

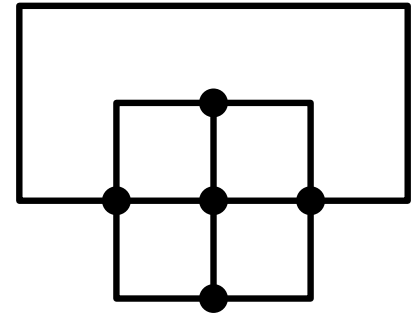
Smooth Orthogonal Drawings of Planar Graphs

Philipp Kindermann
Chair of Computer Science I
Universität Würzburg

Joint work with
Md. Jawaherul Alam, Michael A. Bekos, Michael Kaufmann,
Stephen G. Kobourov & Alexander Wolff

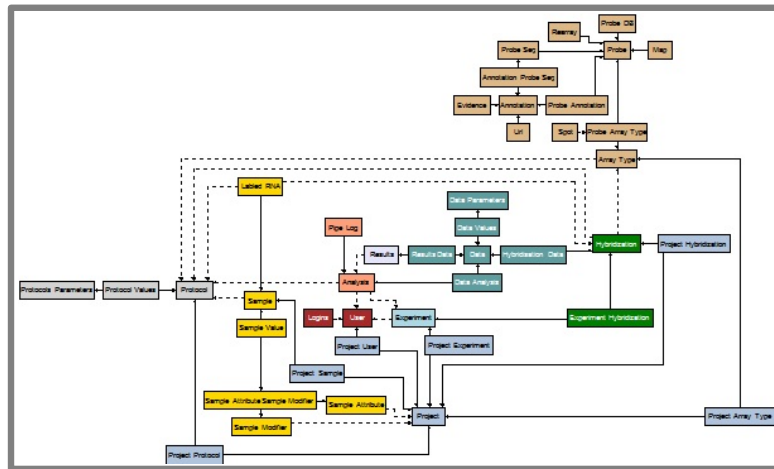
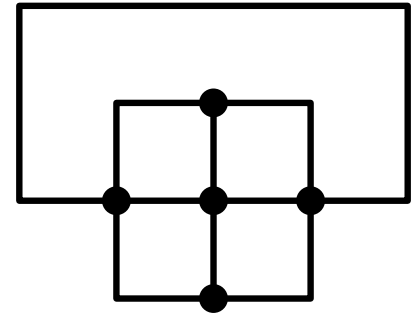
Orthogonal Layouts

- all edge segments are horizontal or vertical
- a well-studied drawing convention
- many examples in applications



Orthogonal Layouts

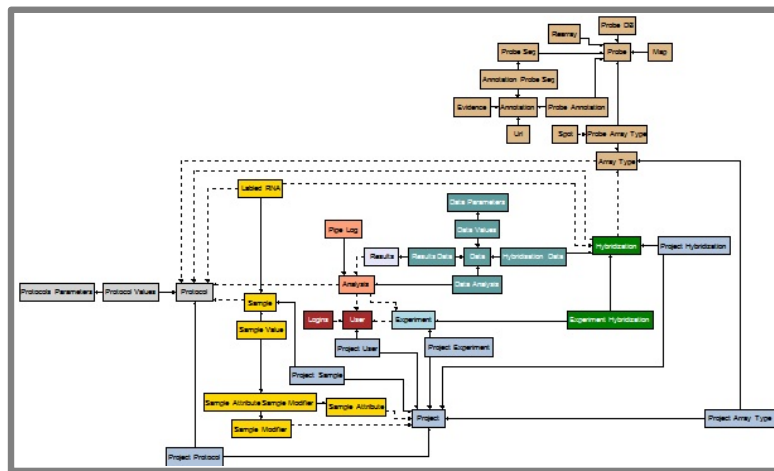
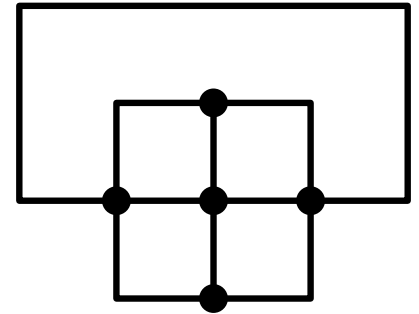
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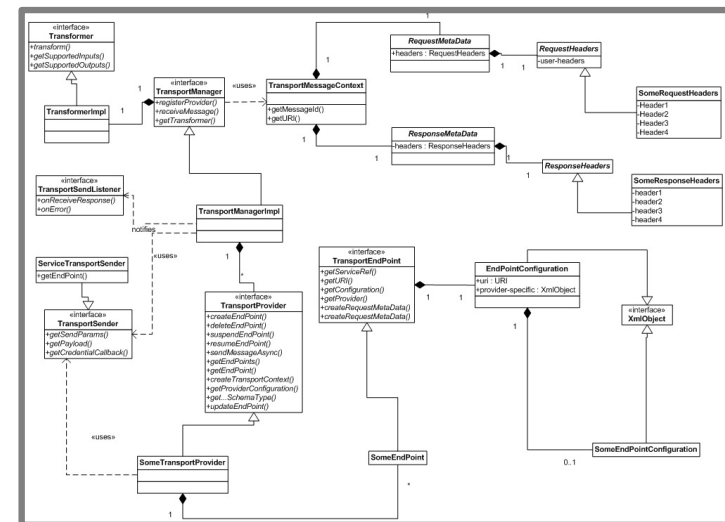
ER diagram in OGDF

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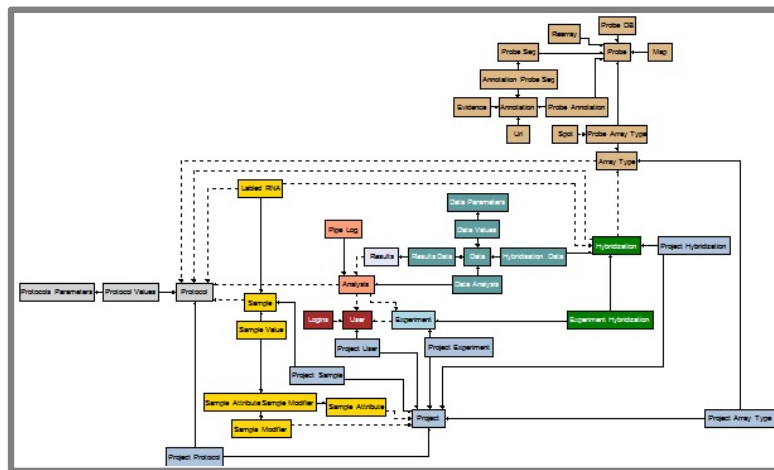
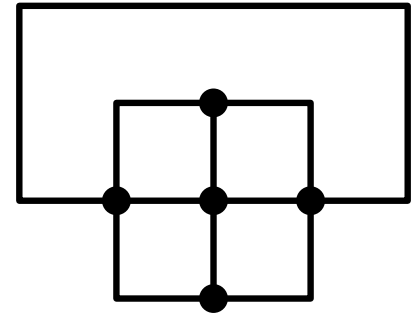
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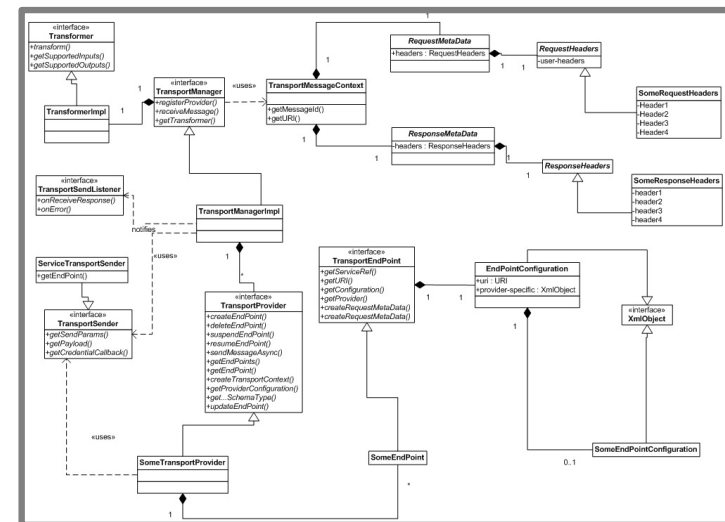
UML diagram by Oracle

Orthogonal Layouts

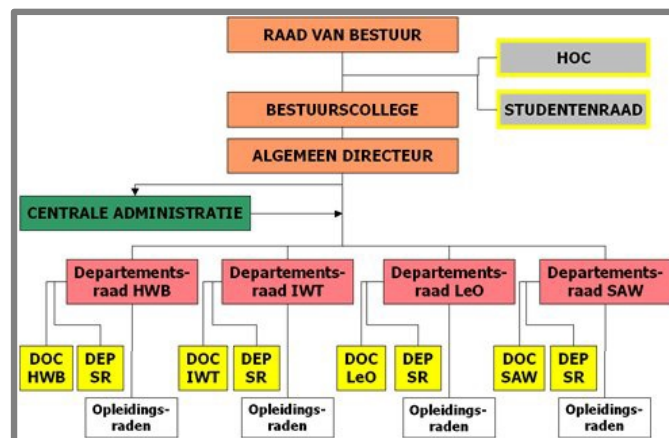
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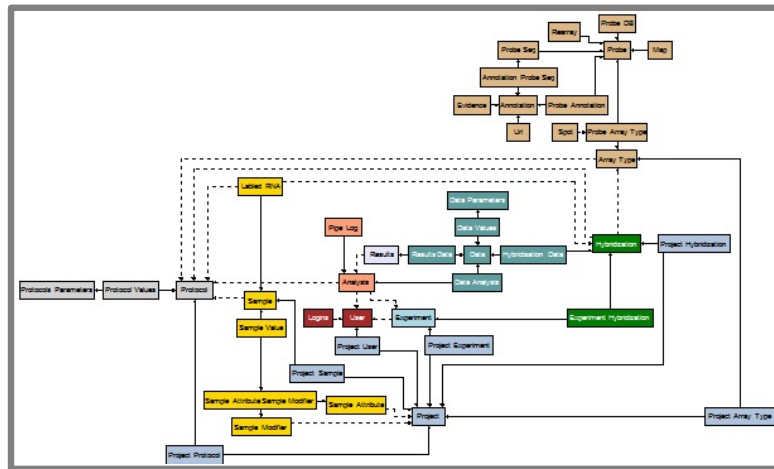
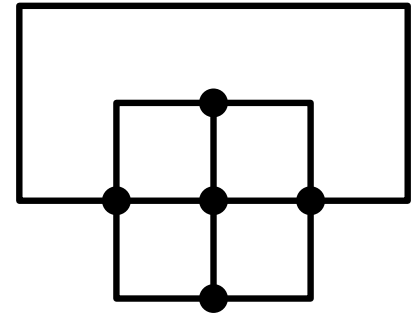
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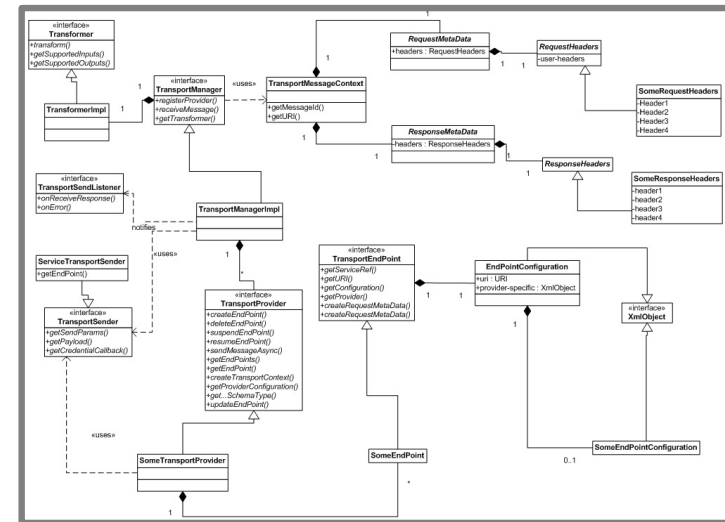
Organigram of HS Limburg

Orthogonal Layouts

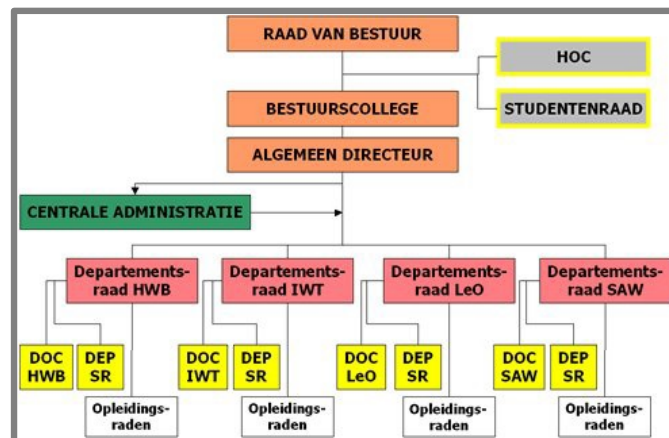
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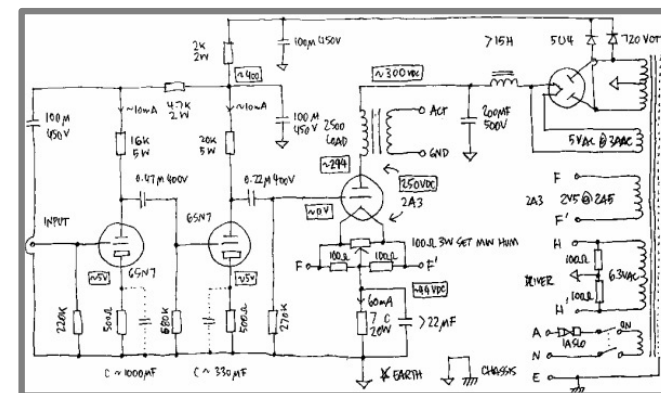
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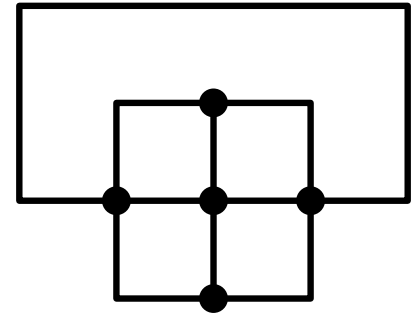
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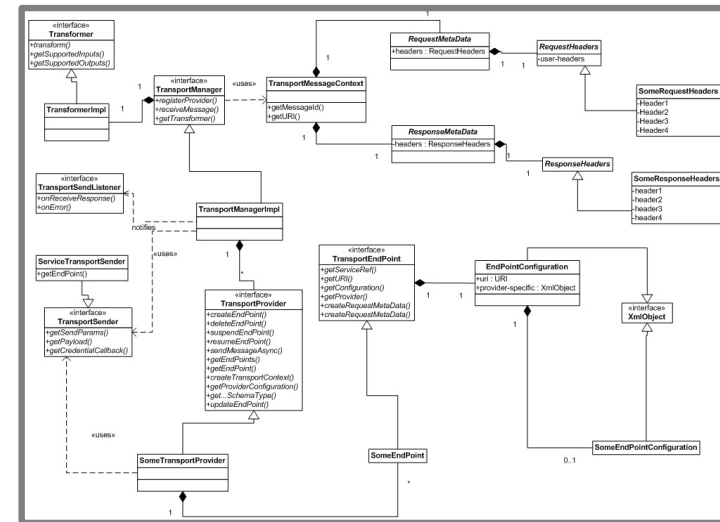
Circuit diagram by Jeff Atwood

Orthogonal Layouts

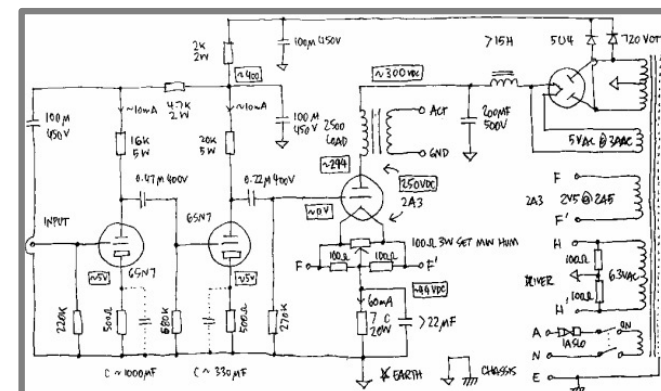
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Fused Grid city layouts [By Fgrammen, via Wikimedia Commons]



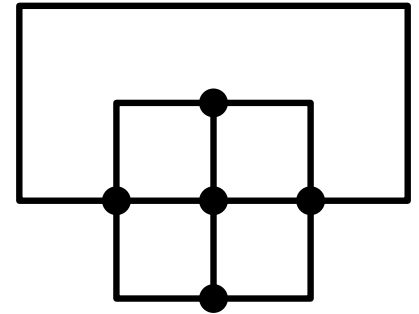
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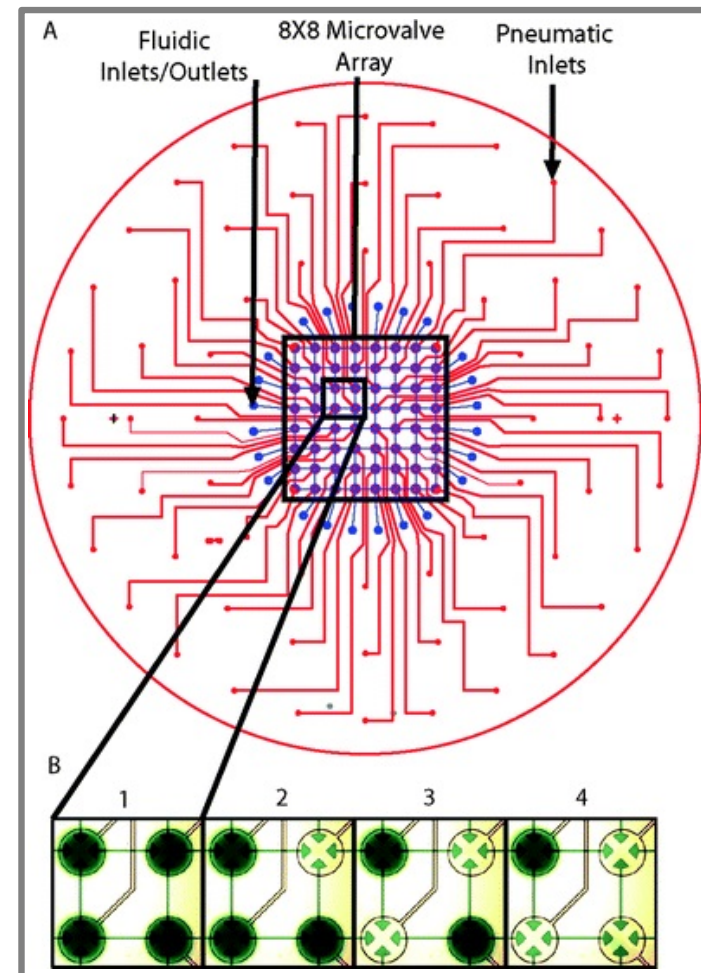
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Fused Grid city layouts [By Fgrammen, via Wikimedia Commons]



VLSI/PCI chip design [Mora et al. 2013]

Orthogonal Layouts – Well-Known Results

[Tamassia, SIAM J Comp'87]

Can minimize number of bends for fixed embedding.

[Garg & Tamassia, SIAM J Comp'01]

Without fixed embedding, bend minimization is hard to approx.

[Biedl & Kant, CGTA'98], [Liu et al., DAM'98]

Can compute drawing on the $(n \times n)$ -grid with $\leq 2n + 2$ bends for any embedding (and ≤ 2 bends/edge – except octahedron)

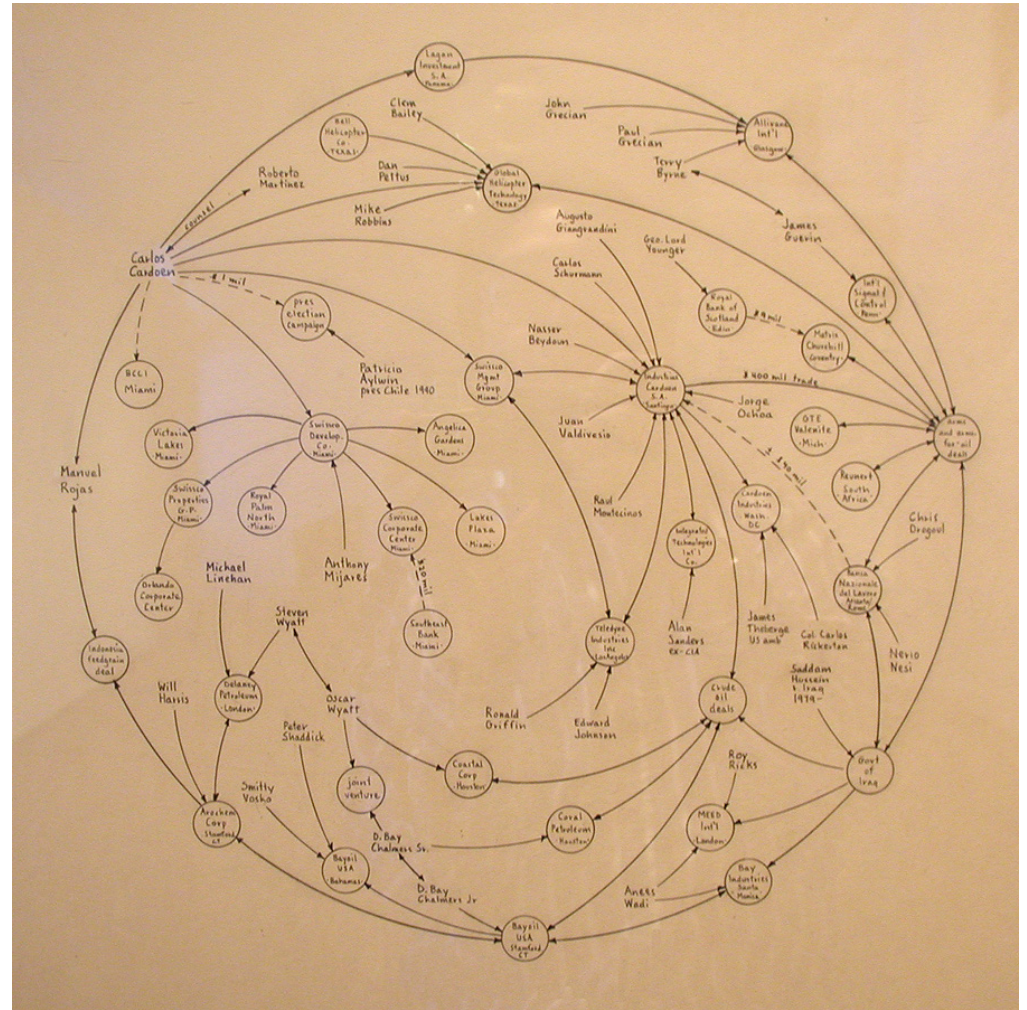
[Bläsius et al., '11]

Given an embedding and a function flex: $E \rightarrow \mathbb{N}_{\geq 1}$, can compute a drawing with $\leq \text{flex}(e)$ bends/edge (if one exists).

Smooth Drawings

Lombardi drawings

- circular arc edges
- perfect angular resolution



Mark Lombardi (1951–2000)

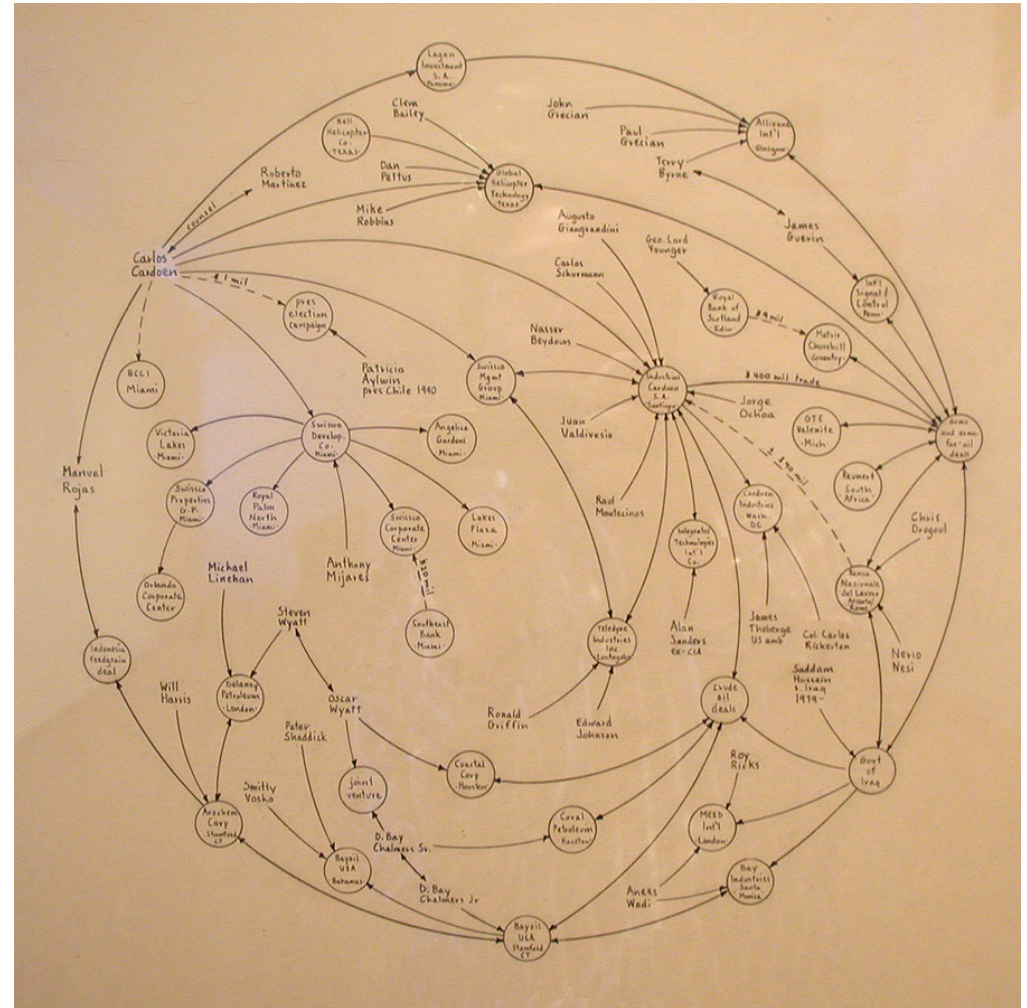
Smooth Drawings

Lombardi drawings

- circular arc edges
- perfect angular resolution

k -Lombardi drawings

- each edge sequence of k circular arcs



Mark Lombardi (1951–2000)

Smooth Drawings

FOLLOW THE MONEY

The New Global Wealth Machine

Sovereign wealth funds have emerged in recent months as the world's power brokers. They have used their tremendous wealth to make big cross-border investments and prop up some of Wall Street's best-known firms. The increased activity comes as other kinds of acquirers have been sidelined by the credit crisis. These funds are state-sponsored investment vehicles and have combined assets of \$2 trillion. With that much dry powder, sovereign funds dwarf the formerly booming private equity industry — and in some cases, compete directly with it. The Government of Singapore Investment Corporation has been the most active among the world's sovereign funds, making its deputy chairman, Tony Tan, a major center of gravity. Wall Street veterans always follow the money, so many of the big-name advisers in New York and London have found themselves traveling the globe playing international matchmaker to these funds. But sovereign funds have also learned the downside of deal-making: some of their blockbuster transactions have been big money losers so far. The question is where all that money will go next. **ANDREW ROSS SORKIN**

The Advisers

Selected financial advisers who worked on more than one of the top 20 deals.

CITIGROUP

DEALS THIS ADVISER WAS INVOLVED IN



Michael Klein, Chairman, institutional clients group
One of the firm's highest-profile investment bankers, he advised Carlyle in its stake sale to Mubadala, as well as Citigroup in both of its deals with sovereign wealth funds.

