



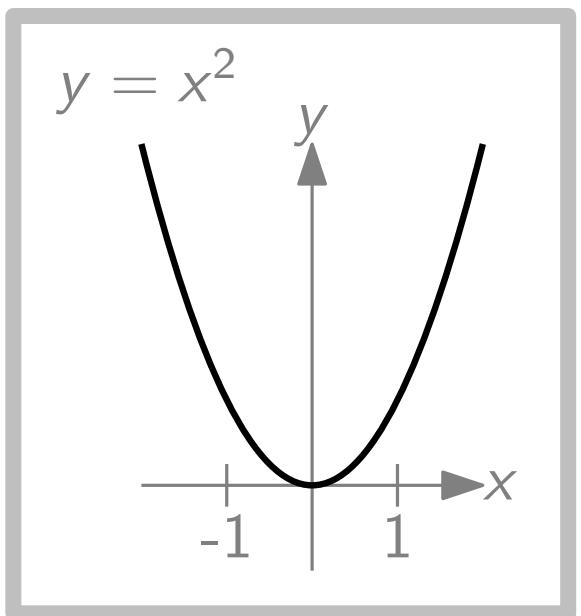
Winkelschematisierung im Graphenzeichnen

Angular Schematization
in Graph Drawing

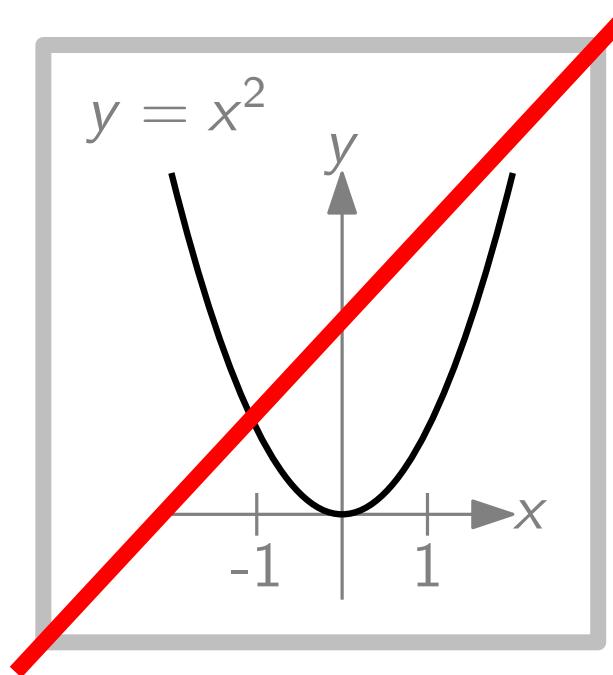
Philipp Kindermann
Lehrstuhl für Informatik I
Universität Würzburg

Größtenteils finanziert durch das ERC/DFG-Projekt
„Graph Drawing and Geometric Representations“ (Wo 758/5-1)

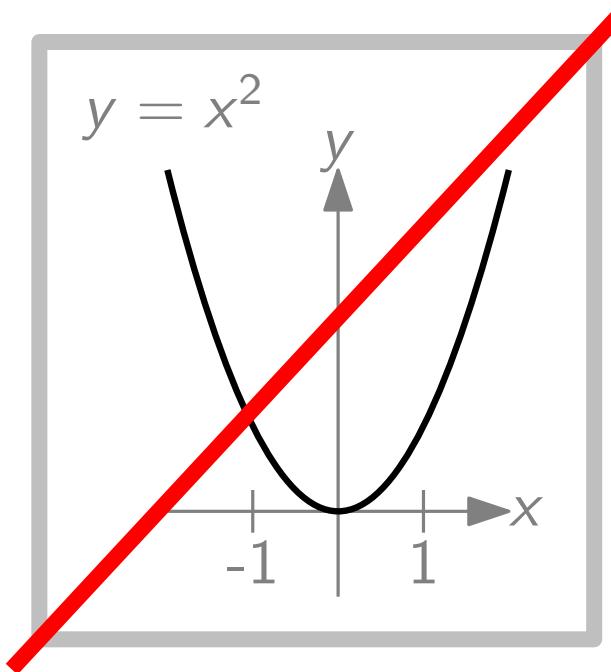
Graphenzeichnen?



Graphenzeichnen?



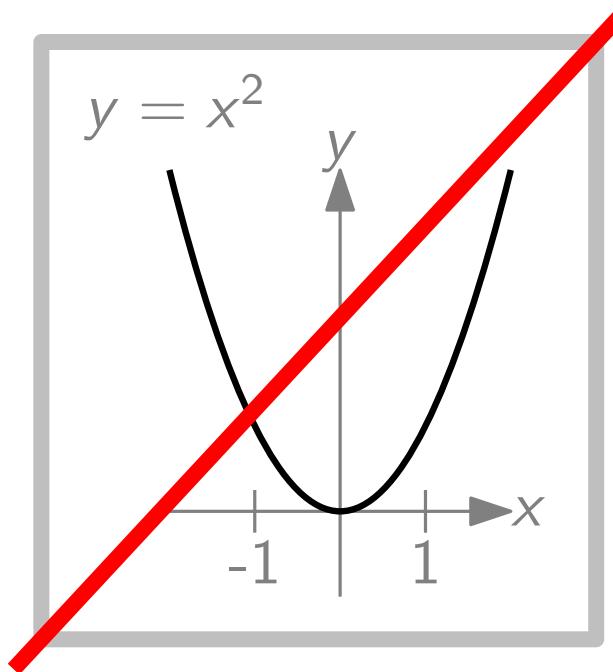
Graphenzeichnen?



Ein *Graph* ist ein Paar $G = (V, E)$

- V *Knotenmenge* und
- $E \subseteq \binom{V}{2}$ *Kantenmenge*

Graphenzeichnen?



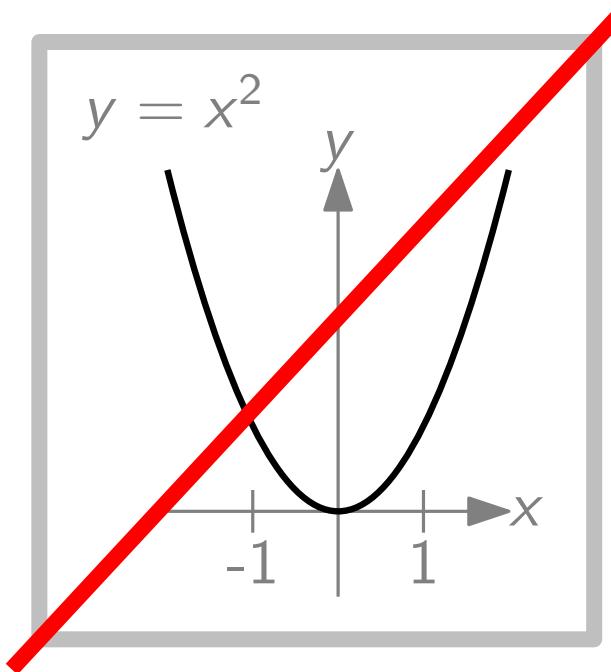
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Bsp. $V = \{1, 2, 3, 4, 5\}$

$$E = \{ \{1, 2\}, \{1, 4\}, \{2, 3\}, \{2, 5\}, \\ \{3, 4\}, \{3, 5\}, \{4, 5\} \}$$

Graphenzeichnen?



Ein *Graph* ist ein Paar $G = (V, E)$

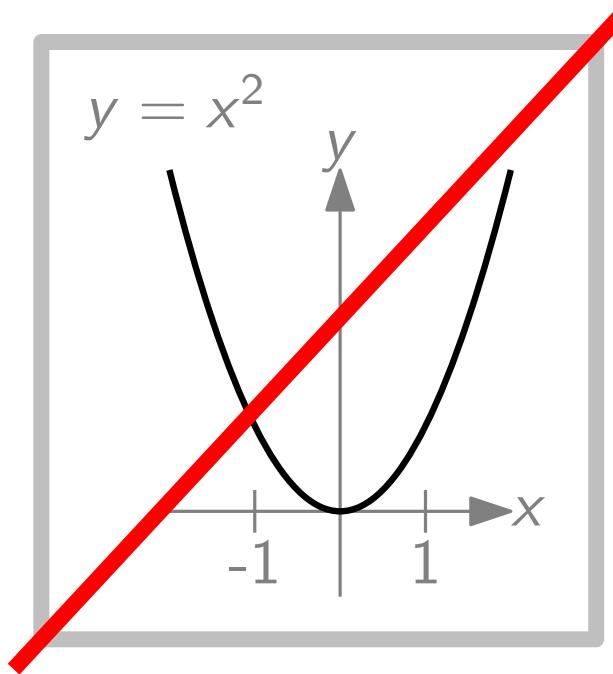
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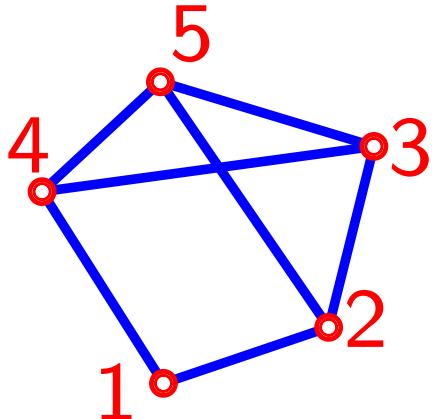
Graphenzeichnen?



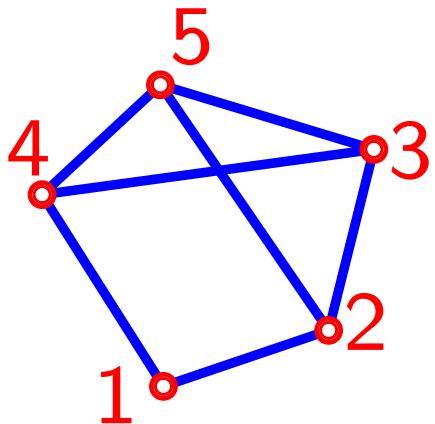
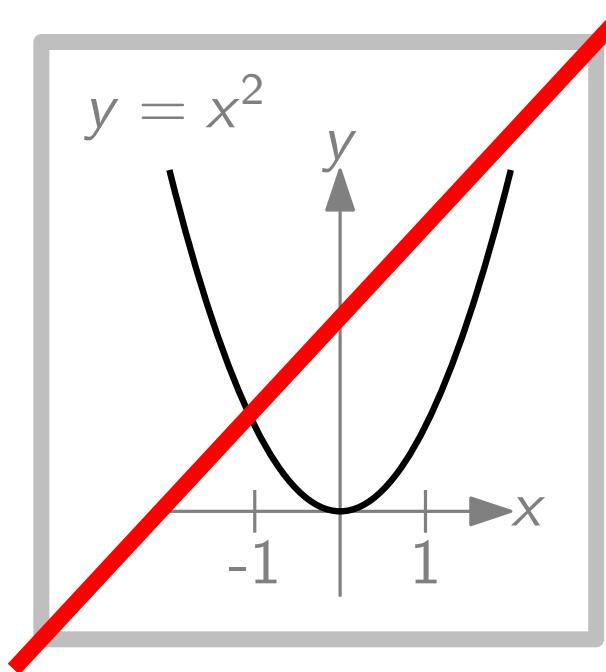
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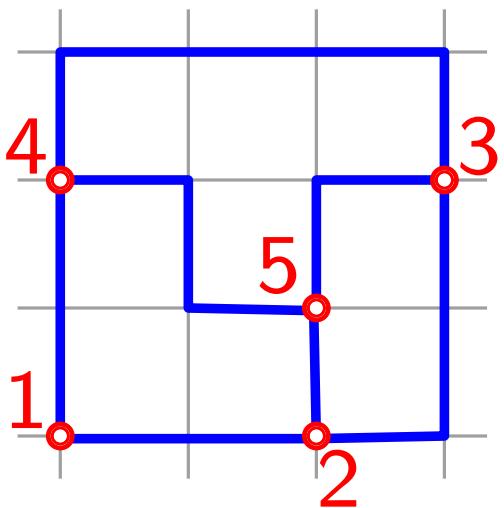


Graphenzeichnen?

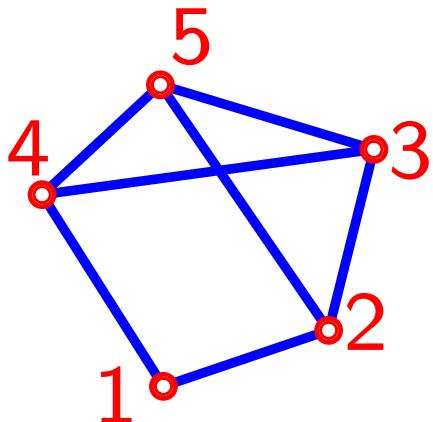
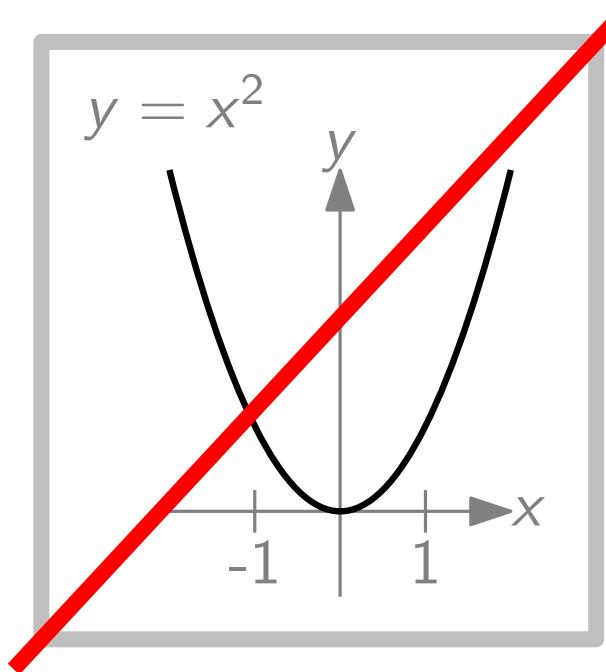


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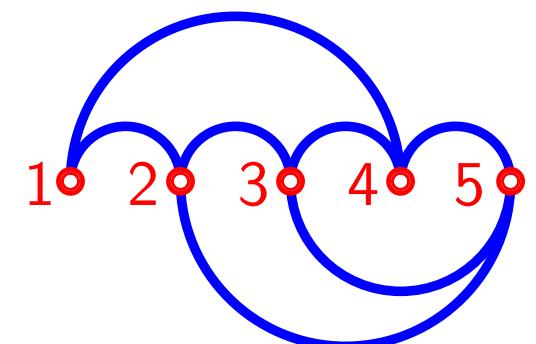
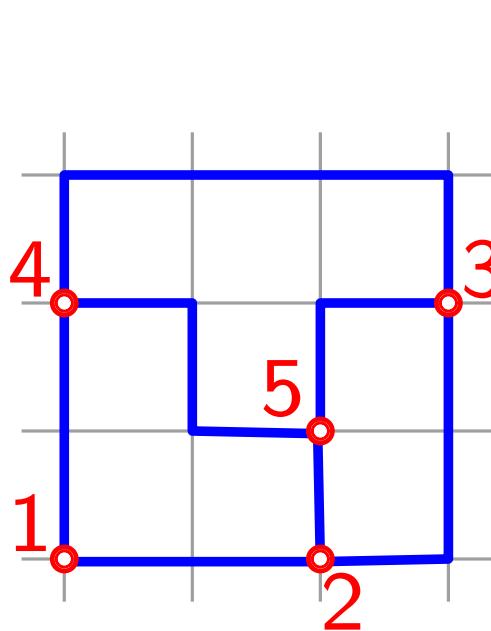


Graphenzeichnen?

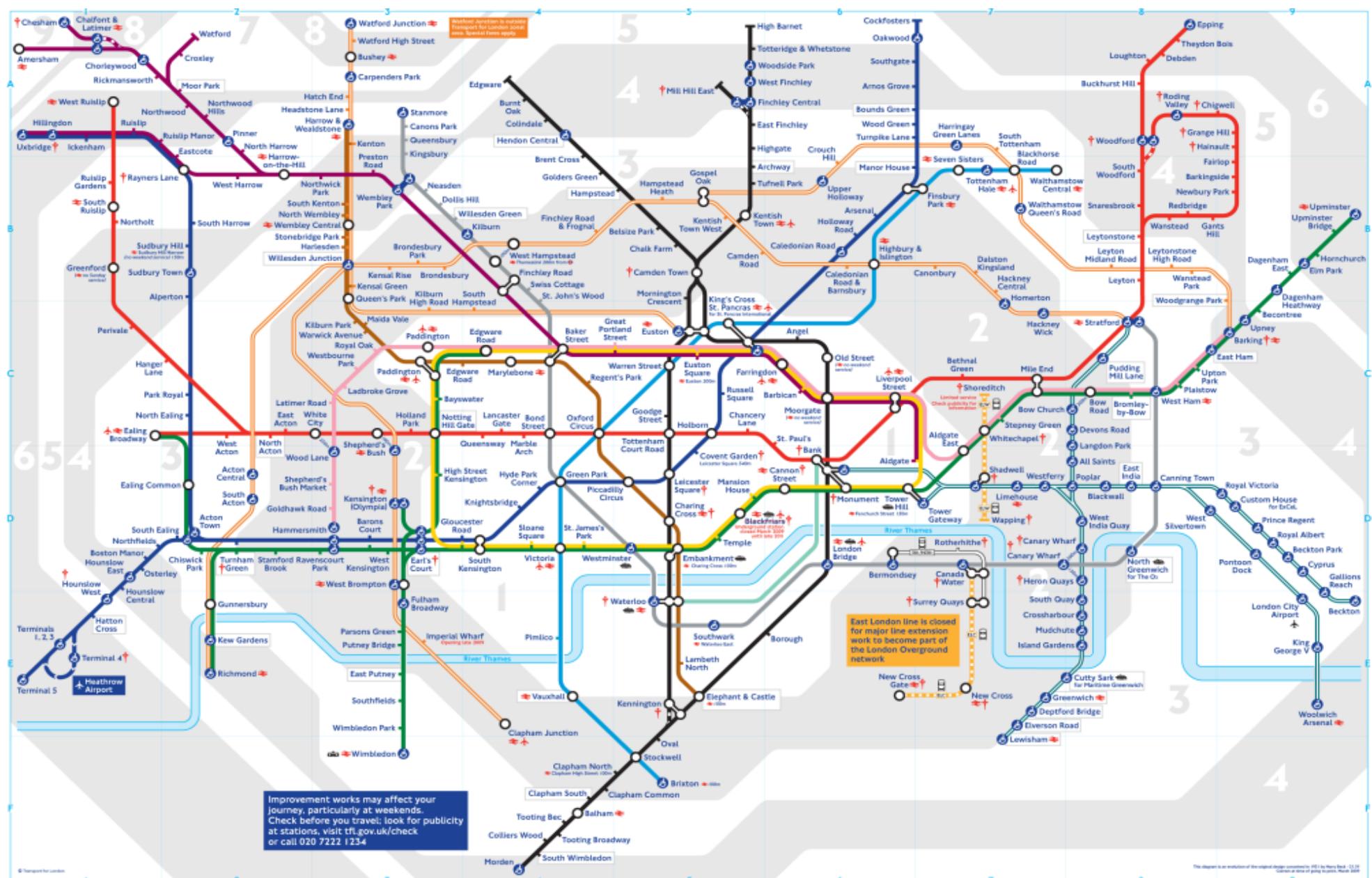


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Graphenzeichnen?

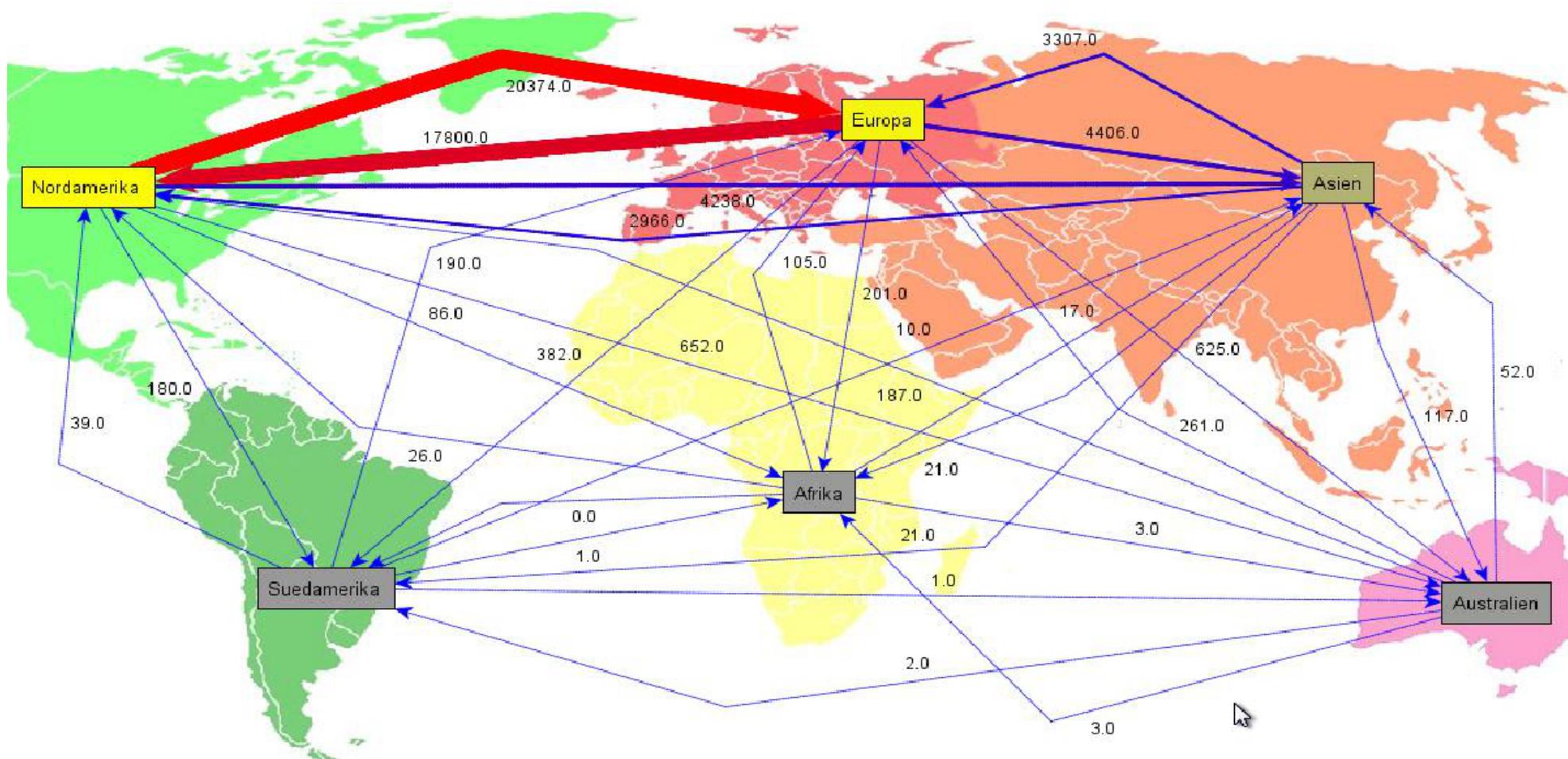


Londoner U-Bahn Liniennetzplan von Transport for London

Graphenzeichnen?

Kontinentale Aufteilung

2001 - 2005



Legende:

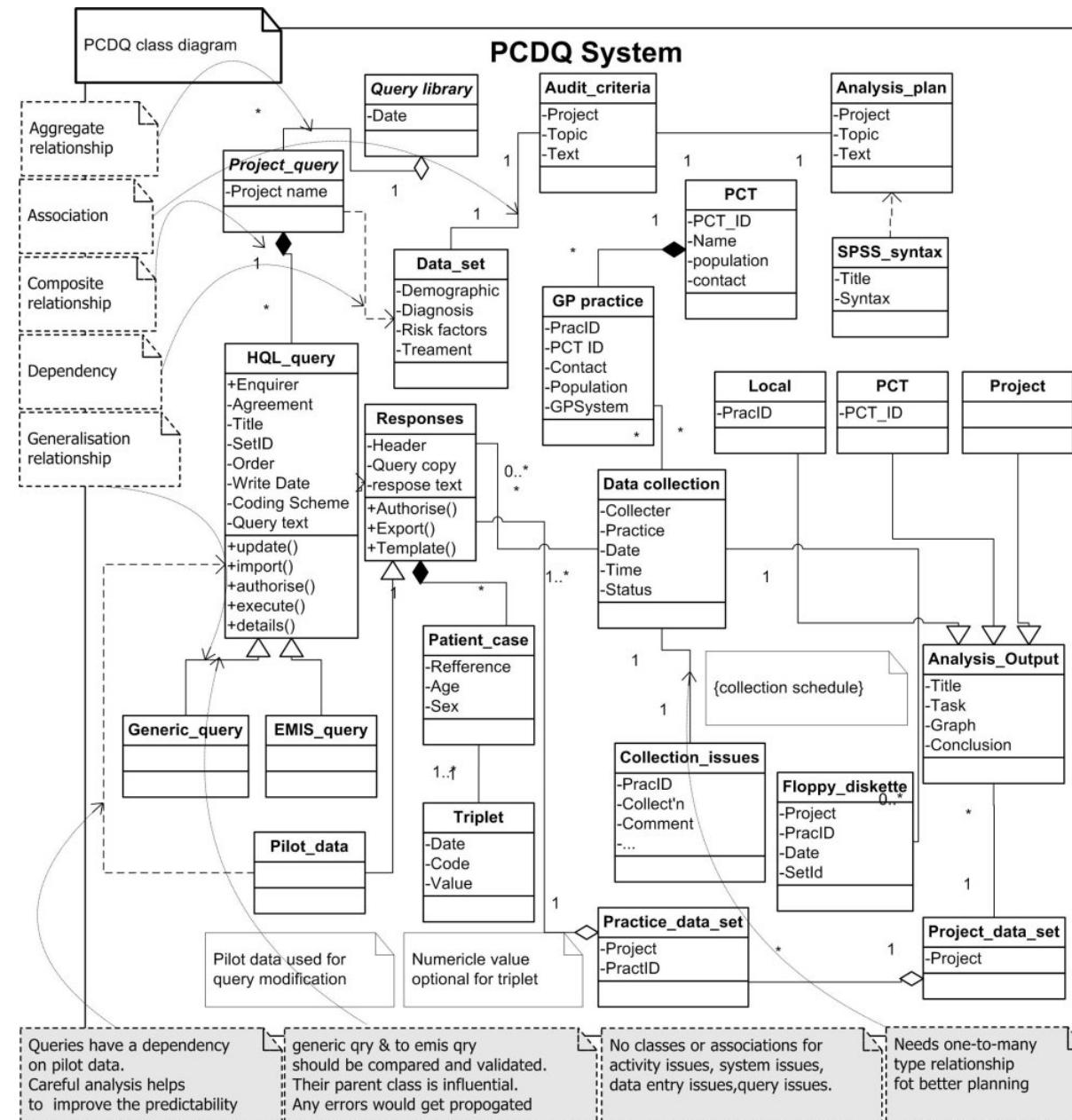
Kantengewicht:

- Maximum
- Minimum

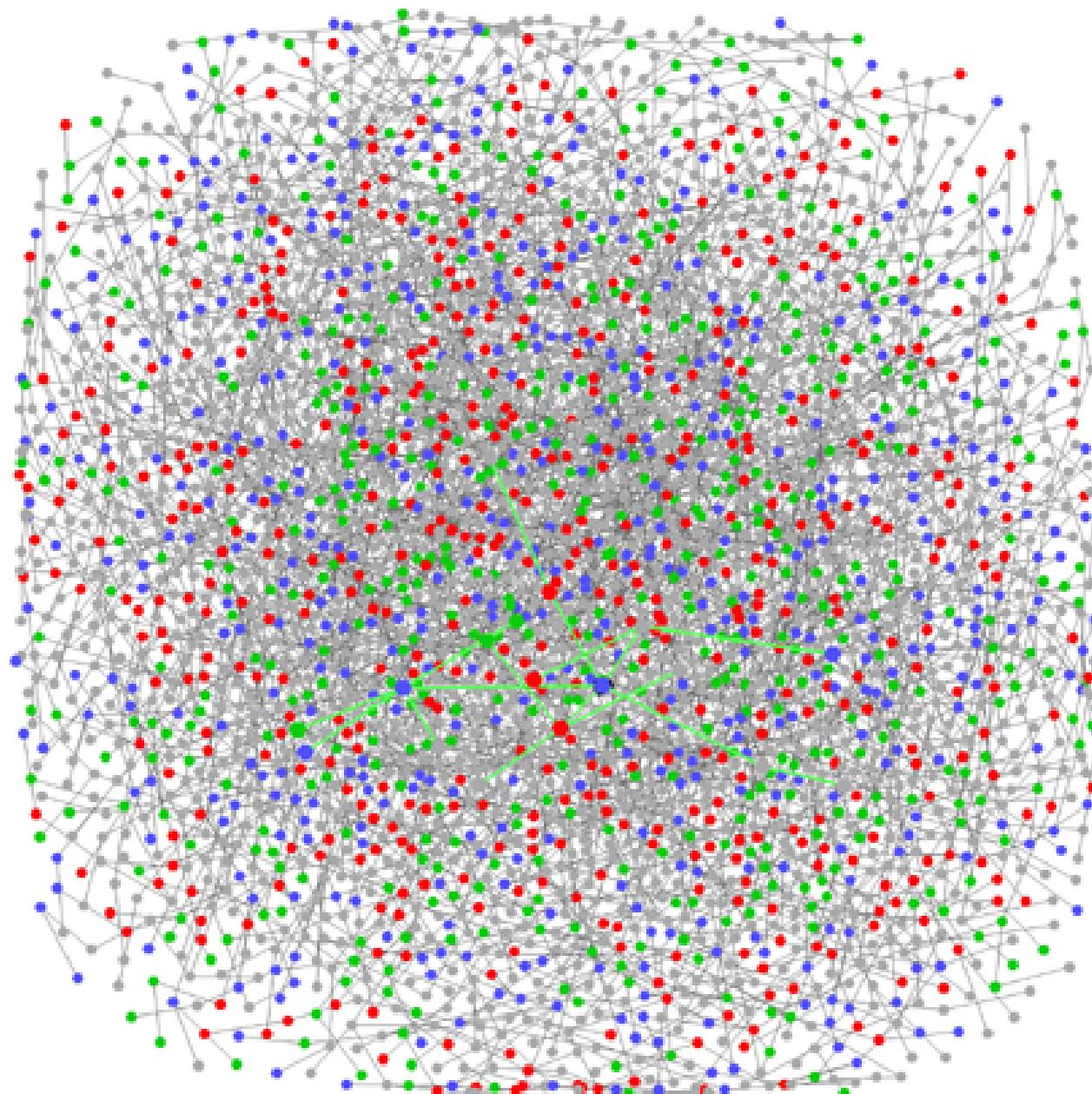
Ausgangsgrad:

- Maximum
- Minimum

Graphenzeichnen?



Graphenzeichnen?



[Wave & Bobrow, InfoVis 2005]

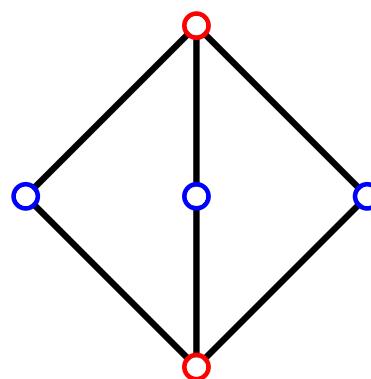
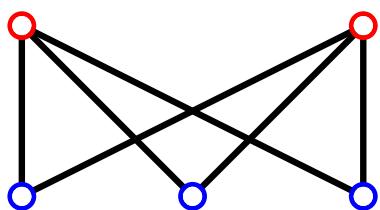
Winkelschematisierung

Schöne Zeichnungen?

Winkelschematisierung

Schöne Zeichnungen?

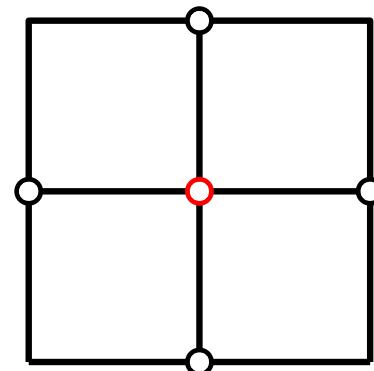
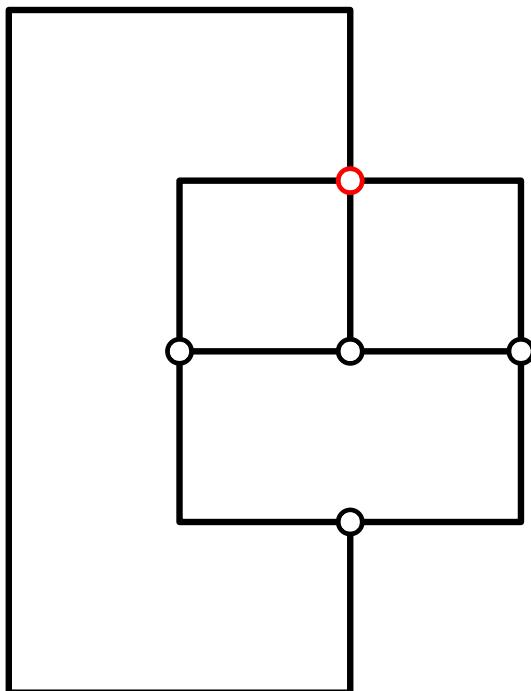
– Kreuzungsminimierung



Winkelschematisierung

Schöne Zeichnungen?

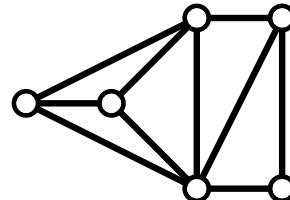
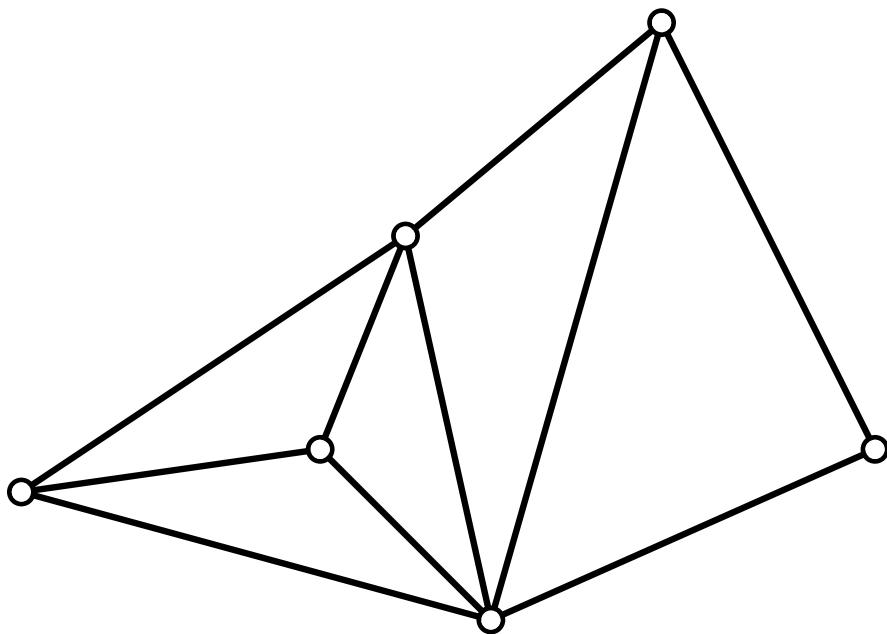
- Kreuzungsminimierung
- Knickminimierung



Winkelschematisierung

Schöne Zeichnungen?

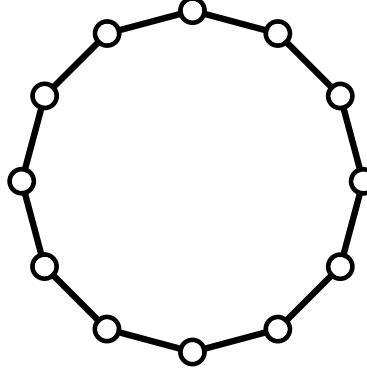
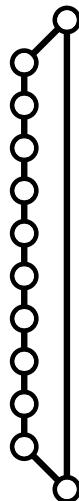
- Kreuzungsminimierung
- Knickminimierung
- minimale Fläche



Winkelschematisierung

Schöne Zeichnungen?

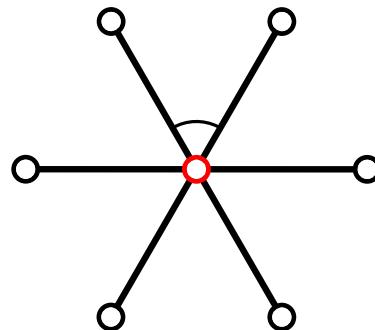
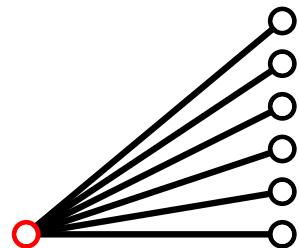
- Kreuzungsminimierung
- Knickminimierung
- minimale Fläche
- gleichmäßige Kantenlängen



Winkelschematisierung

Schöne Zeichnungen?

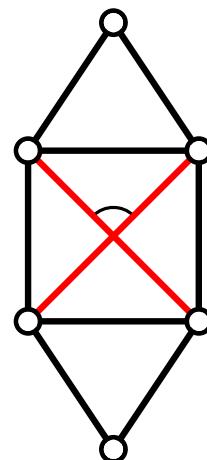
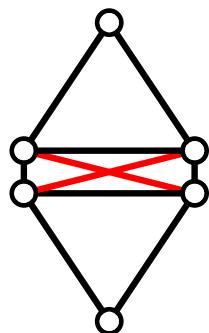
- Kreuzungsminimierung
- Knickminimierung
- minimale Fläche
- gleichmäßige Kantenlängen
- große Winkel
 - an Knoten



Winkelschematisierung

Schöne Zeichnungen?

