Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG
Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems
Institute for Informatics

Smooth Orthogonal Drawings of Planar Graphs

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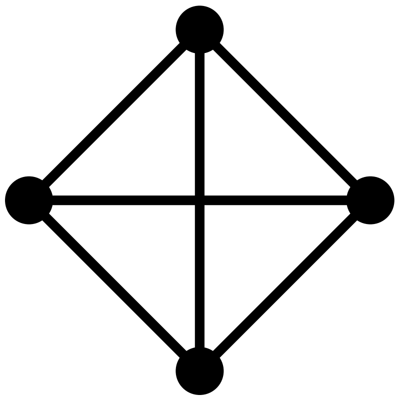
Chair of Computer Science I
Universität Würzburg

Joint work with
Md. Jawaherul Alam, Michael A. Bekos, Michael Kaufmann,
Stephen G. Kobourov & Alexander Wolff

Drawings of Graphs



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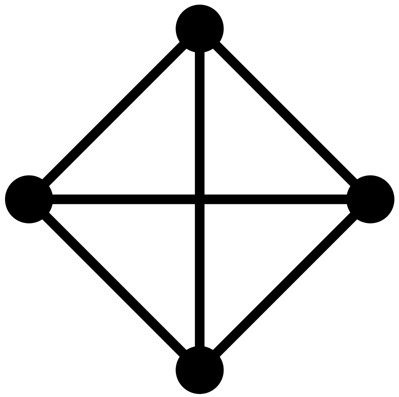

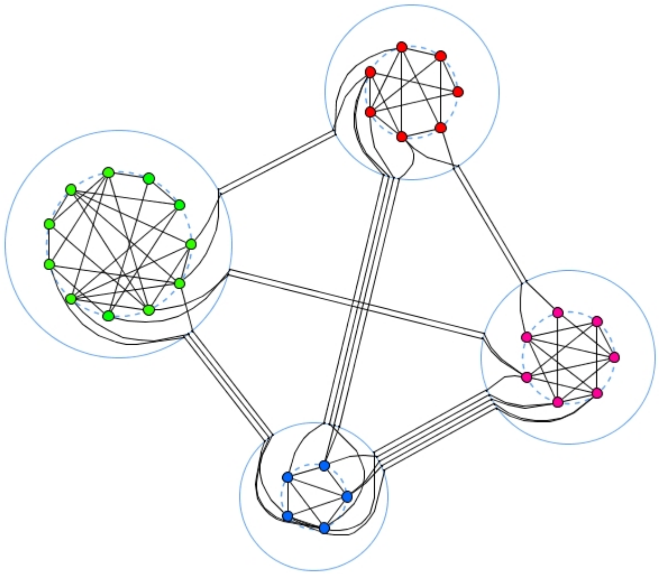

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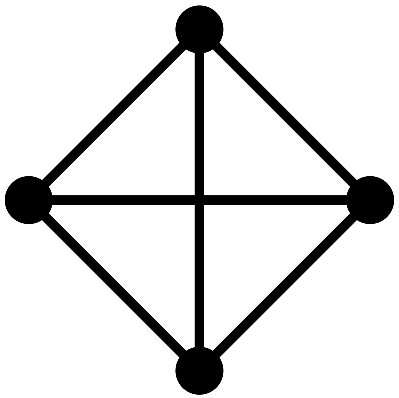

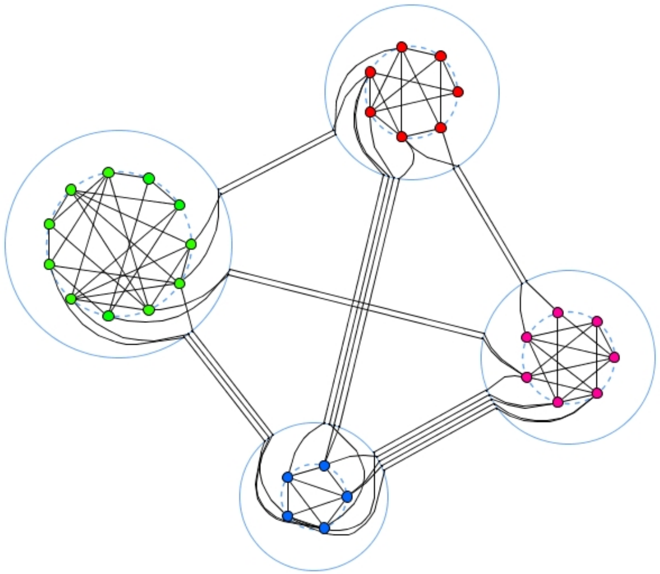

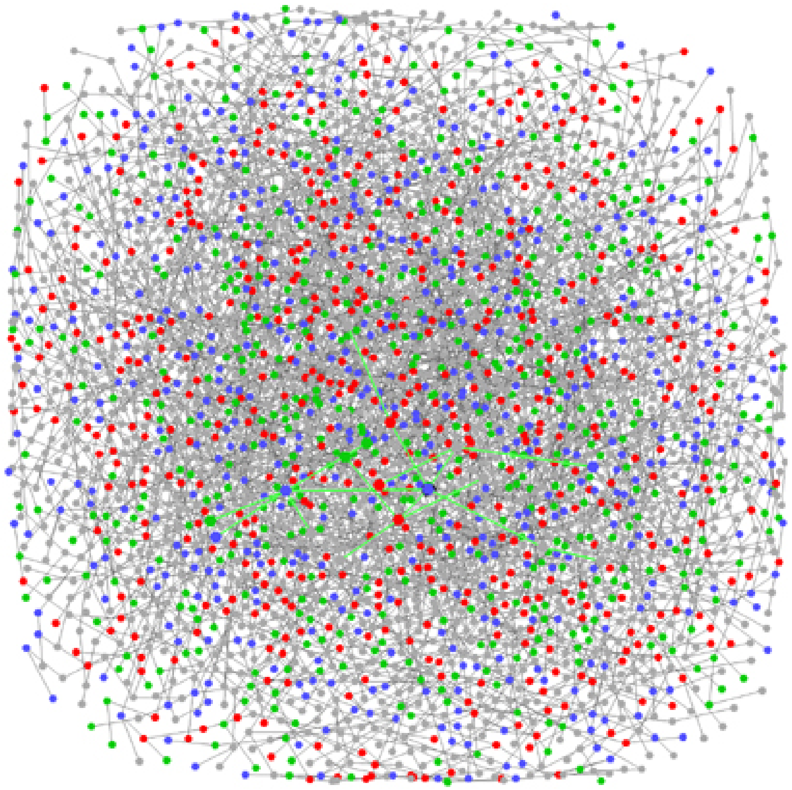

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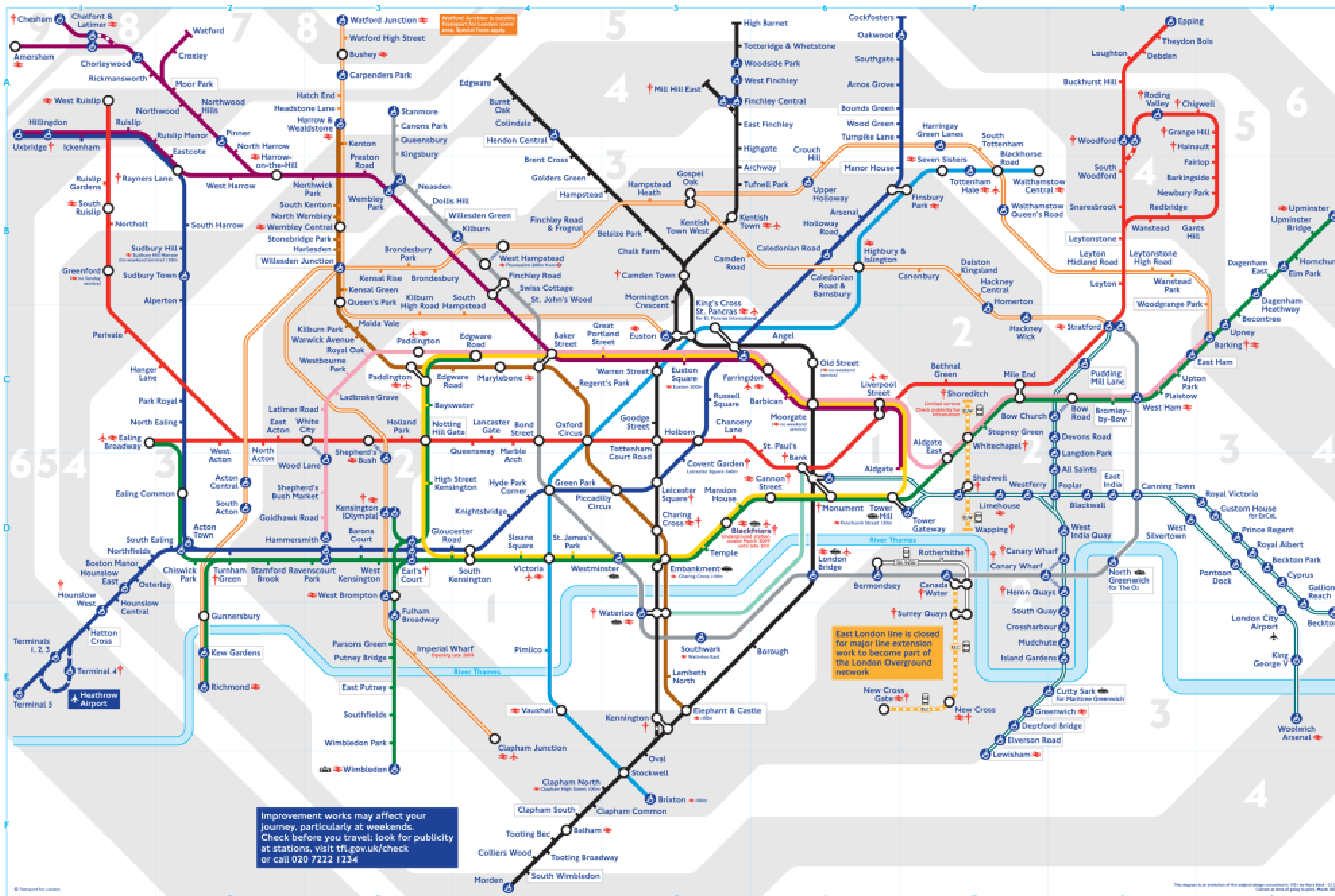

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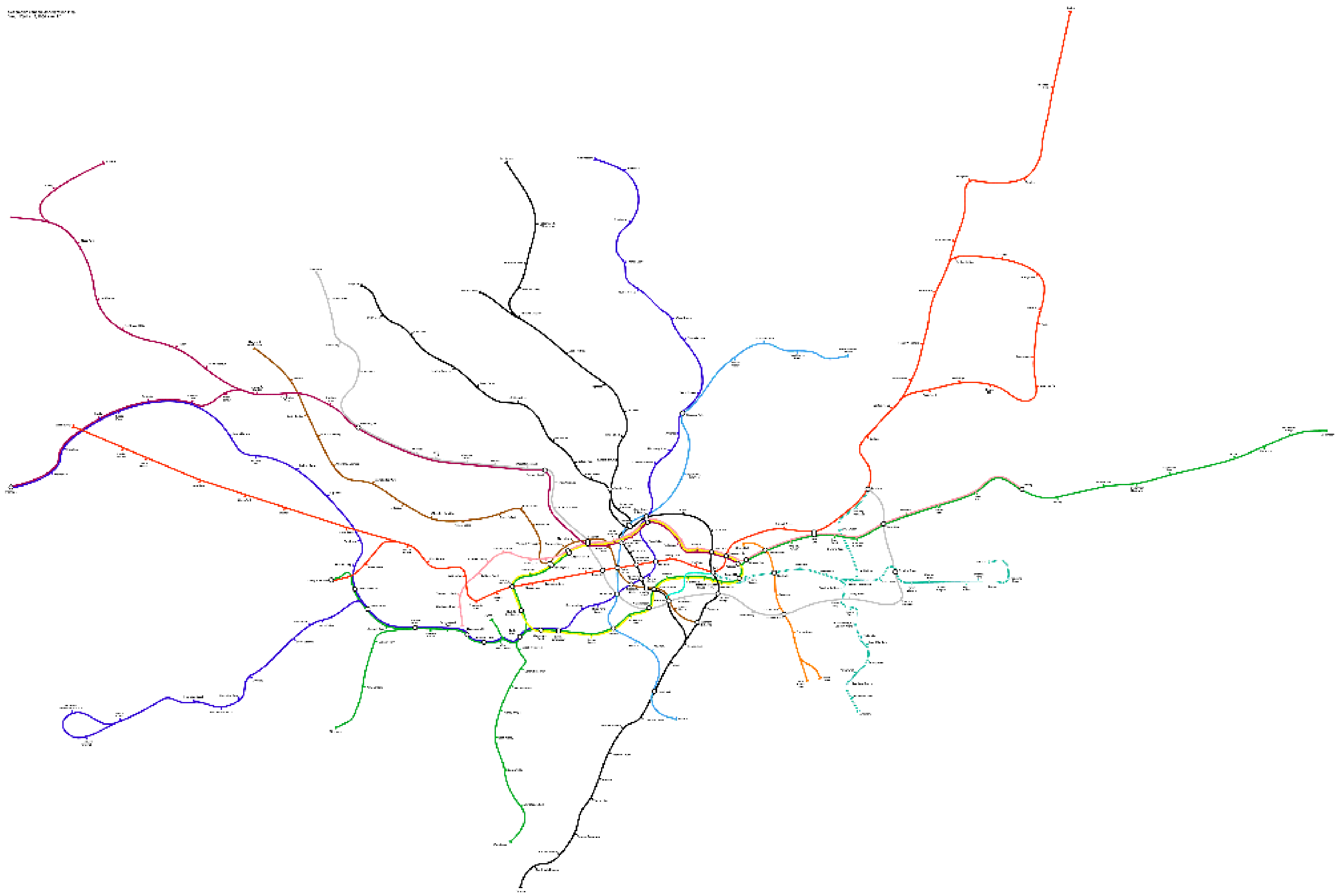

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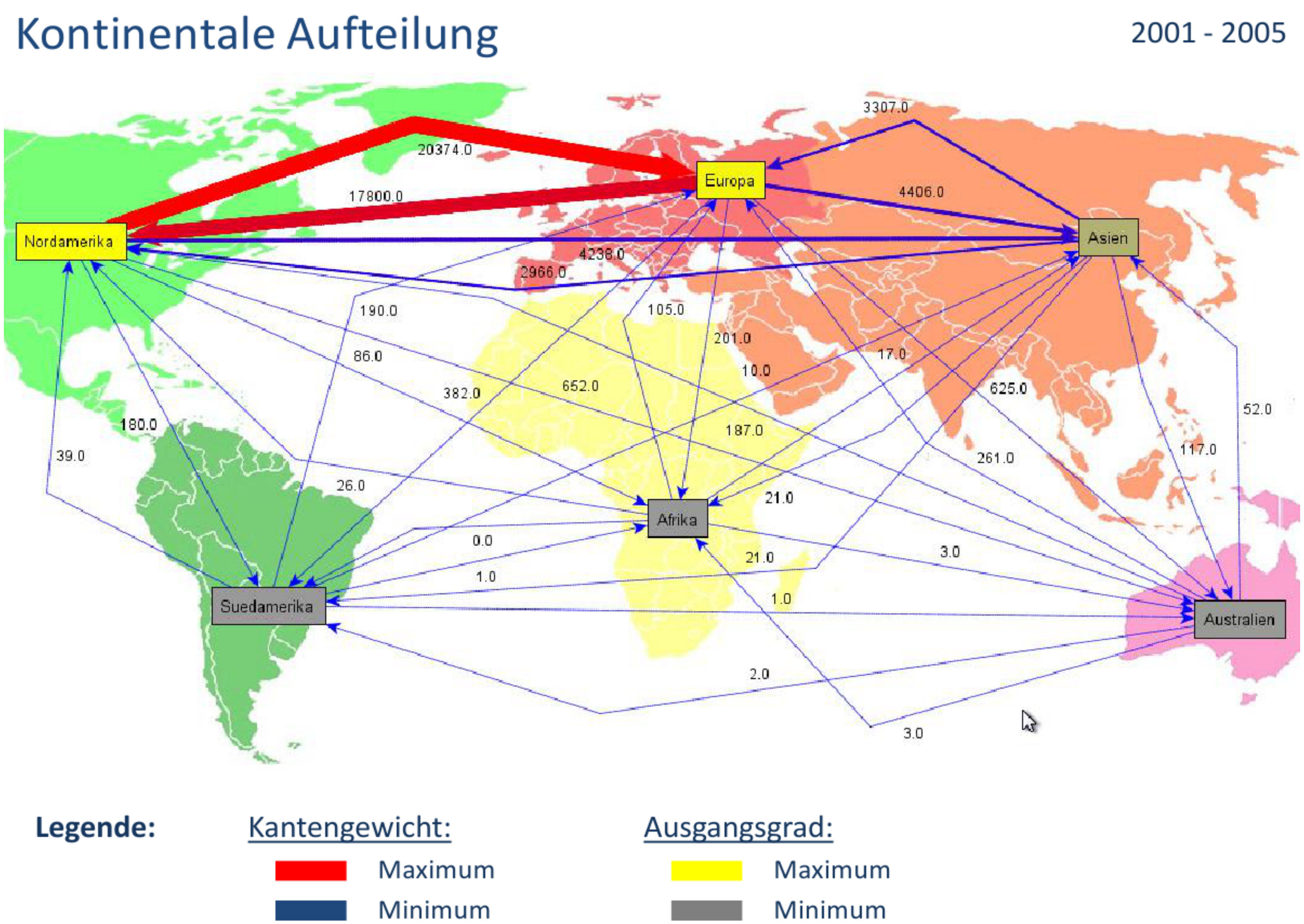

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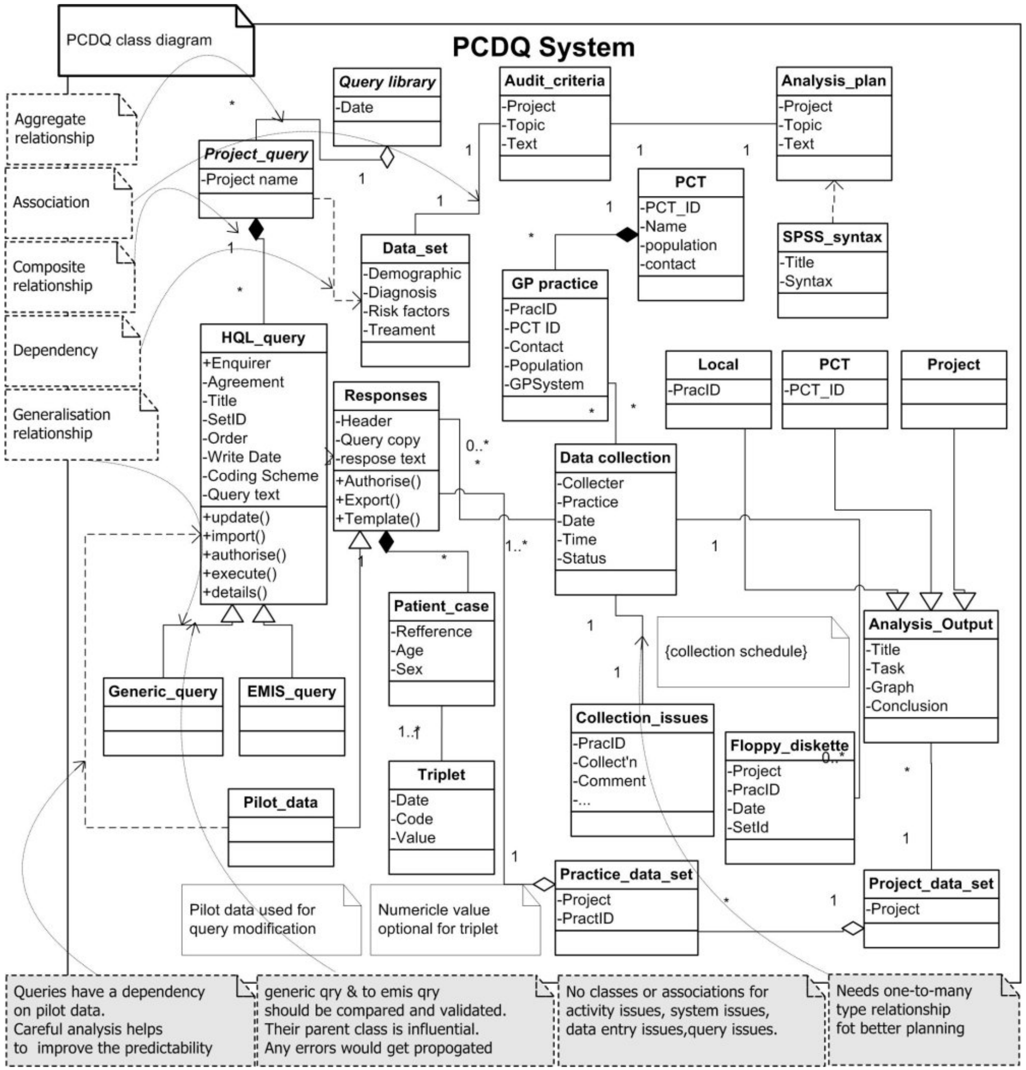

Drawing Styles

$$G = (V, E)$$

nodes $v \in V$

edge $\{u, v\}$ connecting u and v

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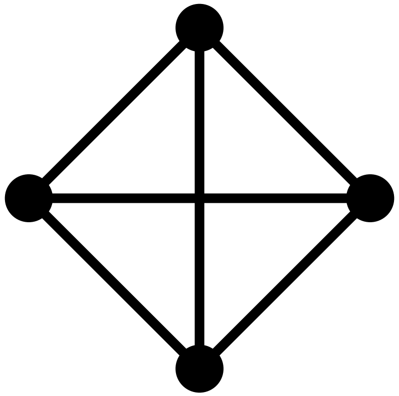

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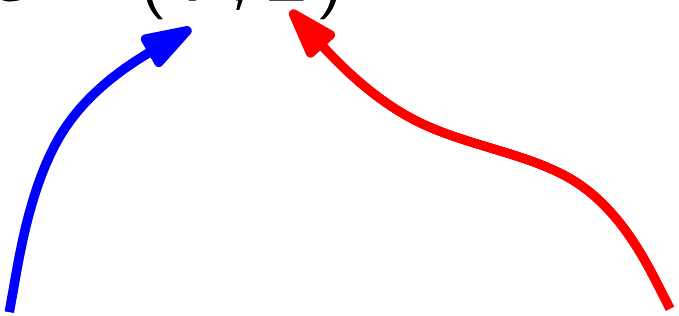

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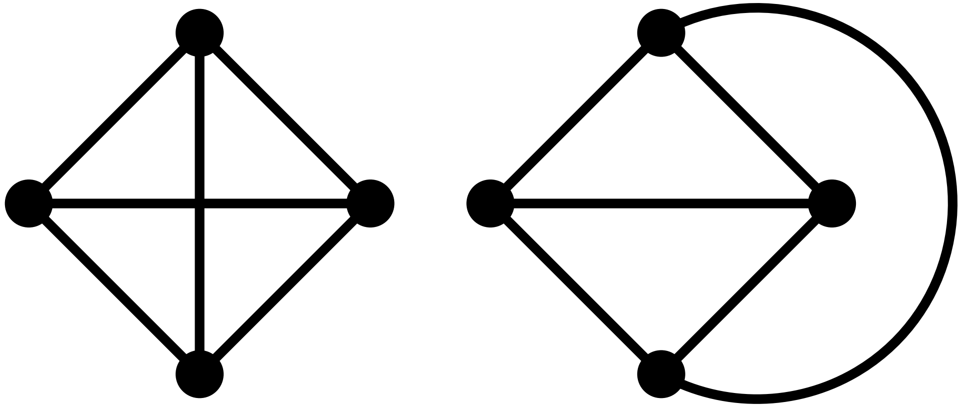
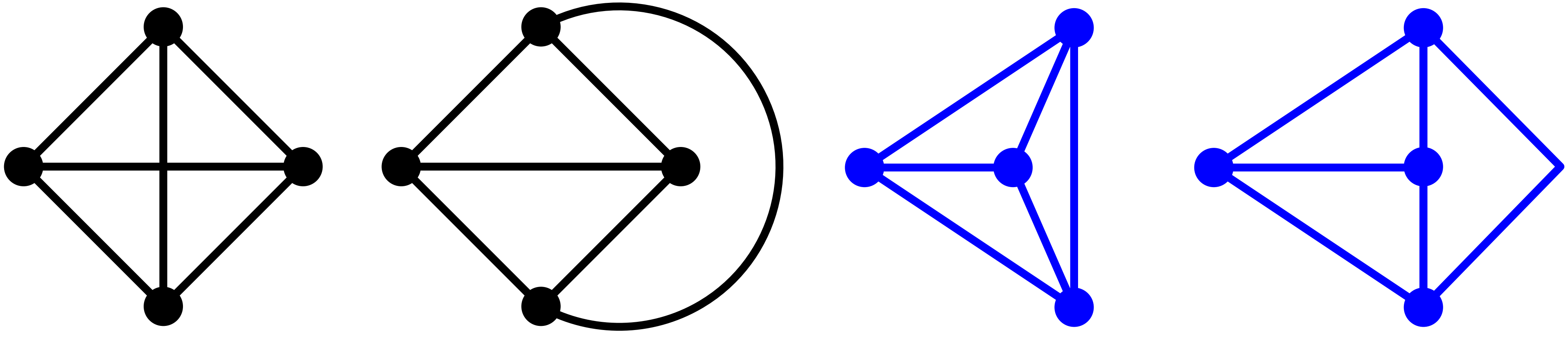


$G = (V, E)$

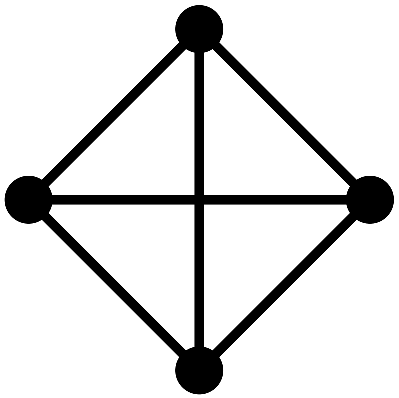
with/without crossings

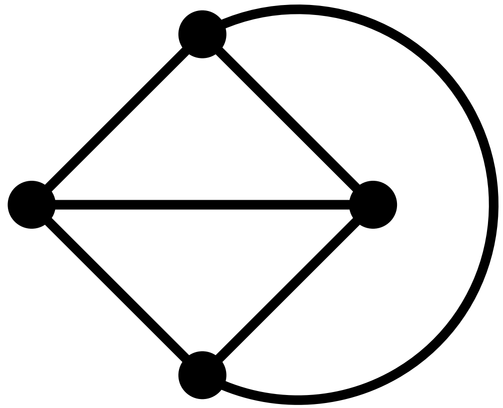
Drawing Styles

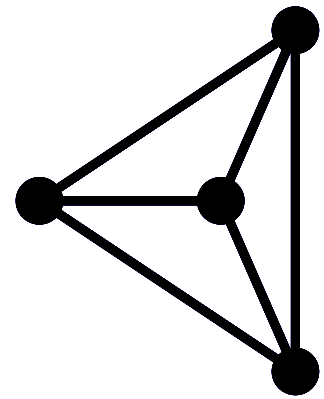


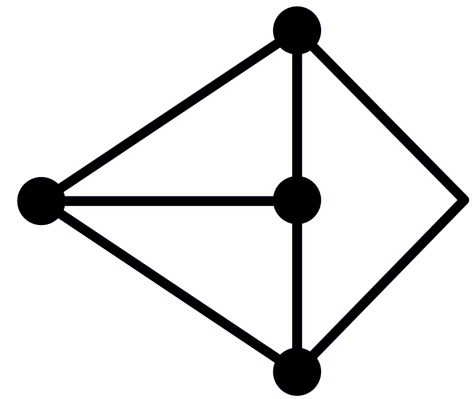
straight-line edges or with *bends*

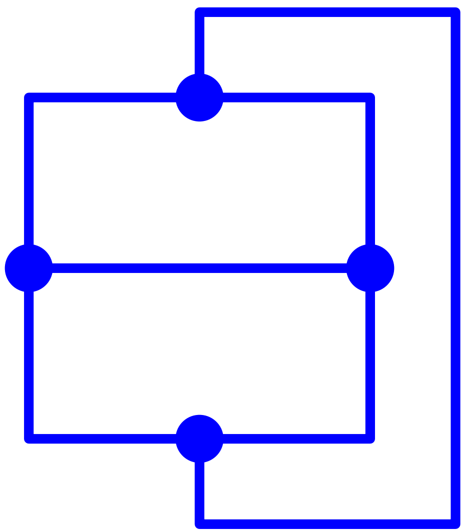
Drawing Styles





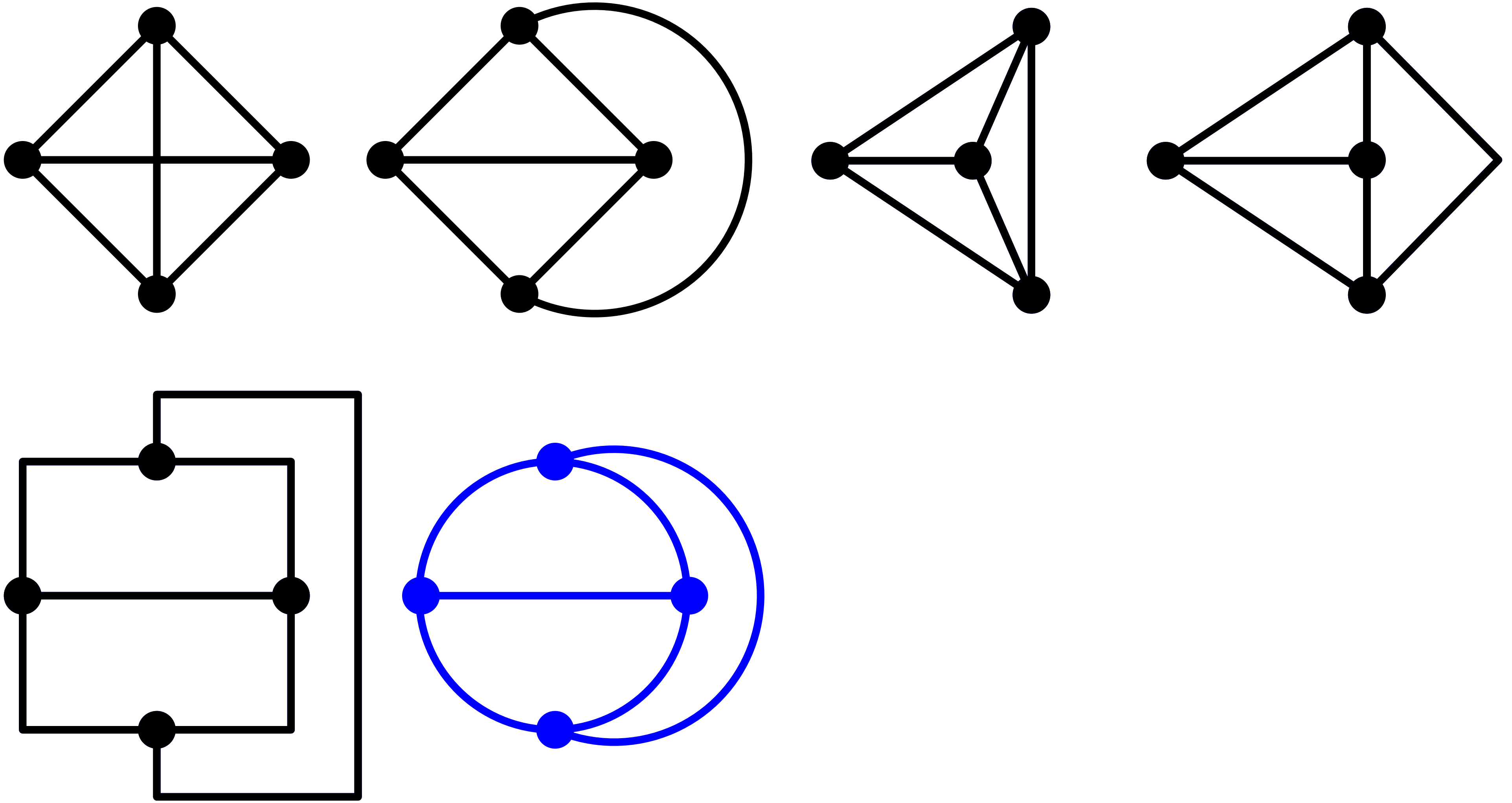






orthogonal

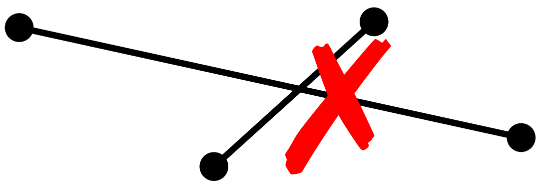
Drawing Styles



curvy edges

Planar Graphs

[Def] graph G *planar* $\Leftrightarrow$ G can be drawn without crossings



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Let G be a graph with n nodes.
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Any planar graph can be drawn with *straight-line* edges.

