Computational Geometry

Lecture 3:
Guarding Art Galleries
or
Triangulating Polygons

Part I:
The Art Gallery Problem

Philipp Kindermann

Winter Semester 2020

Guarding an Art Gallery

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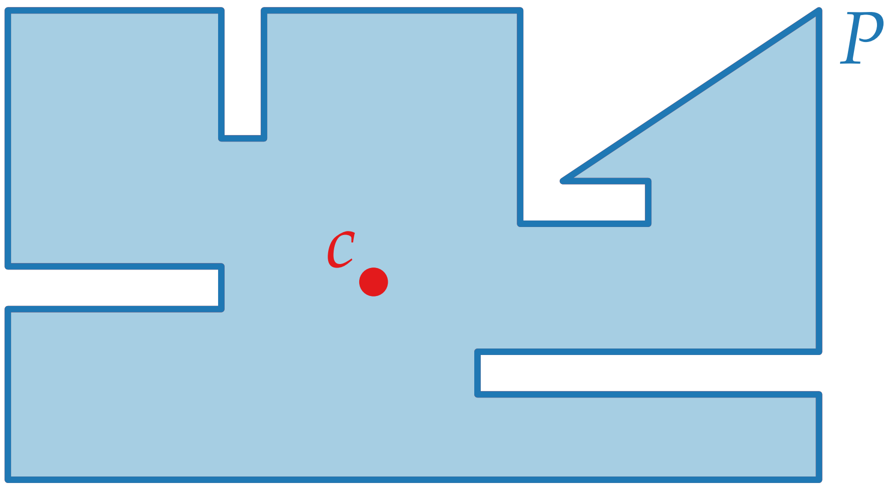

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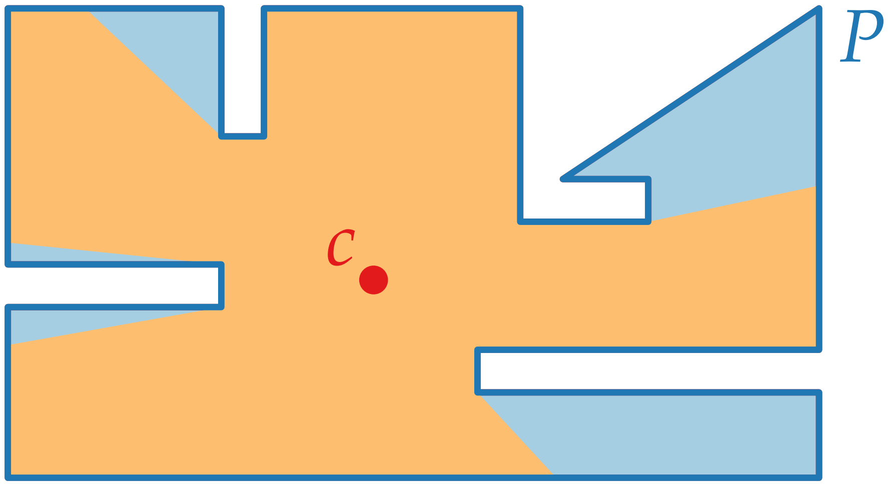

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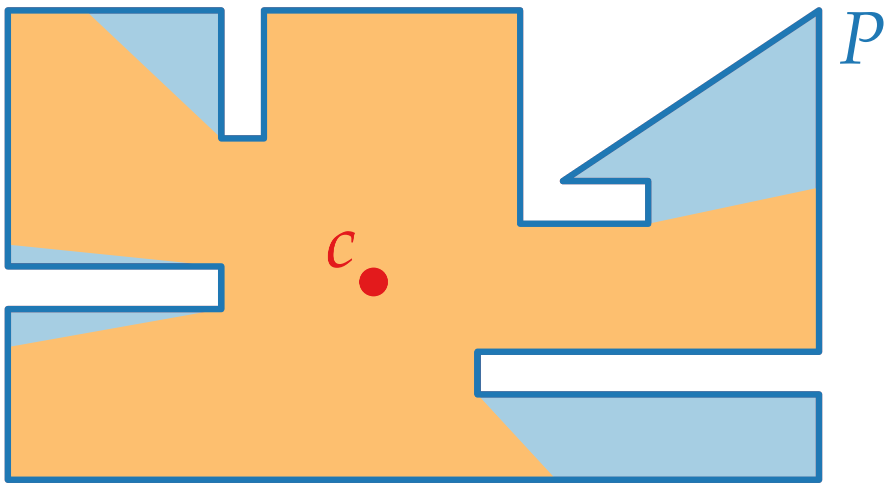

Observation. Camera c "sees" a star-shaped region

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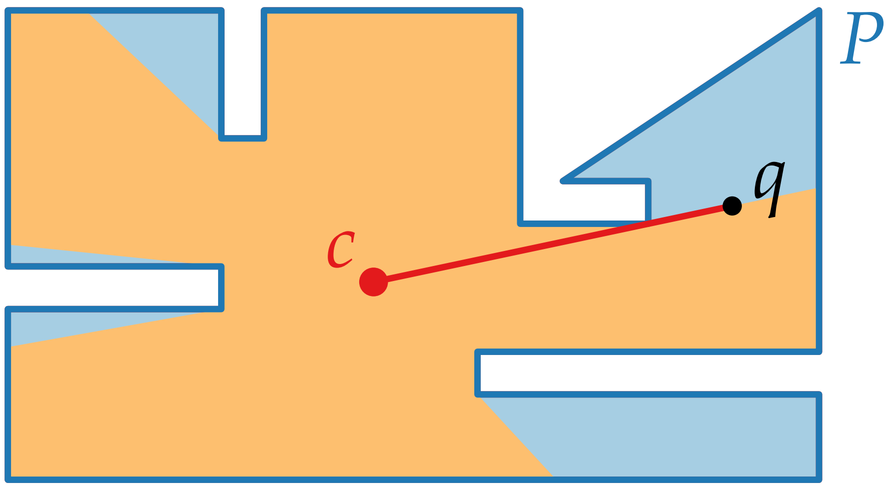

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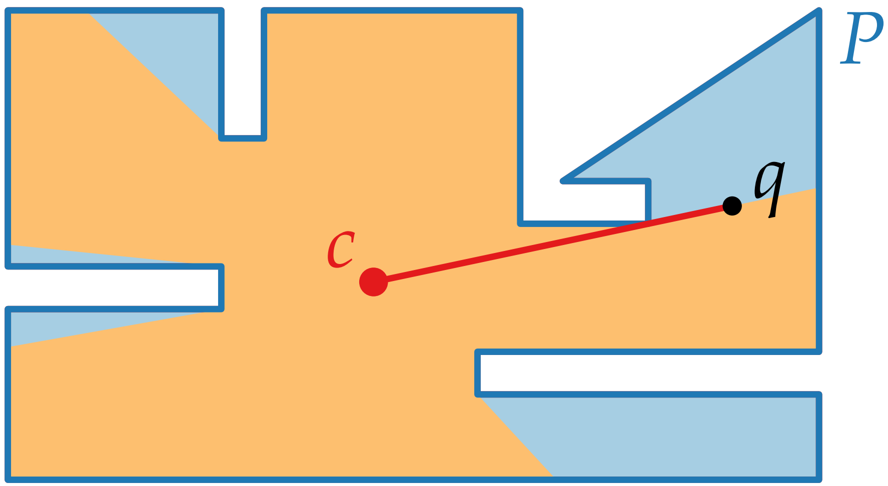

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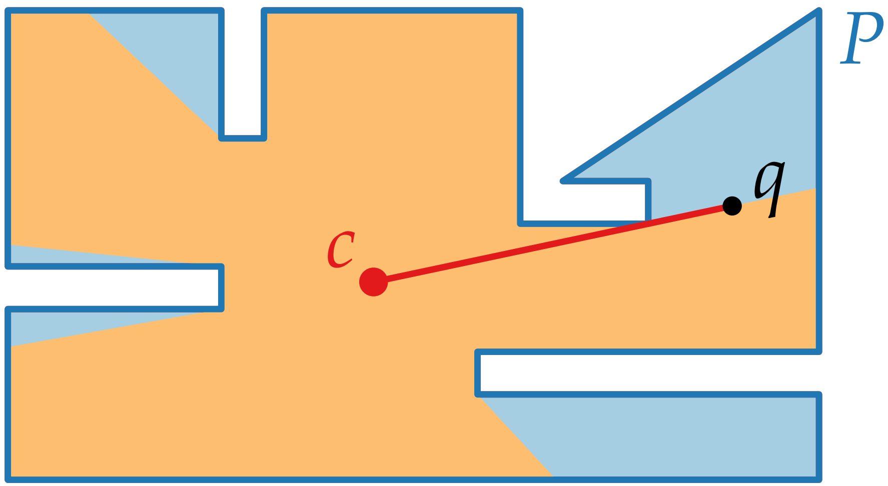

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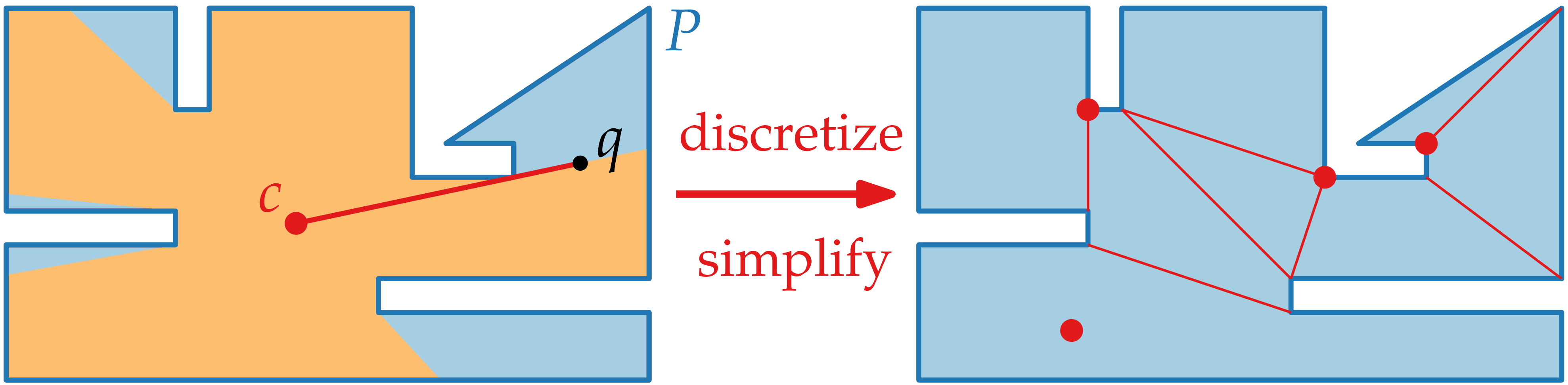

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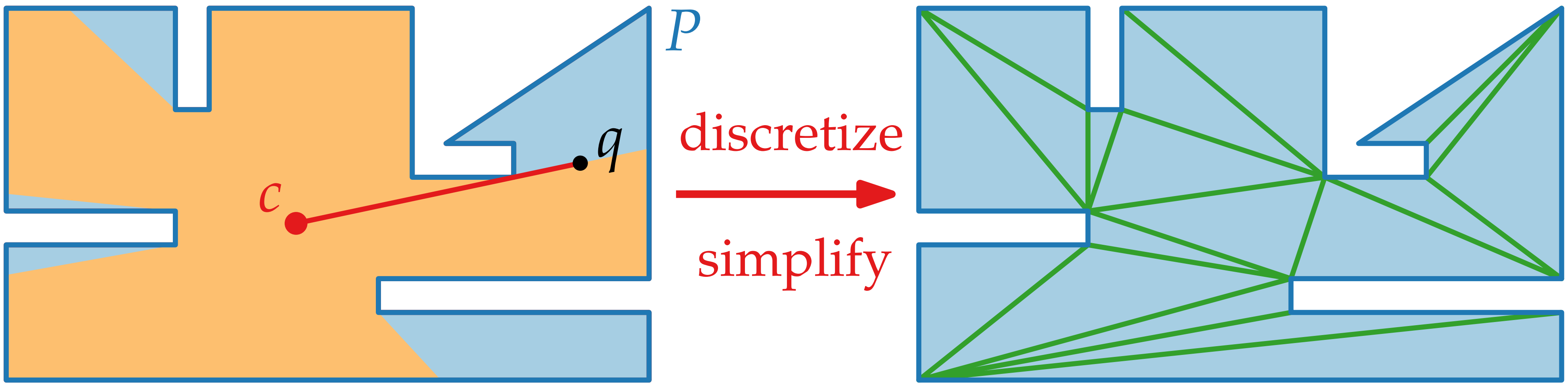

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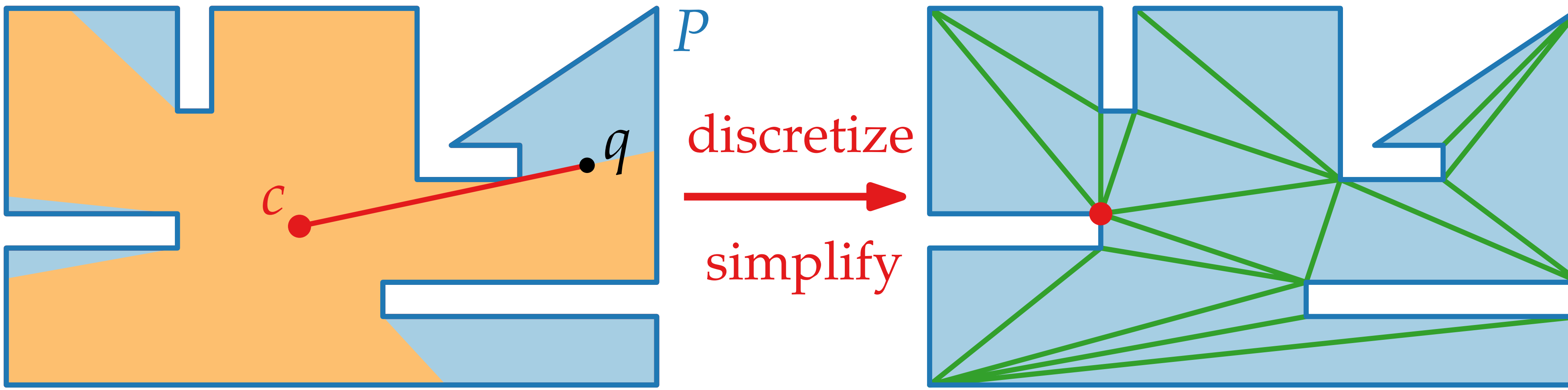

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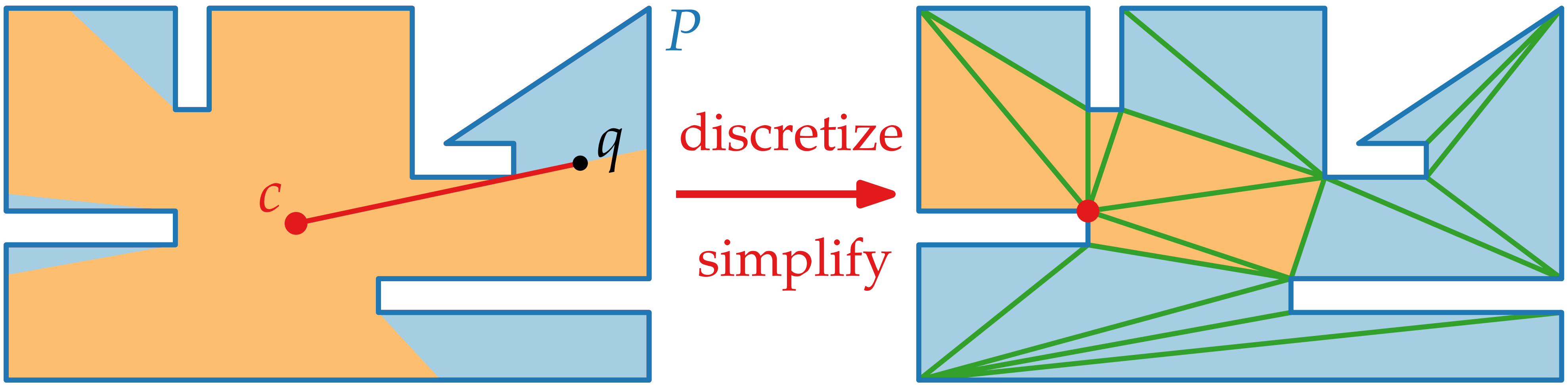

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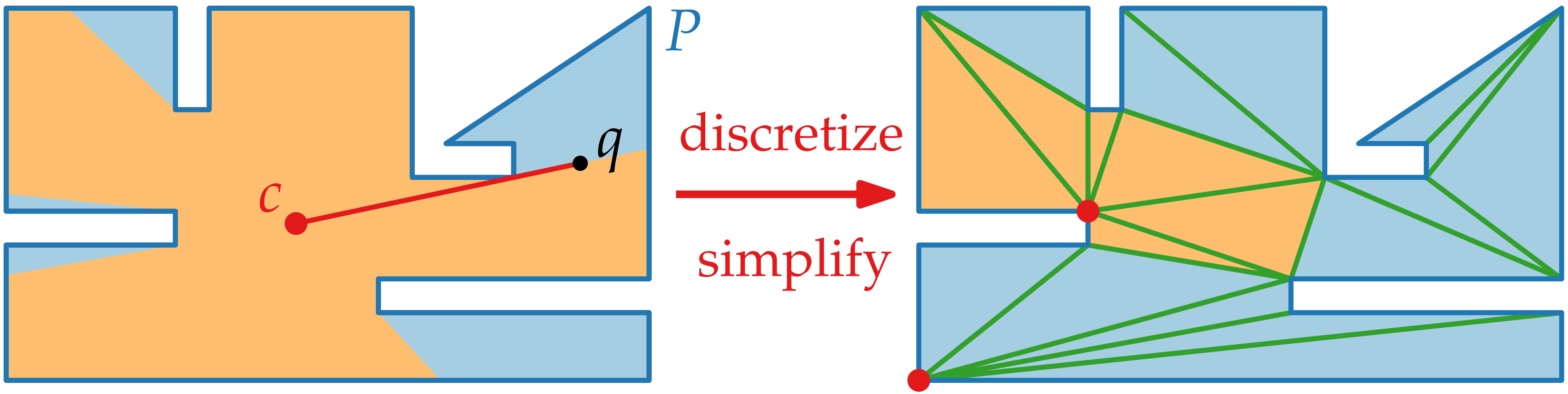

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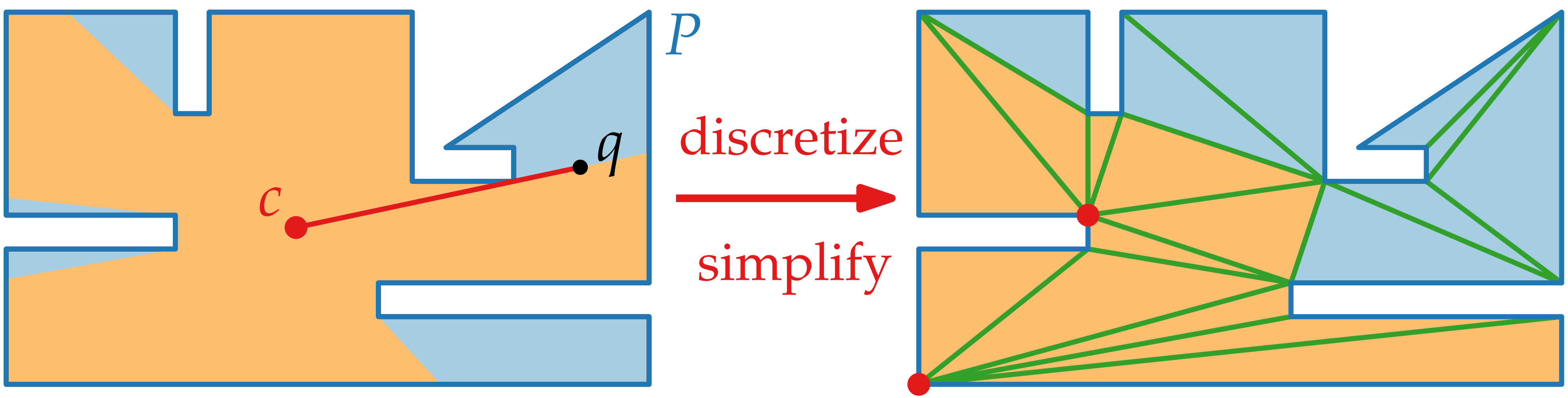

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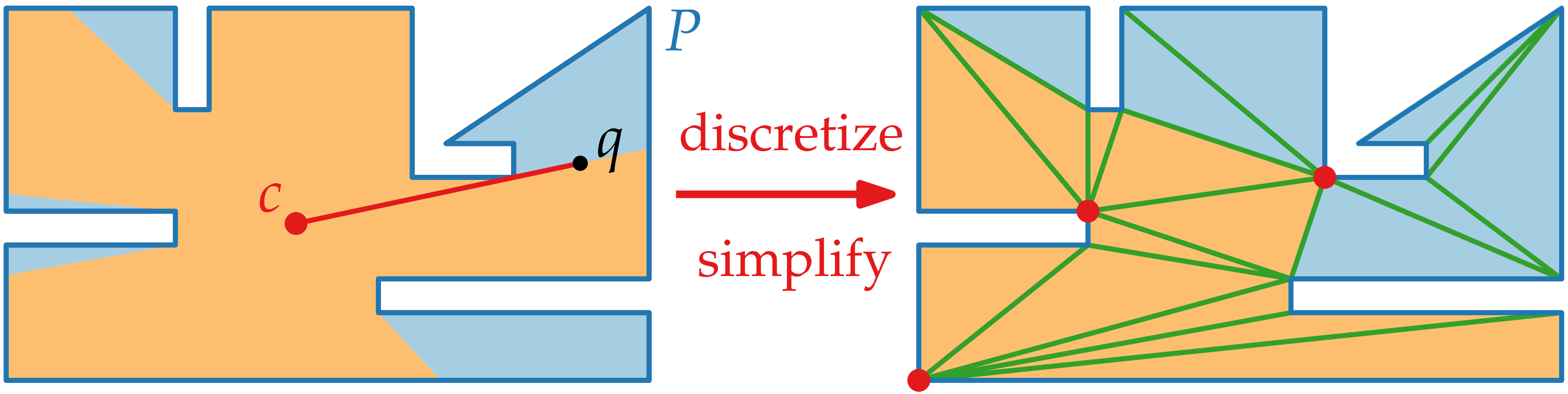

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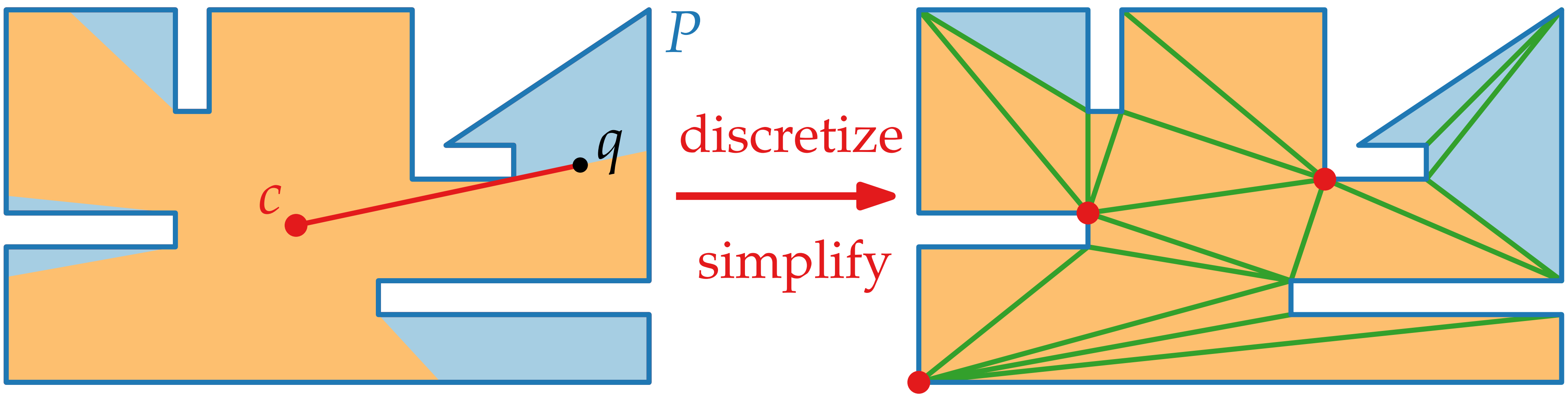

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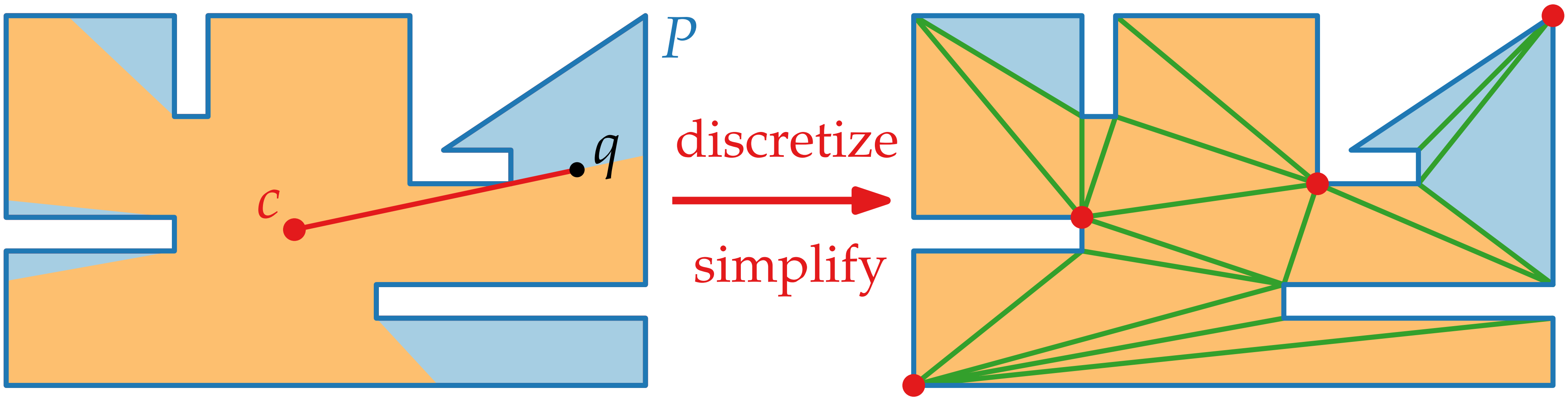

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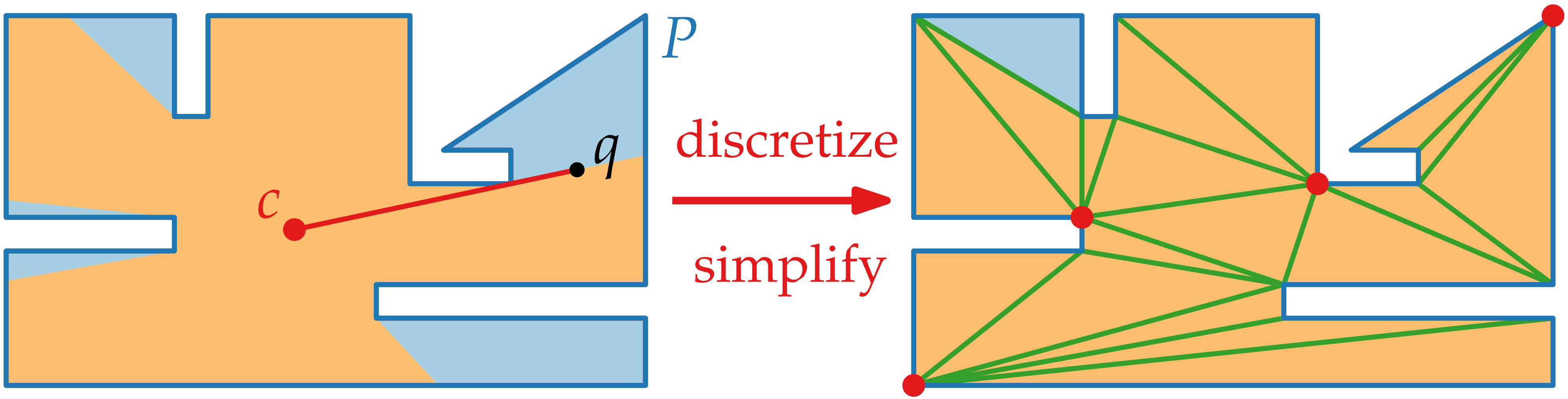

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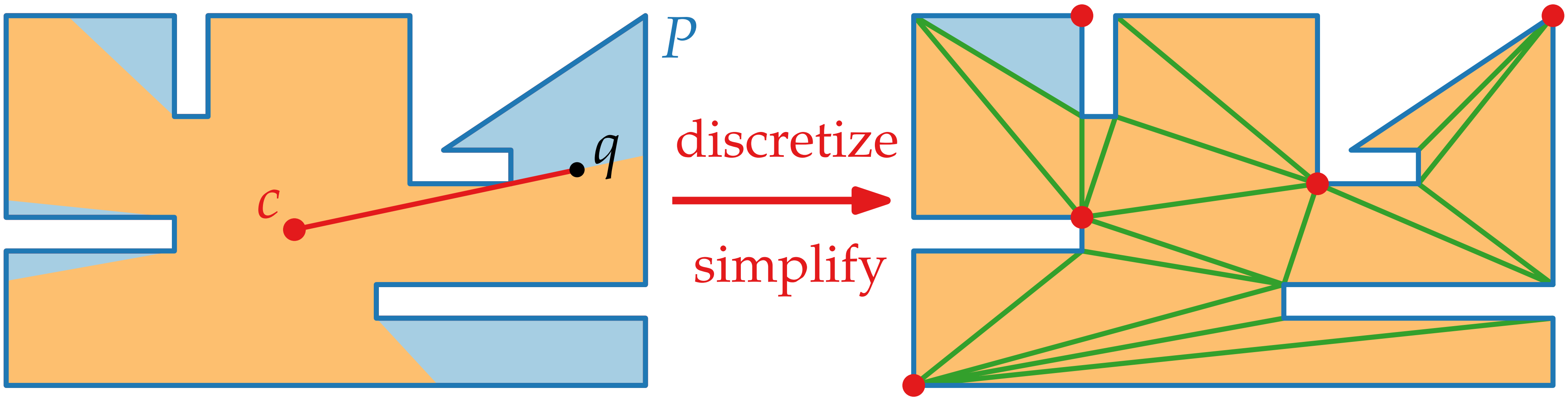

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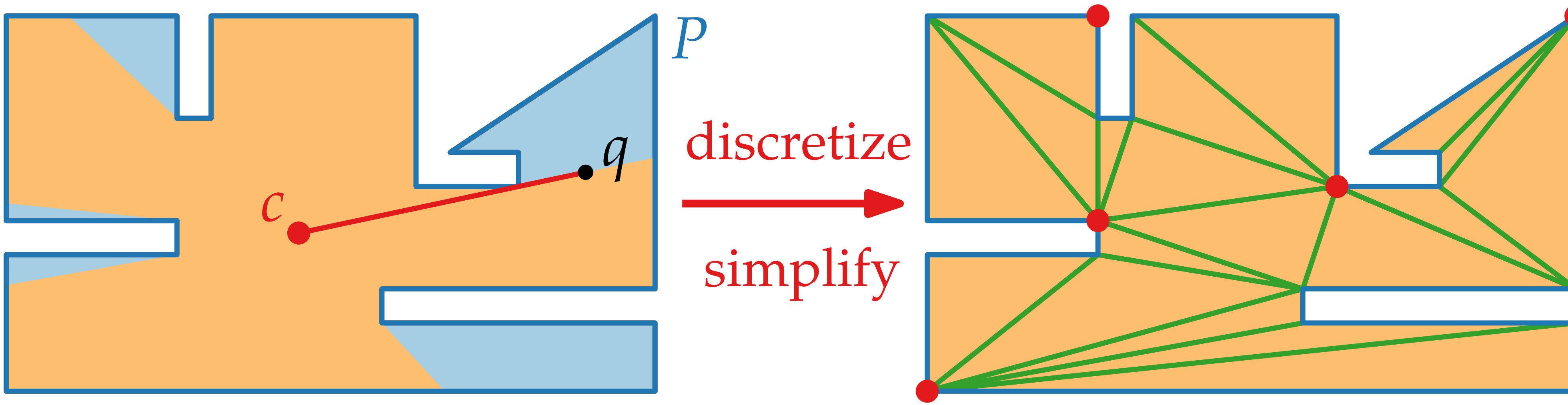

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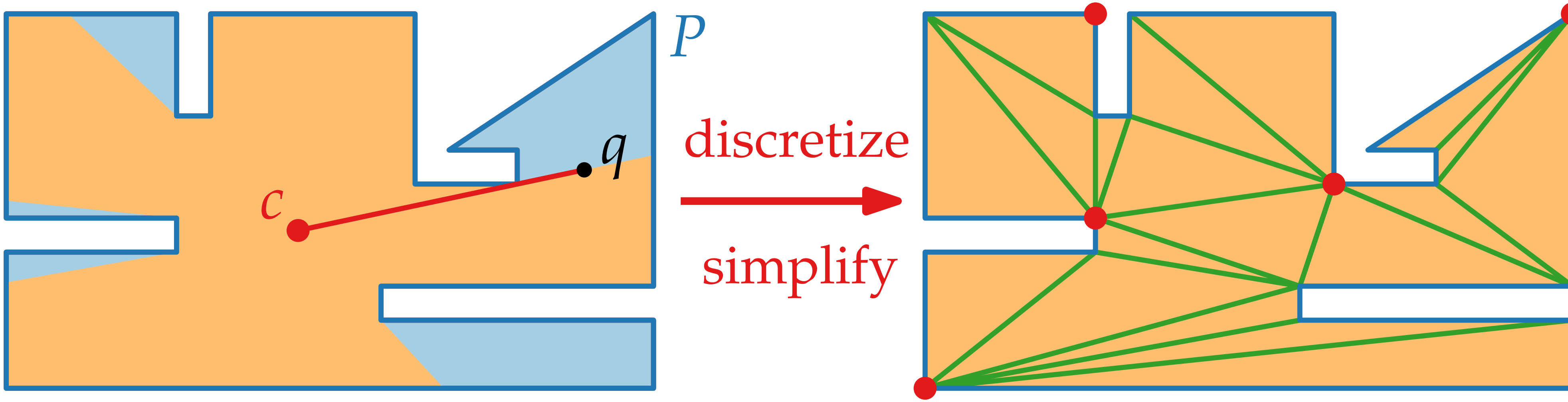

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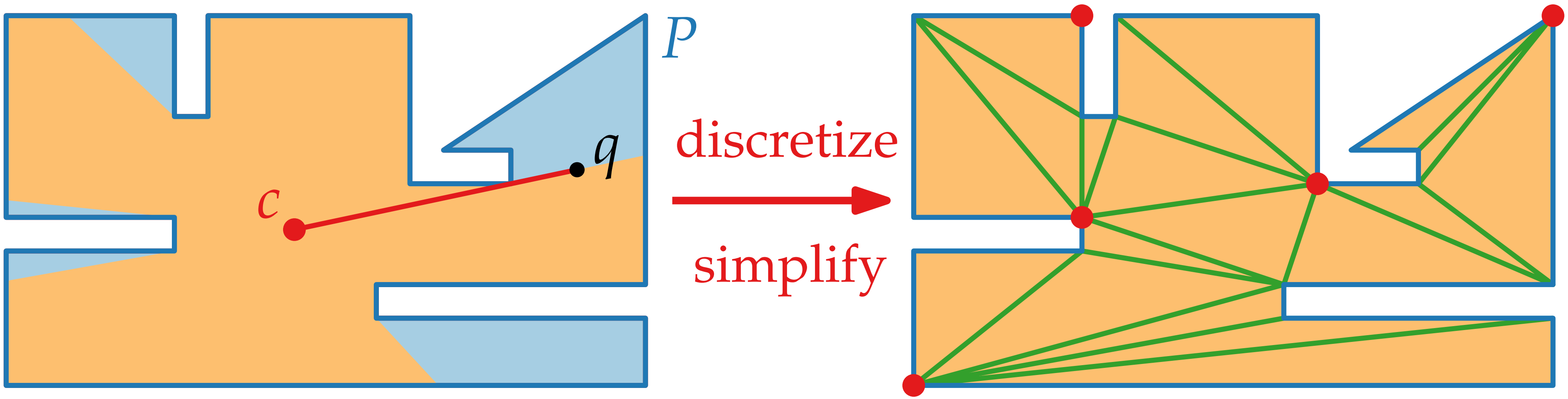

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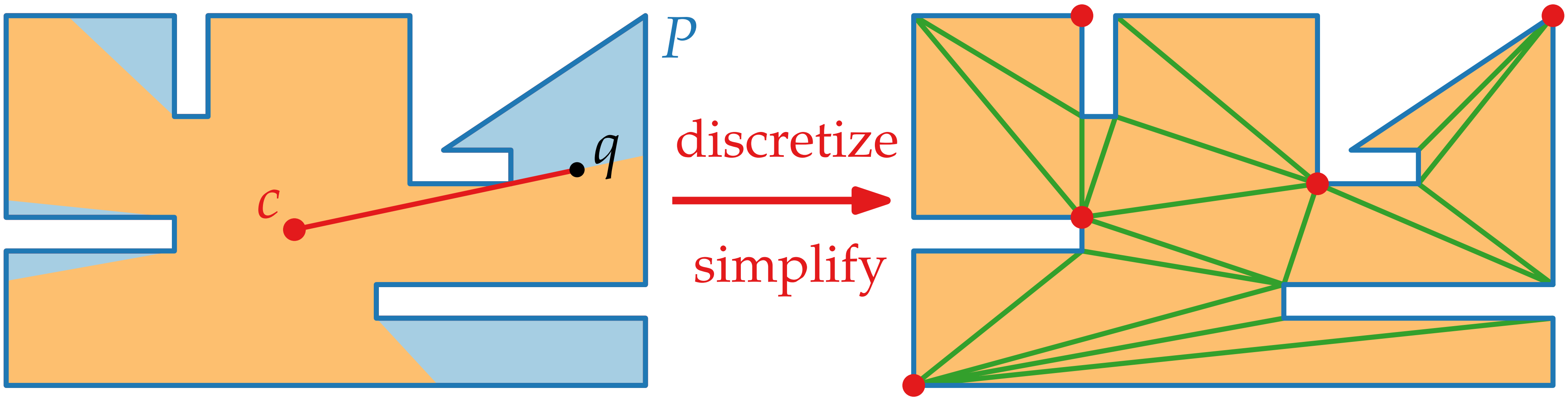

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How can we prove these?

