Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Netzwerk-Design-Probleme in Graphen und in der Ebene

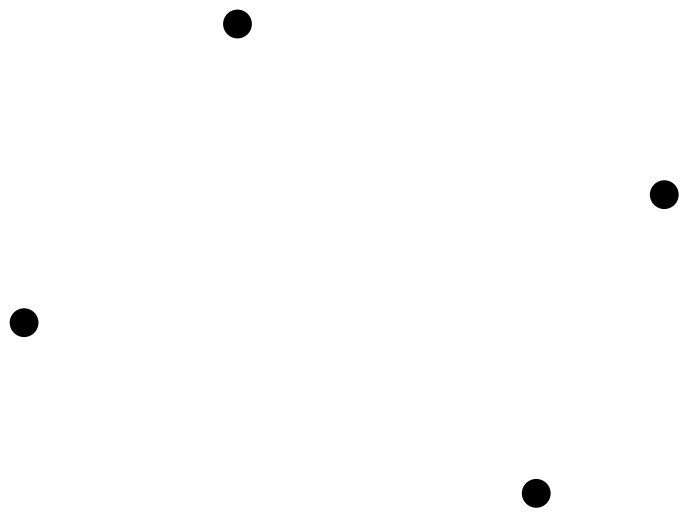
Network-Design Problems
in Graphs and on the Plane

Krzysztof Fleszar

Lehrstuhl für Informatik I
Universität Würzburg

Depart. Math. Engineering
Universidad de Chile

Zwei Beispiele



Verbinde alle Punkte!

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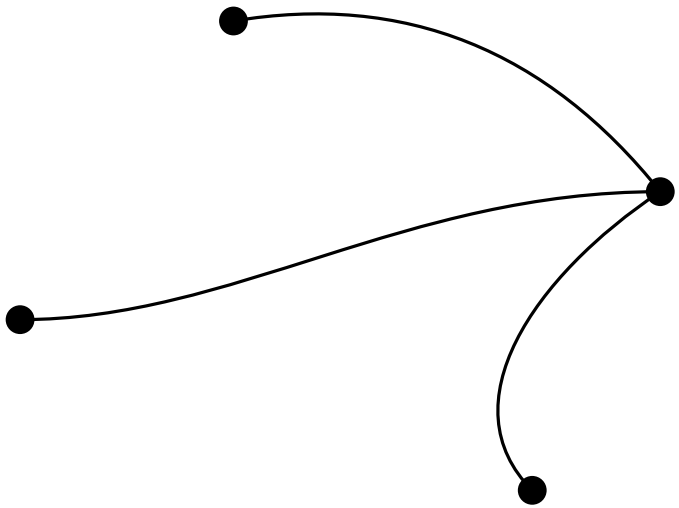

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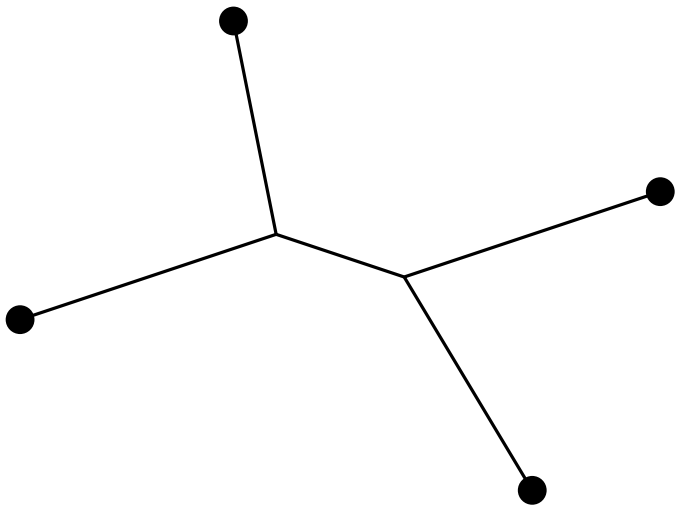

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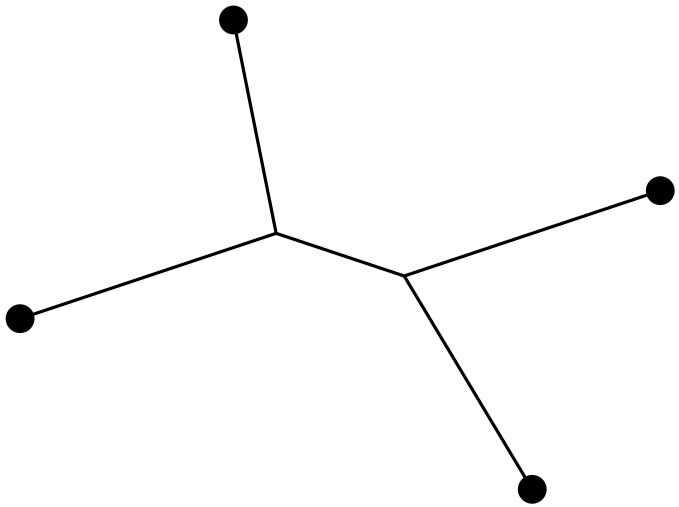

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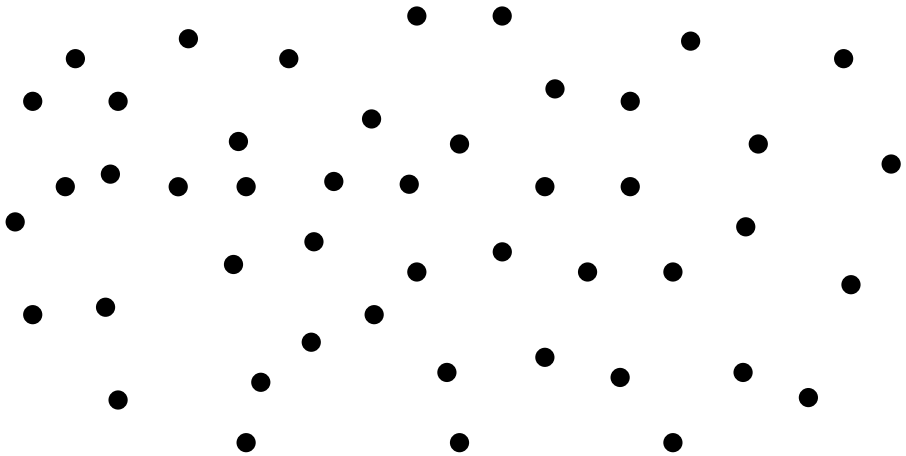

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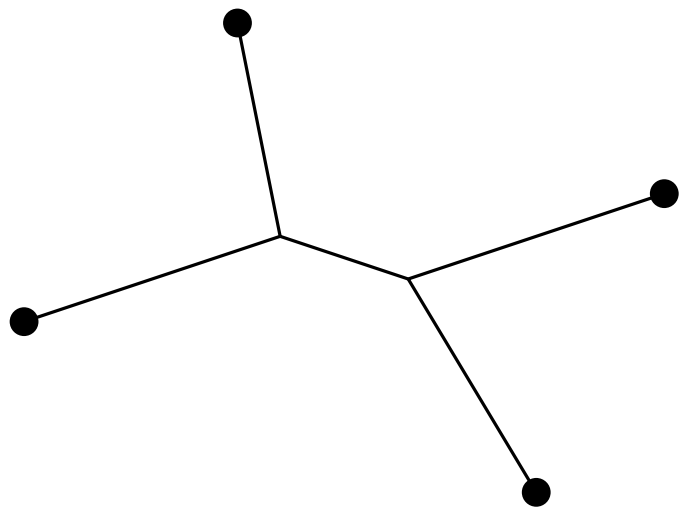

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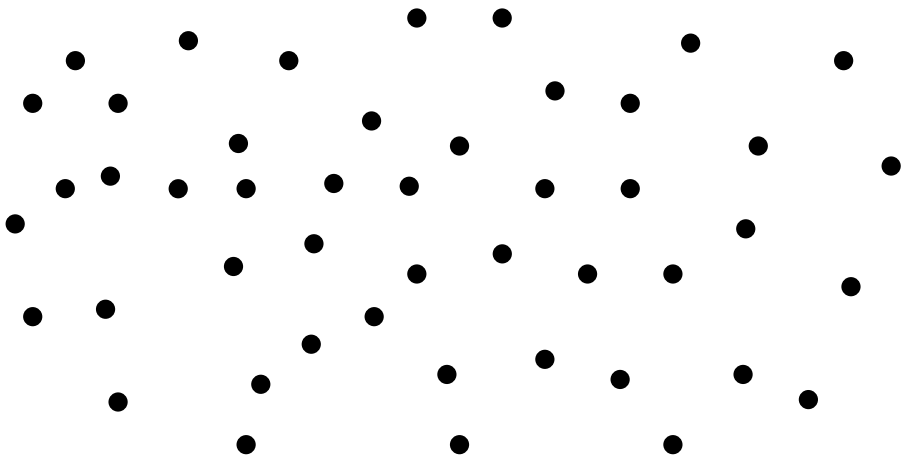
NP-schwer



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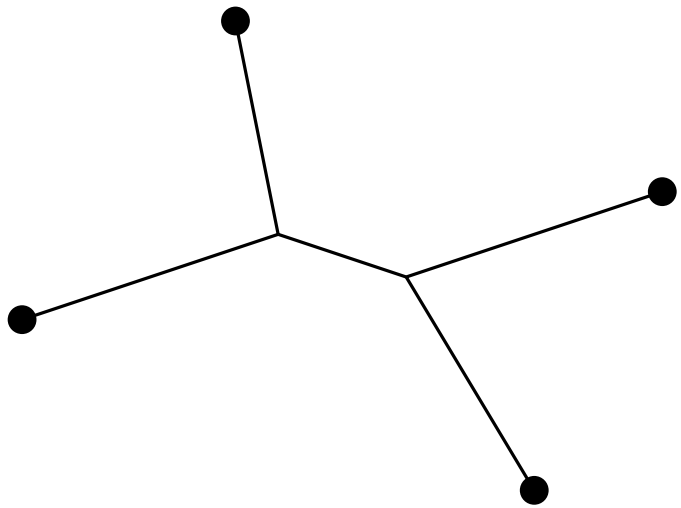

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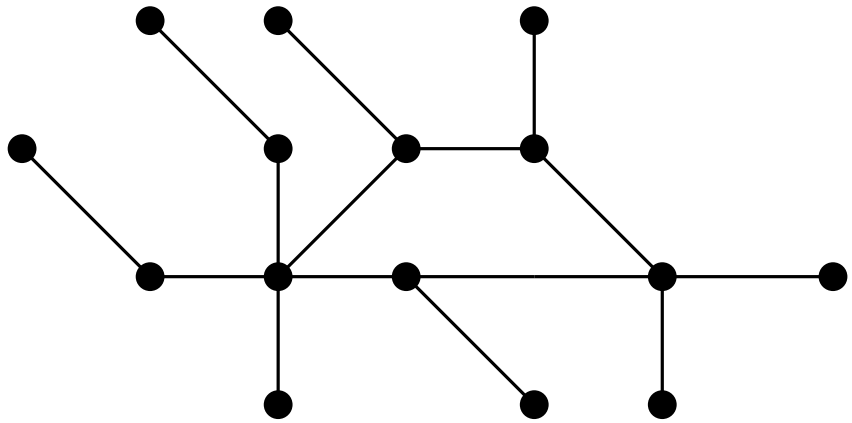

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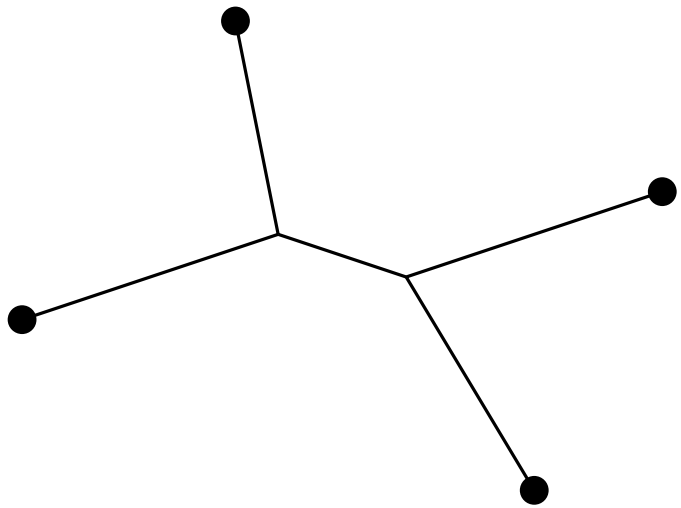

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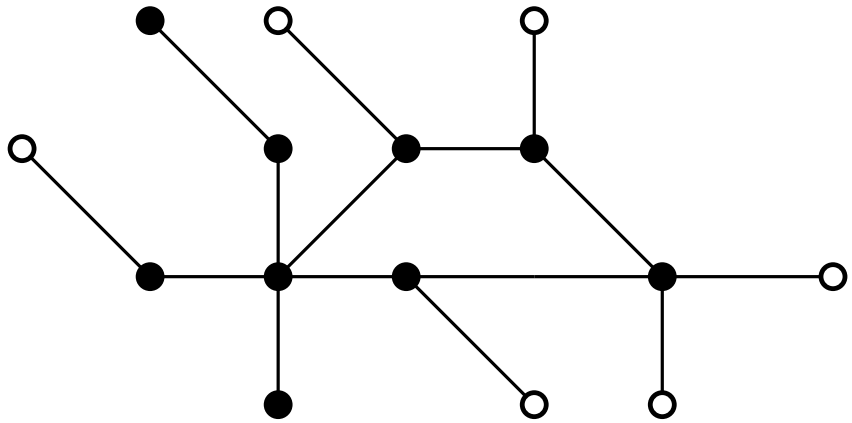

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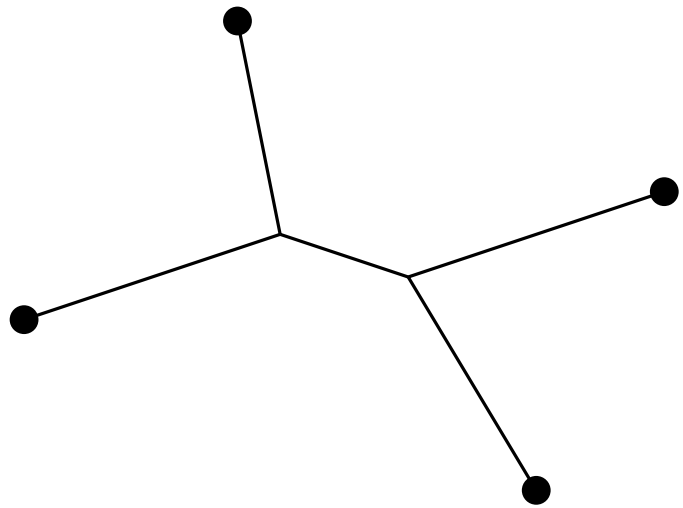

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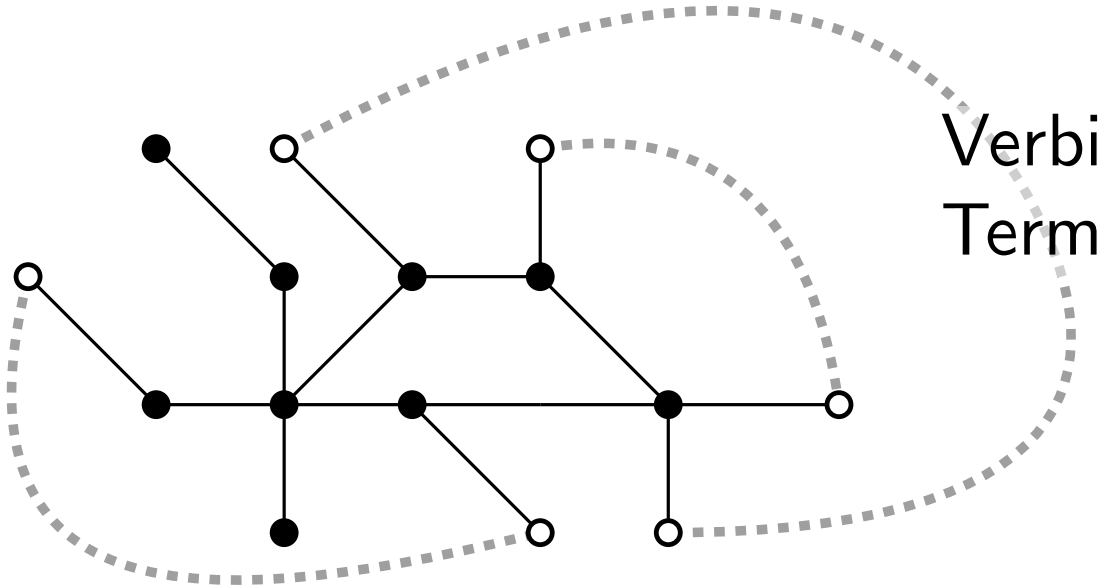
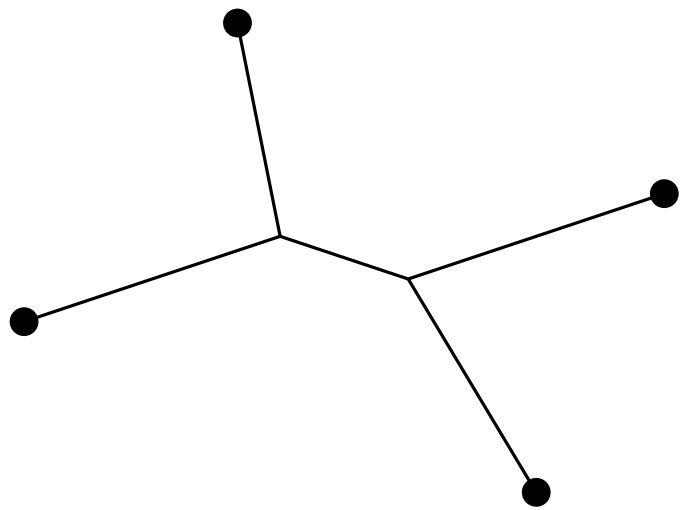


Verbinde möglichst viele Terminalpaare!

Zwei Beispiele



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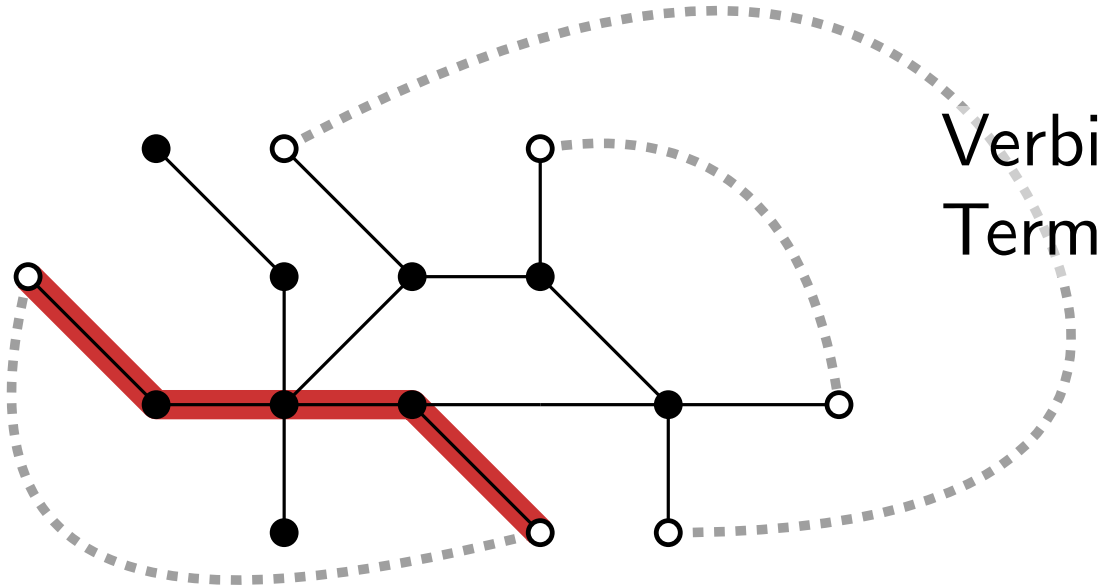

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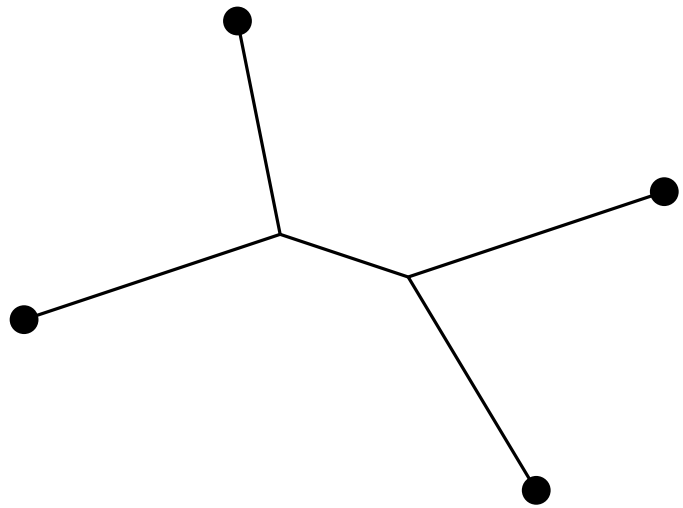

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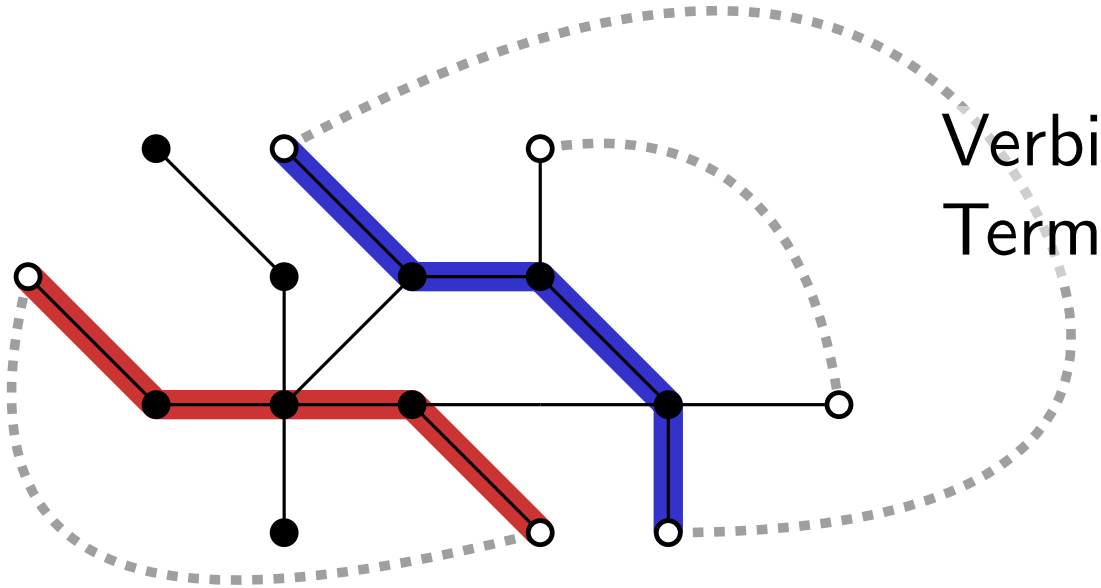

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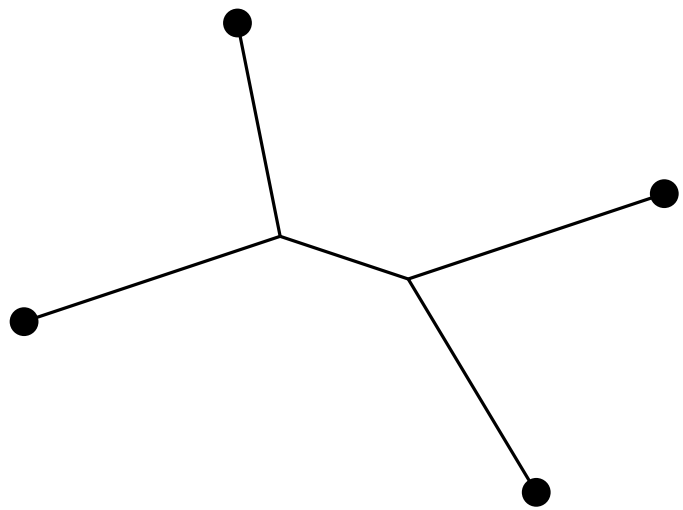

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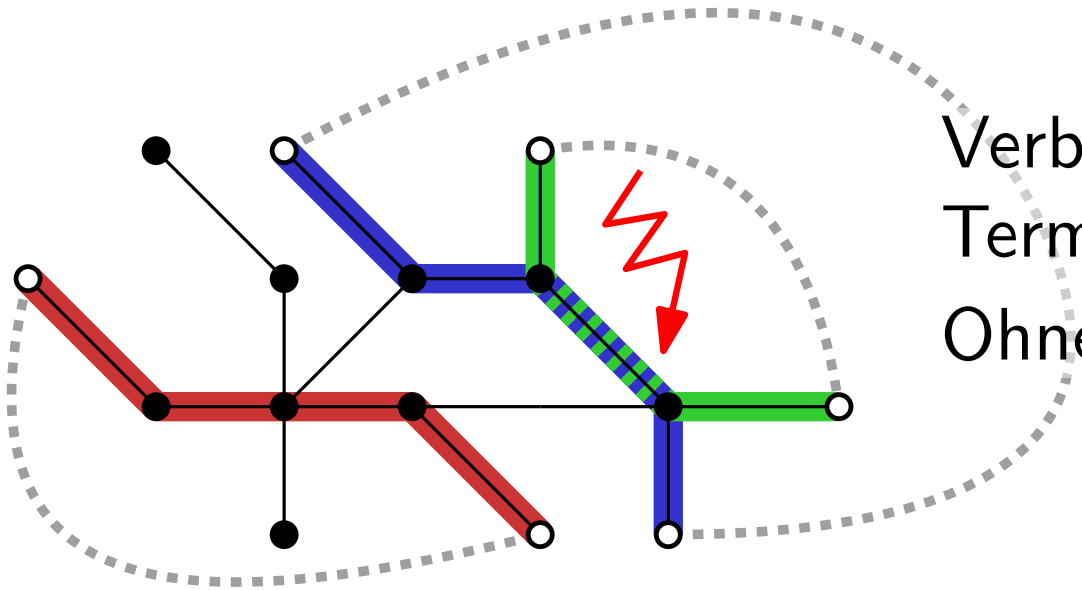

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Ohne Überschneidungen!

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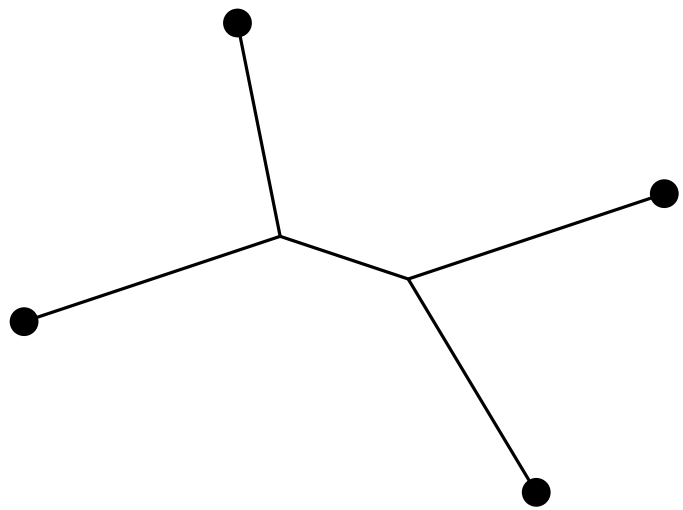

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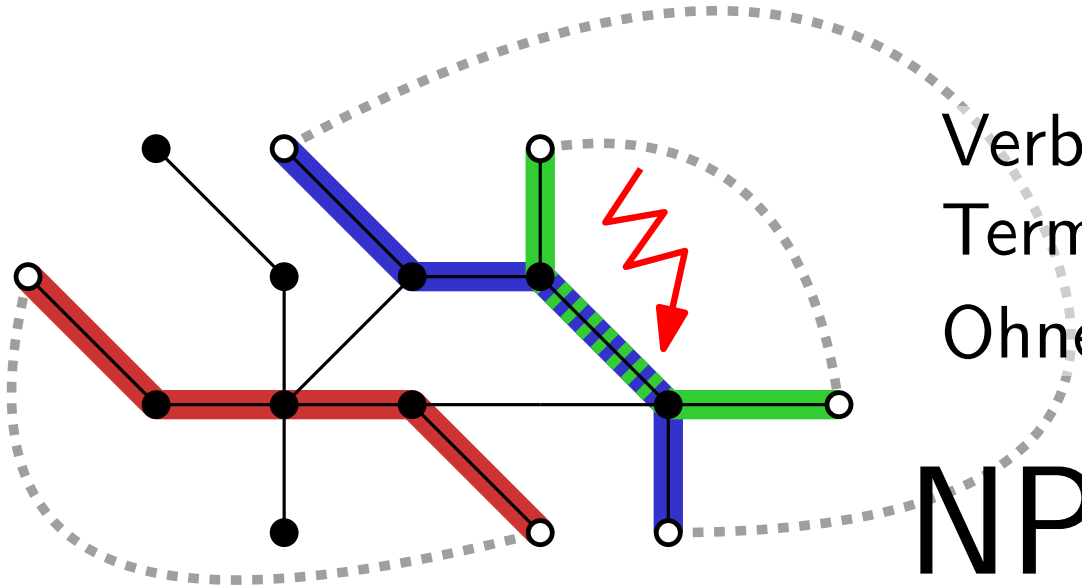

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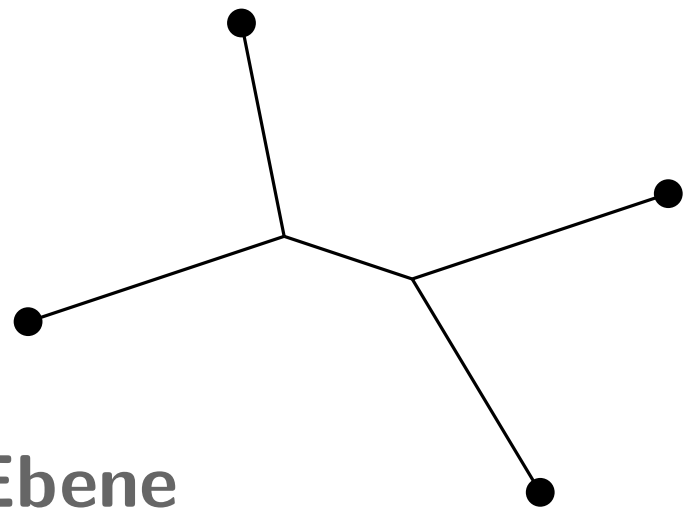

Ebene

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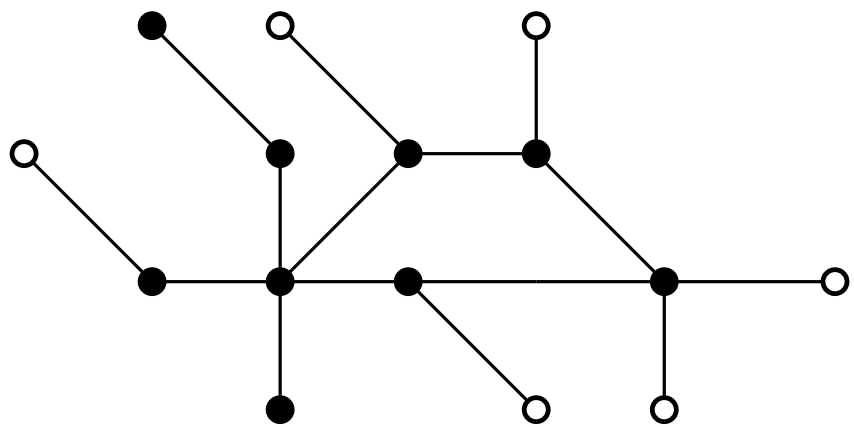


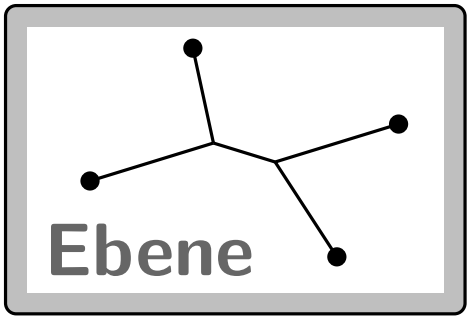
Graph

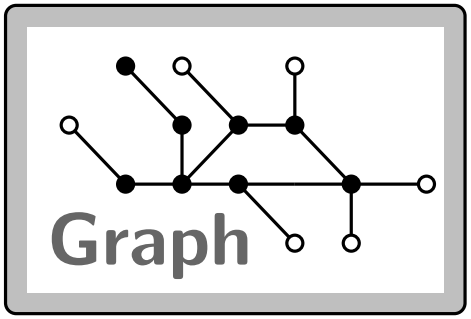
Verbinde möglichst viele Terminalpaare!
Ohne Überschneidungen!

NP-schwer

Probleme der Arbeit

Optimierungsprobleme





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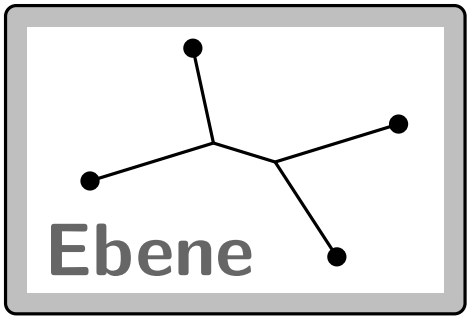

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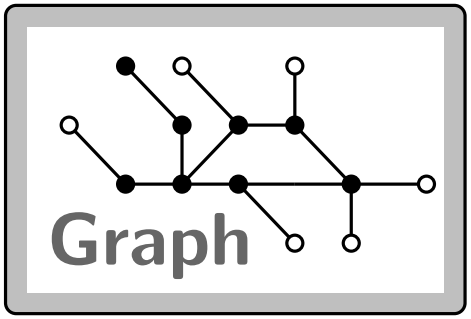

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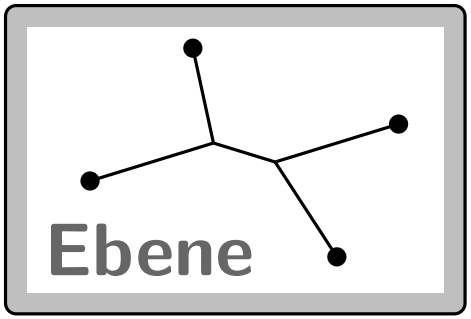

NP-schwer

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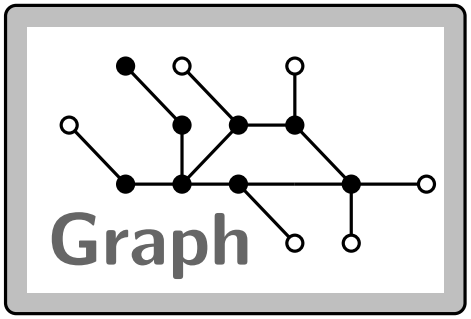
Approximation



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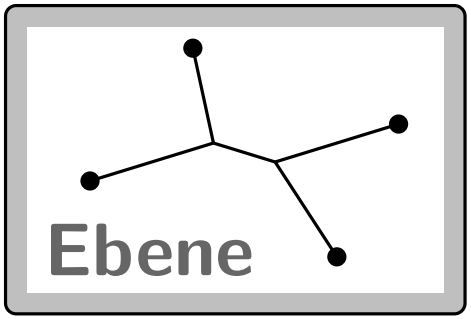

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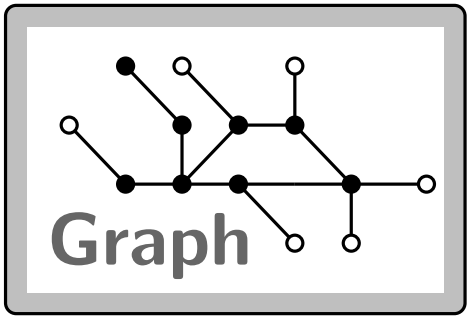

Güte α

Probleme der Arbeit

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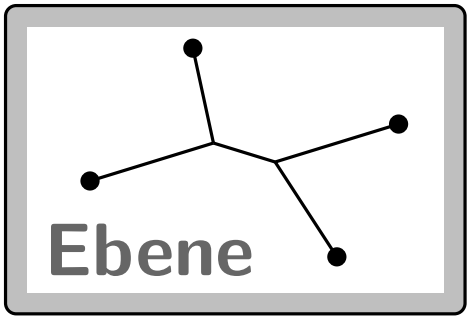

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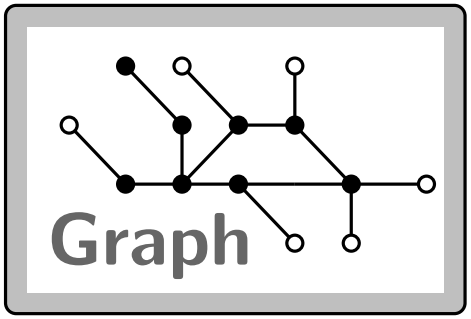

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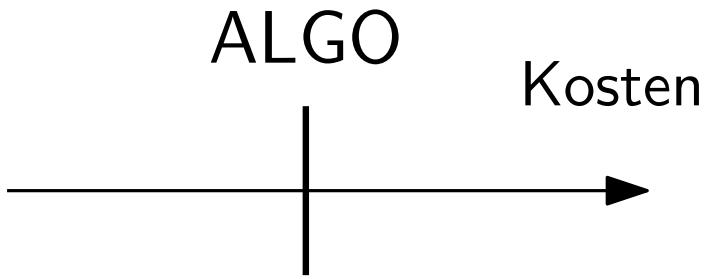

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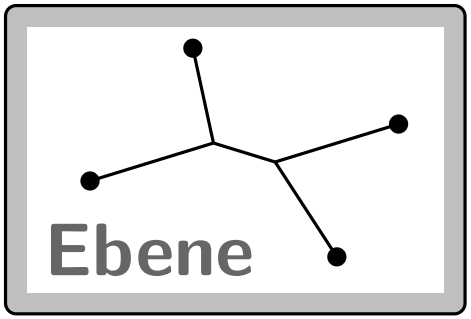

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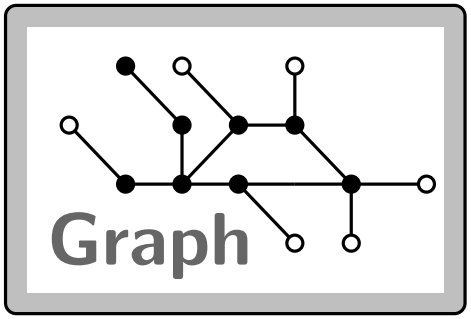

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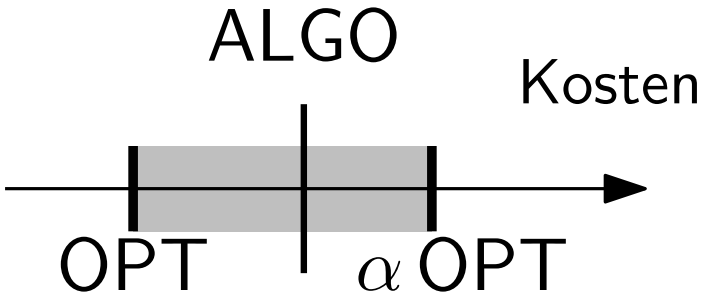

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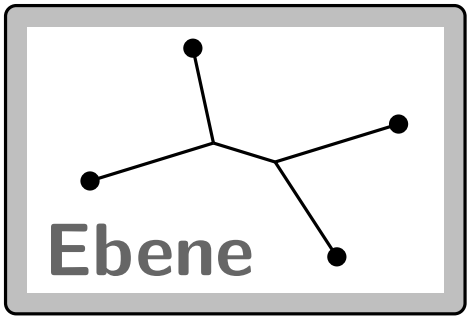

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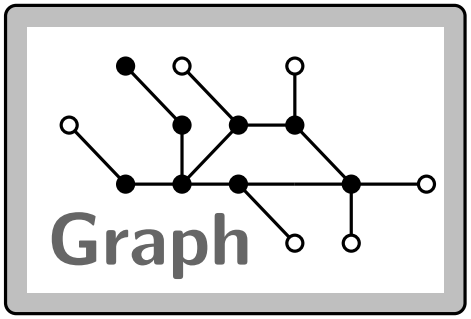

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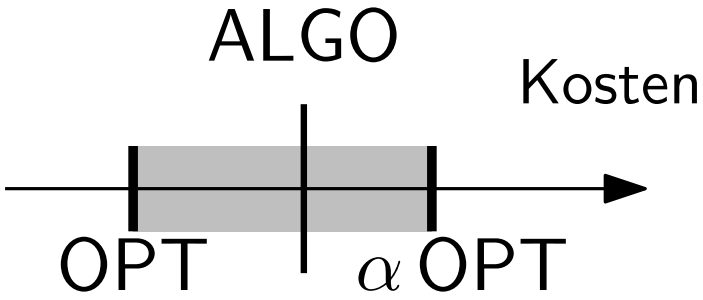

$$\text{Güte} \geq \frac{\text{ALGO}}{\text{OPT}}$$

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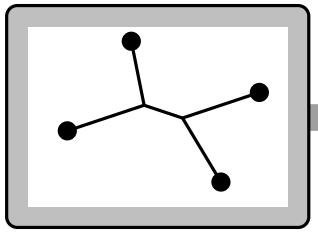
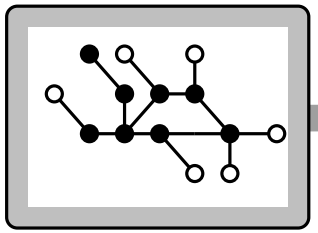


