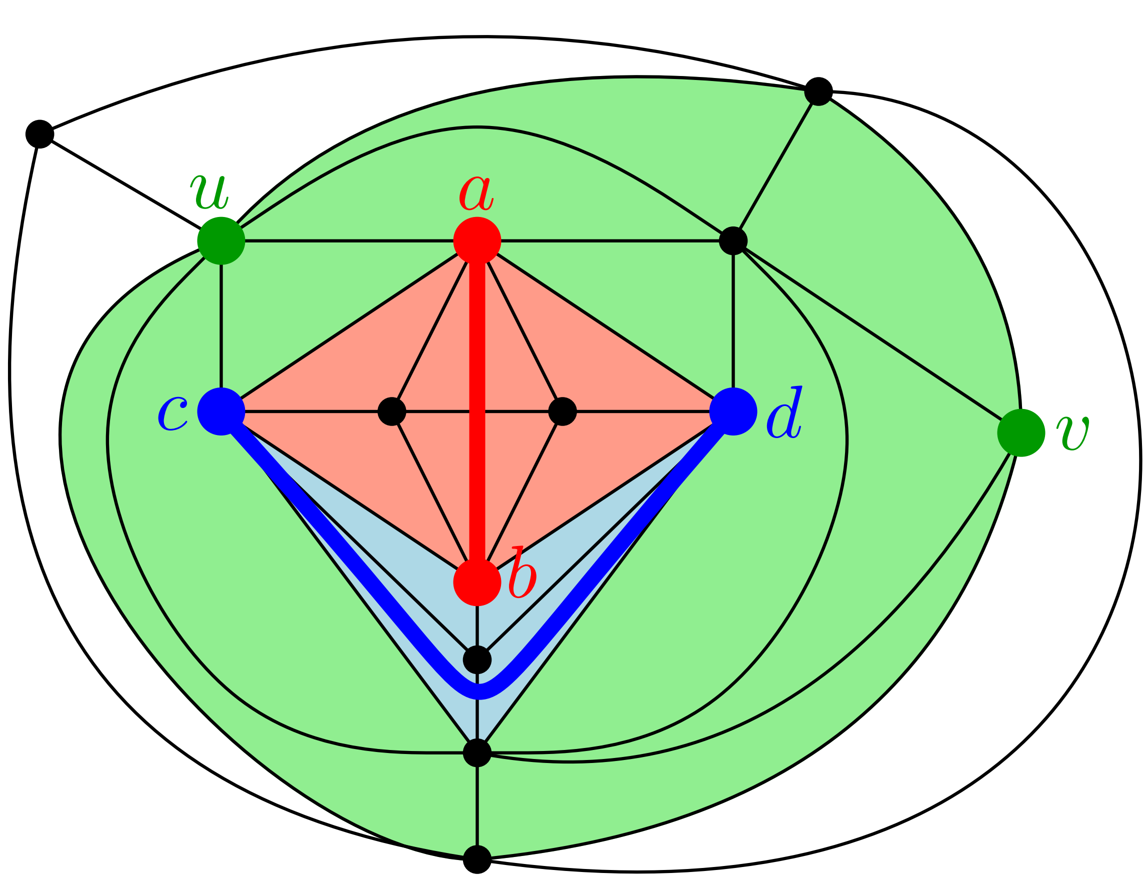
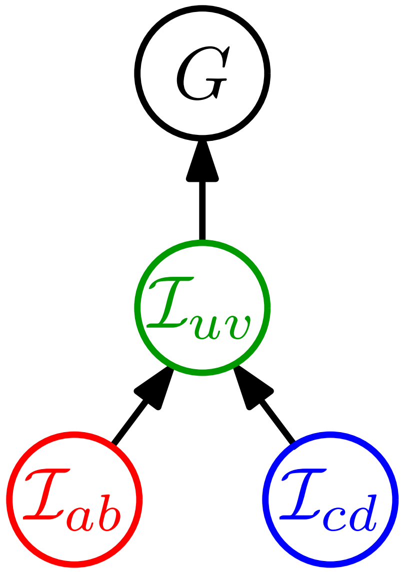
Triangulation + Matching





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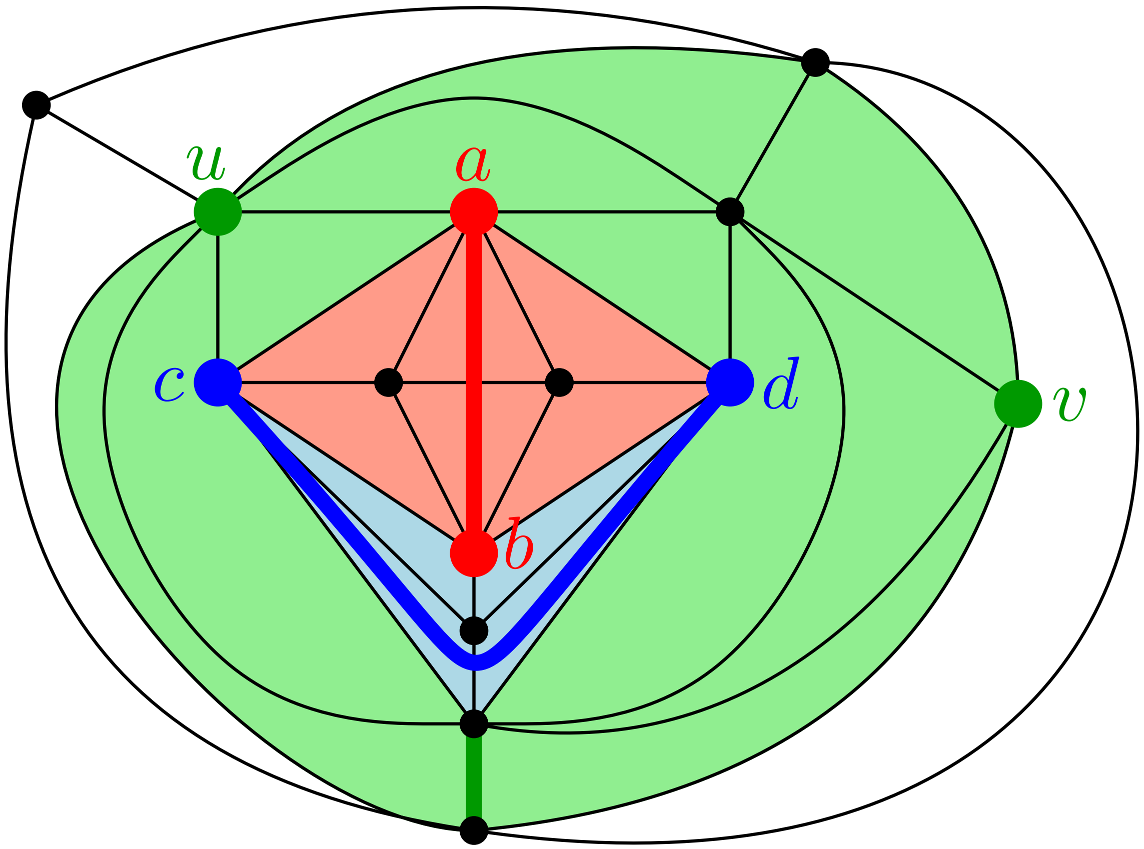
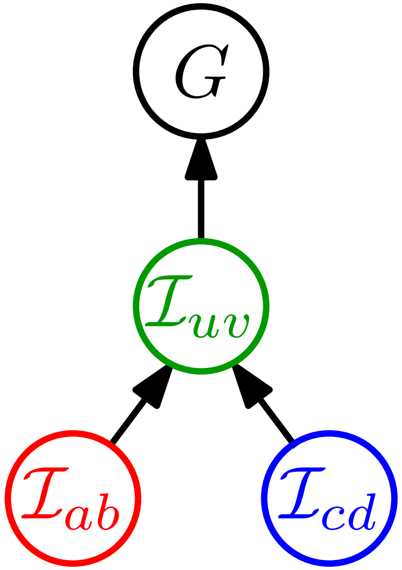
Triangulation + Matching





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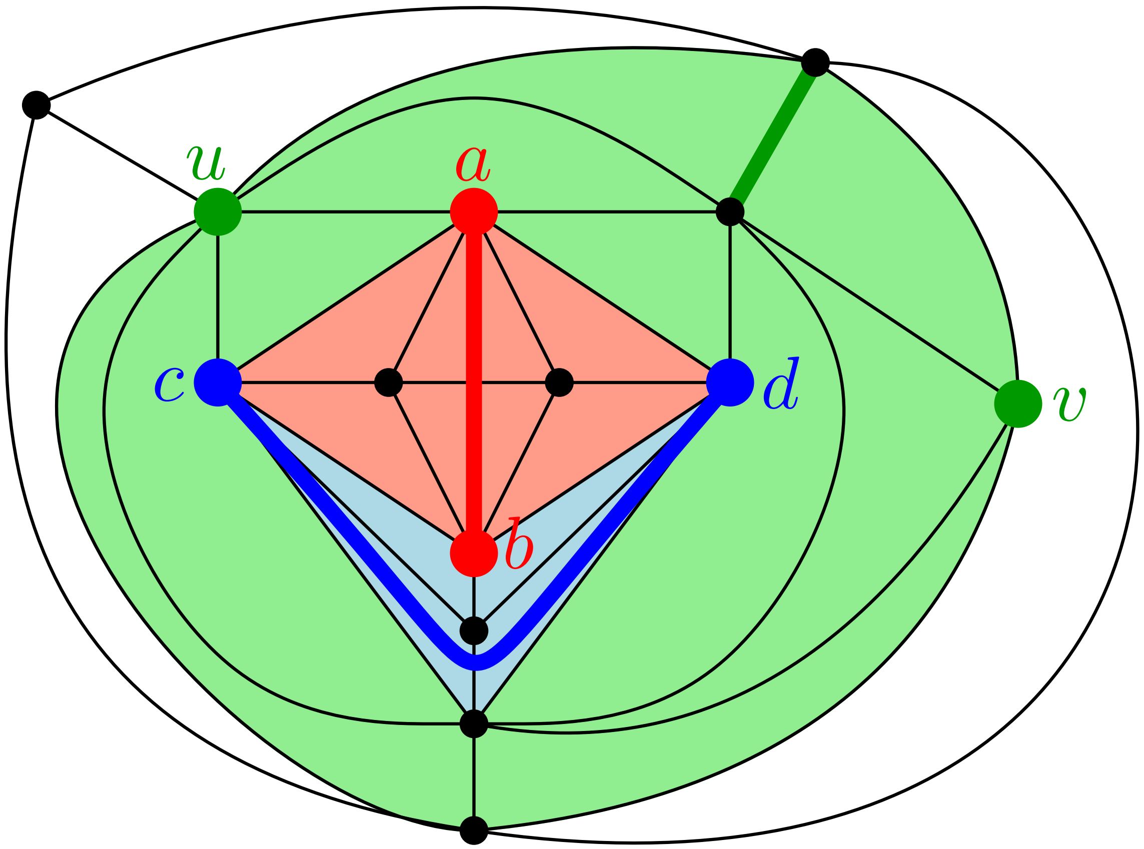

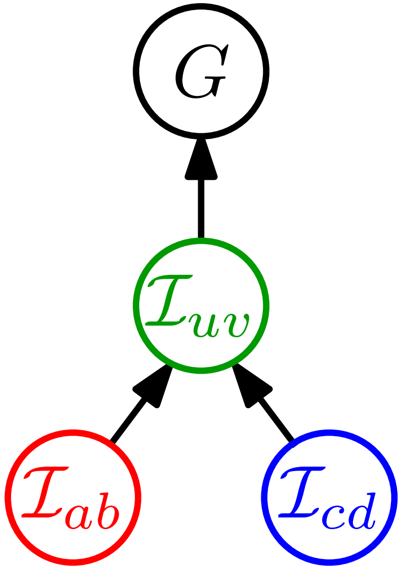

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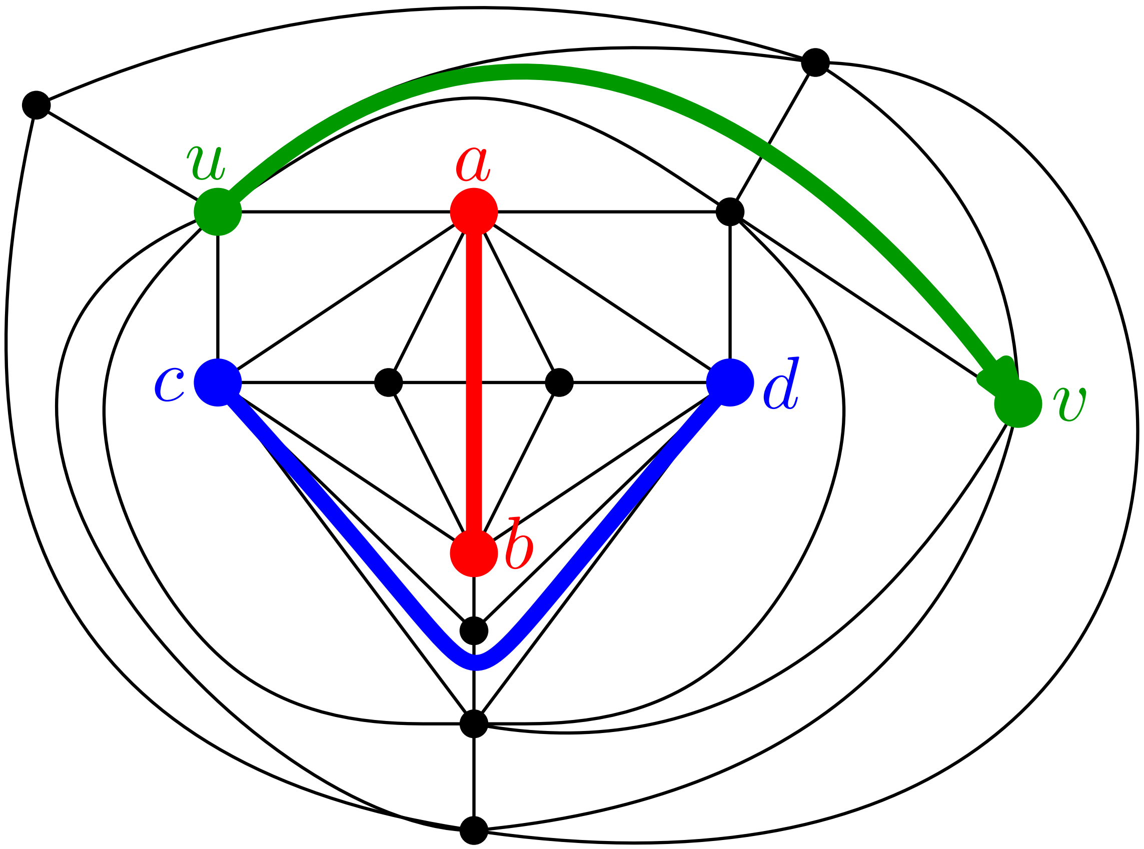

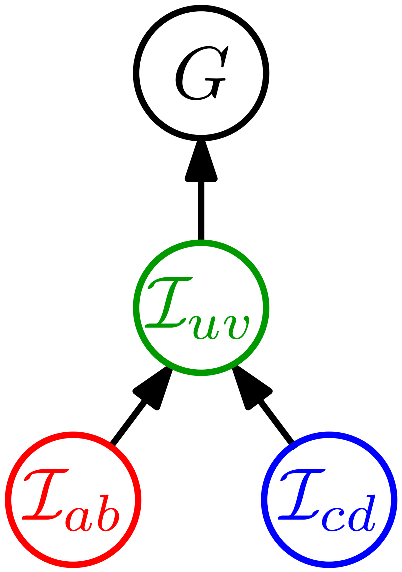

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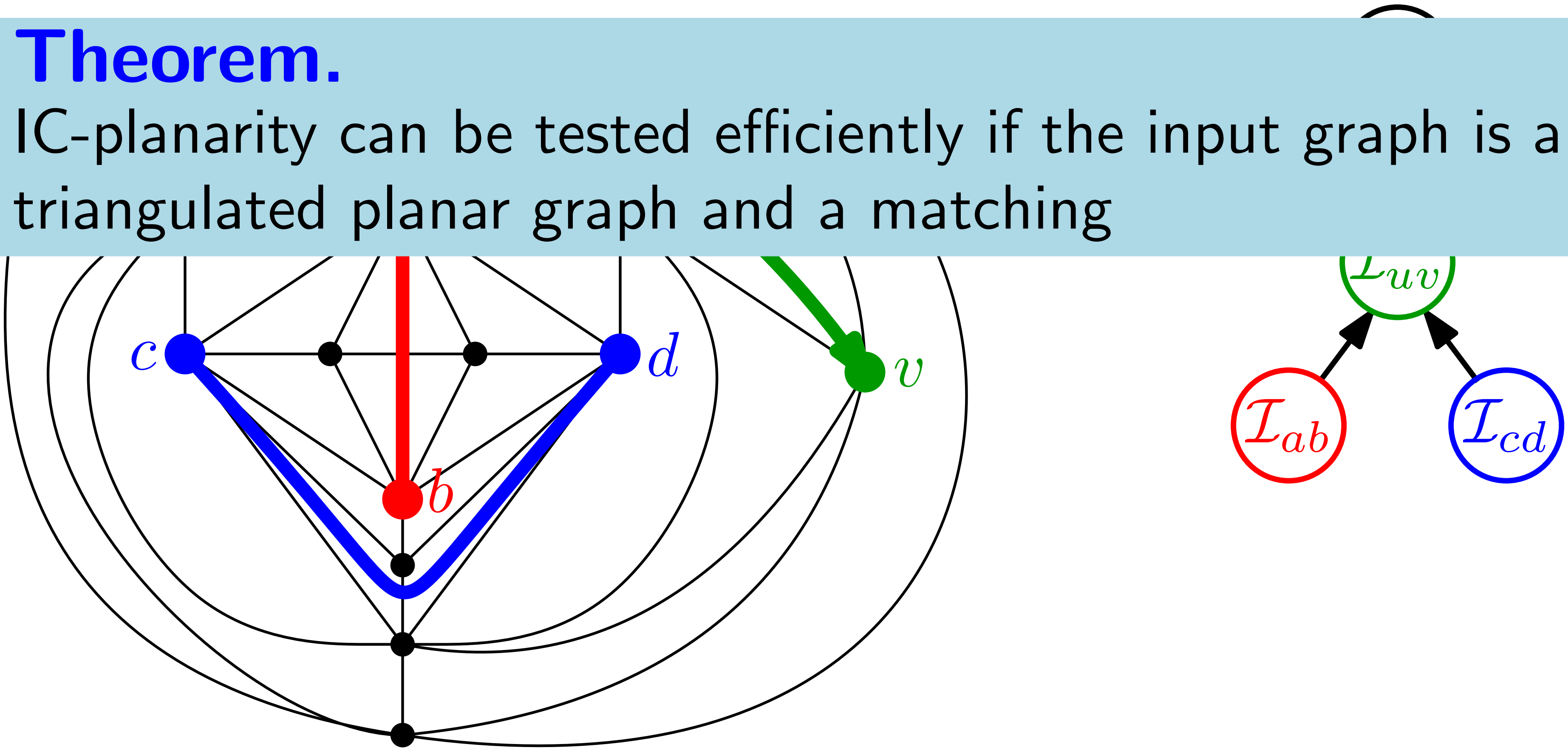
Triangulation + Matching

Theorem.
IC-planarity can be tested efficiently if the input graph is a triangulated planar graph and a matching



- Always pick "middle" routing
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Straight-Line Drawings

Theorem.
IC-plane graphs can be drawn straight-line on the $O(n) \times O(n)$ grid in $O(n)$ time.

