Julius-Maximilians-UNIVERSITÄT WÜRZBURG

Chair for INFORMATICS I
Efficient Algorithms and Knowledge-Based Systems

Institute for Informatics

Smooth Orthogonal Drawings of Planar Graphs

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Joint work with
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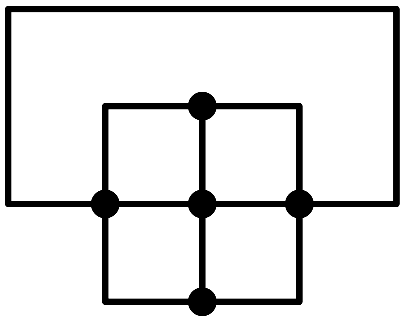
Stephen G. Kobourov & Alexander Wolff

Orthogonal Layouts

- all edge segments are horizontal or vertical
- a well-studied drawing convention
- many examples in applications



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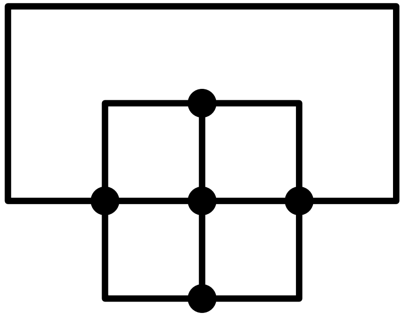

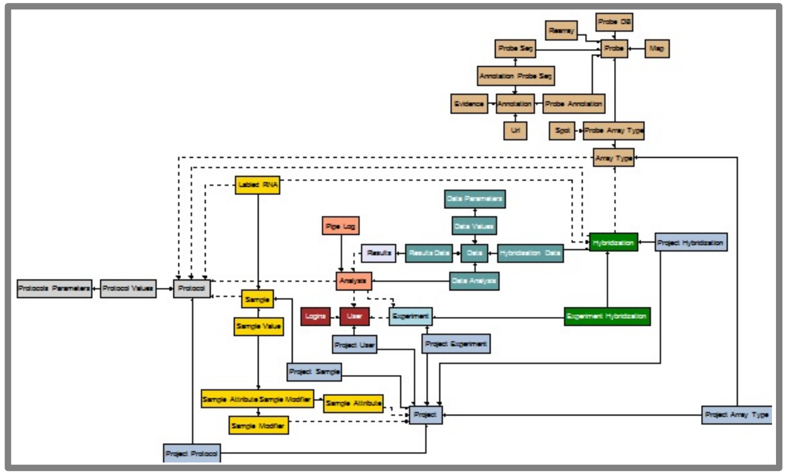

ER diagram in OGDF

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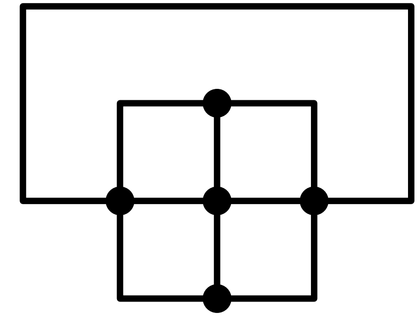

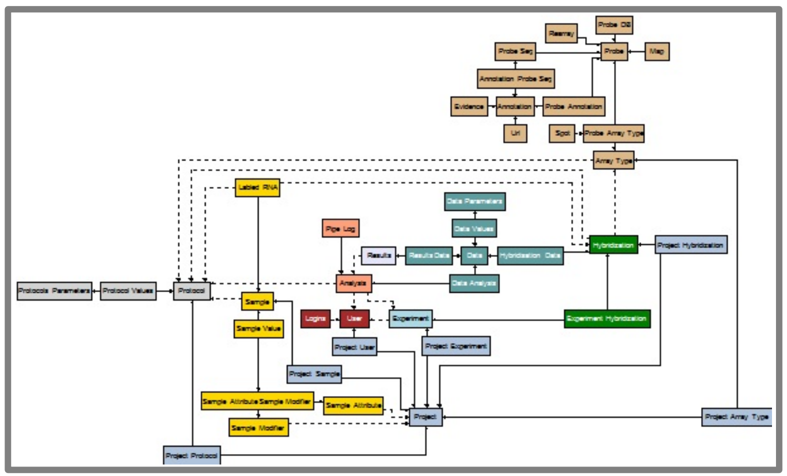

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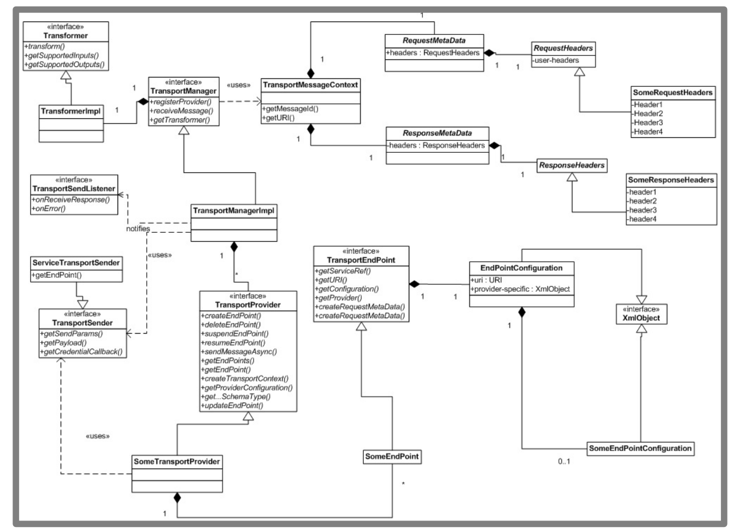

UML diagram by Oracle

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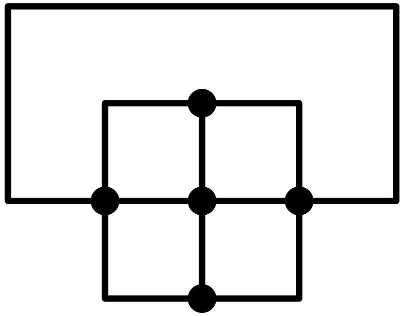

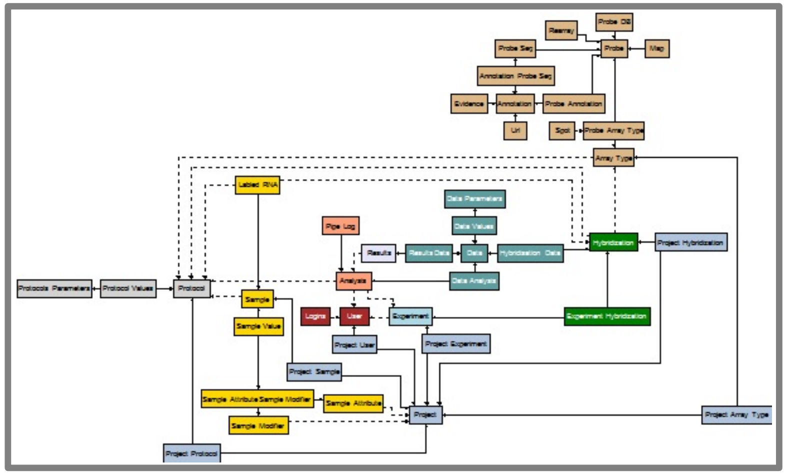

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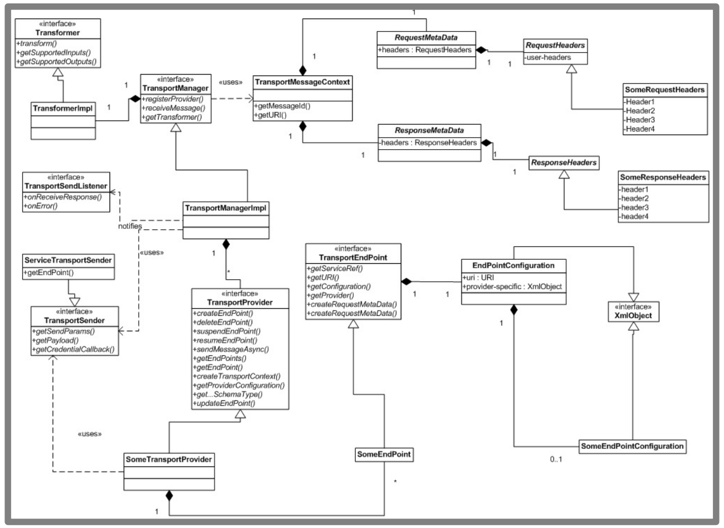

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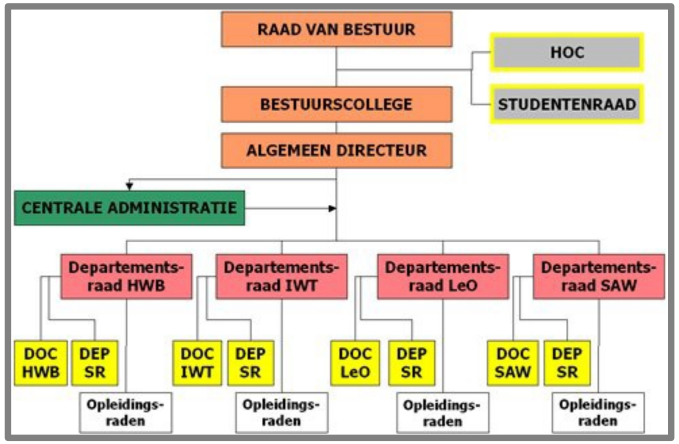

Organigram of HS Limburg

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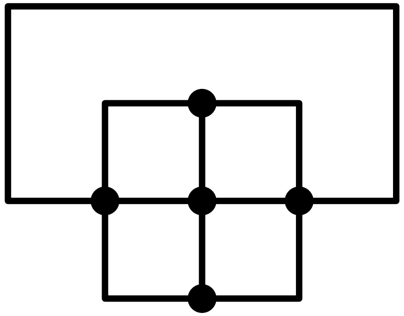

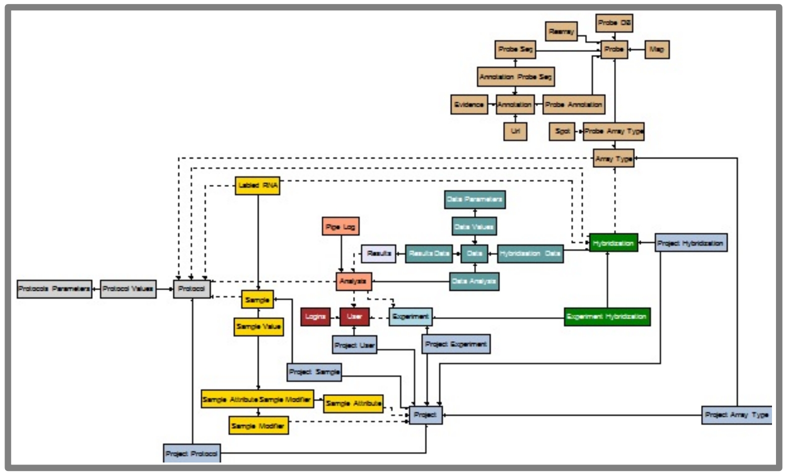

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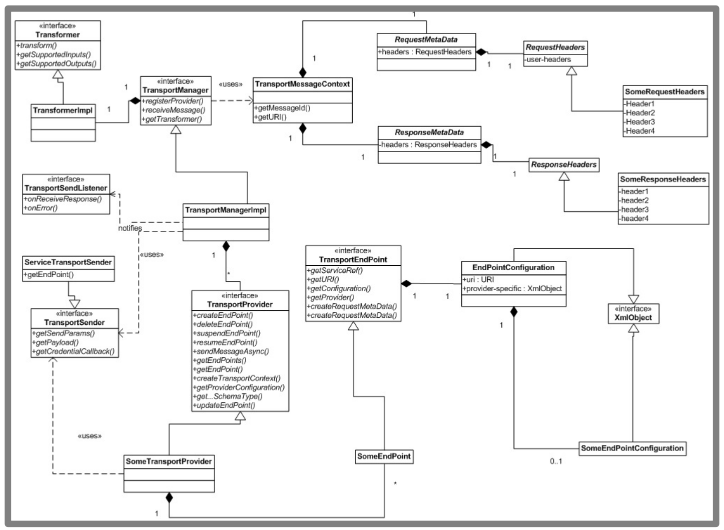

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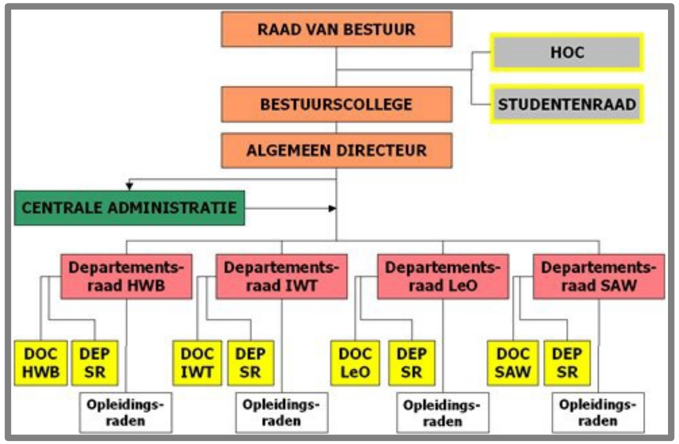

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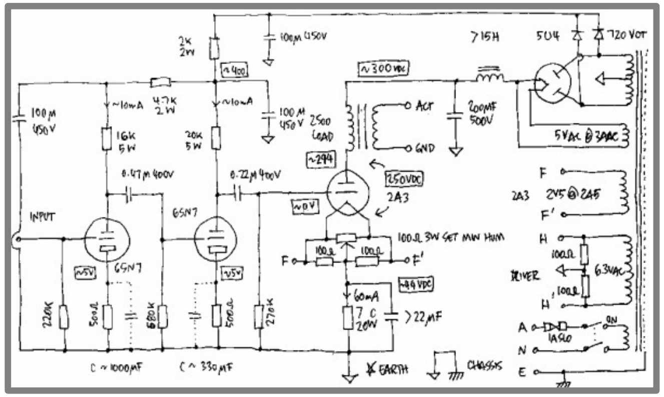
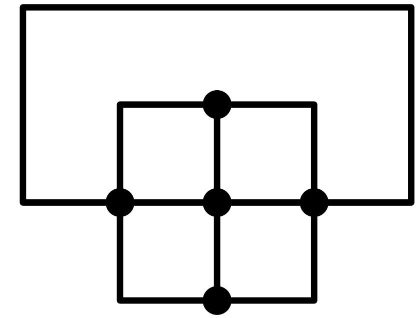


Circuit diagram by Jeff Atwood

Orthogonal Layouts



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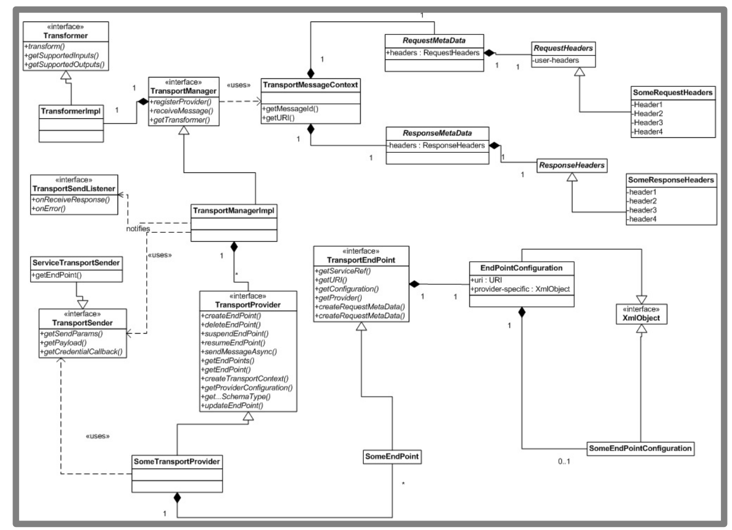
Fused Grid city layouts [By Fgrammen, via Wikimedia Commons]



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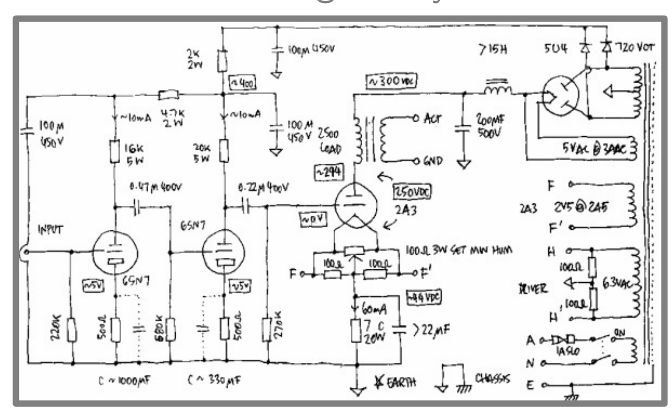

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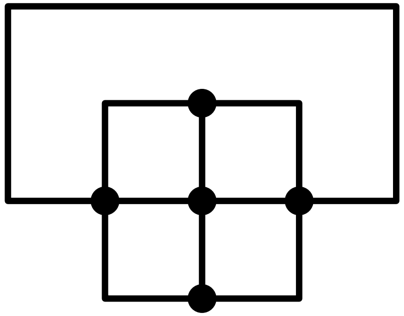

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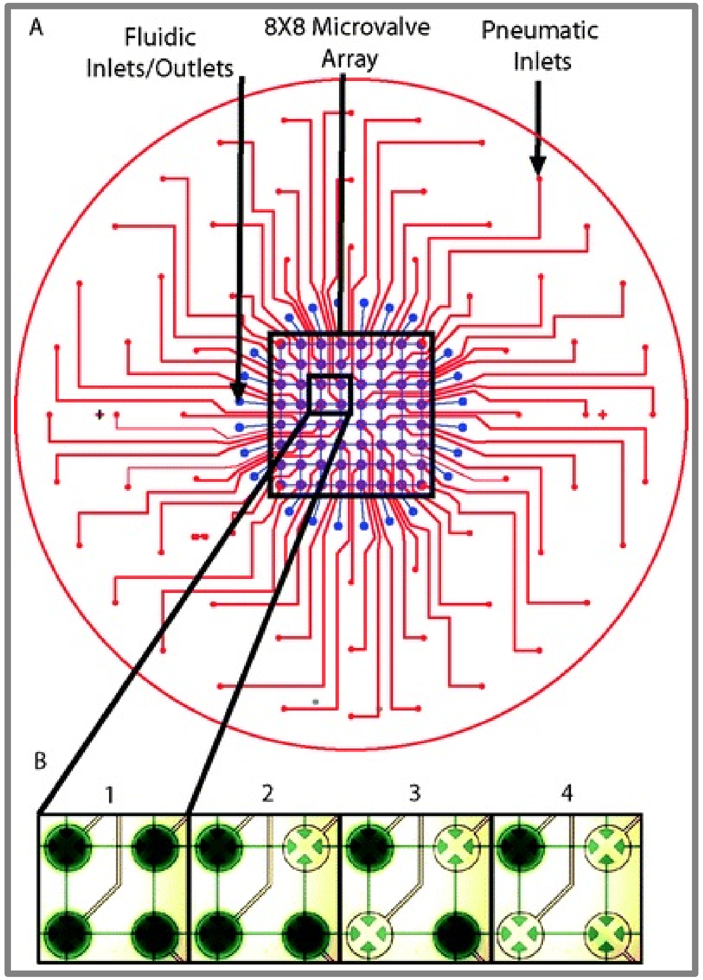


VLSI/PCI chip design [Mora et al. 2013]

Orthogonal Layouts – Well-Known Results

[Tamassia, SIAM J Comp'87]
Can minimize number of bends for fixed embedding.

[Garg & Tamassia, SIAM J Comp'01]
Without fixed embedding, bend minimization is hard to approx.

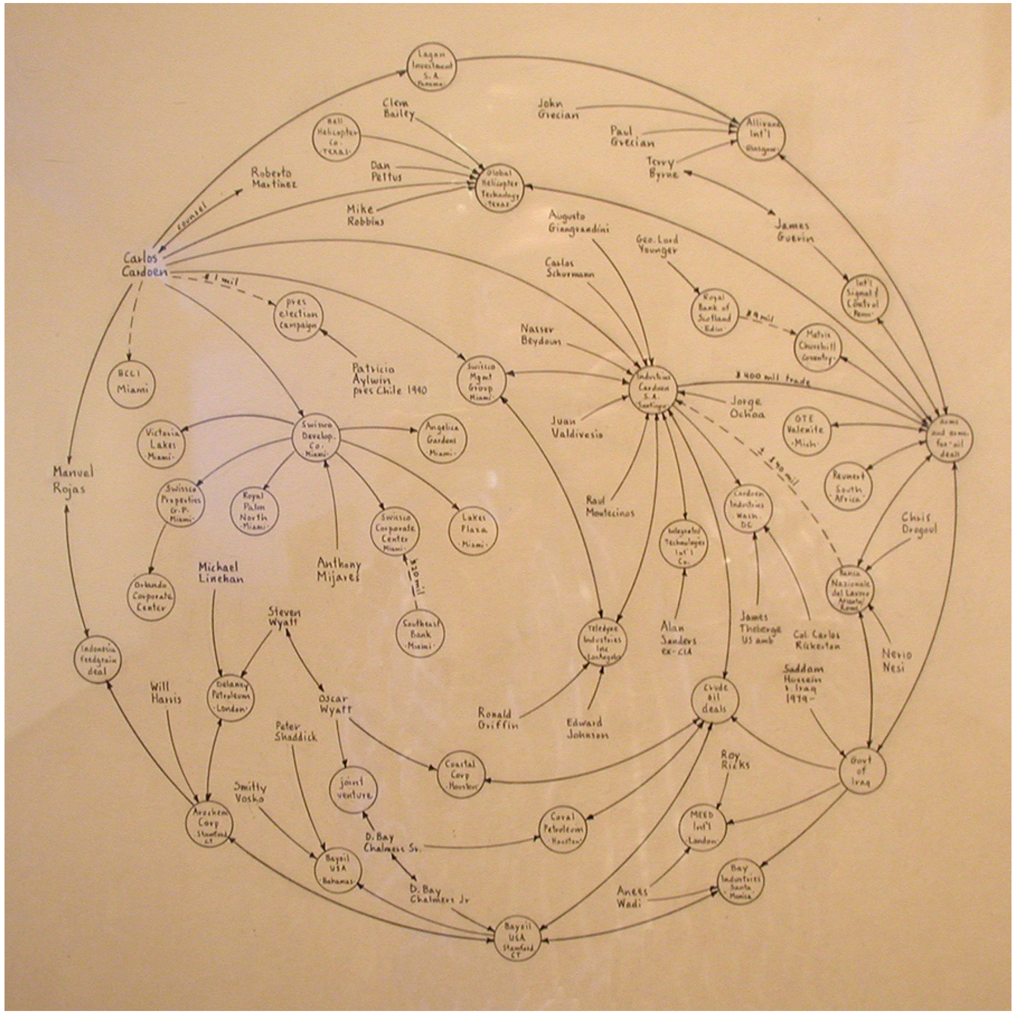
[Biedl & Kant, CGTA'98], [Liu et al., DAM'98]
Can compute drawing on the $(n \times n)$-grid with $\leq 2n + 2$ bends for any embedding (and ≤ 2 bends/edge – except octahedron)

[Bläsius et al., '11]
Given an embedding and a function flex: $E \to \mathbb{N}_{\geq 1}$, can compute a drawing with $\leq$ flex(e) bends/edge (if one exists).

Smooth Drawings

Lombardi drawings

- circular arc edges
- perfect angular resolution



Mark Lombardi (1951–2000)

