Two-Sided Boundary Labeling with Adjacent Sides

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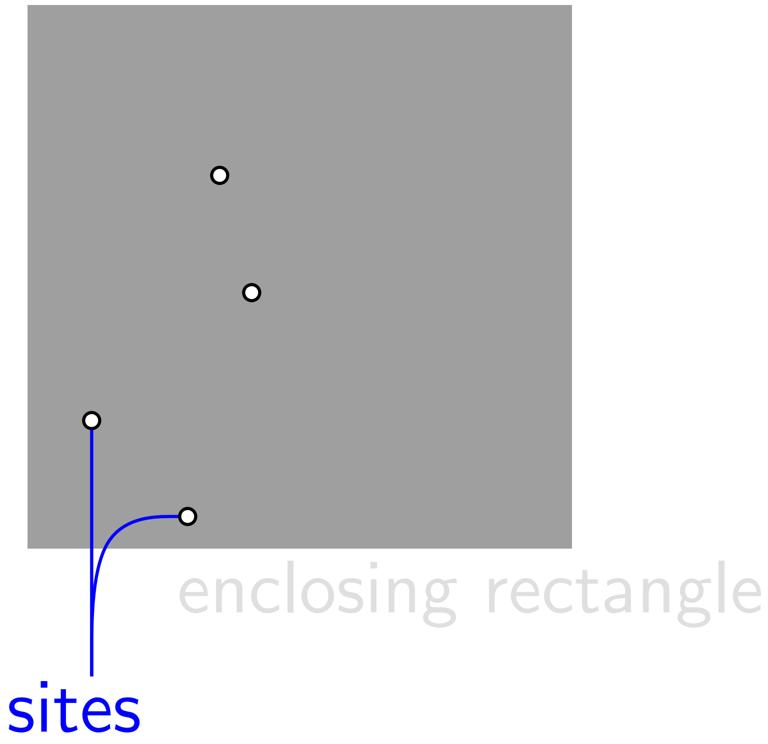
Joint work with
Benjamin Niedermann, Ignaz Rutter, Marcus Schaefer,
André Schulz & Alexander Wolff

Problem Statement

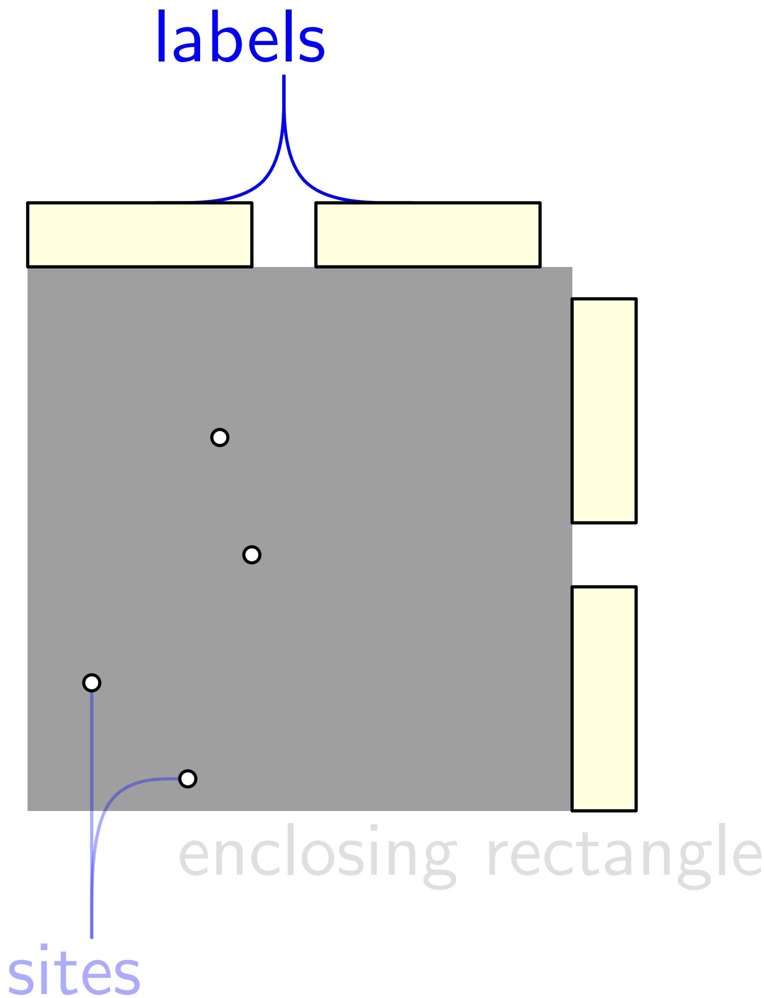
enclosing rectangle

Problem Statement



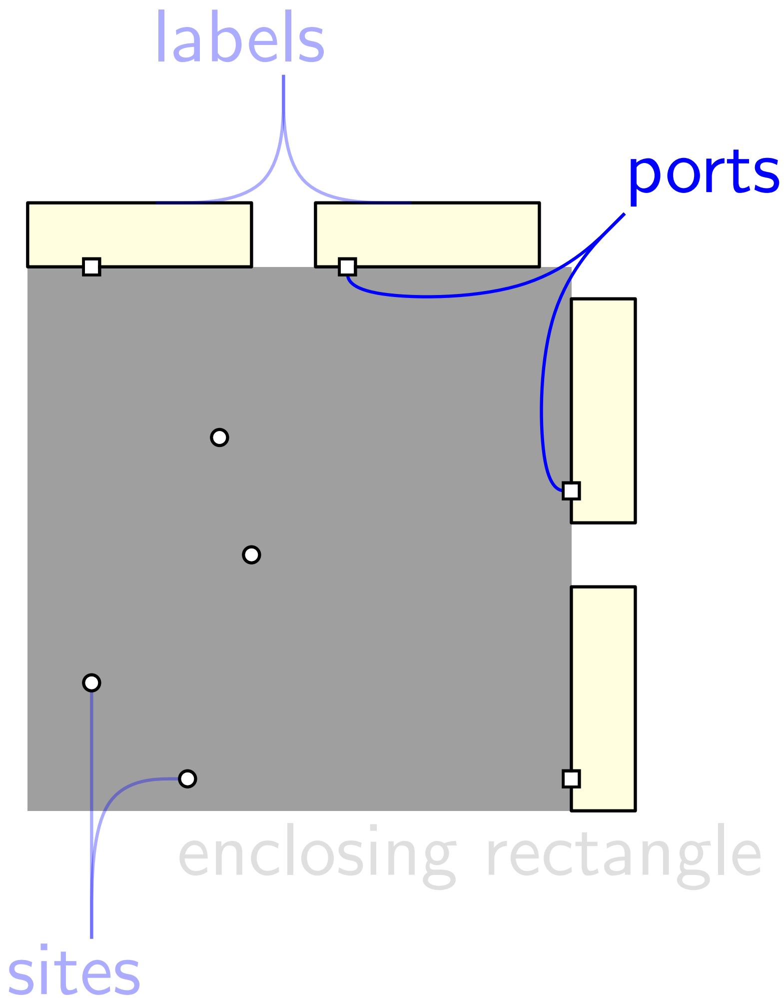
- *n sites* in general position

Problem Statement



- *n sites* in general position
- *labels* on the boundary

Problem Statement



- *n sites* in general position
- *labels* on the boundary
- *ports* are fixed or sliding

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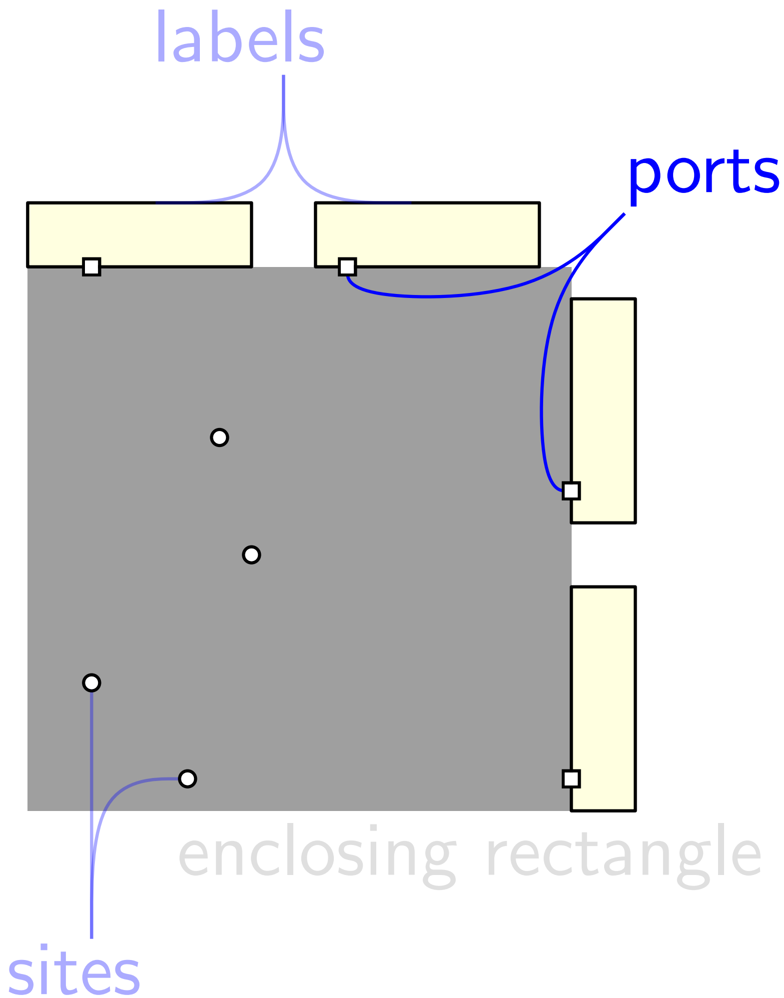

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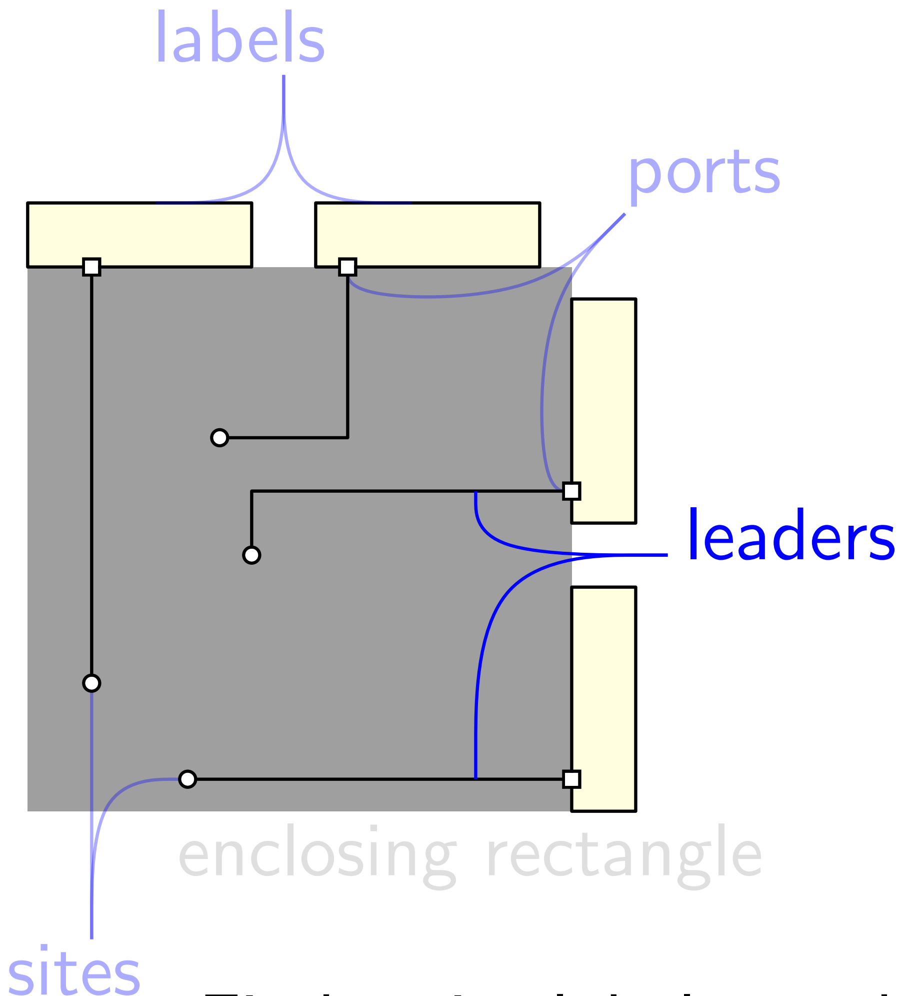
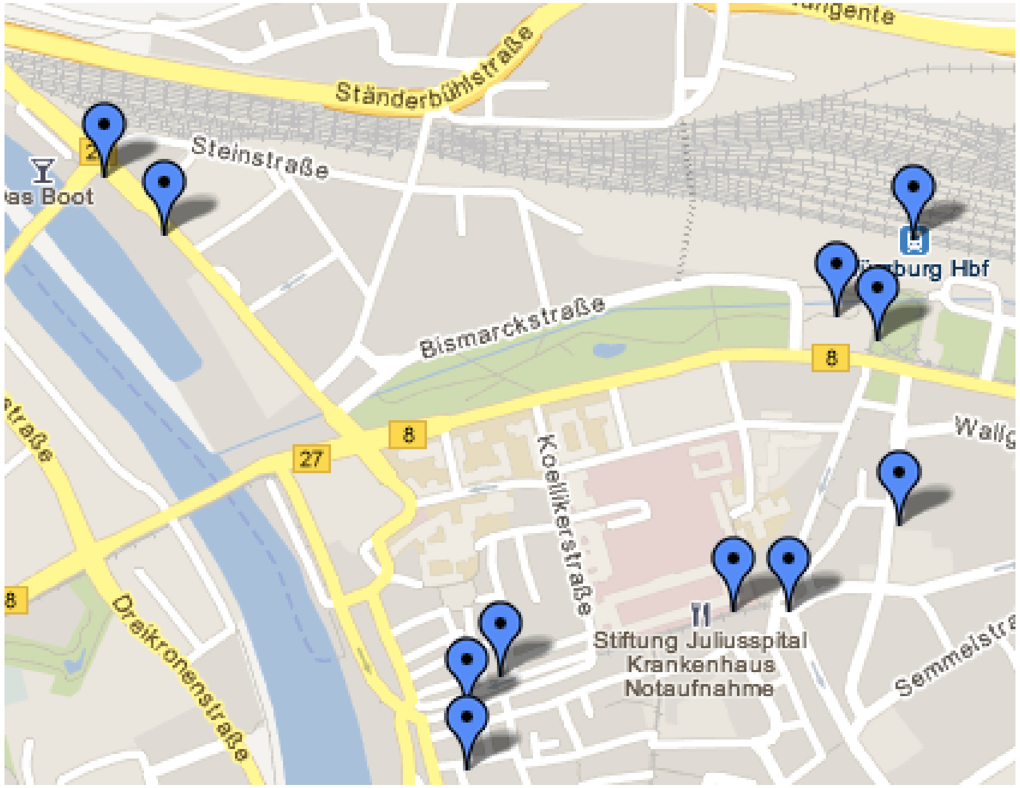