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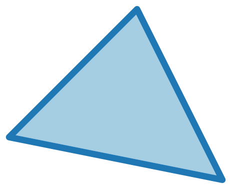
$n = 3$:  1 triangle ✓

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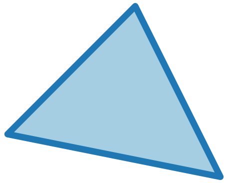
$n = 3$:  1 triangle ✓

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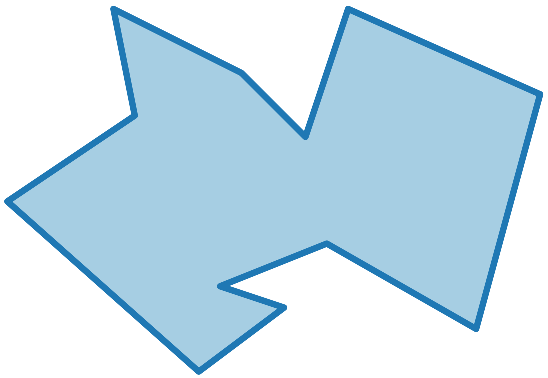

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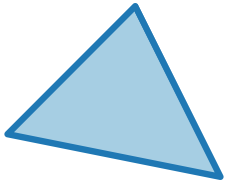

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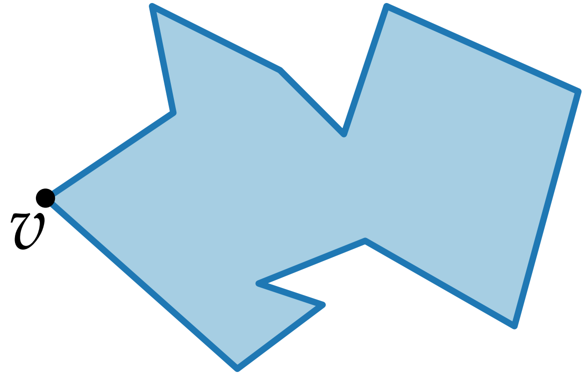

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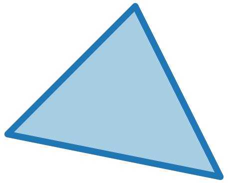
$n = 3$:  1 triangle ✓

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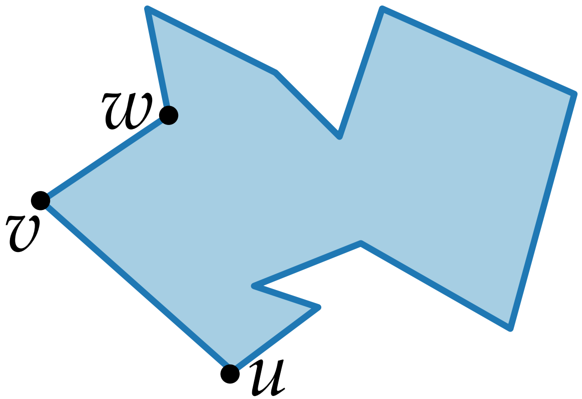

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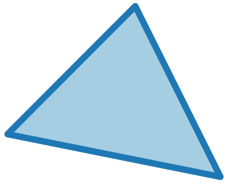
$n = 3$:  1 triangle ✓

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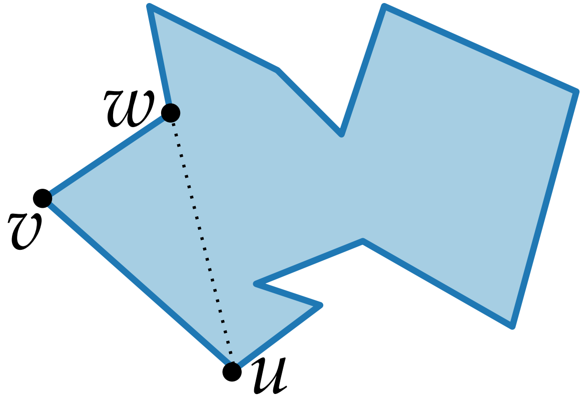

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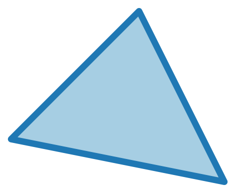
$n = 3$:  1 triangle ✓

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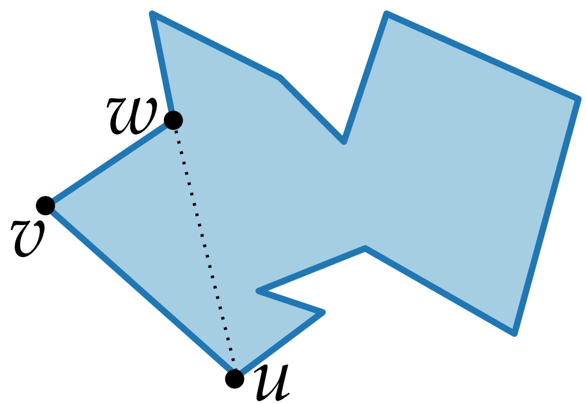

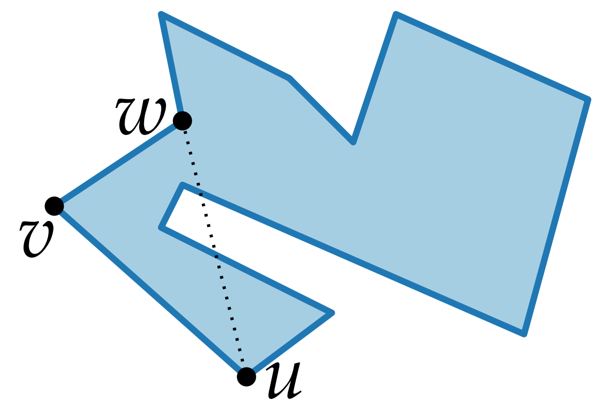

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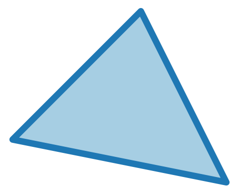

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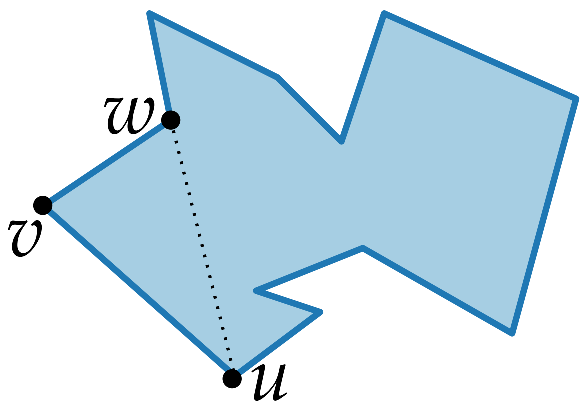
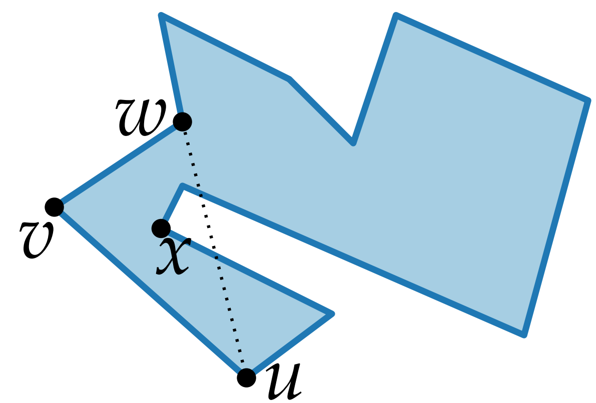


x furthest from uw



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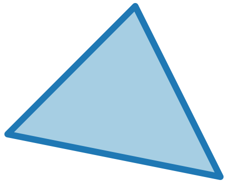
$n = 3$:  1 triangle ✓

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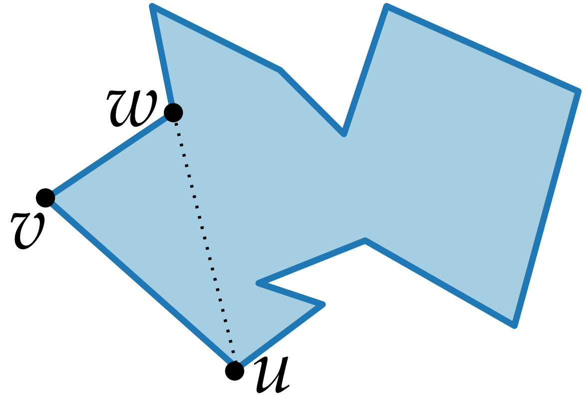

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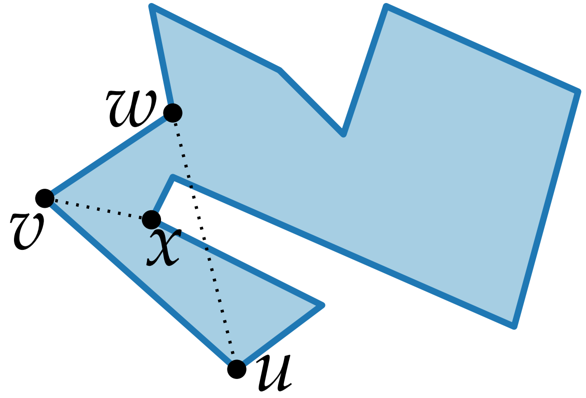

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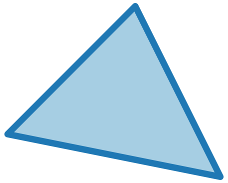
$n = 3$:  1 triangle ✓

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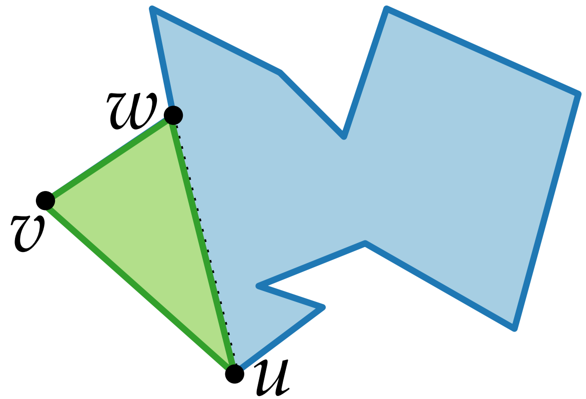

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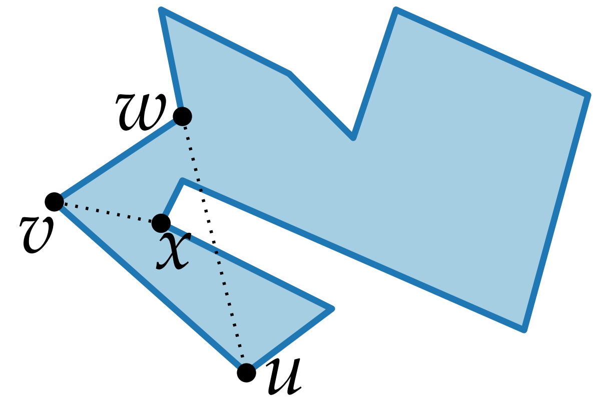

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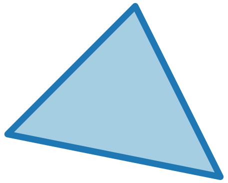

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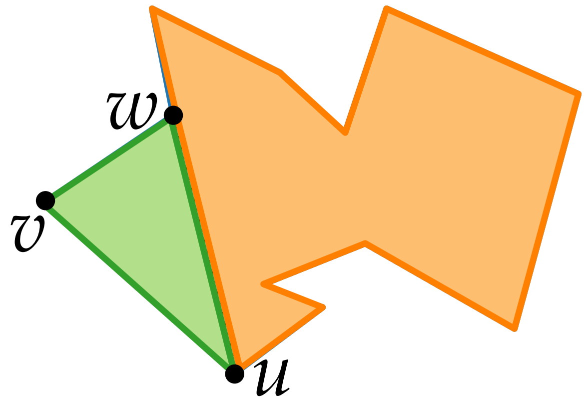

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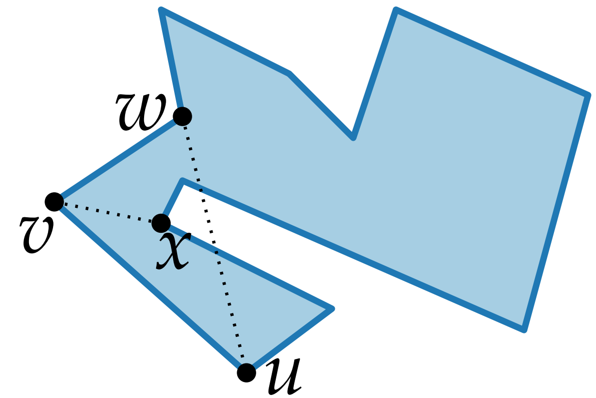

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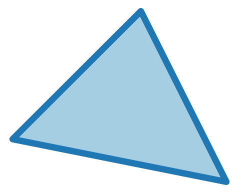

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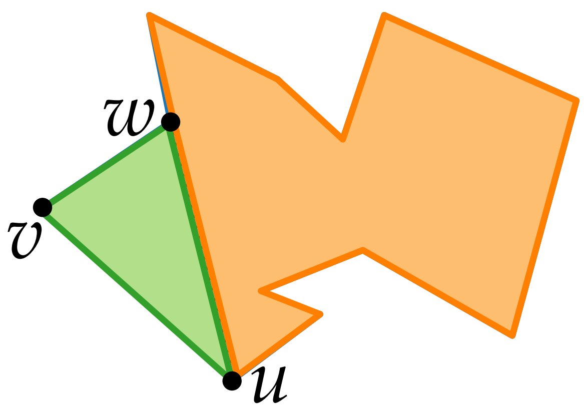

3 vtcs $\Rightarrow$ 1 triangle

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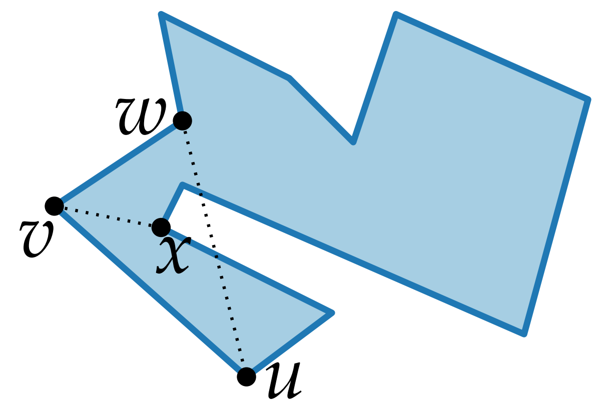

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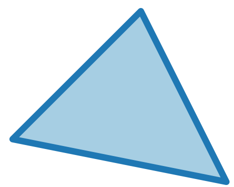
$n = 3$:  1 triangle ✓

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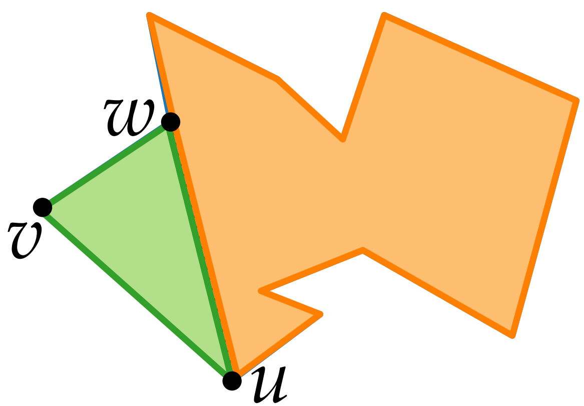
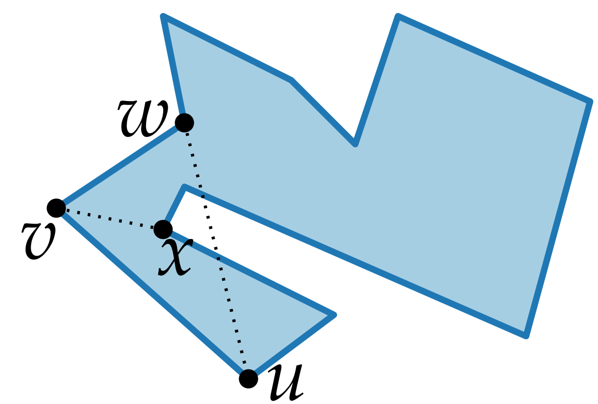


3 vtcs $\Rightarrow$ 1 triangle

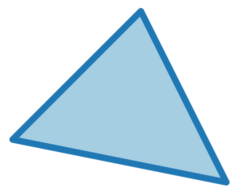
$n-1$ vtcs $\Rightarrow$ $n-3$ triangles

x furthest from uw



Existence of Triangulation

Theorem.
1. Every simple polygon can be triangulated.
2. Any triangulation of a simple polygon with n vertices consists of $n-2$ triangles.

$n = 3$:  1 triangle ✓

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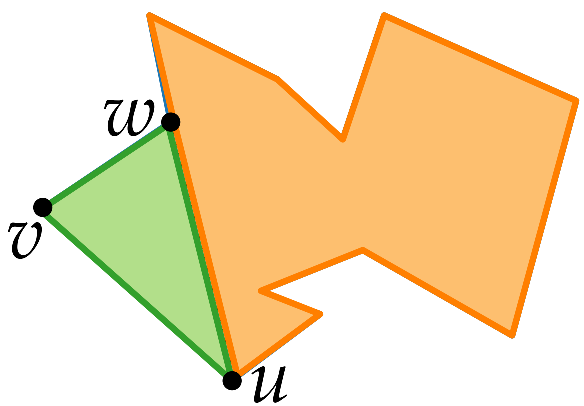

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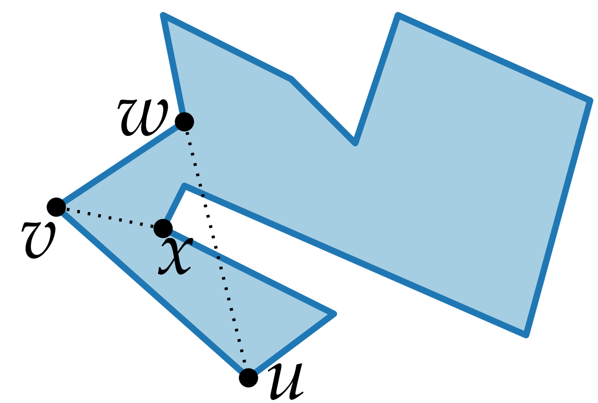

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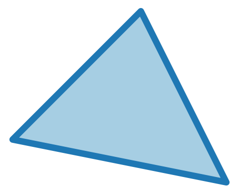
$n = 3$:  1 triangle ✓

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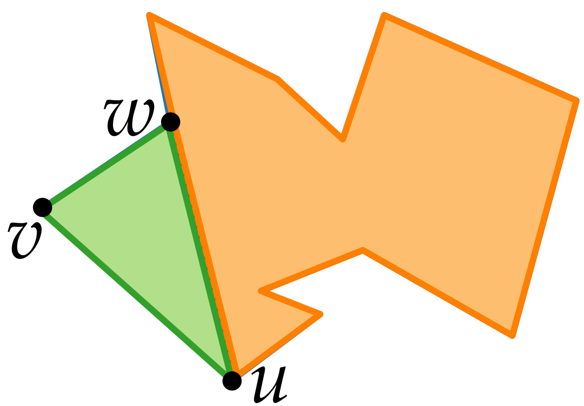

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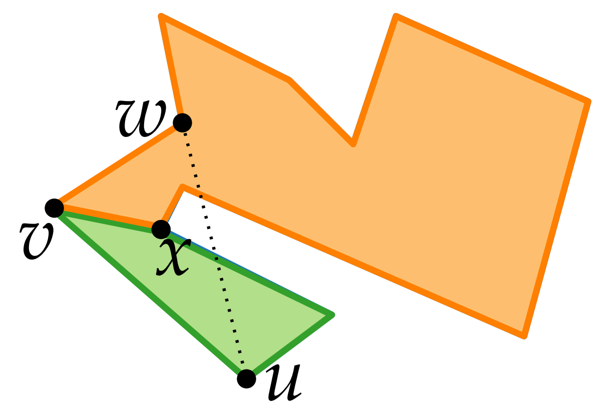

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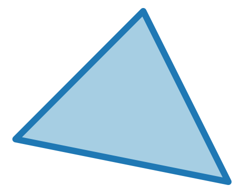
$n = 3$:  1 triangle ✓

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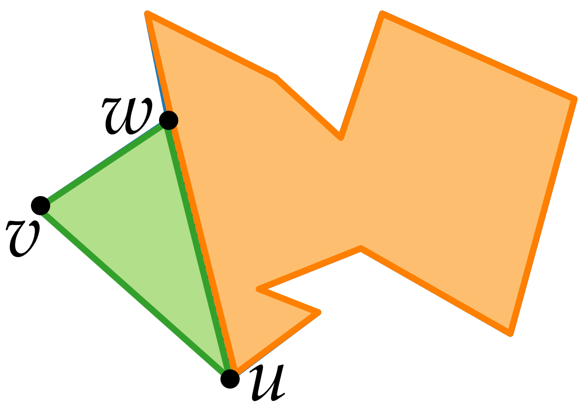

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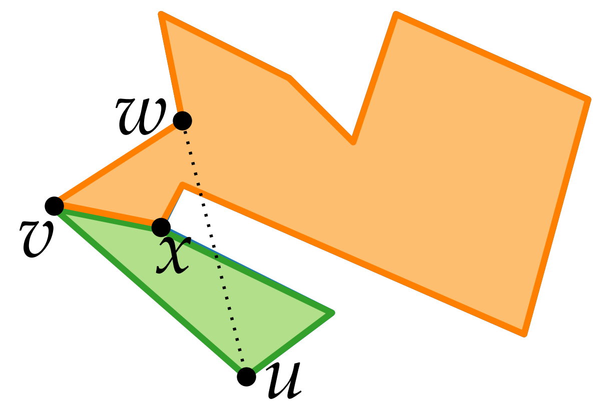
✓

x furthest from uw



m vtcs $\Rightarrow$ $m-2$ triangles

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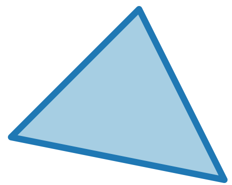
$n = 3$:  1 triangle ✓

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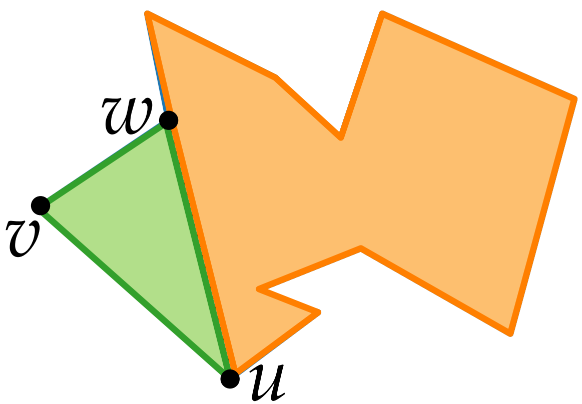

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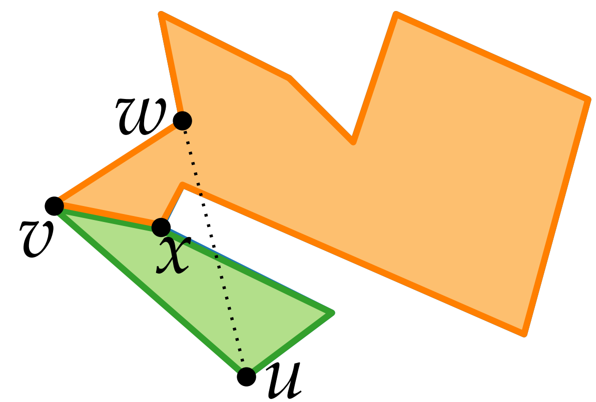

m vtcs $\Rightarrow$ $m-2$ triangles
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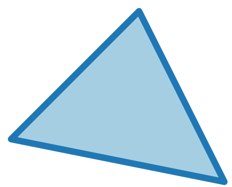
$n = 3$:  1 triangle ✓

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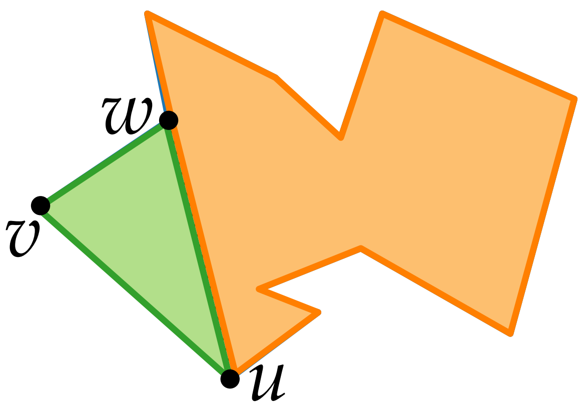

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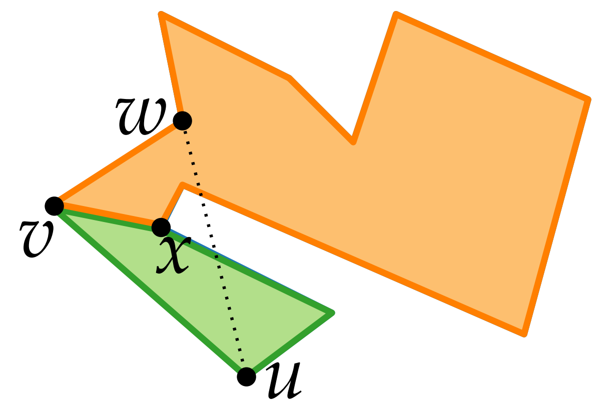

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Computational Geometry

Lecture 3:
Guarding Art Galleries
or
Triangulating Polygons

Part II:
The Art Gallery Theorem

Philipp Kindermann

Winter Semester 2020

The Art Gallery Theorem

[Chvátal '75]

Theorem. For surveilling a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient.

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Exercise. Find, for arbitrarily large n, a polygon with n vertices, where $\approx n/3$ cameras are necessary.

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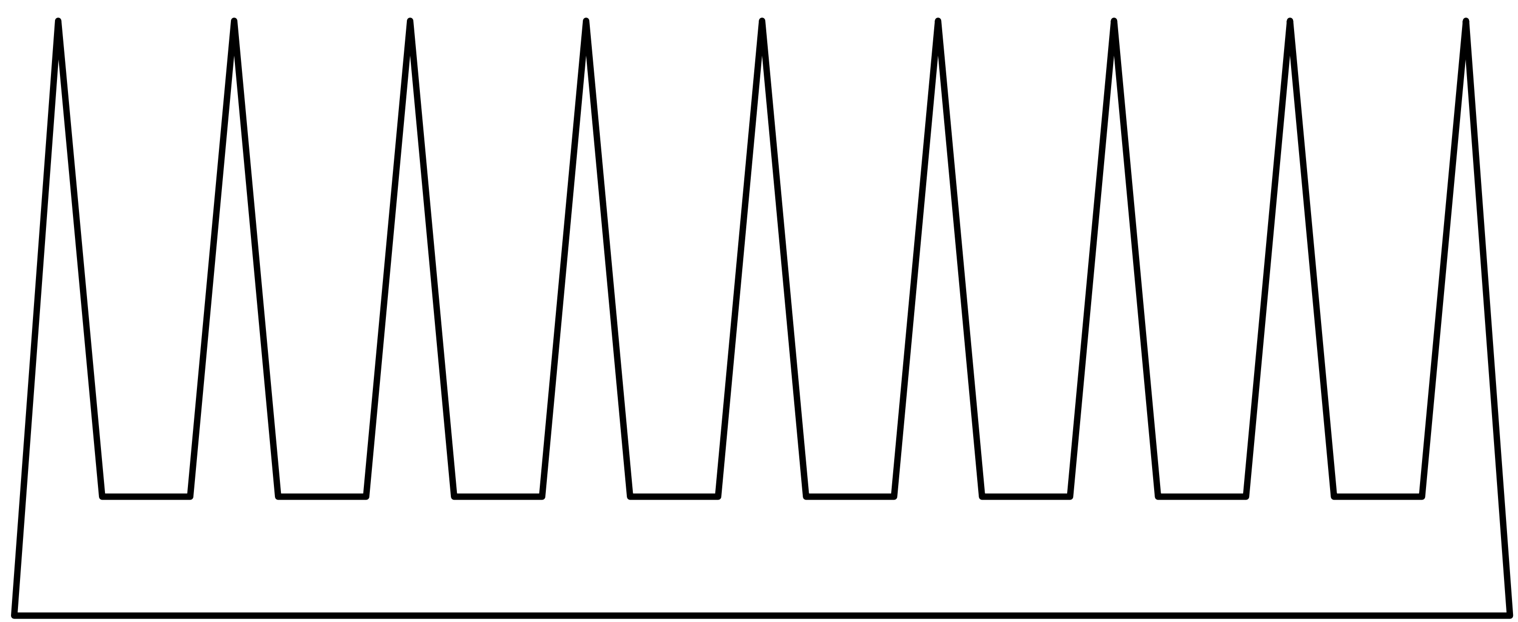

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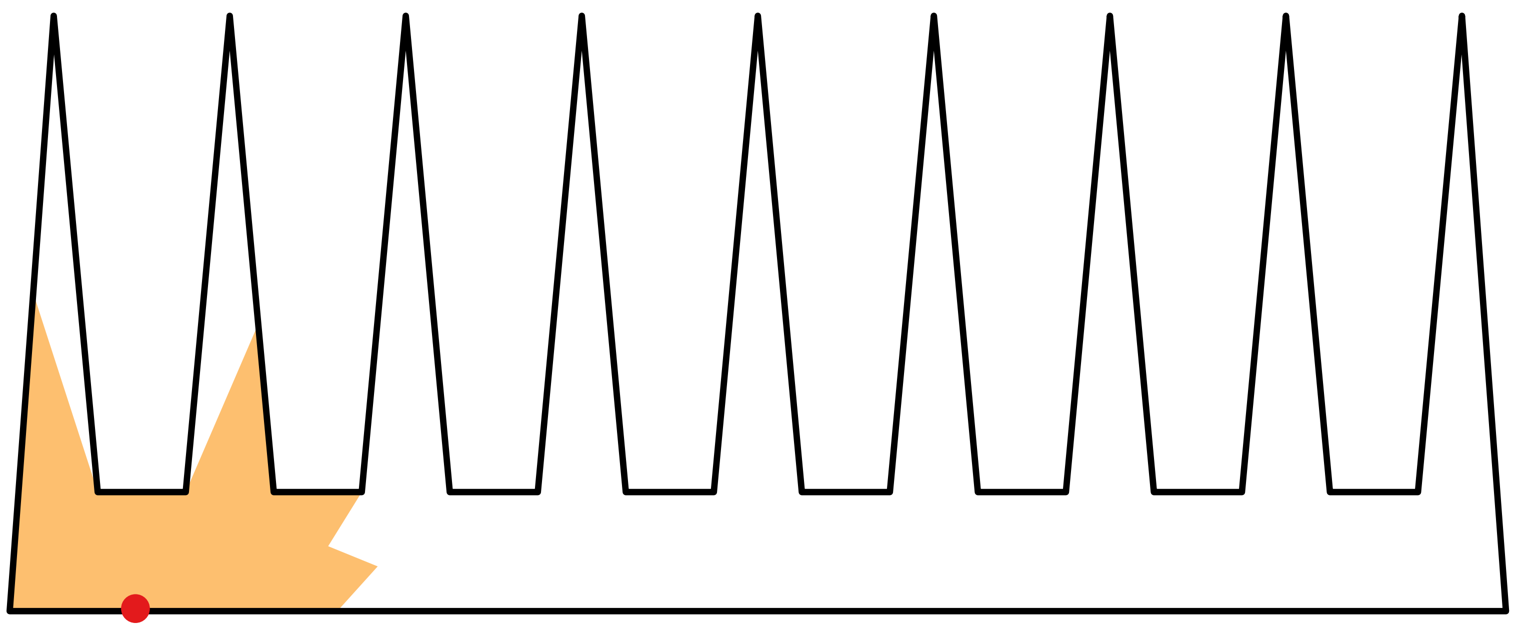

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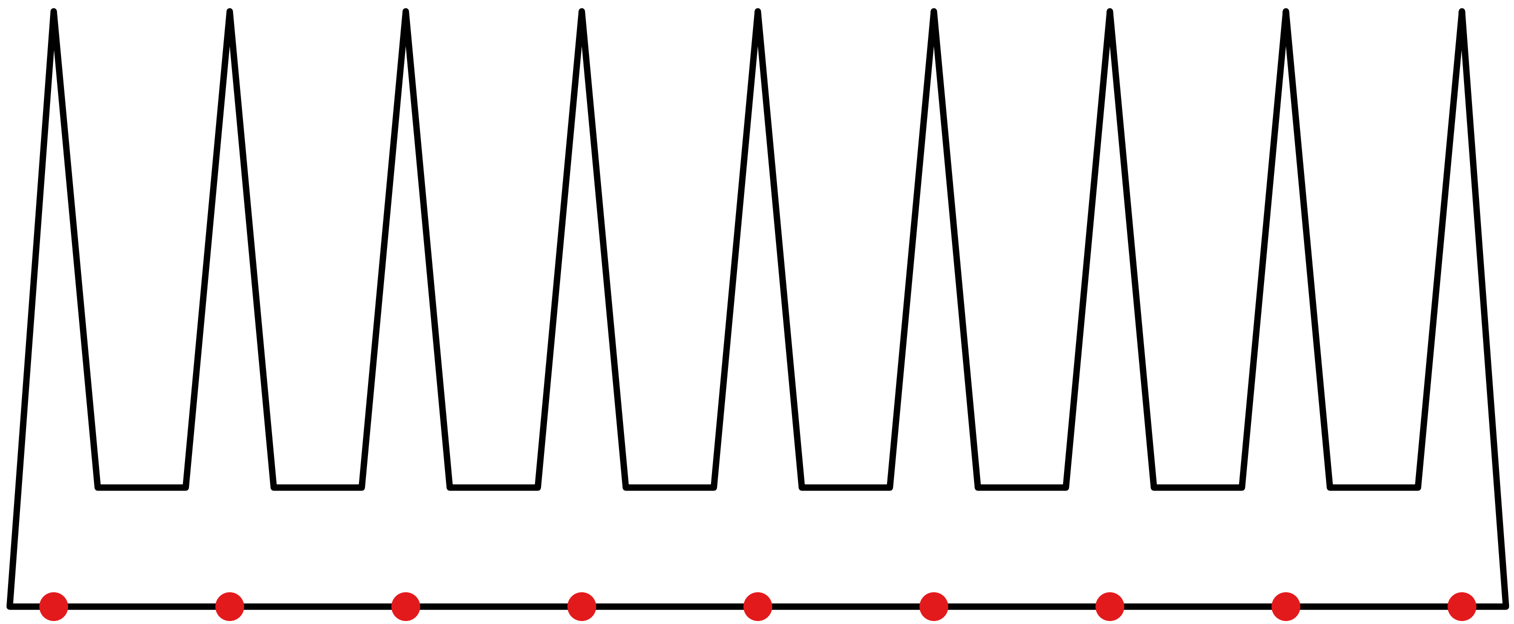

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rectilinear

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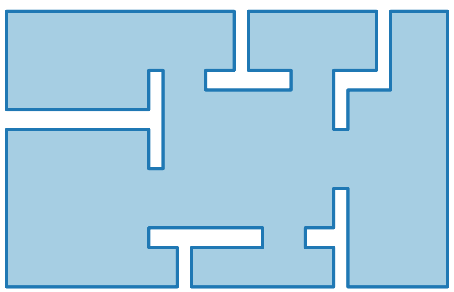
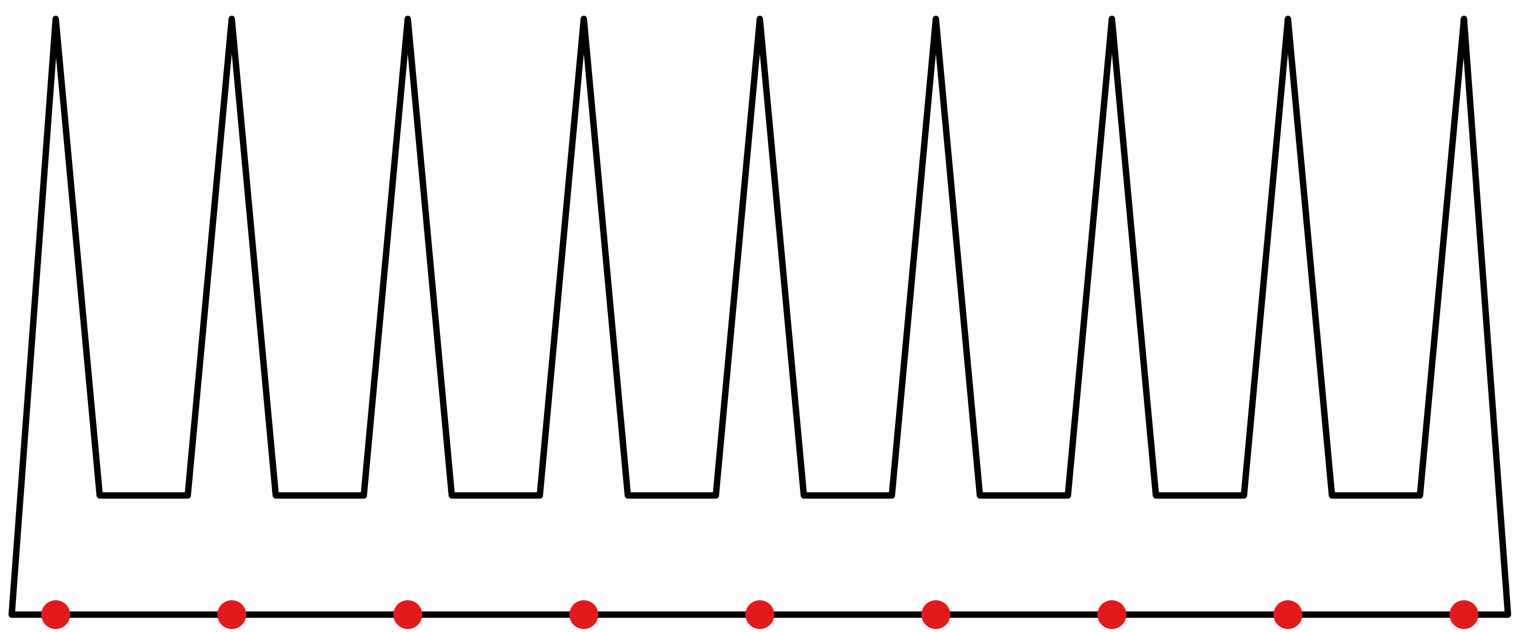
$n/4$





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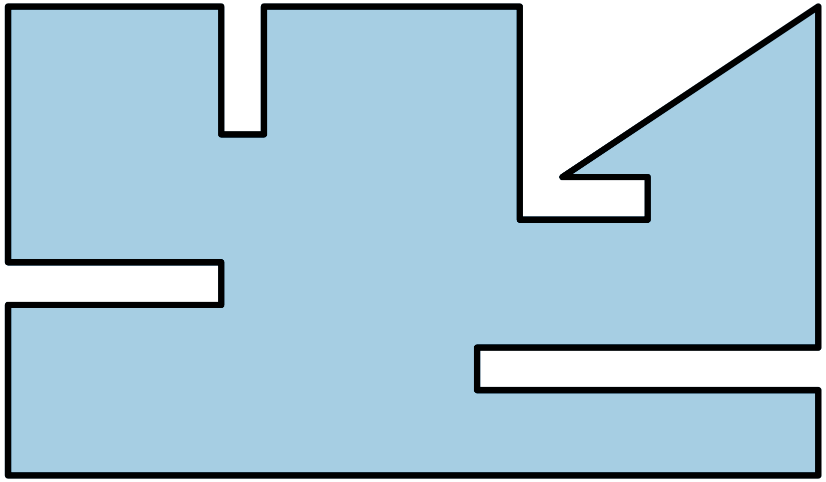

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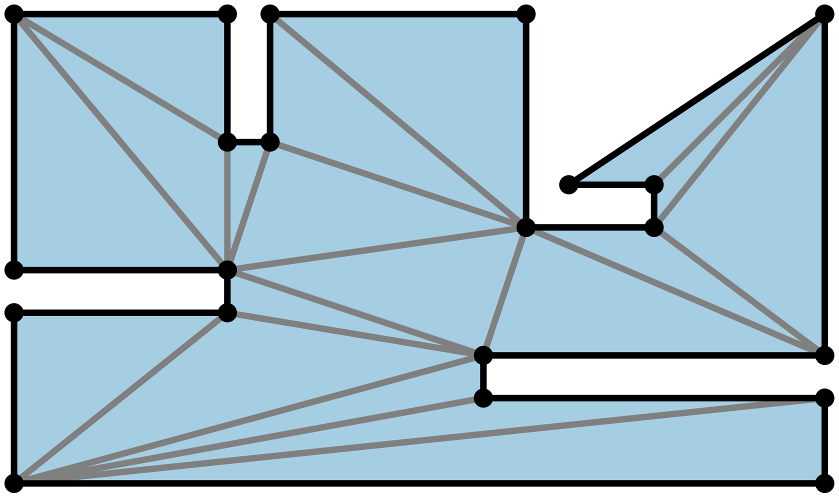

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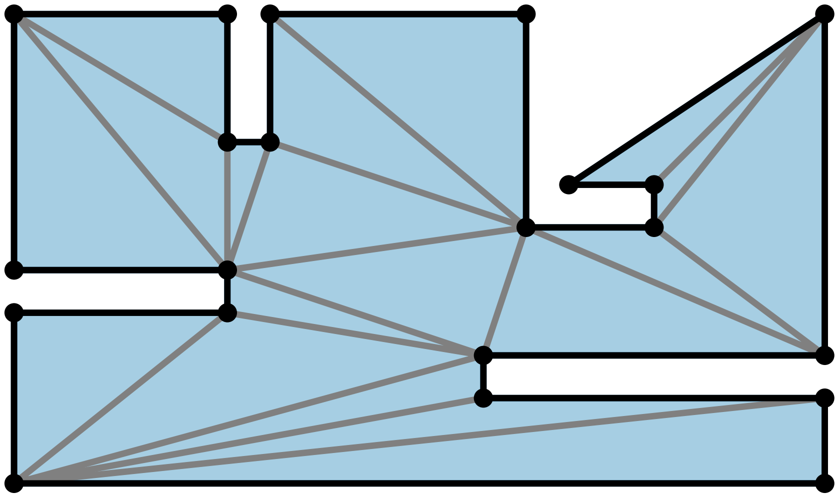
3-color the vtcs



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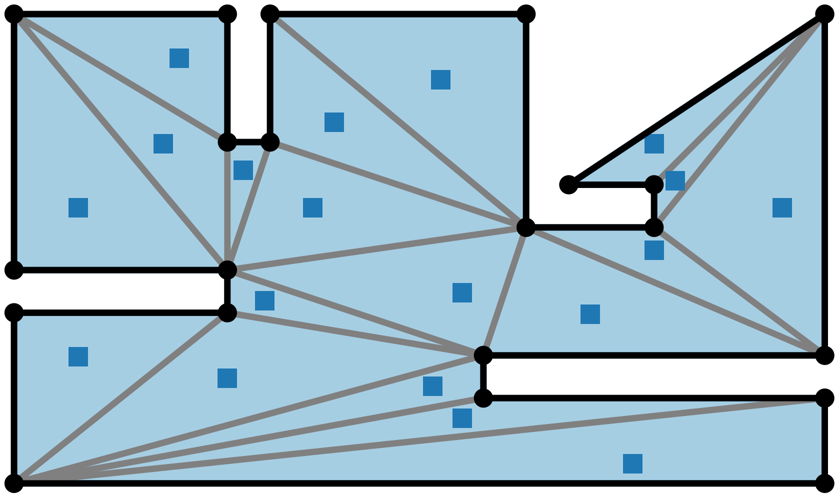

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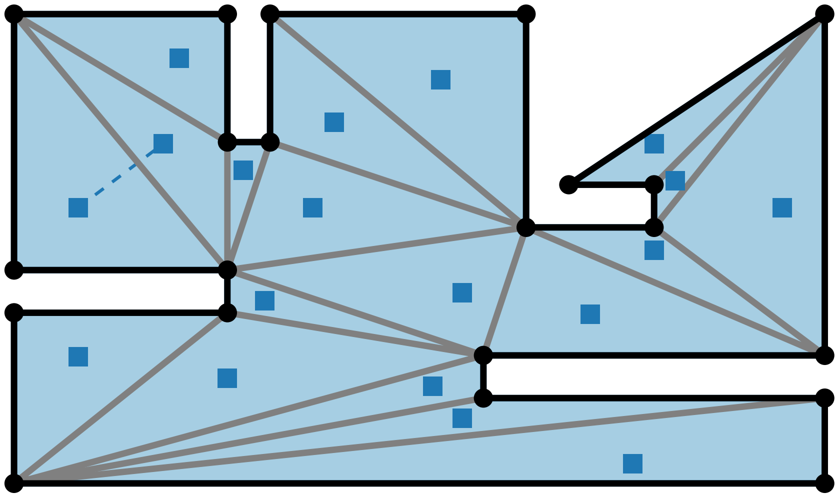

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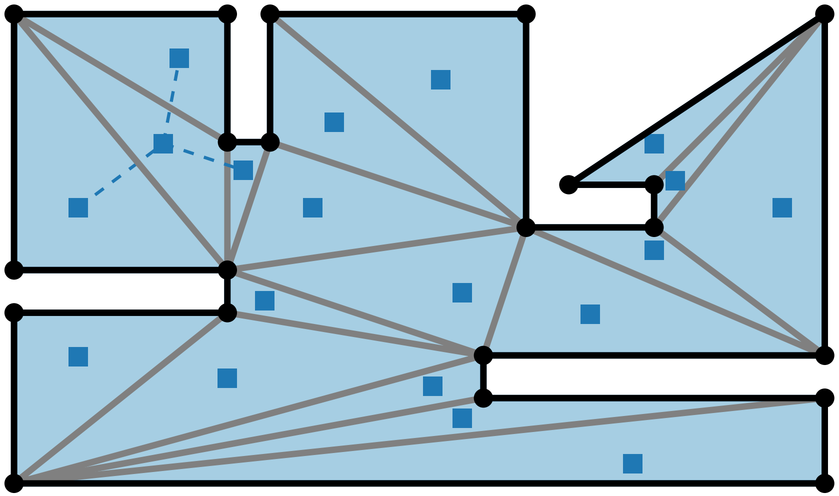

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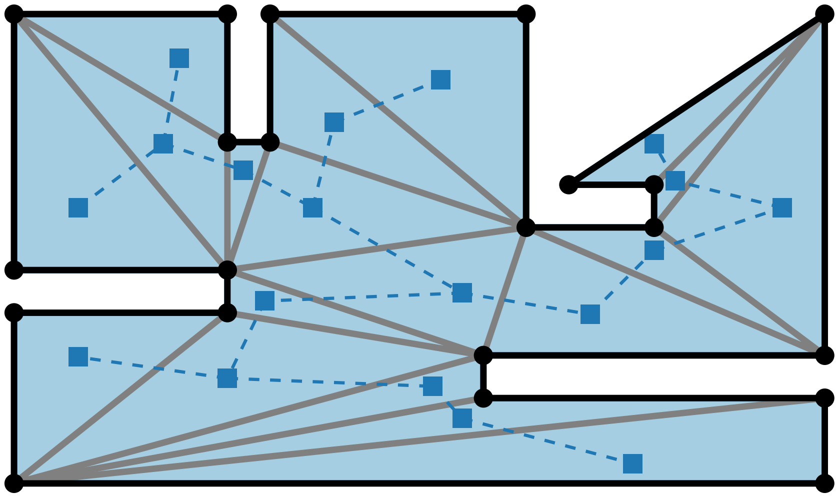
dual tree