GOLDMAN SACHS GROUP

DEALS THIS ADVISER WAS INVOLVED IN



Richard Ong, Former managing director
Mr. Ong left Goldman early this year after the Chinese government refused to allow the firm to promote him to run its Beijing office. Mr. Ong's brother, Charles, was the chief investment officer of Temasek Holdings until 2006.

LAZARD

DEALS THIS ADVISER WAS INVOLVED IN



Gary Parr, Deputy chairman
In addition to becoming the key adviser on many of the biggest sovereign wealth deals, Mr. Parr helped advise Bear Stearns on its distressed sale to JPMorgan Chase.

MORGAN STANLEY

DEALS THIS ADVISER WAS INVOLVED IN



Kate Richdale, Managing director
The head of Morgan Stanley's Asian general industries group, based in Hong Kong. She previously held a senior position in the investment bank's Southeast Asia group.

The Targets

MORGAN STANLEY
John J. Mack, Chairman and C.E.O.

BLACKSTONE GROUP
Stephen A. Schwarzman, Chairman and co-founder

CITIGROUP
Robert E. Rubin, Chairman

MERRILL LYNCH
John A. Thain, Chairman and C.E.O.

STANDARD CHARTERED BANK
Peter Sands, Chief executive

UBS
Marcel Rohner, Chief executive

Qatar Investment Authority
Baker al-Saad, Managing director

Mubadala Development Co.
Khudhor Khalifa al-Mubarak, C.E.O. and managing director

China Investment Corp.
Lou Jiwei, Chairman

CHINA CONSTRUCTION BANK
Chang Zhenming, (then president)

BANK OF CHINA
Li Lihui, President and vice chairman

JAPAN BANKING CORP.
Toshio Masui, (then president)

HONGKONG & KOWLOON
John E. Meredith, Group managing director

HONGKONG INT. TERMINALS
Eric Ip, Managing director

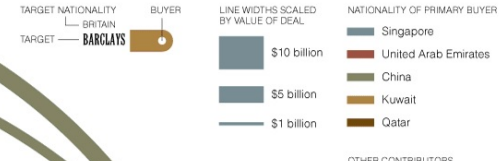
THAILAND SHIN CORP.
Borissak Panagorn, Director and chairman of the group executive committee

Temasek Holdings
Ho Ching, Executive director and C.E.O.

Govt. of Singapore Investment Corp.
Tony Tan, Deputy chairman and executive director

Saudi Arabian Monetary Agency

The 20 Biggest Cross-Border Sovereign Wealth Fund Deals Since 2005



The Buyers

DEALS THIS BUYER WAS INVOLVED IN



The Lawyers

Selected lawyers who worked on more than one of the top 20 deals.

CLIFFORD CHANCE

DEALS THIS LAWYER WAS INVOLVED IN



James Baird, Partner and global head of private equity
Mr. Baird's firm, based in London, was one of the early firms to make a bet on Asia by staffing up there before some of the traditional white-shoe Wall Street firms ventured there.

DAVIS POLK & WARDWELL

DEALS THIS LAWYER WAS INVOLVED IN



Randall D. Guynn, Partner
As head of the firm's financial institutions group, he has advised on many international deals in Europe and Asia. He also worked on the team that advised Morgan Stanley in its \$5.5 billion stake sale to China's sovereign wealth fund.

LINKLATERS

DEALS THIS LAWYER WAS INVOLVED IN



Richard Good, Partner
Based in Singapore, Mr. Good is the firm's man-on-the-ground in Asia. He has worked for Linklaters in Asia since 2000.

SHEARMAN & STERLING

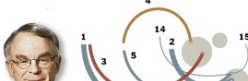
DEALS THIS LAWYER WAS INVOLVED IN



Stephen M. Besen, Partner
A longtime hand in the Middle East, Mr. Besen's deep relationships have helped his firm carve out one of the strongest niches in the region.

SULLIVAN & CROMWELL

DEALS THIS LAWYER WAS INVOLVED IN



H. Rodgin Cohen, Chairman
The world's go-to lawyer for sovereign wealth investments in financial services firms. He worked on twice as many sovereign wealth related deals than any other individual.

Source: Dealogic; the companies

RESEARCH BY MICHAEL DE LAURENCE; GRAPHIC BY GILBERT GATES FOR THE NEW YORK TIMES

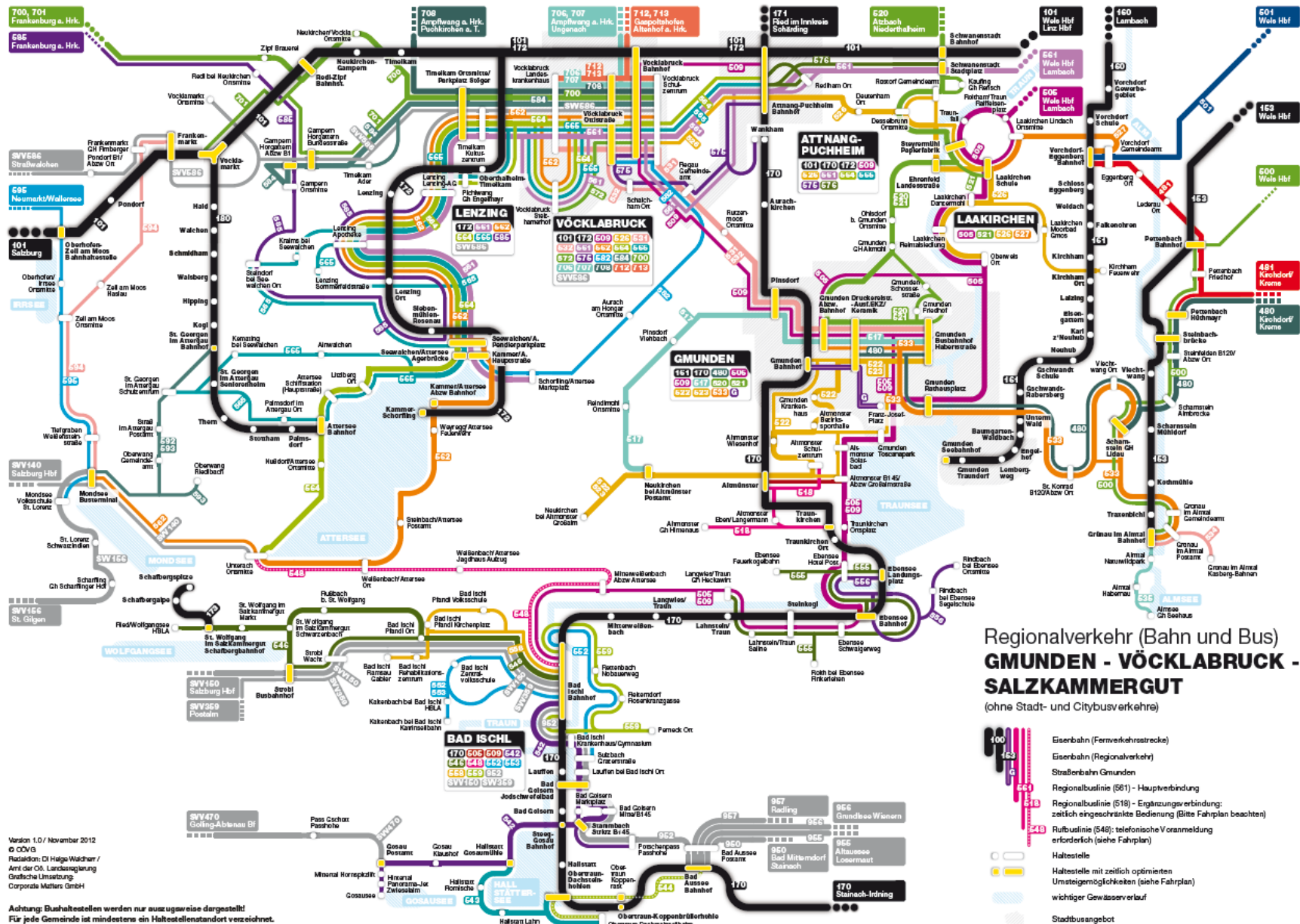
The New York Times info diagram

Smooth Drawings



Terra Nova - Design Overdrive by Carlos Barbosa

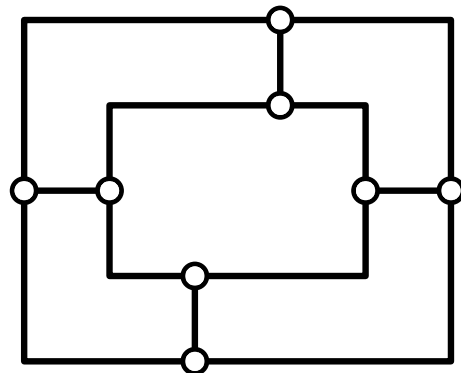
Smooth Drawings



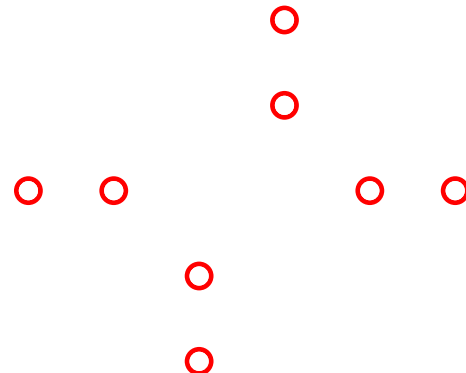
Public transportation map by ÖÖVG

Smooth Orthogonal Layouts

Combine both worlds:



orthogonal

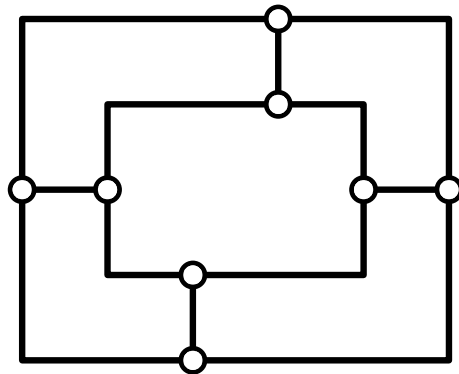


smooth orthogonal

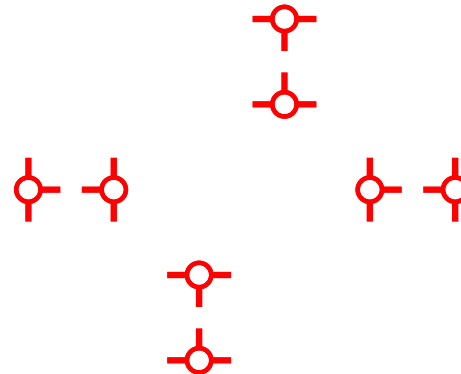
Smooth Orthogonal Layouts

Combine both worlds:

- edges leave and enter vertices horizontally or vertically



orthogonal

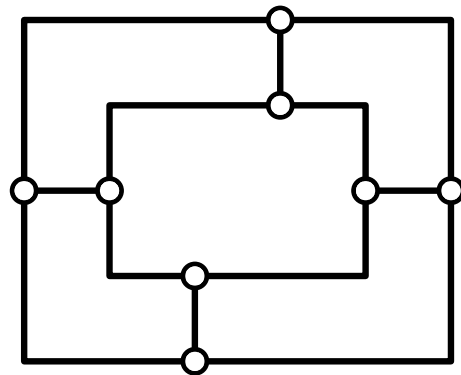


smooth orthogonal

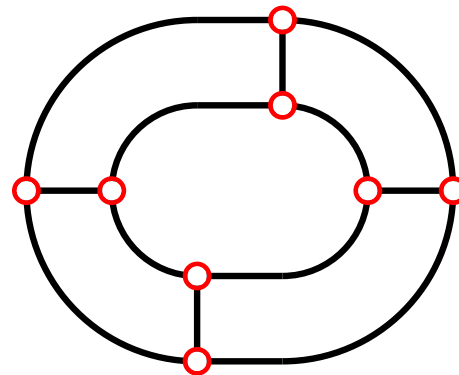
Smooth Orthogonal Layouts

Combine both worlds:

- edges leave and enter vertices horizontally or vertically
- each edge is drawn as a sequence of axis-aligned line segments and circular-arc segments without bends



orthogonal

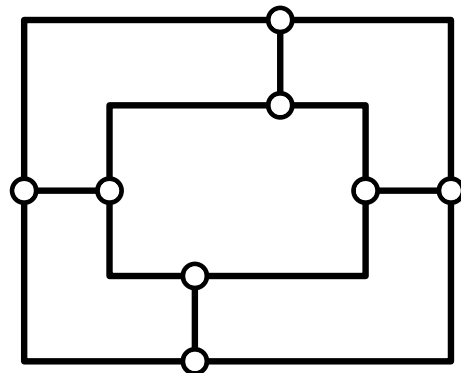


smooth orthogonal

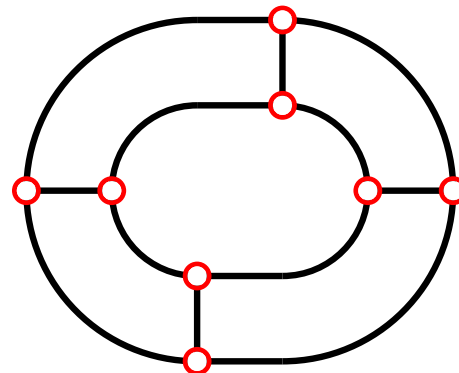
Smooth Orthogonal Layouts

Combine both worlds:

- edges leave and enter vertices horizontally or vertically
- each edge is drawn as a sequence of axis-aligned line segments and circular-arc segments without bends
- there are no edge-crossings (for planar graphs)

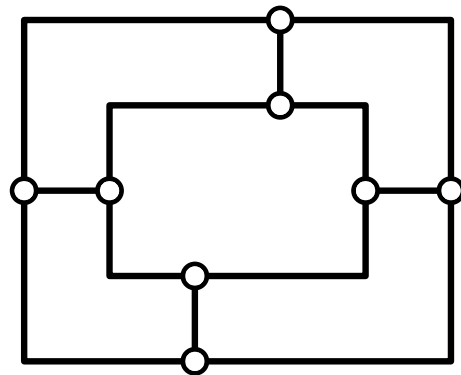


orthogonal

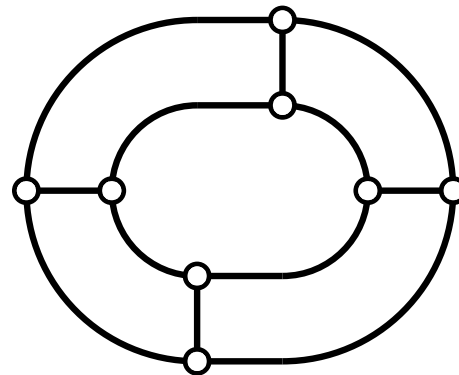


smooth orthogonal

Edge Complexity



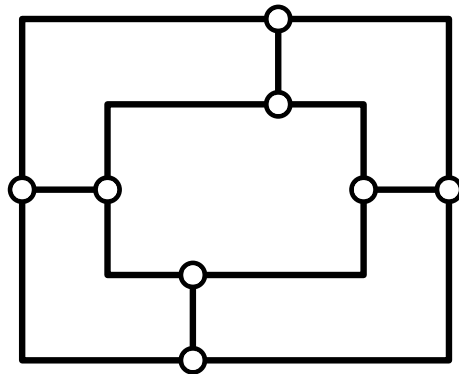
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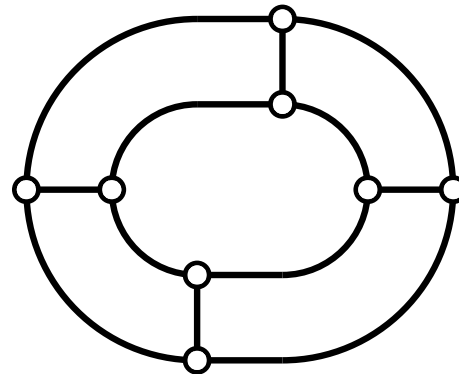
smooth orthogonal

Edge Complexity

complexity of an edge: number of arcs



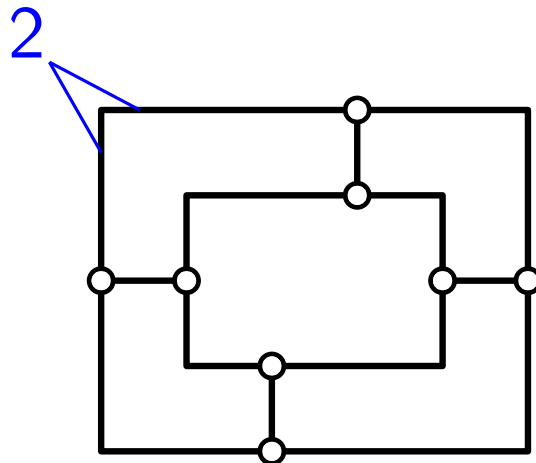
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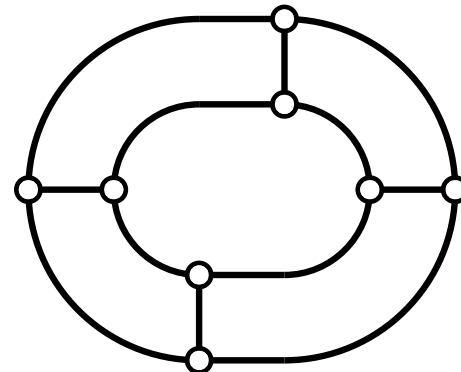
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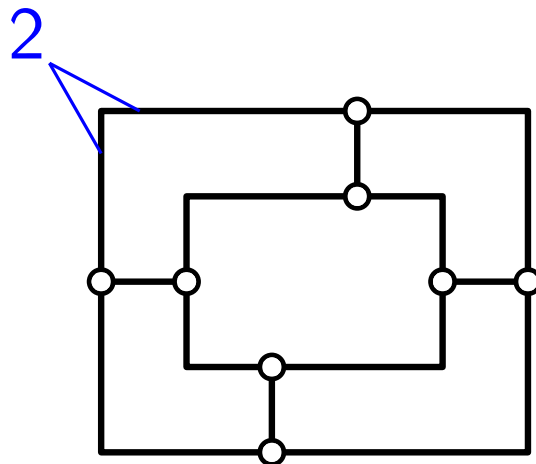
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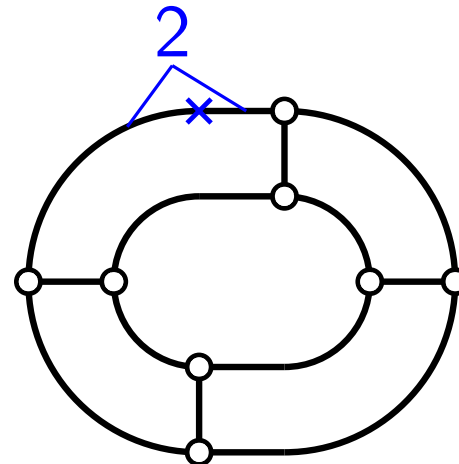
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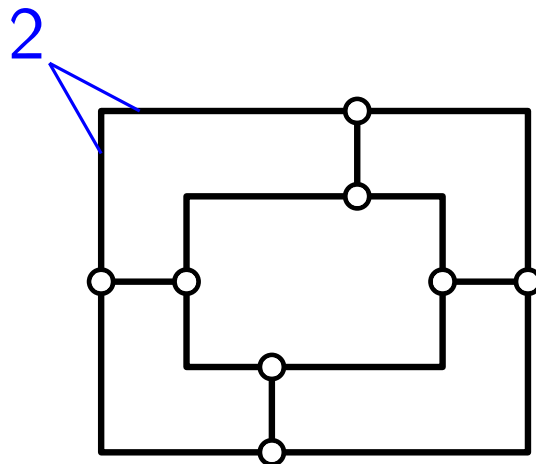


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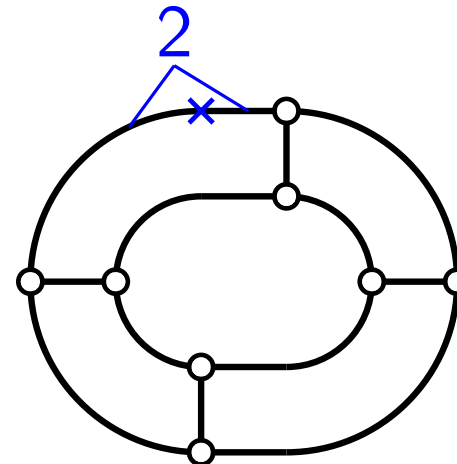
Edge Complexity

complexity of an edge: number of arcs

edge complexity: maximum complexity over all edges



orthogonal

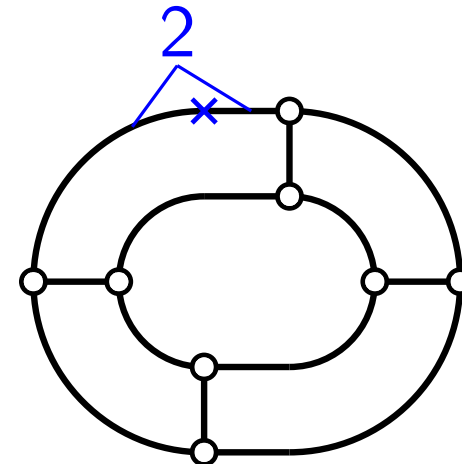
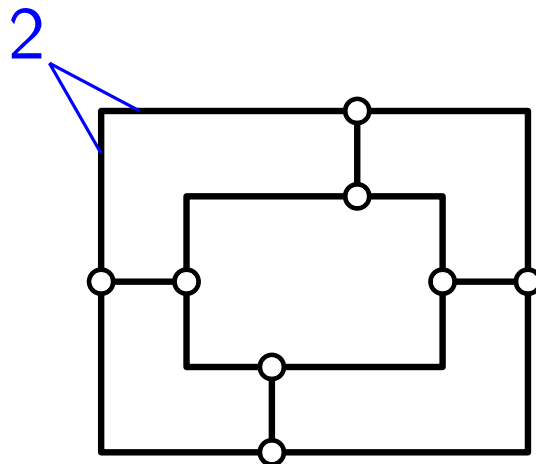
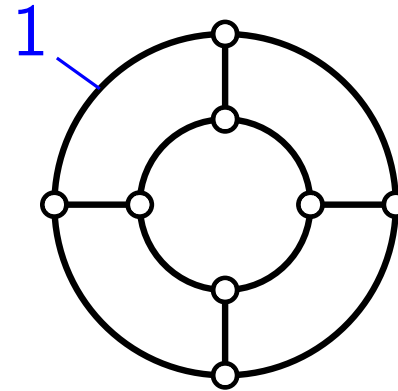
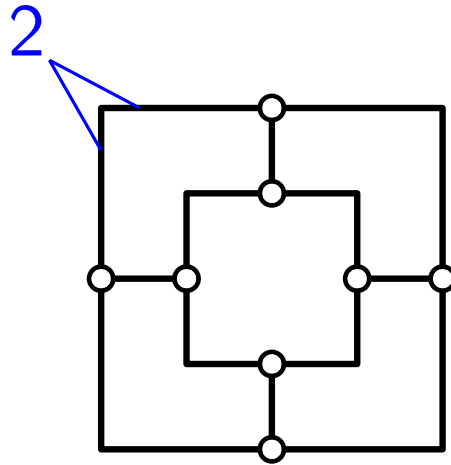


smooth orthogonal

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orthogonal

smooth orthogonal

Liu et al. Algorithm

biconnected 4-planar graph \rightarrow orthogonal complexity-3 layout

Liu et al. Algorithm

biconnected 4-planar graph \rightarrow orthogonal complexity-3 layout

- choose vertices s and t

Liu et al. Algorithm

biconnected 4-planar graph \rightarrow orthogonal complexity-3 layout

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- place vertices by their st -numbering

Liu et al. Algorithm

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- Invariant:
When we fix an endpoint of edge e , we associate e with a column.

Liu et al. Algorithm

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- Use ports in this order:
out: \uparrow \rightarrow \leftarrow
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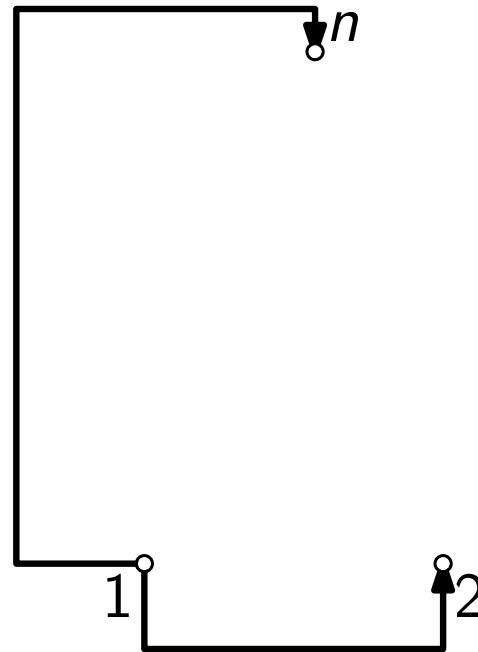
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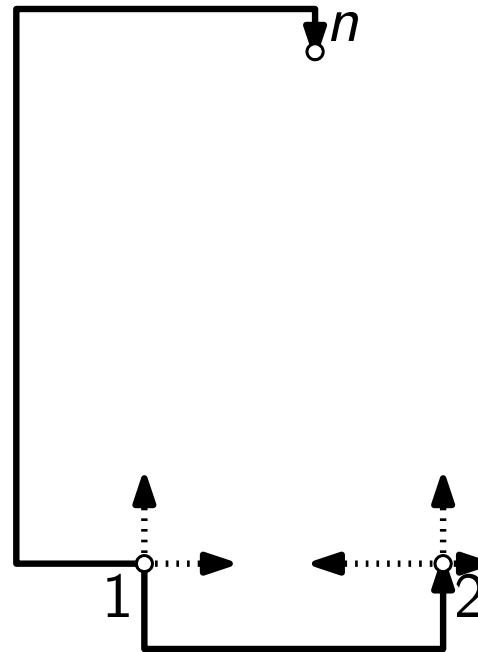
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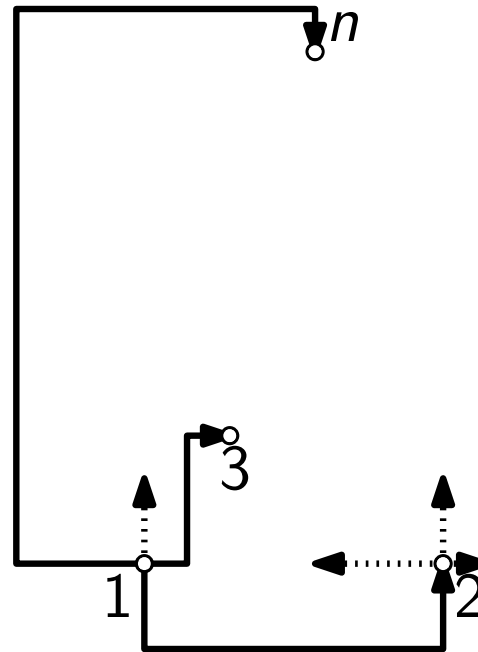


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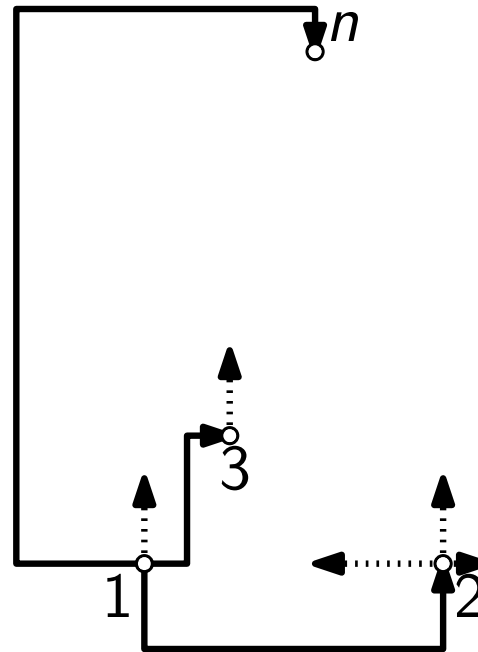
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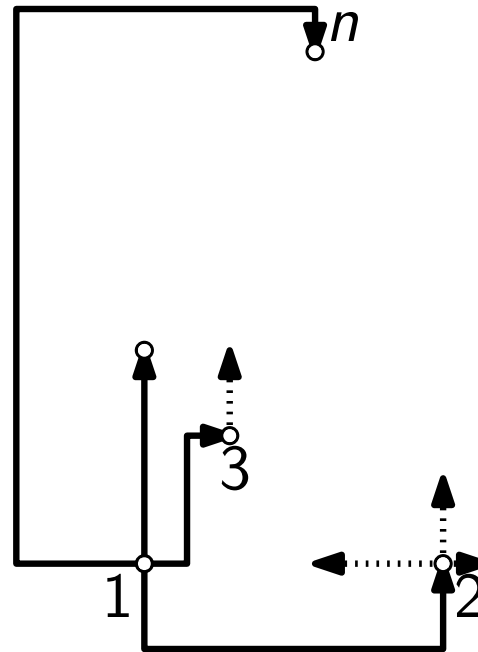


