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Vehicle Localization by Matching Triangulated Point Patterns

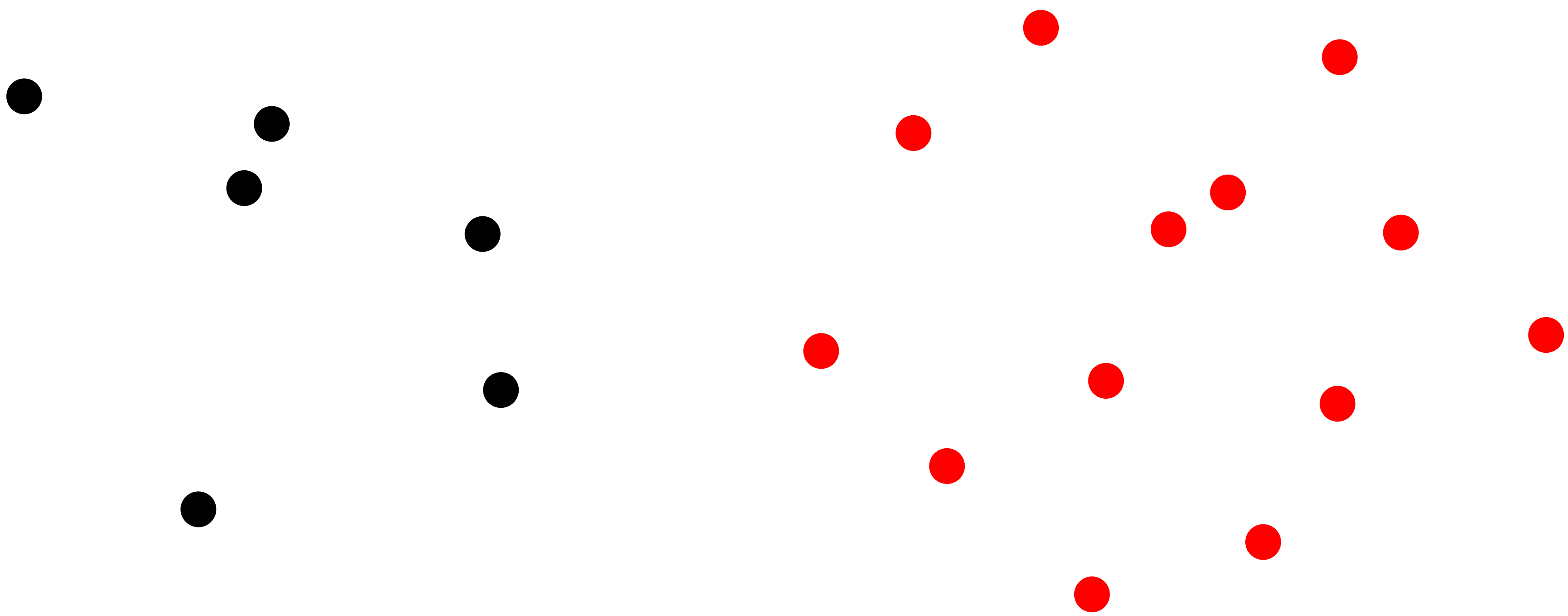
Jan-Henrik Haunert
Institut für Informatik
Universität Würzburg

Claus Brenner
Institut für Kartographie
und Geoinformatik
Universität Hannover

Introduction

Point pattern matching

- given two point sets
- find corresponding points based on geometric configuration



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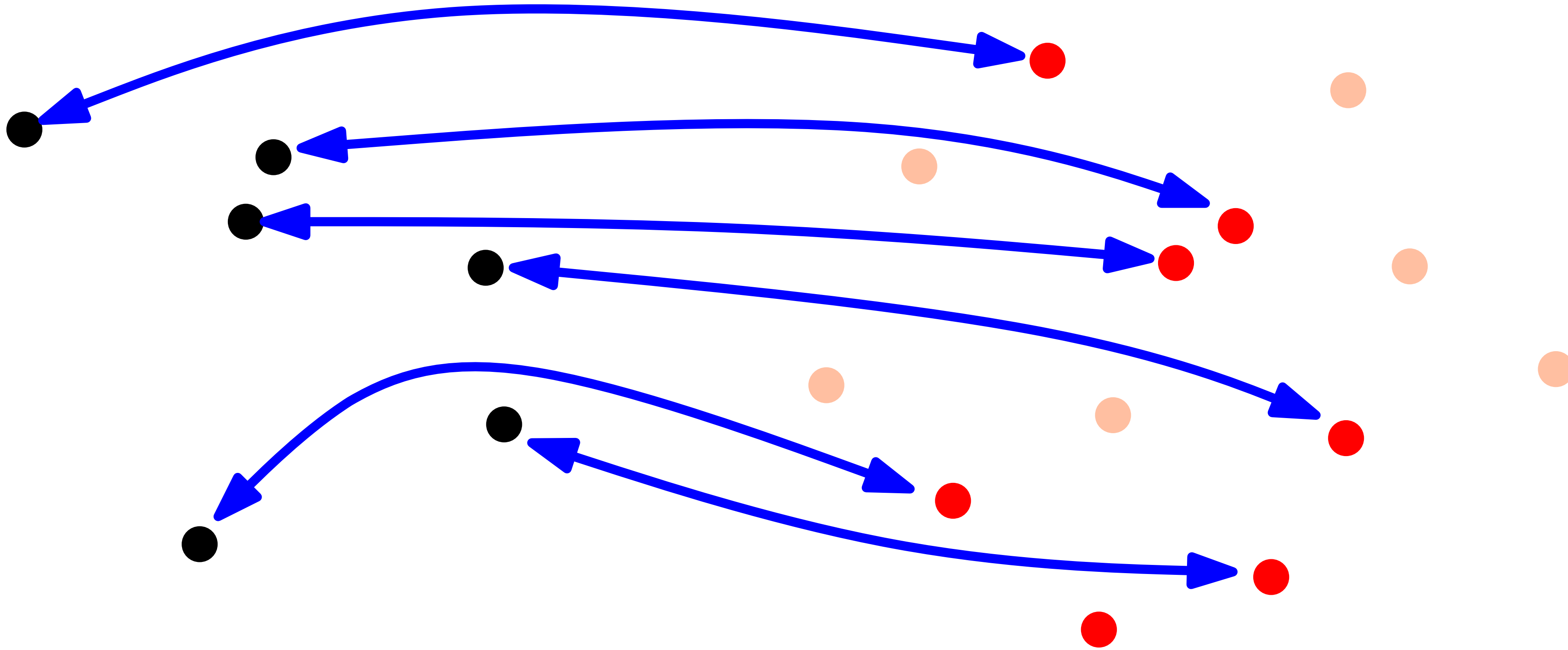

Introduction

Applications of point pattern matching

- fingerprint verification

Introduction

Applications of point pattern matching

- orientation of star cameras

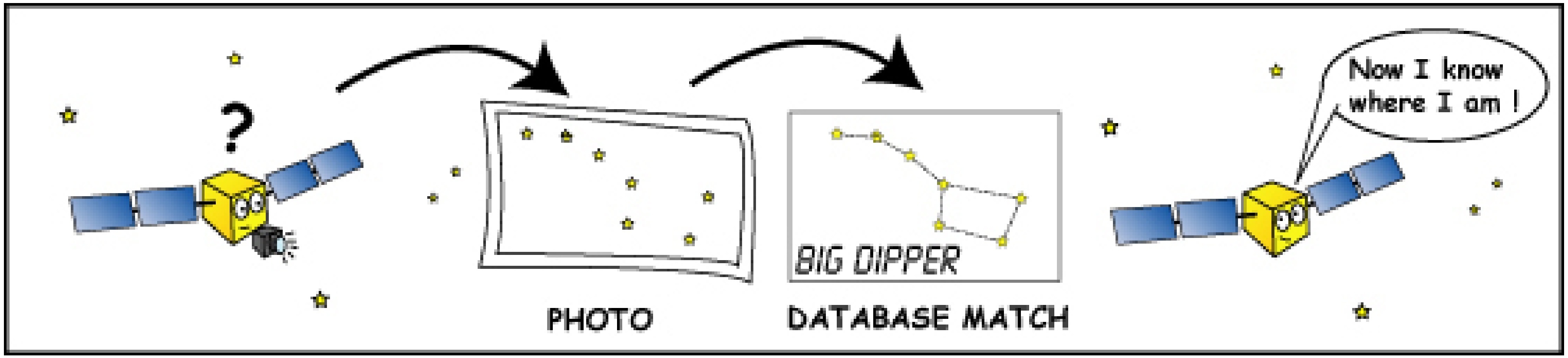

image source: NASA

Introduction

Applications of point pattern matching

- **here:** vehicle positioning

?

Introduction

Applications of point pattern matching

- **here:** vehicle positioning
  - GPS is not always/everywhere available
  - positioning a vehicle with only one system (GPS) is risky if it drives autonomously

?

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Applications of point pattern matching

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- points may represent any kind of landmarks
- **here:** poles (e.g., of traffic signs) observed with a vehicle-mounted laser scanner

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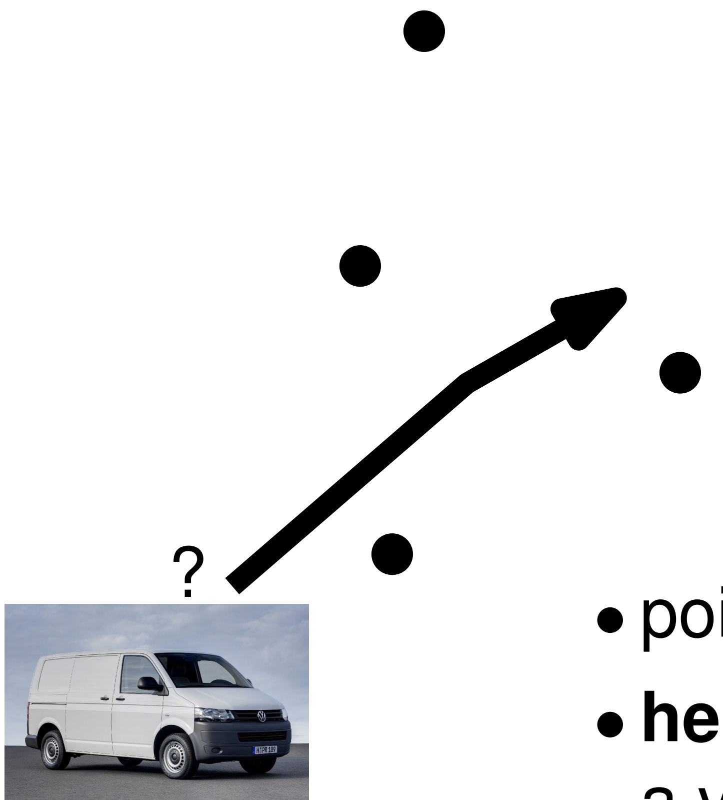

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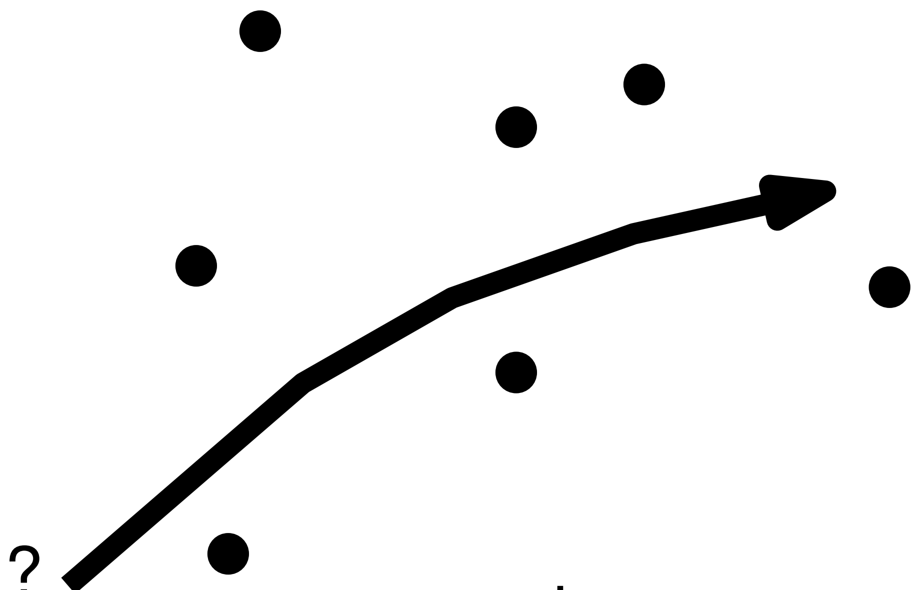

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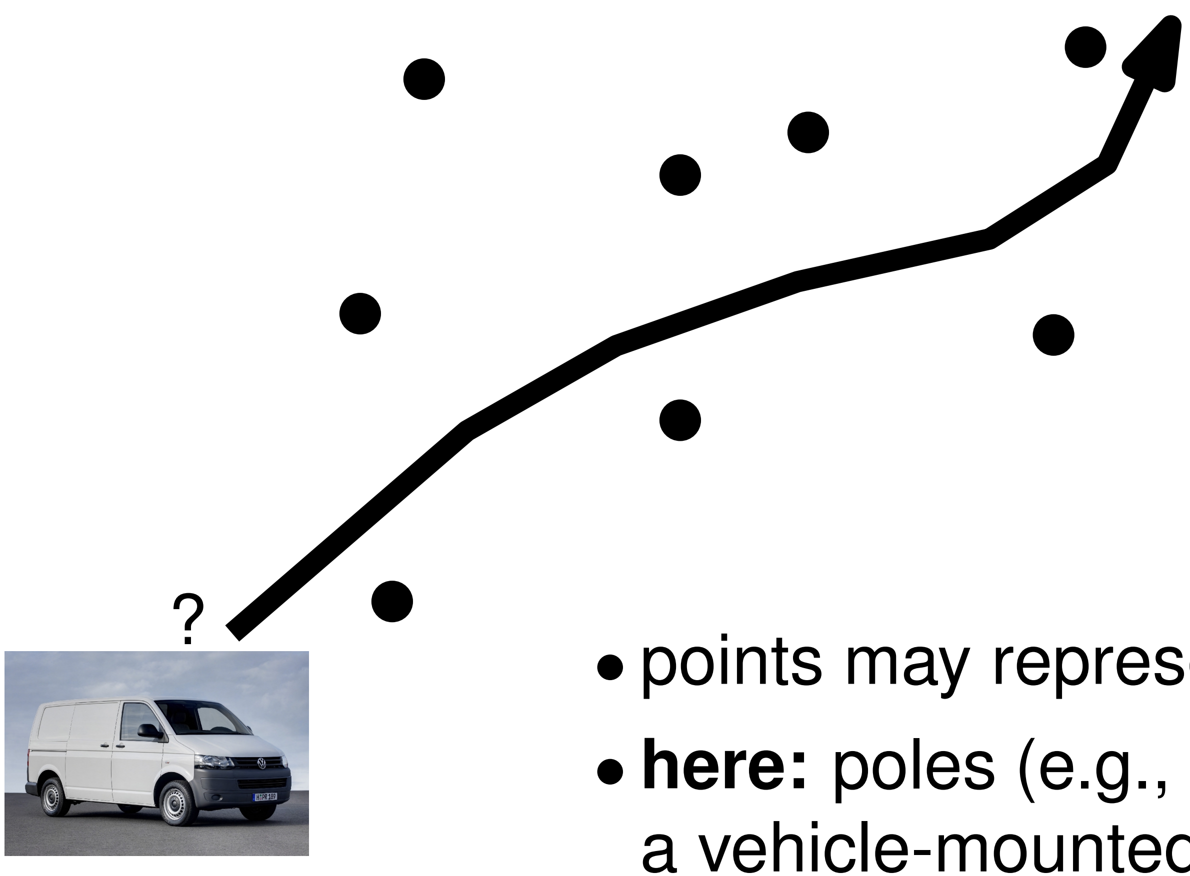