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draw?

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Any planar graph with n nodes can be drawn on the $(n-2) \times (n-2)$ grid in $O(n)$ time.

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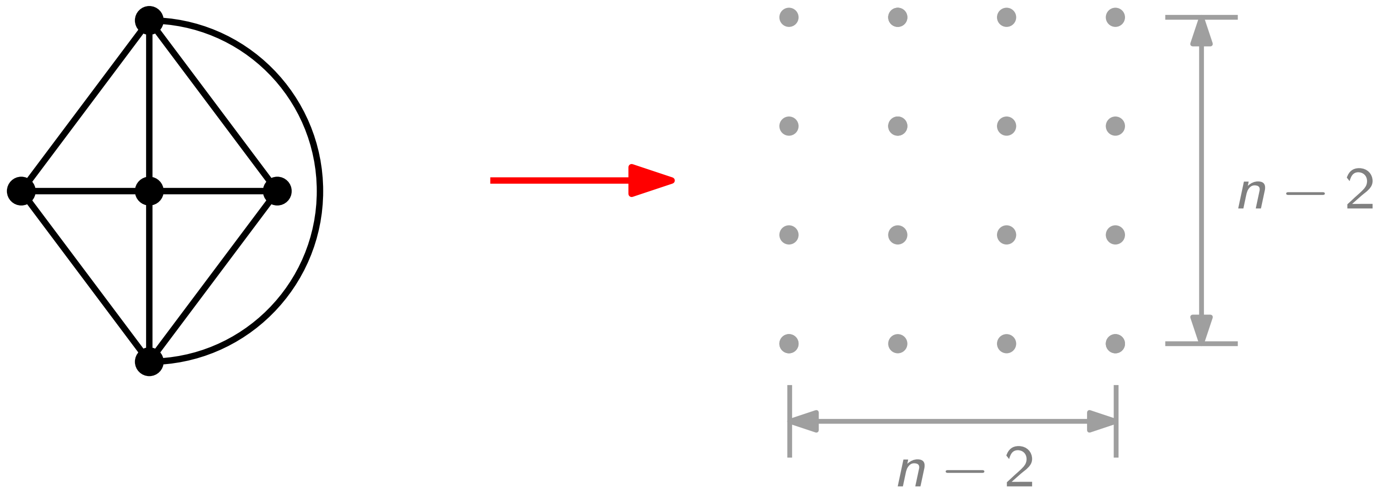

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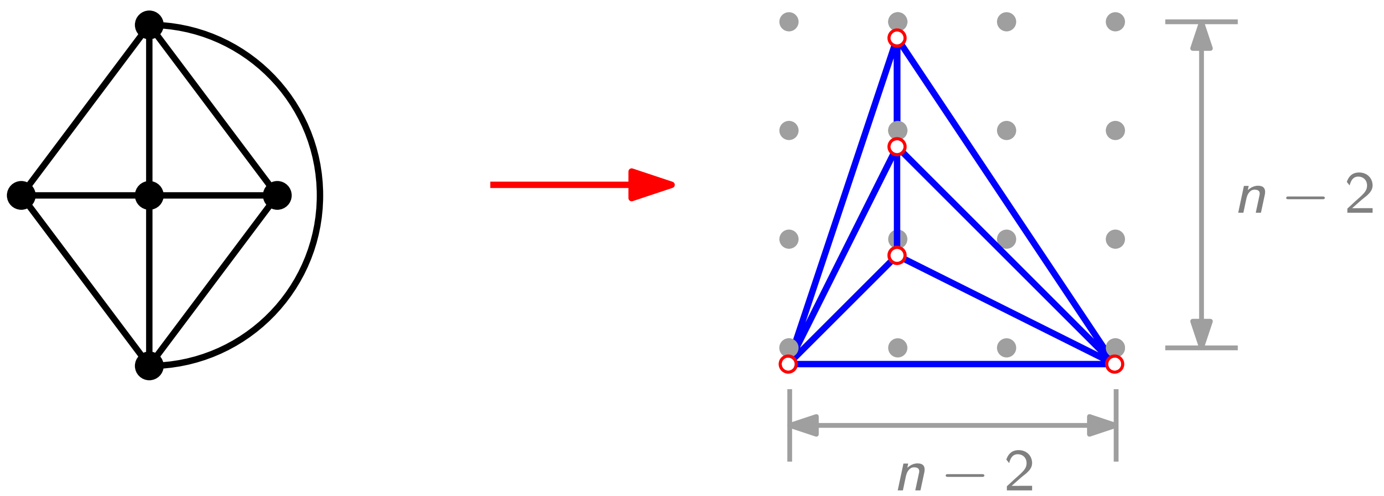
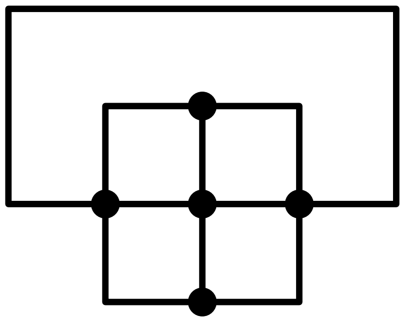


- nodes on grid points
- compact drawing

Orthogonal Layouts

- all edge segments are horizontal or vertical
- a well-studied drawing convention
- many examples in applications



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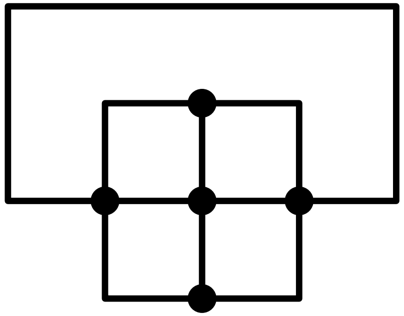

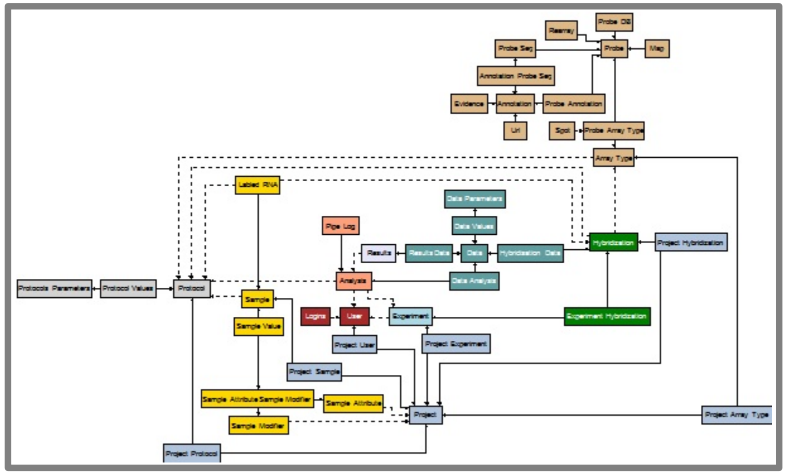

ER diagram in OGDF

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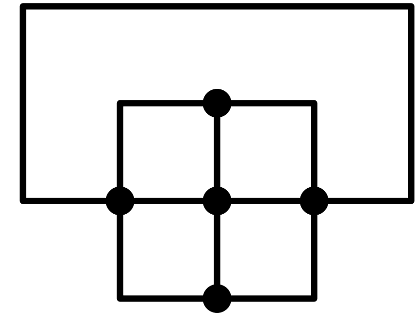

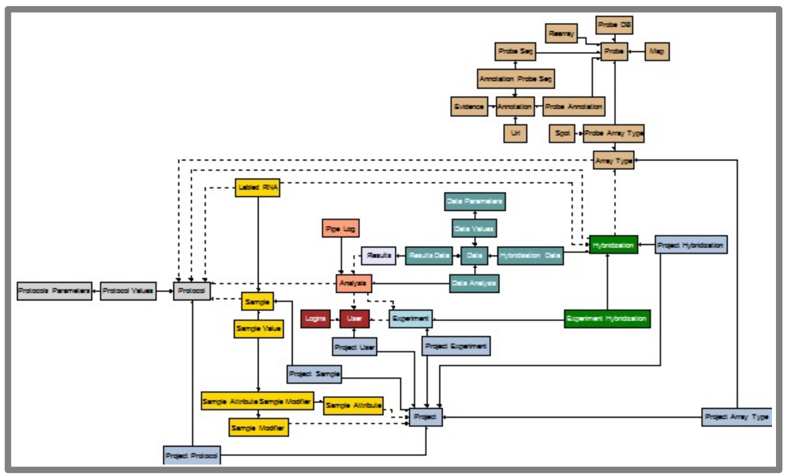

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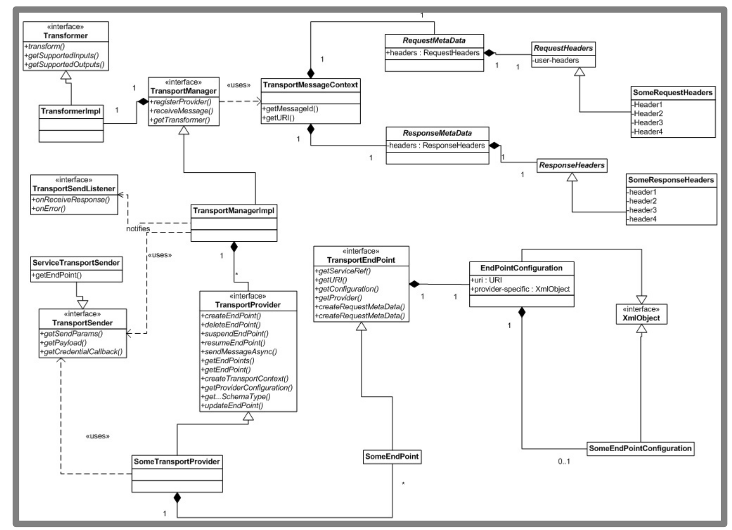

UML diagram by Oracle

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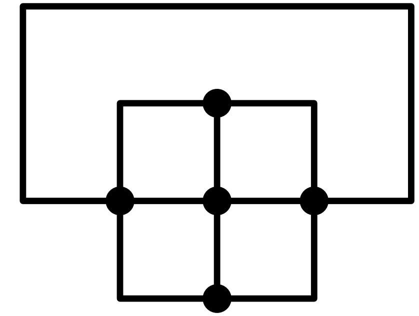

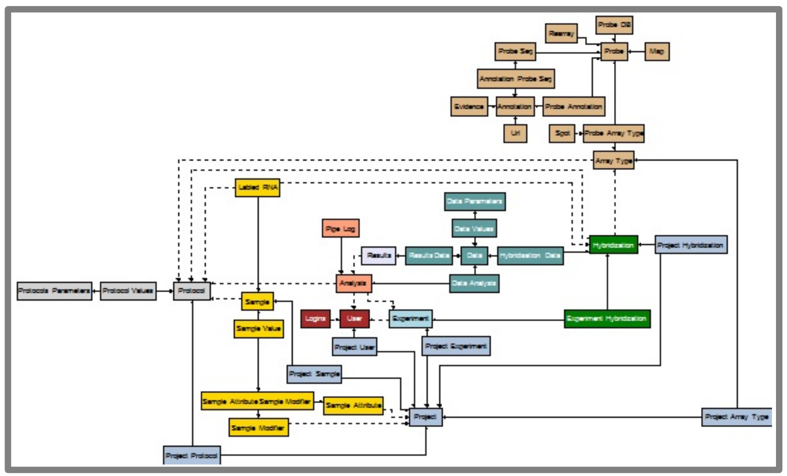

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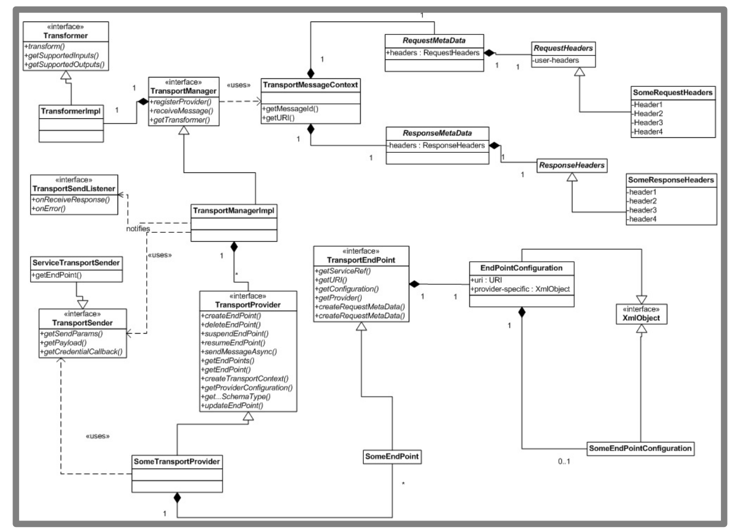

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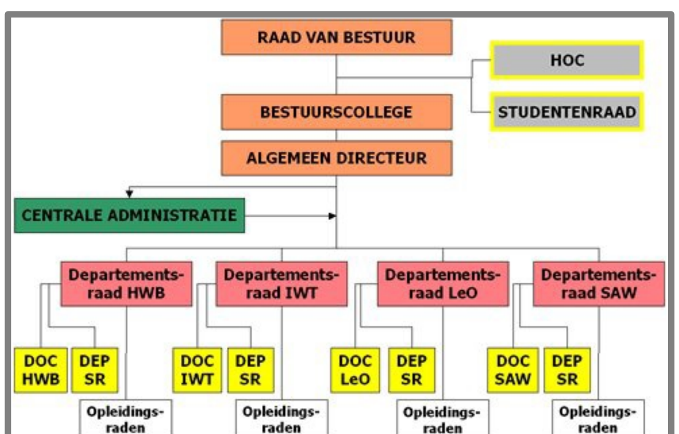

Organigram of HS Limburg

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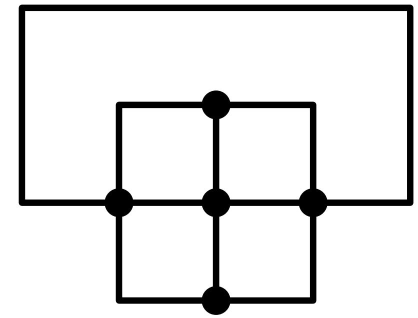

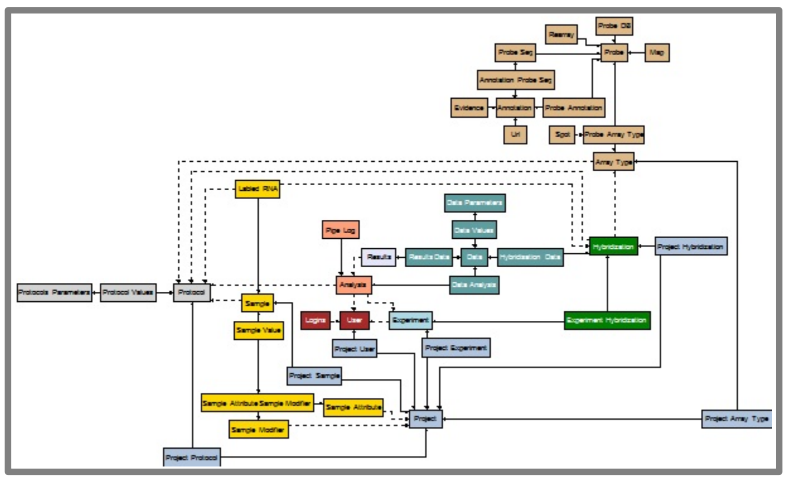

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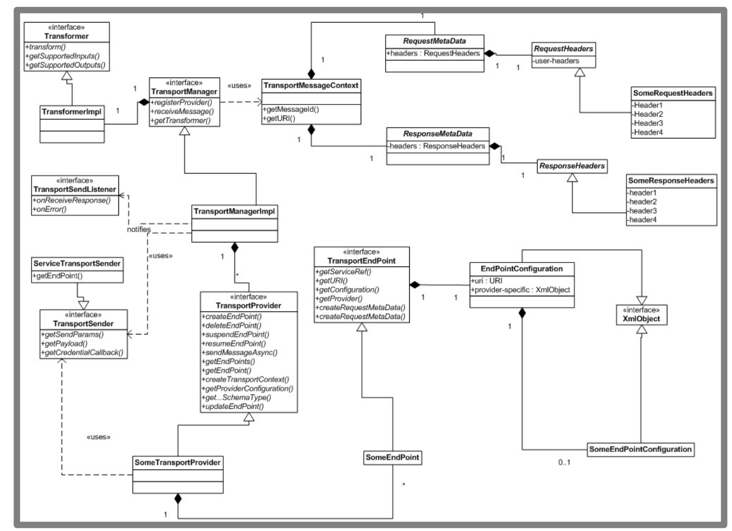

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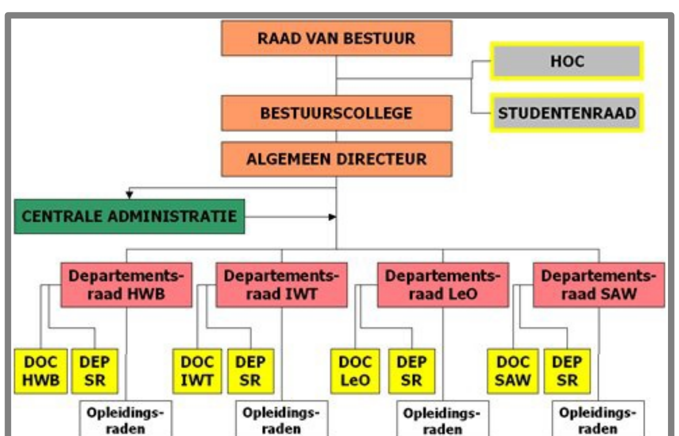

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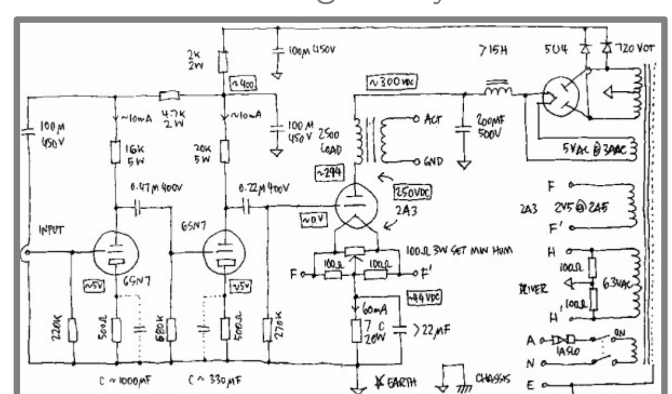

Circuit diagram by Jeff Atwood

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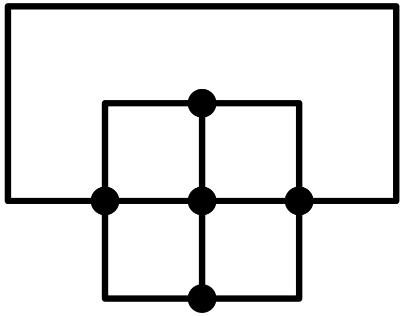
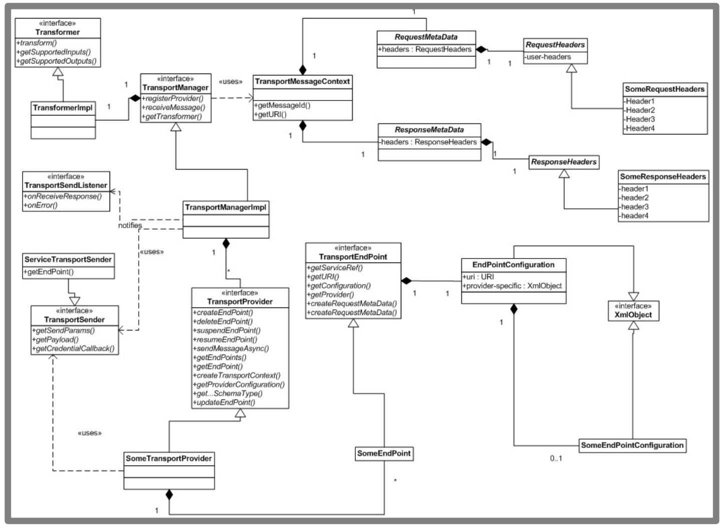


Fused Grid city layouts



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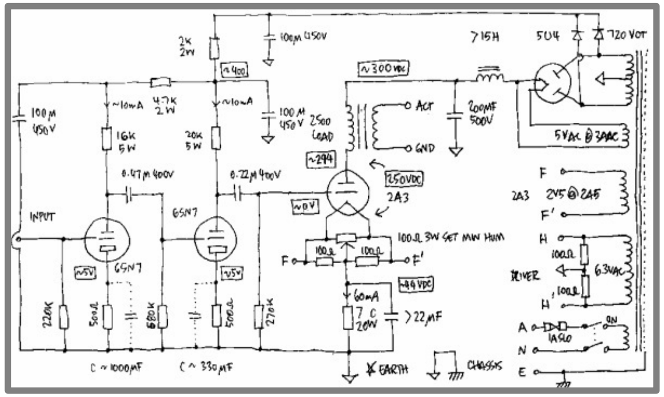

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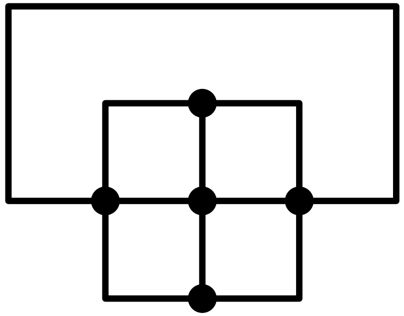

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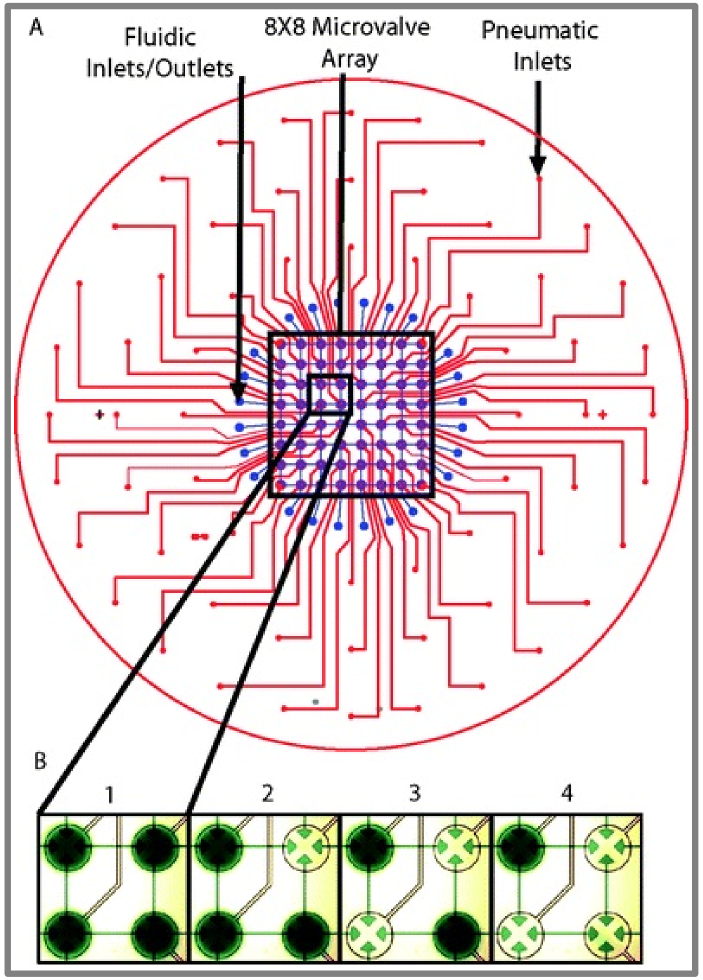

VLSI/PCI chip design

Orthogonal Layouts – Well-Known Results

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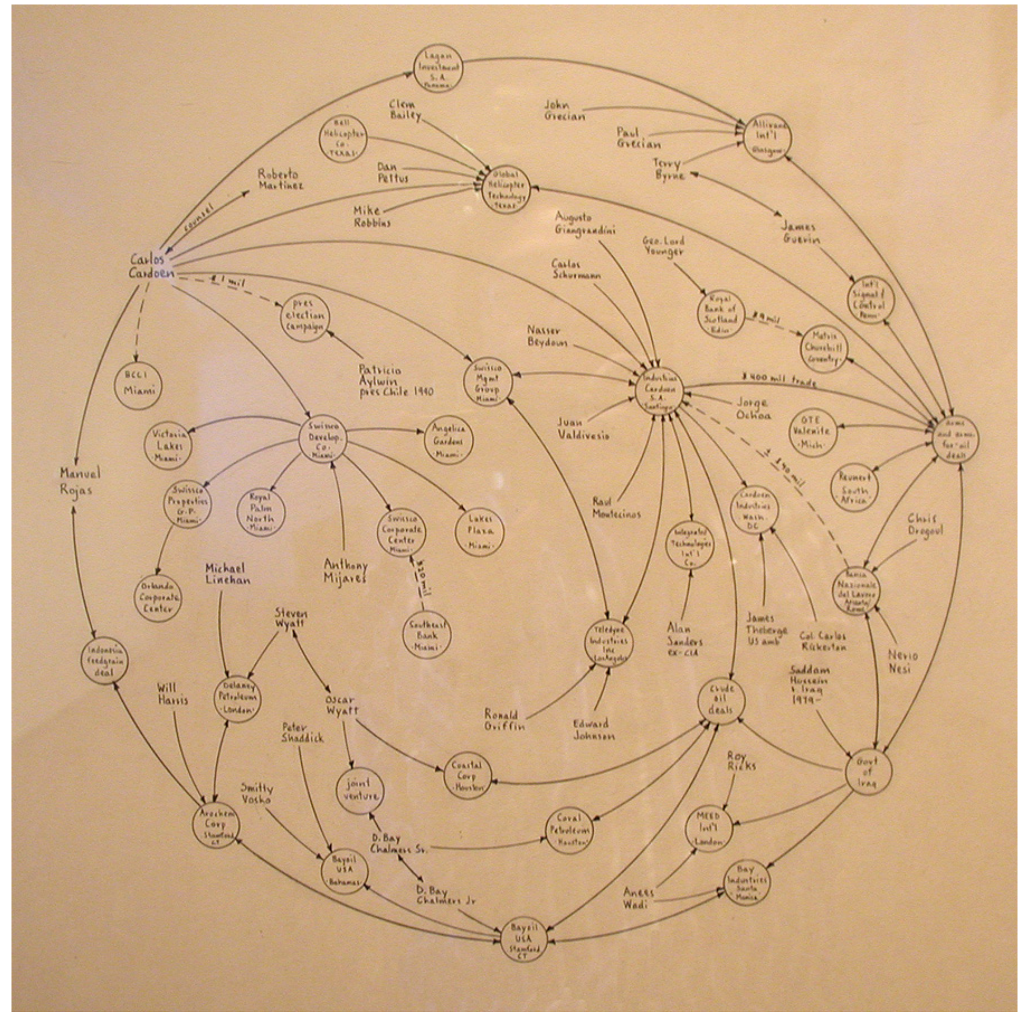
Smooth Drawings

Lombardi drawings

- circular arc edges
- perfect angular resolution



Mark Lombardi
(1951–2000)

