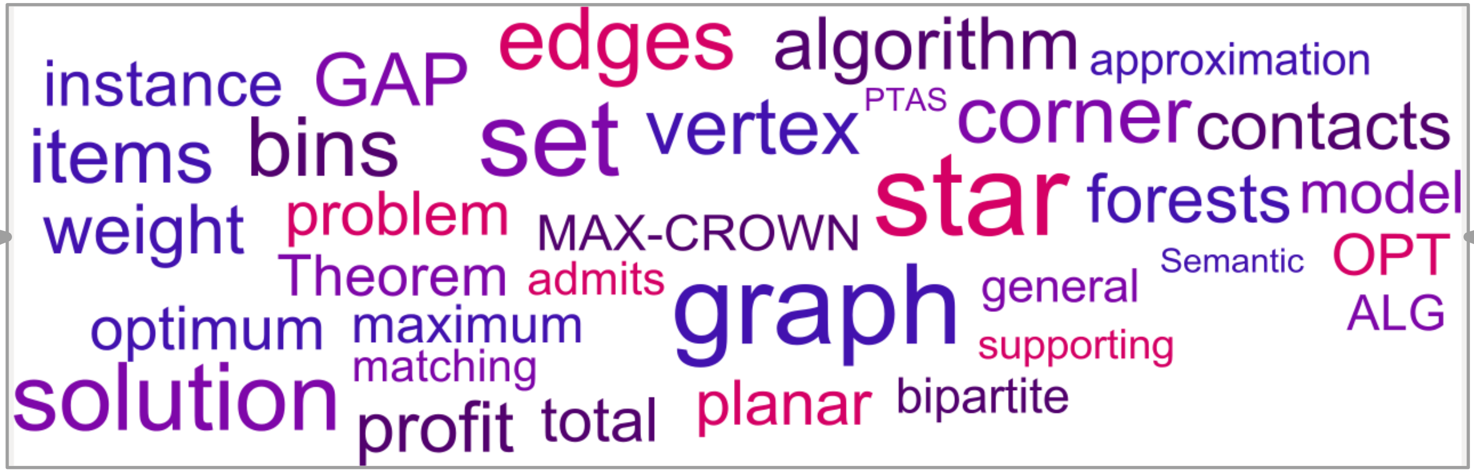

Approximation Algorithms for Contact Representations of Rectangles

Michalis Bekos Thomas van Dijk
Martin Fink Philipp Kindermann
Stephen Kobourov Sergey Pupyrev
Joachim Spoerhase Alexander Wolff

Universität Tübingen
University of Arizona
Universität Würzburg

2008 U.S. Presidential Elections

Heather Williams, June 2008

Coalition treaty 2013

Spiegel
Online
Nov. 2013

Coalition treaty 2013

Words that were more important in the 2013 than in the 2009 treaty

Spiegel Online Nov. 2013

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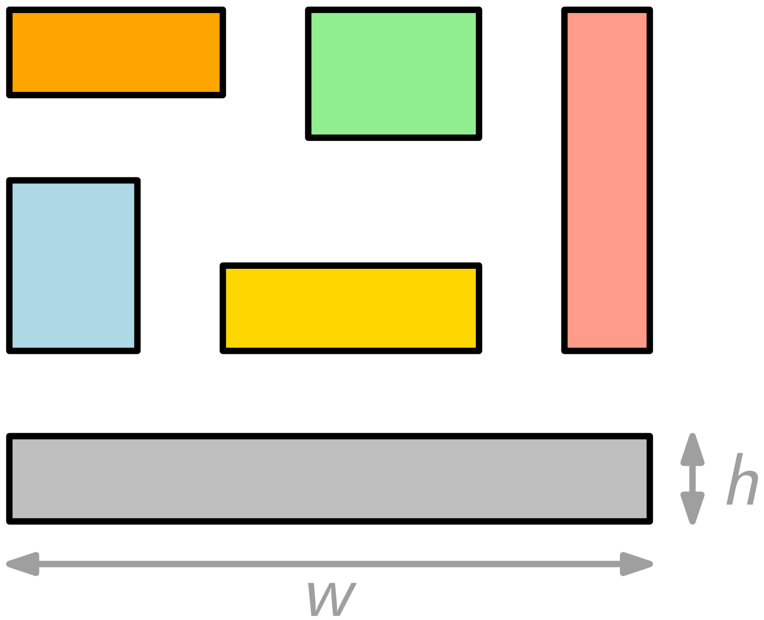
Coalition treaty 2013

Words that were more important in the 2013 than in the 2009 treaty

Spiegel Online
Nov. 2013

*C*ontact *R*epresentation *O*f *W*ord *N*etworks

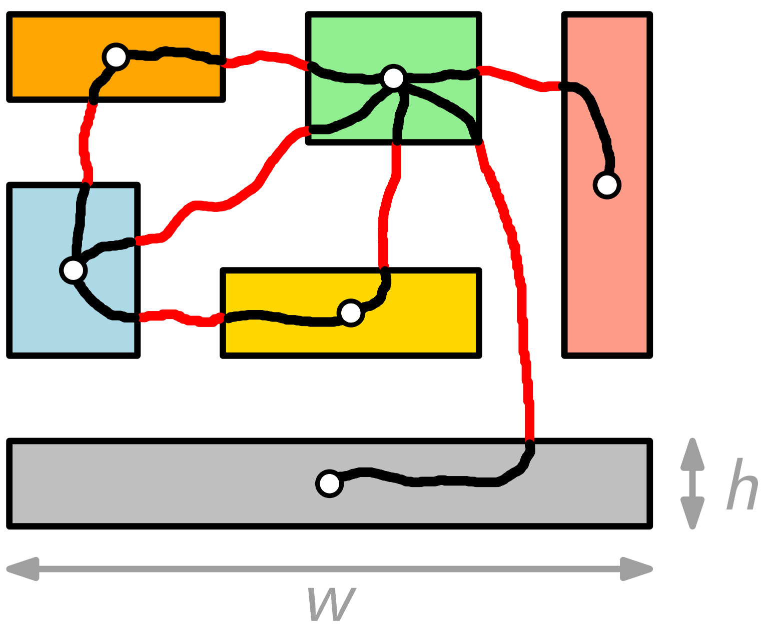
Input



– (integral) box dimensions

Contact Representation Of Word Networks

Input

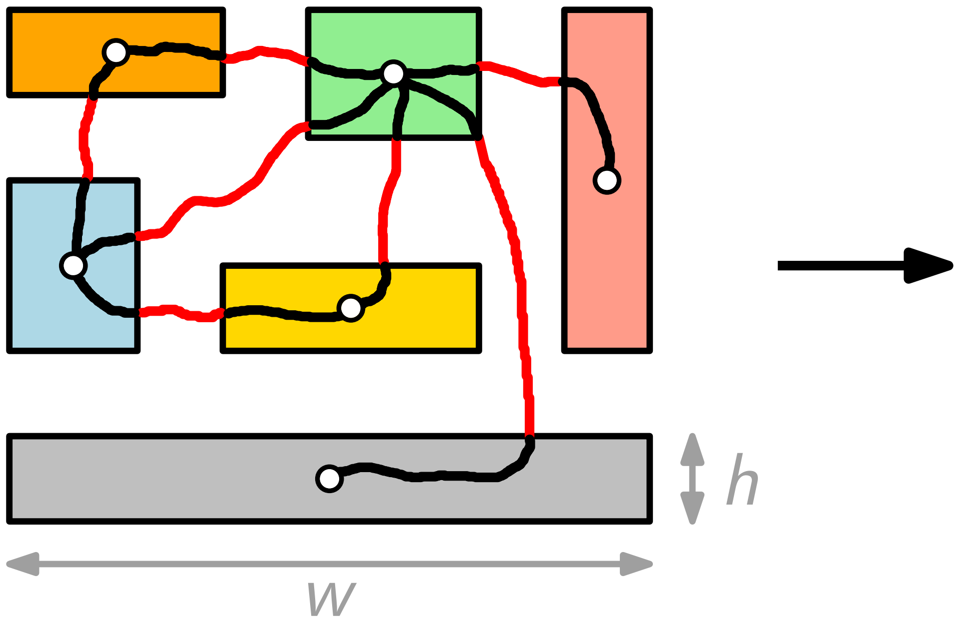


– (integral) box dimensions

– desired contact graph

*C*ontact *R*epresentation *O*f *W*ord *N*etworks

Input **Output**



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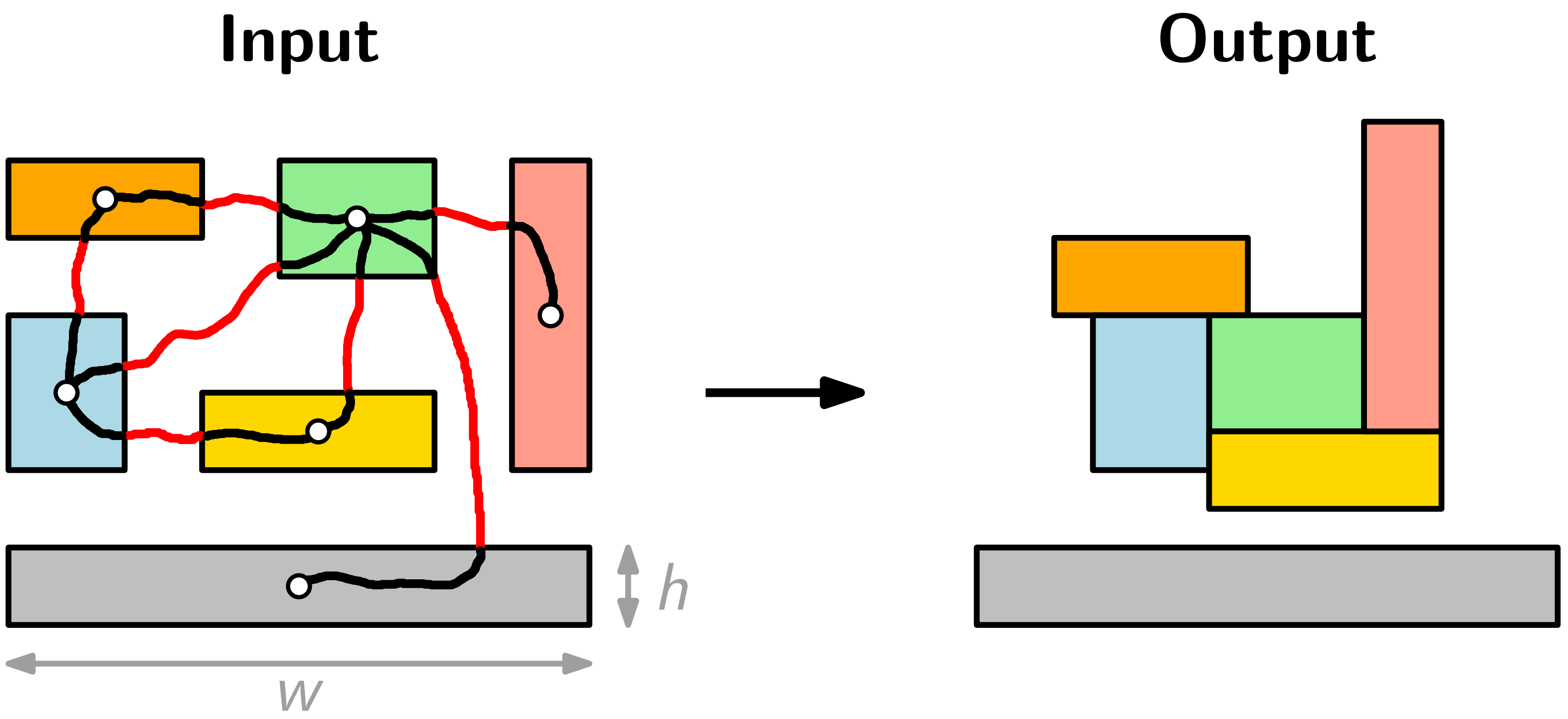
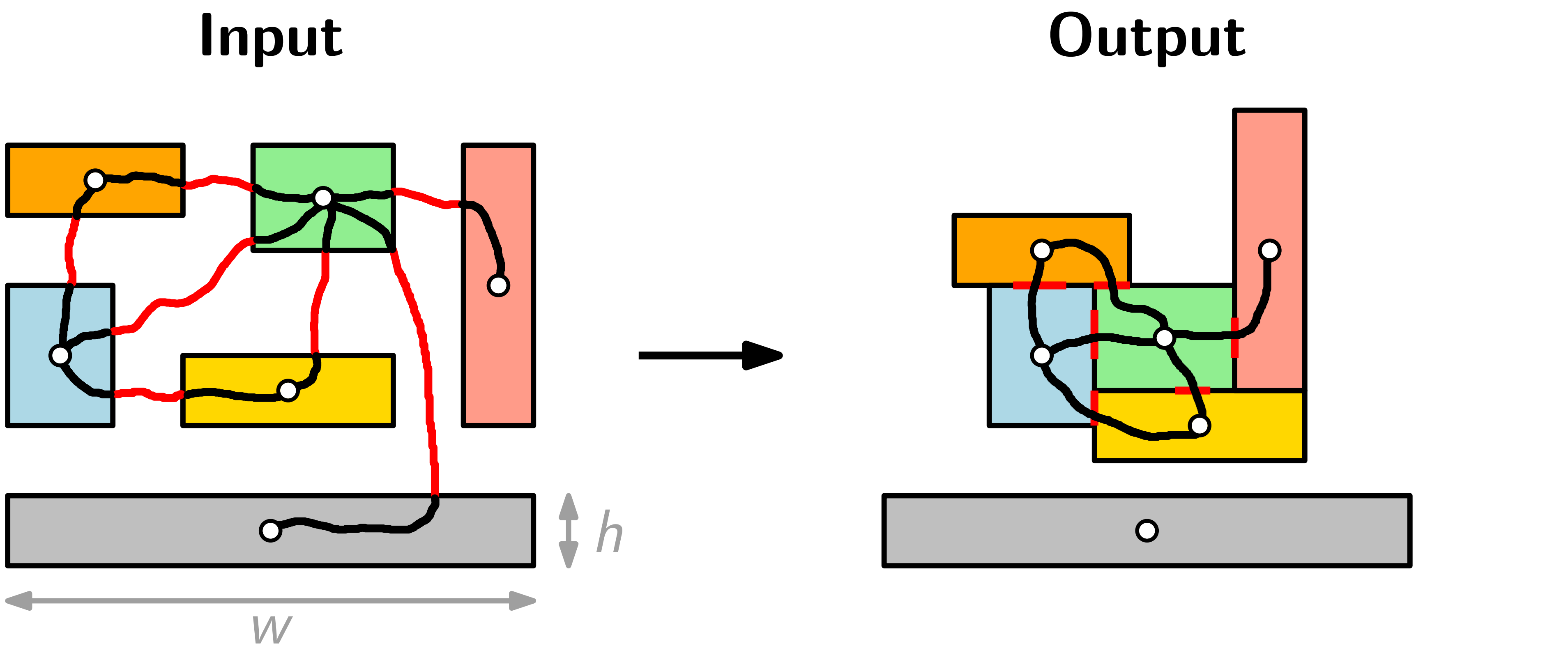