- Kreuzungsminimierung
- Knickminimierung
- minimale Fläche
- gleichmäßige Kantenlängen
- große Winkel
 - an Knoten
 - an Kreuzungen



Gliederung

I. Platzieren von Boxen

- 1) Mehrseitige Randbeschriftungen
- 2) Approximationsalgorithmen für Rechtecks-Kontakt-Repräsentationen

II. Visuelle Führung

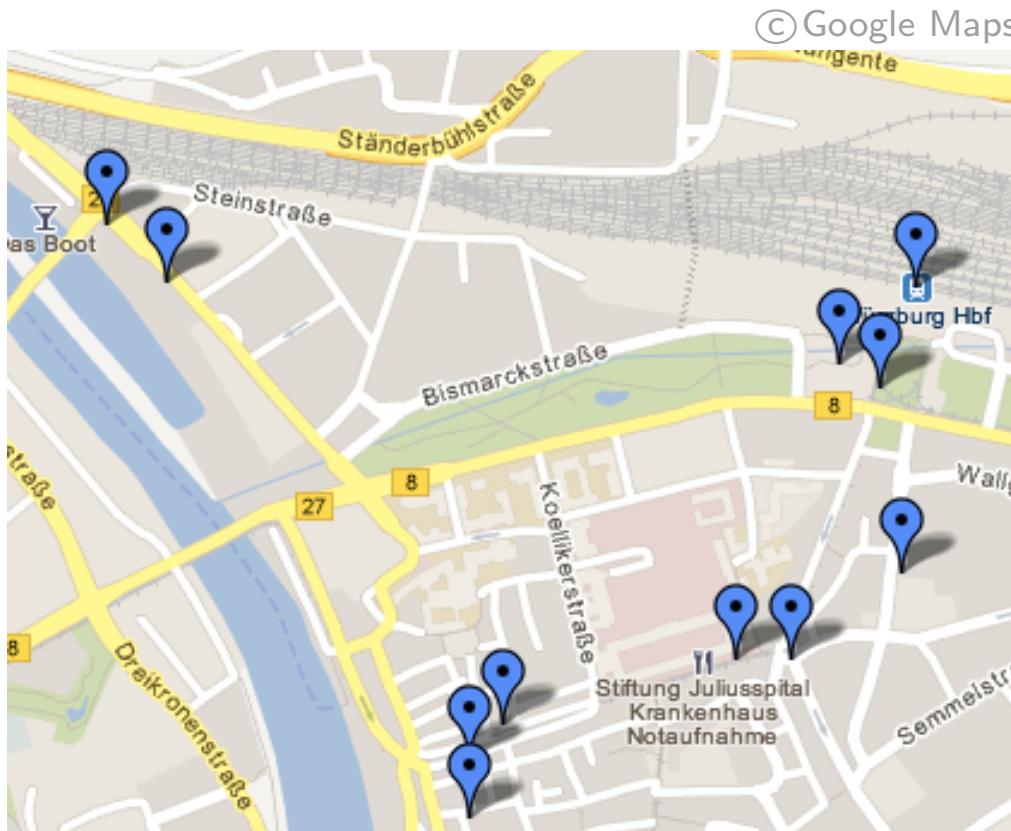
- 3) Glatt-orthogonale Darstellungen planarer Graphen
- 4) Monotone Zeichnungen von Bäumen

III. Kreuzungen mit großen Winkeln

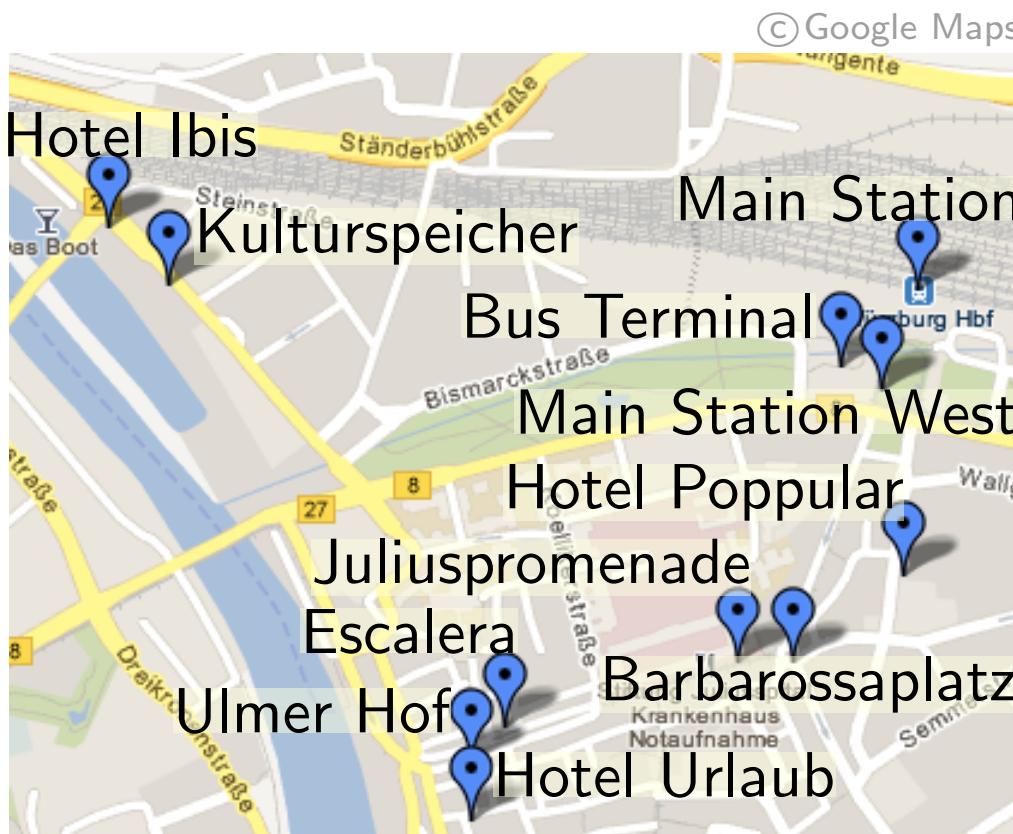
- 5) Gleichzeitiges Zeichnen planarer Graphen mit rechtwinkligen Kreuzungen und wenig Knicken
- 6) Erkennen und Zeichnen von Graphen mit unabhängigen Kreuzungen

Teil 1: Mehrseitige Randbeschriftungen

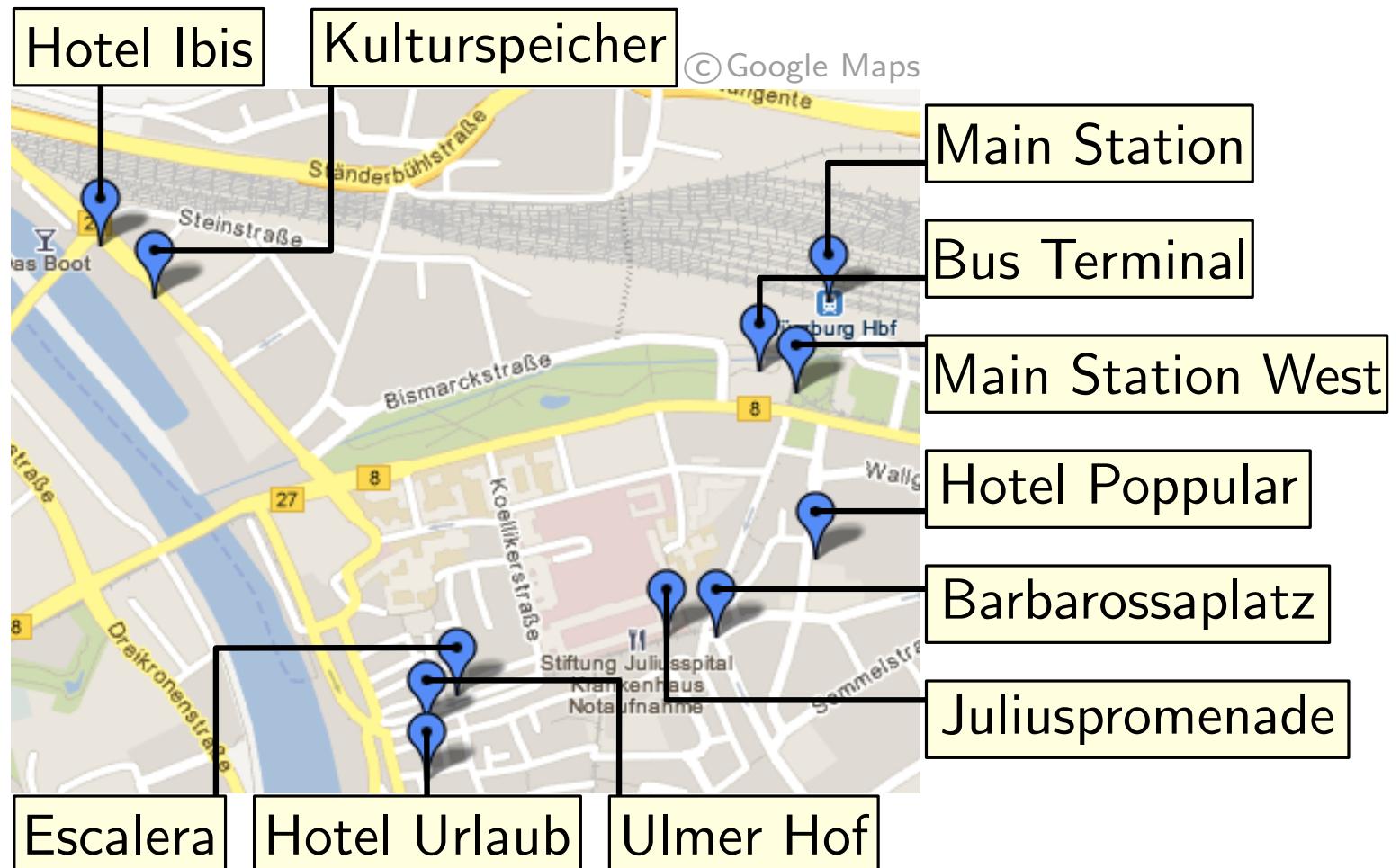
Randbeschriftungen



Randbeschriftungen



Randbeschriftungen



Randbeschriftungen

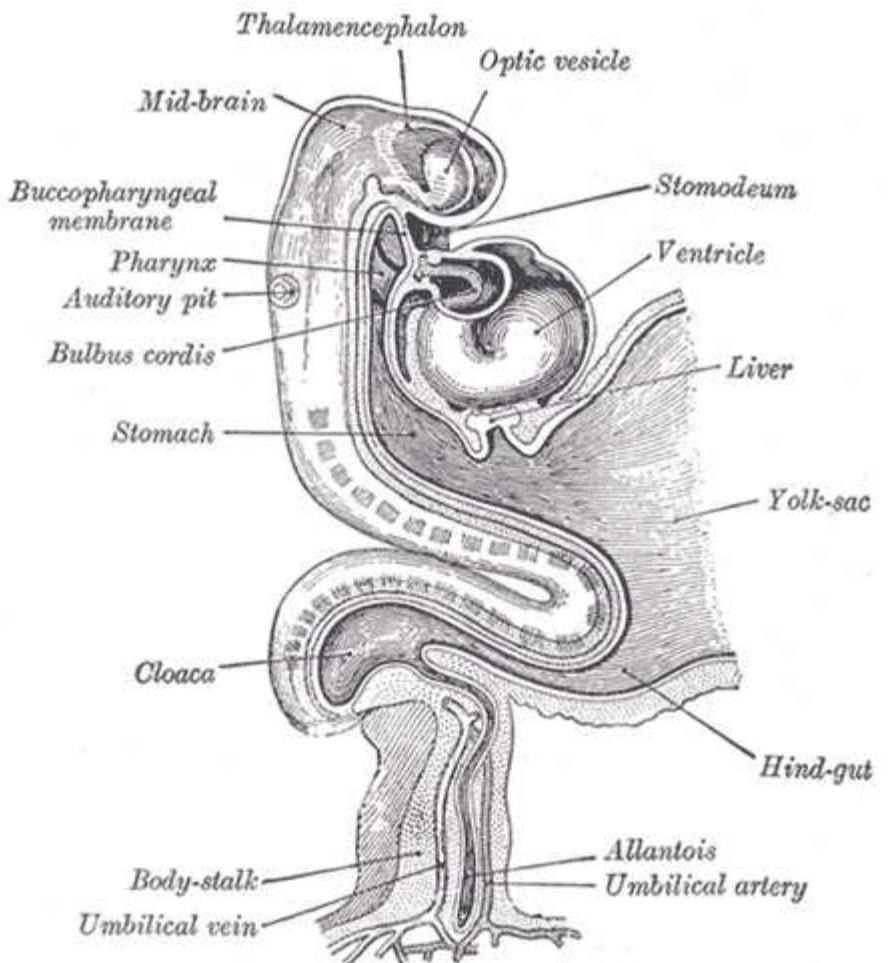


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Randbeschriftungen



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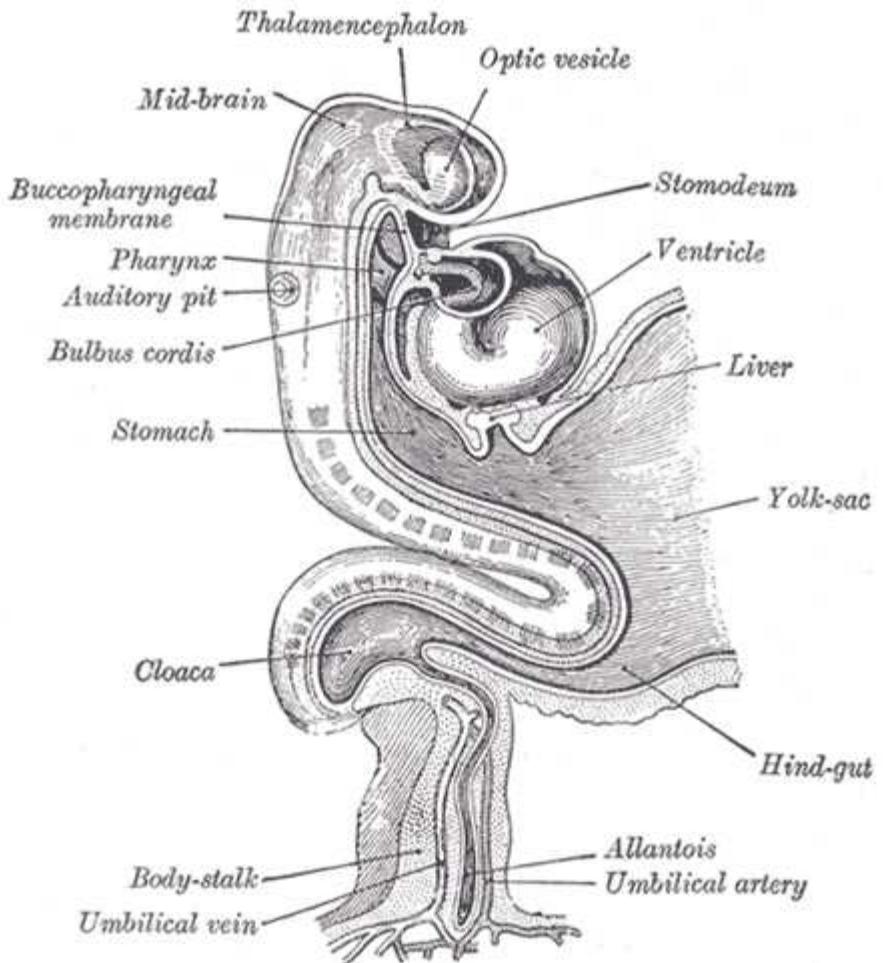
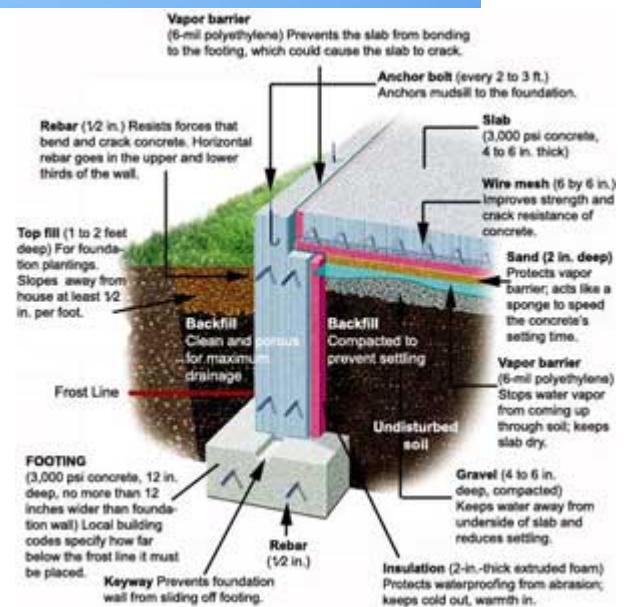


Henry Vandyke Carter, via Wikimedia Commons

Randbeschriftungen



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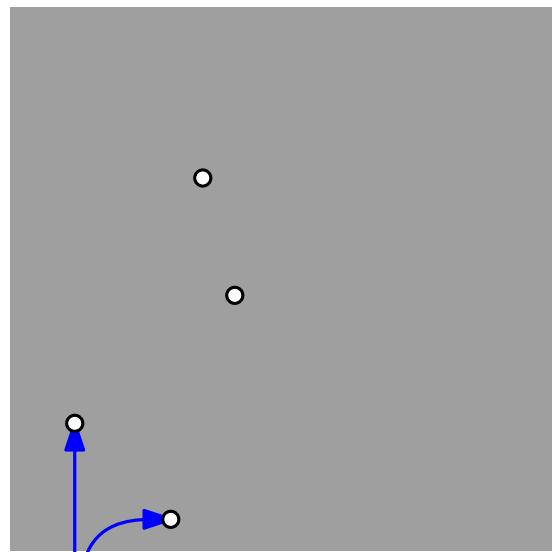
Henry Vandyke Carter, via Wikimedia Commons

Randbeschriftungen – Problembeschreibung



umschließendes Rechteck

Randbeschriftungen – Problembeschreibung

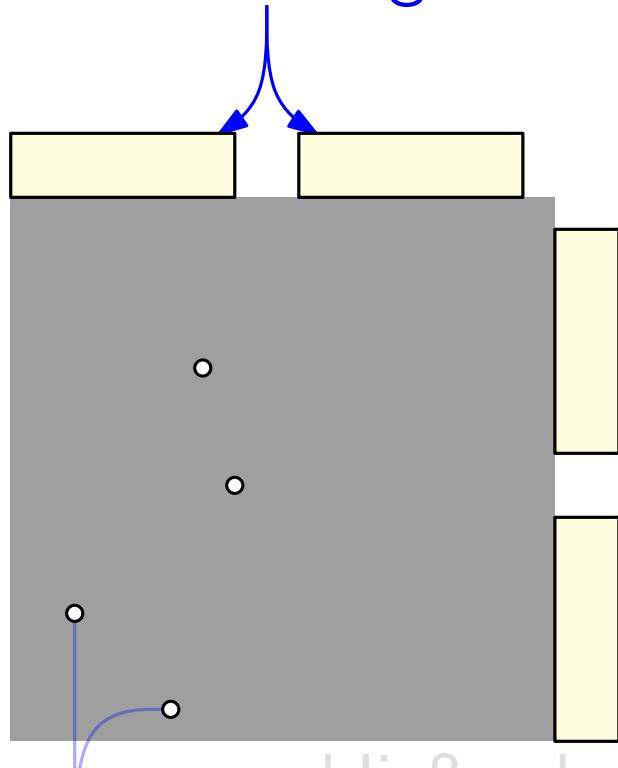


umschließendes Rechteck
Standorte

- *Standorte im Rechteck*

Randbeschriftungen – Problembeschreibung

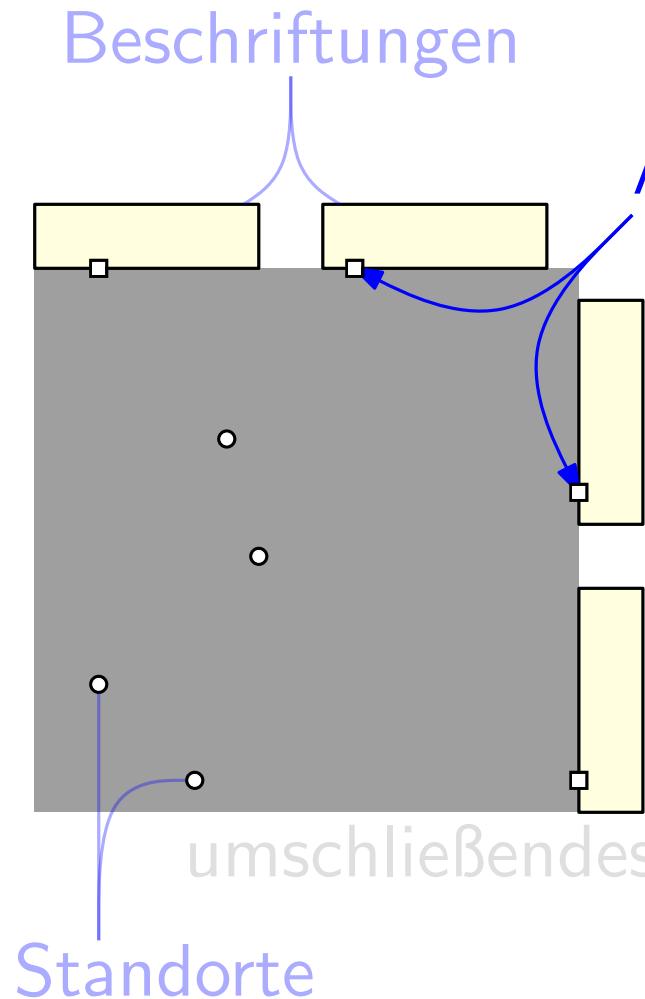
Beschriftungen



Standorte

- *Standorte im Rechteck*
- *Beschriftungen am Rand*

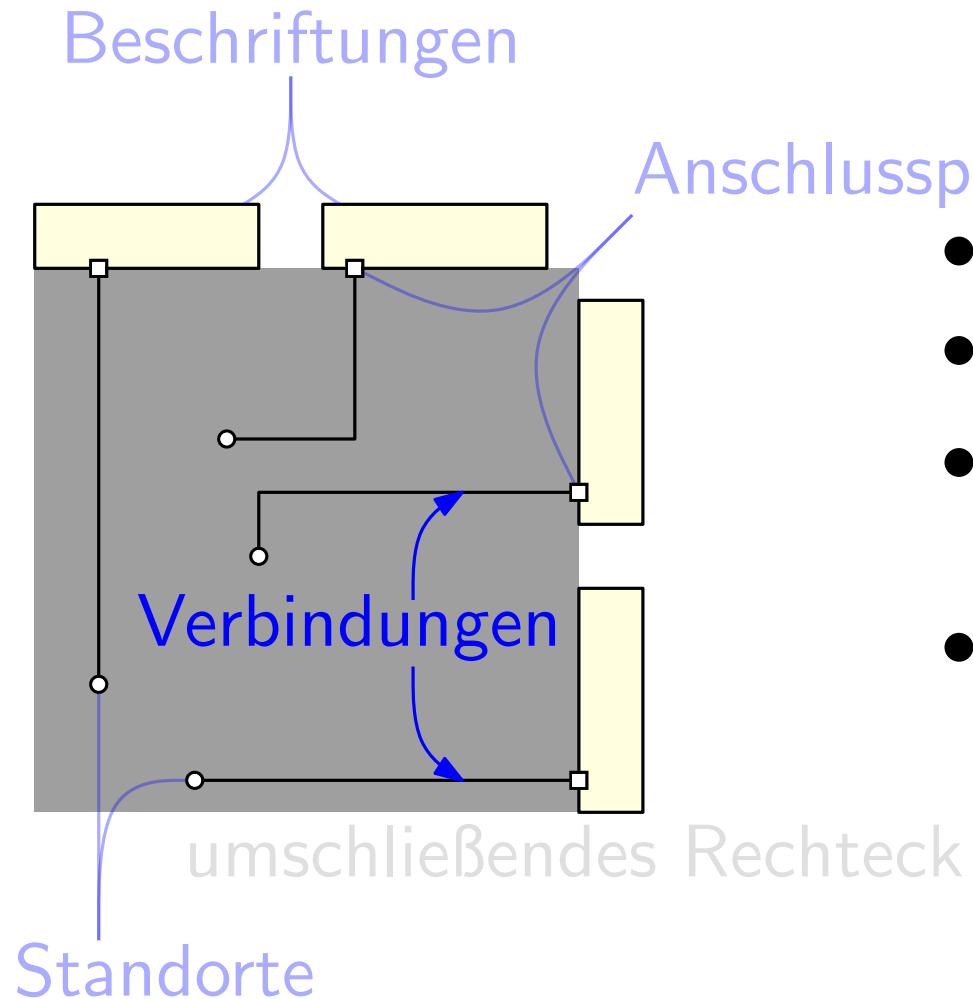
Randbeschriftungen – Problembeschreibung



Anschlusspunkte

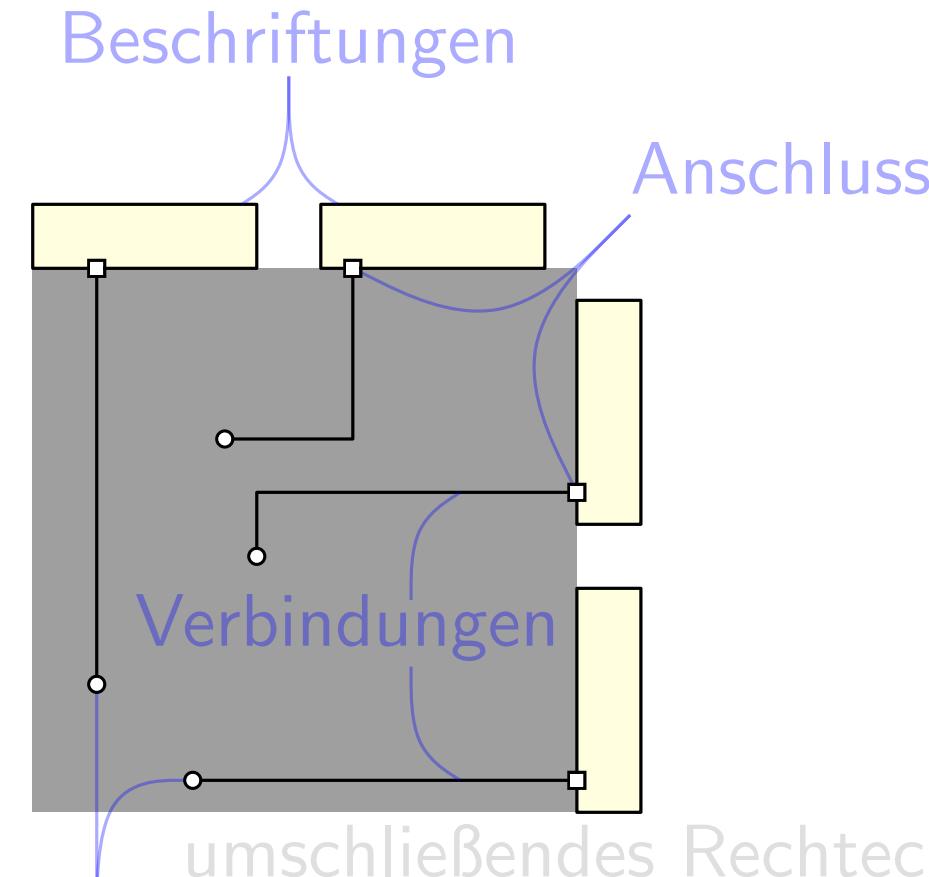
- Standorte im Rechteck
- Beschriftungen am Rand
- Anschlusspunkte sind fest oder verschiebbar

Randbeschriftungen – Problembeschreibung



- *Standorte* im Rechteck
- *Beschriftungen* am Rand
- *Anschlusspunkte* sind fest oder verschiebbar
- *Verbindungen* haben vorgeschriebenen Stil

Randbeschriftungen – Problembeschreibung



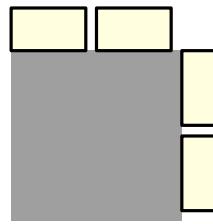
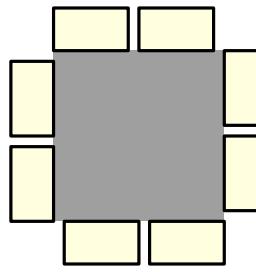
Standorte

Anschlusspunkte

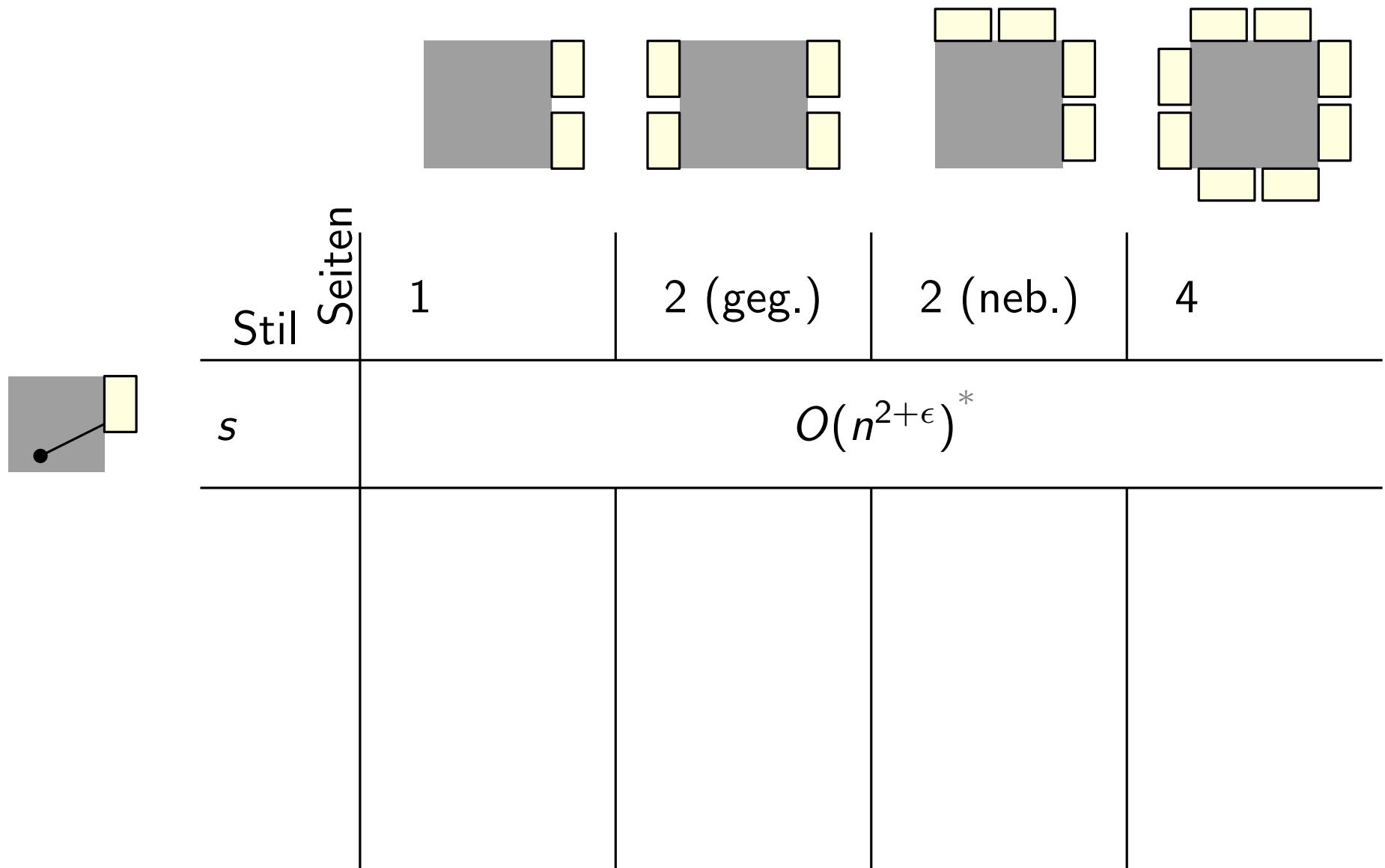
- Standorte im Rechteck
- Beschriftungen am Rand
- Anschlusspunkte sind fest oder verschiebbar
- Verbindungen haben vorgeschriebenen Stil

Finde eine Paarung von Standorten und Beschriftungen mit kreuzungsfreien Verbindungen

Randbeschriftungen – Bekannte Resultate

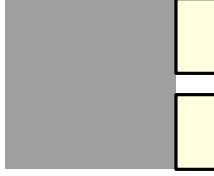
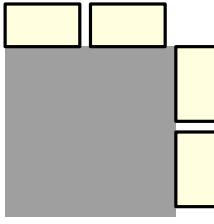
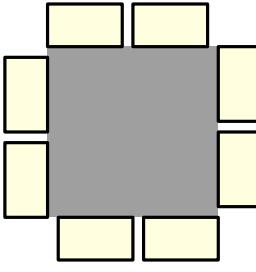
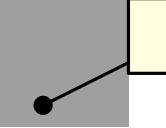
Stil	Seiten	1	2 (geg.)	2 (neb.)	4
					

Randbeschriftungen – Bekannte Resultate



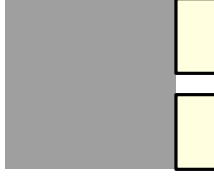
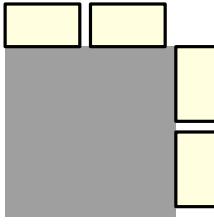
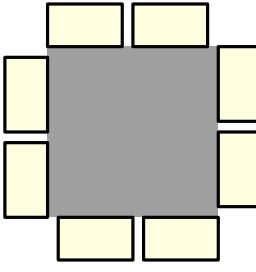
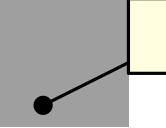
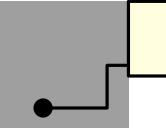
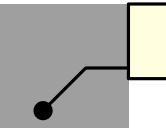
* Bekos et al. [CGTA'07]

Randbeschriftungen – Bekannte Resultate

		Seiten	1	2 (geg.)	2 (neb.)	4
Stil	s					
opo		$O(n \log n)^*$	$O(n^2)^*$	$O(n^2 \log^3 n)^*$		

* Bekos et al. [CGTA'07]

Randbeschriftungen – Bekannte Resultate

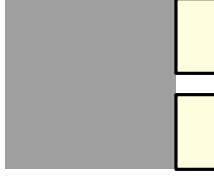
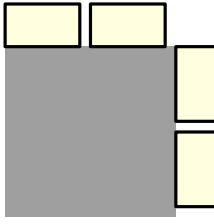
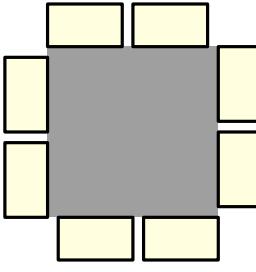
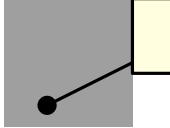
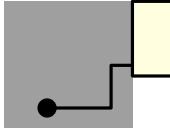
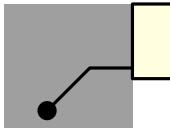
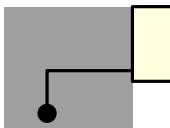
		Seiten	1	2 (geg.)	2 (neb.)	4
Stil						
	s			$O(n^{2+\epsilon})^*$		
	opo	$O(n \log n)^*$	$O(n^2)^*$		$O(n^2 \log^3 n)^*$	
	do/pd	$O(n^2)^\dagger$			$O(n^3)^\P$	

* Bekos et al. [CGTA'07]

† Benkert et al. [JGAA'09]

¶ Bekos et al. [Algorithmica'10]

Randbeschriftungen – Bekannte Resultate

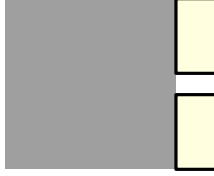
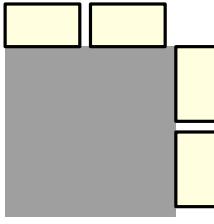
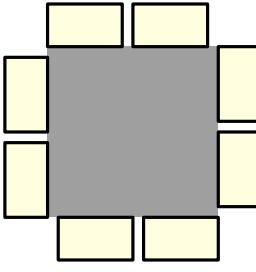
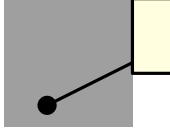
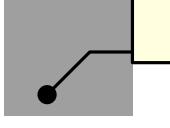
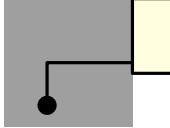
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Stil					
	s			$O(n^{2+\epsilon})^*$	
	opo	$O(n \log n)^*$	$O(n^2)^*$		$O(n^2 \log^3 n)^*$
	do/pd	$O(n^2)^\dagger$		$O(n^3)^\P$	
	po	$O(n \log n)^\dagger$	$O(n^2)^*$		

* Bekos et al. [CGTA'07]

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Randbeschriftungen – Bekannte Resultate

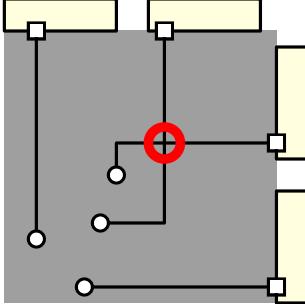
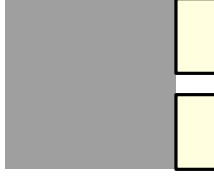
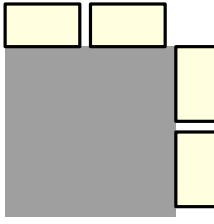
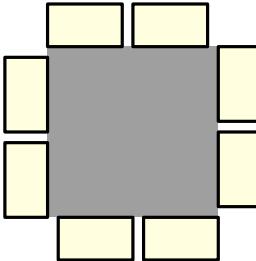
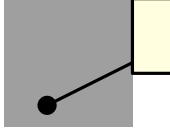
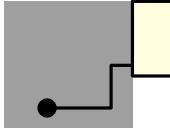
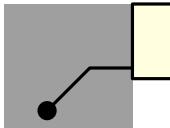
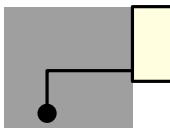
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Stil					
<i>s</i>			$O(n^{2+\epsilon})^*$		
<i>opo</i>		$O(n \log n)^*$	$O(n^2)^*$	$O(n^2 \log^3 n)^*$	
<i>do/pd</i>		$O(n^2)^\dagger$		$O(n^3)^\P$	
<i>po</i>		$O(n \log n)^\dagger$	$O(n^2)^*$?	?

* Bekos et al. [CGTA'07]

† Benkert et al. [JGAA'09]

¶ Bekos et al. [Algorithmica'10]

Randbeschriftungen – Bekannte Resultate

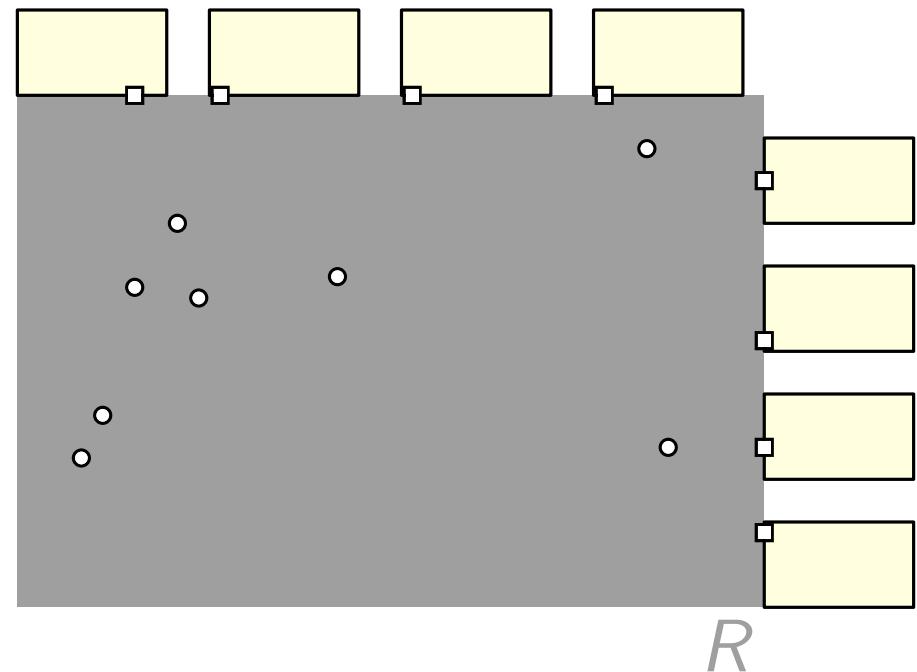
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	Stil					
	s				$O(n^{2+\epsilon})^*$	
	opo		$O(n \log n)^*$	$O(n^2)^*$		$O(n^2 \log^3 n)^*$
	do/pd		$O(n^2)^\dagger$			$O(n^3)^\P$
	po		$O(n \log n)^\dagger$	$O(n^2)^*$?	?

* Bekos et al. [CGTA'07]

† Benkert et al. [JGAA'09]

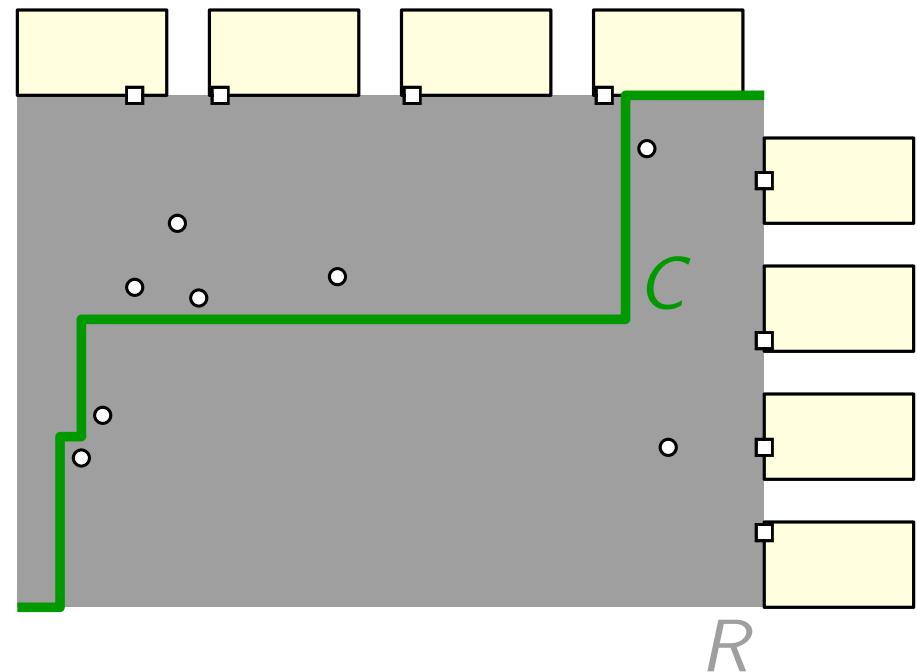
¶ Bekos et al. [Algorithmica'10]

Randbeschriftungen – Struktur



Randbeschriftungen – Struktur

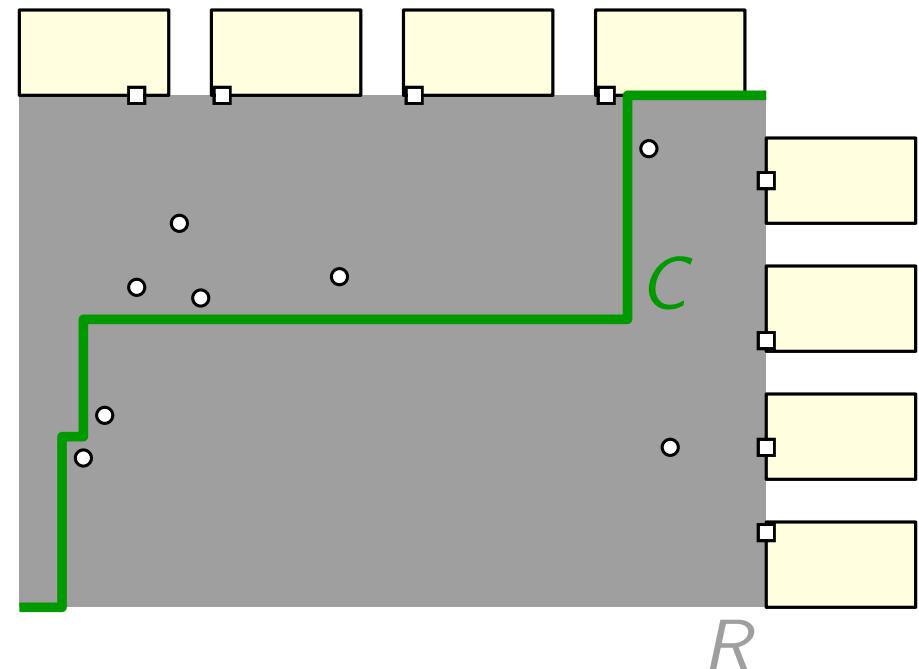
xy-teilende Kurve C



Randbeschriftungen – Struktur

xy-teilende Kurve C

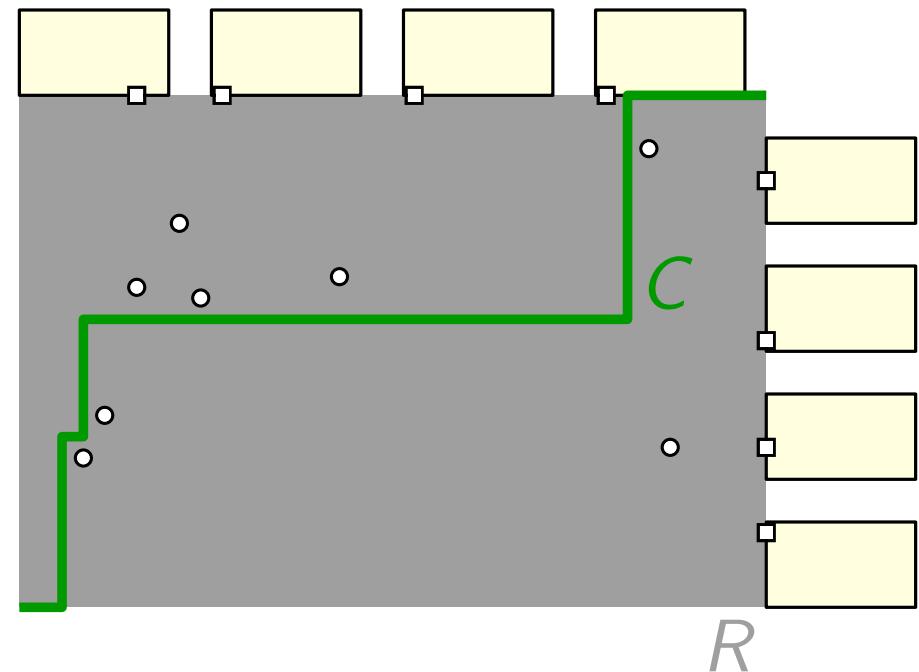
- xy-monoton



Randbeschriftungen – Struktur

xy-teilende Kurve C

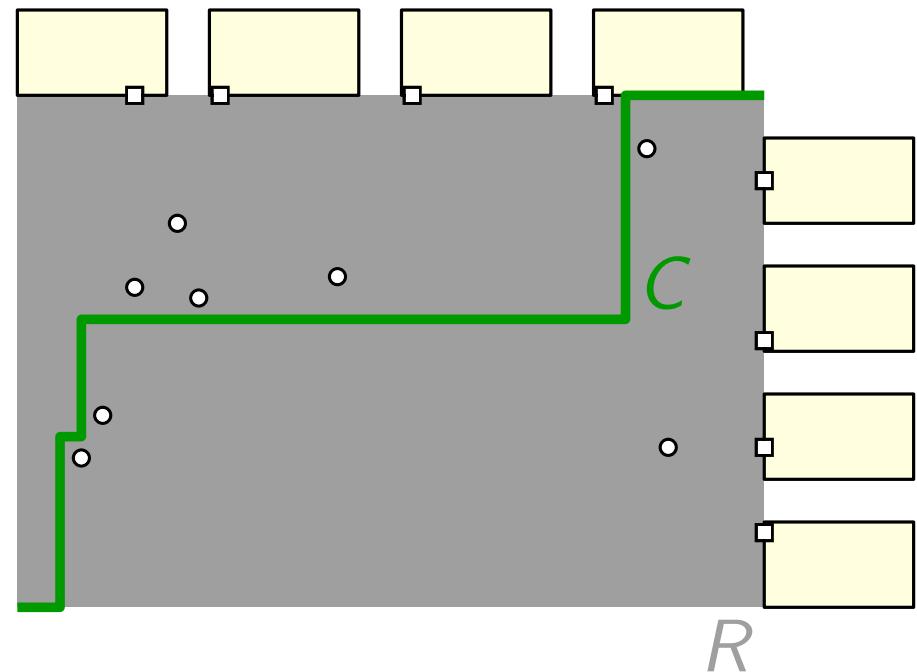
- xy-monoton
- rektilinear



Randbeschriftungen – Struktur

xy-teilende Kurve C

- xy-monoton
- rektilinear
- Verbindet obere rechte mit unterer linker Ecke von R