Straight-Line Drawings

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IC-plane graphs can be drawn straight-line on the $O(n) \times O(n)$ grid in $O(n)$ time.

Using a special 1-planar drawing...

[Alam et al. GD'13]

Straight-Line Drawings

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IC-plane graphs can be drawn straight-line **RAC?**
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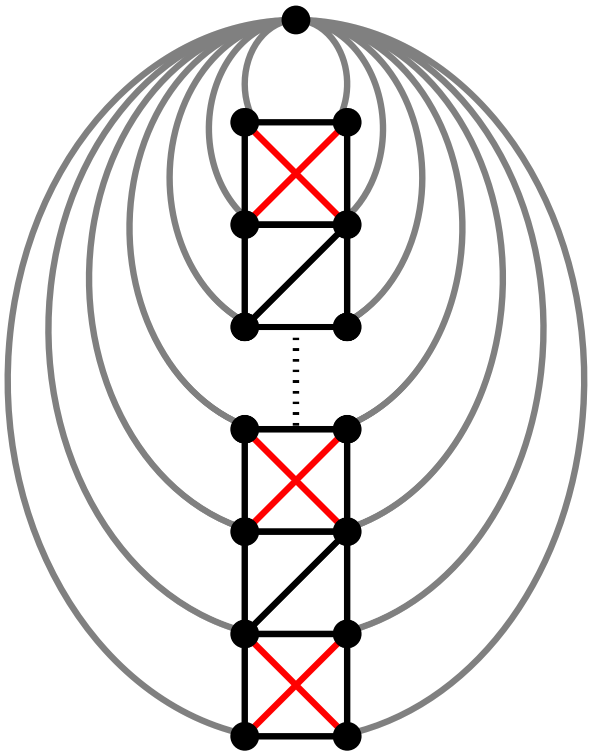

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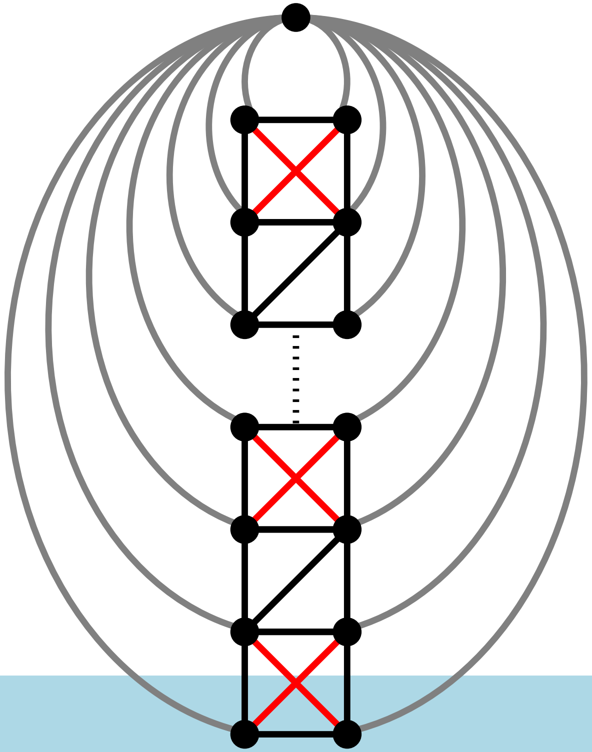

Theorem.
Straight-line RAC drawings of IC-planar graphs may require exponential area.

Straight-Line Drawings

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Straight-Line RAC Drawings

Adjust Shift-Algorithm for planar graphs
[de Fraysseix, Pach & Pollack Comb'90]

Straight-Line RAC Drawings

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↳ • Augment to 3-connected planar graph

Straight-Line RAC Drawings

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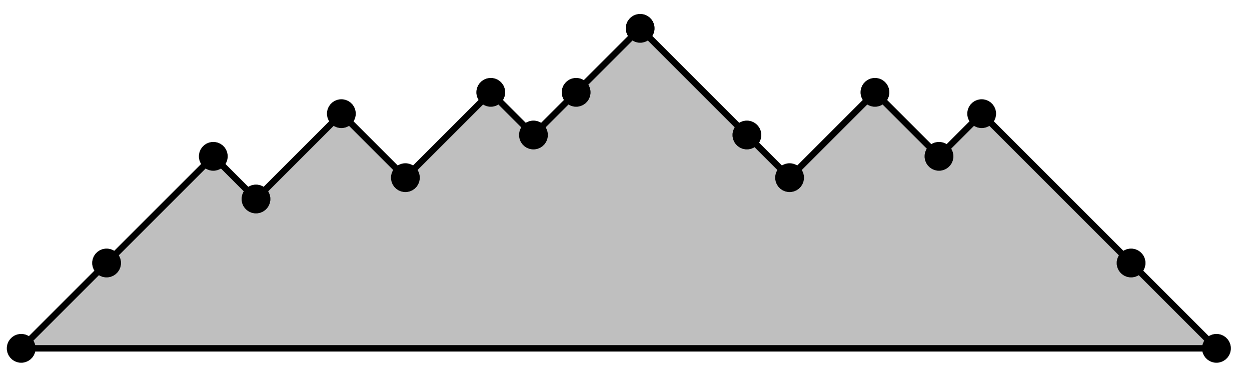

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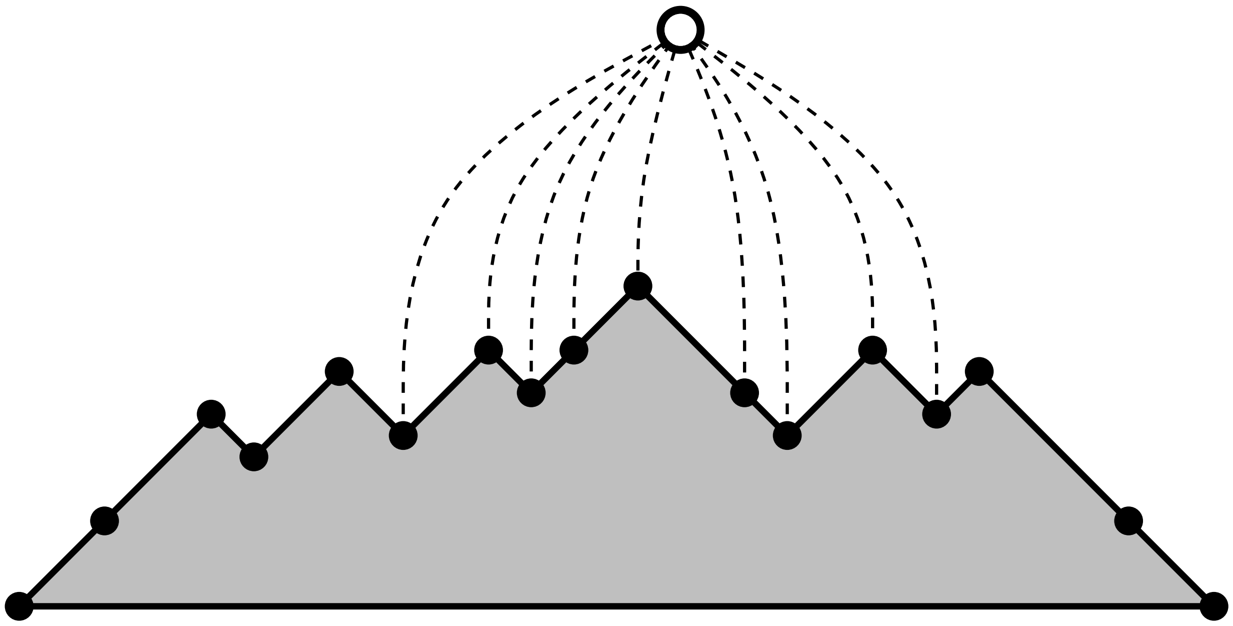

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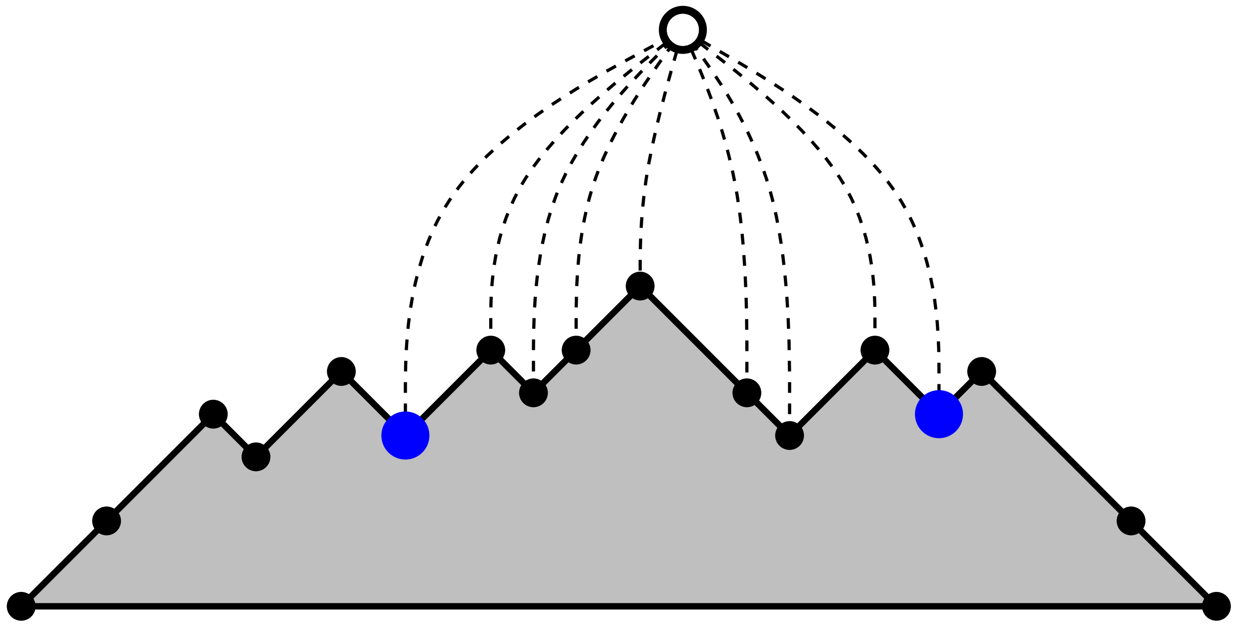

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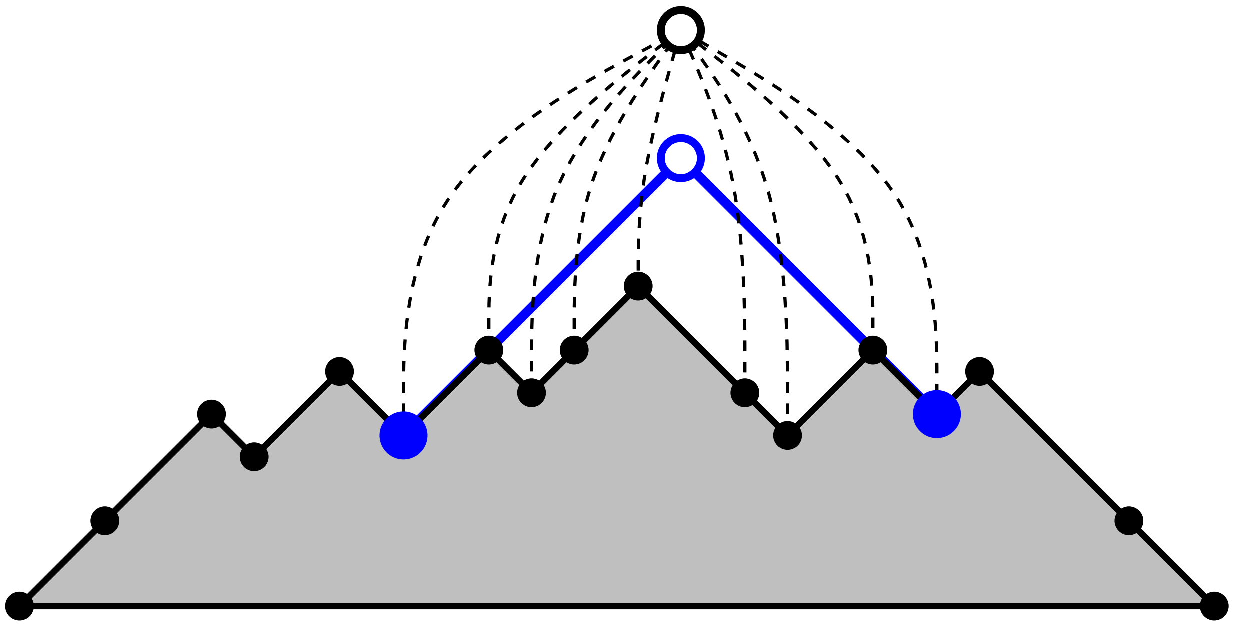

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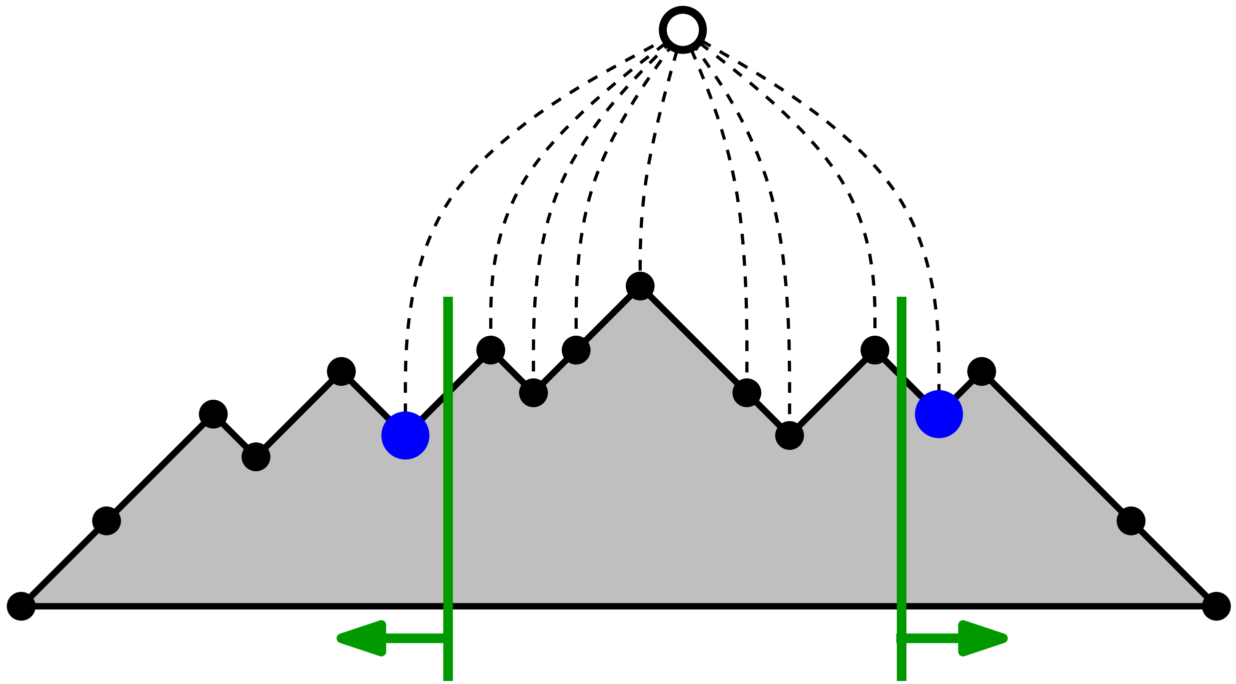

