Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG
Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems
Institute for Informatics

On Monotone Drawings of Trees

Philipp Kindermann
Chair of Computer Science I
Universität Würzburg

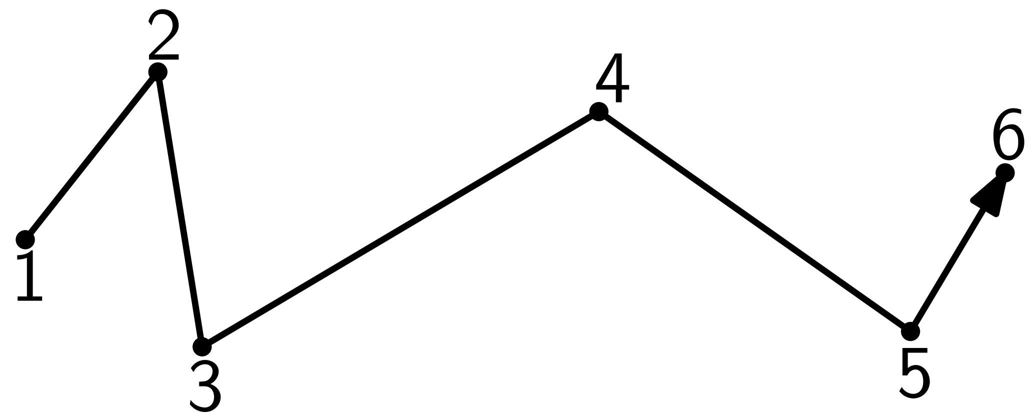
Joint work with
André Schulz, Joachim Spoerhase & Alexander Wolff

Monotone Drawings

A path is *monotone*: $\exists$ direction d such that vertex-order in d = vertex-order along the path.

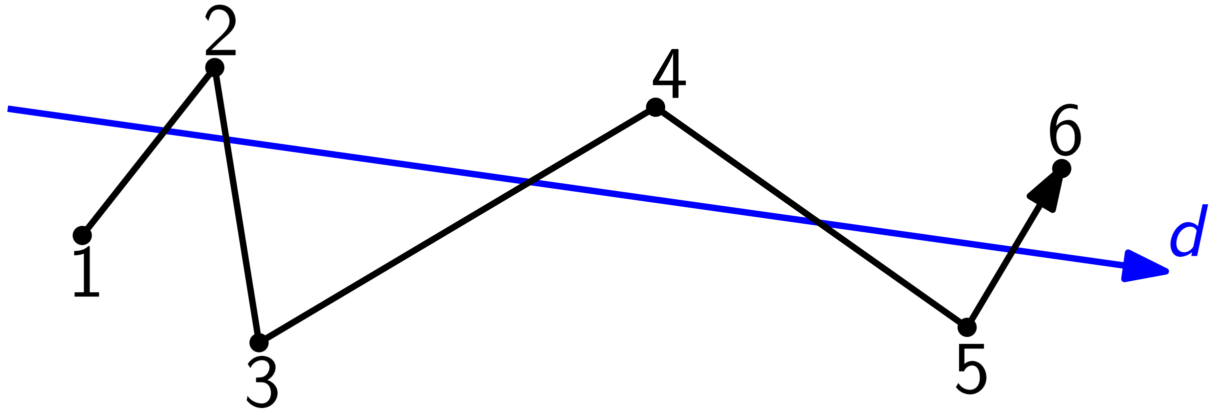
Monotone Drawings

A path is *monotone*: $\exists$ direction d such that vertex-order in d = vertex-order along the path.



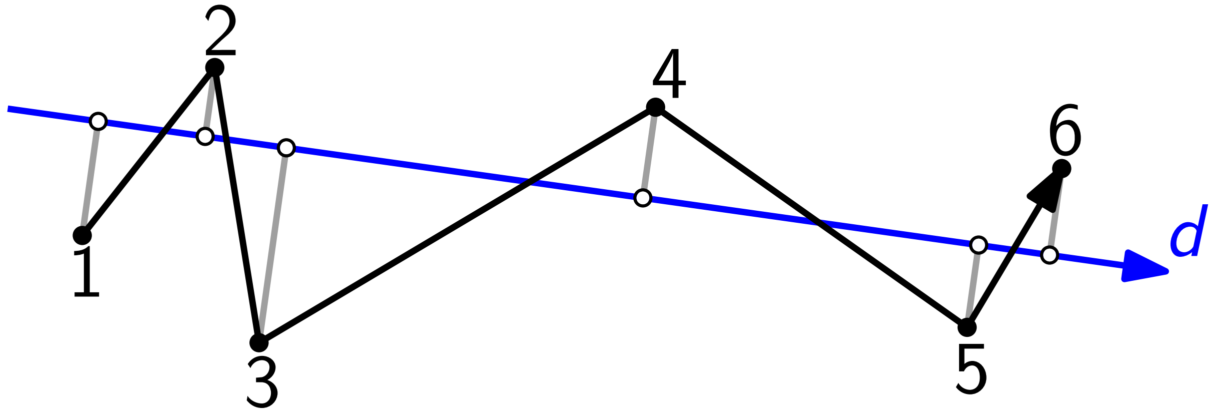
Monotone Drawings

A path is *monotone*: $\exists$ direction d such that vertex-order in d = vertex-order along the path.



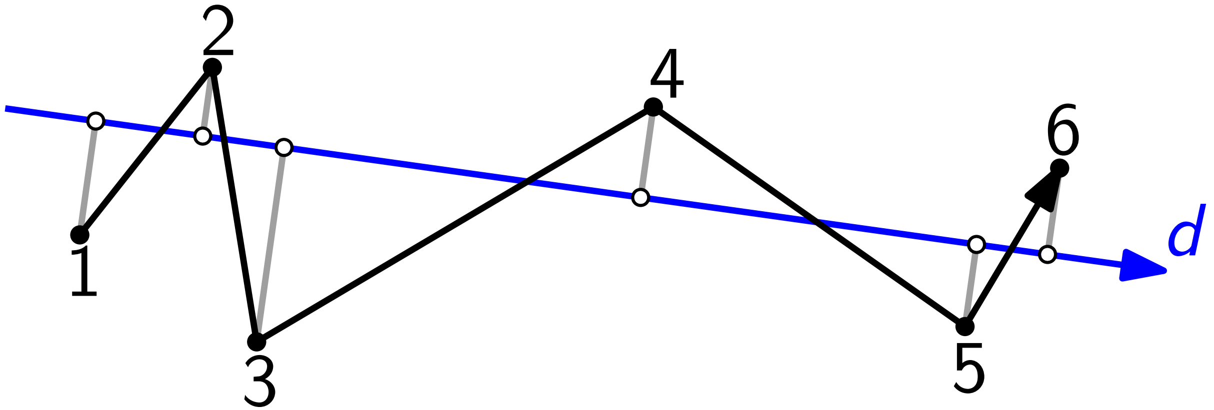
Monotone Drawings

A path is *monotone*: $\exists$ direction d such that vertex-order in d = vertex-order along the path.



Monotone Drawings

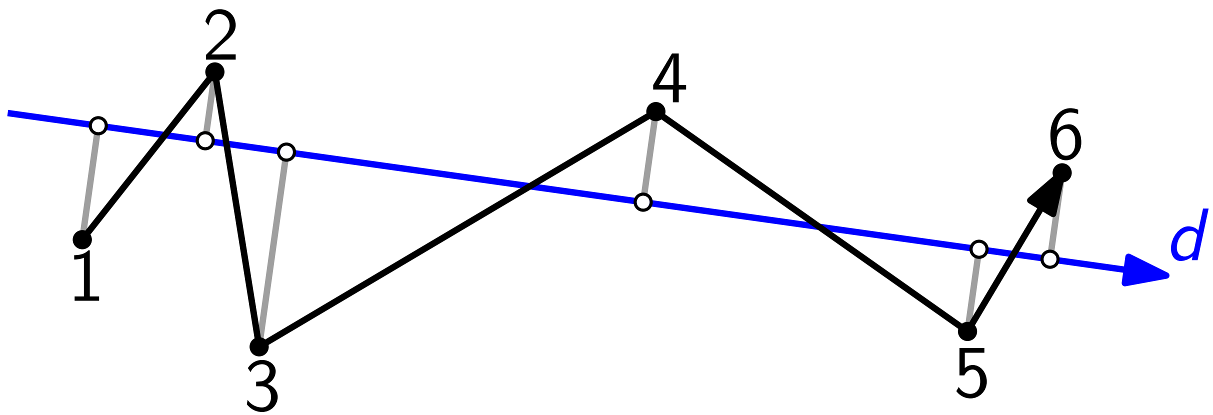
A path is *monotone*: $\exists$ direction d such that vertex-order in d = vertex-order along the path.



A graph is monotone: $\exists$ monotone path for every vertex-pair.

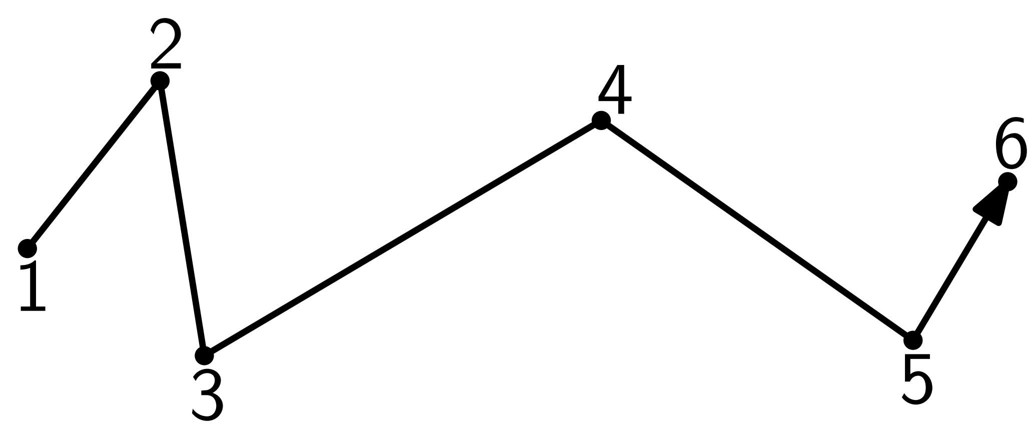
Monotone Drawings

A path is *monotone*: $\exists$ direction d such that vertex-order in d = vertex-order along the path.



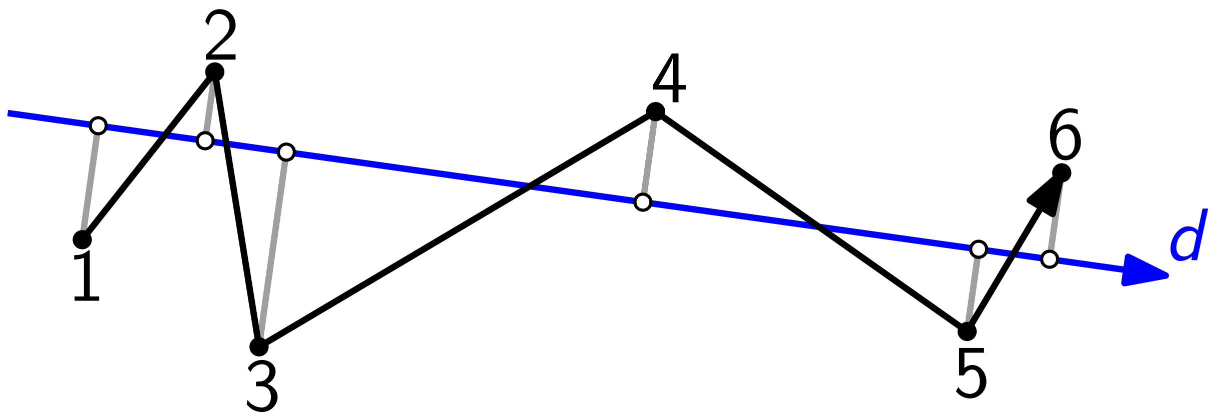
A graph is monotone: $\exists$ monotone path for every vertex-pair.

A u–v path is *strongly monotone* if $d = \overrightarrow{uv}$.



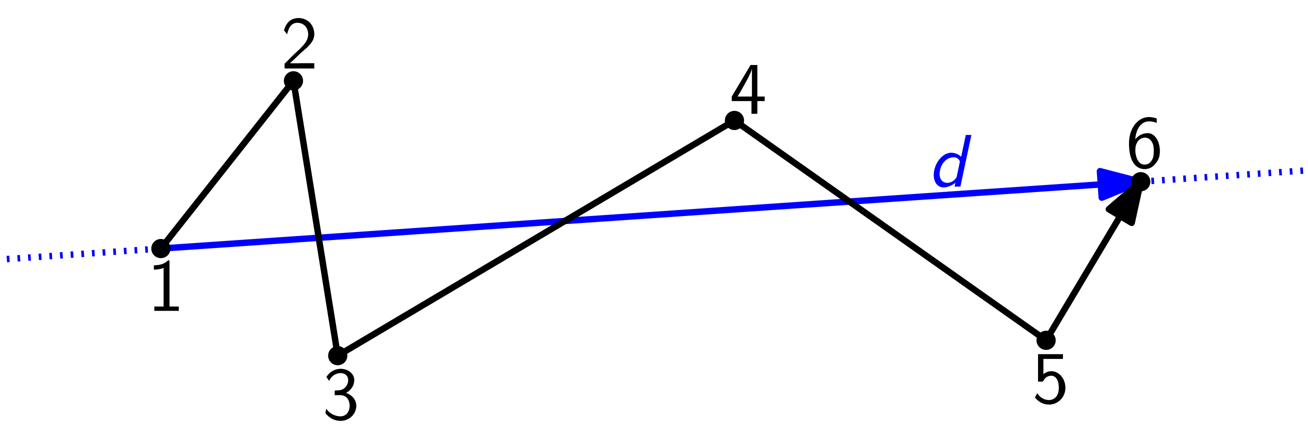
Monotone Drawings

A path is *monotone*: $\exists$ direction d such that vertex-order in d = vertex-order along the path.



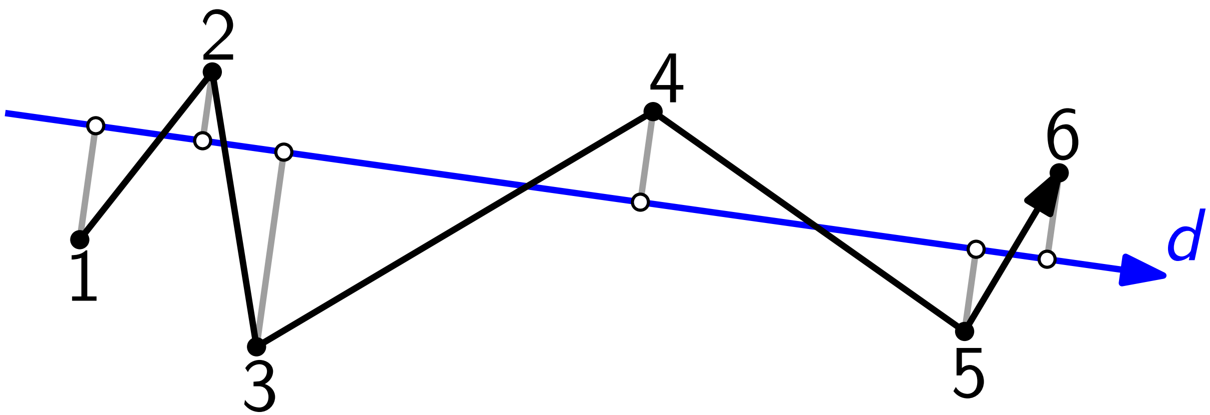
A graph is monotone: $\exists$ monotone path for every vertex-pair.

A u–v path is *strongly monotone* if $d = \overrightarrow{uv}$.



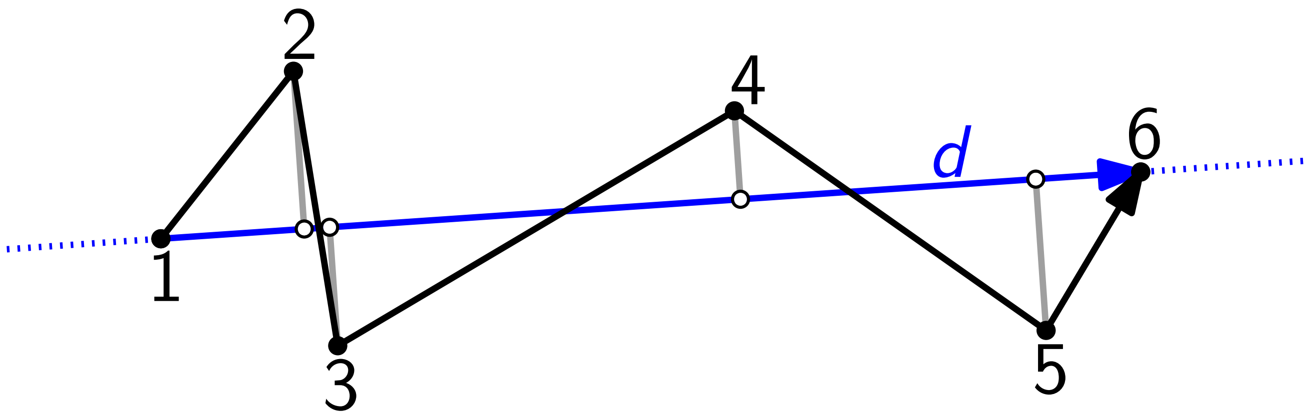
Monotone Drawings

A path is *monotone*: $\exists$ direction d such that vertex-order in d = vertex-order along the path.



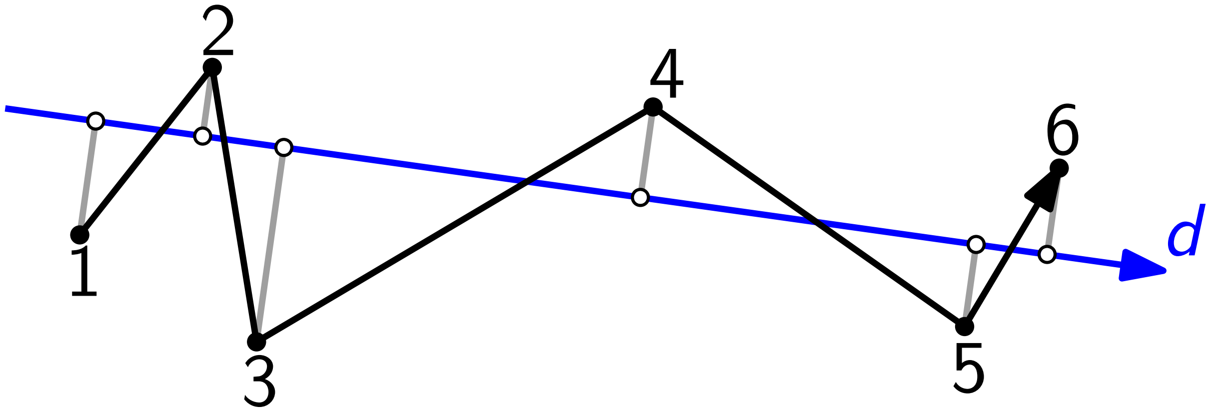
A graph is monotone: $\exists$ monotone path for every vertex-pair.

A u–v path is *strongly monotone* if $d = \overrightarrow{uv}$.



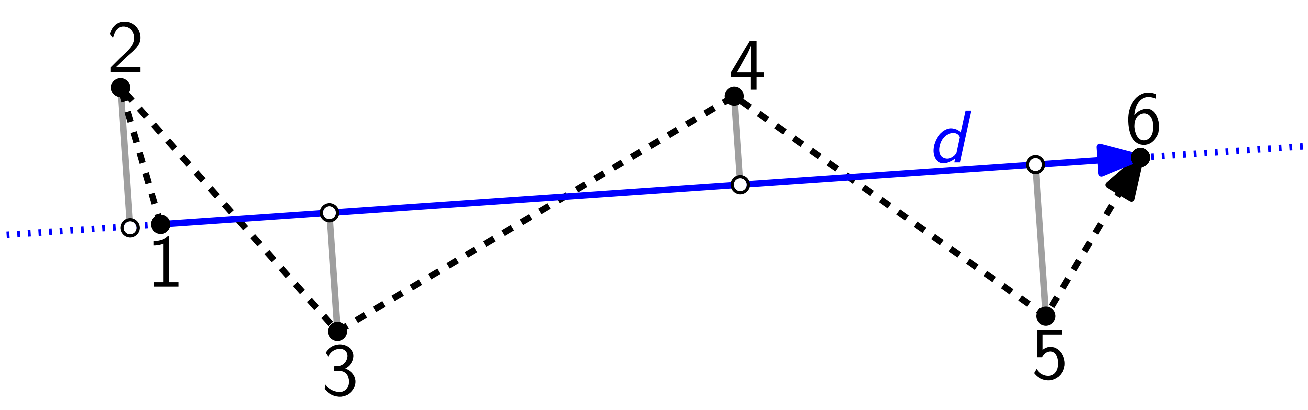
Monotone Drawings

A path is *monotone*: $\exists$ direction d such that vertex-order in d = vertex-order along the path.



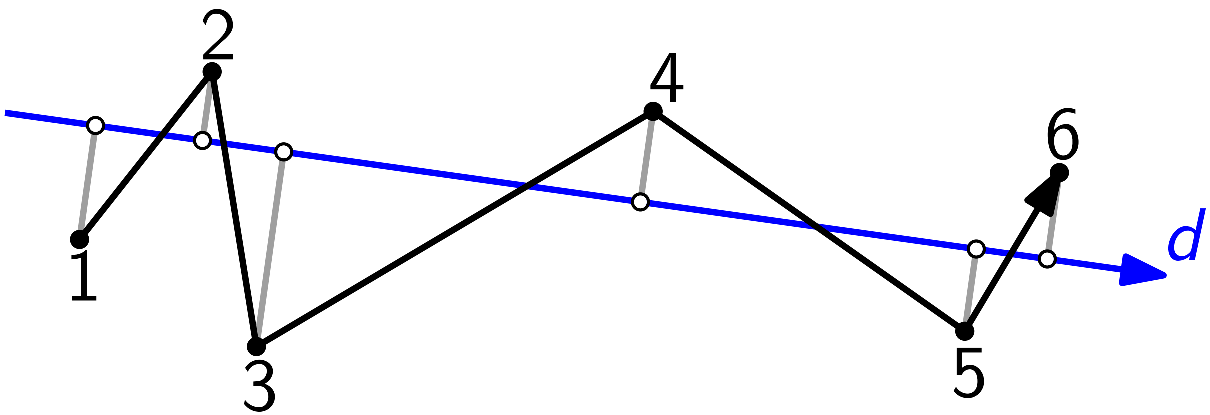
A graph is monotone: $\exists$ monotone path for every vertex-pair.

A u–v path is *strongly monotone* if $d = \overrightarrow{uv}$.



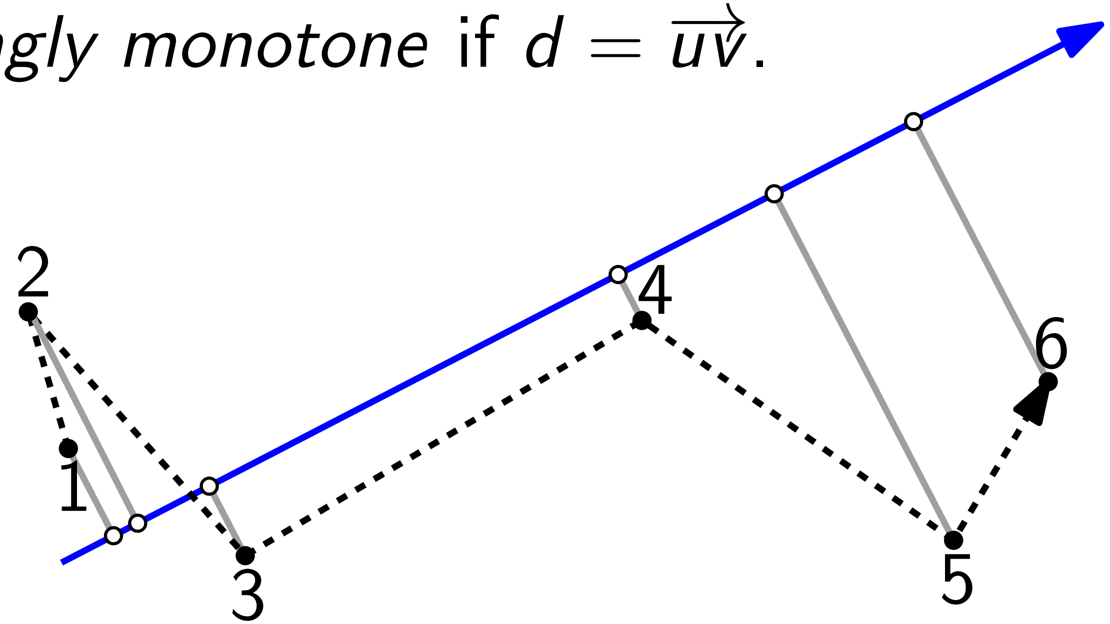
Monotone Drawings

A path is *monotone*: $\exists$ direction d such that vertex-order in d = vertex-order along the path.



A graph is monotone: $\exists$ monotone path for every vertex-pair.

A u–v path is *strongly monotone* if $d = \overrightarrow{uv}$.



Known Results

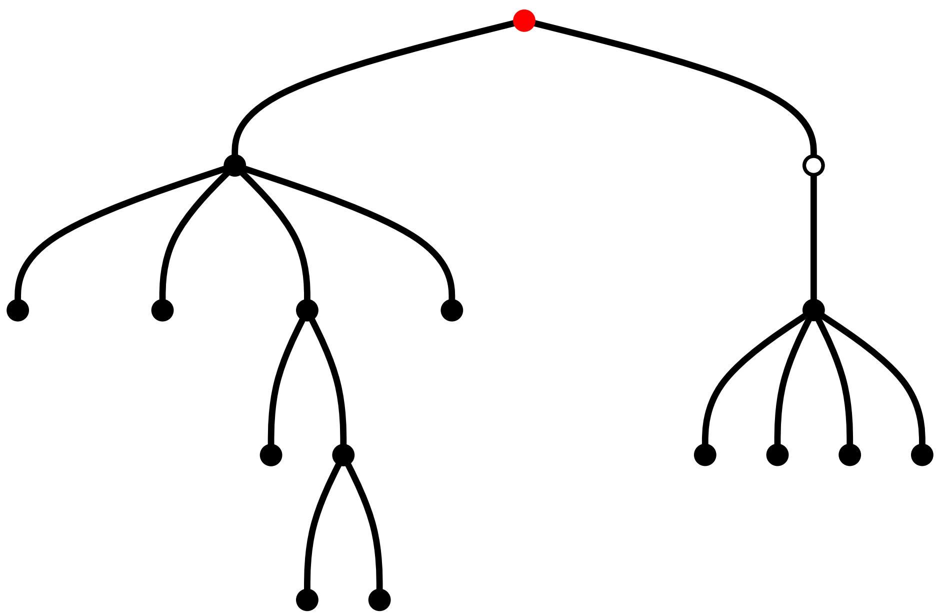
[Angelini et al. JGAA'12]

- Any *n-vertex tree* admits a straight-line monotone drawing on a grid of size $O(n^{1.6}) \times O(n^{1.6})$ or $O(n) \times O(n^2)$.

Known Results

[Angelini et al. JGAA'12]

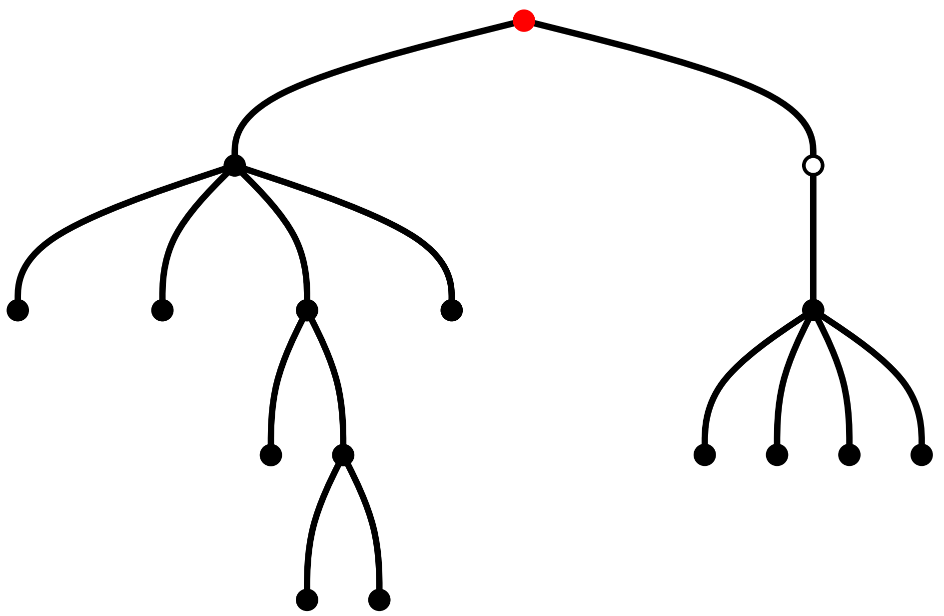
- Any *n-vertex tree* admits a straight-line monotone drawing on a grid of size $O(n^{1.6}) \times O(n^{1.6})$ or $O(n) \times O(n^2)$.

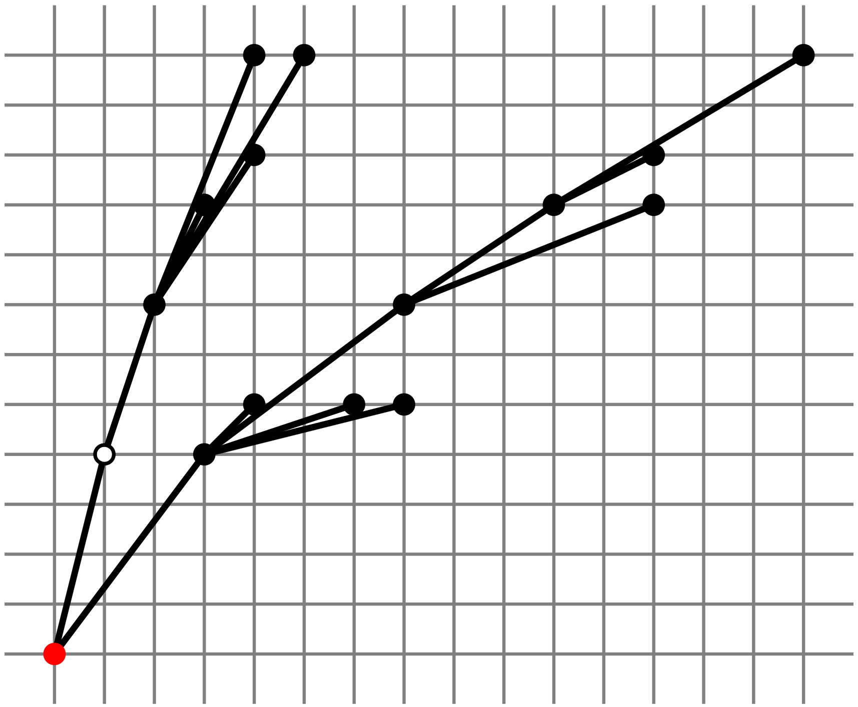


Known Results

[Angelini et al. JGAA'12]

- Any *n-vertex tree* admits a straight-line monotone drawing on a grid of size $O(n^{1.6}) \times O(n^{1.6})$ or $O(n) \times O(n^2)$.





Known Results

[Angelini et al. JGAA'12]

- Any *n-vertex tree* admits a straight-line monotone drawing on a grid of size $O(n^{1.6}) \times O(n^{1.6})$ or $O(n) \times O(n^2)$.
- Any *biconnected planar* graph has a monotone drawing (using exponential area).

Known Results

[Angelini et al. JGAA'12]

- Any *n-vertex tree* admits a straight-line monotone drawing on a grid of size $O(n^{1.6}) \times O(n^{1.6})$ or $O(n) \times O(n^2)$.
- Any *biconnected planar* graph has a monotone drawing (using exponential area).

[Angelini et al. Algorithmica'13]

Any connected *outerplane* graph admits a straight-line monotone drawing on a grid of size $O(n) \times O(n^2)$.

Known Results

[Angelini et al. JGAA'12]

- Any *n-vertex tree* admits a straight-line monotone drawing on a grid of size $O(n^{1.6}) \times O(n^{1.6})$ or $O(n) \times O(n^2)$.
- Any *biconnected planar* graph has a monotone drawing (using exponential area).

[Angelini et al. Algorithmica'13]

Any connected *outerplane* graph admits a straight-line monotone drawing on a grid of size $O(n) \times O(n^2)$.

[Hossain and Rahman FAW'14]

Any connected *planar* graph admits a straight-line monotone drawing on a grid of size $O(n) \times O(n^2)$.

Convex Drawings

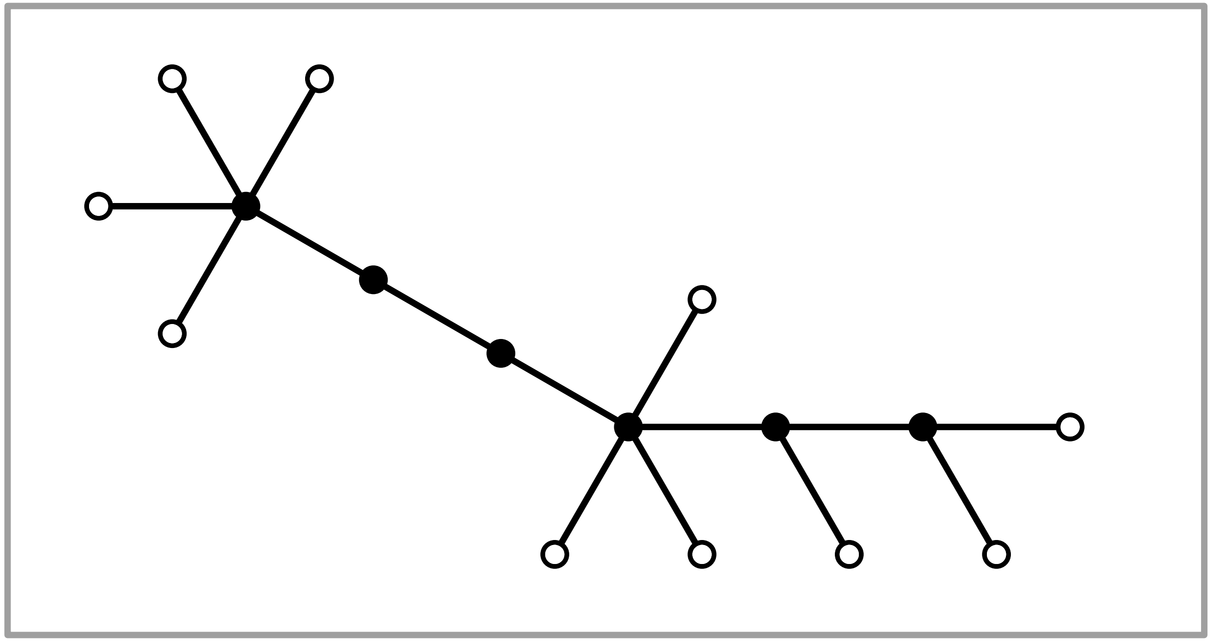
Convex drawing: Every face is convex.

Convex Drawings

Convex drawing: Every face is convex.

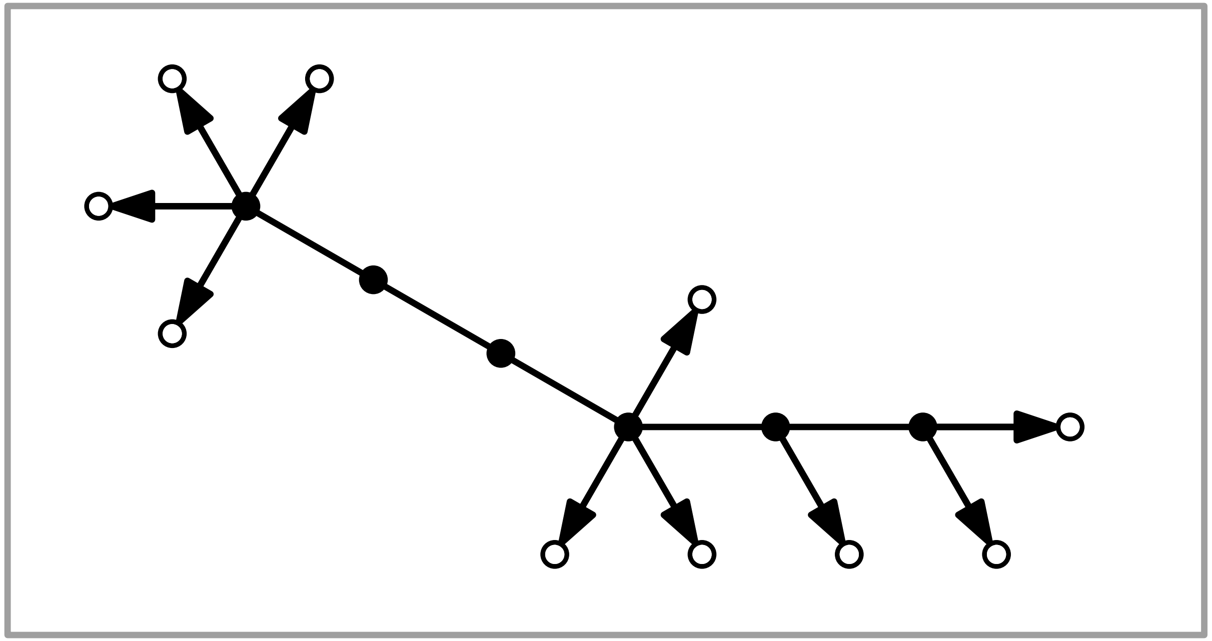
Convex Drawings

Convex drawing: Every face is convex.



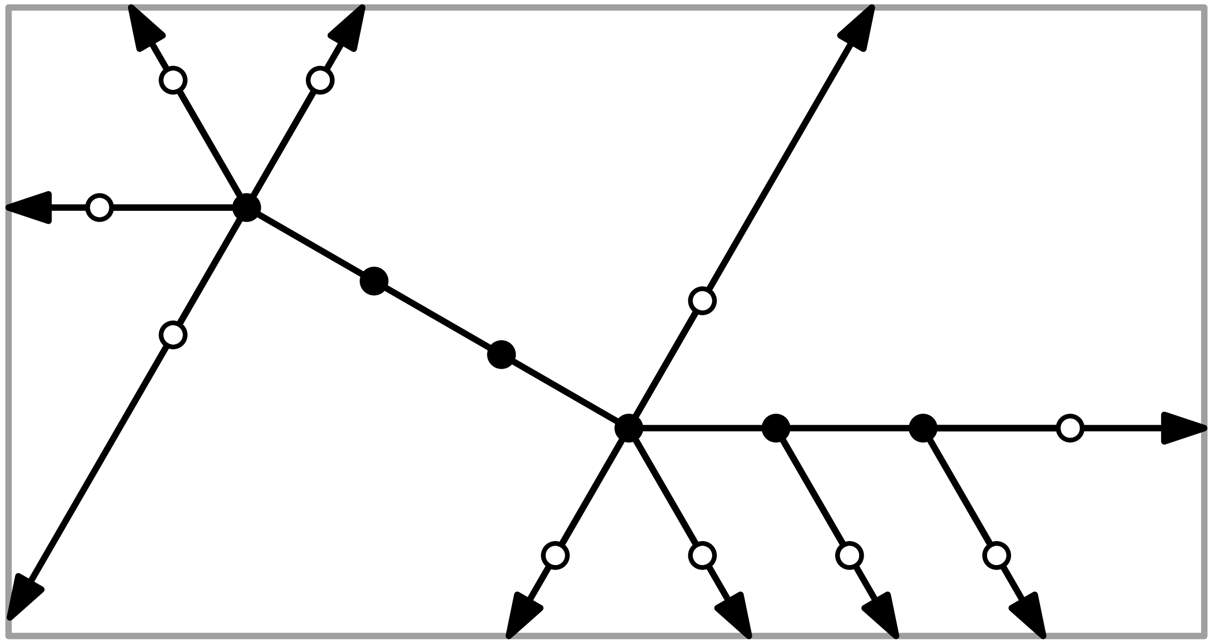
Convex Drawings

Convex drawing: Every face is convex.