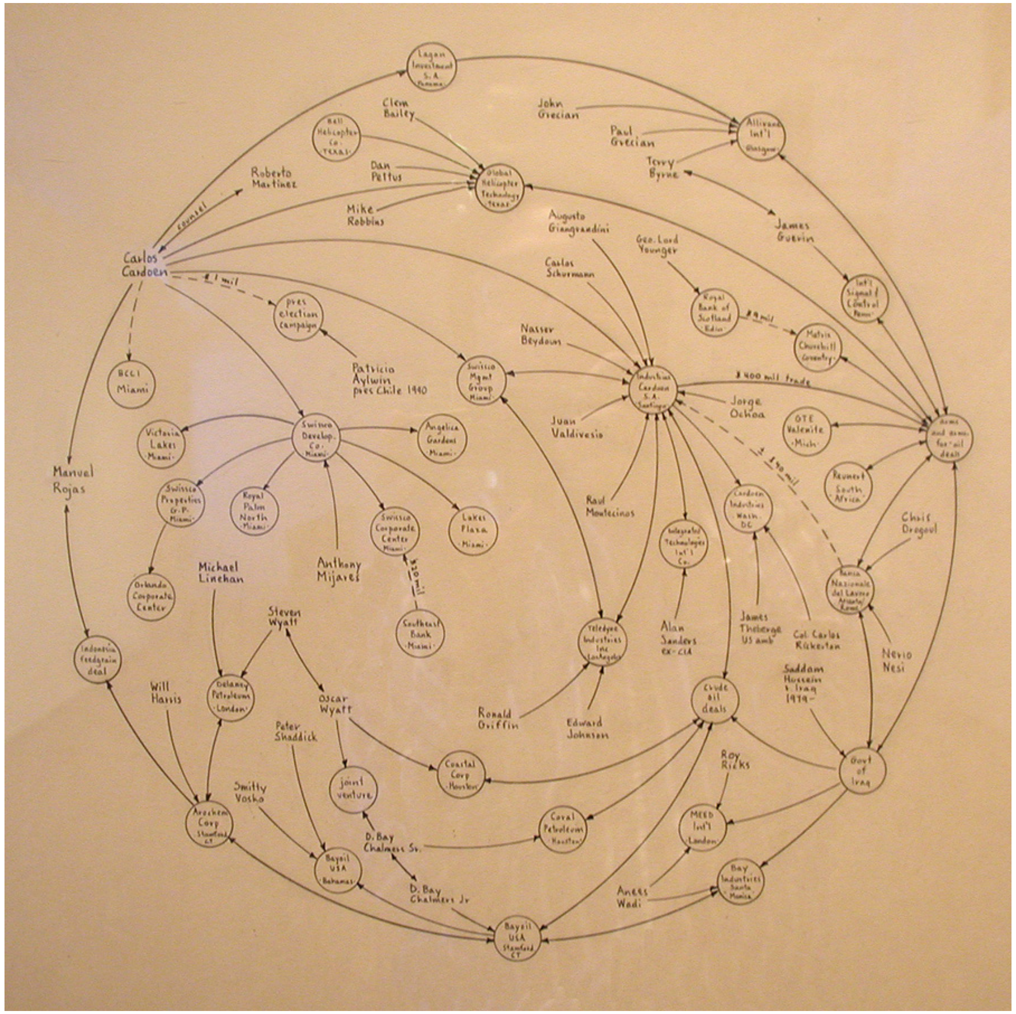
Smooth Drawings

Lombardi drawings

- circular arc edges
- perfect angular resolution

k-Lombardi drawings

- each edge sequence of k circular arcs



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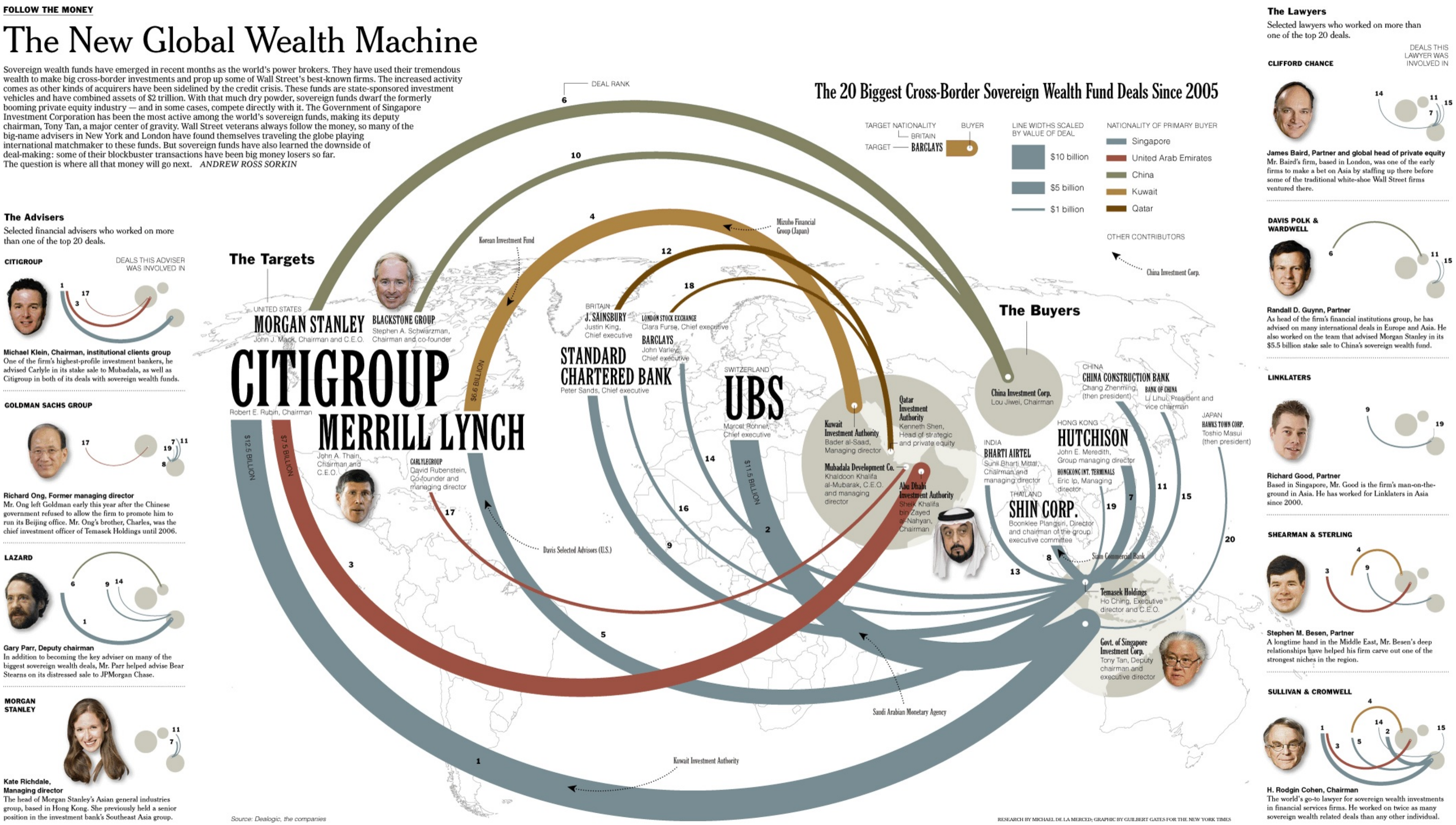
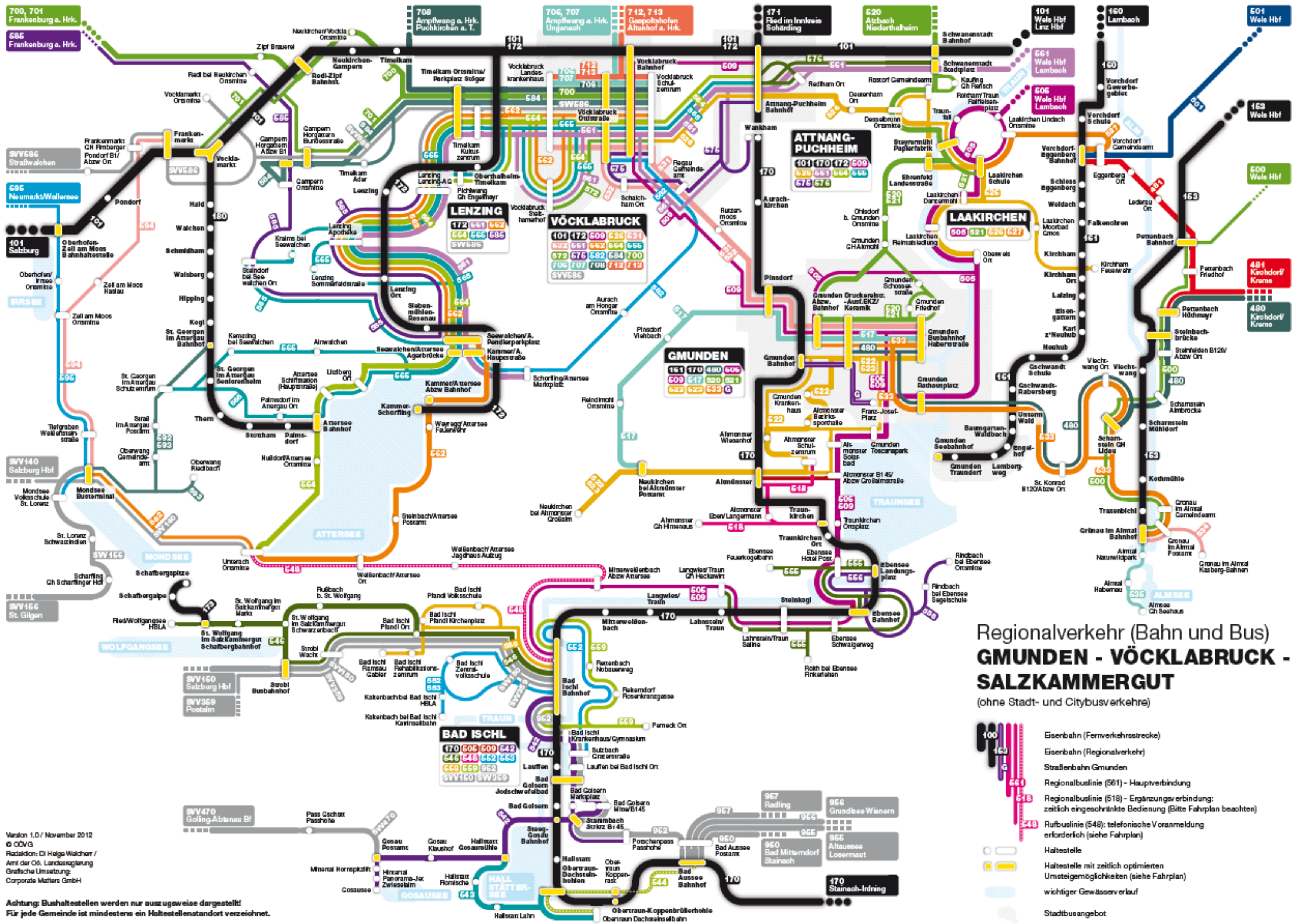


The New York Times info diagram

Smooth Drawings

Terra Nova - Design Overdrive by Carlos Barbosa

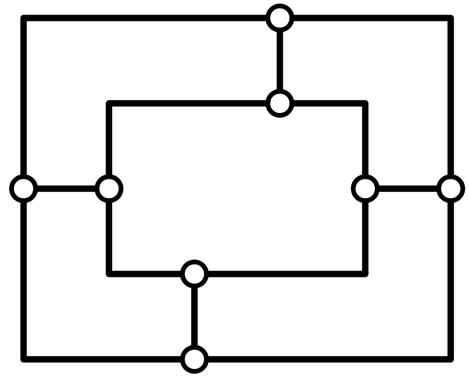
Smooth Drawings



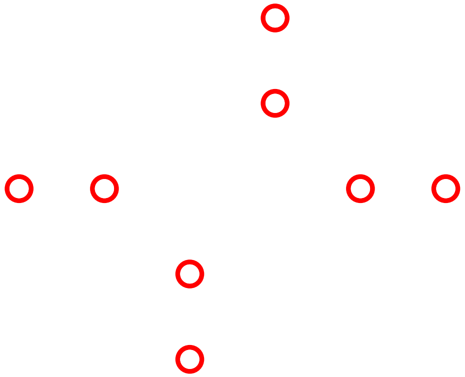
Public transportation map by OÖVG

Smooth Orthogonal Layouts

Combine both worlds:



orthogonal



smooth orthogonal

Smooth Orthogonal Layouts

Combine both worlds:

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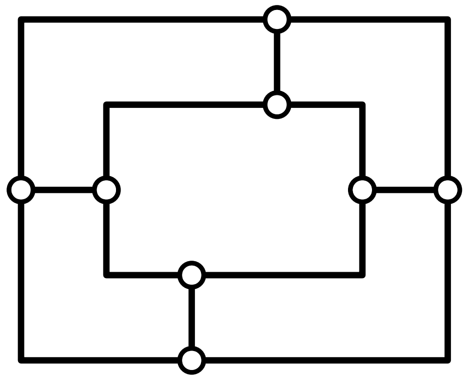

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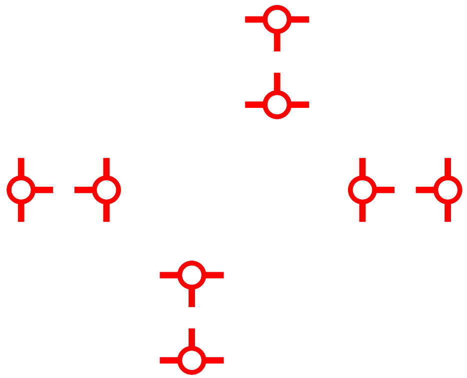

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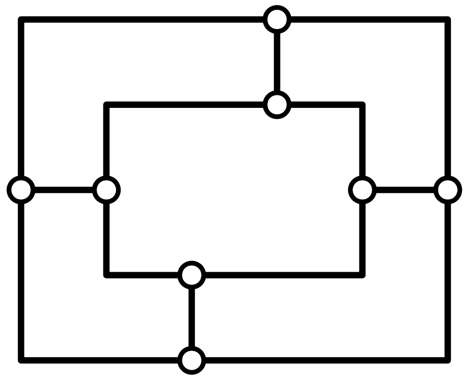
Smooth Orthogonal Layouts

Combine both worlds:

- edges leave and enter vertices horizontally or vertically
- each edge is drawn as a sequence of axis-aligned line segments and circular-arc segments without bends



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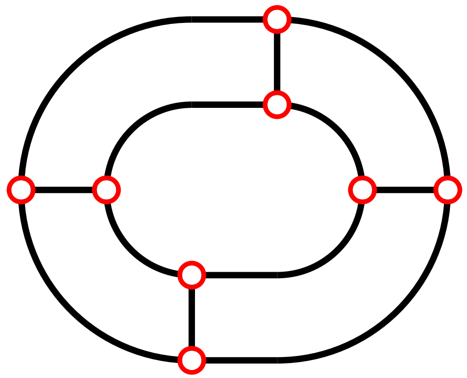

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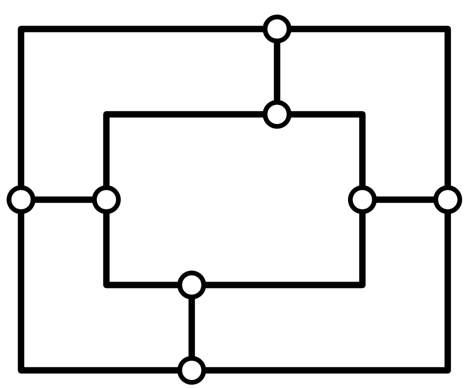
Smooth Orthogonal Layouts

Combine both worlds:

- edges leave and enter vertices horizontally or vertically
- each edge is drawn as a sequence of axis-aligned line segments and circular-arc segments without bends
- there are no edge-crossings (for planar graphs)



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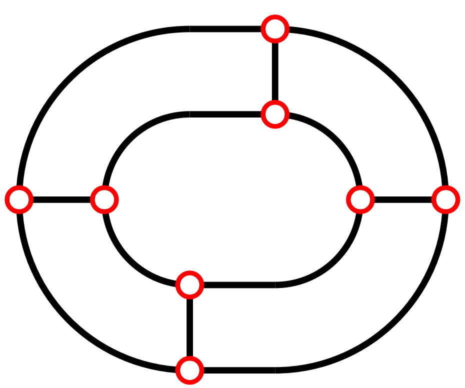
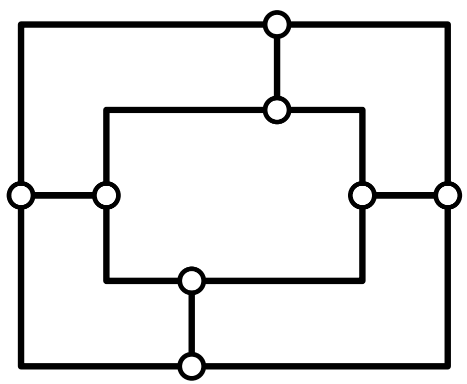


smooth orthogonal

Edge Complexity



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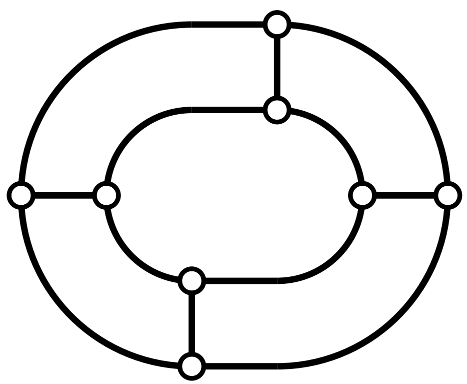

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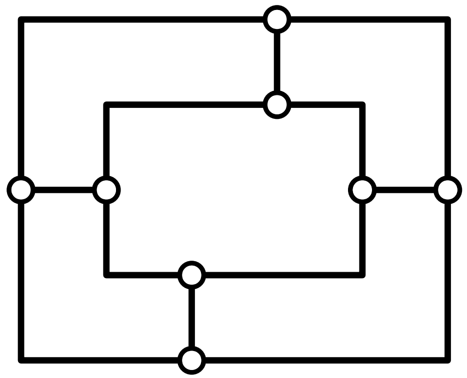
Edge Complexity

complexity of an edge: number of arcs



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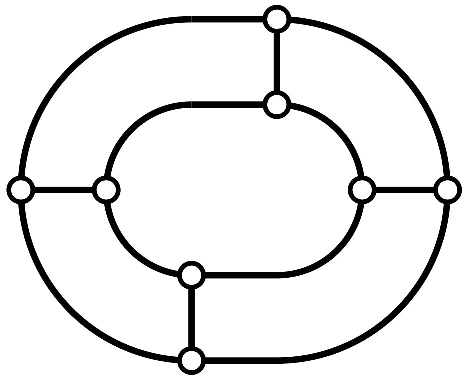

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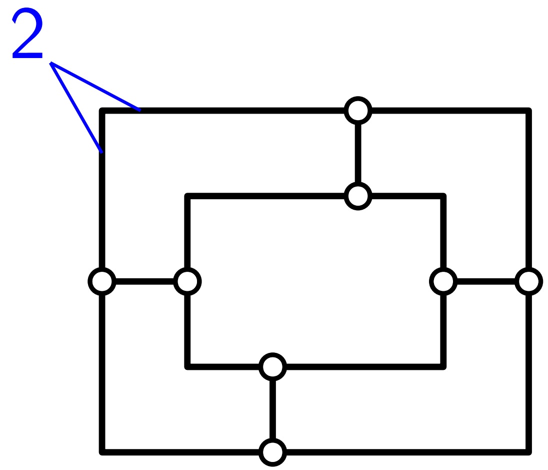

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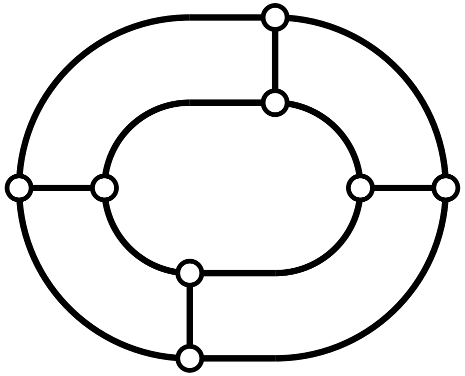

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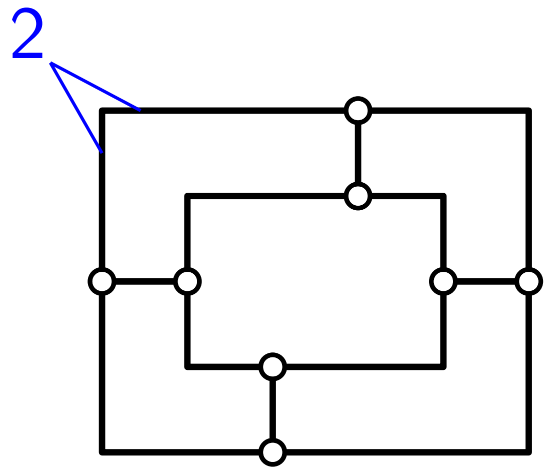

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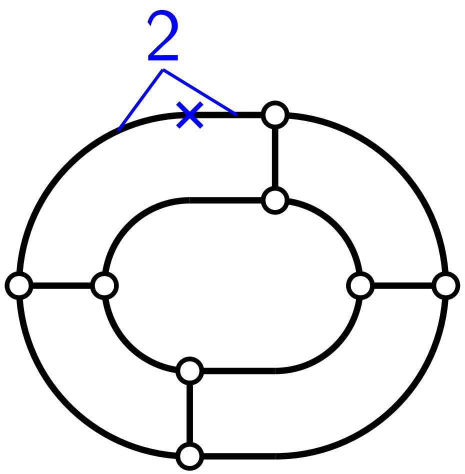

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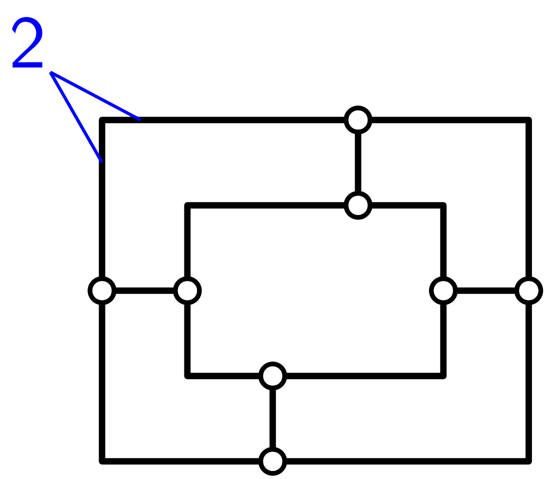
Edge Complexity

complexity of an edge: number of arcs

edge complexity: maximum complexity over all edges



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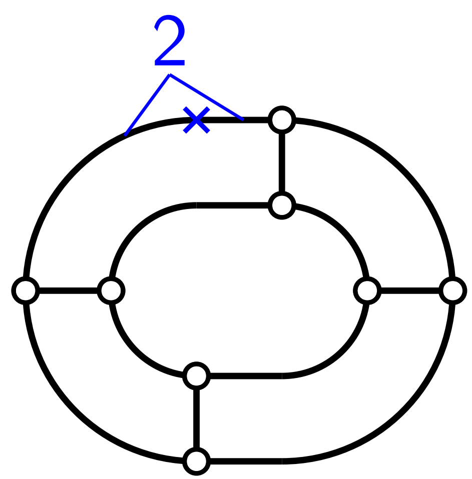

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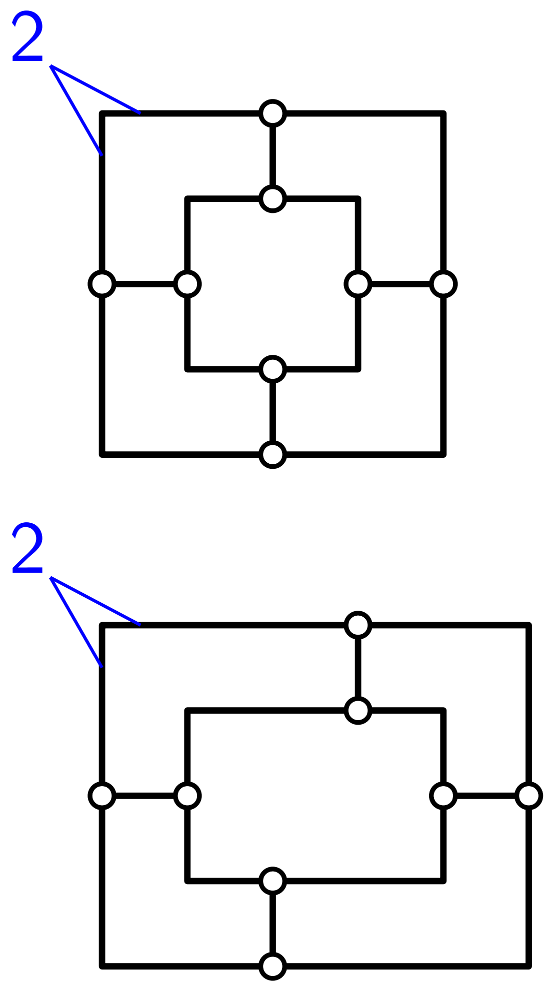

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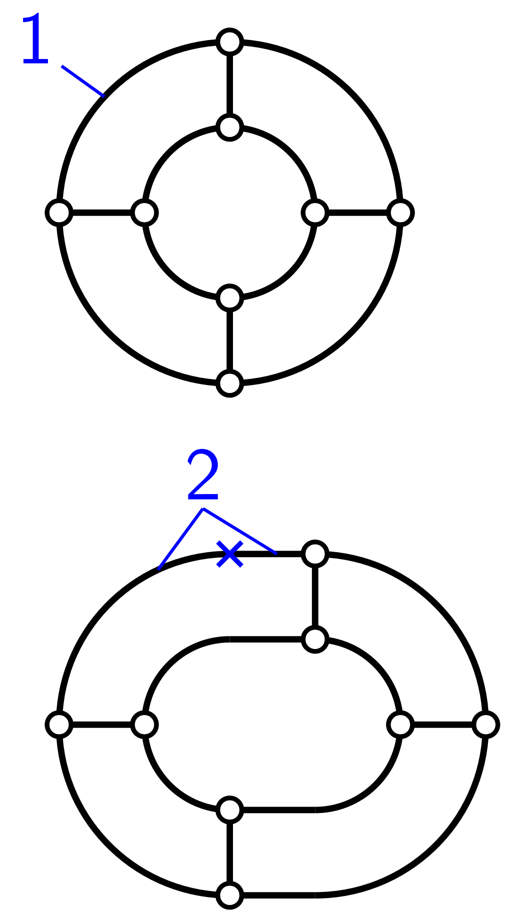

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Liu et al. Algorithm

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-3 layout

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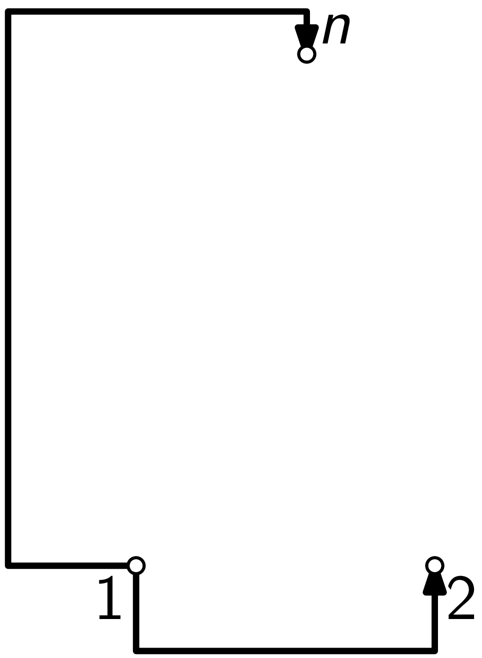

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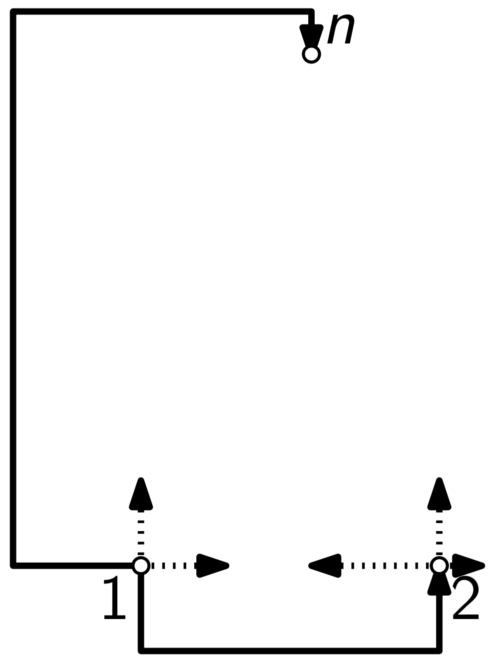

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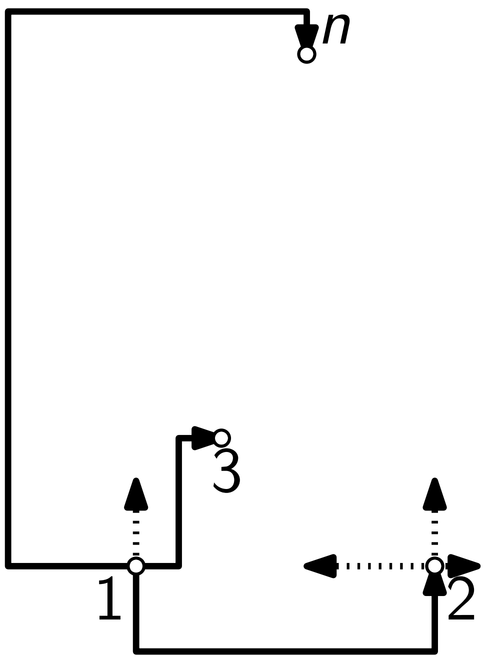

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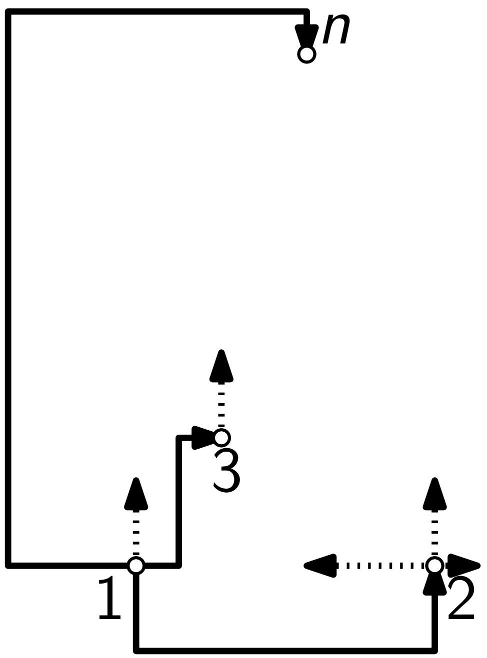

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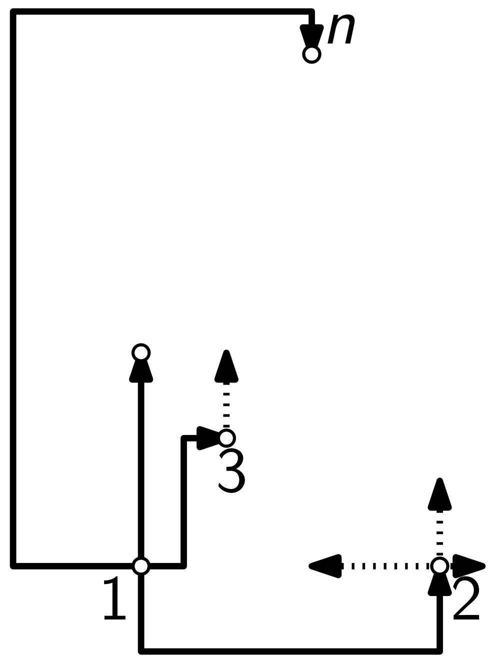

