Using the Straight Skeleton for Generalisation in a Multiple Representation Environment

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1 INTRODUCTION

In recent time a lot of research has been done towards the comprehensive administration of geographic data of different scales and thematic domains. A promising approach is the use of a Multiple Representation Database (MRDB) in which links between corresponding objects are explicitly modelled. The advantage of this approach is a limitation of redundancies and inconsistencies. It can be expected that this comprehensive administration of different data sets supports complex analysis and eases the updating process (Sester et al. 1998).
Our aim is to build up a framework enabling the automatic update of features in an MRDB. We envision a system which is able to propagate information into the other representations in the database when a feature is added to an arbitrary level. To meet this demand algorithms are needed which are able to perform the generalisation of the added data considering the embedding context.
The algorithms needed for generalisation depend on the relationships between objects in adjacent scale levels. The main relation is the aggregation, however, there are also more complex ones. Two of these cases are area collapses and geometry type changes. This also includes partial geometry type changes.
The paper is organized as follows: after a review of related work, the relationships between objects in different scales are briefly analyzed in section 2. The main focus of the paper lies, however, in the presentation of a skeleton algorithm that can be applied to solve this task for the generalisation cases area collapse and geometry type changes from area to line. In this work the Straight Skeleton is used. The algorithm and its application to generalisation is presented in section 3. In section 4 first ideas are presented how this generalisation method can be used for the propagation of updates in MRDB. A summary concludes the paper.

1.1 RELATED WORK

The geometry type change from area to line is a classical problem in automatic map generalisation. Apart from early works, which typically demanded a specific arrangement of polygon points, different alternatives of a similar skeleton have become dominating.
Besides the Straight Skeleton, there are different forms of skeletons, e.g. the Medial Axis (Chin et al., 1995) and the Chordal Axis (Prasad, 1997), which will be compared in section 3. Skeletons are widely used for geometry type change, collapse and plastic merges of disjoint polygons (Bundy et al., 1995). Within map generalisation they are also applied to line simplification and smoothing (Gold & Thibault, 2001), (Christensen, 2000).
Skeletons have been used by Ai & van Oosterom (2002) to extend the model of the Generalized Area Partitioning tree (GAP-tree). This data model is used for 'on-the-fly' map generalisation (van Oosterom, 1995). In its original form the GAP-tree represents a hierarchical subdivision of areas, allowing only 1:n relationships. With the skeleton based extension, it is possible to create also more complex relationships. Applying a skeleton for the propagation of features in an MRDB can be seen as a similar approach.
Cecconi (2003) uses a skeleton for the generalisation of lakes for Web Mapping. In his work it is applied to Multi-Scale Databases.
Strategies for the automatic propagation of updates in a multiple representation environment or 'incremental generalisation' were introduced by Kilpeläinen & Sarjakoski (1995). Harrie & Hellström (1999) describe a prototype system which fulfils a variant of this approach.

2 RELATIONSHIPS OF GEOGRAPHIC OBJECTS INVOLVED IN GEOMETRY TYPE CHANGES

For the development of strategies for the update of a multiple representation database it is valuable to examine the nature of the relationships between corresponding objects in different database levels. We concentrate here on the aggregation and some more complex cases. A detailed compilation of transformations that can be performed on area features is shown in (Galanda, 2003).
Going from large to small scale, one would expect that only n:1-relationships dominate, as objects are generally aggregated. However, mainly due to different modelling issues and assumptions, also all other possible relationships occur, namely 1:n, and the general n:m-relation.
The different cardinalities of the relations will be illustrated using the example of identifying the relation between two data sets in Germany: the digital cadastral map (ALK, approx. 1:500) and the large scale topographic data set ATKIS (approx. 1:25.000).
The common relationship between corresponding geographic objects with a different scale is the aggregation, which means that objects are represented by multiple objects when zooming in. Typical is the aggregation of adjacent parcels to areas of the same use. An example which addresses this case can be found in (Sester et al. 1998): Here the mentioned datasets were linked.
The ALK is fragmented into parcels which constitute the smallest administrative areas. In contrast, the areas in ATKIS are defined by land use and limited in their extension to the areas enclosed by roads. Normally the boundaries of the areas in ATKIS are coincident with the boundaries in the ALK enclosing a set of parcels. Actually, the rule for the generation of ATKIS areas is to follow the borders of the parcels in the ALK.
However, this does not apply generally. The most important exception are parcels with the use “road”. While these objects (as all parcels) are represented as areas in the ALK, the roads in ATKIS are represented as lines. Therefore, road objects in the smaller scale are not a geometrical union of polygons. This applies also to the adjacent areas, which have to grow beyond the border of the parcels to fill the gaps left by the collapsed roads.
Figure 1 shows the representation of the correspondences between ALK and ATKIS as links in the simplest case. It is worth to mention, that the relationships expressed by links can have the cardinality n:1 even though the purely geometrical overlay would result in more complex relationships. This results from the fact that links are only established between objects with similar semantics.