– (integral) box dimensions

– desired contact graph

– placement of boxes

*C*ontact *R*epresentation *O*f *W*ord *N*etworks



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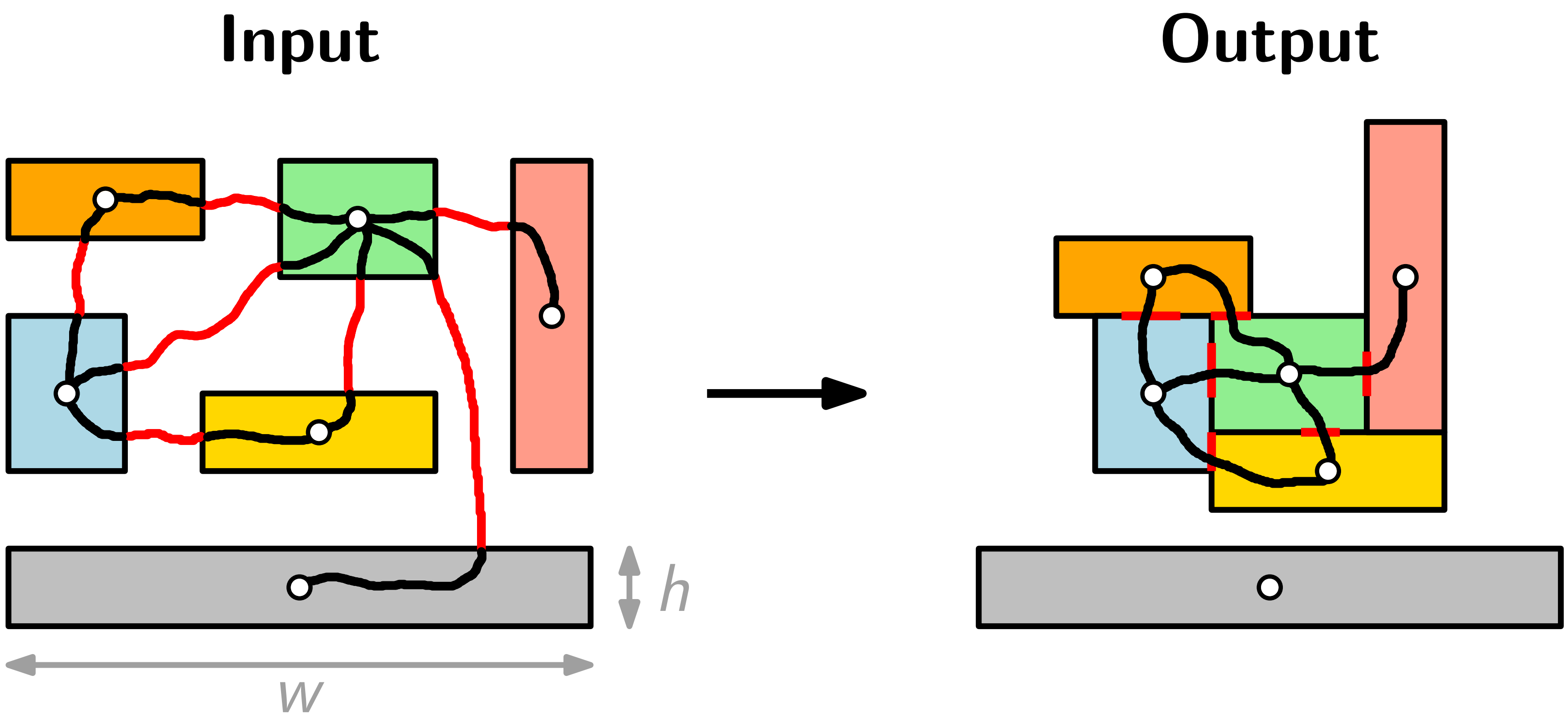
– placement of boxes
– realized desired contacts

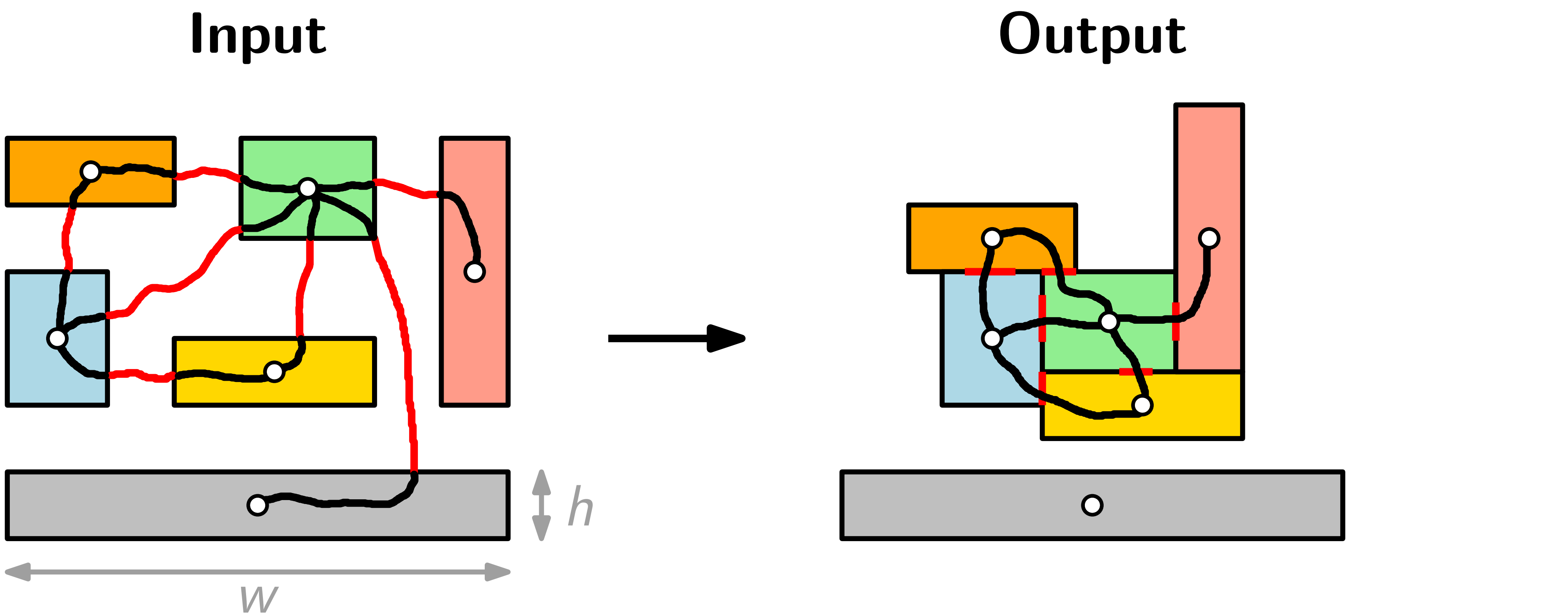
Contact *R*epresentation *O*f *W*ord *N*etworks



– (integral) box dimensions
– desired contact graph

– placement of boxes
– realized desired contacts
– profit: 1 unit / desired edge

*C*ontact *R*epresentation *O*f *W*ord *N*etworks



– (integral) box dimensions
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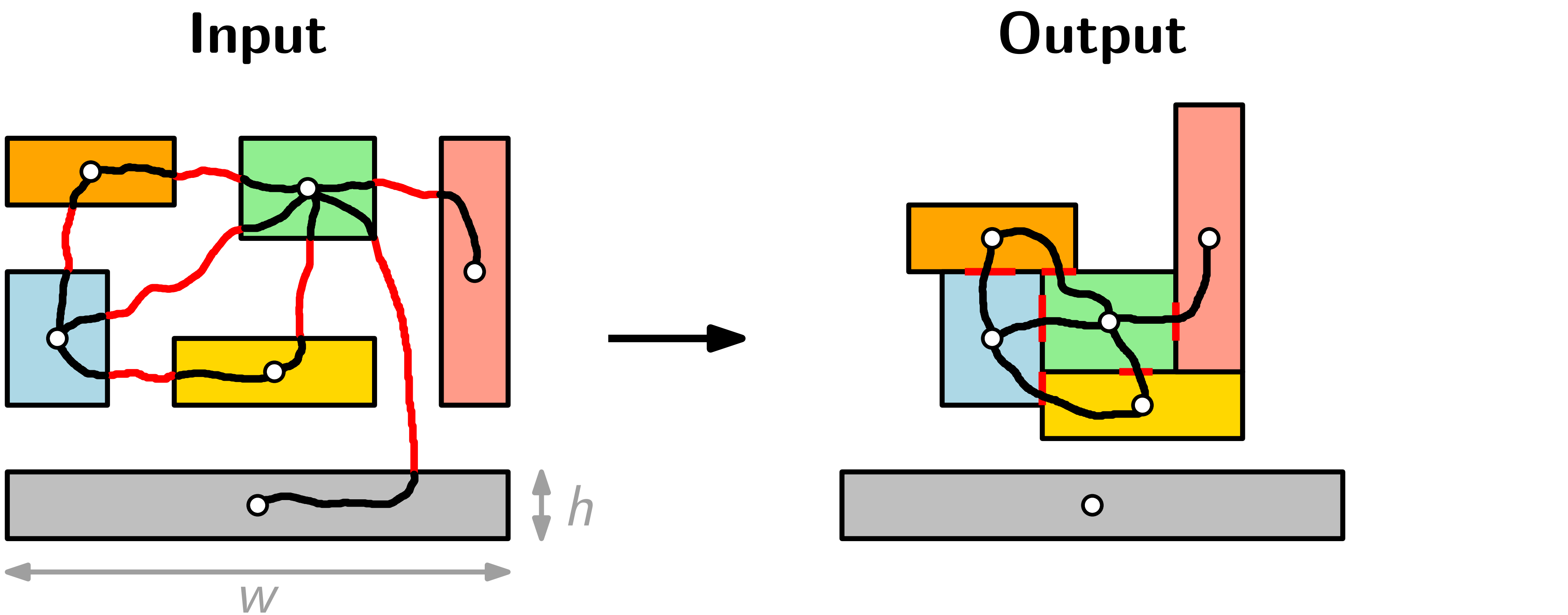
– placement of boxes
– realized desired contacts
– profit: 1 unit / desired edge

MAX-CROWN:

Contact Representation Of Word Networks



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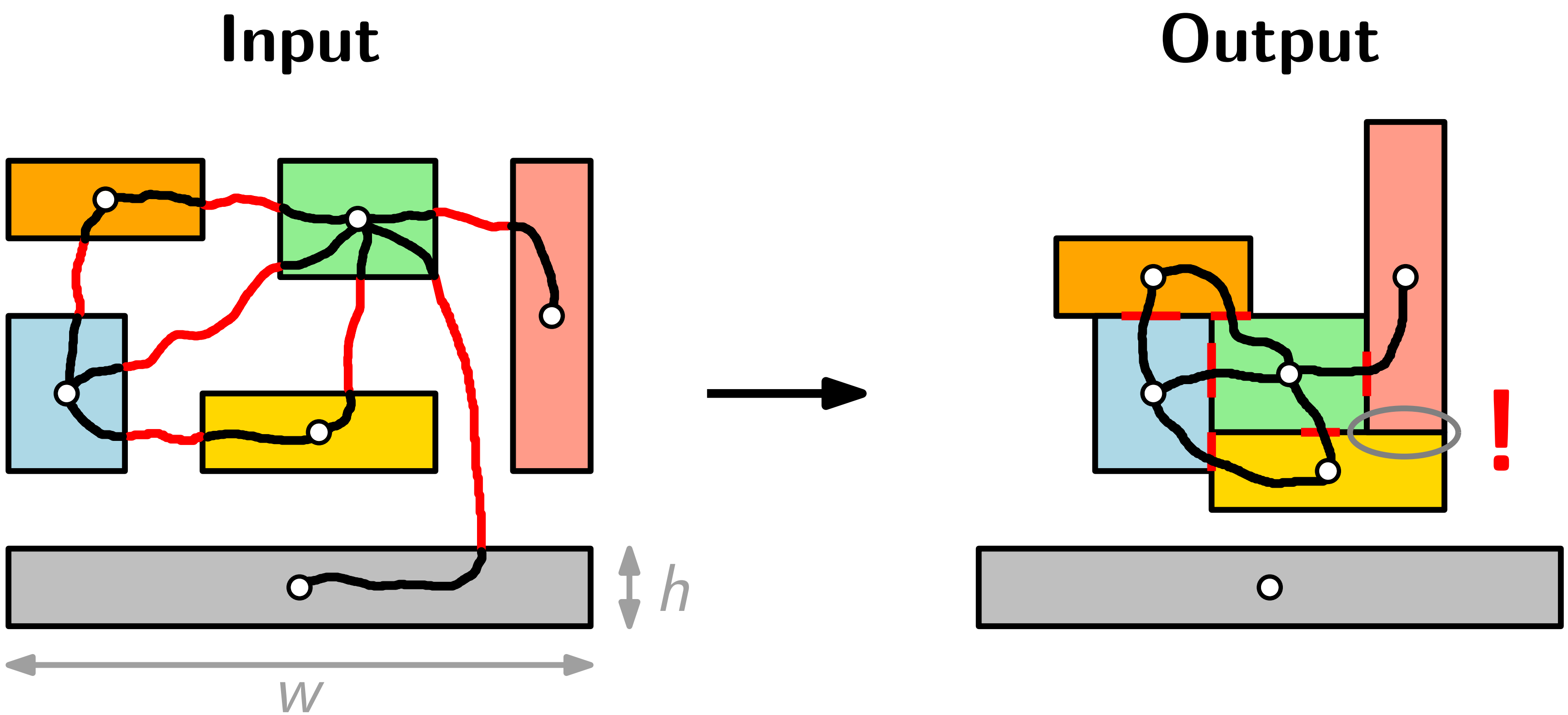
Contact *R*epresentation *Of* *W*ord *N*etworks