- *n sites* in general position
- *labels* on the boundary
- *ports* are fixed or sliding

Find a site-label matching with *crossing-free* leaders.

Why Boundary Labeling?



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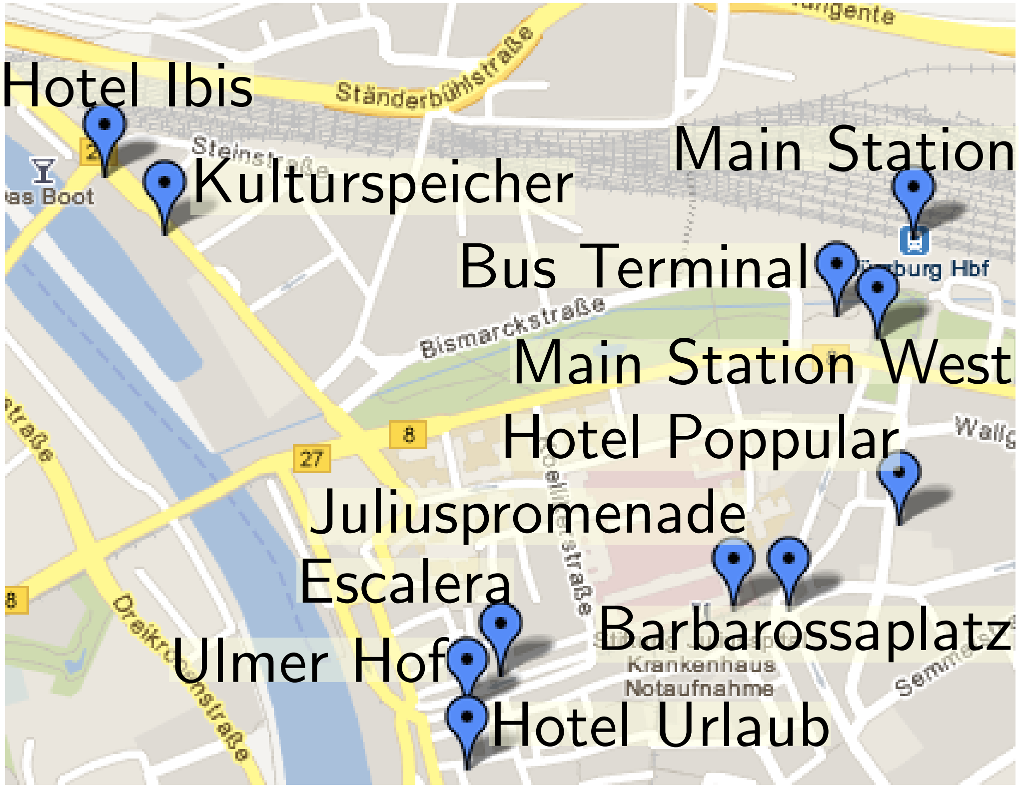

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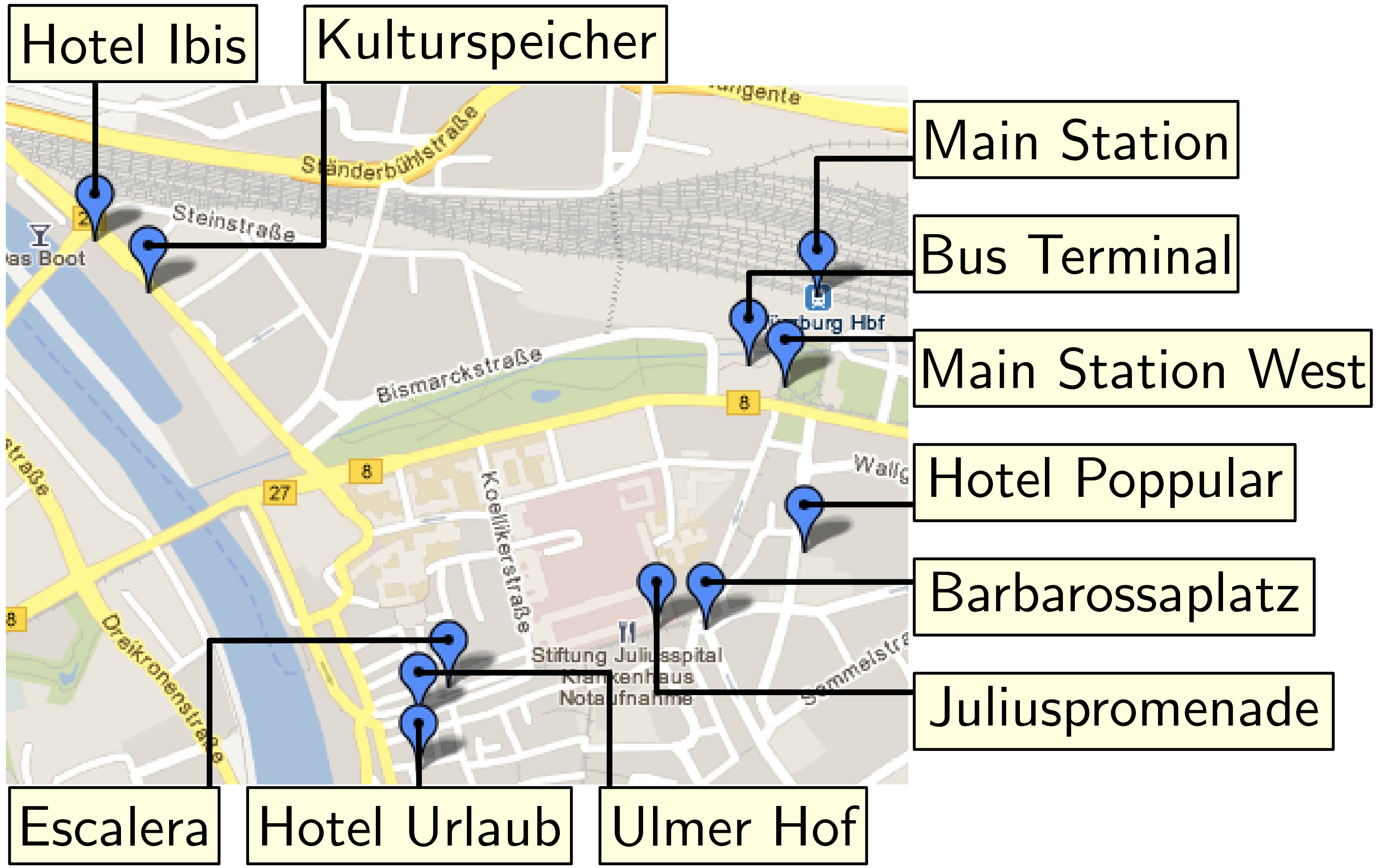

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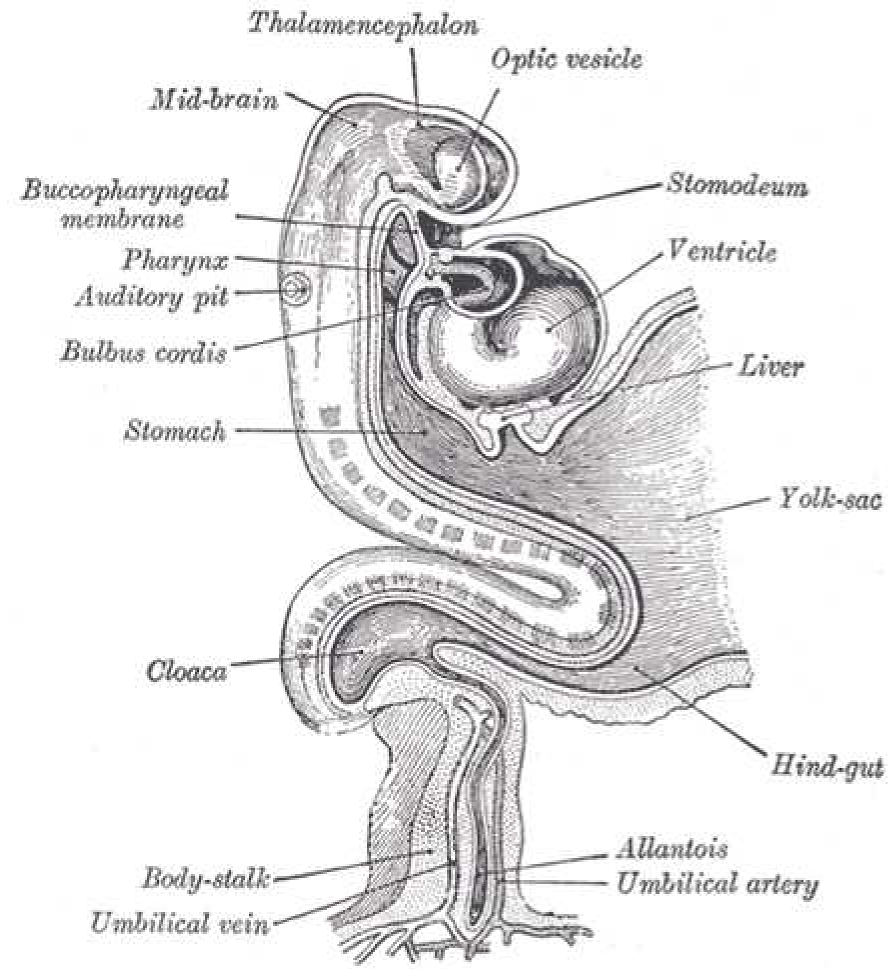