Ebene

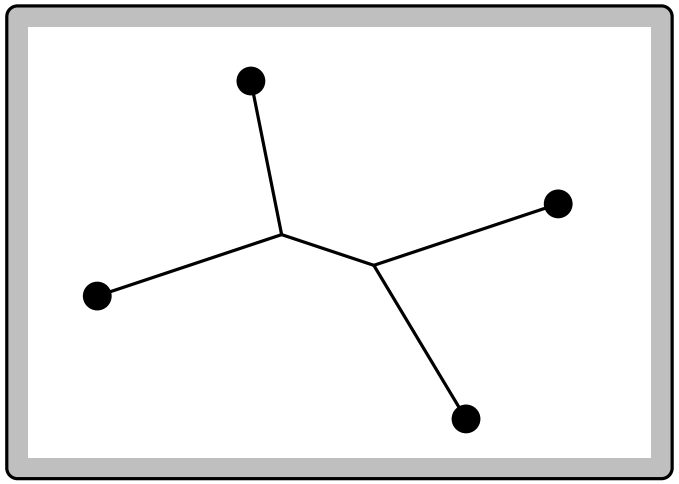
- Colored-Steiner-Forest



Graph

- Kantendisjunkte Pfade

Colored-Steiner-Forest



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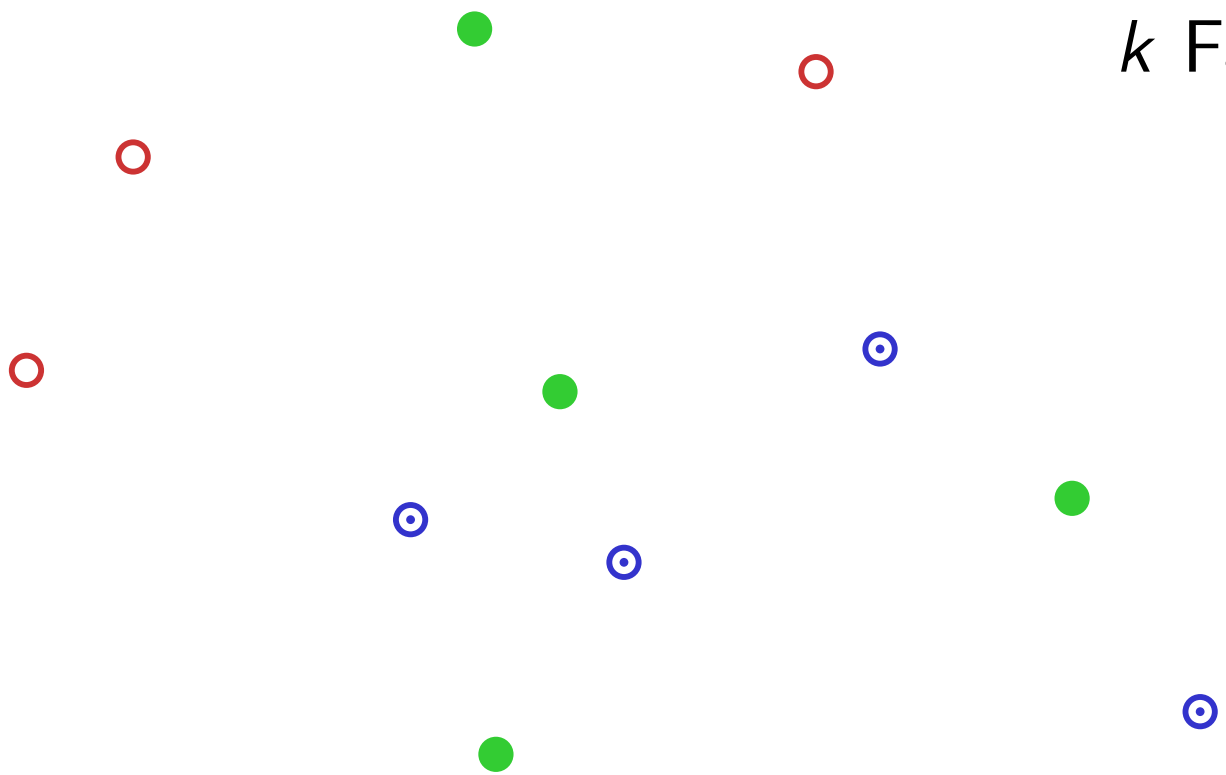
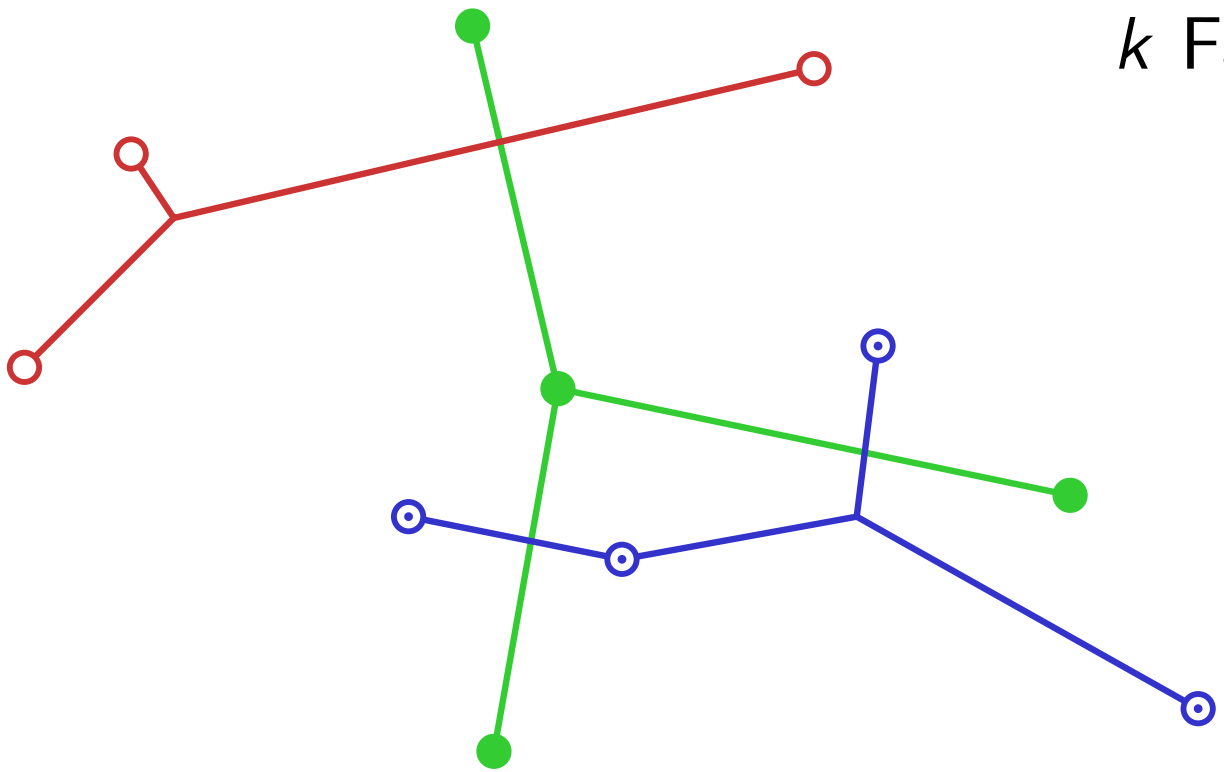


n Punkte
k Farben

Colored-Steiner-Forest



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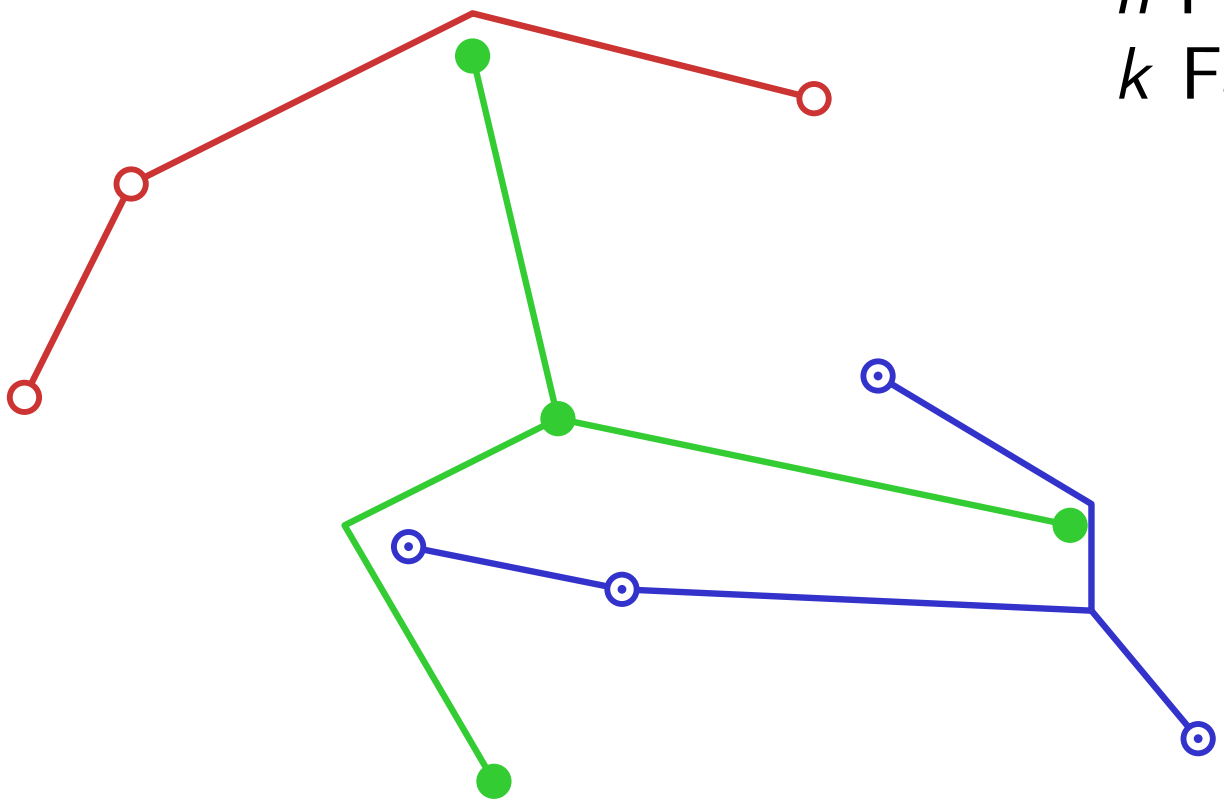
Colored-Steiner-Forest

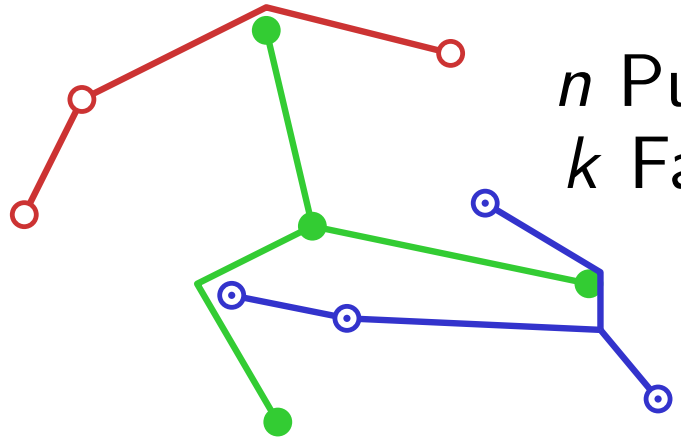


n Punkte
k Farben

Colored-Steiner-Forest

Bekannt

NP-schwer
[Garey et al. '77]



n Punkte
k Farben

Colored-Steiner-Forest

Bekannt

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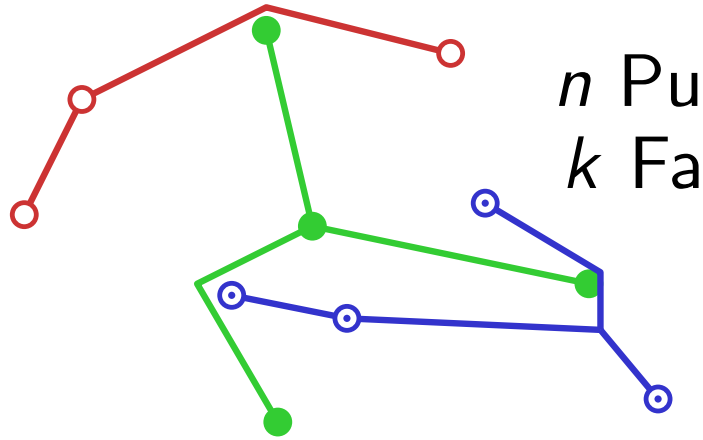
[Garey et al. '77]

$1{,}21k$
[Efrat et al. '15]

$$\text{Güte} \geq \frac{\text{ALGO}}{\text{OPT}}$$



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k Farben

Colored-Steiner-Forest

Bekannt

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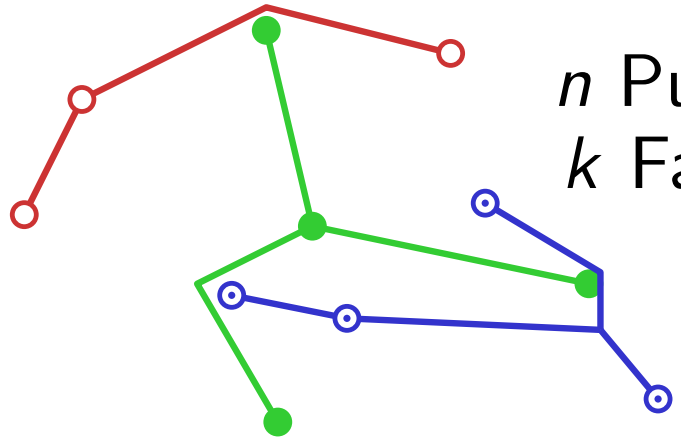
[Garey et al. '77]

$1{,}21k$
[Efrat et al. '15]

für $k = 1$

$1 + \varepsilon$
[Arora '98, Mitchell '99]

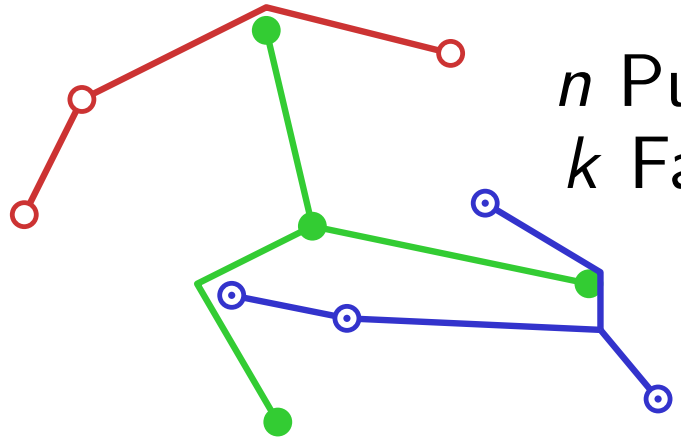
$$\text{Güte} \geq \frac{\text{ALGO}}{\text{OPT}}$$



n Punkte
k Farben

Colored-Steiner-Forest

Bekannt		Neu
NP-schwer [Garey et al. '77]		
$1{,}21k$ [Efrat et al. '15]	Güte $\geq \frac{\text{ALGO}}{\text{OPT}}$	$k + \varepsilon$
für $k = 1$ $1 + \varepsilon$ [Arora '98, Mitchell '99]		



n Punkte
k Farben

Colored-Steiner-Forest

Bekannt		Neu
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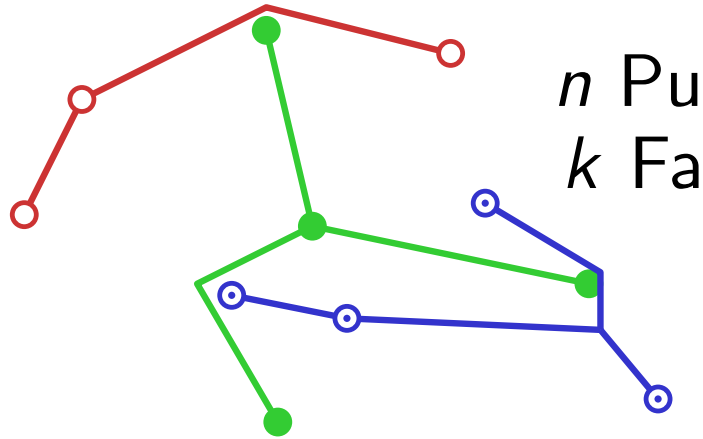
$1 + \varepsilon$ für $k = 1$ [Arora '98, Mitchell '99]		$1 + \varepsilon$ für $k = 2$



n Punkte
k Farben

Colored-Steiner-Forest

Bekannt		Neu
NP-schwer [Garey et al. '77]		
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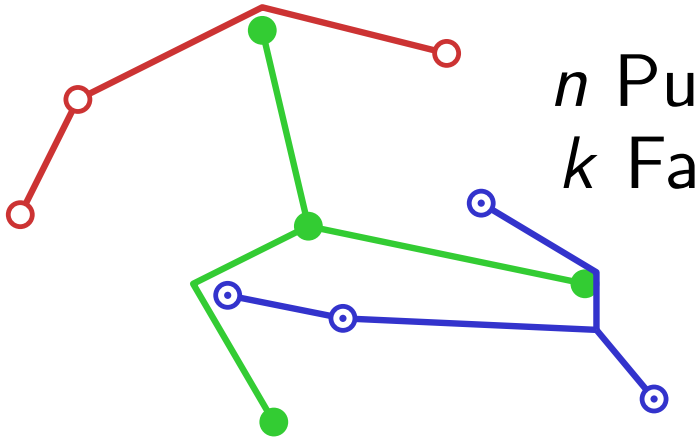
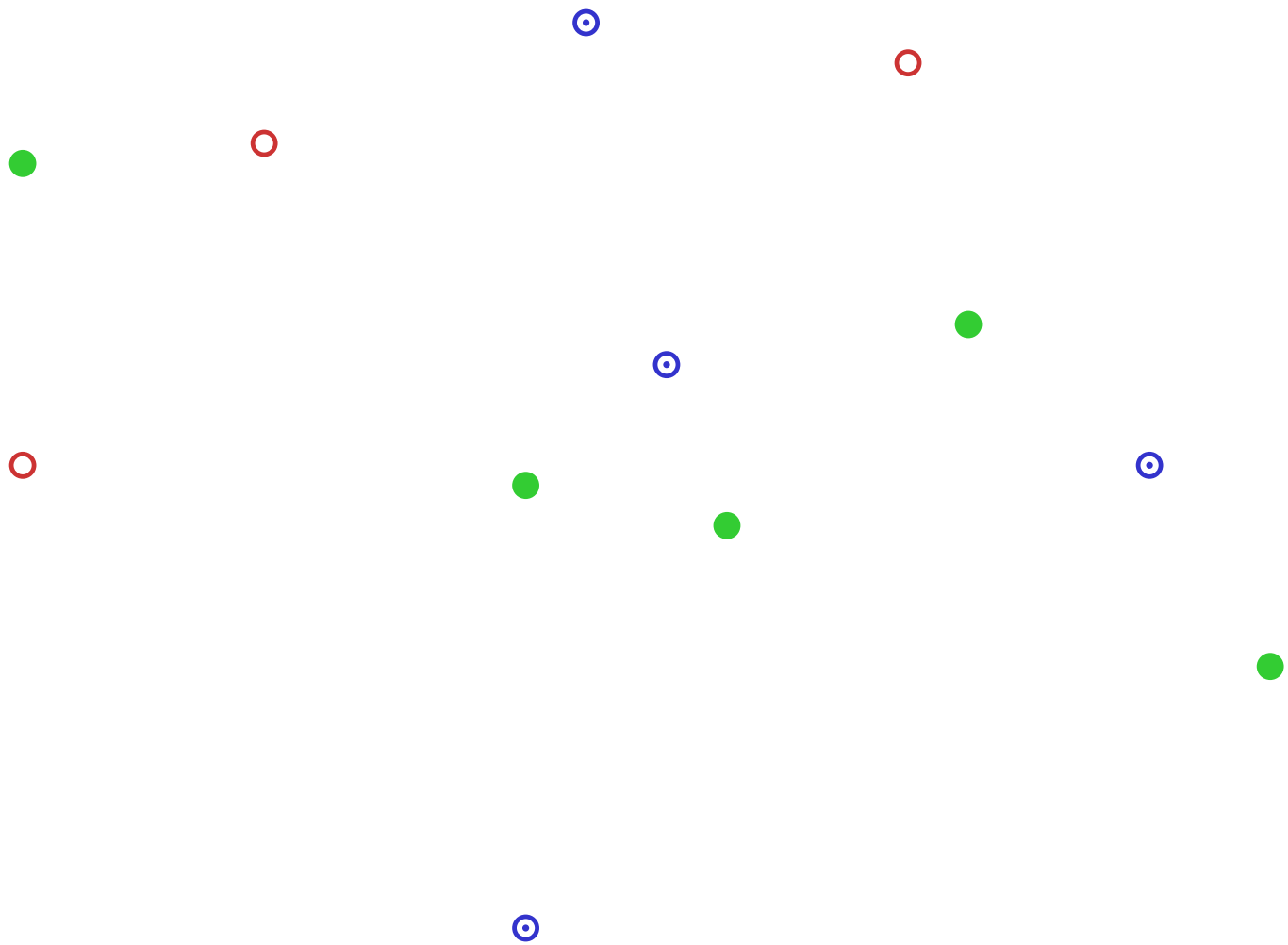
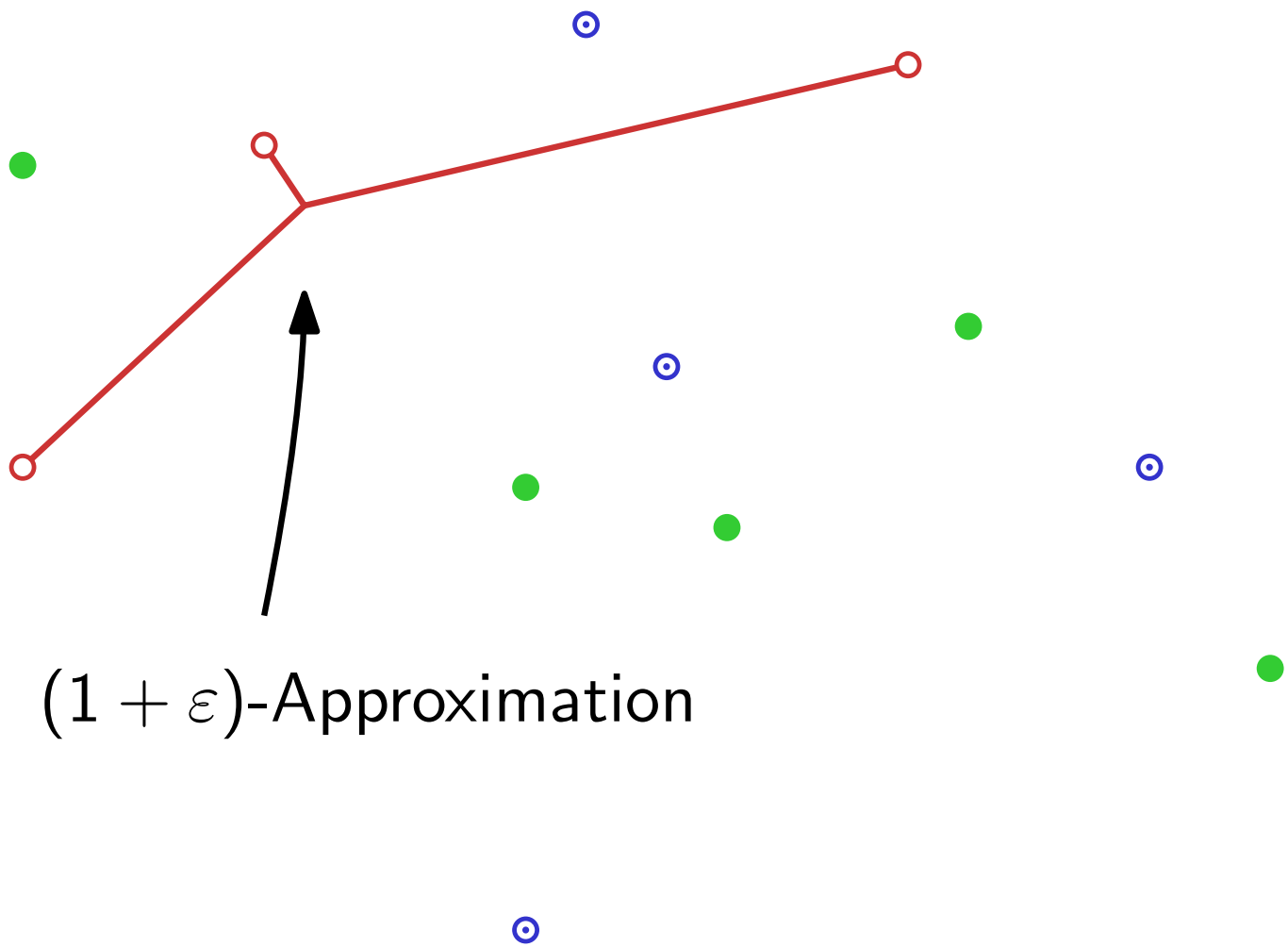