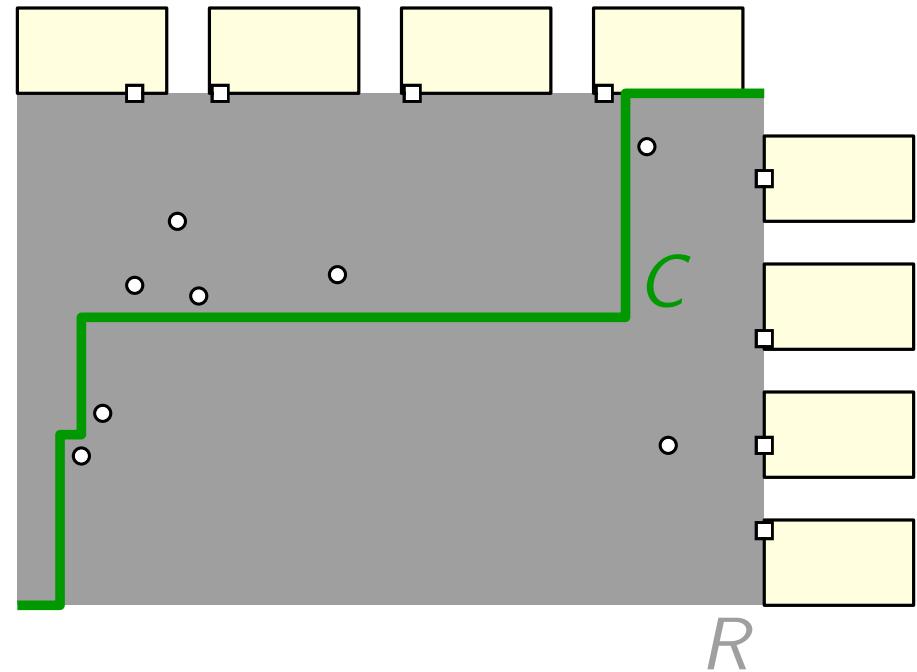


Randbeschriftungen – Struktur

xy-teilende Kurve C

- xy-monoton
- rektilinear
- Verbindet obere rechte mit unterer linker Ecke von R

xy-geteilte Lösung



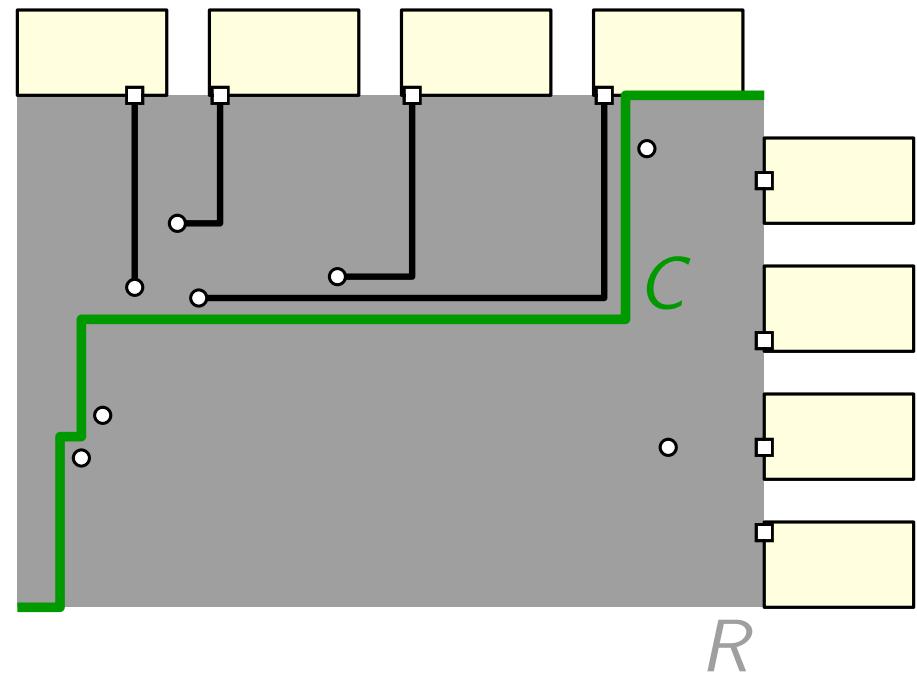
Randbeschriftungen – Struktur

xy-teilende Kurve C

- xy-monoton
- rektilinear
- Verbindet obere rechte mit unterer linker Ecke von R

xy-geteilte Lösung

- Obere Standorte und Verbindungen liegen über C



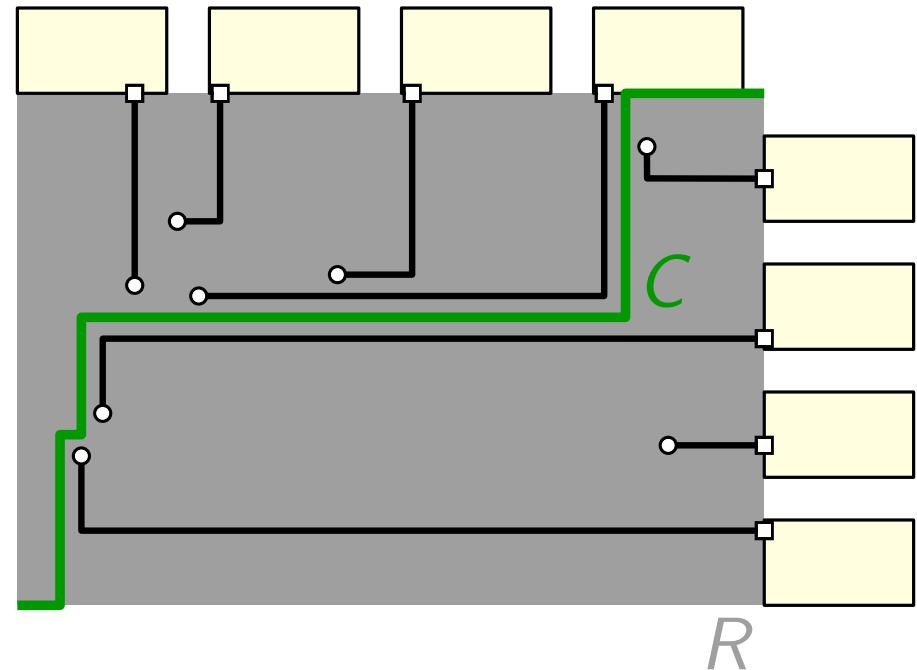
Randbeschriftungen – Struktur

xy-teilende Kurve C

- xy-monoton
- rektilinear
- Verbindet obere rechte mit unterer linker Ecke von R

xy-geteilte Lösung

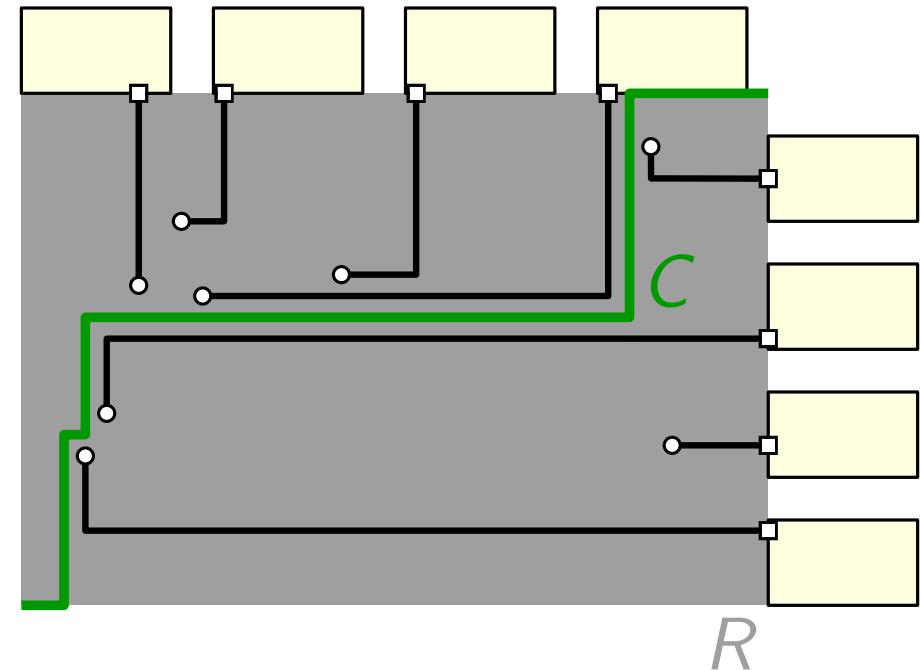
- Obere Standorte und Verbindungen liegen über C
- Rechte Standorte und Verbindungen liegen unter C



Randbeschriftungen – Struktur

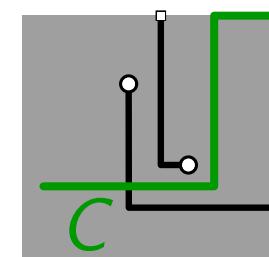
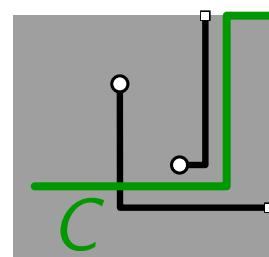
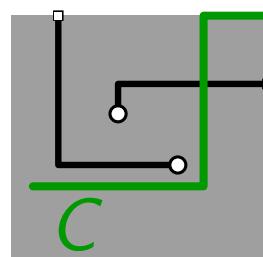
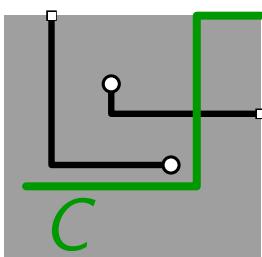
xy-teilende Kurve C

- xy-monoton
- rektilinear
- Verbindet obere rechte mit unterer linker Ecke von R



xy-geteilte Lösung

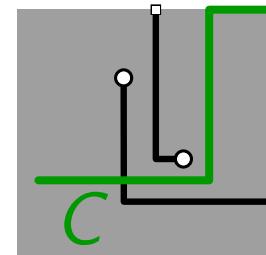
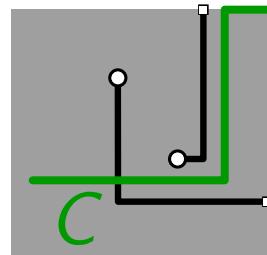
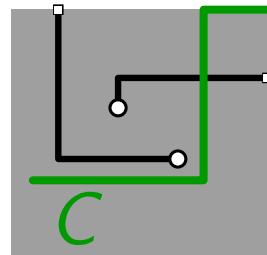
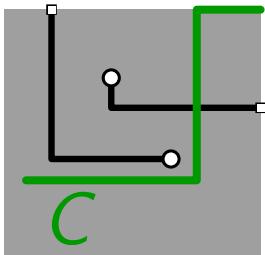
- Obere Standorte und Verbindungen liegen über C
- Rechte Standorte und Verbindungen liegen unter C
- Enthält keine der folgenden Muster



Randbeschriftungen – Struktur

xy-geteilte Lösung

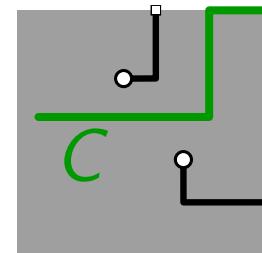
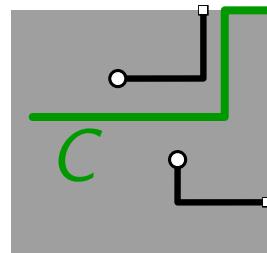
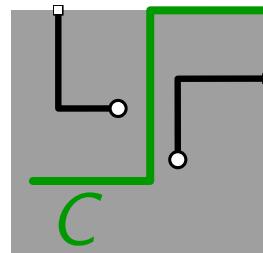
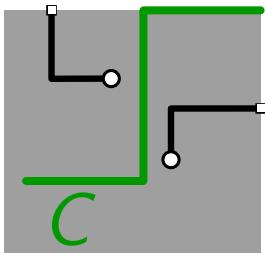
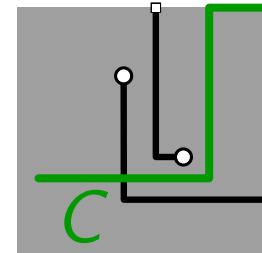
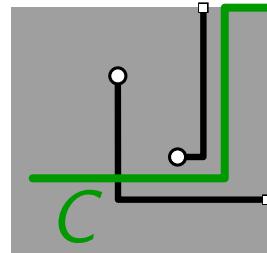
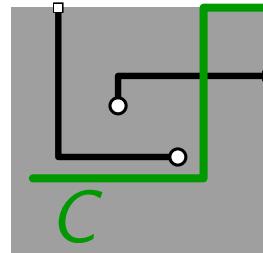
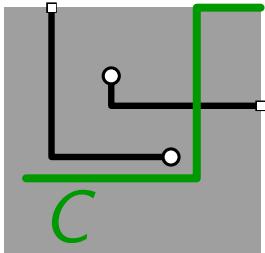
- Enthält keine der folgenden Muster



Randbeschriftungen – Struktur

xy-geteilte Lösung

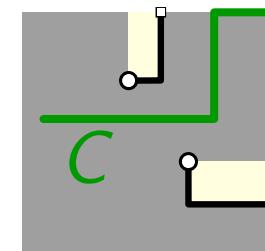
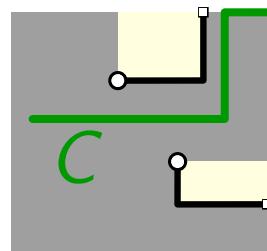
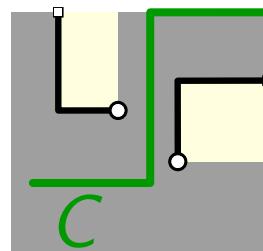
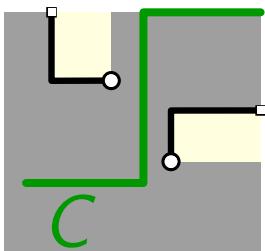
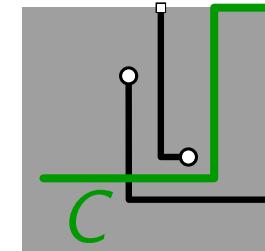
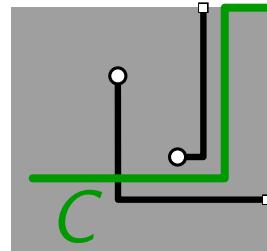
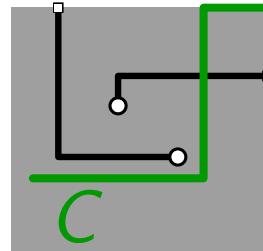
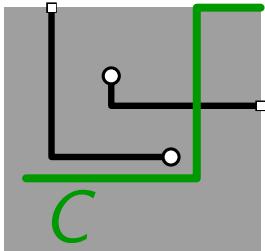
- Enthält keine der folgenden Muster



Randbeschriftungen – Struktur

xy-geteilte Lösung

- Enthält keine der folgenden Muster

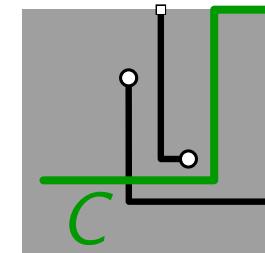
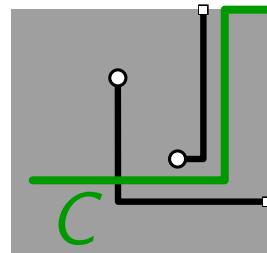
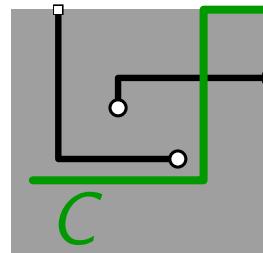
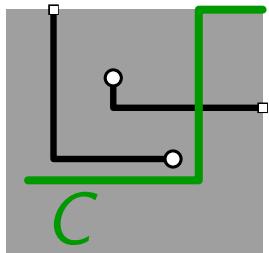


Eliminiere lokale Kreuzungen

Randbeschriftungen – Struktur

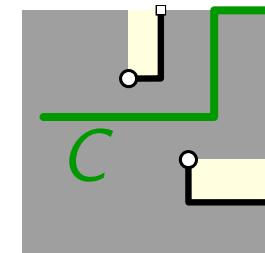
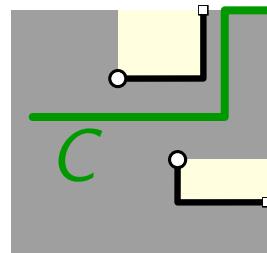
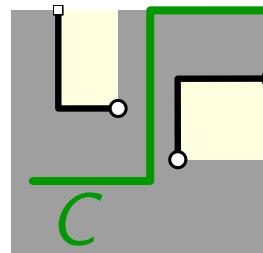
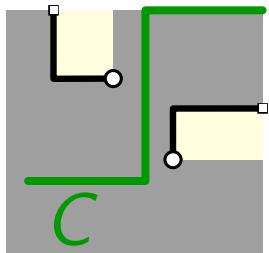
xy-geteilte Lösung

- Enthält keine der folgenden Muster



Charakterisierung: Planare Lösung

⇒ xy-geteilte planare Lösung

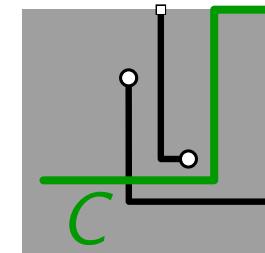
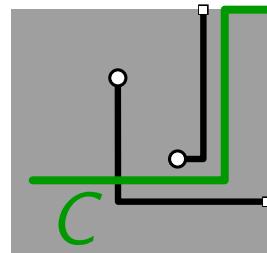
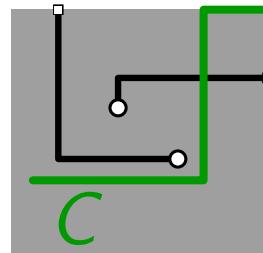
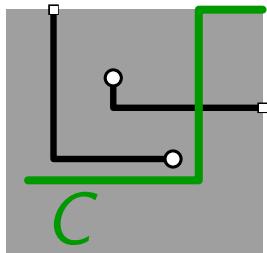


Eliminiere lokale Kreuzungen

Randbeschriftungen – Struktur

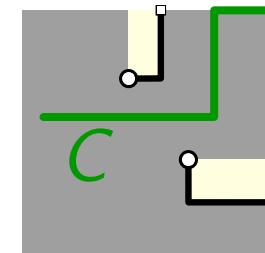
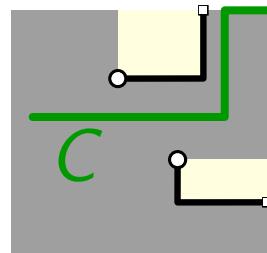
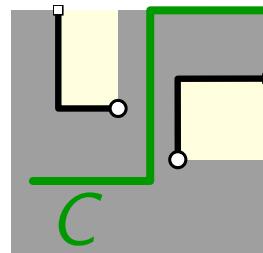
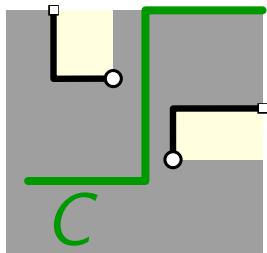
xy-geteilte Lösung

- Enthält keine der folgenden Muster



Charakterisierung: Planare Lösung
⇒ xy-geteilte planare Lösung

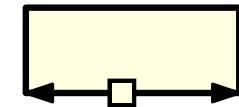
Dynamisches Programm!



Eliminiere lokale Kreuzungen

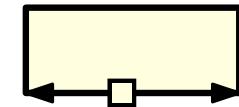
Randbeschriftungen – Ergebnisse

Zweiseitig, verschiebbare Anschlusspunkte:
 $O(n^2)$ Zeit, $O(n)$ Platz



Randbeschriftungen – Ergebnisse

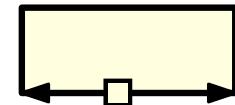
Zweiseitig, verschiebbare Anschlusspunkte:
 $O(n^2)$ Zeit, $O(n)$ Platz



Maximiere Anzahl der Beschriftungen:
 $O(n^3 \log n)$ Zeit, $O(n)$ Platz

Randbeschriftungen – Ergebnisse

Zweiseitig, verschiebbare Anschlusspunkte:
 $O(n^2)$ Zeit, $O(n)$ Platz

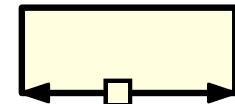


Maximiere Anzahl der Beschriftungen:
 $O(n^3 \log n)$ Zeit, $O(n)$ Platz

Minimiere Gesamtlänge der Verbindungen:
 $O(n^8 \log n)$ Zeit, $O(n^6)$ Platz

Randbeschriftungen – Ergebnisse

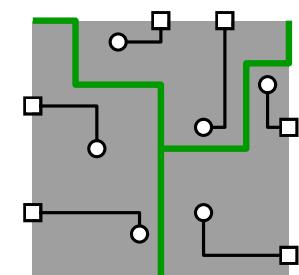
Zweiseitig, verschiebbare Anschlusspunkte:
 $O(n^2)$ Zeit, $O(n)$ Platz



Maximiere Anzahl der Beschriftungen:
 $O(n^3 \log n)$ Zeit, $O(n)$ Platz

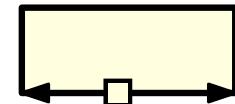
Minimiere Gesamtlänge der Verbindungen:
 $O(n^8 \log n)$ Zeit, $O(n^6)$ Platz

Dreiseitige Randbeschriftungen:
 $O(n^4)$ Zeit, $O(n)$ Platz



Randbeschriftungen – Ergebnisse

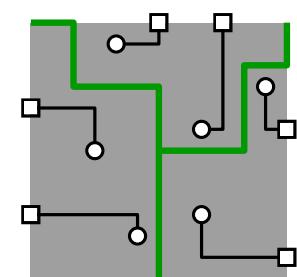
Zweiseitig, verschiebbare Anschlusspunkte:
 $O(n^2)$ Zeit, $O(n)$ Platz



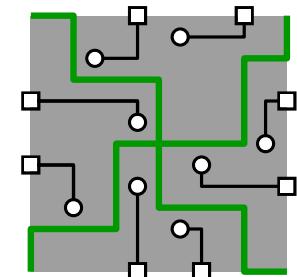
Maximiere Anzahl der Beschriftungen:
 $O(n^3 \log n)$ Zeit, $O(n)$ Platz

Minimiere Gesamtlänge der Verbindungen:
 $O(n^8 \log n)$ Zeit, $O(n^6)$ Platz

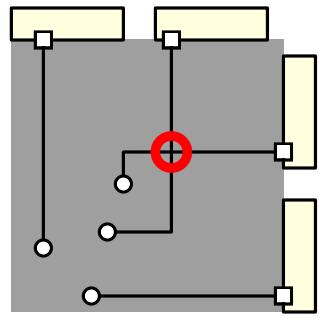
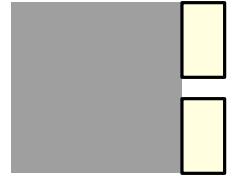
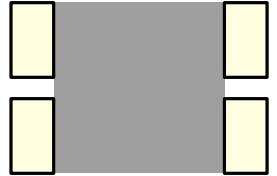
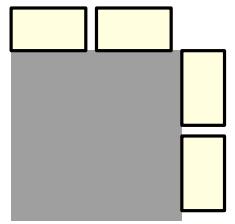
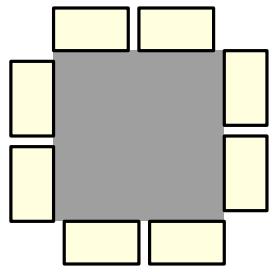
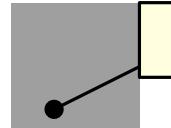
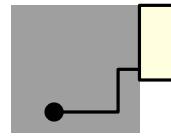
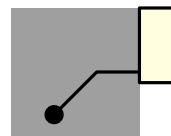
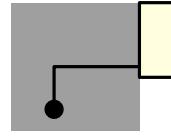
Dreiseitige Randbeschriftungen:
 $O(n^4)$ Zeit, $O(n)$ Platz



Vierseitige Randbeschriftungen:
 $O(n^9)$ Zeit, $O(n)$ Platz



Randbeschriftungen – Ergebnisse

		Seiten	1	2 (geg.)	2 (neb.)	4
	Stil					
	s				$O(n^{2+\epsilon})^*$	
	opo		$O(n \log n)^*$	$O(n^2)^*$		$O(n^2 \log^3 n)^*$
	do/pd		$O(n^2)^\dagger$			$O(n^3)^\P$
	po		$O(n \log n)^\dagger$	$O(n^2)^*$	$O(n^2)$	$O(n^9)$

* Bekos et al. [CGTA'07]

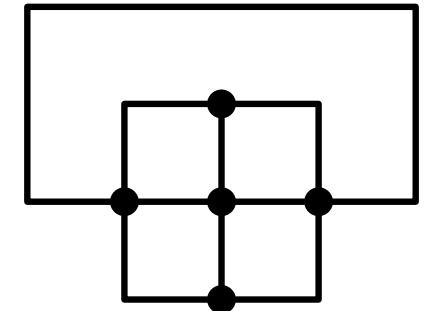
† Benkert et al. [JGAA'09]

¶ Bekos et al. [Algorithmica'10]

Teil 2: Glatt-orthogonale Darstellungen planarer Graphen

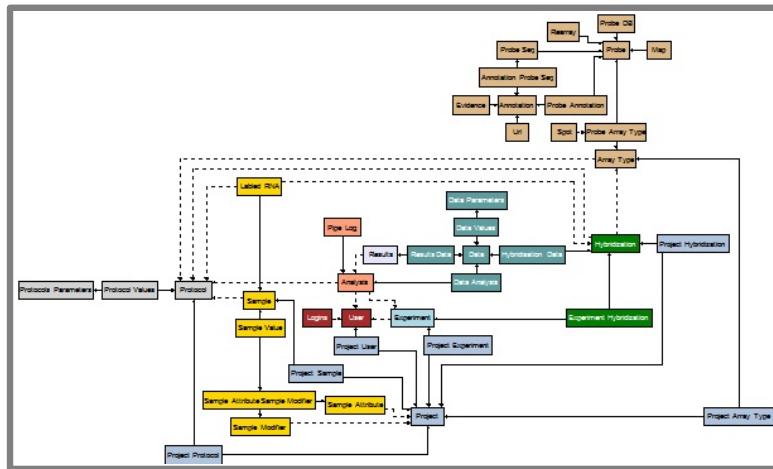
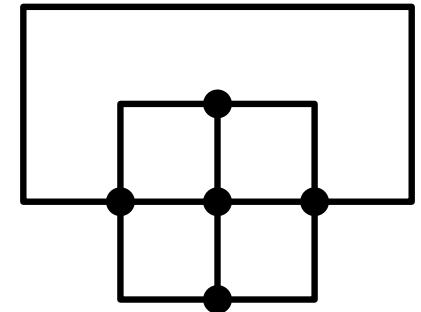
Orthogonale Darstellungen

- Alle Segmente sind horizontal oder vertikal
- Intensiv untersuchter Zeichenstil
- Viele Anwendungsbeispiele

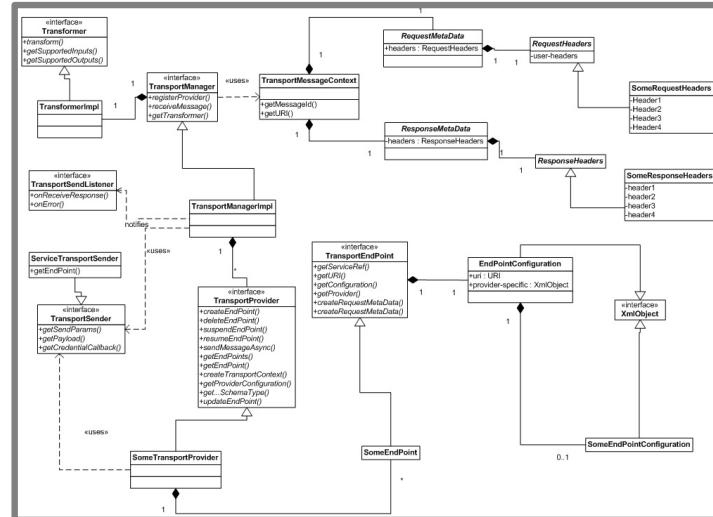


Orthogonale Darstellungen

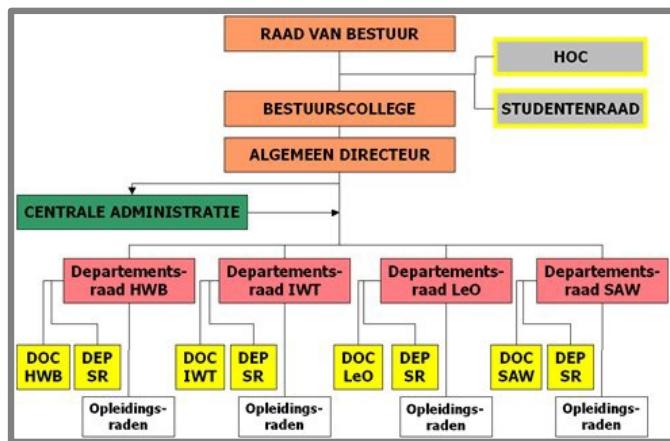
- Alle Segmente sind horizontal oder vertikal
 - Intensiv untersuchter Zeichenstil
 - Viele Anwendungsbeispiele



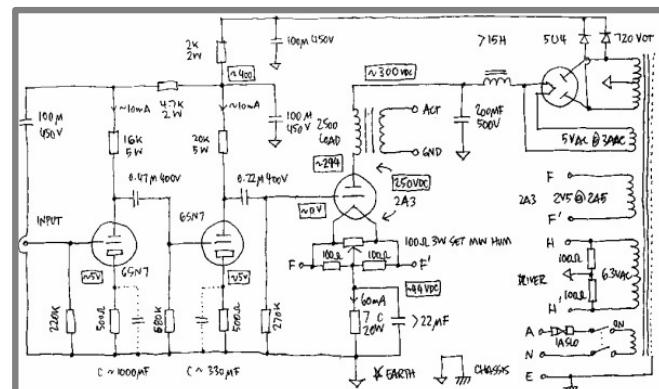
ER-Diagramm in OGDF



UML-Diagramm von Oracle



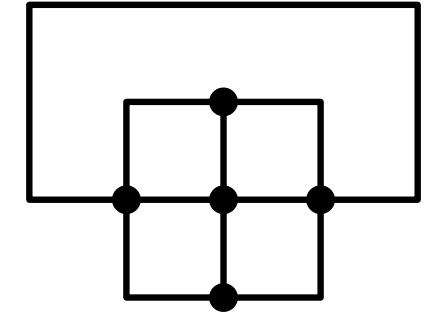
Organigramm von HS Limburg



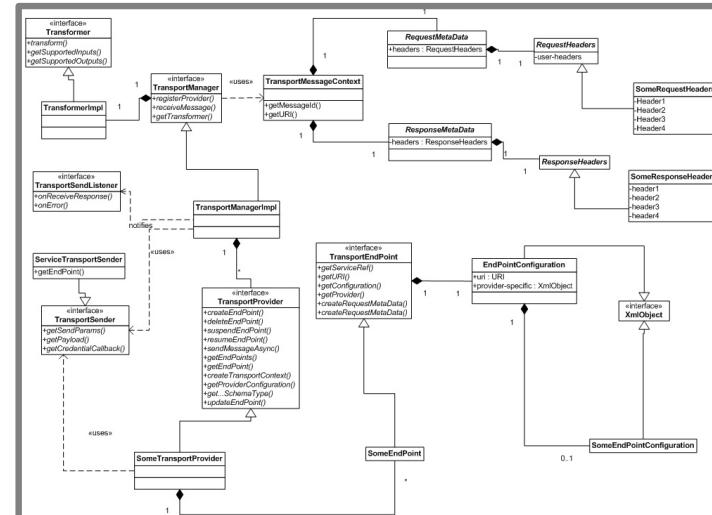
Schaltkreis-Diagramm von Jeff Atwood

Orthogonale Darstellungen

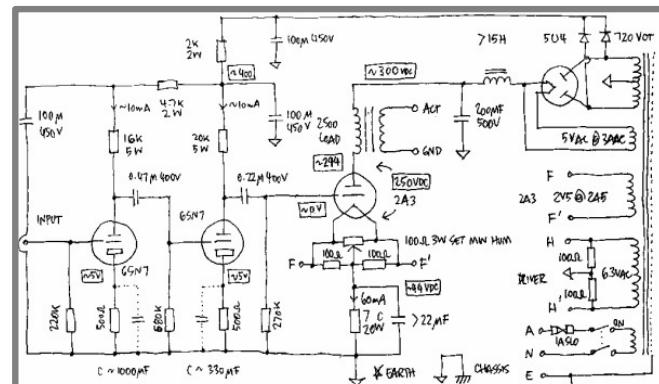
- Alle Segmente sind horizontal oder vertikal
- Intensiv untersuchter Zeichenstil
- Viele Anwendungsbeispiele



“Fused Grid” Stadtentwurf [Fgrammen, via Wikimedia Commons]



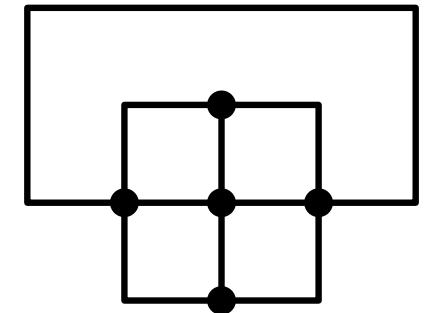
UML-Diagramm von Oracle



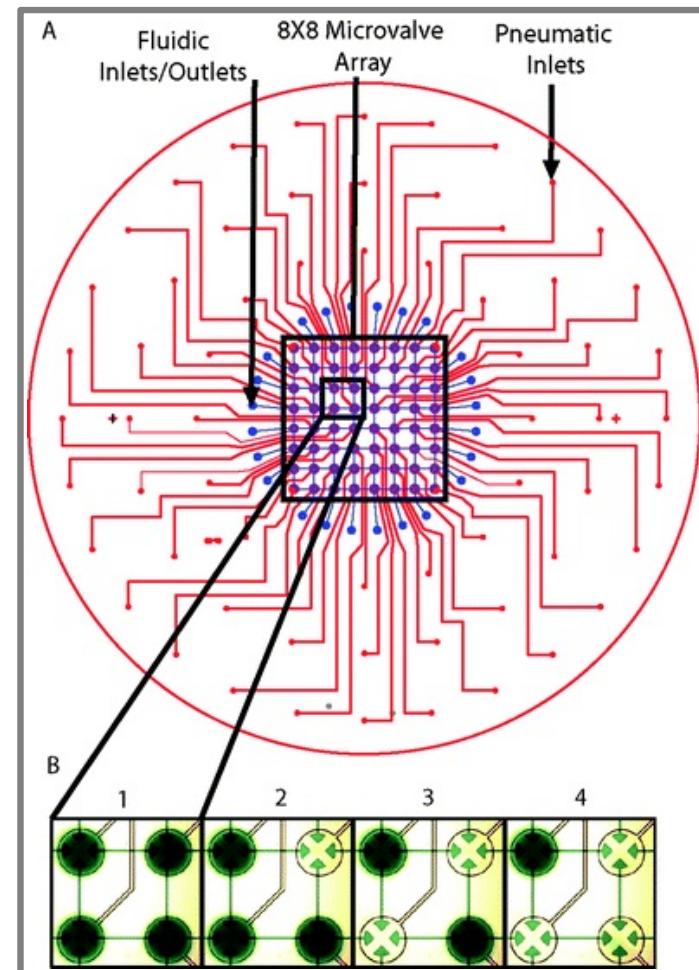
Schaltkreis-Diagramm von Jeff Atwood

Orthogonale Darstellungen

- Alle Segmente sind horizontal oder vertikal
- Intensiv untersuchter Zeichenstil
- Viele Anwendungsbeispiele



“Fused Grid” Stadtentwurf [Fgrammen, via Wikimedia Commons]



VLSI/PCI Chip Entwurf [Mora et al. 2013]

Glatte Zeichnungen



Mercedes W 114, 1967

[Daduschu, via Wikimedia Commons]

Nissan Friend-ME, 2013



© Nissan

Glatte Zeichnungen

FOLLOW THE MONEY

The New Global Wealth Machine

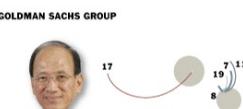
Sovereign wealth funds have emerged in recent months as the world's power brokers. They have used their tremendous wealth to make big cross-border investments and prop up some of Wall Street's best-known firms. The increased activity comes as other kinds of acquirers have been sidelined by the credit crisis. These funds are state-sponsored investment vehicles and have combined assets of \$2 trillion. With that much dry powder, sovereign funds dwarf the formerly boozing private equity industry — and in some cases, compete directly with it. The Government of Singapore Investment Corporation, controlled by Prime Minister Lee Hsien Loong and his deputy chairman, Tony Tan, a major center of gravity, Wall Street veterans always follow the money, so many of the big-name advisers in New York and London have found themselves traveling the globe playing international matchmaker to these funds. But sovereign funds have also learned the downside of deal-making: some of their blockbuster transactions have been big money losers so far. The question is where all that money will go next. **ANDREW ROSS SORKIN**

The Advisers

Selected financial advisers who worked on more than one of the top 20 deals.



Michael Klein, Chairman, institutional clients group
One of the firm's highest-profile investment bankers, he advised Carlyle in its stake sale to Mubadala, as well as Citigroup in both of its deals with sovereign wealth funds.



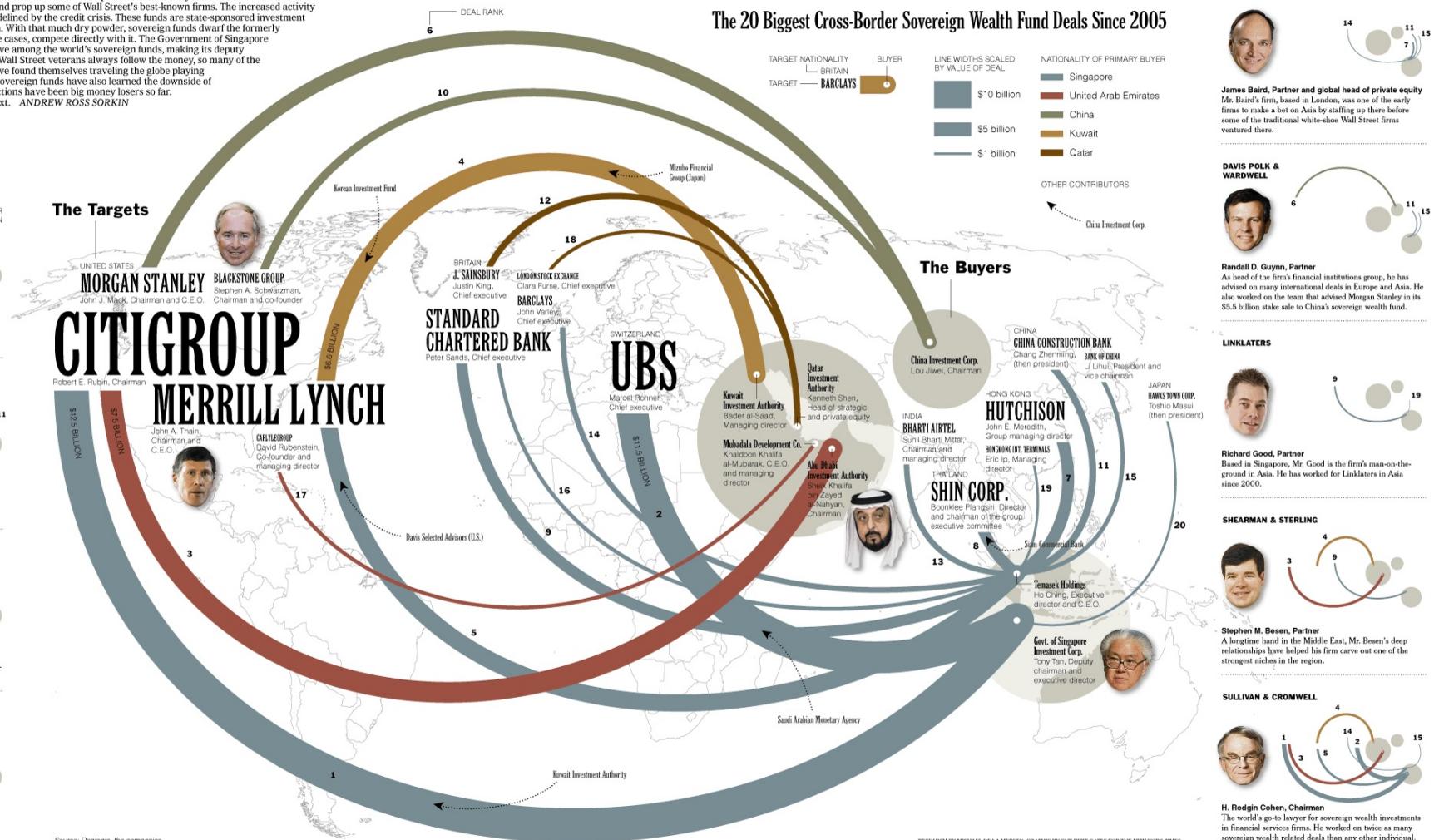
Richard Ong, Former managing director
Mr. Ong left Goldman early this year after the Chinese government refused to allow the firm to promote him to run its Beijing office. Mr. Ong's brother, Charles, was the chief investment officer of Temasek Holdings until 2006.



Gary Parr, Deputy chairman
In addition to becoming the key adviser on many of the biggest sovereign wealth deals, Mr. Parr helped advise Bear Stearns on its distressed sale to JPMorgan Chase.



**Kate Richdale,
Managing director**



The New York Times Infodiagramm

RESEARCH BY MICHAEL DE LA MERCIER; GRAPHIC BY GAIL JUIT GATES FOR THE NEW YORK TIMES

The Lawyer

The Lawyers
Selected lawyers who worked on more than one of the top 20 deals.

CLIFFORD CHANCE



James Baird, Partner and global head of private equity
Mr. Baird's firm, based in London, was one of the early firms to make a bet on Asia by staffing up there before some of the traditional white-shoe Wall Street firms ventured there.

DAVIS POLK
WARDWELL



Randall D. Guynn, Partner
As head of the firm's financial institutions group, he has advised on many international deals in Europe and Asia. He also worked on the team that advised Morgan Stanley in its \$3.5 billion stake sale to China's sovereign wealth fund.

LINKLATE



Richard Good, Partner
Based in Singapore, Mr. Good is the firm's man-on-the-ground in Asia. He has worked for Linklaters in Asia since 2000.

SHEARMAN & STERLING



Stephen M. Besen, Partner
A longtime hand in the Middle East, Mr. Besen's deep relationships have helped his firm carve out one of the strongest niches in the region.

SULLIVAN & CROMWE



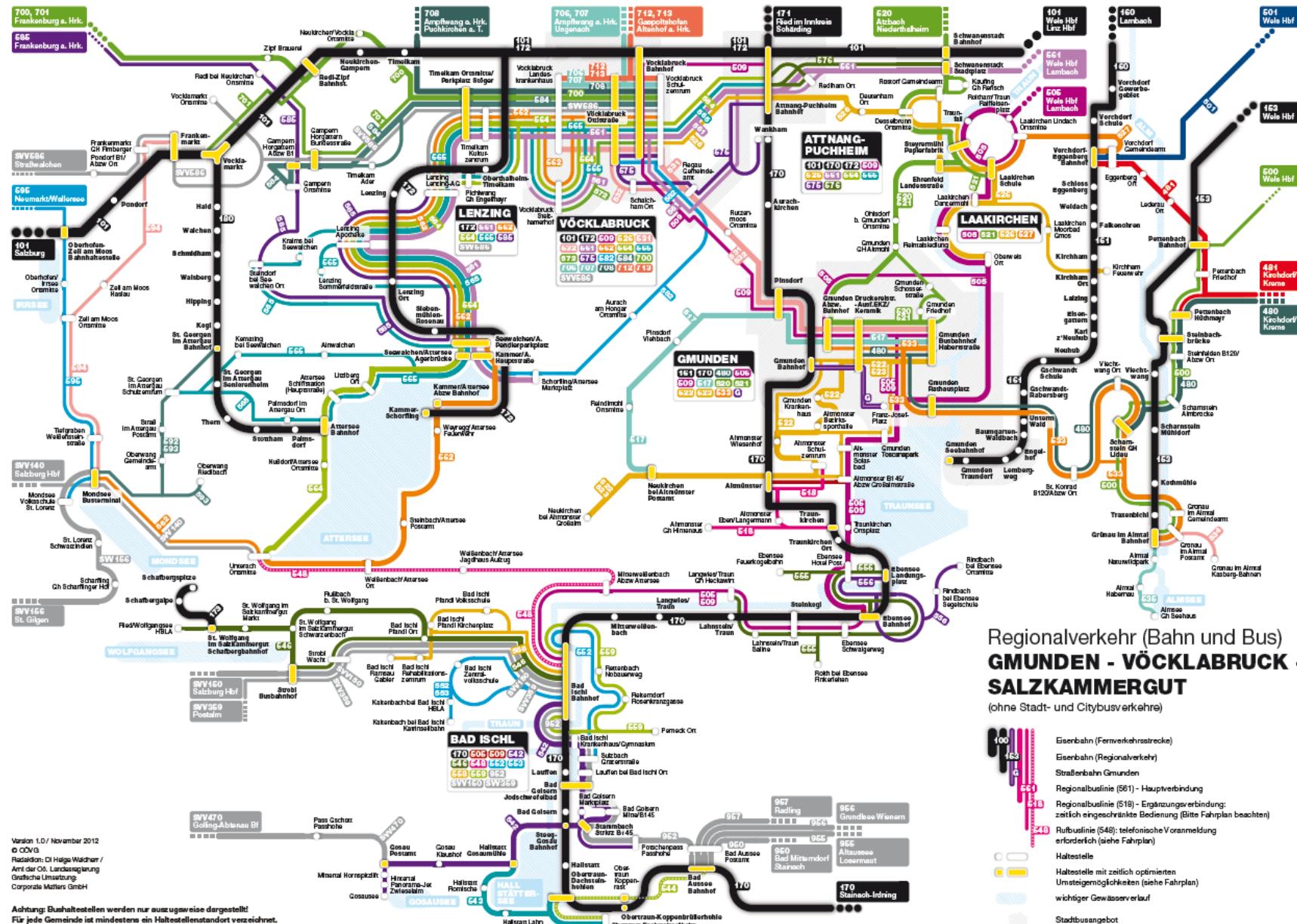
H. Rodgin Cohen, Chairman
The world's go-to lawyer for sovereign wealth investments in financial services firms. He worked on twice as many sovereign wealth related deals than any other individual.