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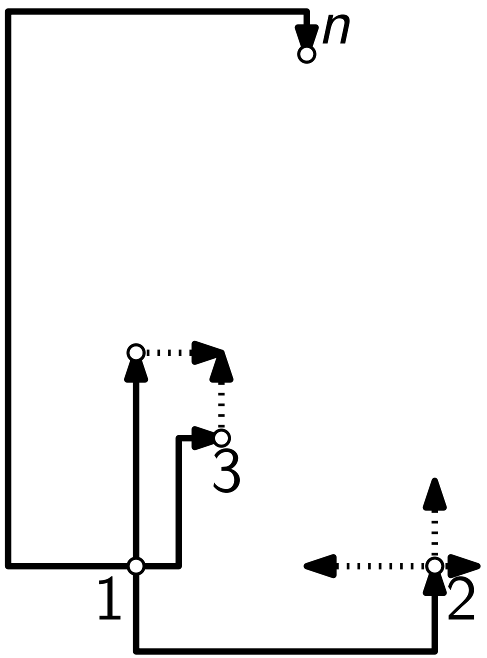

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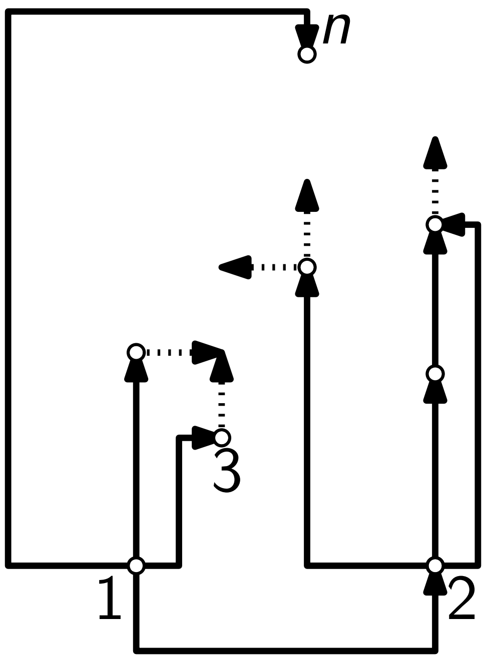

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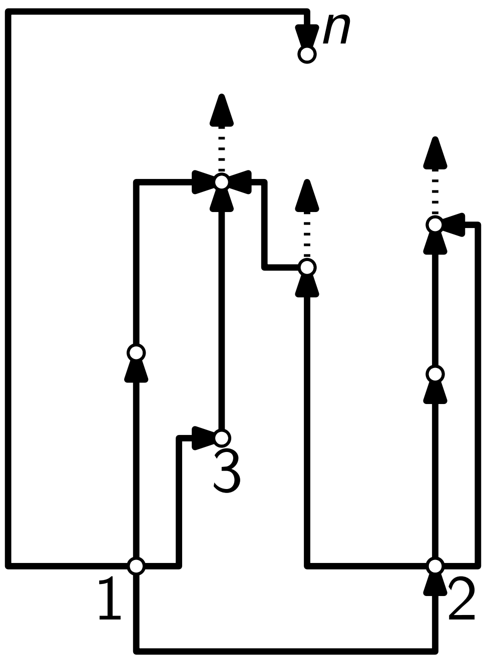

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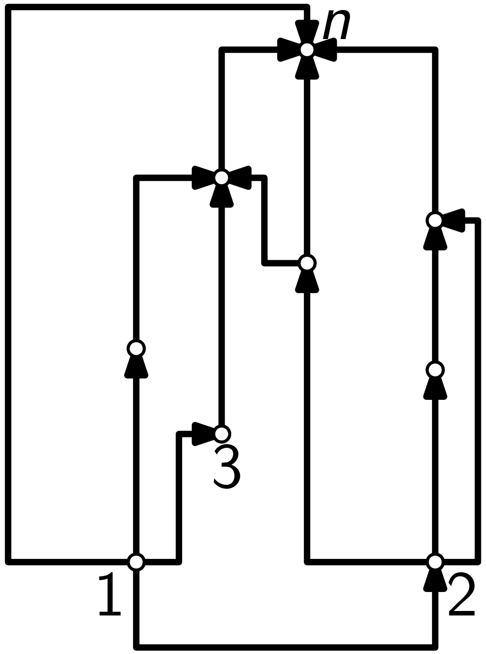

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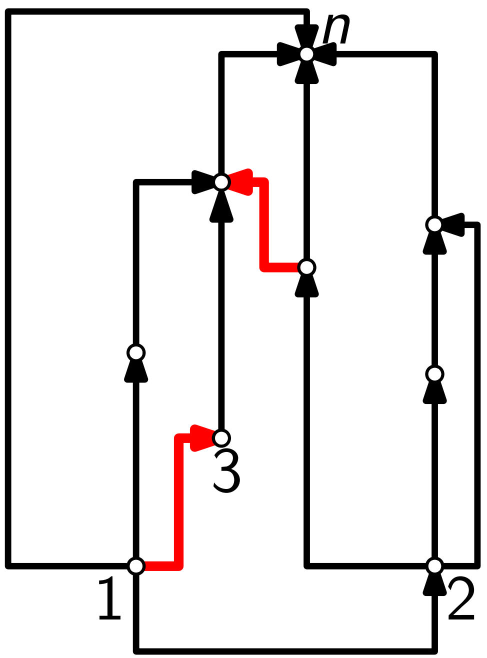
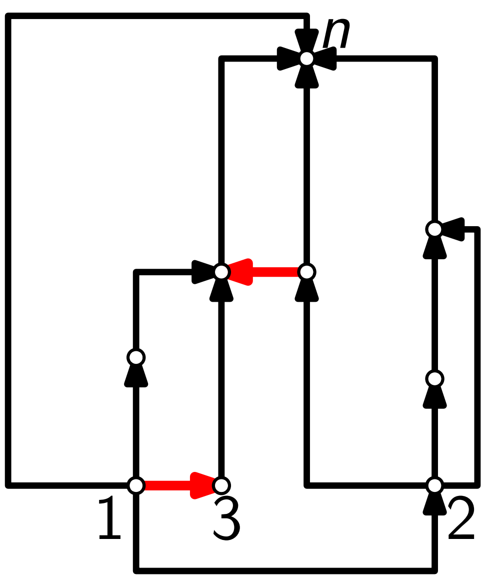


- Eliminate S-shapes

Liu et al. Algorithm

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-3 layout

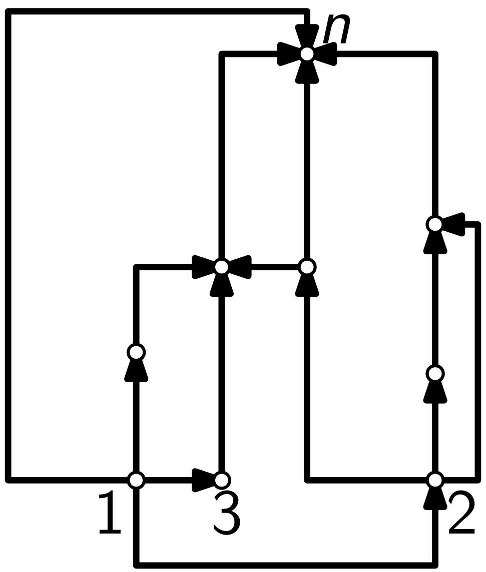
- choose vertices s and t
- place vertices by their st-numbering
- Invariant:
  When we fix an endpoint of edge e, we associate e with a column.
- Use ports in this order:
  out: $\uparrow \quad \rightarrow \quad \leftarrow$
  in: $\downarrow \quad \leftarrow \quad \rightarrow$
- Eliminate S-shapes



SC_2-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

smooth

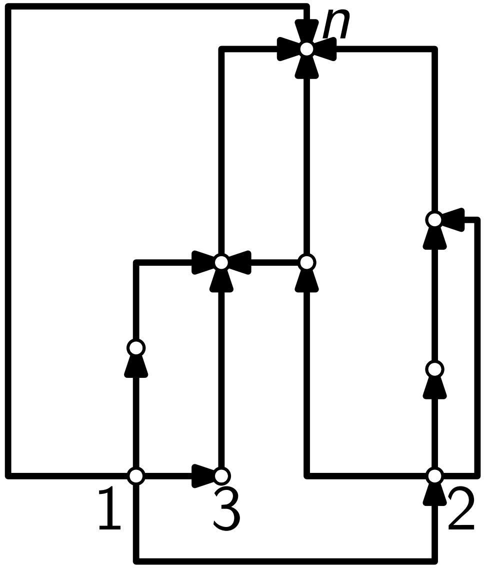


SC_2-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

smooth

| $\rightarrow$ |

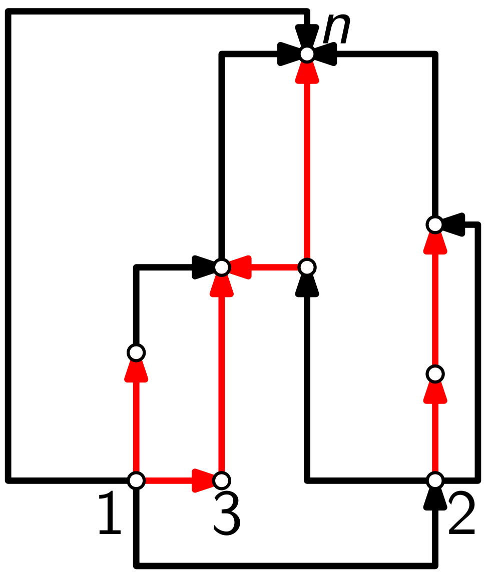


SC_2-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

smooth

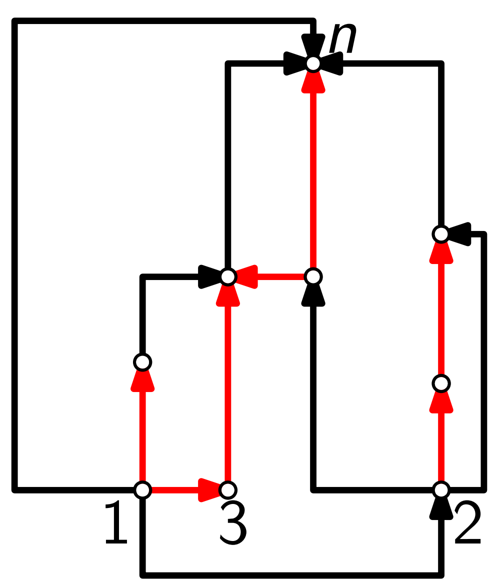
$| \rightarrow |$



SC_2-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

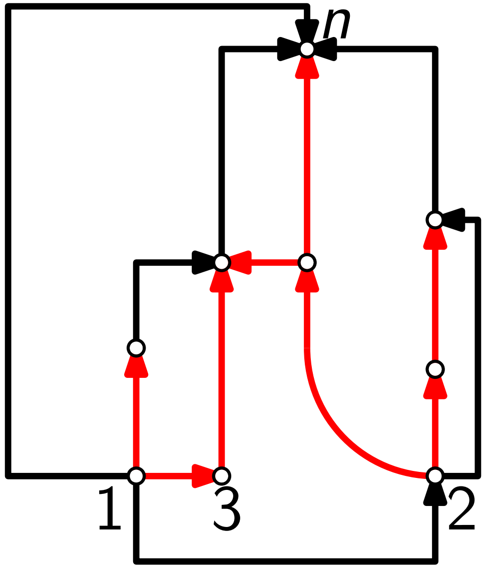
smooth



SC_2-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

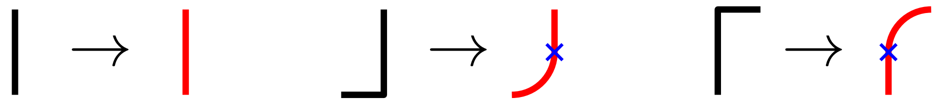
smooth

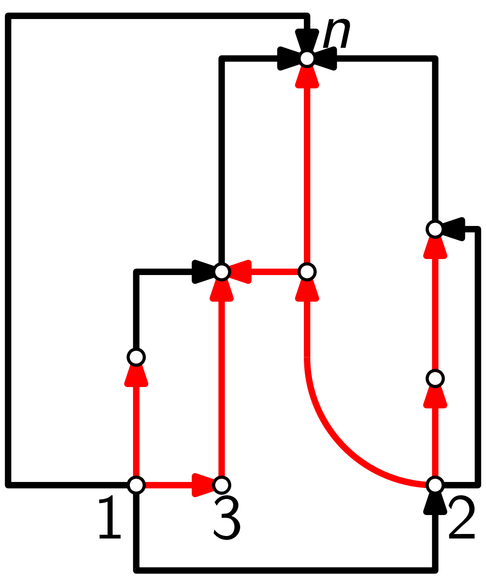


SC_2-Layouts

biconnected 4-planar graph → orthogonal complexity-2 layout

smooth

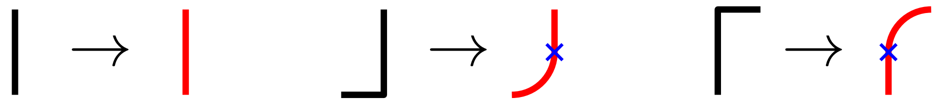


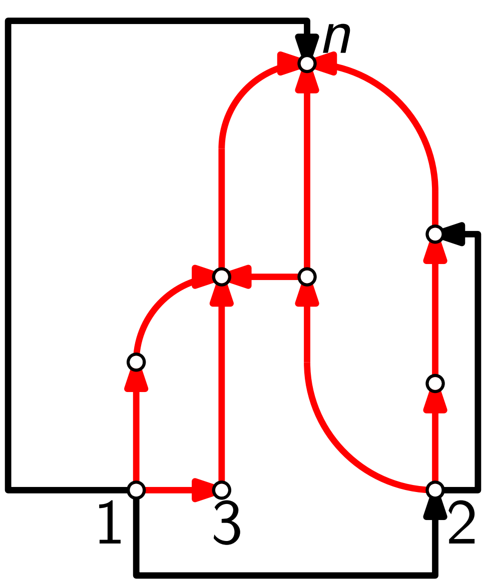


SC_2-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

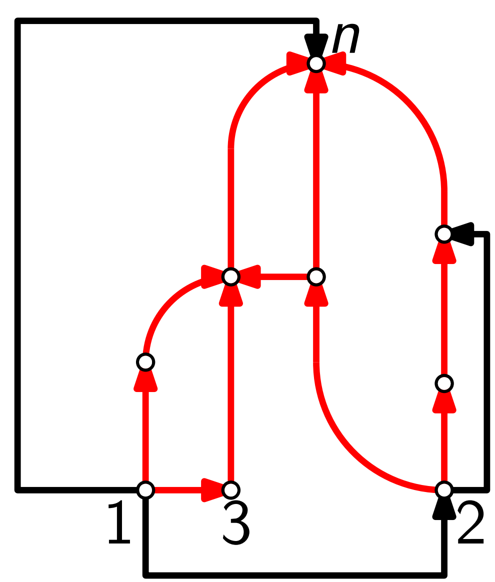
smooth





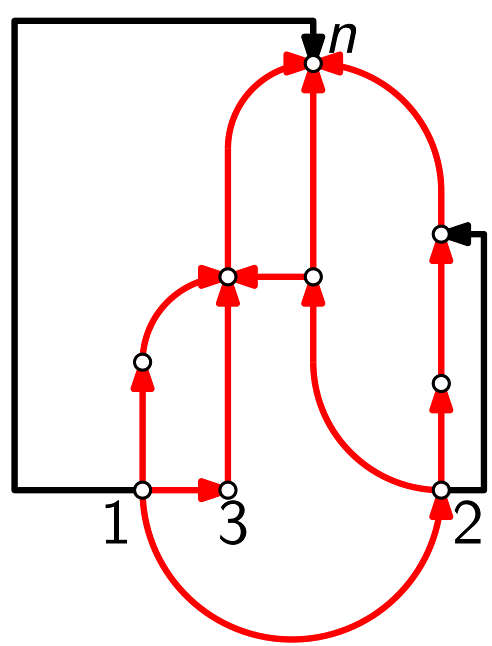
SC_2-Layouts

biconnected 4-planar graph $\rightarrow \underbrace{\ }_{\text{smooth}}$ orthogonal complexity-2 layout



SC_2-Layouts

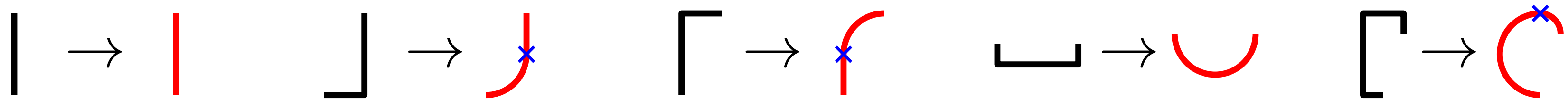
biconnected 4-planar graph $\rightarrow$ $\underbrace{\ }_{\text{smooth}}$ orthogonal complexity-2 layout

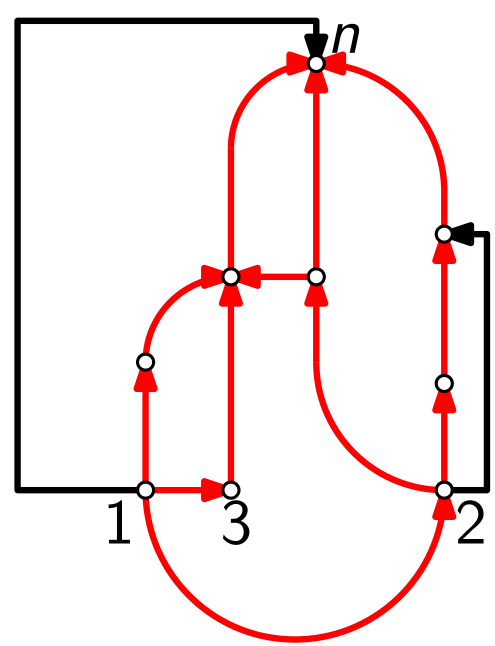


SC_2-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

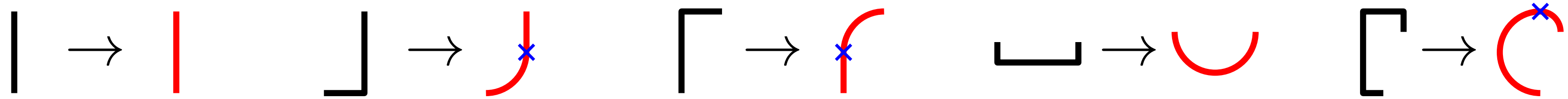
smooth

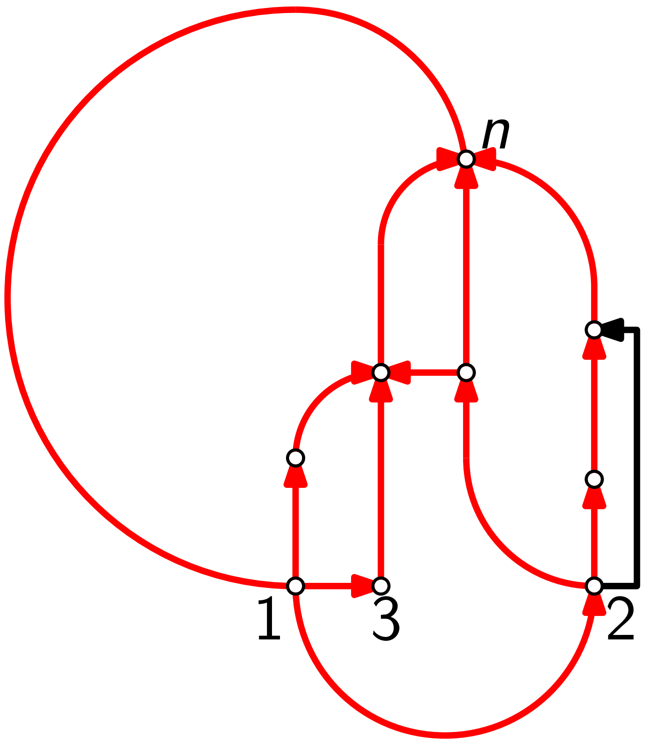




SC_2-Layouts

biconnected 4-planar graph $\rightarrow \underbrace{}_{\text{smooth}}$ orthogonal complexity-2 layout