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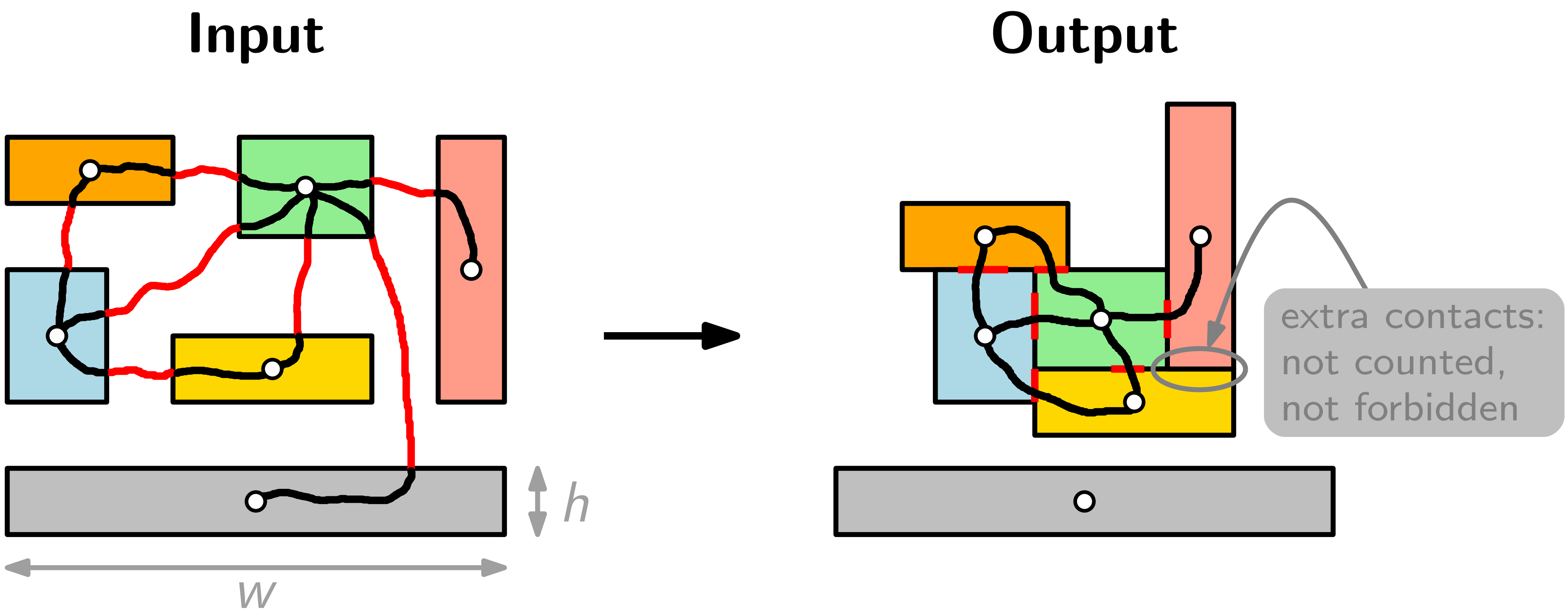
Contact *R*epresentation *O*f *W*ord *N*etworks



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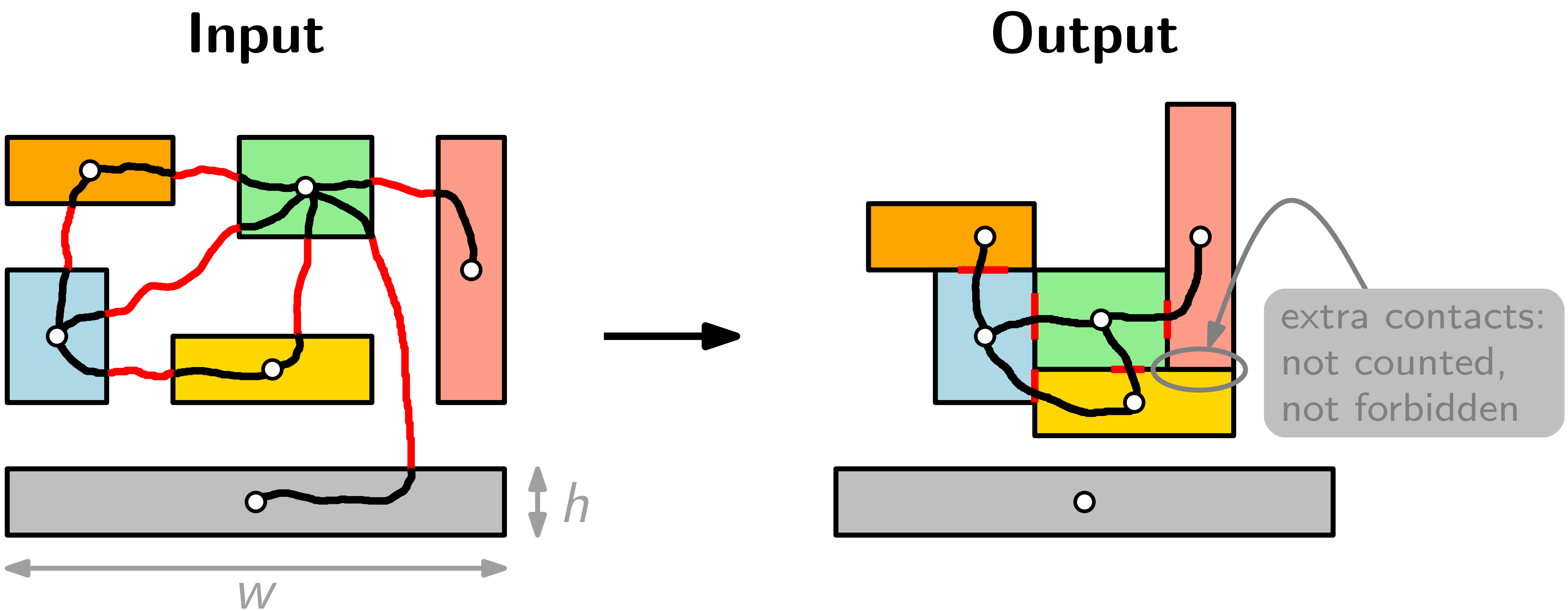
Max-Crown: Maximize profit!

Contact Representation Of Word Networks



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Contact *R*epresentation *O*f *W*ord *N*etworks

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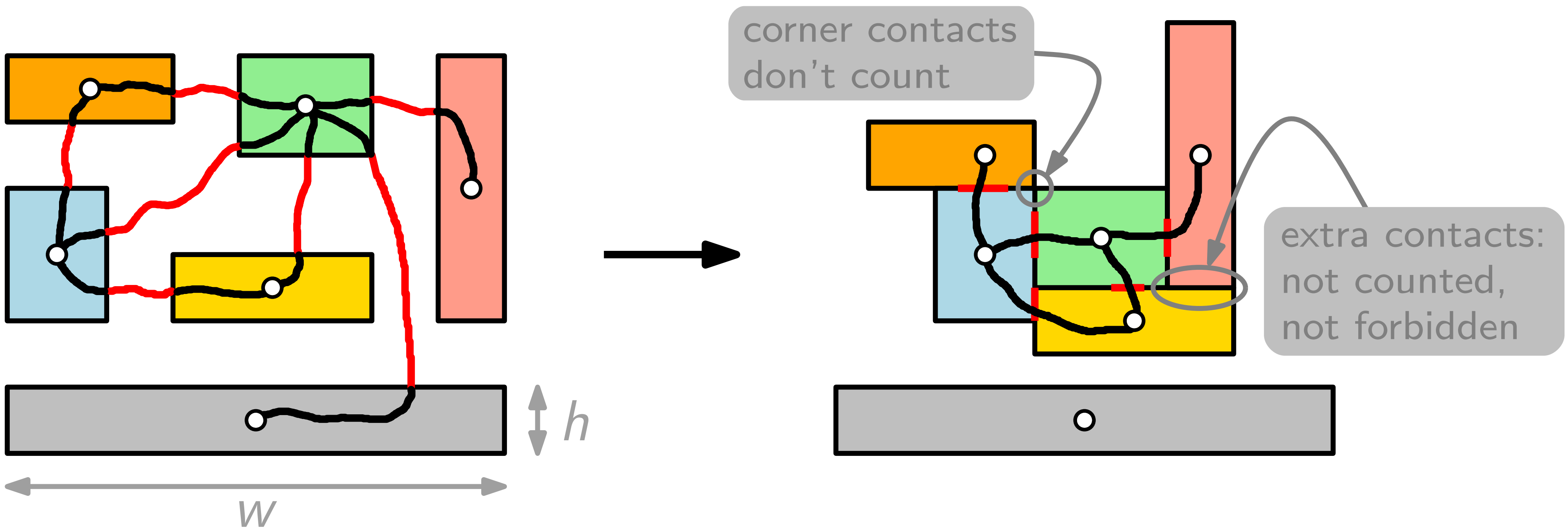

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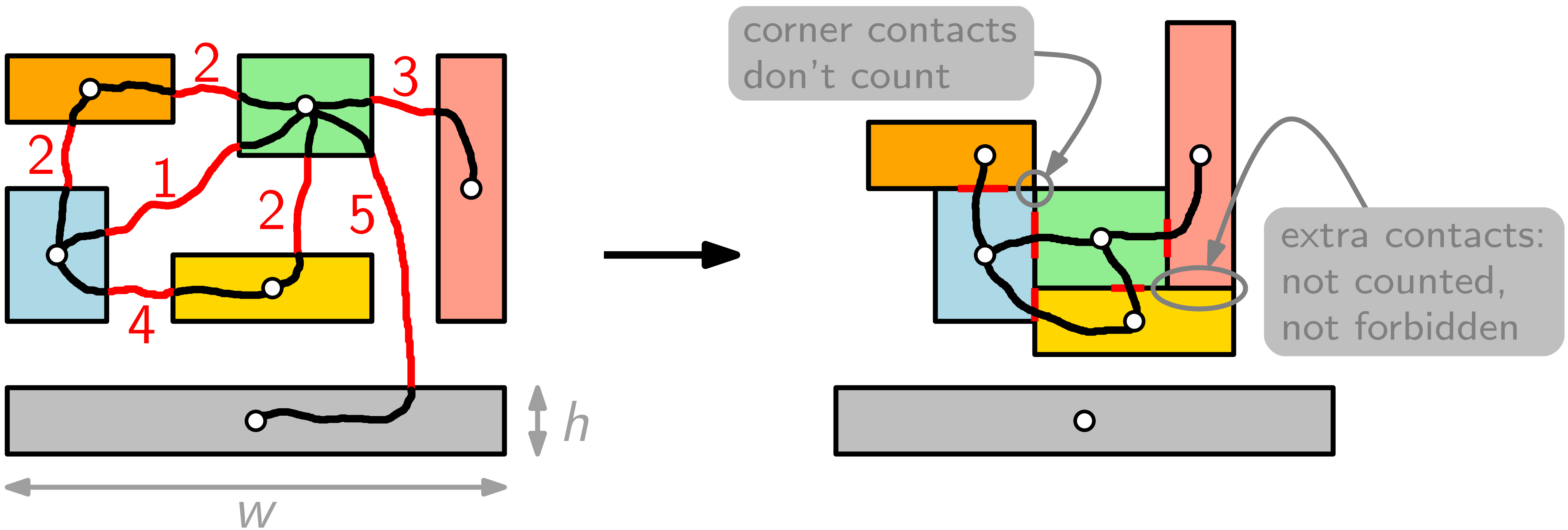

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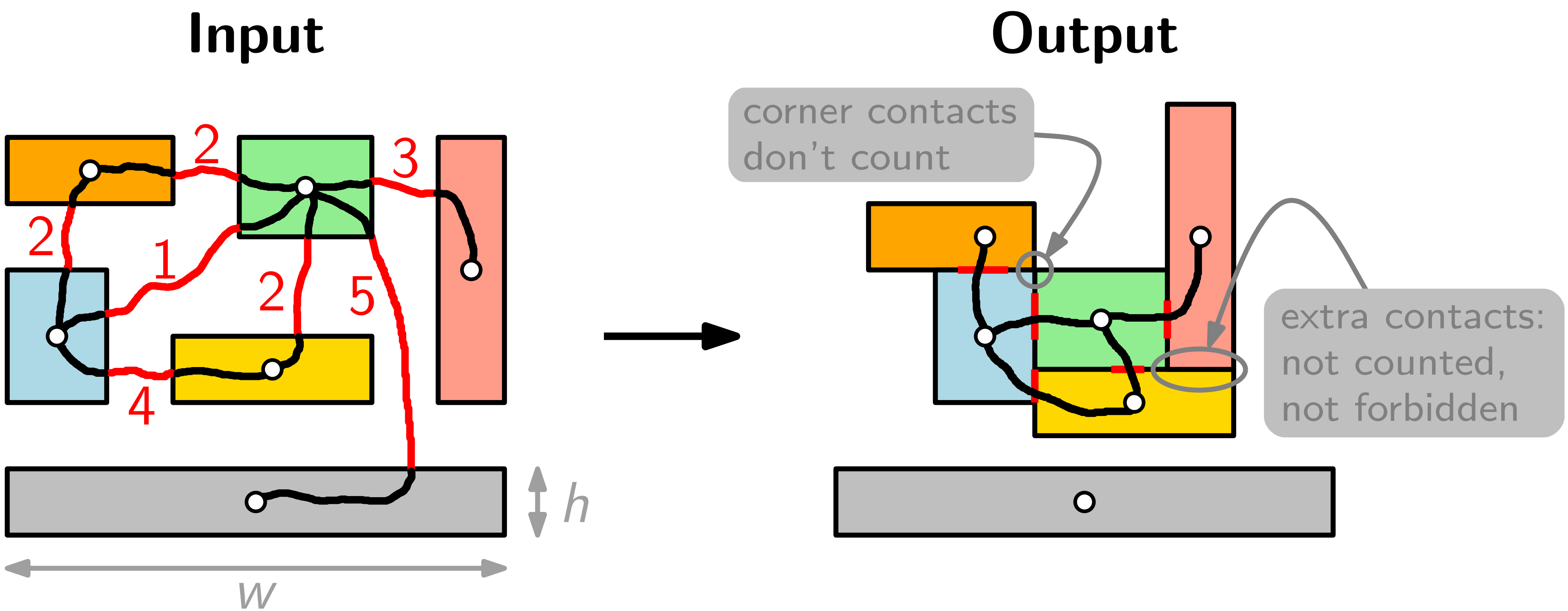
Contact *R*epresentation *O*f *W*ord *N*etworks



- (integral) box dimensions
- desired contact graph

- placement of boxes
- realized desired contacts
- profit: ~~1~~ $p(e)$ units / desired edge e

MAX-CROWN: Maximize profit!