Glatte Zeichnungen



Terra Nova - Design Overdrive von Carlos Barbosa

Glatte Zeichnungen



Regionalverkehr (Bahn und Bus) GMUNDEN - VÖCKLABRUCK - SALZKAMMERTGUT

(ohne Stadt- und Citybusverkehre)

Version 1.0 / November 2012
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Ressort: Ch Heinz Walcher /
Amt der OÖ Landesregierung
Graphische Umsetzung:
Corporate Matters GmbH

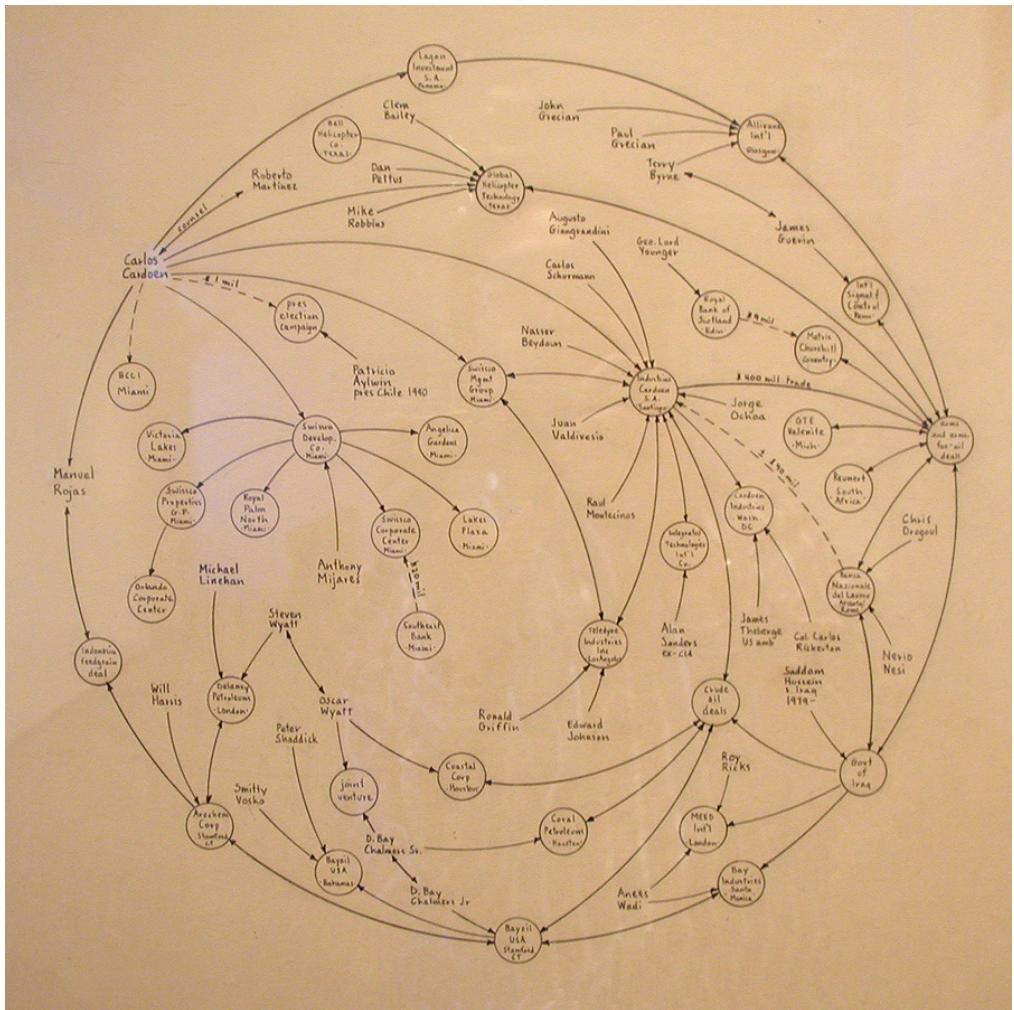
Achtung: Bushaltestellen werden nur auszugsweise dargestellt!
Für jede Gemeinde ist mindestens ein Haltestellenstandort verzeichnet.

Regionalverkehrskarte von OÖVG

Glatte Zeichnungen

Lombardi-Zeichnungen

- Kreissegmente
 - Perfekte Winkelauflösung



Mark Lombardi (1951–2000)

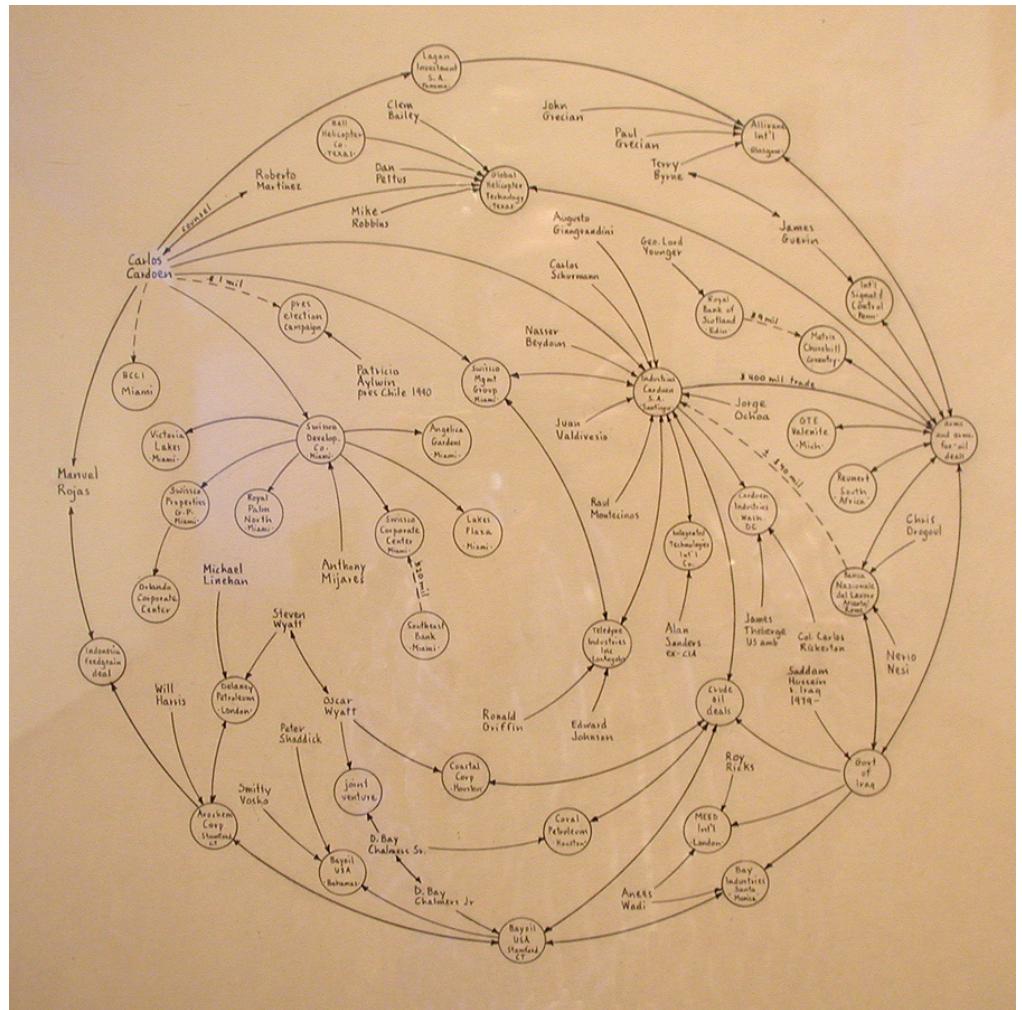
Glatte Zeichnungen

Lombardi-Zeichnungen

- Kreissegmente
- Perfekte
Winkelauflösung

k-Lombardi-Zeichnungen

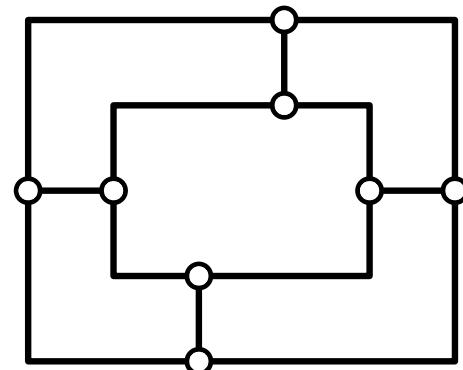
- Kanten bestehen aus k
Kreissegmenten



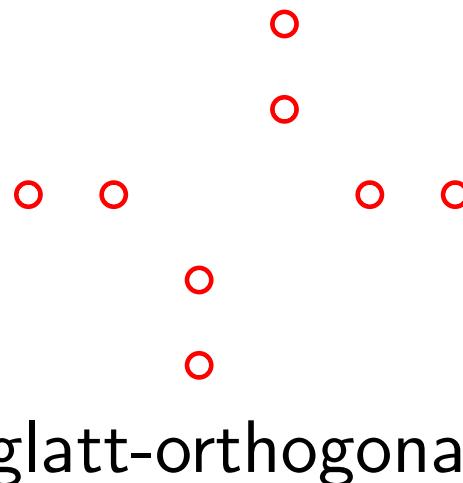
Mark Lombardi
(1951–2000)

Glatt-orthogonale Darstellungen

Vereine beide Welten:



orthogonal

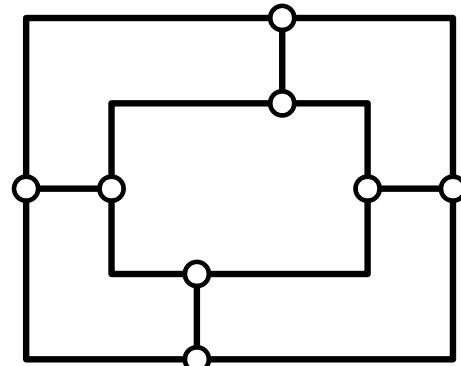


glatt-orthogonal

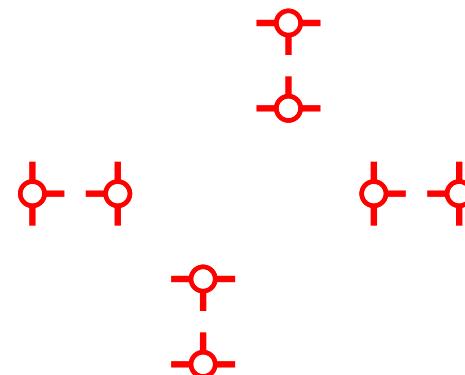
Glatt-orthogonale Darstellungen

Vereine beide Welten:

- Kanten treffen horizontal oder vertikal auf Knoten



orthogonal

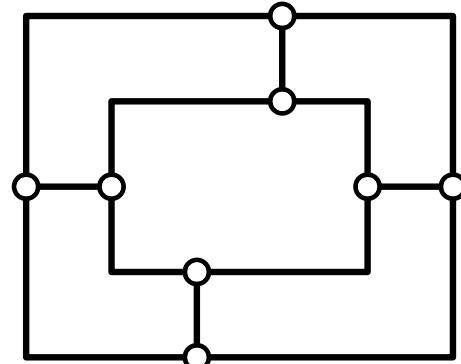


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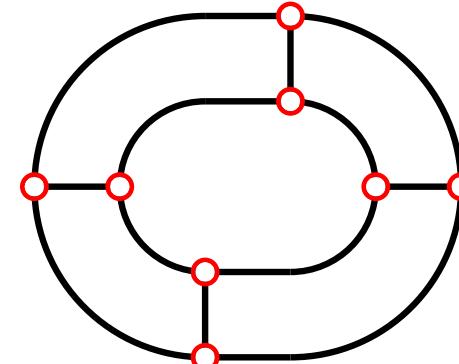
Glatt-orthogonale Darstellungen

Vereine beide Welten:

- Kanten treffen horizontal oder vertikal auf Knoten
- Kanten bestehen aus achsen-parallelens Segmenten und Kreissegmenten ohne Knicke



orthogonal

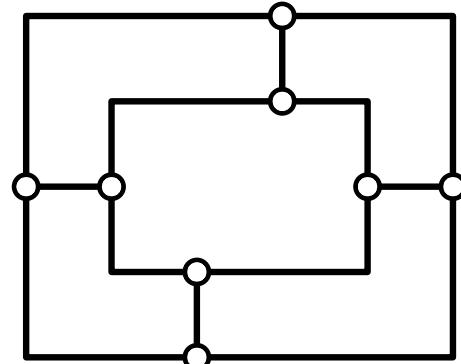


glatt-orthogonal

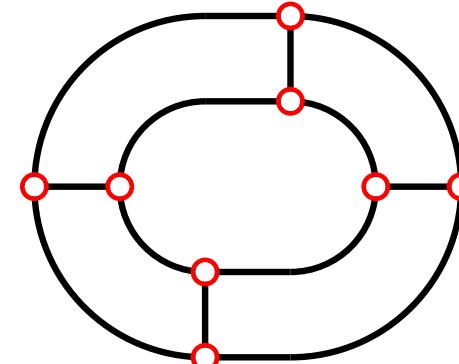
Glatt-orthogonale Darstellungen

Vereine beide Welten:

- Kanten treffen horizontal oder vertikal auf Knoten
- Kanten bestehen aus achsen-parallel Segmenten und Kreissegmenten ohne Knicke
- Es gibt keine Kantenkreuzungen (für planare Graphen)

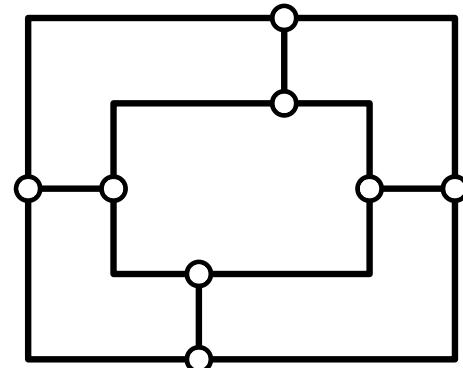


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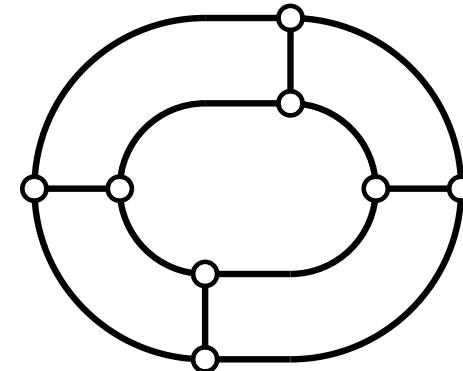


glatt-orthogonal

Kantenkomplexität



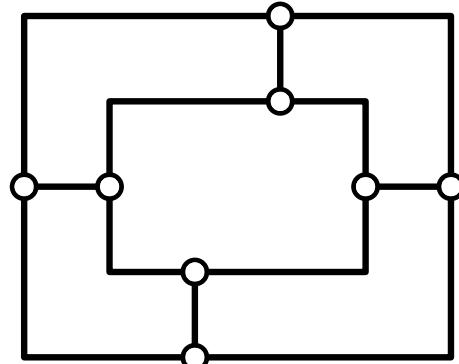
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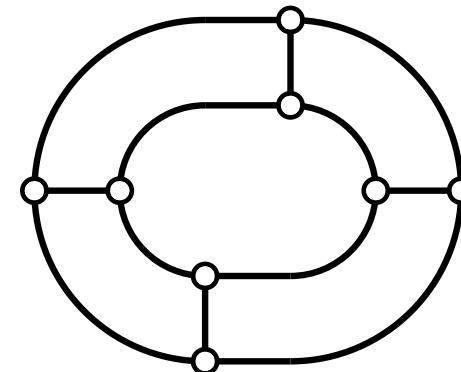
glatt-orthogonal

Kantenkomplexität

Komplexität einer Kante: Anzahl der Segmente



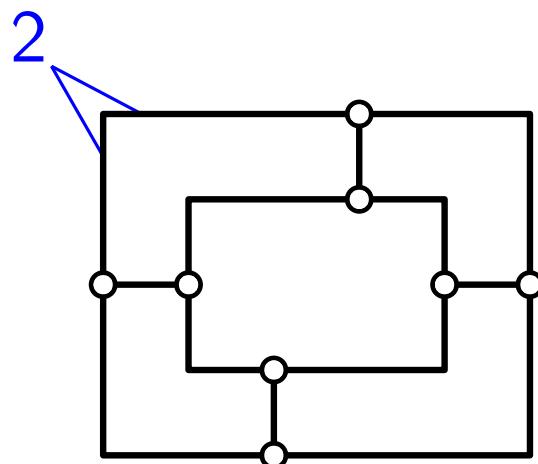
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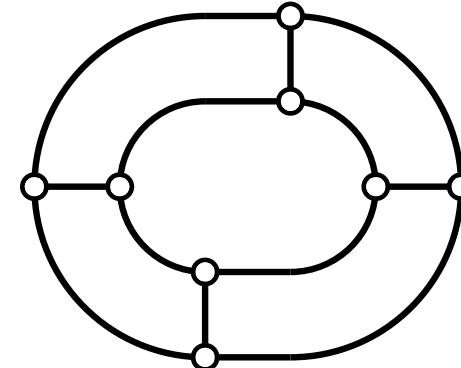
glatt-orthogonal

Kantenkomplexität

Komplexität einer Kante: Anzahl der Segmente



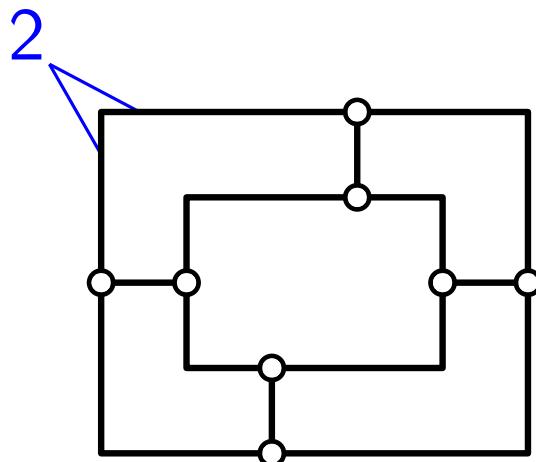
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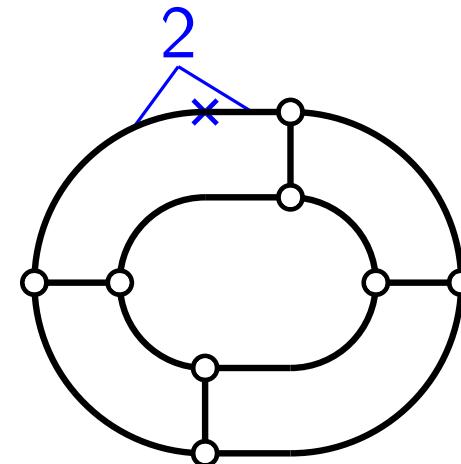
glatt-orthogonal

Kantenkomplexität

Komplexität einer Kante: Anzahl der Segmente



orthogonal

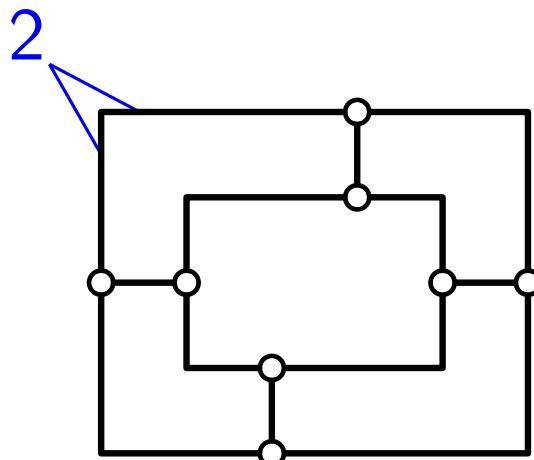


glatt-orthogonal

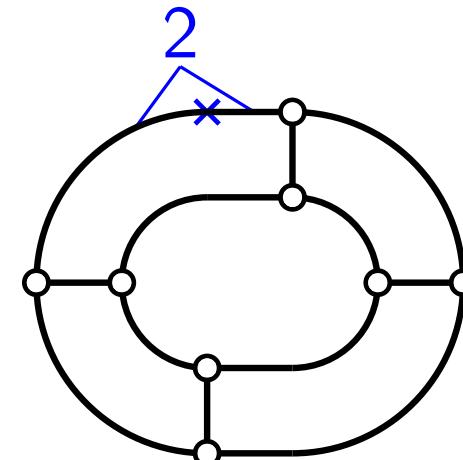
Kantenkomplexität

Komplexität einer Kante: Anzahl der Segmente

Komplexität einer Zeichnung: Größte Komplexität aller Kanten



orthogonal

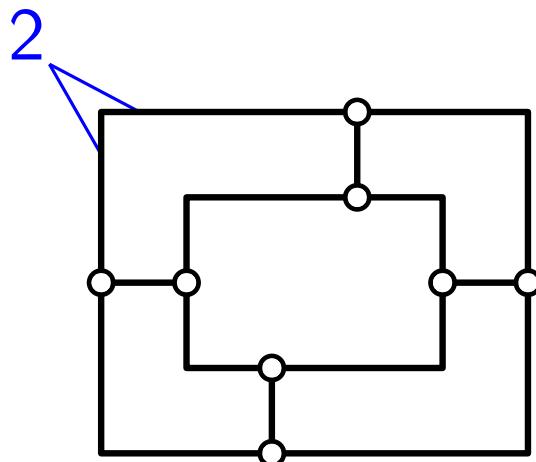
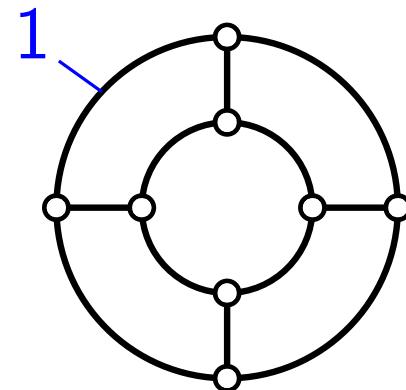
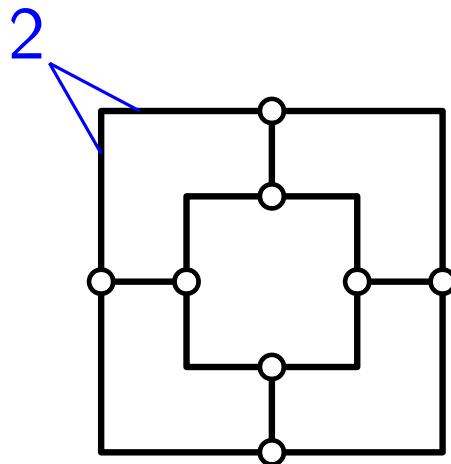


glatt-orthogonal

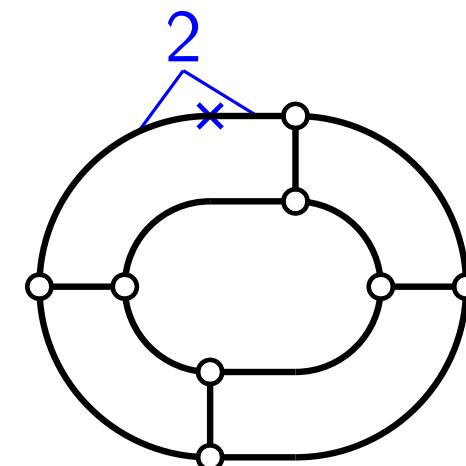
Kantenkomplexität

Komplexität einer Kante: Anzahl der Segmente

Komplexität einer Zeichnung: Größte Komplexität aller Kanten



orthogonal



glatt-orthogonal

Bekannte Resultate

Biedl & Kant [CGTA'98],

Liu et al. [DAM'98]

4-planarer Graph \Rightarrow orth. Komplexität 3, $(n \times n)$ -Gitter

Bekannte Resultate

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Bekos et al.

[GD'12]

- Zweifach zusammenhängender 4-planarer Graph \Rightarrow glatt-orth. Komplexität 3

Bekannte Resultate

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4-planarer Graph \Rightarrow orth. Komplexität 3, $(n \times n)$ -Gitter

Bekos et al.

[GD'12]

- Zweifach zusammenhängender 4-planarer Graph \Rightarrow glatt-orth. Komplexität 3
- 4-planarer Graph $\not\Rightarrow$ glatt-orth. Komplexität 1

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Biedl & Kant [CGTA'98],

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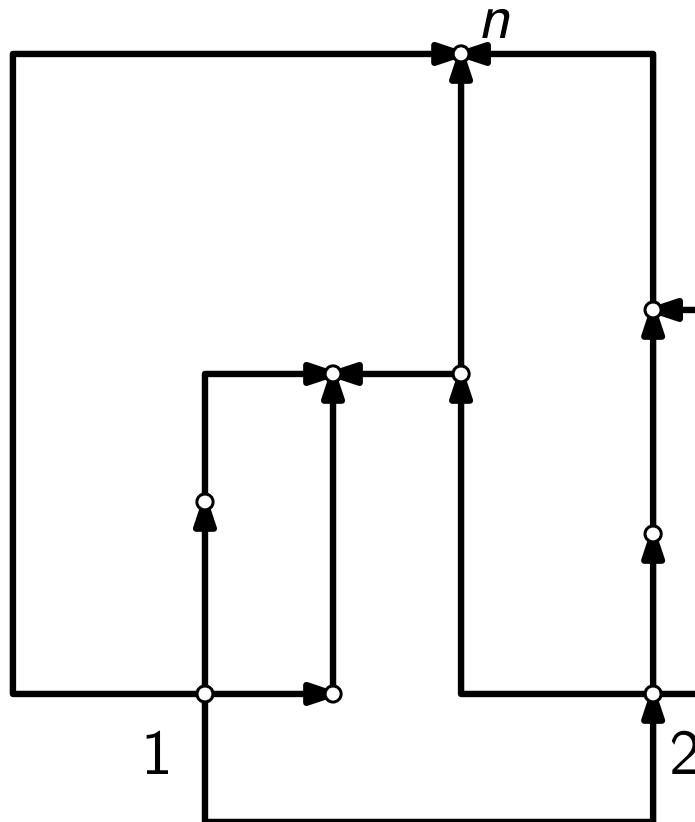
4-planarer Graph \Rightarrow orth. Komplexität 3, $(n \times n)$ -Gitter

Bekos et al.

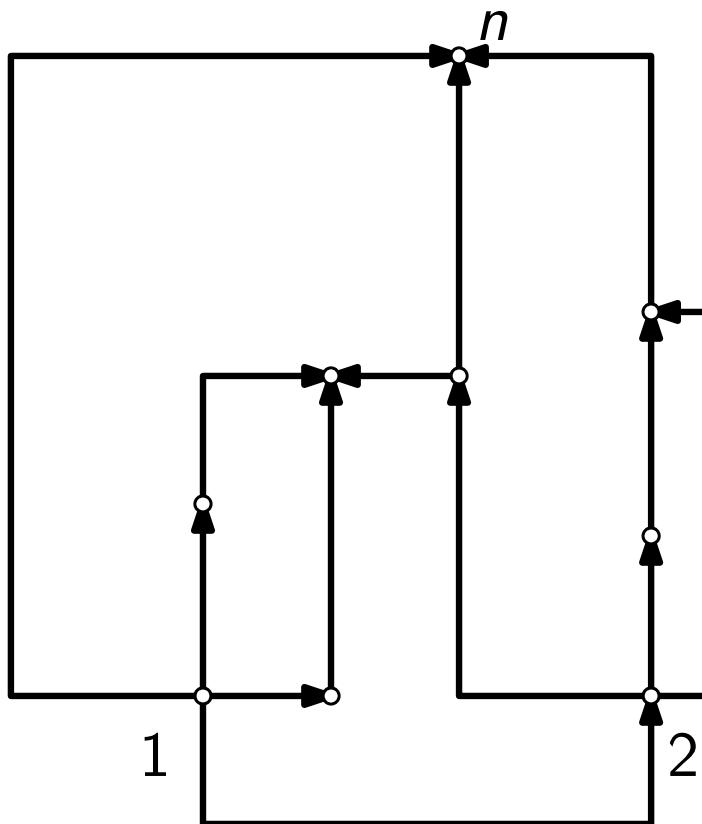
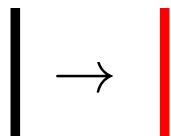
[GD'12]

- Zweifach zusammenhängender 4-planarer Graph \Rightarrow glatt-orth. Komplexität 3
- 4-planarer Graph $\not\Rightarrow$ glatt-orth. Komplexität 1

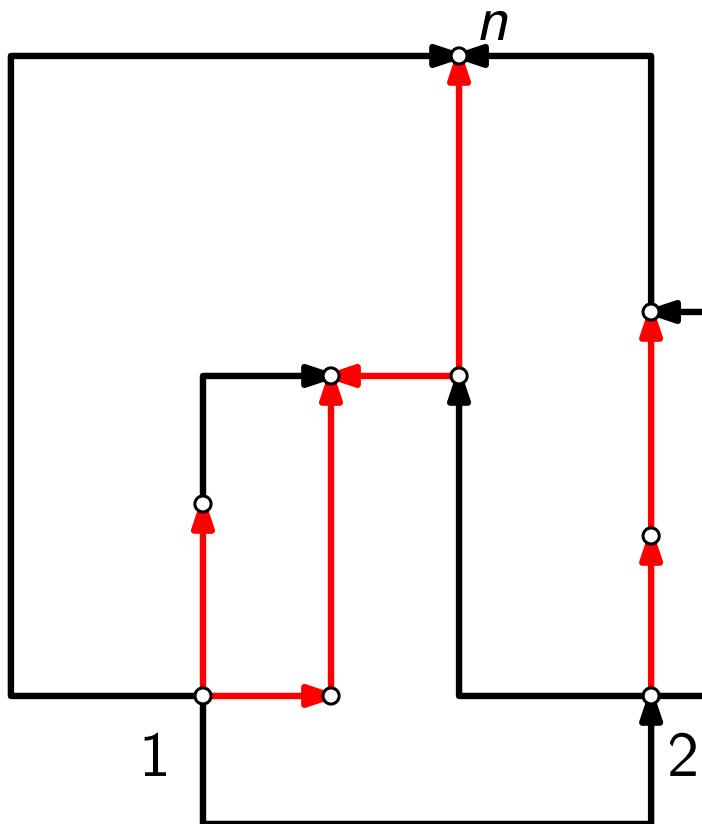
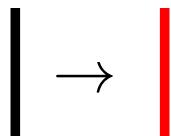
Komplexität-2 Zeichnungen



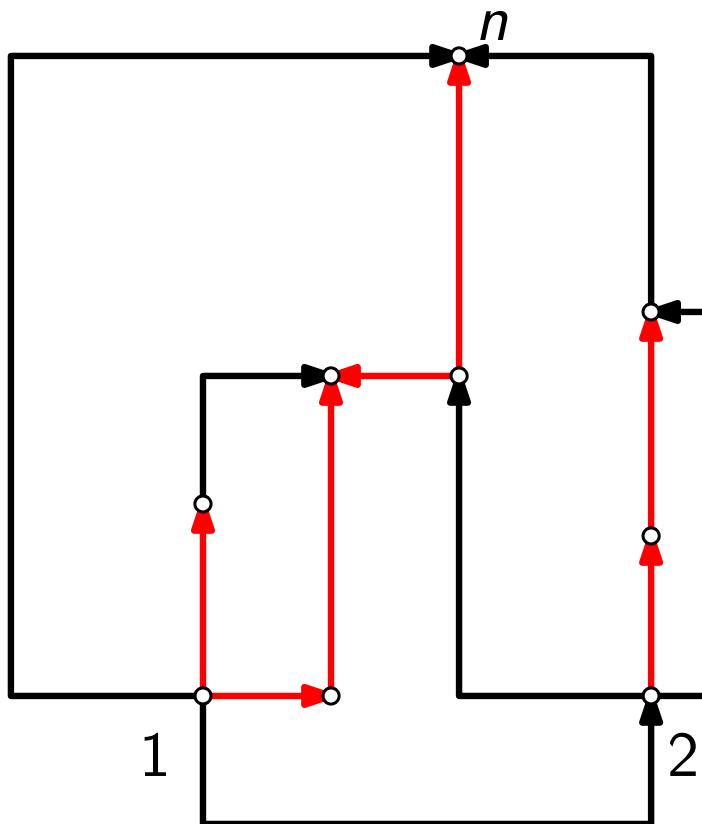
Komplexität-2 Zeichnungen



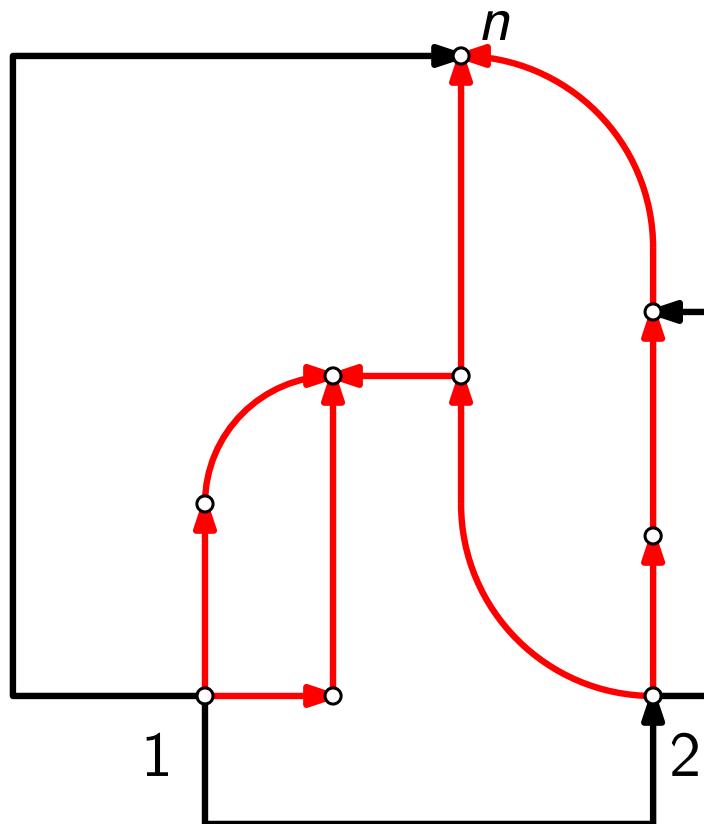
Komplexität-2 Zeichnungen



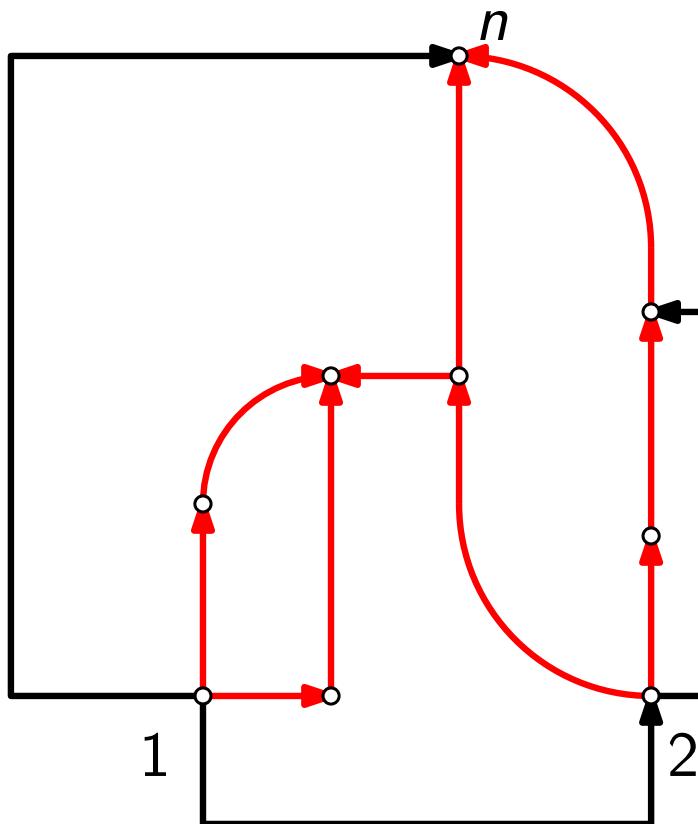
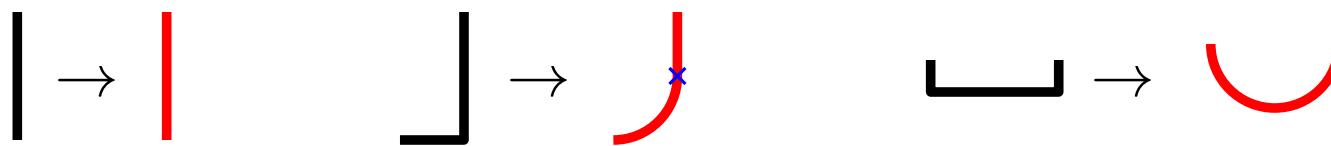
Komplexität-2 Zeichnungen



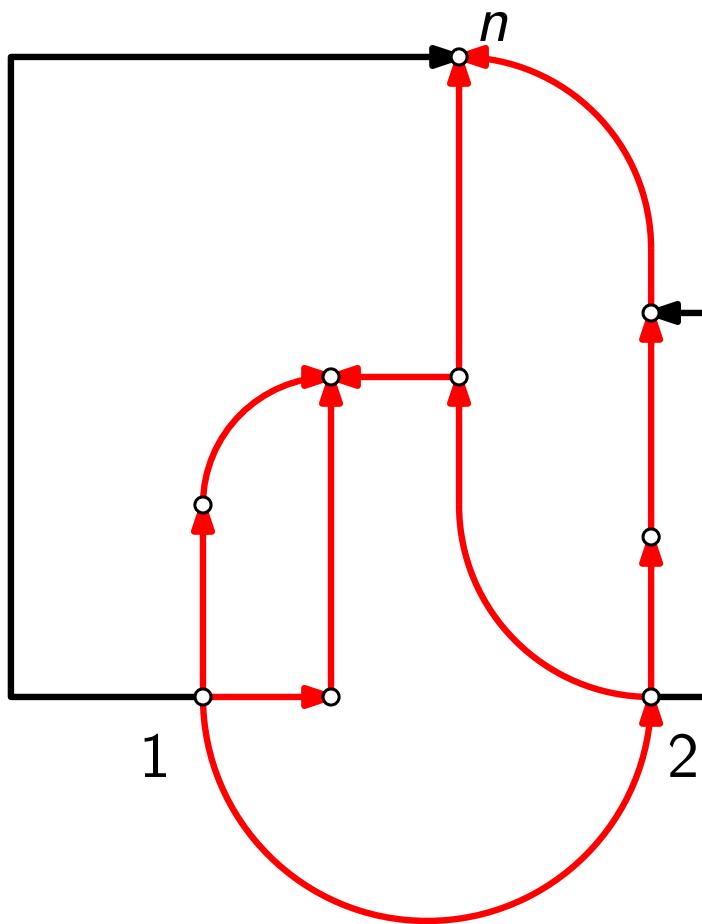
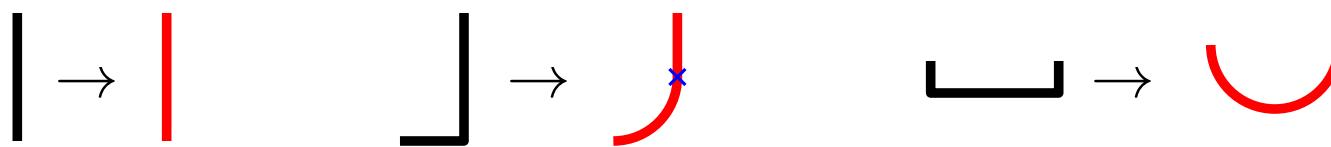
Komplexität-2 Zeichnungen



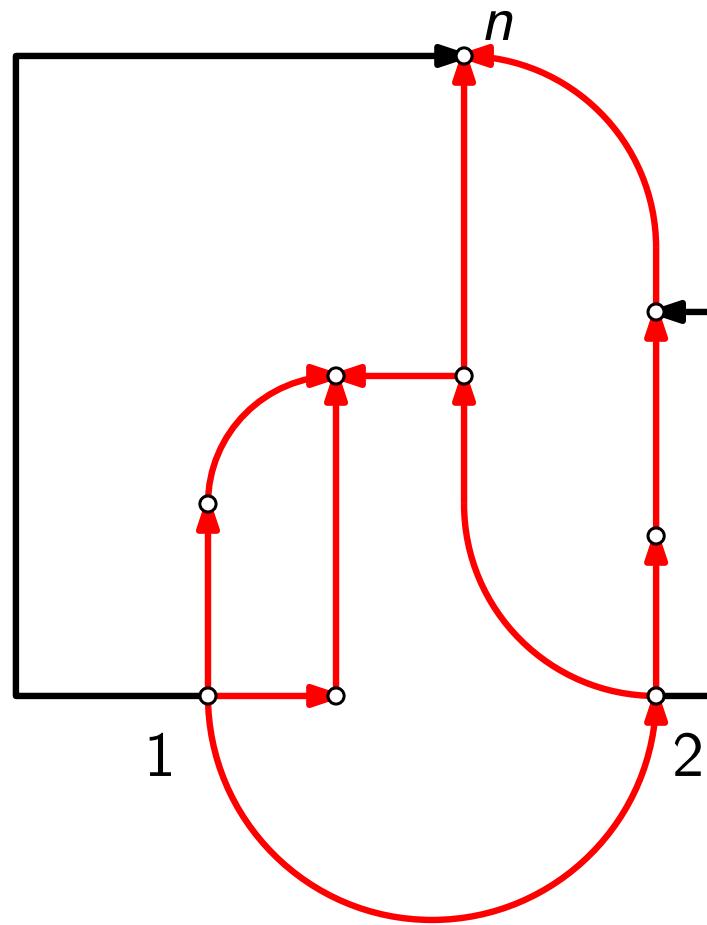
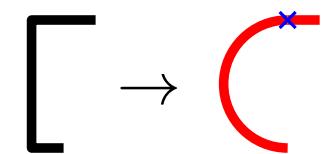
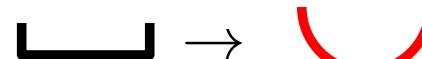
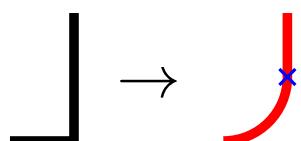
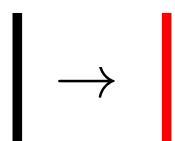
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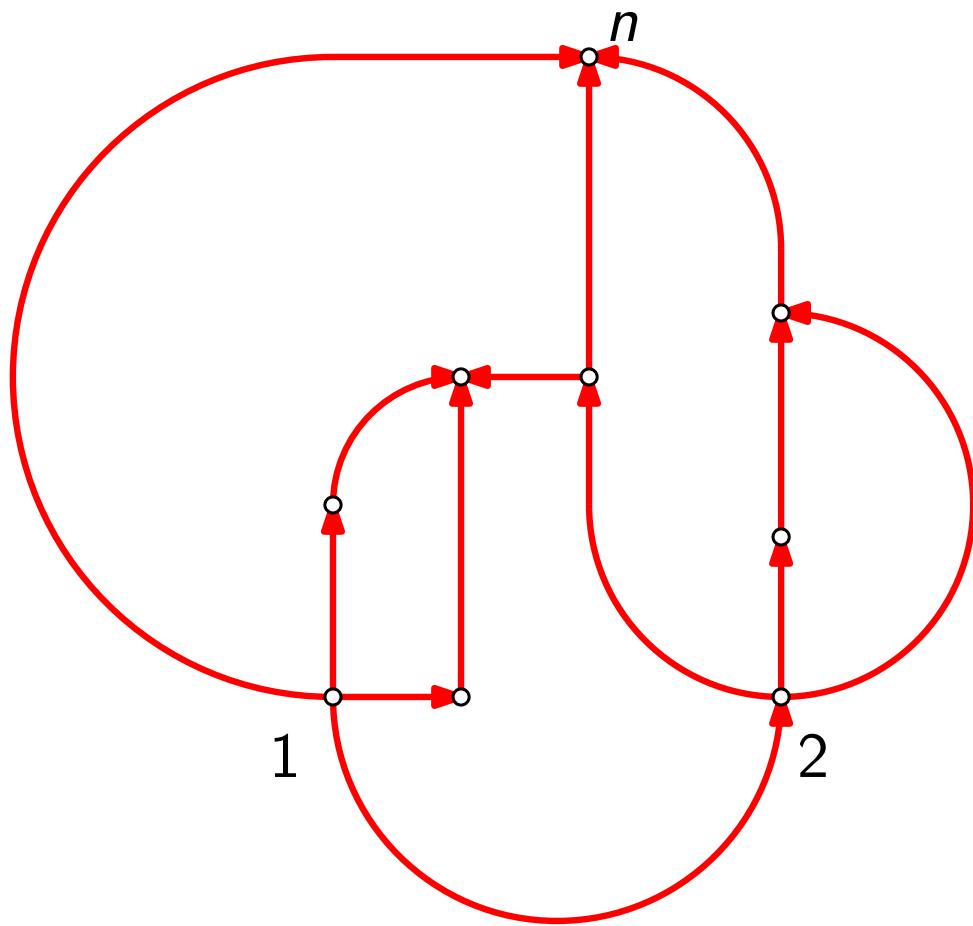
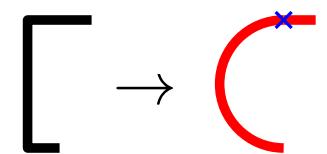
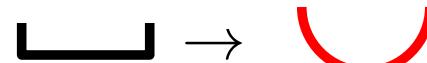
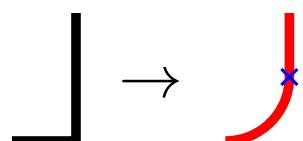
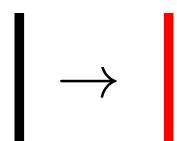
Komplexität-2 Zeichnungen