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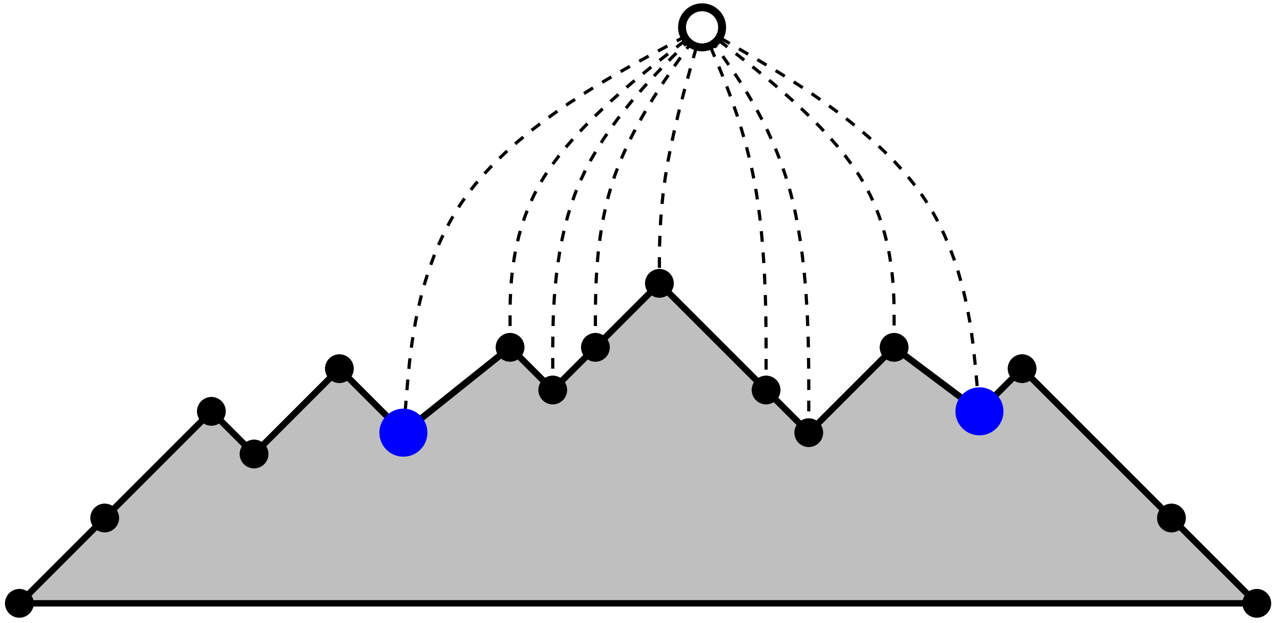

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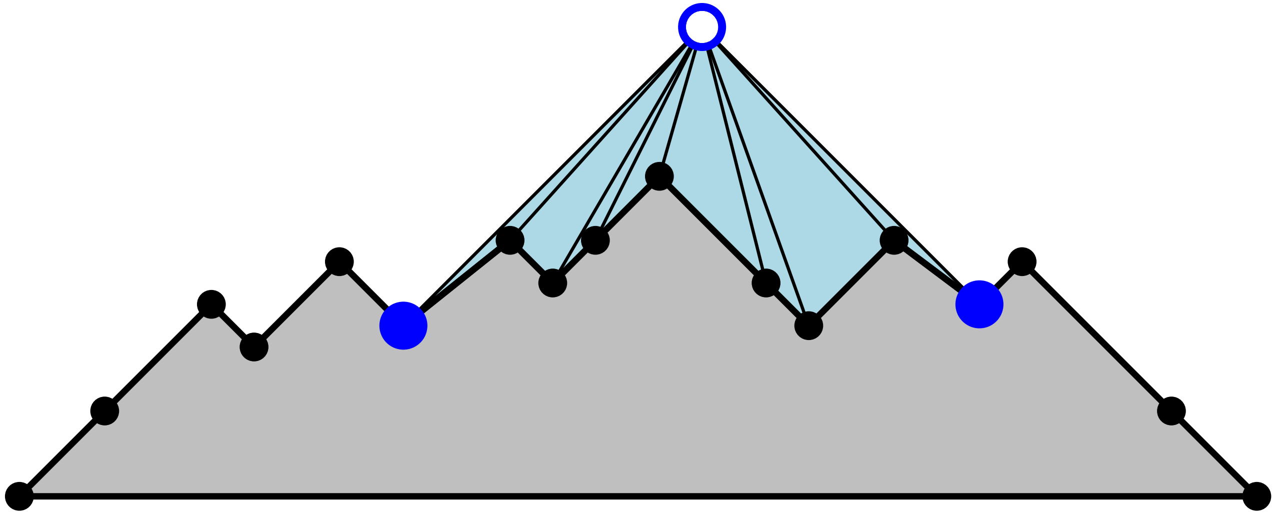

Straight-Line RAC Drawings

Adjust Shift-Algorithm for planar graphs
[de Fraysseix, Pach & Pollack Comb'90]

- Augment to planar-maximal IC-planar graph
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Straight-Line RAC Drawings

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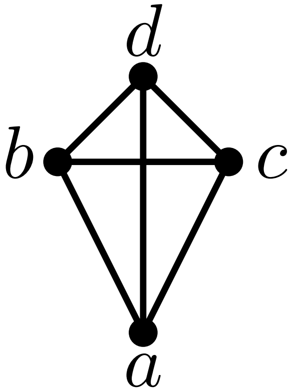

Straight-Line RAC Drawings

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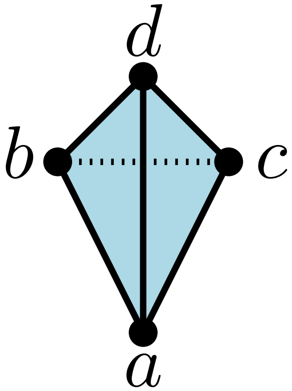

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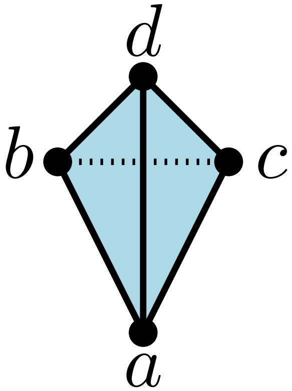

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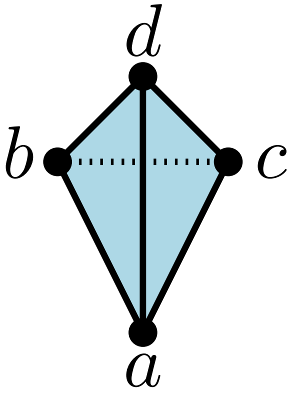

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Highest number in canonical order

Straight-Line RAC Drawings

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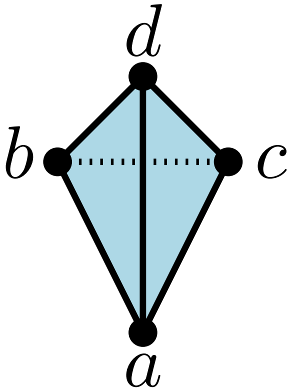

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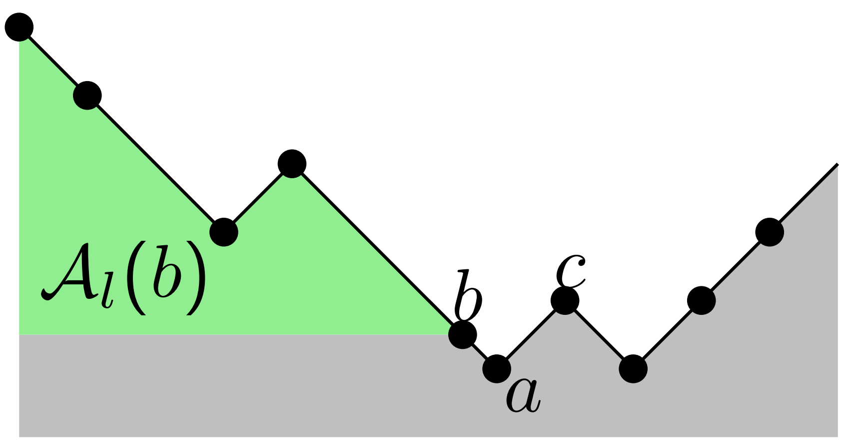

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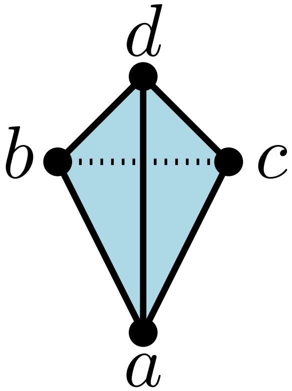

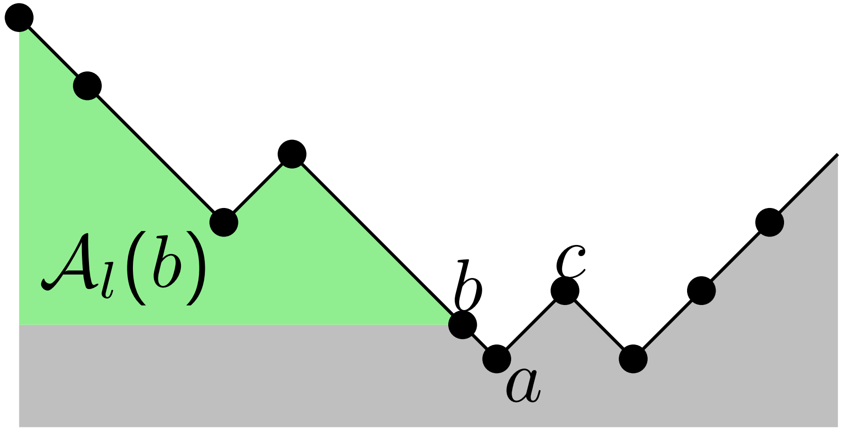

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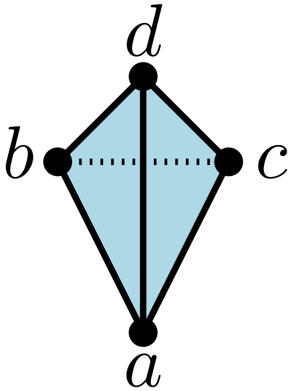

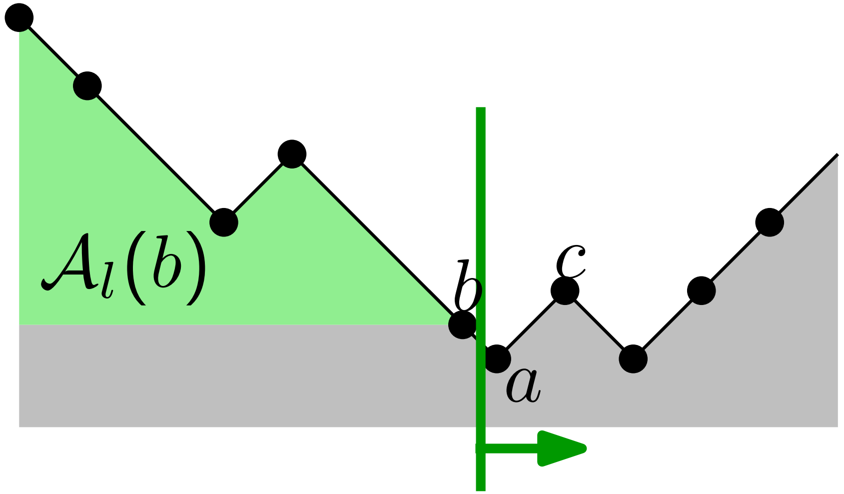

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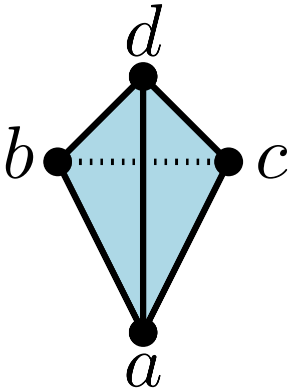

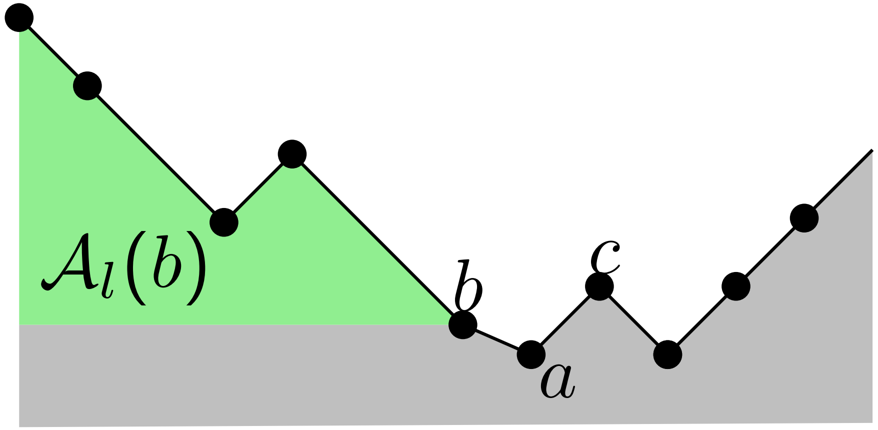

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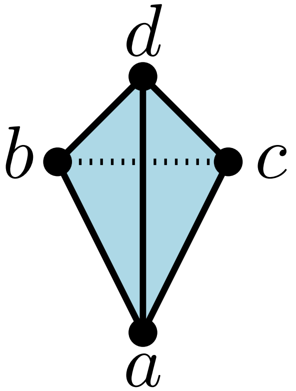

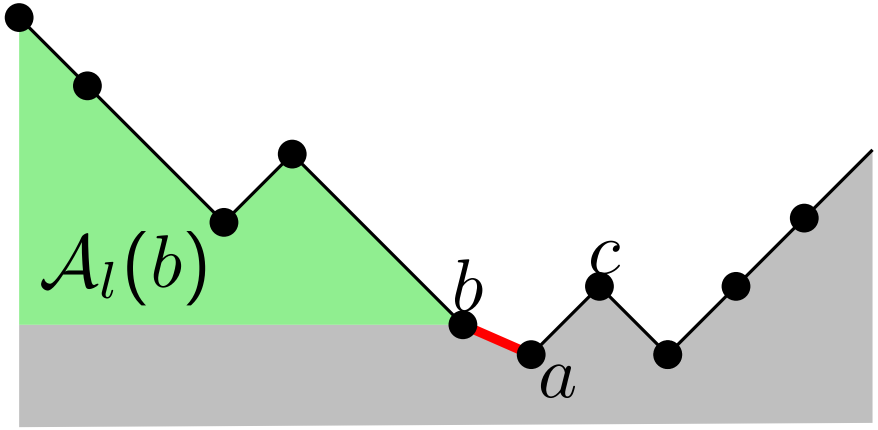

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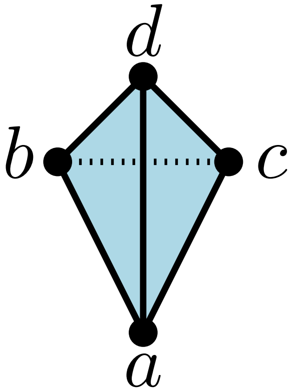

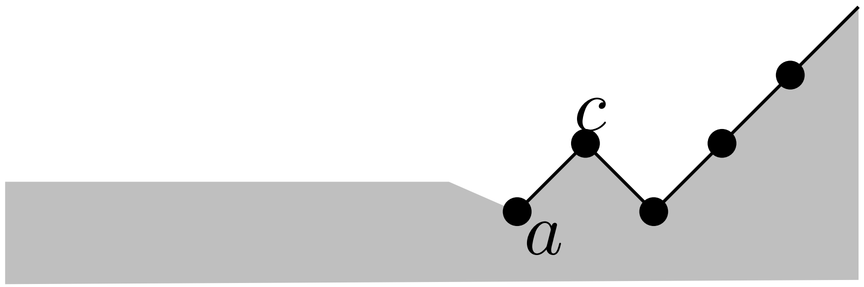

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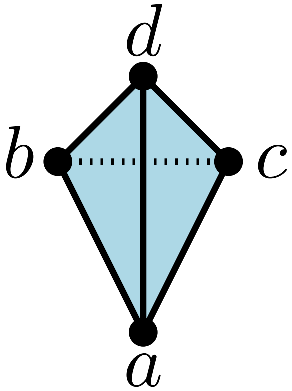