Liu et al. Algorithm

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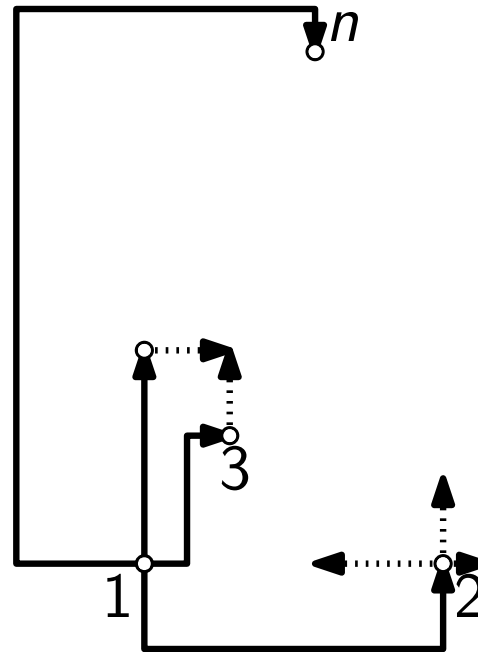


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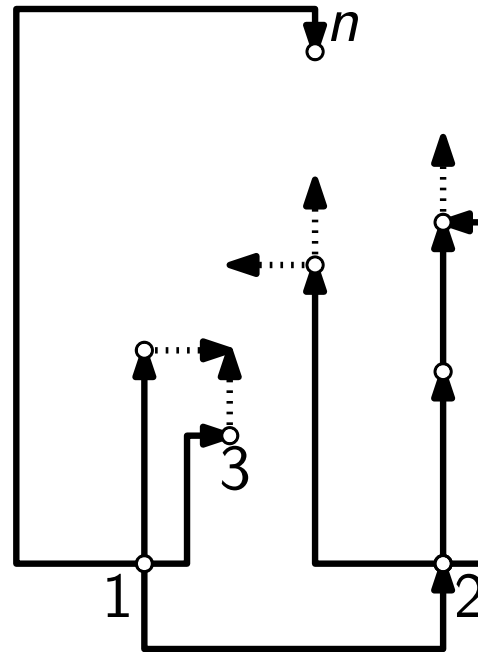


Liu et al. Algorithm

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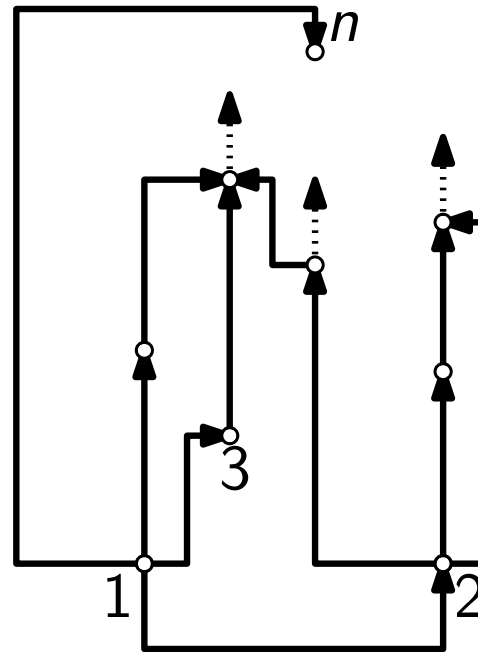


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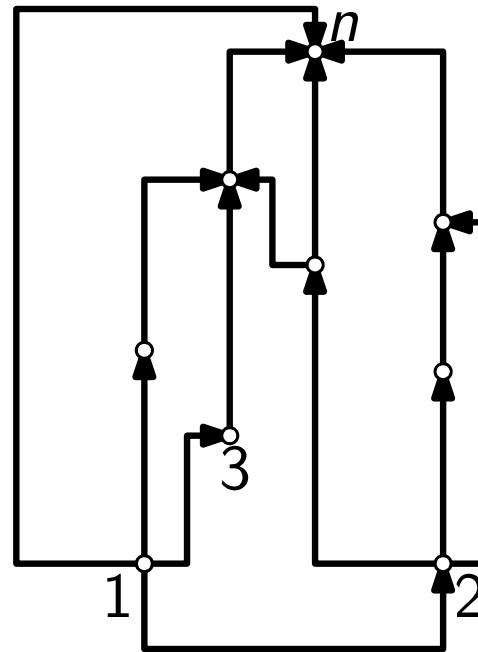


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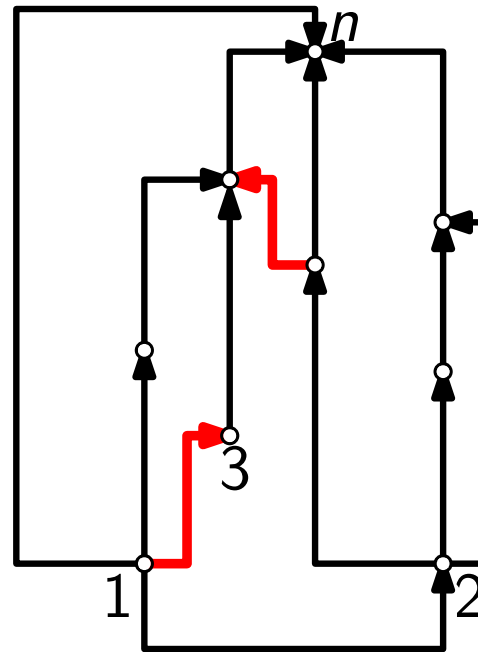
out:	\uparrow	\rightarrow	\leftarrow
in:	\downarrow	\leftarrow	\rightarrow



Liu et al. Algorithm

biconnected 4-planar graph \rightarrow orthogonal complexity-3 layout

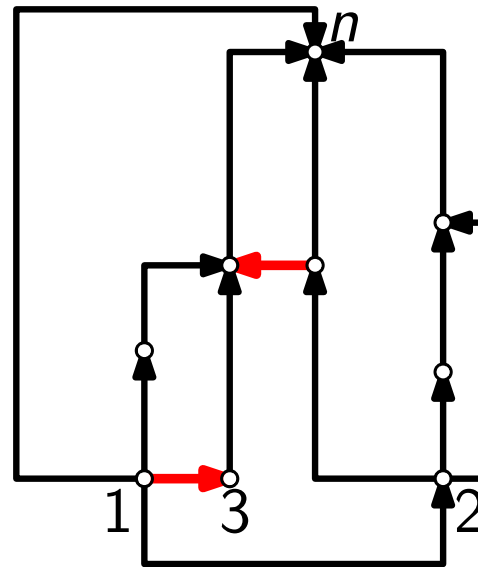
- choose vertices s and t
- place vertices by their st -numbering
- Invariant:
When we fix an endpoint of edge e , we associate e with a column.
- Use ports in this order:
out: \uparrow \rightarrow \leftarrow
in: \downarrow \leftarrow \rightarrow
- Eliminate S-shapes



Liu et al. Algorithm

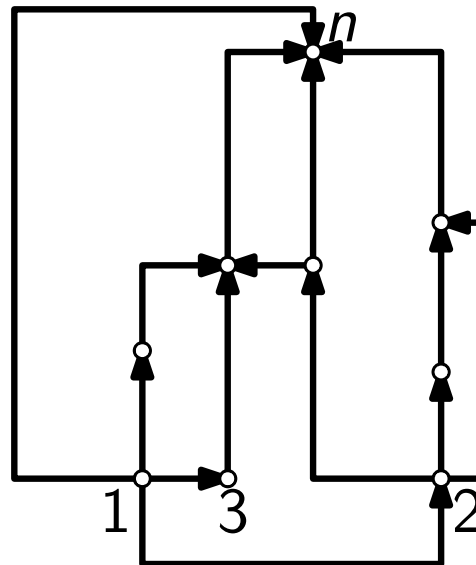
biconnected 4-planar graph \rightarrow orthogonal complexity-3 layout

- choose vertices s and t
- place vertices by their st -numbering
- Invariant:
When we fix an endpoint of edge e , we associate e with a column.
- Use ports in this order:
out: $\uparrow \quad \rightarrow \quad \leftarrow$
in: $\downarrow \quad \leftarrow \quad \rightarrow$
- Eliminate S-shapes



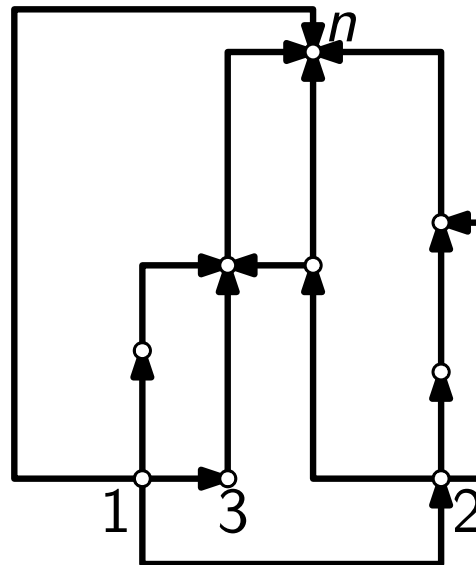
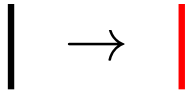
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout



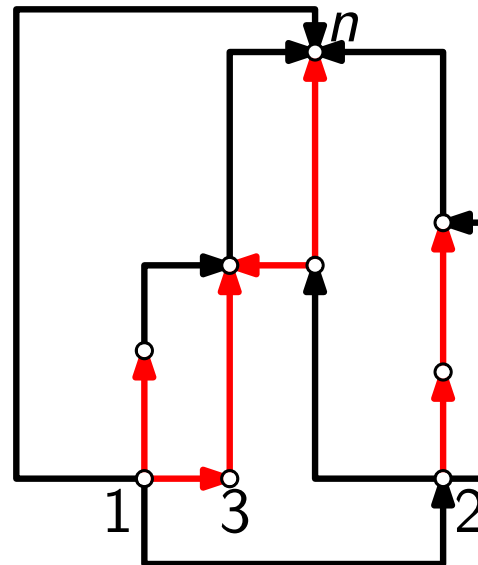
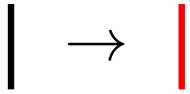
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout



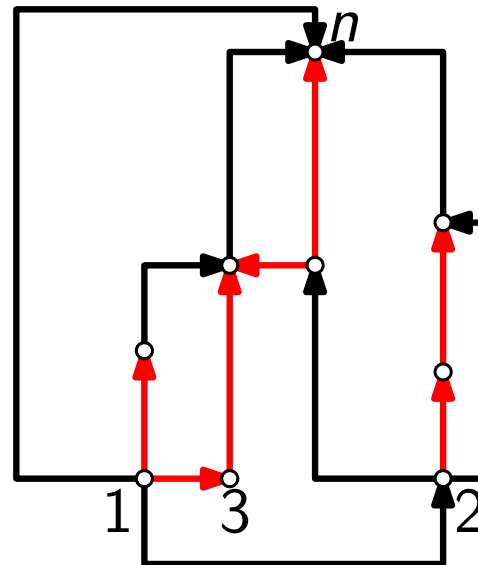
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout



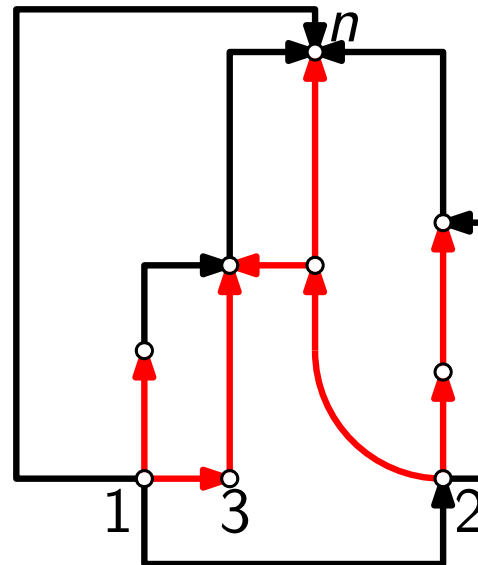
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout



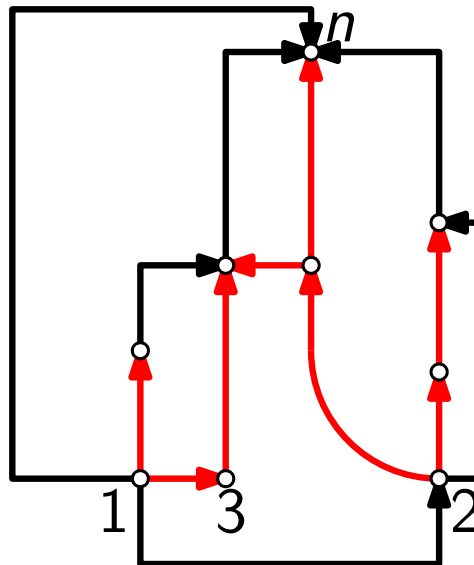
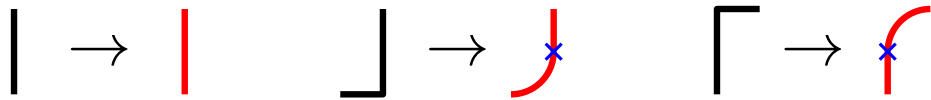
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout



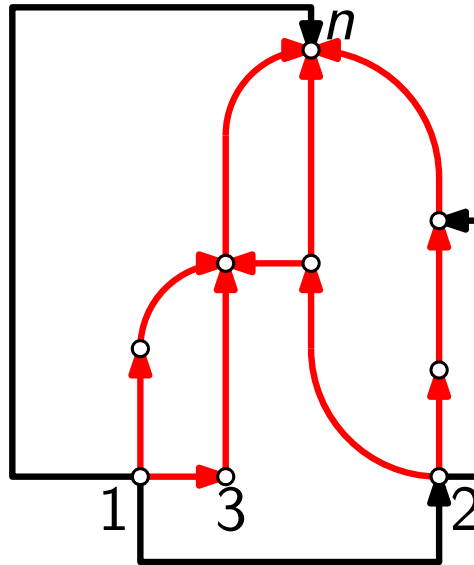
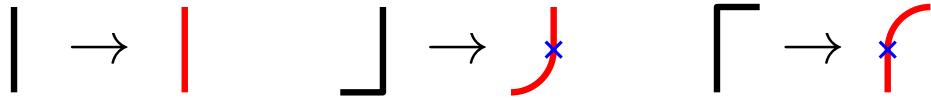
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout



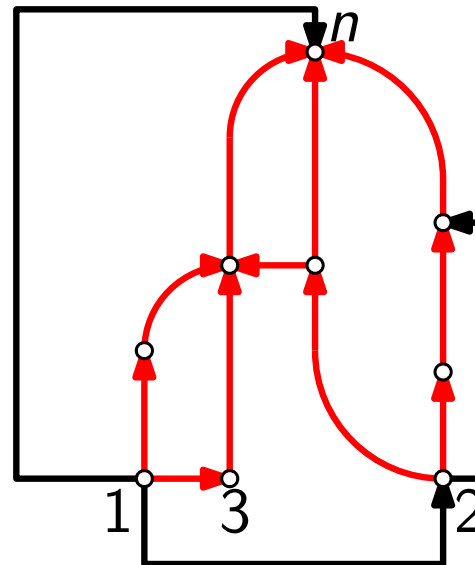
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout



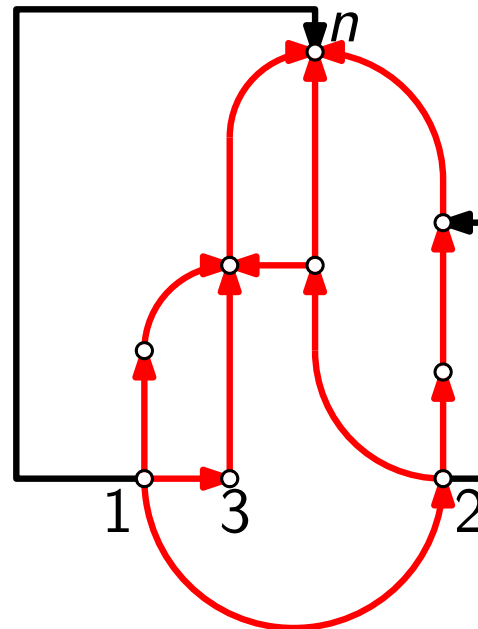
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout



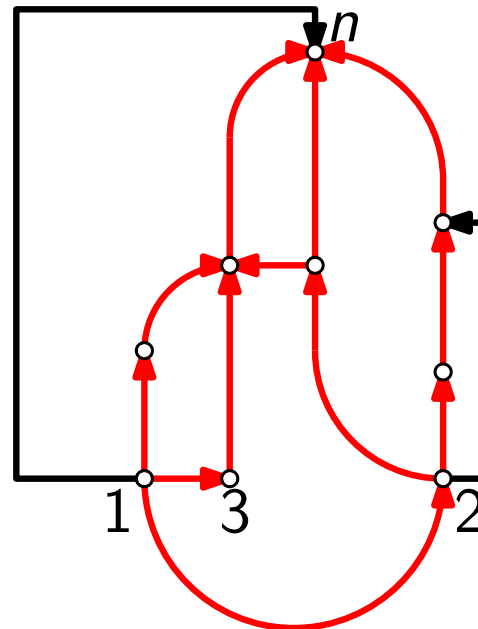
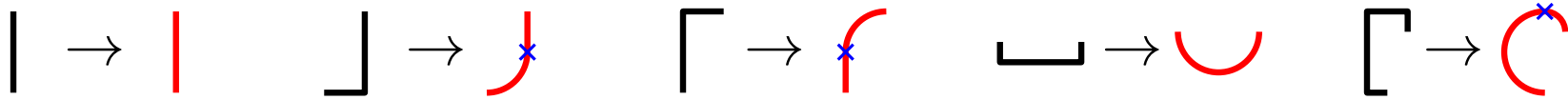
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout



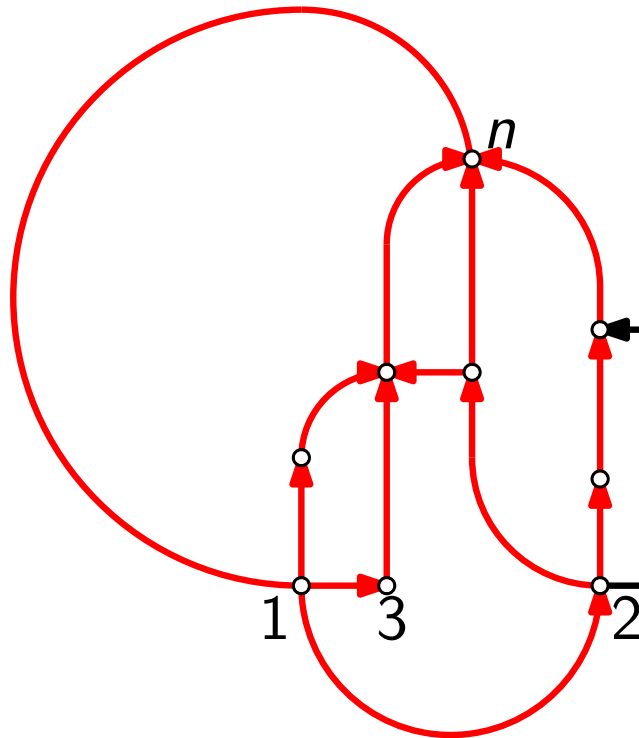
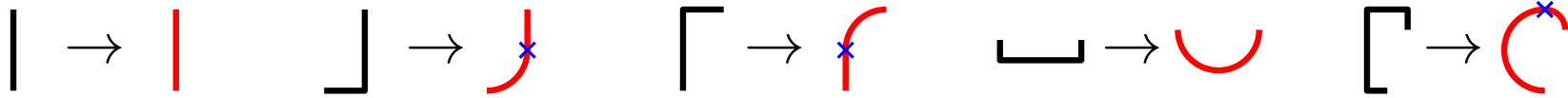
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout



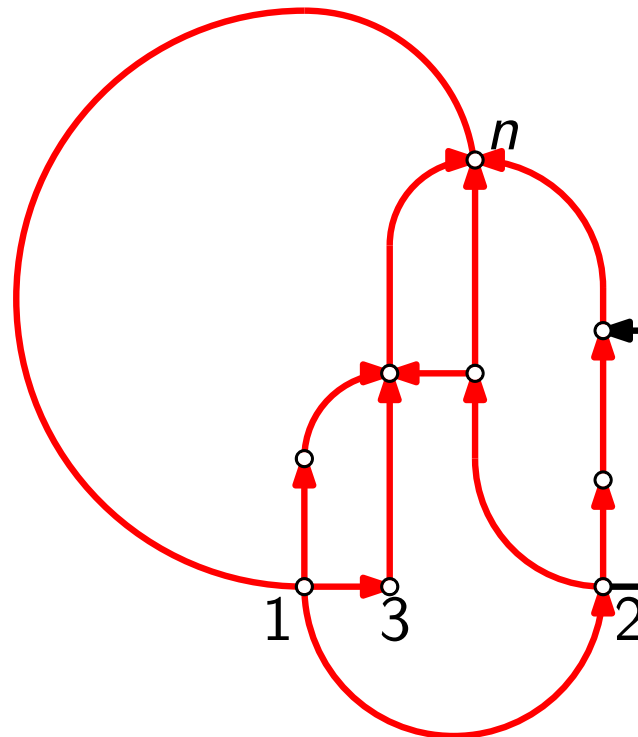
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout



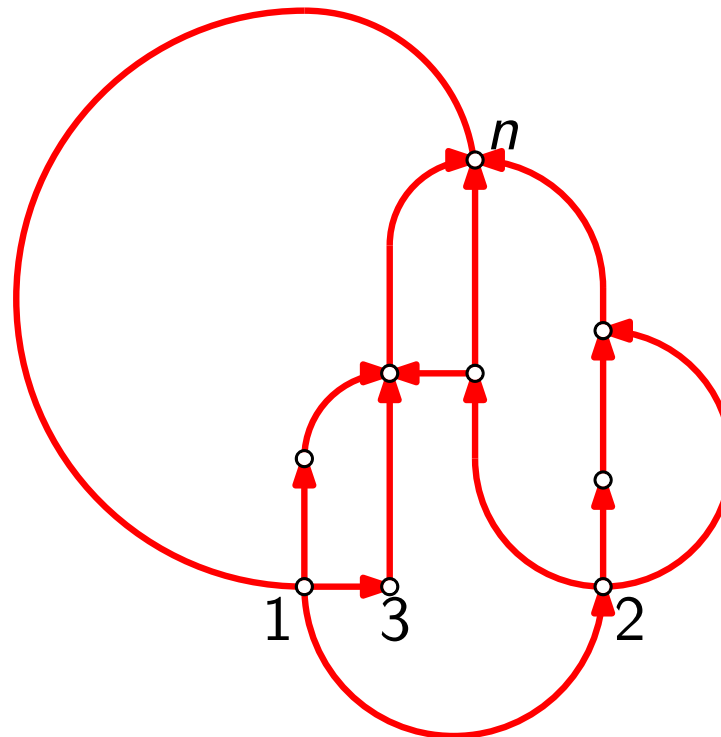
SC₂-Layouts

biconnected 4-planar graph \rightarrow ^{smooth} orthogonal complexity-2 layout

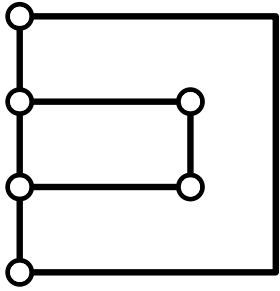


SC₂-Layouts

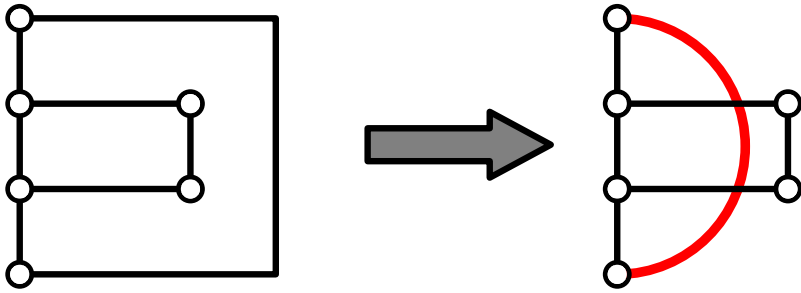
biconnected 4-planar graph $\xrightarrow{\text{smooth}}$ orthogonal complexity-2 layout



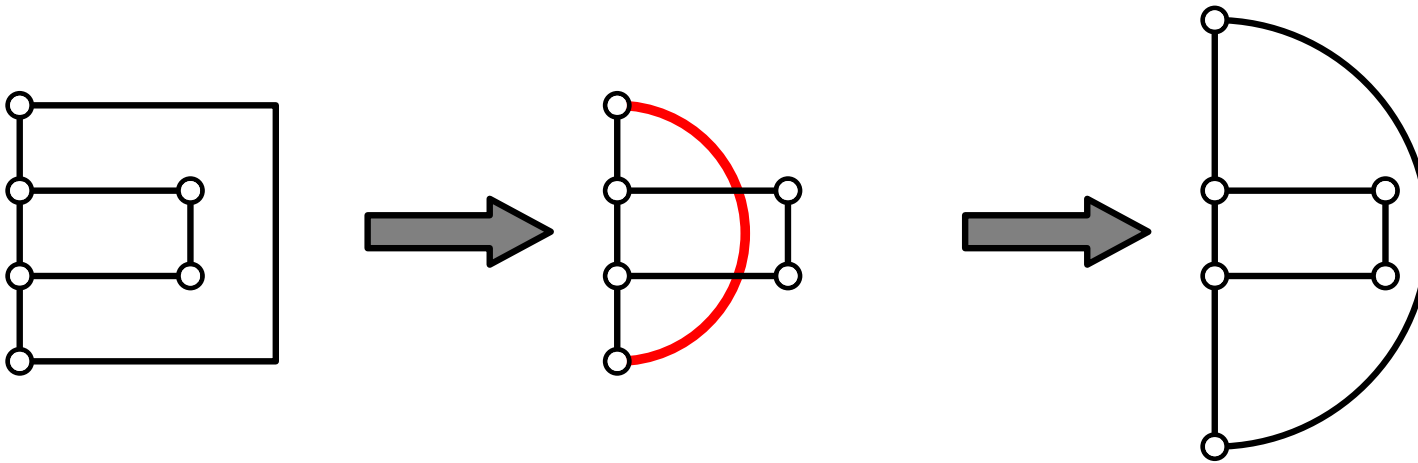
Crossings



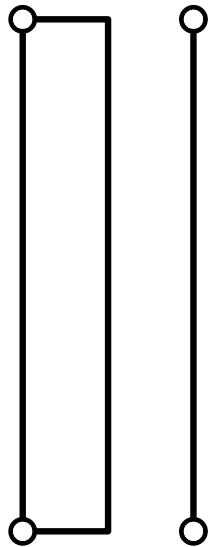
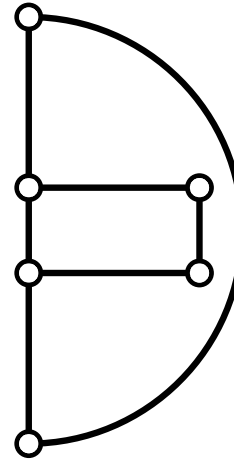
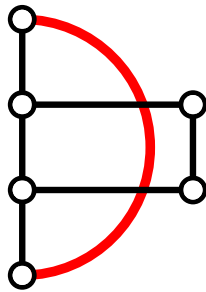
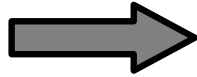
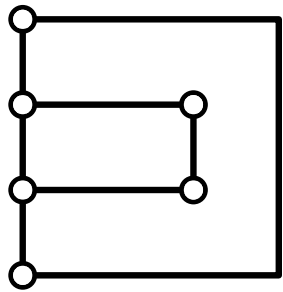
Crossings



