Computational Geometry

Lecture 2:
Line-Segment Intersection
or
Map Overlay

Part I:
Map Overlay

Philipp Kindermann

Winter Semester 2020


Dawson
Whitehorse
Echo Bay
Ft. Simpson
Yellowknife
Uranium City
Prince Rupert
Dawson Creek
Lynn Lake
Edmonton
Vancouver
Victoria
Saskatoon
Calgary
Regina


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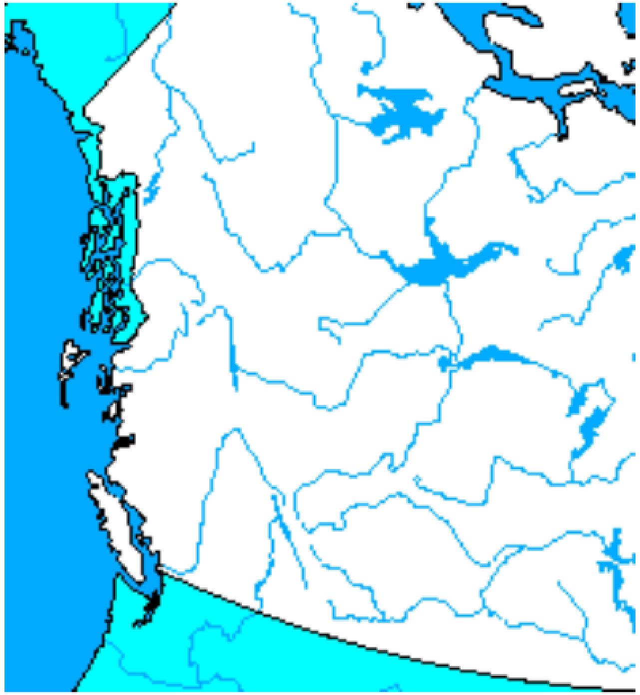


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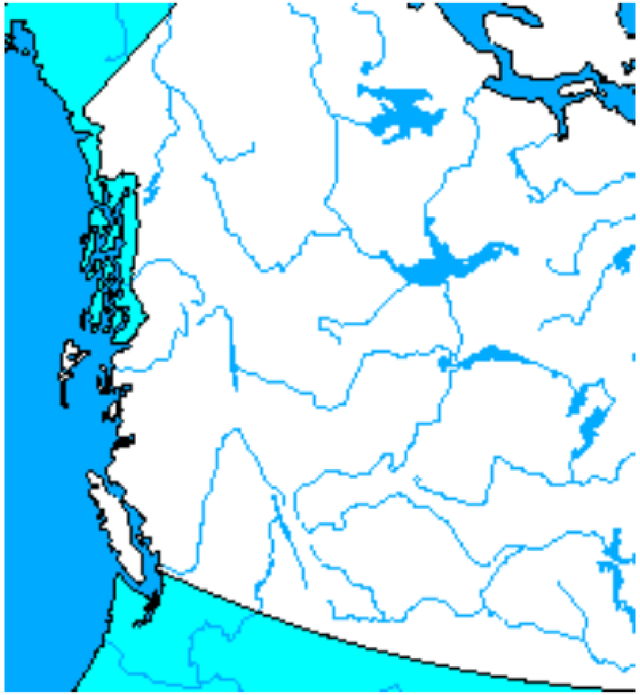
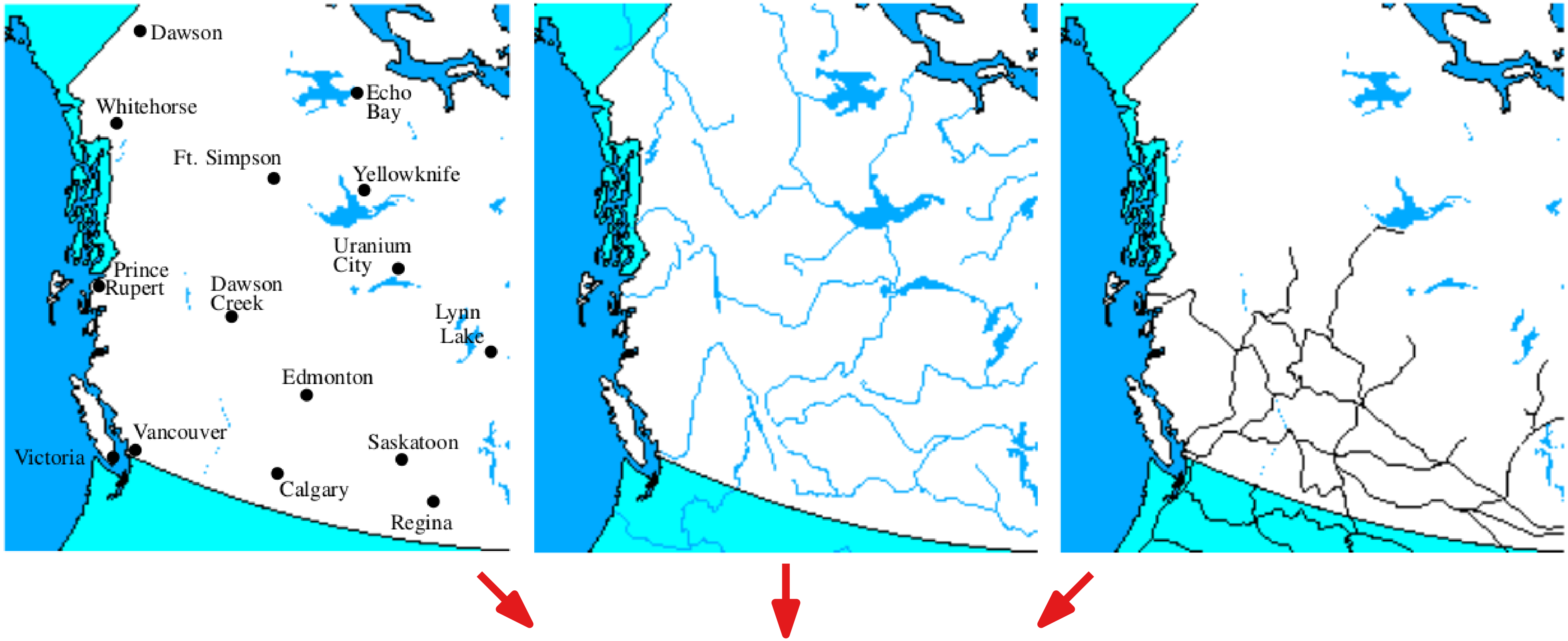
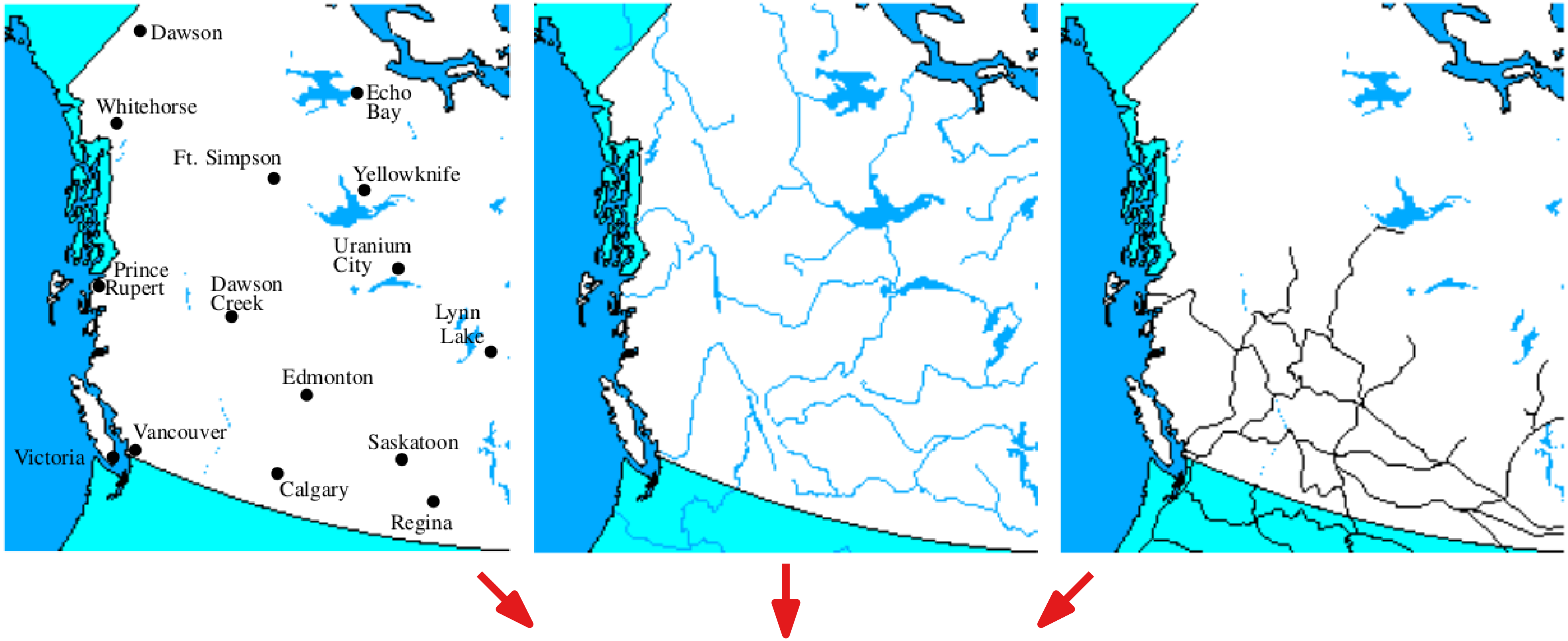




Map Overlay in Geographic Information Systems (GIS)



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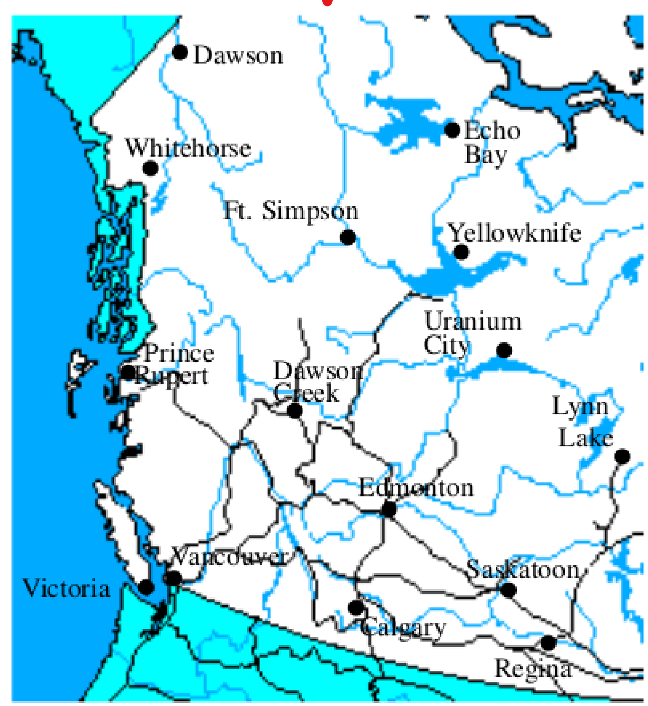

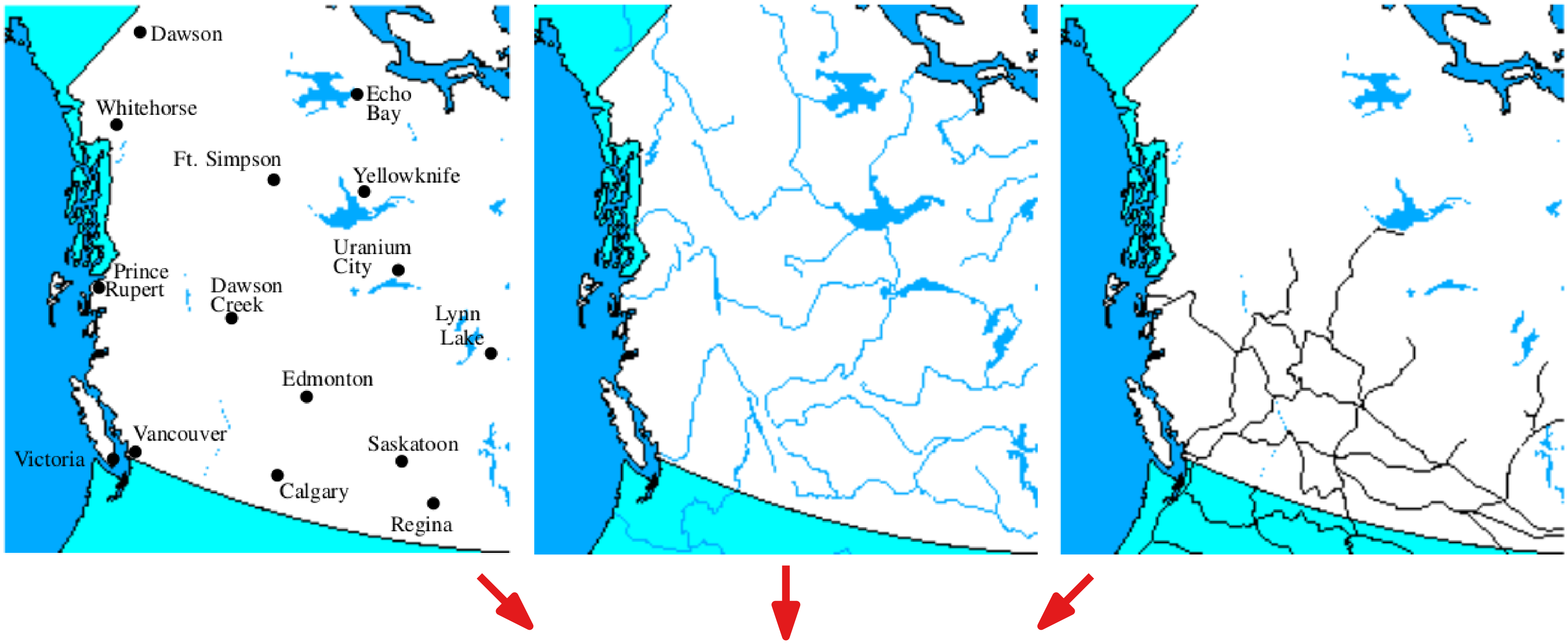

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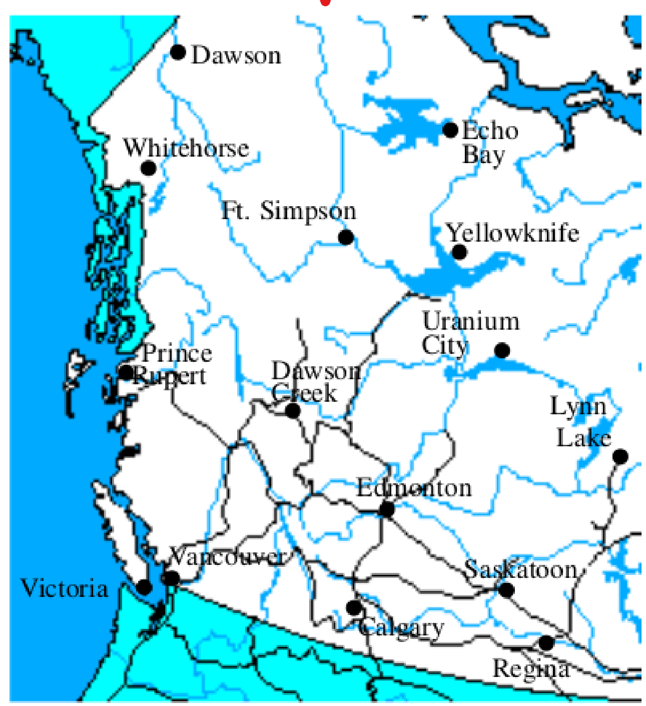

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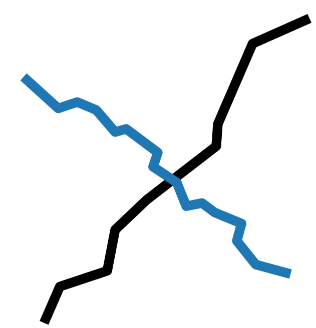

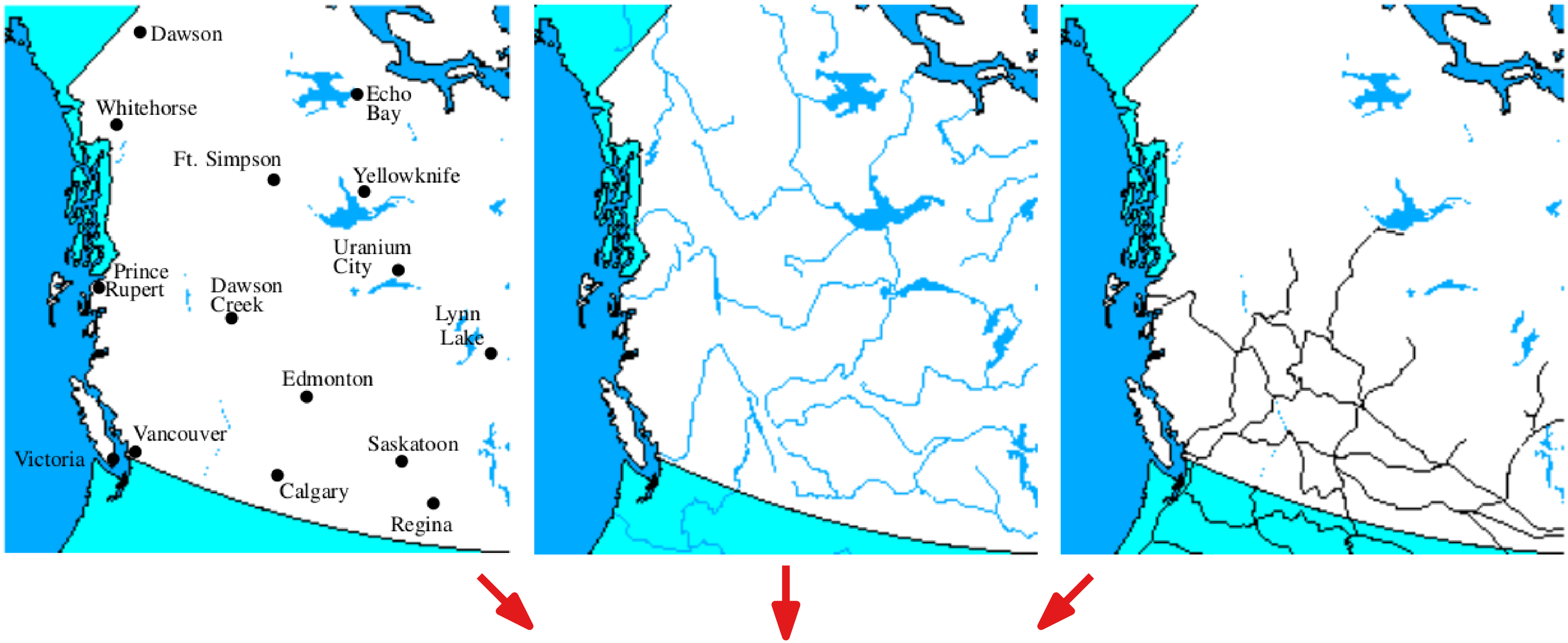

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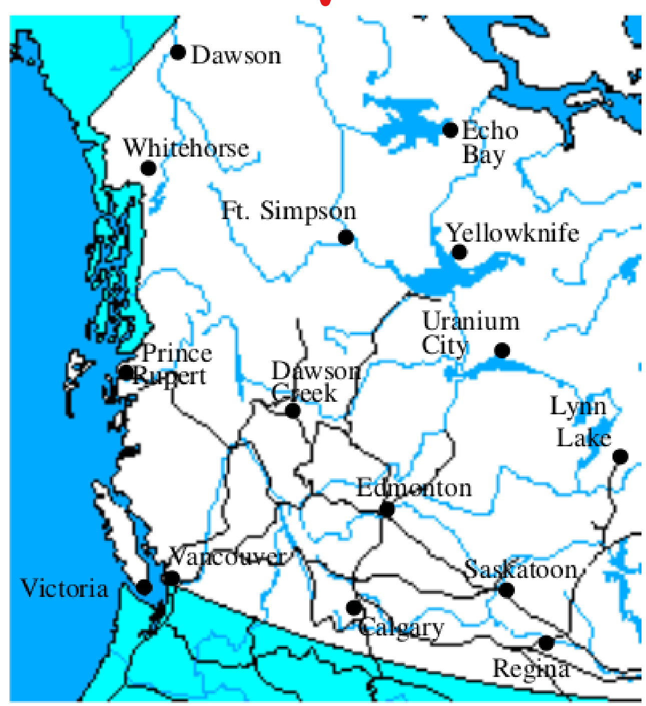

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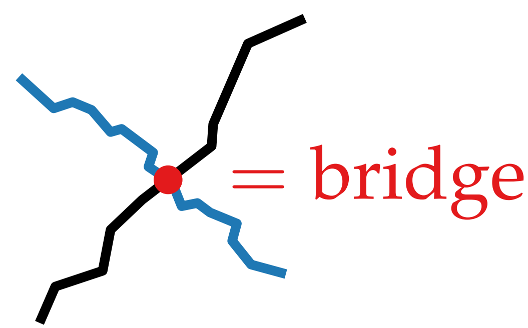
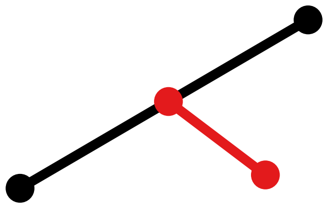


Line-Segment Intersection

Definition:

Line-Segment Intersection

Definition: Is  an intersection?

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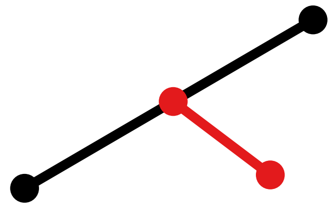
Definition: Is  an intersection?

Answer: Depends. . .

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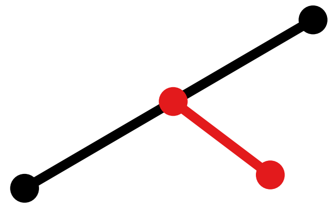
Definition: Is  an intersection?

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Problem: Given a set S of n *closed* non-overlapping line segments in the plane, compute. . .

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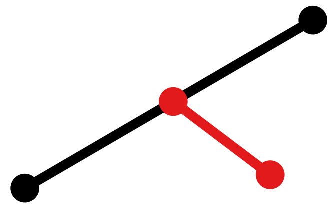
Definition: Is  an intersection?

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Problem: Given a set S of n *closed* non-overlapping line segments in the plane, compute. . .

- all points where at least two segments intersect and
- for each such point report all segments that contain it.

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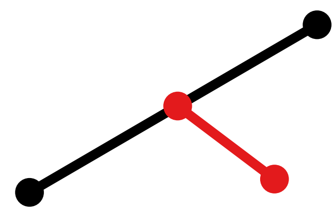
Definition: Is  an intersection?

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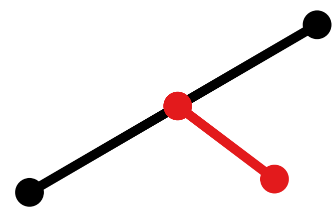
Definition: Is  an intersection?

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yes!

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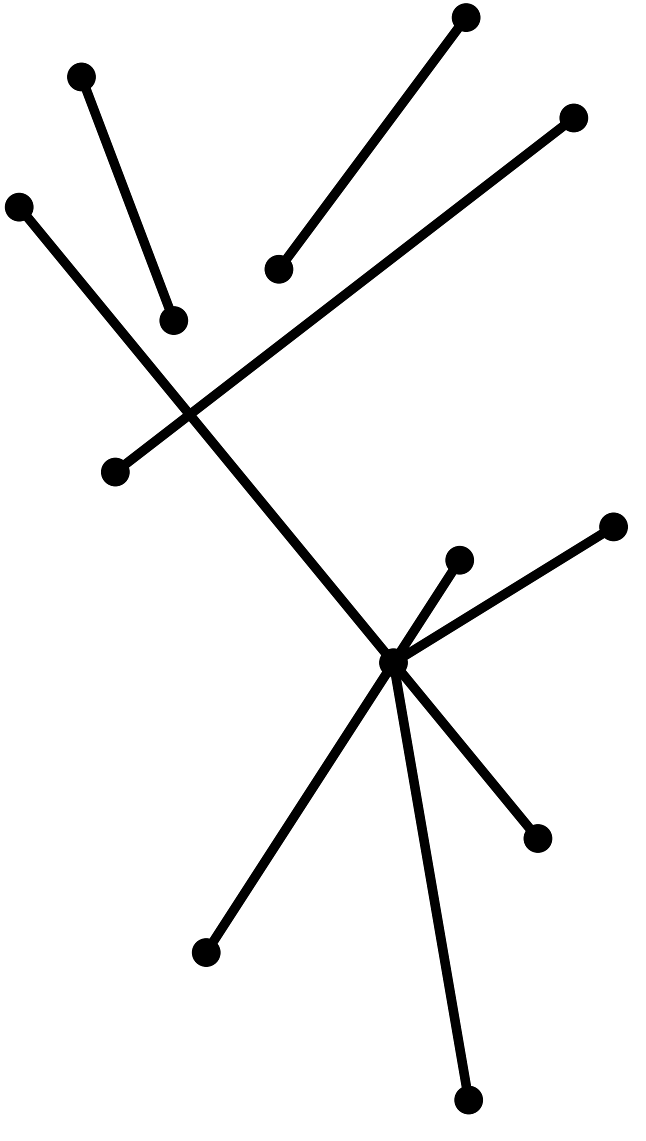
– all points where at least two segments intersect and
– for each such point report all segments that contain it.