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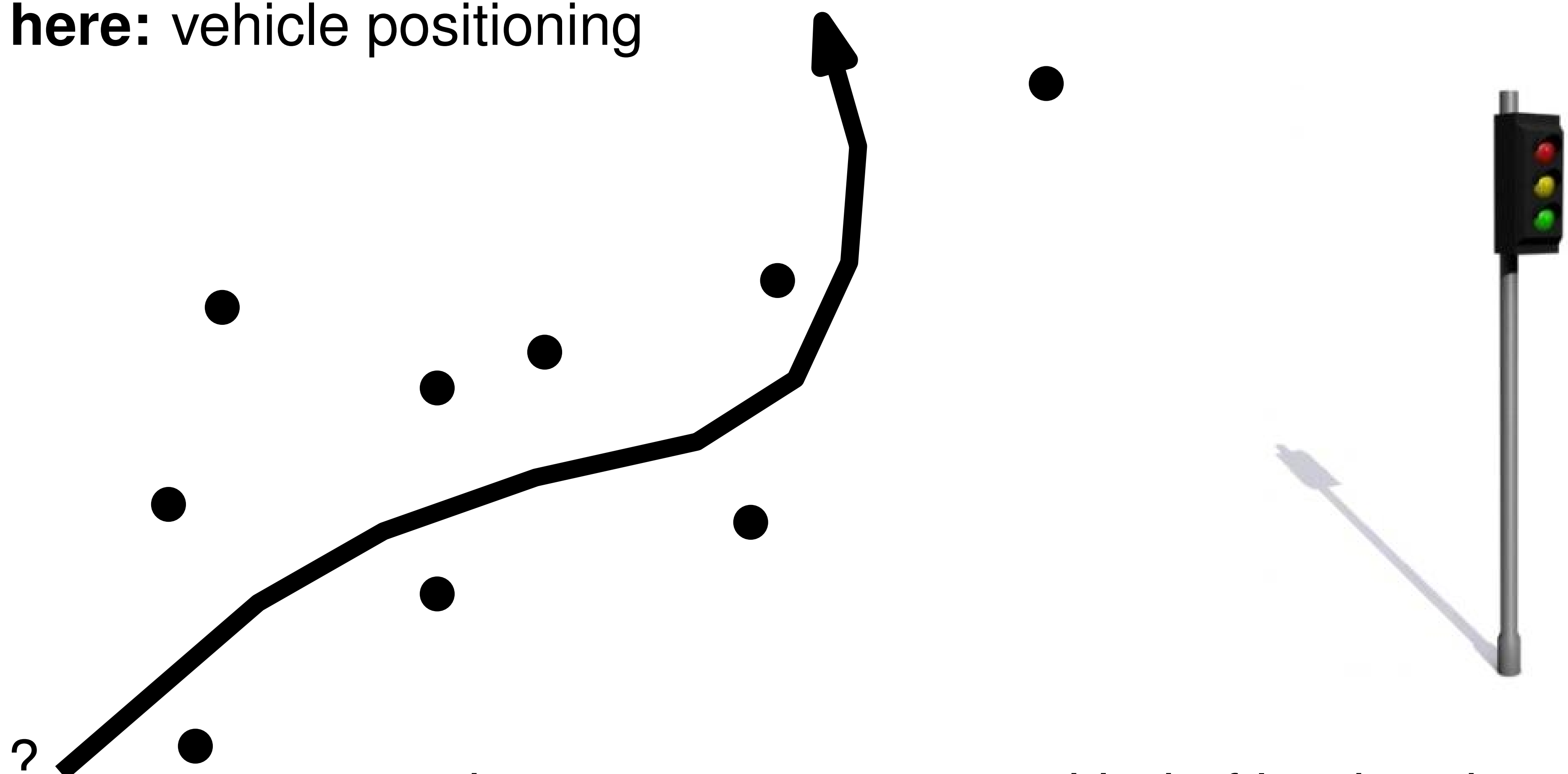

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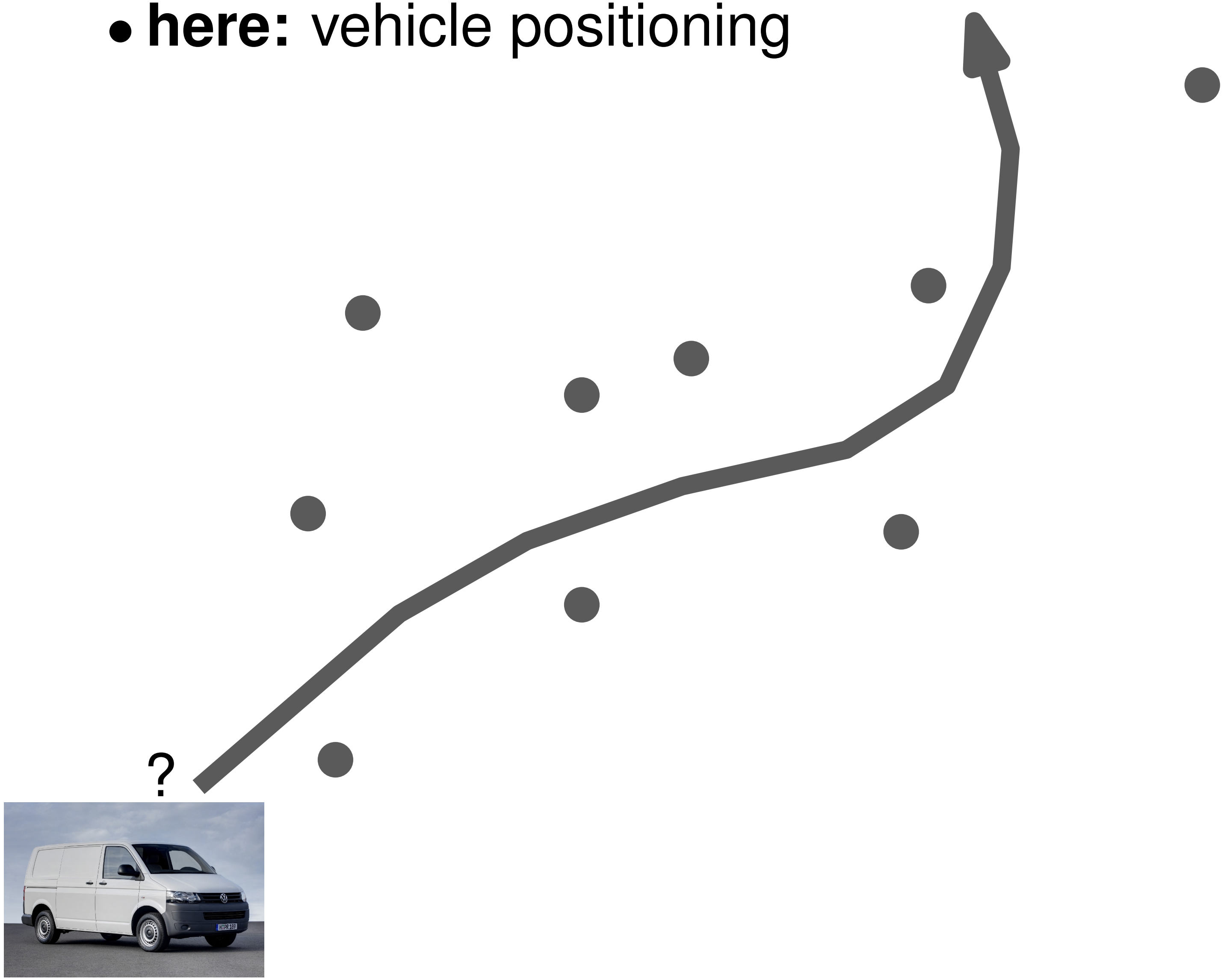

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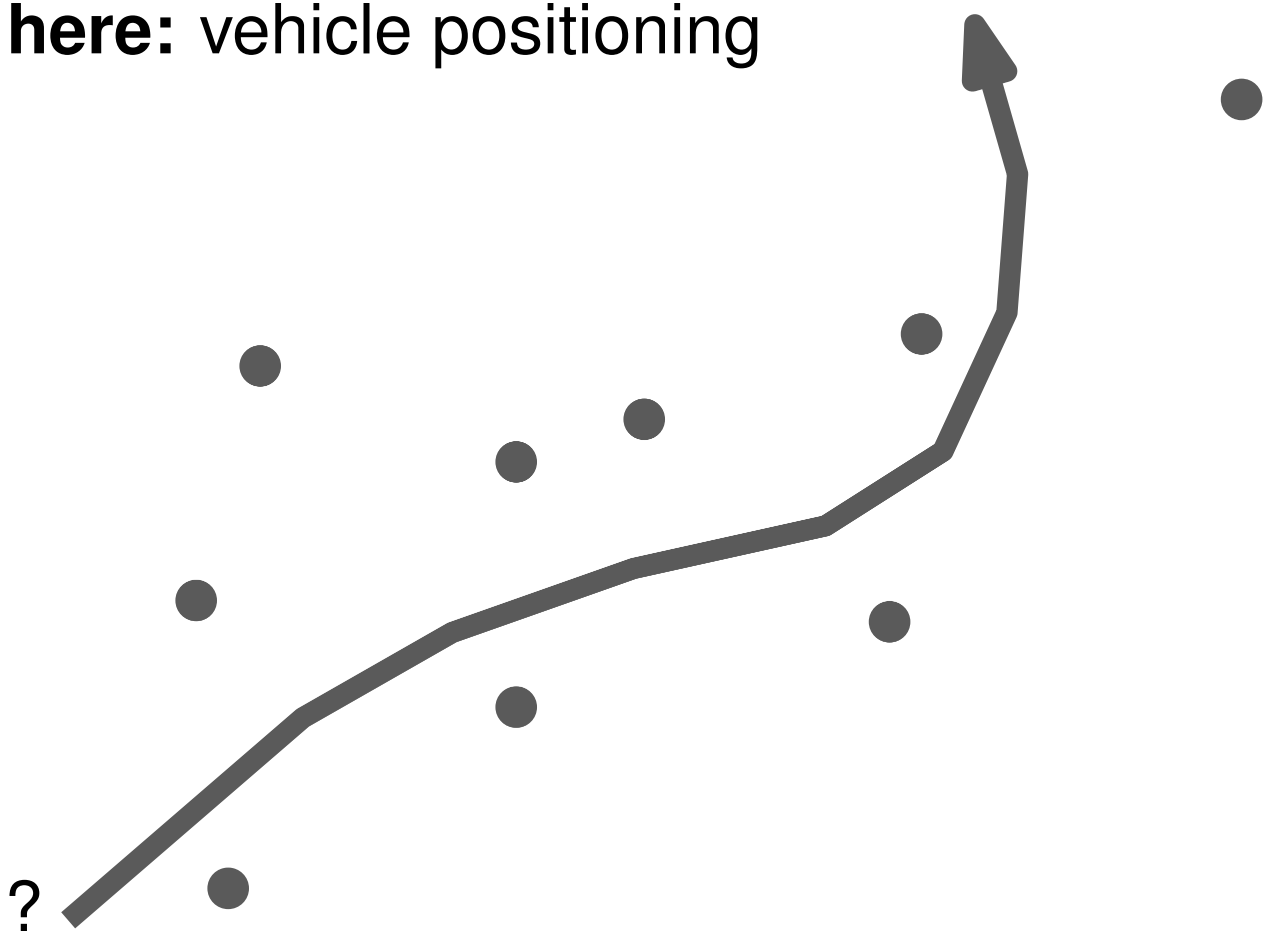

- coordinates may be erroneous
- global rigid transformation does not exist

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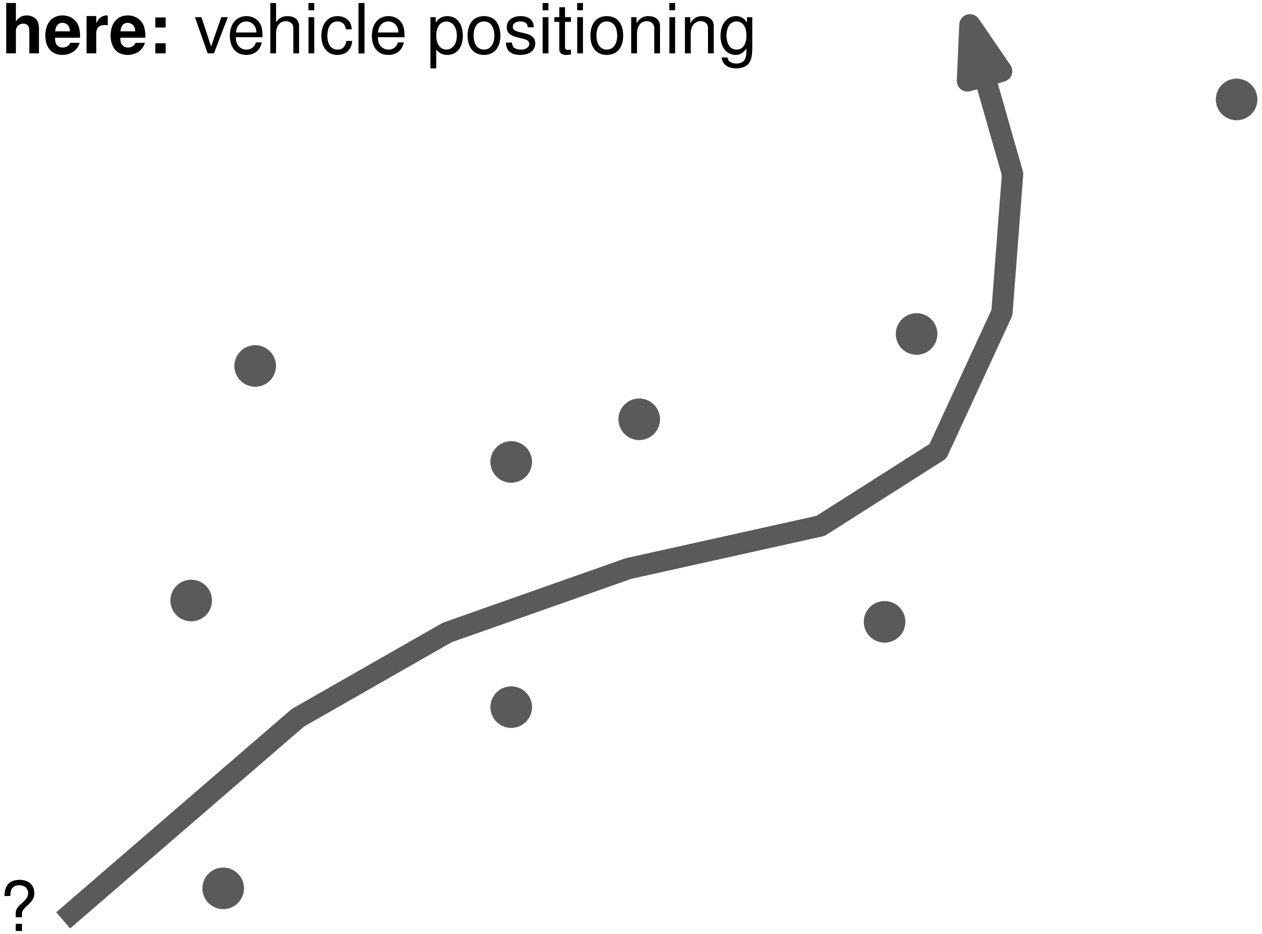