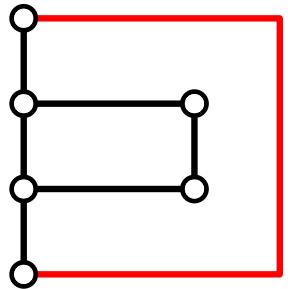
Komplexität-2 Zeichnungen



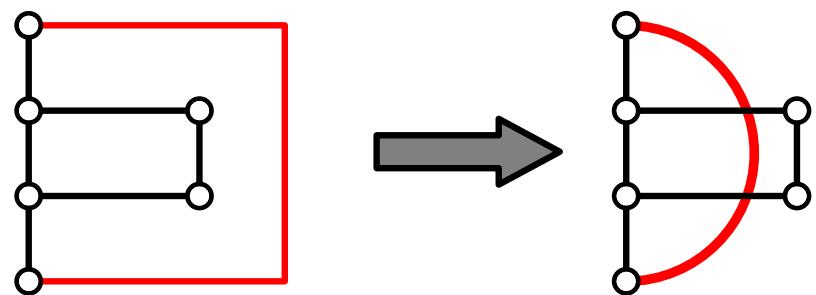
Komplexität-2 Zeichnungen



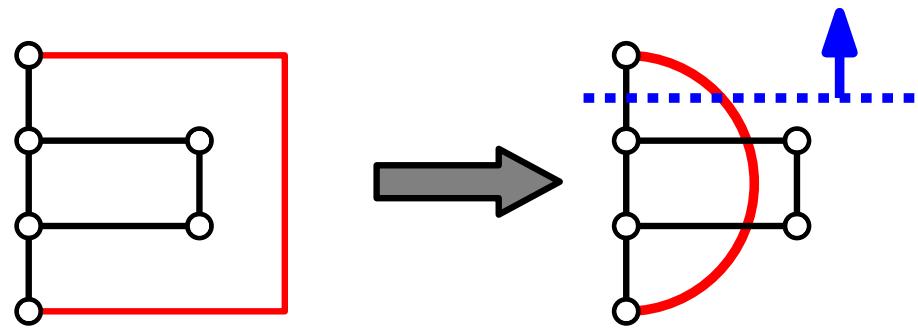
Kreuzungen



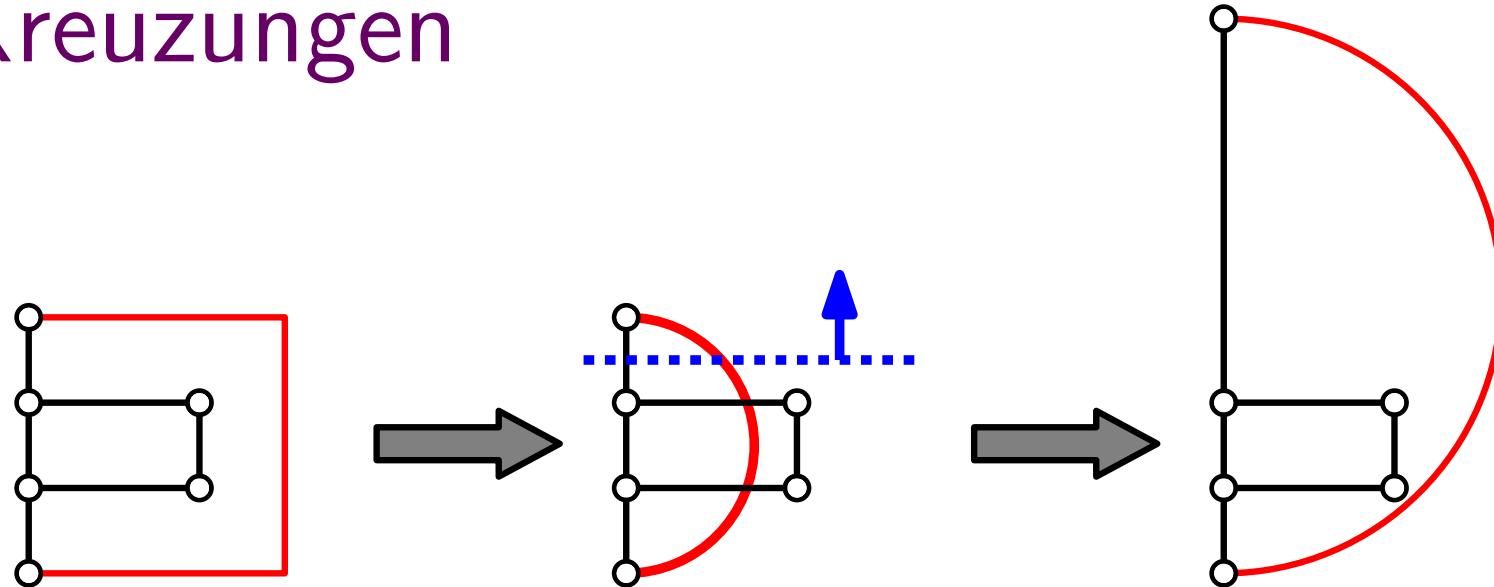
Kreuzungen



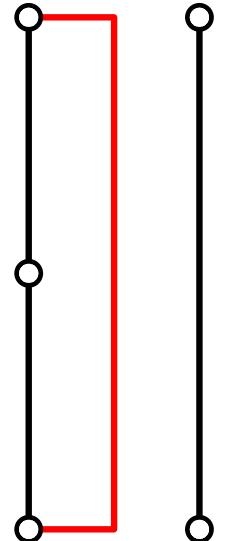
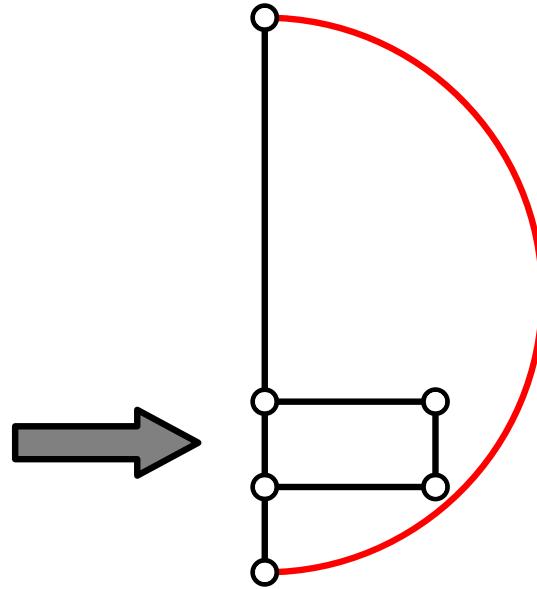
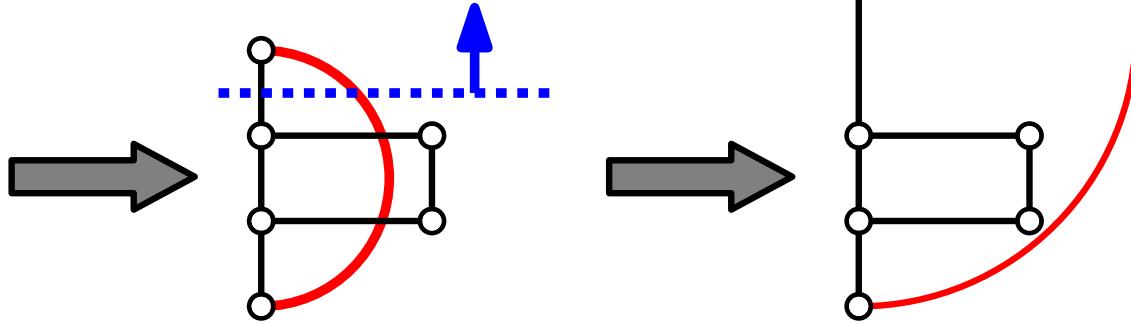
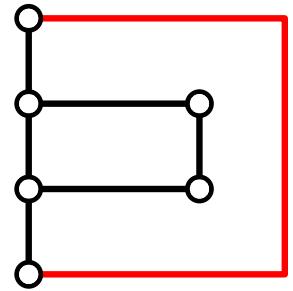
Kreuzungen



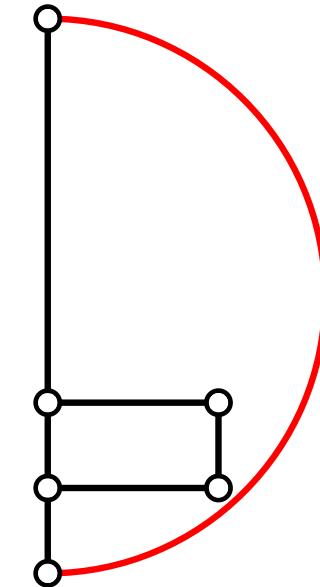
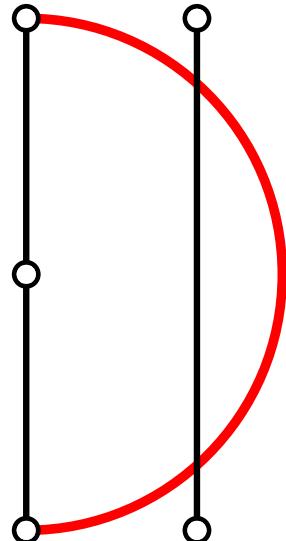
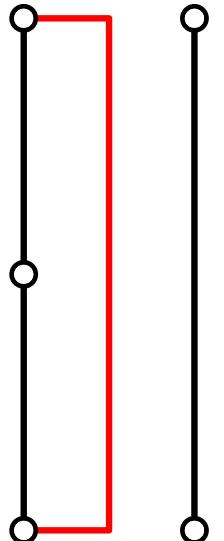
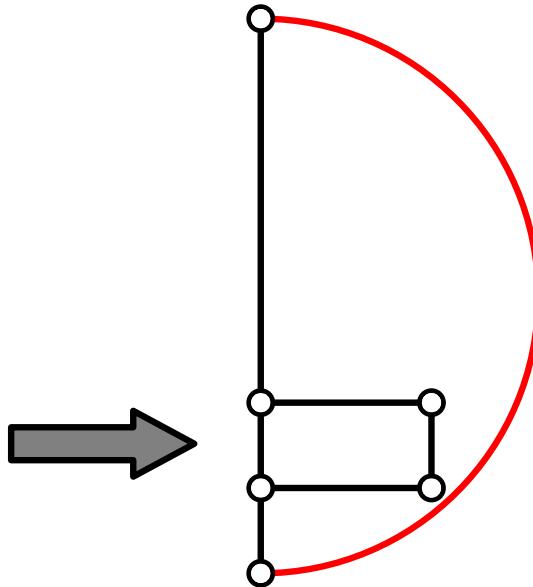
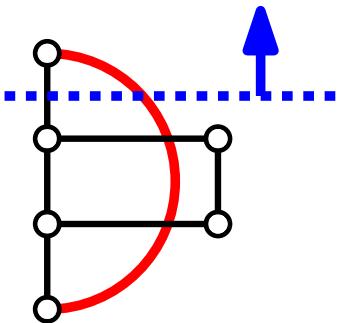
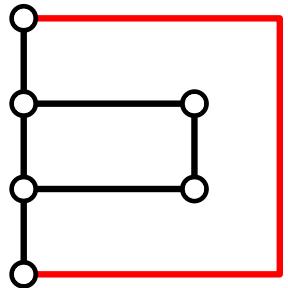
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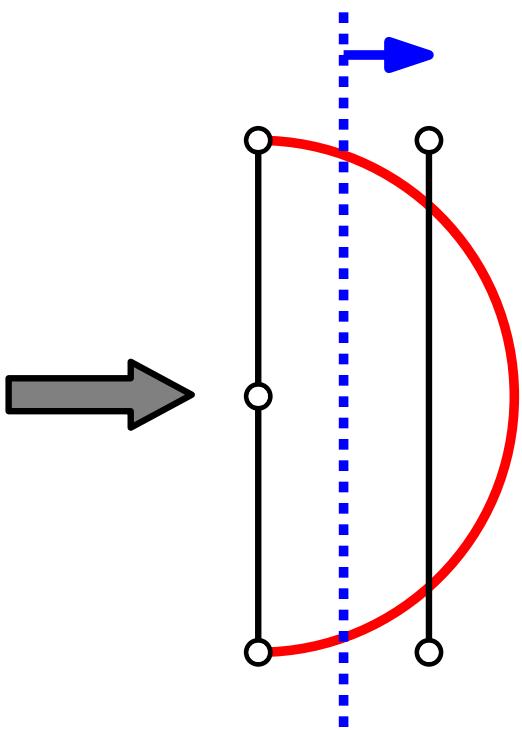
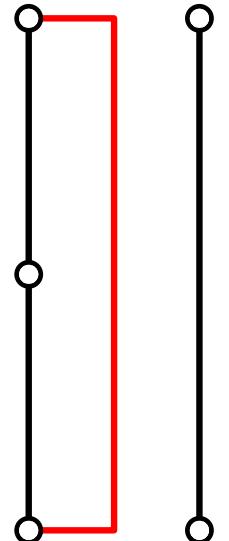
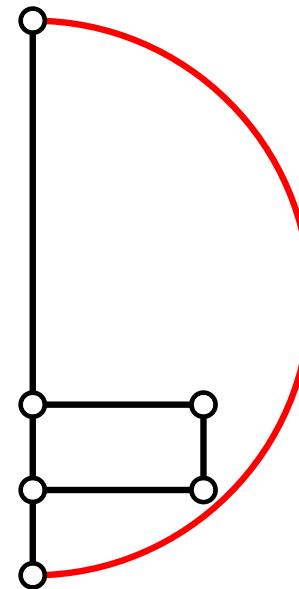
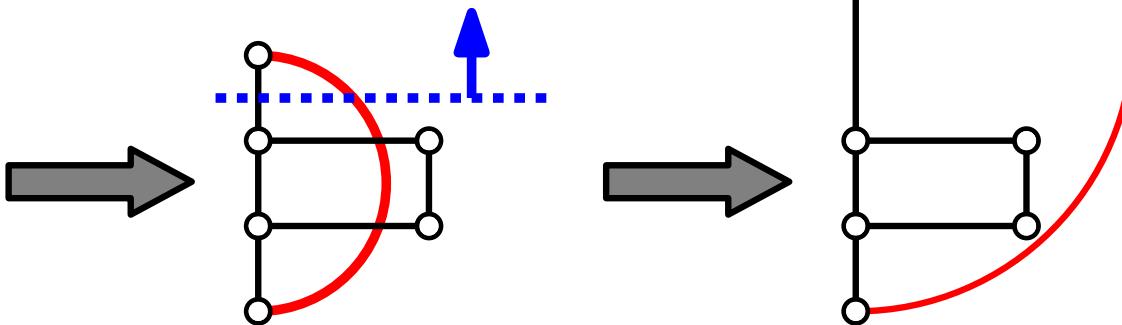
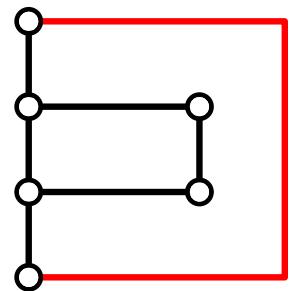
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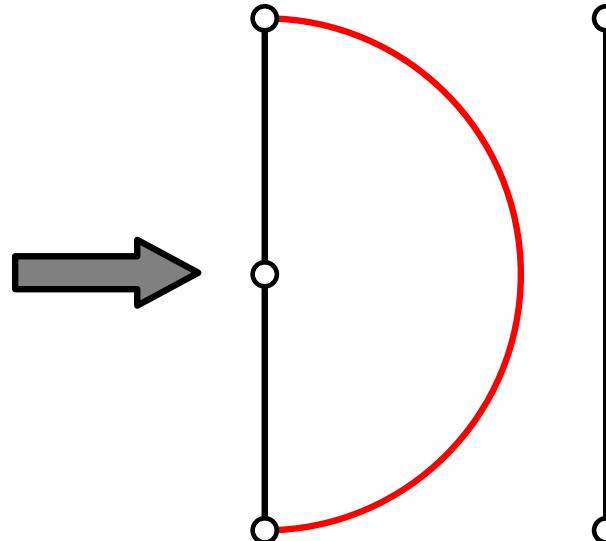
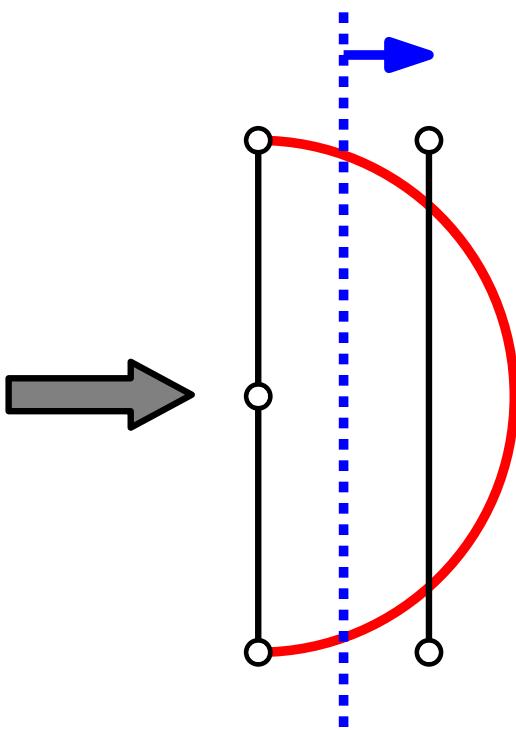
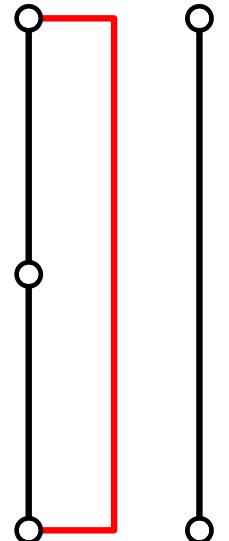
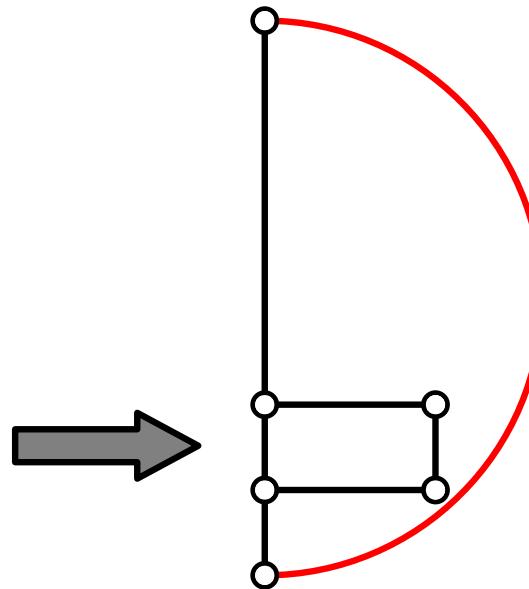
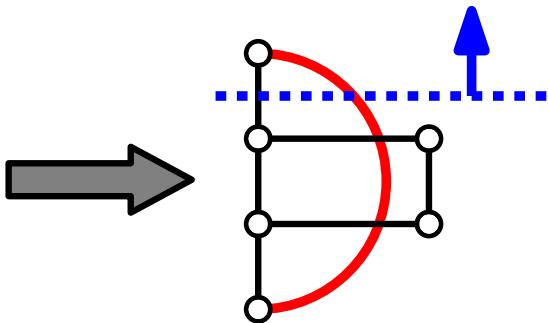
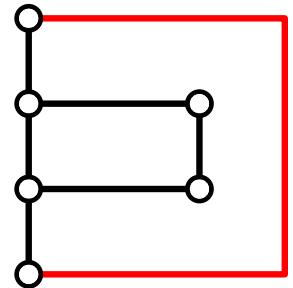
Kreuzungen



Kreuzungen



Kreuzungen



Glatt-orthogonale Zeichungen – Ergebnisse

Zweifach zusammenhängender 4-planarer Graph \Rightarrow
glatt-orth. Komplexität 3

Glatt-orthogonale Zeichungen – Ergebnisse

~~Zweifach zusammenhängender~~ 4-planarer Graph \Rightarrow
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 - dreifach zusammenhängend 3-planar
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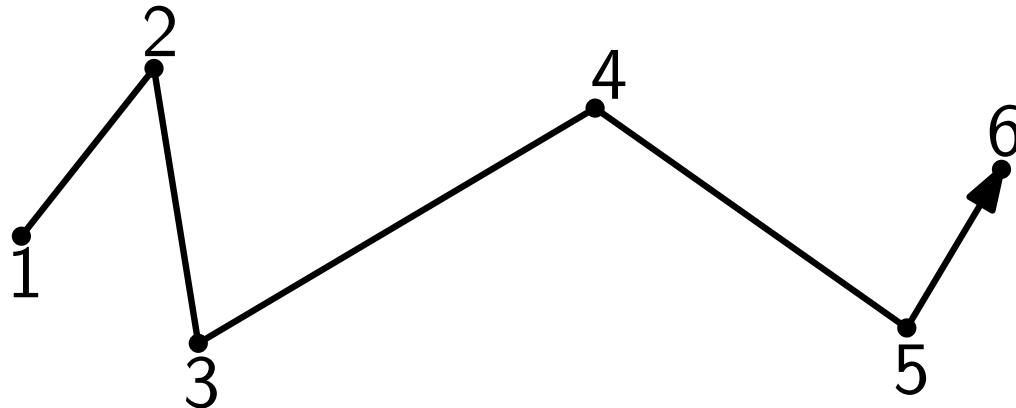
Teil 3: Monotone Zeichnungen von Bäumen

Monotone Zeichnungen

Monotoner Pfad: \exists Richtung d , so dass
Knoten-Reihenfolge in d = Reihenfolge entlang des Pfades.

Monotone Zeichnungen

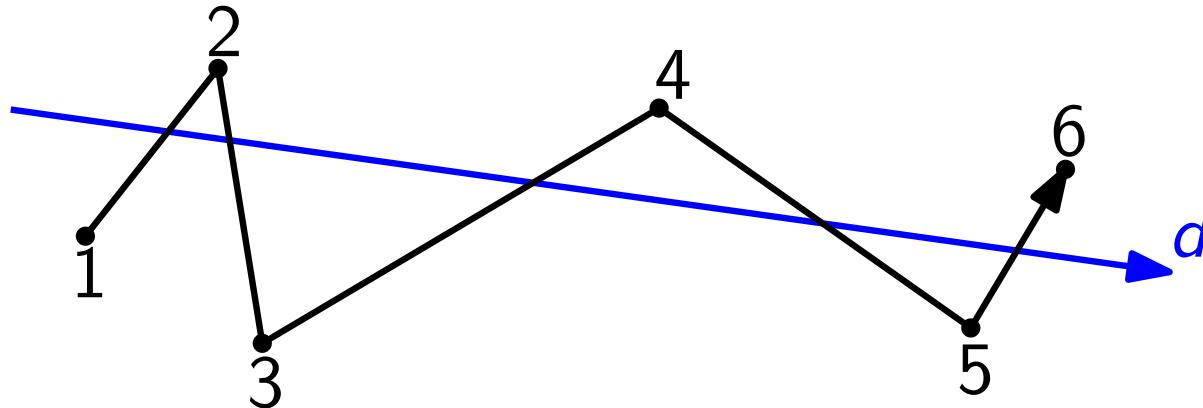
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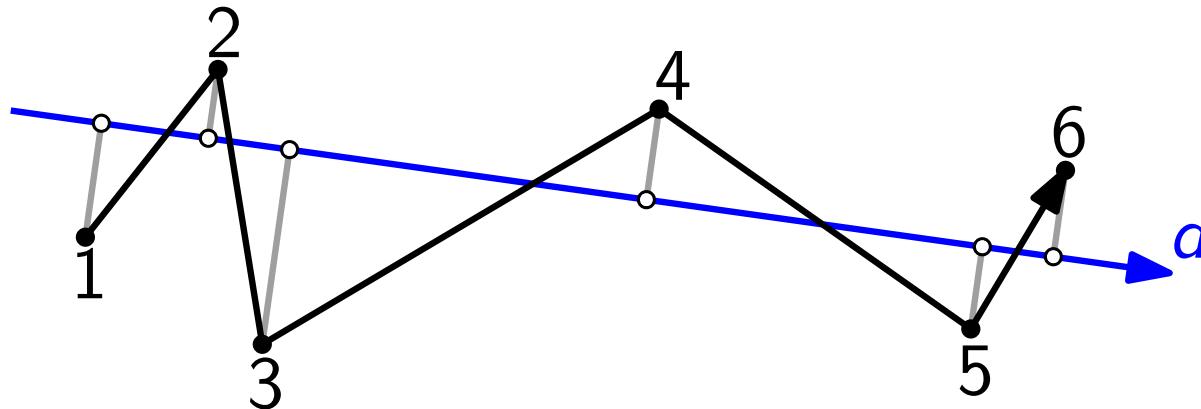
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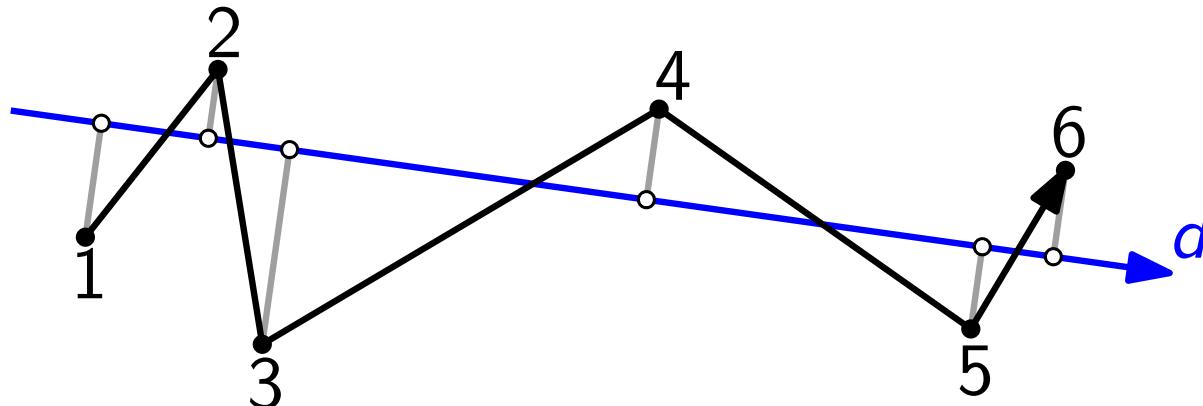
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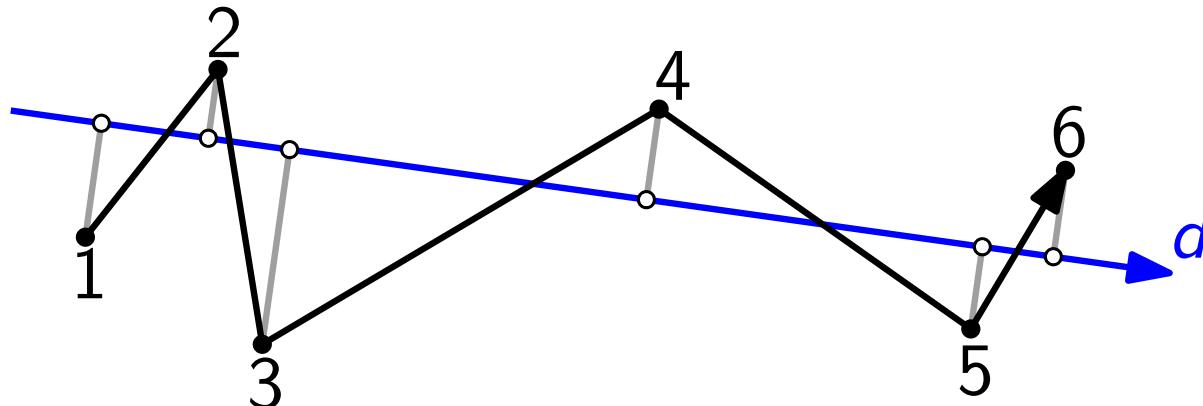
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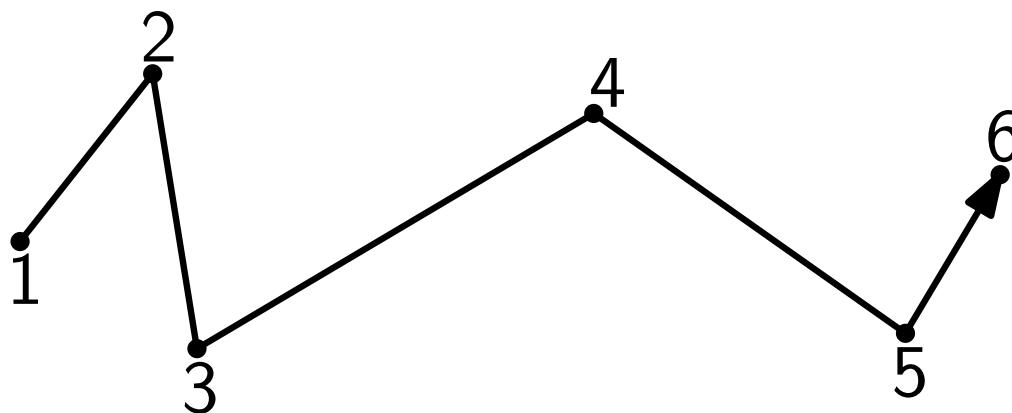
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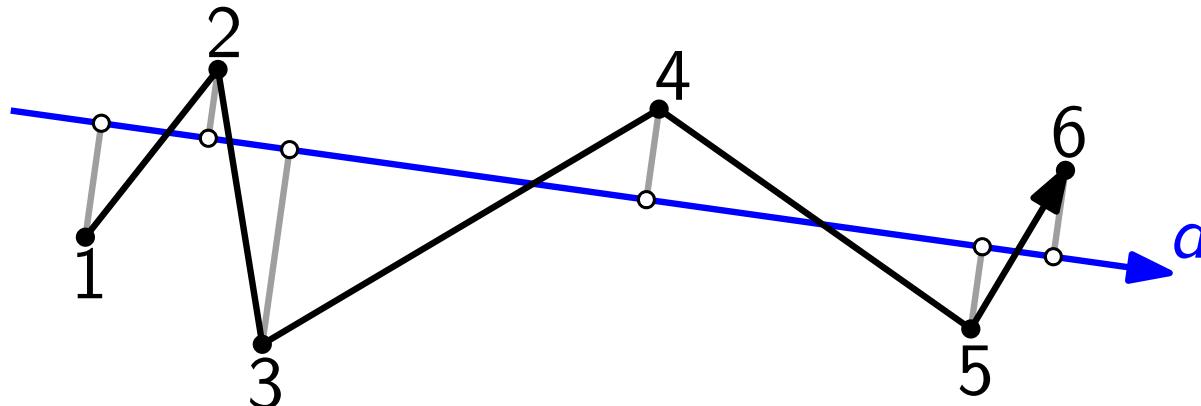
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Streng monotoner $u-v$ Pfad: $d = \overrightarrow{uv}$.



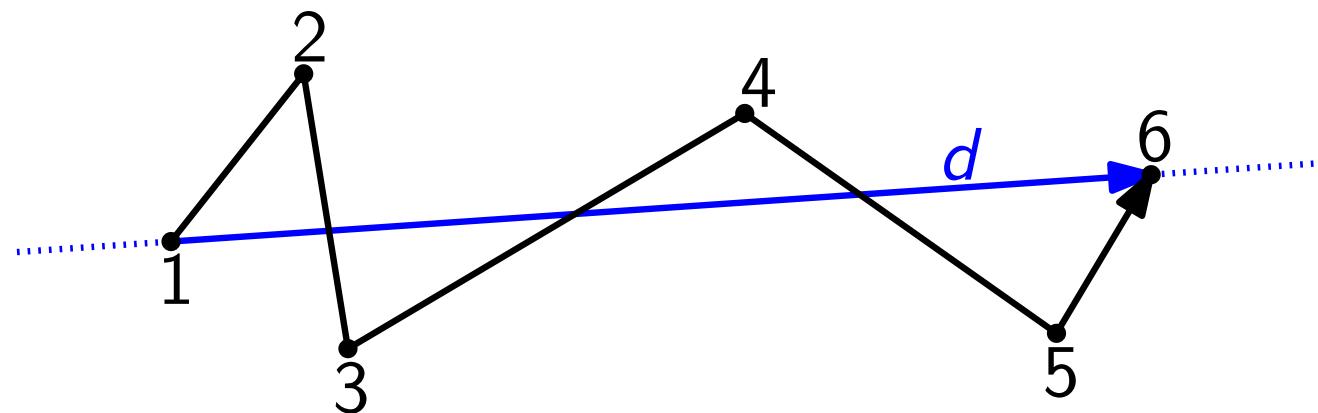
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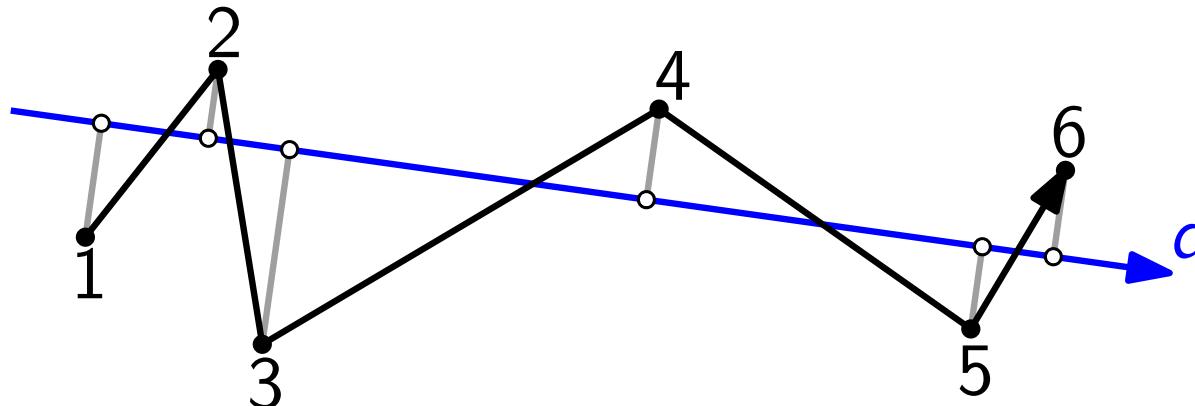
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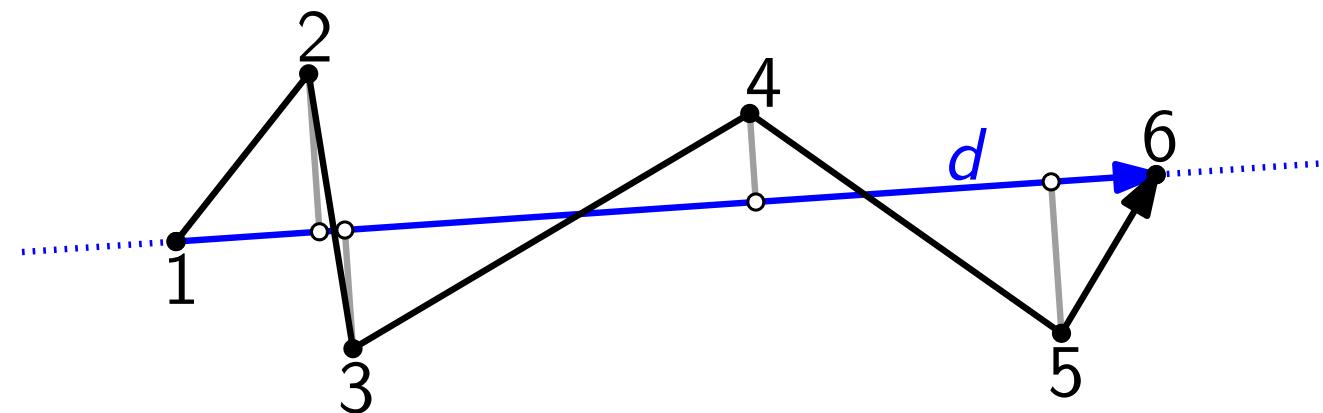
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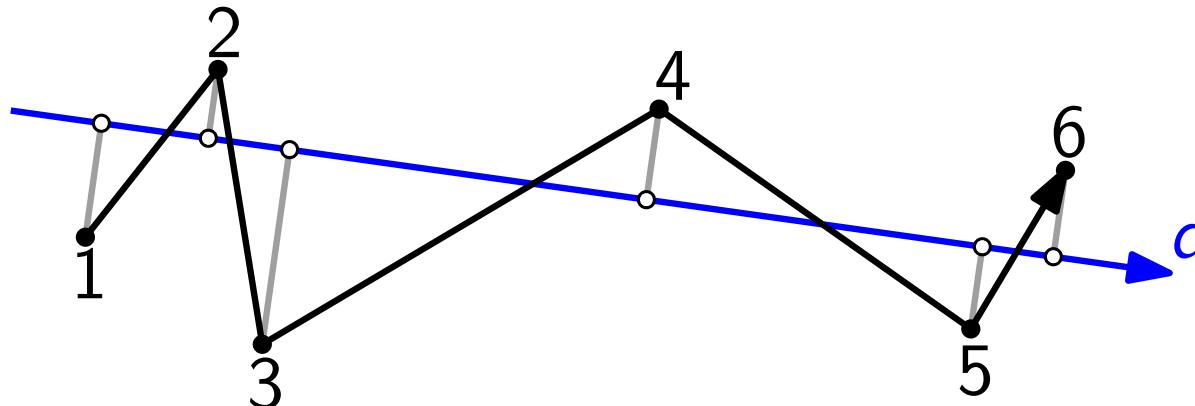
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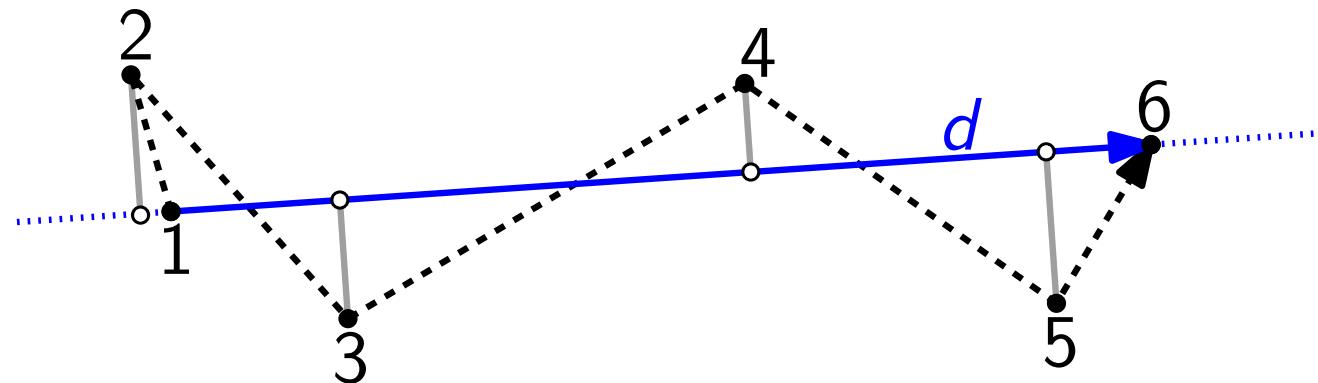
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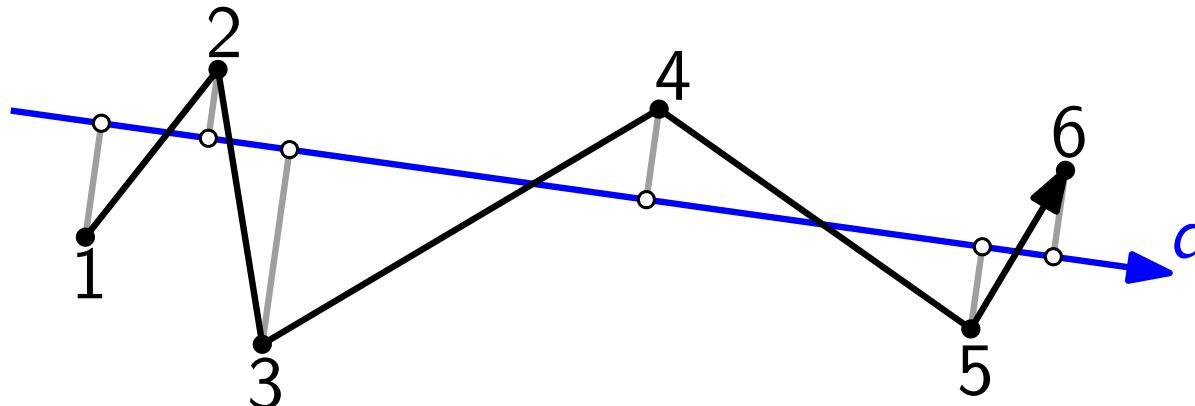
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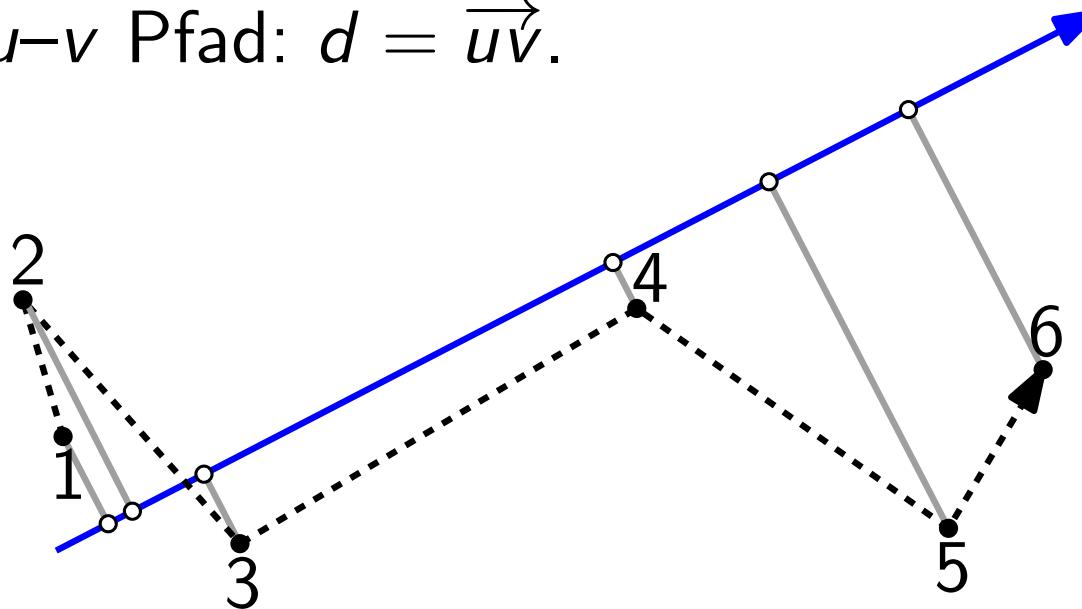
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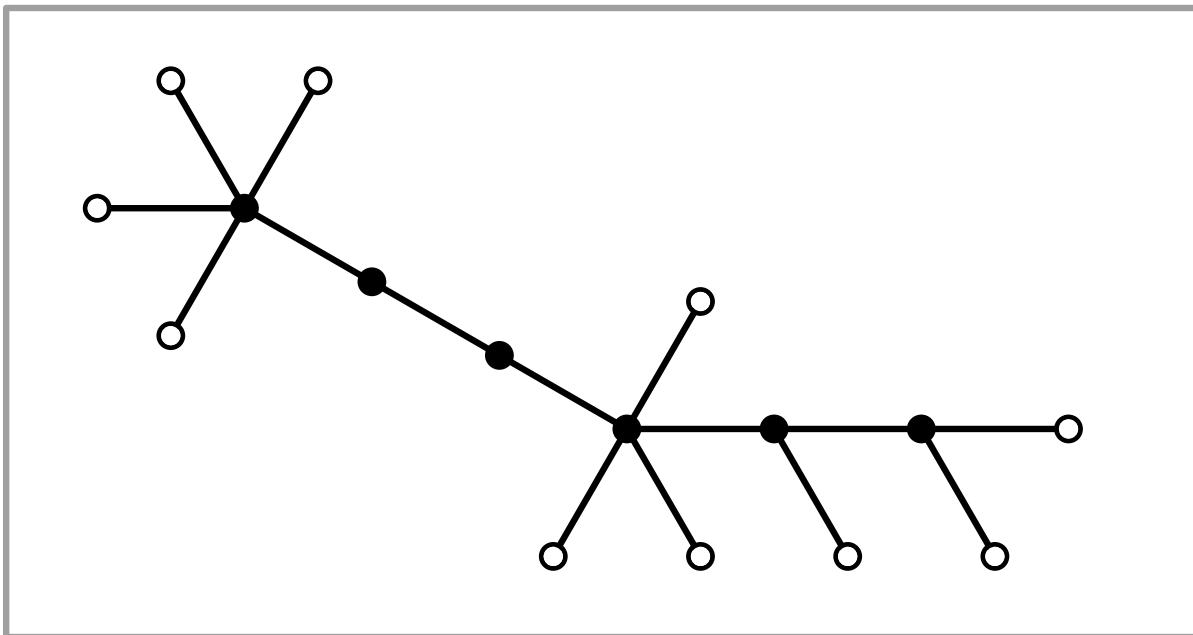


Monotone Zeichnungen

Konvexe Zeichnung: Jede Facette ist konvex.