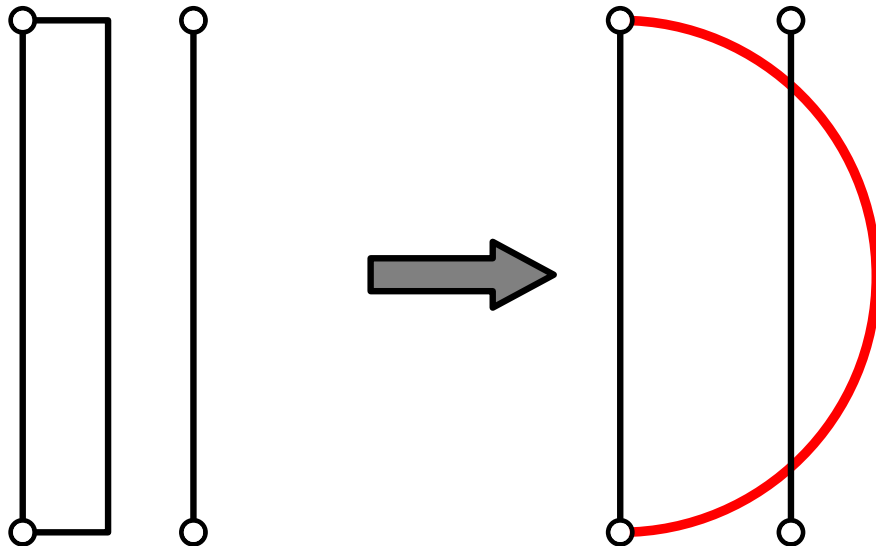
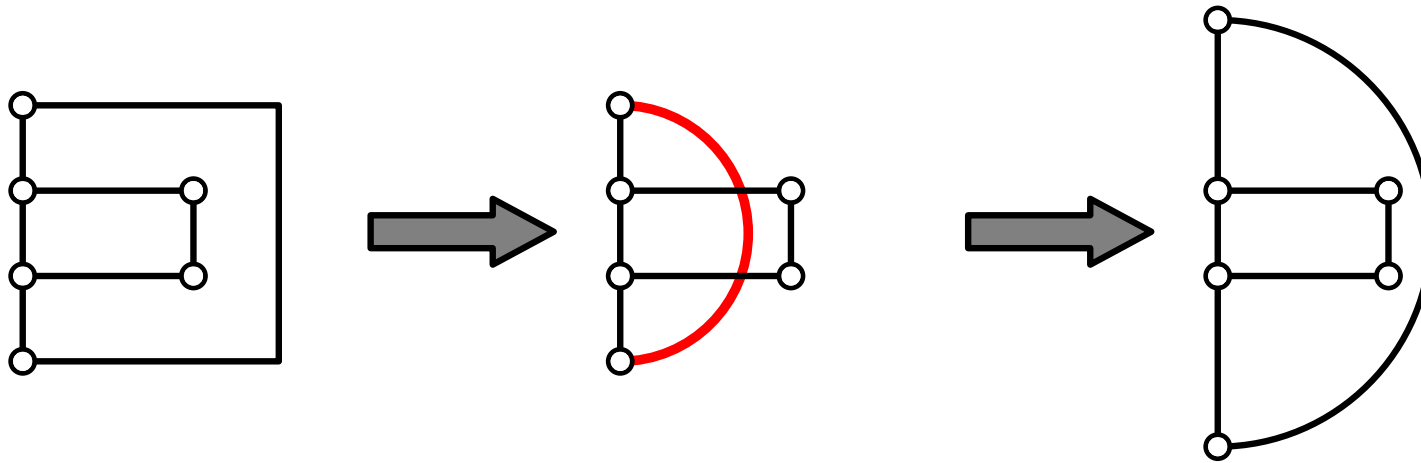
Crossings



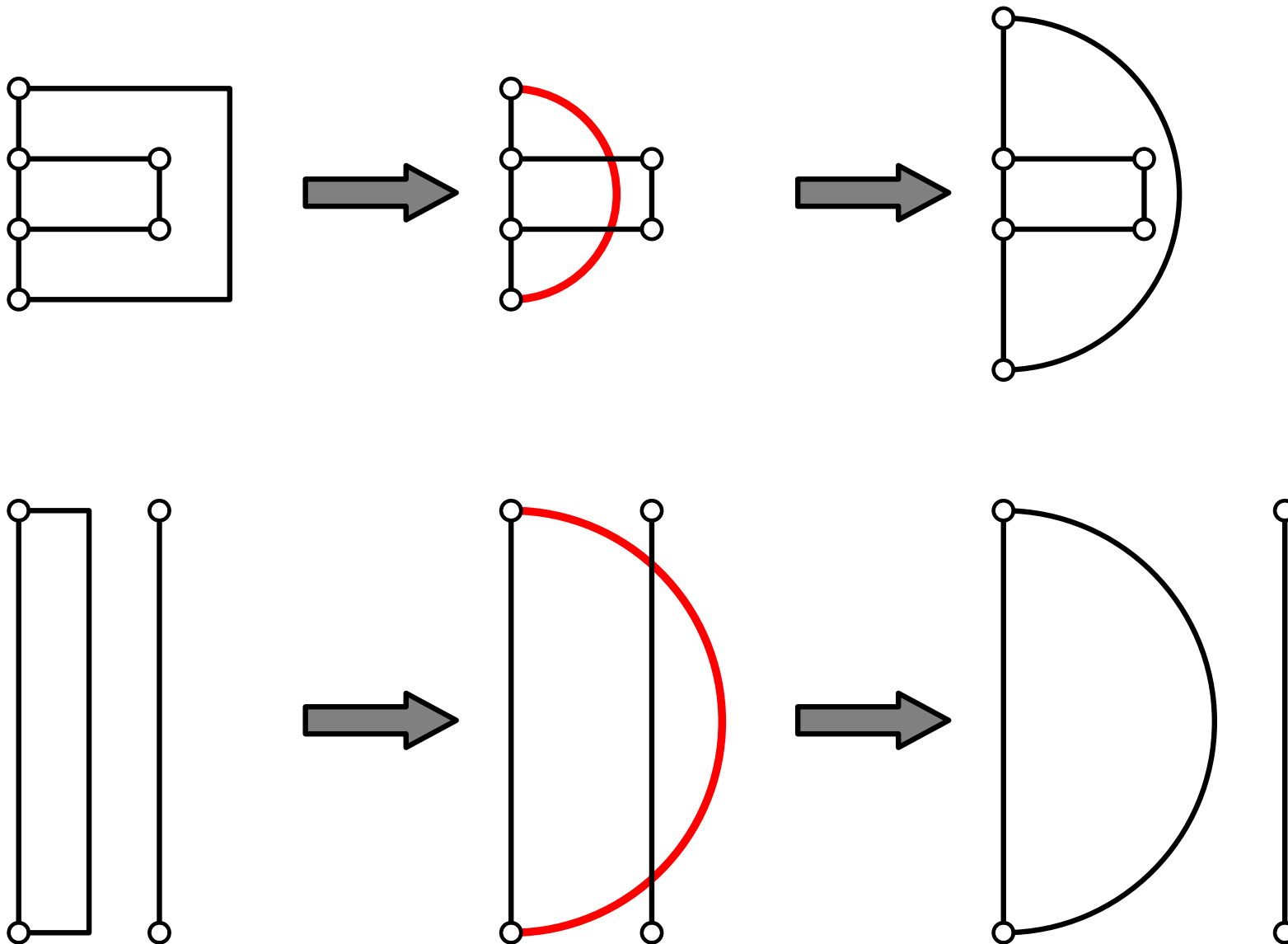
Crossings



Crossings



Crossings

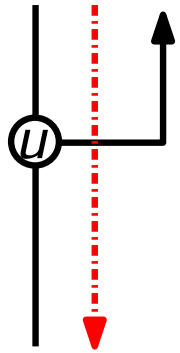


Cut

Def.

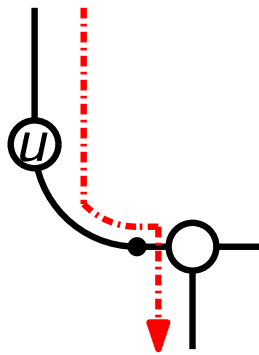
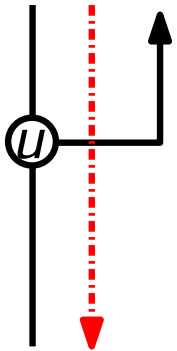
Cut

Def. • y -monotone curve



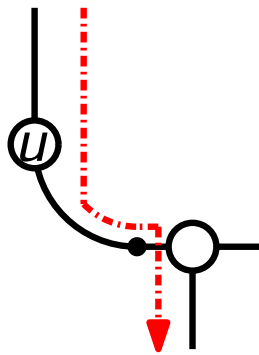
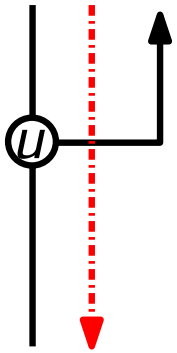
Cut

- Def.
- y -monotone curve
 - consists of horizontal, vertical and circular segments



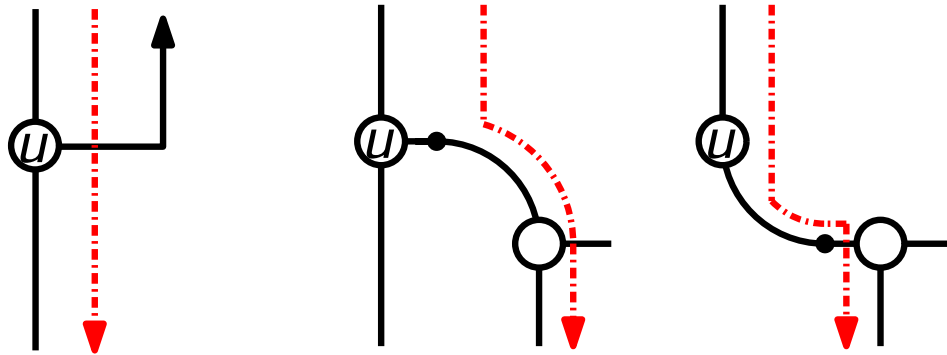
Cut

- Def.
- y -monotone curve
 - consists of horizontal, vertical and circular segments
 - divides the current drawing into a left and a right part



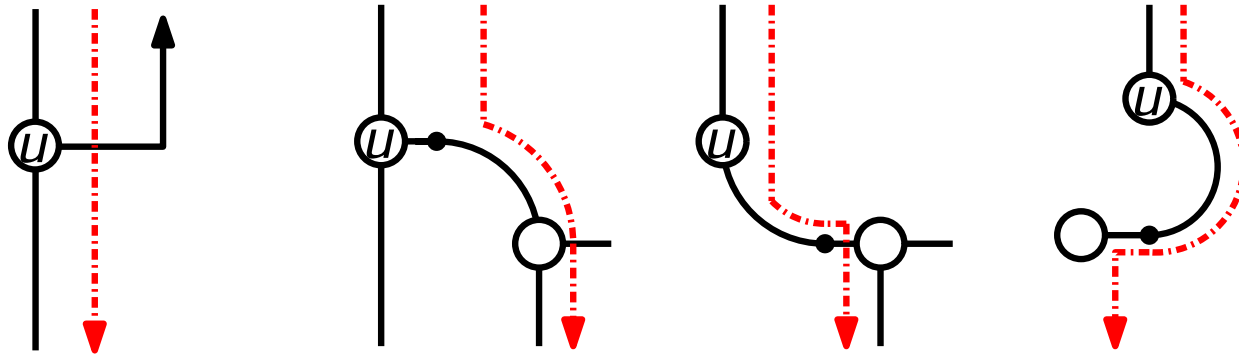
Cut

- Def.
- y-monotone curve
 - consists of horizontal, vertical and circular segments
 - divides the current drawing into a left and a right part
 - intersects only horizontal segments



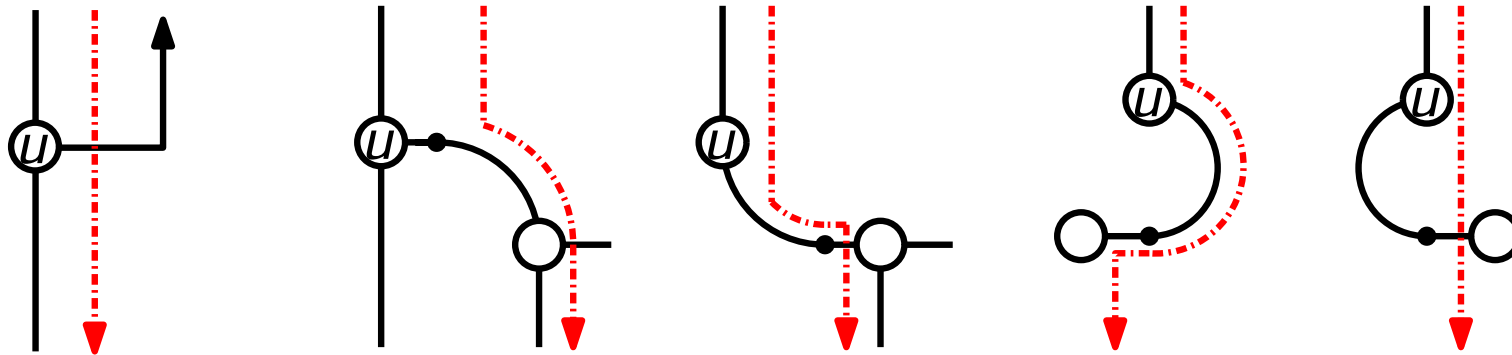
Cut

- Def.
- y-monotone curve
 - consists of horizontal, vertical and circular segments
 - divides the current drawing into a left and a right part
 - intersects only horizontal segments



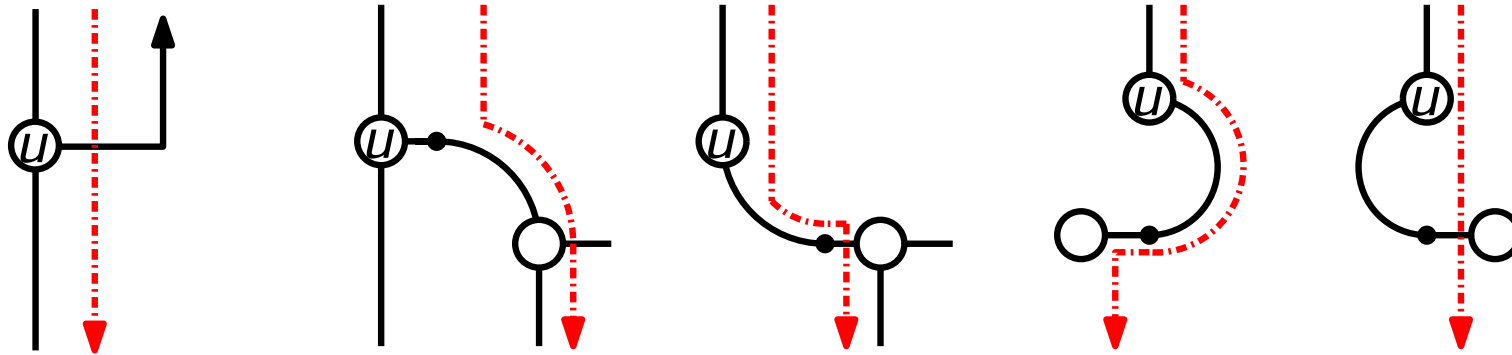
Cut

- Def.
- y -monotone curve
 - consists of horizontal, vertical and circular segments
 - divides the current drawing into a left and a right part
 - intersects only horizontal segments



Cut

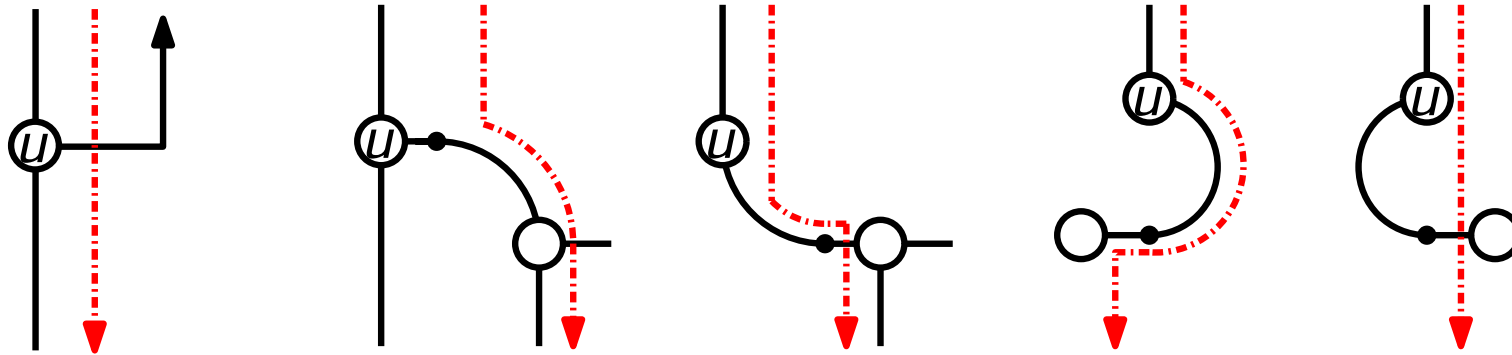
- Def.
- y-monotone curve
 - consists of horizontal, vertical and circular segments
 - divides the current drawing into a left and a right part
 - intersects only horizontal segments



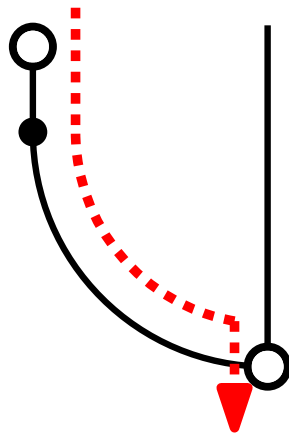
Problems:

Cut

- Def.
- y-monotone curve
 - consists of horizontal, vertical and circular segments
 - divides the current drawing into a left and a right part
 - intersects only horizontal segments

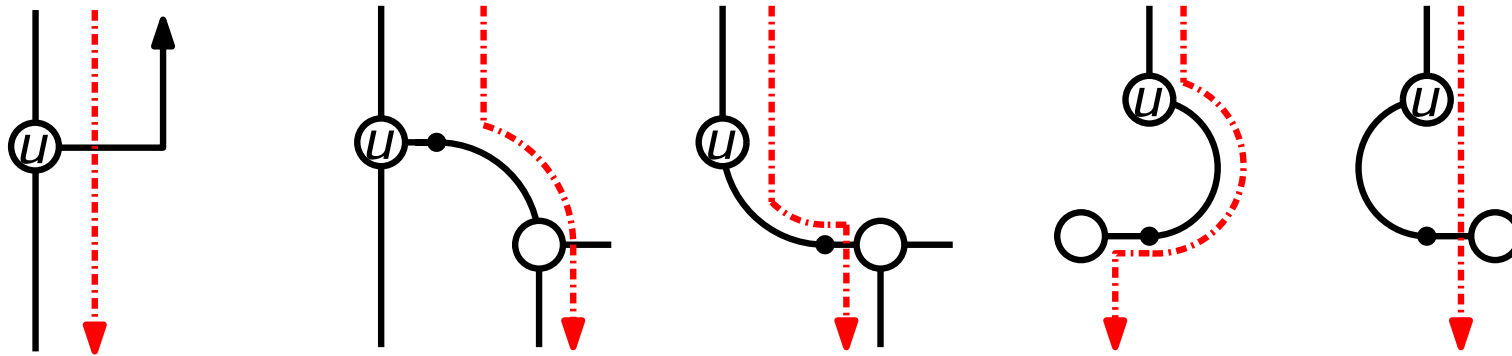


Problems:

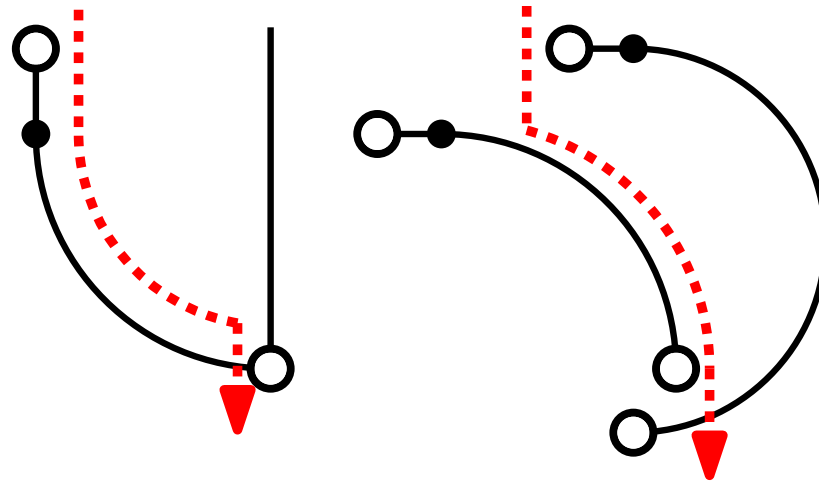


Cut

- Def.
- y-monotone curve
 - consists of horizontal, vertical and circular segments
 - divides the current drawing into a left and a right part
 - intersects only horizontal segments

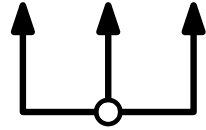


Problems:



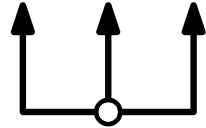
Invariants

(I_1) Every open edge is associated with a column

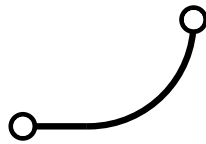


Invariants

(I_1) Every open edge is associated with a column

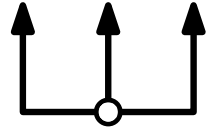


(I_2) An L-shape always contains a horizontal segment;
it never contains a vertical segment.

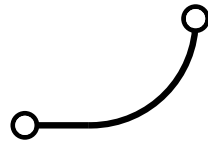


Invariants

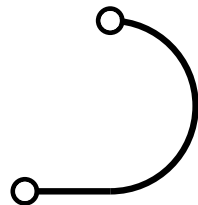
(I_1) Every open edge is associated with a column



(I_2) An L-shape always contains a horizontal segment; it never contains a vertical segment.

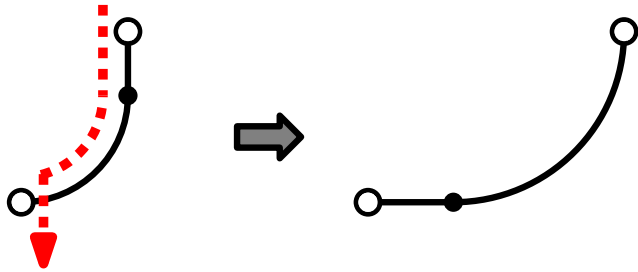


(I_3) A C-shape always has a horizontal segment incident to its bottom vertex.