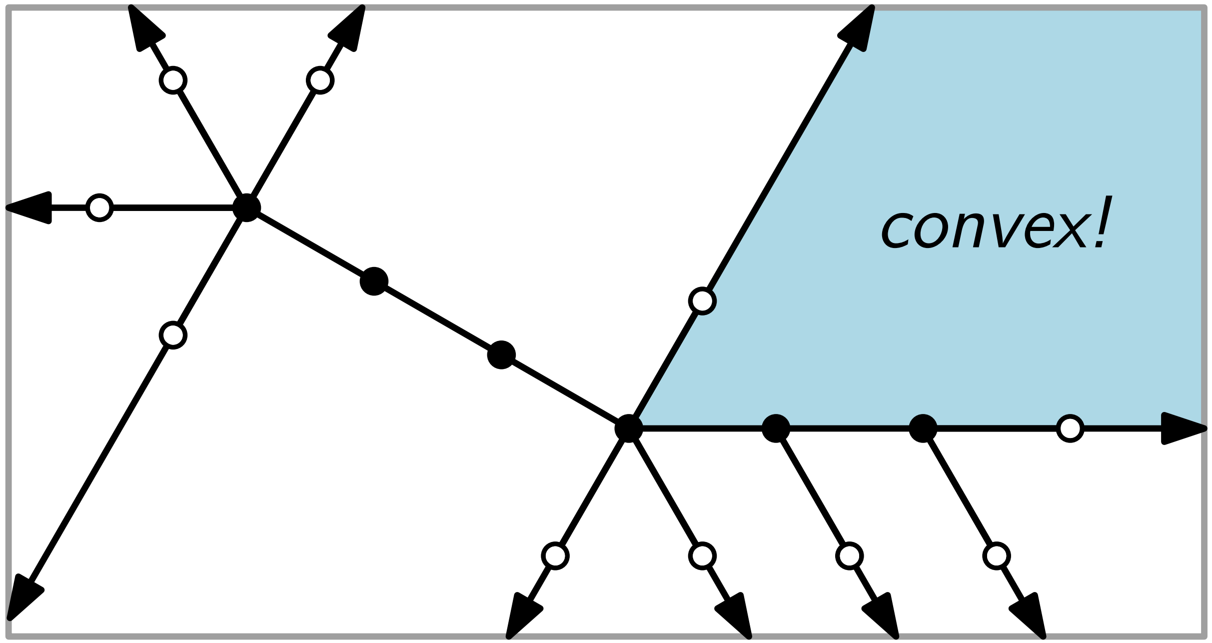
Convex Drawings

Convex drawing: Every face is convex.



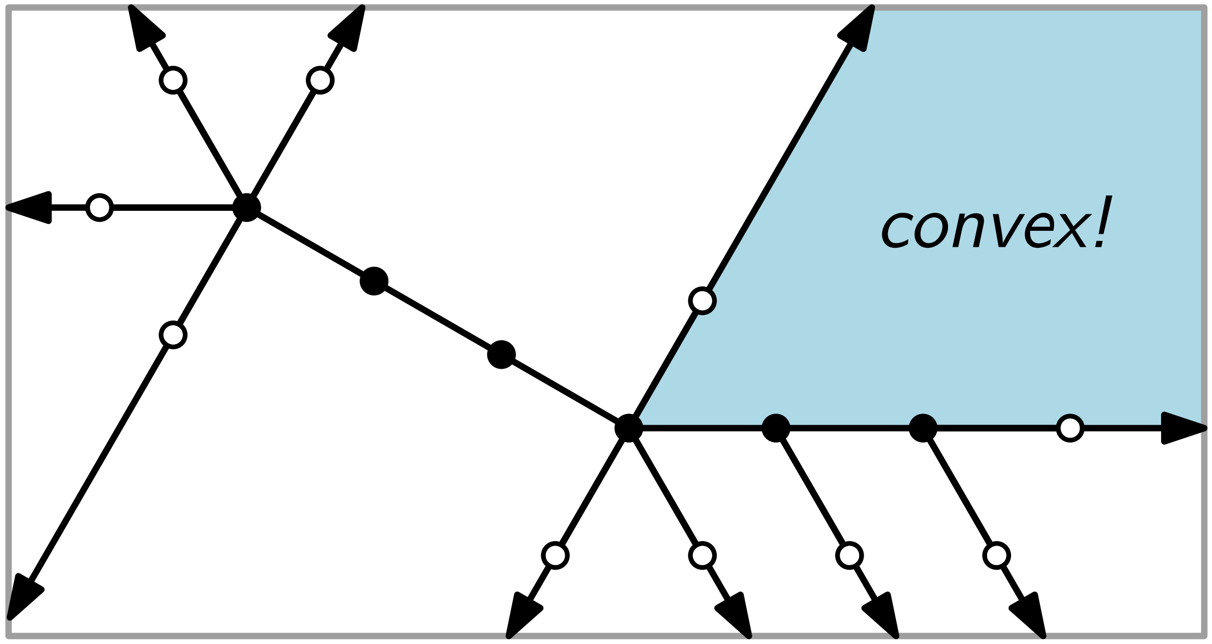
Convex Drawings

Convex drawing: Every face is convex.



Convex Drawings

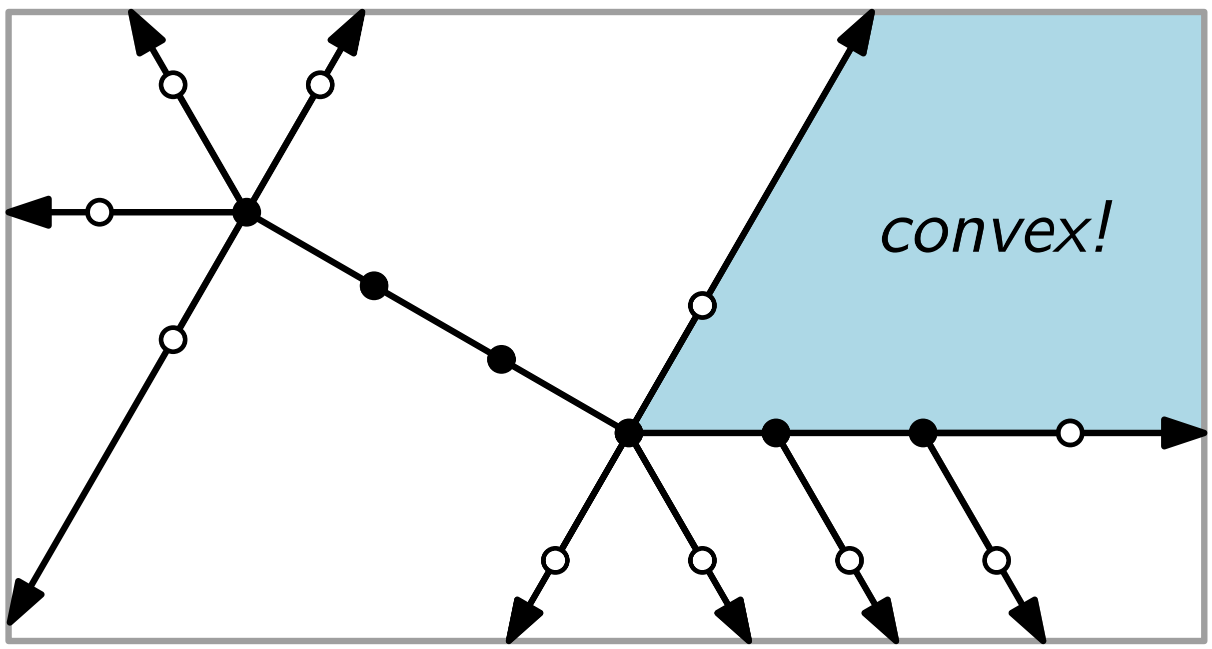
Convex drawing: Every face is convex.



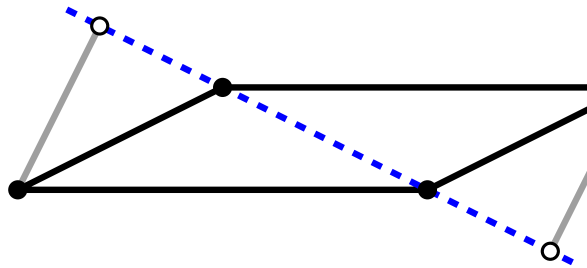
Any convex straight-line drawing is crossing-free and monotone

Convex Drawings

Convex drawing: Every face is convex.

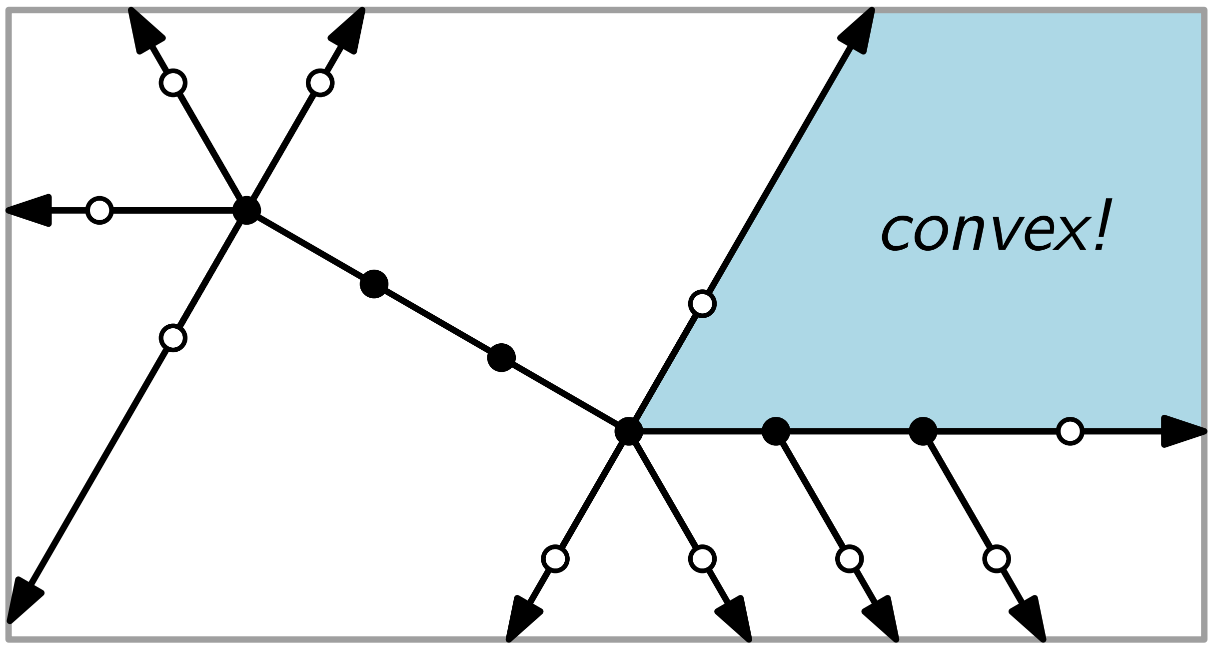


Any convex straight-line drawing is crossing-free and monotone – but in general not strongly monotone.



Convex Drawings

Convex drawing: Every face is convex.



Any convex straight-line drawing is crossing-free and monotone – but in general not strongly monotone.

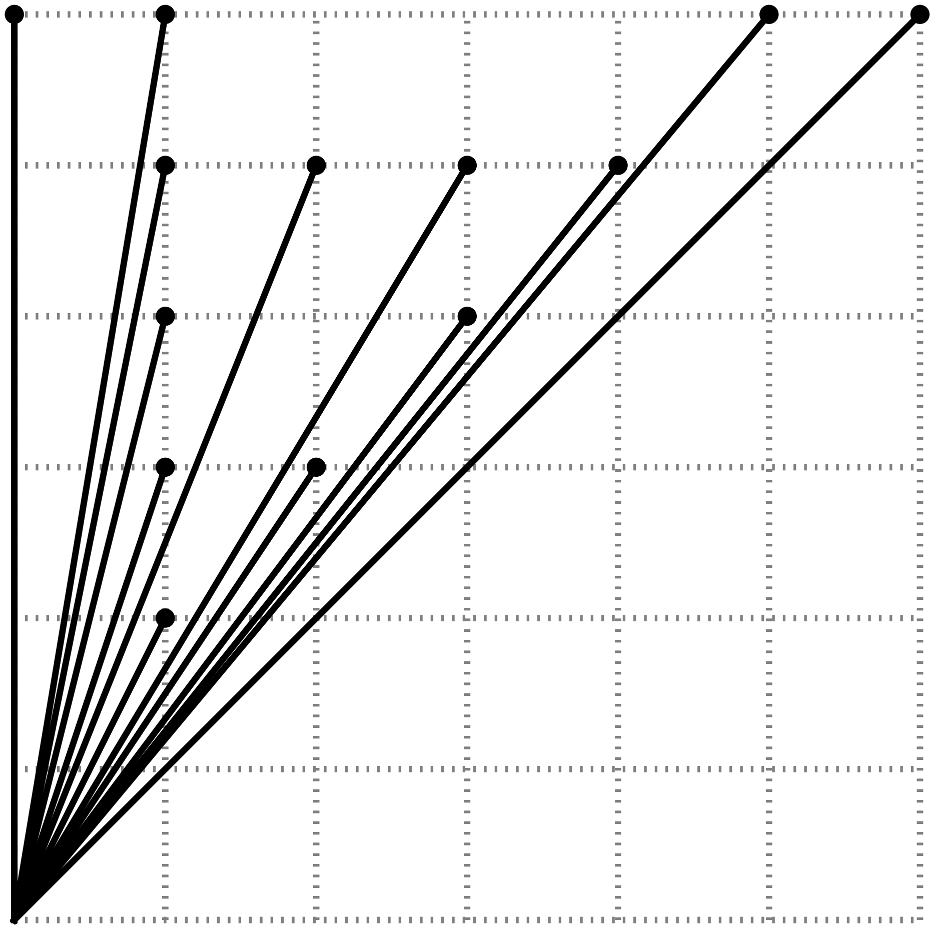
[Carlson & Eppstein GD'06]
Can compute convex drawings of trees,
with optimal *angular resolution*.

Our Main Tool: Primitive Vectors

Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \leq x \leq y \leq d\}$.

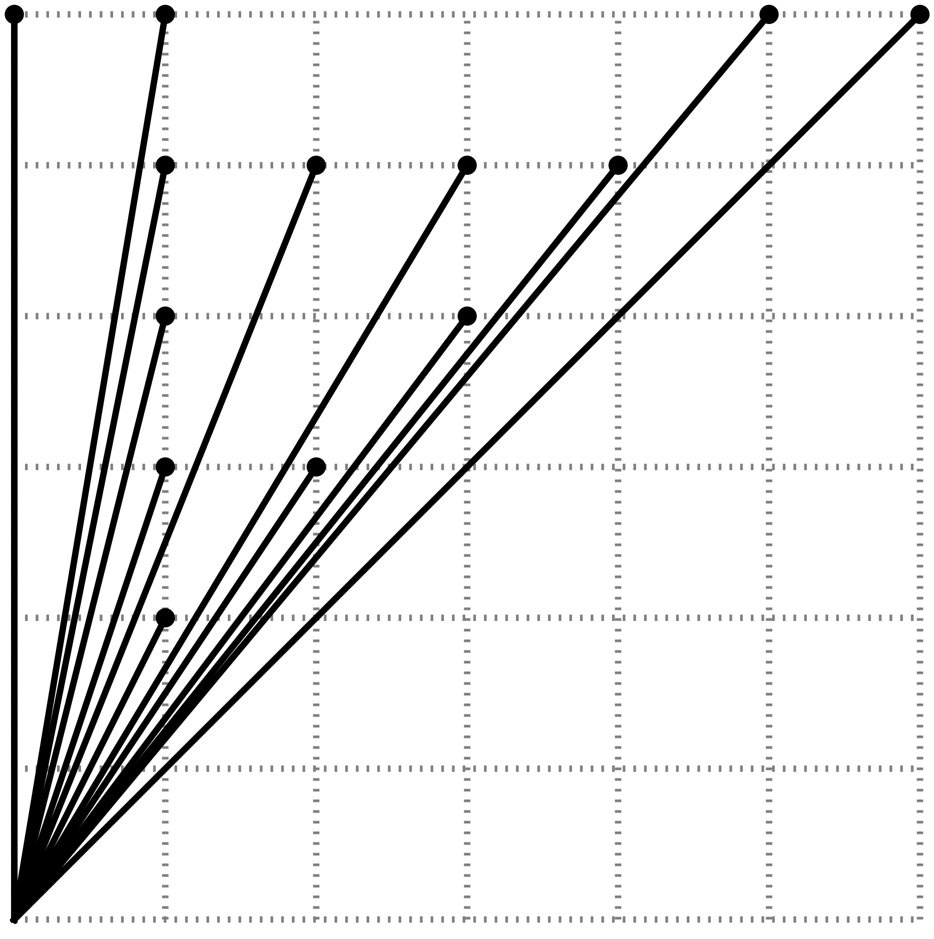
Our Main Tool: Primitive Vectors

Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \le x \le y \le d\}$.



Our Main Tool: Primitive Vectors

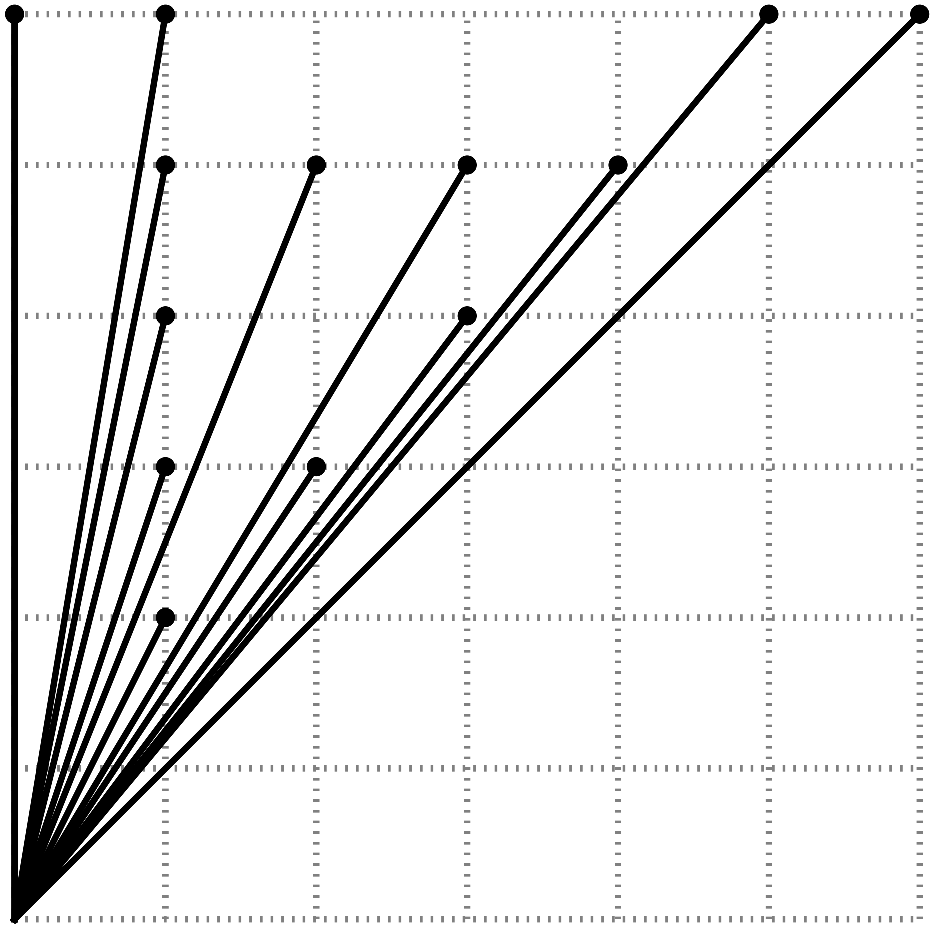
Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \leq x \leq y \leq d\}$.



Lemma. Any two vectors of P_d are separated by an angle of $\Omega(1/|P_d|)$.

Our Main Tool: Primitive Vectors

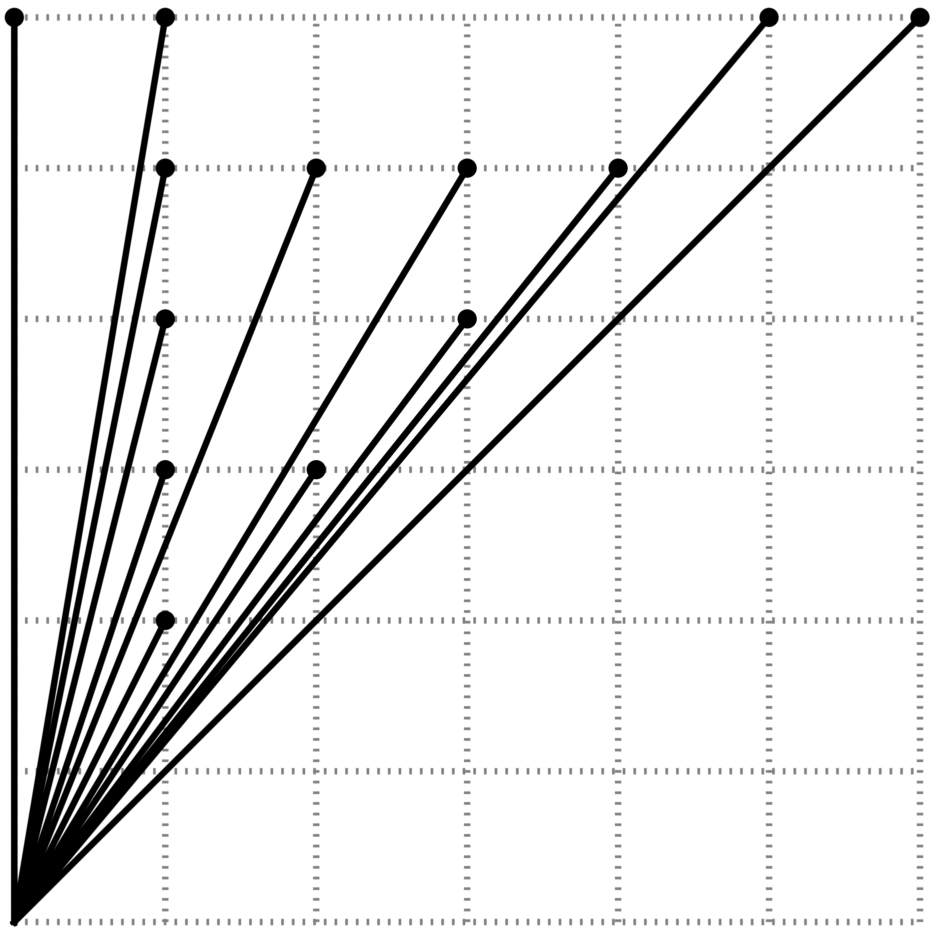
Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \leq x \leq y \leq d\}$.



Lemma. Any two vectors of P_d are separated by an angle of $\Omega(1/|P_d|)$. For $|P_d| \geq n - 1$, choose $d \approx 4\sqrt{n}$.

Our Main Tool: Primitive Vectors

Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \leq x \leq y \leq d\}$.

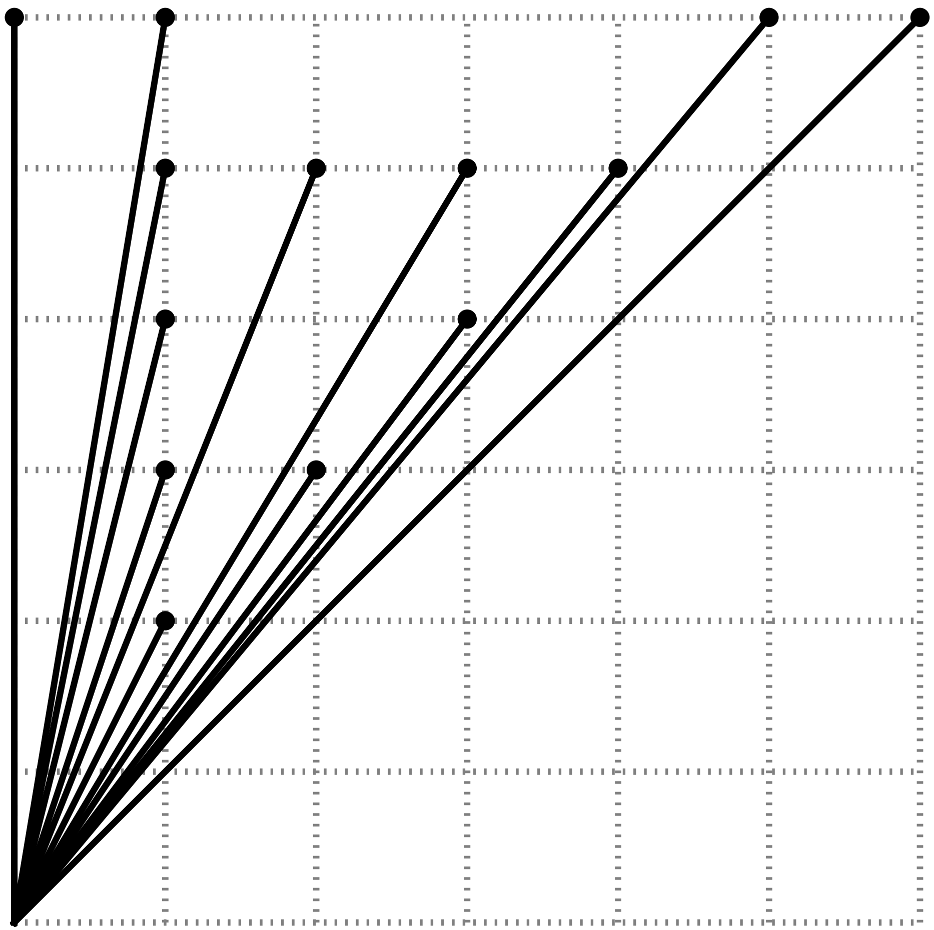


Farey sequence

Lemma. Any two vectors of P_d are separated by an angle of $\Omega(1/|P_d|)$. For $|P_d| \geq n - 1$, choose $d \approx 4\sqrt{n}$.

Our Main Tool: Primitive Vectors

Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \le x \le y \le d\}$.



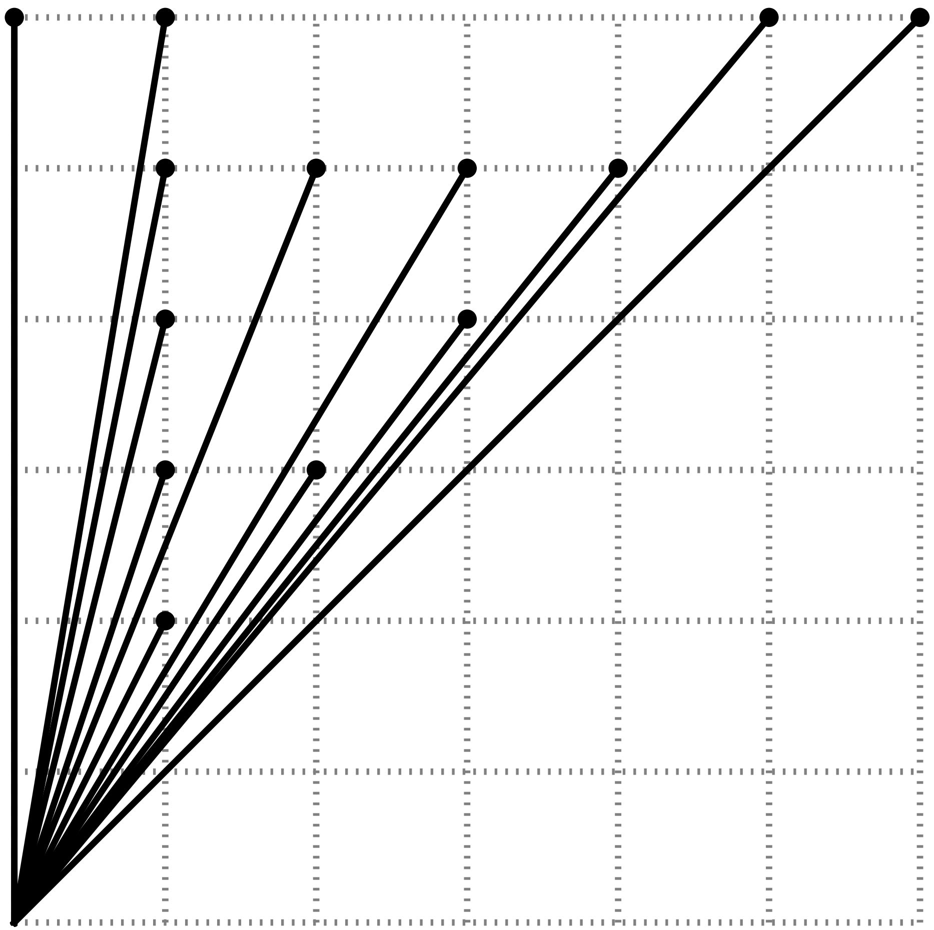
Farey sequence

$F_1 : \frac{0}{1} \qquad \frac{1}{1}$

Lemma. Any two vectors of P_d are separated by an angle of $\Omega(1/|P_d|)$. For $|P_d| \ge n - 1$, choose $d \approx 4\sqrt{n}$.

Our Main Tool: Primitive Vectors

Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \leq x \leq y \leq d\}$.



Farey sequence

$F_1 : \frac{0}{1} \qquad \frac{1}{1}$

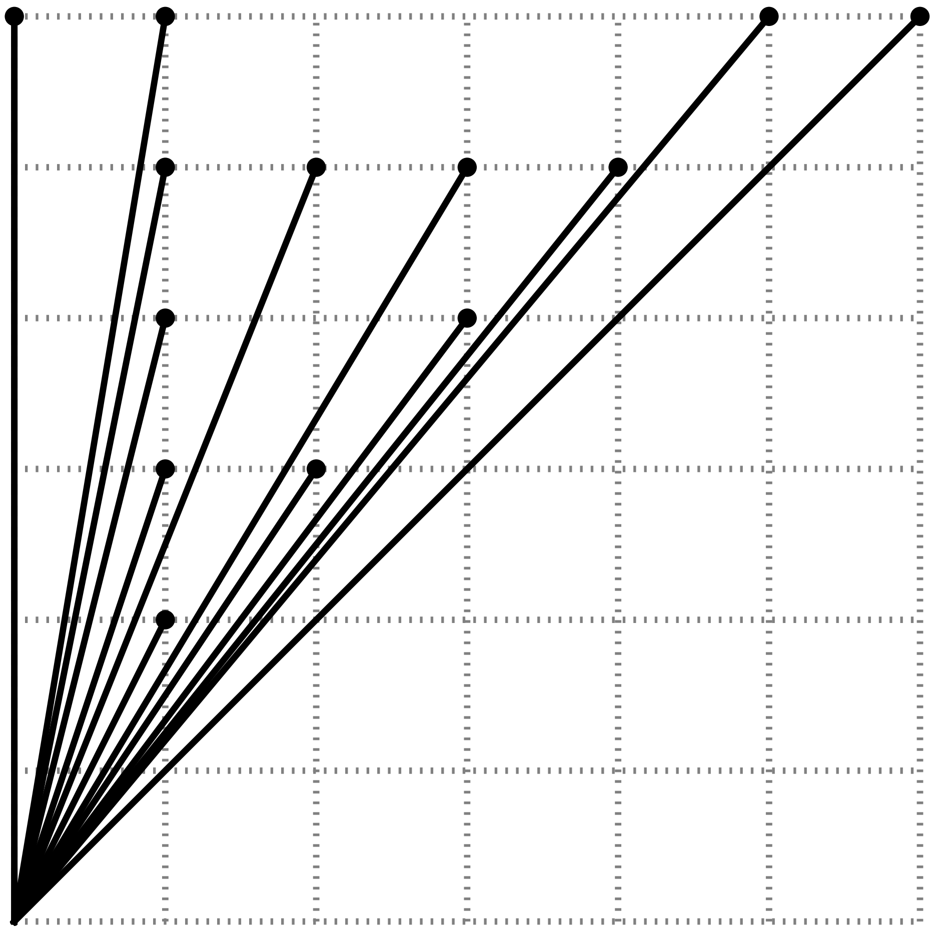
$\frac{a}{b} \qquad \frac{c}{d}$

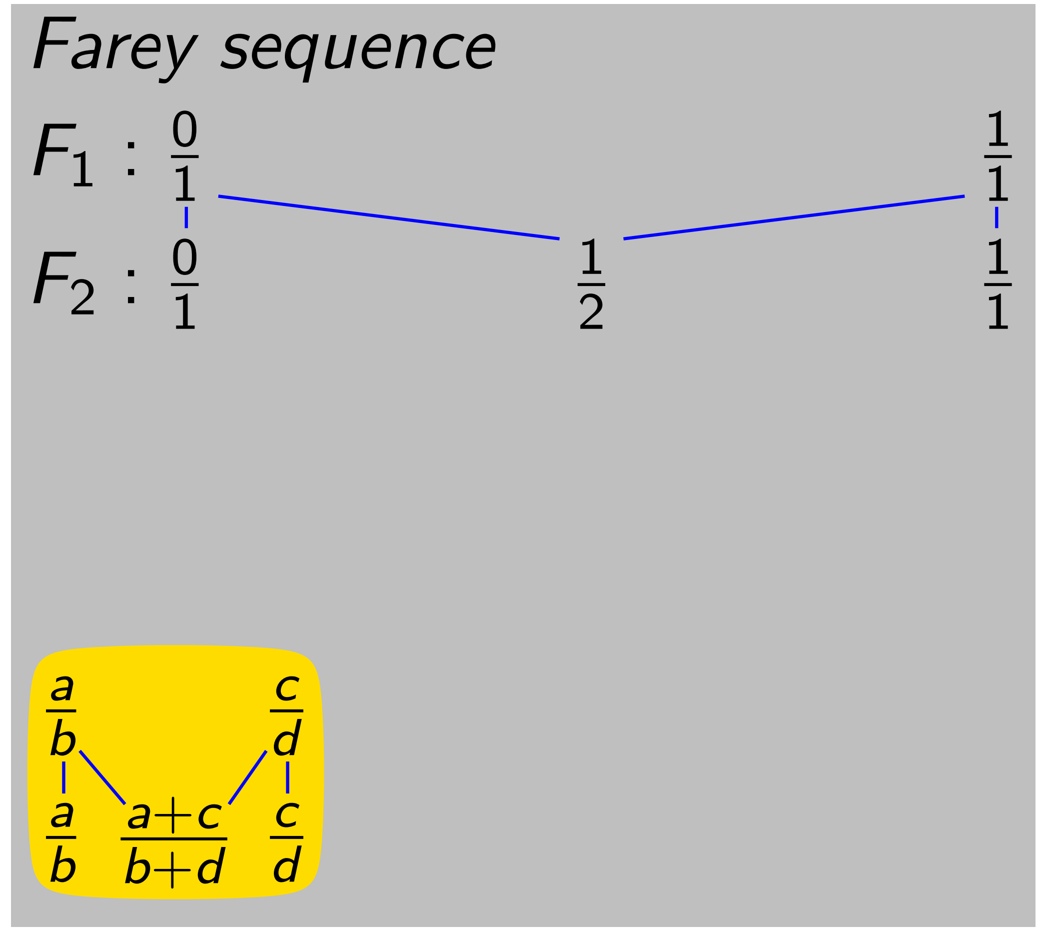
$\frac{a}{b} \quad \frac{a+c}{b+d} \quad \frac{c}{d}$

Lemma. Any two vectors of P_d are separated by an angle of $\Omega(1/|P_d|)$. For $|P_d| \geq n-1$, choose $d \approx 4\sqrt{n}$.

Our Main Tool: Primitive Vectors

Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \leq x \leq y \leq d\}$.

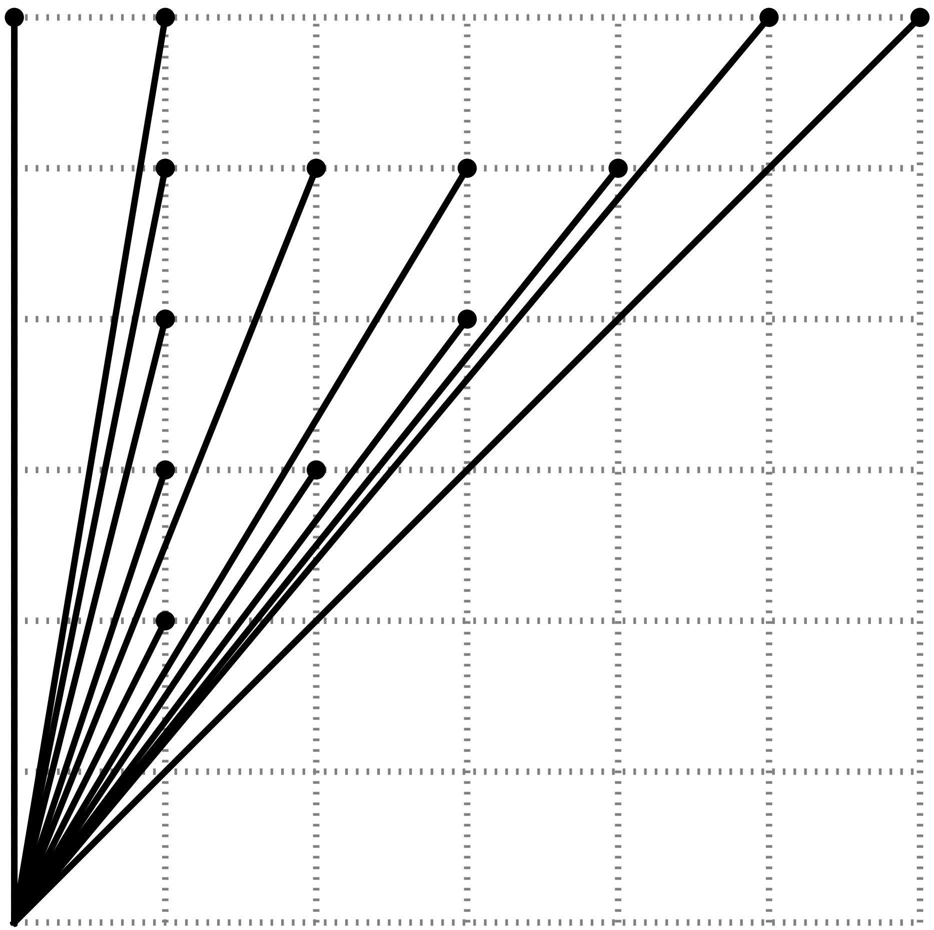




Lemma. Any two vectors of P_d are separated by an angle of $\Omega(1/|P_d|)$. For $|P_d| \geq n - 1$, choose $d \approx 4\sqrt{n}$.

Our Main Tool: Primitive Vectors

Let $P_d = \{(x,y) \mid \gcd(x,y) = 1,\ 0 \le x \le y \le d\}$.

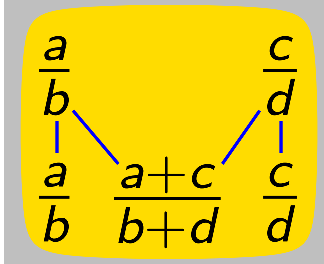


Farey sequence

$F_1 : \frac{0}{1} \quad \frac{1}{1}$

$F_2 : \frac{0}{1} \quad \frac{1}{2} \quad \frac{1}{1}$

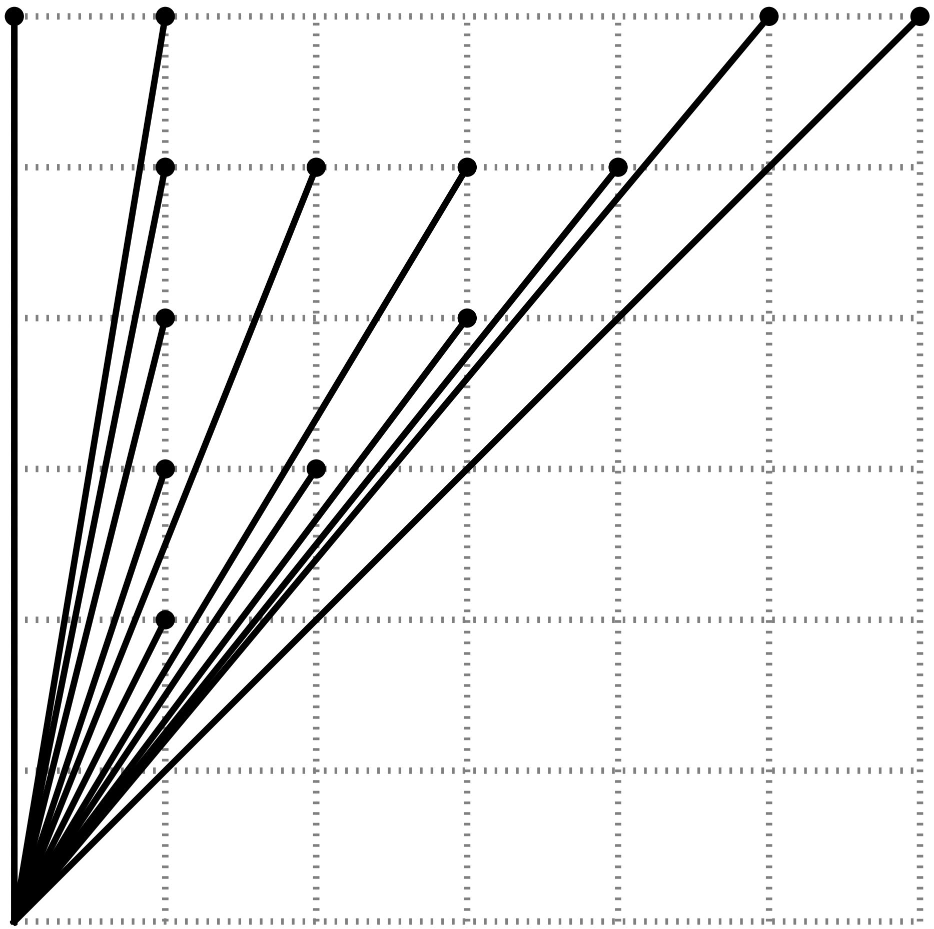
$F_3 : \frac{0}{1} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{1}{1}$

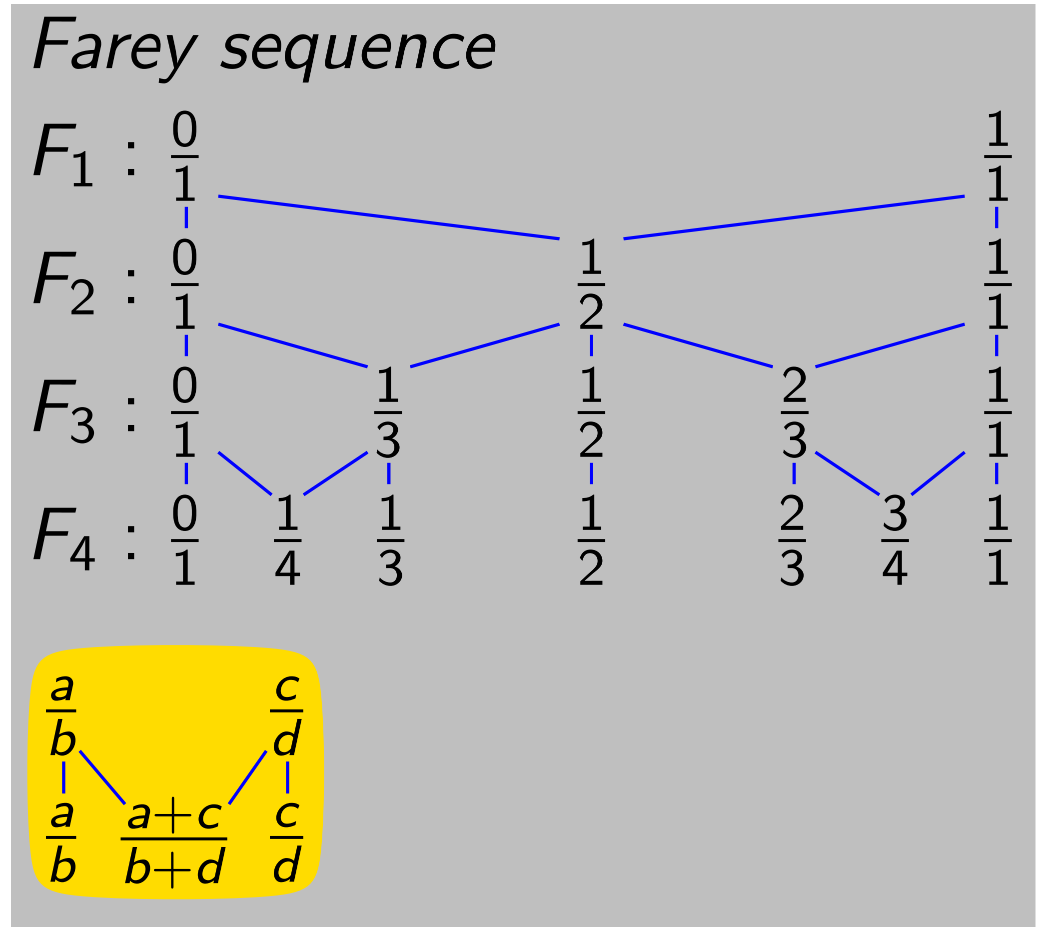


Lemma. Any two vectors of P_d are separated by an angle of $\Omega(1/|P_d|)$. For $|P_d| \ge n-1$, choose $d \approx 4\sqrt{n}$.