Task: How would *you* do it?

Example



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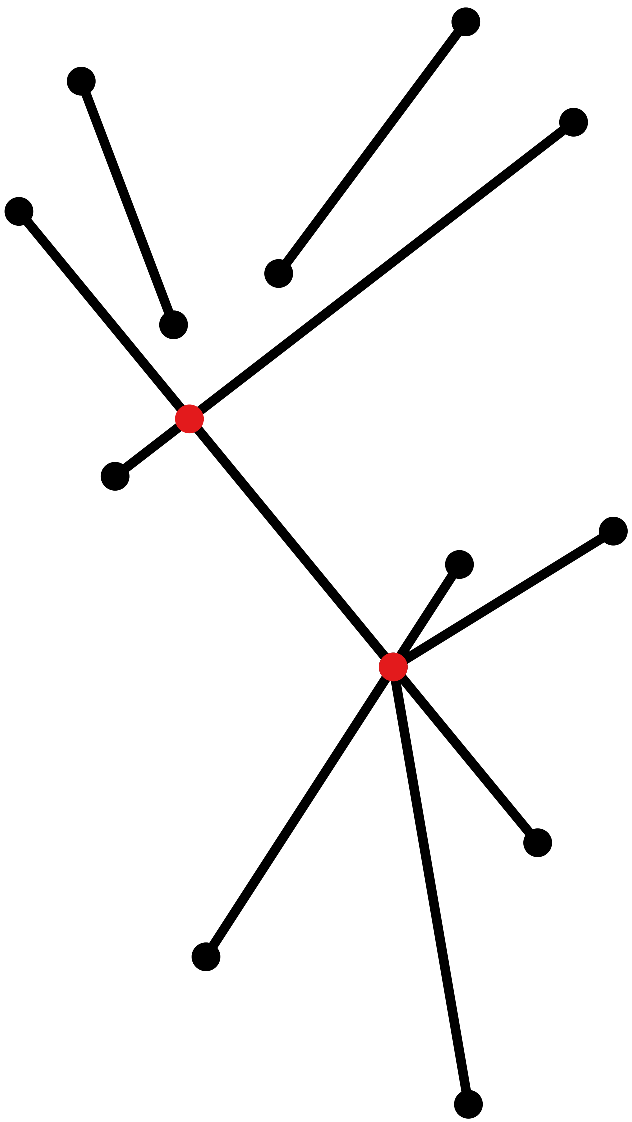

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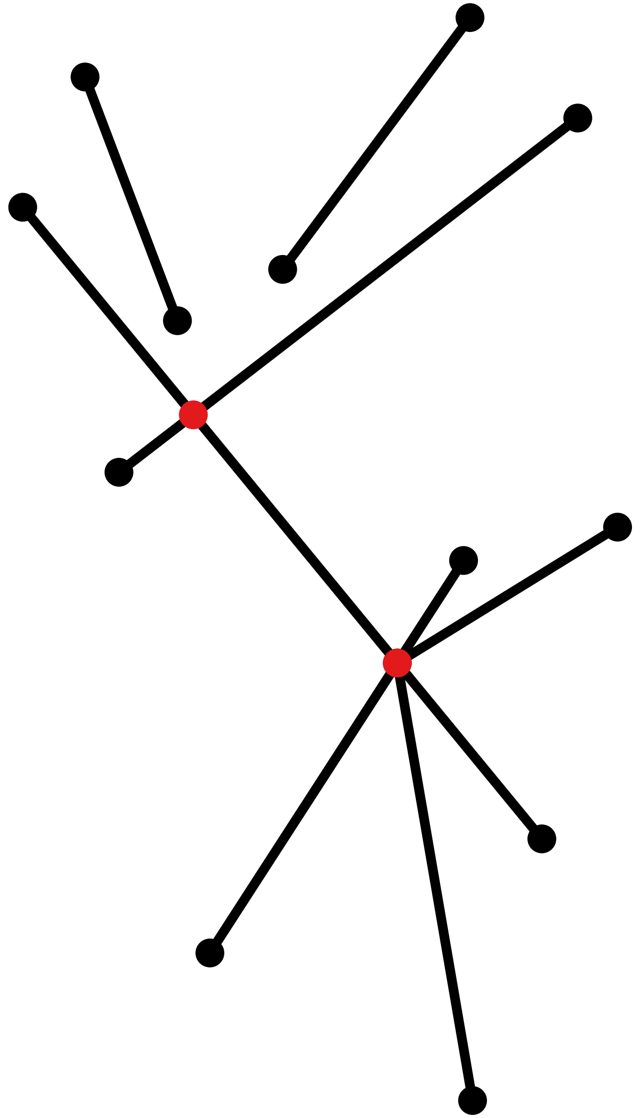
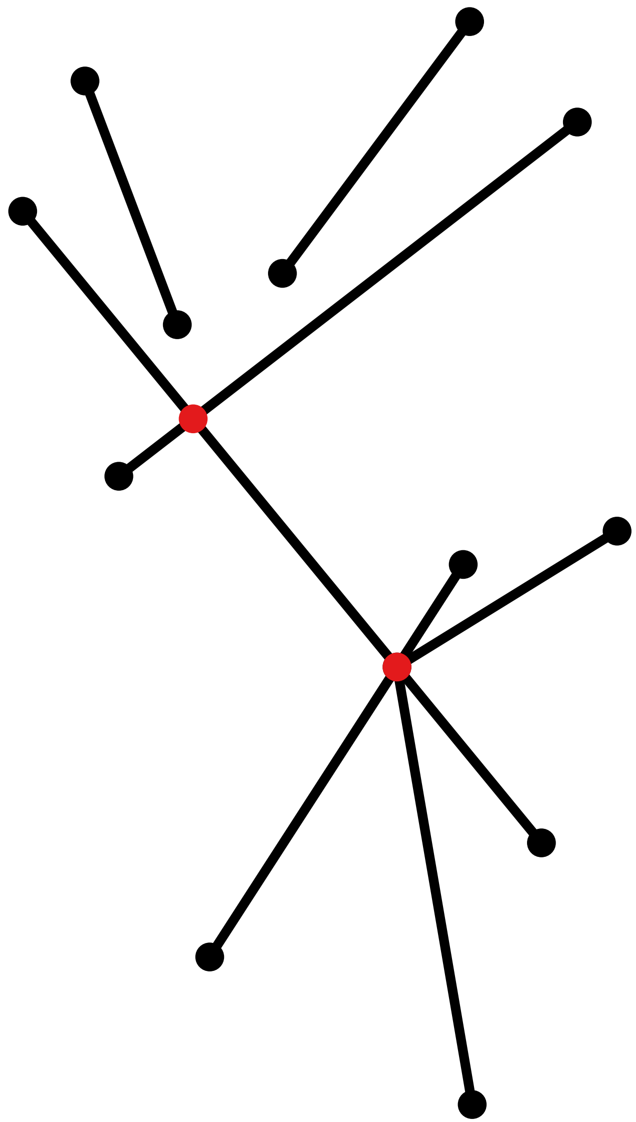


Brute Force?

$O(n^2)$ ... can we do better?

Example



Brute Force?

$O(n^2)$ ... can we do better?

Idea:
Process segments top-to-bottom using a "sweep line".

Computational Geometry

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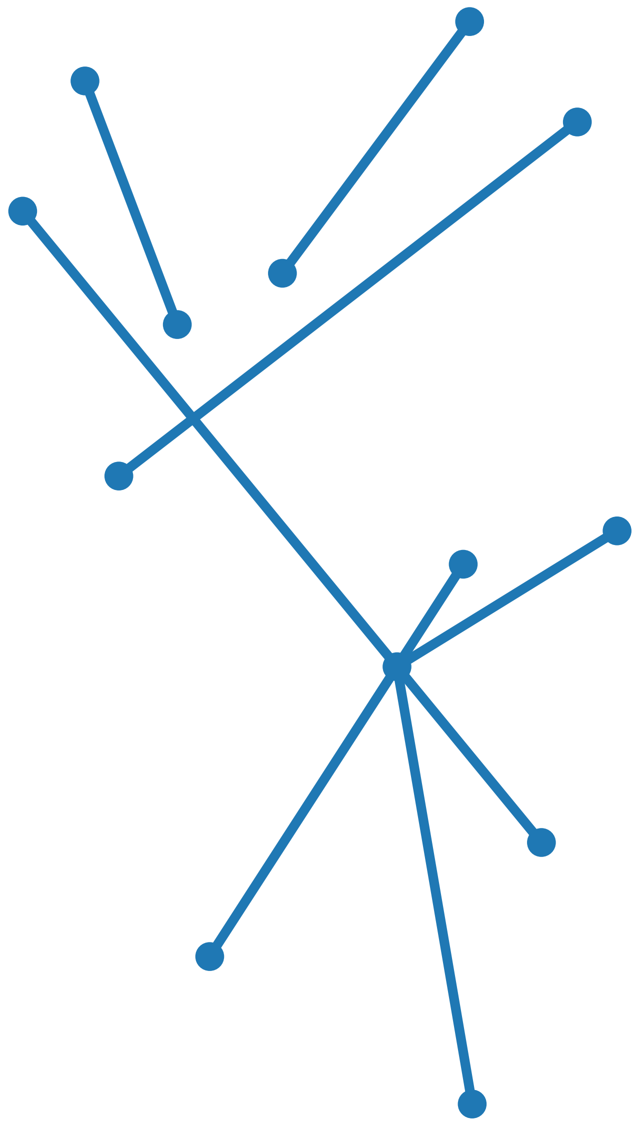
Part II:
Sweep-Line Algorithm

Philipp Kindermann

Winter Semester 2020

Sweep-Line Algorithm



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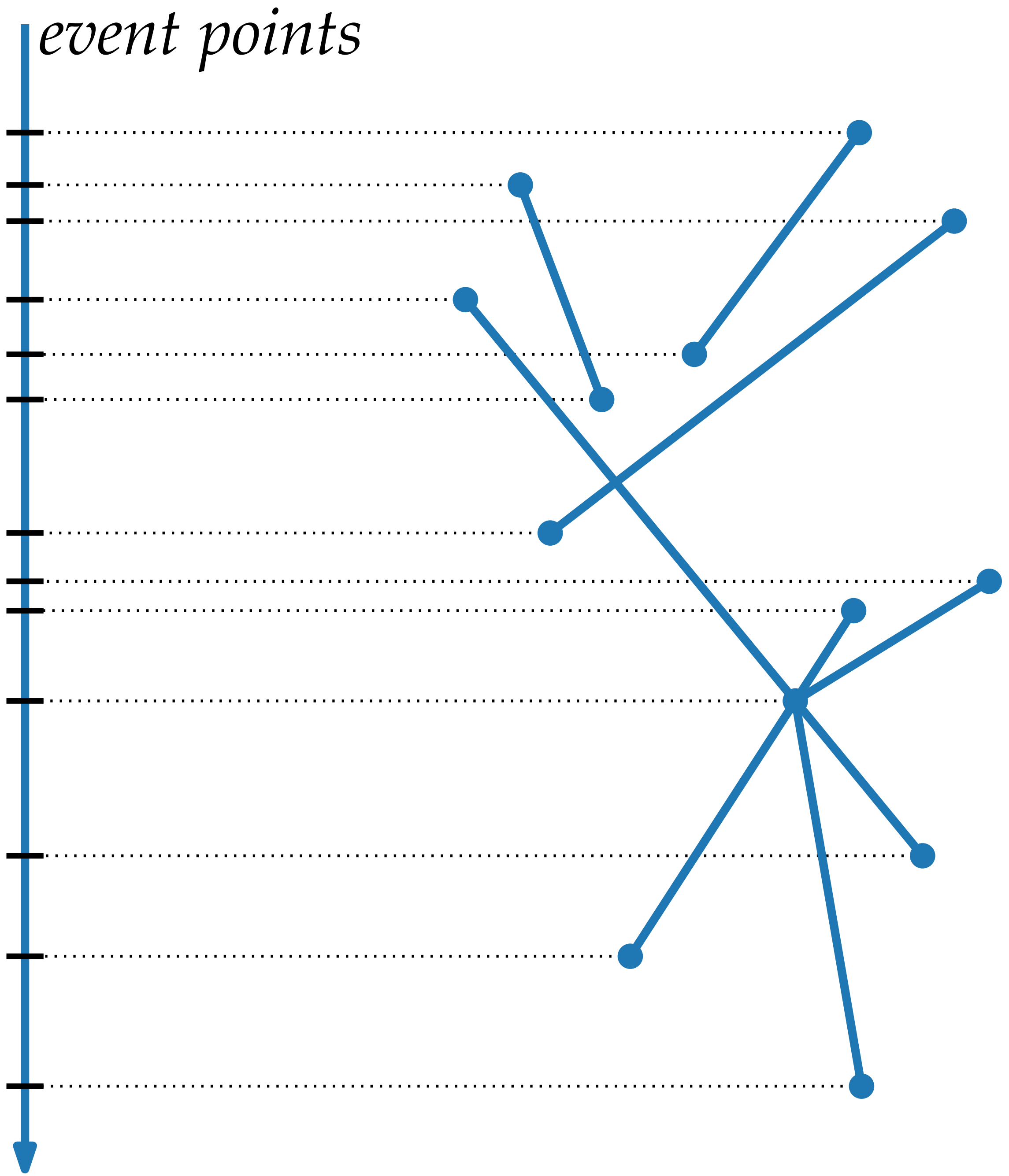
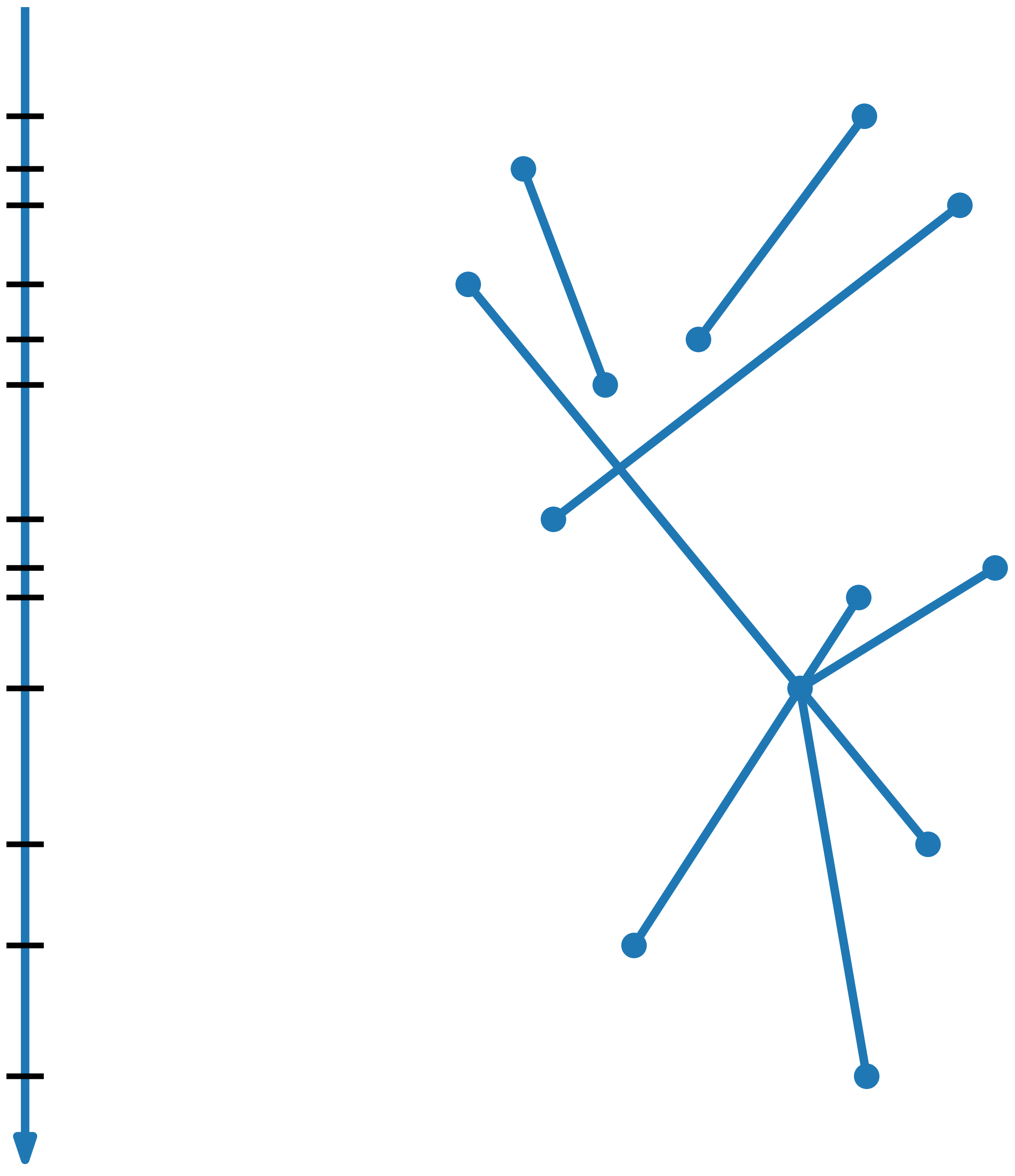


Which active segments should be compared?

Sweep-Line Algorithm



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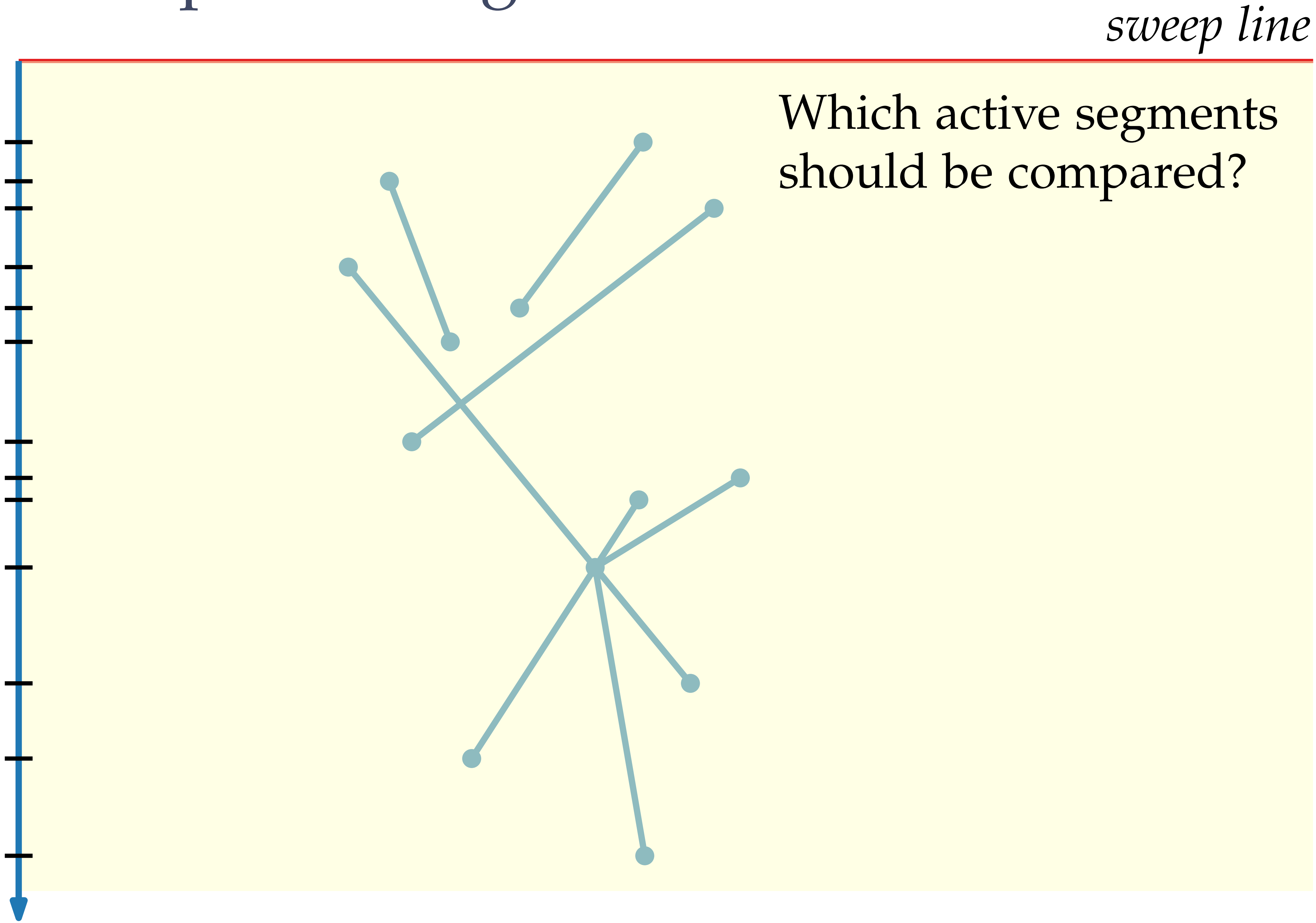

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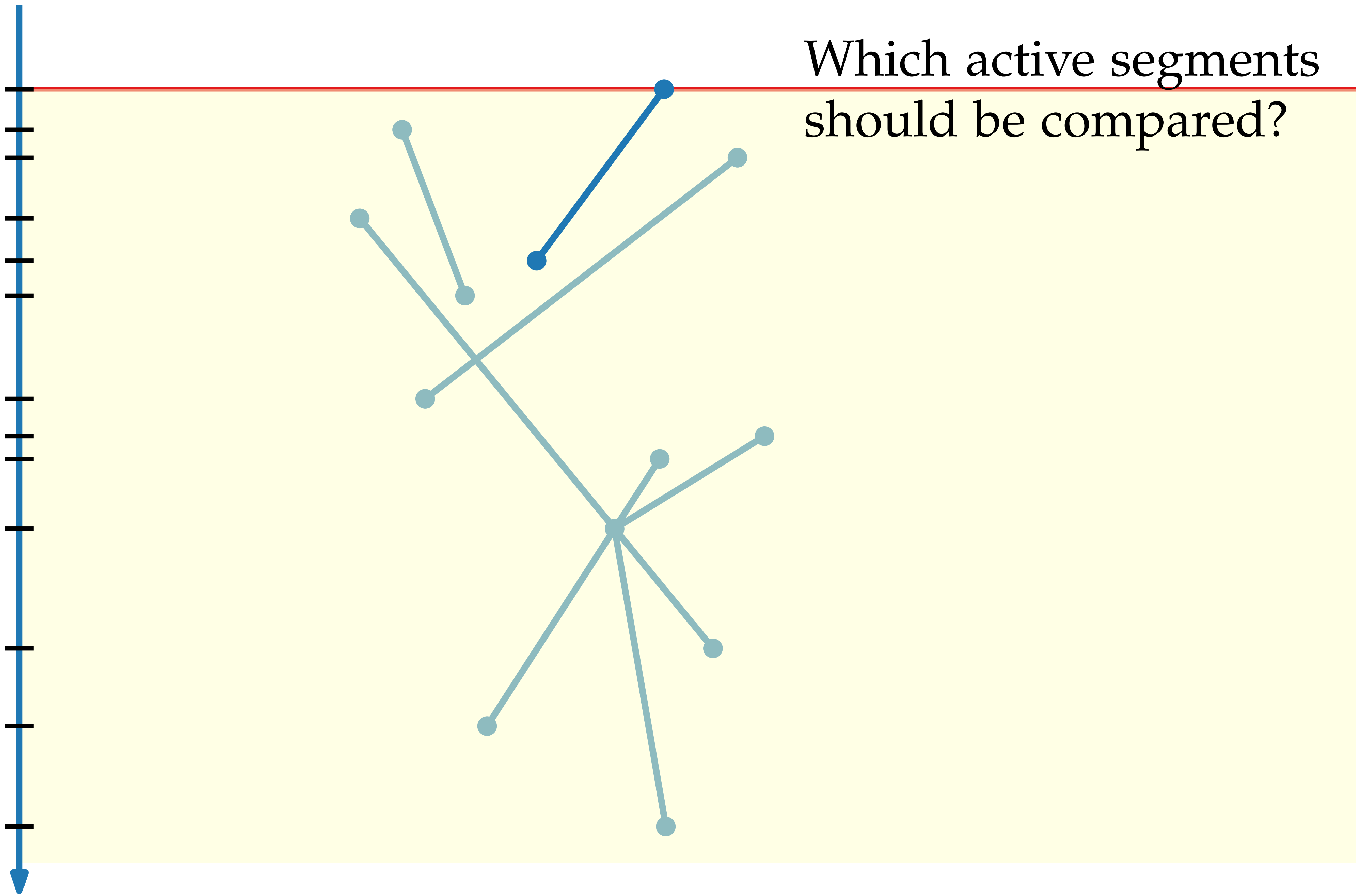