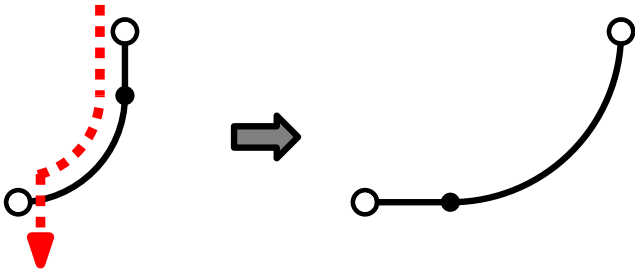
Maintain invariants

L-shape

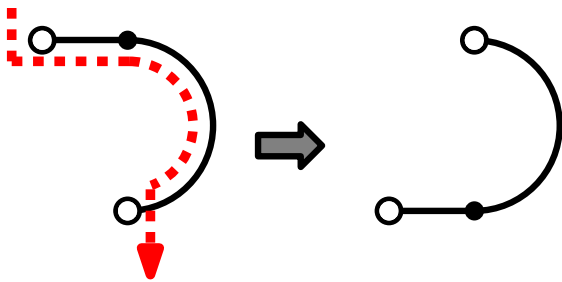


Maintain invariants

L-shape

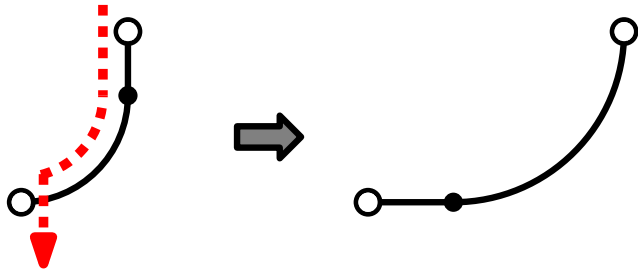


C-shape

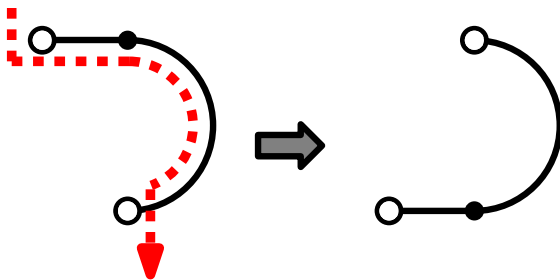


Maintain invariants

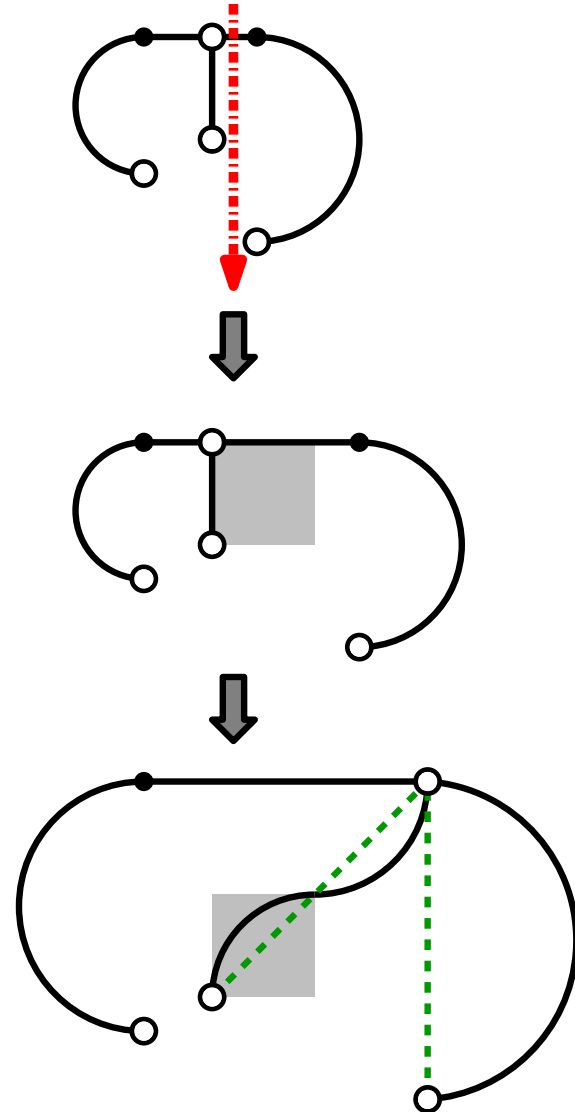
L-shape



C-shape

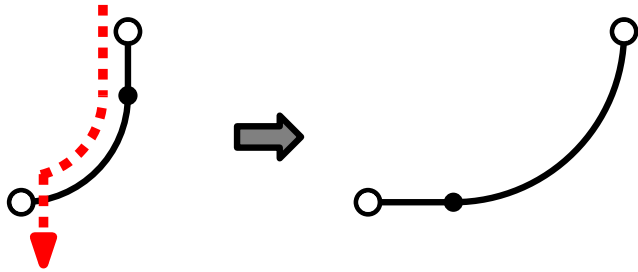


Double C-shape

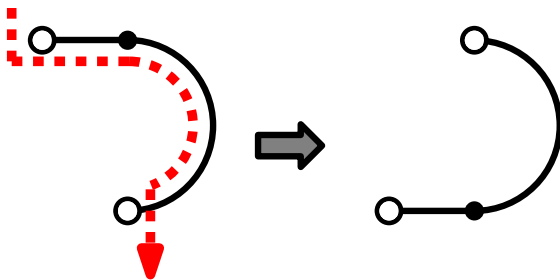


Maintain invariants

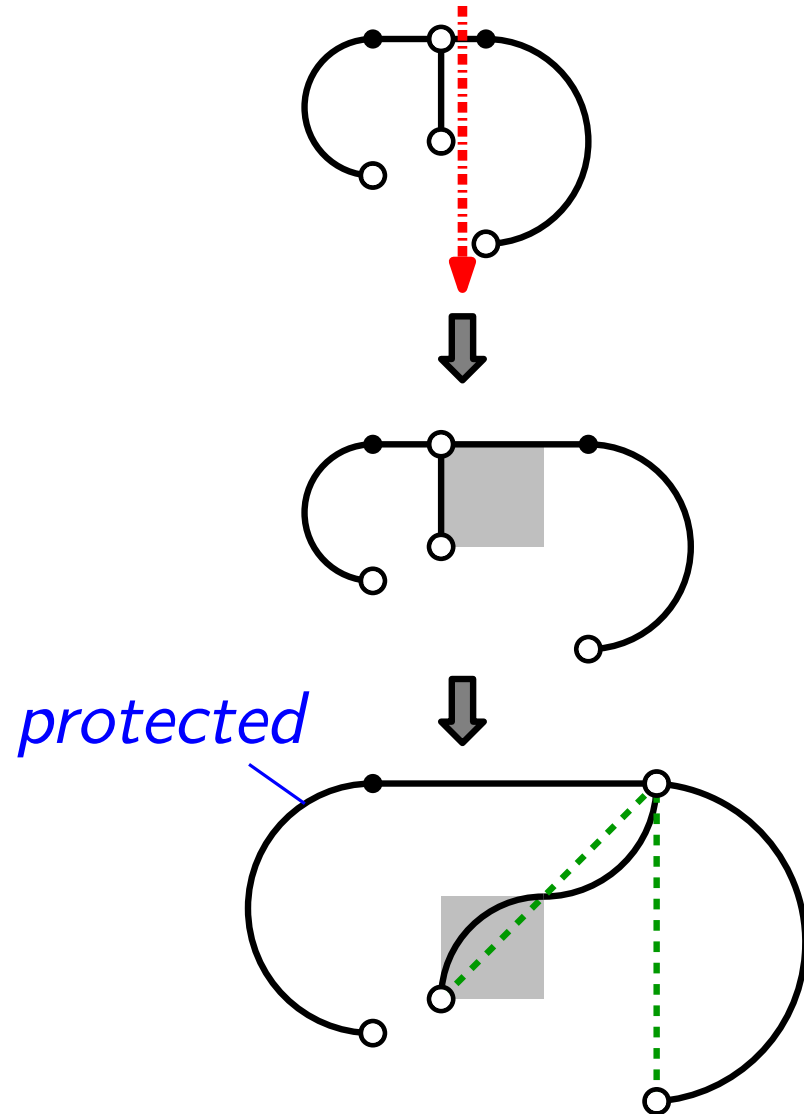
L-shape



C-shape

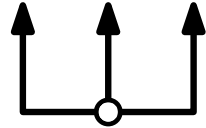


Double C-shape

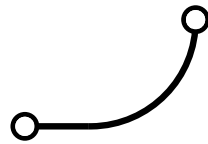


Invariants, updated

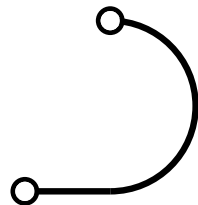
(I_1) Every open edge is associated with a column



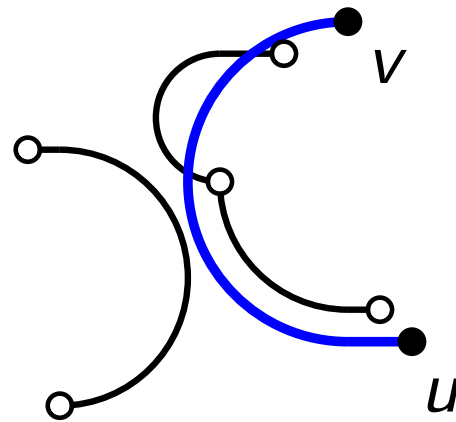
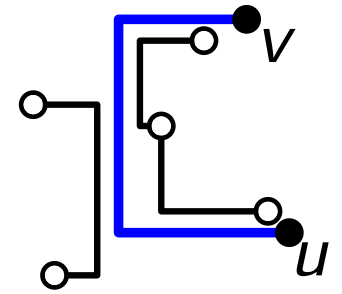
(I_2) An L-shape always contains a horizontal segment;
it never contains a vertical segment.



(I_3) An *unprotected* C-shape always has a horizontal segment
incident to its bottom vertex.

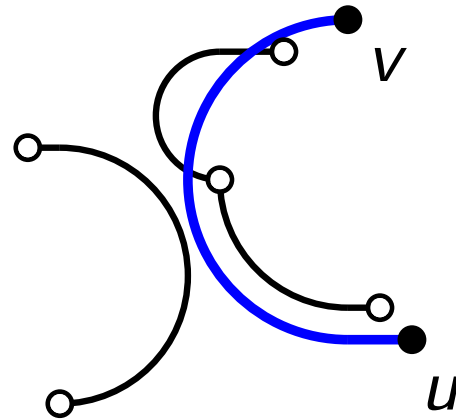
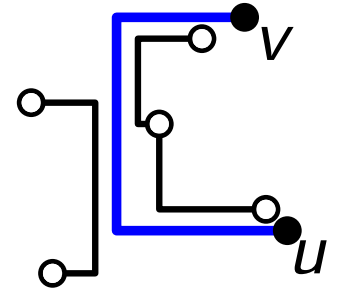


Eliminate crossings



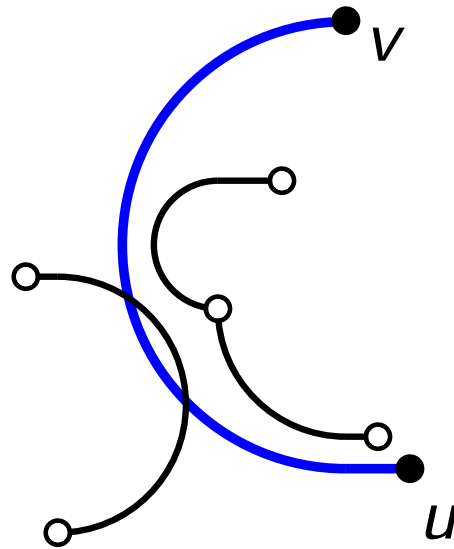
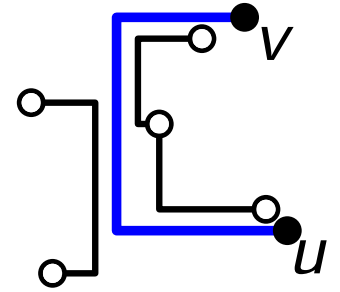
Eliminate crossings

1. move v up



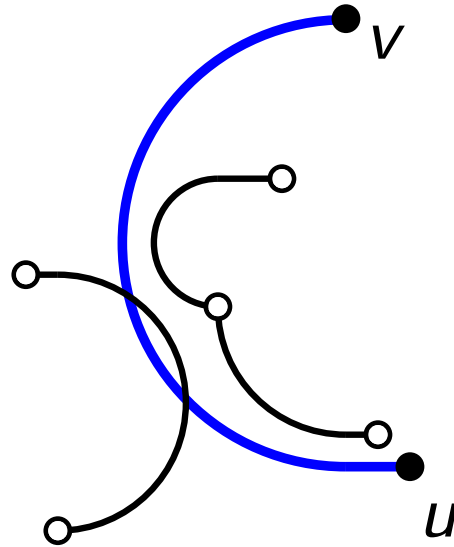
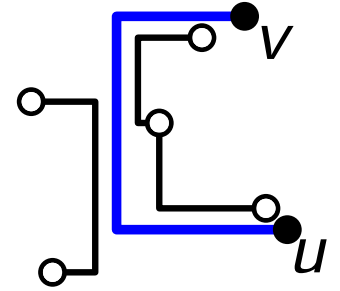
Eliminate crossings

1. move v up



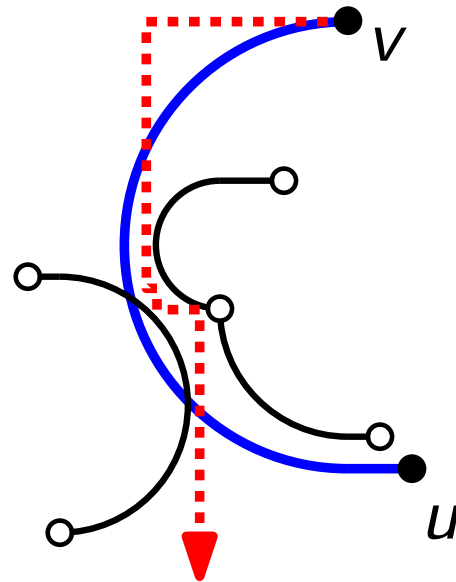
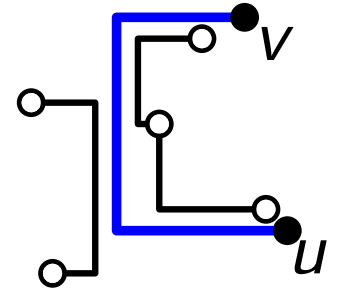
Eliminate crossings

1. move v up
2. find a cut



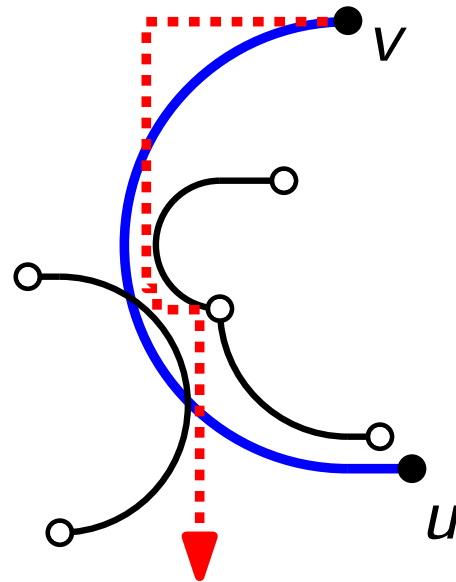
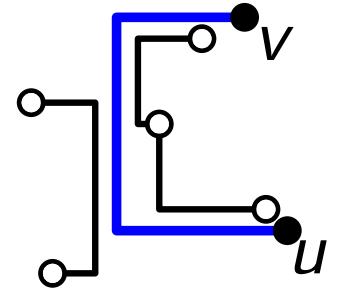
Eliminate crossings

1. move v up
2. find a cut



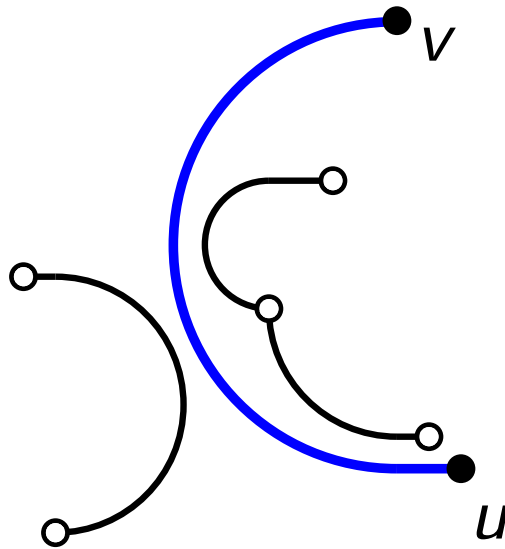
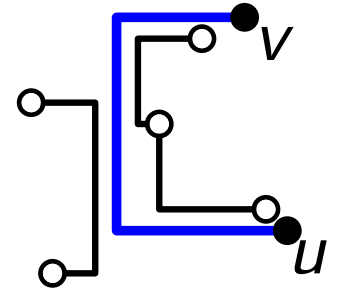
Eliminate crossings

1. move v up
2. find a cut
3. move vertices to the left

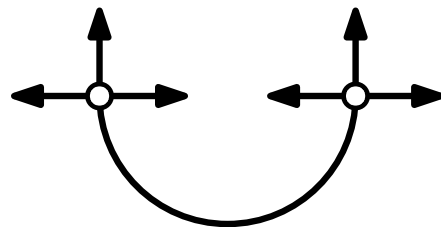
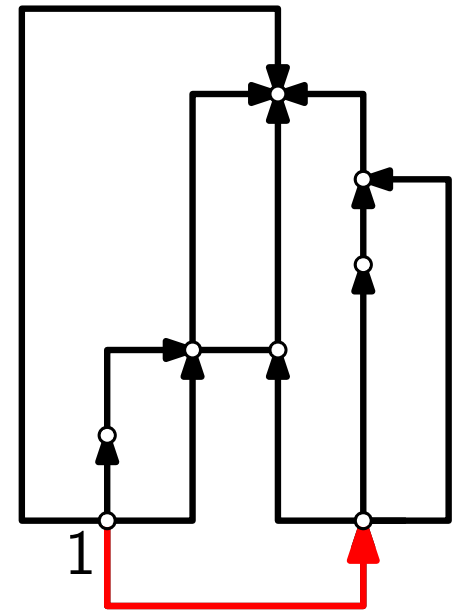


Eliminate crossings

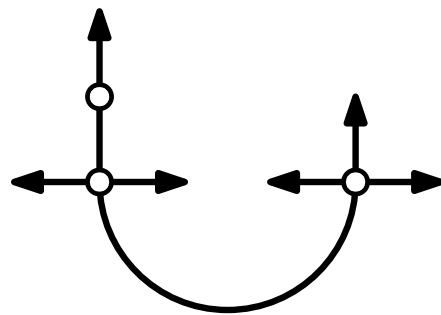
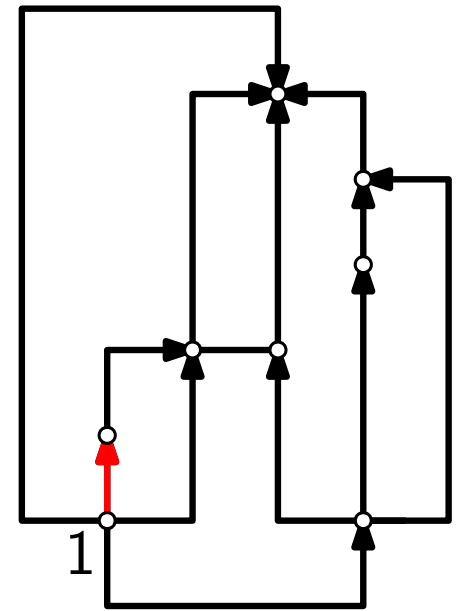
1. move v up
2. find a cut
3. move vertices to the left



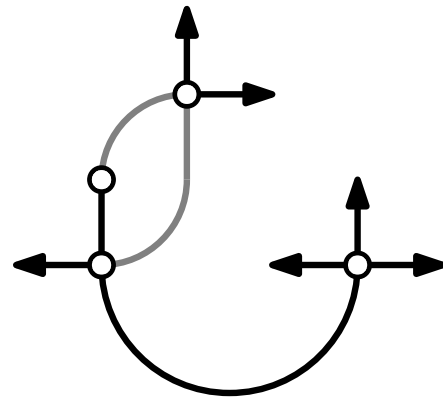
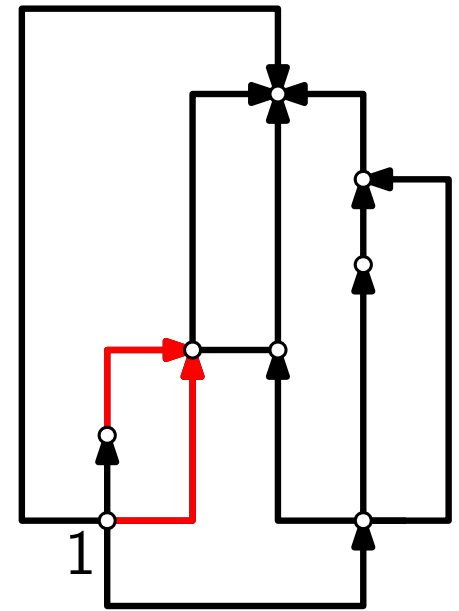
Example Run



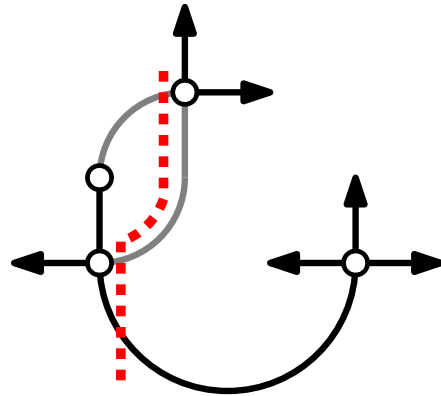
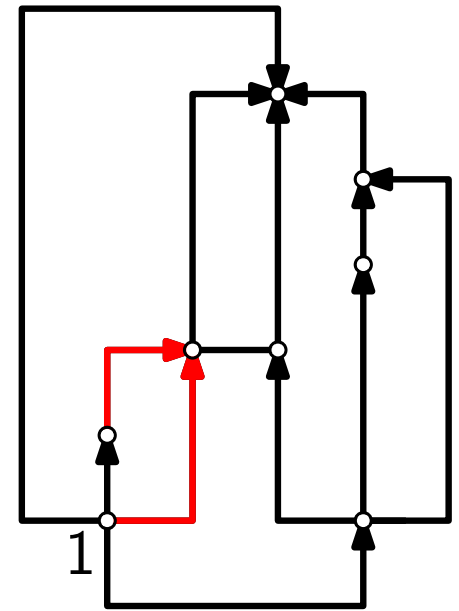
Example Run



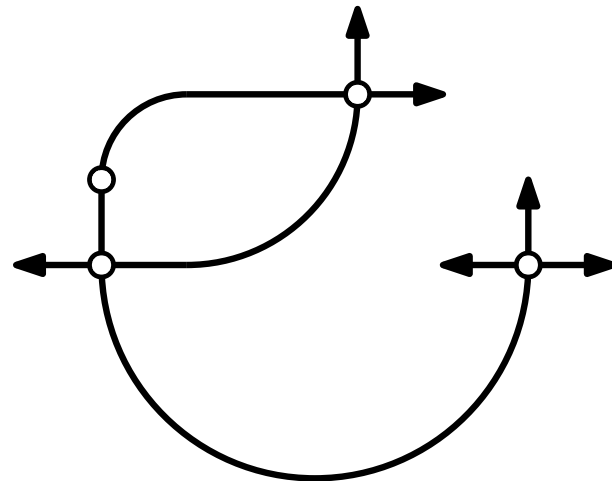
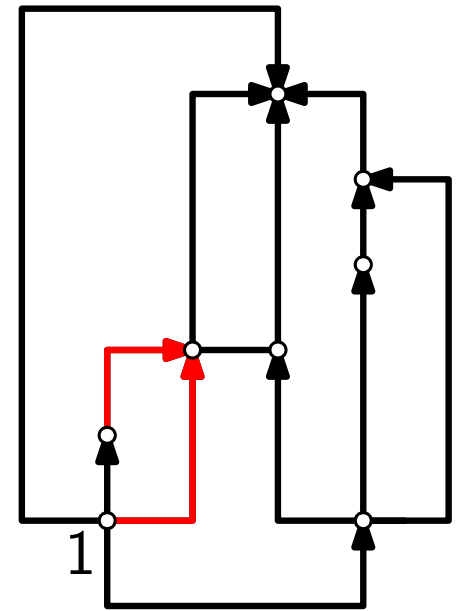
Example Run



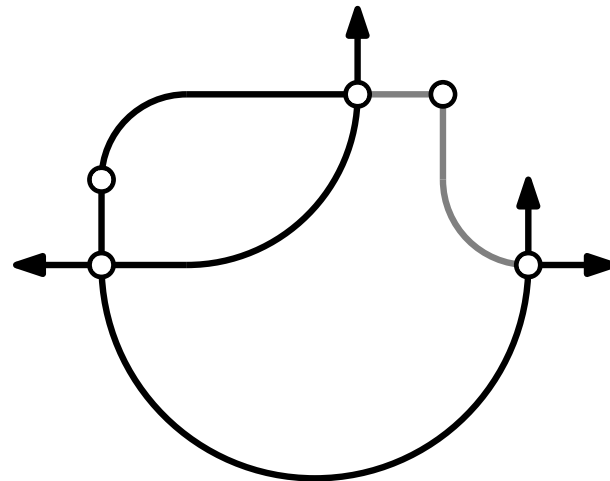
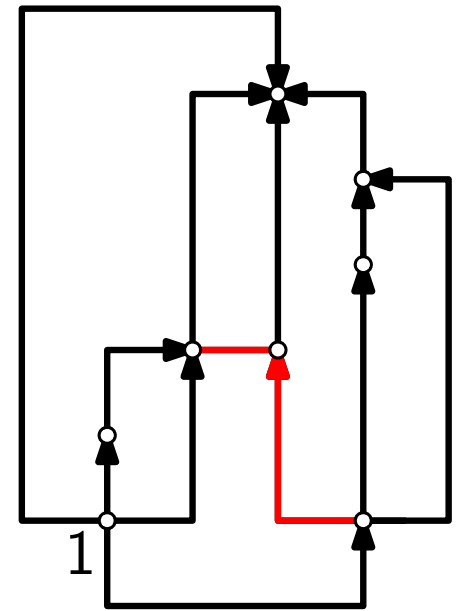
Example Run



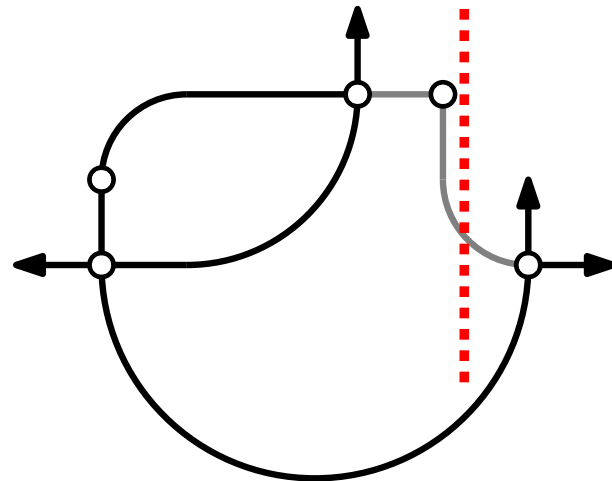
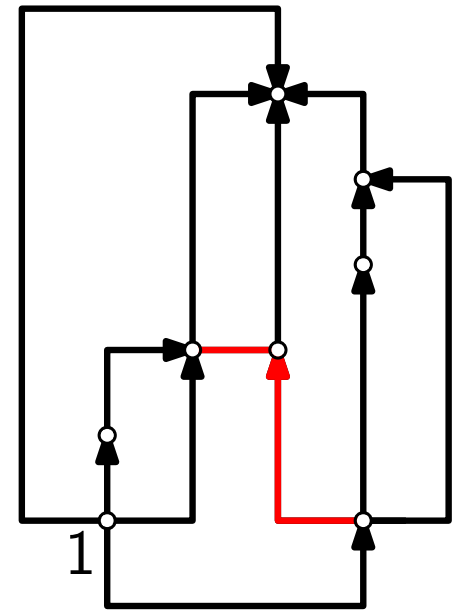
Example Run



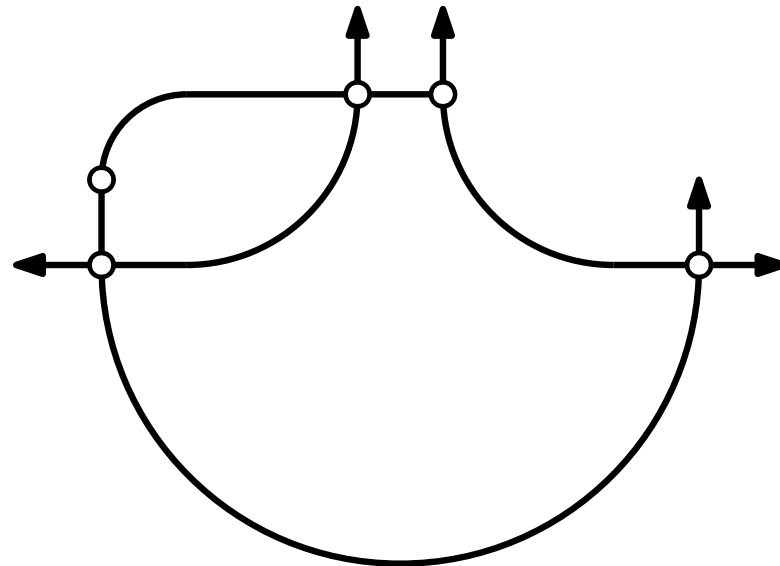
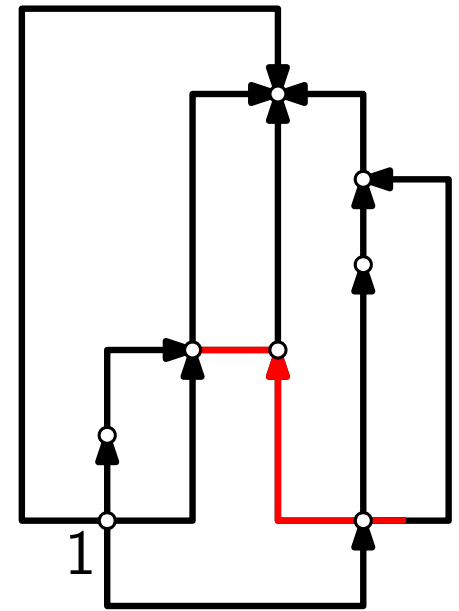
Example Run



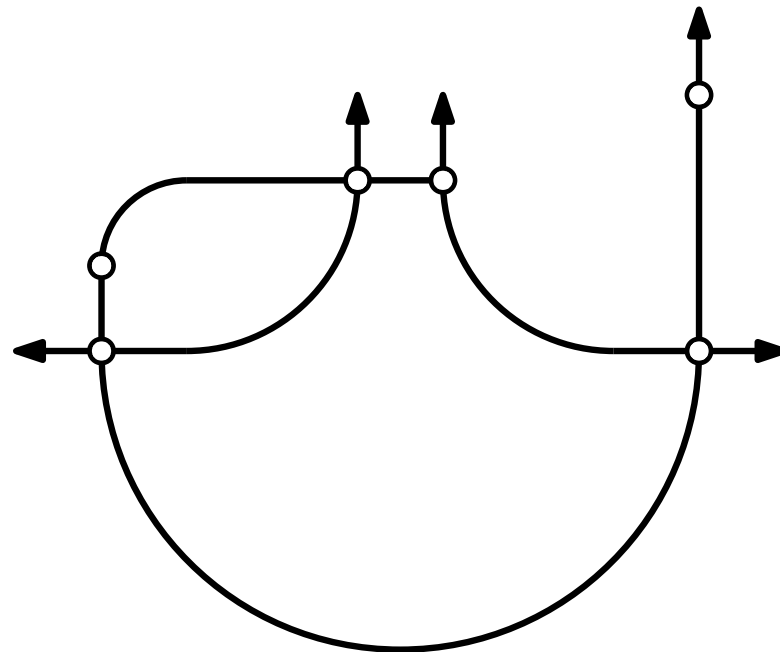
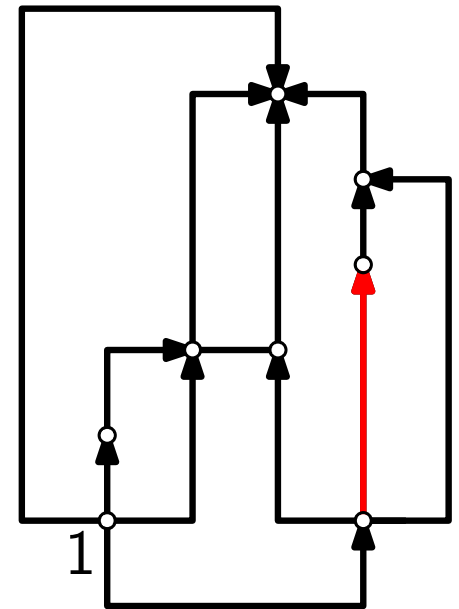
Example Run