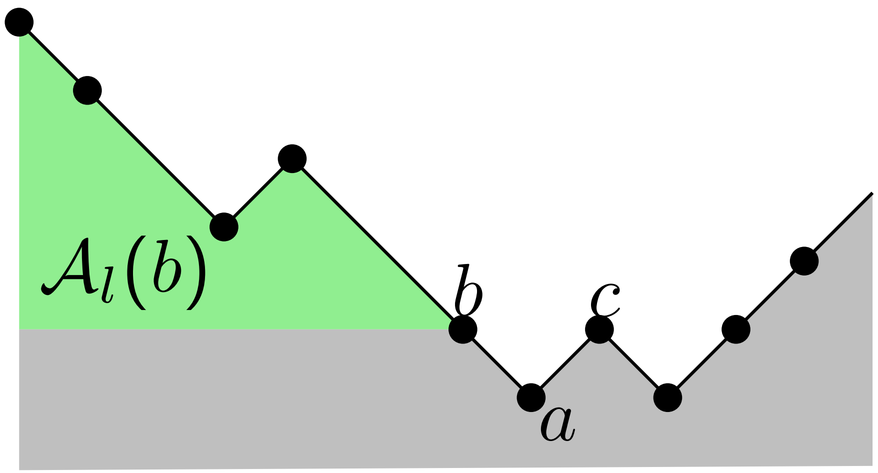

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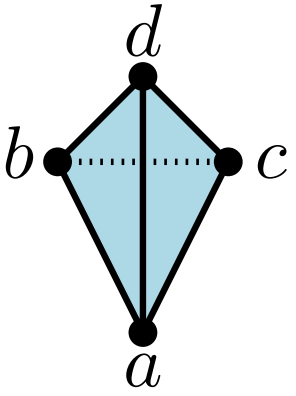

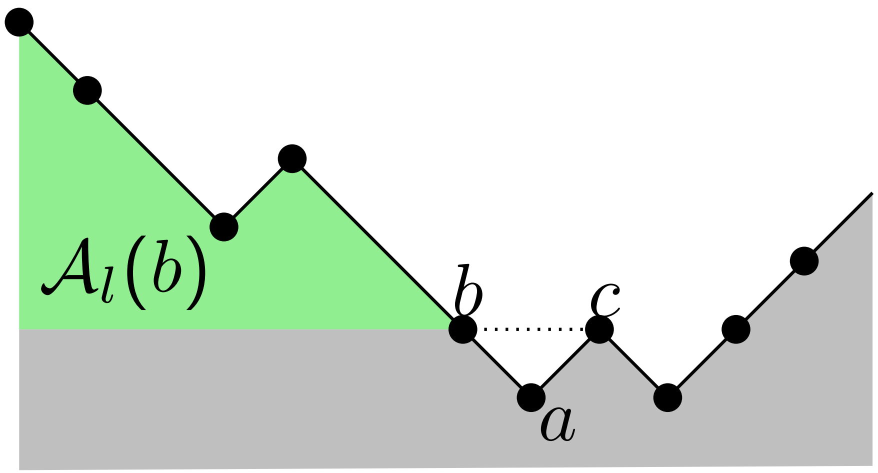

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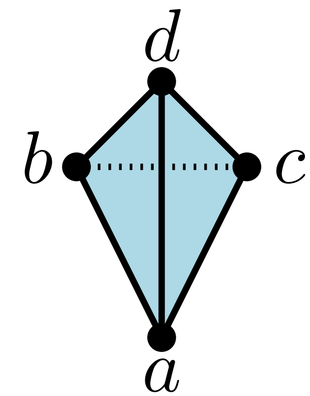

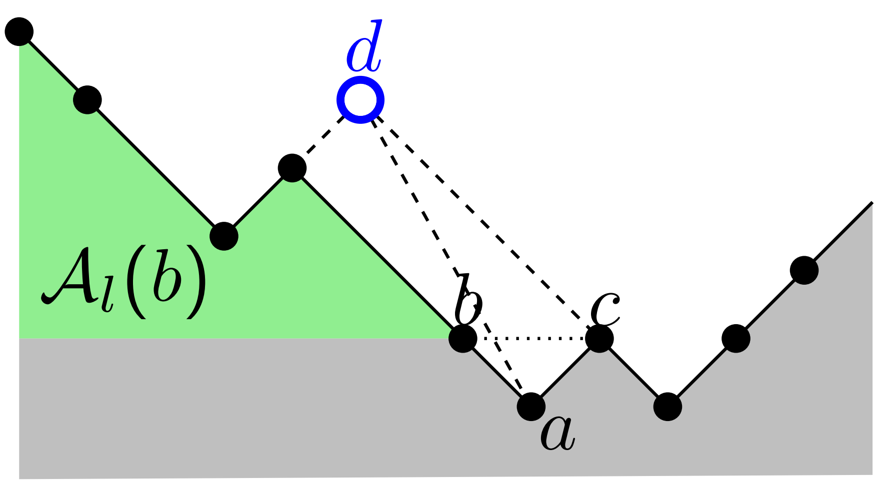

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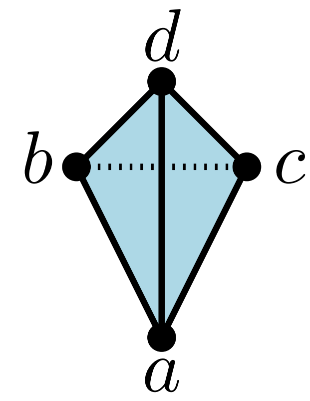

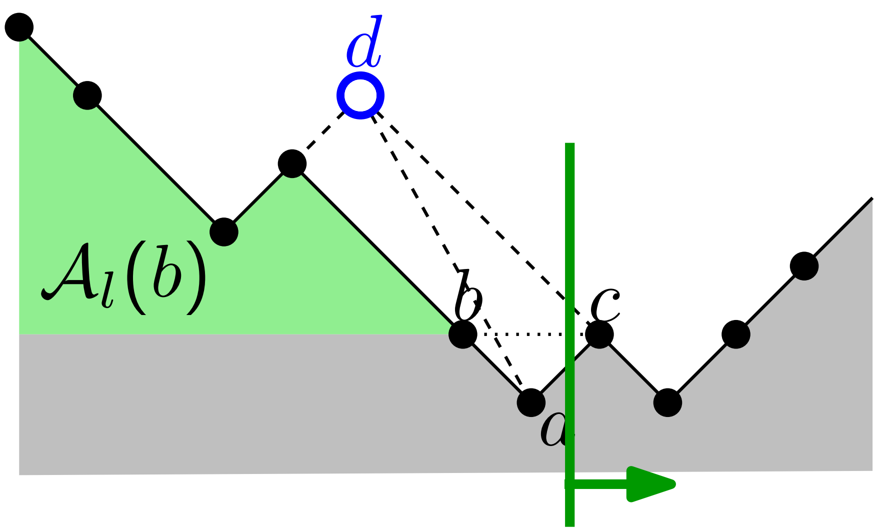

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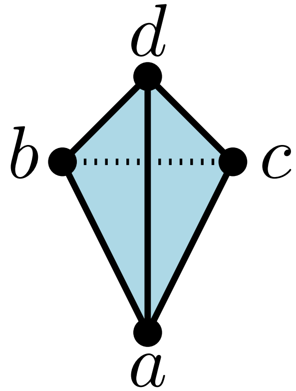

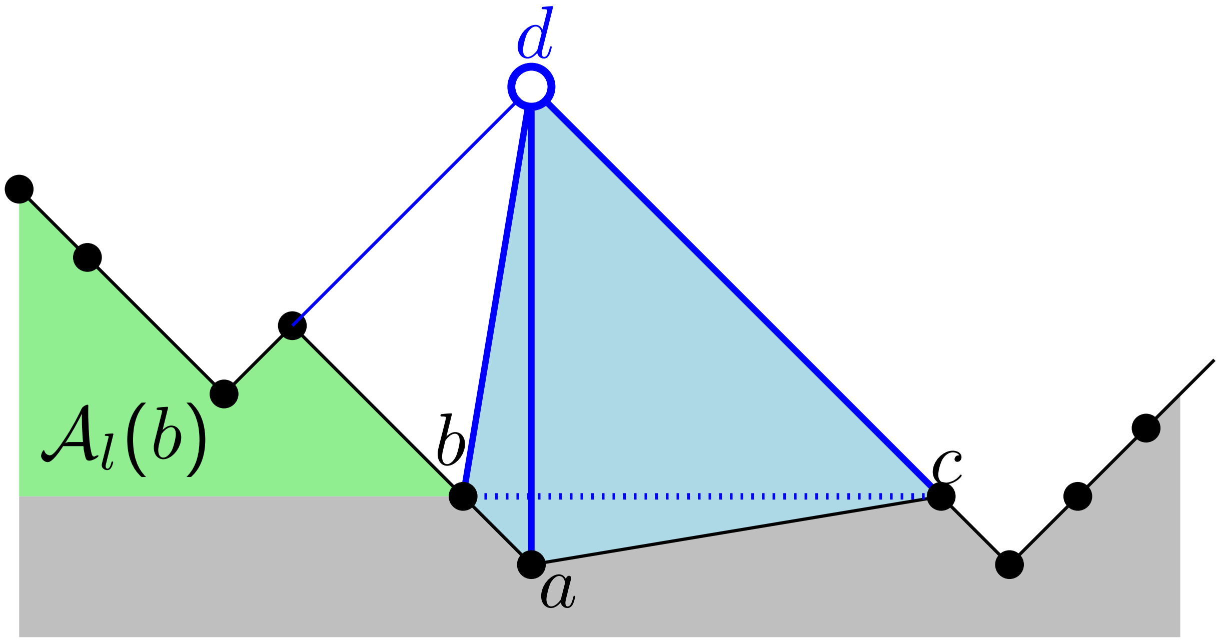

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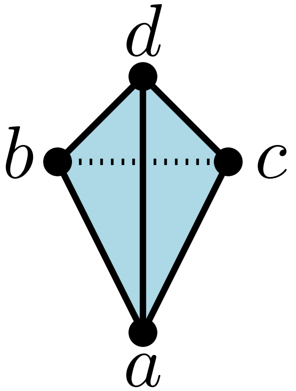

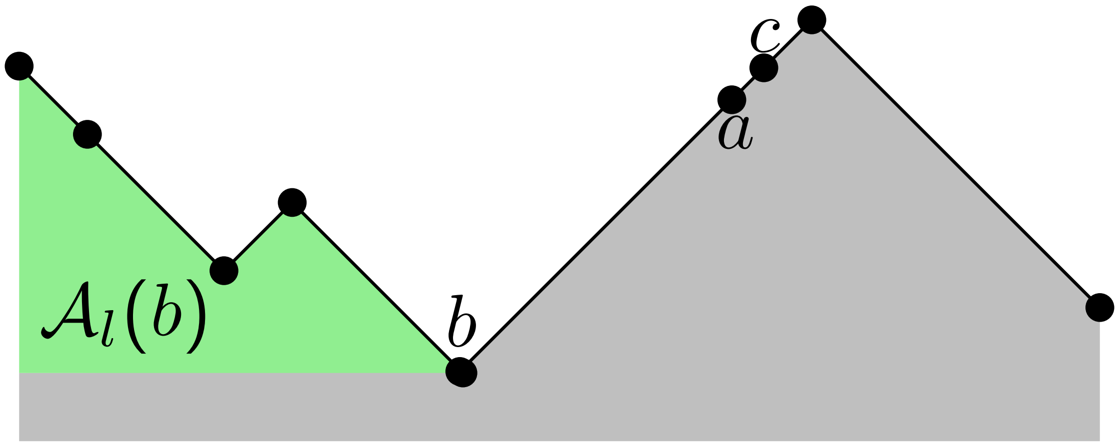

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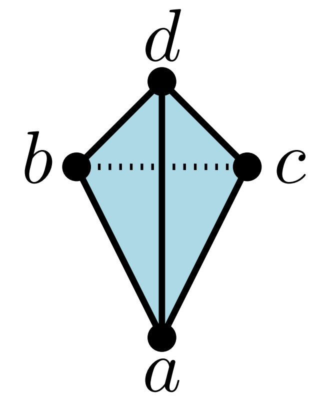

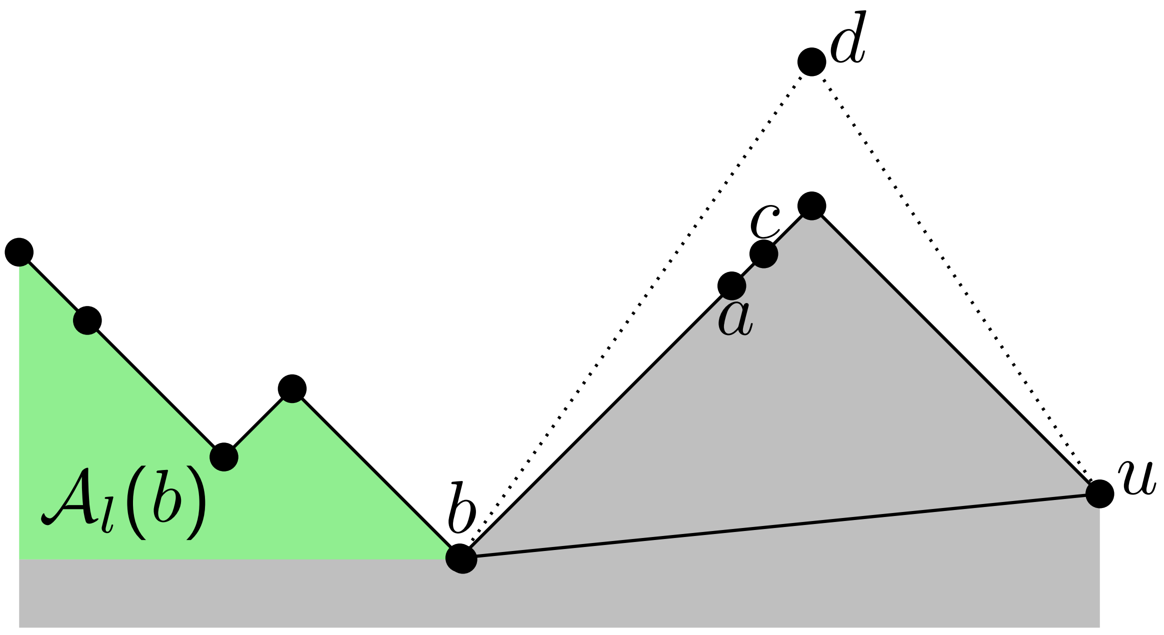

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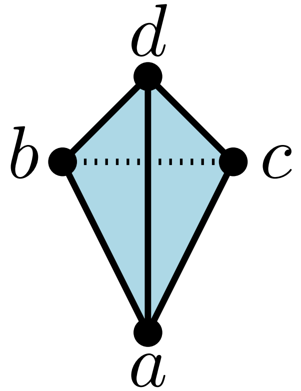

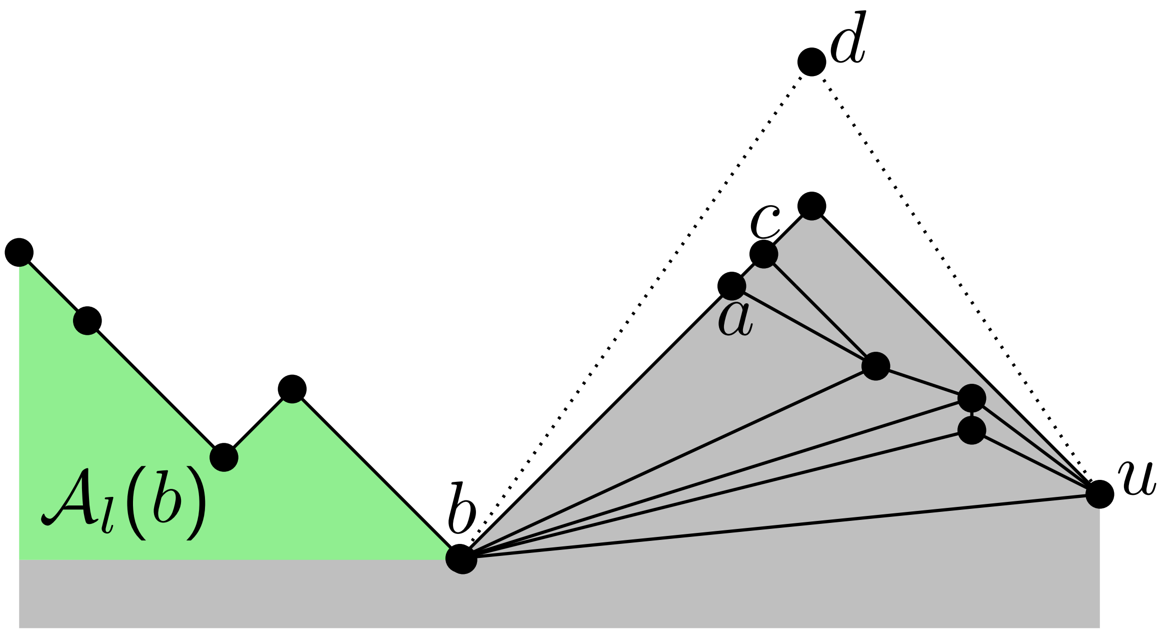

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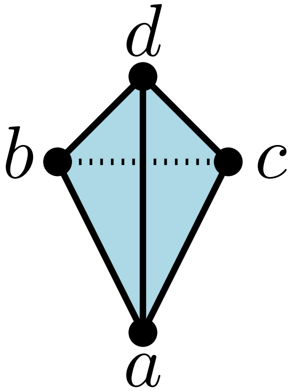

