Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

Institute for Informatics

Simultaneous Drawing of Planar Graphs with Right-Angle Crossings and Few Bends

Philipp Kindermann
Chair of Computer Science I
Universität Würzburg

Joint work with
Michael A. Bekos, Thomas C. van Dijk & Alexander Wolff

Simultaneous Embedding

Given: Two planar graphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$
on same vertex set

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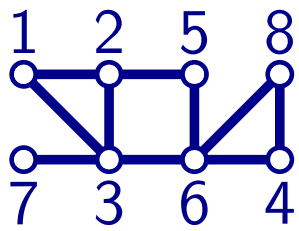

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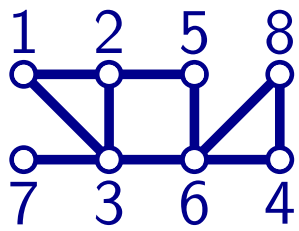

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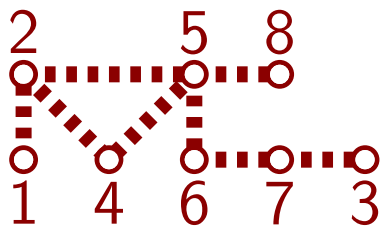

G_2

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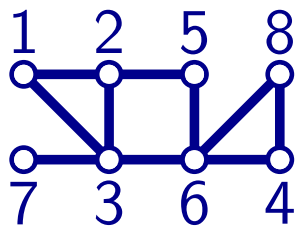

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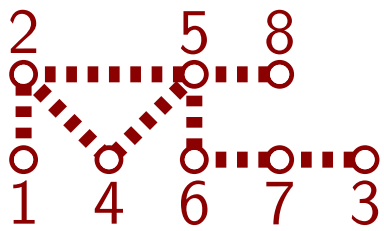

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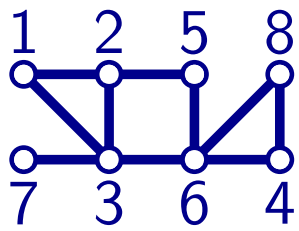

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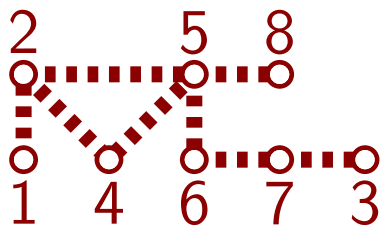

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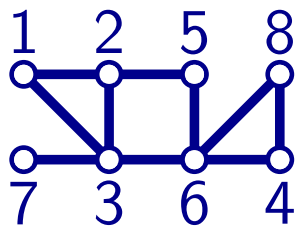

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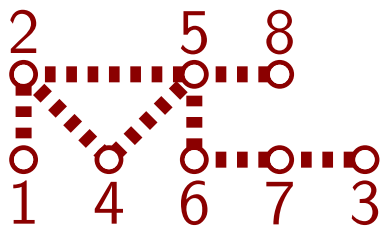

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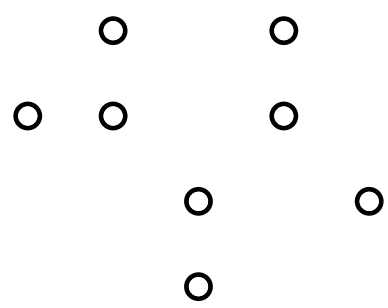

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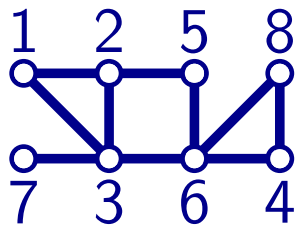

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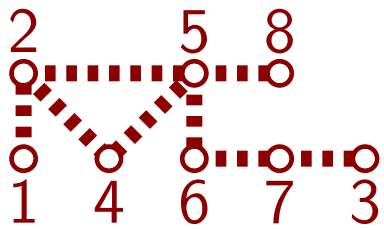

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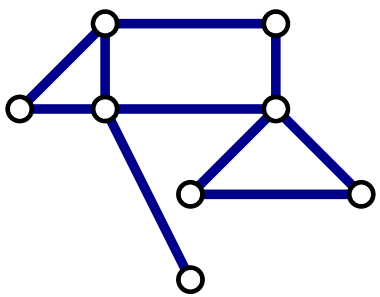

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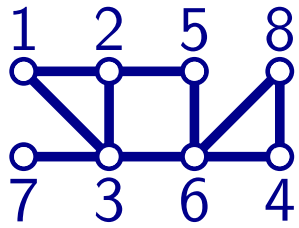

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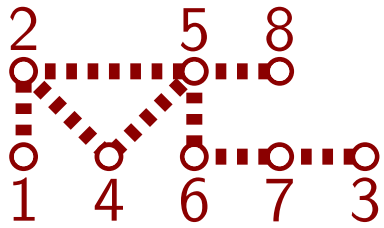

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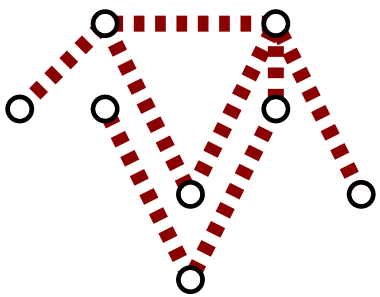

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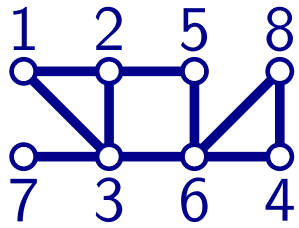

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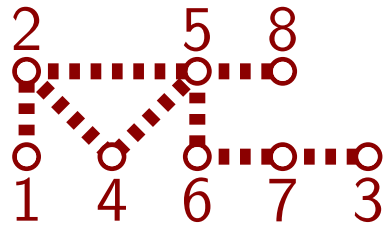

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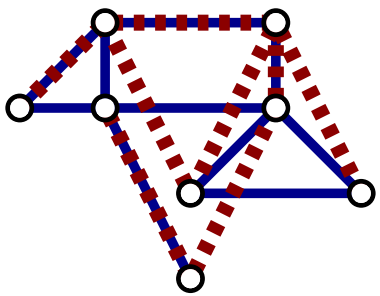

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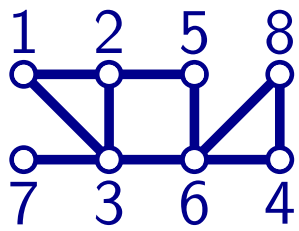

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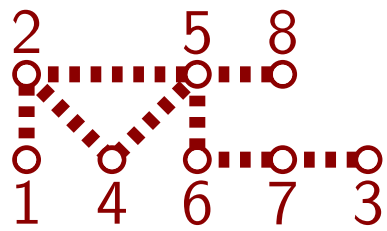

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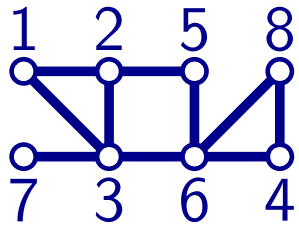

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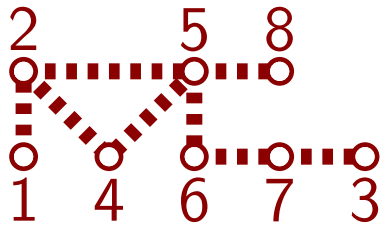

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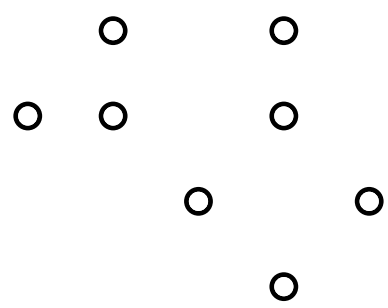

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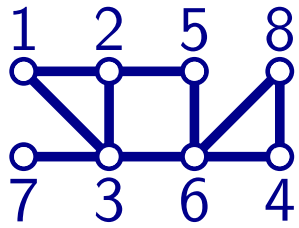

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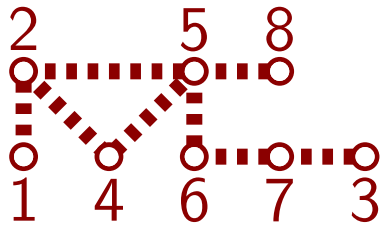

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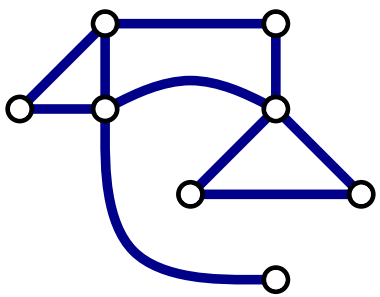

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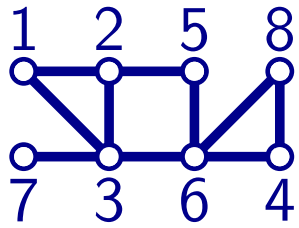

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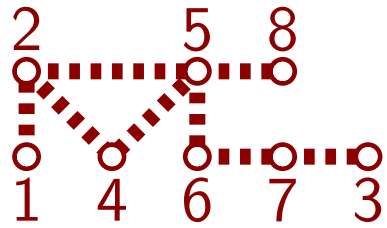

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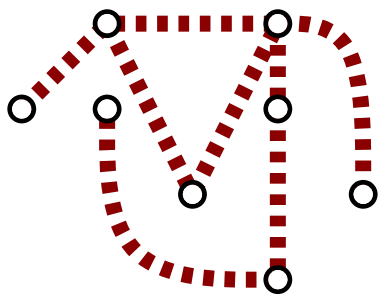

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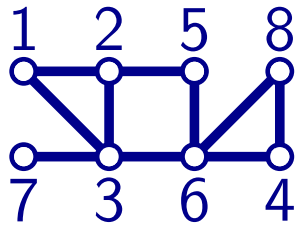

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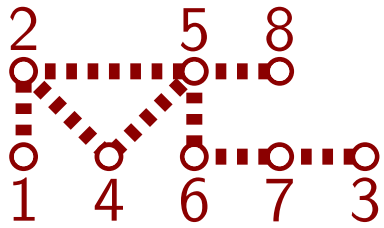

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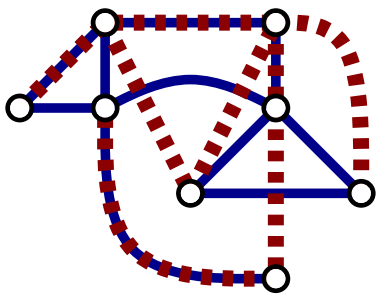

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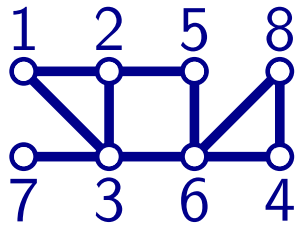

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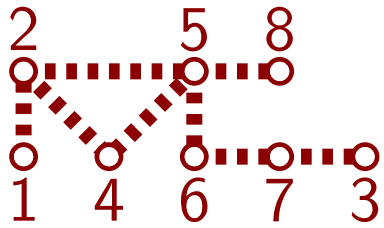

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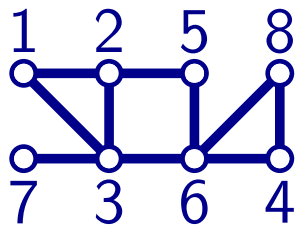

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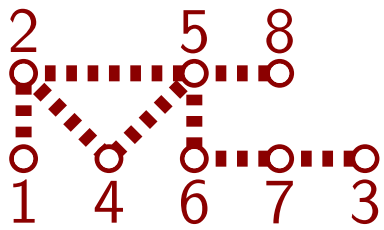

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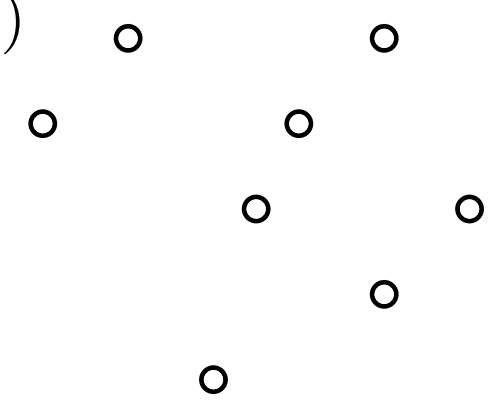

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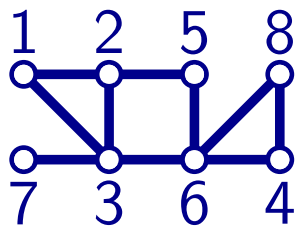

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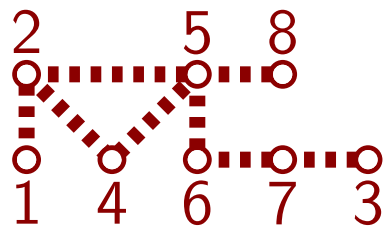

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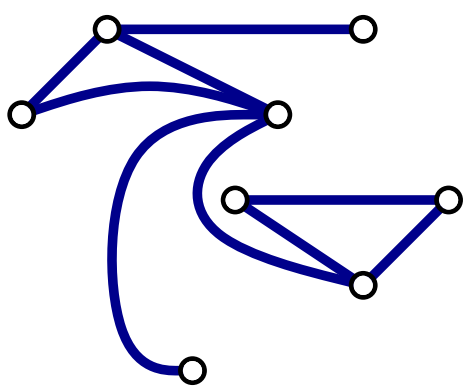

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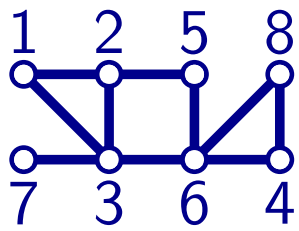

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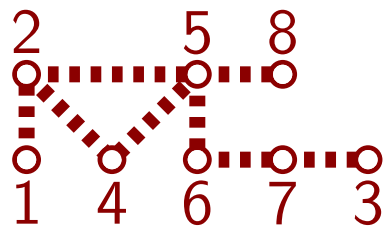

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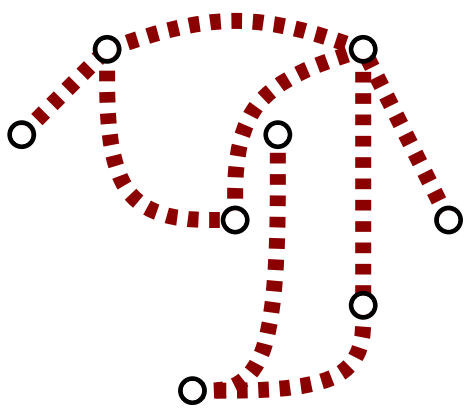

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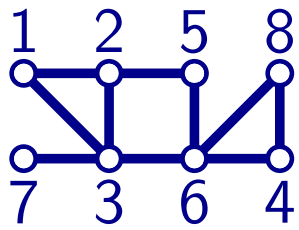

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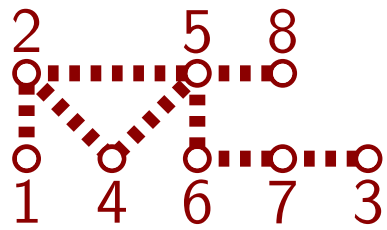

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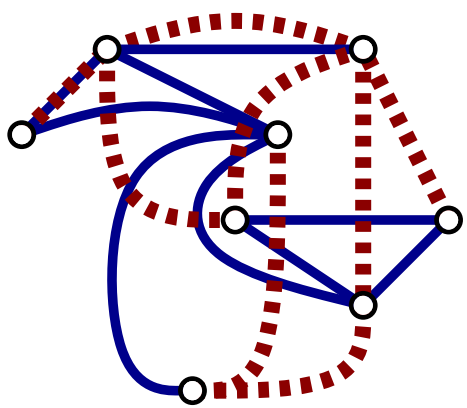

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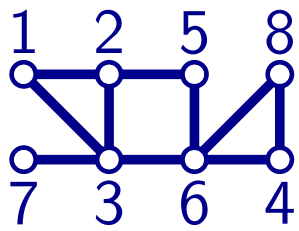
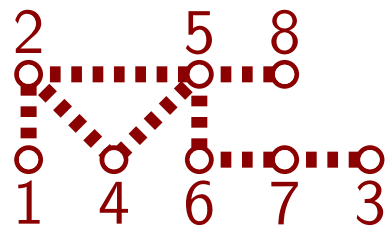
Given: Two planar graphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$
on same vertex set



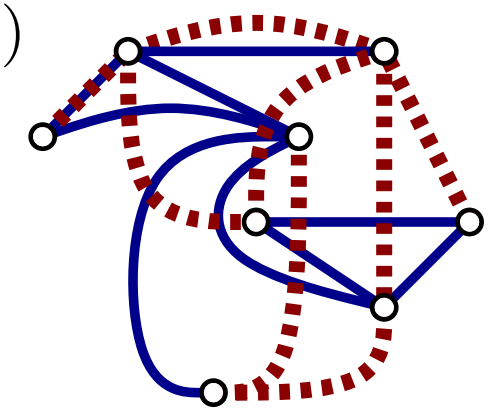
G_1



G_2

Embed both graphs in a planar way
– edges of one graph may intersect edges of the other graph

- Sim. Geometric Emb. (SGE)
- Sim. Emb. with Fixed Edges (SEFE)
- Sim. Emb. (SE)

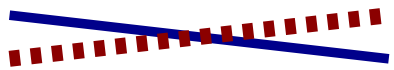


RacSim Drawings

RacSim: Simultaneous Embedding with Right-Angle Crossings

RacSim Drawings

RacSim: Simultaneous Embedding with Right-Angle Crossings



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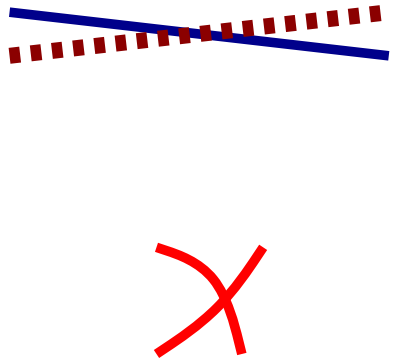

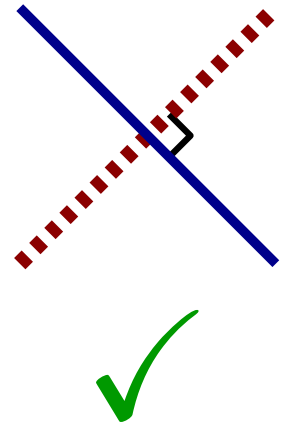

RacSim Drawings

RacSim: Simultaneous Embedding with Right-Angle Crossings

Geometric RacSim: RacSim with straight-line edges

Known results

[Angelini et al. '12]
There exist a tree and a path
which don't admit an SGE drawing

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Any pair of trees admits an SE drawing
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} low angle

Known results

[Argyrious et al. '13]
A cycle and a matching always admit a geometric RacSim drawing in quadratic area

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} low angle

Known results

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There exist a wheel and a cycle which don't admit a geometric RacSim drawing

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[Kammer '06]
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} low angle

Our results

Graph classes			Number of bends
Cycle	$\times$	Cycle	1×1
Caterpillar	$\times$	Cycle	1×1
Four Matchings			$1 \times 1 \times 1 \times 1$
Tree	$\times$	Matching	1×0
Wheel	$\times$	Matching	2×0
Outerpath	$\times$	Matching	2×1
Outerplanar	$\times$	Outerplanar	3×3
2-page book emb.	$\times$	2-page book emb.	4×4
Planar	$\times$	Planar	6×6

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Path $\times$ Path

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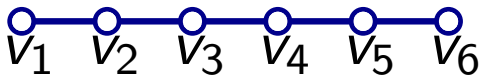
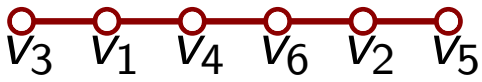
Path $\times$ Path

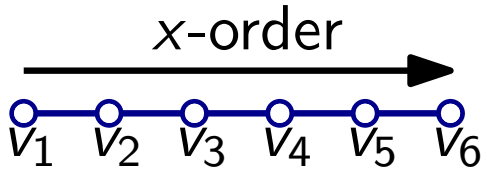
Path $\times$ Path

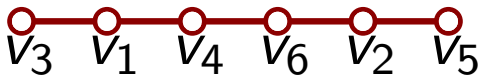


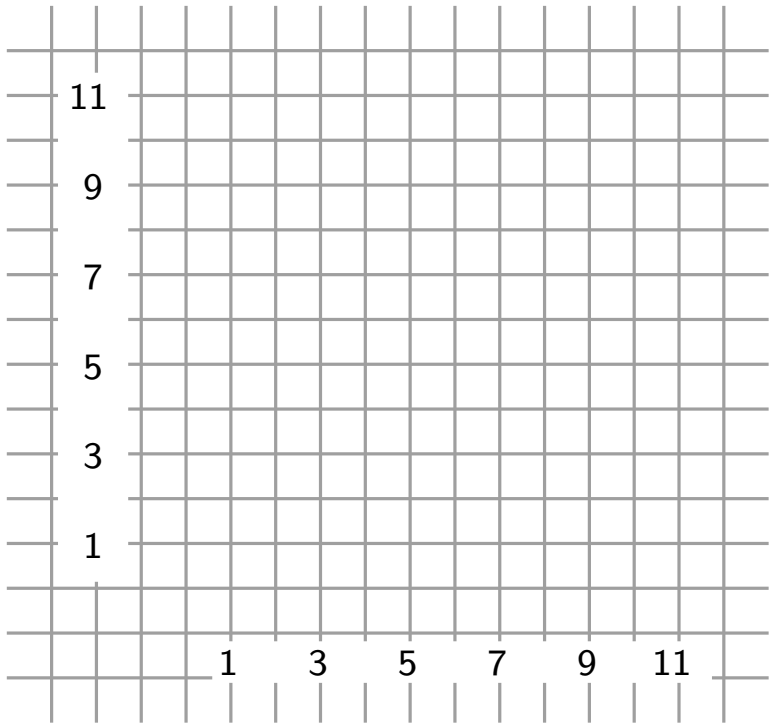


Path $\times$ Path

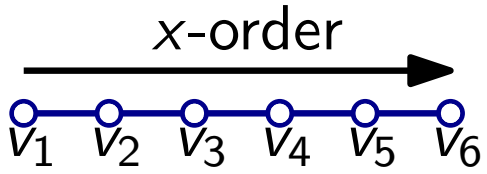
Path $\times$ Path







Path × Path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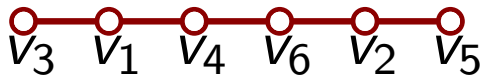
v_1 v_2 v_3 v_4 v_5 v_6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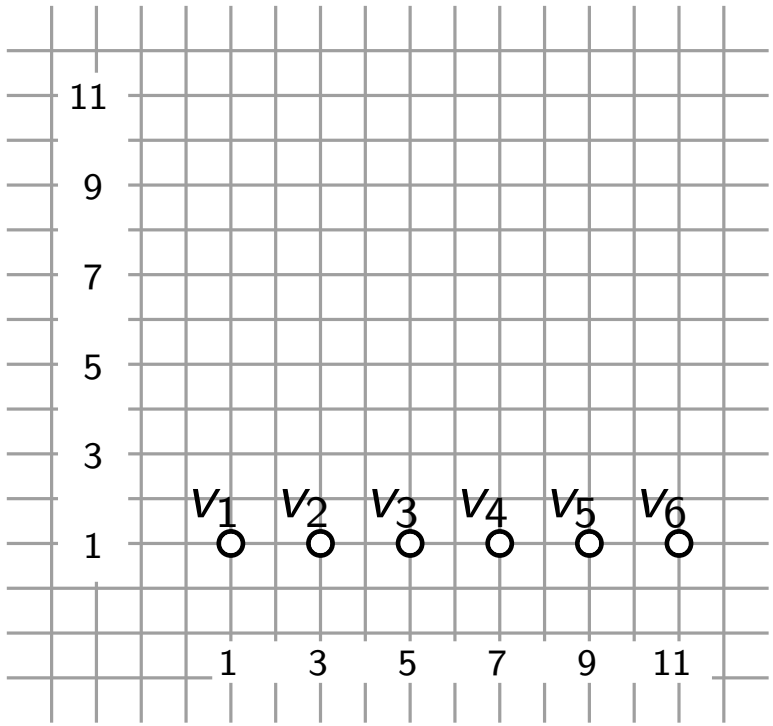
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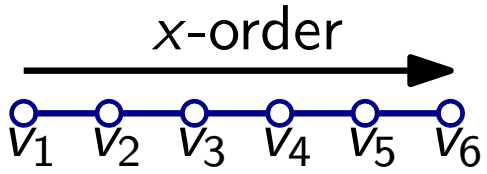
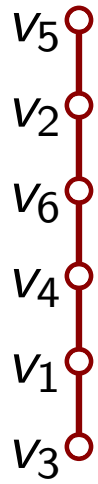
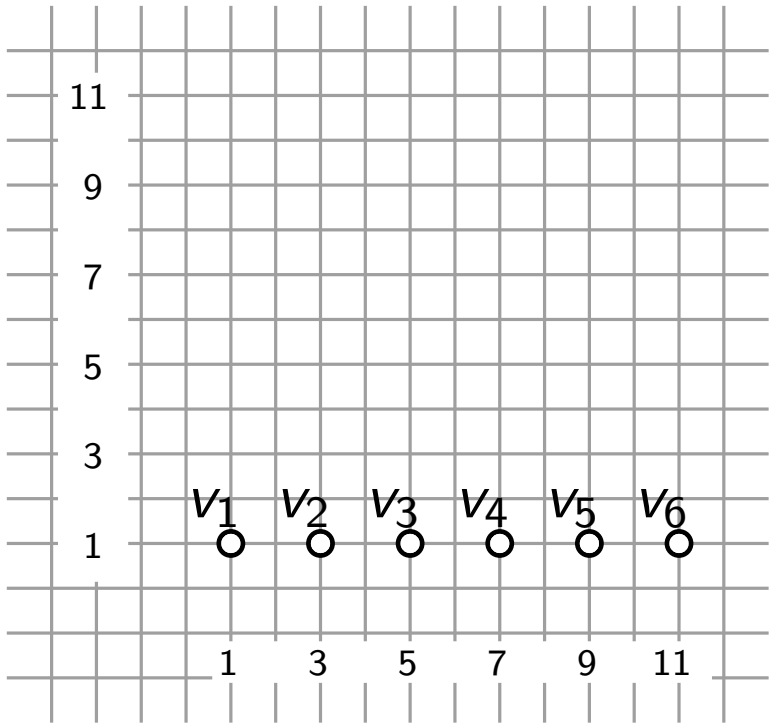
v_1 v_2 v_3 v_4 v_5 v_6
1 3 5 7 9 11

Path × Path


x-order
v1 v2 v3 v4 v5 v6


v5
v2
v6
v4
v1
v3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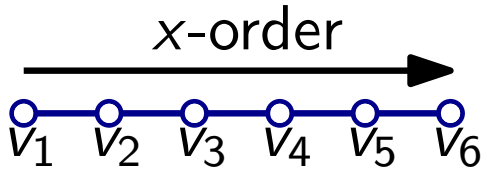
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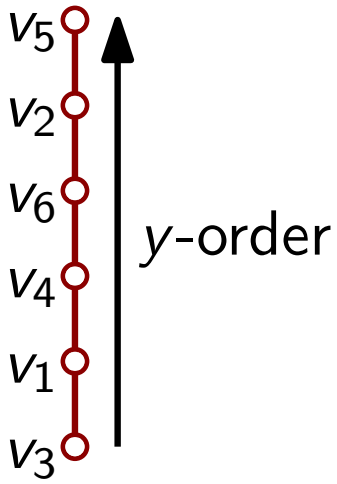
v5
v2
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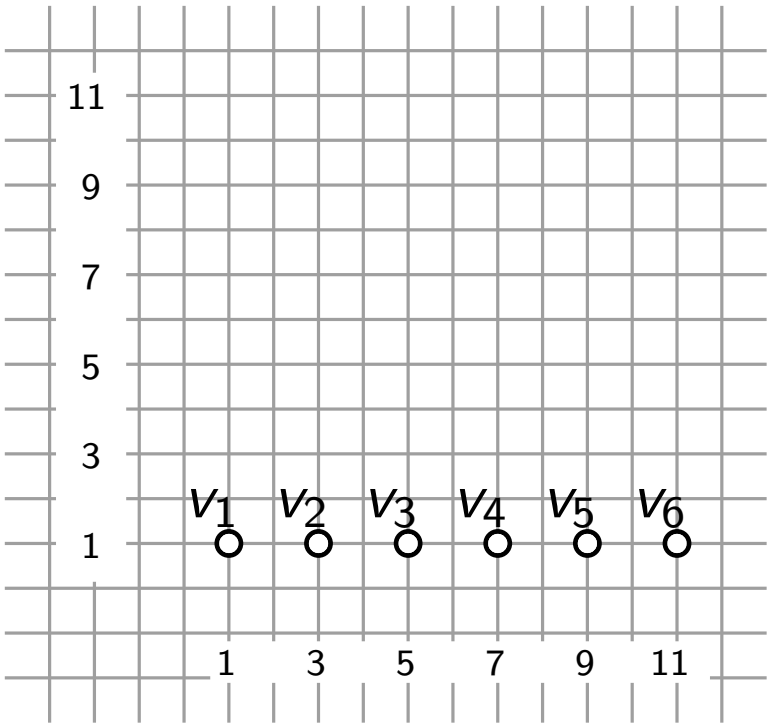
v4
v1
v3
y-order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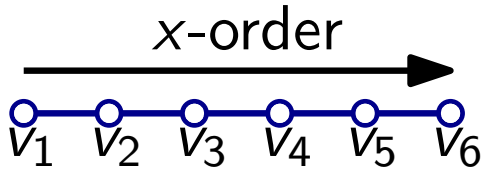
v1 v2 v3 v4 v5 v6
1 3 5 7 9 11

Path × Path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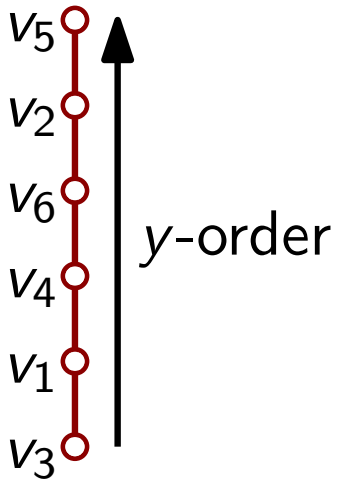
v5
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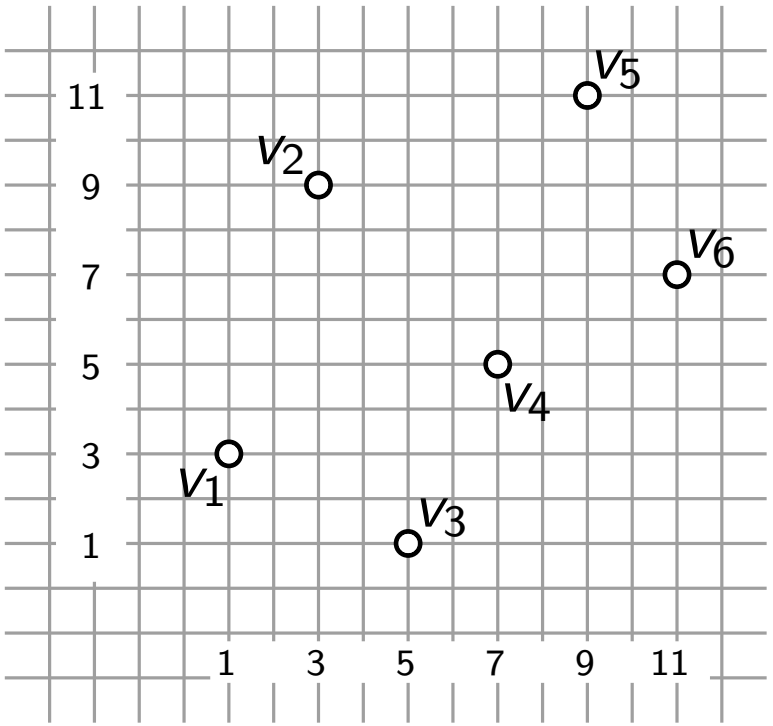
y-order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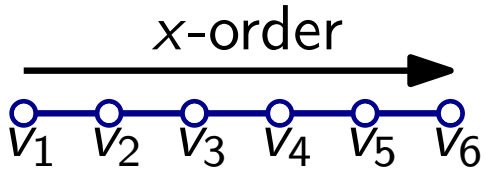
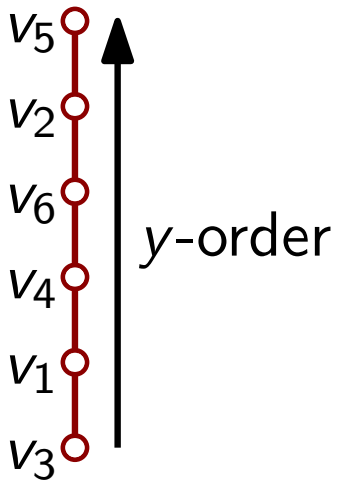
Path × Path





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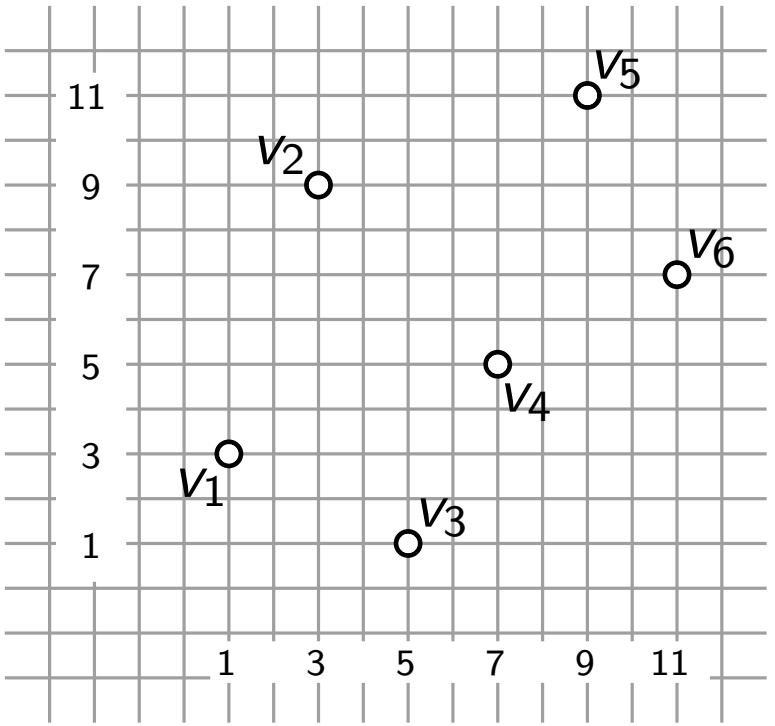

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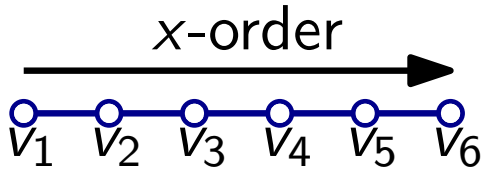

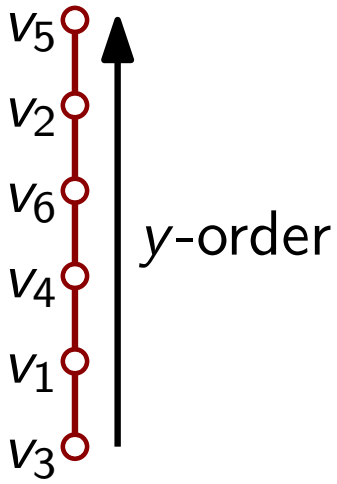

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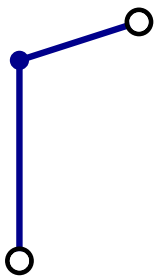

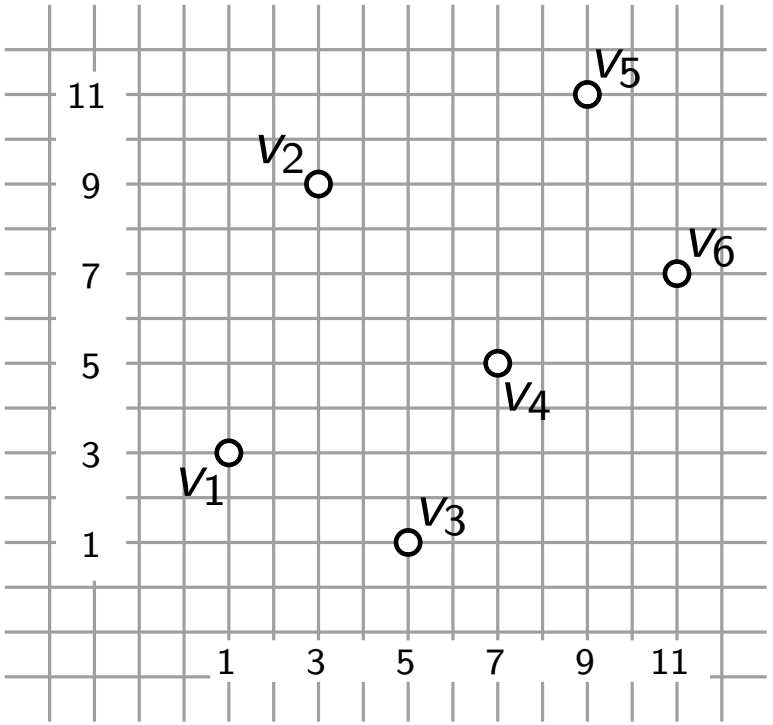

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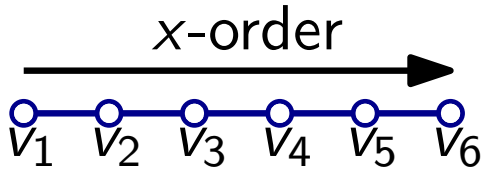

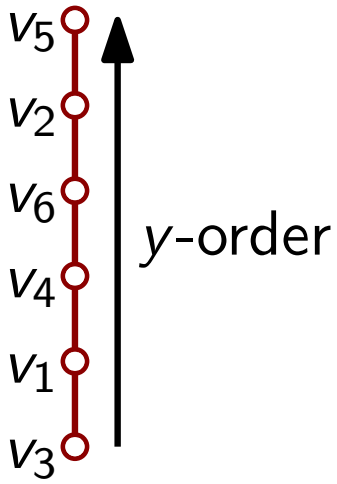

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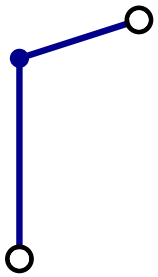

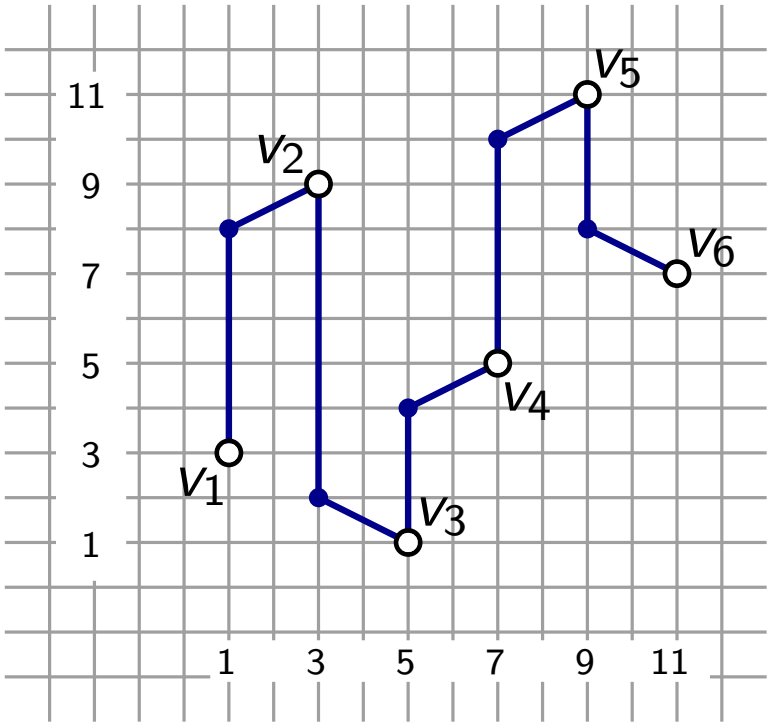

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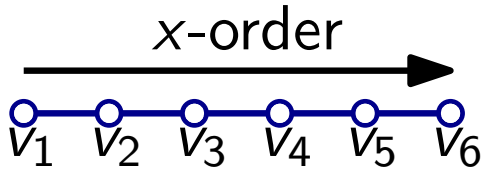

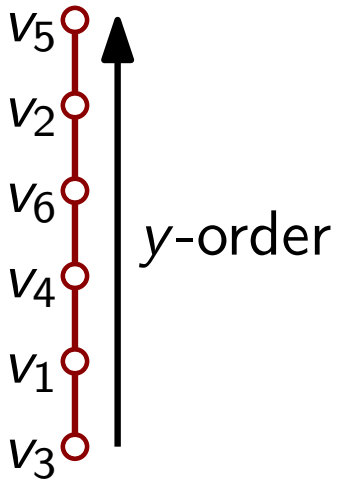

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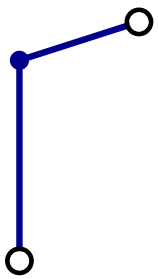
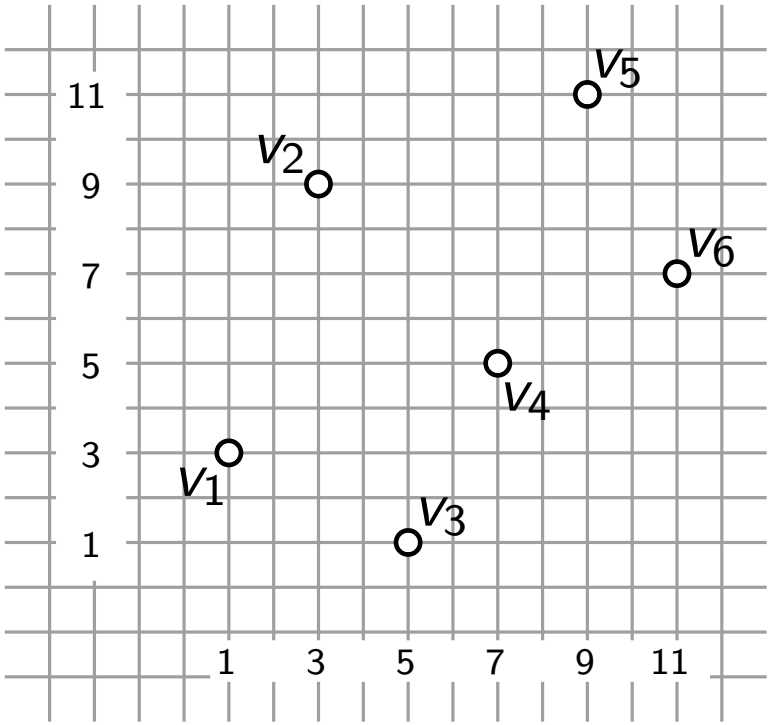
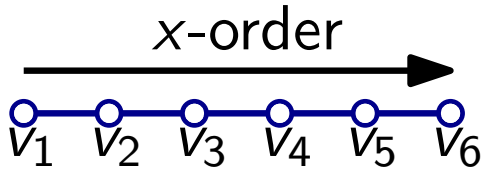
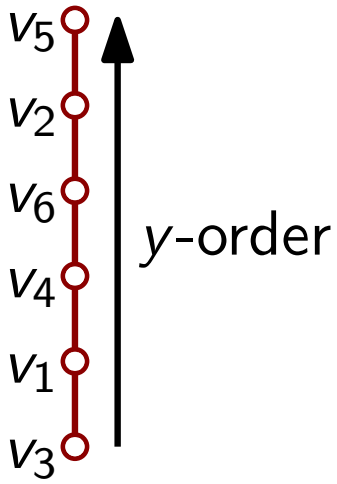




Path × Path





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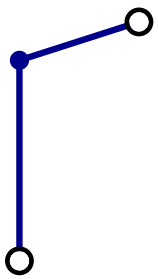
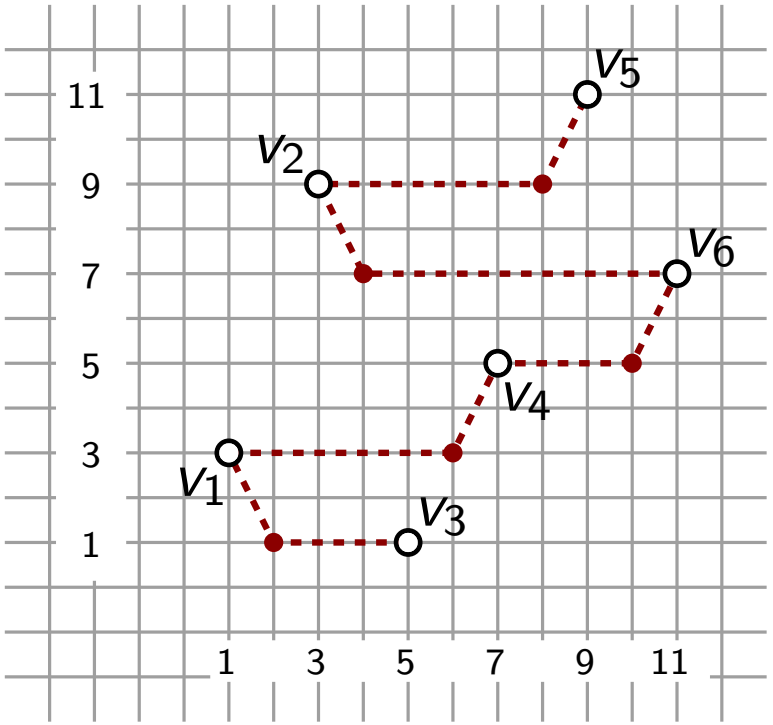
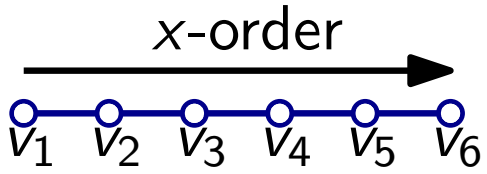
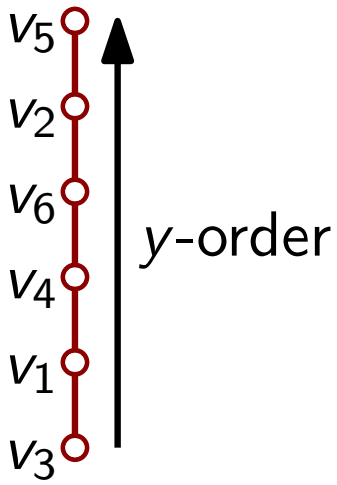




Path × Path





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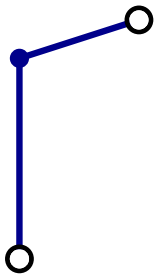
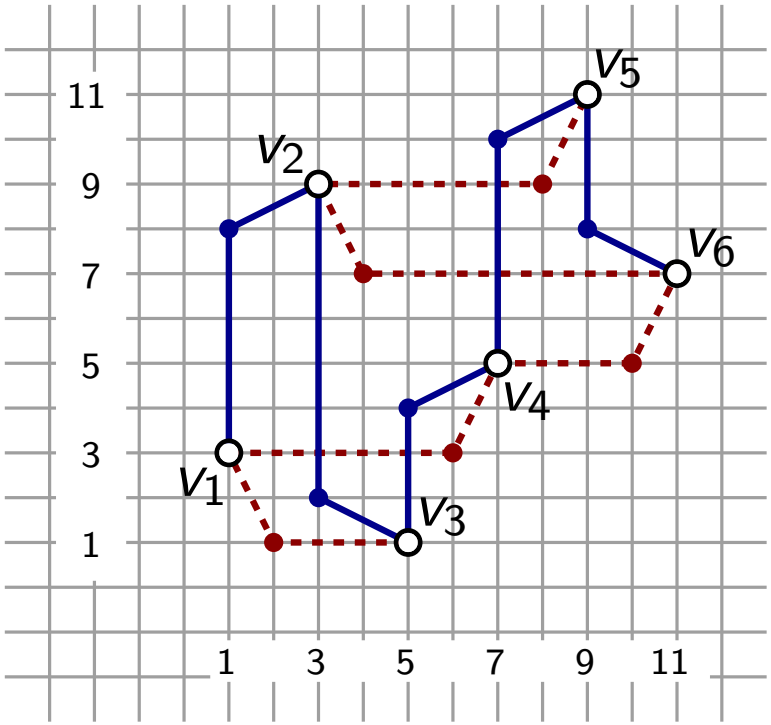
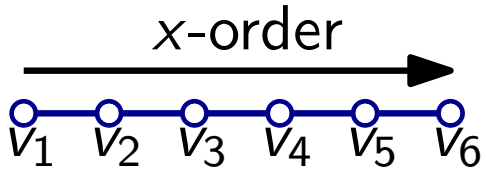
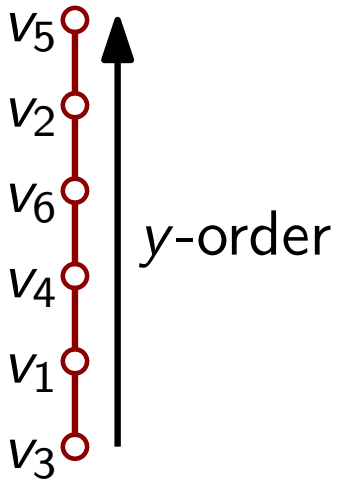




Cycle × Cycle





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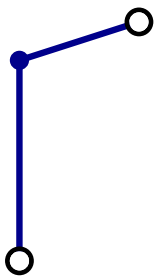
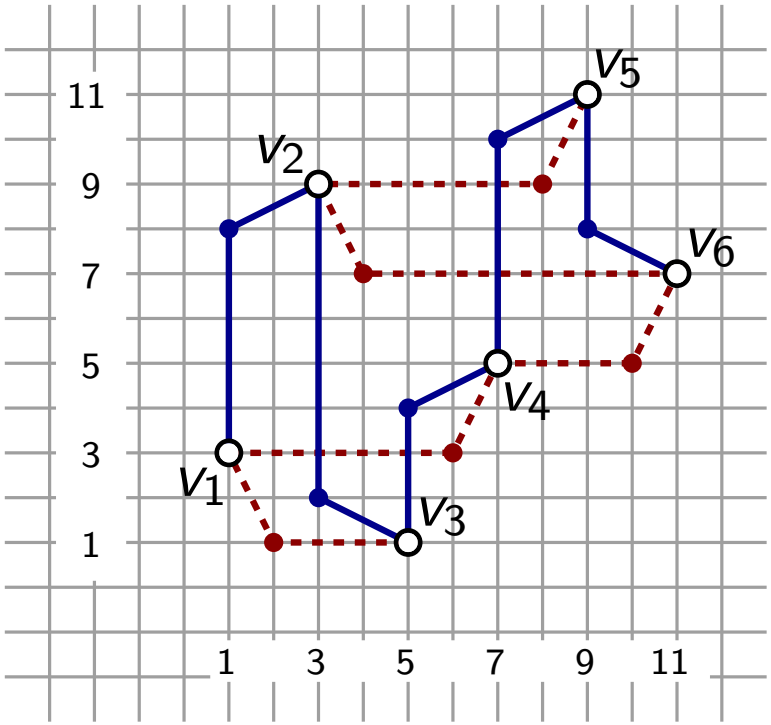
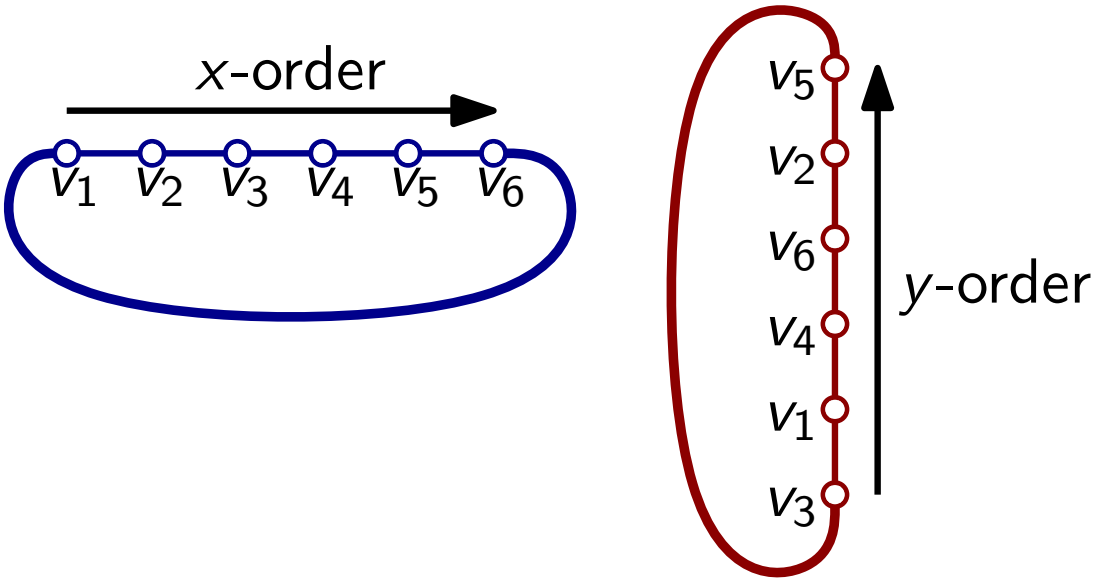




Cycle × Cycle



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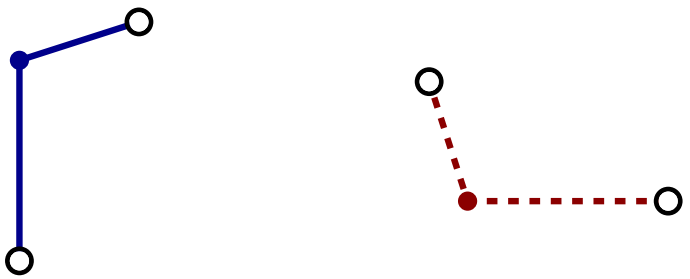

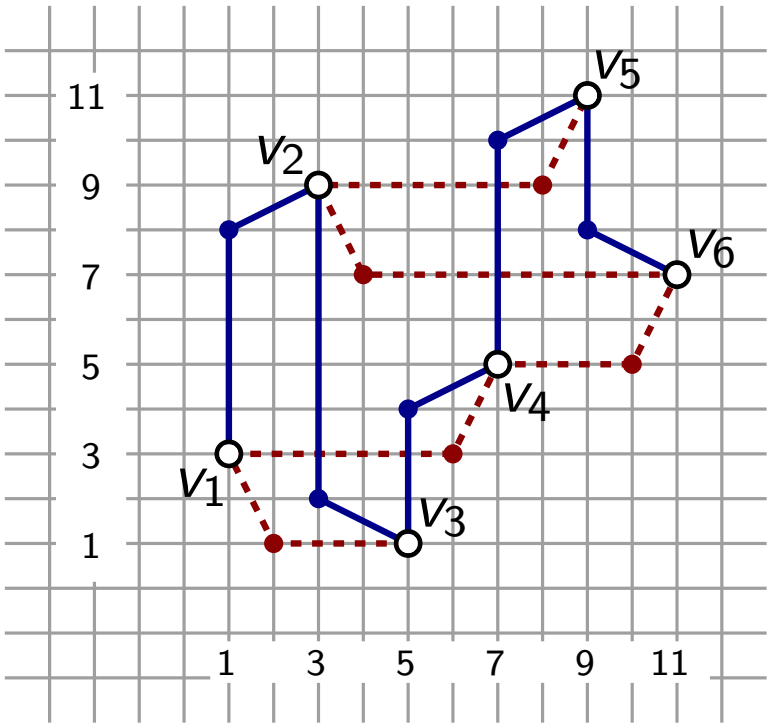

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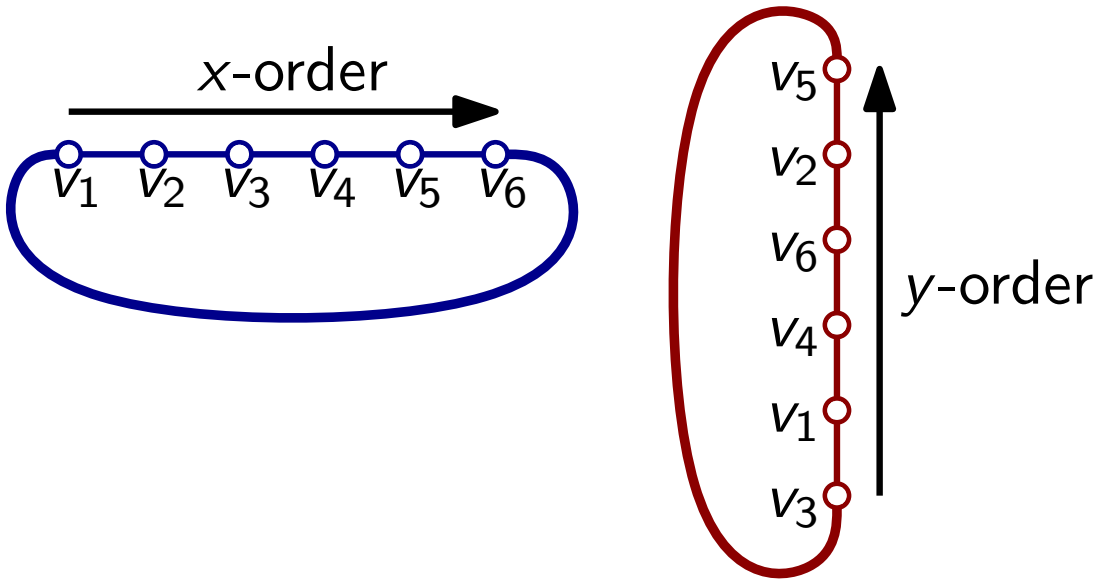

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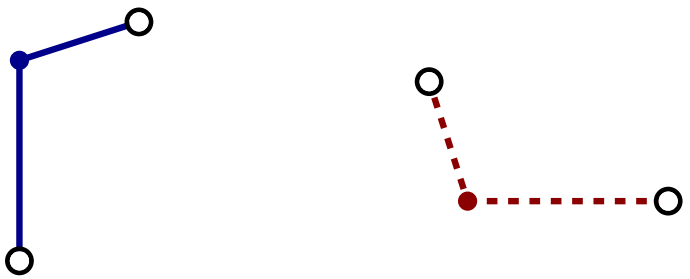

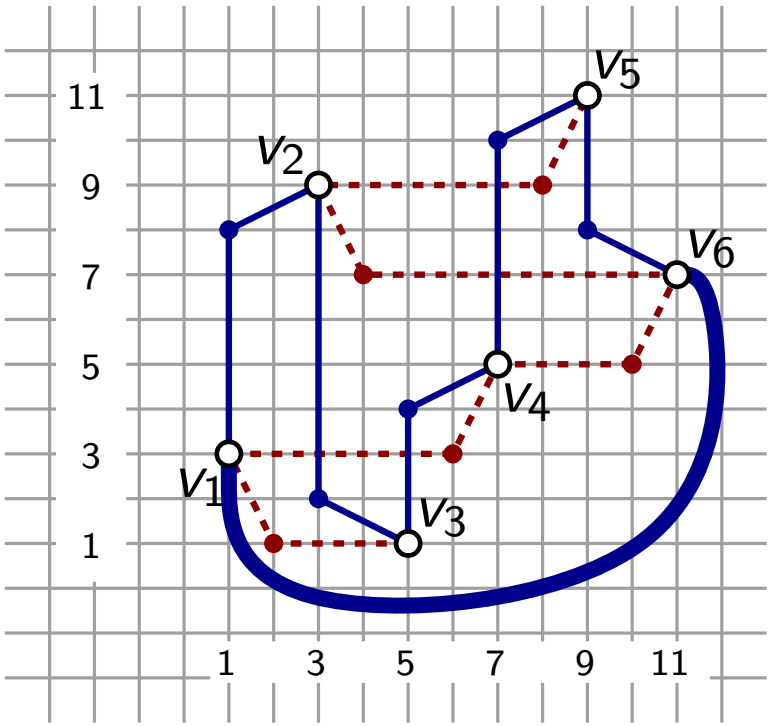

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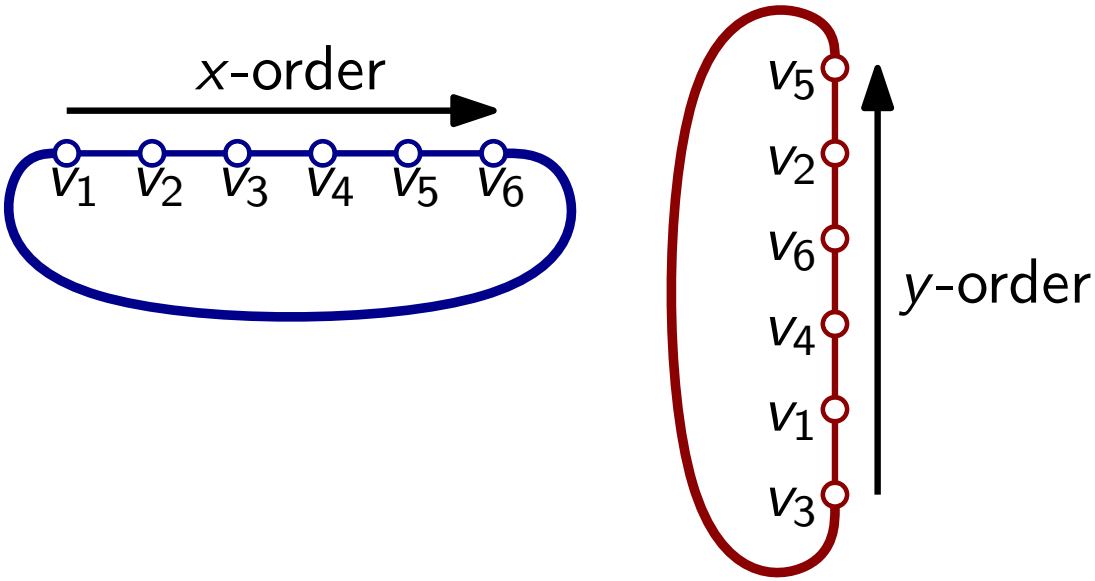