Our Main Tool: Primitive Vectors

Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \le x \le y \le d\}$.

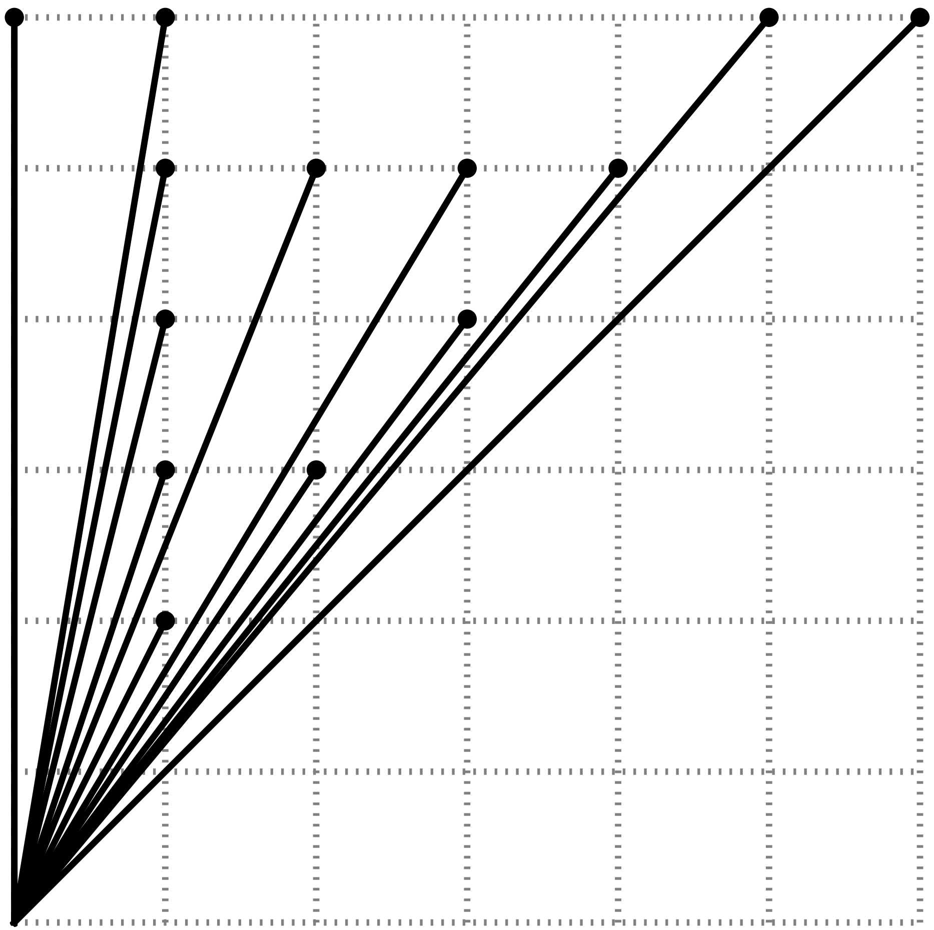




Lemma. Any two vectors of P_d are separated by an angle of $\Omega(1/|P_d|)$. For $|P_d| \ge n-1$, choose $d \approx 4\sqrt{n}$.

Our Main Tool: Primitive Vectors

Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \leq x \leq y \leq d\}$.



Farey sequence

$F_1 : \frac{0}{1}\ \frac{1}{1}$

$F_2 : \frac{0}{1}\ \frac{1}{2}\ \frac{1}{1}$

$F_3 : \frac{0}{1}\ \frac{1}{3}\ \frac{1}{2}\ \frac{2}{3}\ \frac{1}{1}$

$F_4 : \frac{0}{1}\ \frac{1}{4}\ \frac{1}{3}\ \frac{1}{2}\ \frac{2}{3}\ \frac{3}{4}\ \frac{1}{1}$

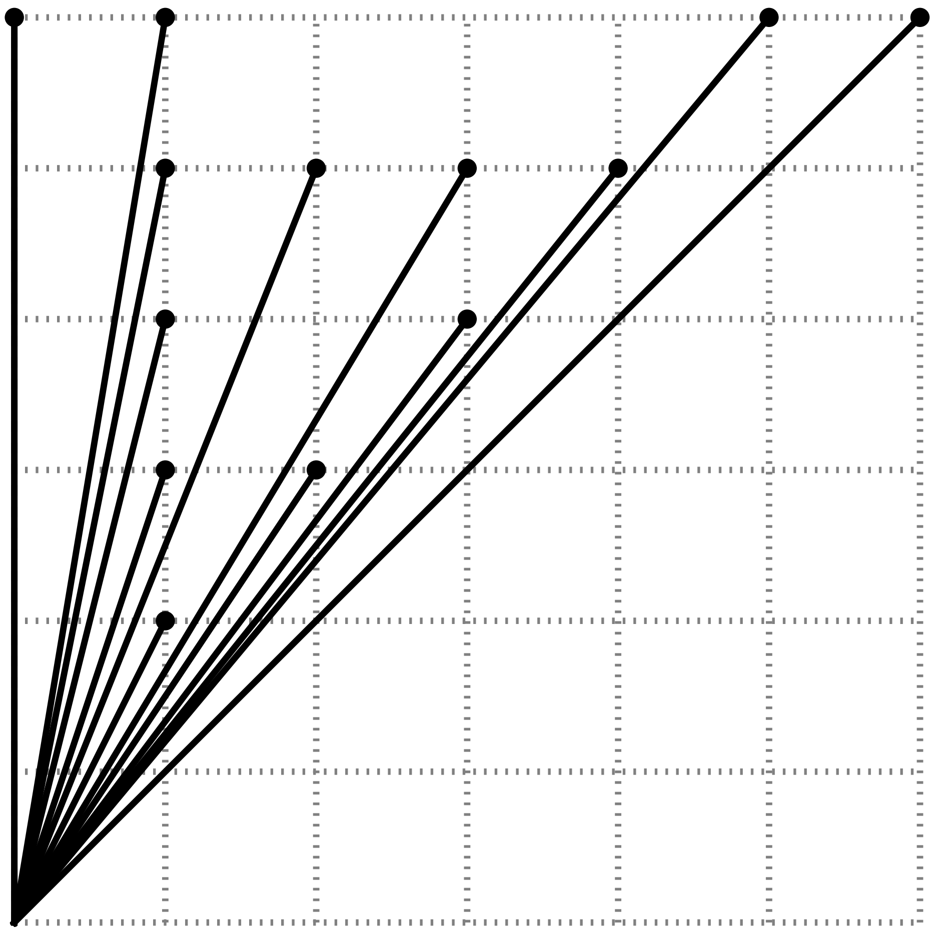
$\frac{a}{b}\ \frac{c}{d} \;\to\; \frac{a}{b}\ \frac{a+c}{b+d}\ \frac{c}{d}$

Set $P_d = F_d$

Lemma. Any two vectors of P_d are separated by an angle of $\Omega(1/|P_d|)$. For $|P_d| \geq n - 1$, choose $d \approx 4\sqrt{n}$.

Our Main Tool: Primitive Vectors

Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \leq x \leq y \leq d\}$.



Farey sequence

$F_1 : \frac{0}{1} \quad \frac{1}{1}$

$F_2 : \frac{0}{1} \quad \frac{1}{2} \quad \frac{1}{1}$

$F_3 : \frac{0}{1} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{1}{1}$

$F_4 : \frac{0}{1} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{1}{1}$

$\frac{a}{b} \quad \frac{c}{d}$

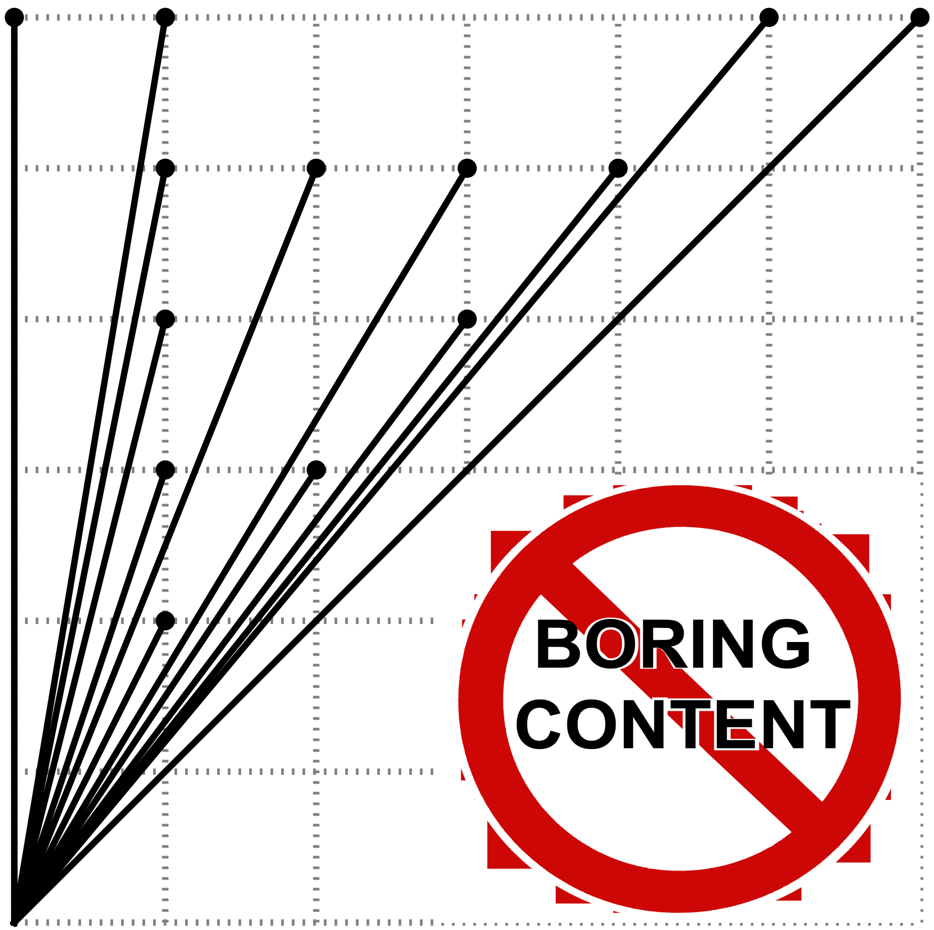
$\frac{a}{b} \quad \frac{a+c}{b+d} \quad \frac{c}{d}$

Set $P_d = F_d$ can be computed in linear time

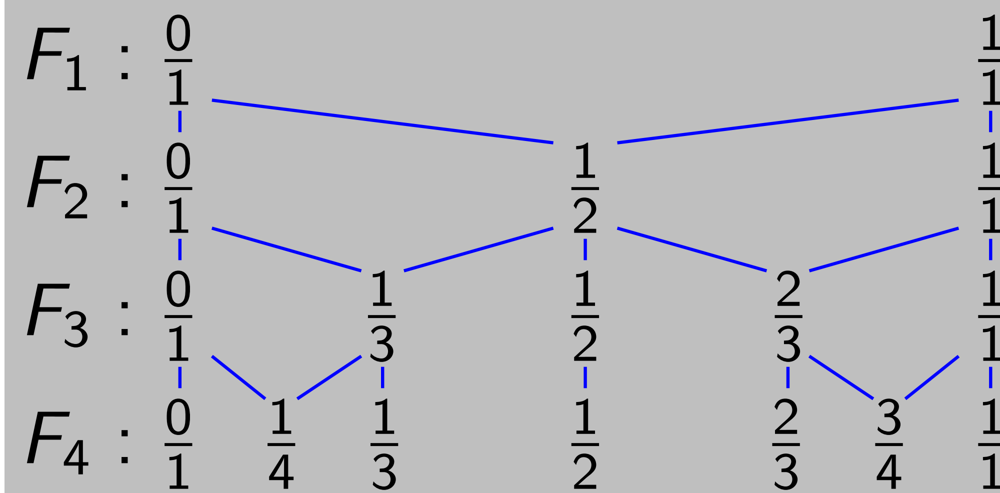
Lemma. Any two vectors of P_d are separated by an angle of $\Omega(1/|P_d|)$. For $|P_d| \geq n - 1$, choose $d \approx 4\sqrt{n}$.

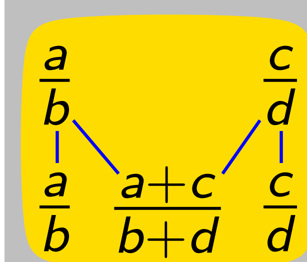
Our Main Tool: Primitive Vectors

Let $P_d = \{(x, y) \mid \gcd(x, y) = 1,\ 0 \le x \le y \le d\}$.



Farey sequence

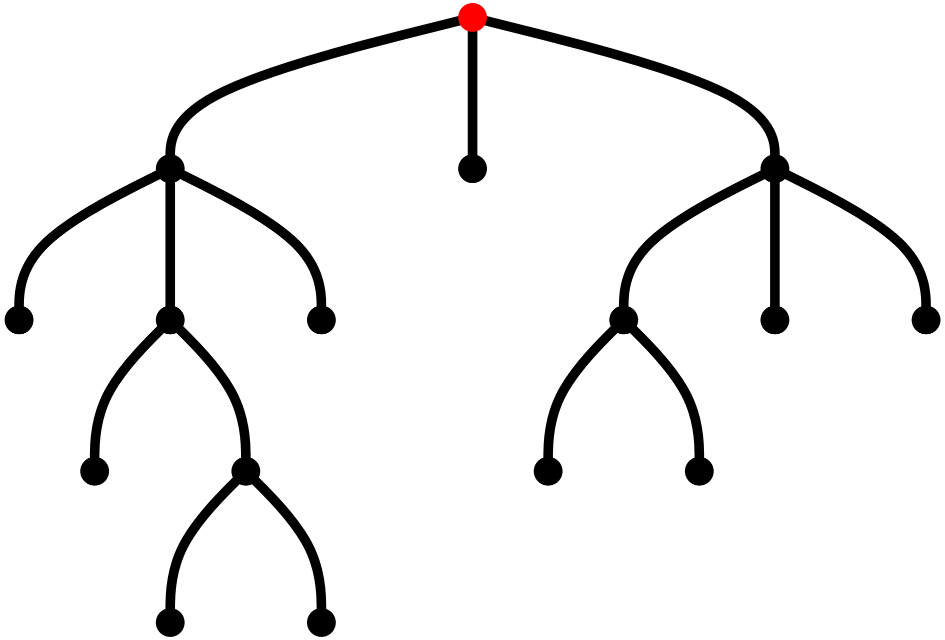




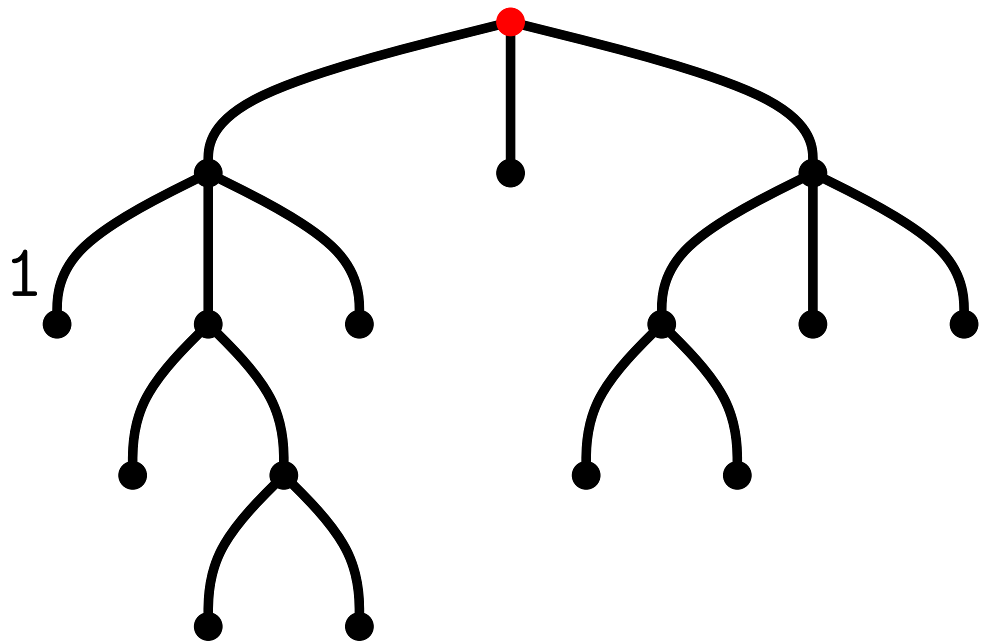
Set $P_d = F_d$ can be computed in linear time

Lemma. Any two vectors of P_d are separated by an angle of $\Omega(1/|P_d|)$. For $|P_d| \ge n-1$, choose $d \approx 4\sqrt{n}$.