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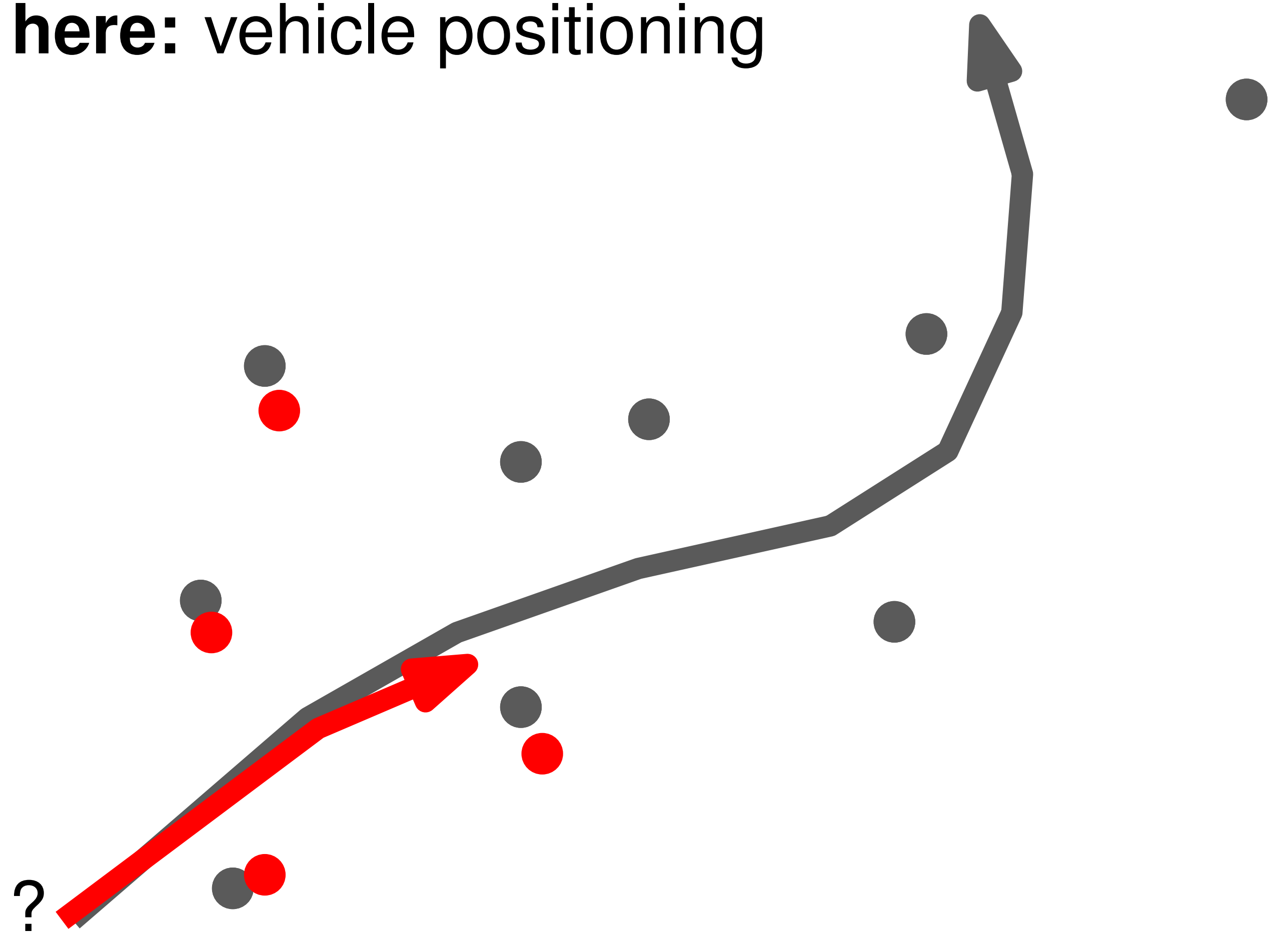

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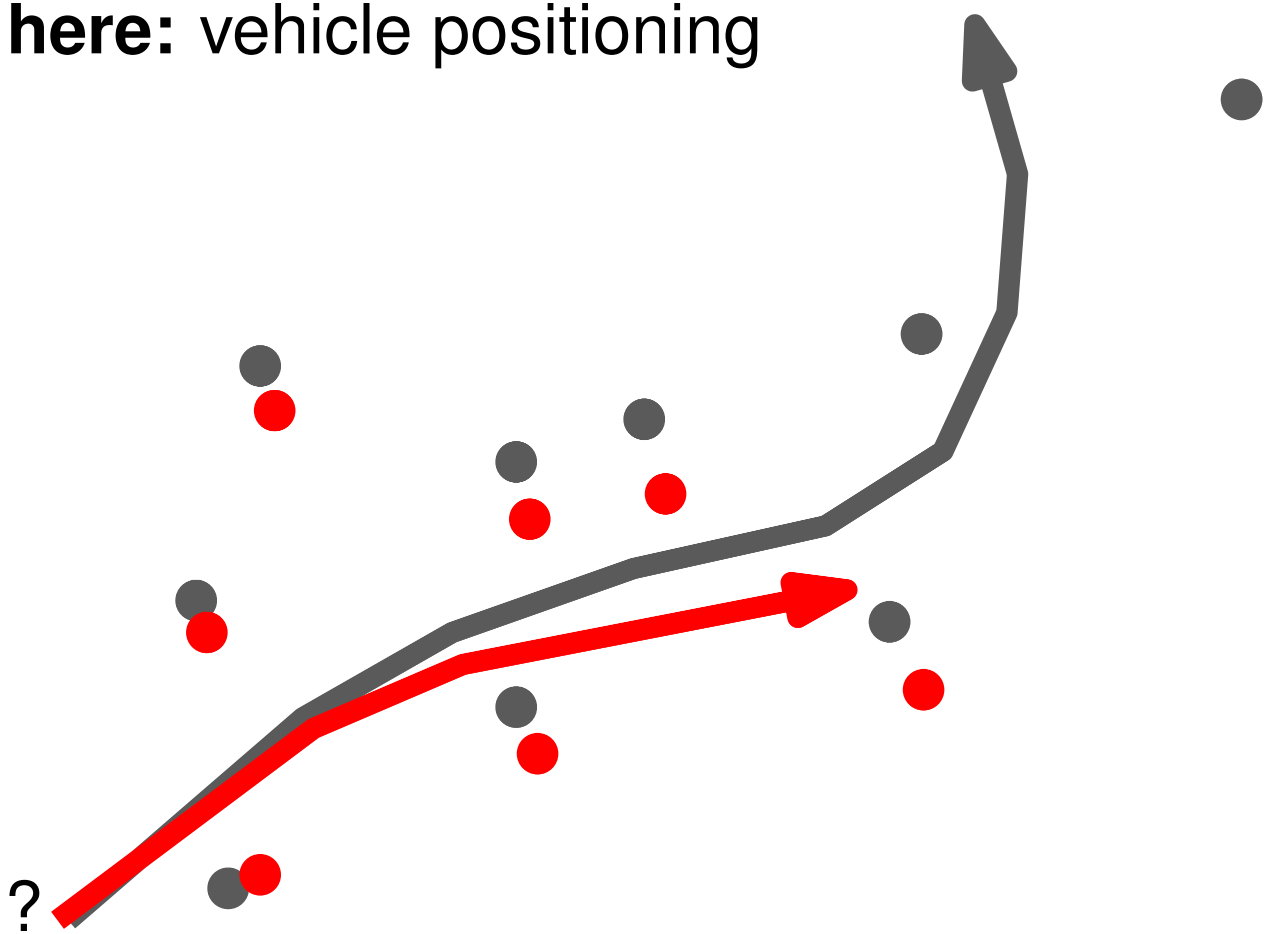

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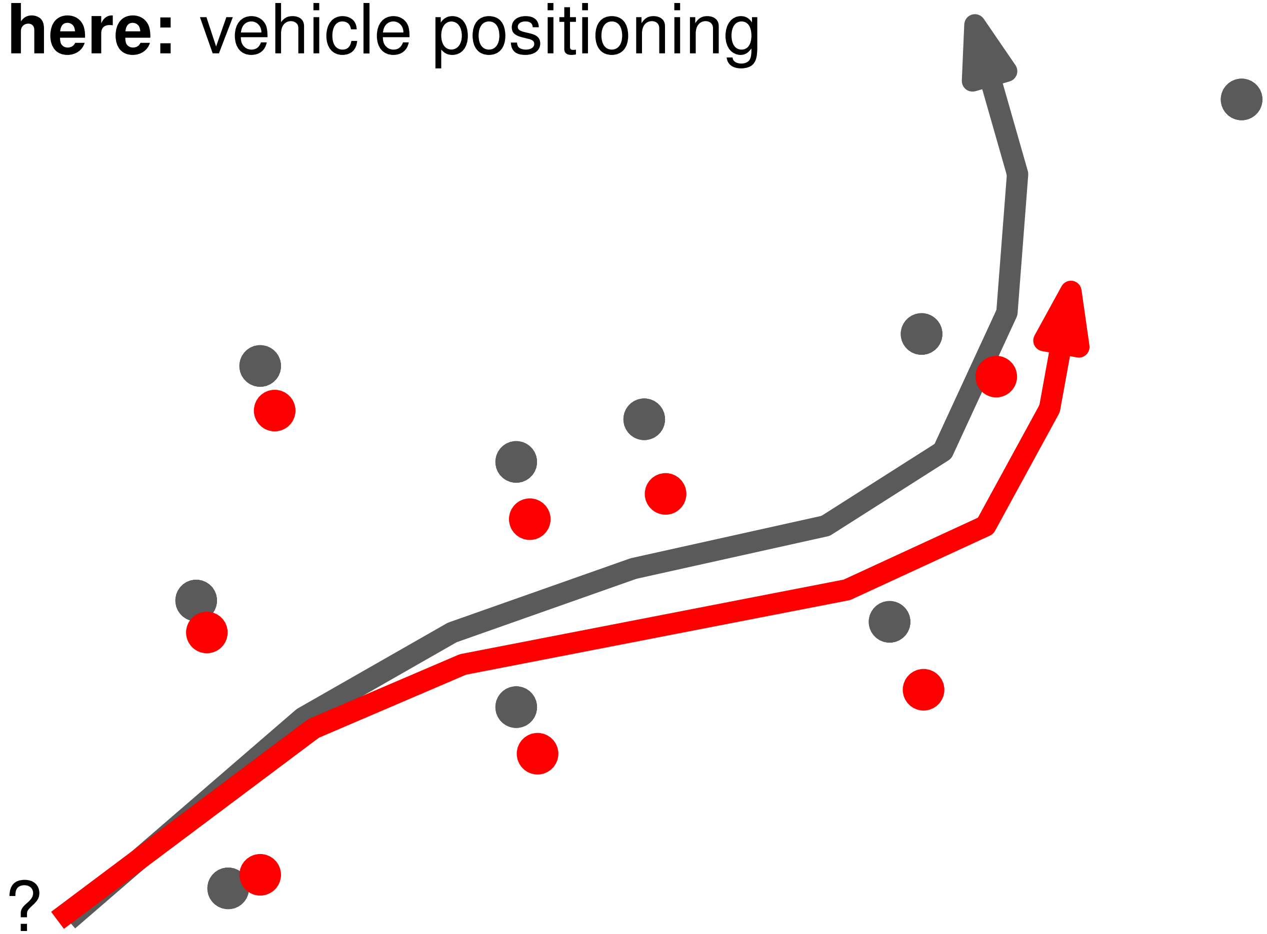

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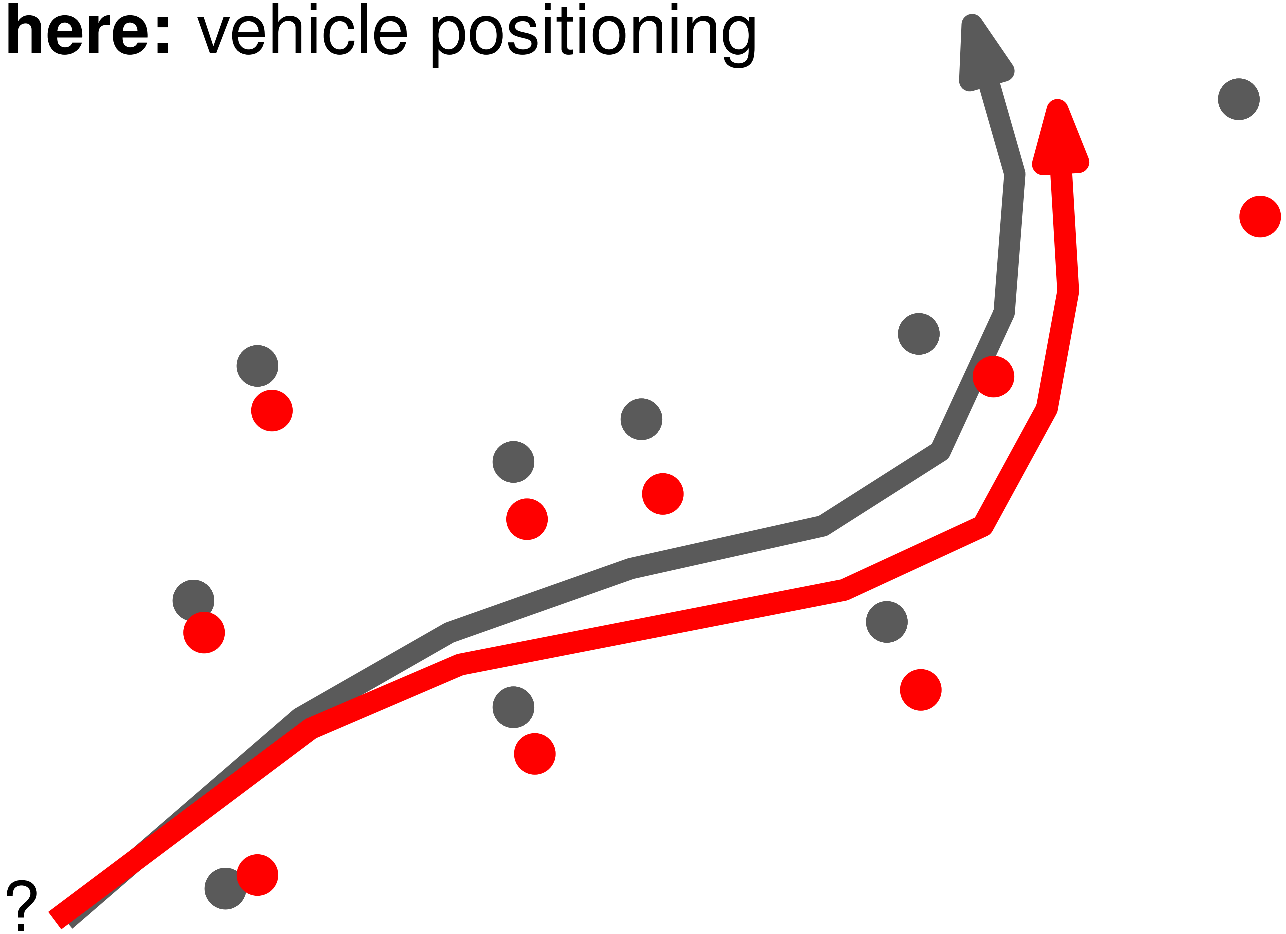