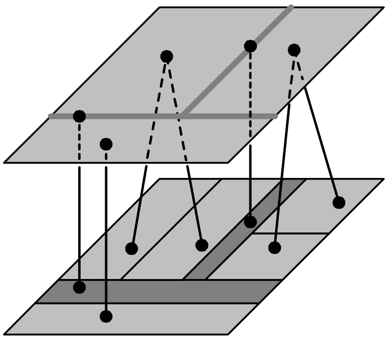
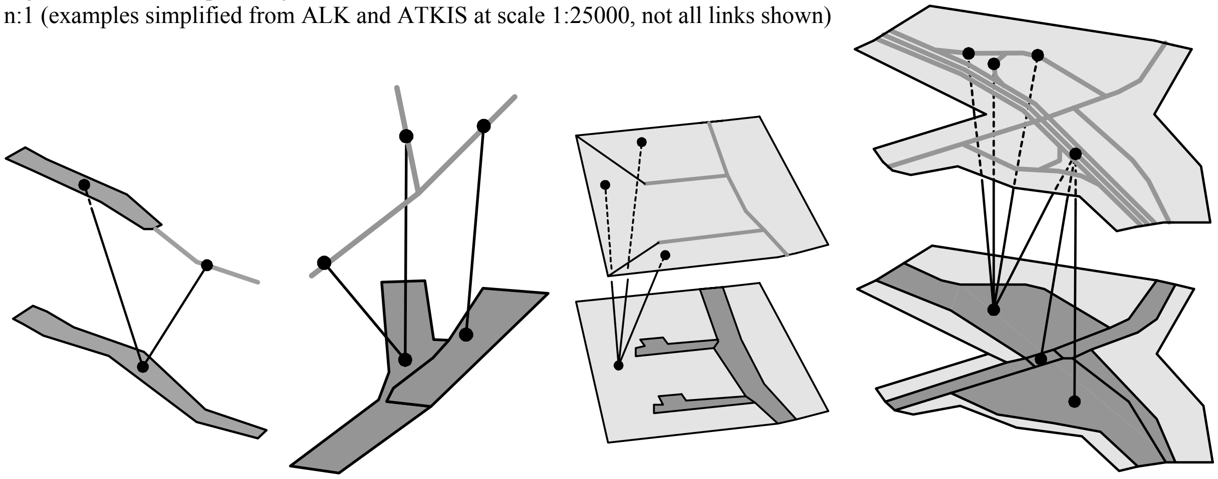


Figure 1. Representation of correspondences by links. Relationships with cardinality n:1.

However, relationships have in general the cardinality n:m. This is due to different reasons.
One reason is that sometimes the geometry type of a part of an object needs to be changed while other parts are not modified. Therefore the object needs to be split into multiple objects even though the scale becomes lower. An example for this are rivers. In ATKIS rivers up to a certain width are represented as lines (12m at the scale 1:25.000). In contrast they are represented as areas when their width goes beyond this threshold (see figure 2a).
Secondly, boundaries between areas are sometimes rather arbitrary than in accordance with a well defined rule, or the rules in the different representations are not the same. For instance for the fragmentation of roads into parcels in the ALK only administrative borders are relevant. Therefore, different roads leading to a junction may fall into the same parcel, while they are separately represented in ATKIS, which mainly reflects the function of a road (see figure 2b). This can also lead to general n:m-relationships. Also borders between ATKIS objects are sometimes formed arbitrarily: Because a build-up area is not supposed to include roads, it is sometimes split into two objects when a dead-end road leads to its interior (see figure 2c).
Another reason for differences to the common aggregation is that the linked data sets do not only carry information of different scales, but also serve different purposes and hence focus on different topics. This means that sometimes the data set with the lower resolution carries more information about a certain topic, which can result in a relationship with the cardinality n:m. Since the ALK displays information about administrative borders and ATKIS focuses on topographic objects many of these cases can be found. While in ATKIS highway ramps are represented as separate road elements, they are included in the same parcel as the highway itself and hence do not have their own representation in the ALK (see figure 2d). Also areas enclosed by ramps and the highway do have their own representation in ATKIS, but do not in the ALK.

Research on the relationship of spatial data sets which have slightly different contents has been done by Walter & Fritsch (1999).
These examples show that the relationships of corresponding geographic objects are often very complex. The number of these complex cases decreases when the different datasets simply show objects at different scales with well defined and harmonized rules for the closure of objects. Certainly, many of the complexities in the example of ALK and ATKIS would not exist in such an ideal case, but at least the example shown in figure 2a needs to be considered. In the next section the skeleton operator will be introduced and applied on examples to show that it can be used for the generalisation in most of the presented cases.