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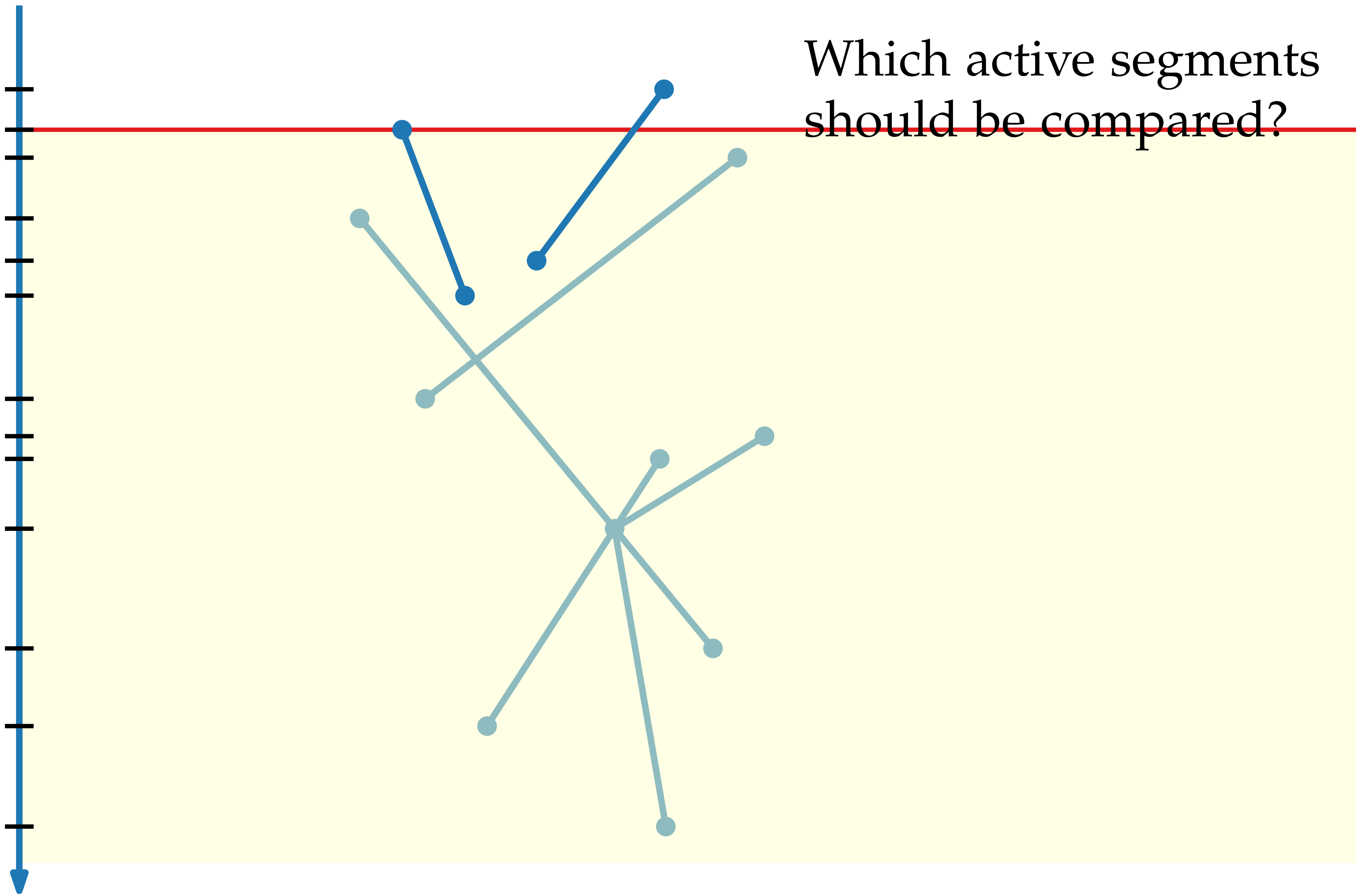

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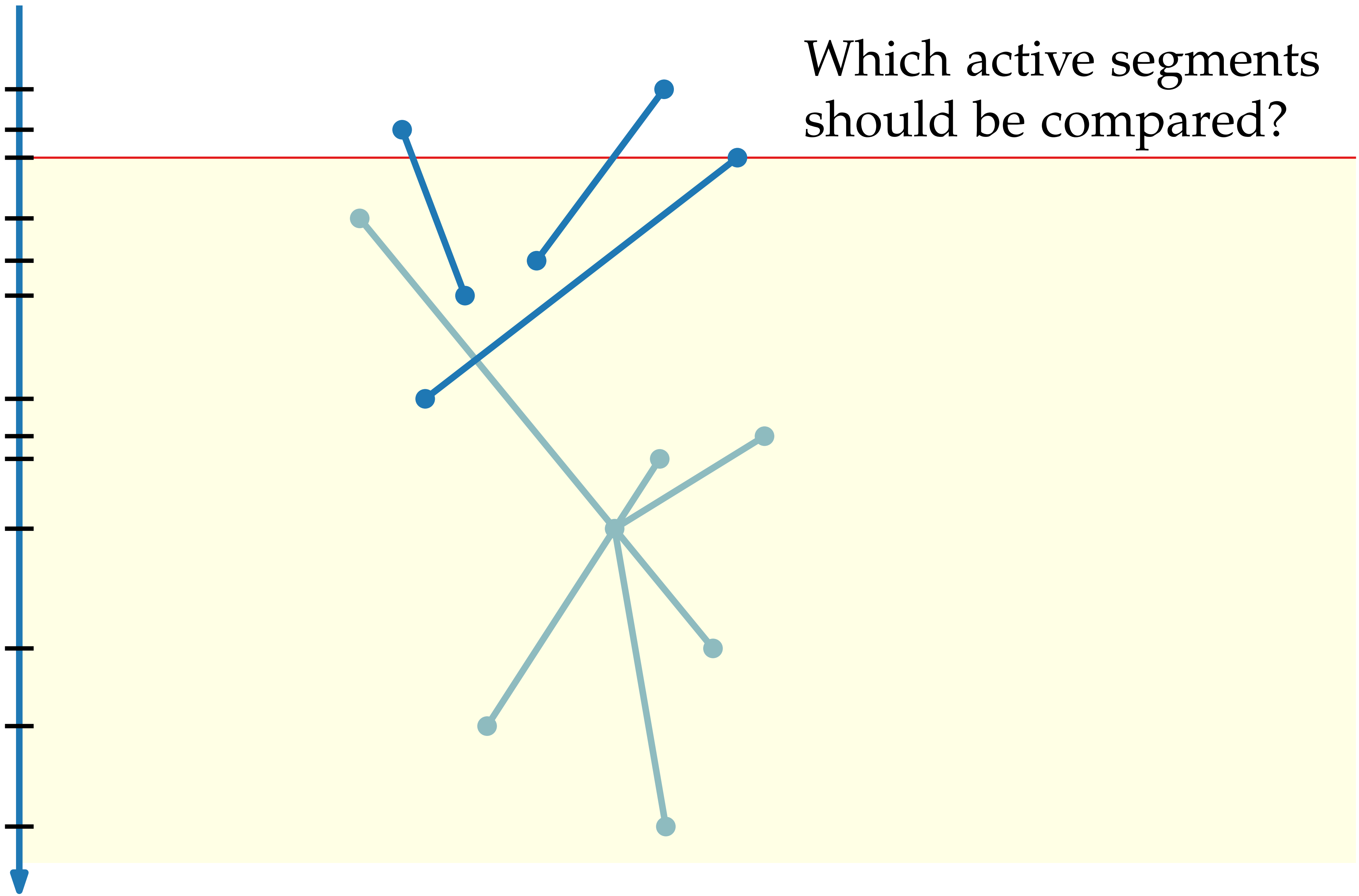

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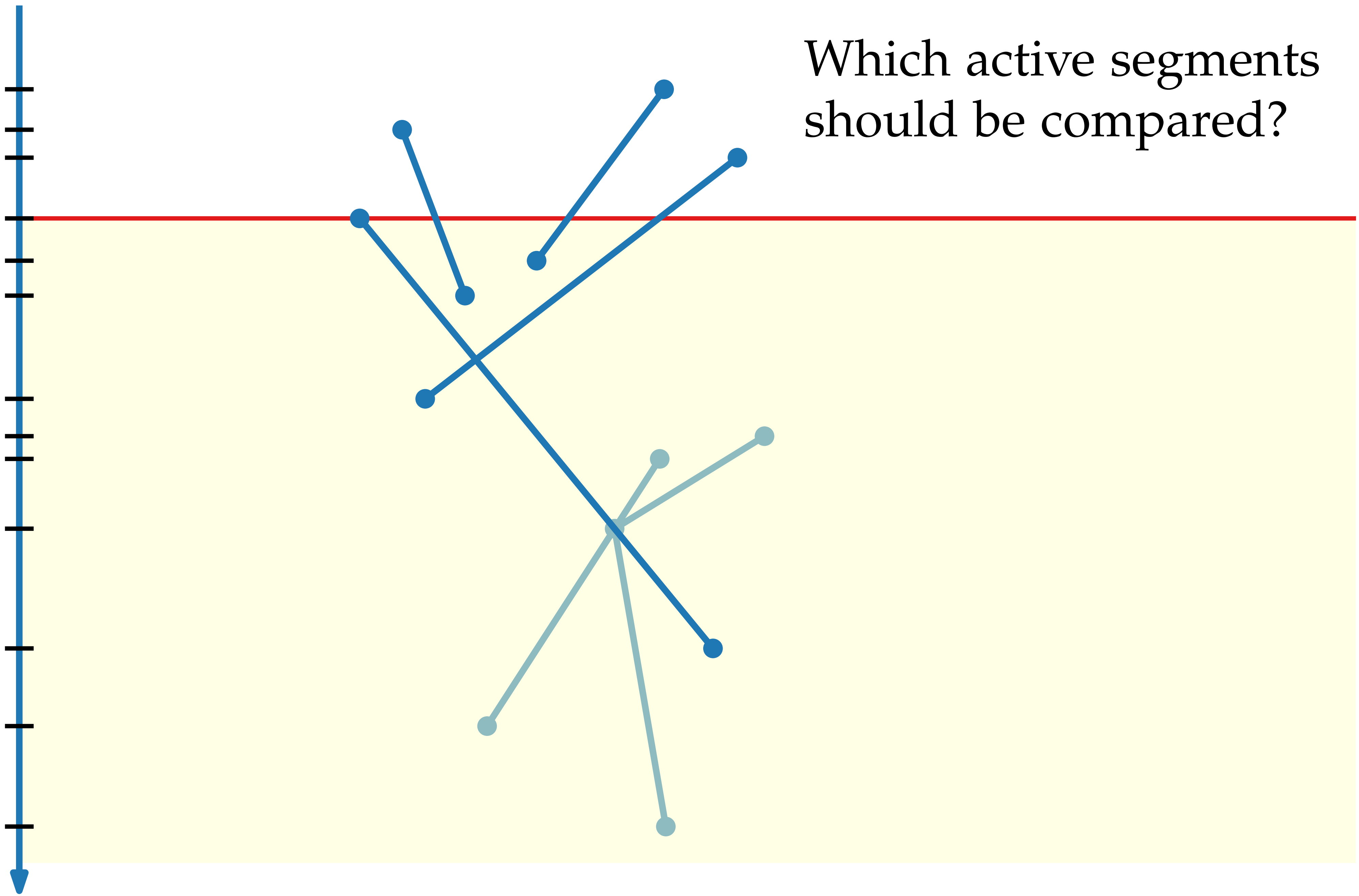

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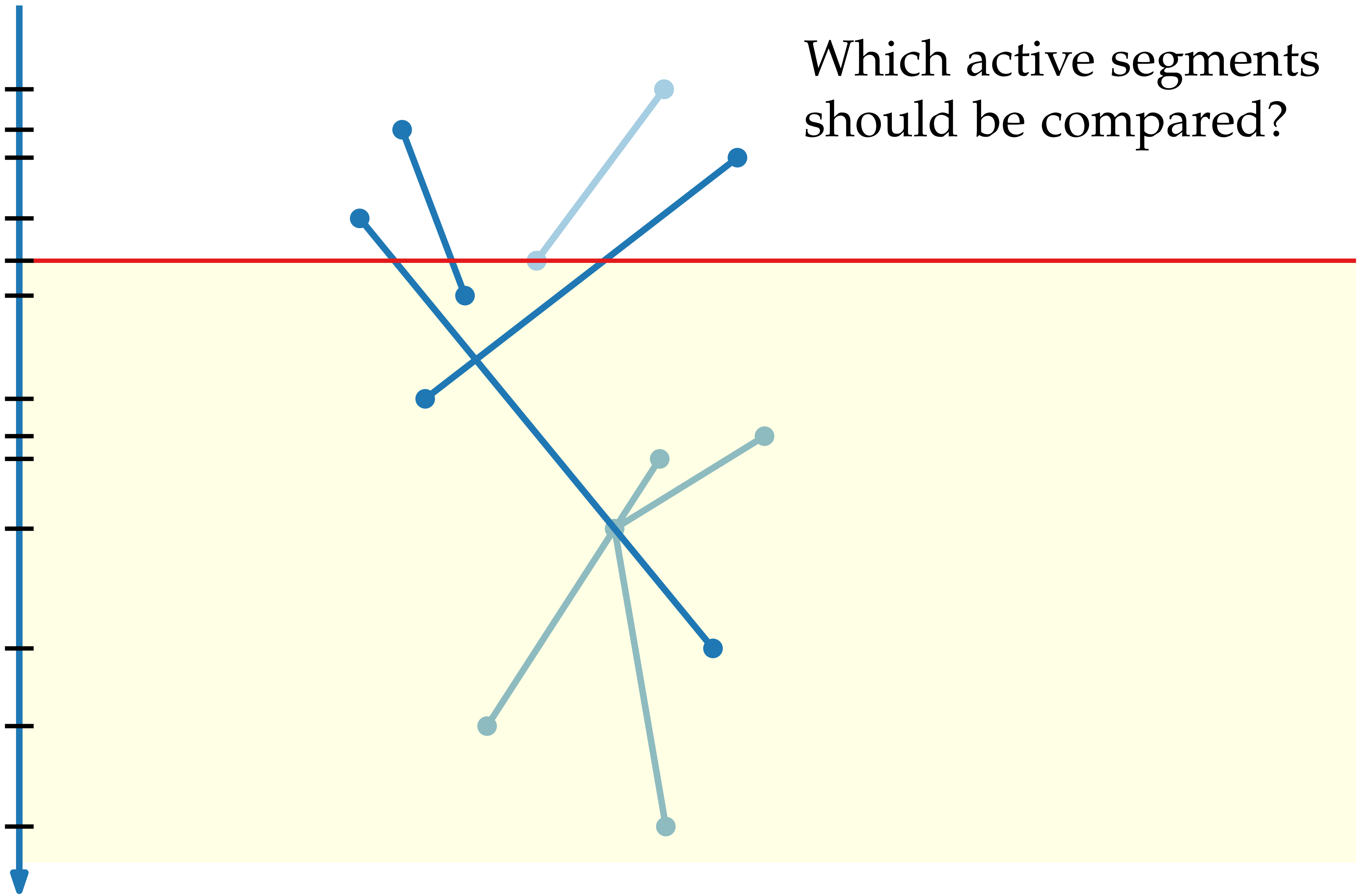

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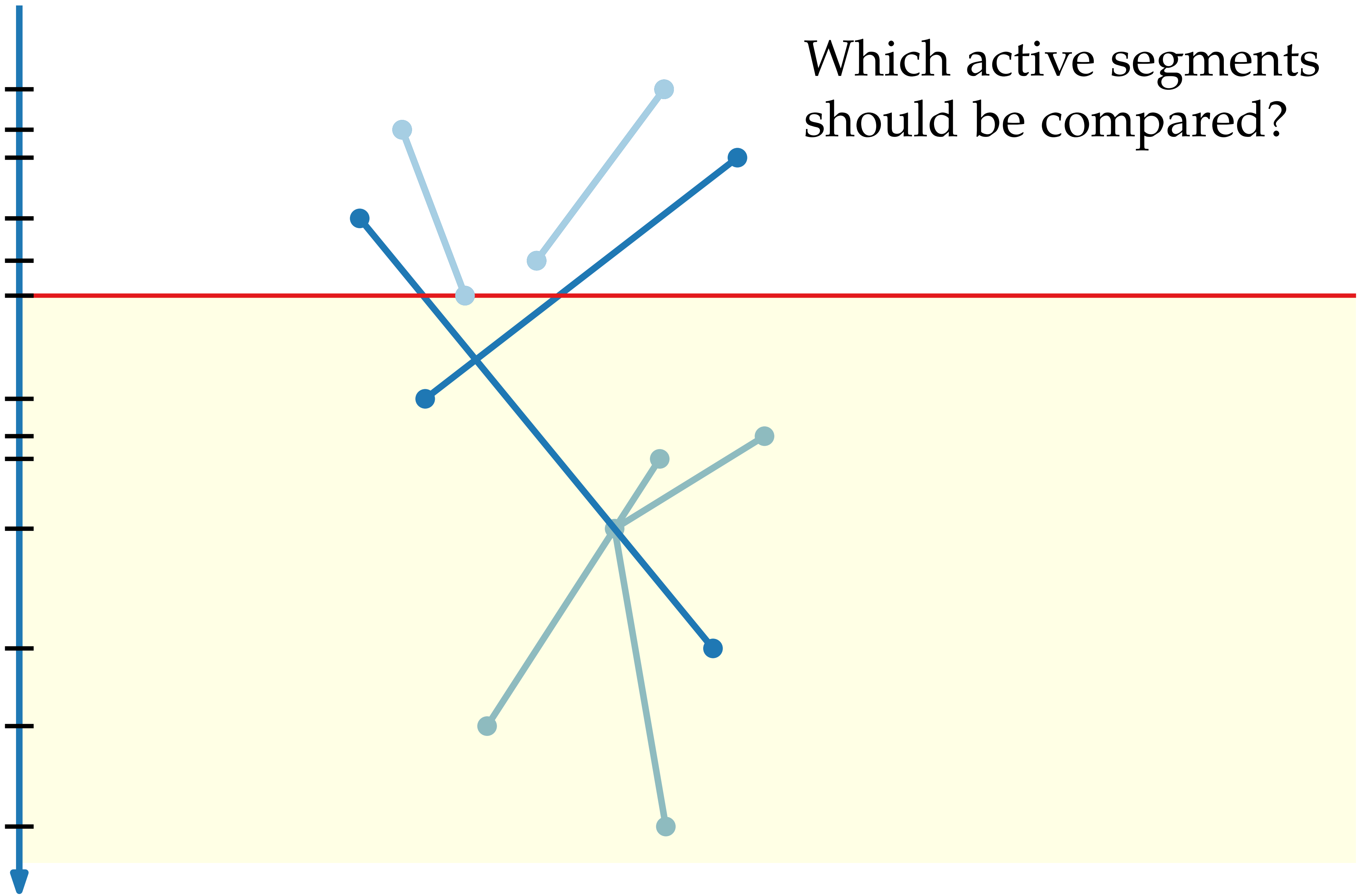

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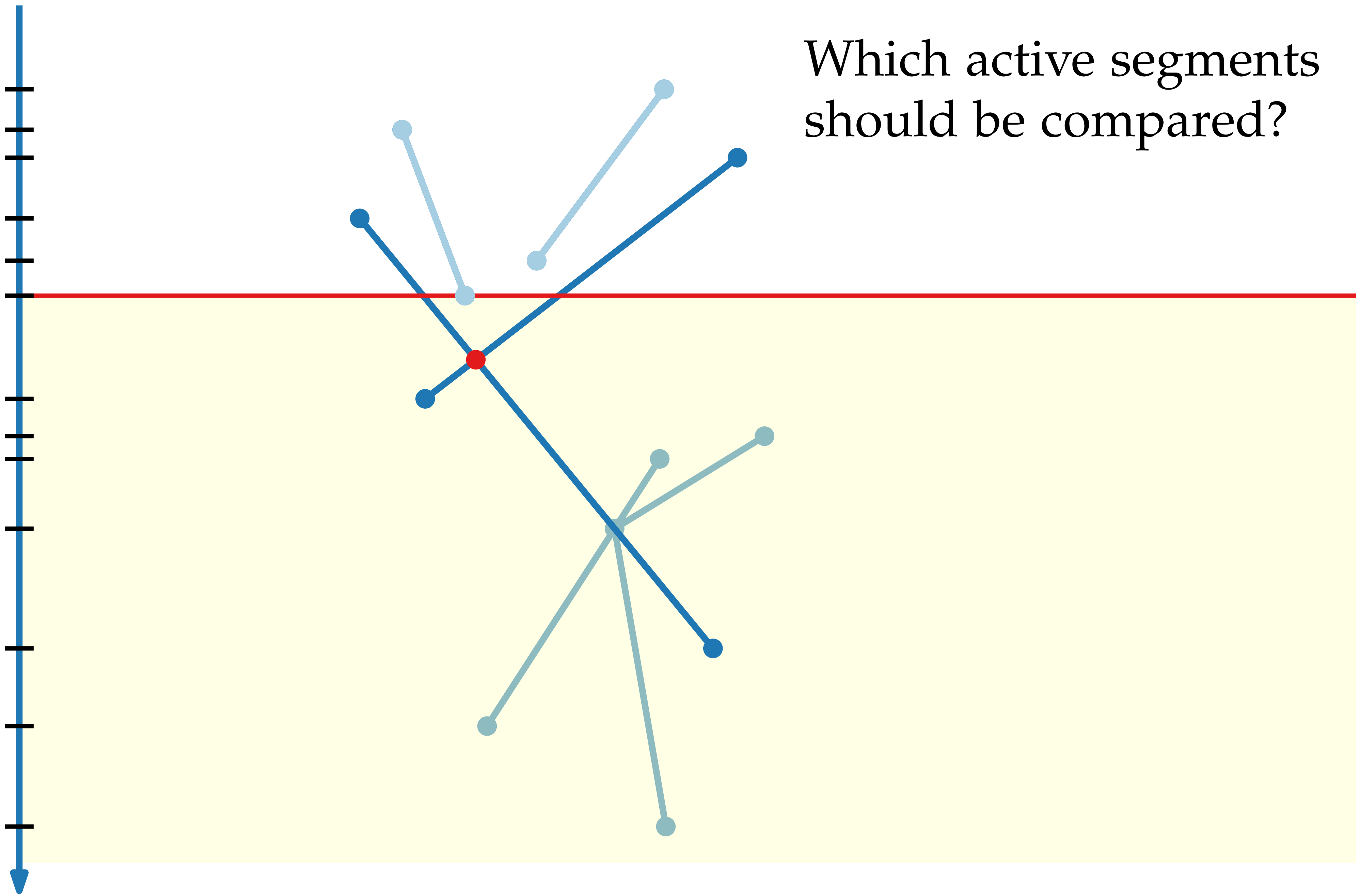