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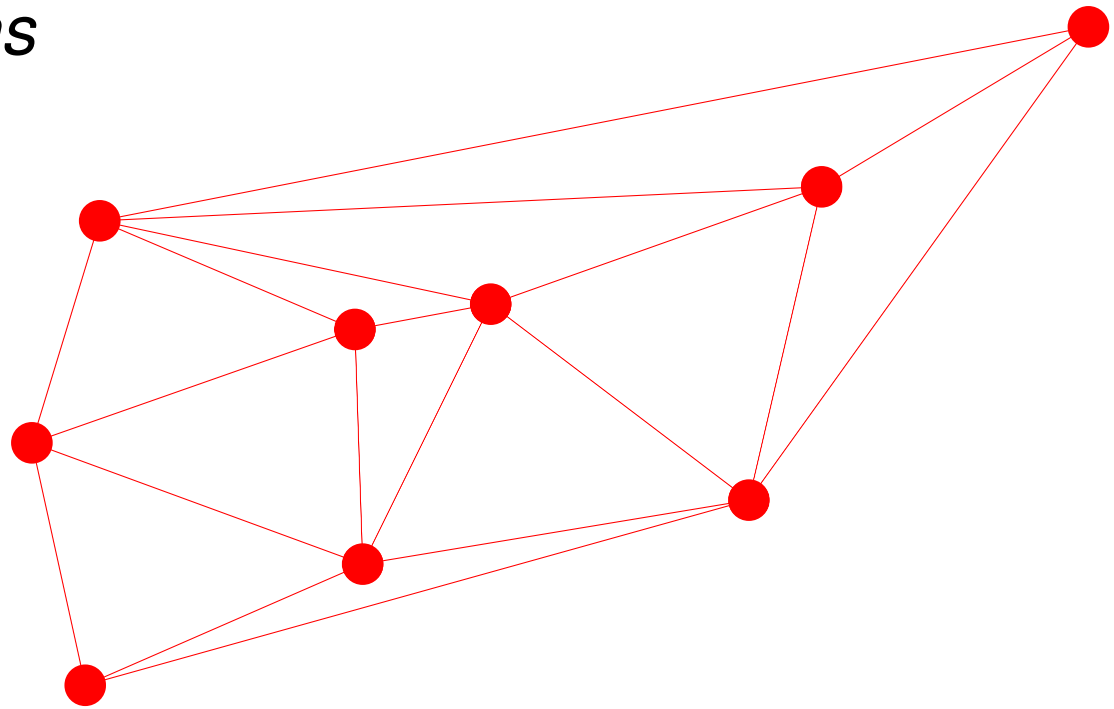
Introduction

Our approach:

- triangulate observed points
- graph matching: match triangles with triangles in a reference database based on *geometric similarity* and *neighbourhood relations*



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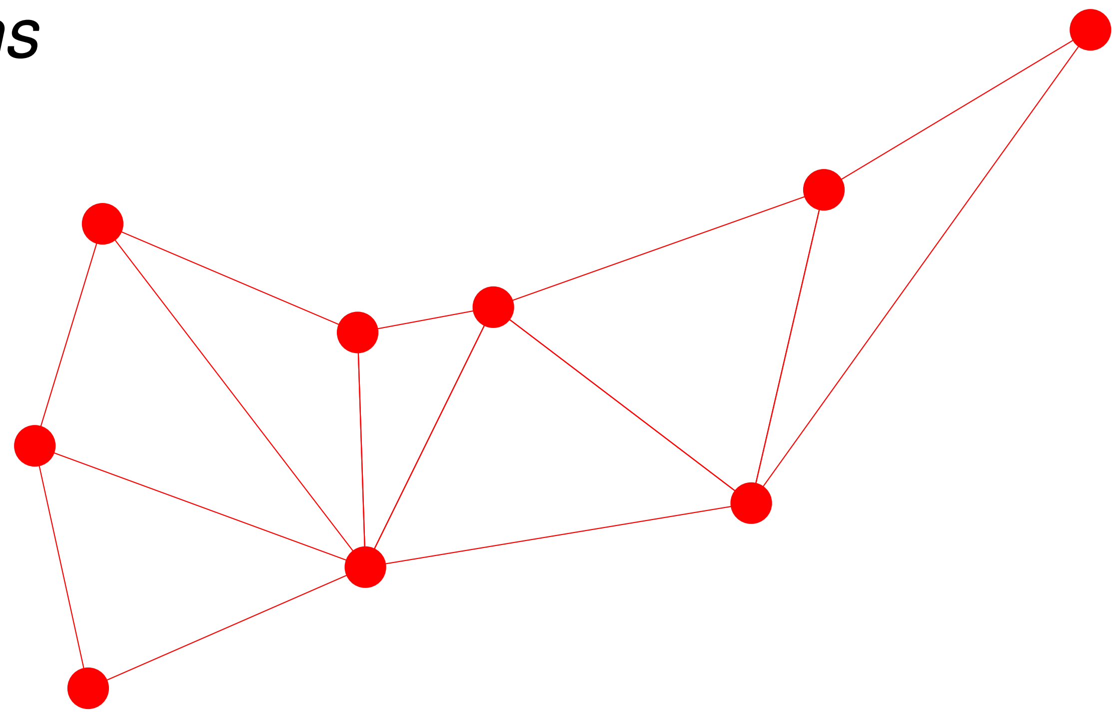
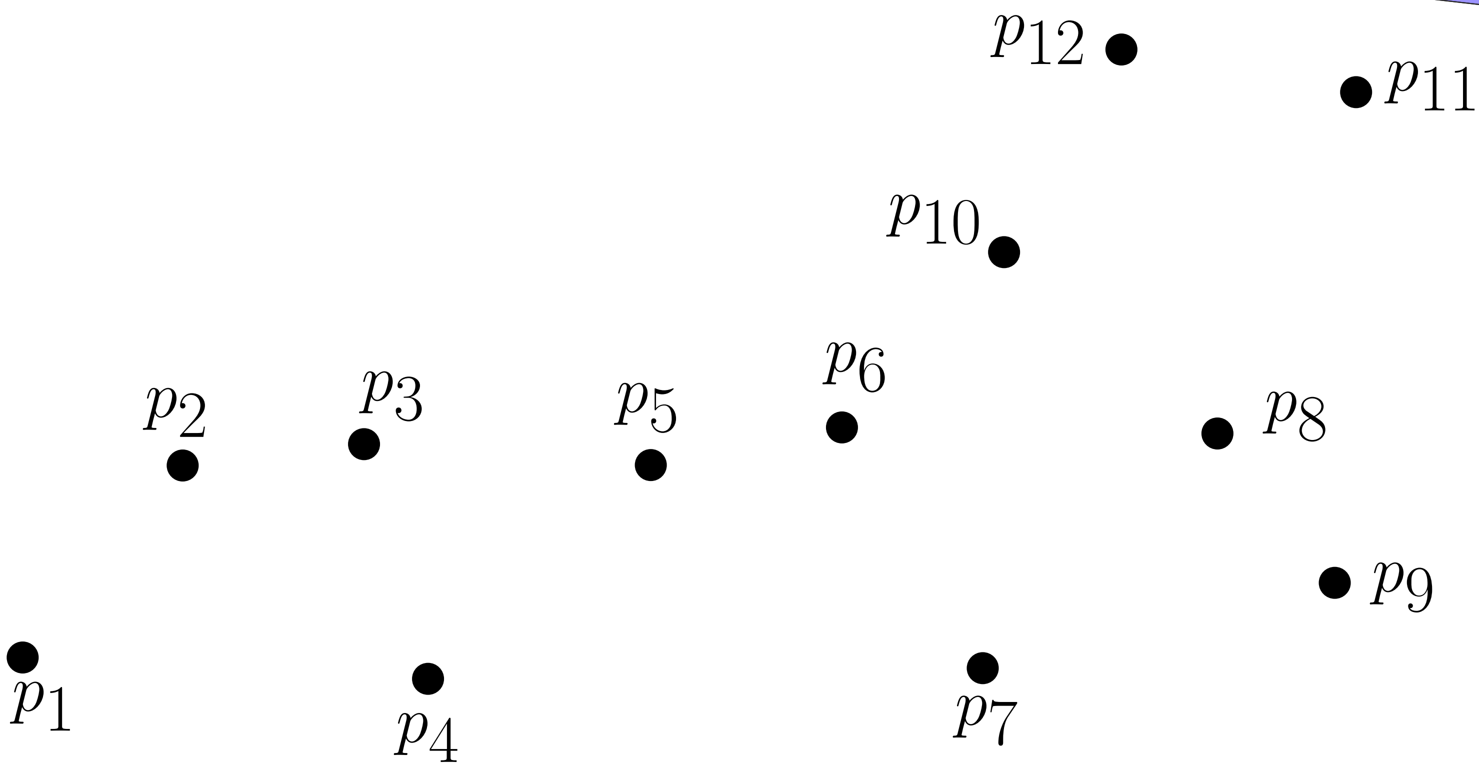


- idea to avoid NP-hard graph matching problem (e.g., subgraph isomorphism problem): only use a triangle strip

Outline

- Triangulation Algorithm
- Matching Problem
- Matching Algorithm
- Experimental Results
- Conclusion/Outlook

Triangulation Algorithm



Input:

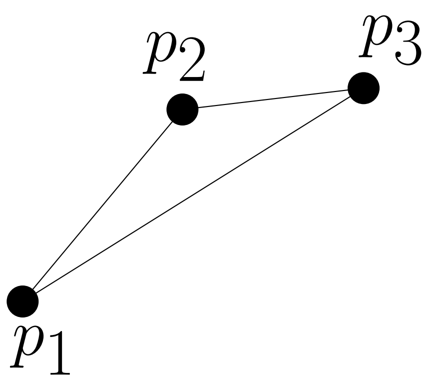
- point sequence $(p_1, p_2, \ldots, p_m)$

Output:

- triangle sequence $(t_1, t_2, \ldots, t_{m-2})$

Triangulation Algorithm

- Define first triangle as (p_1, p_2, p_3)
- For $i = 4$ to m append triangle strip by a triangle including p_i and one of the two edges that were added last.



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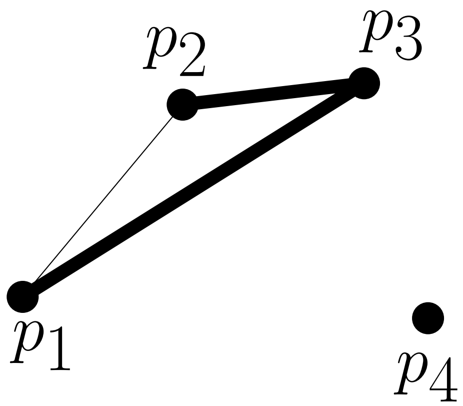

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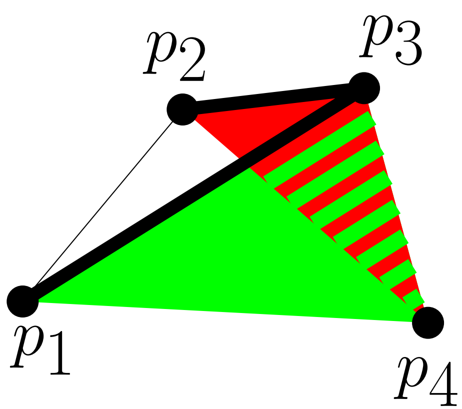

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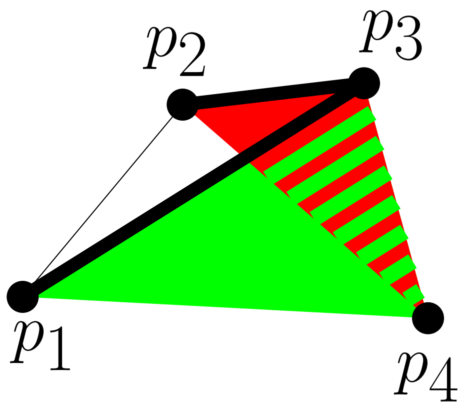

If exactly one candidate triangle overlaps the last triangle then select the other candidate triangle

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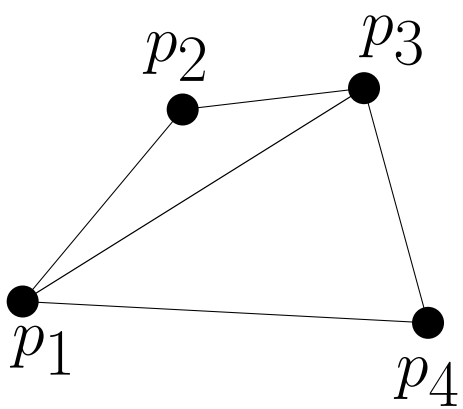

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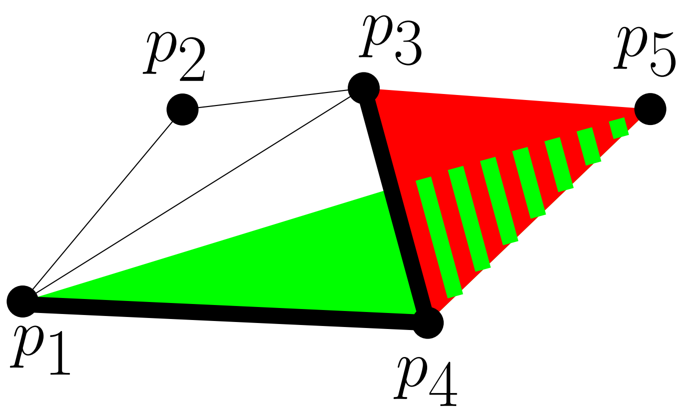

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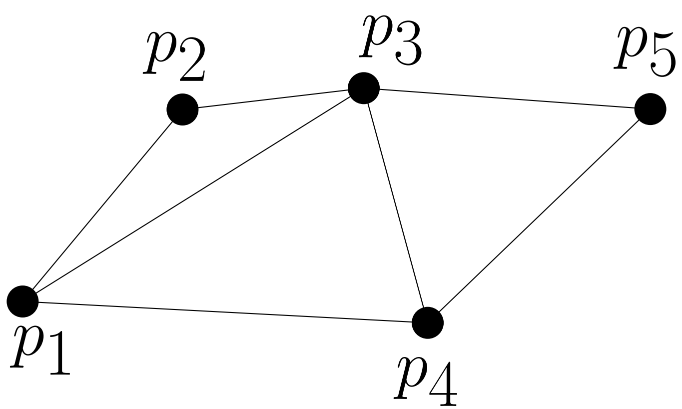