Related Work

- rectangle / cube representation of graphs

Related Work

- rectangle / cube representation of graphs
  - Every planar graph w/o sep. triangles has a touching rectangle representation

[Koźminński & Kinnen, Networks'85; He, SICOMP'93; He & Kant, TCS'97]

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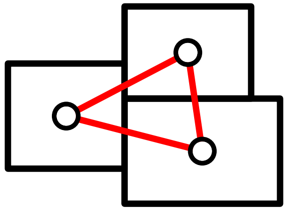

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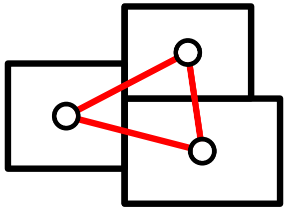

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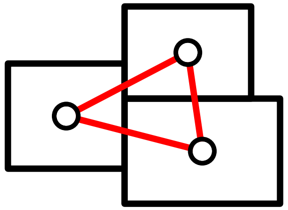

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    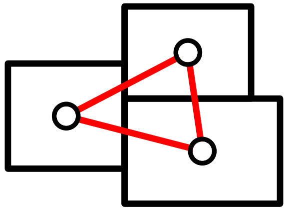

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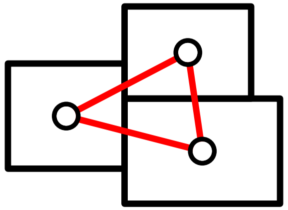

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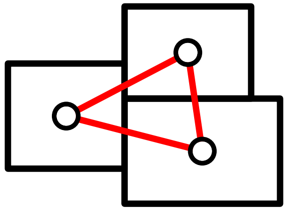

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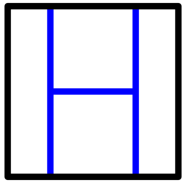

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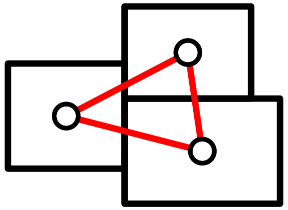

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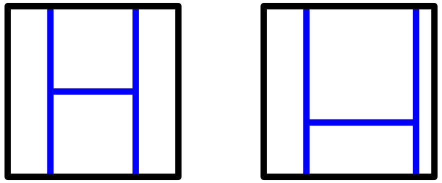

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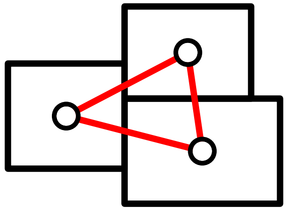

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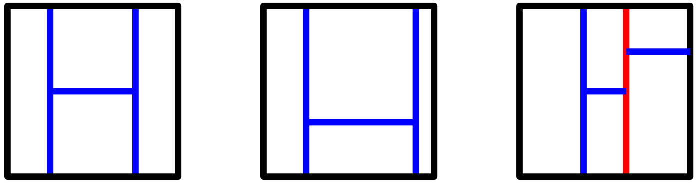

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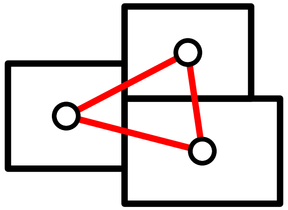

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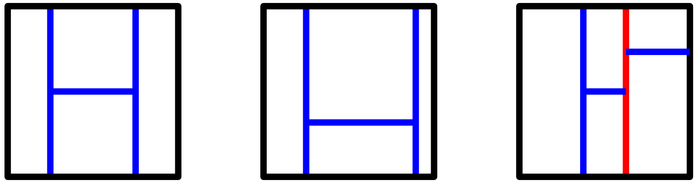

- rectangle representations with edge weights

Related Work

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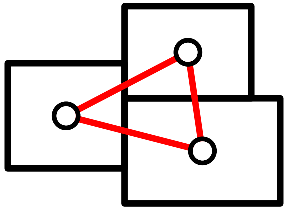

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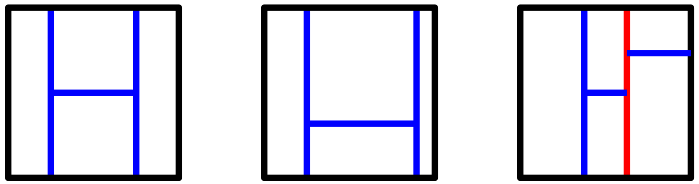

- rectangle representations with edge weights
  - edge weights prescribe length of contact [Nöllenburg et al., GD'12]

Our Results – Approximation Factors

	Weighted	
Graph class	old*	new°
cycle, path	1	
star	α	$1+\varepsilon$
tree	2α, NP-hard	$2+\varepsilon$
max-degree Δ	$\lfloor(\Delta+1)/2\rfloor$	
planar max-deg. Δ		
outerplanar		$3+\varepsilon$
planar	5α	$5+\varepsilon$
bipartite		$16\alpha/3 \approx 8.4$ APX-hard
general		rand.: $32\alpha/3 \approx 16.9$ det.: $40\alpha/3 \approx 21.1$

*) [Barth, Fabrikant, Kobourov, Lubiw, Nöllenburg, Okamoto, Pupyrev, Squarcella, Ueckerdt, Wolff – LATIN'14]

°) [Bekos, van Dijk, Fink, Kindermann, Kobourov, Pupyrev, Spoerhase, Wolff – submitted]

$$\alpha = e/(e-1) \approx 1.58$$

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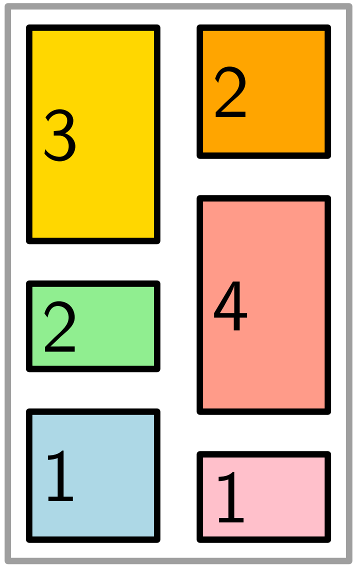
Tool #1: GAP

Tool #1: Gap

Knapsack

items

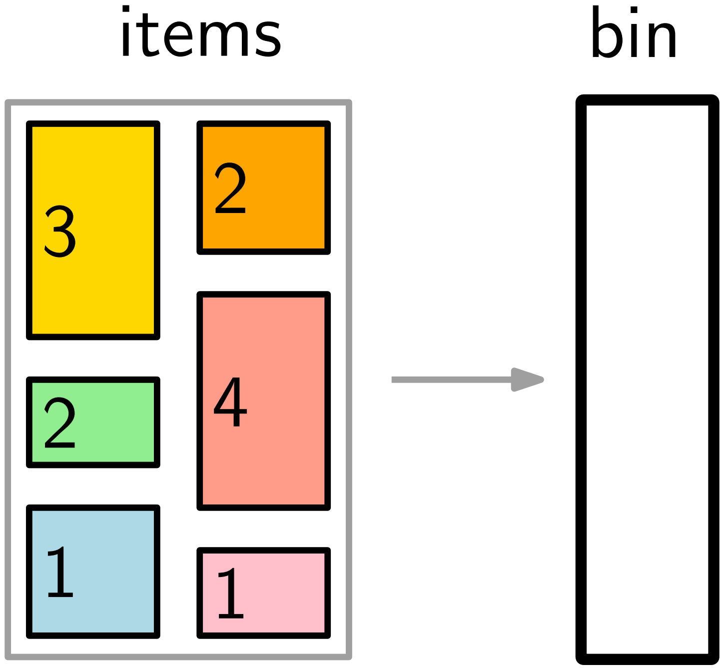


– size s_i

– value v_i

Tool #1: GAP

KNAPSACK



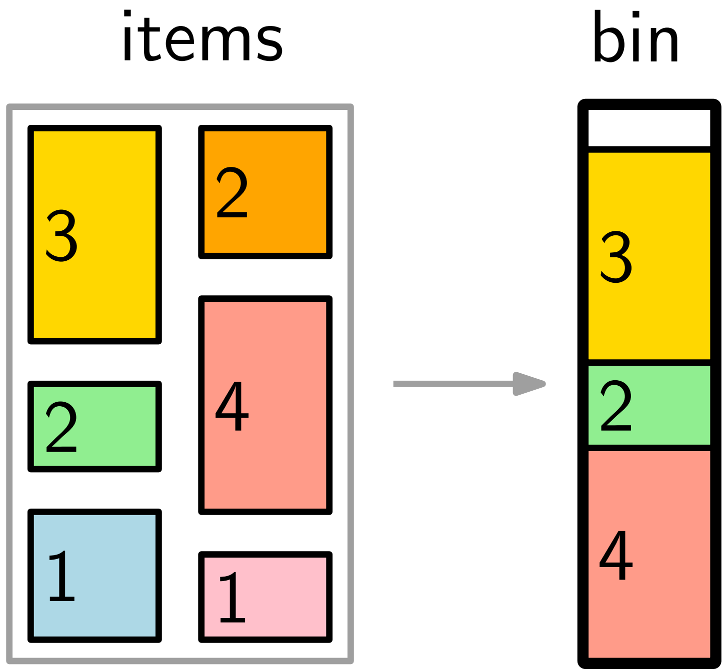
– size s_i

– value v_i

– bin has capacity c

Tool #1: Gap

Knapsack



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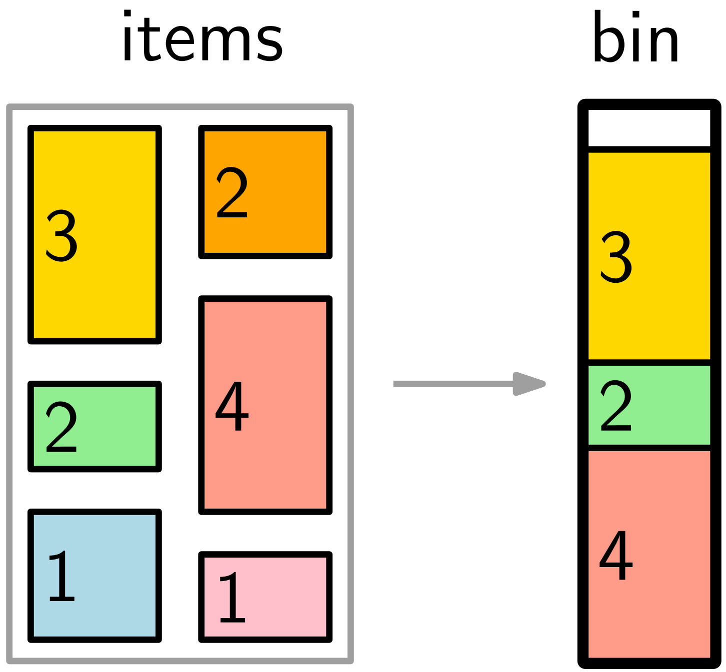

– size s_i

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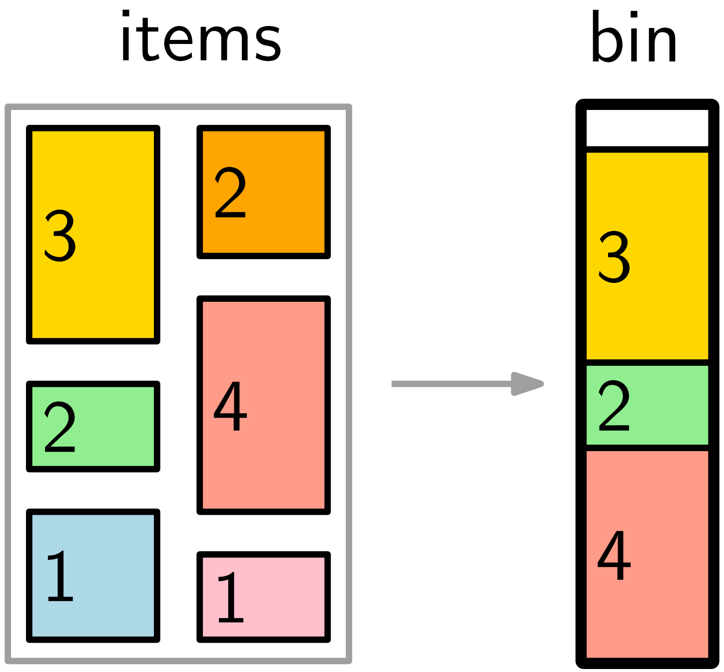
– maximize total value packed

Tool #1: Gap

Knapsack

Generalized Assignment Prob.