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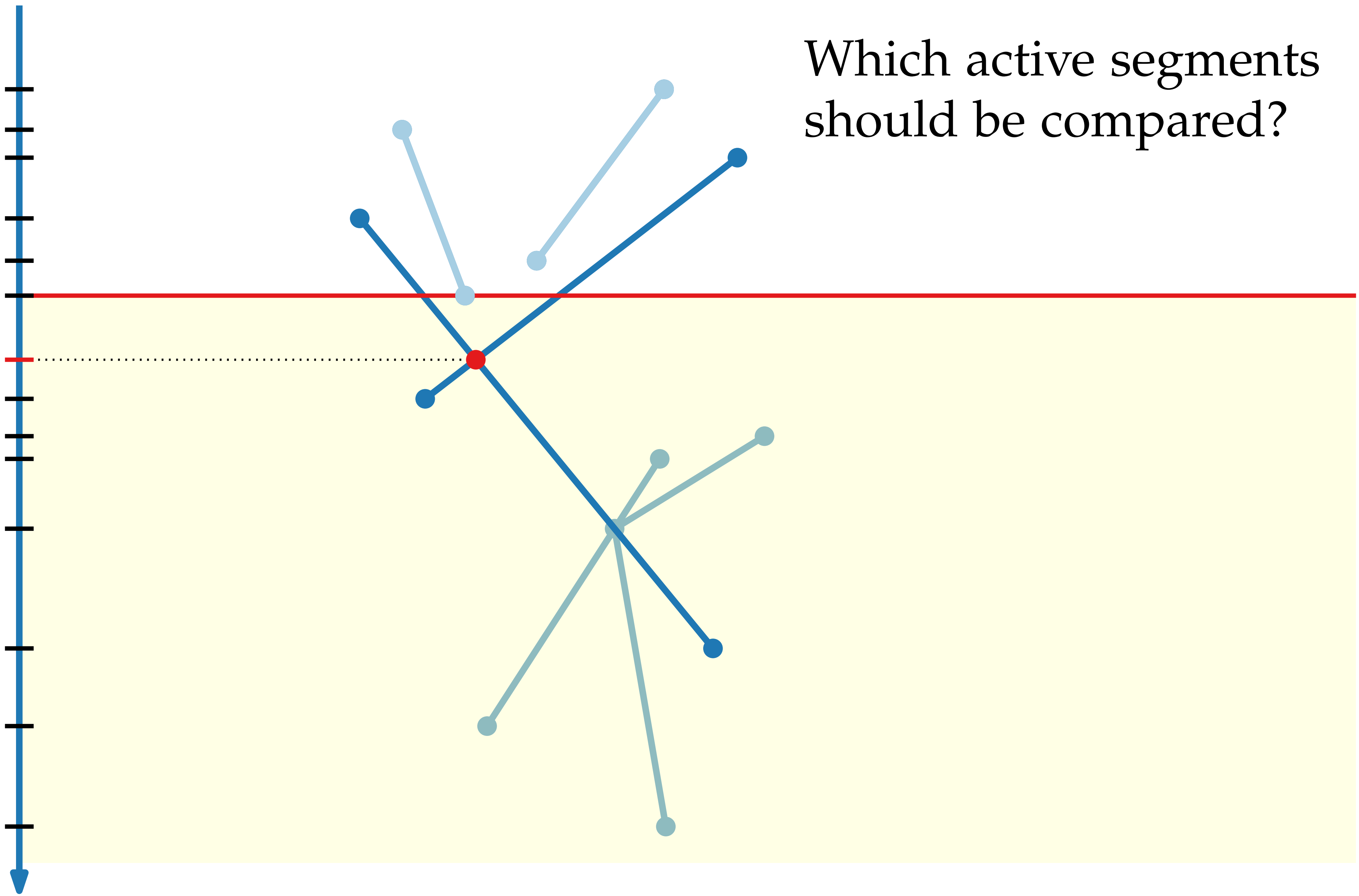

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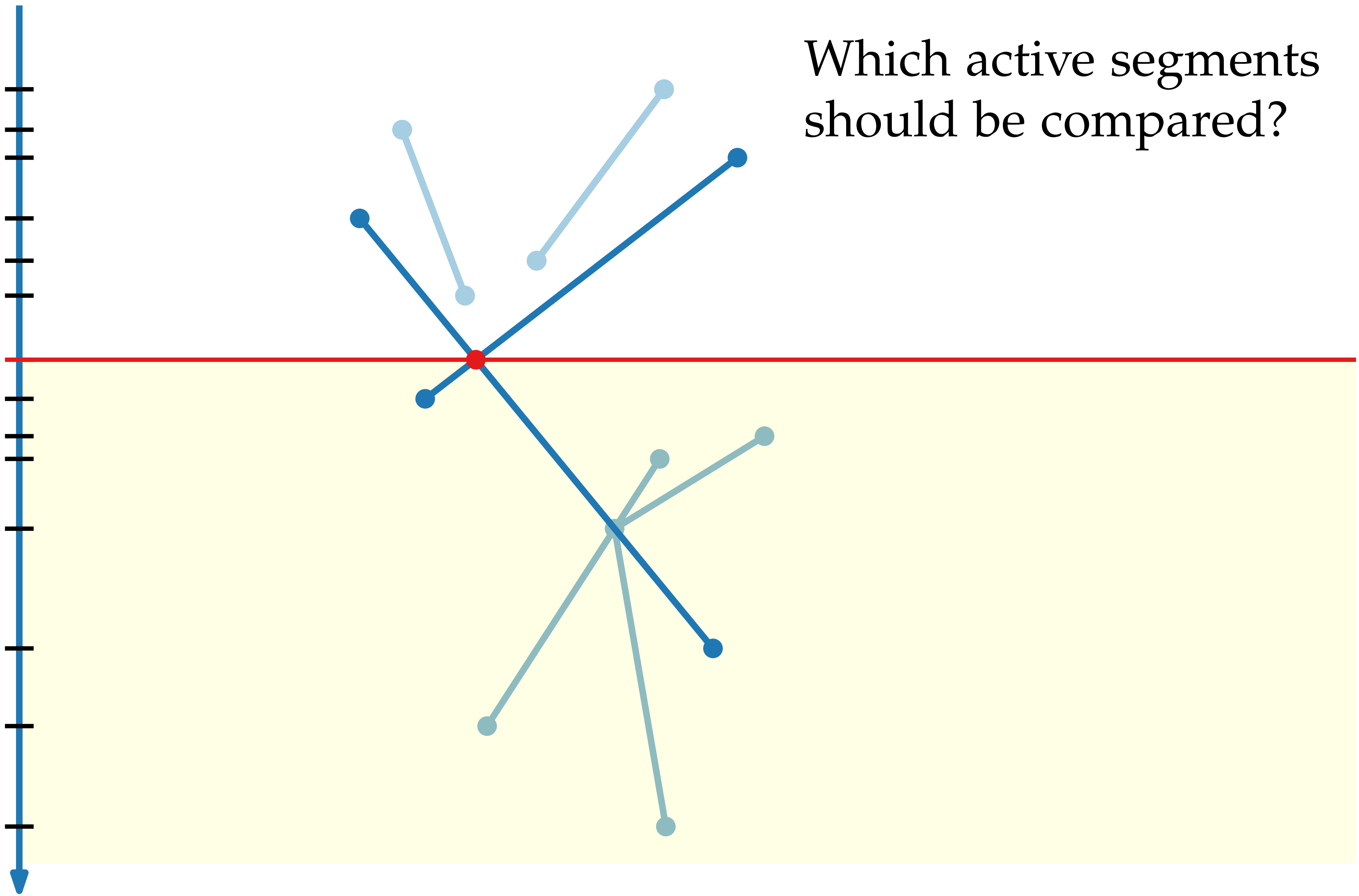

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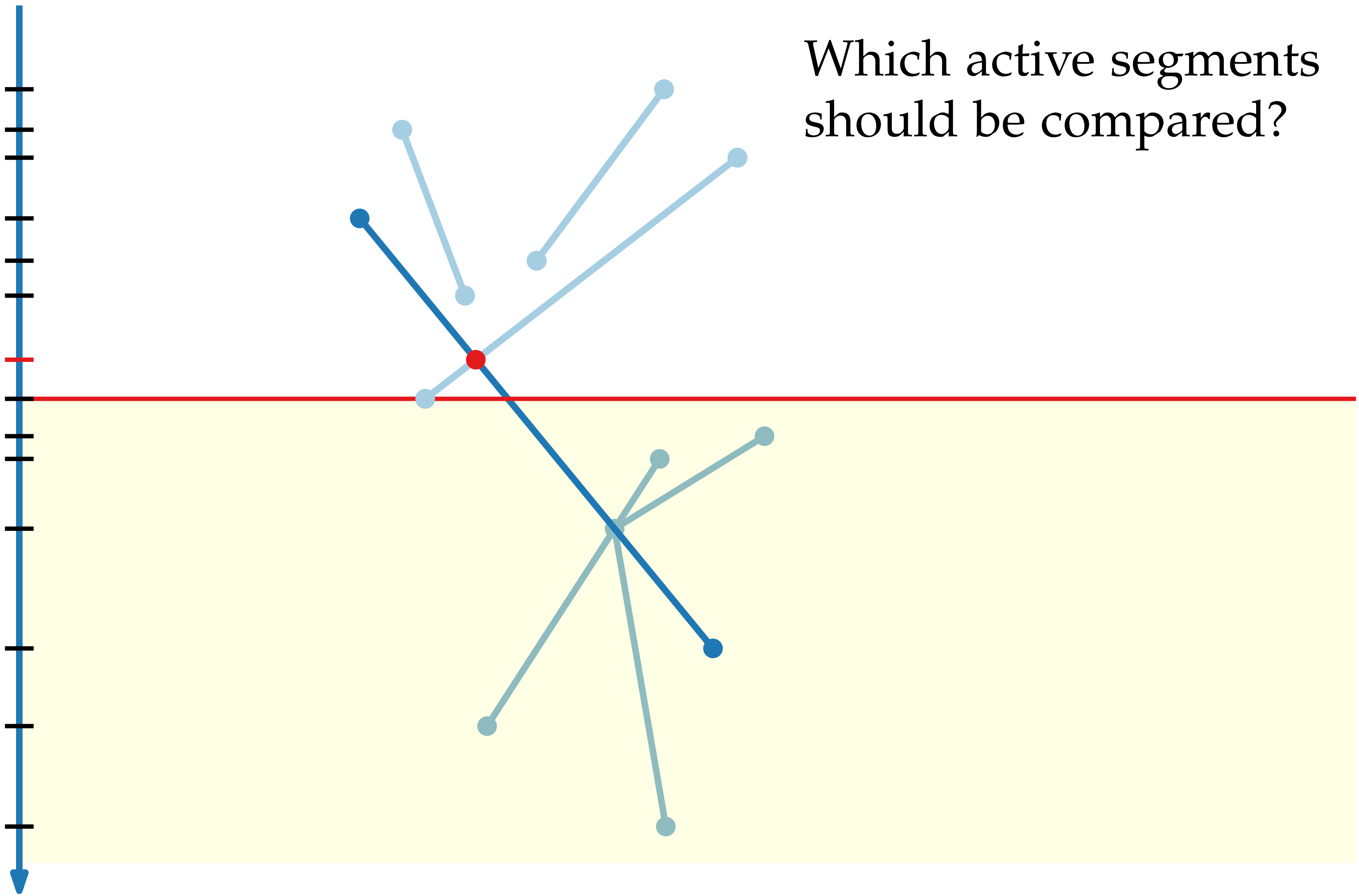

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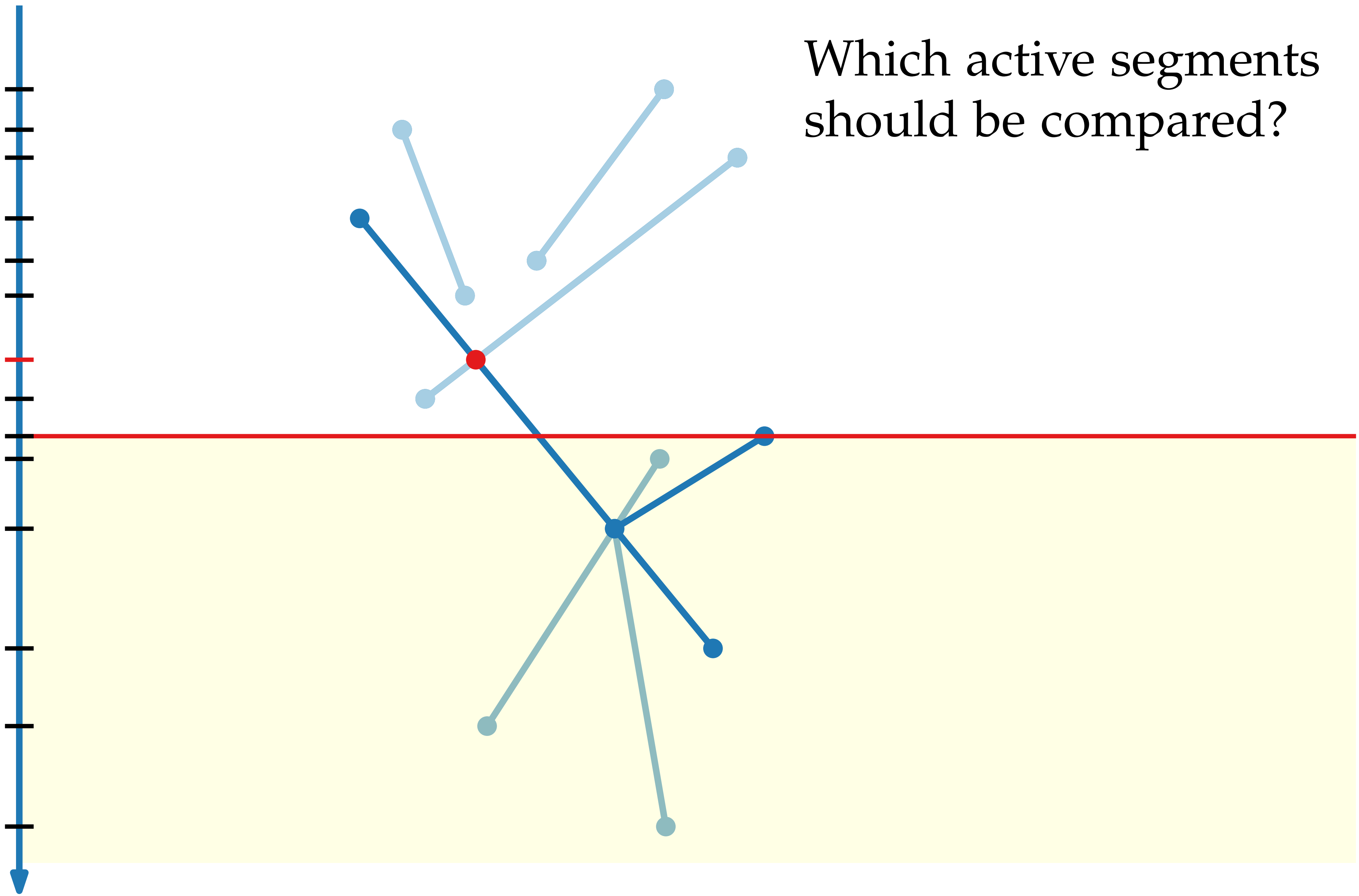

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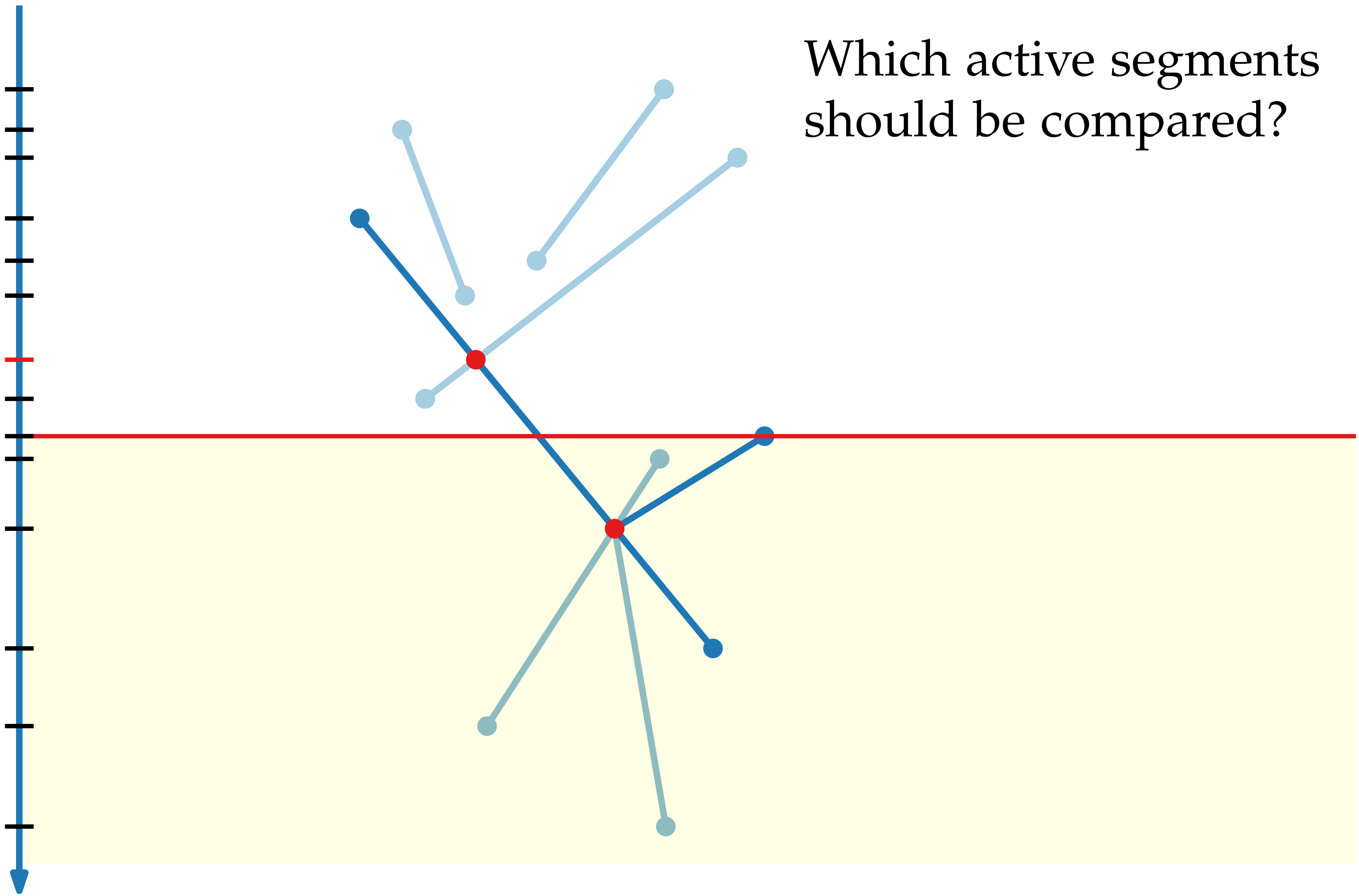

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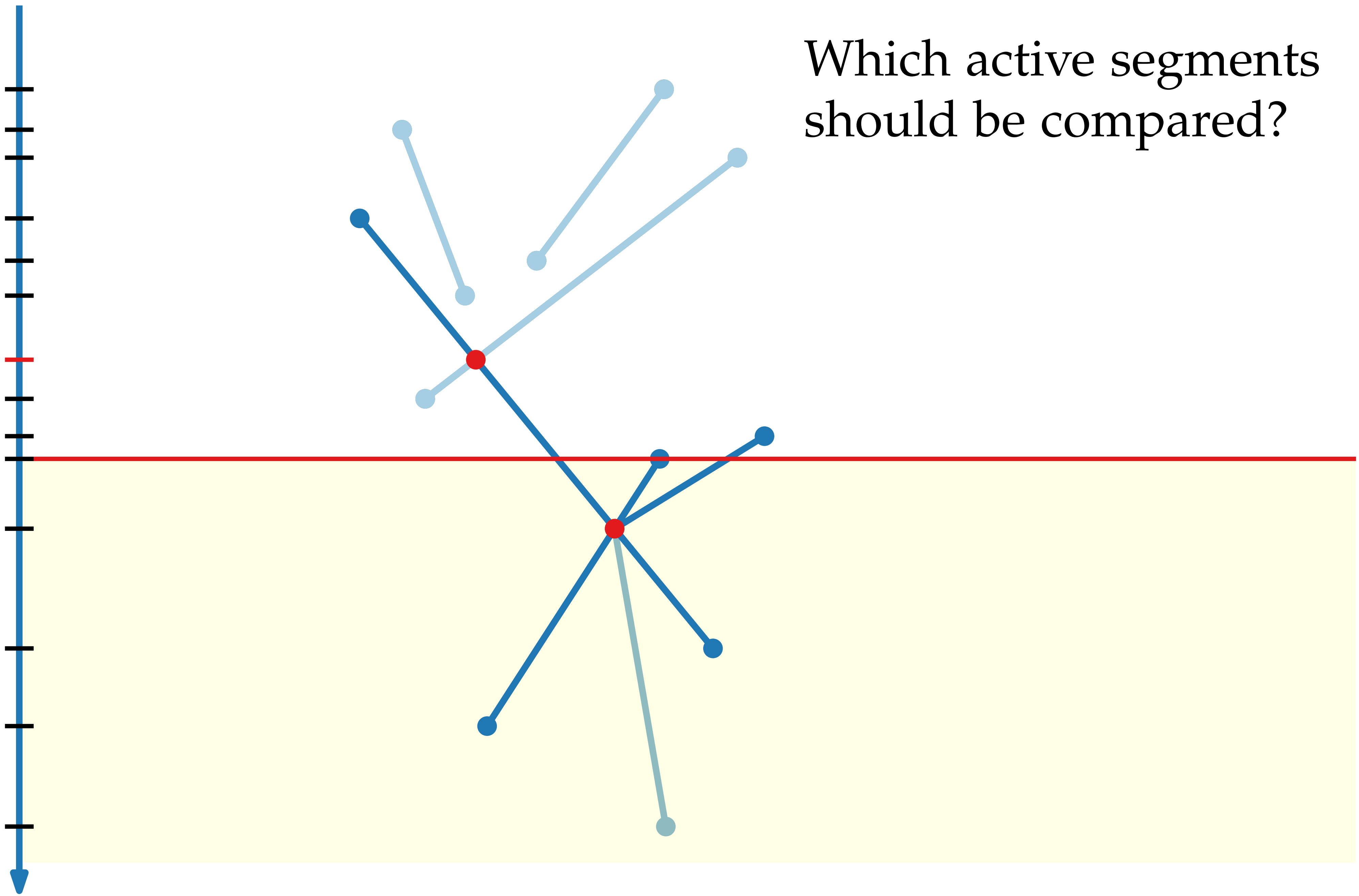

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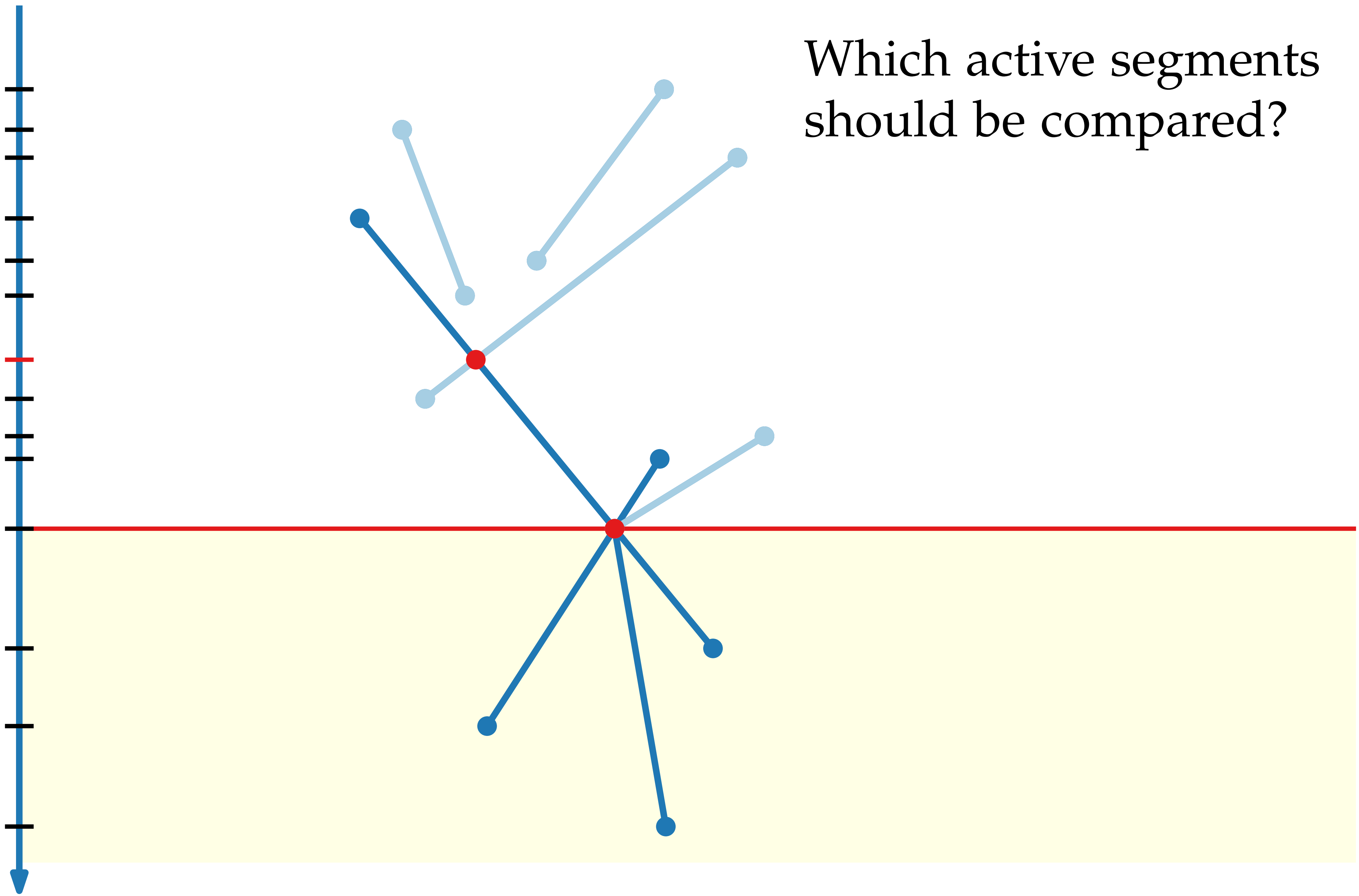