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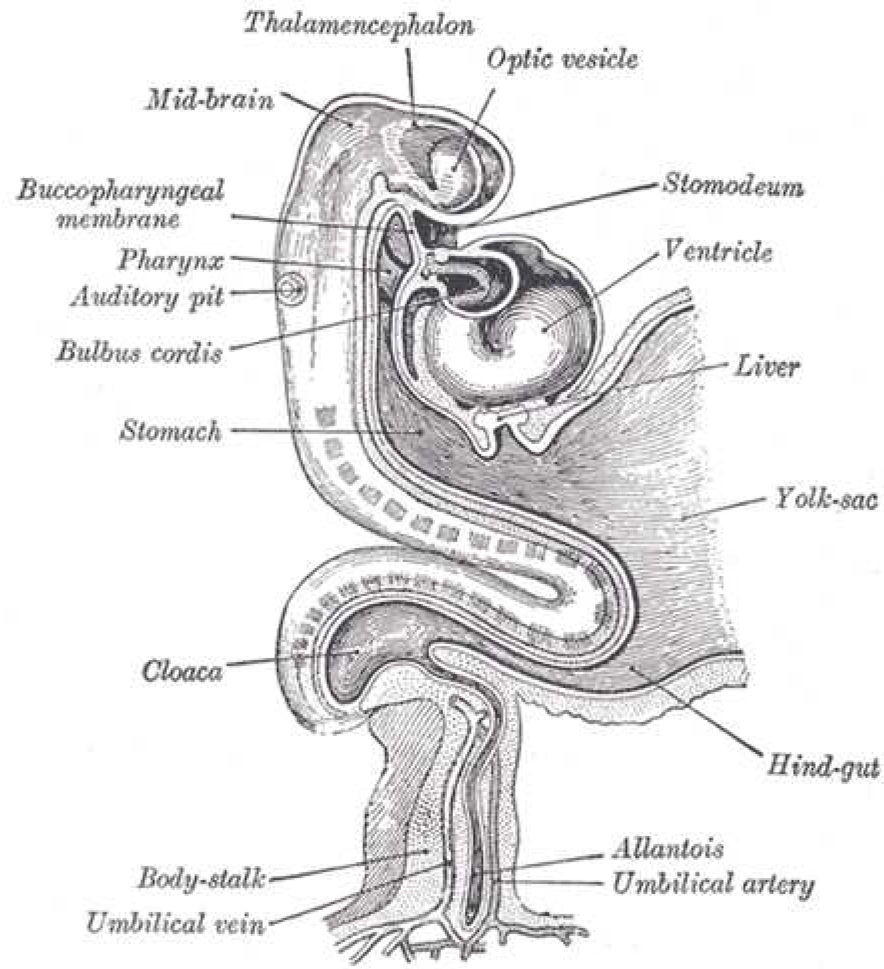
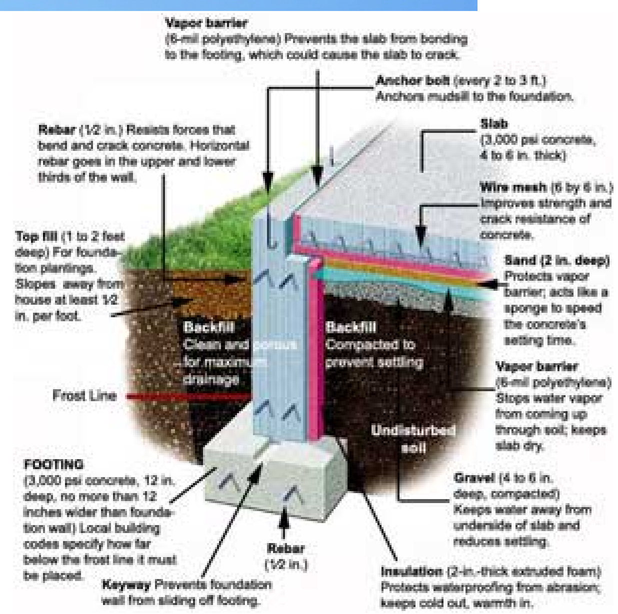
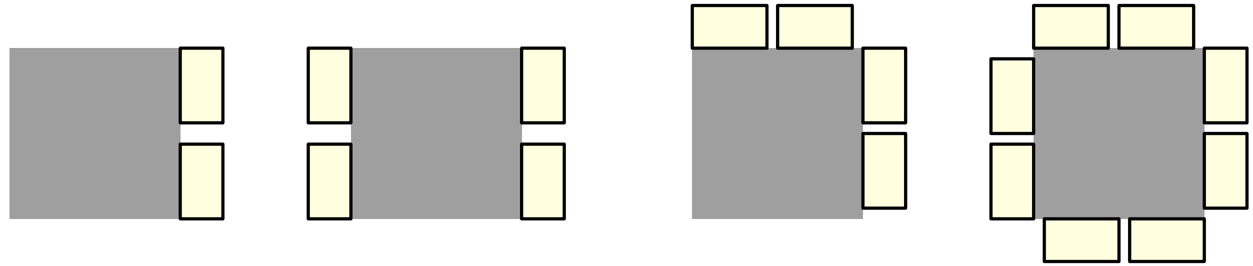




Previous Work



style \ sides	1	2 (opp.)	2 (adj.)	4

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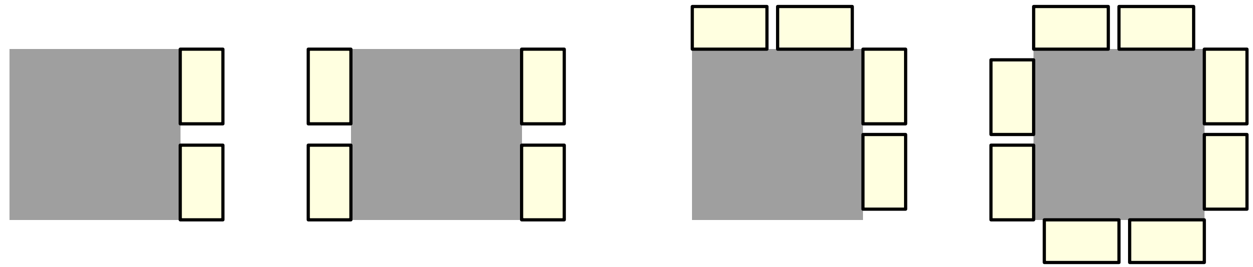

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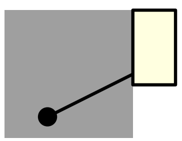 s	$O(n^{2+\epsilon})$ [1]			

[1] Bekos et al. CGTA'07

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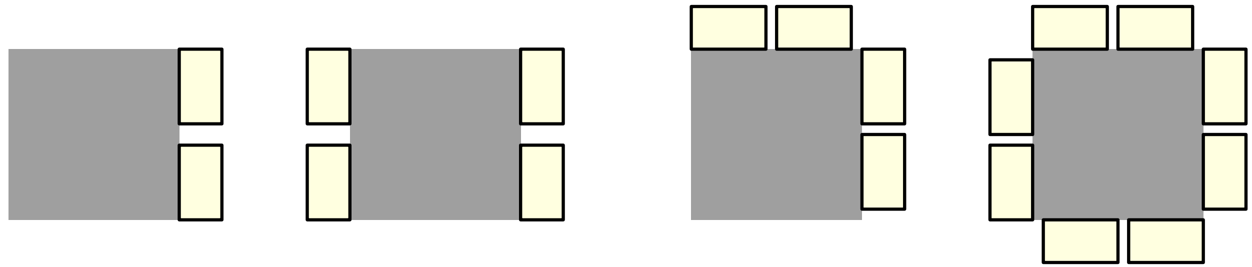

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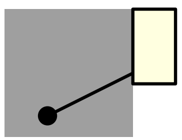
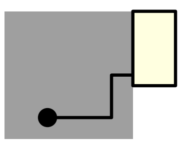
 *s*	$O(n^{2+\epsilon})$ [1]			
 *opo*	$O(n \log n)$ [1]	$O(n^2)$ [1]	$O(n^2 \log^3 n)$ [1]	

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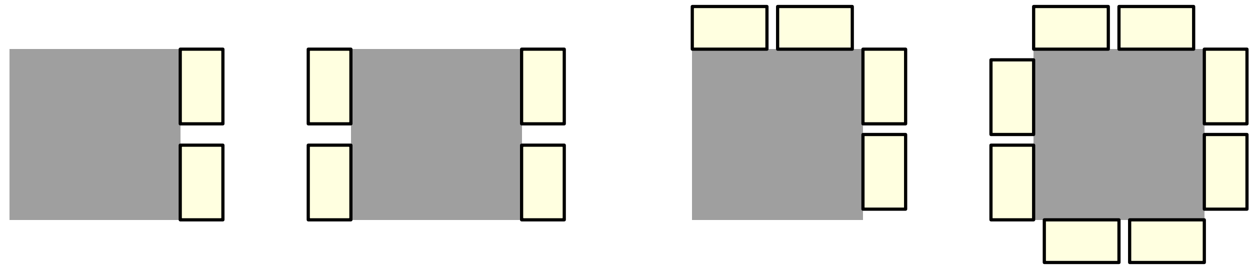

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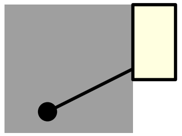
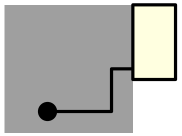
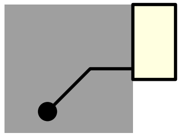
	*s*	$O(n^{2+\epsilon})$ [1]			
	*opo*	$O(n \log n)$ [1]	$O(n^2)$ [1]	$O(n^2 \log^3 n)$ [1]	
	*do/pd*	$O(n^2)$ [2]	$O(n^3)$ [3]		

[1] Bekos et al. CGTA'07 [2] Benkert et al. JGAA'09 [3] Bekos et al. Algorithmica'10

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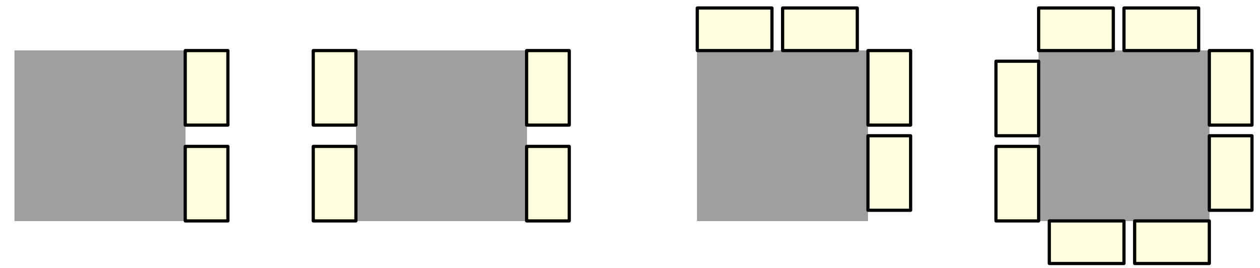

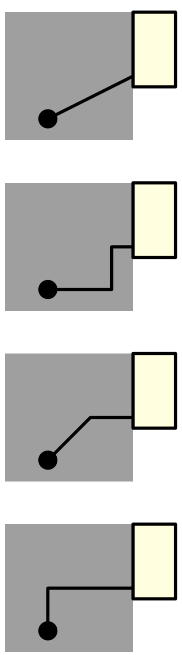

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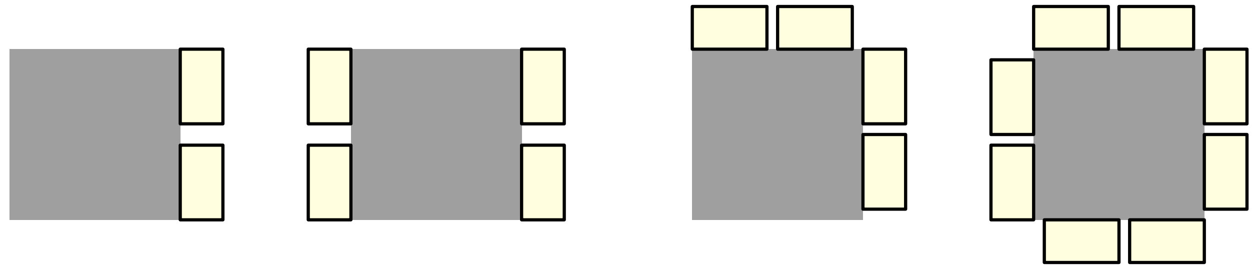

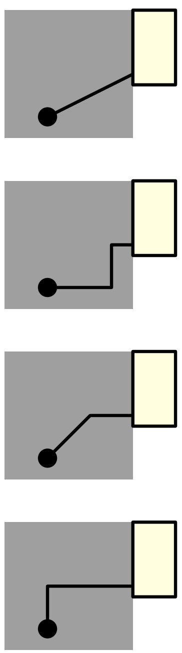

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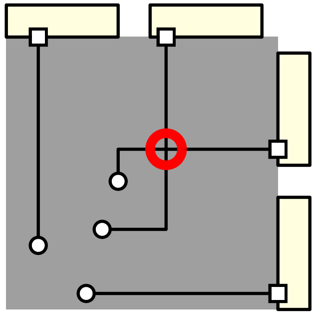

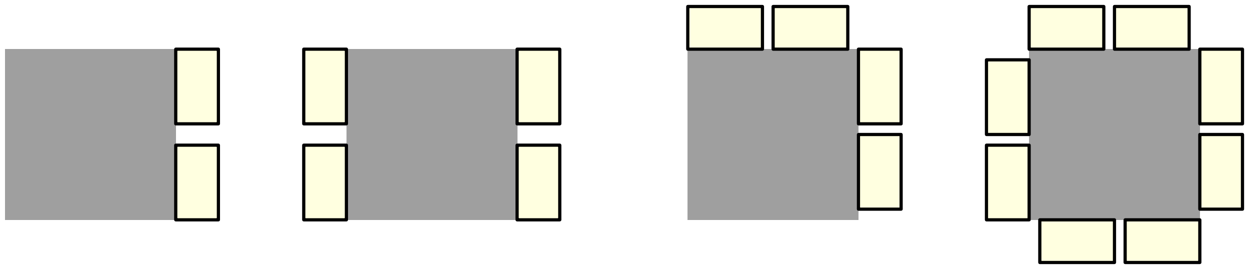

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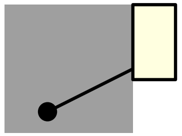
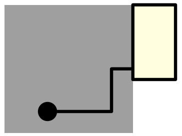
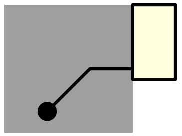
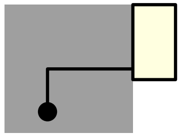
 *s*	$O(n^{2+\epsilon})$ [1]			
 *opo*	$O(n \log n)$ [1]	$O(n^2)$ [1]	$O(n^2 \log^3 n)$ [1]	
 *do/pd*	$O(n^2)$ [2]	$O(n^3)$ [3]		
 *po*	$O(n \log n)$ [2]	$O(n^2)$ [1]	?	?