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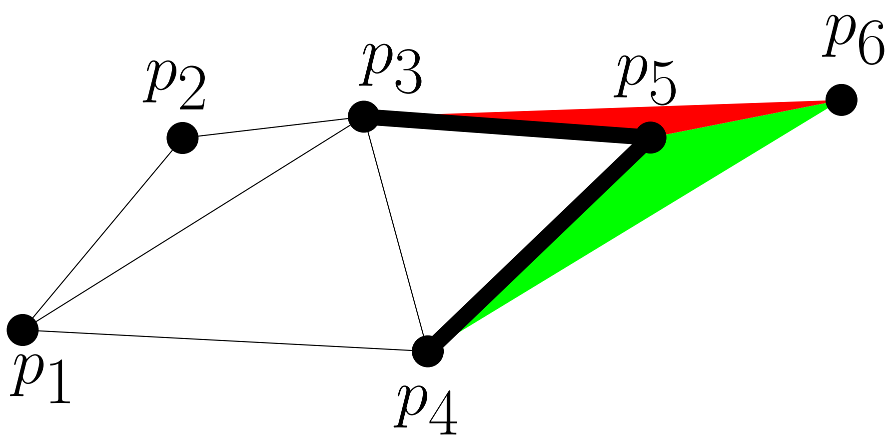

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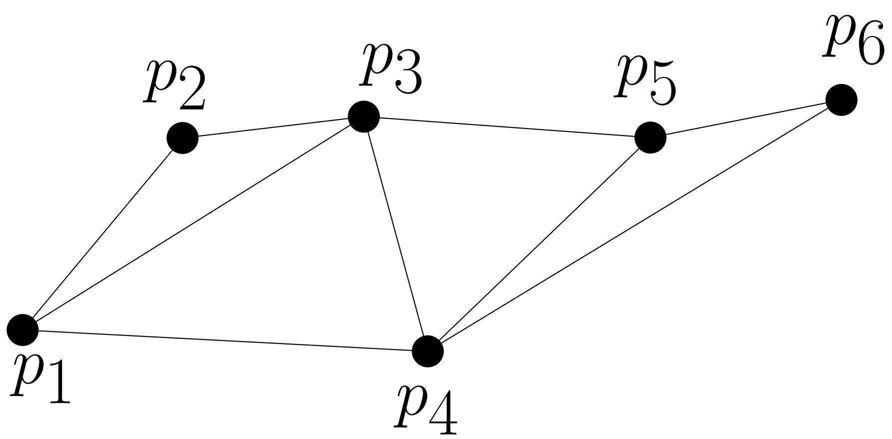

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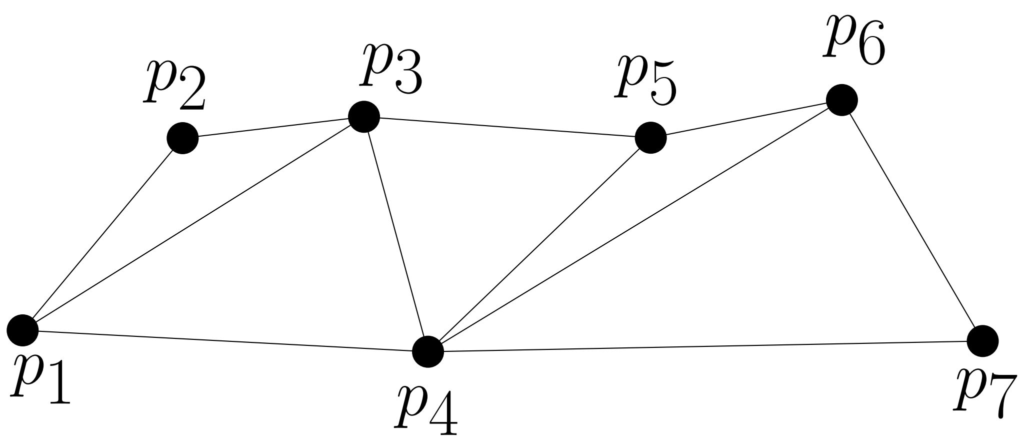

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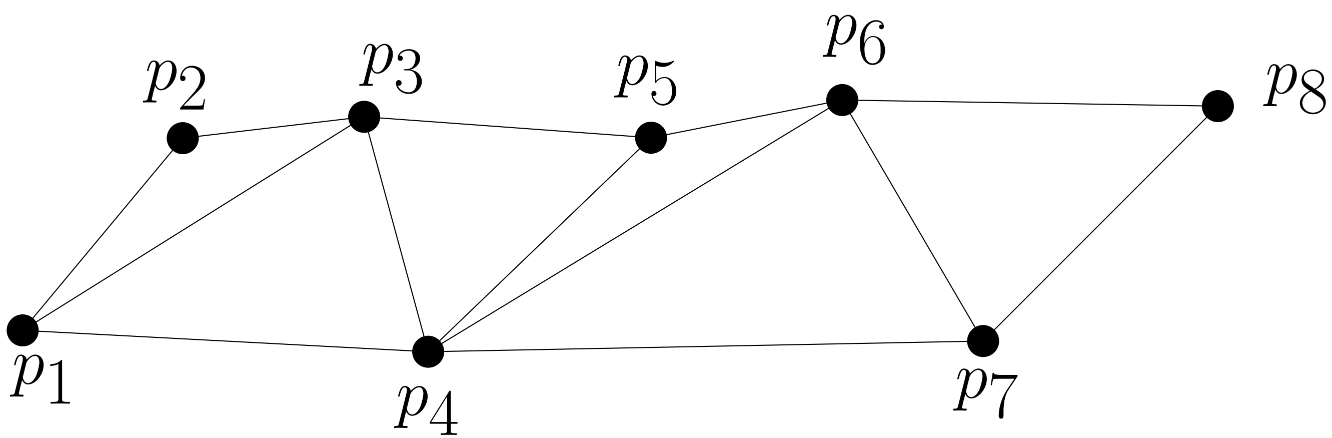

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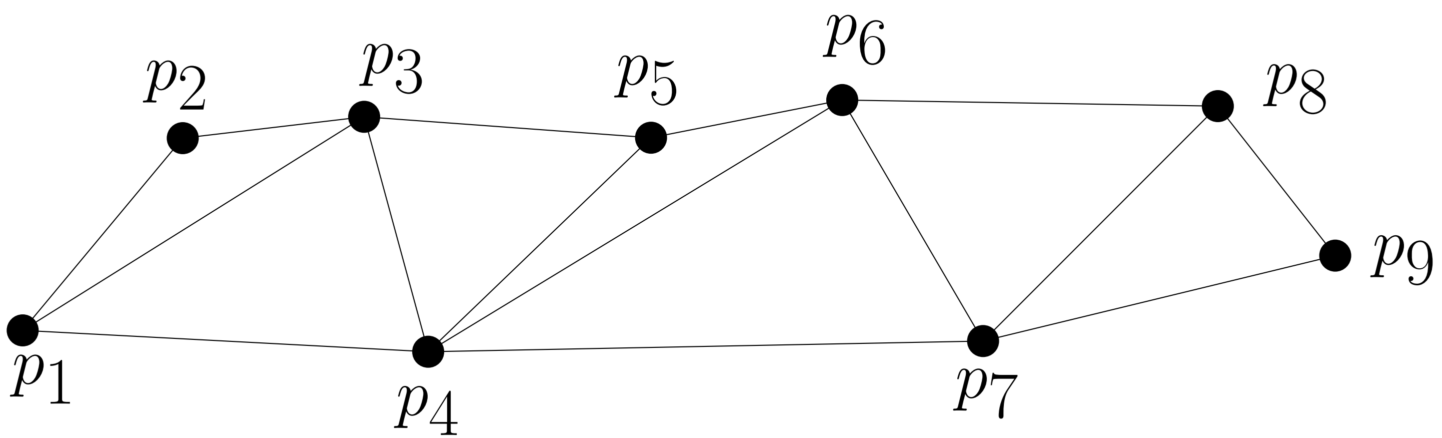

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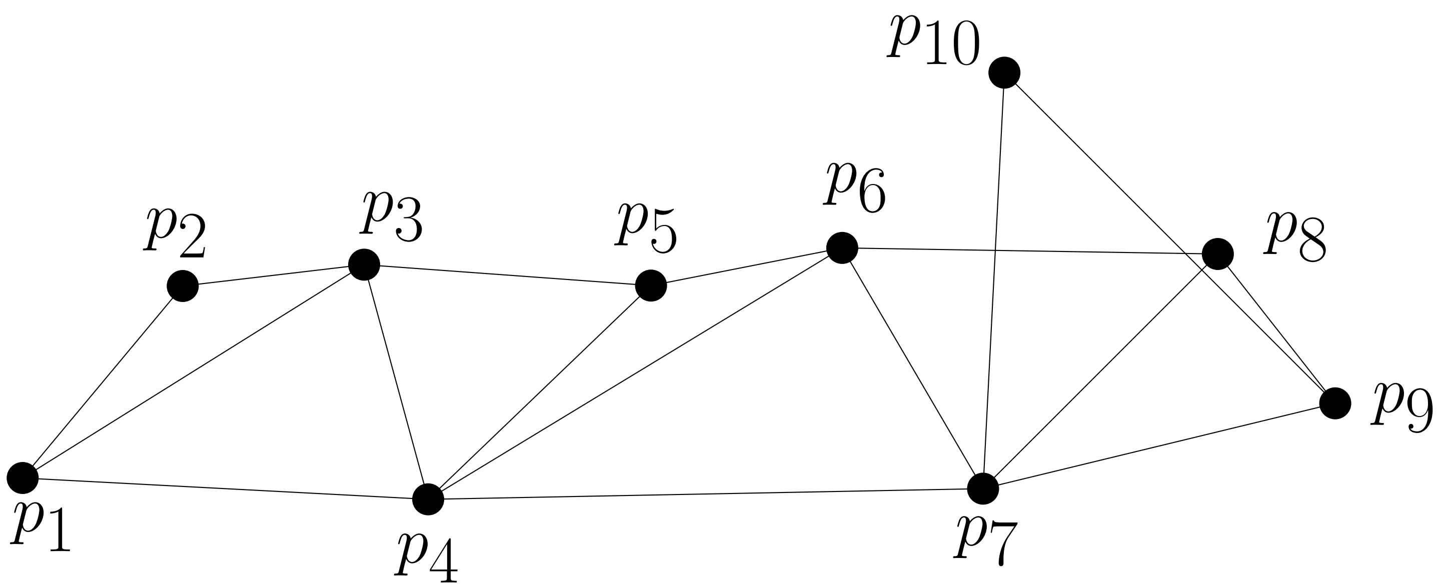

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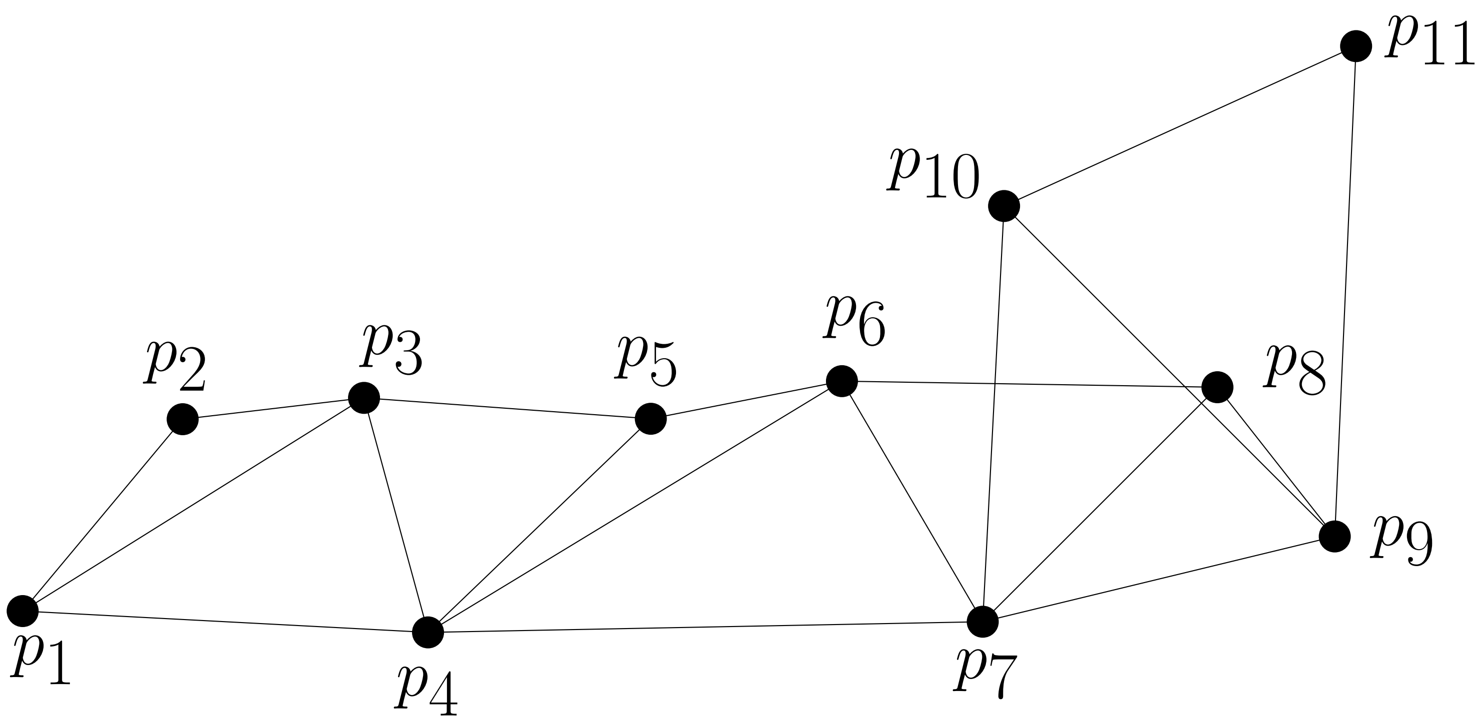