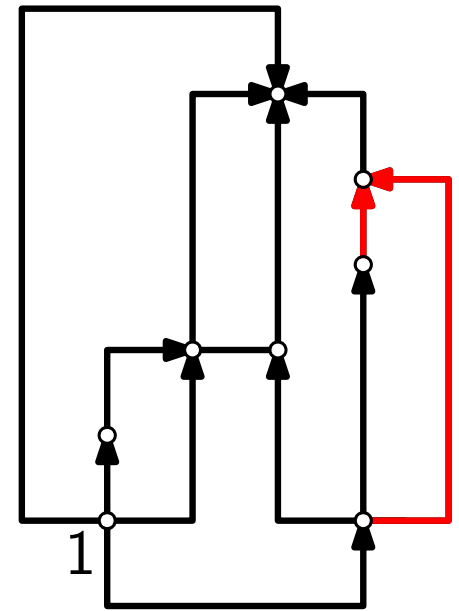
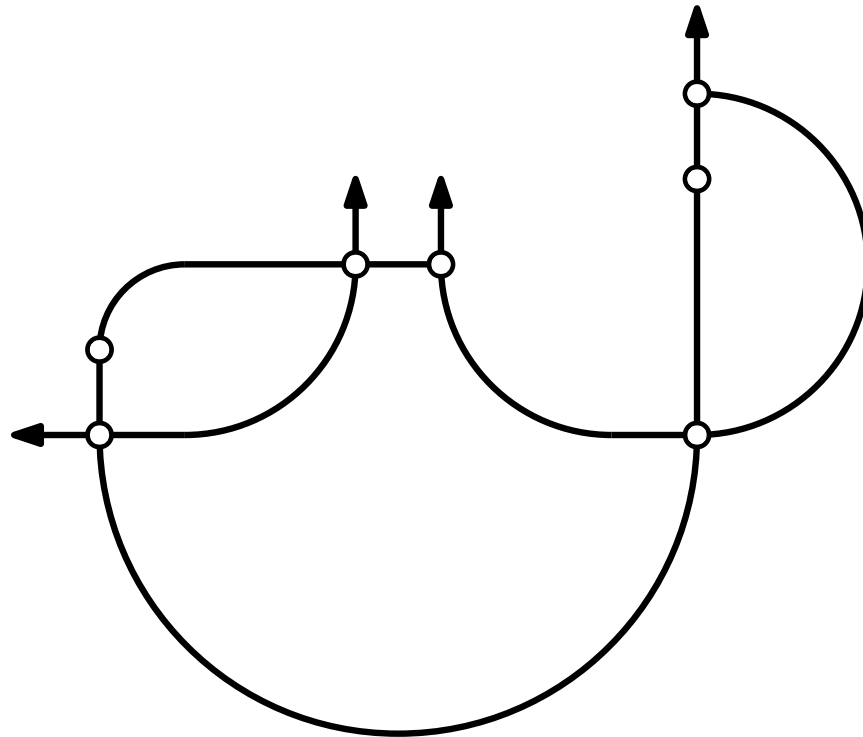
Example Run



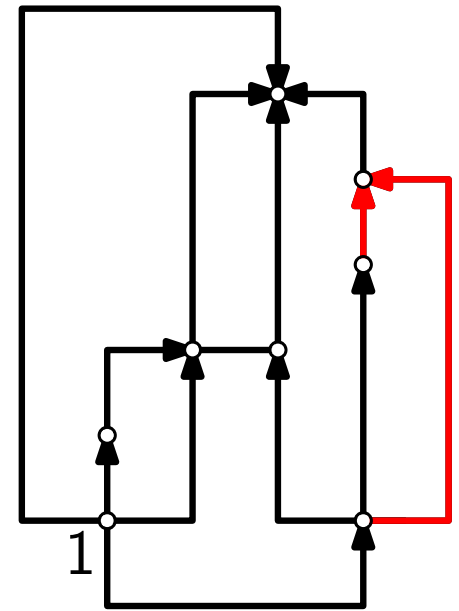
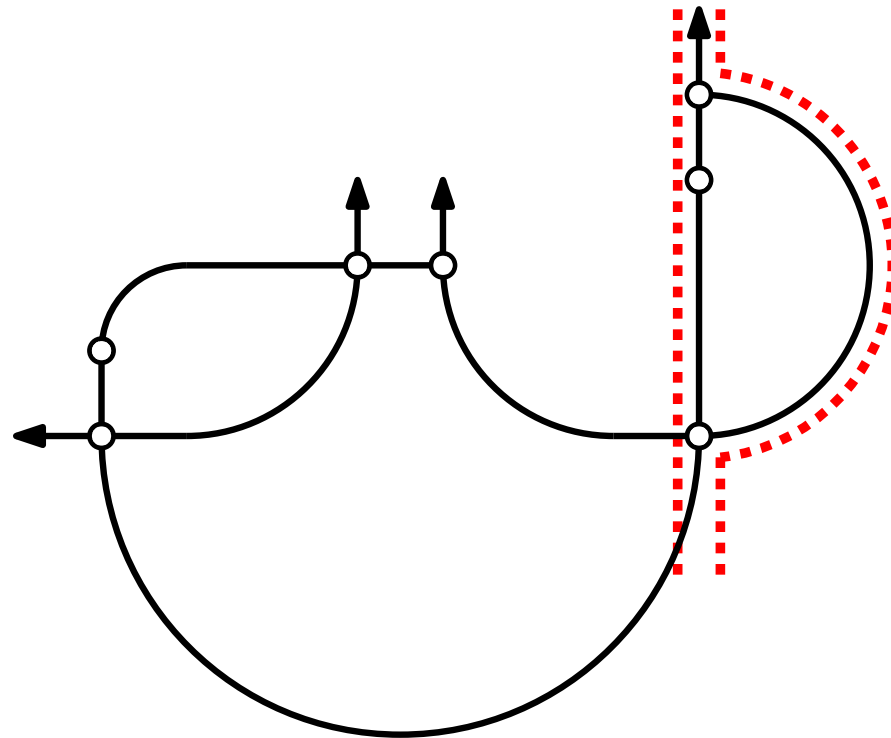
Example Run



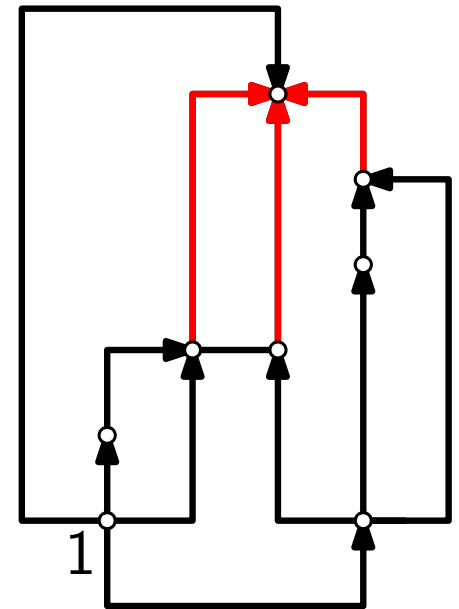
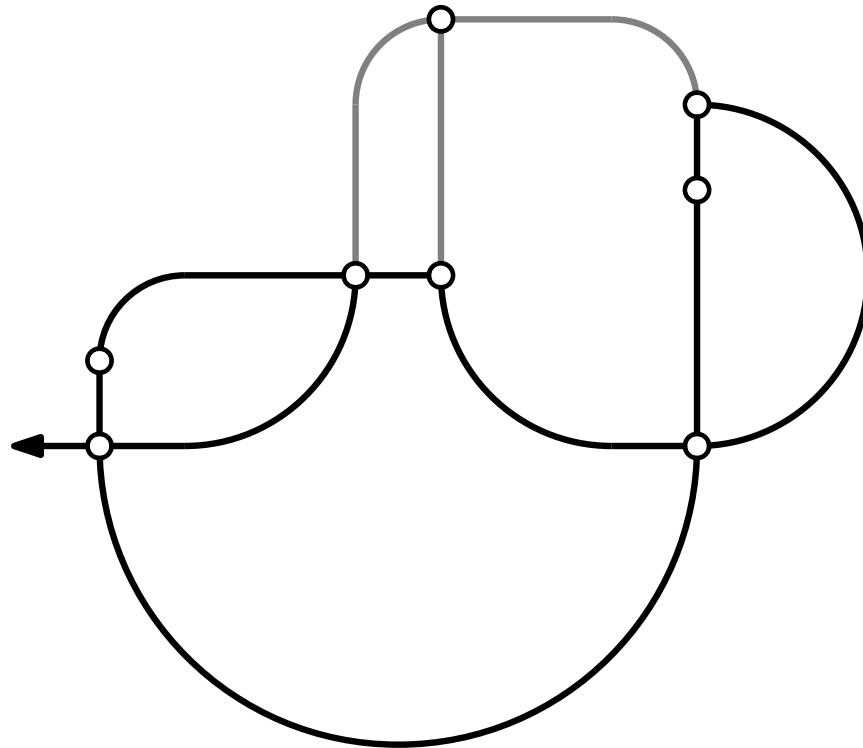
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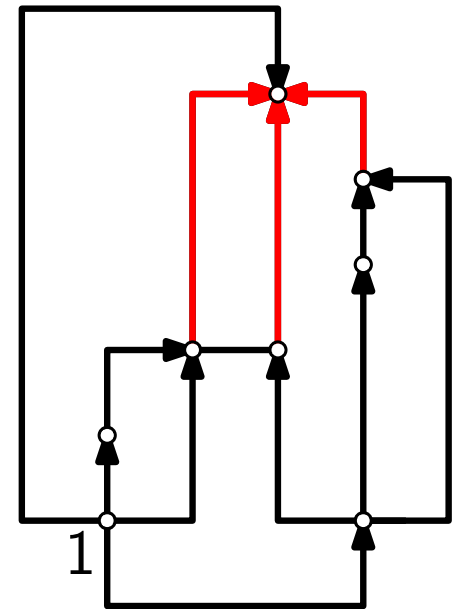
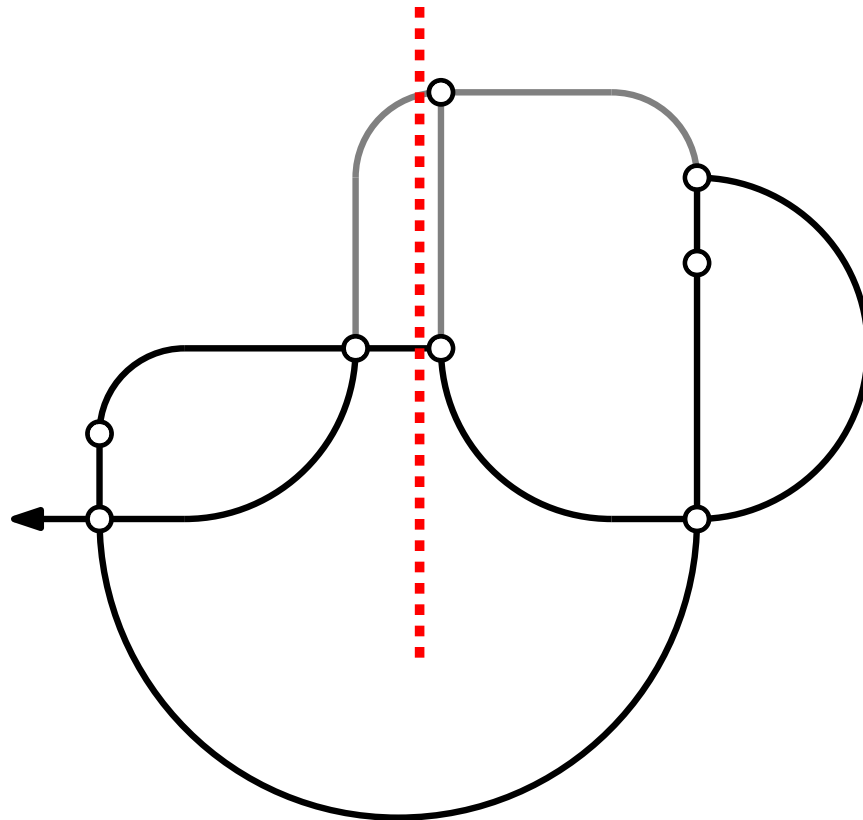
Example Run



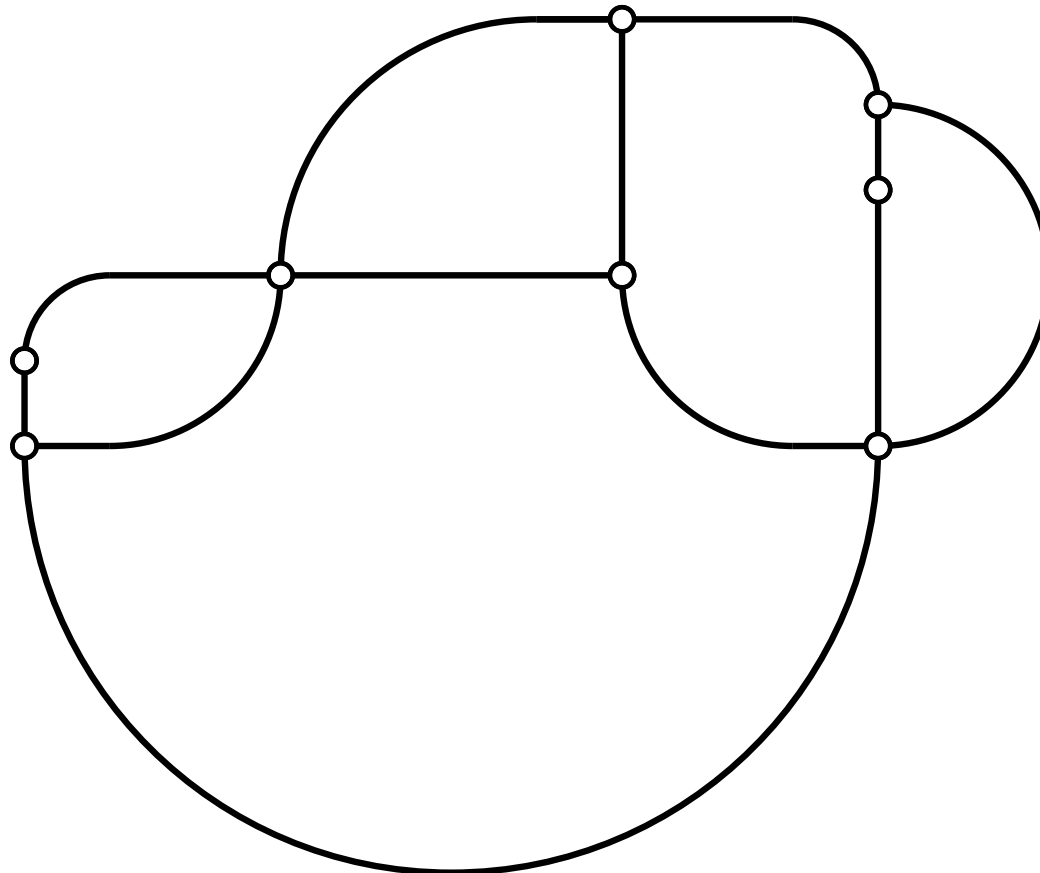
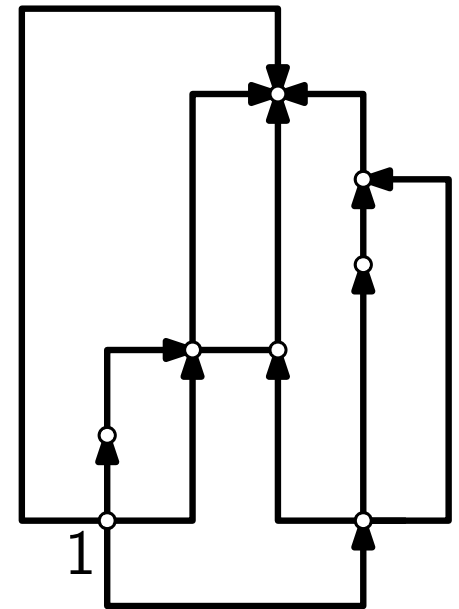
Example Run



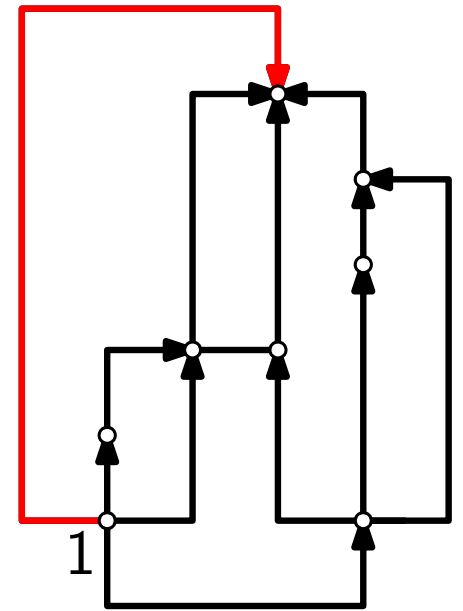
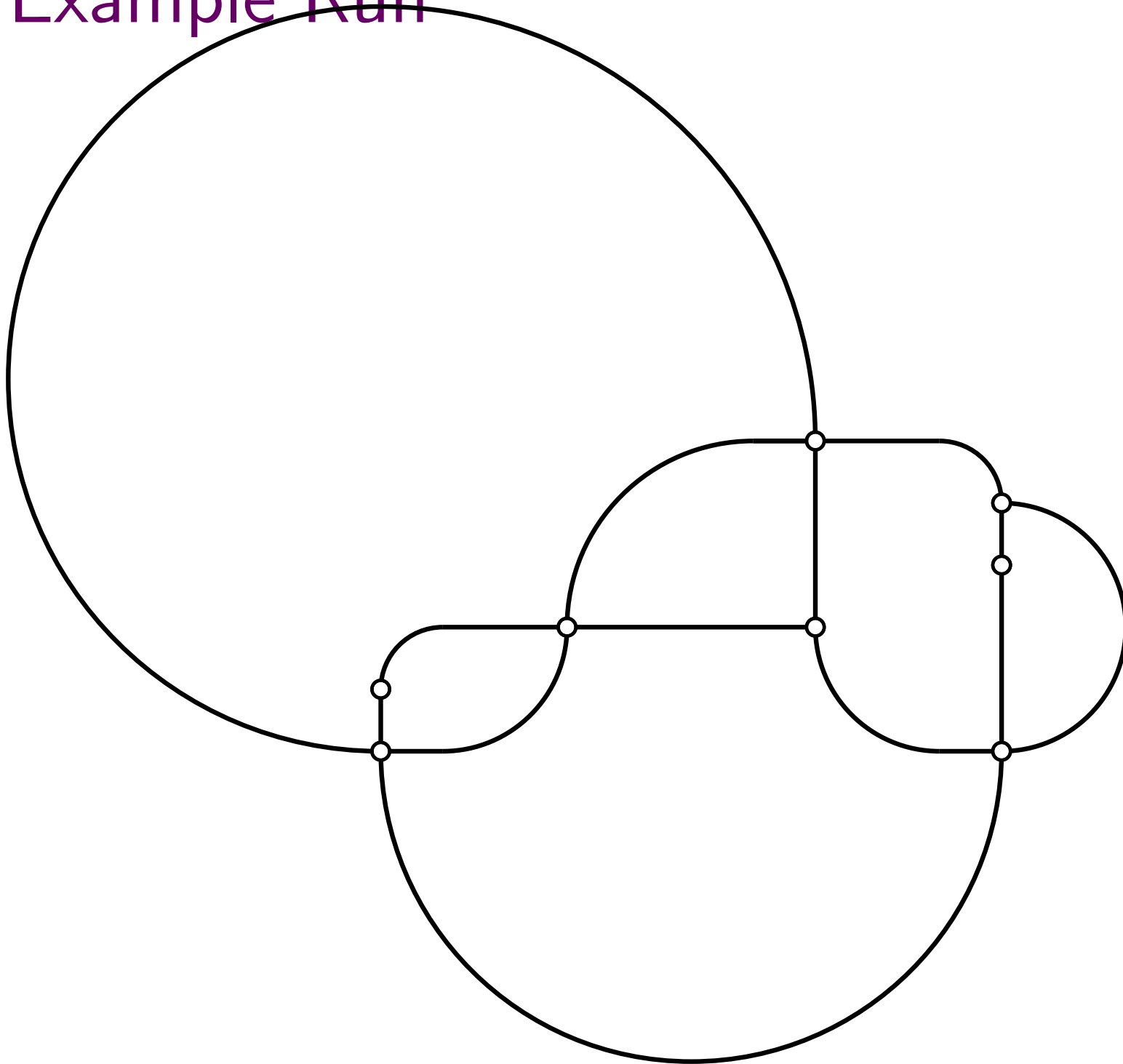
Example Run



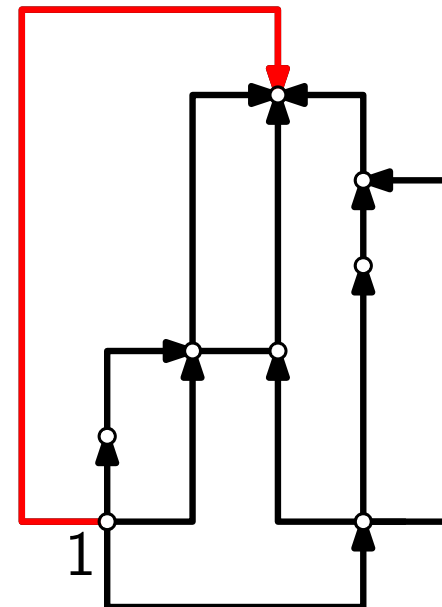
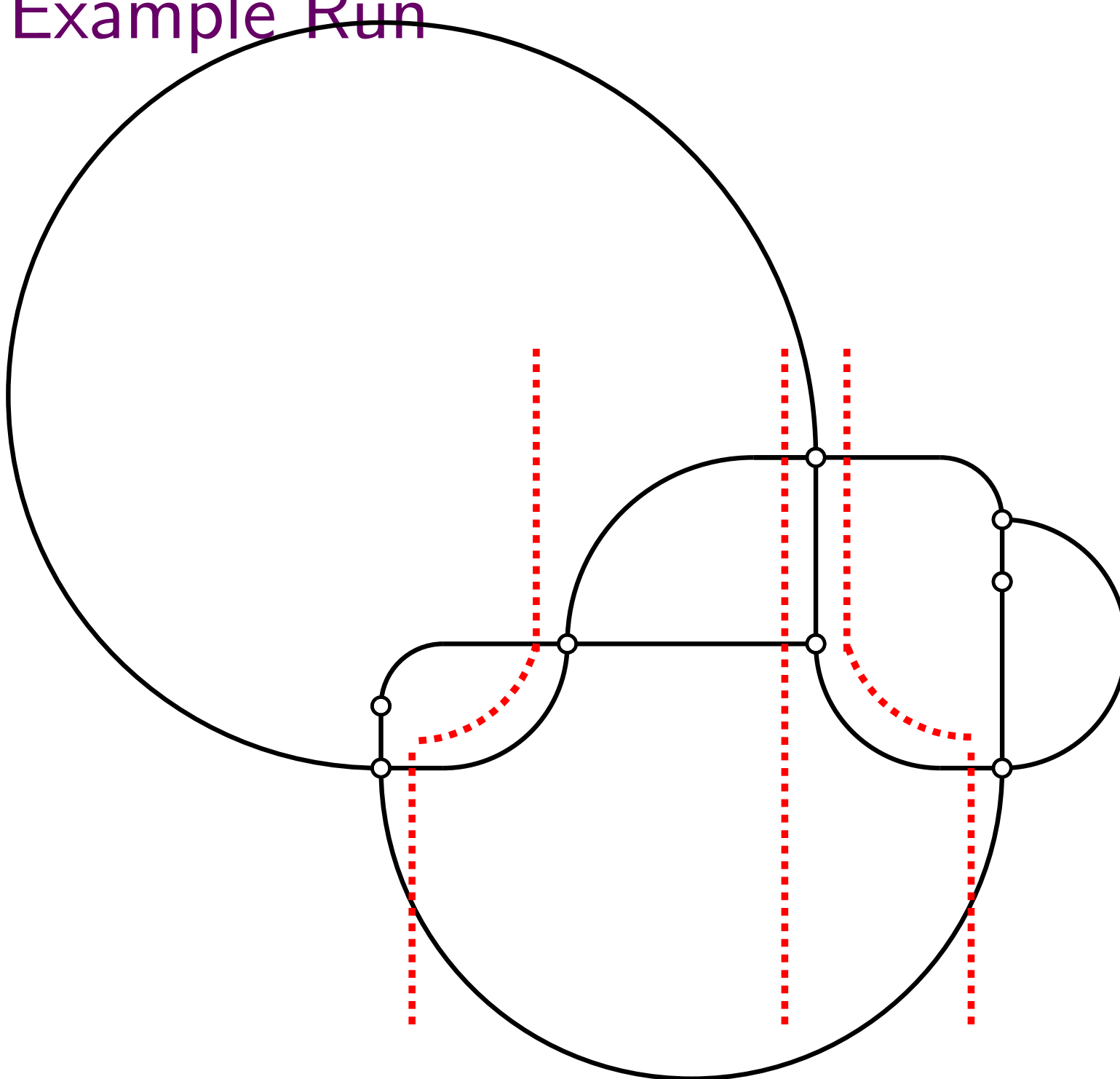
Example Run