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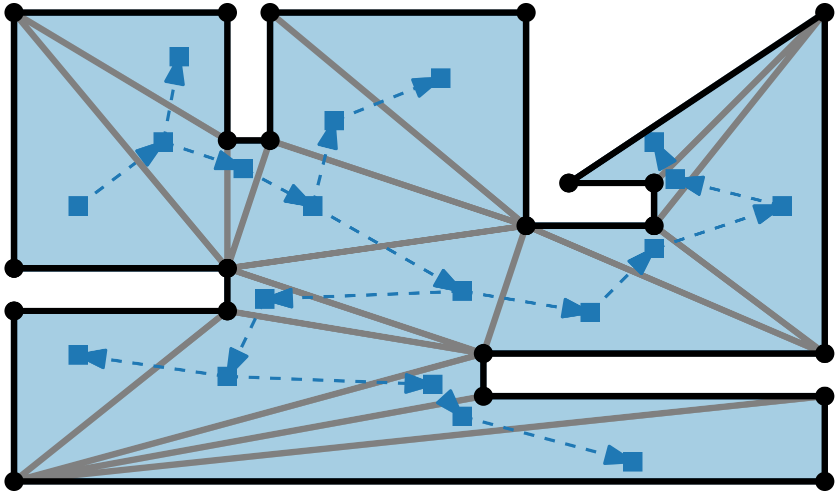

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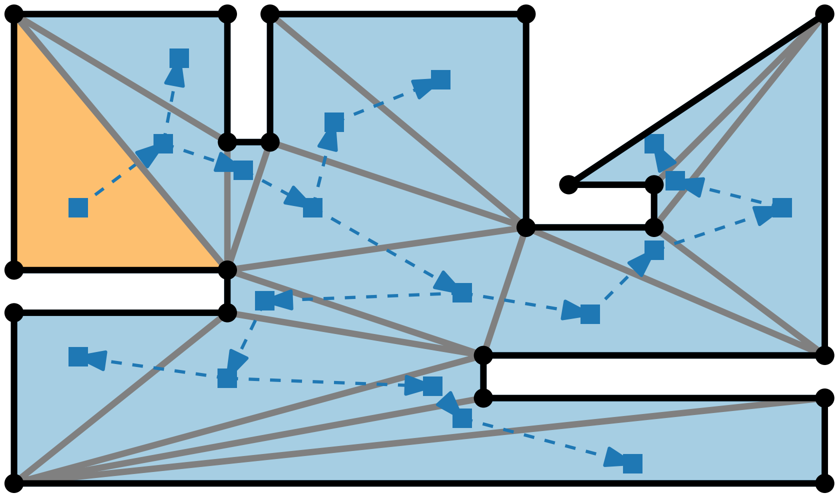

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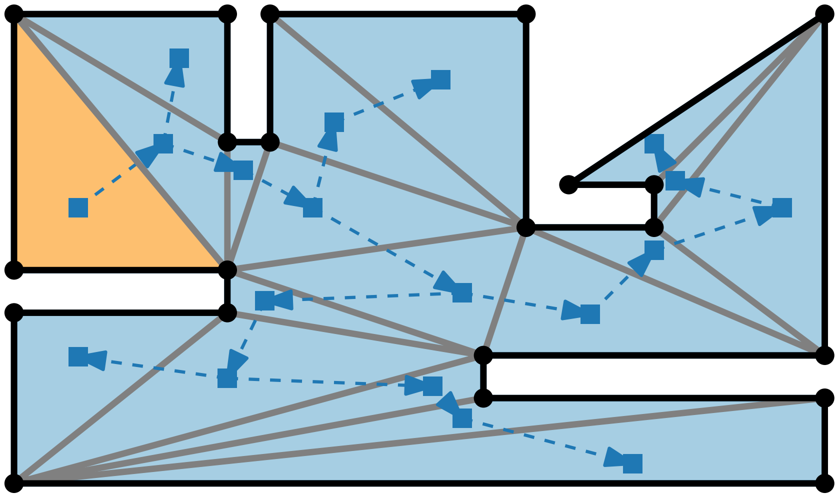

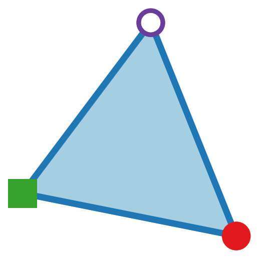

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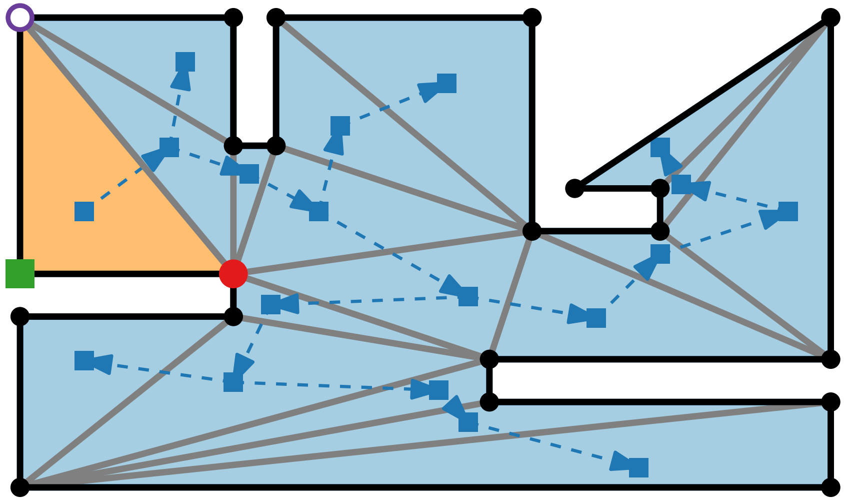

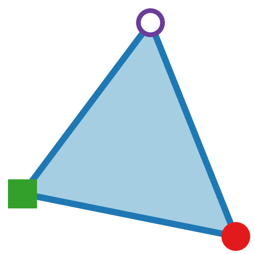

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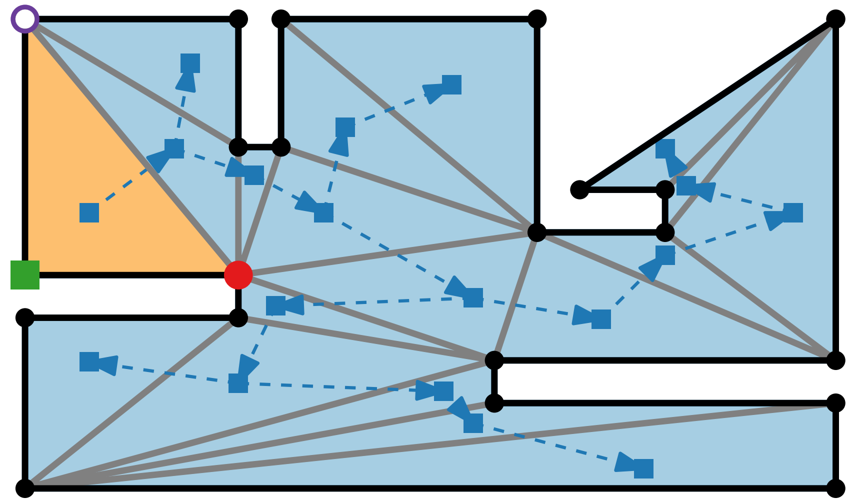

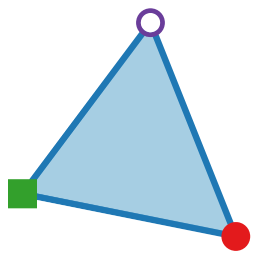

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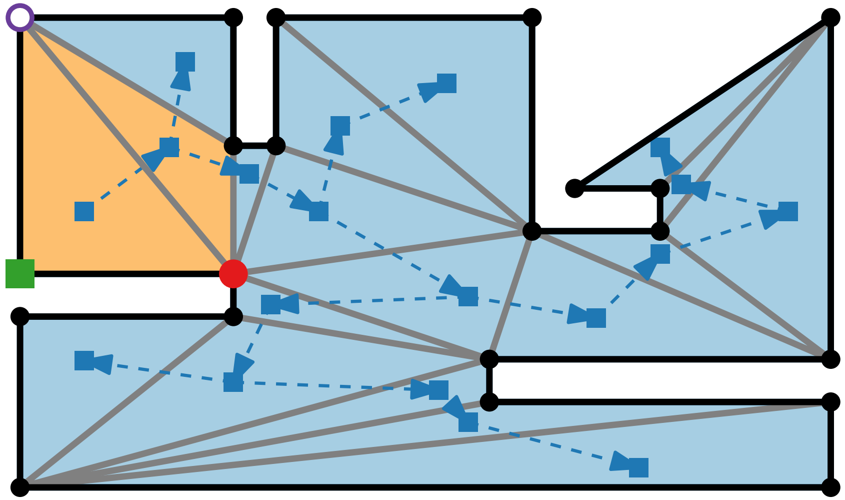

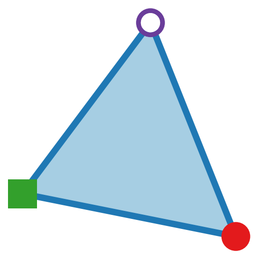

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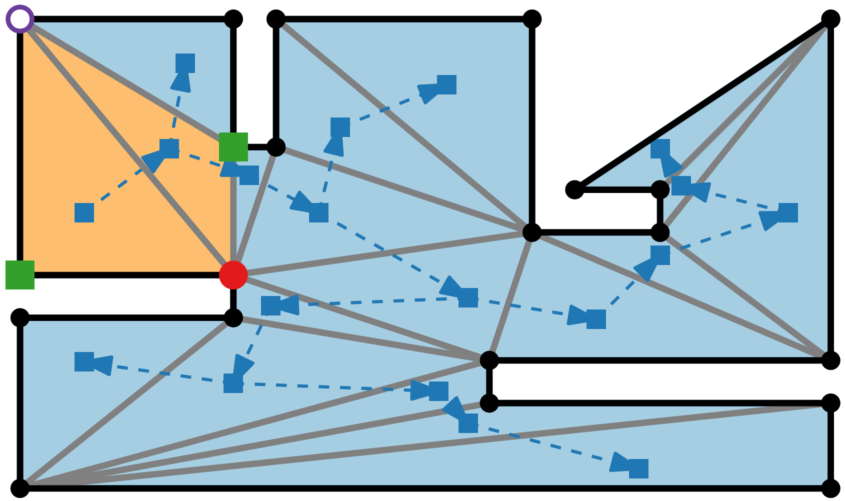

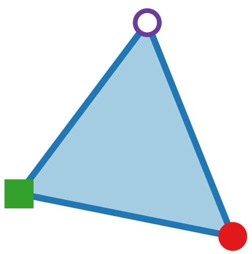

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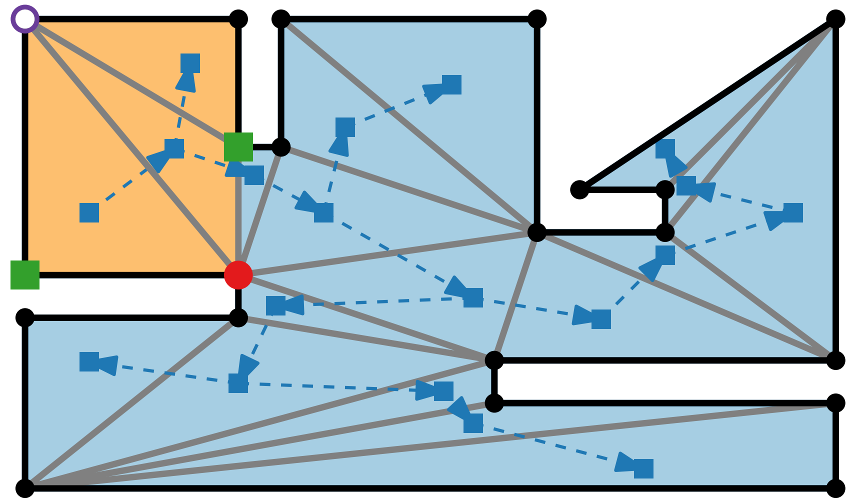

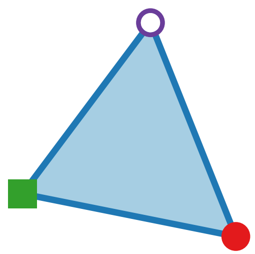

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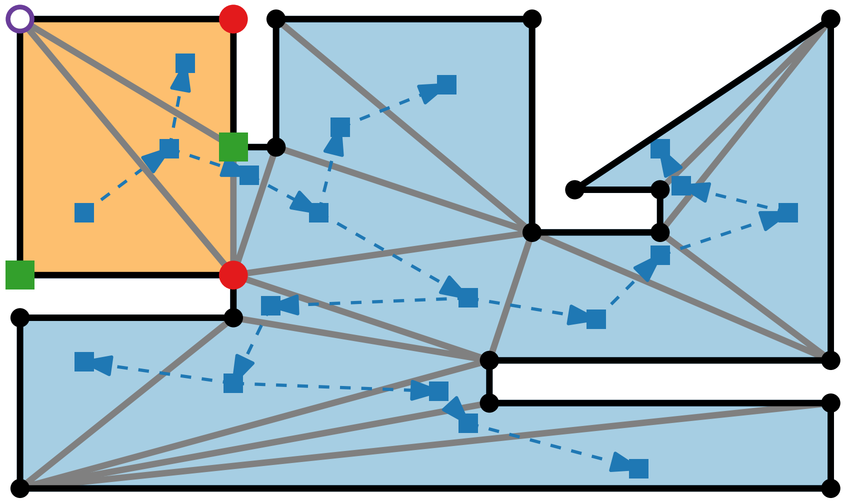

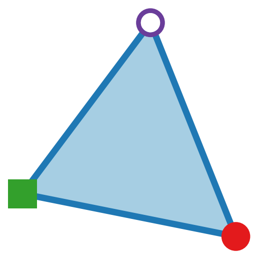

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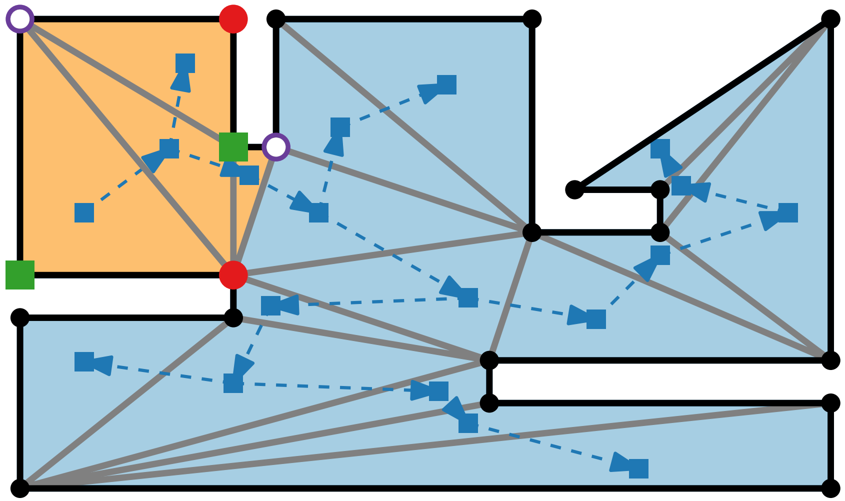

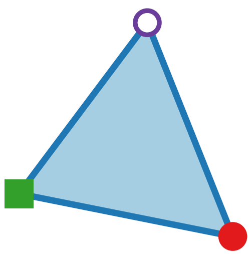

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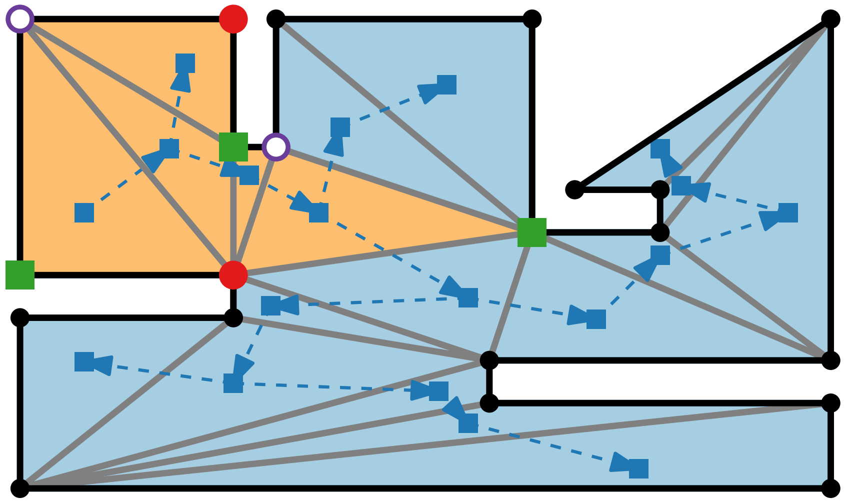

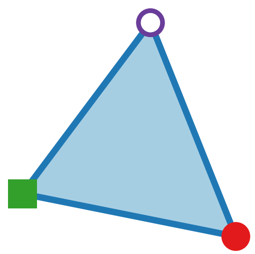

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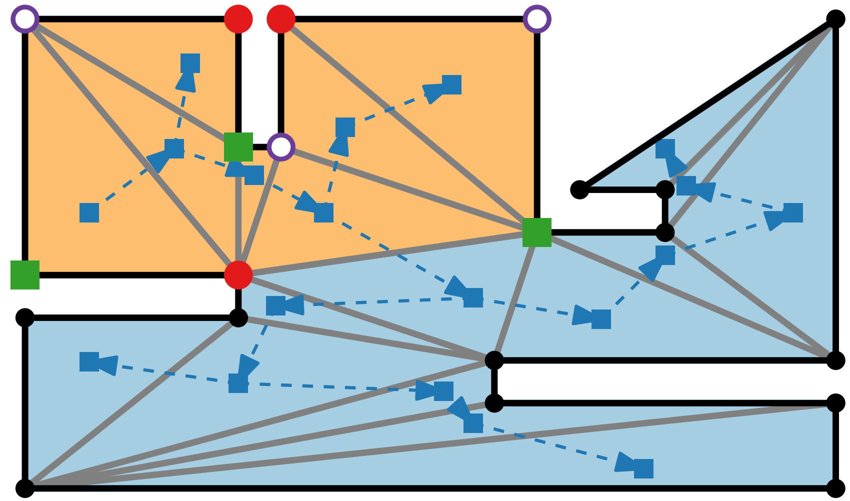

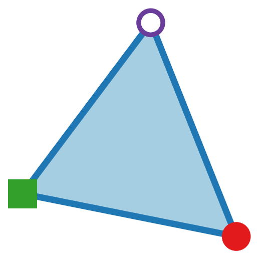

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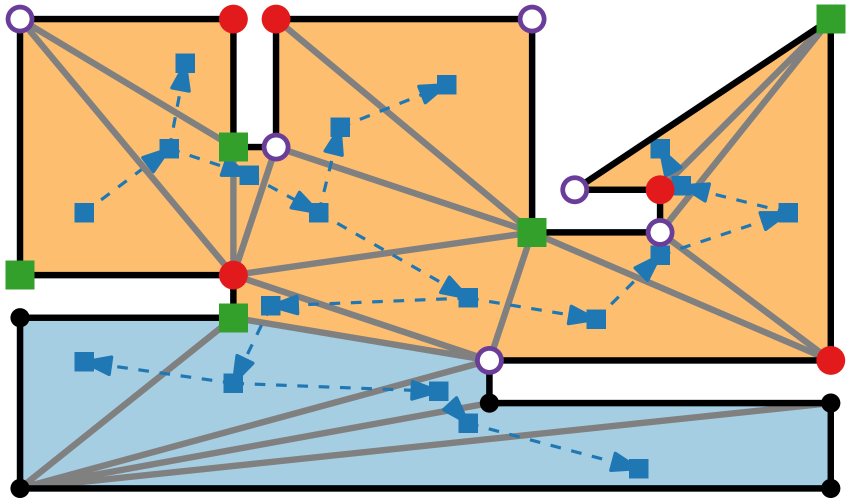

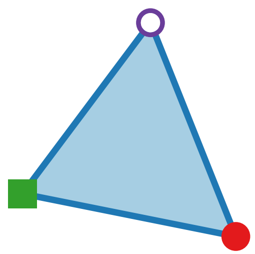

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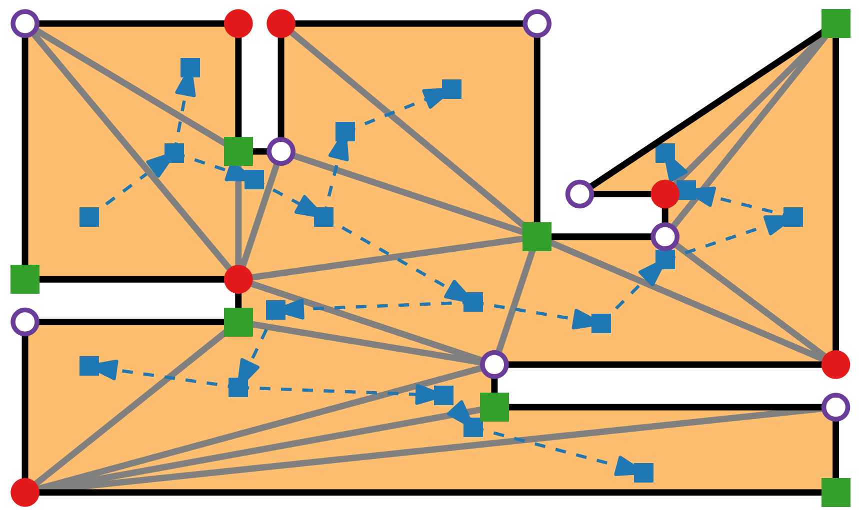

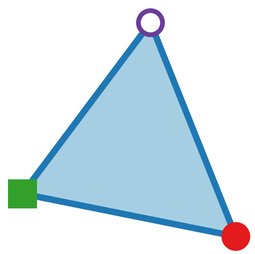

The Art Gallery Theorem

[Chvátal '75]

Theorem. For surveilling a simple polygon with n vertices, $\lfloor n/3 \rfloor$ cameras are sometimes necessary and always sufficient.

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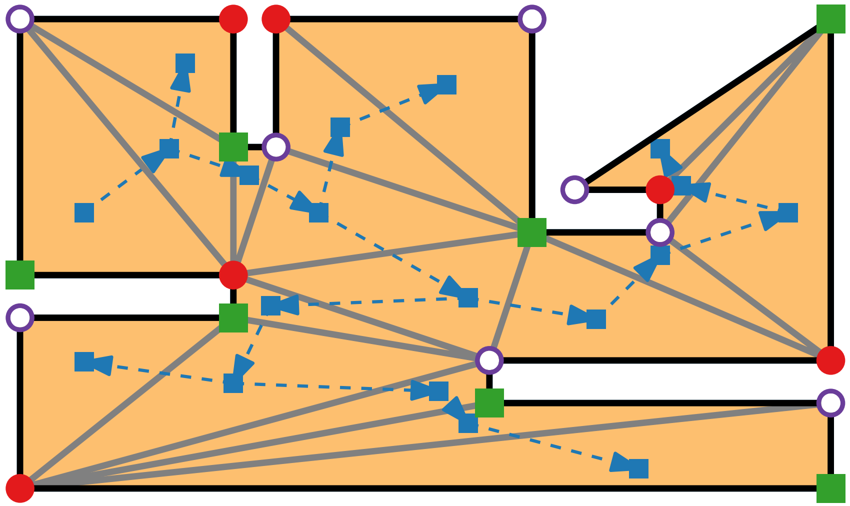
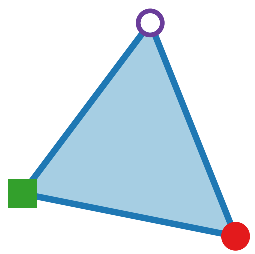


Pick "smallest" color



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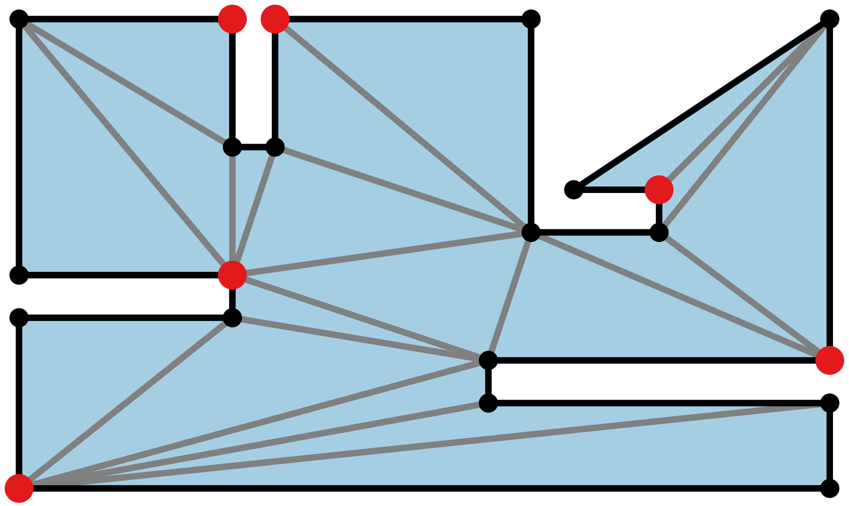

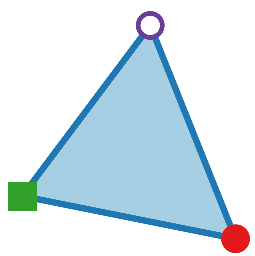

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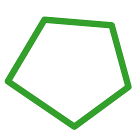

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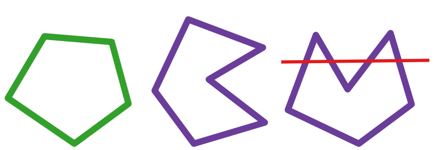

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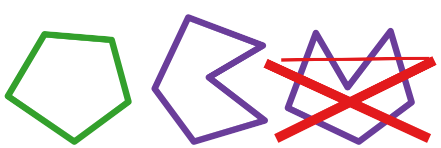

Computational Geometry

Lecture 3:
Guarding Art Galleries
or
Triangulating Polygons

Part III:
Partitioning a Polygon into y-monotone Pieces

Philipp Kindermann

Winter Semester 2020

Part. a Polygon into Monotone Pieces

Idea: Classify vertices of given simple polygon P

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– *turn* vertices:

– *regular* vertices

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vertical component of walking direction changes

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– *turn* vertices:

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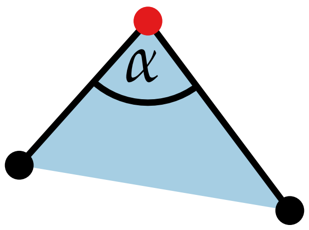

if $\alpha < 180^\circ$

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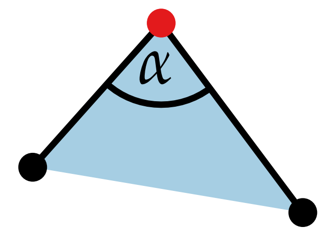
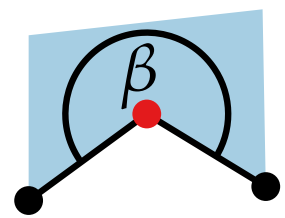


- *split* vertex if $\beta > 180^\circ$



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- *start* vertex — if $\alpha < 180^\circ$
- *split* vertex — if $\beta > 180^\circ$
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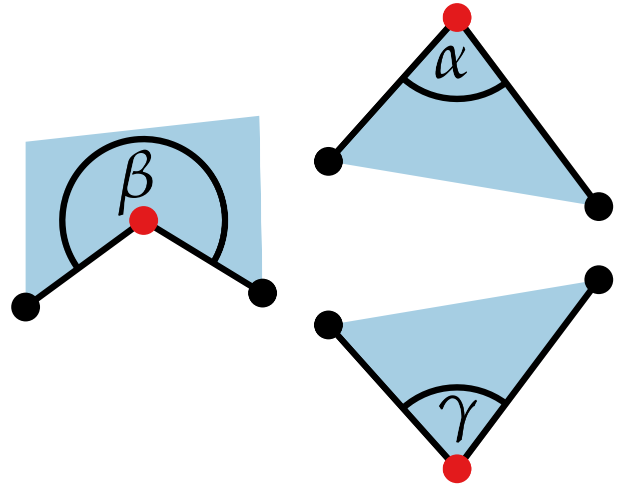

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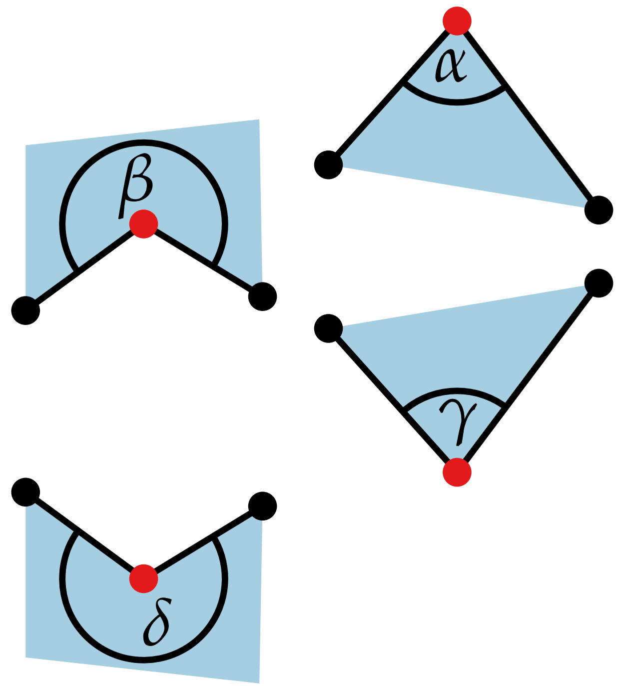

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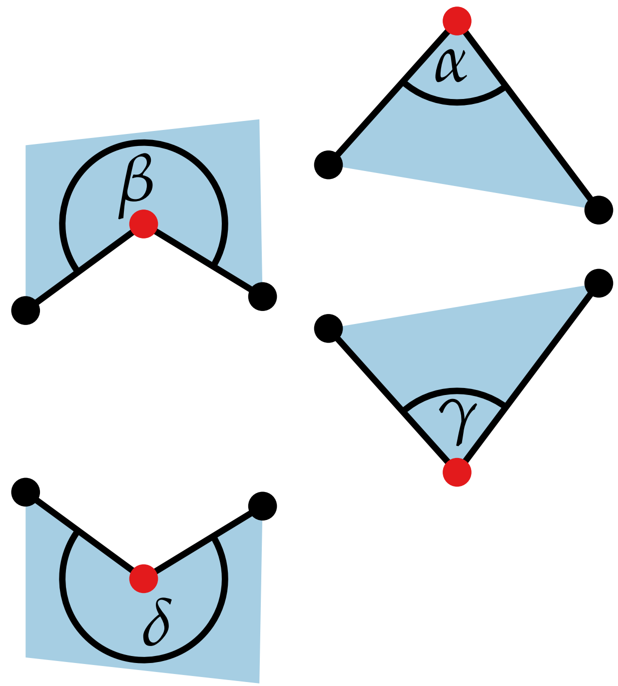
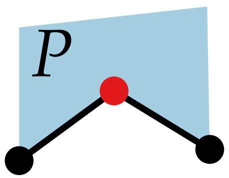
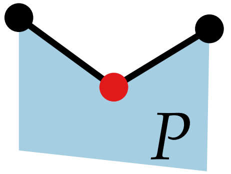


– *regular* vertices

Lemma. Let P be a simple polygon. Then P is y-monotone $\Leftrightarrow$ P has neither split vertices nor merge vertices.

Towards an Algorithm





Idea: Add diagonals to "destroy" split and merge vtcs.

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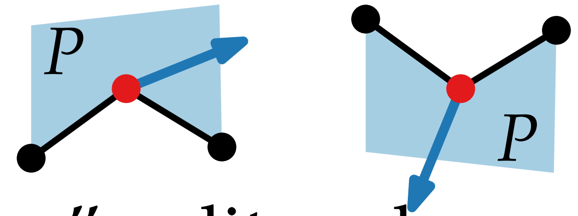

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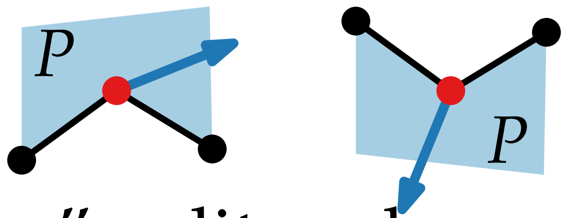

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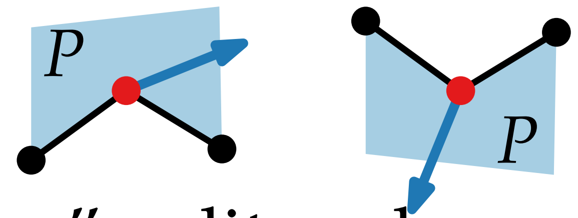
Problem: Diagonals must not cross

Towards an Algorithm



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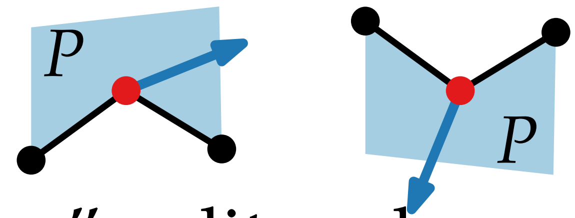
Problem: Diagonals must not cross – each other
– edges of P

Towards an Algorithm



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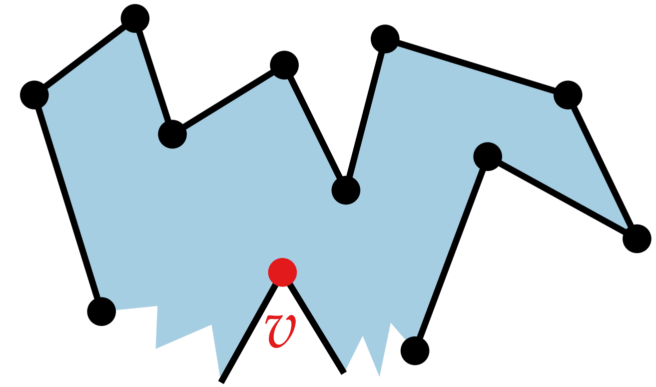
1) Treating split vertices



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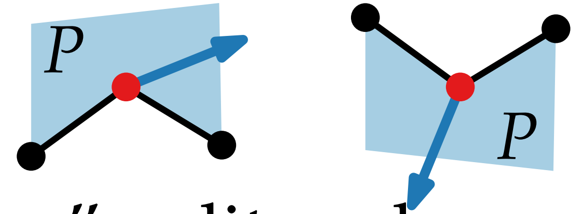

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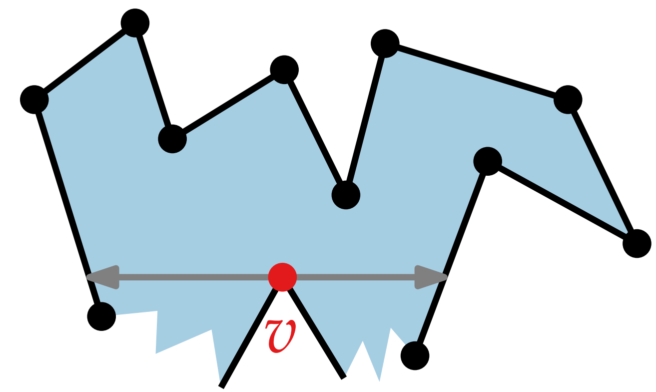

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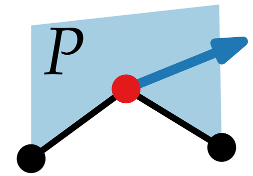

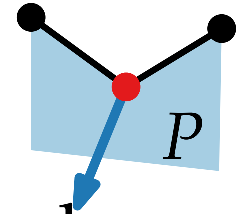

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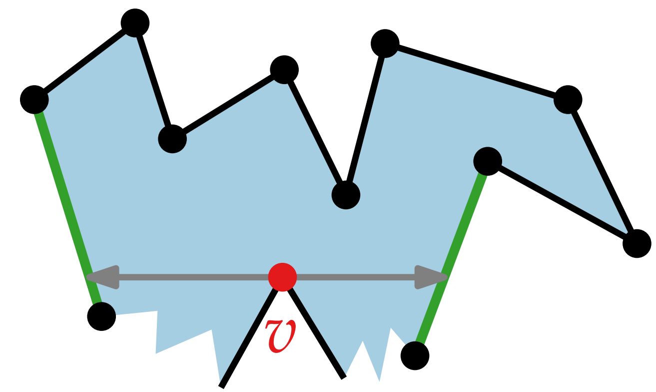

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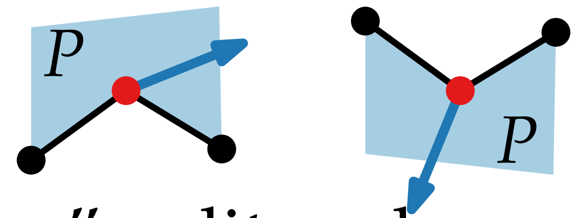

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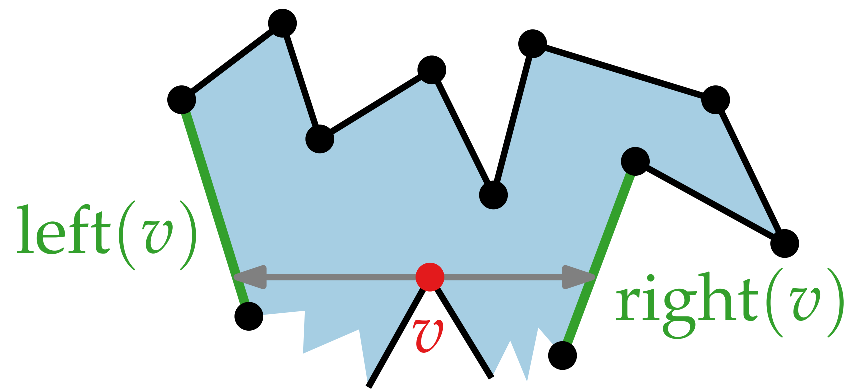

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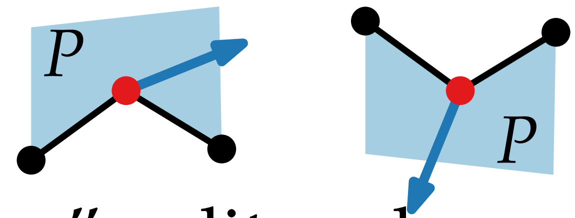

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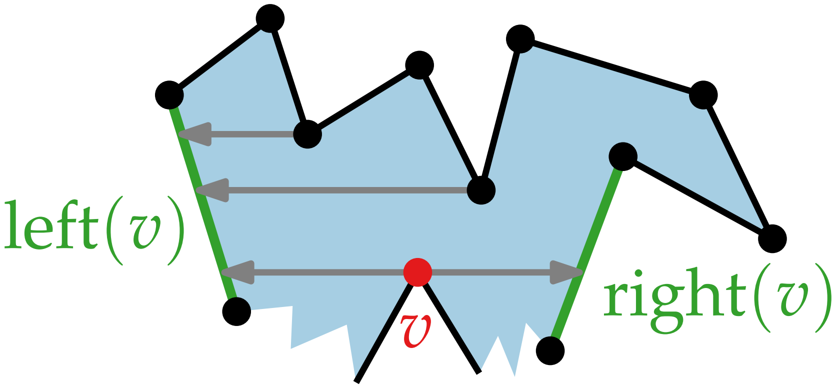

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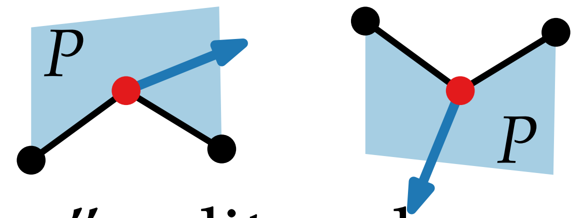

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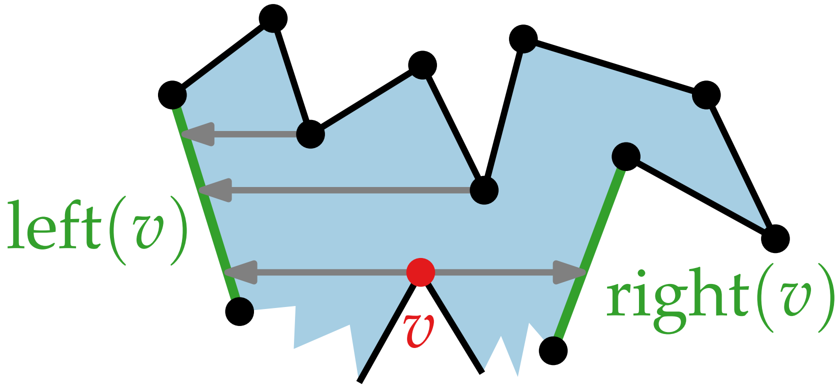
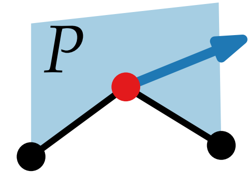
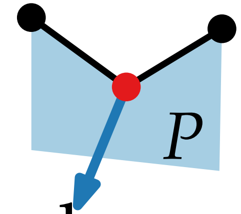


Connect v to vertex $w^\star$ having minimum y-coordinate among all vertices w above v and with $\text{left}(w) = \text{left}(v)$.