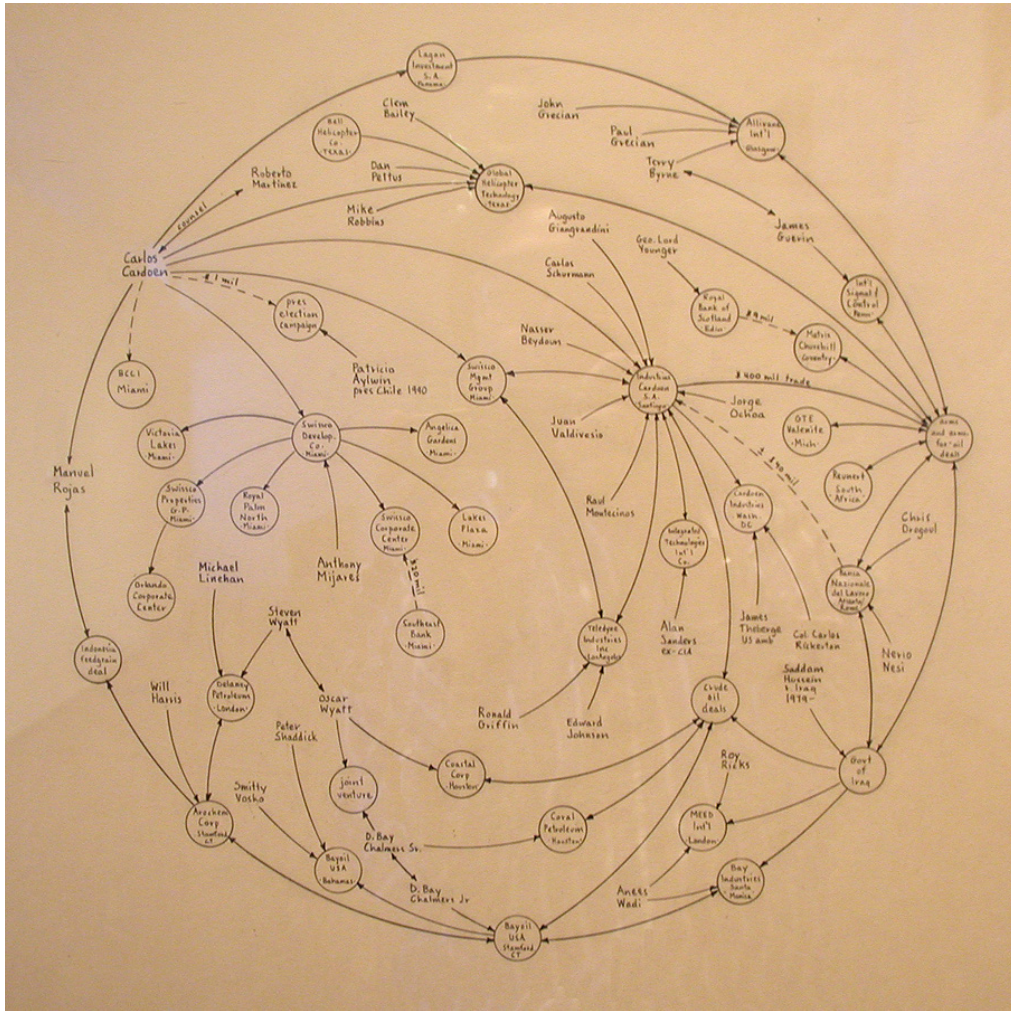
Smooth Drawings

Lombardi drawings

- circular arc edges
- perfect angular resolution

k-Lombardi drawings

- each edge sequence of k circular arcs



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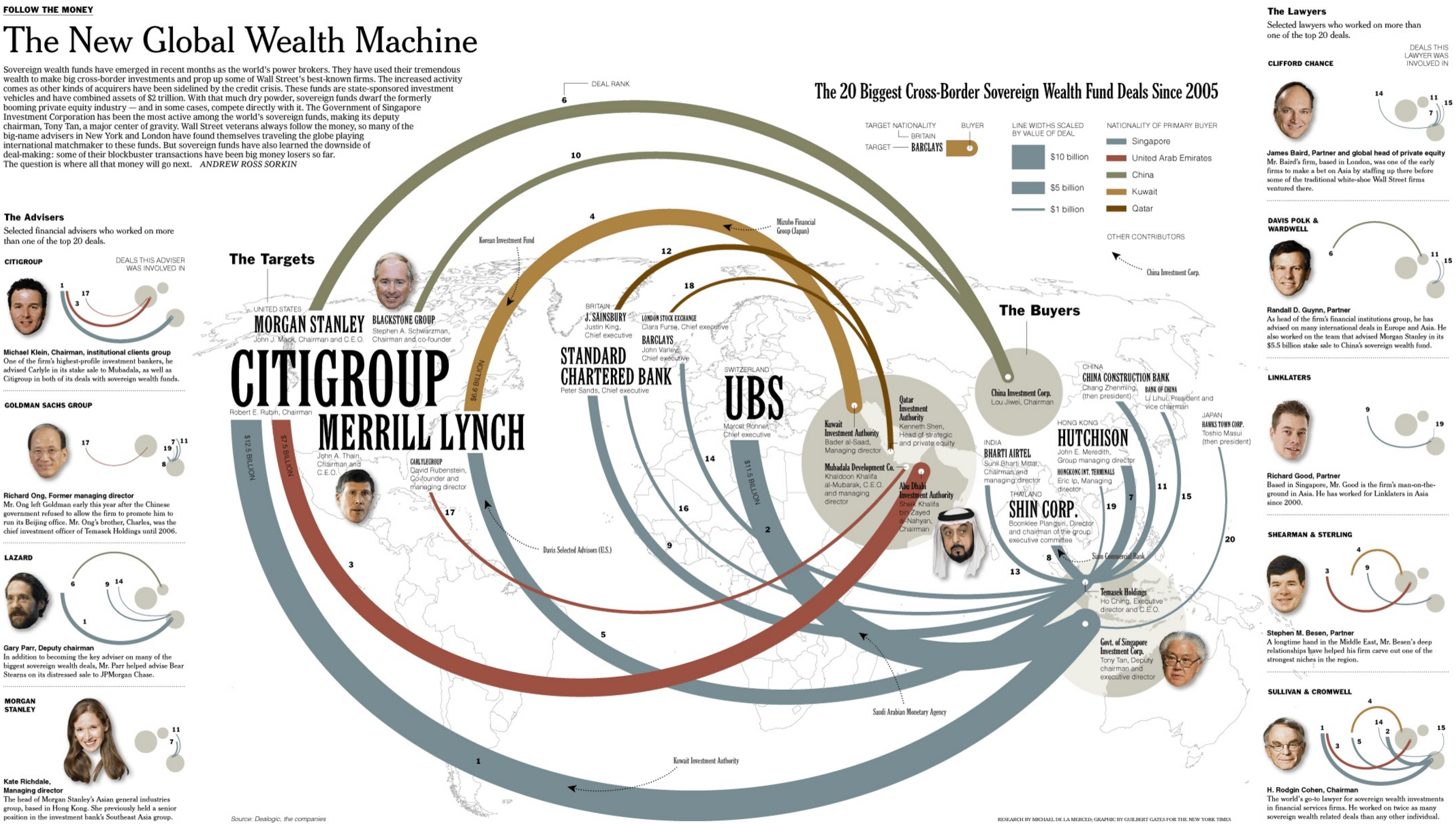
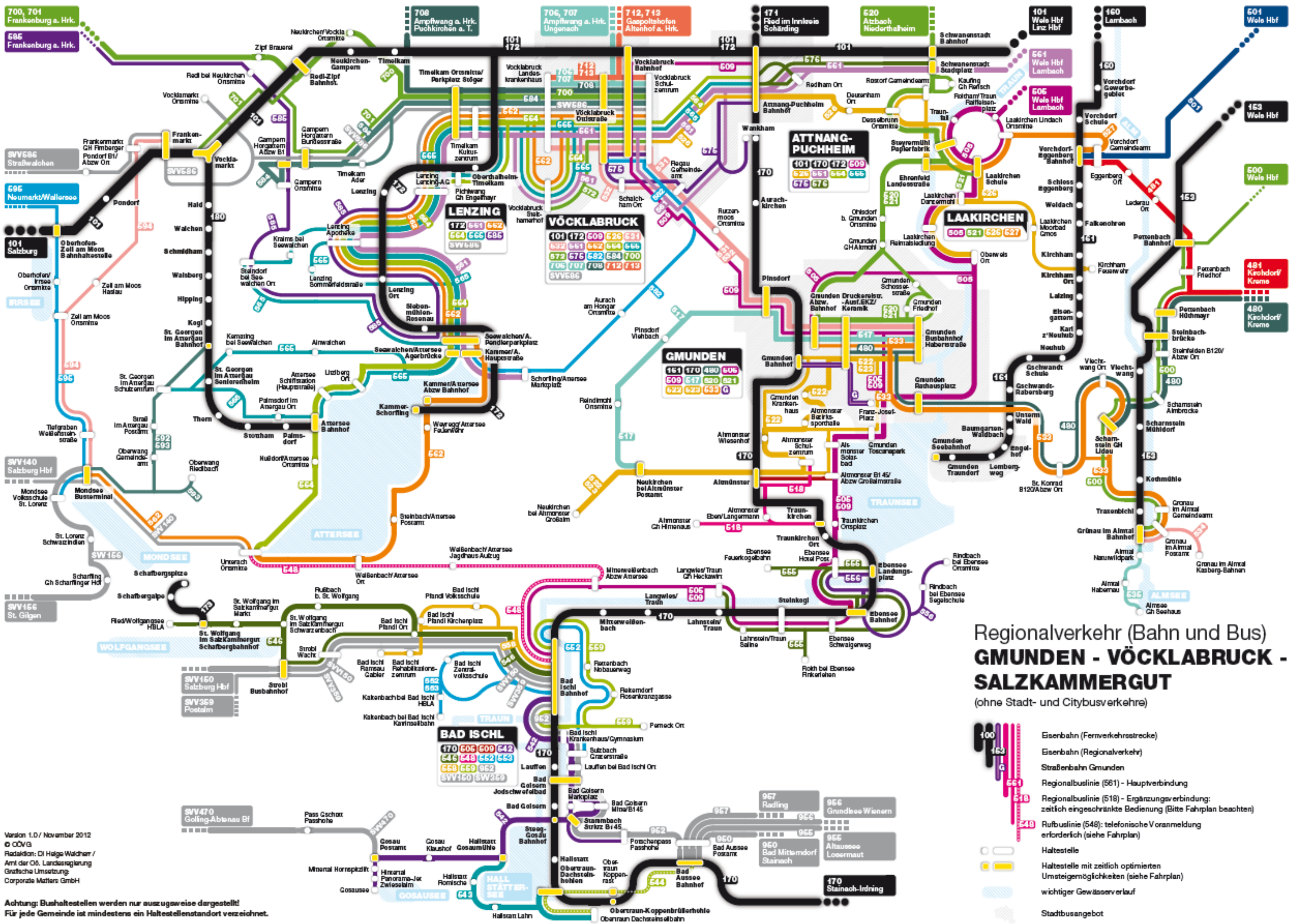


Smooth Drawings

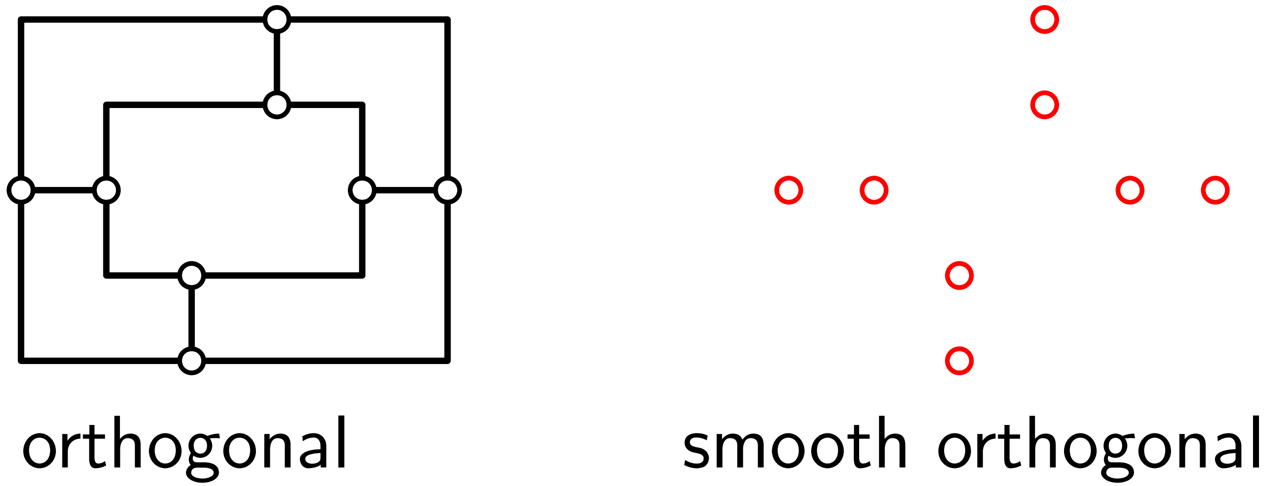
City model; plan

Smooth Drawings



Smooth Orthogonal Layouts

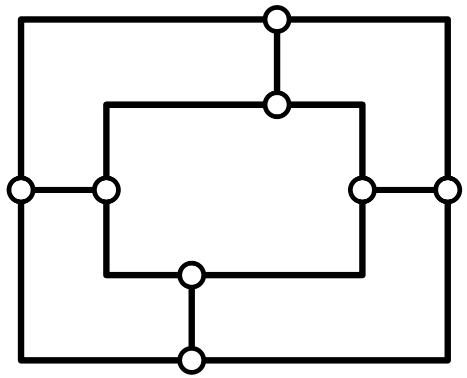
Combine both worlds:



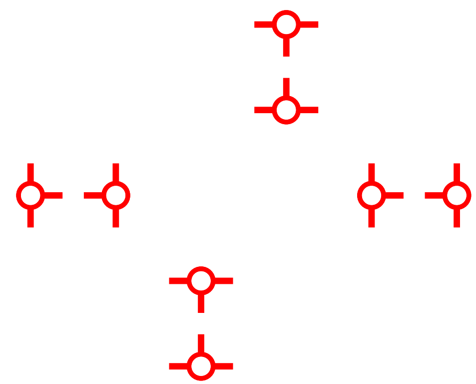
Smooth Orthogonal Layouts

Combine both worlds:

- edges leave and enter vertices horizontally or vertically



orthogonal

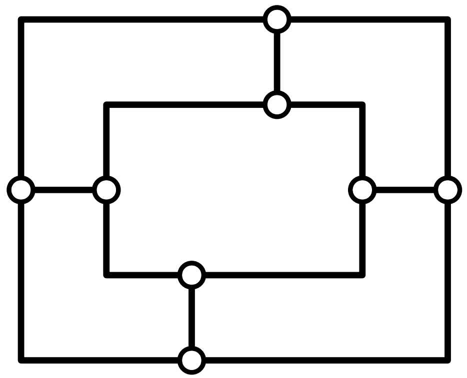


smooth orthogonal

Smooth Orthogonal Layouts

Combine both worlds:

- edges leave and enter vertices horizontally or vertically
- each edge is drawn as a sequence of axis-aligned line segments and circular-arc segments without bends



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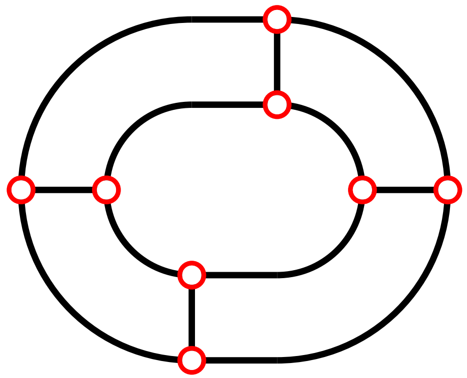
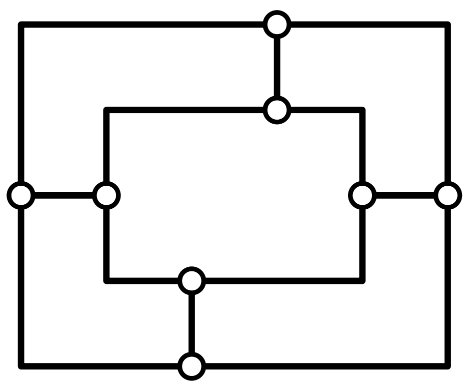


smooth orthogonal

Smooth Orthogonal Layouts

Combine both worlds:

- edges leave and enter vertices horizontally or vertically
- each edge is drawn as a sequence of axis-aligned line segments and circular-arc segments without bends
- there are no edge-crossings (for planar graphs)



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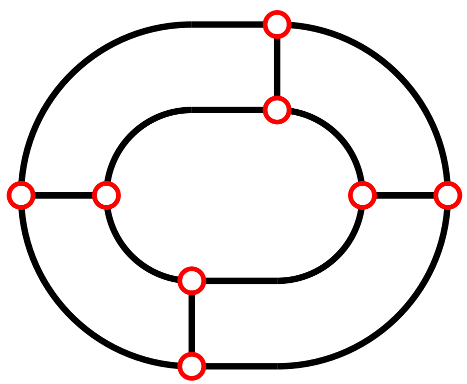
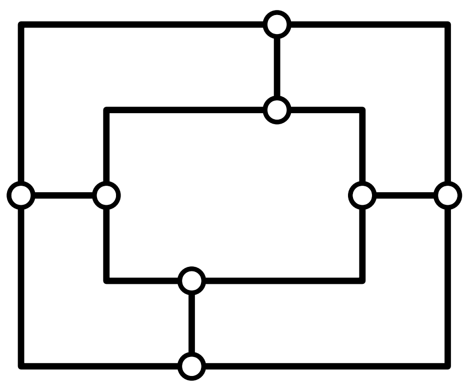
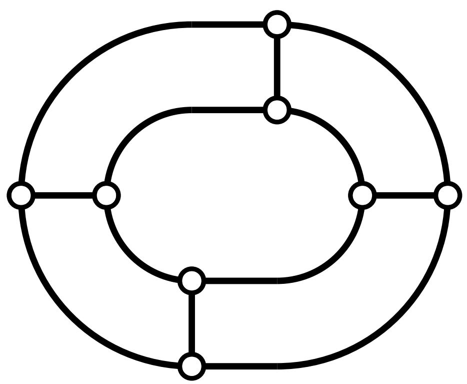


smooth orthogonal

Edge Complexity



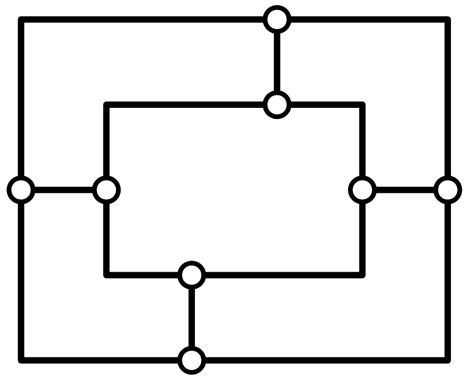
orthogonal



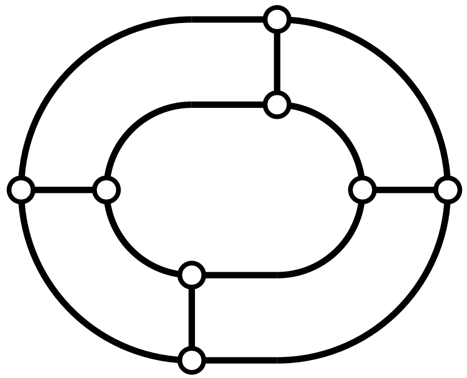
smooth orthogonal

Edge Complexity

complexity of an edge: number of arcs



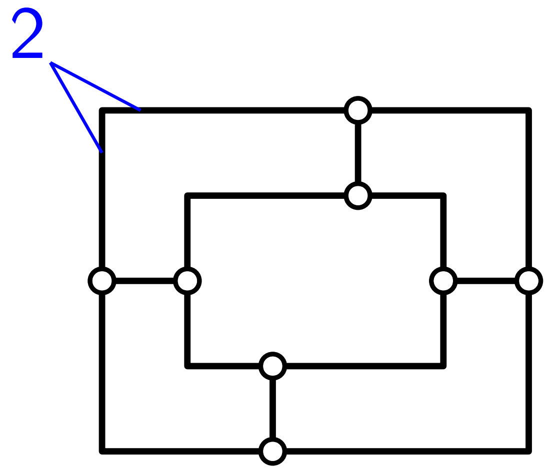
orthogonal



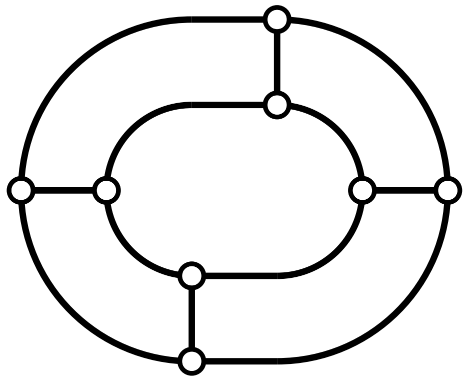
smooth orthogonal

Edge Complexity

complexity of an edge: number of arcs



orthogonal



smooth orthogonal

Edge Complexity

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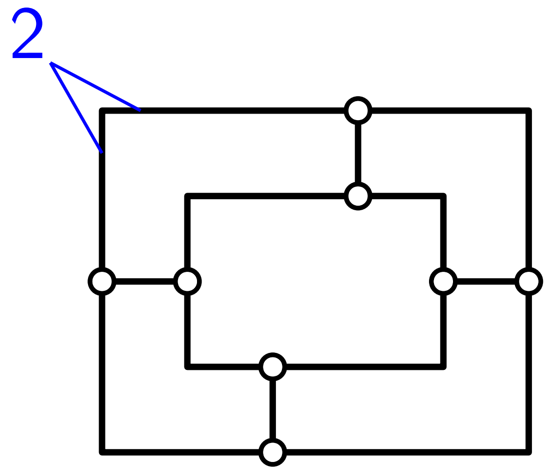
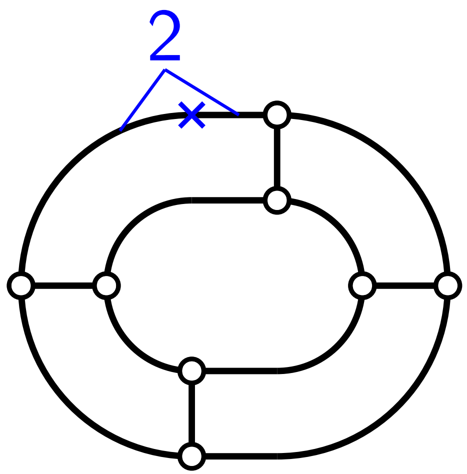


orthogonal

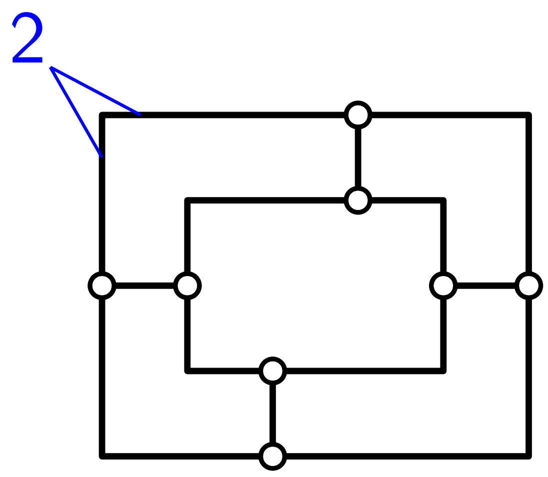


smooth orthogonal

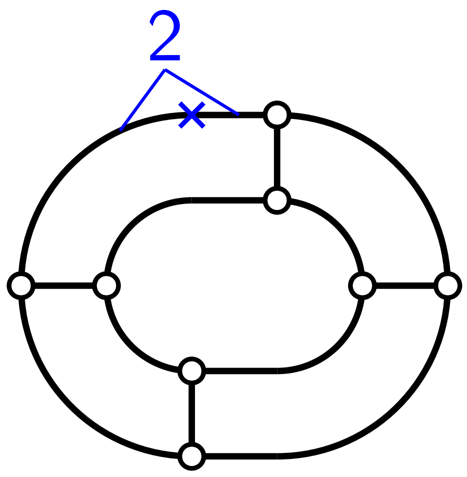
Edge Complexity

complexity of an edge: number of arcs

edge complexity: maximum complexity over all edges



orthogonal

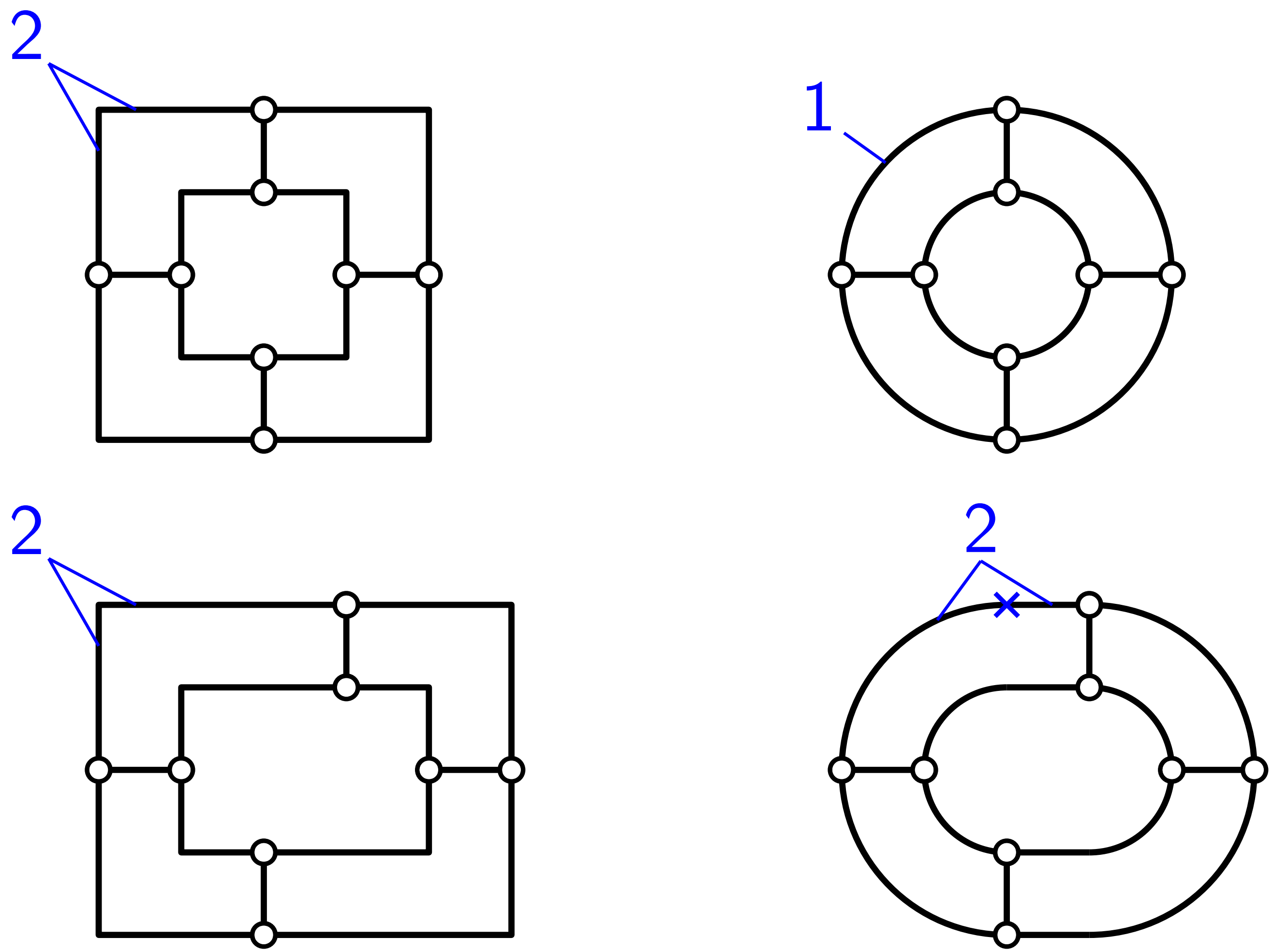


smooth orthogonal

Edge Complexity

complexity of an edge: number of arcs

edge complexity: maximum complexity over all edges



orthogonal smooth orthogonal

Liu et al. Algorithm

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-3 layout

Liu et al. Algorithm

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-3 layout

- choose vertices s and t

Liu et al. Algorithm

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-3 layout

- choose vertices s and t
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out:	$\uparrow$	$\rightarrow$	$\leftarrow$
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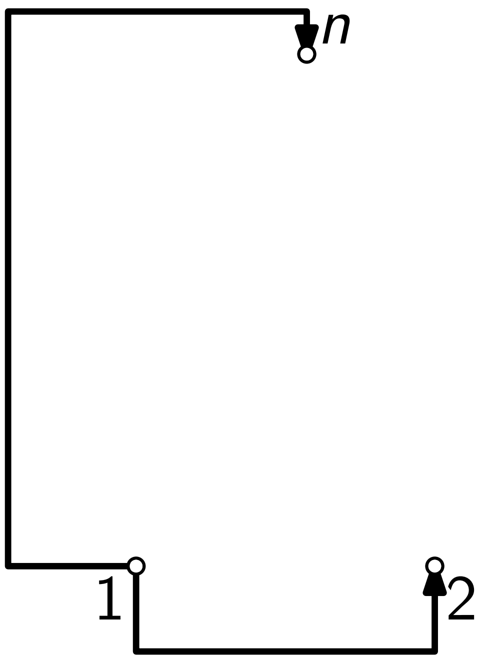

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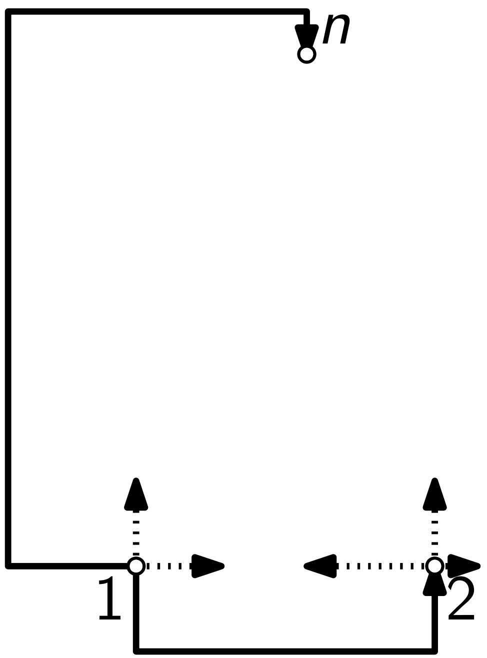

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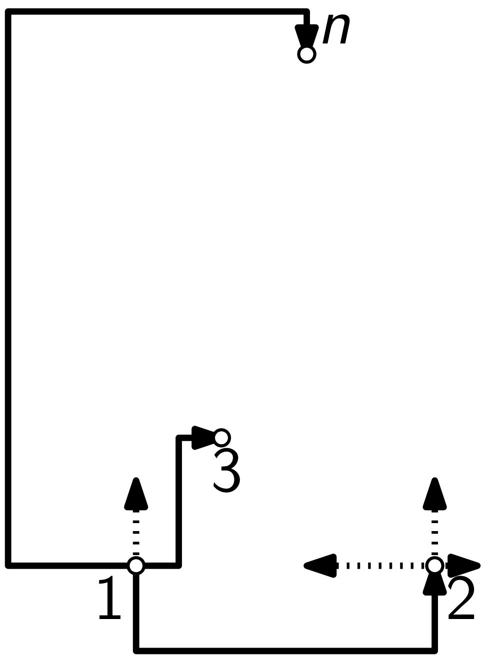

Liu et al. Algorithm

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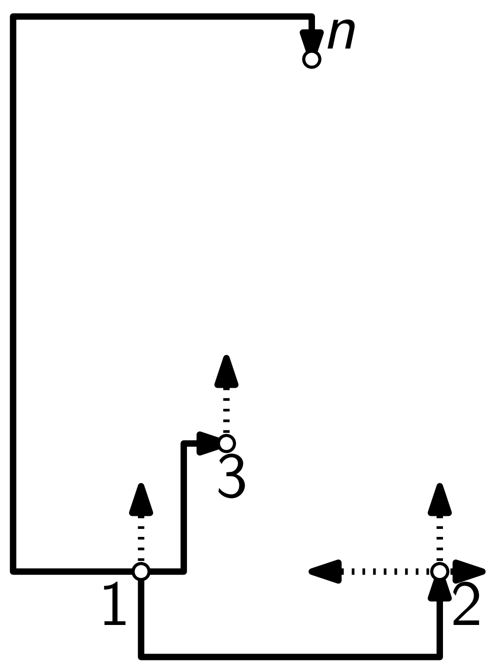

Liu et al. Algorithm

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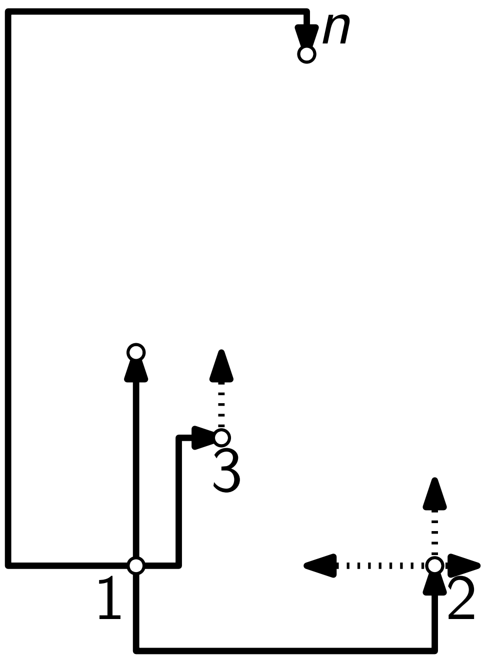

Liu et al. Algorithm

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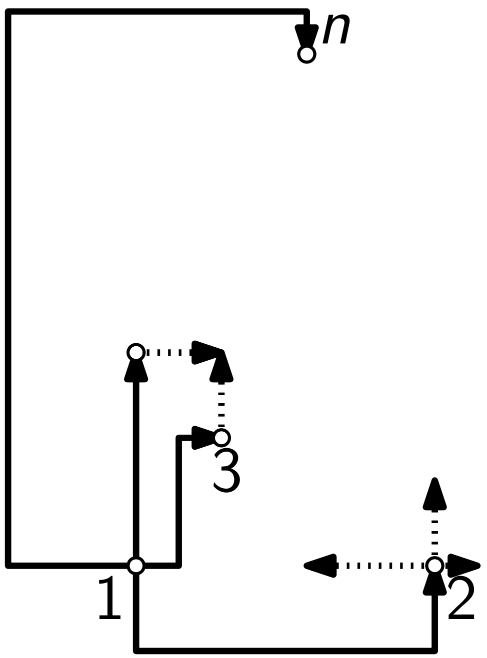

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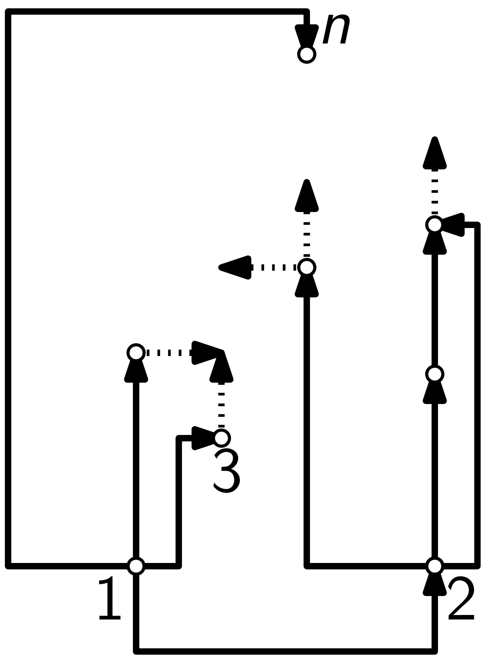

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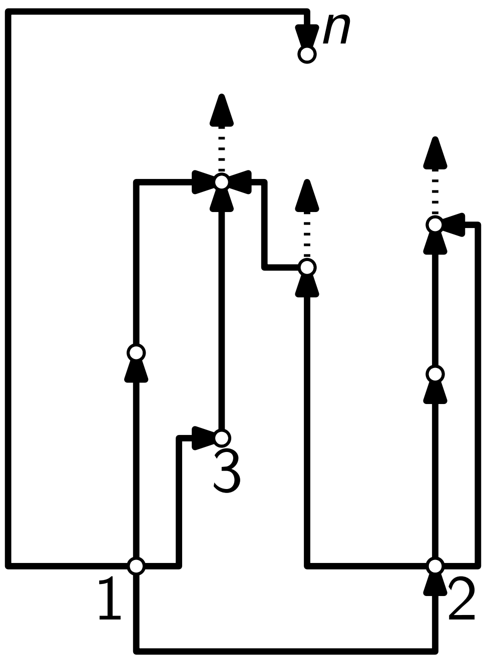

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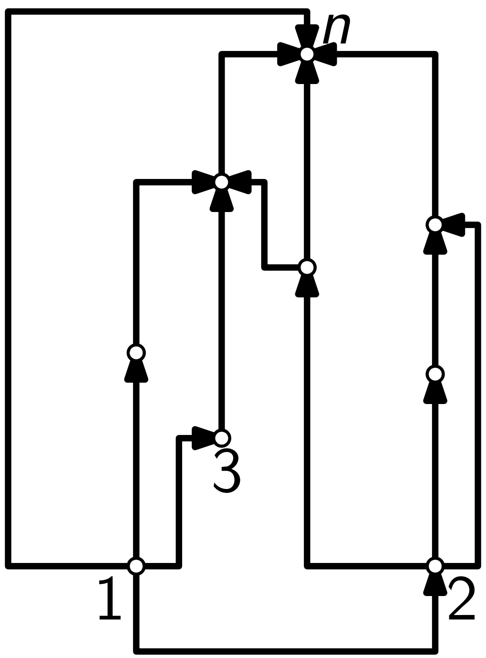