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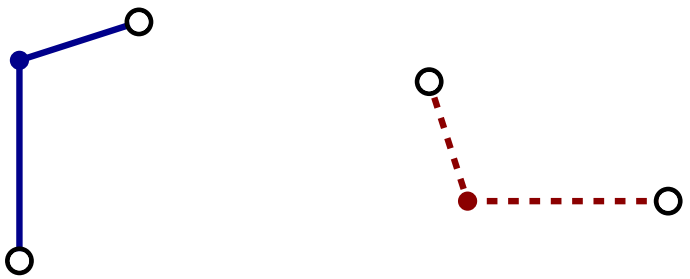

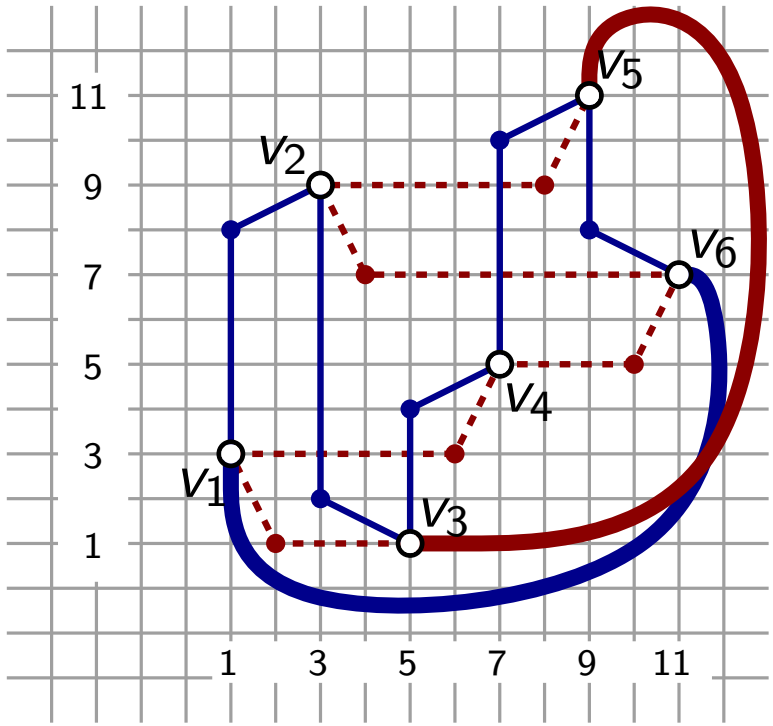

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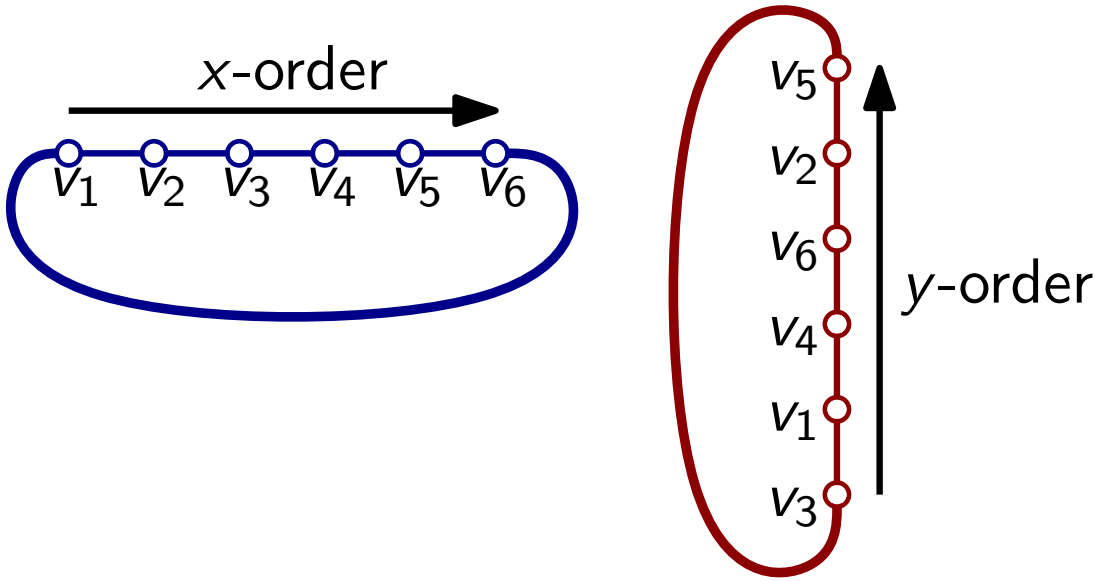

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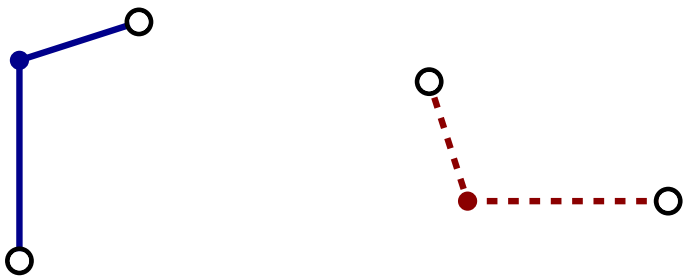

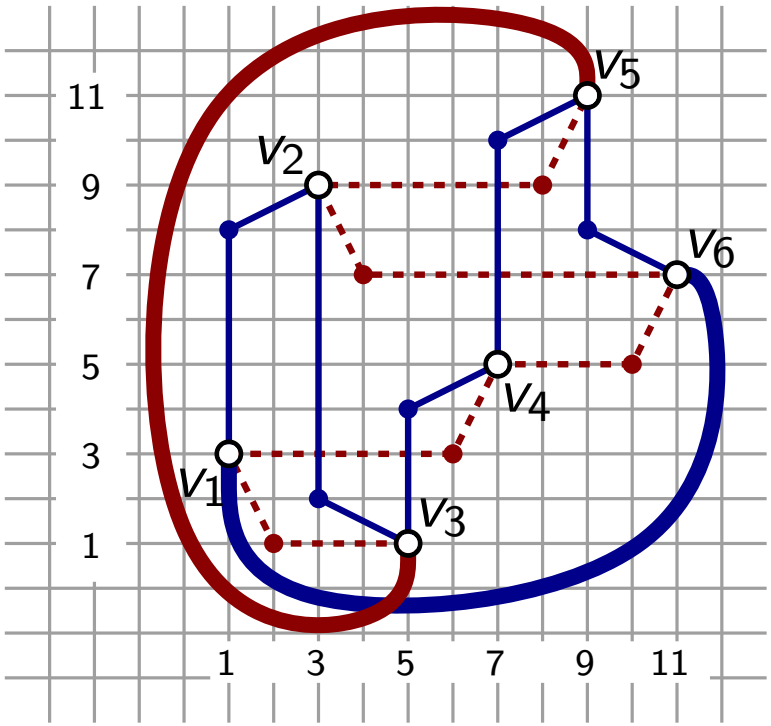

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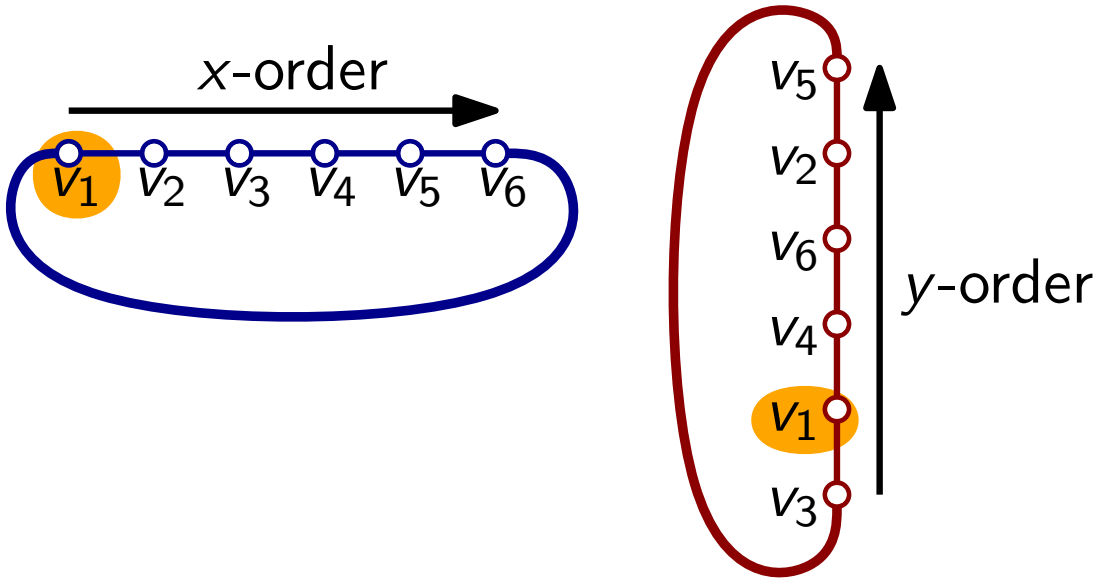

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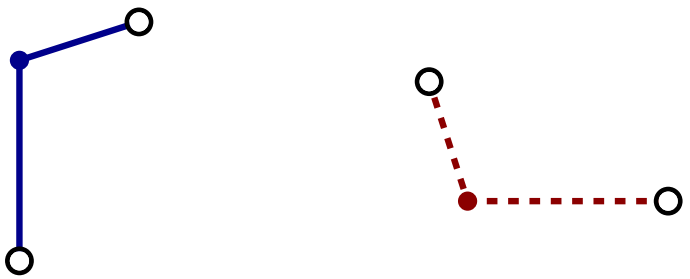

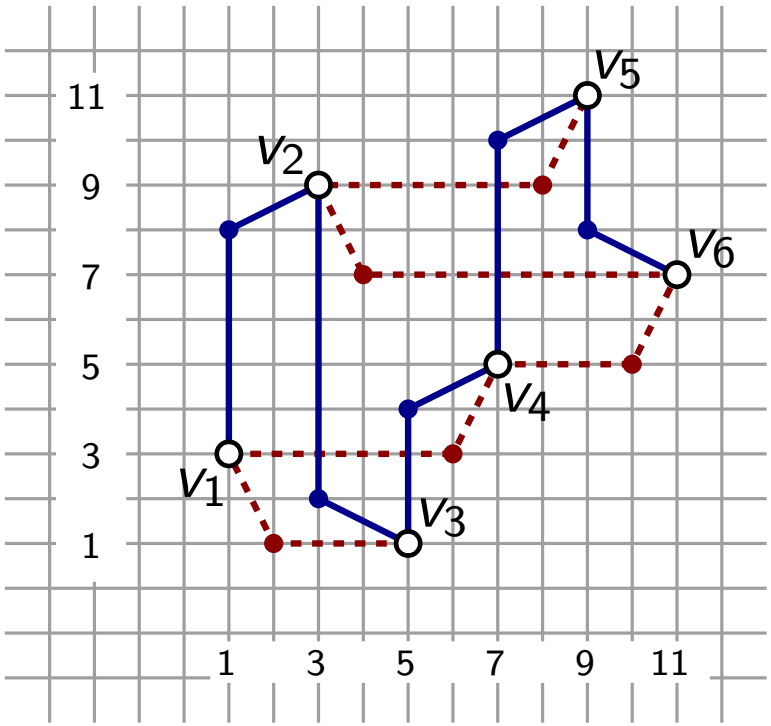

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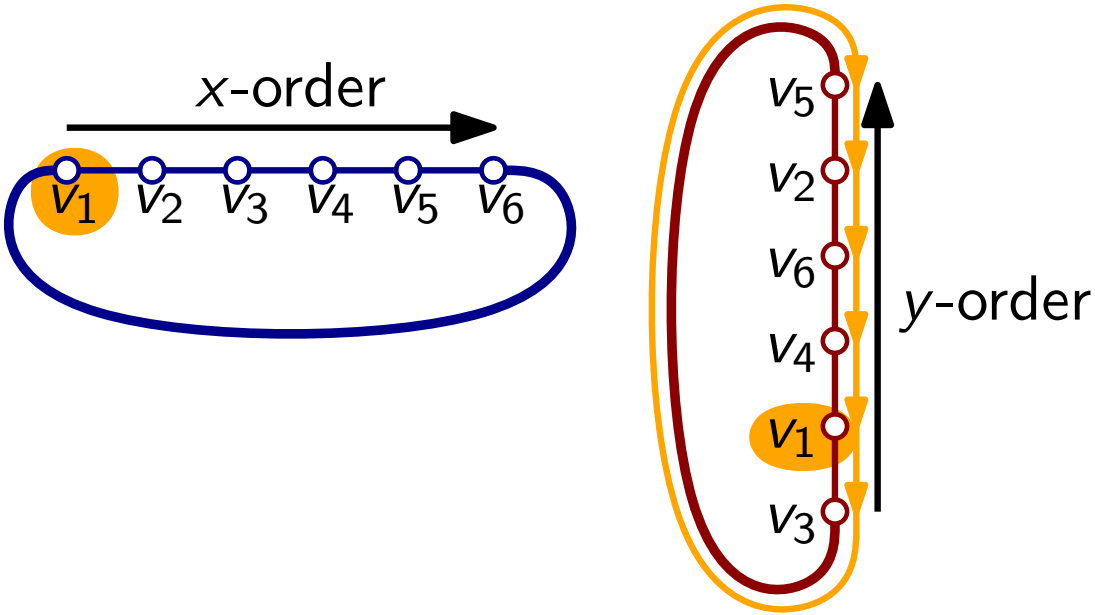

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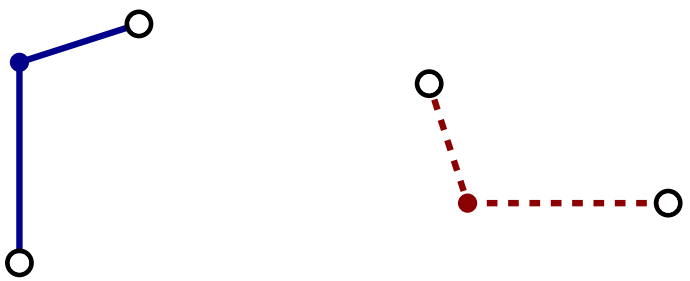

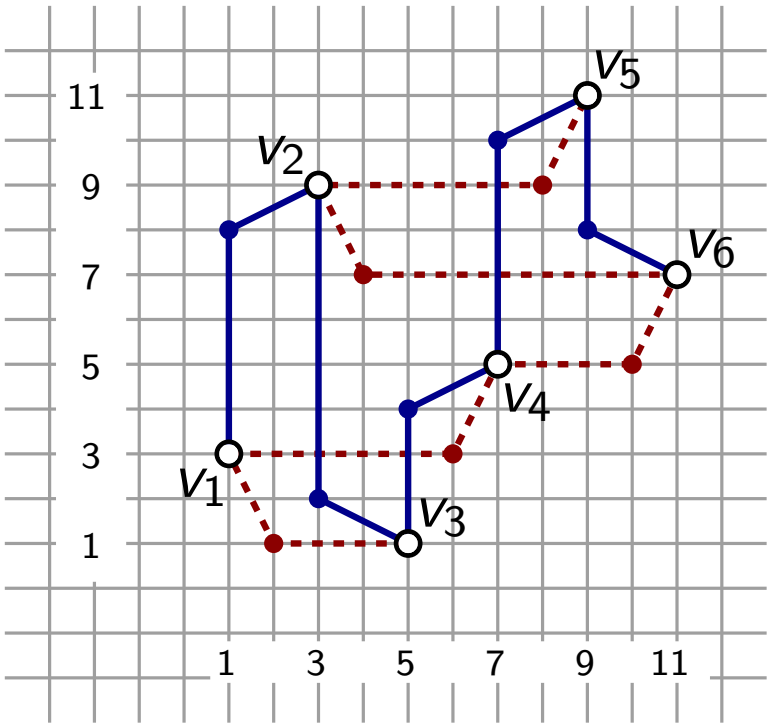

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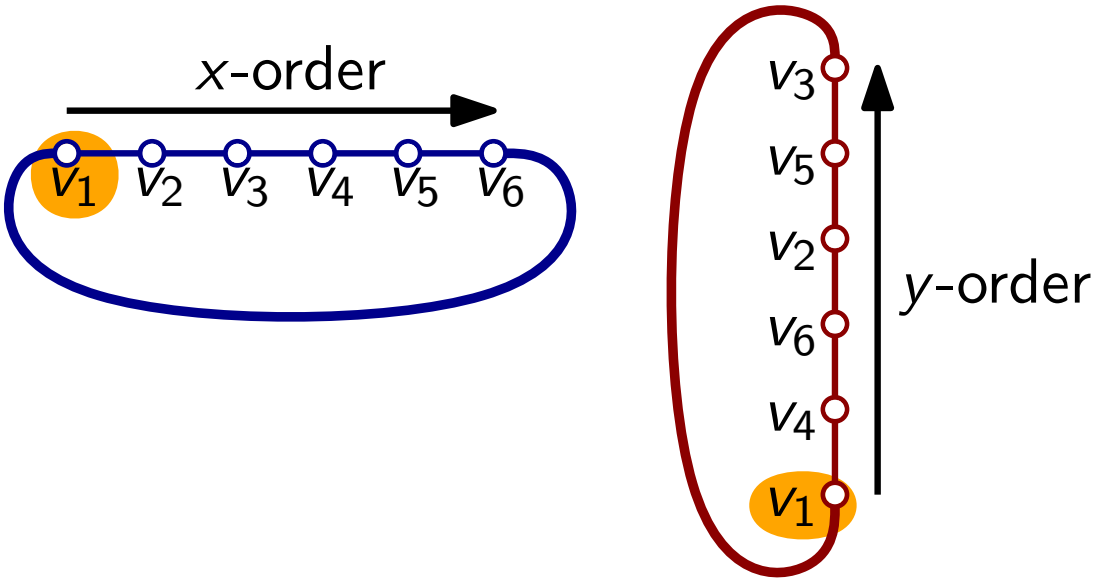

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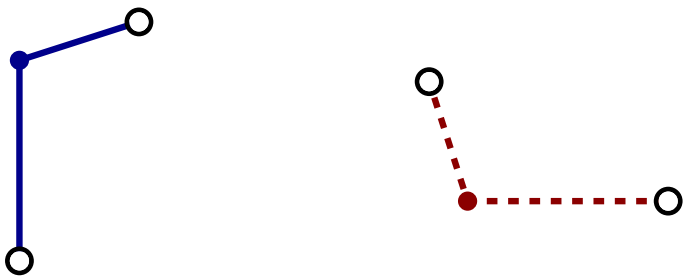

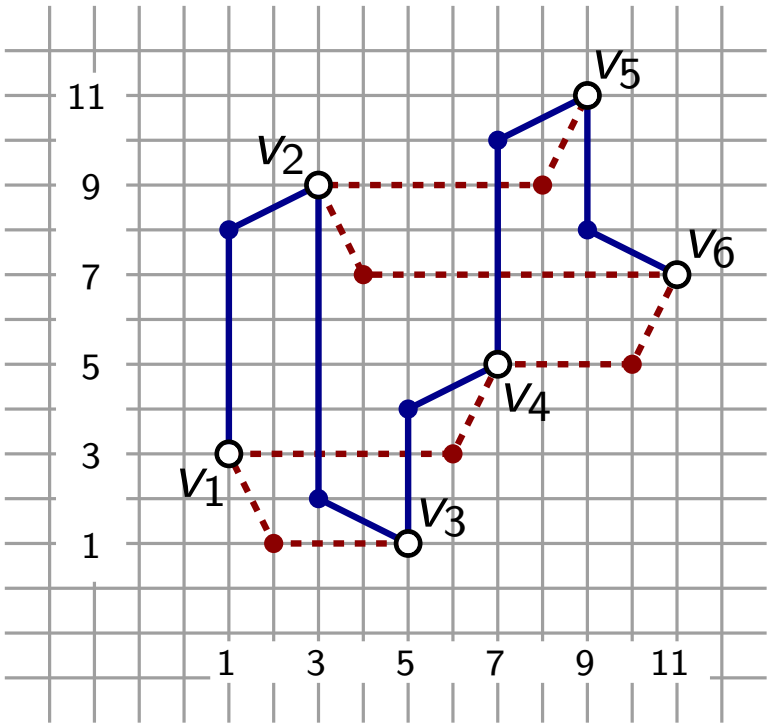

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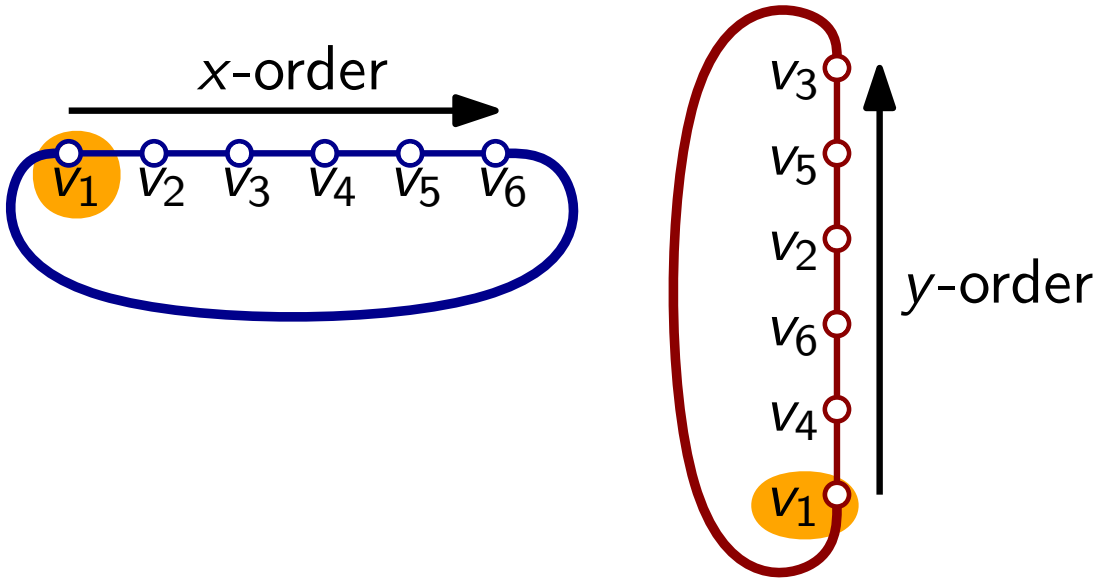

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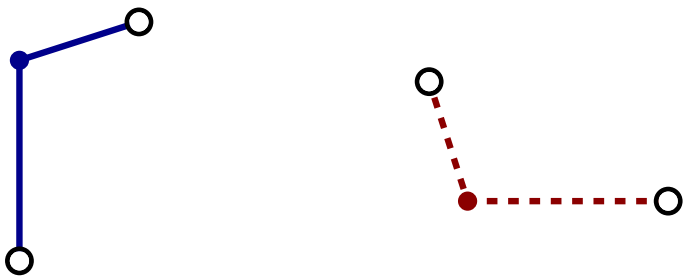

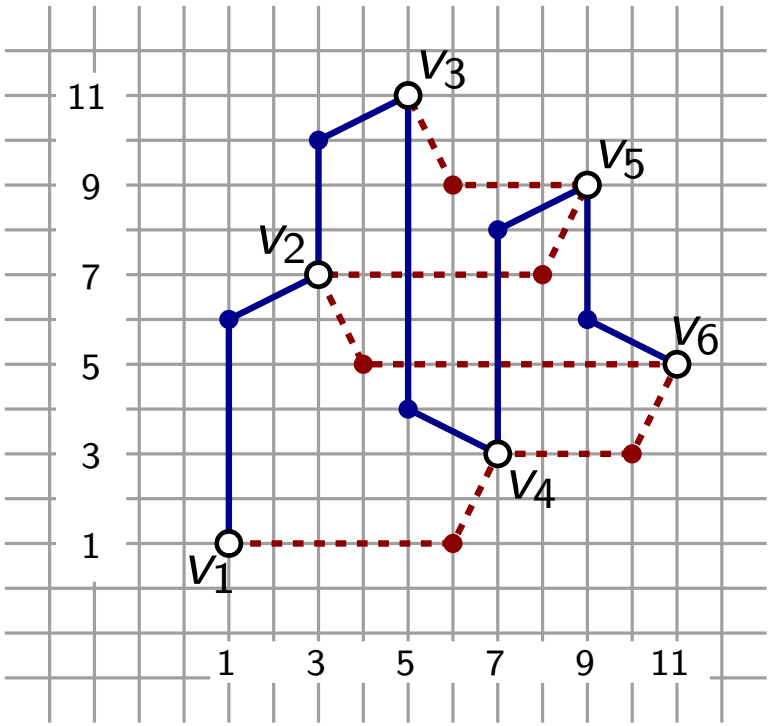

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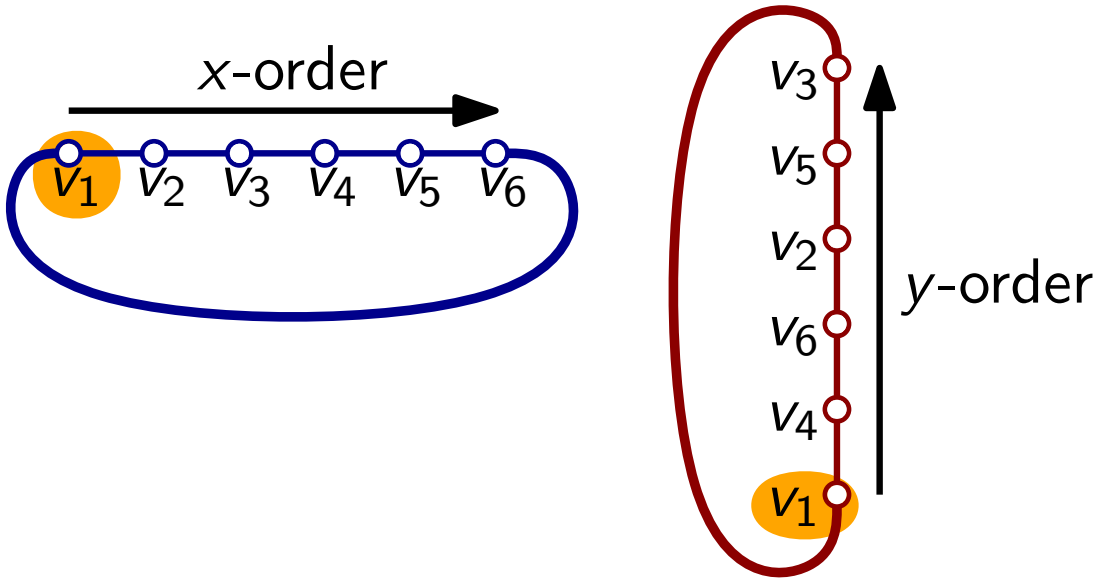

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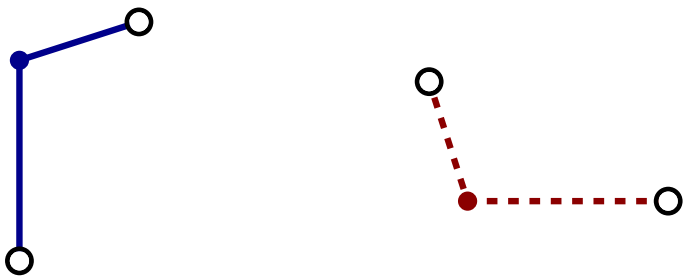

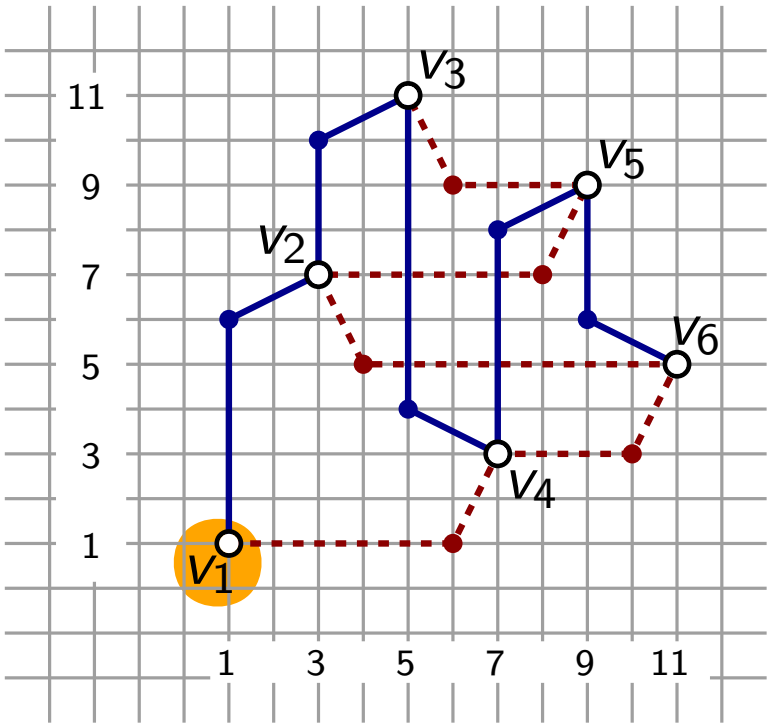

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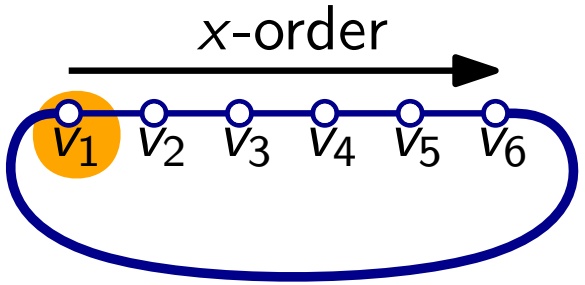

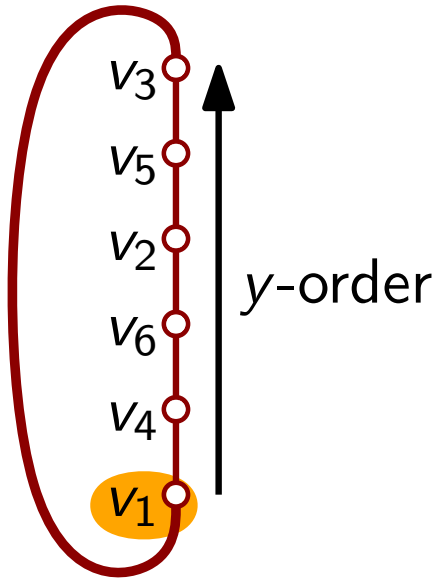

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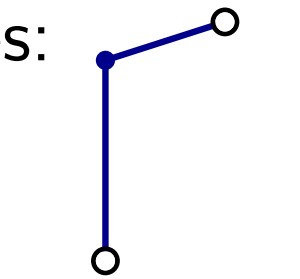

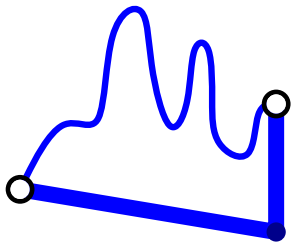

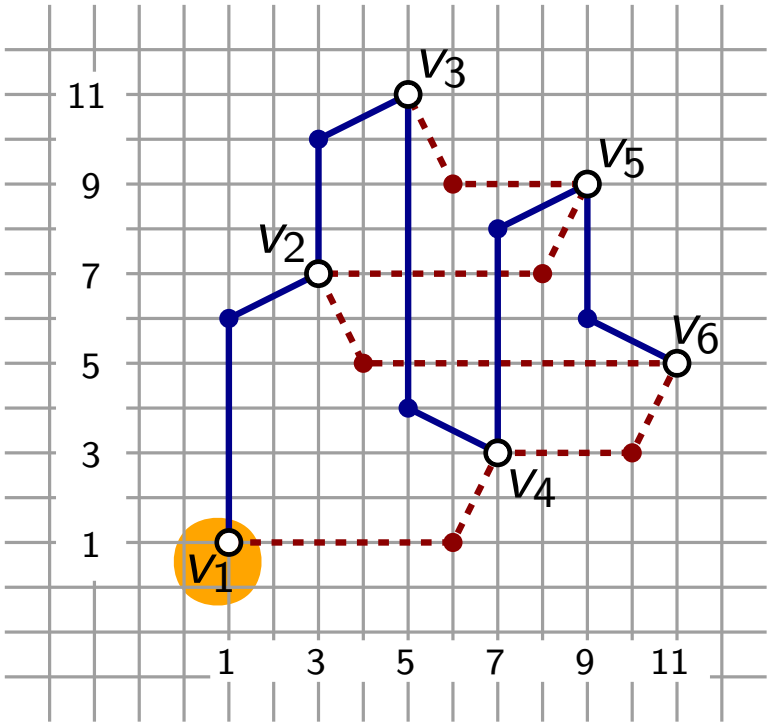

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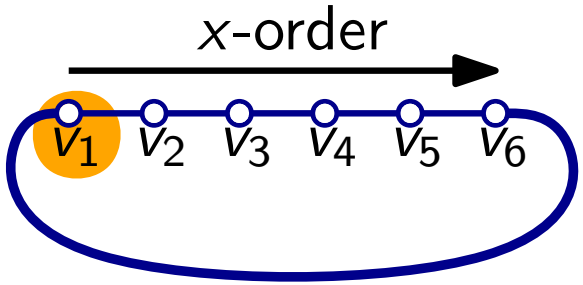

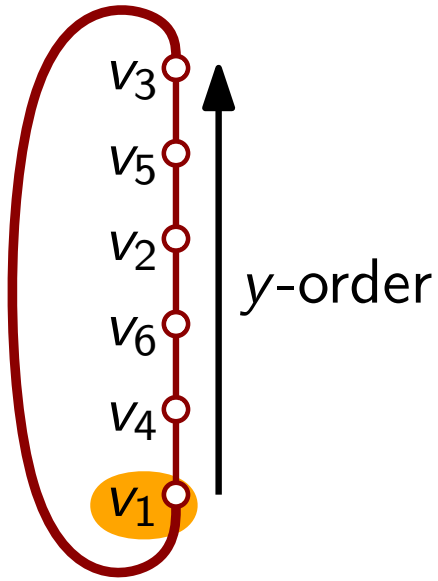

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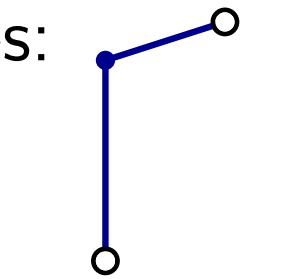

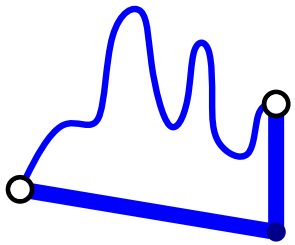

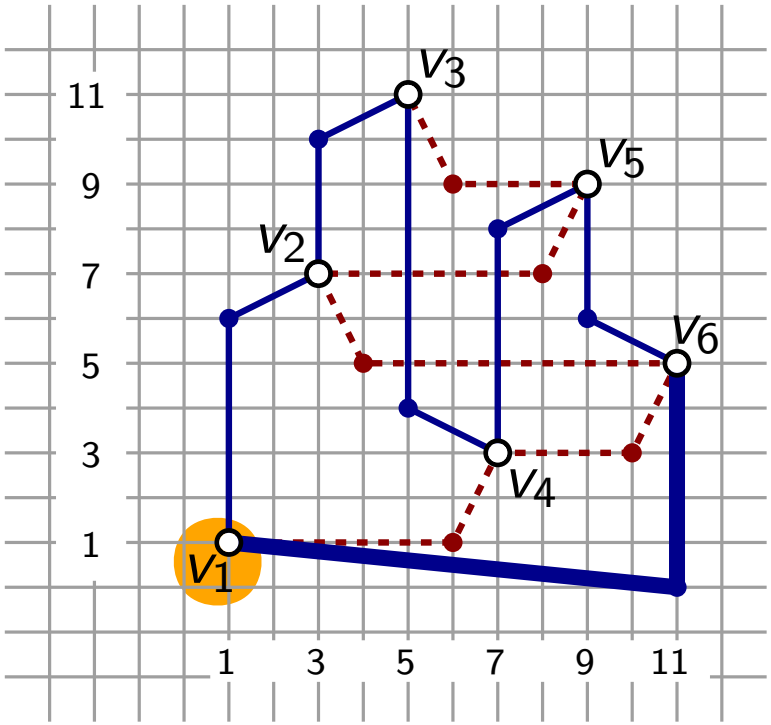

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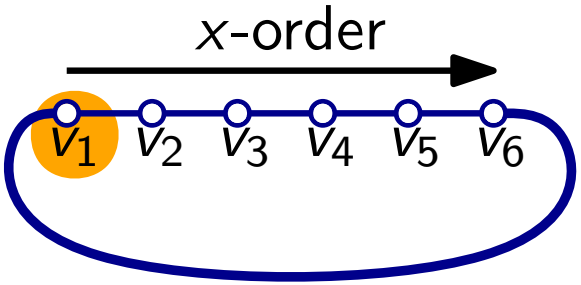

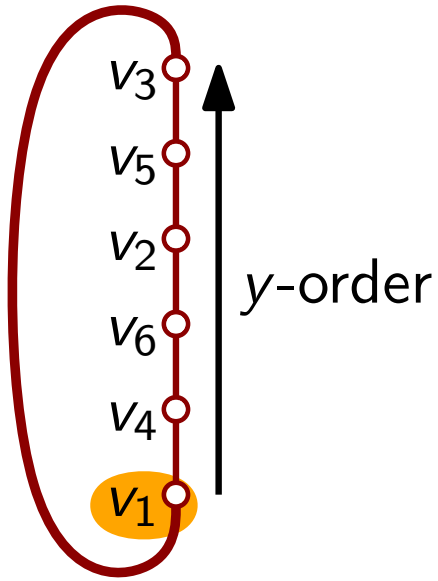

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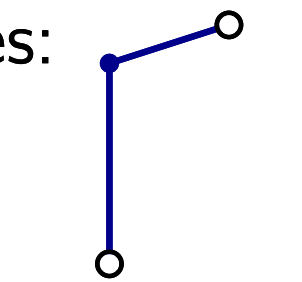

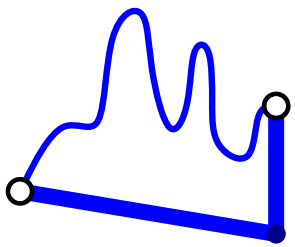

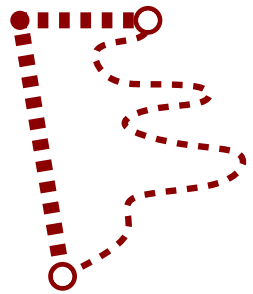

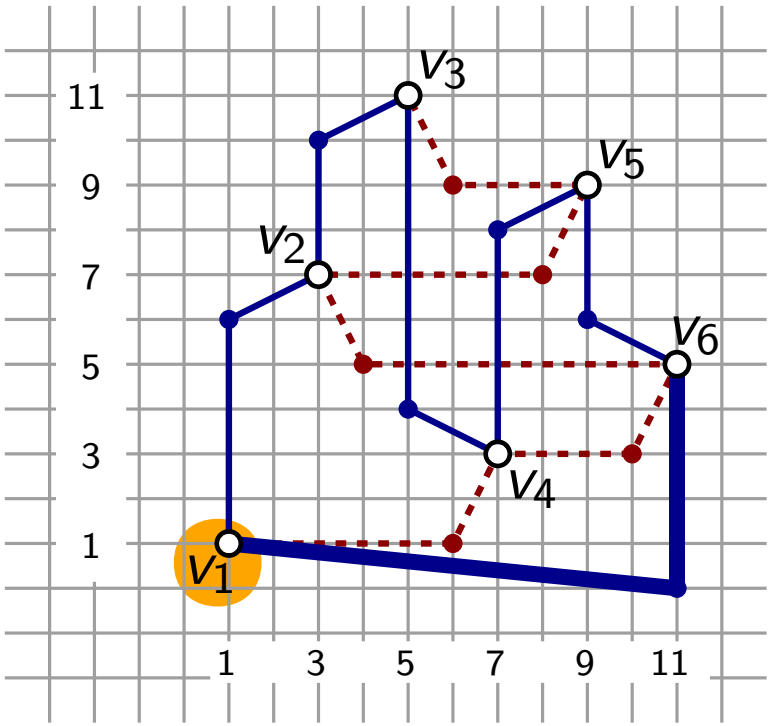

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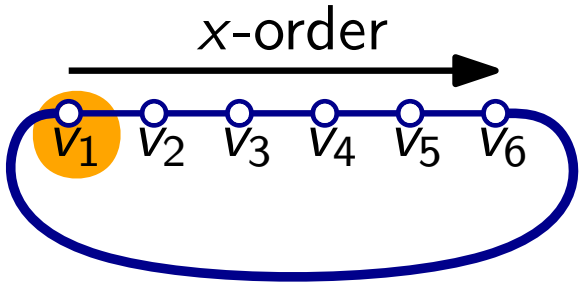

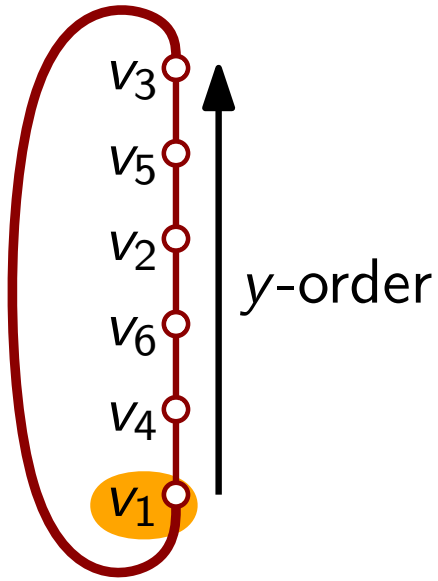

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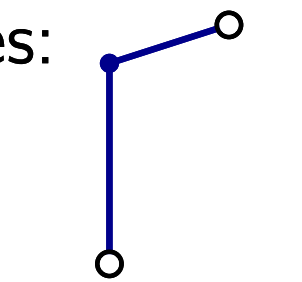

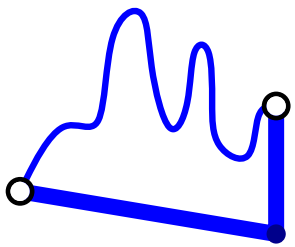

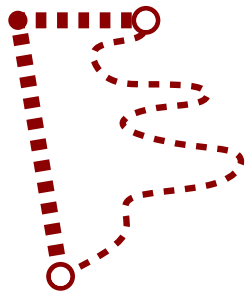

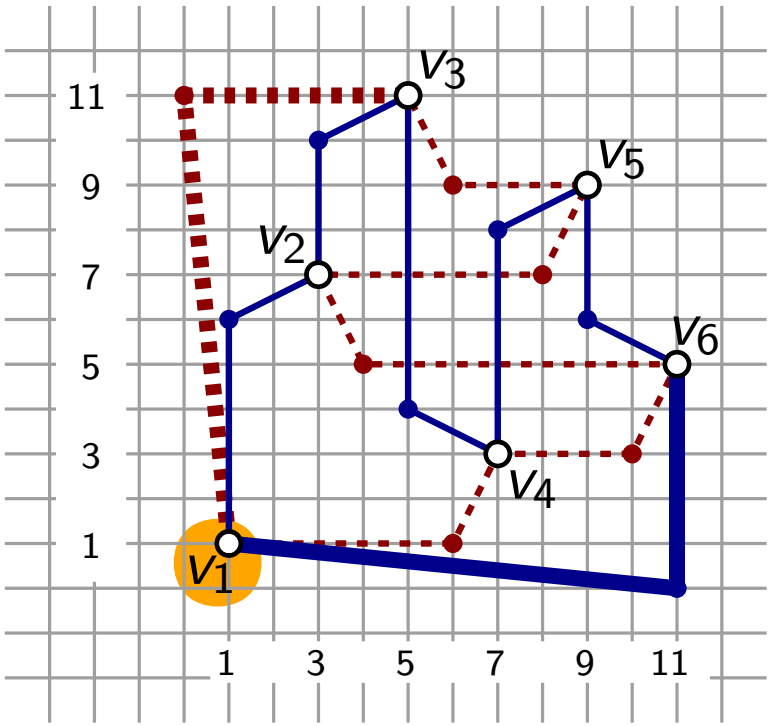

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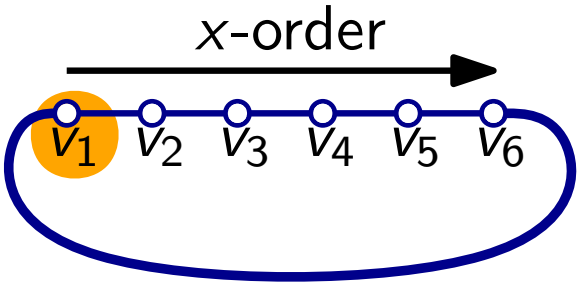
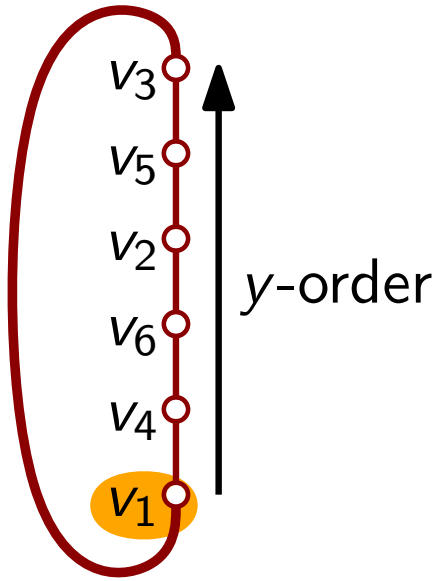
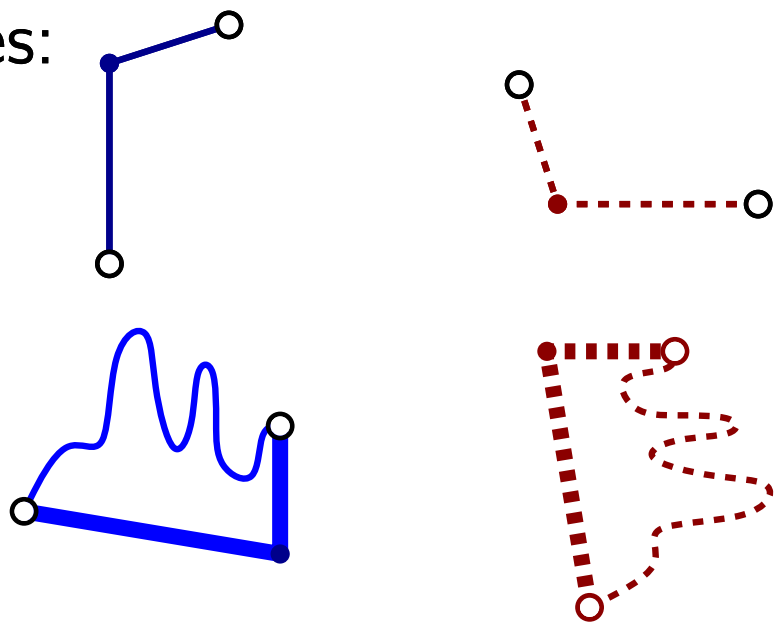
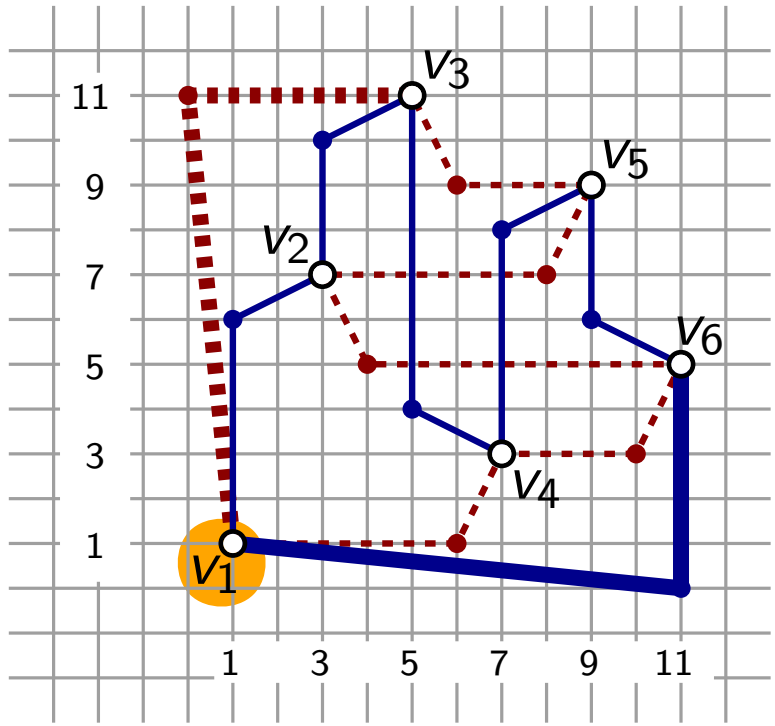




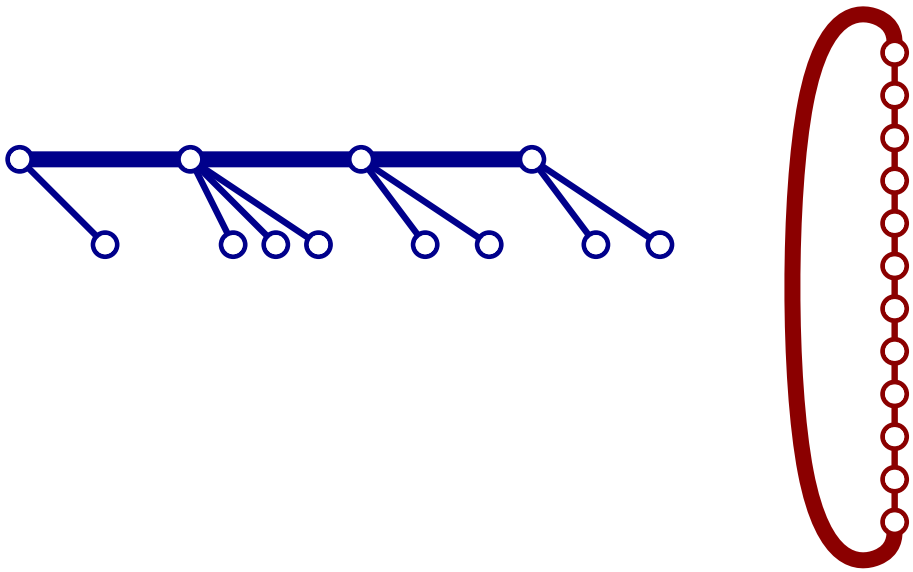
Bends: 1×1
Grid size:
$(2n-1) \times (2n-1)$

Edges:

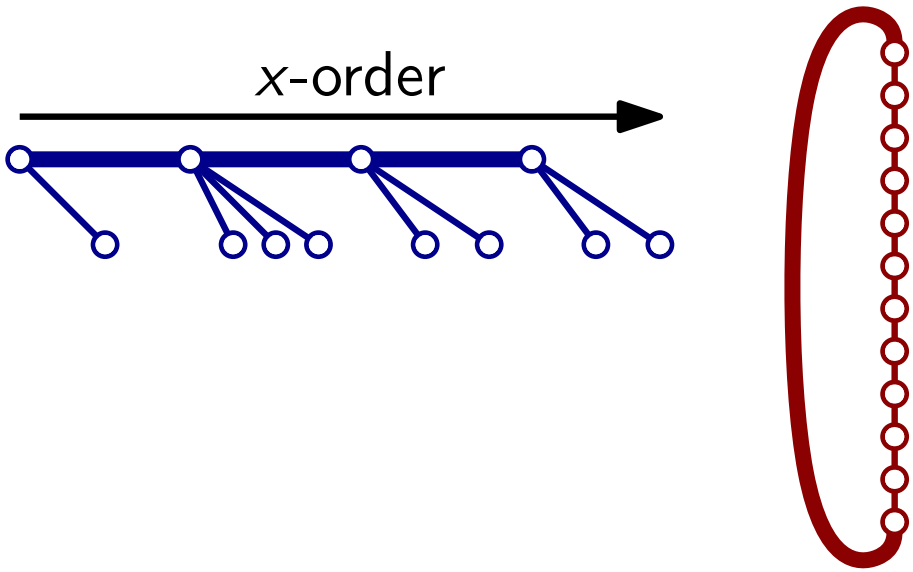




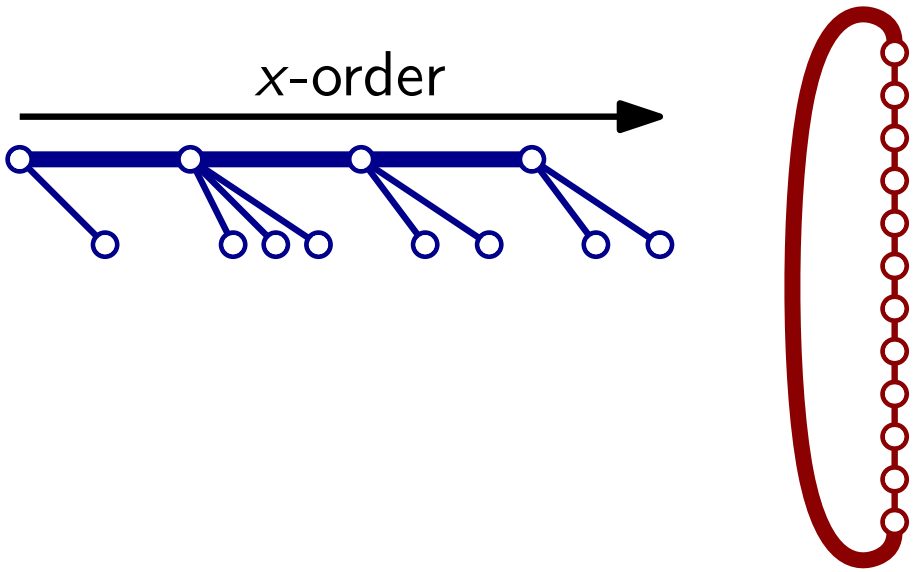
Caterpillar $\times$ Cycle

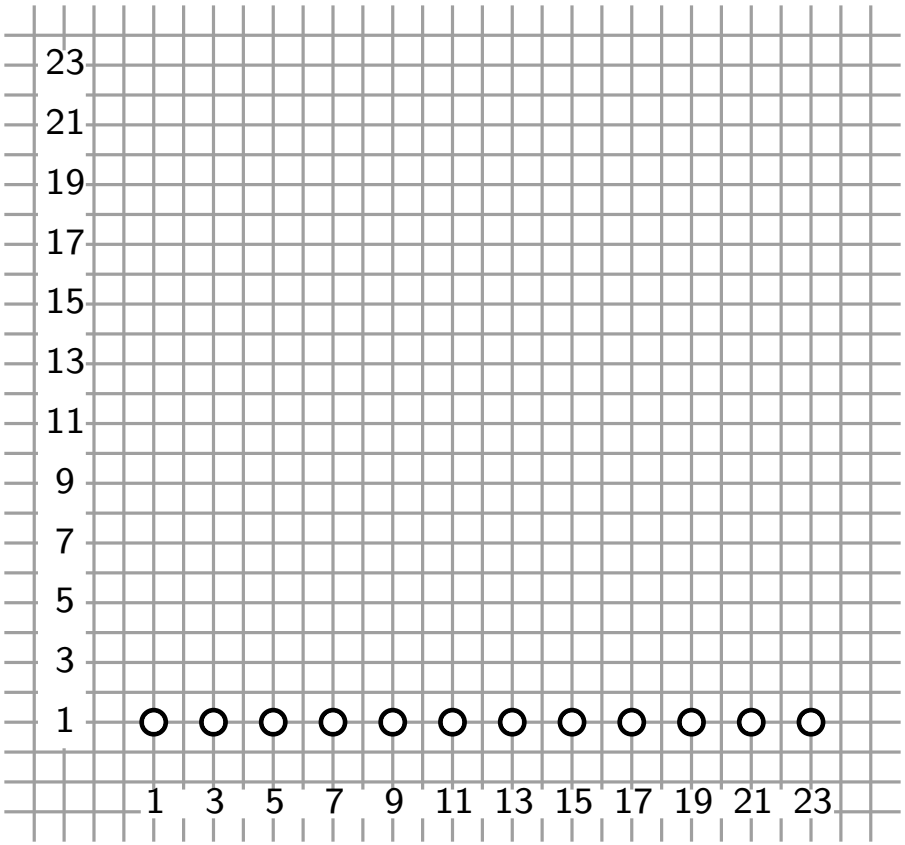


Caterpillar $\times$ Cycle

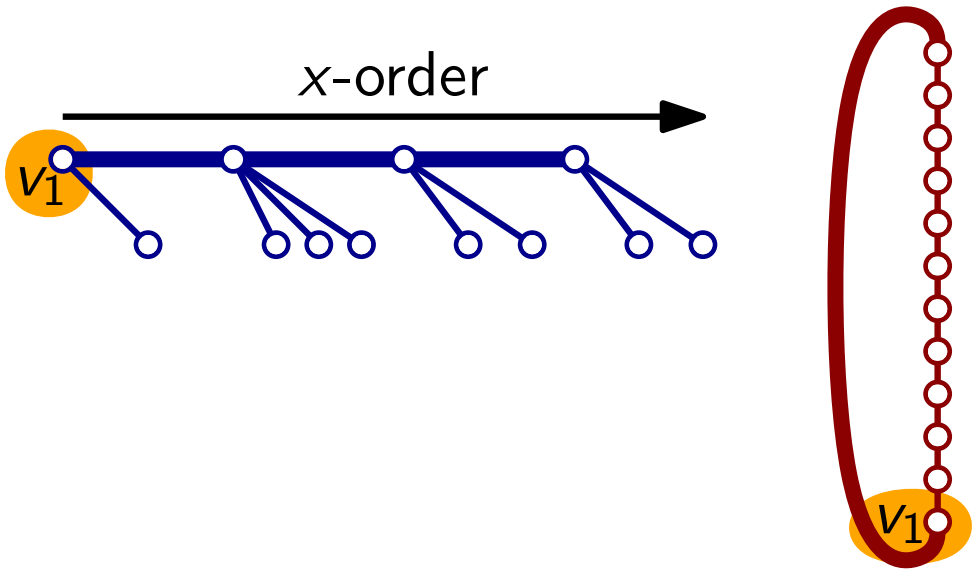


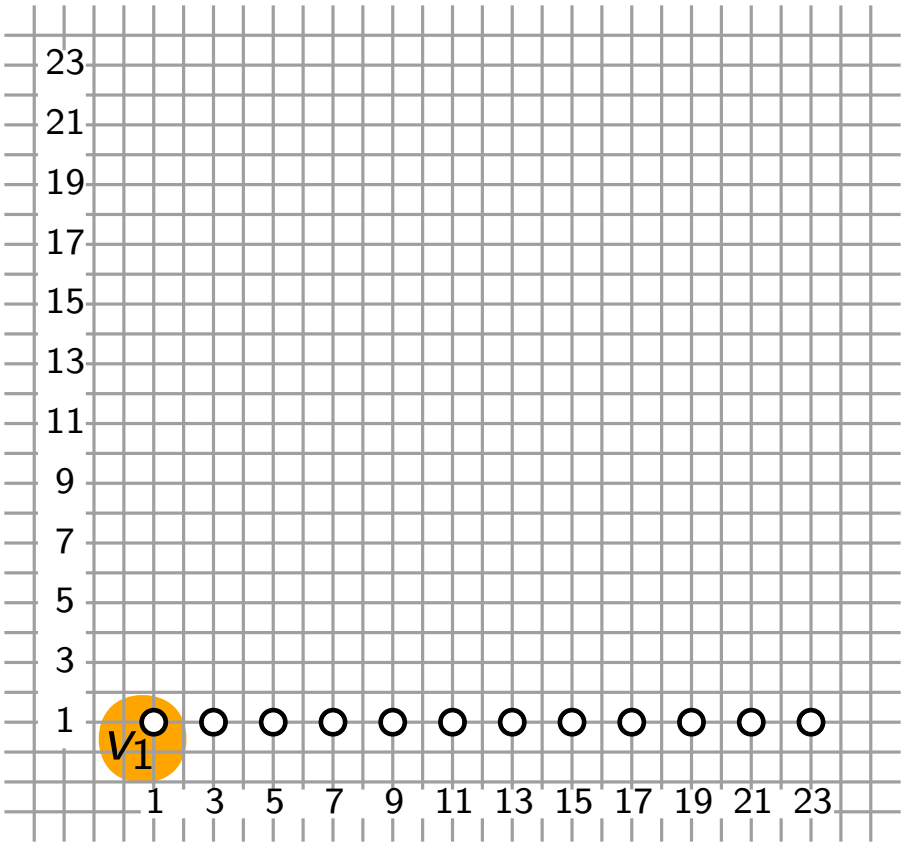
Caterpillar × Cycle



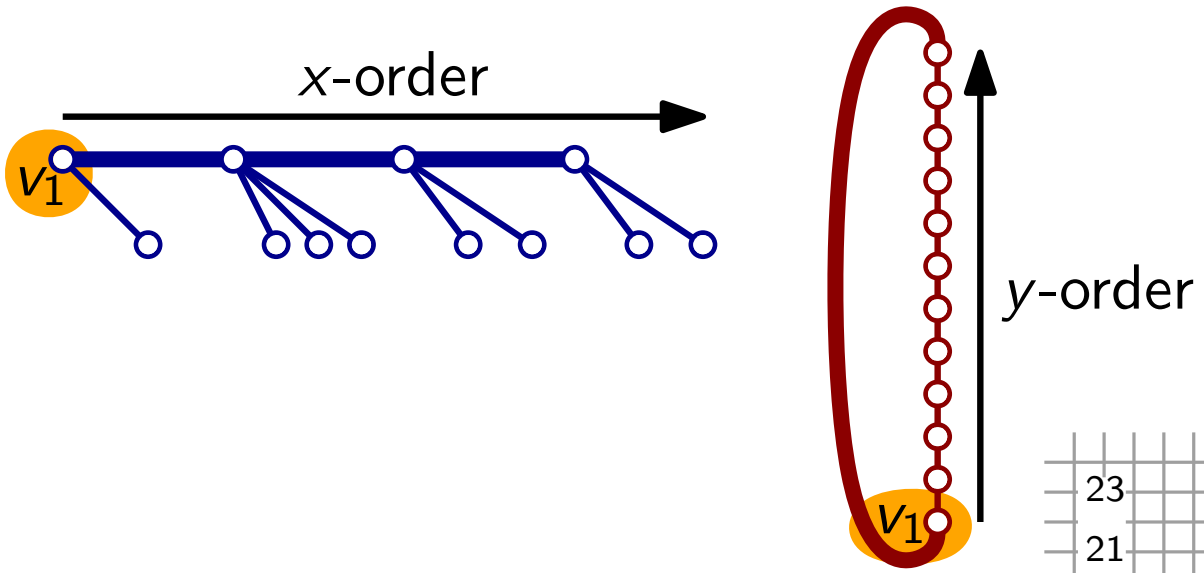


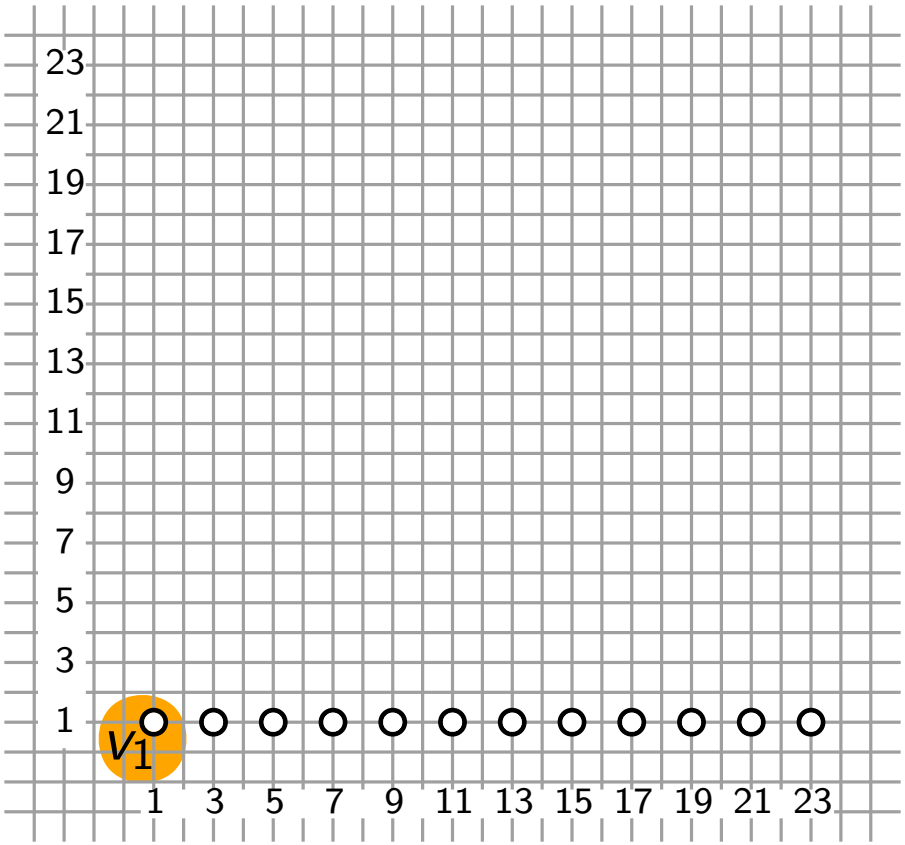
Caterpillar × Cycle



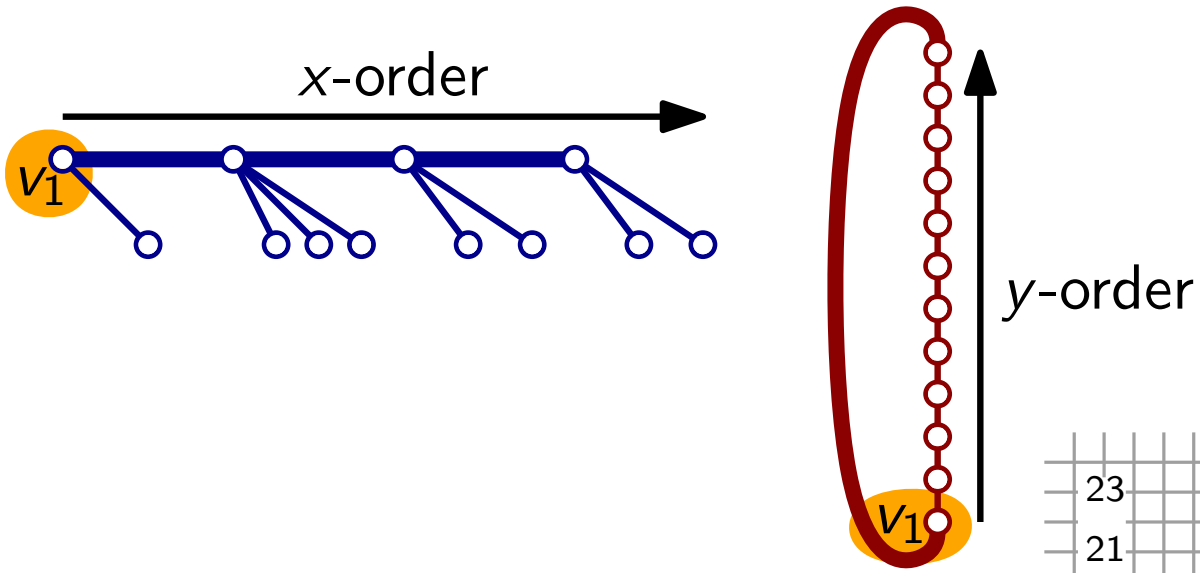


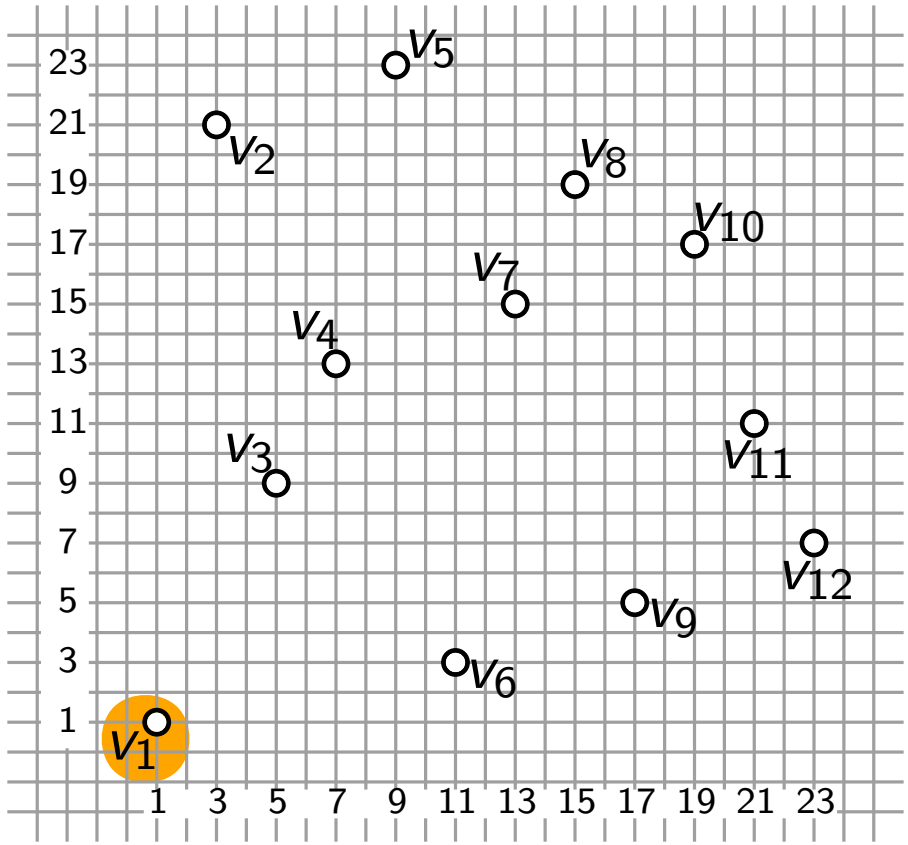
Caterpillar × Cycle



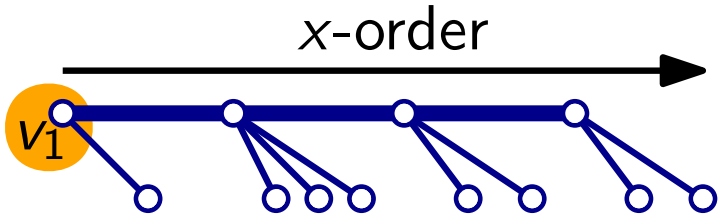


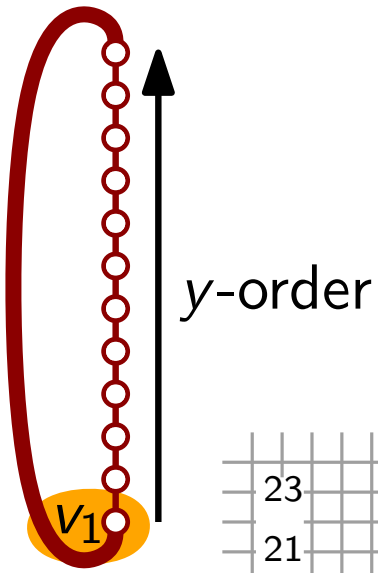
Caterpillar × Cycle





Caterpillar × Cycle





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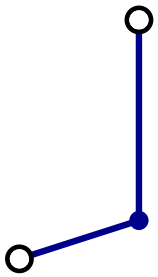
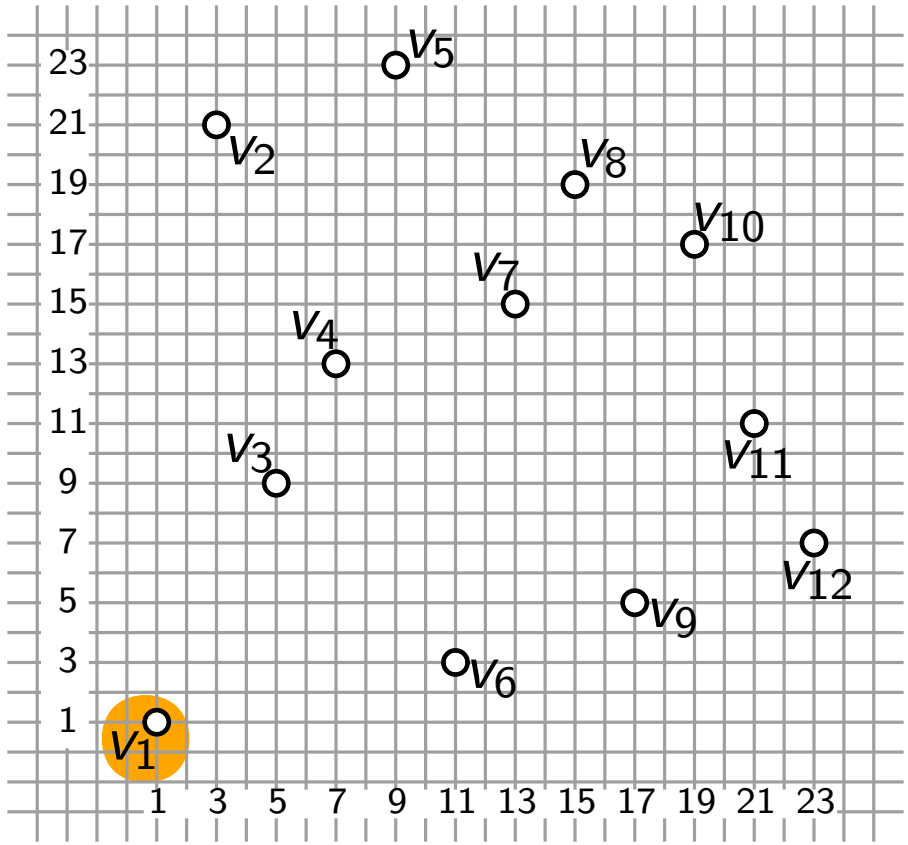
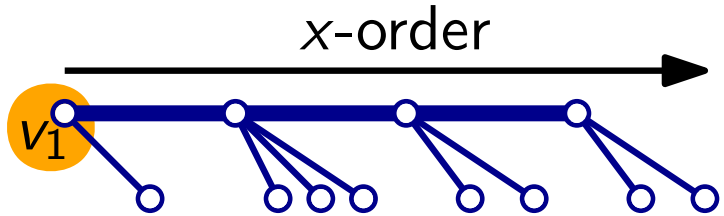
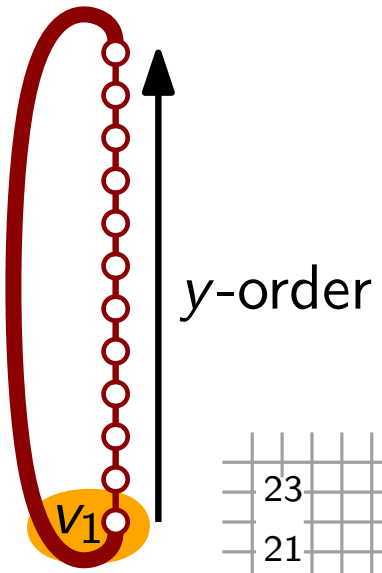




Caterpillar × Cycle





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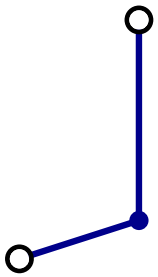
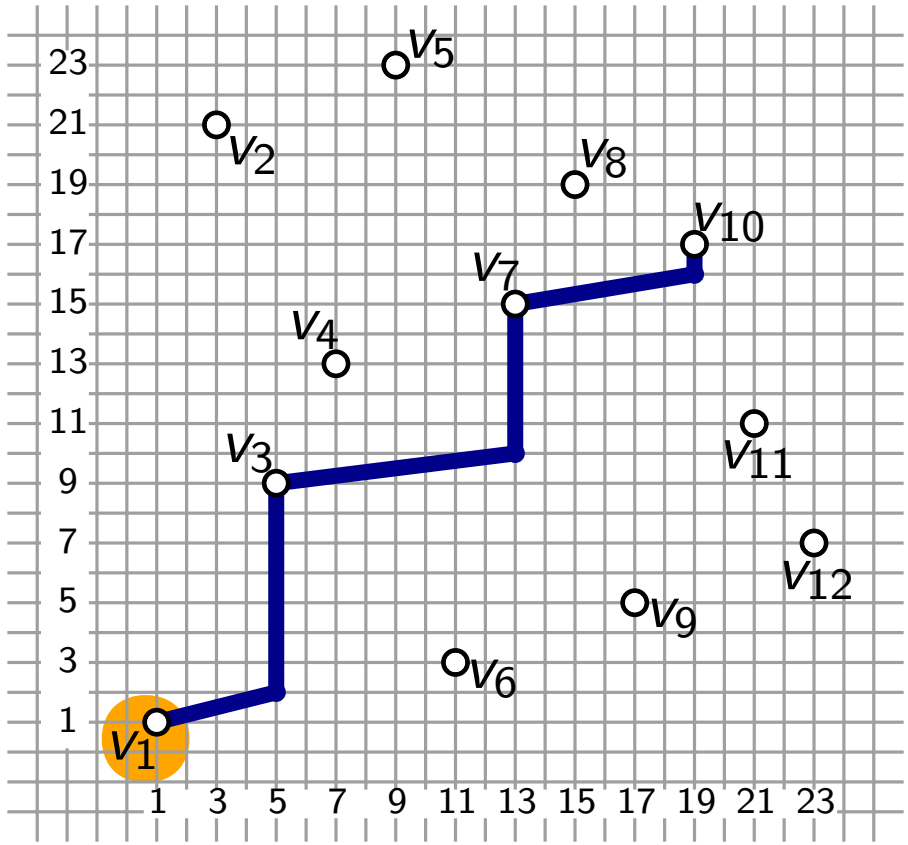
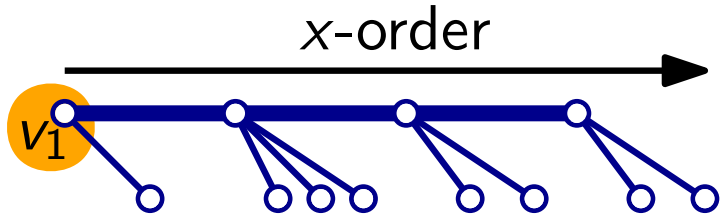
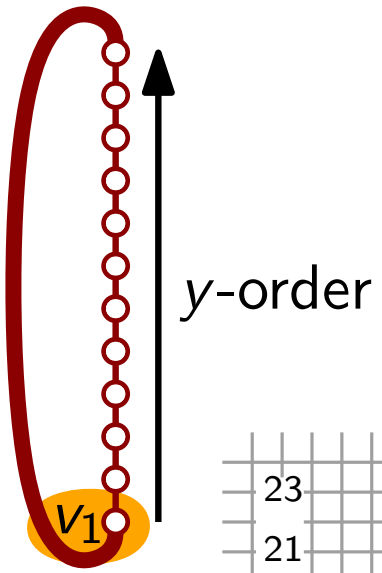




Caterpillar × Cycle





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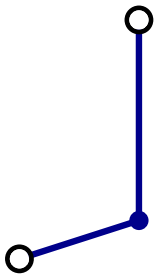
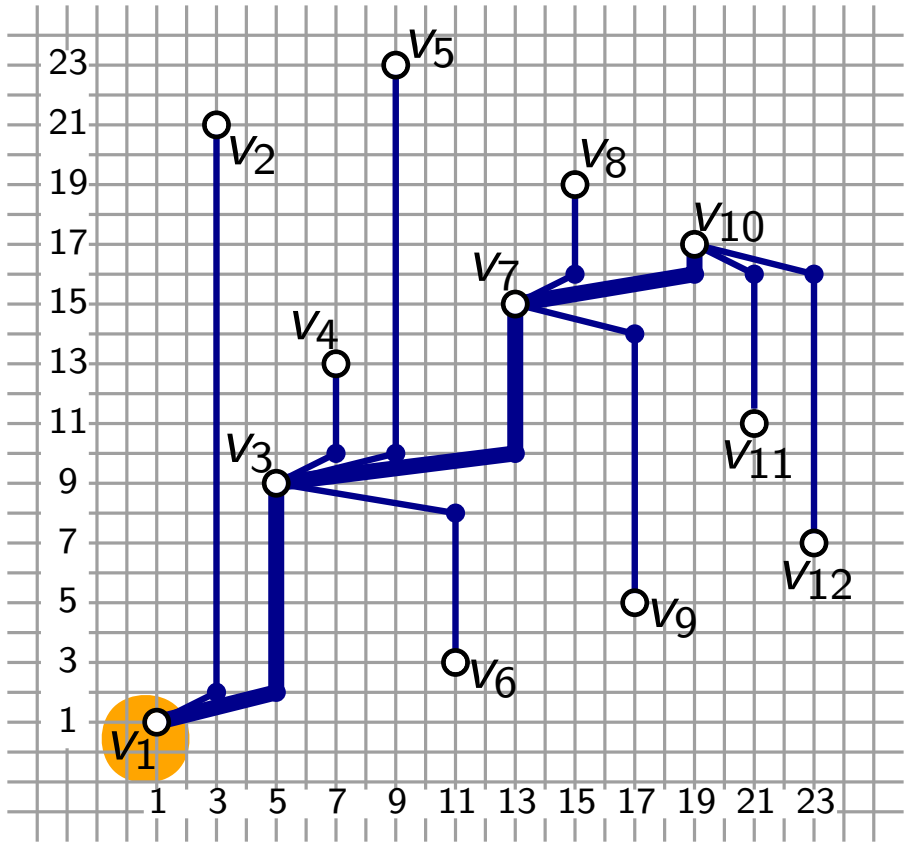
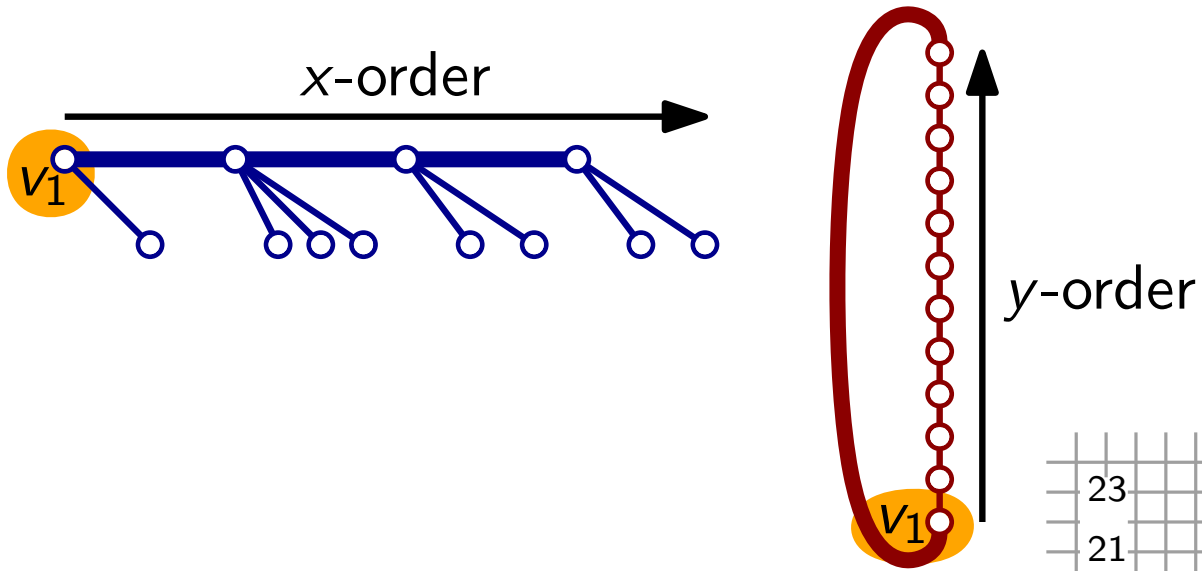




Caterpillar × Cycle



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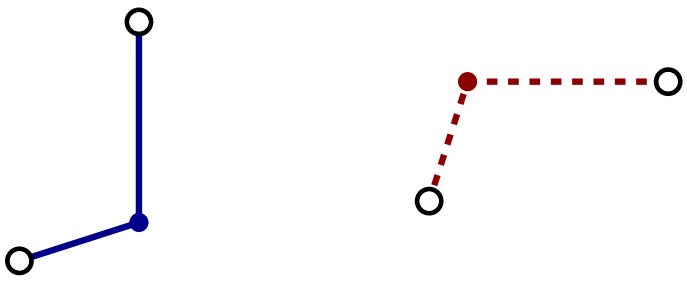
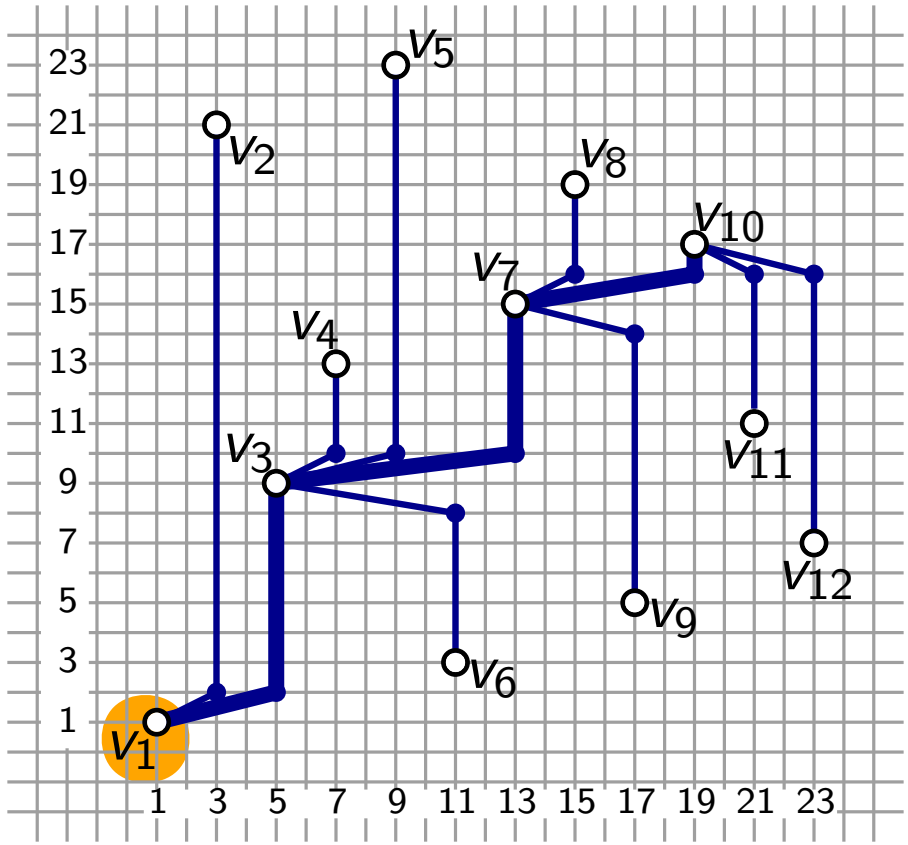
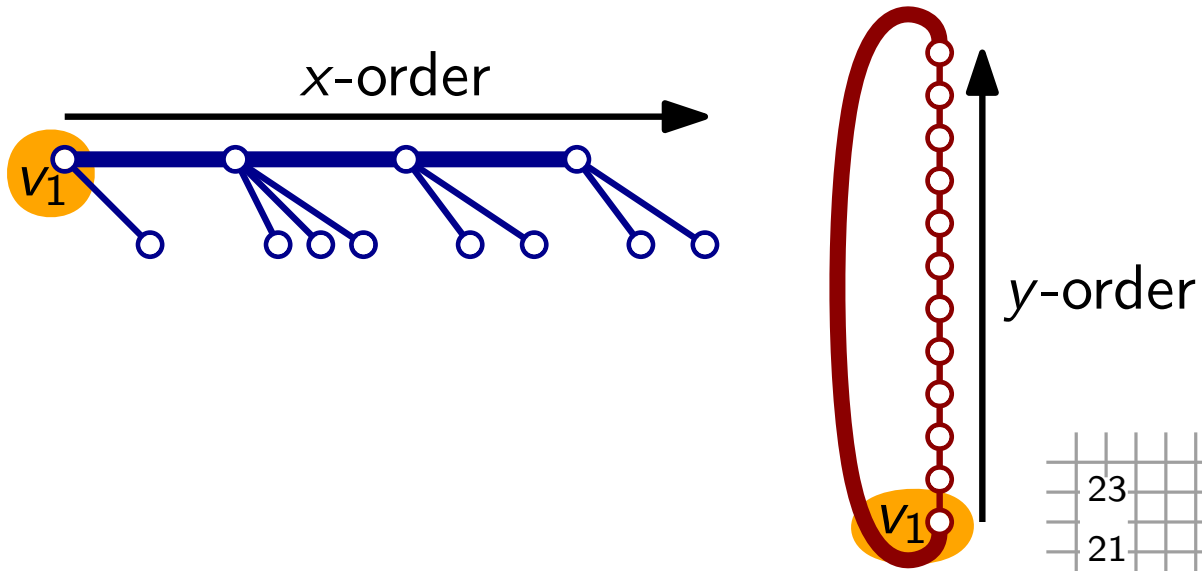




Caterpillar × Cycle



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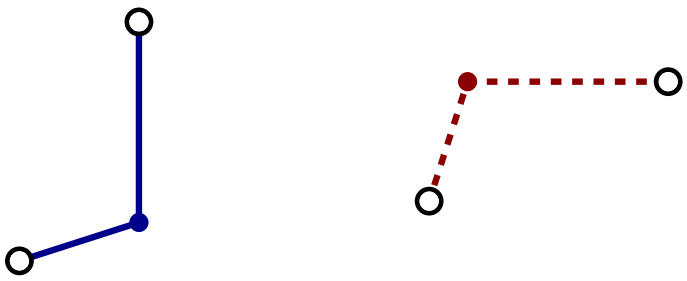
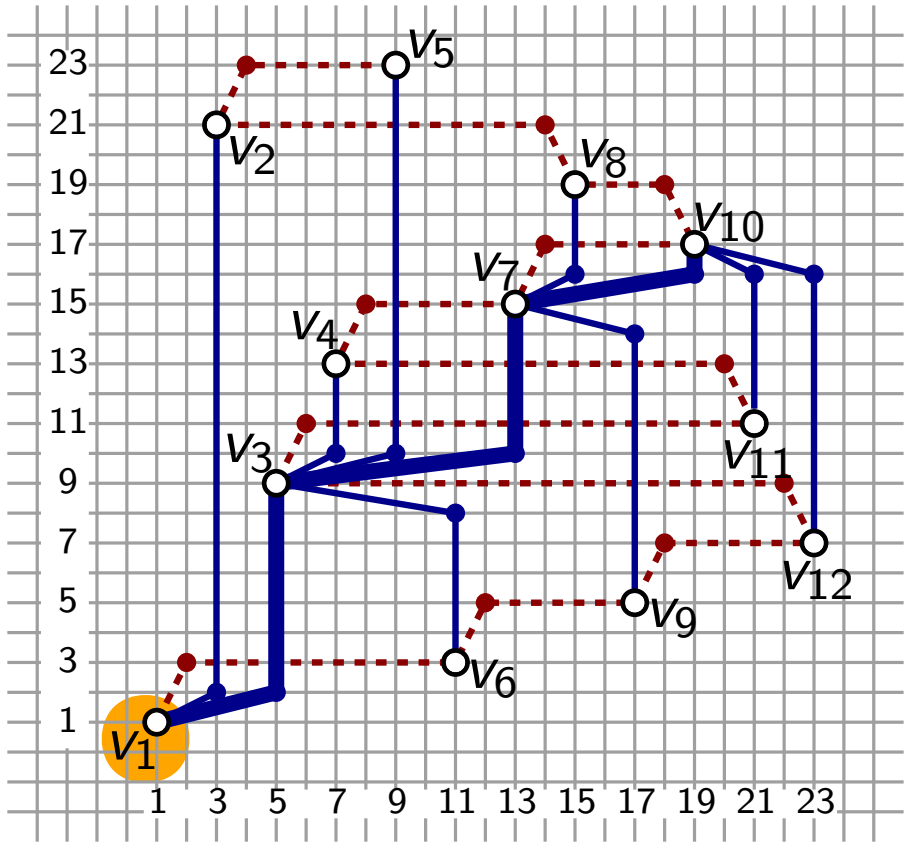
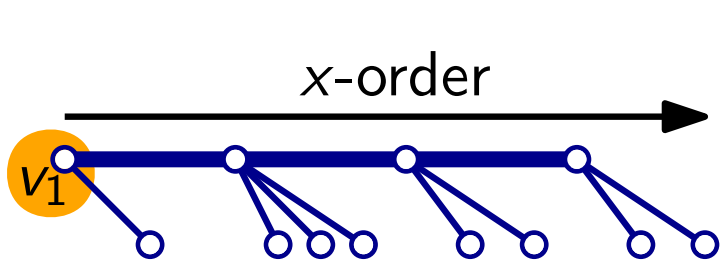
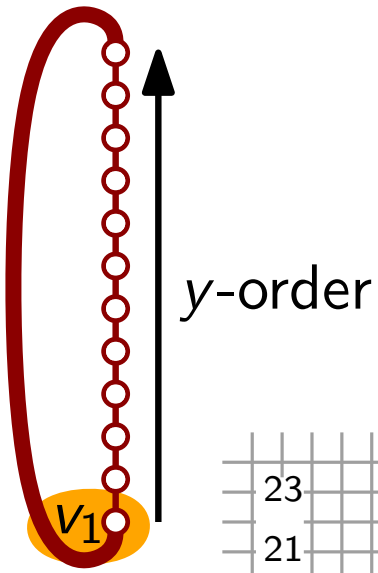




Caterpillar × Cycle





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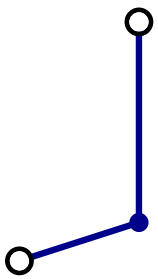
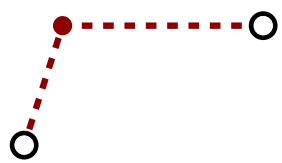
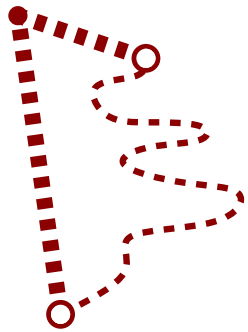
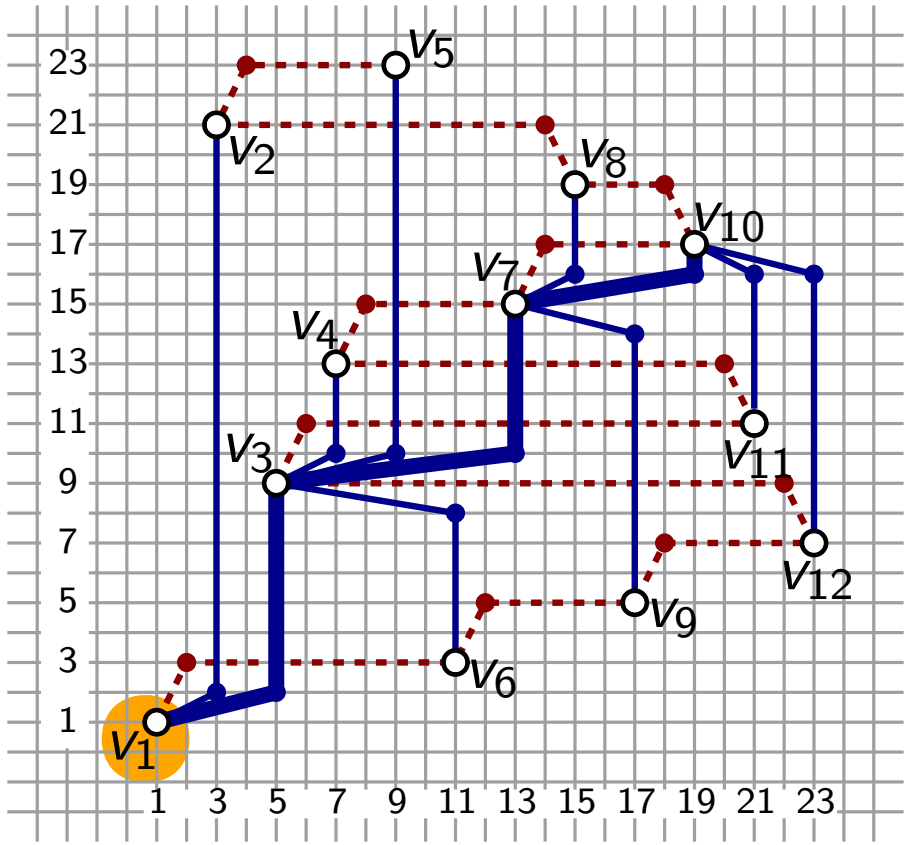
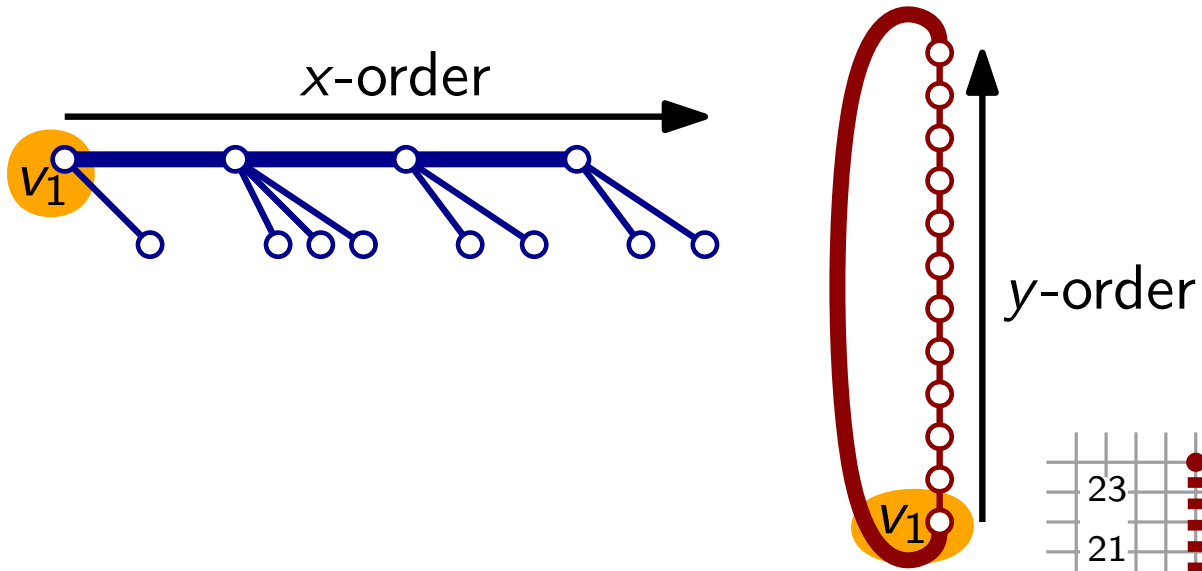








Caterpillar × Cycle



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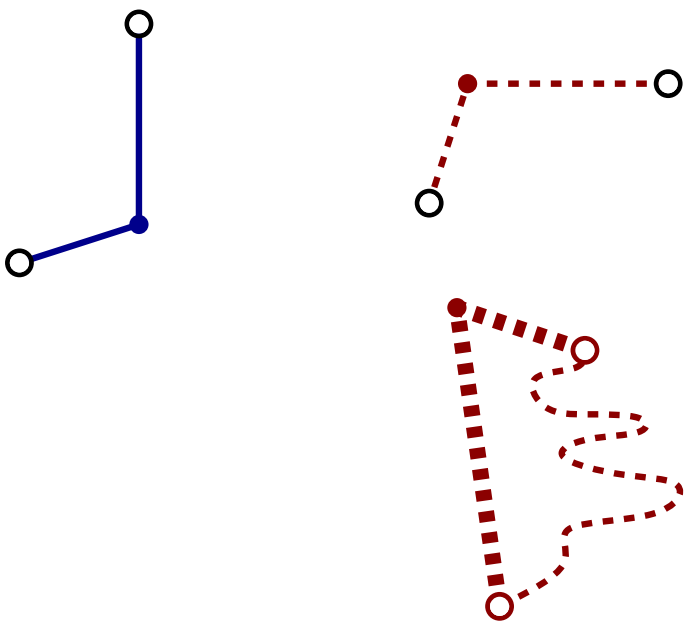
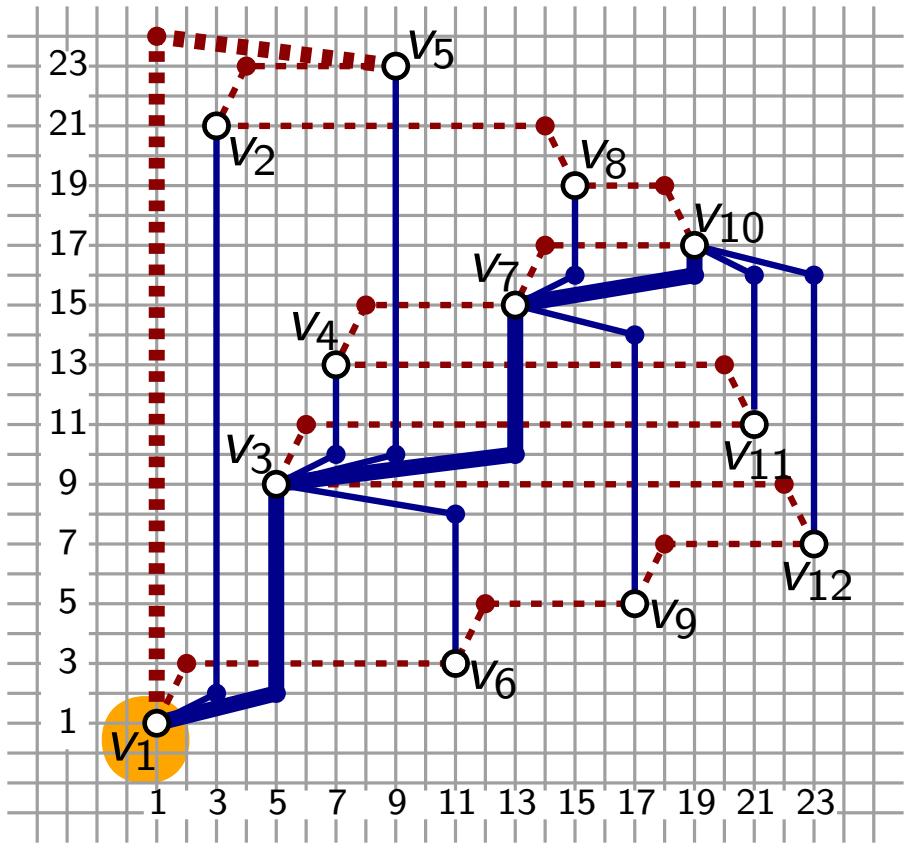
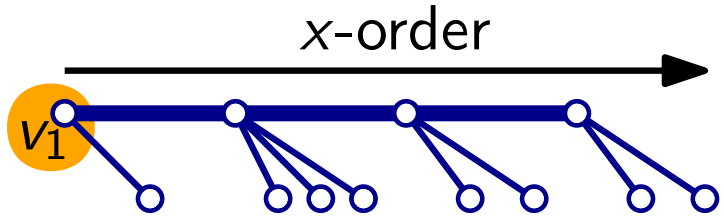
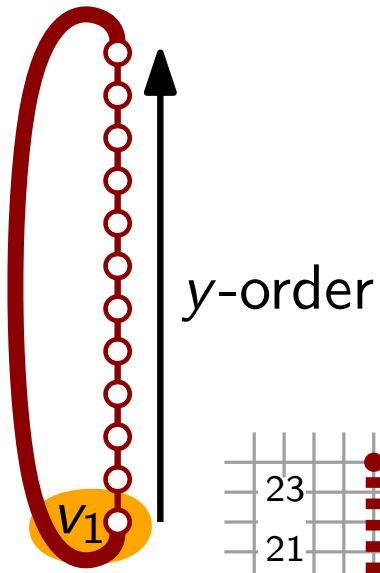




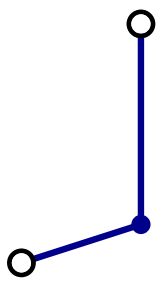
Caterpillar × Cycle

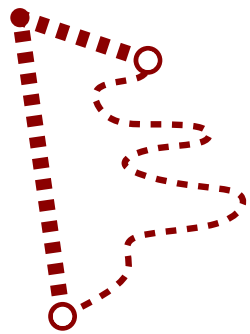


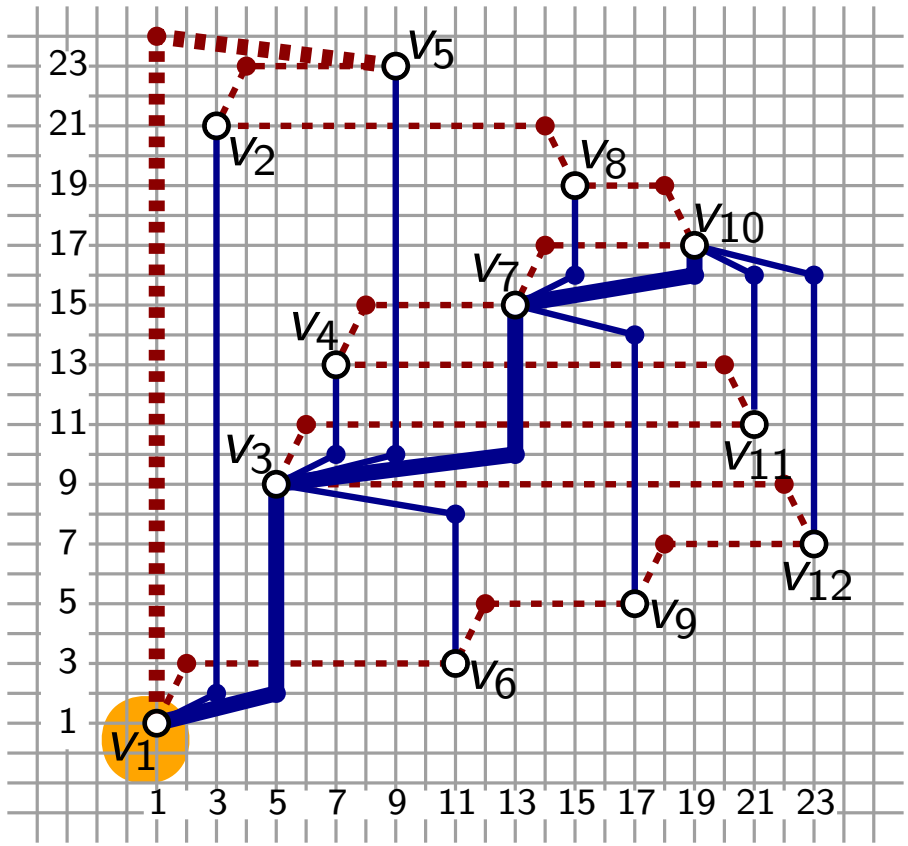


Bends: 1×1
Grid size:
$(2n-1) \times 2n$

Edges:



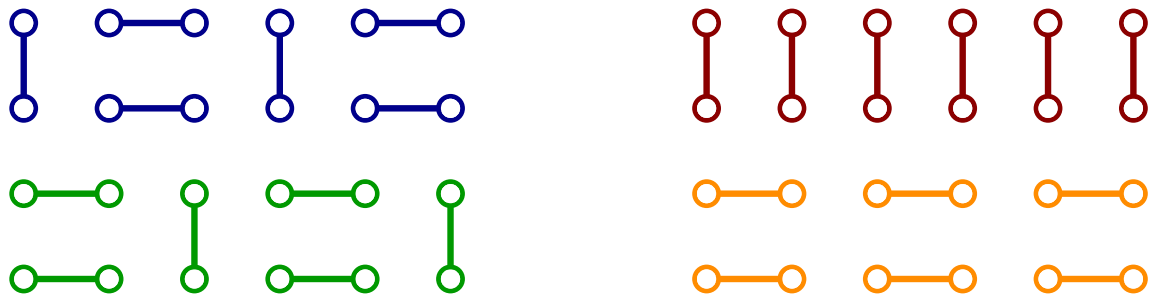




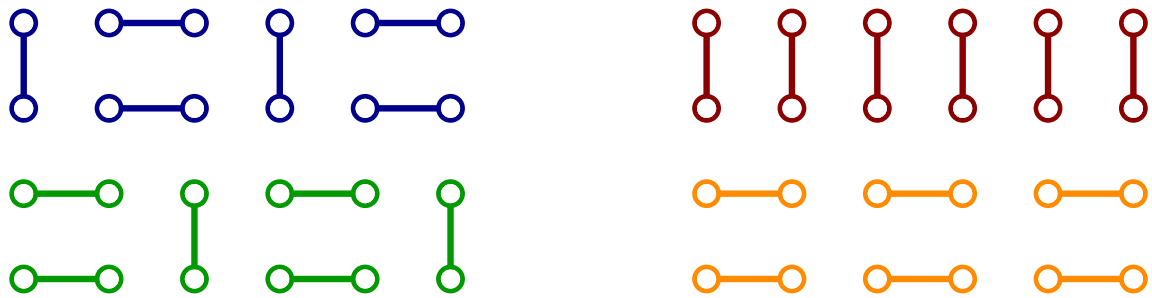
Overview

Graph classes			Number of bends
Cycle	$\times$	Cycle	1×1
Caterpillar	$\times$	Cycle	1×1
Four Matchings			$1 \times 1 \times 1 \times 1$
Tree	$\times$	Matching	1×0
Wheel	$\times$	Matching	2×0
Outerpath	$\times$	Matching	2×1
Outerplanar	$\times$	Outerplanar	3×3
2-page book emb.	$\times$	2-page book emb.	4×4
Planar	$\times$	Planar	6×6

Four Matchings

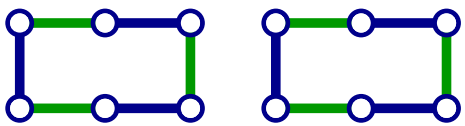


Four Matchings

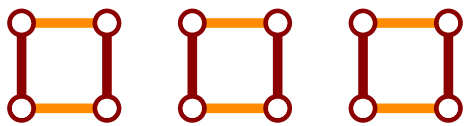


Combine $\Rightarrow$ two sets of cycles $\mathcal{C}_1$, $\mathcal{C}_2$

Four Matchings



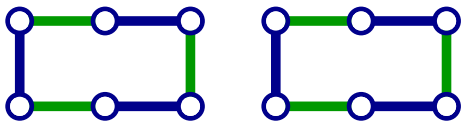
$\mathcal{C}_1$



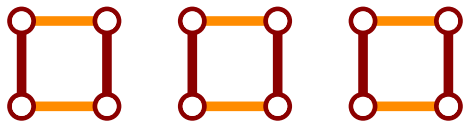
$\mathcal{C}_2$

Combine $\Rightarrow$ two sets of cycles $\mathcal{C}_1$, $\mathcal{C}_2$

Four Matchings

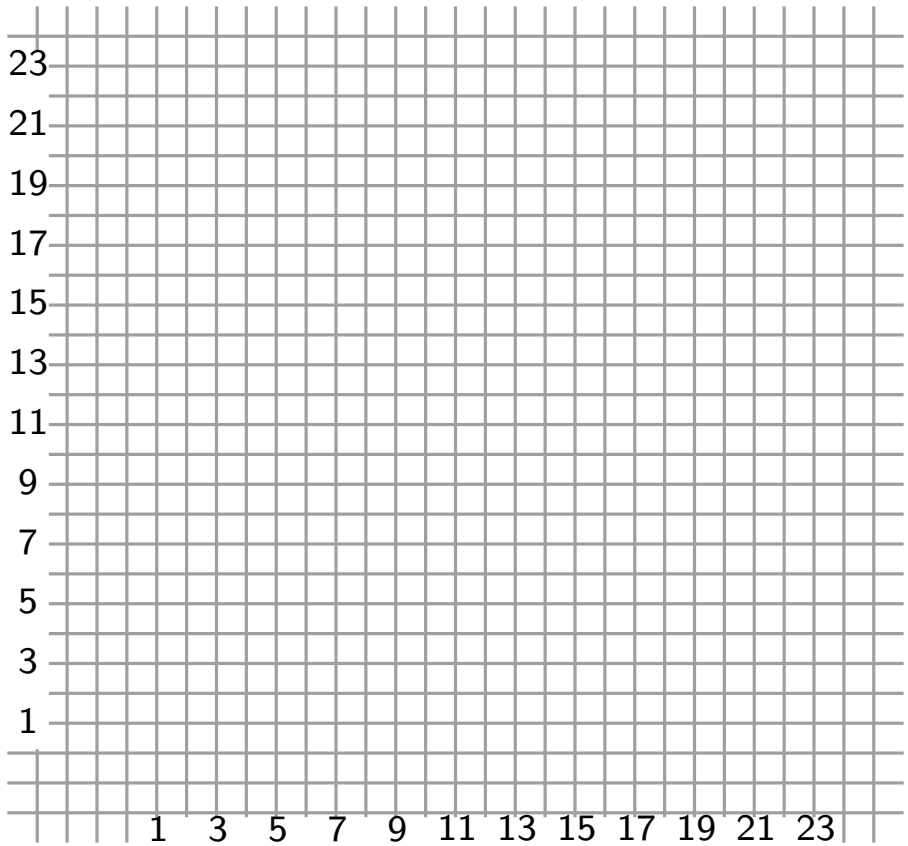


$\mathcal{C}_1$

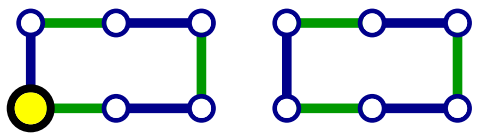


$\mathcal{C}_2$

Placement algorithm:



Four Matchings

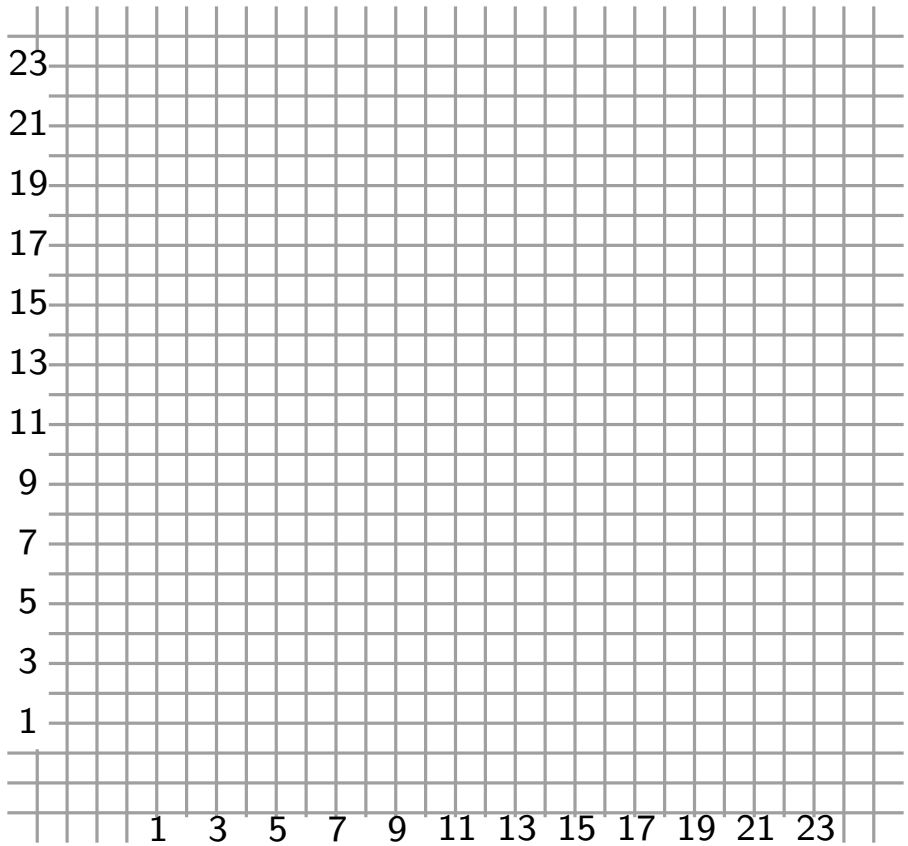


$\mathcal{C}_1$

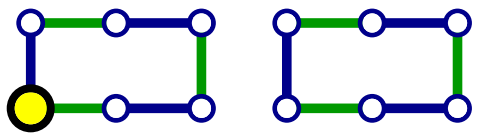
$\mathcal{C}_2$

Placement algorithm:

- Pick v_1



Four Matchings

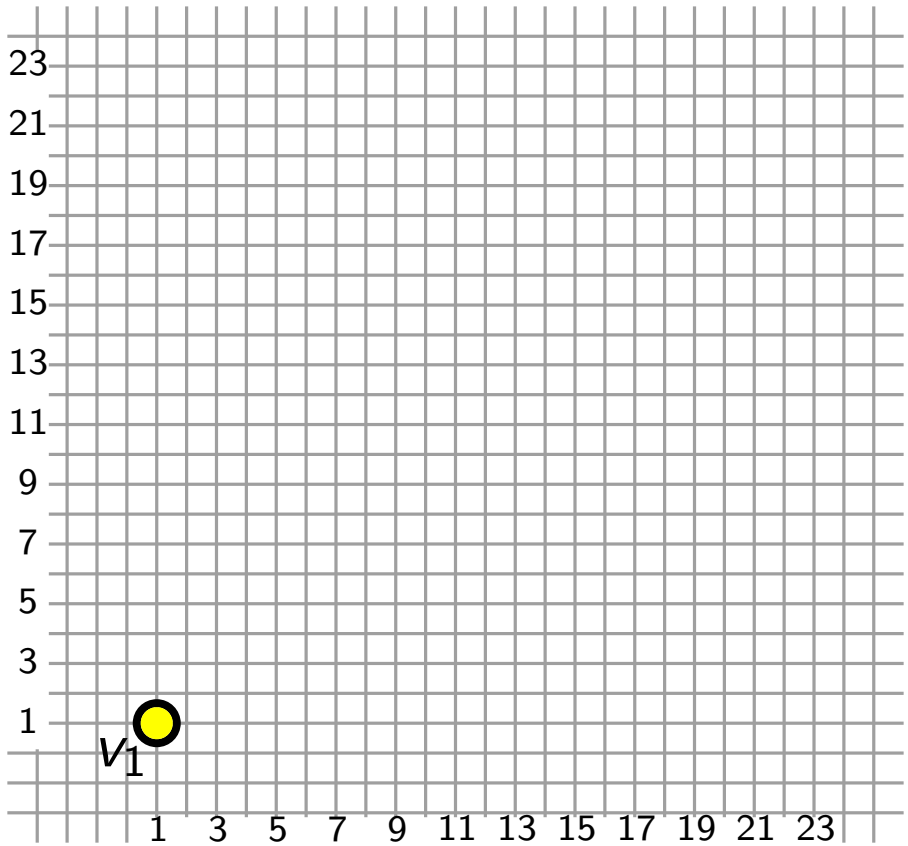


$\mathcal{C}_1$

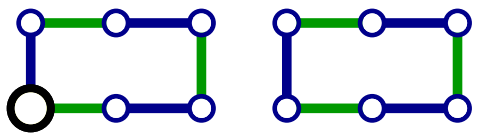
$\mathcal{C}_2$

Placement algorithm:

- Pick v_1 , place it



Four Matchings

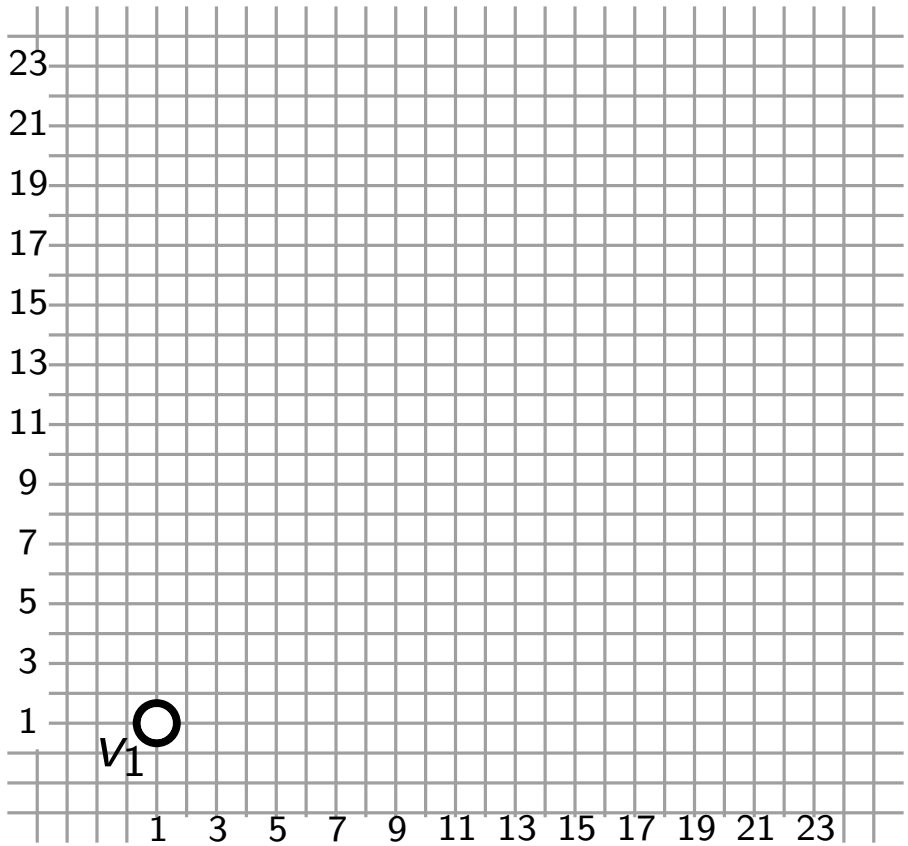


$\mathcal{C}_1$

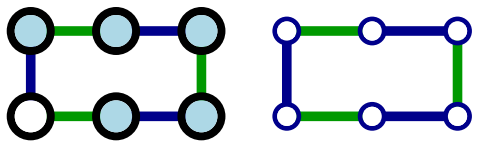
$\mathcal{C}_2$

Placement algorithm:

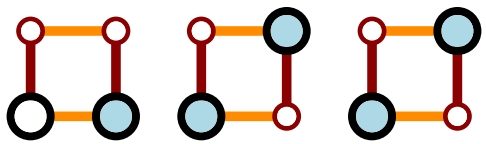
- Pick v_1 , place it



Four Matchings



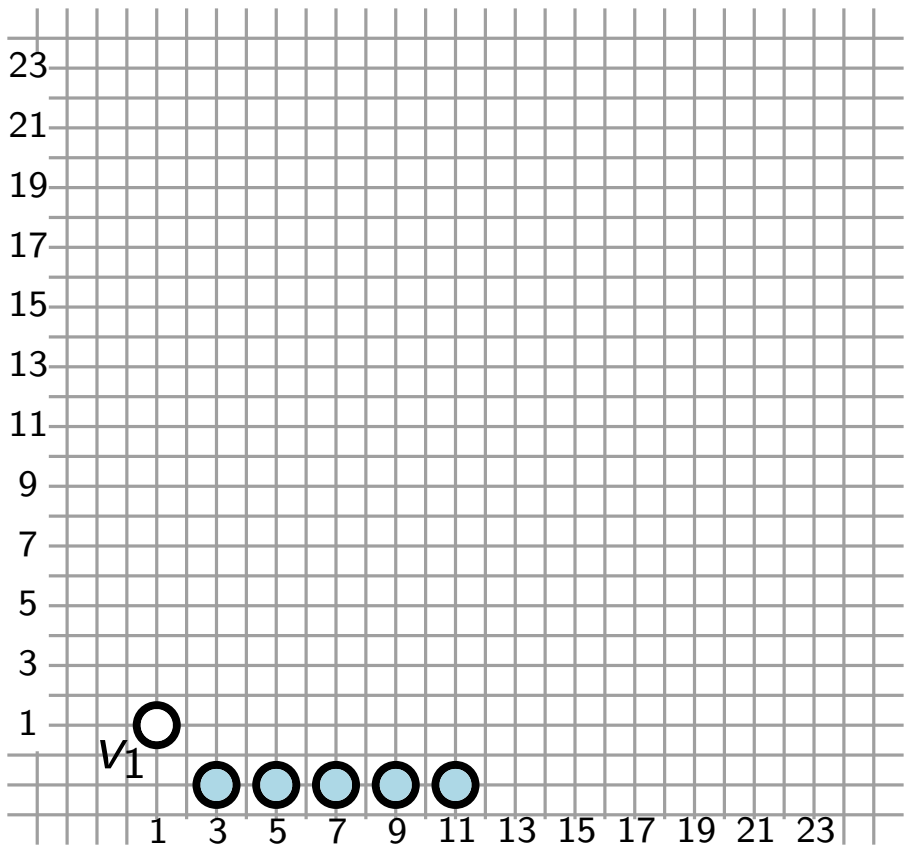
$\mathcal{C}_1$



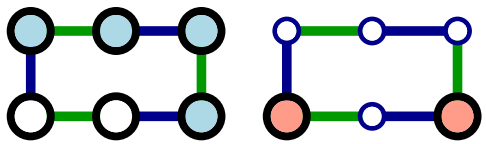
$\mathcal{C}_2$

Placement algorithm:

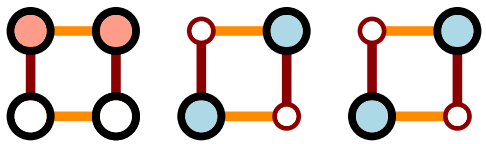
- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$



Four Matchings



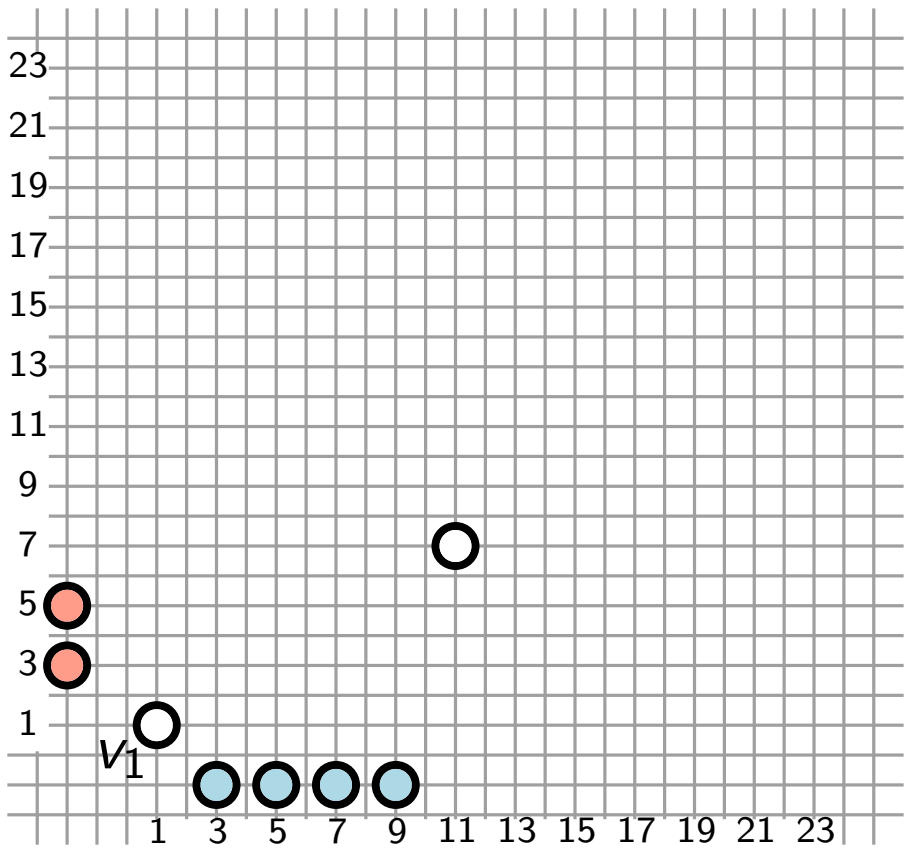
$\mathcal{C}_1$



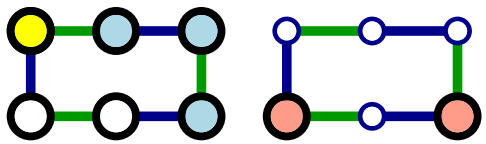
$\mathcal{C}_2$

Placement algorithm:

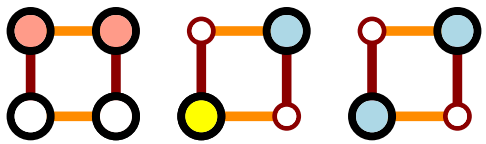
- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$



Four Matchings



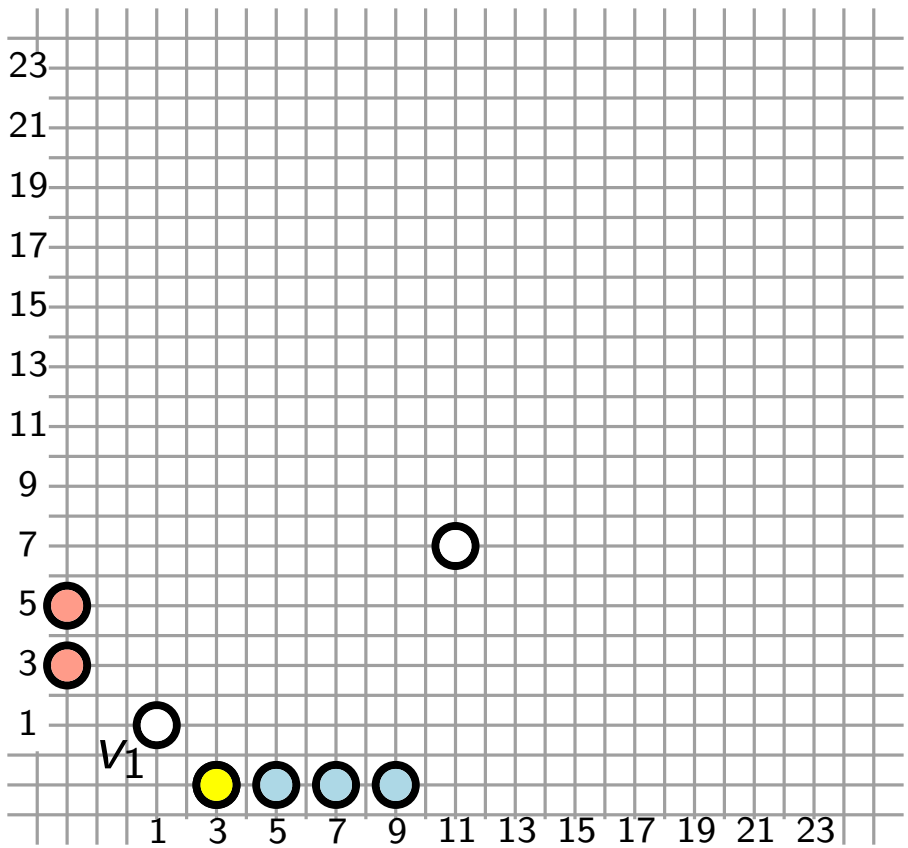
$\mathcal{C}_1$



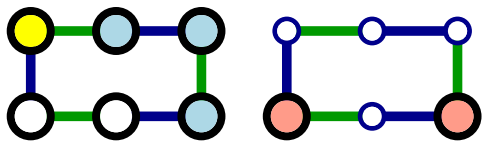
$\mathcal{C}_2$

Placement algorithm:

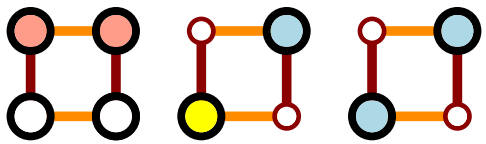
- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.



Four Matchings



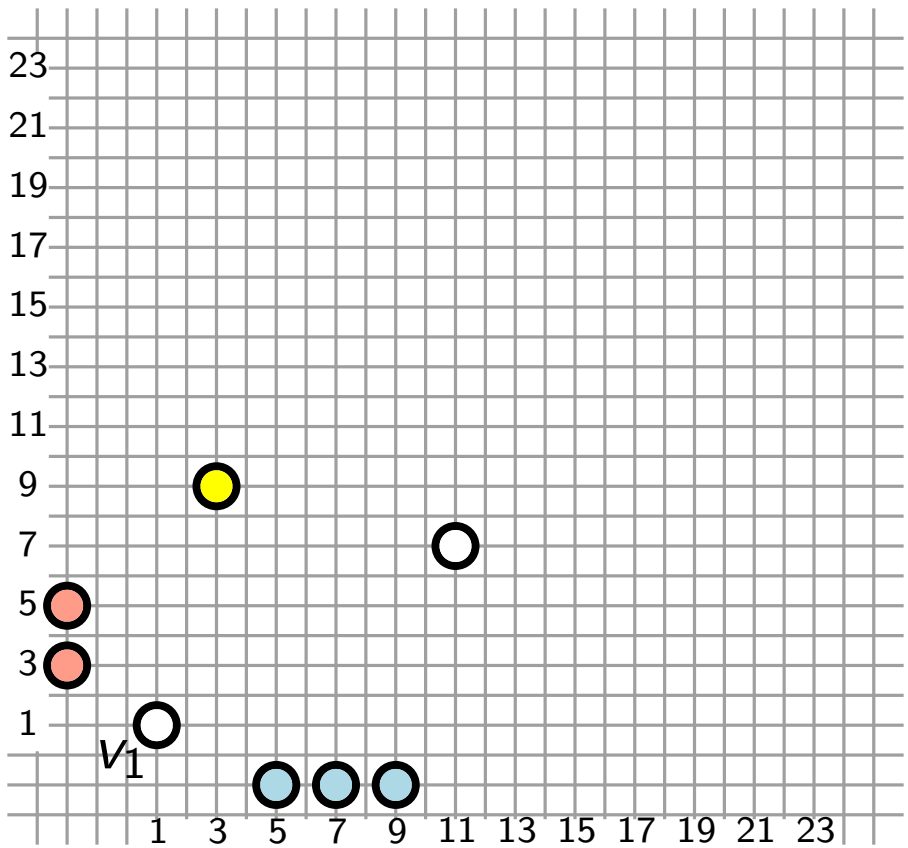
$\mathcal{C}_1$



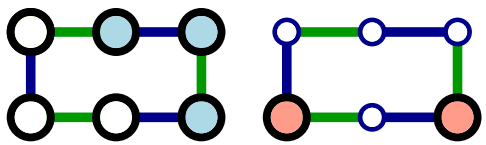
$\mathcal{C}_2$

Placement algorithm:

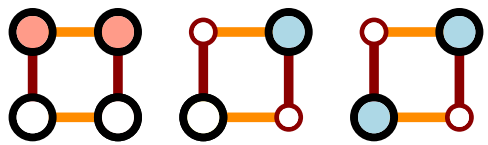
- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



Four Matchings



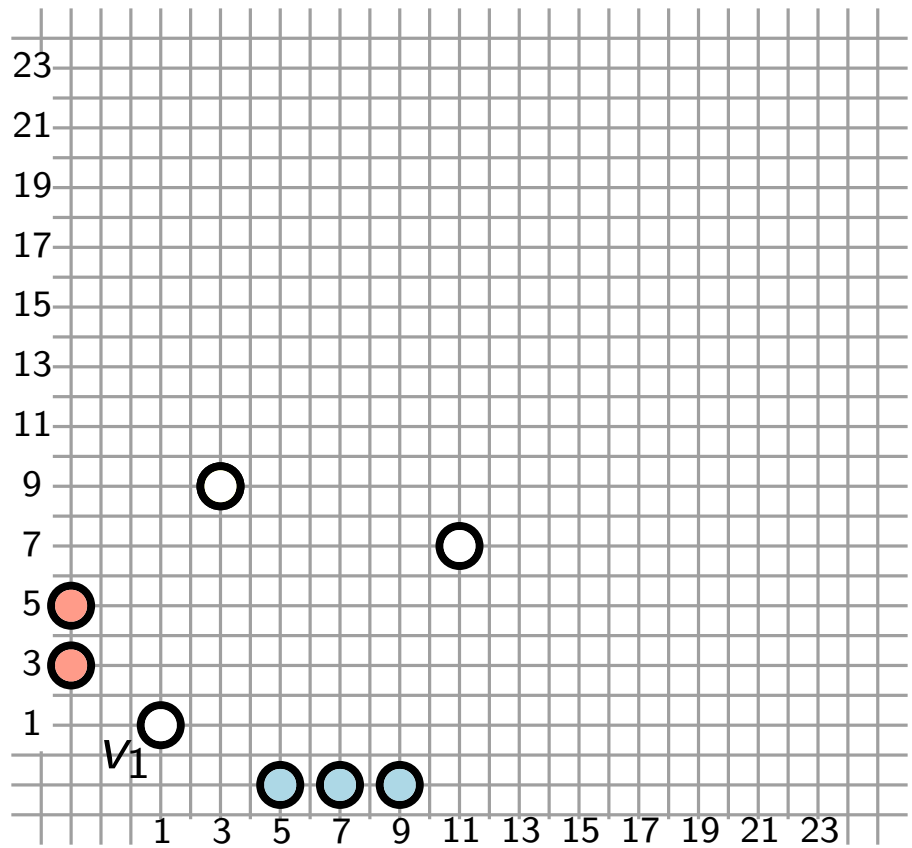
$\mathcal{C}_1$



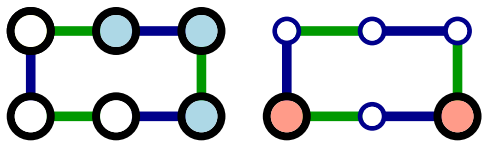
$\mathcal{C}_2$

Placement algorithm:

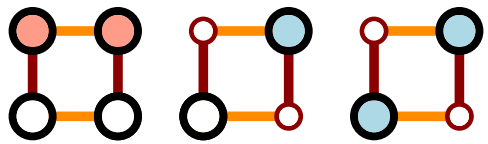
- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



Four Matchings



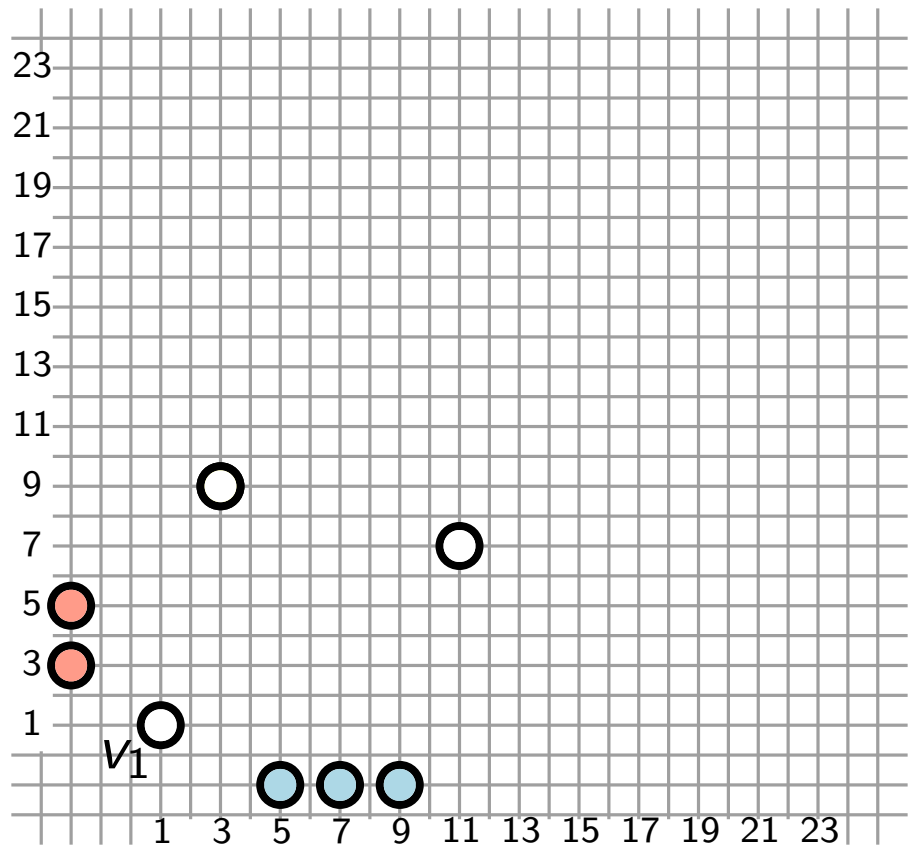
$\mathcal{C}_1$



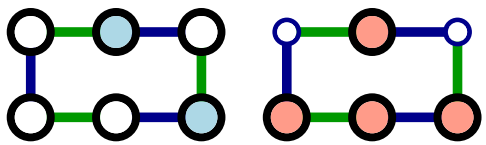
$\mathcal{C}_2$

Placement algorithm:

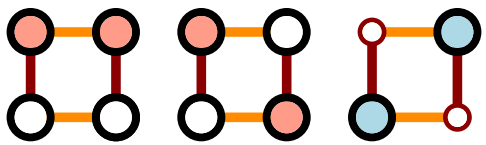
- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



Four Matchings



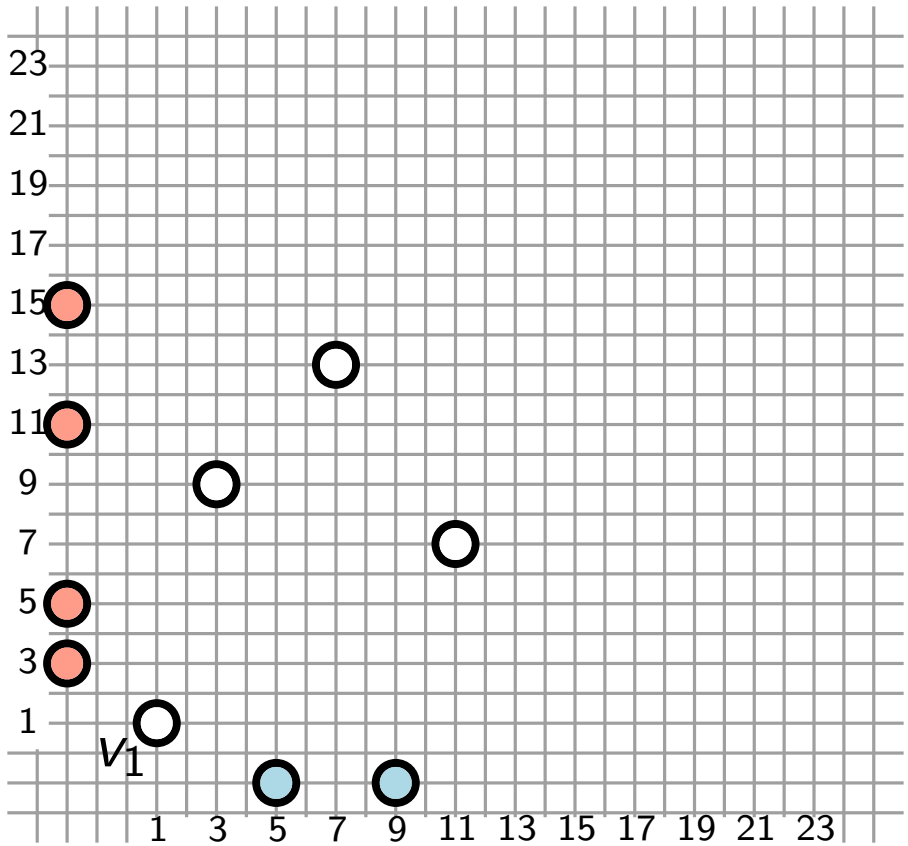
$\mathcal{C}_1$



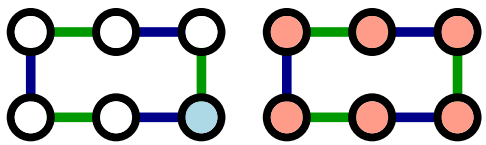
$\mathcal{C}_2$

Placement algorithm:

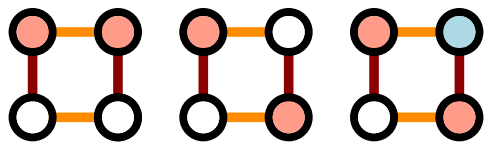
- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



Four Matchings



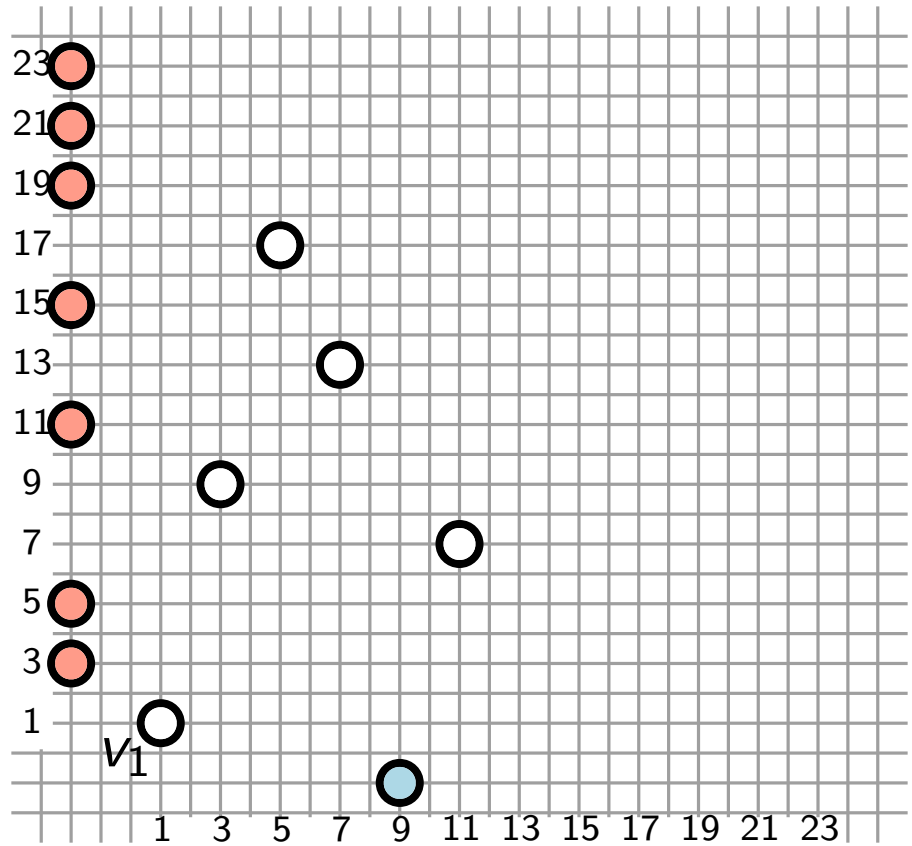
$\mathcal{C}_1$



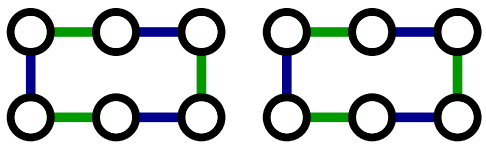
$\mathcal{C}_2$

Placement algorithm:

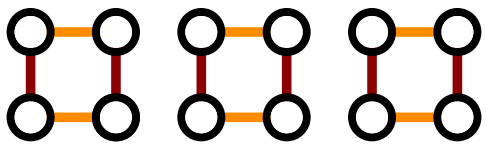
- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



Four Matchings



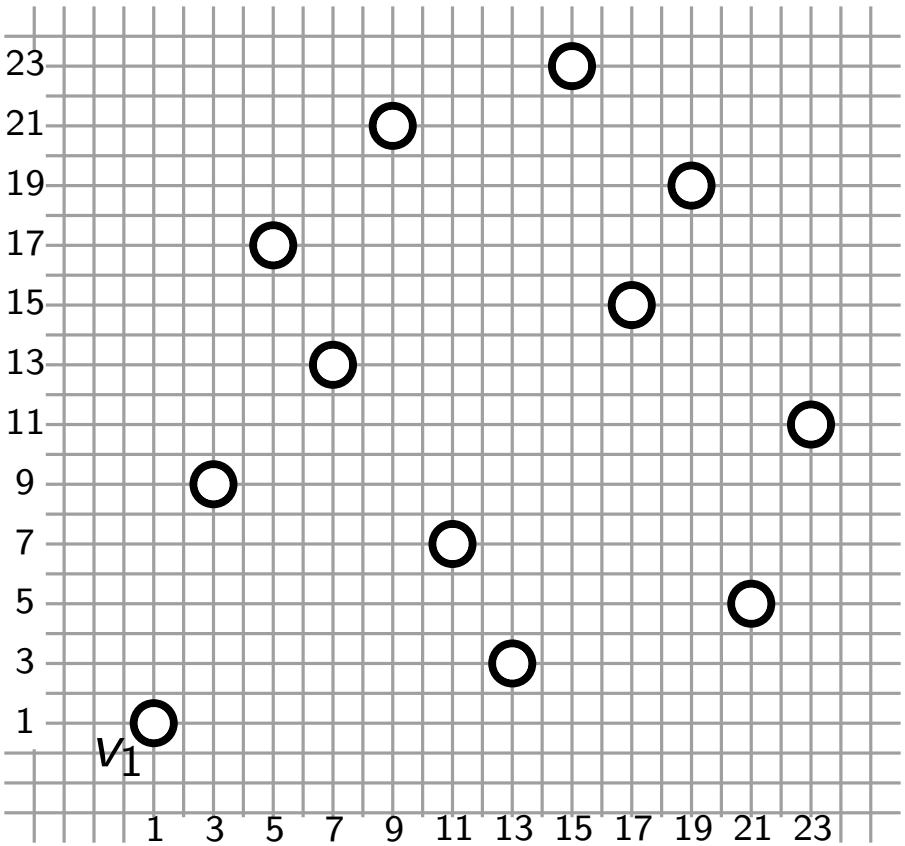
$\mathcal{C}_1$



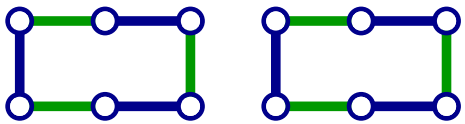
$\mathcal{C}_2$

Placement algorithm:

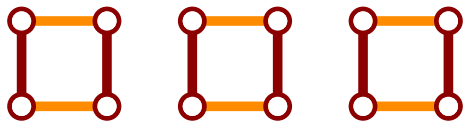
- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



Four Matchings



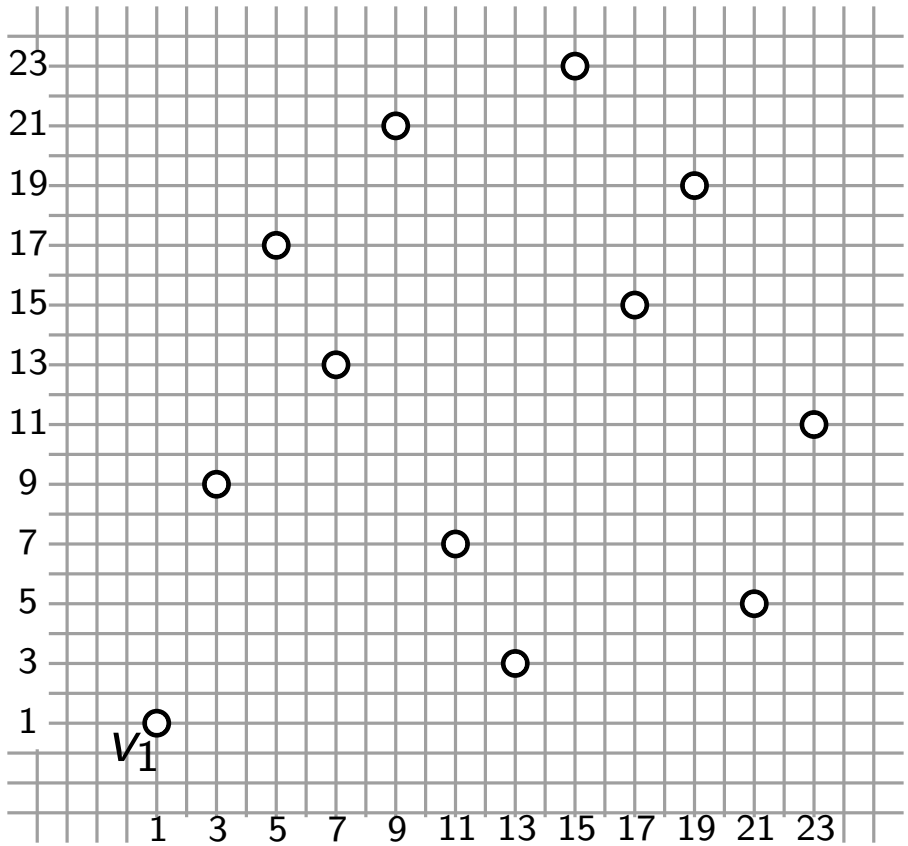
$\mathcal{C}_1$



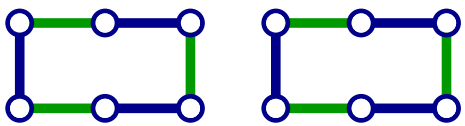
$\mathcal{C}_2$

Placement algorithm:

- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



Four Matchings



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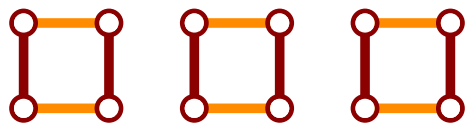
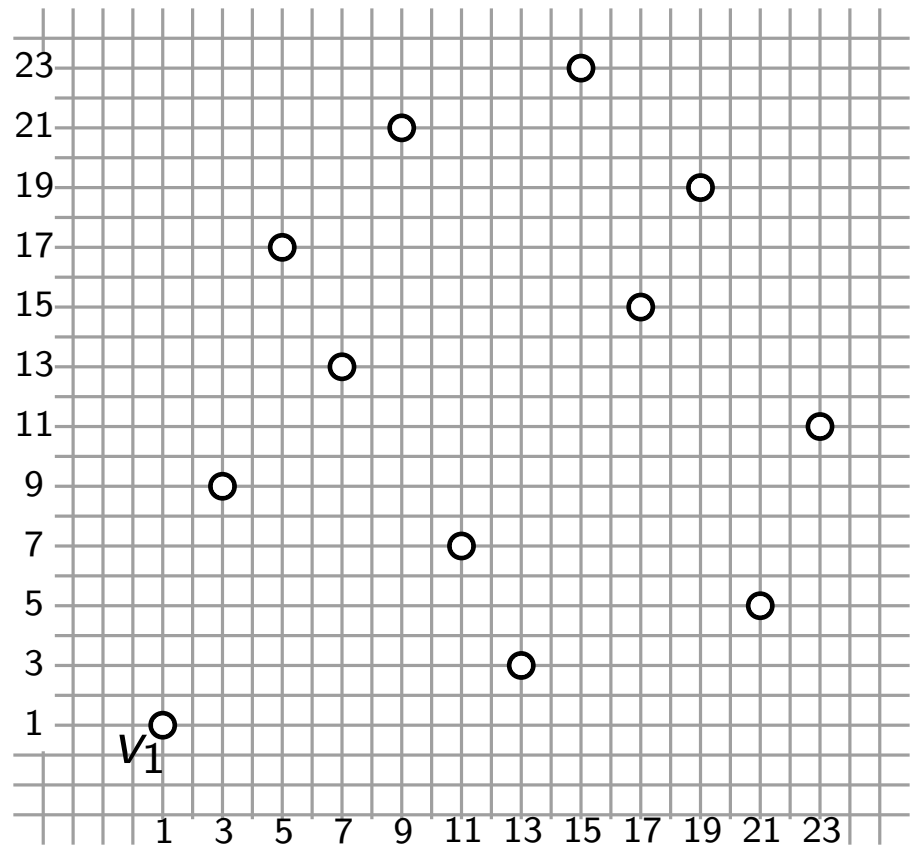


$\mathcal{C}_2$

Placement algorithm:

- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v



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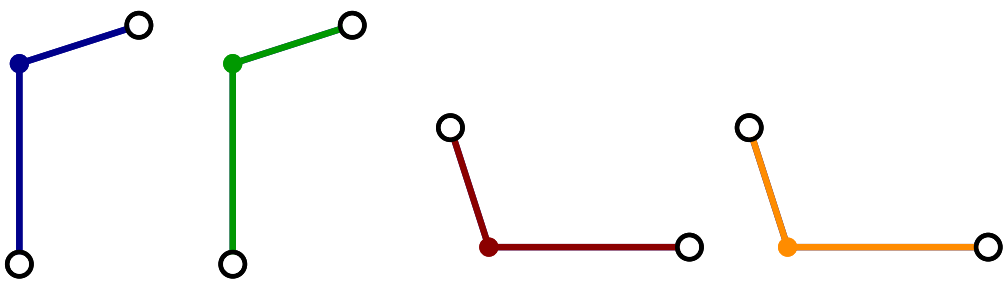
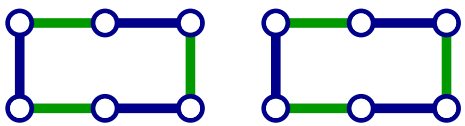
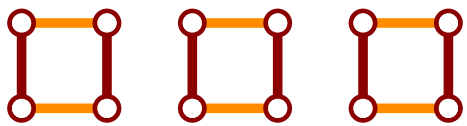


Four Matchings



$\mathcal{C}_1$

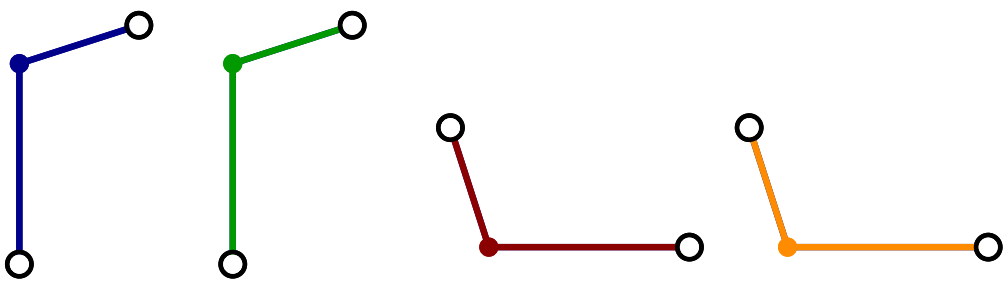


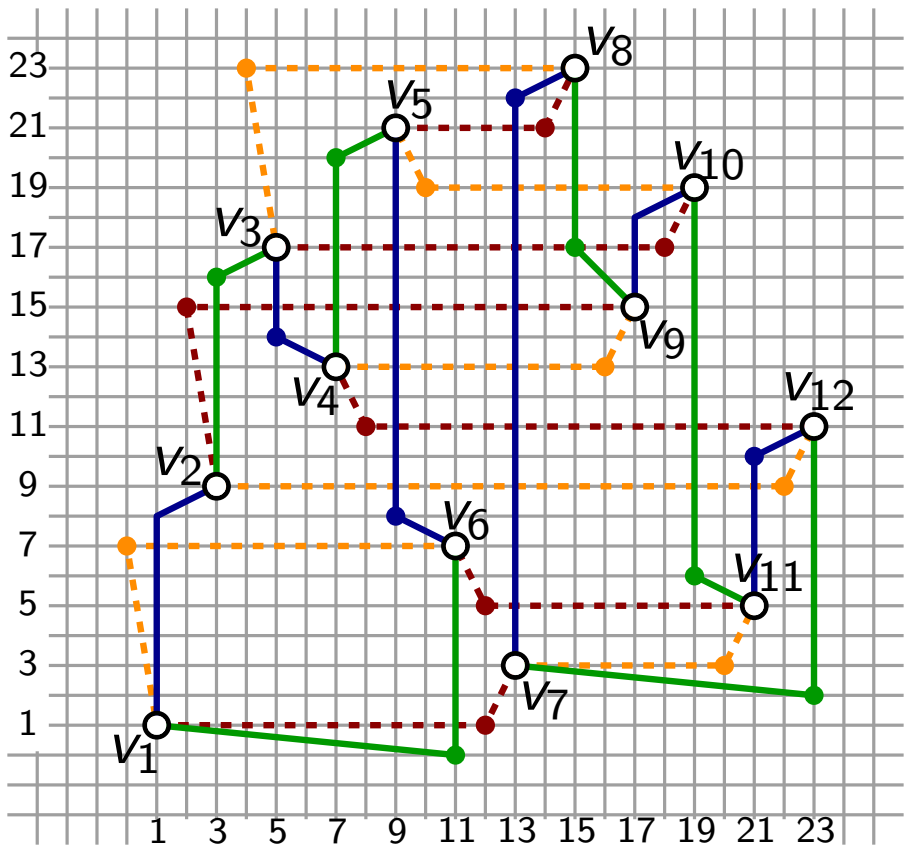
$\mathcal{C}_2$

Placement algorithm:

- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v

Edges:





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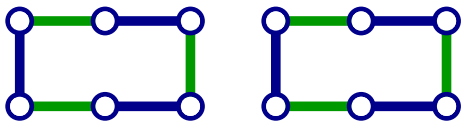

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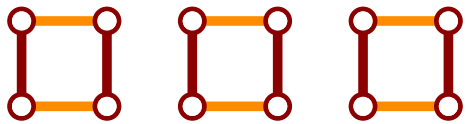

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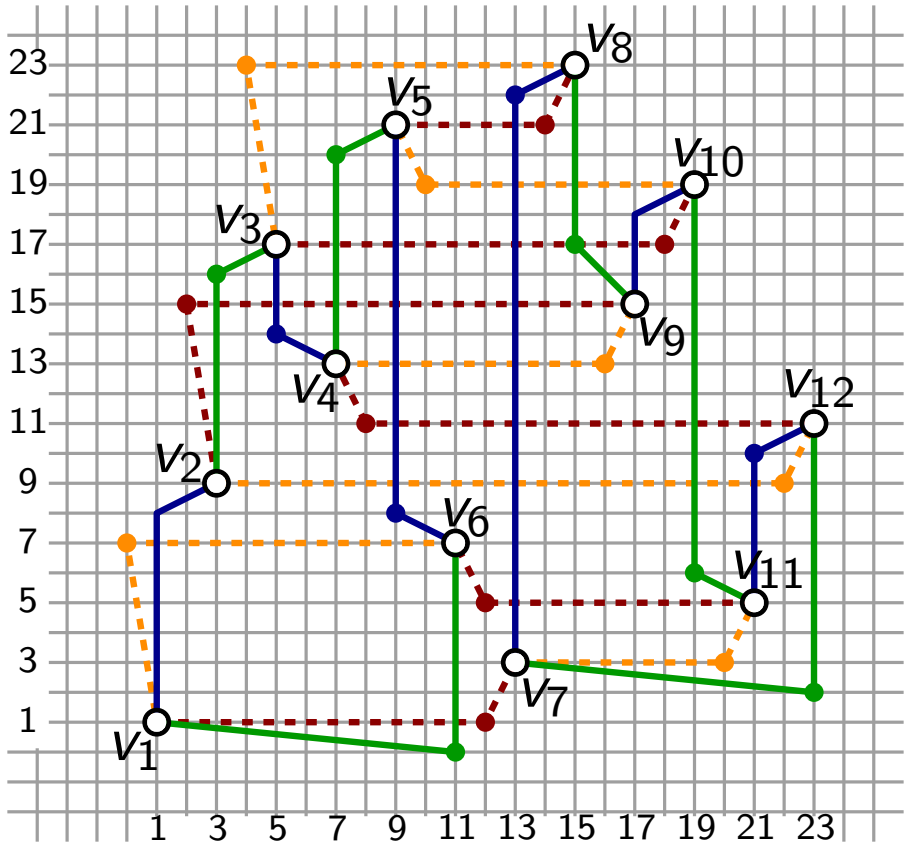
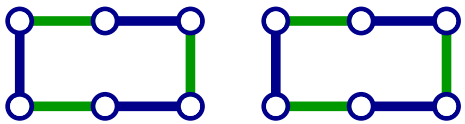


Four Matchings



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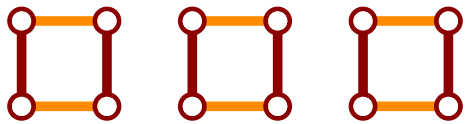

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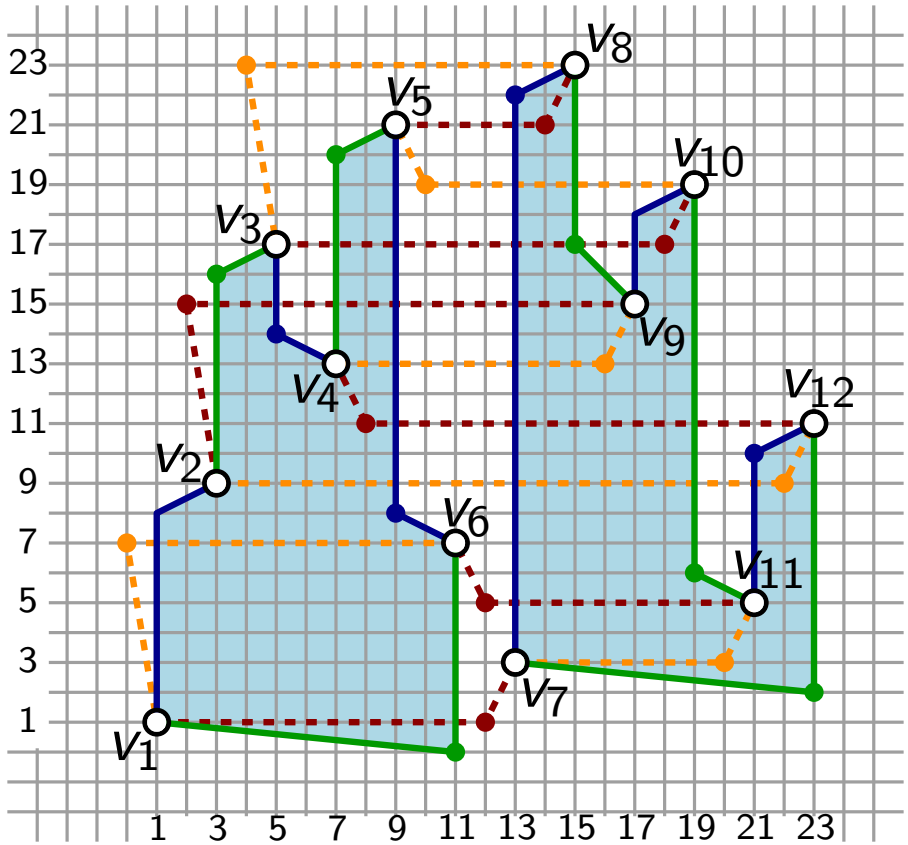
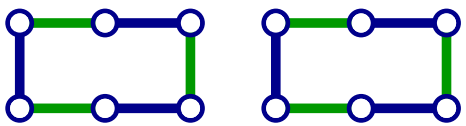


Four Matchings



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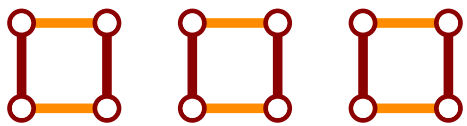

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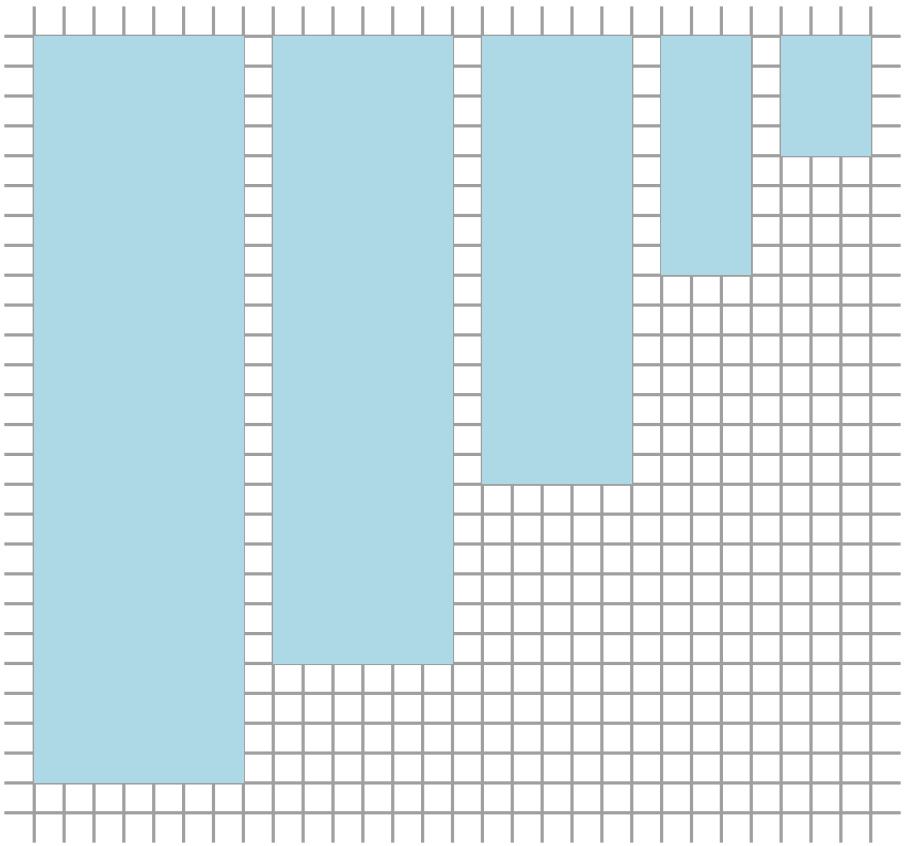
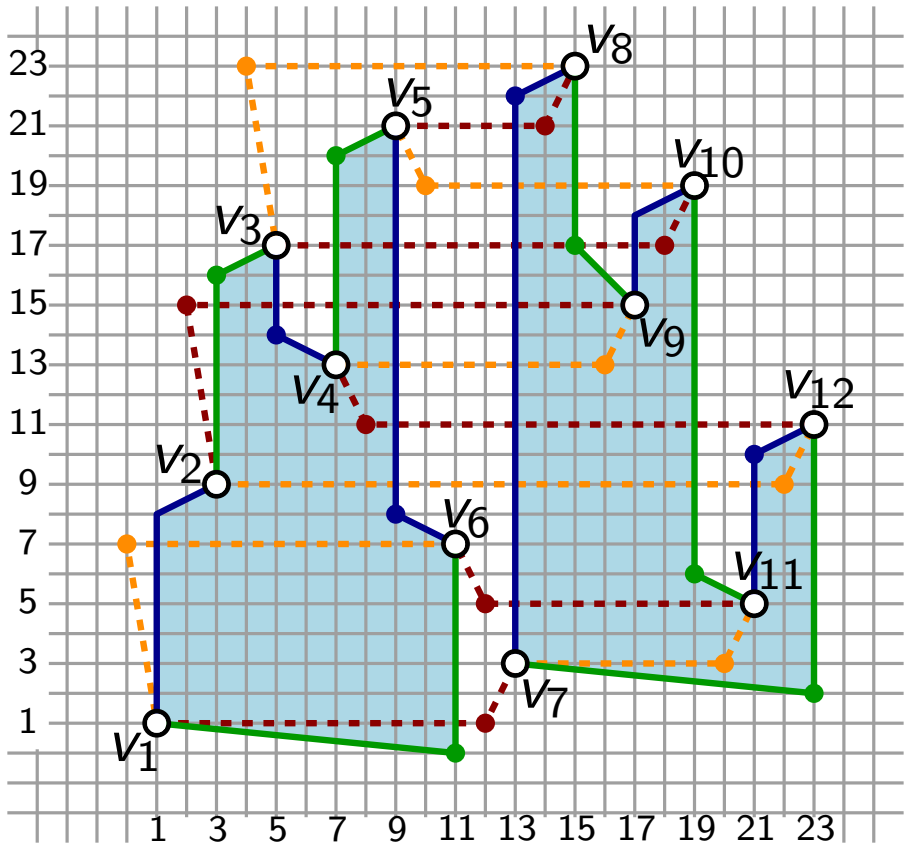
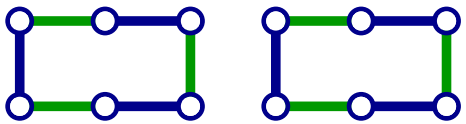




Four Matchings



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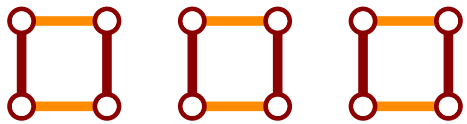

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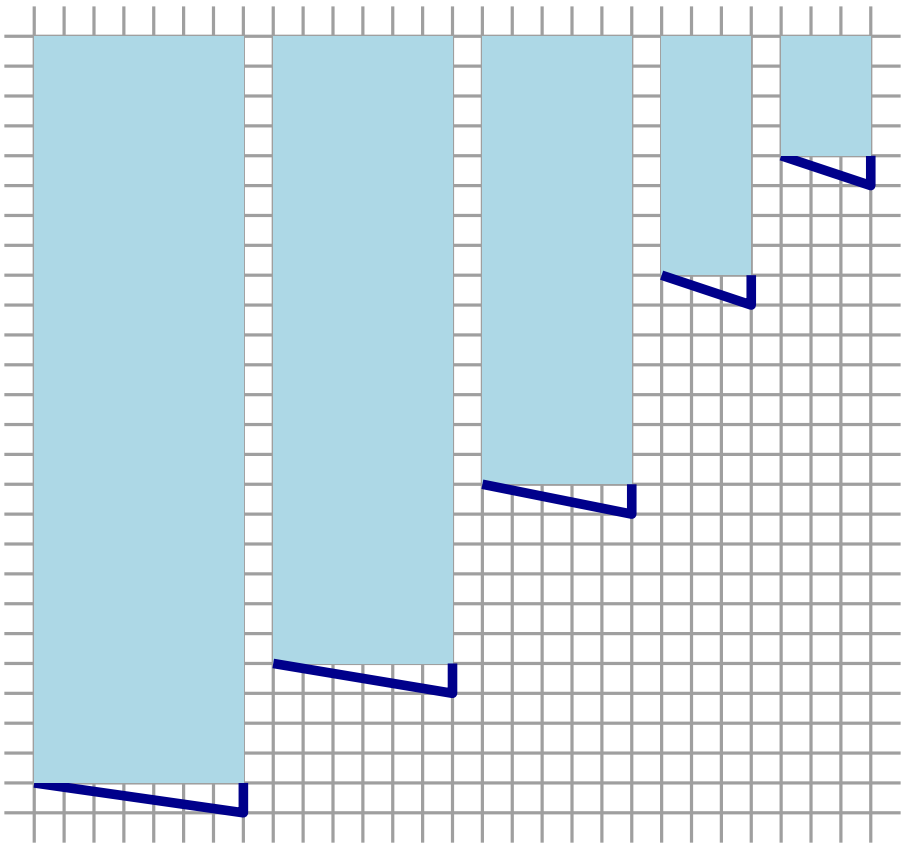

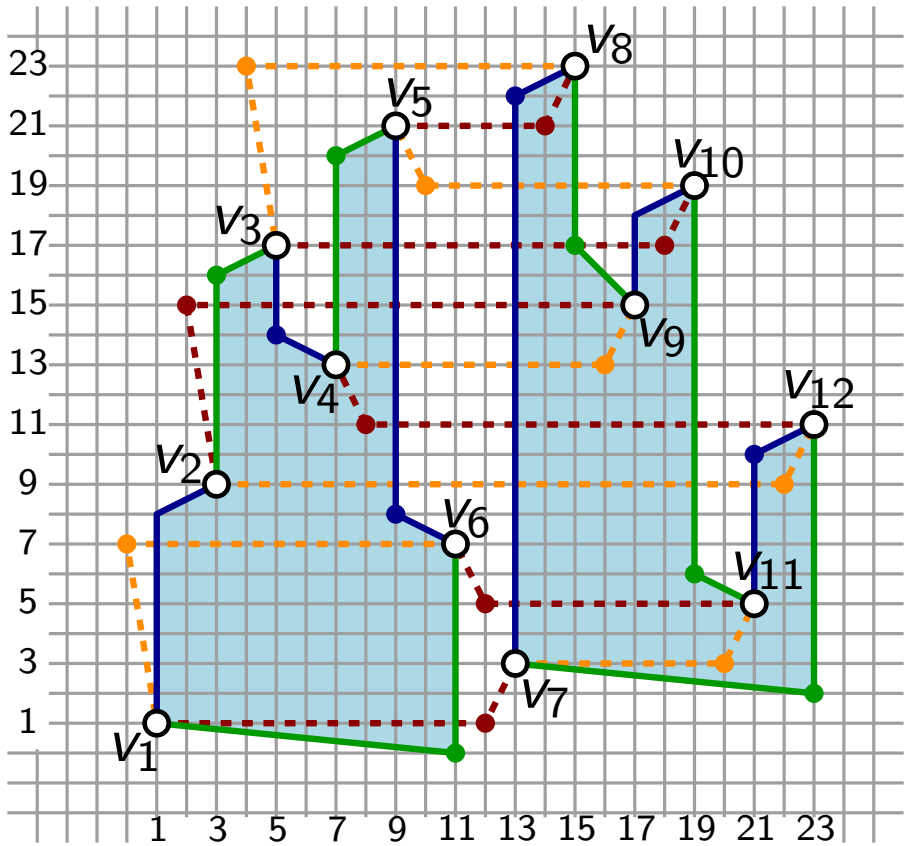

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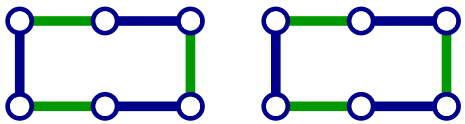
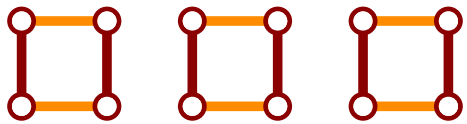


$\mathcal{C}_1$



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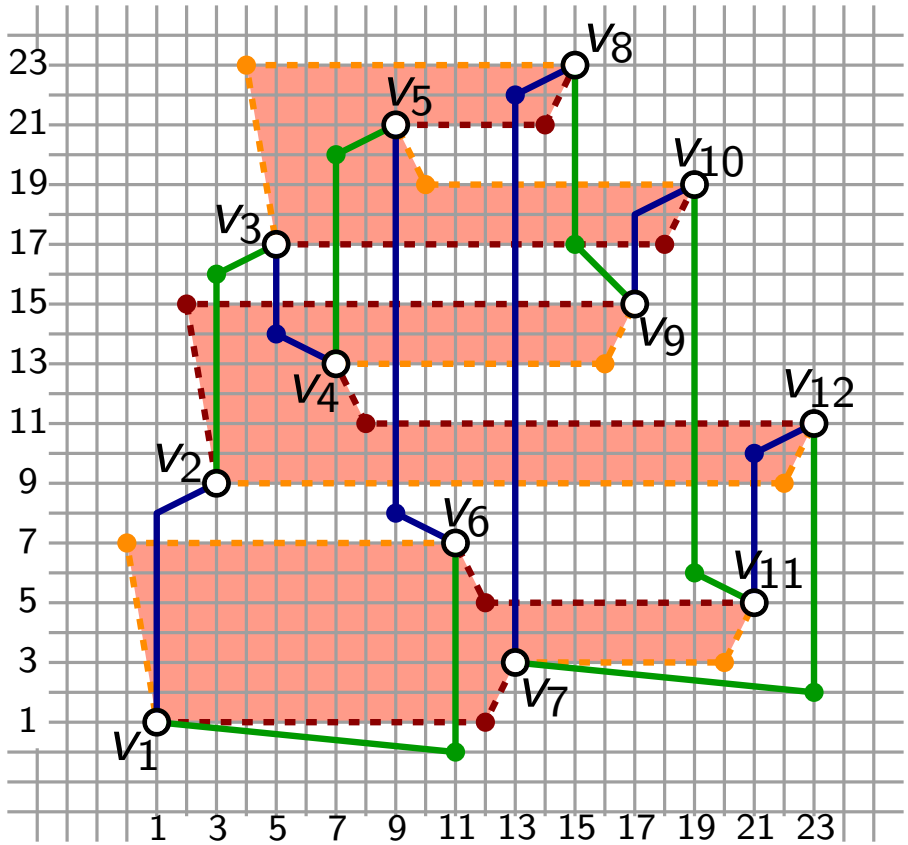