Monotone Zeichnungen

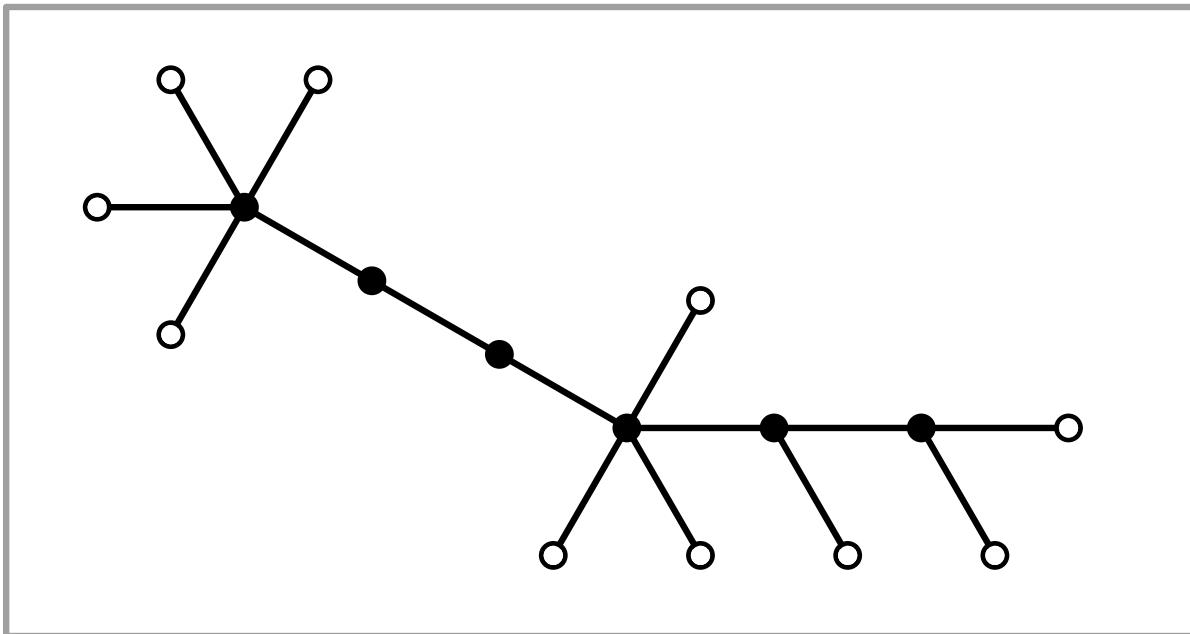
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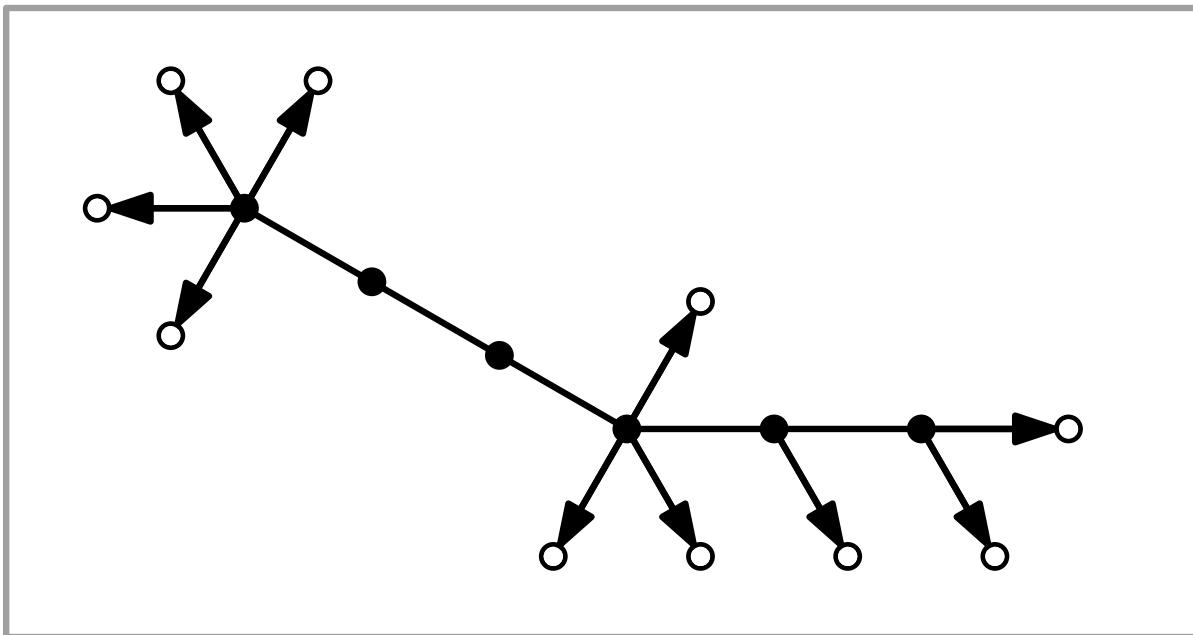
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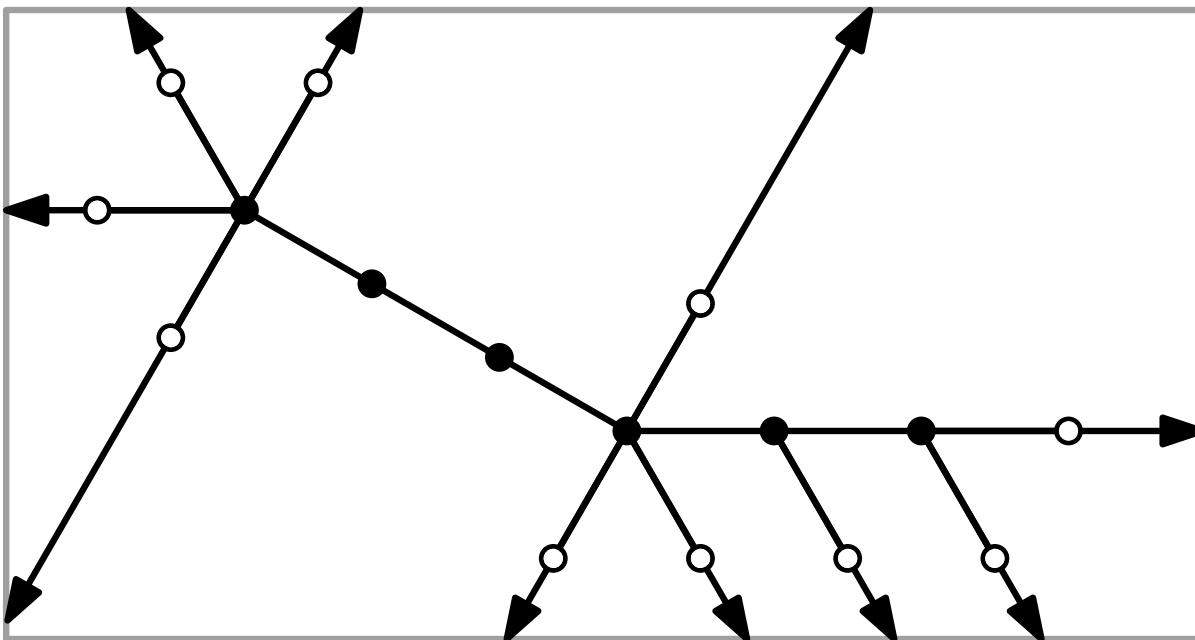
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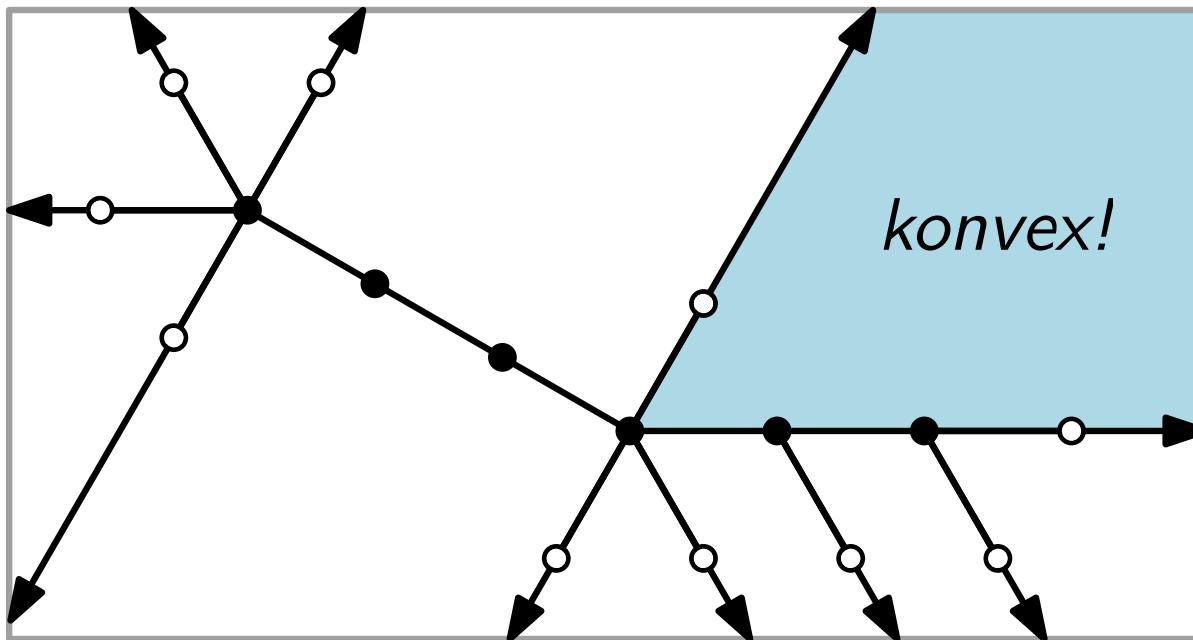
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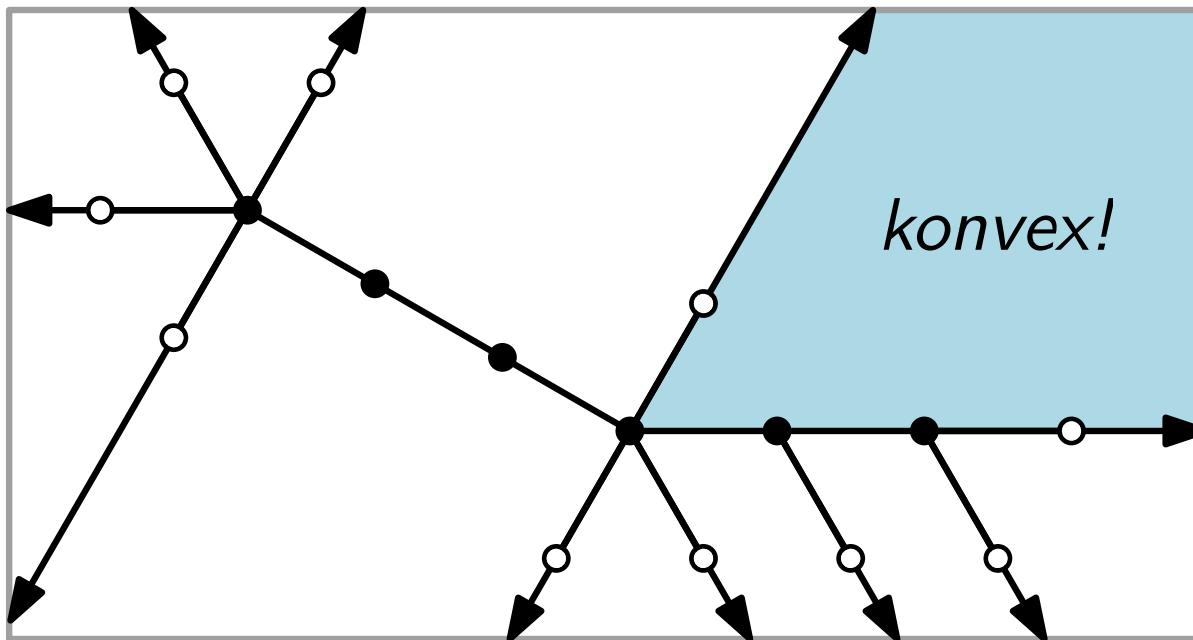
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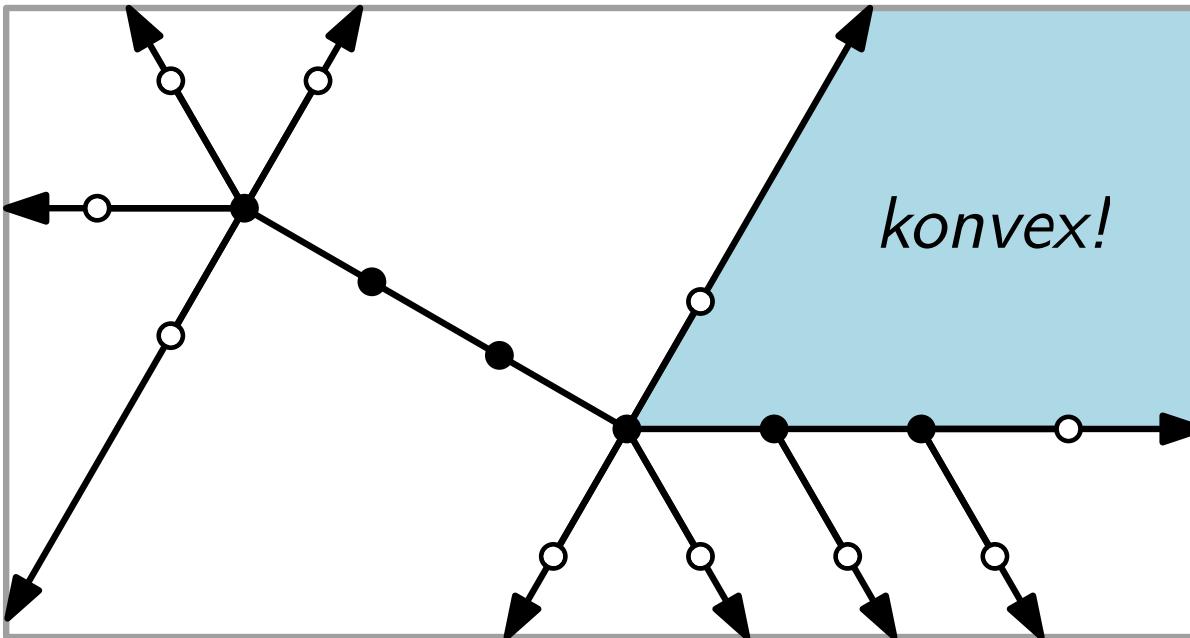
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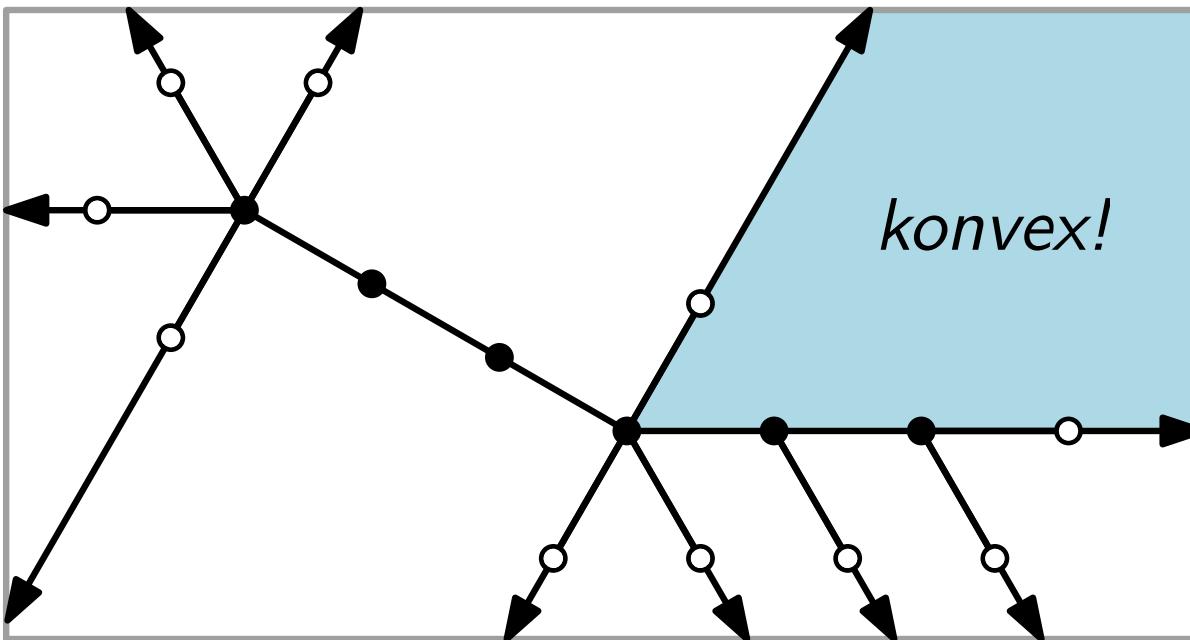


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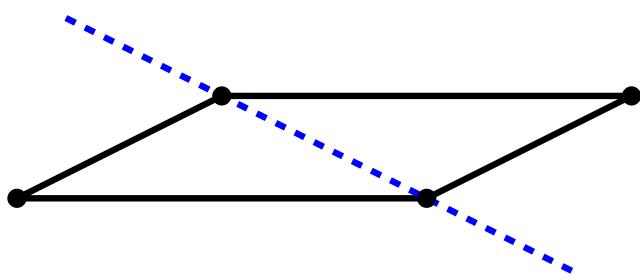


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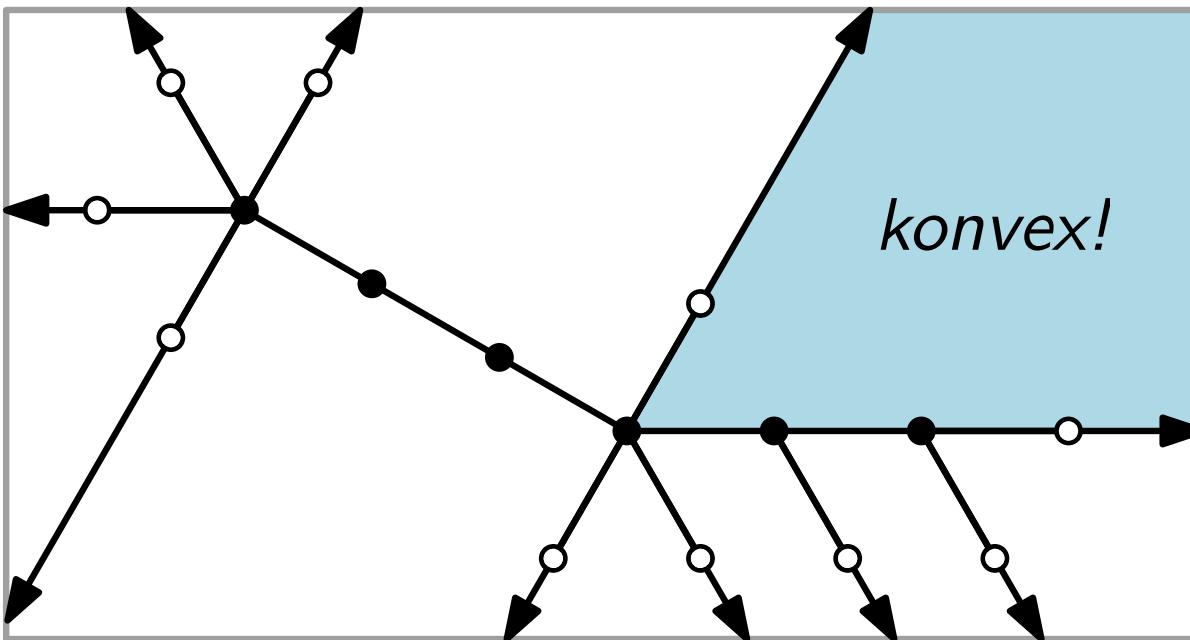


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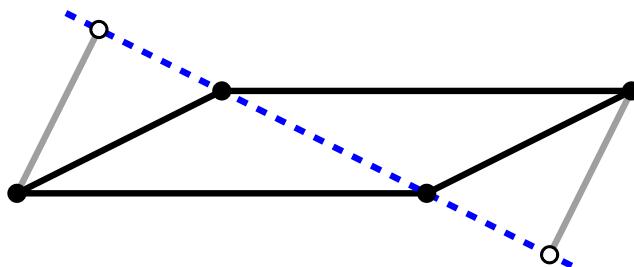


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Monotone Zeichnungen – Bekannte Resultate

Angelini et al.

[JGAA'12]

- *Baum* \Rightarrow monotone Zeichnung, $O(n^{1.6}) \times O(n^{1.6})$ Gitter

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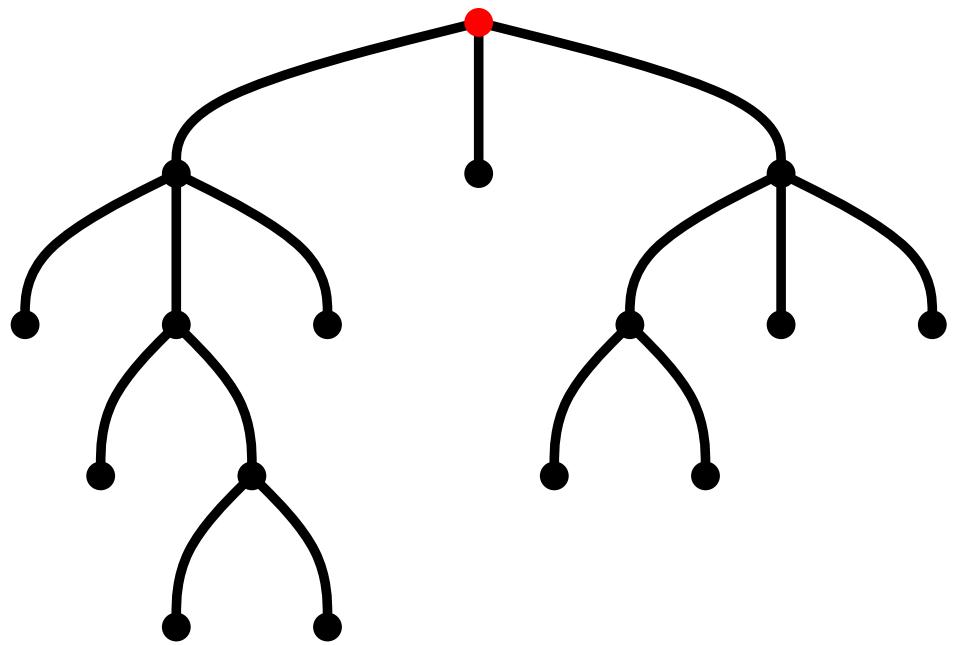
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Carlson & Eppstein

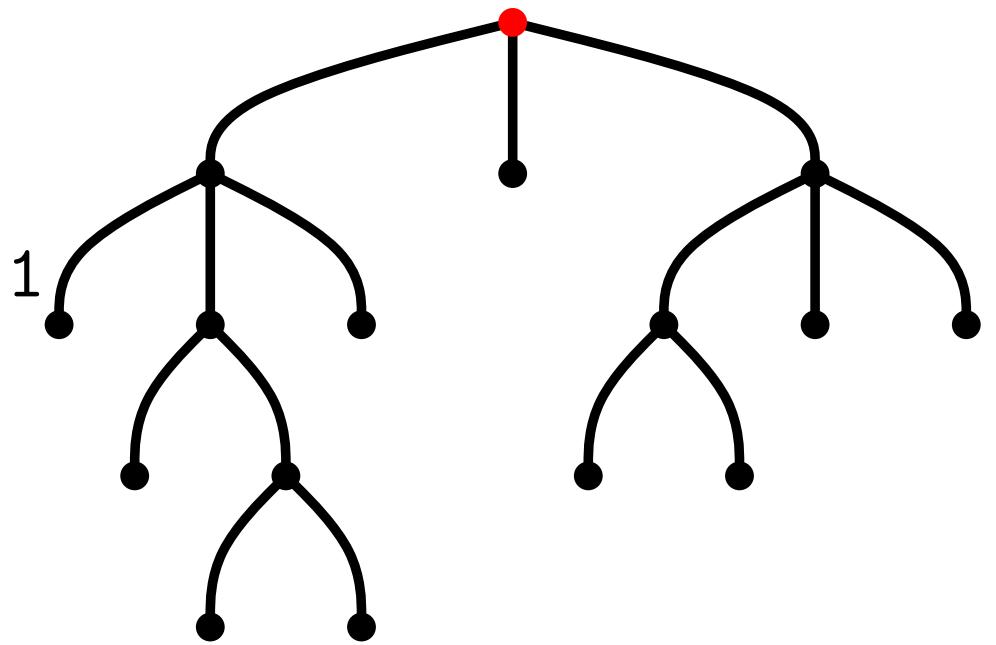
[GD'06]

Baum \Rightarrow konvexe Zeichnung, *optimaler Winkelauflösung*

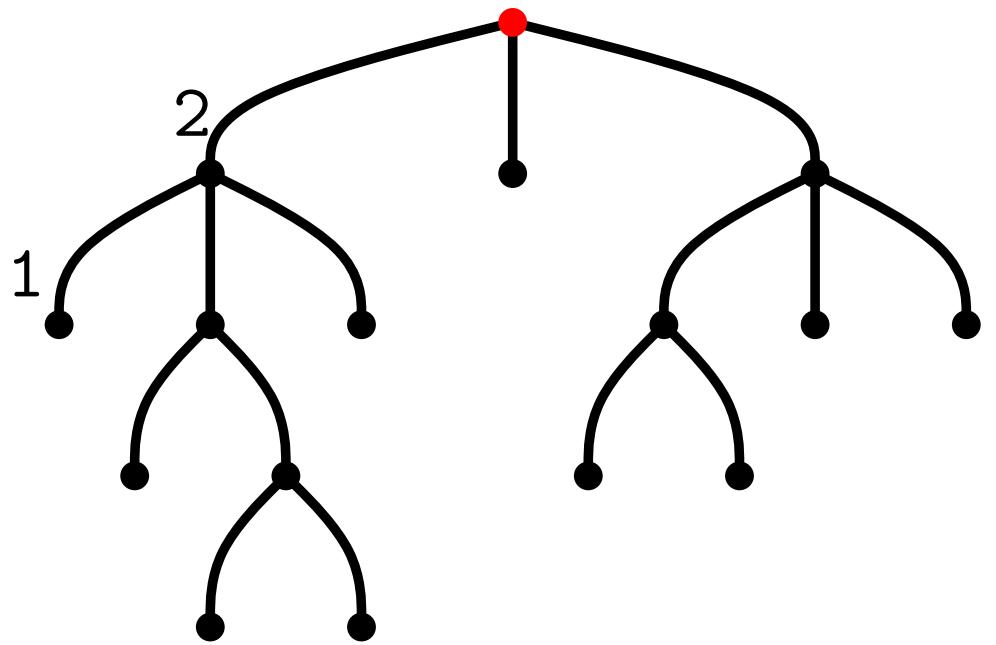
Schritt I: Beschrifte Kanten



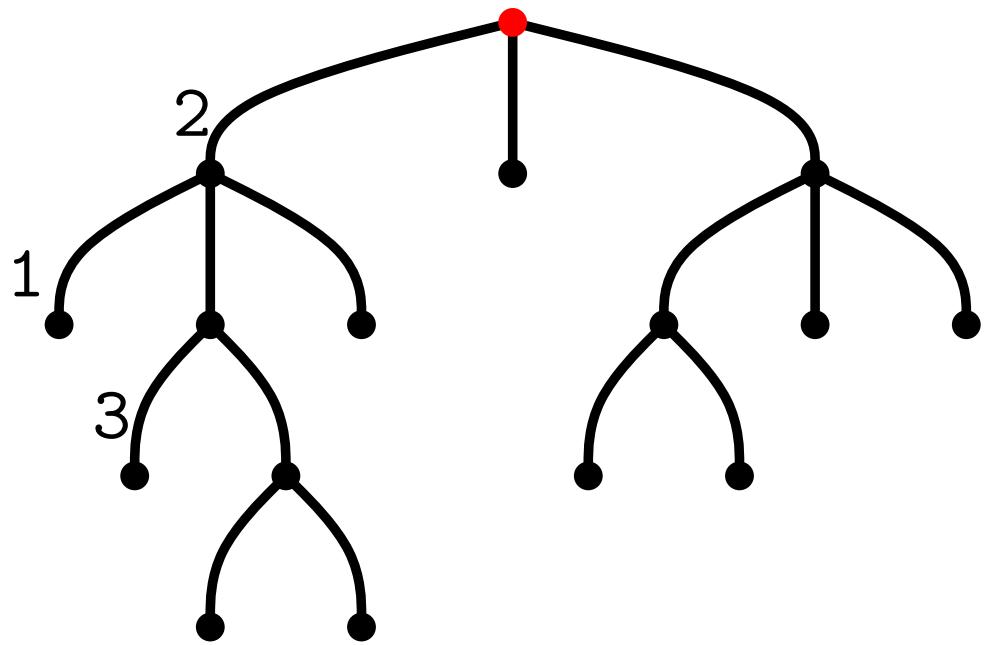
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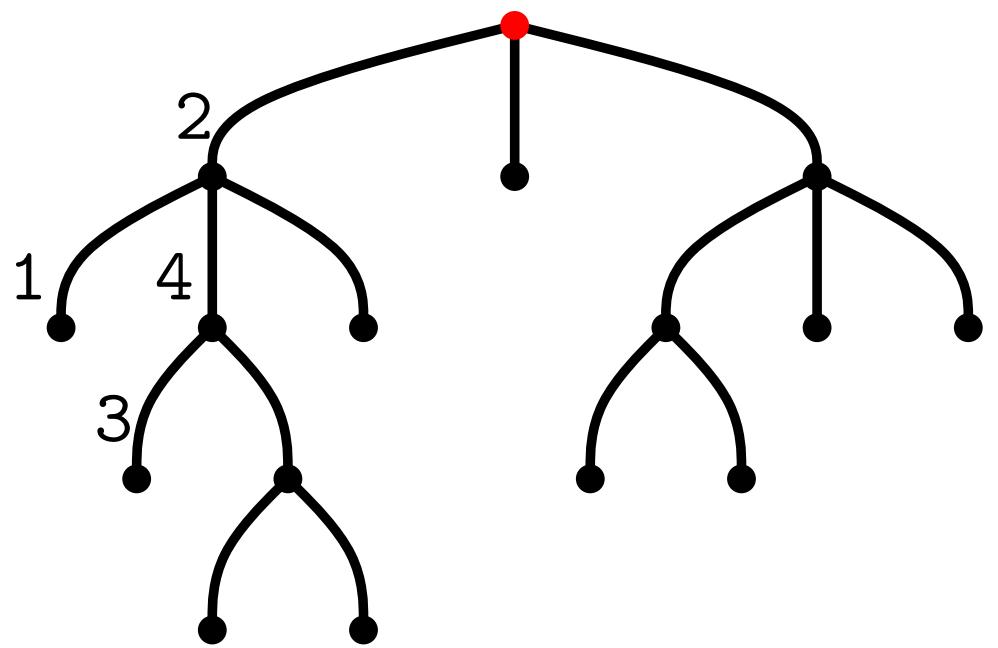
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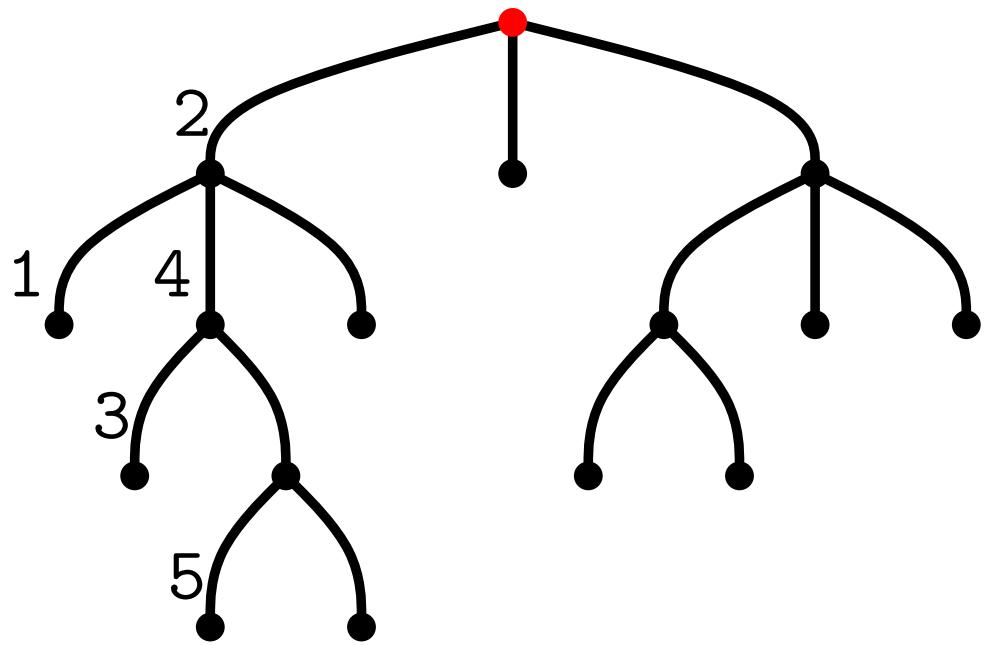
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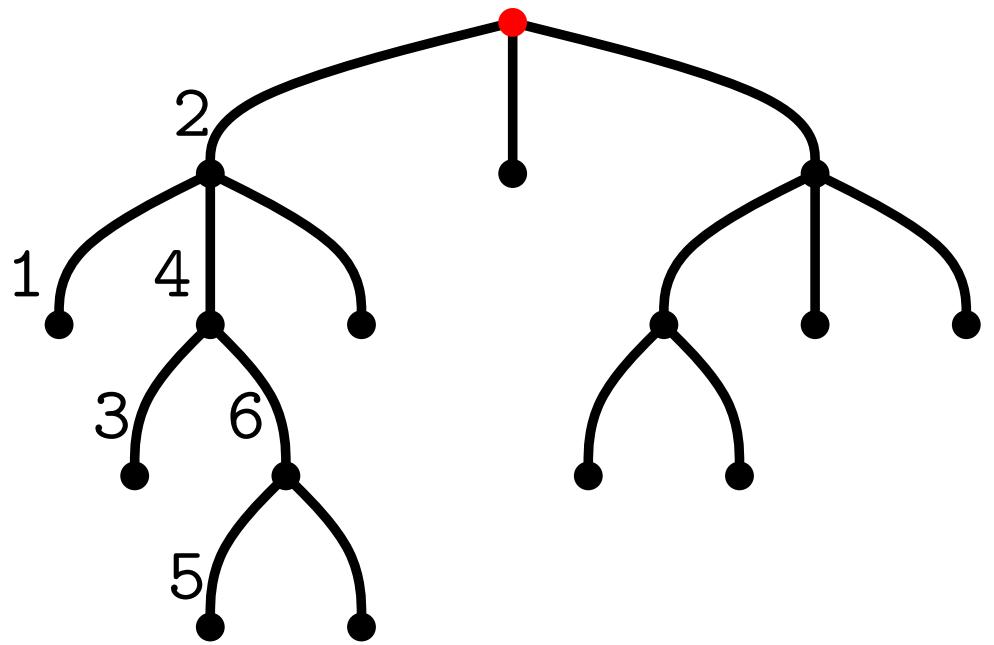
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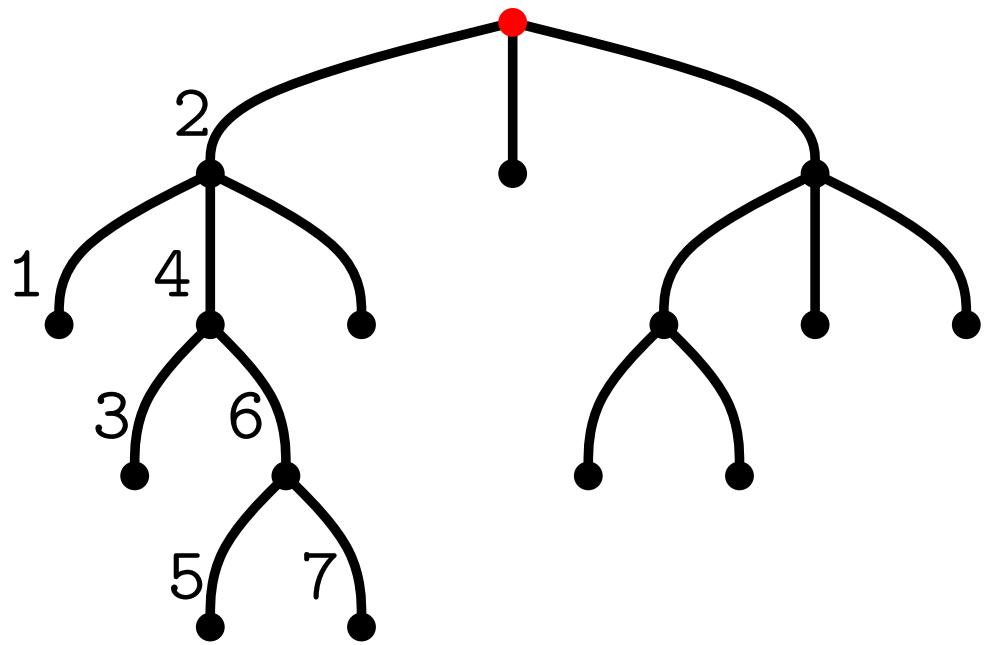
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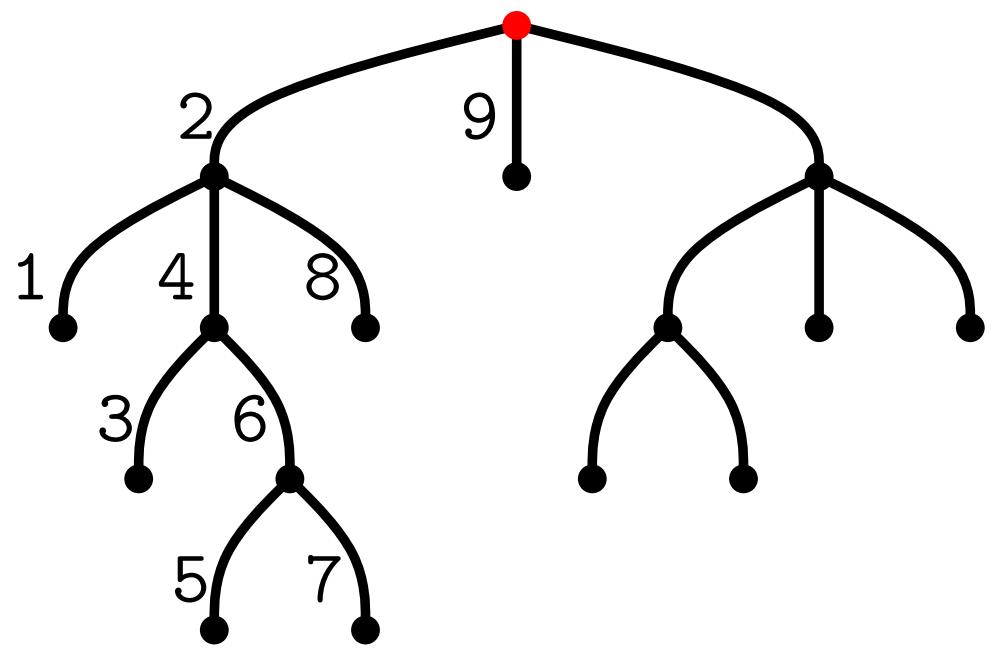
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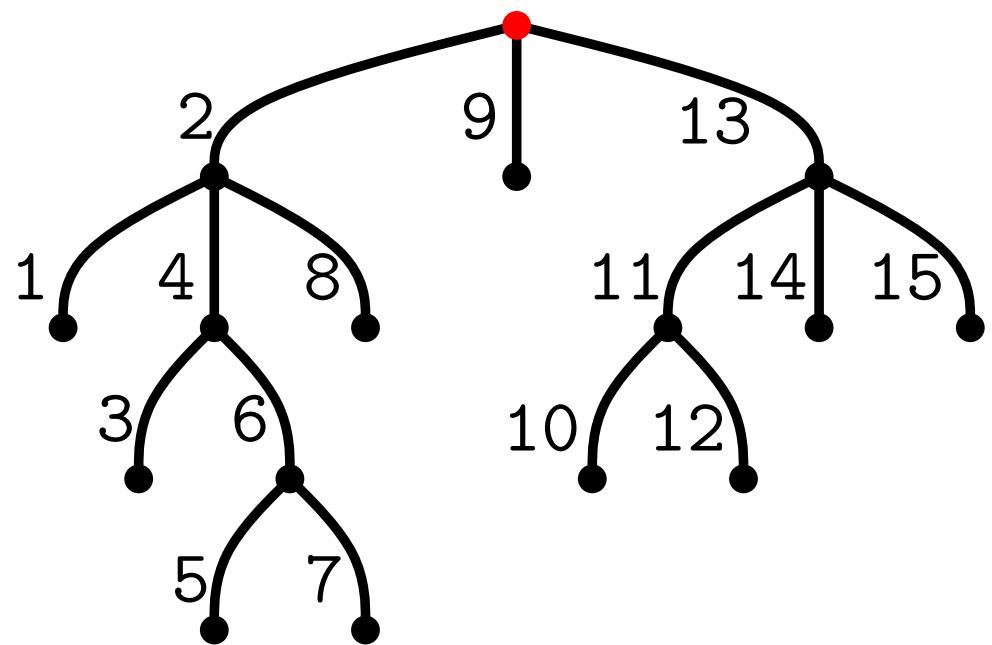
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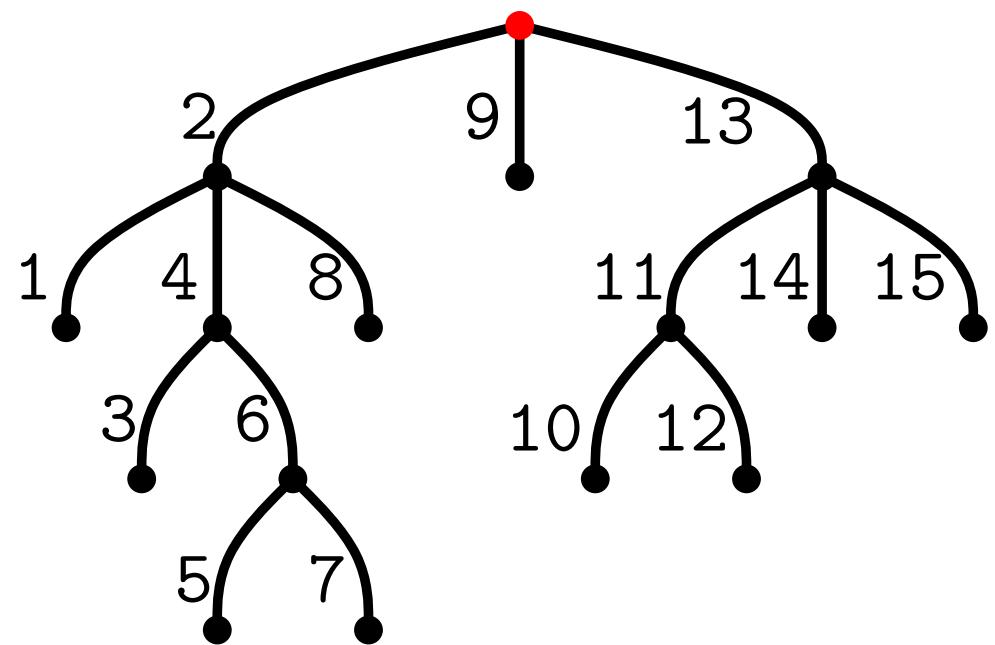
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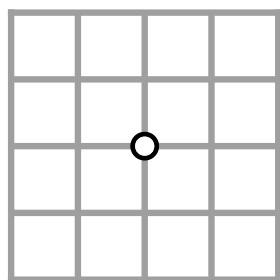
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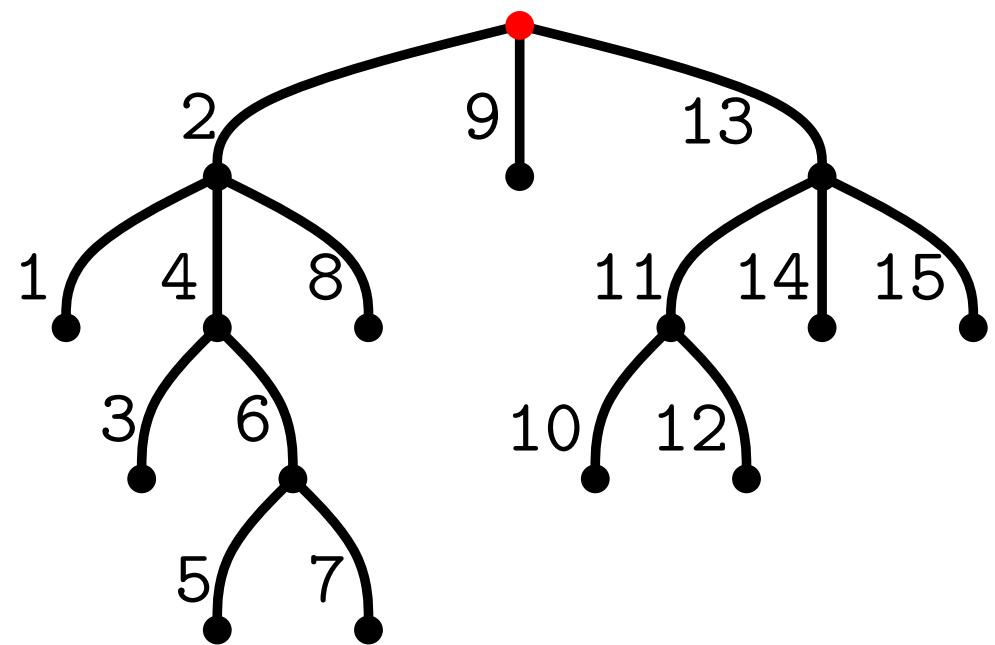
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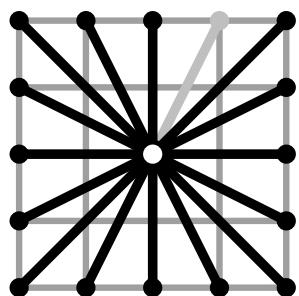
Schritt II: Wähle Vektoren