n Punkte
k Farben

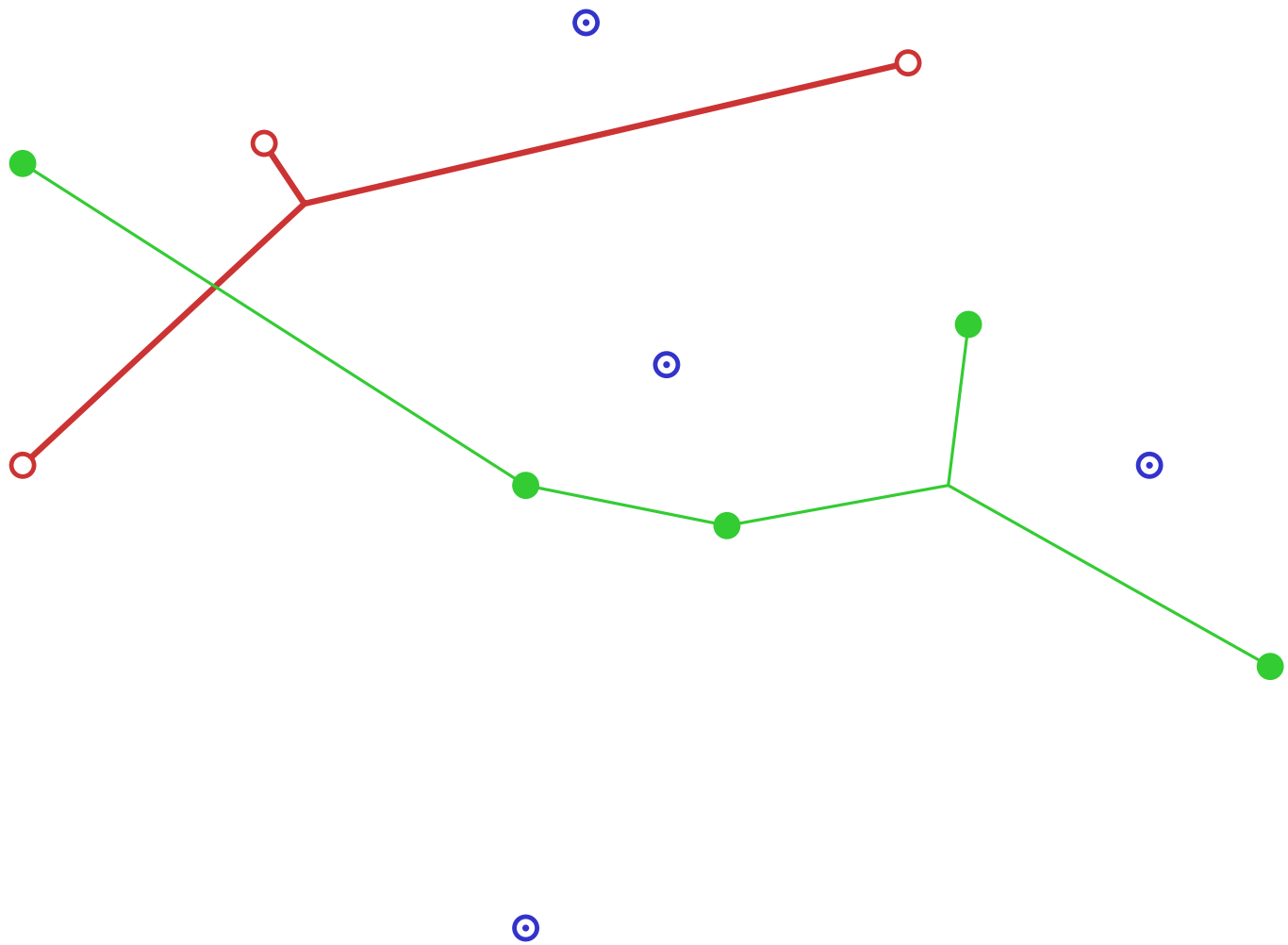
$(k + \varepsilon)$-Approximation



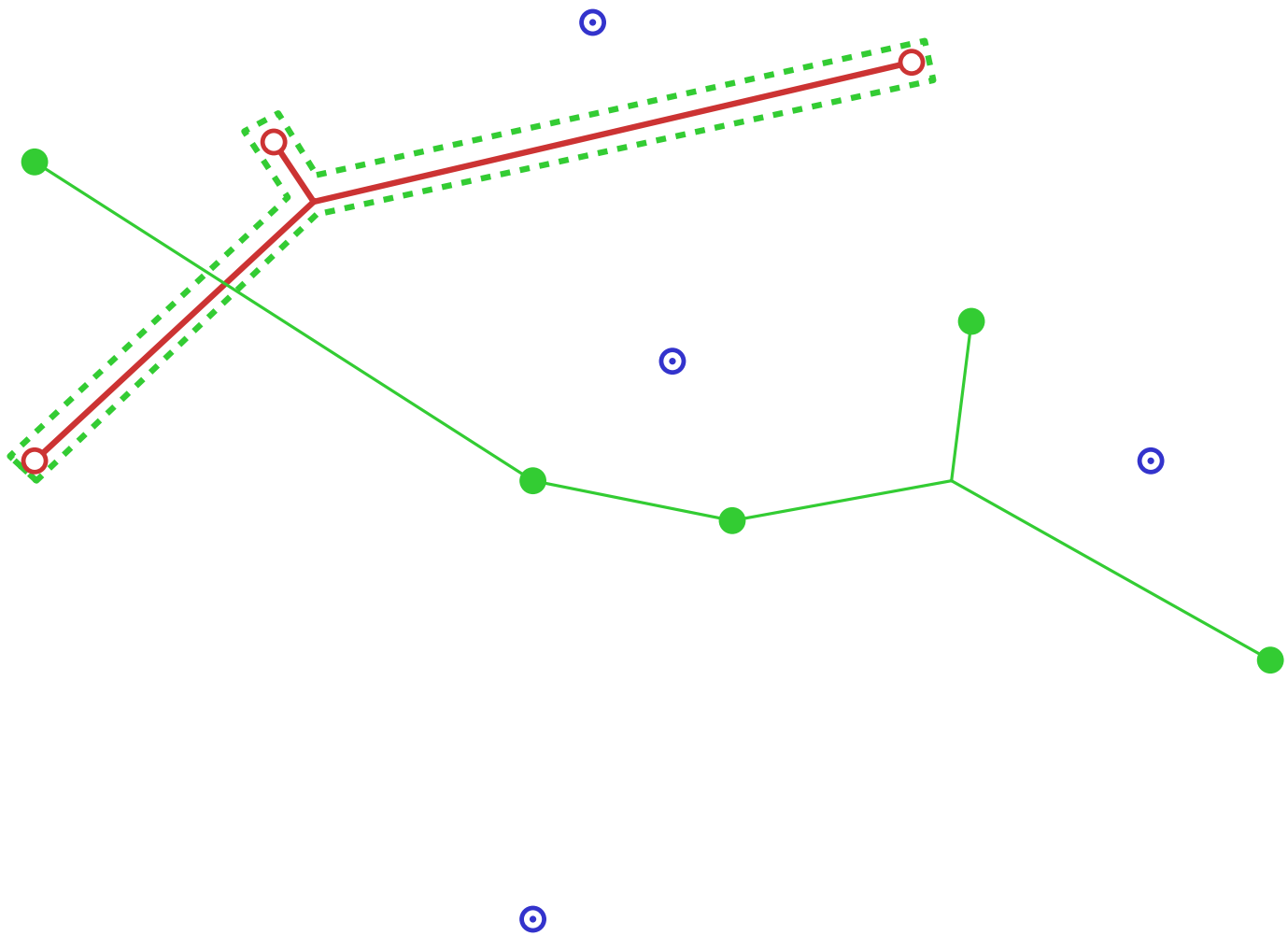
$(k + \varepsilon)$-Approximation



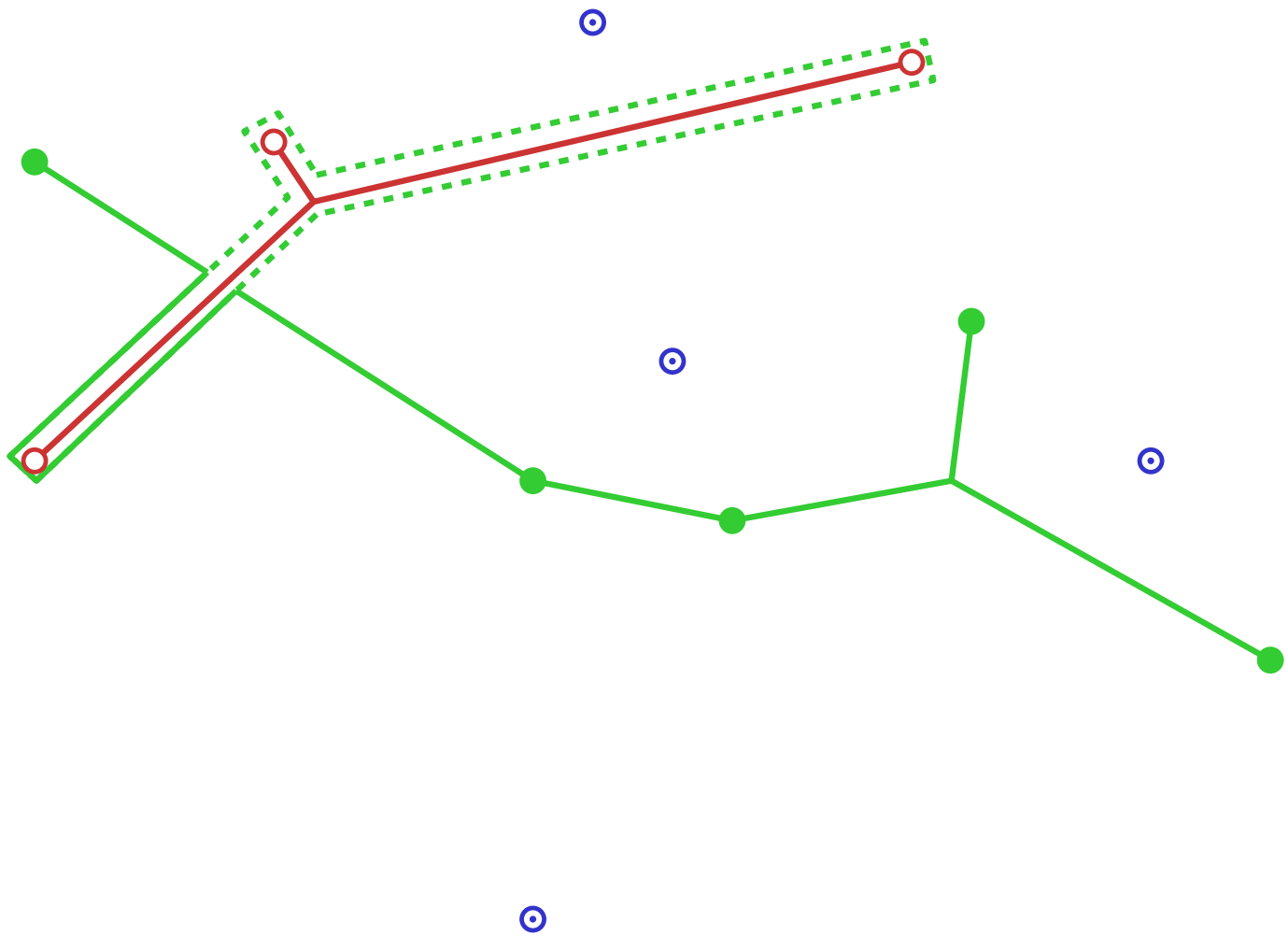
$(k + \varepsilon)$-Approximation



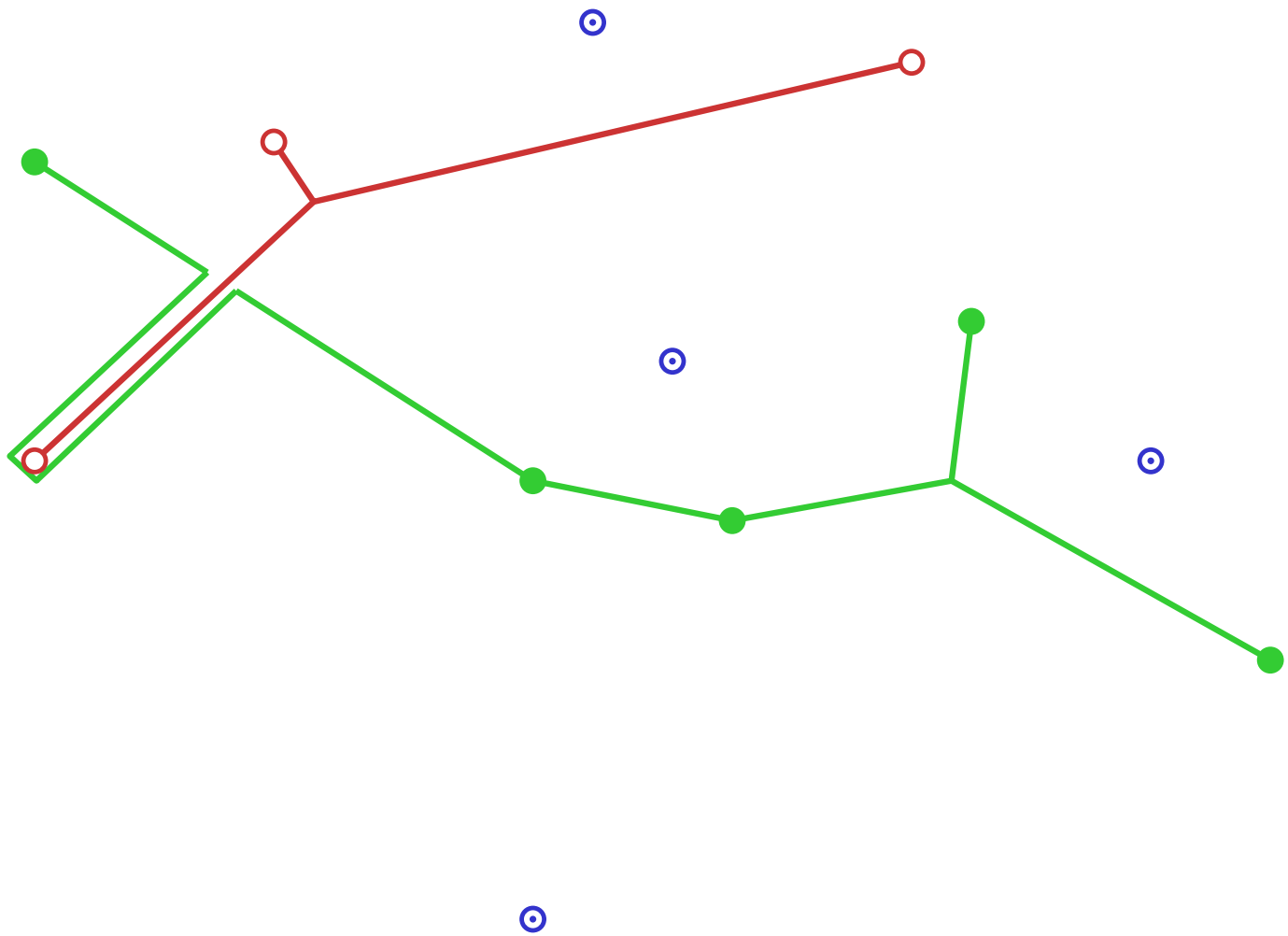
$(k + \varepsilon)$-Approximation



$(k + \varepsilon)$-Approximation



$(k + \varepsilon)$-Approximation



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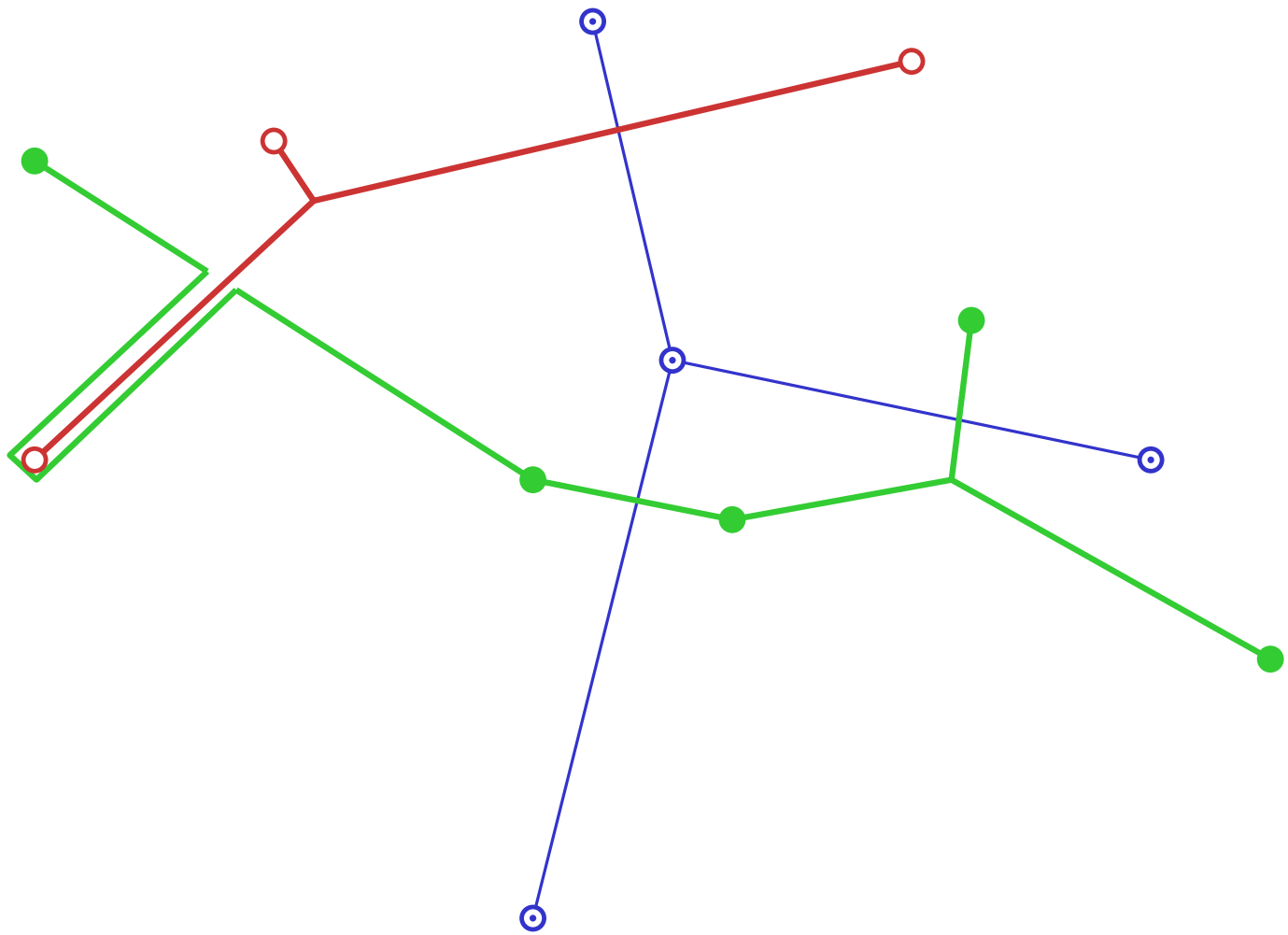
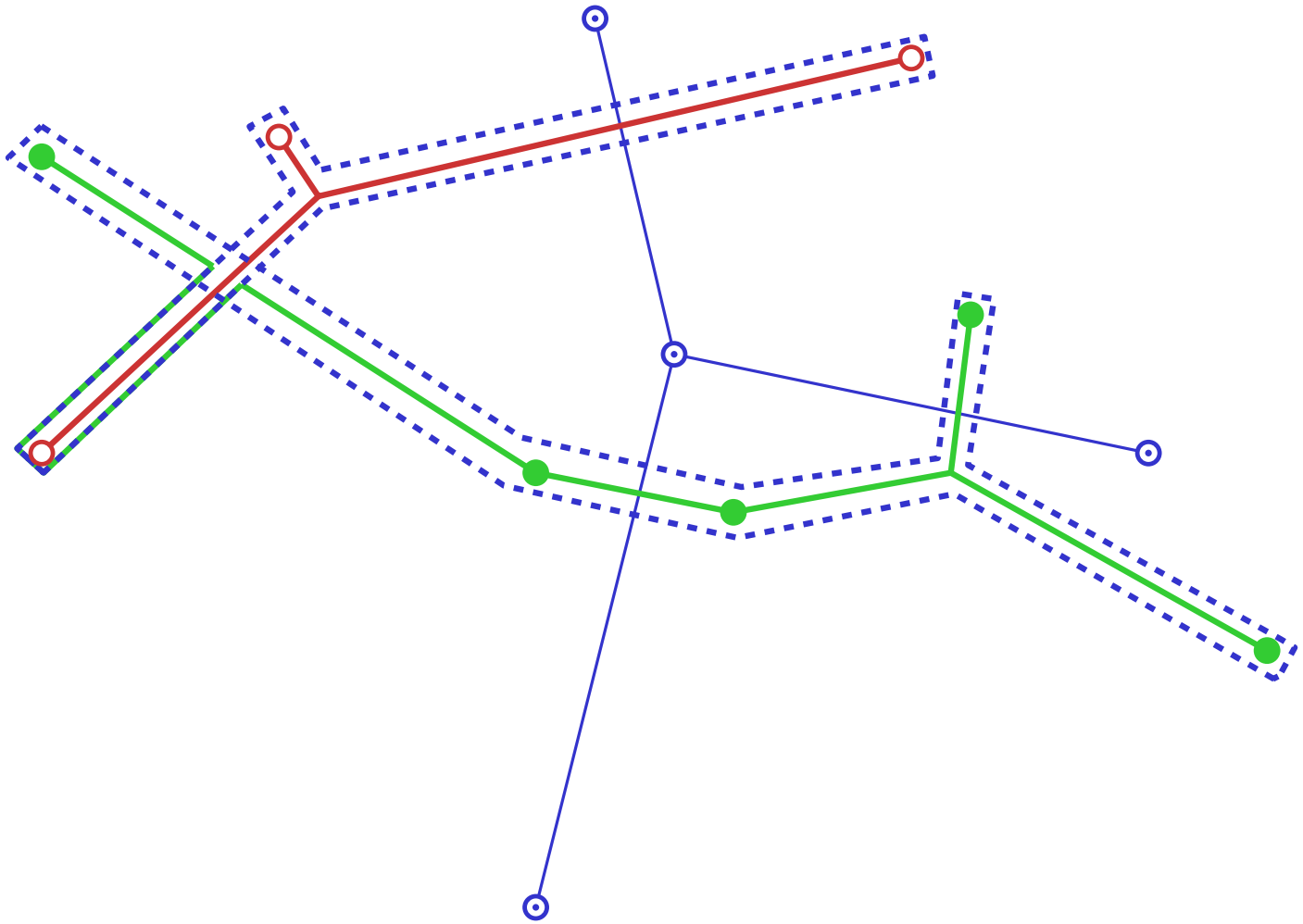
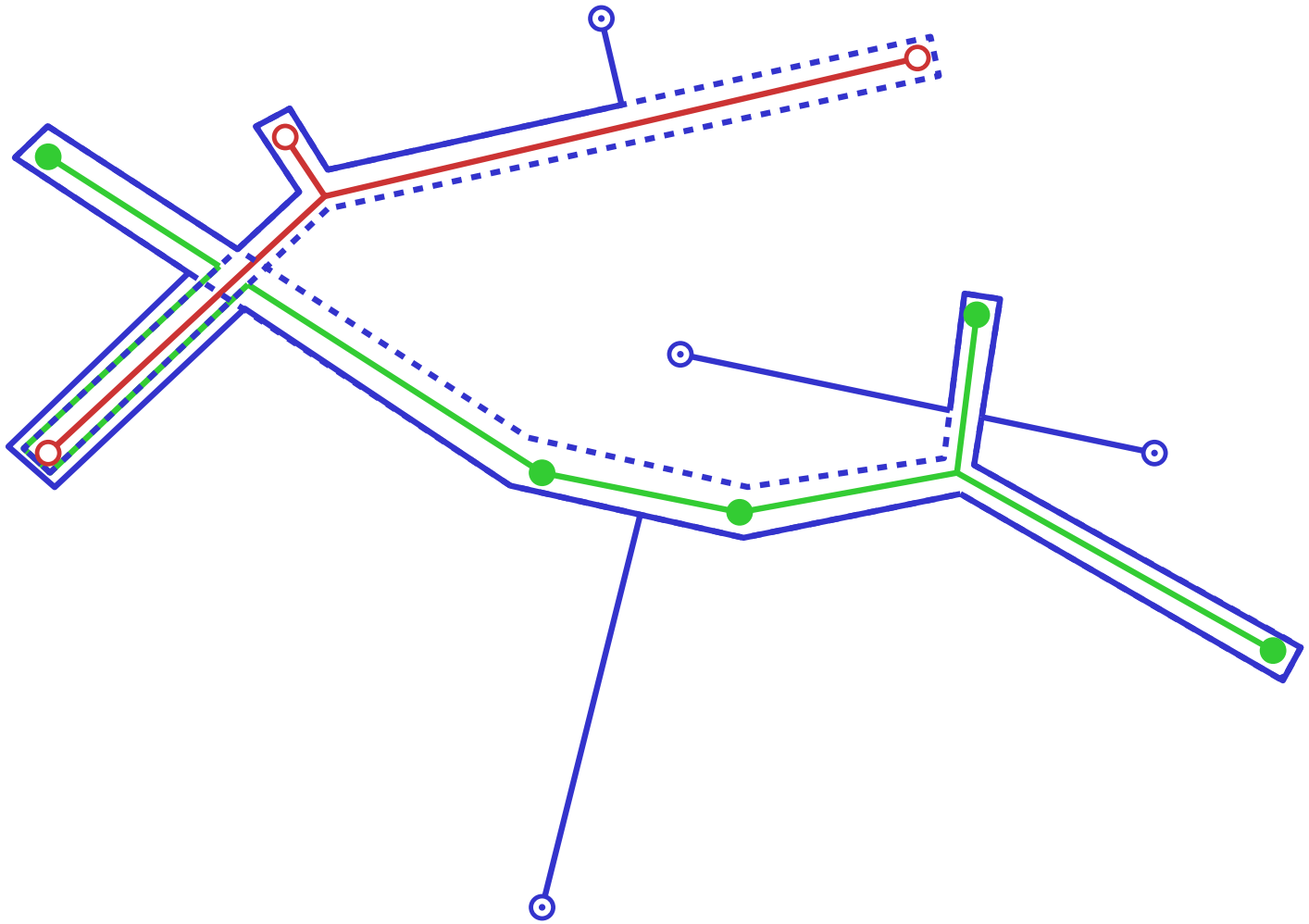


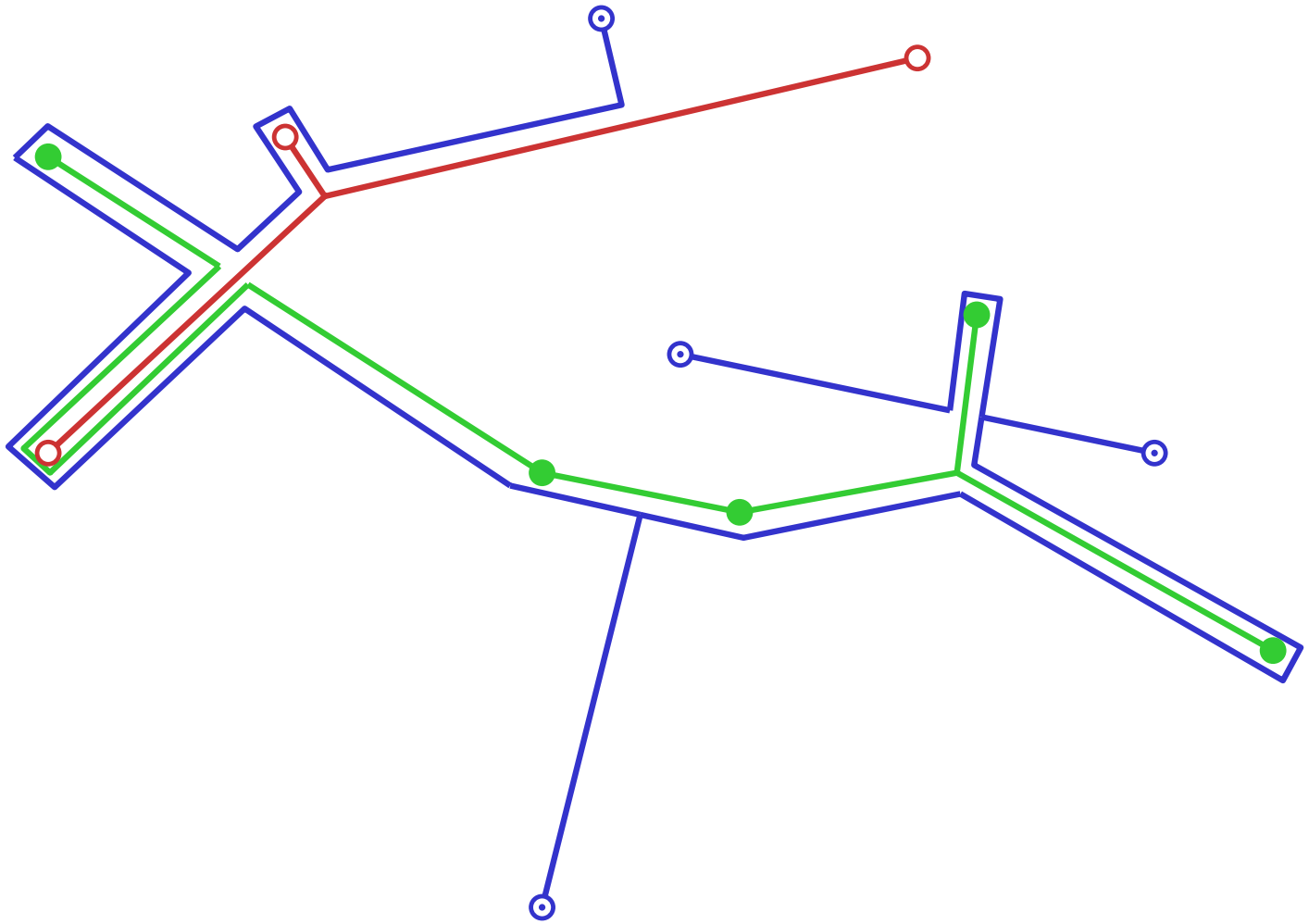
$(k + \varepsilon)$-Approximation



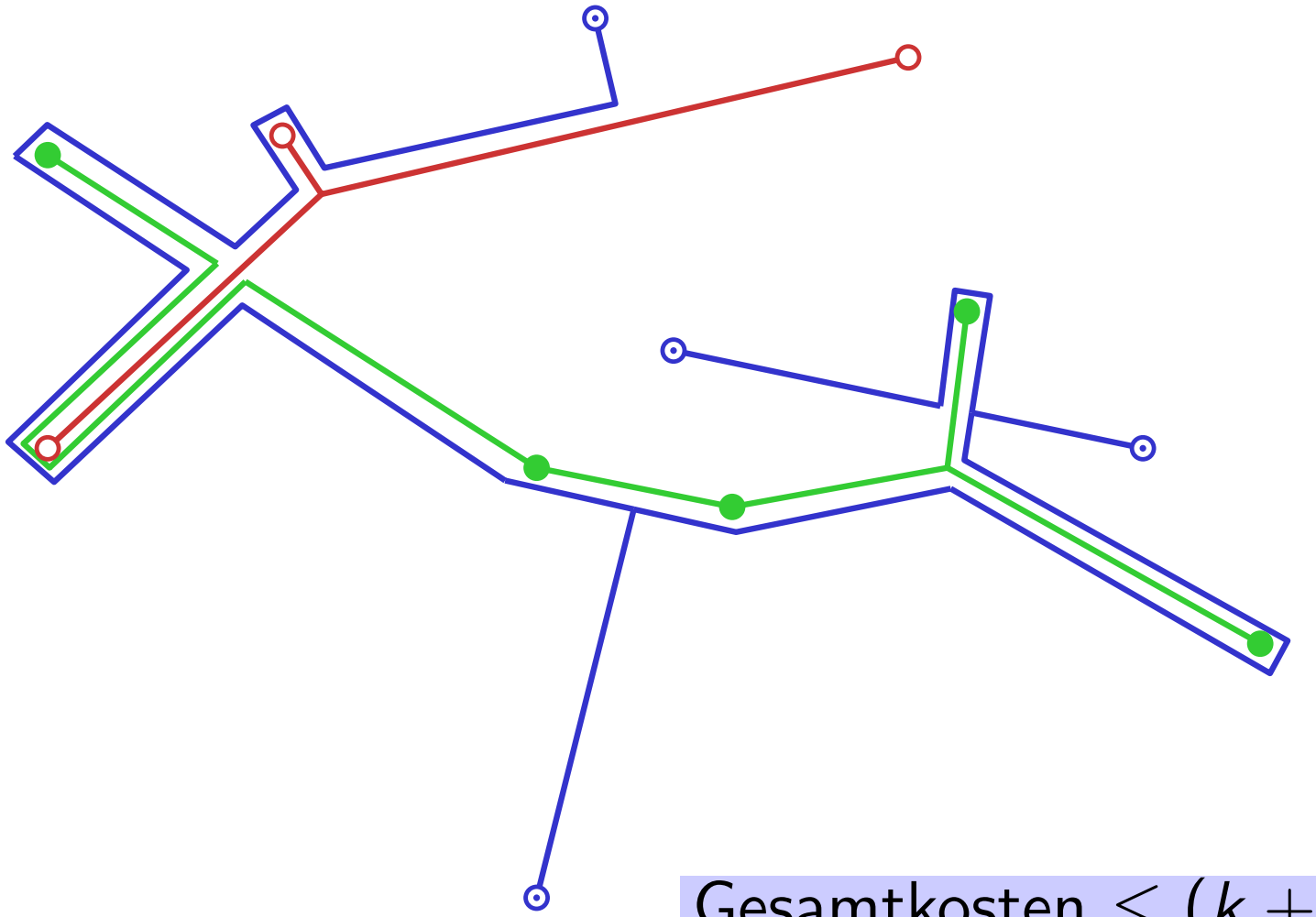
$(k + \varepsilon)$-Approximation



$(k + \varepsilon)$-Approximation

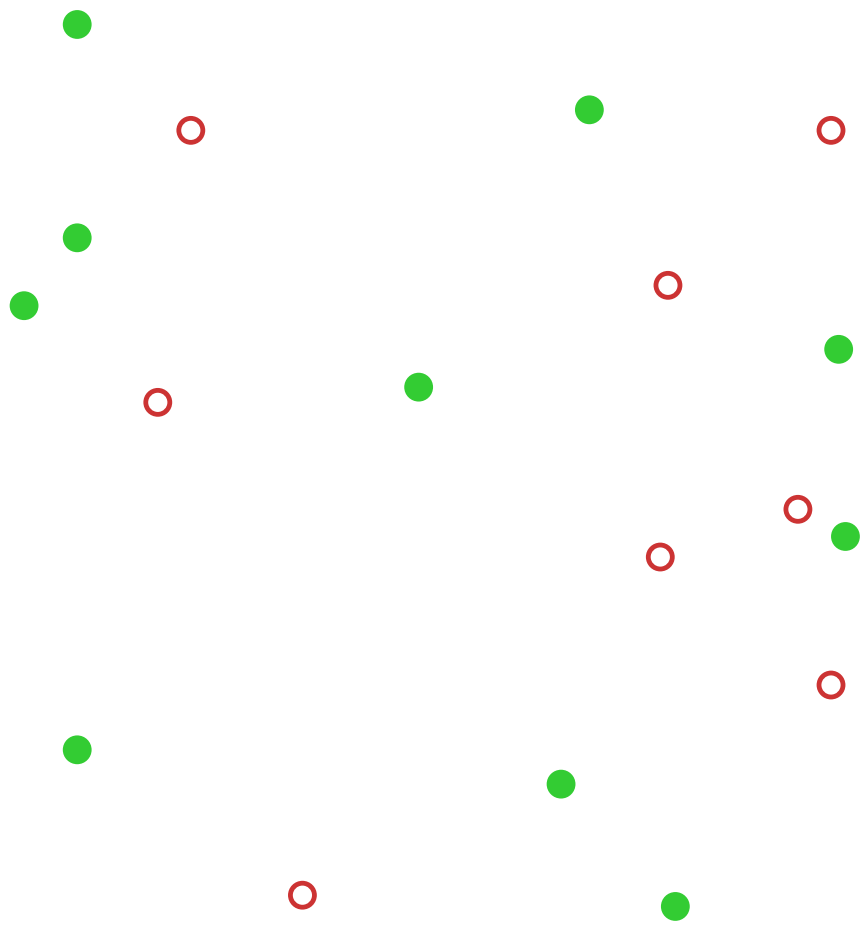


$(k + \varepsilon)$-Approximation

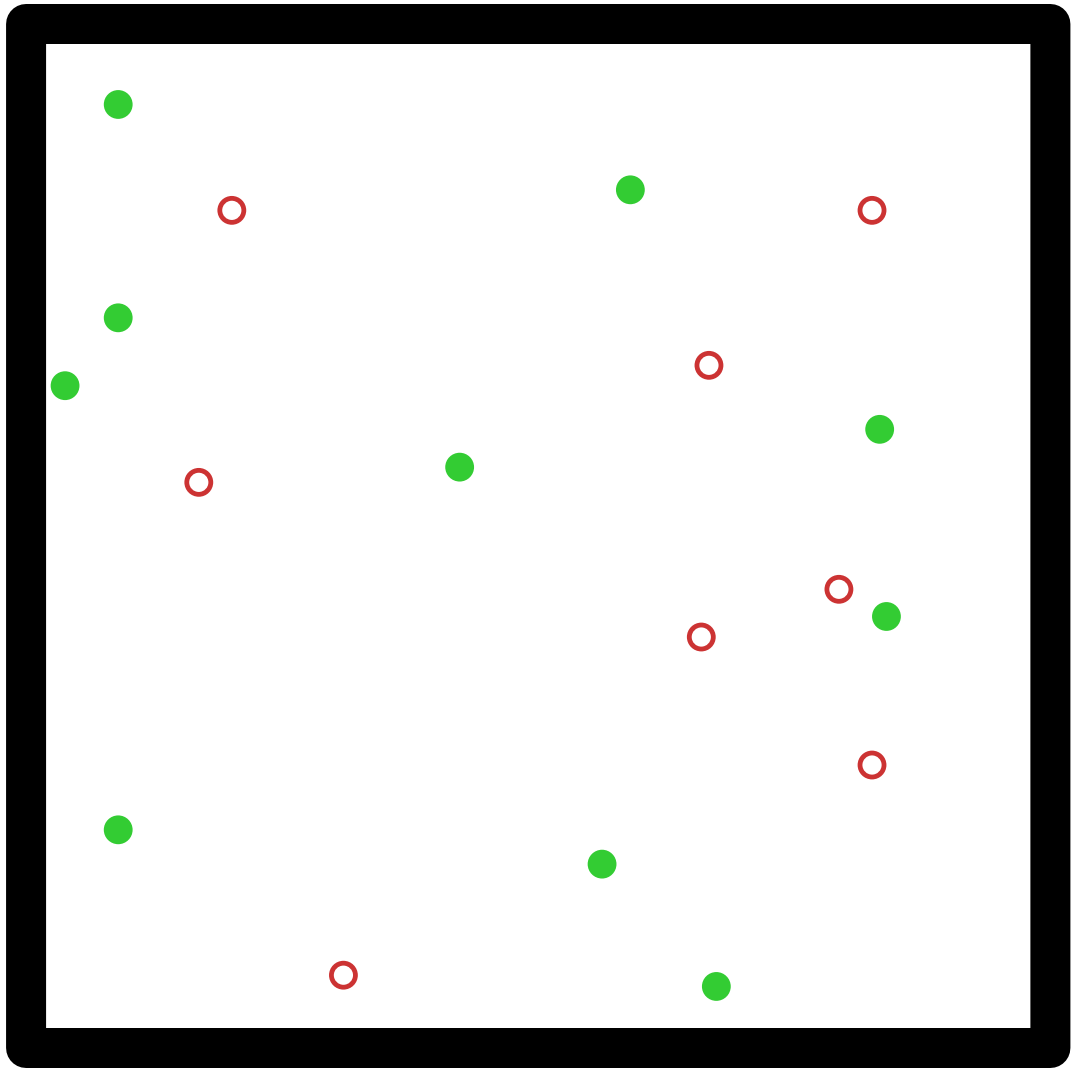


Gesamtkosten $\leq (k + \varepsilon)$ OPT

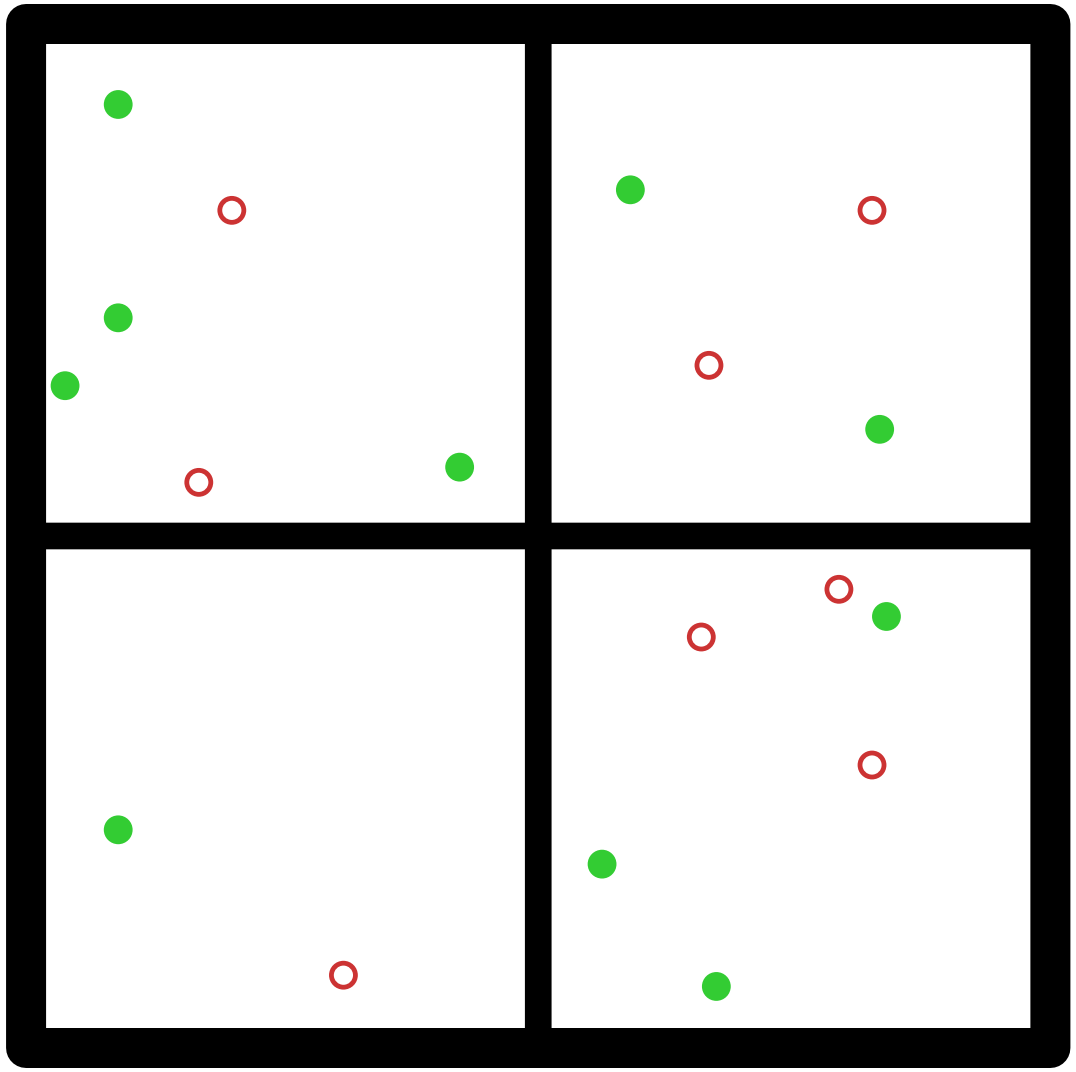
$(1+\varepsilon)$-Approximation für $k = 2$



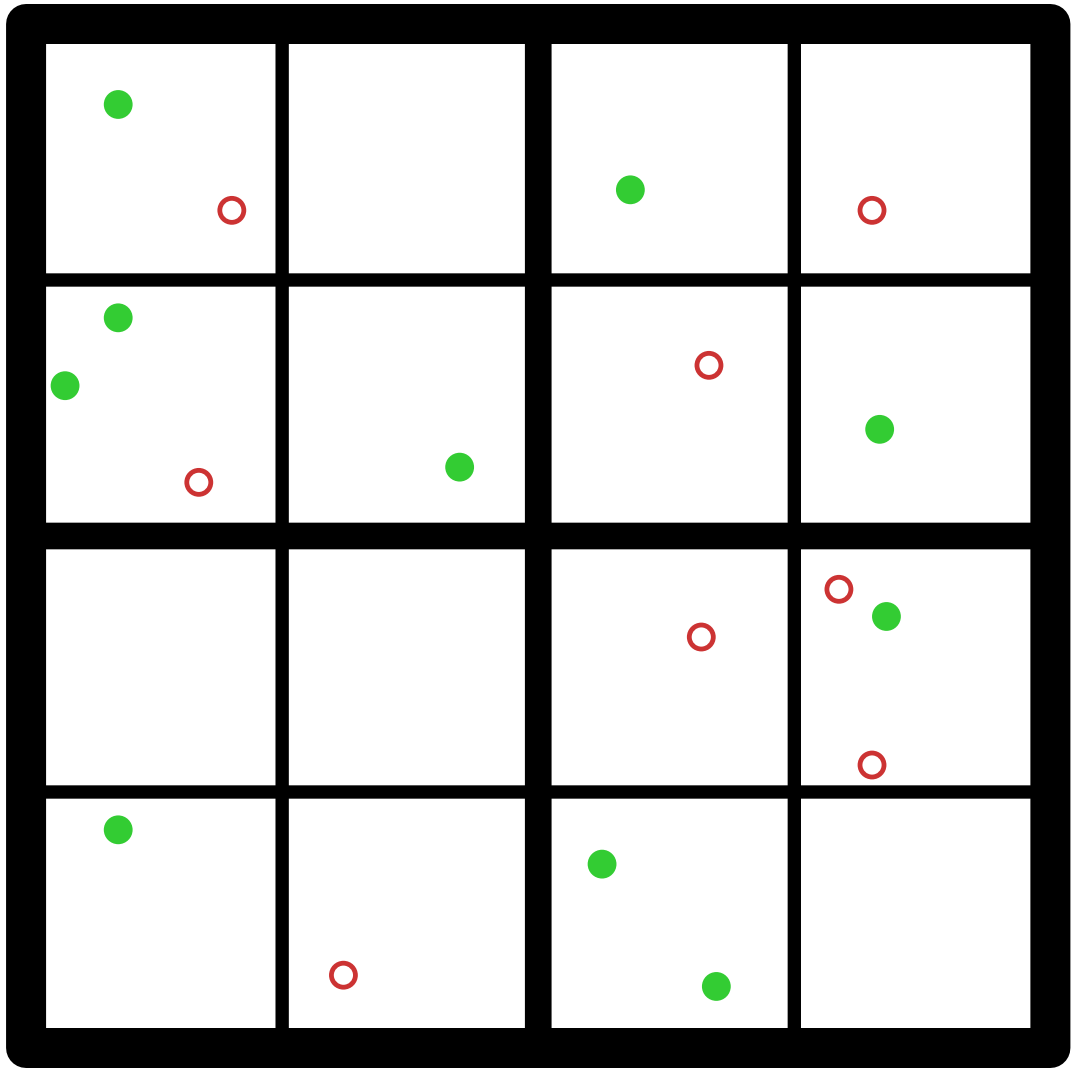
$(1+\varepsilon)$-Approximation für $k = 2$



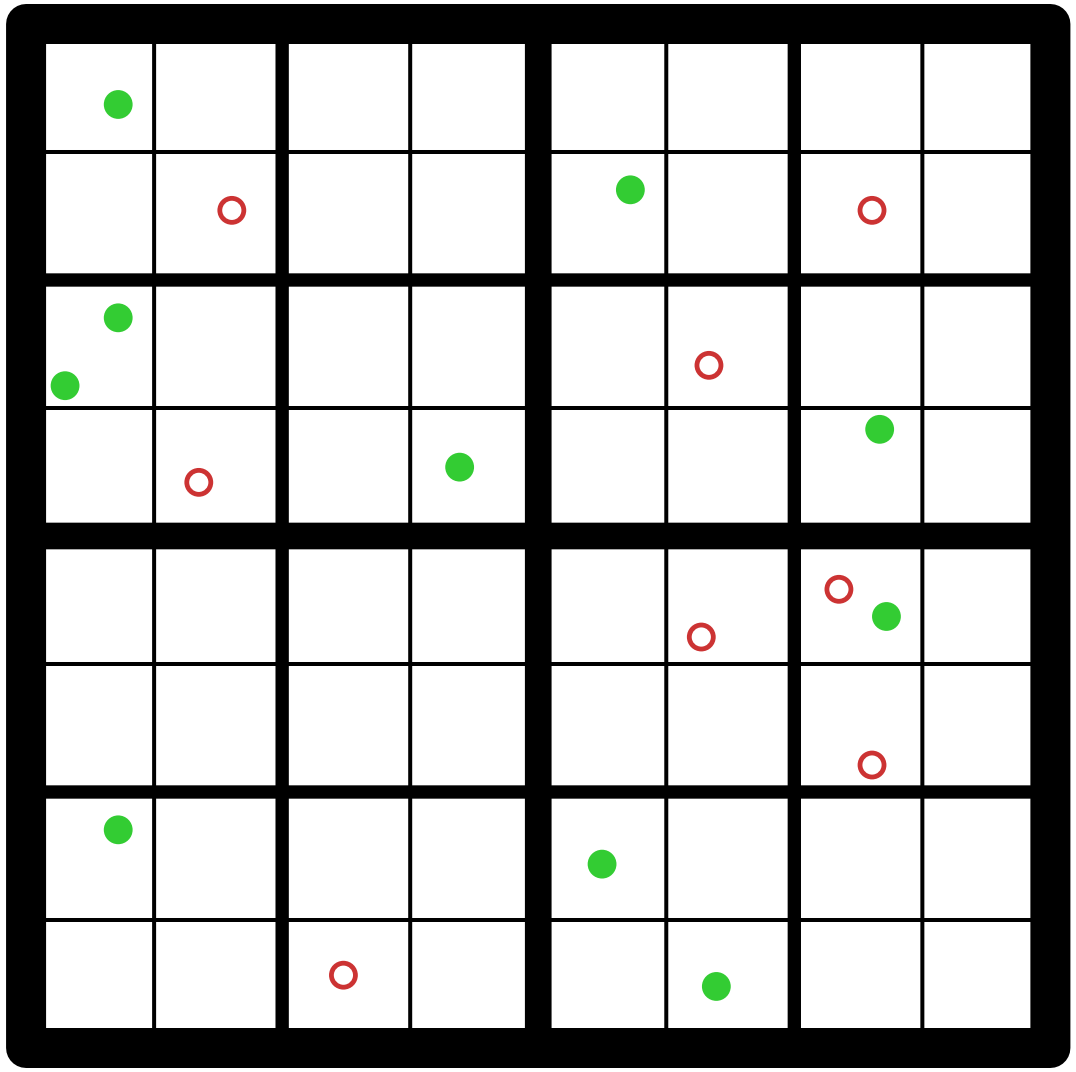
$(1+\varepsilon)$-Approximation für $k = 2$



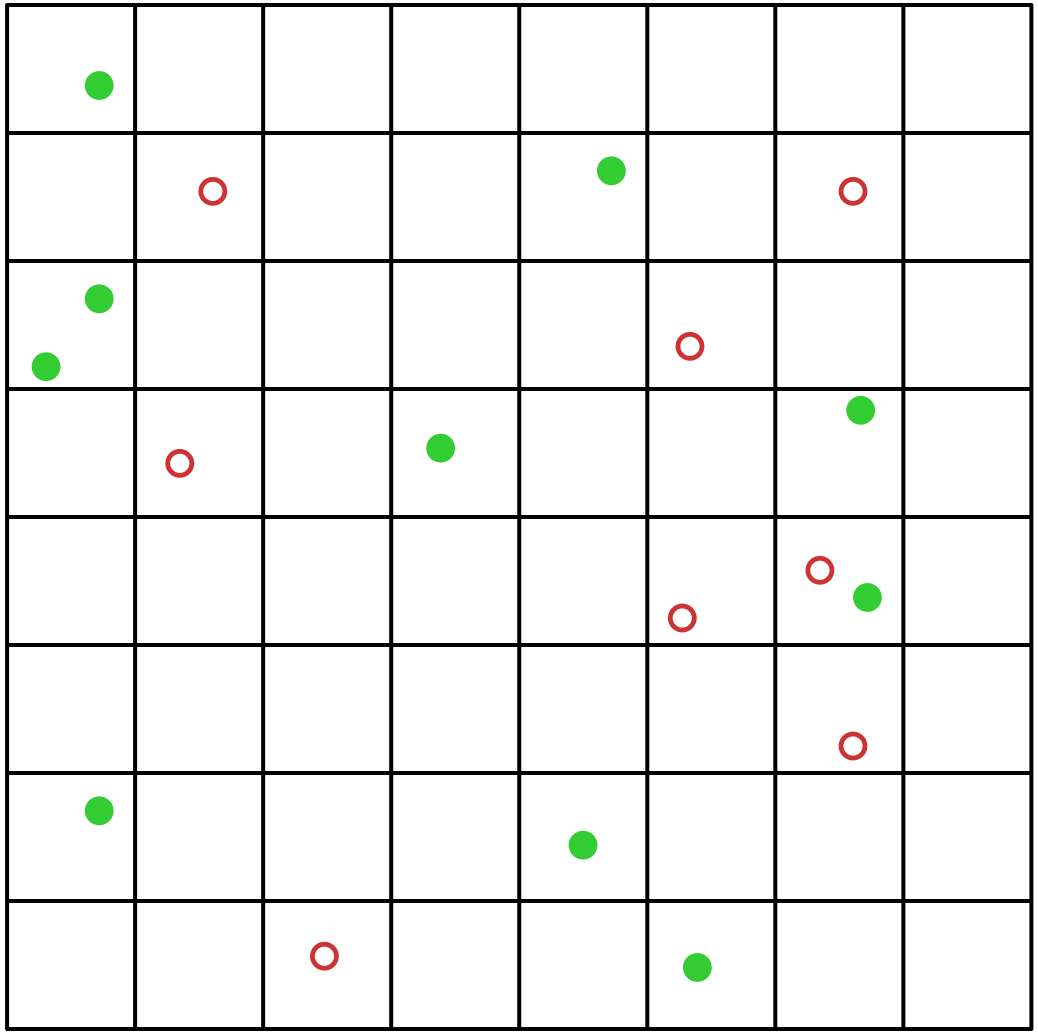
$(1+\varepsilon)$-Approximation für $k = 2$



$(1+\varepsilon)$-Approximation für $k = 2$

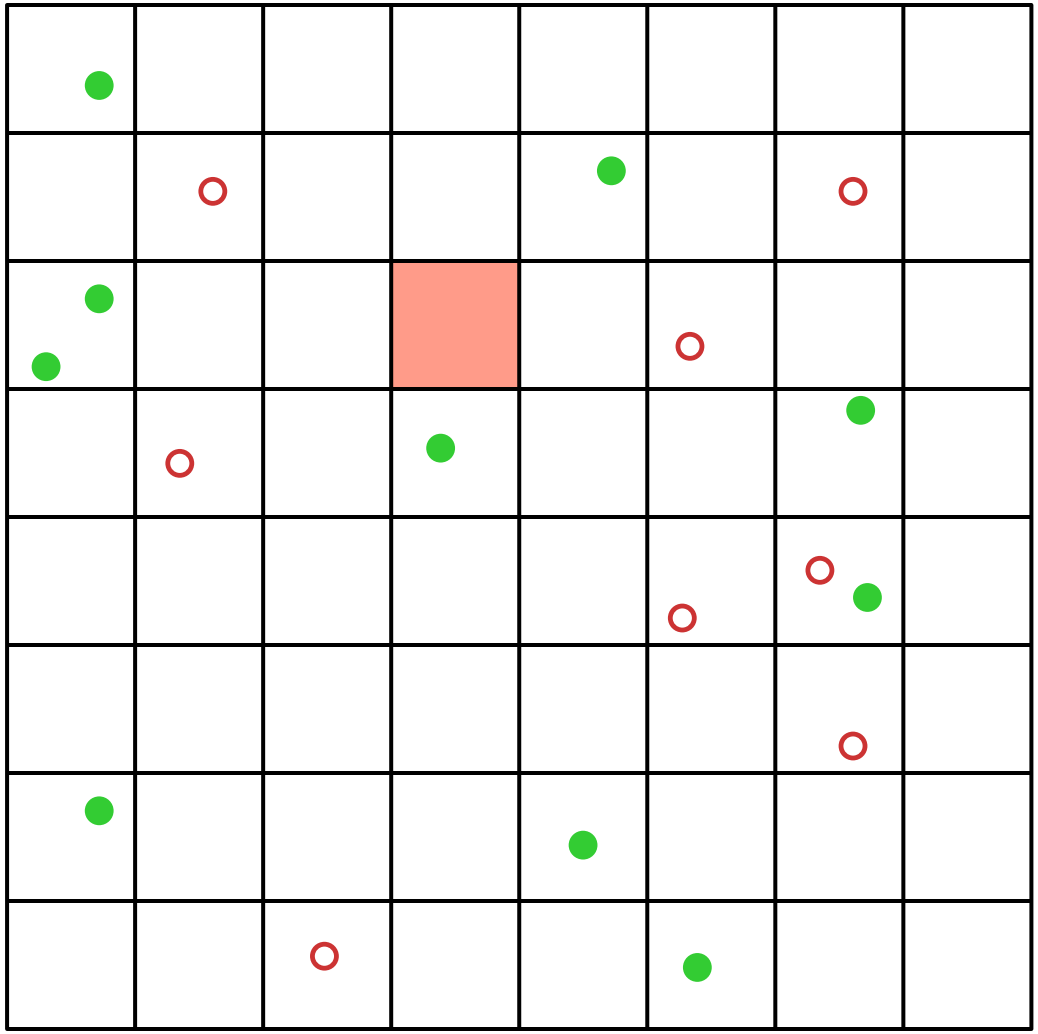


$(1+\varepsilon)$-Approximation für $k = 2$



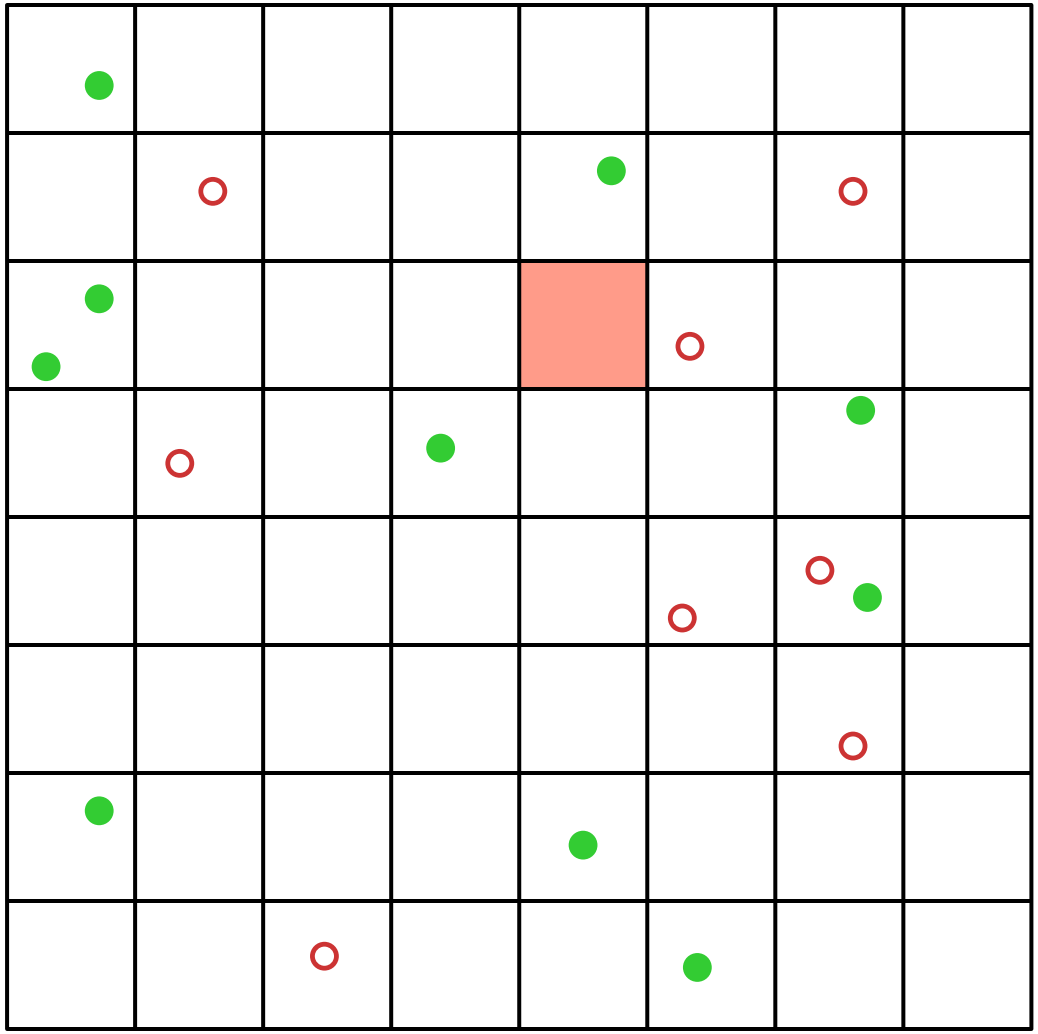
$\mathcal{O}(n) \times \mathcal{O}(n)$ Gitter