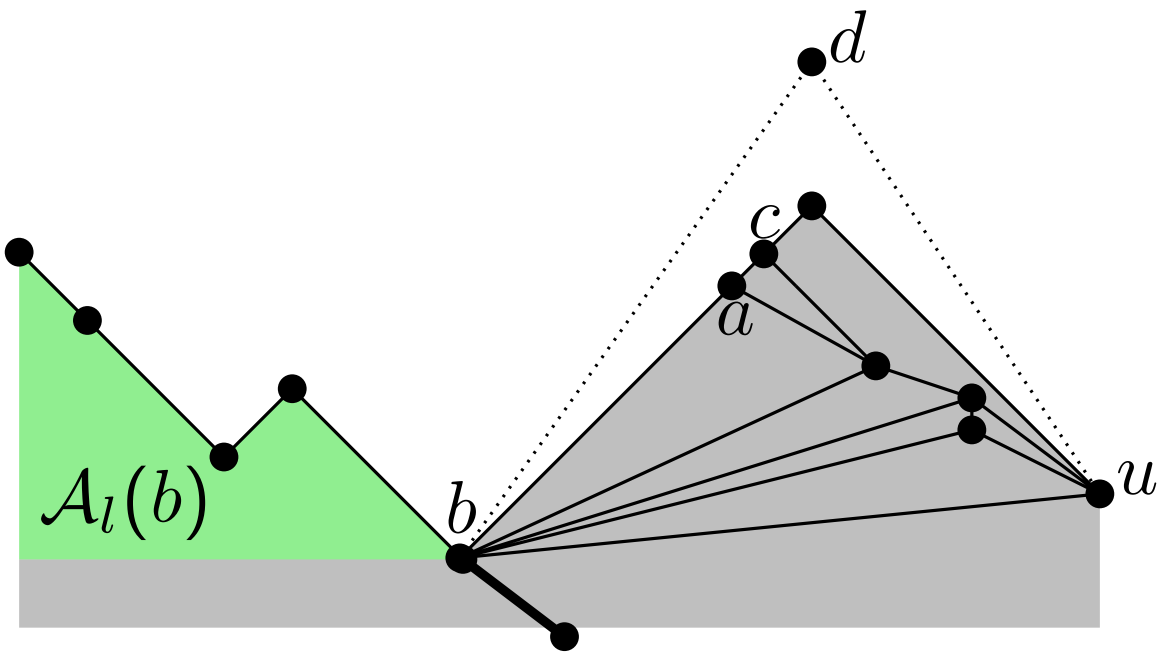

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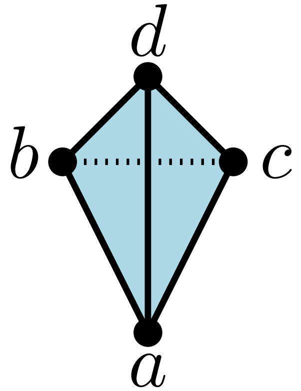
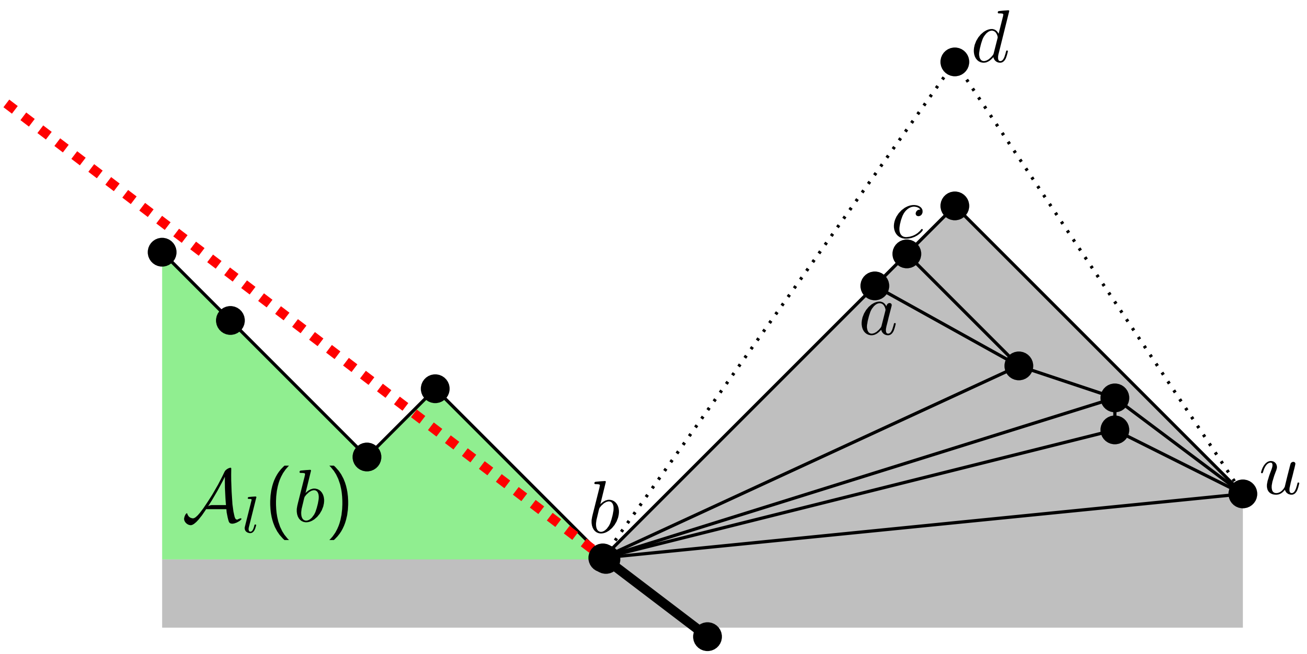
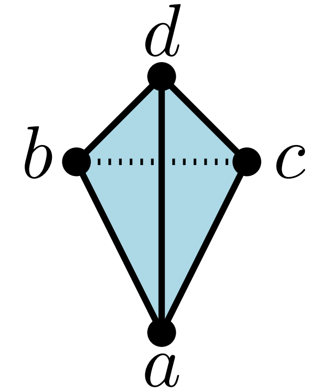
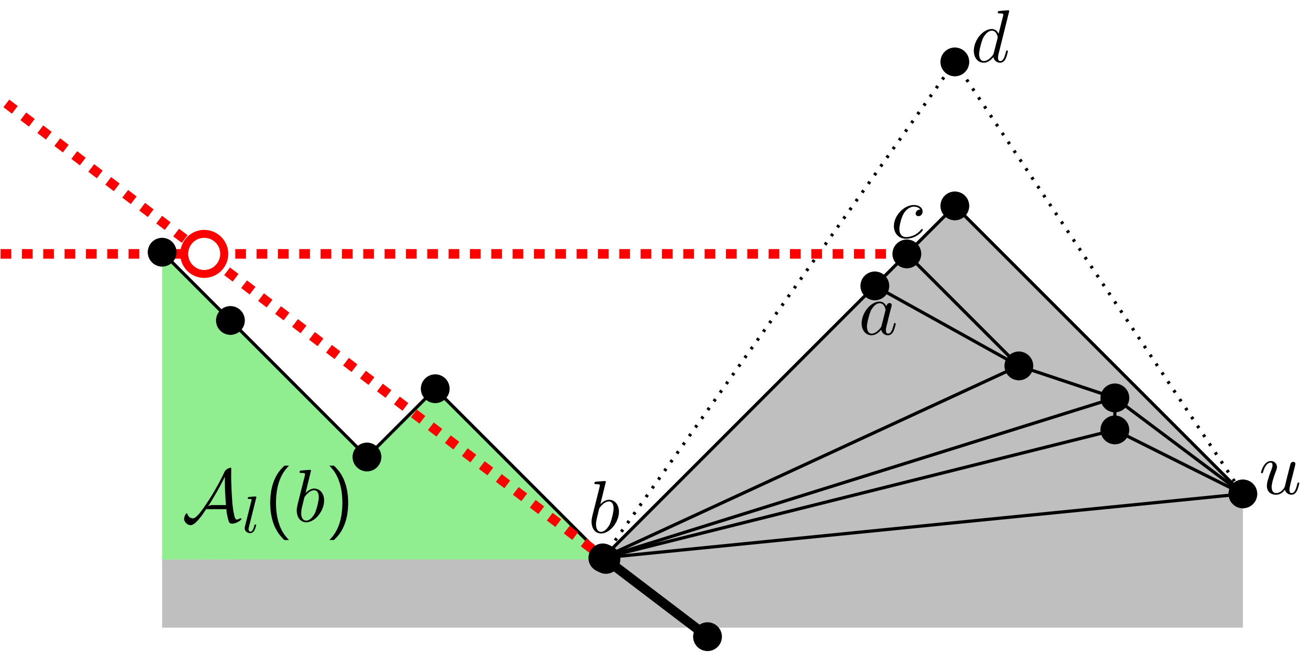




Straight-Line RAC Drawings

Adjust Shift-Algorithm for planar graphs
[de Fraysseix, Pach & Pollack Comb'90]





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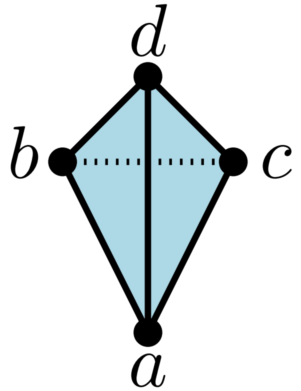
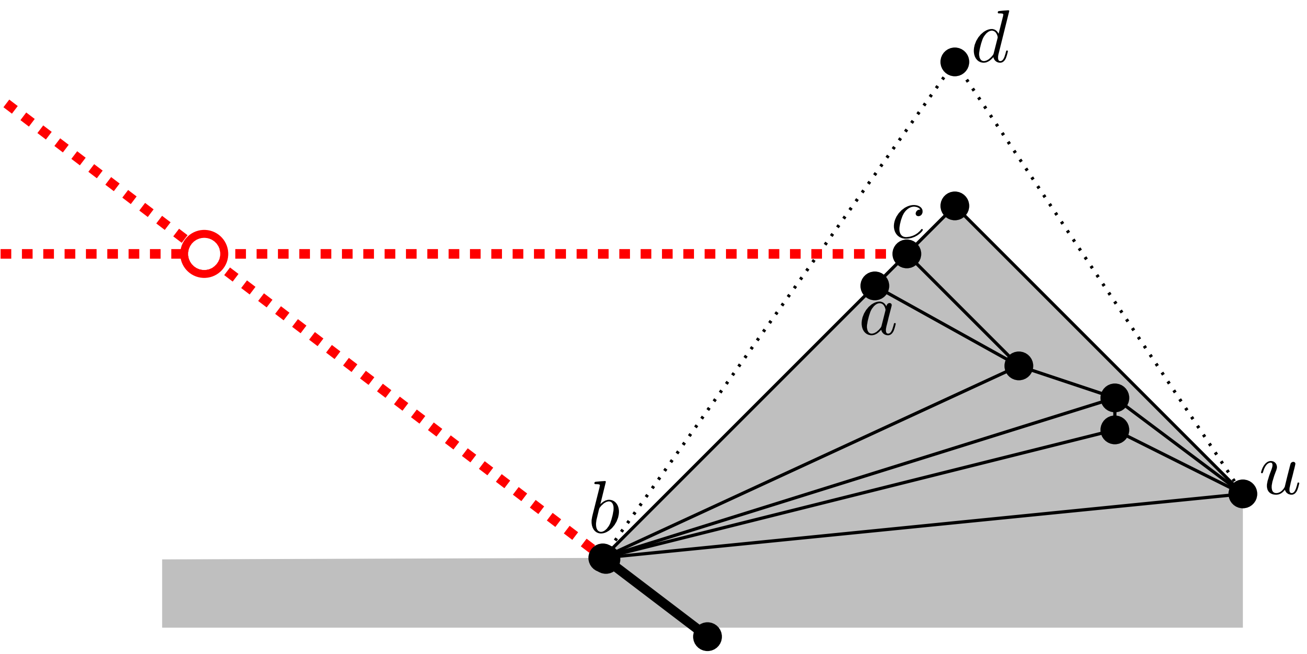
Adjust Shift-Algorithm for planar graphs
[de Fraysseix, Pach & Pollack Comb'90]





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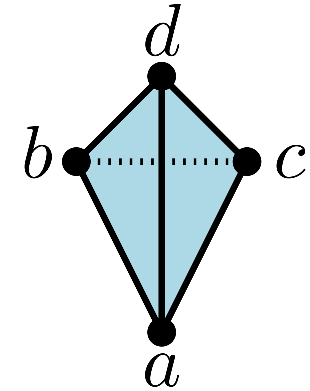
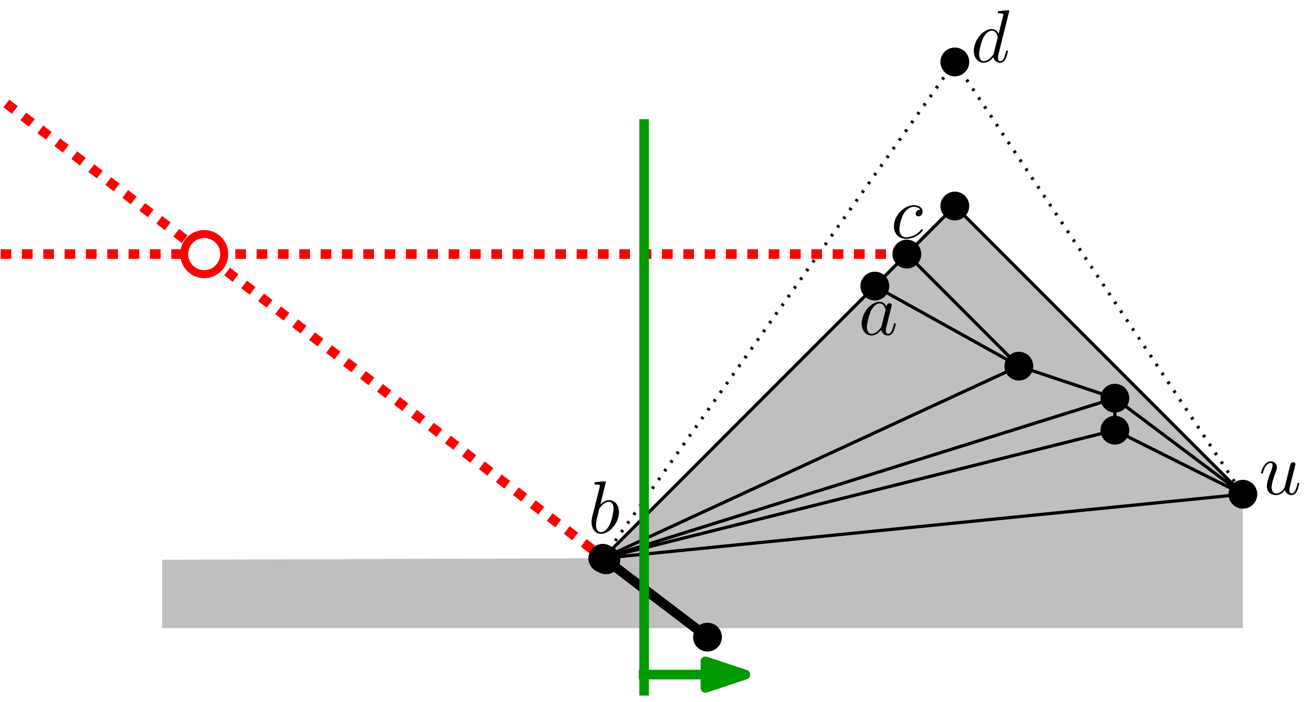
Adjust Shift-Algorithm for planar graphs
[de Fraysseix, Pach & Pollack Comb'90]