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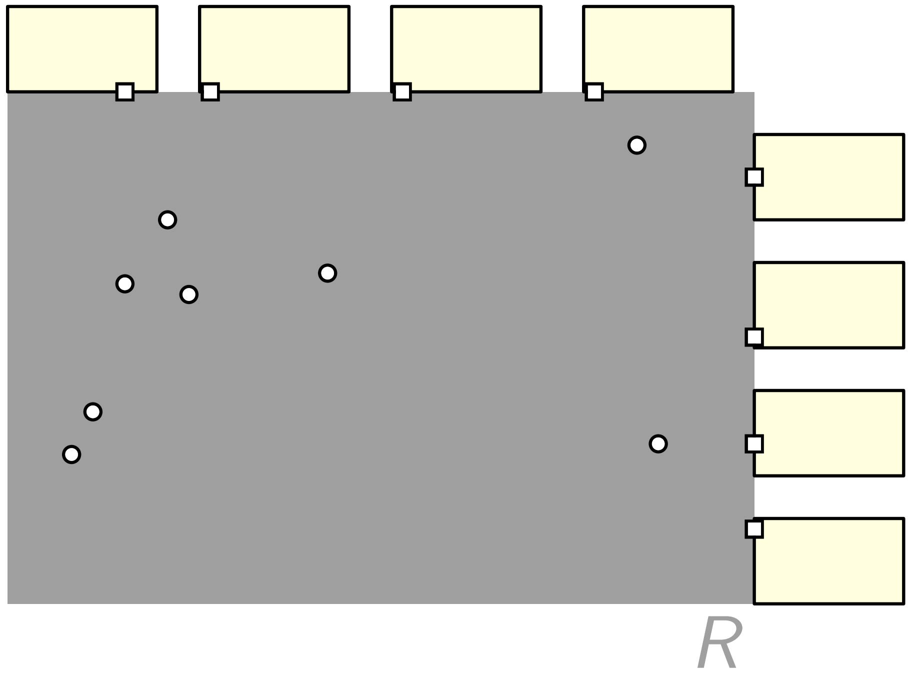
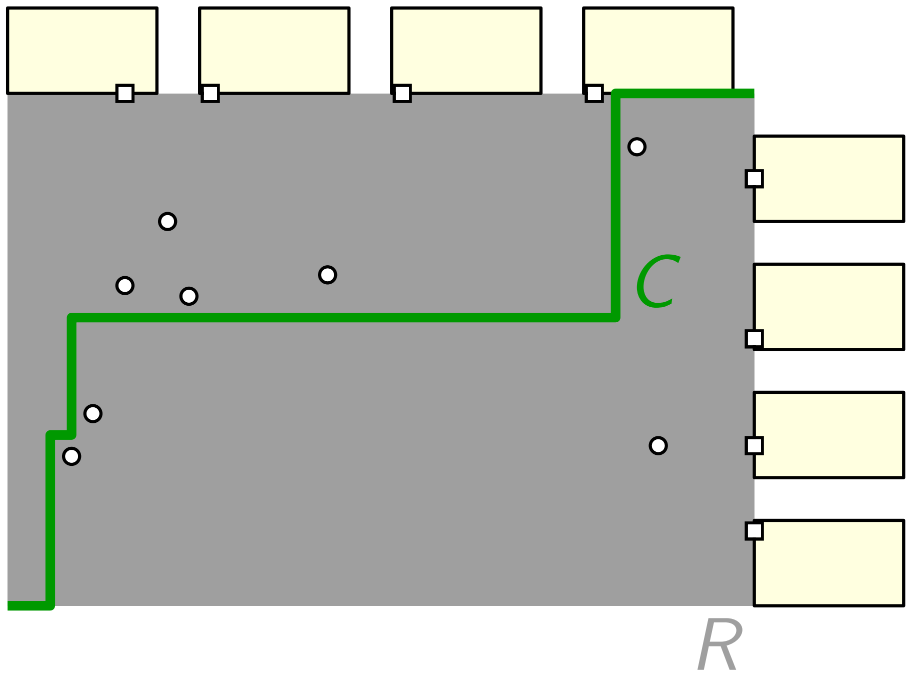
Structure of Planar Solutions



Structure of Planar Solutions

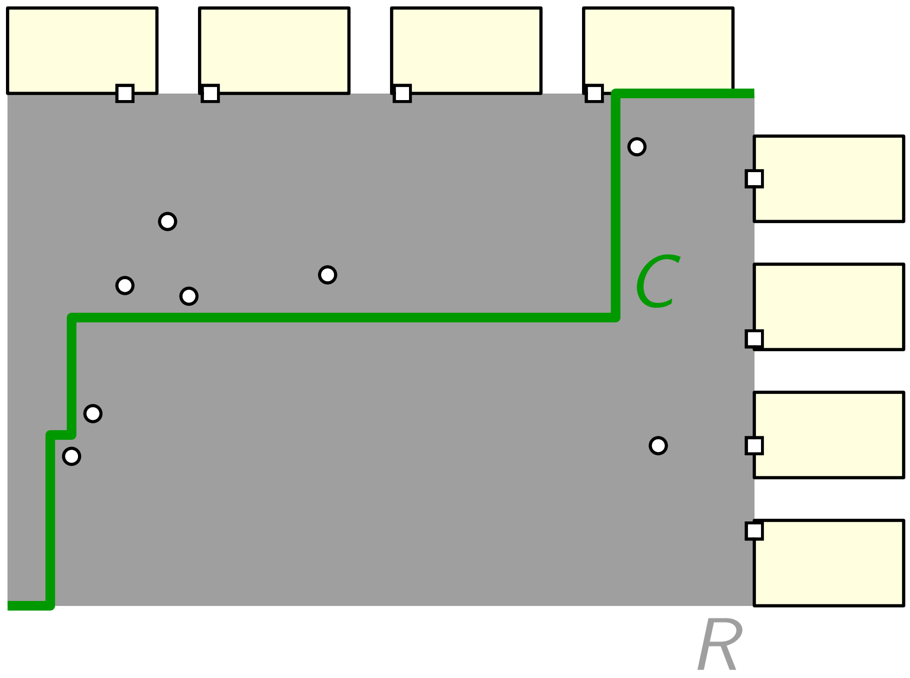
xy-separating curve C



Structure of Planar Solutions

xy-separating curve *C*

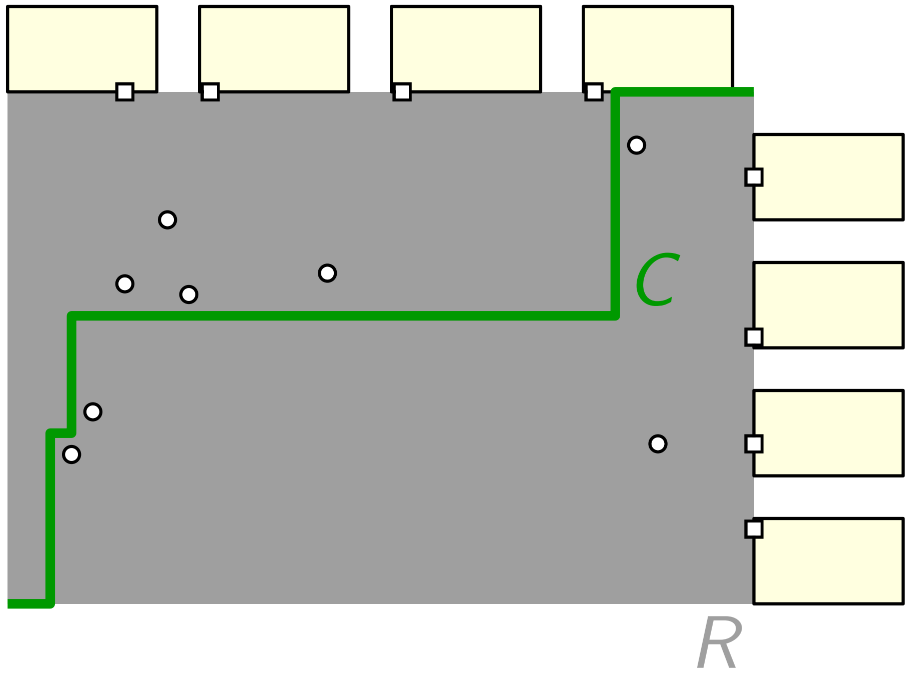
- *xy*-monotone



Structure of Planar Solutions

xy-separating curve C

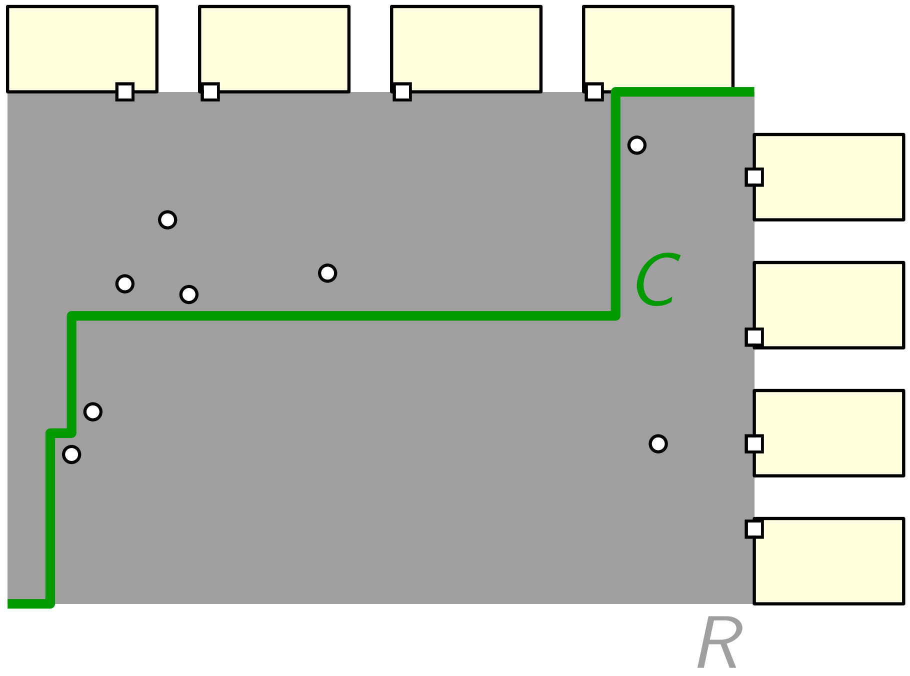
- xy-monotone
- rectilinear



Structure of Planar Solutions

xy-separating curve C

- *xy*-monotone
- rectilinear
- connects the top-right to the bottom-left corner of R



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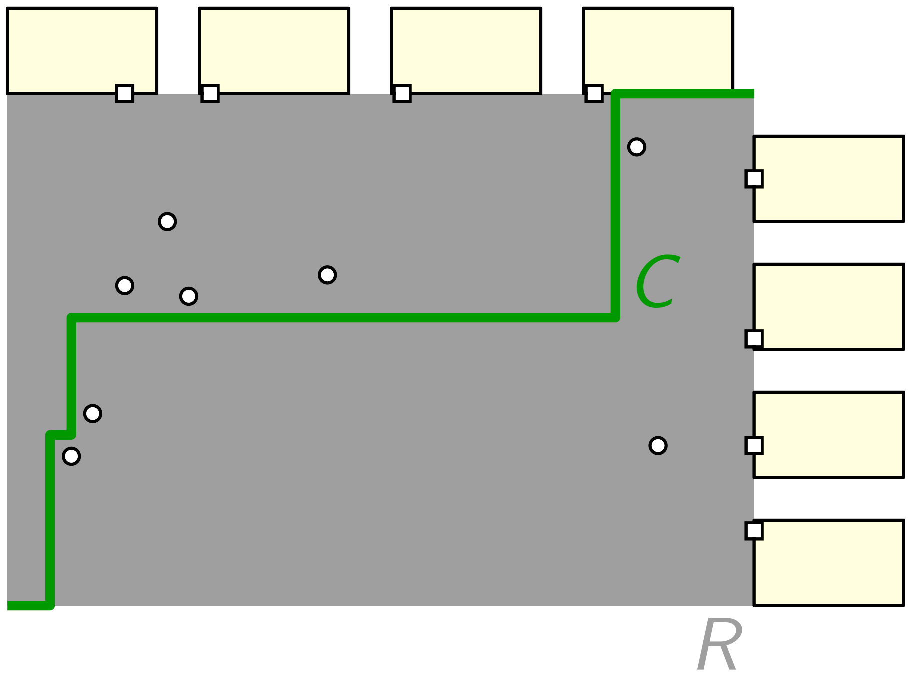
xy-separated solution