Figure 2. Relationships of objects at different scales with cardinalities different from n:1 (examples simplified from ALK and ATKIS at scale 1:25000, not all links shown)



(a) caused by partial geometry type change of a river (cardinality 1:n)

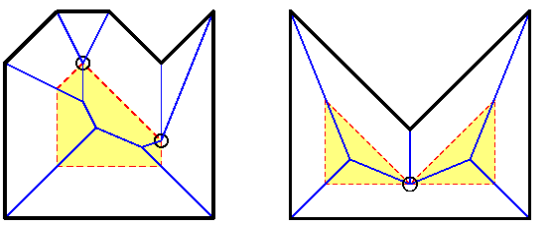
(b) caused by different rules for closure of road objects (cardinality 1:n)

(c) caused by different rules for closure of built-up areas (cardinality 1:n)

(d) caused by different topics of maps (highway ramp, cardinality n:m)

3 THE STRAIGHT SKELETON OF A POLYGON

Skeletons or medial axes can be derived using different approaches (see Table 1). The medial axis is defined as being the locus of all centres of circles inside the polygon that touch the polygon in two or more points, thus it can be composed of straight lines and second order lines. It can be derived from the Voronoi-Diagram (Chin et al., 1995). Another way of constructing it is in raster space using raster operations, namely a grassfire algorithm, that derives the distance of all pixels inside the object from the boundary. The connection of the points with local maximum distance is the medial axis. The chordal axis is an approximation and can be derived using the Delaunay triangulation. It is set up of only straight lines (Prasad, 1997). The same holds for another type of skeleton - the straight skeleton (Eppstein & Erickson 1999) - which also produces a kind of medial axis, yet with a different application focus: An illustrative example is the construction of a roof on a given ground plan, where all the roof parts have to intersect in straight lines.



(a) (b)

Figure 3. (a) Two simultaneous edge events. (b) A split event. (Eppstein et al., 1999)

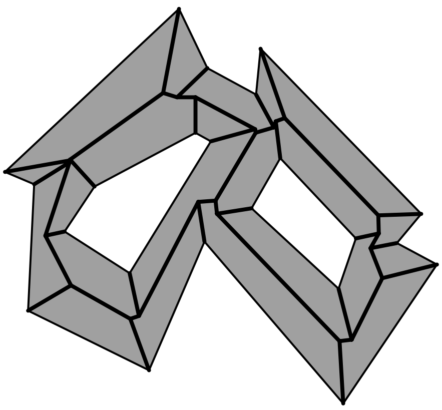


Figure 4. The Straight Skeleton of a polygon with holes.

The construction of the Straight Skeleton of a polygon, which is not self-intersecting, can be illustrated as a shrinking process. The polygon shrinks by simultaneous parallel shifts of all edges to the interior. The offset rate for each edge is predefined and fixed. In this illustration the polygon shrinks with the time. It is also possible to illustrate the construction with a raising roof on a ground plane of a floor. In this case the temporal process can be interpreted as a third spatial dimension.
During this shrinking process two different types of events can be observed. An “Edge Event” means that an edge is omitted due to a collision of the two adjacent edges. A “Split Event” happens when an Edge collides with a point which is shared by two other edges. In this case the polygon is split in this point and two new polygons are generated (fig. 3).
To improve the performance of the algorithm it is possible to handle only those geometries which are affected by the next event. The Events are treated as objects and are maintained in a set. These objects are composed of bisector rays which result from the intersection of two adjacent roof planes and roof triangles which are defined by the two bisectors which belong to an edge. At the beginning all possible events of the polygon are generated. Then the events are processed in the correct chronological order. Processing one event has effects on other events, i. e., new events have to be inserted to the set of maintained events and some events need to be deleted. Furthermore, the algorithm can be extended in order to take different weights of the contributing polygon edges into account (Eppstein & Erickson, 1999).
In table 1 a comparison of the straight skeleton with other forms of skeletons is given.

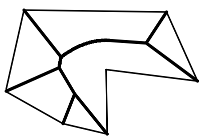
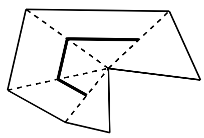
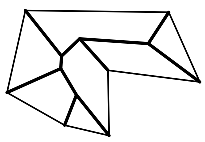
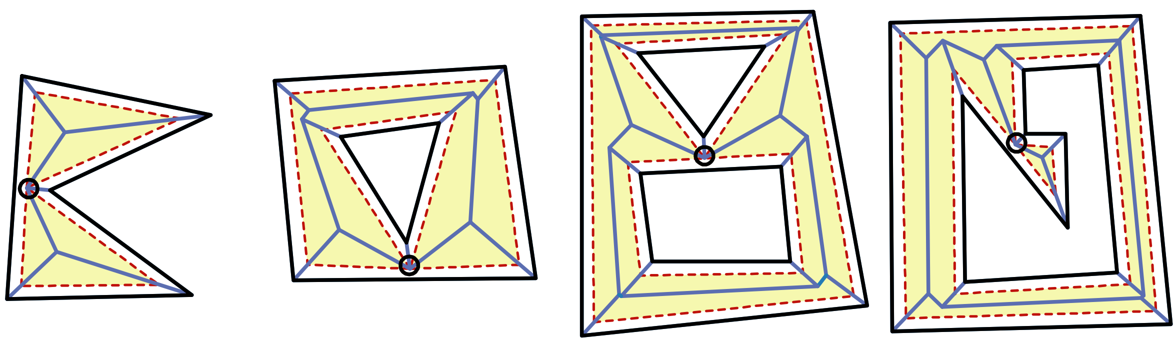
Skeleton based on Medial Axis Transformation (Chin et al., 1995)	Skeleton based on Chordal Axis Transformation (Prasad, 1997)	Straight Skeleton (Eppstein & Erickson, 1999)
		

