Windrose Planarity
Embedding Graphs with Direction-Constrained Edges

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Upward Planarity

An undirected graph is planar: no crossings
An undirected graph is *planar*: no crossings
Upward Planarity

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A *directed* graph is *upwards planar*:
- no crossings
- all edges are $y$-monotone curves directed upwards
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planar
acyclic
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Upward Planarity: Testing

Testing Upward Planarity is...
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- NP-complete in general

[Garg & Tamassia '95]
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planar ➔ acyclic ➔ bimodal
Testing Upward Planarity is...

- NP-complete in general \[\text{[Garg & Tamassia '95]}\]
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- planar
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Windrose Planarity

$q$-constrained graph $(G, Q)$:
q-constrained graph \((G, Q)\):
- \(G\): undirected planar graph
Windrose Planarity

$q$-constrained graph $(G, Q)$:

- $G$: undirected planar graph
- $Q$: partition of all neighbors of $v$ into $\uparrow v$, $\downarrow v$, $\leftarrow v$, and $\rightarrow v$. 
Windrose Planarity

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Windrose Planarity

Two directions:

$q$-constrained graph \((G, Q)\):
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Windrose Planarity

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A $q$-constrained graph is windrose planar:
Windrose Planarity

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Windrose Planarity

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\[
\begin{align*}
\text{\( \uparrow v \)} & \quad \text{\( \downarrow v \)} \\
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Windrose Planarity

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A \(q\)-constrained graph is \textit{windrose planar}:
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- all edges are \(xy\)-monotone curves
- \(u \in \uparrow v \Rightarrow u\) lies in the \(\circ\)-quadrant of \(v\)
Relationship to Upwards Planarity

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Relationship to Upwards Planarity

Two directions:

\[ q\text{-constrained graph } (G, Q): \]
- \( G \): undirected planar graph
- \( Q \): partition of all neighbors of \( v \) into \( v \) and \( \bar{v} \)

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One direction:

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Relationship to Upwards Planarity

One direction:

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Relationship to Upwards Planarity

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Theorem. Testing Windrose Planarity is NP-complete
Angle categories: $0^\circ$, $90^\circ$, $180^\circ$, $270^\circ$, and $360^\circ$
Angular Drawing

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Labeled graph \((G, A)\): \(G\) plane graph, \(A\) labeling of angles
Angular Drawing

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Labeled graph $(G, A)$: $G$ plane graph, $A$ labeling of angles

Angular drawing: end of segments have slopes $\approx \pm 1$
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$(G, A)$ admits angular drawing if:
Angular Drawing

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$(G, A)$ admits angular drawing if:

- **Vertex condition**: sum of angle cat. at vertex is $360^\circ$
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$(G, A)$ admits angular drawing if:

- **Vertex condition**: sum of angle cat. at vertex is $360^\circ$
- **Cycle condition**: sum of angle cat. at (int.) face of length $k$ is $k \cdot 180^\circ - 360^\circ$
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Angular labeling $A$
$\Rightarrow$ q-constraints $Q_A$
unique
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Angular labeling \(A\)

\(\Rightarrow\) q-constraints \(Q_A\)

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q-constraints \(Q\)

+ large-angle assignment \(L\)

\(\Rightarrow\) angular labeling \(A_{Q,L}\)

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Angular Drawing

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q-constraints $Q$

+ large-angle assignment $L$

$\Rightarrow$ angular labeling $A_{Q,L}$ unique

Angular drawing $\hat{=} \triangleleft$ windrose planar drawing
Triangulated Graphs
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- No (int.) $> 180^\circ$ angle categories
Triangulated Graphs

- No (int.) $> 180^\circ$ angle categories
- At least one $0^\circ$ angle category per (int.) face
Triangulated Graphs

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Triangulated Graphs

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• At least one $0^\circ$ angle category per (int.) face

Lemma.
Let $(G, A_Q)$ be a triangulated angular labeled graph. Then, $G^\uparrow$ is acyclic.
Triangulated Graphs

- No (int.) > 180° angle categories
- At least one 0° angle category per (int.) face

Lemma.
Let \((G, A_Q)\) be a triangulated angular labeled graph. Then, \(G^\uparrow\) is acyclic and has no internal sources or sinks.
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**Lemma.** Let \((G, A_Q)\) be a triangulated angular labeled graph. Then, \(G^\uparrow\) and \(G^\rightarrow\) are acyclic and have no internal sources or sinks.

What if there are no (int.) 180° angle categories?
Quasi-triangulated Graphs

What if there are no (int.) $180^\circ$ angle categories?
Quasi-triangulated Graphs

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- topological order on $G^\rightarrow$: $x$-coordinates
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What if there are no (int.) $180^\circ$ angle categories?

- topological order on $G^{\rightarrow}$: $x$-coordinates
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• topological order on $G^\rightarrow$: x-coordinates
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Quasi-triangulated Graphs

What if there are no (int.) $180^\circ$ angle categories?