Liu et al. Algorithm

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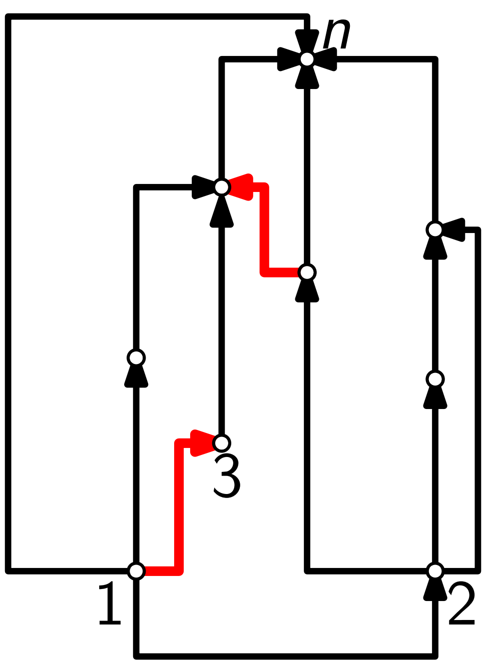
  in: $\downarrow \quad \leftarrow \quad \rightarrow$
- Eliminate S-shapes



Liu et al. Algorithm

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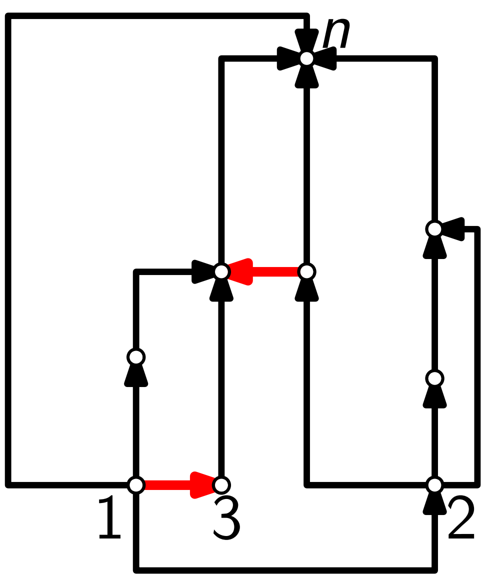
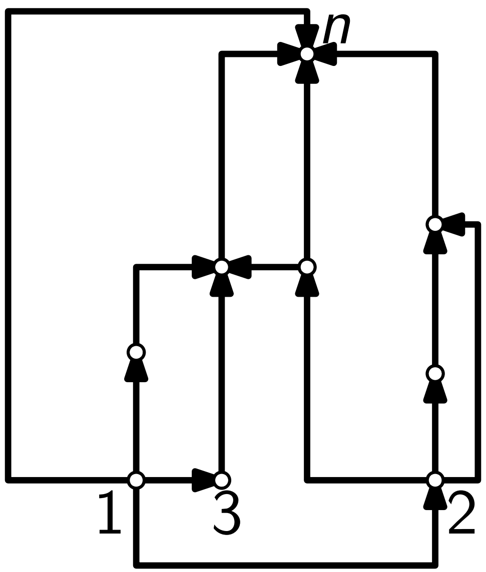


- Eliminate S-shapes

SC_2-Layouts

biconnected 4-planar graph → orthogonal complexity-2 layout

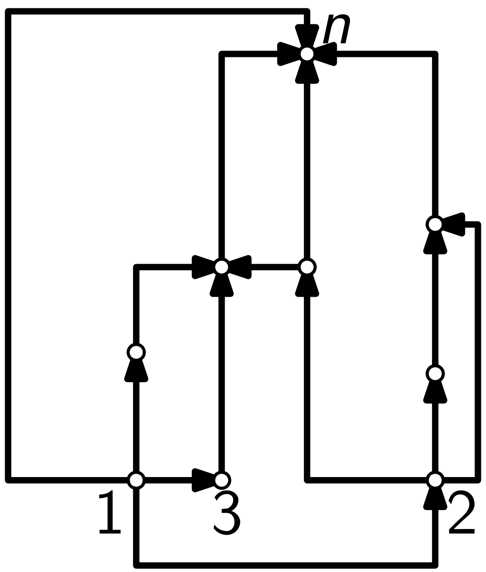
smooth



SC_2-Layouts

biconnected 4-planar graph → orthogonal complexity-2 layout
smooth

| → |

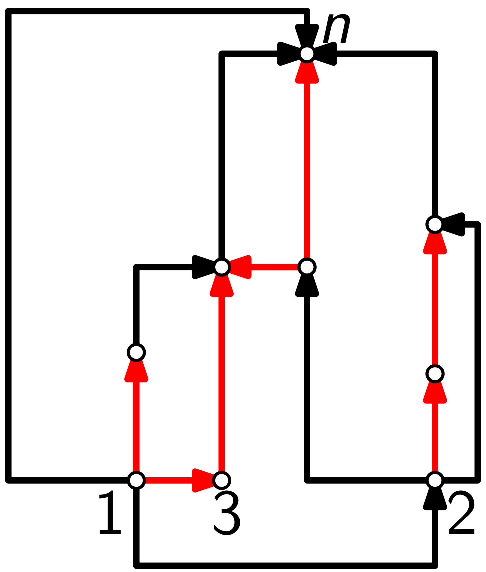


SC_2-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

smooth

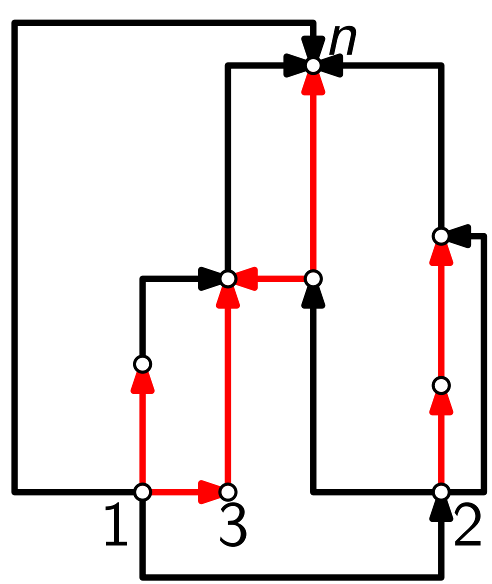
| $\rightarrow$ |



SC$_2$-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

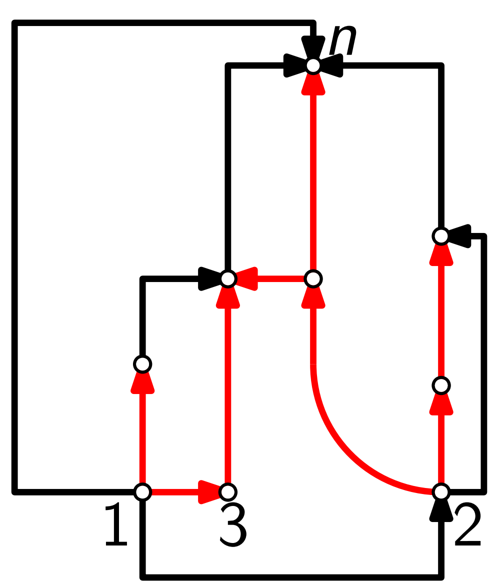
smooth



SC_2-Layouts

biconnected 4-planar graph → orthogonal complexity-2 layout

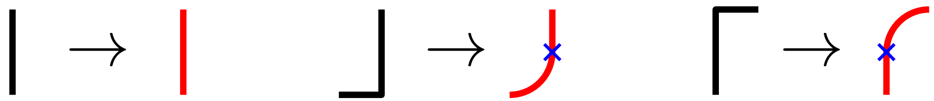
smooth

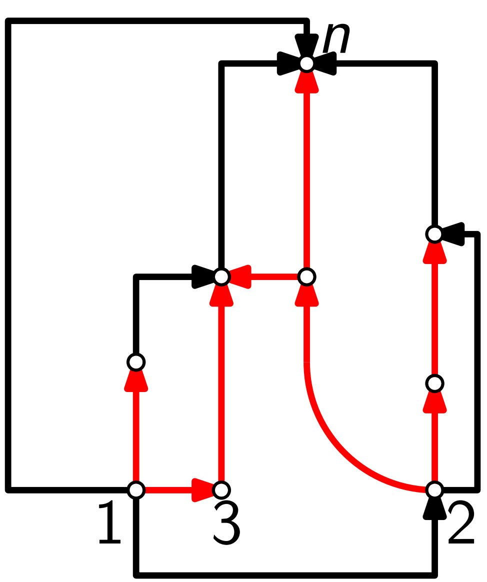


SC_2-Layouts

biconnected 4-planar graph → orthogonal complexity-2 layout

smooth





SC$_2$-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

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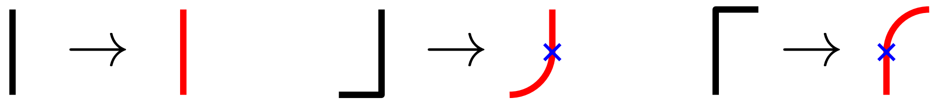

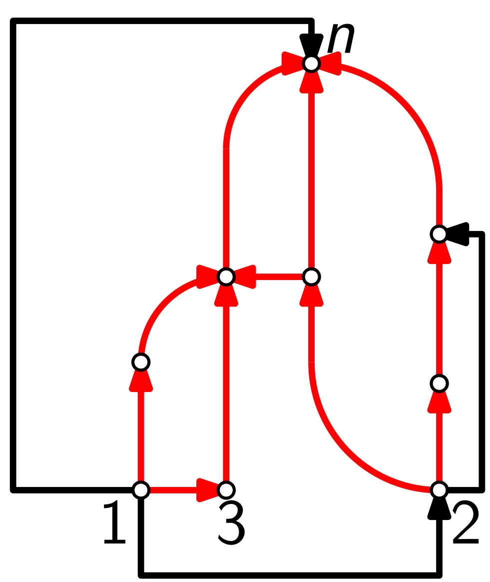

SC_2-Layouts

biconnected 4-planar graph → orthogonal complexity-2 layout

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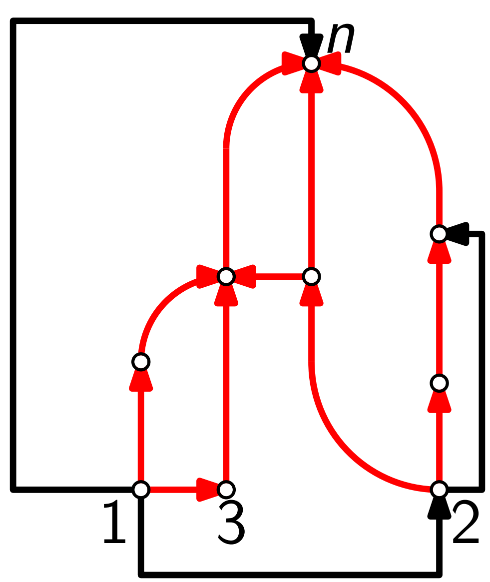

SC_2-Layouts

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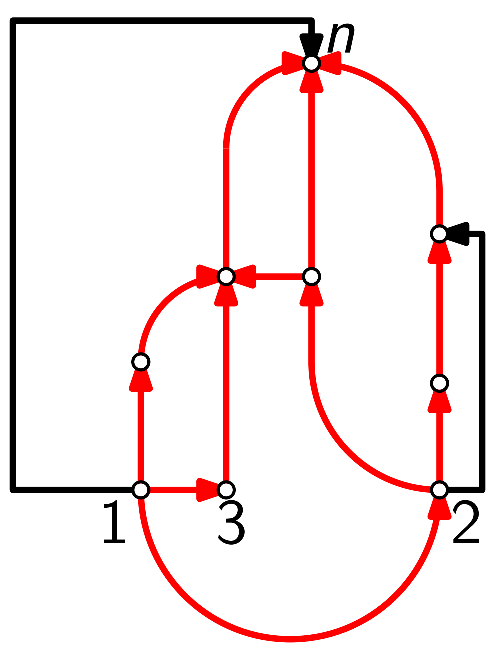
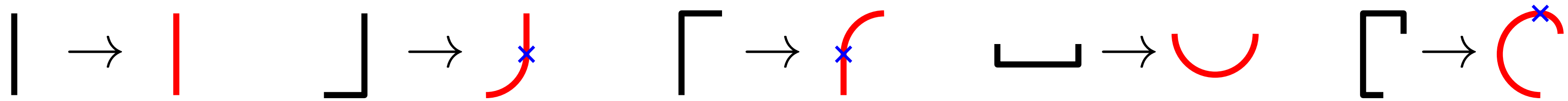
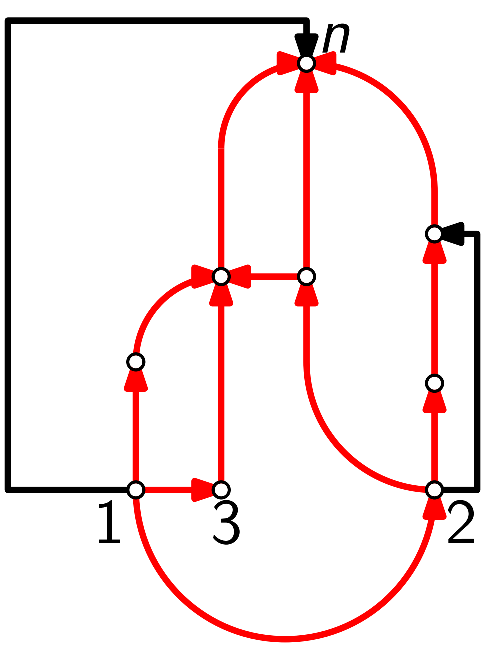


SC_2-Layouts

biconnected 4-planar graph → orthogonal complexity-2 layout

smooth





SC_2-Layouts

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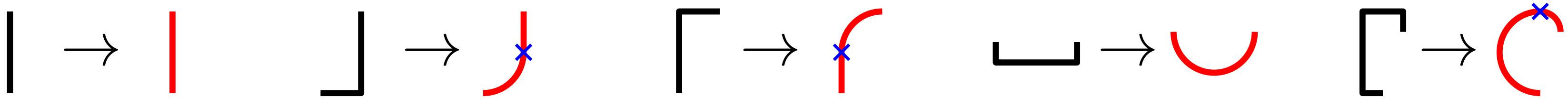

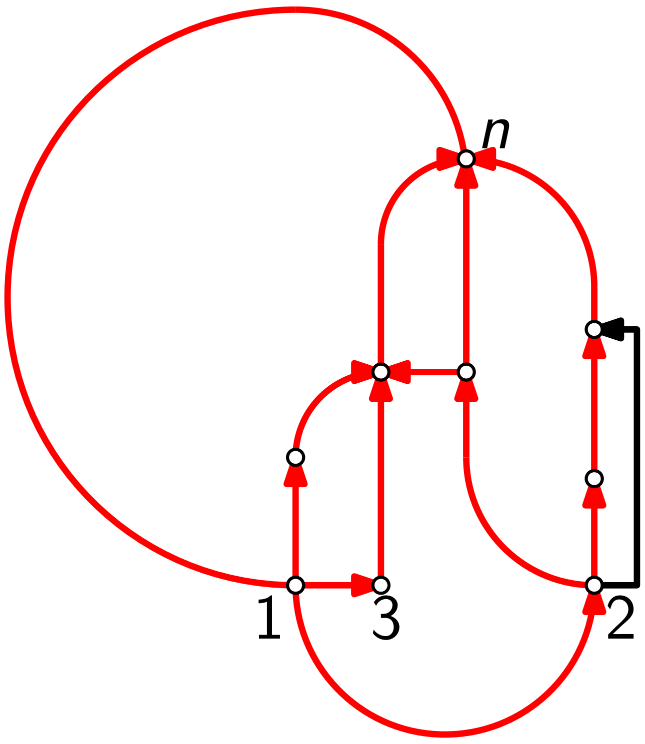

SC_2-Layouts

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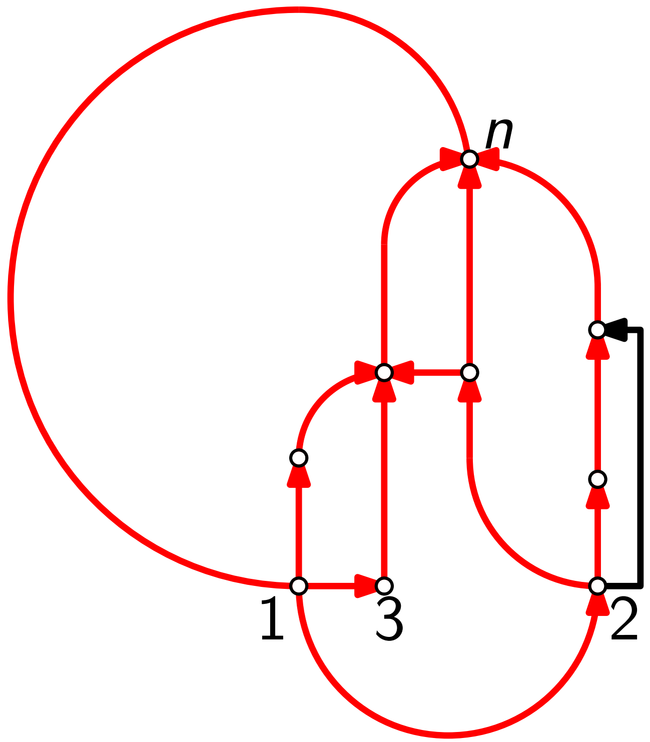
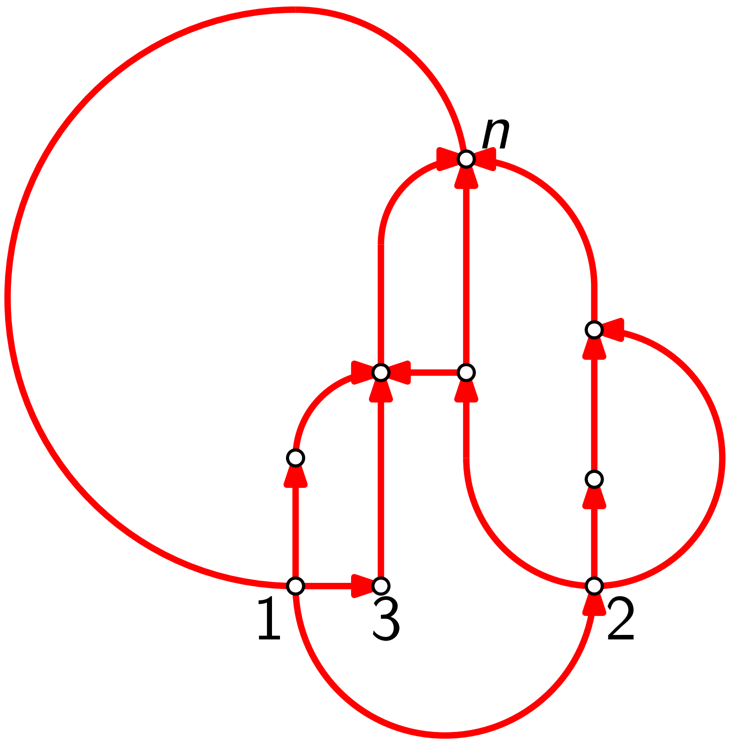


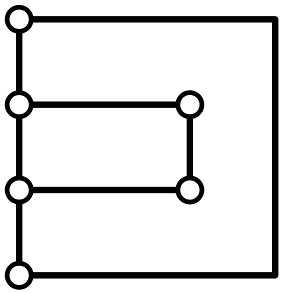
SC$_2$-Layouts

biconnected 4-planar graph $\rightarrow$ orthogonal complexity-2 layout

smooth



Crossings



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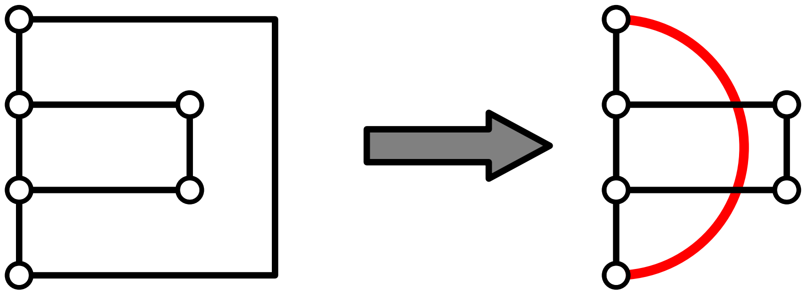

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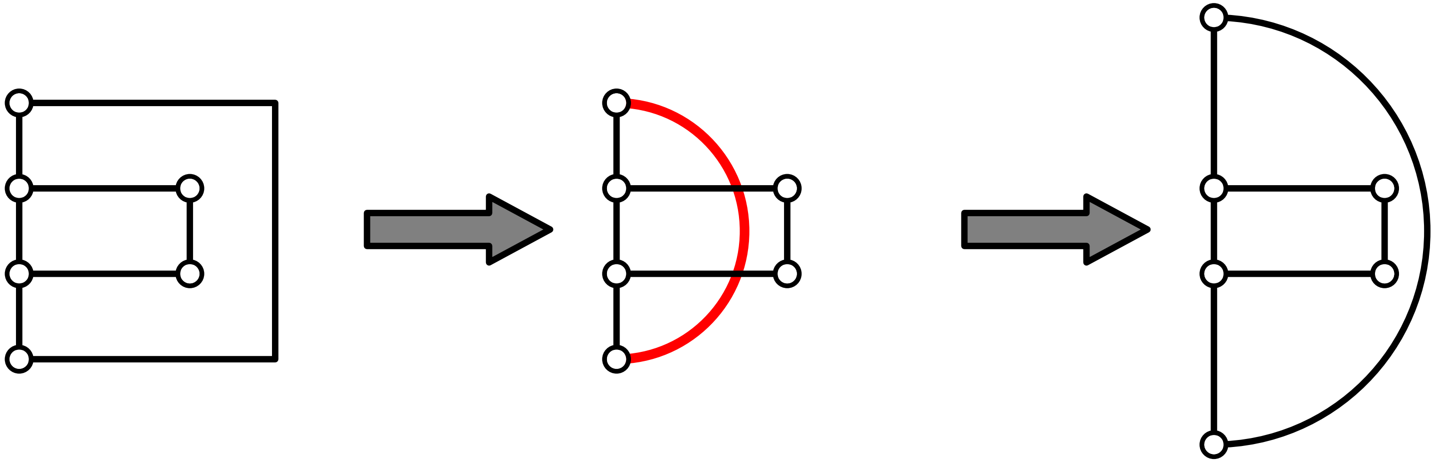
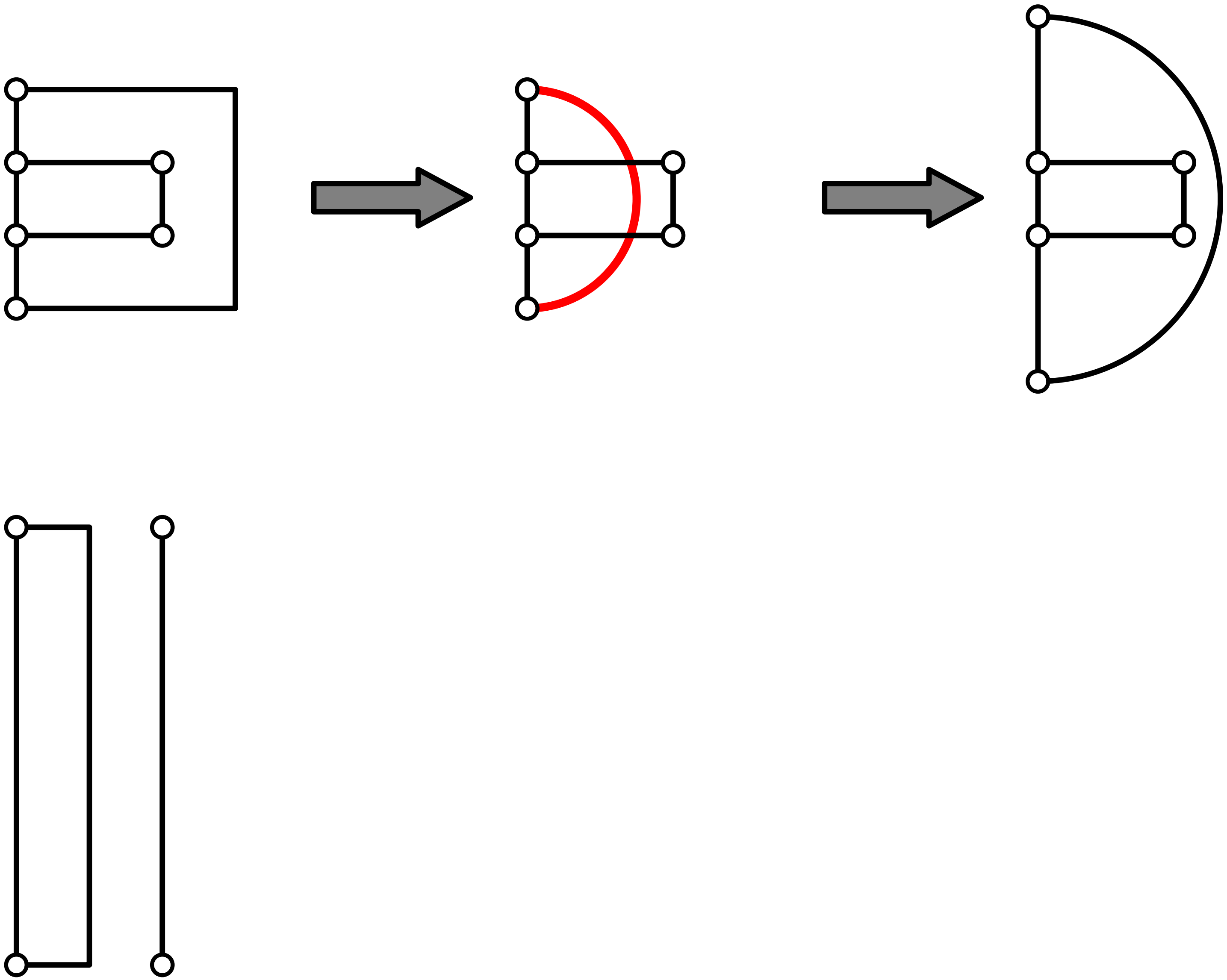


Crossings



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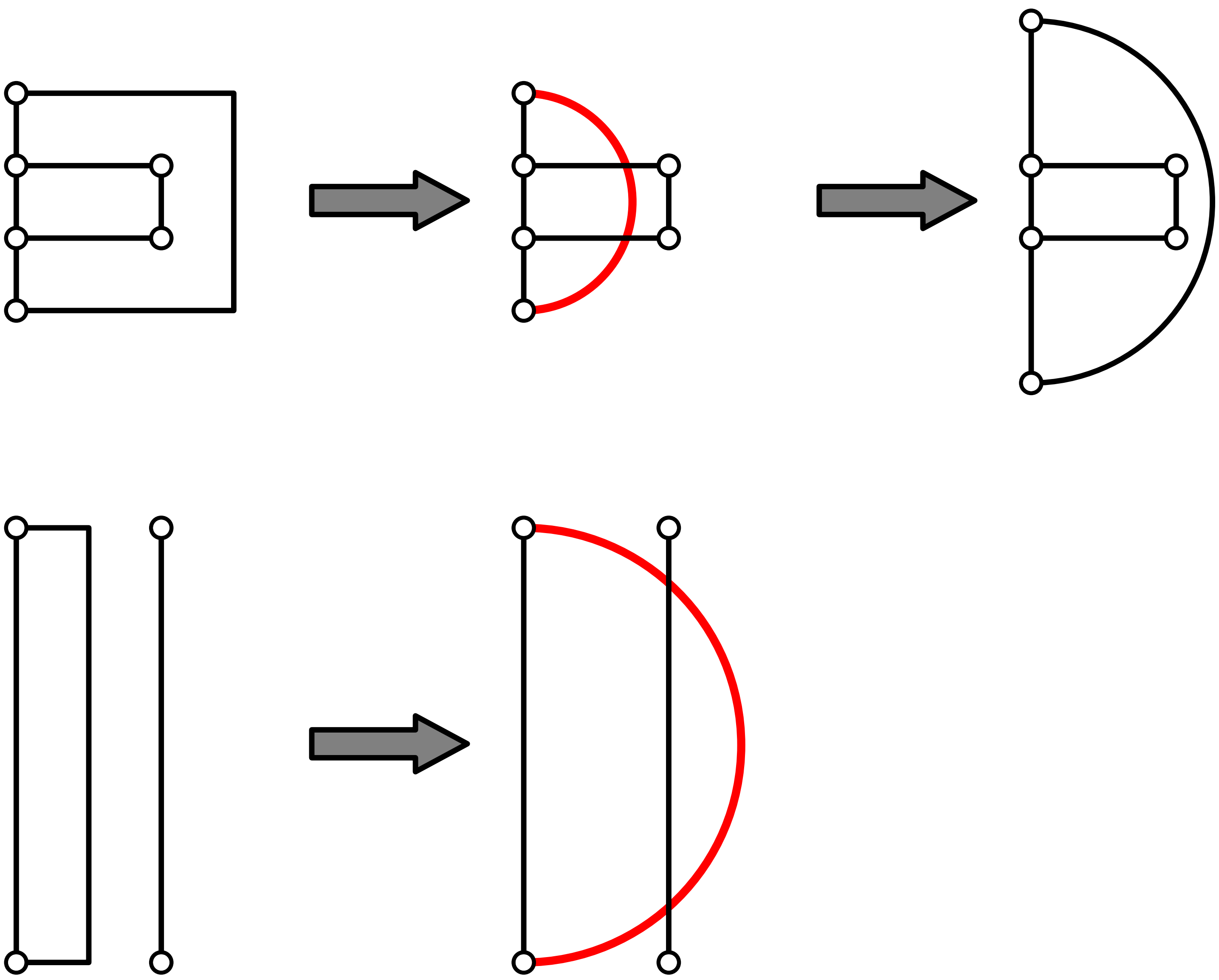
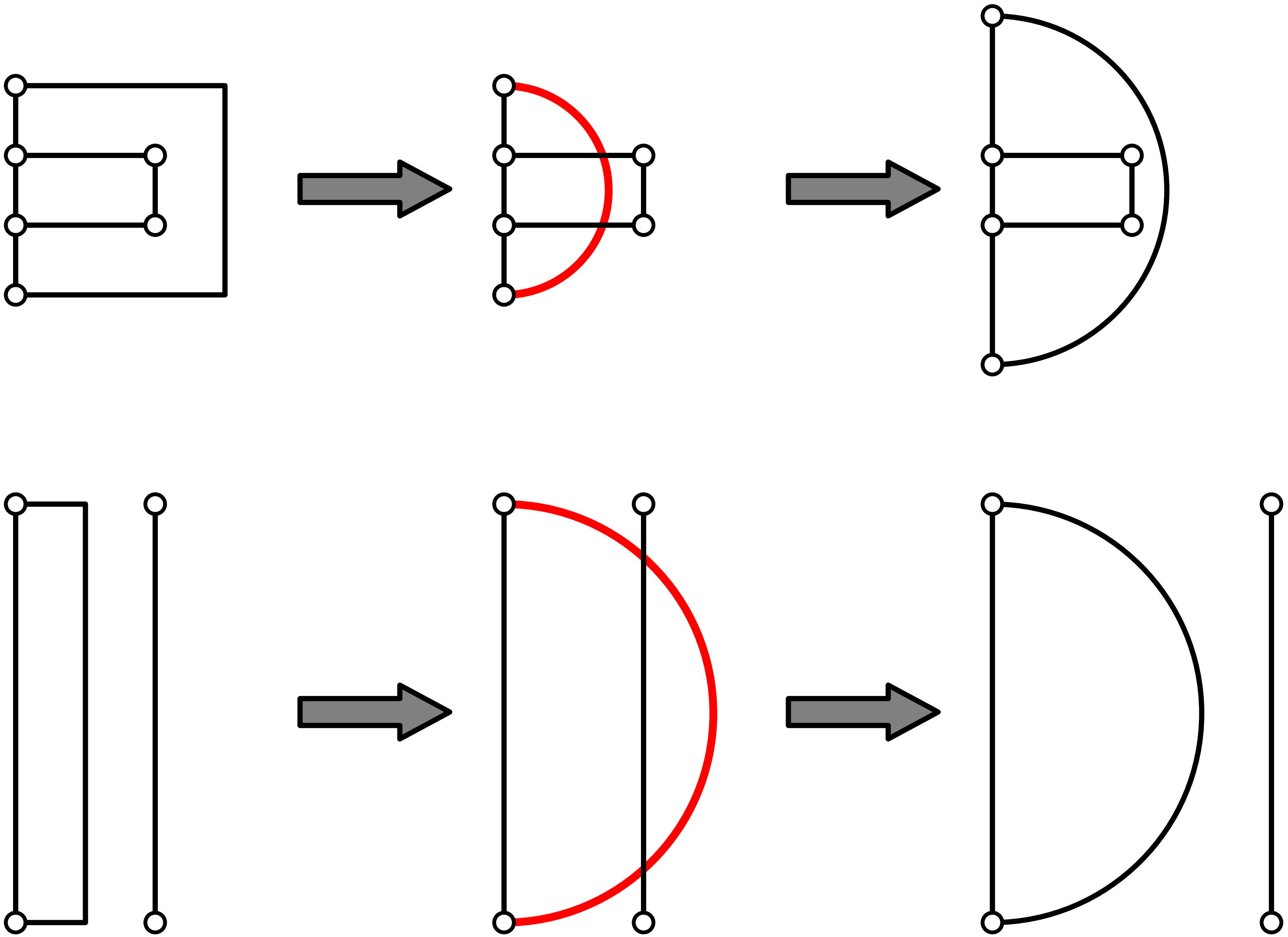


Crossings

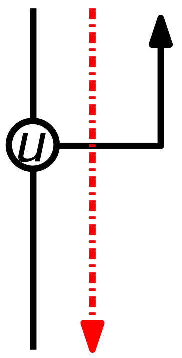


Cut

Def.