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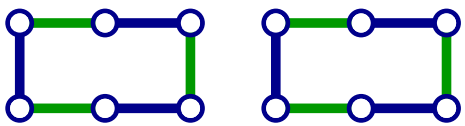
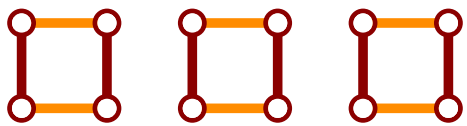


$\mathcal{C}_1$



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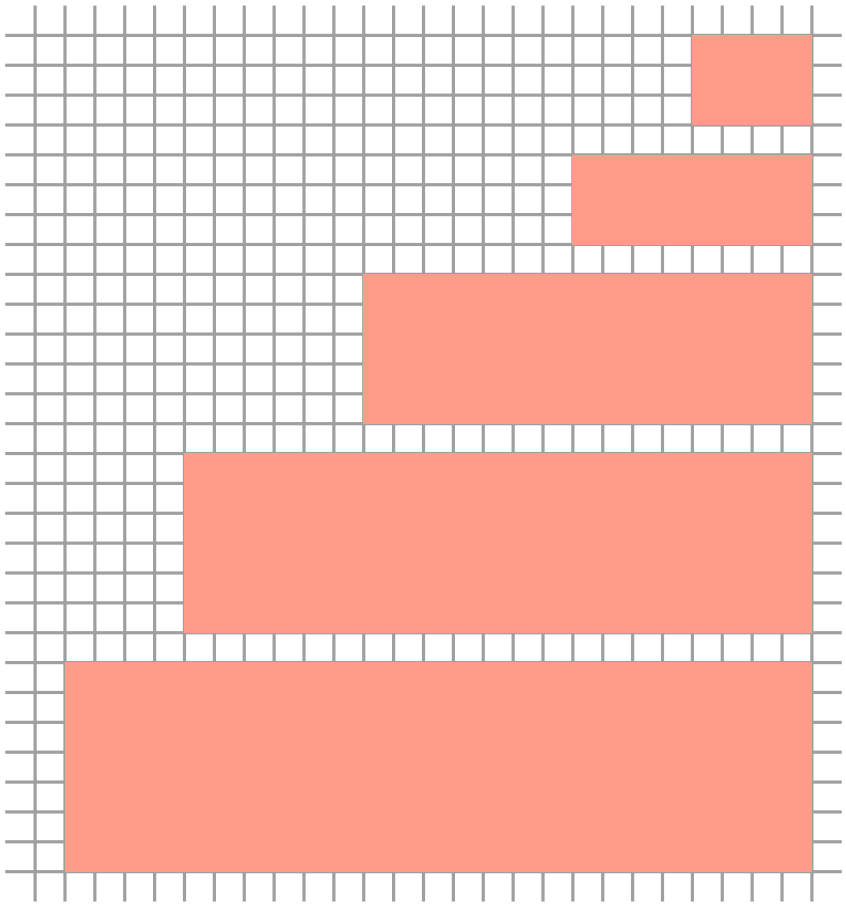
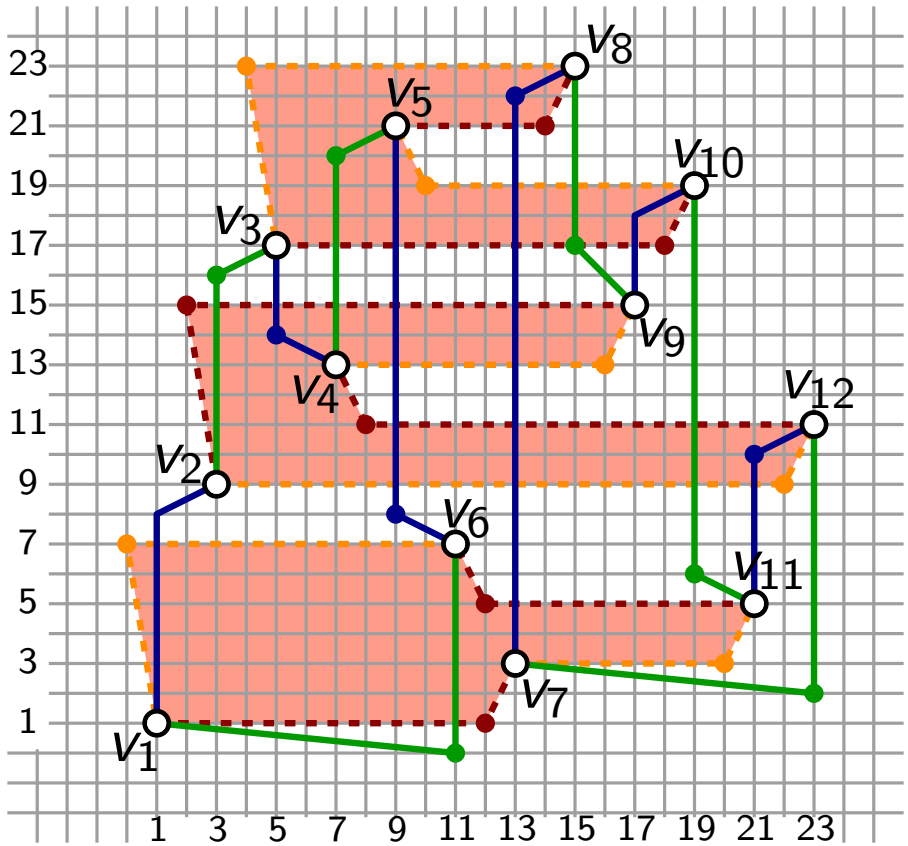
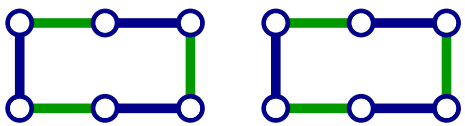




Four Matchings



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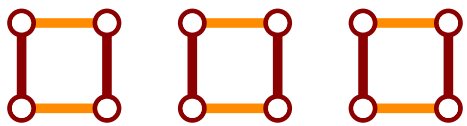

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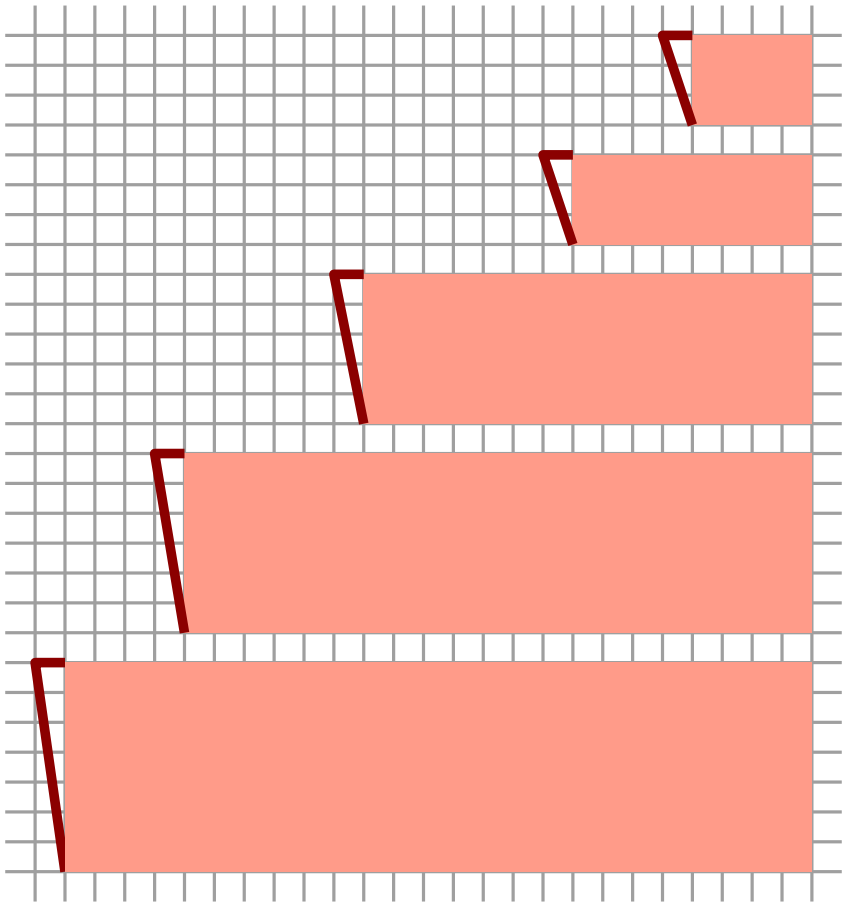
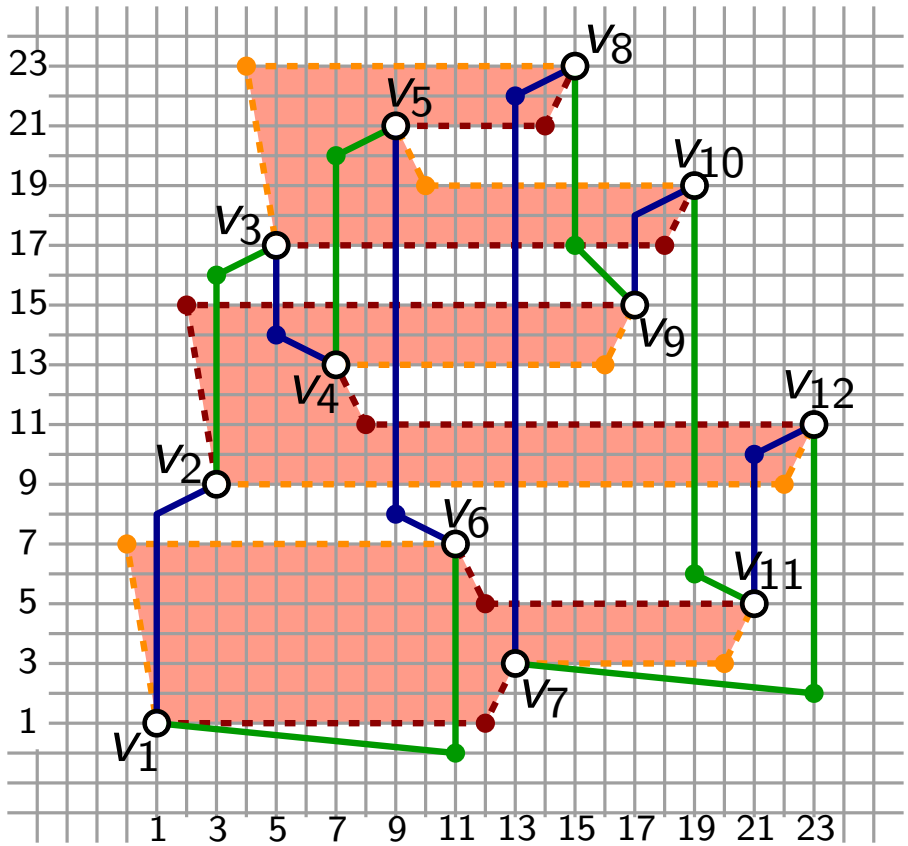
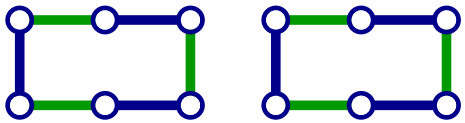
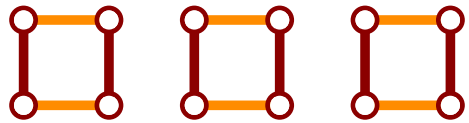




Four Matchings



$\mathcal{C}_1$



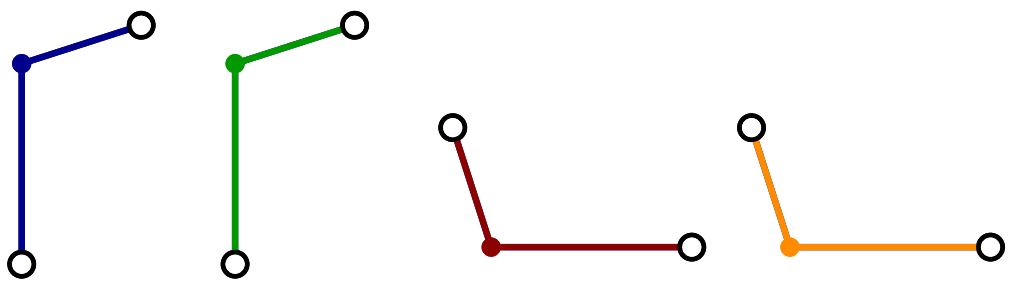
$\mathcal{C}_2$

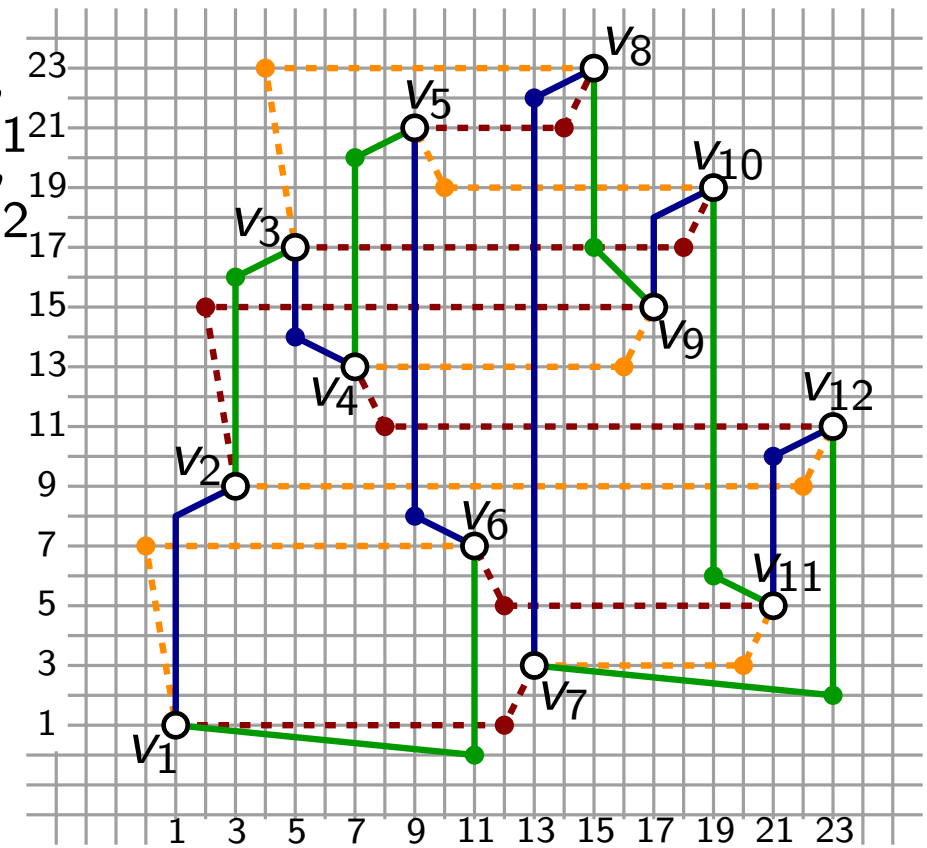
Bends: 1×1
Grid size:
$2n \times 2n$

Placement algorithm:

- Pick v_1 , place it
- Assign x-coords. to cycle in $\mathcal{C}_1$
- Assign y-coords. to cycle in $\mathcal{C}_2$
- Pick v with 1 assigned coord.
- Place v

Edges:





Overview

Graph classes			Number of bends
Cycle	$\times$	Cycle	1×1
Caterpillar	$\times$	Cycle	1×1
Four Matchings			$1 \times 1 \times 1 \times 1$
Tree	$\times$	Matching	1×0
Wheel	$\times$	Matching	2×0
Outerpath	$\times$	Matching	2×1
Outerplanar	$\times$	Outerplanar	3×3
2-page book emb.	$\times$	2-page book emb.	4×4
Planar	$\times$	Planar	6×6

Tree $\times$ Matching

Idea:
- Matching as horizontal line, Tree with 1 bend

Tree $\times$ Matching

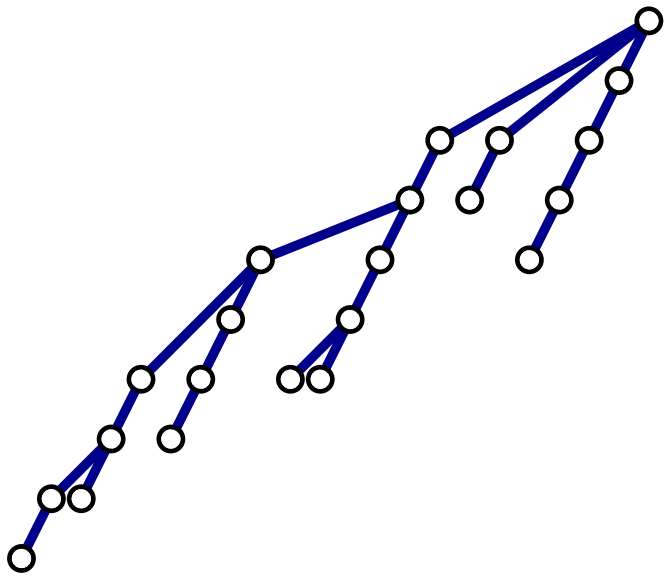
Idea:
- Matching as horizontal line, Tree with 1 bend
- Place Matching edges inductively $\Rightarrow$ y-coord.

Tree $\times$ Matching

Idea:
- Matching as horizontal line, Tree with 1 bend
- Place Matching edges inductively $\Rightarrow$ y-coord.
- Use post-order on Tree $\Rightarrow$ x-coord.

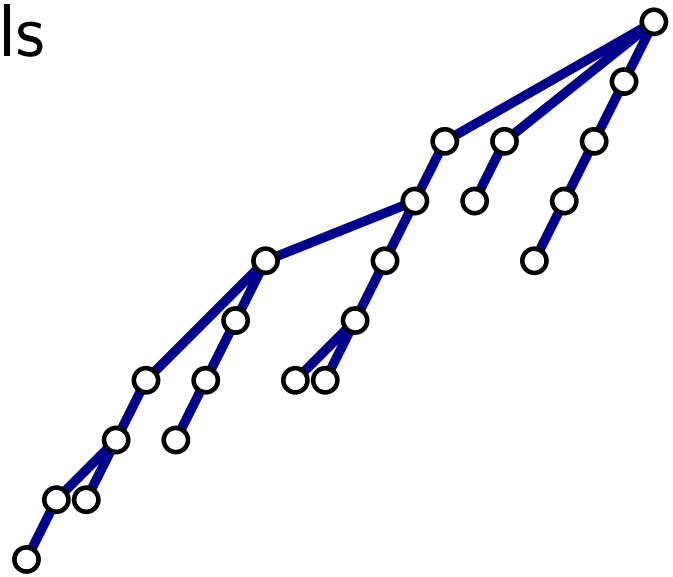
Tree $\times$ Matching

Idea:
- Matching as horizontal line, Tree with 1 bend
- Place Matching edges inductively $\Rightarrow$ y-coord.
- Use post-order on Tree $\Rightarrow$ x-coord.



Tree $\times$ Matching

Idea:
- Matching as horizontal line, Tree with 1 bend
- Place Matching edges inductively $\Rightarrow$ y-coord.
- Use post-order on Tree $\Rightarrow$ x-coord.
- $\Rightarrow$ Subtrees in disjoint x-intervals



Tree $\times$ Matching

Idea:
- Matching as horizontal line, Tree with 1 bend
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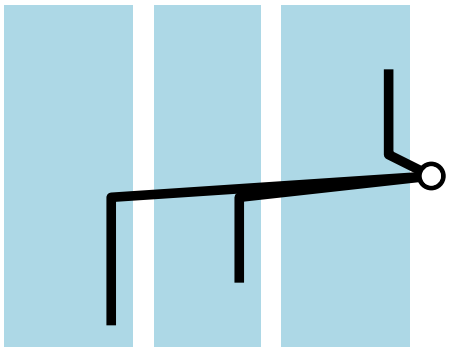
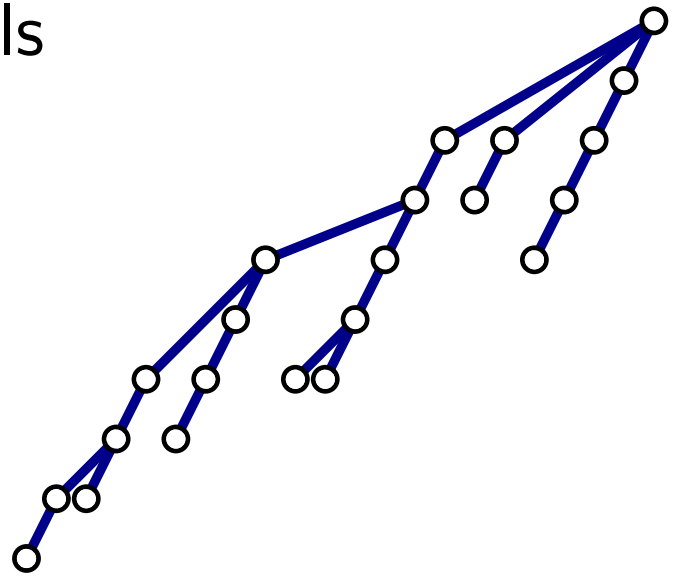
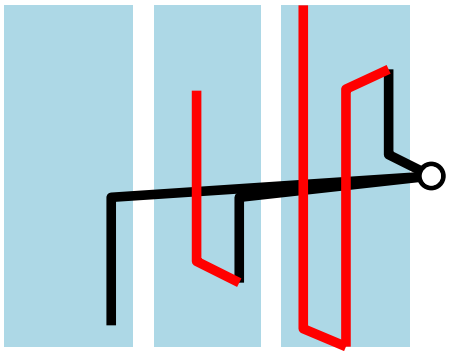


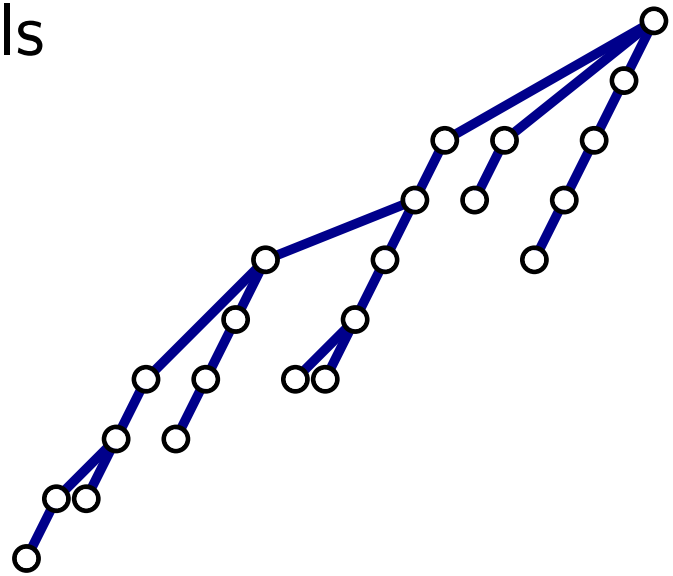


Tree $\times$ Matching

Idea:
- Matching as horizontal line, Tree with 1 bend
- Place Matching edges inductively $\Rightarrow$ y-coord.
- Use post-order on Tree $\Rightarrow$ x-coord.
- $\Rightarrow$ Subtrees in disjoint x-intervals

Problem:

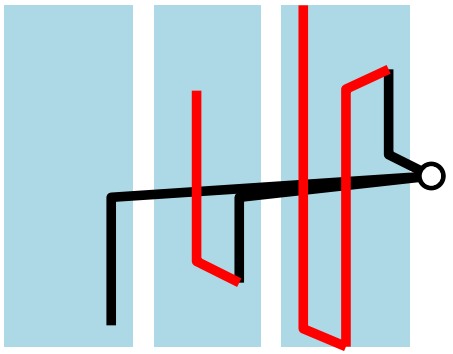


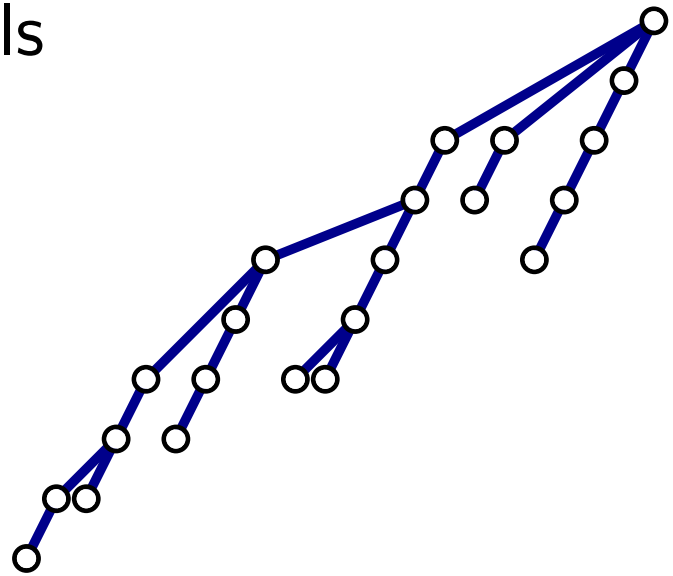


Tree $\times$ Matching

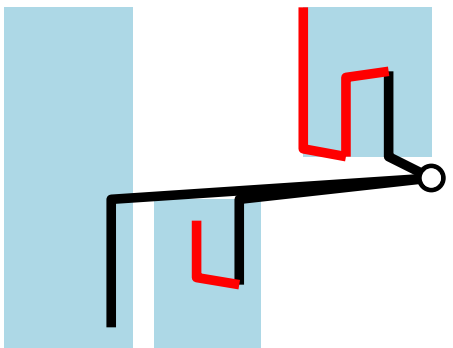
Idea:
- Matching as horizontal line, Tree with 1 bend
- Place Matching edges inductively $\Rightarrow$ y-coord.
- Use post-order on Tree $\Rightarrow$ x-coord.
- $\Rightarrow$ Subtrees in disjoint x-intervals

Problem:





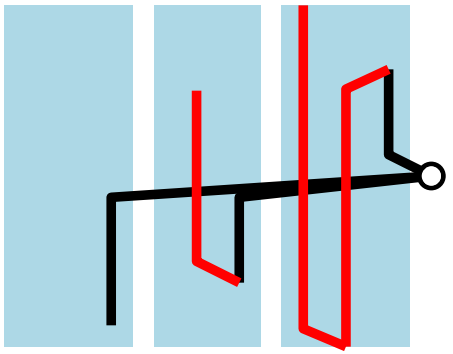
Solution:

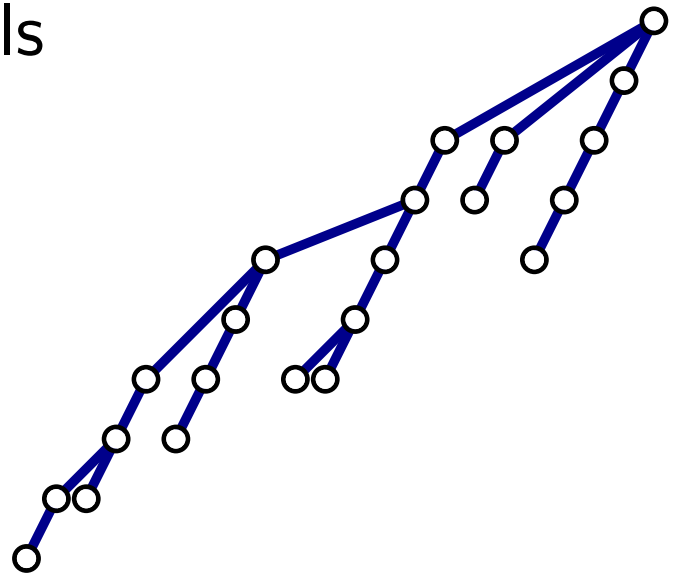


Tree $\times$ Matching

Idea:
- Matching as horizontal line, Tree with 1 bend
- Place Matching edges inductively $\Rightarrow$ y-coord.
- Use post-order on Tree $\Rightarrow$ x-coord.
- $\Rightarrow$ Subtrees in disjoint x-intervals

Problem:





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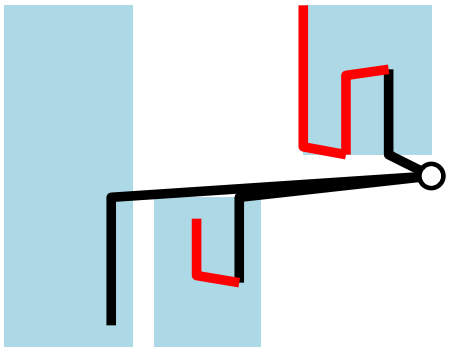
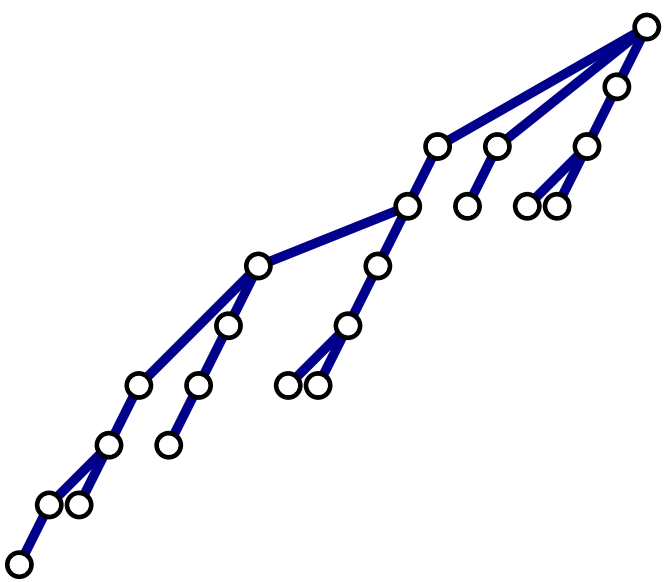
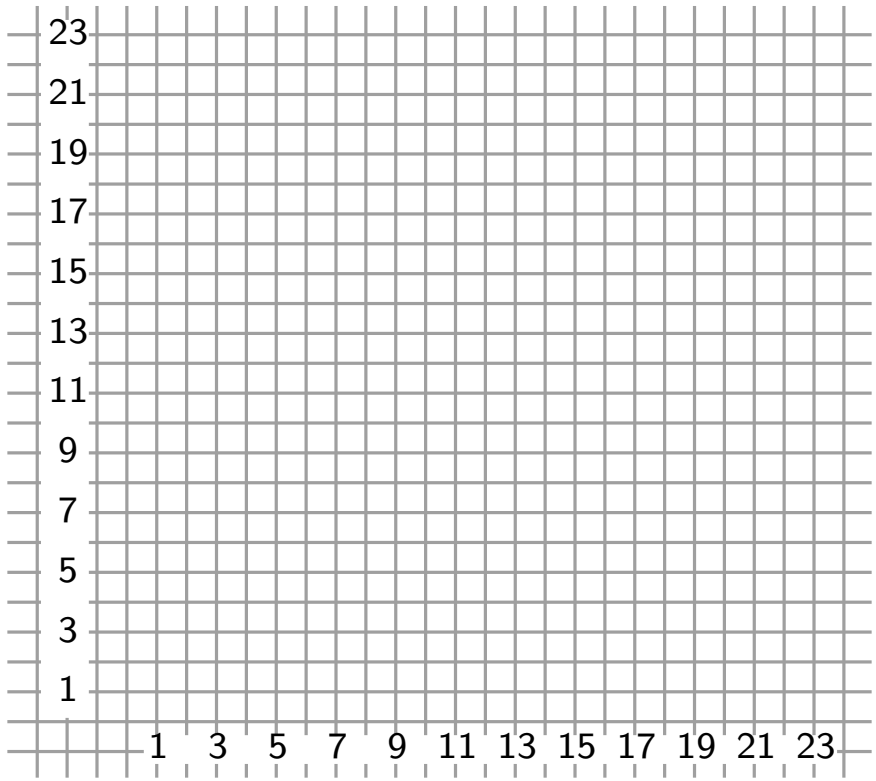


All but one subtree completely above or completely below

Tree $\times$ Matching

- Place root + matching at the top





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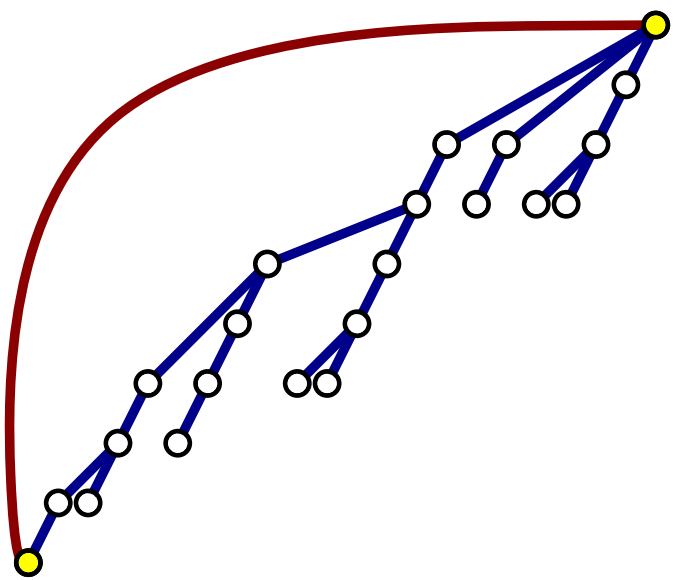

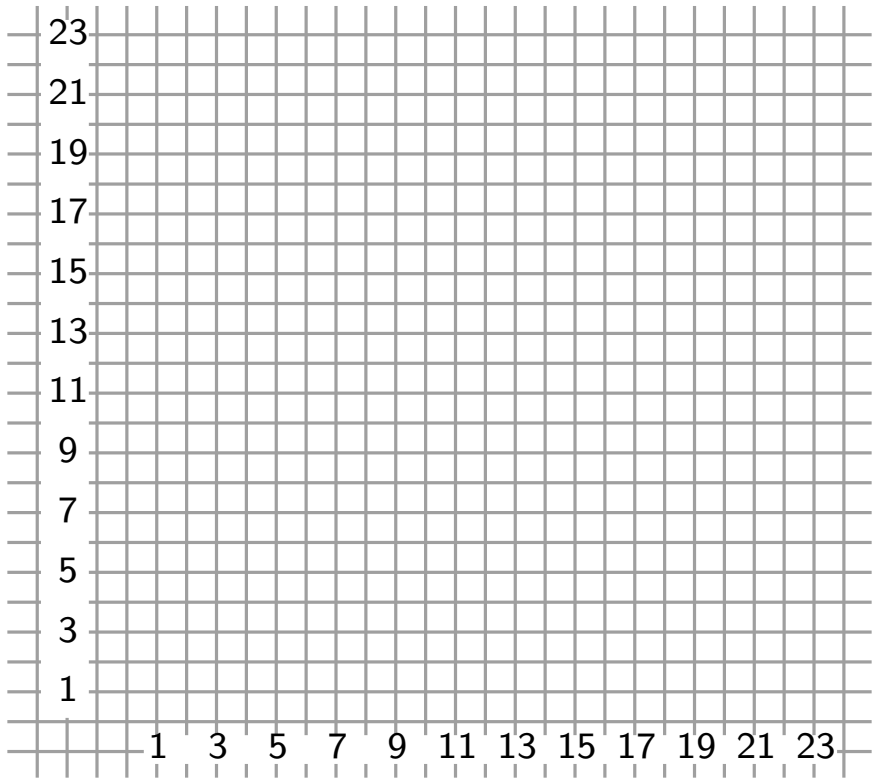

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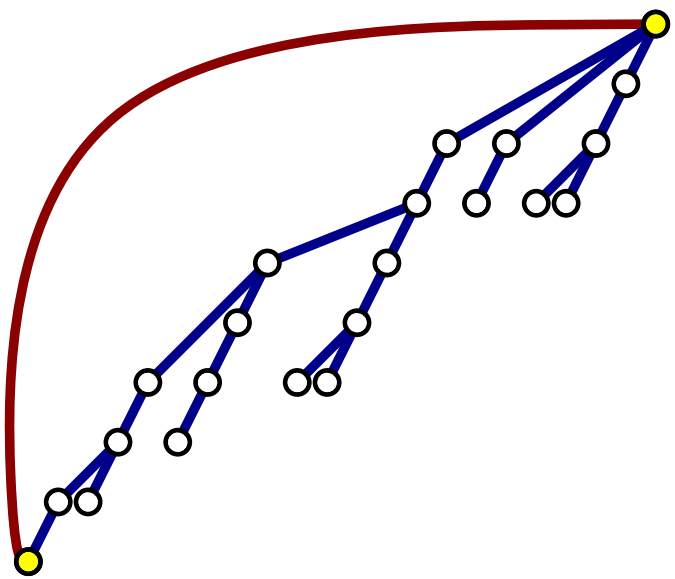
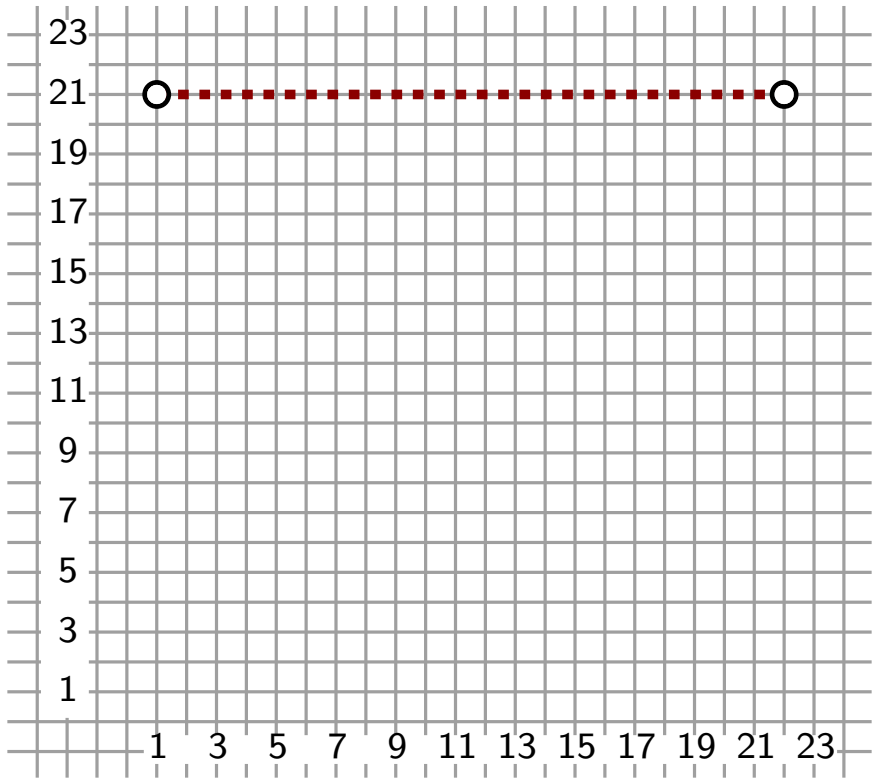
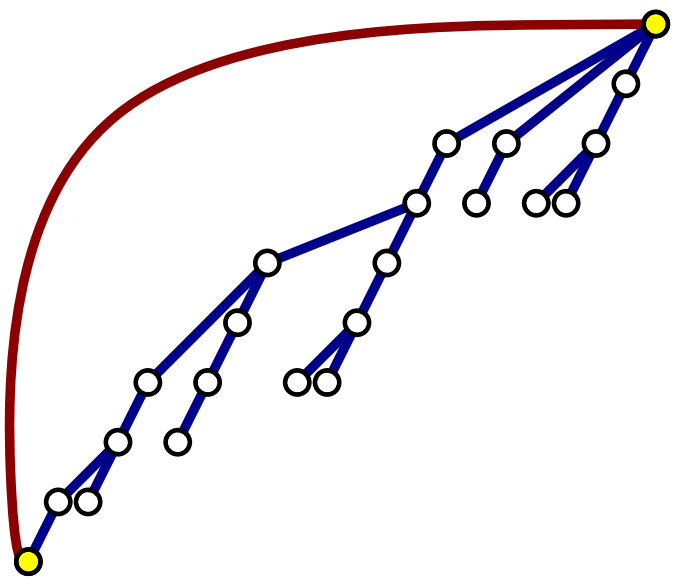
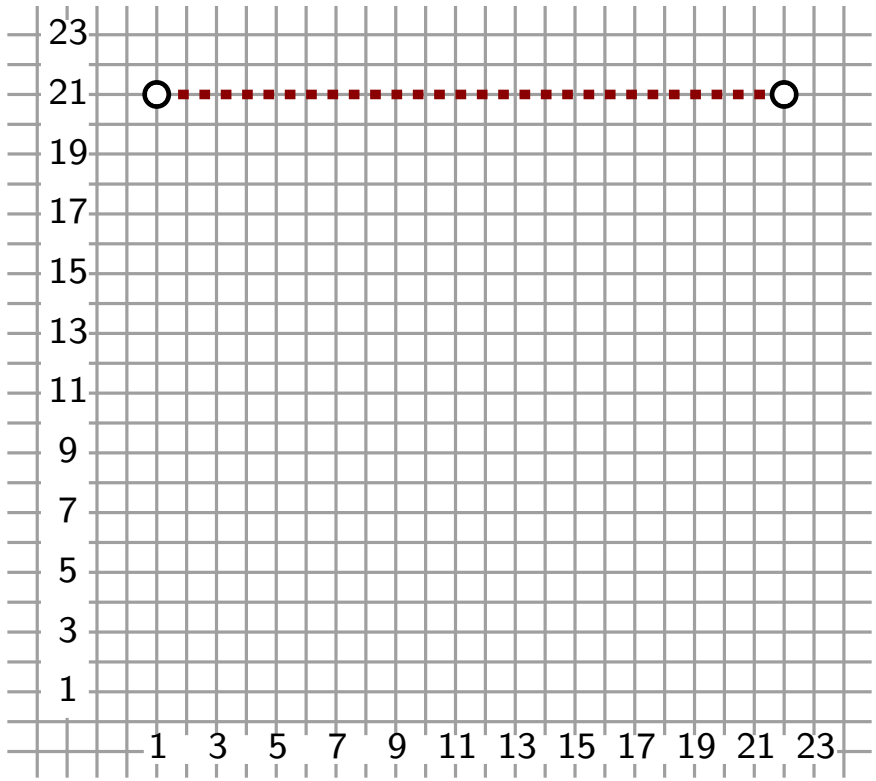




Tree $\times$ Matching

- Place root + matching at the top
- Split the tree





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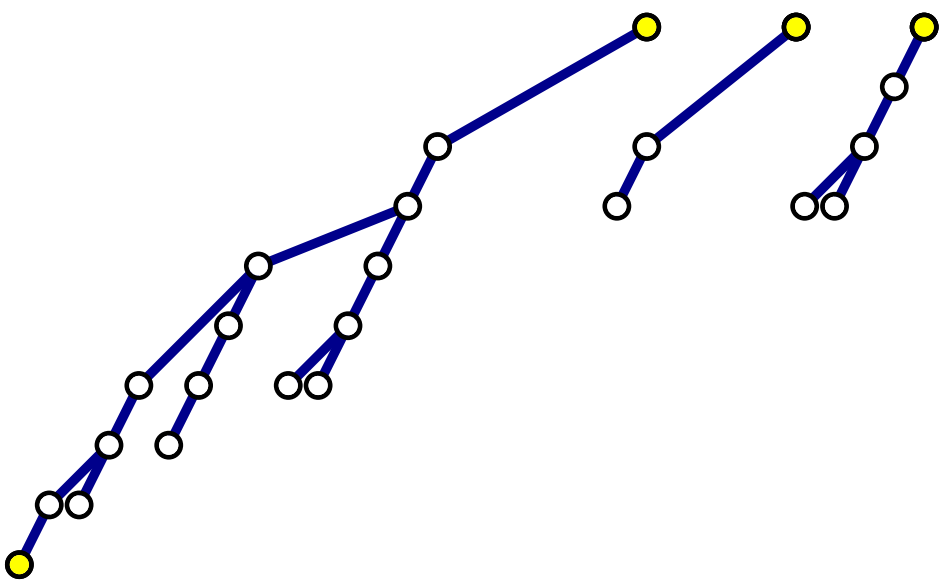
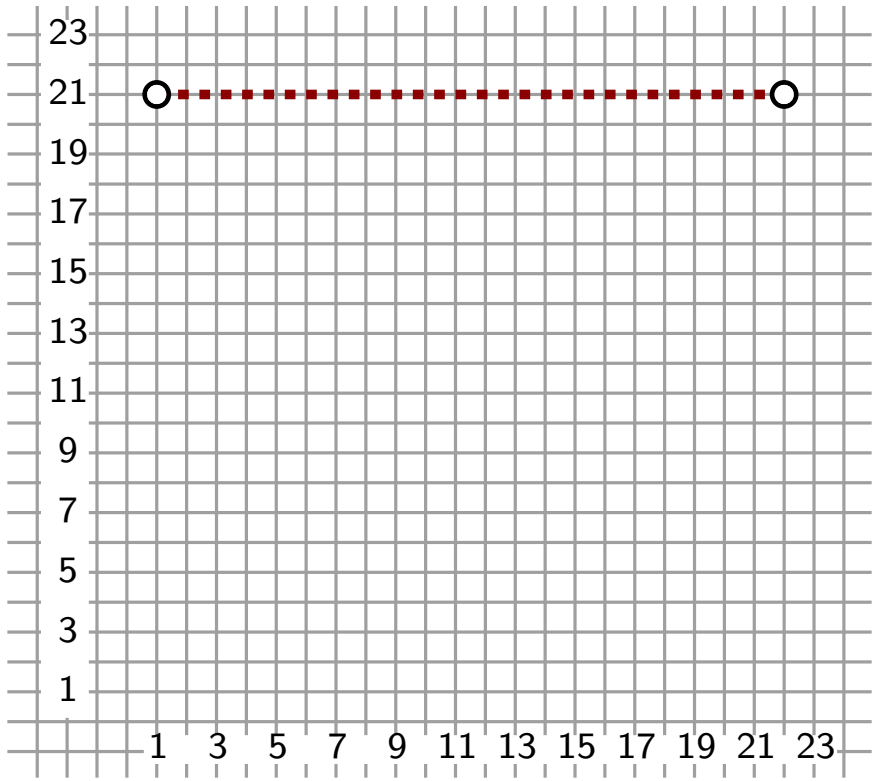
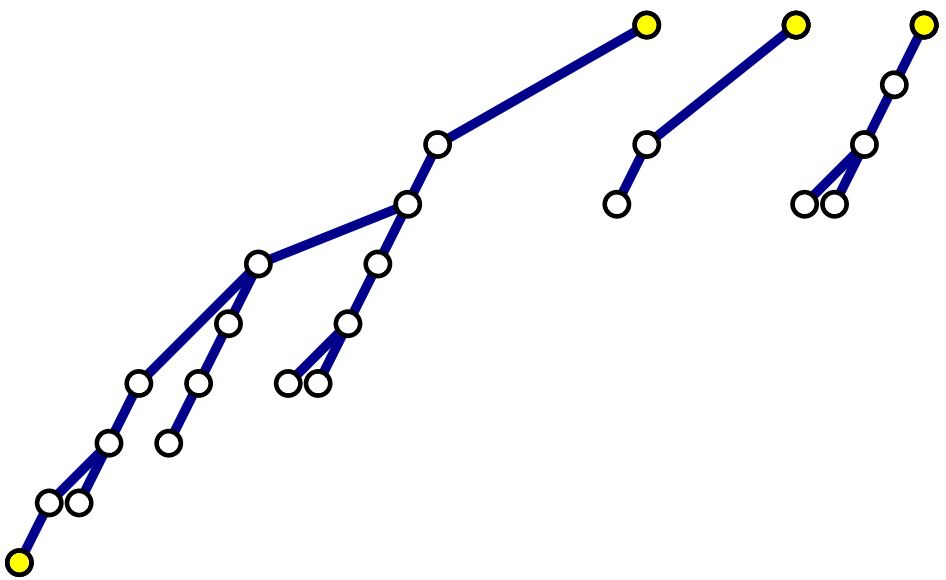
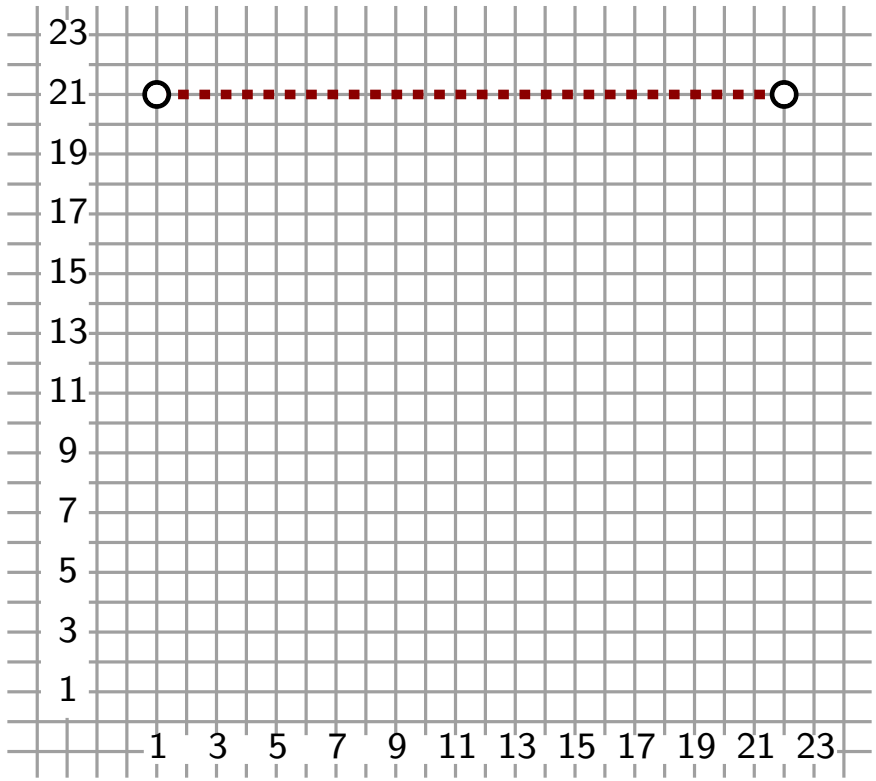




Tree $\times$ Matching

- Place root + matching at the top
- Split the tree
- Place vertex adj. to placed vertex (+ matching) at the top





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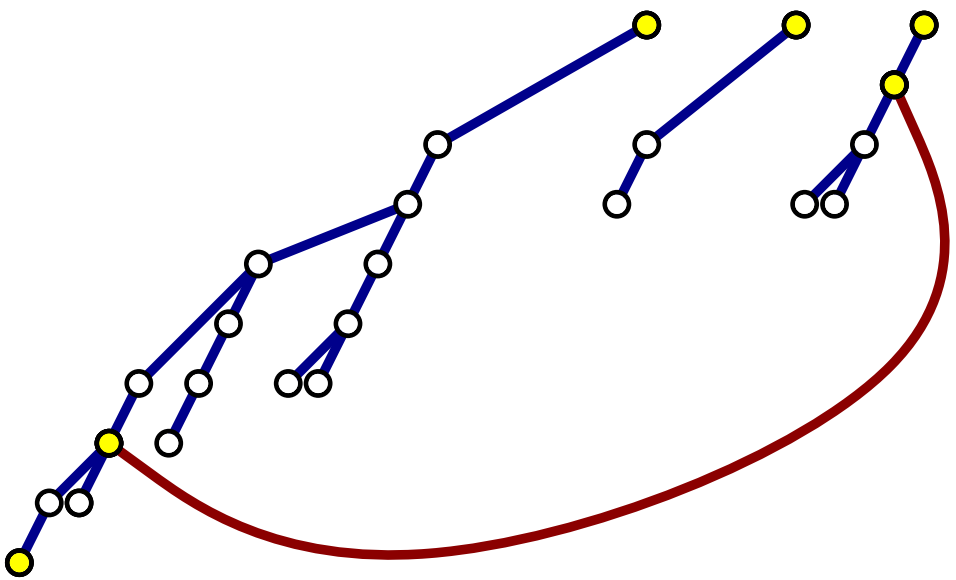

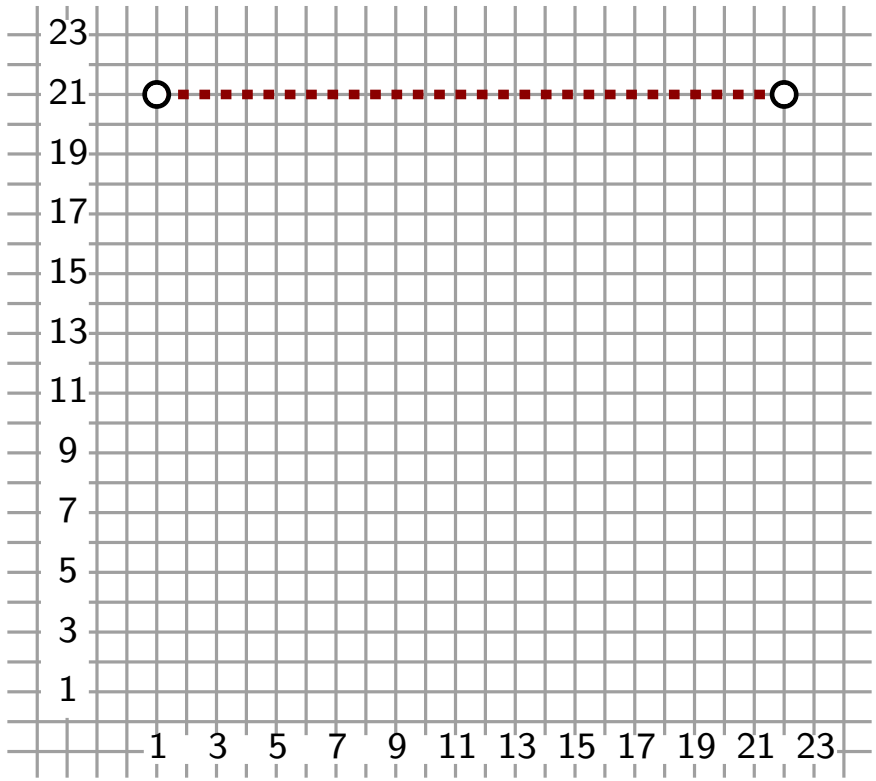

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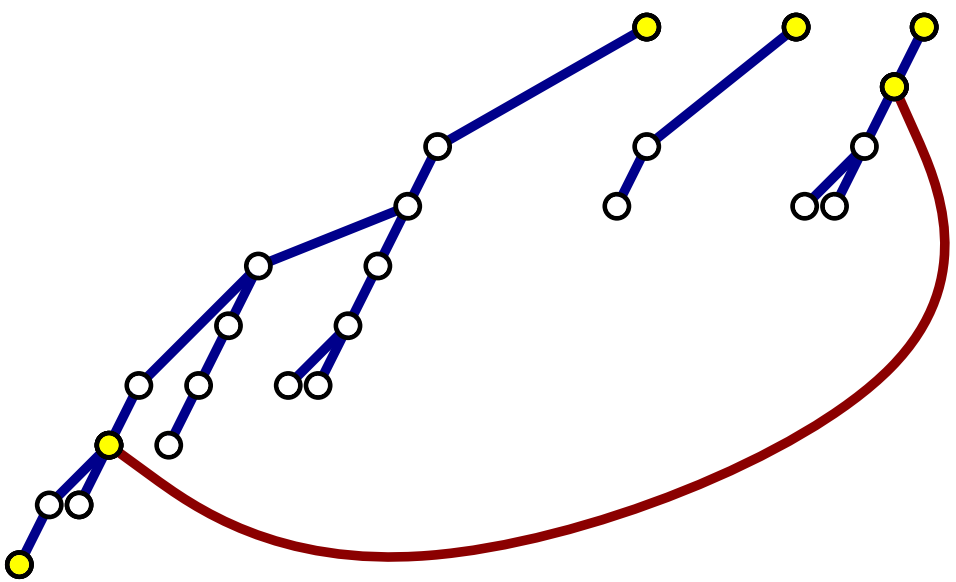

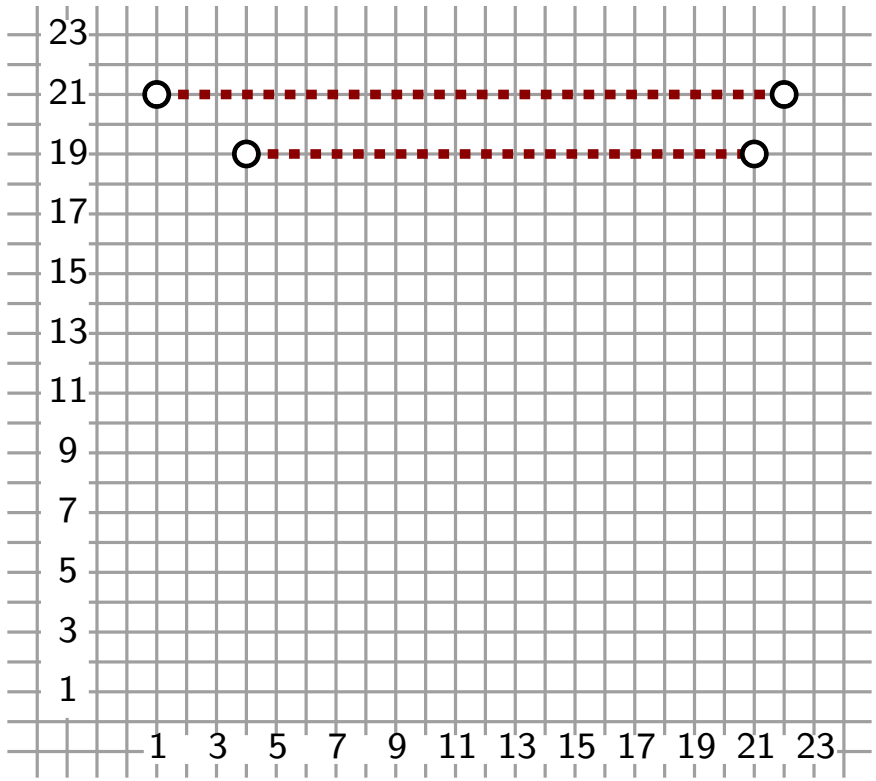

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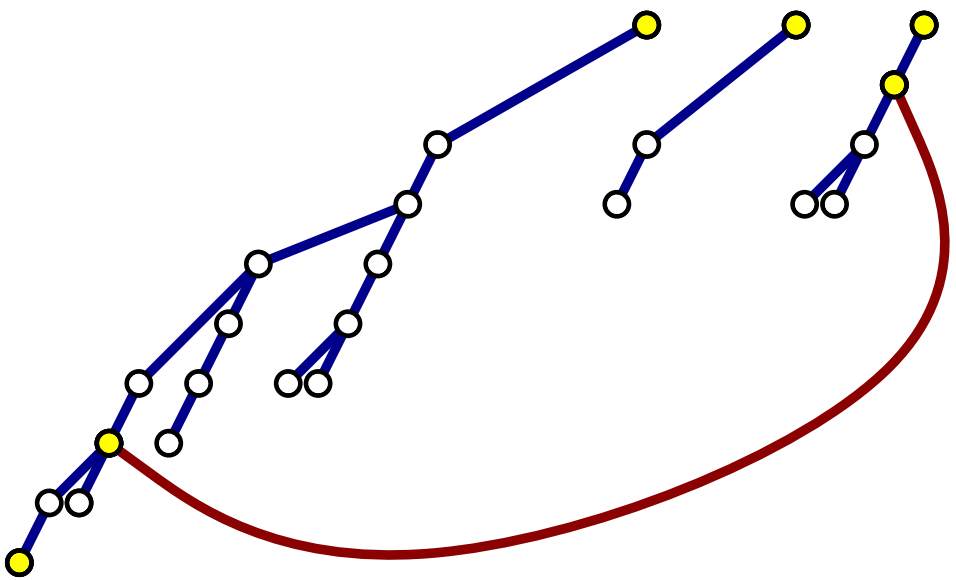