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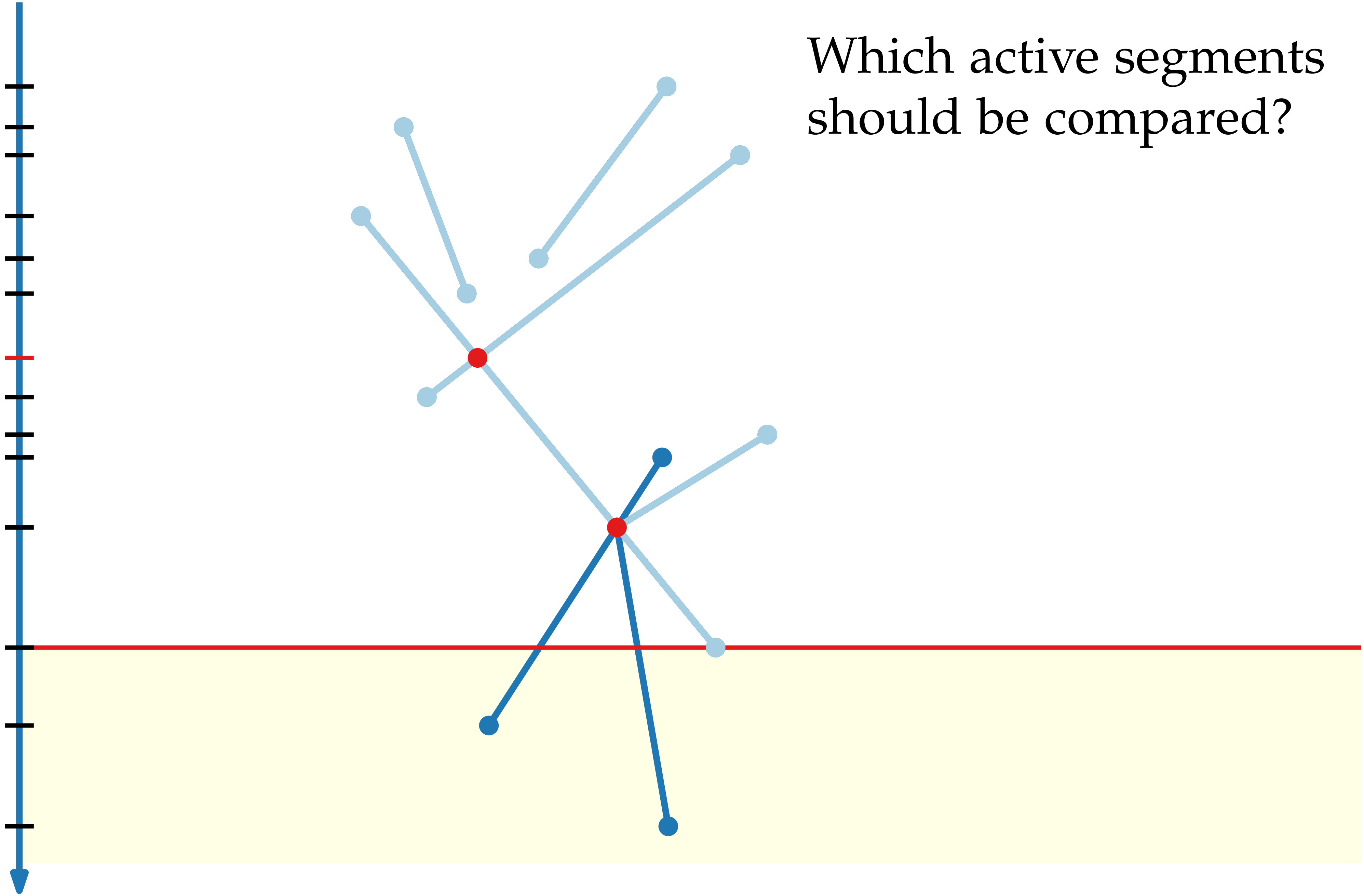

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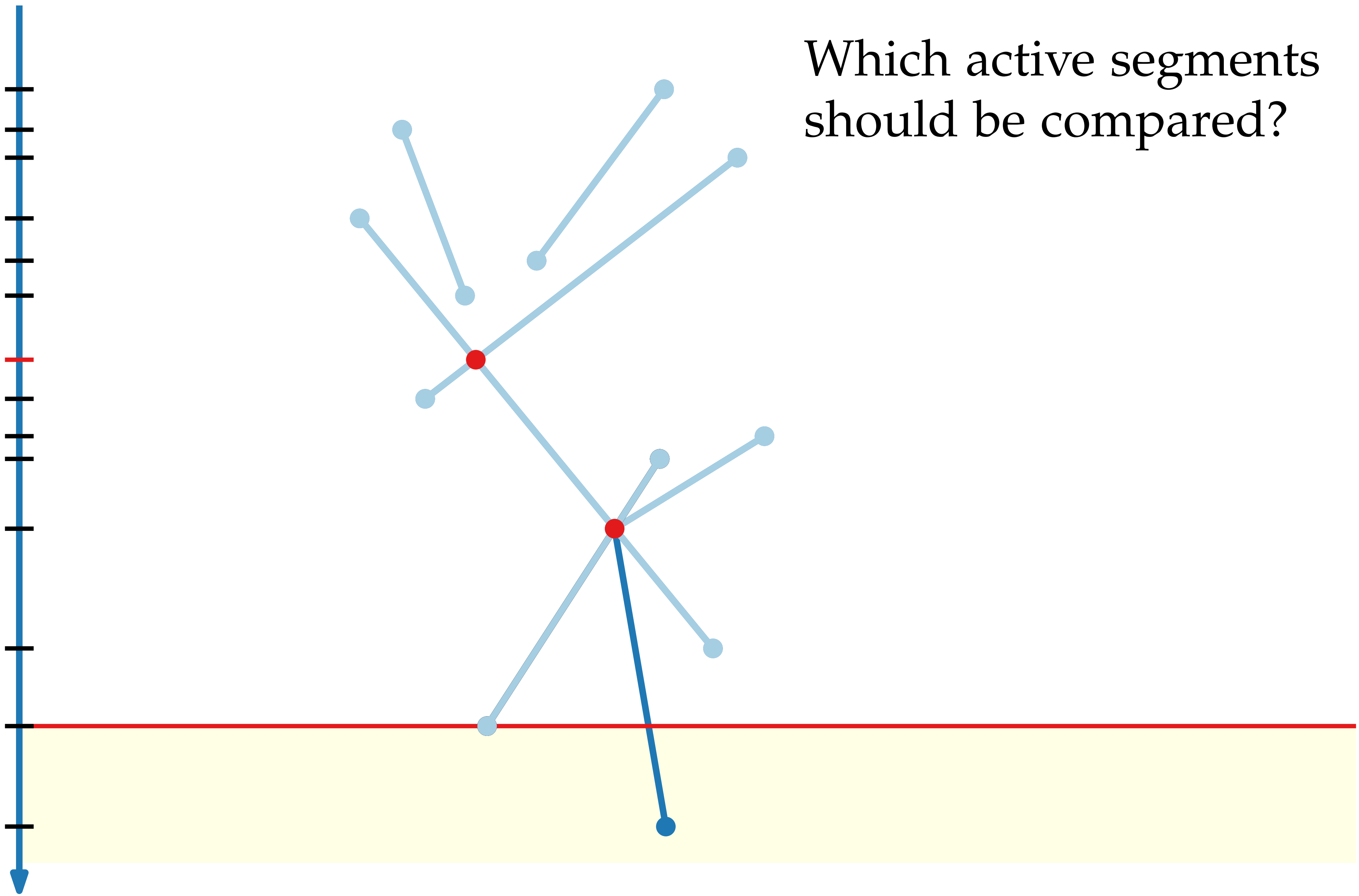

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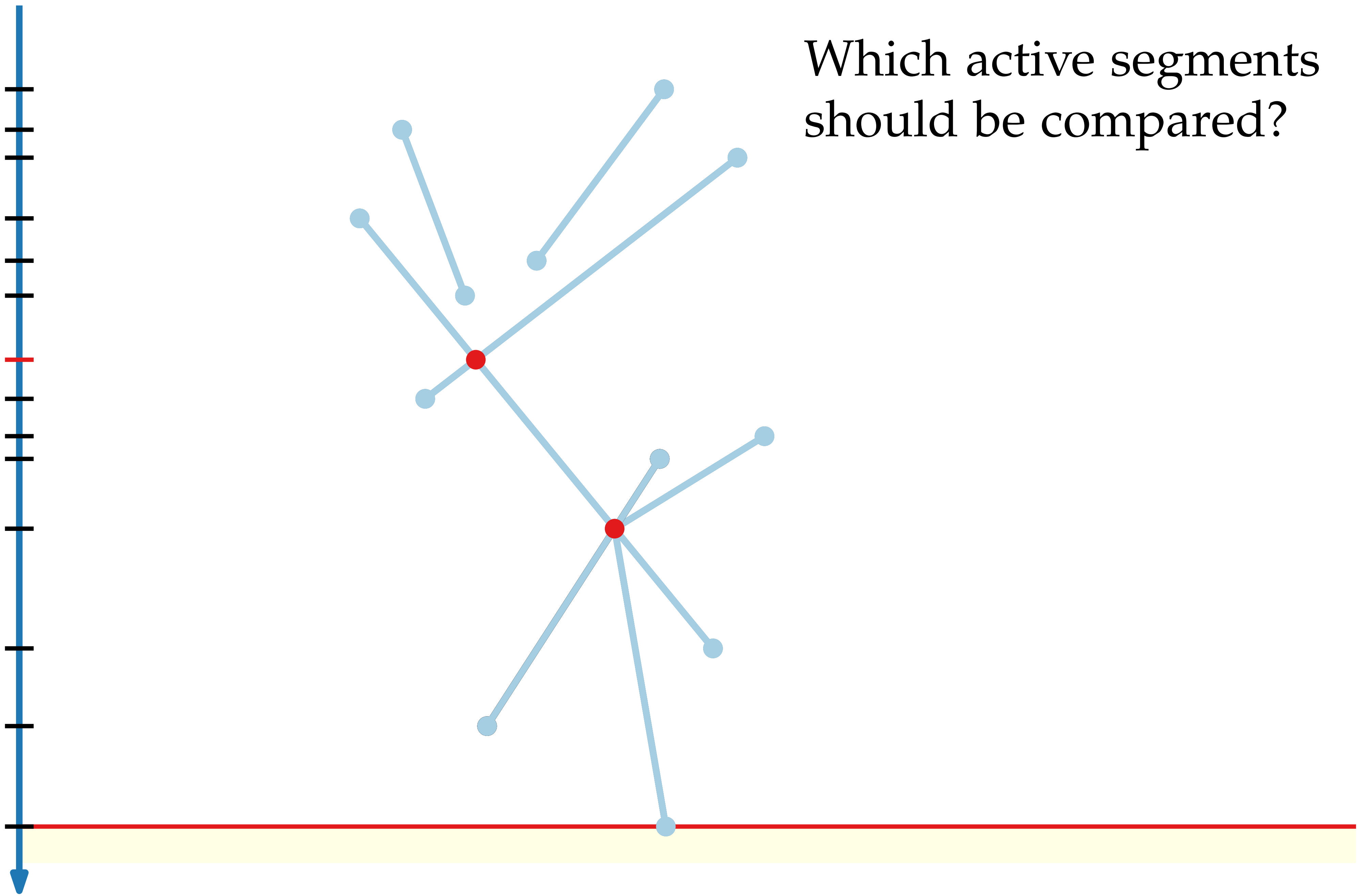

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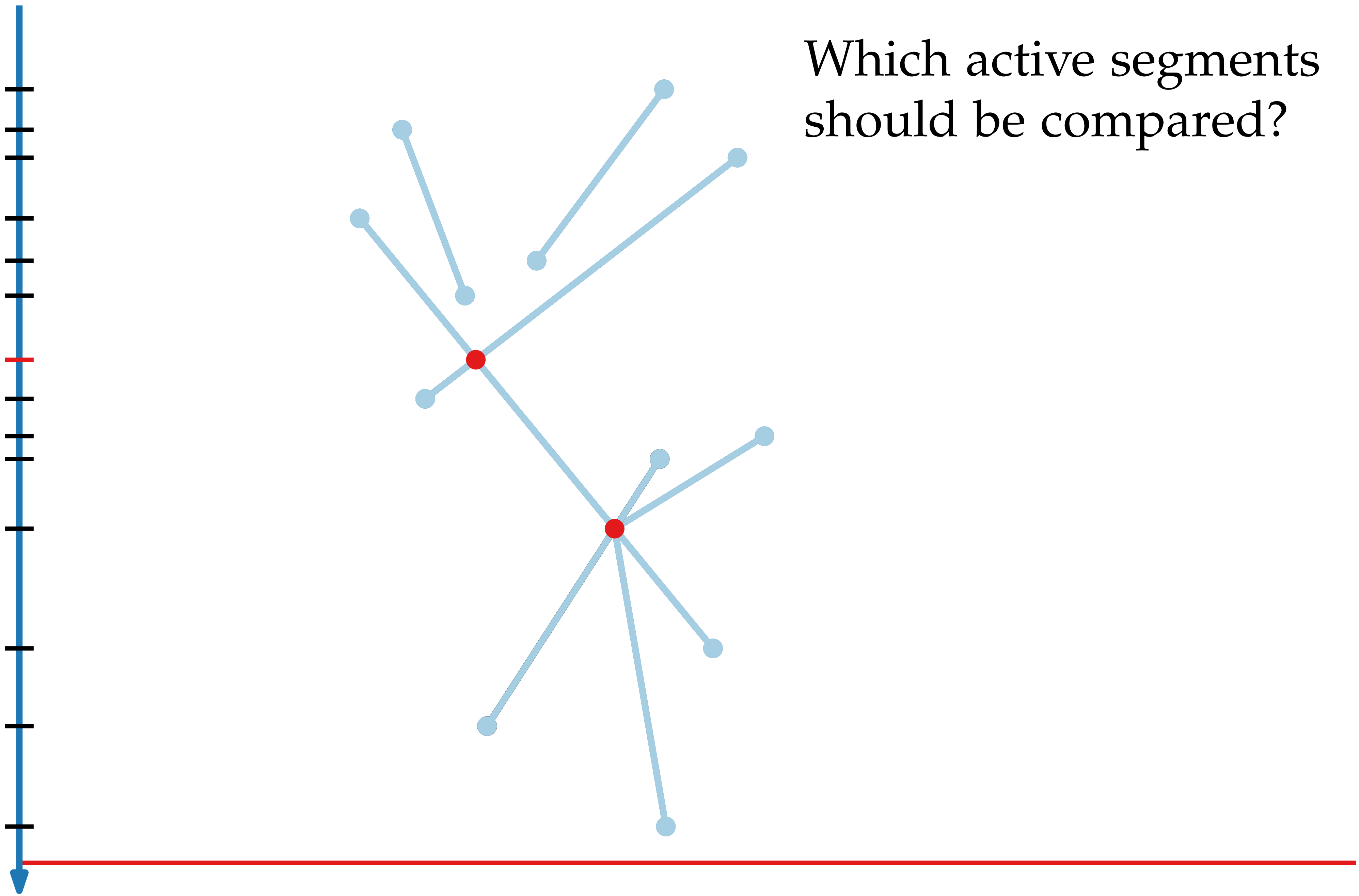

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Data Structures

1) event (-point) queue $\mathcal{Q}$

2) (sweep-line) status $\mathcal{T}$

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$$p \prec q \quad \Leftrightarrow_{\text{def.}}$$

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$$p \prec q \quad \Leftrightarrow_{\text{def.}} \quad y_p > y_q$$

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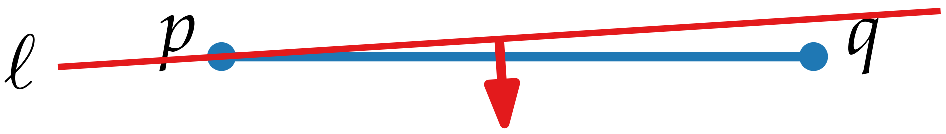

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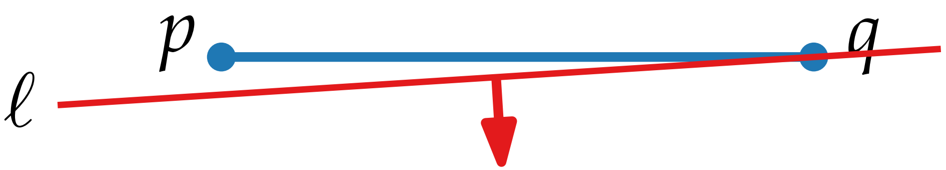

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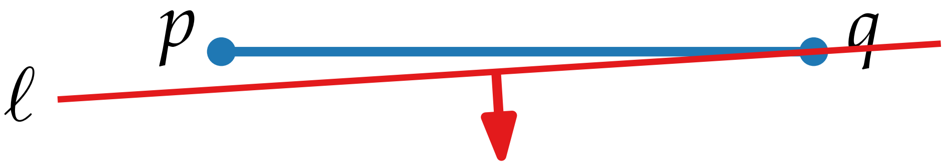

Store event pts in *balanced binary search tree* acc. to $\prec$

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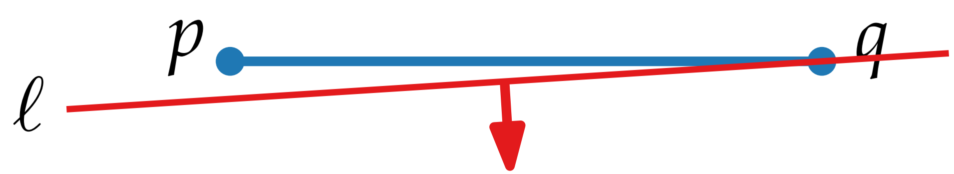

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$\Rightarrow$ nextEvent() and del/insEvent() take $O(\log|\mathcal{Q}|)$ time

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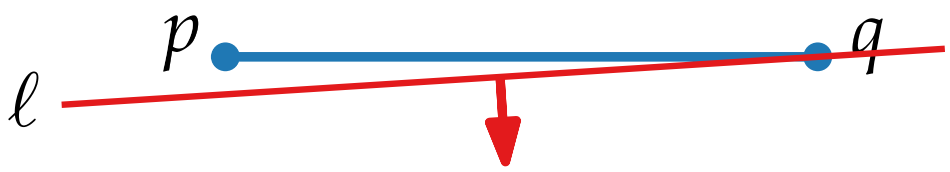

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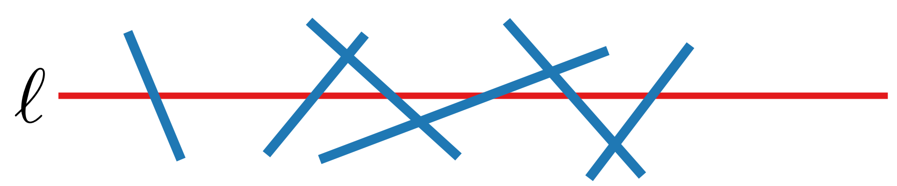

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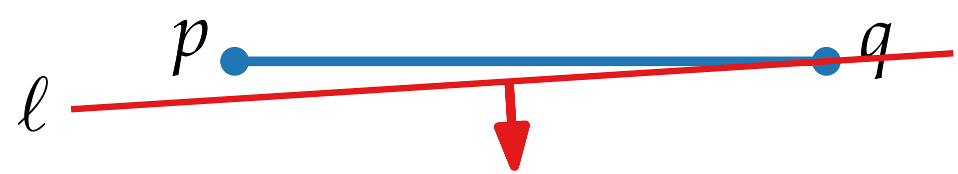

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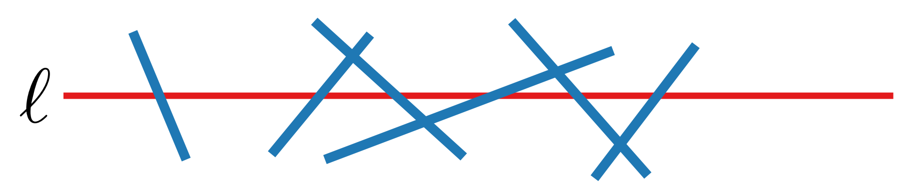

Store the segments intersected by ℓ in left-to-right order.

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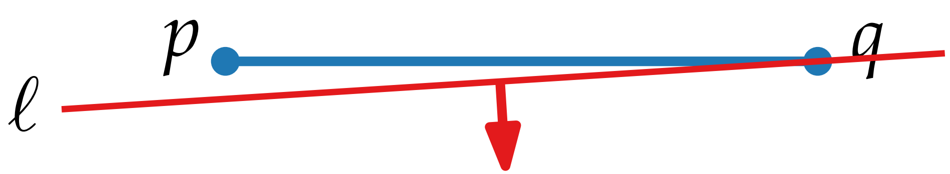

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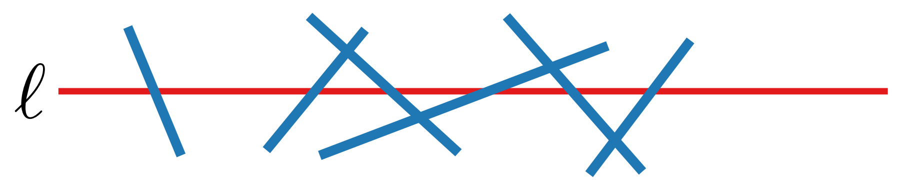

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How?

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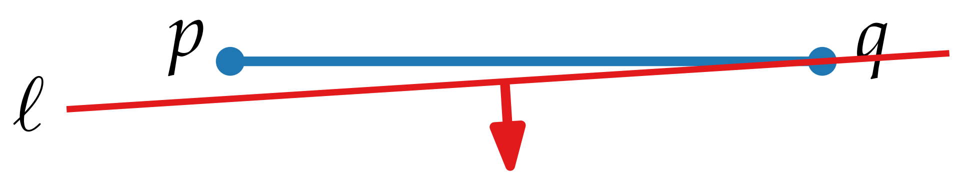

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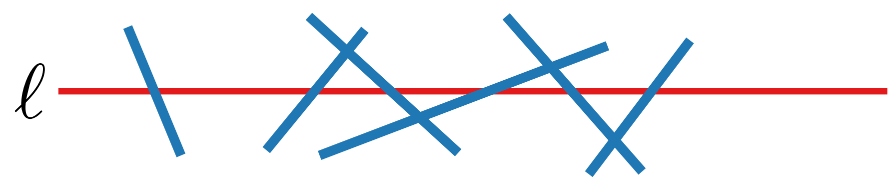
$\Rightarrow$ nextEvent() and del/insEvent() take $O(\log |\mathcal{Q}|)$ time

2) (sweep-line) status $\mathcal{T}$



Store the segments intersected by ℓ in left-to-right order.

How? In a balanced binary search tree!