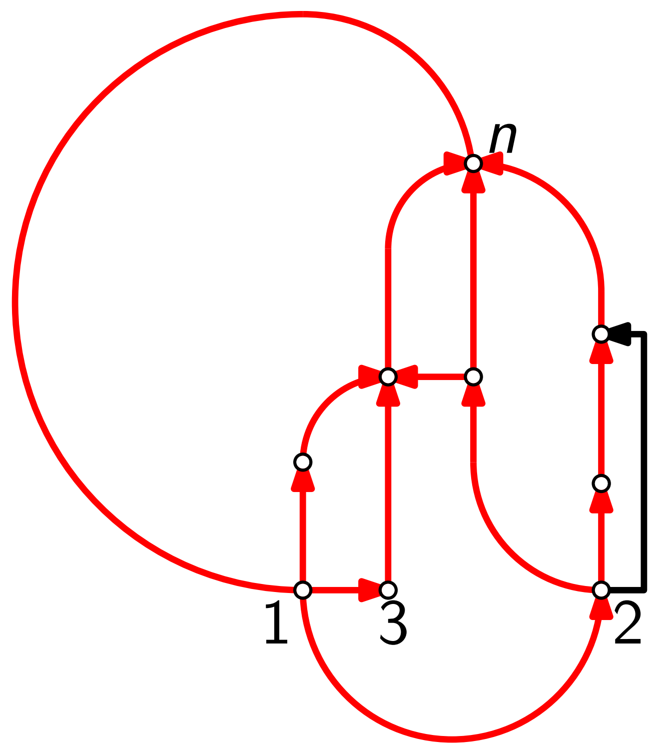




SC_2-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

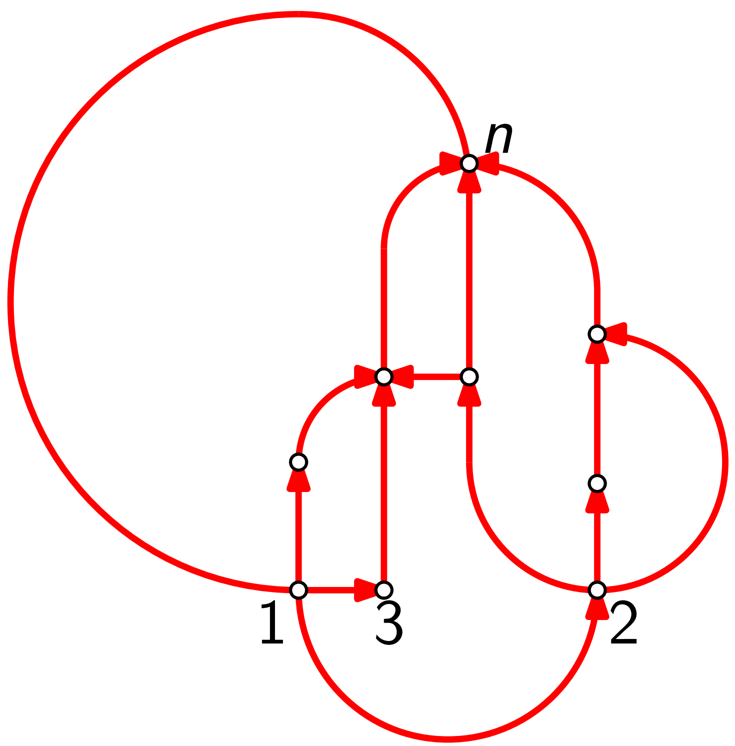
smooth



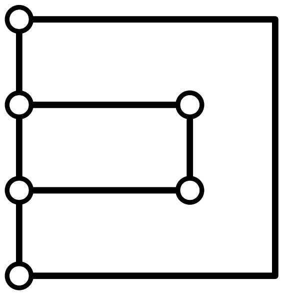
SC_2-Layouts

biconnected 4-planar graph → orthogonal complexity-2 layout

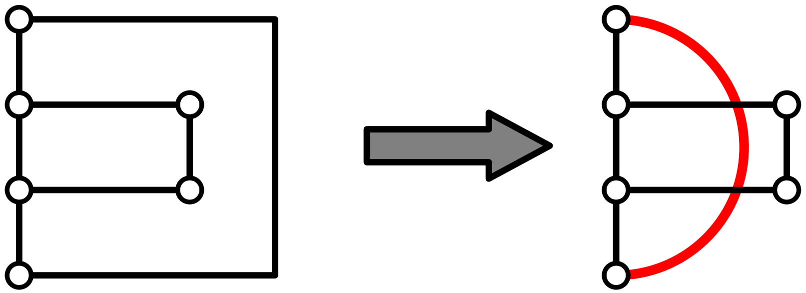
smooth



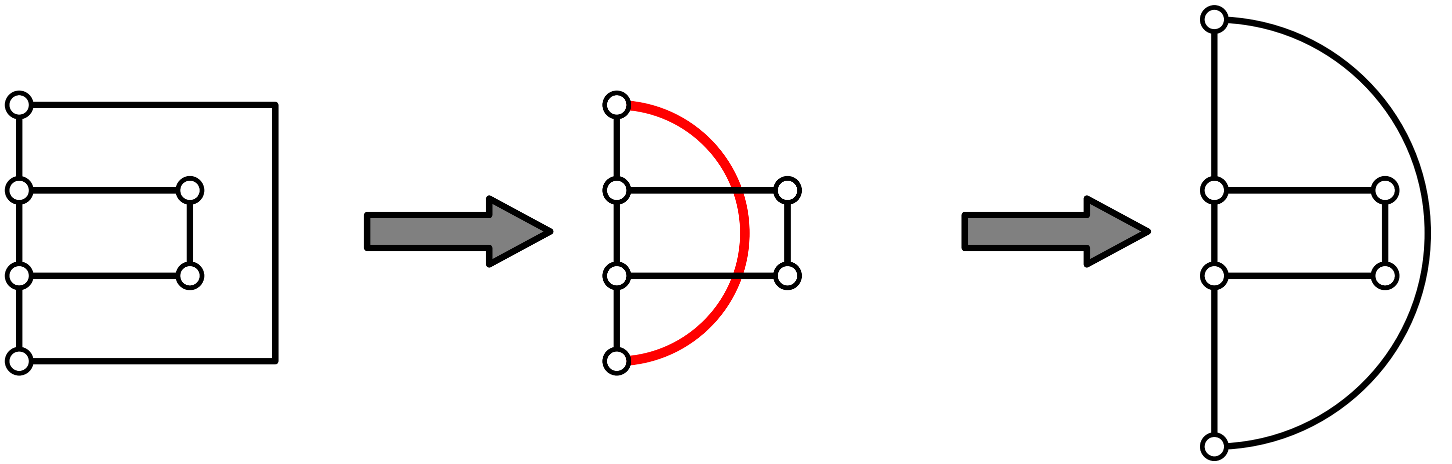
Crossings



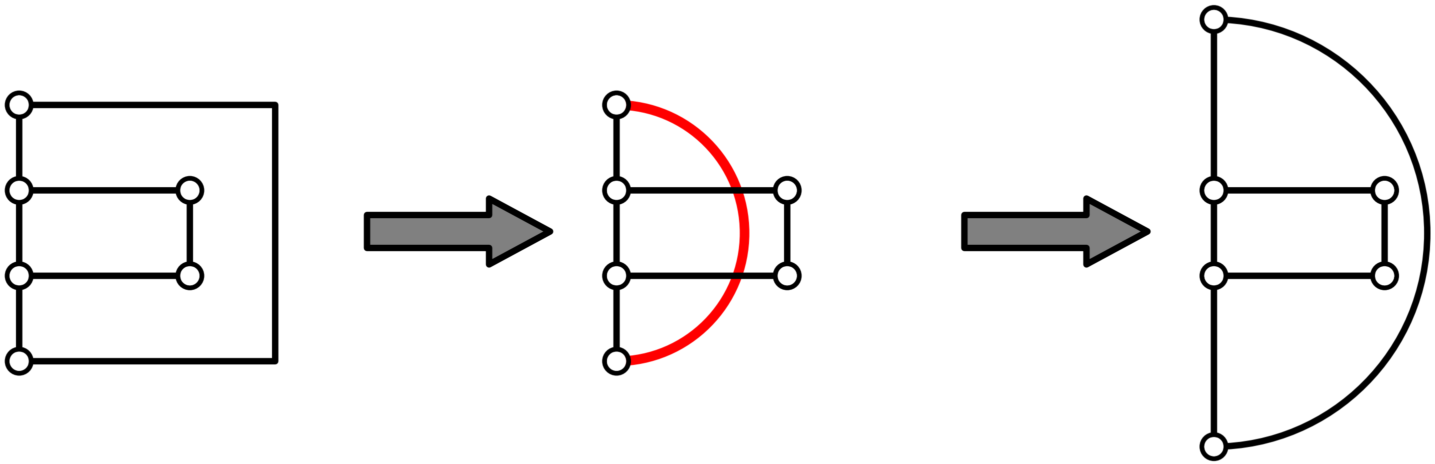
Crossings

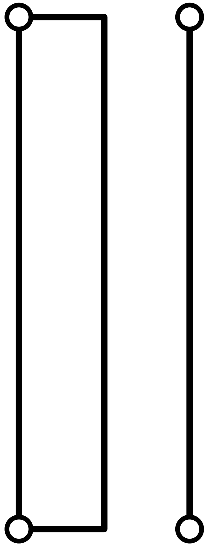


Crossings

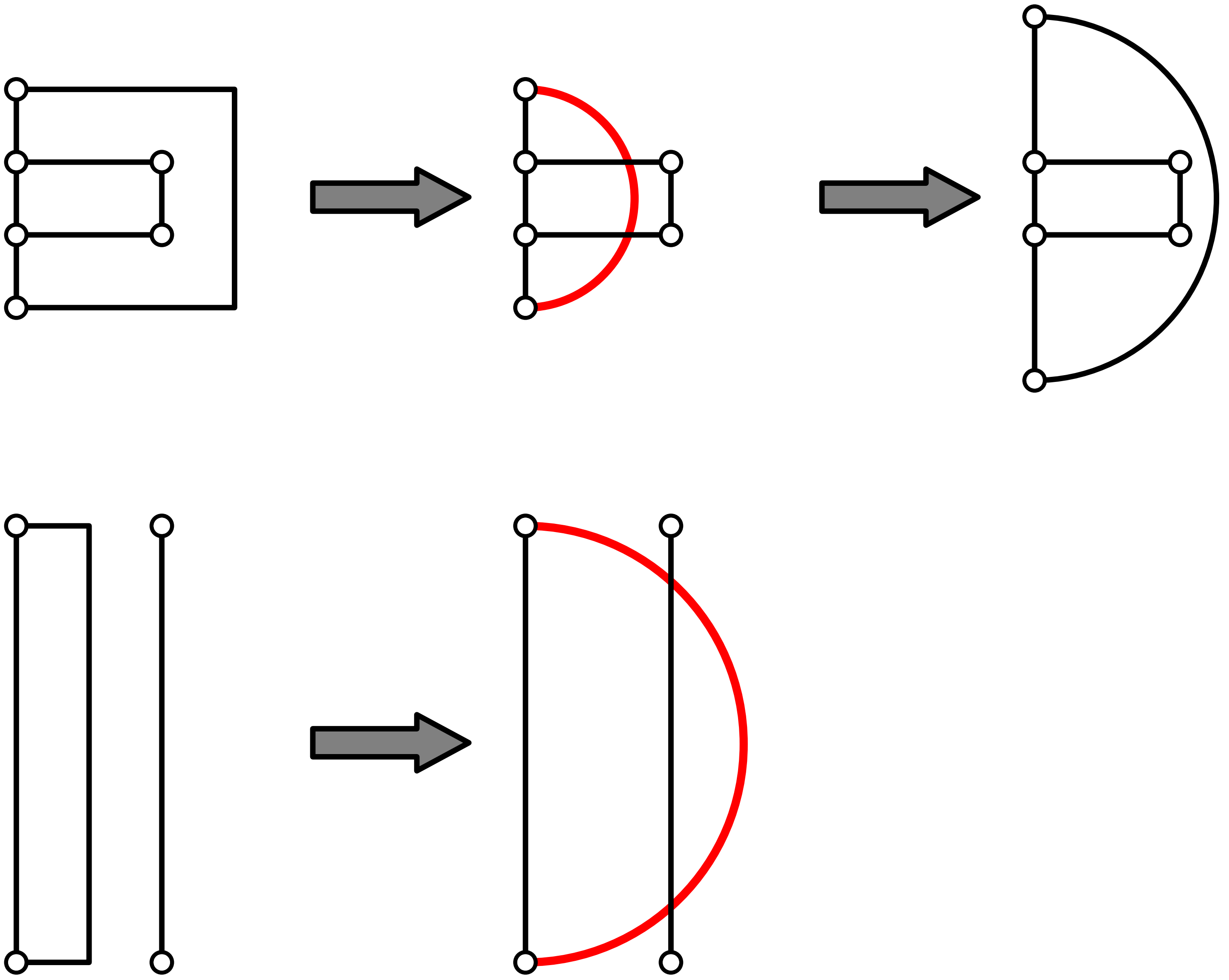


Crossings

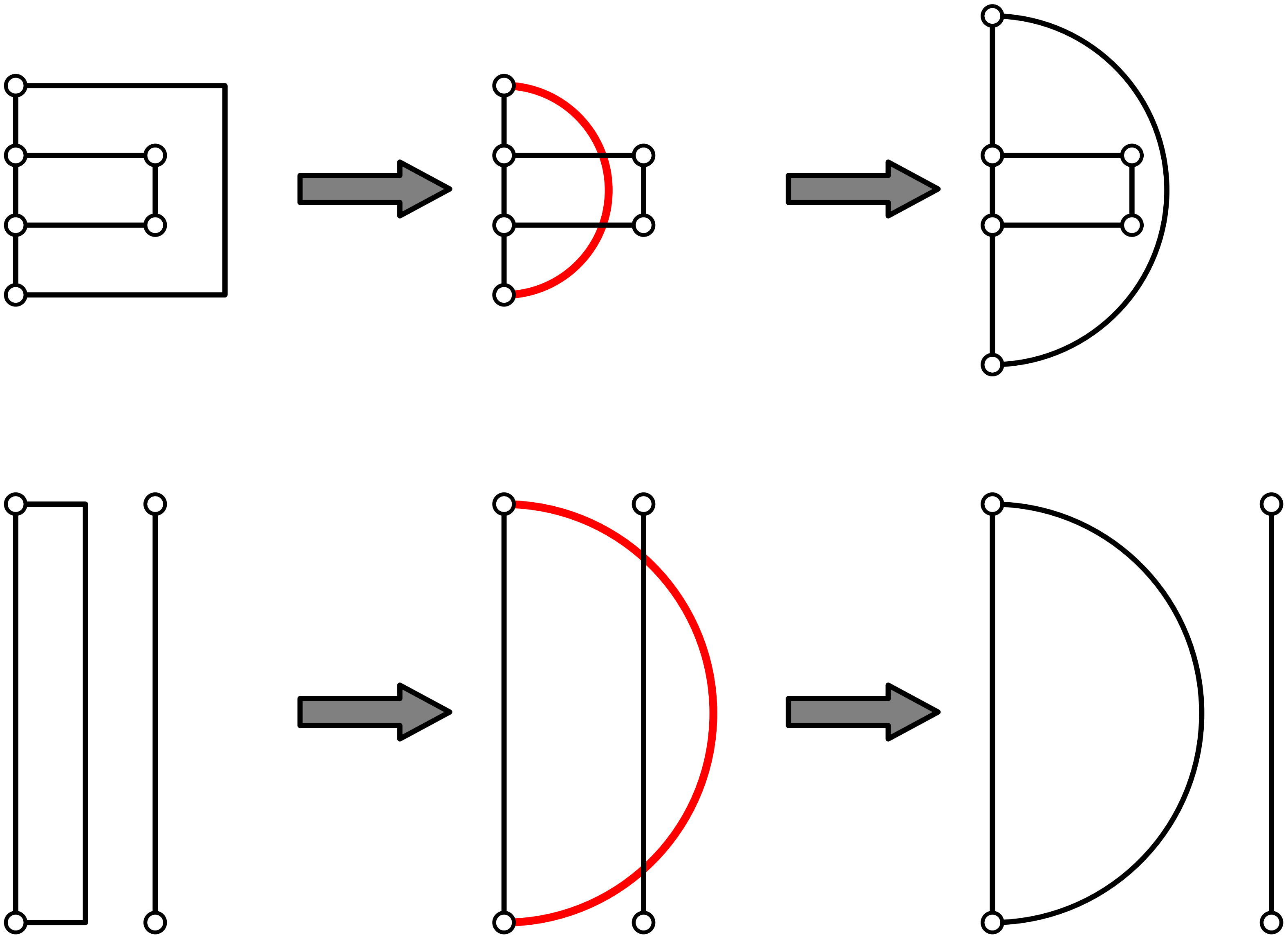




Crossings



Crossings

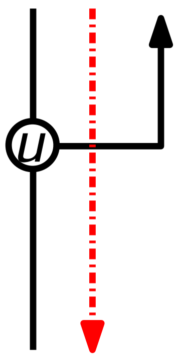


Cut

Def.

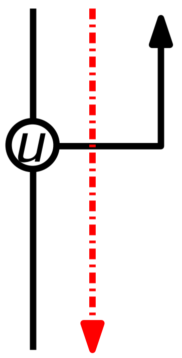
Cut

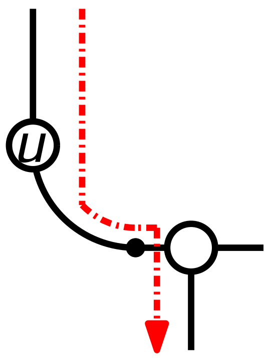
Def. • y-monotone curve



Cut

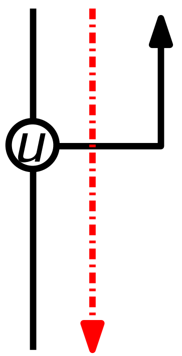
Def. • y-monotone curve
• consists of horizontal, vertical and circular segments

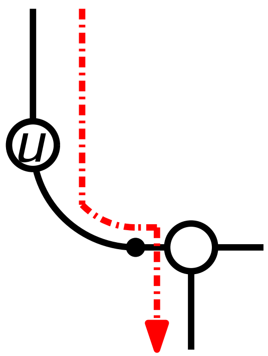




Cut

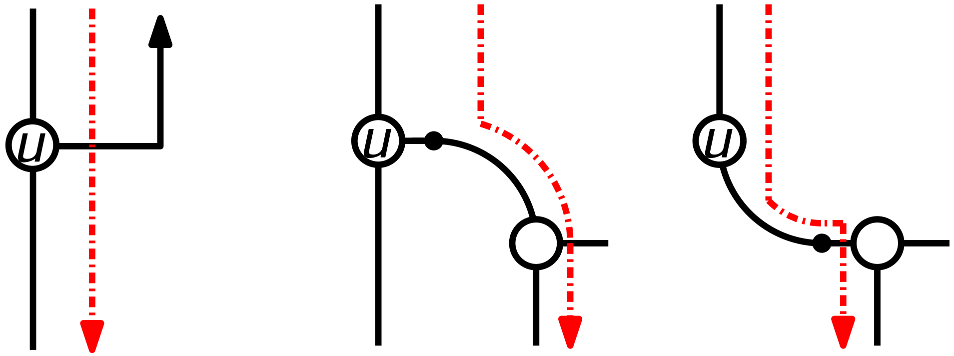
Def.
- y-monotone curve
- consists of horizontal, vertical and circular segments
- divides the current drawing into a left and a right part





Cut

Def.
- y-monotone curve
- consists of horizontal, vertical and circular segments
- divides the current drawing into a left and a right part
- intersects only horizontal segments



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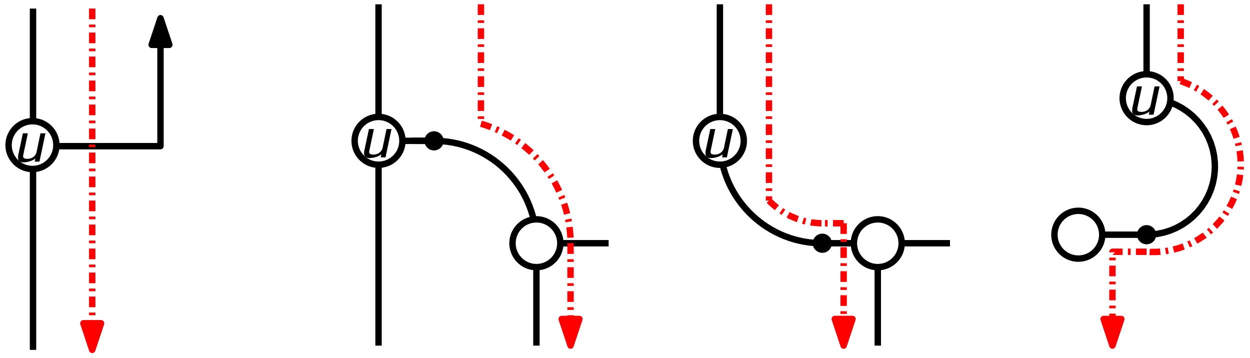

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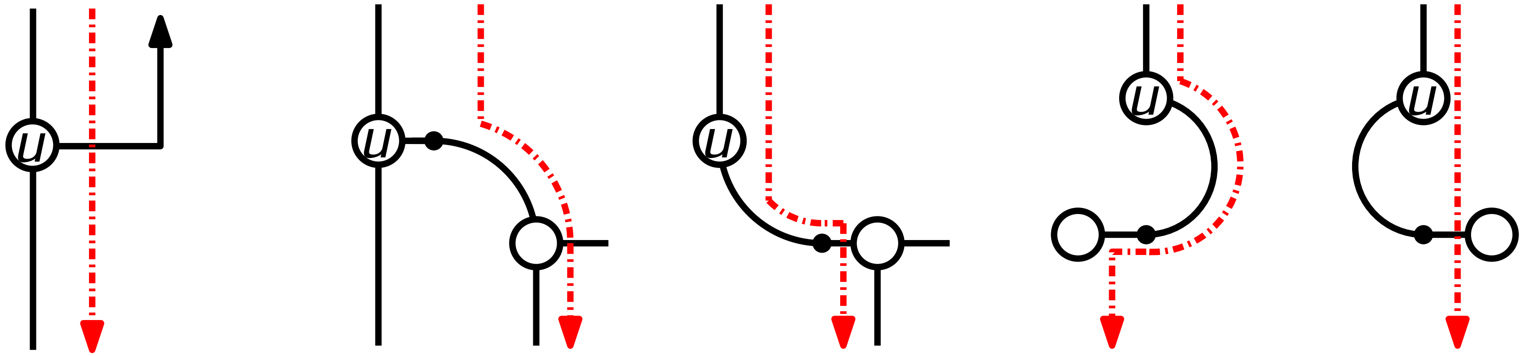

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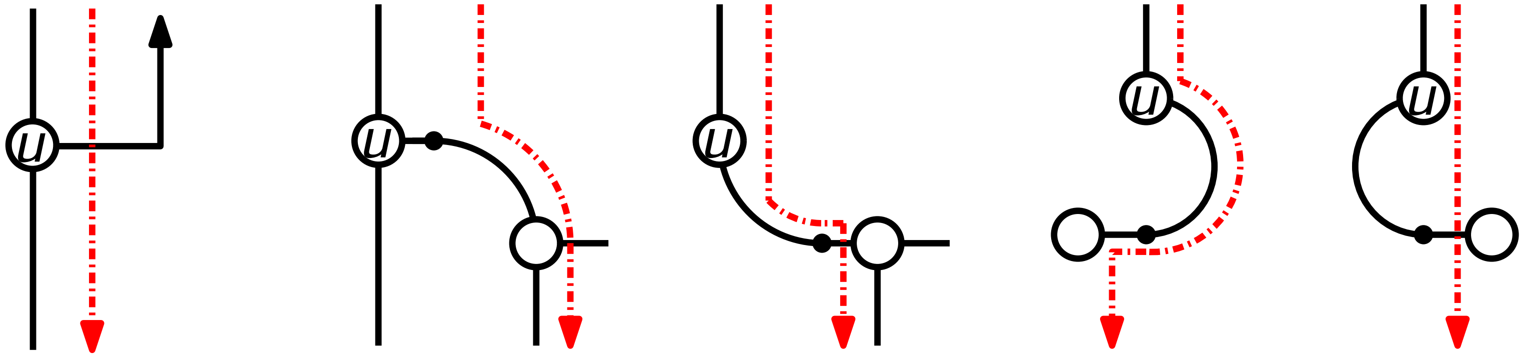
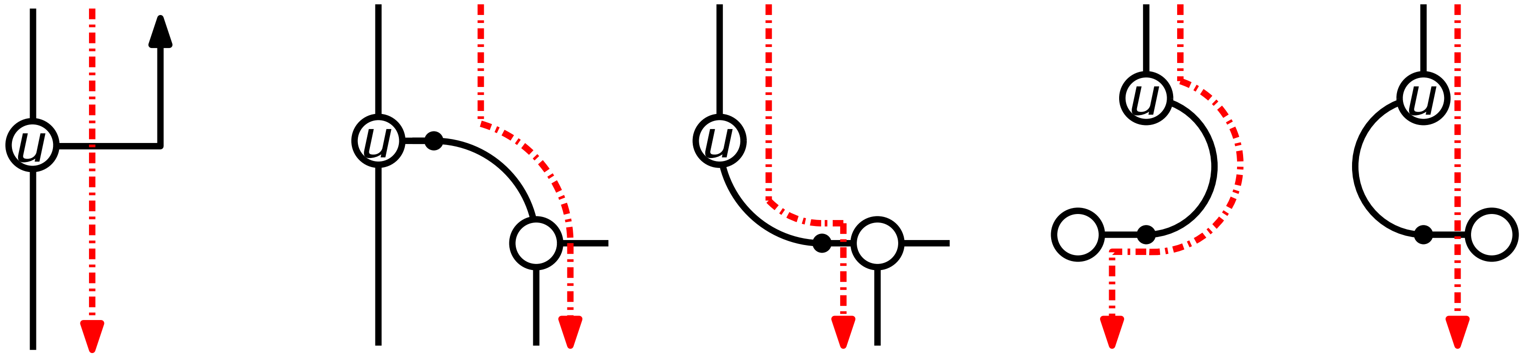


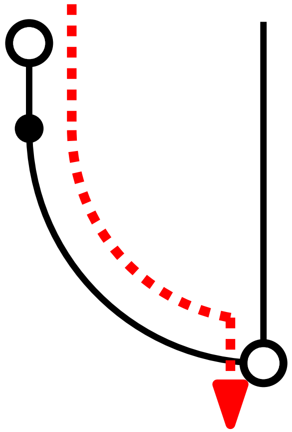
Problems:

Cut

Def.
- y-monotone curve
- consists of horizontal, vertical and circular segments
- divides the current drawing into a left and a right part
- intersects only horizontal segments



Problems:



Cut

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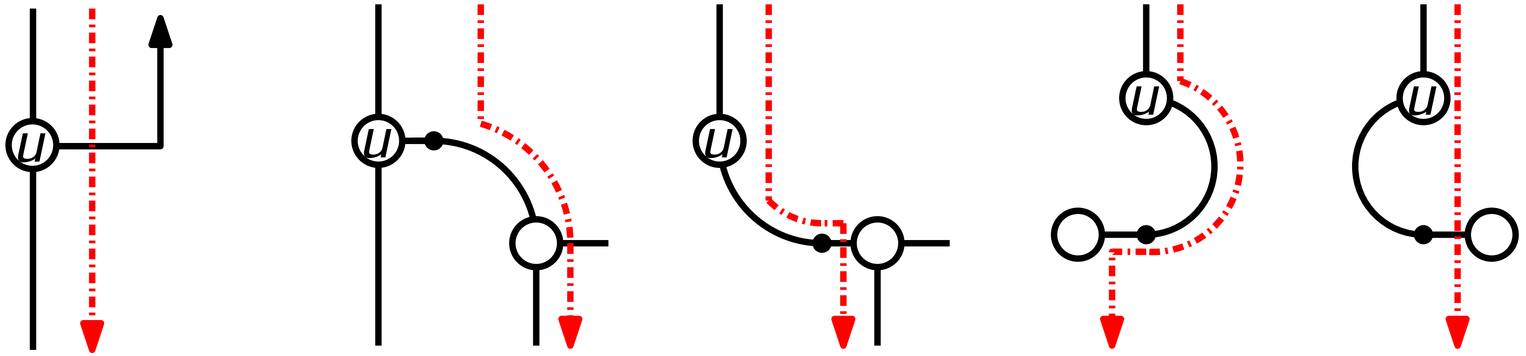
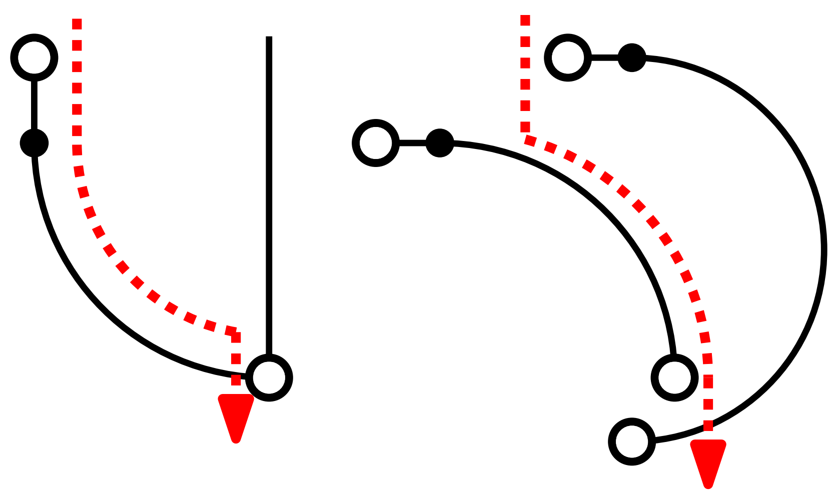
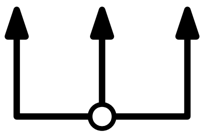


Problems:



Invariants

(I_1) Every open edge is associated with a column



Invariants

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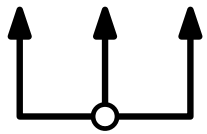
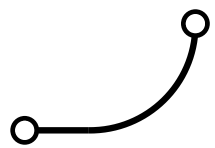


(I_2) An L-shape always contains a horizontal segment;
it never contains a vertical segment.



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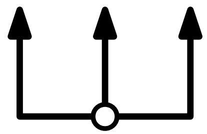

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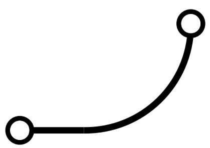
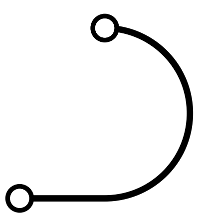
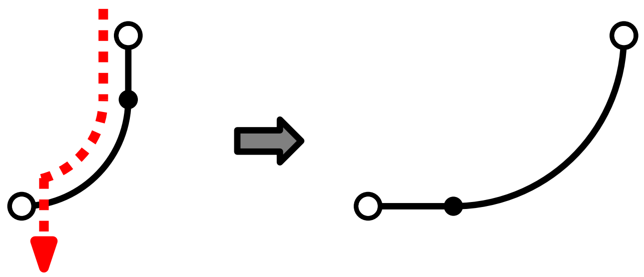


(I_3) A C-shape always has a horizontal segment incident to its bottom vertex.