Towards an Algorithm





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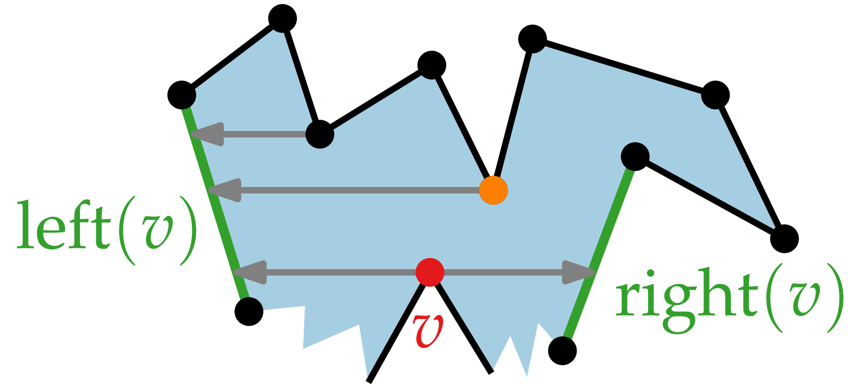

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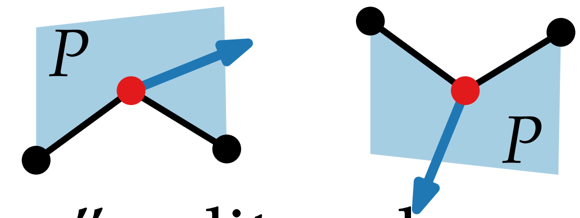

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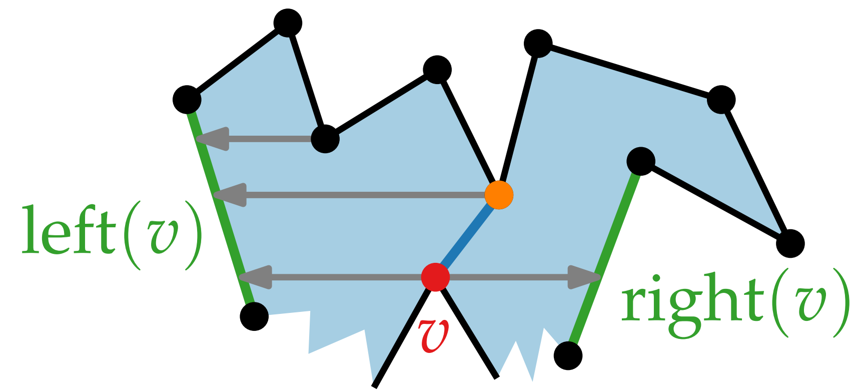

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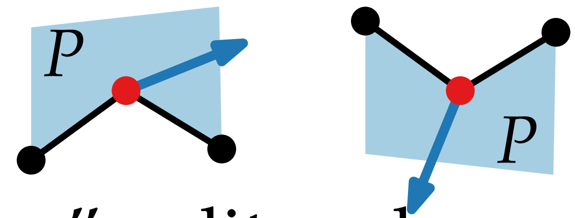

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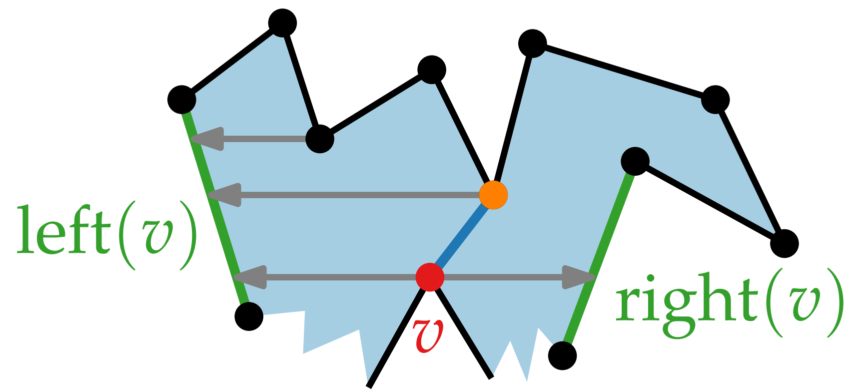

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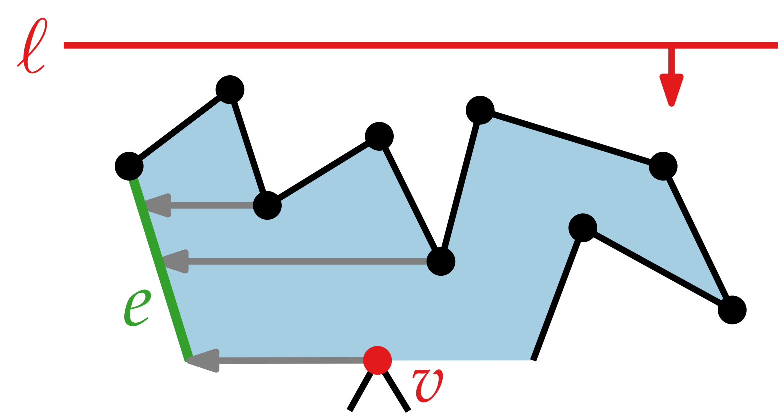
Think of a sweep-line algorithm:



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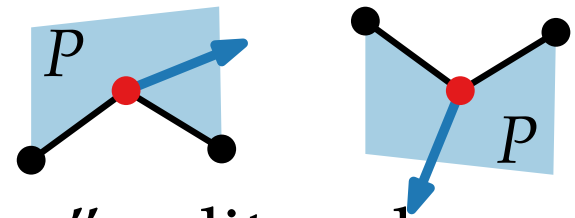

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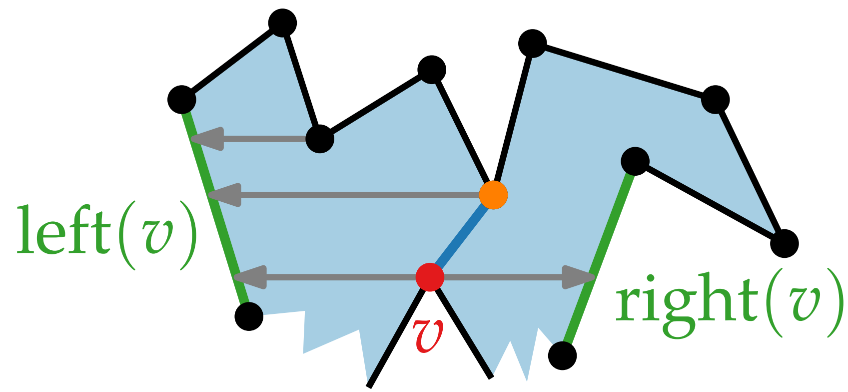

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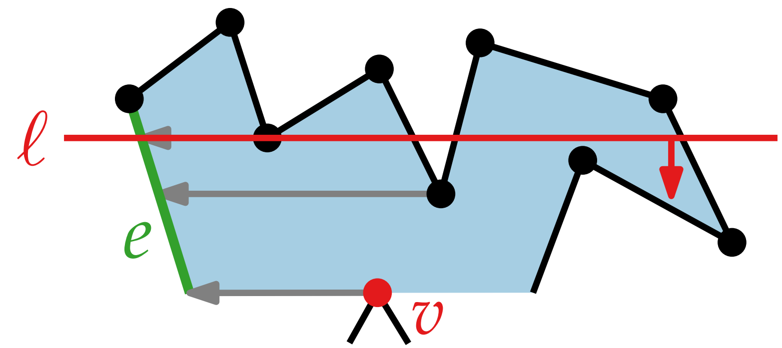
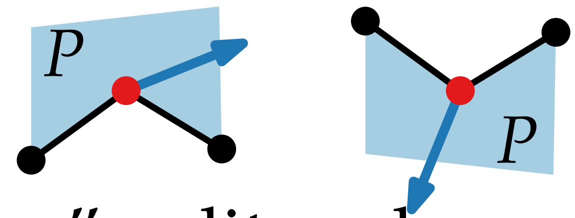


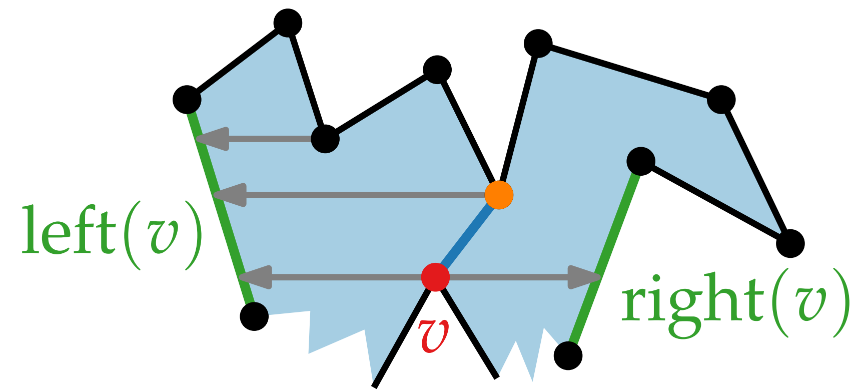
Towards an Algorithm



Idea: Add diagonals to “destroy” split and merge vtcs.

Problem: Diagonals must not cross – each other
– edges of P

1) Treating split vertices



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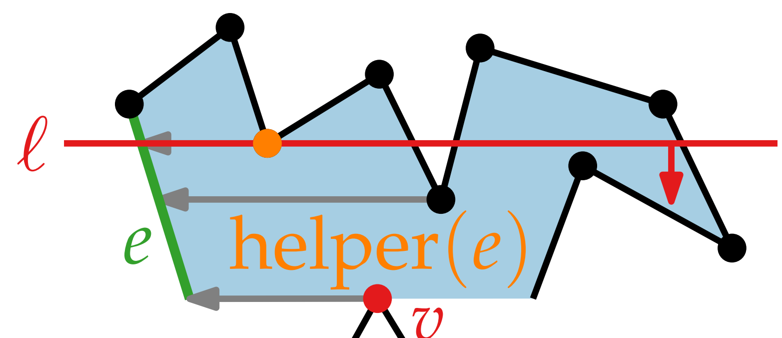

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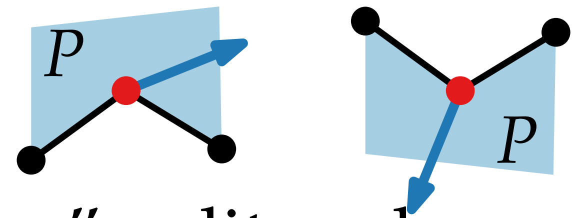

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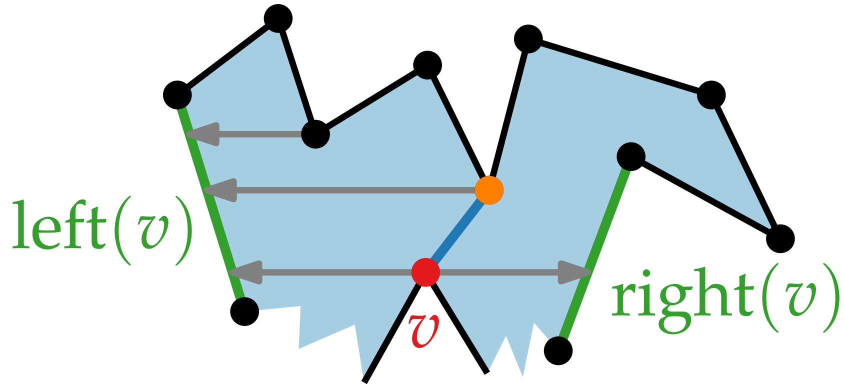

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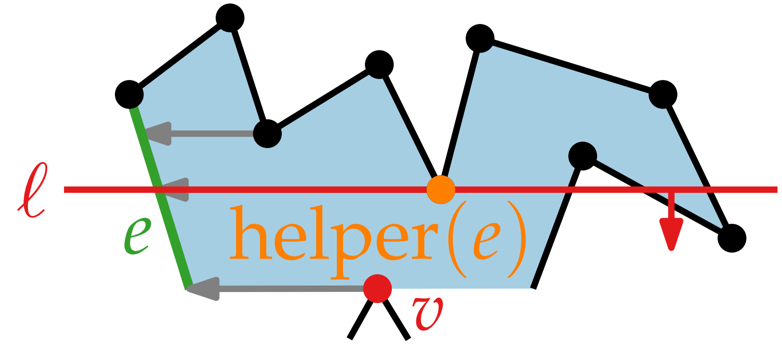
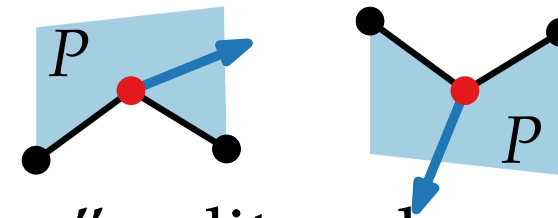


Towards an Algorithm



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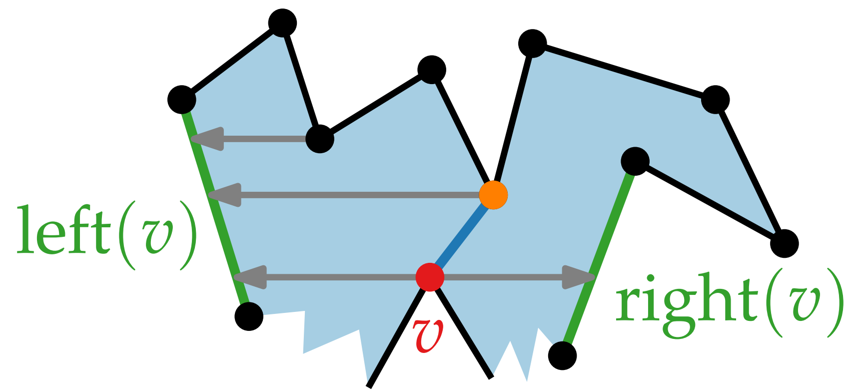

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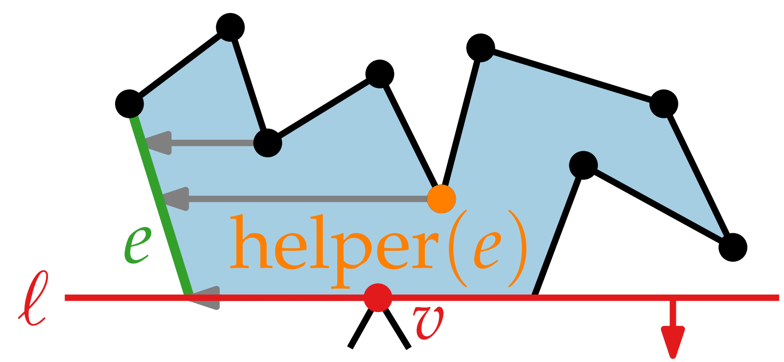
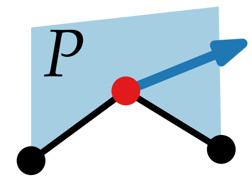
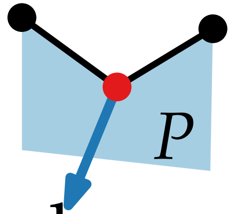


Towards an Algorithm





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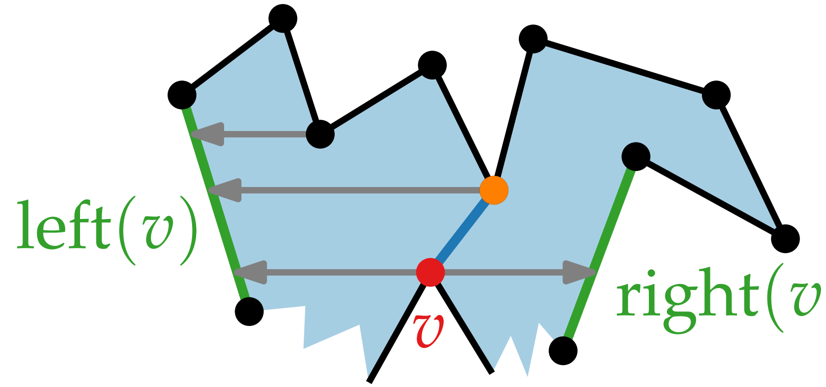

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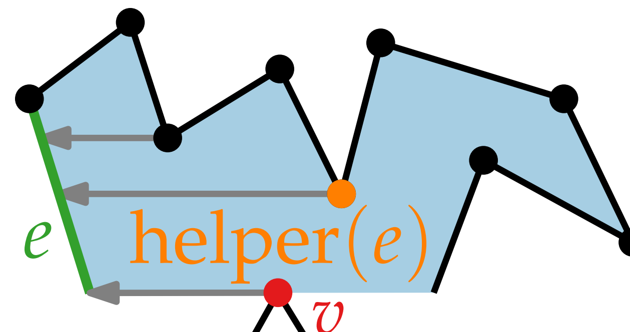
Think of a sweep-line algorithm:

Connect v to helper(left(v)).



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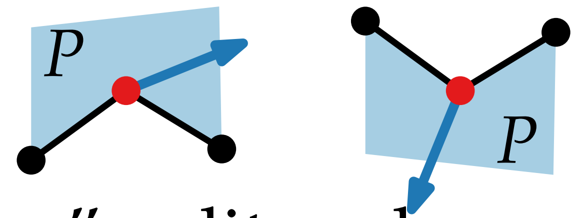

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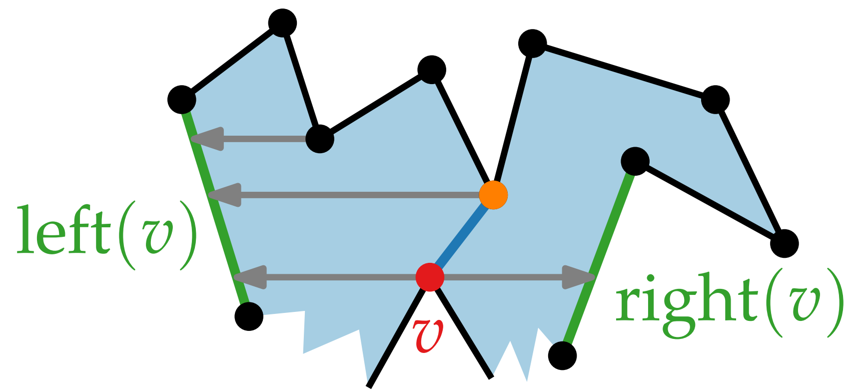

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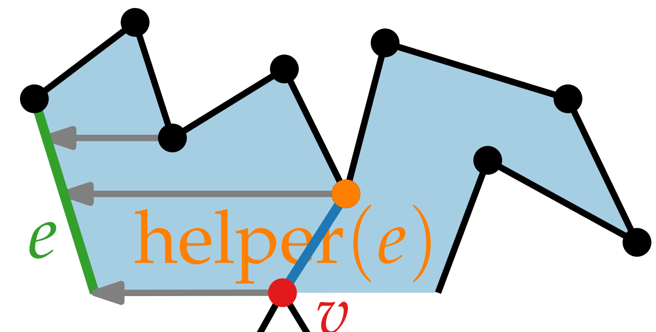
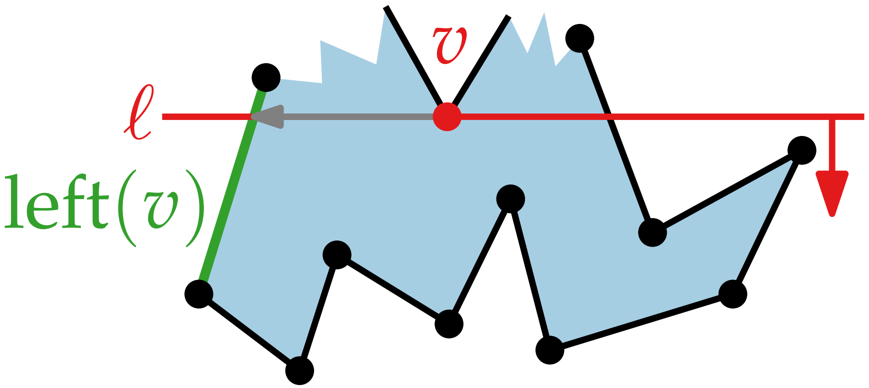


An Algorithm

2) Treating merge vertices



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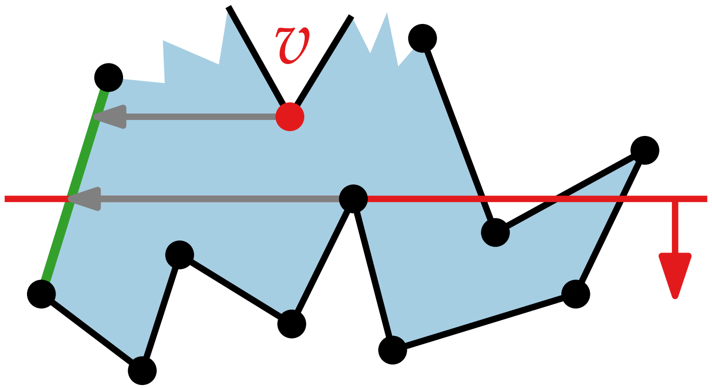

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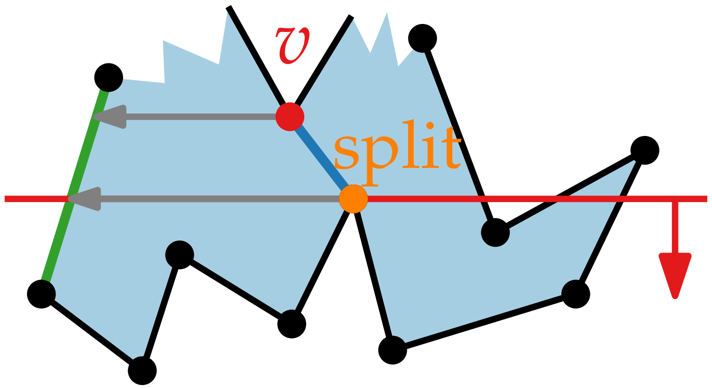

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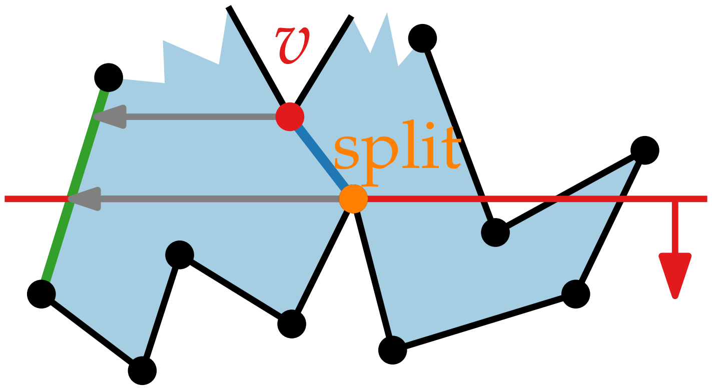

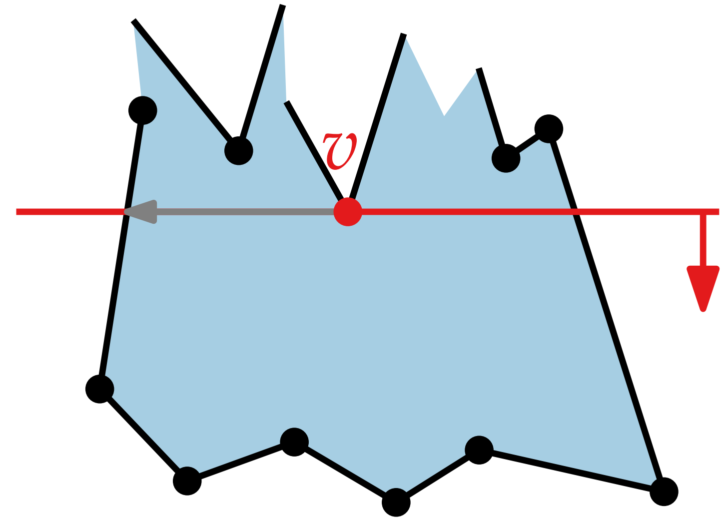

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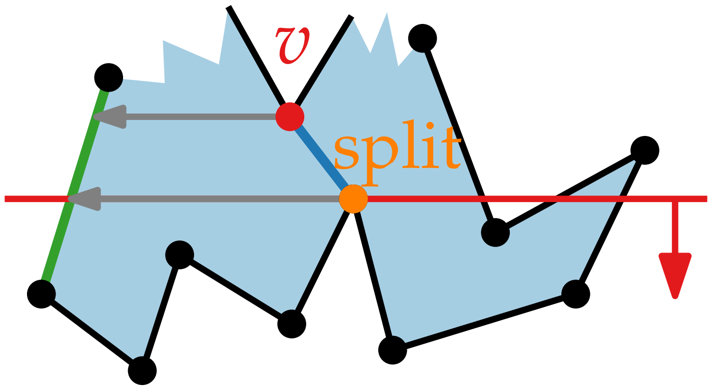

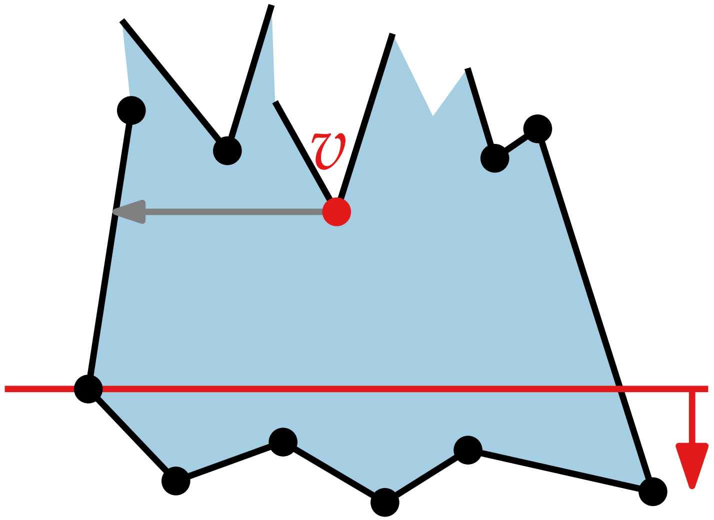

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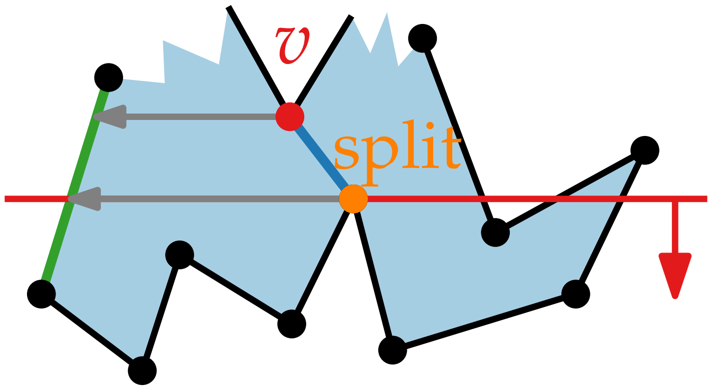

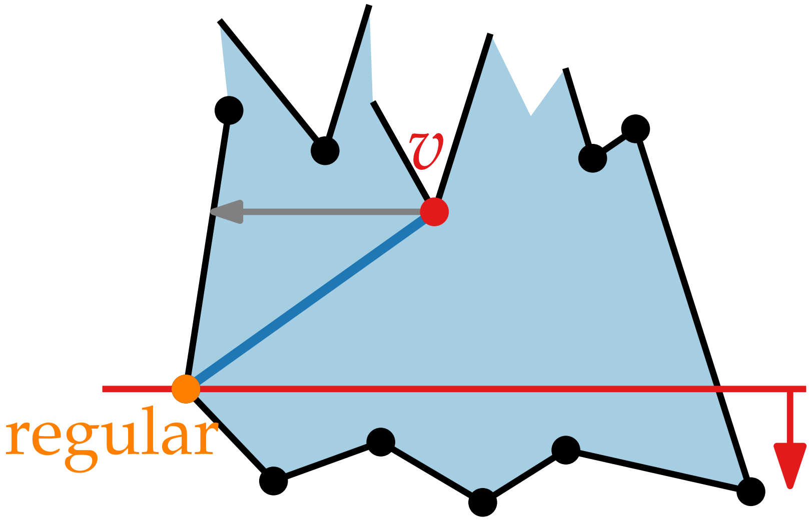

An Algorithm

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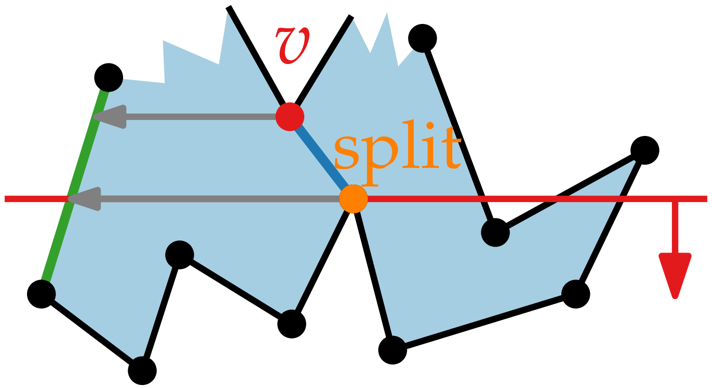

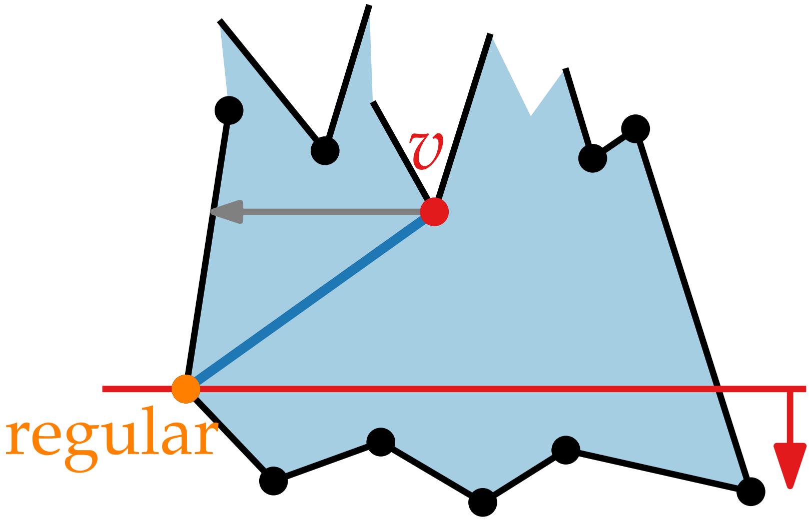

makeMonotone(polygon P**)**
$\mathcal{D} \leftarrow \text{DCEL}(V(P), E(P))$
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An Algorithm

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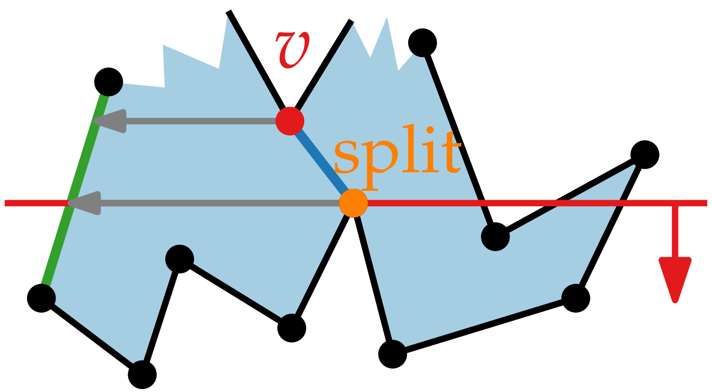

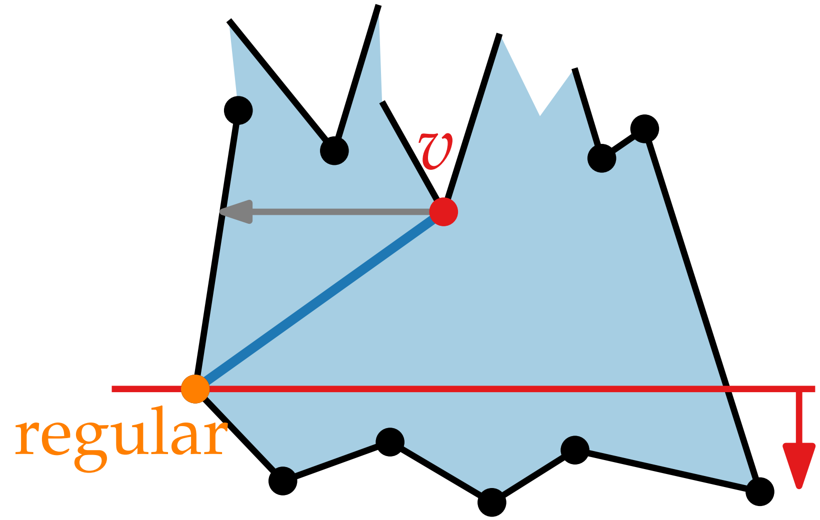

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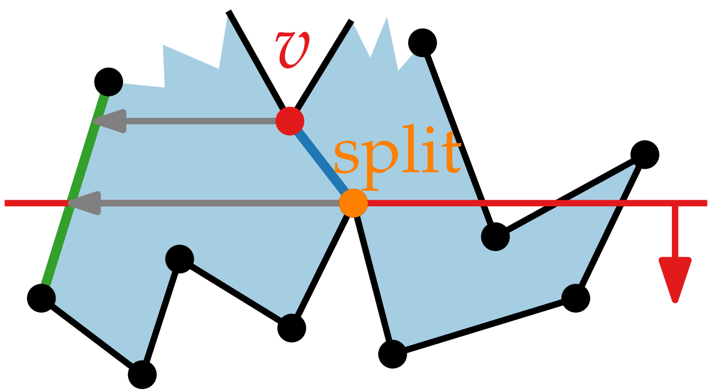
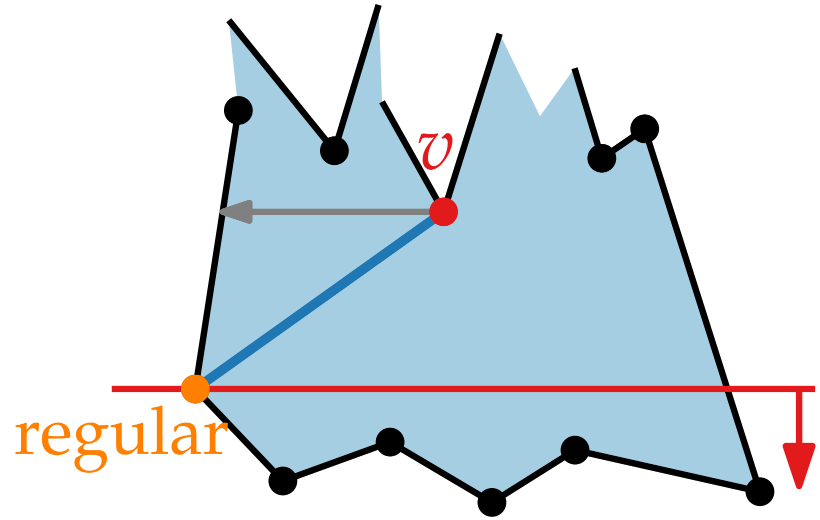
doubly-connected edge list:
data structure for planar subdivisions

An Algorithm

2) Treating merge vertices





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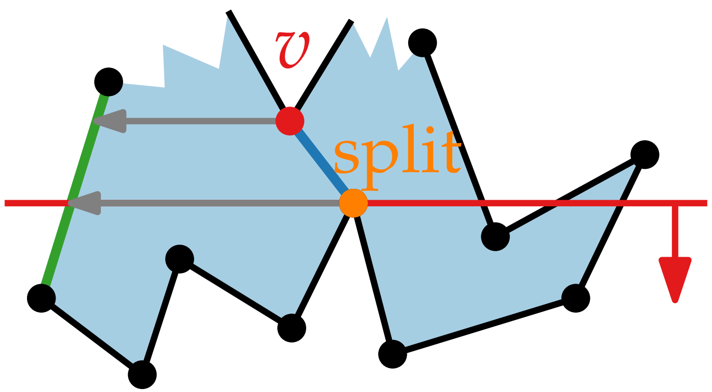
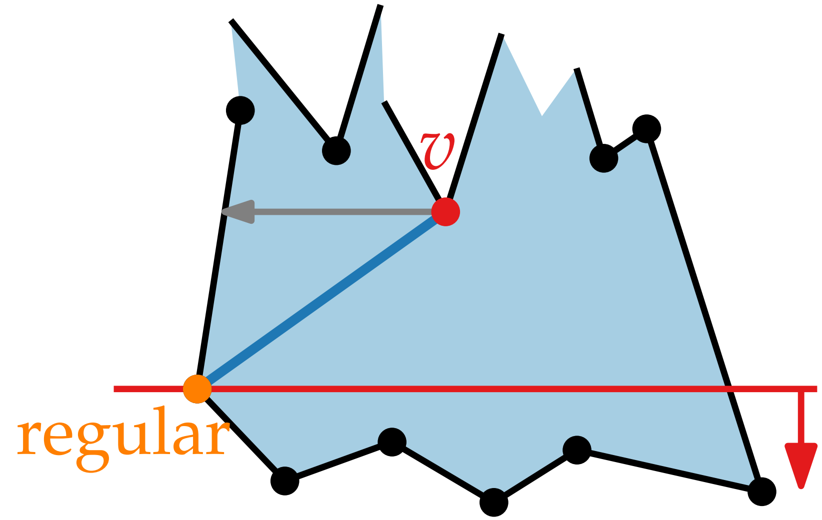
$(x, y) \prec (x', y') \;:\Leftrightarrow$
$y > y' \vee (y = y' \wedge x < x')$

An Algorithm

2) Treating merge vertices





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  type $\leftarrow$ type of vertex $v \in$ start, split, end, merge, regular
  handleTypeVertex(v)
return DCEL $\mathcal{D}$

doubly-connected edge list:
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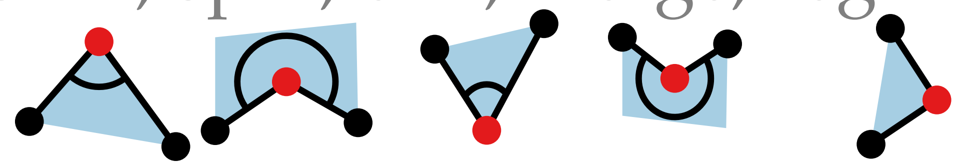
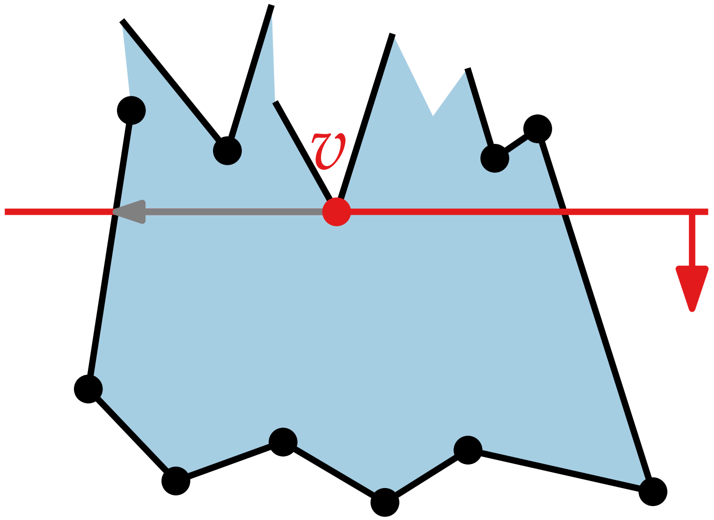


An Algorithm

2) Treating merge vertices



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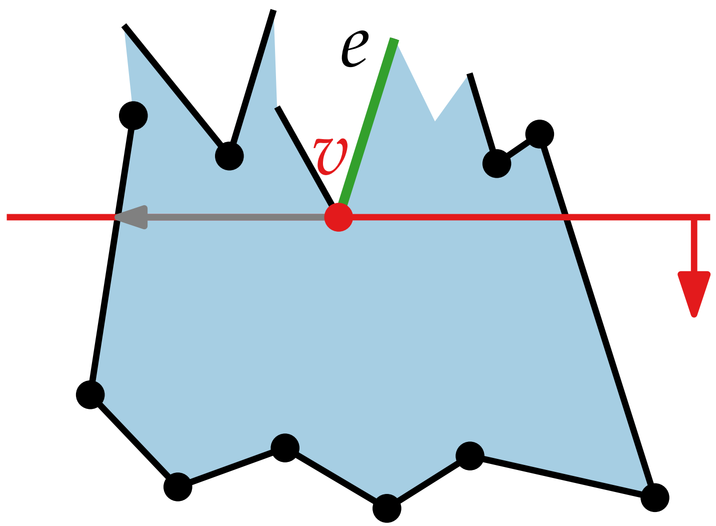
An Algorithm

2) Treating merge vertices



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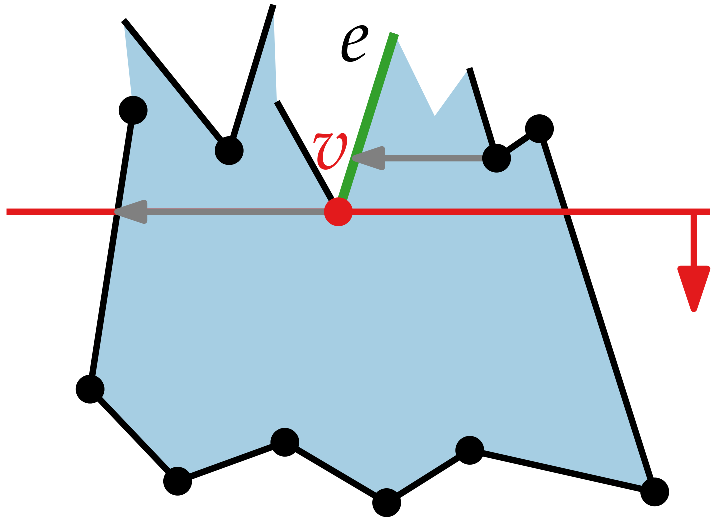
$e \leftarrow$ edge following v cw

An Algorithm

2) Treating merge vertices



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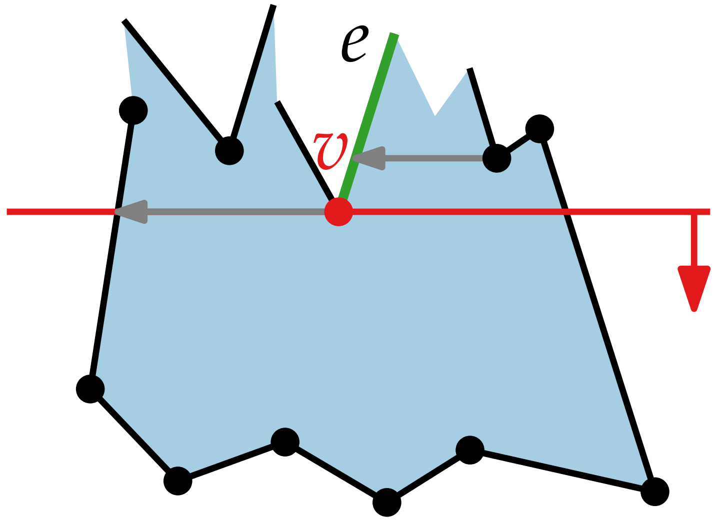
An Algorithm

2) Treating merge vertices



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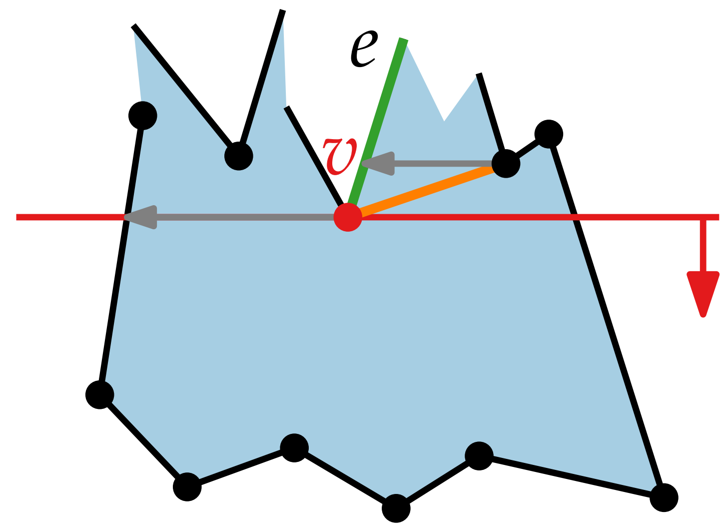
handleMergeVertex(vertex v**)**
$e \leftarrow$ edge following v cw
if helper(e) merge vtx **then**
    $\mathcal{D}$.insert(diag(v, helper(e)))