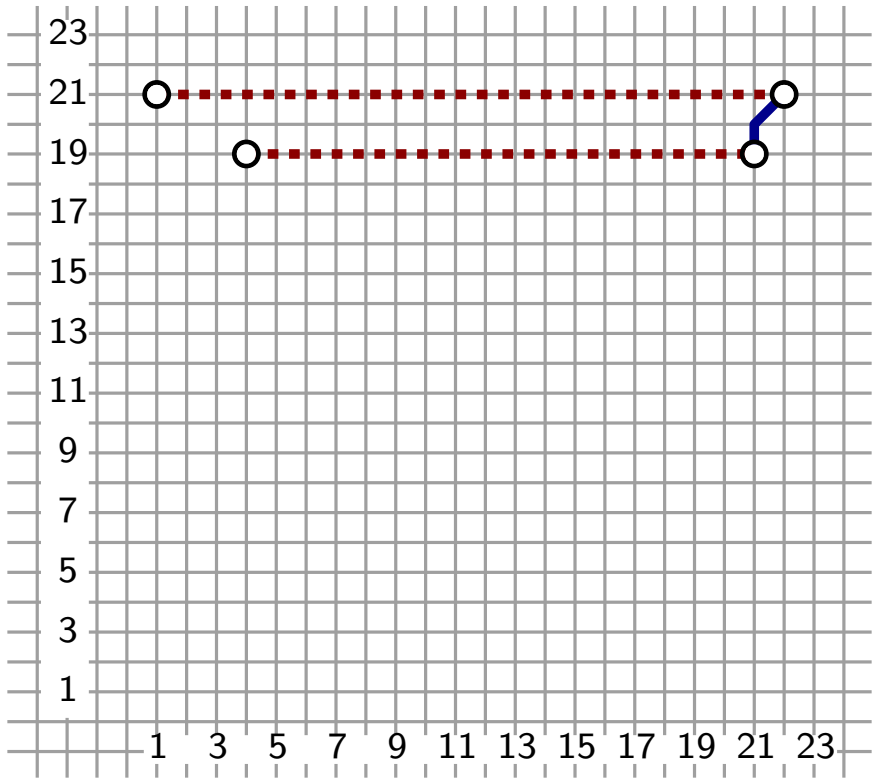

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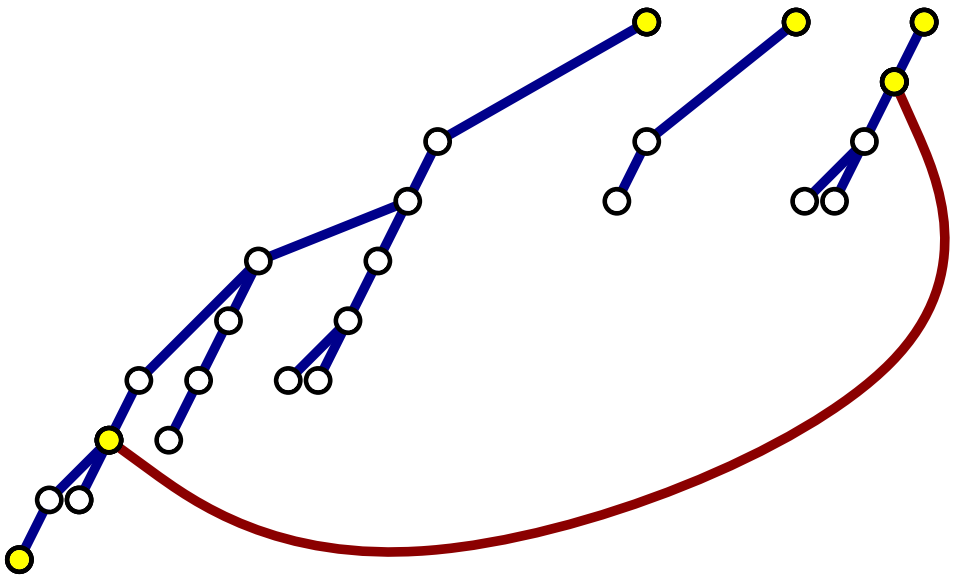

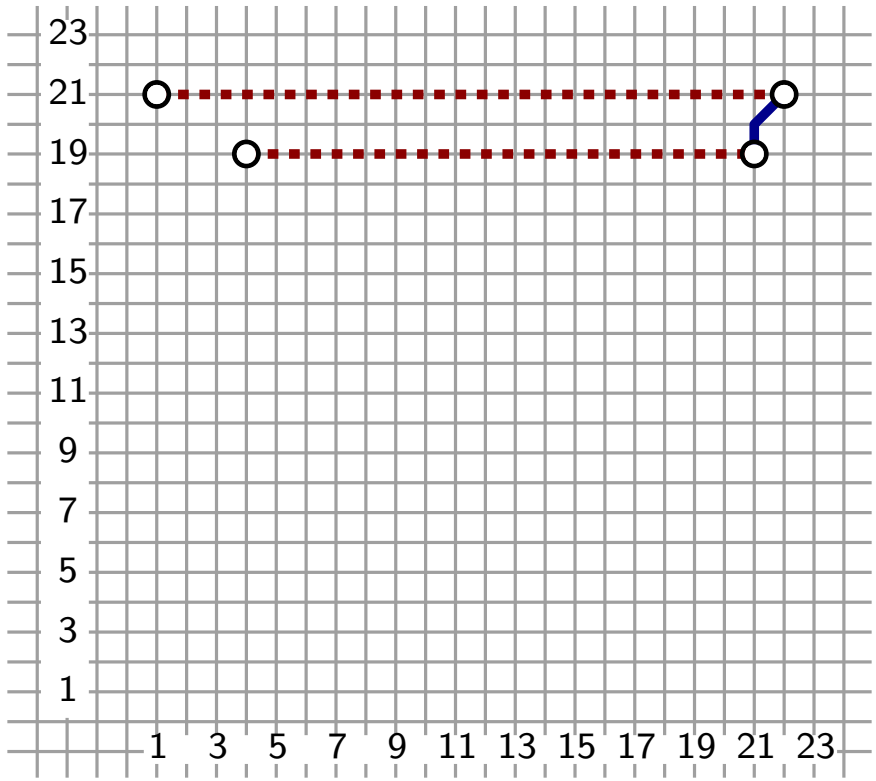

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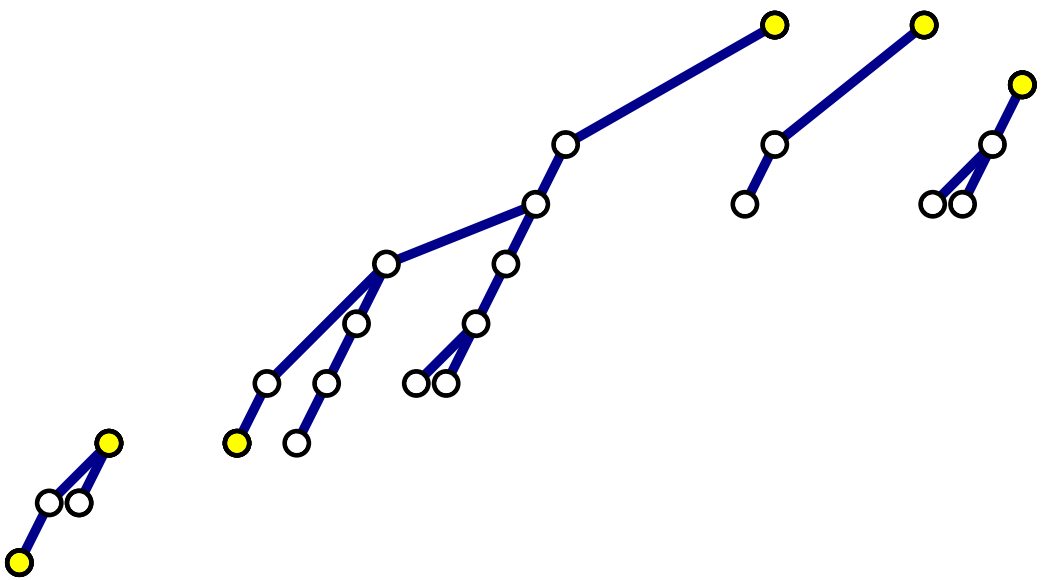

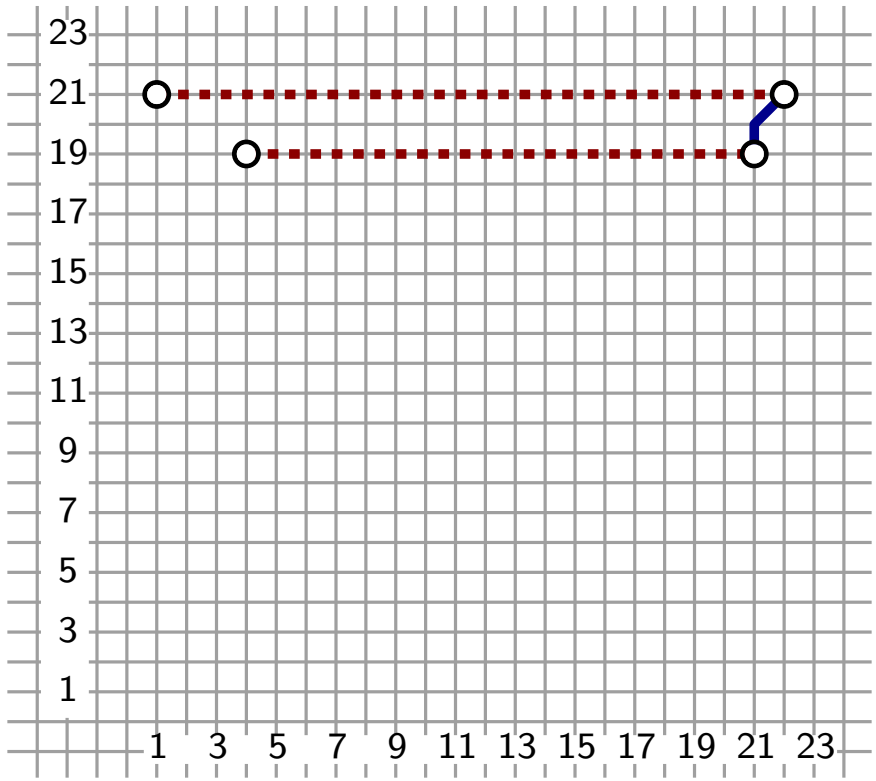

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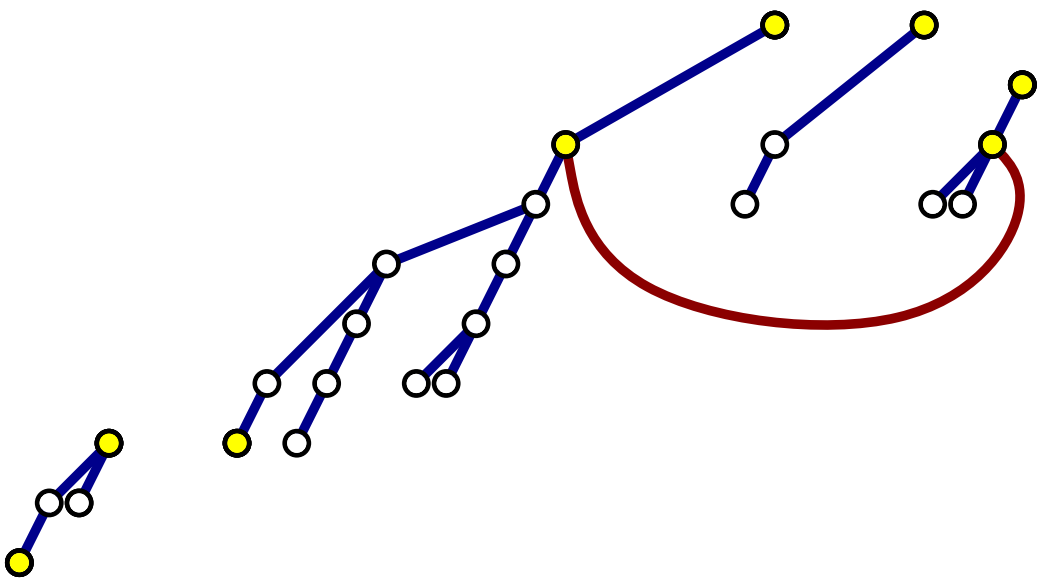

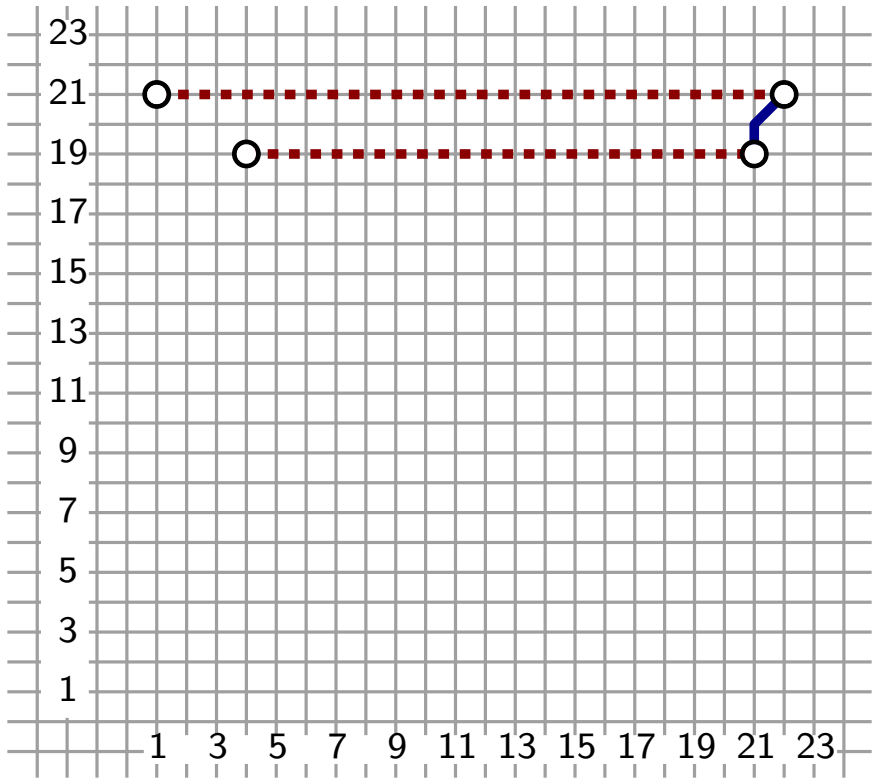

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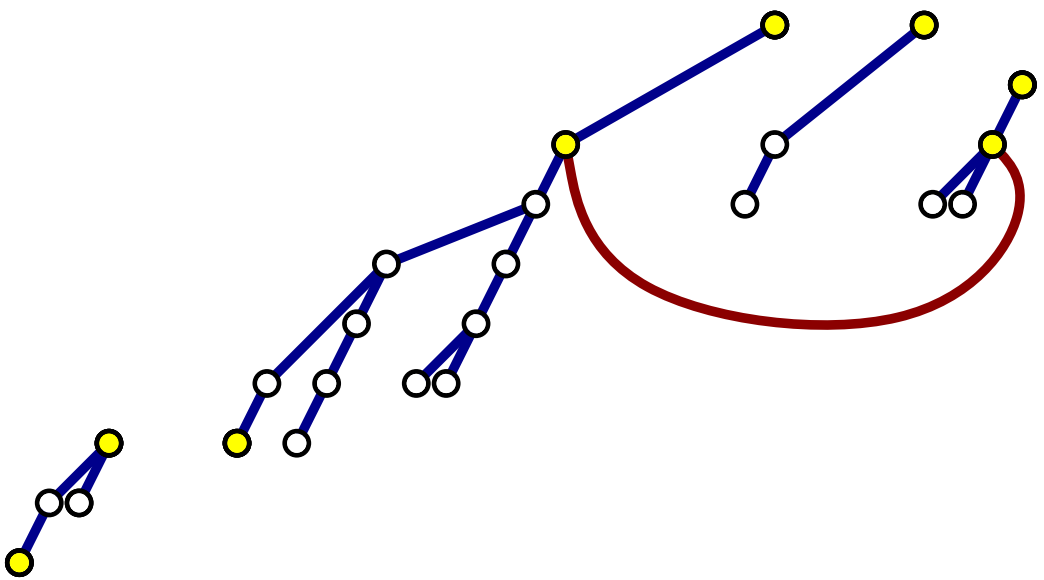

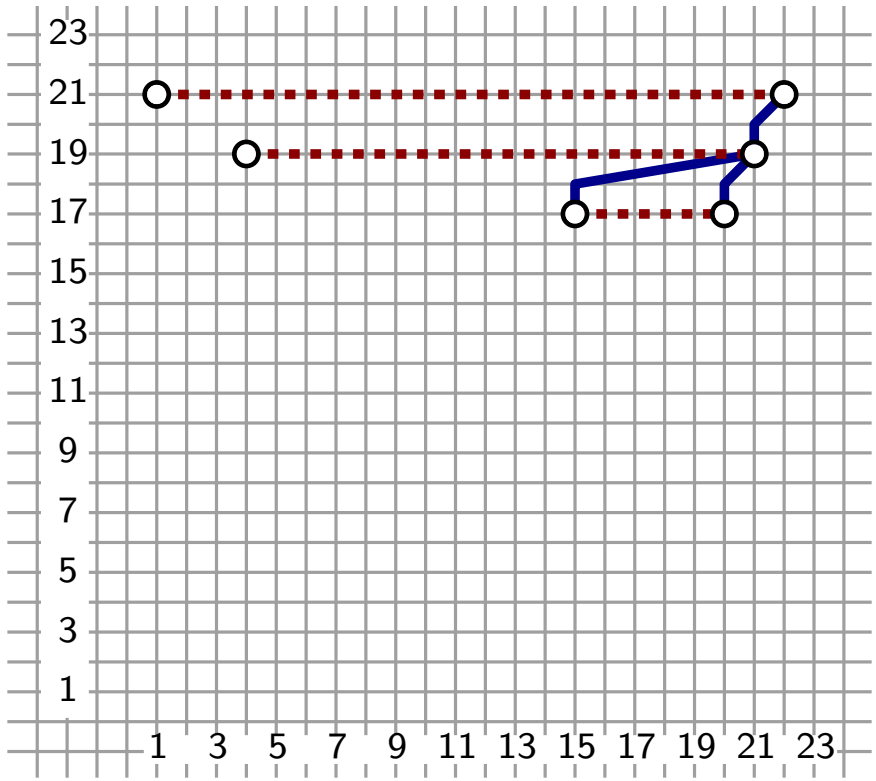

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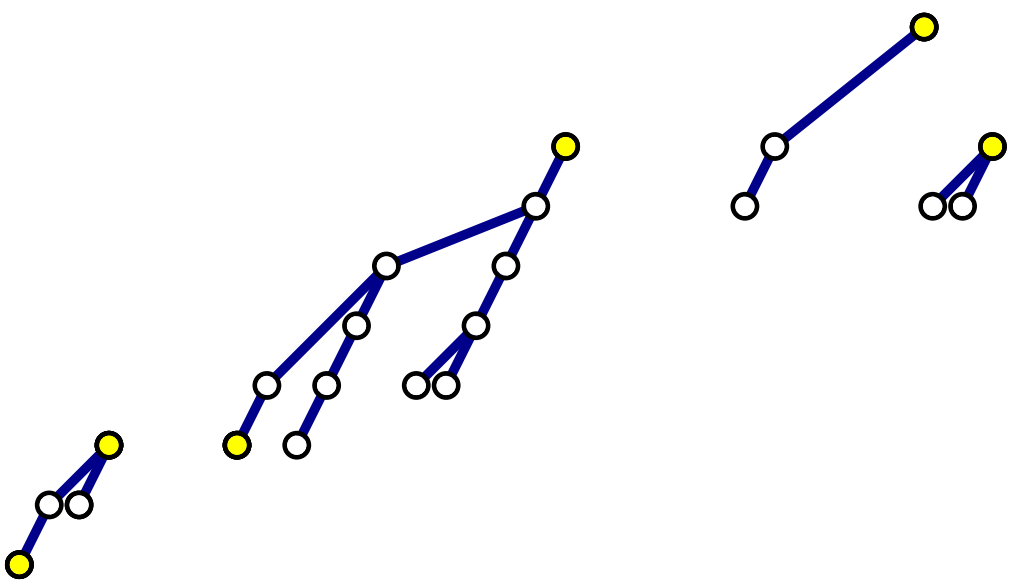

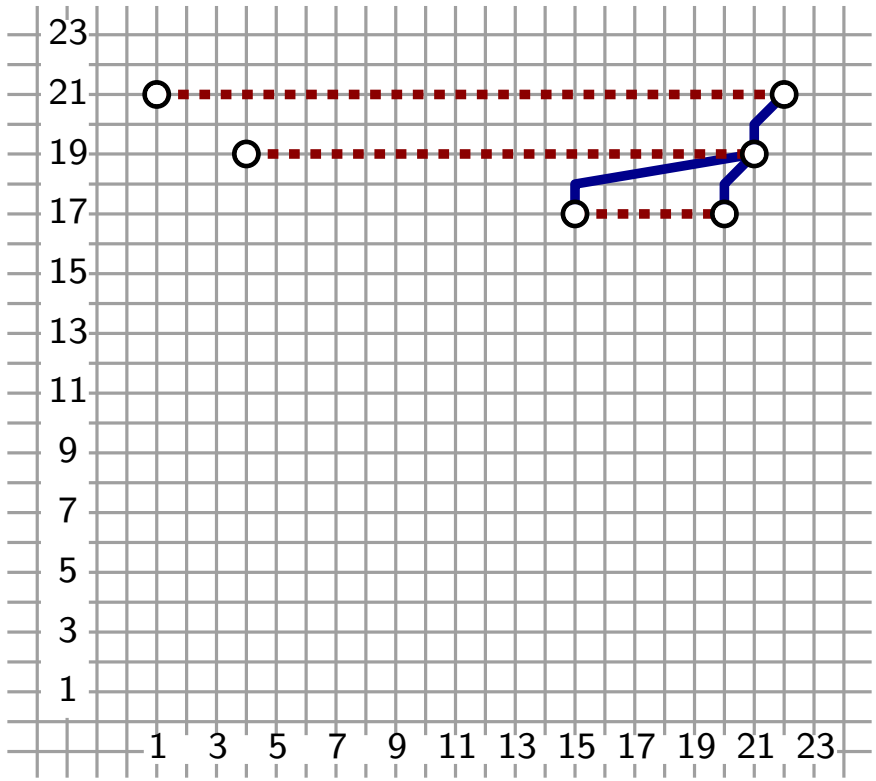

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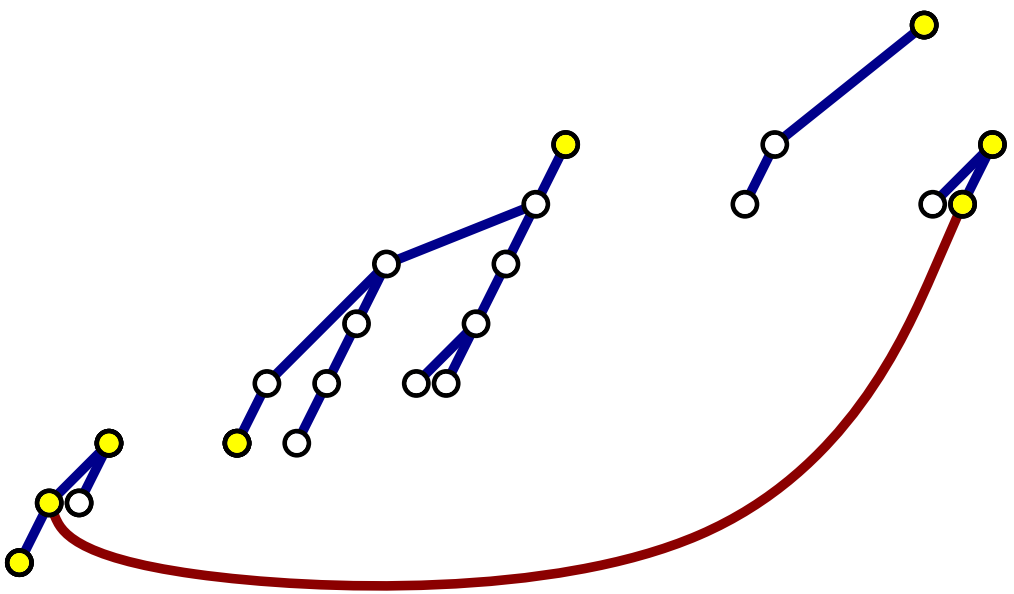

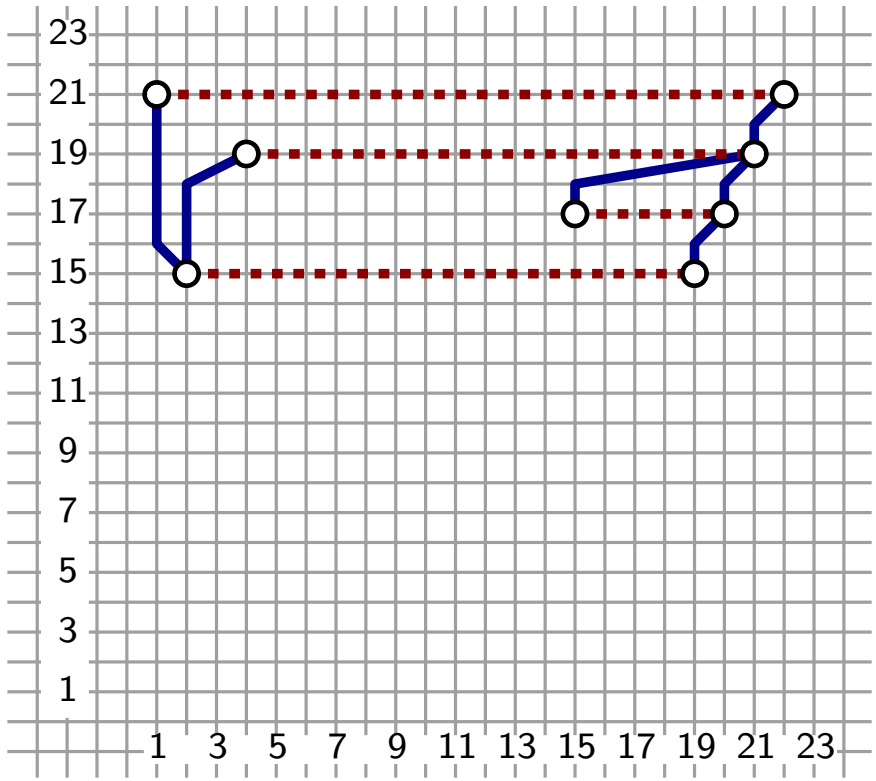

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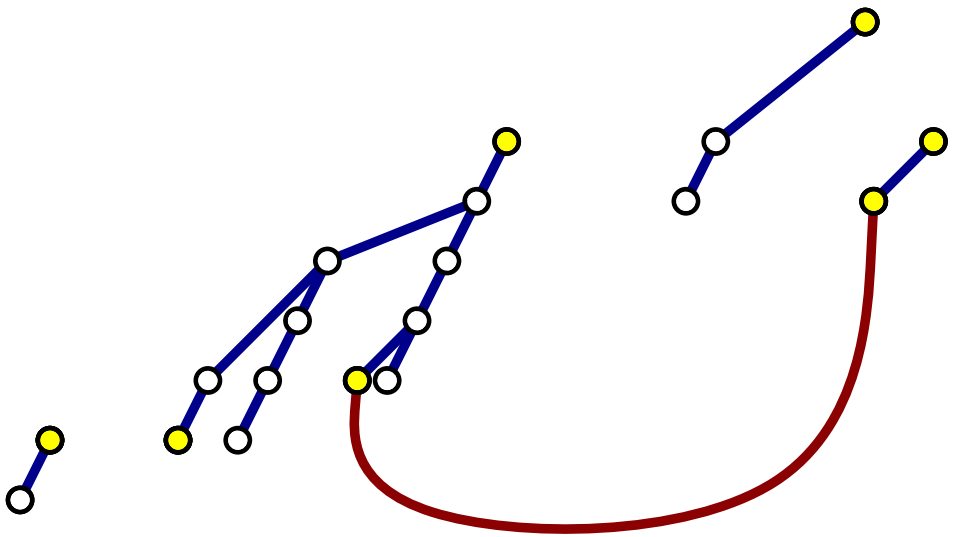

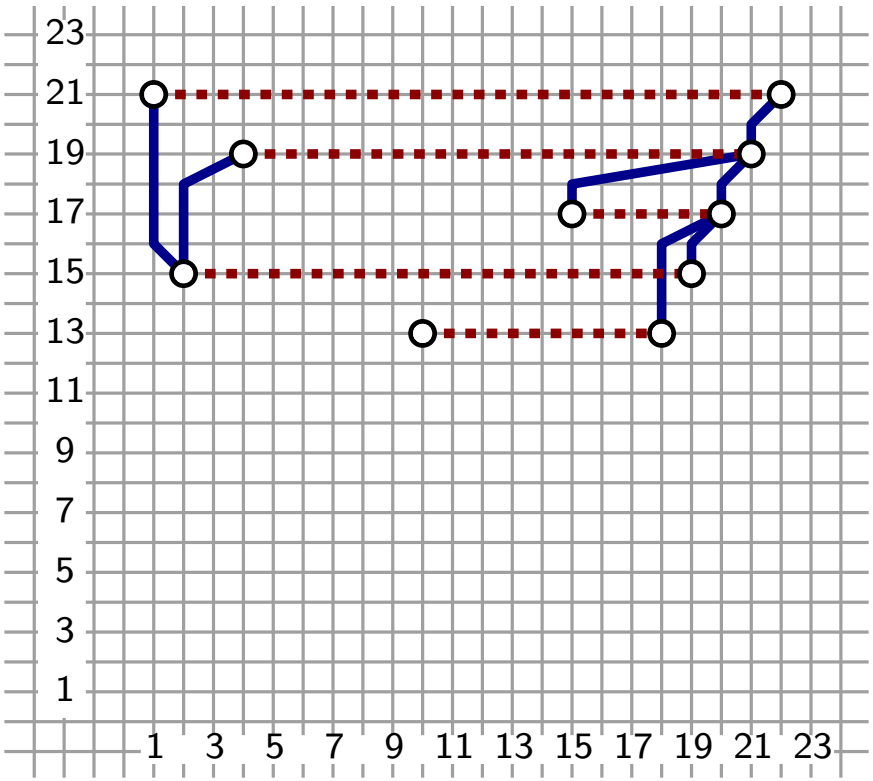

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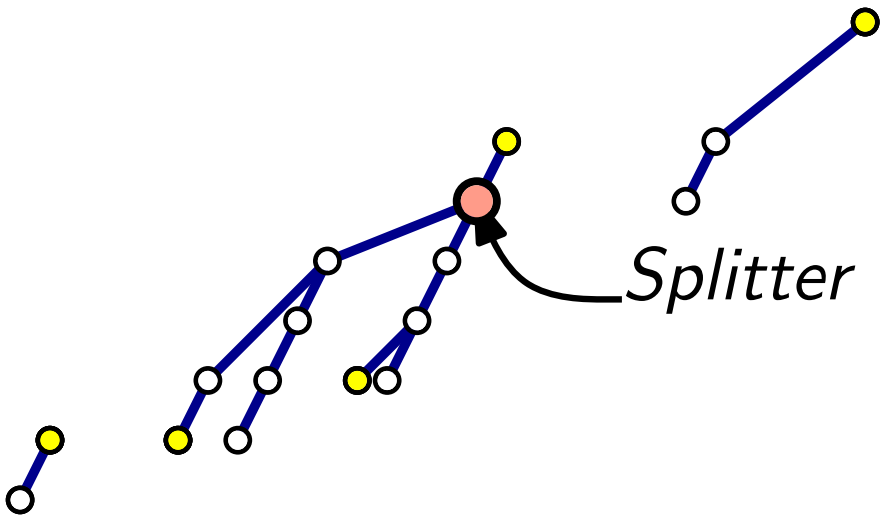

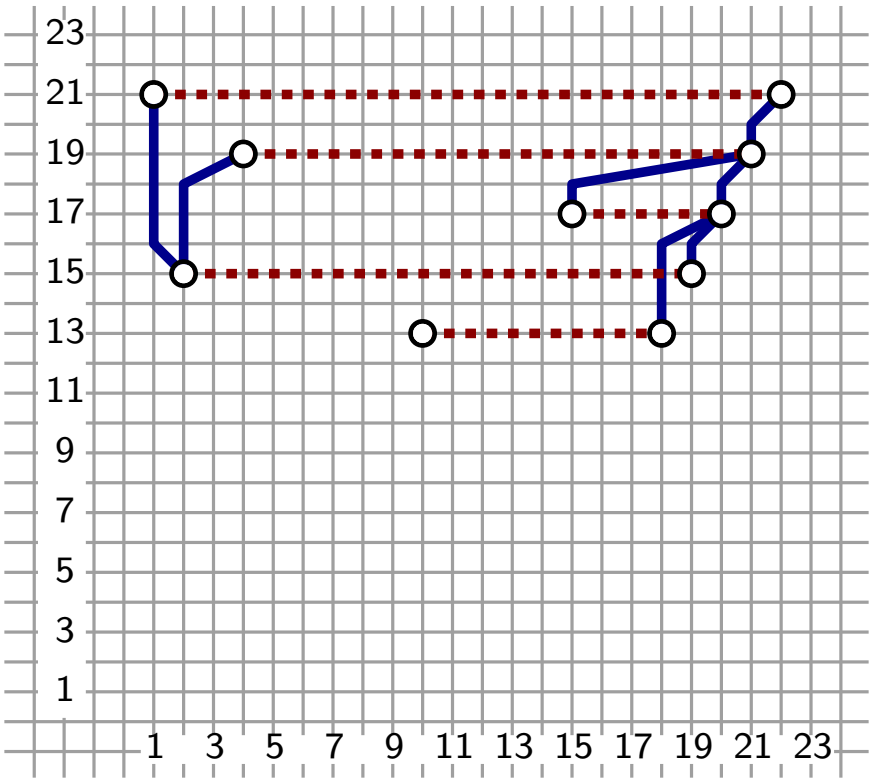

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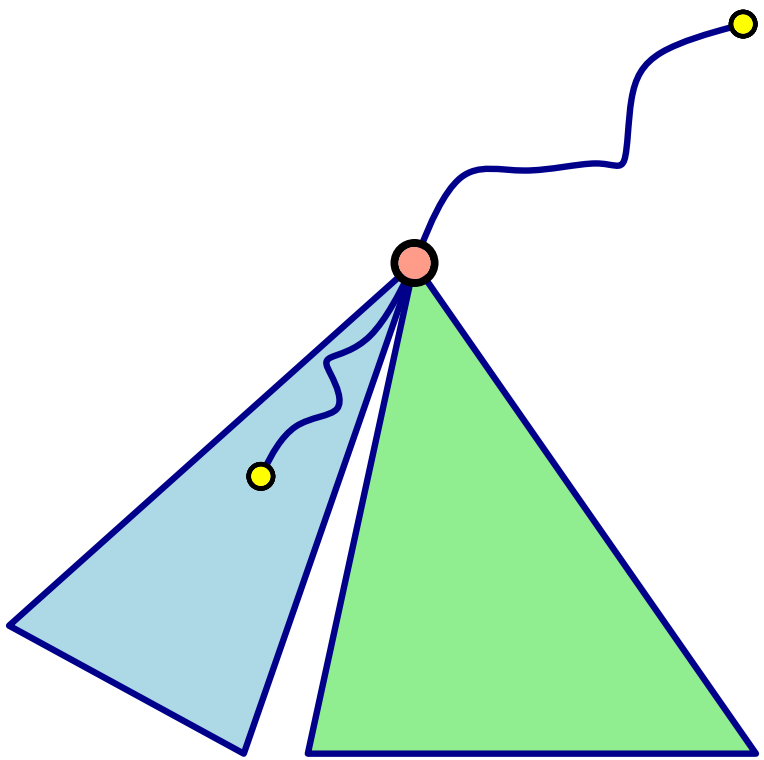
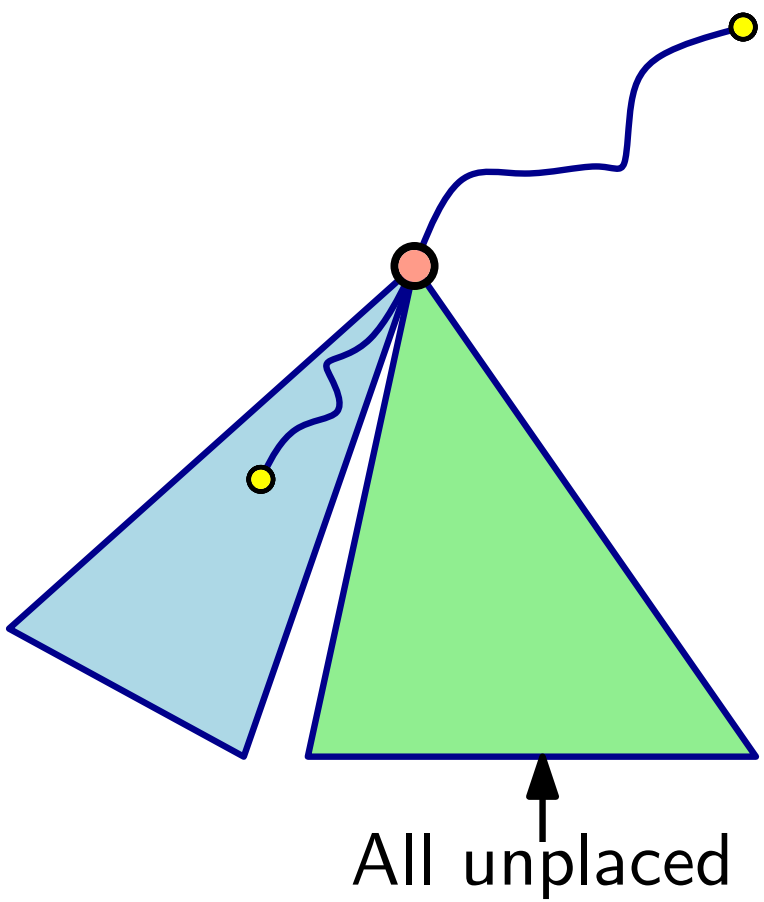


Tree $\times$ Matching

- Place root + matching at the top
- Split the tree
- Place vertex adj. to placed vertex (+ matching) at the top



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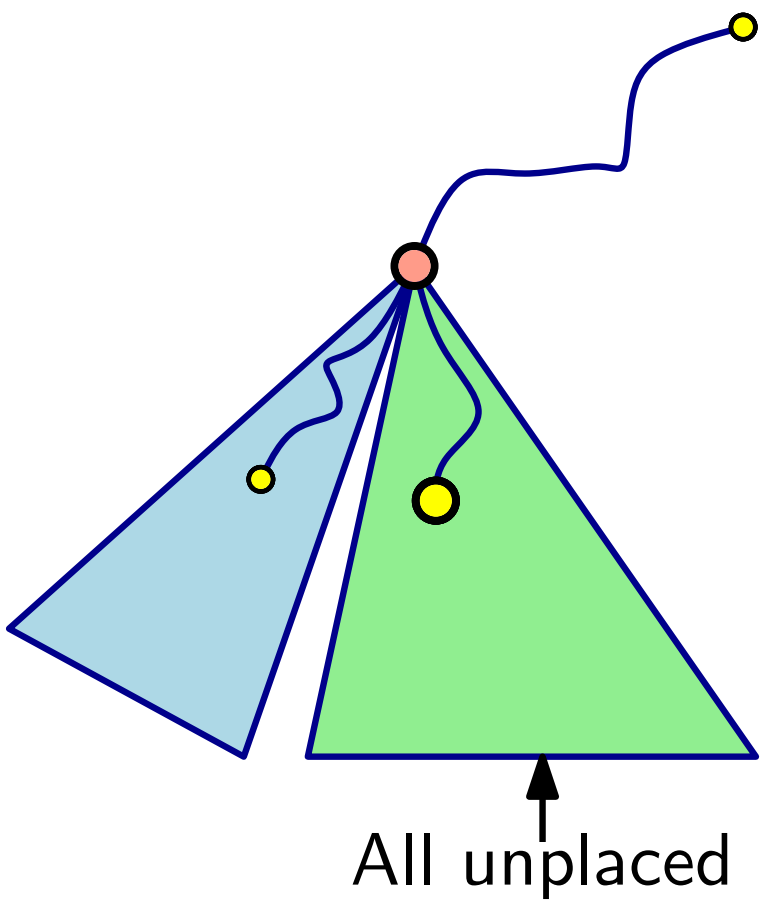

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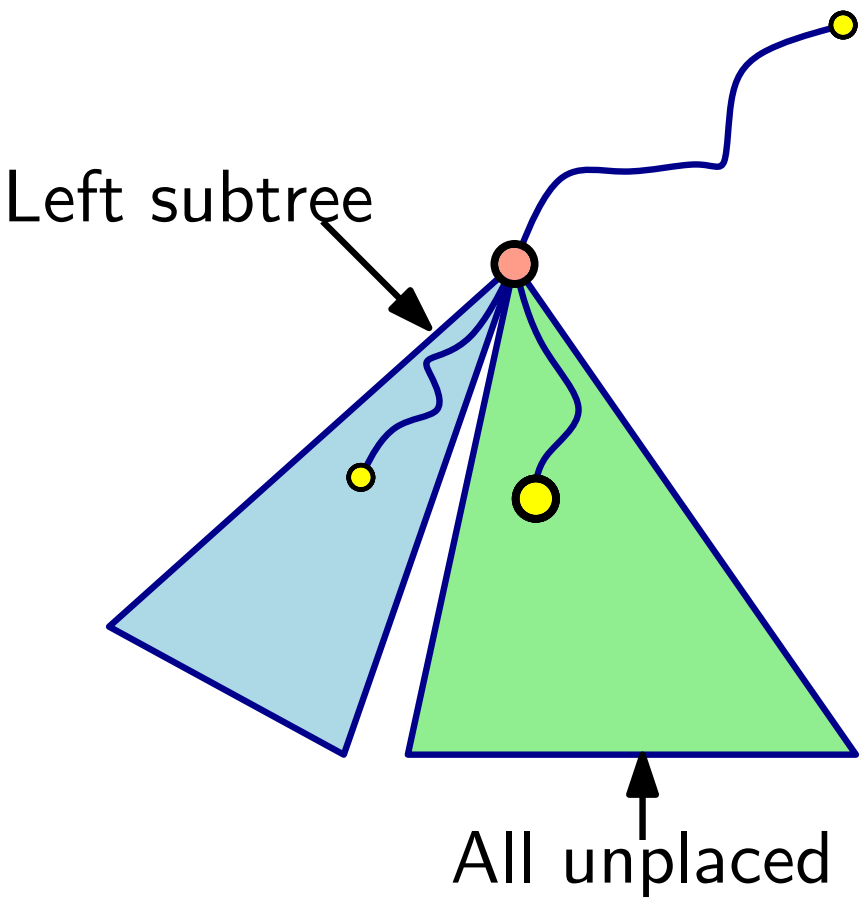

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- Place root + matching at the top
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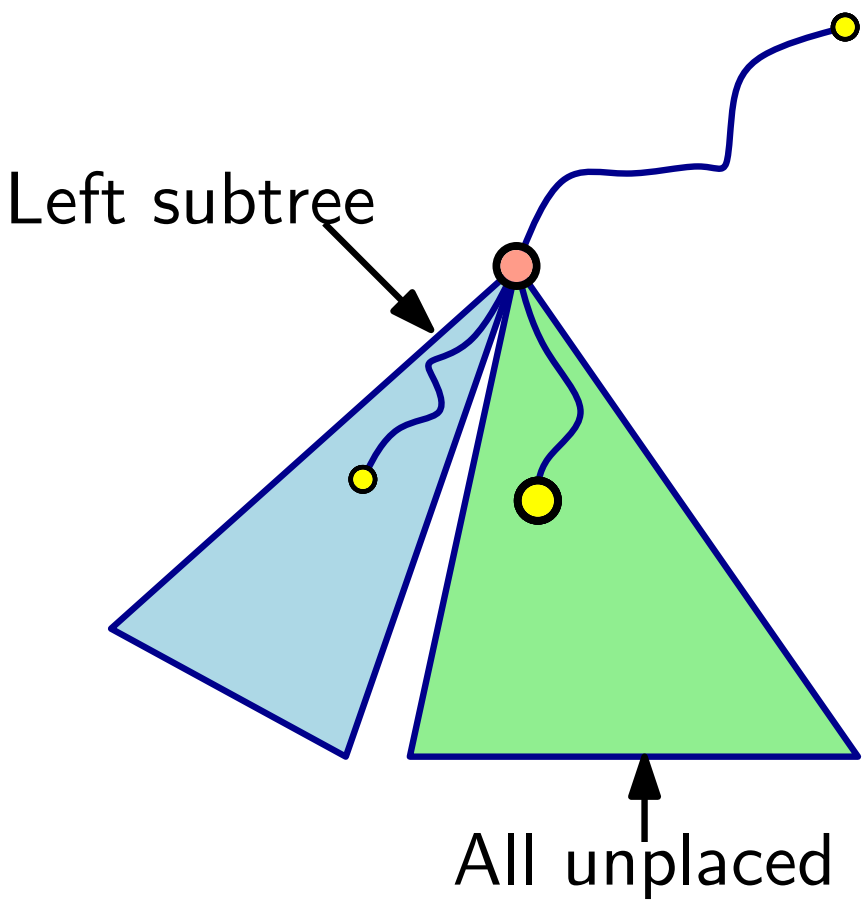
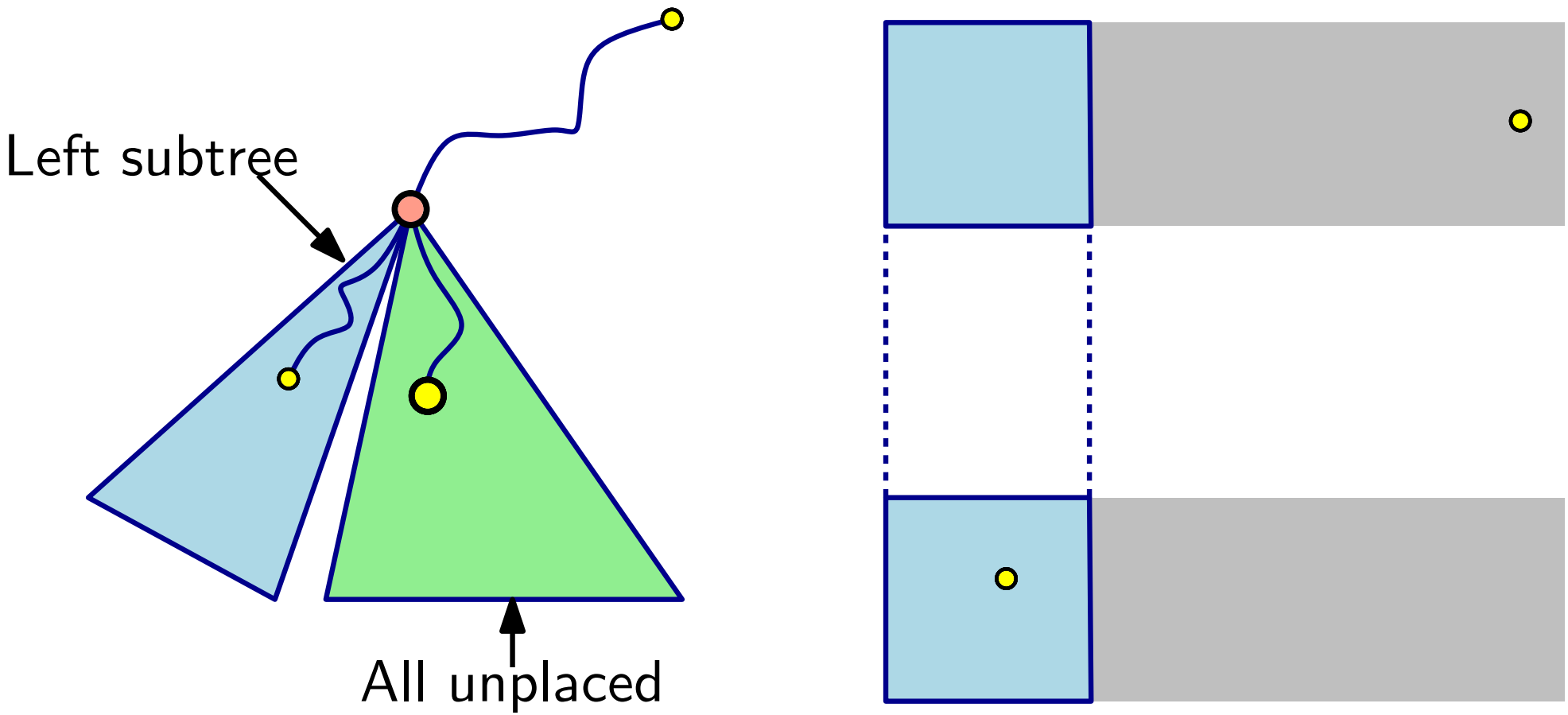


Placed vertices

Placed vertices

Tree × Matching

- Place root + matching at the top
- Split the tree
- Place vertex adj. to placed vertex (+ matching) at the top



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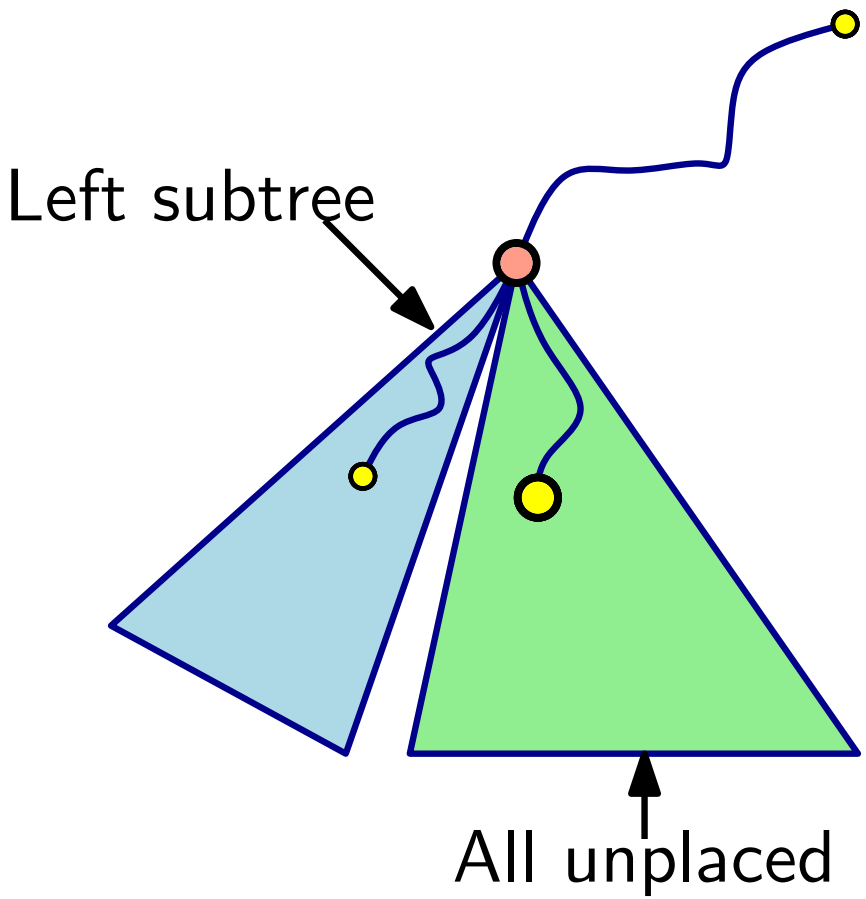

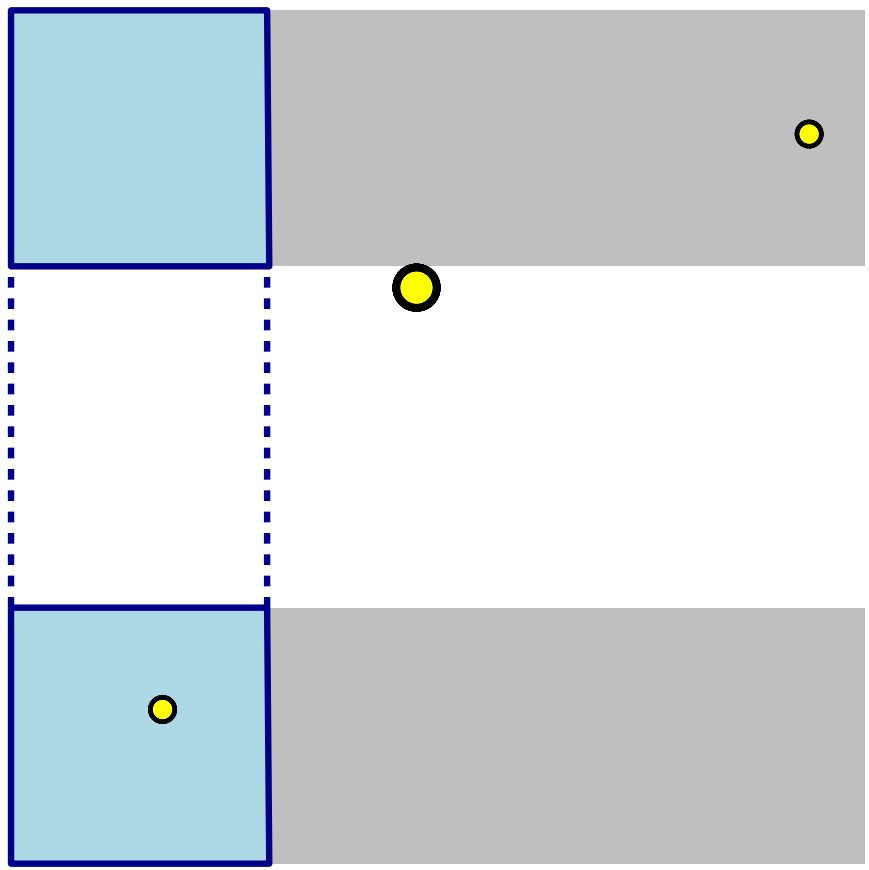

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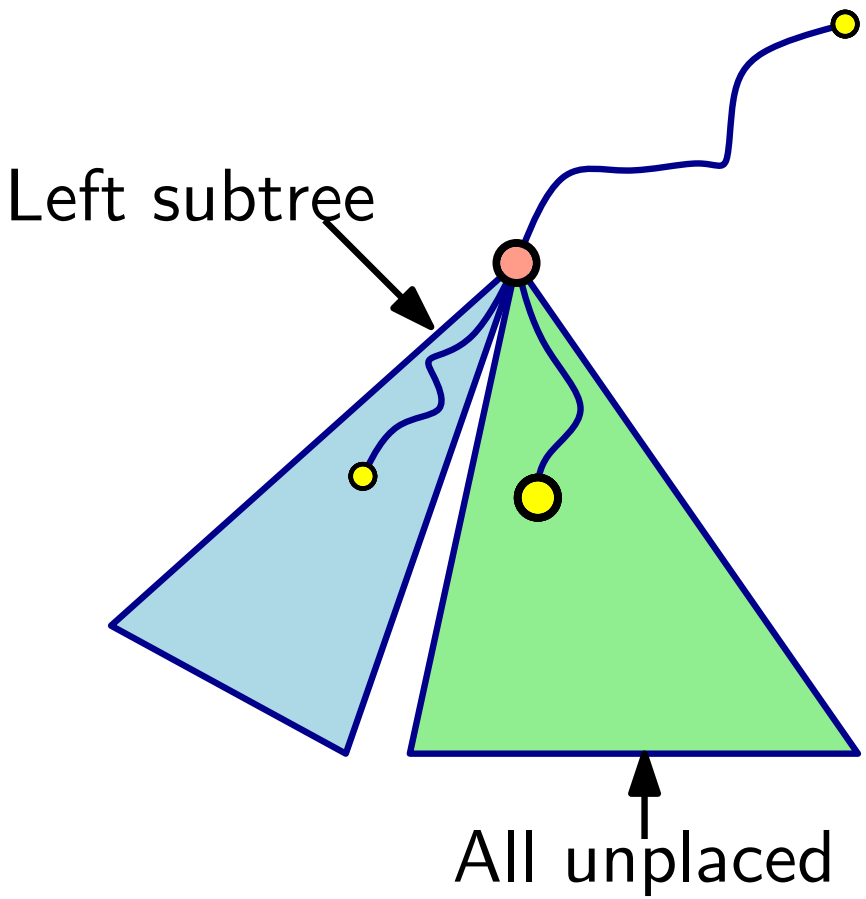

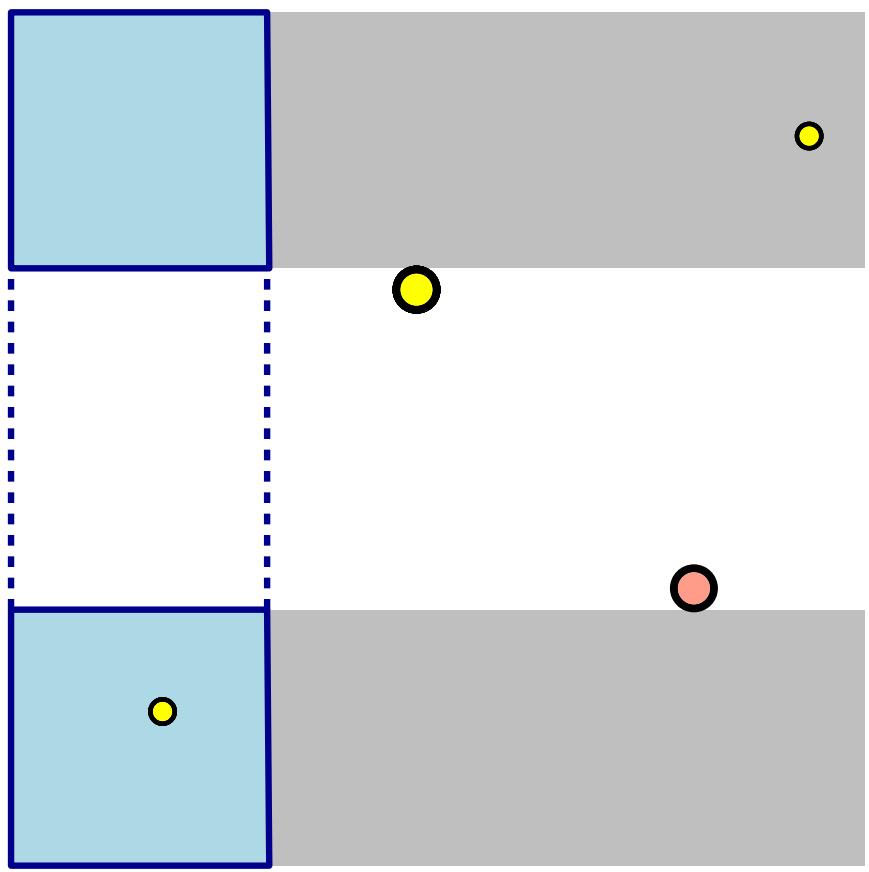

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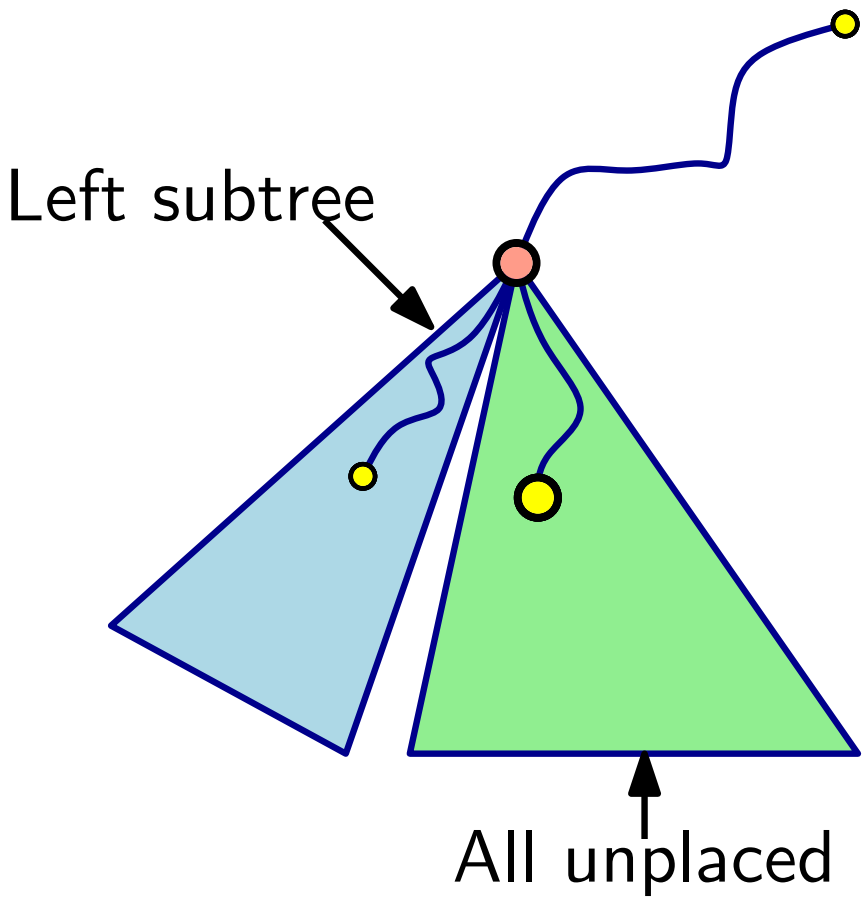