Structure of Planar Solutions

xy-separating curve *C*

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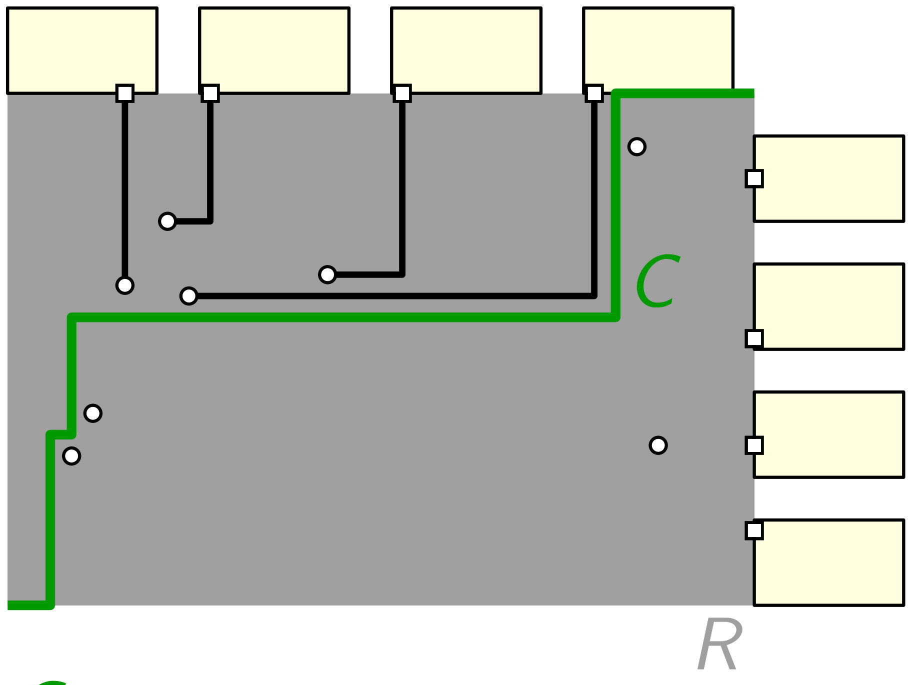
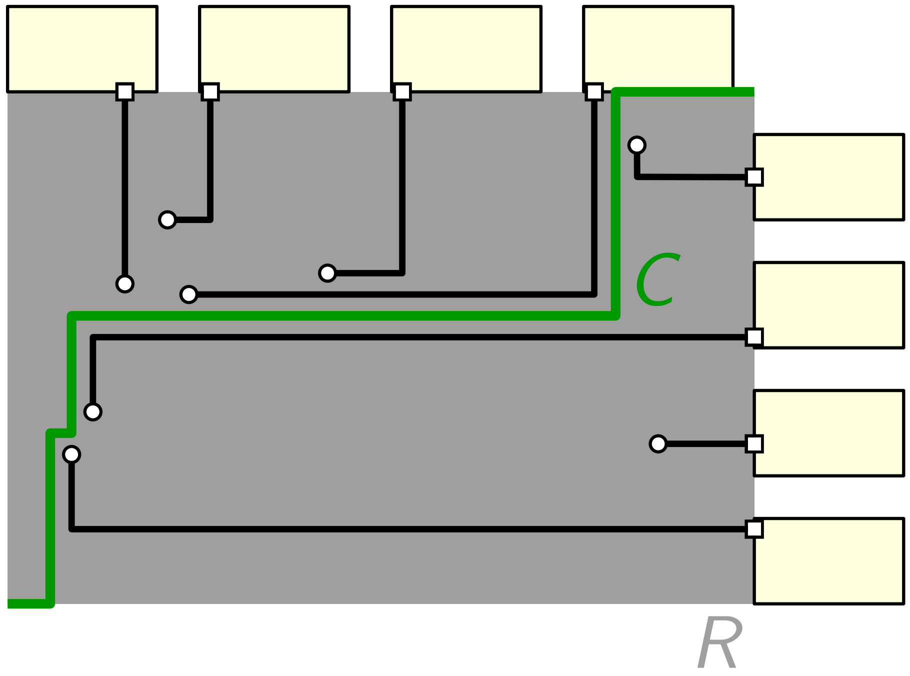


xy-separated solution

- top sites and leaders lie above *C*

Structure of Planar Solutions



xy-separating curve C

- *xy*-monotone
- rectilinear
- connects the top-right to the bottom-left corner of R

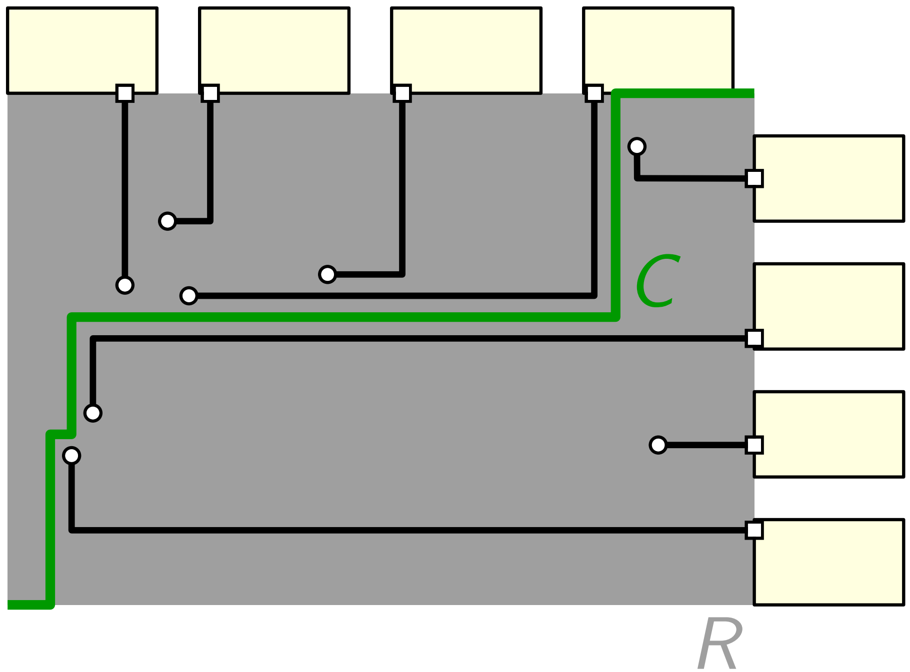
xy-separated solution

- top sites and leaders lie above C
- right sites and leaders lie below C

Structure of Planar Solutions

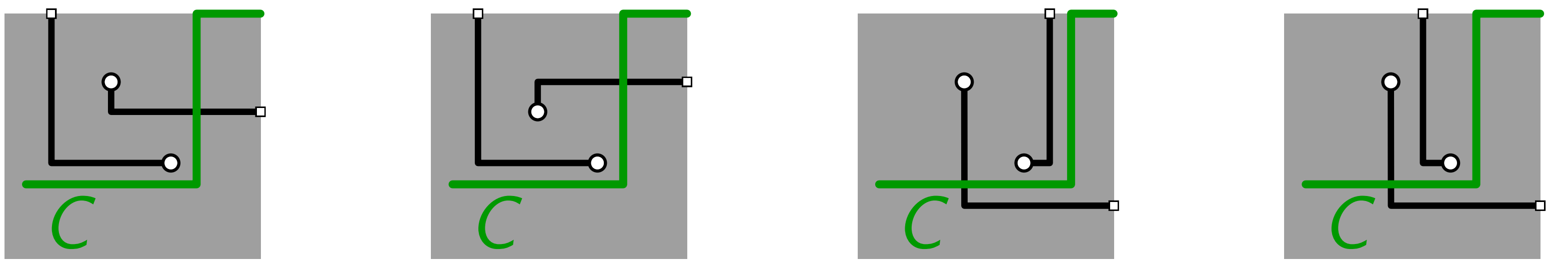
xy-separating curve *C*

- *xy*-monotone
- rectilinear
- connects the top-right to the bottom-left corner of *R*



xy-separated solution

- top sites and leaders lie above *C*
- right sites and leaders lie below *C*
- does not contain any of the following patterns



Structure of Planar Solutions

xy-separated solution

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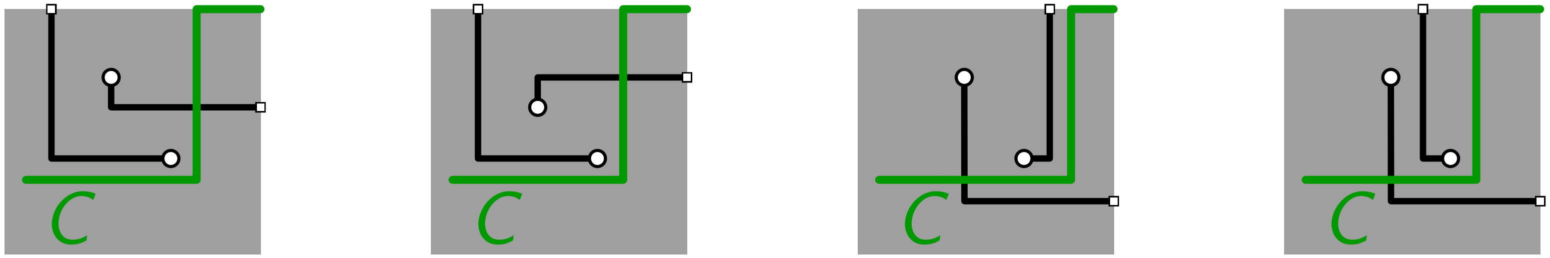

Structure of Planar Solutions

xy-separated solution

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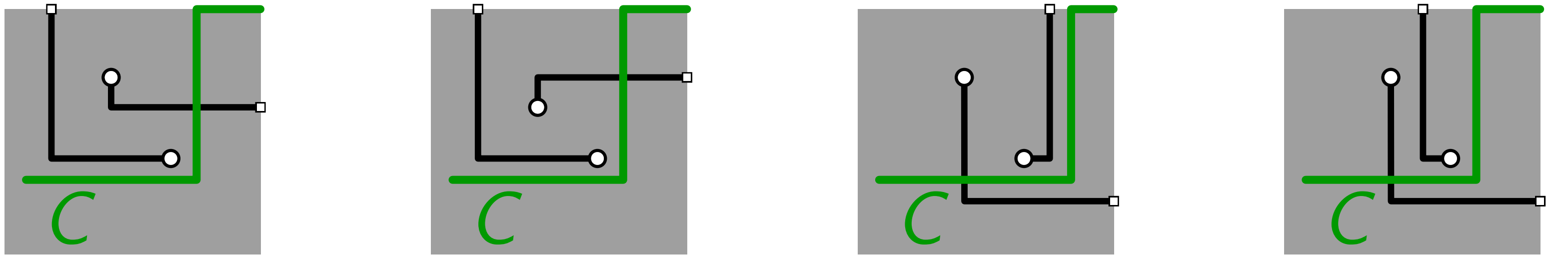

Planar solution $\Rightarrow$ xy-separated planar solution

Structure of Planar Solutions

xy-separated solution

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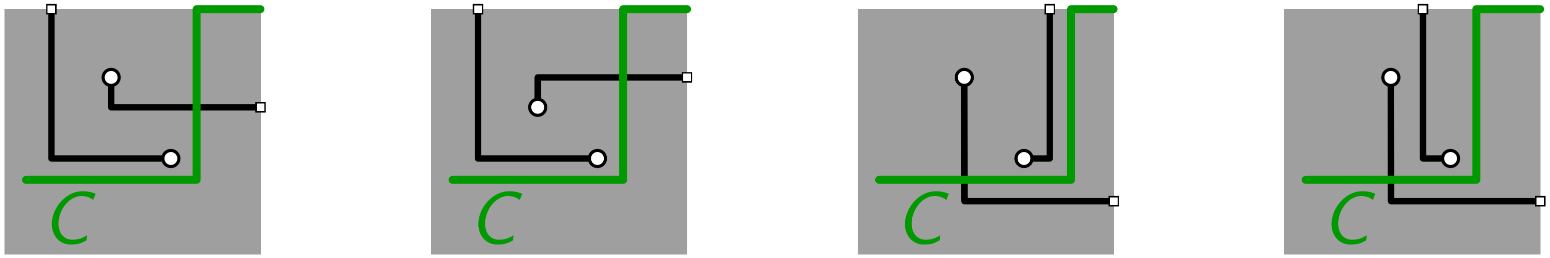
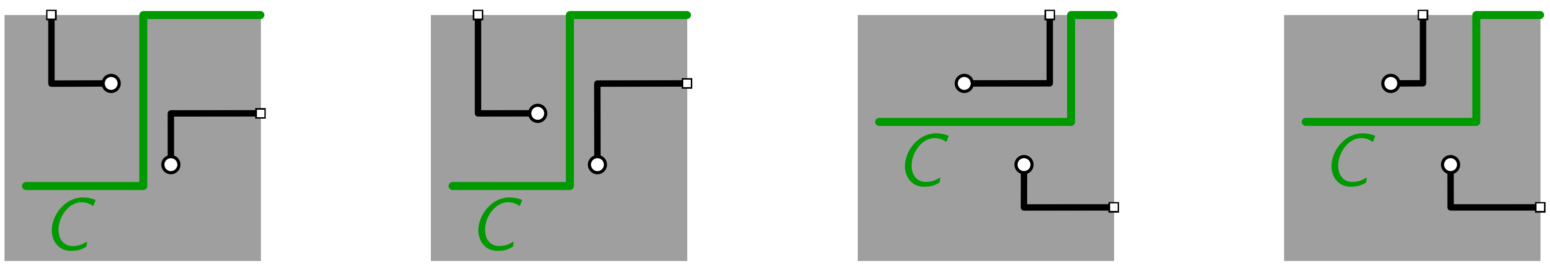