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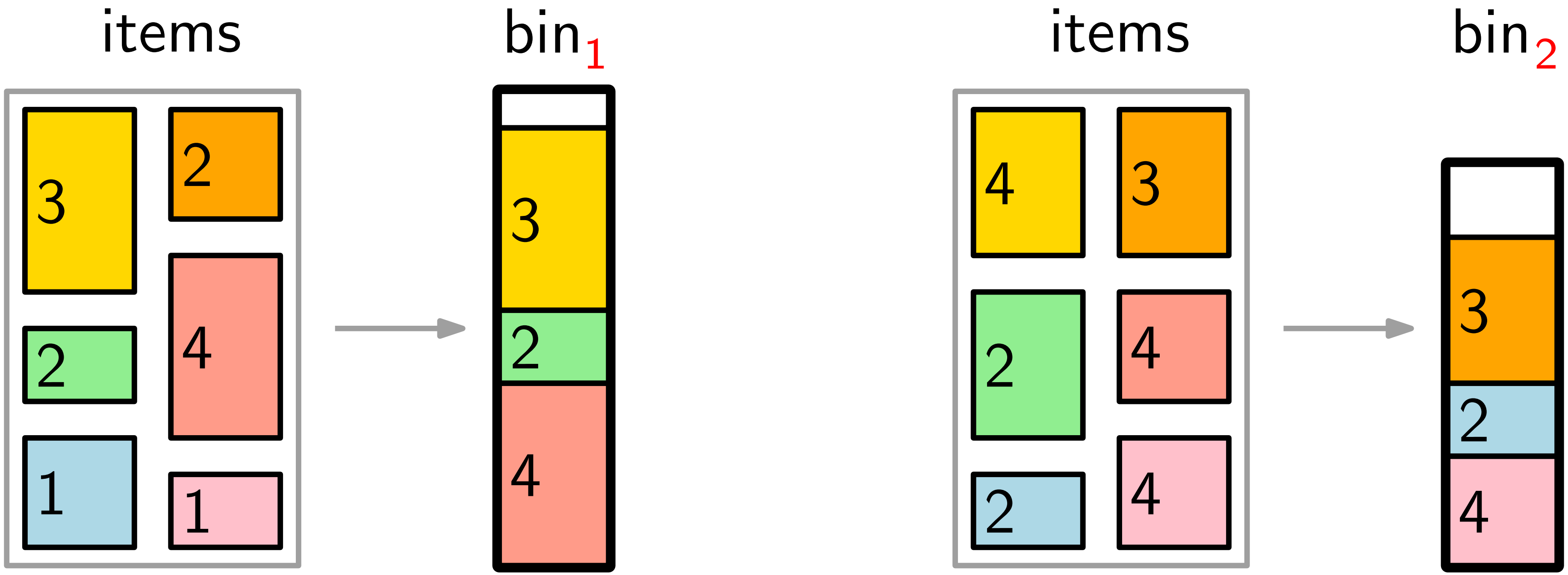


– size s_{ij}

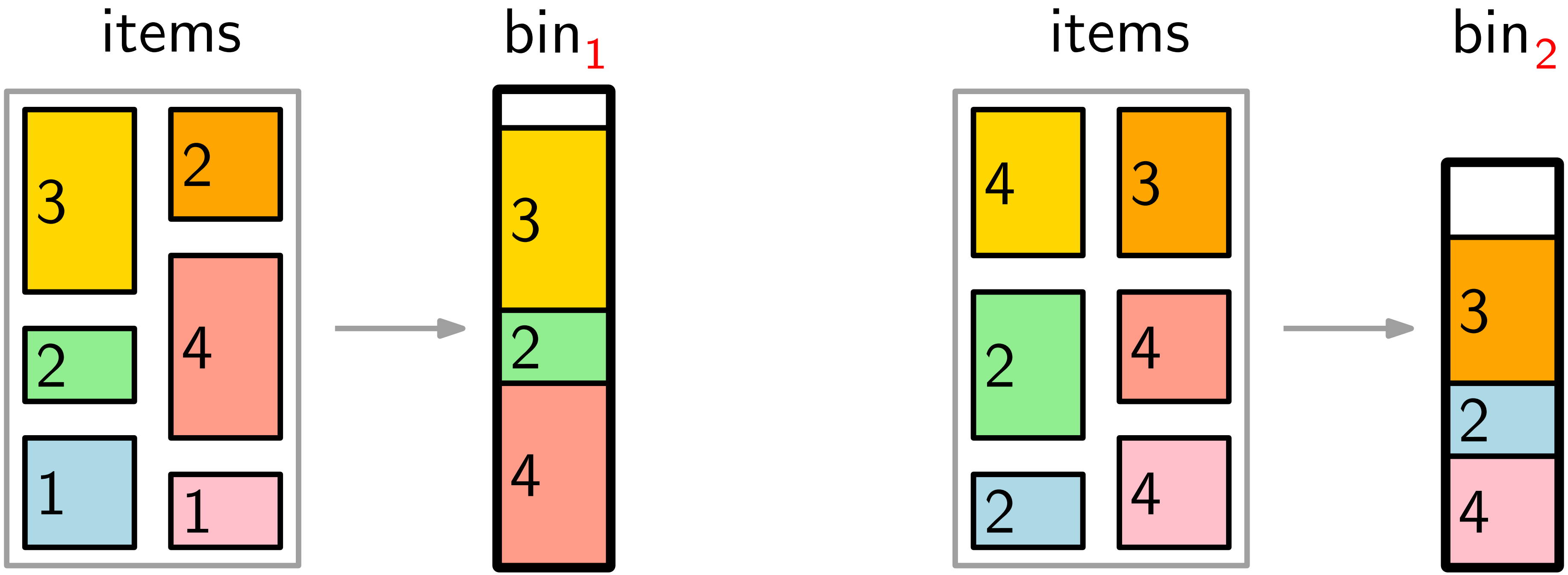
– value v_{ij}

– bin$_j$ has capacity c_j

– maximize total value packed

Tool #1: Gap

Knapsack Generalized Assignment Prob.

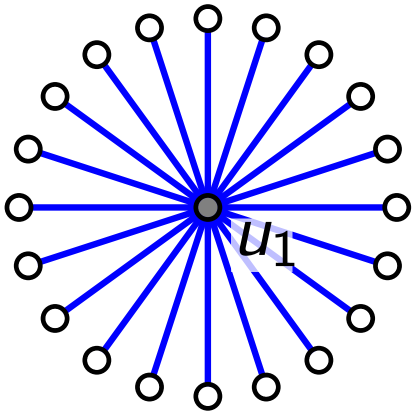


– size s_{i_j}
– value v_{i_j}

– bin$_j$ has capacity c_j
– maximize total value packed

Theorem. Gap admits an approximation algorithm with ratio $\alpha = e/(e-1) \approx 1.58$. [Fleischer et al., MOR'11]

Max-Crown for Stars



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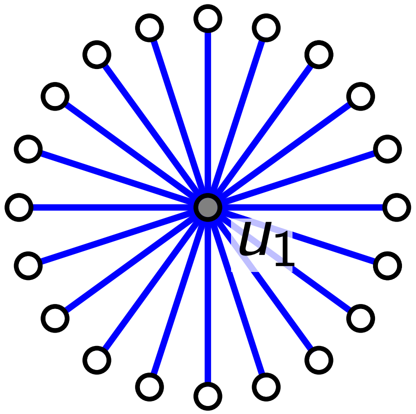
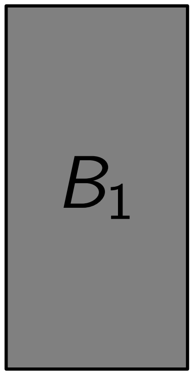
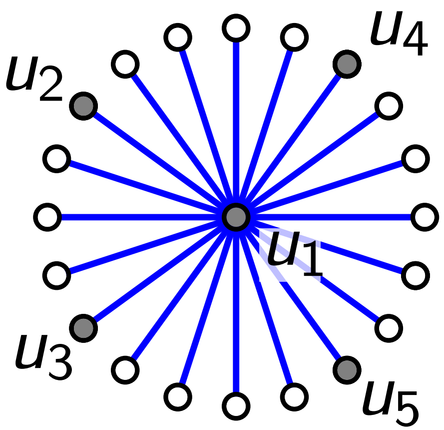
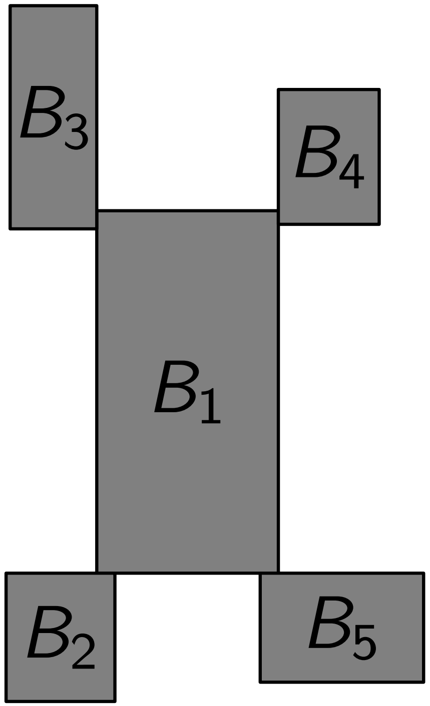




Max-Crown for Stars





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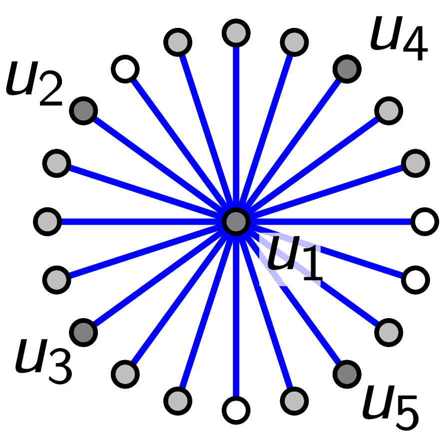

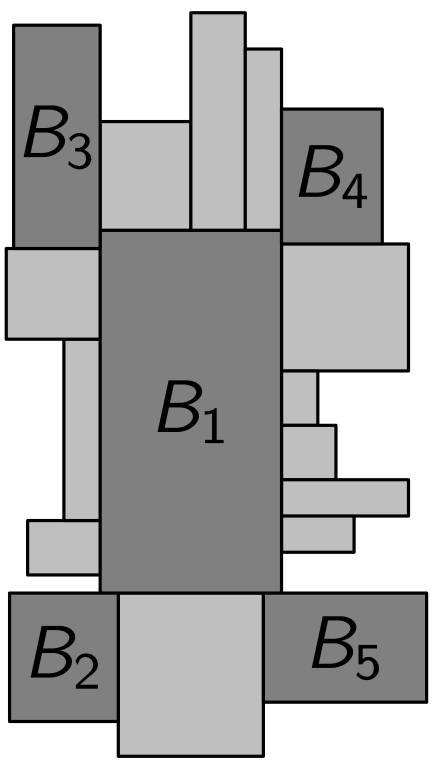

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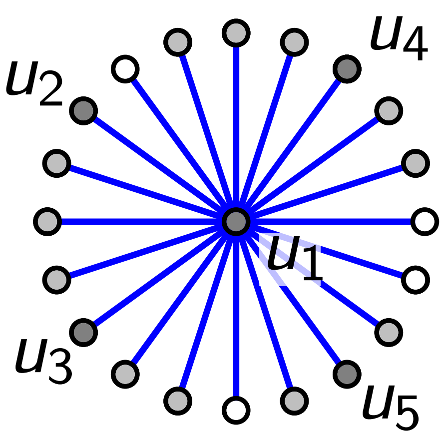

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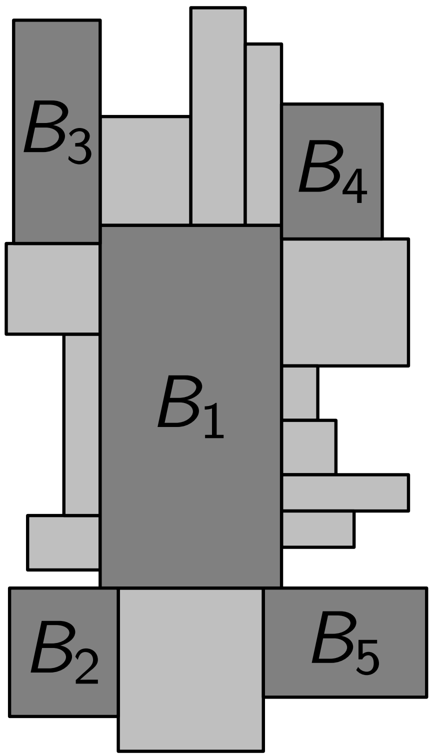

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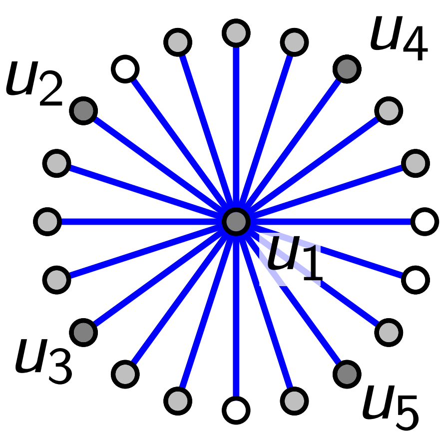

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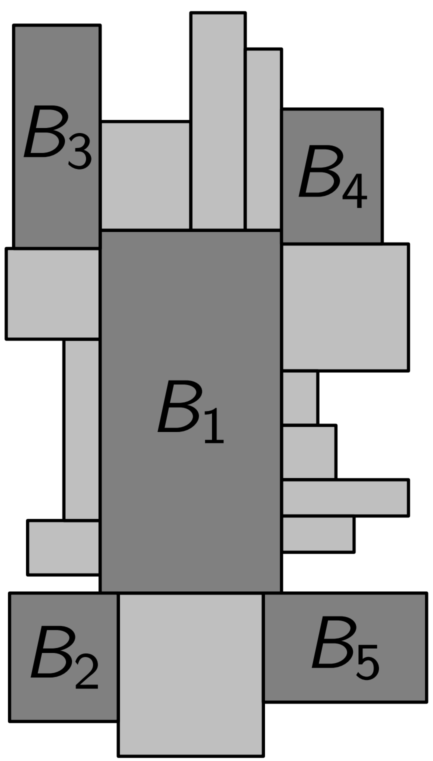

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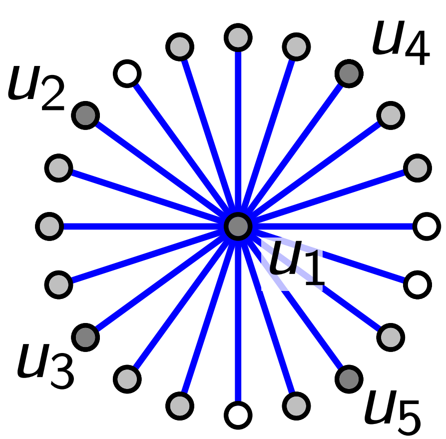

Set up Gap:

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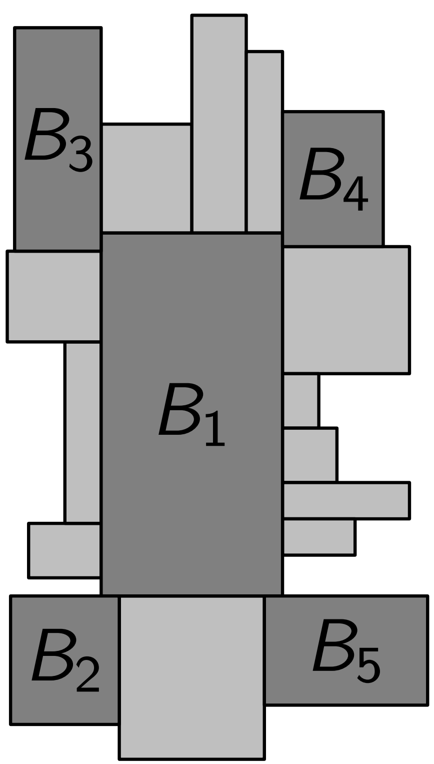
- corner bins have capacity $1/2$



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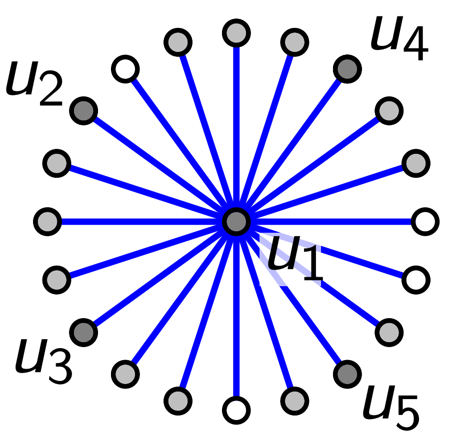

Set up GAP:

- eight bins (for the 4 sides and the 4 corners of B_1)
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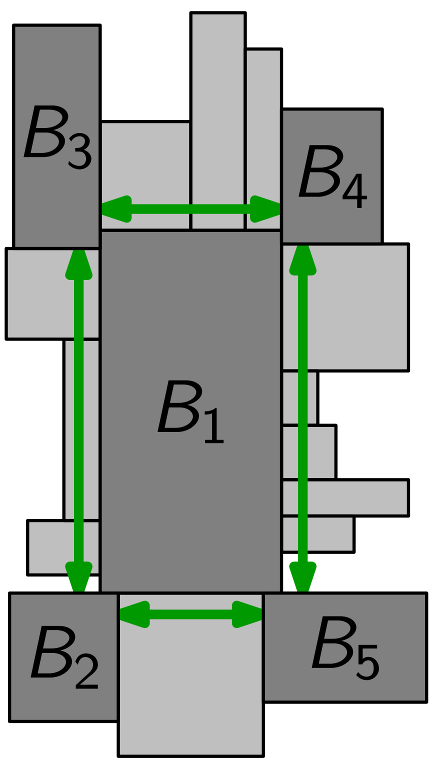
- the capacity of side bins is their "free" length



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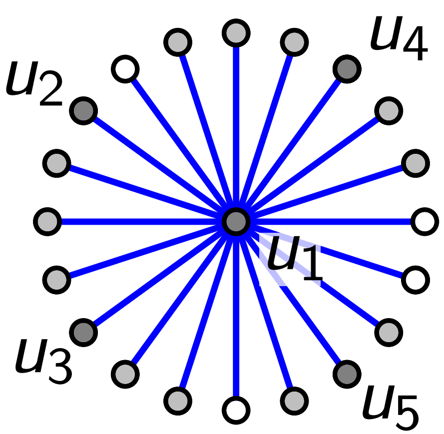

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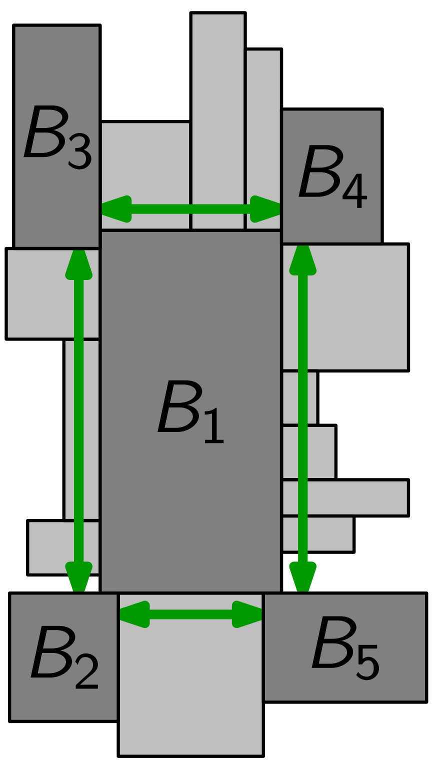

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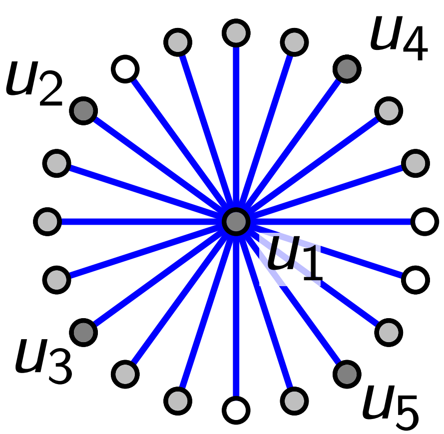

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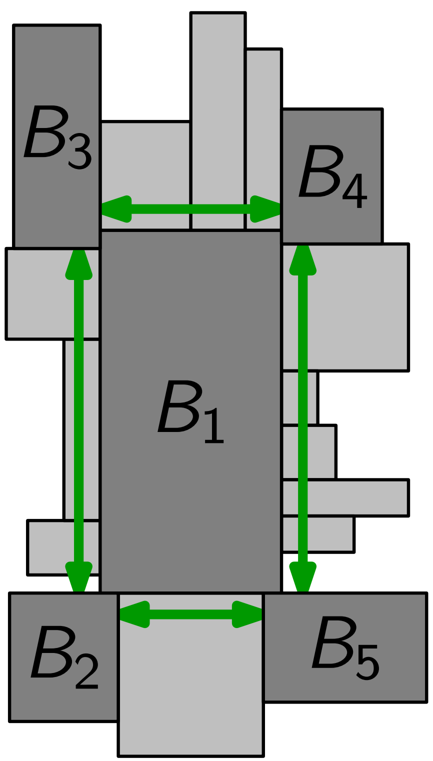

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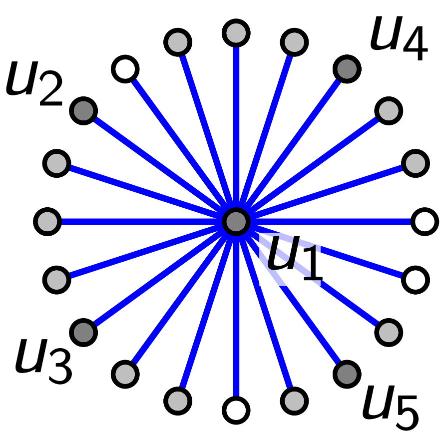

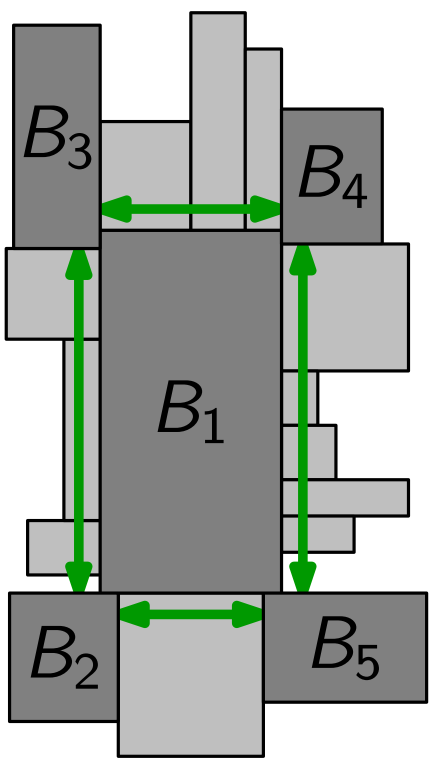

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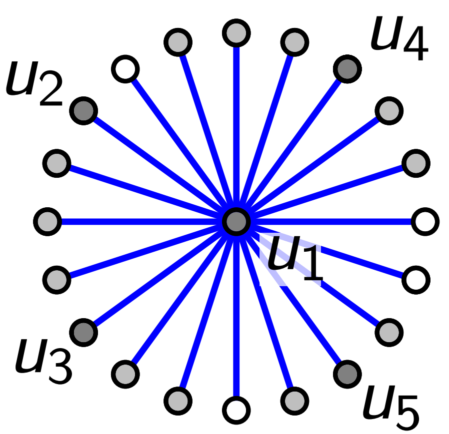
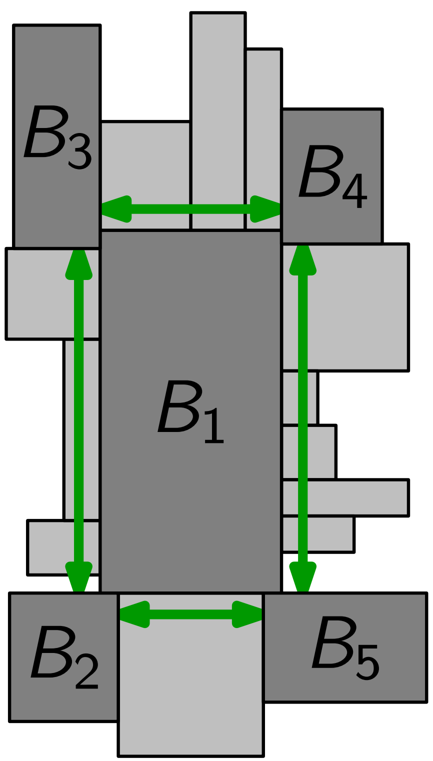
$\textsc{Max-Crown}$ for Stars





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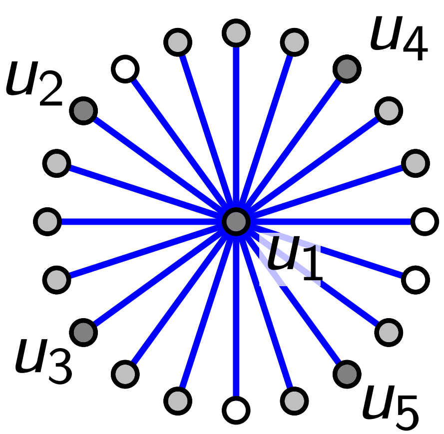
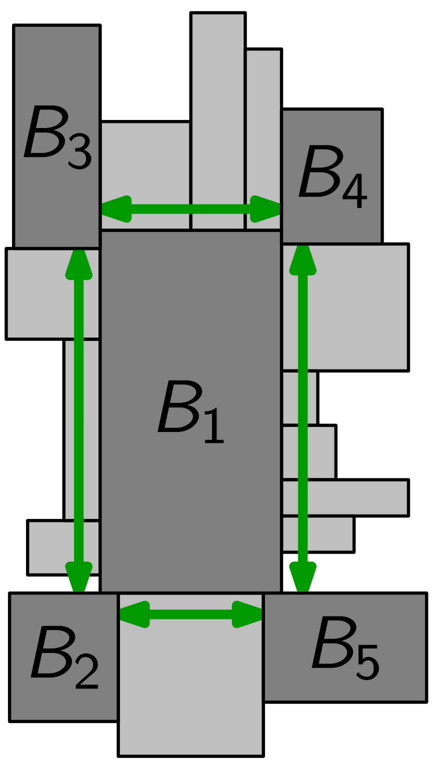
Algorithm:

Max-Crown for Stars





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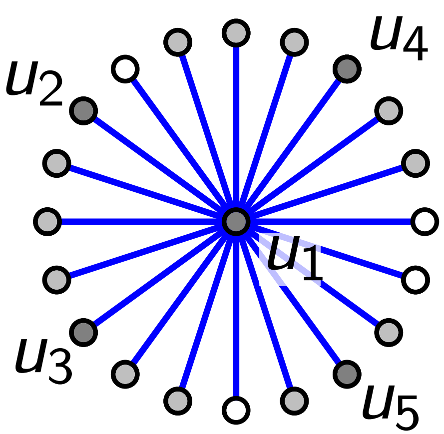

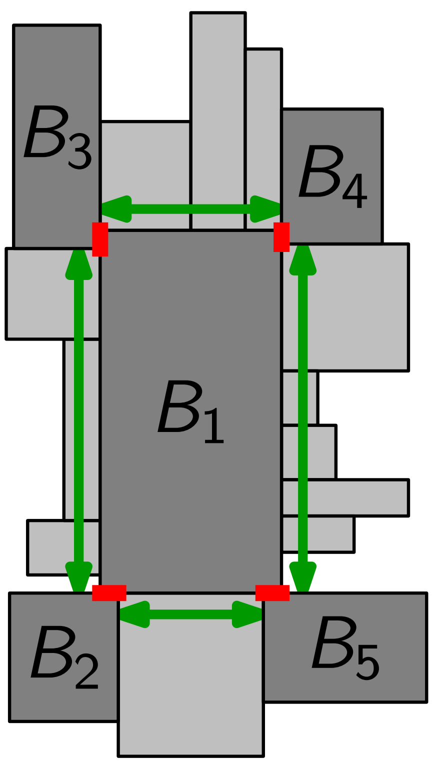

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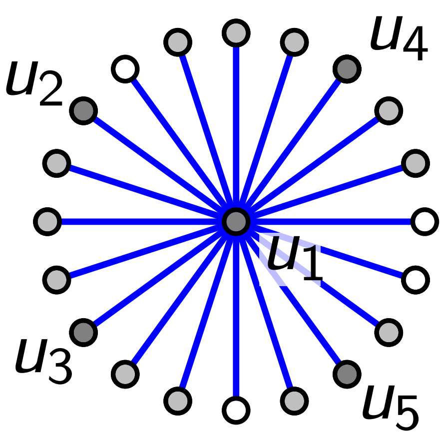