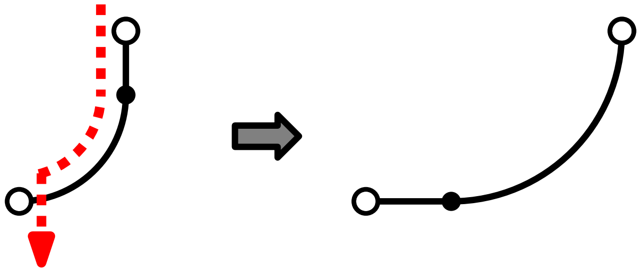
Maintain invariants

L-shape

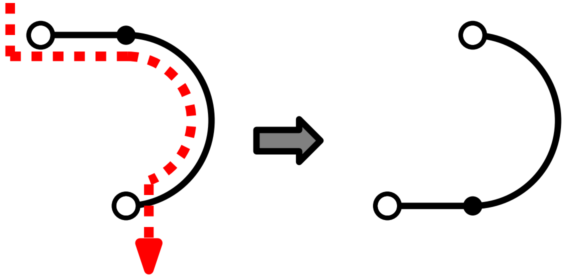


Maintain invariants

L-shape

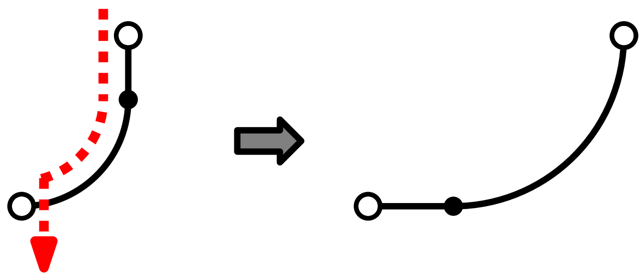


C-shape

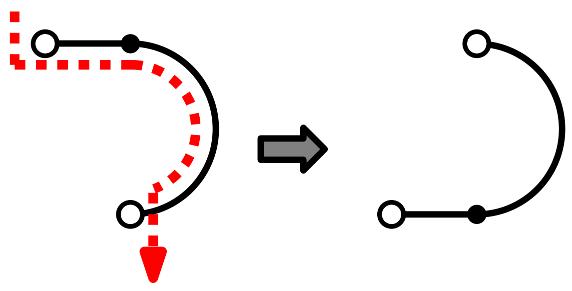


Maintain invariants

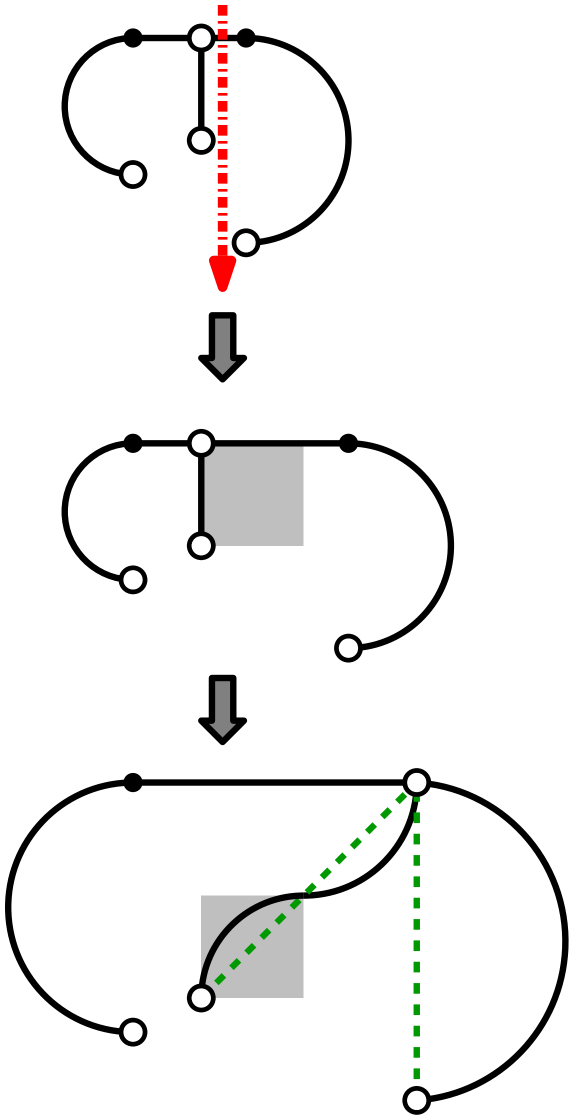
L-shape



C-shape

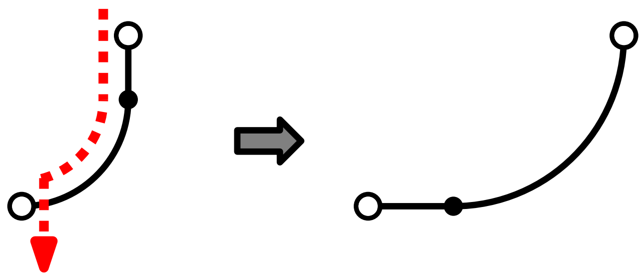


Double C-shape

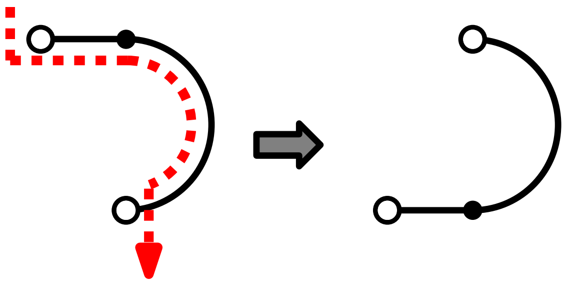


Maintain invariants

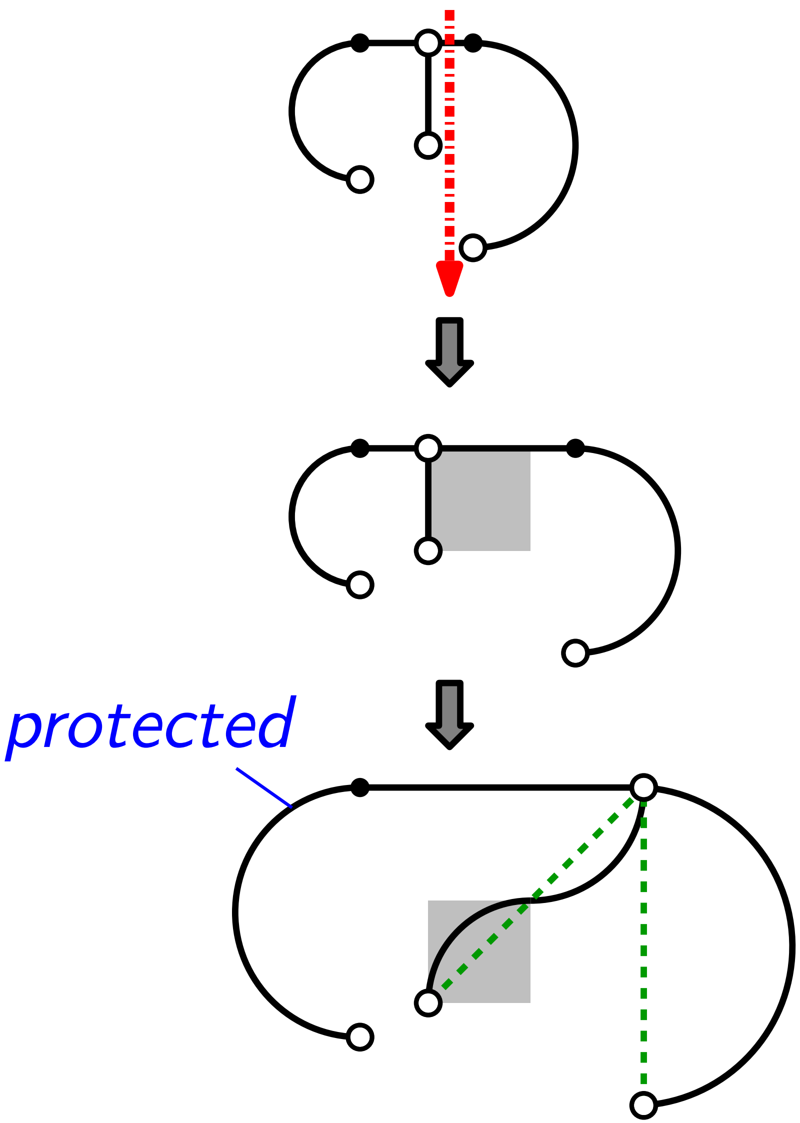
L-shape



C-shape

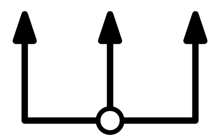


Double C-shape

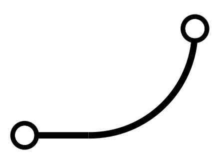


Invariants, updated

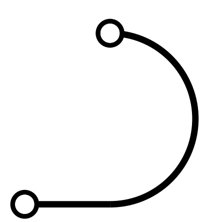
(I_1) Every open edge is associated with a column



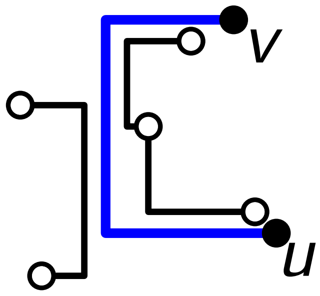
(I_2) An L-shape always contains a horizontal segment;
it never contains a vertical segment.

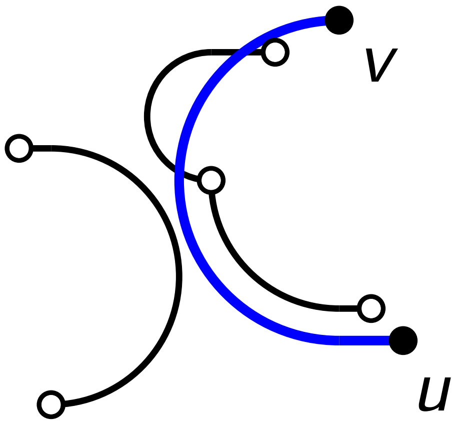


(I_3) An *unprotected* C-shape always has a horizontal segment incident to its bottom vertex.



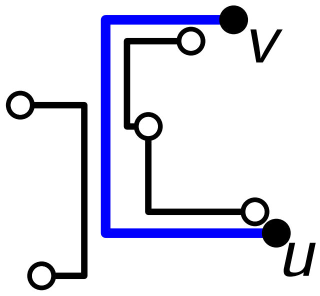
Eliminate crossings

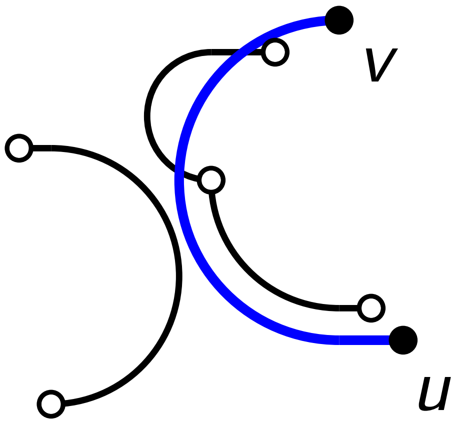




Eliminate crossings

1. move v up





Eliminate crossings

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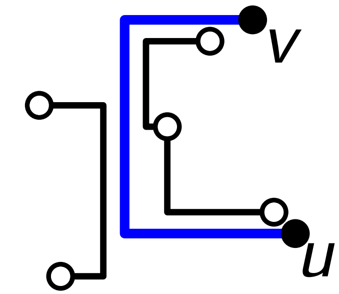
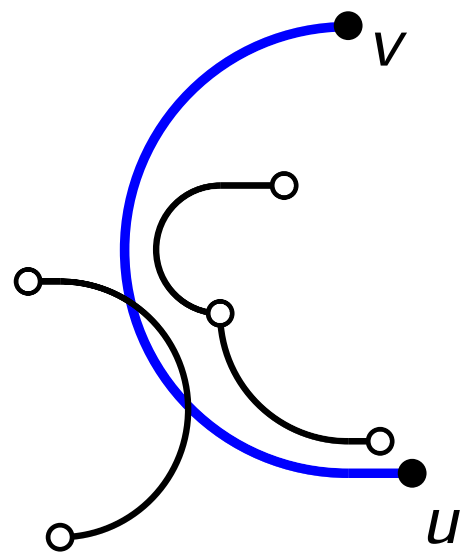
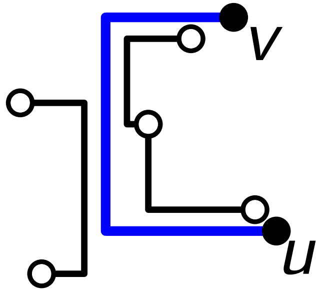
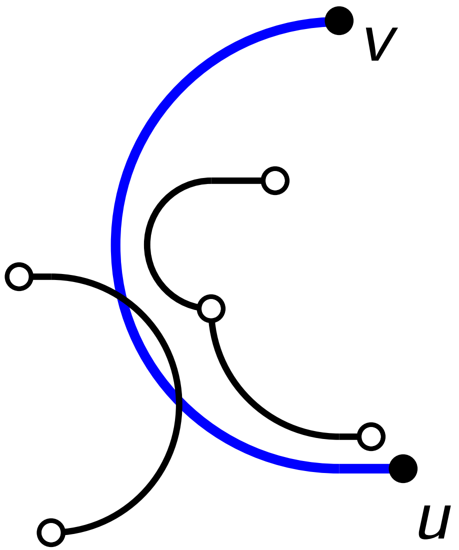




Eliminate crossings

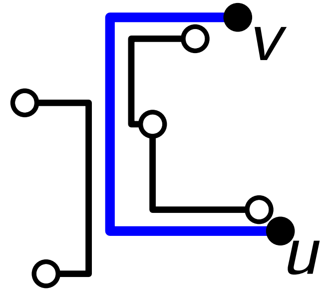
1. move v up
2. find a cut

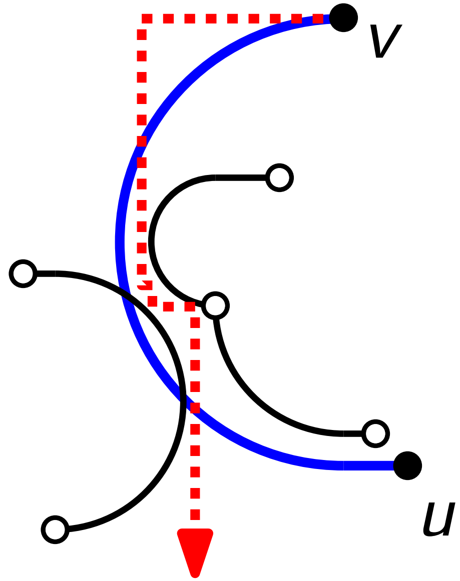




Eliminate crossings

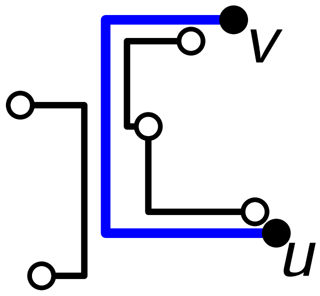
1. move v up
2. find a cut

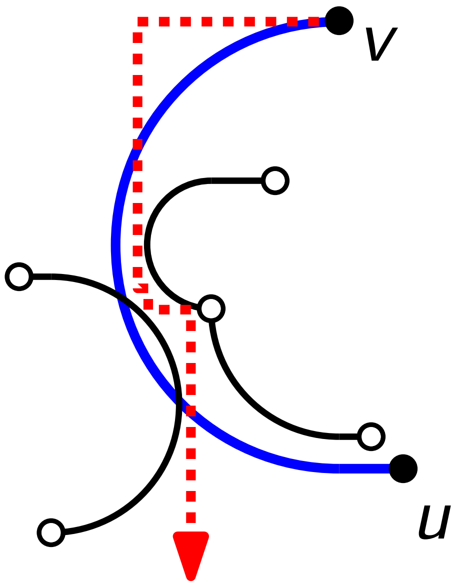




Eliminate crossings

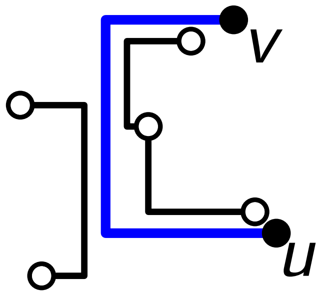
1. move v up
2. find a cut
3. move vertices to the left

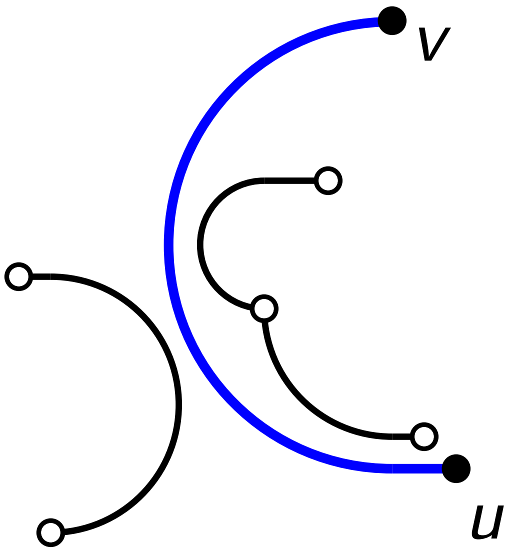




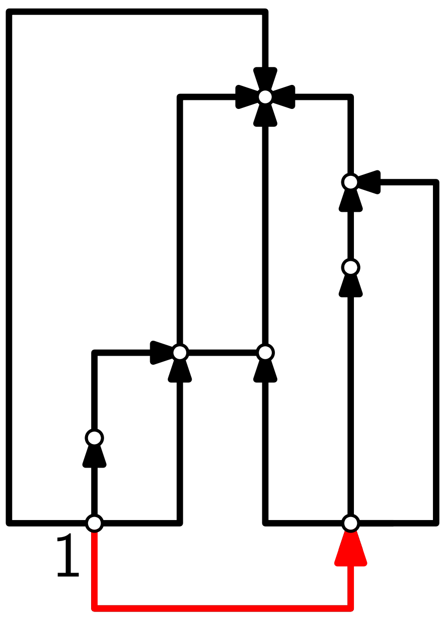
Eliminate crossings

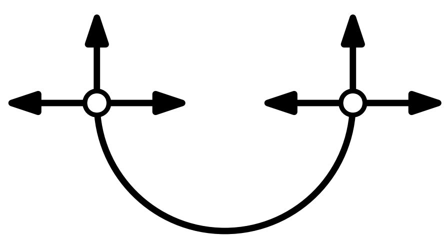
1. move v up
2. find a cut
3. move vertices to the left



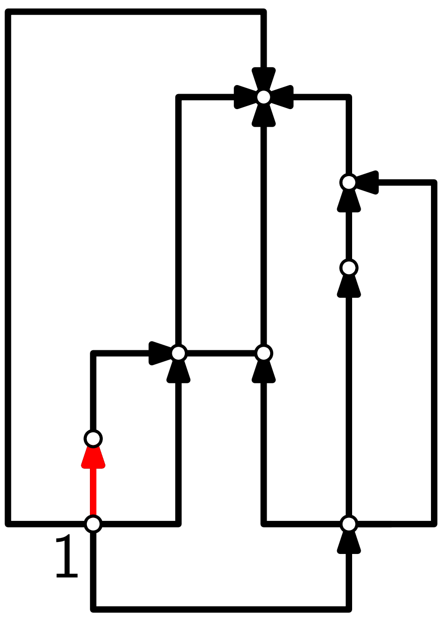


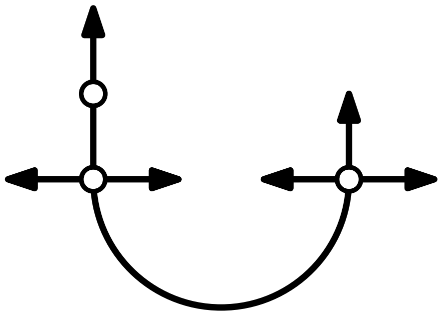
Example Run



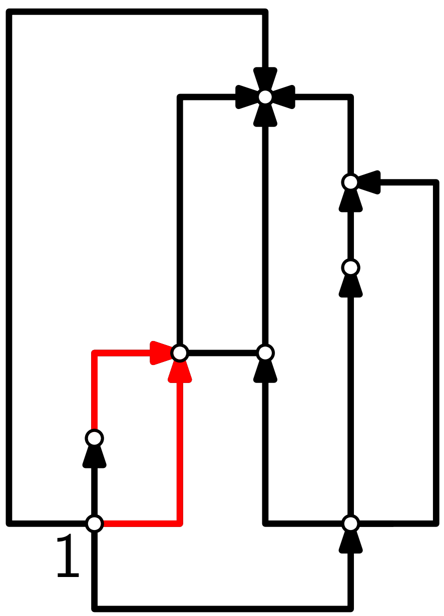


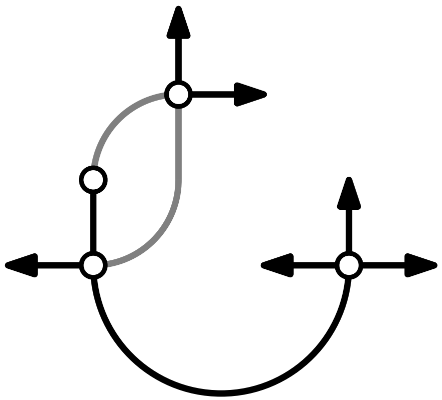
Example Run



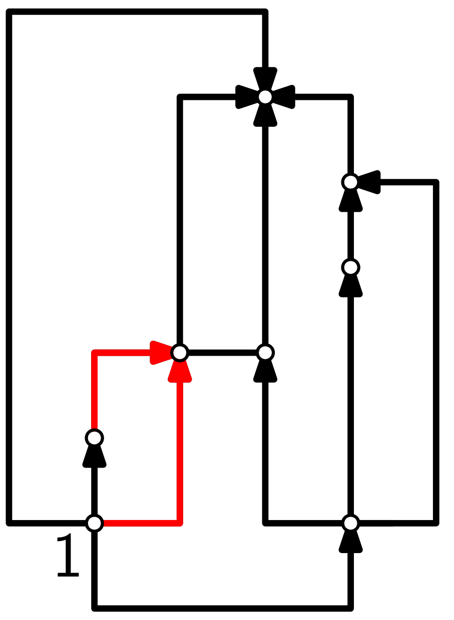


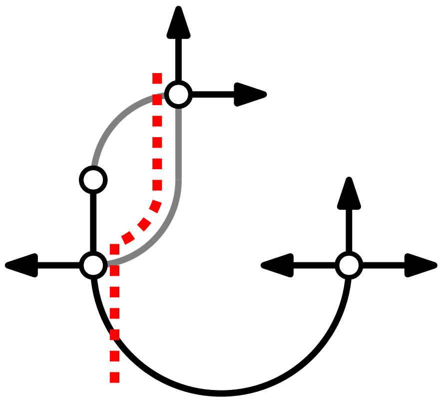
Example Run



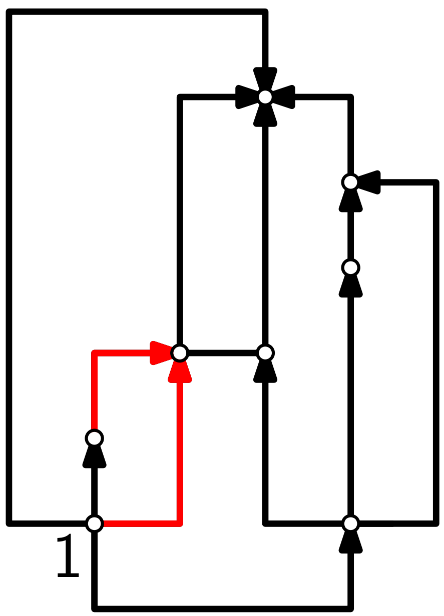


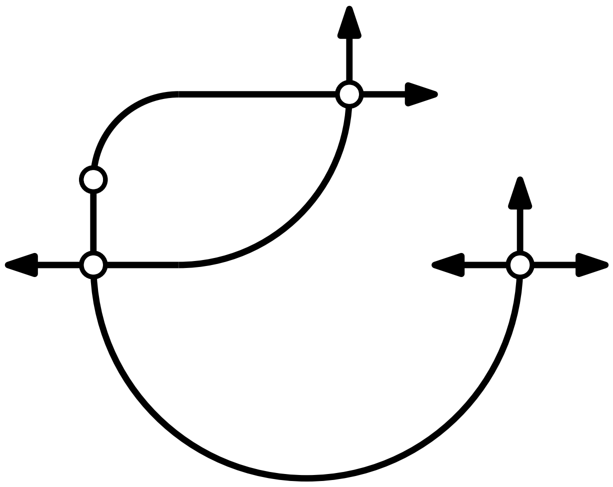
Example Run



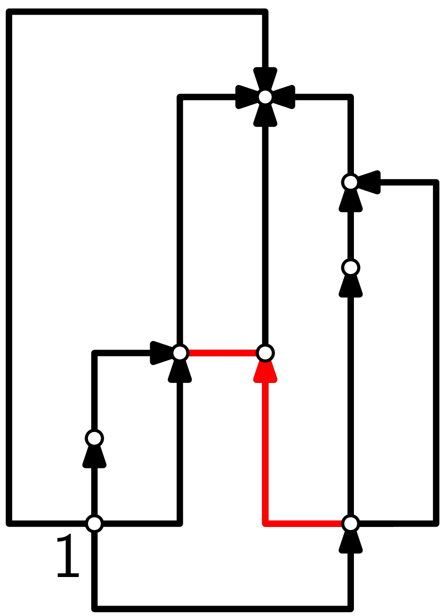


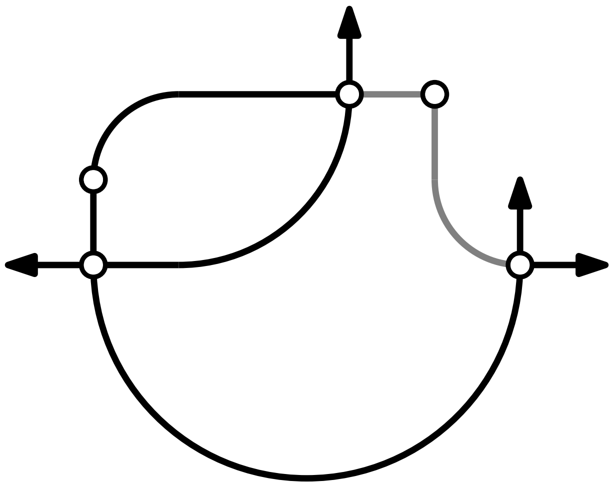
Example Run



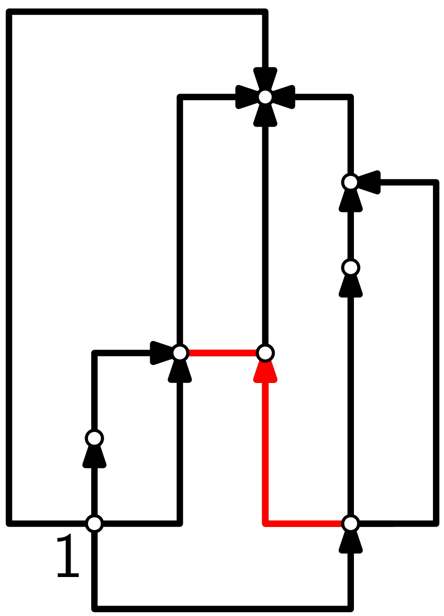


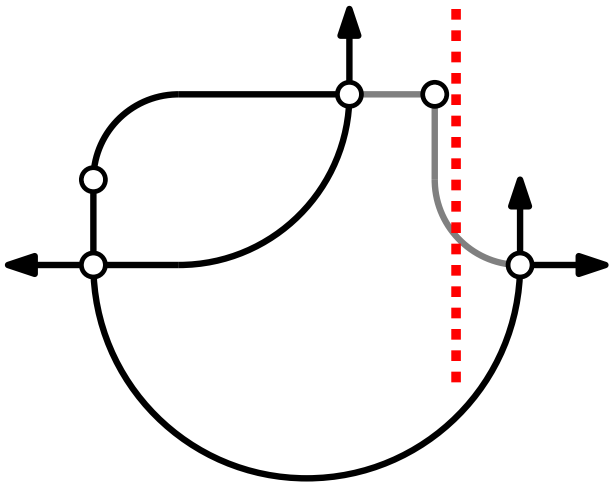
Example Run





Example Run





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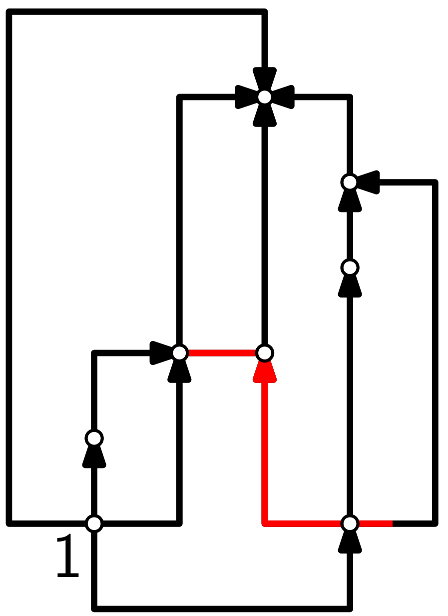
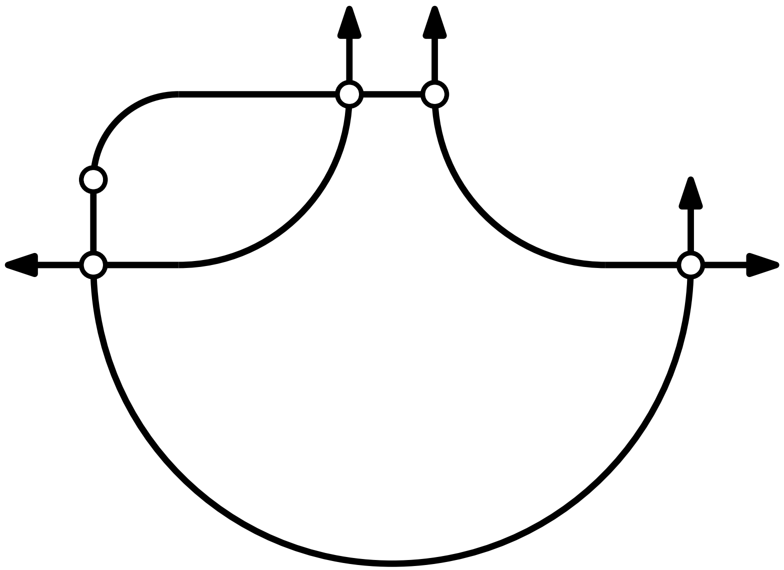
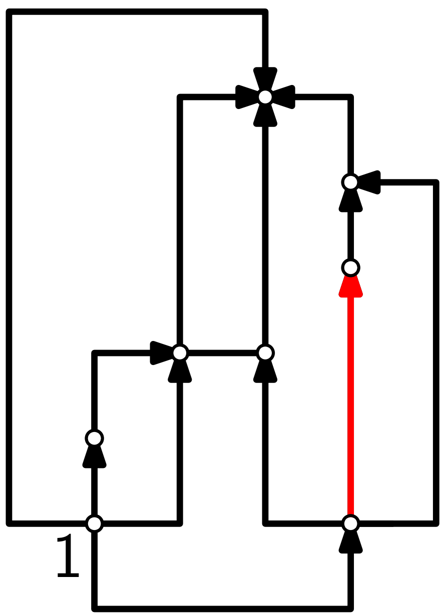
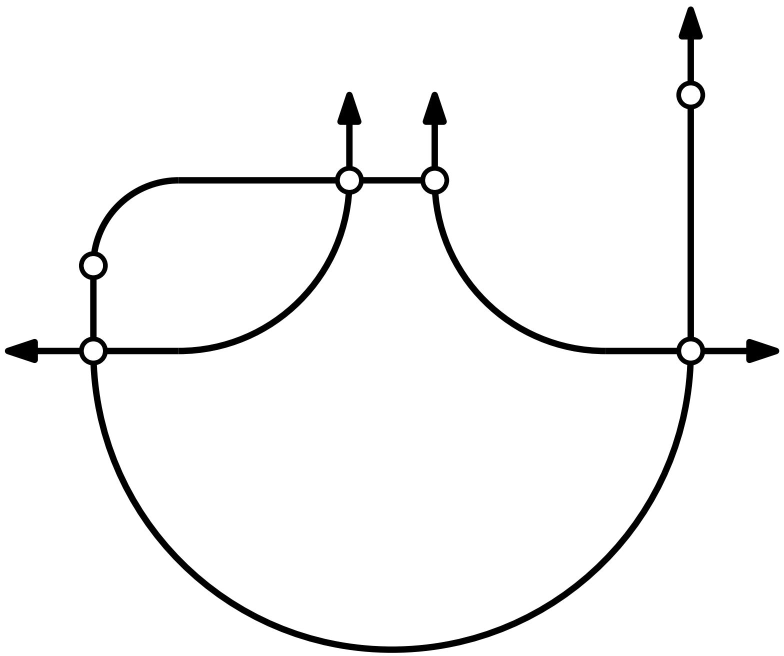