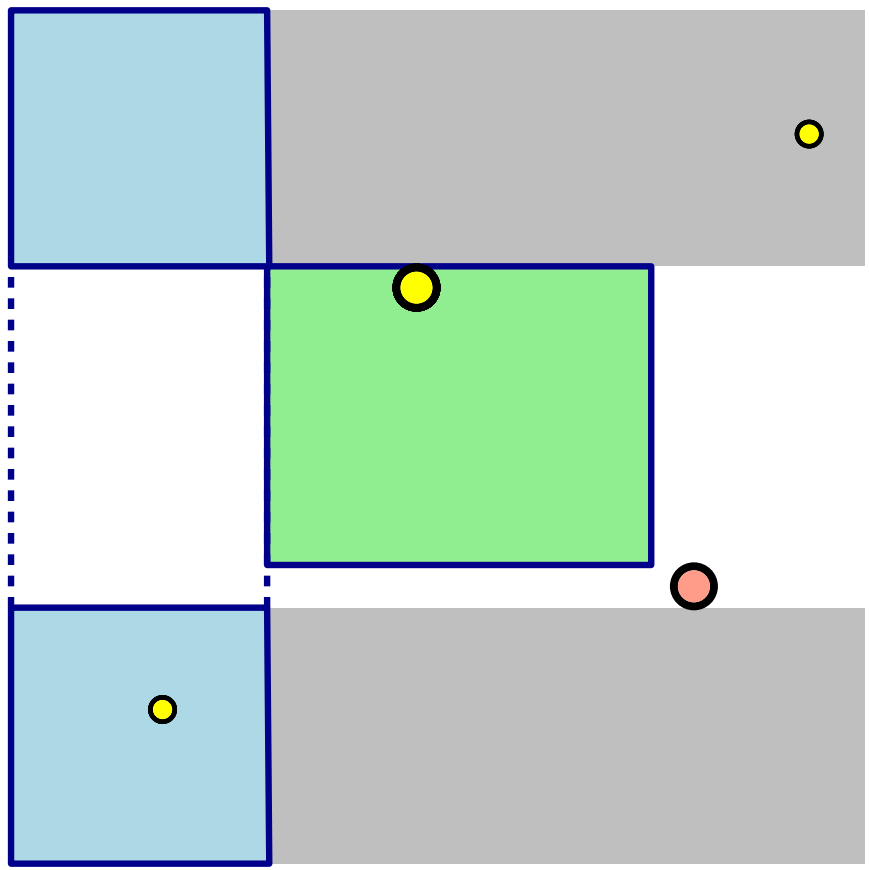

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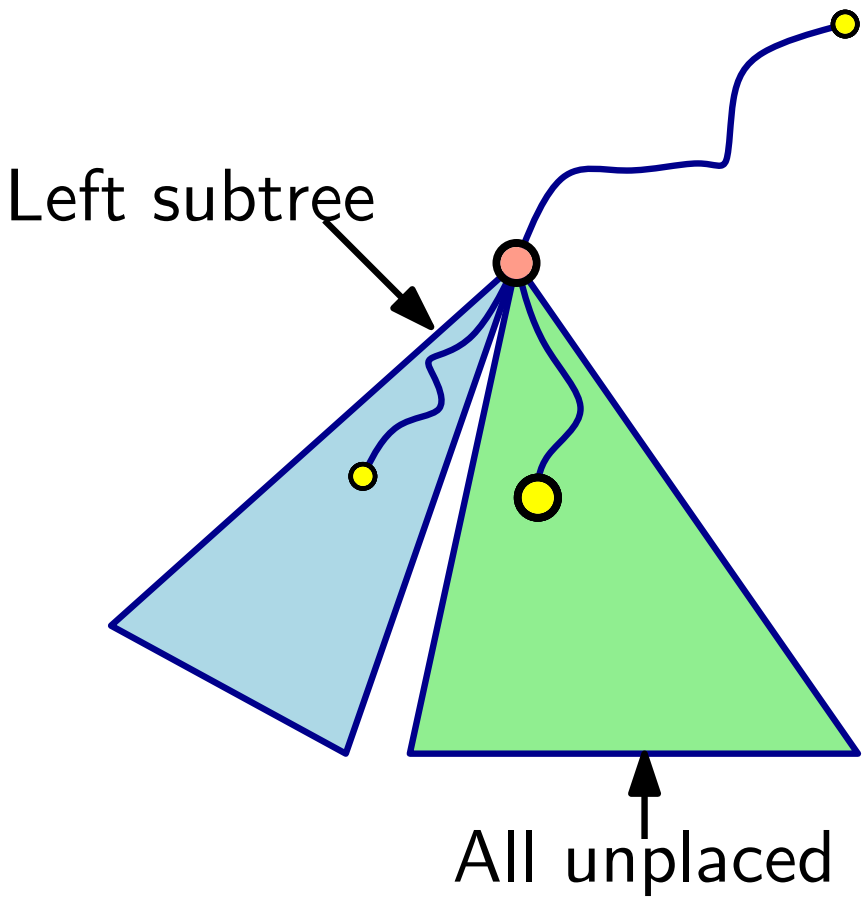
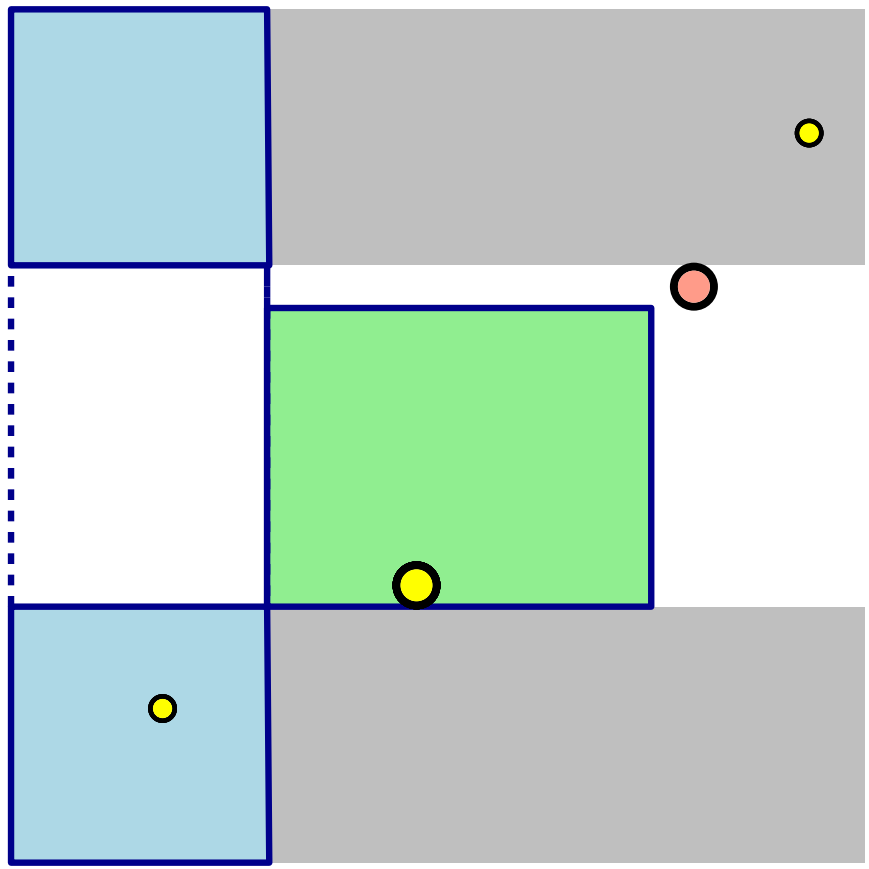
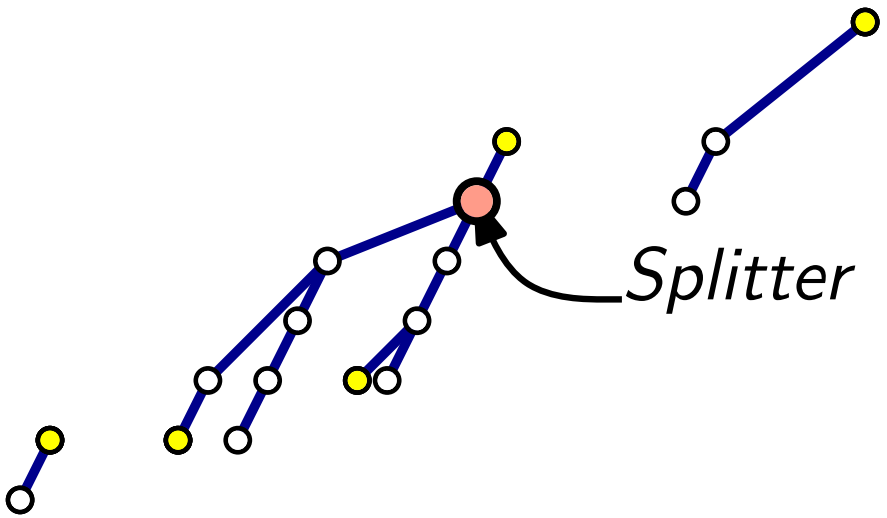
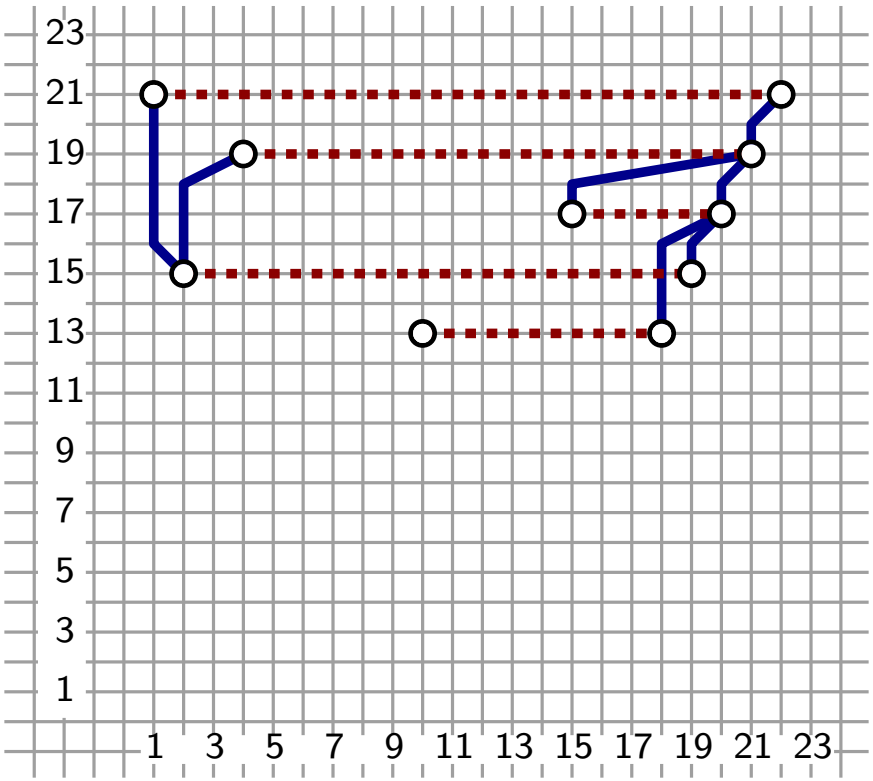




Tree × Matching

- Place root + matching at the top
- Split the tree
- Place vertex adj. to placed vertex (+ matching) at the top
- If splitter: place on opposite side





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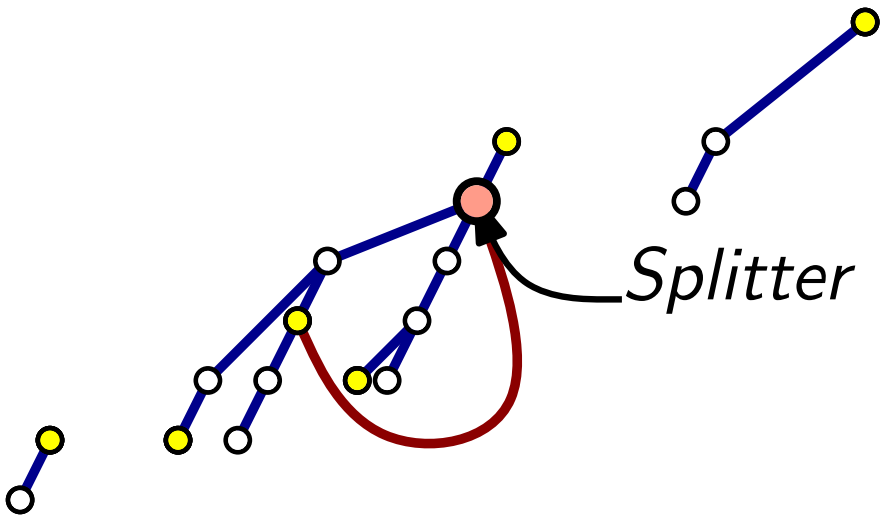

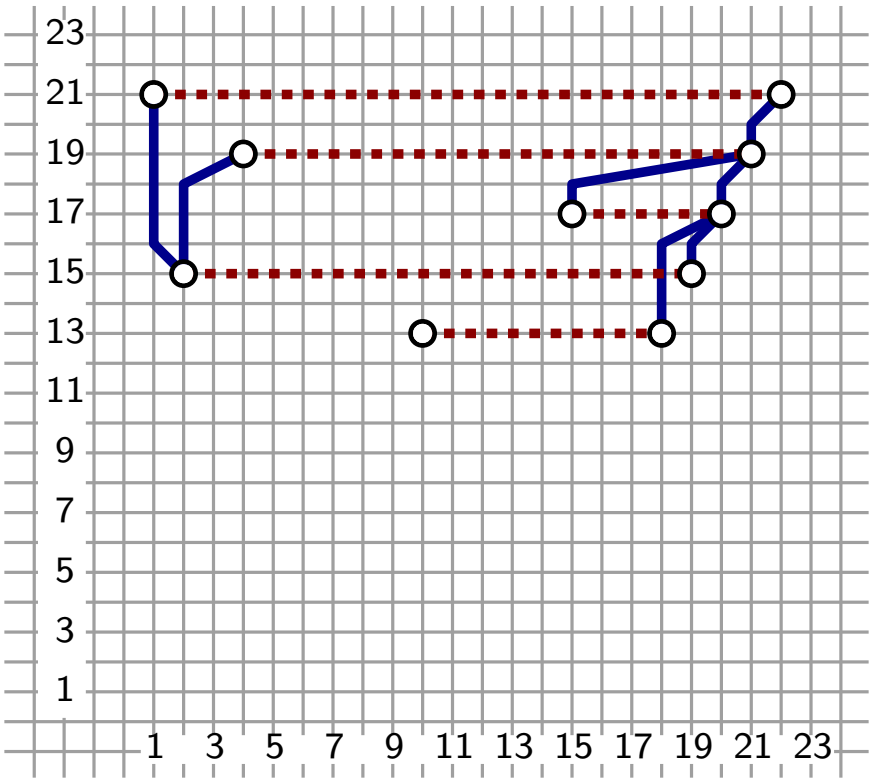

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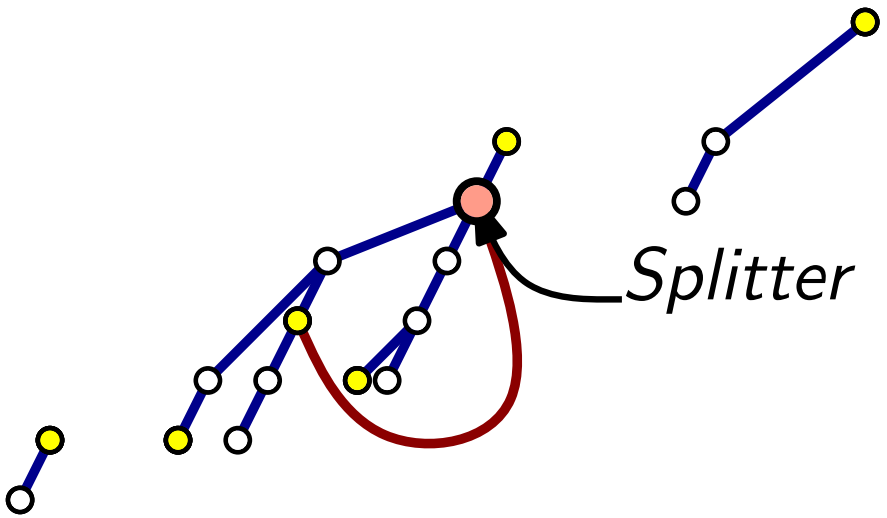

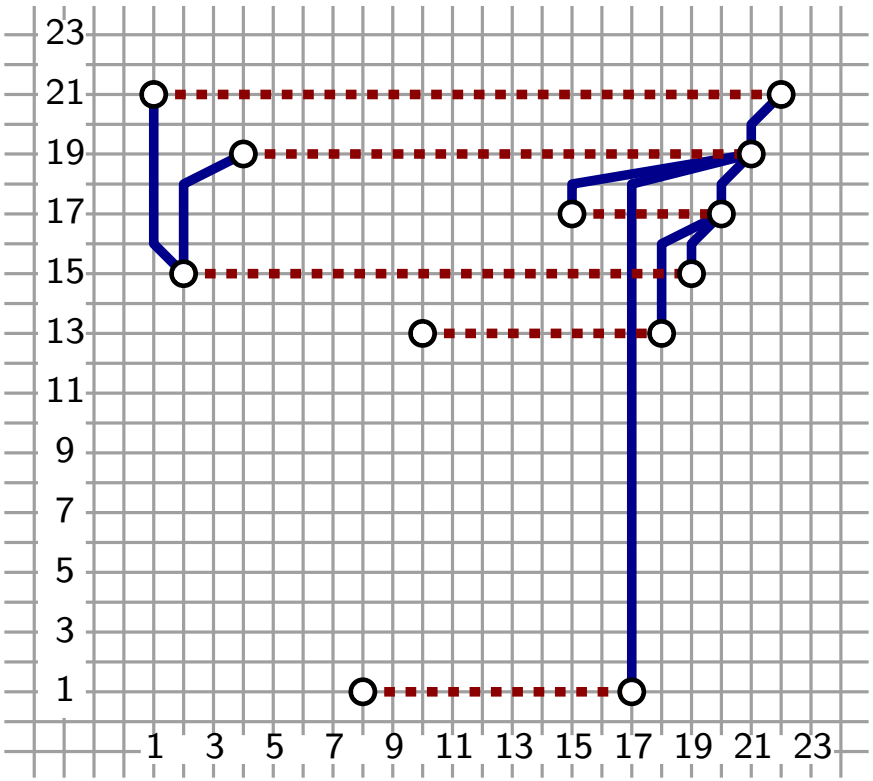

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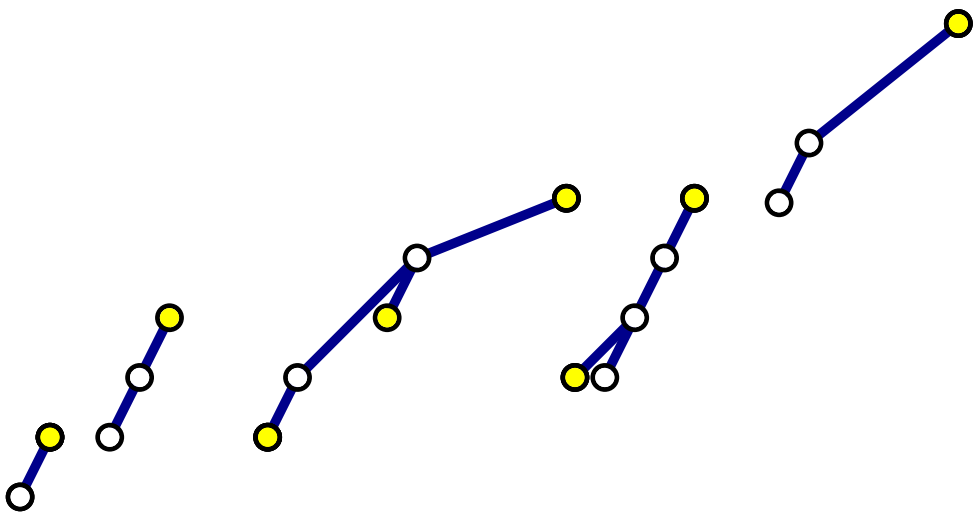

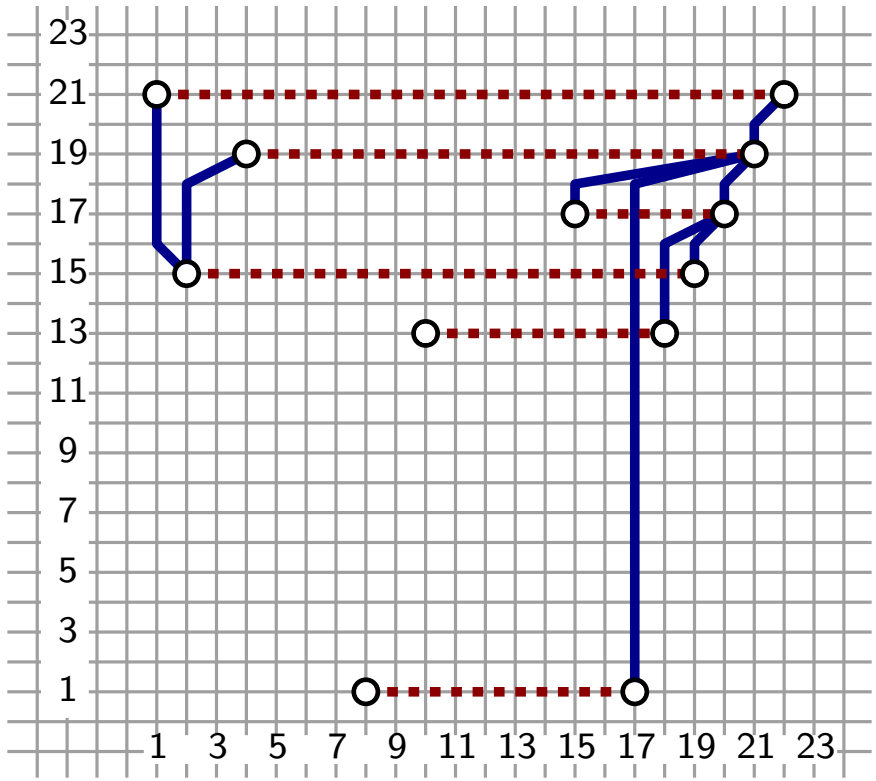

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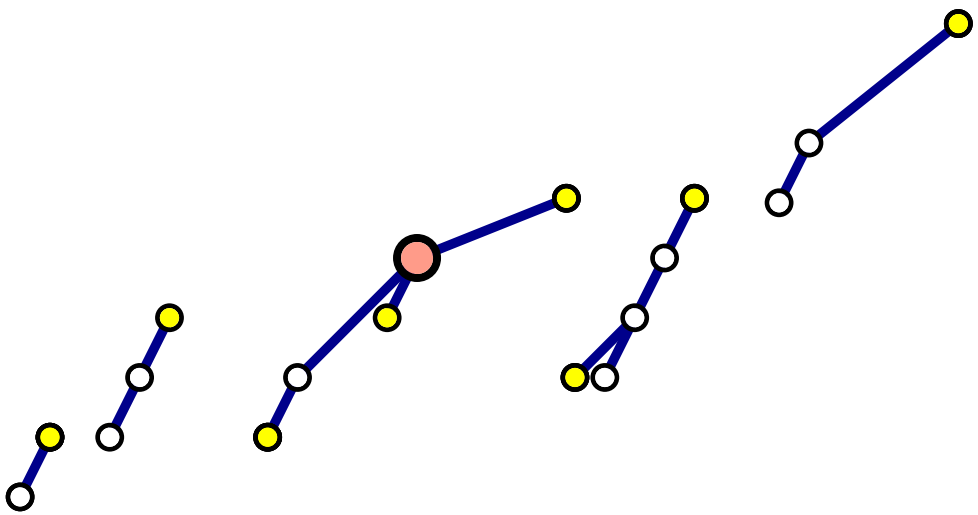

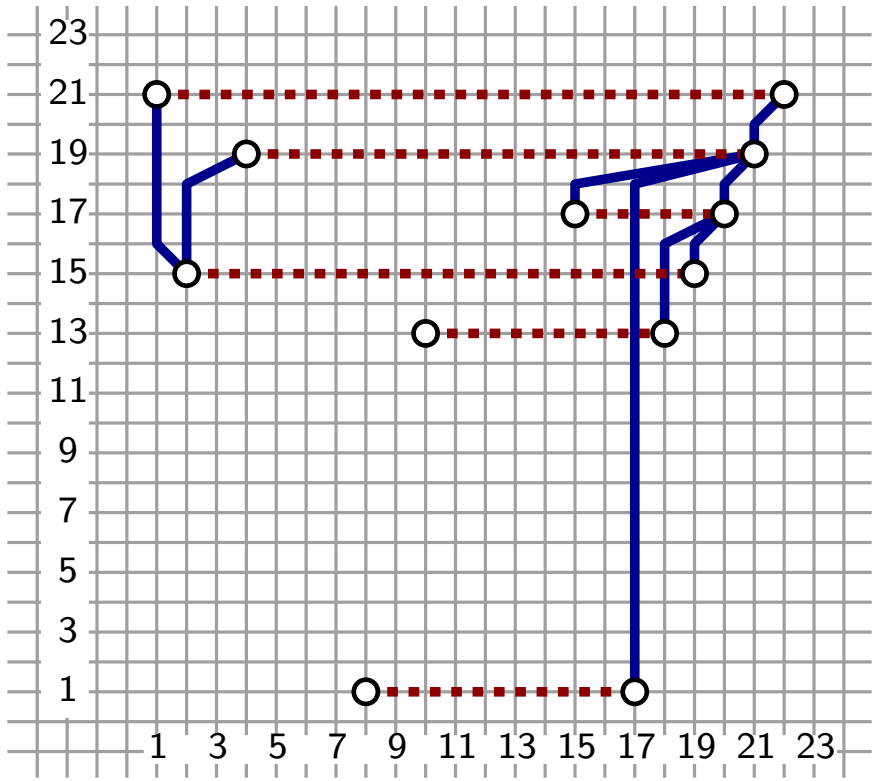

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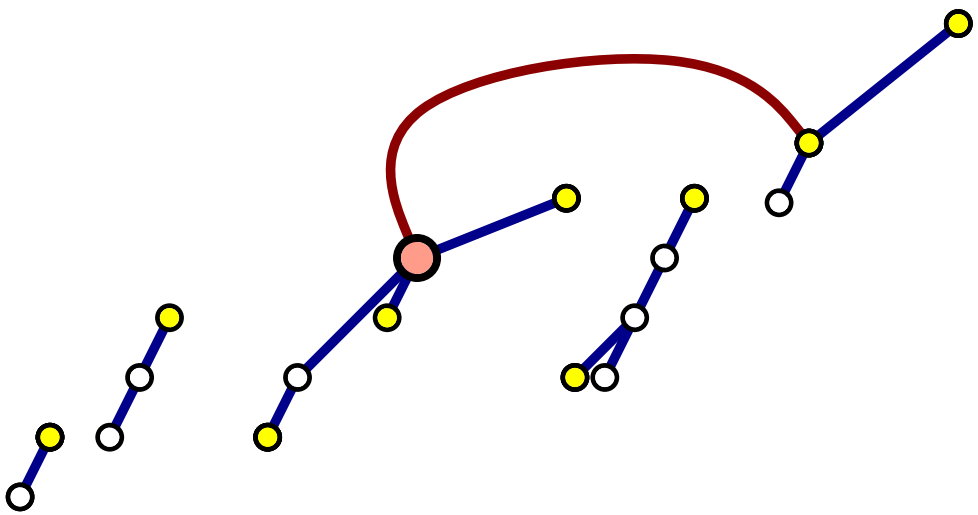

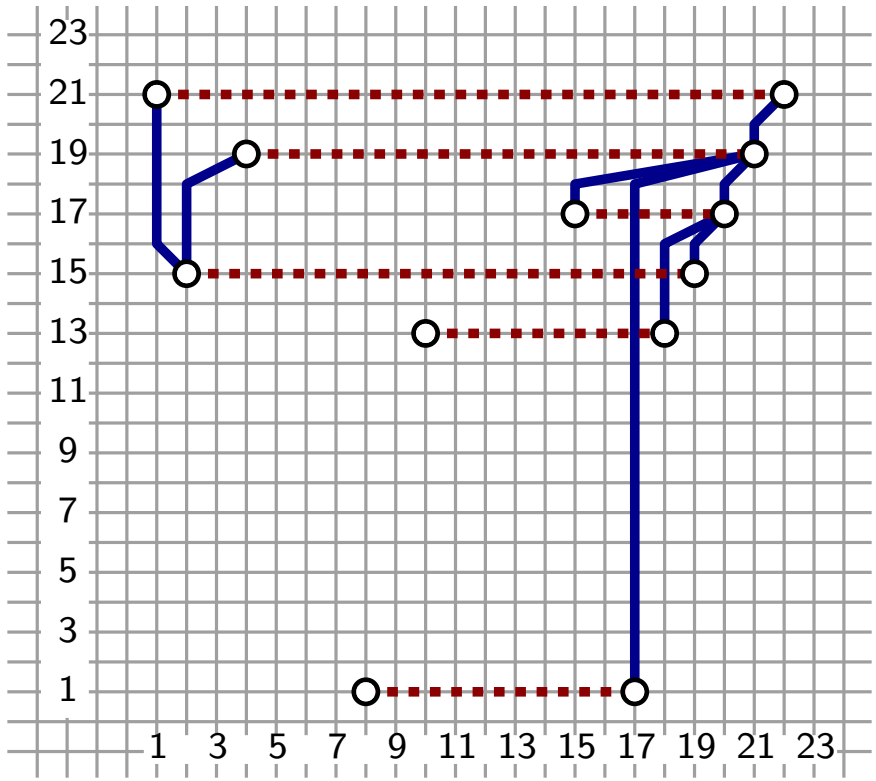

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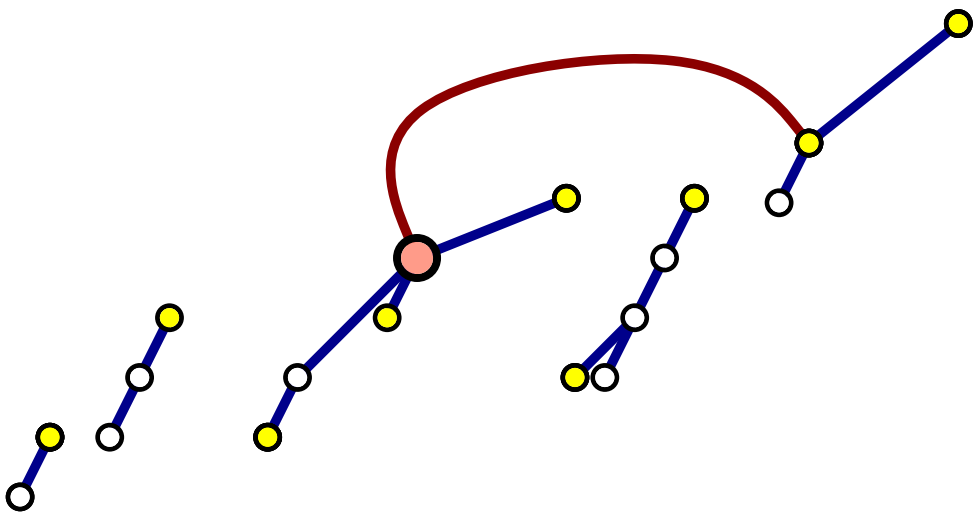

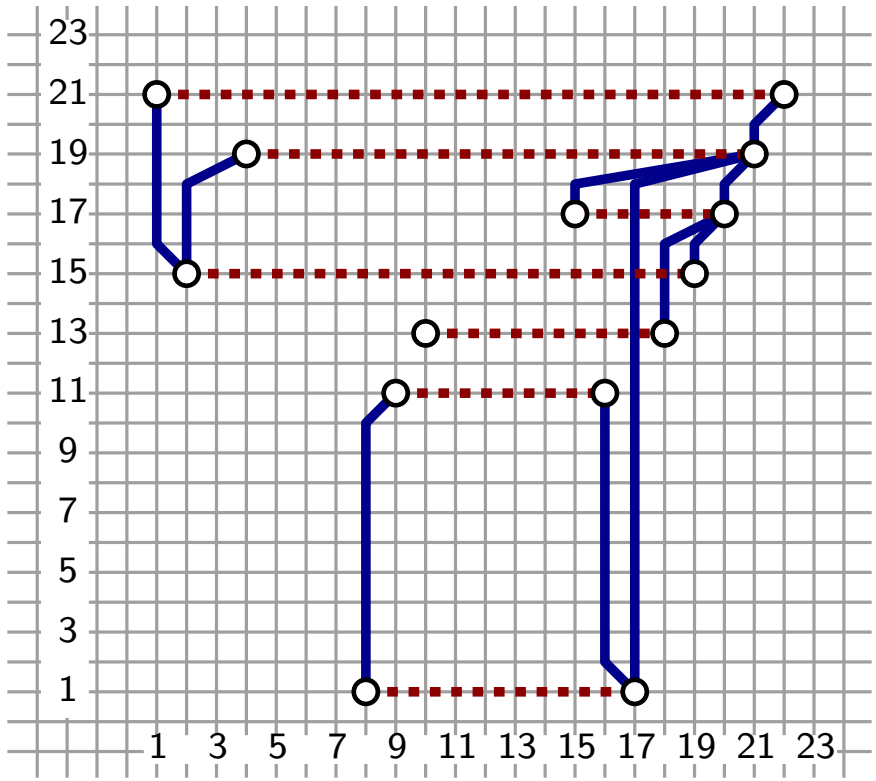

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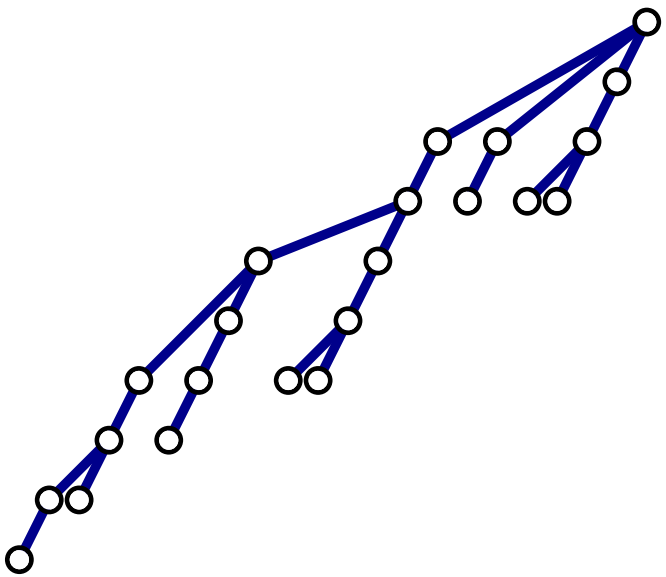

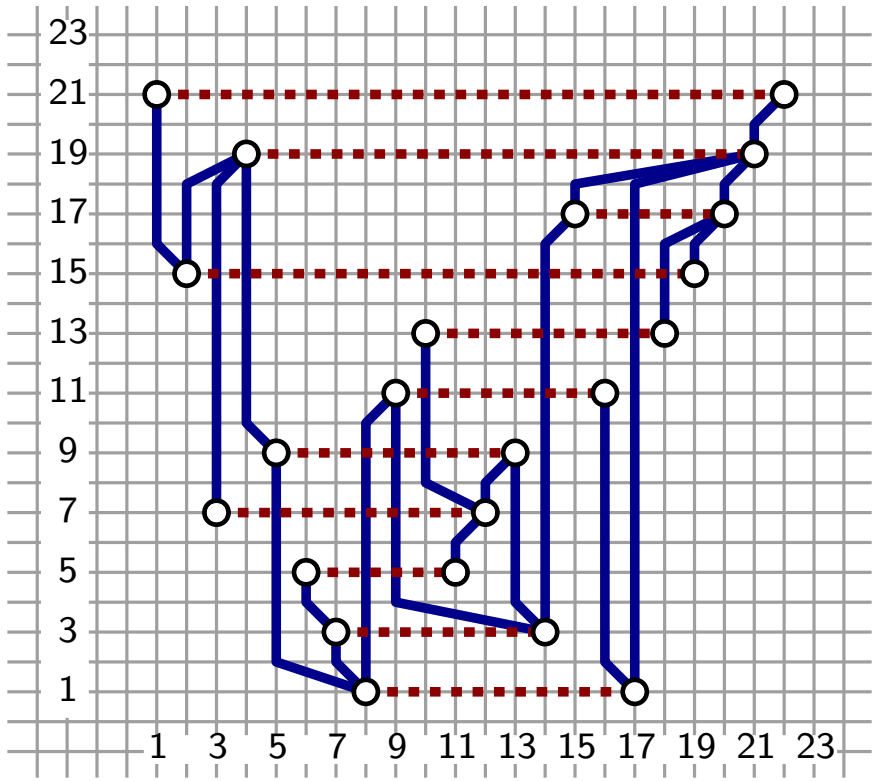

Tree × Matching

- Place root + matching at the top
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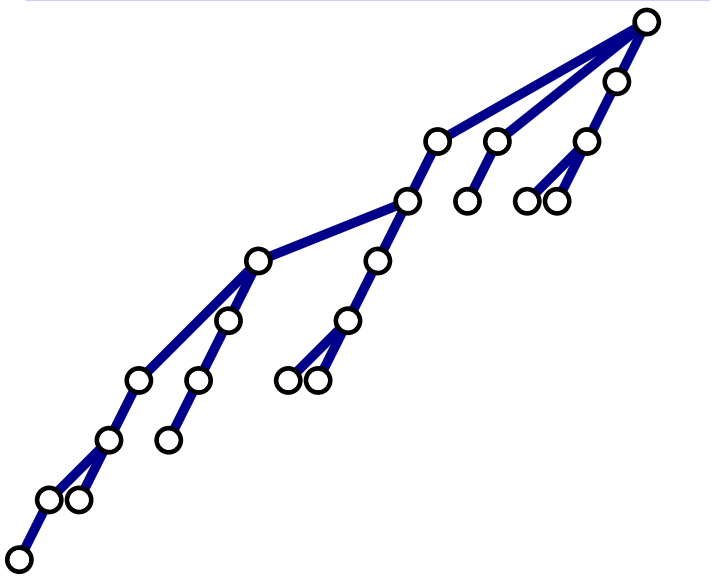
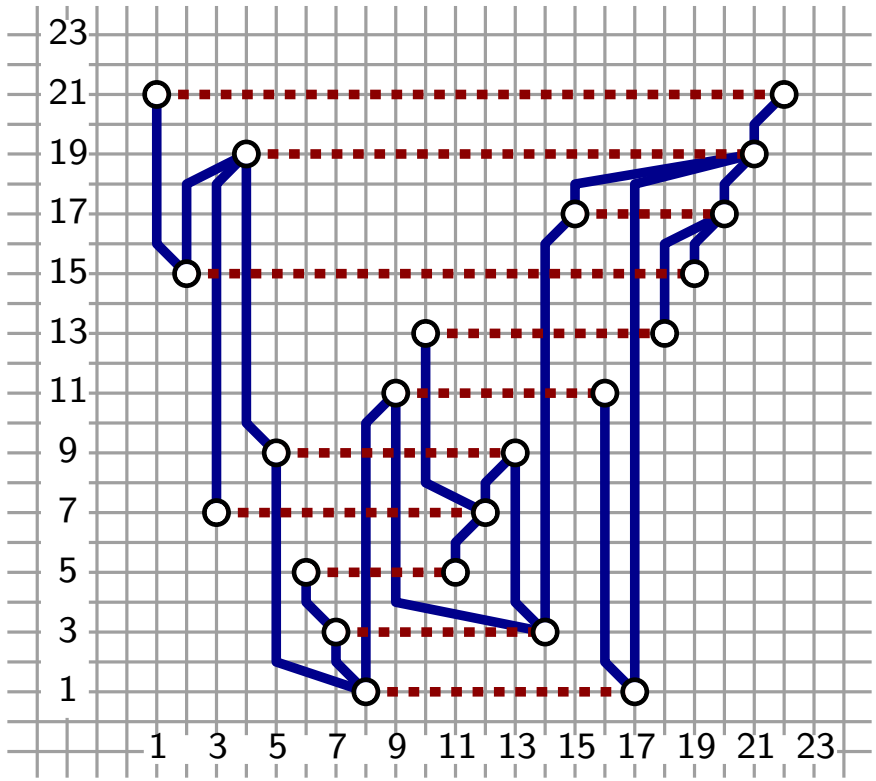
- Place vertex adj. to placed vertex (+ matching) at the top
- If splitter: place on opposite side

Bends: 1×0
Grid size:
$n \times (n-1)$

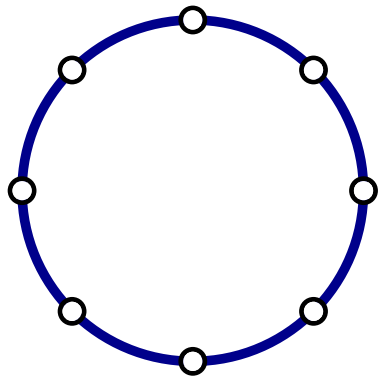




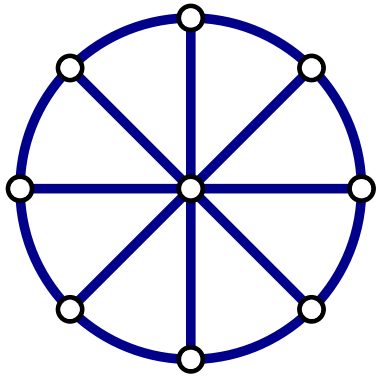
Overview

Graph classes			Number of bends
Cycle	$\times$	Cycle	1×1
Caterpillar	$\times$	Cycle	1×1
Four Matchings			$1 \times 1 \times 1 \times 1$
Tree	$\times$	Matching	1×0
Wheel	$\times$	Matching	2×0
Outerpath	$\times$	Matching	2×1
Outerplanar	$\times$	Outerplanar	3×3
2-page book emb.	$\times$	2-page book emb.	4×4
Planar	$\times$	Planar	6×6

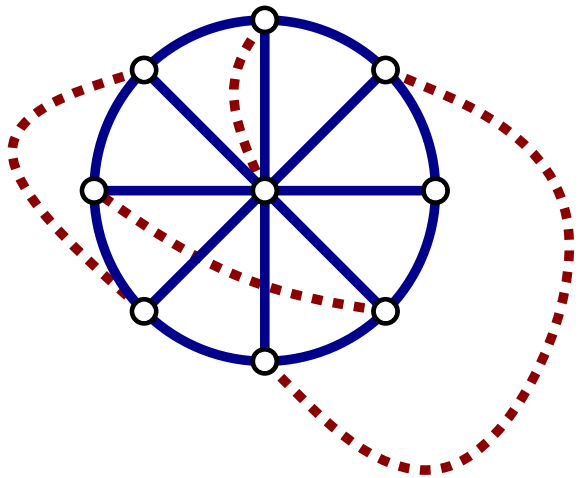
Wheel $\times$ Matching



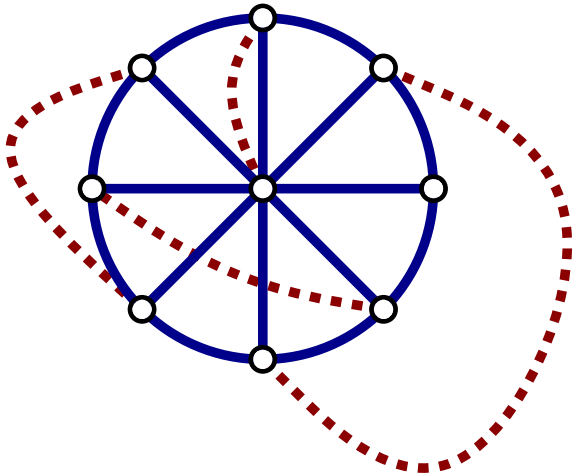
Wheel $\times$ Matching

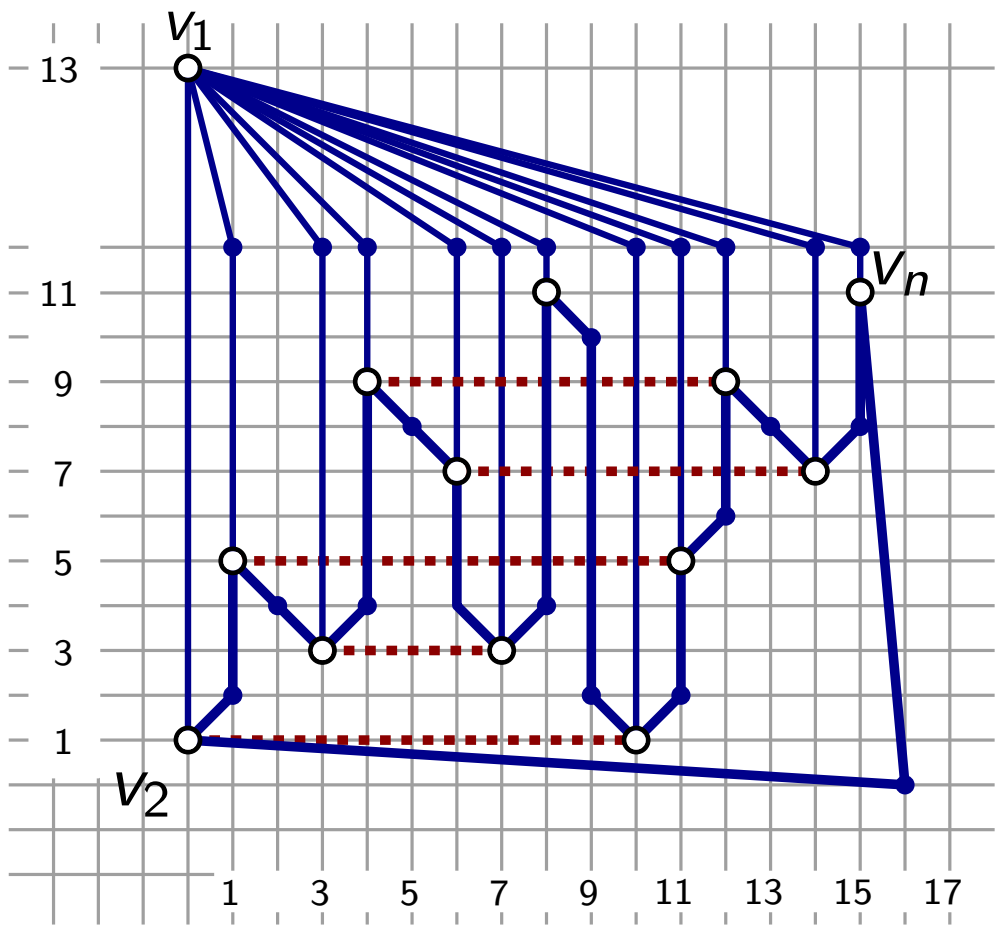


Wheel $\times$ Matching



Wheel $\times$ Matching





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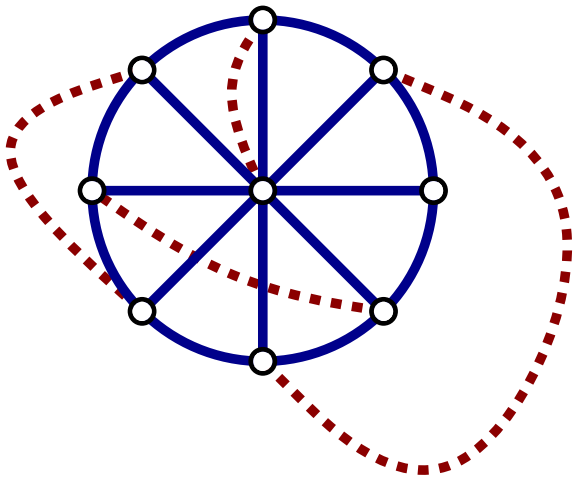
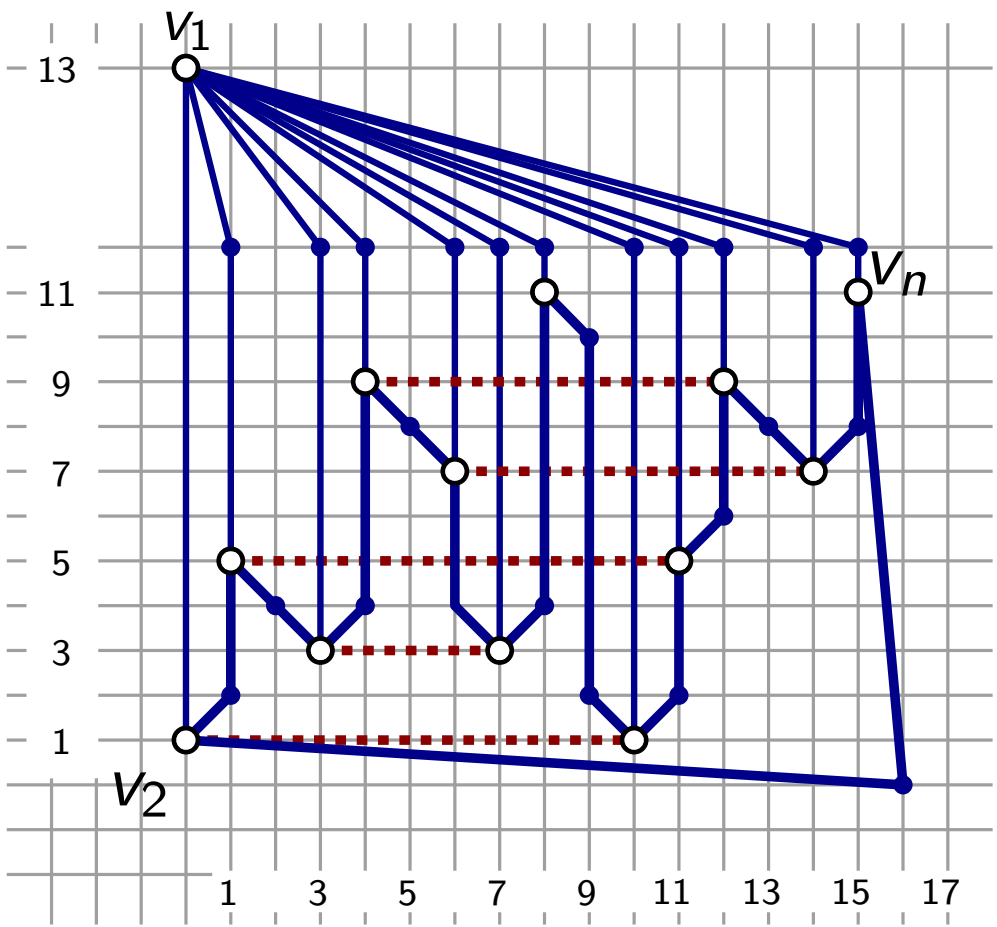
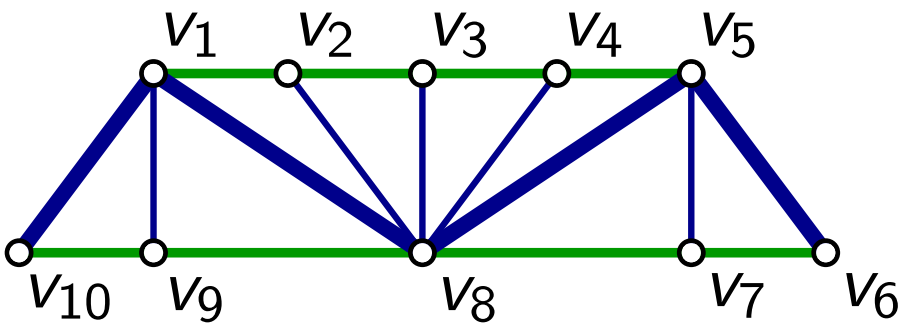


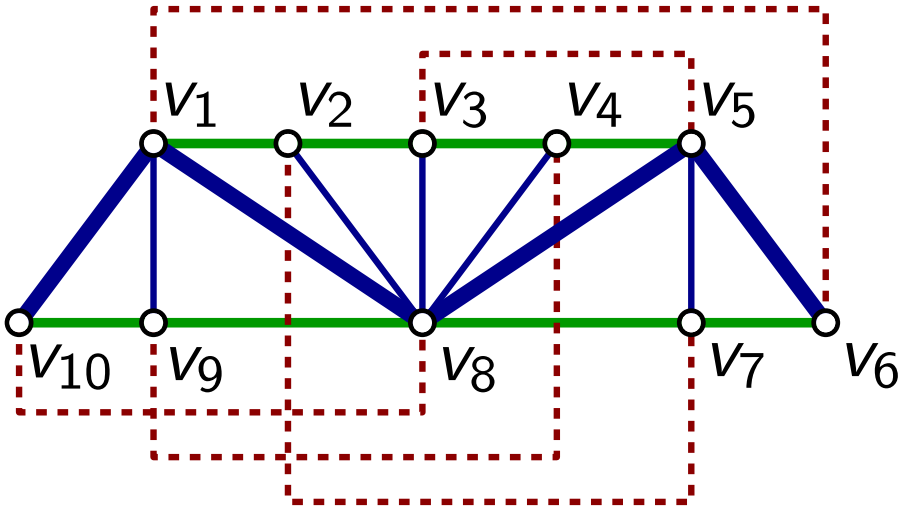
Bends: 2×0
Grid size:
$(1.5n - 1) \times (n + 2)$



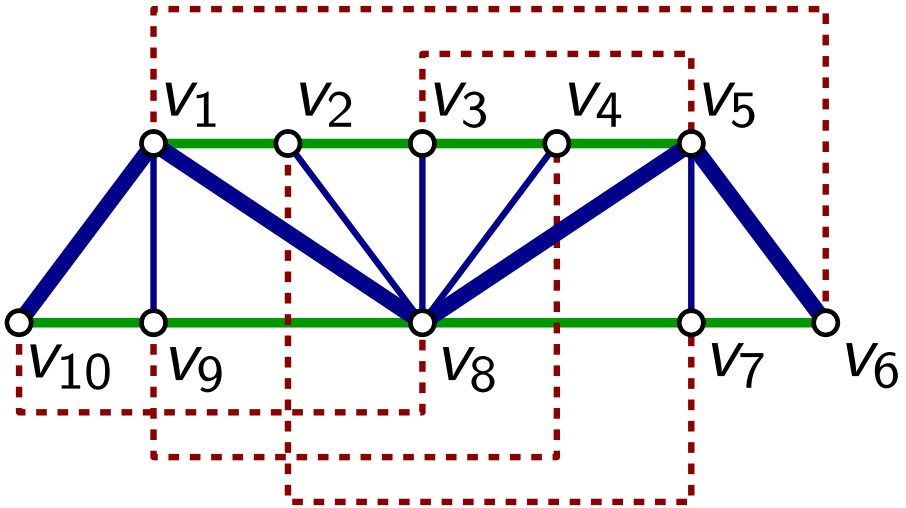
Outerpath $\times$ matching

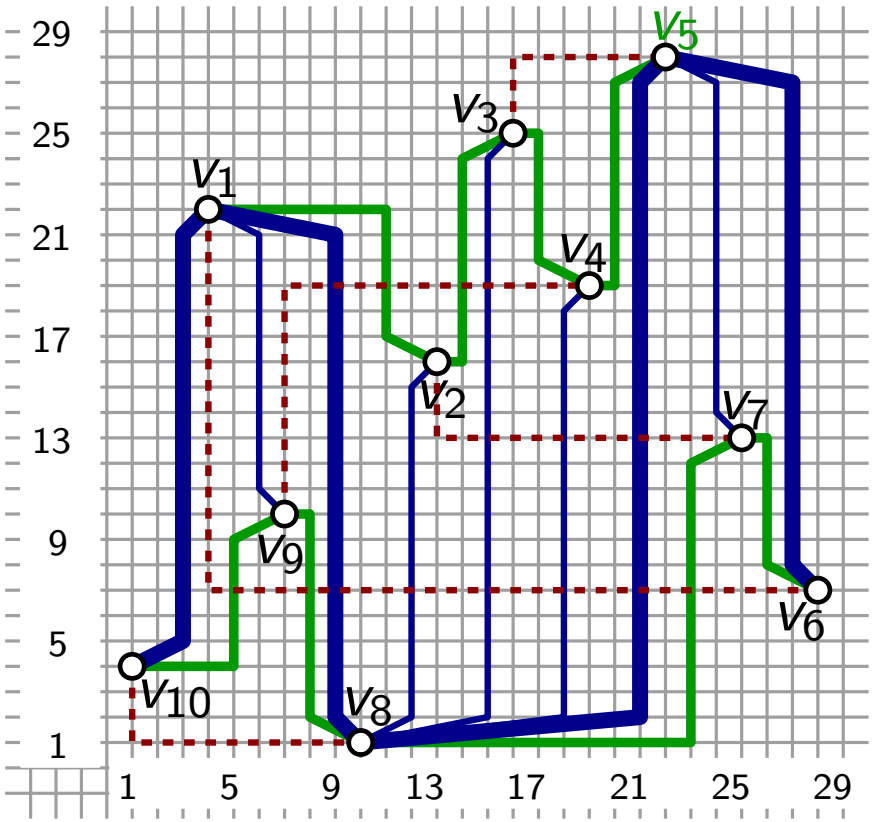


Outerpath $\times$ matching

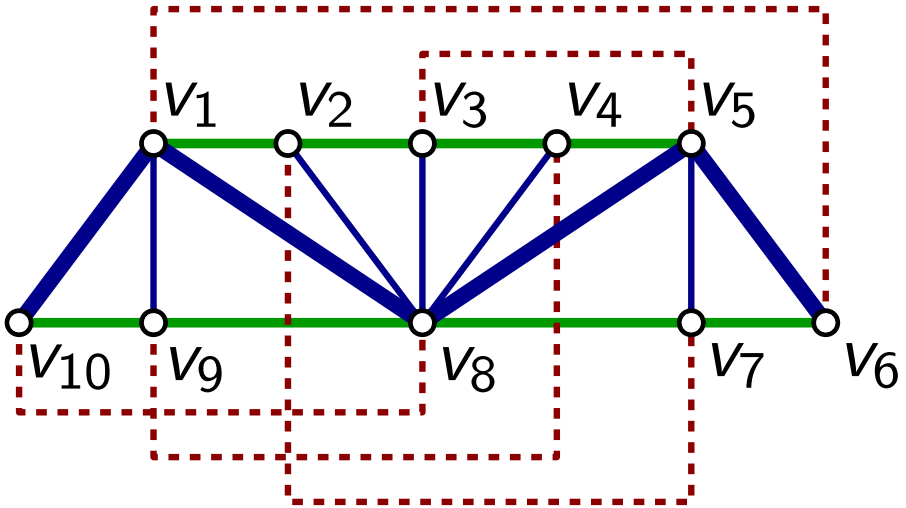


Outerpath × matching

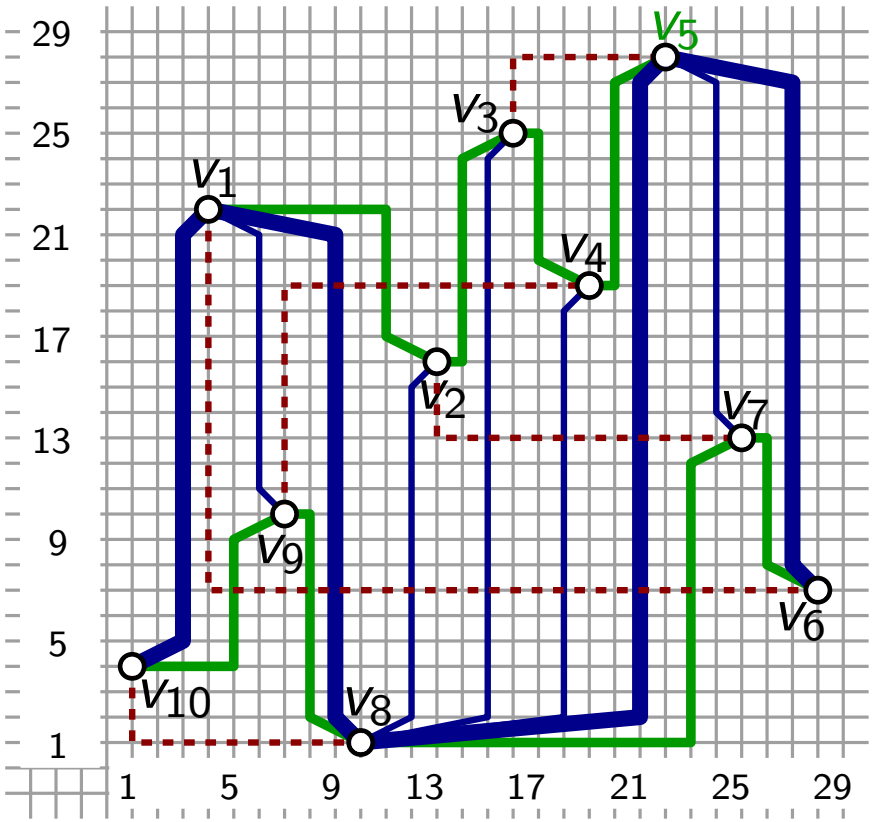




Outerpath × matching



Bends: 2×1
Grid size:
$(3n-2) \times (3n-2)$



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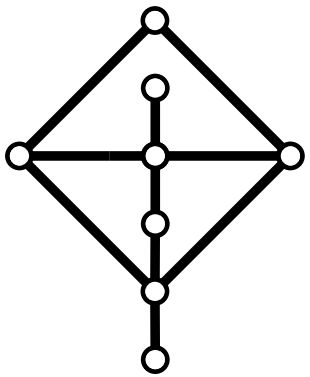
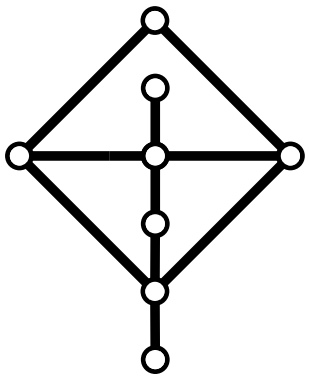
Wheel	$\times$	Matching	2×0 ✓
Outerpath	$\times$	Matching	2×1 ✓
Outerplanar	$\times$	Outerplanar	3×3
2-page book emb.	$\times$	2-page book emb.	4×4
Planar	$\times$	Planar	6×6