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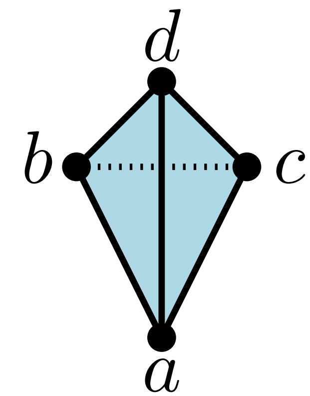
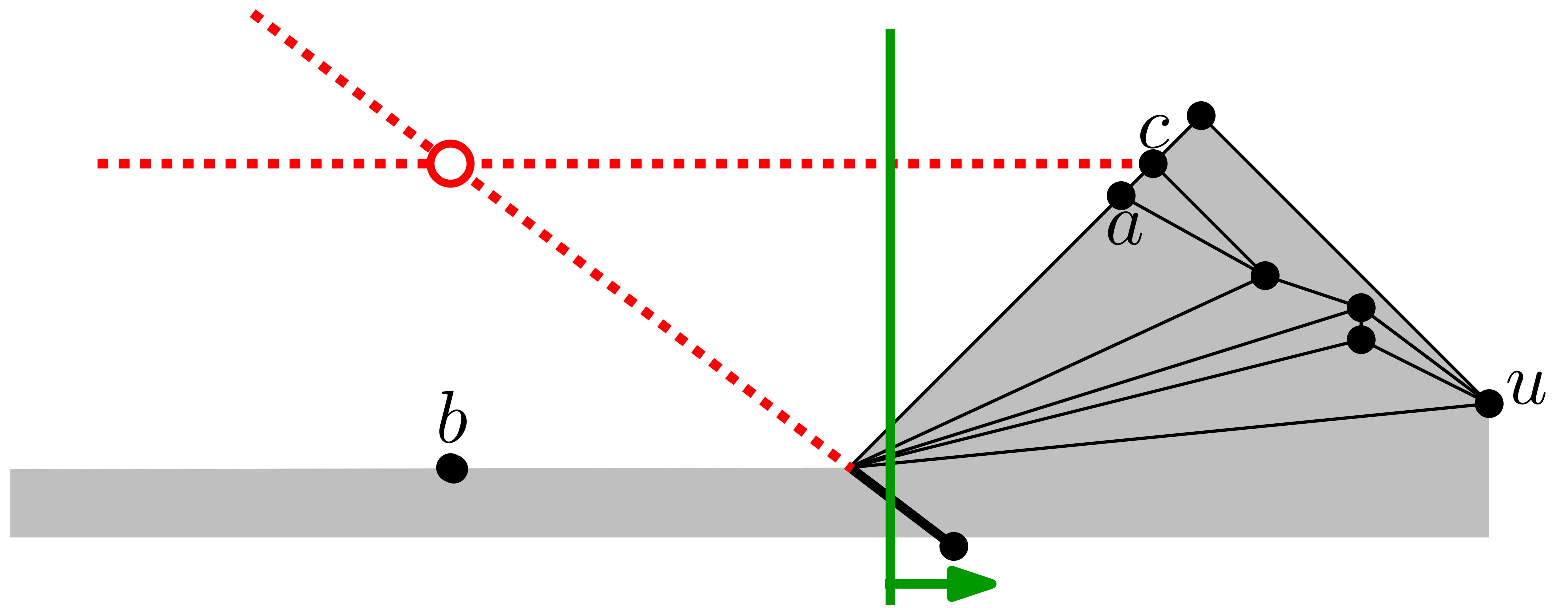
[de Fraysseix, Pach & Pollack Comb'90]





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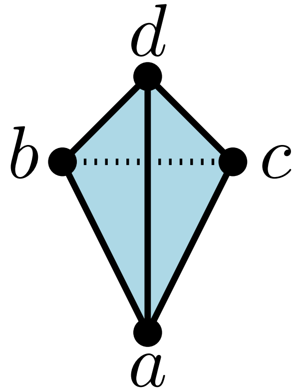

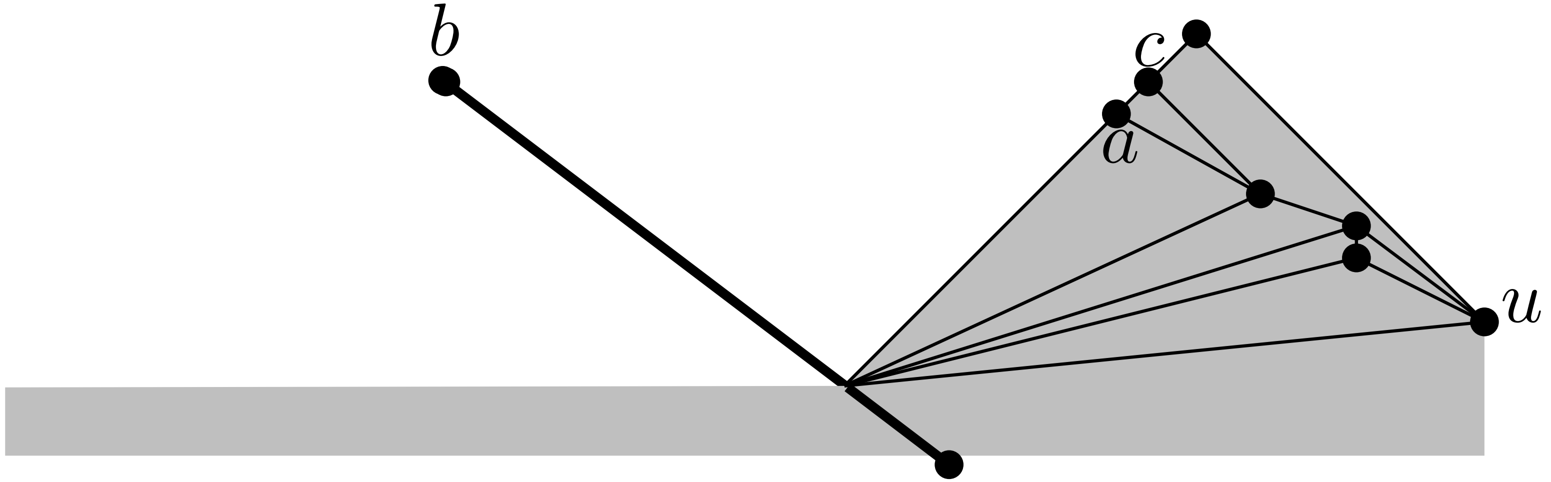

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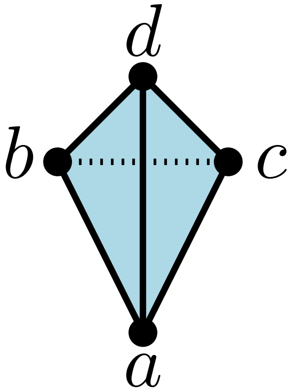
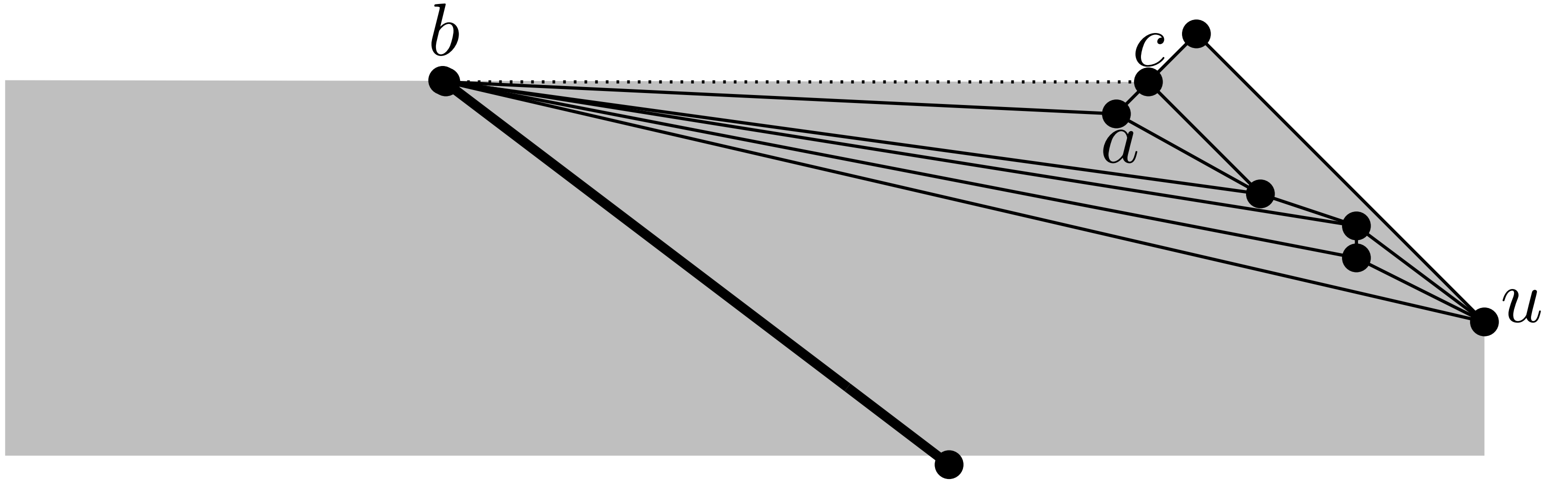
[de Fraysseix, Pach & Pollack Comb'90]

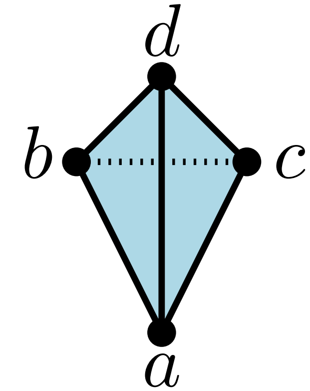


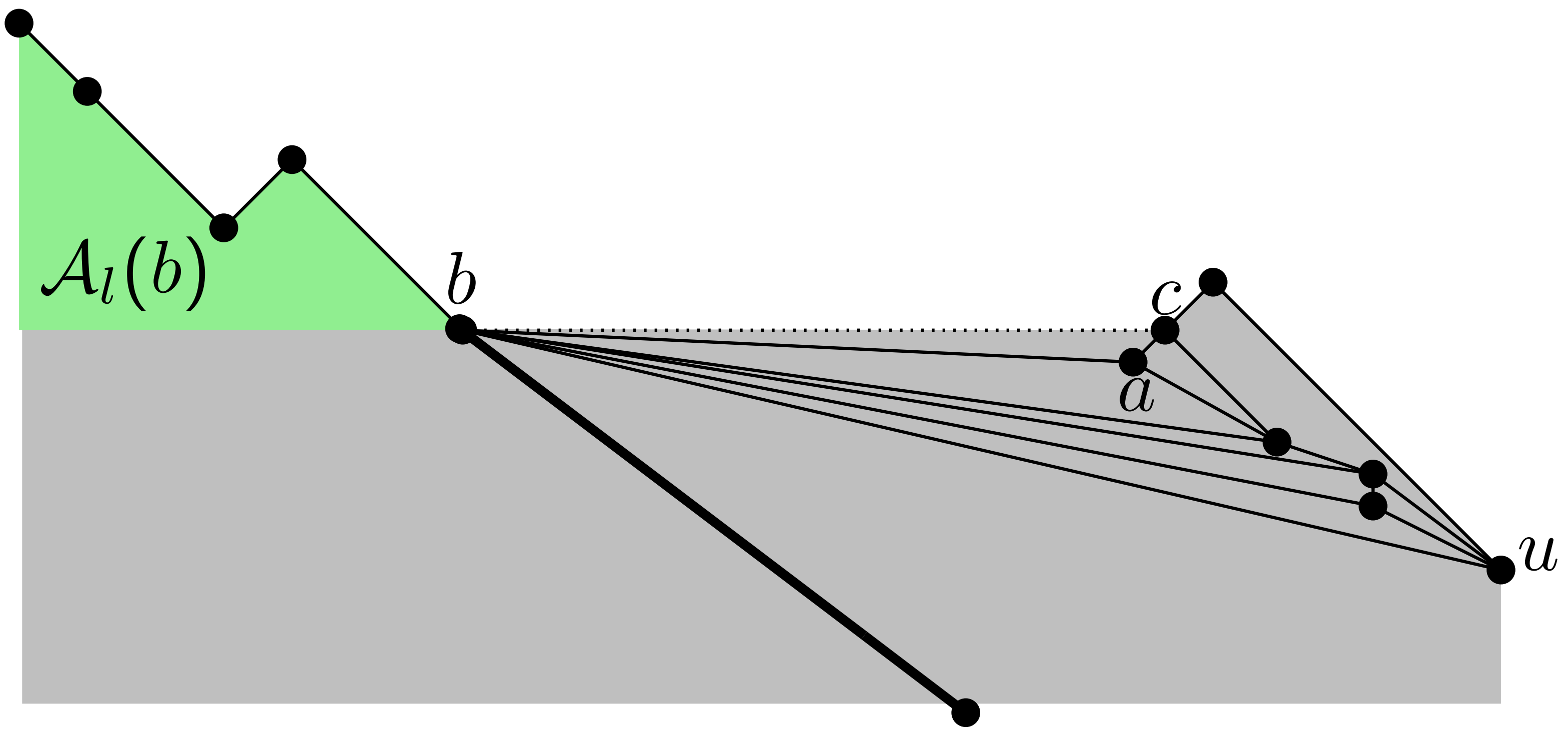


Straight-Line RAC Drawings

Adjust Shift-Algorithm for planar graphs

[de Fraysseix, Pach & Pollack Comb'90]





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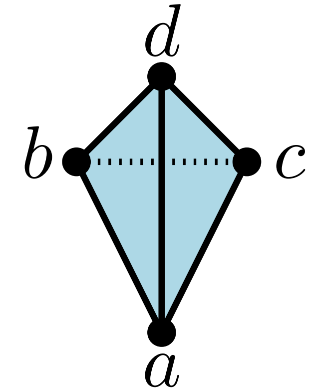
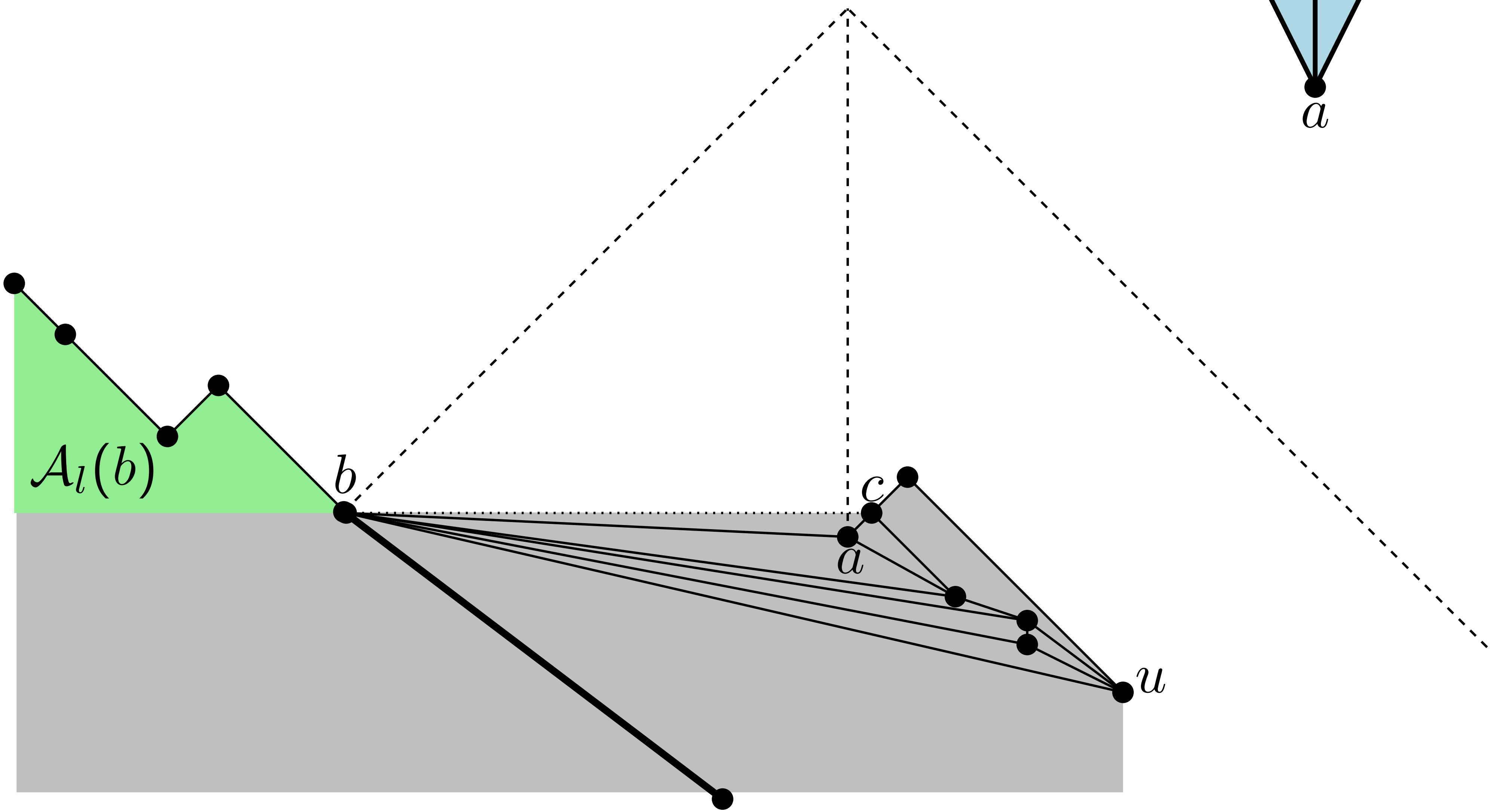
[de Fraysseix, Pach & Pollack Comb'90]





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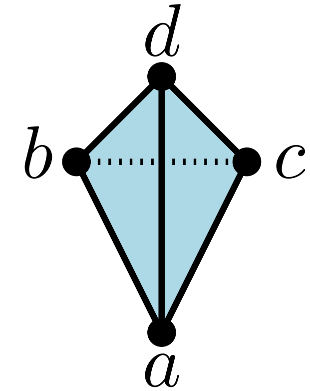