- topological order on $G\rightarrow$: $x$-coordinates
- topological order on $G\uparrow$: $y$-coordinates
What if there are no (int.) 180° angle categories?

- Topological order on $G^\rightarrow$: $x$-coordinates
- Topological order on $G^\uparrow$: $y$-coordinates
Quasi-triangulated Graphs

What if there are no (int.) 180° angle categories?

- topological order on $G^{-}$: $x$-coordinates
- topological order on $G^{↑}$: $y$-coordinates
Quasi-triangulated Graphs

What if there are no (int.) 180° angle categories?

- topological order on $G^\rightarrow$: $x$-coordinates
- topological order on $G^\uparrow$: $y$-coordinates
Quasi-triangulated Graphs

What if there are no (int.) 180° angle categories?

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Quasi-triangulated Graphs

What if there are no (int.) $180^\circ$ angle categories?

• topological order on $G^\rightarrow$: $x$-coordinates
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Quasi-triangulated Graphs

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- topological order on $G^{\uparrow}$: $y$-coordinates
Quasi-triangulated Graphs

What if there are no (int.) $180^\circ$ angle categories?

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Quasi-triangulated Graphs

What if there are no (int.) 180° angle categories?

• topological order on $G^\rightarrow$: $x$-coordinates
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Lemma.
quasi-triangulated angular labeled graph $(G, A_Q)$,
all internal angles have category 0° or 90°
⇒ straight-line windrose planar drawing
on $n \times n$ grid in $O(n)$ time
Triangulated graphs

Let \((G, A_Q)\) be a triangulated angular labeled graph.
Triangulated graphs

Let \((G, A_Q)\) be a triangulated angular labeled graph. Task: Augment \((G, A_Q)\) to a quasi-triangulated angular labeled graph \((G^*, A_{Q^*})\) without internal angle category \(180^\circ\).
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Triangulated graphs

Let \((G, A_Q)\) be a triangulated angular labeled graph.

- Assume that \(\uparrow u \neq \emptyset, \uparrow w \neq \emptyset\) (otherwise, set \(v = u/w\))
Triangulated graphs

Let \((G, A_Q)\) be a triangulated angular labeled graph.

- Assume that \(\uparrow u \neq \emptyset, \uparrow w \neq \emptyset\) (otherwise, set \(v = u/w\))
- if \((u, w)\) not on outer face:
Let \((G, A_Q)\) be a triangulated angular labeled graph.

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![Diagram of triangulated graph with vertices u, v, and w]

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- Assume that \(\uparrow u \neq \emptyset, \downarrow w \neq \emptyset\) (otherwise, set \(v = u/w\))
- if \((u, w)\) not on outer face:
- if \((u, w)\) on outer face:

- Add \(w_N, w_E, w_S, \text{ and } w_W\)
Triangulated graphs

Let $(G, A_Q)$ be a triangulated angular labeled graph.

- Assume that $\uparrow u \neq \emptyset, \uparrow w \neq \emptyset$ (otherwise, set $v = u/w$)
- if $(u, w)$ not on outer face:
  - if $(u, w)$ on outer face:

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Let \((G, A_Q)\) be a triangulated angular labeled graph.

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- if \((u, w)\) not on outer face:  
  - if \((u, w)\) on outer face:

\[
\begin{align*}
  v &= u/w, \\
  w &= w_{N}, w_{E}, w_{S}, \text{ and } w_{W}
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- if \((u, w)\) not on outer face:  
  - if \((u, w)\) on outer face:
    - Add \(w_N, w_E, w_S, \) and \(w_W\)

\[\begin{array}{c}
\text{u} \\
\downarrow \\
\text{v} \\
\downarrow \\
\text{w}
\end{array}\]
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\[\text{Theorem.}\]

A triangulated q-constrained graph \((G, Q)\) is windrose planar

\[\Leftrightarrow AQ\] is angular

\[\rightarrow\] draw with 1 bend per edge

on an \(O(n) \times O(n)\) grid in \(O(n)\) time
Lemma.
plane q-constrained graph \((G, Q)\)
\[\Rightarrow \text{find a large-angle assignment } L \text{ such that } A_{Q,L} \text{ is angular (if it exists) in } O(n \log^3 n) \text{ time}\]
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**Lemma.**
Plane angular labeled graph \((G, A)\)
⇒ augment in \(O(n)\) time to a
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**Lemma.**
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Plane angular labeled graph \((G, A)\)
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**Theorem.**
Plane q-constrained Graph
\(\Rightarrow\) test windrose planarity in \(O(n \log^3 n)\) time
\(\rightarrow\) draw with 1 bend per edge on \(O(n) \times O(n)\) grid
Further Results

**Theorem.**
Windrose planar q-constrained graph \((G, Q)\) whose blocks are either edges or planar 3-trees
\[ \Rightarrow \text{straight-line windrose planar drawing} \]
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Straight-line windrose planar drawings require exponential area.
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Open problems

• Draw windrose planar graphs straight-line?
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• Generalizations: each edge has a set of possible directions or allow more than two directions
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