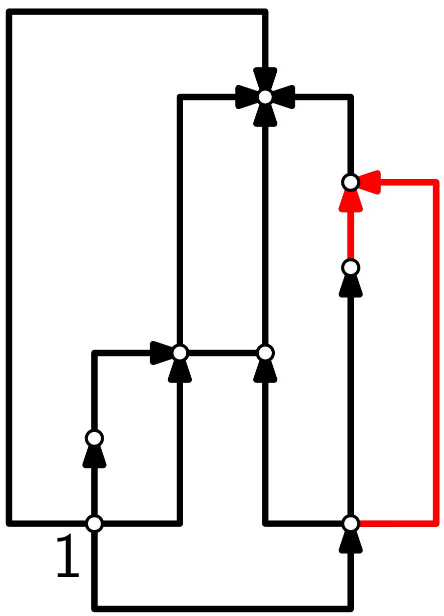


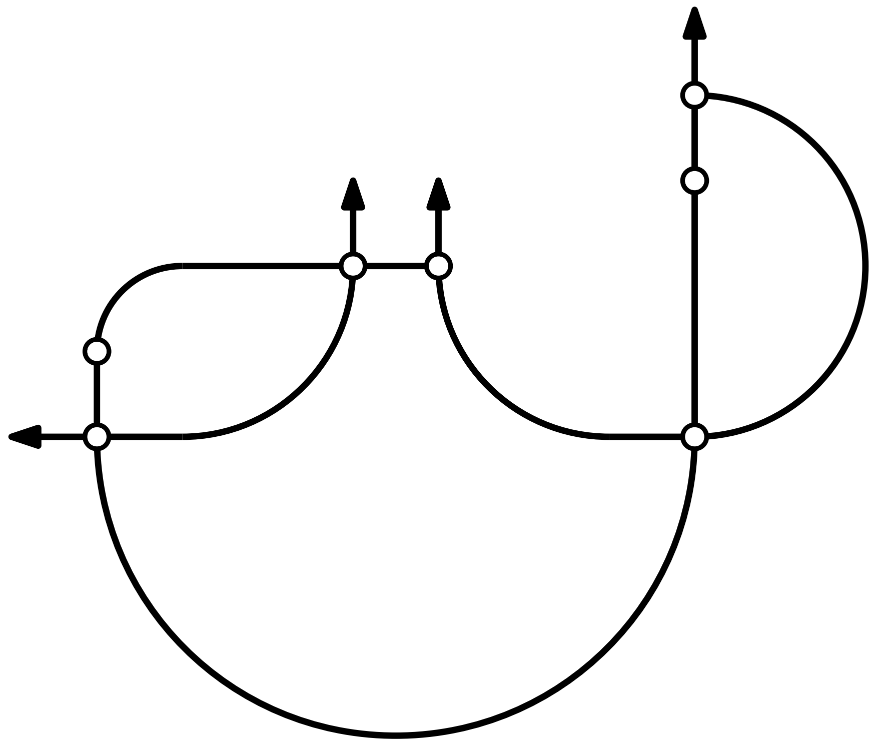
Example Run



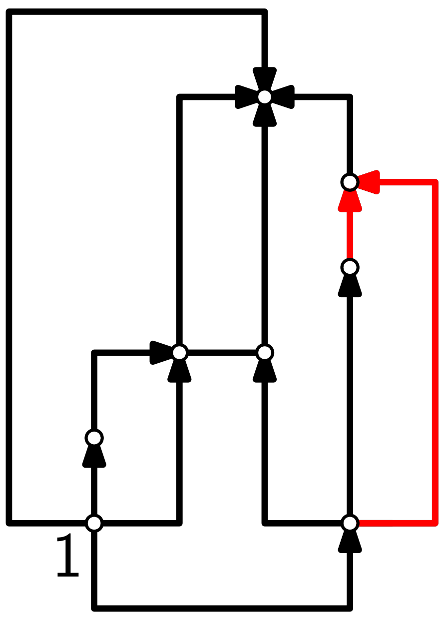


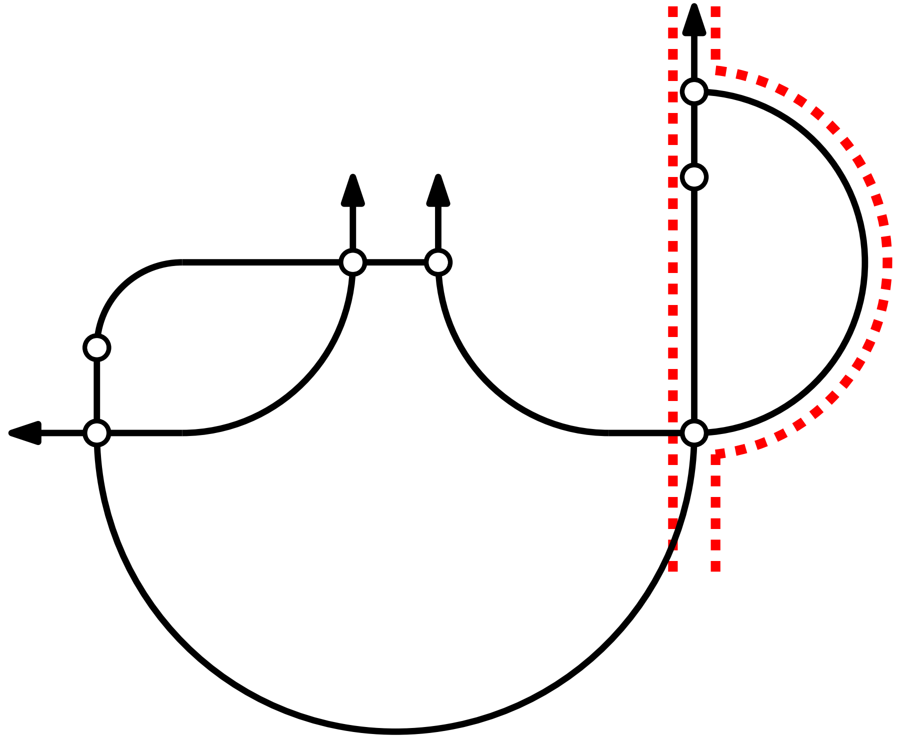
Example Run





Example Run





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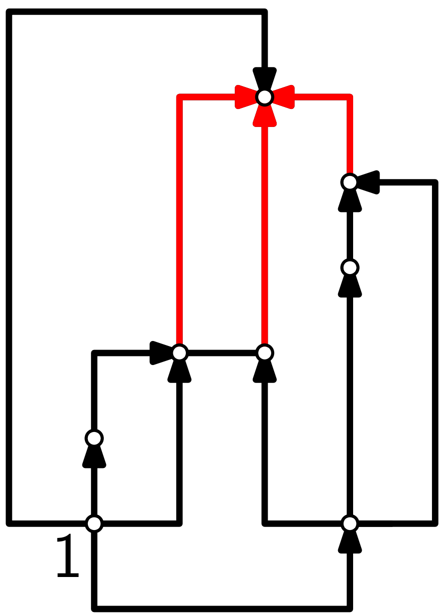
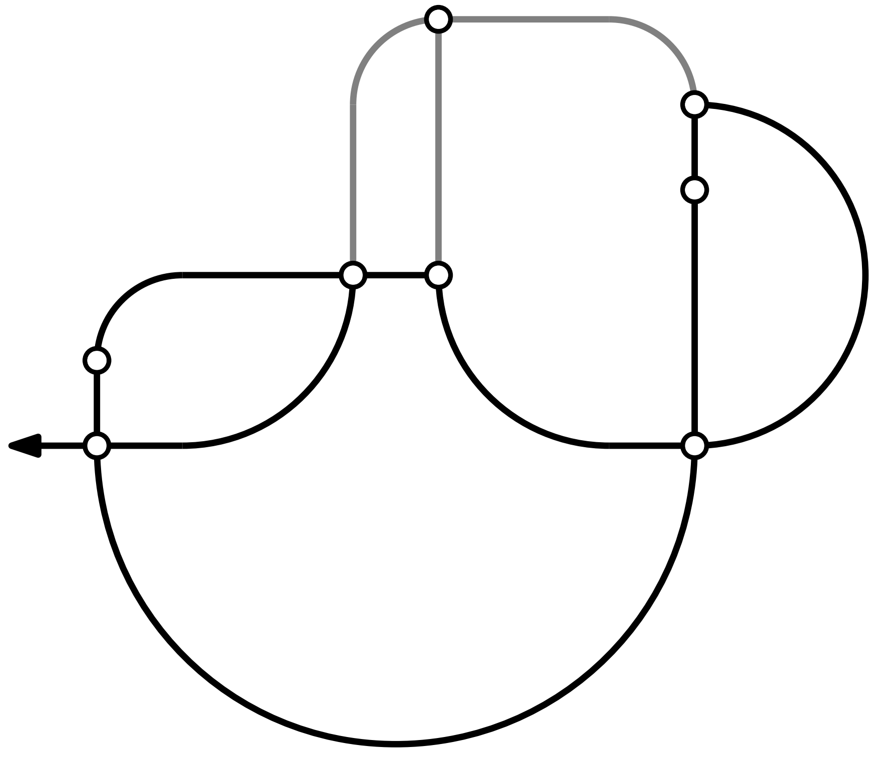
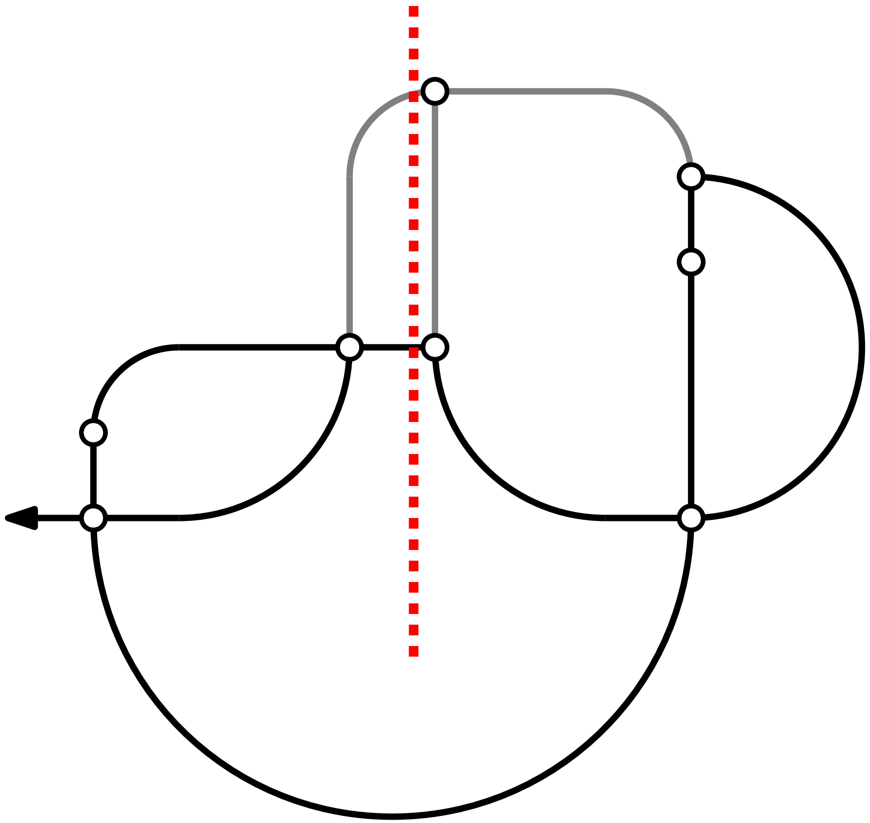
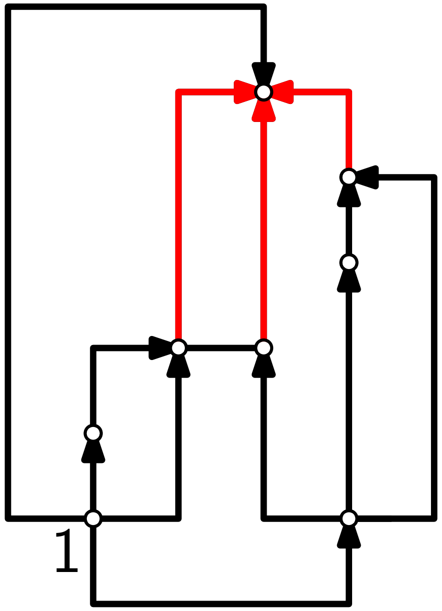


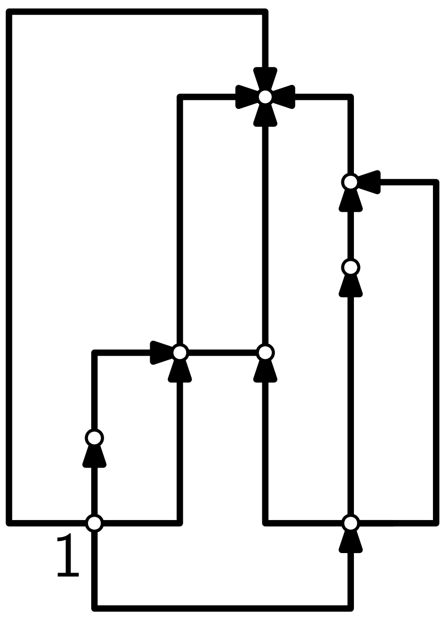


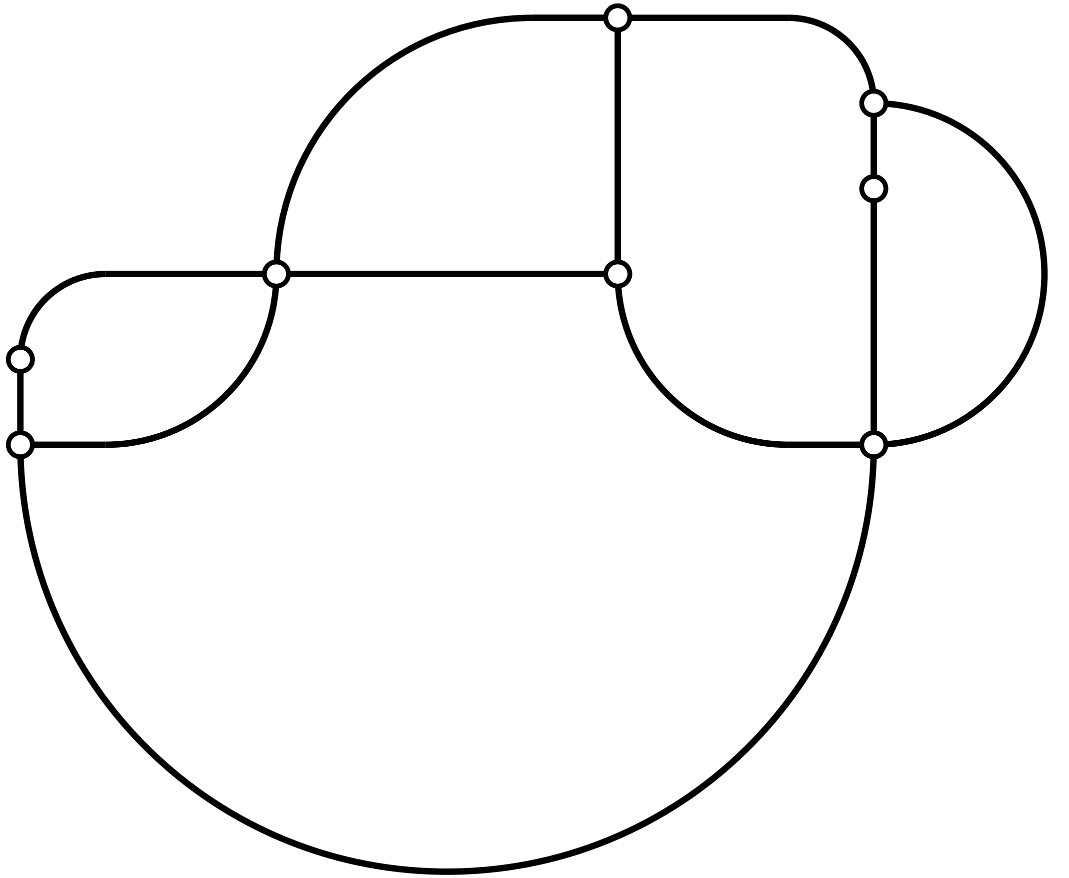
Example Run



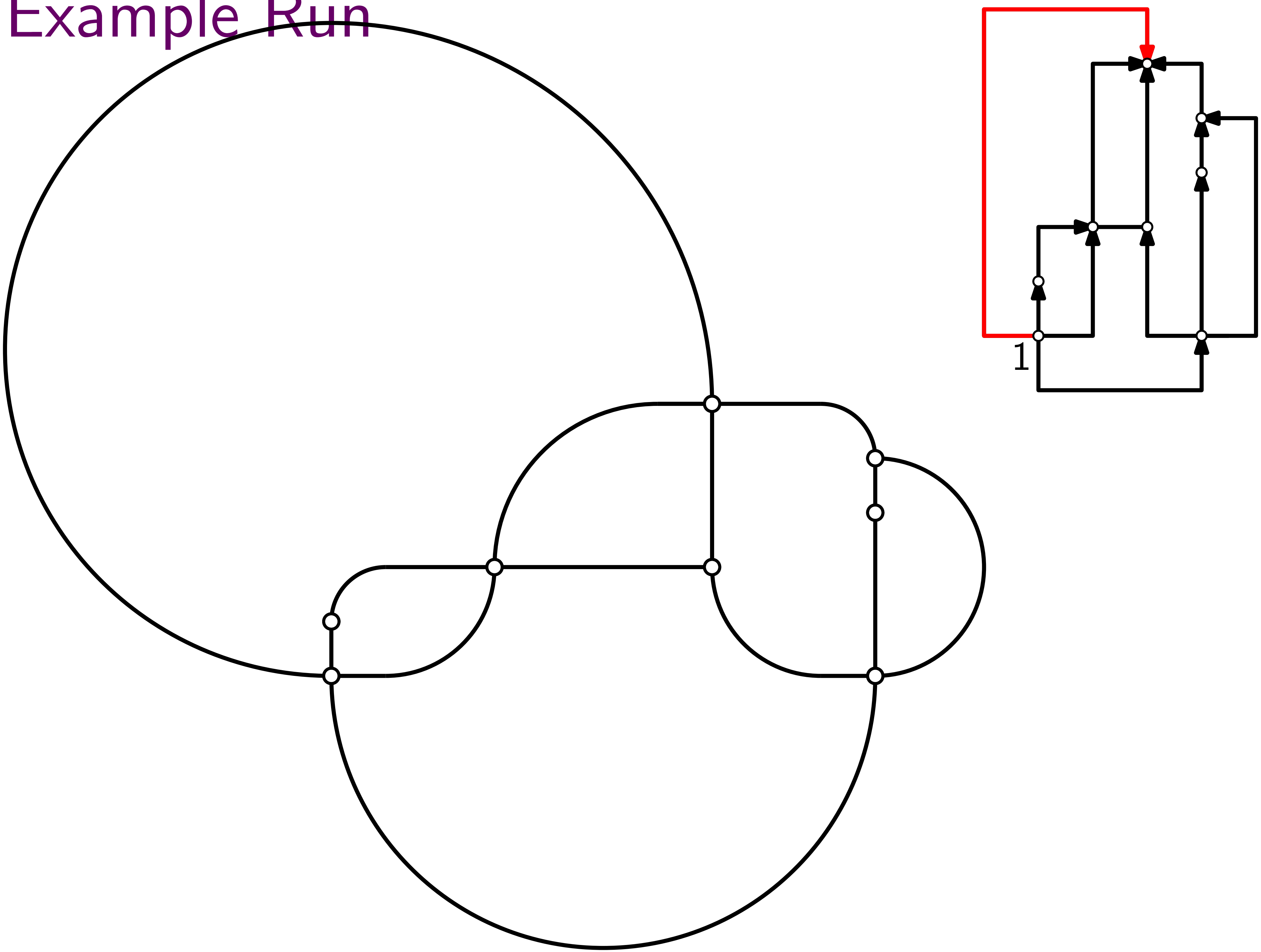


Example Run





Example Run



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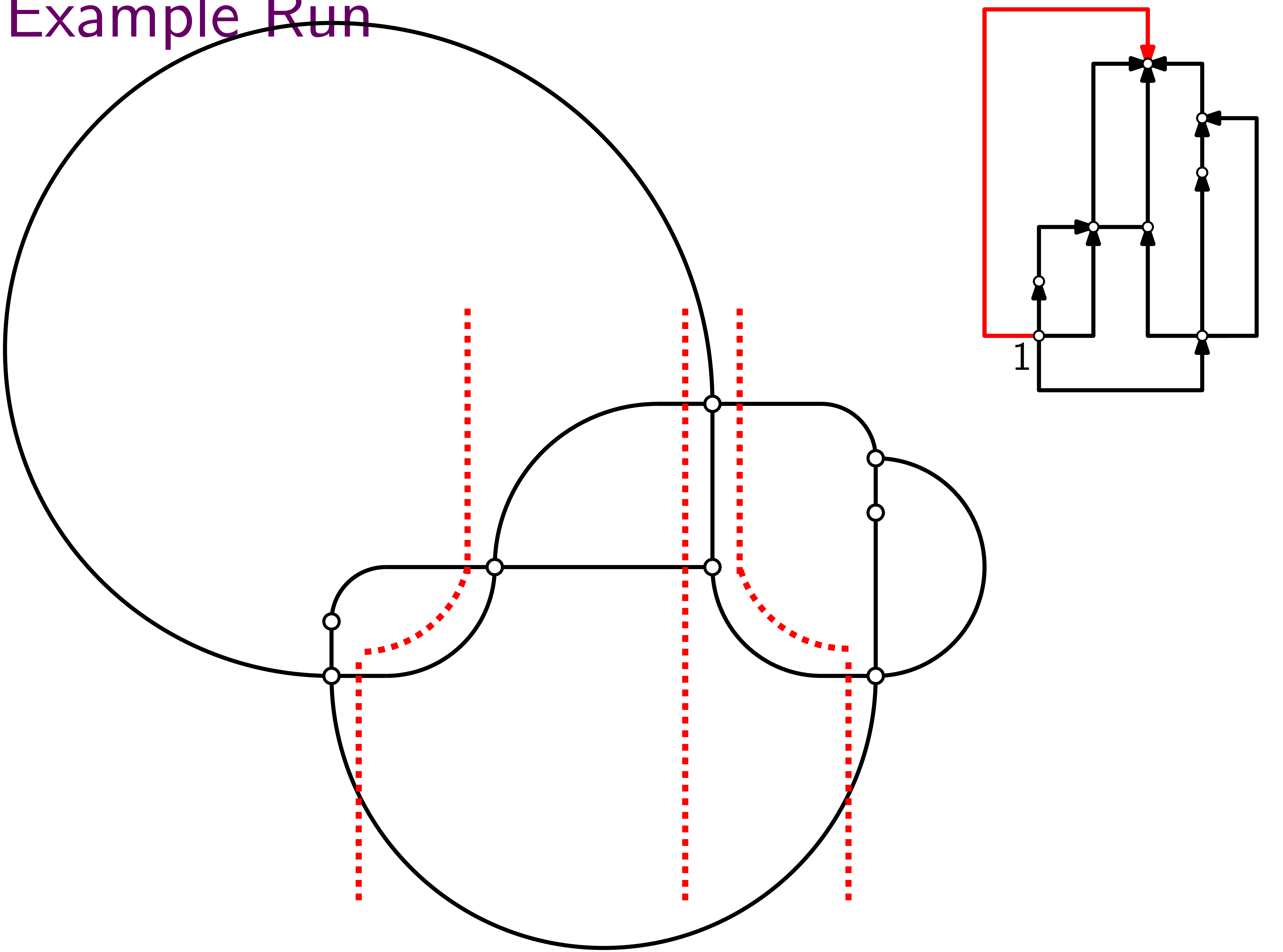