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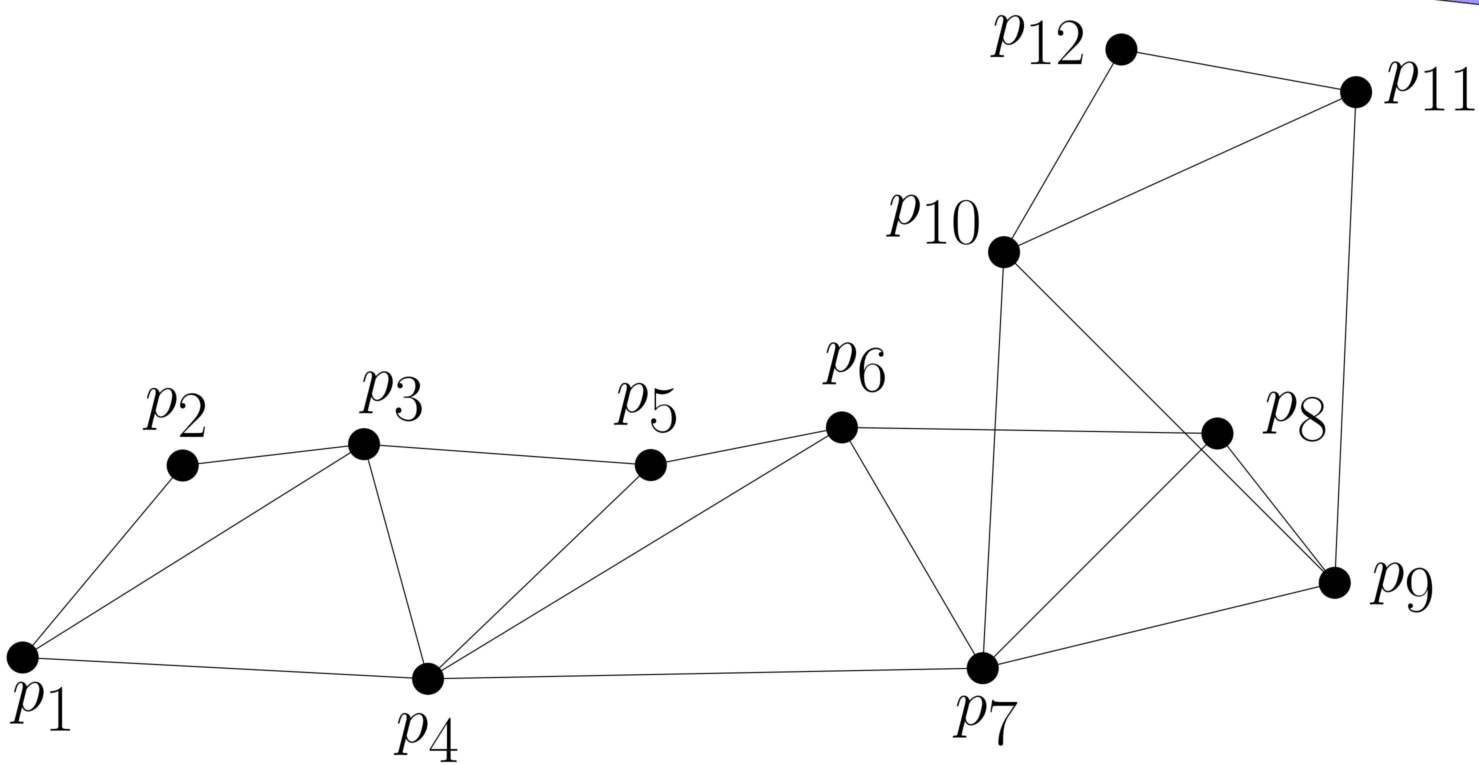

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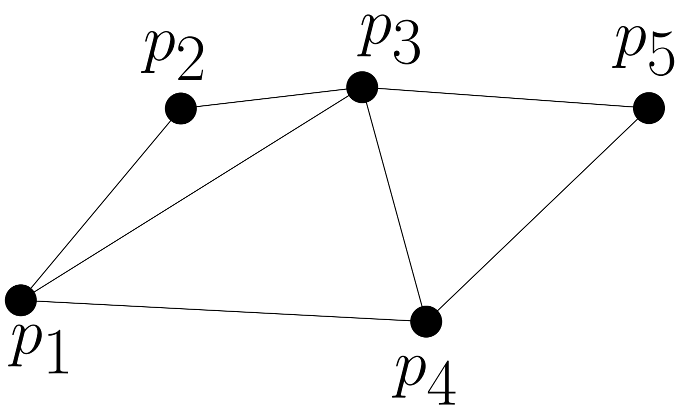
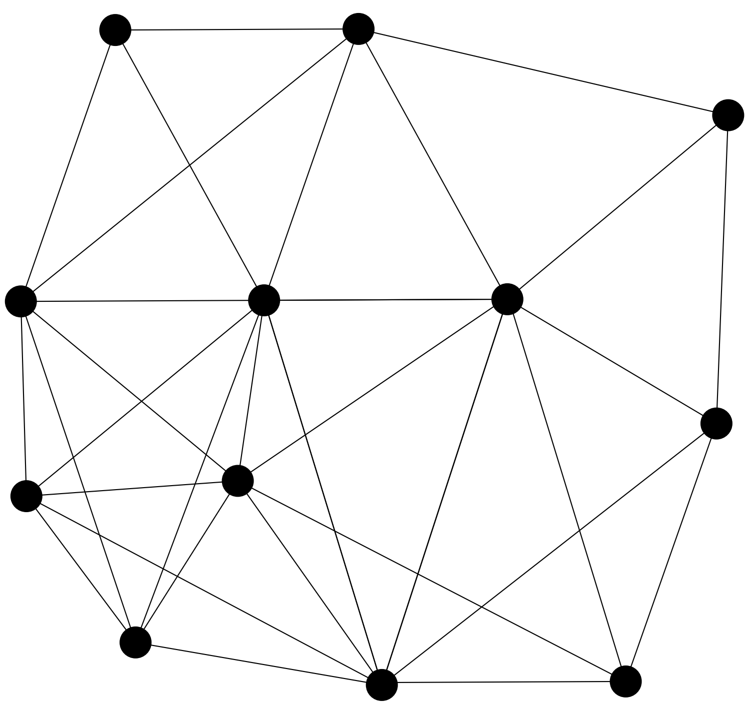
Matching Problem

observed triangles T



reference triangles T'



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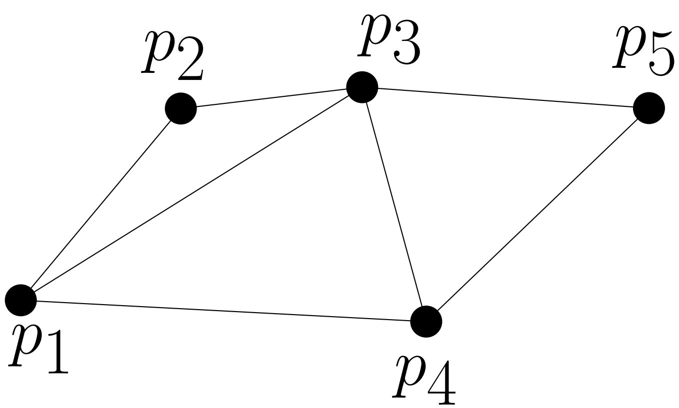
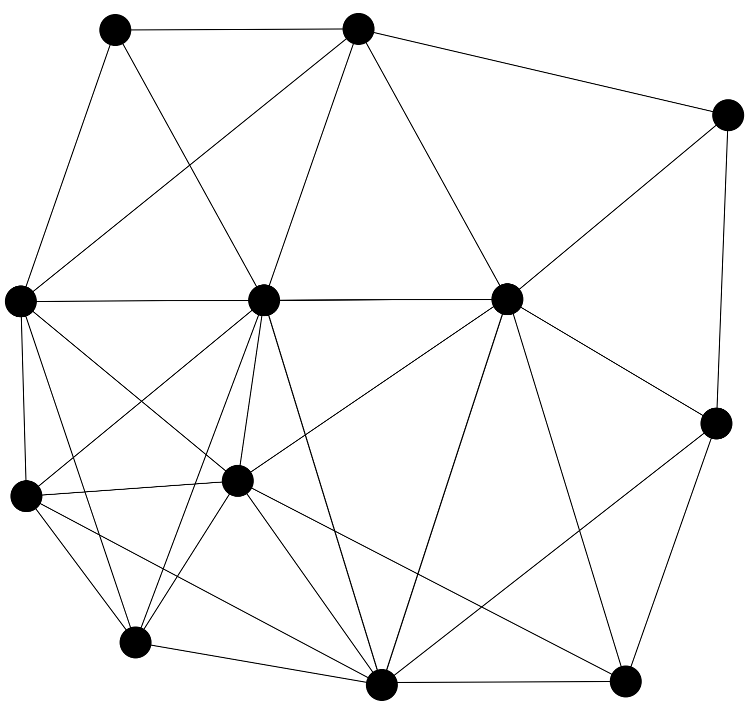


reference triangles T' $=$ all possible triangles of three reference points



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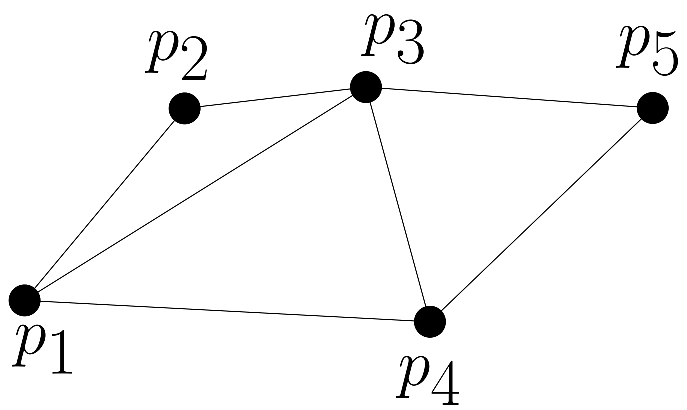

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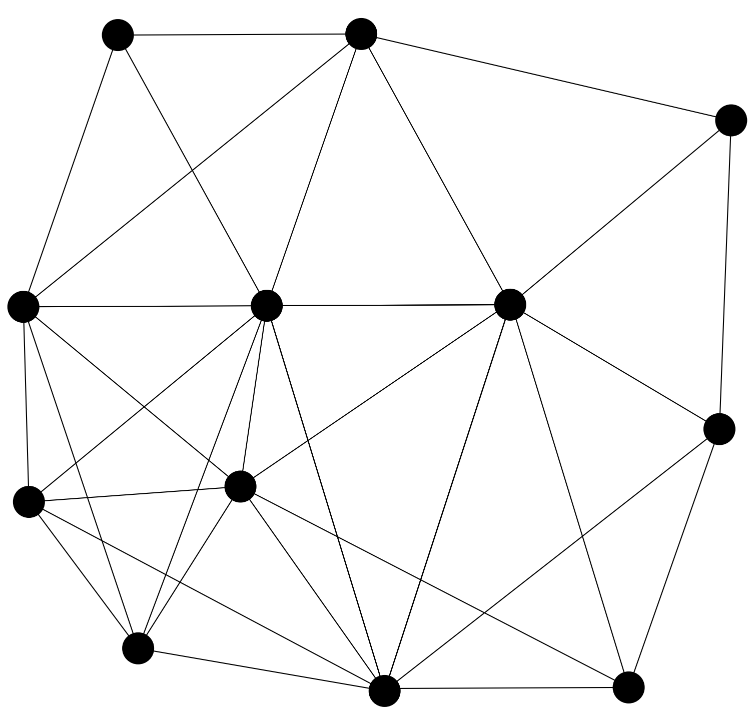
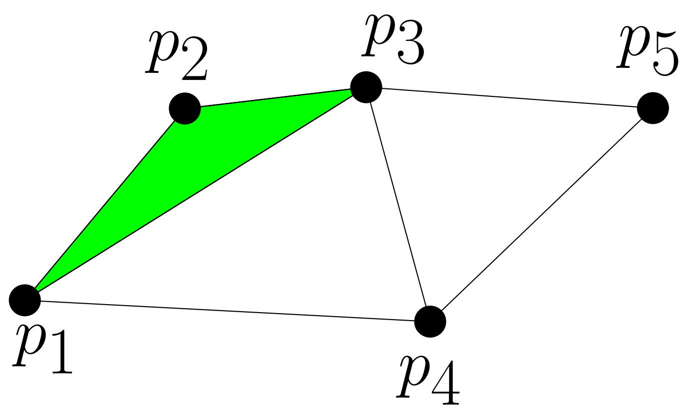


Find a set of triangle matches $\theta \in T \times T'$.

Matching Problem

observed triangles T



Find a set of triangle matches $\theta \in T \times T'$.

Constraint 1:

For each match $(t, t') \in \theta$ the triangles t and t' must be sufficiently similar.

$$\left| \text{longest side of } t - \text{ longest side of } t' \right| \leq \varepsilon$$

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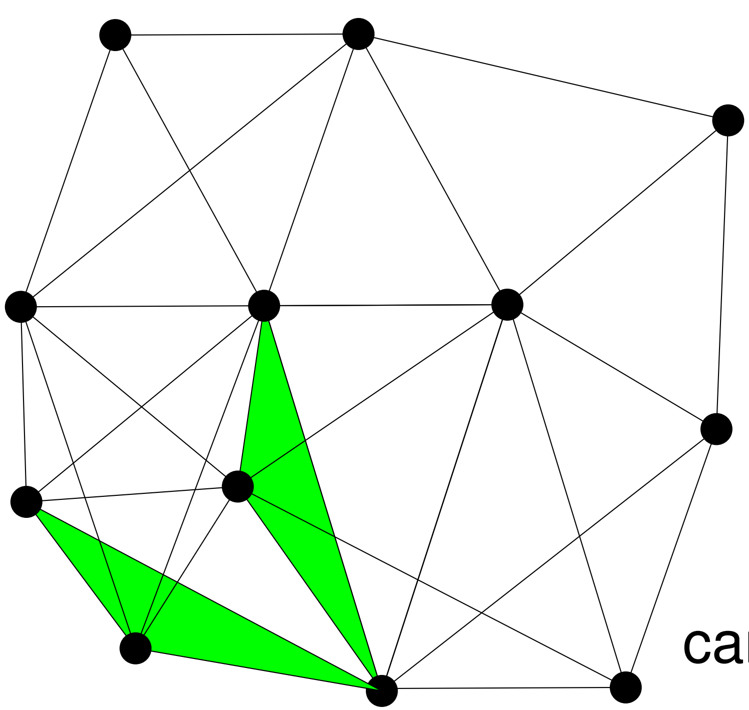

candidate matches

Matching Problem

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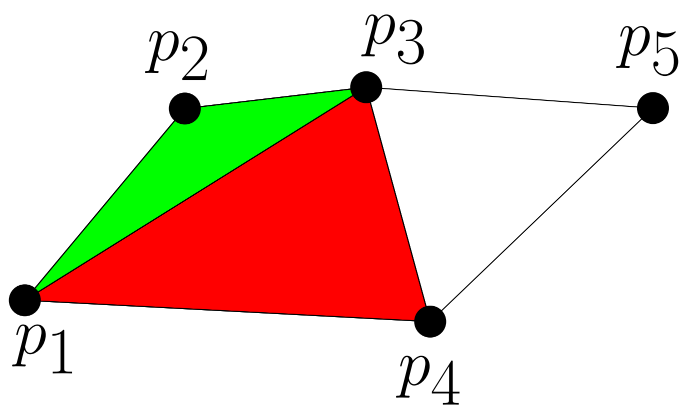

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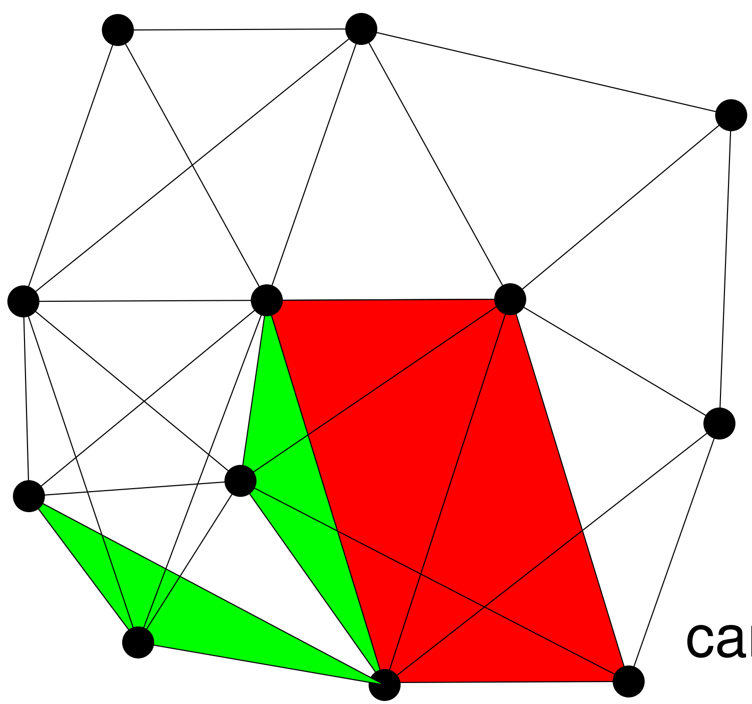