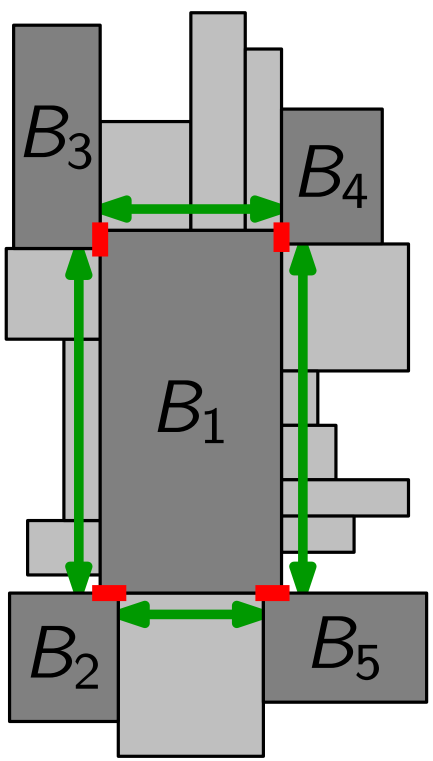

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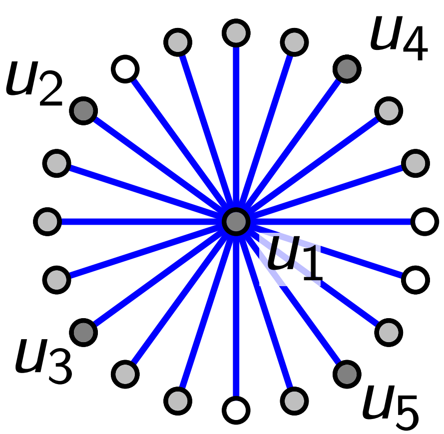

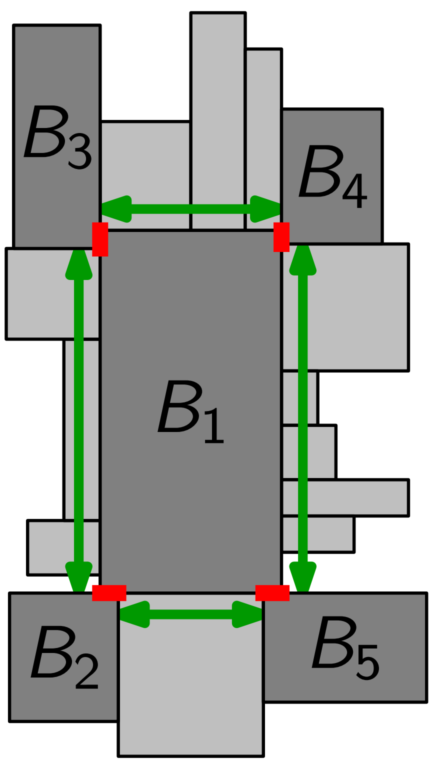

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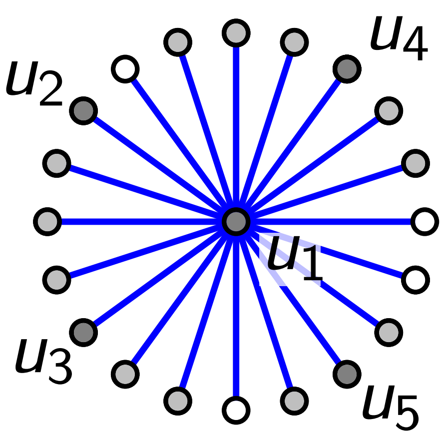
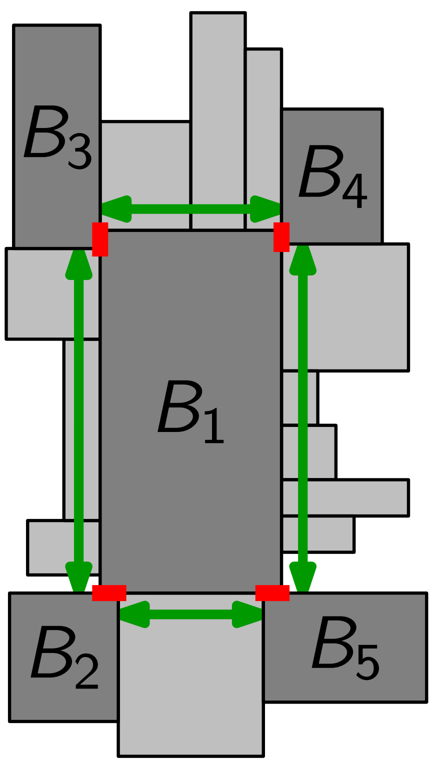
Max-Crown for Stars





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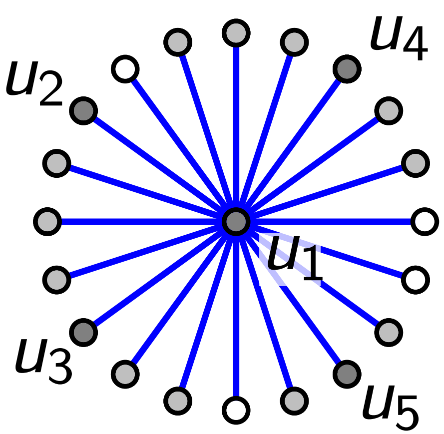
Max-Crown for Stars



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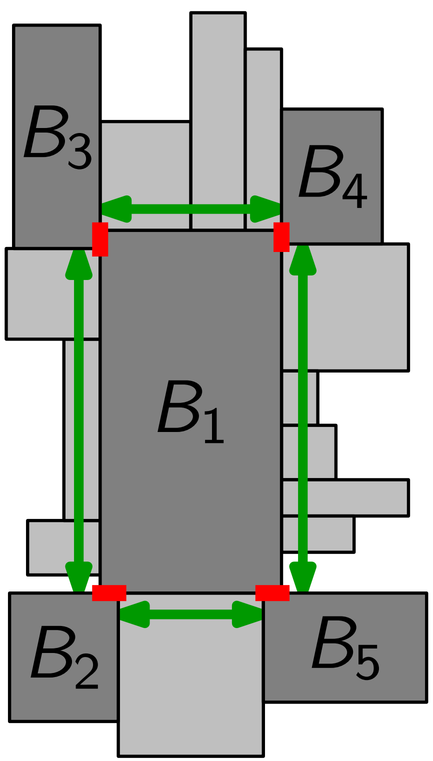

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$\Rightarrow$ α-approx. algorithm for Max-Crown on stars $\square$

Overview

Graph class	Weighted old*	Weighted new°	Unweighted new°
cycle, path	1		
star	$\boxed{\alpha}$	$1+\varepsilon$	
tree	$\boxed{2\alpha}$, NP-hard	$2+\varepsilon$	2
max-degree Δ	$\lfloor(\Delta+1)/2\rfloor$		
planar max-deg. Δ			$1+\varepsilon$
outerplanar		$3+\varepsilon$	
planar	$\boxed{5\alpha}$	$5+\varepsilon$	
bipartite		$\boxed{16\alpha/3} \approx 8.4$ APX-hard	
general		rand.: $\boxed{32\alpha/3} \approx 16.9$ det.: $\boxed{40\alpha/3} \approx 21.1$	$5+16\alpha/3$ ≈ 13.4

*) [Barth, Fabrikant, Kobourov, Lubiw, Nöllenburg, Okamoto, Pupyrev, Squarcella, Ueckerdt & Wolff, LATIN'14]

°) [Bekos, van Dijk, Fink, Kindermann, Kobourov, Pupyrev, Spoerhase, Wolff – submitted]

$$\alpha = e/(e-1) \approx 1.58$$

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Lemma. Let $G_1 = (V, E_1)$, $G_2 = (V, E_2)$, $G = (V, E_1 \cup E_2)$.

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Proof. Algorithm.

Analysis.

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Return result with larger profit for G.

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By def., $\mathrm{ALG}_i > \mathrm{OPT}_i / \alpha_i$.

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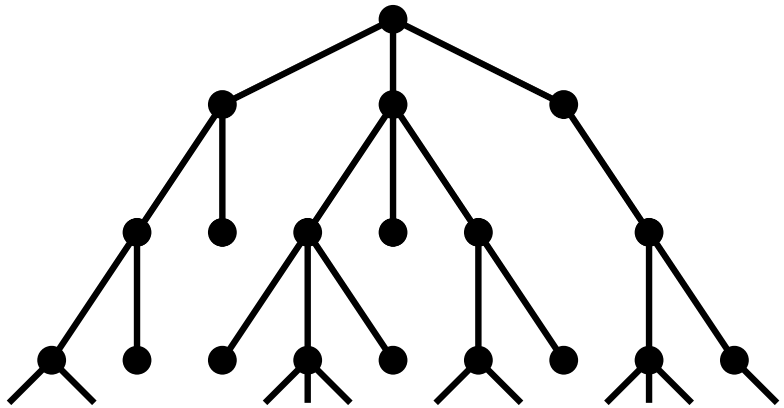

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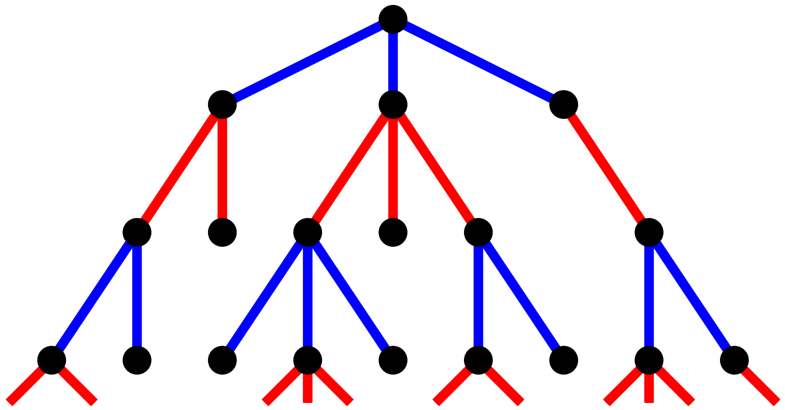

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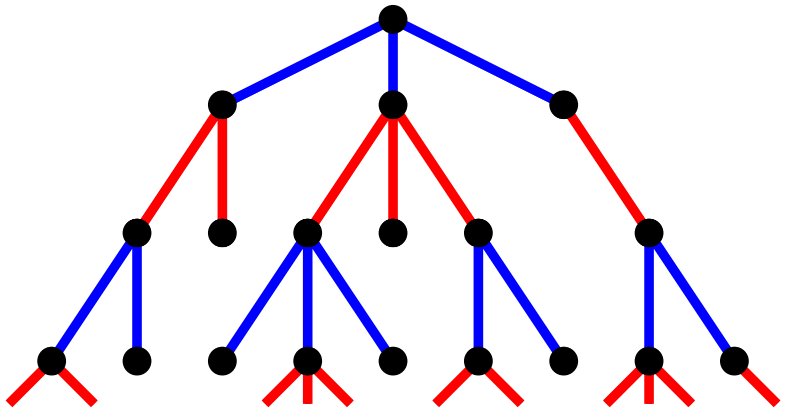

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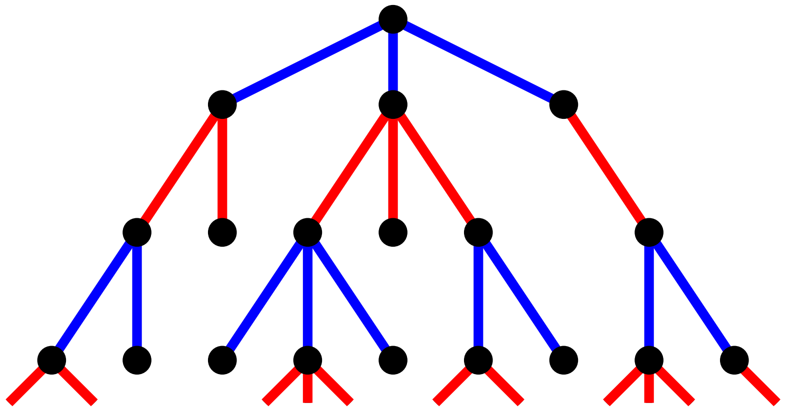

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Outerplanar | planar graphs have star arboricity 3|5. [Hakimi et al., DM'96]

$\square$

Overview

Graph class	Weighted: old*	Weighted: new°	Unweighted: new°
cycle, path	1		
star	$\boxed{\alpha}$ ✓	$1+\varepsilon$	
tree	$\boxed{2\alpha}$ NP-hard	$2+\varepsilon$	2
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Tool #1^{++}: PTAS for GAP with $O(1)$ bins

Theorem. GAP with $O(1)$ bins admits a PTAS.
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Theorem. GAP with $O(1)$ bins does not admit an FPTAS (unless...).
[Chekuri & Khanna: SIAM J. Comput.'05]

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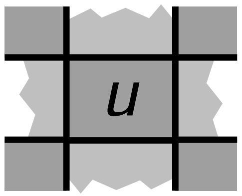

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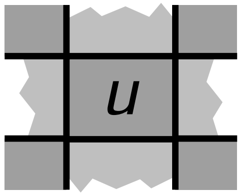

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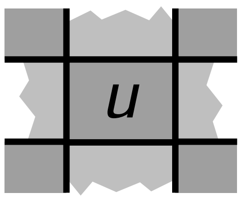

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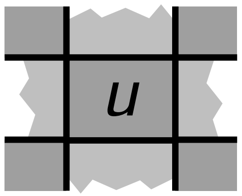

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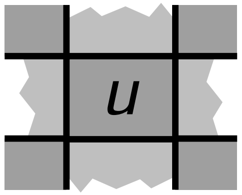

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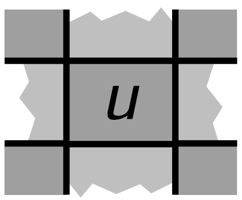

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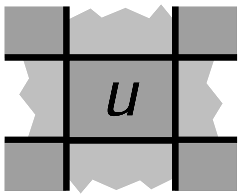

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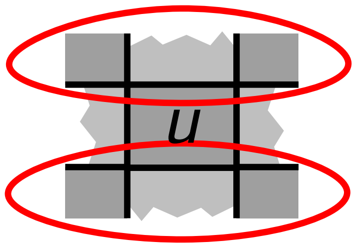

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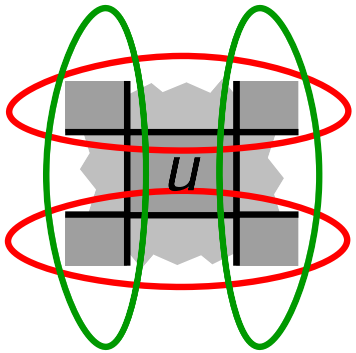