Table 1. Commonly used skeletons in cartographic applications

In order to also cope with polygons with holes, an extension of the original algorithm was done. The construction of the Straight Skeleton of a polygon with holes is possible when each hole is represented as an interior ring, associated with an exterior polygon boundary. The result is shown in figure 4. Computing the straight skeleton of a polygon with holes the split event can be observed in several versions which are shown in Figure 5. Also here only local changes are computed. To be able to identify and process these different cases the components of the events (roof triangles and bisector rays) are associated with a ring. Performing a split event means that the association of the components in the involved rings may change. In the cases 5a and 5d it is also necessary to associate the existing interior rings to the new exterior rings which result from the split.

Figure 5. Shrinking polygons lead to different versions of the split event in a polygon with holes. The labelling states the characteristic of the involved rings.



(a) Exterior-Exterior. (b) Exterior-Interior. (c) Interior1-Interior2. (d) Interior1-Interior1.

3.1 APPLYING THE STRAIGHT SKELETON ON GENERALISATION

The Straight Skeleton is used here for the extraction of a one-dimensional representation of an object, i. e., a set of connected lines, from its two-dimensional representation (area). Also it is used for the collapse of an area and its substitution by extensions of the adjacent areas.

3.1.1 GEOMETRY TYPE CHANGE

The geometry type change will be illustrated with examples of road data from the ALK and ATKIS (at scale 1:25.000) which have been introduced in section 2. There it was extensively discussed how road areas are formed in the ALK and how they are related to ATKIS road data.
Considering the complexity of the relationships between objects of both data sets displayed in figure 2d, it is certainly not possible to derive the ATKIS roads totally from the cadastral ALK data, as detailed information about ramps and lanes is missing. This would mean to increase the information by Generalisation, what is not possible. However, it will be shown that a propagation of the information included in the ALK is possible.
From figure 2b it can be derived, that a generalisation of single road parcels will fail. Here it is necessary to generate an aggregation which meets the idea of a road or road network. That is a combination of areas which are expanded along a path and meet in several junction areas (see figure 6).
Performing the skeleton algorithm on this aggregated polygon will lead to a network which consists of linear elements. Figure 7 shows the typical appearance of the Straight Skeleton. The edges form a graph whose leaves are the vertices of the polygon.
Apparently, it is necessary to modify this graph to get the appropriate representation of the roads for ATKIS. The first step is to eliminate the edges which are incident to the branch vertices, thus the first rule reads as follows:

Rule 1: IF an edge is incident to a branch vertex THEN eliminate it.

Occasionally, small branches remain which do not correspond to a road. To eliminate these branches a rule has been established which is based on the incircles of the polygon with the centre points in the junctions of the skeleton.

Rule 2: IF a branch that leaves from a junction point does not leave the circle around its origin THEN eliminate it.

Applying these rules finally reduces the skeleton to the centrelines (see figure 8).