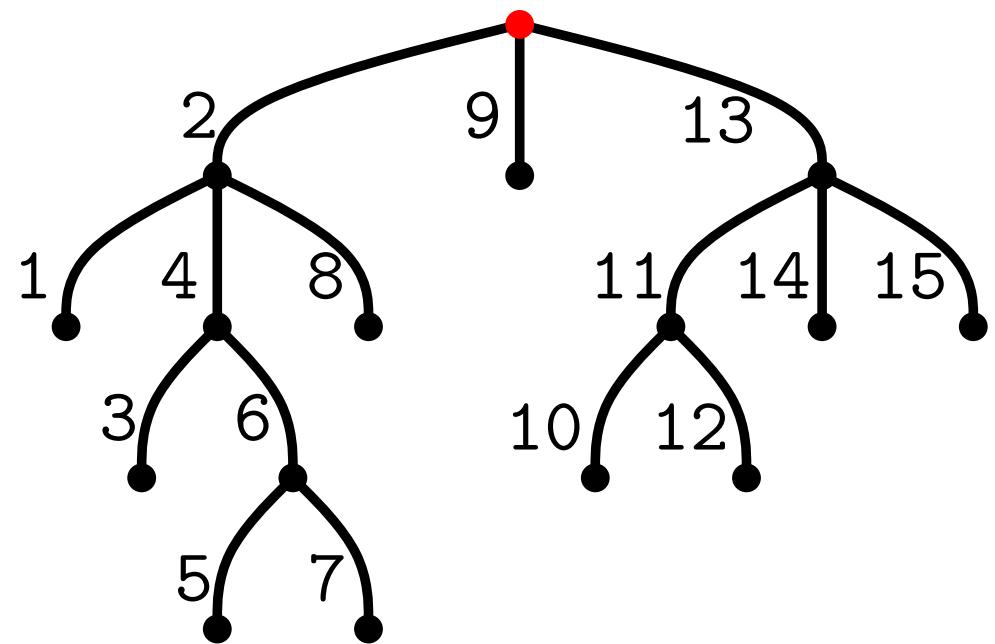
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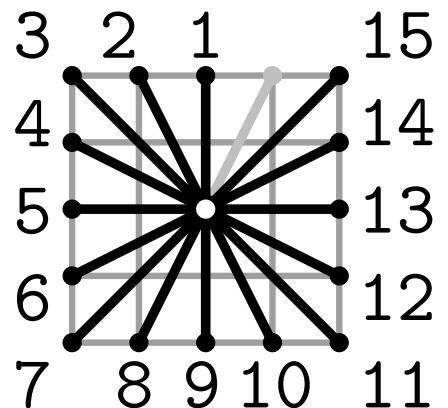
Schritt II: Wähle Vektoren



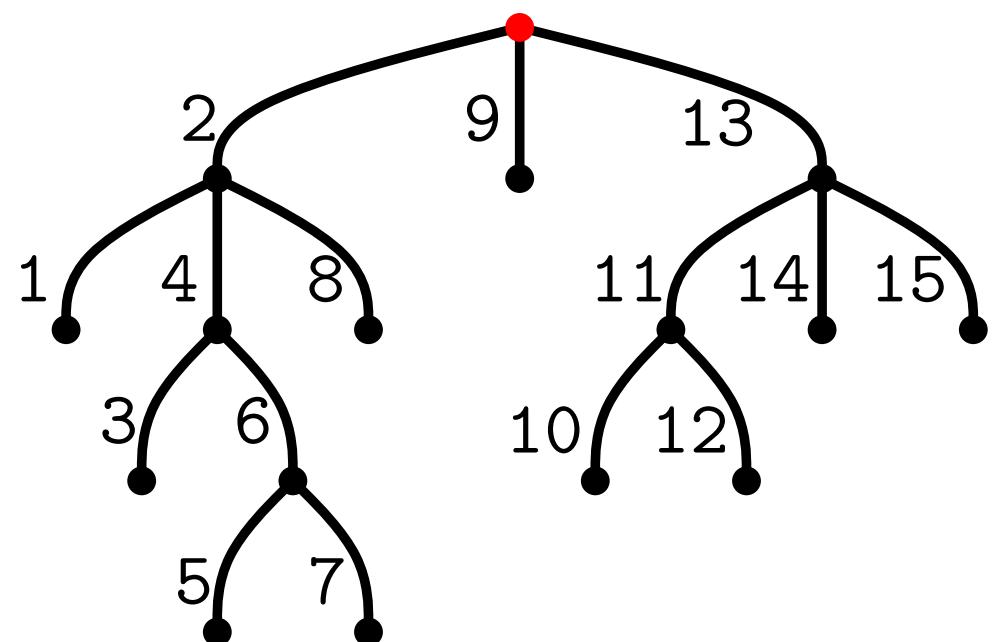
Schritt I: Beschrifte Kanten



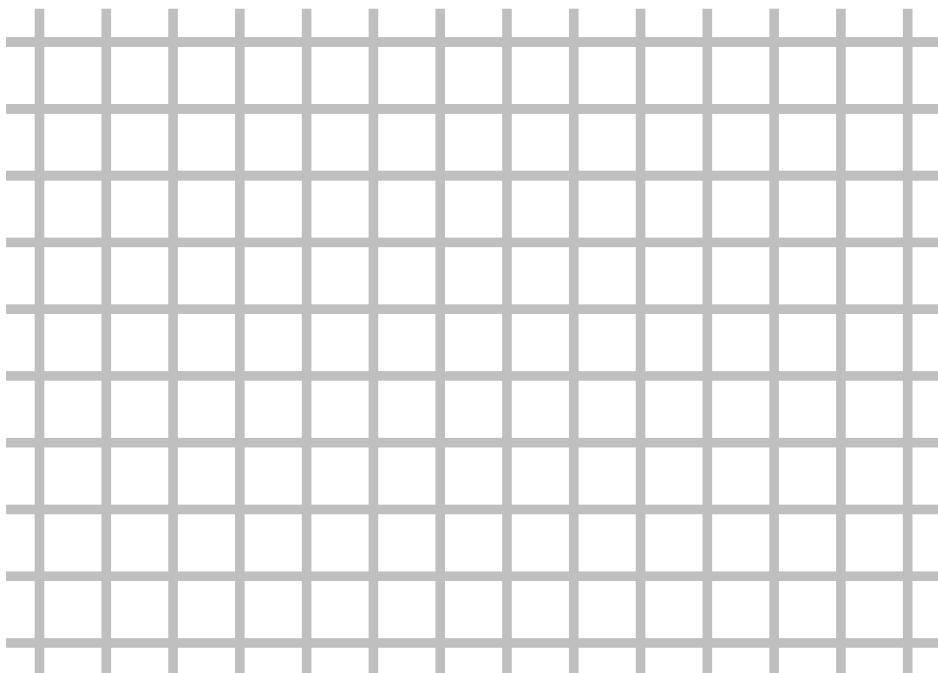
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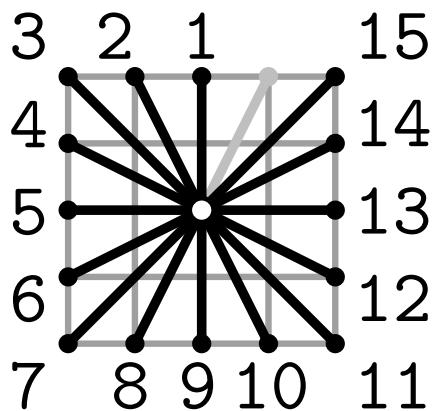
Schritt I: Beschrifte Kanten



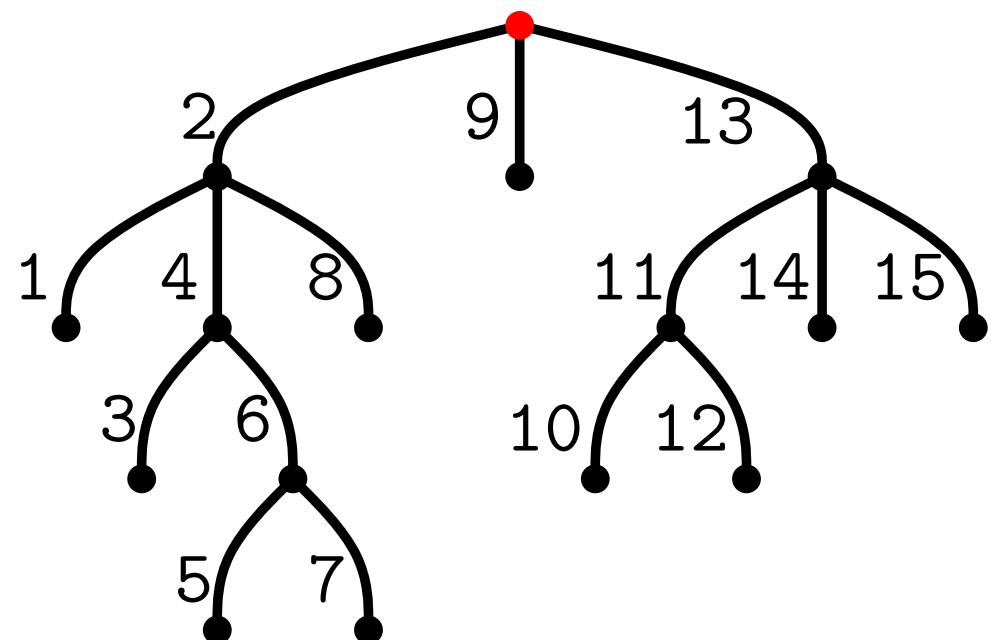
Schritt III: Zeichne Baum



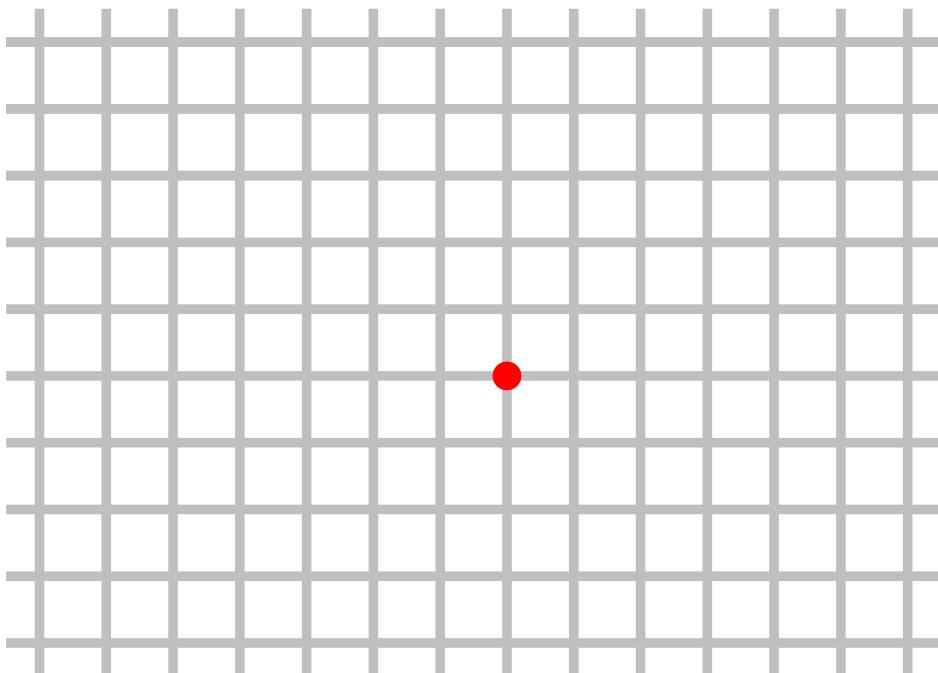
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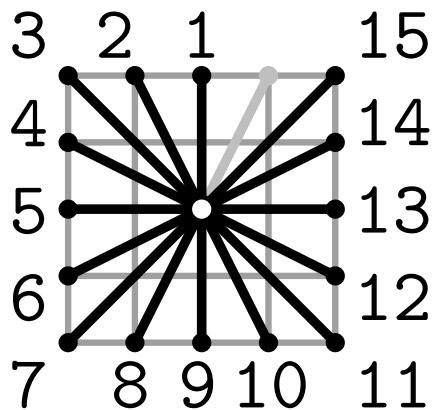
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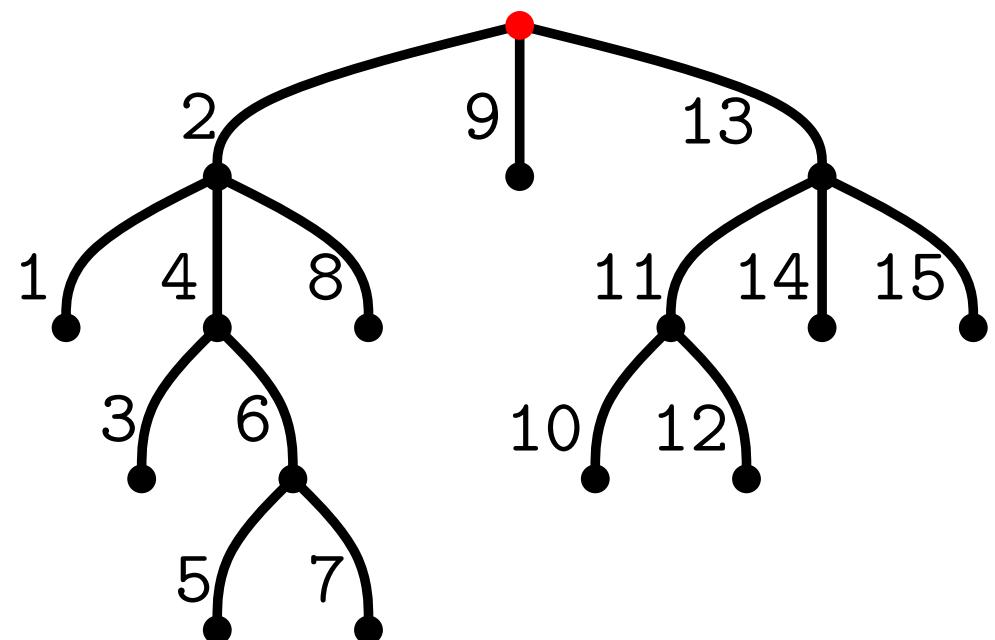
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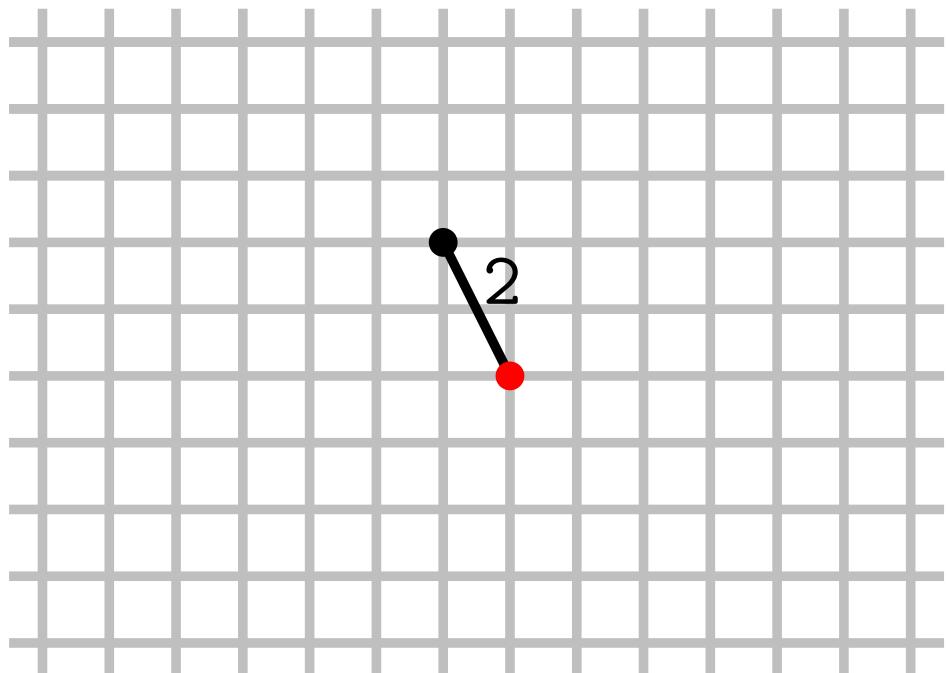
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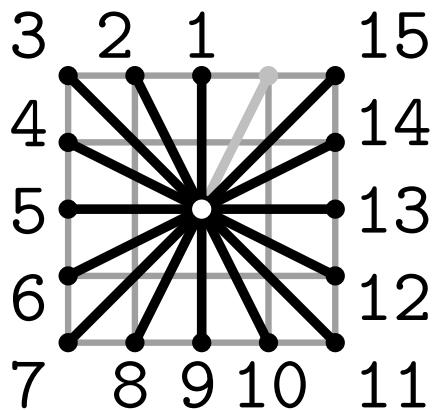
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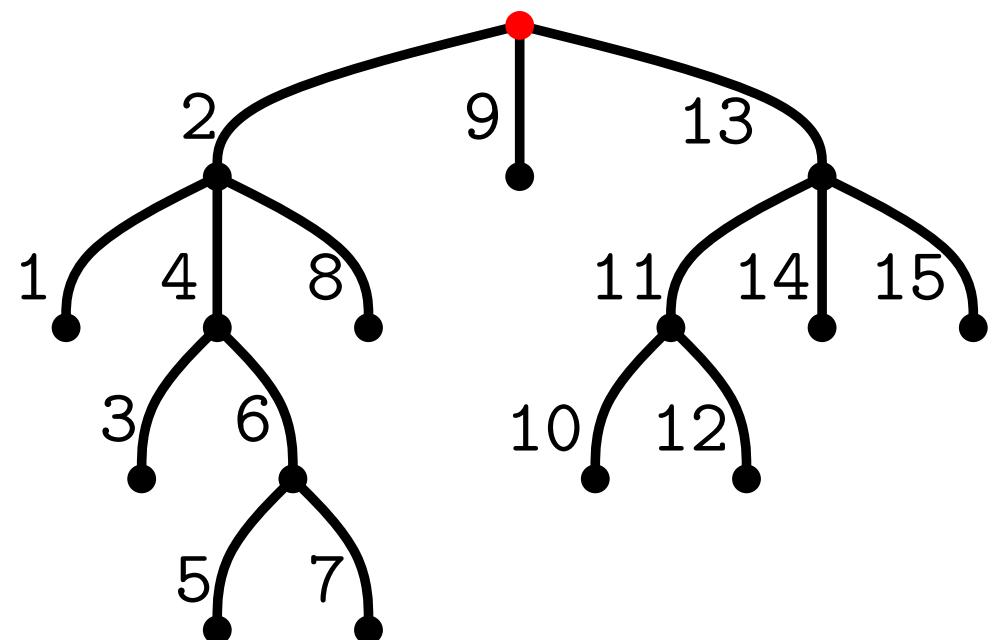
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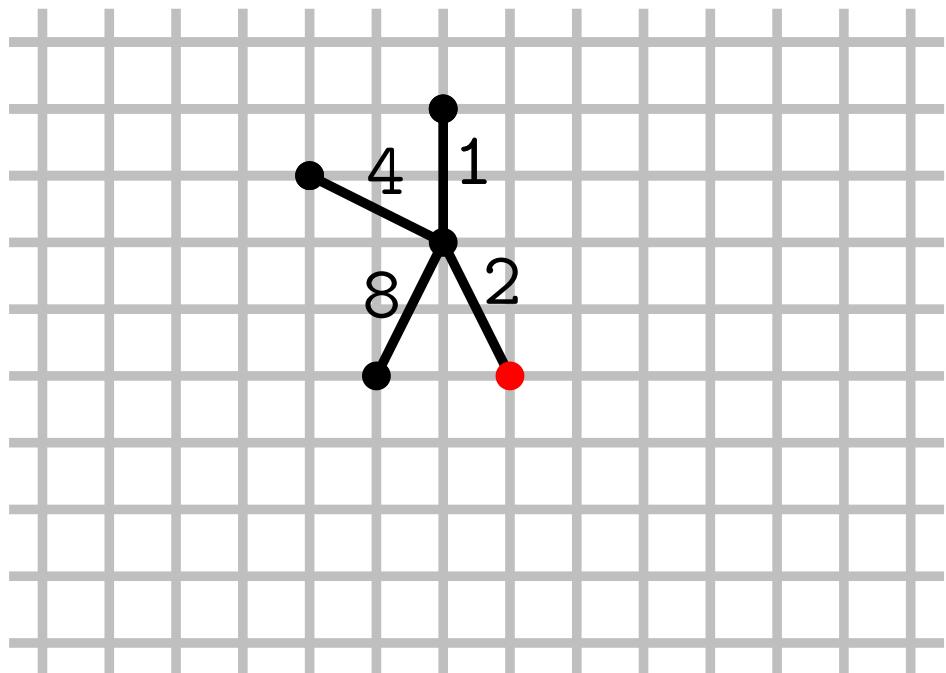
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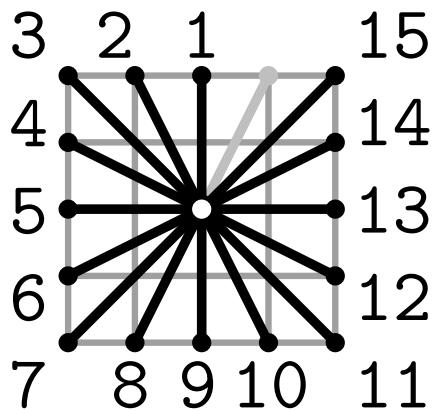
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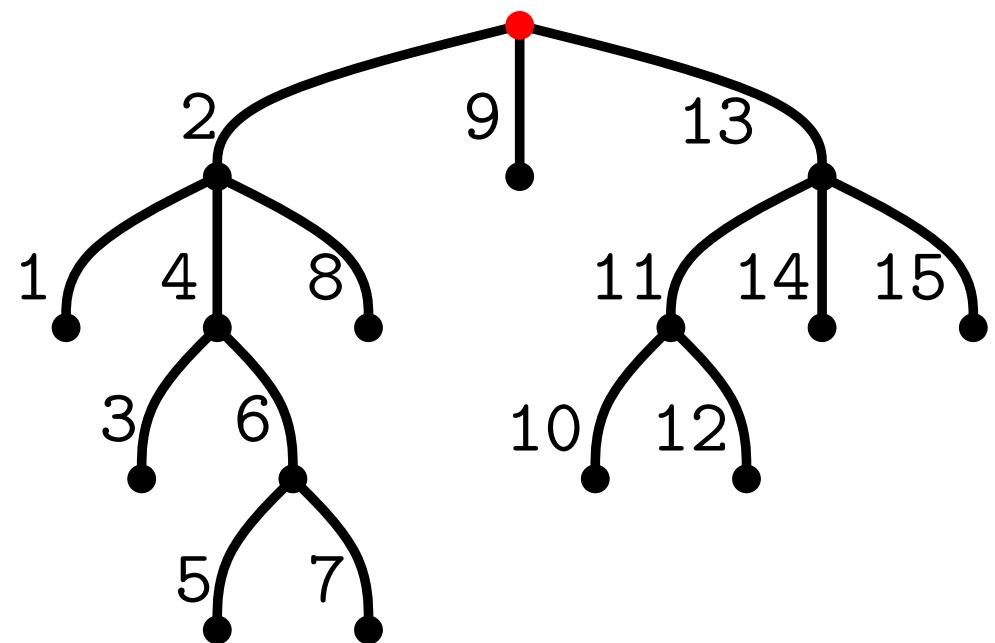
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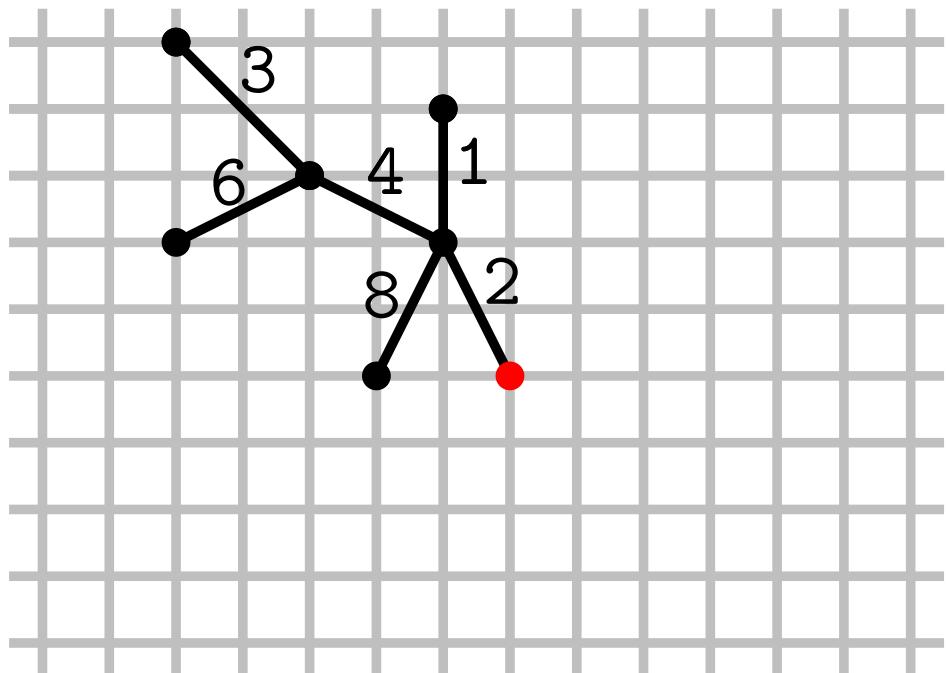
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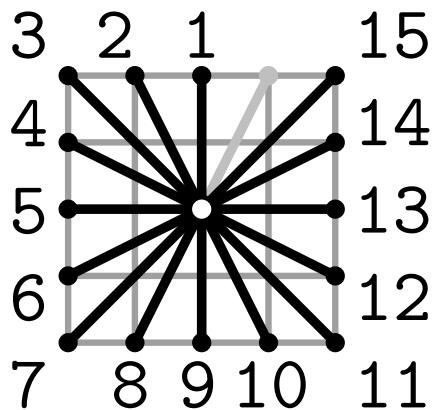
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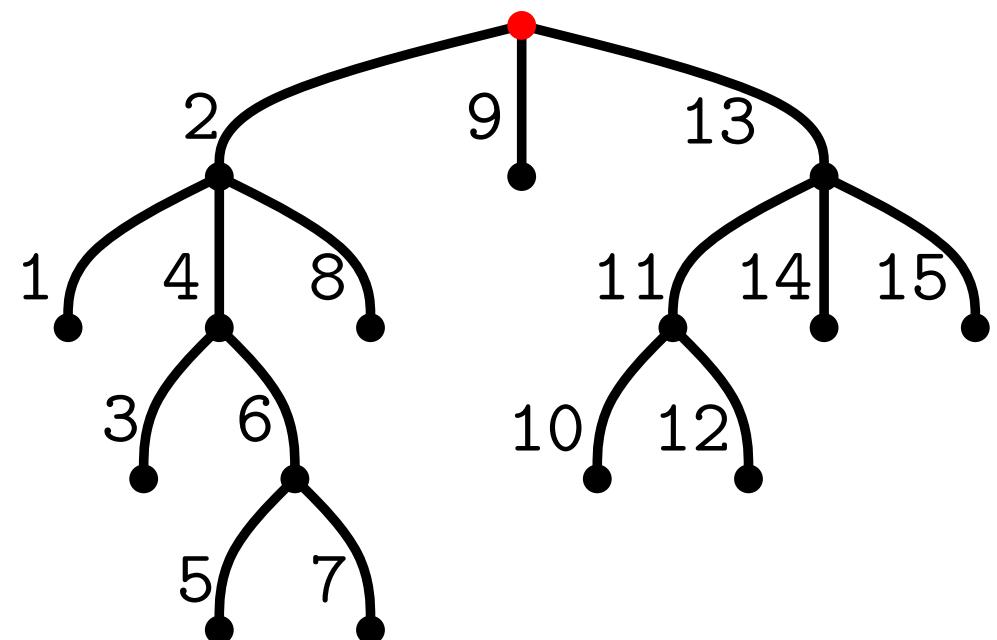
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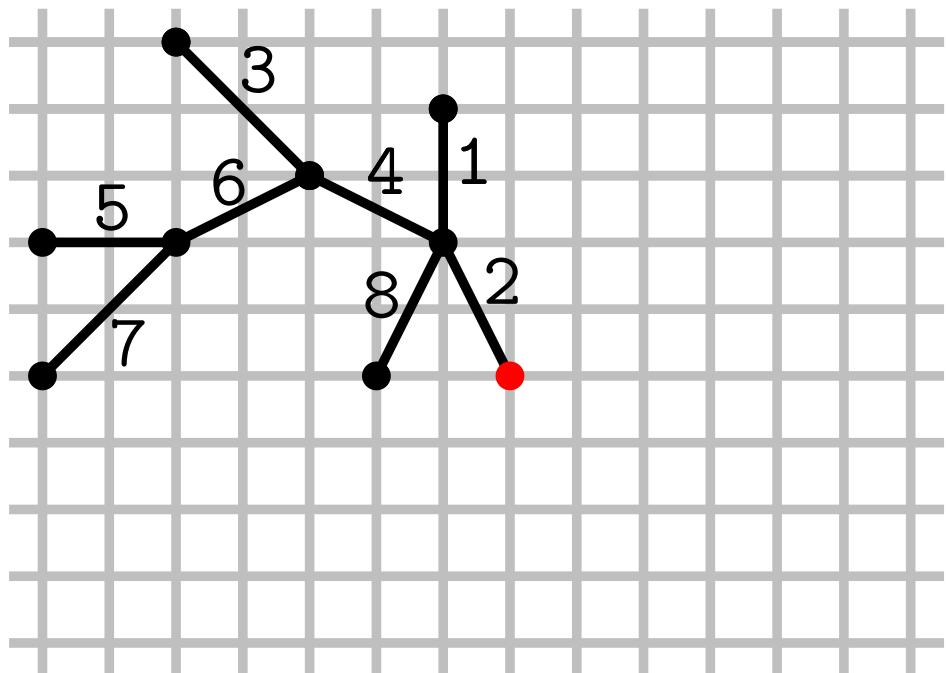
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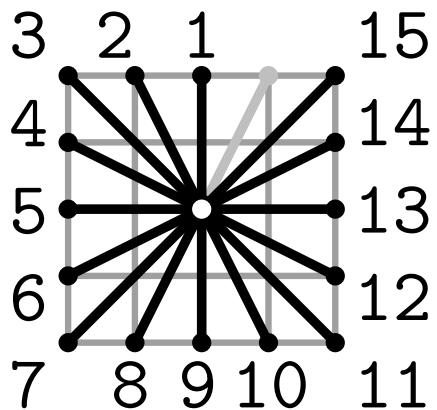
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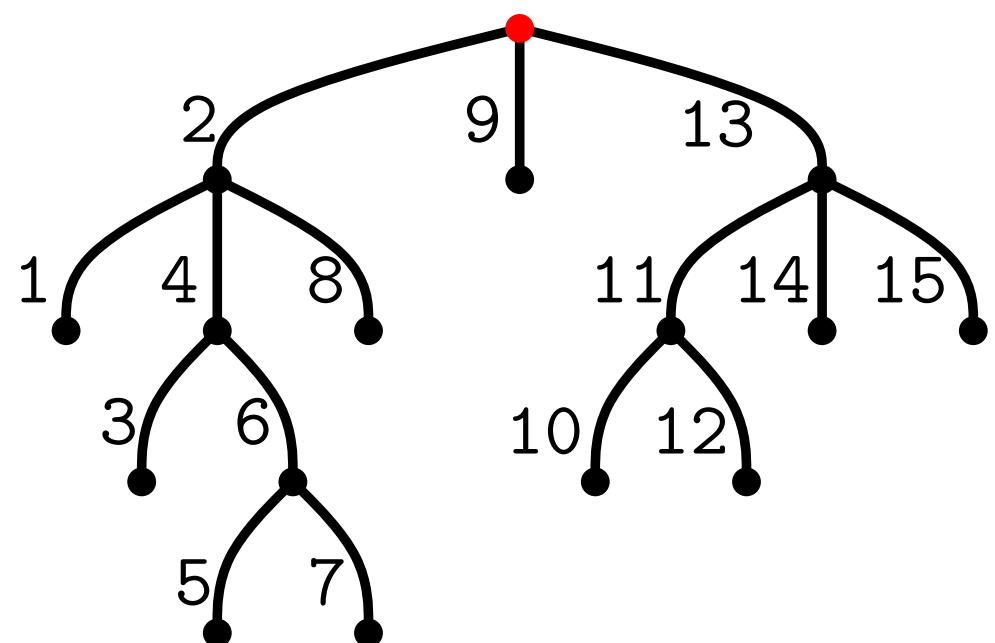
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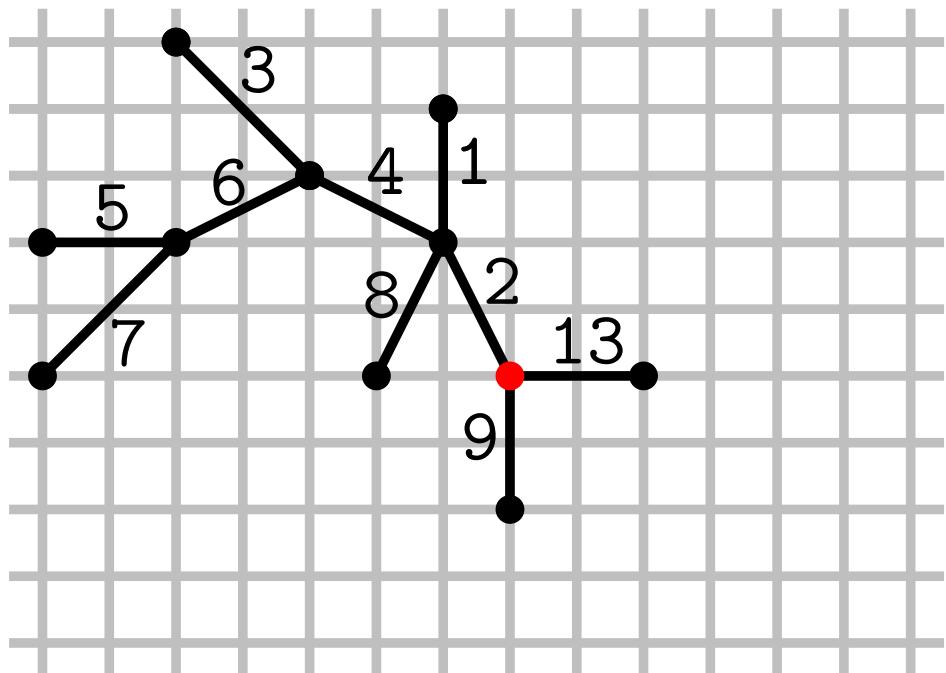
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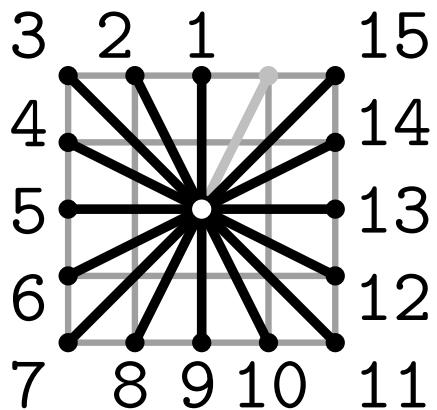
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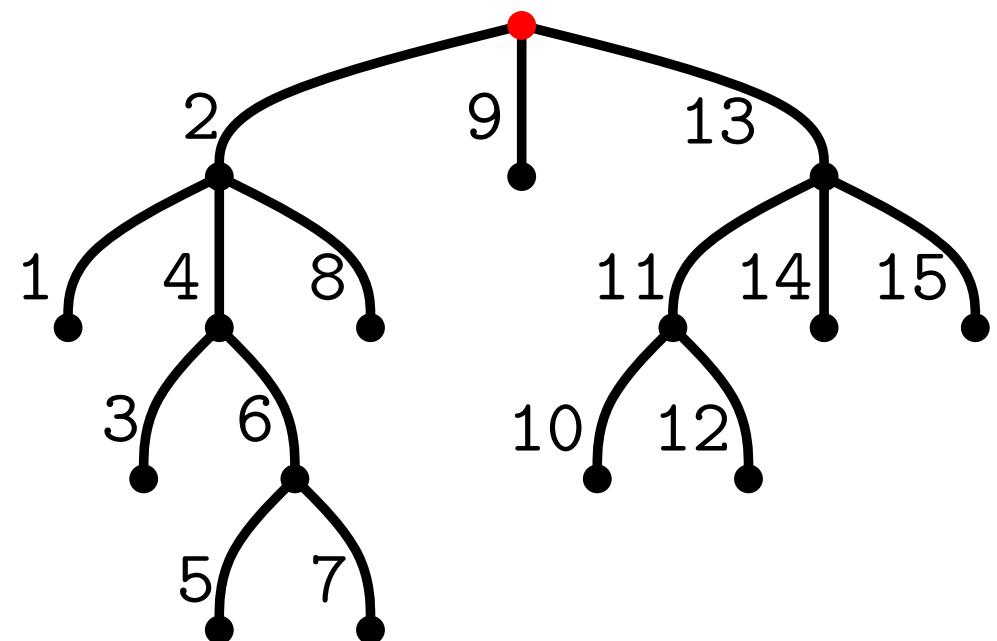
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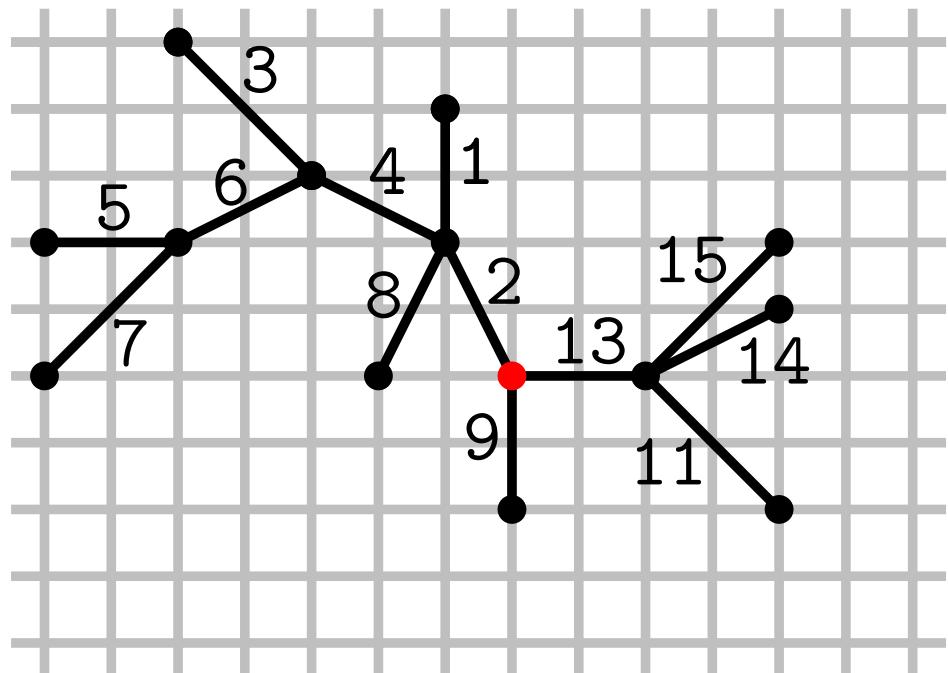
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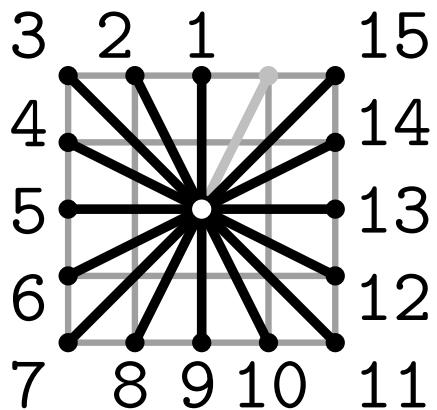
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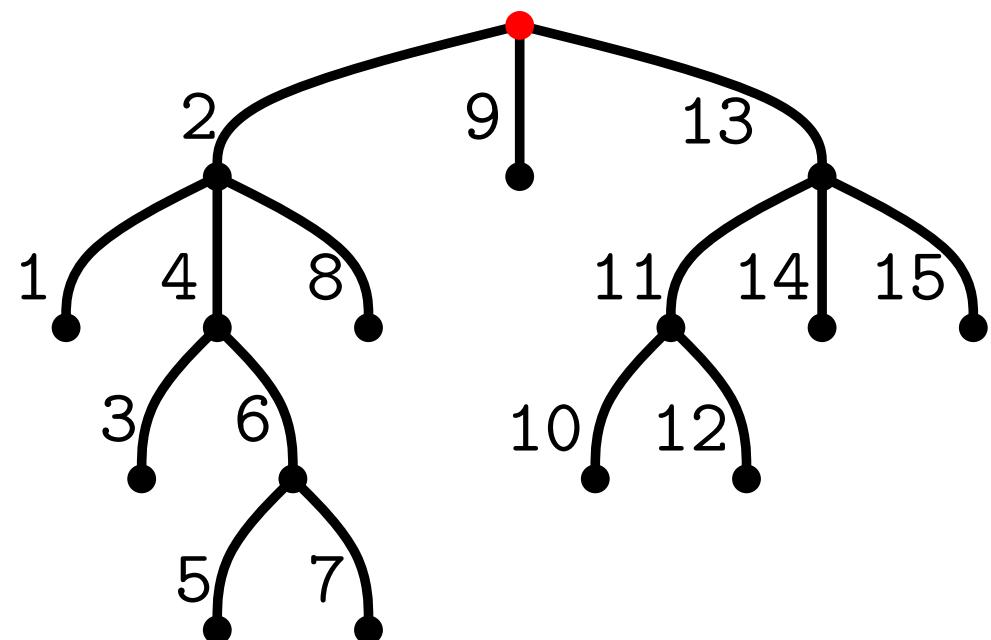
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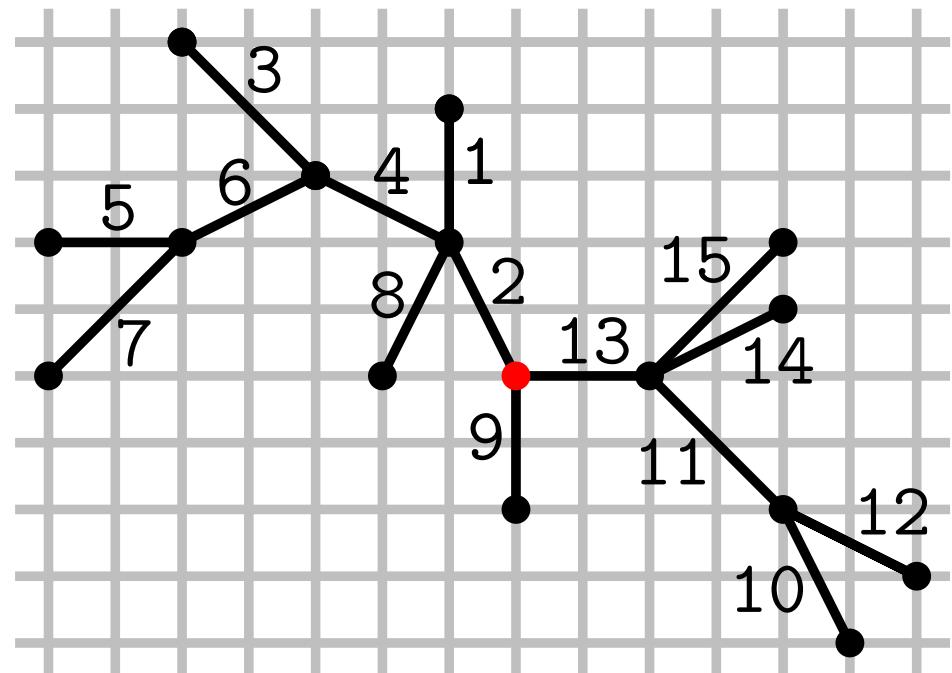
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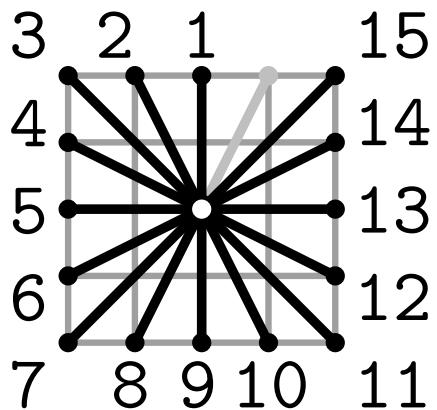
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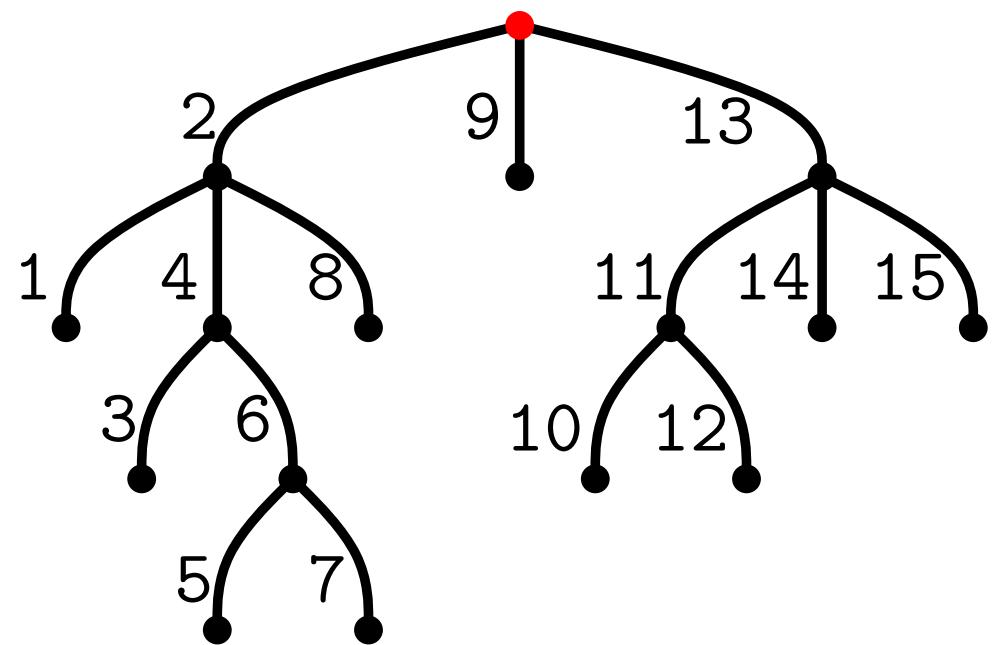
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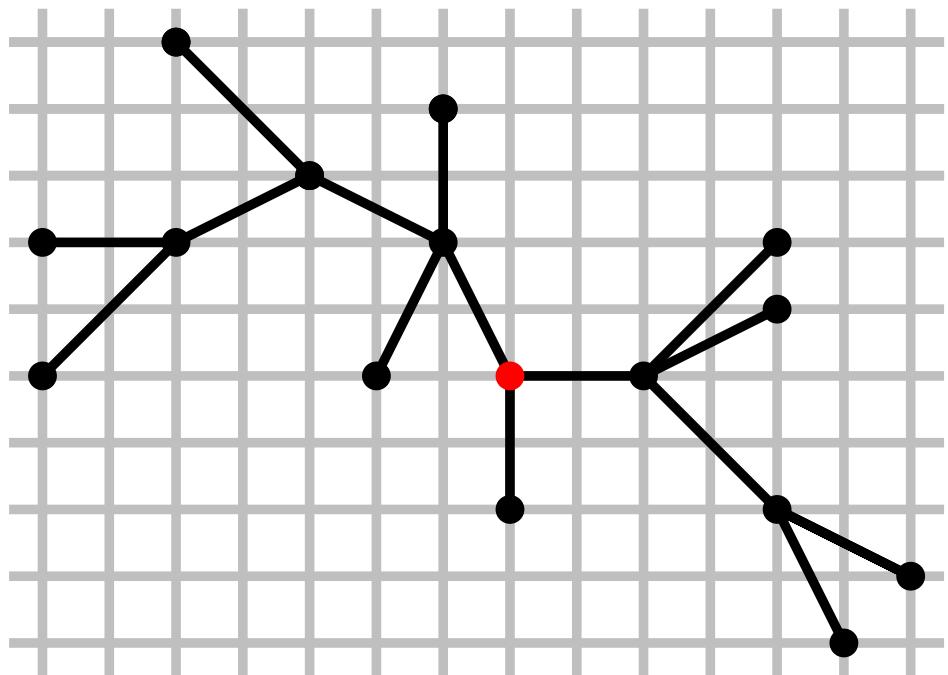
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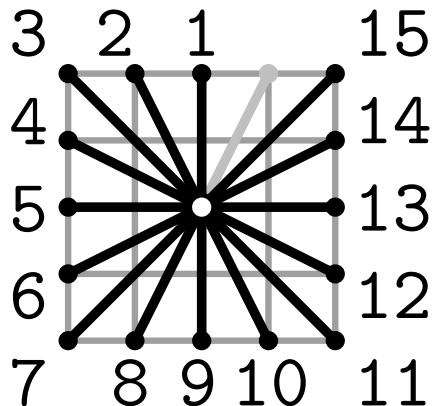
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Schritt III: Zeichne Baum