An Algorithm

2) Treating merge vertices



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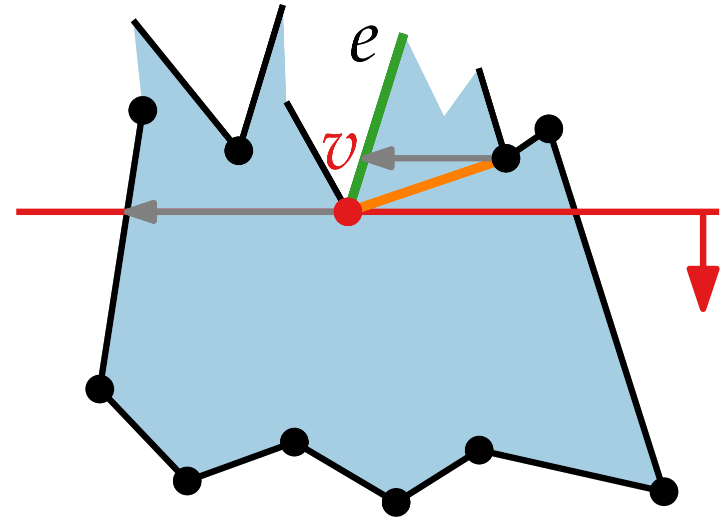

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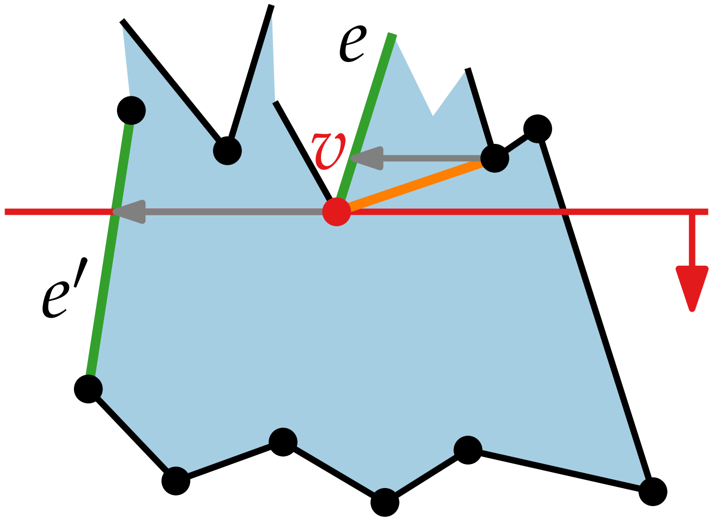
$\mathcal{T}$.delete(e)

An Algorithm

2) Treating merge vertices



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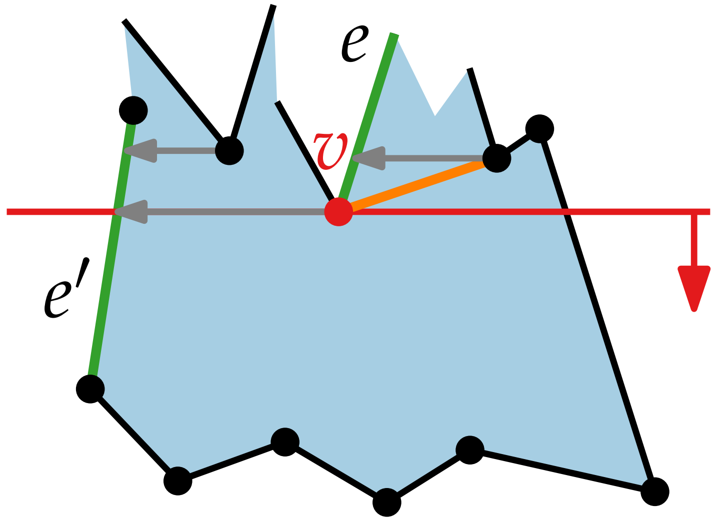
$e' \leftarrow \mathcal{T}$.edgeLeftOf$(v)$

An Algorithm

2) Treating merge vertices



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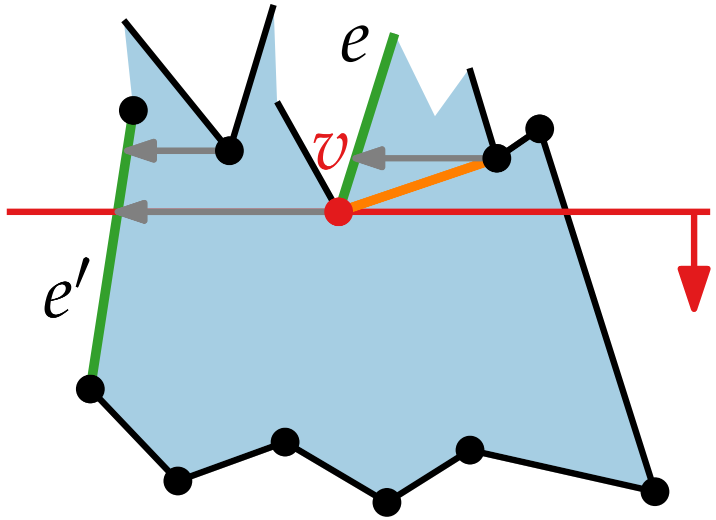
An Algorithm

2) Treating merge vertices



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An Algorithm

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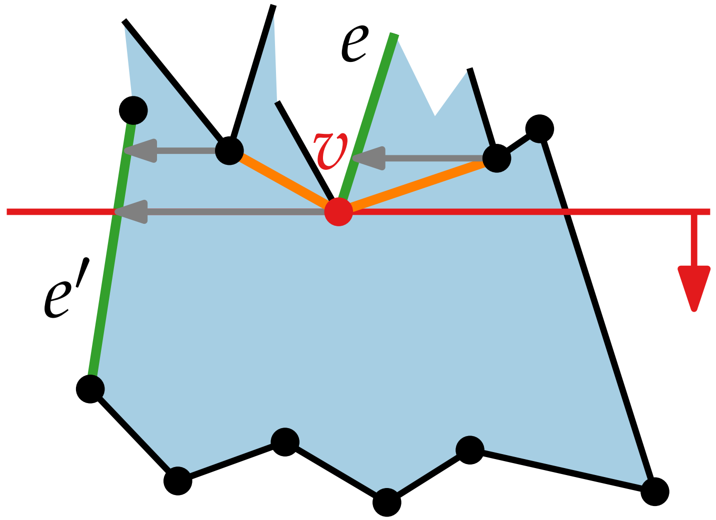

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An Algorithm

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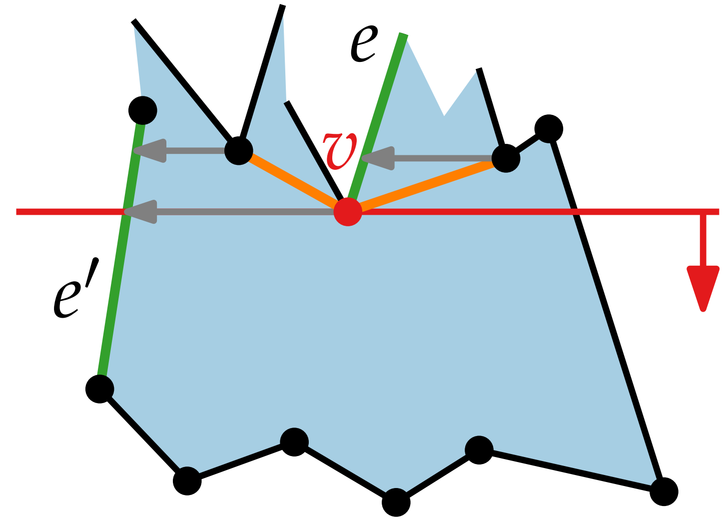

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if helper(e) merge vtx **then**
    $\mathcal{D}$.insert(diag(v, helper(e)))
$\mathcal{T}$.delete(e)
$e' \leftarrow \mathcal{T}$.edgeLeftOf$(v)$
if helper(e') merge vtx **then**
    $\mathcal{D}$.insert(diag(v, helper(e')))
helper$(e') \leftarrow v$

Analysis

Lemma. makeMonotone() adds a set of non-intersecting diagonals to P such that P is partitioned into y-monotone subpolygons.

Analysis

Lemma. makeMonotone() adds a set of non-intersecting diagonals to P such that P is partitioned into y-monotone subpolygons.

Lemma. A simple polygon with n vertices can be subdivided into y-monotone polygons in $O(n \log n)$ time.

Computational Geometry

Lecture 3:
Guarding Art Galleries
or
Triangulating Polygons

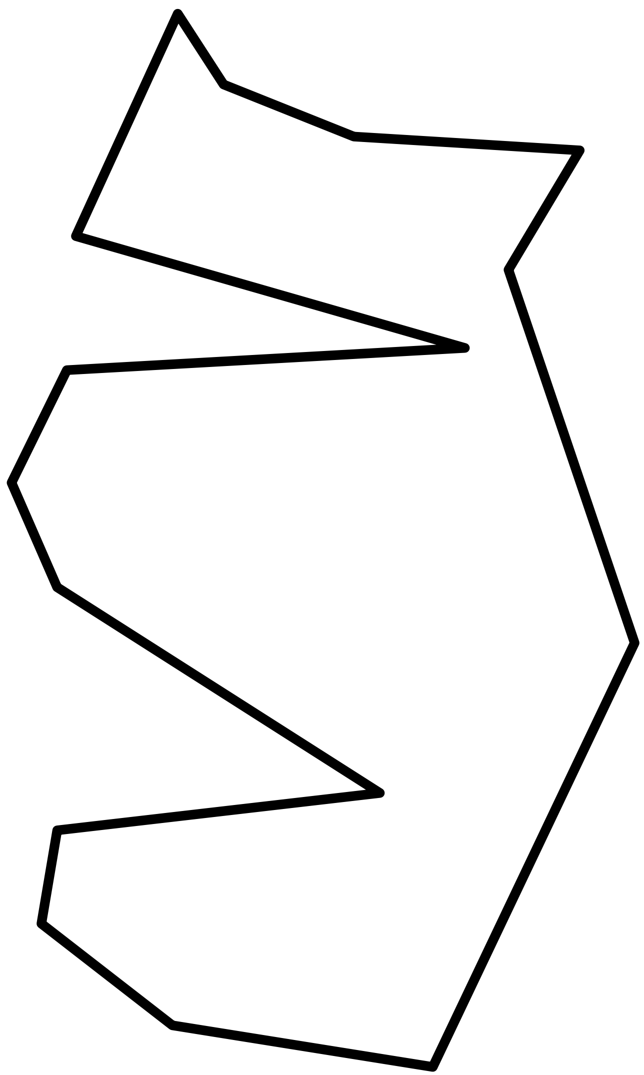
Part IV:
Triangulating a y-monotone Polygon

Philipp Kindermann

Winter Semester 2020

Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



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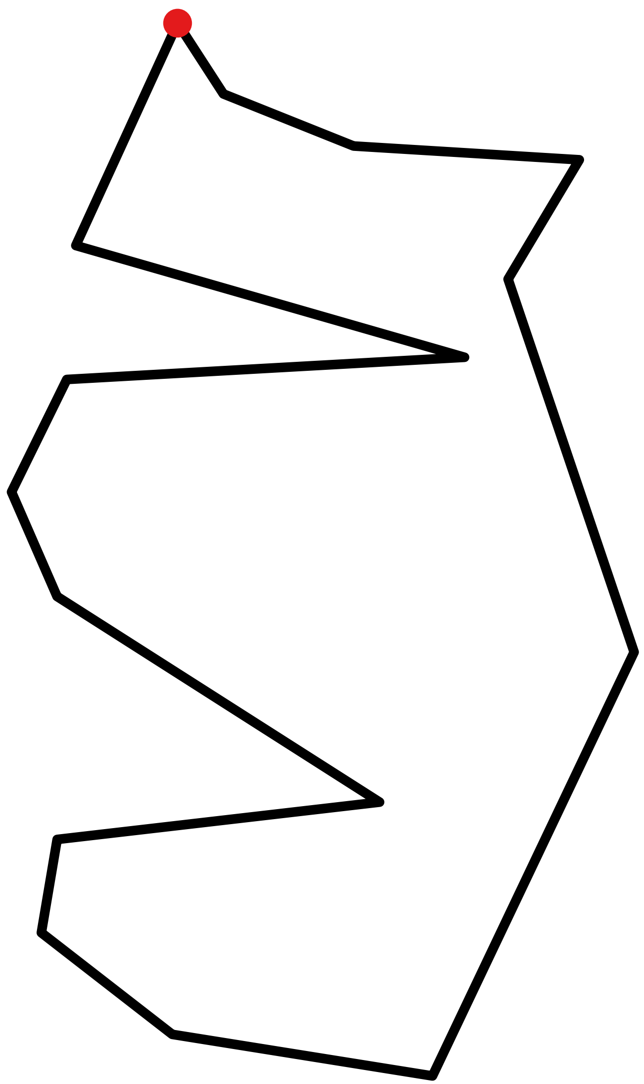

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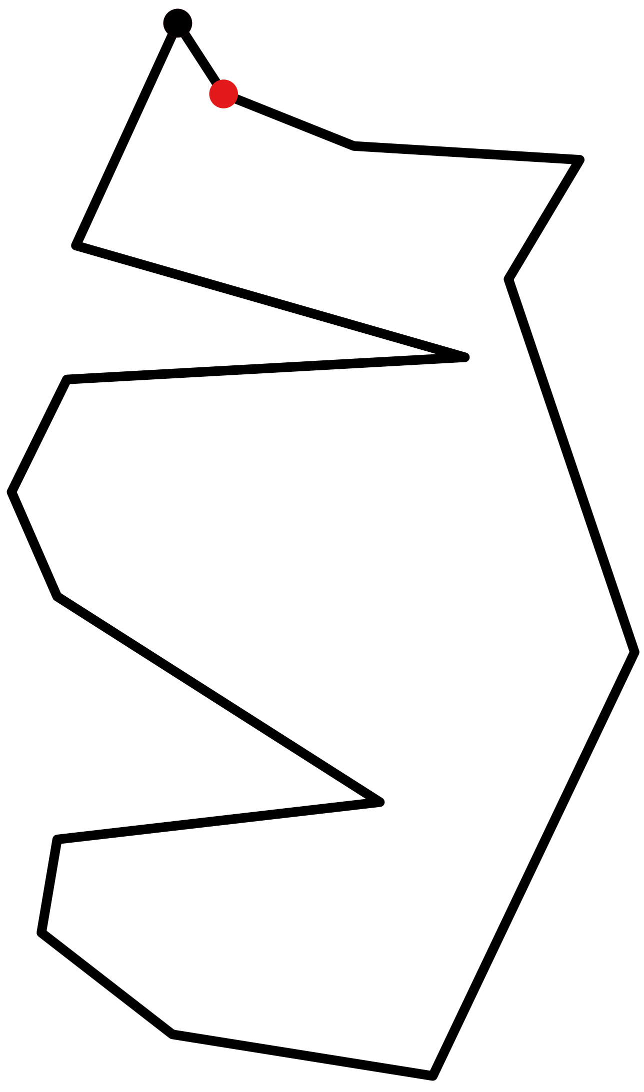

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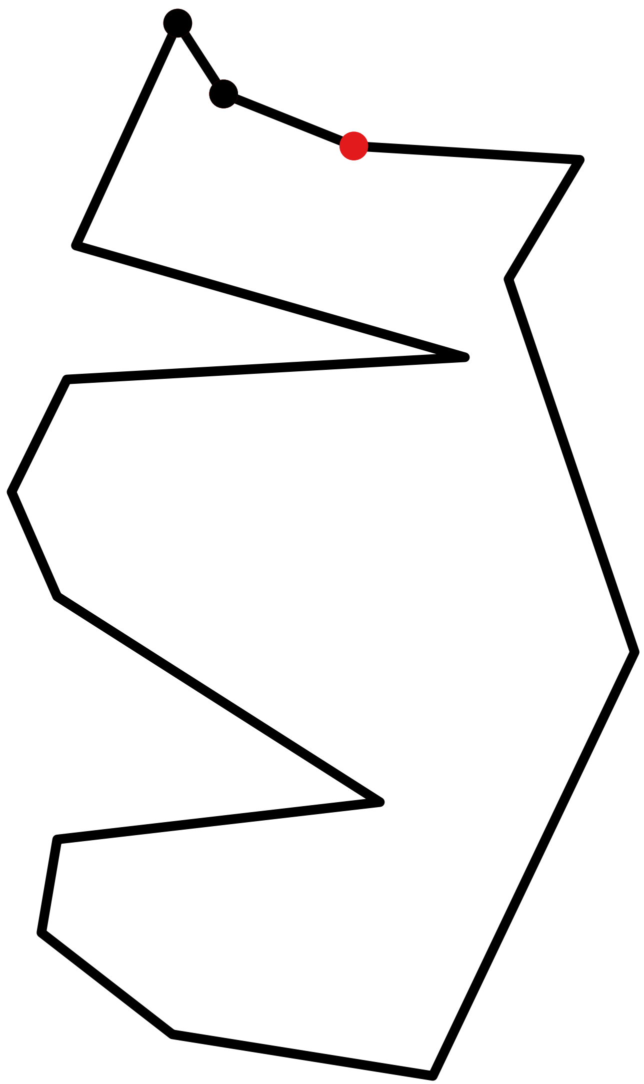

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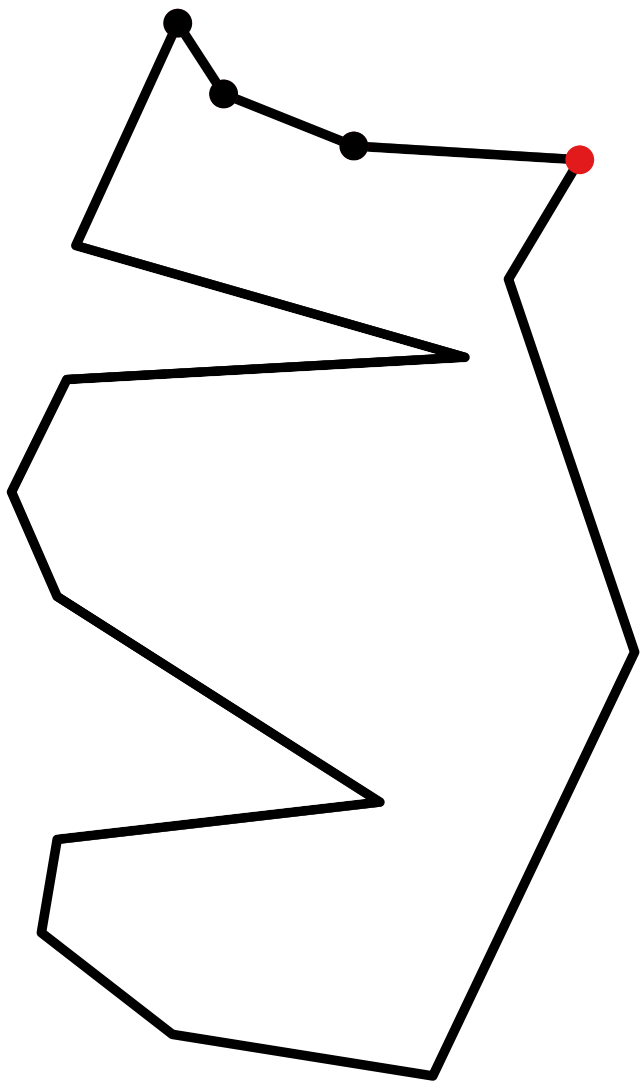

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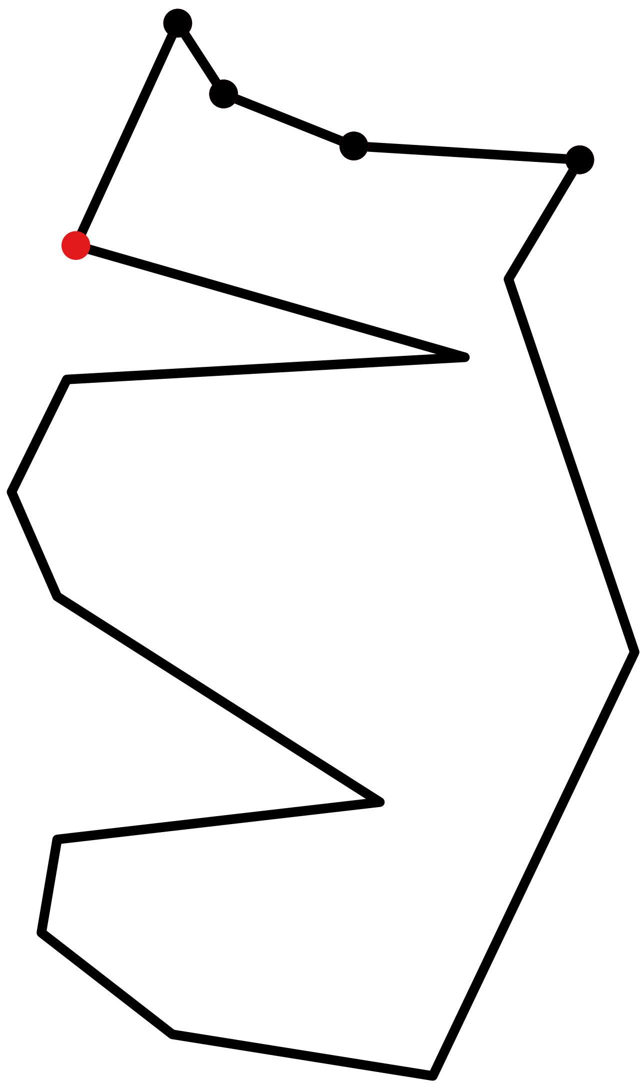

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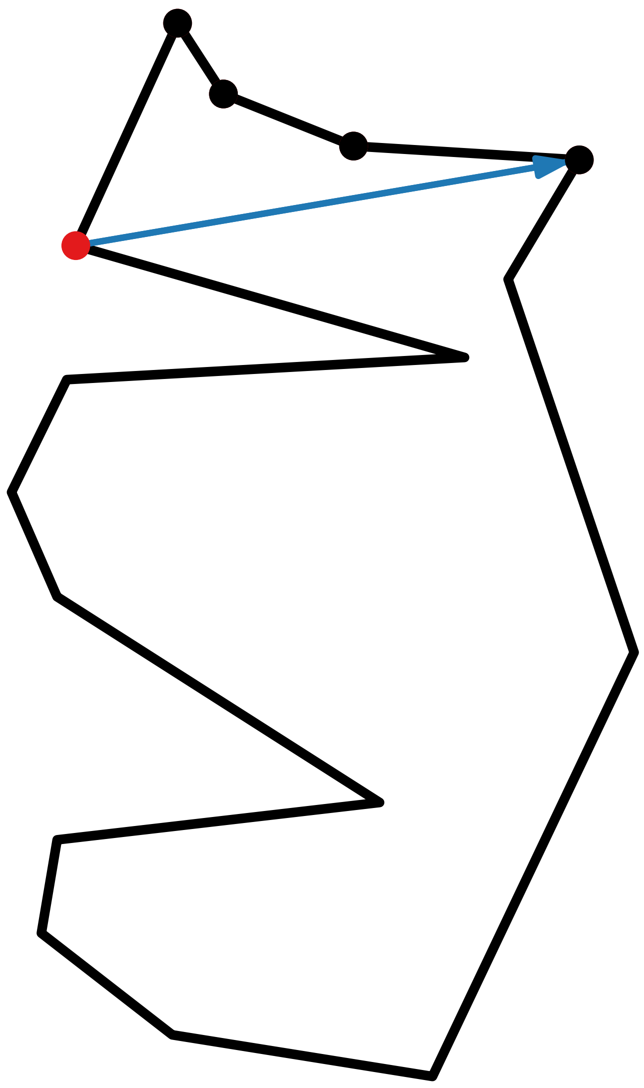

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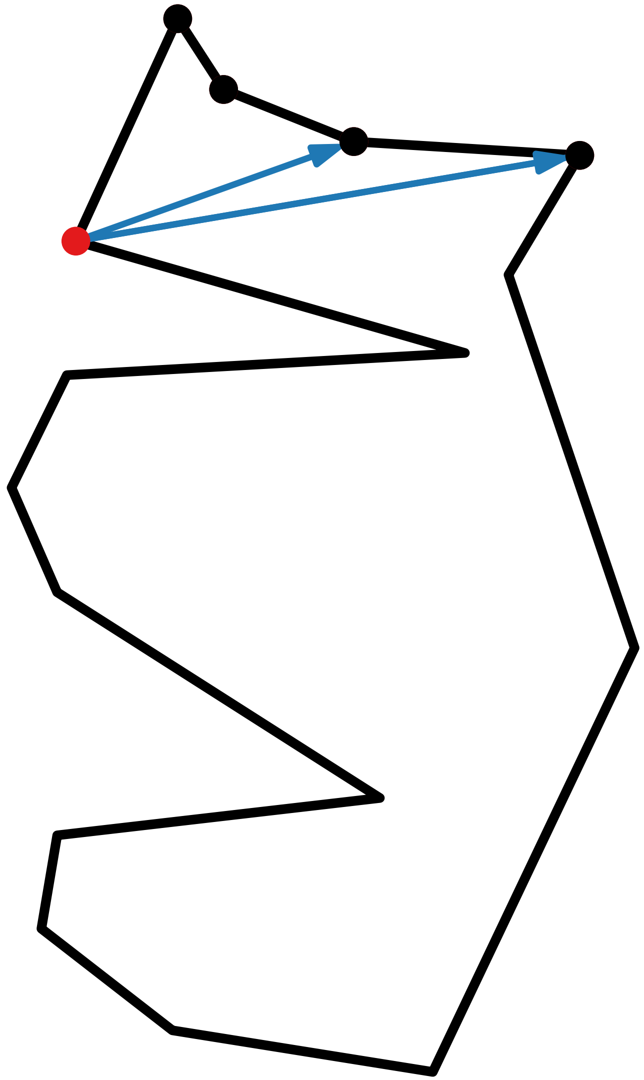

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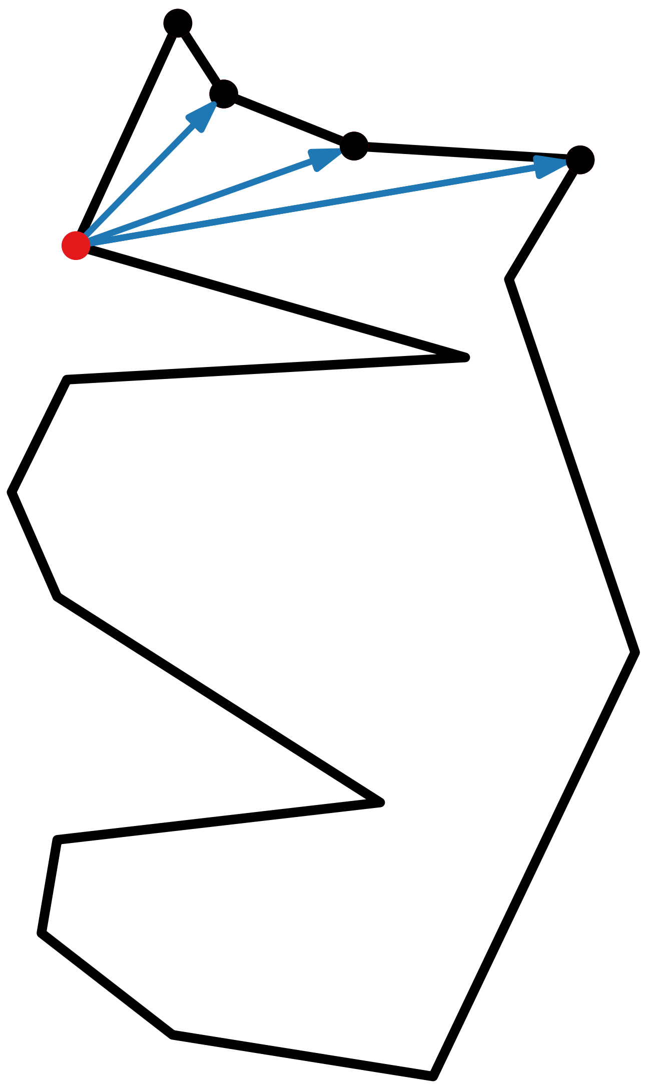

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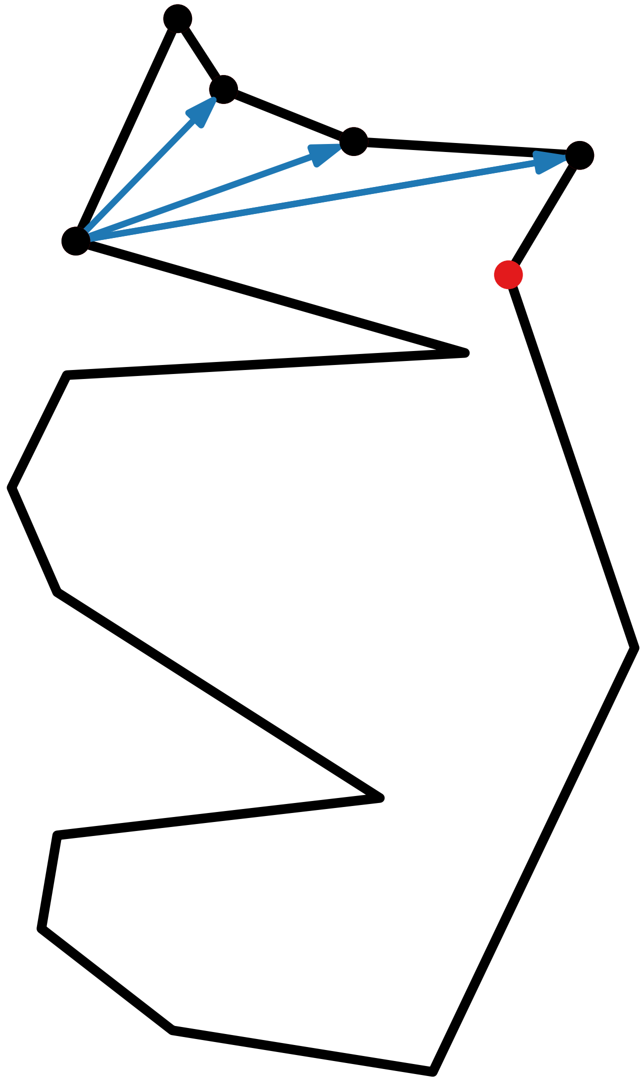

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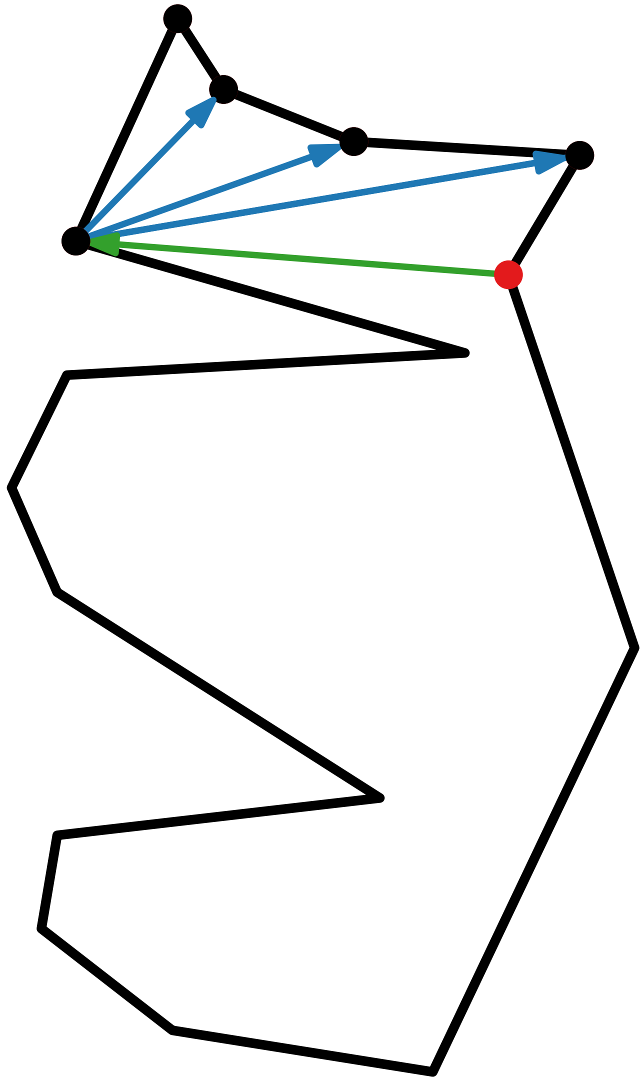

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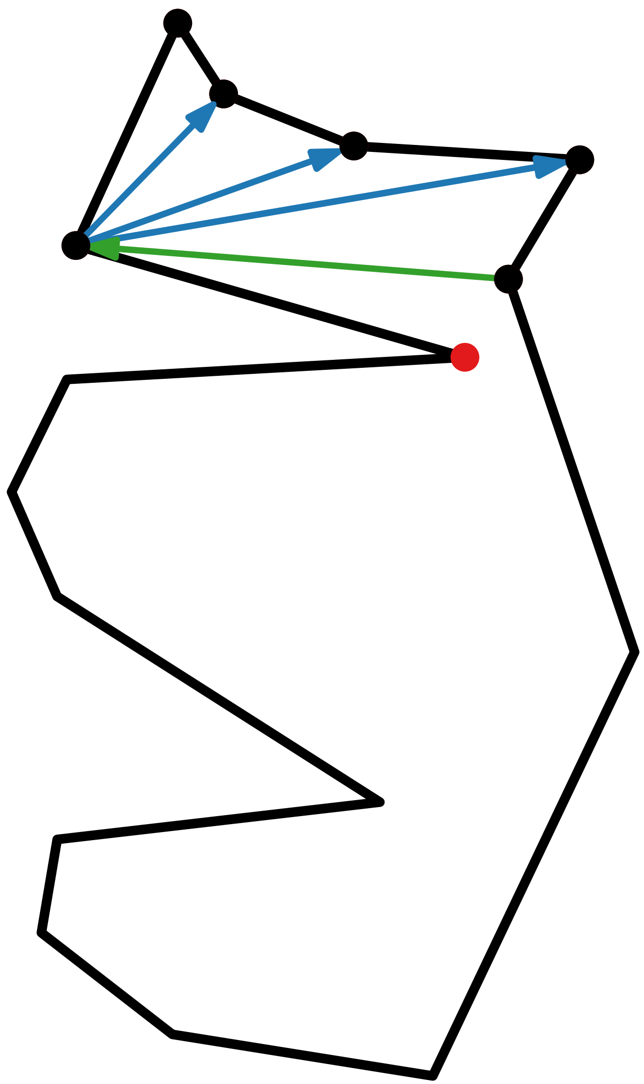

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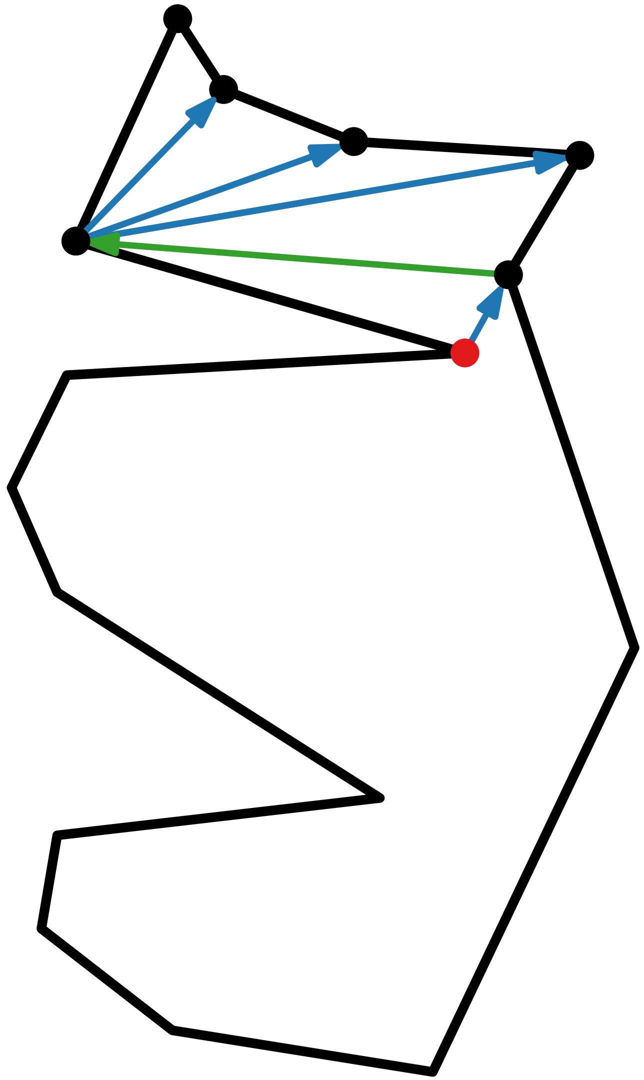

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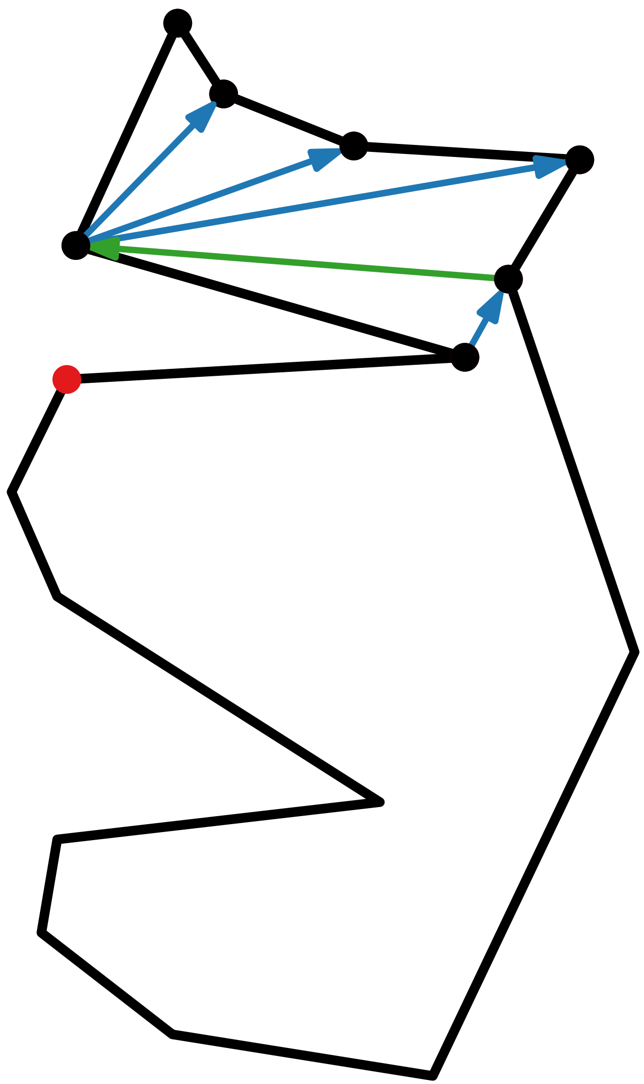

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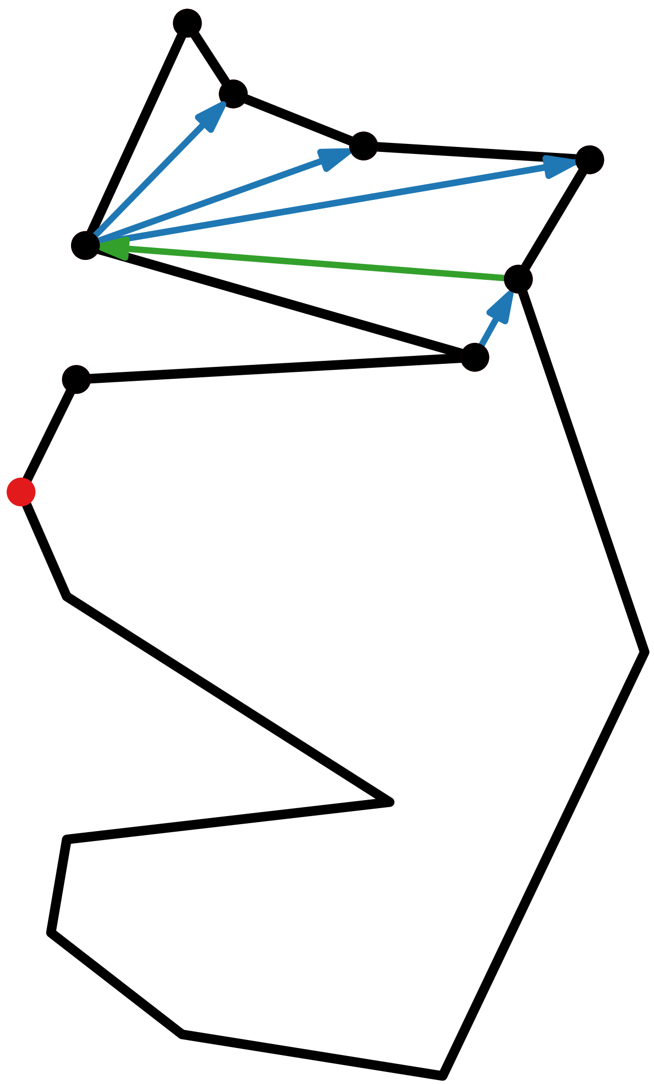

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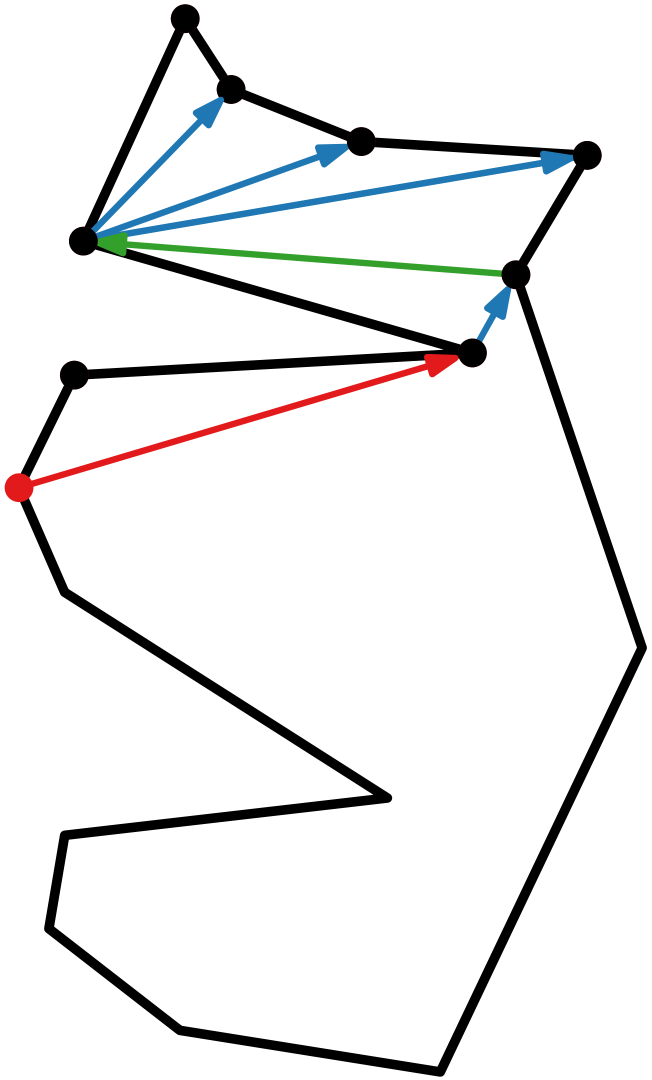