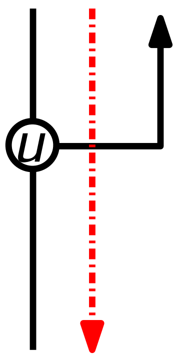
Cut

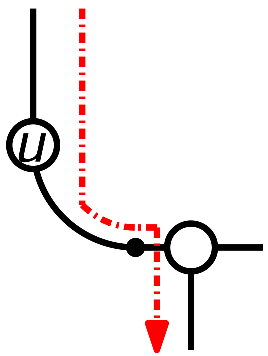
Def. • y-monotone curve



Cut

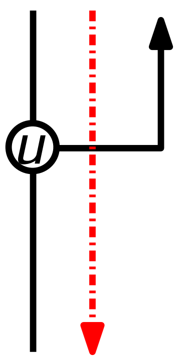
Def.
- y-monotone curve
- consists of horizontal, vertical and circular segments

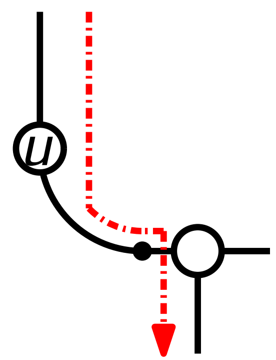




Cut

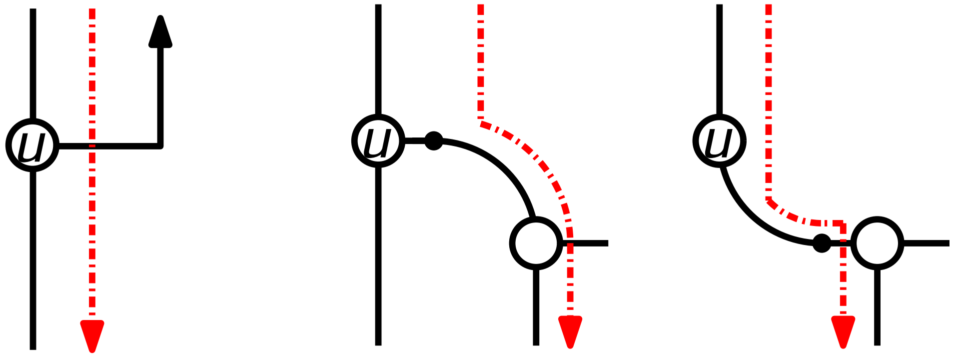
Def.
- *y*-monotone curve
- consists of horizontal, vertical and circular segments
- divides the current drawing into a left and a right part





Cut

Def.
- y-monotone curve
- consists of horizontal, vertical and circular segments
- divides the current drawing into a left and a right part
- intersects only horizontal segments



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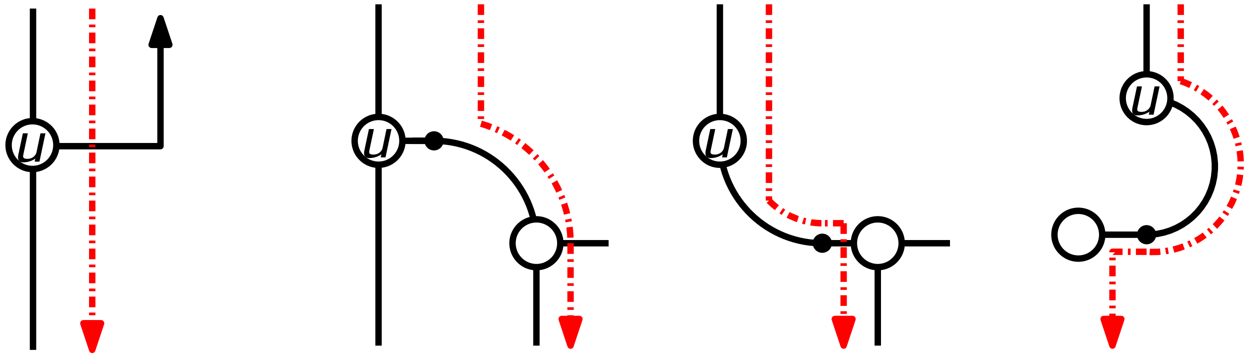

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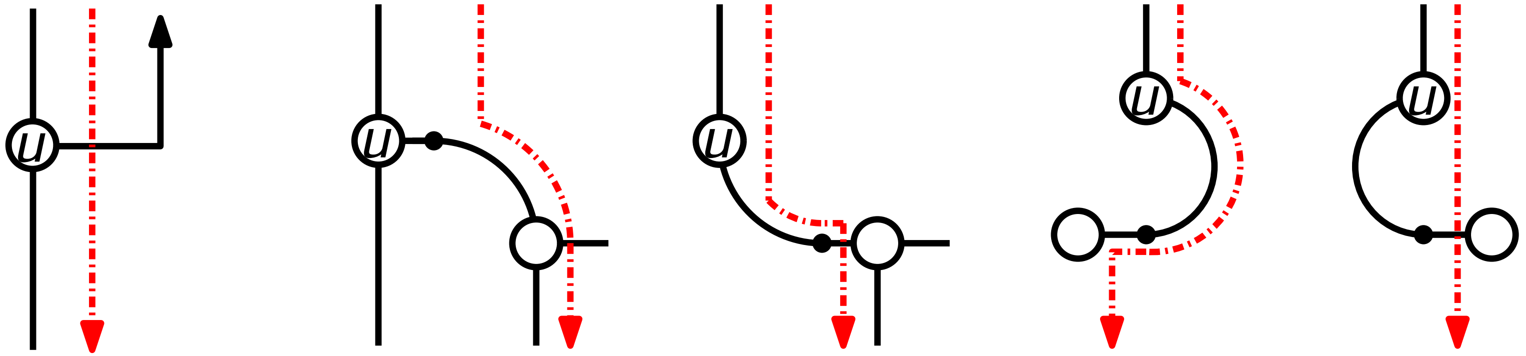

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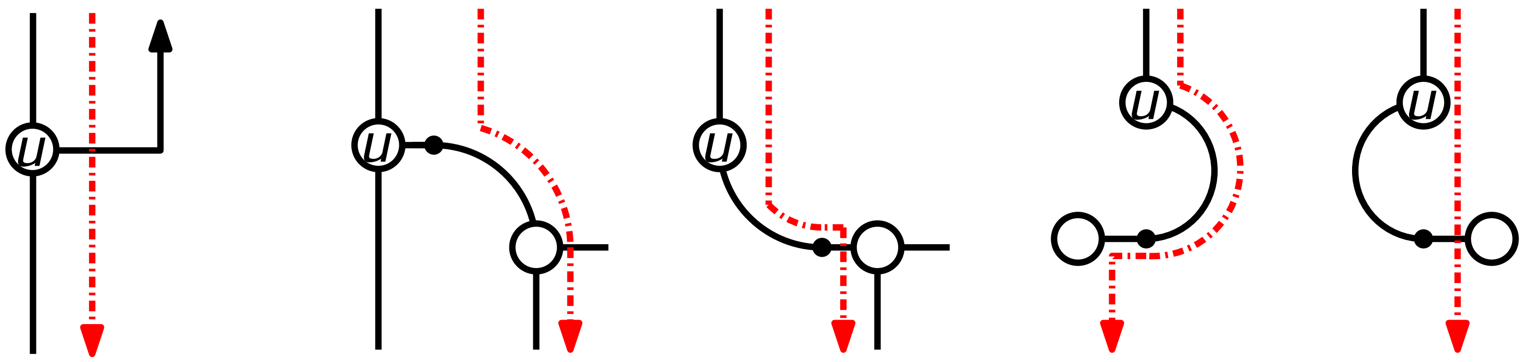

Problems:

Cut

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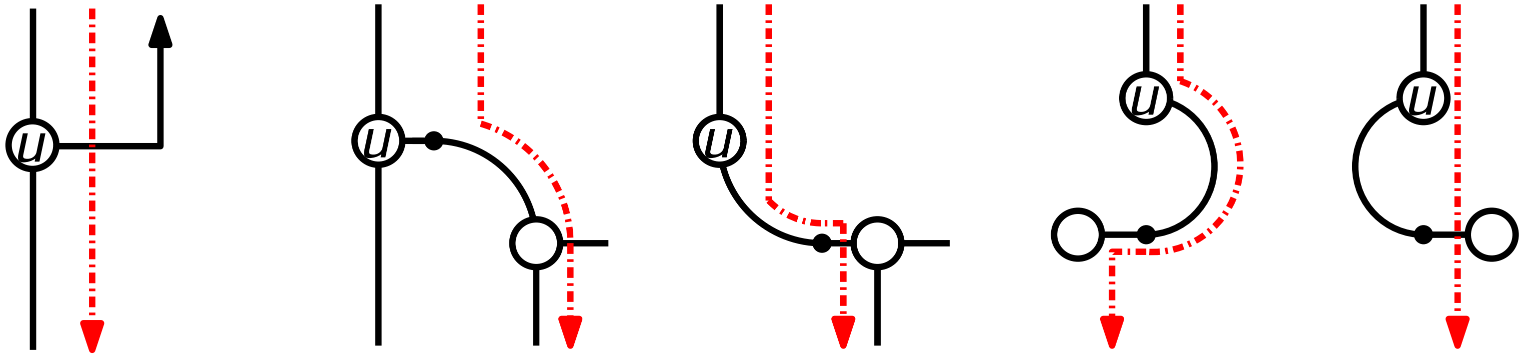

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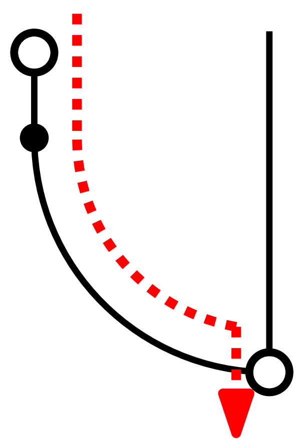

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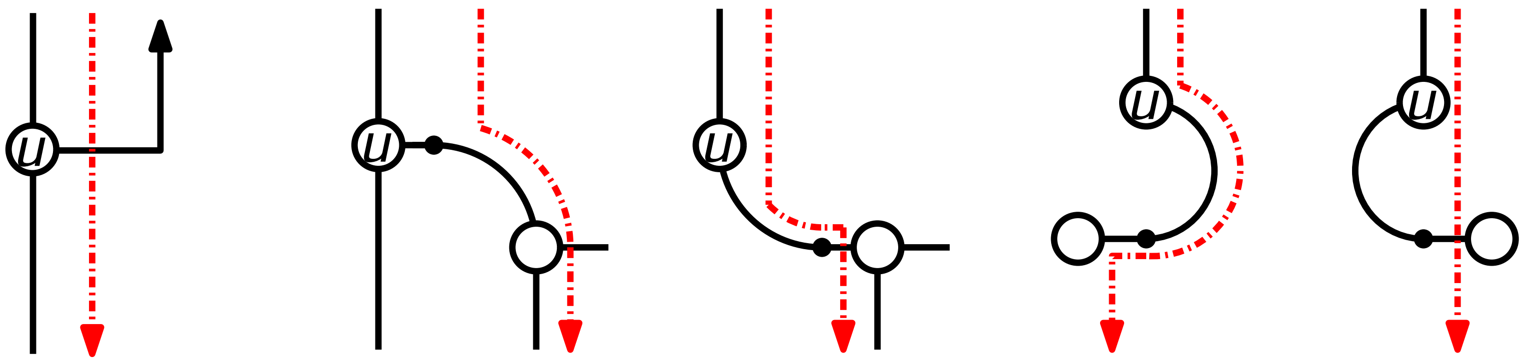
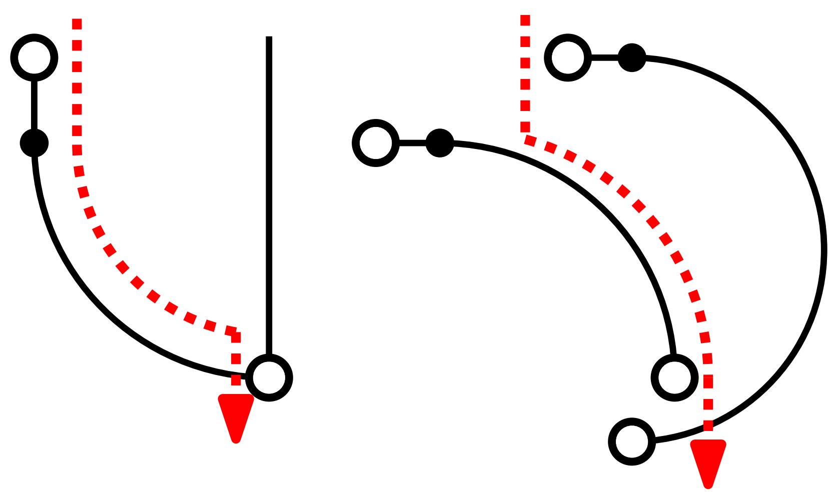
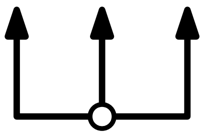


Problems:



Invariants

(I_1) Every open edge is associated with a column



Invariants

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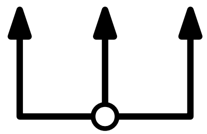
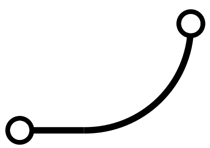


(I_2) An L-shape always contains a horizontal segment;
it never contains a vertical segment.



Invariants

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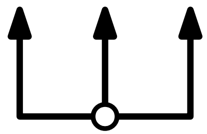

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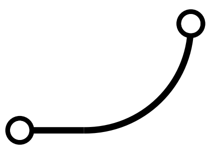
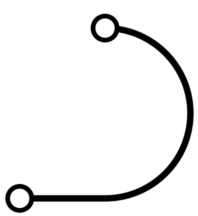
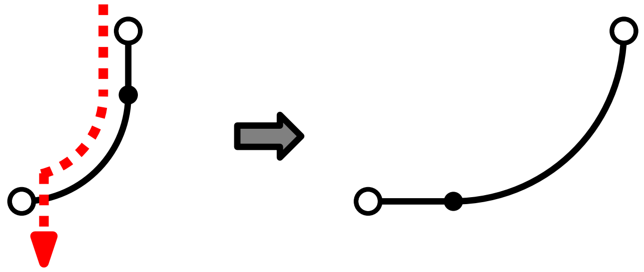


(I_3) A C-shape always has a horizontal segment incident to its bottom vertex.



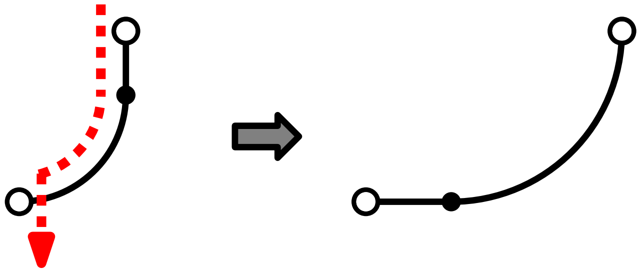
Maintain invariants

L-shape

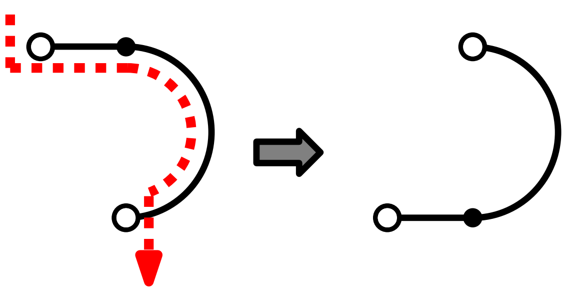


Maintain invariants

L-shape

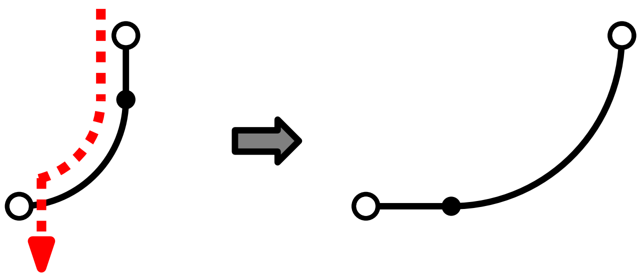


C-shape

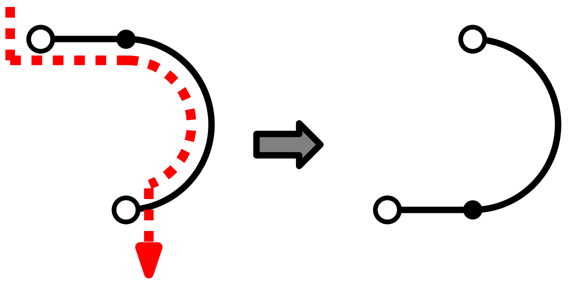


Maintain invariants

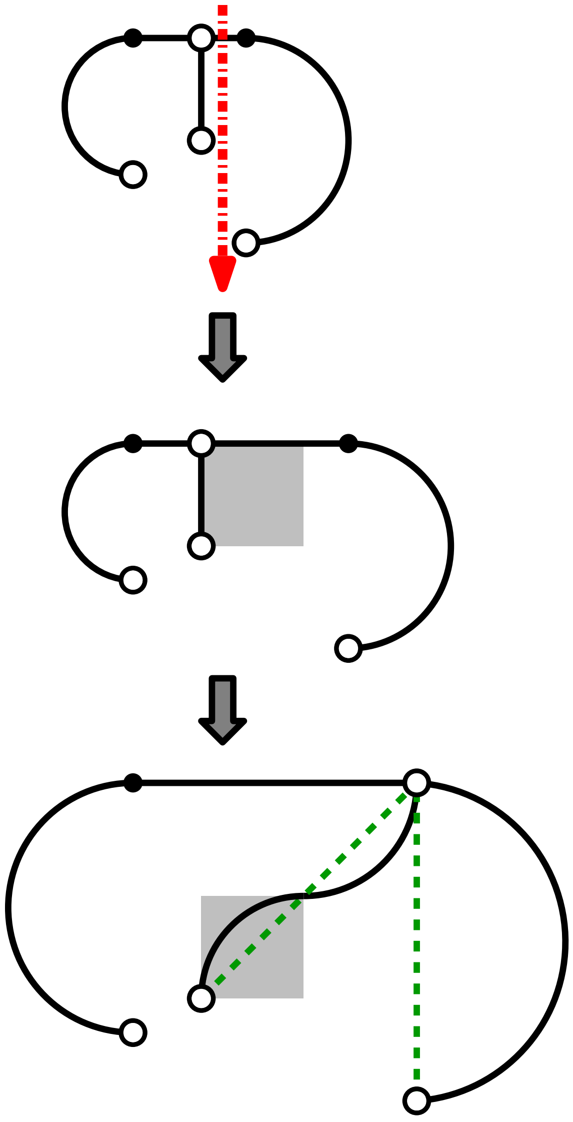
L-shape



C-shape

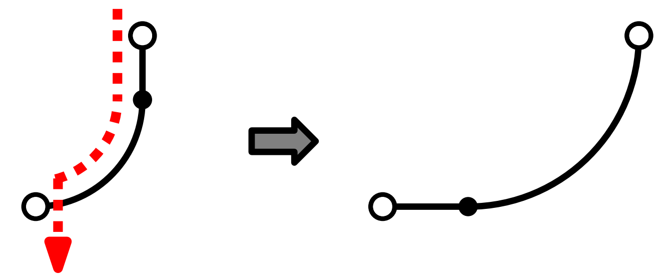


Double C-shape

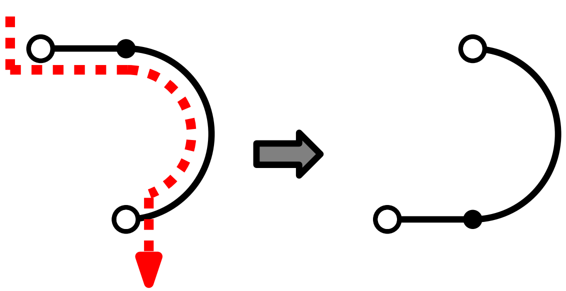


Maintain invariants

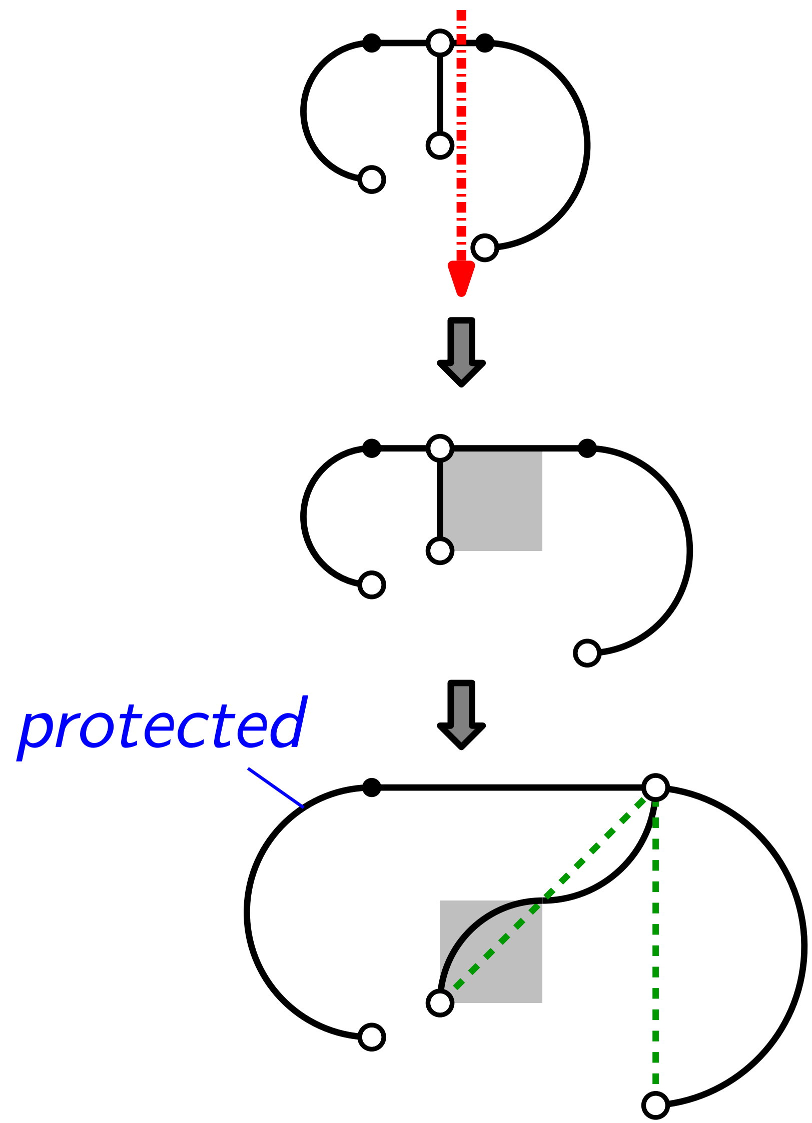
L-shape



C-shape

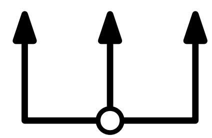


Double C-shape



Invariants, updated

(I_1) Every open edge is associated with a column



(I_2) An L-shape always contains a horizontal segment;
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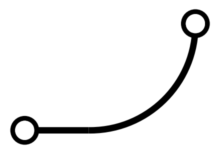
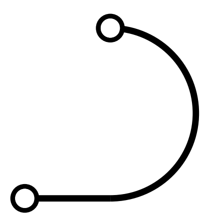
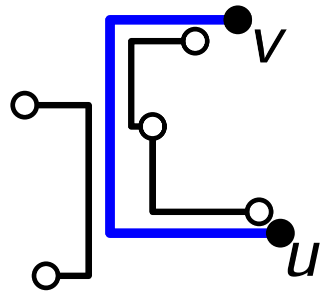
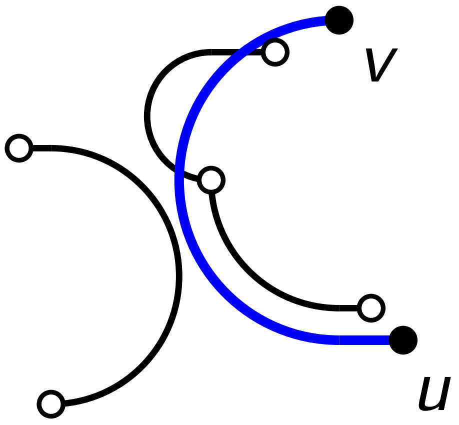


(I_3) An *unprotected* C-shape always has a horizontal segment incident to its bottom vertex.



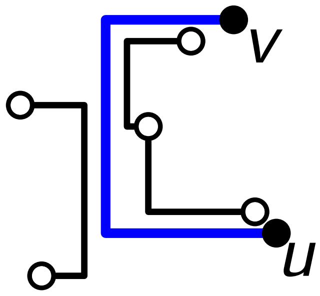
Eliminate crossings

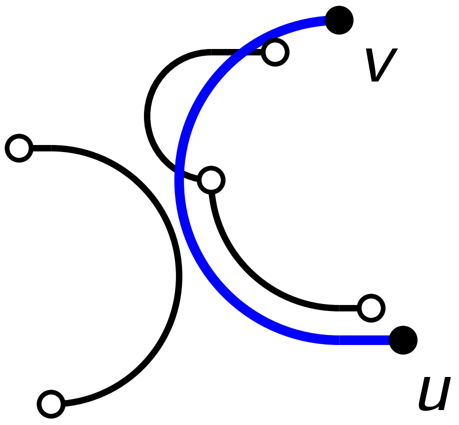




Eliminate crossings

1. move v up





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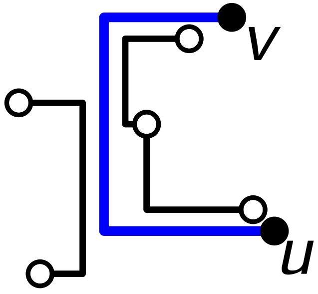
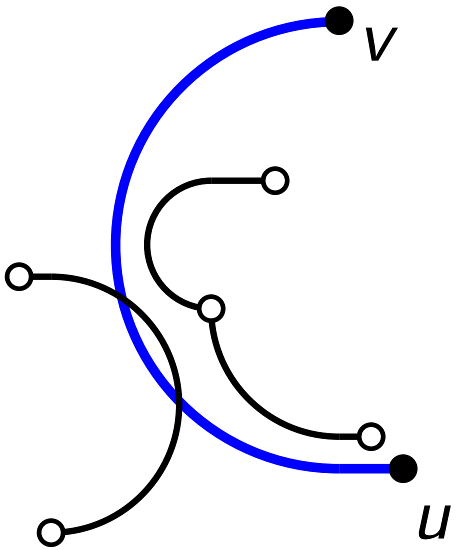
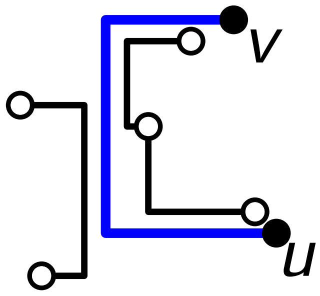
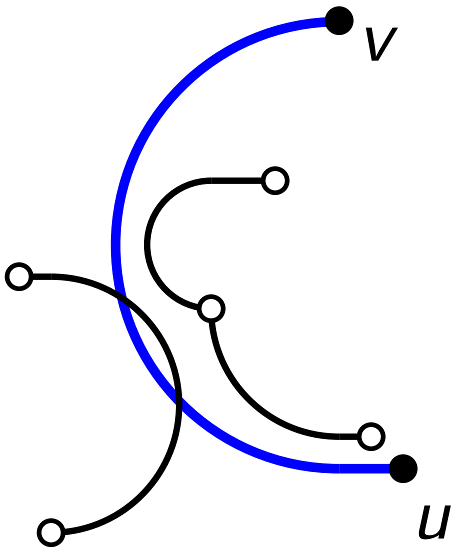




Eliminate crossings

1. move v up
2. find a cut





Eliminate crossings

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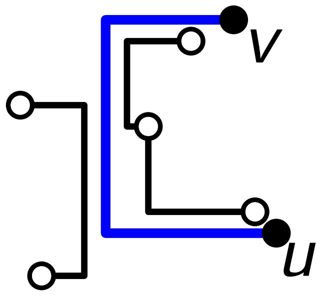
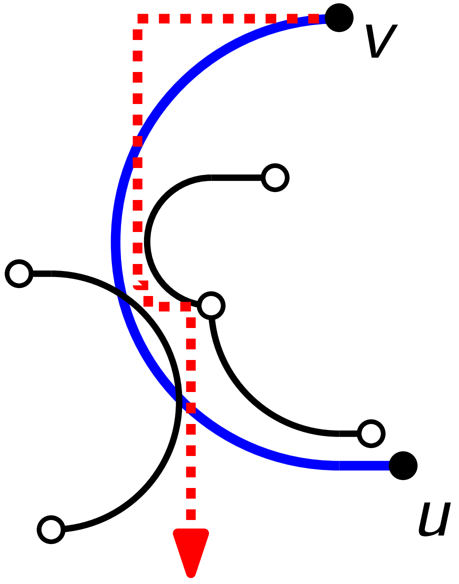
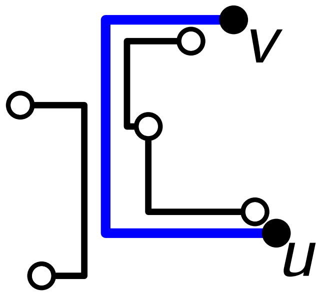
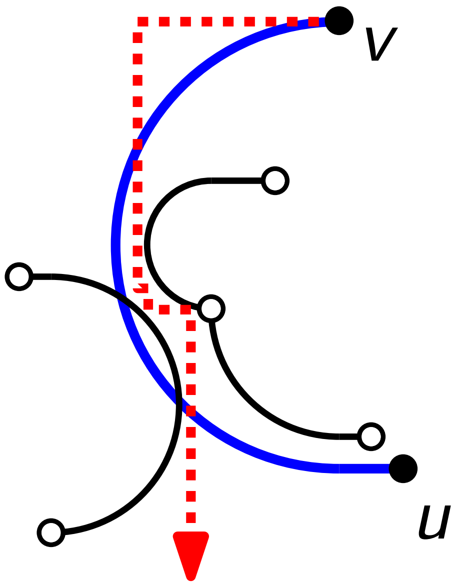