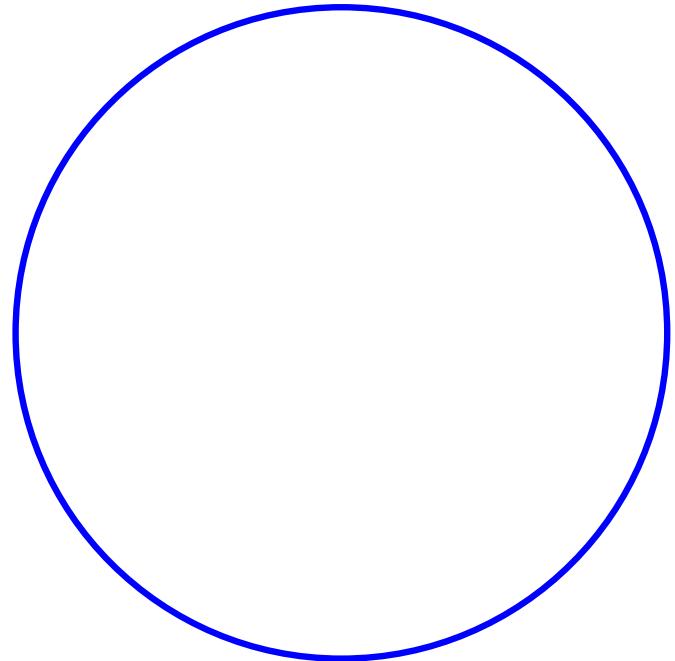
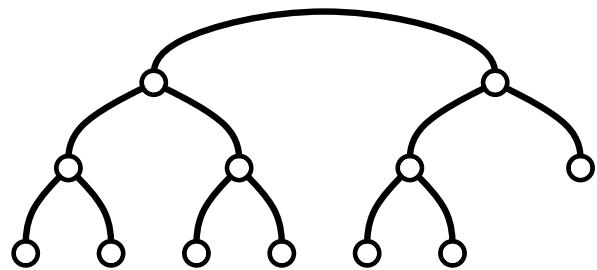
Schritt II: Wähle Vektoren



Baum \Rightarrow
monotone konvexe Zeichnung,
 $O(n^{1.5}) \times O(n^{1.5})$ Gitter

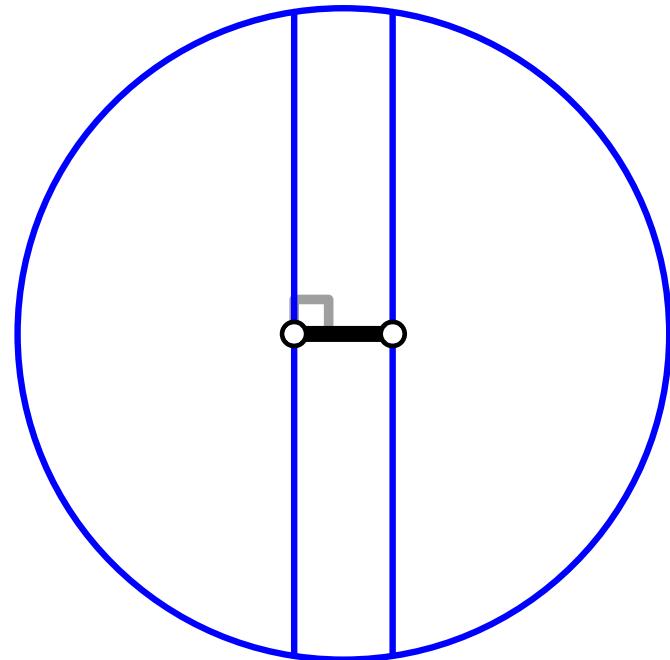
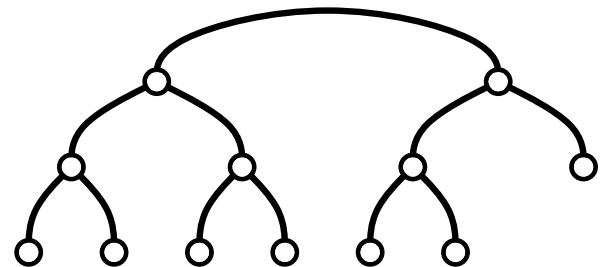
Streng Monotone Zeichnungen

Ordentlicher Binärbaum:
Kein Grad-2 Knoten



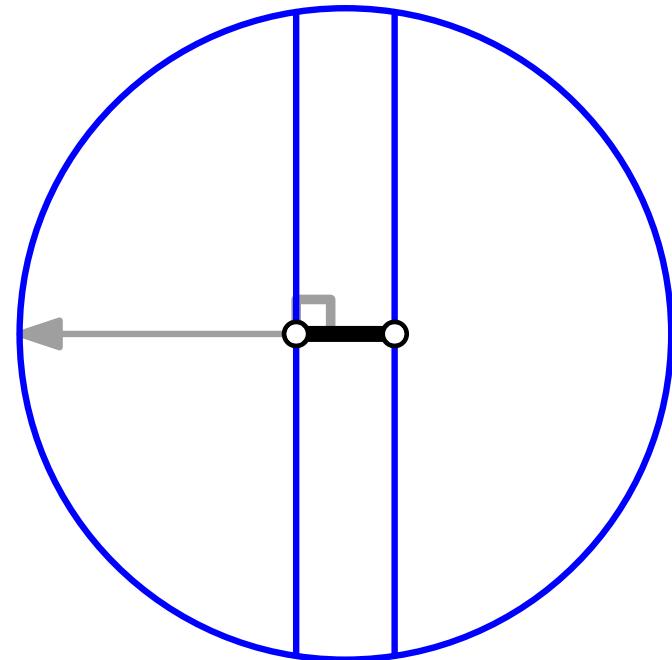
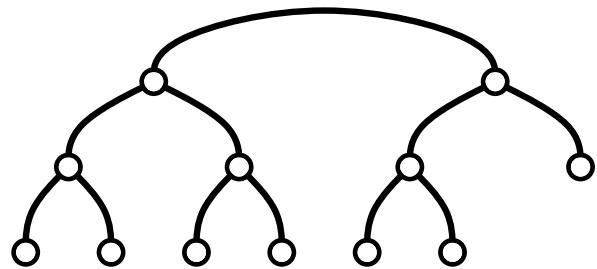
Streng Monotone Zeichnungen

Ordentlicher Binärbaum:
Kein Grad-2 Knoten



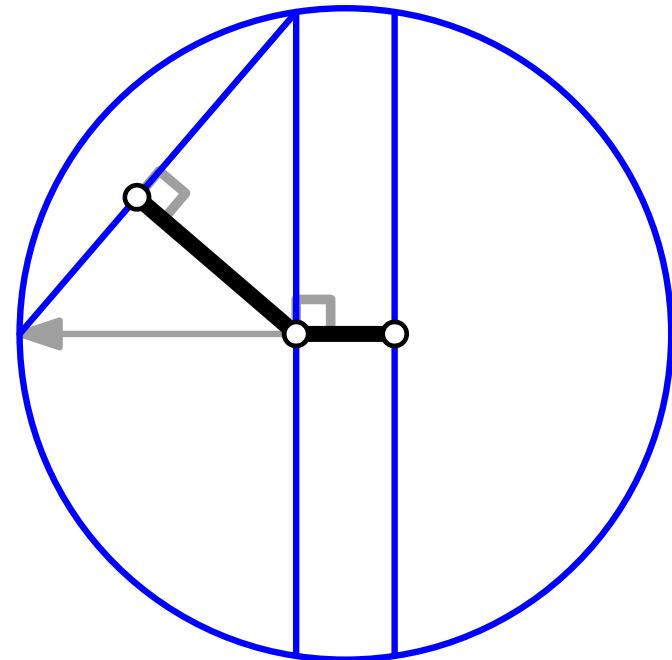
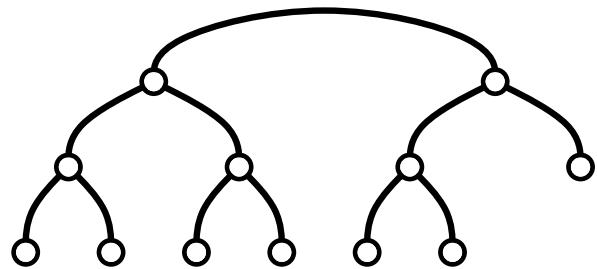
Streng Monotone Zeichnungen

Ordentlicher Binärbaum:
Kein Grad-2 Knoten



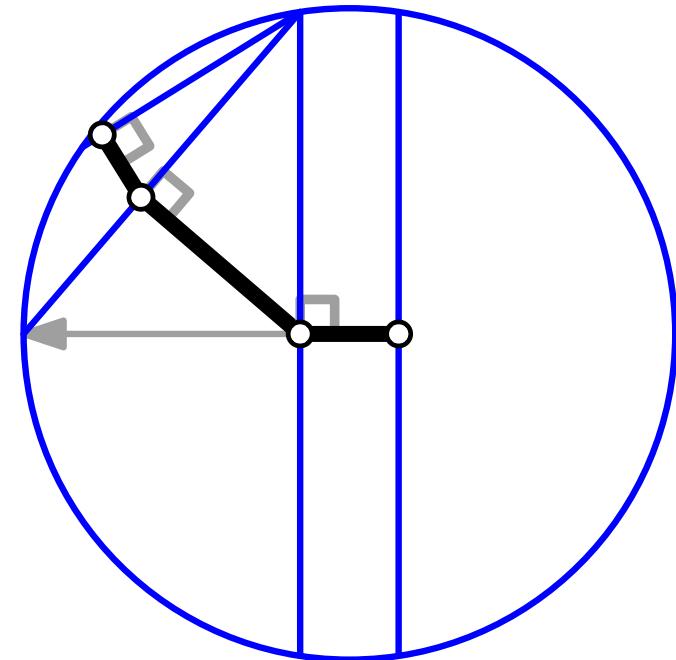
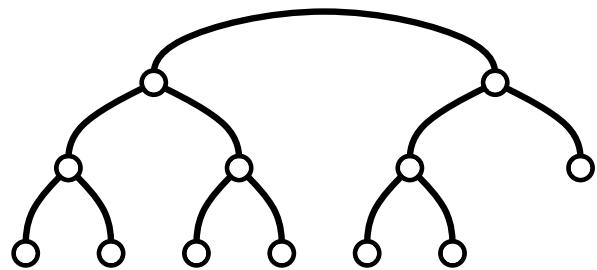
Streng Monotone Zeichnungen

Ordentlicher Binärbaum:
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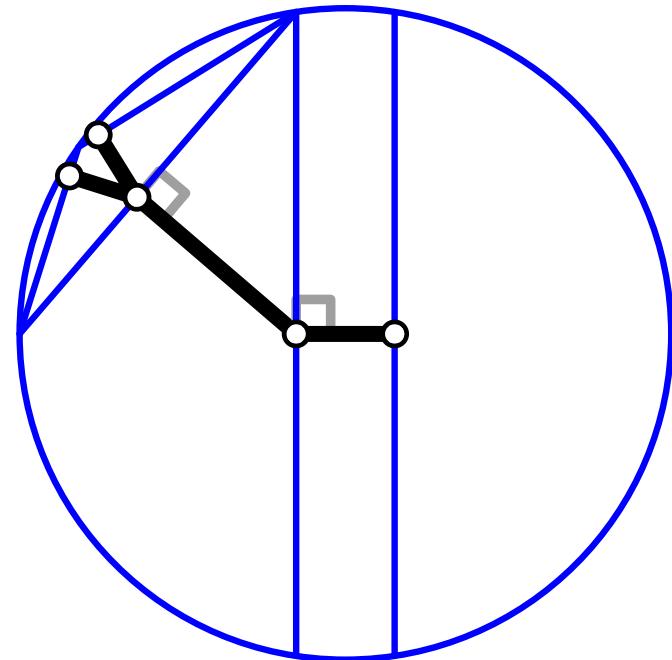
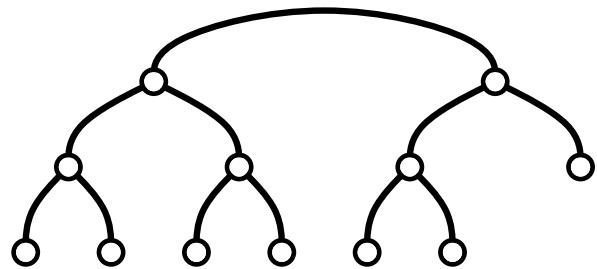
Streng Monotone Zeichnungen

Ordentlicher Binärbaum:
Kein Grad-2 Knoten



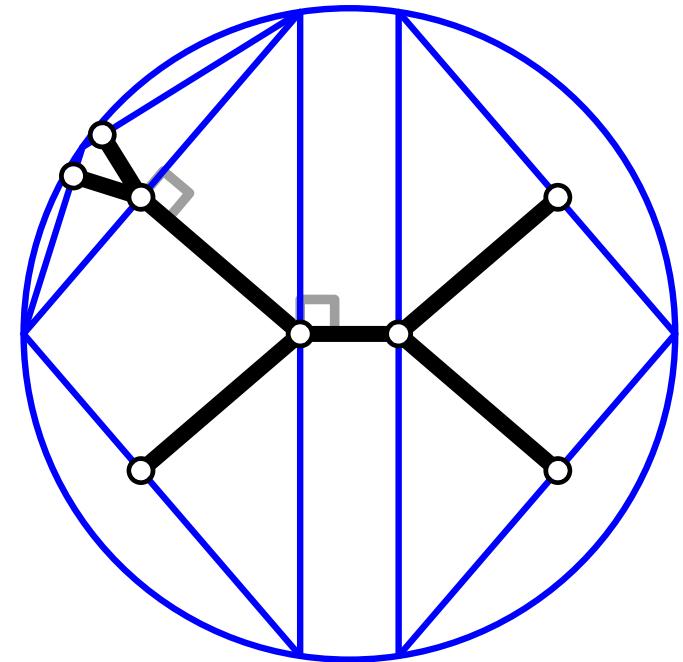
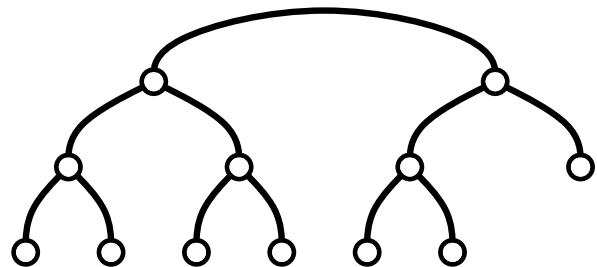
Streng Monotone Zeichnungen

Ordentlicher Binärbaum:
Kein Grad-2 Knoten



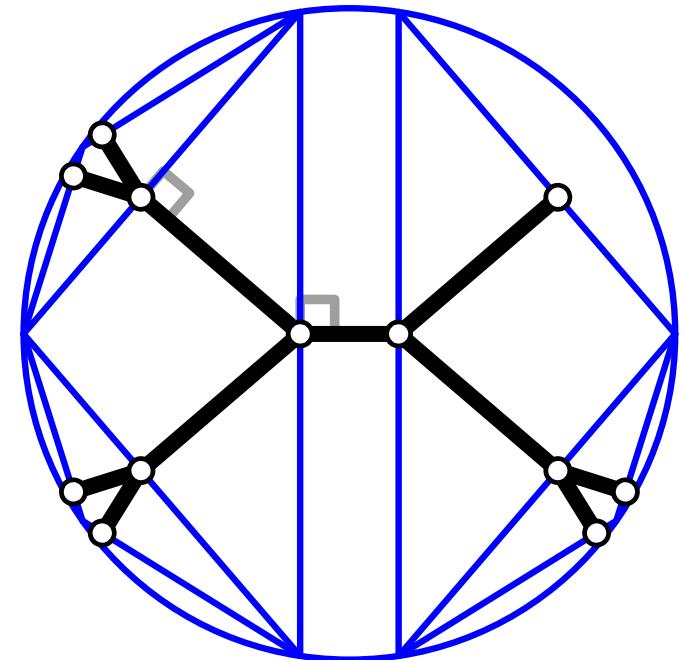
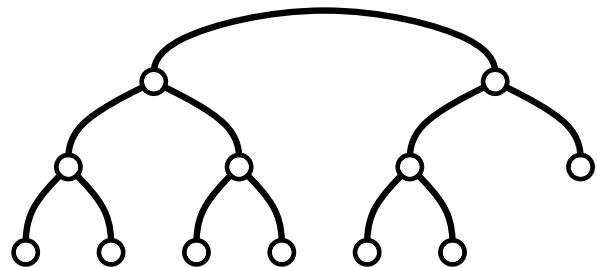
Streng Monotone Zeichnungen

Ordentlicher Binärbaum:
Kein Grad-2 Knoten



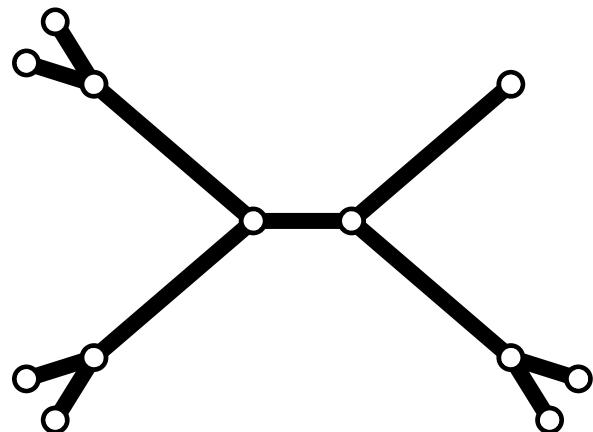
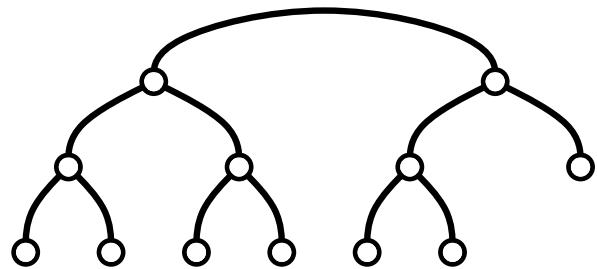
Streng Monotone Zeichnungen

Ordentlicher Binärbaum:
Kein Grad-2 Knoten



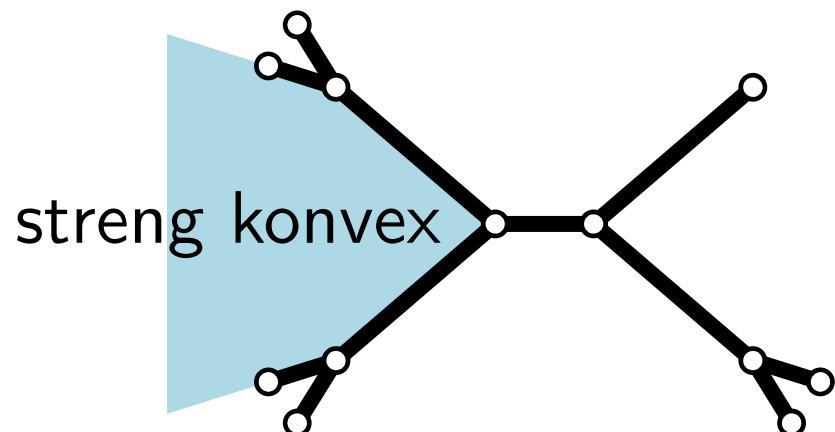
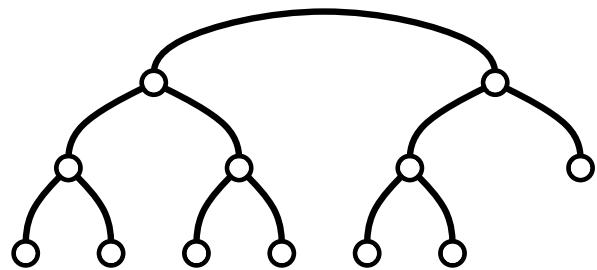
Streng Monotone Zeichnungen

Ordentlicher Binärbaum:
Kein Grad-2 Knoten



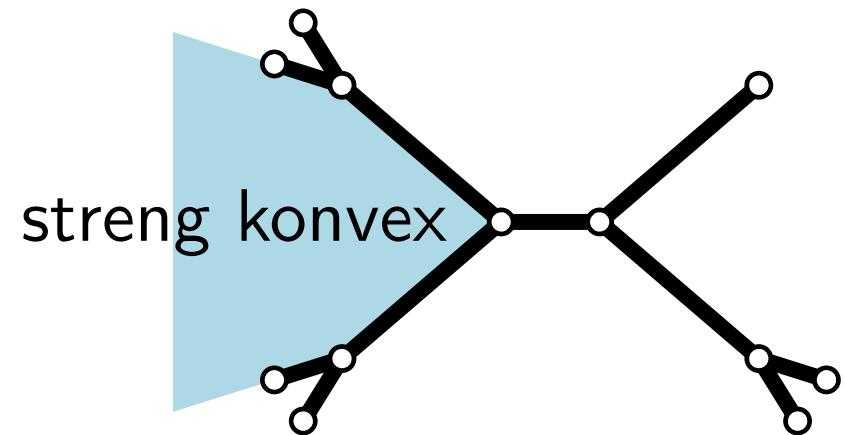
Streng Monotone Zeichnungen

Ordnlicher Binärbaum:
Kein Grad-2 Knoten



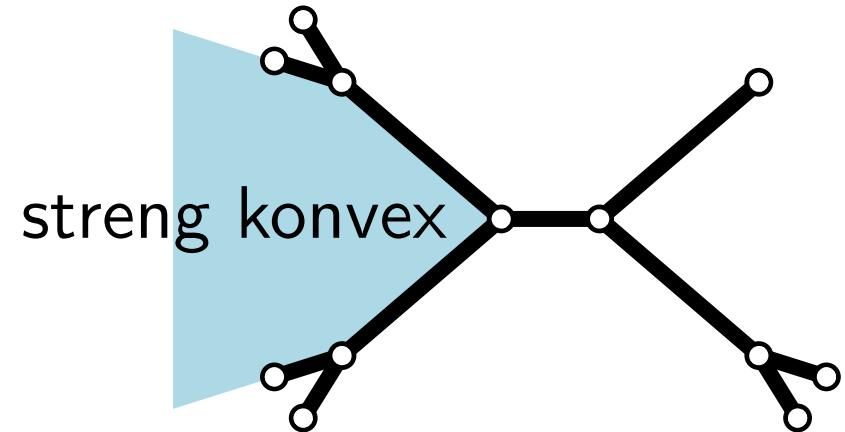
Streng Monotone Zeichnungen

Ordnlicher Binärbaum \Rightarrow
streng monotone,
streng konvexe Zeichnung,
exponentielle Fläche



Streng Monotone Zeichnungen

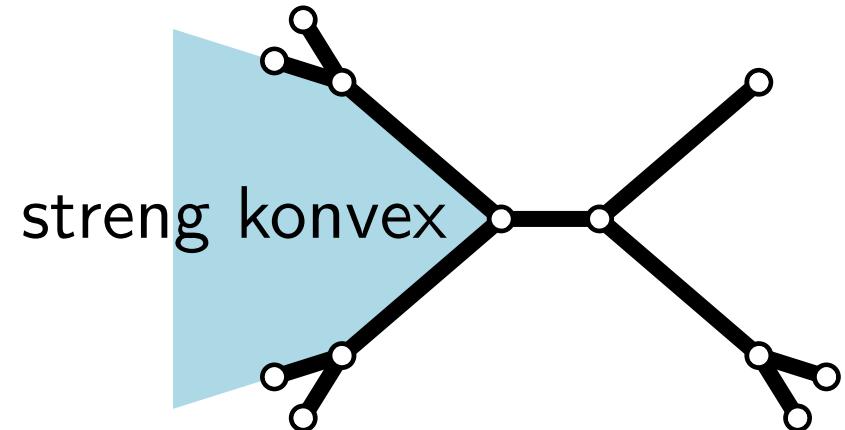
Ordentlicher Binärbaum \Rightarrow
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exponentielle Fläche



Baum \Rightarrow streng monotone Zeichnung, exponentielle Fläche

Streng Monotone Zeichnungen

Ordentlicher Binärbaum \Rightarrow
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exponentielle Fläche



Baum \Rightarrow streng monotone Zeichnung, exponentielle Fläche

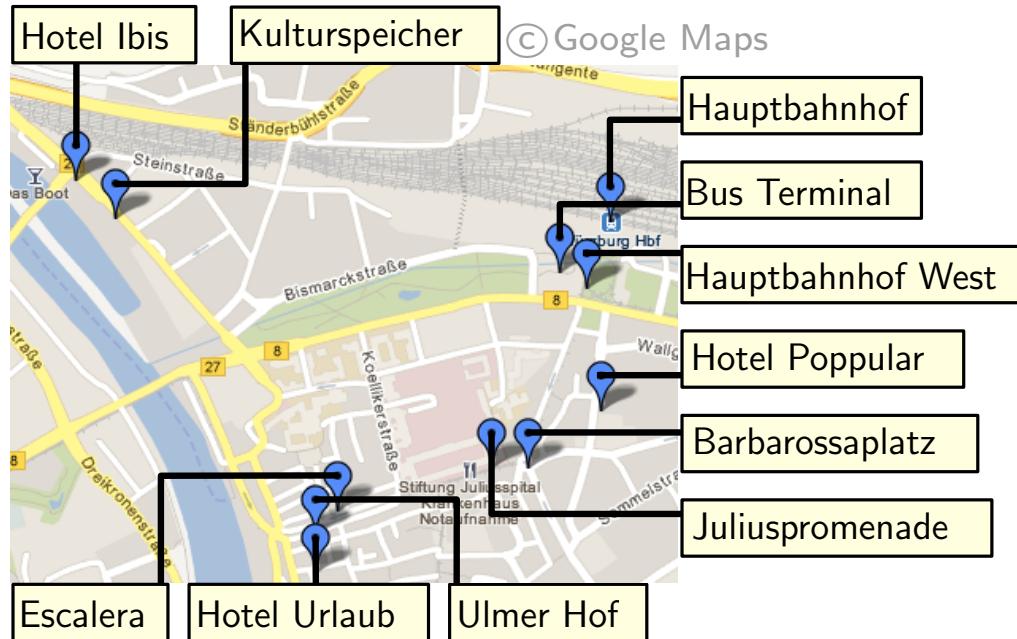
Nöllenburg et al.

Exponentielle Fläche wird benötigt.

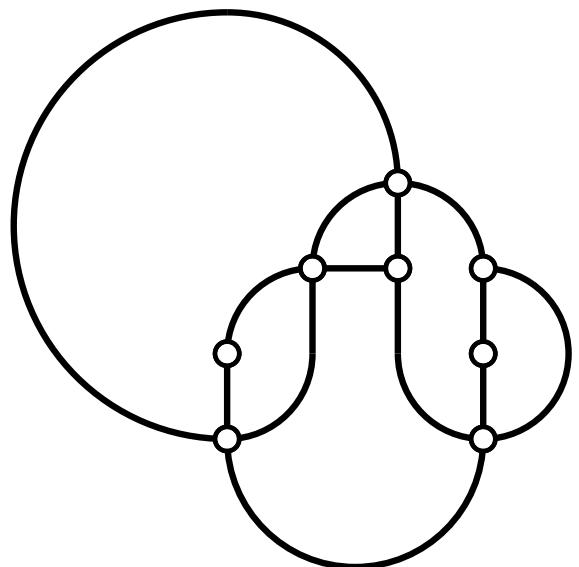
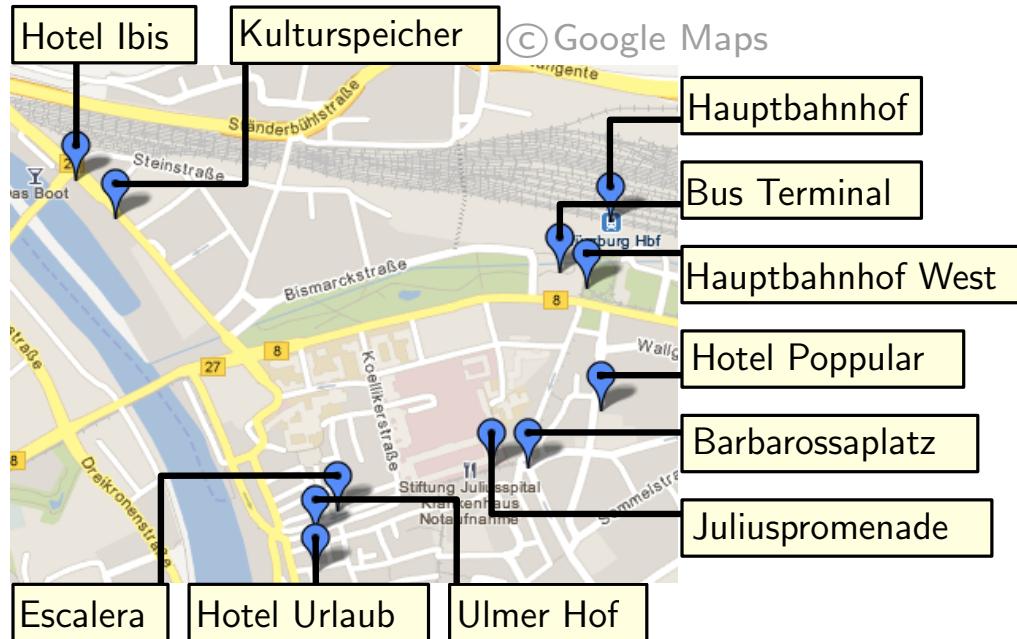
[arXiv'14]

Zusammenfassung

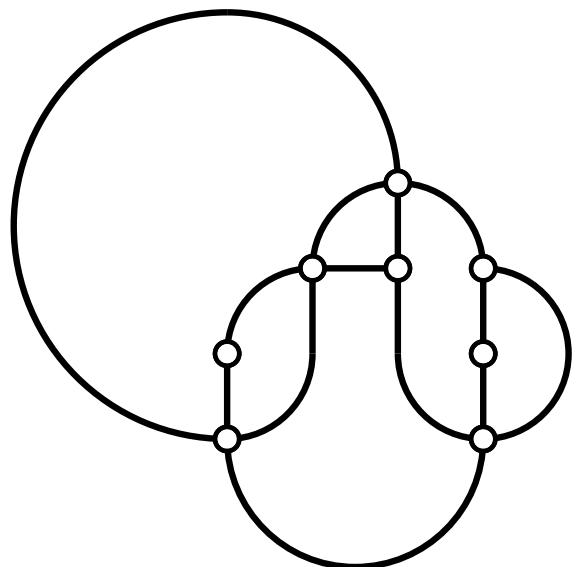
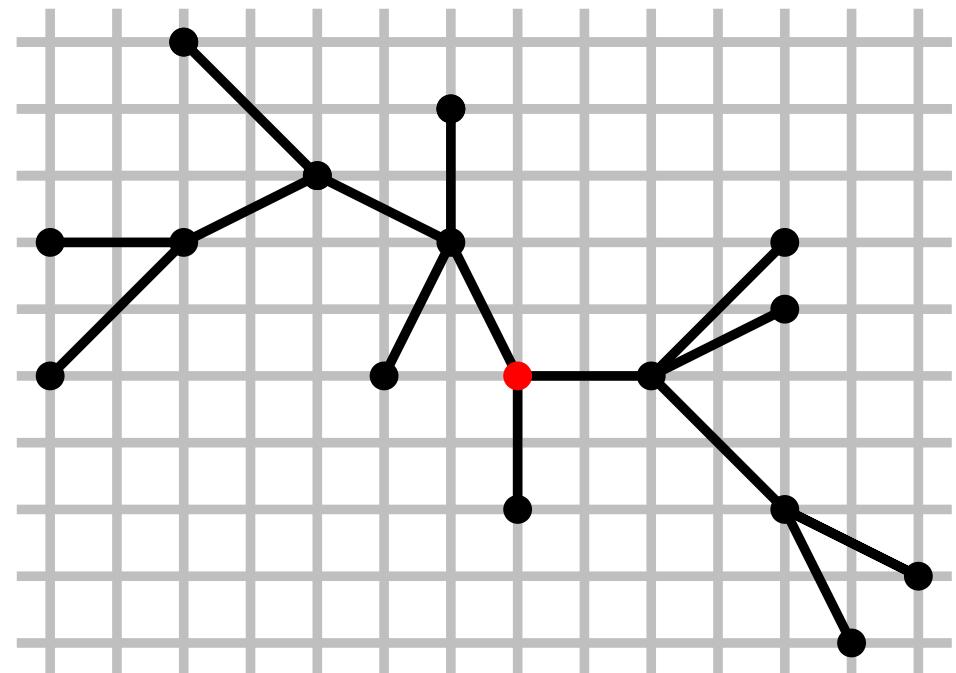
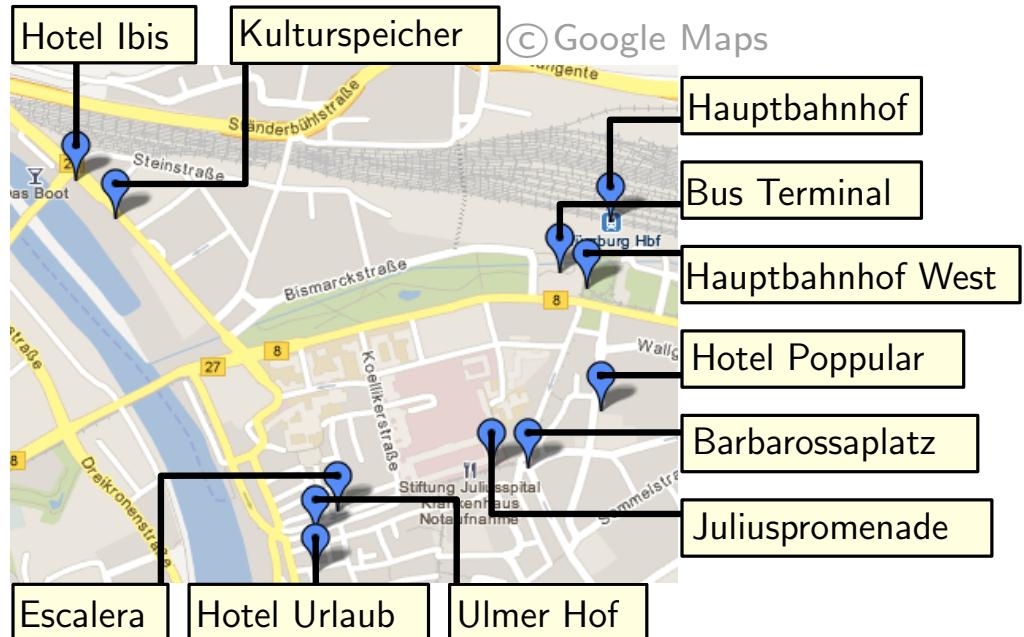
Zusammenfassung



Zusammenfassung



Zusammenfassung



Zusammenfassung