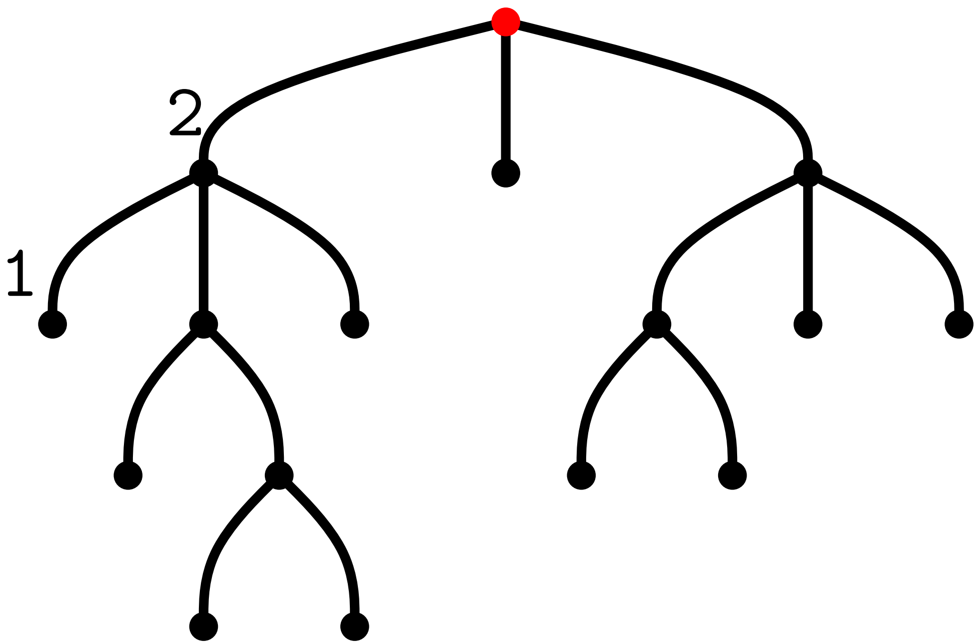
Step I: Rank Edges



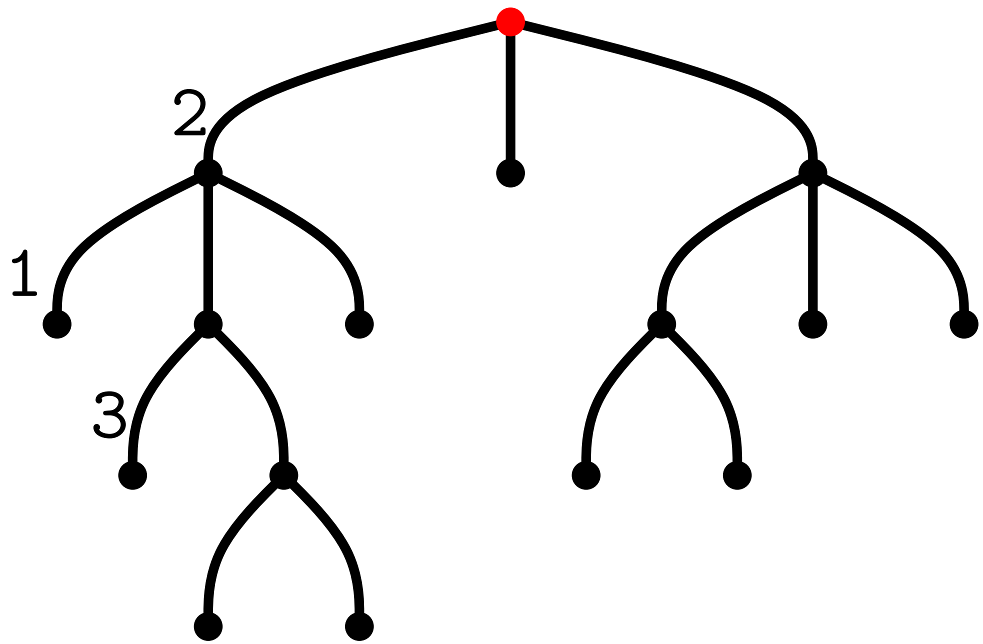
Step I: Rank Edges



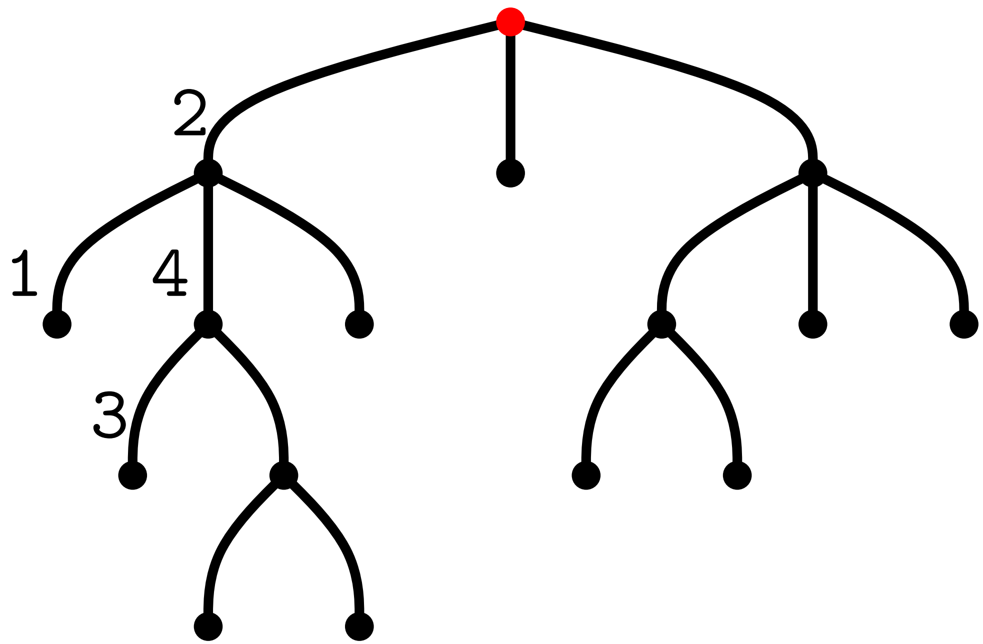
Step I: Rank Edges



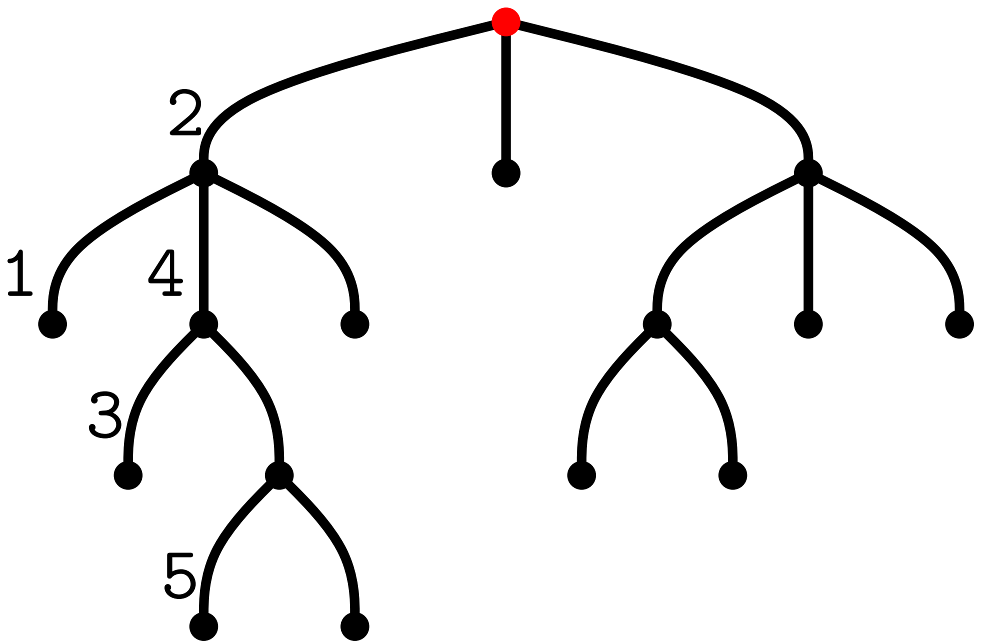
Step I: Rank Edges



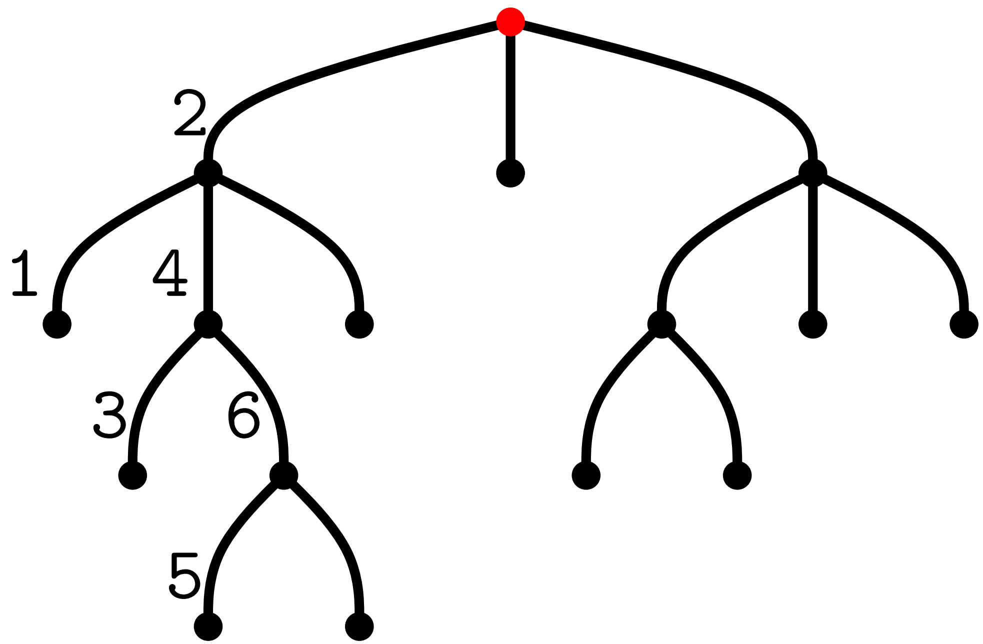
Step I: Rank Edges



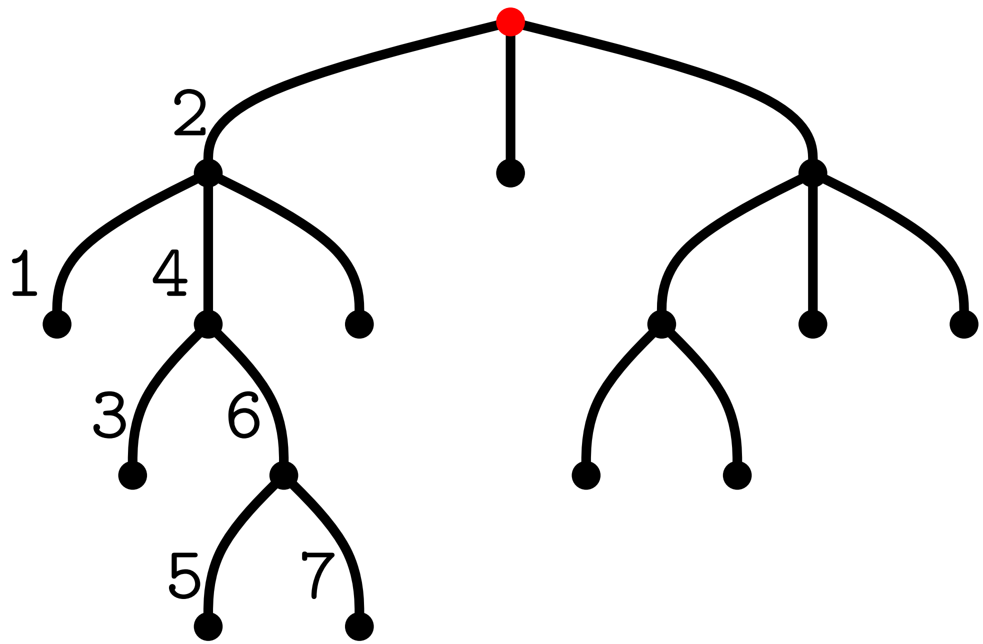
Step I: Rank Edges



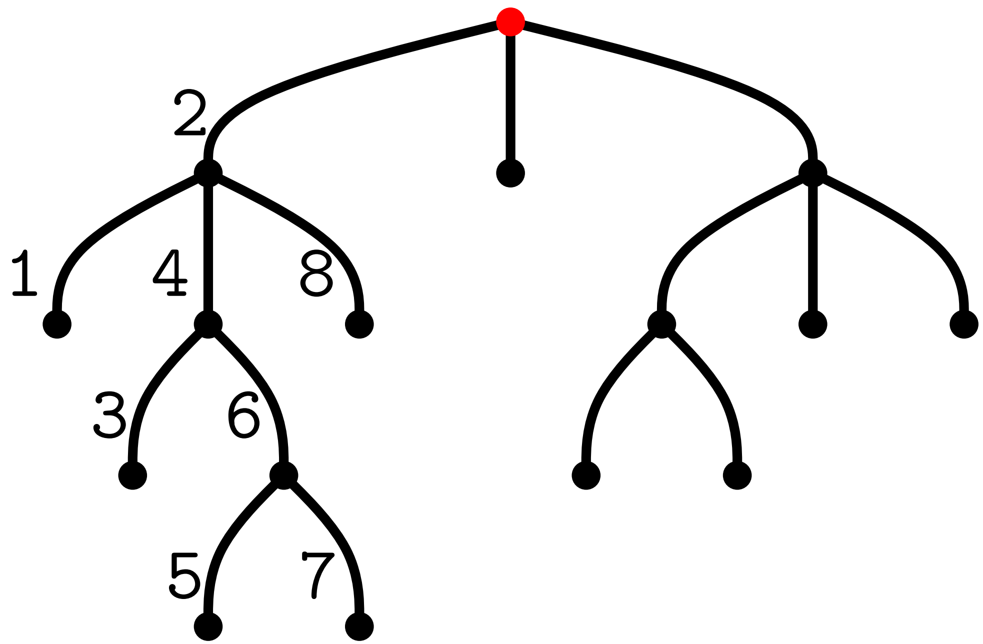
Step I: Rank Edges



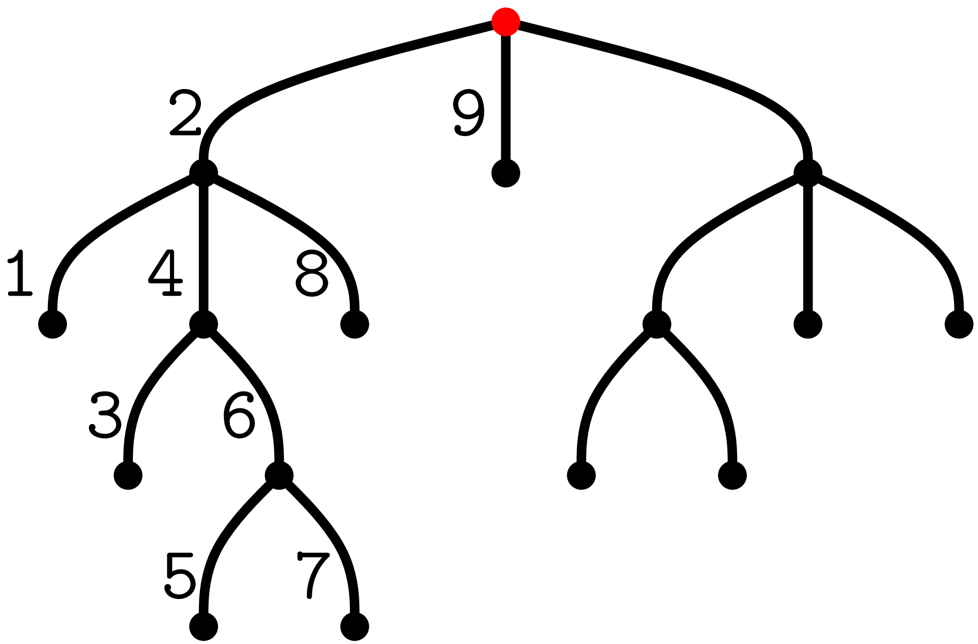
Step I: Rank Edges



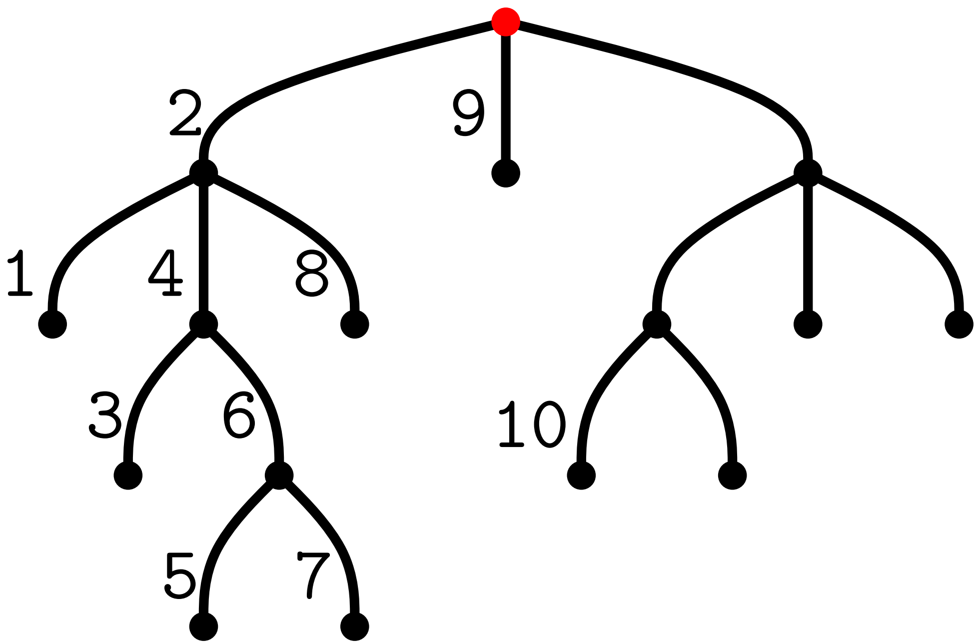
Step I: Rank Edges



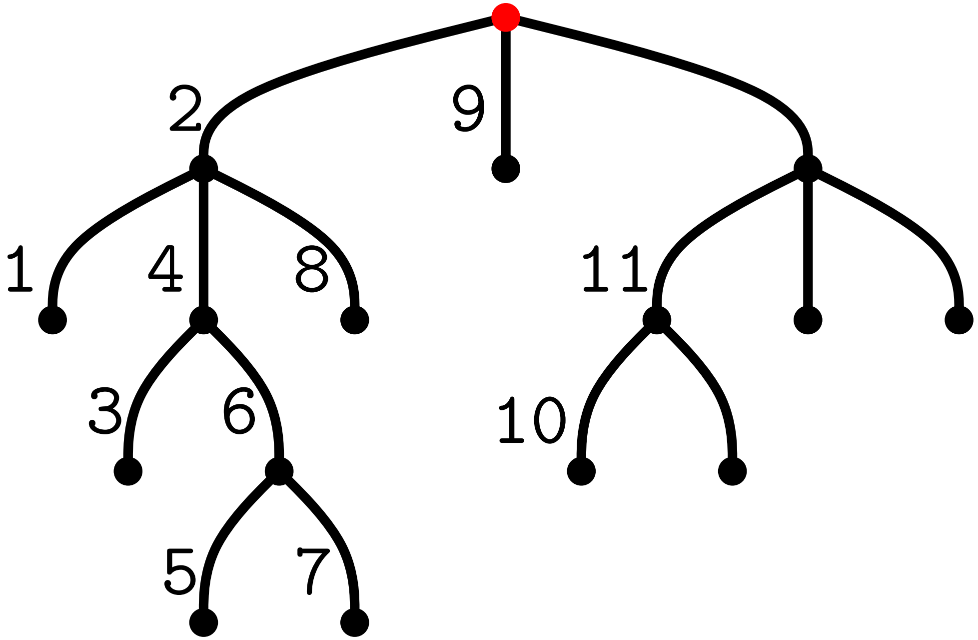
Step I: Rank Edges



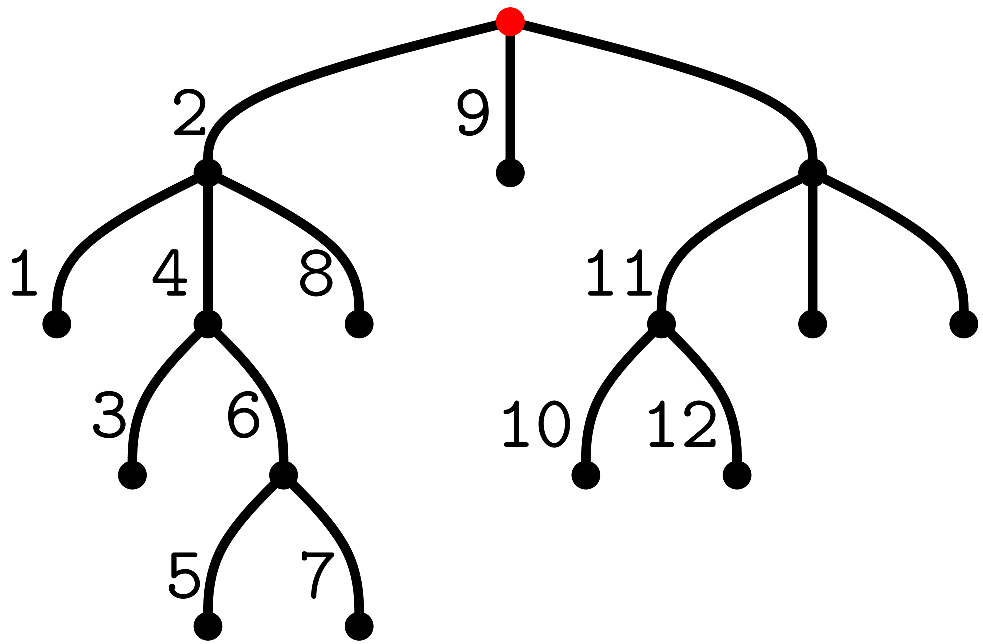
Step I: Rank Edges



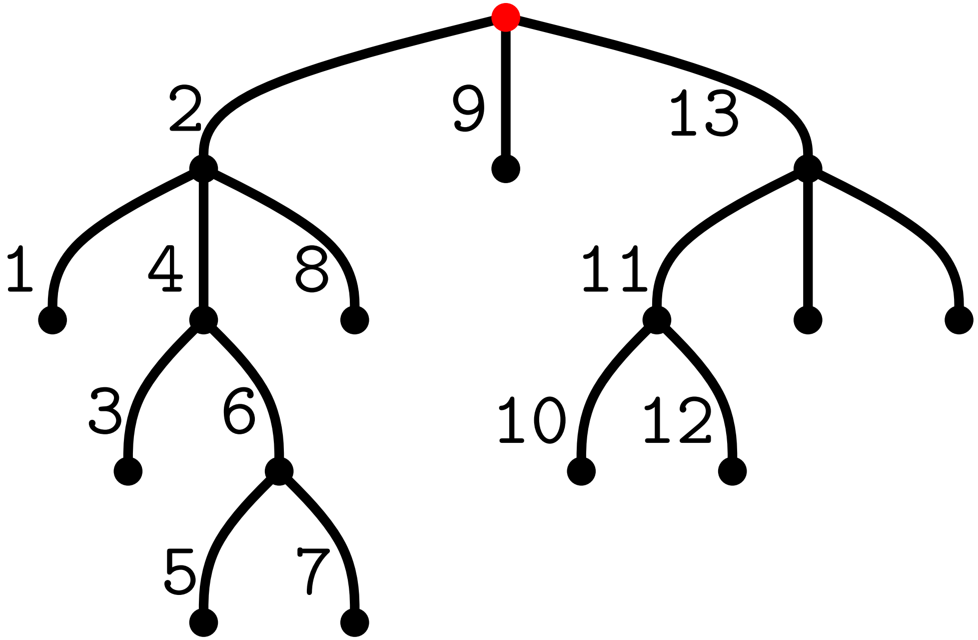
Step I: Rank Edges



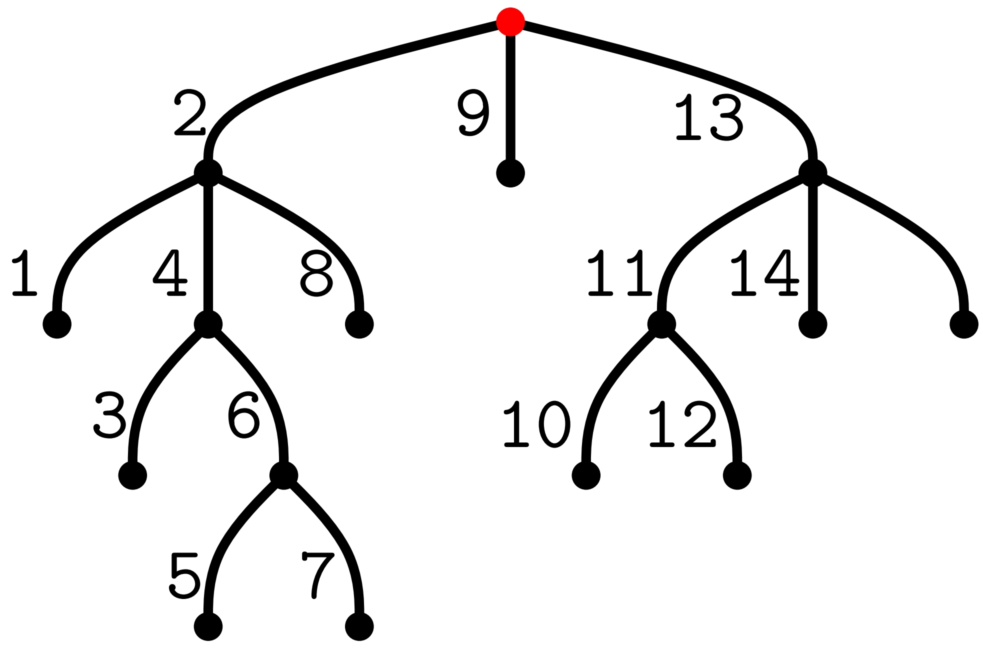
Step I: Rank Edges



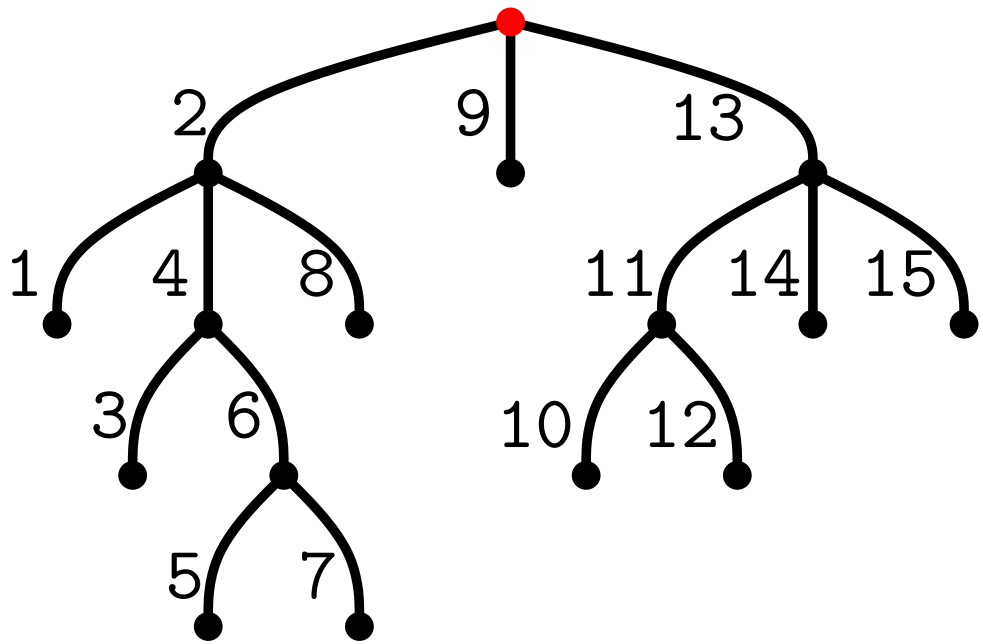
Step I: Rank Edges



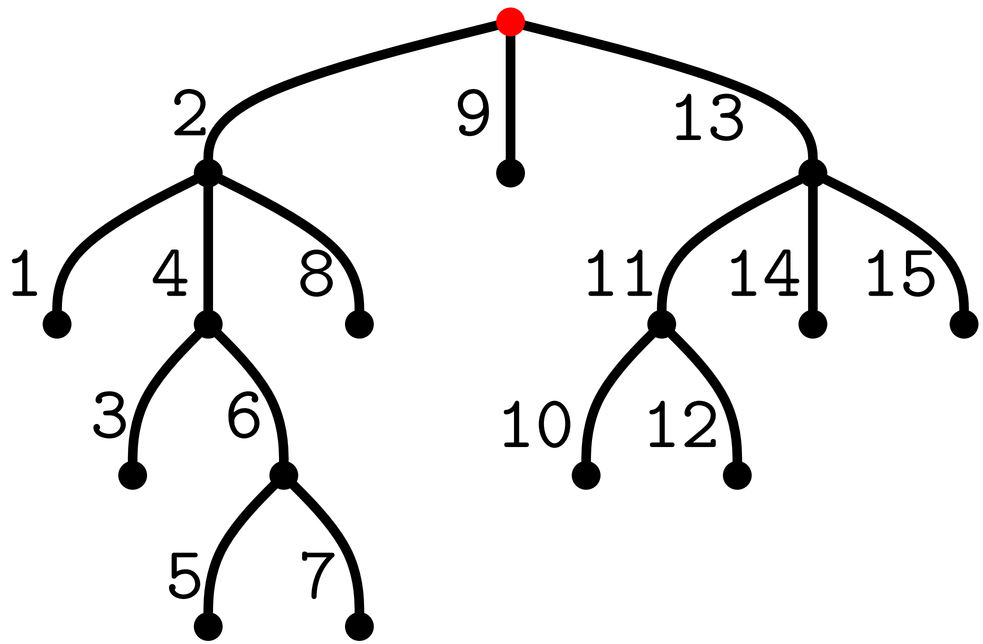
Step I: Rank Edges



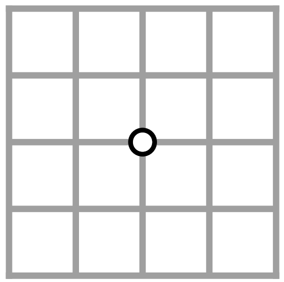
Step I: Rank Edges



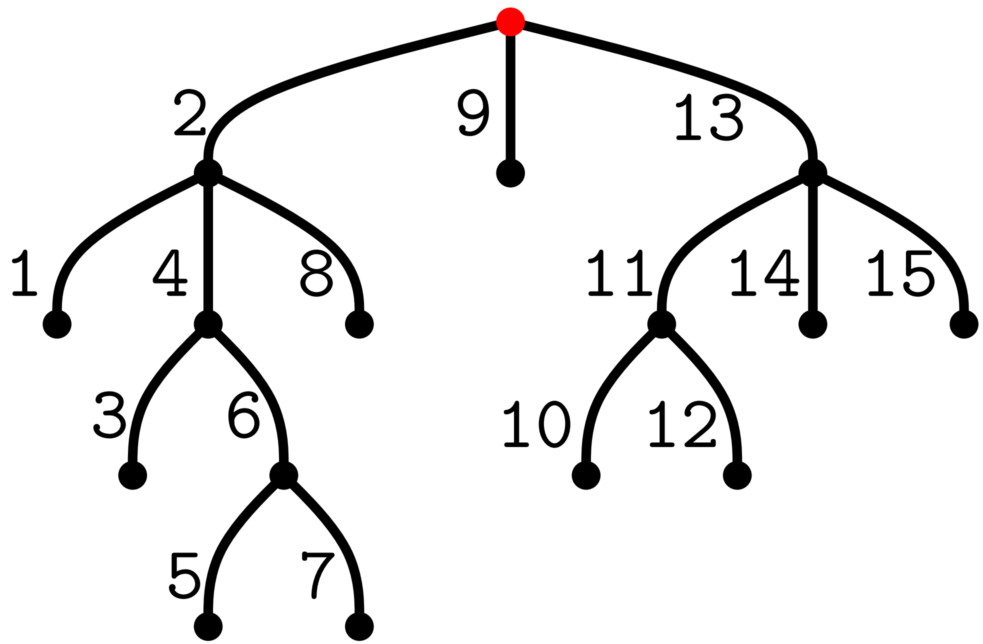
Step I: Rank Edges



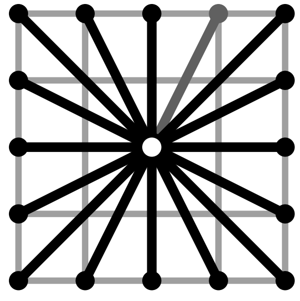
Step II: Primitive Vectors



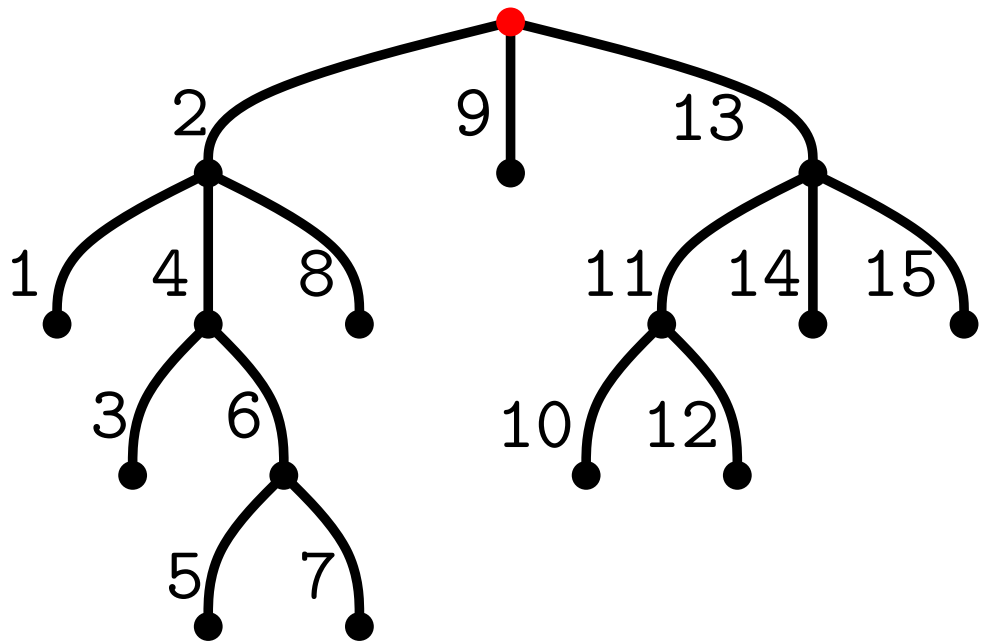
Step I: Rank Edges



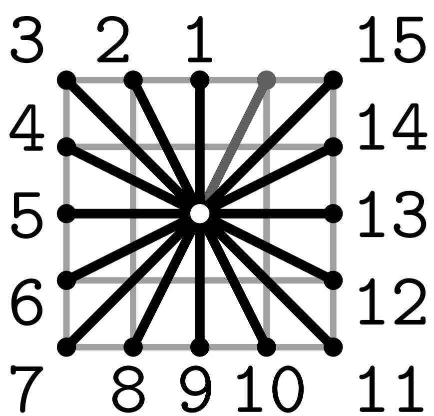
Step II: Primitive Vectors



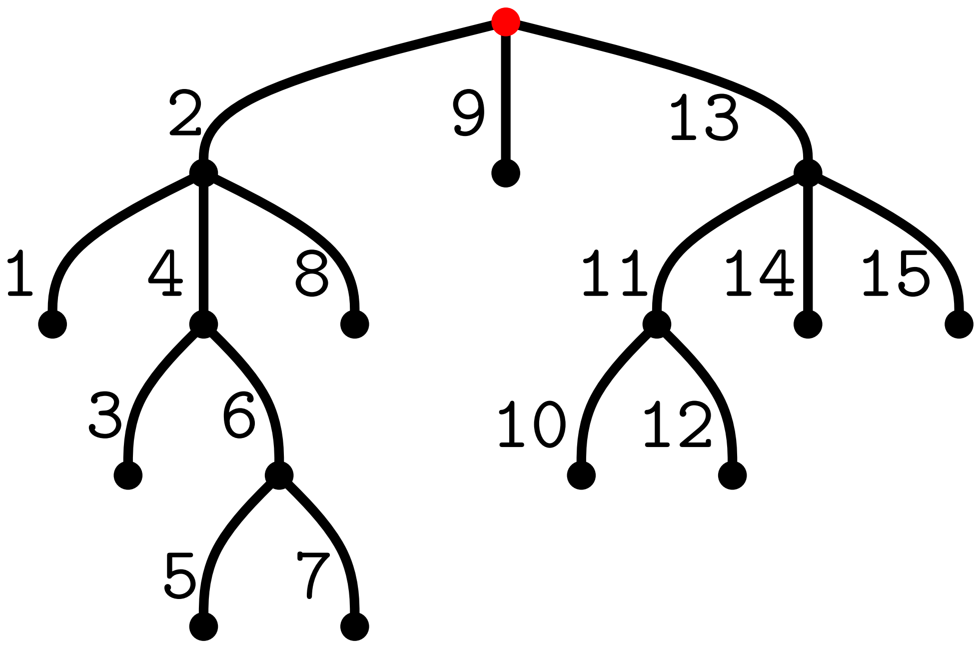
Step I: Rank Edges



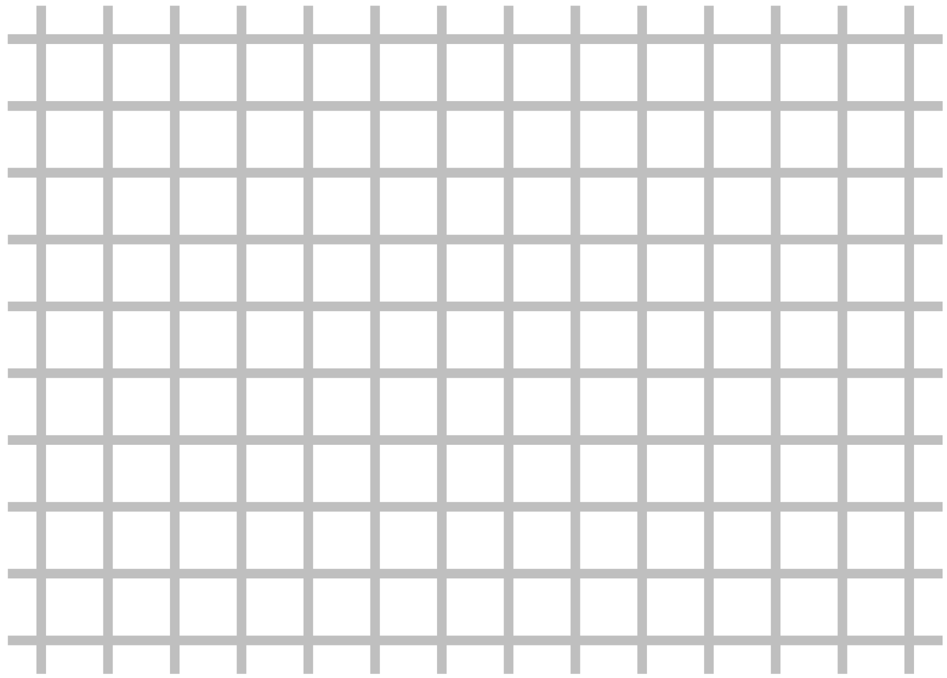
Step II: Primitive Vectors



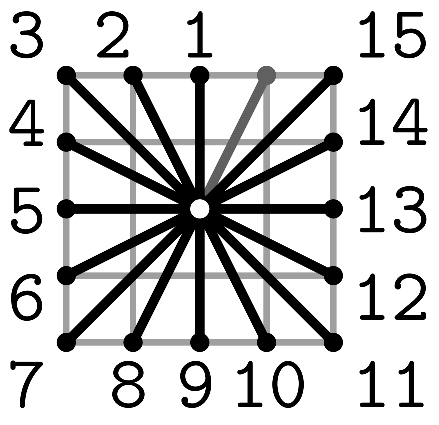
Step I: Rank Edges



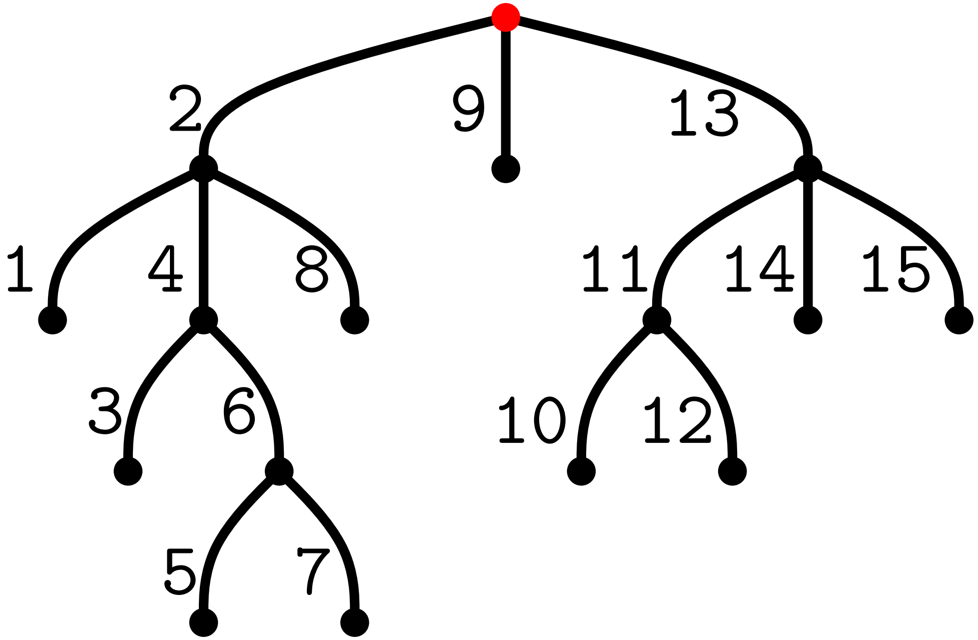
Step III: Draw Tree



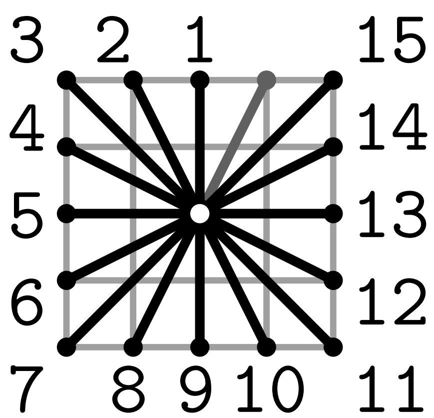
Step II: Primitive Vectors



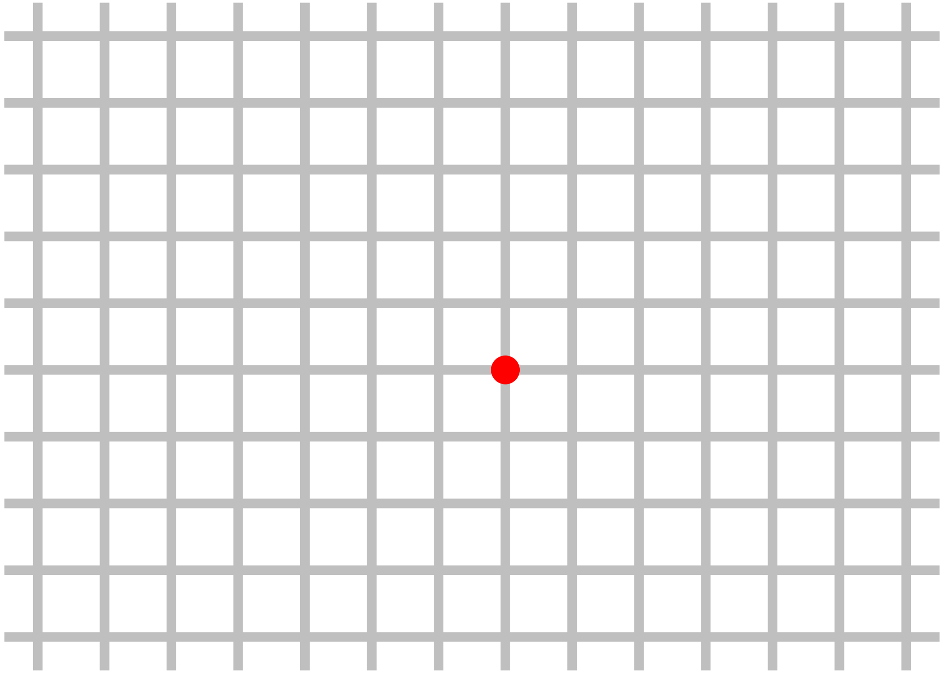
Step I: Rank Edges



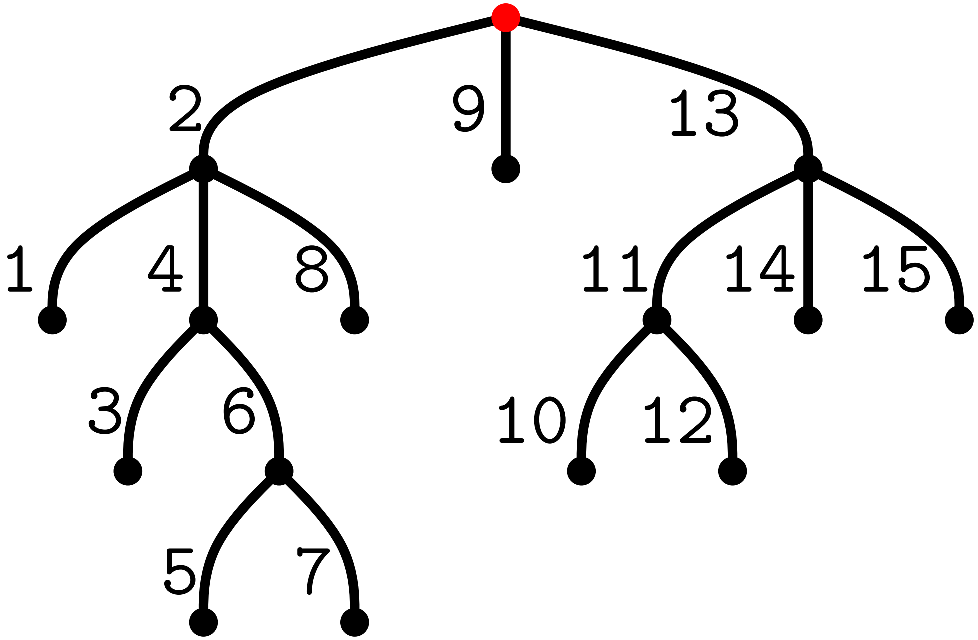
Step II: Primitive Vectors



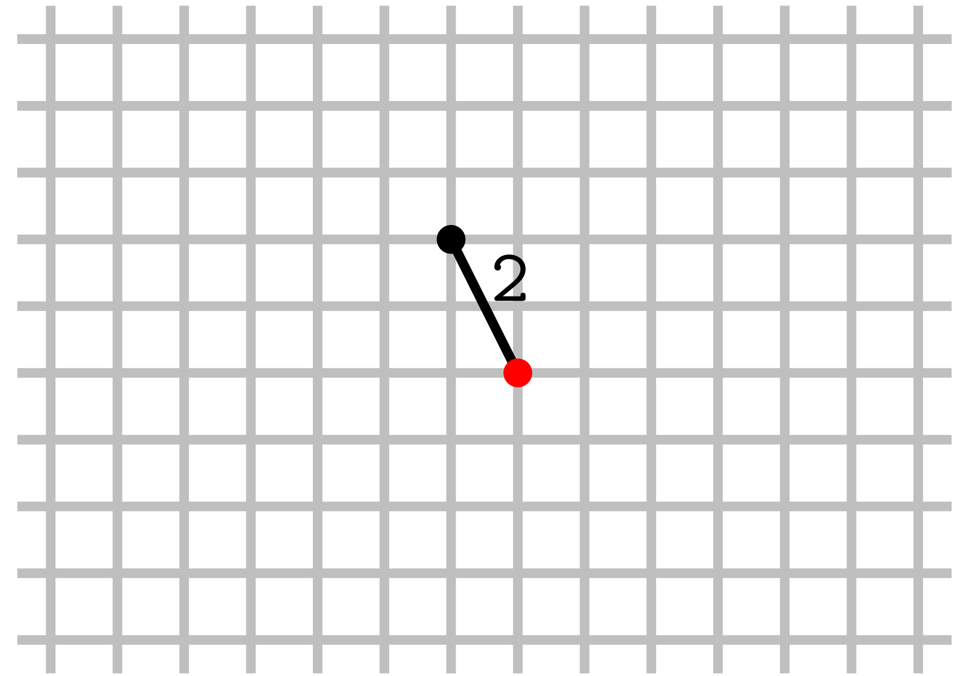
Step III: Draw Tree



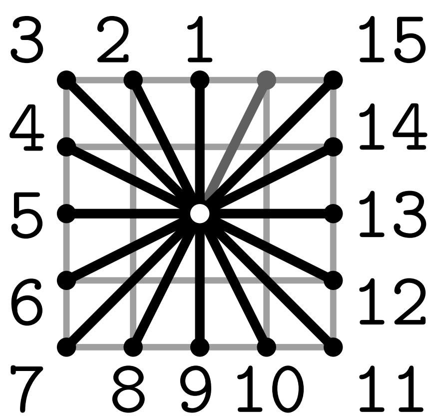
Step I: Rank Edges



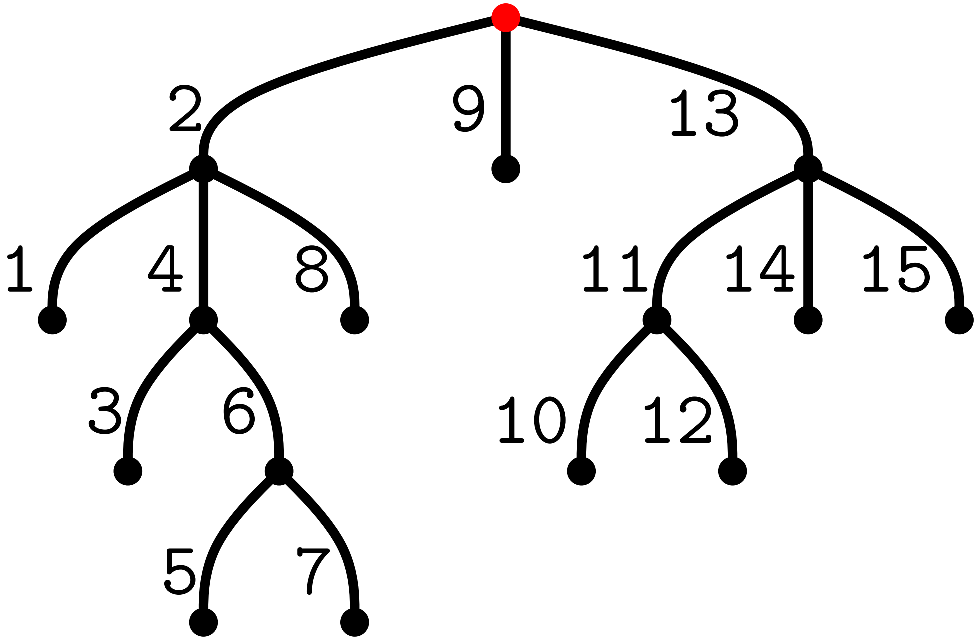
Step III: Draw Tree



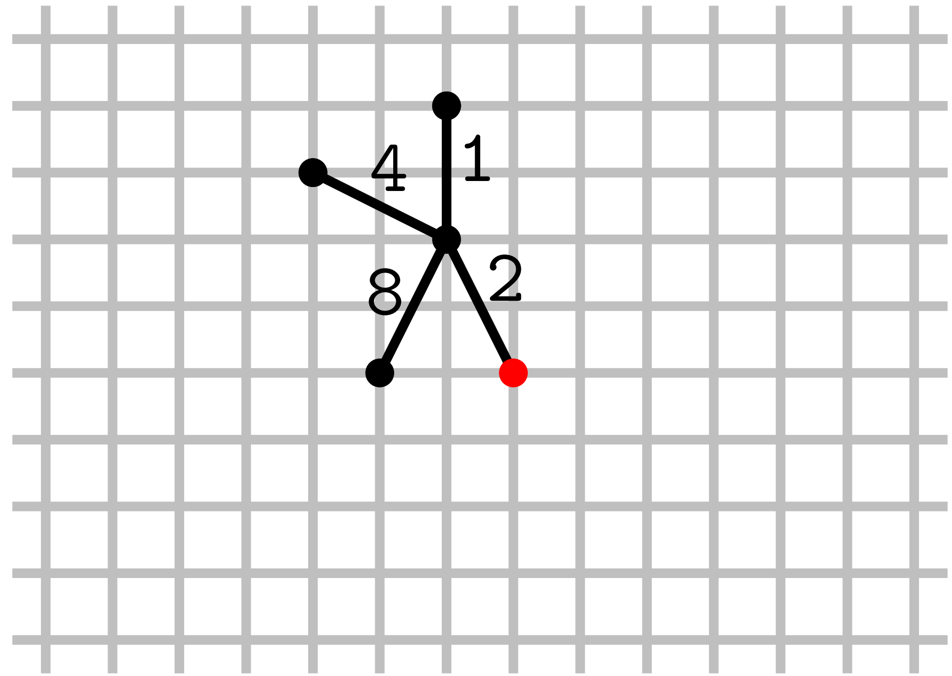
Step II: Primitive Vectors