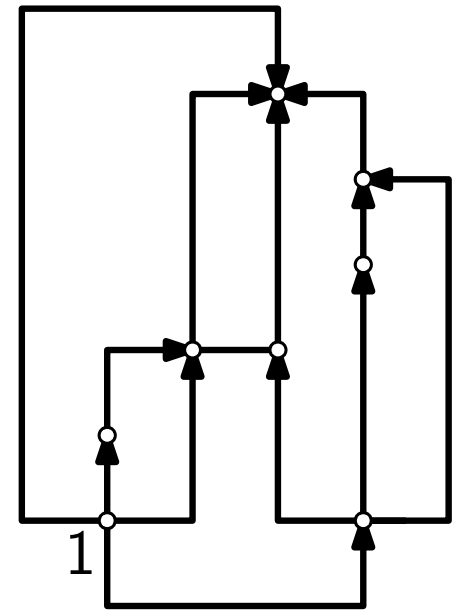
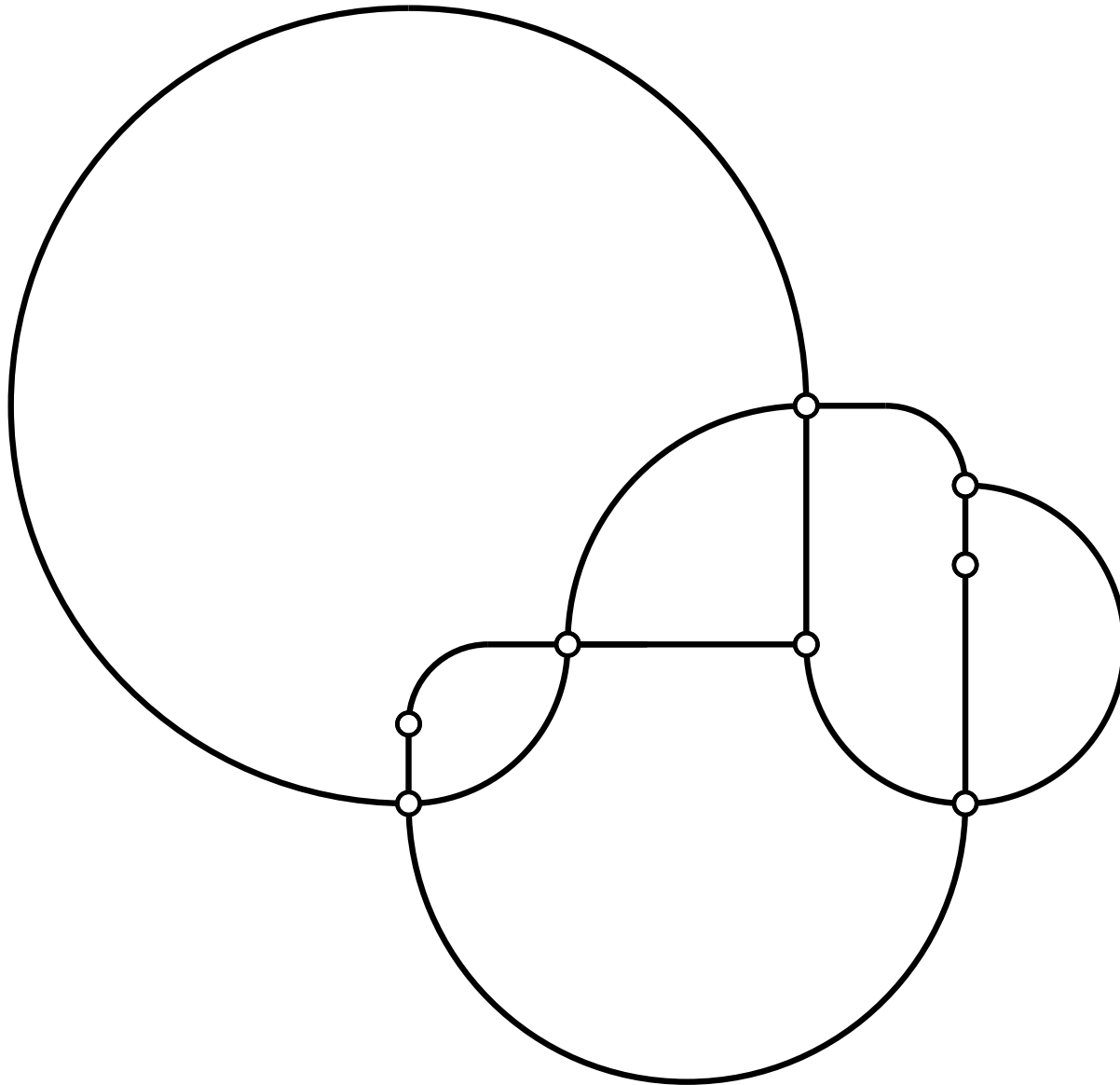
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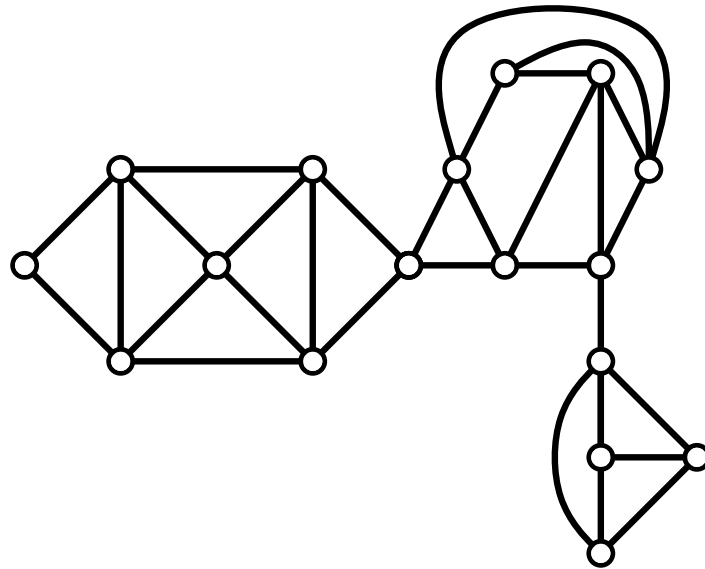
Example Run