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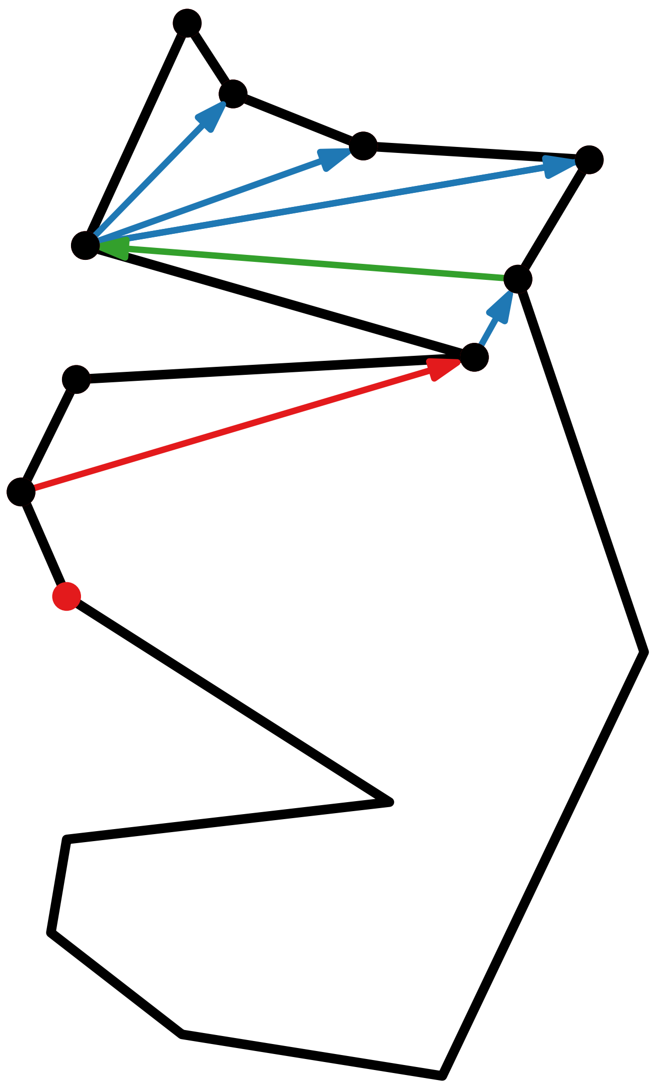

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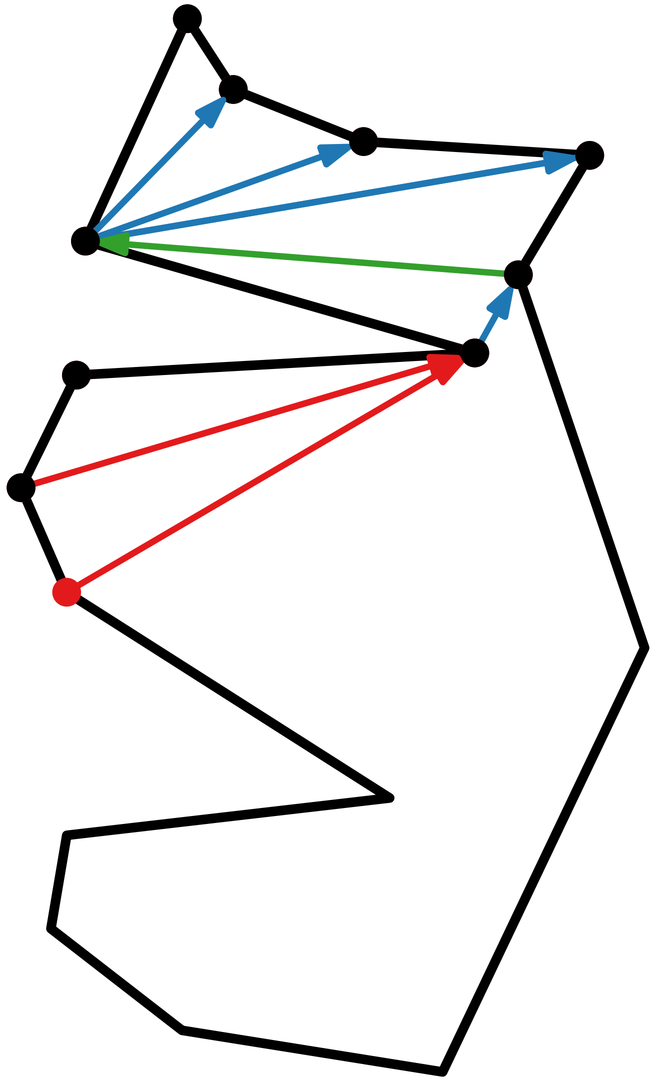

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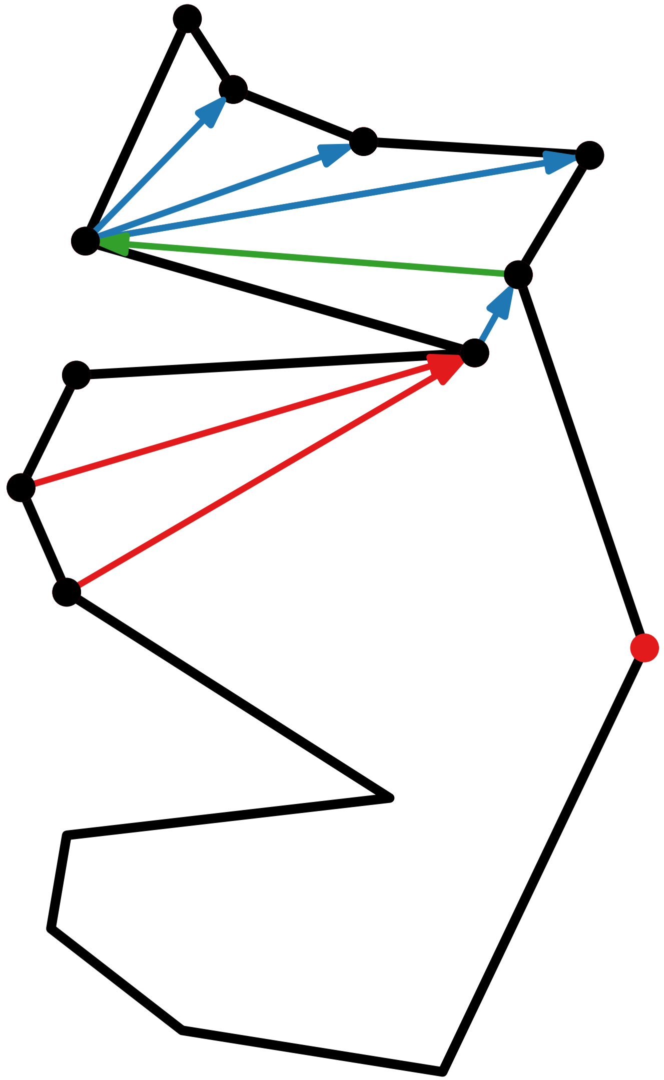

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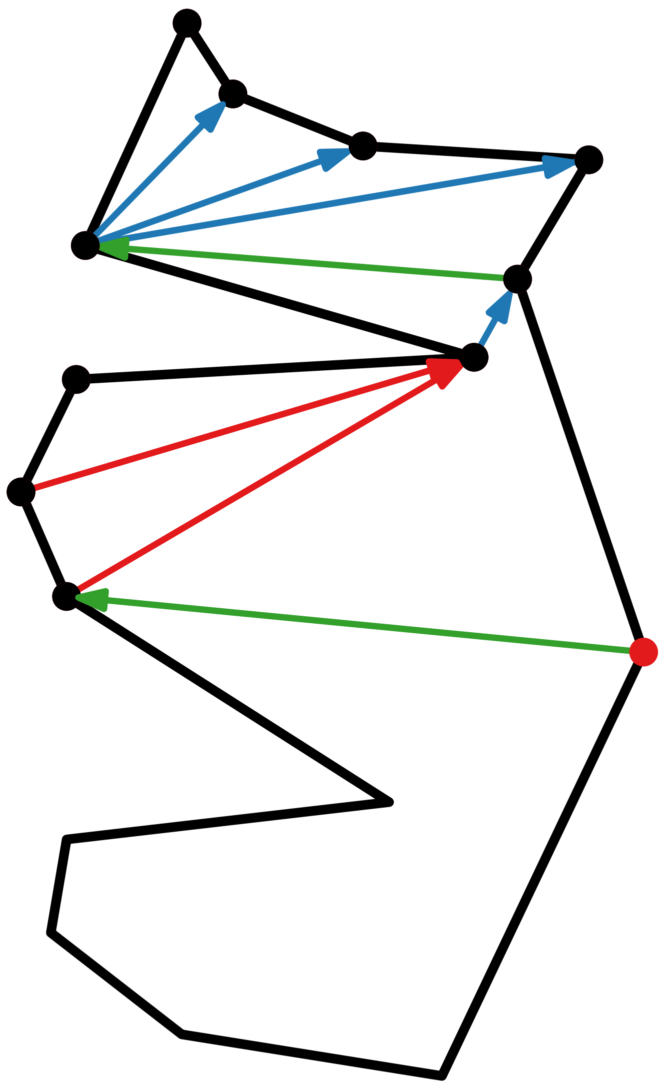

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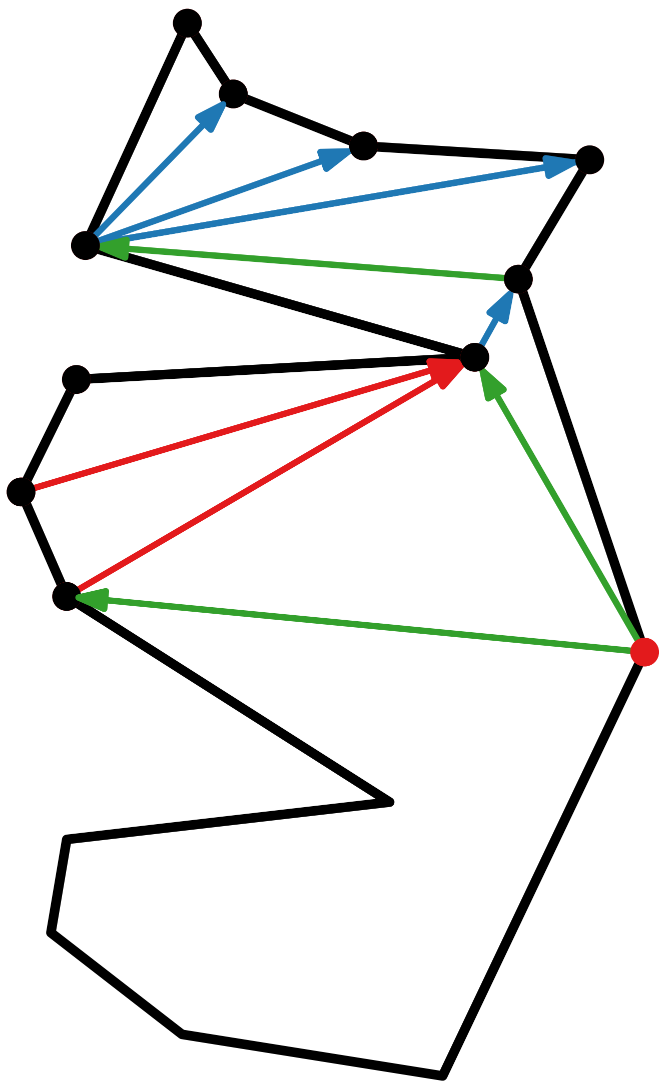

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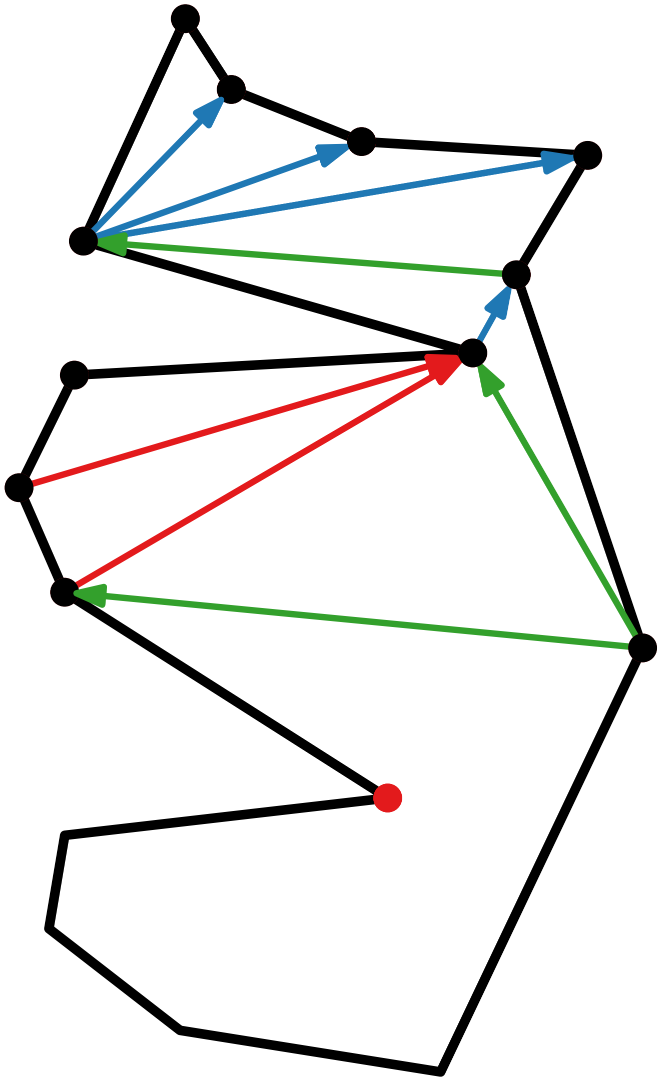

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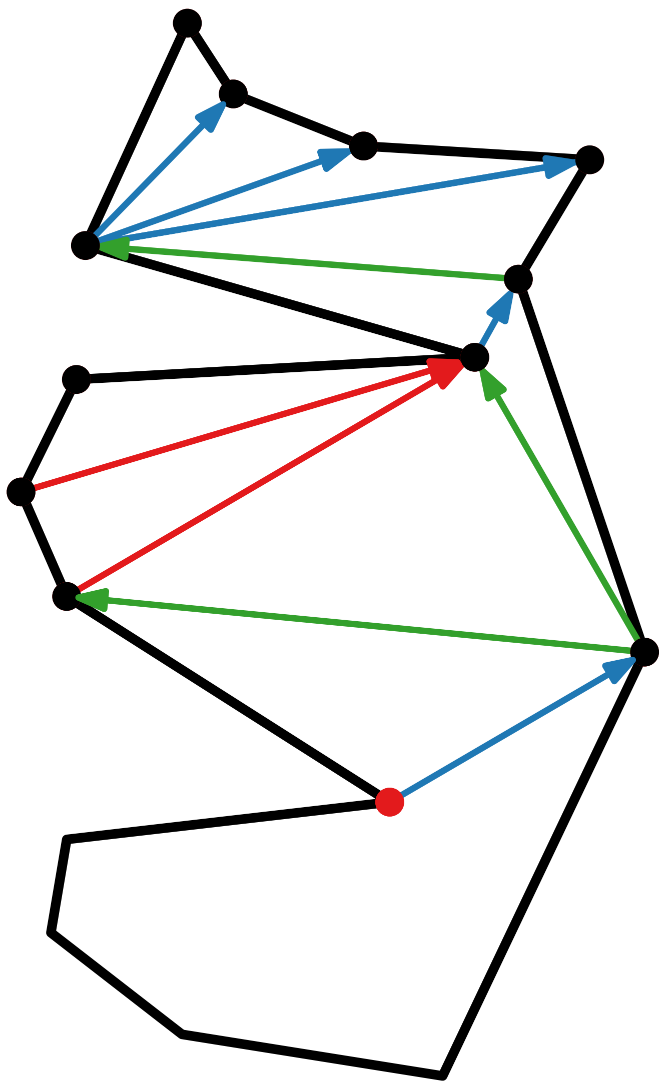

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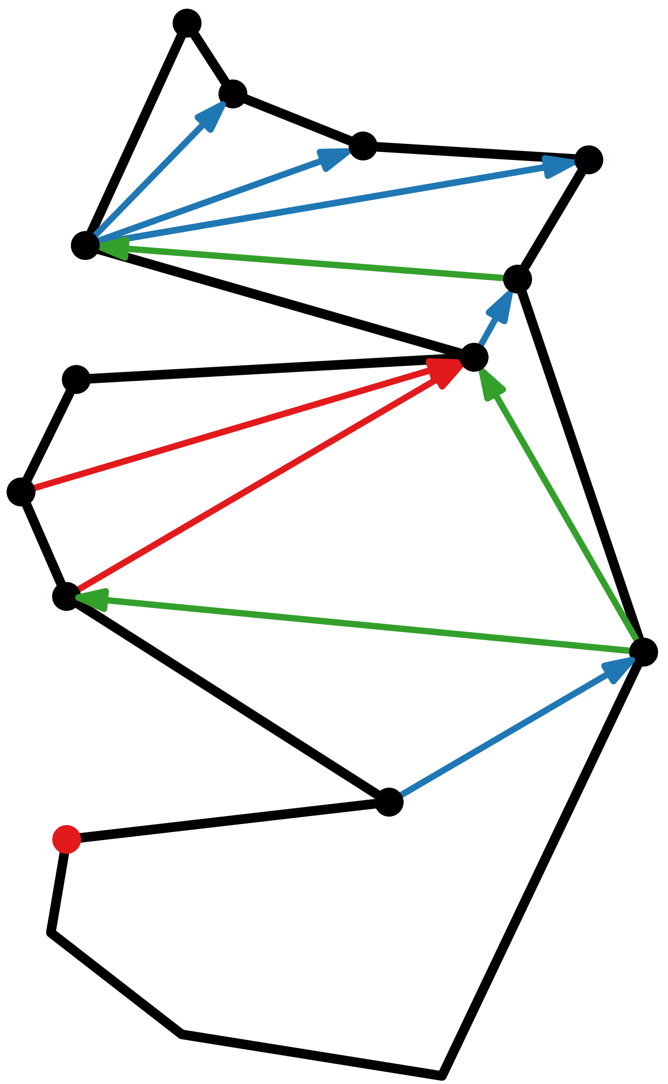

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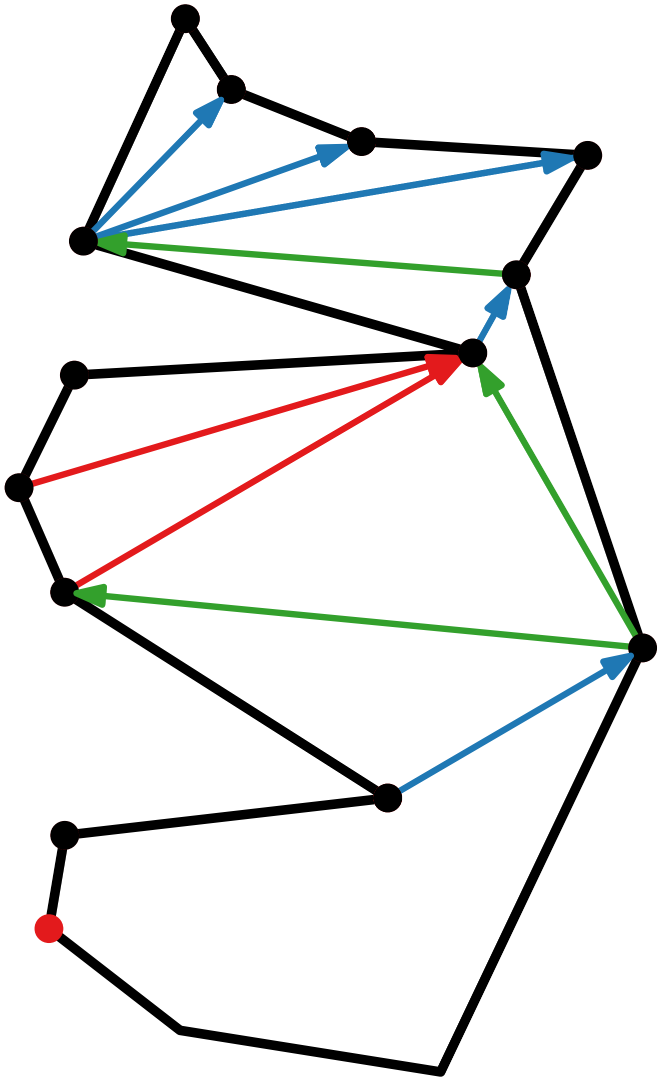

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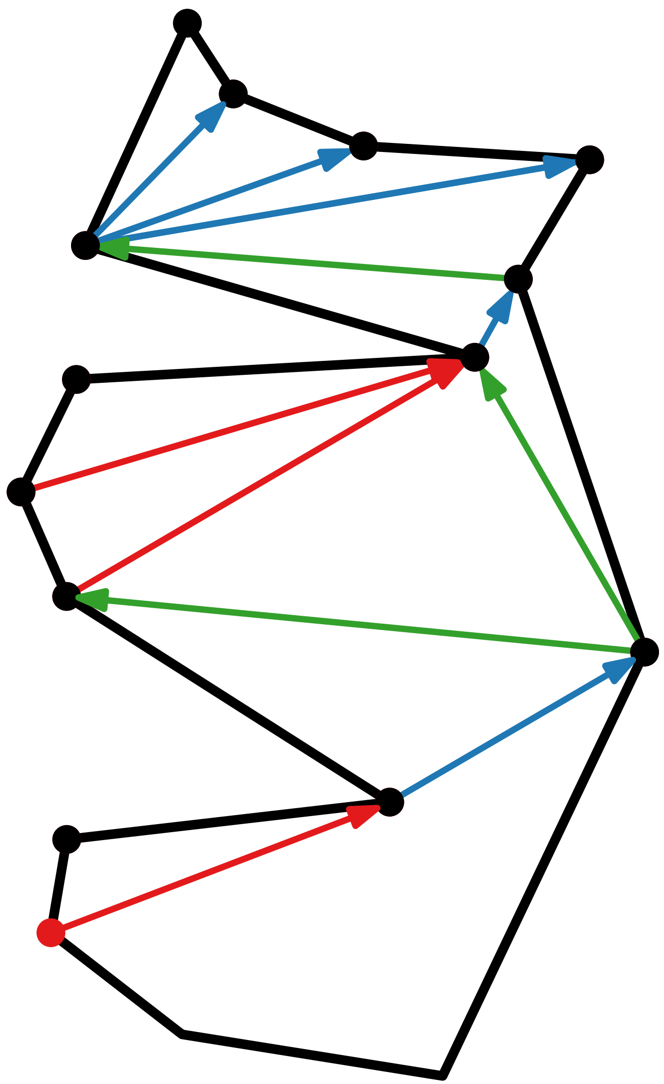

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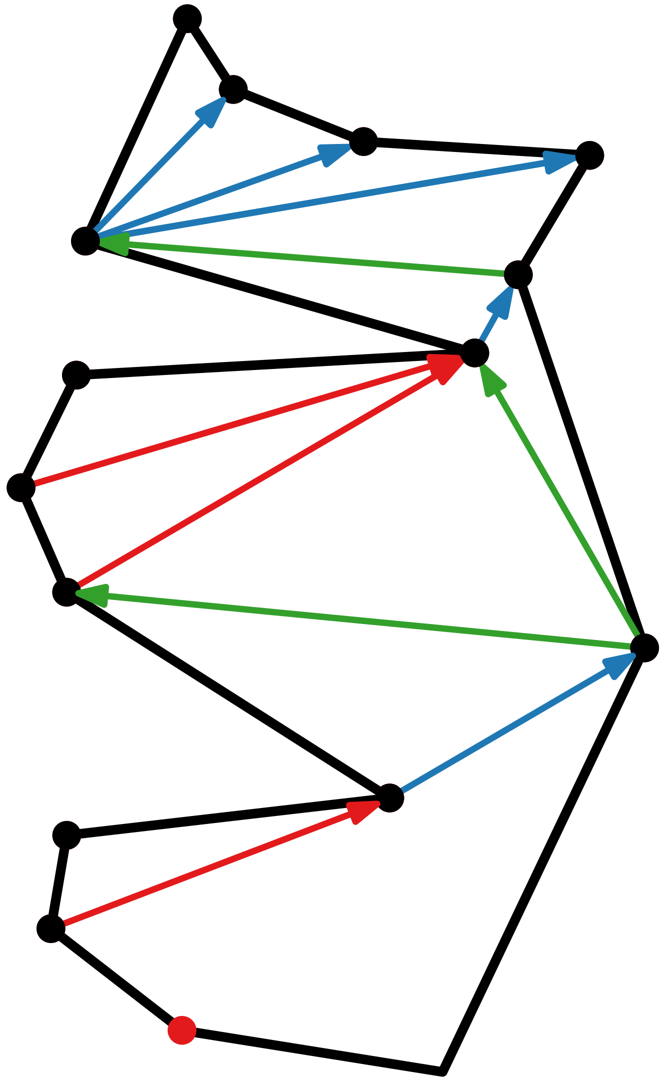

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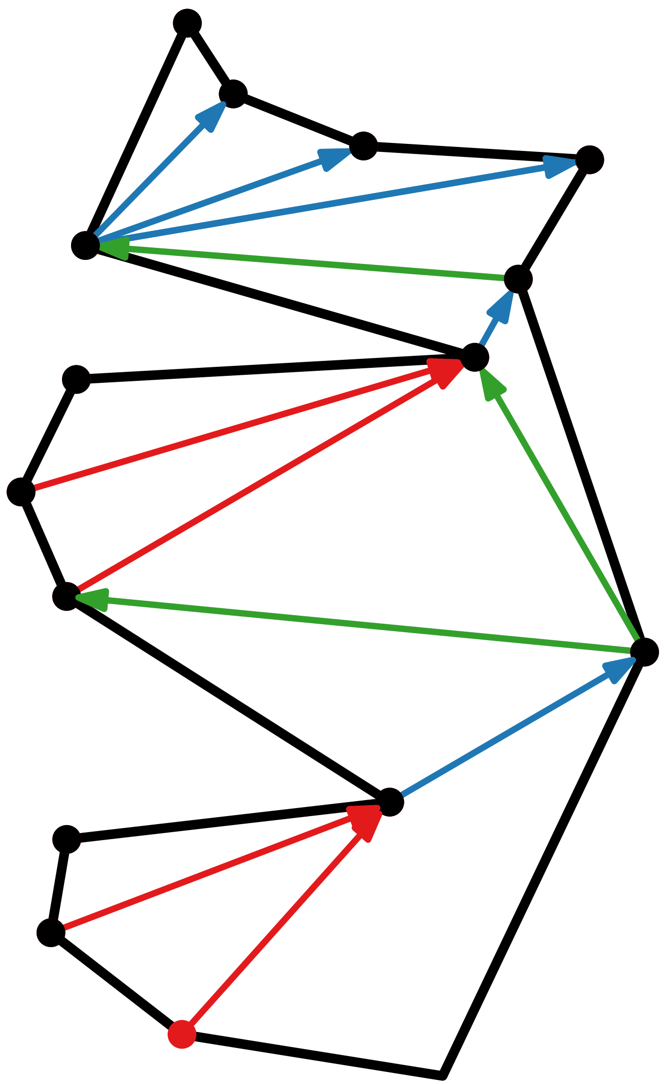

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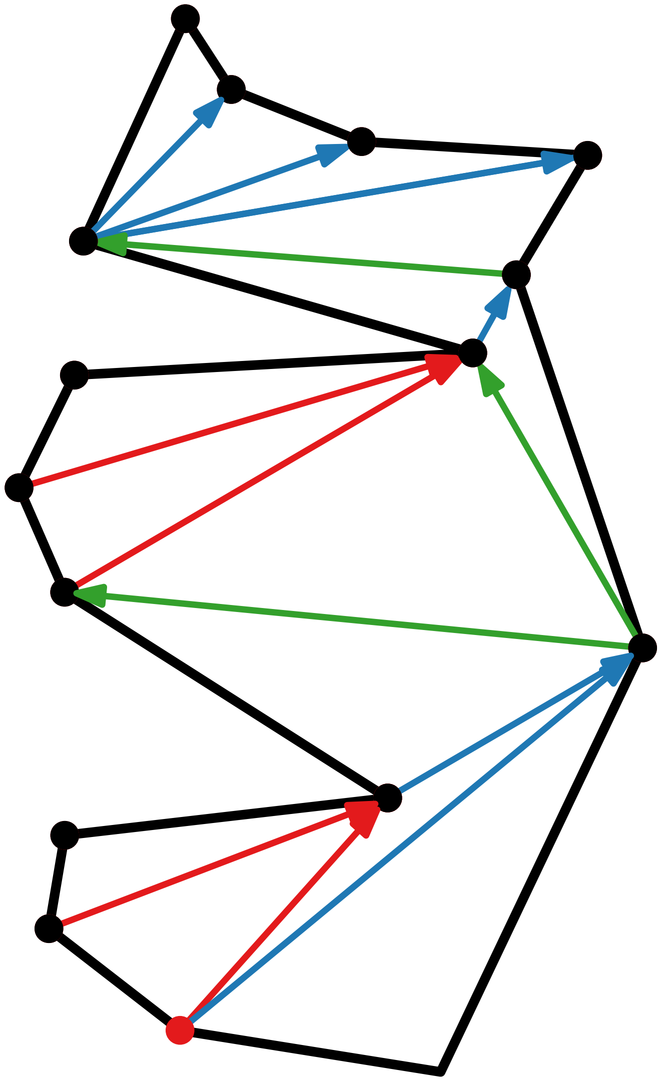

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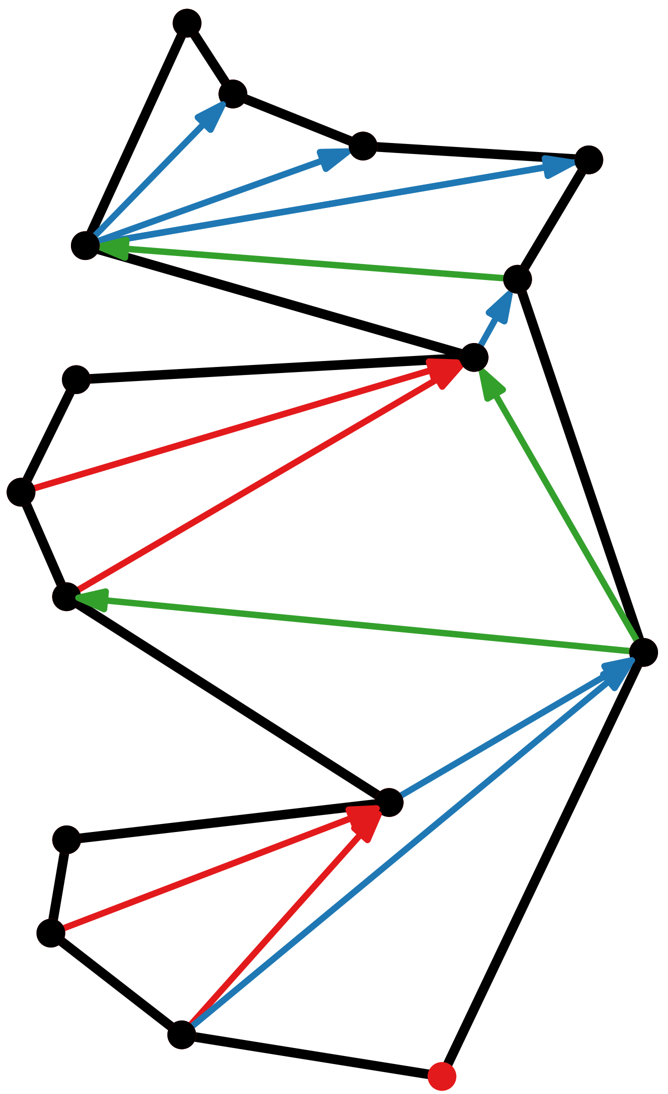

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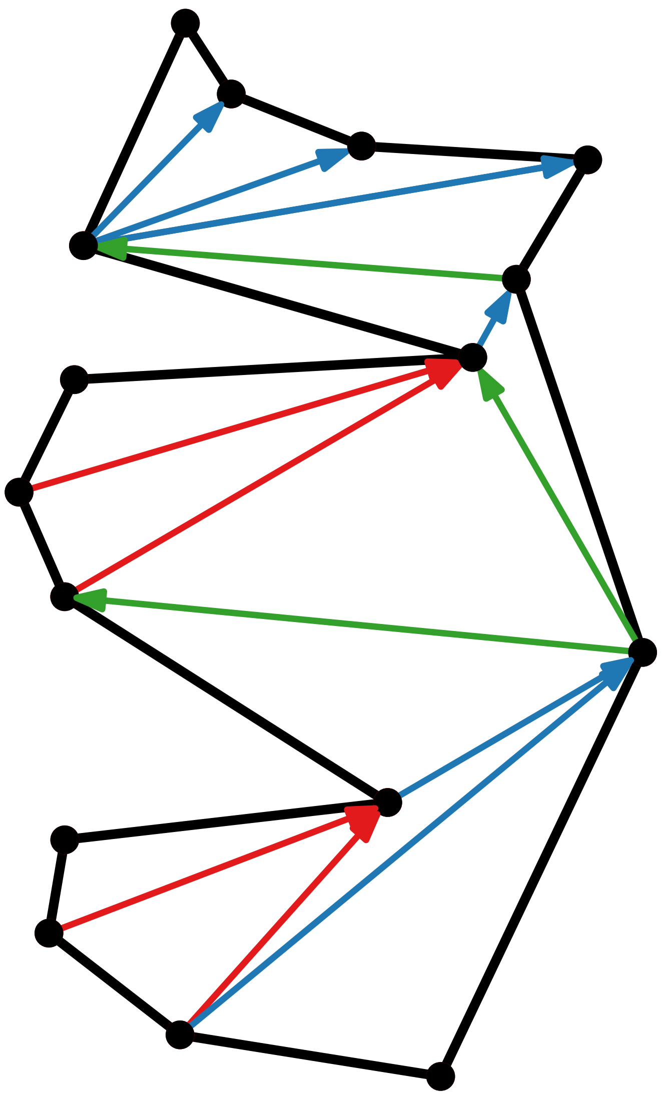
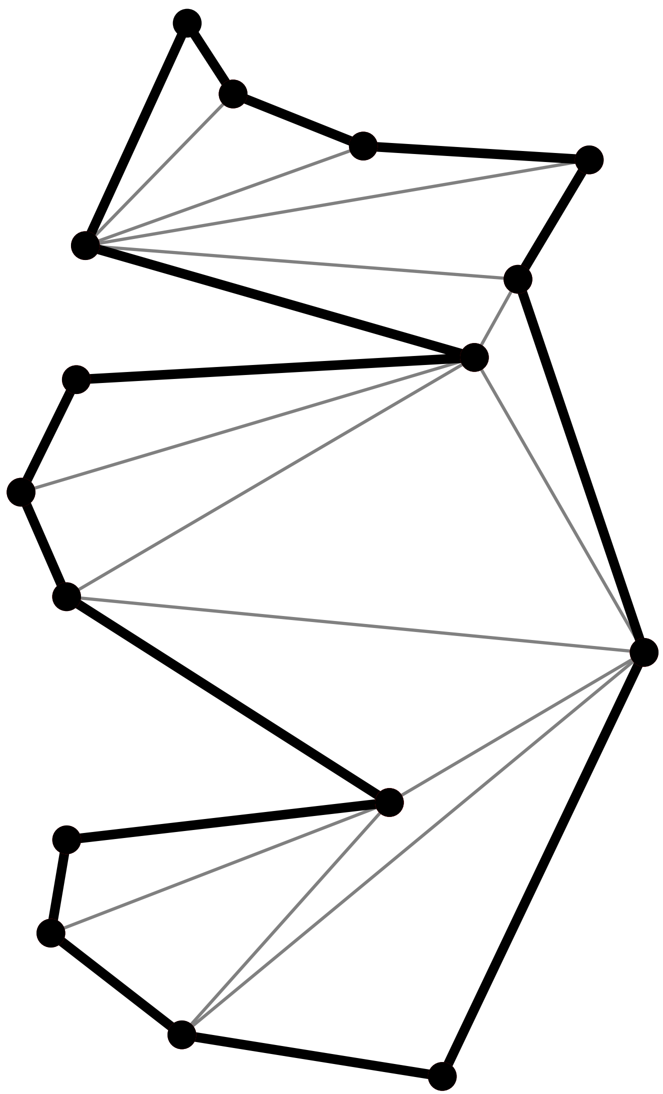


Invariant?