Computational Geometry

Lecture 2:
Line-Segment Intersection
or
Map Overlay

Part III:
Algorithmic Details

Philipp Kindermann

Winter Semester 2020

Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts

Pseudo-code

findIntersections(*S*)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

$\mathcal{Q} \leftarrow \emptyset$;

Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

$\mathcal{Q} \leftarrow \emptyset$; $\mathcal{T} \leftarrow \langle$ vertical lines at $x = -\infty$ and $x = +\infty \rangle$ // sentinels

Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

```
Q ← ∅;  T ← ⟨vertical lines at x = −∞ and x = +∞⟩   // sentinels
foreach s ∈ S do                                   // initialize event queue Q
```

Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

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Q ← ∅;  T ← ⟨vertical lines at x = −∞ and x = +∞⟩   // sentinels
foreach s ∈ S do                                     // initialize event queue Q
    foreach endpoint p of s do
```

Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output:
- set I of intersection pts
- for each $p \in I$ every $s \in S$ with $p \in s$

```
Q ← ∅;  T ← ⟨ vertical lines at x = −∞ and x = +∞ ⟩   // sentinels
foreach s ∈ S do                                       // initialize event queue Q
    foreach endpoint p of s do
        if p ∉ Q then Q.insert(p);
```

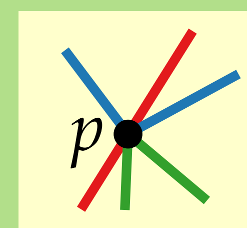
Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

$\mathcal{Q} \leftarrow \emptyset$; $\mathcal{T} \leftarrow \langle$ vertical lines at $x = -\infty$ and $x = +\infty \rangle$ // sentinels
foreach $s \in S$ **do** // initialize event queue $\mathcal{Q}$
  foreach endpoint p of s **do**
    if $p \notin \mathcal{Q}$ **then** $\mathcal{Q}$.insert(p);



Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
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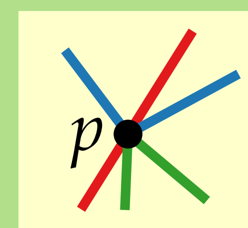
  foreach endpoint p of s **do**
    if $p \notin \mathcal{Q}$ **then** $\mathcal{Q}$.insert(p); $L(p) = U(p) = C(p) = \emptyset$



Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
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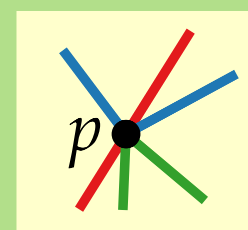
  foreach endpoint p of s **do**
    if $p \notin \mathcal{Q}$ **then** $\mathcal{Q}$.insert(p); $L(p) = U(p) = C(p) = \emptyset$
    if p lower endpt of s **then** $L(p)$.append(s)



Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

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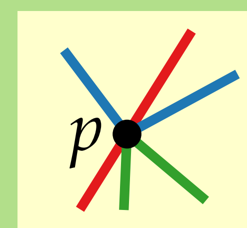
    if p lower endpt of s **then** $L(p)$.append(s)
    if p upper endpt of s **then** $U(p)$.append(s)



Pseudo-code

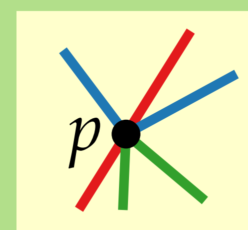
findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

```
Q ← ∅;  T ← ⟨vertical lines at x = −∞ and x = +∞⟩   // sentinels
foreach s ∈ S do                                   // initialize event queue Q
    foreach endpoint p of s do
        if p ∉ Q then Q.insert(p); L(p) = U(p) = C(p) = ∅
        if p lower  endpt of s then L(p).append(s)
        if p upper endpt of s then U(p).append(s)

while Q ≠ ∅ do
```



Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

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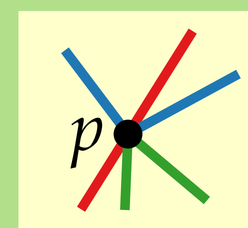
        if p lower endpt of s then L(p).append(s)
        if p upper endpt of s then U(p).append(s)

while Q ≠ ∅ do
    p ← Q.nextEvent()
```



Pseudo-code

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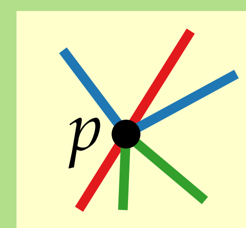
        if p lower endpt of s then L(p).append(s)
        if p upper endpt of s then U(p).append(s)

while Q ≠ ∅ do
    p ← Q.nextEvent()
    Q.deleteEvent(p)
```



Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output:
– set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

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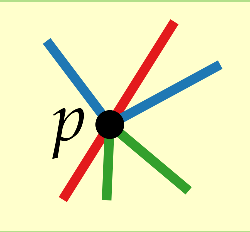
Q ← ∅;  T ← ⟨ vertical lines at x = −∞ and x = +∞ ⟩   // sentinels
foreach s ∈ S do                                      // initialize event queue Q
    foreach endpoint p of s do
        if p ∉ Q then Q.insert(p); L(p) = U(p) = C(p) = ∅
        if p lower  endpt of s then L(p).append(s)
        if p upper endpt of s then U(p).append(s)

while Q ≠ ∅ do
    p ← Q.nextEvent()
    Q.deleteEvent(p)
    handleEvent(p)
```



Pseudo-code

findIntersections(S)

Input: set S of n non-overlapping closed line segments

Output: – set I of intersection pts
– for each $p \in I$ every $s \in S$ with $p \in s$

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foreach $s \in S$ **do** // initialize event queue $\mathcal{Q}$
  foreach endpoint p of s **do**
    if $p \notin \mathcal{Q}$ **then** $\mathcal{Q}$.insert(p); $L(p) = U(p) = C(p) = \emptyset$
    if p lower endpt of s **then** $L(p)$.append(s)
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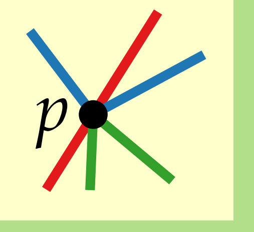
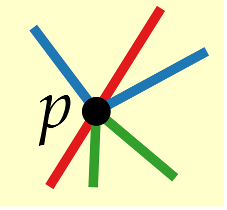


while $\mathcal{Q} \neq \emptyset$ **do**
  $p \leftarrow \mathcal{Q}$.nextEvent()
  $\mathcal{Q}$.deleteEvent(p)
  handleEvent(p)

This subroutine does the real work.
How would you implement it?

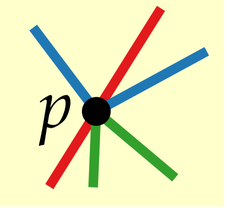
Handling an Event



$C(p), L(p), U(p)$

handleEvent(event p)

Handling an Event

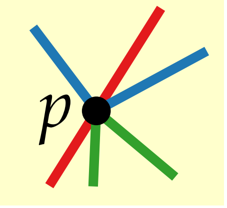


$C(p)$, $L(p)$, $U(p)$

handleEvent(event p)

if $|U(p) \cup L(p) \cup C(p)| > 1$ **then**

Handling an Event



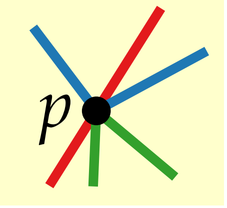
$C(p), L(p), U(p)$

handleEvent(event p)

if $|U(p) \cup L(p) \cup C(p)| > 1$ **then**

report intersection in p, report segments in $U(p) \cup L(p) \cup C(p)$

Handling an Event



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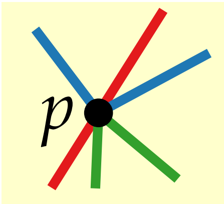
delete $L(p) \cup C(p)$ from $\mathcal{T}$ // consecutive in $\mathcal{T}$!

Handling an Event



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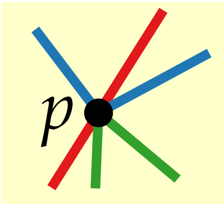
insert $U(p) \cup C(p)$ into $\mathcal{T}$ in their order slightly below ℓ

Handling an Event



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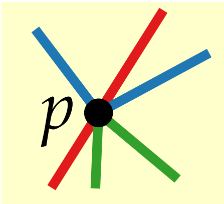
insert $U(p) \cup C(p)$ into $\mathcal{T}$ in their order slightly below ℓ
if $U(p) \cup C(p) = \emptyset$ **then**

else

Handling an Event



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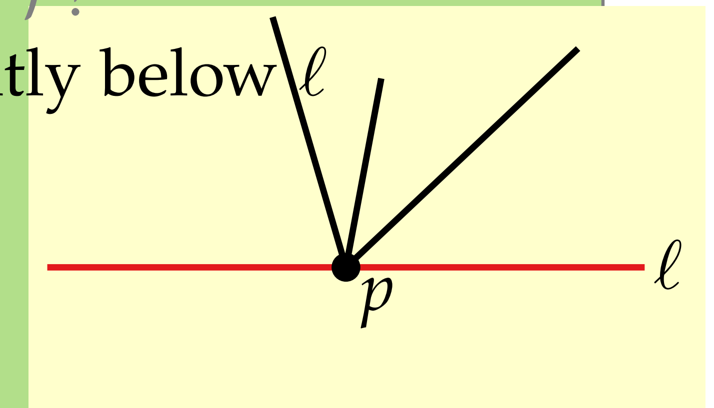
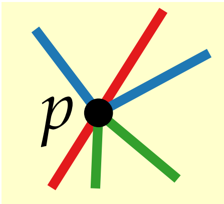


Handling an Event



$C(p)$, $L(p)$, $U(p)$

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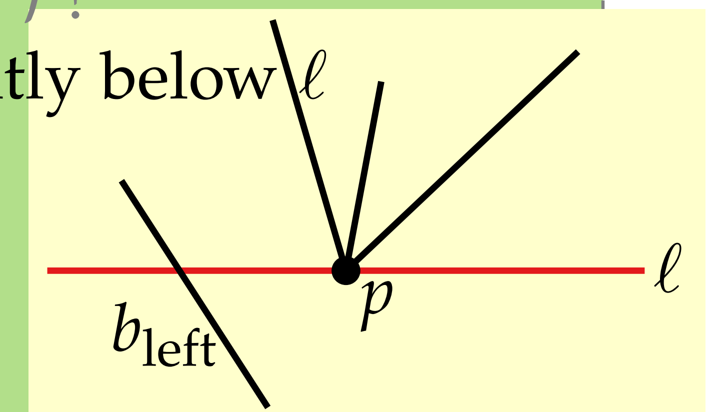
delete $L(p) \cup C(p)$ from $\mathcal{T}$ // consecutive in $\mathcal{T}$!

insert $U(p) \cup C(p)$ into $\mathcal{T}$ in their order slightly below ℓ

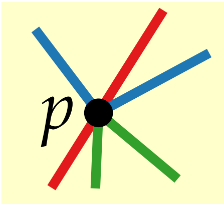
if $U(p) \cup C(p) = \emptyset$ **then**

  $b_{\text{left}}/b_{\text{right}}$ = left/right neighbor of p in $\mathcal{T}$

else



Handling an Event



$C(p)$,$L(p)$,$U(p)$

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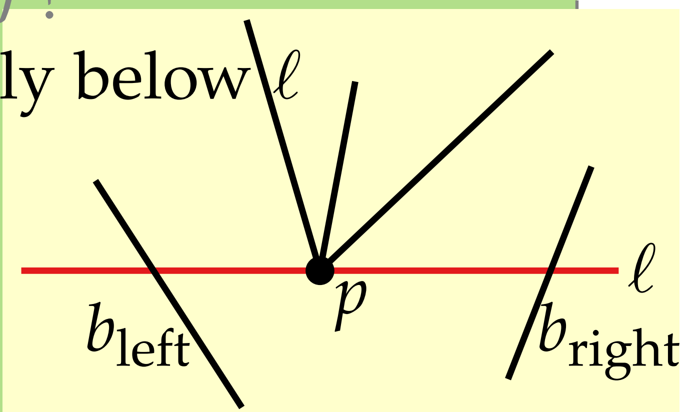
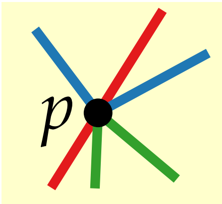


Handling an Event



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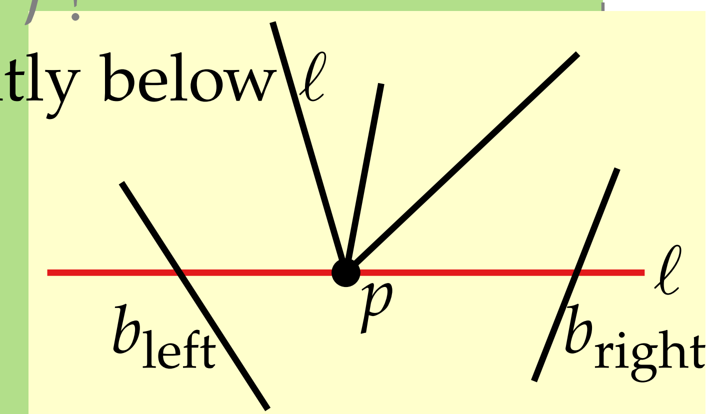
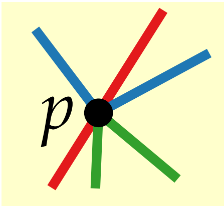
delete $L(p) \cup C(p)$ from $\mathcal{T}$ // consecutive in $\mathcal{T}$!
insert $U(p) \cup C(p)$ into $\mathcal{T}$ in their order slightly below ℓ
if $U(p) \cup C(p) = \emptyset$ **then**
  $b_{\text{left}}/b_{\text{right}}$ = left/right neighbor of p in $\mathcal{T}$
  findNewEvent(b_{left}, b_{right}, p)
else



Handling an Event



$C(p)$, $L(p)$, $U(p)$

findNewEvent(s, s', p)

handleEvent(event p)
if $|U(p) \cup L(p) \cup C(p)| > 1$ **then**
  report intersection in p, report segments in $U(p) \cup L(p) \cup C(p)$
delete $L(p) \cup C(p)$ from $\mathcal{T}$ // consecutive in $\mathcal{T}$!
insert $U(p) \cup C(p)$ into $\mathcal{T}$ in their order slightly below ℓ
if $U(p) \cup C(p) = \emptyset$ **then**
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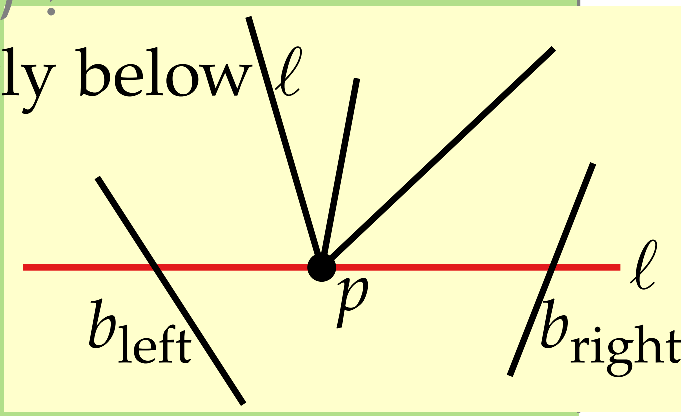
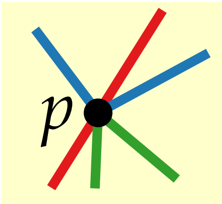


Handling an Event



$C(p)$, $L(p)$, $U(p)$

findNewEvent(s, s', p)
if $s \cap s' = \emptyset$ **then return**

handleEvent(event p)
if $|U(p) \cup L(p) \cup C(p)| > 1$ **then**
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 $b_{\text{left}}/b_{\text{right}}$ = left/right neighbor of p in $\mathcal{T}$
 findNewEvent(b_{left}, b_{right}, p)
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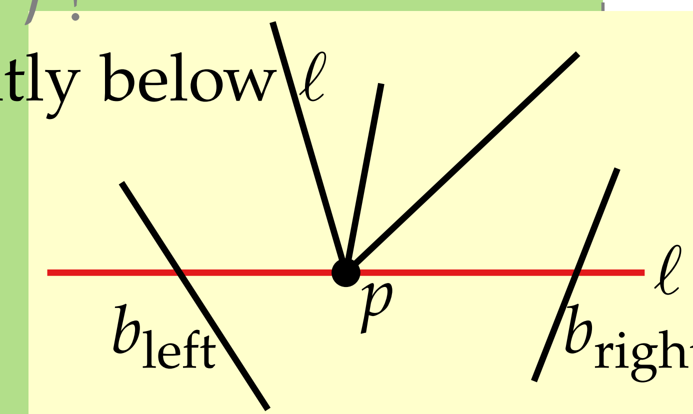
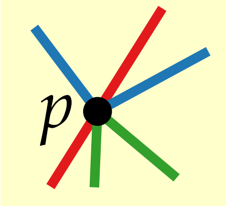


Handling an Event



$C(p)$, $L(p)$, $U(p)$

findNewEvent(s, s', p)
if $s \cap s' = \emptyset$ **then return**
$\{x\} = s \cap s'$

handleEvent(event p)
if $|U(p) \cup L(p) \cup C(p)| > 1$ **then**
  report intersection in p, report segments in $U(p) \cup L(p) \cup C(p)$
delete $L(p) \cup C(p)$ from $\mathcal{T}$ // consecutive in $\mathcal{T}$!
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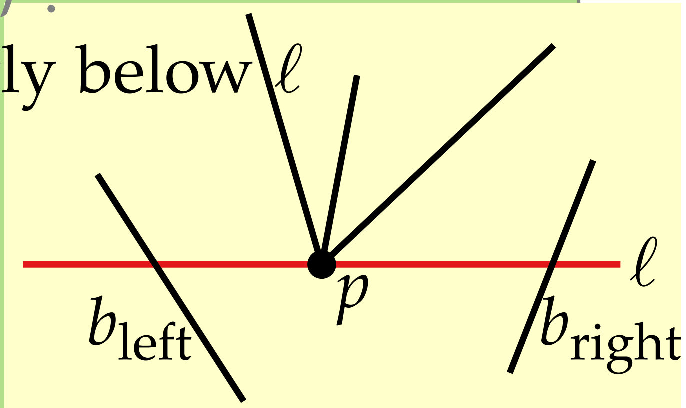
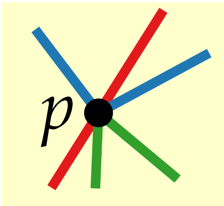


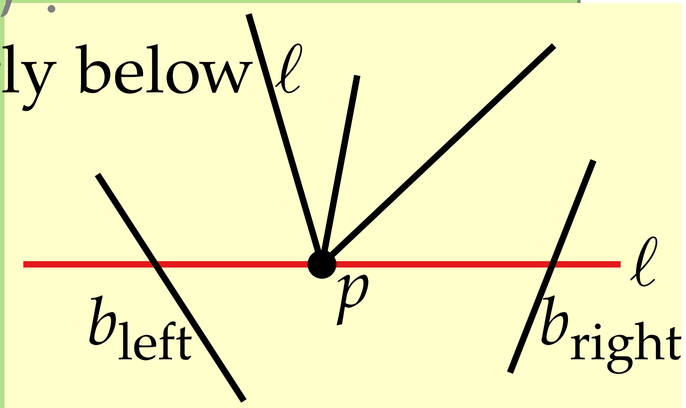
Handling an Event



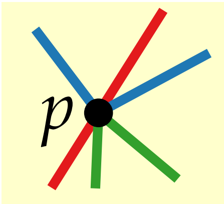
$C(p)$, $L(p)$, $U(p)$

findNewEvent(s, s', p)
if $s \cap s' = \emptyset$ **then return**
$\{x\} = s \cap s'$
if x below ℓ or on ℓ to the right of p **then**

handleEvent(event p)
if $|U(p) \cup L(p) \cup C(p)| > 1$ **then**
  report intersection in p, report segments in $U(p) \cup L(p) \cup C(p)$
delete $L(p) \cup C(p)$ from $\mathcal{T}$ // consecutive in $\mathcal{T}$!
insert $U(p) \cup C(p)$ into $\mathcal{T}$ in their order slightly below ℓ
if $U(p) \cup C(p) = \emptyset$ **then**
  $b_{\text{left}}/b_{\text{right}}$ = left/right neighbor of p in $\mathcal{T}$
  findNewEvent(b_{left}, b_{right}, p)
else



Handling an Event



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if x below ℓ or on ℓ to the right of p **then**
  if $x \notin \mathcal{Q}$ **then** $\mathcal{Q}$.add(x)

handleEvent(event p)
if $|U(p) \cup L(p) \cup C(p)| > 1$ **then**
  report intersection in p, report segments in $U(p) \cup L(p) \cup C(p)$
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  findNewEvent$(b_{\text{left}}, b_{\text{right}}, p)$
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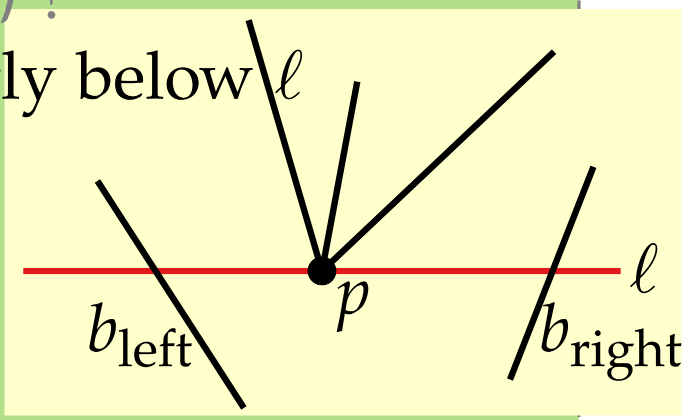
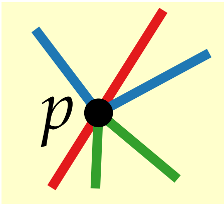


Handling an Event



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  if $x \in$ rel-int(s) **then** $C(x) \leftarrow C(x) \cup \{s\}$

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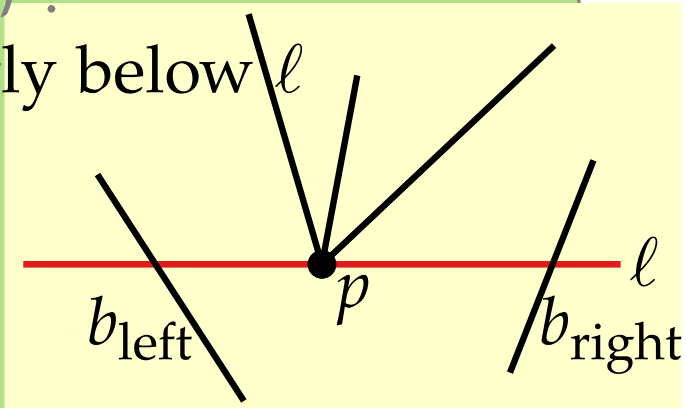

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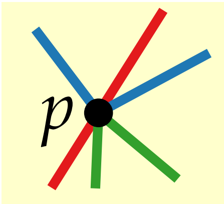

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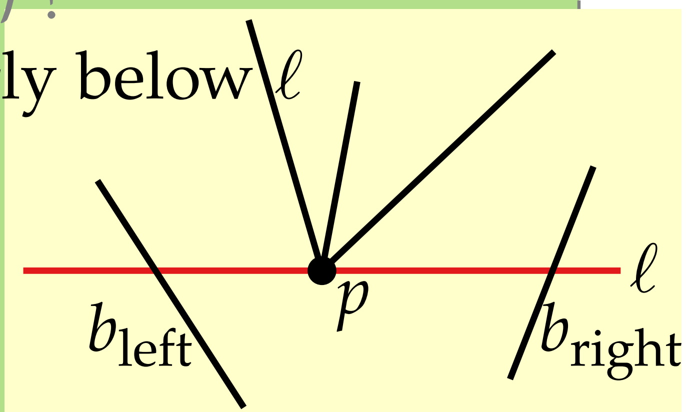
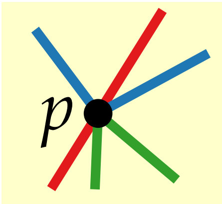


Handling an Event



$C(p)$, $L(p)$, $U(p)$

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if $|U(p) \cup L(p) \cup C(p)| > 1$ **then**
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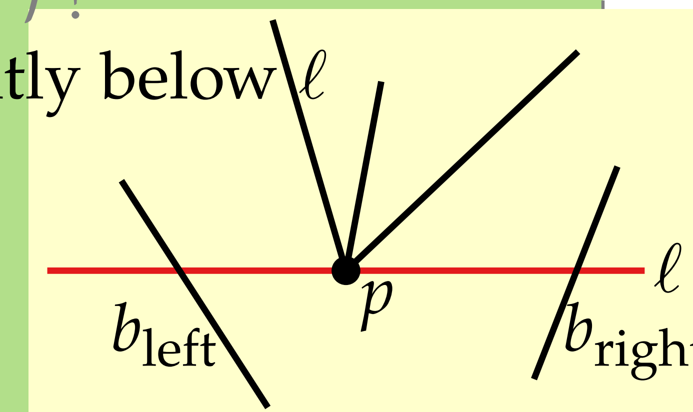
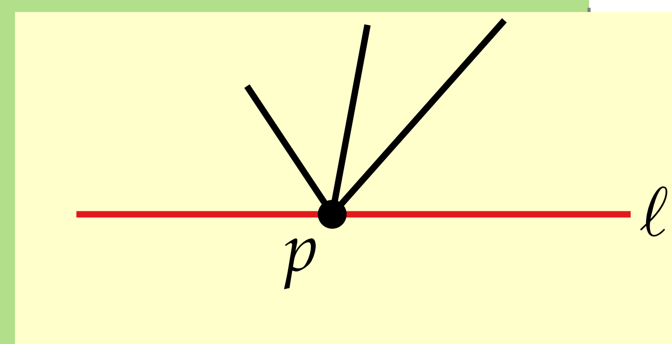
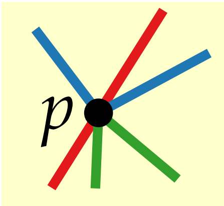
if $U(p) \cup C(p) = \emptyset$ **then**
  $b_{\text{left}} / b_{\text{right}}$ = left/right neighbor of p in $\mathcal{T}$
  findNewEvent$(b_{\text{left}}, b_{\text{right}}, p)$
else