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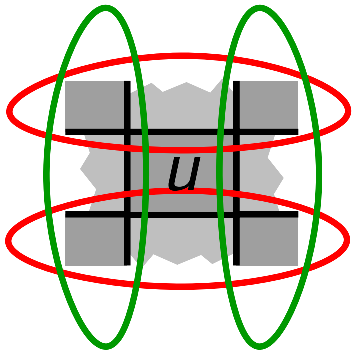

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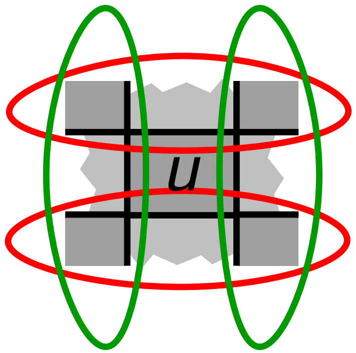

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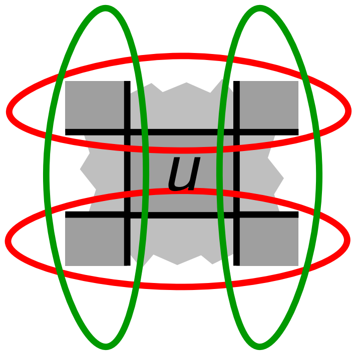

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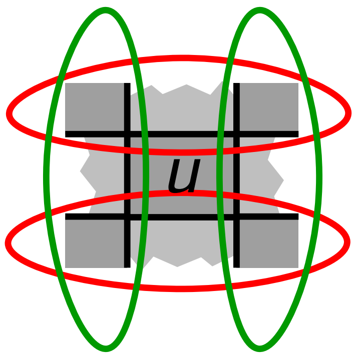

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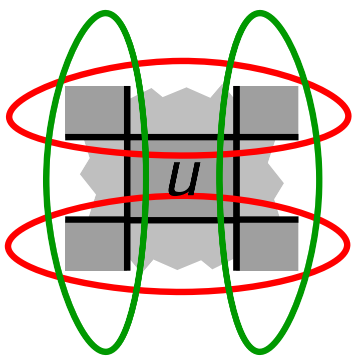

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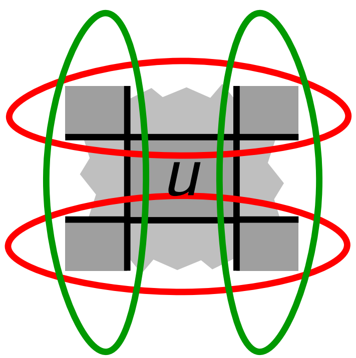

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Bipartite Graphs, Proof cont'd

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Overview

	Weighted		Unweighted
Graph class	old*	new°	new°
cycle, path	1		
star	$\boxed{\alpha}$ ✓	$1+\varepsilon$ ✓	
tree	✓ $\boxed{2\alpha}$, NP-hard	$2+\varepsilon$ ✓	2
max-degree Δ	$\lfloor(\Delta+1)/2\rfloor$		
planar max-deg. Δ			$1+\varepsilon$
outerplanar	3α ✓	$3+\varepsilon$ ✓	
planar	$\boxed{5\alpha}$ ✓	$5+\varepsilon$ ✓	
bipartite		$\boxed{16\alpha/3}$ ≈ 8.4 APX-hard	
general		rand.: $\boxed{32\alpha/3}$ ≈ 16.9 det.: $\boxed{40\alpha/3}$ ≈ 21.1	$5+16\alpha/3$ ≈ 13.4

*) [Barth, Fabrikant, Kobourov, Lubiw, Nöllenburg, Okamoto, Pupyrev, Squarcella, Ueckerdt & Wolff, LATIN'14]

°) [Bekos, van Dijk, Fink, Kindermann, Kobourov, Pupyrev, Spoerhase, Wolff – submitted]

$$\alpha = e/(e-1) \approx 1.58$$

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Overview

Graph class	Weighted old*	Weighted new$^\circ$	Unweighted new$^\circ$
cycle, path	1		
star	$\boxed{\alpha}$ ✓	$1+\varepsilon$ ✓	
tree	✓ $\boxed{2\alpha}$, NP-hard	$2+\varepsilon$ ✓	2
max-degree Δ	$\lfloor(\Delta+1)/2\rfloor$		
planar max-deg. Δ			$1+\varepsilon$
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planar	$\boxed{5\alpha}$ ✓	$5+\varepsilon$ ✓	
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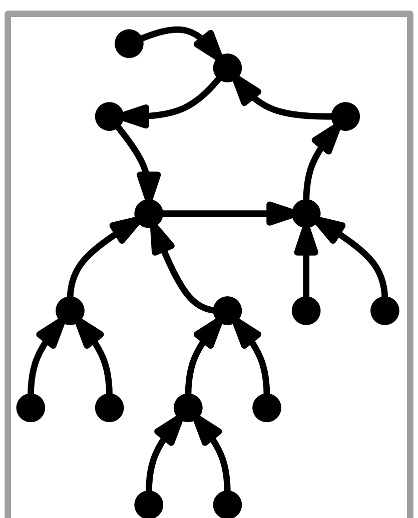

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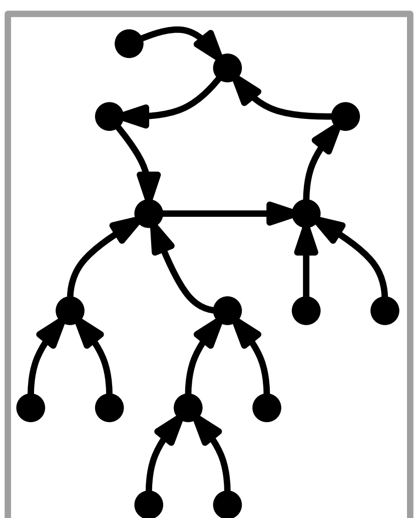

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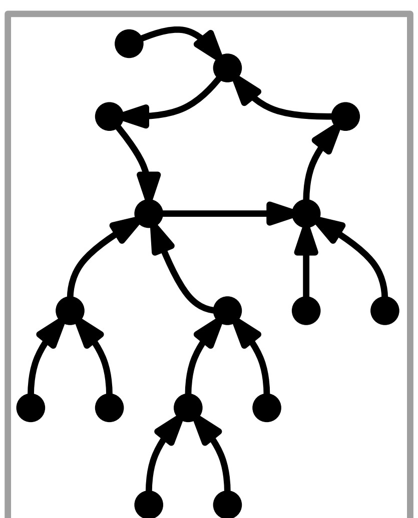

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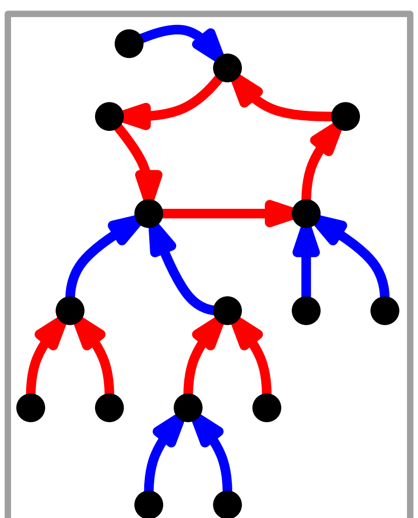

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Partition each into star forest S_1 and star forest + cycle S_2.

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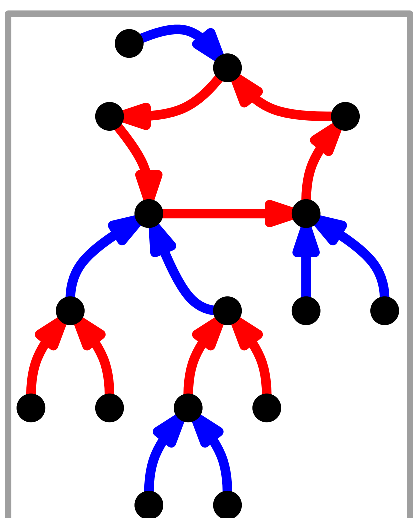

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All contacts in S_i can be realized – with corner contacts!

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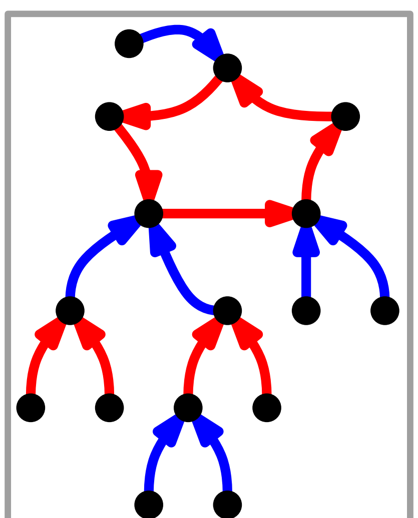

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All contacts in S_i can be realized – with corner contacts!
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Thm. Max-Crown admits a deterministic $40\alpha/3$-approx.

Proof. Let $G = (V, E)$ be any graph. Use Gap – as in bipartite case!

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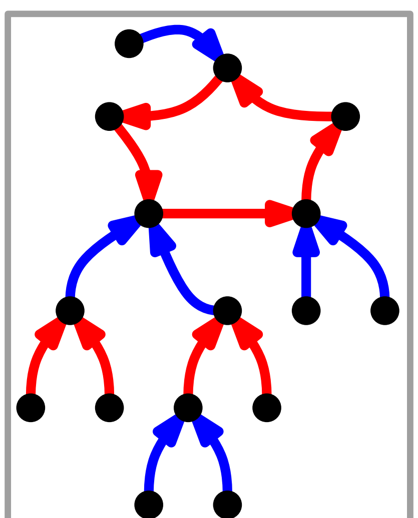

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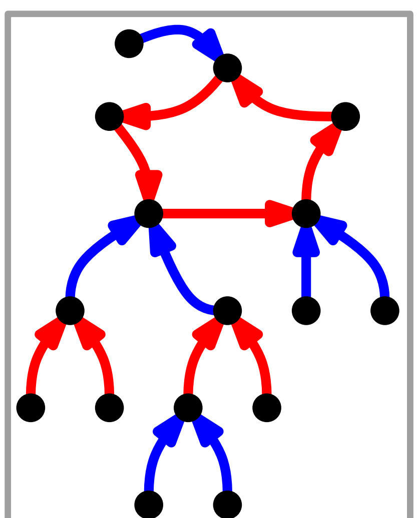

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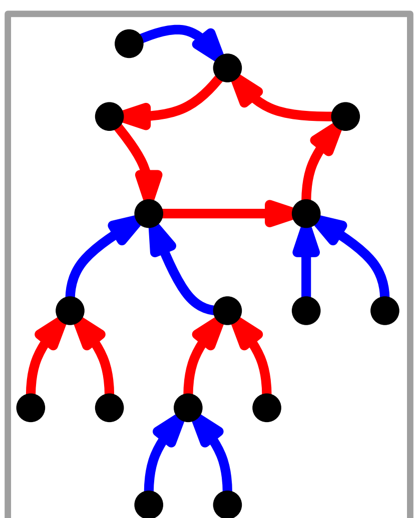

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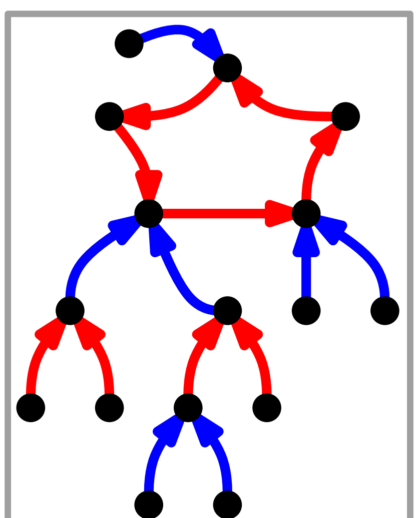

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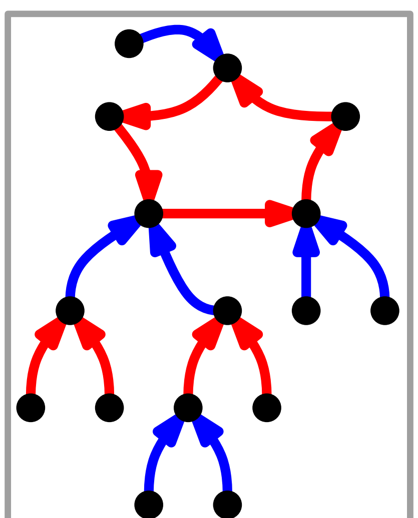

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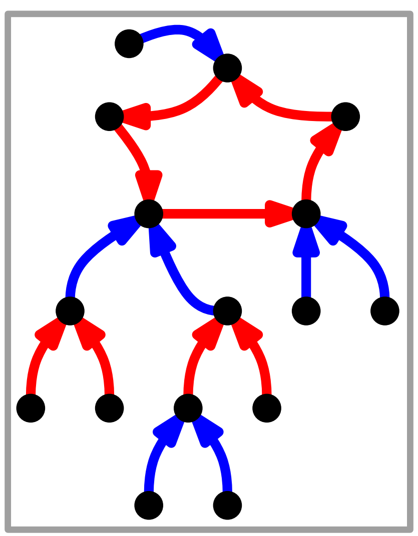

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Graph class	Weighted old*	Weighted new°	Unweighted new°
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tree	✓ $\boxed{2\alpha}$, NP-hard	$2+\varepsilon$ ✓	2
max-degree Δ	$\lfloor(\Delta+1)/2\rfloor$		
planar max-deg. Δ			$1+\varepsilon$
outerplanar	3α ✓	$3+\varepsilon$ ✓	
planar	$\boxed{5\alpha}$ ✓	$5+\varepsilon$ ✓	
bipartite		$\boxed{16\alpha/3}$ ✓ ≈ 8.4 APX-hard	
general		rand.: $\boxed{32\alpha/3}$ ✓ ≈ 16.9 det.: $\boxed{40\alpha/3} \approx 21.1$	$5+16\alpha/3$ ≈ 13.4

*) [Barth, Fabrikant, Kobourov, Lubiw, Nöllenburg, Okamoto, Pupyrev, Squarcella, Ueckerdt & Wolff, LATIN'14]

°) [Bekos, van Dijk, Fink, Kindermann, Kobourov, Pupyrev, Spoerhase, Wolff – submitted]

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  What other problems have been solved combinatorially, but are interesting to optimize when we add more constraints?