Triangulating a y-Monotone Polygon P

Approach: greedy, going from top to bottom



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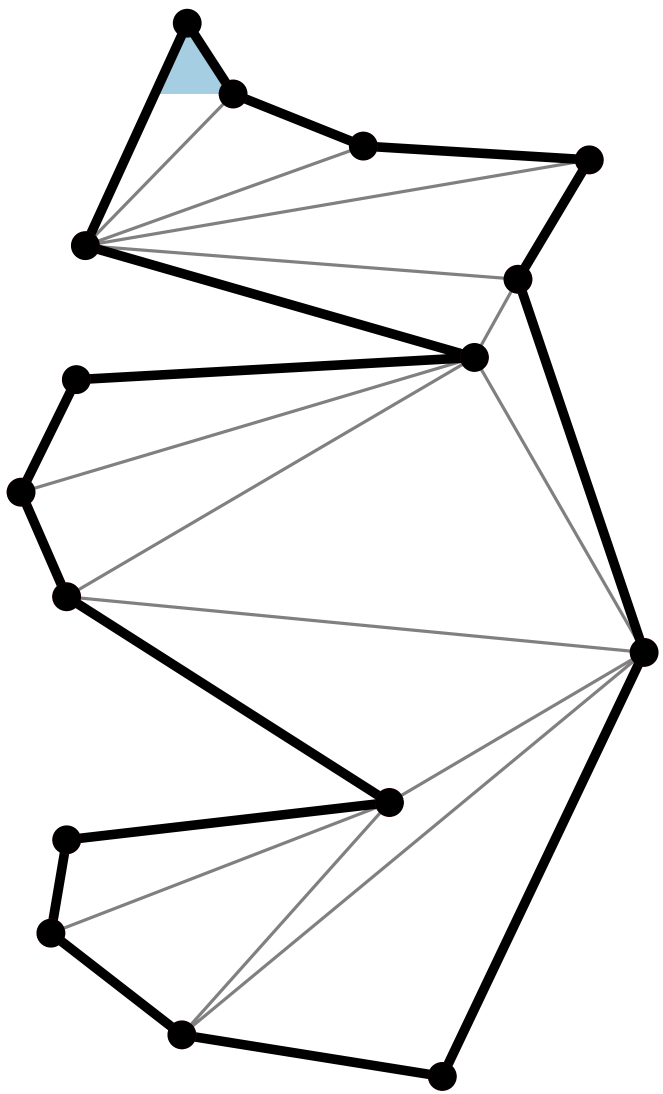

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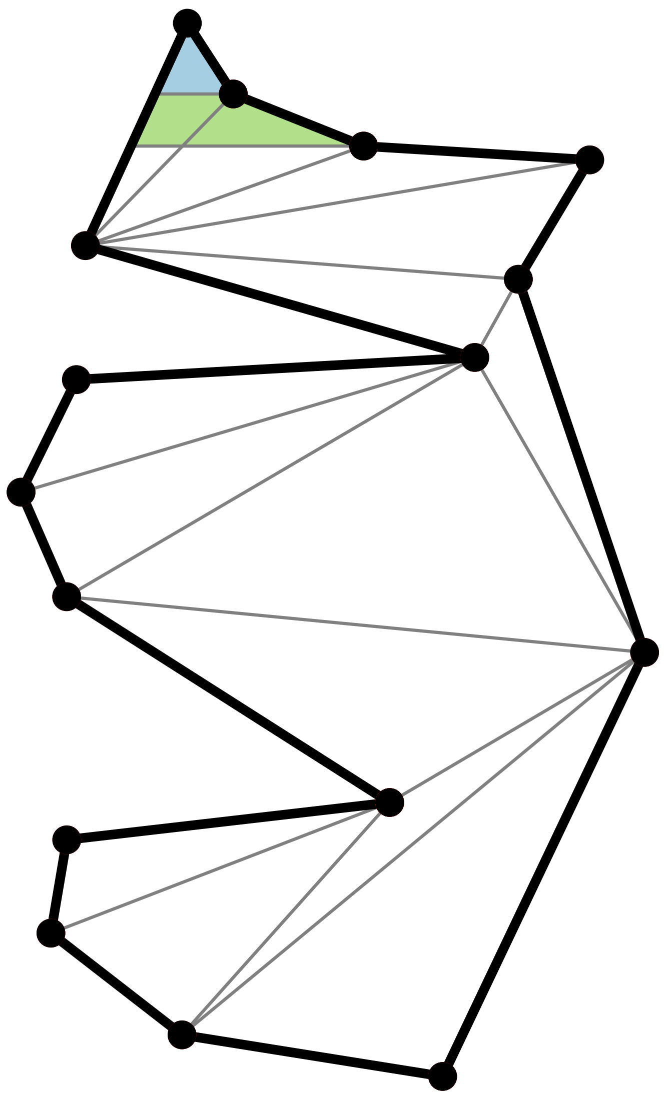

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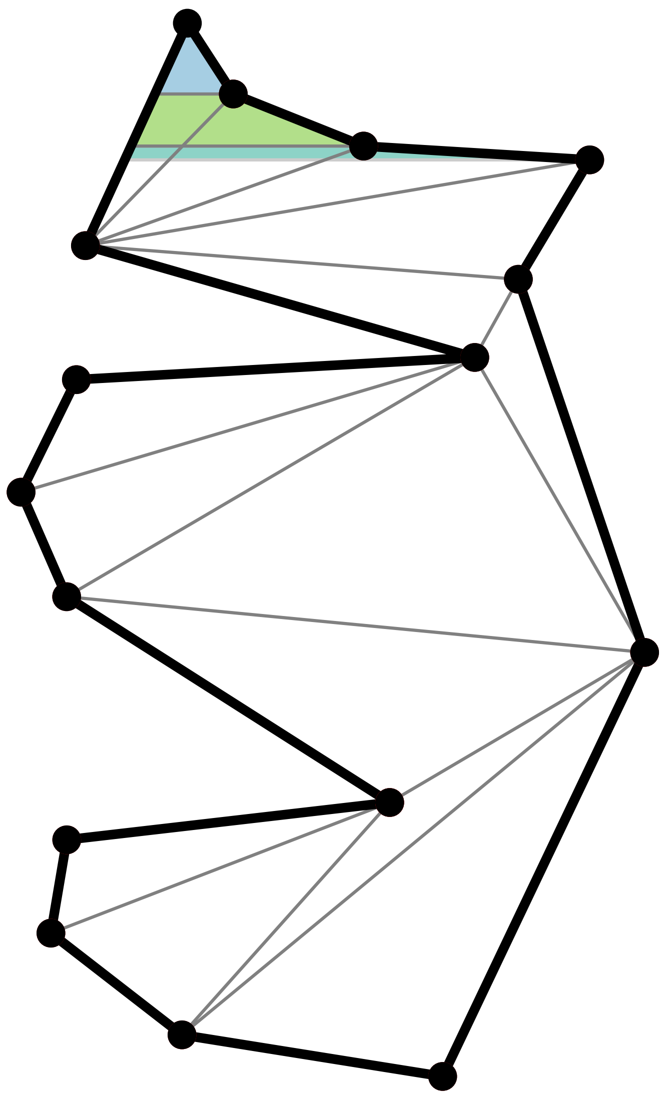

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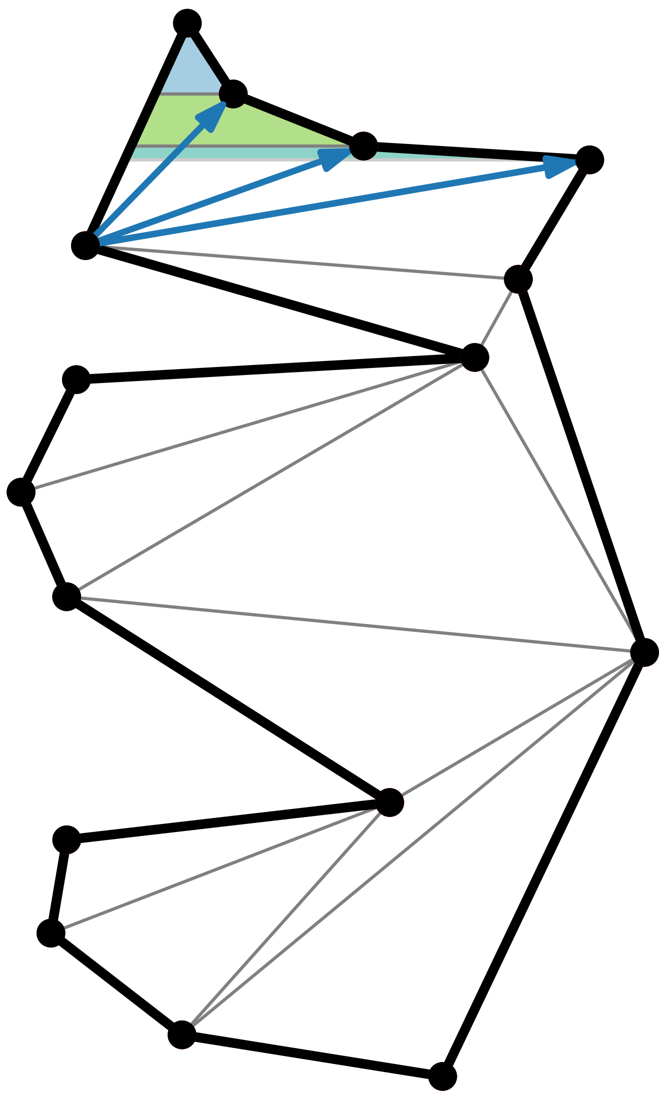

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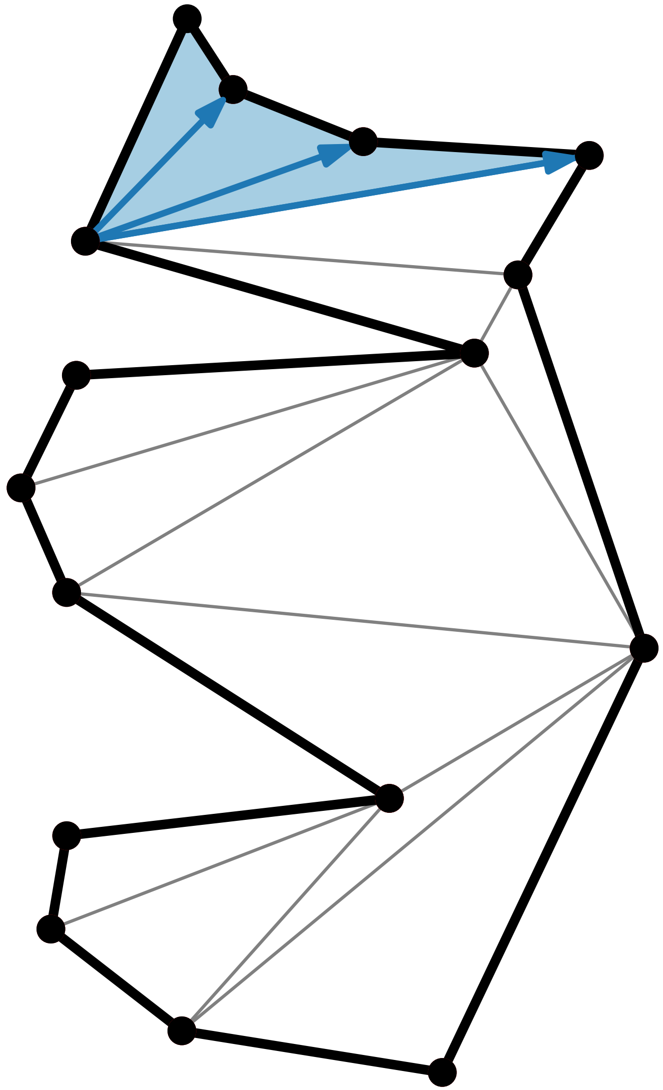

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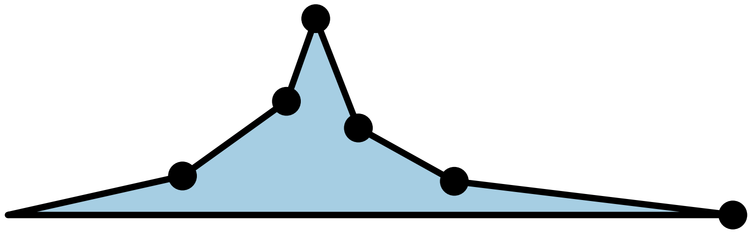
The part of P that we have seen but not yet triangulated is a *funnel*.



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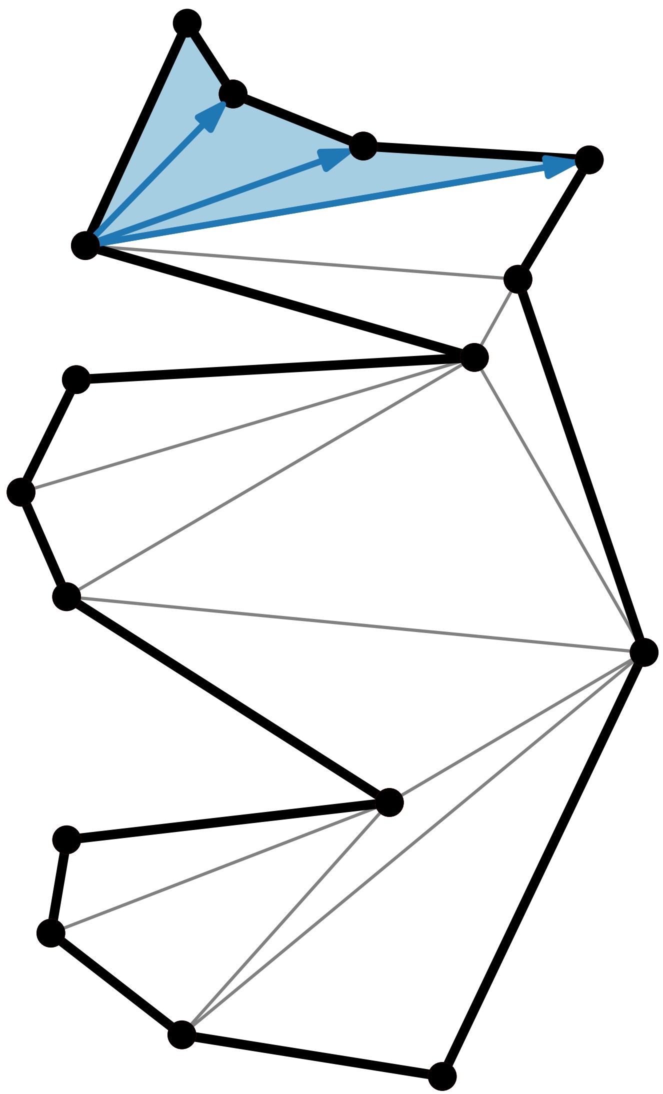

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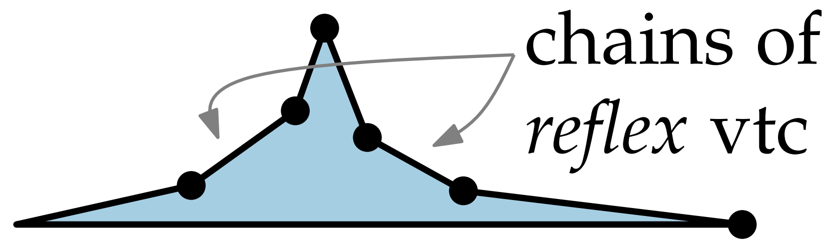

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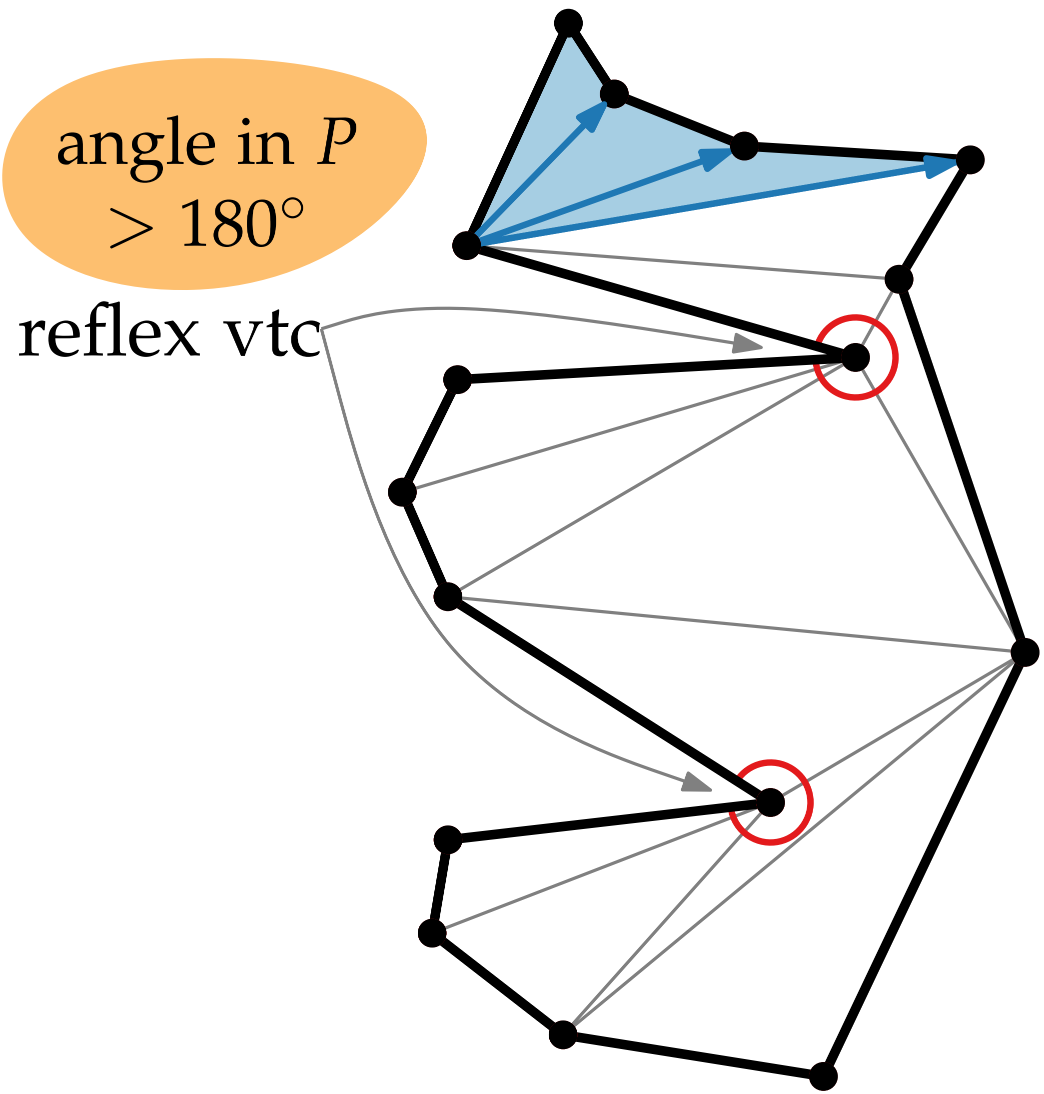

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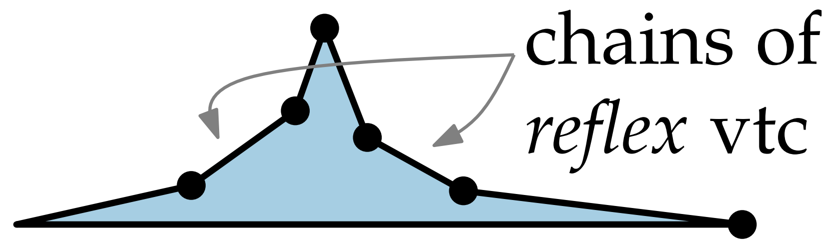

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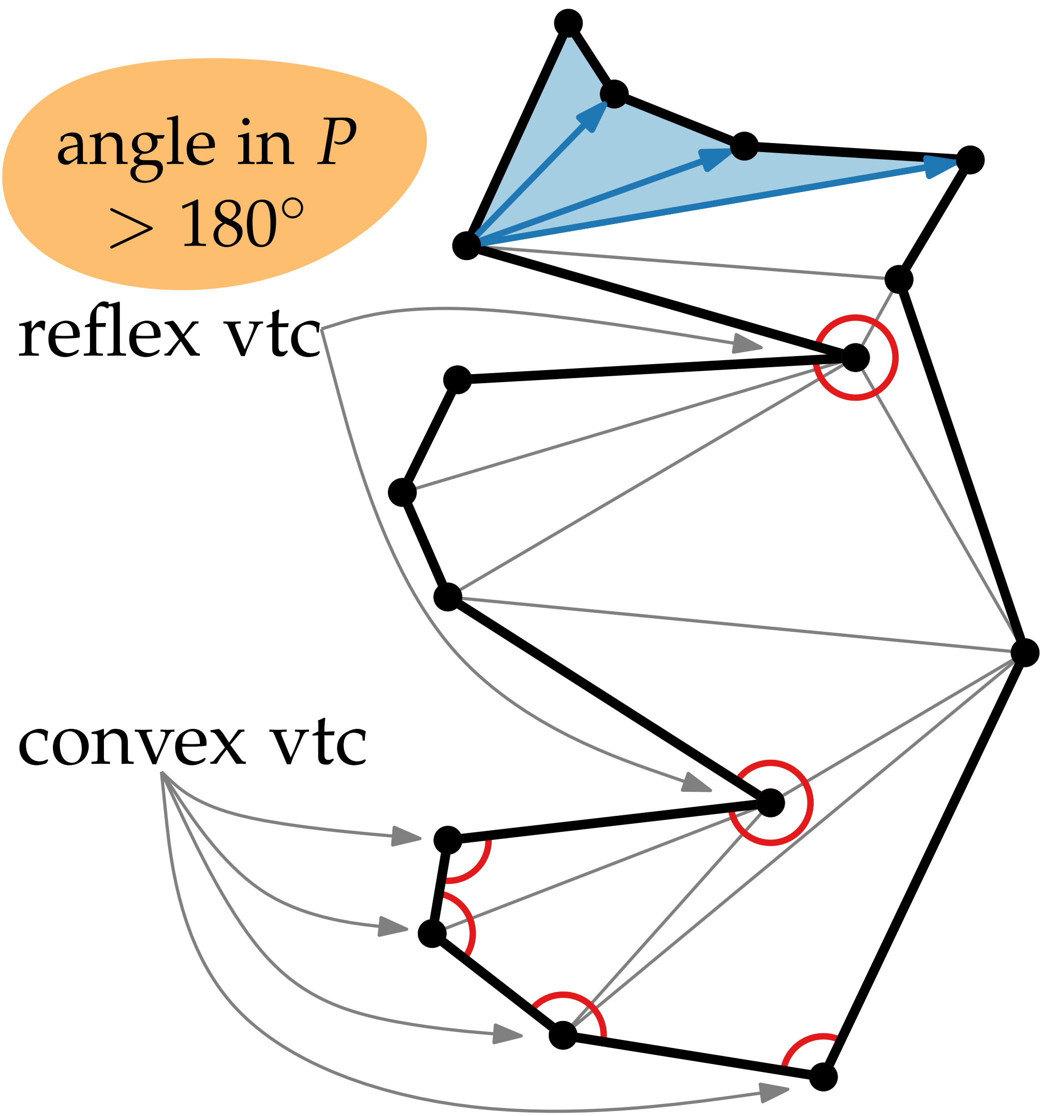

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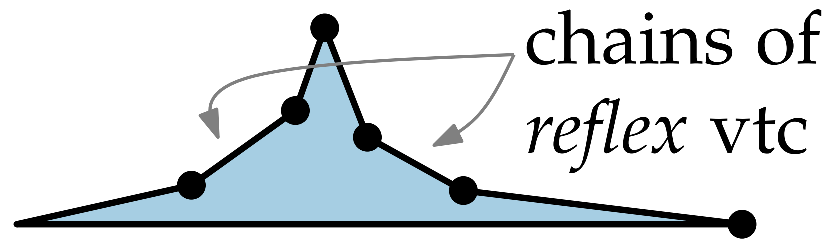

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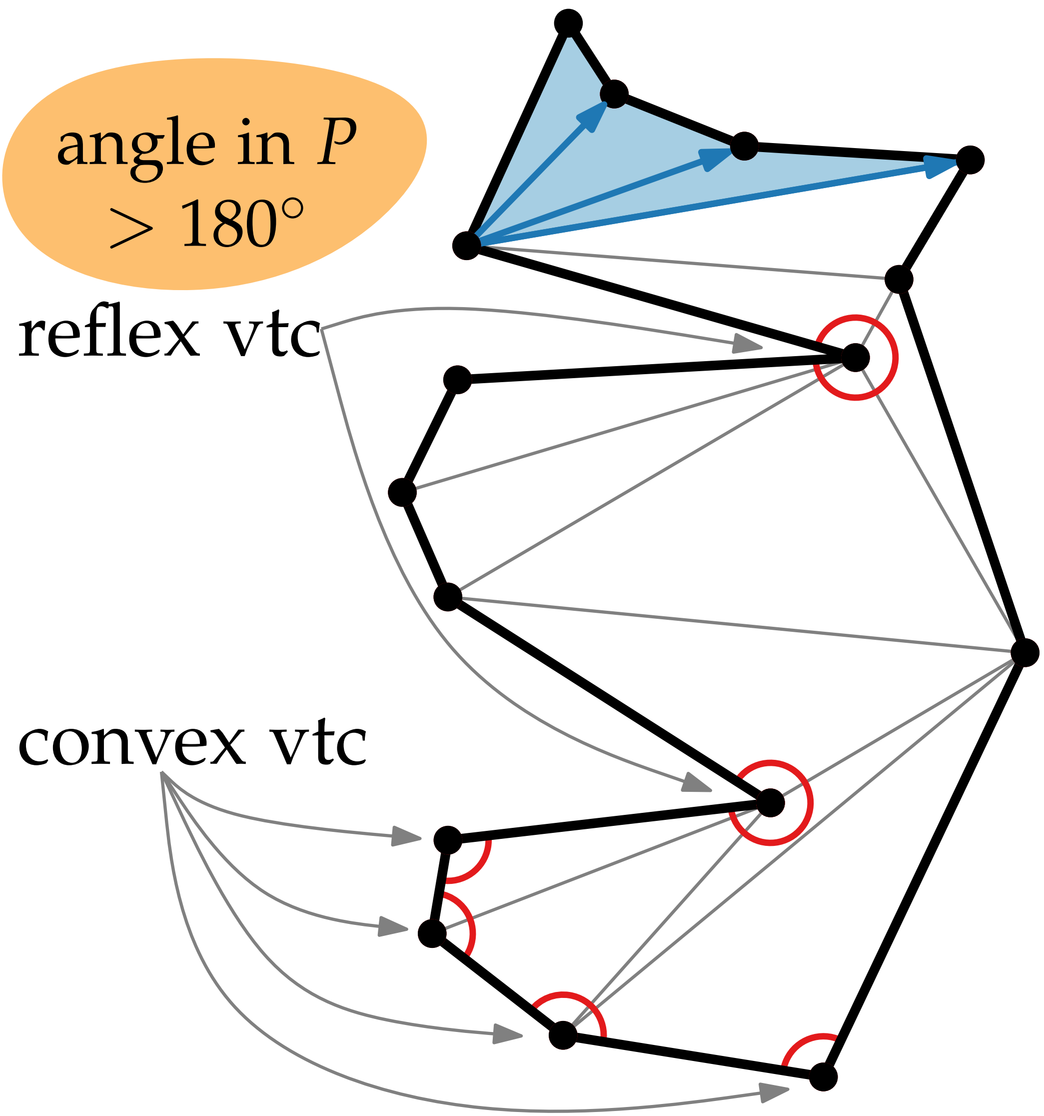

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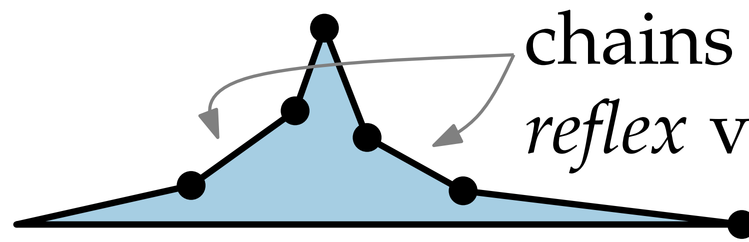

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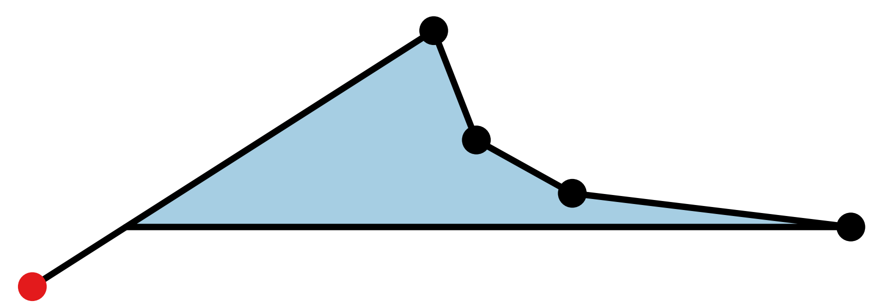

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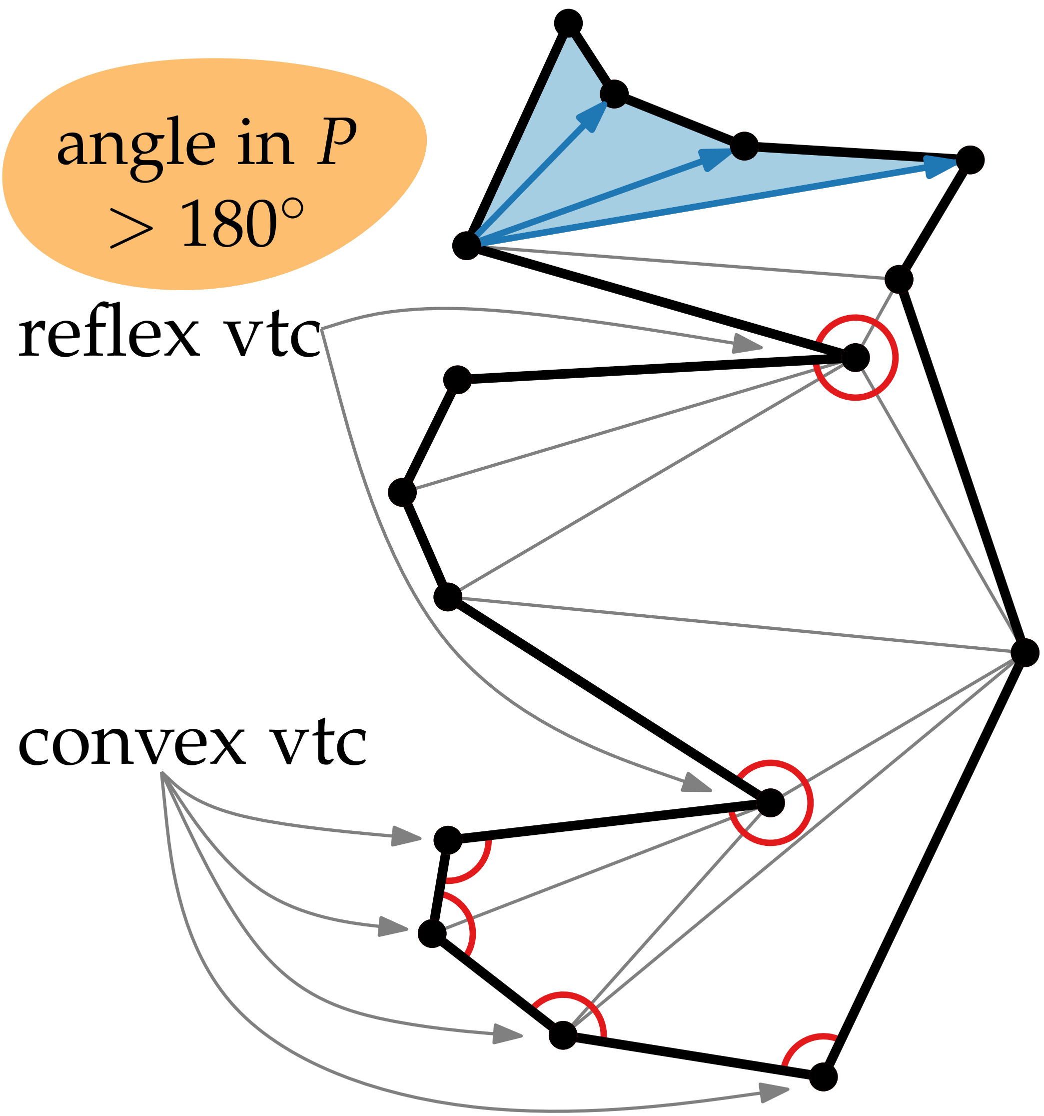

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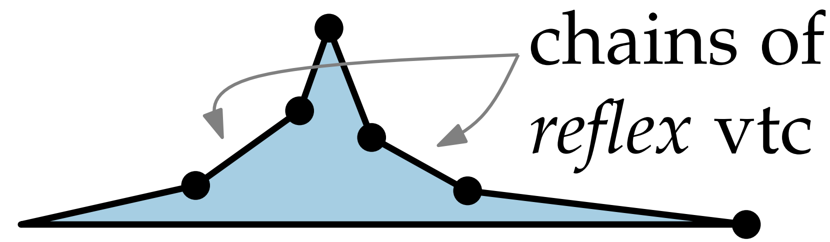

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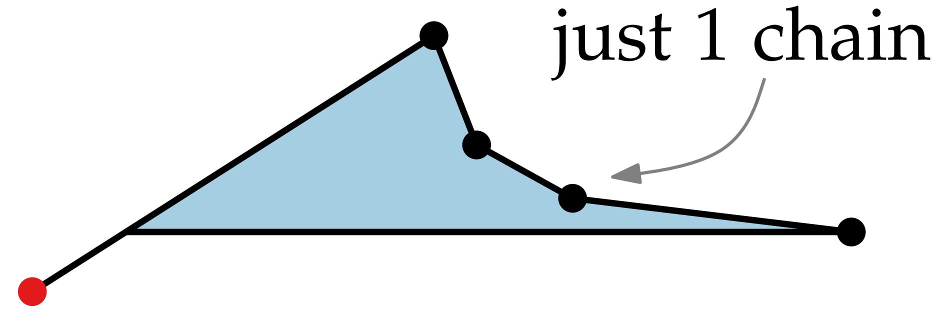

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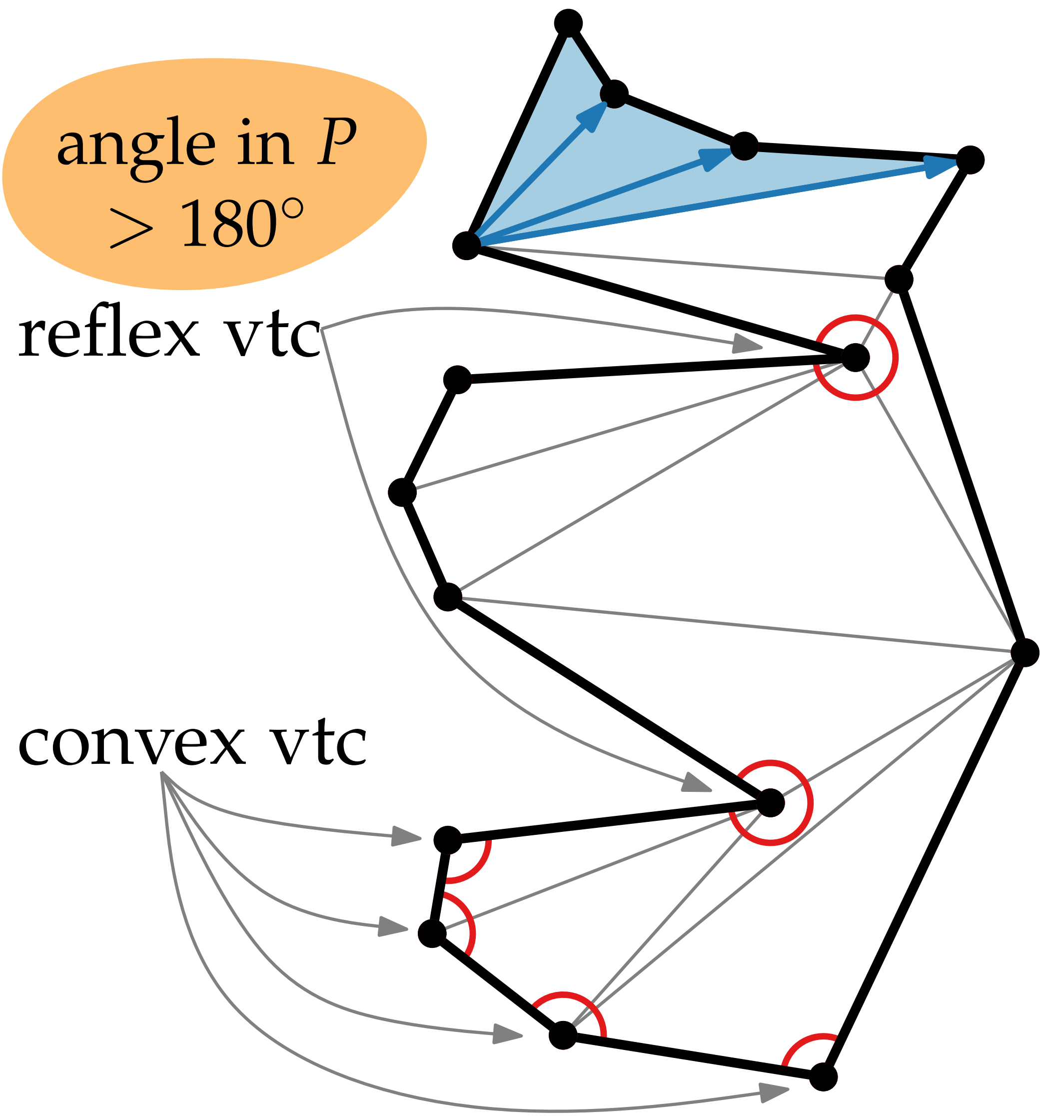

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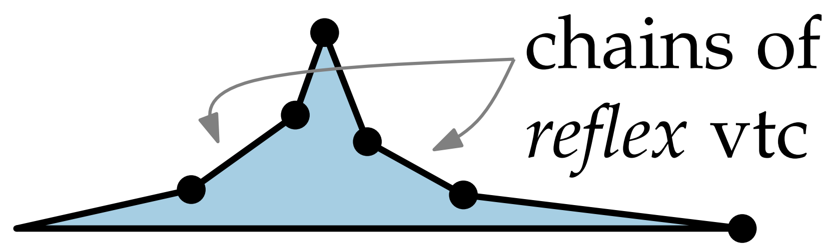

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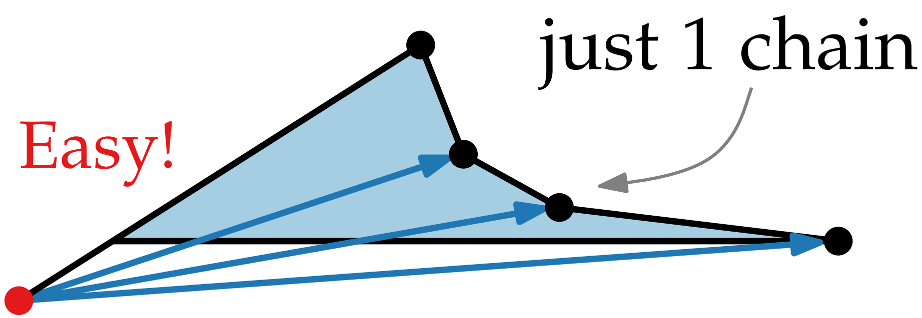

Algorithm

TriangulateMonotonePolygon(Polygon P as circular vertex list)
merge left and right chain $\rightarrow$ seq. $u_1, \ldots, u_n$ with $y_1 \geq \ldots \geq y_n$
Stack S; S.push(u_1); S.push(u_2)
for $j \leftarrow 3$ **to** $n - 1$ **do**

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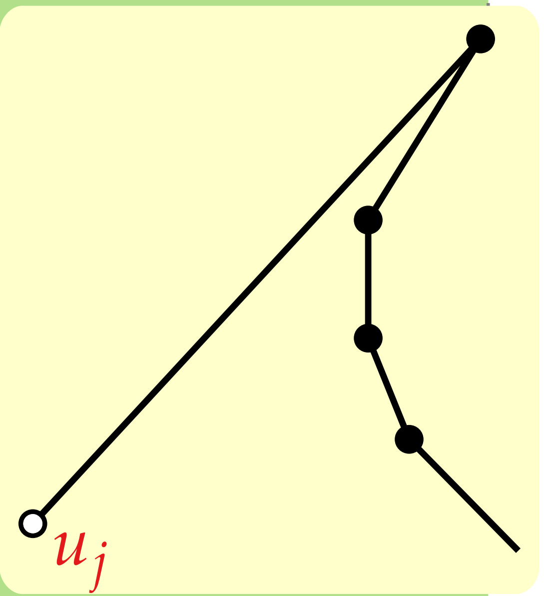
Stack S; S.push(u_1); S.push(u_2)
for $j \leftarrow 3$ **to** $n-1$ **do**
  if u_j and S.top() lie on different chains **then**

  else



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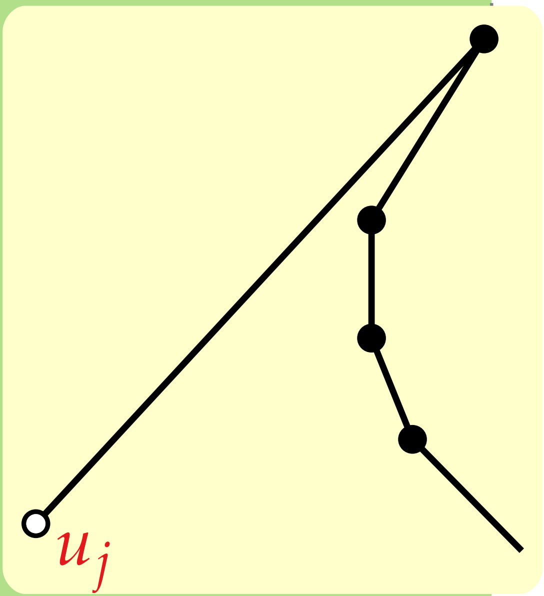

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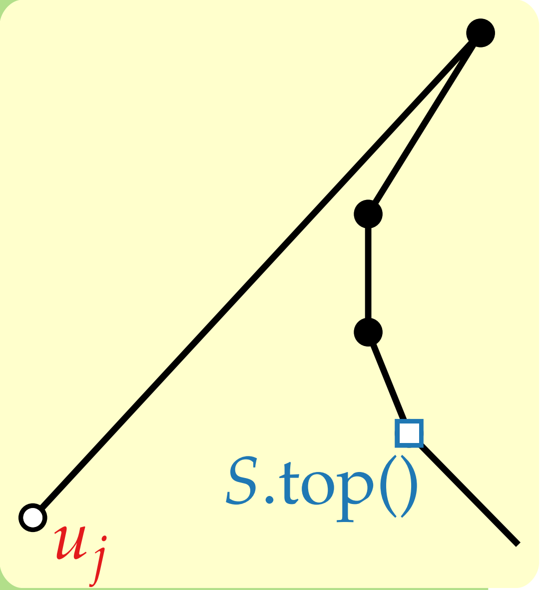

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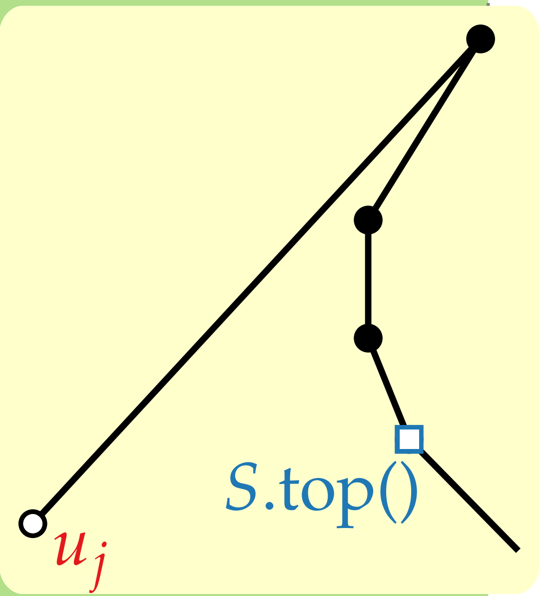
Stack S; S.push(u_1); S.push(u_2)
for $j \leftarrow 3$ **to** $n-1$ **do**
  if u_j and S.top() lie on different chains **then**
    while not S.empty() **do**
      $v \leftarrow S$.pop()
      if not S.empty() **then** draw diag. (u_j, v)
  else



Algorithm

TriangulateMonotonePolygon(Polygon P as circular vertex list)
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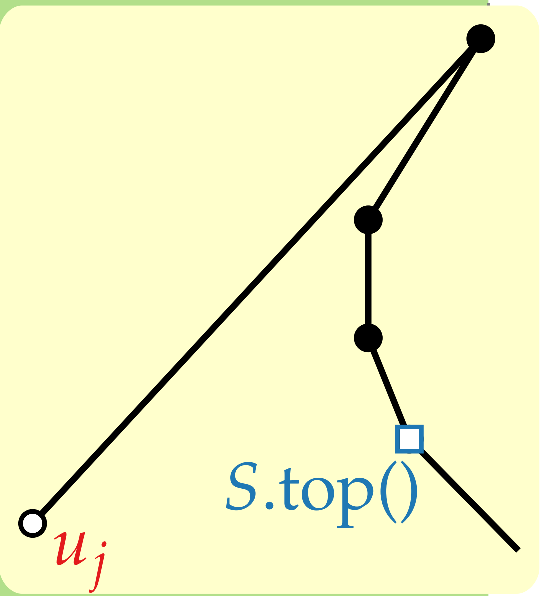

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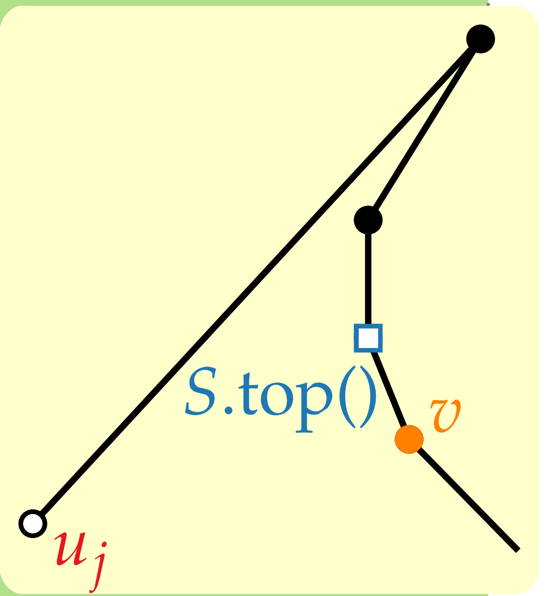

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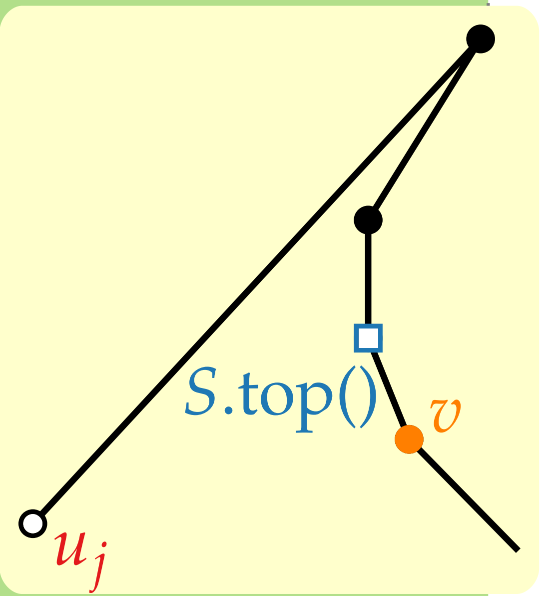

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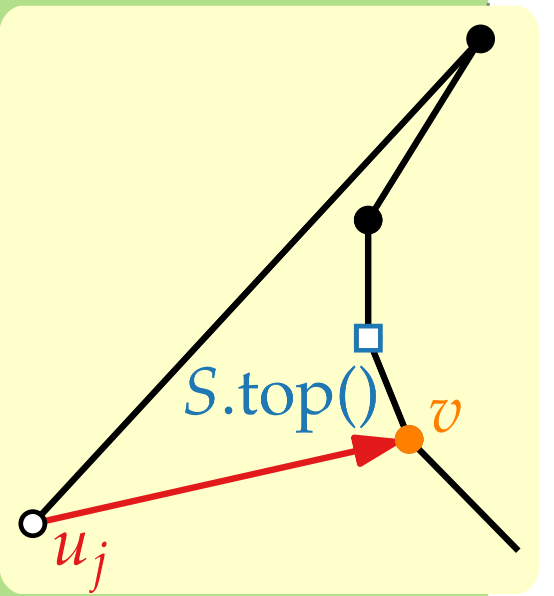