Handling an Event



$C(p)$, $L(p)$, $U(p)$

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if $s \cap s' = \emptyset$ **then return**
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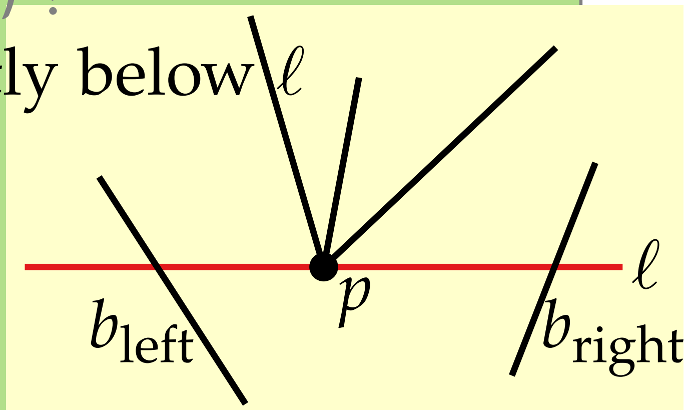
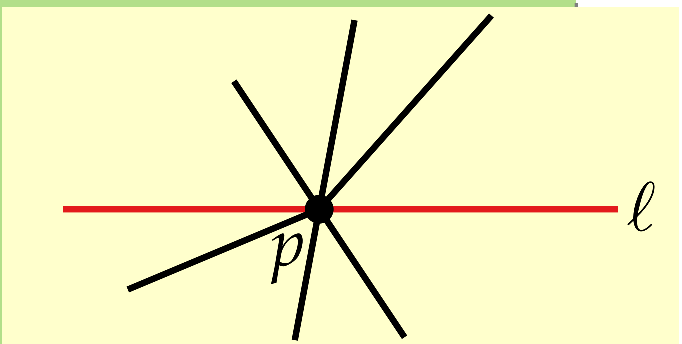
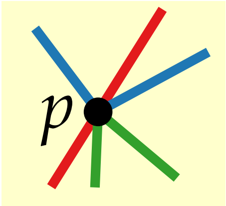




Handling an Event



$C(p)$, $L(p)$, $U(p)$

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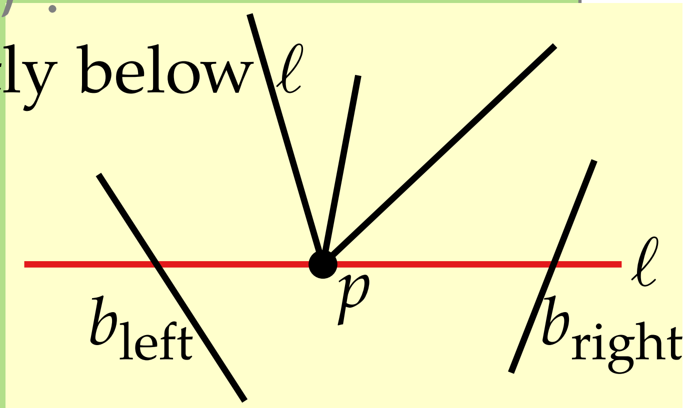
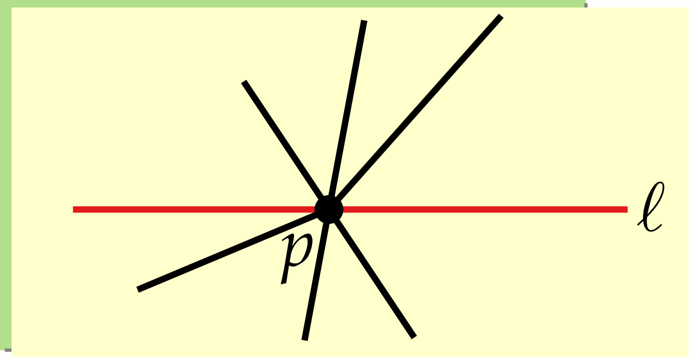
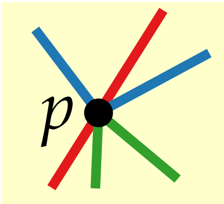




Handling an Event



$C(p)$, $L(p)$, $U(p)$

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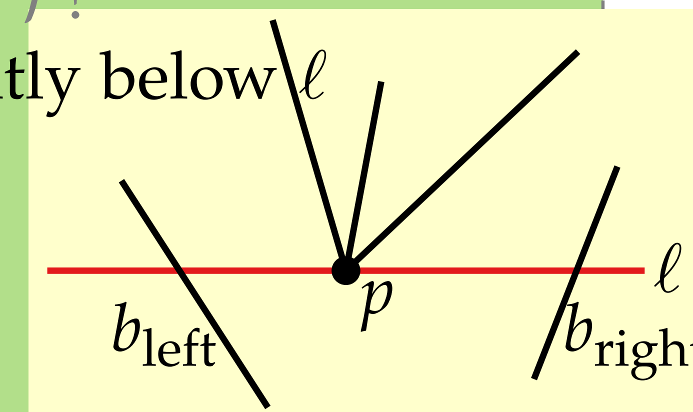
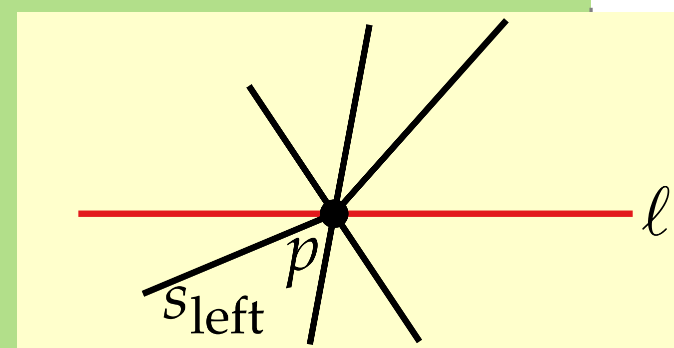
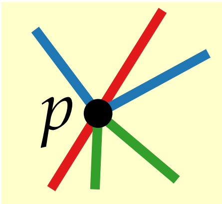




Handling an Event



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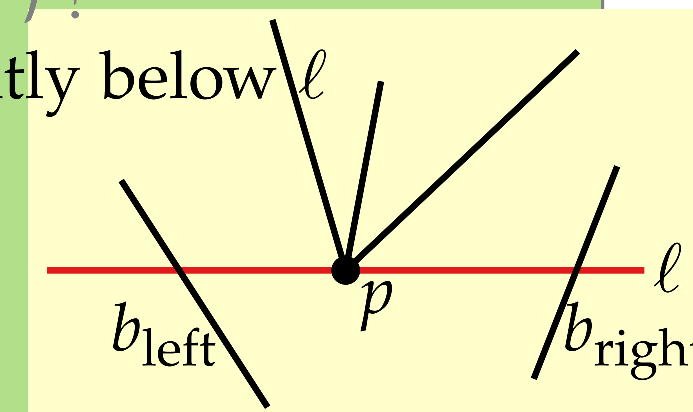
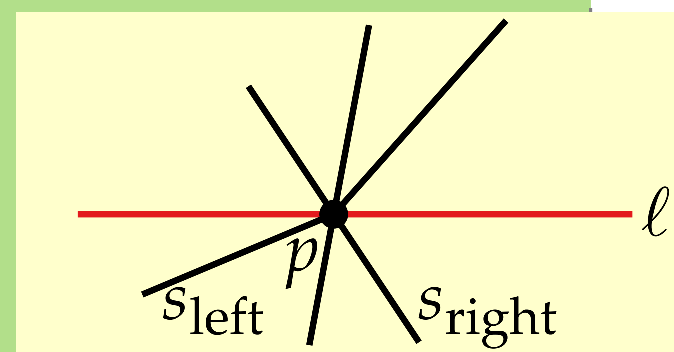
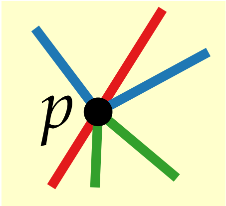




Handling an Event



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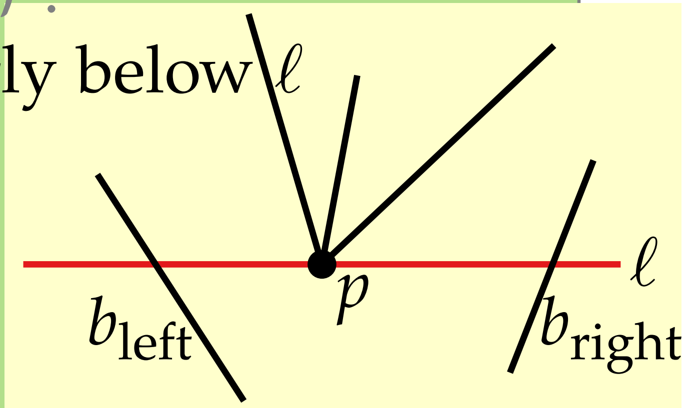
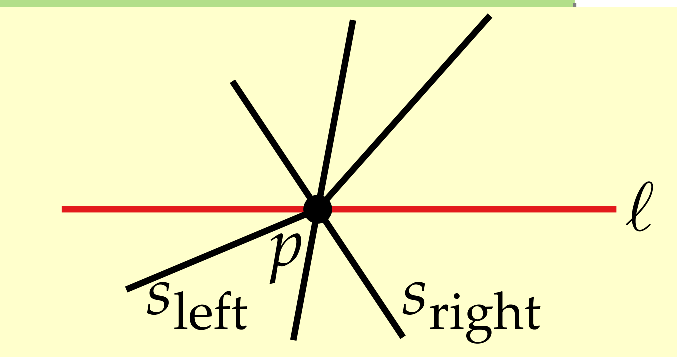
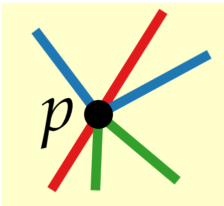




Handling an Event



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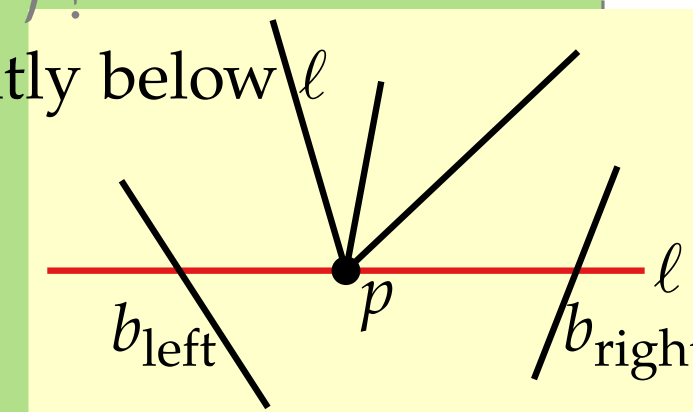

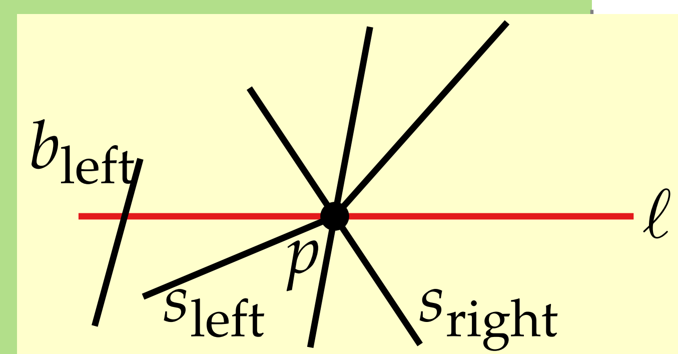

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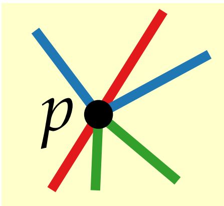

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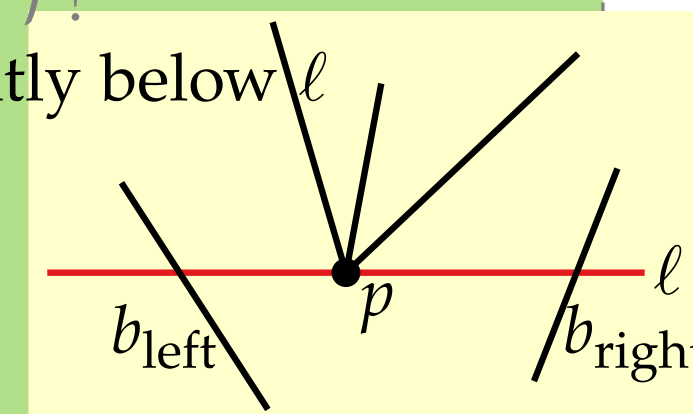

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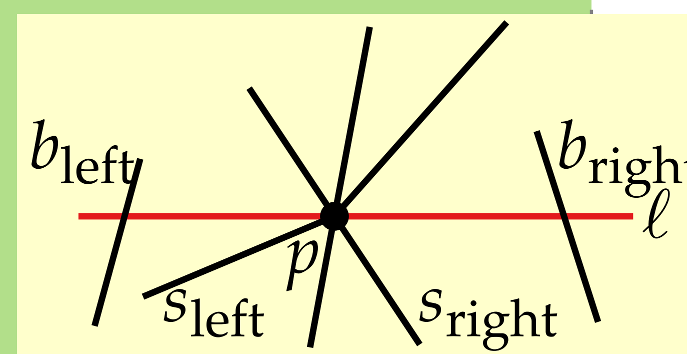

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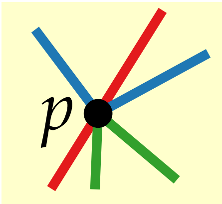

$C(p), L(p), U(p)$

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if $U(p) \cup C(p) = \emptyset$ **then**
  $b_{\text{left}} / b_{\text{right}}$ = left/right neighbor of p in $\mathcal{T}$
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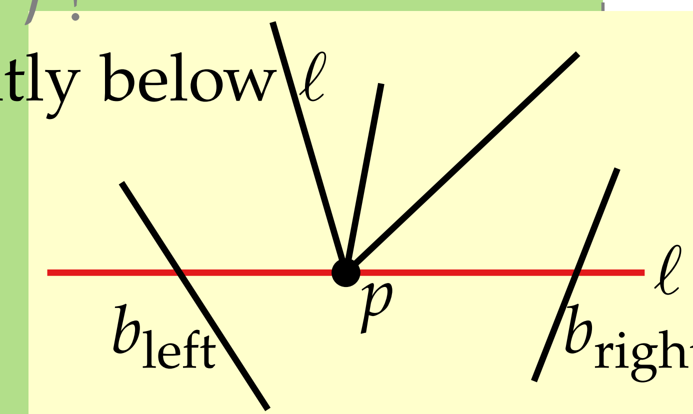
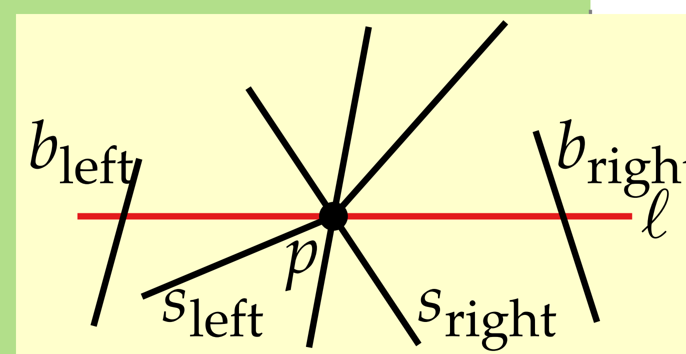
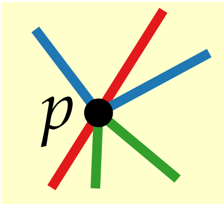




Handling an Event



$C(p)$, $L(p)$, $U(p)$

findNewEvent(s, s', p)
if $s \cap s' = \emptyset$ **then return**
$\{x\} = s \cap s'$
if x below ℓ or on ℓ to the right of p **then**
  if $x \notin \mathcal{Q}$ **then** $\mathcal{Q}$.add(x)
  if $x \in$ rel-int(s) **then** $C(x) \leftarrow C(x) \cup \{s\}$
  if $x \in$ rel-int(s') **then** $C(x) \leftarrow C(x) \cup \{s'\}$

handleEvent(event p)
if $|U(p) \cup L(p) \cup C(p)| > 1$ **then**
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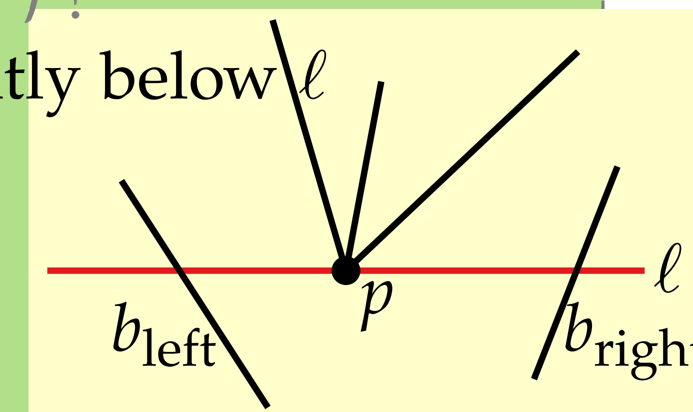

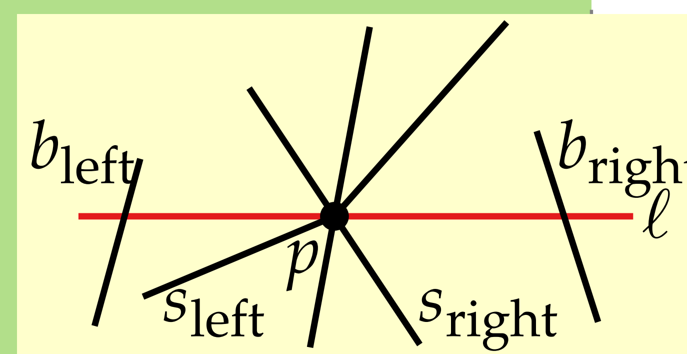

Computational Geometry

Lecture 2:
Line-Segment Intersection
or
Map Overlay

Part IV:
Correctness

Philipp Kindermann

Winter Semester 2020

Correctness

Lemma. findIntersections(S) correctly computes all intersection points & the segments that contain them.

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Correctness (Case II)

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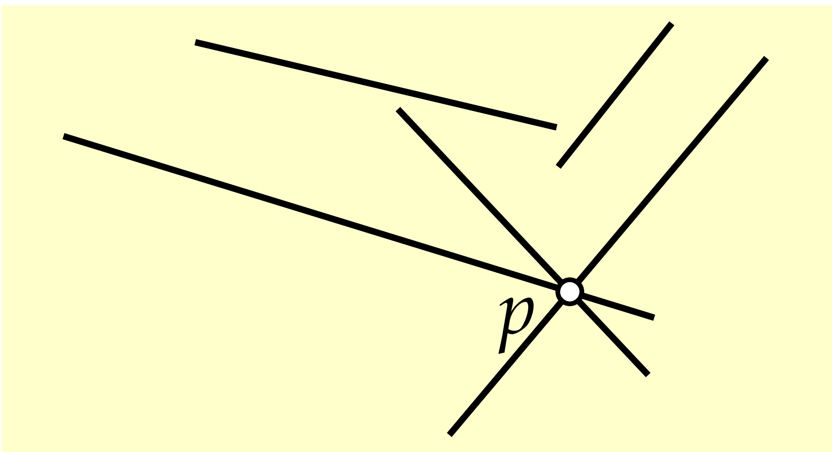

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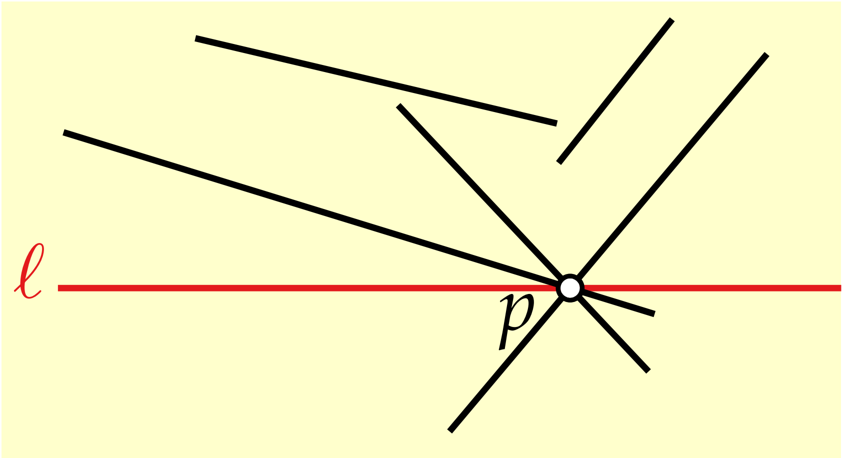

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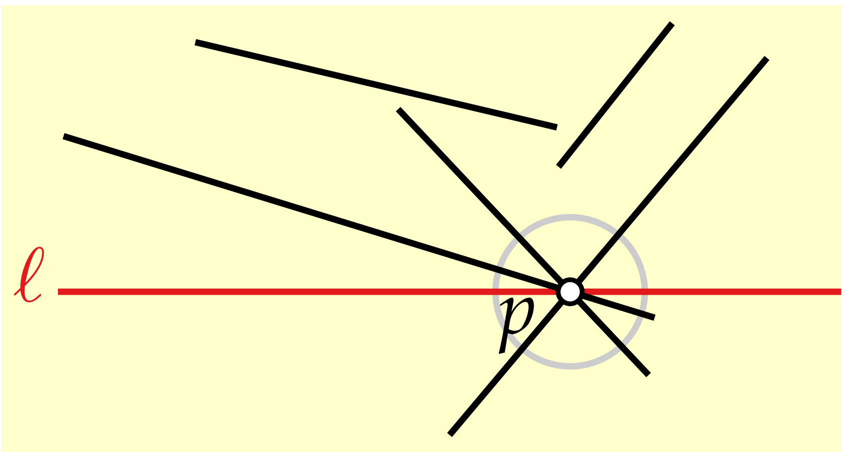

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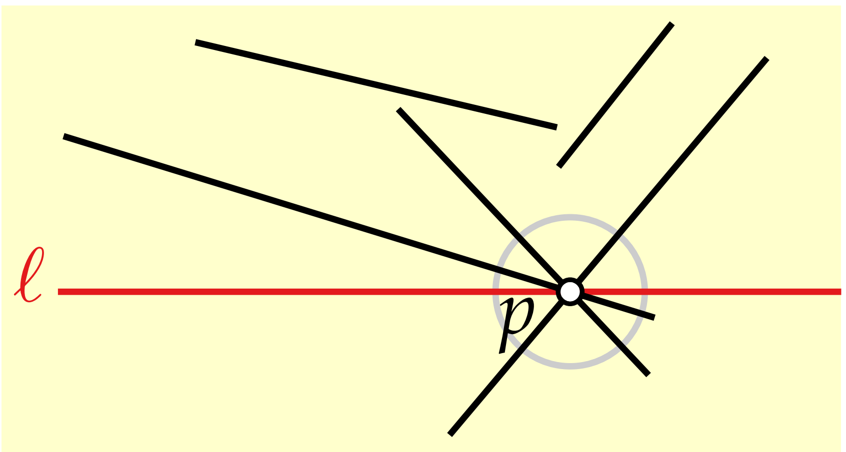

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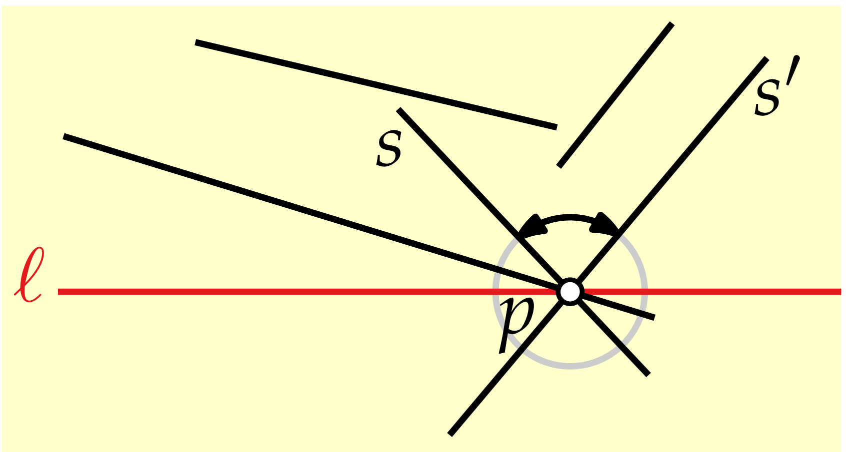

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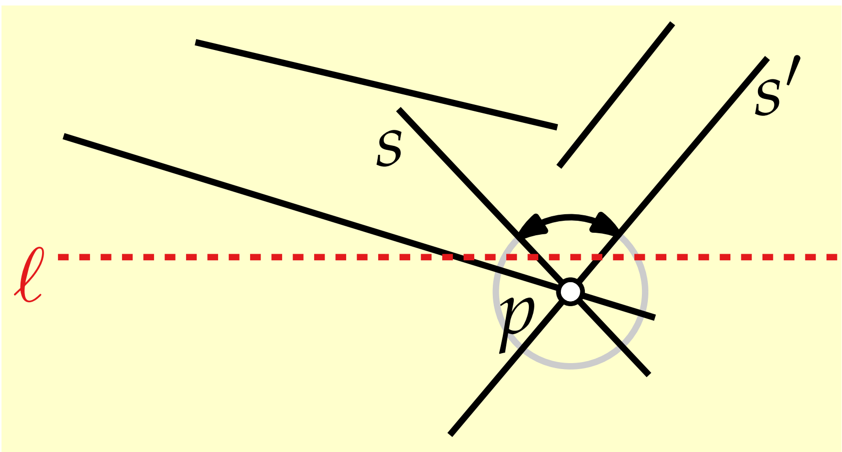

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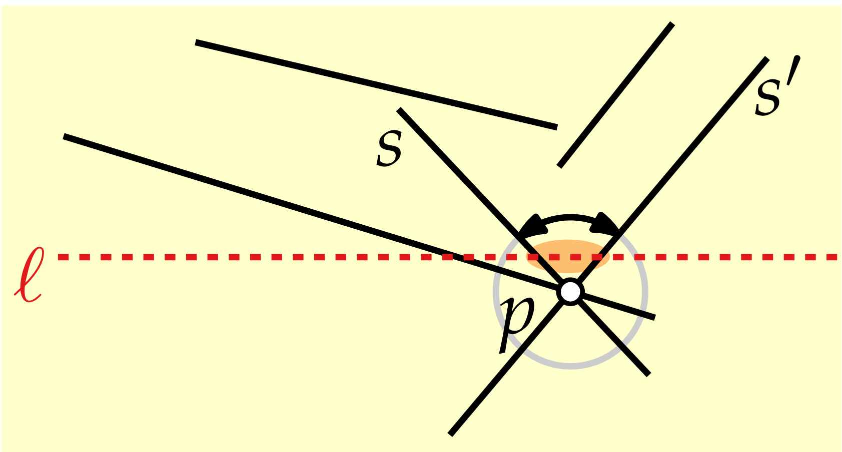

Let $s, s' \in C(p)$ be neighbors in the circular ordering of $C(p) \cup \{\ell\}$ around p. Imagine moving ℓ slightly back in time.
Then s, s' were neighbors in the left-to-right order on ℓ (in $\mathcal{T}$).