Planar solution $\Rightarrow$ *xy*-separated planar solution



Structure of Planar Solutions

xy-separated solution

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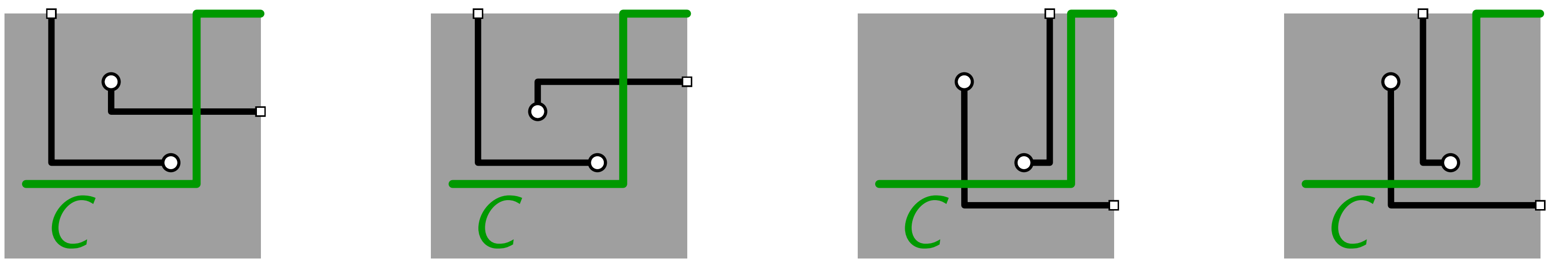
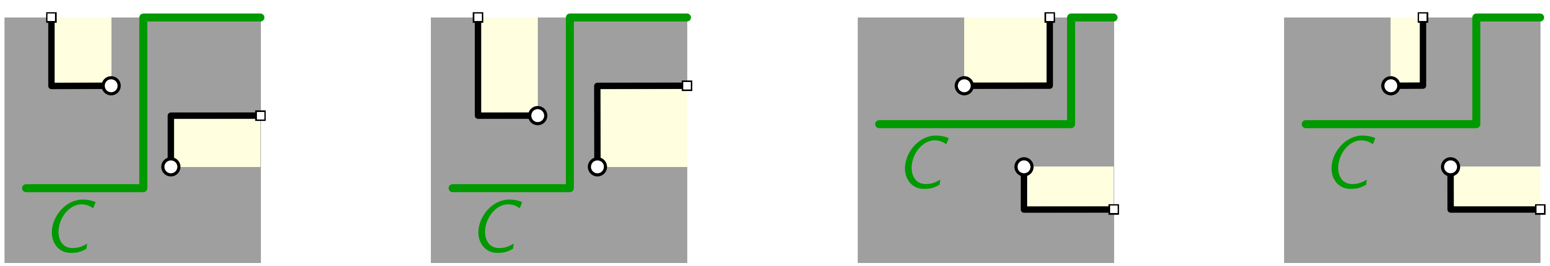


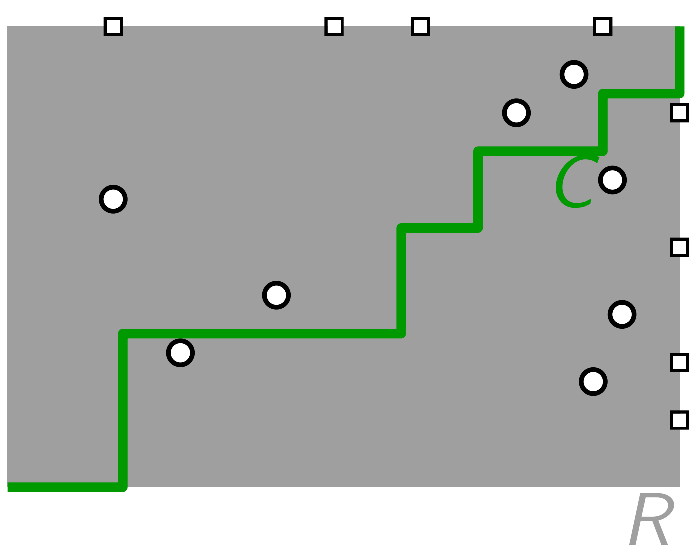
Planar solution ⇒ *xy*-separated planar solution



Eliminate local crossings

The Strip Condition

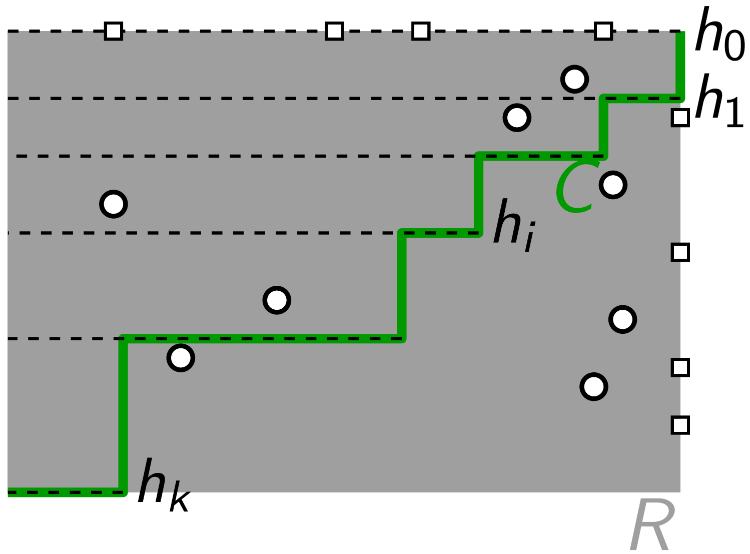
Given: xy-monotone curve C



The Strip Condition

Given: xy-monotone curve C

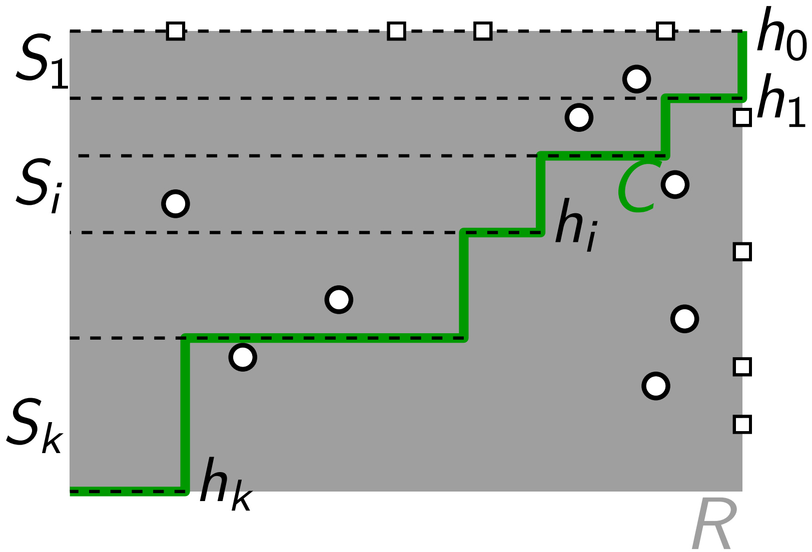
- $h_0, \ldots, h_k$: horizontal segments of C extended to the left



The Strip Condition

Given: xy-monotone curve C

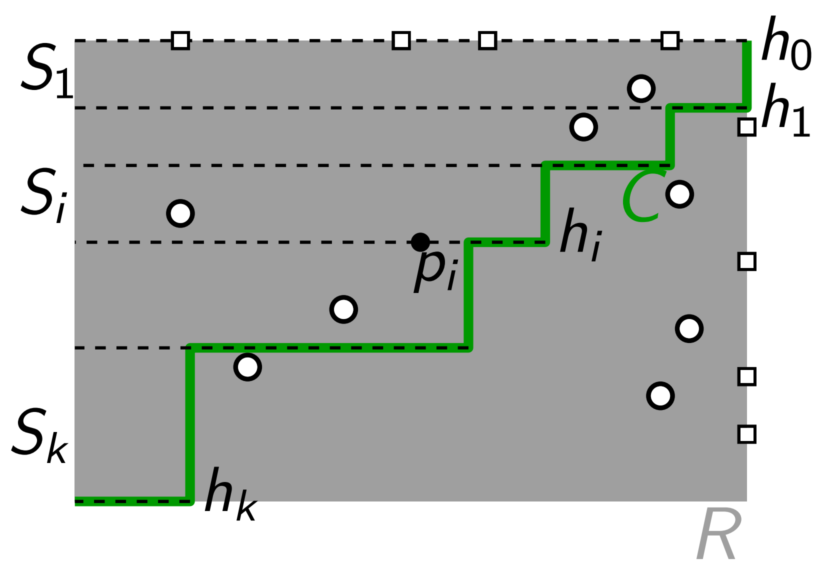
- $h_0, \ldots, h_k$: horizontal segments of C extended to the left
- $S_1, \ldots, S_k$: strips of R partitioned by $h_0, \ldots, h_k$



The Strip Condition

Given: xy-monotone curve C

- $h_0, \ldots, h_k$: horizontal segments of C extended to the left
- $S_1, \ldots, S_k$: strips of R partitioned by $h_0, \ldots, h_k$
- p_i: any point on $h_i \setminus C$



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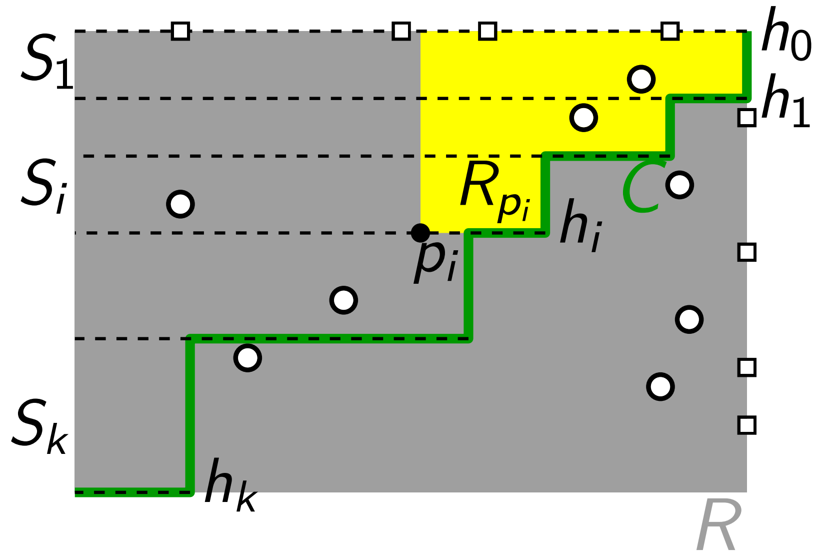
- p_i: any point on $h_i \setminus C$
- R_{p_i}: polygon spanned by p_i, h_i and C



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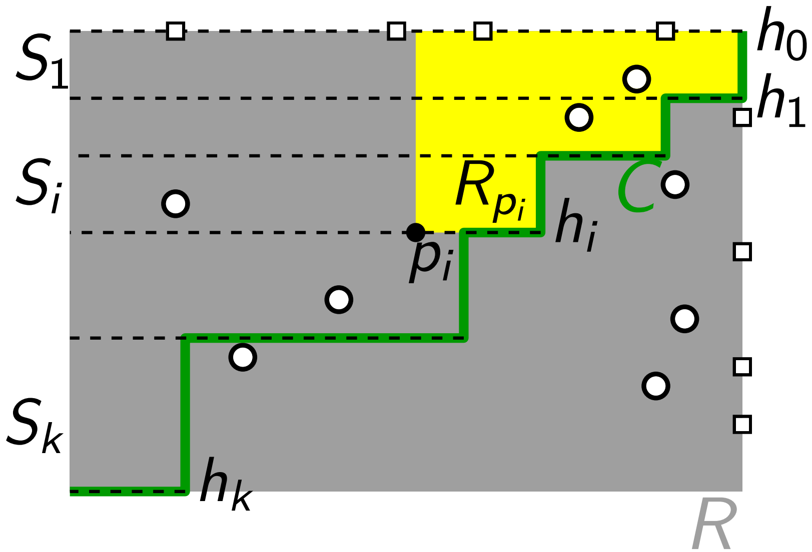
- R_{p_i} is *valid*: number of sites $\geq$ number of ports



The Strip Condition

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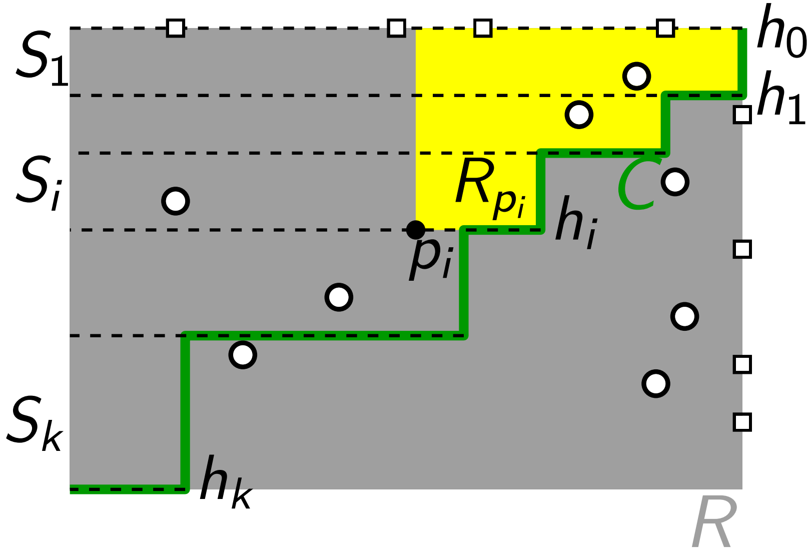
- p_i: any point on $h_i \setminus C$
- R_{p_i}: polygon spanned by p_i, h_i and C
- R_{p_i} is *valid*: number of sites $\geq$ number of ports

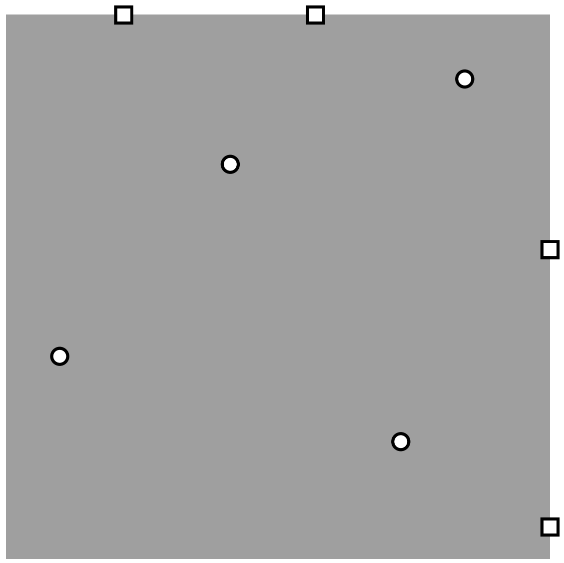
Condition. *strip condition* of S_i is satisfied

$$\Leftrightarrow \exists p_i \in h_i \setminus C : R_{p_i} \text{ is valid.}$$



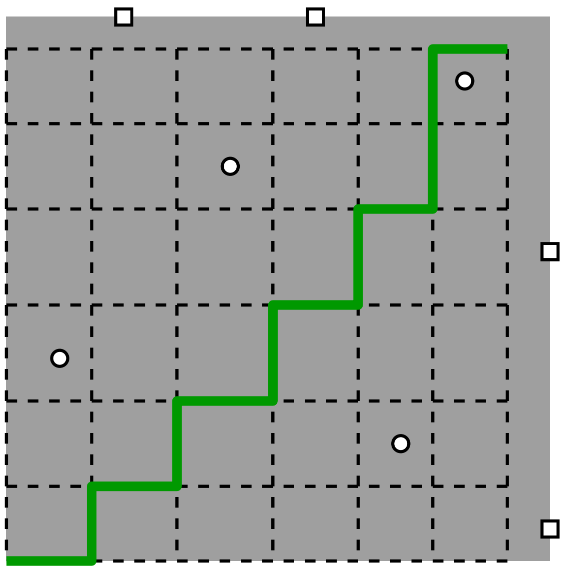
The Algorithm

- Find an xy-separating curve C that satisfies the strip conditions.