Figure 6. Aggregation of parcels declared as roads in the ALK

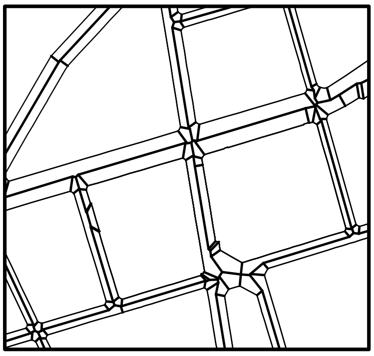


Figure 7. Straight Skeleton of aggregated road parcels

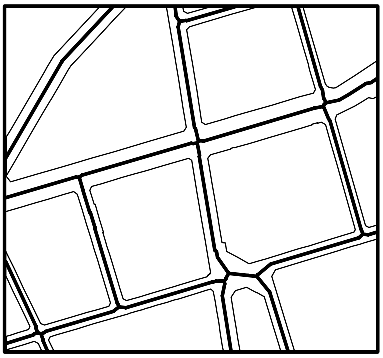


Figure 8. Centrelines of polygon

Still these centrelines differ from the optimal solution, since a junction does not appear as an intersection point of road axes. Instead of that, a centreline enters the surrounding of a junction in a rather unnatural arc, because road parcels widen in these areas. Also, at T-junctions, the straight line is bent into the direction of the incoming road.
To solve this problem an algorithm has been developed which is able to change most of the junctions to the desired appearance. This algorithm mainly relies on rules which define when a junction point has to be repositioned and in which cases multiple junction points can be replaced by a single point. These rules are essentially based on a definition of junction areas. Basically, the new locations of the junction points are calculated by intersecting the extrapolated centrelines which enter a certain junction area. It came out that the result of this method highly depends on the definition of the shape and size of the area around a junction point which is taken into account for the change of the skeleton. A good solution was found using the incircles of the polygon in the junction points also for the definition of these areas (see figure 9). This has the advantage that no parameter has to be set manually and the area is automatically adjusted to different road widths. The final shape of the skeleton was achieved by applying the two following rules:

Rule 3: IF two junction points are each located within the incircle around the other point THEN replace them by a single point.

*Rule 4: IF a junction point has more than one incident branch which leaves a group of intersecting incircles THEN generate a junction by extrapolating and intersecting these branches.
IF the new junction does not cause any conflict THEN replace the old junction point by the new one.*

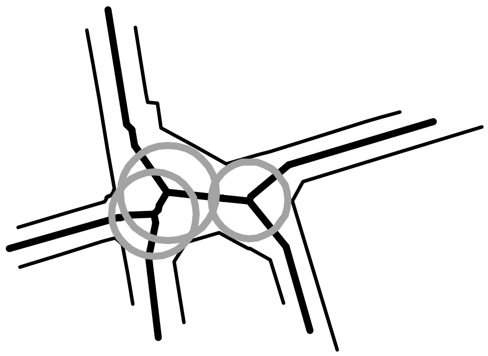


Figure 9. Incircles for the reconfiguration of road junctions: the two junctions on the left can be merged to one, as their counterpoints lie in each others' incircle (see text).

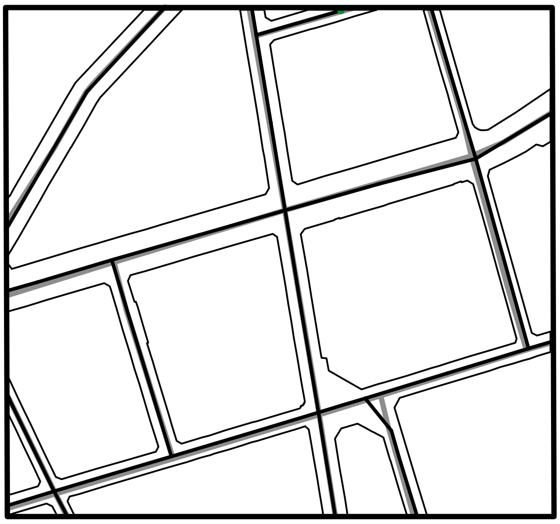


Figure 10. Results from geometry type change of ALK areas (black) compared with ATKIS roads (grey).

The last statement satisfies the fact that not all special cases of junctions can be covered by the rules. It is based on the consideration that the centreline of the straight skeleton is a better result than a wrongly remodelled junction. The check for conflicts includes the test if the new road axes lie within the road polygon and do not intersect outside of the junction points.
However, this heuristic method generates the correct appearance for all common junctions such as T junctions and crossroads and also copes with many special cases. Intersections of roads with different widths as well as different intersection angles were tested and verified. These achieved results are visually pleasing, and conform with the specifications of the topographic maps (figure 10). However, complex junctions, such as motorway junctions with many ramps or squares in a city with many roads leading in where mostly modelled incorrectly. A possibility to approach these difficult cases will be mentioned in the end of the next section. The test of the method for a large connected road network is missing.

3.1.2 PARTIAL GEOMETRY TYPE CHANGE

Often by definition the geometry type of an object depends on its properties. As it was discussed in section 2 this means that an object needs to be split when different parts of an object need to be represented differently. Figure 11a shows a lake represented with its incoming and outgoing rivers as one area. The task is to split this area into lines and areas according to a given threshold.
To solve this task it is possible to utilize the fact that the construction of the skeleton is based on a shrinking process. The events described in section 4.1 will be performed only if they happen within the defined threshold. Events which happen later are not to be processed.
Figure 11b shows the skeleton after its termination at an early stage. The polygons which are left are defined by the points which are located at the same level as the last processed event. The next step is to extend these polygons until they touch the boundary of the input polygon. This can be done by a simple buffering. The buffer intersects some edges of the skeleton. These edges are simply cut to preserve the connectivity of the skeleton's graph to the generated polygons. Also the skeleton is reduced to its centreline according to section 3.1.1. The result is shown in figure 11c. Figure 11d and e show the result after altering the threshold width. The method works similar to the morphologic opening operator which is widely used in image processing (Gonzalez & Woods 2002). The setting of the parameters can e.g. be taken from the ATKIS specifications, which determine, when a river has to be modelled as linear feature or as areal one – depending on its width. To generate line features of all parts of a river where the width does not reach a specified value, the half of the value is used as threshold for the algorithm. Often the rule can not be formulated in such a simple way. For example, to distinguish lakes from rivers it is normally not sufficient to define a threshold width. Here more information like the connectivity to a network or the ratio of the width to the length is needed. In these cases it is the challenging task to find rules for the abort of the shrinking.