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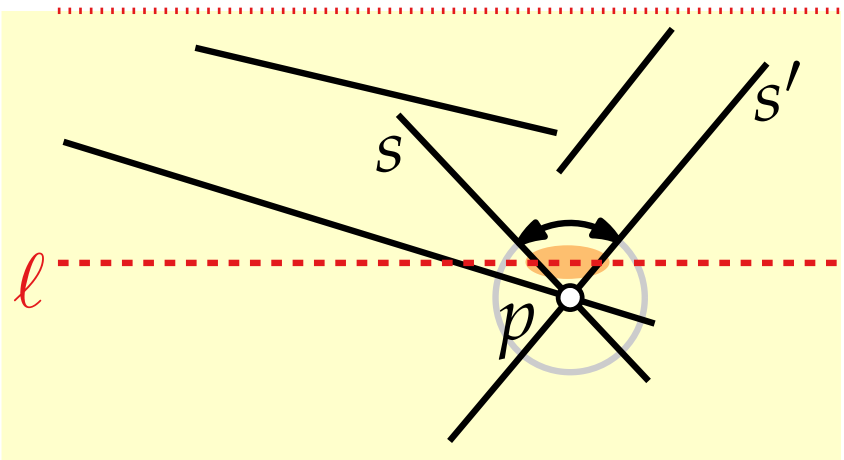

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At the beginning of the alg., they weren't neighbors in $\mathcal{T}$.

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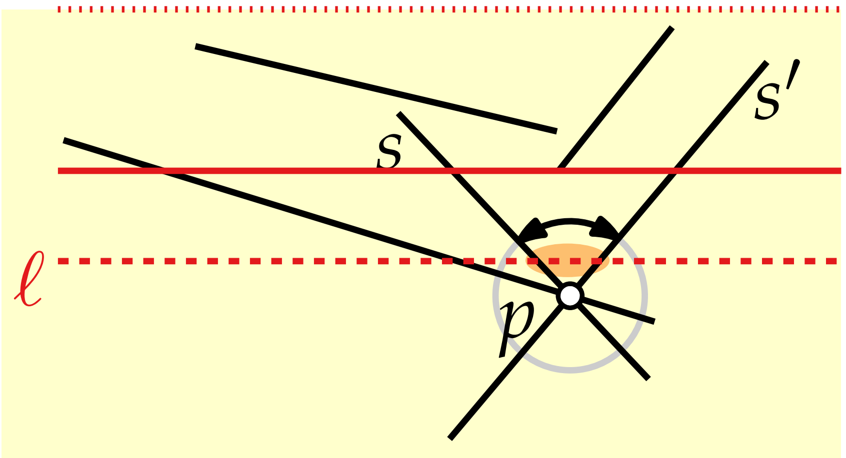

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At the beginning of the alg., they weren't neighbors in $\mathcal{T}$.
$\Rightarrow$ There was some moment when they became neighbors!

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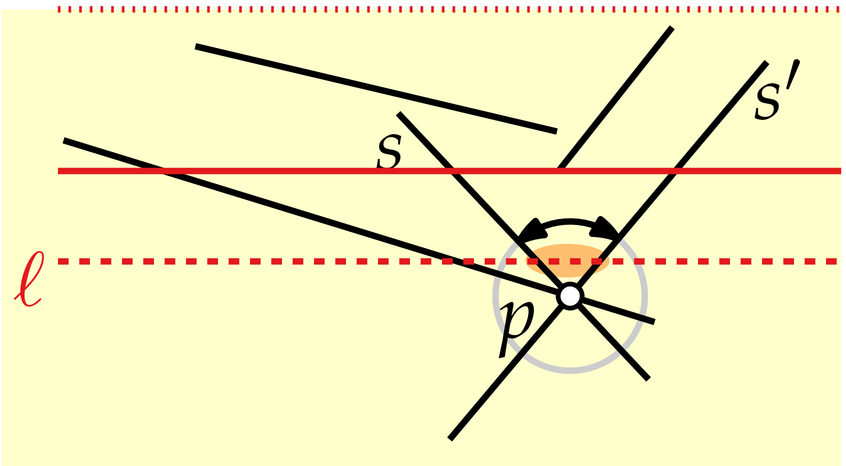

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This is when $\{p\} = s \cap s'$ was inserted into $\mathcal{Q}$.

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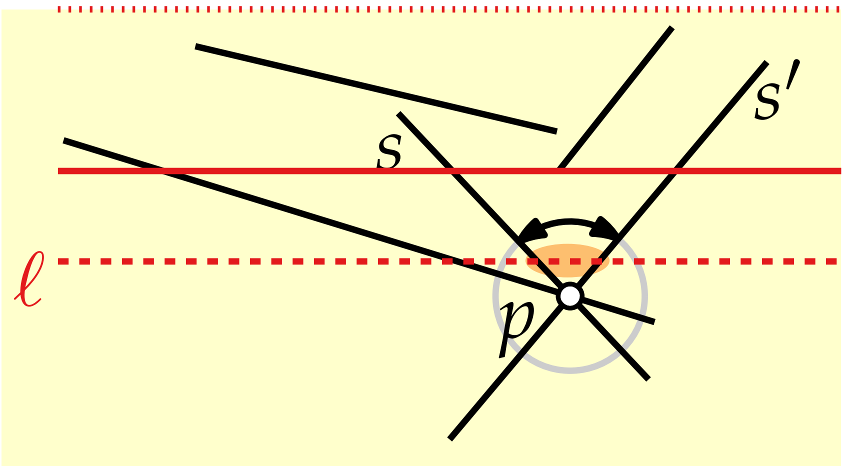

We also need that *every* segment with p as an interior point is added to $C(p)$.

Let $s, s' \in C(p)$ be neighbors in the circular ordering of $C(p) \cup \{\ell\}$ around p. Imagine moving ℓ slightly back in time. Then s, s' were neighbors in the left-to-right order on ℓ (in $\mathcal{T}$).
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This is when $\{p\} = s \cap s'$ was inserted into $\mathcal{Q}$. $\square$

Computational Geometry

Lecture 2:
Line-Segment Intersection
or
Map Overlay

Part V:
Running Time

Philipp Kindermann

Winter Semester 2020

```
Q ← ∅;  T ← ⟨ vertical lines at x = −∞ and x = +∞ ⟩   // sentinels
foreach s ∈ S do                                     // initialize event queue Q
    foreach endpoint p of s do
        if p ∉ Q then Q.insert(p); L(p) = U(p) = C(p) = ∅
        if p lower  endpt of s then L(p).append(s)
        if p upper endpt of s then U(p).append(s)
while Q ≠ ∅ do
    p ← Q.nextEvent()
    Q.deleteEvent(p)
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Running time?

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delete L(p) ∪ C(p) from T   // consecutive in T!
insert U(p) ∪ C(p) into T in their order slightly below ℓ
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    b_left/b_right = left/right neighbor of p in T
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$\{x\} = s \cap s'$

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    b_right = right neighbor of s_right in T
    findNewEvent(b_left, s_left, p)
    findNewEvent(b_right, s_right, p)
```

Running time?

$\mathcal{Q} \leftarrow \emptyset$; $\mathcal{T} \leftarrow \langle$ vertical lines at $x = -\infty$ and $x = +\infty \rangle$ // sentinels

foreach $s \in S$ **do** // initialize event queue $\mathcal{Q}$

  foreach endpoint p of s **do**

    if $p \notin \mathcal{Q}$ **then** $\mathcal{Q}$.insert(p); $L(p) = U(p) = C(p) = \emptyset$

    if p lower endpt of s **then** $L(p)$.append(s)

    if p upper endpt of s **then** $U(p)$.append(s)

while $\mathcal{Q} \neq \emptyset$ **do**

  $p \leftarrow \mathcal{Q}$.nextEvent()

  $\mathcal{Q}$.deleteEvent(p)

  handleEvent(p)

handleEvent(event p)

if $|U(p) \cup L(p) \cup C(p)| > 1$ **then**

  report int. in p, report segments in $U(p) \cup L(p) \cup C(p)$

delete $L(p) \cup C(p)$ from $\mathcal{T}$ // consecutive in $\mathcal{T}$!

insert $U(p) \cup C(p)$ into $\mathcal{T}$ in their order slightly below ℓ

if $U(p) \cup C(p) = \emptyset$ **then**

  $b_{\text{left}}/b_{\text{right}}$ = left/right neighbor of p in $\mathcal{T}$

  findNewEvent(b_{left}, b_{right}, p) → $\{x\} = s \cap s'$; **if** $x \notin \mathcal{Q}$ **then** $\mathcal{Q}$.insert(x)

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Q ← ∅;  T ← ⟨ vertical lines at x = −∞ and x = +∞ ⟩   // sentinels
foreach s ∈ S do                                    // initialize event queue Q
    foreach endpoint p of s do
        if p ∉ Q then Q.insert(p); L(p) = U(p) = C(p) = ∅
        if p lower endpt of s then L(p).append(s)
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while Q ≠ ∅ do
    p ← Q.nextEvent()
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handleEvent(event p)
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    report int. in p, report segments in U(p) ∪ L(p) ∪ C(p)
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insert U(p) ∪ C(p) into T in their order slightly below ℓ
if U(p) ∪ C(p) = ∅ then
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findNewEvent →

```
{x} = s ∩ s'
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Check your knowledge about planar graphs!

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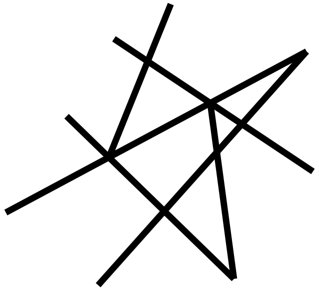

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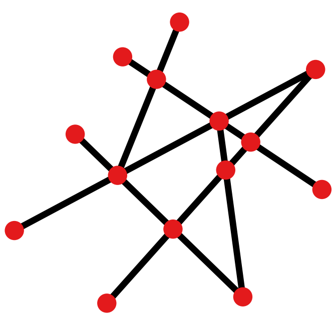

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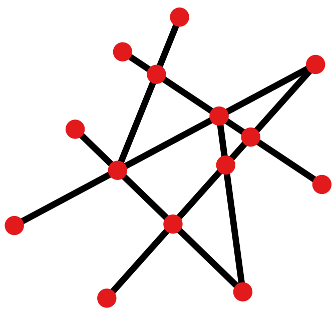

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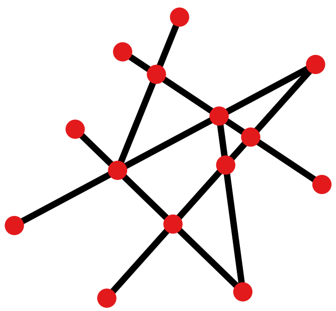

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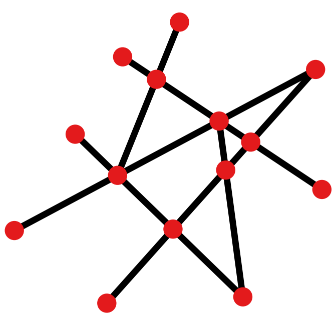

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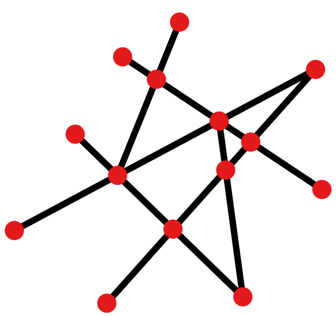

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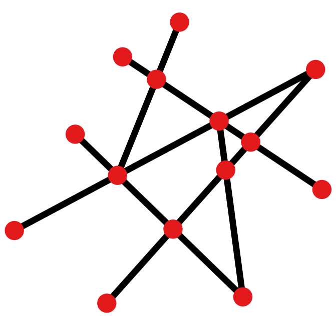

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Euler (G is planar!!)

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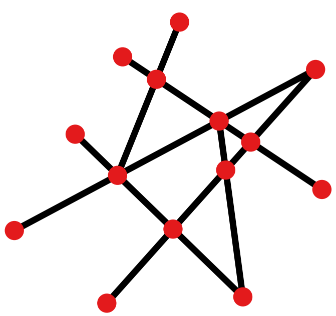

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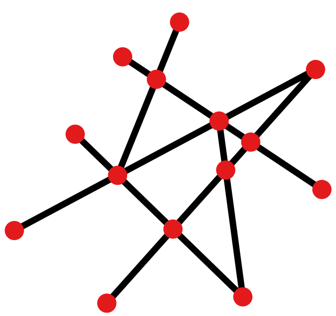

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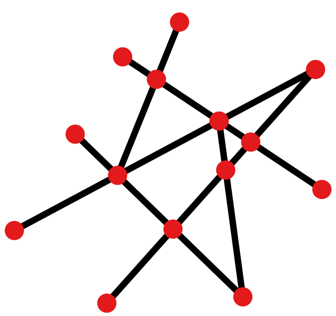

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$\in O(n+I)$ $\square$

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Theorem. We can report all I intersection points among n non-overlapping line segments in the plane and report the segments involved in the intersections in $O((n + I) \log n)$ time and $O(n)$ space.

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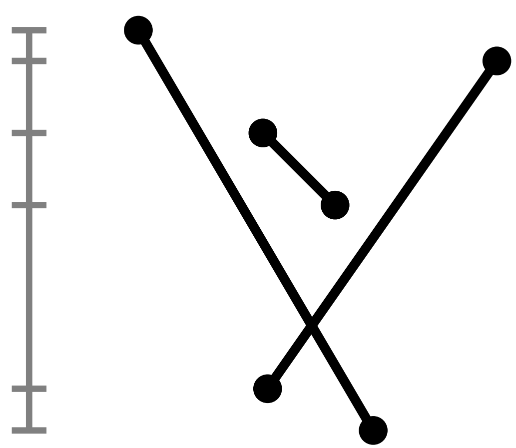

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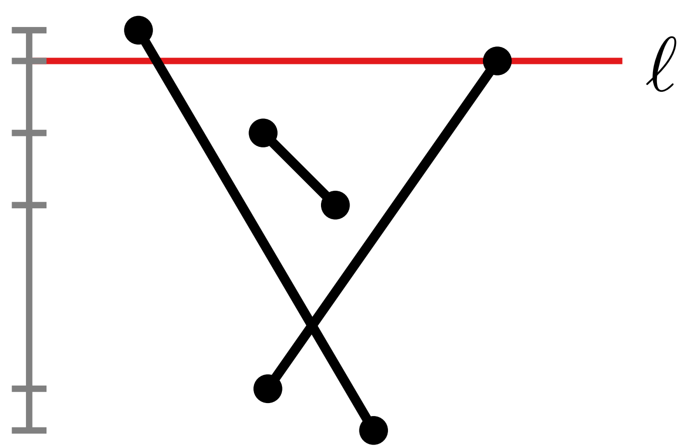

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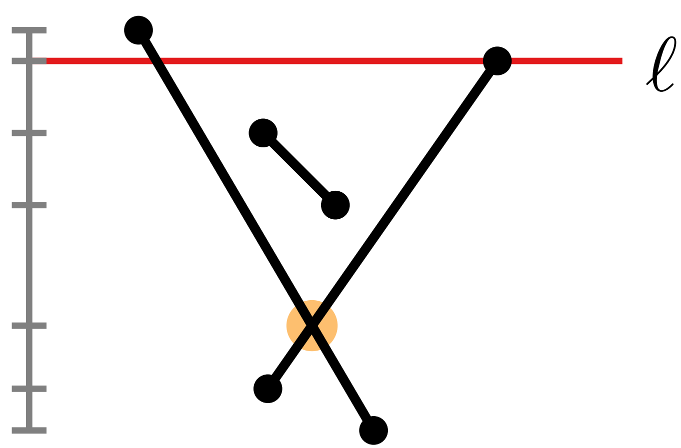

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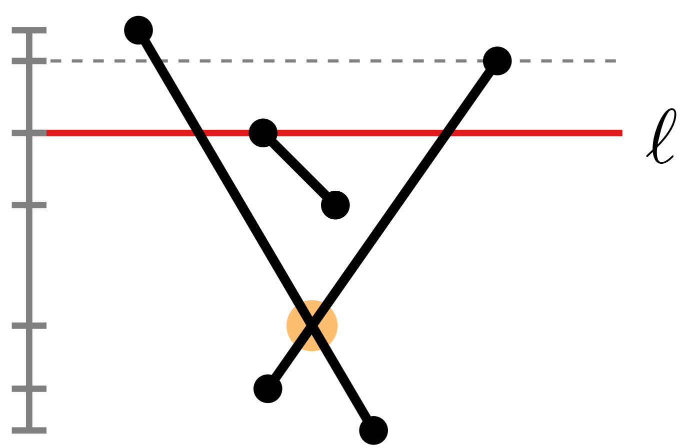

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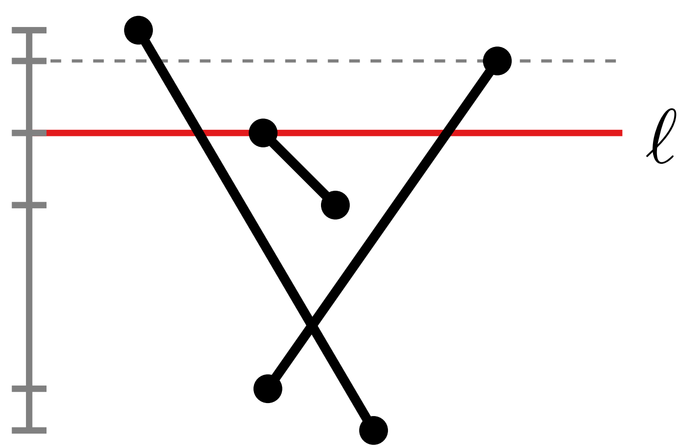

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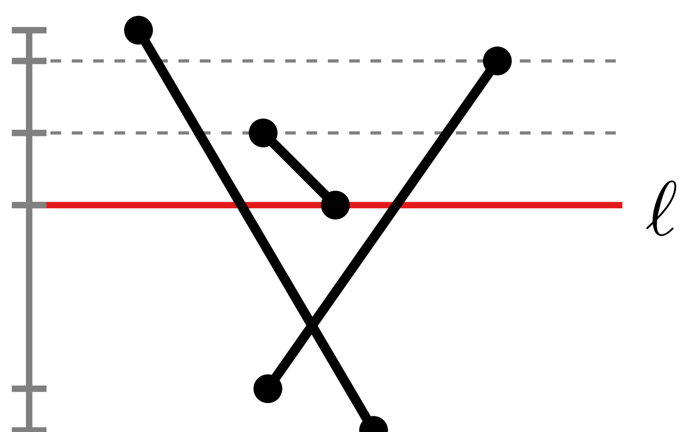

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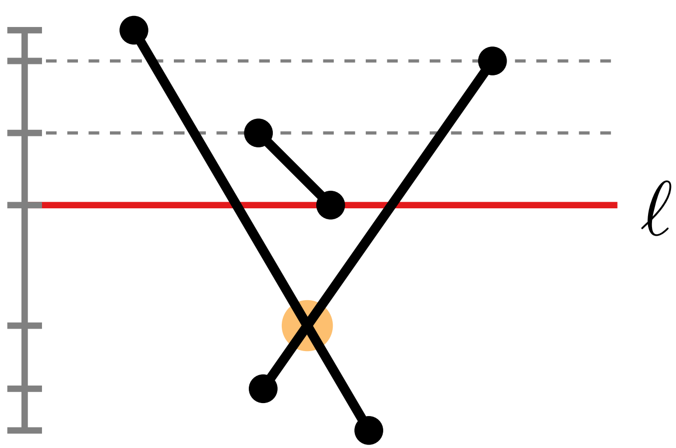

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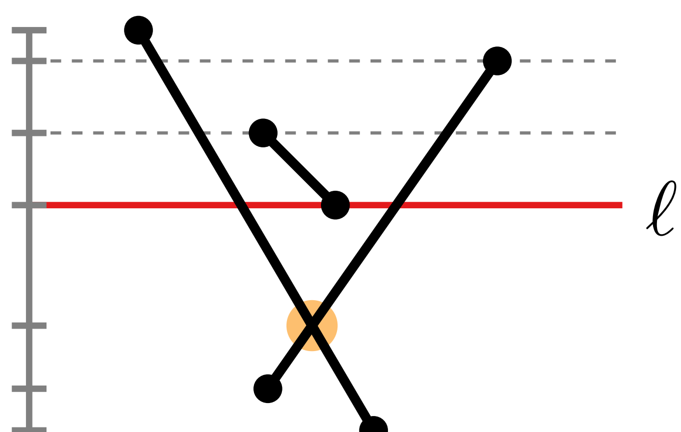

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Today's Main Result

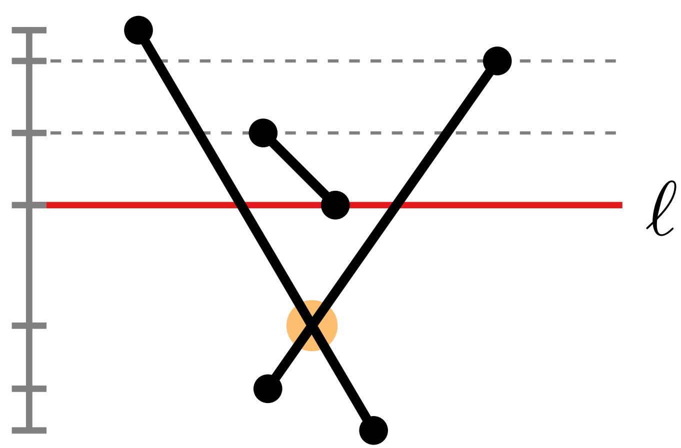
Theorem. We can report all I intersection points among n non-overlapping line segments in the plane and report the segments involved in the intersections in $O((n+I)\log n)$ time and $O(n)$ space.

Sure? The event-point queue $\mathcal{Q}$ contains

- all segment end pts below the sweep line
- all intersection pts below the sweep line

$\Rightarrow$ (worst-case) space consumption $\in \Theta(n+I)$:-(

Can we do better?



– insert $s \cap s'$ into $\mathcal{Q}$

– remove $s \cap s'$ from $\mathcal{Q}$

– re-insert $s \cap s'$ into $\mathcal{Q}$

$\Rightarrow$ need just $O(n)$ space; (asymptotic) running time doesn't change $\square$