Eliminate crossings

1. move v up
2. find a cut
3. move vertices to the left





Eliminate crossings

1. move v up
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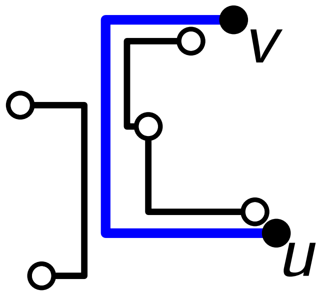
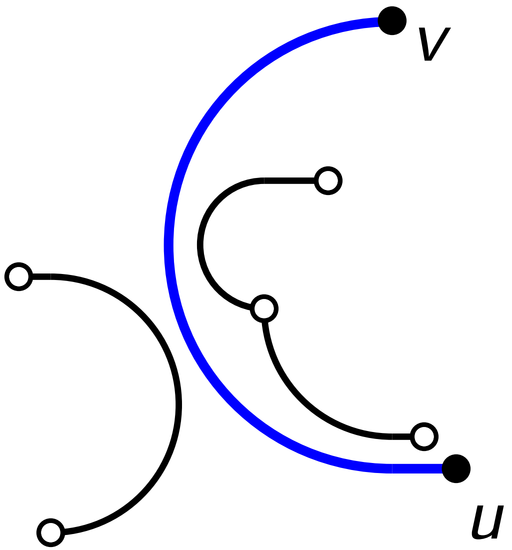
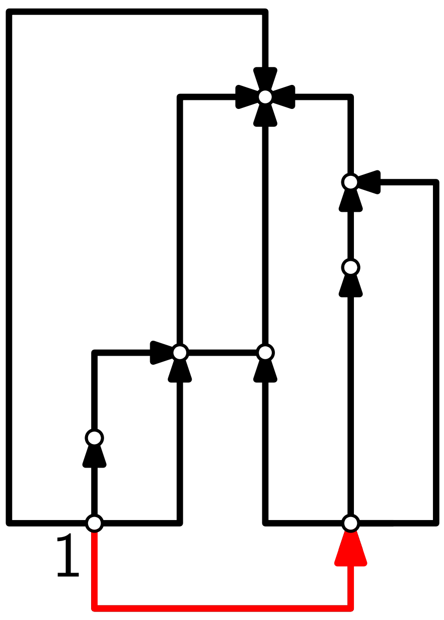
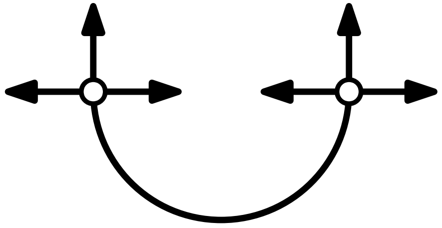