$(1+\varepsilon)$-Approximation für $k = 2$



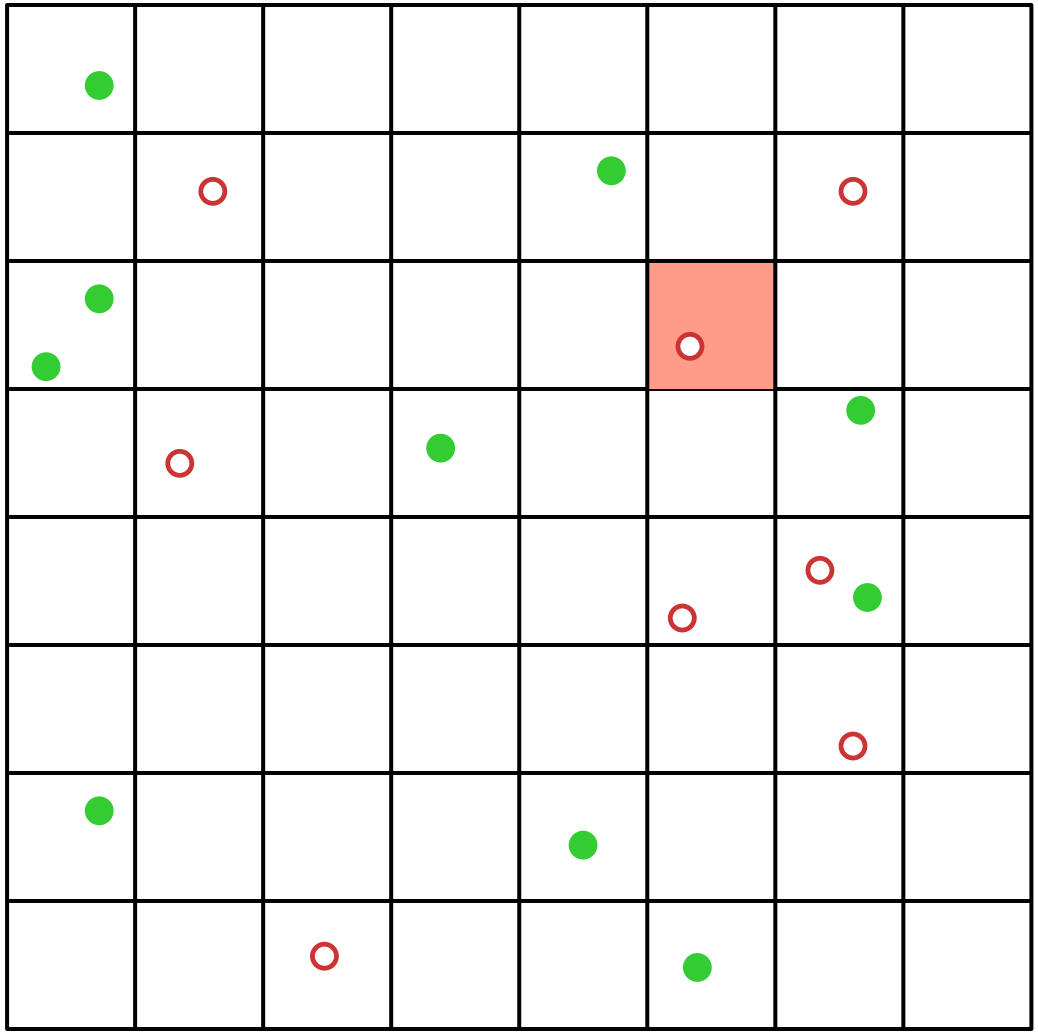
$\mathcal{O}(n) \times \mathcal{O}(n)$ Gitter

$(1+\varepsilon)$-Approximation für $k = 2$



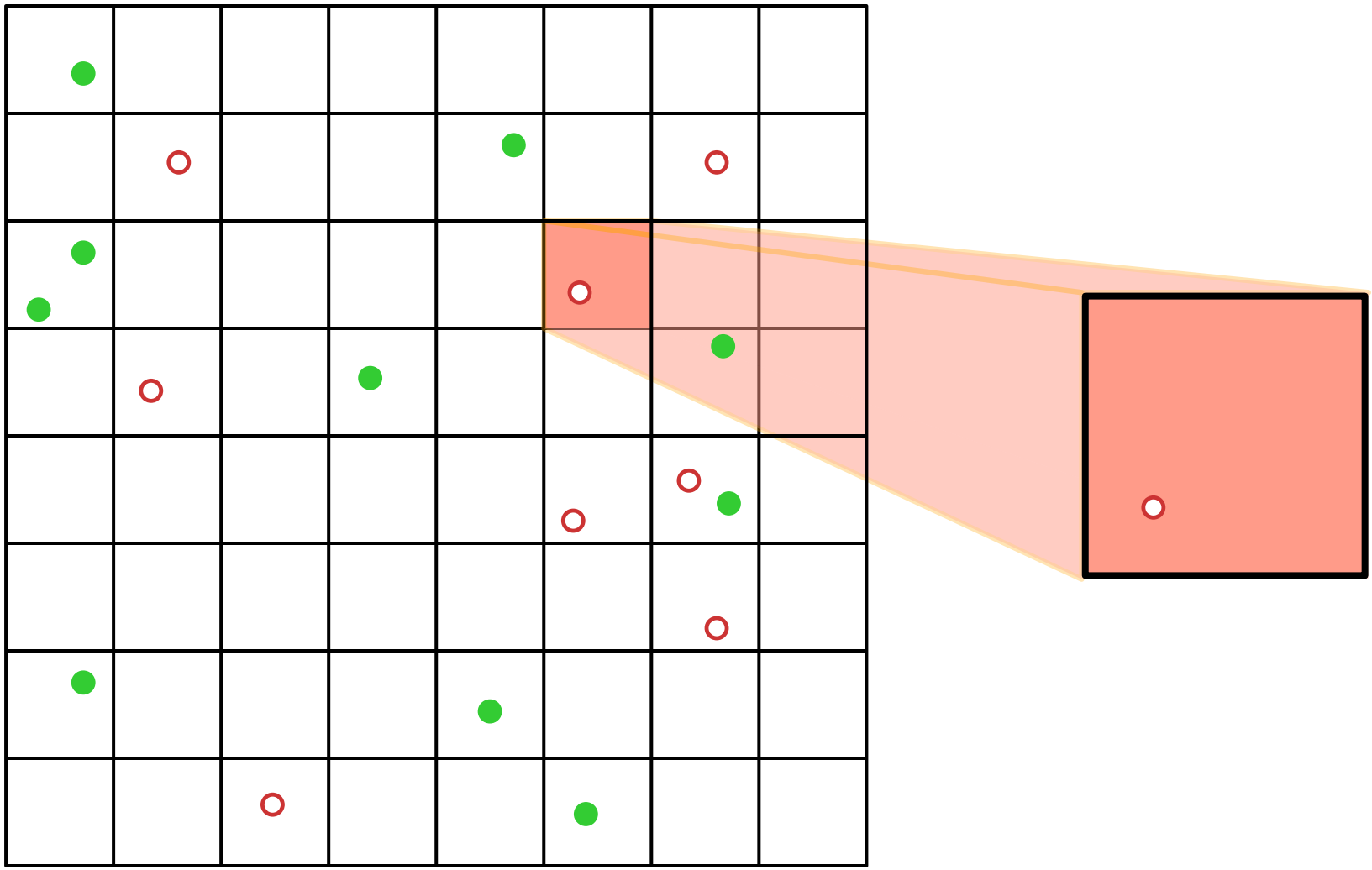
$\mathcal{O}(n) \times \mathcal{O}(n)$ Gitter

$(1+\varepsilon)$-Approximation für $k = 2$

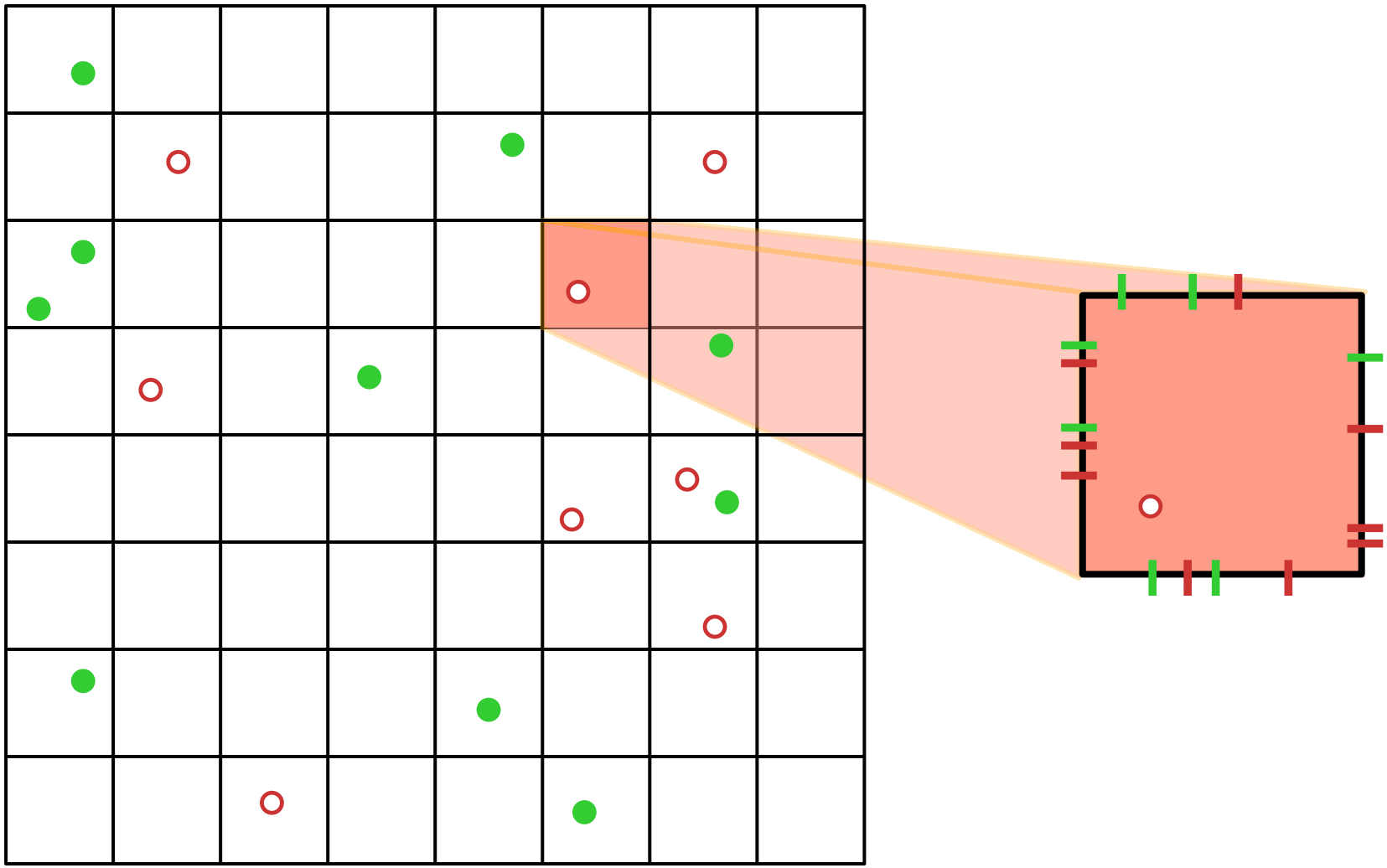


$\mathcal{O}(n) \times \mathcal{O}(n)$ Gitter

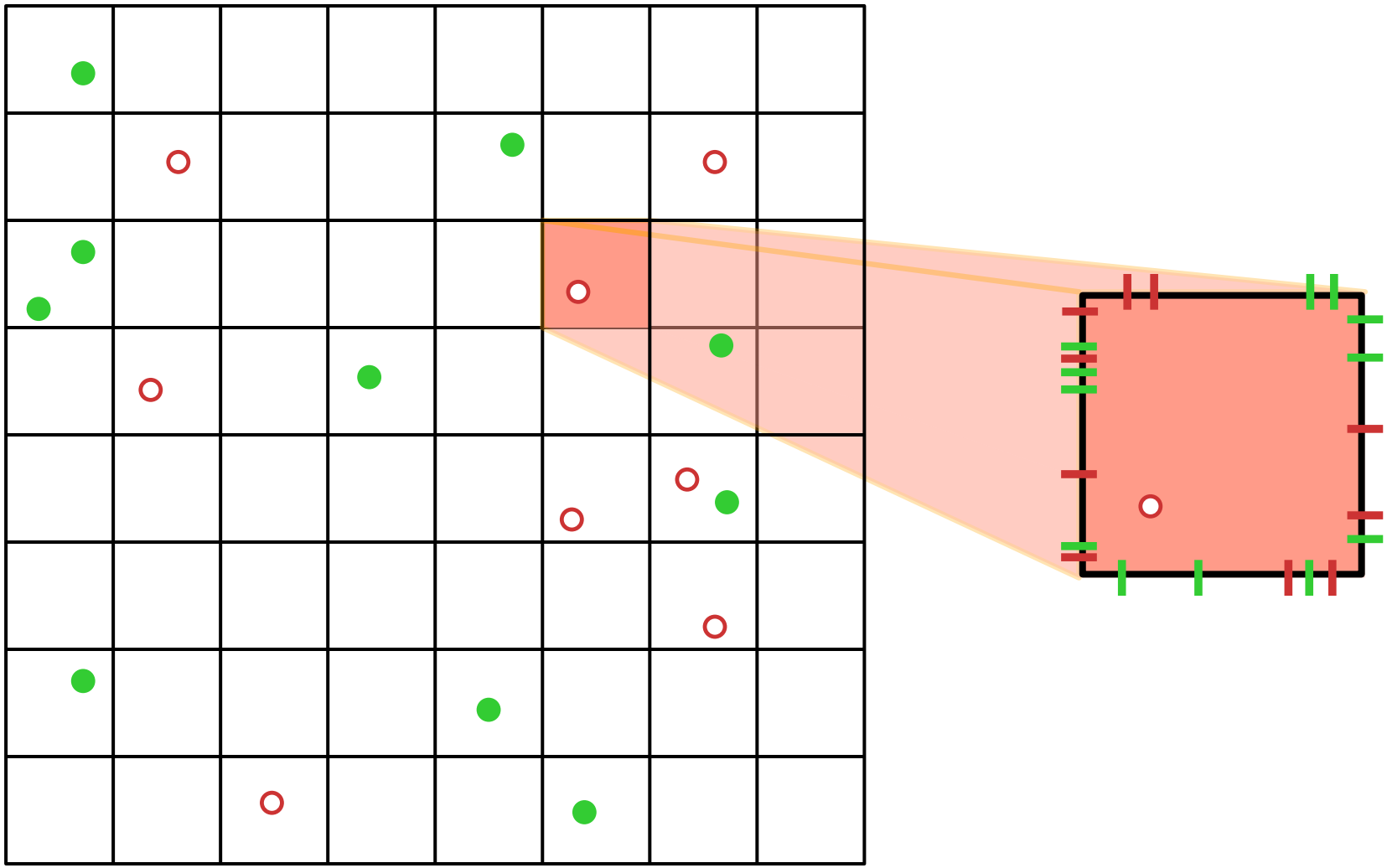
$(1+\varepsilon)$-Approximation für $k = 2$



$(1+\varepsilon)$-Approximation für $k = 2$



$(1+\varepsilon)$-Approximation für $k = 2$



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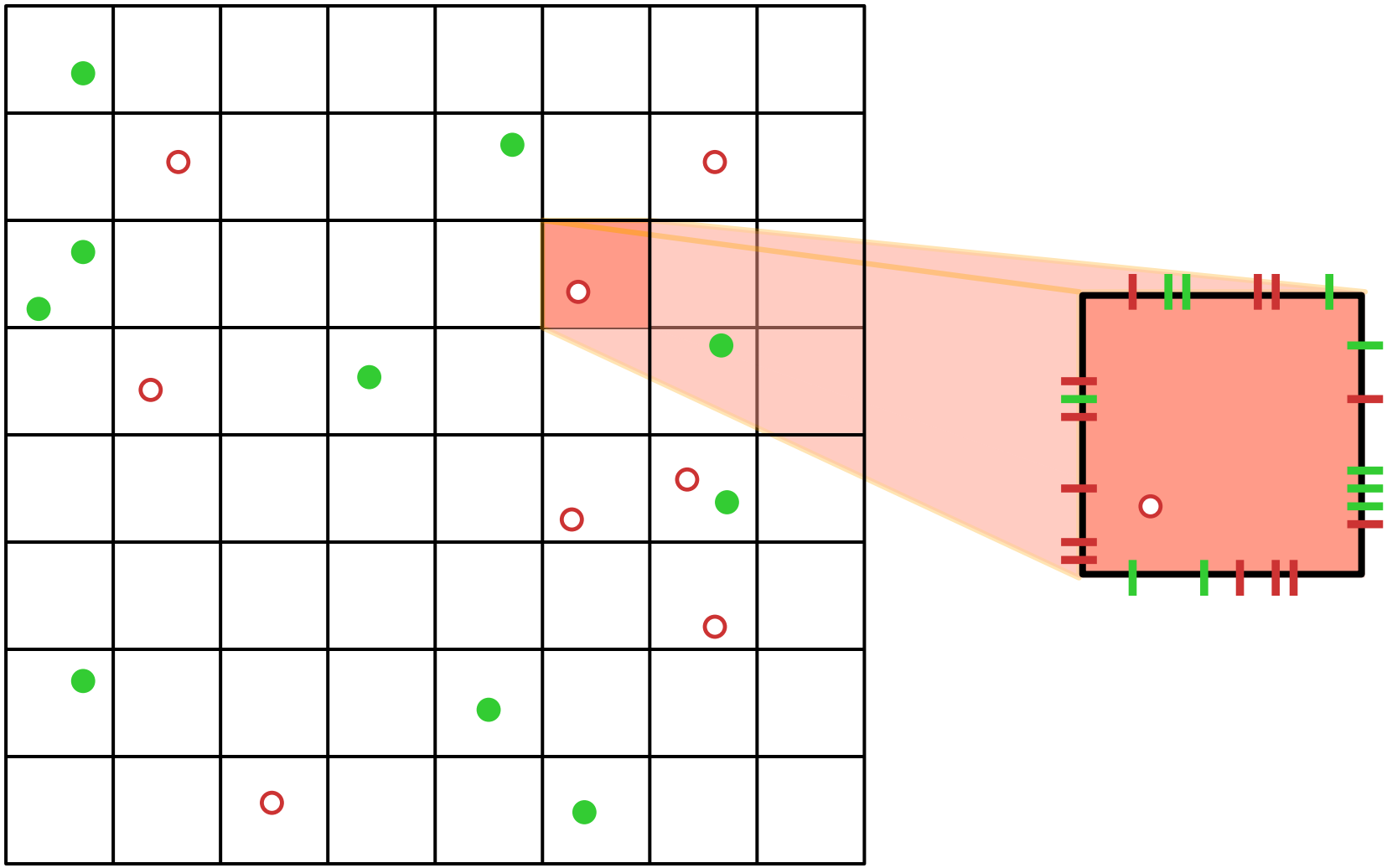
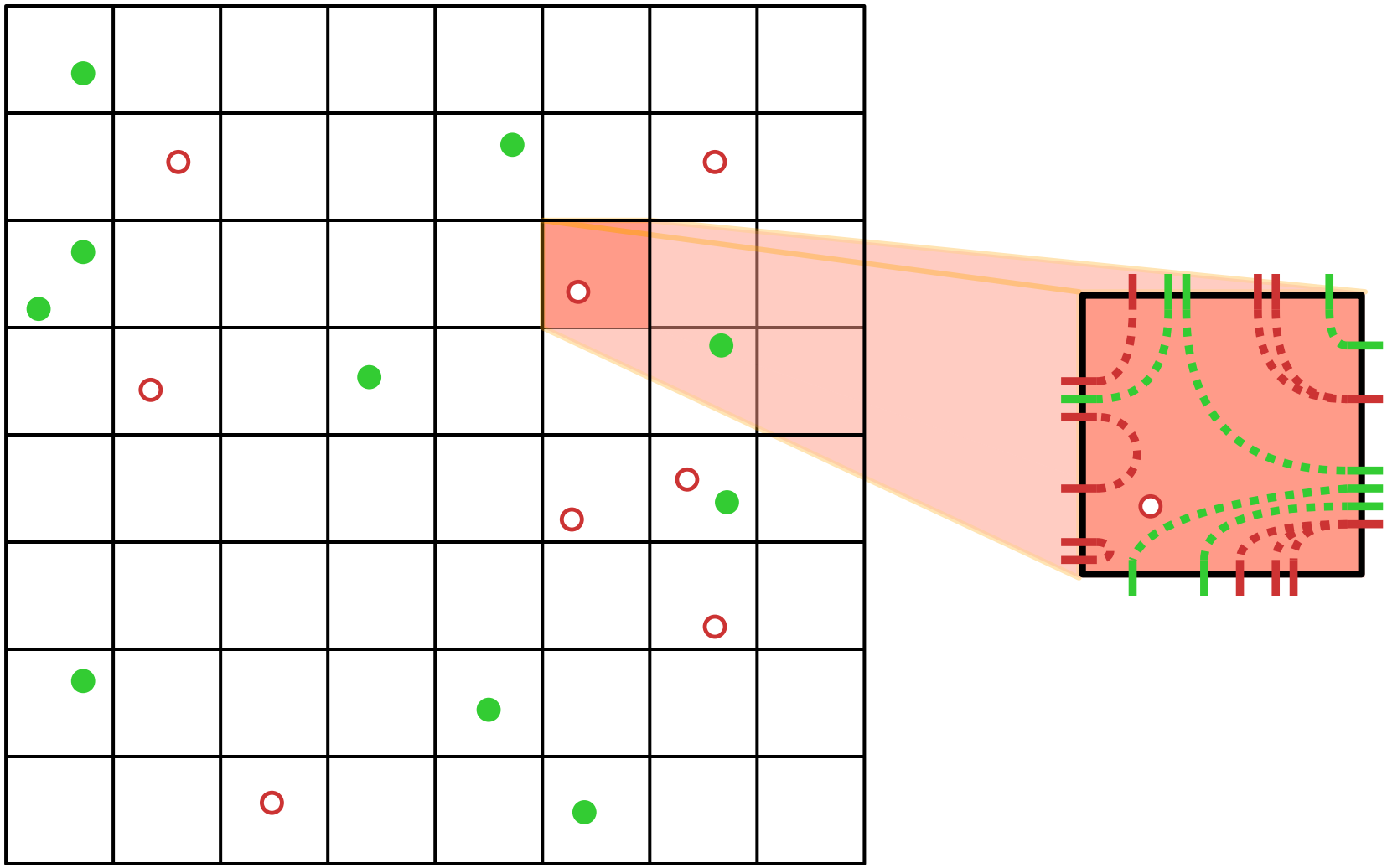
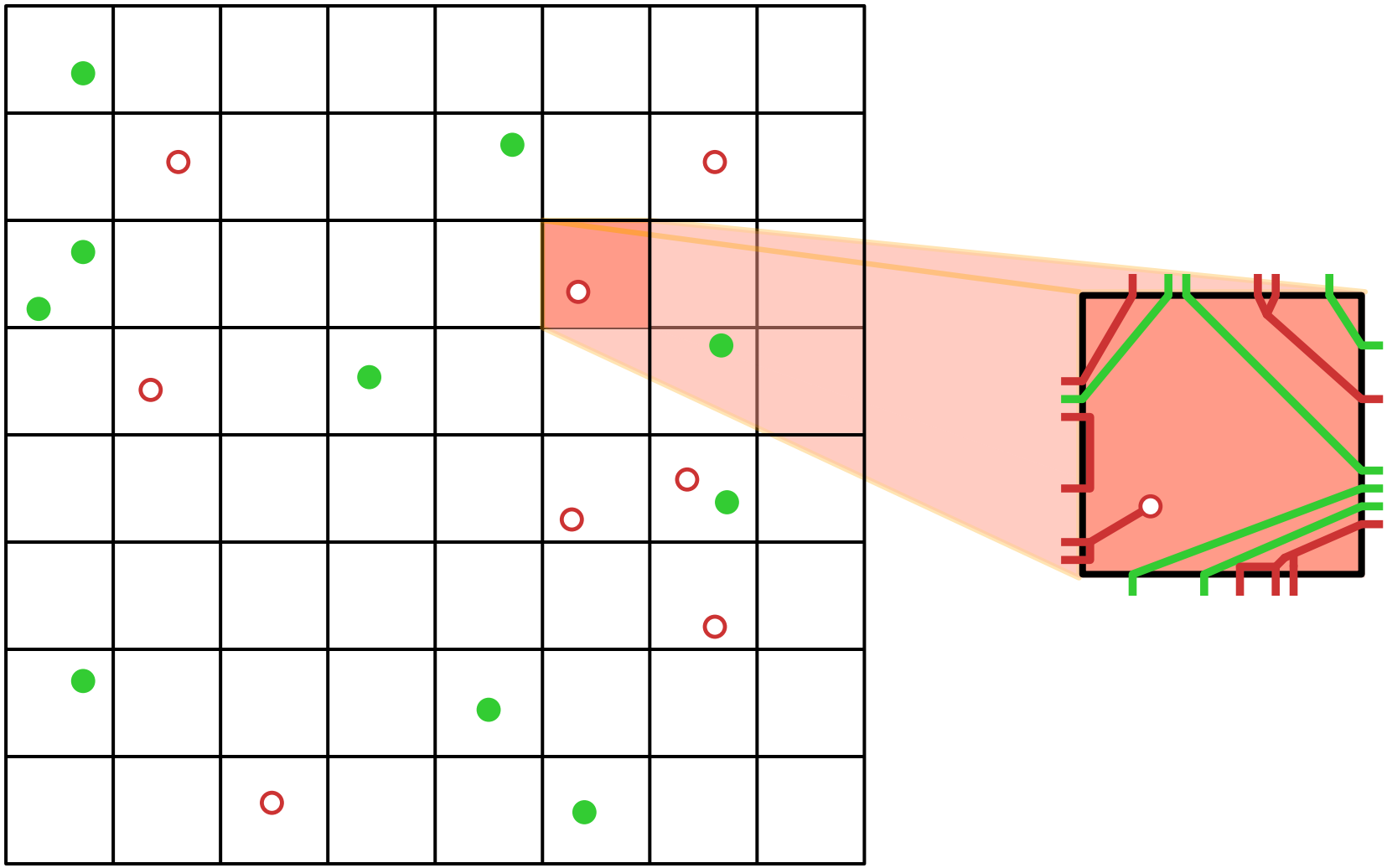


$(1+\varepsilon)$-Approximation für $k = 2$



$(1+\varepsilon)$-Approximation für $k = 2$



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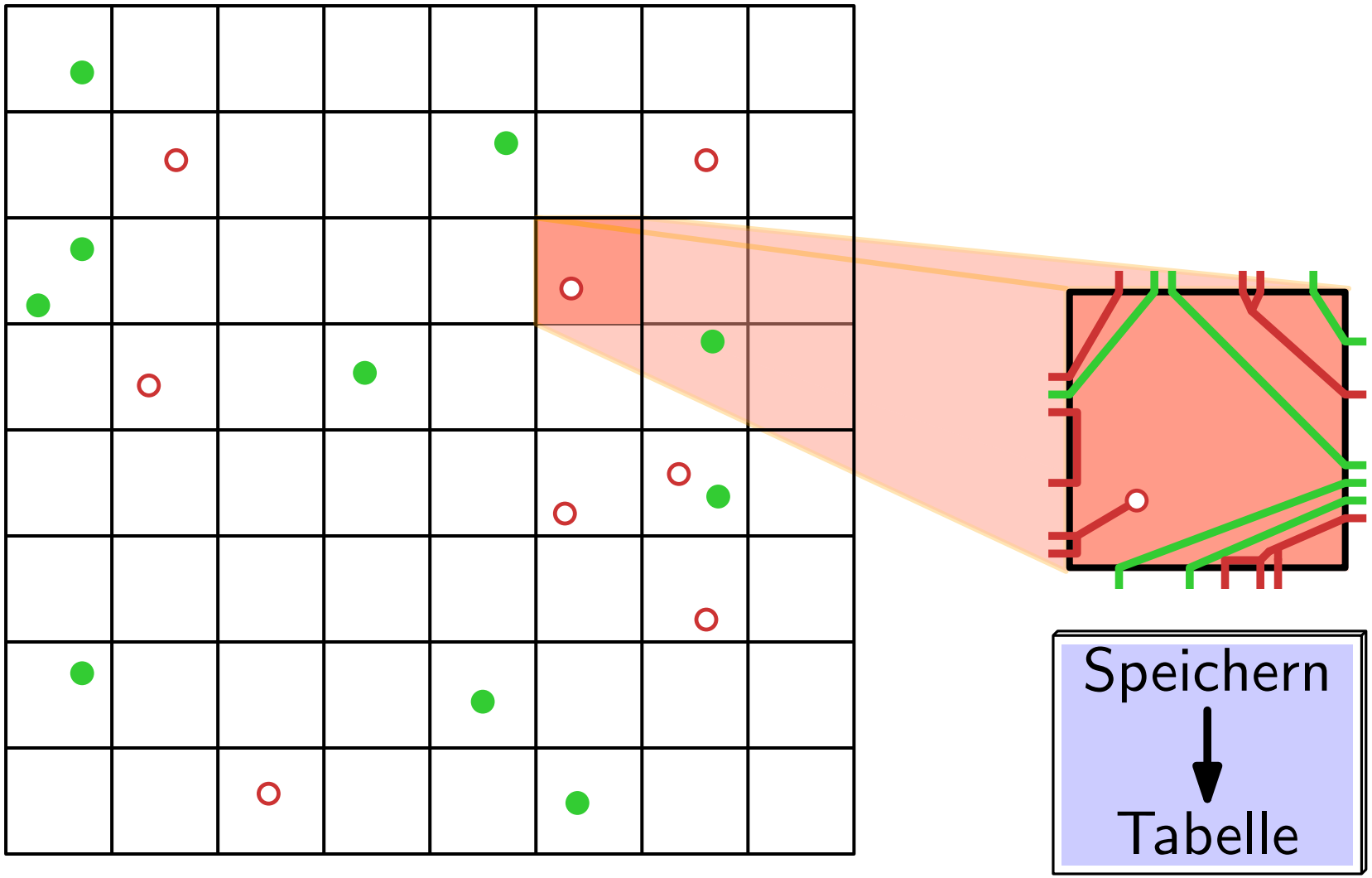

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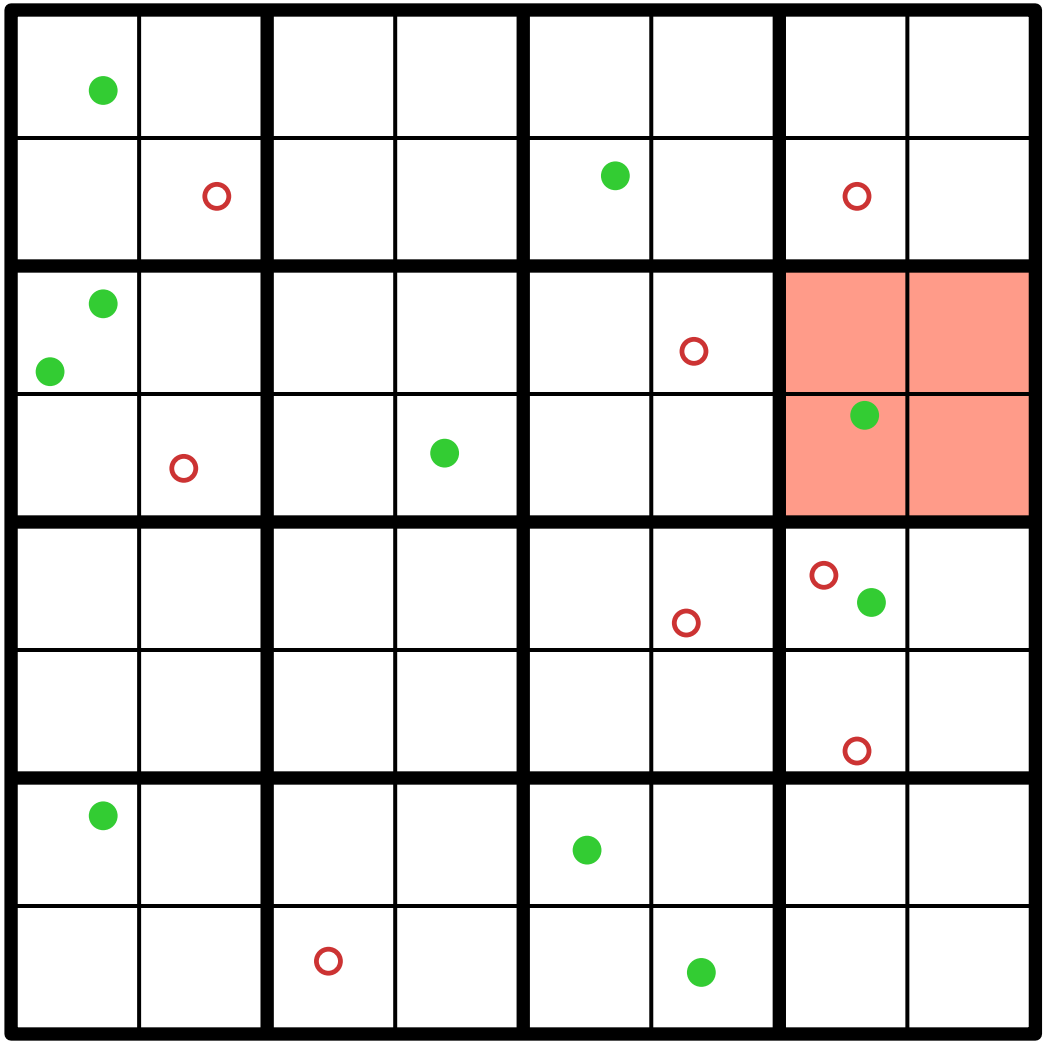

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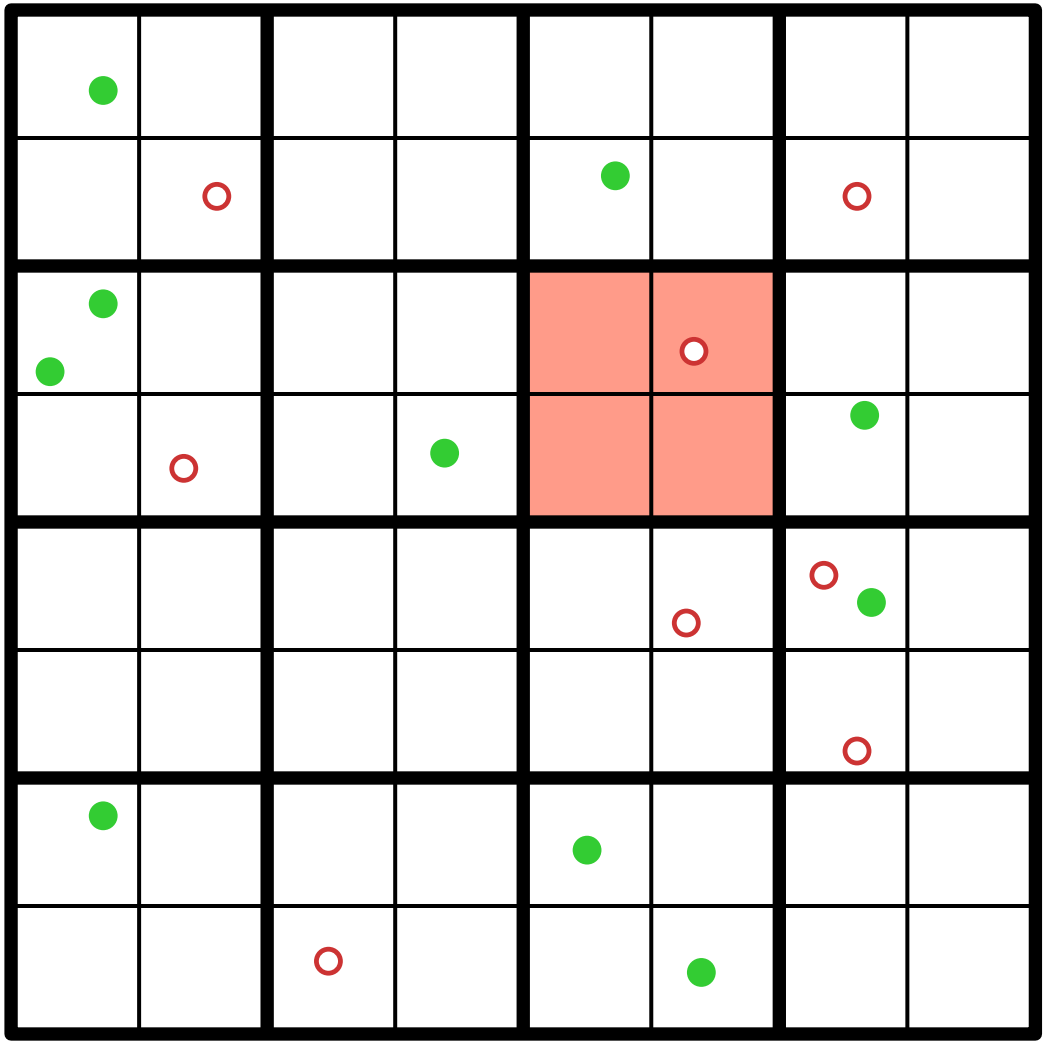

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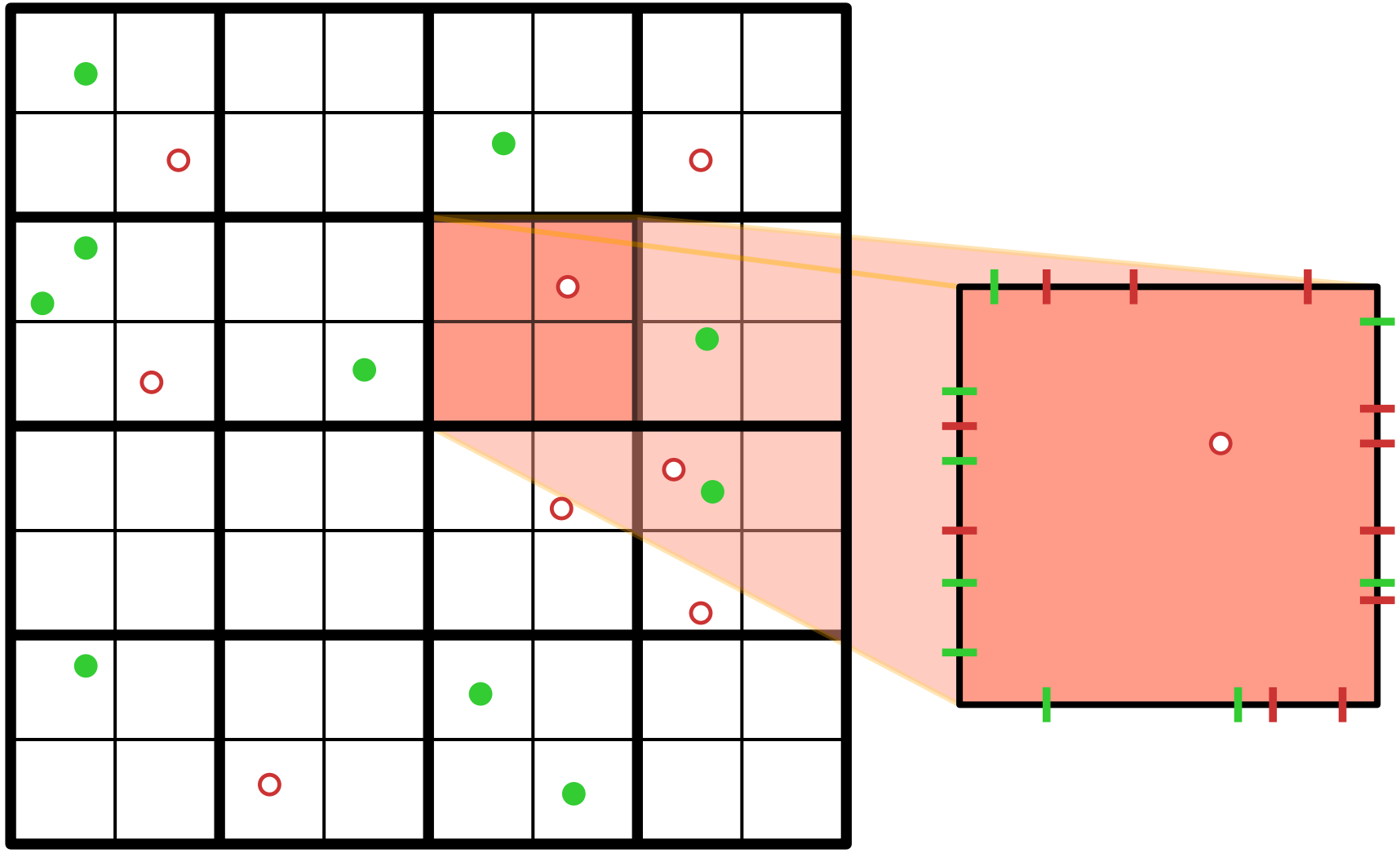