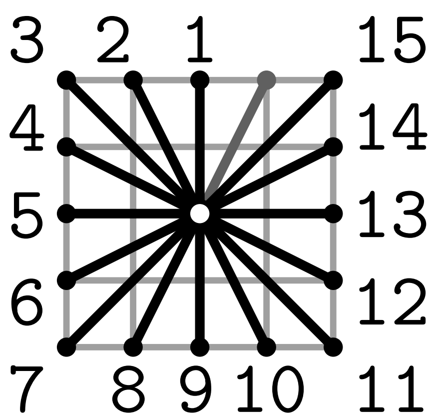
Step I: Rank Edges



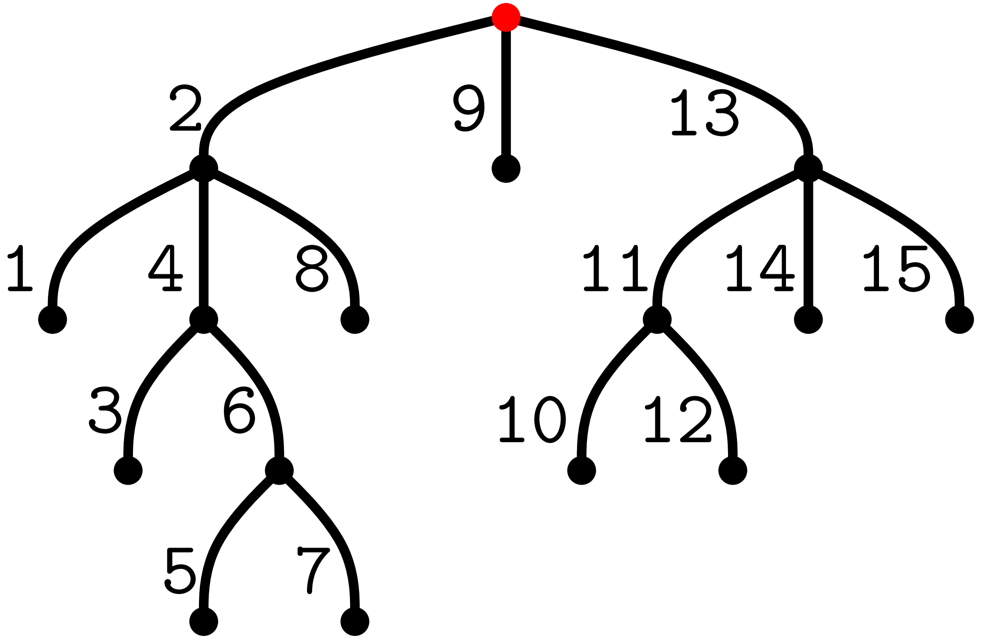
Step III: Draw Tree



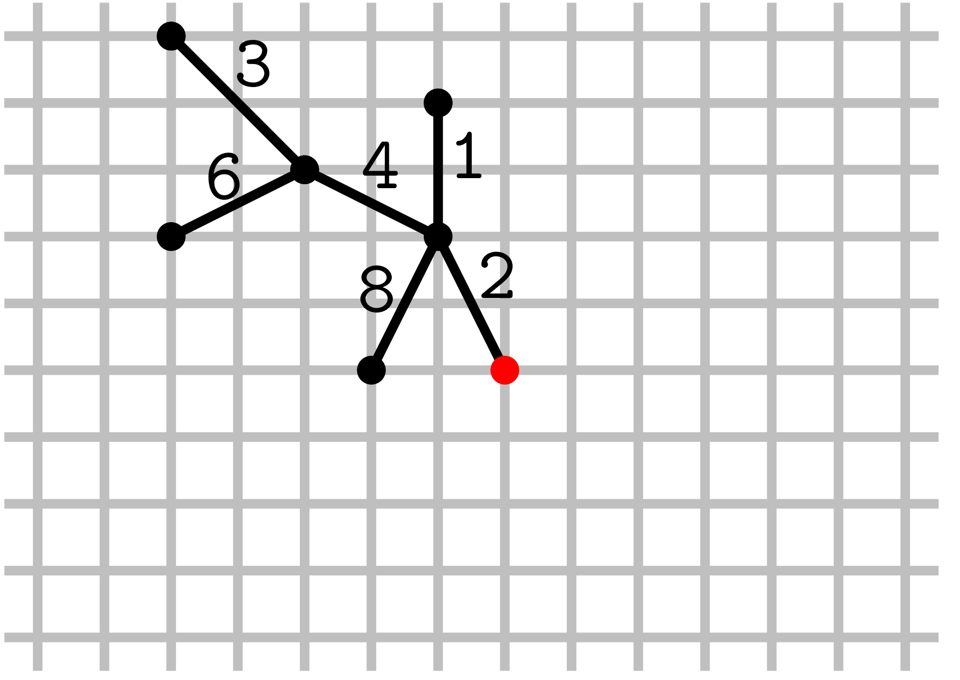
Step II: Primitive Vectors



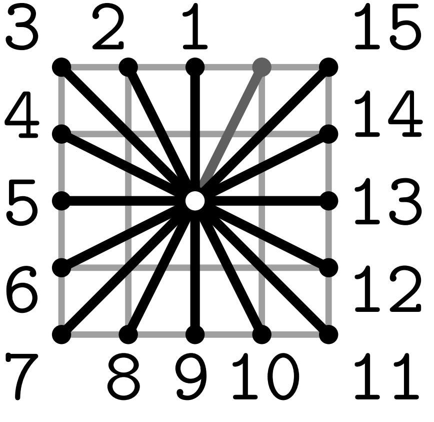
Step I: Rank Edges



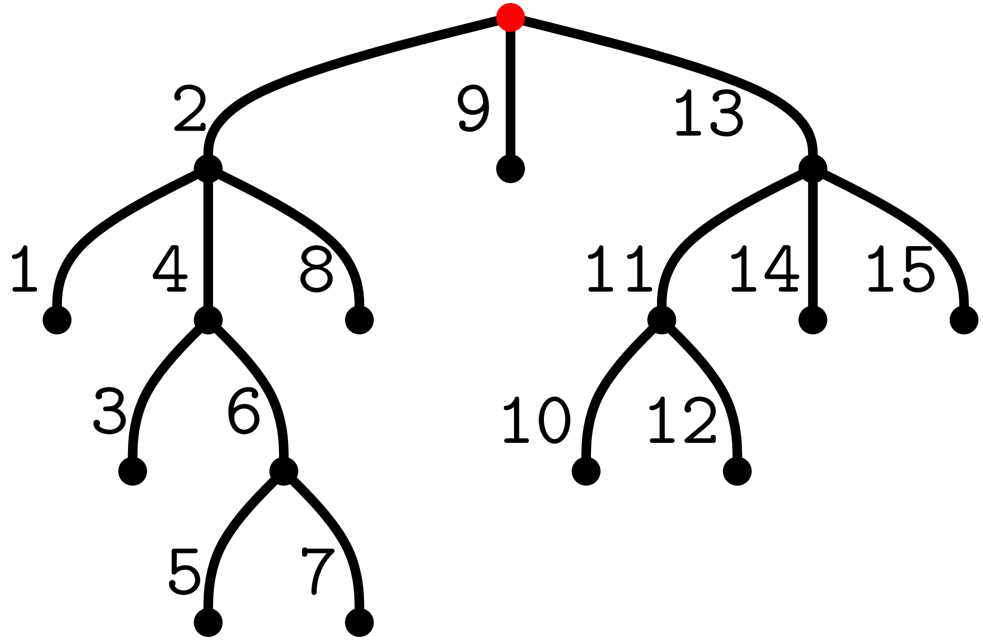
Step III: Draw Tree



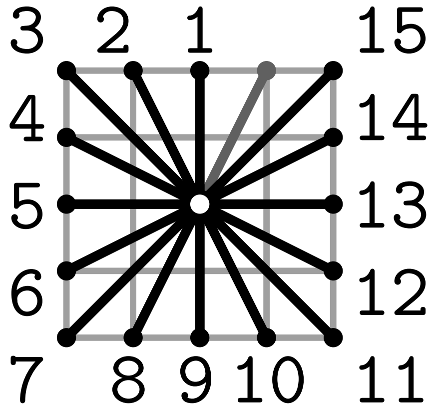
Step II: Primitive Vectors



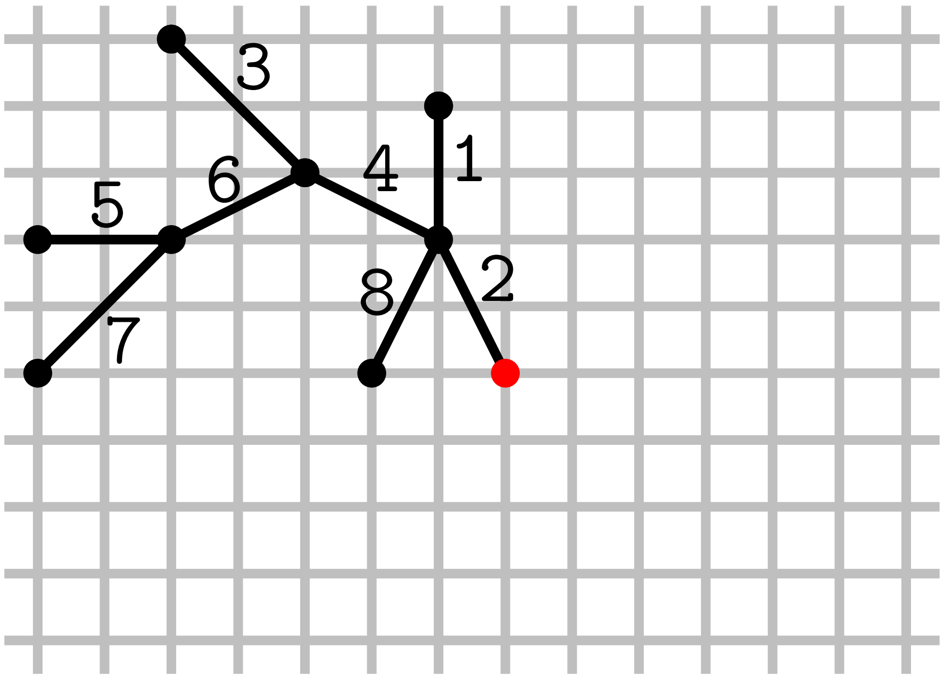
Step I: Rank Edges



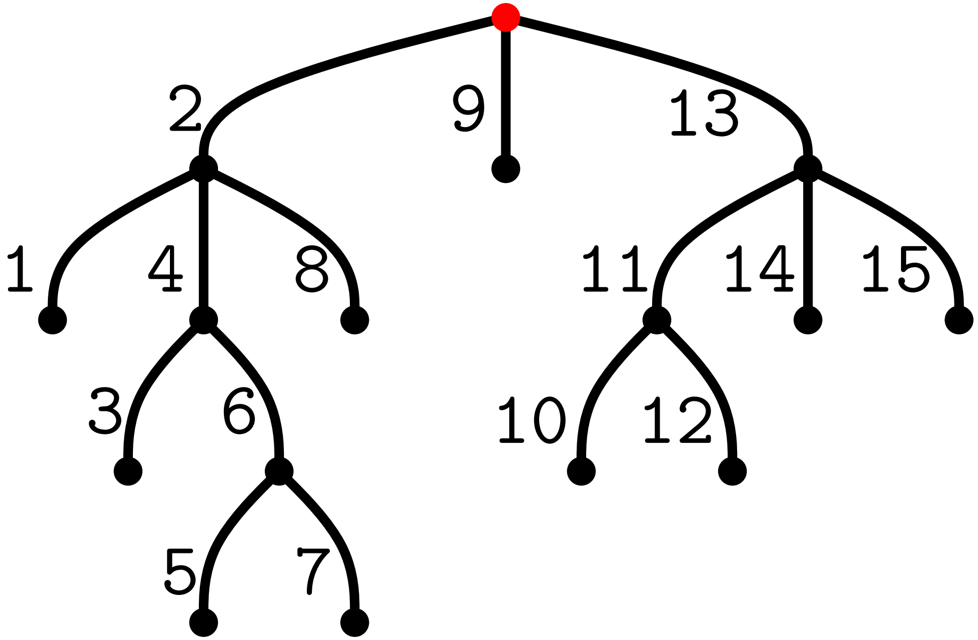
Step II: Primitive Vectors



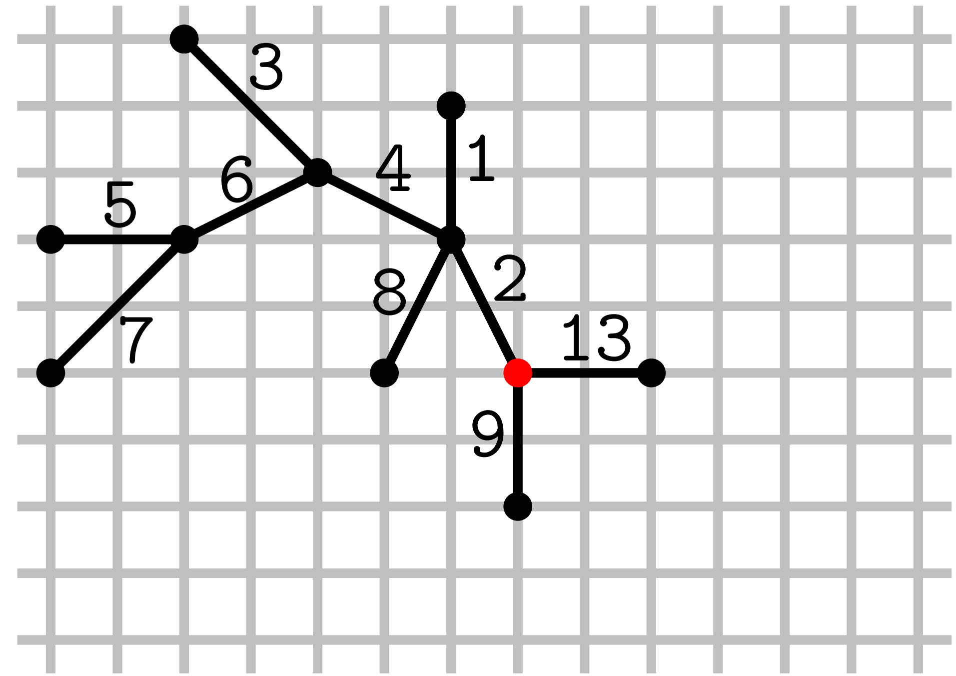
Step III: Draw Tree



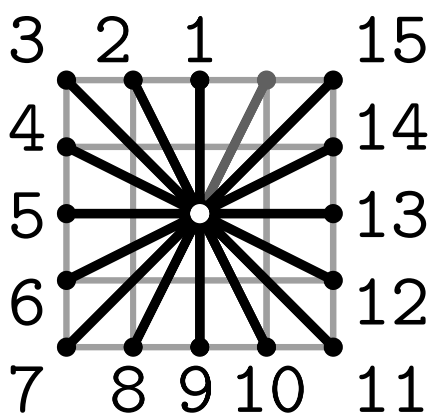
Step I: Rank Edges



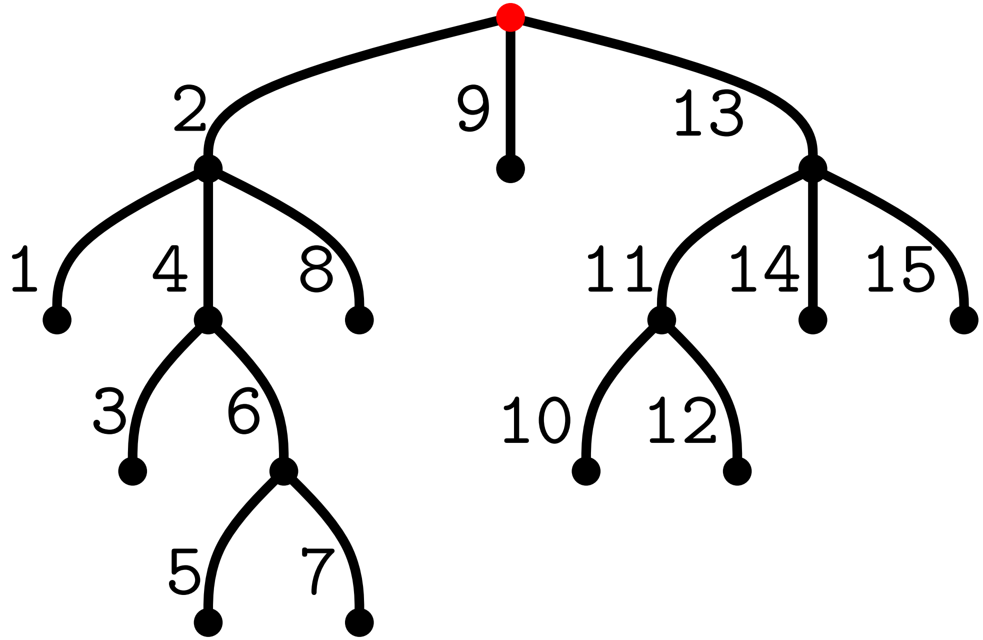
Step III: Draw Tree



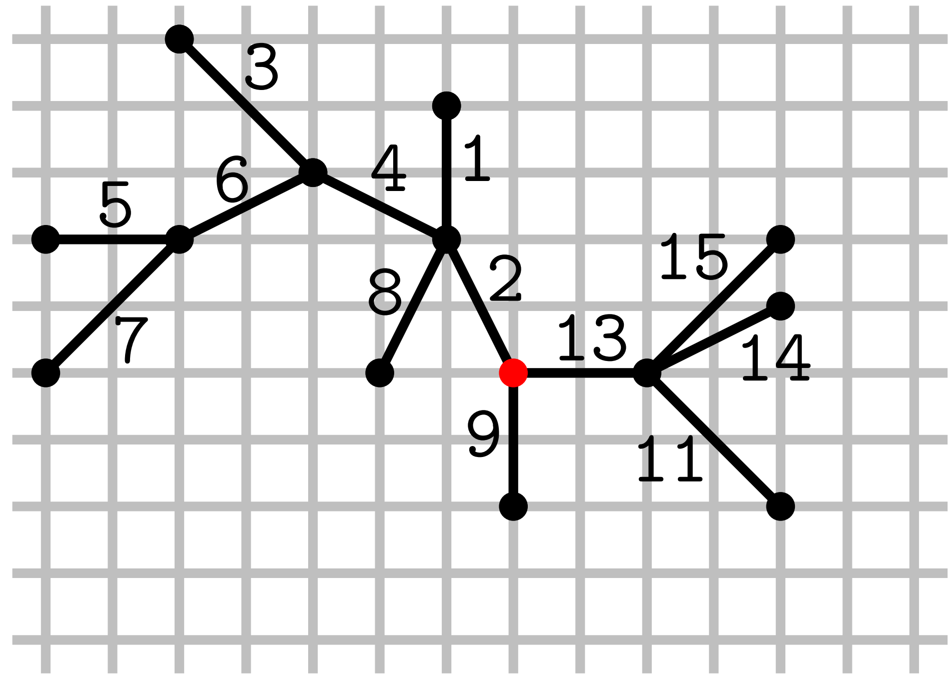
Step II: Primitive Vectors



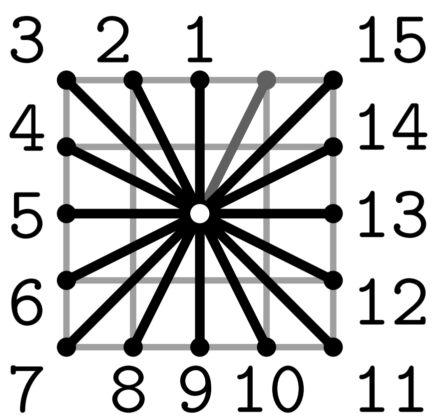
Step I: Rank Edges



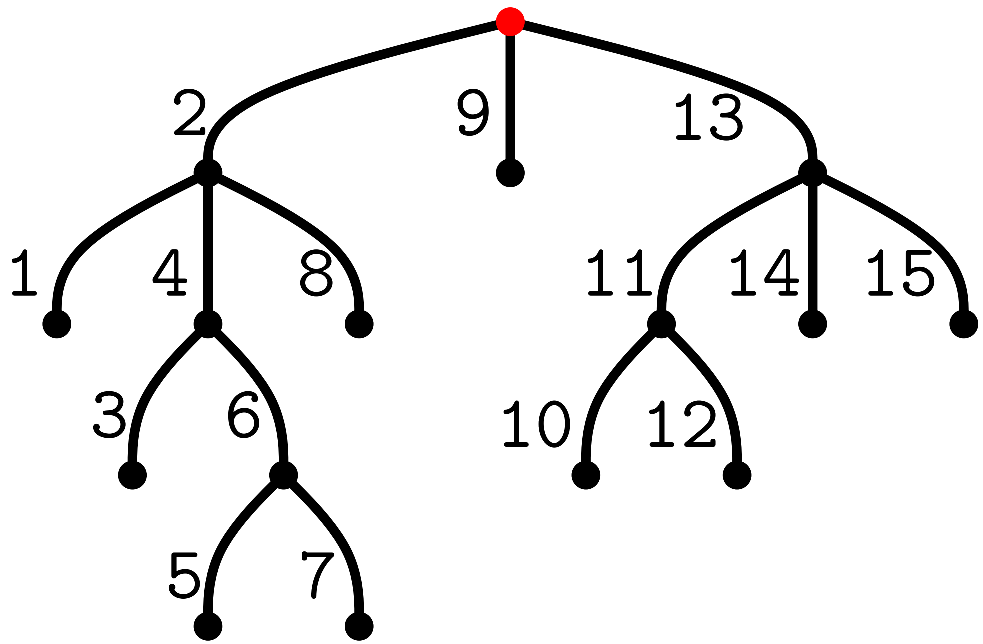
Step III: Draw Tree



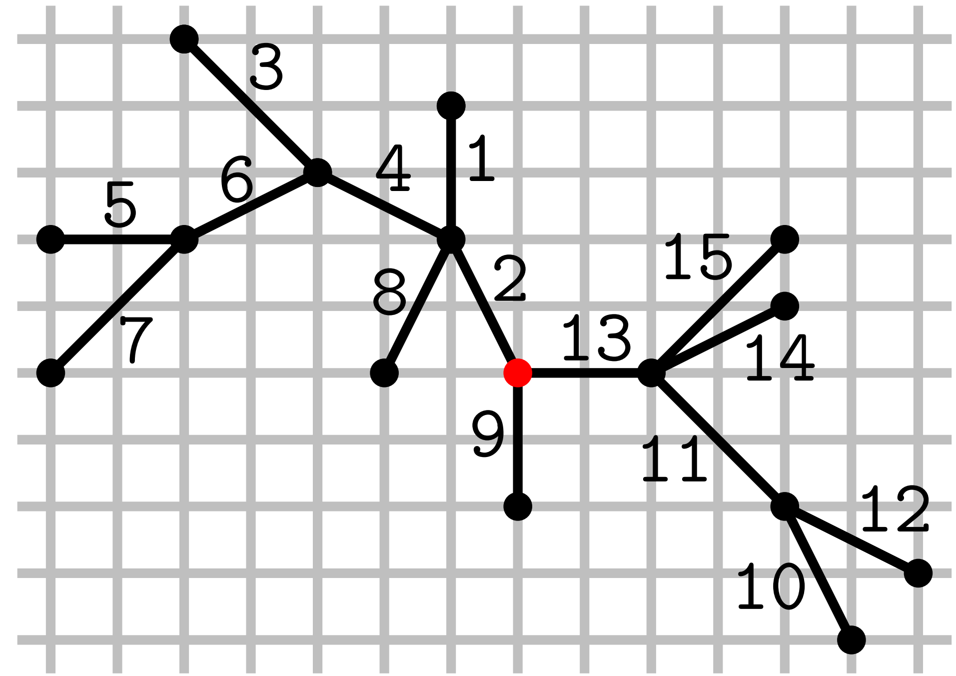
Step II: Primitive Vectors



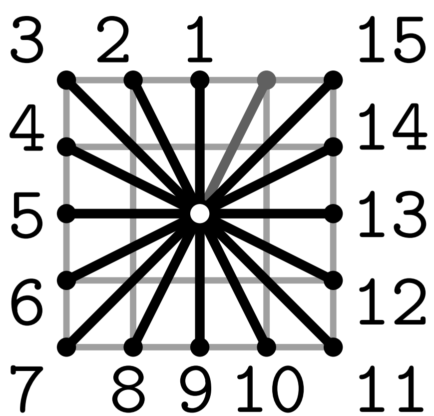
Step I: Rank Edges



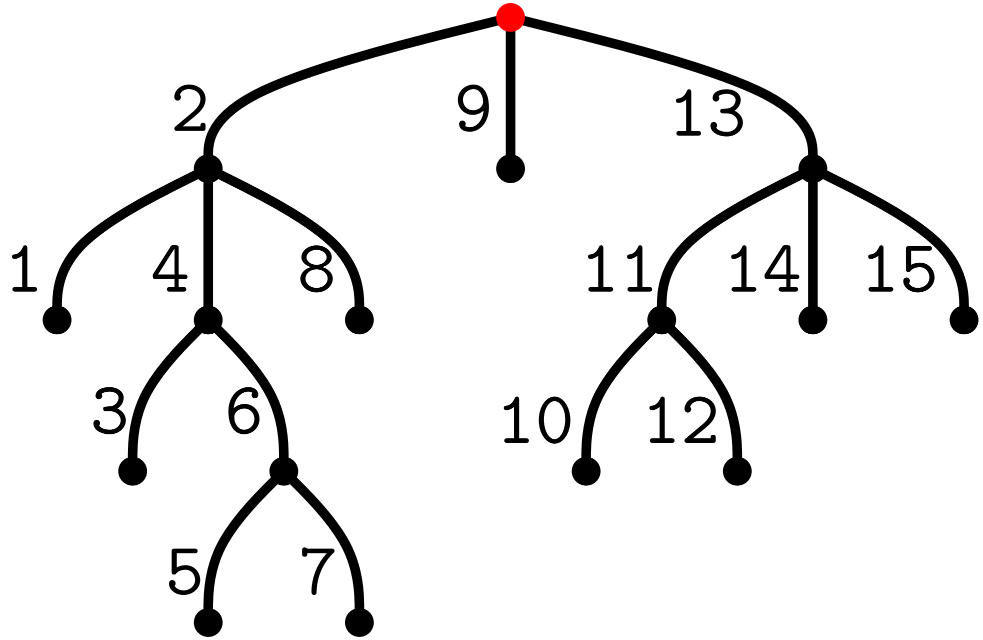
Step III: Draw Tree



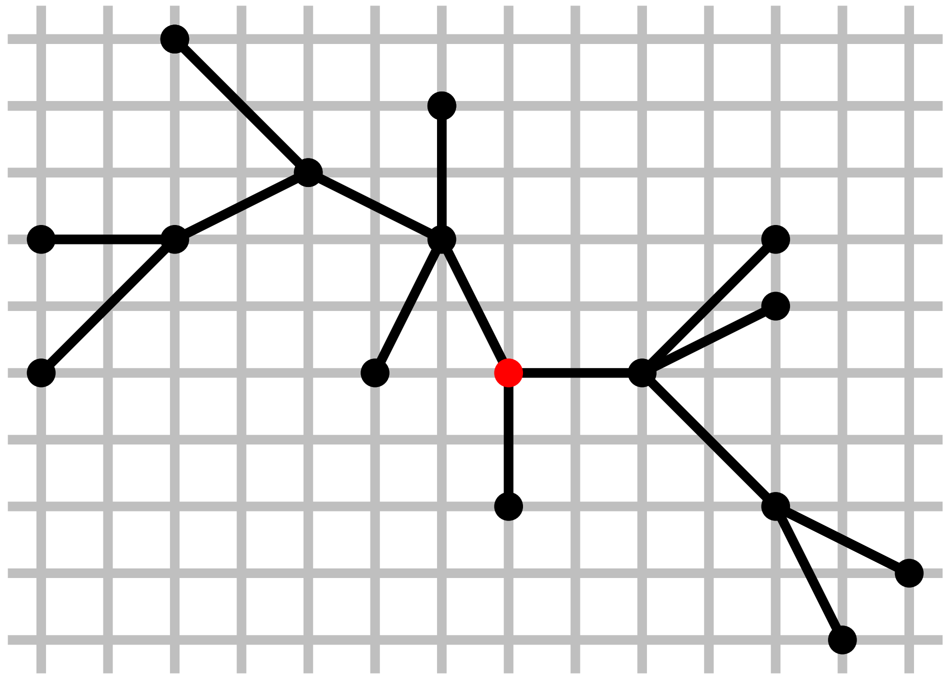
Step II: Primitive Vectors



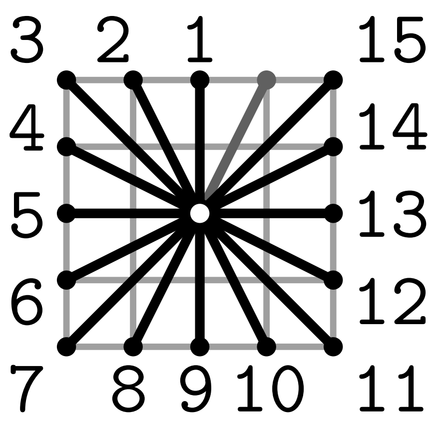
Step I: Rank Edges



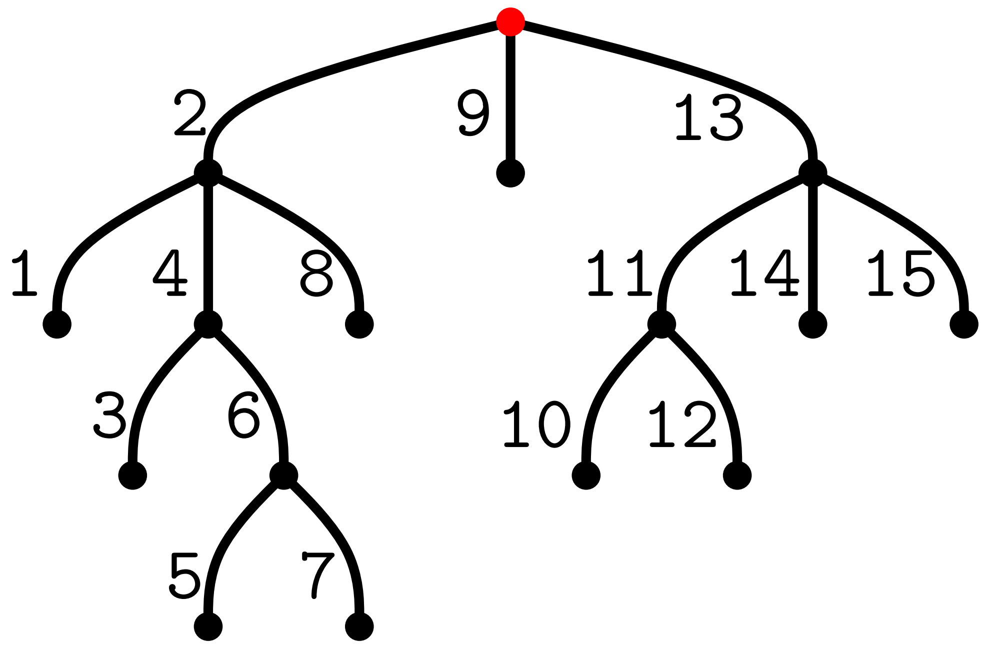
Step III: Draw Tree



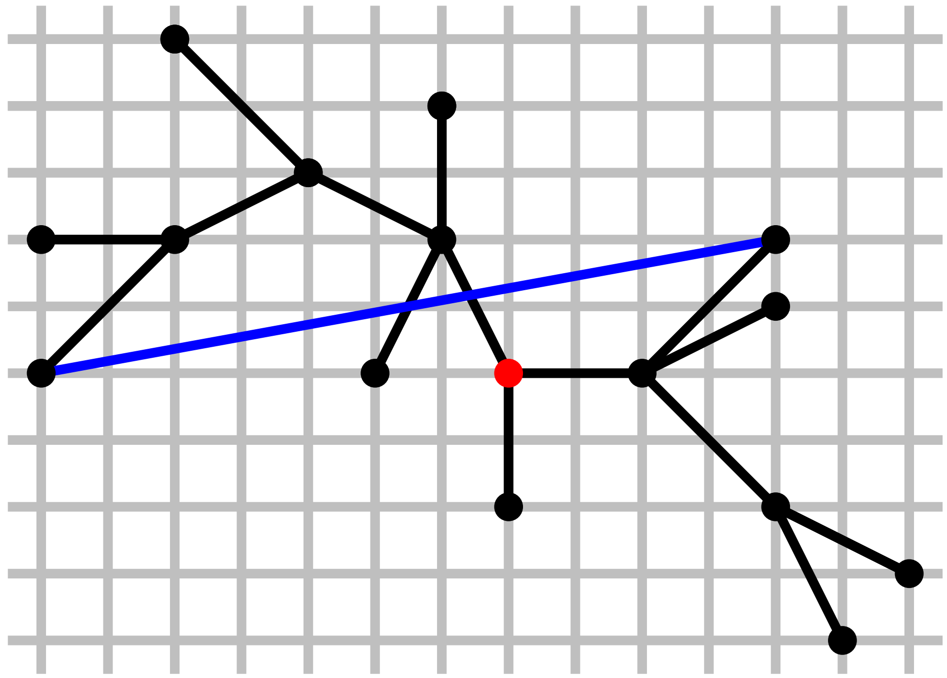
Step II: Primitive Vectors



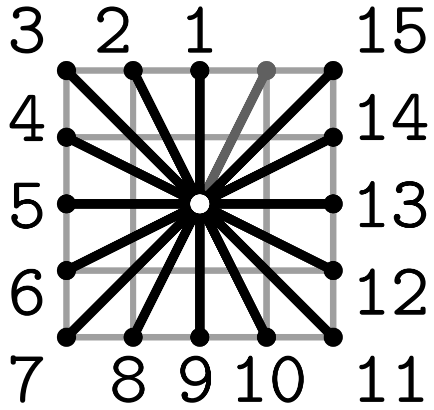
Step I: Rank Edges



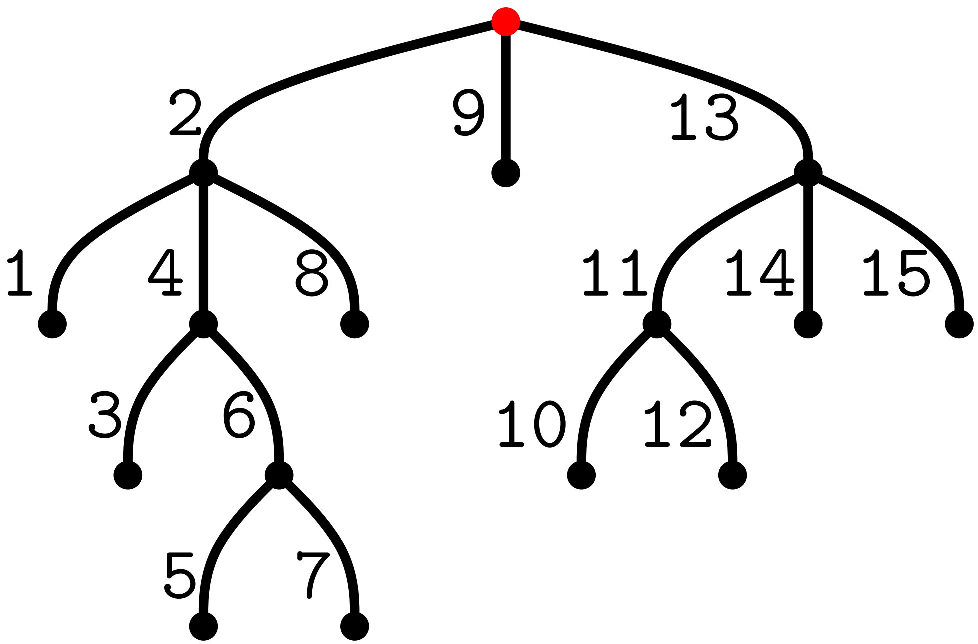
Step III: Draw Tree



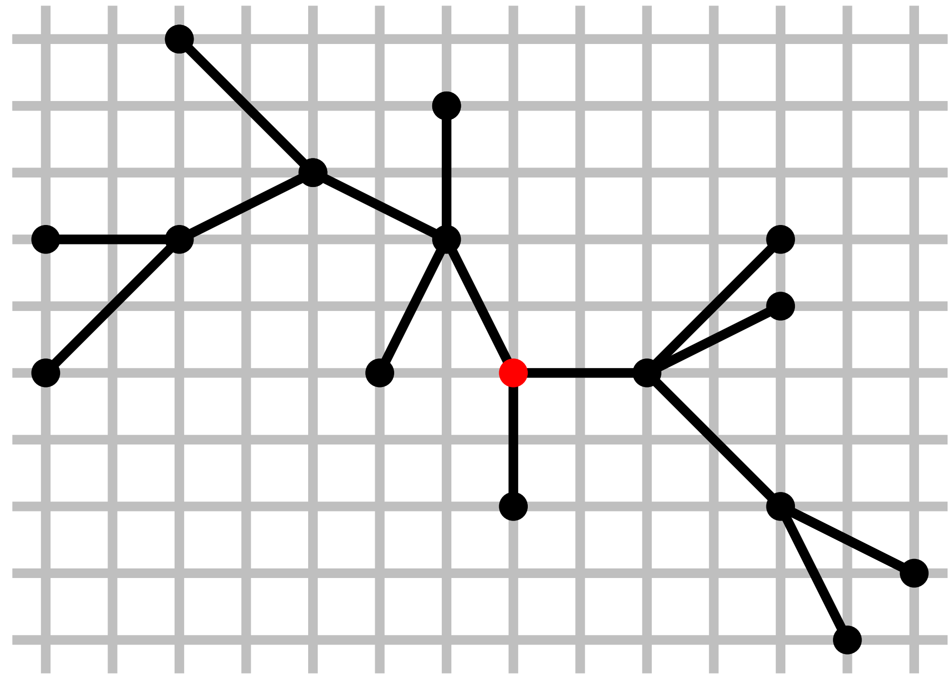
Step II: Primitive Vectors



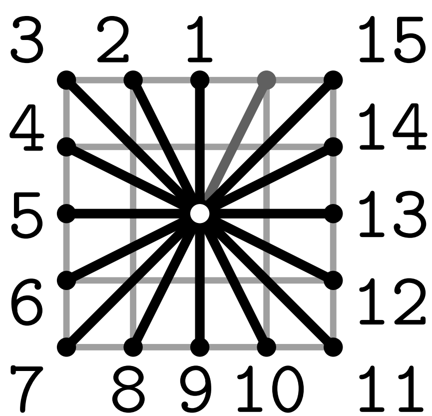
Step I: Rank Edges



Step III: Draw Tree



Step II: Primitive Vectors



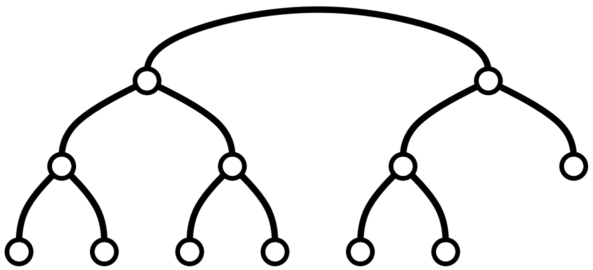
Theorem.
Every tree has a monotone and convex drawing on a grid of size $O(n^{1.5}) \times O(n^{1.5})$.

Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

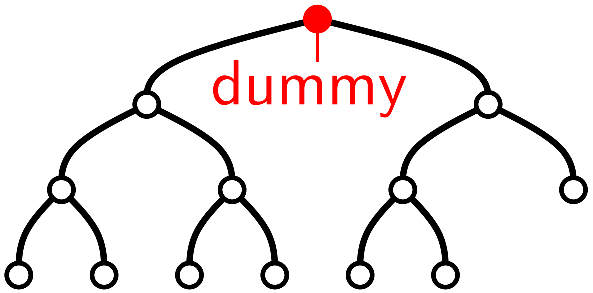
Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex



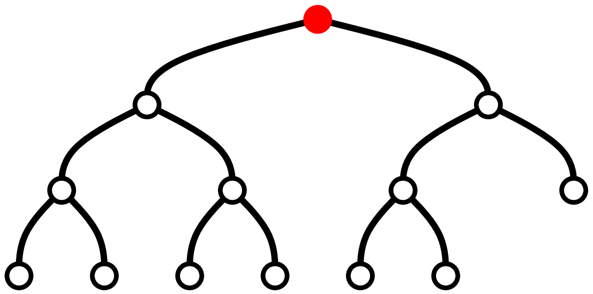
Strongly Monotone Drawings

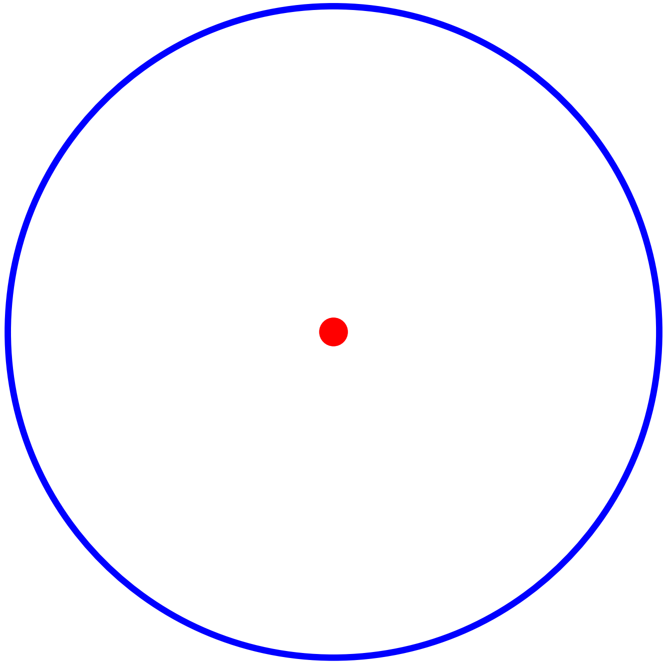
Proper Binary Trees: No degree-2 vertex



Strongly Monotone Drawings

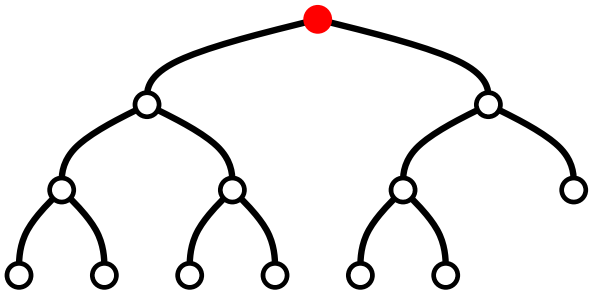
Proper Binary Trees: No degree-2 vertex

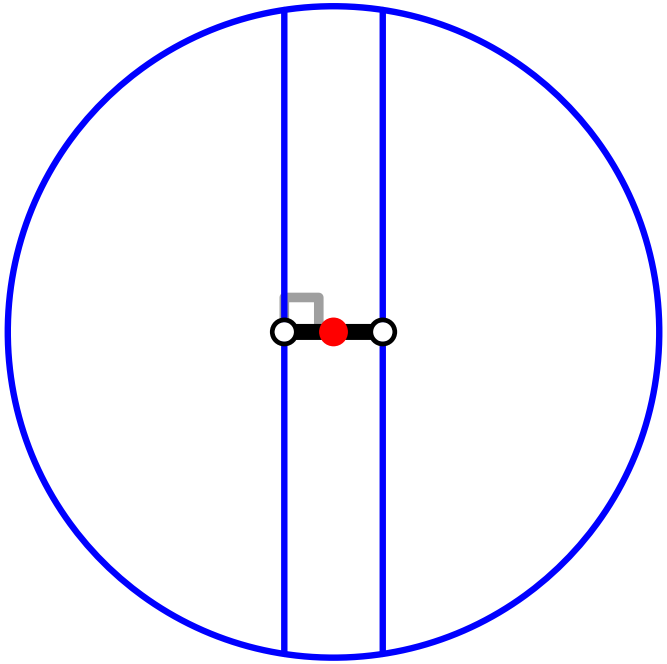




Strongly Monotone Drawings

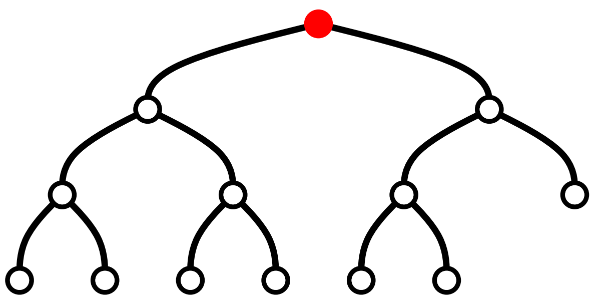
Proper Binary Trees: No degree-2 vertex

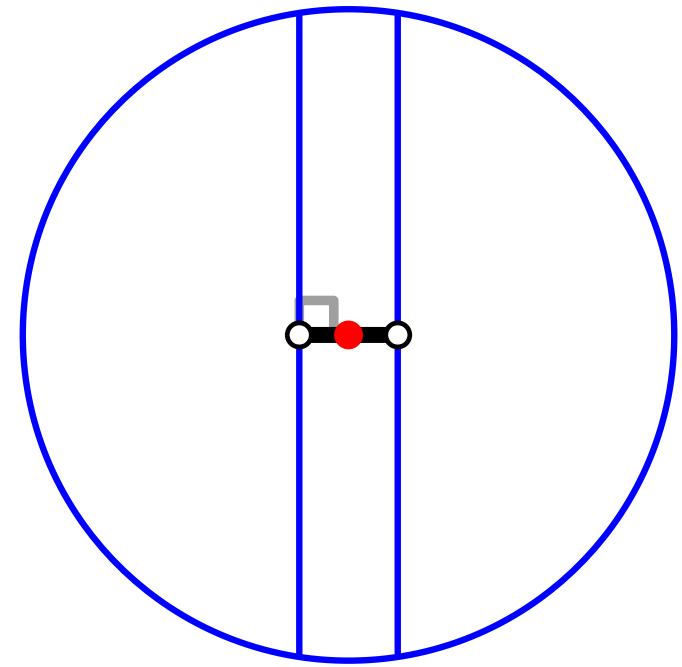


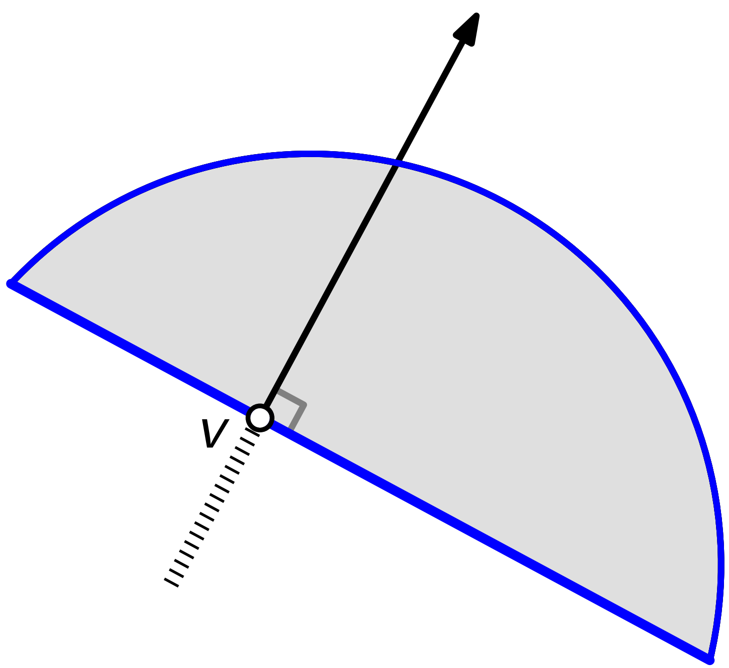


Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

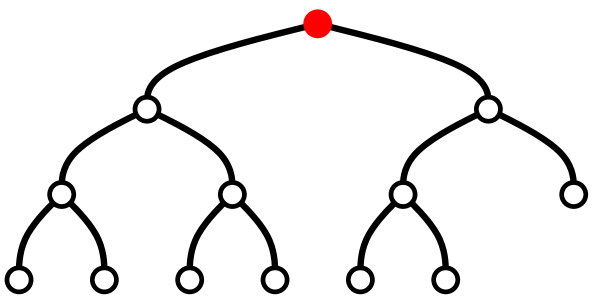


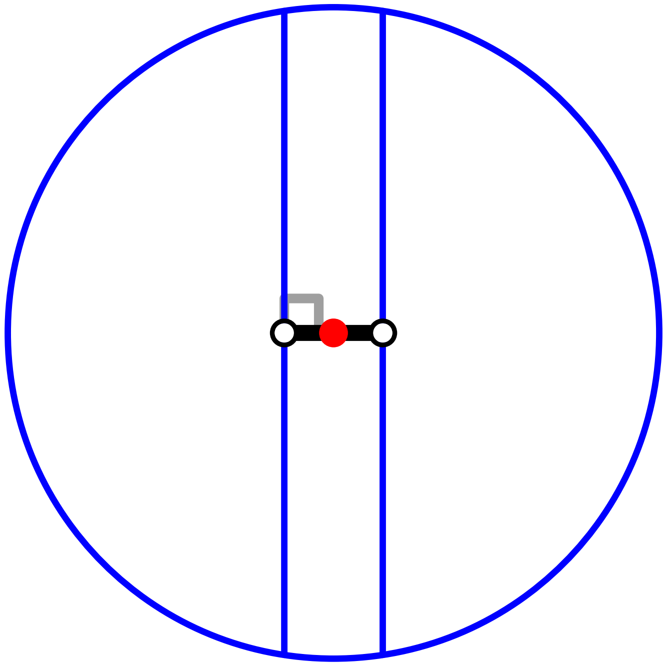


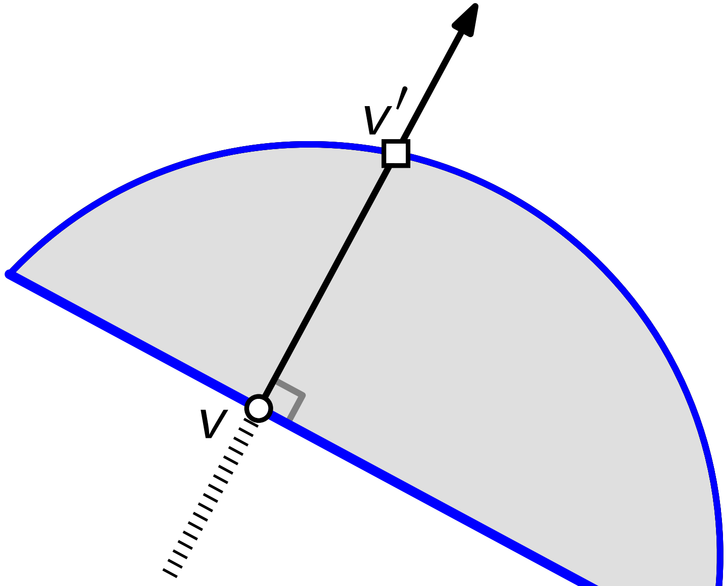


Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

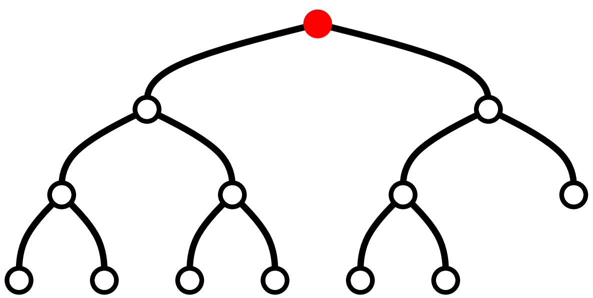


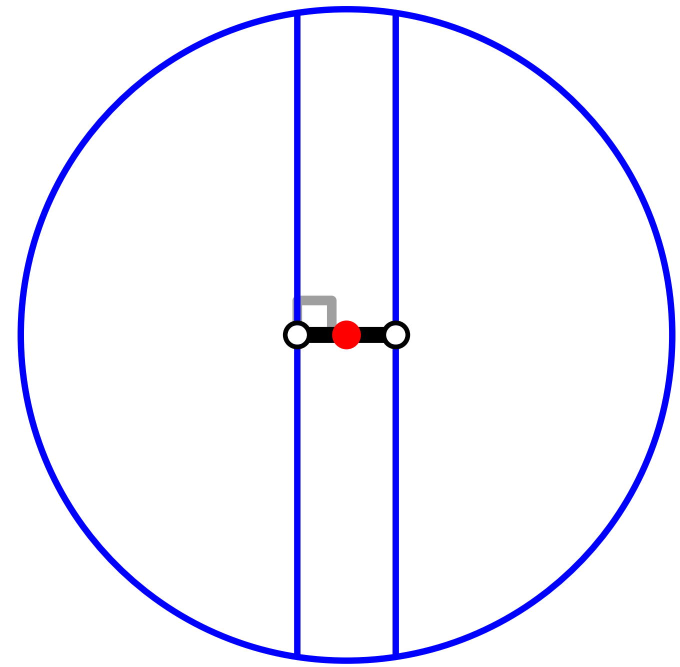


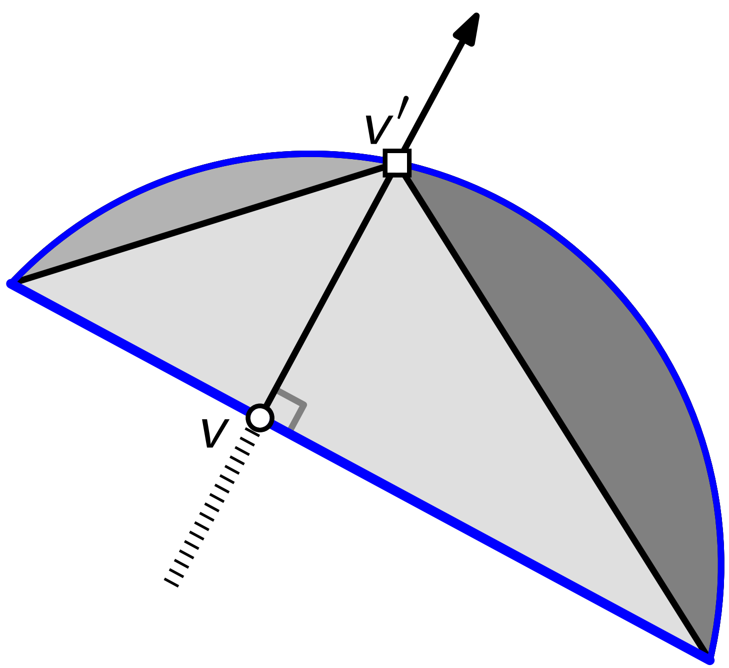


Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

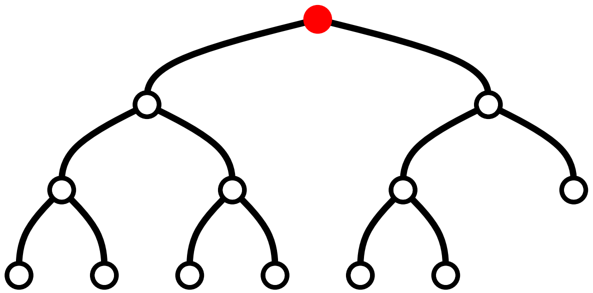


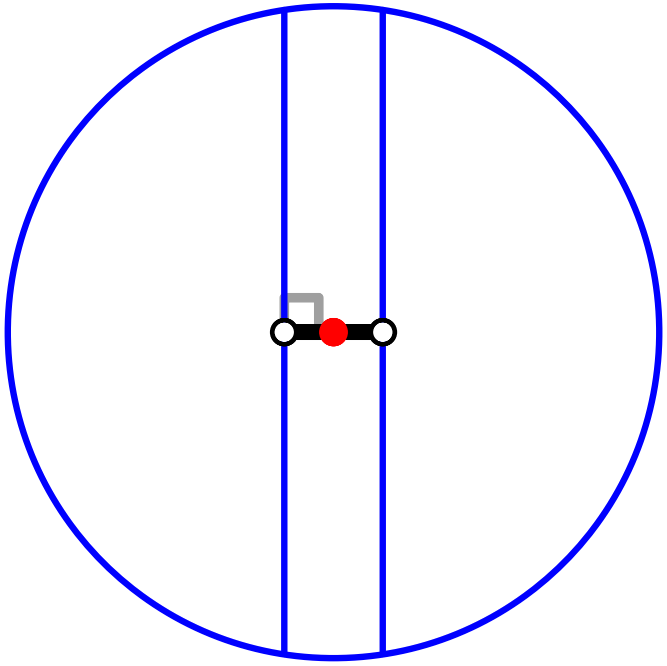


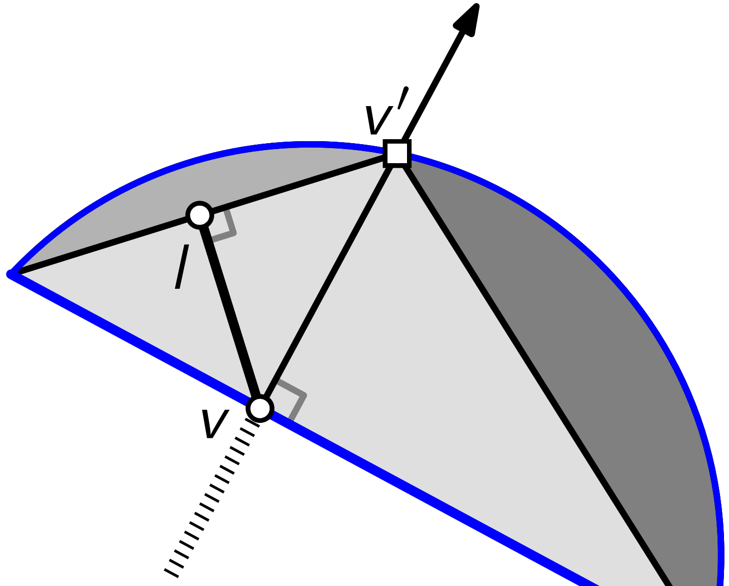


Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

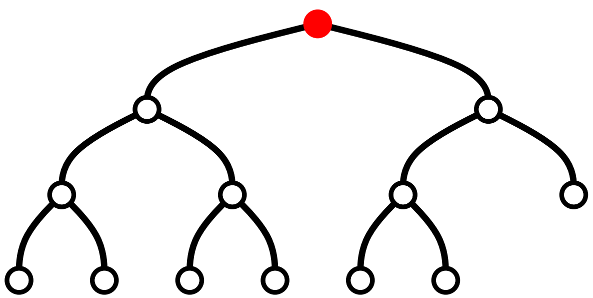


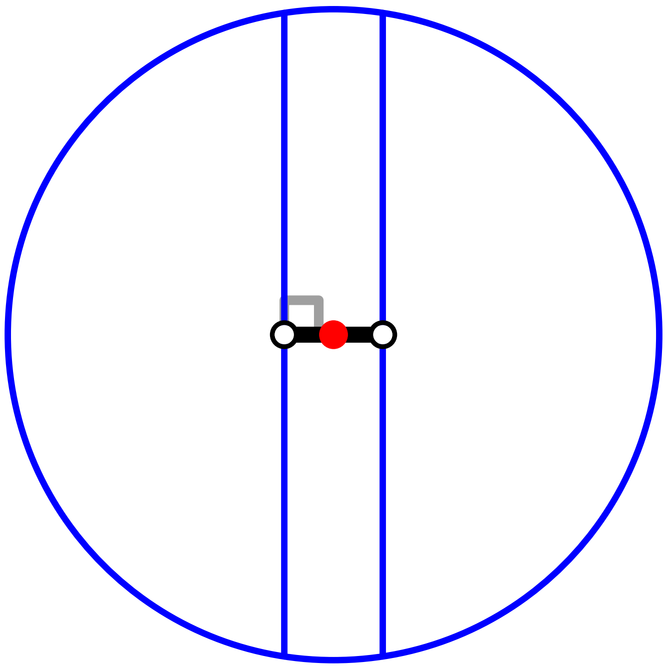


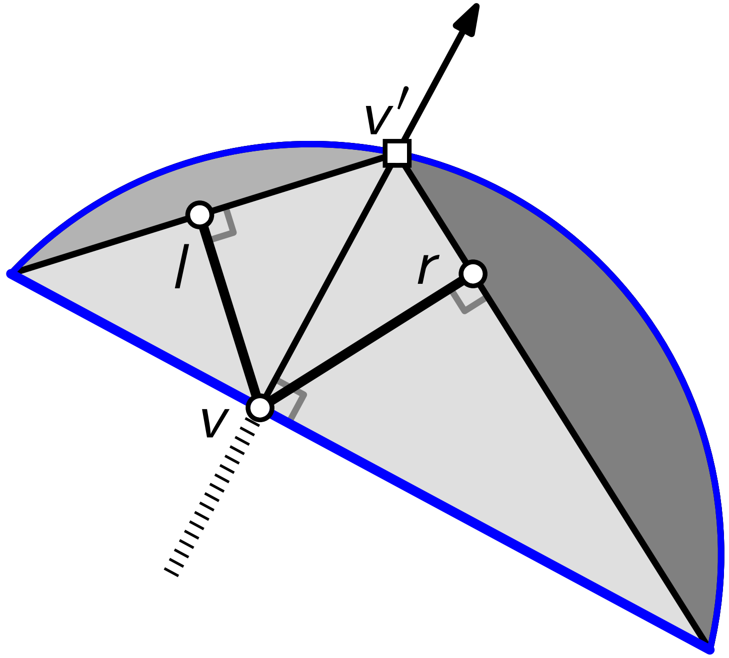


Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

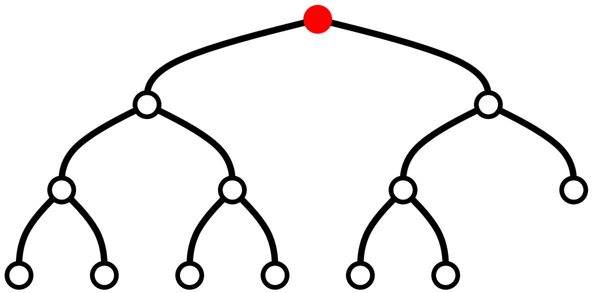


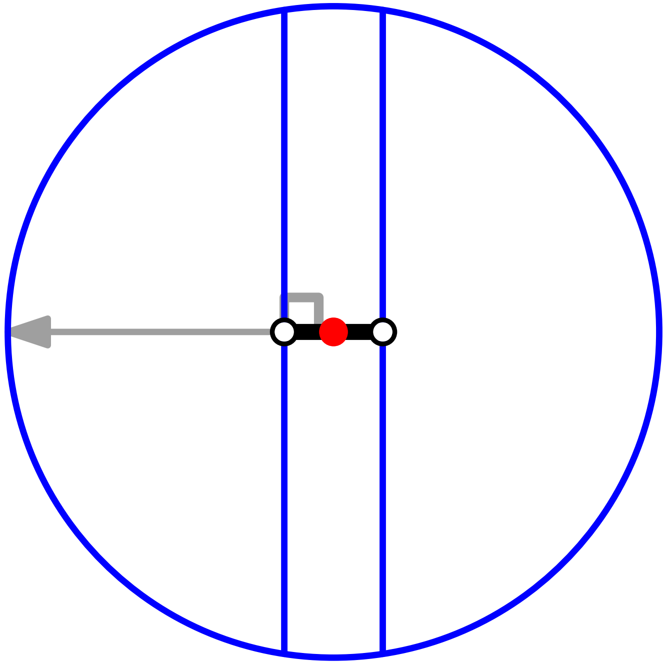


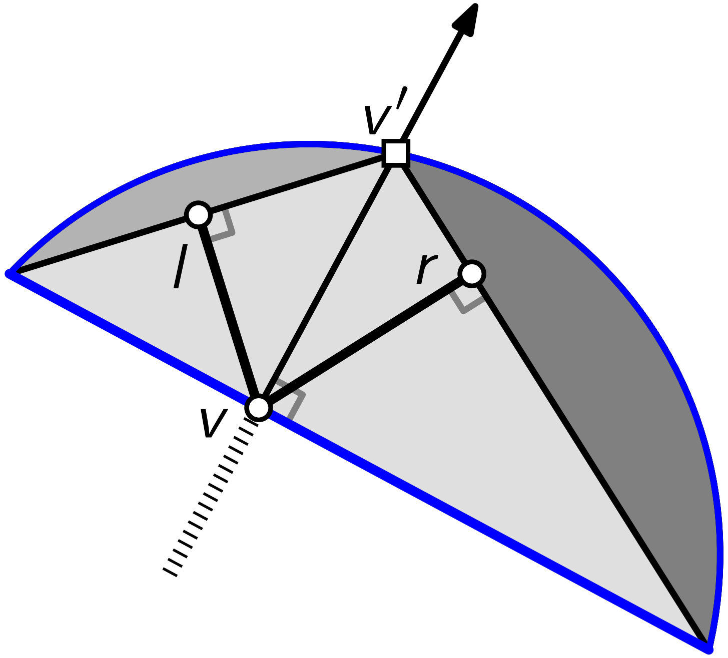


Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

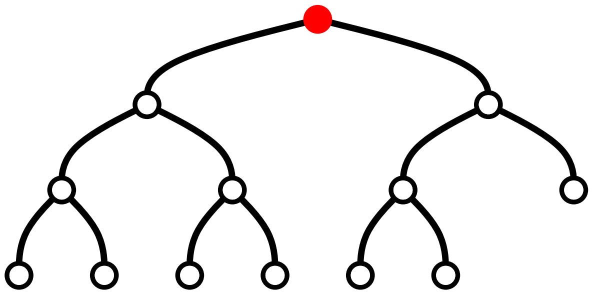


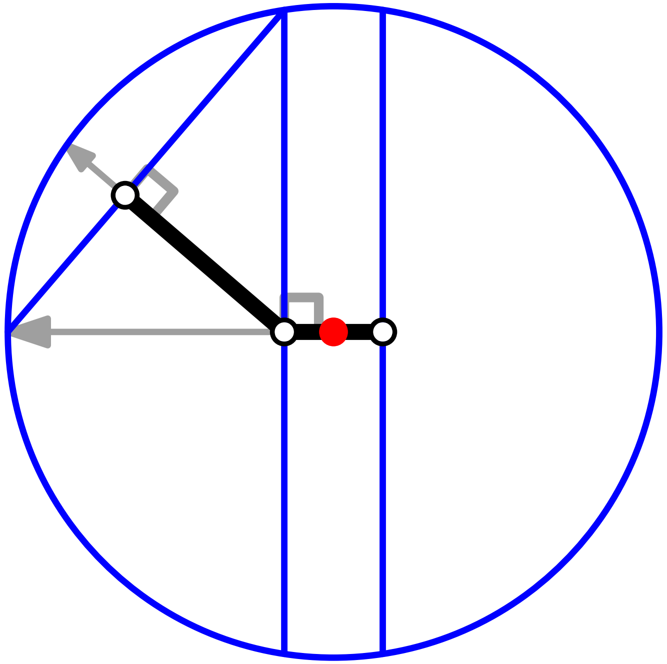


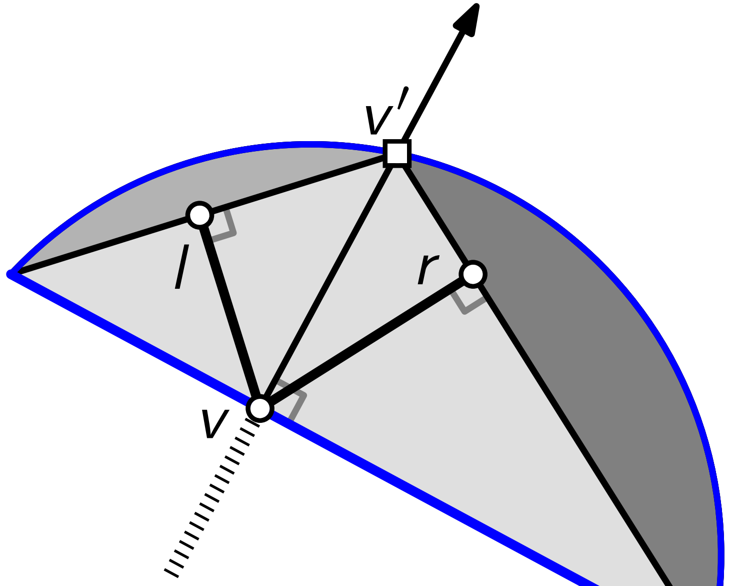


Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

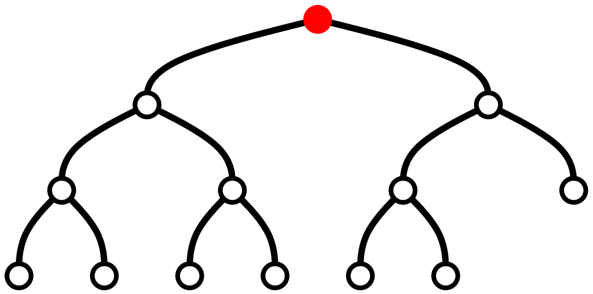


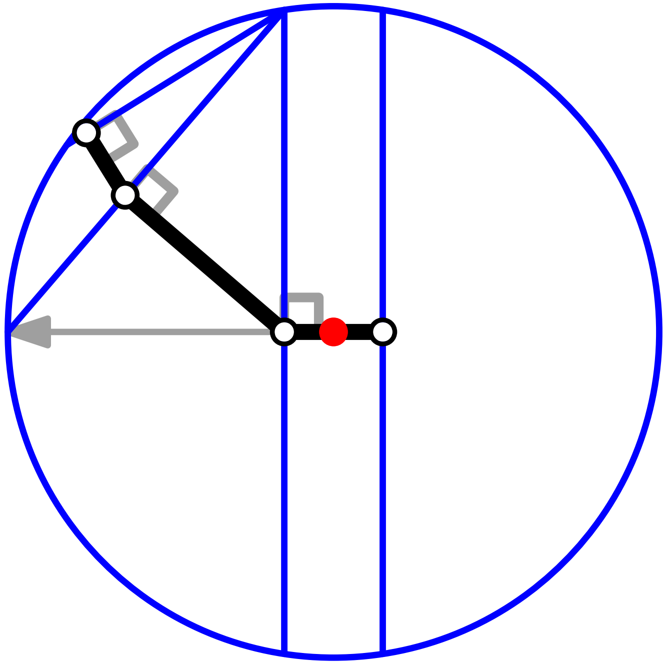


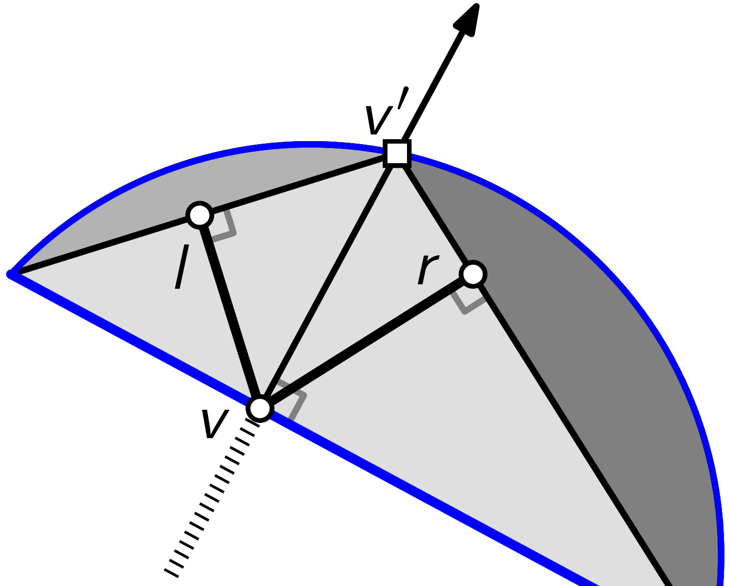


Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

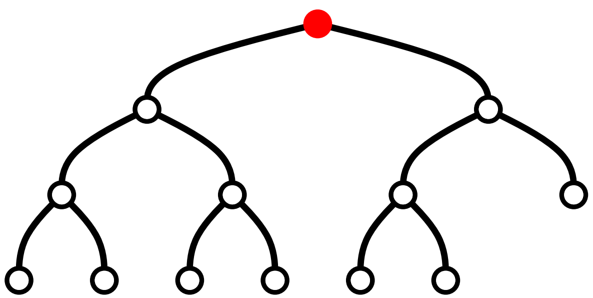


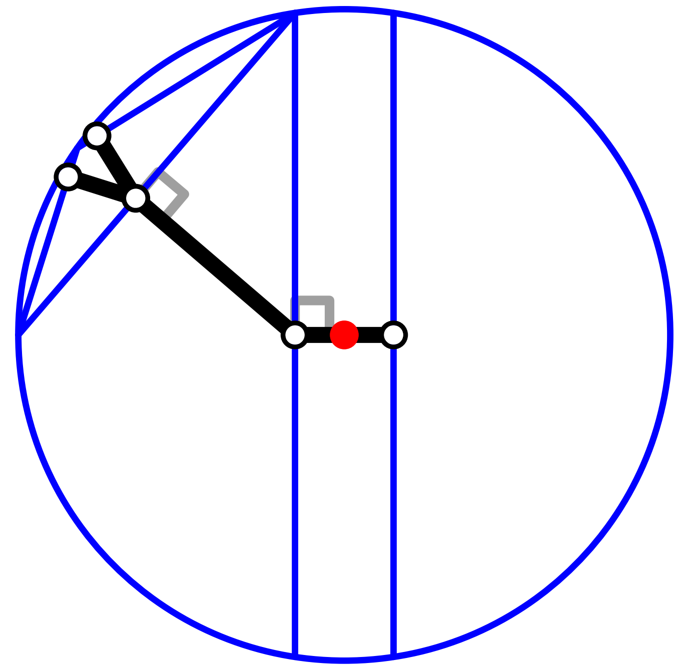


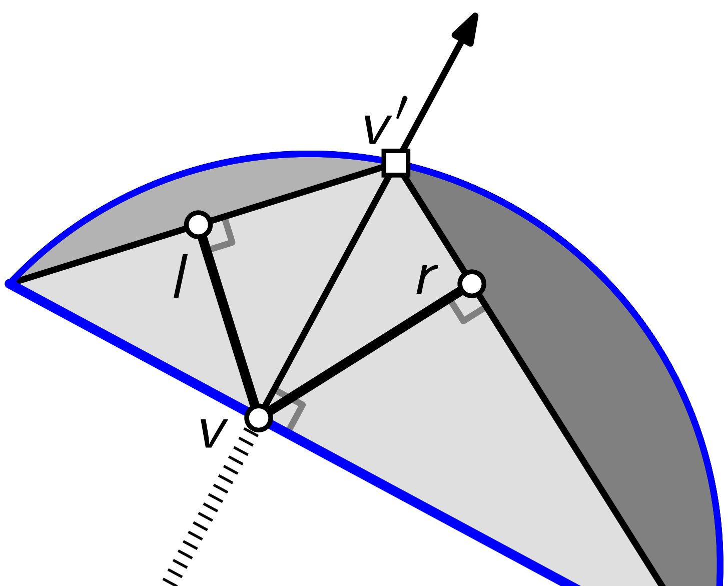


Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

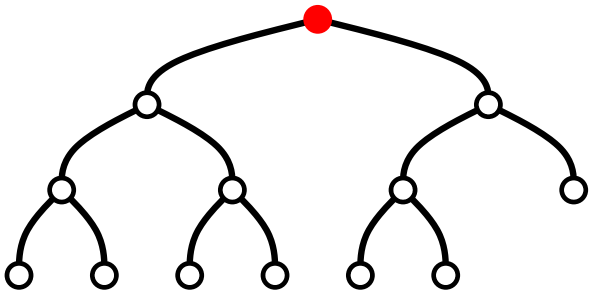


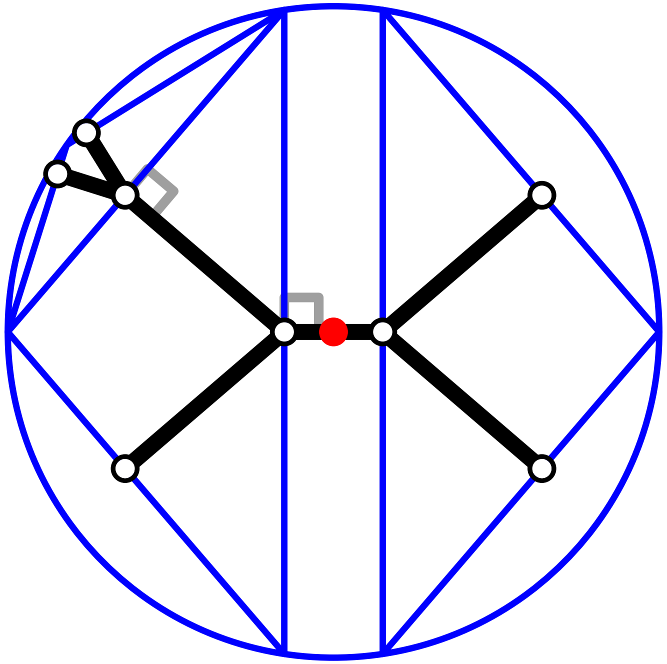


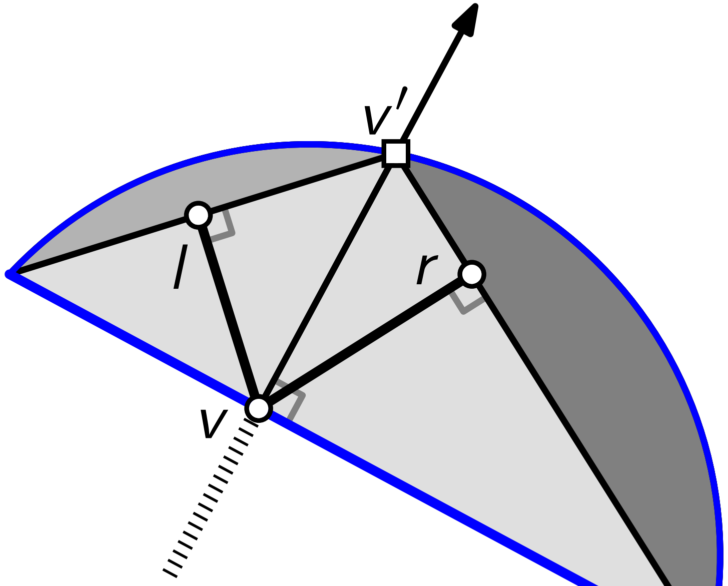


Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

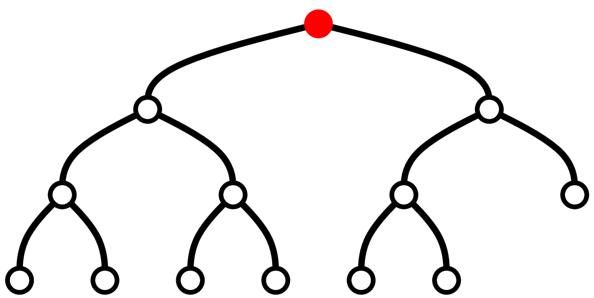


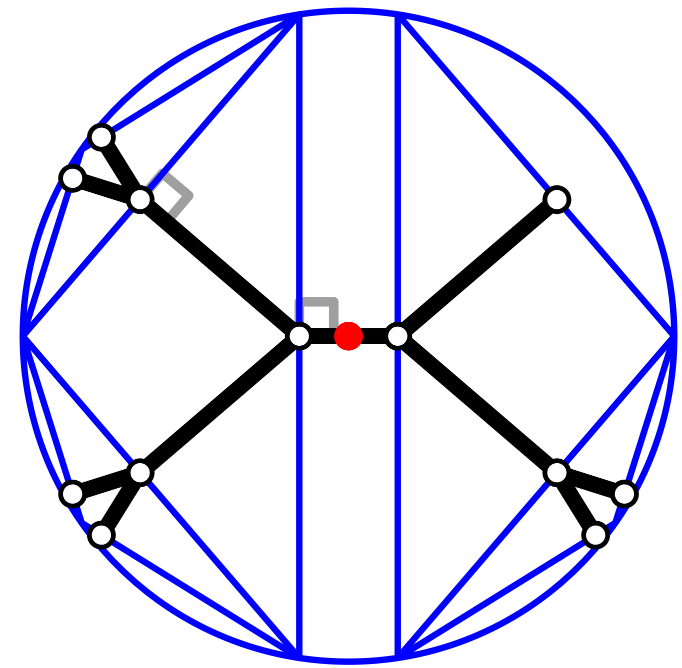


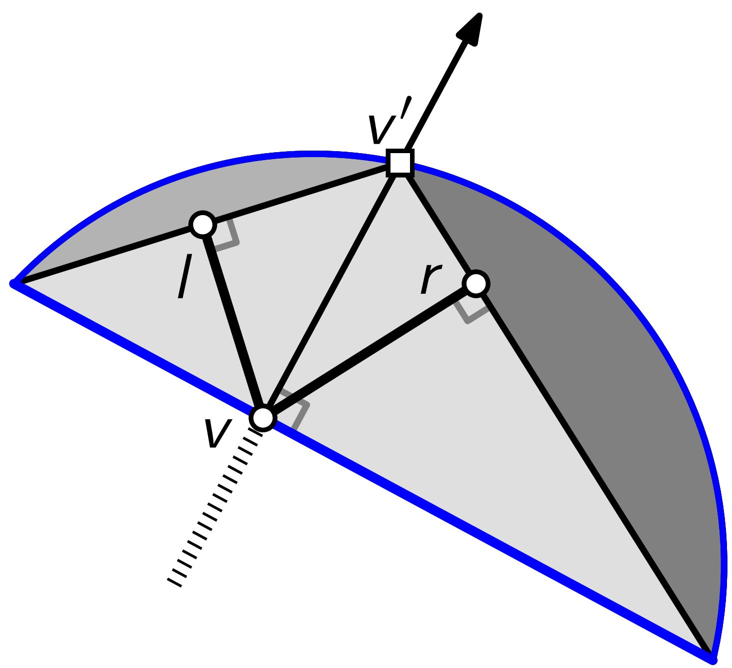


Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

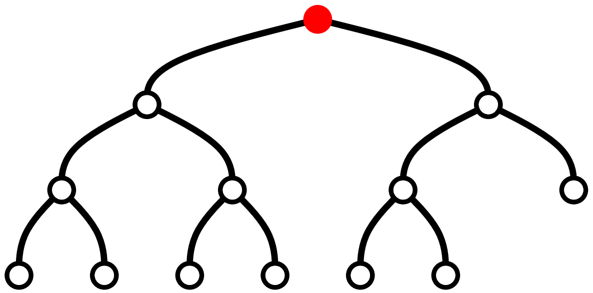


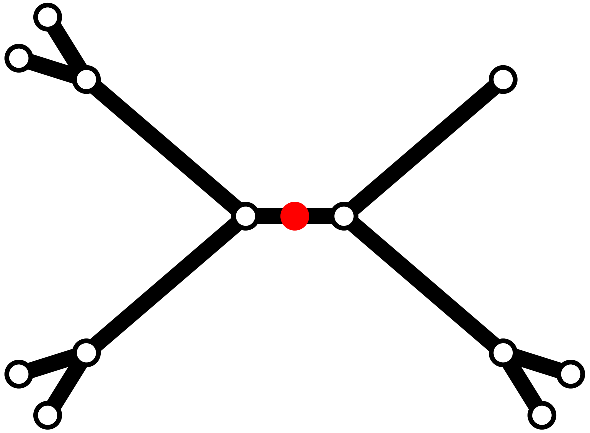




Strongly Monotone Drawings

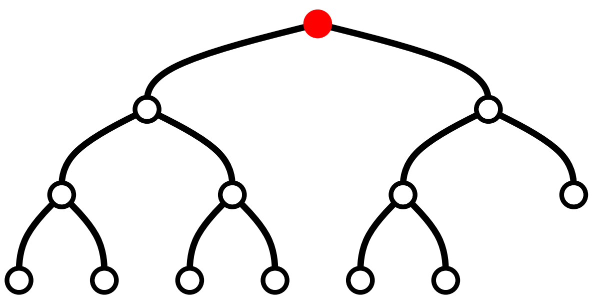
Proper Binary Trees: No degree-2 vertex

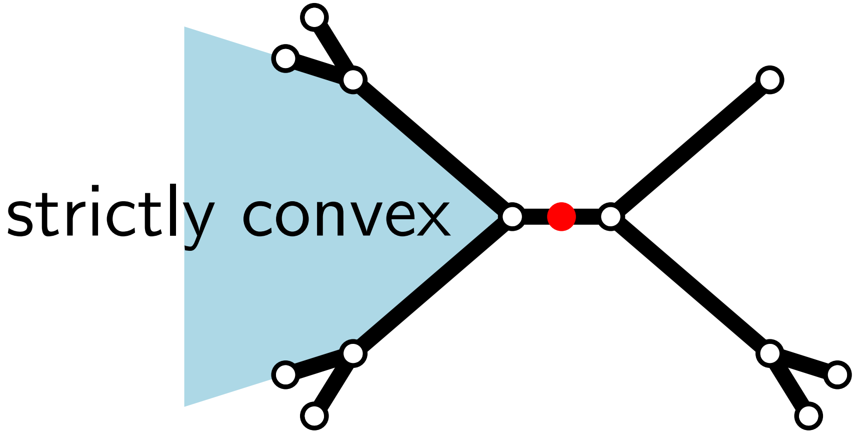




Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex

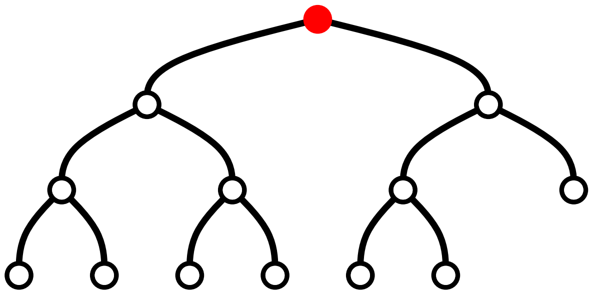


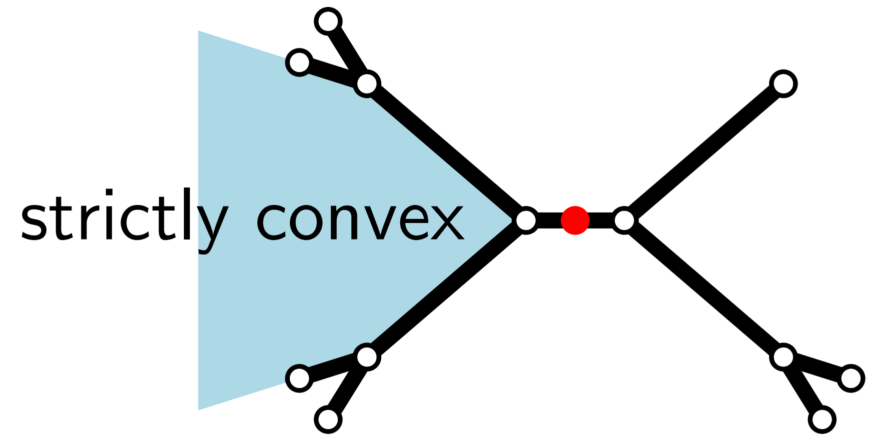


- All angles $< \pi \Rightarrow$ strictly convex ✓

Strongly Monotone Drawings

Proper Binary Trees: No degree-2 vertex





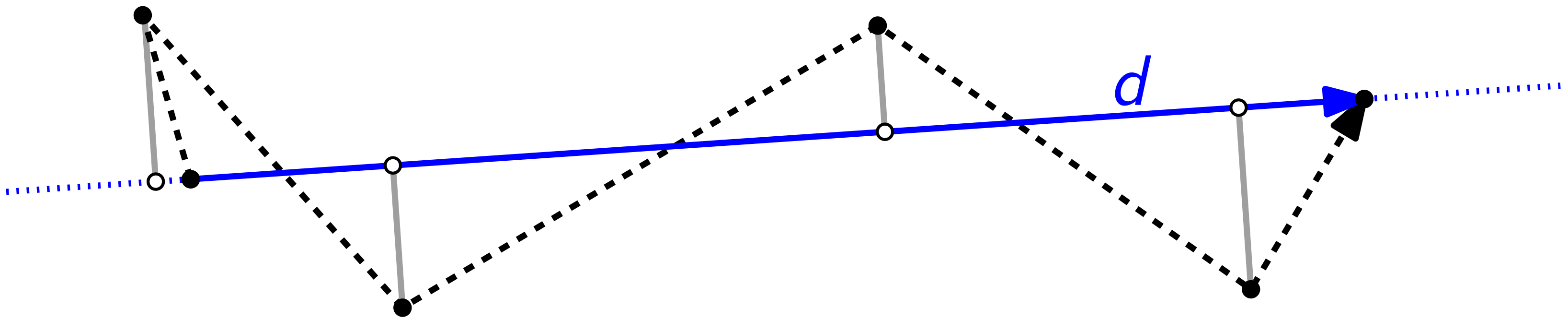
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly monotone?

Properties

Observation.
A *u*-*v*-path is *not* strongly monotone
$\Leftrightarrow \exists$ an edge e with $\measuredangle(\vec{e}, \overrightarrow{uv}) > \pi/2$.

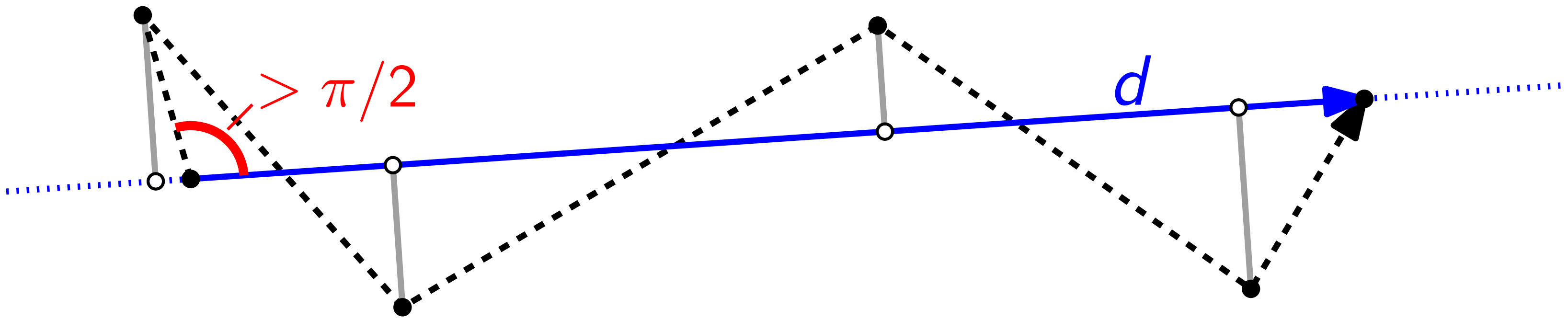
Properties

Observation.
A u-v-path is *not* strongly monotone
$\Leftrightarrow \exists$ an edge e with $\measuredangle(\vec{e}, \overrightarrow{uv}) > \pi/2$.



Properties

Observation.
A u-v-path is *not* strongly monotone
$\Leftrightarrow \exists$ an edge e with $\measuredangle(\vec{e}, \overrightarrow{uv}) > \pi/2$.



Properties

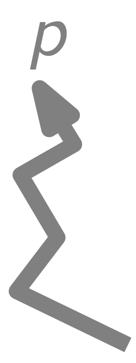
Observation.
A u-v-path is *not* strongly monotone
$\Leftrightarrow \exists$ an edge e with $\measuredangle(\vec{e}, \overrightarrow{uv}) > \pi/2$.

Lemma.
If a path is monotone to $\vec{v}_1$ and $\vec{v}_2$,
then it is monotone to $\vec{v}_3$ *between* $\vec{v}_1$ and $\vec{v}_2$.

Properties

Observation.
A u-v-path is *not* strongly monotone
$\Leftrightarrow \exists$ an edge e with $\measuredangle(\vec{e}, \overrightarrow{uv}) > \pi/2$.

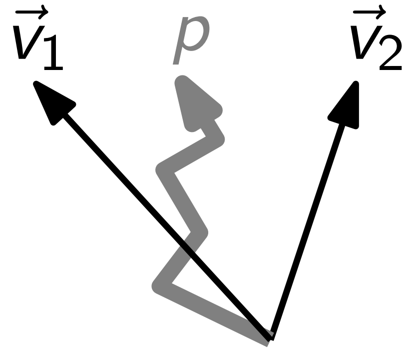
Lemma.
If a path is monotone to $\vec{v}_1$ and $\vec{v}_2$,
then it is monotone to $\vec{v}_3$ *between* $\vec{v}_1$ and $\vec{v}_2$.



Properties

Observation.
A u-v-path is *not* strongly monotone
$\Leftrightarrow \exists$ an edge e with $\measuredangle(\vec{e}, \overrightarrow{uv}) > \pi/2$.