Planar $\times$ Planar

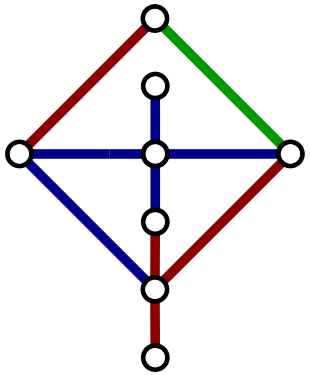


Planar $\times$ Planar

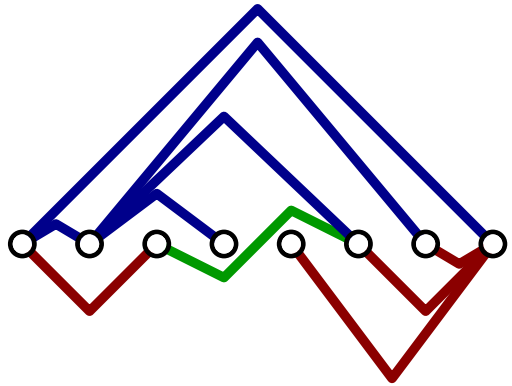


[Kaufmann & Wiese '02]:

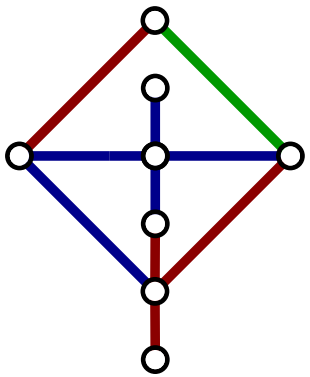
Planar $\times$ Planar



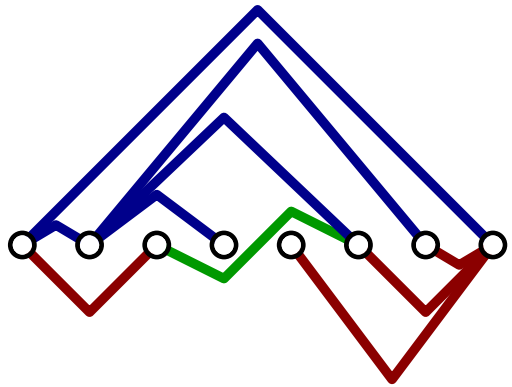
[Kaufmann & Wiese '02]:



Planar $\times$ Planar

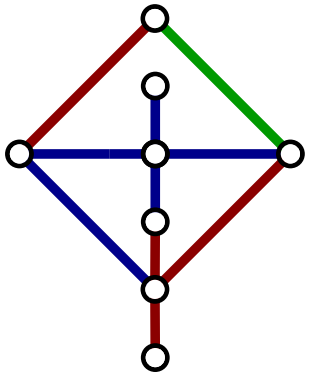


[Kaufmann & Wiese '02]:

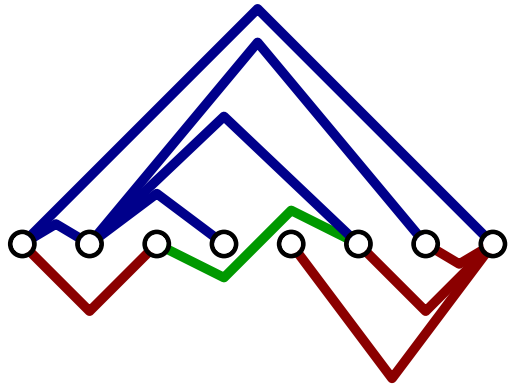


Graph 1: x-coordinates
Graph 2: y-coordinates

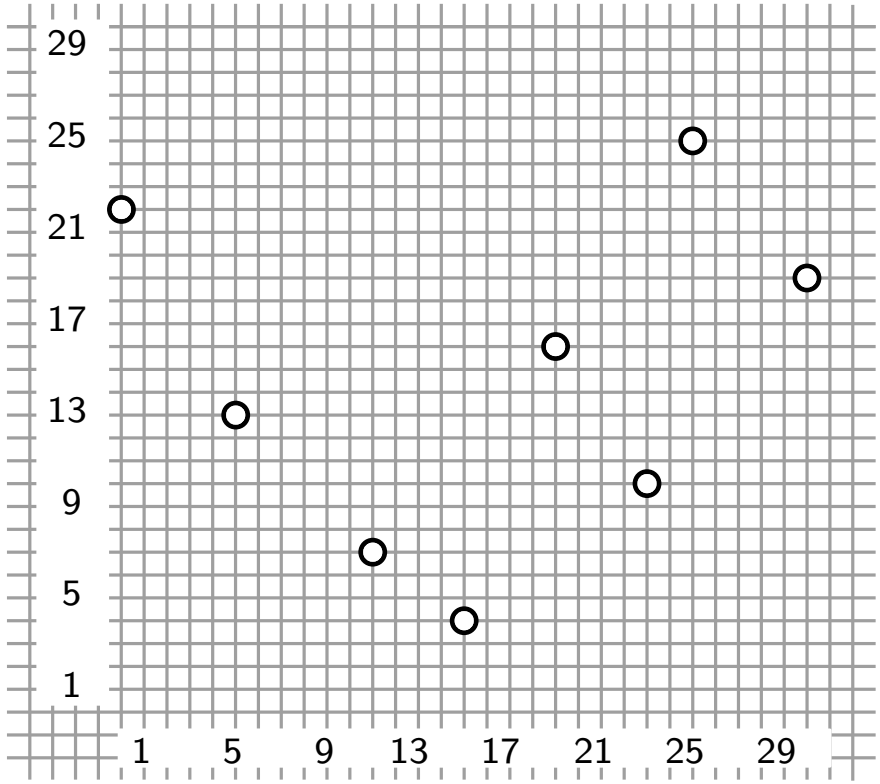
Planar × Planar



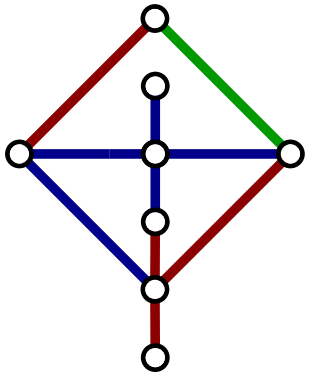
[Kaufmann & Wiese '02]:



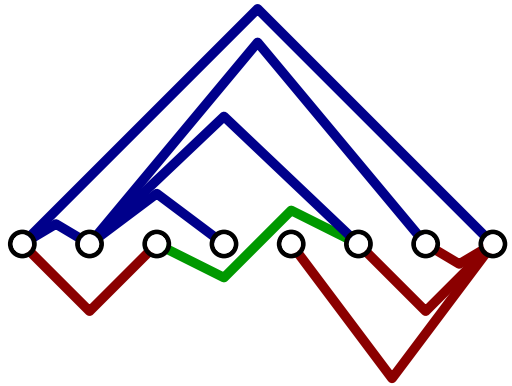
Graph 1: x-coordinates
Graph 2: y-coordinates



Planar × Planar



[Kaufmann & Wiese '02]:



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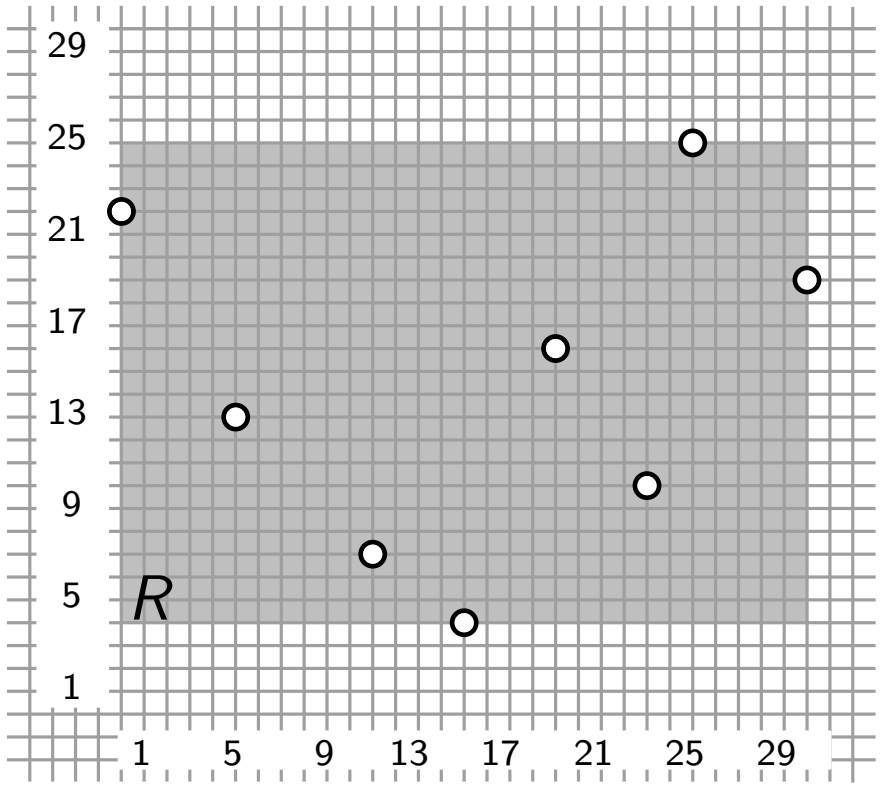
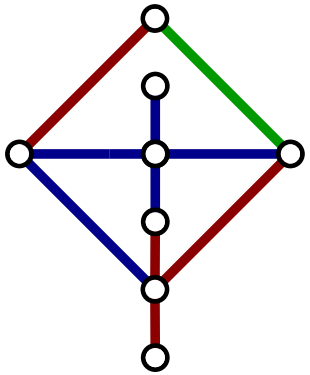


Planar × Planar



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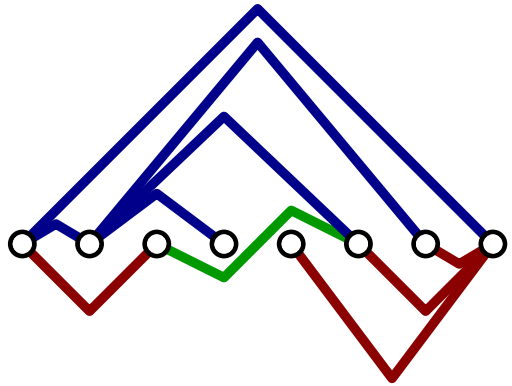
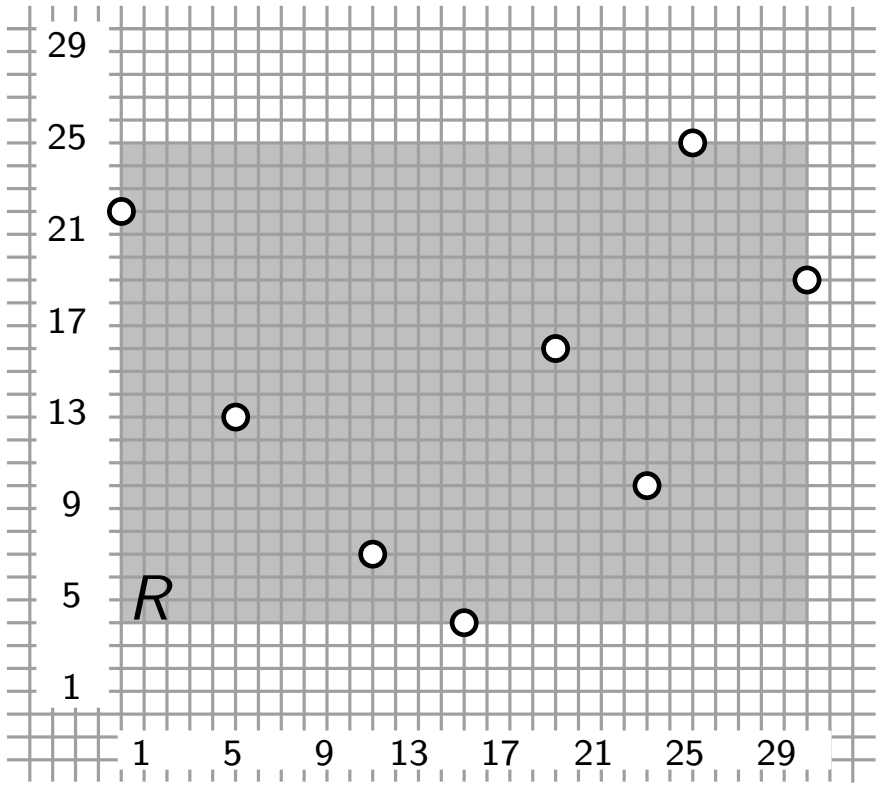
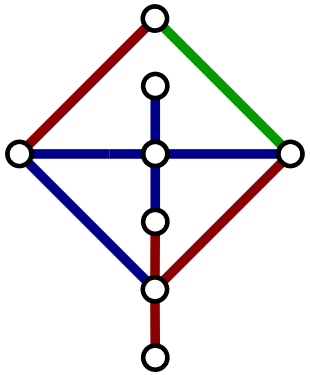


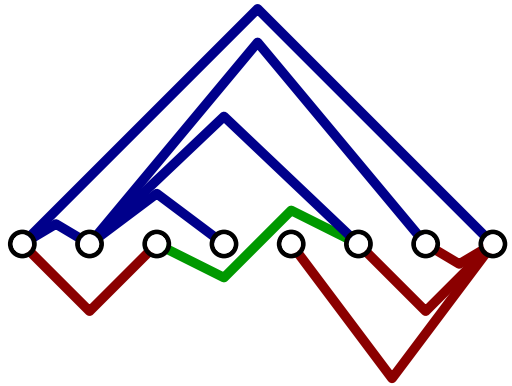
Graph 1: x-coordinates
Graph 2: y-coordinates
In R: all segments vertical
or of y-length 1



Planar × Planar



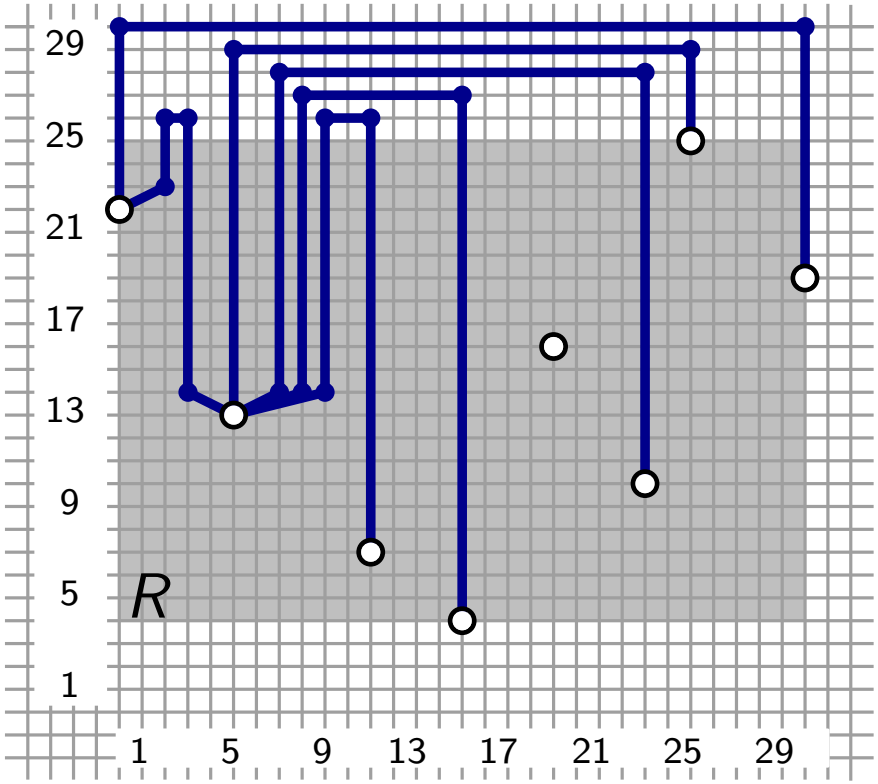
[Kaufmann & Wiese '02]:



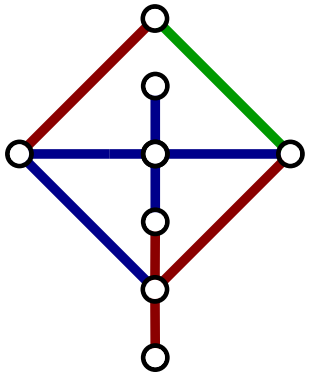
Graph 1: x-coordinates
Graph 2: y-coordinates
In R: all segments vertical
or of y-length 1

Edges:

4



Planar × Planar



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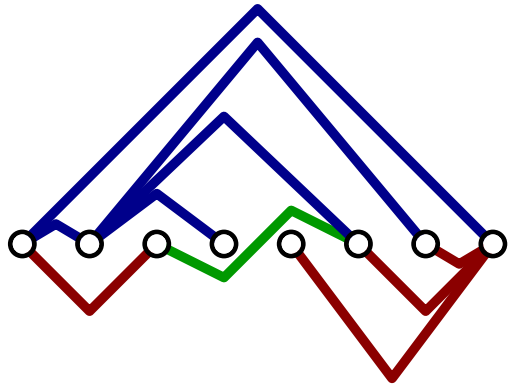
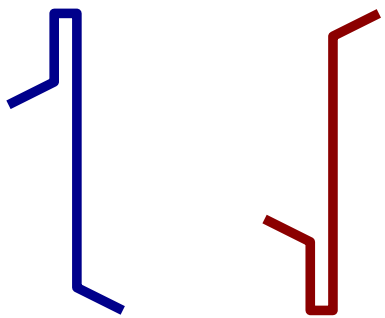
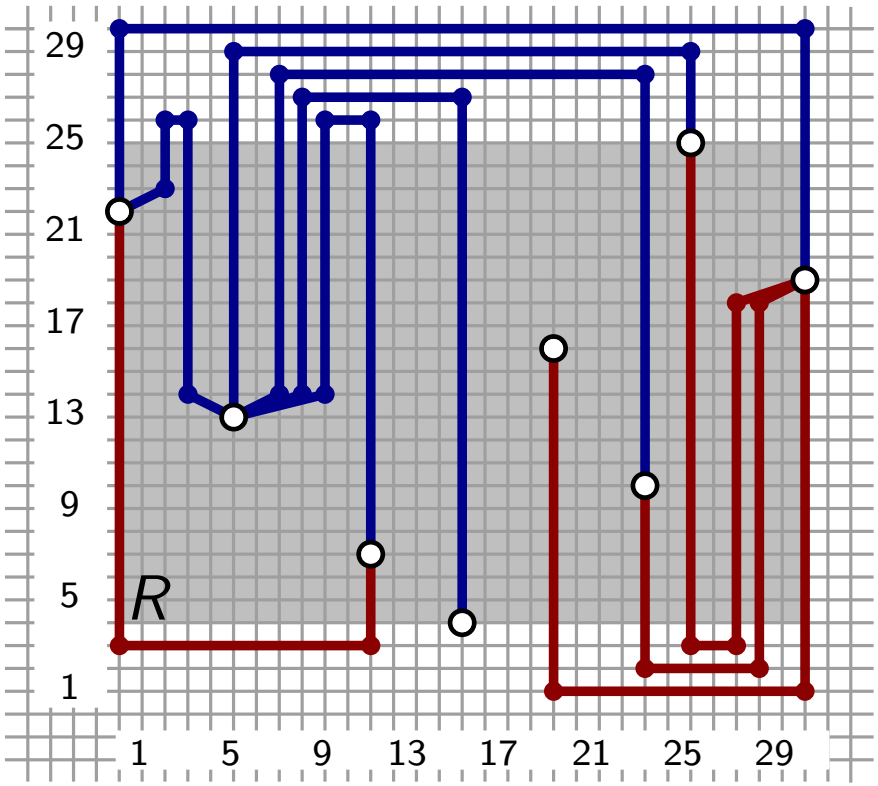


Graph 1: x-coordinates
Graph 2: y-coordinates
In R: all segments vertical
or of y-length 1

Edges:





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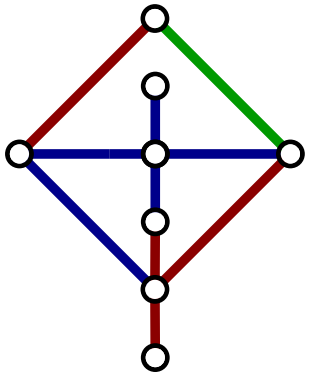

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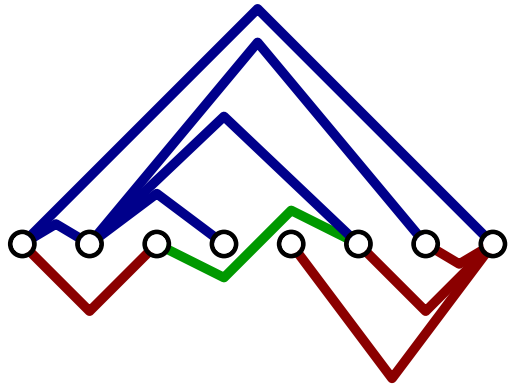

Graph 1: x-coordinates
Graph 2: y-coordinates
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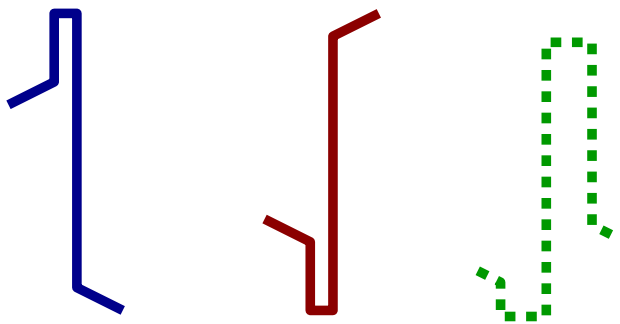
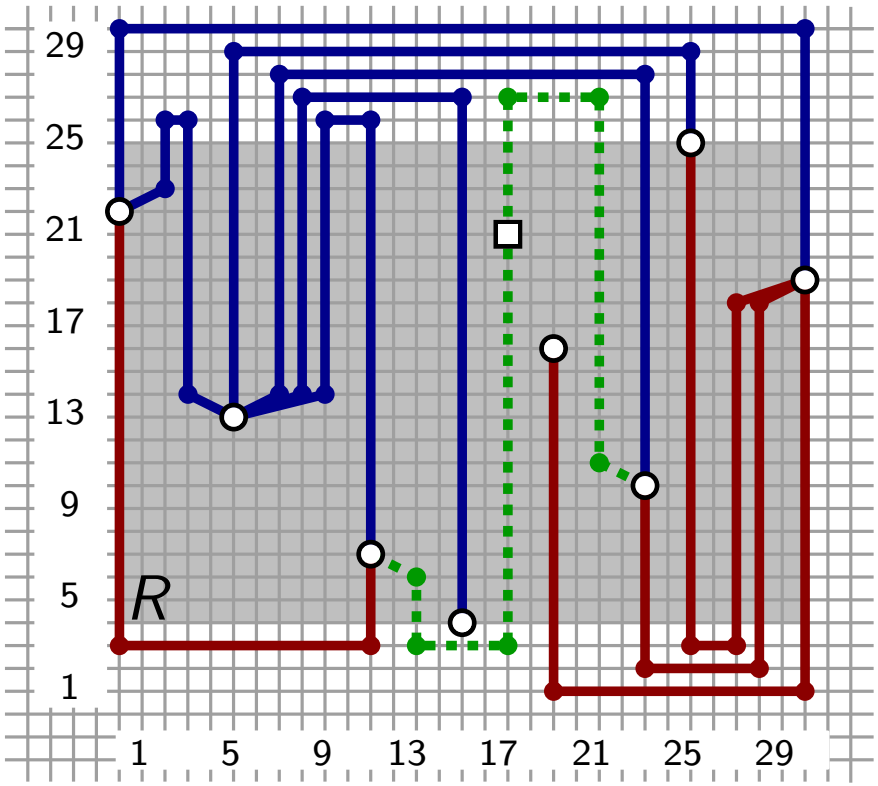
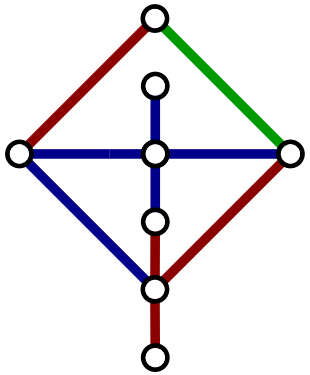




Planar × Planar



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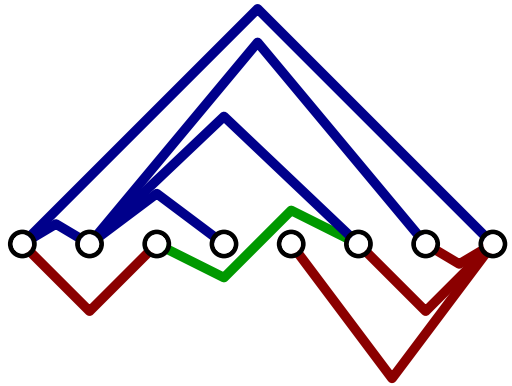

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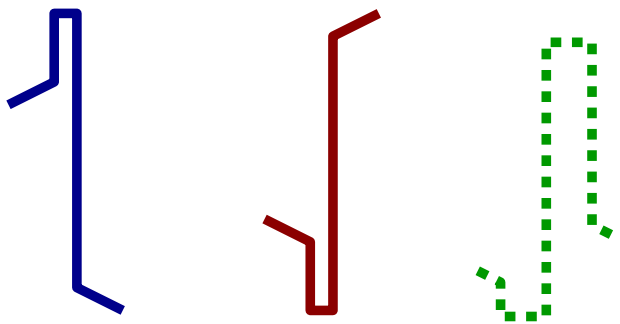
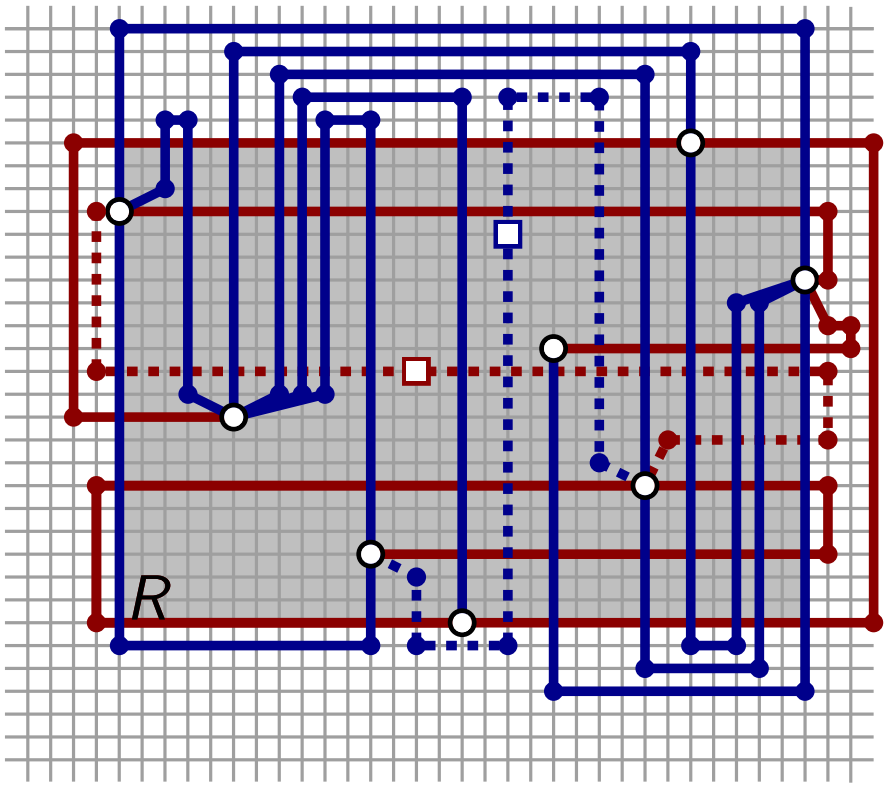
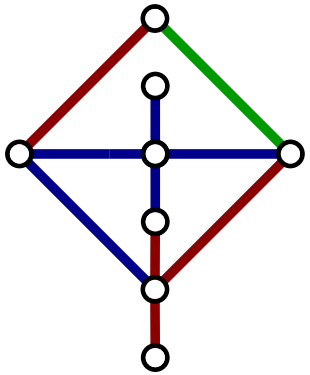
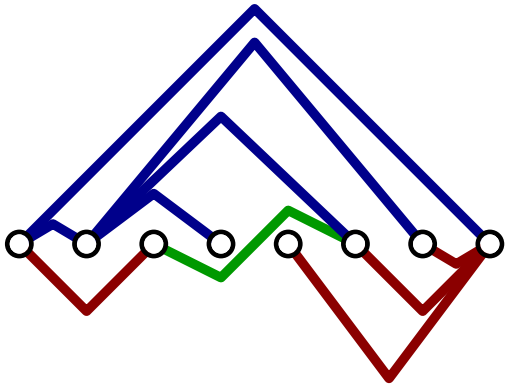




Planar × Planar



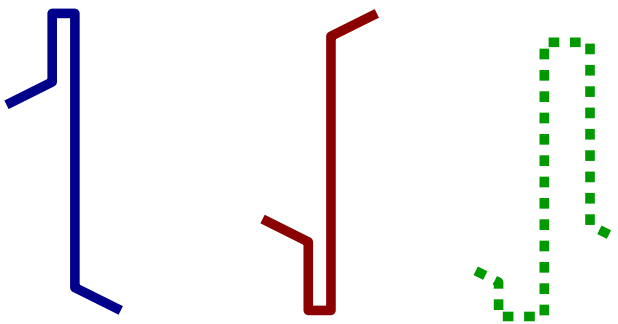
[Kaufmann & Wiese '0

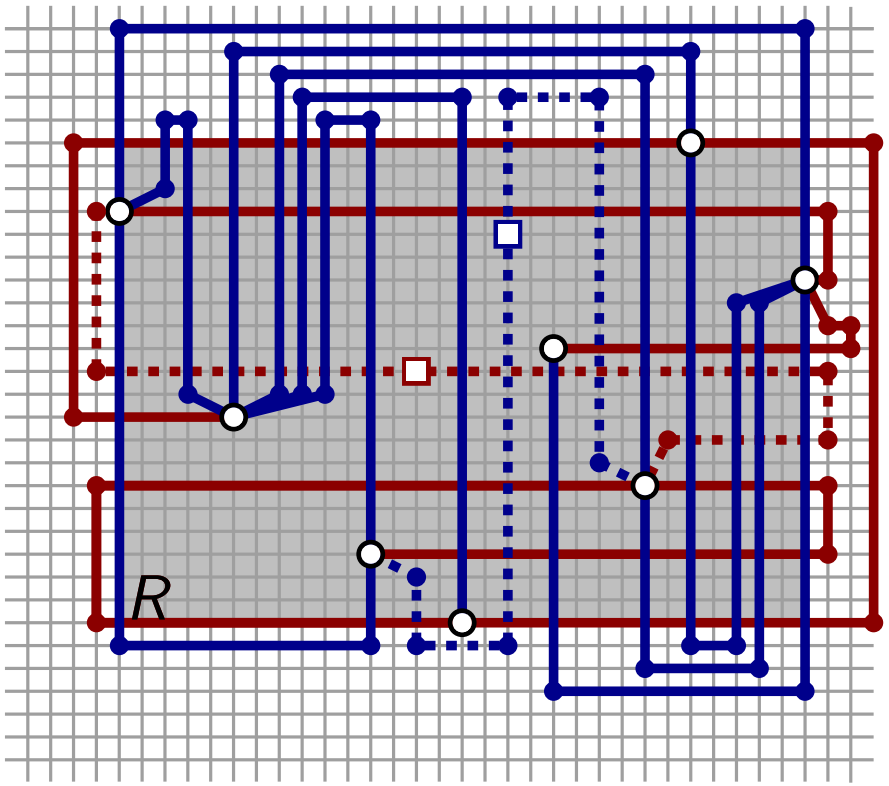


Bends: 6×6
Grid size:
$(14n - 26) \times (14n - 26)$

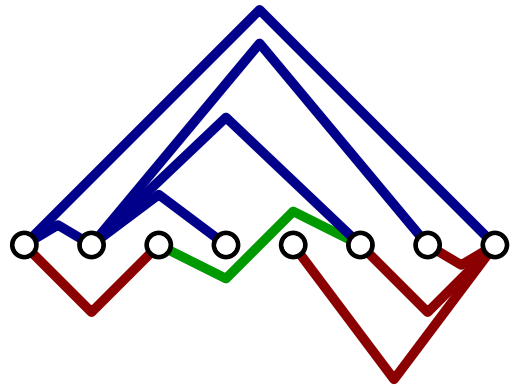
Graph 1: x-coordinates
Graph 2: y-coordinates
In R: all segments vertical
or of y-length 1

Edges:



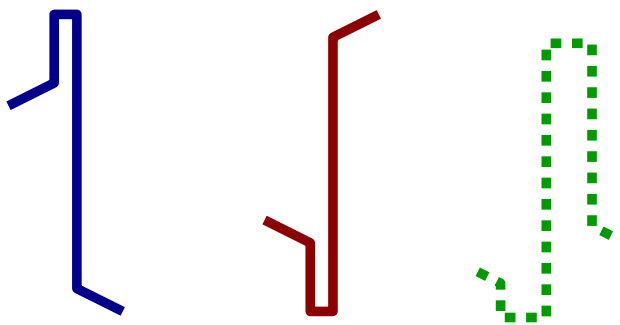


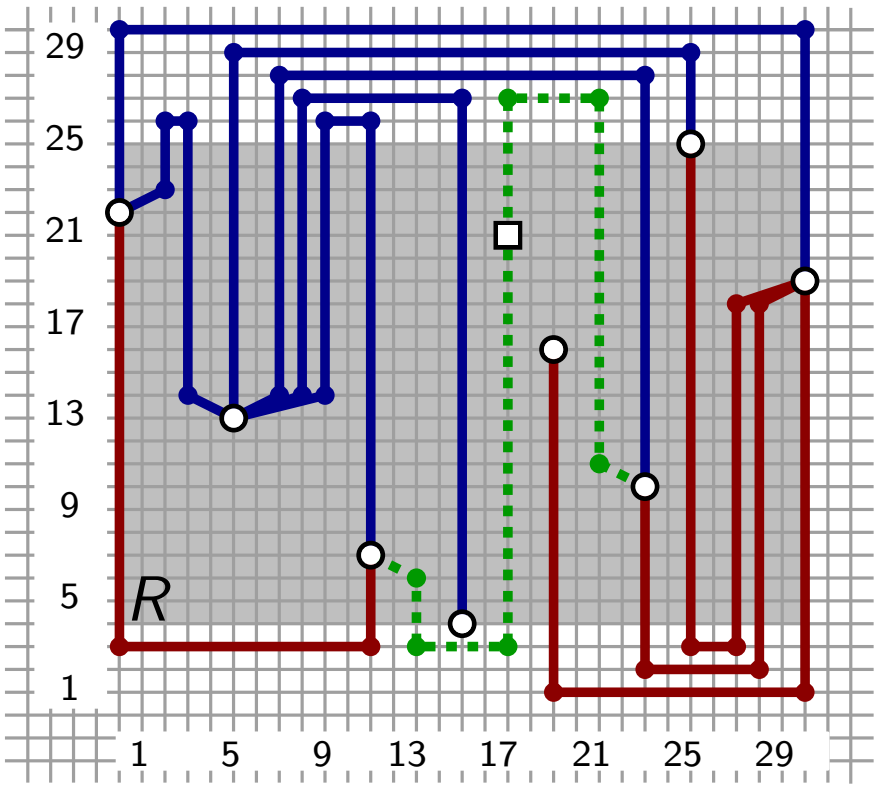
2-page book embeddable



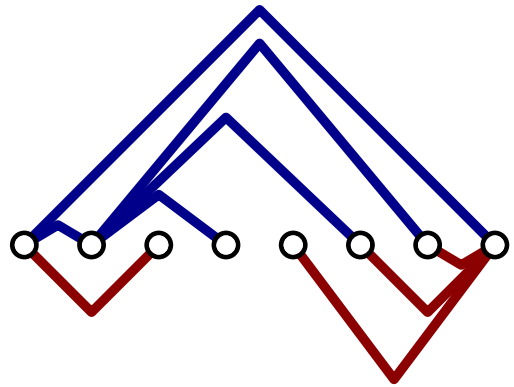
Graph 1: x-coordinates
Graph 2: y-coordinates
In R: all segments vertical
or of y-length 1

Edges:



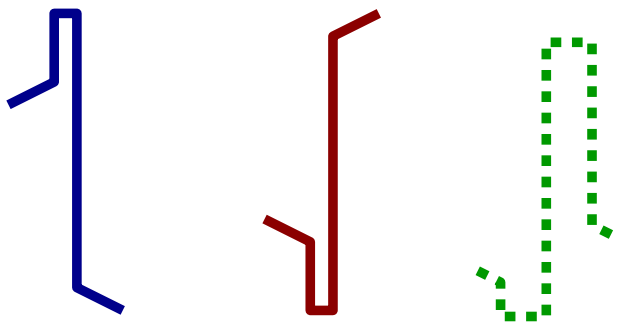


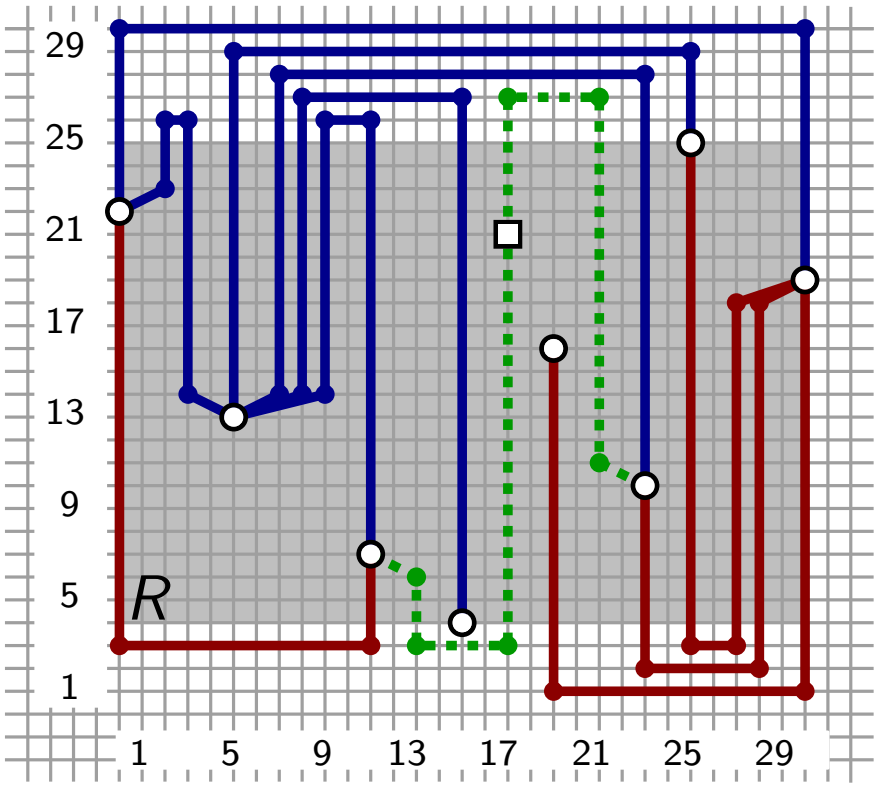
2-page book embeddable



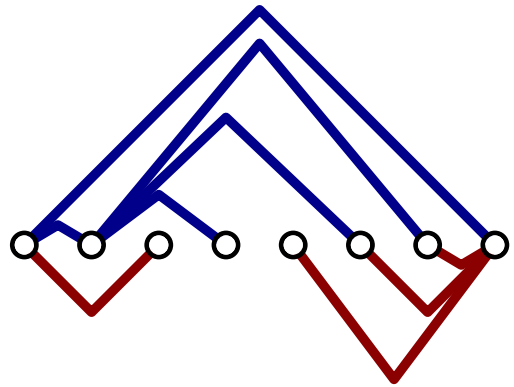
Graph 1: x-coordinates
Graph 2: y-coordinates
In R: all segments vertical
or of y-length 1

Edges:



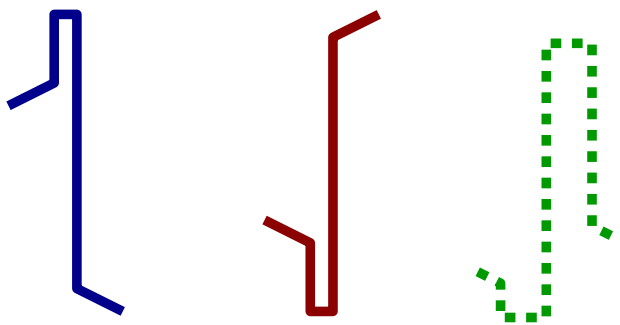


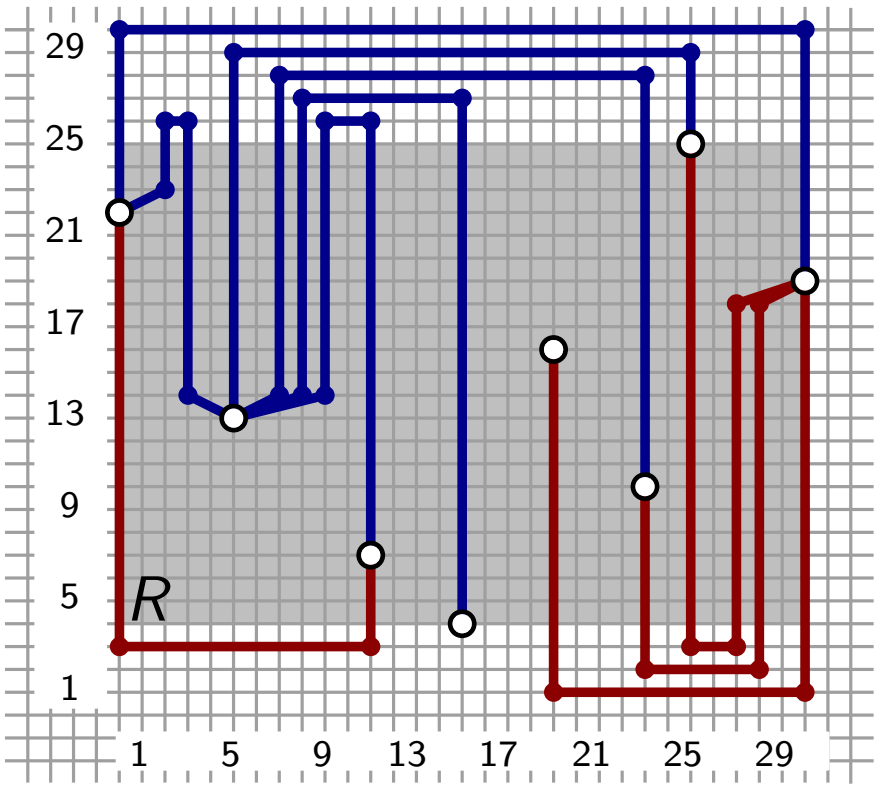
2-page book embeddable



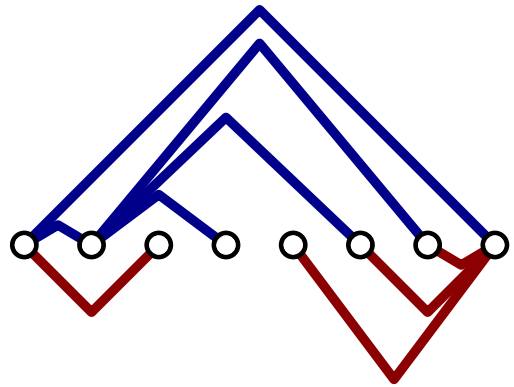
Graph 1: x-coordinates
Graph 2: y-coordinates
In R: all segments vertical
or of y-length 1

Edges:



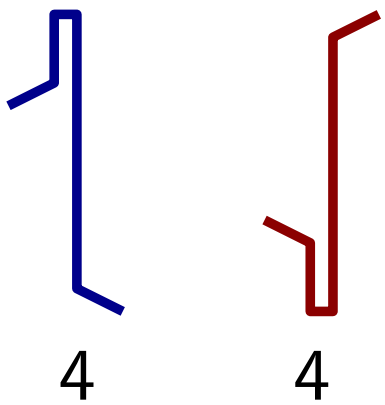


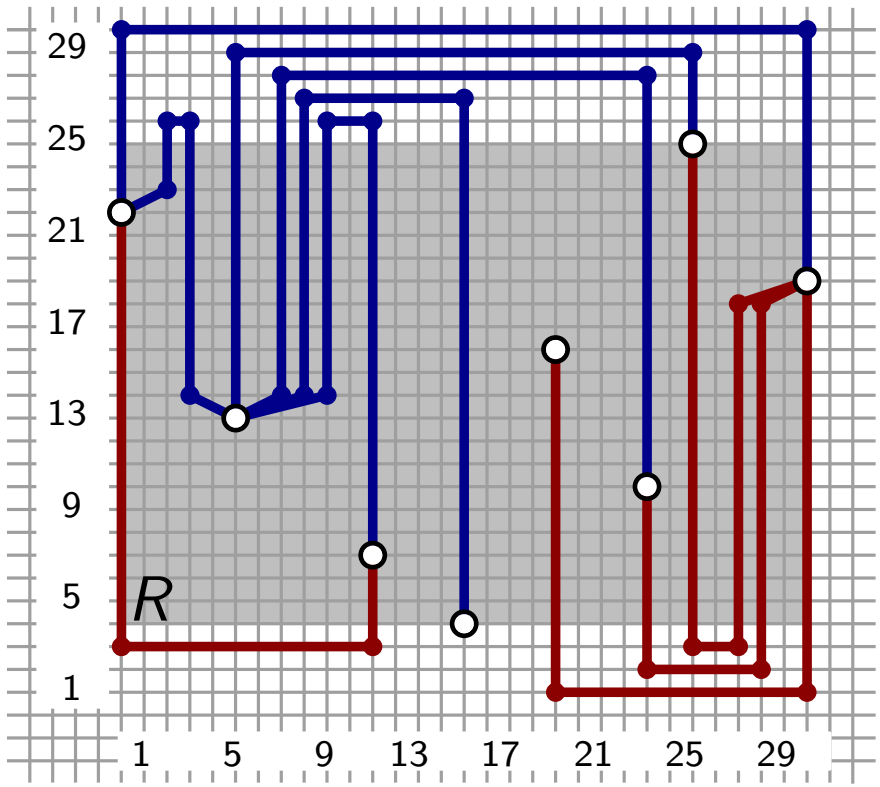
2-page book embeddable



Graph 1: x-coordinates
Graph 2: y-coordinates
In R: all segments vertical
or of y-length 1

Edges:





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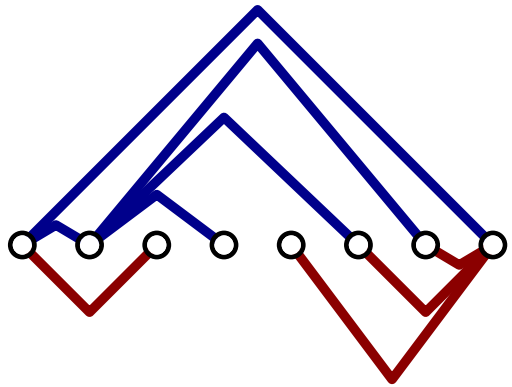
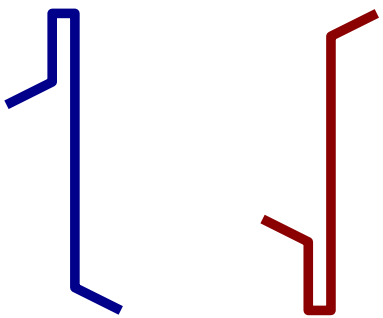
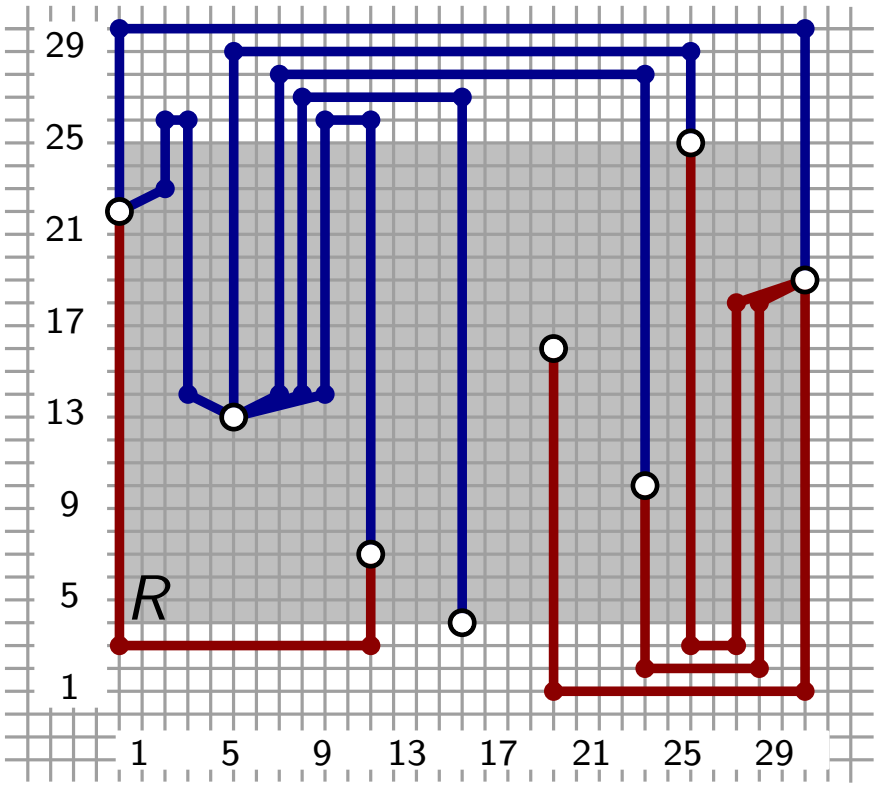


Bends: 4×4
Grid size:
$(11n - 32) \times (11n - 32)$

Graph 1: x-coordinates
Graph 2: y-coordinates
In R: all segments vertical
or of y-length 1

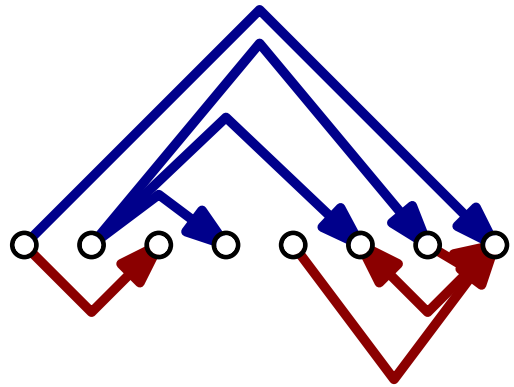
Edges:





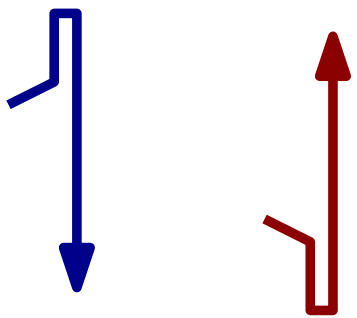
Outerplanar × Outerplanar

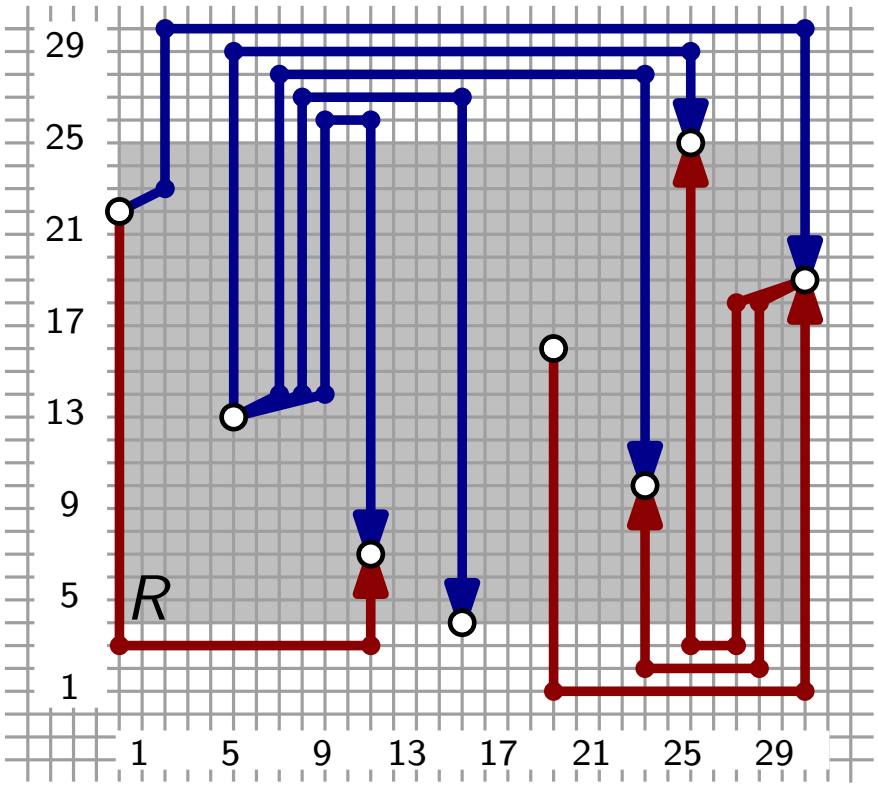
Decompose
into two
directed trees



Graph 1: x-coordinates
Graph 2: y-coordinates
In R: all segments vertical
or of y-length 1

Edges:





Outerplanar × Outerplanar

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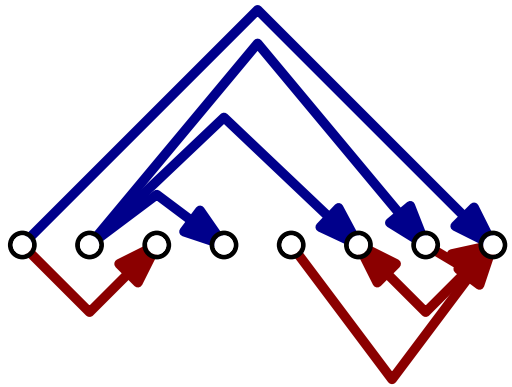
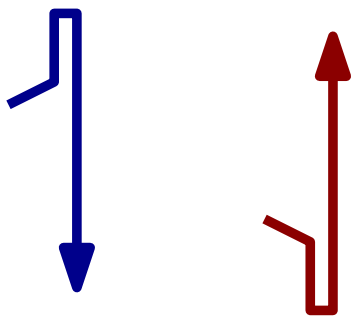
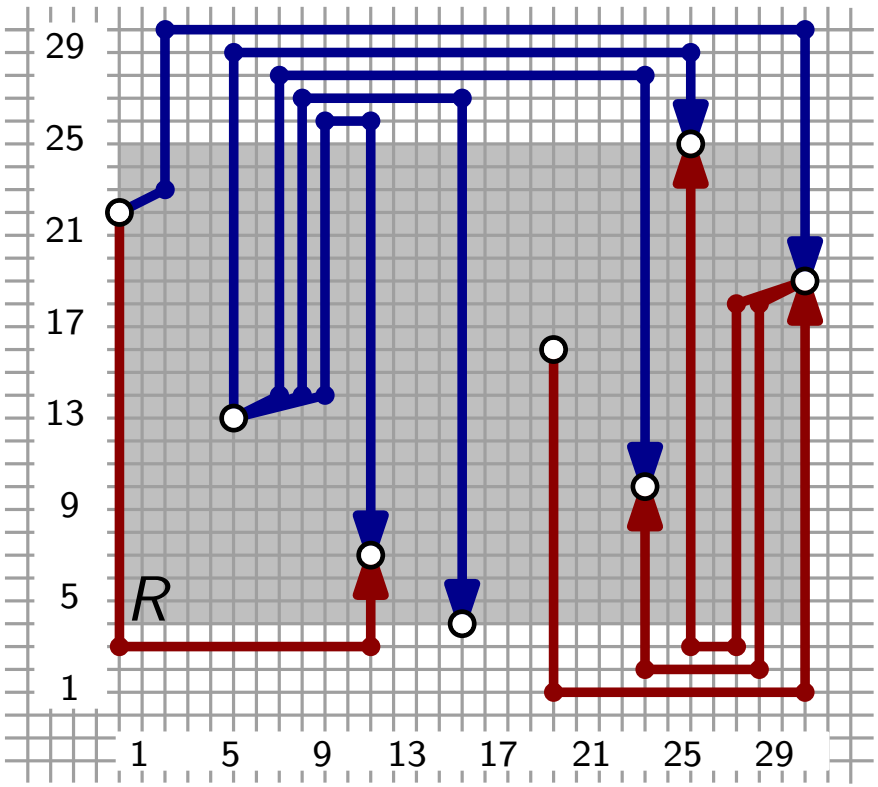


Bends: 3×3
Grid size:
$(7n - 10) \times (7n - 10)$

Graph 1: x-coordinates
Graph 2: y-coordinates
In R: all segments vertical
or of y-length 1

Edges:





Overview

Graph classes			Number of bends	
Cycle	$\times$	Cycle	1×1	
Caterpillar	$\times$	Cycle	1×1	
Four Matchings			$1 \times 1 \times 1 \times 1$	
Tree	$\times$	Matching	1×0	
Wheel	$\times$	Matching	2×0	
Outerpath	$\times$	Matching	2×1	
Outerplanar	$\times$	Outerplanar	3×3	✓
2-page book emb.	$\times$	2-page book emb.	4×4	✓
Planar	$\times$	Planar	6×6	✓

Overview

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Cycle	$\times$	Cycle	1×1
Caterpillar	$\times$	Cycle	1×1
Four Matchings			$1 \times 1 \times 1 \times 1$
Tree	$\times$	Matching	1×0
Wheel	$\times$	Matching	2×0
Outerpath	$\times$	Matching	2×1
Outerplanar	$\times$	Outerplanar	3×3
2-page book emb.	$\times$	2-page book emb.	4×4
Planar	$\times$	Planar	6×6

- All graphs are drawn on the $O(n) \times O(n)$-grid

Overview

Graph classes			Number of bends
Cycle	$\times$	Cycle	1×1
Caterpillar	$\times$	Cycle	1×1
Four Matchings			$1 \times 1 \times 1 \times 1$
Tree	$\times$	Matching	1×0
Wheel	$\times$	Matching	2×0
Outerpath	$\times$	Matching	2×1
Outerplanar	$\times$	Outerplanar	3×3
2-page book emb.	$\times$	2-page book emb.	4×4
Planar	$\times$	Planar	6×6

- All graphs are drawn on the $O(n) \times O(n)$-grid
- All algorithms run in $O(n)$ time

Open Problems

- Relax constraints on crossing resolution
  $\Rightarrow$ LAC (*Large Angle Crossings*)

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  $\Rightarrow$ LAC (*Large Angle Crossings*)

- Comp. Complexity of general problem:
  Given two graphs, is there a RACSIM drawing with at most k bends per edge

- SIMRAC: Each graph is drawn RAC