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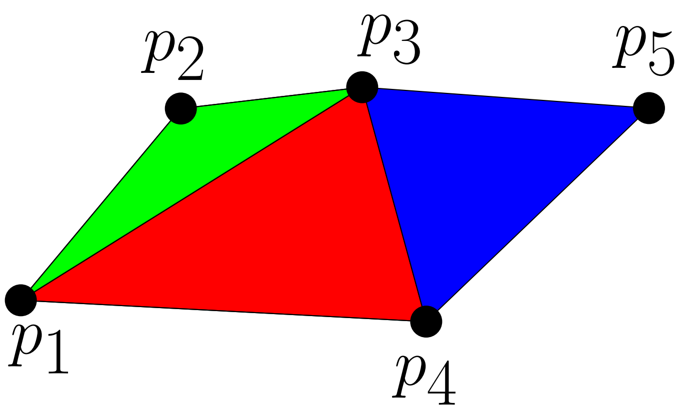

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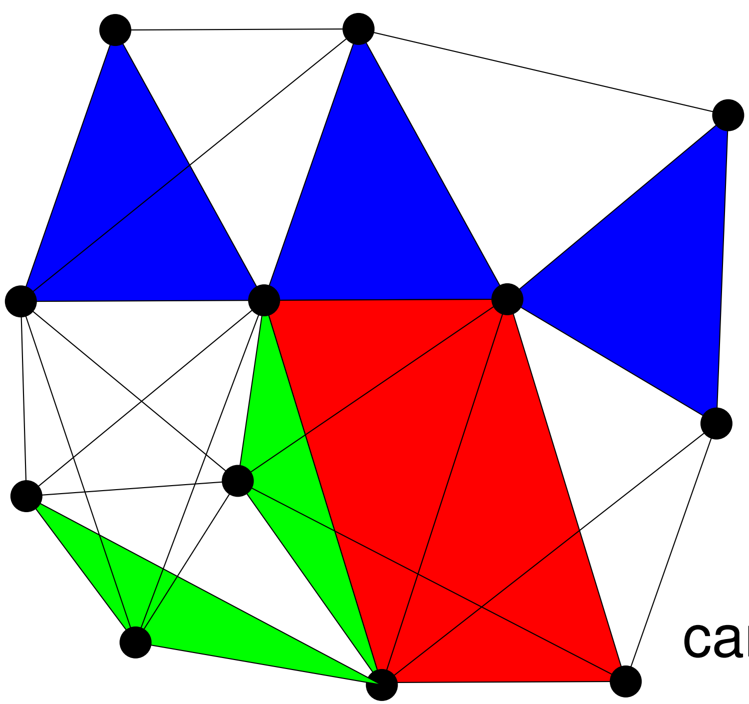

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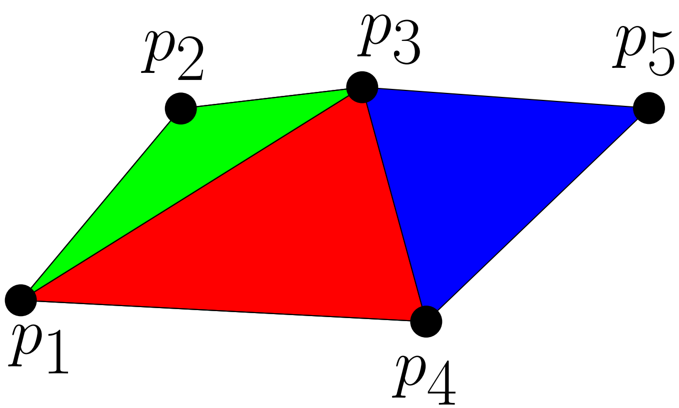

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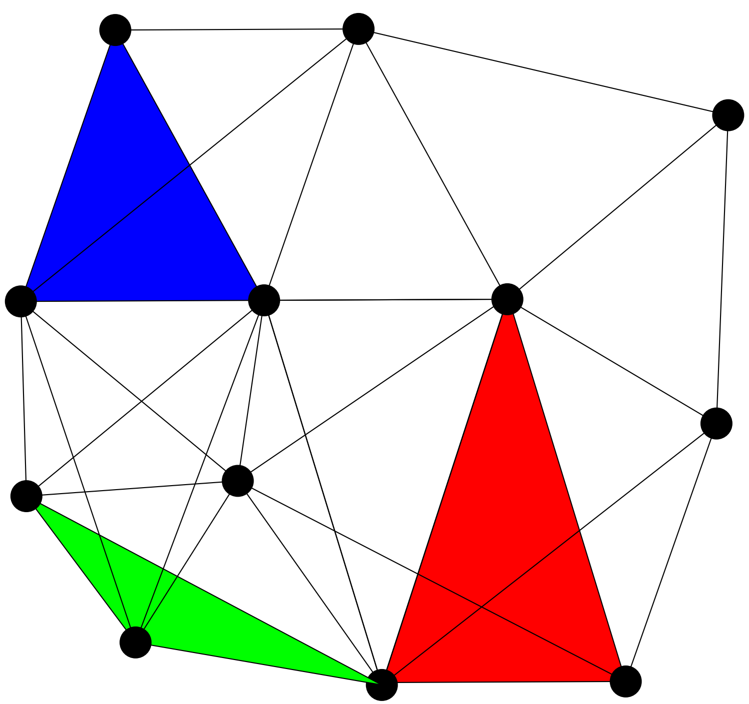
Constraint 2:
A triangle $t \in T$ must not be matched to more than one reference triangle.

reference triangles T'



Matching Problem

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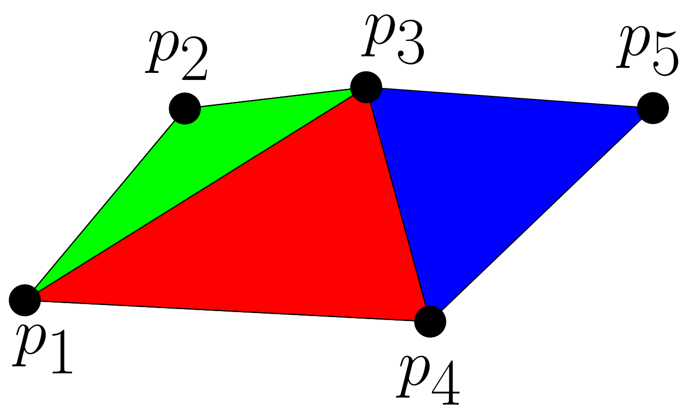
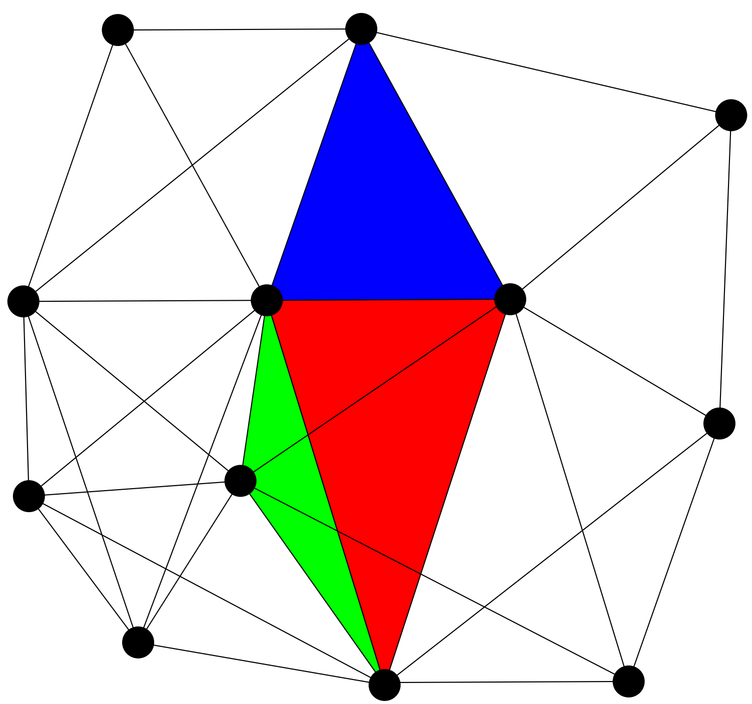


Find a set of triangle matches $\theta \in T \times T'$.

Constraint 3:
For each two matches $(a, a') \in \theta$ and $(b, b') \in \theta$ the triangles a' and b' must share an edge if a and b share an edge.

reference triangles T'



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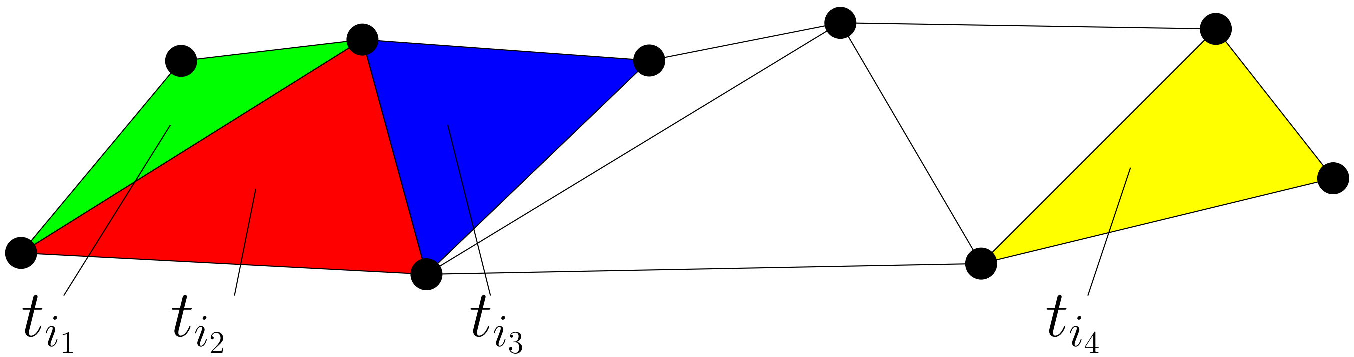

triangles in T that are matched

Some triangles in T cannot be matched, therefore:

- maximize $|\theta|$ (= number of matches)
- among solutions maximizing $|\theta|$ maximize quality of matches
- additional constraints to ensure that solutions for different components "fit together"

Matching Problem

Constraint 4:

t_{i_j} and $t_{i_{j+1}}$ must not be matched to the same reference triangle.

Constraint 5:

If t_{i_j} and $t_{i_{j+1}}$ do not share an edge then
the matched reference triangles must not share an edge.

Constraint 6:

If t_{i_j} and $t_{i_{j+1}}$ do not share an edge then
the distances between t_{i_j} and $t_{i_{j+1}}$ must be sufficiently similar to
the distances between the matched reference triangles.

Matching Algorithm

Offline:

- build an index (a three-dimensional kd-tree) that references each triangle in T' by its side lengths

Online:

- triangulate observed point set $\to T$
- set up directed acyclic graph G_{match} based on T and T'
- search path of maximum weight in $G_{\mathrm{match}} \to \theta$

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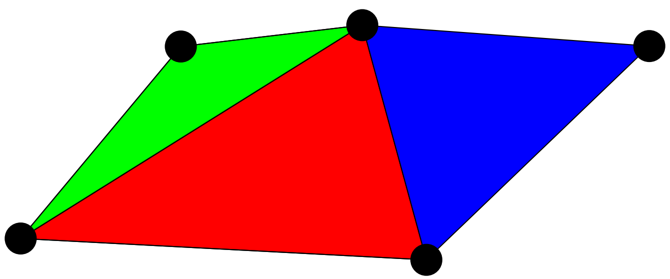
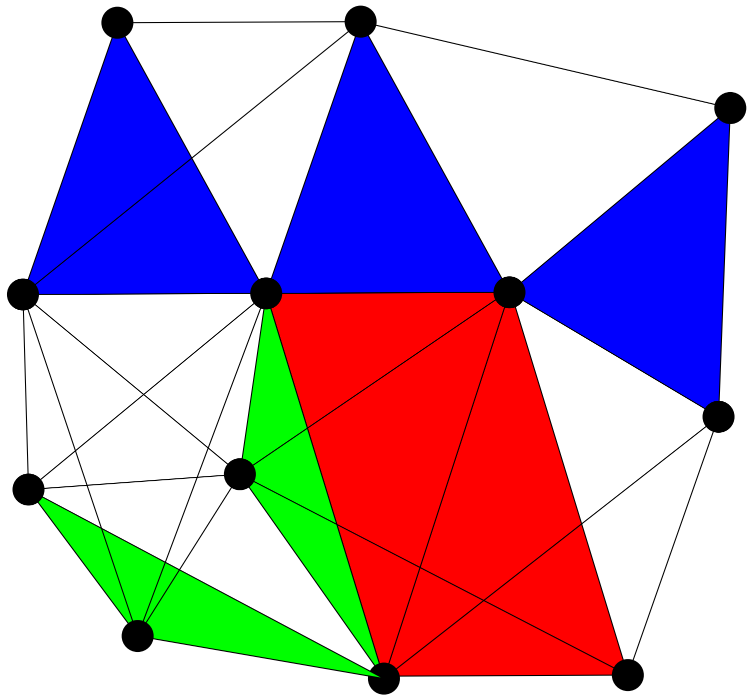
- triangulate observed point set $\rightarrow T$
- set up directed acyclic graph G_{match} based on T and T'
- search path of maximum weight in $G_{\text{match}} \rightarrow \theta$

Matching Algorithm

Set up directed acyclic graph $G_{\mathrm{match}}(V_{\mathrm{match}}, A_{\mathrm{match}})$:

- V_{match} contains a node for each candidate match
- V_{match} can be found by applying range queries to kd-tree (one query for each triangle in T)