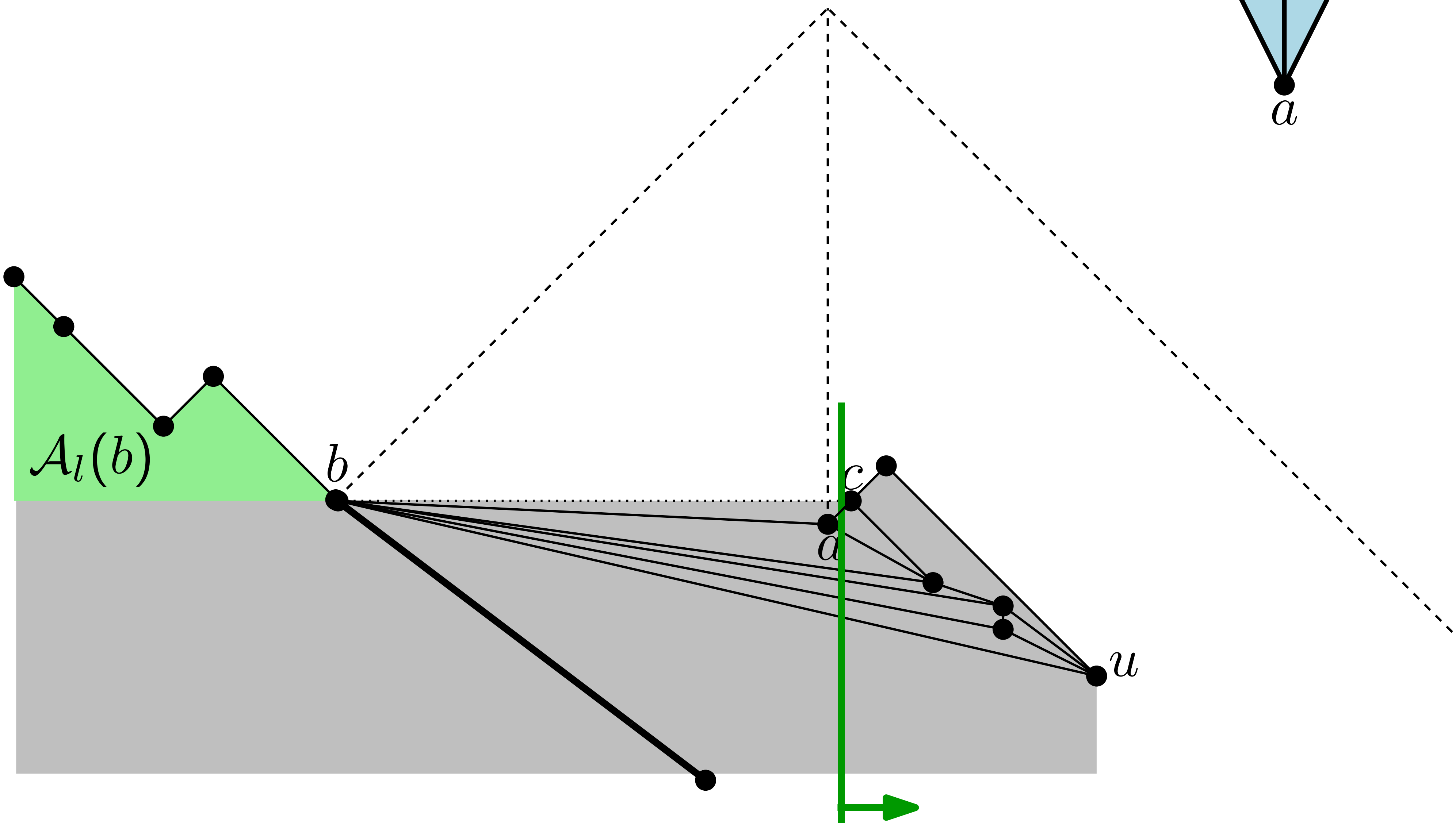

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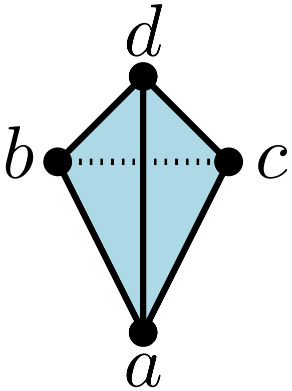

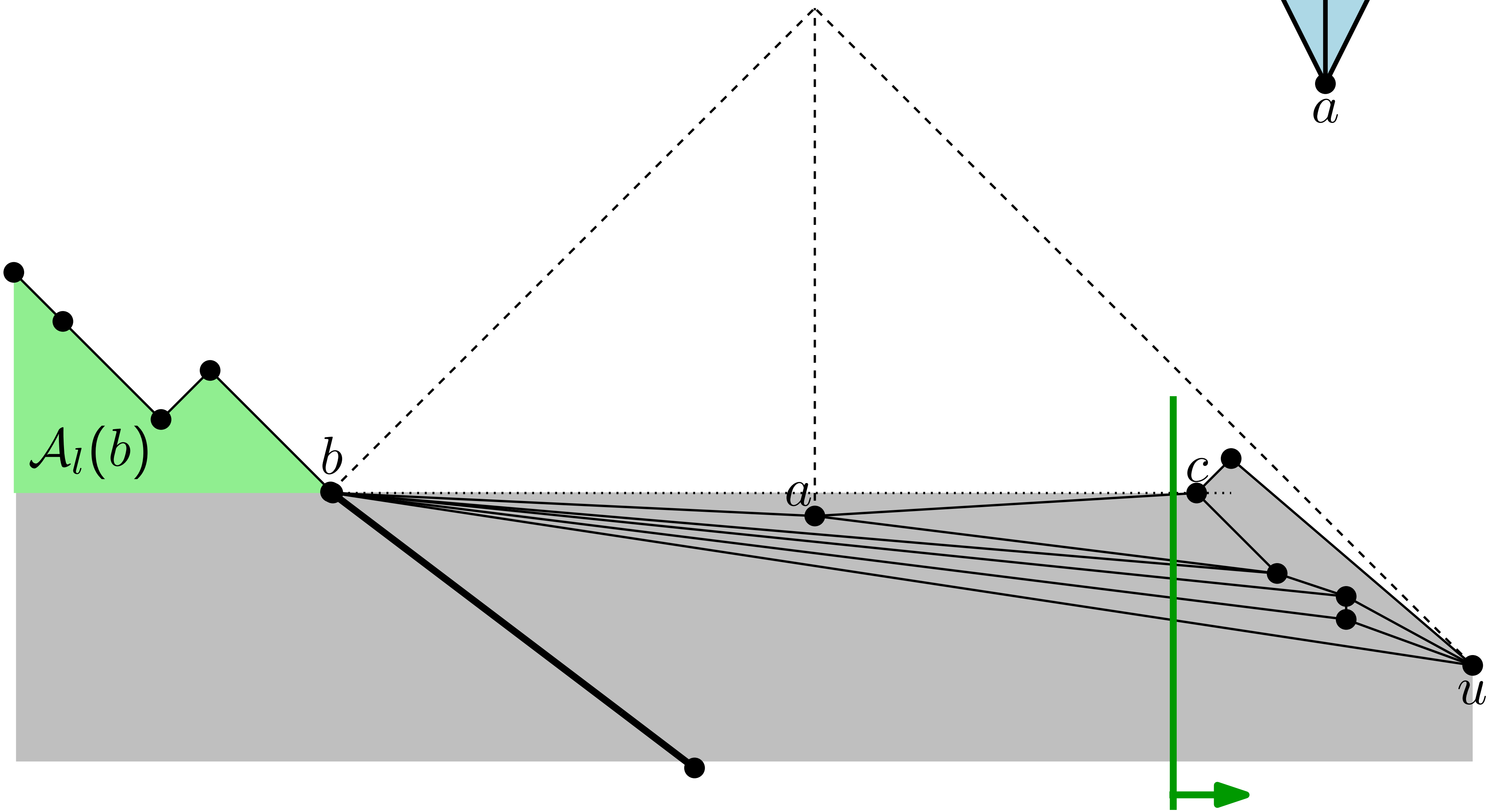

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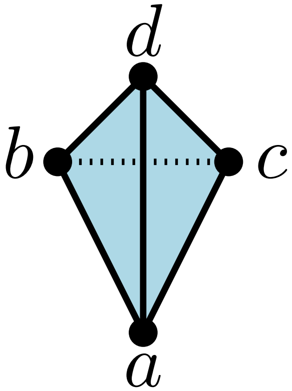

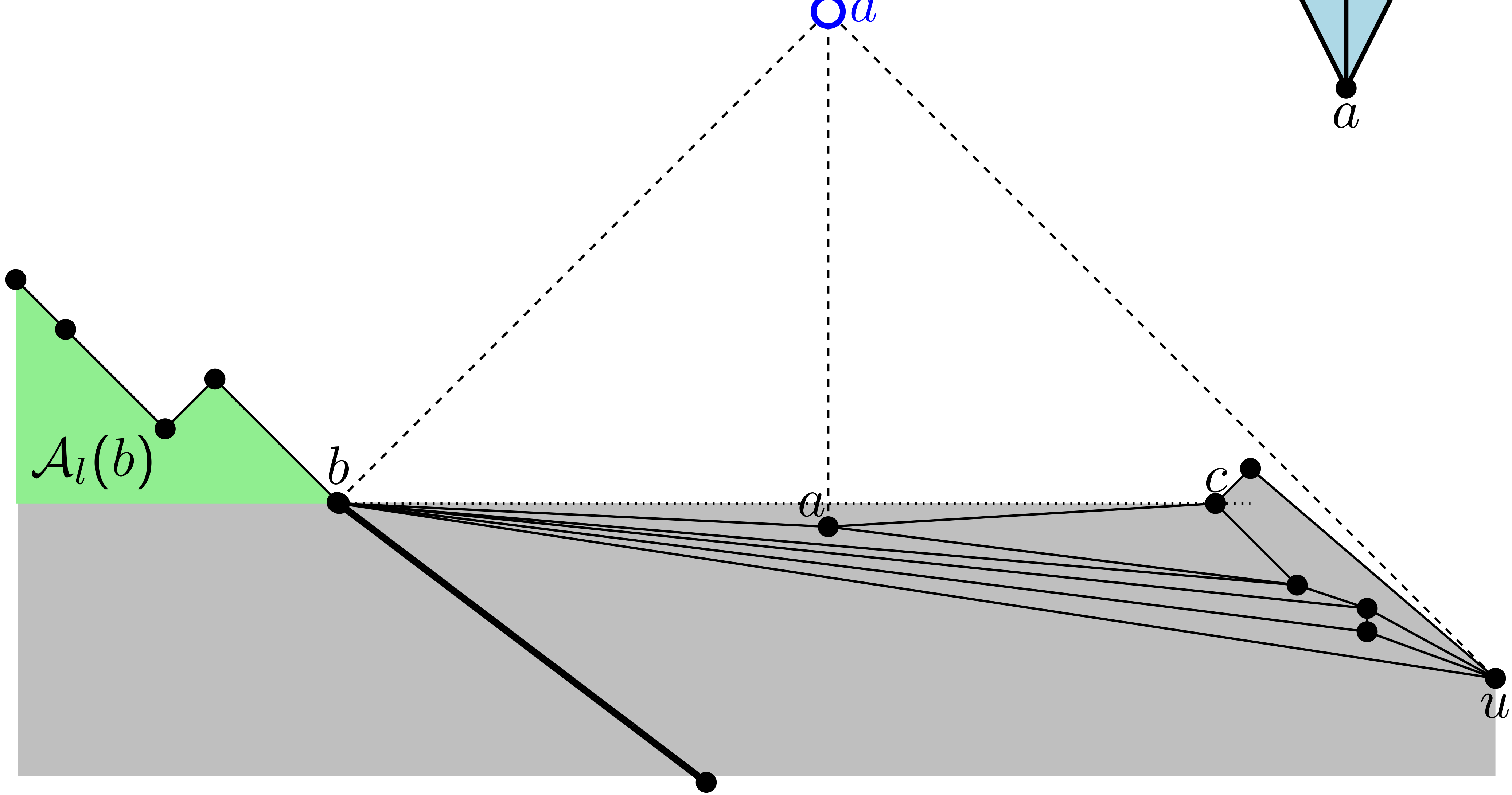

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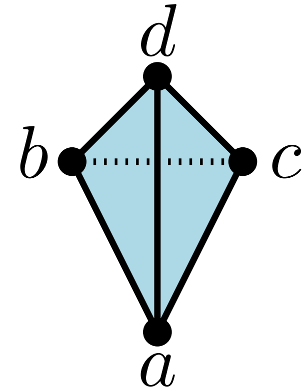