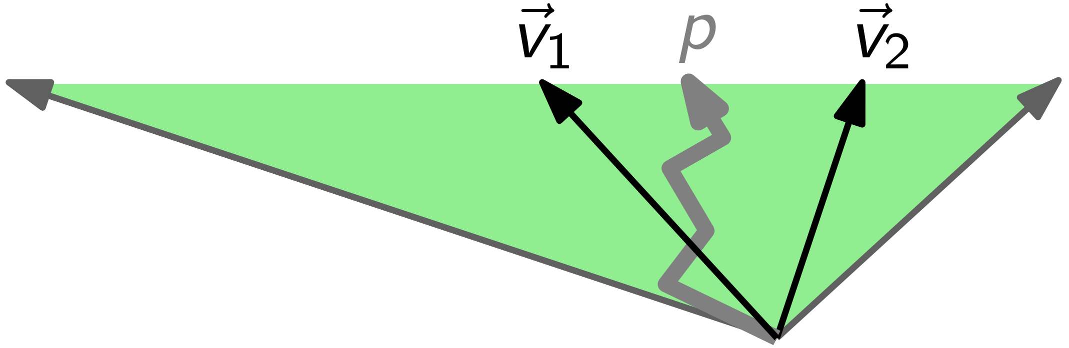
Lemma.
If a path is monotone to $\vec{v}_1$ and $\vec{v}_2$,
then it is monotone to $\vec{v}_3$ *between* $\vec{v}_1$ and $\vec{v}_2$.



Properties

Observation.
A u-v-path is *not* strongly monotone
$\Leftrightarrow \exists$ an edge e with $\measuredangle(\vec{e}, \overrightarrow{uv}) > \pi/2$.

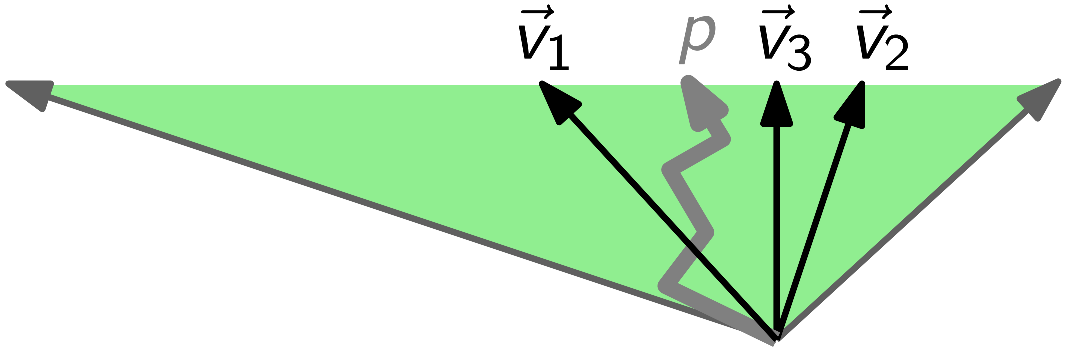
Lemma.
If a path is monotone to $\vec{v}_1$ and $\vec{v}_2$,
then it is monotone to $\vec{v}_3$ *between* $\vec{v}_1$ and $\vec{v}_2$.



Properties

Observation.
A u-v-path is *not* strongly monotone
$\Leftrightarrow \exists$ an edge e with $\measuredangle(\vec{e}, \overrightarrow{uv}) > \pi/2$.

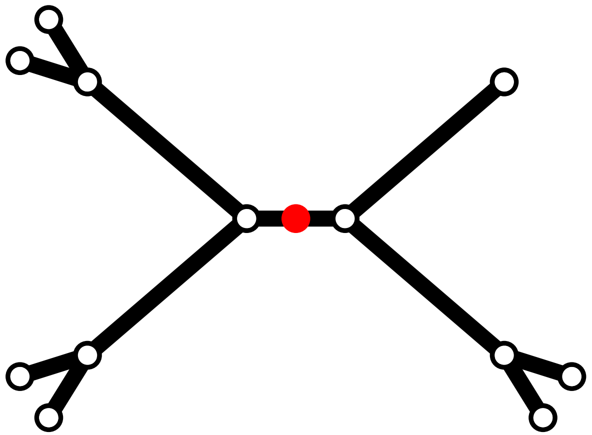
Lemma.
If a path is monotone to $\vec{v}_1$ and $\vec{v}_2$,
then it is monotone to $\vec{v}_3$ *between* $\vec{v}_1$ and $\vec{v}_2$.



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

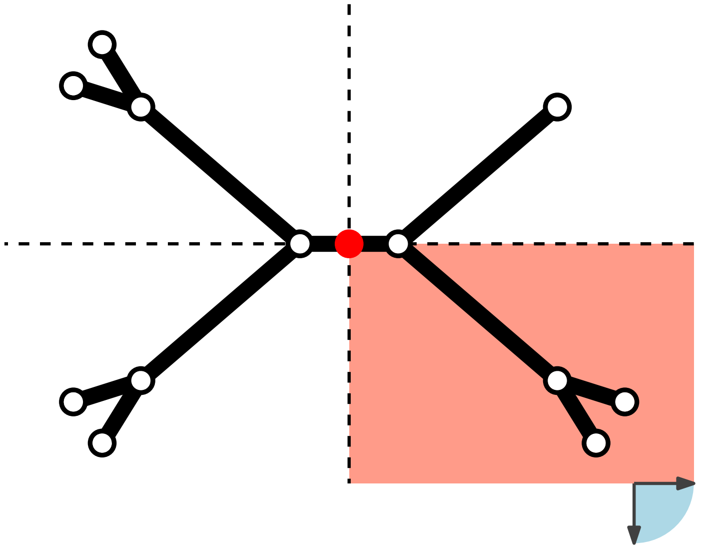
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

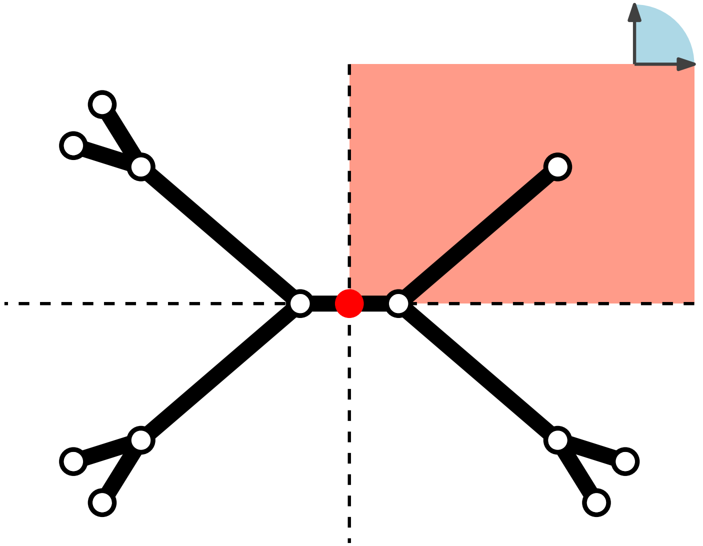
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

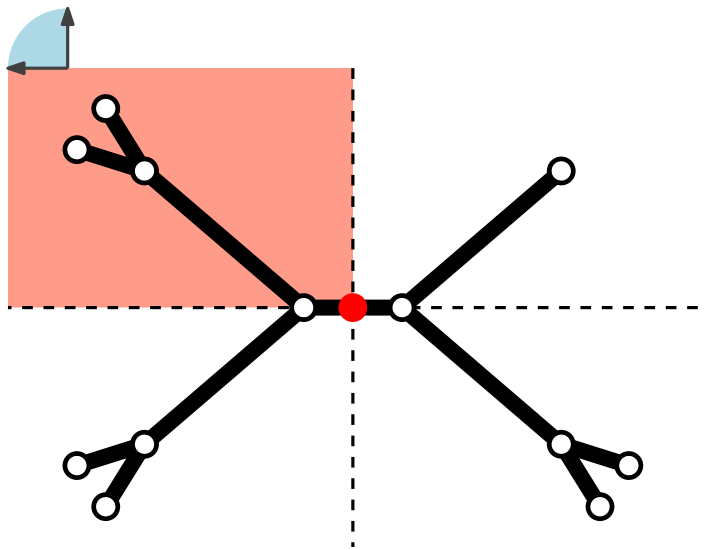
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

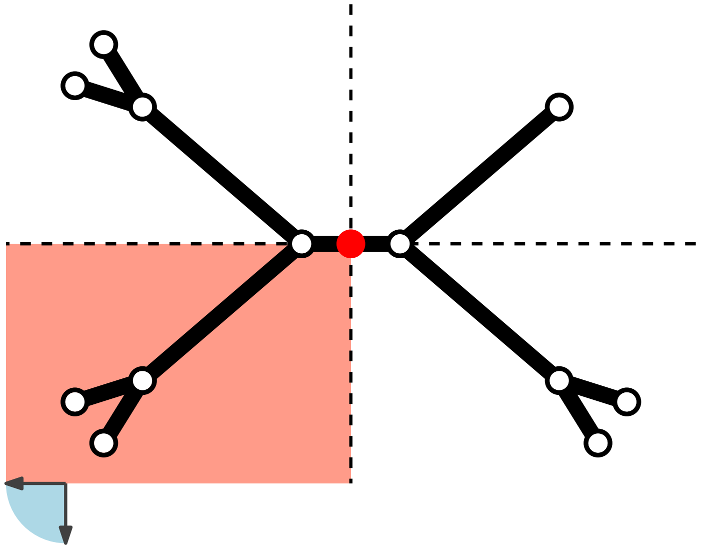
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

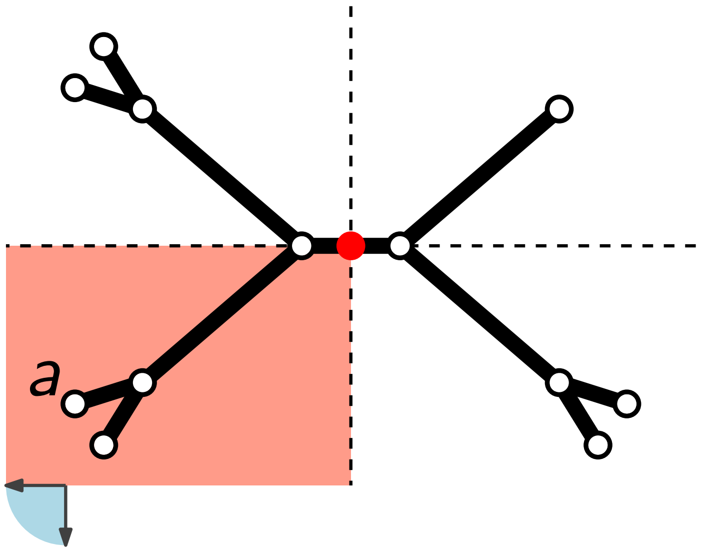


Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

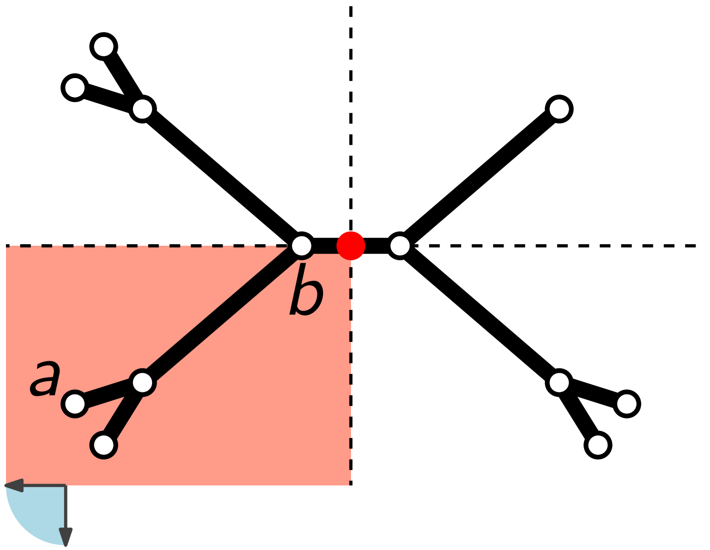
W.l.o.g. assume *a* lies bottom-left



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?



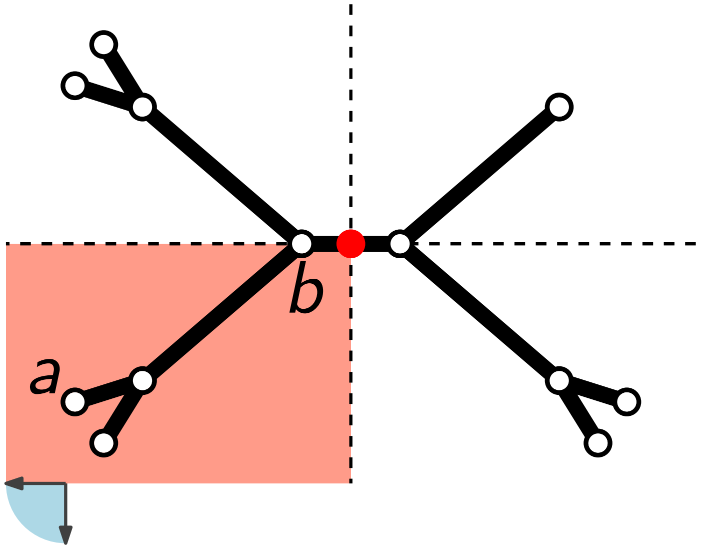
W.l.o.g. assume *a* lies bottom-left

Case 1: *a* and *b* on common root-leaf path

Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

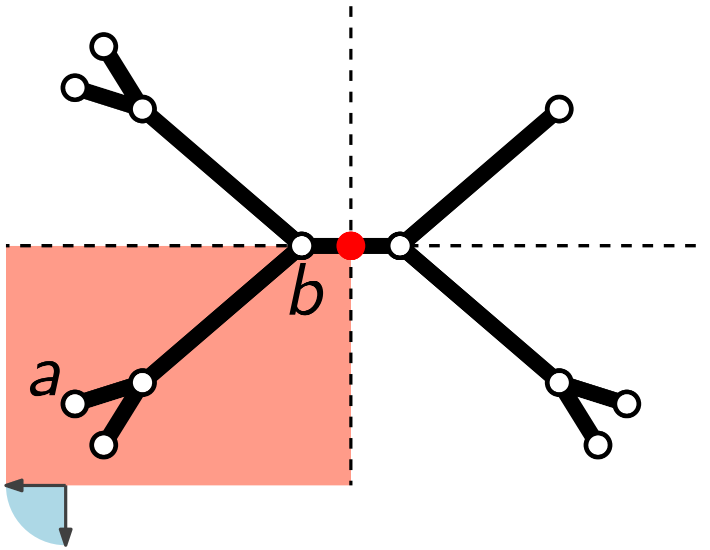


W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path

$\Rightarrow \overrightarrow{ba} \in$ 

Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume a lies bottom-left

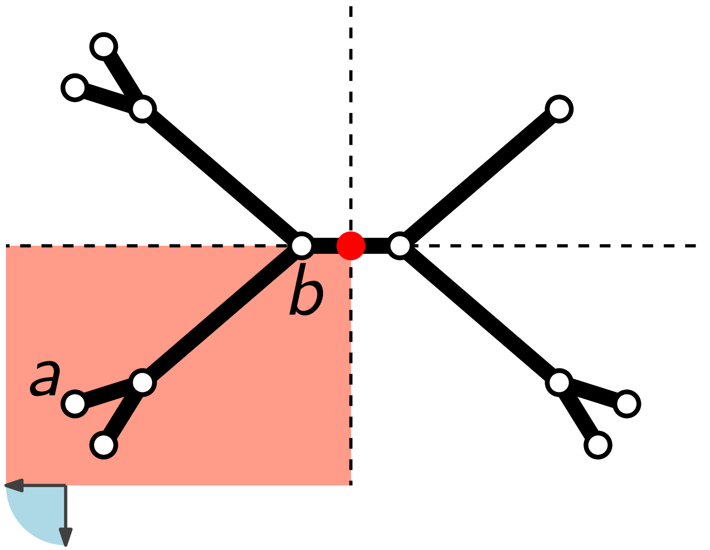
Case 1: a and b on common root-leaf path

$\Rightarrow \overrightarrow{ba} \in$ , path from b to $a \in$ 

Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?



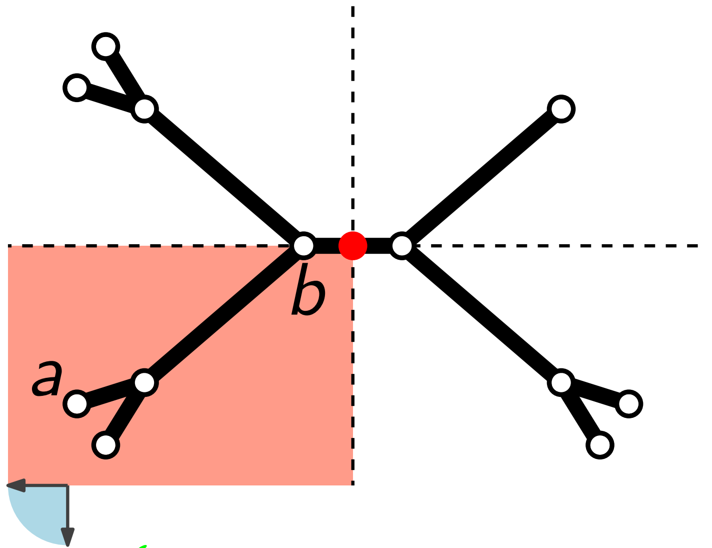
W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path

$\Rightarrow \overrightarrow{ba} \in$ , path from b to $a \in$ 

$\Rightarrow$ path from b to a is strongly monotone

Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

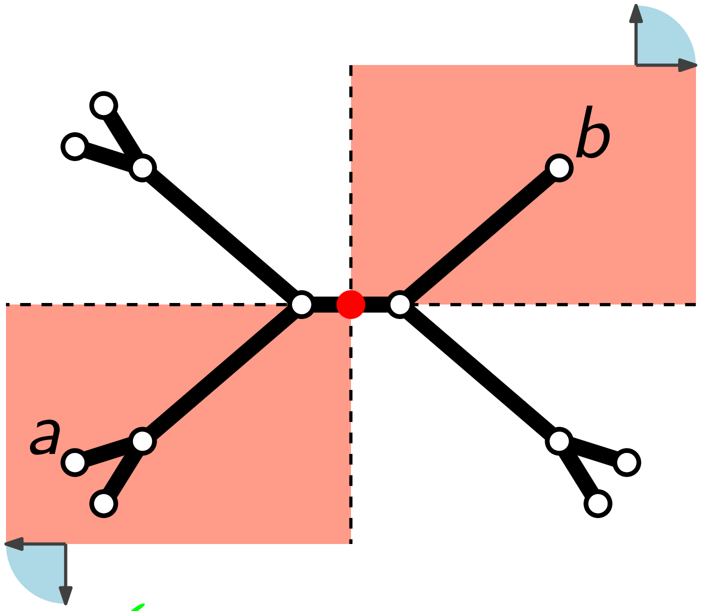
W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path ✓

$\Rightarrow \overrightarrow{ba} \in$ , path from b to $a \in$ 

$\Rightarrow$ path from b to a is strongly monotone

Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

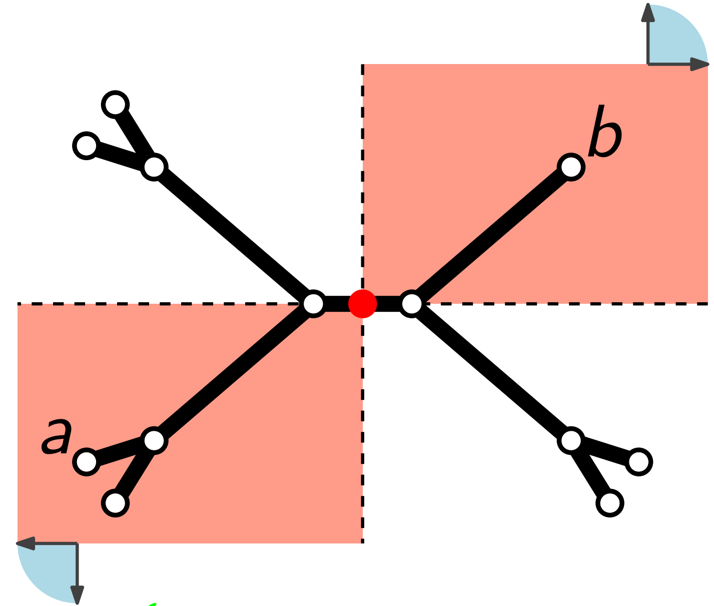
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume *a* lies bottom-left

Case 1: *a* and *b* on common root-leaf path ✓

Case 2: *a* and *b* in opposite sectors

Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

$\Rightarrow \overrightarrow{ba} \in$ , path from b to $a \in$ 

$\Rightarrow$ path from b to a is strongly monotone

Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

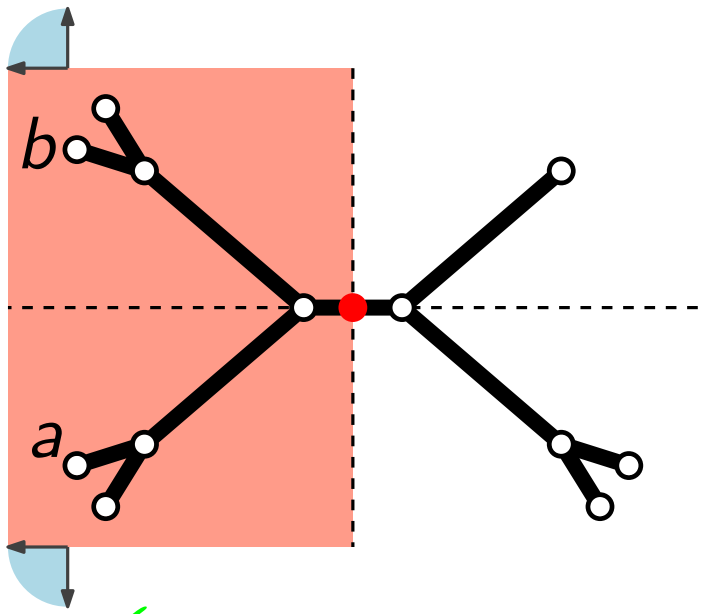
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume *a* lies bottom-left

Case 1: *a* and *b* on common root-leaf path ✓

Case 2: *a* and *b* in opposite sectors ✓

Case 3: else



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

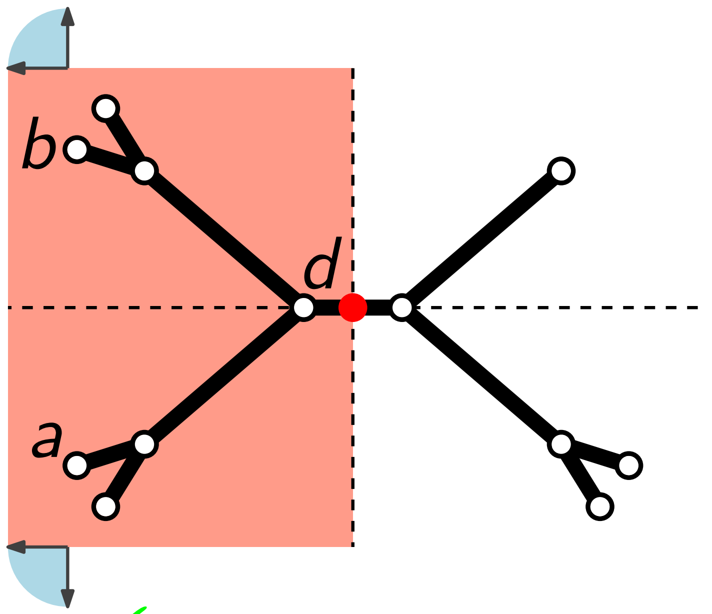
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume a lies bottom-left

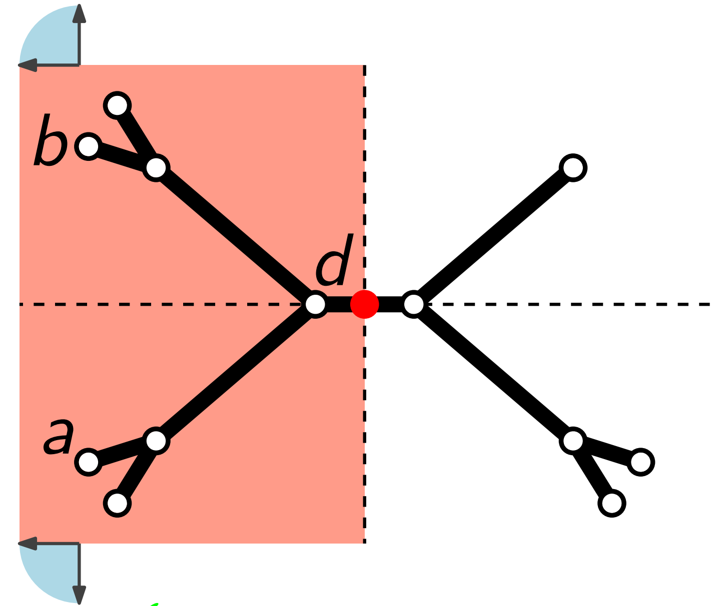
Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

Case 3: else



Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

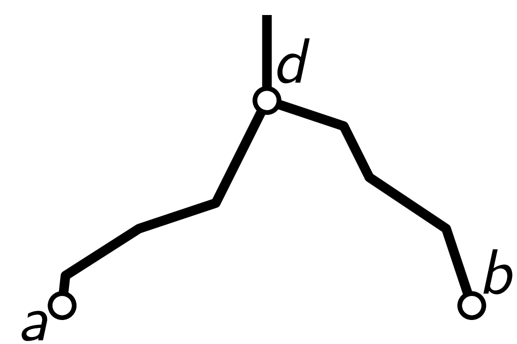
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

Case 3: else



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

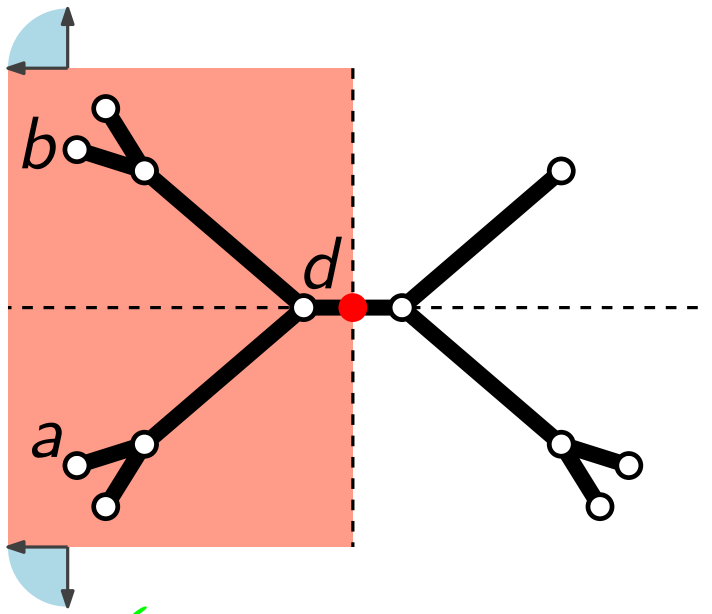
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

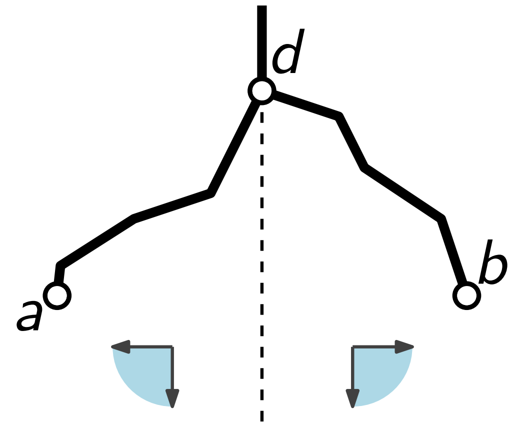
W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path ✓

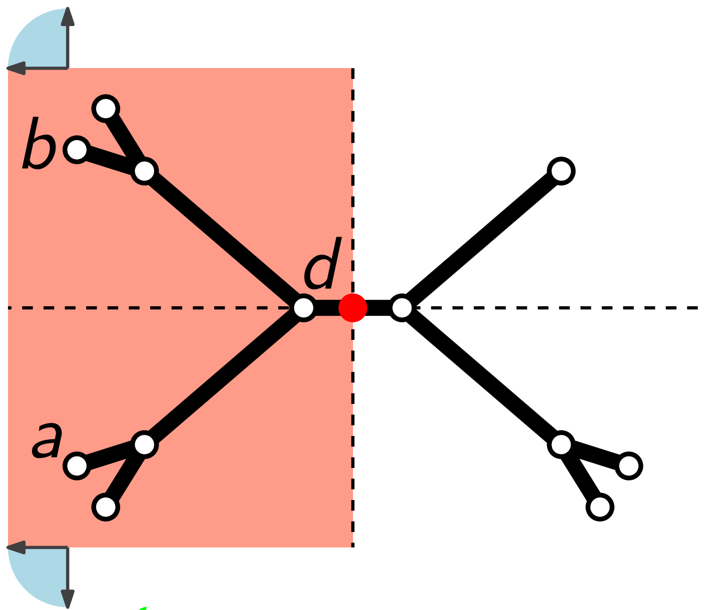
Case 2: a and b in opposite sectors ✓

Case 3: else





Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

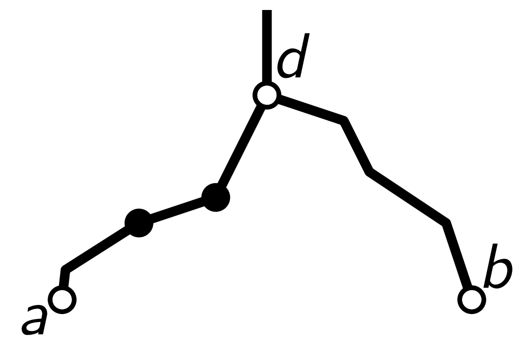
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume *a* lies bottom-left

Case 1: *a* and *b* on common root-leaf path ✓

Case 2: *a* and *b* in opposite sectors ✓

Case 3: else



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

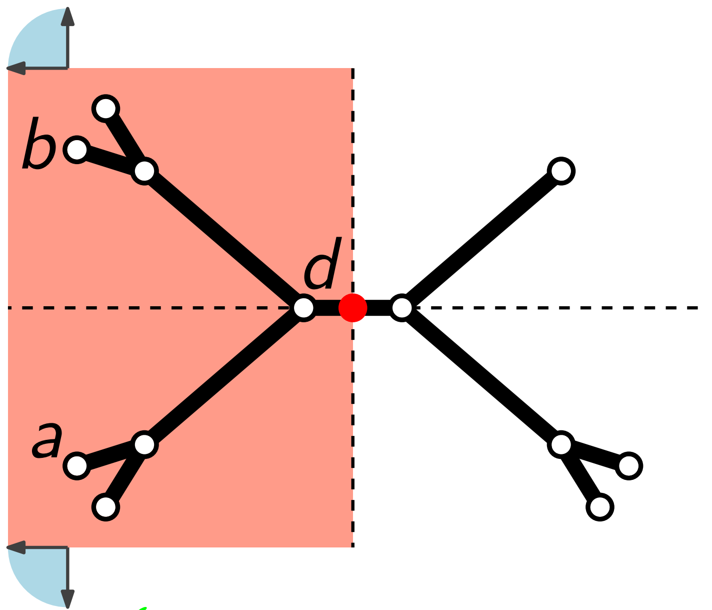
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

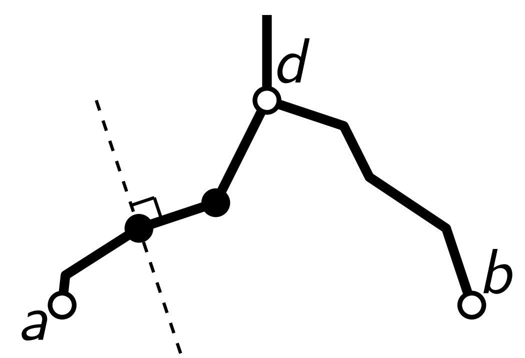
W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

Case 3: else





Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

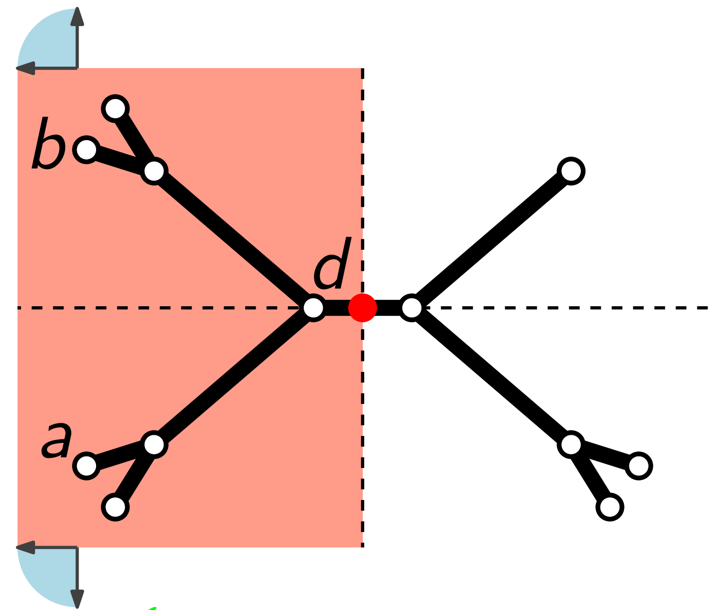
- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

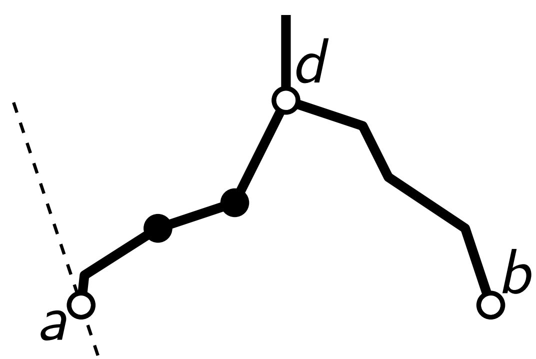
W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

Case 3: else

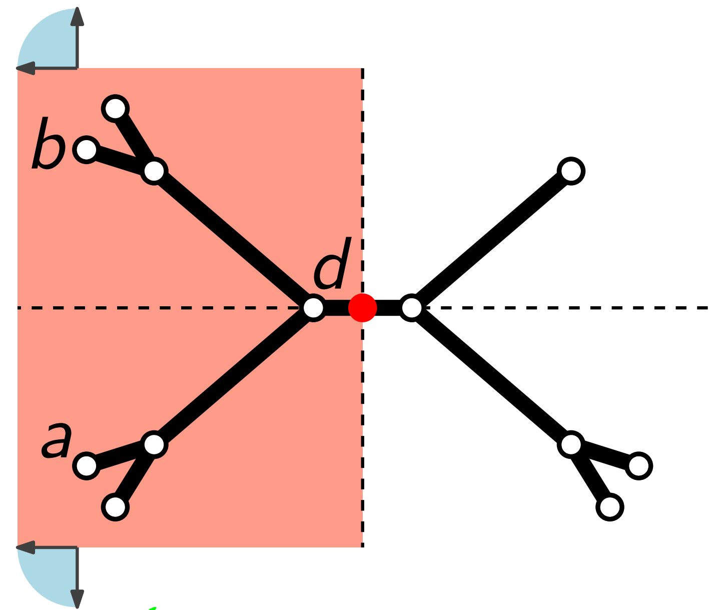




Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

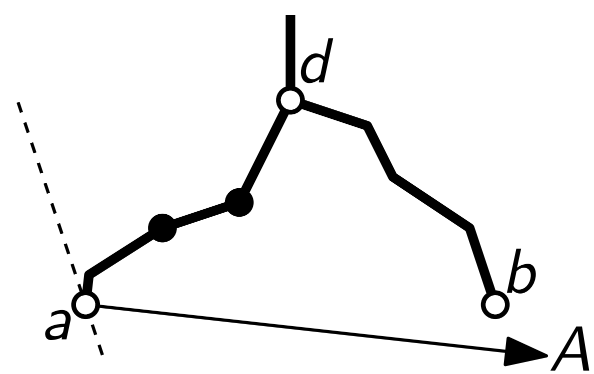


W.l.o.g. assume a lies bottom-left

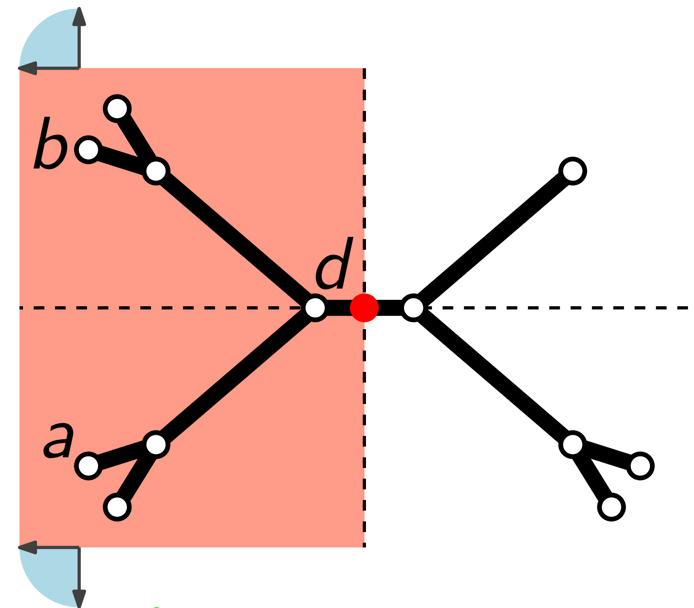
Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

Case 3: else



Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

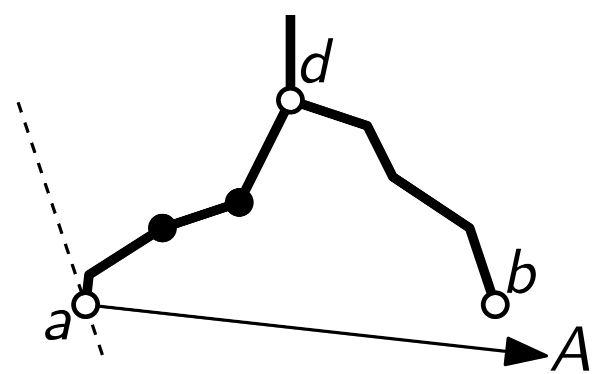
W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

Case 3: else

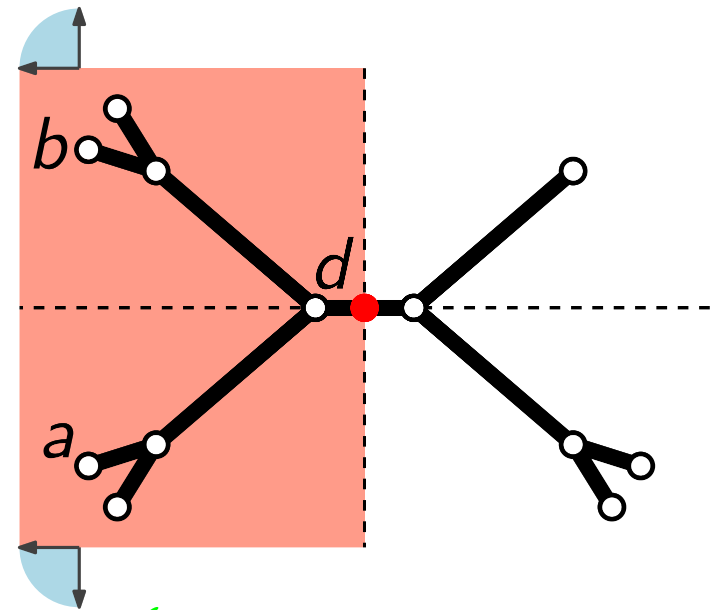
a-d-path monotone to A



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?