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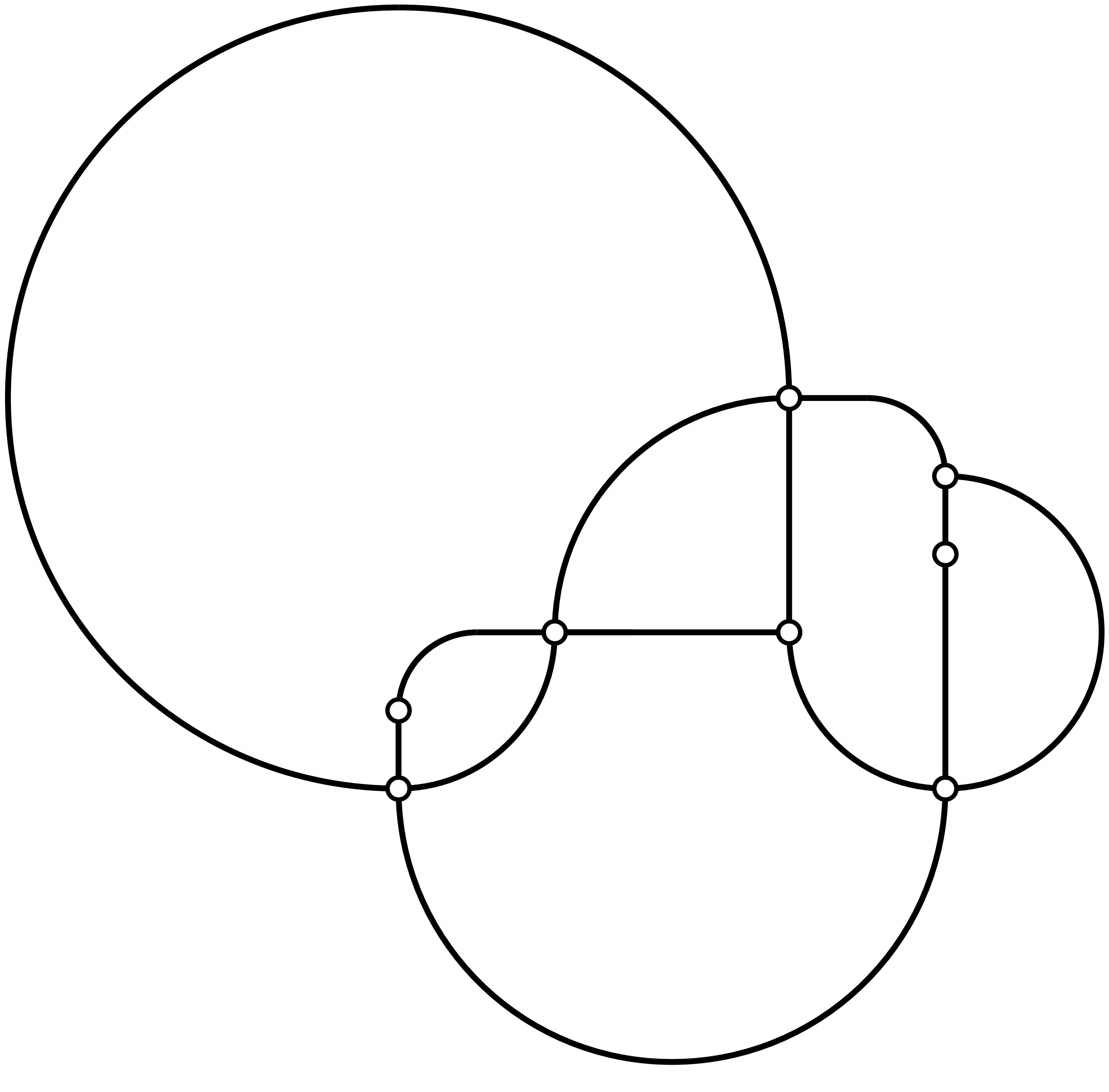
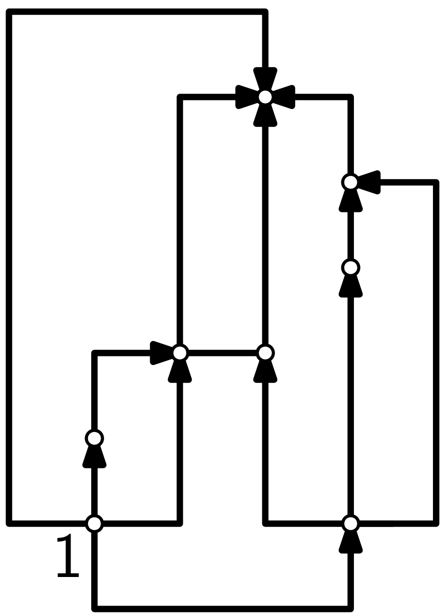
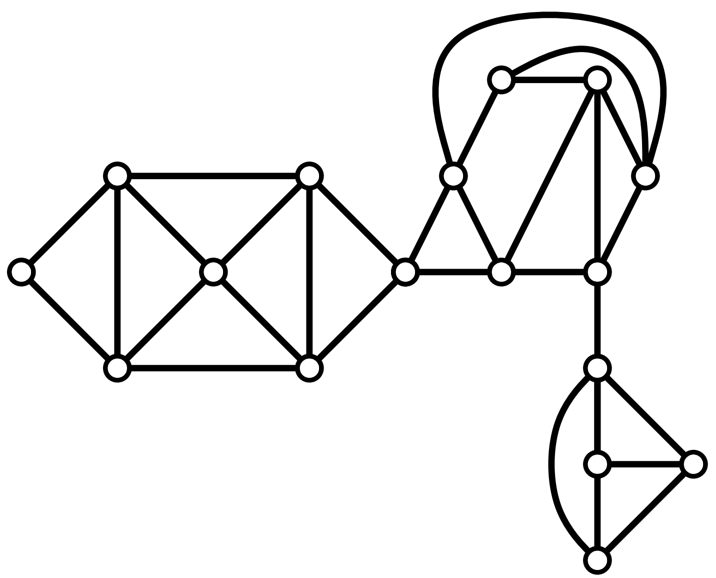


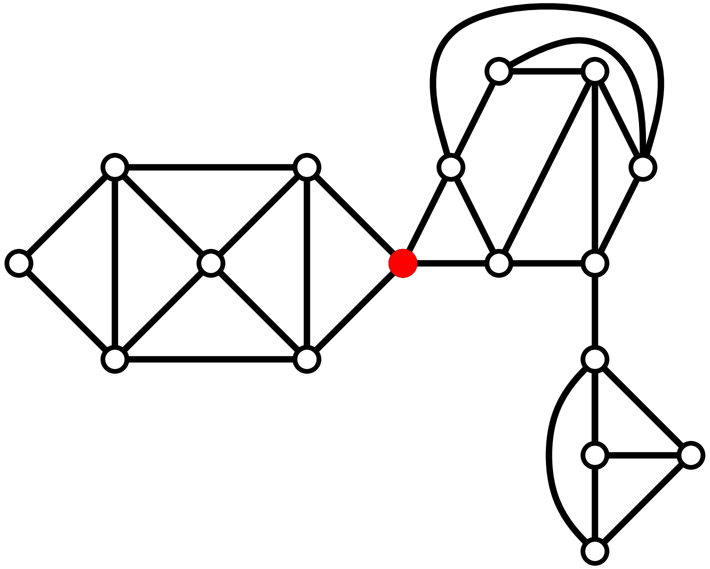


Extension to Arbitrary Graphs



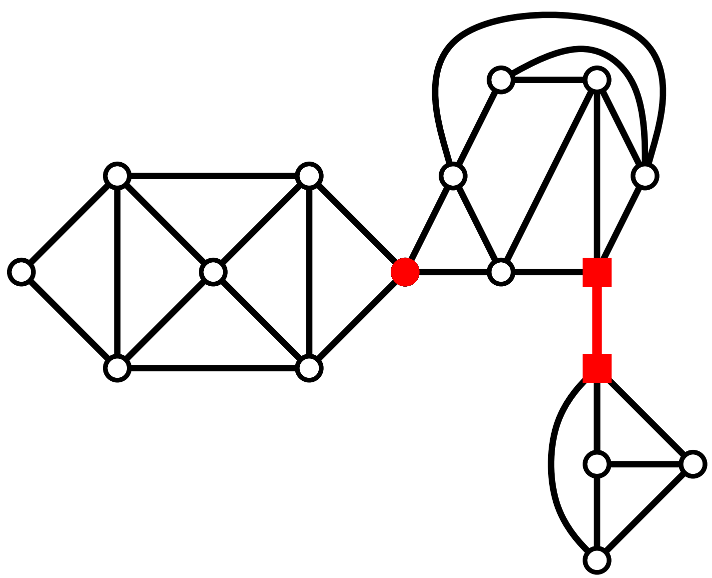
Extension to Arbitrary Graphs

- cutvertices



Extension to Arbitrary Graphs

- cutvertices
- bridges



Extension to Arbitrary Graphs

- cutvertices
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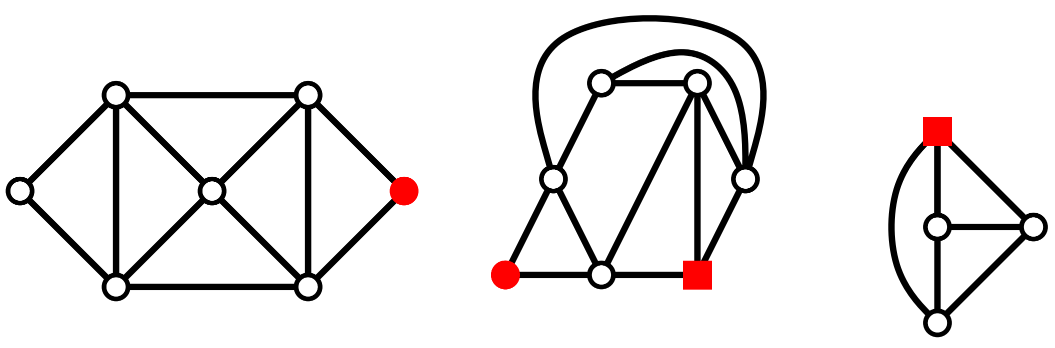
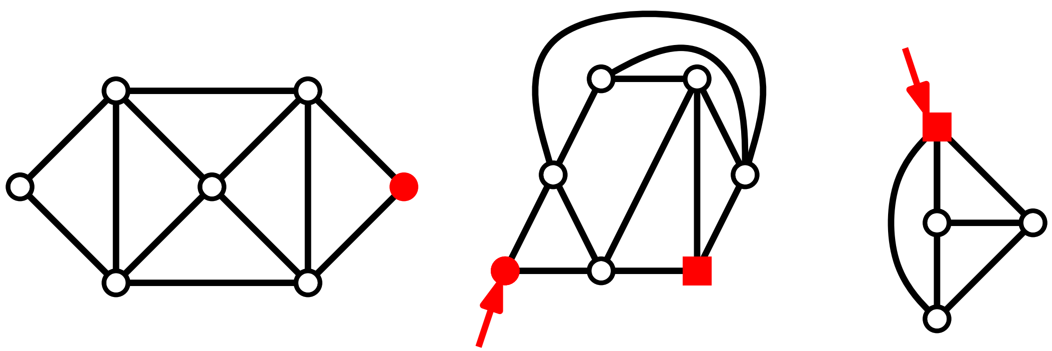


Extension to Arbitrary Graphs

- cutvertices
- bridges

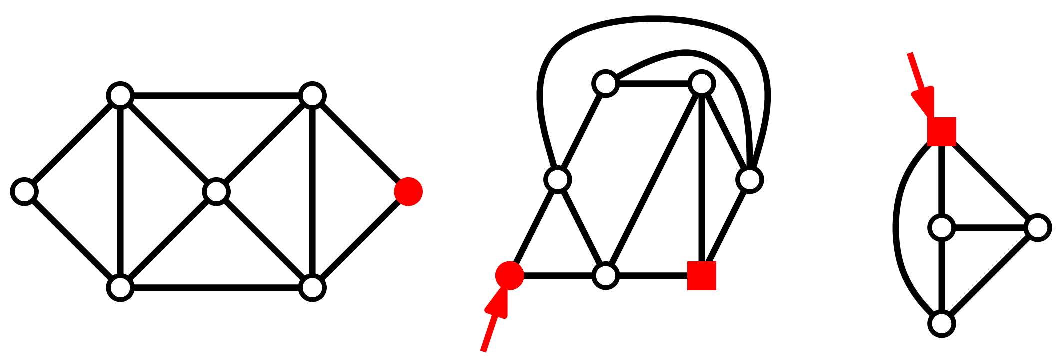
Draw specific vertex on outer face



Extension to Arbitrary Graphs

- cutvertices
- bridges

Draw specific vertex on outer face
Draw cut vertices with right angles



Extension to Arbitrary Graphs

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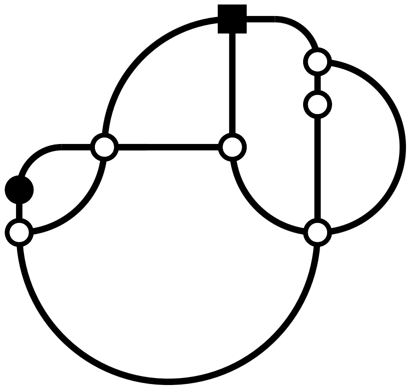

Extension to Arbitrary Graphs

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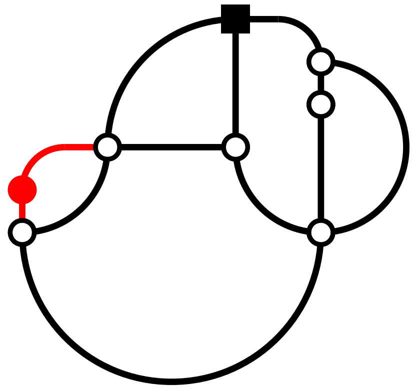

Extension to Arbitrary Graphs

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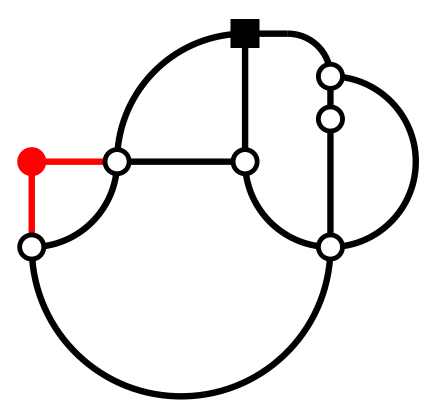

Extension to Arbitrary Graphs

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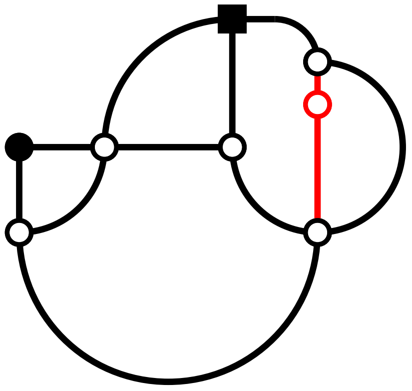


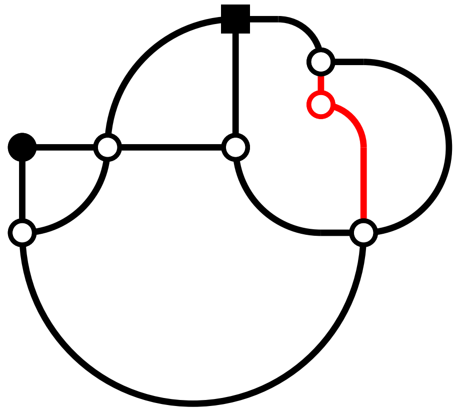
Extension to Arbitrary Graphs

cutvertices

bridges

Draw specific vertex on outer face

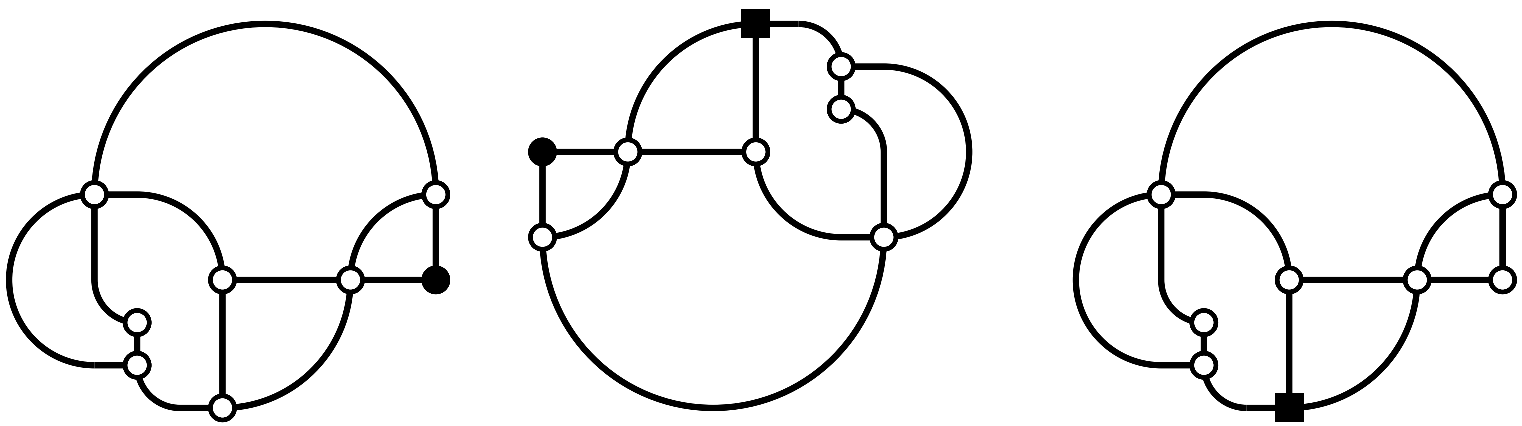
Draw cut vertices with right angles



Extension to Arbitrary Graphs

- cutvertices
- bridges

Draw specific vertex on outer face
Draw cut vertices with right angles
Connect the pieces



Extension to Arbitrary Graphs

- cutvertices
- bridges

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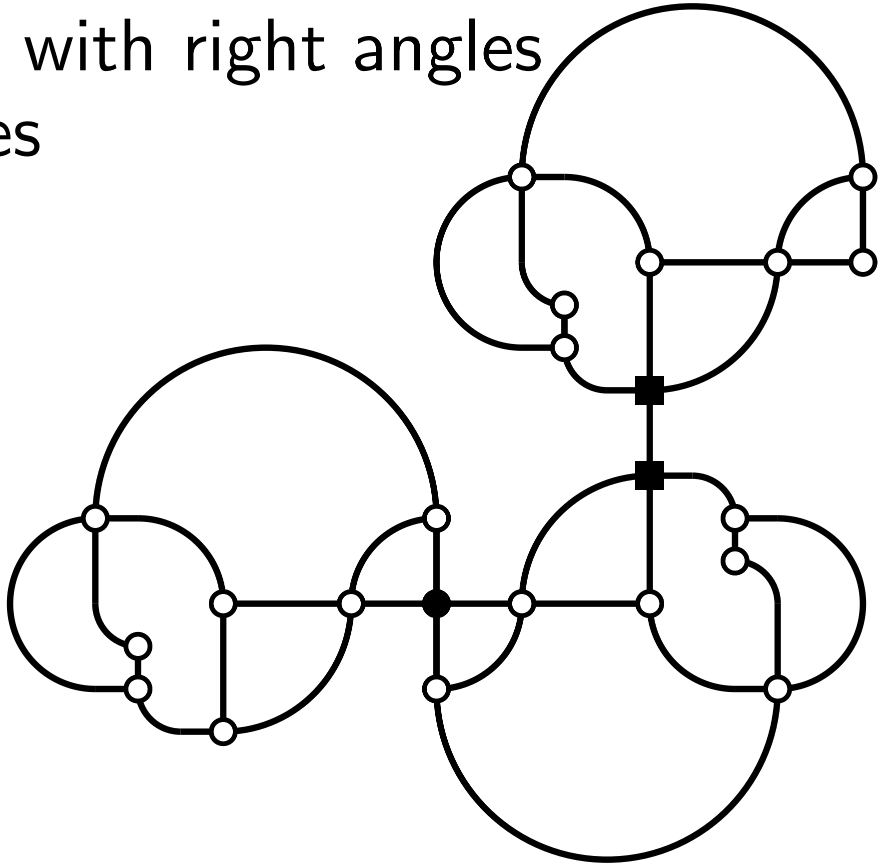

Extension to Arbitrary Graphs

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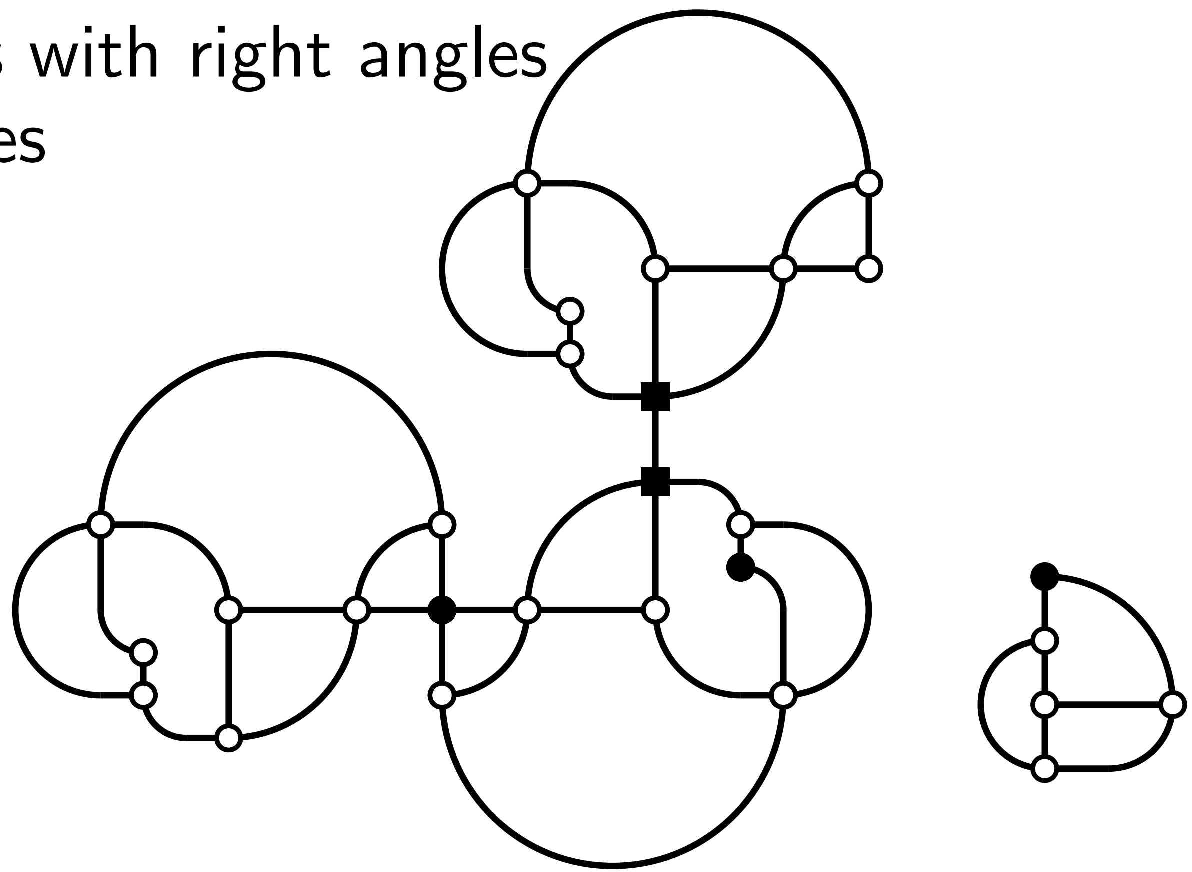

Extension to Arbitrary Graphs

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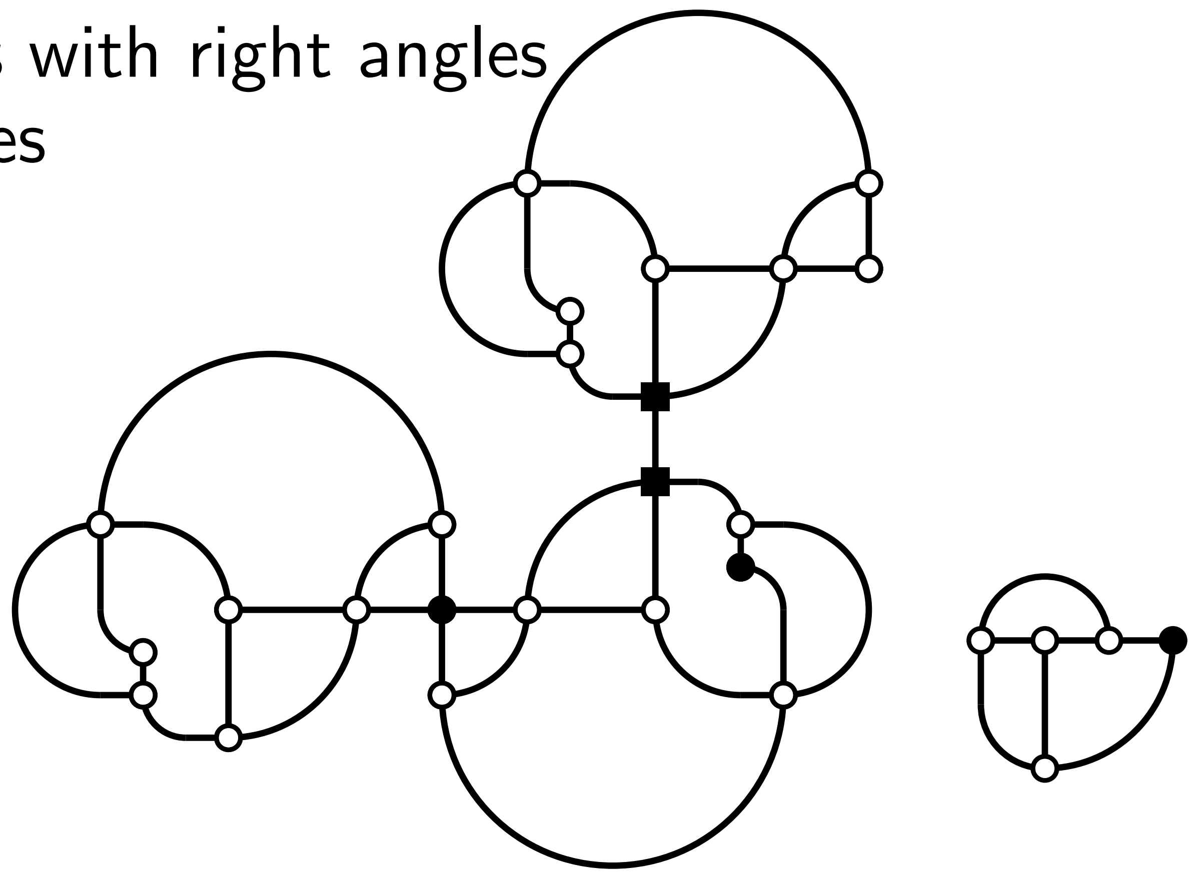

Extension to Arbitrary Graphs

 cutvertices

 bridges

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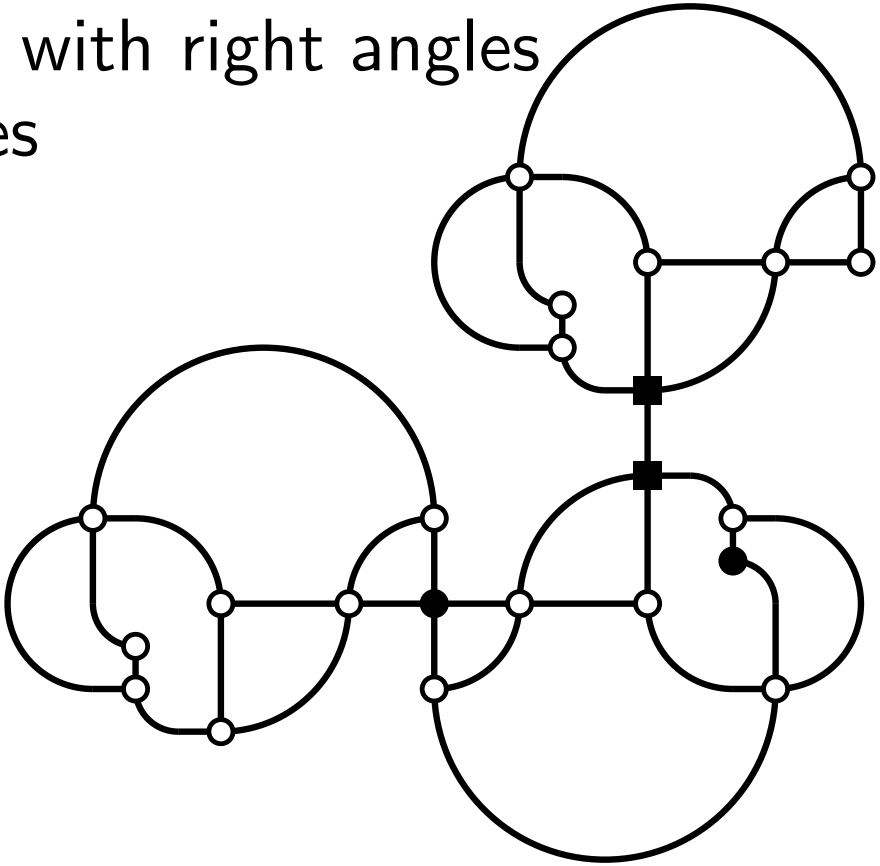