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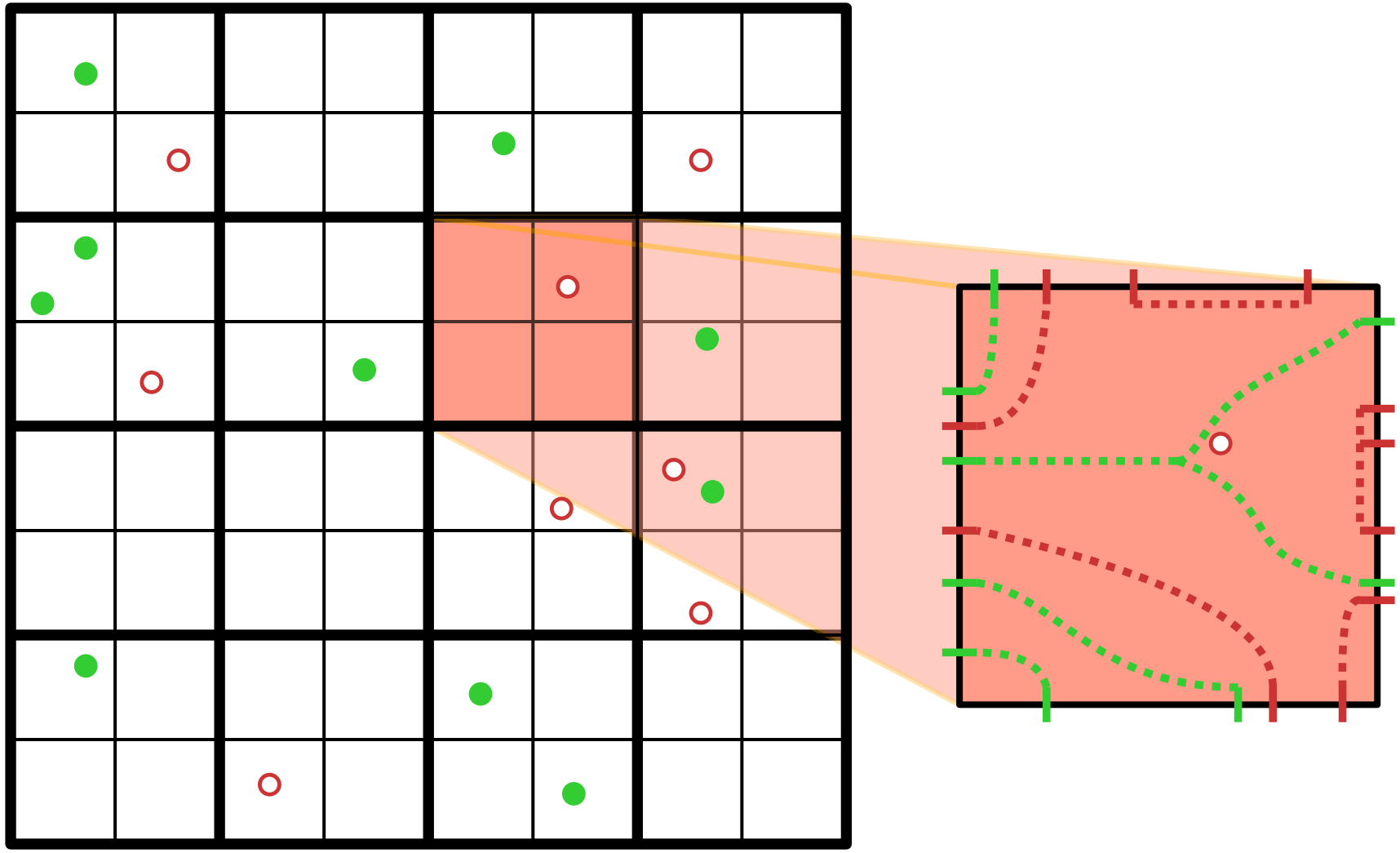
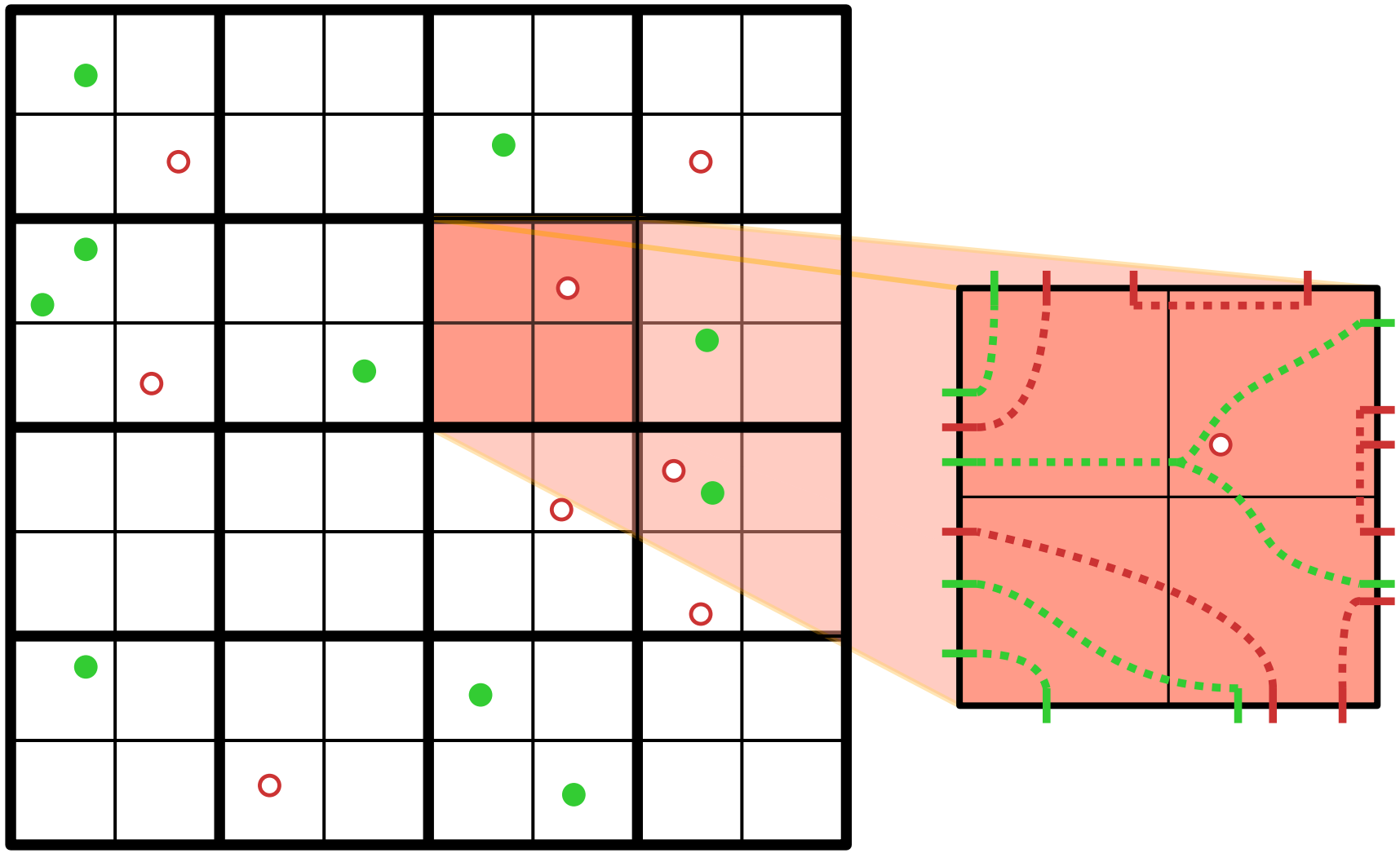
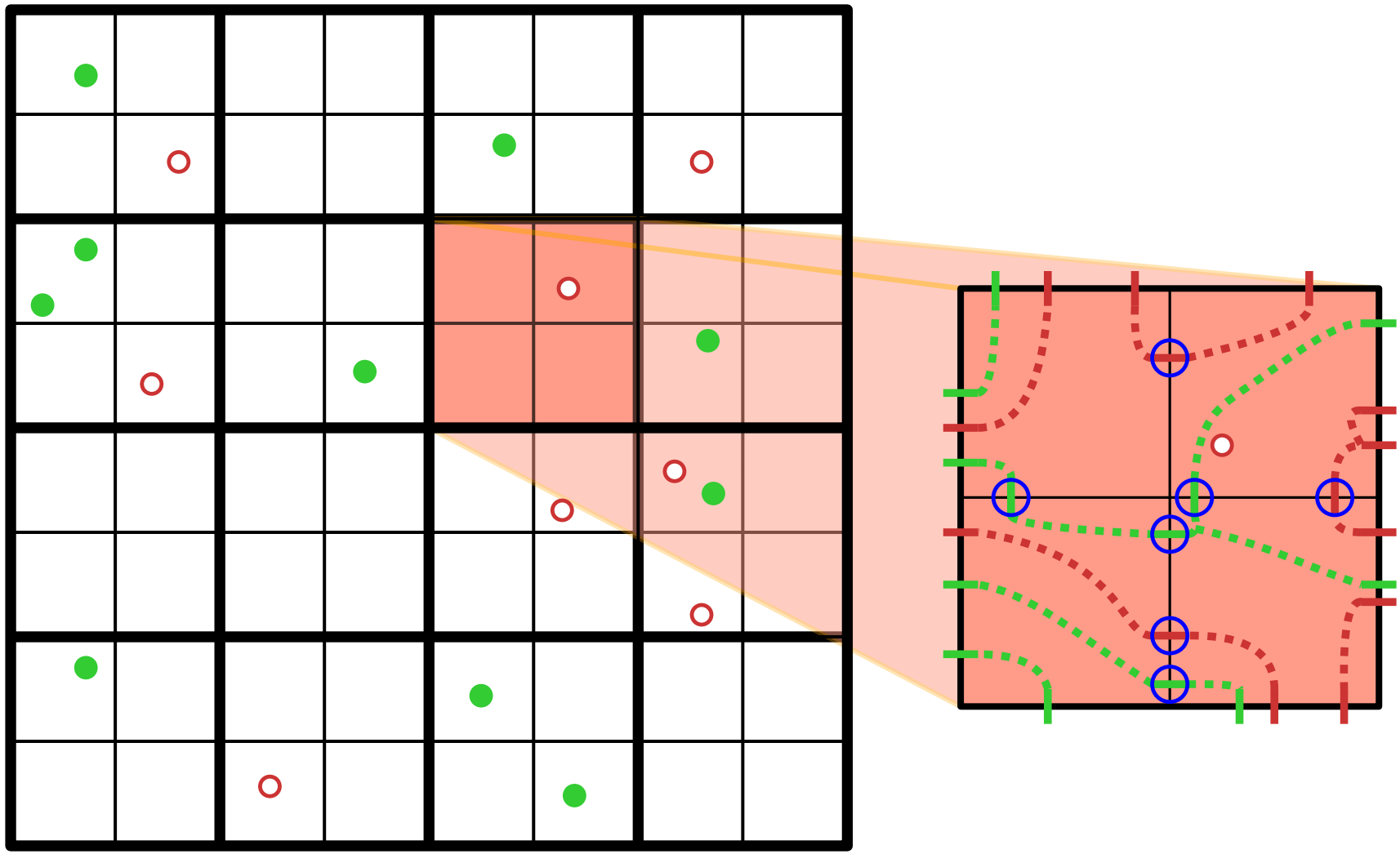


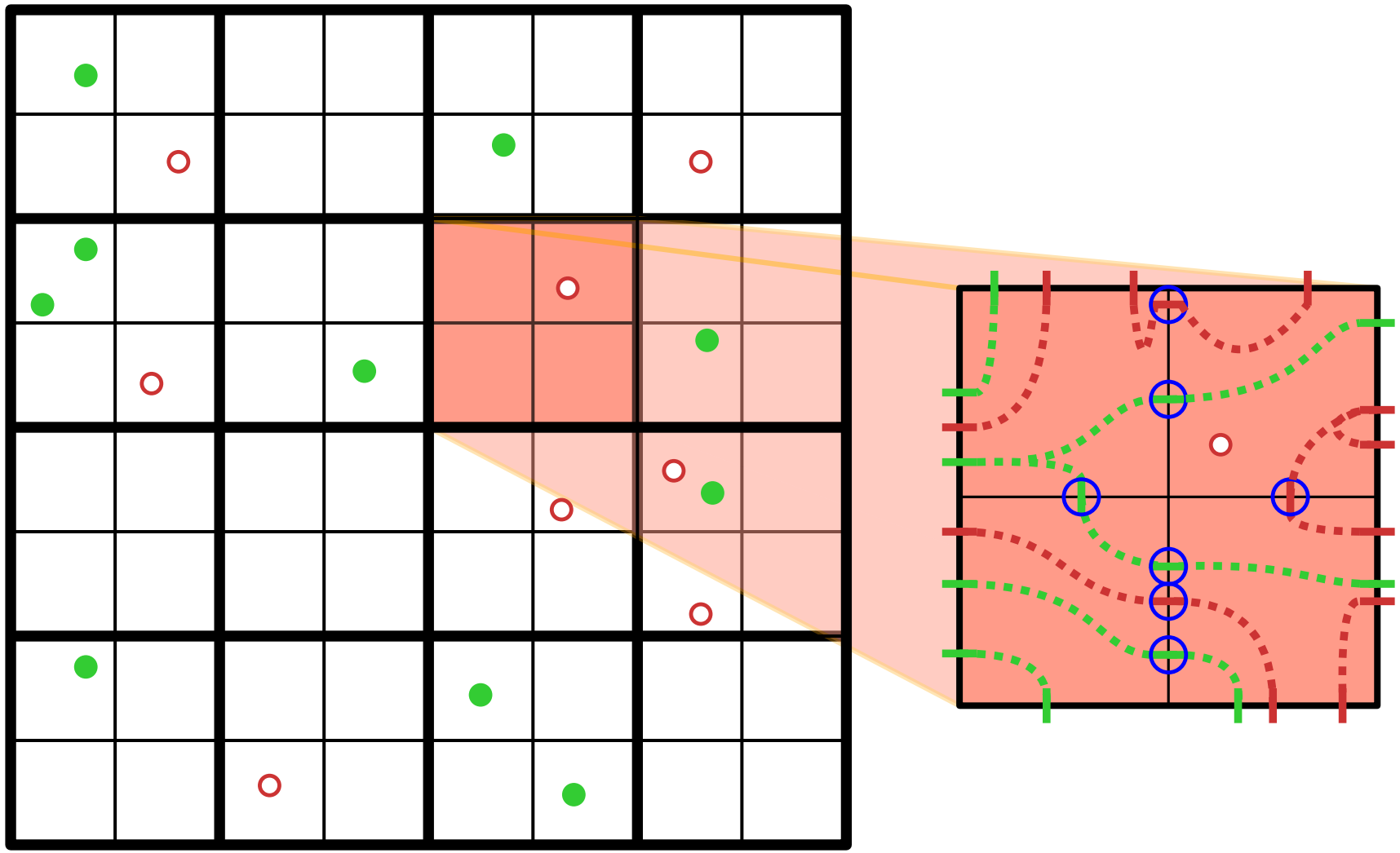
$(1+\varepsilon)$-Approximation für $k = 2$



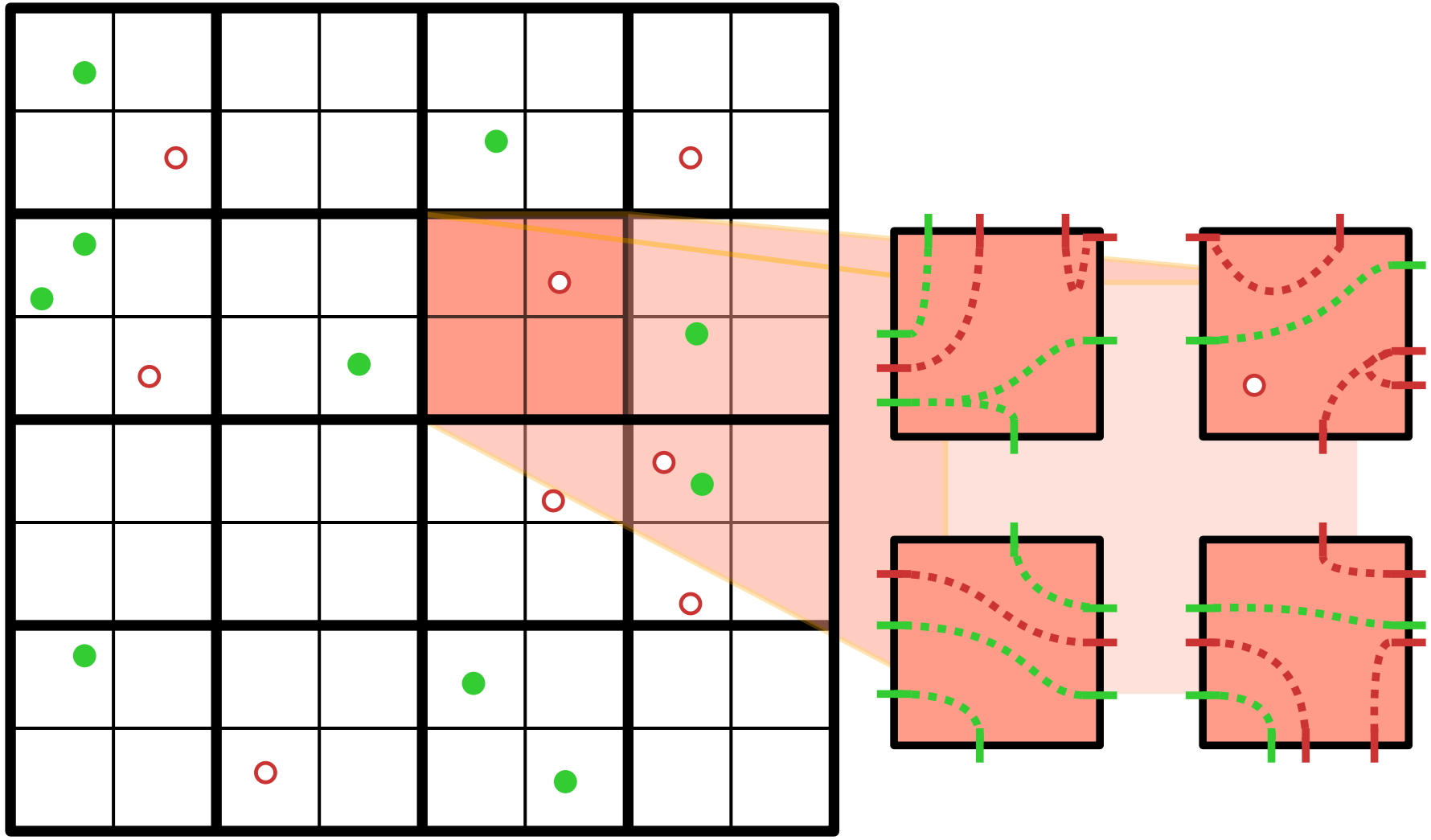
$(1+\varepsilon)$-Approximation für $k = 2$



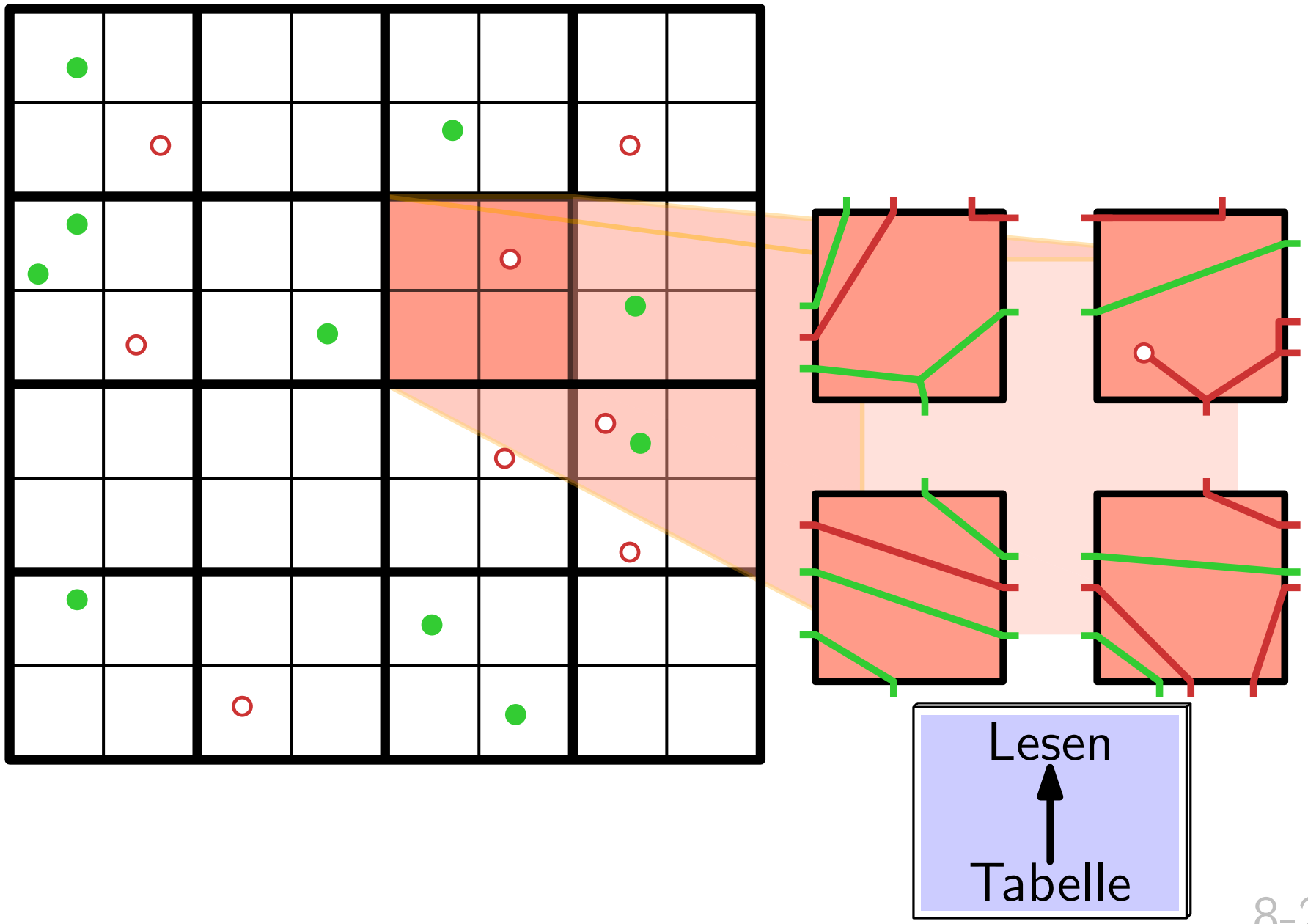
$(1+\varepsilon)$-Approximation für $k = 2$



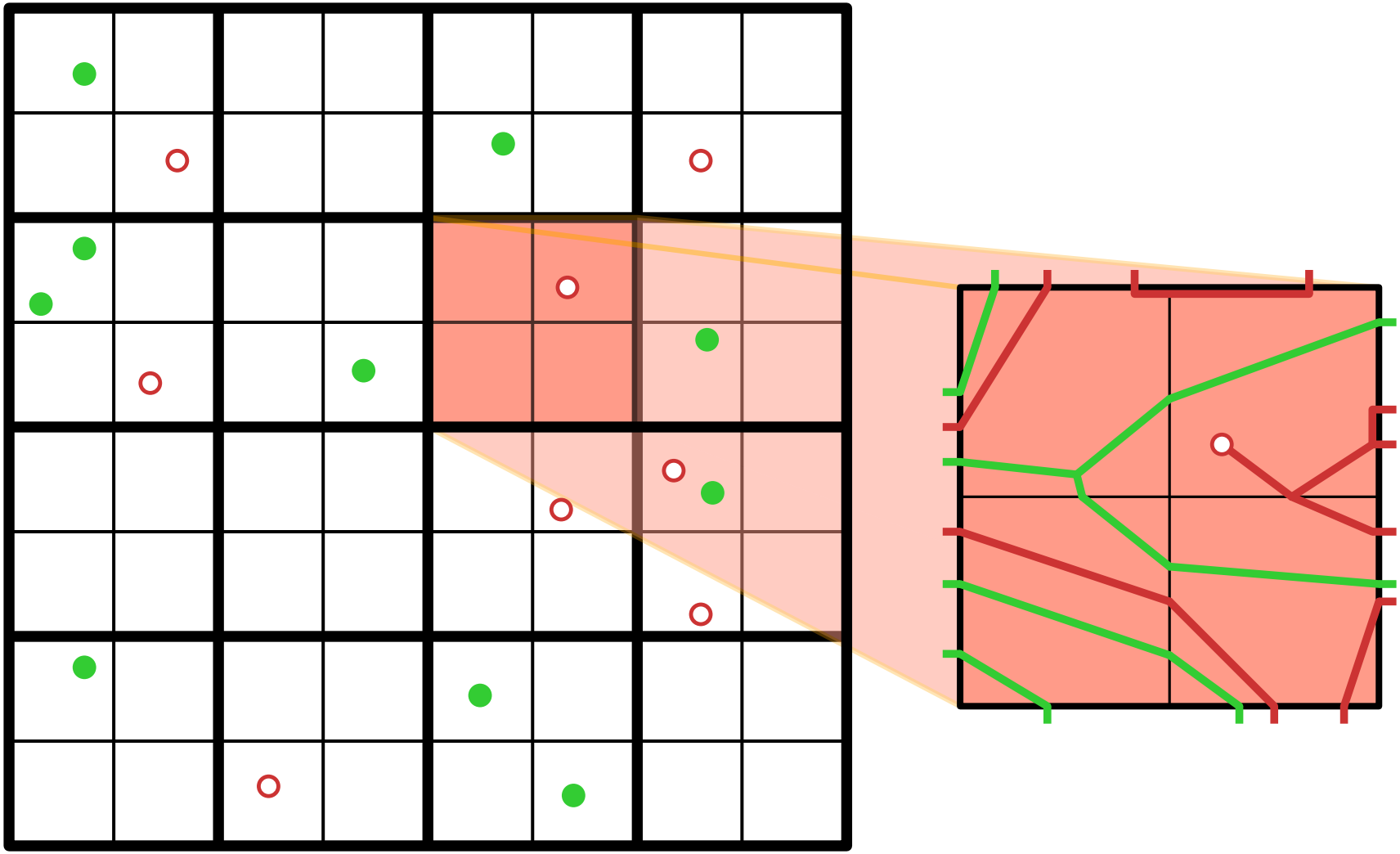
$(1+\varepsilon)$-Approximation für $k = 2$



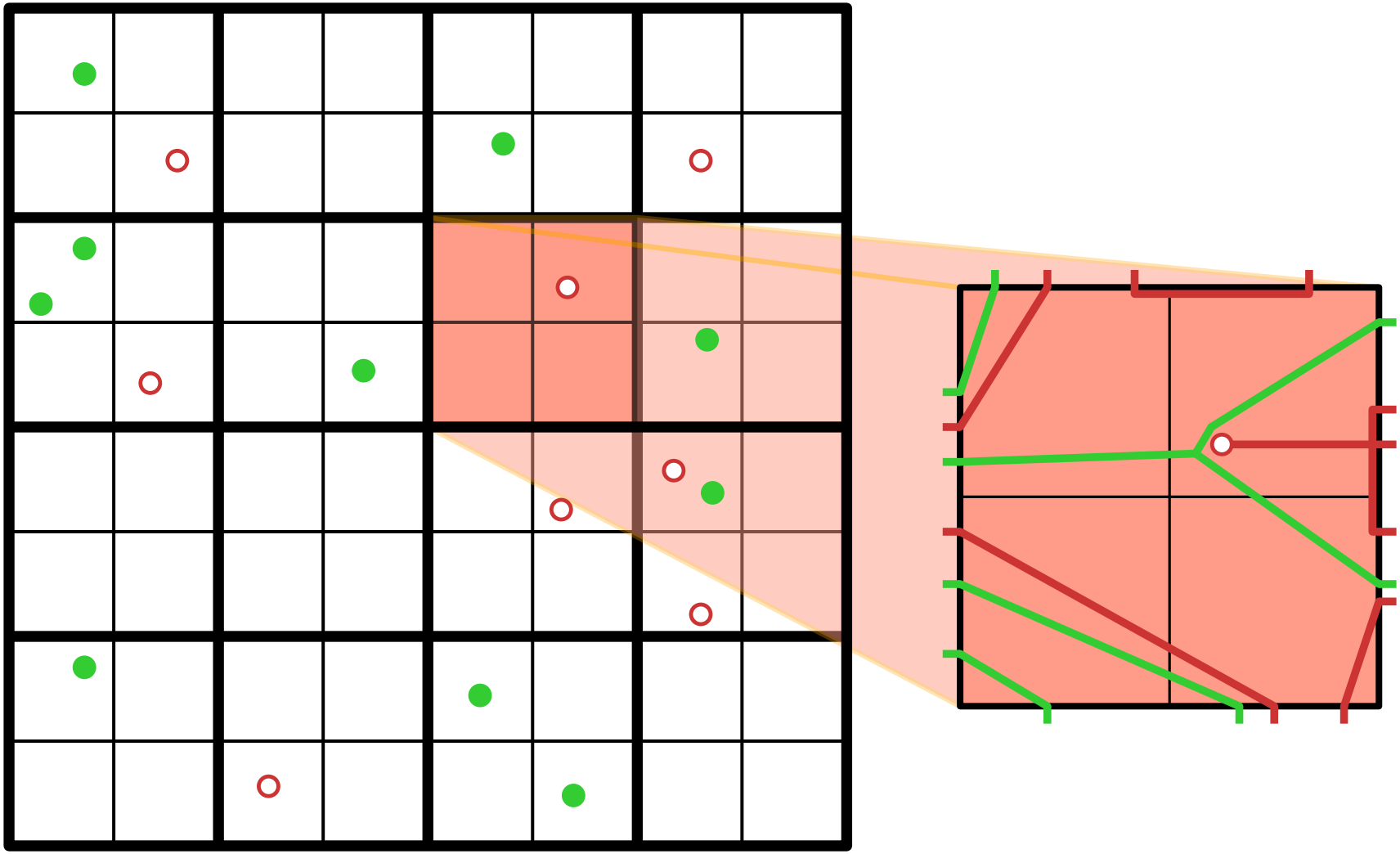
$(1+\varepsilon)$-Approximation für $k = 2$



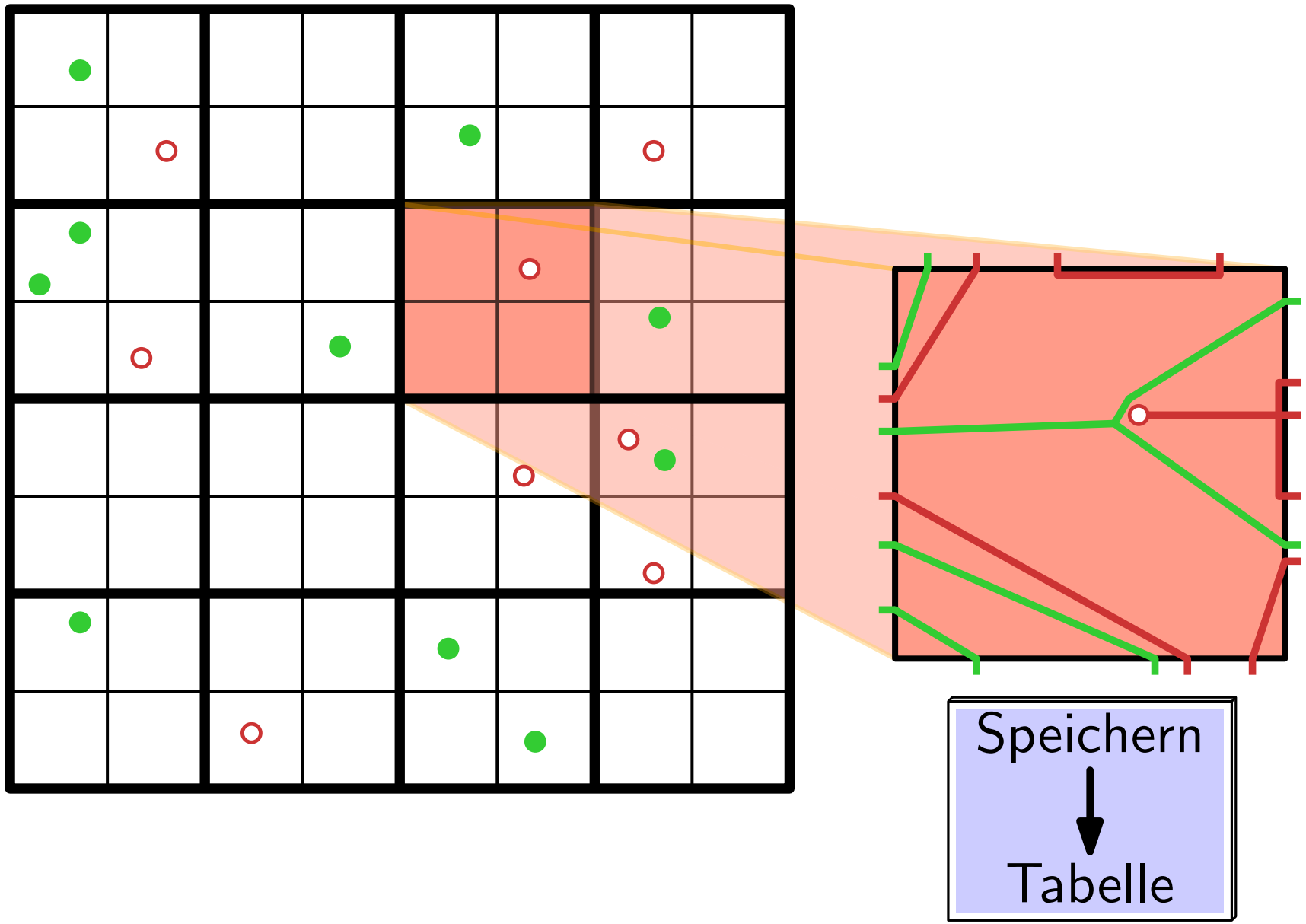
$(1+\varepsilon)$-Approximation für $k = 2$



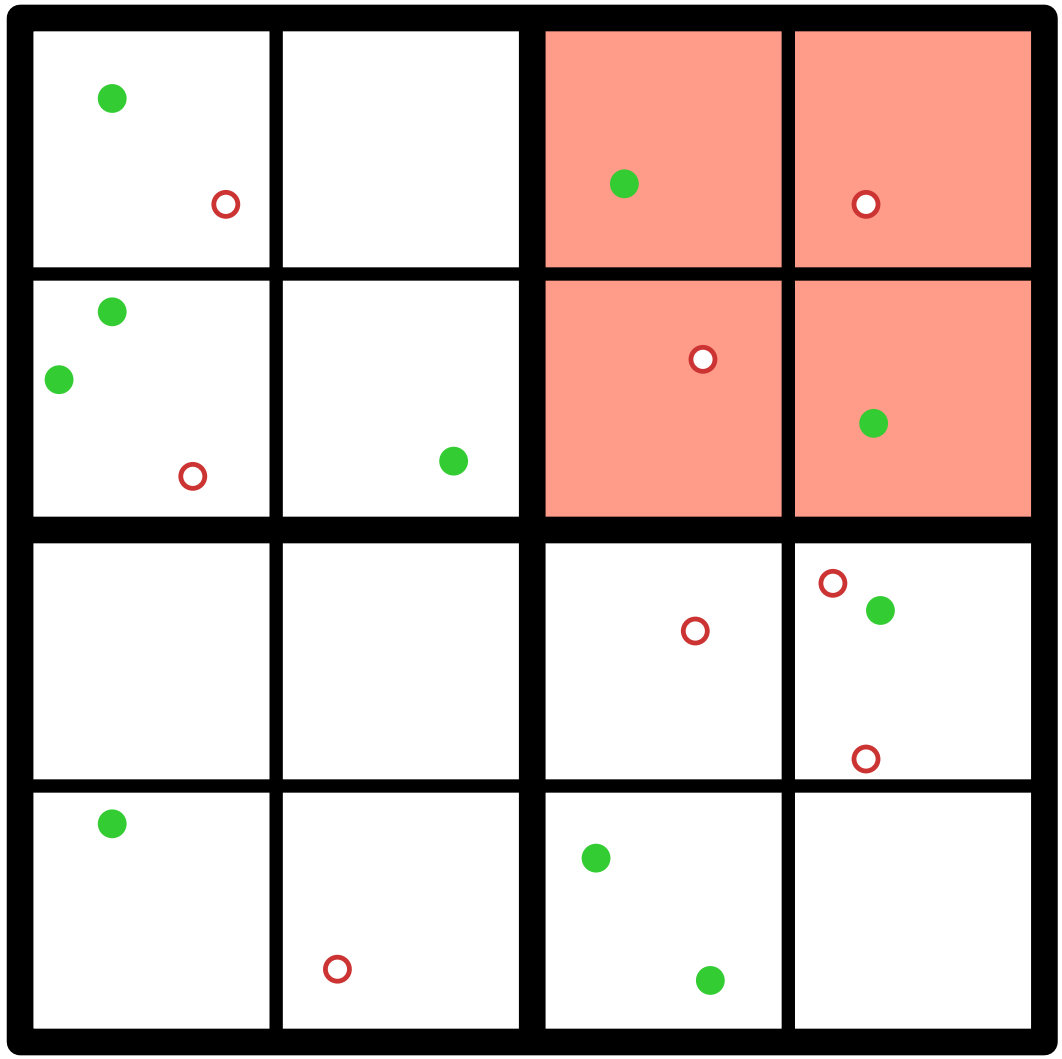
$(1+\varepsilon)$-Approximation für $k = 2$



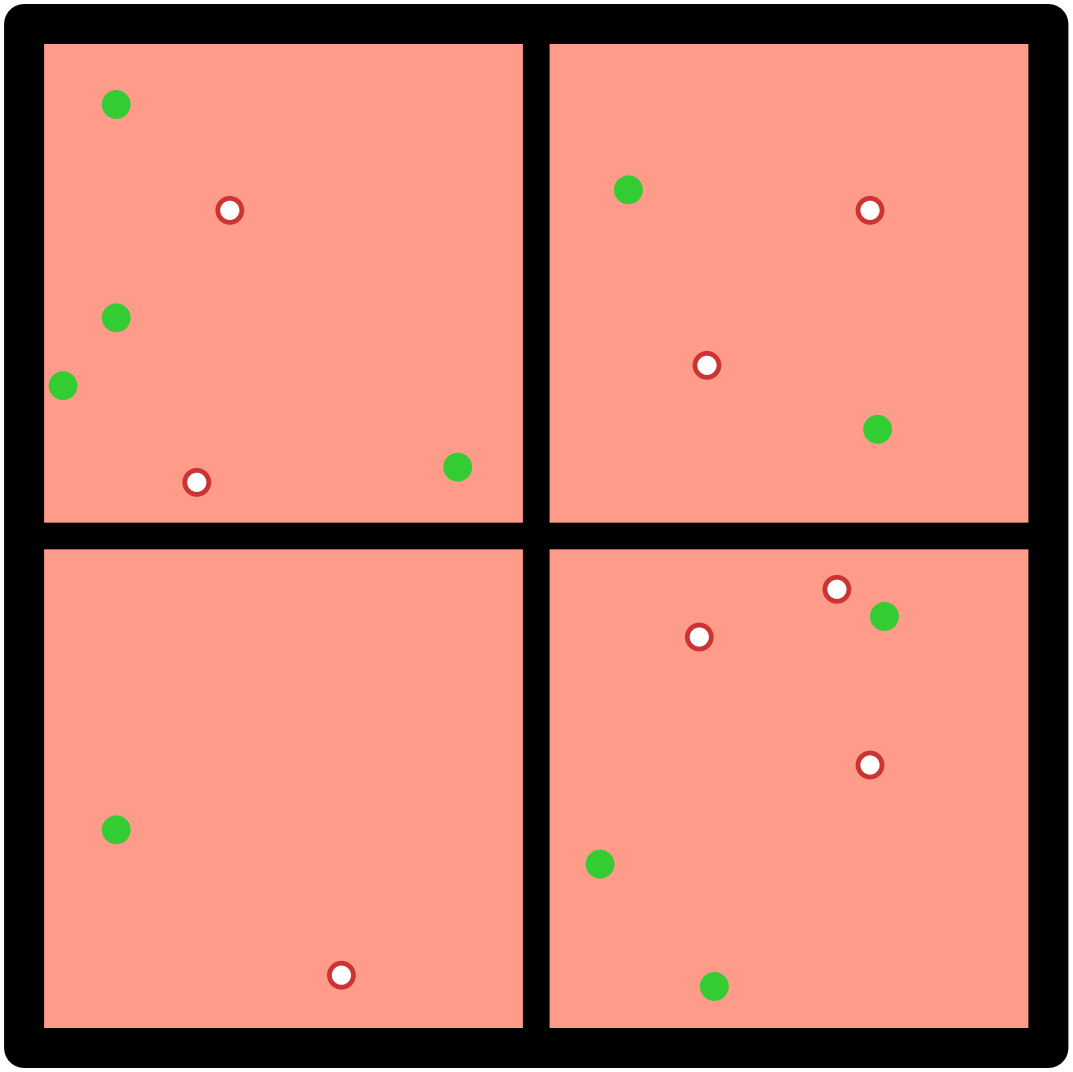
$(1+\varepsilon)$-Approximation für $k = 2$



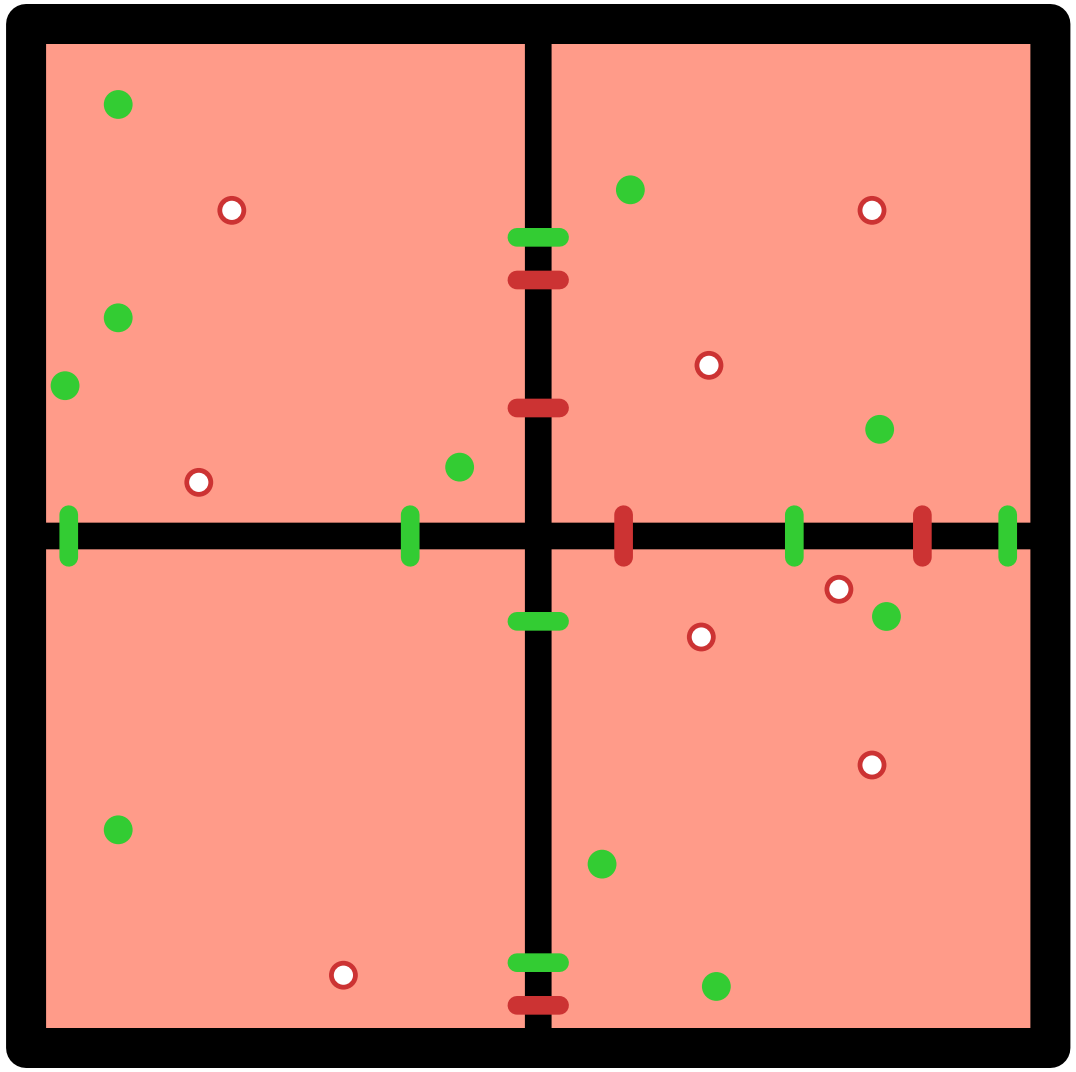
$(1+\varepsilon)$-Approximation für $k = 2$



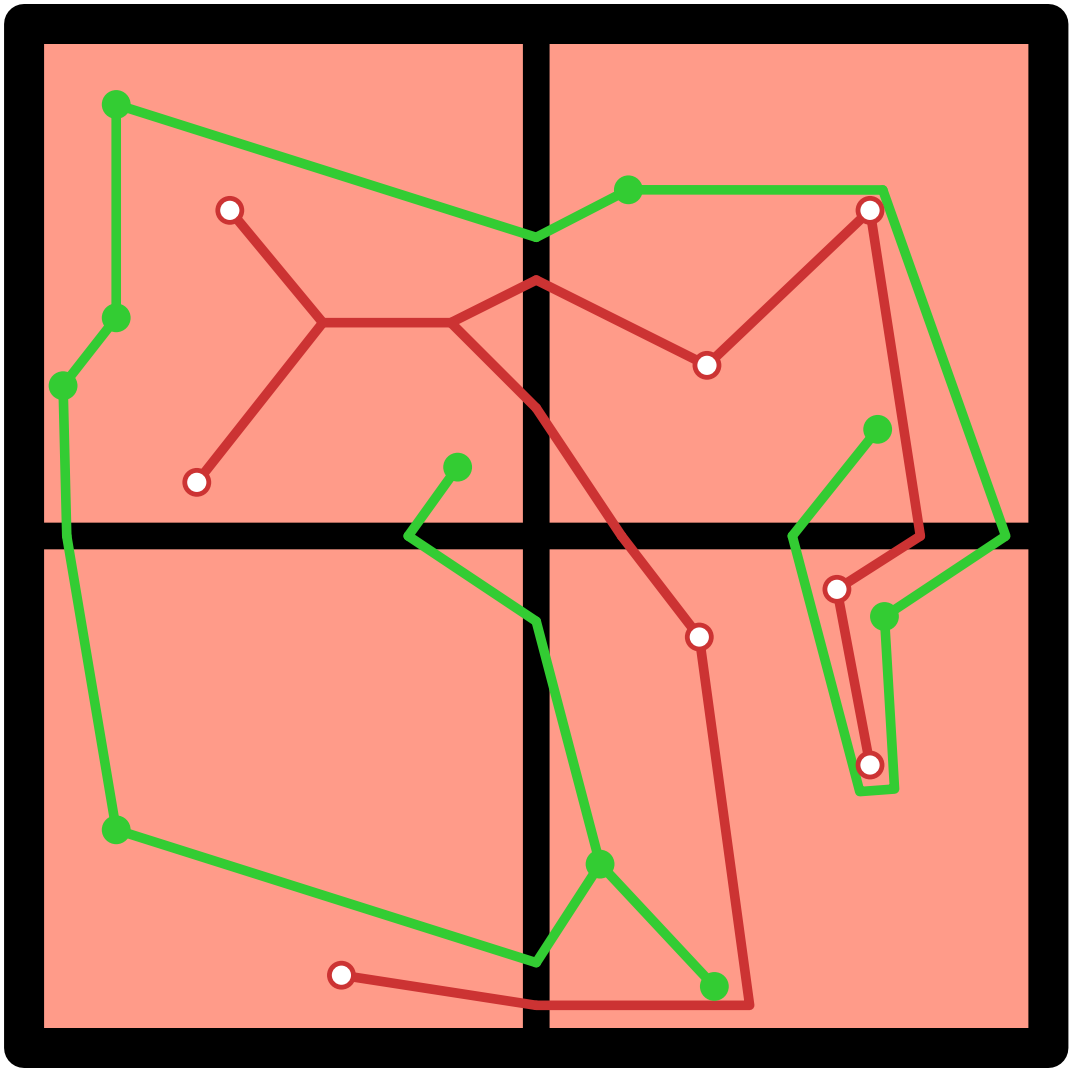
$(1+\varepsilon)$-Approximation für $k = 2$



$(1+\varepsilon)$-Approximation für $k = 2$

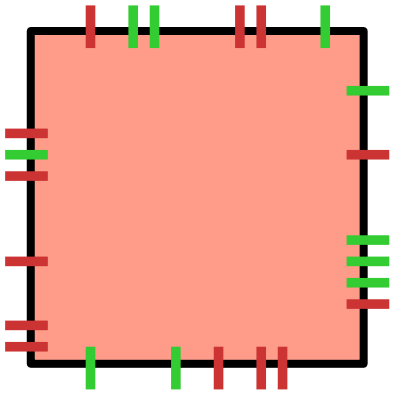


$(1+\varepsilon)$-Approximation für $k = 2$

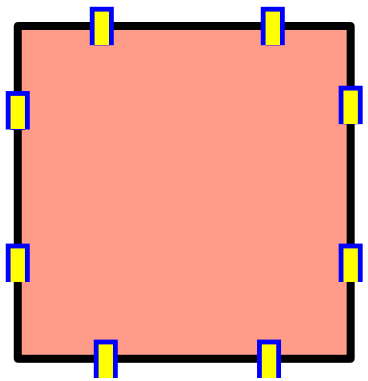


Details: $(1+\varepsilon)$-Approximation für $k = 2$

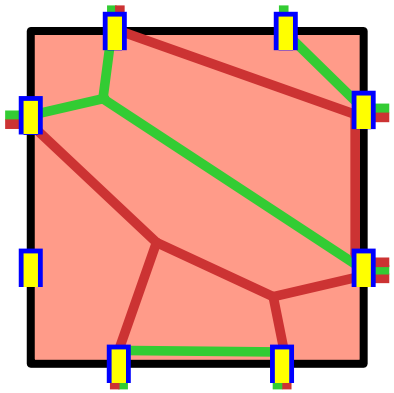
Zu viele Möglichkeiten!