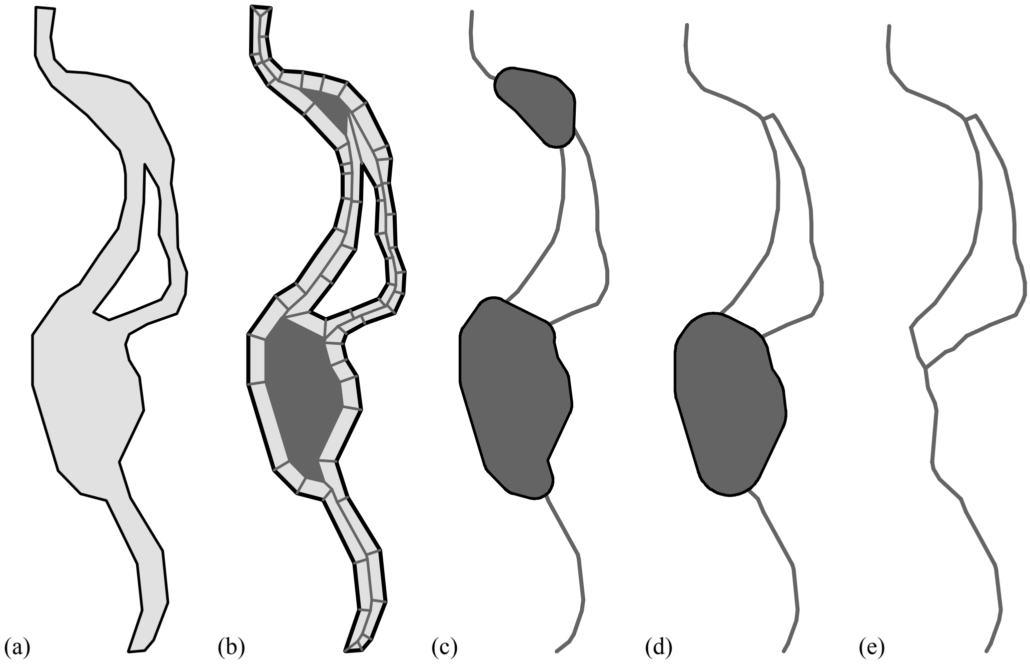


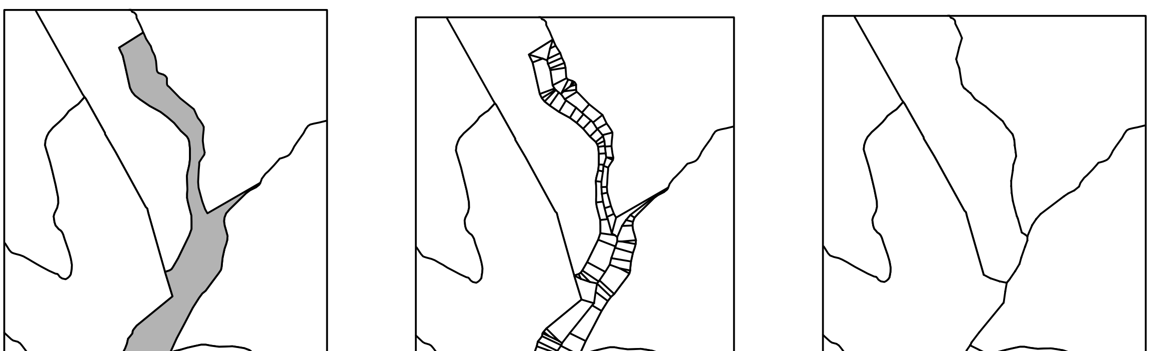
Figure 11. Fragmentation of a river into polygons and lines with different thresholds leading to different results (c, d, e).

Applying this extension on the example shown in figure 2d it is possible to identify those areas, which belong to highway ramps, because the parcels are much wider in these areas than the maximum width of a road. Since a detailed model of the ramps and lanes can not be achieved, it might be beneficial to define these regions as area features. The same applies to broad squares in cities which were mentioned in section 3.1.1.

3.1.3 COLLAPSE

The collapse operator is used when a feature is no longer represented in the generalized data set. This applies to an object whose area is smaller than a certain threshold. Also an object whose class is not represented in the generalized map needs to be replaced.