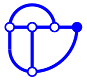

Extension to Arbitrary Graphs

- cutvertices
- bridges

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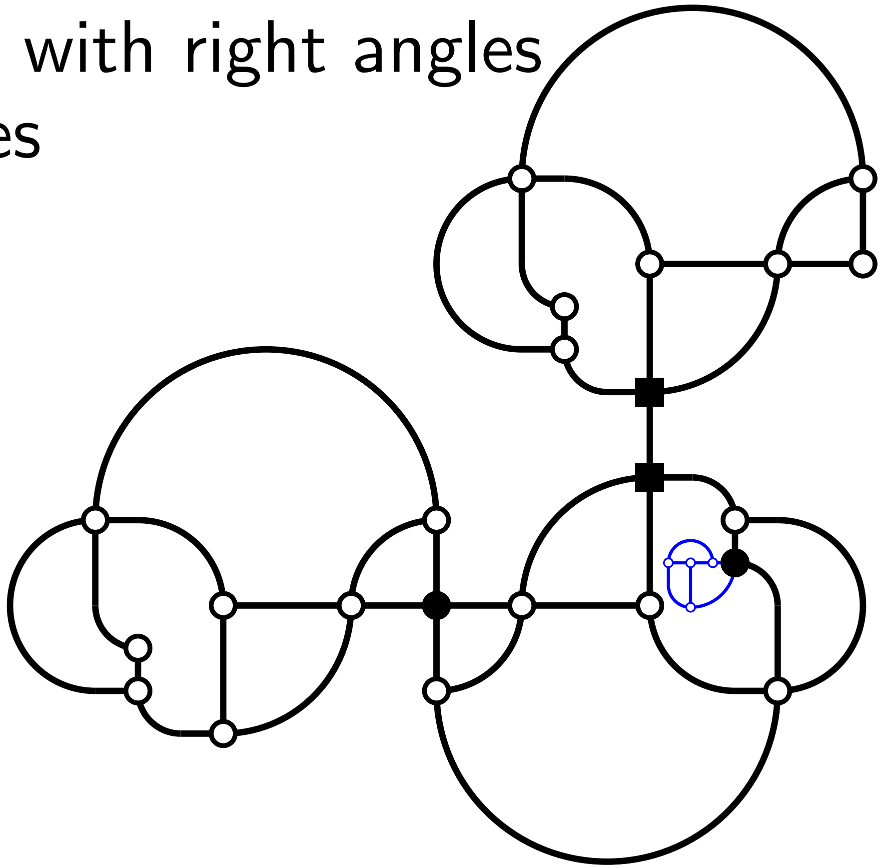
Connect the pieces

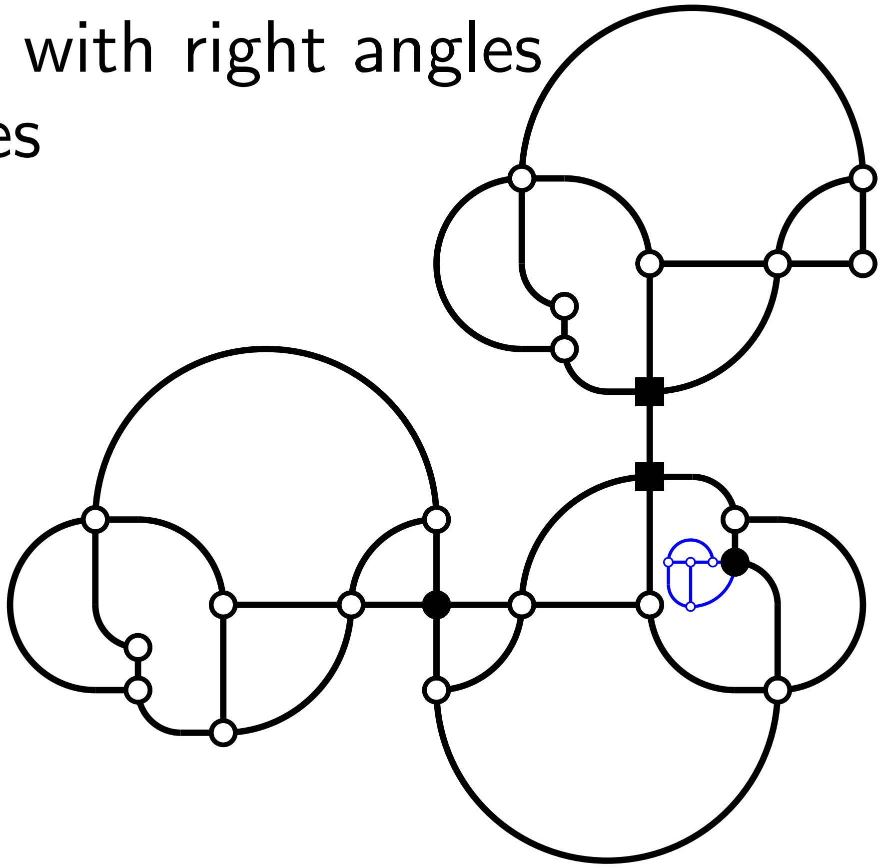


Extension to Arbitrary Graphs

4-planar graph → smooth orthogonal complexity-2 layout

- cutvertices
- bridges

Draw specific vertex on outer face
Draw cut vertices with right angles
Connect the pieces



SC_1-Layouts

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Any biconnected 4-outerplanar graph admits an SC_1-layout.

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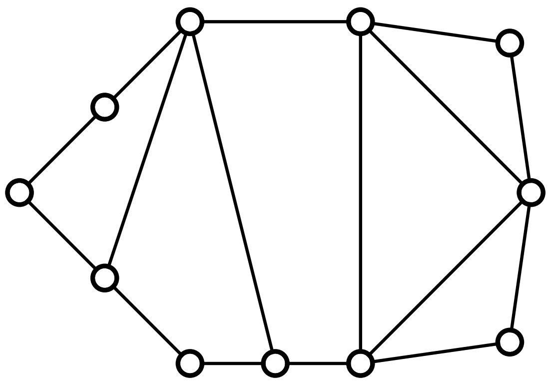
Any biconnected 4-outerplanar graph admits an SC_1-layout.



SC_1-Layouts

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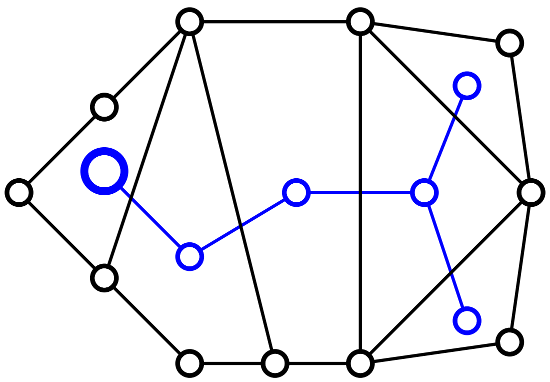
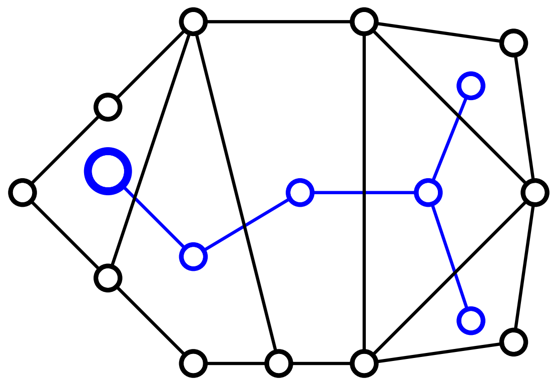


Consider the dual tree

SC_1-Layouts

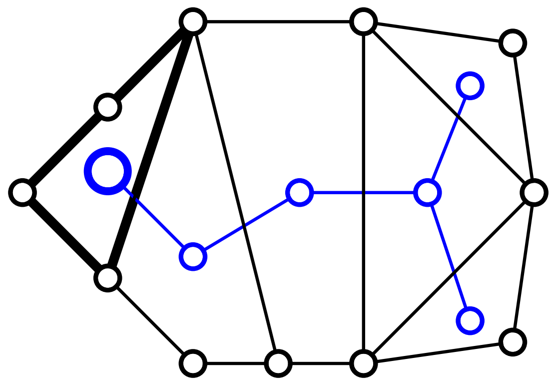
Any biconnected 4-outerplanar graph admits an SC_1-layout.

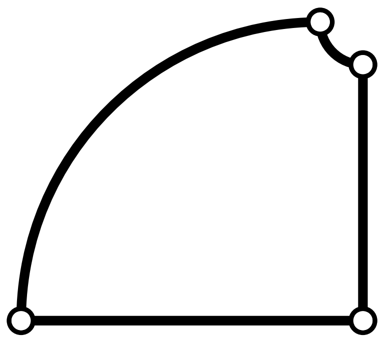


Consider the dual tree
Pick a root, traverse the tree

SC_1-Layouts

Any biconnected 4-outerplanar graph admits an SC_1-layout.

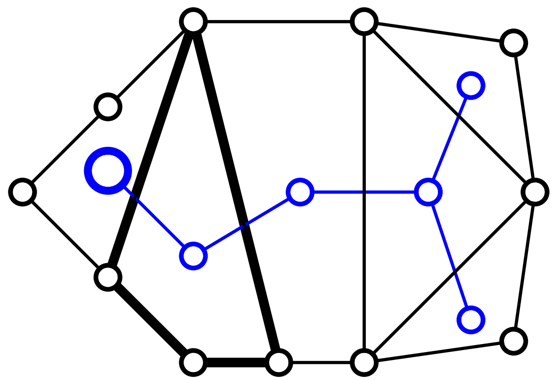


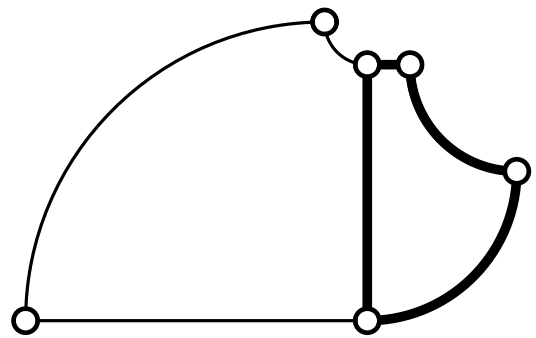


Consider the dual tree
Pick a root, traverse the tree

SC$_1$-Layouts

Any biconnected 4-outerplanar graph admits an SC$_1$-layout.





Consider the dual tree
Pick a root, traverse the tree

SC_1-Layouts

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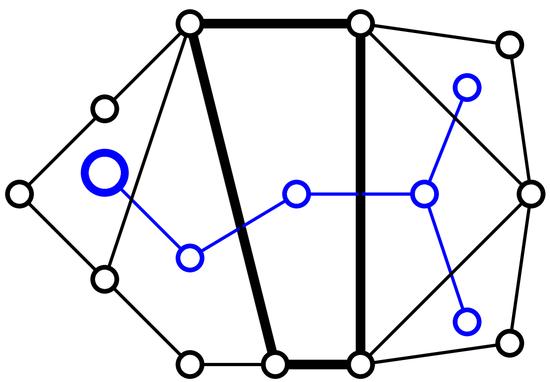

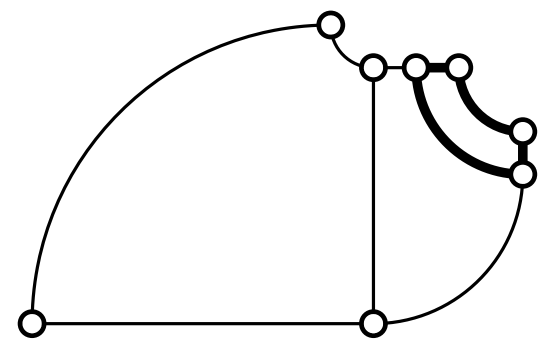

Consider the dual tree
Pick a root, traverse the tree

SC_1-Layouts

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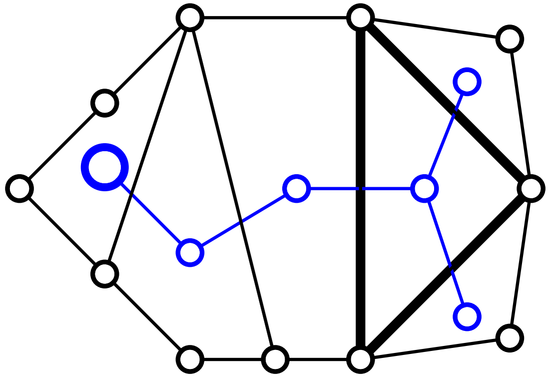
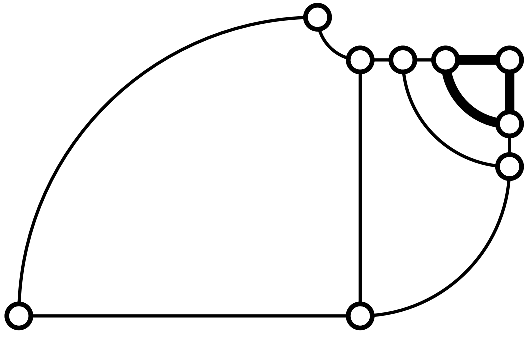
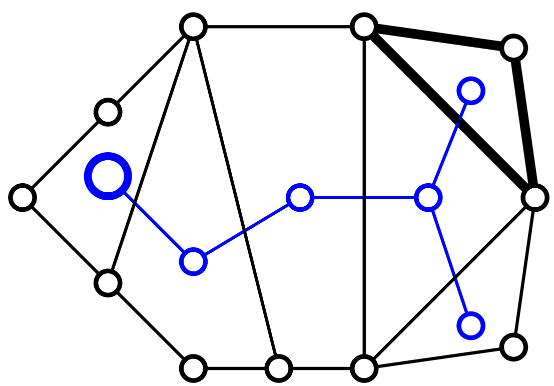


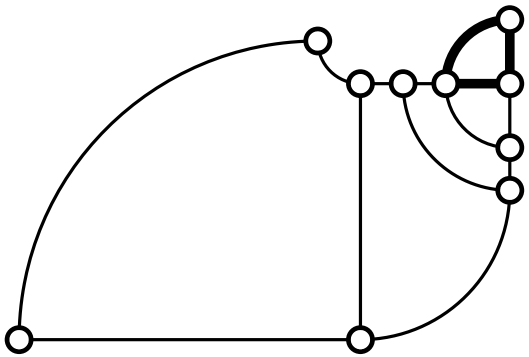


Consider the dual tree
Pick a root, traverse the tree

SC_1-Layouts

Any biconnected 4-outerplanar graph admits an SC_1-layout.





Consider the dual tree
Pick a root, traverse the tree

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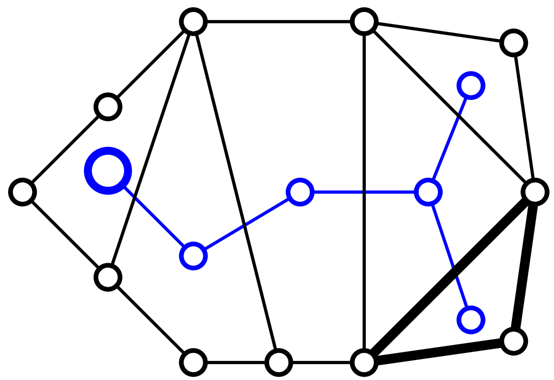

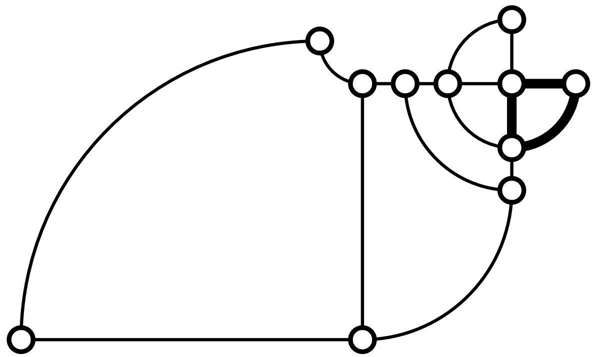

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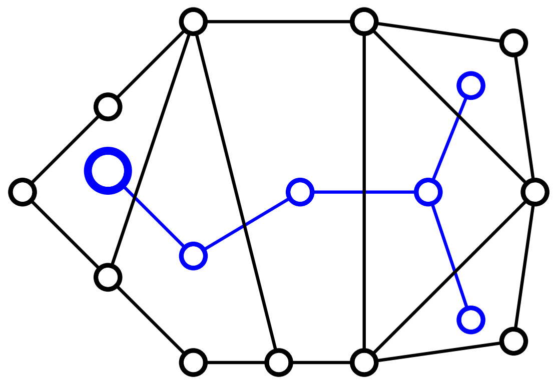

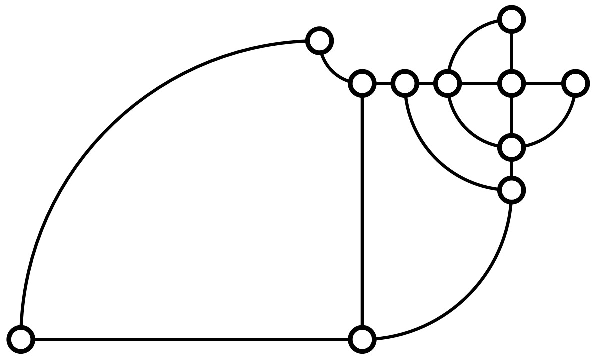

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There is an infinite class of graphs that require exponential area if they are drawn with SC_1.

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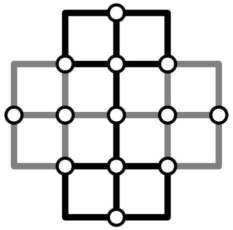

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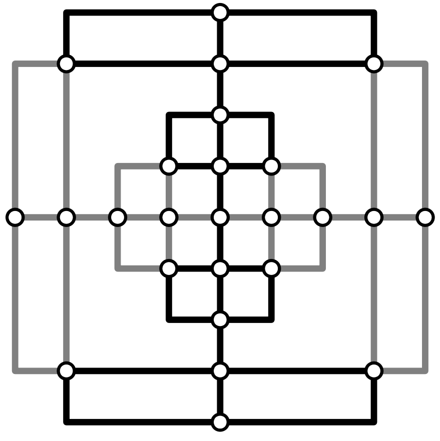

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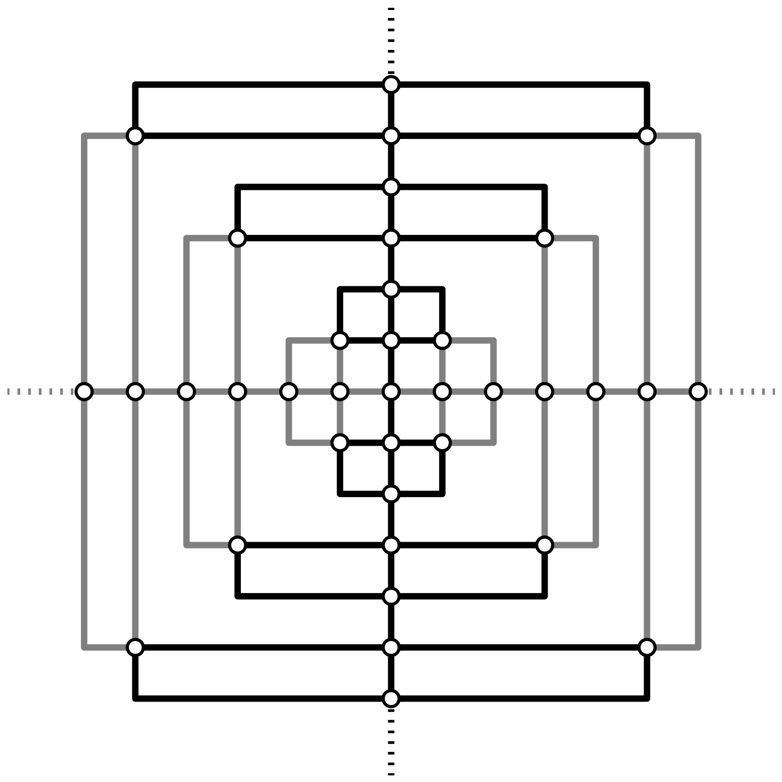

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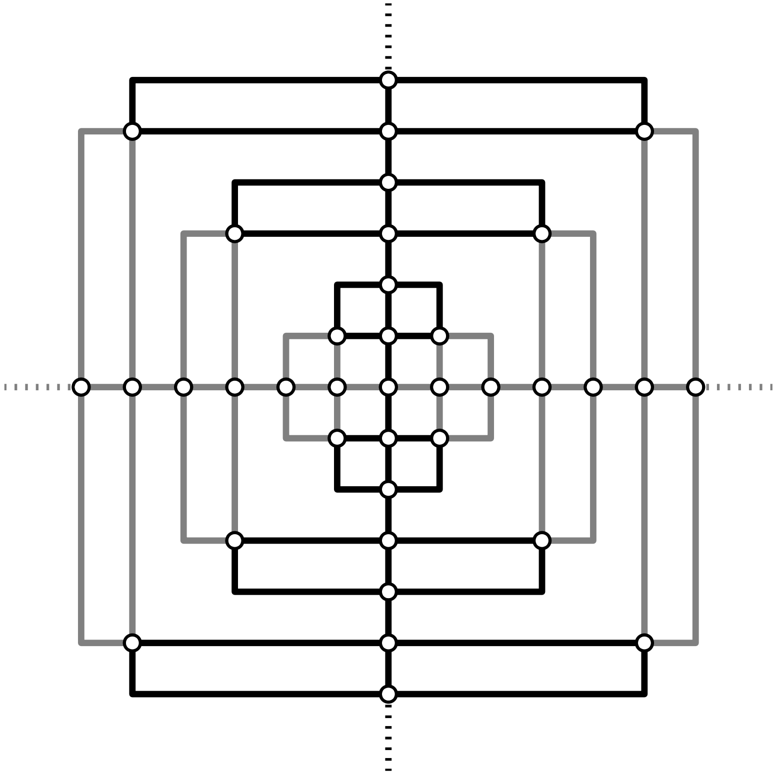

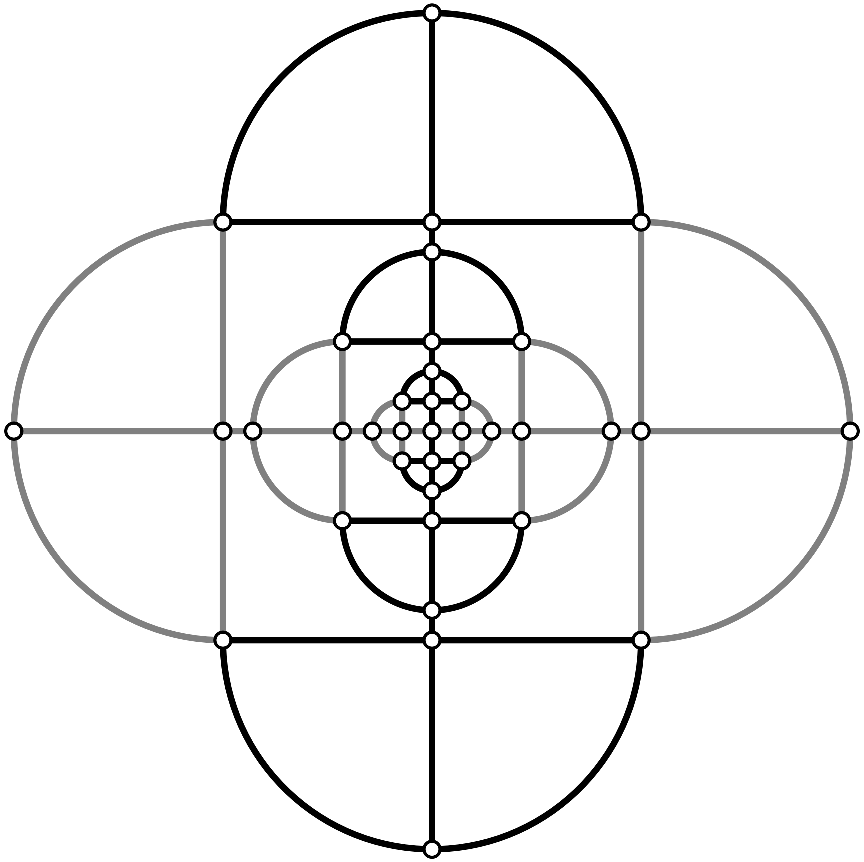

Biconnected Graphs without SC_1-Layout

There exists a biconnected 4-planar graph that admits an OC_2-layout, but does not admit an SC_1-layout.

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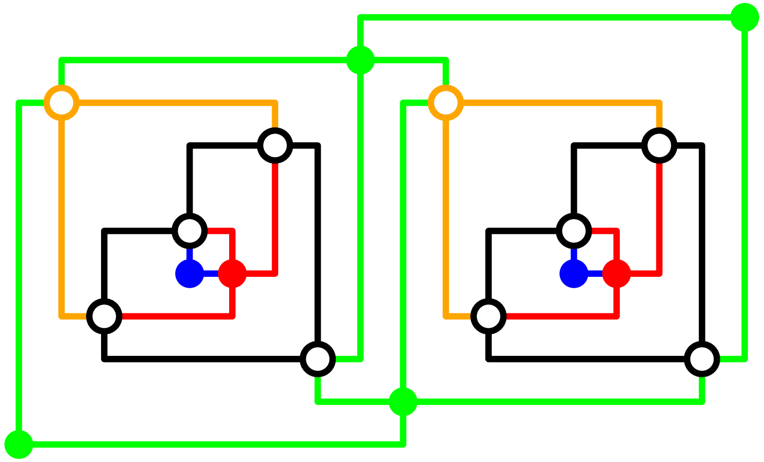

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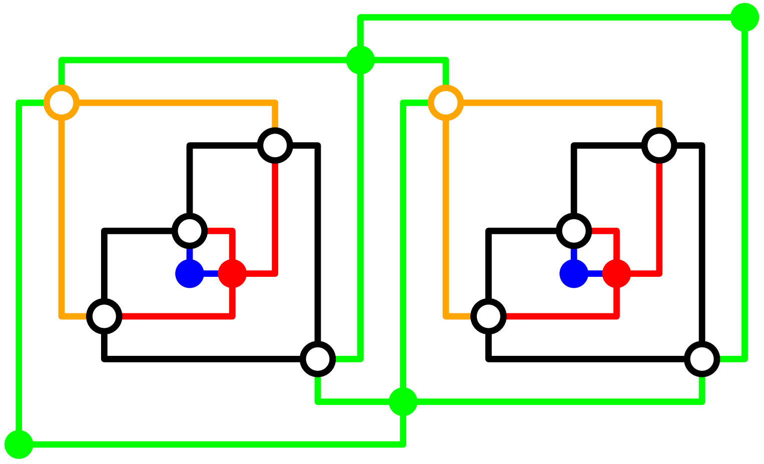

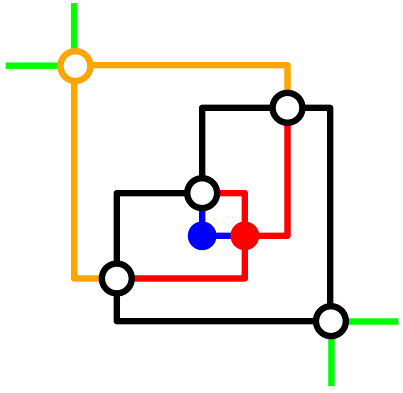

Open Problems

- Do all 4-planar graphs admit an SC_2-layout *in polynomial area*?

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- Do all 4-planar graphs admit an SC_2-layout *in polynomial area*?

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- Do all 4-planar graphs admit an SC_2-layout *in polynomial area*?
- Do all 4-outerplanar graphs admit an SC_1-layout?
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Open Problems

- Do all 4-planar graphs admit an SC_2-layout *in polynomial area*?

- Do all 4-outerplanar graphs admit an SC_1-layout?

- Do all 3-planar graphs admit an SC_1-layout?

- Is it NP-hard to decide whether a 4-planar graph admits an SC_1-layout?