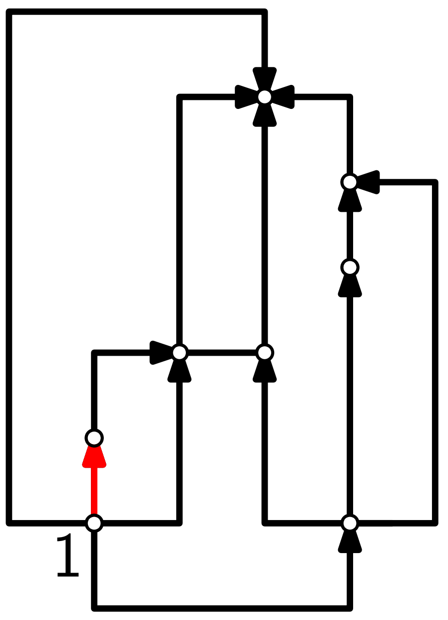


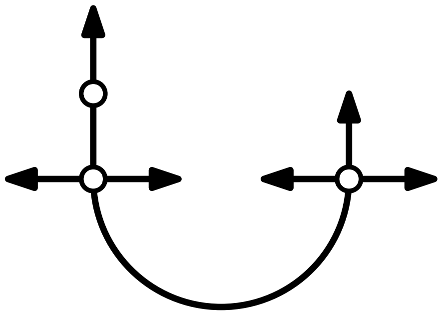
Example Run



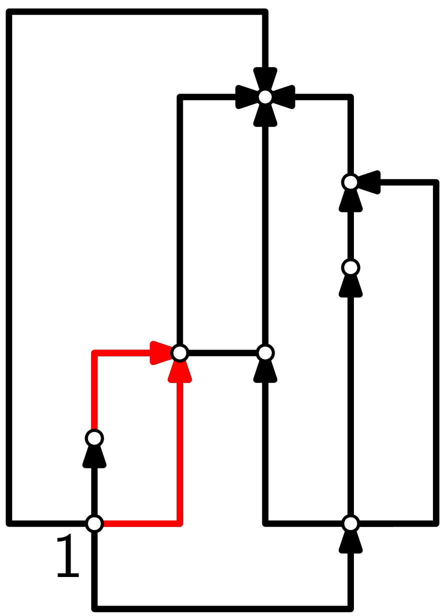


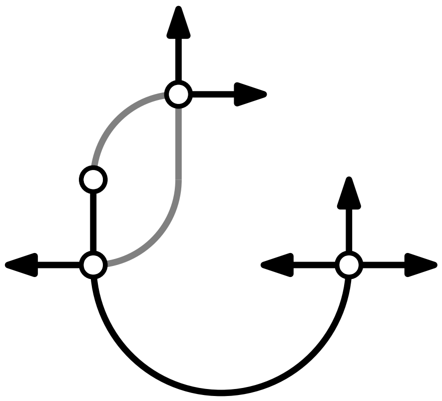
Example Run



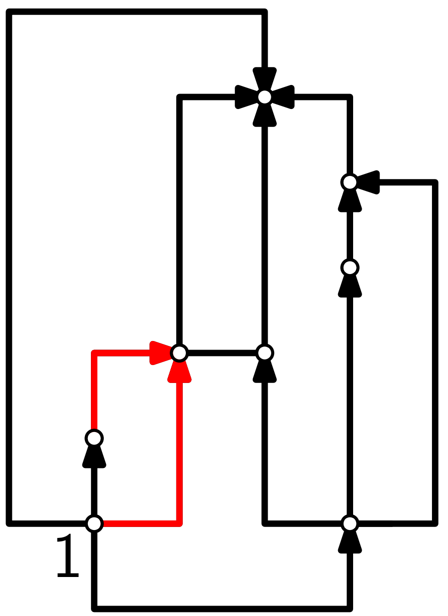


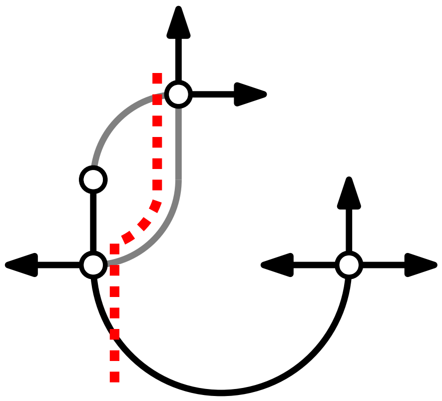
Example Run



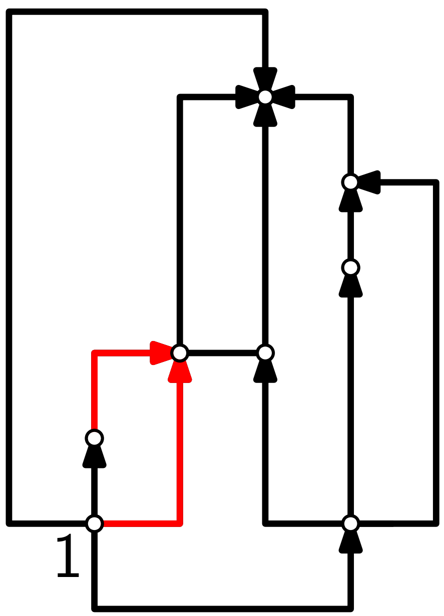


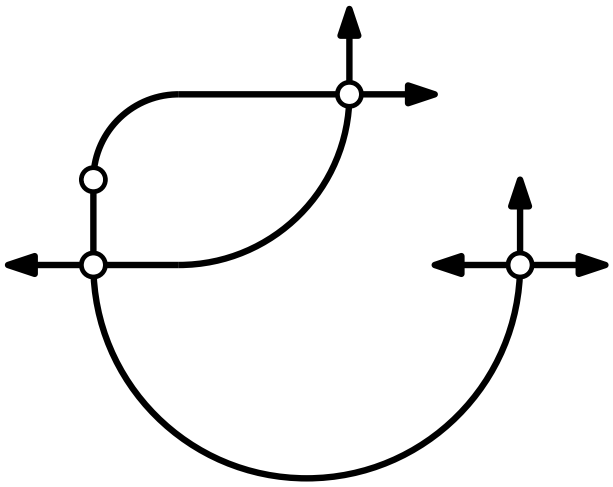
Example Run



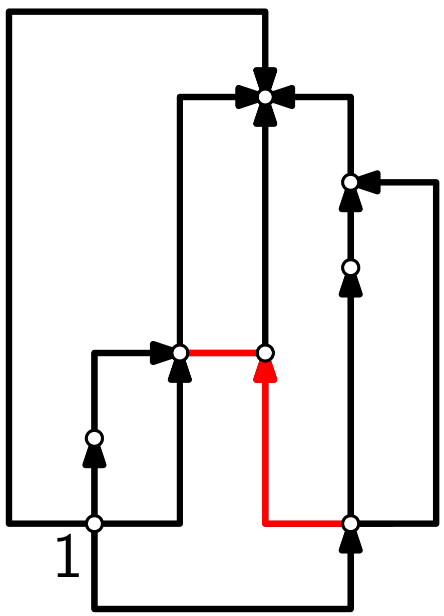


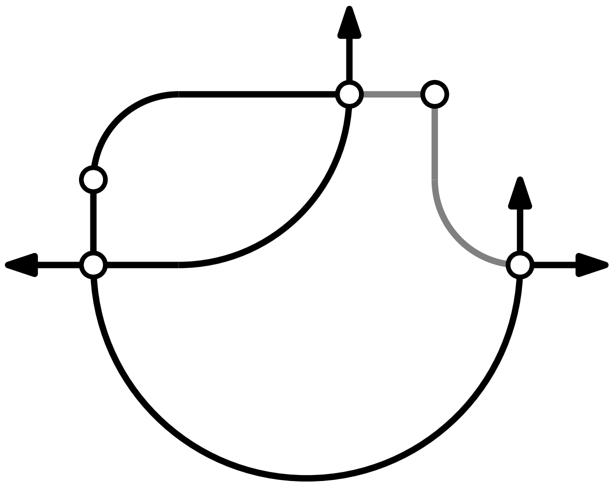
Example Run



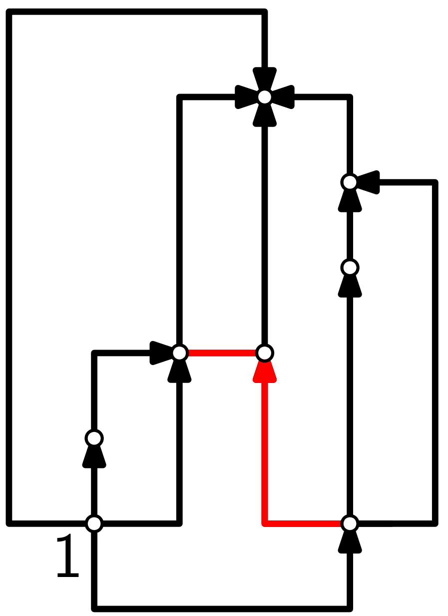


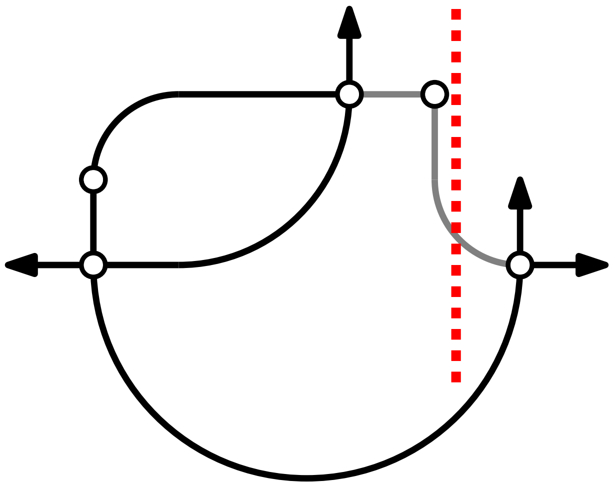
Example Run



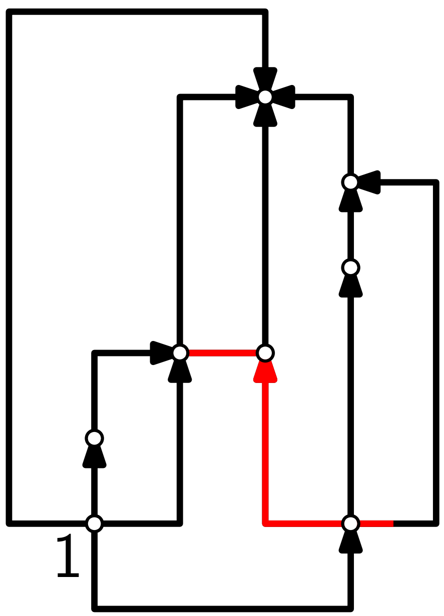


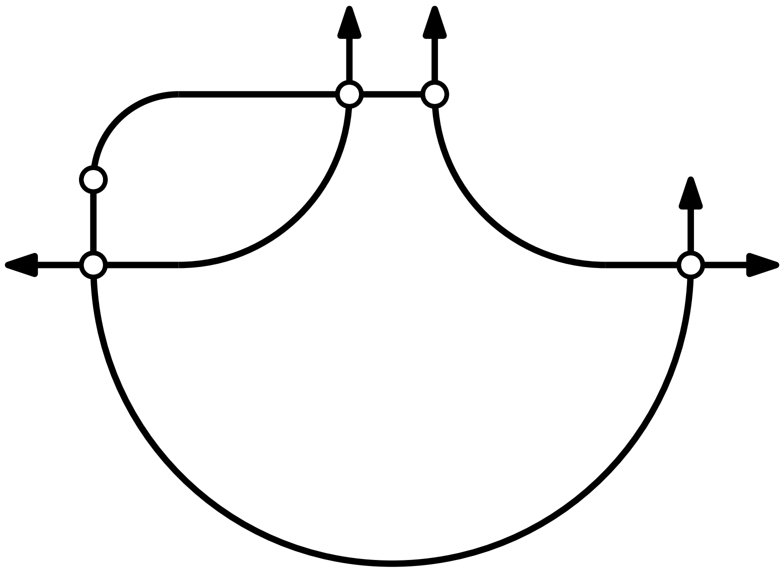
Example Run



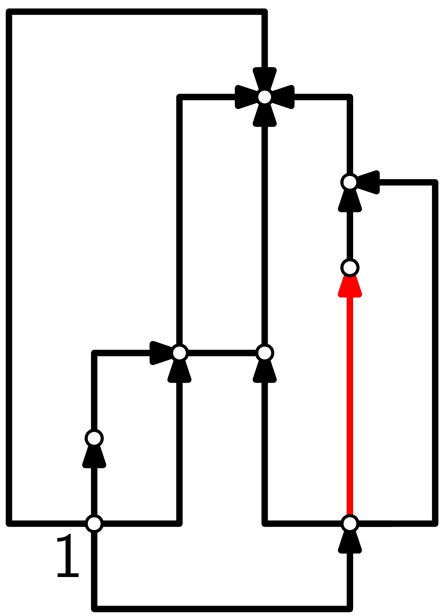


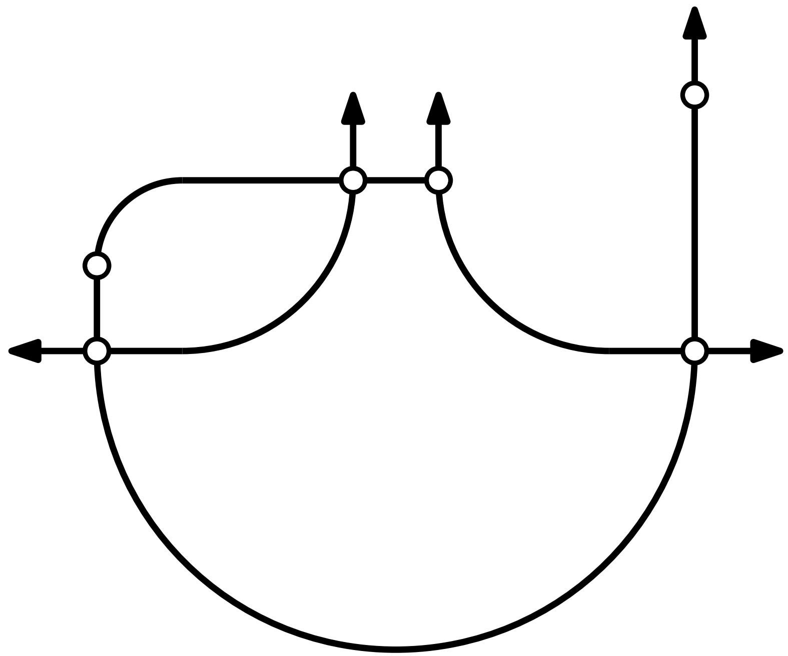
Example Run



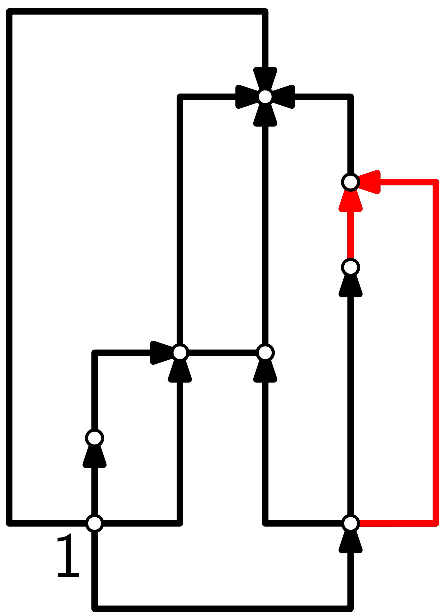


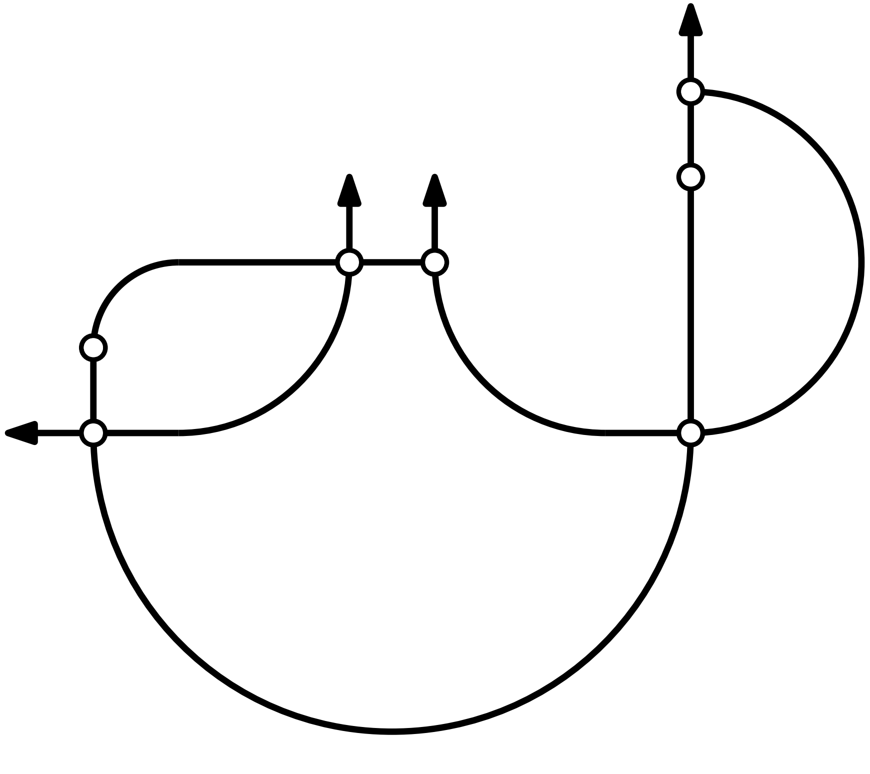
Example Run



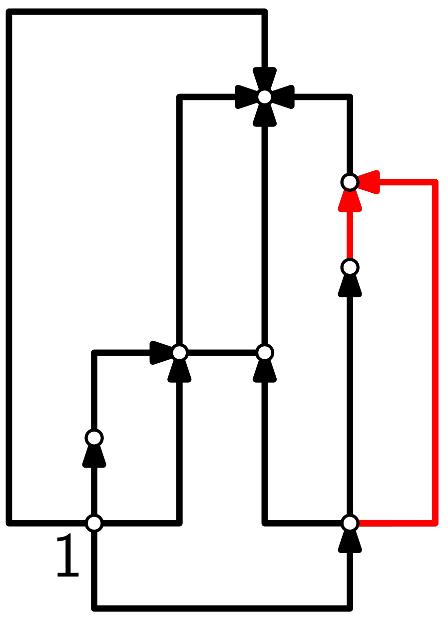


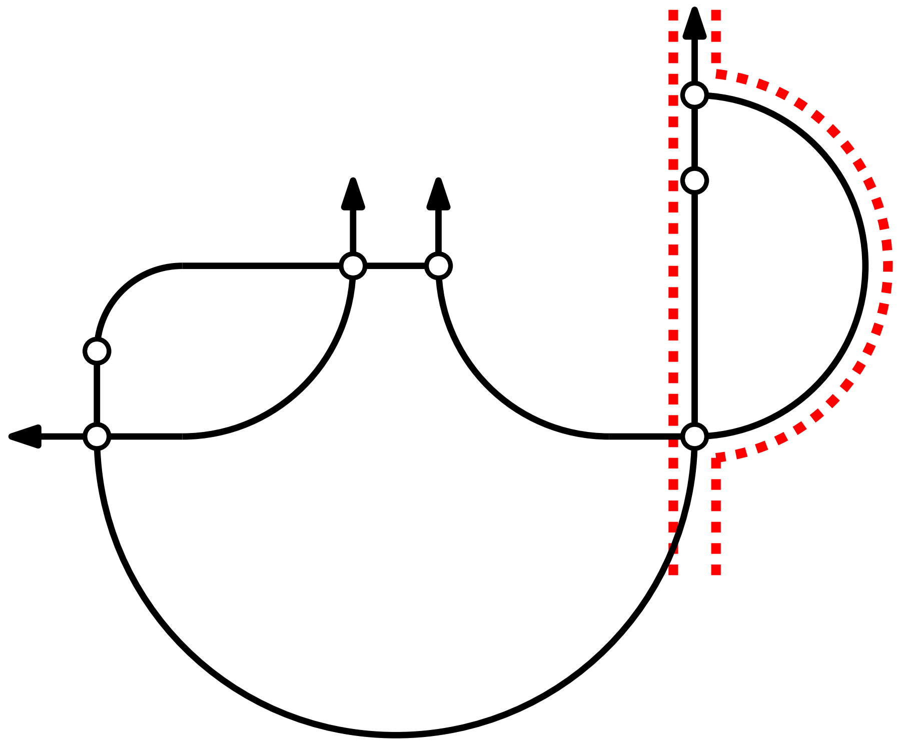
Example Run



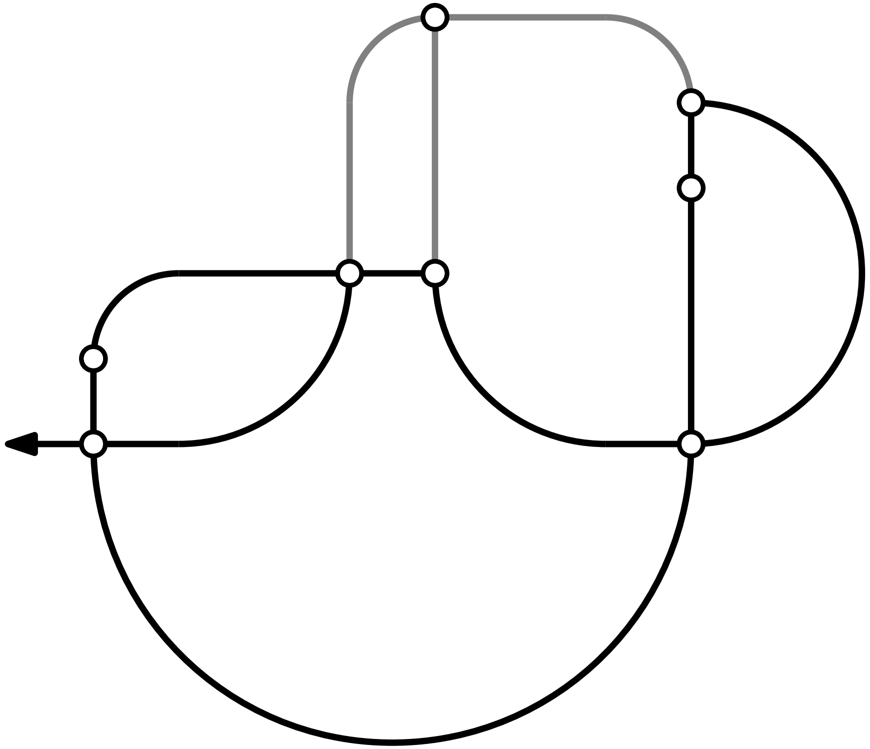


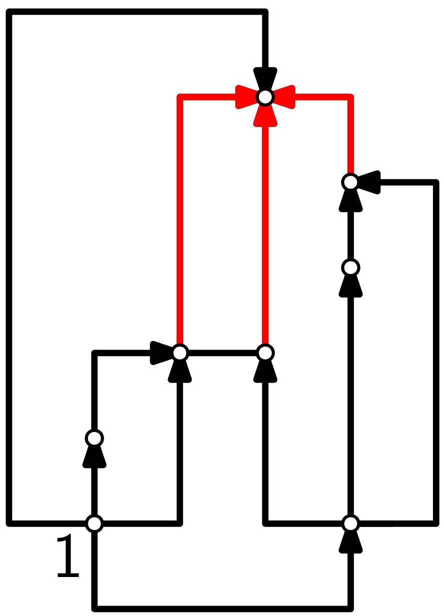
Example Run



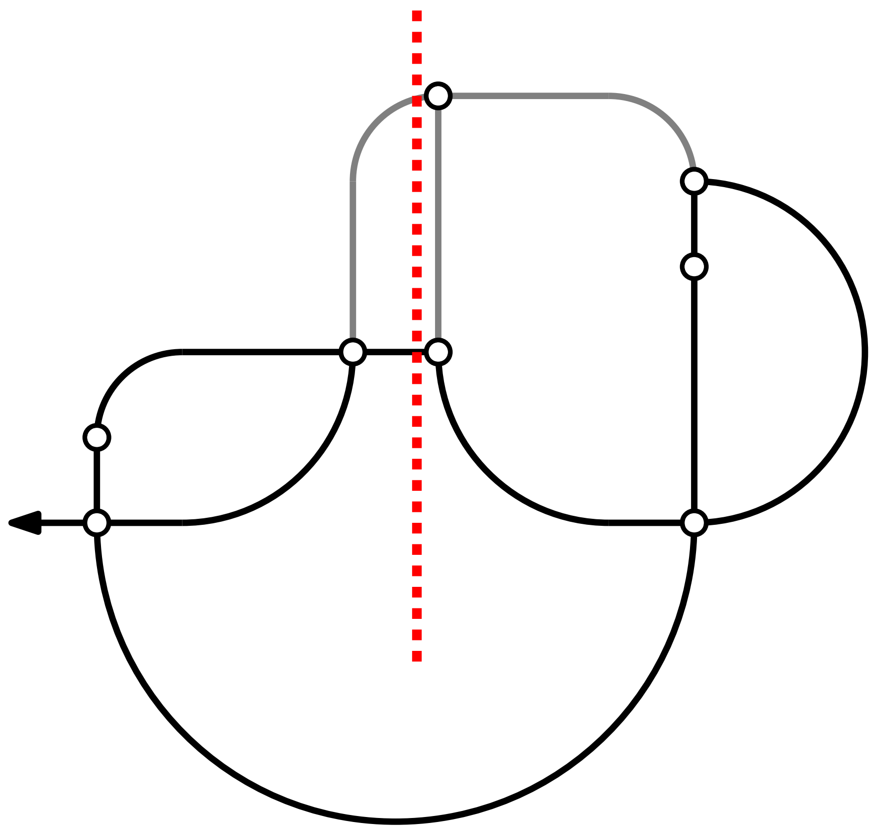


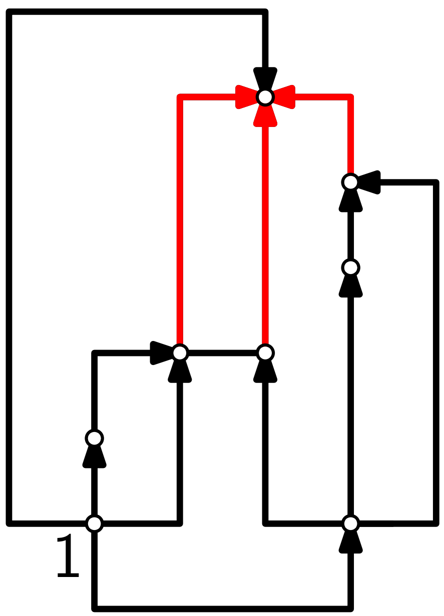
Example Run



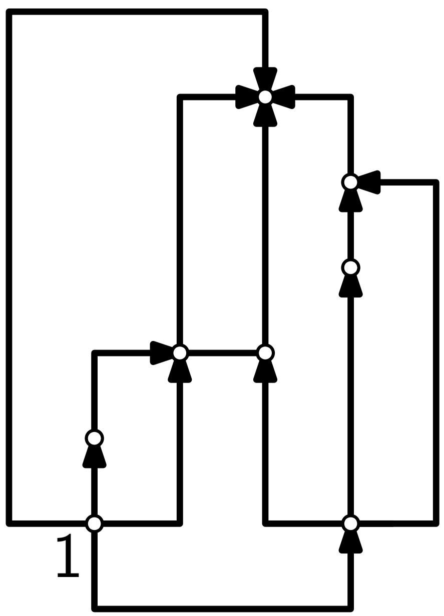


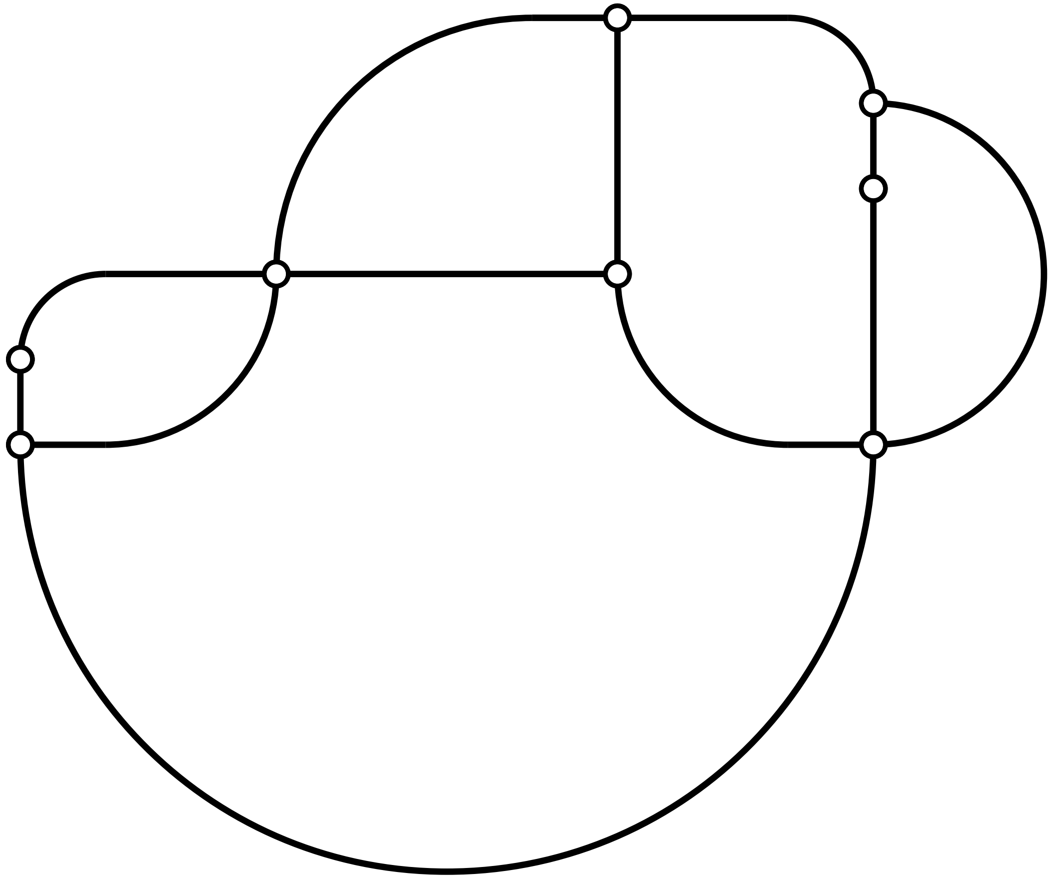
Example Run



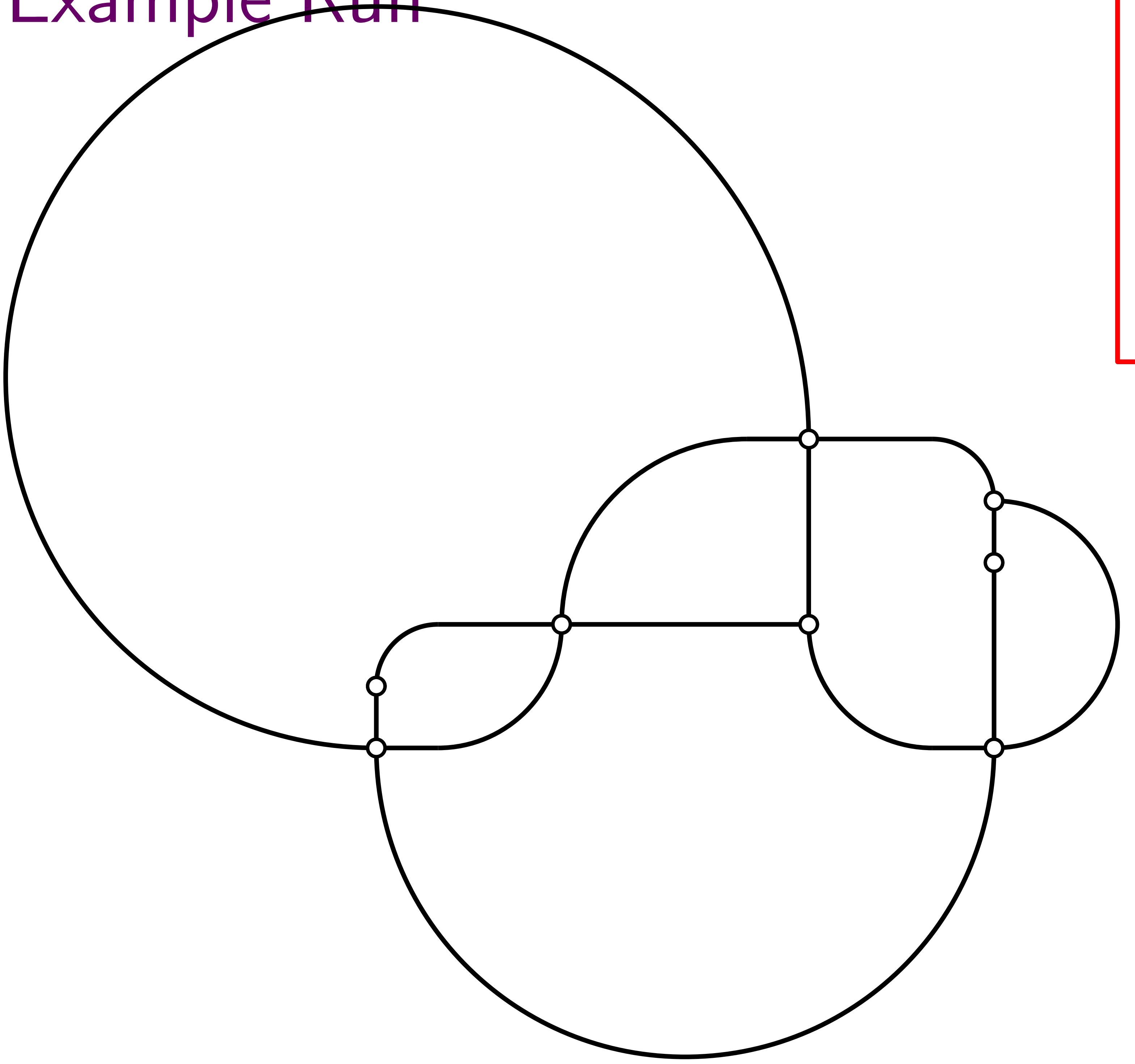


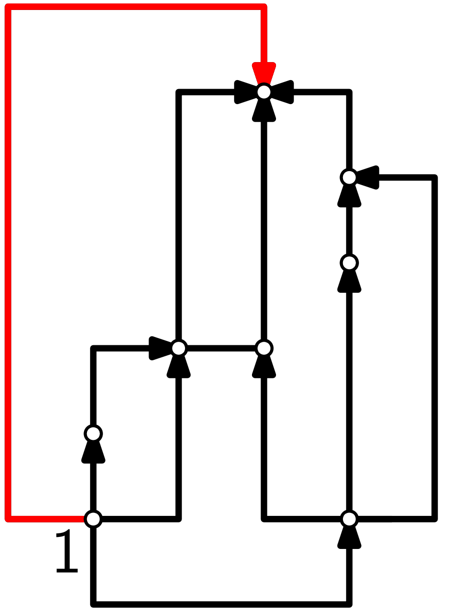
Example Run



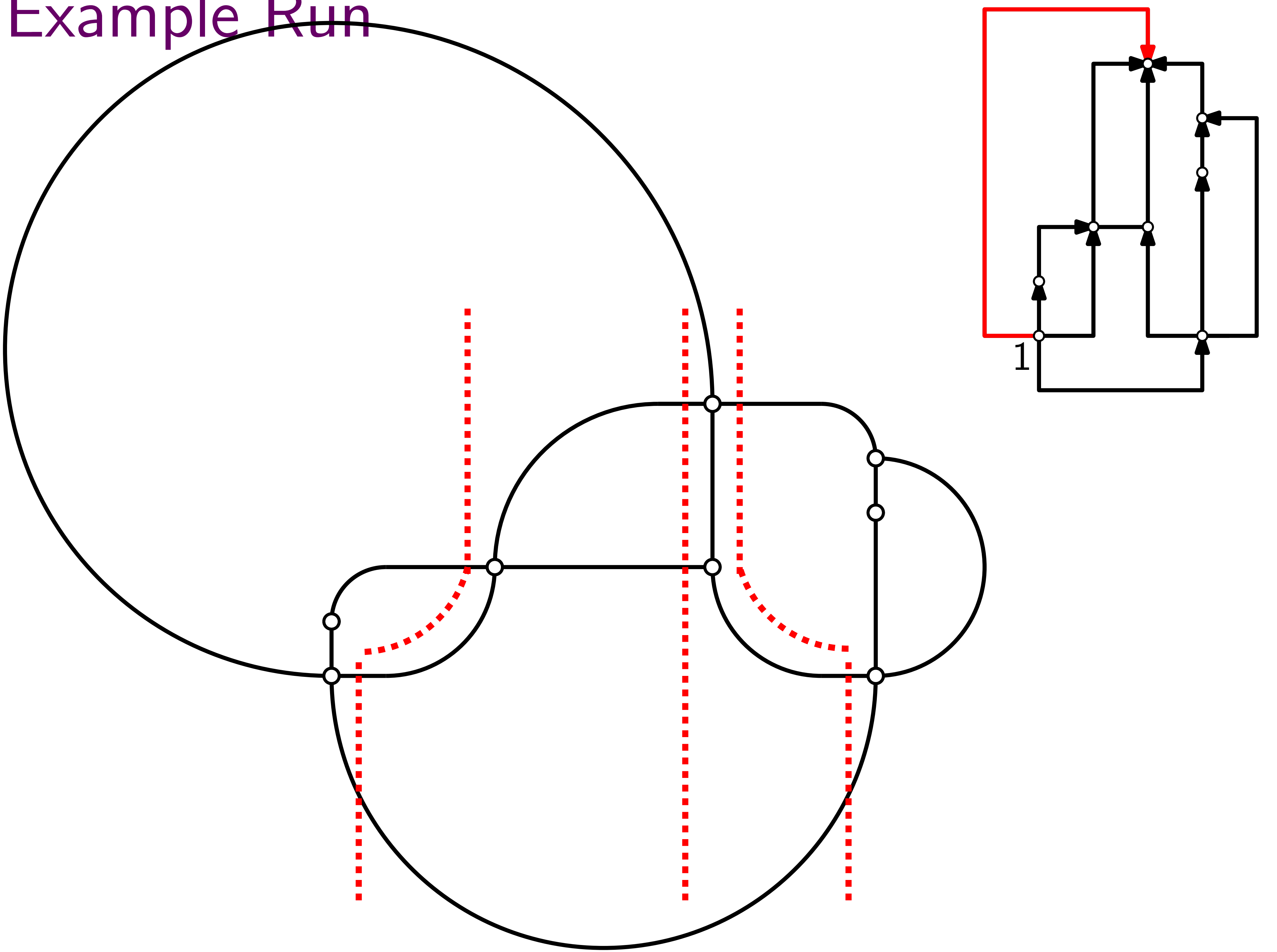


Example Run

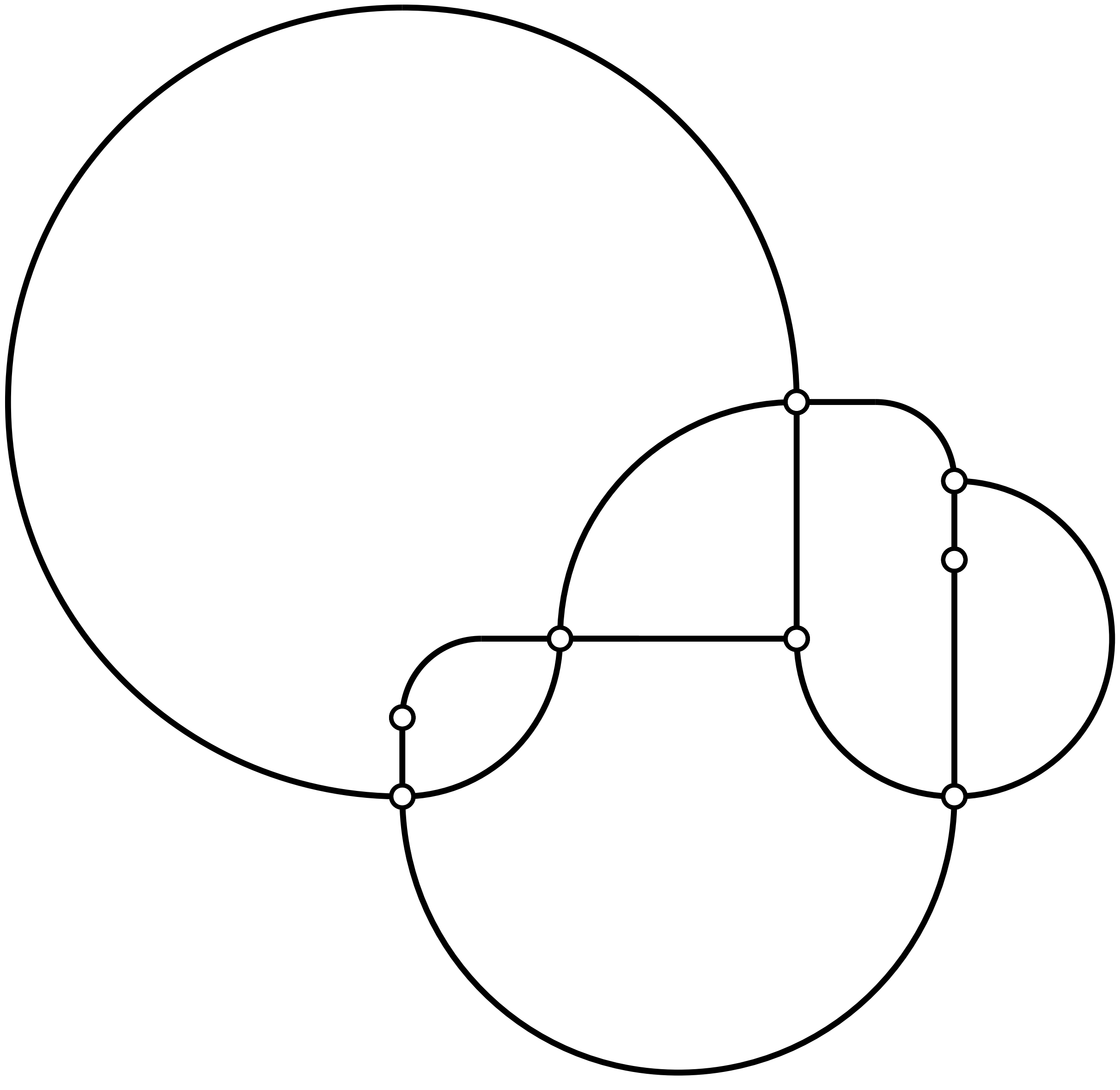


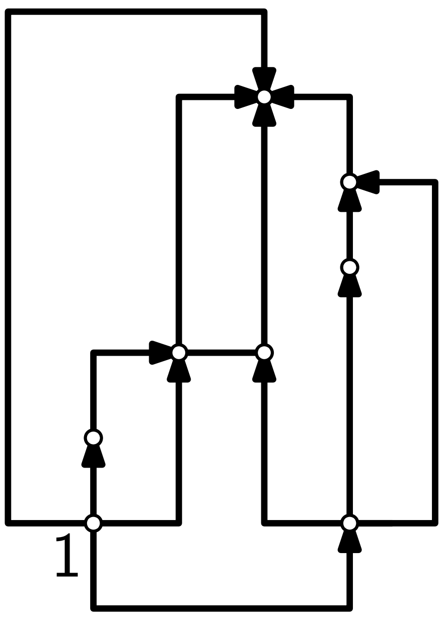


Example Run

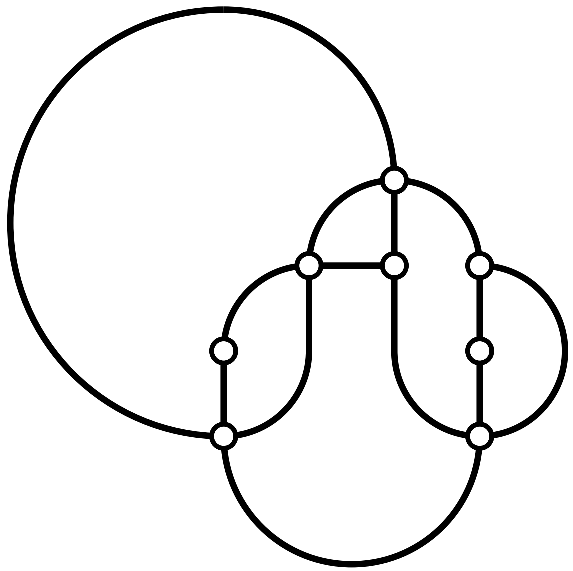


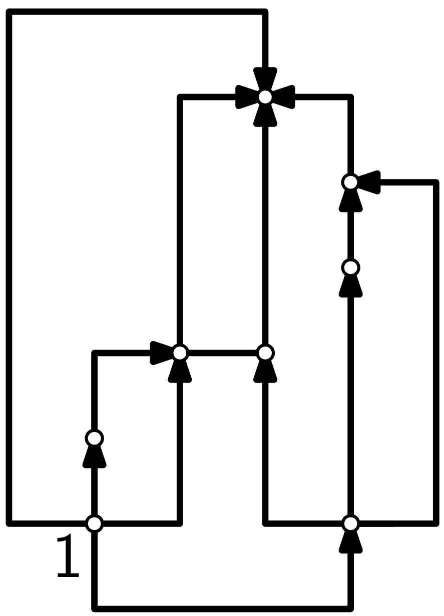
Example Run





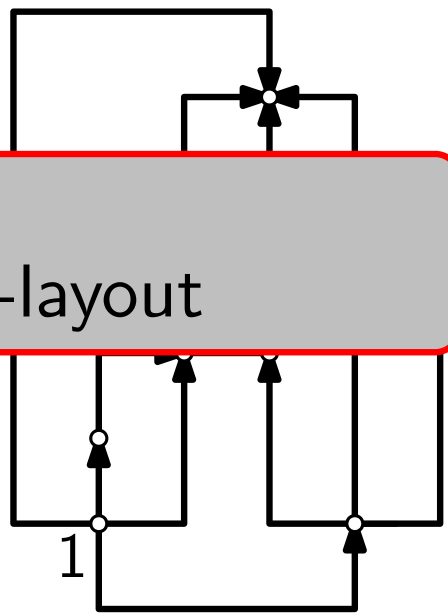
Example Run

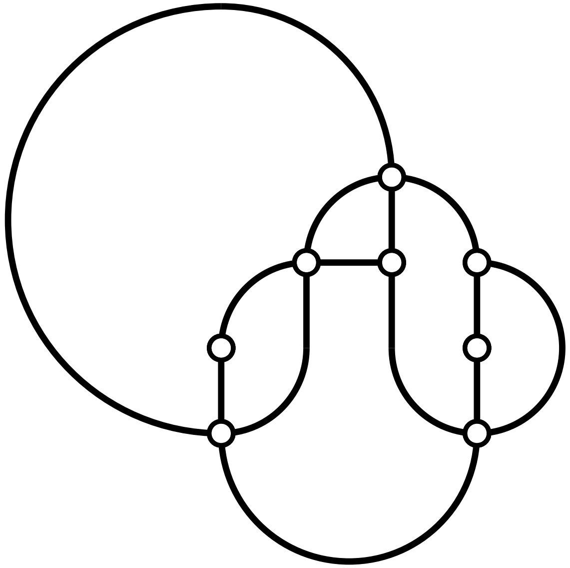




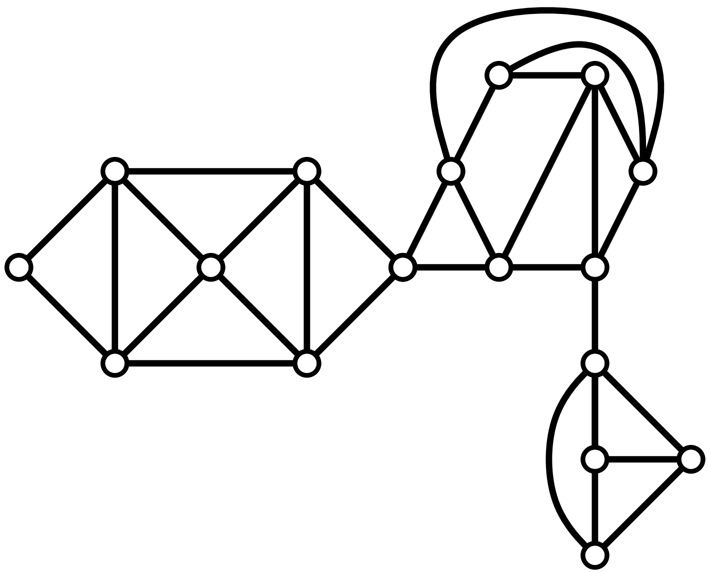
Example Run

[Theorem]
Every biconnected 4-planar graph admits an SC_2-layout



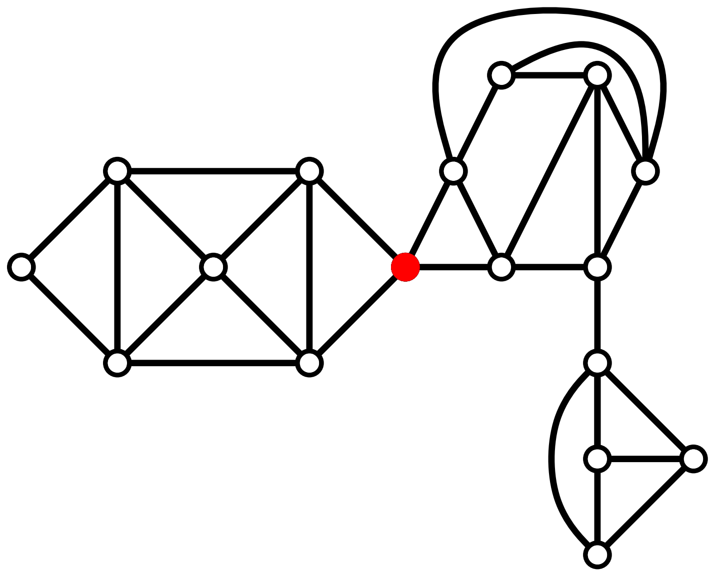


Extension to Arbitrary Graphs



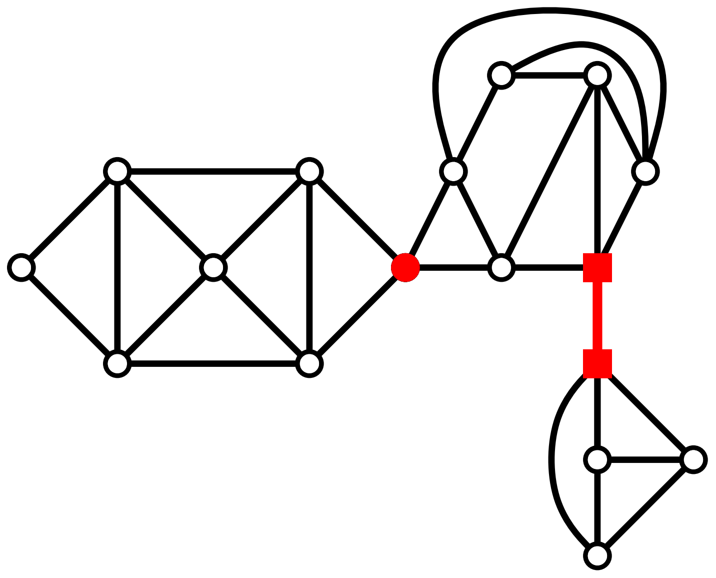
Extension to Arbitrary Graphs

- cutvertices



Extension to Arbitrary Graphs

- cutvertices
- bridges



Extension to Arbitrary Graphs

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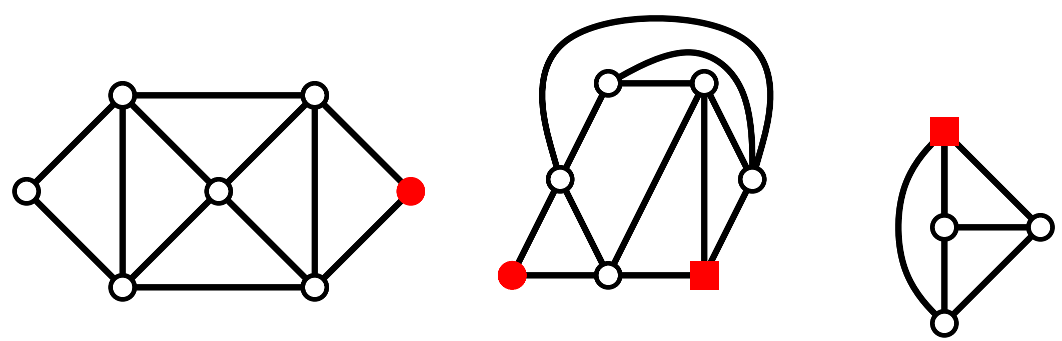

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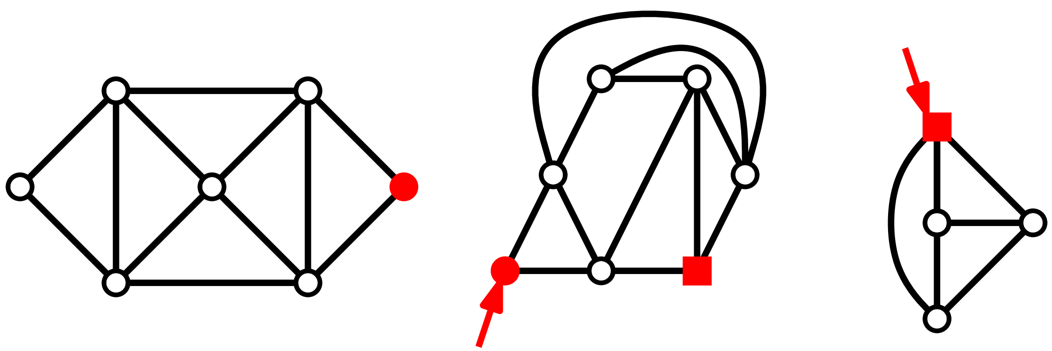
Draw specific vertex on outer face



Extension to Arbitrary Graphs

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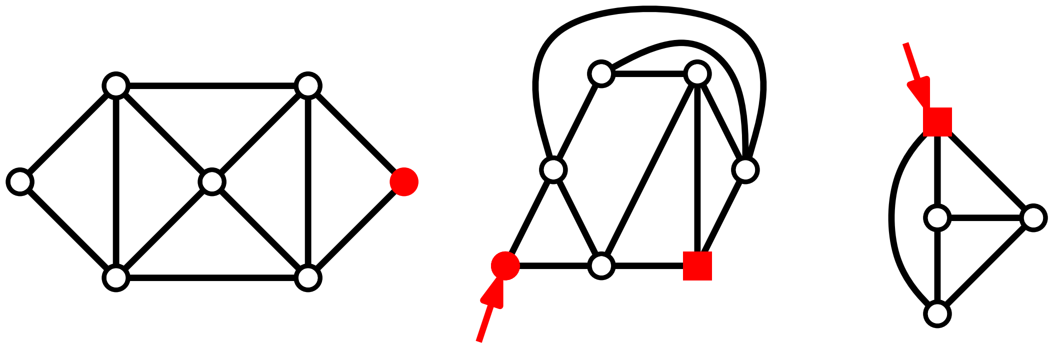

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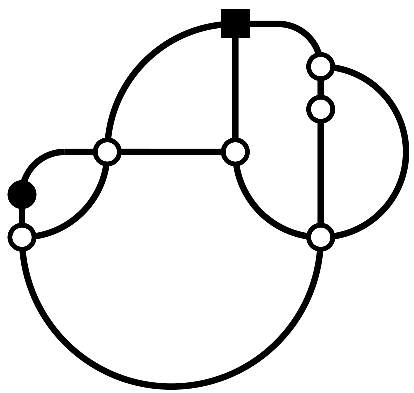

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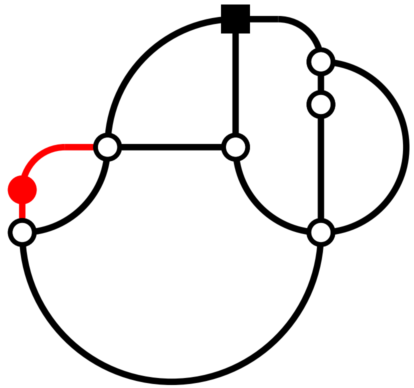

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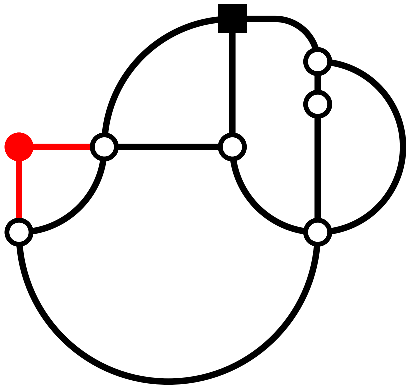

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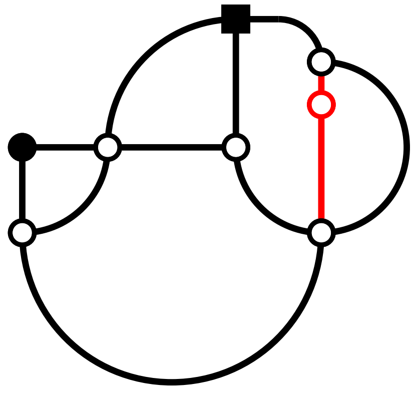

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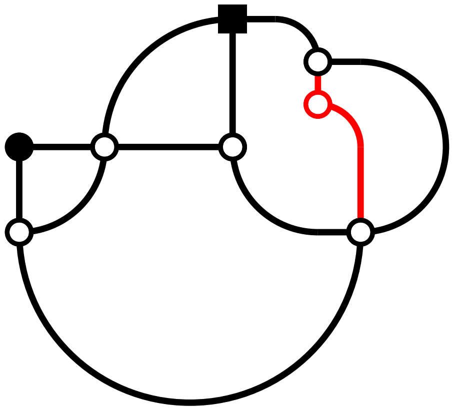

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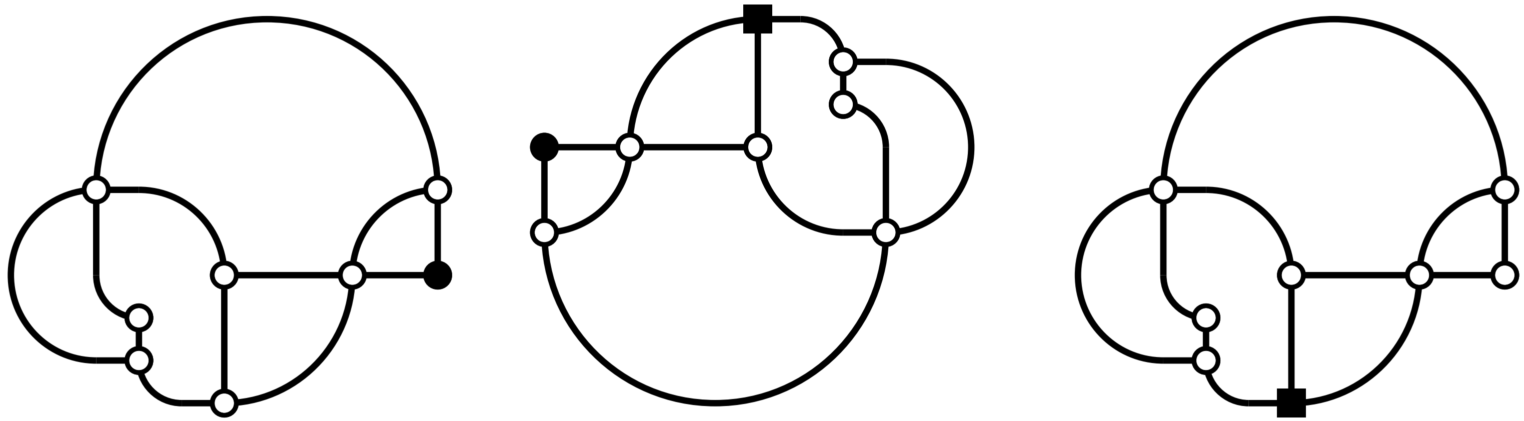

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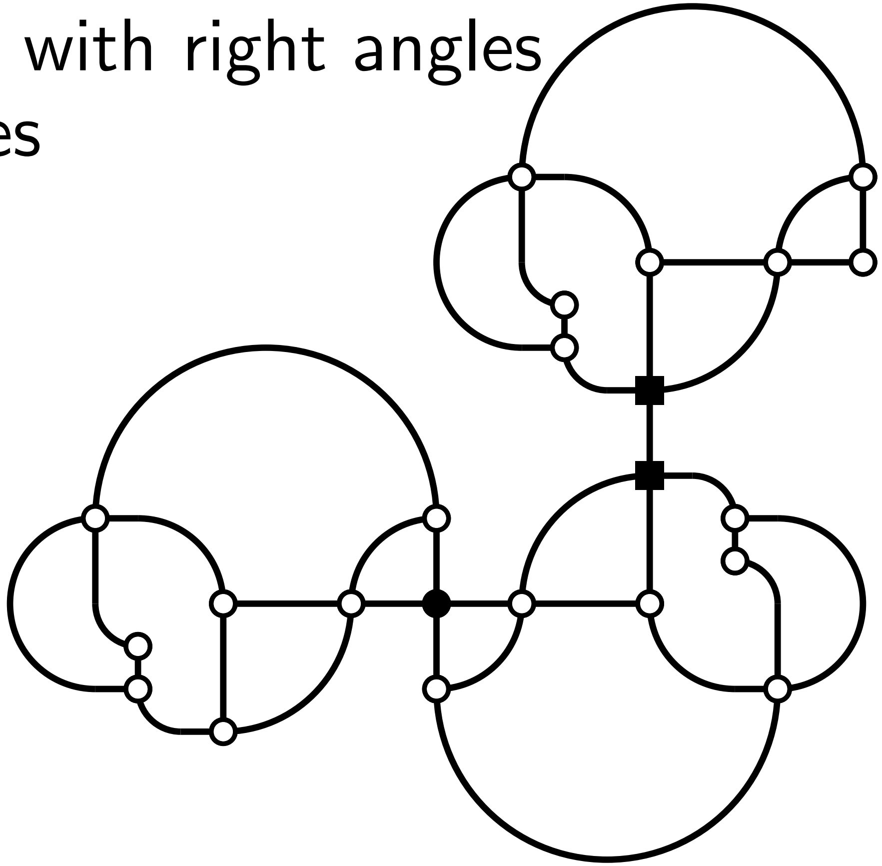