Often this problem is simply solved by adding the area of the object to the adjacent object with the largest area. Also it is common to take the length of the common boundary into account or to define similarity measures between object classes (van Oosterom, 1995). Then the object can be added to the neighbour whose class has the highest similarity.



(a) (b) (c)

Figure 12. Collapse of an area using the Straight Skeleton.

Unfortunately, this approach often results in area boundaries which are rather unnatural. Figure 12a shows a forest area. Here it is not possible to add the area completely to a neighbour without a dramatic change of the neighbour's shape. Performing the skeleton operator on the area leads to the result in figure 12b. Each area enclosed by the skeleton edges can be associated with a neighbour of the polygon. Figure 12c shows the result after the union of these areas with the corresponding neighbour. In this result the typical shapes of the neighbours remain. Generally, the skeleton works fine for collapses of areas which have an organic shape. For the elimination of artificial areas such as a block of flats it is more appropriate to perform an aggregation.
While the collapse of an area can be performed with the skeleton algorithm, other functions are needed to decide which features need to be eliminated. In Peter & Weibel (1999) solutions to this task are discussed.

3.2 PRESERVING TOPOLOGICAL RELATIONSHIPS AND CONSTRAINTS

Bobzien & Morgenstern (2002) point out, that a geometry-type change involves also topological problems. To meet this demand the approach is to identify those constraints and preserve them by modifying the settings for the skeleton prior to its construction. In this section it will be described how this is done.
It is possible to influence the skeleton's shape on two ways. The first possibility is to add points to the polygon before the skeleton is computed. Since each polygon point is a leaf of the skeleton's graph the connectivity to the point will be kept. To preserve this topologic relationship it is important not to delete the branch ending at this point when reducing the skeleton to its centreline. Figure 13 shows the result of a geometry type change without and with the consideration of the connection to a point or line.
The second possibility is to modify the "inclination of the roof" which raises upward from each edge and defines the skeleton. With a growing inclination of a roof plane the skeleton moves in the direction of the edge from which the plane raises. Defining a vertical roof plane means, that the skeleton touches the edge.
This can be utilized to preserve the adjacency of two areas or the adjacency to a line. Figure 14a shows a river which meets the sea. The change to the linear geometry type can destroy this relationship (see figure 14b). It can be preserved by defining the roof to be vertical in the shared edge (see figure 14c).
Also vertical slopes can be used to extend the collapse operator. Often it is wished that an area is replaced by some of the adjacent areas but not by all. Figure 15a shows features classified as built-up areas inside an urban area which is delimited with a bold line from rural area. The task is to perform the collapse operator on a marginal built-up area (marked with X) without modifying the prior borderline. Figure 15b shows that the application of the collapse operator in its simple version fails. The important border is destroyed since it is replaced by the centreline of the polygon's skeleton. By assigning a vertical inclination to the external edges the original bold border can be preserved (see figure 15c).

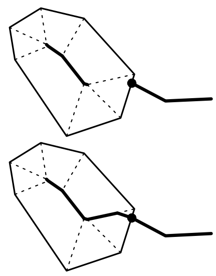
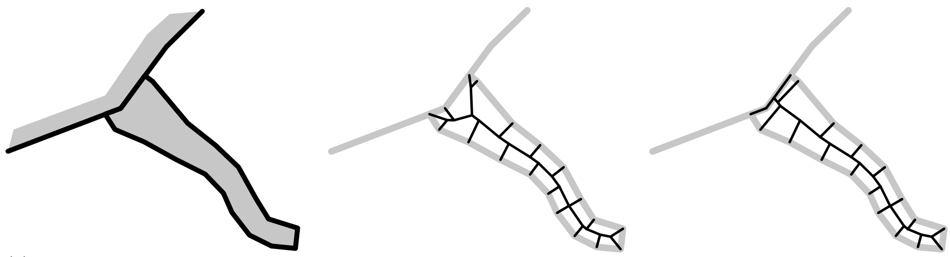
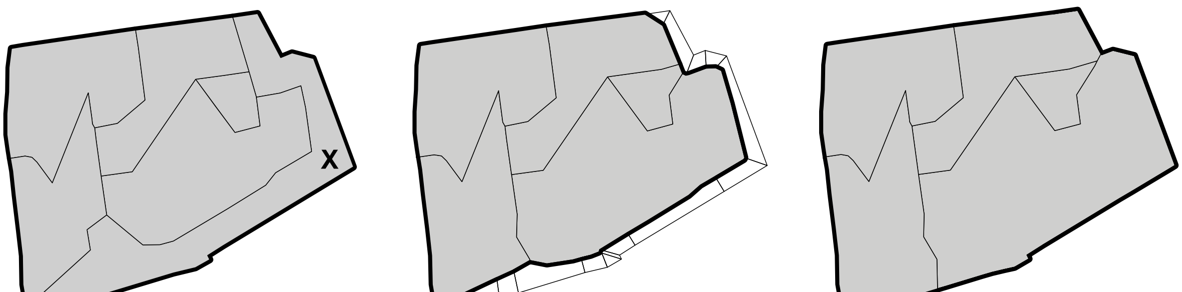


Figure 13. Preserving connectivity to a line or point



(a) (b) (c)

Figure 14. Skeleton which preserves the adjacency to polygons. Roof planes raising from shared edges were assigned to be quasi-vertical.