W.l.o.g. assume a lies bottom-left

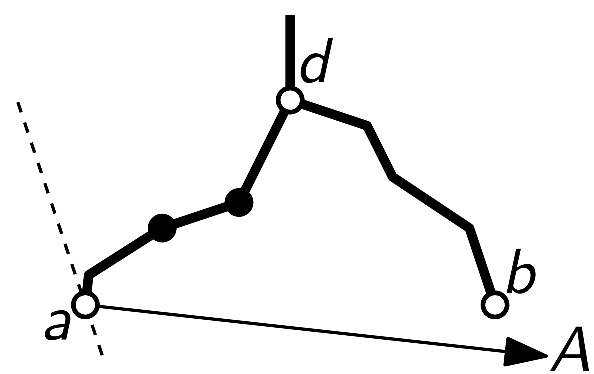
Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

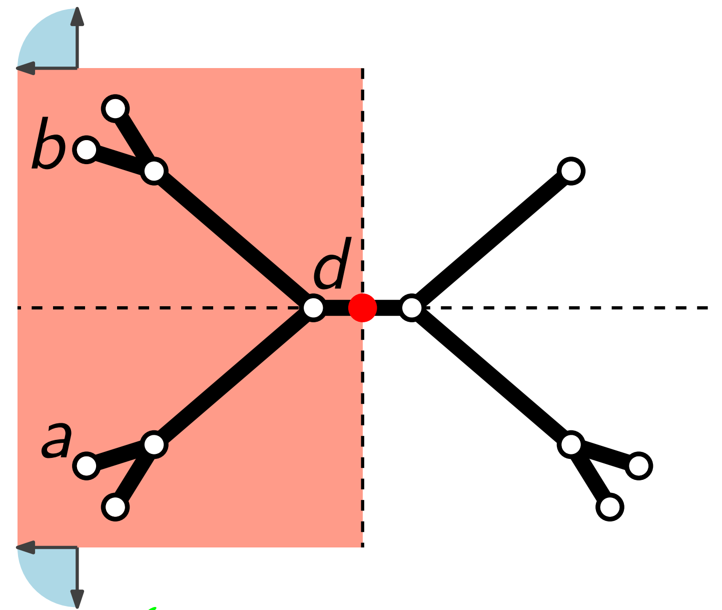
Case 3: else

a-d-path monotone to A

d-b-path monotone to A



Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path ✓

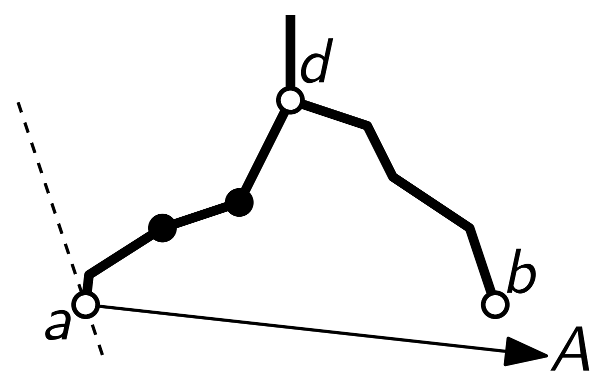
Case 2: a and b in opposite sectors ✓

Case 3: else

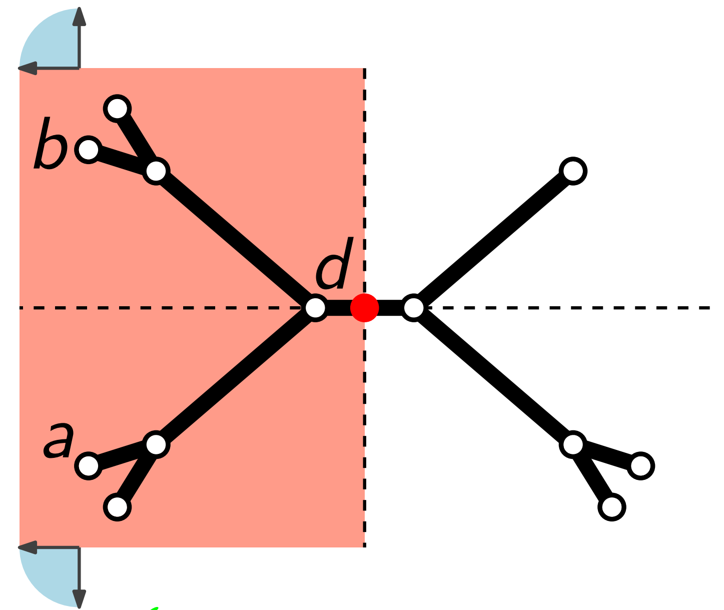
a-d-path monotone to A

d-b-path monotone to A

$\Rightarrow$ a-b-path monotone to A



Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

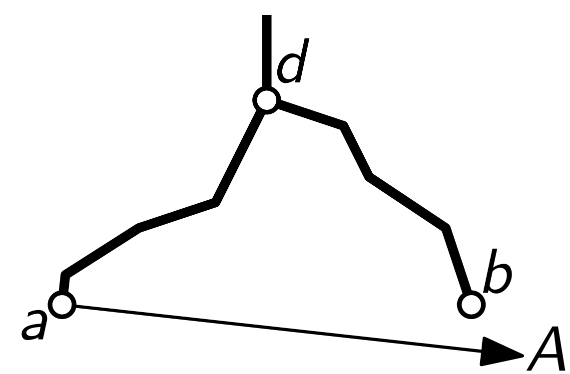
W.l.o.g. assume *a* lies bottom-left

Case 1: *a* and *b* on common root-leaf path ✓

Case 2: *a* and *b* in opposite sectors ✓

Case 3: else

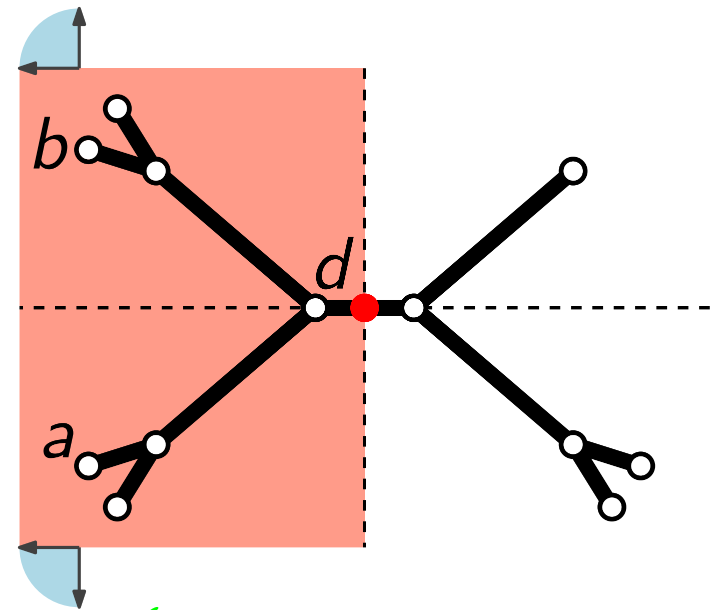
a-*b*-path monotone to *A*



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?



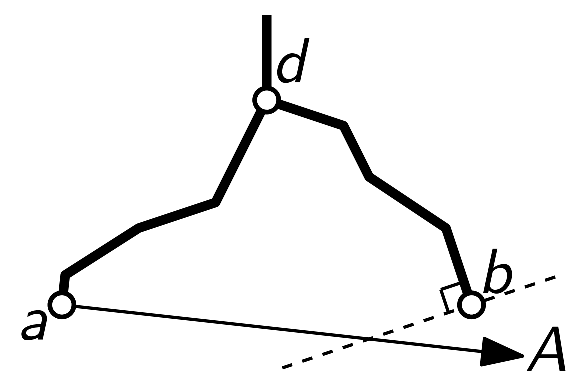
W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path ✓

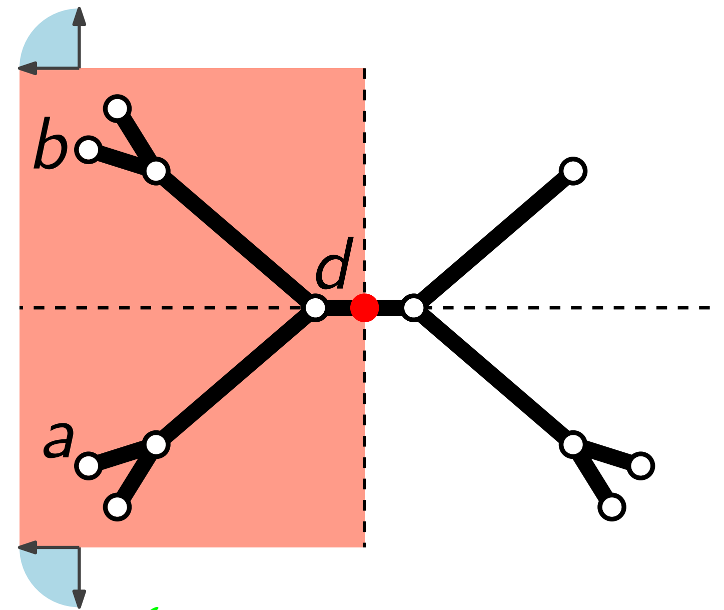
Case 2: a and b in opposite sectors ✓

Case 3: else

a-b-path monotone to A



Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

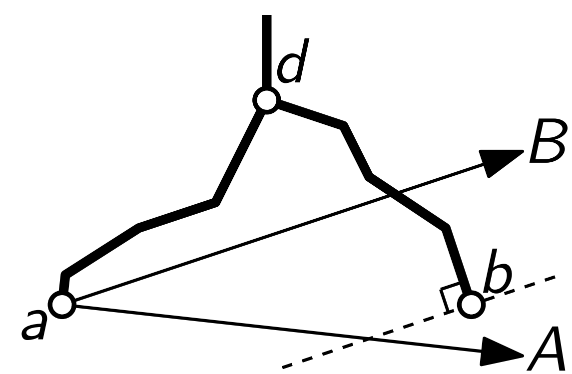
W.l.o.g. assume a lies bottom-left

Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

Case 3: else

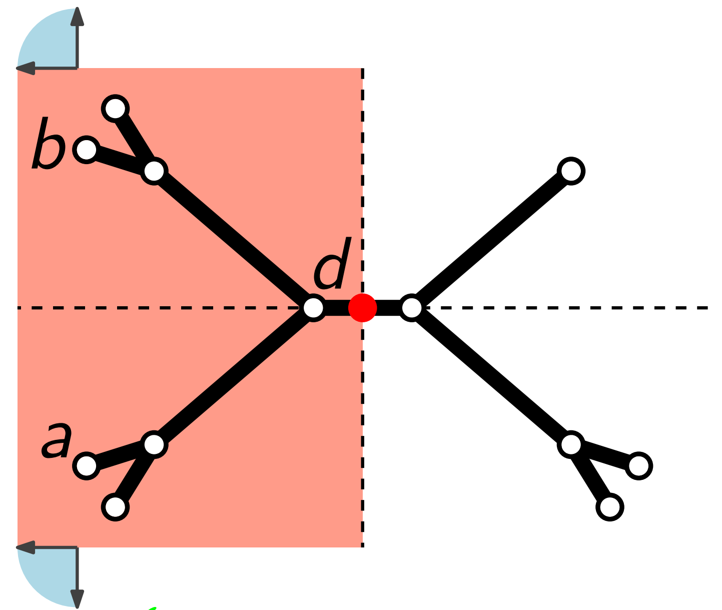
a-b-path monotone to A



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?



W.l.o.g. assume a lies bottom-left

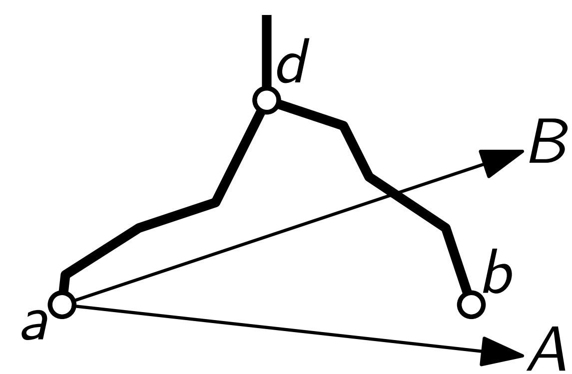
Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

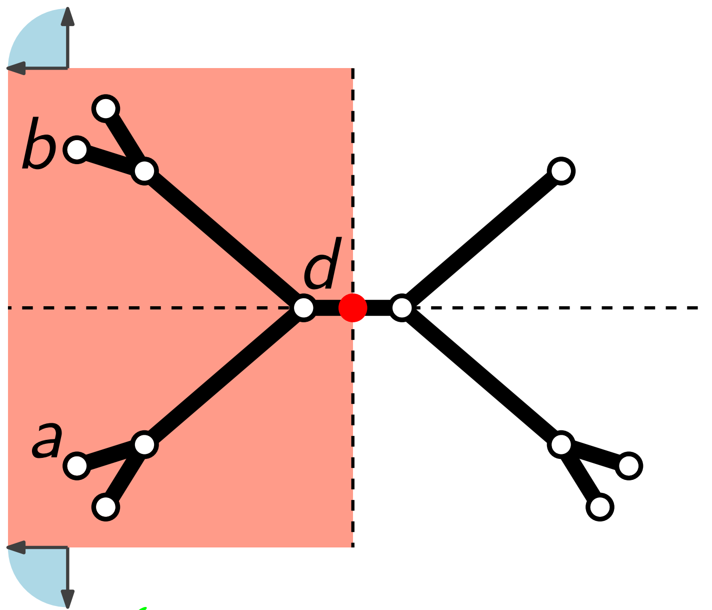
Case 3: else

a-b-path monotone to A

a-b-path monotone to B



Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume a lies bottom-left

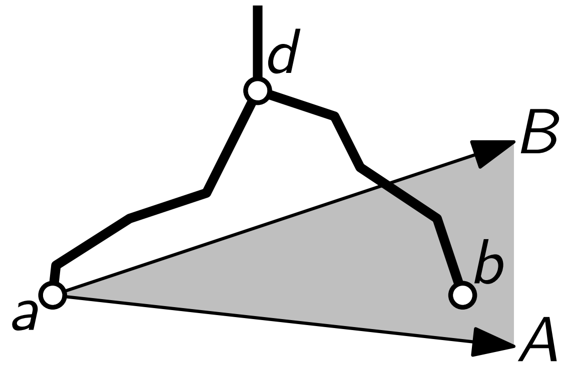
Case 1: a and b on common root-leaf path ✓

Case 2: a and b in opposite sectors ✓

Case 3: else

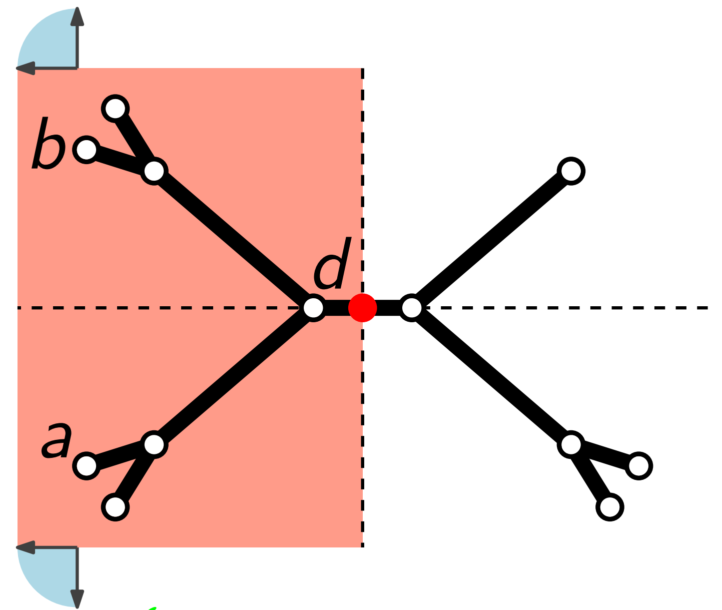
a-b-path monotone to A

a-b-path monotone to B



Proper Binary Trees

Proper Binary Trees: No degree-2 vertex



- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume *a* lies bottom-left

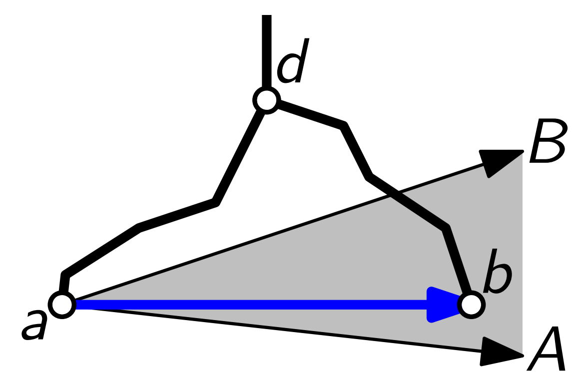
Case 1: *a* and *b* on common root-leaf path ✓

Case 2: *a* and *b* in opposite sectors ✓

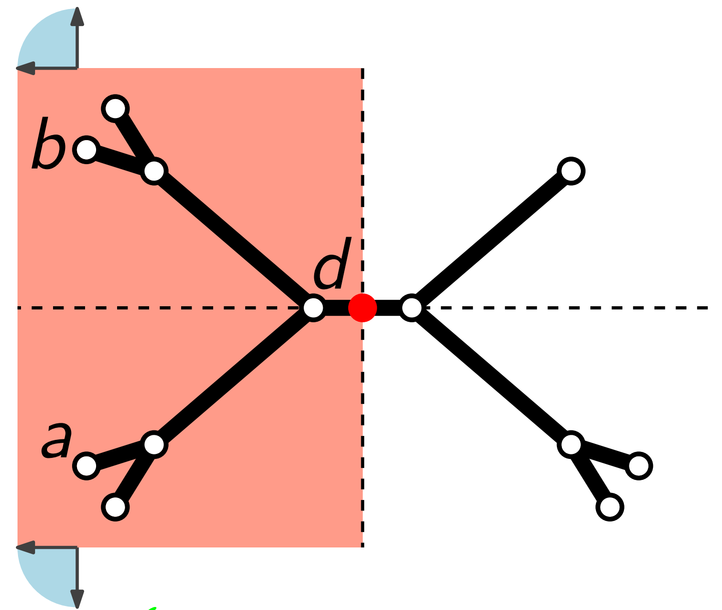
Case 3: else

a-*b*-path monotone to *A*

a-*b*-path monotone to *B*



Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume *a* lies bottom-left

Case 1: *a* and *b* on common root-leaf path ✓

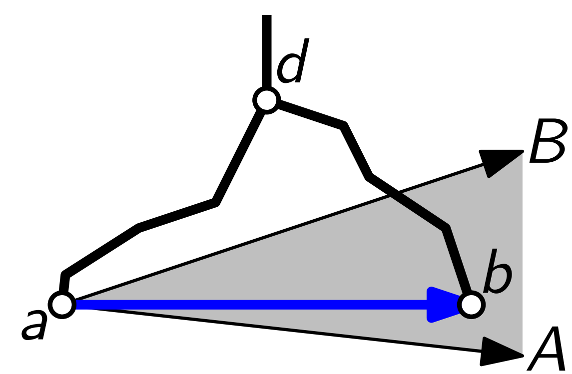
Case 2: *a* and *b* in opposite sectors ✓

Case 3: else

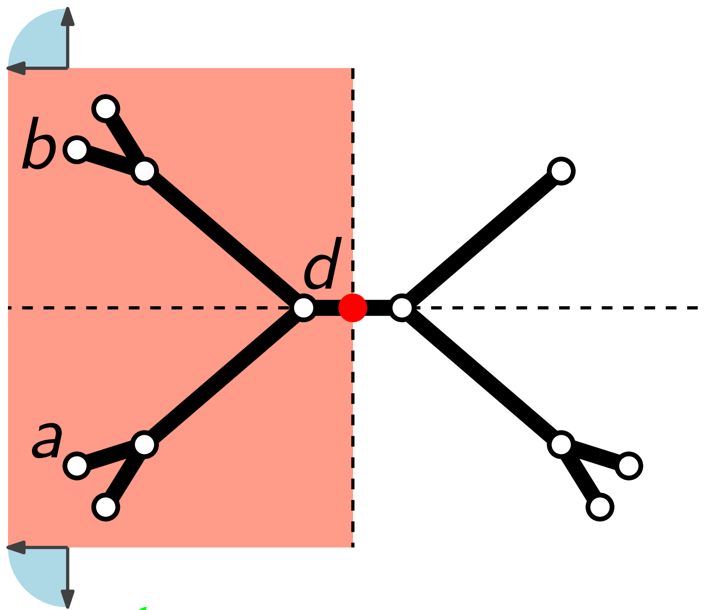
a-*b*-path monotone to *A*

a-*b*-path monotone to *B*

a-*b*-path *strongly* monotone



Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

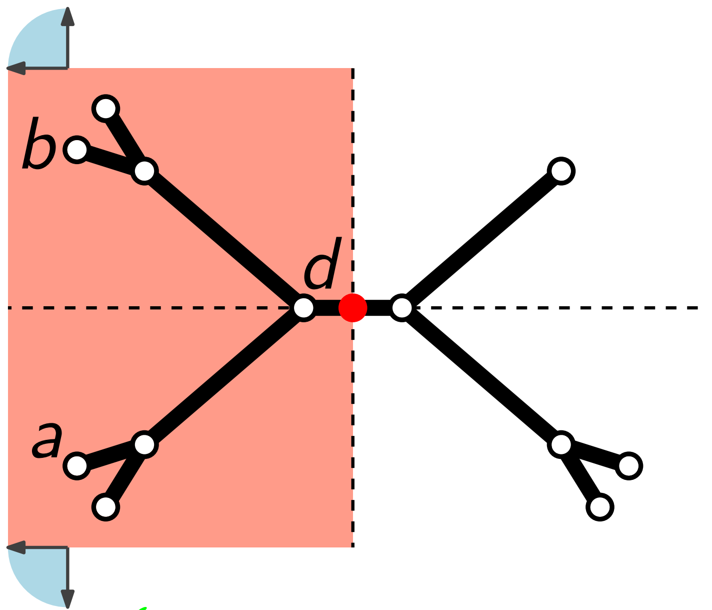
W.l.o.g. assume *a* lies bottom-left

Case 1: *a* and *b* on common root-leaf path ✓

Case 2: *a* and *b* in opposite sectors ✓

Case 3: else ✓

Proper Binary Trees



Proper Binary Trees: No degree-2 vertex

- All angles $< \pi \Rightarrow$ strictly convex ✓
- Strongly Monotone?

W.l.o.g. assume *a* lies bottom-left

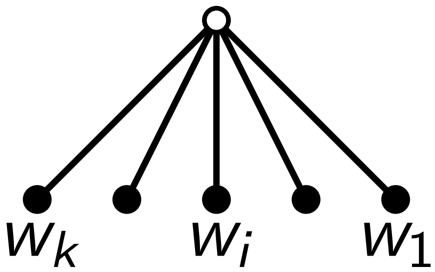
Case 1: *a* and *b* on common root-leaf path ✓

Case 2: *a* and *b* in opposite sectors ✓

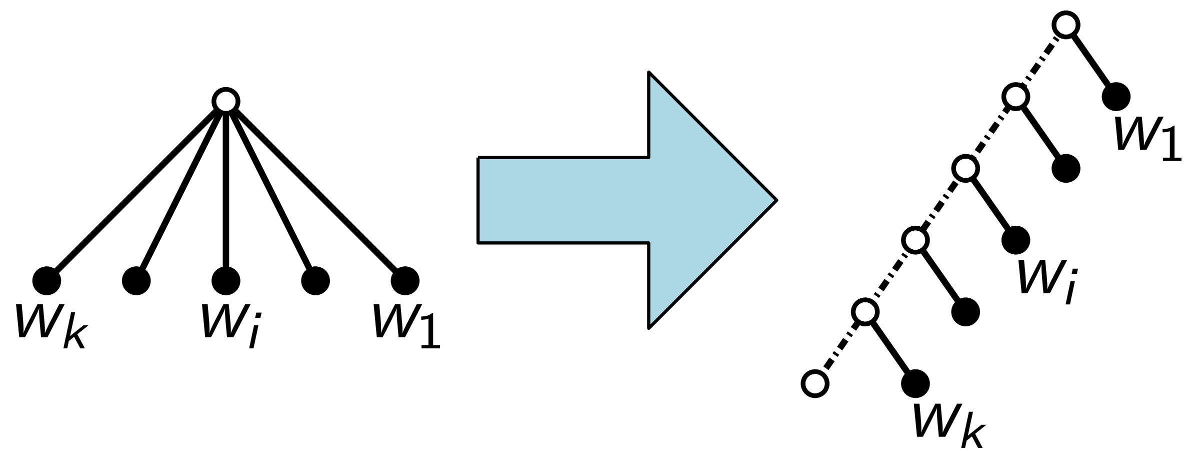
Case 3: else ✓

Theorem.
Any proper binary tree has a strongly monotone and strictly convex drawing.

General Trees

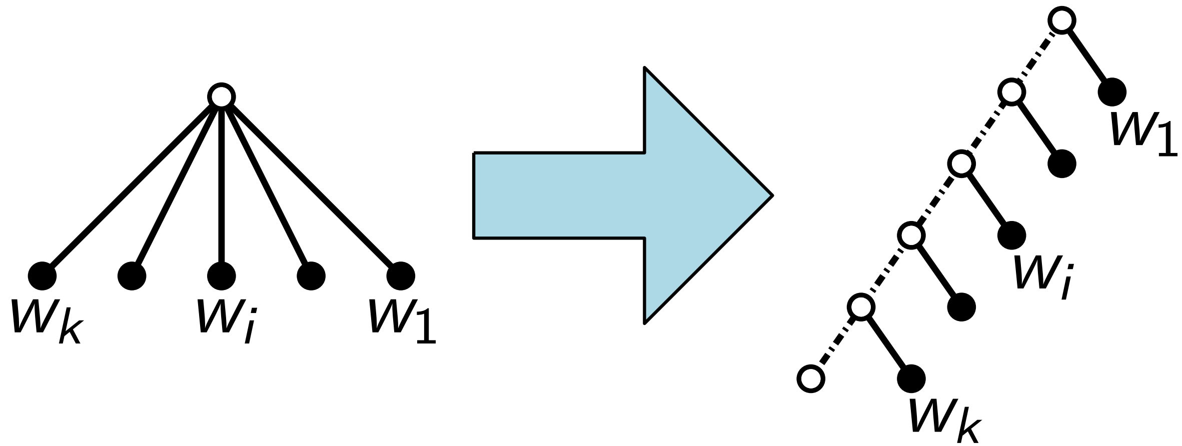


General Trees

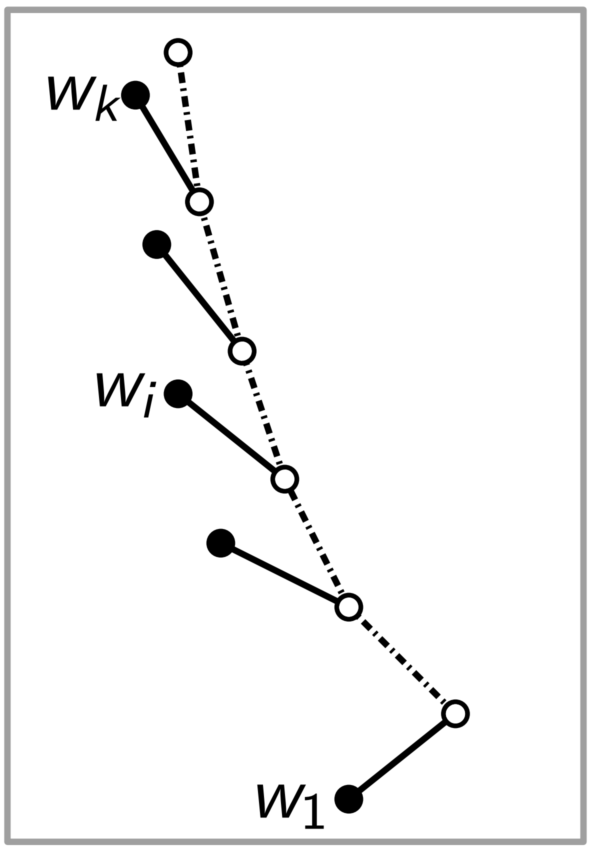


1. Substitute high-degree vertices by paths

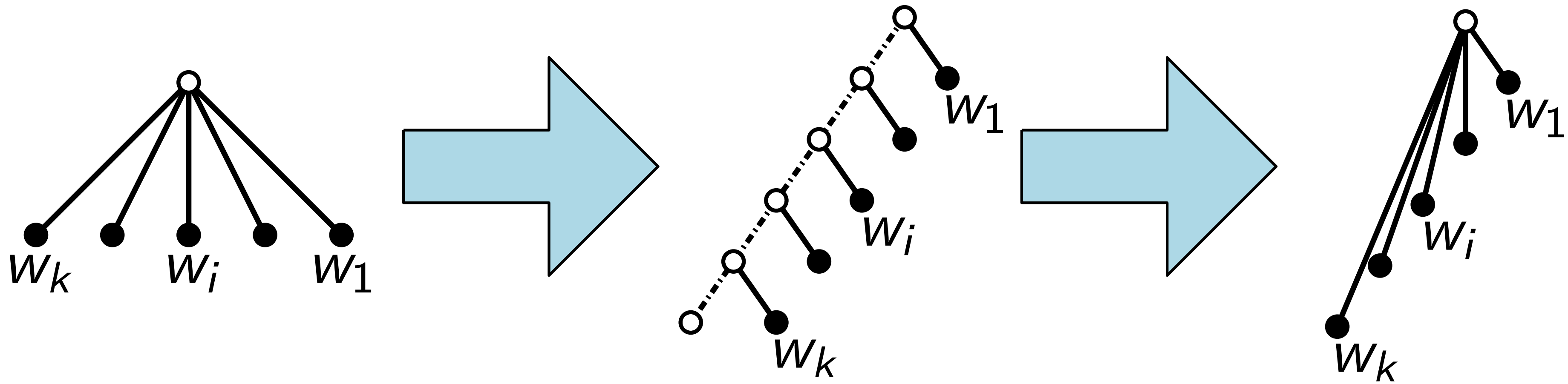
General Trees



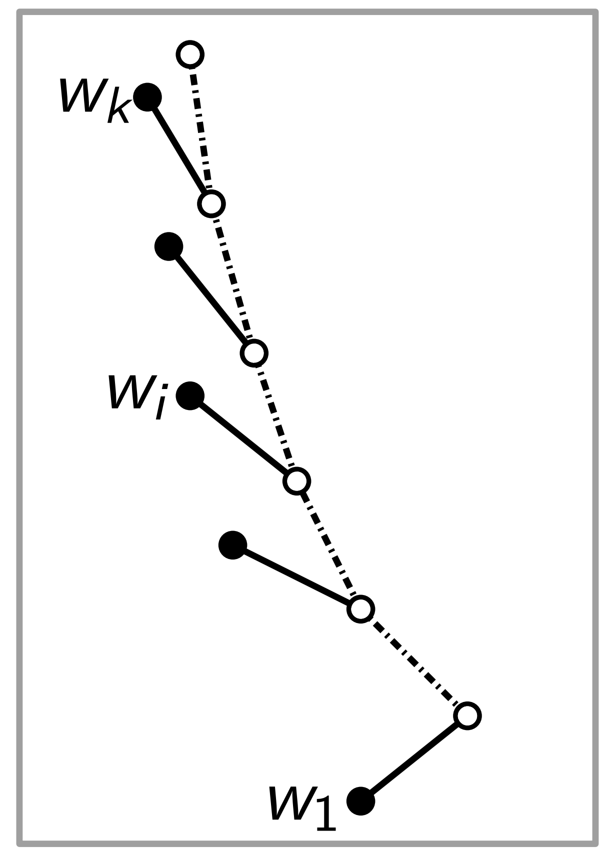
1. Substitute high-degree vertices by paths
2. Draw Proper Binary Tree



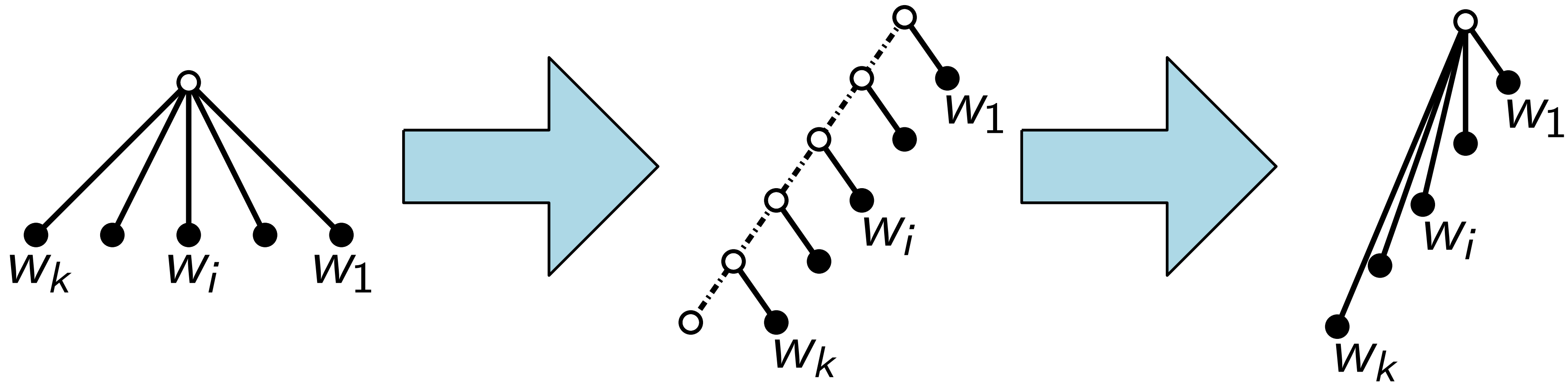
General Trees



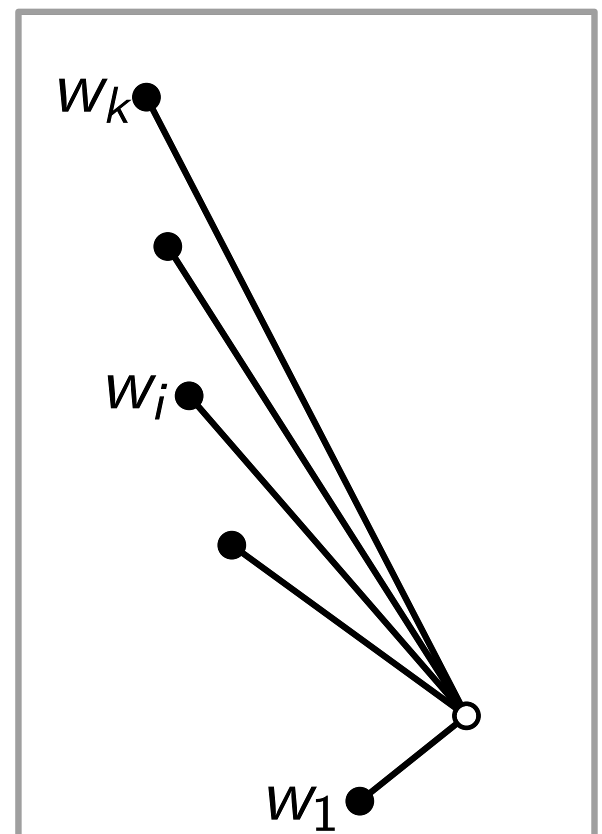
1. Substitute high-degree vertices by paths
2. Draw Proper Binary Tree
3. Shortcut edges



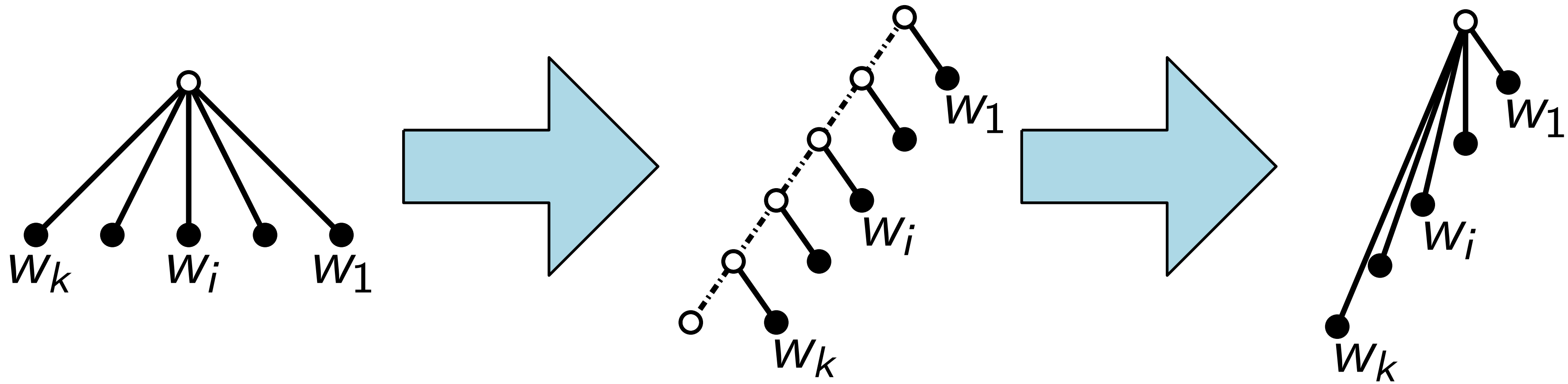
General Trees



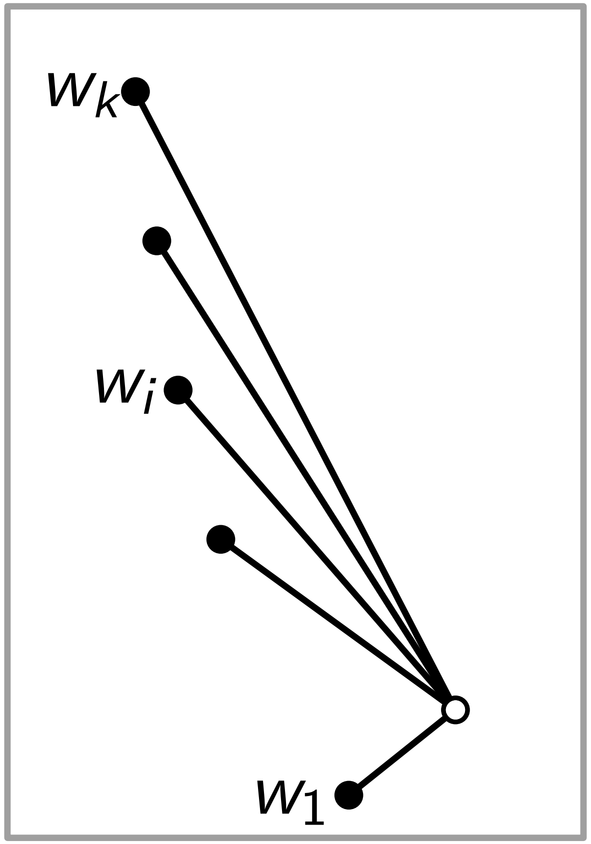
1. Substitute high-degree vertices by paths
2. Draw Proper Binary Tree
3. Shortcut edges



General Trees



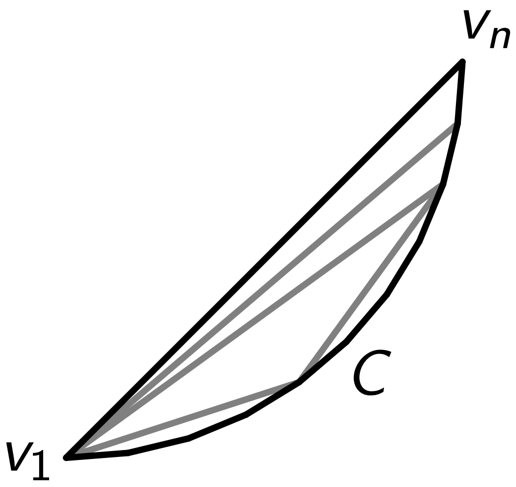
1. Substitute high-degree vertices by paths
2. Draw Proper Binary Tree
3. Shortcut edges



Theorem.
Any tree has a strongly monotone drawing.

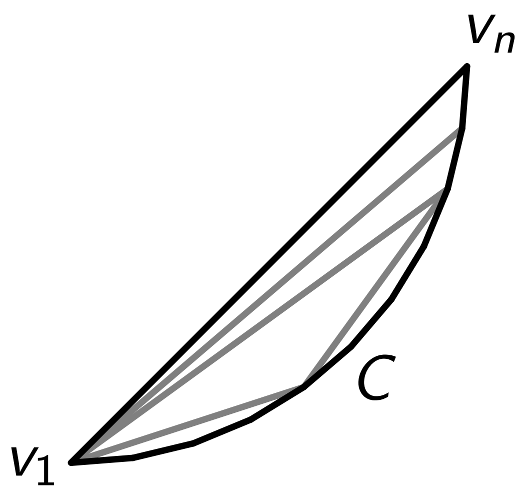
Planar Graphs

Theorem.
Any biconnected outerplanar graph has a strongly monotone and strictly convex drawing.

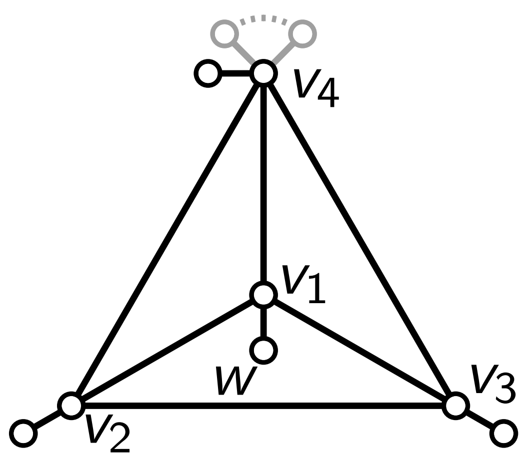


Planar Graphs

Theorem.
Any biconnected outerplanar graph has a strongly monotone and strictly convex drawing.



Theorem.
There is an infinite family of connected planar graphs that do not have a strongly monotone drawing in any combinatorial embedding.



Open Problems

- Does any tree have a strongly monotone drawing on a grid of polynomial size?

Open Problems

- Does any tree have a strongly monotone drawing on a grid of polynomial size?
- Is there a triconnected (or biconnected) planar graph that does not have any strongly monotone drawing?

Open Problems

- Does any tree have a strongly monotone drawing on a grid of polynomial size?
- Is there a triconnected (or biconnected) planar graph that does not have any strongly monotone drawing?
  If yes, can this be tested efficiently?

Open Problems

- Does any tree have a strongly monotone drawing on a grid of polynomial size?
- Is there a triconnected (or biconnected) planar graph that does not have any strongly monotone drawing?
  If yes, can this be tested efficiently?
- Are our drawings for general trees also convex?

Open Problems

- Does any tree have a strongly monotone drawing on a grid of polynomial size?
- Is there a triconnected (or biconnected) planar graph that does not have any strongly monotone drawing?
  If yes, can this be tested efficiently?
- Are our drawings for general trees also convex?
  If yes, then all Halin graphs would automatically have convex and strictly monotone drawings, too.