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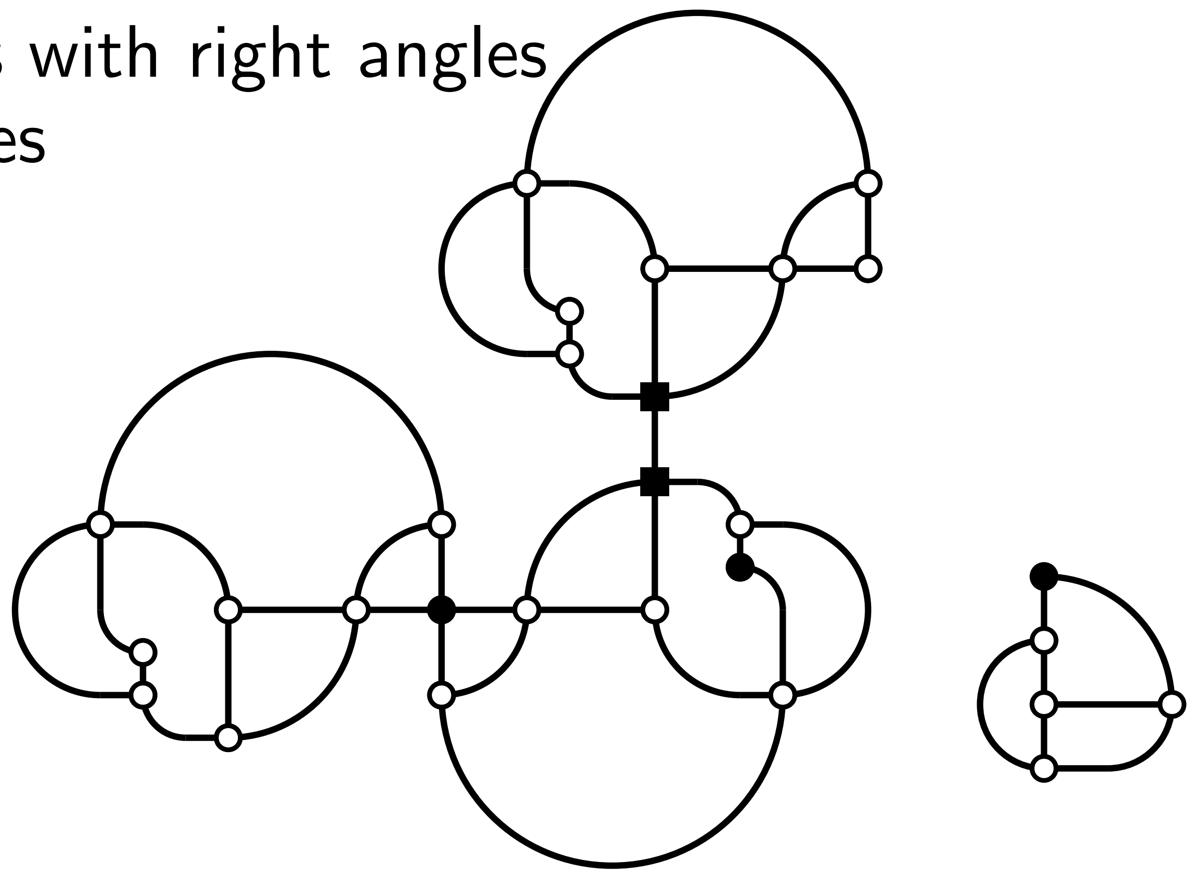

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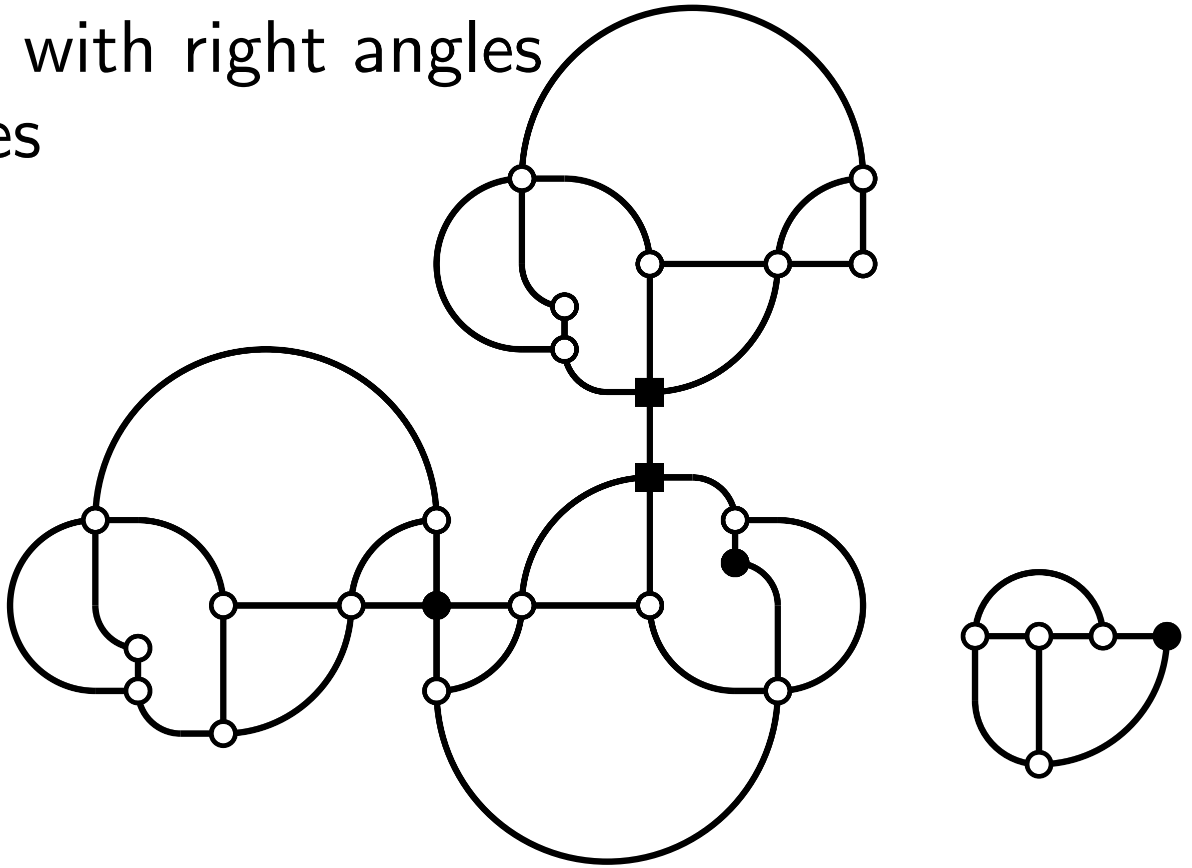

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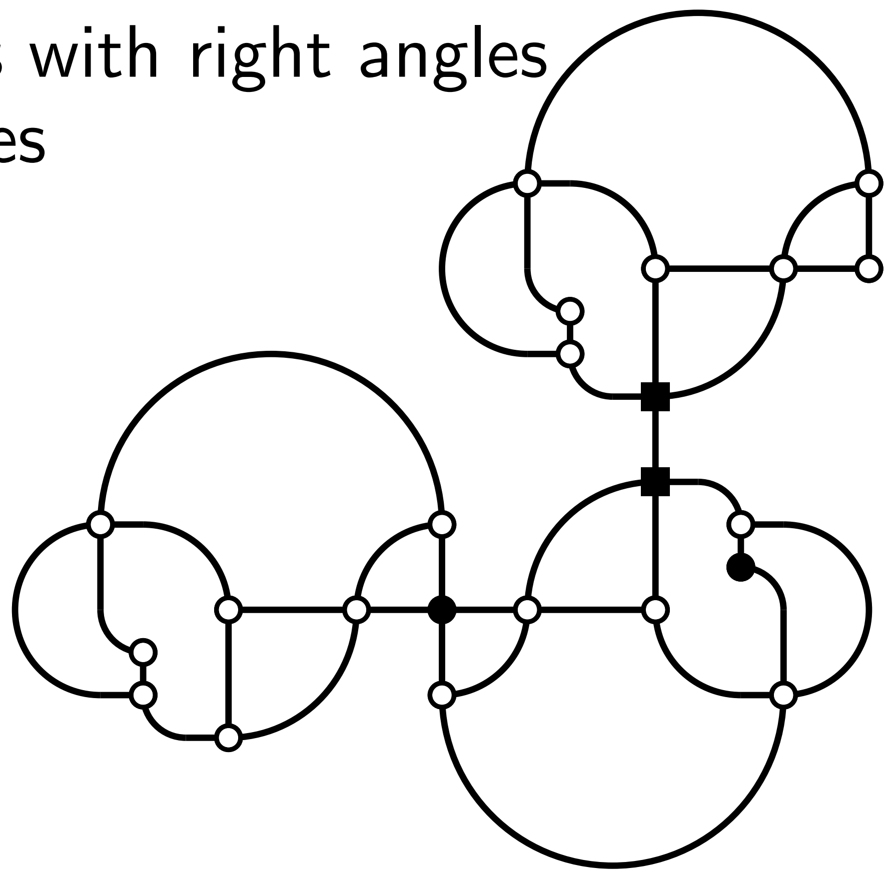

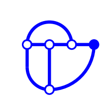

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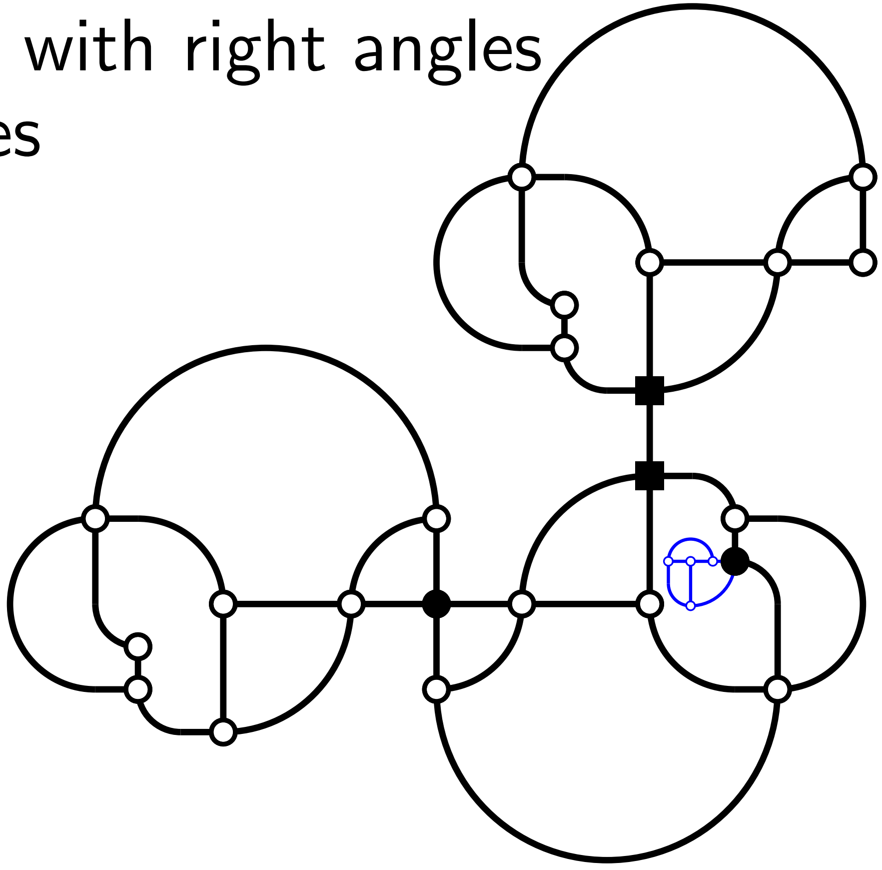

Extension to Arbitrary Graphs

[Theorem]
Every 4-planar graph admits an SC_2-layout

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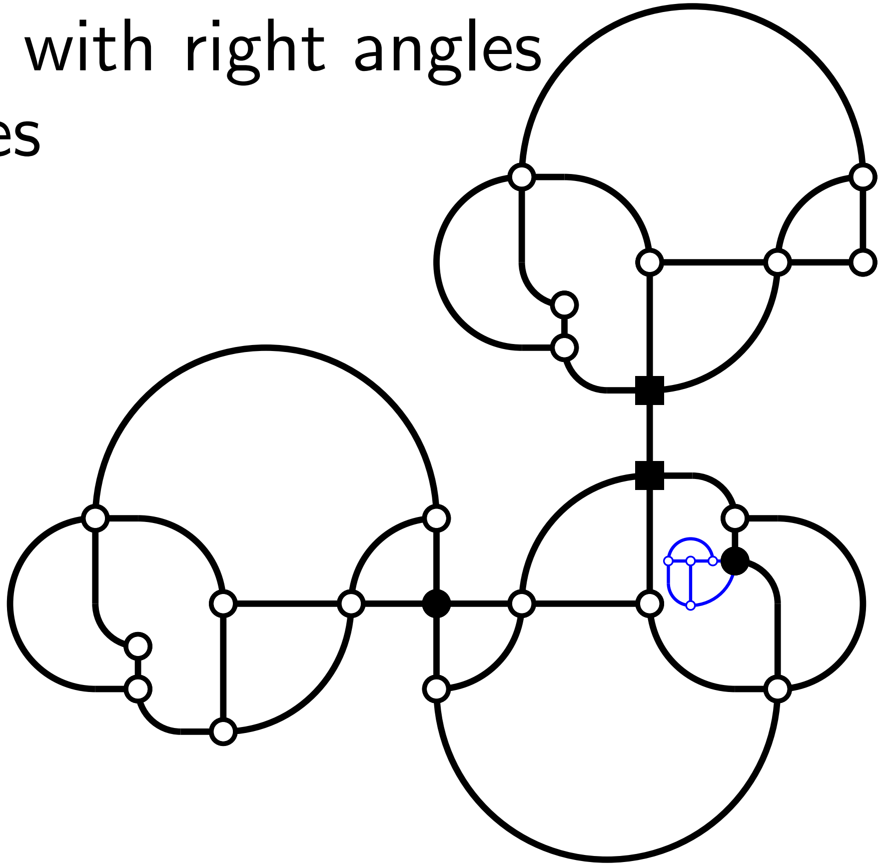

SC_1-Layouts

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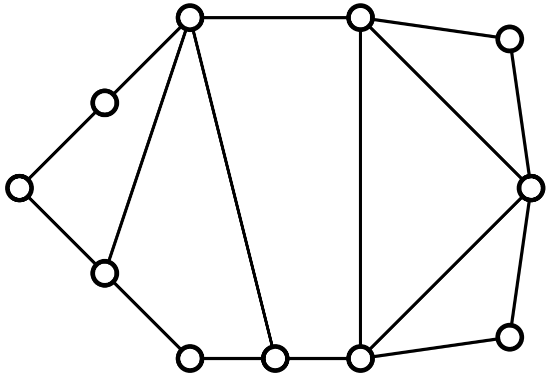

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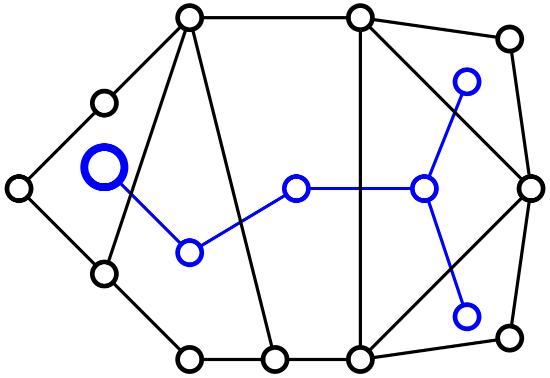

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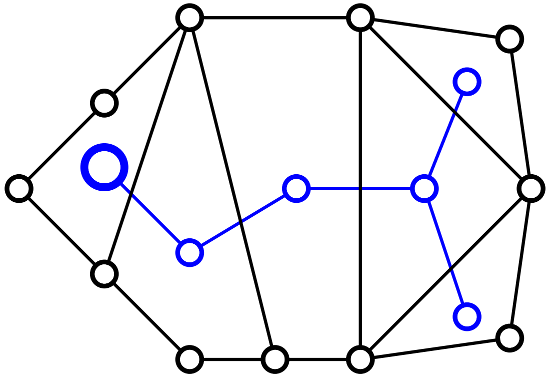

Consider the dual tree
Pick a root, traverse the tree

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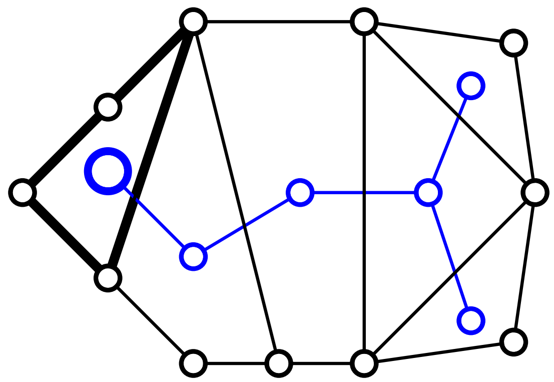

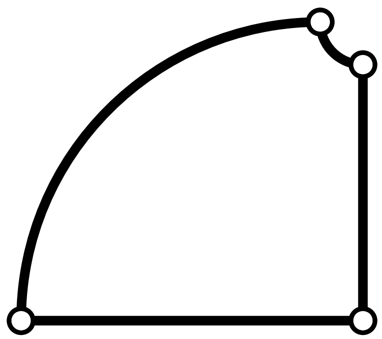

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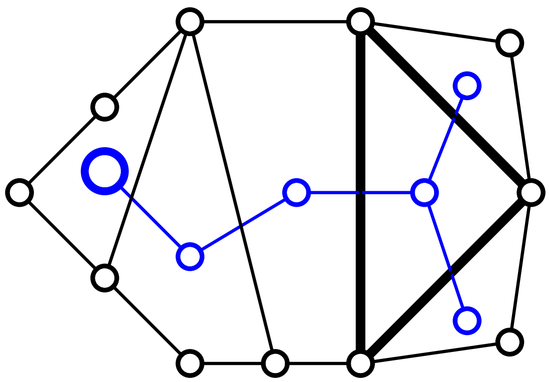

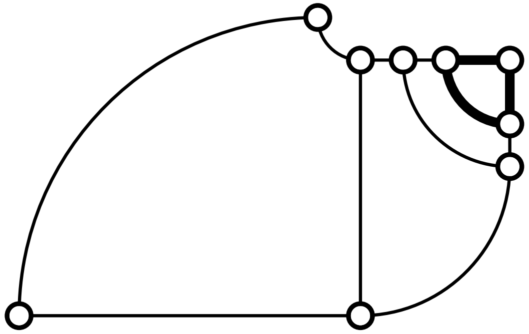

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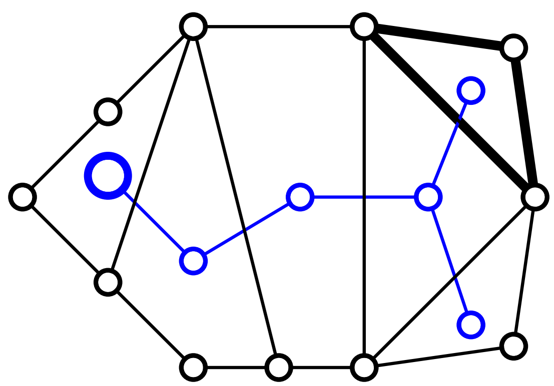

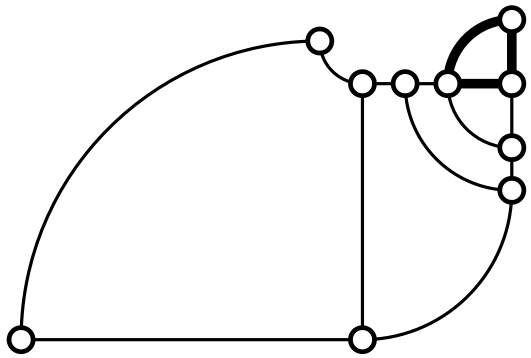

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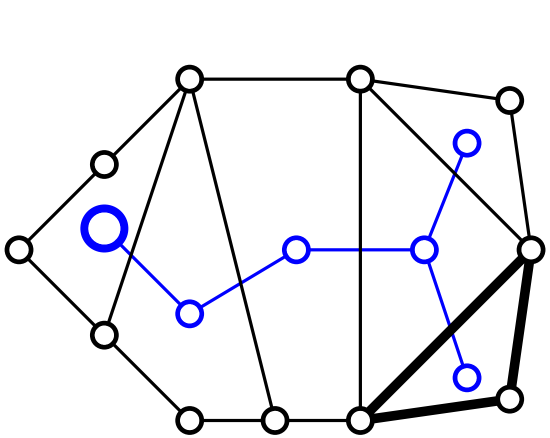

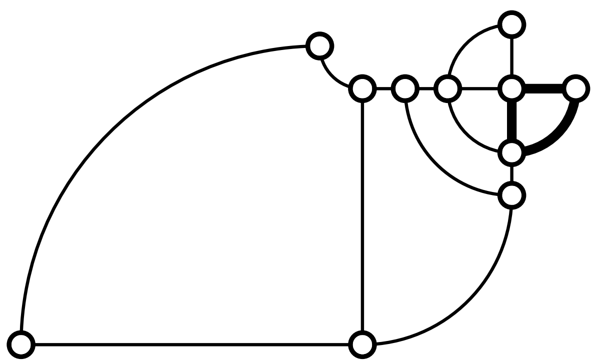

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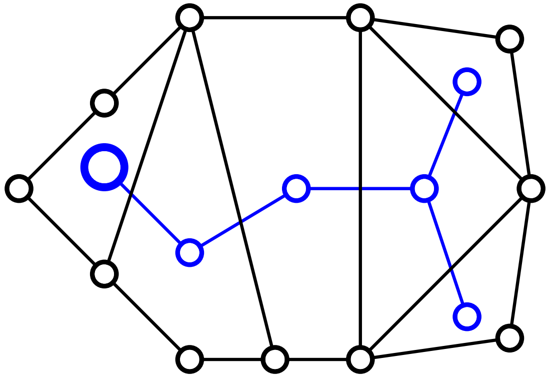

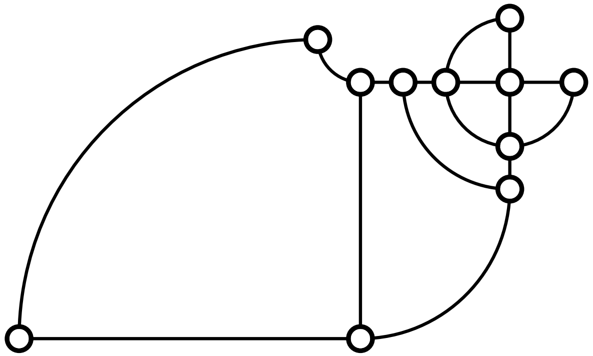

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Any Hamiltonian 3-planar graph admits an SC_1-layout.

Area Requirement of SC_1-Layouts

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There is an infinite class of graphs that require exponential area if they are drawn with SC_1.

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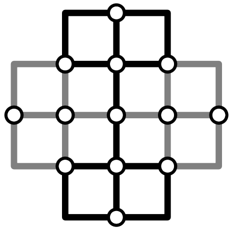

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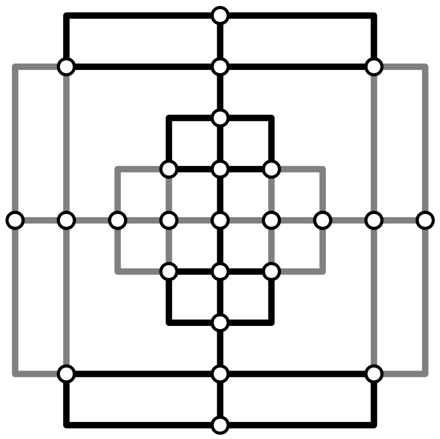

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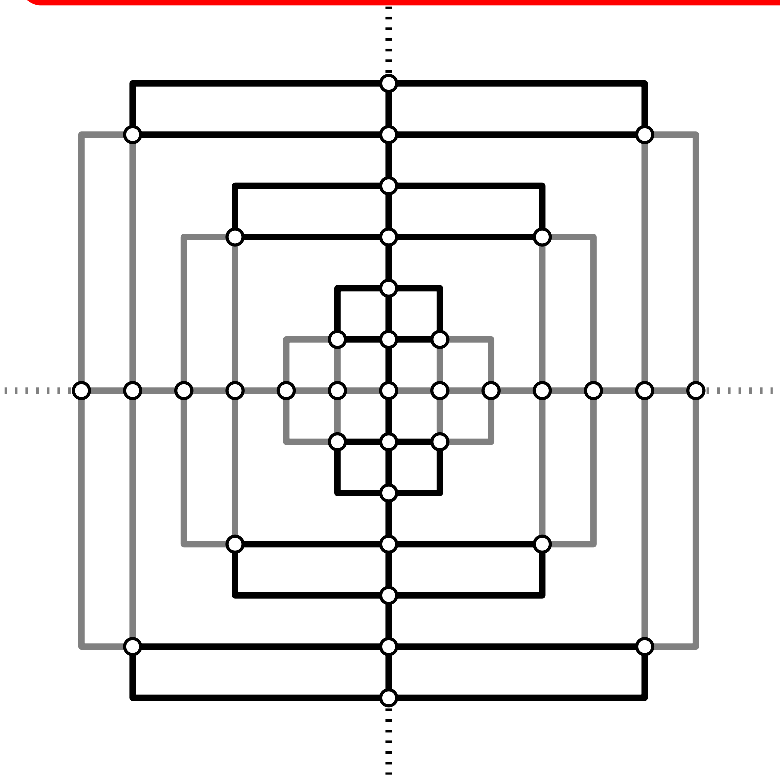

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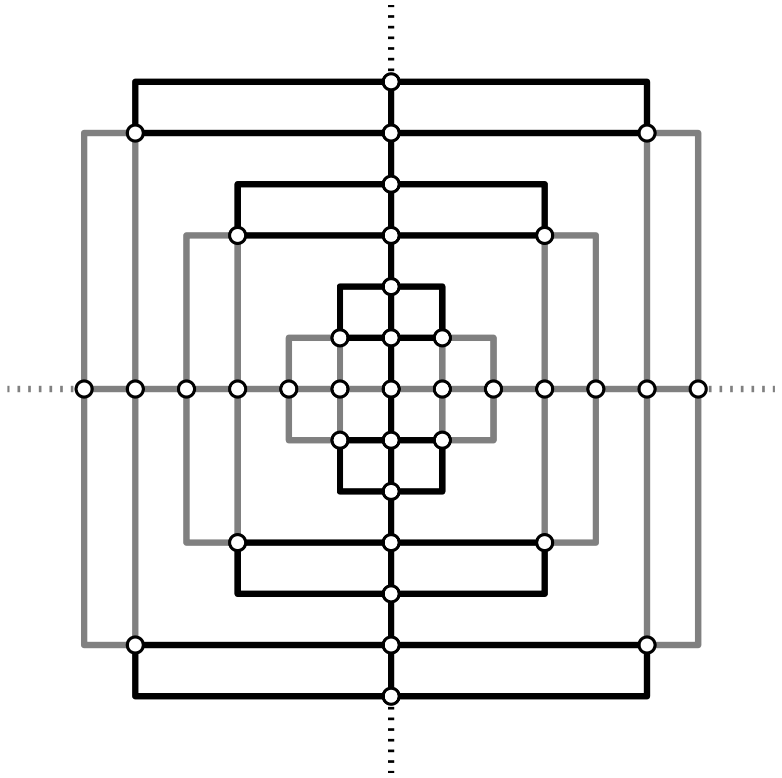

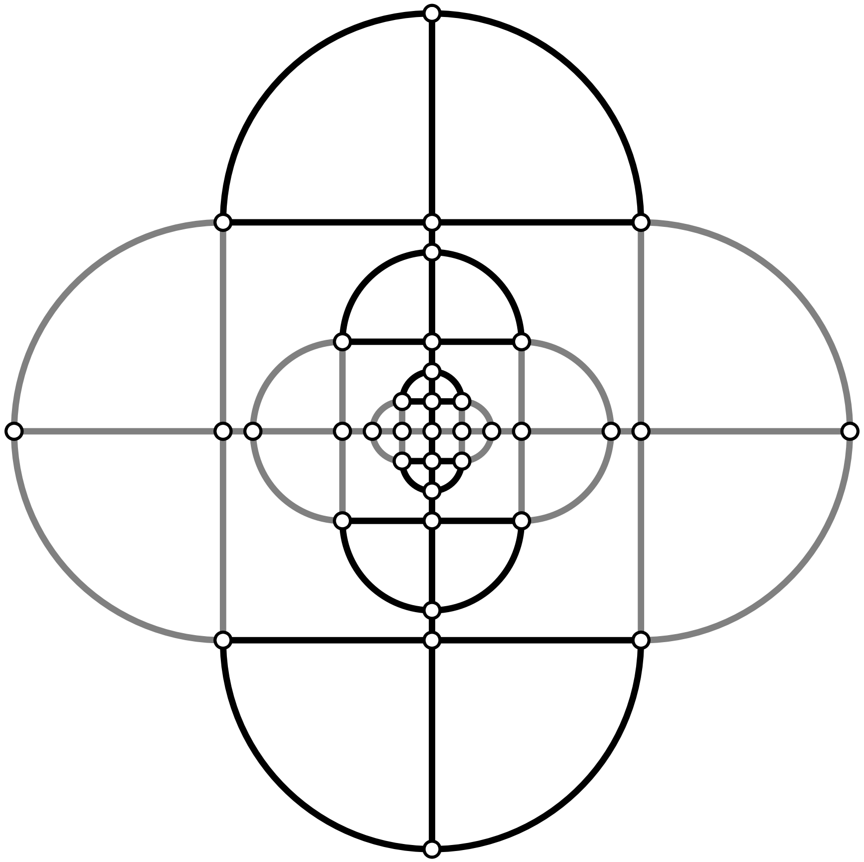

Biconnected Graphs without SC_1-Layout

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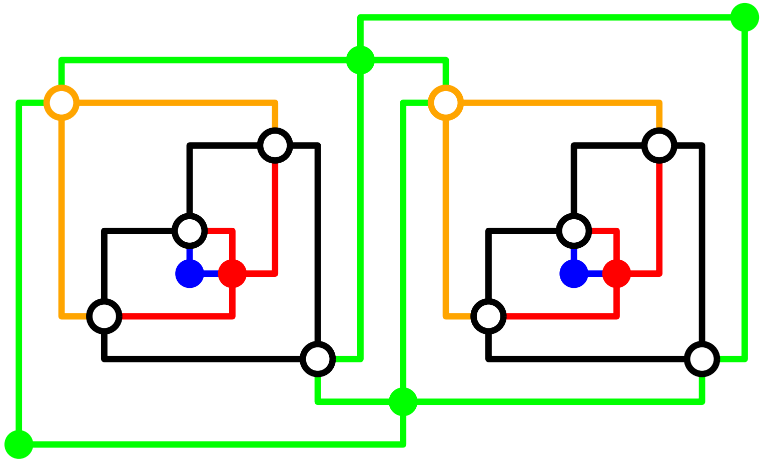

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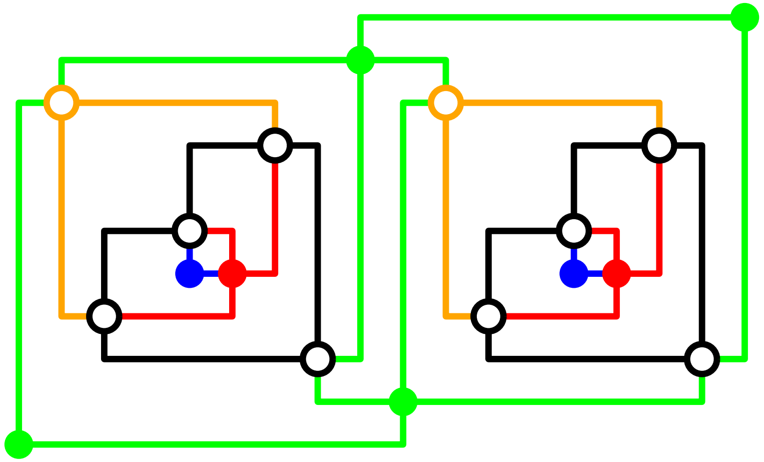

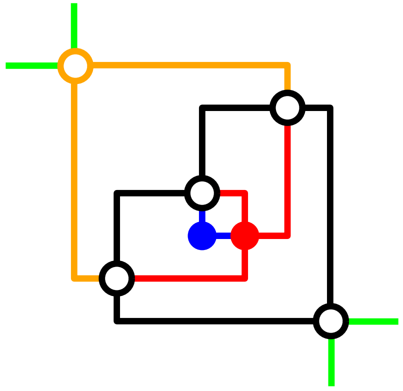

Open Problems

- Do all 4-planar graphs admit an SC_2-layout *in polynomial area*?

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Open Problems

- Do all 4-planar graphs admit an SC_2-layout *in polynomial area*?
- Do all 4-outerplanar graphs admit an SC_1-layout?
- Do all 3-planar graphs admit an SC_1-layout?
- Is it NP-hard to decide whether a 4-planar graph admits an SC_1-layout?