Example Run

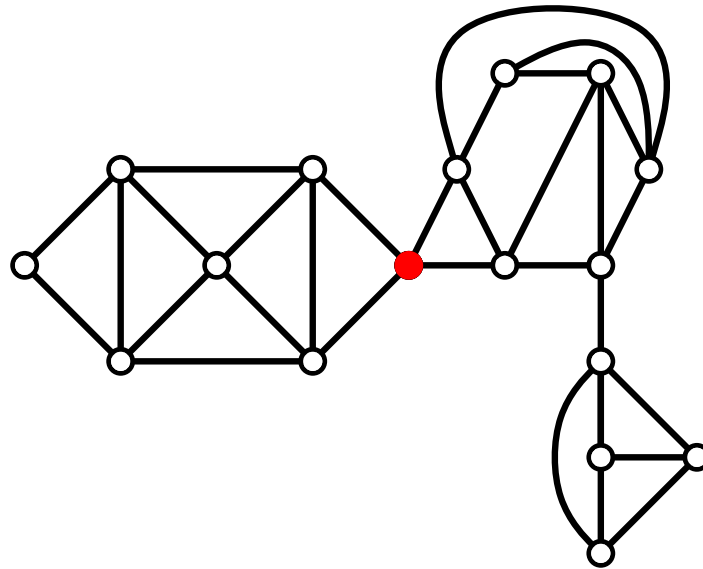


Extension to Arbitrary Graphs



Extension to Arbitrary Graphs

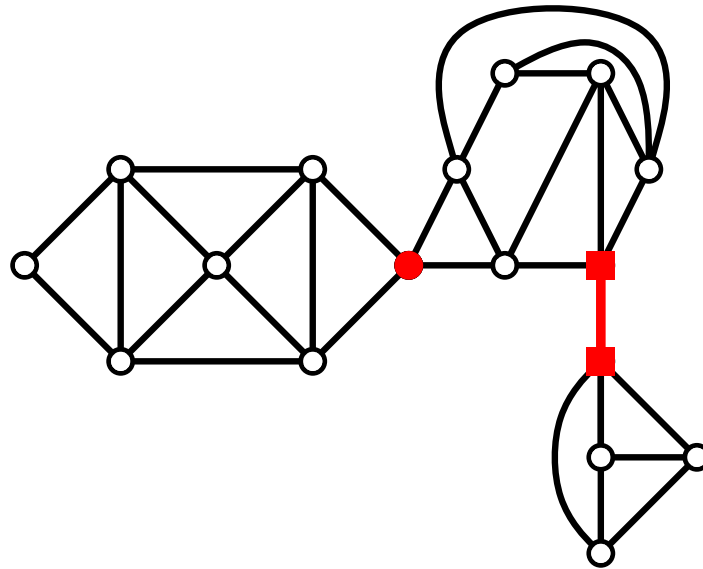
- cutvertices



Extension to Arbitrary Graphs

- cutvertices

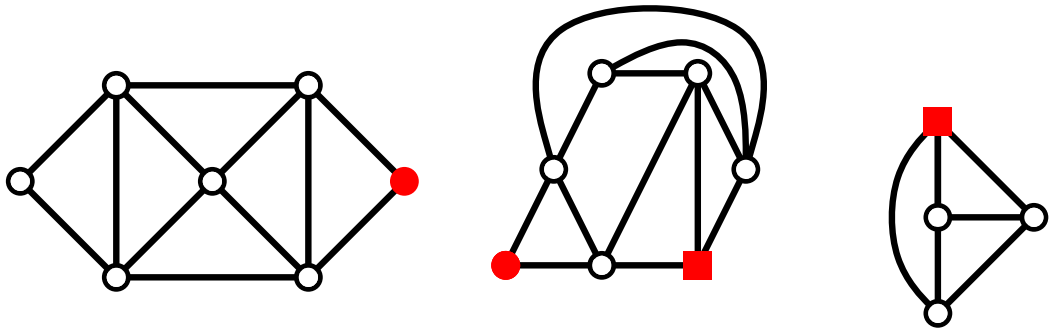
 bridges



Extension to Arbitrary Graphs

● cutvertices

■—■ bridges

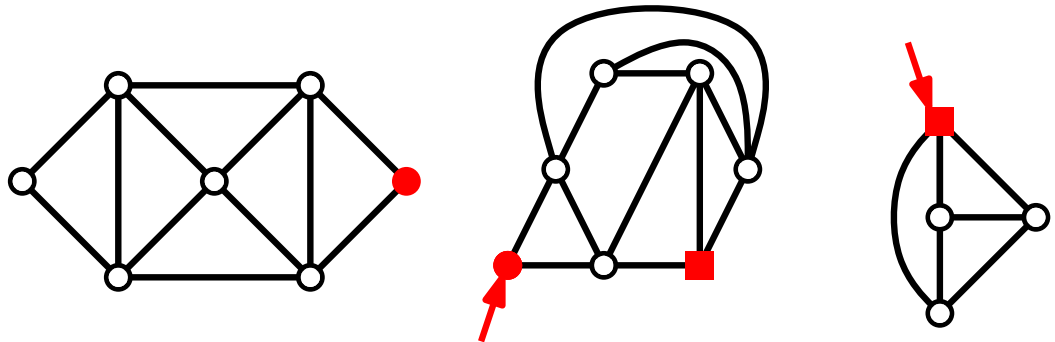


Extension to Arbitrary Graphs

● cutvertices

■—■ bridges

Draw specific vertex on outer face



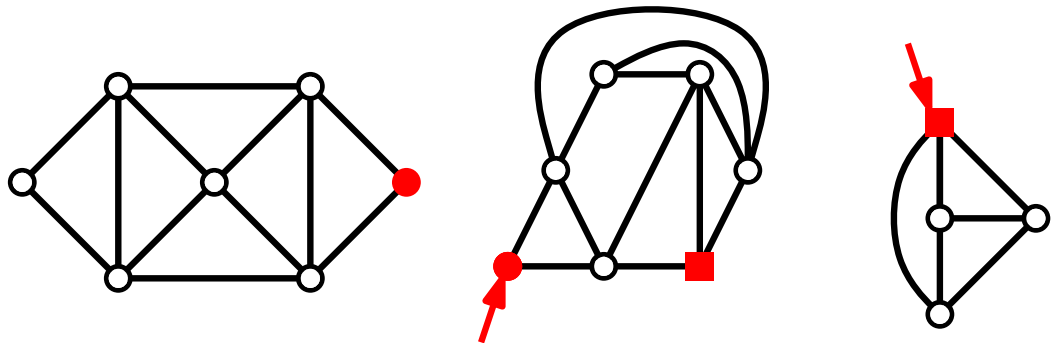
Extension to Arbitrary Graphs

● cutvertices

■—■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles



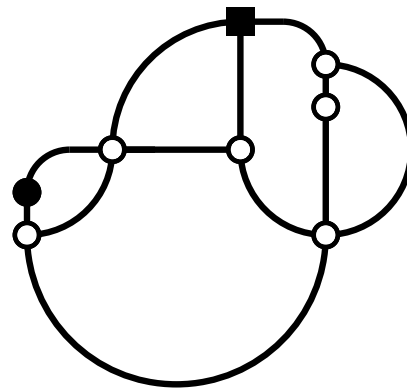
Extension to Arbitrary Graphs

● cutvertices

■—■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles



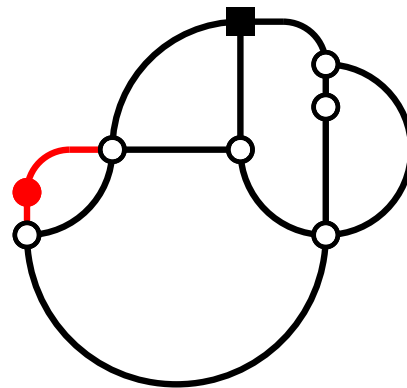
Extension to Arbitrary Graphs

● cutvertices

■—■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles



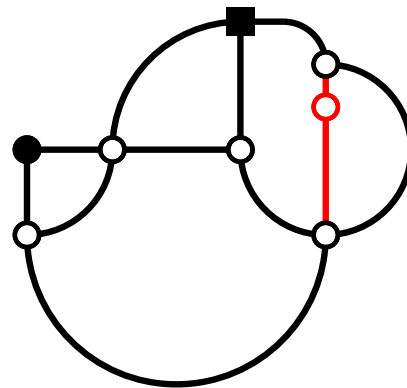
Extension to Arbitrary Graphs

● cutvertices

■—■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles



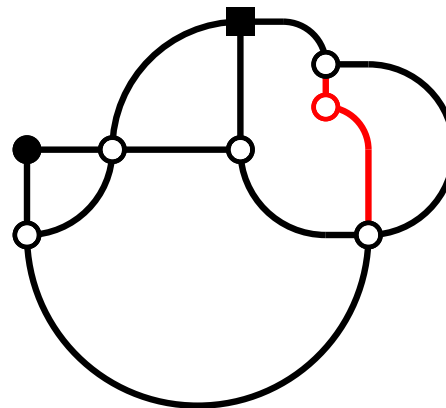
Extension to Arbitrary Graphs

● cutvertices

■—■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles



Extension to Arbitrary Graphs

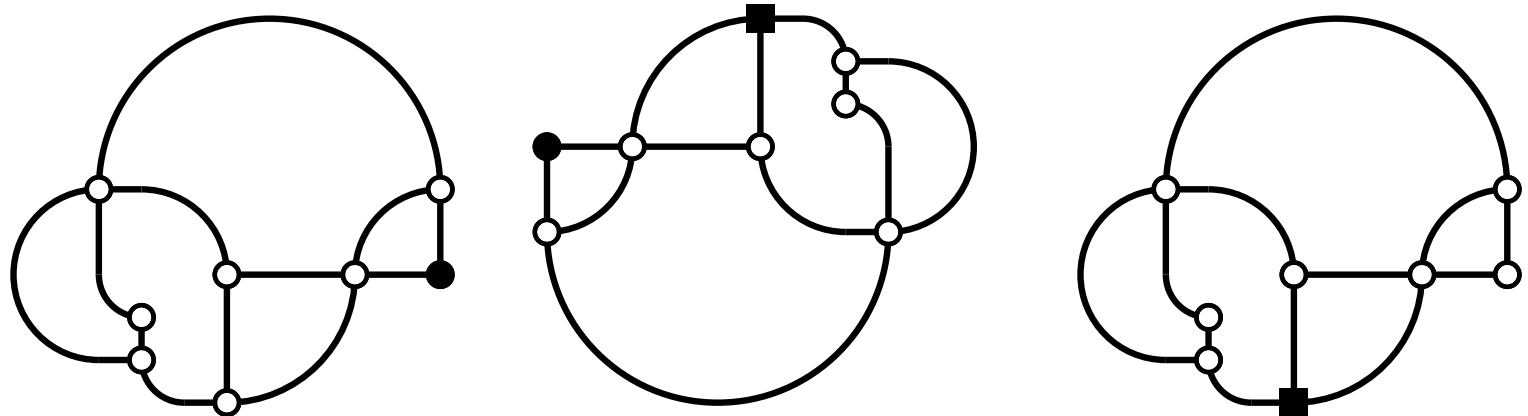
● cutvertices

■—■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles

Connect the pieces



Extension to Arbitrary Graphs

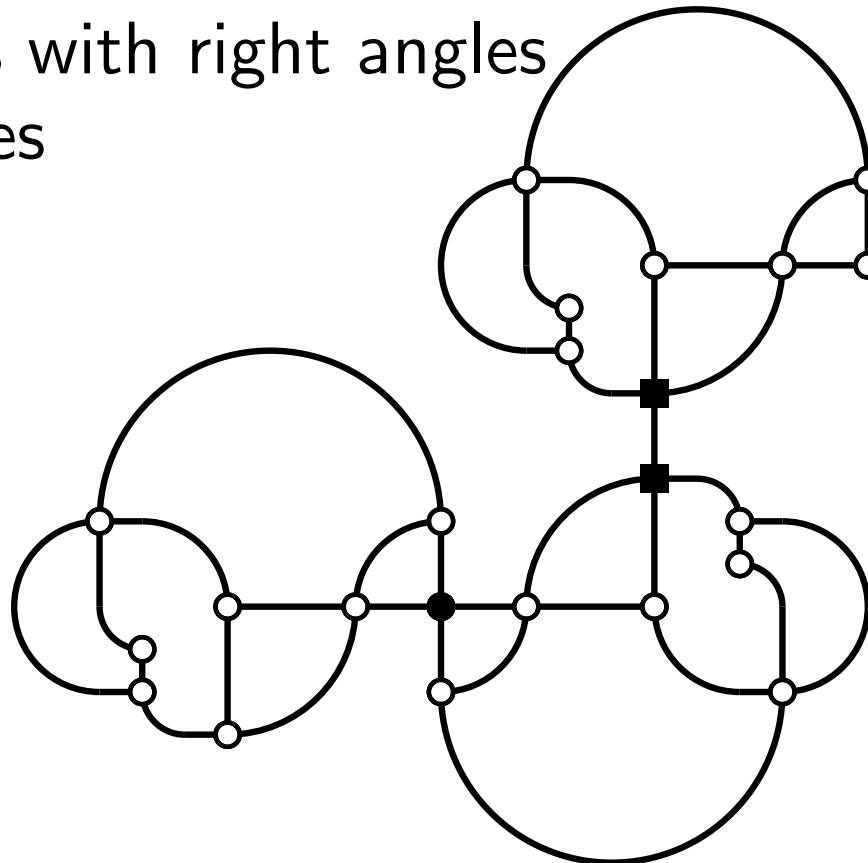
● cutvertices

■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles

Connect the pieces



Extension to Arbitrary Graphs

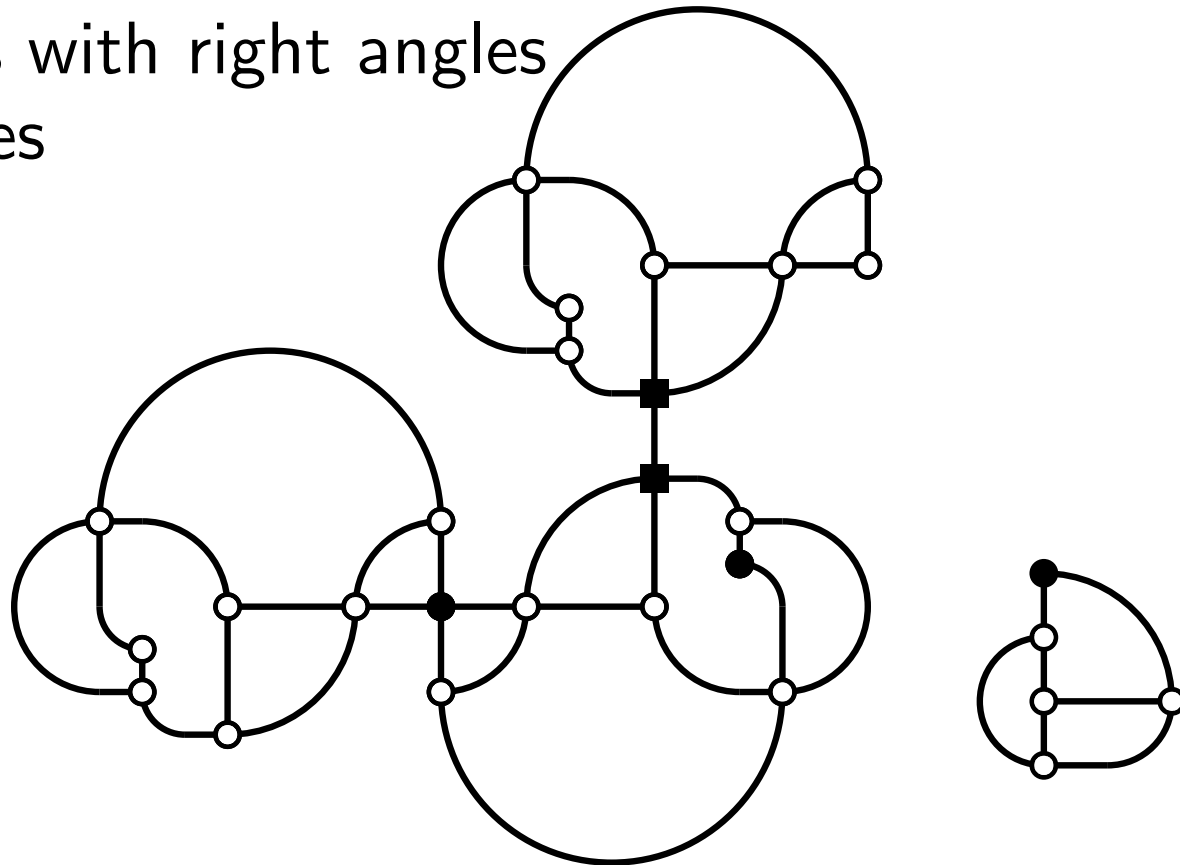
● cutvertices

■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles

Connect the pieces



Extension to Arbitrary Graphs

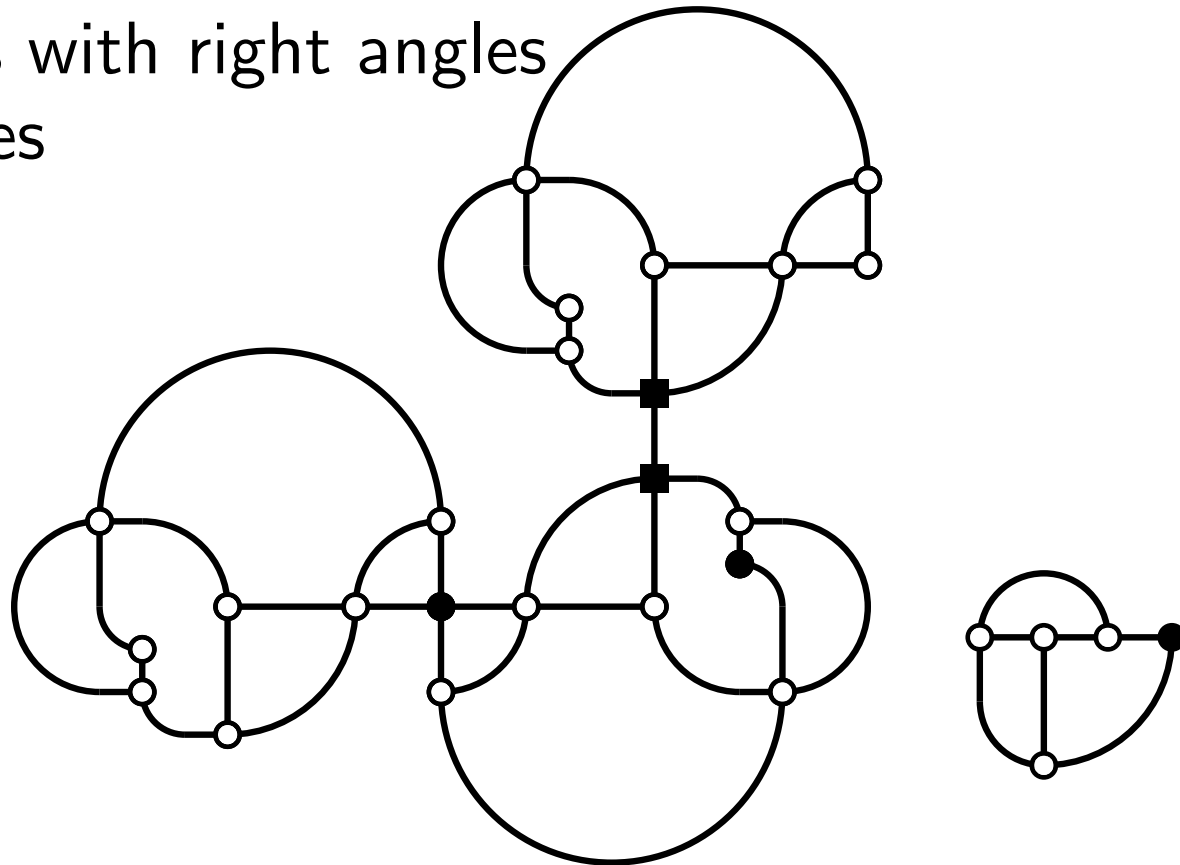
● cutvertices

■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles

Connect the pieces



Extension to Arbitrary Graphs

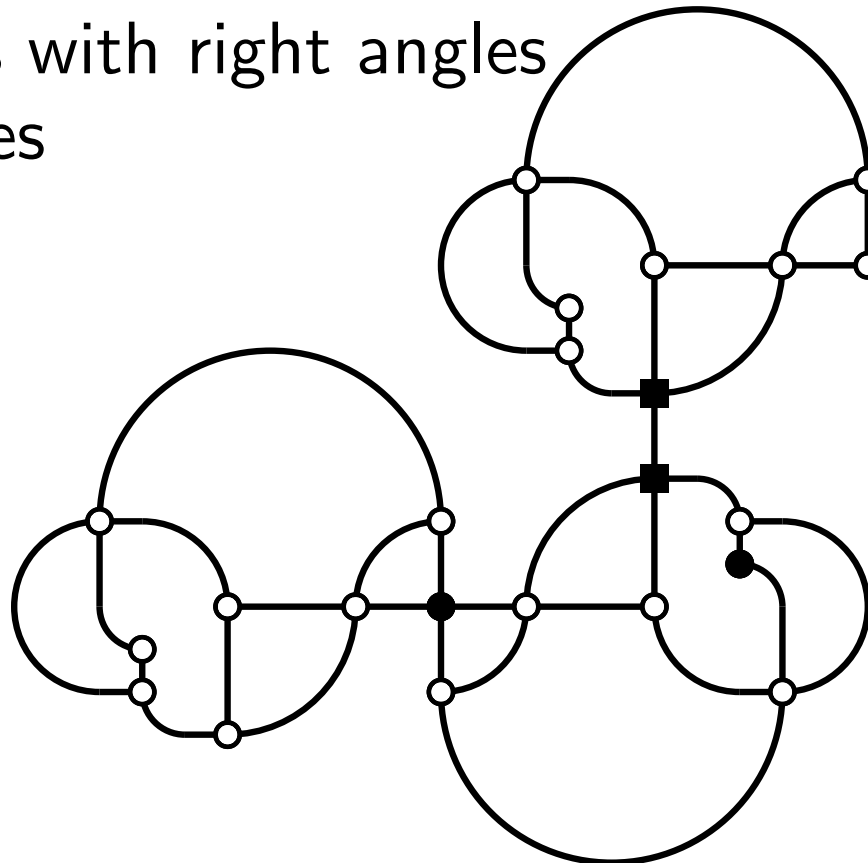
● cutvertices

■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles

Connect the pieces



Extension to Arbitrary Graphs

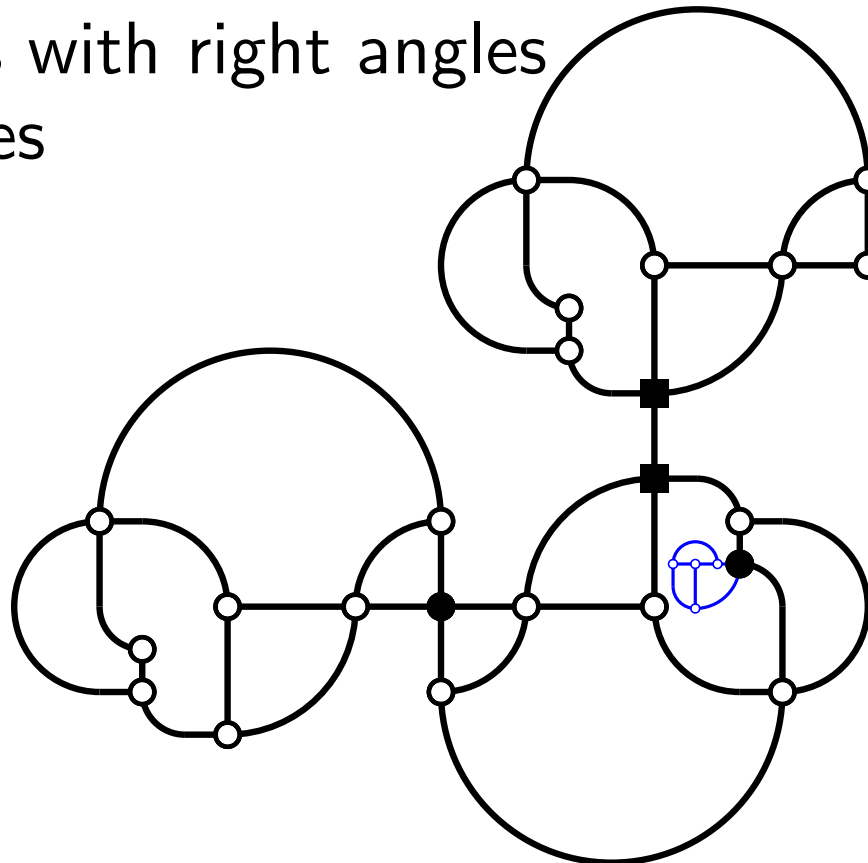
● cutvertices

■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles

Connect the pieces



Extension to Arbitrary Graphs

4-planar graph \rightarrow smooth orthogonal complexity-2 layout

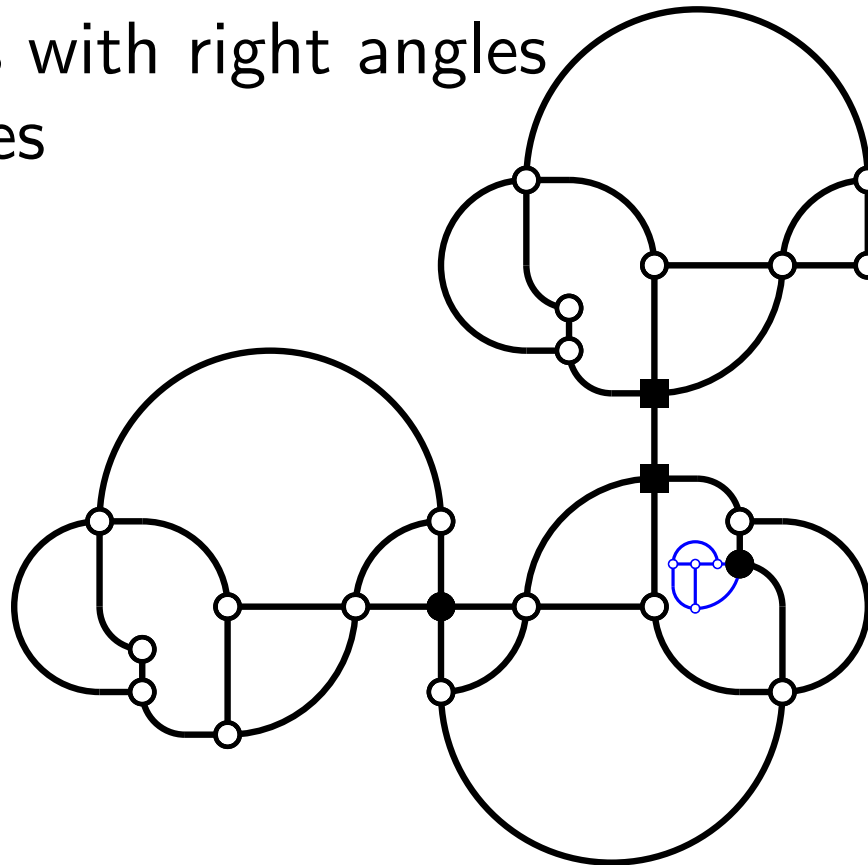
● cutvertices

■ bridges

Draw specific vertex on outer face

Draw cut vertices with right angles

Connect the pieces



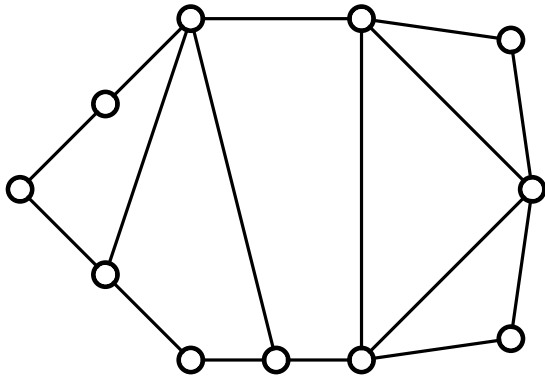
SC₁-Layouts

SC_1 -Layouts

Any biconnected 4-outerplanar graph admits an SC_1 -layout.

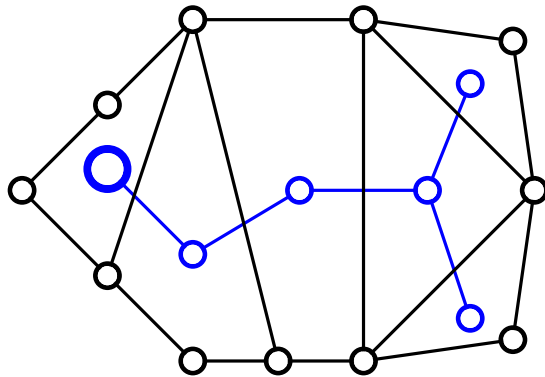
SC_1 -Layouts

Any biconnected 4-outerplanar graph admits an SC_1 -layout.



SC_1 -Layouts

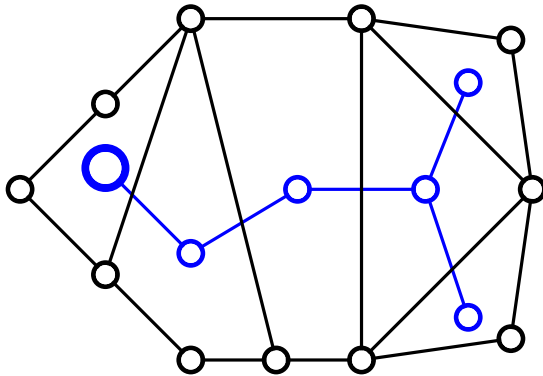
Any biconnected 4-outerplanar graph admits an SC_1 -layout.



Consider the dual tree

SC_1 -Layouts

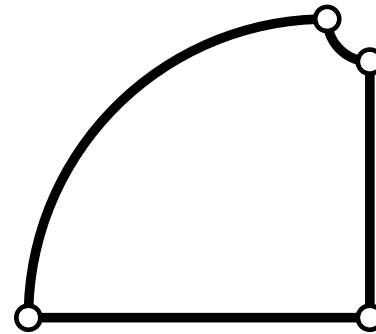
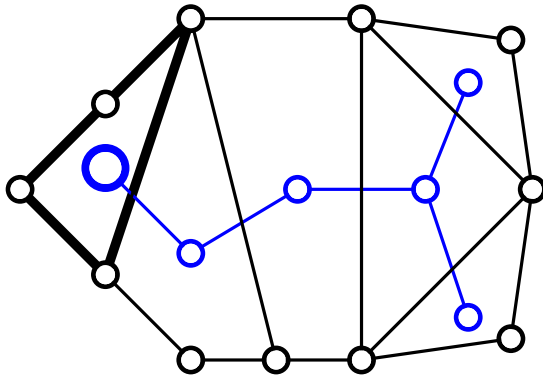
Any biconnected 4-outerplanar graph admits an SC_1 -layout.



Consider the dual tree
Pick a root, traverse the tree

SC_1 -Layouts

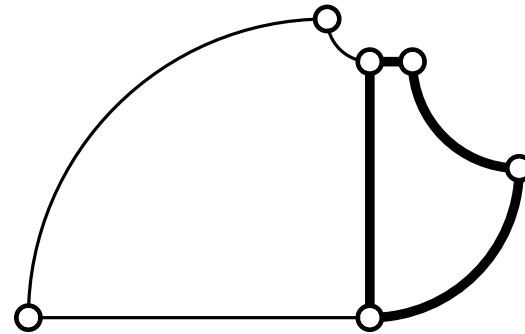
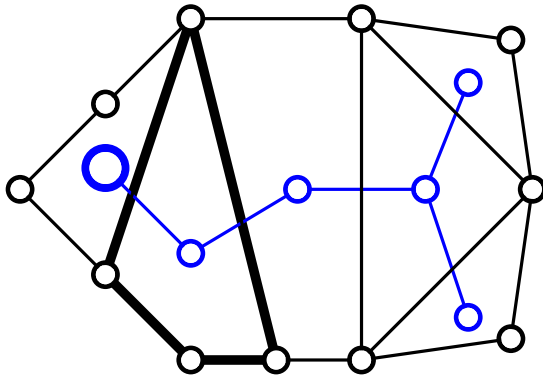
Any biconnected 4-outerplanar graph admits an SC_1 -layout.



Consider the dual tree
Pick a root, traverse the tree

SC_1 -Layouts

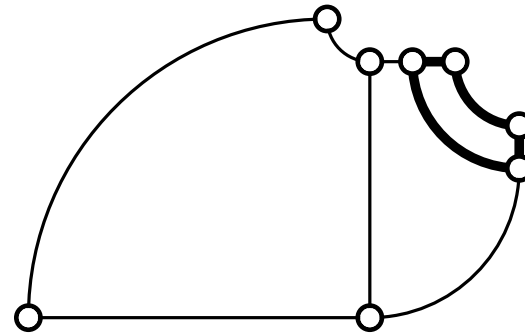
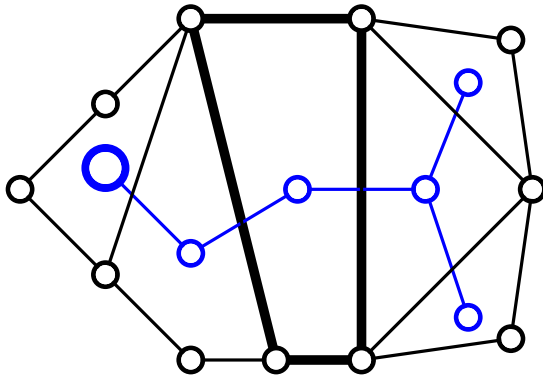
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Consider the dual tree
Pick a root, traverse the tree

SC_1 -Layouts

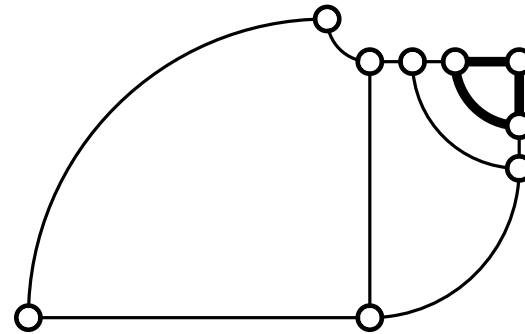
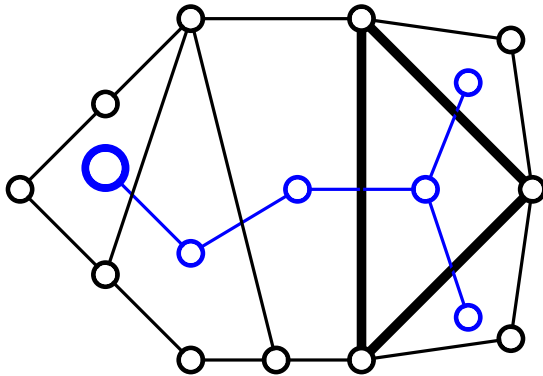
Any biconnected 4-outerplanar graph admits an SC_1 -layout.



Consider the dual tree
Pick a root, traverse the tree

SC_1 -Layouts

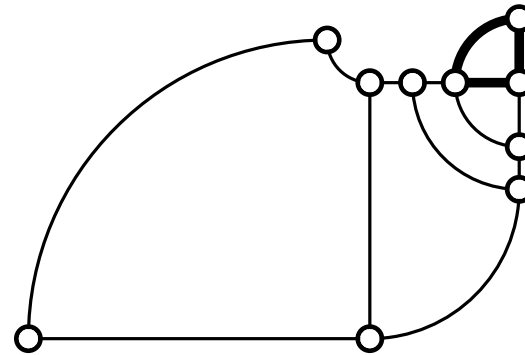
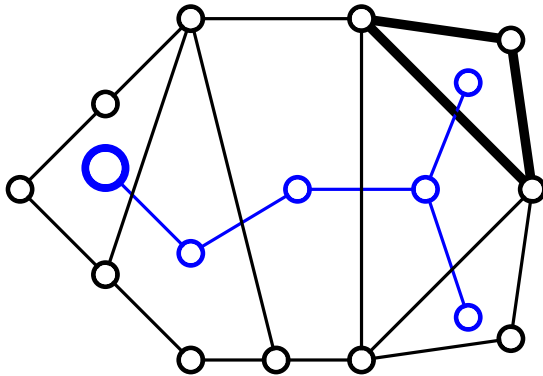
Any biconnected 4-outerplanar graph admits an SC_1 -layout.



Consider the dual tree
Pick a root, traverse the tree

SC_1 -Layouts

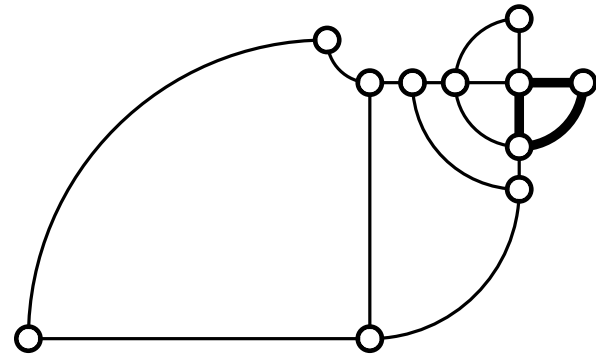
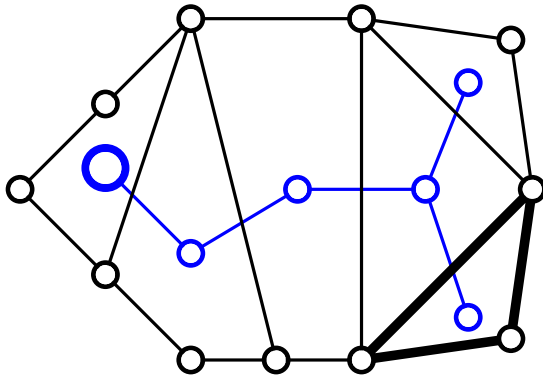
Any biconnected 4-outerplanar graph admits an SC_1 -layout.



Consider the dual tree
Pick a root, traverse the tree

SC_1 -Layouts

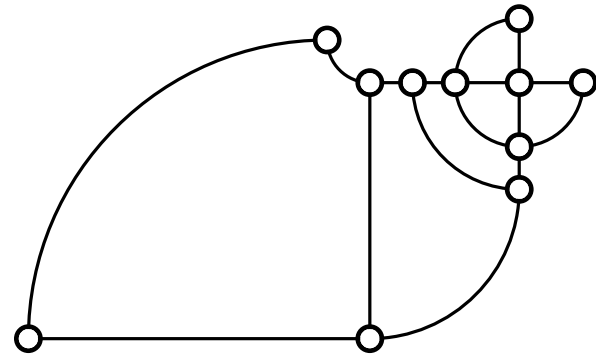
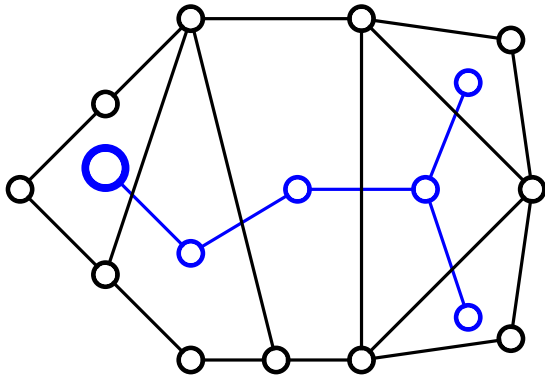
Any biconnected 4-outerplanar graph admits an SC_1 -layout.



Consider the dual tree
Pick a root, traverse the tree

SC_1 -Layouts

Any biconnected 4-outerplanar graph admits an SC_1 -layout.



Consider the dual tree
Pick a root, traverse the tree

SC_1 -Layouts

Any biconnected 4-outerplanar graph admits an SC_1 -layout.

Any triconnected 3-planar graph admits an SC_1 -layout.

SC_1 -Layouts

Any biconnected 4-outerplanar graph admits an SC_1 -layout.

Any triconnected 3-planar graph admits an SC_1 -layout.

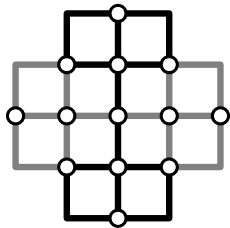
Any Hamiltonian 3-planar graph admits an SC_1 -layout.

Area Requirement of SC_1 -Layouts

There is an infinite class of graphs that require exponential area if they are drawn with SC_1 .

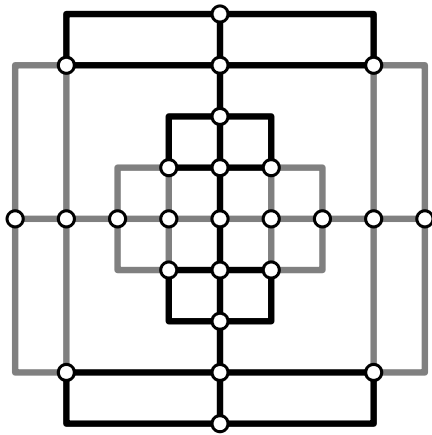
Area Requirement of SC_1 -Layouts

There is an infinite class of graphs that require exponential area if they are drawn with SC_1 .



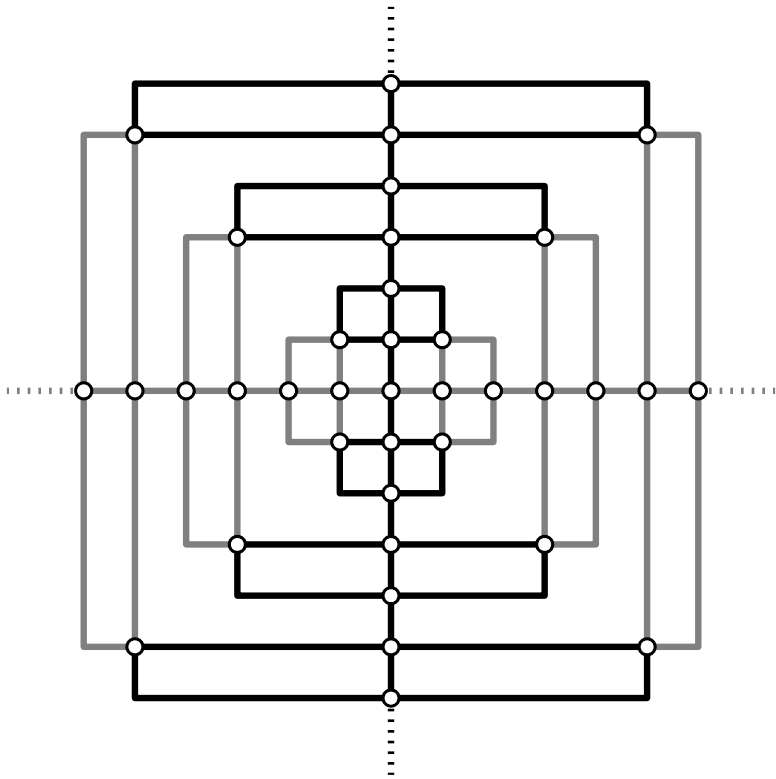
Area Requirement of SC_1 -Layouts

There is an infinite class of graphs that require exponential area if they are drawn with SC_1 .



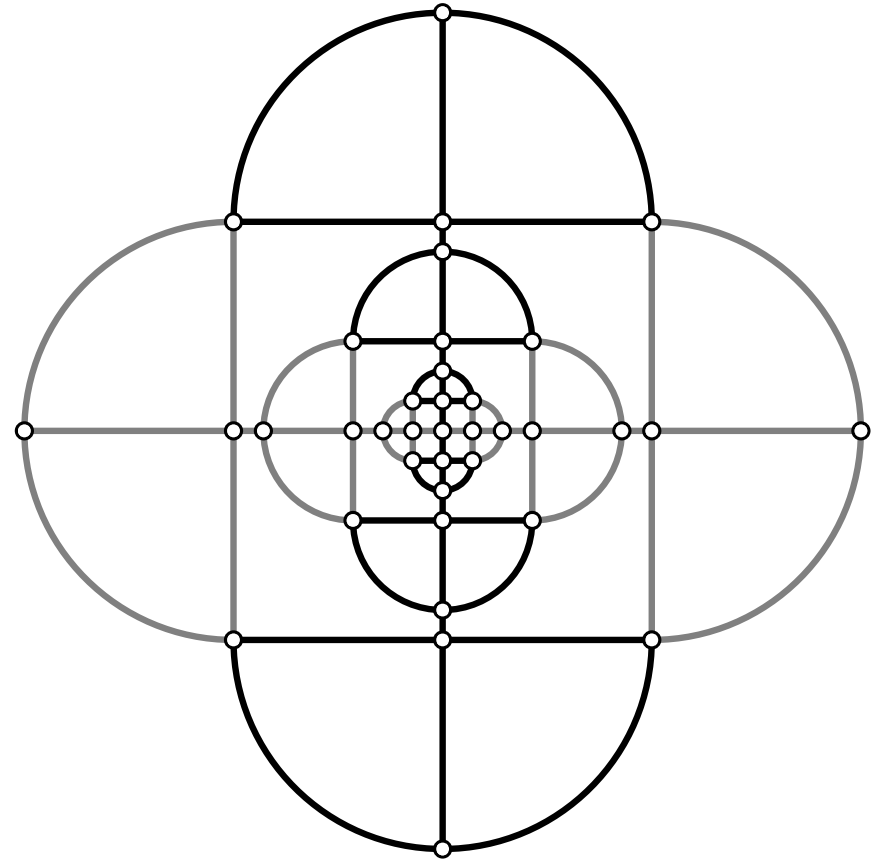
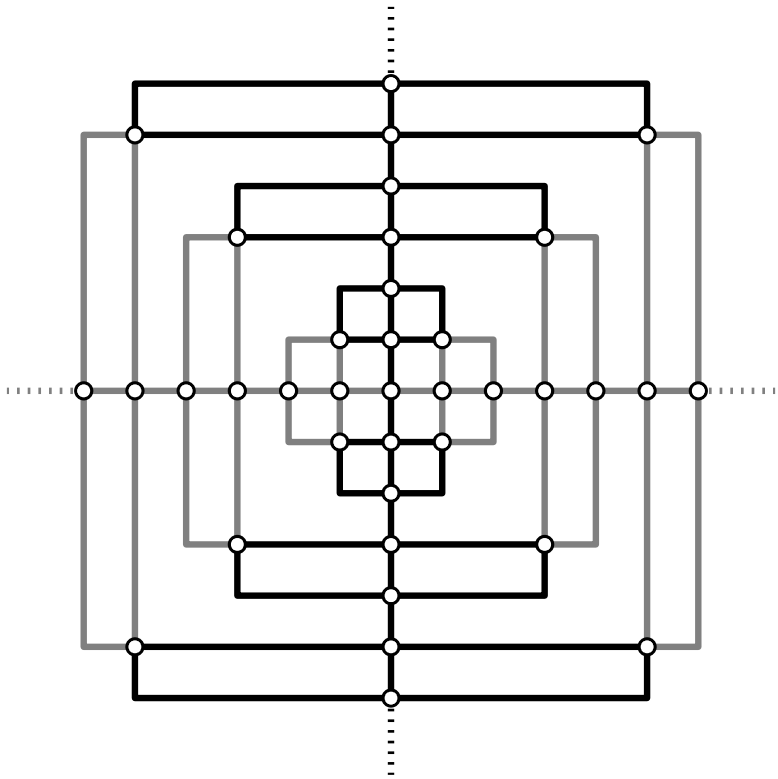
Area Requirement of SC_1 -Layouts

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Area Requirement of SC_1 -Layouts

There is an infinite class of graphs that require exponential area if they are drawn with SC_1 .

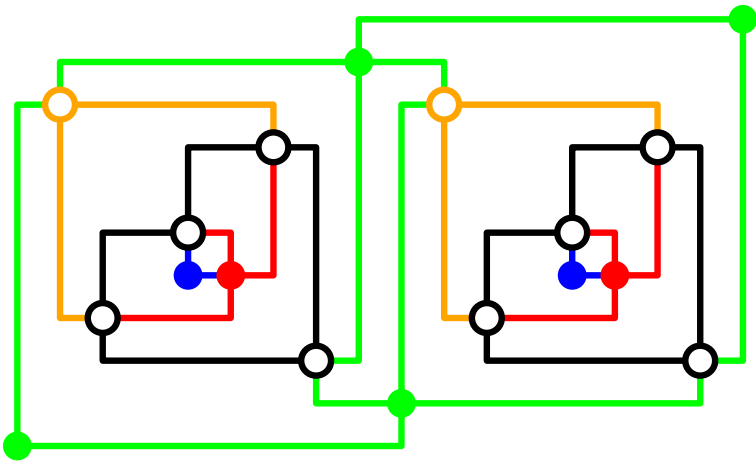


Biconnected Graphs without SC_1 -Layout

There exists a biconnected 4-planar graph that admits an OC_2 -layout, but does not admit an SC_1 -layout.

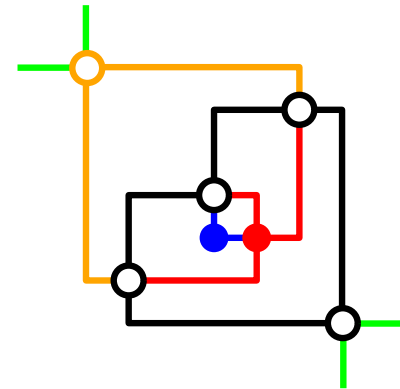
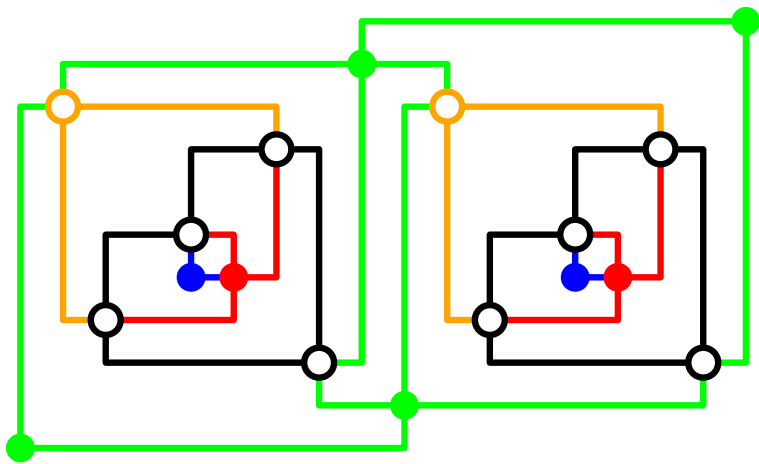
Biconnected Graphs without SC_1 -Layout

There exists a biconnected 4-planar graph that admits an OC_2 -layout, but does not admit an SC_1 -layout.



Biconnected Graphs without SC_1 -Layout

There exists a biconnected 4-planar graph that admits an OC_2 -layout, but does not admit an SC_1 -layout.



Open Problems

- Do all 4-planar graphs admit an SC_2 -layout *in polynomial area*?

Open Problems

- Do all 4-planar graphs admit an SC_2 -layout *in polynomial area*?
- Do all 4-outerplanar graphs admit an SC_1 -layout?

Open Problems

- Do all 4-planar graphs admit an SC_2 -layout *in polynomial area*?
- Do all 4-outerplanar graphs admit an SC_1 -layout?
- Do all 3-planar graphs admit an SC_1 -layout?

Open Problems

- Do all 4-planar graphs admit an SC_2 -layout *in polynomial area*?
- Do all 4-outerplanar graphs admit an SC_1 -layout?
- Do all 3-planar graphs admit an SC_1 -layout?
- Is it NP-hard to decide whether a 4-planar graph admits an SC_1 -layout?