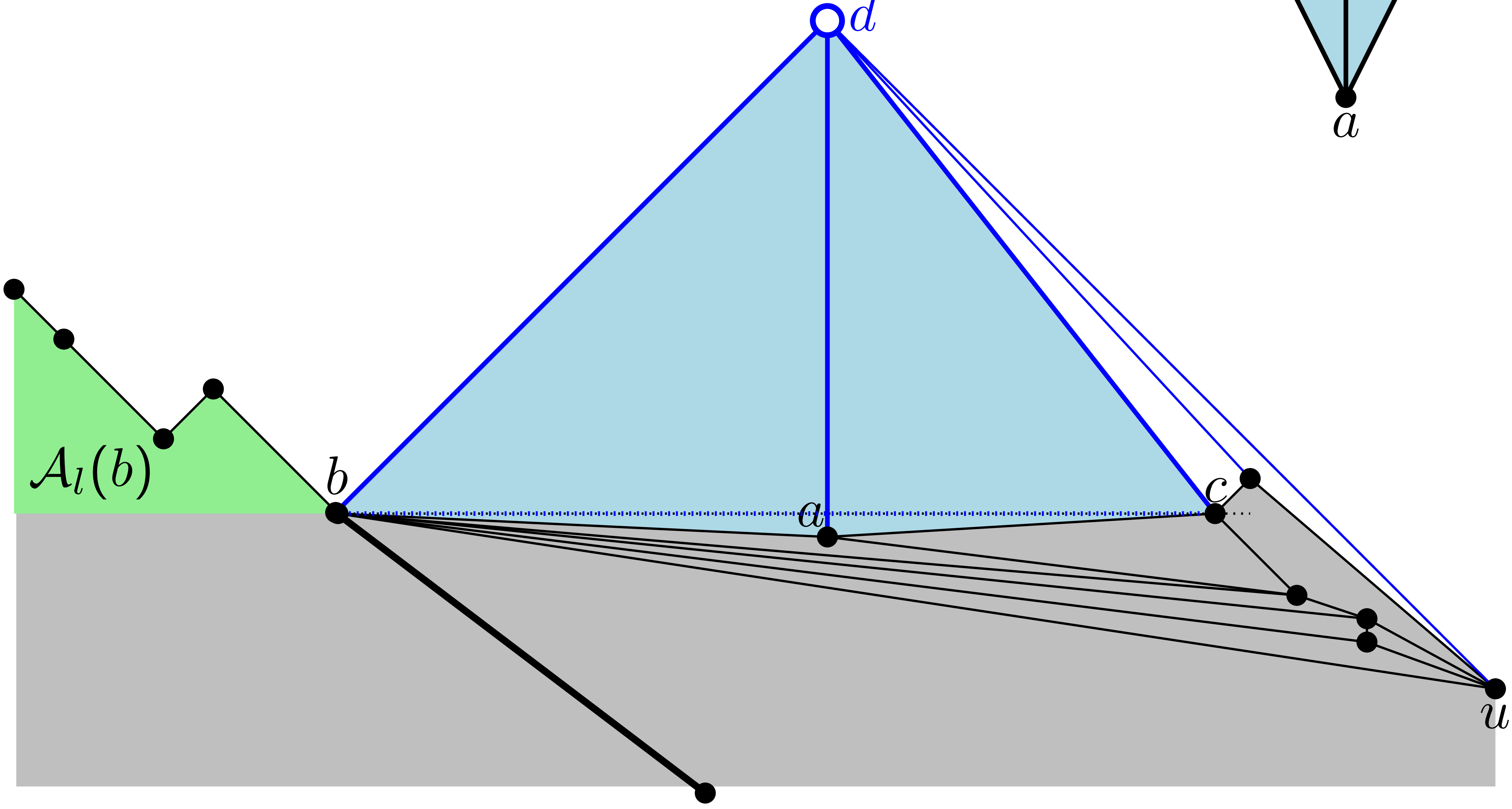

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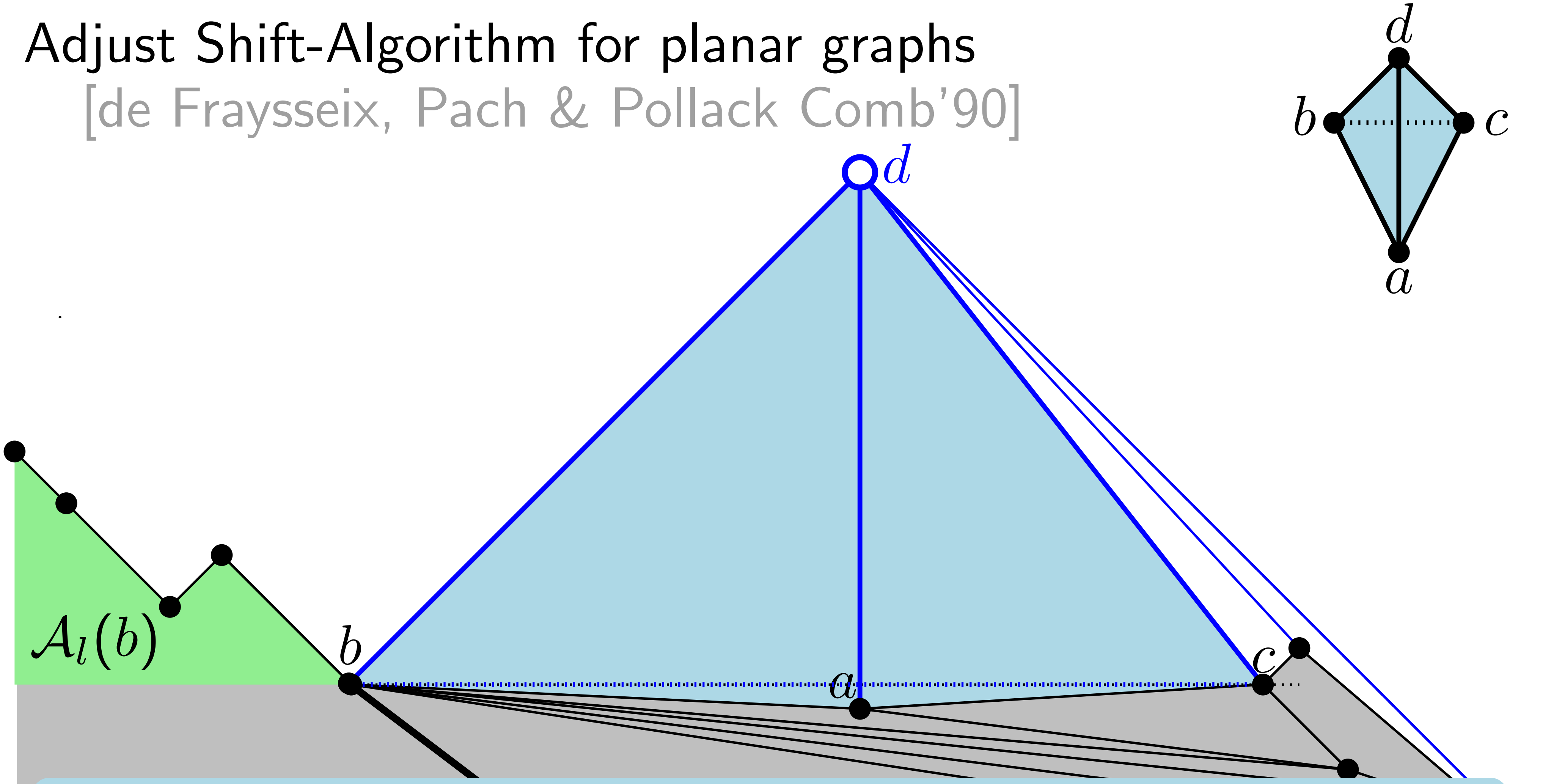
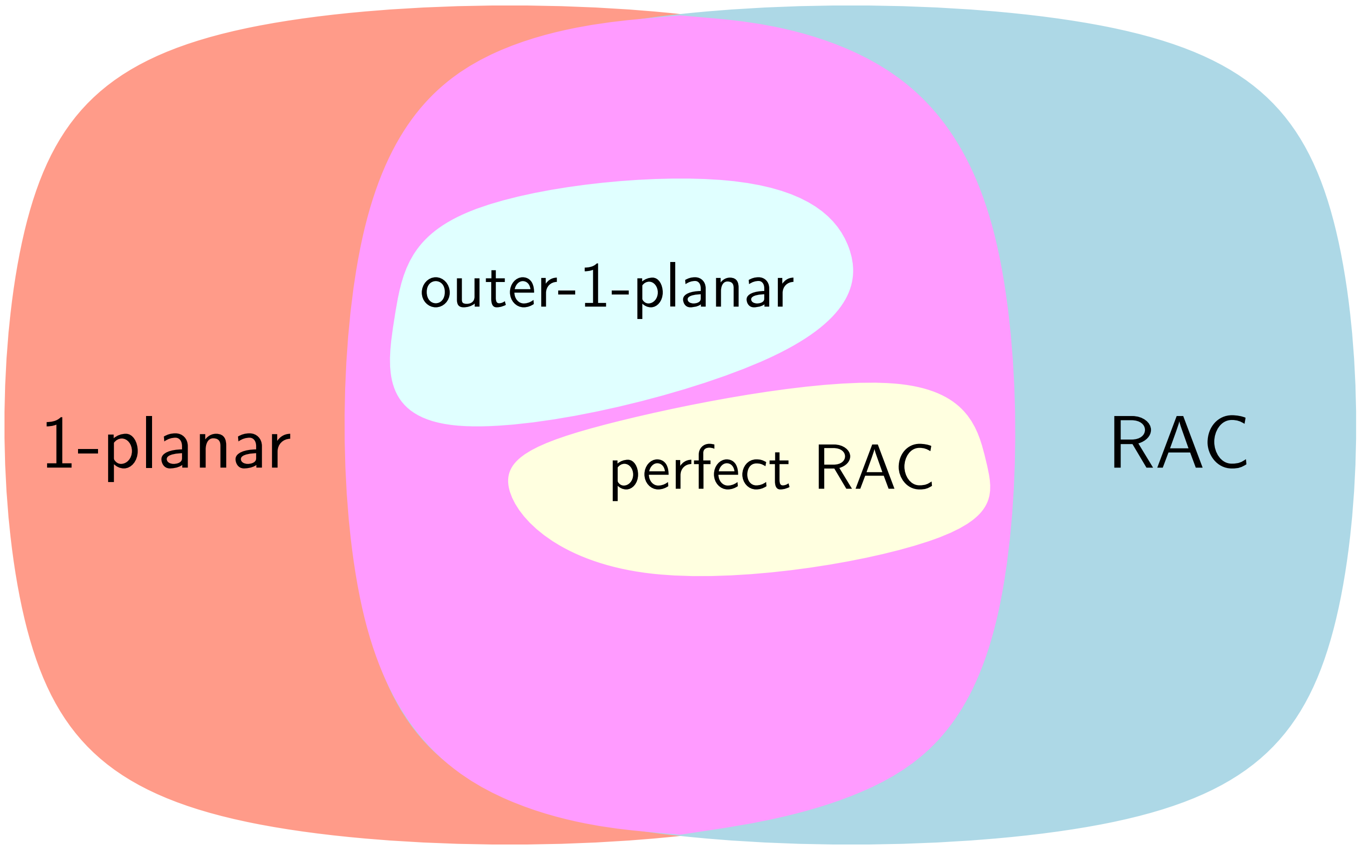
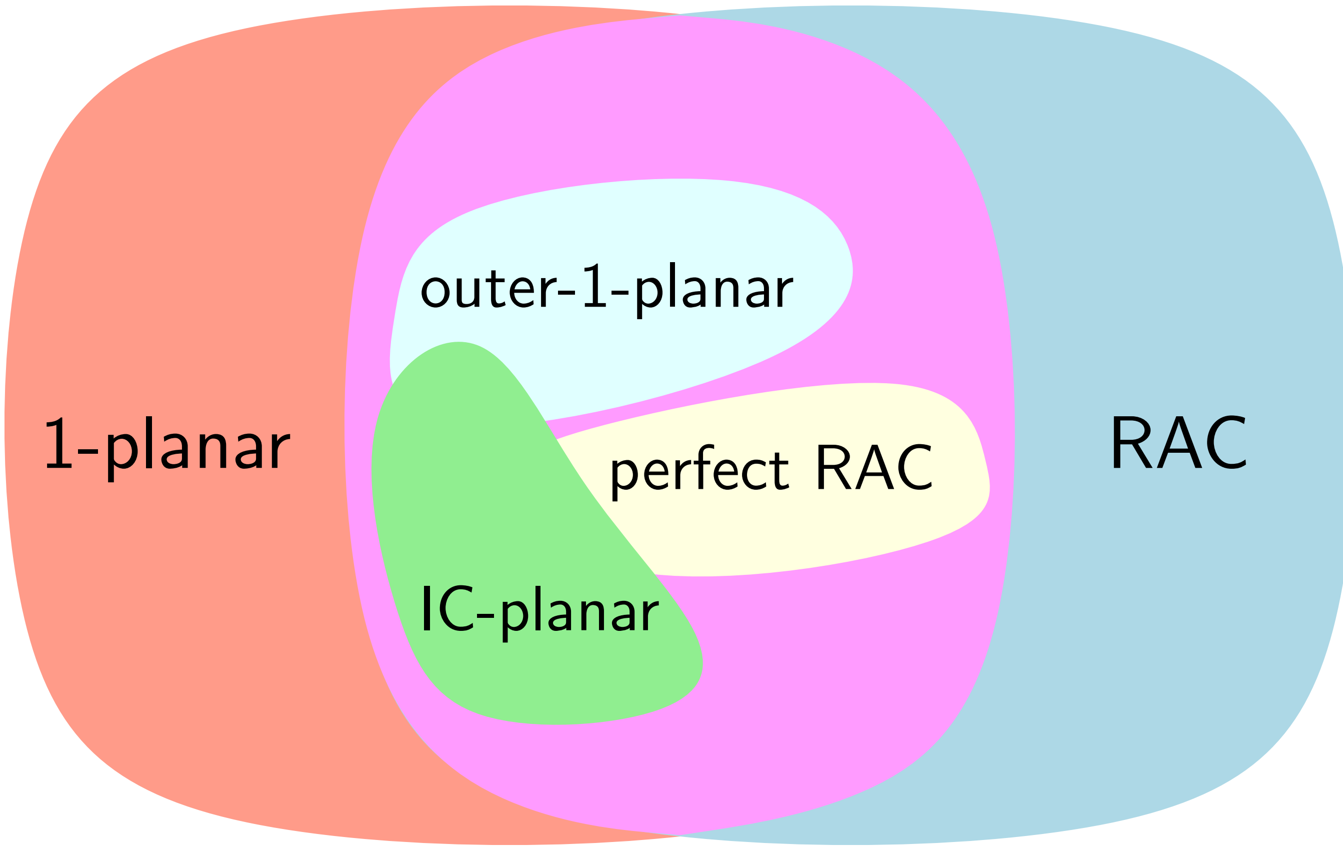


Theorem.
IC-planar graphs can be drawn straight-line RAC in exponential area in $O(n^3)$ time.

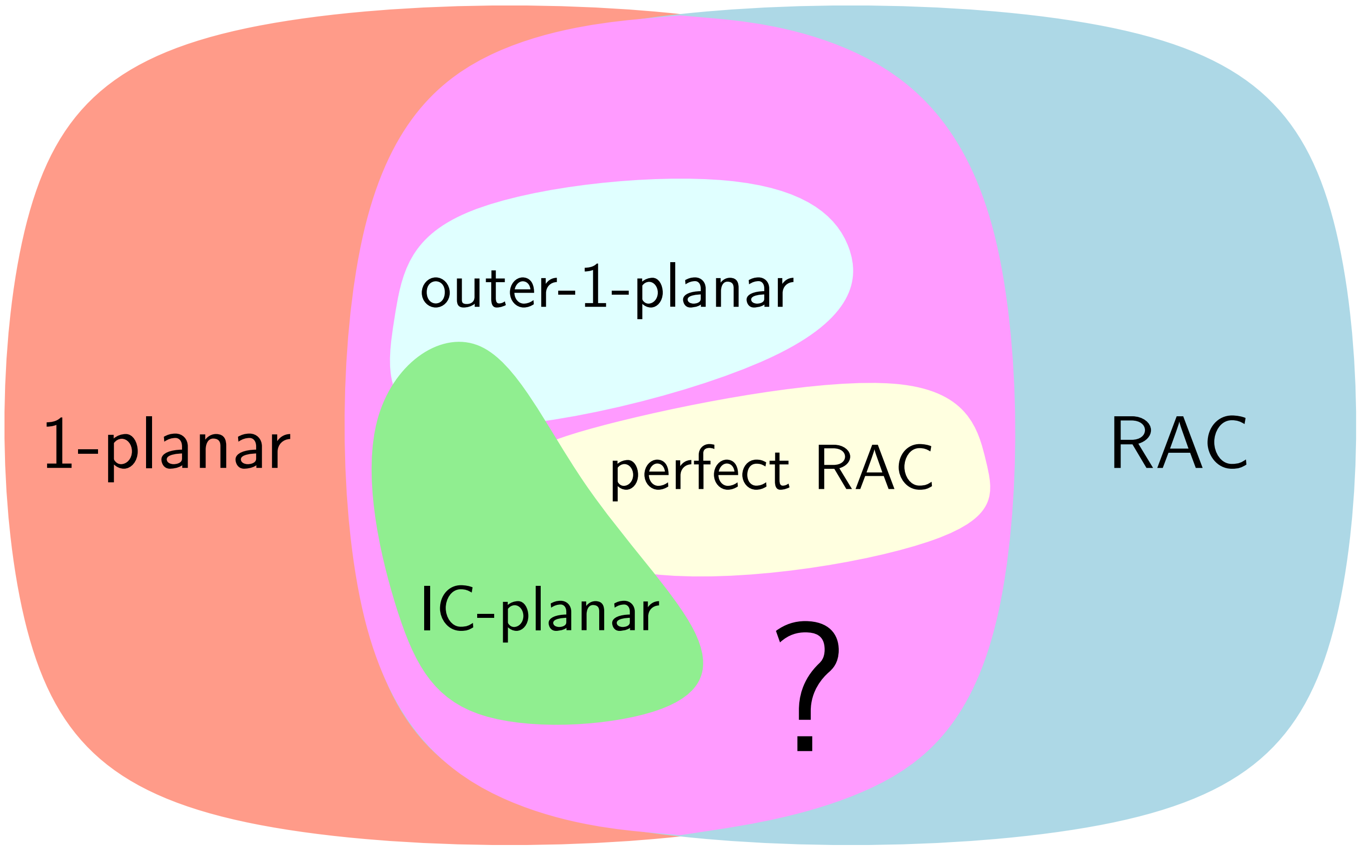
Conclusion



Conclusion

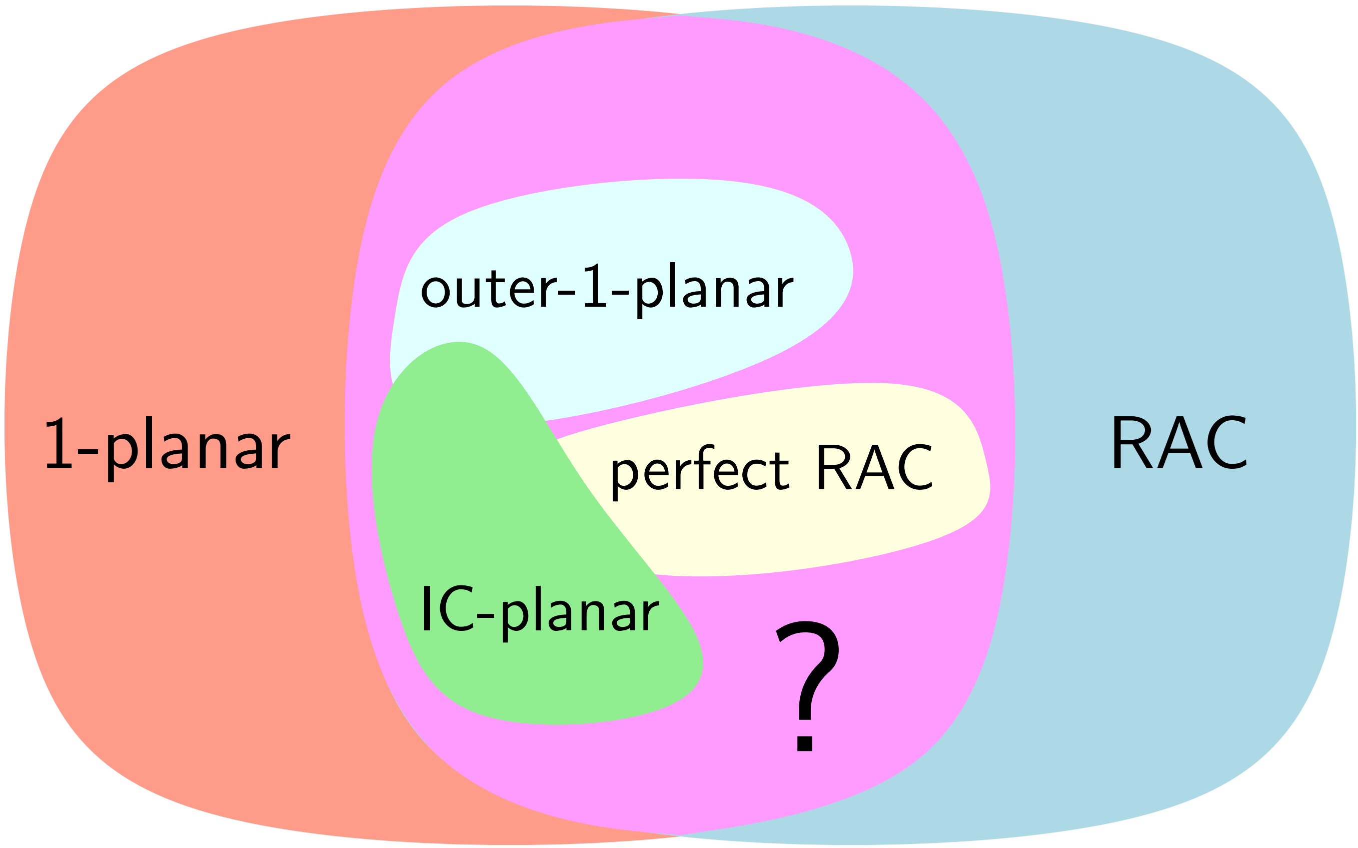


Conclusion



Conclusion

Draw in polynomial area with good crossing resolution?



Conclusion

Draw in polynomial area with good crossing resolution?

What about maximal IC-planar graphs?