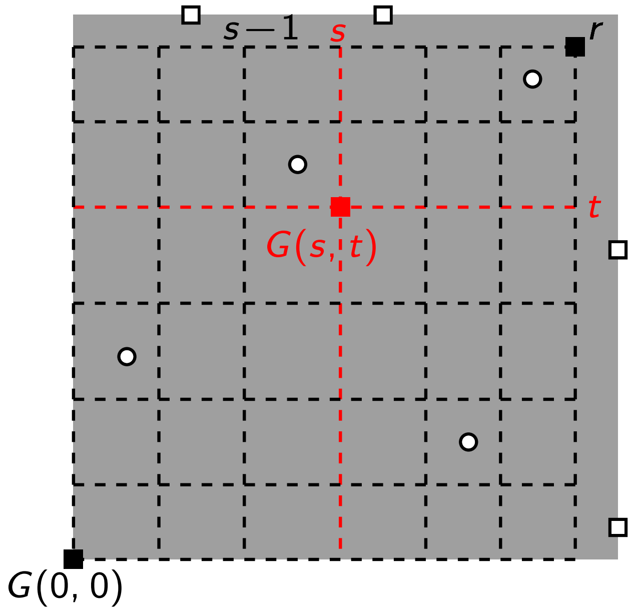
The Algorithm

- Find an xy-separating curve C that satisfies the strip conditions.
- Consider the dual of the grid induced by sites and ports



The Algorithm

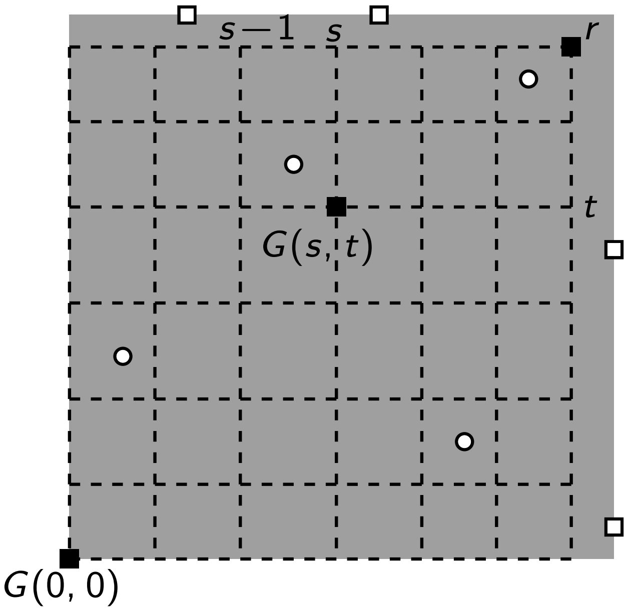
- Find an xy-separating curve C that satisfies the strip conditions.
- Consider the dual of the grid induced by sites and ports
- Define grid points $G(s, t)$ and top-right corner r



The Algorithm

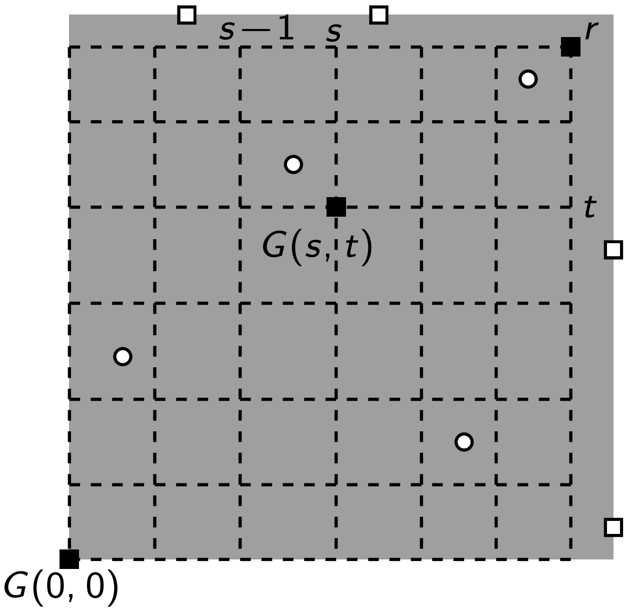
Dynamic Program:

Compute table $T[(s,t),u]$.



The Algorithm

Dynamic Program:
Compute table $T[(s,t),u]$.
$T[(s,t),u] = \texttt{true}$
$\Leftrightarrow \exists xy$-monotone chain C:



The Algorithm

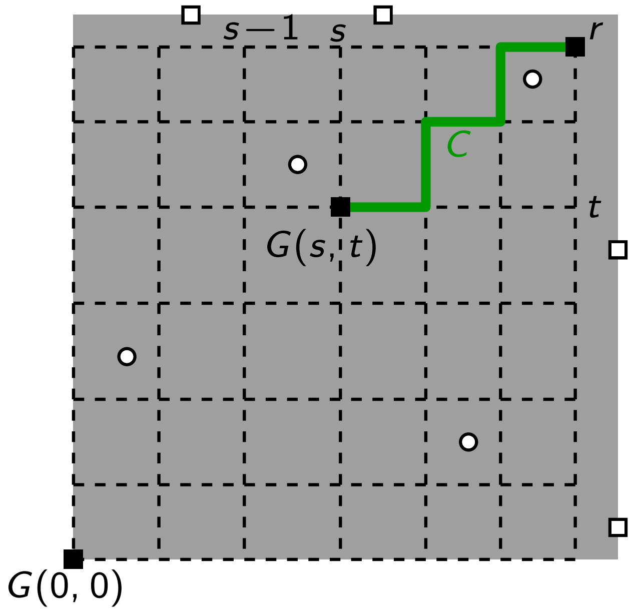
Dynamic Program:

Compute table $T[(s,t),u]$.

$T[(s,t),u] = \texttt{true}$

$\Leftrightarrow \exists xy$-monotone chain C:

- C starts at r and ends at $G(s,t)$



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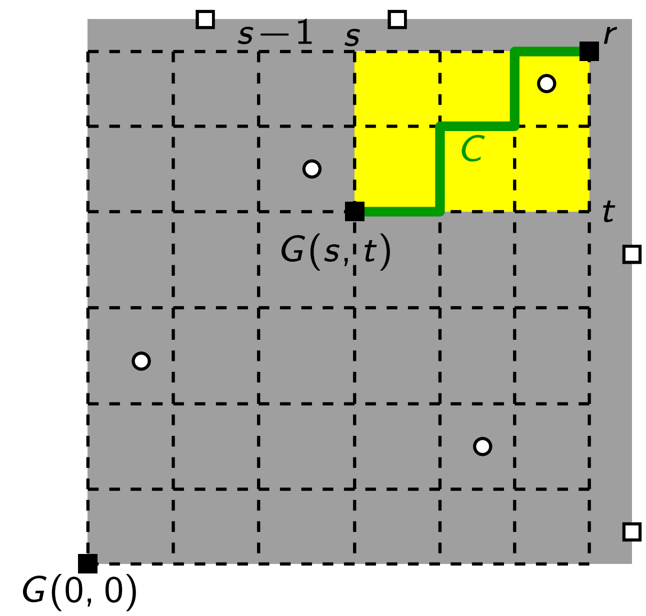

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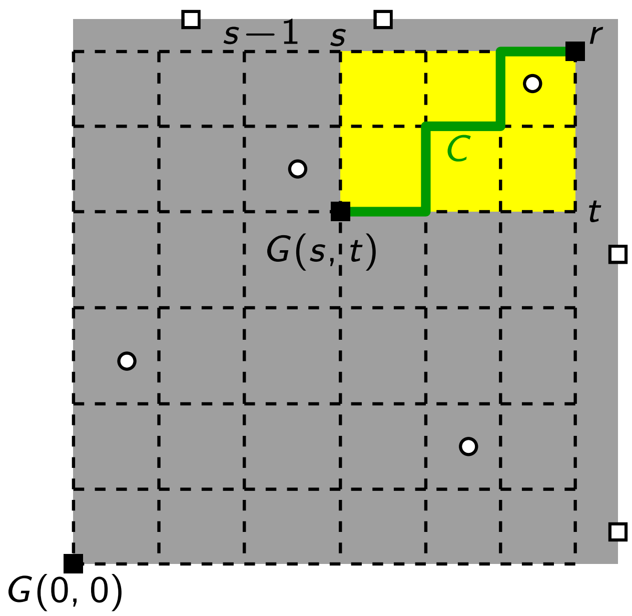

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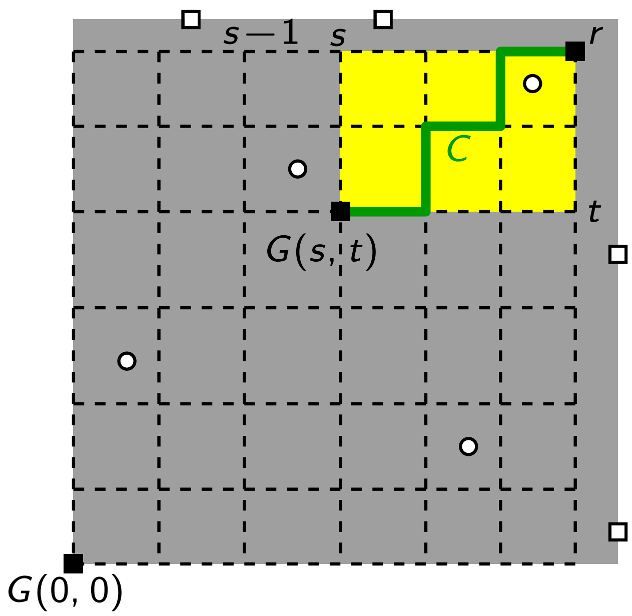
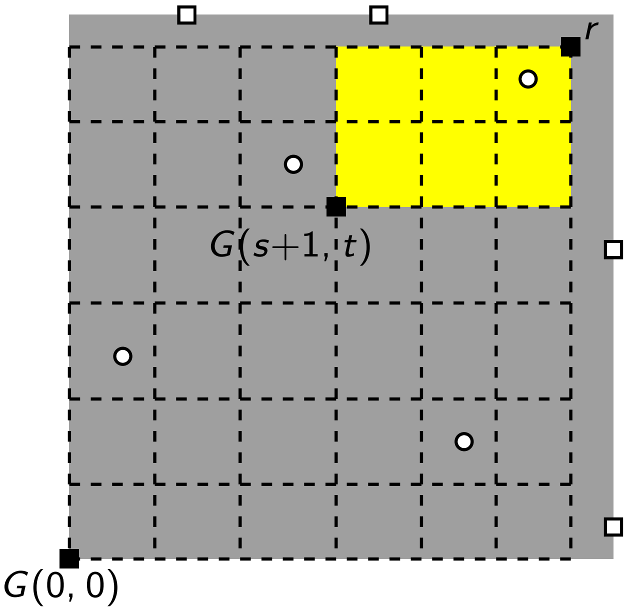


Instance solvable $\Leftrightarrow T[(0,0),u] = \texttt{true}$ for some u.