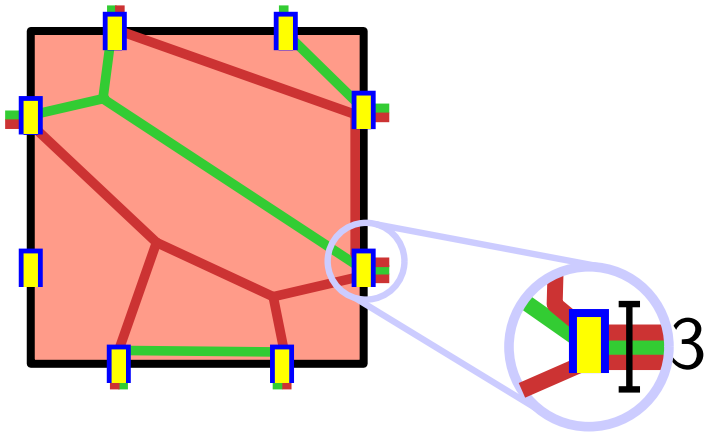
Details: $(1+\varepsilon)$-Approximation für $k = 2$



Details: $(1+\varepsilon)$-Approximation für $k = 2$



Details: $(1+\varepsilon)$-Approximation für $k = 2$



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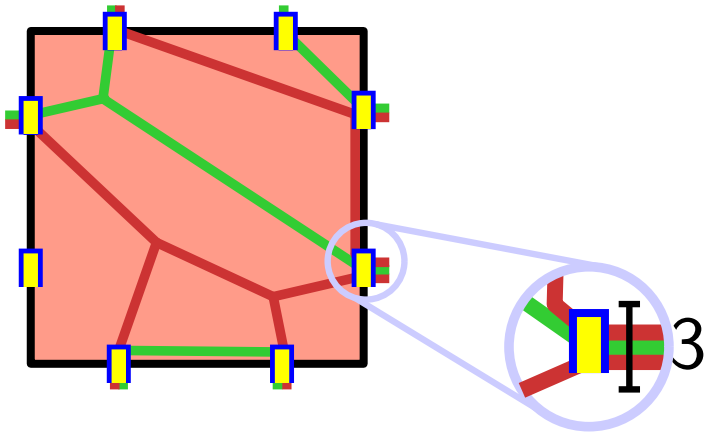
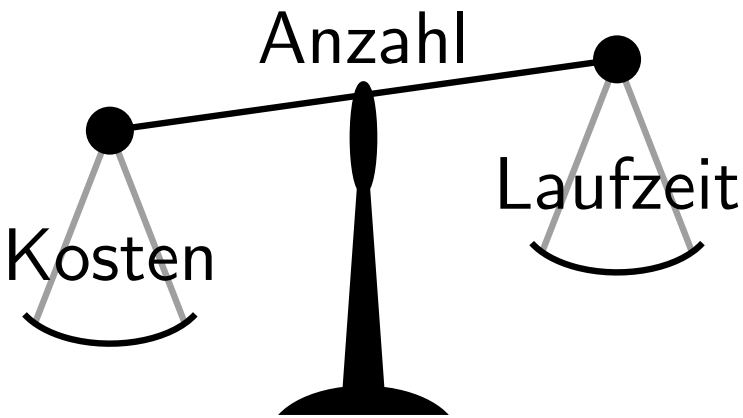
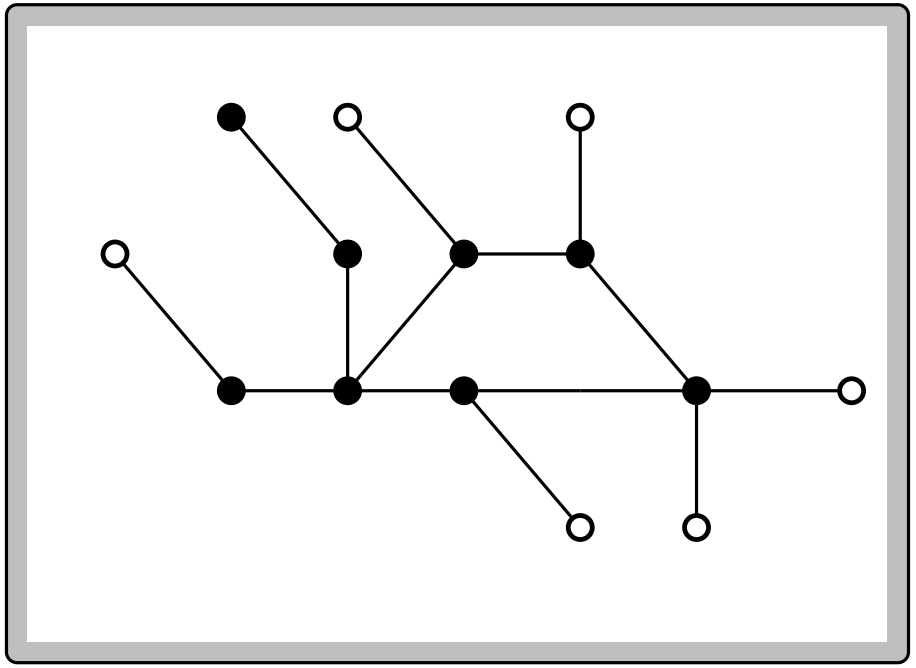


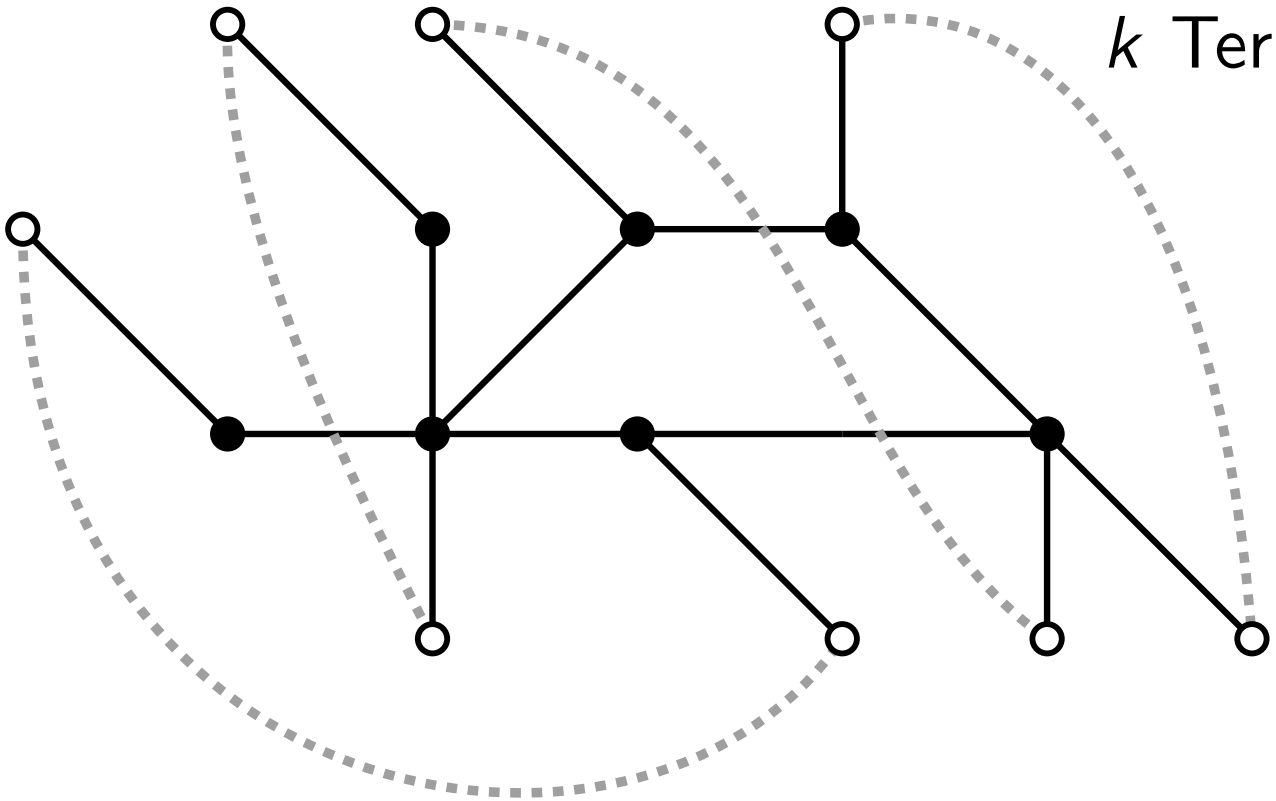


Kantendisjunkte Pfade



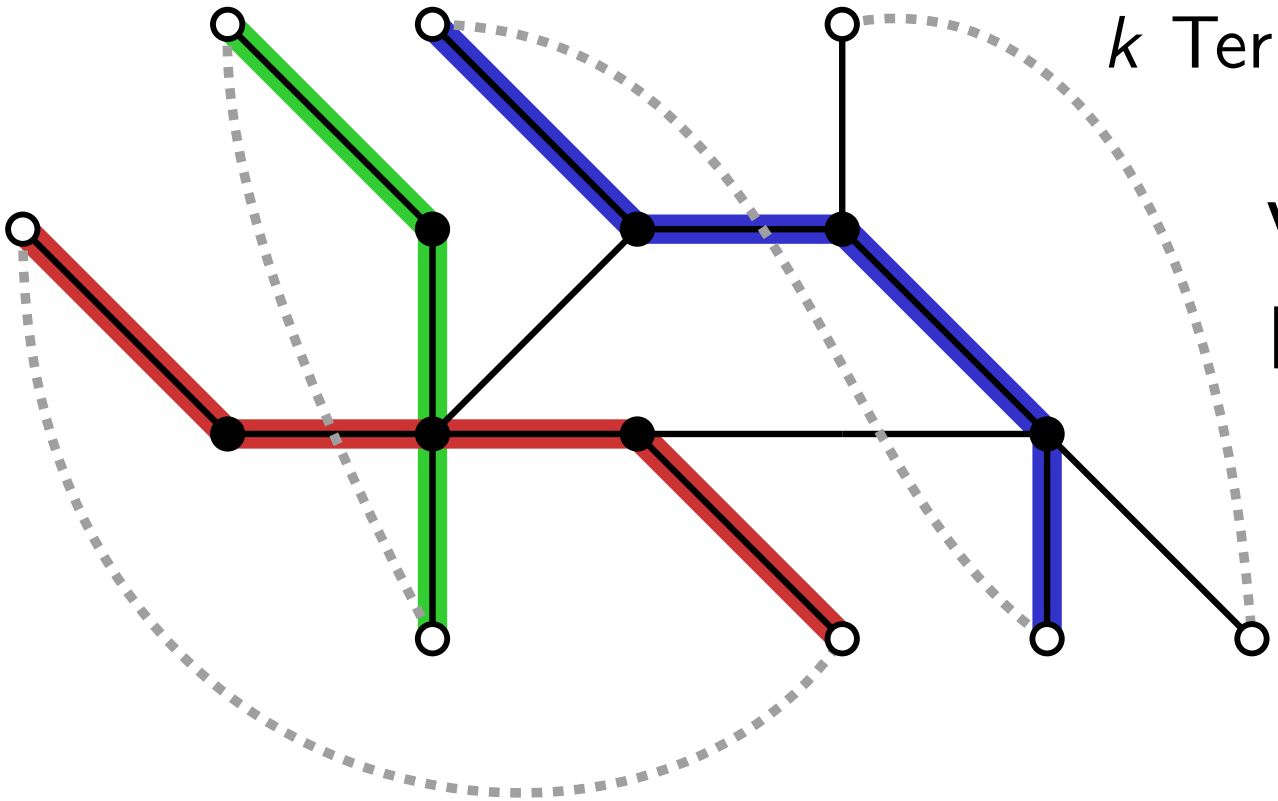
Graph

Kantendisjunkte Pfade



Graph	~~Ebene~~
n Knoten	~~Punkte~~
k Terminalpaare	~~Farben~~

Kantendisjunkte Pfade



Graph	~~Ebene~~
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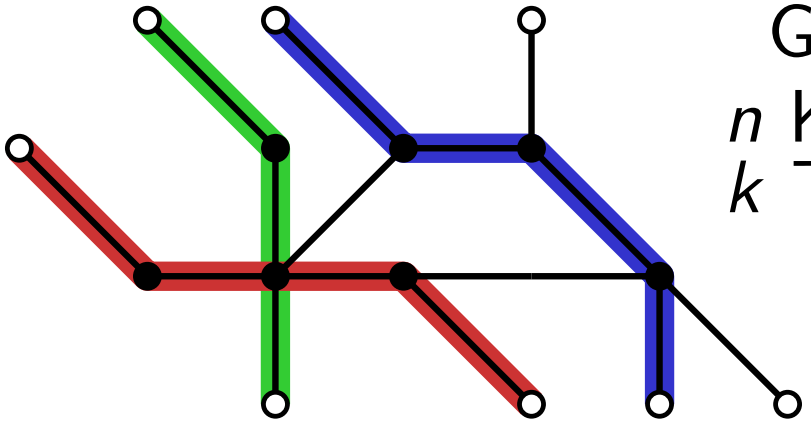
Verbinde so viele Paare wie möglich!

Kantendisjunkte Pfade



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n Knoten
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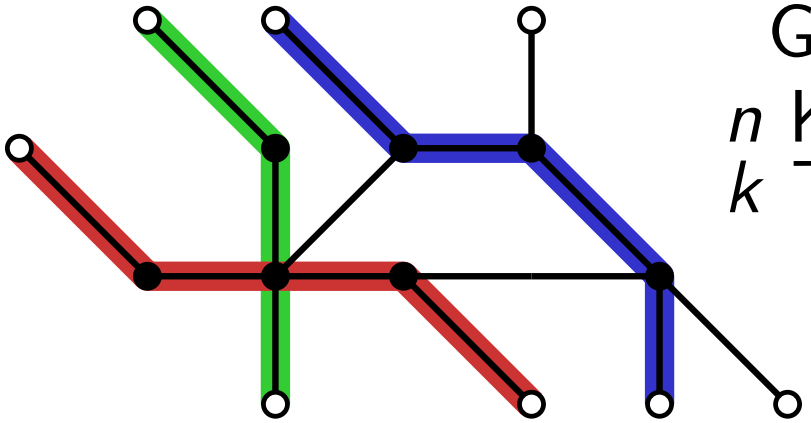
Kantendisjunkte Pfade

Bekannt

NP-schwer
[Karp '75]



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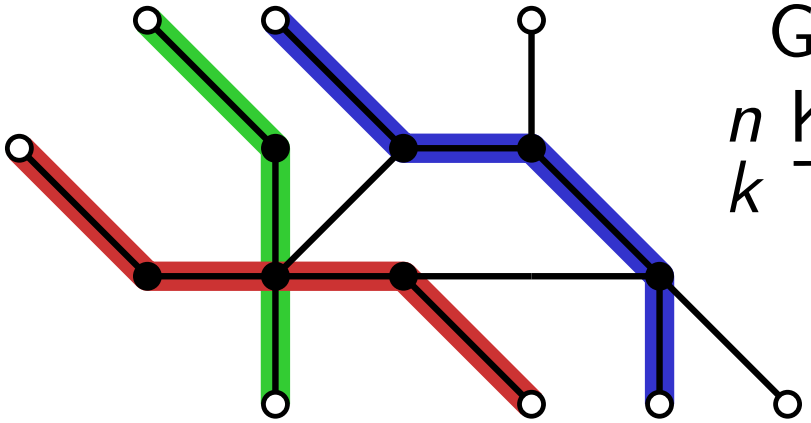
$\mathcal{O}(\sqrt{n})$
[Chekuri et al. '06]

Güte $= \frac{\text{OPT}}{\text{ALGO}}$

$\Omega(\frac{\text{OPT}}{\sqrt{n}})$ Pfade



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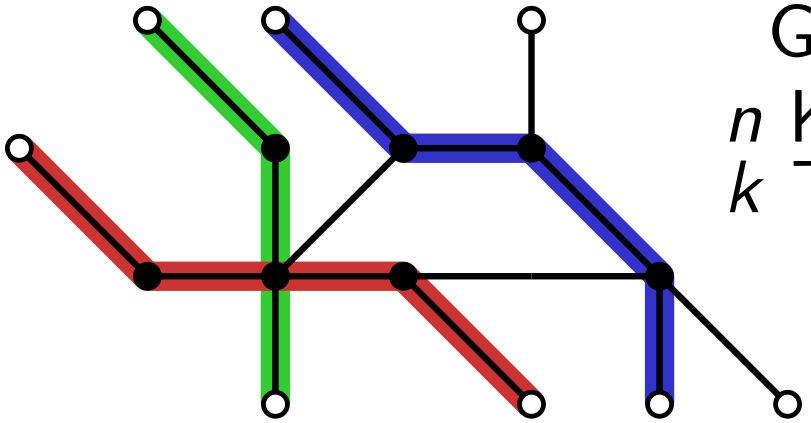
Einfach in Wäldern
[Garg et al. '97]

Wald = Graph ohne Kreise



Graph
n Knoten
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Kantendisjunkte Pfade

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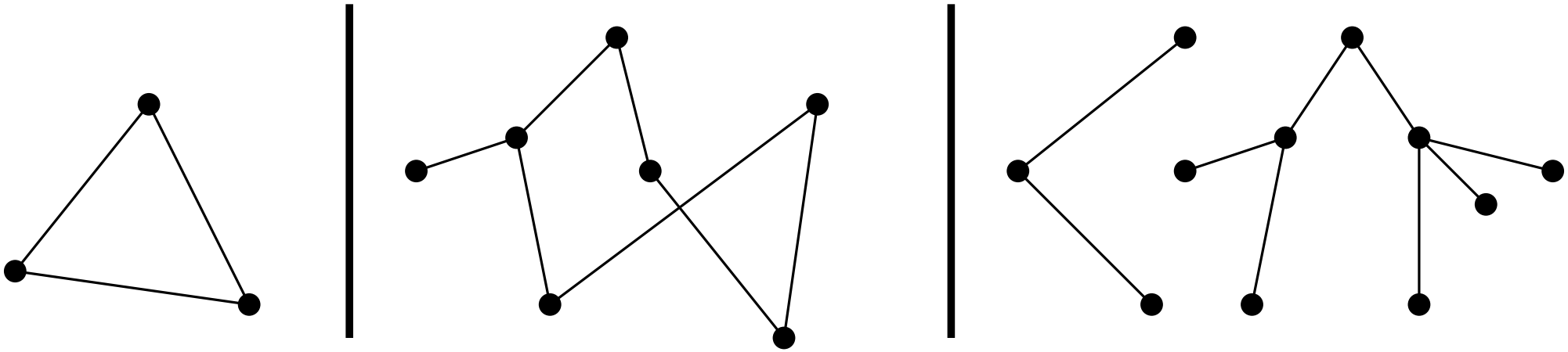
Wald = Graph ohne Kreise

Beispiel



Kantendisjunkte Pfade

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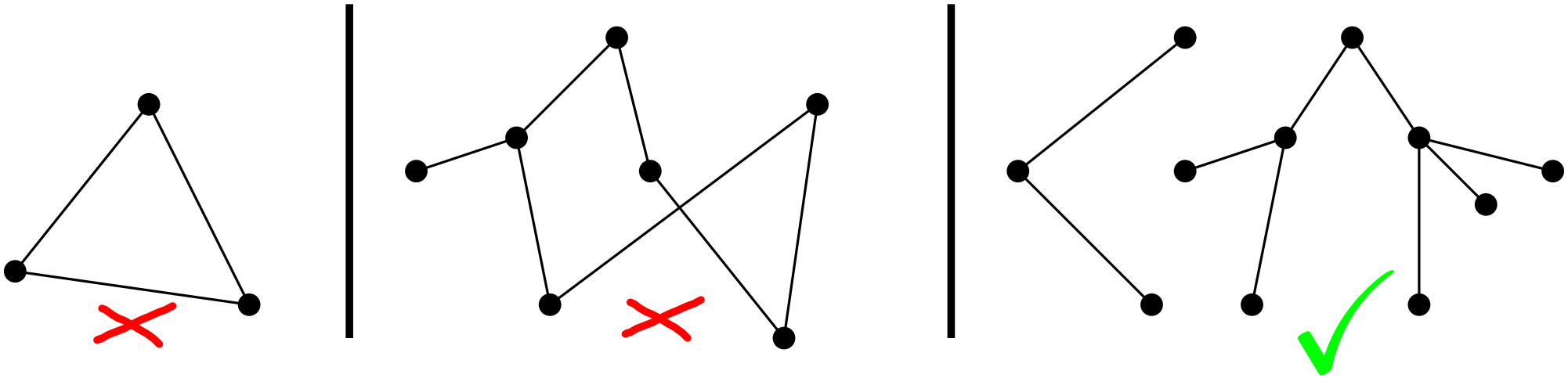

Kantendisjunkte Pfade

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„Abstand“ zum Wald?

Graph $\xrightarrow{\text{Lösche Knoten}}$ Wald

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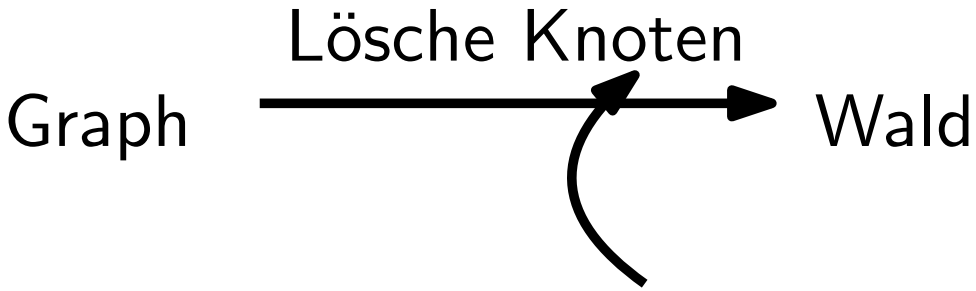

Parameter r = min. Anzahl

Kantendisjunkte Pfade

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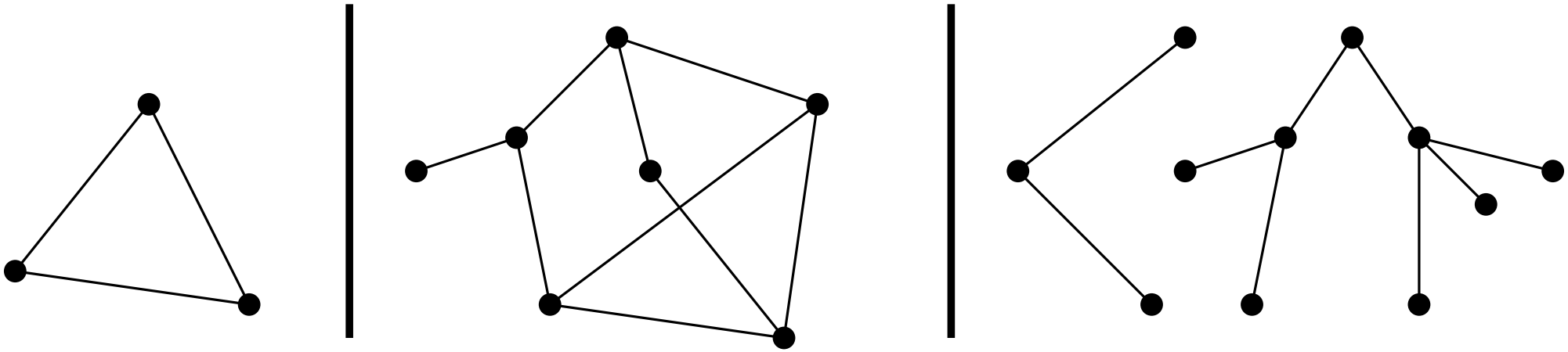
Beispiel



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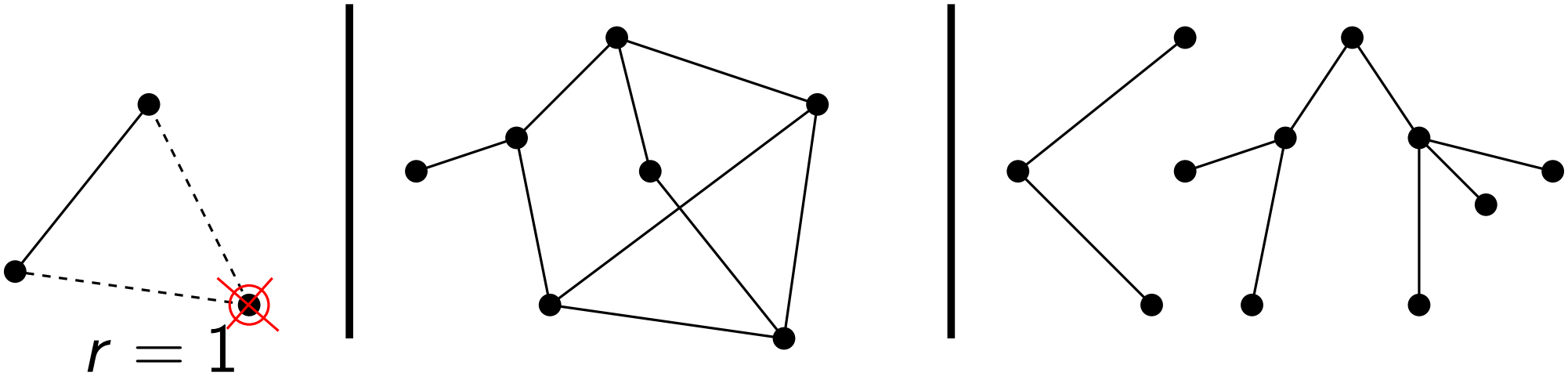

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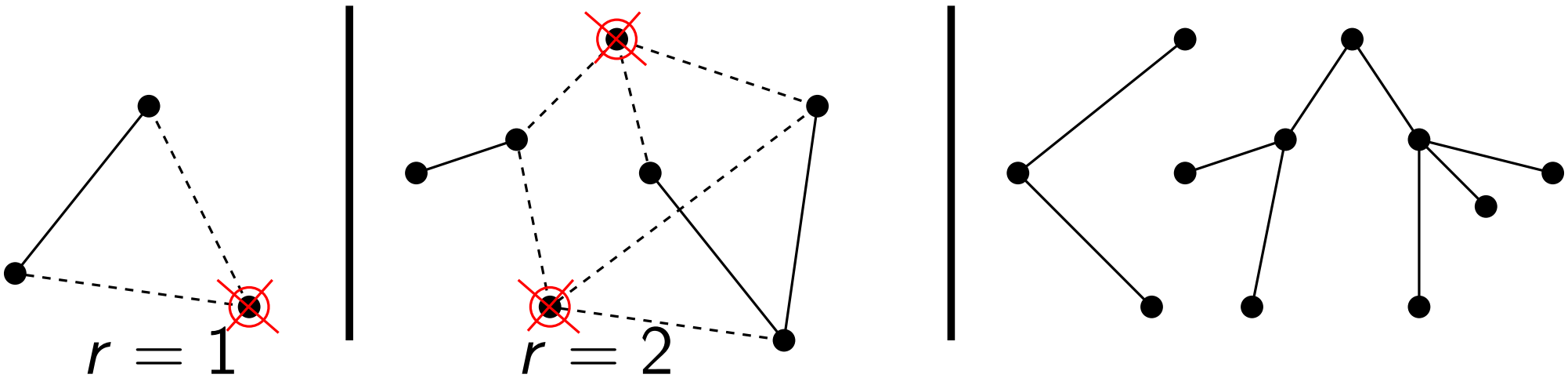

Kantendisjunkte Pfade

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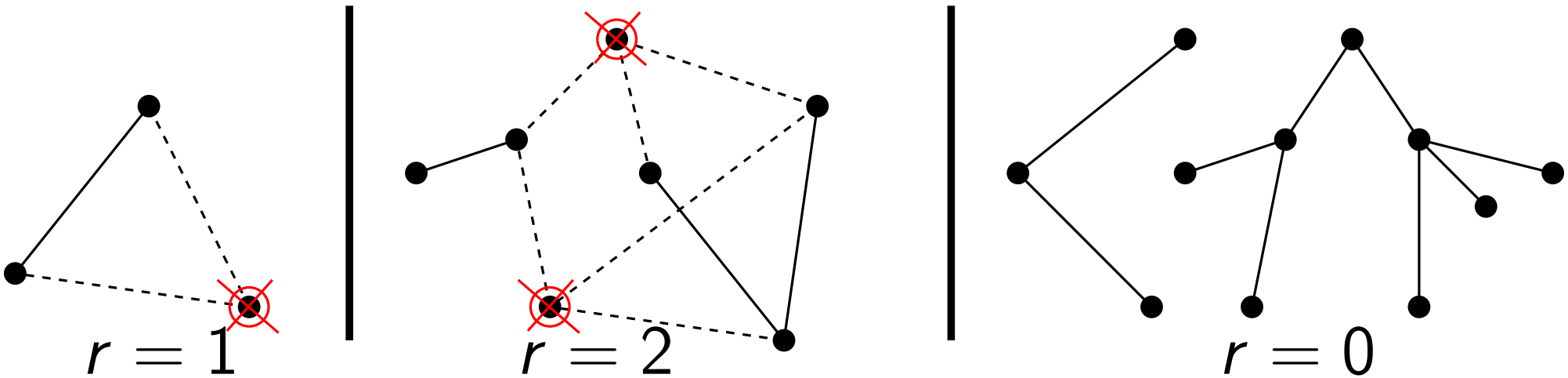

Kantendisjunkte Pfade

Bekannt

NP-schwer
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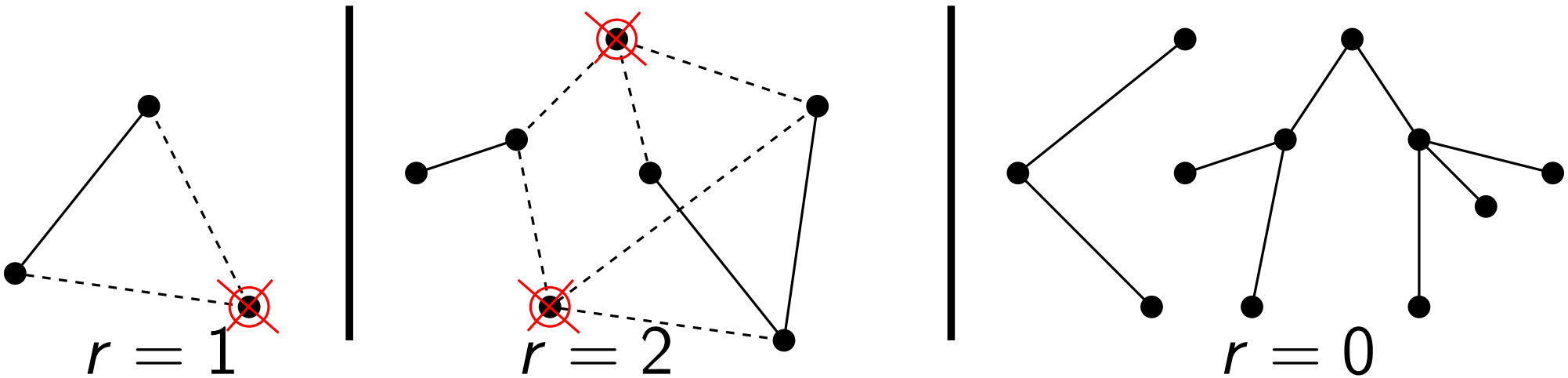
Einfach für $r = 0$
[Garg et al. '97]

„Abstand" zum Wald?