(a) (b) (c)

Figure 15. Collapse Operator without (b) and with the consideration of geometric constraints (c).

4 FUTURE WORK

For the future it is aimed to integrate the presented generalisation methods into a framework for the maintenance of an MRDB. The skeleton algorithm proved to be a powerful tool which can be applied to different problems. Yet, with the implementation of a proper generalisation algorithm the aim of an automatic database updating system has not been reached. At least two important components are missing.

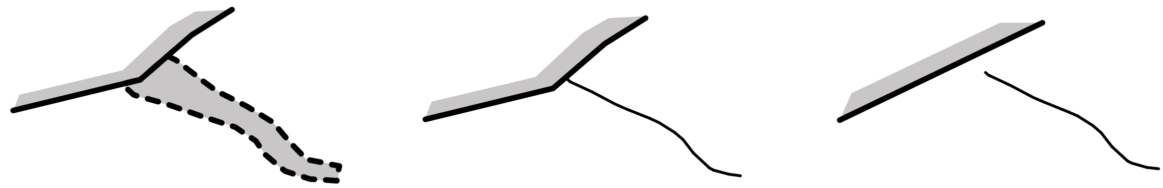
The first task for an automatic updating would be to identify and formalize the relationships of a newly added feature to other features. These relationships can result in constraints and dependencies. Hence, the generalisation of a single feature can not be formulated as an isolated task. Kilpeläinen & Sarjakoski (1995) introduced the concept of incremental generalisation. The main idea of this approach is to divide the generalisation of the whole map into modules. After modifying a level of the MRDB those modules need to be recomputed which are influenced by the update. Buttenfield (1995) introduced an object-oriented model for the generalisation task. In this representation also relationships between features of the same scale can be modelled. Possibly, it is beneficial also to establish links between features of the same MRDB level which are able to express these relationships.

In chapter 3.2 possibilities of influencing the skeleton's shape were introduced, to guaranty constraints and a correct topology. However, the more crucial task is to identify these constraints in advance. Also for the introduced algorithms which require parameters as the partial geometry type change a formalization is needed to find the proper parameters automatically.

After the analysis of the relationships and the formalization of constraints the generalisation can be performed as it was presented in section 3. However, the task of inserting the new feature to a target level in the MRDB is still not solved. This problem is illustrated in figure 16. While the relationship of a coastline to a river can be preserved within the generalisation, it is lost when the river is added to a target level, where the coast line has a different shape. This means that after the generalisation process an integration process needs to be performed. This integration task is a map conflation problem. With the knowledge about the transformation which maps the coastline from the base level onto the coastline in the target level it is possible to transform also the new river. A promising approach which uses a rubber-sheeting transformation was introduced by Doytsher et al. (2001). There the map conflation problem was divided into four sub tasks: Finding counterpart features, partitioning the map into sub-regions, transforming counterpart features and finally transforming the remaining new features. The advantage which arises from the link structure in an MRDB is that the correspondences are given by the links and hence the difficult matching problem does not need to be solved.

We aim to develop a system which brings the here discussed parts together: Formalization of relationships between features in a map, generalisation of features to get the proper representation for every MRDB level and integration of new features into the database.

Figure 16. Generalisation and integration of a newly added feature.



1. A river is added to the base level of the MRDB.
2. Generalisation: The river is generalized considering the context. Here: Relationship to coastline
3. Integration: The generalized feature is inserted to the target level of the MRDB. The relationship is lost if it is not considered in this process, since the shape of the coast line differs from its shape in the base level.

5 CONCLUSION

After presenting typical relationships of corresponding objects in different representations it was shown that the Straight Skeleton is a powerful tool for the generalisation of areas. The skeleton was used to generate linear representations of features with a two dimensional shape. Also the application on a collapse operator was introduced. Especially the capabilities to perform partial geometry type changes and to preserve constraints are advantages which make it superior compared to other skeletons.

The extraction of road axes from parcels was introduced as a typical example for the problem of a geometry type change. Here it was shown, that the skeleton needs to be modified to achieve appropriate results. The required representation was generated by heuristic rules for the remodelling of junction areas.

Finally, it was shown that a correct generalisation of a solitary feature does not ensure the correct integration of a new feature into the MRDB. To solve this task it is necessary to formalize the relationships to other features, determine an influence region and perform an integration process.

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