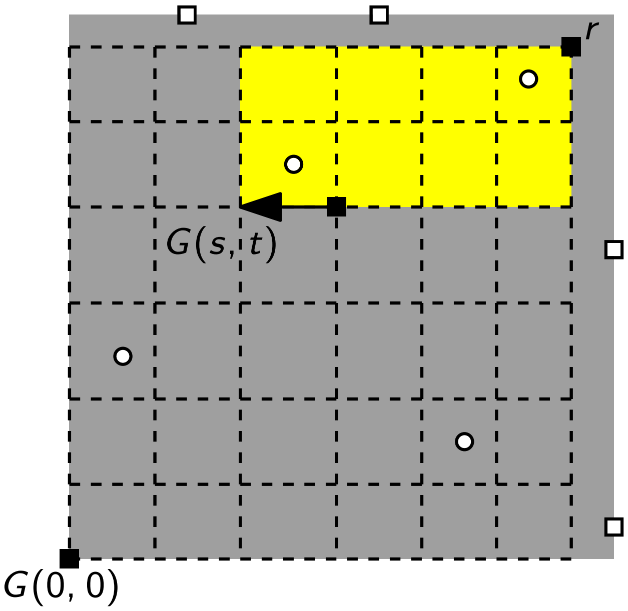
One Step of the Algorithm

Assume $T[(s+1, t), u] = \texttt{true}$.



One Step of the Algorithm

Assume $T[(s+1,t),u] = \texttt{true}$.
Go from $s+1$ to s.

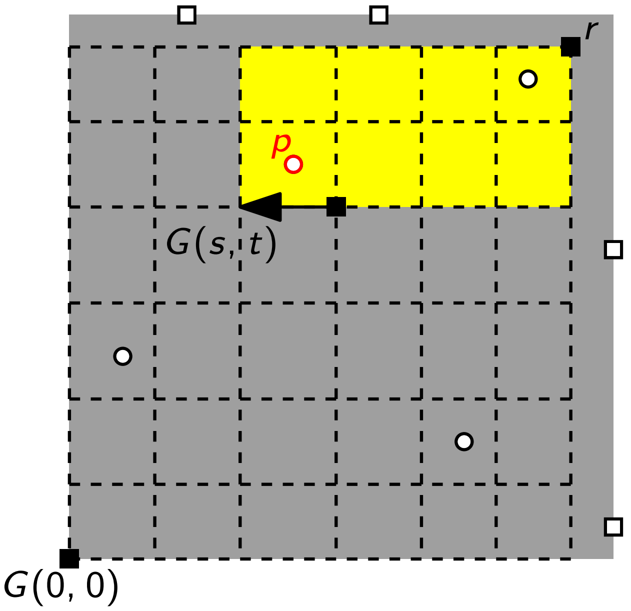


One Step of the Algorithm

Assume $T[(s+1,t),u] = \texttt{true}$.

Go from $s+1$ to s.

Case 1: *site event*



One Step of the Algorithm

Assume $T[(s+1,t),u] = \texttt{true}$.

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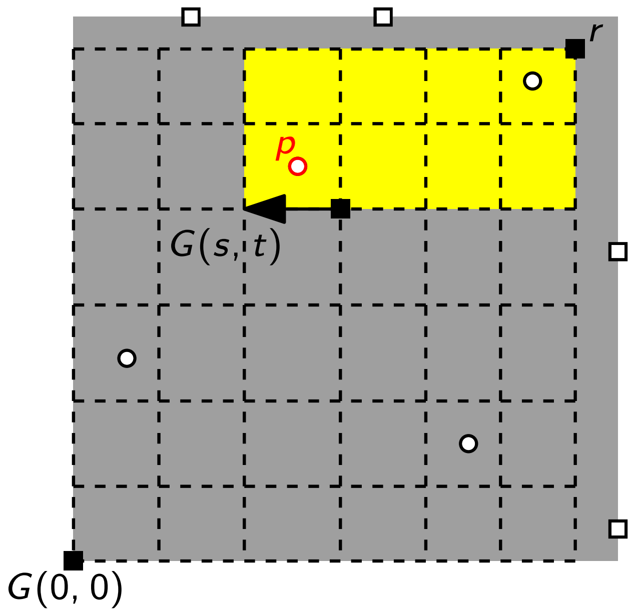
Case 1: *site event*

$y(p) > G_y(t)$



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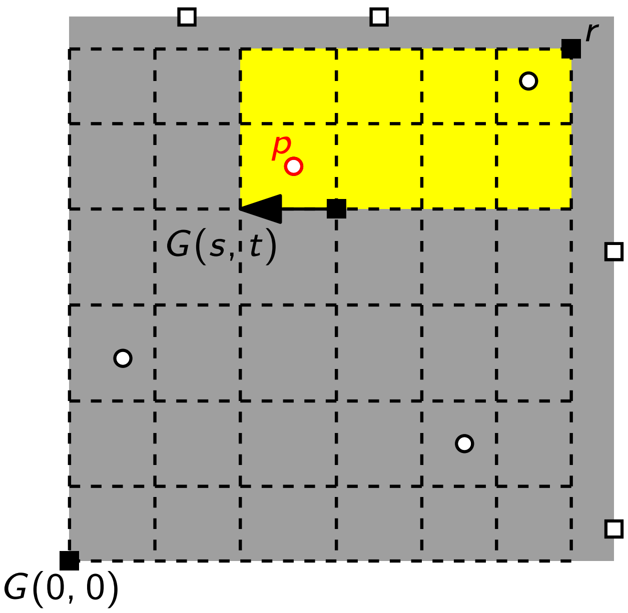

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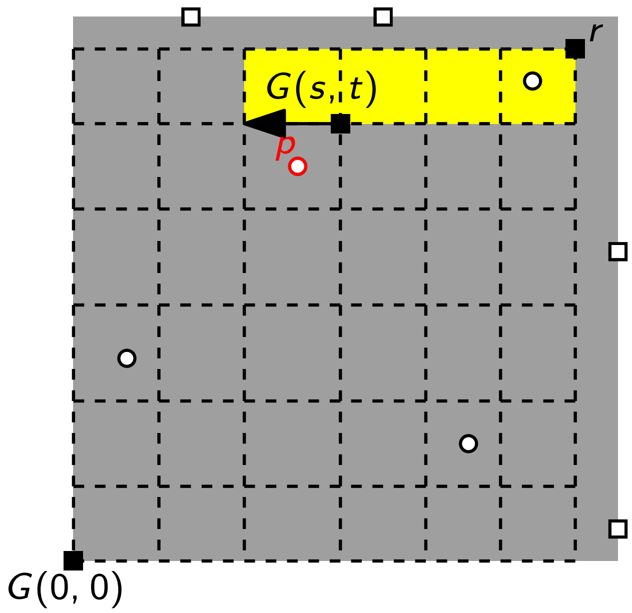

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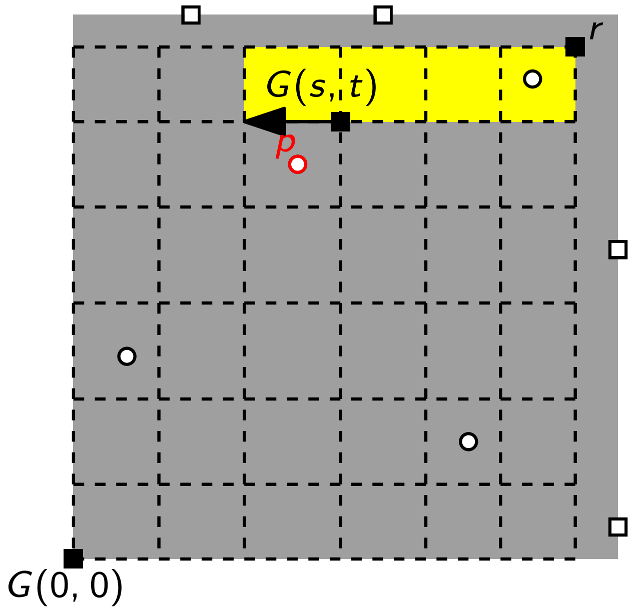

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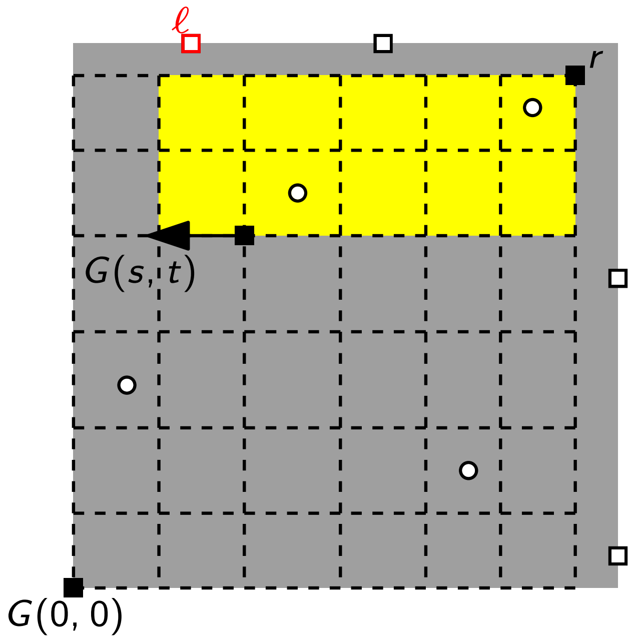

One Step of the Algorithm

Assume $T[(s+1,t),u] = \texttt{true}$.

Go from $s+1$ to s.

Case 1: *site event*

$y(p) > G_y(t)$

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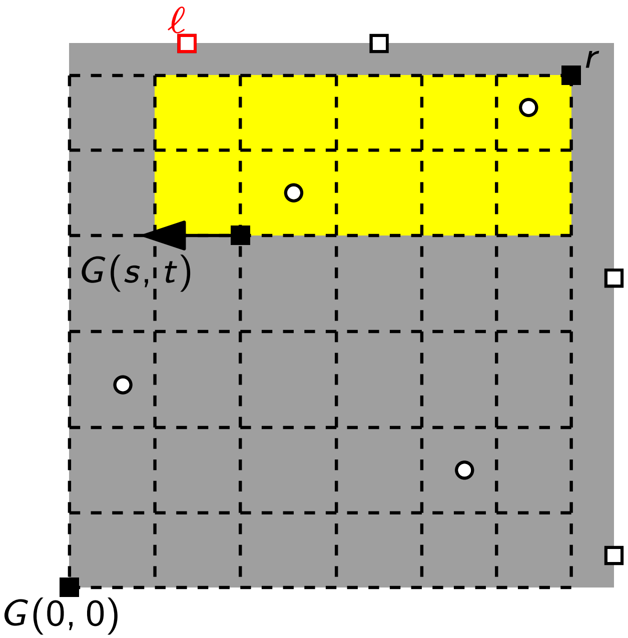
$y(p) < G_y(t)$

$\Rightarrow T[(s,t),u] = \texttt{true}$.

Case 2: *port event*

Check strip condition



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Go from $s+1$ to s.

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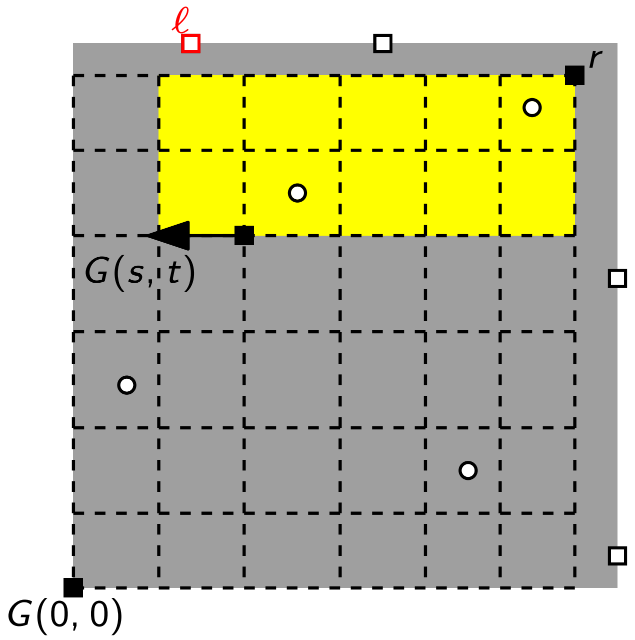
Case 2: *port event*

Check strip condition

satisfied $\Rightarrow T[(s,t),u] = \texttt{true}$



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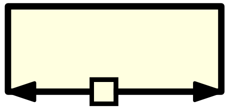
Extensions

- Sliding Ports:
  $O(n^2)$ time, $O(n)$ space



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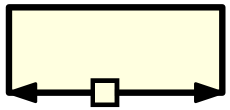

- Maximize number of labeled sites:
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Extensions

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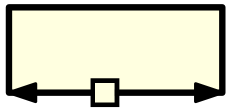

- Maximize number of labeled sites:
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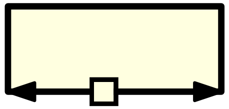

- Maximize number of labeled sites:
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- Leader-length minimization:
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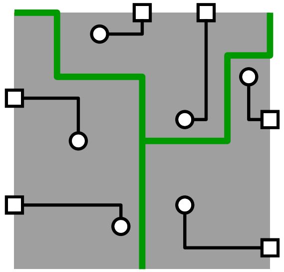
- Three-Sided Boundary Labeling:
  $O(n^5)$ time, $O(n)$ space



Extensions

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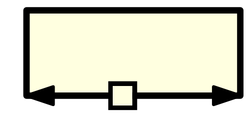

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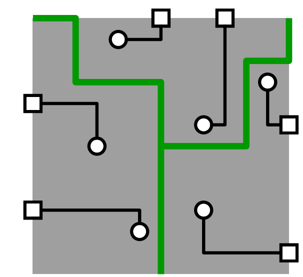

- Four-Sided Boundary Labeling:
  $O(n^{10})$ time, $O(n)$ space