Graph $\xrightarrow{\text{Lösche Knoten}}$ Wald

Parameter r = min. Anzahl

Beispiel



Kantendisjunkte Pfade

Bekannt
NP-schwer [Karp '75] $\mathcal{O}(\sqrt{n})$ [Chekuri et al. '06] Einfach für $r = 0$ [Garg et al. '97]

Neu
NP-schwer ab $r = 1$

Kantendisjunkte Pfade

Bekannt	Neu
NP-schwer [Karp '75]	NP-schwer ab $r = 1$
$\mathcal{O}(\sqrt{n})$ [Chekuri et al. '06]	$\tilde{\mathcal{O}}(\sqrt{r})$
Einfach für $r = 0$ [Garg et al. '97]	

Kantendisjunkte Pfade

Bekannt	Neu
NP-schwer [Karp '75]	NP-schwer ab $r = 1$
$\mathcal{O}(\sqrt{n})$ [Chekuri et al. '06]	$\tilde{\mathcal{O}}(\sqrt{r})$
Einfach für $r = 0$ [Garg et al. '97]	

Beachte: $r \leq n$

Kantendisjunkte Pfade

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NP-schwer [Karp '75]	NP-schwer ab $r = 1$
$\mathcal{O}(\sqrt{n})$ [Chekuri et al. '06]	$\tilde{\mathcal{O}}(\sqrt{r})$
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Beachte: $r \leq n$

$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

1. Löse LP.

$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

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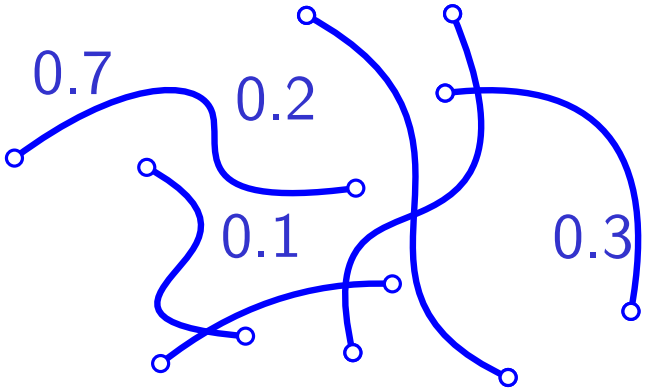
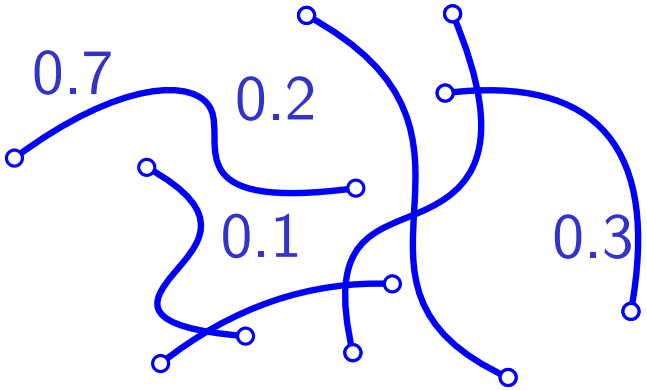


Lineares Programm

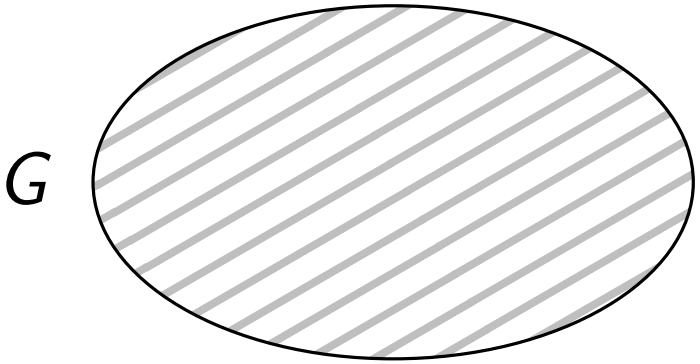
- $x_p \in [0,1] \quad \forall$ Pfad p
- $\sum_{p \ni e} x_p \leq 1 \quad \forall$ Kante e
- $\sum x_p \geq \mathrm{OPT}$

$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

1. Löse LP.



2. Bestimme R.



$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

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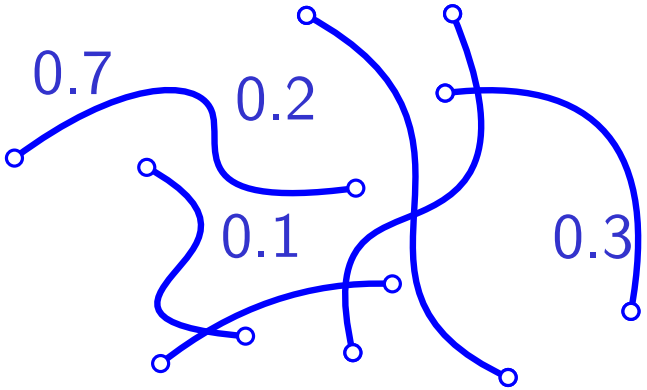

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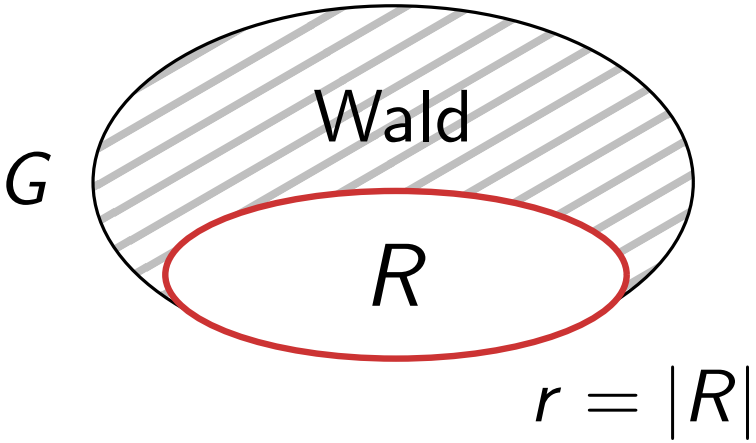

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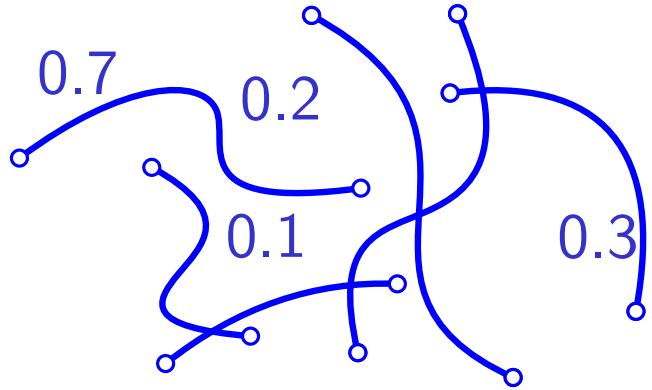

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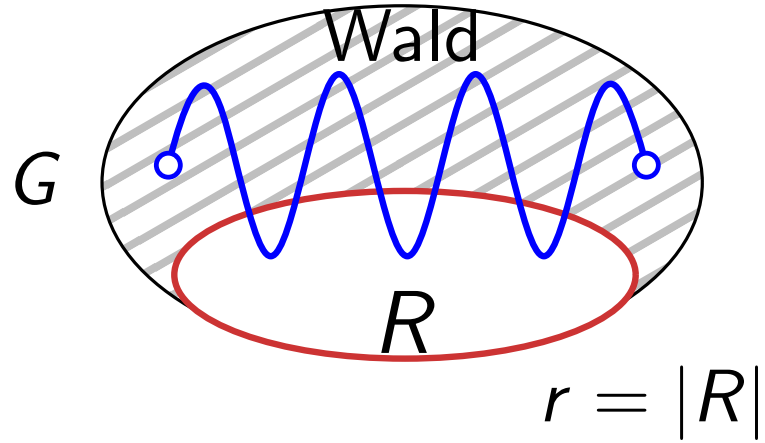

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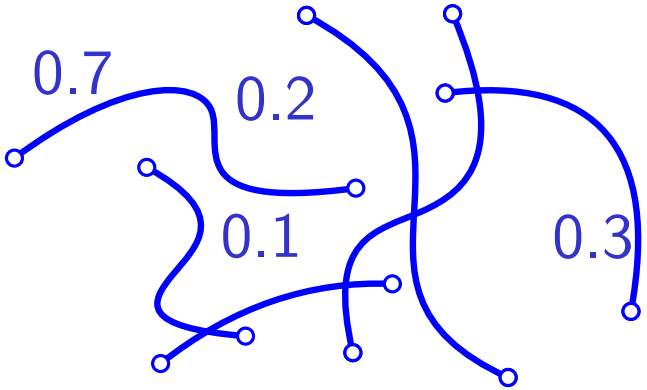

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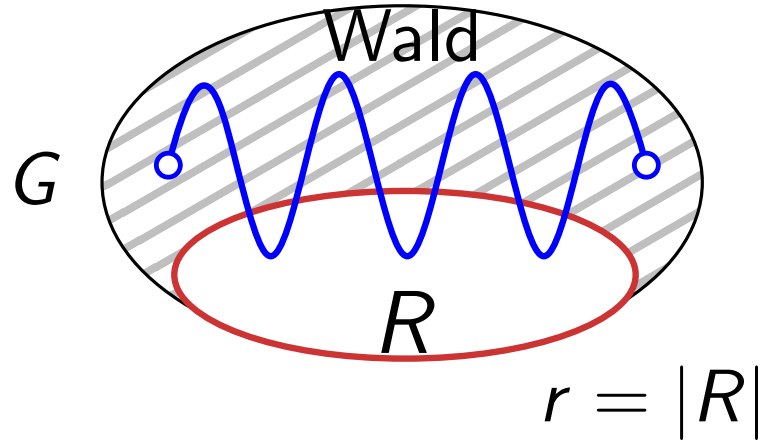
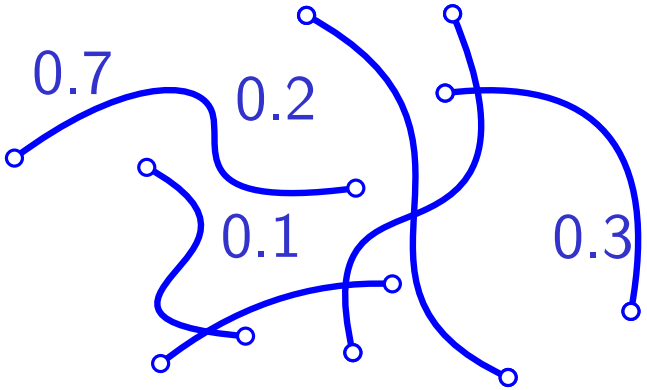


3. (a) „Viele“ R-lange Pfade:

(b) „Viele“ R-kurze Pfade:

$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

1. Löse LP.



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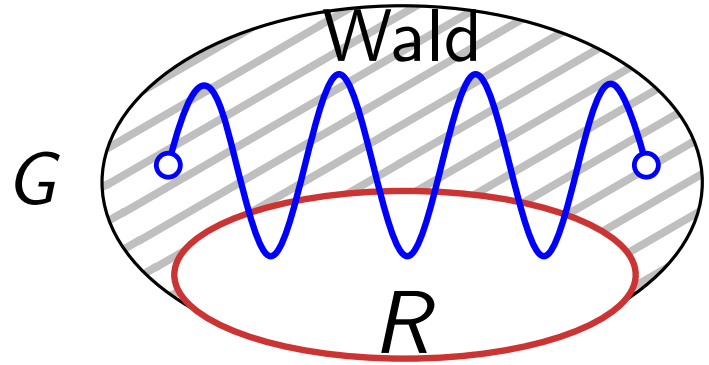
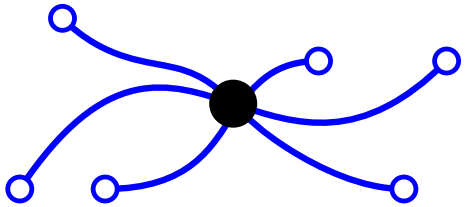


$r = |R|$

3. (a) „Viele“ R-lange Pfade:



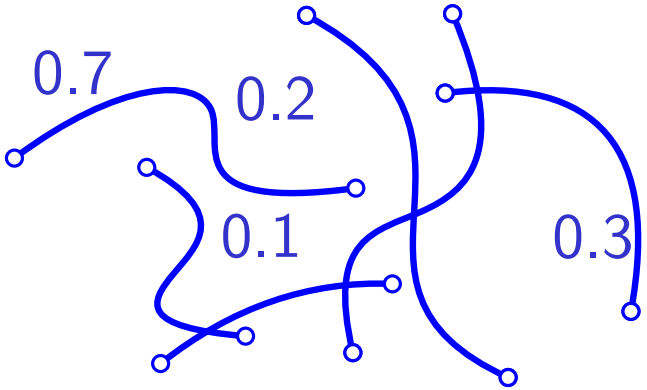
Lemma
[Chekuri et al. '06]

$\approx \frac{\text{OPT}}{\sqrt{r}}$

(b) „Viele“ R-kurze Pfade:

$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

1. Löse LP.



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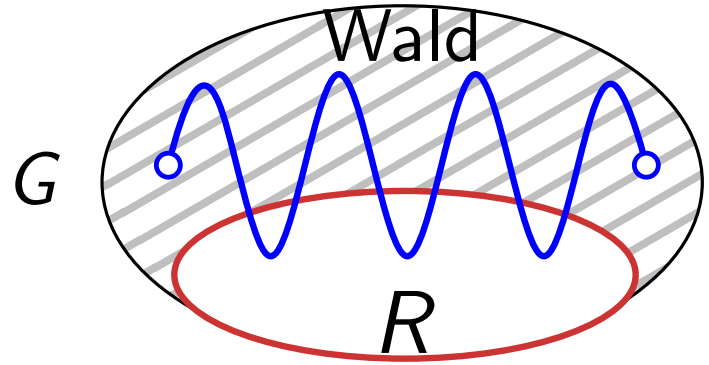

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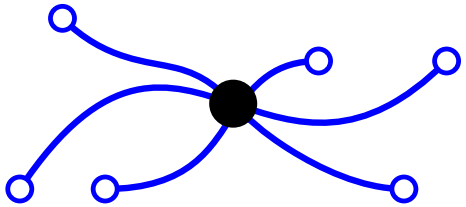

$\xrightarrow[\text{[Chekuri et al. '06]}]{\text{Lemma}}$ $\approx \frac{\text{OPT}}{\sqrt{r}}$

(b) „Viele“ R-kurze Pfade:

$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

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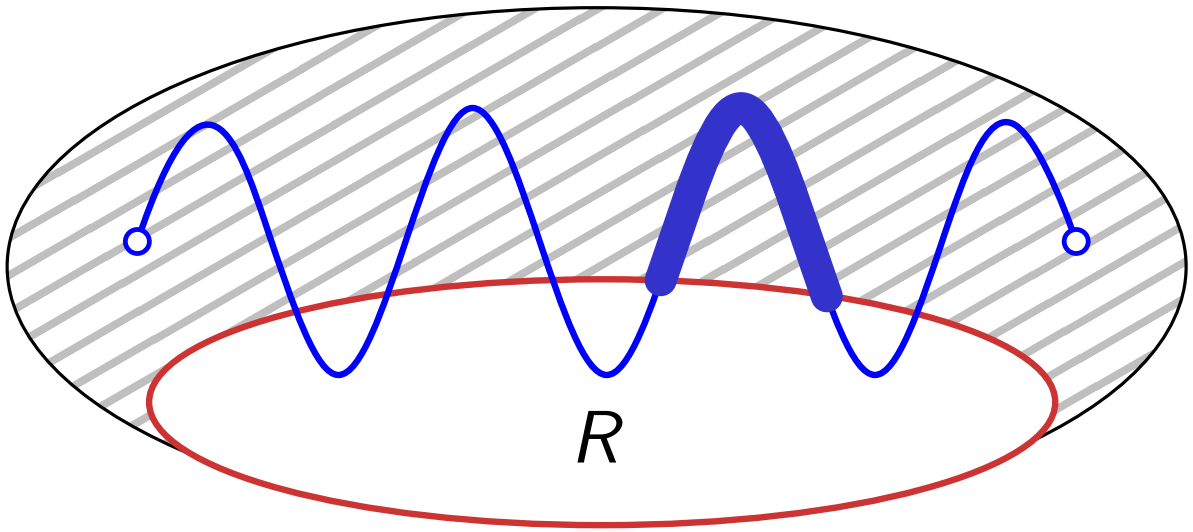

$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

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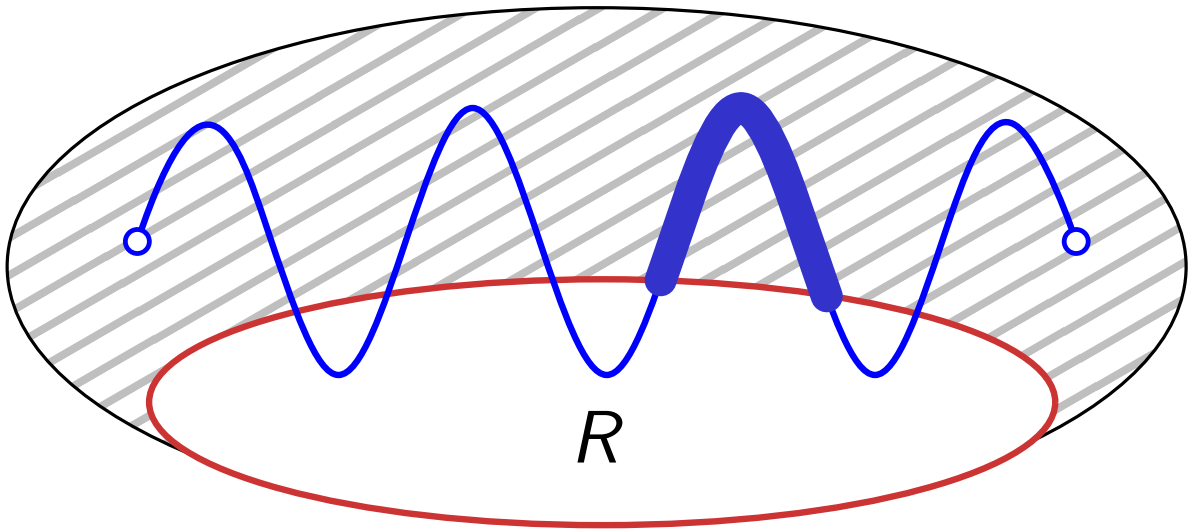

- wenige Teilpfade je Pfad

$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

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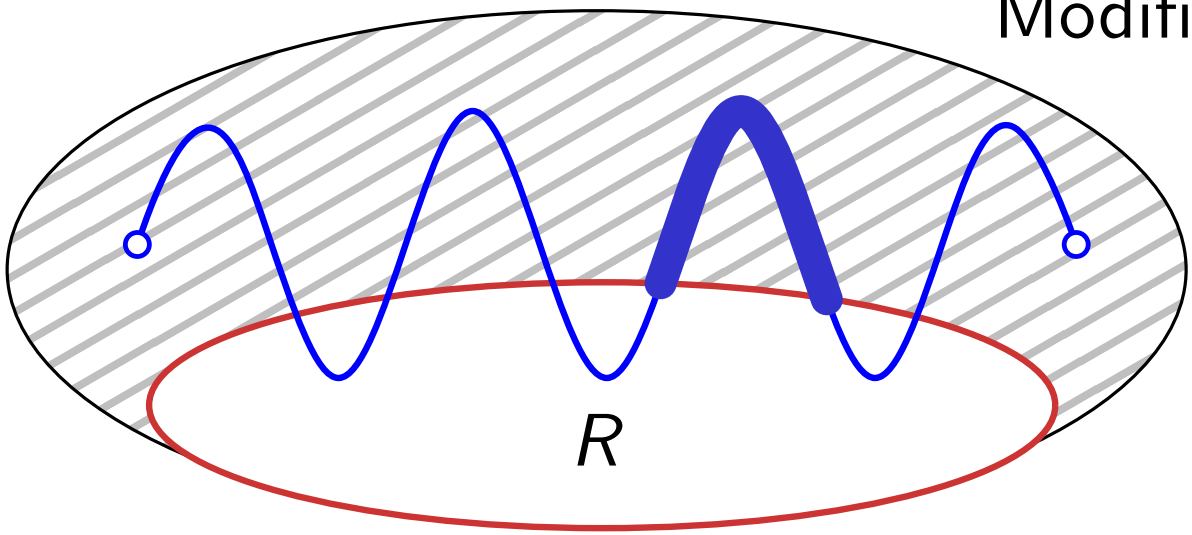

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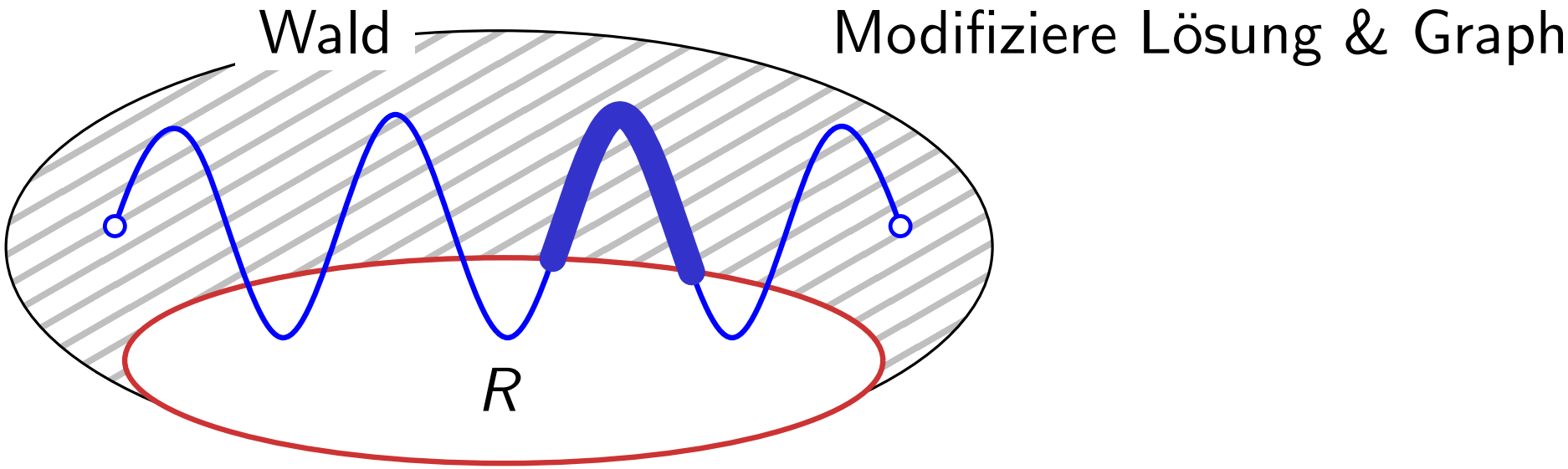

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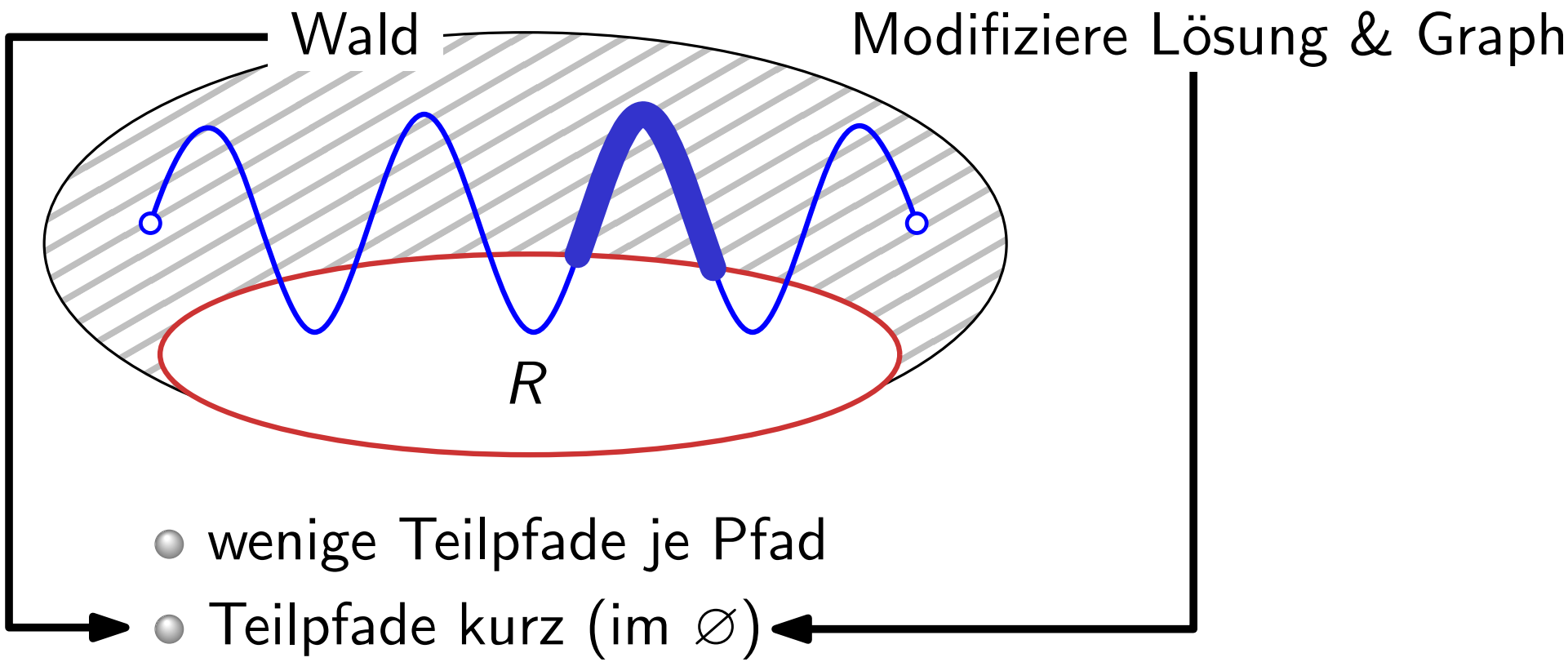
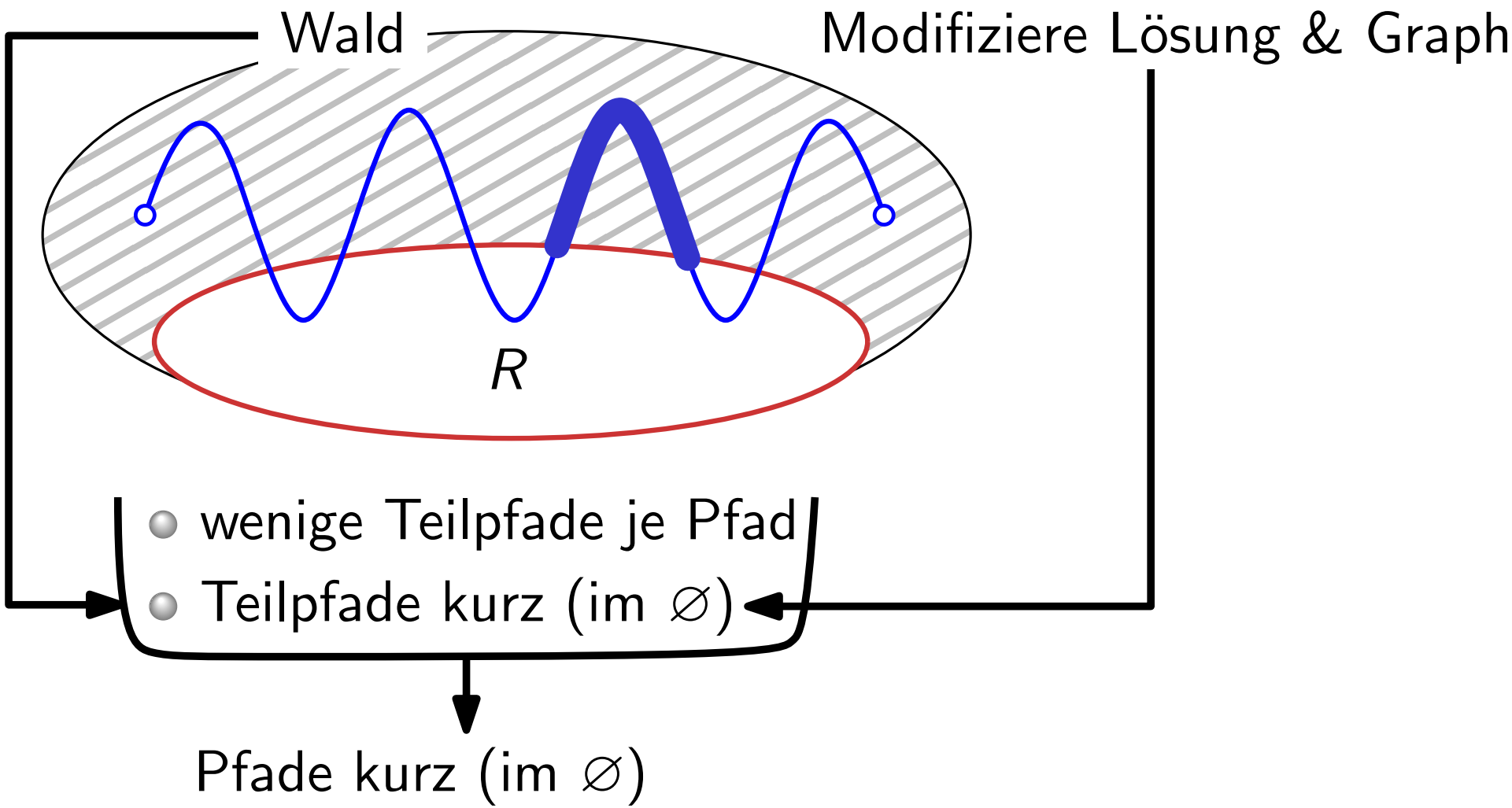
$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

(b) „Viele“ R-kurze Pfade:



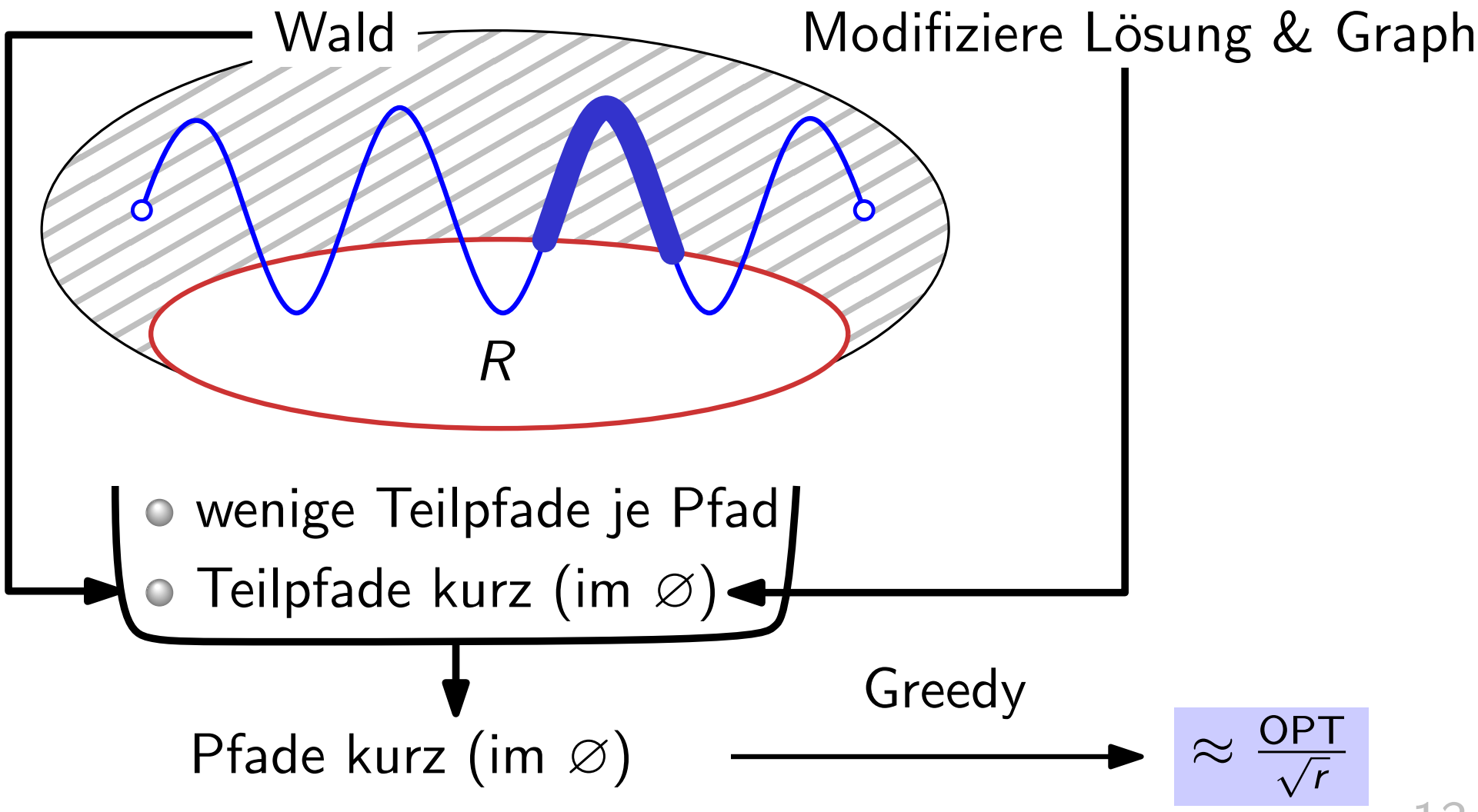
$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

(b) „Viele“ R-kurze Pfade:

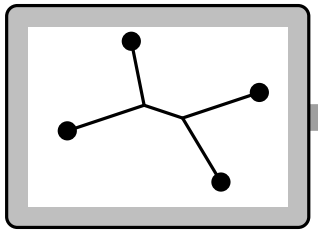


$\tilde{\mathcal{O}}(\sqrt{r})$-Approximation

(b) „Viele“ R-kurze Pfade:

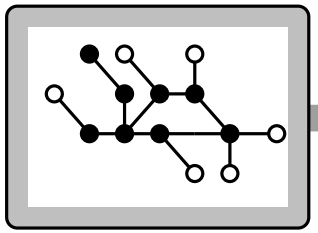


Probleme der Arbeit



Ebene

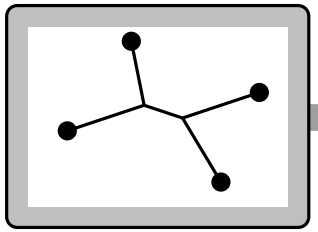
- Colored-Steiner-Forest



Graph

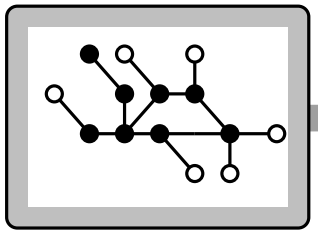
- Kantendisjunkte Pfade

Probleme der Arbeit



Ebene

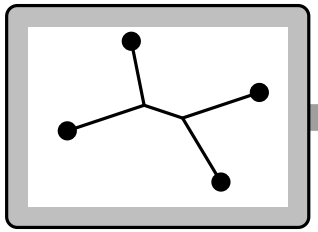
- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen



Graph

- Kantendisjunkte Pfade
- k-Median mit Kapazitäten

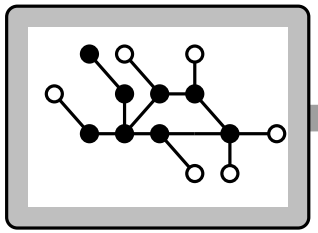
Probleme der Arbeit



Ebene

neu

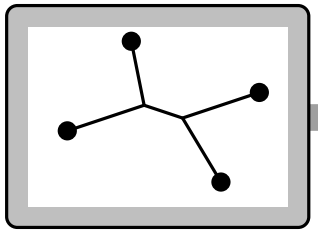
- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen



Graph

- Kantendisjunkte Pfade
- k-Median mit Kapazitäten

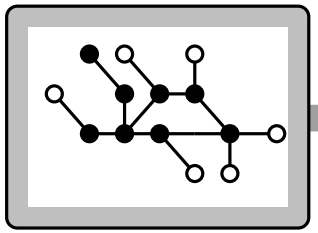
Probleme der Arbeit



Ebene

neu

- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen



Graph

klassisch

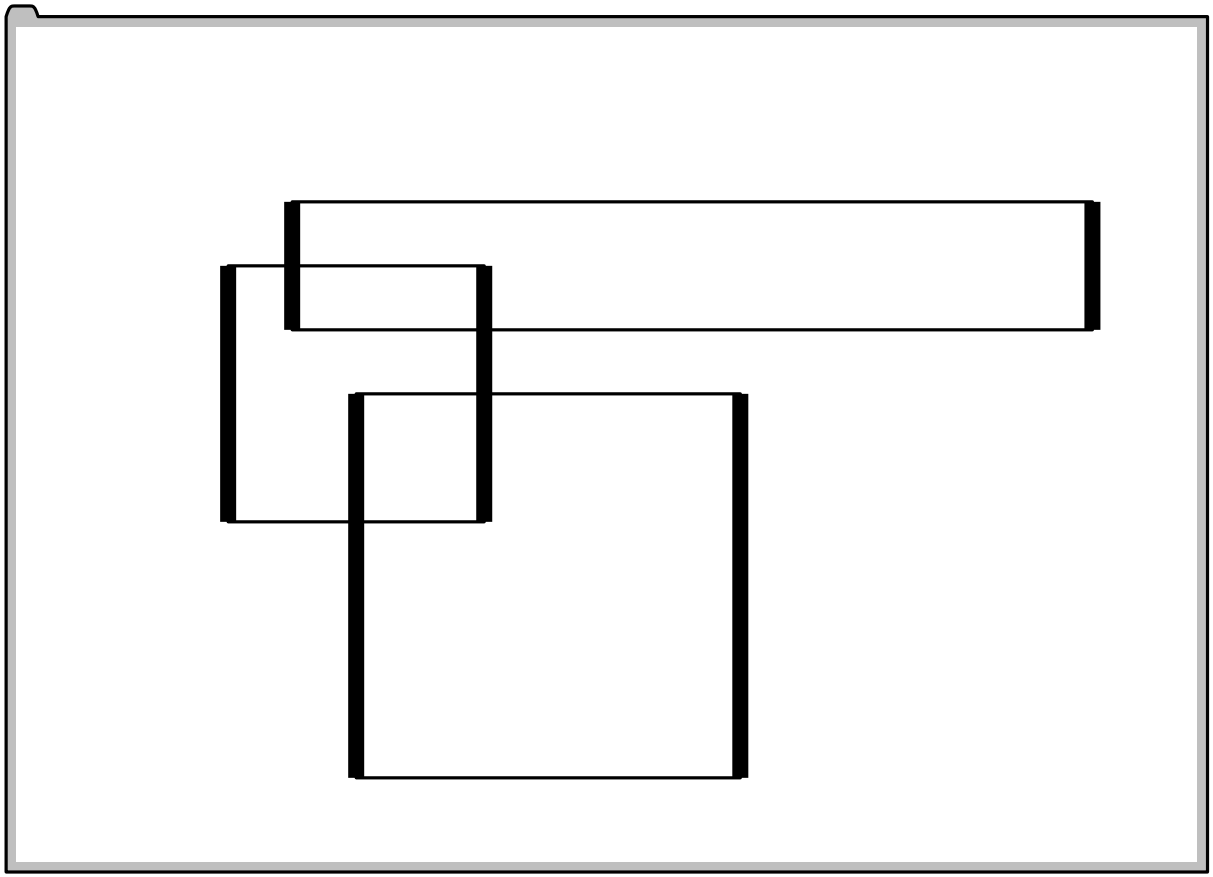
- Kantendisjunkte Pfade
- k-Median mit Kapazitäten

Supplement

Probleme der Arbeit (Details)

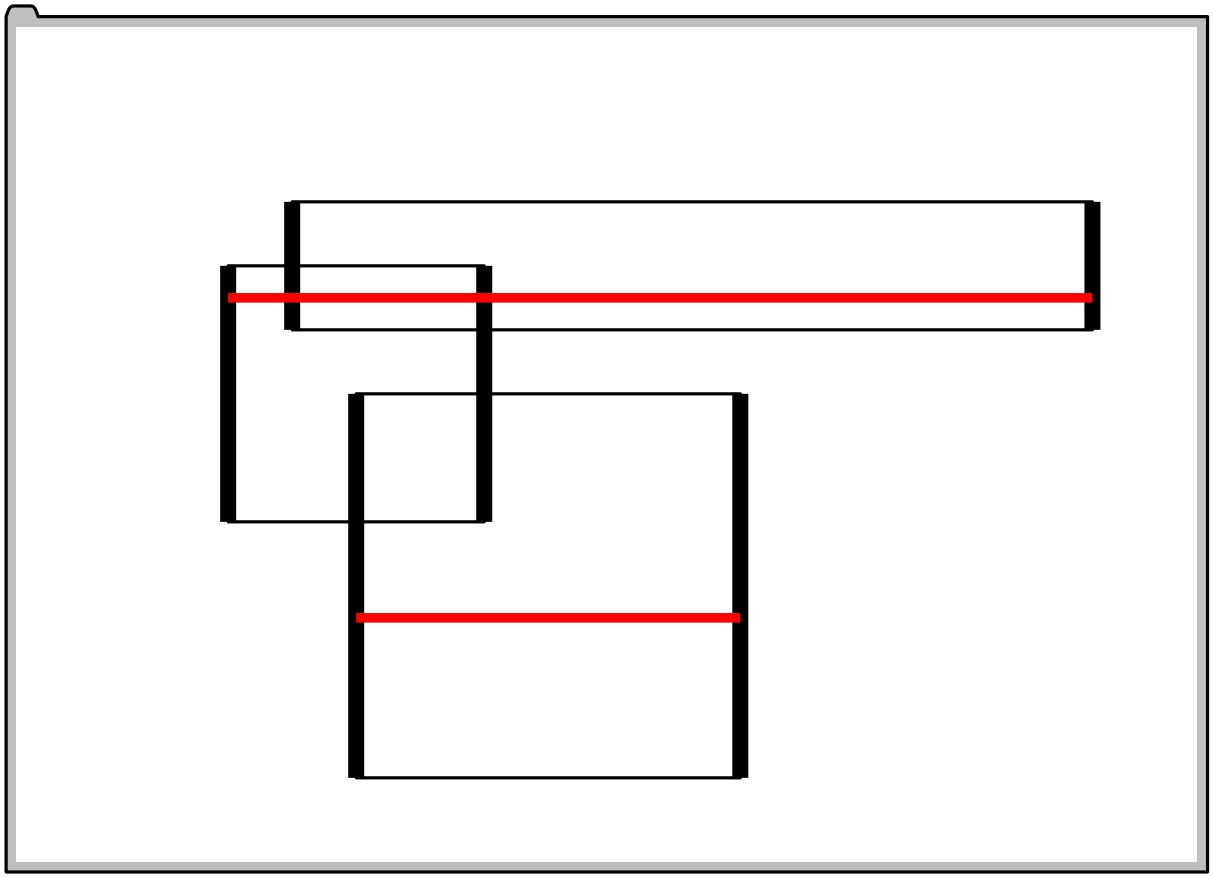
- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen
- Kantendisjunkte Pfade
- k-Median mit Kapazitäten

Probleme der Arbeit (Details)



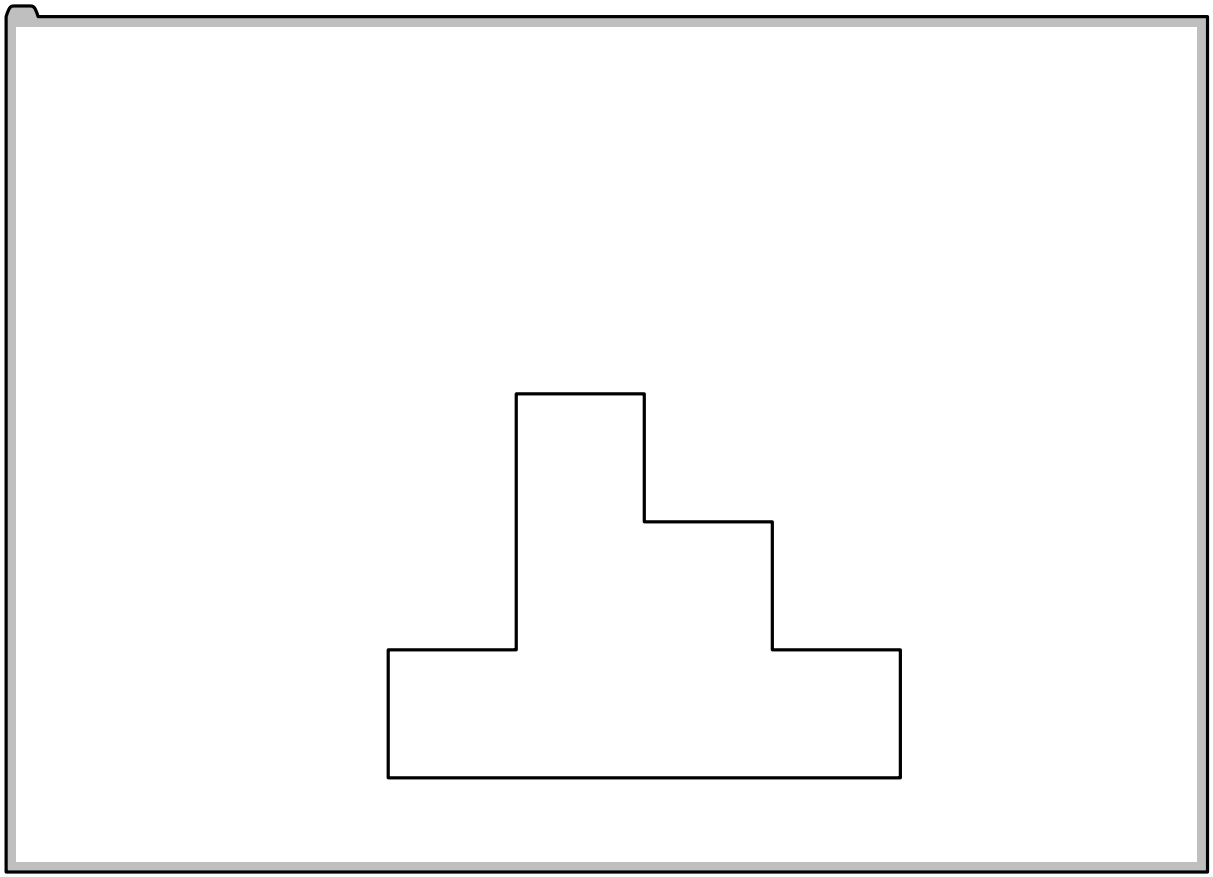
- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen
- Kantendisjunkte Pfade
- k-Median mit Kapazitäten

Probleme der Arbeit (Details)



- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen
- Kantendisjunkte Pfade
- k-Median mit Kapazitäten

Probleme der Arbeit (Details)



- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen
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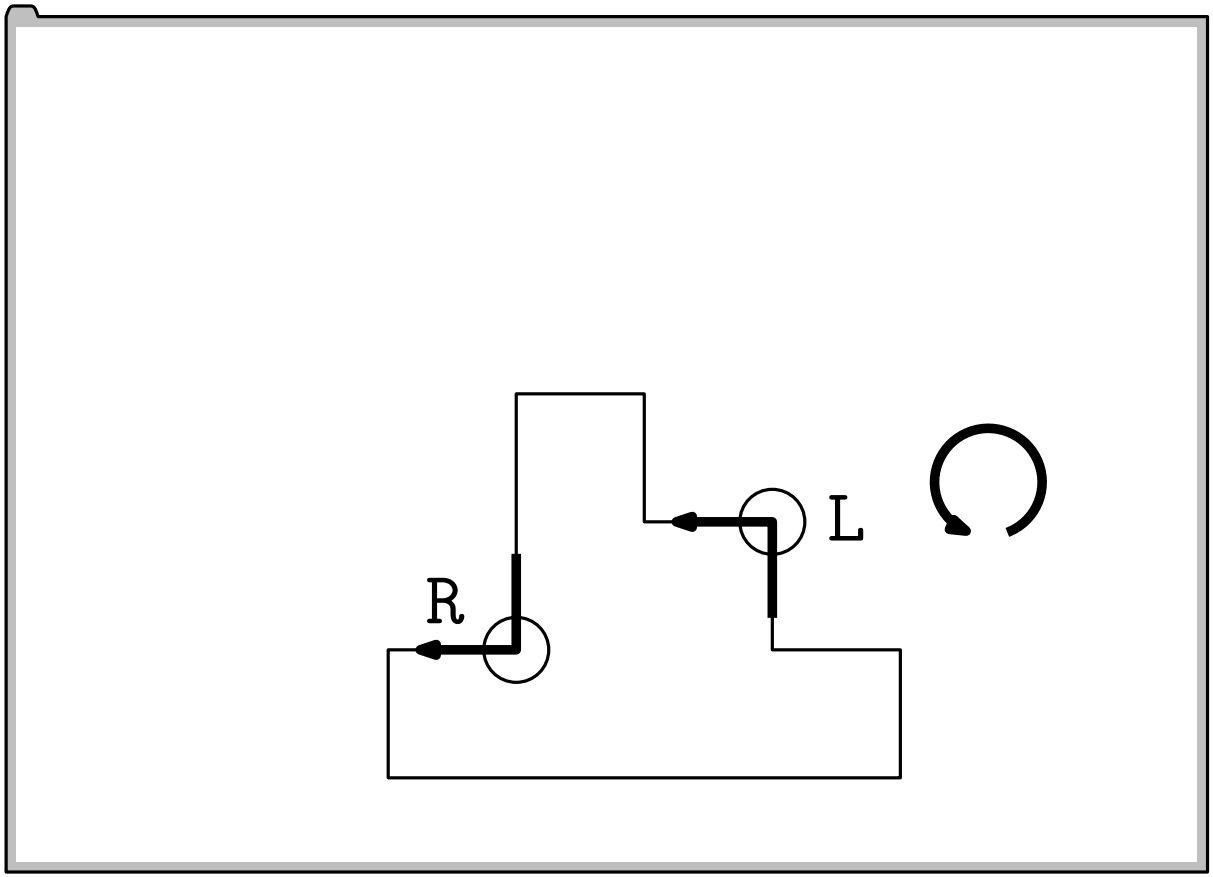
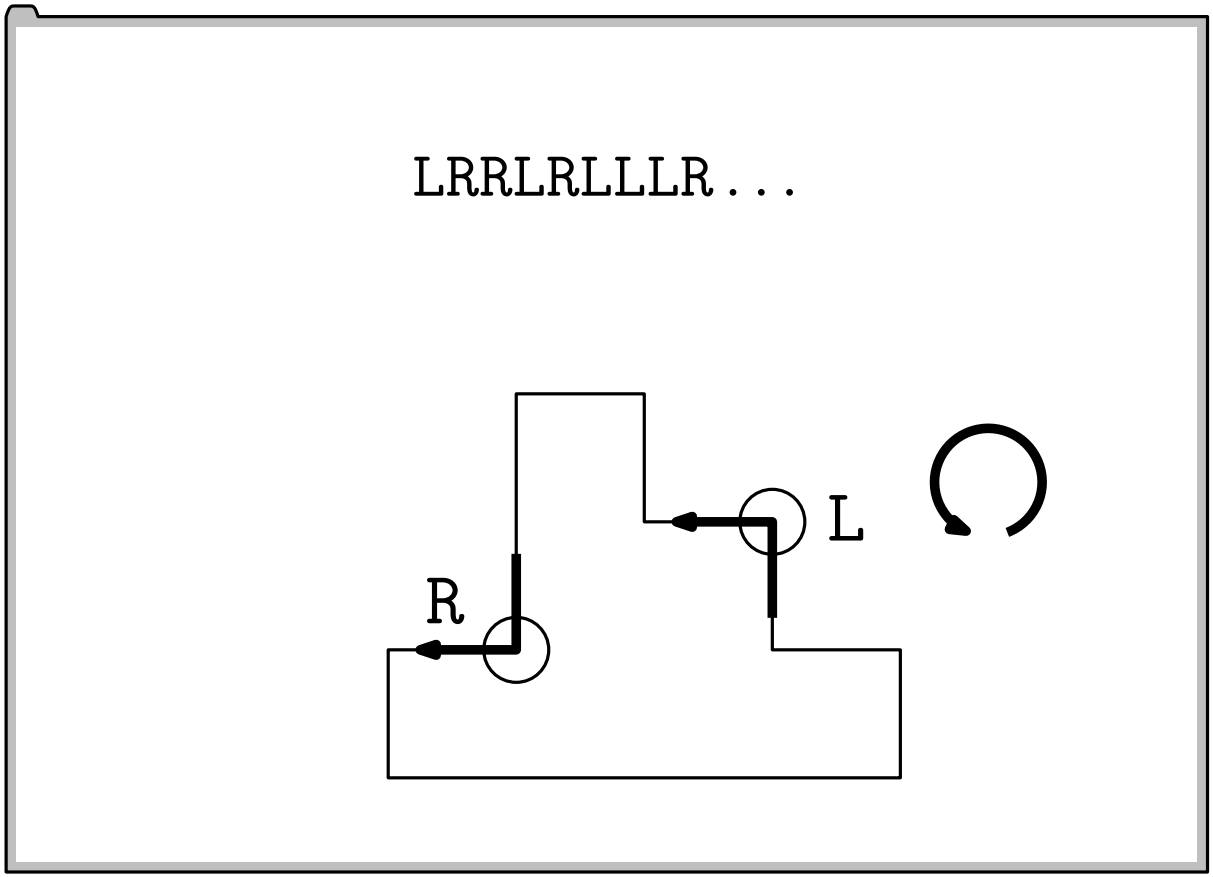
- k-Median mit Kapazitäten

Probleme der Arbeit (Details)



- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen
- Kantendisjunkte Pfade
- k-Median mit Kapazitäten

Probleme der Arbeit (Details)



- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen
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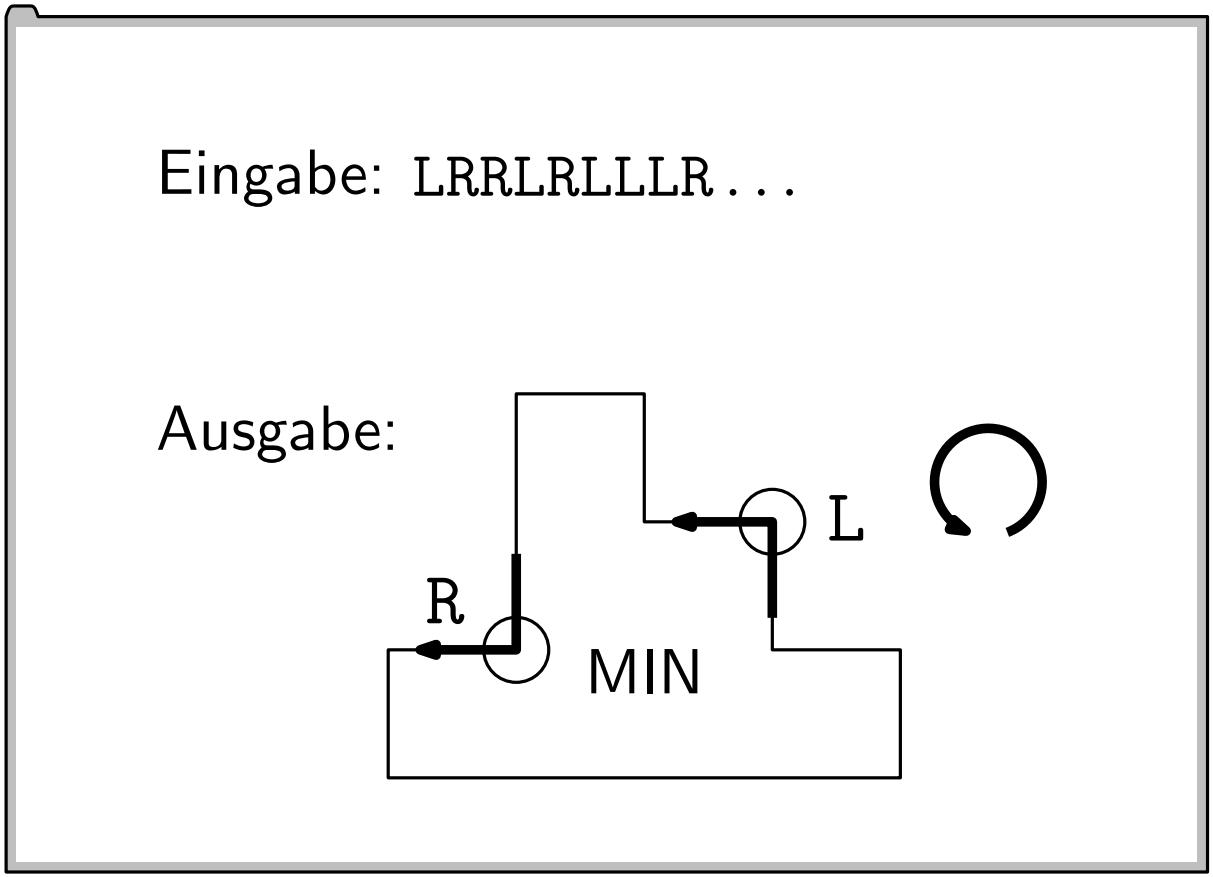
- k-Median mit Kapazitäten

Probleme der Arbeit (Details)



- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen
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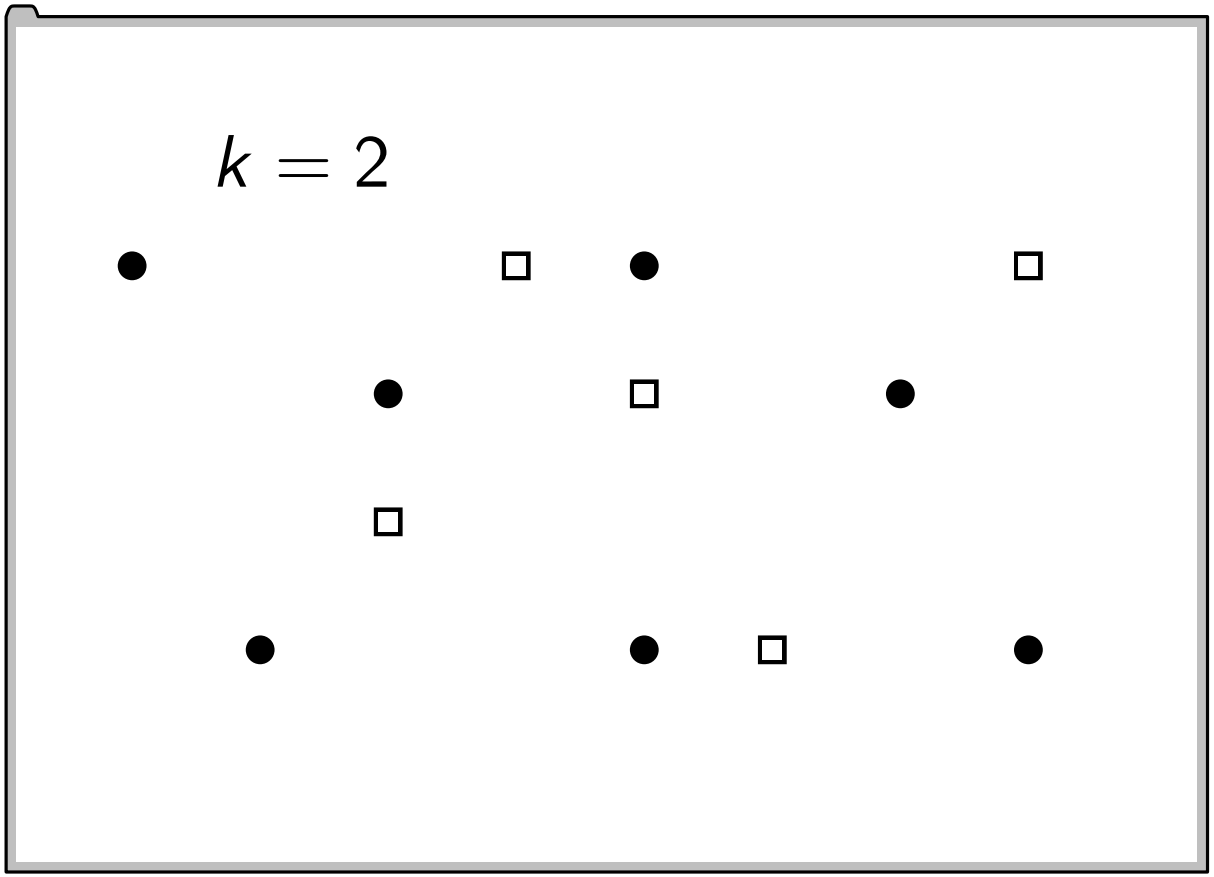
- k-Median mit Kapazitäten

Probleme der Arbeit (Details)



- Rechtecke schneiden
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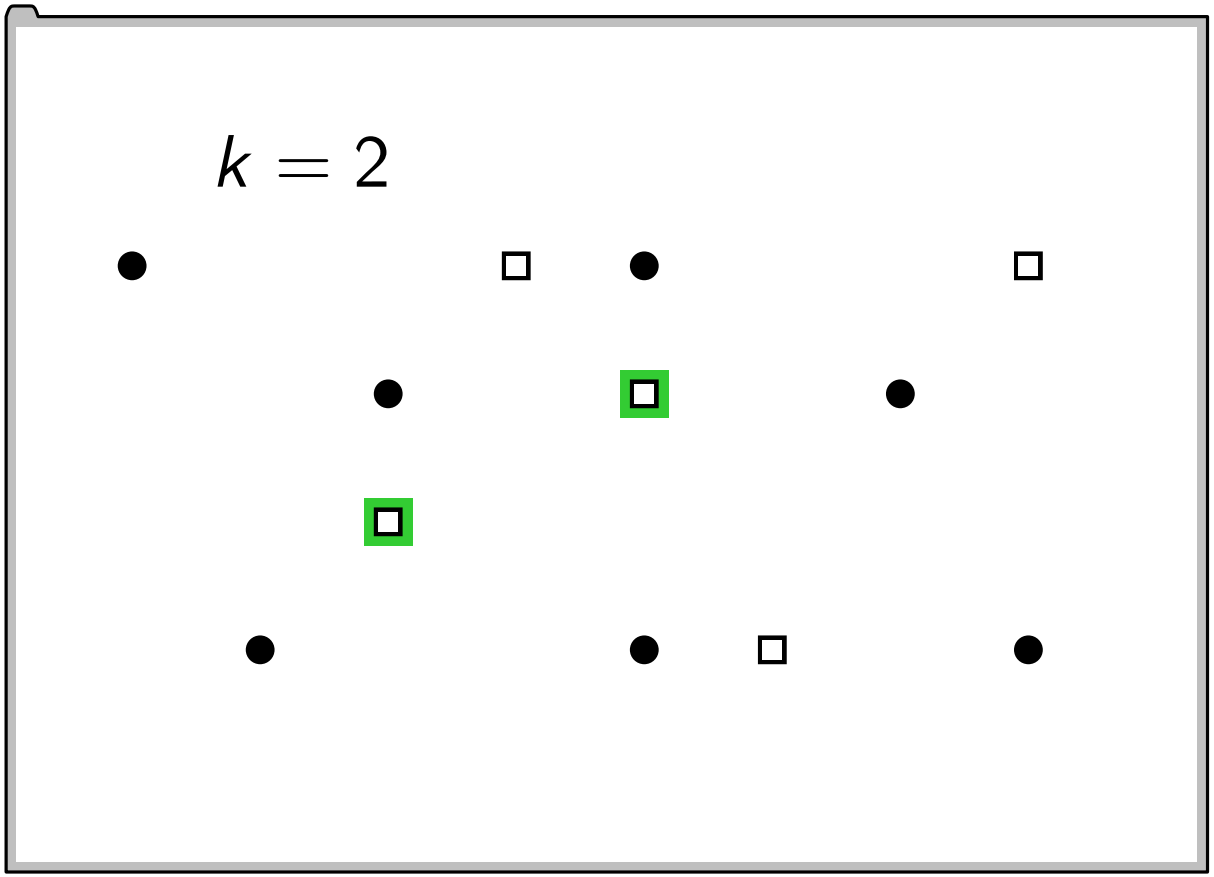
Probleme der Arbeit (Details)



- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen
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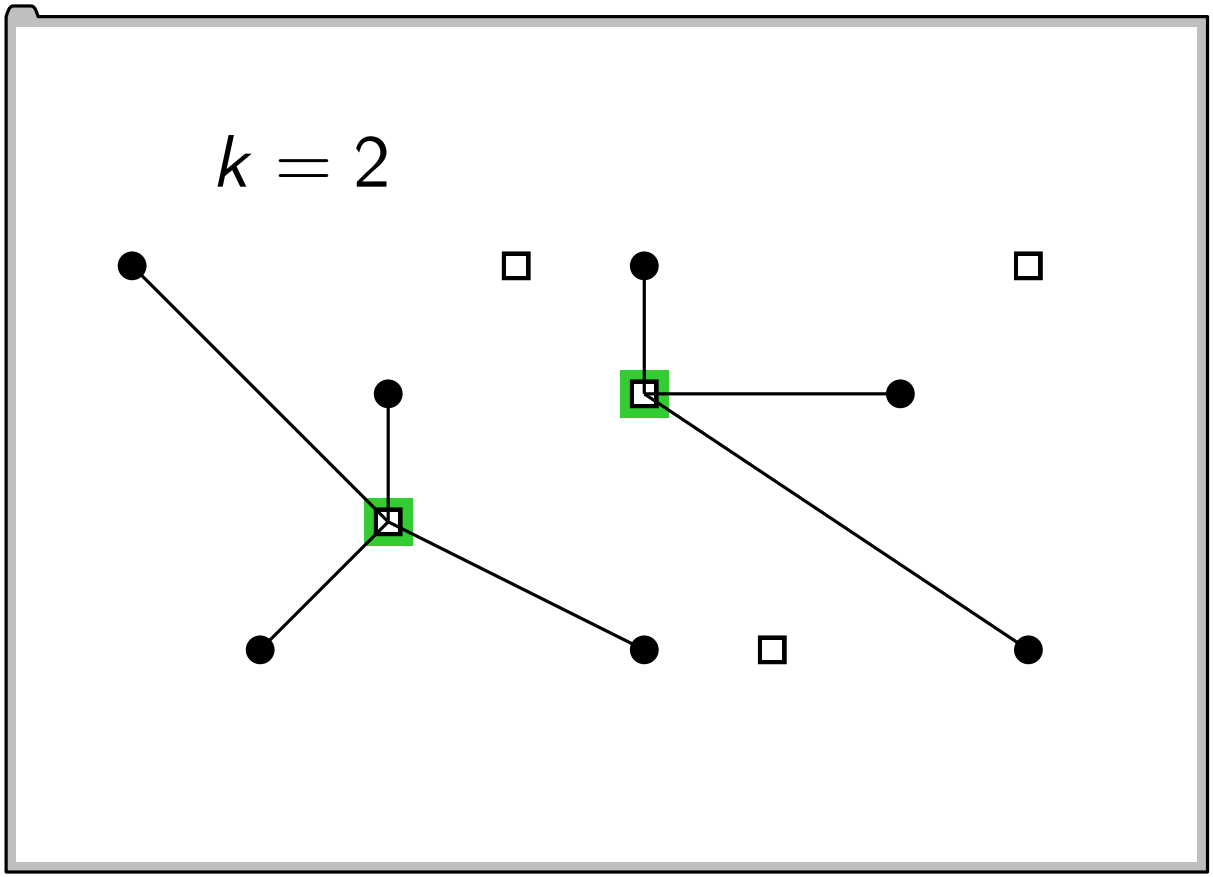
- k-Median mit Kapazitäten

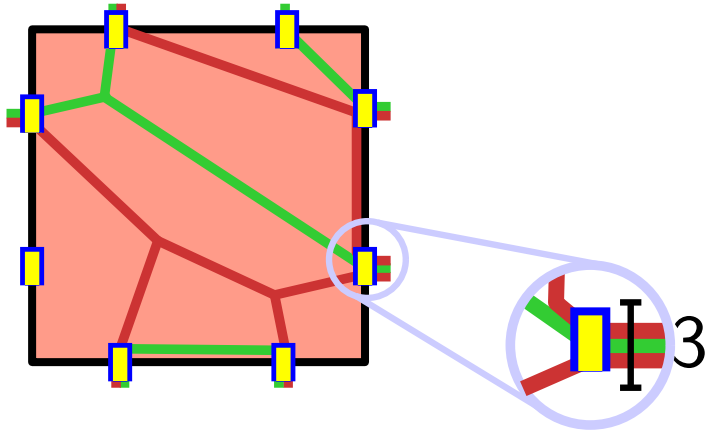
Probleme der Arbeit (Details)

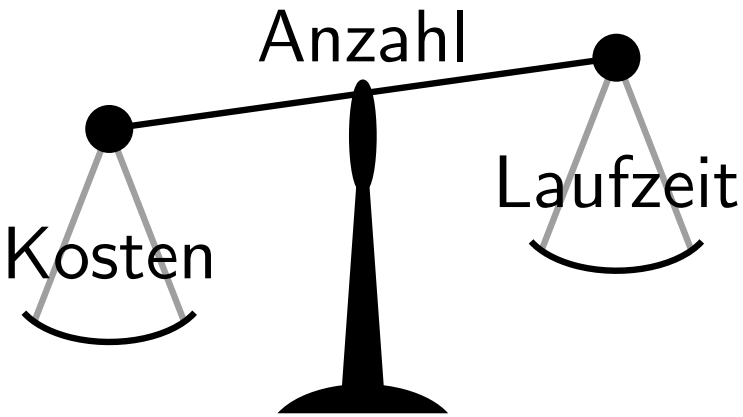


- Rechtecke schneiden
- Colored-Steiner-Forest
- Minimale Polygone zeichnen
- Kantendisjunkte Pfade
- k-Median mit Kapazitäten

Details: $(1+\varepsilon)$-Approximation für $k = 2$

1





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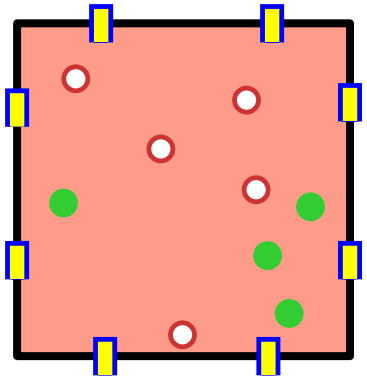

Details: $(1+\varepsilon)$-Approximation für $k = 2$

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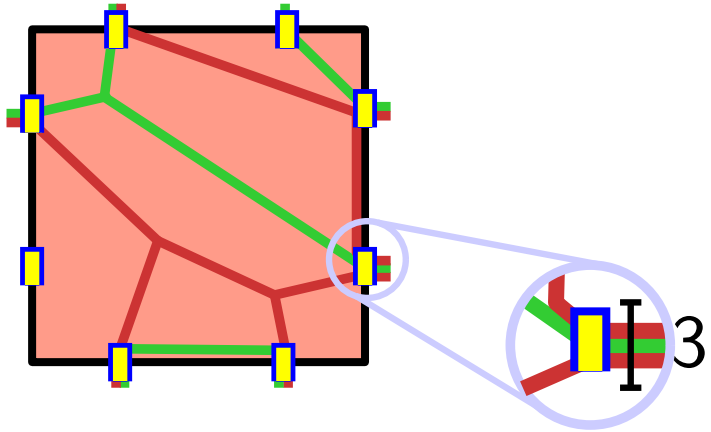

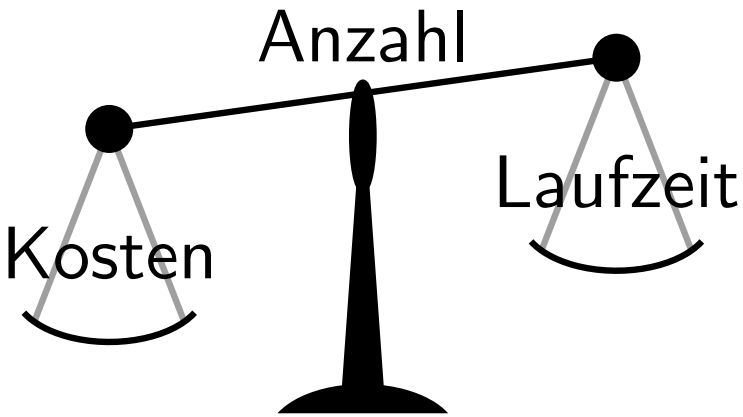

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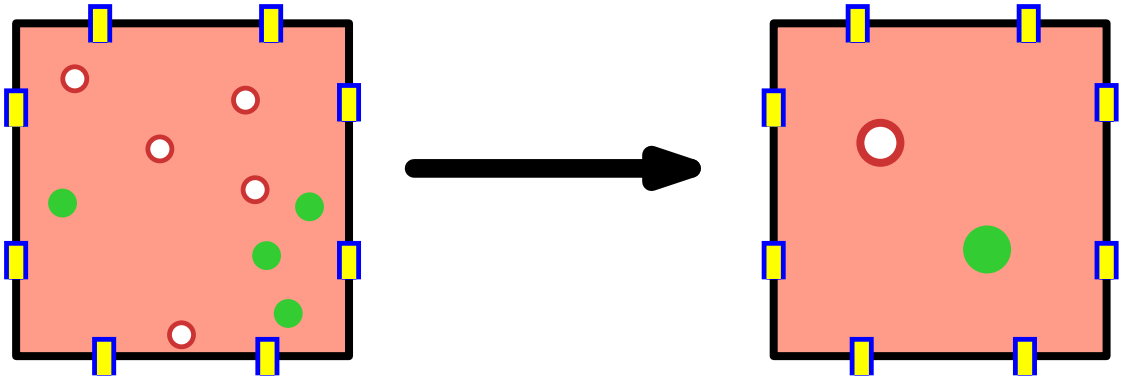

Details: $(1+\varepsilon)$-Approximation für $k = 2$

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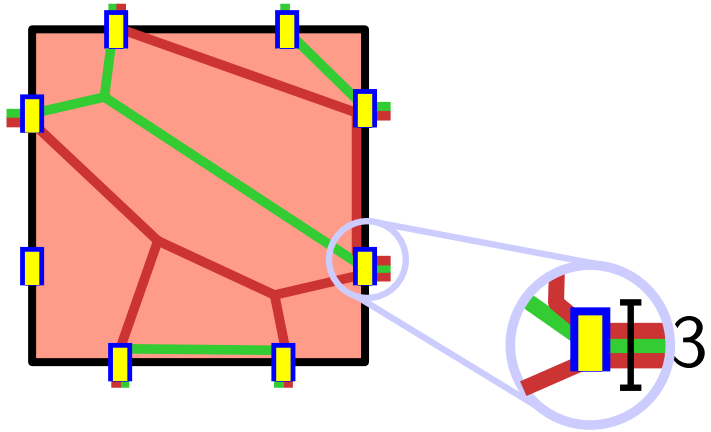

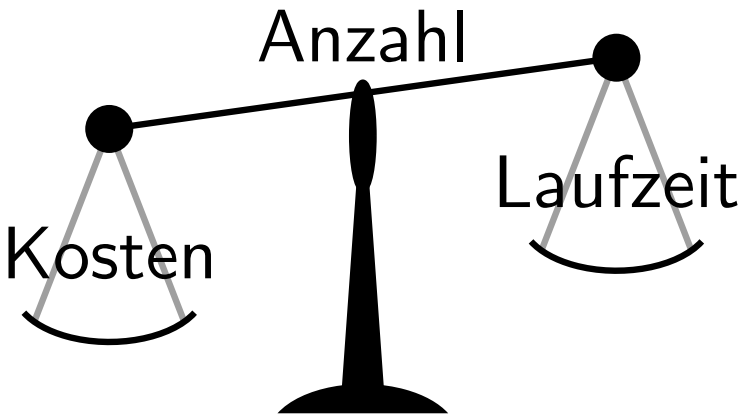

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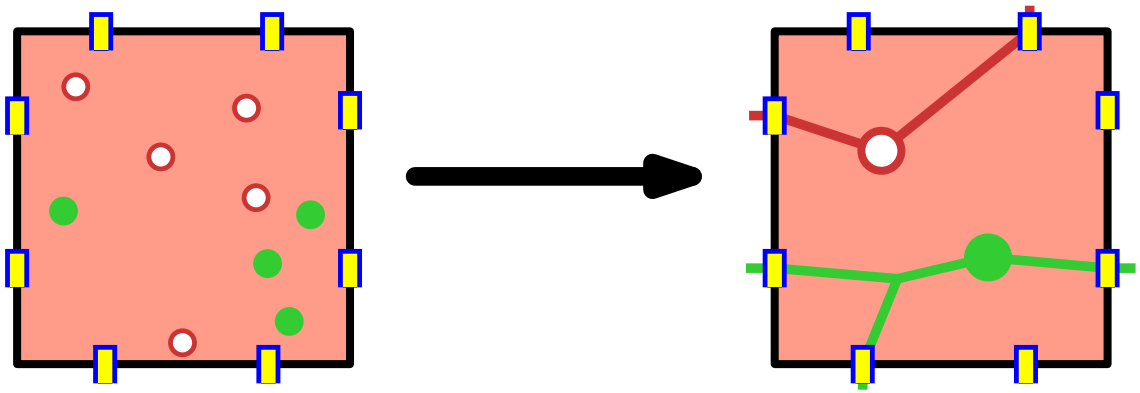

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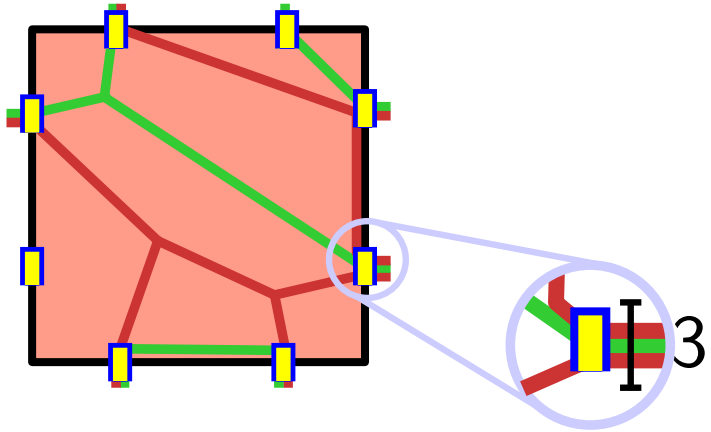

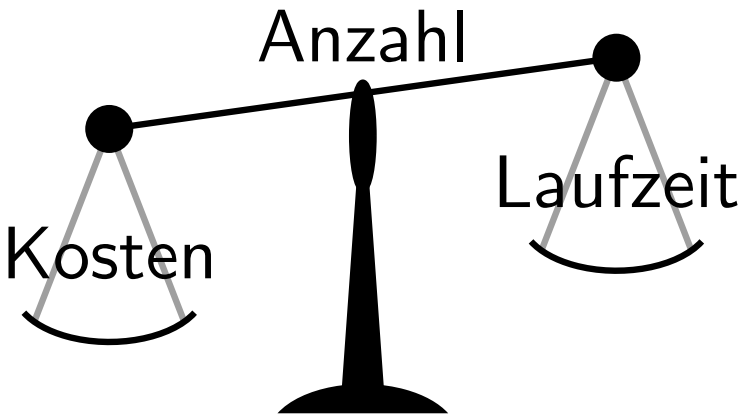

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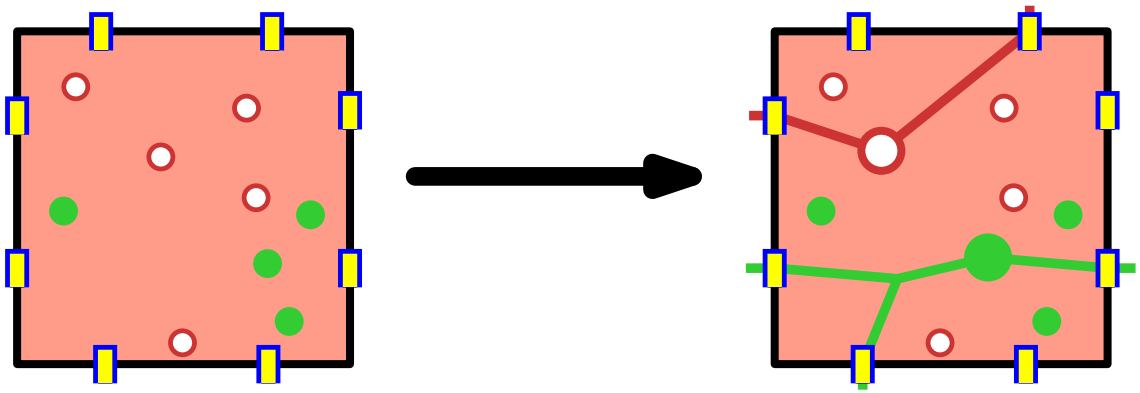

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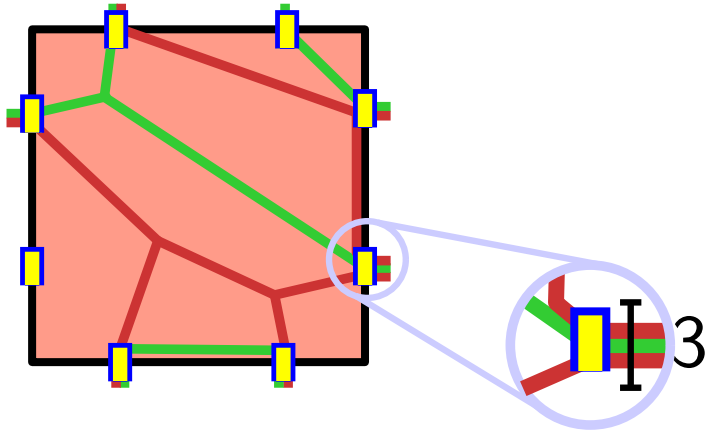

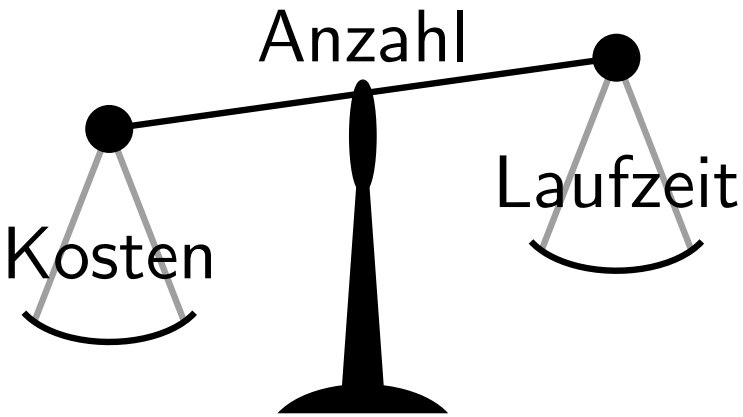

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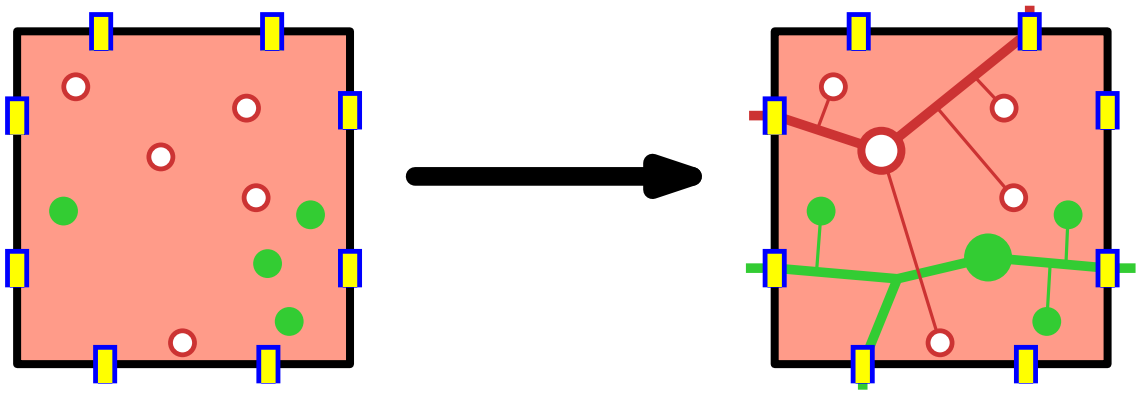

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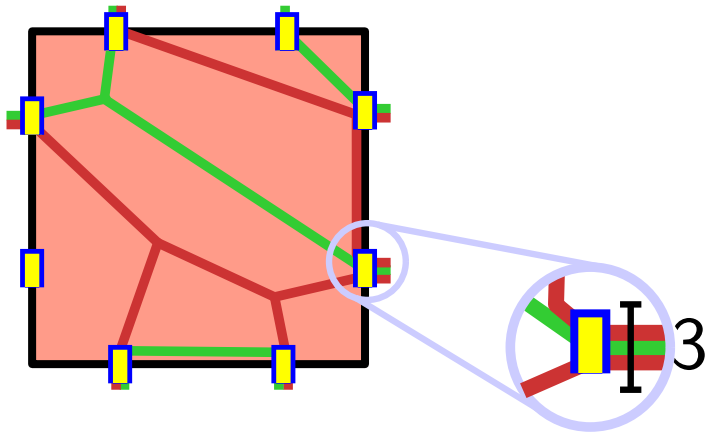

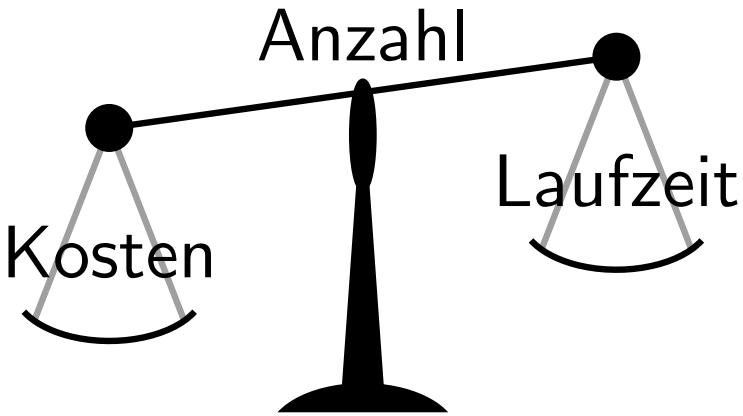

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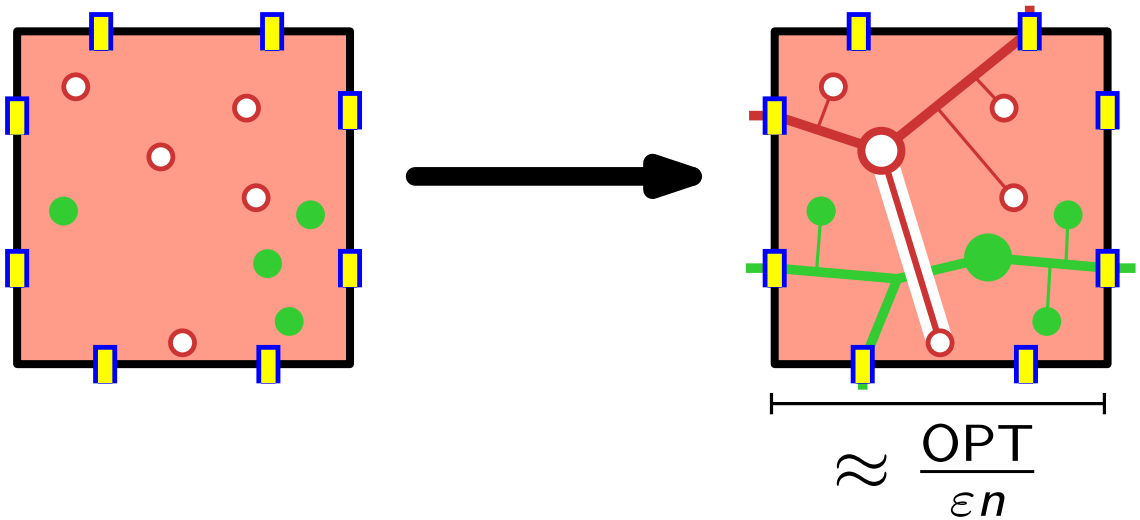

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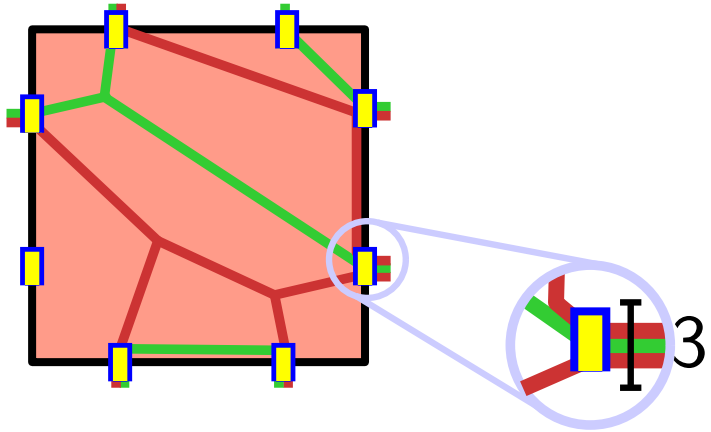

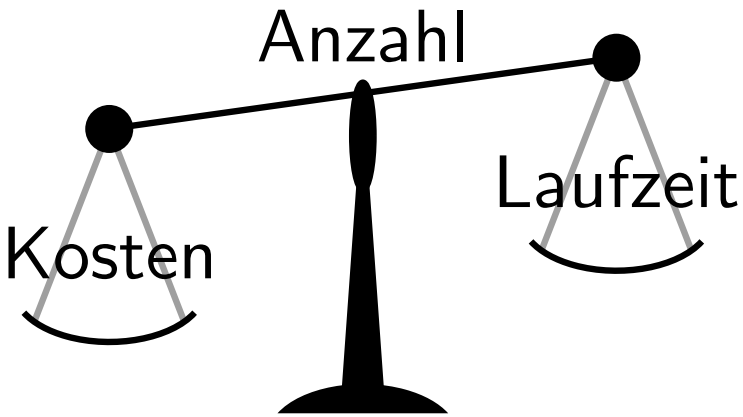

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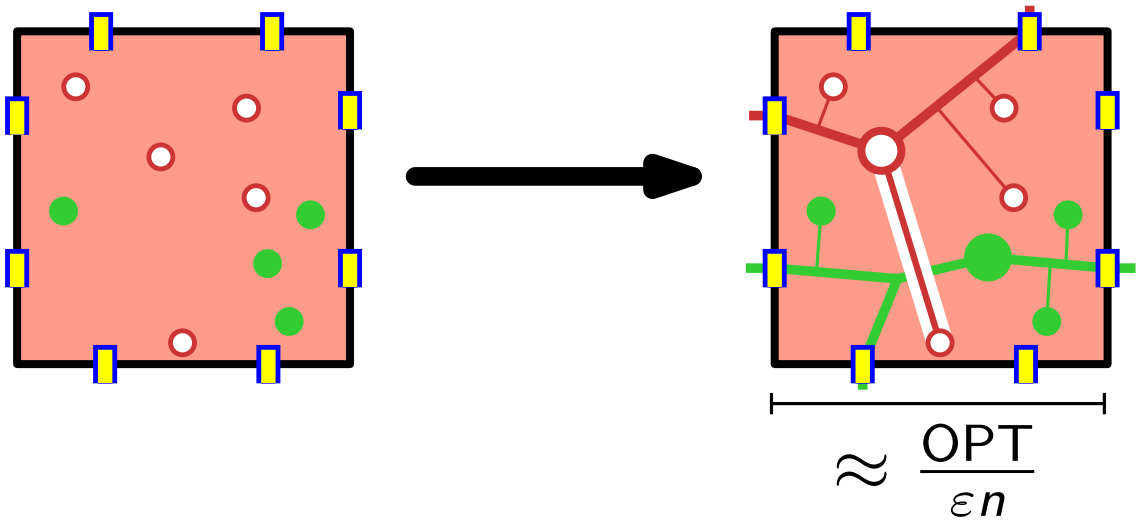

$$\cancel{n} \cdot \frac{\mathrm{OPT}}{\varepsilon \cancel{n}} = \frac{\mathrm{OPT}}{\varepsilon}$$