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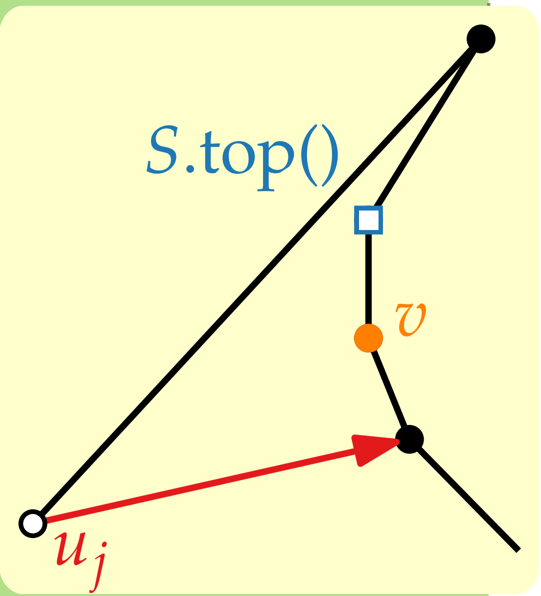

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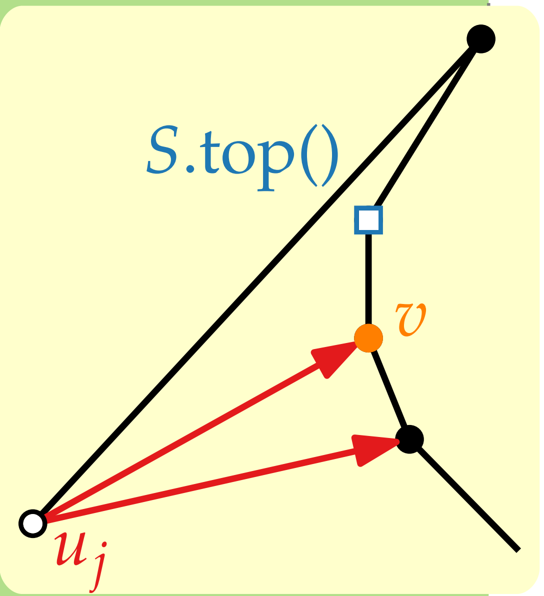

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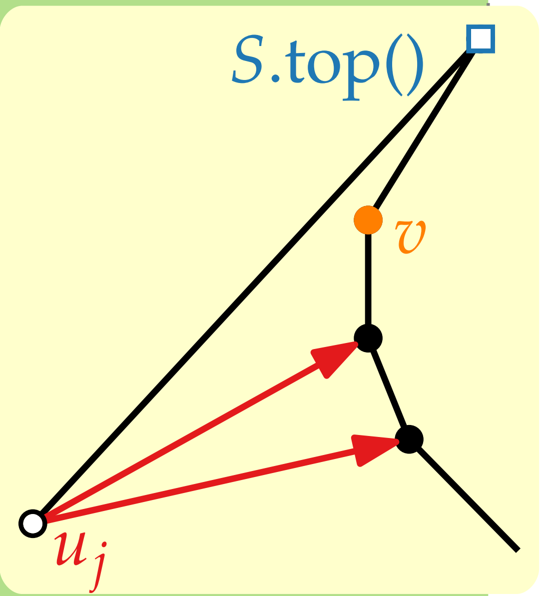

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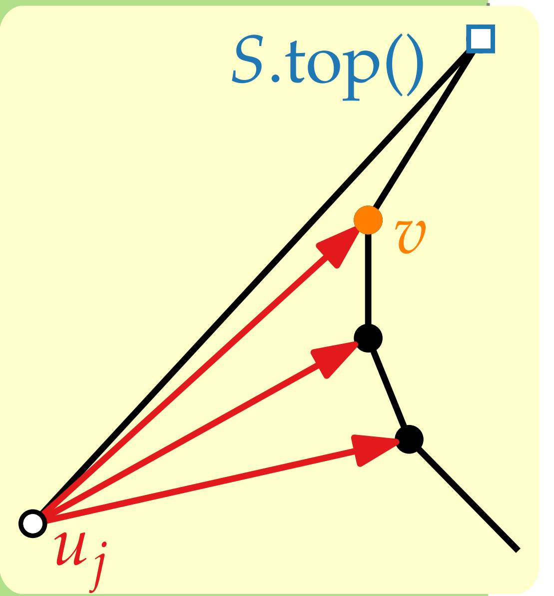

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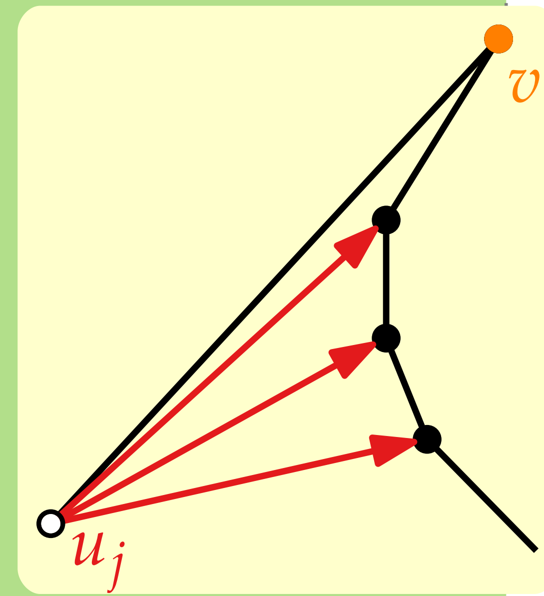

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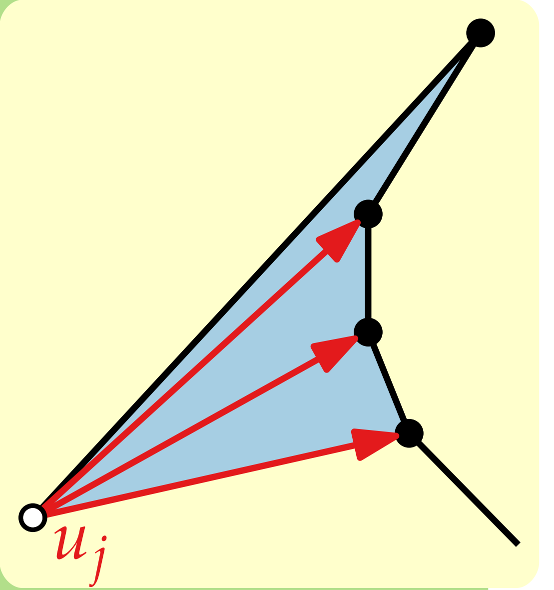

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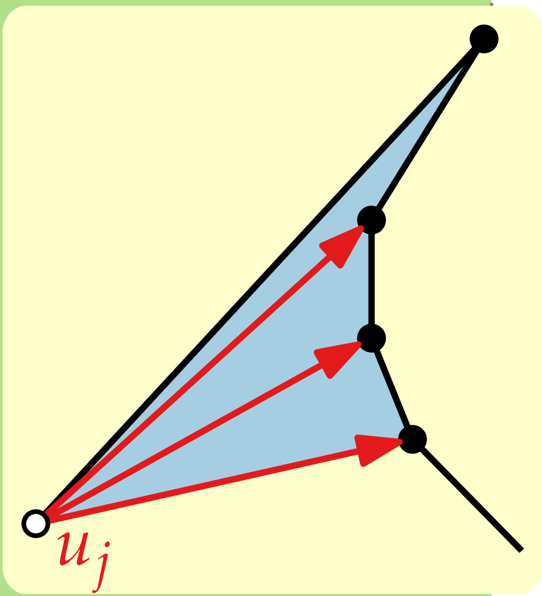

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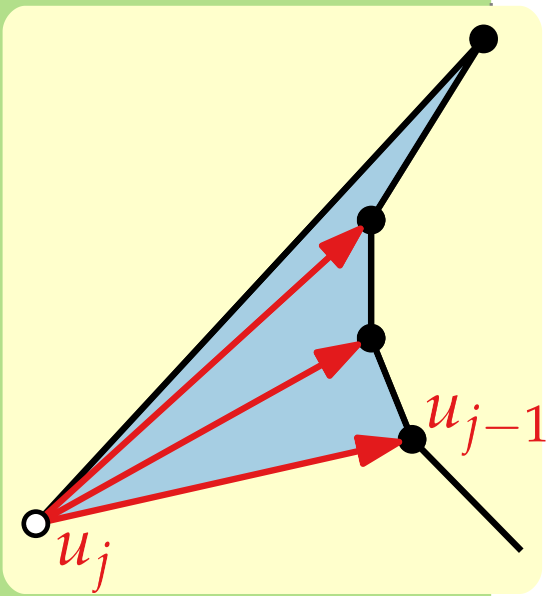

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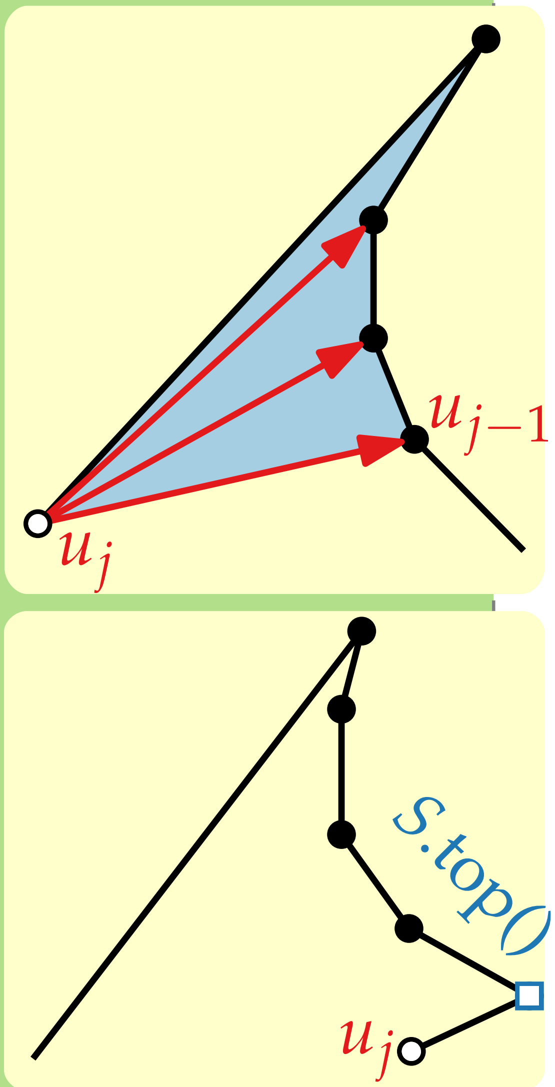

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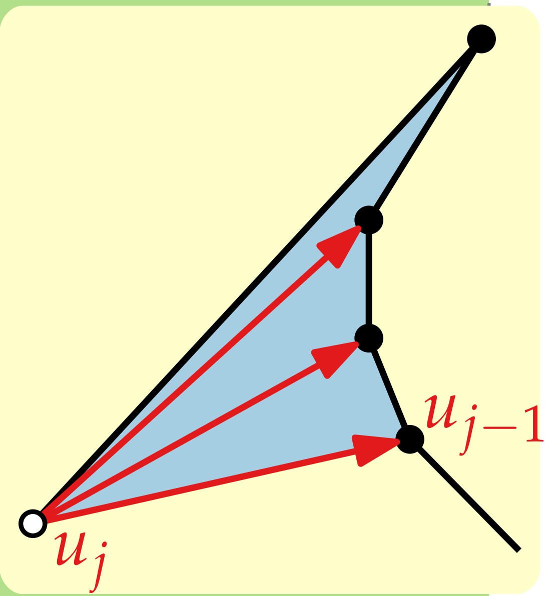

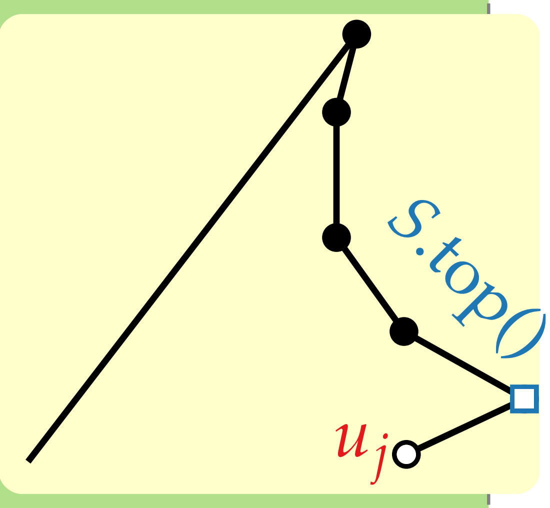

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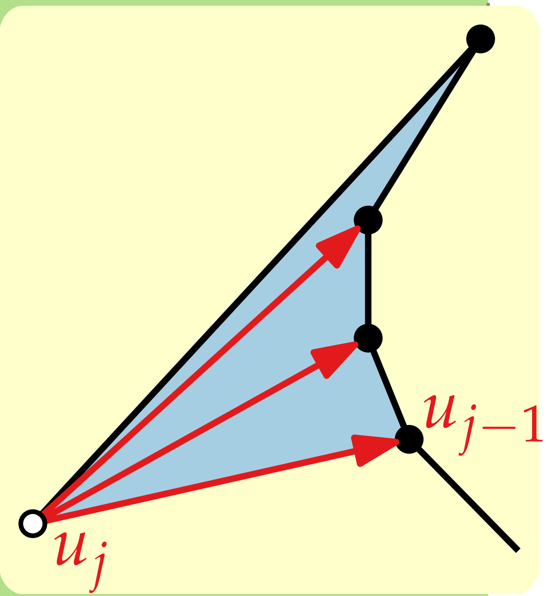

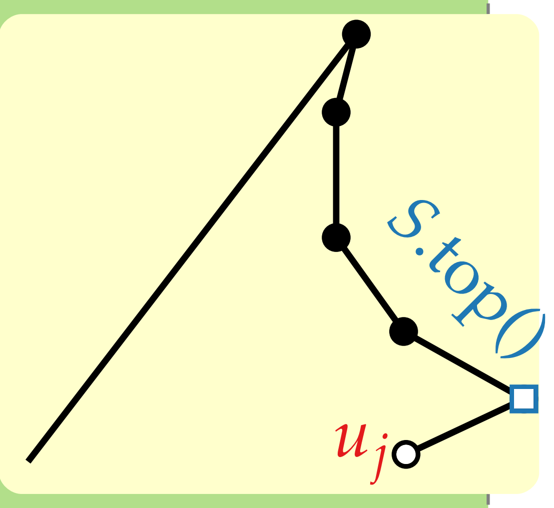

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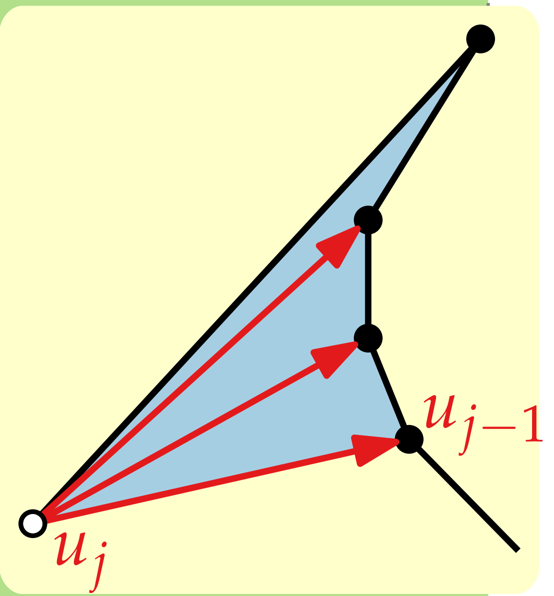

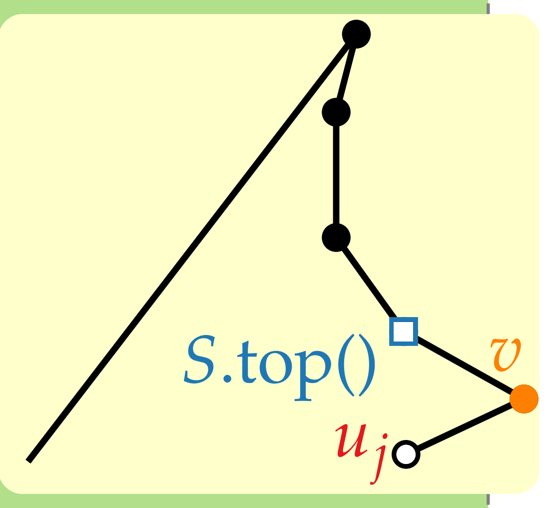

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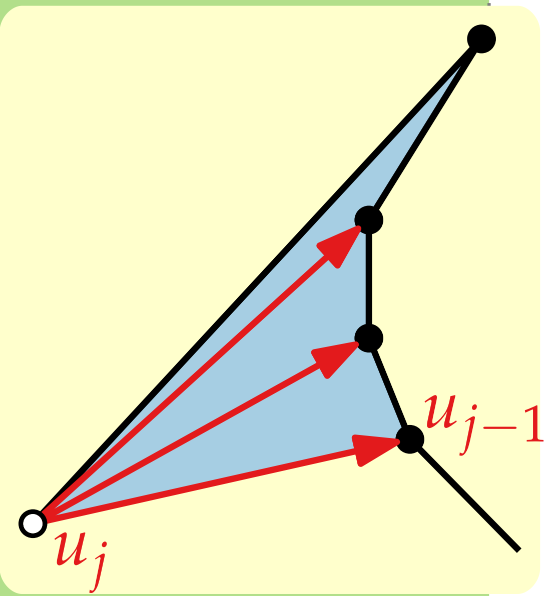

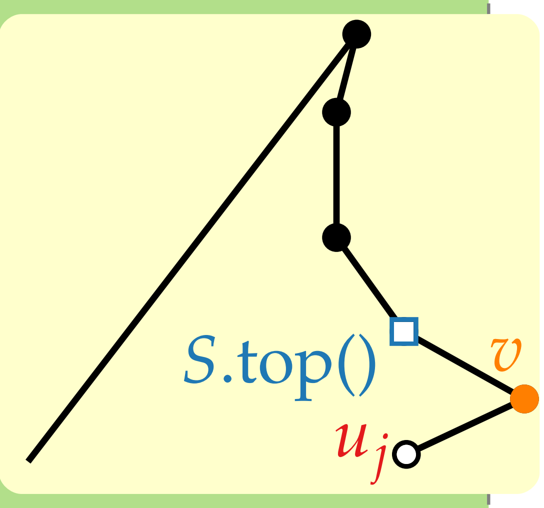

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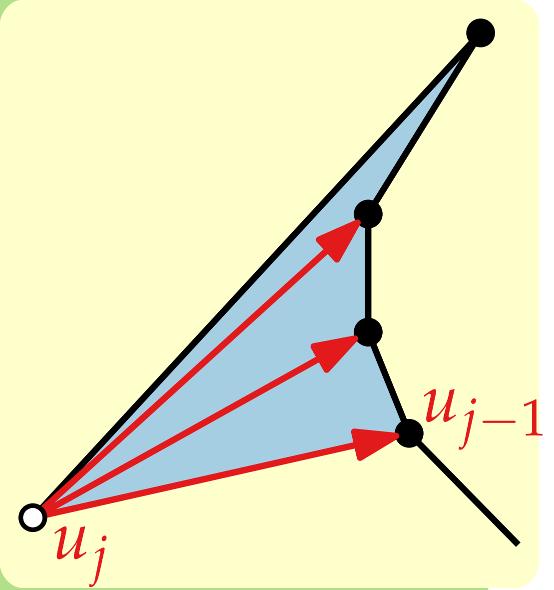

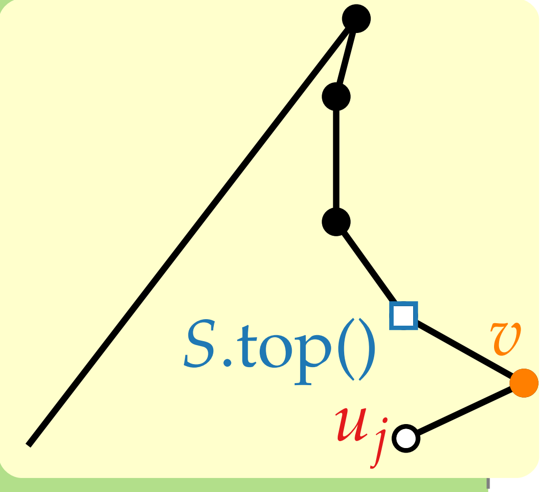

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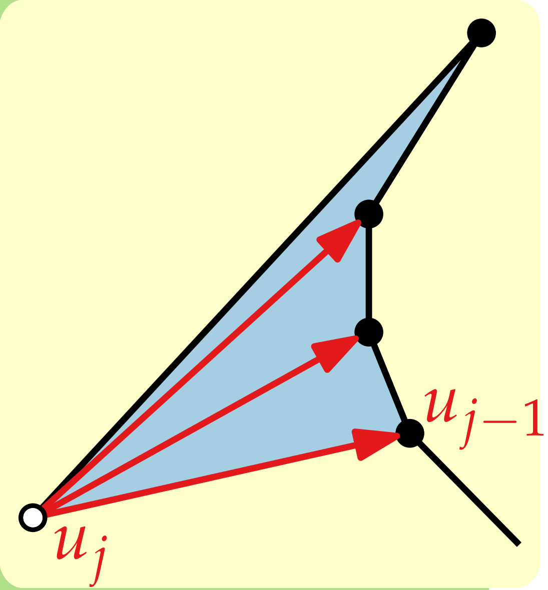

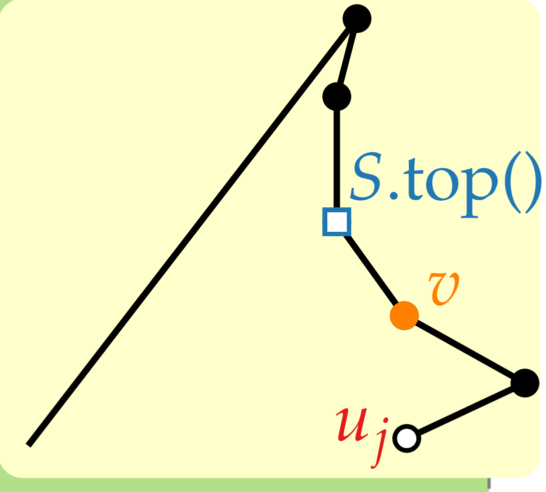

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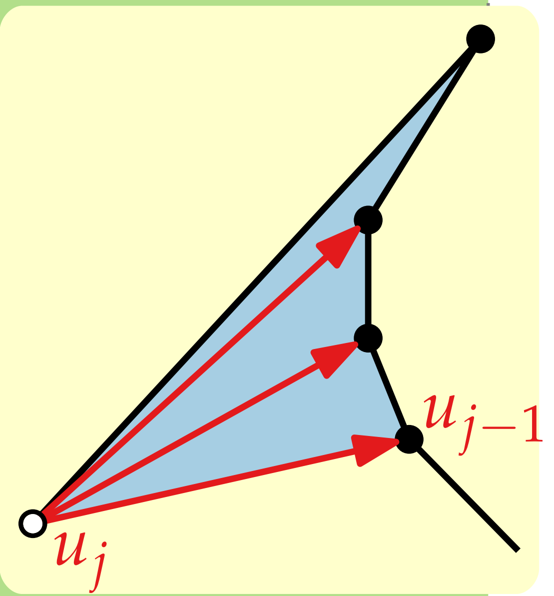

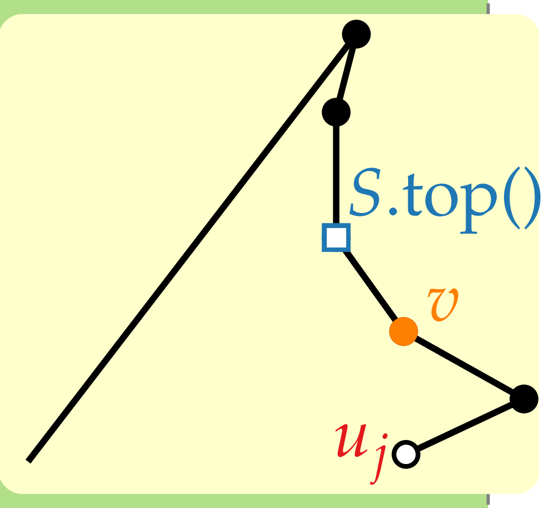

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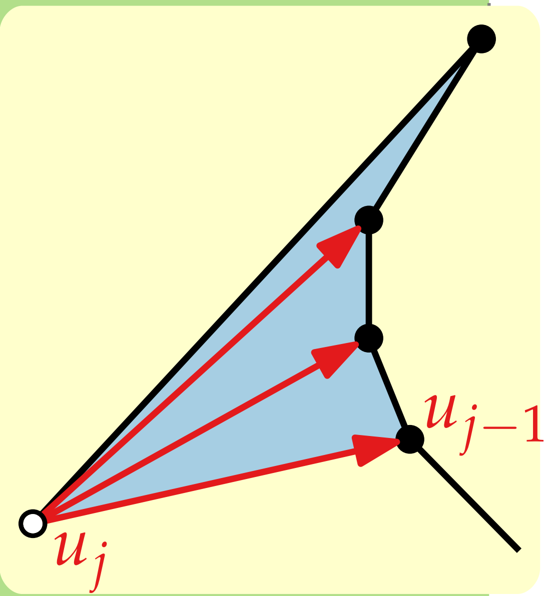

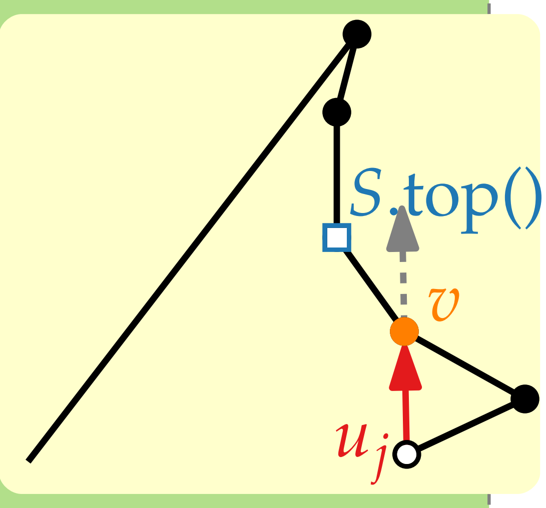

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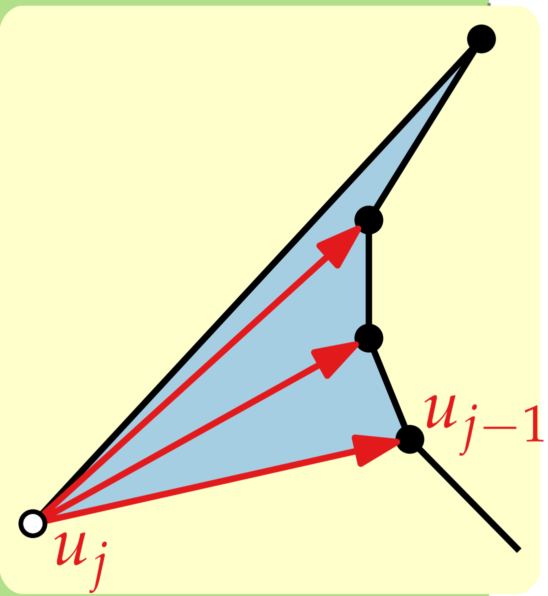

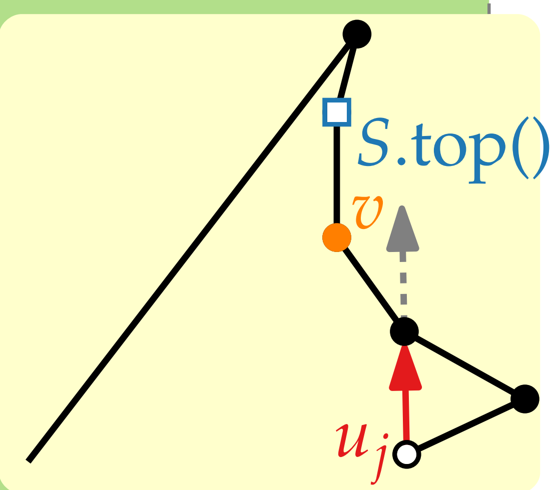

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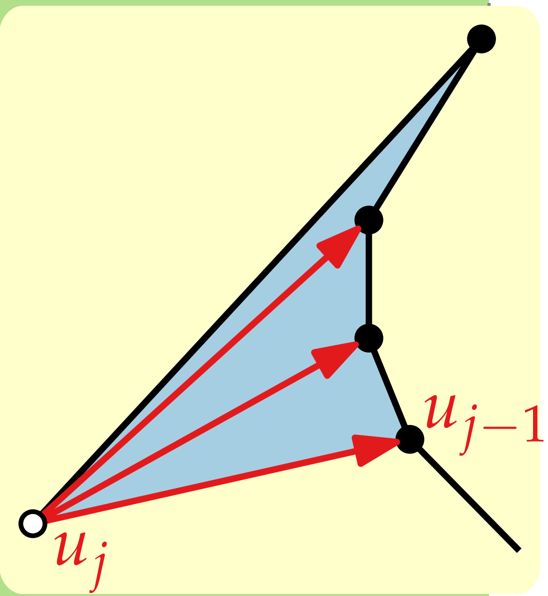

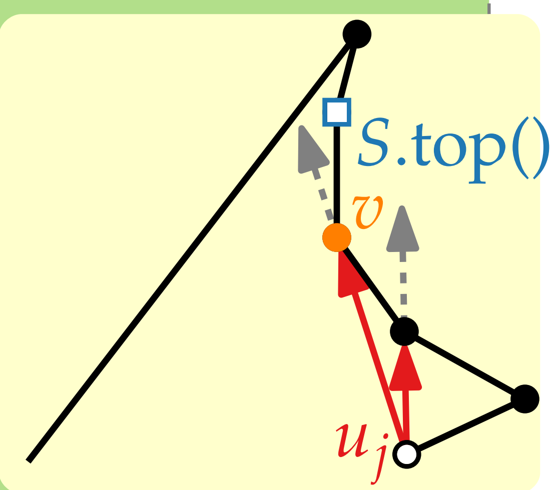

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TriangulateMonotonePolygon(Polygon P as circular vertex list)
merge left and right chain $\rightarrow$ seq. $u_1, \ldots, u_n$ with $y_1 \geq \ldots \geq y_n$
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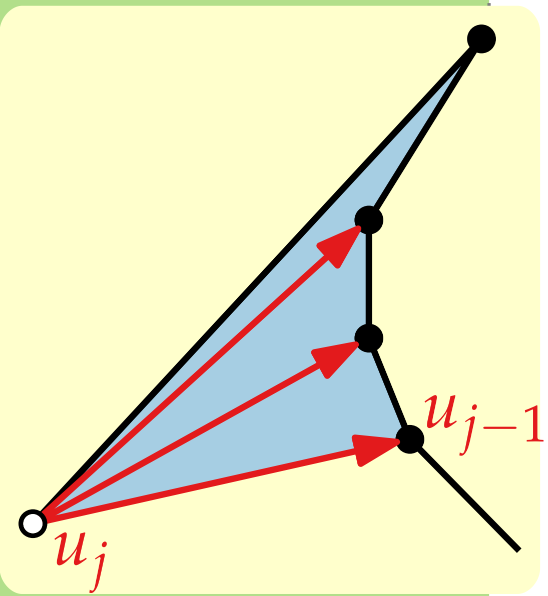

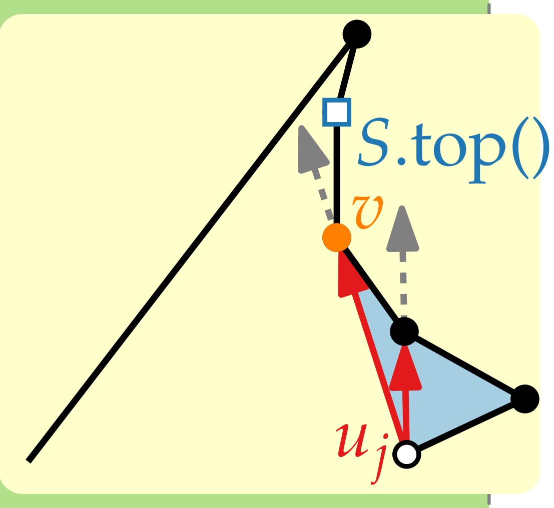

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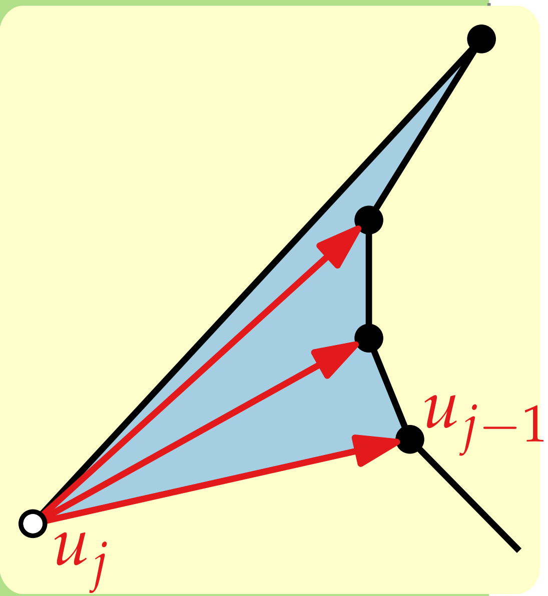

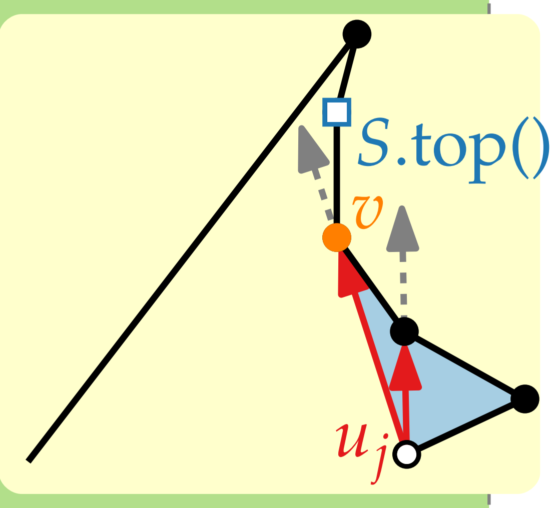

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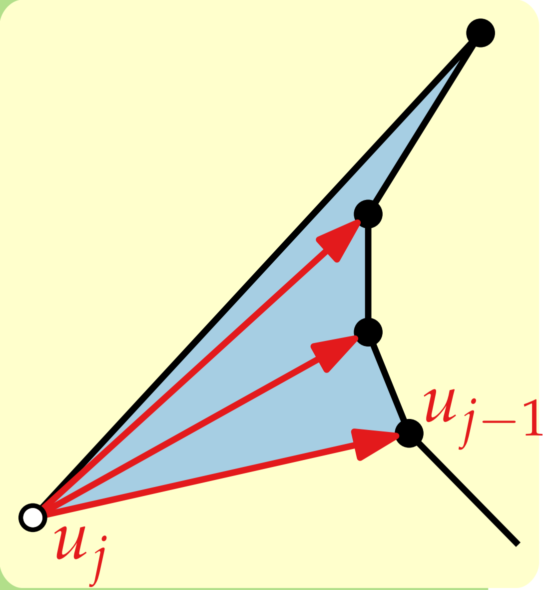

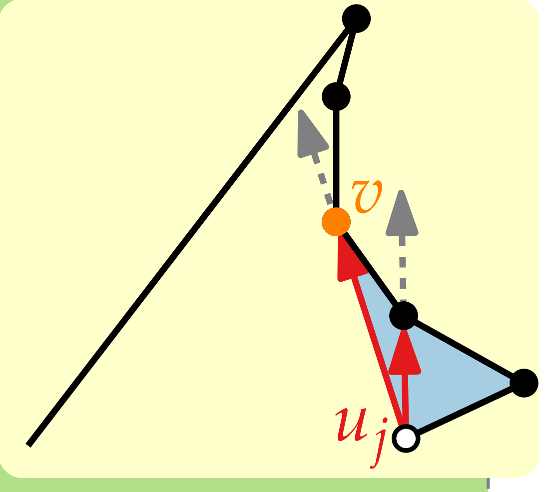

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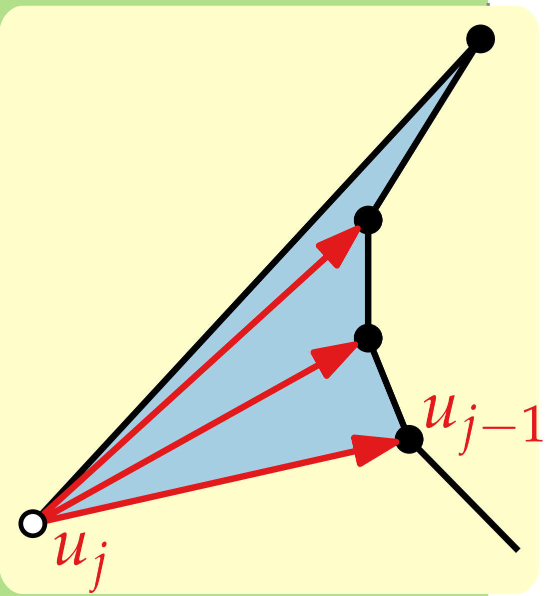

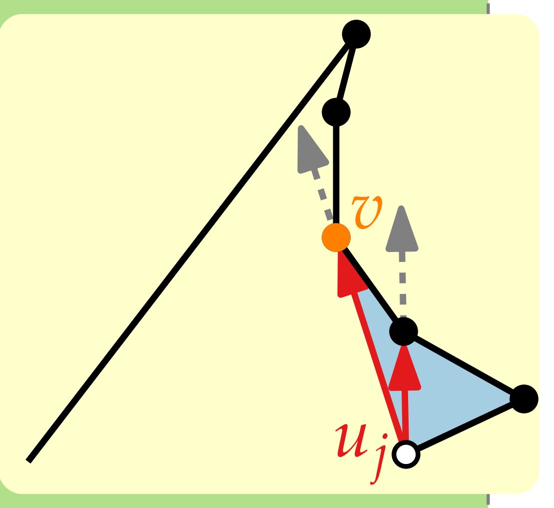

Algorithm

Running time?

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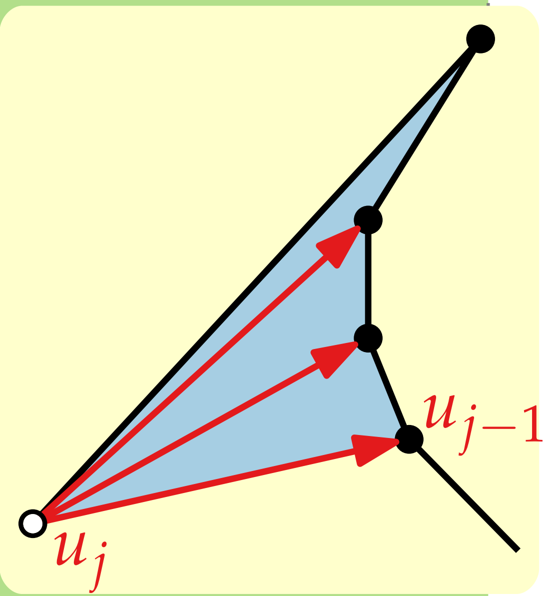

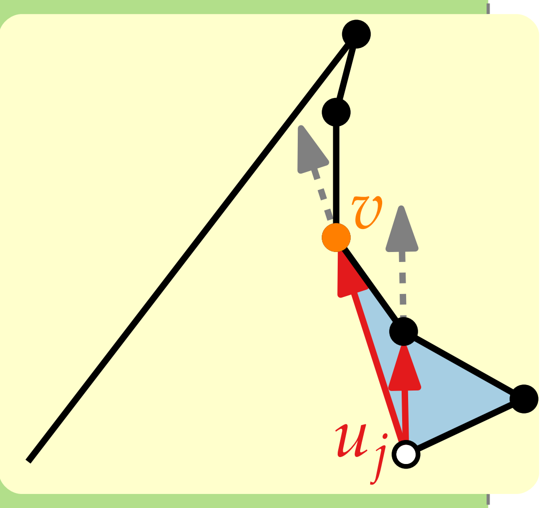

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Running time? $\Theta(n)$

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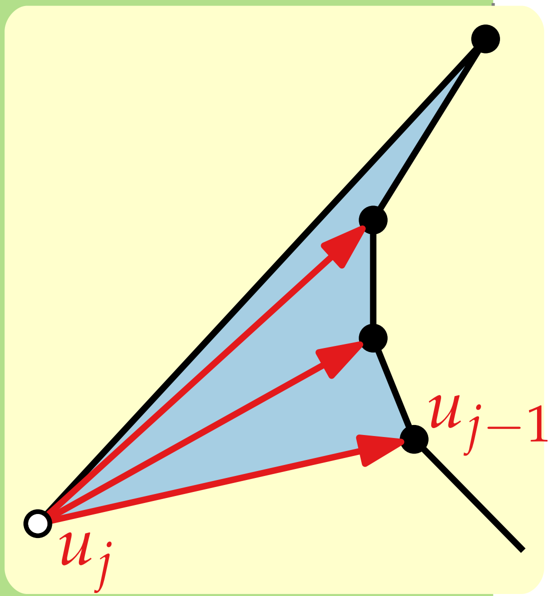

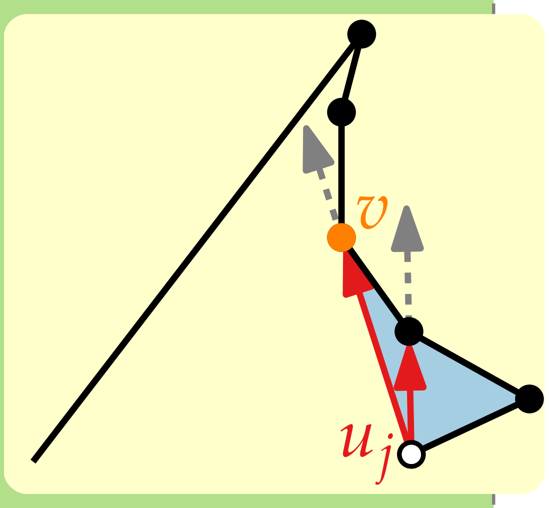

Summary

n-vtx polygon $\xrightarrow[O(n \log n)]{}$ "nice" pieces, n' vtc $\xrightarrow[O(n')]{}$ n'' triangles

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