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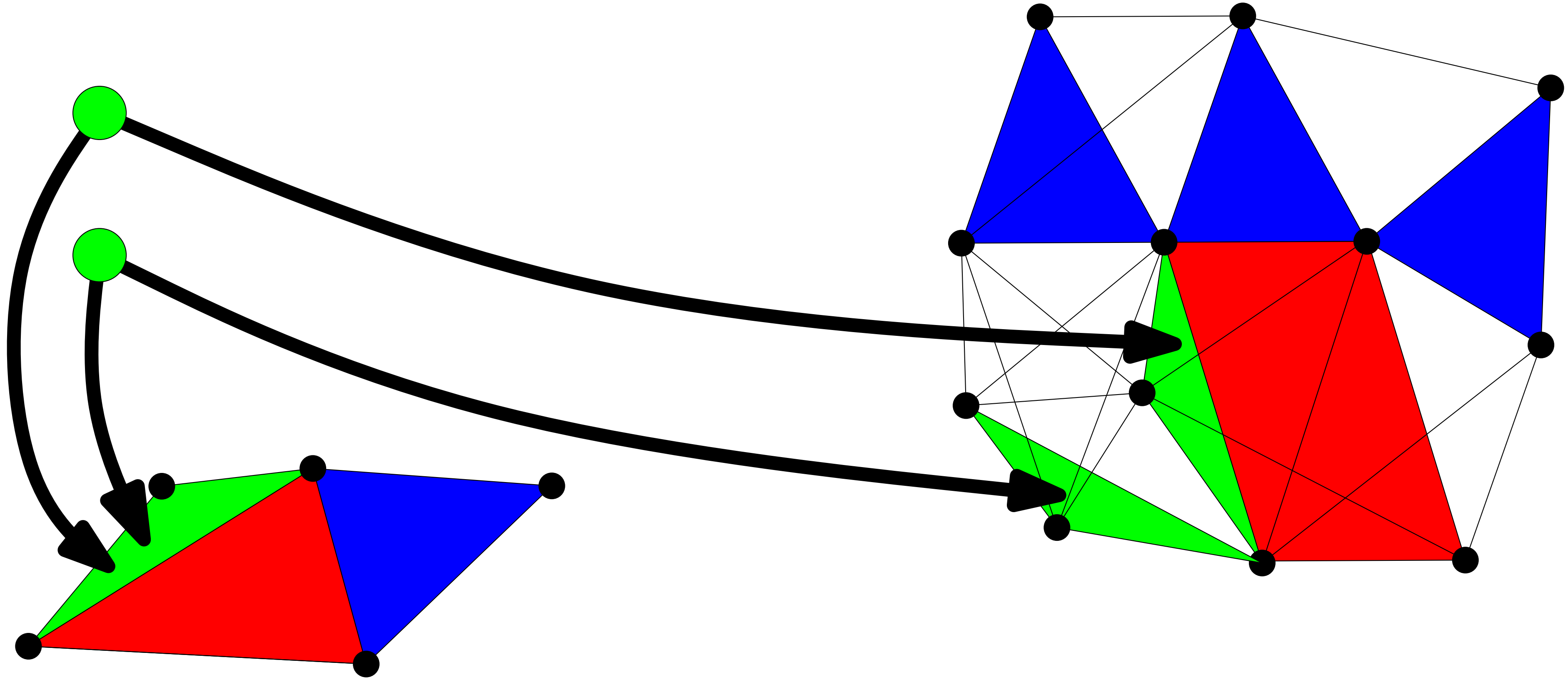

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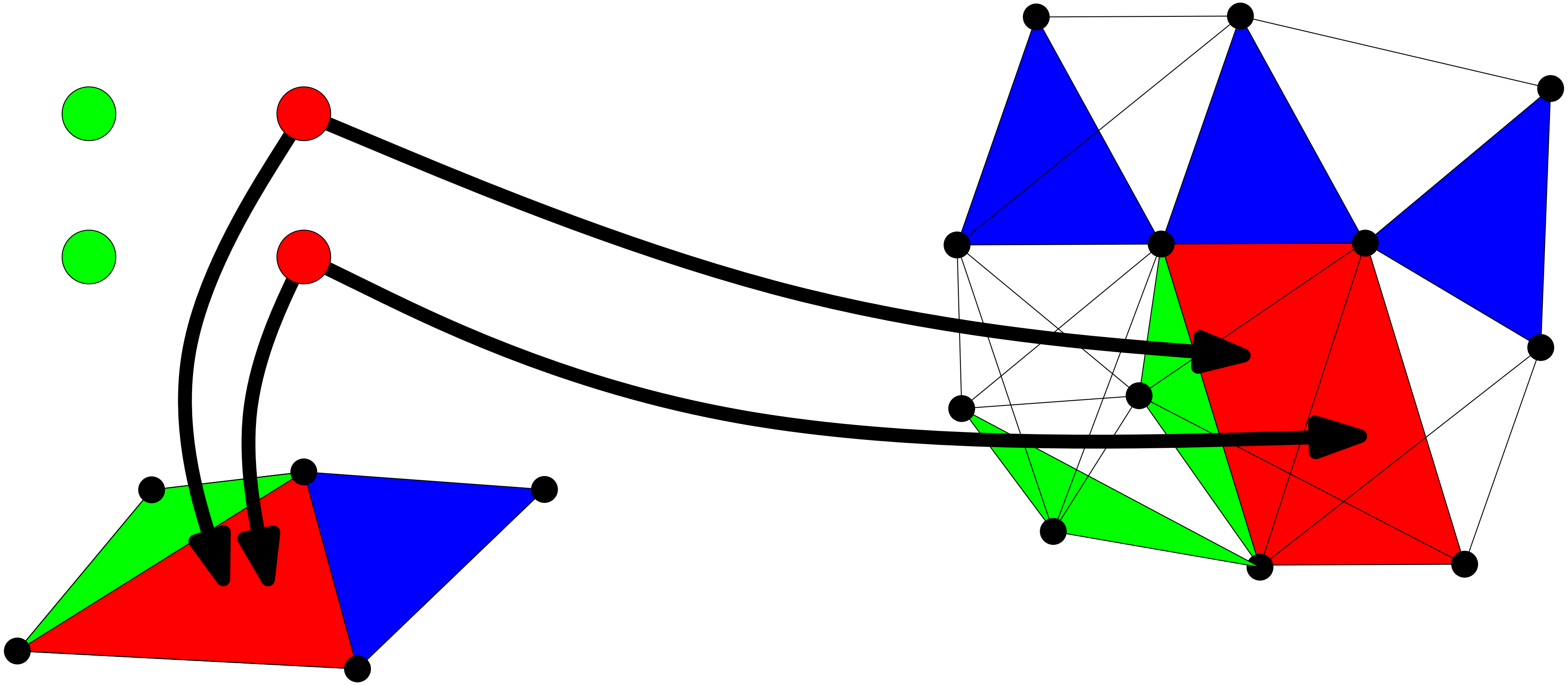

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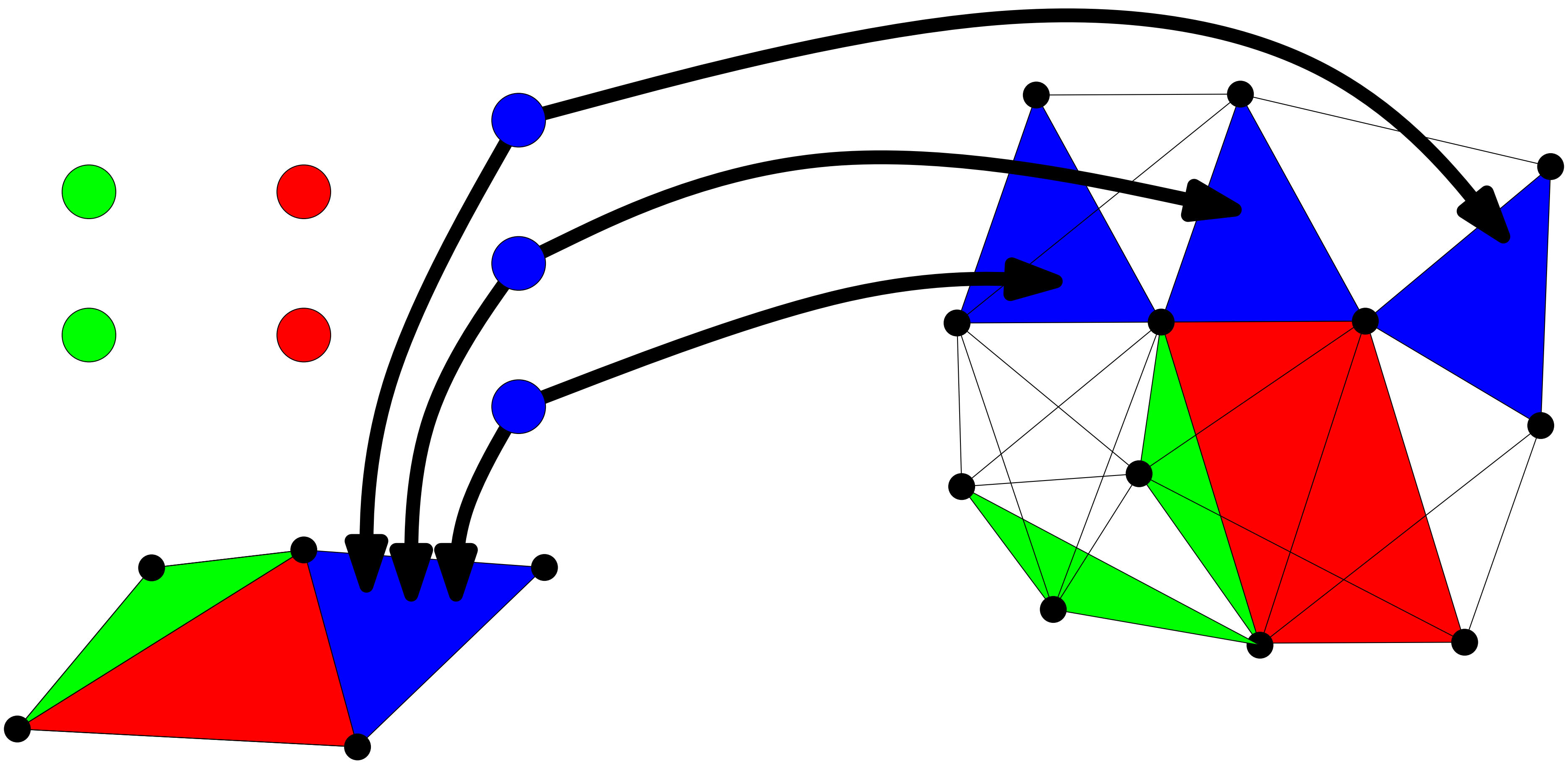

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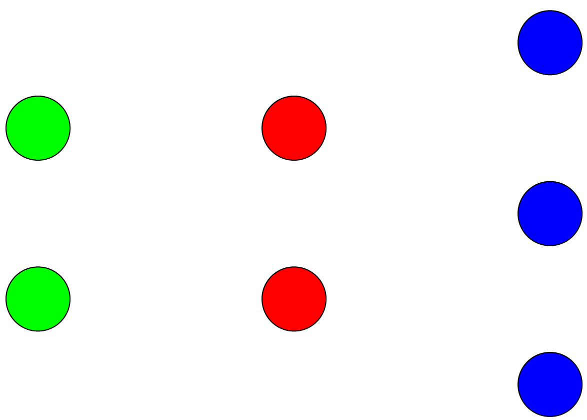

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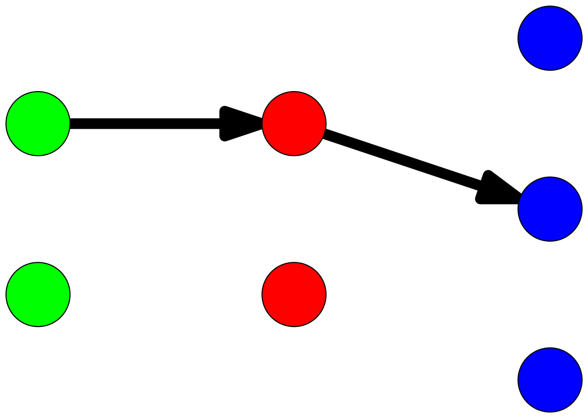

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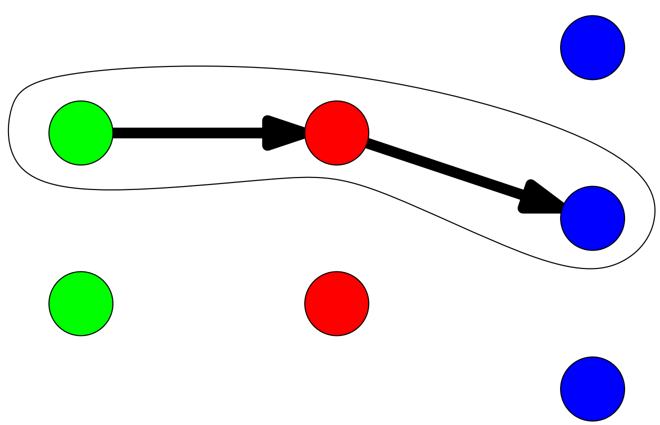


Search path of maximum weight in G_{match}:

- solution by dynamic programming in $\mathcal{O}(|V_{\mathrm{match}}| + |A_{\mathrm{match}}|)$ time

Experimental Results

Streetmapper system:

- 4 laser scanners
- GPS
- odometer
- IMU
- used to create reference point set

Experimental Results

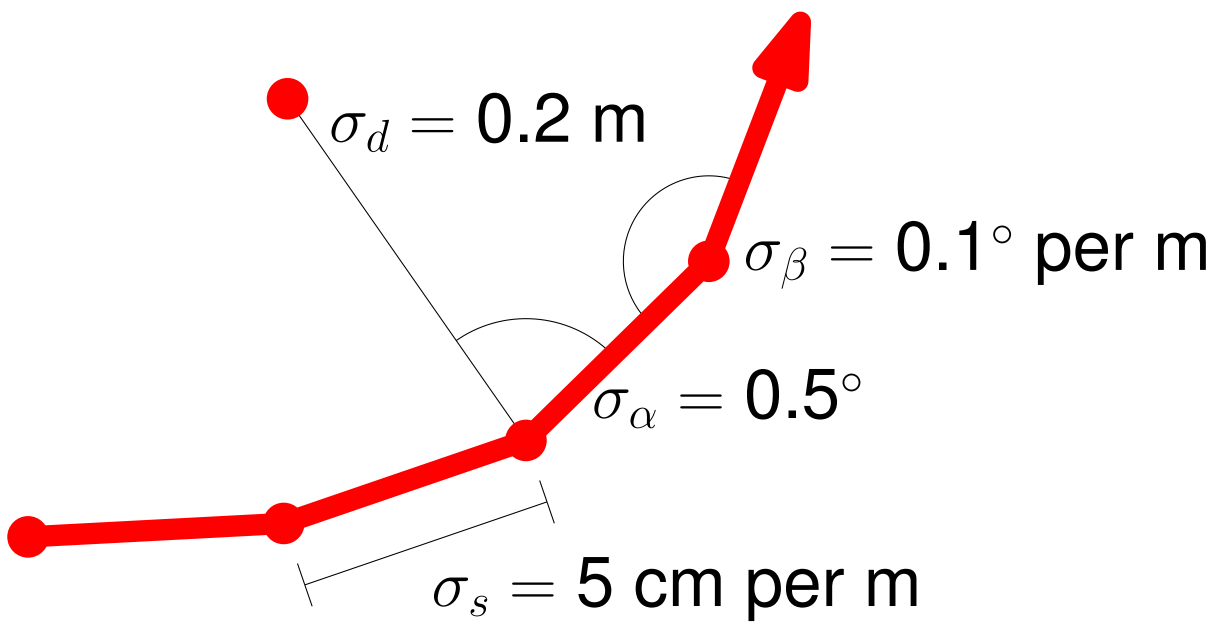
Reference dataset:

- 22 km track in Hannover, Germany
- 2658 reference points
- 643247 reference triangles

Experimental Results

Test samples matched with reference set:

- 88 sub-tracks of the whole track
- noise added



Experimental Results

Experiments with different error tolerances:

ε	0.25 m	0.50 m	0.75 m	1.00 m

All experiments run on Windows PC with 64 bits, 8 GB RAM, 2.93 GHz CPU

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unmatched triangles	91.5%	55.6%	12.7%	3.5%
correctly matched triangles	7.8%	44.1%	86.9%	96.0%
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very high success rate in reasonable time

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Conclusion

- new deterministic and efficient method for point pattern matching
- robust against different errors, e.g., trajectory deformation
- geometric configurations of observed landmarks are unique, i.e., they allow us to unambiguously determine our location

Outlook

- tests with low-cost sensors
- consider more objects than poles, i.e., other point features, planes, road markings

Outlook